

ABSTRACT

Title of Dissertation: UNRAVELING THE NUCLEON 3D STRUCTURE
FROM EXPERIMENT, LATTICE,
AND GLOBAL ANALYSIS

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In this work, we study the nucleon spin structures and derive a new twist-3 partonic sum rule for the transverse spin of the proton, which involves the well-known quark spin structure function $g_T(x)$, the less-studied but known transverse gluon polarization density $\Delta G_T(x)$, and quark and gluon canonical orbital angular momentum densities associated with transverse polarization. This is the counterpart of the sum rule for the longitudinal spin of the proton derived by Jaffe and Manohar previously. We discuss in detail an important technicality related to the transverse polarization in the infinite momentum frame, i.e., separation of intrinsic contributions from the extrinsic ones. We apply our finding to the transverse-space distributions of partons, angular momentum, and magnetic moment, respectively, in a transversely polarized proton.

For the experimental access to these structures, we consider the deeply virtual Compton scattering (DVCS) in twist-two generalized parton distribution (GPD) approximation. We show that different choices of light-cone vectors and gauges for the final photon polarization will lead

to different higher-order kinematical corrections. The choice of light-cone vectors affects kinematic corrections at the twist-three level, accounting for the differences between the cross-section formulas in the literature. On the other hand, kinematical corrections from higher-twist GPDs eliminate the light-cone dependence at twist three. Those light-cone dependencies are studied systematically at JLab 12 GeV and future EIC kinematics. They serve as the intrinsic systematic uncertainties in extracting the Compton form factors. More importantly, they are also necessary for understanding cross-section measurements with higher-twist precision and to reconstruct higher-order Compton form factors.

As DVCS is mostly sensitive to the quark structure, for the gluon, we also consider the near threshold photoproduction of heavy vector quarkonium off the proton. We show that the production amplitude can be factorized in terms of gluonic GPDs and the quarkonium distribution amplitude, similar to the high-energy limit. At the threshold, the threshold kinematics has a large skewness parameter ξ , leading to the dominance of the spin-2 contribution over higher-spin twist-2 operators. Thus, threshold production data are useful to extract the gluonic gravitational form factors, allowing studying the gluonic contributions to the quantum anomalous energy, mass radius, spin and mechanical pressure in the proton. We use the recent GlueX data on the J/ψ photoproduction to illustrate the potential physics impact from the high-precision data from future JLab 12 GeV and EIC physics program.

To put various constraints together and obtain the GPDs, we present the first global analysis of GPDs combining lattice quantum chromodynamics (QCD) calculations and experiment measurements including global parton distribution functions (PDFs), form factors (FFs) and DVCS measurements. Together with the constraints at zero skewness, we fit GPDs to global DVCS measurements from both the recent JLab and the earlier Hadron-Electron Ring Accelerator (HERA)

experiments with two active quark flavors and leading order QCD evolution. With certain choices of empirical constraints, both sea and valence quark distributions are extracted with the combined inputs, and we present the quark distributions in the proton correspondingly. We also discuss how to extend the framework to accommodate more off-forward constraints beyond the small ξ expansion, especially the lattice calculated GPDs.

UNRAVELING THE NUCLEON 3D STRUCTURE
FROM EXPERIMENT, LATTICE AND GLOBAL ANALYSIS

by

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List of Abbreviations

3D	3-Dimensional
AM	Angular Momentum
BH	Bethe-Heitler
BKM	Belitsky-Kirchner-Müller
BMJ	Belitsky-Müller-Ji
BMMP	Braun-Manashov-Müller-Pirnay
BMP	Braun-Manashov-Pirnay
BNL	Brookhaven National Lab
CEBAF	Continuous Electron Beam Accelerator Facility
CFF	Compton Form Factor
DA	Distribution Amplitude
DDA	Double Distribution Ansatz
DEP	Deep Exclusive Process
DIS	Deep Inelastic Scattering
DGLAP	Dokshitzer-Gribov-Lipatov-Altarelli-Parisi
DVCS	Deeply Virtual Compton Scattering
DVMP	Deeply Virtual Meson Production
EIC	Electron-Ion Collider
EMC	European Muon Collaboration
EMT	Energy Momentum Tensor
ERBL	Efremov-Radyushkin-Brodsky-Lepage
FF	Form Factor
GFF	Gravitational Form Factor
GK	Goloskokov-Kroll
GPD	Generalized Parton Distribution
GUMP	GPDs through Universal Moment Parameterization

HERA	Hadron-Electron Ring Accelerator
JLab	Jefferson Lab
KM	Kumerički-Müller
LaMET	Large Momentum Effective Theory
LHS	Left-Hand Side
NRQCD	Non-Relativistic QCD
OAM	Orbital Angular Momentum
OPE	Operator Product Expansion
PDF	Parton Distributions Function
QCD	Quantum Chromodynamics
RHS	Right-Hand Side
SLAC	Stanford Linear Accelerator Center
TCS	Time-like Compton Scattering
VGG	Vanderhaeghen-Guichon-Guidal
WW	Wandzura-Wilczek

Chapter 1: Introduction

Among the four fundamental forces in nature: gravitational, electromagnetic, weak and strong forces, the strong force is particularly mysterious. The theory of quantum chromodynamics (QCD) is widely accepted to be the mathematical description of the strong force, which states that strongly interacting particles, the so-called hadrons, are made of fundamental particles called quarks and the strong force is simply the exchange of the mediators, the gluons, between them. Nevertheless, a further understanding of how exactly the hadrons are formed of these elementary particles has not yet been obtained, making it extremely challenging to interpret even the most basic properties of the hadrons, e.g., how does the mass or spin of hadrons arise. Since the visible matter is made of mostly hadrons such as the proton and neutron, together with the much lighter electron, the study of hadron structures with QCD is crucial for the comprehension of the visible world at the most fundamental level.

A salient property possessed by the strong interaction is the asymptotic freedom [1, 2] that the strong interaction gets weaker as the length scale decreases (and the energy scale increases correspondingly), and therefore particles approach complete freedom under strong interaction as they get infinitely close. Such a property makes it exceedingly complicated to study the hadron structure, since the hadron typically has a large scale (comparing to the scale required for asymptotic freedom) where all the constituent particles within are strongly coupled together.

The physics in this case is non-perturbative, compared to the case with small length scale and weak coupling between particles that can be perturbatively solved. The non-perturbative physics can not be easily solved theoretically. Consequently, one will have to rely on the experiments or numerical method like lattice QCD to access them.

Tremendous efforts had been devoted to the exploration of the non-perturbative hadron structure via high energy scattering processes like deep inelastic scattering (DIS) that dissect the hadron or nucleon in the direction of the injected high energy particle. By tuning the injected particle, one can obtain the one-dimensional structure of the nucleon connected to the so-called parton distributions functions (PDFs) that give the probability densities of finding a high-energy quark or gluon (collectively denoted as partons) of different momentum fraction in the nucleon. The PDFs together with perturbative QCD calculations show great success in the description of the worldwide high energy scattering measurements, providing strong evidence for the justification of QCD as the theory of the strong interaction. Meanwhile, these quantities allow the establishment of a high-quality picture of the longitudinal structure of the nucleon.

Over the last two decades, studies of the non-perturbative structure of the protons and neutrons, and thereby nuclei, have undergone several major paradigm shifts. One common feature of these shifts is the extension of the mathematical description of nucleons and nuclei to higher-dimensional spaces. Whereas the spatial and momentum structures of the proton used to be described in terms of distinct, one-dimensional distributions, the elastic form factors (FFs) and quark and gluon PDFs, respectively, the concept of generalized parton distributions (GPDs) [3–5] unified these distributions into a single set of three-dimensional functions.

Mapping the 3D images of the nucleon in terms of GPDs has been one of the most important goals in modern nuclear physics, which will provide, among other insights, an intuitive

understanding of how the fundamental properties of the nucleon, such as its mass and spin, arise from the underlying quark and gluon degrees of freedom. Nevertheless, obtaining high-quality images from experimental data has remained a dream for more than two decades. First, persisting efforts are needed for an adequate amount of data of specific processes, namely the deep exclusive processes (DEP), from high-energy electron-nucleon collisions. Second, extraction of the critical information from experimental data has also been challenging.

However, several recent breakthroughs have made the 3D imaging possible. Large data sets of DEP are generated at Jefferson Lab (JLab) with the continuous electron beam accelerator facility (CEBAF), and more will come from the planned Electron-Ion Collider (EIC) [6, 7] at Brookhaven National Lab (BNL). Additionally, novel development in lattice QCD allows the access to the nucleon structures from first principle calculation [8–10], providing information almost impossible to get with current and future experiments.

In this dissertation, we will present researches aiming at the nucleon 3D structures from various perspectives following the above development.

1.1 Deep inelastic scattering, quark model, and QCD

One crucial discovery that lays a solid foundation for the QCD interpretation of the strong force is the Bjorken scaling in the measurement of the electron-nucleon deep inelastic scattering (DIS), as illustrated in Figure. 1.1. It was found that the scattering cross-section becomes independent of the scale, namely the virtuality Q of the exchanged virtual photon in a certain limit [11]. As the virtuality of the photon represents the spatial resolution of the probe where higher virtuality corresponds to finer spatial resolution, the observation of the scale-independence

indicates that a point-like constituent particle without any internal structures has been knocked out at high energies free of interaction.

Such a picture is first interpreted by Feynman's parton model [12], suggesting that in a fast-moving proton the interaction among the constituent particles, or partons, is suppressed due to Lorentz time contraction, and therefore the probe detects almost-free constituent particles. Then, the experimental validation of the Callan-Gross relation stating that the two structure functions $F_1(x)$ and $F_2(x)$ in the DIS process satisfy the relation $2xF_1(x) = F_2(x)$ [13] proves that these partons must have $1/2$ spin. These pioneering discoveries in the hadron structures were eventually justified by the theory of QCDs — the spin- $1/2$ partons are identified as the fundamental particles quarks, and the interaction-free behavior in the high energy limit is nothing but the asymptotic freedom in QCD.

However, it is interesting to note that the original quark model was proposed earlier than

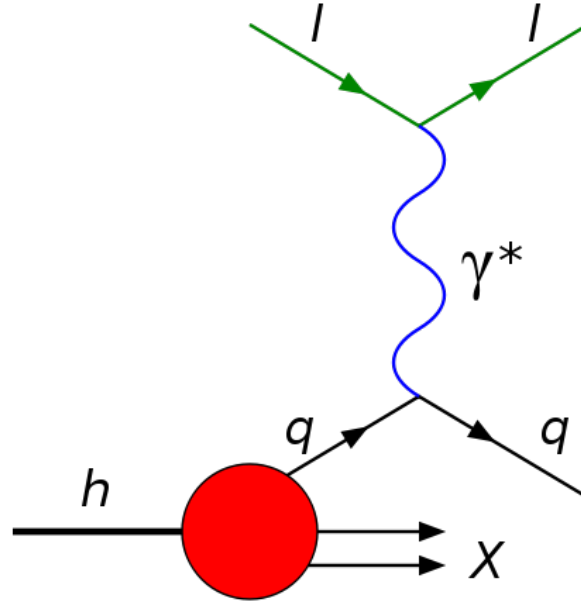


Figure 1.1: The electron-nucleon deep inelastic scattering process. Picture from Wikipedia.

the very first DIS experiment at Stanford linear accelerator center (SLAC) in 1968 [14], and therefore independently of the above discoveries in DIS. It was invented as the tool to classify hadrons in terms of their quark components in the flavor space [15, 16], i.e., in terms of their different combinations of up (u), down (d), and strange (s) quarks as shown in Figure. 1.2.

It was later realized that the existence of the spin-3/2 $\Delta^{++}(uuu)$ baryon that contains three up quarks with parallel spins and zero orbital angular momentum (OAM) contradicts the Pauli exclusion principle that requires antisymmetric wave functions [17]. Therefore, a hidden degree of freedom which is later named as color is proposed to provide the required antisymmetric wave function in the color space and resolve this problem [18], upon which the modern quark model with three colors as the charge of strong interaction is developed [19] under the framework of the $SU(3)$ Yang-Mills gauge theory [20]. This long journey from the rather unrelated discovery of Bjorken scaling and quark model to the unified theory of QCD with asymptotic freedom, as Winston Churchill says, “is not the end. It is not even the beginning of the end. But it is, perhaps,

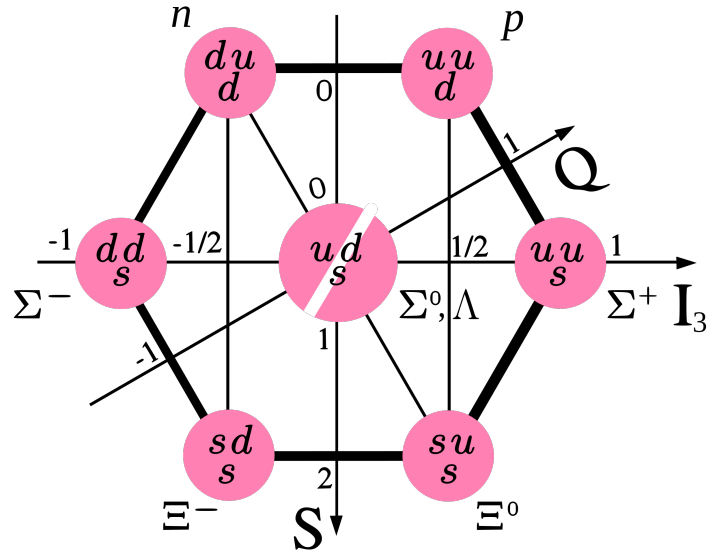


Figure 1.2: Baryon octet with up, down, and strong quark of spin 1/2. Picture from Wikipedia.

the end of the beginning” in the studies of the strong interaction and hadron structures.

1.2 Proton spin crisis

Moving forward from the establishment of the QCD, the spin structure of the proton or nucleon has been an important topic in hadronic physics for over the past 30 years [21–23]. With the naive quark model, the proton or neutron spin seems rather intuitive. Since each of the three constituent quarks have 1/2 spin, the lowest possible spin they can make is just 1/2 which coincides with the spin of the proton or neutron. Therefore, one might expect the quark spin to contribute a significant part to the proton spin even in the presence of strong interaction.

As natural as such an argument might sound, what the European muon collaboration (EMC) experiment found triggered the so-called “spin crisis”, as their measured quark spin contribution to the proton spin is [24]:

$$\Delta\Sigma(Q^2 = 10.7 \text{ GeV}^2) = 0.060 \pm 0.047 \pm 0.069, \quad (1.1)$$

which is consistent with zero. This result shocked the whole community, as it forces everyone to face the cruel fact that the non-perturbative physics can be utterly different from the perturbative one, and there seems to be no key to that but brutal force.

To fully understand the proton spin structures, one has to invoke the theory of QCD, which dictates that gluons must be considered to mediate the strong force between quarks (and the strong force among themselves). Accordingly, the candidates that account for the missing spin in the proton are intuitively the quark and gluon OAM and the gluon spin. However, the sophistication of QCD especially in the non-perturbative region seems to defy a simple description, and thus

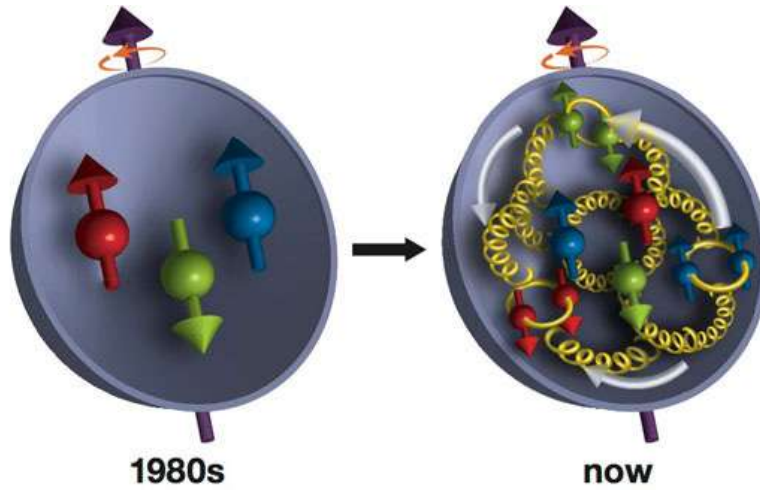


Figure 1.3: Modern understanding of the proton spin structure. Credit to Brookhaven National Laboratory.

one can not, or at least has not been able to, find proper wave function descriptions of quarks and gluons that allow one to access various observables including angular momentum (AM) in a straightforward way. Alternatively, one can only look at each observable individually and decompose it into pieces with different physical interpretations, from which a so-called sum rule can be derived. For instance, various sum rules for the nucleon spin are derived in the literatures [4, 25], as well as for the mass [26]. Therefore, the measurement of the different components in these sum rules become a crucial part in the understanding of the hadron structure and strong interaction.

A more careful treatment of the nucleon AM decomposition requires one to look into the QCD energy momentum tensor (EMT) and AM operators which will be discussed in the following chapter, where we will derive a new sum rule for the transverse polarized nucleon as well. The clear message one can take from here is, on the other hand, fairly straightforward that the proton spin must be built up in a much more sophisticated way with the necessary involvement of the gluons as shown in Figure 1.3.

1.3 Nucleon 3D structures, tomography and generalized parton distributions

Emerging from the proton spin crisis, the higher-dimensional structure of the nucleon beyond the one-dimensional longitudinal one has received rising attention. Since the complete proton spin decomposition requires one to consider the orbital motions of quarks and gluons as mentioned above, one inevitably has to consider the dynamics in the transverse space where the orbit exists. Correspondingly, the spin structure of the nucleon will be encoded in the higher-dimensional structure functions.

Indeed, it was shown that the nucleon spin structures can be accessed through the 3-dimensional parton distributions known as the generalized parton distributions (GPDs) [4]. Moreover, GPDs enabled us to access new quantities, such as the gravitational form factors (GFFs) parameterizing the partonic EMT [4], fundamental to the distributions of mass, AM, and partonic forces [4, 26, 27]. These new perspectives shed a refreshing new light on the problem of proton structure and confinement within a unified framework.

As indicated by the name, GPDs are the generalization of ordinary PDFs to the off-forward case, i.e., the nucleon recoils with a non-zero momentum transfer when the partons are probed. Analogous to the relation between the nucleon charge form factors and its charge distributions, the Fourier transformation of the non-zero transferred momentum of GPDs also defines certain parton distributions in the coordinate space. This picture is simplest when the momentum transfer is completely transverse to the direction of the parton momentum. In this case, the Fourier transformation of GPDs represent the transverse space (also known as the impact parameter space) distributions of the parton moving in the longitudinal direction, as first pointed out in ref. [28].

Compared to the interpretation of the PDFs which concerns the longitudinal structures

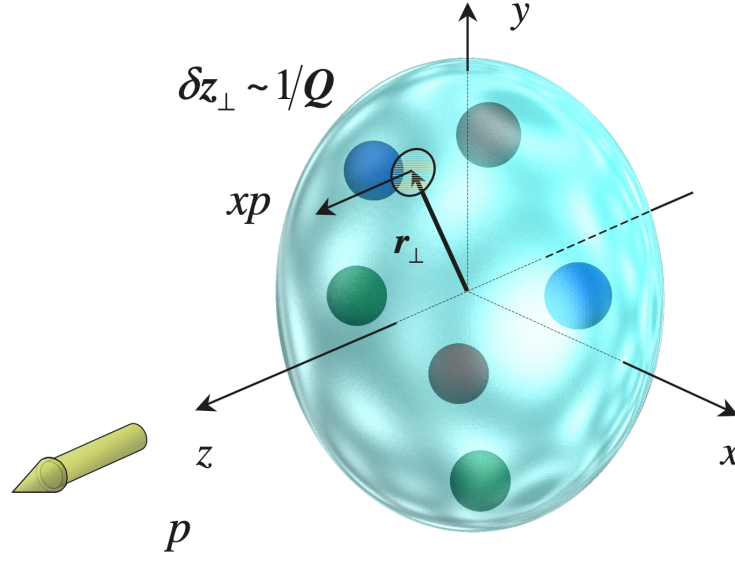


Figure 1.4: The interpretation as impact parameter space distributions of GPDs at zero skewness. Credits to ref. [29].

only, the transverse space distributions represent the probability densities of finding a parton with momentum fraction x that is localized in the transverse coordinate space with a displacement $\mathbf{r}_\perp$, as illustrated in Figure 1.4, providing extra information in the transverse directions.

Such interpretations of GPDs stimulate the concept of the nucleon tomography: by sectioning the nucleon into partons of different momentum fraction and localizing them in the transverse space, the nucleon 3D structures can be equivalently represented by the collection of all the two-dimensional transverse space distributions, which are given by the Fourier transformation of the GPDs. These transverse space distributions serve as semi-classic pictures of the quarks and gluons in the nucleon despite that we cannot obtain their wave functions. It can also be shown that these semi-classic pictures can generate the sum rules e.g., the transverse spin sum rule, manifestly, as we will discuss more in the next chapter. These 3D distributions are also part of the even higher-dimensional Wigner distributions, which represent the quantum phase space distributions of the parton in both momentum and coordinate space.

1.4 Lattice QCD for hadron structures

Another possibility to access the non-perturbative hadron structure is through lattice simulation. While QCD itself is a quantum field theory living in the continuous four-dimensional Minkowski spacetime, lattice QCD was proposed for the numerical simulation of QCD on a four-dimensional but discretized Euclidean spacetime [31], where quarks live on discretized spacetime points and gluons exist as the gauge links between them as illustrated in Figure 1.5.

Benefiting from decades of theoretical development towards the simulation of field theory on discretized lattice as well as the rapidly growing computational power, lattice QCD has become a unique tool in the determination of the non-perturbative hadron structures, allowing one to obtain physical observables ranging from the local nucleon matrix elements that give the nucleon charges and form factors to the non-local ones that lead to the parton structures with the novel development of the large momentum effective theory (LaMET) [8–10] from first principle.

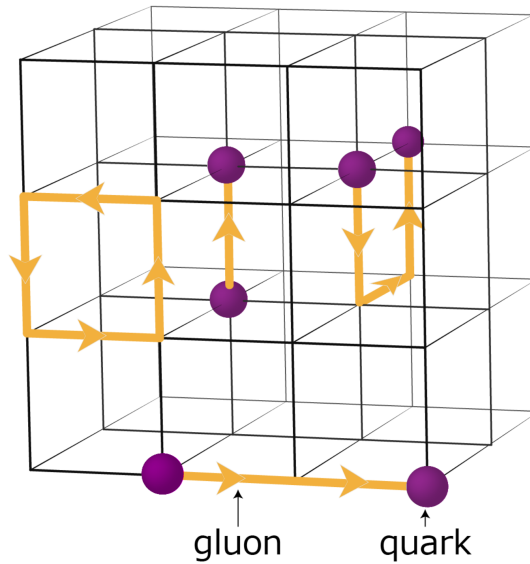


Figure 1.5: Lattice QCD discretization illustration with quarks living on the site and gluons living on the link. Credits to ref. [30].

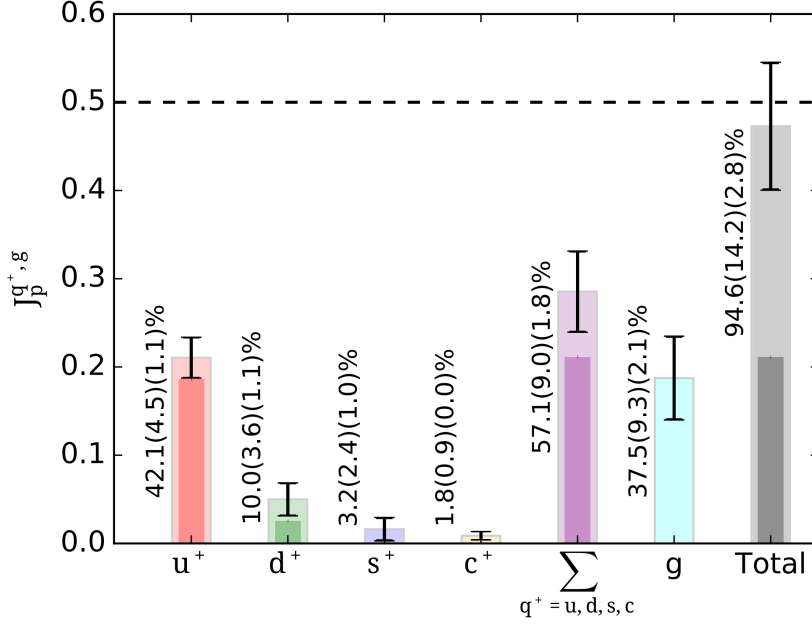


Figure 1.6: Proton spin decomposition from lattice QCD simulation by the ETMC collaboration [32].

