TECHNICAL RESEARCH REPORT

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T.R. 96-12

INSTITUTE FOR SYSTEMS RESEARCH

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the National Science Foundation
Engineering Research Center Program,
the University of Maryland,
Harvard University,
and Industry

Blind Adaptive Equalization of SIMO Channels based on Second-Order Statistics *

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ABSTRACT: *This article investigates blind adaptive equalization for single-input/multiple-output (SIMO) channels. A second-order statistics based algorithm (SOSA) and a modified second-order statistics based algorithm (MSOSA) for equalization of SIMO channels are presented. Computer simulation demonstrates that the new algorithms converge faster than fractionally spaced constant-modulus algorithm (FS-CMA). The proposed algorithms can be applied in wireless communication systems with antenna arrays to combat the multipath fading.*

SPL EDICS: SPL.SP.3.4, SPL.SP.3.6, and SPL.SP.2.6

I. Introduction

To improve the quality of wireless communication systems, antenna arrays are used for diversity reception. Under the assumption that each signal occupies different band, the digital communication systems with antenna arrays can be modeled as single-input/multiple-output (SIMO) systems. The SIMO systems can also be viewed as the oversampled digital communication systems with single sensor. In the past, the blind adaptive equalization algorithms are based on the higher-order statistics of the channel outputs. Since the SIMO channels satisfying certain conditions can be identified using second-order statistics or correlation function of the channel outputs[1], [2], [4], [5], there should exist a second-order statistics based blind adaptive equalization algorithm for those SIMO systems satisfying some conditions. In this article, we will derive blind adaptive equalization algorithms for the SIMO systems using second-order statistics of the channel outputs. The fast convergence of the new algorithms are illustrated by a computer simulation example.

II. Problem Formulation

An SIMO system can be described as in Figure 1. The input sequence $\{s[n]\}$ is sent through M different linear channels with impulse response $\{h_m[n]\}$ for $m = 1, 2, \cdots, M$. Therefore, each channel output can be expressed as

$$x_m[n] = h_m[n] * s[n], \text{ or } \mathbf{x}[n] = \mathbf{h}[n] * s[n], \tag{1}$$

where $*$ denotes the convolution of sequences (or vectors), and $\mathbf{h}[n]$ and $\mathbf{x}[n]$ are respectively defined as

$$\mathbf{h}[n] \triangleq \begin{pmatrix} h_1[n] \\ \vdots \\ h_M[n] \end{pmatrix}, \text{ and } \mathbf{x}[n] \triangleq \begin{pmatrix} x_1[n] \\ \vdots \\ x_M[n] \end{pmatrix}. \tag{2}$$

In this article, we will assume that the SIMO channels are of finite impulse response (FIR) with length L, and furthermore, they satisfy the *length-and-zero condition* [3], i.e. the M subchannels satisfy the following two conditions:

* The work was supported in part by the NSF grants MIP9309506 and MIP9457397.
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1. $h_{m_1}[0] \neq 0$ *and* $h_{m_2}[L-1] \neq 0$ *for some* $1 \leq m_1,\ m_2 \leq M$*, where* L *is the largest length of the* M *subchannels.*

2. $\{H_m(z)\}_1^M$ *have no common zeros, where* $H_m(z)$ *is the* Z*-transform of* $\{h_m[n]\}$*.*

For FIR SIMO channels, the channel outputs can be written in matrix form as

$$\mathbf{x}_K[n] = \mathcal{H}_K \mathbf{s}_K[n], \quad \text{or} \quad \mathcal{X}_K[n] = \mathcal{H}_K \mathcal{S}_K[n], \tag{3}$$

where we have used the definitions

$$\mathcal{H}_K \triangleq \begin{pmatrix} \mathbf{h}[L-1] & \cdots & \mathbf{h}[0] & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{h}[L-1] & \cdots & \mathbf{h}[0] \end{pmatrix}, \tag{4}$$

$$\mathbf{x}_K[n] \triangleq \begin{pmatrix} \mathbf{x}[n] \\ \vdots \\ \mathbf{x}[n+K-1] \end{pmatrix}, \quad \mathcal{X}_K[n] \triangleq (\mathbf{x}_K[n], \cdots, \mathbf{x}_K[n+N-1]), \tag{5}$$

and

$$s_K[n] \triangleq \begin{pmatrix} s[n-L+1] \\ \vdots \\ s[n+K-1] \end{pmatrix}, \quad \mathcal{S}_K[n] \triangleq (\mathbf{s}_K[n], \cdots, \mathbf{s}_K[n+N-1]). \tag{6}$$