Lattice QCD not only provides a different perspective to the studies of the non-perturbative nucleon structures, in some cases it is the only practical approach to certain nucleon properties that are too hard to extract from experiments. One representative example is the mass and spin decomposition of the nucleon. Despite the importance of the nucleon mass and spin structure, acquiring them from experiments has been a major challenge. Since the mass and spin are parts of the mechanic properties of the nucleon, they are related to the nucleon EMT which naturally couples to the gravitational interaction. Given the masses of the nucleons as microscopic particles, experimentally probing the gravitational scattering of them is merely outrageous. Lattice QCD, on the other hand, does not suffer from such suppression due to the gravitational coupling — the nucleon matrix elements of the EMT can be directly simulated on the lattice. Thus, the matrix elements of the EMT can be obtained much more easily with lattice QCD. In Figure 1.6, an example of the proton spin decomposition with lattice QCD simulation by the ETMC collab-

oration [32] is shown. The proton mass decomposition can be done similarly with the nucleon EMT as presented in ref. [33].

1.5 3D global analysis program

In the end, we would like to bring attention to the necessity of a global analysis program for the higher-dimensional hadron structures. The global analysis of the one-dimensional PDFs has been a crucial part in the analysis of worldwide high energy scattering measurements [34–36]. As mentioned above, it shows great success and strongly justifies the theory of QCD. For the higher-dimensional hadron structure functions such as the GPDs for nucleon 3D structures, a global analysis program is even more demanded as the higher dimension simply means much more data and much more complicated structure functions to extract.

Not only that, another challenge appears in the extraction of these higher-dimensional structure functions, known to be the inverse problem. Since each single probe commonly has a lower dimension, e.g., a two-dimensional probe of three-dimensional structure functions like GPDs, reconstructing the structure functions from each of these lower-dimensional probe individually suffers from uncertainties in the missing dimension. Therefore, a global analysis program that could combine all the lower-dimensional measurements to synthesize the higher-dimensional structure functions is required to overcome this inverse problem.

With such a motivation, lots of GPD parameterization methods are proposed in the literature and many efforts have been put into extracting the critical information from various measurements and obtaining GPDs from them [37–47]. However, a general parameterization strategy that combines various physical constraints on GPDs with the lattice calculations still seems to be

The isovector GPD H_{u-d} at $-t = 0.69 \text{ GeV}^2$ tuned with DA terms

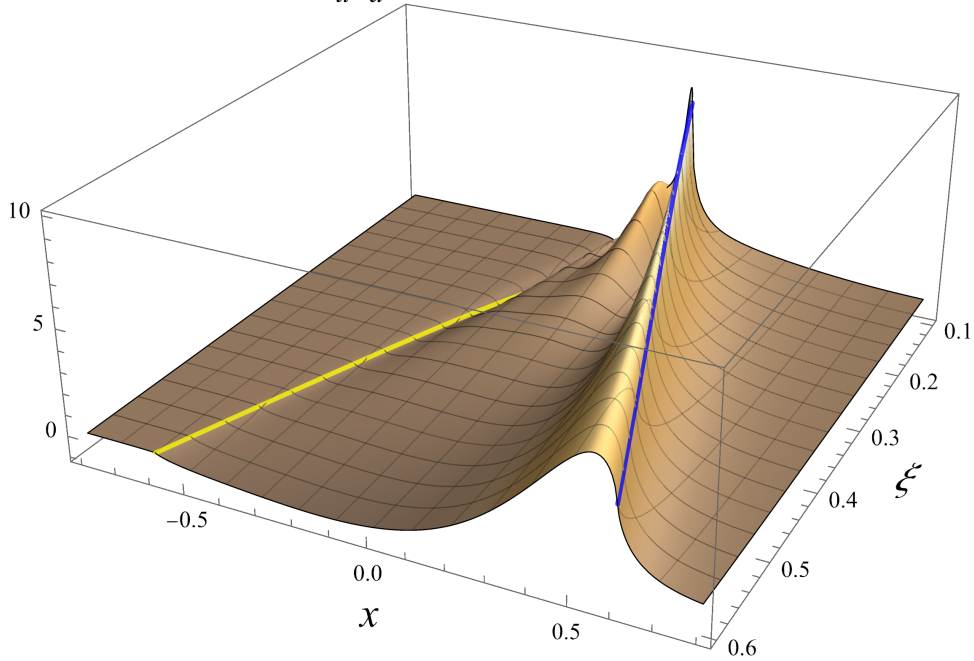


Figure 1.7: The isovector GPD H_{u-d} on the (x, ξ) plane obtained with combined inputs of lattice and experiments. More details will be discussed in chapter 5.

lacking. As the lattice QCD provides unique information complimentary to experimental inputs, the state-of-art higher-dimensional structure functions can only be obtained considering both. In Figure 1.7, we show an example of the GPD H_{u-d} obtained with combined inputs of lattice and experiments. More details of the global analysis of GPDs and the GPDs through Universal Moment Parameterization (GUMP) program will be discussed in chapter 5.

1.6 Outline

The outline of this work is as follows. After this introduction, there are four main chapters discussing various topics mentioned above. In chapter 2, we start by studying the nucleon spin structures. We will derive a new twist-3 partonic sum rule for the transverse spin of the proton, which involves the well-known quark spin structure function $g_T(x)$, the less-studied but known

transverse gluon polarization density $\Delta G_T(x)$, and quark and gluon canonical OAM densities associated with transverse polarization. For the experimental access to GPDs, we will study the deeply virtual Compton scattering (DVCS) cross-section in twist-two GPD approximation in chapter 3. We will focus on clarifying the higher-order kinematical effects in the DVCS cross-section due to different choices of light-cone vectors and gauges for the final photon polarization. In chapter 4, we will also study the near threshold photo or electroproduction of heavy vector quarkonium off the proton with QCD factorization. We will show that, similar to the high-energy limit, the production amplitude can be factorized in terms of gluonic GPDs and the quarkonium distribution amplitude. We will also show that with the near threshold kinematics, the spin-2 contribution dominates, allowing studying the gluonic contributions to the quantum anomalous energy, mass radius, spin and mechanical pressure in the proton. In order to eventually obtain the GPDs from various constraints, in chap 5 we present the first global analysis of GPDs combining lattice QCD calculations and experiment measurements including global PDFs, FFs, and DVCS measurements with the GUMP program. In the end, we present the summary and outlook in chapter 6.

Chapter 2: A new transverse spin sum rule and transverse space distributions

In the introduction, we discussed the proton spin crisis and argued that the nucleon spin structure can be identified by decomposing the full AM operator of QCD. However, the decomposition of the spin into various physical parts, also known as the spin sum rules, is not unique. For instance, the sum rule for the longitudinal spin can be derived by decomposing the full QCD AM operator into [48],

$$\mathbf{J}_{\text{QCD}} = \int d^3\mathbf{x} \left[\psi_q^\dagger \frac{\boldsymbol{\Sigma}}{2} \psi_q + \psi_q^\dagger \mathbf{x} \times (-i\boldsymbol{\partial}) \psi_q + \mathbf{E}_a \times \mathbf{A}_a + \mathbf{E}_a (\mathbf{x} \times \boldsymbol{\partial}) \mathbf{A}_a \right], \quad (2.1)$$

where ψ_q are the quark fields, $\mathbf{E}_a$ and $\mathbf{A}_a$ are the gluon fields and $\boldsymbol{\Sigma} \equiv (\boldsymbol{\sigma}, \boldsymbol{\sigma})$ with $\boldsymbol{\sigma}$ the Pauli matrices. The quark flavor (q) and color (a) indices are always summed over. Taking the matrix element of the above decomposition of the AM operator, one obtain the Jaffe-Manohar spin sum rule that reads,

$$\frac{1}{2} \Delta \Sigma + \Delta G + l_q + l_g = \frac{\hbar}{2}. \quad (2.2)$$

Straightforwardly, the four terms corresponds to the partonic quark helicity, gluon helicity, quark canonical OAM, and gluon canonical OAM in the infinite momentum frame and light-front gauge. On the other hand, the AM operator can also be decomposed in a gauge-invariant way

as [4],

$$\mathbf{J}_{\text{QCD}} = \int d^3\mathbf{x} \left[\psi_q^\dagger \frac{\boldsymbol{\Sigma}}{2} \psi_q + \psi_q^\dagger \mathbf{x} \times (-i\boldsymbol{\partial} - g\mathbf{A}_a) \psi_q + \mathbf{x} \times (\mathbf{E}_a \times \mathbf{B}_a) \right], \quad (2.3)$$

where each term on the right-hand side is manifestly gauge-invariant. Again, taking the matrix element of the above decomposition, one get instead a covariant spin sum rule that reads,

$$\frac{1}{2}\Delta\Sigma + L_q^z + J_g^z = \frac{\hbar}{2}, \quad (2.4)$$

which is valid for any reference frame, and applies to both longitudinal and transverse polarizations. Details of comments and the comparison of the two sum rules can be found in ref. [23].

Both sum rules above provide the desired $\hbar/2$ spin of the proton after summing over all components and can guide us to find out the missing pieces of proton spin. The quark and gluon spin contributions are related to the polarized operators of quark $\bar{\psi}_q \gamma^\mu \gamma^5 \psi_q$ and gluon $G^{\mu\alpha} \tilde{G}_\alpha^\nu$ which can be accessed from the first-moment of their polarized PDFs [49]. On the other hand, recent developments [50–53] have shown that the parton OAM is closely connected to the quantum phase space Wigner function or distribution [54, 55], which are essentially GPDs when integrating over the momentum space. Therefore, the task of resolving the proton spin structure is intrinsically related to the higher-dimensional structures of the proton.

In this chapter, we start with a overview of the nucleon spin sum rules, which have been studied extensively in the literatures in the case of longitudinal polarization. As for the transverse polarization, much controversy has been generated in the literature [50, 56–59] due to the contribution from the center-of-mass motion when the nucleon is transversely polarized. The issue has

been recently resolved through by an explicit elimination of the center-of-mass contribution [60]. Following this observation, we show that how the extrinsic contributions to transverse AM from the center-of-mass motion can be removed and derive a new transverse AM sum rule that involves the well-know quark spin structure function $g_T(x) = g_1(x) + g_2(x)$, the less-studied but known transverse gluon polarization density $\Delta G_T(x)$, and quark and gluon canonical transverse OAM densities, which are expressible in terms of the quark and gluon GPDs [61]. The result is also crucial for the proper understanding of the transverse-space distributions of partons in a transversely polarized nucleon.

In order to access the spin sum rules, GPDs of different twist will be involved. Twist is a quantity invented for the power counting in high energy experiments which also corresponds to the short distance in the case of e.g., short-distance operator product expansion (OPE). Dynamically, the matrix elements of operators of twist- t will be suppressed by Q^{2-t} , with Q the typical scale of the system, and therefore the leading order quantities that are not suppressed correspond to $t = 2$ and will be denoted as twist-two accordingly. Another definition of twist is based on the operators themselves, which defines

$$\text{twist}(t) = \text{Dimension of the operator}(d) - \text{Lorentz spin}(s) . \quad (2.5)$$

This so-called geometric twist is slightly different from the dynamic twist defined above, but the two definitions are consistent.

It is well-known that the partonic AM densities in a transversely polarized nucleon are of leading twist, whereas the longitudinal AM densities are twist-three. This can be understood

intuitively by their different behaviors under Lorentz boost:

$$J'_{\parallel} = J_{\parallel} \quad , \quad J'_{\perp} = \gamma (J_{\perp} - \beta K_{\perp'}) \quad (2.6)$$

with β the boost velocity $\beta \equiv v/c$, $\gamma \equiv (1 - \beta^2)^{-1/2}$ and K the boost operator that mixes with the angular momentum operators J under Lorentz boost. In the high energy limit $\gamma \rightarrow \infty$, the transverse AM $J'_{\perp}$ is clearly enhanced compared to the longitudinal one $J'_{\parallel}$.

Therefore, sum rules for the transverse and longitudinal AM will involve parton distributions of different twist. For instance, the sum rule in eq. (3.1) describes the AM densities in transversely polarized nucleons [28], and the two GPDs H and E are twist-two GPDs, whereas for the longitudinal Jaffe-Manohar sum rules, the orbital parts will be given by twist-three GPDs [50–53]. Consequently, both the leading twist and next-to-leading twist are important to complete the understanding of the partonic orbital motions in the nucleon, besides the practical consideration that the higher-twist effects can be significant in experiments and need to be considered anyway.

We start with the full QCD AM operator which can be derived from the EMT. The EMT is a fundamental object which plays an important role in both classical and quantum field theory. It can be derived from the Lagrangian density:

$$T^{\mu\nu}(r) = \frac{\partial \mathcal{L}}{\partial(\partial_{\mu}\phi)} \partial^{\nu}\phi(r) - g^{\mu\nu} \mathcal{L}(r) . \quad (2.7)$$

The resulting EMT operator will be conserved: $\partial_{\mu} T^{\mu\nu} = 0$, and the corresponding conserved

charge is the 4-momentum P^μ :

$$P^\mu \equiv \int d^3\mathbf{r} T^{0\mu}(\mathbf{r}) . \quad (2.8)$$

However, the EMT has two standard definitions. Besides the canonical one in eq. (2.7), there is also the Belinfante improved one, which is constructed to be both conserved and symmetric, while still leads to the same conserved charge as the canonical definition

$$T_{\text{Bel}}^{\mu\nu}(r) = T^{\mu\nu}(r) + \partial_\lambda G^{[\lambda\mu]\nu}(r) , \quad (2.9)$$

where the $G^{[\lambda\mu]\nu}$ term is sometimes referred to as a *super potential*. Its specific form is not important, as in observables this term gets discarded as a surface term, where some care has been taken in the past to ensure this is valid in a wave packet analysis [62, 63]. The Belinfante one in eq. (2.9) will be used hereafter for its nice symmetry properties.

From the EMT we can then define the AM density operator as follows

$$M^{\mu\nu\rho}(r) = r^\nu T_{\text{Bel}}^{\mu\rho}(r) - r^\rho T_{\text{Bel}}^{\mu\nu}(r) , \quad (2.10)$$

which is again a conserved quantity $\partial_\mu M^{\mu\nu\lambda} = 0$ and the corresponding conserved charge will be the relativistic AM operators:

$$J^{\mu\nu} = \int d^3\mathbf{r} M^{0\mu\nu}(\mathbf{r}) . \quad (2.11)$$

These operators are key ingredients in the so-called Pauli-Lubanski vector, that can be used to

represent the spin state of a relativistic particle. The Pauli-Lubanski vector is formally defined by

$$W^\mu = -\frac{1}{2}\epsilon^{\mu\alpha\lambda\sigma}\frac{J_{\alpha\lambda}P_\sigma}{M}, \quad (2.12)$$

where $\epsilon^{0123} = 1$ and P_σ and $J_{\alpha\lambda}$ are the momentum and AM operators above. With the Pauli-Lubanski vector, one could construct a scalar operator $W^\mu W_\mu$ that commutes with all the generators of Poincaré group, also known as the Casimir operator. Thus, all the particles can be constructed to be the eigenstate of the Casimir operator, and the eigenvalue corresponds to the spin of such state:

$$W^\mu W_\mu \xrightarrow{\text{eigenvalues}} -s(s+1)\hbar^2, \quad (2.13)$$

which describe the intrinsic AM of the particles. Therefore, the intrinsic AM according to eq. (2.13) involves not only the 3D AM operators J^i , but also the 4-momentum P^μ and the boost operators K^i , where J^i and K^i are defined as,

$$J^i = \frac{1}{2}\epsilon^{0ikl}J_{kl}, \quad (2.14)$$

$$K^i = J^{0i}. \quad (2.15)$$

To make this more intuitive, consider a moving particle of mass M and momentum P^μ ,

$$P^\mu = \gamma M(1, \boldsymbol{\beta}), \quad (2.16)$$

where $\gamma = (1 - |\boldsymbol{\beta}|^2)^{-\frac{1}{2}}$, $\boldsymbol{\beta} = \mathbf{v}/c$. For such a particle, the Pauli-Lubanski vector W^μ can

alternatively be expressed as

$$W^\mu = \gamma(\mathbf{J} \cdot \boldsymbol{\beta}, \mathbf{J} - \boldsymbol{\beta} \times \mathbf{K}) , \quad (2.17)$$

which clearly involves both $\mathbf{J}$ and $\mathbf{K}$. To find out the spin of the particle, we first define the 4-dimensional polarization vector S^μ which is simply the 4-dimensional polarization vector s in the rest frame:

$$S^\mu = (0, \mathbf{s}) \quad \text{for} \quad \boldsymbol{\beta} = 0 , \quad (2.18)$$

and satisfies the condition $P \cdot S = 0$ obviously. Then the polarization vector S^μ in an arbitrary frame can be obtained by boosting the rest-frame S^μ , which gives:

$$S^\mu = (\gamma \mathbf{s} \cdot \boldsymbol{\beta} , \mathbf{s} + (\gamma - 1)(\mathbf{s} \cdot \hat{\boldsymbol{\beta}})\hat{\boldsymbol{\beta}}) . \quad (2.19)$$

The spin operator S^p of the particle is then the projection of Pauli-Lubanski vector W^μ to the polarization vector S^μ :

$$S_p = -W^\mu S_\mu = \gamma \mathbf{J} \cdot \mathbf{s} - (\gamma - 1) \mathbf{s} \cdot \boldsymbol{\beta} \mathbf{J} \cdot \boldsymbol{\beta} - \gamma \mathbf{s} \cdot (\mathbf{K} \times \boldsymbol{\beta}) , \quad (2.20)$$

that satisfies

$$S_p |P, S\rangle = \frac{\hbar}{2} |P, S\rangle , \quad (2.21)$$

for a spin $\frac{1}{2}$ particle with momentum P^μ and polarization vector S^μ .

Although the spin operator S_p operator provides the desired $\frac{\hbar}{2}$ spin, it includes both AM and boost operators as shown in eq. (2.20), which are not easily disentangled. Therefore, we

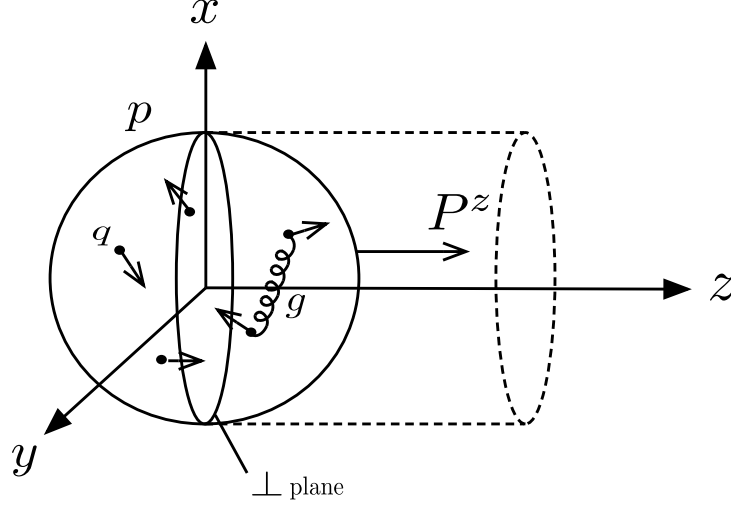


Figure 2.1: A proton moving along the z -axis with an intrinsic AM that could be in any direction generally due to the internal dynamics of its quark and gluon constituents (both their motion and their spin orientation).

will be looking directly at the matrix elements of the AM operator J^i , which on the other hand indicates that extra care must be taken to define the intrinsic AM since the AM operator J^i is not a scalar operator like the spin operator S_p .

Without loss of generality, we take the proton to be traveling in the $\hat{z}$ direction with $P^\mu = (P^0, 0, 0, P^z)$ as shown in Fig. 2.1. Transverse then refers to any direction in the xy plane, while longitudinal is in the z direction. Using the definitions above, the expectation value of $J^x(r^0 = 0)$ is then,

$$\langle P, S | J^x(r^0 = 0) | P, S \rangle = \int d^3\mathbf{r} \langle P, S | r^y T_{\text{Bel}}^{0z}(0, \mathbf{r}) - r^z T_{\text{Bel}}^{0y}(0, \mathbf{r}) | P, S \rangle, \quad (2.22)$$

which can be re-written as the limit of the off-forward matrix element,

$$\langle P, S | J^x(r^0 = 0) | P, S \rangle = \lim_{\Delta \rightarrow 0} \int d^3\mathbf{r} e^{-i\Delta \cdot \mathbf{r}} \langle P + \Delta, S' | r^y T_{\text{Bel}}^{0z}(0) - r^z T_{\text{Bel}}^{0y}(0) | P, S \rangle. \quad (2.23)$$

With the replacement $r^j e^{-i\Delta \cdot \mathbf{r}} \rightarrow i \frac{\partial}{\partial \Delta^j} e^{-i\Delta \cdot \mathbf{r}}$, we can then integrate by parts to get [4]

$$\langle P, S | J^x(r^0 = 0) | P, S \rangle = i(2\pi)^3 \delta^{(3)}(0) \lim_{\Delta \rightarrow 0} \left[\frac{\partial}{\partial \Delta^y} \langle P + \Delta, S' | T_{\text{Bel}}^{0z}(0) | P, S \rangle - (z \leftrightarrow y) \right], \quad (2.24)$$

up to a total derivative term. The total derivative term was shown to be the AM of the nucleon's wave packet about the origin [63] and we will consequently discard it. The divergent prefactor will be cancelled in a properly normalized expectation value.

The above results show clearly that the matrix element of the AM operator J^i is related to the off-forward matrix elements of the EMT, which can be parameterized in terms of the so-called GFFs as [4]

$$\begin{aligned} \langle P' | T_{q,g,\text{Bel}}^{\mu\nu} | P \rangle = & \bar{u}(P') \left[A_{q,g}(\Delta^2) \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g}(\Delta^2) \bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha / 2M \right. \\ & \left. + C_{q,g}(\Delta^2) (\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2) / M + \bar{C}_{q,g}(\Delta^2) g^{\mu\nu} M \right] u(P), \end{aligned} \quad (2.25)$$

where the $A_{q,g}(\Delta^2)$, $B_{q,g}(\Delta^2)$, $C_{q,g}(\Delta^2)$, and $\bar{C}_{q,g}(\Delta^2)$ are the four GFFs of quarks and gluons.

Accordingly, the AM can be expressed in terms of these GFF with eq. (2.24) as

$$J_{q,g} = \frac{1}{2} [A_{q,g}(0) + B_{q,g}(0)], \quad (2.26)$$

which are quark and gluon contributions to the AM of the the nucleon. These GFFs also contain other interesting properties such as the mass and mechanical properties of the nucleon [4, 26, 27].

2.1 Twist-two and twist-three GPDs

The measurement of these GFFs, however, is quite challenging. Since the EMT naturally coupled to spin-2 particle, it can be probed by the elastic scattering with such particles like the graviton, which is not realistic at all. Therefore, we consider another type of off-forward matrix elements, namely the GPDs to access these GFFs.

GPDs are defined as the matrix elements of non-local light-front correlators. For instance, the simplest GPDs for involving quark fields can be written as

$$\int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P', S' | \bar{\psi} \left(-\frac{\lambda n}{2} \right) W \left(-\frac{\lambda n}{2}, \frac{\lambda n}{2} \right) \Gamma \psi \left(\frac{\lambda n}{2} \right) | P, S \rangle, \quad (2.27)$$

where $W(y^\mu, x^\mu) \equiv P \exp \left(-ig \int_x^y dz^\mu A_\mu(z) \right)$ is the gauge link required to keep the expression gauge-invariant and Γ is a certain Dirac matrix. The hadron states are normalized as $\langle P', S' | P, S \rangle = 2E_P (2\pi)^3 \delta^{(3)}(\mathbf{P}' - \mathbf{P})$ and all matrix elements will be normalized with a factor of $1/\langle P, S | P, S \rangle$ implicitly hereafter.

The light front vectors p^μ and n^μ are auxiliary vectors whose spatial parts are in the z direction and can thus be written as $p^\mu = \frac{1}{\sqrt{2}}(\mathcal{P}, 0, 0, \mathcal{P})$ and $n^\mu = \frac{1}{\sqrt{2}}(1/\mathcal{P}, 0, 0, -1/\mathcal{P})$ with $\mathcal{P}$ an arbitrary scale. Consider the frame in which the average momentum of initial and final states, $\bar{P}^\mu \equiv \frac{1}{2}(P^\mu + P'^\mu)$, is in the longitudinal direction, so the average momentum $\bar{P}^\mu$ can be expressed with light cone vectors p^μ and n^μ . Commonly we choose the normalization $n \cdot \bar{P} = 1$ and therefore $\mathcal{P} = \bar{P}^+$ where we define the light front coordinate $x^\pm = (x^0 \pm x^3)/\sqrt{2}$. Then we can express $\bar{P}^\mu$ as $\bar{P}^\mu = p^\mu + \frac{1}{2}\tilde{M}^2 n^\mu$ with $\tilde{M}^2 \equiv M^2 - \Delta^2/4$ and $\Delta^\mu \equiv P'^\mu - P^\mu$. We also define the momentum transfer squared $t \equiv \Delta^2$ and S^μ the polarization vector of the particle

satisfying $S \cdot P = 0$ and normalized according to $S^2 = -1$.

2.1.1 Parameterization of GPDs

Similar to the case of GFFs, it is helpful to parameterize the matrix elements with a set of scalar functions. Consider the matrix element of bi-local quark operators defined as the following

$$\begin{aligned} \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P', S' | \bar{\psi} \left(-\frac{\lambda n}{2} \right) W_{-\frac{\lambda}{2}, \frac{\lambda}{2}} \Gamma \psi \left(\frac{\lambda n}{2} \right) | P, S \rangle \\ = \bar{u}(P', S') \mathcal{F}_{q,\Gamma}(x, \bar{P}, \Delta, n) u(P, S) , \end{aligned} \quad (2.28)$$

where $\mathcal{F}_{q,\Gamma}(x, \bar{P}, \Delta, n)$ indicates a set of independent and complete functions of different Dirac structures preserving all the symmetries such as the Lorentz and parity symmetry and q represents the quark flavor. For $\Gamma = \gamma^+$, it is well-known that it can be written as [4]

$$\mathcal{F}_{q,\gamma^+} = H_q(x, \xi, t) \gamma^+ + E_q(x, \xi, t) \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} , \quad (2.29)$$

where $\xi \equiv -\Delta \cdot n/2$. More generally for $\Gamma = \gamma^\mu$, it can be parameterized as

$$\begin{aligned} \mathcal{F}_{q,\gamma^\mu} = & \gamma_+^\mu H_q(x, \xi, t) + \frac{i\sigma_+^{\mu\alpha} \Delta_\alpha}{2M} E_q(x, \xi, t) \\ & + \frac{\Delta_\perp^\mu}{M} G_{q,1}(x, \xi, t) + \Delta_\perp^\mu \not{n} G_{q,2}(x, \xi, t) + \frac{i\sigma_\perp^{\mu\alpha} \Delta_\alpha}{2M} G_{q,3}(x, \xi, t) \\ & + M i\sigma_\perp^{\mu\nu} n_\nu G_{q,4}(x, \xi, t) \\ & + M n_-^\mu F_{q,1}(x, \xi, t) + M^2 n_-^\mu \not{n} F_{q,2}(x, \xi, t) , \end{aligned} \quad (2.30)$$

where each μ comes with a subscript indicating that it is non-zero when μ takes the value of the subscripts only. More details can be found in appendix A.

The above parameterization indicates that there are two twist-two GDPs for $\Gamma = \gamma^+$, four twist-three GDPs for $\Gamma = \gamma^\perp$ and two twist-4 GDPs for $\Gamma = \gamma^-$. The counting of the total number of independent terms in these expansions can be achieved by counting the physical degrees of freedom as stated in refs. [64–67]. For each operator $\hat{O}$ the matrix element $\langle P', S' | \hat{O} | P, S \rangle$ includes 4 different helicity amplitudes. However, a half of them are not independent due to the parity symmetry. Thus, for each independent operator, or in this case for each free index in the operator, there will be 2 independent matrix elements.

The arguments also apply to the tri-local correlators. For instance, consider the matrix element of the tri-local correlator

$$\int \frac{d\lambda d\mu}{(2\pi)^2} e^{i\frac{\lambda}{2}(x+y)+i\mu(y-x)} \langle P', S' | \bar{\psi} \left(-\frac{\lambda n}{2} \right) W_{-\lambda/2, \mu} \gamma^\alpha i \overleftrightarrow{D}^\beta (\mu n) W_{\mu, \lambda/2} \psi \left(\frac{\lambda n}{2} \right) | P, S \rangle , \quad (2.31)$$

where the double arrow covariant derivative is defined as $\overleftrightarrow{D} = \frac{1}{2} (\overrightarrow{D} - \overleftarrow{D})$ with $\overrightarrow{D} = \overrightarrow{\partial} + igA$ and $\overleftarrow{D} = \overleftarrow{\partial} - igA$. These correspond to the so-called D-type GDPs [68] and we also have

$$\int \frac{d\lambda d\mu}{(2\pi)^2} e^{i\frac{\lambda}{2}(x+y)+i\mu(y-x)} \langle P', S' | \bar{\psi} \left(-\frac{\lambda n}{2} \right) W_{-\lambda/2, \mu} \gamma^\alpha ig G^{+\beta} (\mu n) W_{\mu, \lambda/2} \psi \left(\frac{\lambda n}{2} \right) | P, S \rangle , \quad (2.32)$$

which correspond to the F-type GDPs. The D- and F-type GDPs are accompanied by two free indices α, β , and thus a more general parameterization is needed. Attempting to parameterize those matrix elements, they can be expressed as

$$\text{D/F-type tri-local correlators} = \bar{u}(P', S') \mathcal{F}_{q, D/F}^{\alpha\beta}(x, y, \bar{P}, \Delta, n) u(P, S) . \quad (2.33)$$

It can be shown that the following expression provide a parameterization (See Appendix A for more details)

$$\begin{aligned} \mathcal{F}_{q,D}^{\alpha\beta} \sim & M g_{\perp}^{\alpha\beta} A_1(x, y, \xi, t) + M^2 g_{\perp}^{\alpha\beta} \not{n} A_2(x, y, \xi, t) + M \sigma_{\perp}^{\alpha\beta} A_3(x, y, \xi, t) \\ & + M V_S^{\alpha} \otimes V_S^{\beta} + M^2 \not{n} V_S^{\alpha} \otimes V_S^{\beta} + M V_S^{\alpha} \otimes V_T^{\beta} + M V_T^{\alpha} \otimes V_S^{\beta}, \end{aligned} \quad (2.34)$$

where V_S^{α} is the linear space expanded by vectors with Dirac scalar

$$V_S^{\alpha} = \left\{ \frac{\bar{P}_{+}^{\alpha}}{M}, \frac{\Delta_{\perp}^{\alpha}}{M}, M n_{-}^{\alpha} \right\}, \quad (2.35)$$

V_T^{α} is the linear space expanded by vectors with a Dirac tensor

$$V_T^{\alpha} = \left\{ M i \sigma_{\perp}^{\alpha\beta} n_{\beta}, \frac{i \sigma_{\perp}^{\alpha\beta} \Delta_{\beta}}{2M} \right\}, \quad (2.36)$$

and $\otimes$ in eq. (2.34) represents the direct product of the two linear spaces. Each base vector will be associated with an independent function $A_n(x, y, \xi, t)$. It can be shown that there are 32 independent structures and as a result 32 independent functions $A_n(x, y, \xi, t)$. Equivalent structures are first derived in ref. [69] for the matrix elements of the EMT. One can check its consistency with other parameterization by taking the antisymmetric part of $F_{q,D/F}^{\alpha\beta}(x, y, \bar{P}, \Delta, n)$. It turns out there are 12 independent functions for antisymmetric $F_{q,D/F}^{\alpha\beta}(x, y, \bar{P}, \Delta, n)$ which are equivalent to the results in ref. [70].

Though the parameterization seems to be complicated, the number of independent coefficients is consistent with the parameter counting method. Since there are $4 \times 4 = 16$ free indices for a rank-2 tensor and $(4 \times 3)/2 = 6$ free indices for an antisymmetric rank-2 tensor, there

should be at most 32 and 12 independent parameters for each of them respectively.

The twist-three part of the tri-local GPDs is essential when the AM is of interest after the twist-two terms, so consider eq. (2.34) with $\alpha = +, \beta = \perp$ which gives

$$\begin{aligned} \frac{1}{\bar{P}^+} \mathcal{F}_{q,D}^{+\perp} &= \frac{\Delta^\perp}{M} G_{q,D,1}(x, y, \xi, t) + \Delta^\perp \not{n} G_{q,D,2}(x, y, \xi, t) + \frac{i\sigma^{\perp\rho} \Delta_\rho}{2M} G_{q,D,3}(x, y, \xi, t) \\ &+ M i\sigma^{\perp\rho} n_\rho G_{q,D,4}(x, y, \xi, t) . \end{aligned} \quad (2.37)$$

The parameterization of the F-type GPDs will also be defined as

$$\begin{aligned} \frac{1}{(\bar{P}^+)^2} \mathcal{F}_{q,F}^{+\perp} &= \frac{\Delta^\perp}{M} G_{q,F,1}(x, y, \xi, t) + \Delta^\perp \not{n} G_{q,F,2}(x, y, \xi, t) \\ &+ \frac{i\sigma^{\perp\rho} \Delta_\rho}{2M} G_{q,F,3}(x, y, \xi, t) + M i\sigma^{\perp\rho} n_\rho G_{q,F,4}(x, y, \xi, t) . \end{aligned} \quad (2.38)$$

If one instead takes $\beta = +$ inside $\mathcal{F}_{q,D/F}^{\alpha\beta}$, the GPDs will be much simpler in the light front gauge. As with $A^+ = 0$, $\overleftrightarrow{D}^+(\mu n)$ reduces to $\overleftrightarrow{\partial}^+$ where the μ dependence is removed, and it becomes xP^+ up to an integration by parts. As a result, one has the following general substitution for $\beta = +$:

$$\begin{aligned} \int \frac{d\lambda d\mu}{(2\pi)^2} e^{i\frac{\lambda}{2}(x+y) + i\mu(y-x)} \bar{\psi} \left(-\frac{\lambda n}{2} \right) \gamma^\alpha i\overleftrightarrow{D}(\mu n)^+ \psi \left(\frac{\lambda n}{2} \right) . \\ \rightarrow x\mathcal{P}\delta(y-x) \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \bar{\psi} \left(-\frac{\lambda n}{2} \right) \gamma^\alpha \psi \left(\frac{\lambda n}{2} \right) , \end{aligned} \quad (2.39)$$

where the gauge links reduce to unity in the light front gauge, while the F-type GPDs simply vanish since $G^{++} = 0$. Then the off-forward matrix elements of the relations above would imply

$$\mathcal{F}_{q,D}^{\mu+}(x, y, \xi, t) \rightarrow x\bar{P}^+ \delta(y-x) \mathcal{F}_{q,\gamma^\mu}(x, \xi, t) . \quad (2.40)$$

Thus all D-type GPDs with $\beta = +$ can be expressed by the 2-field GPDs in the light front gauge, while the F-type GPDs simply vanish.

The gluon GPDs can be defined very similarly as the quark ones. For instance, GPDs defined with a gluon-gluon correlation can be expressed as

$$\int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P', S' \left| 2\text{Tr} \left\{ G^{(\alpha i} \left(-\frac{\lambda n}{2} \right) W_{-\frac{\lambda}{2}, \frac{\lambda}{2}} G^{\beta)}_i \left(\frac{\lambda n}{2} \right) \right\} \right| P, S \right\rangle, \quad (2.41)$$

where $()$ in the indices indicate a symmetrization. This expression can be parameterized in much the same way as in eq. (2.34), with the only new constraint being the need to symmetrize the α, β indices. With the expansion of eq. (2.41) as

$$\text{Gluon correlators} = \bar{u}(P', S') \mathcal{F}_g^{\alpha\beta}(x, \bar{P}, \Delta, n) u(P, S), \quad (2.42)$$

the following parameterization is chosen

$$\begin{aligned} \mathcal{F}_g^{\alpha\beta} = & M g_{\perp}^{\alpha\beta} A_1(x, \xi, t) + M^2 g_{\perp}^{\alpha\beta} \not{n} A_2(x, \xi, t) \\ & + M V_S^{(\alpha} \otimes V_S^{\beta)} + M^2 \not{n} V_S^{(\alpha} \otimes V_S^{\beta)} + M V_S^{(\alpha} \otimes V_T^{\beta)}. \end{aligned} \quad (2.43)$$

There are 20 independent structures, and the twist-two part with $\alpha = \beta = +$ can be written as [4]

$$\frac{1}{\bar{P}^+} \mathcal{F}_g^{++} = \gamma^+ x H_g(x, \xi, t) + \frac{i\sigma^{+\alpha} \Delta_{\alpha}}{2M} x E_g(x, \xi, t), \quad (2.44)$$

in which a Gordon identity was used and a factor of x included so that the parameterization

matches with the common result. For the twist-three GPDs with $\alpha = +, \beta = \perp$, one has

$$\begin{aligned} \mathcal{F}_g^{+\perp} = \bar{P}^{(+)} & \left[\frac{\Delta^\perp}{M} x G_{g,1}(x, \xi, t) + \Delta^\perp \not{x} G_{g,2}(x, \xi, t) + \frac{i\sigma^{\perp\rho}\Delta_\rho}{2M} x G_{g,3}(x, \xi, t) \right. \\ & \left. + M i\sigma^{\perp\rho} n_\rho x G_{g,4}(x, \xi, t) \right], \end{aligned} \quad (2.45)$$

where a factor of x is multiplied again for consistency. The GPDs defined here are symmetric under exchange of α and β , so the parameterization of $\alpha = \perp, \beta = +$ will be the same.

Besides the GPDs with a gluon-gluon correlation above, three-gluon correlation functions can also be parameterized. Consider the D-type gluon GPDs defined with

$$\int \frac{d\lambda d\mu}{(2\pi)^2} e^{i\frac{\lambda}{2}(x+y) + i\mu(y-x)} \left\langle P', S' \left| 2\text{Tr} \left\{ G^{\alpha i} \left(-\frac{\lambda n}{2} \right) iD^\beta(\mu n) G_i^+ \left(\frac{\lambda n}{2} \right) \right\} \right| P, S \right\rangle, \quad (2.46)$$

and the F-type GPDs defined with

$$\int \frac{d\lambda d\mu}{(2\pi)^2} e^{i\frac{\lambda}{2}(x+y) + i\mu(y-x)} \langle P', S' | \sum_i 2\text{Tr} \left\{ G^{\alpha i} \left(-\frac{\lambda n}{2} \right) i g G^{+\beta}(\mu n) G_i^+ \left(\frac{\lambda n}{2} \right) \right\} | P, S \rangle, \quad (2.47)$$

where all the gauge links are removed in the light cone gauge. Then with their parameterization written as

$$\text{D/F-type gluon tri-local correlators} = \bar{u}(P', S') \mathcal{F}_{g,D/F}^{\alpha\beta}(x, y, \bar{P}, \Delta, n) u(P, S), \quad (2.48)$$

they will follow analogously to the quark D/F-type GPDs. For instance, the twist-two case of

$\alpha = +, \beta = +$ gives us

$$\mathcal{F}_{g,D}^{\alpha+}(x, y, \bar{P}, \Delta, n) = x\bar{P}^+\delta(y-x)\mathcal{F}_g^{\alpha+}(x, \bar{P}, \Delta, n), \quad (2.49)$$

$$\mathcal{F}_{g,F}^{\alpha+}(x, y, \bar{P}, \Delta, n) = 0, \quad (2.50)$$

where these relations hold for any α , so one has the same relation for the twist-three GPDs with

$\alpha = \perp, \beta = +$. As for the twist-three case with $\alpha = +, \beta = \perp$, the D-type GPDs are

$$\begin{aligned} \frac{1}{(\bar{P}^+)^2} \mathcal{F}_{g,D}^{+\perp} &= \frac{\Delta^\perp}{M} G_{g,D,1}(x, y, \xi, t) + \Delta^\perp \not{n} G_{g,D,2}(x, y, \xi, t) \\ &+ \frac{i\sigma^{\perp\rho}\Delta_\rho}{2M} G_{g,D,3}(x, y, \xi, t) + Mi\sigma^{\perp\rho}n_\rho G_{g,D,4}(x, y, \xi, t), \end{aligned} \quad (2.51)$$

and the F-type GPDs are

$$\begin{aligned} \frac{1}{(\bar{P}^+)^3} \mathcal{F}_{g,F}^{+\perp} &= \frac{\Delta^\perp}{M} G_{g,F,1}(x, y, \xi, t) + \Delta^\perp \not{n} G_{g,F,2}(x, y, \xi, t) \\ &+ \frac{i\sigma^{\perp\rho}\Delta_\rho}{2M} G_{g,F,3}(x, y, \xi, t) + Mi\sigma^{\perp\rho}n_\rho G_{g,F,4}(x, y, \xi, t). \end{aligned} \quad (2.52)$$

These twist-two and twist-three GPDs will be considered in the study of nucleon spin sum rules, as we will show later.

2.1.2 Discrete symmetry of the GPDs

The matrix elements defined above must also satisfy discrete symmetries such as those of parity and time reversal invariance. Also, the result should be valid under Hermitian conjugation. These conditions can relate them into constraints of the expansion coefficient functions themselves. The derivation is somewhat tedious and only the results are shown here for future

convenience (see for instance refs. [29, 64]). The Hermiticity condition combined with time reversal symmetry require that

$$G_{q/g,3}^*(x, -\xi, t) = G_{q/g,3}(x, \xi, t) , \quad (2.53)$$

$$G_{q/g,3}(x, -\xi, t) = G_{q/g,3}(x, \xi, t) , \quad (2.54)$$

$$G_{q/g,j}^*(x, -\xi, t) = -G_{q/g,j}(x, \xi, t) , \quad (2.55)$$

$$G_{q/g,j}(x, -\xi, t) = -G_{q/g,j}(x, \xi, t) , \quad (2.56)$$

with $j = \{1, 2, 4\}$. Notice that under both Hermitian conjugation and time reversal, the initial and final states will be exchanged which means $\Delta \rightarrow -\Delta$ and $\xi \rightarrow -\xi$. The equations above simply imply that $G_{q/g,i}$ for $i = \{1, 2, 3, 4\}$ are all real. Meanwhile, $G_{q/g,3}$ is an even function of ξ , and $G_{q/g,j}$ with $j = \{1, 2, 4\}$ are odd functions of ξ , so in the forward limit: $G_{q/g,j}(x, t = 0, \xi = 0) = 0$.

The discrete symmetry of D-type and F-type GPDs can be derived as well and those GPDs are functions of both x and y which makes it more complicated. For the D-type 3-field GPDs defined in eq. (2.37) and eq. (2.51), the Hermiticity condition combined with the time reversal symmetry gives

$$G_{q/g,D,3}^*(x, y, -\xi, t) = G_{q/g,D,3}(y, x, \xi, t) , \quad (2.57)$$

$$G_{q/g,D,3}(x, y, -\xi, t) = G_{q/g,D,3}(y, x, \xi, t) , \quad (2.58)$$

$$G_{q/g,D,j}^*(x, y, -\xi, t) = -G_{q/g,D,j}(y, x, \xi, t) , \quad (2.59)$$

$$G_{q/g,D,j}(x, y, -\xi, t) = -G_{q/g,D,j}(y, x, \xi, t) , \quad (2.60)$$

with $j = \{1, 2, 4\}$. Again all the GPDs defined here are real. Also, if one expands those GPDs into powers of ξ via

$$f(x, y, \xi, t) = \sum_{n=0}^{\infty} f^{(n)}(x, y, t) \frac{\xi^n}{n!} , \quad (2.61)$$

then it follows that all the $G_{q/g,D,3}^{(n)}(x, y, -\xi, t)$ are symmetric when n is even and antisymmetric when n is odd which is the opposite for $G_{q/g,D,j}^{(n)}(x, y, -\xi, t)$.

There remains the F-type GPDs for which the constraints can be derived in the same way.

Combining the Hermiticity condition and the time reversal symmetry one has

$$G_{q/g,F,3}^*(x, y, -\xi, t) = -G_{q/g,F,3}(y, x, \xi, t) , \quad (2.62)$$

$$G_{q/g,F,3}(x, y, -\xi, t) = -G_{q/g,F,3}(y, x, \xi, t) , \quad (2.63)$$

$$G_{q/g,F,j}^*(x, y, -\xi, t) = G_{q/g,F,j}(y, x, \xi, t) , \quad (2.64)$$

$$G_{q/g,F,j}(x, y, -\xi, t) = G_{q/g,F,j}(y, x, \xi, t) , \quad (2.65)$$

with $j = \{1, 2, 4\}$. These relations indicate that $G_{q/g,F,3}^{(n)}(x, y, -\xi, t)$ are antisymmetric when n is even and symmetric when n is odd and the opposite applies to $G_{q/g,F,j}^{(n)}(x, y, -\xi, t)$ with $j = \{1, 2, 4\}$.

2.2 Revisiting longitudinal spin sum rules

The spin sum rules for a longitudinally polarized nucleon have been thoroughly discussed in the literature [48, 50, 53, 68]. As discussed above, the AM can be expressed in terms of the

off-forward matrix elements of the EMT. For example, J_q^z and J_g^z can be in general written as

$$J_{q,g}^z = \langle P, S | J_{q,g}^z | P, S \rangle = \frac{1}{2E_P} \left(\lim_{\Delta \rightarrow 0, S' \rightarrow S} i \frac{\partial}{\partial \Delta_x} \langle P + \Delta, S' | T_{q,g,\text{Bel}}^{0y}(0) | P, S \rangle \right. \\ \left. - \lim_{\Delta \rightarrow 0, S' \rightarrow S} i \frac{\partial}{\partial \Delta_y} \langle P + \Delta, S' | T_{q,g,\text{Bel}}^{0x}(0) | P, S \rangle \right), \quad (2.66)$$

which also one to derive the AM sum rules for the longitudinal spin.

2.2.1 Quark AM decomposition

We start with the quark contributions to the AM, which is related to the quark EMT that can be written as,

$$T_{q,\text{Bel}}^{\mu\nu} = \bar{\psi} \gamma^{(\mu} i \overleftrightarrow{D}^{\nu)} \psi. \quad (2.67)$$

For longitudinal AM J^z , the contributions are from $T_{q,\text{Bel}}^{0y}$ and $T_{q,\text{Bel}}^{0x}$ equivalently. Without loss of generality, we consider $T_{q,\text{Bel}}^{+y}$, the leading twist contribution to $T_{q,\text{Bel}}^{0y}$. The matrix element of $T_{q,\text{Bel}}^{+y}$ is obviously related to the D-type correlators, which can be written as

$$T_{q,LF,\text{Bel}}^{+y}(x, y) = \int \frac{d\lambda d\mu}{(2\pi)^2} e^{i\frac{\lambda}{2}(x+y) + i\mu(y-x)} \bar{\psi} \left(-\frac{\lambda n}{2} \right) \gamma^{(+} W_{-\frac{\lambda}{2}, \mu} i \overleftrightarrow{D} (\mu n)^y W_{\mu, \frac{\lambda}{2}} \psi \left(\frac{\lambda n}{2} \right). \quad (2.68)$$

The EMT is simply the corresponding local operator: $T_{q,\text{Bel}}^{+y}(0) = \int dx dy T_{q,LF,\text{Bel}}^{+y}(x, y)$. Such a relation immediately indicates the following constraints on the D-type GPDs as,

$$\int dx dy G_{q,D,3}(x, y) + \int dx x G_{q,3}(x) = A_q(0) + B_q(0), \quad (2.69)$$

$$\int dx dy G_{q,D,j}(x, y) = \int dx x G_{q,j}(x) = 0, \quad (2.70)$$

where $j = \{1, 2, 4\}$ and we define the notation $G_{q,i}(x) \equiv \lim_{\Delta \rightarrow 0} G_{q,i}(x, t, \xi)$. However, they do not have a simple parton picture, since it involves three partons interacting with each other.

With the goal of decomposing the quark AM in order to derive a partonic sum rule, one could decompose the quark spin contribution [48]. Since the Belinfante EMT for QCD is simply the symmetrized canonical EMT: $T_{\text{Bel}}^{\mu\nu} = T^{(\mu\nu)}$, it can alternatively be written as: $T_{\text{Bel}}^{\mu\nu} = T^{\mu\nu} - T^{[\mu\nu]}$. Then the AM density $M^{\mu\nu\rho}(r)$ can be expressed with the canonical EMT as

$$\begin{aligned} M^{\mu\nu\rho}(r) &= r^\nu (T^{\mu\rho}(r) - T^{[\mu\rho]}(r)) - r^\rho (T^{\mu\nu} - T^{[\mu\nu]}(r)) , \\ &= r^\nu T^{\mu\rho}(r) - r^\rho T^{\mu\nu} + S^{\mu\nu\rho}(r) , \end{aligned} \quad (2.71)$$

where we define

$$S^{\mu\nu\rho}(r) = - (r^\nu T^{[\mu\rho]}(r) - r^\rho T^{[\mu\nu]}(r)) . \quad (2.72)$$

With the equations of motion of quark, the following identities can be useful

$$\bar{\psi}(r)[\gamma^\alpha \vec{D}^\beta(r) - \gamma^\beta \vec{D}^\alpha(r)]\psi(r) = i\epsilon^{\alpha\beta\rho\sigma} \bar{\psi}(r)\gamma_\rho\gamma^5 \vec{D}_\sigma(r)\psi(r) , \quad (2.73)$$

$$\bar{\psi}(r)[\gamma^\alpha \overleftarrow{D}^\beta(r) - \gamma^\beta \overleftarrow{D}^\alpha(r)]\psi(r) = -i\epsilon^{\alpha\beta\rho\sigma} \bar{\psi}(r)\gamma_\rho\gamma^5 \overleftarrow{D}_\sigma(r)\psi(r) . \quad (2.74)$$

With these relations it follows that

$$S_q^{\mu\nu\rho}(r) = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma} \bar{\psi}(r)\gamma_\sigma\gamma^5\psi(r) + \frac{1}{2}\partial_\beta (x^{[\nu}\epsilon^{\rho]\mu\beta\sigma} \bar{\psi}(r)\gamma_\sigma\gamma^5\psi(r)) , \quad (2.75)$$

where the second term is a total derivative term called a super potential whose forward matrix

element is shown to vanish with proper wave packet treatment in refs. [62,63].