It has been shown in [1] that if the SIMO channels satisfy the length-and-zero condition, then $\mathcal{H}_K$ for any $K \geq L-1$ is of full column rank, which is also the identifiable condition [5] of SIMO channels using second-order statistics of the channel outputs. Hence, from (3), there exists a $KM \times (K+L-1)$ matrix $\mathcal{F}$ (not unique) such that

$$\mathbf{s}_K[n] = \mathcal{F}^H \mathbf{x}_K[n]. \tag{7}$$

Let

$$\mathcal{F} = (\mathbf{f}_1,\ \mathbf{f}_2, \cdots,\ \mathbf{f}_{K+L-1}), \tag{8}$$

where $\mathbf{f}_k$ for $k = 1, \cdots, K+L-1$ are column vectors with KM elements.

The task of blind adaptive equalization of SIMO channels is to find algorithms to adaptively adjust the parameters $\mathbf{f}_k$ such that

$$\mathcal{F}^H \mathcal{H}_K = cI_{K+L-1}, \tag{9}$$

for some non-zero constant c.

III. Algorithm Development

Having described the problem of blind adaptive equalization for SIMO channels, we now develop blind adaptive equalization algorithms for SIMO channels using second-order statistics.

A. Basic Principle

From the above definitions, we have

$$\mathbf{f}_k^H \mathbf{x}_K[n] = \mathbf{f}_{k+1}^H \mathbf{x}_K[n-1], \tag{10}$$

for $k = 1, \cdots, K+L-2$, and every integer n.

It can be proved that if the channel satisfies the length-and-zero condition and the channel input in *the* $K+1$*-th oreder persistently exciting* [6] then the $\mathbf{f}_k$'s satisfying (10) will satisfy (9).

If the channel noise is considered, $\mathbf{f}_k$ for $k = 1, \cdots, K+L-1$ can be estimated by minimizing the cost function

$$\mathcal{C} = \sum_{k=1}^{K+L-2} E|\mathbf{f}_k^H \mathbf{x}_K[n] - \mathbf{f}_{k+1}^H \mathbf{x}_K[n-1]|^2, \tag{11}$$

subject to

$$\sum_{k=1}^{K+L-1} |\mathbf{f}_k^H \mathbf{x}_K[n]|^2 = c_o, \tag{12}$$

where c_o is a non-negative constant. The constraint is added here to prevent from the trivial solution of $\mathbf{f}_k = \mathbf{0}$ for all $k = 1, \cdots, K+L-1$.

Based on the basic principle presented here, we are able to develop a second-order statistics based algorithm (SOSA) and a modified second-order statistics based algorithm (MSOSA).

B. Second-Order Statistics based Algorithm

From (11), a direct calculation yields that

$$\mathcal{C} = \sum_{k=1}^{K+L-2} (\mathbf{f}_k^H R_x[0]\mathbf{f}_k - \mathbf{f}_k^H R_x[1]\mathbf{f}_{k+1} - \mathbf{f}_{k+1}^H R_x^H[1]\mathbf{f}_k + \mathbf{f}_{k+1}^H R_x[0]\mathbf{f}_{k+1}), \tag{13}$$

where we have used the definition

$$R_x[m] \triangleq E\{\mathbf{x}_K[n]\mathbf{x}_K^H[n-m]\}, \tag{14}$$

and the identity

$$R_x[-m] = R_x^H[m]. \tag{15}$$

Hence,

$$\frac{\partial \mathcal{C}}{\partial \mathbf{f}_k} = \begin{cases} R_x[0]\mathbf{f}_1 - R_x[1]\mathbf{f}_2, & k = 1, \\ 2R_x[0]\mathbf{f}_k - R_x[1]\mathbf{f}_{k+1} - R_x^H[1]\mathbf{f}_{k-1}, & k = 1, \cdots, K+L-2, \\ R_x[0]\mathbf{f}_{K+L-1} - R_x^H[1]\mathbf{f}_{K+L-2}, & k = K+L-1. \end{cases} \tag{16}$$