Then the spin contribution can be derived with eq. (2.75) as

$$J_{q,S}^z = -\frac{1}{2}\epsilon^{0xy\sigma} \int d^3\mathbf{r} \langle P, S | \bar{\psi}(\mathbf{r}) \gamma_\sigma \gamma^5 \psi(\mathbf{r}) | P, S \rangle . \quad (2.76)$$

The matrix element of the axial current can be expressed as

$$\langle P, S | \bar{\psi}(0) \gamma_\sigma \gamma^5 \psi(0) | P, S \rangle = 2MS_\sigma \Delta q , \quad (2.77)$$

where the quantity Δq is the singlet axial charge with all quark flavors summed over. The axial charge Δq can be related to the PDF defined in ref. [25] with the axial current as

$$\begin{aligned} & \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \gamma_\mu \gamma_5 \psi(\lambda n) | PS \rangle \\ & \equiv 2 \left[g_1(x) M p_\mu (S \cdot n) + M g_T(x) S_{\perp\mu} + M^3 g_3(x) n_\mu (S \cdot n) \right] , \end{aligned} \quad (2.78)$$

where we have slightly rewritten it with our normalization condition $S^2 = -1$. Then one has the relation (see also ref. [71])

$$\Delta q = \int dx g_1(x) = \int dx g_T(x) , \quad (2.79)$$

The relation that $\int dx g_1(x) = \int dx g_T(x)$ is the well-known Burkhardt-Cottingham sum rule [72].

The spin decomposition and the axial charge also put extra constraints on the GPDs. With eqs.

(2.73) and (2.74), it can be shown that (see also ref. [59])

$$\int \mathrm{d}x \, x G_{q,3}(x) - \int \mathrm{d}x \mathrm{d}y G_{q,D,3}(x, y) = \Delta q . \quad (2.80)$$

Such a relation can be used to reexpress the AM. By eliminating $G_{q,D,3}(x, y)$, we have

$$J_q^z = \int \mathrm{d}x \left(x G_{q,3}(x) - \frac{1}{2} g_1(x) \right) , \quad (2.81)$$

as shown in the literature [50, 73, 74]. Alternatively, one could eliminate the $G_{q,3}(x)$ to get

$$\int \mathrm{d}x \mathrm{d}y G_{q,D,3}(x, y) = J_q^z - \frac{1}{2} \Delta q = L_q^z , \quad (2.82)$$

which motivates the definition of a gauge-invariant covariant OAM density,

$$L_q^z(x) = \frac{1}{2} \int \mathrm{d}y [G_{q,D,3}(x, y) + G_{q,D,3}(y, x)] . \quad (2.83)$$

However, the partonic interpretation of such covariant OAM density is not simple.

To further decompose the above covariant OAM density which does not have a simple partonic interpretation, we consider decomposing the covariant derivative $D^\nu(r)$ which involves interaction with gluon field. Therefore, the quark EMT can be further written as

$$T_q^{\mu\nu}(r) = \bar{\psi}(r) \gamma^\mu i \overleftrightarrow{D}^\nu \psi(r) = \bar{\psi}(r) \gamma^\mu i \overleftrightarrow{\partial}^\nu \psi(r) - g \bar{\psi}(r) \gamma^\mu A^\nu \psi(r) . \quad (2.84)$$

Then with following definitions of AM densities

$$M_{q,L}^{\mu\nu\rho}(r) \equiv r^\nu T_q^{\mu\rho}(r) - r^\rho T_q^{\mu\nu}(r) , \quad (2.85)$$

$$M_{q,l}^{\mu\nu\rho}(r) \equiv r^\nu \bar{\psi}(r) \gamma^\mu i \overleftrightarrow{\partial}^\nu \psi(r) - (\nu \leftrightarrow \rho) , \quad (2.86)$$

$$M_{q,\text{pot}}^{\mu\nu\rho}(r) \equiv r^\nu \bar{\psi}(r) \gamma^\mu i g A^\nu \psi(r) - (\nu \leftrightarrow \rho) , \quad (2.87)$$

as well as their matrix elements in the longitudinally polarized nucleon,

$$L_q^z \equiv \int d^3\mathbf{r} \langle P, S | M_{q,L}^{0xy}(\mathbf{r}) | P, S \rangle , \quad (2.88)$$

$$l_q^z \equiv \int d^3\mathbf{r} \langle P, S | M_{q,l}^{0xy}(\mathbf{r}) | P, S \rangle , \quad (2.89)$$

$$l_{q,\text{pot}}^z \equiv \int d^3\mathbf{r} \langle P, S | M_{q,\text{pot}}^{0xy}(\mathbf{r}) | P, S \rangle , \quad (2.90)$$

the canonical OAM can be written as

$$l_q^z = L_q^z + l_{q,\text{pot}}^z . \quad (2.91)$$

Unlike the covariant OAM L_q^z , the canonical OAM l_q^z only involves two quark fields, so it has a clear partonic interpretation, which can be also be seen through its connection with TMD's and Wigner distributions [51–53]. An extra term $l_{q,\text{pot}}^z$ is invented to related these two different OAM, see also refs. [75–78]. Rearranging the $l_{q,\text{pot}}^z$ term from quarks to gluons will transform the covariant AM sum rule into the canonical Jaffe-Manohar sum rule.

Then the canonical OAM l_q^z can be expressed with the corresponding light-front canonical

OAM density $l_q^z(x)$ which is defined as [79–81],

$$l_q^z(x) = \frac{1}{P^+} \int \frac{d\lambda d^2\mathbf{r}^\perp}{2\pi} e^{i\lambda x} \langle P, S | \bar{\psi}(0, \mathbf{r}^\perp) \gamma^+ (r^x i\partial^y - r^y i\partial^x) \psi(\lambda n, \mathbf{r}^\perp) | P, S \rangle , \quad (2.92)$$

with gauge links implicit between fields. Though the moment of $r^\perp$ indicates that this forward matrix element can only be defined as the limit of an off-forward matrix element, this expression indeed has a simple partonic interpretation. Since $l_q^z(x)$ is the combination of the OAM operator $(\xi^x i\partial^y - \xi^y i\partial^x)$ and the parton density operator $\bar{\psi}(0) \gamma^+ \psi(\lambda n)$, it can be considered as the OAM density of partons with momentum fraction x .

However, this decomposition breaks the gauge invariance, so neither the expression for $l_{q,\text{pot}}^z$ nor l_q^z is gauge-invariant. If one defines a new type of GPD as

$$\int \frac{d\lambda d\mu}{(2\pi)^2} e^{i\frac{\lambda}{2}(x+y) + i\mu(y-x)} \langle P', S' | \bar{\psi} \left(-\frac{\lambda n}{2} \right) W_{-\lambda/2, \mu} \gamma^\alpha A^\beta(\mu n) W_{\mu, \lambda/2} \psi \left(\frac{\lambda n}{2} \right) | P, S \rangle , \quad (2.93)$$

it will not be measurable since it is not gauge-invariant. To obtain measurable OAM densities, equivalent GPDs with manifest gauge-invariance is needed. Given that the light front gauge $A^+ = 0$ will always be used, we have the following relation derived from $A^\alpha = \frac{G^{+\alpha}}{\partial^+}$,

$$A^\mu(y) = A^\mu(x) + \int_x^y dz G^{+\mu}(z) , \quad (2.94)$$

where the gauge links reduce to unity in the light front gauge. This leads to the following identity

$$\begin{aligned} \bar{\psi} \left(-\frac{\lambda n}{2} \right) \gamma^\alpha i g A^\beta(\mu n) \psi \left(\frac{\lambda n}{2} \right) = & \bar{\psi} \left(-\frac{\lambda n}{2} \right) \gamma^\alpha i g \int_\nu^\mu dt G^{+\beta}(tn) \psi \left(\frac{\lambda n}{2} \right) \\ & + \bar{\psi} \left(-\frac{\lambda n}{2} \right) \gamma^\alpha i g A^\beta(\nu n) \psi \left(\frac{\lambda n}{2} \right) . \end{aligned} \quad (2.95)$$

On the Left-Hand Side (LHS) we have the potential term $l_{q,\text{pot}}^z$ in quark OAM, which is then expressed as the off-forward matrix element of the Right-Hand Side (RHS) that corresponds to the F-type GPDs defined before. The extra boundary term in the second line does not depend on μ and corresponds to a zero-mode contribution [82], and will be omitted for the discussion here. Thus, the potential term $l_{q,\text{pot}}^z$ can be expressed with the F-type GPDs, and we have the following result with the parameterization of the F-type GPDs in eq. (2.38),

$$l_{q,\text{pot}}^z = \int dx dy \mathbf{P} \frac{1}{y-x} G_{q,F,3}(x, y) . \quad (2.96)$$

Since the covariant quark OAM L_q^z is expressible in terms of the D-type GPDs as shown already, the canonical OAM density $l_q^z(x)$ can then be expressed as

$$l_q^z(x) = \int dy G_{q,D,3}(x, y) + \int dy \mathbf{P} \frac{1}{y-x} G_{q,F,3}(x, y) , \quad (2.97)$$

and the canonical OAM is simply the integral of the canonical OAM density:

$$l_q^z = \int dx l_q^z(x) . \quad (2.98)$$

Though we note that the expression for l_q^z might be affected by the light front zero mode as in

ref. [82]. The expression for the canonical OAM density $l_q^z(x)$ was first derived in refs. [50, 68], which also show that the D-type and F-type twist-three GPDs are related to each other [83, 84].

2.2.2 Gluon AM and longitudinal spin sum rule

The gluon AM decomposition follows similar processes. Consider the gluon EMT:

$$T_g^{\mu\nu}(\xi) = 2\text{Tr} \left\{ -G^{\mu\rho}(\xi) G^\nu{}_\rho(\xi) + \frac{1}{4} g^{\mu\nu} G^{\rho\sigma}(\xi) G_{\rho\sigma}(\xi) \right\} . \quad (2.99)$$

When the longitudinal AM is concerned, only the $+$ $\perp$ components are involved, so we can ignore the $g^{\mu\nu}$ term. Then, the gluon EMT can be expressed with the following light front correlator

$$T_{g,LF}^{+\perp}(x) = - \int \frac{d\lambda}{2\pi} e^{i\lambda x} 2\text{Tr} \left\{ G^{+\rho} \left(-\frac{\lambda n}{2} \right) W_{-\frac{\lambda}{2}, \frac{\lambda}{2}} G^\perp{}_\rho \left(\frac{\lambda n}{2} \right) \right\} , \quad (2.100)$$

with $T_g^{+\perp}(0) = \int dx T_{g,LF}^{+\perp}(x)$. Again, this relation immediately indicates:

$$\int dx \, x \, G_{g,3}(x) = A_g(0) + B_g(0) , \quad (2.101)$$

$$\int dx \, x \, G_{g,j}(x) = 0 , \quad (2.102)$$

with $j = \{1, 2, 4\}$, similar to the quark case.

Unlike the quark AM, the gluon AM consists only of the 2-field correlation. However, this does not mean that it has a simple partonic interpretation, since $G_a^{\mu\nu}$ contains also quadratic terms in the gluon field A_a^μ in non-Abelian gauge theory. To find a partonic interpretation for the gluon

contribution, further decomposition of the gluon AM is needed and one has [48]

$$\begin{aligned}
M_g^{\mu\nu\lambda}(r) &= (r^\nu 2\text{Tr} \{ G^{\mu\alpha} G^\lambda{}_\alpha \} - (\nu \leftrightarrow \lambda)) \\
&= 2\text{Tr} \left\{ G^{\mu\alpha} (r^\lambda \partial^\nu - r^\nu \partial^\lambda) A_\alpha + G^{\mu\lambda} A^\nu + G^{\nu\mu} A^\lambda \right\} \\
&\quad + g \bar{\psi} \gamma^\mu (r^\nu A^\lambda - r^\lambda A^\nu) \psi .
\end{aligned} \tag{2.103}$$

Accordingly, the following quantities are defined

$$M_{g,l}^{\mu\nu\lambda}(r) \equiv 2\text{Tr} \{ G^{\mu\alpha} (r^\lambda \partial^\nu - r^\nu \partial^\lambda) A_\alpha \} , \tag{2.104}$$

$$M_{g,S}^{\mu\nu\lambda}(r) \equiv 2\text{Tr} \{ G^{\mu\lambda} A^\nu + G^{\nu\mu} A^\lambda \} , \tag{2.105}$$

$$M_{q,\text{pot}}^{\mu\nu\lambda}(r) \equiv g \bar{\psi} \gamma^\mu (r^\nu A^\lambda - r^\lambda A^\nu) \psi . \tag{2.106}$$

Though each of the terms here is not gauge-invariant, the total contribution will be [85]. We note that the $M_{q,\text{pot}}^{\mu\nu\rho}$ term has exactly the same form as the $l_{q,\text{pot}}$ term defined before for the quark AM decomposition except for a minus sign, so they will cancel once we add the quark and gluon AM together. Thus, the potential term will not be further discussed here.

For the gluon helicity contribution, consider the gluon spin term $M_S^{\mu\nu\rho}$ which actually does not correspond to the $U(1)$ axial current K_σ as discussed in ref. [48]. However, in the light cone gauge $A^+ = 0$ at leading twist where $\mu = +$, they are the same. Since the gluon axial current K_σ can be written as

$$\epsilon^{\mu\nu\lambda\sigma} K_\sigma \equiv 2\text{Tr} \{ G^{\mu\lambda} A^\nu + G^{\nu\mu} A^\lambda + G^{\lambda\nu} A^\mu \} , \tag{2.107}$$

its only difference from $M_{q,\text{pot}}^{\mu\nu\rho}$ is the third term $G^{\lambda\nu} A^\mu$ which vanishes at leading twist with $\mu = +$ and in the light cone gauge $A^+ = 0$. This relation also allows one to relate the gluon spin to the polarized gluon distribution $\Delta G(x)$ defined as [86, 87]

$$\Delta G(x) = \frac{i}{xP^+} \int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle P, S | 2\text{Tr} \left\{ G^{+\alpha} \left(-\frac{\lambda n}{2} \right) W_{-\frac{\lambda}{2}, \frac{\lambda}{2}} \tilde{G}^+_{\alpha} \left(\frac{\lambda n}{2} \right) \right\} | P, S \rangle , \quad (2.108)$$

where $\tilde{G}^{\mu\nu\alpha\beta} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}$. Then we have the following expression for the gluon helicity contribution,

$$J_{g,S}^z = \int d^3\mathbf{r} \langle P, S | M_S^{+xy}(\mathbf{r}) | P, S \rangle = \Delta G \equiv \int dx \Delta G(x) . \quad (2.109)$$

These expressions allow a simple parton picture for the gluon helicity contribution, with $\Delta G(x)$ the contribution from gluon spin of partons with momentum fraction x .

As for the gluon OAM density, consider the gluon OAM given by,

$$l_g^z = \langle P, S | \int d^3\mathbf{r} M_{g,l}^{0xy}(\mathbf{r}) | P, S \rangle = \langle P, S | \int d^3\mathbf{r} 2\text{Tr} \{ G^{0\alpha} (r^y \partial^x - r^x \partial^y) A_\alpha \} | P, S \rangle , \quad (2.110)$$

which can be expressed with a light front correlation defined as

$$l_g^z(x) = \int \frac{d\lambda d^2\mathbf{r}^\perp}{2\pi} e^{i\lambda x} \langle P, S | 2\text{Tr} \{ G^{+\alpha}(0, \mathbf{r}^\perp) (r^x \partial^y - r^y \partial^x) A_\alpha(\lambda n, \mathbf{r}^\perp) \} | P, S \rangle , \quad (2.111)$$

while l_g^z is simply the integral of $l_g^z(x)$ over x . The expression of $l_g^z(x)$ has a simple partonic interpretation, since it is just the OAM operator $(r^x \partial^y - r^y \partial^x)$, combining the gluon density operator $2\text{Tr} \{ G^{+\alpha}(0) A_\alpha(\lambda n) \}$ of partons with momentum fraction x . However, we still need to find out gauge-invariant quantities equivalent to the gluon OAM density $l_g^z(x)$ for them to be

measurable. Thus, we consider the following relation in the light front gauge

$$G^{\mu\alpha} i\partial^\nu A_\alpha = G^{\mu\alpha} iD^\nu \frac{1}{\partial^+} G^+_\alpha + G^{\mu\alpha} g \frac{1}{\partial^+} G^{+\nu} \frac{1}{\partial^+} G^+_\alpha , \quad (2.112)$$

and its non-local version

$$\begin{aligned} G^{\alpha\sigma} \left(-\frac{\lambda}{2}n \right) i\partial^\beta A_\sigma \left(-\frac{\lambda}{2}n \right) = & G^{\alpha\sigma} \left(-\frac{\lambda}{2}n \right) iD^\beta(\mu n) \frac{1}{\partial^+} G^+_\sigma \left(\frac{\lambda}{2}n \right) \\ & + G^{\alpha\sigma} \left(-\frac{\lambda}{2}n \right) g \frac{1}{\partial^+} G^{+\beta}(\mu n) \frac{1}{\partial^+} G^+_\sigma \left(\frac{\lambda}{2}n \right) . \end{aligned} \quad (2.113)$$

The operators on the RHS correspond to the D/F-type gluon GPDs again with zero mode contribution omitted, while the operator on the LHS gives us the gluon OAM operator in the local limit. Therefore, this relation can be used to relate gluon OAM to measurable D-type and F-type gluon GPDs, for which we define two auxiliary quantities,

$$M_{g,\tilde{L}}^{\mu\nu\rho}(r) \equiv 2\text{Tr} \left\{ G^{\mu\alpha} \left(r^\lambda D^\nu - r^\nu D^\lambda \right) A_\alpha \right\} , \quad (2.114)$$

$$M_{g,\text{pot}}^{\mu\nu\rho}(r) \equiv -2ig\text{Tr} \left\{ G^{\mu\alpha} \left(r^\lambda A^\nu - r^\nu A^\lambda \right) A_\alpha \right\} , \quad (2.115)$$

with their matrix element defined as

$$\tilde{L}_g^z \equiv \int d^3\mathbf{r} \langle P, S | M_{g,\tilde{L}}^{0xy}(\mathbf{r}) | P, S \rangle , \quad (2.116)$$

$$l_{g,\text{pot}}^z \equiv \int d^3\mathbf{r} \langle P, S | M_{g,\text{pot}}^{0xy}(\mathbf{r}) | P, S \rangle . \quad (2.117)$$

Similar to the quark case, we have the following expression for the $\tilde{L}_g^z(x)$ and $l_{g,pot}^z(x)$

$$\tilde{L}_g^z(x) = \int dy \mathbf{P} \frac{1}{x+y} G_{g,D,3}(x, y), \quad (2.118)$$

$$l_{g,pot}^z(x) = \int dy \mathbf{P} \frac{1}{y^2 - x^2} G_{g,F,3}(x, y), \quad (2.119)$$

which eventually lead to the following expression of $l_g^z(x)$,

$$l_g^z(x) = \int dy \mathbf{P} \frac{1}{x+y} G_{g,D,3}(x, y) + \int dy \mathbf{P} \frac{1}{y^2 - x^2} G_{g,F,3}(x, y), \quad (2.120)$$

with the total gluon OAM l_g^z as an integral of gluon OAM density $l_g^z(x)$:

$$l_g^z = \int dx l_g^z(x). \quad (2.121)$$

Though again, the integral might suffer from the zero mode problem just like the quark case [82].

Equivalent expressions are first derived in refs. [50, 68].

Combining all the terms derived above, we reach the Jaffe-Manohar sum rule that reads:

$$\frac{1}{2} \Delta q + \Delta G + l_q^z + l_g^z = \frac{\hbar}{2}, \quad (2.122)$$

where each term has a partonic interpretation as well as the corresponding parton densities $\Delta q(x)$,

$\Delta G(x)$, $l_q^z(x)$, and $l_g^z(x)$ defined above.

2.3 Transverse spin sum rules

Spin sum rules for a transversely polarized nucleon have received much less attention in the community, and some of their theoretical results have been hotly debated over the years [23, 28, 53, 56, 57, 59, 63, 88–90]. It turns out that one of the key issues in this case is the separation of the intrinsic spin from the center-of-mass contributions when the nucleon is in motion. For a transversely-polarized nucleon, its overall “center-of-mass motion” will generate a contribution to the AM as well as to any quantity that depends on the polarization. Fortunately, Lorentz symmetry allows the intrinsic and extrinsic contributions to be distinguished through their dependencies on the different Lorentz structures. As explained in ref. [60], the Pauli-Lubanski vector was introduced to exclude extrinsic sources of AM automatically. When one works directly with the Pauli-Lubanski vector, this introduces the boost operator whose matrix elements do not have a clear physical interpretation [60]. Therefore, we choose to work directly with matrix elements of the AM operator.

The transverse AM contains both twist-two and twist-three contributions. To see this, one can write the forward matrix element of transverse AM as

$$J_{q,g}^x \equiv \langle P, S | J_{q,g}^x | P, S \rangle = \frac{1}{2E_P} \left(\lim_{\Delta \rightarrow 0, S' \rightarrow S} i \frac{\partial}{\partial \Delta_y} \langle P + \Delta, S' | T_{q,g,\text{Bel}}^{0z}(0) | P, S \rangle - \lim_{\Delta \rightarrow 0, S' \rightarrow S} i \frac{\partial}{\partial \Delta_z} \langle P + \Delta, S' | T_{q,g,\text{Bel}}^{0y}(0) | P, S \rangle \right), \quad (2.123)$$

where we assumed the transverse polarization is in the x direction while the other transverse

direction is y . Then we can split the transverse AM into

$$J_{q,g}^x = J_{q,g}^{x(2)} + J_{q,g}^{x(3)} , \quad (2.124)$$

where

$$J_{q,g}^{x(2)} \equiv \langle P, S | \hat{J}_{q,g}^{x(2)} | P, S \rangle = \frac{1}{2E_P} \lim_{\Delta \rightarrow 0, S' \rightarrow S} i \frac{\partial}{\partial \Delta_y} \langle P + \Delta, S' | T_{q,g,\text{Bel}}^{0z}(0) | P, S \rangle , \quad (2.125)$$

$$J_{q,g}^{x(3)} \equiv \langle P, S | \hat{J}_{q,g}^{x(3)} | P, S \rangle = -\frac{1}{2E_P} \lim_{\Delta \rightarrow 0, S' \rightarrow S} i \frac{\partial}{\partial \Delta_z} \langle P + \Delta, S' | T_{q,g,\text{Bel}}^{0y}(0) | P, S \rangle . \quad (2.126)$$

The twist counting is made clearer in light front coordinates where we have roughly $0z \rightarrow ++$ and $0y \rightarrow + \perp$. Then it is obvious that the two terms corresponds to twist-two and twist-three respectively. While in the rest frame the system has rotational symmetry and the two terms are related by the exchange of indices, such relation does not hold in an arbitrary frame. Therefore, a more detailed treatment of the twist-two and twist-three terms in moving frames is needed.

2.3.1 Intrinsic transverse AM

To find out the two transverse AM $J_{q,g}^{x(2)}$ and $J_{q,g}^{x(3)}$ in terms of the GFFs, we need to expand the Dirac spinors. With the gamma matrices in the Dirac representation, the Dirac spinor can be written as

$$u(P, \mathbf{s}) = \sqrt{P^0 + M} \begin{pmatrix} \chi^{(s)} \\ \frac{\boldsymbol{\sigma} \cdot \mathbf{P}}{P^0 + M} \chi^{(s)} \end{pmatrix} , \quad (2.127)$$

where σ represents the Pauli matrices, $\mathbf{s}$ is the spin vector in the rest frame and $\chi^{(s)}$ satisfies

$$\chi^{(s)\dagger} \sigma^i \chi^{(s)} = \mathbf{s}^i . \quad (2.128)$$

Two expansions of Dirac bilinears will be used, which are

$$\bar{u}(\bar{P} + \Delta/2, S) u(\bar{P} - \Delta/2, S) = 2M + \epsilon^{0\rho\alpha\beta} \frac{i\Delta_\rho}{\bar{P}^0 + M} S_\alpha \bar{P}_\beta + O(\Delta^2) , \quad (2.129)$$

$$\bar{u}(\bar{P} + \Delta/2, S) \sigma^{\mu\nu} u(\bar{P} - \Delta/2, S) = -2\epsilon^{\mu\nu\alpha\beta} S_\alpha \bar{P}_\beta + O(\Delta) . \quad (2.130)$$

Then the transverse AM can be written with the GFFs as

$$J_{q,g}^{x(2)} = \frac{P^{02} + P^{z2}}{4MP^0} (A_{q,g}(0) + B_{q,g}(0)) - \frac{P^{z2}}{2M(P^0 + M)} A_{q,g}(0) , \quad (2.131)$$

and for the twist-three term,

$$J_{q,g}^{x(3)} = \frac{P^{02} - P^{z2}}{4MP^0} (A_{q,g}(0) + B_{q,g}(0)) . \quad (2.132)$$

We note that with the proton on-shell, the four-momenta satisfy $\bar{P} \cdot \Delta = 0$, which means $\Delta^0 \bar{P}^0 - \Delta^z \bar{P}^z = 0$. Therefore, the two components of Δ^μ are not independent which should be considered when taking the derivative. Then the total transverse AM will be the sum of the these two transverse AM:

$$J_{q,g}^x = \frac{P^0}{2M} (A_{q,g}(0) + B_{q,g}(0)) - \frac{P^{z2}}{2M(P^0 + M)} A_{q,g}(0) . \quad (2.133)$$

This is consistent with the result in ref. [91], where an extra term proportional to P^z is present in the expression.

However, this contribution actually comes from the center-of-mass motion, and shall be removed for intrinsic spin sum rules [60] — terms in the combination of P^i or P^j and J^{ij} do not contribute to the Pauli-Lubanski vector due to the anti-symmetrization in the indices. Thus, for transverse AM $J^x = J^{yz}$, terms proportional to P^z will not contribute to the Pauli-Lubanski vector. It is also clear if one writes the matrix element of AM covariantly as [91]

$$J_{q,g}^{ij} = -\frac{\epsilon^{ij\alpha\beta}}{2M} S_\alpha P_\beta (A_{q,g}(0) + B_{q,g}(0)) - \frac{\mathbf{P}^i(\mathbf{P} \times \mathbf{s})^j - \mathbf{P}^j(\mathbf{P} \times \mathbf{s})^i}{2M(P^0 + M)} A_{q,g}(0) , \quad (2.134)$$

where the last term is apparently proportional to P^i or P^j and thus non-intrinsic. Removing the extrinsic contributions, we have

$$J_{q,g,\text{In}}^{x(2)} = \frac{\gamma}{4} (A_{q,g}(0) + B_{q,g}(0)) , \quad (2.135)$$

$$J_{q,g,\text{In}}^{x(3)} = \frac{\gamma}{4} (A_{q,g}(0) + B_{q,g}(0)) , \quad (2.136)$$

where the subscript “In” indicates taking the intrinsic part of each term respectively and $\gamma \equiv P^0/M$ is the Lorentz boost factor. Then the total transverse AM will be

$$J_{q,g}^x = \frac{\gamma}{2} (A_{q,g}(0) + B_{q,g}(0)) . \quad (2.137)$$

The total transverse AM $J_{q,g}^x$ grows with a factor of γ as the nucleon gets boosted, which is consistent with the Lorentz transformation of the AM operator as discussed in ref. [60].

Also, we find that after removing the extrinsic contribution, each of the twist-two and twist-three parts contributes to half of the total transverse AM. Therefore, an individual sum rule can be derived from each of them, respectively. This argument is consistent with the rest-frame picture where the two terms contribute equally just from rotation symmetry.

On the other hand, the same result will be achieved if one removes the extrinsic contributions in the total AM only. In this case, the twist-two and twist-three parts are

$$J_{q,g}^{x(2)} = \left(\frac{\gamma}{2} - \frac{1}{4\gamma} \right) (A_{q,g}(0) + B_{q,g}(0)) , \quad (2.138)$$

$$J_{q,g}^{x(3)} = \frac{1}{4\gamma} (A_{q,g}(0) + B_{q,g}(0)) . \quad (2.139)$$

Now the twist-two one will dominate in the infinite momentum limit. But still, by requiring that the total AM $J_{q,g}^x$ must have an extra factor γ in any frame as predicted by its Lorentz transformation, each of them should still lead to a distinct transverse AM sum rules, which will be studied in the following subsections.

2.3.2 Twist-two transverse AM sum rule

The twist-two partonic contribution to transverse AM has already been well-established [53, 60, 89]. Here we revisit it to emphasize the procedure to isolate the intrinsic contribution. The covariant sum rule derived in ref. [4] is gauge-invariant and frame independent, but it does not provide a simple parton picture. The lesson from the Jaffe-Manohar sum rule for longitudinal polarization is that the canonical AM can be easily related to partons [48]. However, for the transverse polarization, the covariant sum rule in the infinite momentum frame does provide a simple parton interpretation [53, 88].

Consider the twist-two transverse AM $J_{q,g}^{x(2)}$ defined in eq. (2.125), in which the EMT $T_{q,g,\text{Bel}}^{0z}(0)$ is involved. In the infinite momentum frame, one can generate this term from a light-front correlation defined as

$$T_{q,LF,\text{Bel}}^{0z}(x, y) = \int \frac{d\lambda d\mu}{(2\pi)^2} e^{i\frac{\lambda}{2}(x+y)+i\mu(y-x)} \bar{\psi} \left(-\frac{\lambda n}{2} \right) W_{-\frac{\lambda}{2},\mu} \gamma^{(0)} i \overleftrightarrow{D}^z(\mu n) W_{\mu,\frac{\lambda}{2}} \psi \left(\frac{\lambda n}{2} \right), \quad (2.140)$$

while $T_{q,\text{Bel}}^{0z} = \int dx dy T_{q,LF,\text{Bel}}^{0z}(x, y)$. Now consider the leading twist contribution to $T_{q,\text{Bel}}^{0z}$ for which $T_{q,LF,\text{Bel}}^{++}(x)$ will be concerned and one has

$$T_{q,LF,\text{Bel}}^{++}(x) = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \bar{\psi} \left(-\frac{\lambda n}{2} \right) \gamma^+ i \overleftrightarrow{\partial}^+ \psi \left(\frac{\lambda n}{2} \right), \quad (2.141)$$

where the light front gauge $A^+ = 0$ is chosen, so the covariant derivative $\overleftrightarrow{D}^+(\mu n)$ reduces to a partial derivative $\overleftrightarrow{\partial}^+$ and the gauge links reduces to unity. Then the leading twist EMT can be expressed as $T_{q,\text{Bel}}^{++} = \int dx T_{q,LF,\text{Bel}}^{++}(x)$, which has a simple parton picture with $i\partial^+$ the momentum operator combining the parton density operator $\bar{\psi}(-\lambda n/2) \gamma^+ \psi(\lambda n/2)$.

The above expression leads to the definition of the parton AM density,

$$J_q^{x(2)}(x) = \int \frac{d\lambda d^2\mathbf{r}^\perp}{2\pi} e^{i\lambda x} \langle P, S | \bar{\psi} \left(-\frac{\lambda n}{2}, \mathbf{r}^\perp \right) \gamma^+ r^y i \overleftrightarrow{\partial}^+ \psi \left(\frac{\lambda n}{2}, \mathbf{r}^\perp \right) | P, S \rangle, \quad (2.142)$$

with the total transverse AM given by $J_q^{x(2)} = \int dx J_q^{x(2)}(x)$. Thus, the covariant sum rule has a simple parton picture without decomposing the AM into spin and orbital components. To find out the expression of the transverse AM density $J_q^{x(2)}(x)$, one still need to remove the extrinsic contributions. The RHS of eq. (2.142) involves the moment of r and it can again be written with

the corresponding off-forward matrix element, which reads

$$\begin{aligned} & \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P', S' | \bar{\psi} \left(-\frac{\lambda n}{2} \right) \gamma^+ \psi \left(\frac{\lambda n}{2} \right) | P, S \rangle \\ &= \frac{\bar{P}^+}{M} H_q(x) \left(2M + \epsilon^{0\rho\alpha\beta} \frac{i\Delta_\rho}{\bar{P}^0 + M} S_\alpha \bar{P}_\beta \right) - (H_q(x) + E_q(x)) i \frac{\epsilon^{+\nu\alpha\beta}}{M} \Delta_\nu S_\alpha \bar{P}_\beta . \end{aligned} \quad (2.143)$$

Then by removing terms proportional to P^μ or P^ν in $J^{\mu\nu}$, the intrinsic part of the twist-two transverse AM density is [4]

$$J_q^{x(2)}(x) = \gamma \frac{x}{2} (H_q(x) + E_q(x)) , \quad (2.144)$$

while the gluon one can be derived in the same way [4]. The twist-two transverse AM densities $J_{q,g}^{x(2)}(x)$ represents the AM carried by partons with momentum fraction x , while the extra γ factor corresponds to the boost effect it possesses.

2.3.3 Twist-three transverse AM sum rule

With all the above preparations, the twist-three transverse AM sum rule is rather straightforward to derive. In a very similar way to the longitudinal ones, the twist-three sum rule can be carried out with an extra step of removing the extrinsic contributions. Consider the twist-three transverse AM of quarks, which can be written as

$$\hat{J}_q^{x(3)} = \int d^3\mathbf{r} \left[-\bar{\psi}(\mathbf{r}) \gamma^0 \left(r^z \times i \overleftrightarrow{D}^y(\mathbf{r}) \right) \psi(\mathbf{r}) + \frac{1}{4} \epsilon^{0yzz} \bar{\psi}(\mathbf{r}) \gamma_x \gamma_5 \psi(\mathbf{r}) \right] . \quad (2.145)$$

This allows for further decomposition into canonical terms. Matrix elements similar to the longitudinal ones can then be defined as

$$l_q^{x(3)} \equiv - \int d^3\mathbf{r} \langle P, S | \bar{\psi}(\mathbf{r}) \gamma^0 \left(r^z \times i \overleftrightarrow{\partial}^y \right) \psi(\mathbf{r}) | P, S \rangle , \quad (2.146)$$

$$L_q^{x(3)} \equiv - \int d^3\mathbf{r} \langle P, S | \bar{\psi}(\mathbf{r}) \gamma^0 \left(r^z \times i \overleftrightarrow{D}^y(\mathbf{r}) \right) \psi(\mathbf{r}) | P, S \rangle , \quad (2.147)$$

$$l_{q,\text{pot}}^{x(3)} \equiv - \int d^3\mathbf{r} \langle P, S | \bar{\psi}(\mathbf{r}) \gamma^0 (r^z \times g A^y(\mathbf{r})) \psi(\mathbf{r}) | P, S \rangle . \quad (2.148)$$

And we clearly have $l_q^{x(3)} = L_q^{x(3)} + l_{q,\text{pot}}^{x(3)}$ by the definition of the covariant derivative. Then the twist-three AM $J_q^{x(3)}$ can be expressed with the quantities defined above as

$$J_q^{x(3)} = l_q^{x(3)} - l_{q,\text{pot}}^{x(3)} + \frac{1}{2} \Delta q_T . \quad (2.149)$$

The $l_{q,\text{pot}}^{x(3)}$ term will cancel the term that shows up when decomposing the gluon AM as we discussed in the longitudinal case, whereas the Δq_T term is nothing but the transverse spin term which corresponds to the polarized PDF $g_T(x)$ defined in eq. (2.78) that represents the transverse spin contribution from quarks of momentum fraction x .

As for the canonical OAM $l_q^{x(3)}$, it can be expressed in terms of the parton density defined as

$$l_q^{x(3)}(x) = - \frac{1}{P^+} \int \frac{d\lambda d^2\mathbf{r}^\perp}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0, \mathbf{r}^\perp) \gamma^+ r^z i \partial^y \psi(\lambda n, \mathbf{r}^\perp) | PS \rangle , \quad (2.150)$$

similar to eq. (2.92), and we have $l_q^{x(3)} = \int dx l_q^{x(3)}(x)$. This canonical OAM density has a simple parton picture with $r^z i \partial^y$ the twist-three part of the transverse AM operator with momentum

fraction x . Following the same procedure of defining the gauge-invariant equivalence of the canonical OAM, the following expression can be obtained:

$$l_q^{x(3)}(x) = \int dy \left[(G_{q,D,3}(x, y) + G'_{q,D,4}(x, y)) + \mathbf{P} \frac{1}{y-x} (G_{q,F,3}(x, y) + G'_{q,F,4}(x, y)) \right] , \quad (2.151)$$

where $G'_{q,D/F,4}(x, y) \equiv \lim_{\Delta \rightarrow 0} \frac{d}{d\xi} G_{q,D/F,4}(x, y, t, \xi)$. We can also find for the covariant OAM density $L_q^{x(3)}$ and the potential term $l_{q,\text{pot}}^{x(3)}$ respectively

$$L_q^{x(3)} = \int dx dy (G_{q,D,3}(x, y) + G'_{q,D,4}(x, y)) , \quad (2.152)$$

$$l_{q,\text{pot}}^{x(3)} = \int dx dy \mathbf{P} \frac{1}{y-x} (G_{q,F,3}(x, y) + G'_{q,F,4}(x, y)) . \quad (2.153)$$

Though the integrals of $G'_{q,D/F,4}(x, y)$ vanish after integration over both x and y , they reflect the fact that the transverse and longitudinal AM have different partonic densities. Once integrated over x , $l_q^{x(3)} = l_q^z$ as required by rotational symmetry.

The gluon parts of the twist-three AM sum rule can be derived as well. The twist-three gluon transverse AM can be written as

$$\hat{j}_g^{x(3)} = \int d^3\mathbf{r} \, 2r^z \text{Tr} \{ G^{0\alpha}(\mathbf{r}) G^y_{\alpha}(\mathbf{r}) \} . \quad (2.154)$$

Gluonic matrix elements similar to the longitudinal ones are defined as

$$\tilde{L}_g^{x(3)} \equiv 2 \int d^3\mathbf{r} \langle P, S | \text{Tr} \{ G^{0\alpha} (r^z \times D^y) A_\alpha \} | P, S \rangle , \quad (2.155)$$

$$l_g^{x(3)} \equiv 2 \int d^3\mathbf{r} \langle P, S | \text{Tr} \{ G^{0\alpha} (r^z \times \partial^y) A_\alpha \} | P, S \rangle , \quad (2.156)$$

$$l_{g,\text{pot}}^{x(3)} \equiv -2ig \int d^3\mathbf{r} \langle P, S | \text{Tr} \{ G^{0\alpha} (r^z \times A^y) A_\alpha \} | P, S \rangle , \quad (2.157)$$

with $l_g^{x(3)} = \tilde{L}_g^{x(3)} + l_{g,\text{pot}}^{x(3)}$. Then the twist-three gluon AM can be expressed as

$$J_g^{x(3)} = l_g^{x(3)} + l_{q,\text{pot}}^{x(3)} + \Delta G_T . \quad (2.158)$$

Again, since the potential term $l_{q,\text{pot}}^{x(3)}$ cancels the one in the corresponding quark AM, it will not be discussed here. The last term defined by $\Delta G_T \equiv 4 \int d^3\mathbf{r} \langle P, S | \text{Tr} \{ G^{0z}(\mathbf{r}) A^y(\mathbf{r}) \} | P, S \rangle$ is naturally related to the twist-three polarized gluon distribution $\Delta G_T(x)$ [90, 92, 93] through $\Delta G_T = \int \Delta G_T(x) dx$, where

$$\Delta G_T(x) = \frac{i}{xP^+} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P, S | 2\text{Tr} \left\{ G^{+\alpha} \left(-\frac{\lambda n}{2} \right) W_{-\frac{\lambda}{2}, \frac{\lambda}{2}} \tilde{G}_\alpha^\perp \left(\frac{\lambda n}{2} \right) \right\} | P, S \rangle . \quad (2.159)$$

The $\Delta G_T(x)$ is the transversely-polarized gluon density in a transversely-polarized nucleon, counterpart to the gluon helicity distribution $\Delta G(x)$ in a longitudinally-polarized nucleon.

Then we are left with the gluon canonical OAM $l_g^{x(3)}$ which is related to the canonical OAM density,

$$l_g^{x(3)}(x) = \int \frac{d\lambda d^2\mathbf{r}^\perp}{2\pi} e^{i\lambda x} \langle P, S | 2\text{Tr} \{ G^{+\alpha}(0, \mathbf{r}^\perp) r^z \partial^y A_\alpha(\lambda n, \mathbf{r}^\perp) \} | P, S \rangle , \quad (2.160)$$

and we have $l_g^{x(3)} = \int dx l_g^{x(3)}(x)$. The density $l_g^{x(3)}(x)$ has a simple parton interpretation as the canonical OAM of gluons with momentum fraction x . Again, following the same procedure of defining the gauge-invariant equivalence of the canonical OAM, $l_g^{x(3)}(x)$ can be expressed with D- and F-type GPDs as

$$l_g^{x(3)}(x) = \int dy \left[\mathbf{P} \frac{1}{x+y} (G_{g,D,3}(x, y) + G'_{g,D,4}(x, y)) + \mathbf{P} \frac{1}{y^2 - x^2} (G_{g,F,3}(x, y) + G'_{g,F,4}(x, y)) \right]. \quad (2.161)$$

And also for the covariant OAM density $\tilde{L}_g^{x(3)}$ and the potential term $l_{g,\text{pot}}^{x(3)}$ respectively:

$$\tilde{L}_g^{x(3)} = \int dx dy \mathbf{P} \frac{1}{x+y} (G_{g,D,3}(x, y) + G'_{g,D,4}(x, y)) , \quad (2.162)$$

$$l_{g,\text{pot}}^{x(3)} = \int dx dy \mathbf{P} \frac{1}{y^2 - x^2} (G_{g,F,3}(x, y) + G'_{g,F,4}(x, y)) , \quad (2.163)$$

which finish the process of decomposing the twist-three transverse AM $J_{q,g}^{x(3)}$.

Combining all the twist-three transverse AM densities discussed so far, we arrive at the following sum rule

$$\frac{1}{2} \Delta q_T + \Delta G_T + l_q^{x(3)} + l_g^{x(3)} = \frac{\hbar}{2} , \quad (2.164)$$

with each term resembling the longitudinal ones because of the rotational symmetry, but has different partonic densities: $\Delta q_T(x)/\Delta G_T(x)$ gives the parton transverse spin densities, and $l_{q,g}^{x(3)}(x)$ are the parton transverse canonical OAM densities.

Finally, we mention that all terms in the above equation are renormalization scale depen-

dent. Furthermore, when evolved from a particular scale, renormalization equations are complicated as they do not form a closed set: twist-three evolution generally involves light-cone correlations with two momentum fractions.

2.4 Transverse distributions of partons, AM, and magnetic moment

Besides the AM densities, it was pointed out that GPDs with arbitrary t but $\xi = 0$ can be used to picture the transverse-space distributions of partons [28]. This helps to explore effects of the transverse polarization on the parton distributions in the transverse plane. However, as we learned in the previous section, not all effects related to the transverse polarization are intrinsic, it is critically important to factor out the non-intrinsic contributions which are not related to the internal structure of the nucleon.

In this section, we study the transverse-space distributions of partons inside the nucleon, particularly the effects from the transverse polarization. We find some interesting results which are different from the standard formula in the literature [88,94,95]. Our results help to understand interesting physical observables, such as OAM and the magnetic moment [96] in terms of 3-dimensional images of the nucleon, or nuclear femtography.

2.4.1 Transverse distributions of partons

We start with the relation between GPDs and transverse-space distributions proposed in refs. [28,88]. In order to define transverse coordinates $\mathbf{b}$ for partons, the nucleon state shall be taken as localized in the transverse plane through superimposing plane waves with all transverse

momentum, while taking its z -component going to $P^z = \infty$,

$$|P_\infty^z, \mathbf{R}^\perp = 0\rangle \equiv \mathcal{N} \int \frac{d^2 \mathbf{p}^\perp}{(2\pi)^2} |P_\infty^z, \mathbf{p}^\perp\rangle , \quad (2.165)$$

where $|P_\infty^z, \mathbf{p}^\perp\rangle$ is a momentum eigenstate and $\mathcal{N}$ is a normalization constant. Using such a state, we can define the quark distribution in the transverse space as,

$$\rho_q(x, \mathbf{b}) \equiv \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P_\infty^z, \mathbf{R}^\perp = 0 | \bar{\psi} \left(-\frac{\lambda n}{2}, \mathbf{b} \right) \gamma^+ \psi \left(\frac{\lambda n}{2}, \mathbf{b} \right) | P_\infty^z, \mathbf{R}^\perp = 0 \rangle , \quad (2.166)$$

with gauge links between fields omitted. Then the Fourier transform of $\rho_q(x, \mathbf{b})$,

$$\rho_q(x, \Delta) \equiv \int d^2 \mathbf{b} e^{i\Delta \cdot \mathbf{b}} \rho_q(x, \mathbf{b}) , \quad (2.167)$$

can be expressed as

$$\rho_q(x, \Delta) = \mathcal{N}^2 (2\pi)^2 \delta^{(2)}(\mathbf{0}) \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi} \left(-\frac{\lambda n}{2} \right) \gamma^+ \psi \left(\frac{\lambda n}{2} \right) | P \rangle , \quad (2.168)$$

which corresponds to the twist-two GPDs defined before with $\Gamma = \gamma^+$ up to a normalization constant, except that Δ is in the transverse direction and hence $\xi = 0$. Then, the quark distribution can be expressed as

$$\rho_q(x, \Delta) = \mathcal{N}^2 (2\pi)^2 \delta^{(2)}(\mathbf{0}) \bar{u}(P') \left(H_q(x, -\Delta^2) \gamma^+ + E_q(x, -\Delta^2) \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} \right) u(P) . \quad (2.169)$$

In the infinite momentum frame where $P^z \rightarrow \infty$ and for an unpolarized state, it becomes simply

$$\rho_q(x, \Delta) = H_q(x, -\Delta^2) , \quad (2.170)$$

with proper normalization where the $E_q(x, -\Delta^2)$ term does not contribute to an unpolarized state. Thus, the quark transverse space distribution and $H_q(x, -\Delta^2)$ will be given by

$$\rho_q(x, \mathbf{b}) = \int \frac{d^2 \Delta}{(2\pi)^2} e^{-i\Delta \cdot \mathbf{b}} H_q(x, -\Delta^2) = \mathcal{H}_q(x, \mathbf{b}) , \quad (2.171)$$

where the Fourier transformation of GPD in transverse space $\mathcal{H}_q(x, \mathbf{b})$ is defined. The distribution $\rho_q(x, \mathbf{b})$ describes the the transverse-plane distribution of quarks with momentum fraction x in the localized nucleon state. By summing over all partons: $\rho_q(\mathbf{b}) \equiv \int dx \rho(x, \mathbf{b})$, such a matrix element becomes

$$\rho_q(\mathbf{b}) = \langle P_\infty^z, \mathbf{R}^\perp = 0 | \bar{\psi}(\mathbf{b}) \gamma^+ \psi(\mathbf{b}) | P_\infty^z, \mathbf{R}^\perp = 0 \rangle . \quad (2.172)$$

This is actually the Fourier transformation of the Dirac form factor $F_{1,q}(-\Delta^2)$ of the vector current [97]:

$$\rho_q(\mathbf{b}) = \int \frac{d^2 \Delta}{(2\pi)^2} e^{-i\Delta \cdot \mathbf{b}} \int dx H_q(x, -\Delta^2) = \int \frac{d^2 \Delta}{(2\pi)^2} e^{-i\Delta \cdot \mathbf{b}} F_{1,q}(-\Delta^2) , \quad (2.173)$$

which can be used to define charge distributions of the nucleon [98, 99] without the relativistic effects affecting the standard interpretation of nucleon electromagnetic form factors in the textbook [100, 101].