Let

$$\mathbf{f} \triangleq \begin{pmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_{K+L-1} \end{pmatrix}, \tag{17}$$

then, (16) can be written as

$$\frac{\partial \mathcal{C}}{\partial \mathbf{f}} = R\mathbf{f}, \tag{18}$$

where R is a $KM(K+L-1) \times KM(K+L-1)$ matrix defined as

$$R \triangleq \begin{pmatrix} R_x[0] & -R_x[1] & \mathbf{0} & \cdots & \mathbf{0} \\ -R_x^H[1] & 2R_x[0] & -R_x[1] & \ddots & \mathbf{0} \\ \mathbf{0} & -R_x^H[1] & 2R_x[0] & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \cdots & R_x[0] \end{pmatrix}. \tag{19}$$

Using the gradient-based approach, we can obtain an adaptive algorithm to estimate $\mathbf{f}$ as following:

$$\widehat{\mathbf{f}}^{(n+1)} = \mathbf{f}^{(n)} - \mu R\mathbf{f}^{(n)}, \tag{20}$$

$$\mathbf{f}^{(n+1)} = \frac{c_o}{p^{(n+1)}} \widehat{\mathbf{f}}^{(n+1)}, \quad p^{(n+1)} = (\sum_{k=1}^{K+L-1} (\widehat{\mathbf{f}}_k^{(n+1)})^H R_x^{(n+1)}[0] \widehat{\mathbf{f}}_k^{(n+1)})^{1/2}, \tag{21}$$

and $R_x[m]$ in R can be estimated using

$$R_x^{(n+1)}[m] = \lambda R_x^{(n)}[m] + (1-\lambda)\mathbf{x}_K[n]\mathbf{x}_K^H[n-m], \tag{22}$$

where μ is a step-size and $\lambda \in [0,\ 1]$ is a forgetting factor.

The blind adaptive algorithm for SIMO channel equalization defined by (20)-(22) is called *second-order statistics based algorithm*, or SOSA.

C. Modified Second-Order based Algorithm

If we examine the identities (6) and (7) carefully, we will see that

$$\mathbf{f}_{k_1}^H \mathbf{x}_K[n-k_1] = \mathbf{f}_{k_2}^H \mathbf{x}_K[n-k_2] \tag{23}$$

for $k_1,\ k_2 = 1, 2, \cdots, K+L-1$ and all integer n. Hence, we can modify the cost function (11) as

$$\tilde{C} = \sum_{k_1,k_2=1}^{K+L-1} E|\mathbf{f}_{k_1}^H \mathbf{x}_K[n-k_1] - \mathbf{f}_{k_2}^H \mathbf{x}_K[n-k_2]|^2. \tag{24}$$

From this cost function, we are able to derive a *modified second-order statistics based algorithm* (MSOSA) similar to (20)-(22), except that R is substituted by

$$\tilde{R} = \begin{pmatrix} (K+L-2)R_x[0] & -R_x[1] & \cdots & -R_x[K+L-2] \\ -R_x^H[1] & (K+L-2)R_x[0] & \ddots & -R_x[K+L-3] \\ \vdots & \ddots & \ddots & \vdots \\ -R_x^H[K+L-2] & \cdots & \cdots & (K+L-2)R_x[0] \end{pmatrix}. \tag{25}$$

Since the MSOSA exploits more information about the structure of $\mathbf{s}_K[n]$, as comfirmed by our computer simulations, it is more robust than the SOSA. However, the MSOSA requires a little bit more computation than the SOSA.

IV. Computer Simulation and Conclusion

A Monte Carlo simulation example has been conducted to demonstrate the performance of the new algorithms used in 16-QAM digital communication systems. In our simulation, the channel input sequence $\{s[n]\}$ is i.i.d. and randomly distributed over $\{\pm 1 \pm \jmath\}$, $\{\pm 3 \pm \jmath\}$, $\{\pm 1 \pm 3\jmath\}$ and $\{\pm 3 \pm 3\jmath\}$. It is sent through two time-invariant linear channels with impulse responses $\{0.08427,\ 0.5167,\ 0.01383\}$ and $\{0.3874,\ 0.3132,\ -0.08374\}$ respectively. The complex white Gaussian noise, with zero-mean and variance making $SNR = 30dB$, is added at each channel output. The step-size μ and the forgetting factor λ are chosen to optimize the performance of each equalization algorithm. Figure 2 (a), (b) and (c) are the eye patterns of the SOSA, the MSOSA and the FS-CMA respectively after 1,000 iterations, and Figure 2 (d) illustrates the convergence of ISI of the three algorithms with respect to the number of iterations. From Figure 2, the performance of the SOSA and the MSOSA are much better than that of the FS-CMA with the MSOSA being the best of the three algorithms.