As for transversely-polarized nucleons, the polarized state $|P^z, \mathbf{R}^\perp = 0, S\rangle$ needs a more careful consideration. Since the states on the RHS use different $\mathbf{p}_\perp$, it is unclear how to have the same polarization vector S in a transverse direction while satisfying the constraint $S \cdot P = 0$ for all of them. However, in the infinite momentum limit where $P^z \rightarrow \infty$, all the states $|P^z, \mathbf{p}^\perp\rangle$ with finite $\mathbf{p}^\perp$ can be in a definite helicity state, $h = \pm$. Then the transversely-polarized nucleon can be defined as [28, 96],

$$|P^z, X\rangle \equiv \frac{1}{\sqrt{2}} (|P^z, \mathbf{R}^\perp = 0, +\rangle + |P^z, \mathbf{R}^\perp = 0, -\rangle) , \quad (2.174)$$

where $|P^z, \mathbf{R}^\perp = 0, +(-)\rangle$ are states localized in the transverse plane with definite helicity. The transverse-space quark density distribution in polarized nucleon states can be defined as

$$\rho_q^X(x, \mathbf{b}) \equiv \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P^z, X | \bar{\psi} \left(-\frac{\lambda n}{2}, \mathbf{b} \right) \gamma^+ \psi \left(\frac{\lambda n}{2}, \mathbf{b} \right) | P^z, X \rangle . \quad (2.175)$$

Its Fourier transformation is then

$$\rho_q^X(x, \Delta) = \frac{1}{2} \mathcal{N}^2 (2\pi)^2 \delta^{(2)}(\mathbf{0}) \sum_{h', h} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P', h' | \bar{\psi} \left(-\frac{\lambda n}{2} \right) \gamma^+ \psi \left(\frac{\lambda n}{2} \right) | P, h \rangle , \quad (2.176)$$

which is related to the twist-two GPDs,

$$\rho_q^X(x, \Delta) \propto \frac{1}{2} \sum_{h', h} \bar{u}(P', h') \left(H_q(x, -\Delta^2) \gamma^+ + E_q(x, -\Delta^2) \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} \right) u(P, h) , \quad (2.177)$$

where summations over helicity are performed for the initial and final states, separately. Using

the spinors in the infinite momentum limit $P^z \rightarrow \infty$, one then finds

$$\rho_q^X(x, \Delta) = H_q(x, -\Delta^2) + \frac{i\Delta_y}{2M} E_q(x, -\Delta^2) . \quad (2.178)$$

After inverse Fourier-transformed to the transverse space, this gives the quark distribution in transversely polarized nucleons

$$\begin{aligned} \rho_q^X(x, \mathbf{b}) &= \int \frac{d^2\Delta}{(2\pi)^2} e^{-i\Delta \cdot \mathbf{b}} \left(H_q(x, -\Delta^2) + \frac{i\Delta_y}{2M} E_q(x, -\Delta^2) \right) , \\ &= \mathcal{H}_q(x, \mathbf{b}) - \frac{1}{2M} \frac{\partial}{\partial b^y} \mathcal{E}_q(x, \mathbf{b}) , \end{aligned} \quad (2.179)$$

with the Fourier-transformed GPDs $\mathcal{H}_q(x, \mathbf{b})/\mathcal{E}_q(x, \mathbf{b})$ in transverse space, as first derived in ref. [28].

As discussed in the last section, however, the result above also receives contributions from the center-of-mass motion which needs to be removed to get the intrinsic quark distributions. To show that, we again expand the Dirac spinor in $\rho_q^X(x, \Delta)$ to get

$$\begin{aligned} \rho_q^X(x, \Delta) &= \frac{\bar{P}^\mu}{\bar{P}^0} H_q(x, -\Delta^2) \left(1 + \epsilon^{0\rho\alpha\beta} \frac{i\Delta_\rho}{2M(\bar{P}^0 + M)} S_\alpha \bar{P}_\beta \right) \\ &\quad - (H_q(x, -\Delta^2) + E_q(x, -\Delta^2)) \frac{i\epsilon^{\mu\nu\alpha\beta} \Delta_\nu}{2M\bar{P}^0} S_\alpha \bar{P}_\beta , \end{aligned} \quad (2.180)$$

where clearly the second term in the first line is proportional to the center-of-mass momentum $\bar{P}^\mu$ and does not correspond to the intrinsic contributions. Such a term, if not removed, will cancel the $H_q(x, -\Delta^2)$ term in the second line and produce the transverse space distributions derived above. However, for the intrinsic contributions, it shall be removed to restore the proper γ factor of the transverse AM indicated by its Lorentz transformation [60]. Therefore, we define

the intrinsic quark density distribution in transverse-space as,

$$\rho_{q,\text{In}}^X(x, \Delta) = H_q(x, -\Delta^2) + \frac{i\Delta_y}{2M} (H_q(x, -\Delta^2) + E_q(x, -\Delta^2)) . \quad (2.181)$$

And its Fourier transformation becomes

$$\begin{aligned} \rho_{q,\text{In}}^X(x, \mathbf{b}) &= \int \frac{d^2\Delta}{(2\pi)^2} e^{-i\Delta \cdot \mathbf{b}} \left[H_q(x, -\Delta^2) + \frac{i\Delta_y}{2M} (H_q(x, -\Delta^2) + E_q(x, -\Delta^2)) \right] , \\ &= \mathcal{H}_q(x, \mathbf{b}) - \frac{1}{2M} \frac{\partial}{\partial b^y} (\mathcal{H}_q(x, \mathbf{b}) + \mathcal{E}_q(x, \mathbf{b})) , \end{aligned} \quad (2.182)$$

which differs from eq. (2.179) by the $\mathcal{H}_q(x, \mathbf{b})$ term in the parentheses.

2.4.2 Transverse AM distribution

With the intrinsic parton distribution $\rho_{q,\text{In}}^X(x, \mathbf{b})$, the transverse AM density in the transverse plane can be derived straightforwardly. The twist-two quark transverse AM density in the transverse plane can be defined as

$$J_q^{x(2)}(x, \mathbf{b}) \equiv \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P^z, X | \bar{\psi} \left(-\frac{\lambda n}{2}, \mathbf{b} \right) \gamma^+ b^y i \overleftrightarrow{\partial}^+ \psi \left(\frac{\lambda n}{2}, \mathbf{b} \right) | P^z, X \rangle , \quad (2.183)$$

whose inverse Fourier transformation is

$$\begin{aligned} J_q^{x(2)}(x, \Delta) &= (b^y \times x P^+) \rho_{q,\text{In}}^X(x, \Delta) , \\ &= \frac{\gamma}{2} x (H_q(x, -\Delta^2) + E_q(x, -\Delta^2)) , \end{aligned} \quad (2.184)$$

where b^y becomes the derivative operator $-i \frac{\partial}{\partial \Delta_y}$ in momentum representation and $(b^y \times x P^+)$ is the twist-two transverse AM operator. In the forward limit, it is equivalent to the transverse AM

derived in eq. (2.144). The quark transverse AM distribution in the transverse space $J_q^{x(2)}(x, \mathbf{b})$ is now

$$J_q^{x(2)}(x, \mathbf{b}) = \frac{\gamma}{2} x (\mathcal{H}_q(x, \mathbf{b}) + \mathcal{E}_q(x, \mathbf{b})) , \quad (2.185)$$

through Fourier transformation.

The total quark transverse AM distribution in the transverse space defined as

$$J_q^{x(2)}(\mathbf{b}) \equiv \langle P^z, X | \bar{\psi}(\mathbf{b}) \gamma^+ b^y_i \overleftrightarrow{\partial}^+ \psi(\mathbf{b}) | P^z, X \rangle , \quad (2.186)$$

can then be shown to be simply the sum over all quarks of different x as $J_q^{x(2)}(\mathbf{b}) = \int dx J_q^{x(2)}(x, \mathbf{b})$.

With eq. (2.184), $J_q^{x(2)}(\mathbf{b})$ can be expressed as

$$J_q^{x(2)}(\mathbf{b}) = \frac{\gamma}{2} \int \frac{d^2 \Delta}{(2\pi)^2} e^{-i\Delta \cdot \mathbf{b}} (A_q(-\Delta^2) + B_q(-\Delta^2)) , \quad (2.187)$$

which relates the gravitational form factors $A_q(-\Delta^2)$ and $B_q(-\Delta^2)$ to the AM distribution in the transverse plane of the nucleon. The gluon AM distribution can be defined similarly and the total AM density is $J^{x(2)}(\mathbf{b}) \equiv \sum_i J_i^{x(2)}(\mathbf{b})$ with i summing over quarks and gluons.

2.4.3 Transverse magnetic moment distribution

Distributions of other quantities in transversely-polarized nucleons can be derived similarly, such as the magnetic moment whose property resembles that of AM. The magnetic moment can be written as

$$\mathbf{m} = \frac{1}{2} \int d^3 \mathbf{r} \langle P | \mathbf{r} \times \mathcal{J}_{EM} | P \rangle , \quad (2.188)$$

with $\mathcal{J}_{EM}$ the electromagnetic current of the system which simply sums over all the quark currents of different flavors i and charges e_i in nucleon as

$$\mathcal{J}_{EM}^\mu = \sum_i e_i \mathcal{J}_i^\mu = \sum_i e_i \bar{\psi}_i \gamma^\mu \psi_i . \quad (2.189)$$

The magnetic moment of nucleons can be related to their spin s as

$$\mathbf{m} = \mu \mathbf{s} , \quad (2.190)$$

so the scalar μ also gives the magnetic moment of nucleons. For a transversely polarized nucleon in momentum eigenstates, the magnetic moment can be written as

$$\mathbf{m}^x = \frac{1}{2} \int d^3\mathbf{r} \langle P, S | (\mathbf{r}^y \mathcal{J}_{EM}^z - \mathbf{r}^z \mathcal{J}_{EM}^y) | P, S \rangle . \quad (2.191)$$

The same as AM, in the infinite momentum frame, it can be split into the twist-two $\mathbf{r}^y \mathcal{J}_{EM}^z$ term and twist-three $-\mathbf{r}^z \mathcal{J}_{EM}^y$ term. The twist-two magnetic moment which dominates in the infinite momentum limit is defined as

$$\mu^{(2)} \equiv \langle P^z, S | \int d^3\mathbf{r} \mathbf{r}^y \mathcal{J}_{EM}^z | P^z, S \rangle , \quad (2.192)$$

Just as the AM which can be expressed with the gravitational form factors, the magnetic moment can be related to the electromagnetic form factor which is defined as,

$$\langle P' | \mathcal{J}_{EM}^\mu | P \rangle = e_N \bar{u}(P') \left(F_1(\Delta^2) \gamma^\mu + F_2(\Delta^2) \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} \right) u(P) , \quad (2.193)$$

where e_N is the charge of the nucleon and $F_1(\Delta^2)$ and $F_2(\Delta^2)$ are the Dirac and Pauli form factors of nucleon. Then the magnetic moment μ can be expressed as

$$\mu = \frac{e_N \hbar}{2M} (F_1(0) + F_2(0)) , \quad (2.194)$$

which can also be expressed with the nuclear magneton $\mu_N \equiv e_N \hbar / (2M)$ and the magnetic form factor $G_M(\Delta^2) \equiv F_1(\Delta^2) + F_2(\Delta^2)$ as $\mu = \mu_N G_M(0)$.

The magnetic moment distribution in the transverse space follows from

$$\mu^{(2)}(x, \mathbf{b}) = b^y \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P_\infty^z, X | \sum_i e_i \bar{\psi}_i \left(-\frac{\lambda n}{2}, \mathbf{b} \right) \gamma^+ \psi_i \left(\frac{\lambda n}{2}, \mathbf{b} \right) | P_\infty^z, X \rangle . \quad (2.195)$$

Again, with the intrinsic parton distribution in transverse space $\rho_{q_i, \text{In}}^X(x, \mathbf{b})$, one has the simple relation

$$\mu^{(2)}(x, \Delta) = b^y \sum_i e_i \rho_{q_i, \text{In}}^X(x, \Delta) , \quad (2.196)$$

in momentum representation with again b^y the derivative operator $-i \frac{\partial}{\partial \Delta_y}$. Using eq. (2.182), it can be written with GPDs in transverse space as

$$\mu^{(2)}(x, \mathbf{b}) = \sum_i \frac{e_i \hbar}{2M} (\mathcal{H}_{q_i}(x, \mathbf{b}) + \mathcal{E}_{q_i}(x, \mathbf{b})) , \quad (2.197)$$

where $-i \frac{\partial}{\partial \Delta_y}$ takes the term proportional to Δ_y only. The relation connects the magnetic moment density distribution to GPDs in transverse space, analogous to eq. (2.185) for AM.

The total magnetic moment distribution in the transverse space defined as

$$\mu^{(2)}(\mathbf{b}) = b^y \langle P^z, X | \sum_i e_i \bar{\psi}_i(\mathbf{b}) \gamma^+ \psi_i(\mathbf{b}) | P^z, X \rangle , \quad (2.198)$$

is then a sum over all partons: $\mu^{(2)}(\mathbf{b}) = \int dx \mu^{(2)}(x, \mathbf{b})$. The following relation between $\mu^{(2)}(\mathbf{b})$ and electromagnetic form factors can be derived as

$$\mu^{(2)}(\mathbf{b}) = \sum_i \frac{e_i \hbar}{2M} \int \frac{d^2 \Delta}{(2\pi)^2} e^{-i\Delta \cdot \mathbf{b}} (F_{1,q_i}(-\Delta^2) + F_{2,q_i}(-\Delta^2)) , \quad (2.199)$$

which indicates that the magnetic form factors $G_M(-\Delta^2)$ describe the magnetic moment distribution, analogous to the relation between Dirac form factor and charge distribution. This result agrees with the form factor result in eq. (2.194). It confirms the previous result [96] for the anomalous magnetic momentum distribution in the transverse space.

Chapter 3: DVCS and the kinematical higher twist effects

Experimental observables are essential to access these structures in the real world. As mentioned above, while the parton spin contributions can be accessed with their polarized PDFs, their orbital motions are given by their quantum phase space distributions, or essentially their GPDs. For instance, the quark and gluon AM densities can be expressed as [4],

$$J_{q,g}(x) = \frac{x}{2} (H_{q,g}(x, 0, 0) + E_{q,g}(x, 0, 0)) , \quad (3.1)$$

where $H_{q,g}(x, \xi, t)$ and $E_{q,g}(x, \xi, t)$ are two different types of GPDs. Therefore, experiments that can access these GPDs are crucial for the measurement of nucleon spin structures. GPDs are naturally probed by the DEPs, namely the exclusive productions of particles off nucleons with deeply virtual photon, including deeply virtual Compton scattering (DVCS) [102] and deeply virtual meson production (DVMP) [103, 104] via the collinear factorization of QCD. DVCS is considered the golden channel for the measurements of quark GPDs with its clean final state, whereas DVMP allows more gluon sensitivities as shown in Fig. 3.1.

We focus on the DVCS process [4, 102], namely the process of leptonproduction of a photon off the nucleon with the virtual photon momentum square, or the virtuality, $Q^2 = -q^2$ much larger than the momentum transfer square t . GPDs enter the DVCS cross-section in the form of CFFs, which can be generally expressed in terms of the convolution of GPDs and complex-valued

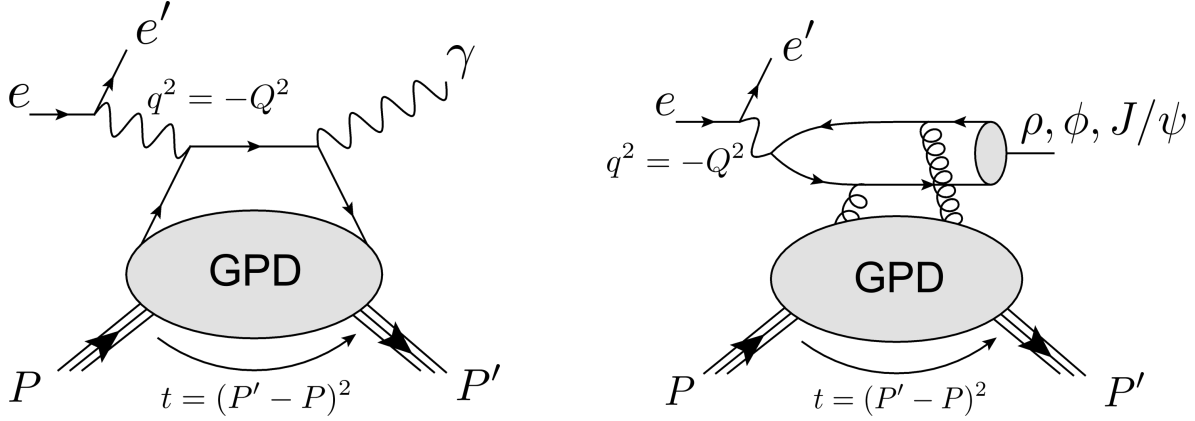


Figure 3.1: Leading-order handbag diagrams for the DVCS (left) and DVMP (right).

Wilson coefficients [29, 64, 105]. Therefore, the DVCS measurements that allow us to measure the CFFs and reconstruct the GPDs play an important role in studying the internal structure of the nucleon. There have been measurements from HERA (H1 [106–109], ZEUS [110, 111] and HERMES [112, 113]) and JLab (CLAS [114, 115] and Hall A [116, 117]) and more programs are planned in the future such as the JLab 24 GeV, EIC [118] and EIC [7].

The theoretical foundation of studying the DVCS process is the collinear factorization proven in QCD to the leading power accuracy of Q [119–121], where twist expansion is introduced. However, the twist expansion depends on the convention one chooses, similar to the case in which one can do a series expansion for a given function at different points and get different series. Those series converge to the same original function that does not depend on the expansion points chosen, but their finite sums do. Therefore, intrinsic uncertainties due to different conventions will enter the cross-section formulas at higher order, which can be significant. As a rough estimation, at JLab 12 GeV kinematics where $Q^2 \sim 5 \text{ GeV}^2$, the twist-four parameter related to the target mass M will be around $M^2/Q^2 \sim 20\%$ which is quite substantial for precision measurements of CFFs, while the twist-three effects are even more considerable. The twist-three effects in the DVCS process have been studied in details [74, 122–125]. Besides, progresses have

been made in understanding the kinematical twist-four effects, such as finite t and target mass correction for both forward and off-forward processes [126–132].

In this chapter, we start with a comprehensive overview of the DVCS cross-section formulas in the literatures. We will address another source of higher-twist effects related to the choice of light-cone vectors and present a more general set-up of the cross-section formulas to study such effects. We will show that the dependence on the choice of light-cone vectors shows up at the twist three level, making it a non-negligible effect one must take into account in order to obtain the cross-section formula to twist-three accuracy. On the other hand, we will also show that the light-cone dependence gets canceled at twist three by considering kinematical corrections from twist-three GPDs.

3.1 Notation and twist expansion in DVCS

In this section, we briefly review the general formalism for the DVCS cross-section, see for instance [29, 102, 105, 133] for more details. Consider the electroproduction of a photon off a proton as,

$$e(k, h) + N(P, S) \rightarrow e(k', h') + N(P', S') + \gamma(q', \Lambda') , \quad (3.2)$$

where k, k', P, P', q' are the momenta of the particles and h, h', S, S', Λ' correspond to their helicities or polarization. The helicities h and h' take the value of $\pm 1/2$, Λ' takes ± 1 while the nucleon polarization vector S satisfies $S^2 = -1$ and $S \cdot P = 0$ and similarly for S' . We also define the virtual photon momentum $q \equiv k - k'$ and its helicity Λ .

Two sub-processes, Bethe-Heitler (BH) and Deeply Virtual Compton Scattering (DVCS),

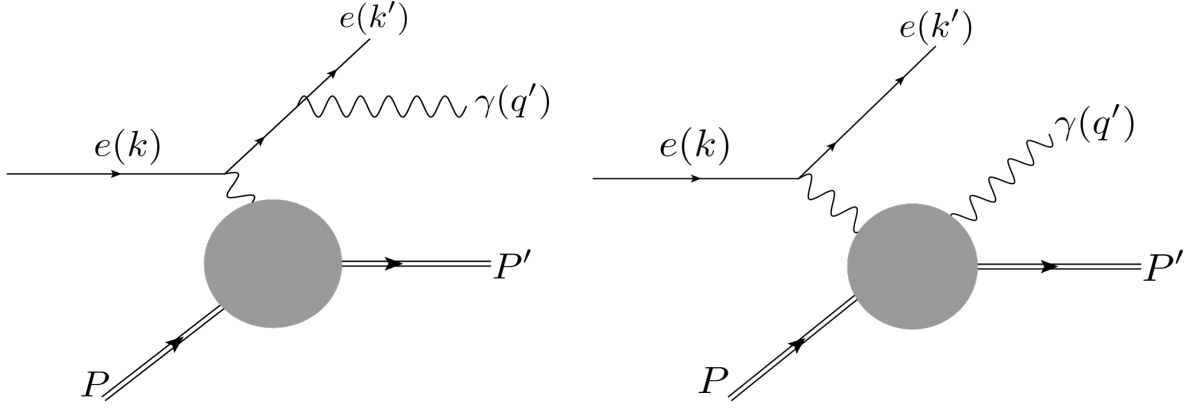


Figure 3.2: The Bethe-Heitler (left) and the DVCS (right) processes that both contribute to the same photon production. Another Bethe-Heitler process where the photon is emitted from the proton will be suppressed by the proton mass, and thus not plotted here.

mainly contribute as shown in Fig. 3.2, and the full amplitude will be the sum of them:

$$\mathcal{T} = \mathcal{T}_{\text{BH}} + \mathcal{T}_{\text{DVCS}} . \quad (3.3)$$

With the coordinate set-up shown in Fig. 3.3, the electroproduction cross-section in the lab frame can be expressed in terms of the squared amplitude combined with some kinematical prefactors as [105, 133],

$$\frac{d^5\sigma}{dx_B dQ^2 dt |d\phi d\phi_S} = \frac{\alpha_{\text{EM}}^3 x_B y^2}{16\pi^2 Q^4 \sqrt{1 + \gamma^2}} |\mathcal{T}|^2 , \quad (3.4)$$

where we have the following definitions: fine structure constant $\alpha_{\text{EM}} \equiv e^2/(4\pi)$, photon virtuality $Q^2 \equiv -q^2$, the Bjorken scaling variable $x_B \equiv Q^2/(2P \cdot q)$, the electron energy loss variable $y \equiv P \cdot q/(P \cdot k)$ and momentum transfer square $t \equiv (P' - P)^2$. ϕ and ϕ_S describe the azimuthal angle between the leptonic plane and the reaction plane and the azimuthal angle between the leptonic plane and the target polarization vector in the case of transversely polarized target, respectively. The notation $\gamma \equiv 2Mx_B/Q$ is introduced with M the target mass. We also define the notation $\bar{P} \equiv (P + P')/2$, $\bar{q} \equiv (q + q')/2$ and $\Delta \equiv P' - P = q - q'$.

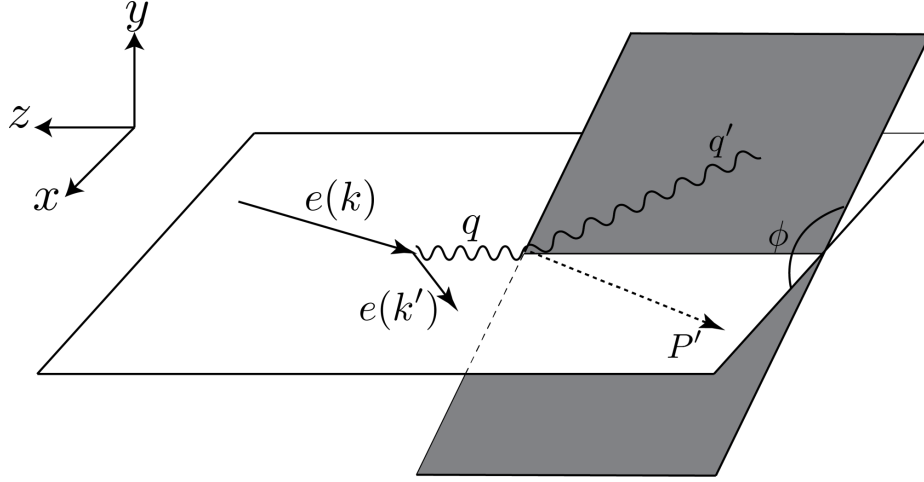


Figure 3.3: The DVCS process in the lab frame where the proton is at rest initially.

The squared amplitude consists of three parts,

$$|\mathcal{T}|^2 = |\mathcal{T}_{\text{BH}}|^2 + |\mathcal{T}_{\text{DVCS}}|^2 + \mathcal{T}_{\text{BH}}^* \mathcal{T}_{\text{DVCS}} + \mathcal{T}_{\text{DVCS}}^* \mathcal{T}_{\text{BH}} . \quad (3.5)$$

The interference contribution defined by the last term

$$\mathcal{I} \equiv \mathcal{T}_{\text{BH}}^* \mathcal{T}_{\text{DVCS}} + \mathcal{T}_{\text{DVCS}}^* \mathcal{T}_{\text{BH}} = 2\text{Re} [\mathcal{T}_{\text{BH}}^* \mathcal{T}_{\text{DVCS}}] , \quad (3.6)$$

is linear in both the BH and DVCS amplitudes, and will be of more interest because of its rich angular structure, whereas the BH amplitude can be carried out relatively easily and will not be discussed in details. The notation and convention for the laboratory frame can be found in Appendix B, which follows from ref. [133].

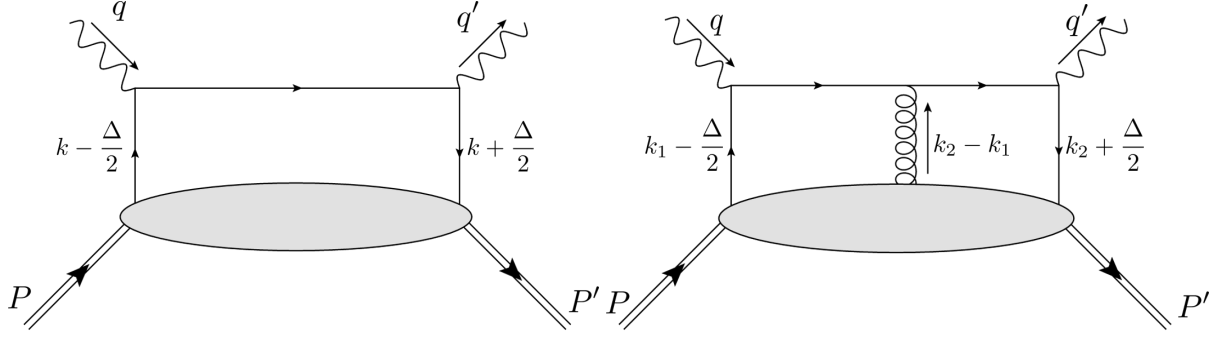


Figure 3.4: Examples of twist-two (left) and twist-three (right) handbag diagrams that contribute to the twist-three Compton tensor. Other diagrams that are related to them by permutation are not shown here.

3.1.1 Compton tensor and light-cone vectors

The DVCS amplitude can be expressed in terms of the Compton tensor as,

$$\mathcal{T}_{\text{DVCS}} = \frac{e_l}{Q^2} \bar{u}(k', h') \gamma^\nu u(k, h) T^{\mu\nu} \varepsilon_\mu^*(q', \Lambda') , \quad (3.7)$$

with e_l the lepton charge in the unit of electron charge that is positive (negative) for electron (positron), while the Compton tensor $T^{\mu\nu}$ is defined as,

$$T^{\mu\nu} \equiv i \int d^4z e^{i(q+q')z/2} \langle P', S' | T \left\{ J^\mu \left(\frac{z}{2} \right) J^\nu \left(-\frac{z}{2} \right) \right\} | P, S \rangle , \quad (3.8)$$

with $J^\mu(z)$ the electromagnetic current operator and T the time order operator.

The Compton tensor $T^{\mu\nu}$ is in general non-perturbative, but with QCD factorization in the Bjorken limit, it can be expressed in terms of the combination of perturbative coefficients and CFFs with Feynman diagrams as shown in Fig. 3.4, see also refs. [29, 102]. Then the Compton

tensor $T^{\mu\nu}$ can be expressed in terms of twist expansion as,

$$T^{\mu\nu} = T_{(2)}^{\mu\nu} + T_{(3)}^{\mu\nu} + \cdots, \quad (3.9)$$

where higher twist contribution will not be considered, and the leading and next-to-leading Compton tensor can be written as [29],

$$\begin{aligned} T_{(2)}^{\mu\nu} &= \int_{-1}^1 dx \sum_q \left(\mathcal{T}_{(2)}^{\mu\nu} C_{(0)}^{q[-]}(x, \xi) n^\rho W^{[\gamma_\rho]}(x, \xi, t) + \widetilde{\mathcal{T}}_{(2)}^{\mu\nu} C_{(0)}^{q[+]}(x, \xi) n^\rho W^{[\gamma_\rho \gamma^5]}(x, \xi, t) \right), \\ T_{(3)}^{\mu\nu} &= \int_{-1}^1 dx \sum_q \left(\mathcal{T}_{(3)}^{\mu\nu, \rho} C_{(0)}^{q[-]}(x, \xi) W^{[\gamma_\rho^\perp]}(x, \xi, t) + \widetilde{\mathcal{T}}_{(3)}^{\mu\nu, \rho} C_{(0)}^{q[+]}(x, \xi) W^{[\gamma_\rho^\perp \gamma^5]}(x, \xi, t) \right), \end{aligned} \quad (3.10)$$

respectively, where $W^{[\Gamma]}$ are GPDs defined as

$$W^{[\Gamma]} \equiv \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P', S' | \bar{\psi} \left(-\frac{\lambda n}{2} \right) \Gamma \psi \left(\frac{\lambda n}{2} \right) | P, S \rangle, \quad (3.11)$$

with Γ certain combination of Dirac matrices: $\Gamma = \{1, \gamma^\mu, \sigma^{\mu\nu}, \gamma^\mu \gamma^5, \gamma^5\}$ and $\sigma^{\mu\nu} \equiv \frac{i}{2} [\gamma^\mu, \gamma^\nu]$.

The tree-level Wilson coefficient functions $C_{(0)}^{q[\pm]}$ read [102, 105],

$$C_{(0)}^{q[\pm]} = -Q_q^2 \left(\frac{1}{x - \xi + i0} \mp \frac{1}{x + \xi - i0} \right), \quad (3.12)$$

where Q_q is the charge of quarks in the unit of proton charge. The (0) subscript stands for the tree-level coefficients, whereas in the next order of the strong coupling, much more complicated coefficients functions will be generated from loop correction. As for the GPDs, we use the

definition in ref. [102],

$$n_\mu W^{[\gamma^\mu]} = \bar{u}(P', S') \left[\not{n} H(x, \xi, t) + \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2M} E(x, \xi, t) \right] u(P, S) , \quad (3.13)$$

$$n_\mu W^{[\gamma^\mu \gamma^5]} = \bar{u}(P', S') \left[\not{n} \gamma^5 \tilde{H}(x, \xi, t) + \frac{n^\mu \Delta_\mu \gamma^5}{2M} \tilde{E}(x, \xi, t) \right] u(P, S) , \quad (3.14)$$

and the corresponding CFFs are defined from the convolution of GPDs and Wilson coefficients as [105, 133],

$$\begin{aligned} \mathcal{H}(\xi, t) &\equiv \int_{-1}^1 dx C_{(0)}^{q[-]}(x, \xi) H(x, \xi, t) , & \mathcal{E}(\xi, t) &\equiv \int_{-1}^1 dx C_{(0)}^{q[-]}(x, \xi) E(x, \xi, t) , \\ \tilde{\mathcal{H}}(\xi, t) &\equiv \int_{-1}^1 dx C_{(0)}^{q[+]}(x, \xi) \tilde{H}(x, \xi, t) , & \tilde{\mathcal{E}}(\xi, t) &\equiv \int_{-1}^1 dx C_{(0)}^{q[+]}(x, \xi) \tilde{E}(x, \xi, t) , \end{aligned} \quad (3.15)$$

where summing over all the quarks is implicit here. Note that we focus on the cross-section formula related to the leading twist CFFs in this paper, so the second line of eq. (3.10) is set to zero hereafter unless mentioned otherwise.

In the Bjorken limit, both q^2 and $P \cdot q$ go to infinity while their ratio is kept finite. Therefore, both initial and final photon momenta reach the light-cone and define the light-cone vector n in the expressions above (mass dimension -1), and the skewness parameter ξ is defined as $\xi \equiv -\Delta \cdot n / (2\bar{P} \cdot n)$. With the light-cone vector n and its conjugate light-cone vector p (mass dimension $+1$) that satisfies $p^2 = 0$ and $n \cdot p = 1$, we could define the transverse projection operator as,

$$g_\perp^{\mu\nu} \equiv g^{\mu\nu} - p^\mu n^\nu - n^\mu p^\nu . \quad (3.16)$$

Then the $\gamma_\rho^\perp$ which indicates terms that are non-zero only in the transverse direction can be expressed with the transverse projection operator as $\gamma_\rho^\perp \equiv g_{\rho\sigma}^\perp \gamma^\sigma$. We also define the following

antisymmetric symbol in the transverse space as,

$$\epsilon_{\perp}^{\mu\nu} \equiv \epsilon^{\mu\nu\rho\sigma} p_{\rho} n_{\sigma} . \quad (3.17)$$

Then the Compton tensor coefficients $\mathcal{T}^{\mu\nu}$ s in the Compton tensor can be expressed in terms of the light-cone vectors as [29],

$$\begin{aligned} \mathcal{T}_{(2)}^{\mu\nu} &= -\frac{1}{2} \left[g_{\perp}^{\mu\nu} - \frac{1}{p \cdot \bar{q}} (p^{\mu} q'_{\perp}{}^{\nu} + q_{\perp}^{\mu} p^{\nu}) \right] , \\ \widetilde{\mathcal{T}}_{(2)}^{\mu\nu} &= \frac{i}{2} \left[\epsilon_{\perp}^{\mu\nu} - \frac{1}{p \cdot \bar{q}} \left(-p^{\mu} \epsilon_{\perp}^{\nu\rho} q'_{\rho}{}^{\perp} + \epsilon_{\perp}^{\mu\rho} q_{\rho}^{\perp} p^{\nu} \right) \right] , \\ \mathcal{T}_{(3)}^{\mu\nu,\rho} &= \frac{1}{2p \cdot \bar{q}} \left[q'^{\mu} g_{\perp}^{\nu\rho} + g_{\perp}^{\mu\rho} (q^{\nu} + 4\xi p^{\nu}) \right] , \\ \widetilde{\mathcal{T}}_{(3)}^{\mu\nu,\rho} &= \frac{i}{2p \cdot \bar{q}} \left[\epsilon^{\mu\nu\rho\sigma} \bar{q}_{\sigma} + \xi (p^{\mu} \epsilon_{\perp}^{\rho\nu} + p^{\nu} \epsilon_{\perp}^{\rho\mu}) \right] , \end{aligned} \quad (3.18)$$

with twist-three accuracy, where the leading terms are first derived in ref. [102]. It is worth noting that $\mathcal{T}^{\mu\nu}$'s are ambiguous unless the light-cone vectors are clearly defined, and we will leave the detailed discussion on this for the next section.

We notice that there are other set-ups without explicit light-cone vectors found in the literature. One of the commonly used is that of Belitsky-Kirchner-Müller (BKM) [105], while a similar set-up also is used in the Belitsky-Müller-Ji (BMJ) results [134]. Instead of the light-cone vectors n and p , the effective light-cone vectors are expressed in terms of the covariant physical momenta as,

$$p_{\text{BKM}}^{\mu} = \bar{P}^{\mu} , \quad n_{\text{BKM}}^{\mu} = \frac{\bar{q}^{\mu}}{\bar{P} \cdot \bar{q}} , \quad (3.19)$$

which are not light-like: $p_{\text{BKM}}^2 \neq 0, n_{\text{BKM}}^2 \neq 0$. Those vectors are not ideal for rigorous twist expansions, and the GPDs defined with them will mix with higher-order GPDs. But still, one can calculate the Compton tensor coefficients with those effective light-cone vectors, which gives [29],

$$\begin{aligned}
\mathcal{T}_{\text{BKM},(2)}^{\mu\nu} &= -\frac{1}{2}\mathcal{P}^{\mu\rho} \left[g_{\rho\sigma} + \frac{\bar{q}^2}{(\bar{P} \cdot \bar{q})^2} \bar{P}_\rho \bar{P}_\sigma \right] \mathcal{P}^{\sigma\nu} , \\
\widetilde{\mathcal{T}}_{\text{BKM},(2)}^{\mu\nu} &= \frac{i}{2\bar{P} \cdot \bar{q}} \mathcal{E}^{\mu\nu\rho\sigma} \bar{P}_\rho \bar{q}_\sigma , \\
\mathcal{T}_{\text{BKM},(3)}^{\mu\nu,\rho} &= -\frac{\bar{q}^2}{2(\bar{P} \cdot \bar{q})^2} \left[\mathcal{P}^{\mu\rho} \bar{P}_\sigma \mathcal{P}^{\sigma\nu} + \mathcal{P}^{\mu\sigma} \bar{P}_\sigma \mathcal{P}^{\rho\nu} \right] , \\
\widetilde{\mathcal{T}}_{\text{BKM},(3)}^{\mu\nu,\rho} &= \frac{i}{2\bar{P} \cdot \bar{q}} \mathcal{E}^{\mu\nu\rho\sigma} \bar{q}_\sigma ,
\end{aligned} \tag{3.20}$$

where the projectors are defined as,

$$\mathcal{P}^{\mu\nu} \equiv g^{\mu\nu} - \frac{q^\mu q'^\nu}{q \cdot q'} , \quad \mathcal{E}^{\mu\nu\rho\sigma} \equiv \left(g^{\mu\alpha} - \frac{\bar{P}^\mu q'^\alpha}{\bar{P} \cdot q'} \right) \left(g^{\nu\beta} - \frac{\bar{P}^\nu q^\beta}{\bar{P} \cdot q} \right) \epsilon_{\alpha\beta}{}^{\rho\sigma} , \tag{3.21}$$

and we rewrite the original expression in terms of our notations. The BKM Compton tensor coefficients are equivalent to the Compton tensor coefficients expressed with light-cone vectors p^μ in the direction of $\bar{P}$ and n^μ in the direction of $\bar{q}$ up to twist-four corrections. Without loss of generality, we will still use our light-cone vectors p and n unless otherwise specified. A careful comparison between those two different set-ups is left to the next section.

3.1.2 Gauge invariance and helicity amplitudes

Up to now, we have not used the explicit photon polarization vectors. It is simple to sum over the final photon polarization, for which we replace,

$$\sum_{\Lambda'=\pm 1} \varepsilon^\mu(q', \Lambda') \varepsilon^{*\nu}(q', \Lambda') \rightarrow -g^{\mu\nu} , \quad (3.22)$$

and then the cross-sections will be independent of the specific form of polarization vectors of the photon. However, the substitution of photon polarization vectors needs more care, since the Compton tensor $T^{\mu\nu}$ only has twist-three accuracy, and it does not satisfy the current conservation condition exactly as $q'_\mu T^{\mu\nu} = T^{\mu\nu} q_\nu = 0$. Instead, it only conserves up to a higher twist term,

$$q'_\mu T^{\mu\nu} \sim T^{\mu\nu} q_\nu \sim \mathcal{O}(\Delta_\perp^2) . \quad (3.23)$$

Therefore, the exact gauge invariance cannot be maintained, and the gauge dependence of the polarization sum is non-trivial.

To be more specific, consider an on-shell photon $A^\mu(q')$, for which one additional gauge fixing condition is needed besides the physical requirement $q'^\mu A_\mu(q') = 0$, since there are only two degrees of freedom. If we choose in general $V^\mu A_\mu(q') = 0$ with V^μ any four-vector, the photon polarization sum can be written as

$$\sum_{\Lambda'=\pm 1} \varepsilon^\mu(q', \Lambda') \varepsilon^{*\nu}(q', \Lambda') = -g^{\mu\nu} + \frac{V^\mu q'^\nu + V^\nu q'^\mu}{V \cdot q'} - \frac{V^2 q'^\mu q'^\nu}{(V \cdot q')^2} . \quad (3.24)$$

Besides the $-g^{\mu\nu}$ term, we also have terms that depend explicitly on the gauge choice V^μ . Those

terms are proportional to q'^μ or q'^ν , so they do not contribute if the Compton tensor conserves to all orders. However, since the Compton tensor only conserves up to some higher order twist-four terms, ignoring those extra gauge-dependent terms in the polarization sum will introduce extra twist-four contributions to the cross-section and is not preferred. It is worth noting that the necessity of using eq. (3.24) results from the unconserved Compton tensor. Thus, one might regard the usage of eq. (3.24) as a gauge improving process, since it eliminates the terms in the Compton tensor that are proportional to q'^μ , which are unphysical. The effect of the gauge dependence of the cross-section will be discussed more carefully later.

Another way of taking care of the gauge dependence is to use the helicity amplitude method. Since the Compton tensor is always connected to two photons, one could rewrite eq. (3.7) in terms of the photon polarization vectors as [133, 135, 136]

$$\mathcal{T}_{\text{DVCS}} = \frac{1}{Q^2} \bar{u}(k', h') \gamma^\rho u(k, h) \sum_{\Lambda} \varepsilon_\rho^*(q, \Lambda) \varepsilon_\nu(q, \Lambda) T^{\mu\nu} \varepsilon_\mu^*(q', \Lambda') , \quad (3.25)$$

where Λ sums over the three possible polarizations of the virtual photon. If we define the helicity amplitude as,

$$T_{\Lambda\Lambda'} \equiv \varepsilon_\nu(q, \Lambda) T^{\mu\nu} \varepsilon_\mu^*(q', \Lambda') , \quad (3.26)$$

the DVCS amplitude can then be expressed in helicity amplitude as,

$$\mathcal{T}_{\text{DVCS}} = \frac{1}{Q^2} \sum_{\Lambda} \bar{u}(k', h') \gamma^\rho u(k, h) \varepsilon_\rho^*(q, \Lambda) T_{\Lambda\Lambda'} . \quad (3.27)$$

The DVCS amplitude expressed this way will be independent of frame, since the helicity is an invariant scalar. *However, the definition of helicity depends on the choice of light-cone vectors, as*

the transverse directions are defined by the light-cone vectors, and thus each helicity amplitude will depend on the convention for the light-cone vectors. Meanwhile, the DVCS amplitude still depends on the gauge choice implicitly, because the photon polarization vectors $\varepsilon_\mu^*(q', \Lambda')$ do. The helicity amplitude method does not resolve the gauge dependence issue, it only makes it implicit. Still, we will need to fix the gauge in order to calculate the helicity amplitude, and the helicity amplitude will be different under different gauge choices, though it will be a higher-twist effect.

In the literature, the radiation gauge for which we set $V^\mu \propto (1, 0, 0, 0)$ in the lab frame is commonly chosen such that the A^0 will be eliminated. In an arbitrary frame, we could simply set $V^\mu = P^\mu$, since eq. (3.24) is invariant under a rescaling of V^μ . In order to calculate the helicity amplitude, we set up our coordinates in the lab frame by setting the virtual photon momentum in the $-z$ direction and the lepton beam momentum in the $x-z$ plane. The details of all the physical momenta are given in Appendix B which follows refs. [105, 133]. Then the photon polarization vectors can be written as

$$\begin{aligned}\varepsilon(q, \Lambda = \pm 1) &= \frac{e^{-i\Lambda\phi}}{\sqrt{2}}(0, 1, i\Lambda, 0), \\ \varepsilon(q, \Lambda = 0) &= \frac{Q}{\sqrt{2x_B M}}(-\sqrt{1 + \gamma^2}, 0, 0, 1), \\ \varepsilon^*(q', \Lambda' = \pm 1) &= \frac{1}{\sqrt{2}}(0, -\cos\theta \cos\phi + i\Lambda' \sin\phi, -i\Lambda' \cos\phi - \cos\theta \sin\phi, \sin\theta),\end{aligned}\tag{3.28}$$

and the completeness relation for the virtual photon reads

$$\sum_{\Lambda=\pm 1} \varepsilon^\mu(q, \Lambda) \varepsilon^{*\nu}(q, \Lambda) - \varepsilon^\mu(q, \Lambda = 0) \varepsilon^{*\nu}(q, \Lambda = 0) = -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}.\tag{3.29}$$

The polarization sum of the virtual photon polarization can be replaced with $-g^{\mu\nu}$, since the leptonic part always satisfies the relation $\bar{u}(k')\gamma^\nu u(k)q_\nu = 0$, and the extra $q^\mu q^\nu$ term will not contribute.

We calculate the helicity amplitudes using the Compton tensor coefficients in eq. (3.20) for the BKM set-up as an example, which reads,

$$\begin{aligned}\mathcal{T}_{\text{BKM},(2)}^{\Lambda\Lambda'=++} &= \mathcal{T}_{\text{BKM},(2)}^{\Lambda\Lambda'=-} = \frac{1}{2} - \frac{x_B^2(t(1-x_B) + M^2x_B^2)}{2(2-x_B)^2Q^2} + \mathcal{O}(Q^{-3}), \\ \mathcal{T}_{\text{BKM},(2)}^{\Lambda\Lambda'=+-} &= \mathcal{T}_{\text{BKM},(2)}^{\Lambda\Lambda'=-+} = -\frac{x_B^2(t(1-x_B) + M^2x_B^2)}{2(2-x_B)^2Q^2} + \mathcal{O}(Q^{-3}), \\ \mathcal{T}_{\text{BKM},(2)}^{\Lambda\Lambda'=0+} &= \mathcal{T}_{\text{BKM},(2)}^{\Lambda\Lambda'=0-} = \frac{\sqrt{-x_B^2(t(1-x_B) + M^2x_B^2)}}{\sqrt{2}(x_B-2)Q} + \mathcal{O}(Q^{-3}),\end{aligned}\tag{3.30}$$

where we defined $\mathcal{T}_{(2)}^{\Lambda\Lambda'} \equiv \varepsilon_\nu(q, \Lambda) \mathcal{T}_{(2)}^{\mu\nu} \varepsilon_\mu^*(q', \Lambda')$ and they are consistent with the results in ref. [135]. We leave the helicity amplitudes with light-cone vectors and the comparison with the BKM ones to the next section, after we define our light-cone vectors there.

3.2 Cross-section formalism with general light-cone vectors

In the last section, we introduce the light-cone vectors n and p without defining them, since their definitions are conventional. In this section, we define those light-cone vectors in a general form such that light-cone vectors under different conventions can be studied systematically. Then we can use them to work out the cross-section formula explicitly.

Note that the BH process does not involve the Compton tensor, so it does not depend on

the light-cone vectors defined. With the BH amplitude which can be written as,

$$\mathcal{T}_{\text{BH}} = -\frac{1}{t}\epsilon_{\mu}^*(q', \Lambda') l_{\text{BH}}^{\mu\nu} \bar{u}(P', S') \left[(F_1 + F_2)\gamma_{\nu} - \frac{\bar{P}_{\nu}}{M} F_2 \right] u(P, S) , \quad (3.31)$$

where the BH leptonic amplitude is defined as,

$$l_{\text{BH}}^{\mu\nu} \equiv \bar{u}(k', h') \left[\frac{\gamma^{\mu}(\not{k}' + \not{q}')\gamma^{\nu}}{(k' + q')^2} + \frac{\gamma^{\nu}(\not{k} - \not{q}')\gamma^{\mu}}{(k - q')^2} \right] u(k, h) , \quad (3.32)$$

one can write down the BH cross-section without light-cone vectors, see for instance refs. [102, 105, 133] for more details. With that in mind, we will focus on the pure DVCS and interference cross-sections and treat the BH contribution as the background in this paper.

3.2.1 Choice of light-cone vectors

Since each four vector has four degrees of freedom, we will need 8 constraints to determine the two light-cone vectors n and p . To start with, we have 3 constraints $n^2 = 0$, $p^2 = 0$ and $p \cdot n = 1$ from the definitions of the light-cone vectors. In addition, since the light-cone vectors should always lie in the space spanned by the four physical momenta P, P', q, q' who only span a three-dimensional subspace due to momentum conservation, we have two extra constraints that

$$n_{\mu} \epsilon^{\mu\nu\rho\sigma} P_{\nu} P'_{\rho} q_{\sigma} = p_{\mu} \epsilon^{\mu\nu\rho\sigma} P_{\nu} P'_{\rho} q_{\sigma} = 0 . \quad (3.33)$$

Therefore, there remain $3 = 2 \times 4 - 5$ degrees of freedom unfixed in defining the light-cone vectors which are totally conventional. First, we could fix the scale of the light-cone vectors by

asking that $\bar{P} \cdot n = 1$, which can always be done by boosting in the n direction. Then we are left with two extra degrees of freedom that describe the direction of the two light-cone vectors n and p . Two parameters, α and β , are introduced to define the directions of two light-cone vectors, with which we define

$$q_L \equiv \alpha q^\mu + (1 - \alpha) q'^\mu, \quad P_L \equiv \beta P^\mu + (1 - \beta) P'^\mu. \quad (3.34)$$

Then our light-cone vectors p and n are defined such that q_L and P_L are longitudinal:

$$q_L^\mu g_{\mu\nu}^\perp = P_L^\mu g_{\mu\nu}^\perp = 0. \quad (3.35)$$

The above two conditions completely fix the light-cone vectors, though implicitly, and thus all the physical momenta can be expressed in terms of light-cone vectors and their transverse components using the general decomposition,

$$V^\mu = p^\mu (V \cdot n) + n^\mu (V \cdot p) + V_\perp^\mu. \quad (3.36)$$

The following expansions can be worked out for all the physical momenta,

$$\begin{aligned}
P^\mu &= (1 + \xi)p^\mu + \frac{M^2 + (\beta - 1)^2|\Delta_\perp|^2}{2(1 + \xi)}n^\mu + (\beta - 1)\Delta_\perp^\mu, \\
P'^\mu &= (1 - \xi)p^\mu + \frac{M^2 + \beta^2|\Delta_\perp|^2}{2(1 - \xi)}n^\mu + \beta\Delta_\perp^\mu, \\
\bar{P}^\mu &= p^\mu + \frac{1}{2} \left(\frac{M^2 + (\beta - 1)^2|\Delta_\perp|^2}{2(1 + \xi)} + \frac{M^2 + \beta^2|\Delta_\perp|^2}{2(1 - \xi)} \right) n^\mu + (\beta - 1/2)\Delta_\perp^\mu, \\
\Delta^\mu &= -2\xi p^\mu - \frac{|\Delta_\perp|^2 + t}{4\xi}n^\mu + \Delta_\perp^\mu, \\
q^\mu &= -2\eta p^\mu - \frac{-Q^2 + (\alpha - 1)^2|\Delta_\perp|^2}{4\eta}n^\mu + (1 - \alpha)\Delta_\perp^\mu, \\
q'^\mu &= -2(\eta - \xi)p^\mu - \left(\frac{-Q^2 + (\alpha - 1)^2|\Delta_\perp|^2}{4\eta} - \frac{|\Delta_\perp|^2 + t}{4\xi} \right) n^\mu - \alpha\Delta_\perp^\mu.
\end{aligned} \tag{3.37}$$

With all the physical constraints on the momenta such as $P^2 = P'^2 = M^2, q^2 = -Q^2, q'^2 = 0$ and $\Delta^2 = t$ and the momentum conservation $\Delta = P' - P = q - q'$, the $|\Delta_\perp|^2$ and the parameter $\eta \equiv -q \cdot n / (2\bar{P} \cdot n)$ can be solved, for which we have

$$|\Delta_\perp|^2 = \frac{-4\xi^2 M^2 + (\xi^2 - 1)t}{[1 + (2\beta - 1)\xi]^2}, \tag{3.38}$$

$$\begin{aligned}
\eta &= \xi \frac{\sqrt{(Q^2 + t)^2 + 4\alpha(Q^2 + (1 - \alpha)t)|\Delta_\perp|^2} - Q^2 + t + 2(1 - \alpha)|\Delta_\perp|^2}{2(t + |\Delta_\perp|^2)} \\
&= \xi \left(1 - \frac{\alpha^2|\Delta_\perp|^2}{Q^2} \right) + \mathcal{O}(Q^{-4}).
\end{aligned} \tag{3.39}$$

The last undetermined constant in the above expansions is the skewness parameter ξ , which can be solved in terms of the Bjorken scaling parameter x_B with $2P \cdot q = Q^2/x_B$, and the solution reads

$$\xi = \frac{x_B}{2 - x_B} - \frac{2x_B \{2\alpha M^2 x_B^2 + (x_B - 1)[1 - 2\alpha + (\beta - 1)x_B]t\}}{(x_B - 2)^2(1 + (\beta - 1)x_B)Q^2} + \mathcal{O}(Q^{-4}). \tag{3.40}$$

An exact solution of ξ is in general inaccessible, so we expand ξ with twist-four accuracy. However, in the case where $\alpha = 0$ we can solve them exactly to get,

$$\eta(\alpha = 0) = \xi, \quad \xi(\alpha = 0) = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}. \quad (3.41)$$

which corresponds to choosing the light-cone vector n in the direction q' . The above definition of ξ with $\alpha = 0$ agrees with the one in ref. [129]. This simplification can be easily understood considering that $q'^2 = 0$, and thus the choice $n \propto q'$ builds a simple connection between the physical momenta and light-cone vectors.