In this article, we have proposed the SOSA and the MSOSA for blind adaptive equalization of SIMO channels using the correlation function of the channel outputs. The new algorithms converge faster than fractionally spaced CMA [3]. We are currently studying how to use the proposed algorithms in IS-54 systems to tracking the fast fading mobile radio channels.

References

[1] Y. Li and Z. Ding, 'Blind channel identification based on second order cyclostationary statistics,' *Proc. of International Conference on Acoustics, Speech and Signal Processing*, pp. IV 81–84, April 1993.

[2] Y. Li and Z. Ding, 'ARMA system identification based on second order cyclostationarity,' *IEEE Transactions on Signal Processing*, vol.42, pp3483-3494, December 1994.

[3] Y. Li and Z. Ding, 'Global convergence of fractionally spaced Godard equalizer,' to appear in *IEEE Trans. on Signal Processing.*

[4] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Transactions on Signal Processing*, vol.43, pp516-525, Feb. 1995.

[5] L. Tong, G. Xu and T. Kailath, "Blind identification and equalization based on second-order statistics: a time domain approach," *IEEE Transactions on Information Theory*, vol.40, pp340-349, March 1994

[6] D. Yellin and B. Porat, "Blind identification of FIR systems excited by discrete-alphabet inputs," *IEEE Transactions on Signal Processing*, vol.41, pp1331-1339, March 1993

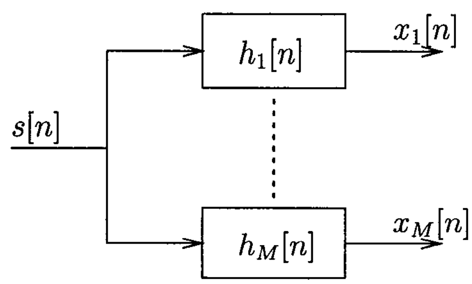

Fig. 1. Single-input/multiple-output channel model.

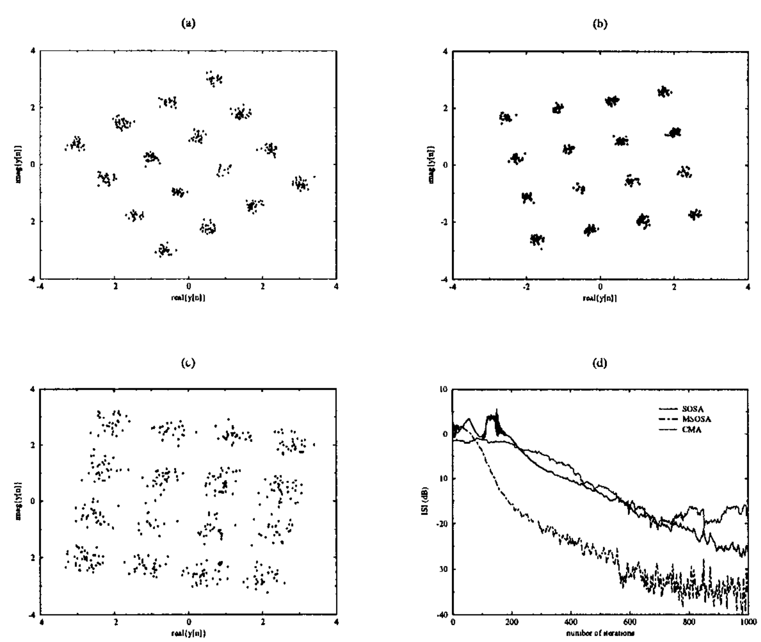

Fig. 2. Comparison of the SOSA, the MSOSA, and the CMA when $SNR = 30db$, 500 symbol eye patterns of (a) the SOSA, (b) the MSOSA, and (c) the CMA after 1000 iterations, and the convergence of the ISI for the three algorithms respectively.