Other different conventions have been used in the literatures. Besides the Braun-Manashov-Pirnay (BMP) convention [129], corresponding to $\alpha = 0, \beta \rightarrow \infty$, in ref. [102] (Ji), $\alpha = 1, \beta = 1/2$, in ref. [105] (BKM), $\alpha = \beta = 1/2$ and in ref. [133] (UVa), $\alpha = \beta = 1$ are chosen respectively. Those different conventions are plotted together in Fig. 3.5.

The above vector expansions on the light-cone also allow us to explicitly express the light-cone vectors in terms of the physical momenta as

$$\begin{aligned} p^\mu &= \frac{-DP_L + Bq_L}{BC - DA}, \\ n^\mu &= \frac{CP_L - Aq_L}{BC - DA}, \end{aligned} \quad (3.42)$$

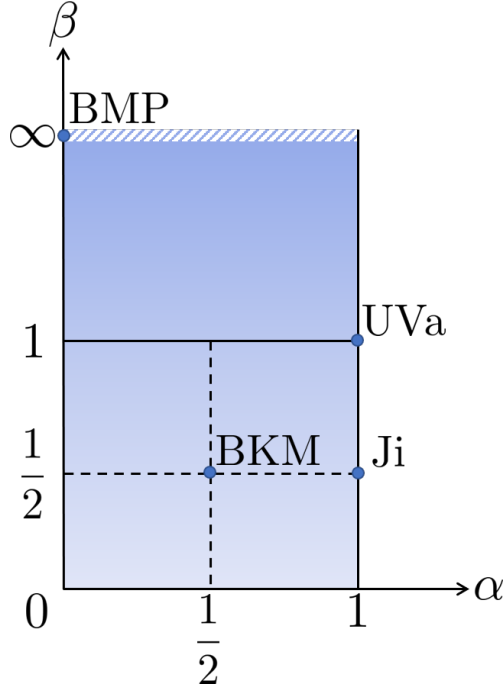


Figure 3.5: A plot illustrating that different conventions in the literatures corresponds to different choice of α and β .

where we define the constants as

$$\begin{aligned}
 A &= (1 - \xi) + 2\xi\beta, \\
 B &= \frac{M^2 + \beta^2|\Delta_\perp|^2}{2(1 - \xi)} + \beta \frac{t + |\Delta_\perp|^2}{4\xi}, \\
 C &= -2(\eta - \xi) - 2\xi\alpha, \\
 D &= -\frac{-Q^2 + (\alpha - 1)^2|\Delta_\perp|^2}{4\eta} + (1 - \alpha) \frac{|\Delta_\perp|^2 + t}{4\xi}.
 \end{aligned} \tag{3.43}$$

Therefore, all the variables in the light-cone coordinates can be expressed in terms of physical momenta and lab frame kinematical variables with twist-four accuracy.

While each set of value of $\{\alpha, \beta\}$ represents a different choice of light-cone vectors, the value of $\{\alpha, \beta\}$ should not be chosen arbitrarily. The twist expansion assumes that the photon momenta q and q' are hard and approach the light-cone n direction in the Bjorken limit, so α

should satisfy $|\alpha|^2|\Delta_\perp|^2 \ll Q^2$ and $|1 - \alpha|^2|\Delta_\perp|^2 \ll Q^2$. Since $|\Delta_\perp|^2/Q^2$ is already twist-four suppressed, we can simply take

$$0 \leq \alpha \leq 1, \quad (3.44)$$

which has very simple physical meaning that when $\alpha = 0$ the light-cone vector n is in the direction of q' and when $\alpha = 1$, it basically aligns with the q .

As for the choice of β , while it is not as constrained, the twist expansion has a singularity at $1 + (2\beta - 1)\xi = 0$. At the singularity, we have $n \cdot P_L = 0$ and thus P_L has to be proportional to n . Then in the Bjorken limit, q, q' and P_L will all approach the n direction and the p vector is not well-defined, whereas all physical quantities are regular when $\beta \rightarrow \infty$ and it is safe to let $\beta \rightarrow \infty$. With that in mind, we require that $|\Delta_\perp|^2 \ll Q^2$, which gives $|1 + (2\beta - 1)\xi| \gg (-4\xi^2 M^2 + (\xi^2 - 1)t)/Q^2$. Again, since t/Q^2 and M^2/Q^2 are already twist-four suppressed, we can simply have $|1 + (2\beta - 1)\xi| \gtrsim \mathcal{O}(1)$. In practice, we choose

$$0 \leq \beta \leq 2. \quad (3.45)$$

For $0 < \xi < 1$, it satisfies $|1 + (2\beta - 1)\xi| \geq 1 - \xi$. Though large β will not cause any singularity, we drop that region from our choice of β since it differs too much from the other choices in the literatures with β from 0 to 1. Apparently, the larger range we choose for α and β , the more conventions will be covered, but the more uncertainties will appear in the final cross-section consequently. The above ranges of α and β define the confidence bands we used in this paper.

With all the light-cone vectors defined, we can then perform the calculation of the helicity

amplitudes in the radiation gauge. For comparison, we first calculate the helicity amplitudes using the Compton tensor coefficients in eq. (3.18) expressed with light-cone vectors, and get

$$\begin{aligned}
\mathcal{T}_{(2)}^{\Lambda\Lambda'=++} &= \mathcal{T}_{(2)}^{\Lambda\Lambda'=-} = \frac{1}{2} - \frac{x_B^2(t(1-x_B) + M^2x_B^2)(\beta-1)^2}{2[1 + (\beta-1)x_B]^2Q^2} + \mathcal{O}(Q^{-3}) , \\
\mathcal{T}_{(2)}^{\Lambda\Lambda'=+-} &= \mathcal{T}_{(2)}^{\Lambda\Lambda'=-+} = -\frac{x_B^2(t(1-x_B) + M^2x_B^2)(\beta-1)^2}{2[1 + (\beta-1)x_B]^2Q^2} + \mathcal{O}(Q^{-3}) , \\
\mathcal{T}_{(2)}^{\Lambda\Lambda'=0+} &= \mathcal{T}_{(2)}^{\Lambda\Lambda'=0-} = \frac{(\beta-1)\sqrt{-x_B^2(t(1-x_B) + M^2x_B^2)}}{\sqrt{2}[1 + (\beta-1)x_B]Q} + \mathcal{O}(Q^{-3}) ,
\end{aligned} \tag{3.46}$$

which indeed reduces to the BKM helicity amplitudes in eq. (3.30) when one takes $\beta = 1/2$, consistent with the choice made in the BKM set-up. The helicity amplitudes turn out to be independent of α at twist four, whereas it depends on β starting at twist three. The dependence of the helicity amplitudes on the choice of light-cone vectors is a mixture, originated from two sources. The obvious one is the Compton tensor coefficients in eq. (3.18) that depends on the light-cone vectors. Besides that, the polarization vectors also depend on the choice of light-cone vectors, because the transverse and longitudinal directions are conventional, as one could see in the definition of $g_{\perp}^{\mu\nu}$ and $\epsilon_{\perp}^{\mu\nu}$ in Eqs. (3.16) and (3.17).

More specifically, in the Bjorken limit all physical momenta reach the light-cone and instead of a three-dimensional subspace, they only span a two-dimensional space expanded by the light-cone vectors n and p . The remaining two-dimensional space orthogonal to the physical momenta defines the transverse space, and the helicity can be defined unambiguously in the two-dimensional transverse space. However, the above picture ceases to work when higher twist effects are taken into account. The four physical momenta will then span a three-dimensional subspace, and only one transverse direction, the direction of $\epsilon^{\mu\nu\rho\sigma} P_{\nu} P'_{\rho} q_{\sigma}$, is well-defined. Then the helicity corresponding to a circular polarization that involves two transverse directions in-

evitably mixes the well-defined transverse direction with another transverse direction that lies within the P, P', q, q' plane, which are totally conventional.

In the end, when we sum over all the polarizations, the dependence of polarization vectors on the light-cone vectors will vanish, since they reduce to the tensor in eq. (3.24), while the dependence of the Compton tensor on the light-cone vectors will persist in the final cross-section formula. On the other hand, the gauge dependence is always there, explicit or not, which cannot be avoided.

With those thoughts in mind, we will not use the helicity amplitude method for the cross-section formula in this paper. Instead, we simply apply the gauge fixing condition directly when summing over the final photon polarization, which has the same effect as the helicity amplitude method in terms of the final cross-section but does not involve specific expressions of the polarization vectors.

3.2.2 Pure DVCS cross-section with polarizations

We start with the pure DVCS cross-section, which is relatively simpler. The DVCS squared amplitude $|\mathcal{T}_{\text{DVCS}}|^2$ in eq. (3.5) can be written into the leptonic and hadronic parts as,

$$|\mathcal{T}_{\text{DVCS}}|^2 = \frac{1}{Q^4} L_{\text{DVCS}}^{\rho\sigma} H_{\rho\sigma}^{\text{DVCS}}, \quad (3.47)$$

where we have

$$L_{\text{DVCS}}^{\rho\sigma} \equiv \sum_{h'} \bar{u}(k, h) \gamma^\rho u(k', h') \bar{u}(k', h') \gamma^\sigma u(k, h) , \quad (3.48)$$

$$H_{\text{DVCS}}^{\rho\sigma} \equiv \sum_{S', \Lambda'} T^{*\mu\rho} T^{\nu\sigma} \varepsilon_\mu(q', \Lambda') \varepsilon_\nu^*(q', \Lambda') . \quad (3.49)$$

The leptonic tensor $L_{\text{DVCS}}^{\rho\sigma}$ can always be expressed in terms of the polarized part and the unpolarized part as,

$$L_{\text{DVCS}}^{\rho\sigma} = L_{\text{DVCS,U}}^{\rho\sigma} + i2h L_{\text{DVCS,L}}^{\rho\sigma} , \quad (3.50)$$

where each term can be solved as [102],

$$L_{\text{DVCS,U}}^{\rho\sigma} = 2 (k^\rho k'^\sigma + k'^\rho k^\sigma - g^{\rho\sigma} k \cdot k') , \quad (3.51)$$

$$L_{\text{DVCS,L}}^{\rho\sigma} = 2\epsilon^{\rho\sigma\alpha\beta} k_\alpha k'_\beta . \quad (3.52)$$

As for the hadronic part, it can be worked out with eq. (3.49) and the Compton tensor given in eq. (3.10). It turns out that the hadronic tensor contains four terms that have a different dependence on the target polarization, which can be written as,

$$H_{\text{DVCS}}^{\rho\sigma} = H_{\text{DVCS,U}}^{\rho\sigma} + \frac{1}{M} H_{\text{DVCS,T}}^{\rho\sigma} \epsilon^{n\bar{P}\Delta S} + M H_{\text{DVCS,n}}^{\rho\sigma} (n \cdot S) + \frac{1}{M} H_{\text{DVCS,\Delta}}^{\rho\sigma} (\Delta \cdot S) , \quad (3.53)$$

where we introduce the notation $\epsilon^{n\bar{P}\Delta S} \equiv \epsilon^{\mu\nu\alpha\beta} n_\mu \bar{P}_\nu \Delta_\alpha S_\beta$ when contracting with the ϵ tensor, which will be used hereafter. The explicit expressions of those $H_{\text{DVCS}}^{\rho\sigma}$ s of different polarization

configurations are not of interest for now.

In order to further study the polarization dependence of the cross-section, it is helpful to write down a basis for the target polarization S and express the cross-section in terms of the base vectors, as the polarized cross-section is linear in S . We define the longitudinal polarization vector to be a spatial vector along the direction of n^μ such that $S_L^\mu \sim n^\mu$, where $\sim$ indicates projecting out the temporal component and normalization. As for the transverse polarization, note that the transverse components of all the physical momenta in eq. (3.37) are in the same direction, the direction of $\Delta_\perp^\mu$. Thus, we define the two transverse polarization vectors as $S_{T,\text{in}}^\mu \sim \Delta_\perp^\mu$ and $S_{T,\text{out}}^\mu \sim \epsilon_\perp^{\mu\nu} \Delta_\nu^\perp$, where $\Delta_\perp^\mu$ is defined as $\Delta_\perp^\mu \equiv g_\perp^{\mu\nu} \Delta_\nu$ and satisfies $\Delta_\perp^2 = -|\Delta_\perp|^2$. The in and out subscripts indicate that the two vectors are parallel and perpendicular to the hadronic plane, respectively. Then we define our polarization vectors as,

$$\begin{aligned} S_L^\mu &= \frac{M}{1+\xi} \left(-n^\mu + \frac{1+\xi}{M^2} P^\mu \right), \\ S_{T,\text{in}}^\mu &= \frac{1}{NM(1+\xi)} \epsilon^{\mu\nu P n} \epsilon^{n P P'}_{\nu}, \\ S_{T,\text{out}}^\mu &= \frac{1}{NM} \epsilon^{n P P' \mu} \propto \epsilon_\perp^{\mu\nu} \Delta_\nu^\perp, \end{aligned} \tag{3.54}$$

where we introduce the notation

$$N = \frac{\sqrt{-4M^2\xi^2 - t(1-\xi^2)}}{M}, \tag{3.55}$$

and an overall plus or minus sign is conventional for all the polarization vectors. Note that while we have the relation $S_{T,\text{out}}^\mu \propto \epsilon_\perp^{\mu\nu} \Delta_\nu^\perp$, it does NOT apply to $S_{T,\text{in}}$ as $\Delta_\perp^\mu$ might have a temporal component depending on the choice of light-cone vectors. Consequently, we use the definition in

eq. (3.54) such that $S_{T,\text{in}} \cdot P = 0$ is manifest. The three polarization vectors form an orthonormal basis for the polarization vector. With a specific choice of parameters $\alpha = \beta = 1$, they reduce to

$$\begin{aligned} S_L^\mu &= (0, 0, 0, 1) , \\ S_{T,\text{in}}^\mu &= (0, \cos \phi, \sin \phi, 0) , \\ S_{T,\text{out}}^\mu &= (0, -\sin \phi, \cos \phi, 0) , \end{aligned} \tag{3.56}$$

in the lab frame, whereas higher twist corrections to those expressions are needed if different α and β are chosen. Then, if we write the polarization vector as [133]

$$S^\mu = 2\Lambda_L(0, 0, 0, 1) + 2\Lambda_T(0, \cos \phi_S, \sin \phi_S, 0) , \tag{3.57}$$

where Λ_L, Λ_T stands for the spin projection in the longitudinal/transverse direction respectively and ϕ_S is the azimuthal angle between the transverse polarization vector and the leptonic plane, it can be decomposed as,

$$S^\mu = (2\Lambda_L)S_L^\mu + 2\Lambda_T \cos(\phi_S - \phi) S_{T,\text{in}}^\mu + 2\Lambda_T \sin(\phi_S - \phi) S_{T,\text{out}}^\mu , \tag{3.58}$$

and thus the hadronic tensor for the pure DVCS cross-section can be written as,

$$\begin{aligned} H_{\text{DVCS}}^{\rho\sigma} &= H_{\text{DVCS,U}}^{\rho\sigma} + (2\Lambda_L)H_{\text{DVCS,L}}^{\rho\sigma} \\ &+ 2\Lambda_T \left[H_{\text{DVCS,T,in}}^{\rho\sigma} \cos(\phi_S - \phi) + H_{\text{DVCS,T,out}}^{\rho\sigma} \sin(\phi_S - \phi) \right] , \end{aligned} \tag{3.59}$$

where we defined

$$H_{\text{DVCS,U}}^{\rho\sigma} \equiv \frac{1}{2} \left[H_{\text{DVCS}}^{\rho\sigma}(S) + H_{\text{DVCS}}^{\rho\sigma}(-S) \right], \quad (3.60)$$

$$H_{\text{DVCS,L}}^{\rho\sigma} \equiv \frac{1}{2} \left[H_{\text{DVCS}}^{\rho\sigma}(S_L) - H_{\text{DVCS}}^{\rho\sigma}(-S_L) \right], \quad (3.61)$$

$$H_{\text{DVCS,T,in}}^{\rho\sigma} \equiv \frac{1}{2} \left[H_{\text{DVCS}}^{\rho\sigma}(S_{\text{T,in}}) - H_{\text{DVCS}}^{\rho\sigma}(-S_{\text{T,in}}) \right], \quad (3.62)$$

$$H_{\text{DVCS,T,out}}^{\rho\sigma} \equiv \frac{1}{2} \left[H_{\text{DVCS}}^{\rho\sigma}(S_{\text{T,out}}) - H_{\text{DVCS}}^{\rho\sigma}(-S_{\text{T,out}}) \right]. \quad (3.63)$$

Plugging the polarization vectors S into the definition of the hadronic tensor above, we have for

$$H_{\text{DVCS,U}}^{\rho\sigma},$$

$$\begin{aligned} H_{\text{DVCS,U}}^{\rho\sigma} = & 4 \left[(1 - \xi^2) \left(\mathcal{H}^{\rho\sigma} \mathcal{H}^* \mathcal{H} + \widetilde{\mathcal{H}}^{\rho\sigma} \widetilde{\mathcal{H}}^* \widetilde{\mathcal{H}} \right) - \frac{t}{4M^2} \left(\mathcal{H}^{\rho\sigma} \mathcal{E}^* \mathcal{E} + \xi^2 \widetilde{\mathcal{H}}^{\rho\sigma} \widetilde{\mathcal{E}}^* \widetilde{\mathcal{E}} \right) \right. \\ & \left. - \xi^2 \left(\mathcal{H}^{\rho\sigma} \mathcal{E}^* \mathcal{E} + \mathcal{H}^{\rho\sigma} (\mathcal{E}^* \mathcal{H} + \mathcal{H}^* \mathcal{E}) + \widetilde{\mathcal{H}}^{\rho\sigma} (\widetilde{\mathcal{E}}^* \widetilde{\mathcal{H}} + \widetilde{\mathcal{H}}^* \widetilde{\mathcal{E}}) \right) \right], \end{aligned} \quad (3.64)$$

and for $H_{\text{DVCS,L}}^{\rho\sigma}$

$$\begin{aligned} H_{\text{DVCS,L}}^{\rho\sigma} = & -8i \mathcal{H}_-^{\rho\sigma} \text{Re} \left\{ (1 - \xi^2) \widetilde{\mathcal{H}}^* \mathcal{H} - \xi^2 \left(\widetilde{\mathcal{H}}^* \mathcal{E} + \widetilde{\mathcal{E}}^* \mathcal{H} \right) - \left(\frac{\xi^2}{1 + \xi} + \frac{t}{4M^2} \right) \xi \widetilde{\mathcal{E}}^* \mathcal{E} \right\} \\ & + 8 \mathcal{H}_+^{\rho\sigma} \text{Im} \left\{ (1 - \xi^2) \widetilde{\mathcal{H}}^* \mathcal{H} - \xi^2 \left(\widetilde{\mathcal{H}}^* \mathcal{E} + \widetilde{\mathcal{E}}^* \mathcal{H} \right) - \left(\frac{\xi^2}{1 + \xi} + \frac{t}{4M^2} \right) \xi \widetilde{\mathcal{E}}^* \mathcal{E} \right\}, \end{aligned} \quad (3.65)$$

and for $H_{\text{DVCS,T,in}}^{\rho\sigma}$

$$H_{\text{DVCS,T,in}}^{\rho\sigma} = -4Ni\mathcal{H}_-^{\rho\sigma}\text{Re}\left\{\tilde{\mathcal{H}}^*\mathcal{E} - \xi\tilde{\mathcal{E}}^*\mathcal{H} - \frac{\xi^2}{1+\xi}\tilde{\mathcal{E}}^*\mathcal{E}\right\} \\ + 4N\mathcal{H}_+^{\rho\sigma}\text{Im}\left\{\tilde{\mathcal{H}}^*\mathcal{E} - \xi\tilde{\mathcal{E}}^*\mathcal{H} - \frac{\xi^2}{1+\xi}\tilde{\mathcal{E}}^*\mathcal{E}\right\}, \quad (3.66)$$

and for $H_{\text{DVCS,T,out}}^{\rho\sigma}$

$$H_{\text{DVCS,T,out}}^{\rho\sigma} = 4N\text{Im}\left[\mathcal{H}^{\rho\sigma}\mathcal{H}^*\mathcal{E} - \xi\widetilde{\mathcal{H}}^{\rho\sigma}\tilde{\mathcal{H}}^*\tilde{\mathcal{E}}\right], \quad (3.67)$$

where four tensor structures emerge in the hadronic tensor, and we define them as

$$\mathcal{H}^{\rho\sigma} \equiv \mathcal{T}_{(2)}^{\mu\rho}\mathcal{T}_{(2)}^{\nu\sigma}\mathcal{G}_{\mu\nu}, \quad \mathcal{H}_+^{\rho\sigma} \equiv \frac{-i}{2}\left(\mathcal{T}_{(2)}^{\mu\rho}\widetilde{\mathcal{T}}_{(2)}^{\nu\sigma} + \mathcal{T}_{(2)}^{\nu\sigma}\widetilde{\mathcal{T}}_{(2)}^{\mu\rho}\right)\mathcal{G}_{\mu\nu}, \\ \widetilde{\mathcal{H}}^{\rho\sigma} \equiv -\widetilde{\mathcal{T}}_{(2)}^{\mu\rho}\widetilde{\mathcal{T}}_{(2)}^{\nu\sigma}\mathcal{G}_{\mu\nu}, \quad \mathcal{H}_-^{\rho\sigma} \equiv \frac{-i}{2}\left(\mathcal{T}_{(2)}^{\mu\rho}\widetilde{\mathcal{T}}_{(2)}^{\nu\sigma} - \mathcal{T}_{(2)}^{\nu\sigma}\widetilde{\mathcal{T}}_{(2)}^{\mu\rho}\right)\mathcal{G}_{\mu\nu}. \quad (3.68)$$

We also define the projection tensor

$$\mathcal{G}^{\mu\nu} \equiv \sum_{\Lambda'=\pm 1} \varepsilon^\mu(q', \Lambda')\varepsilon^{*\nu}(q', \Lambda') = -g^{\mu\nu} + \frac{P^\mu q'^\nu + P^\nu q'^\mu}{P \cdot q'} - \frac{P^2 q'^\mu q'^\nu}{(P \cdot q')^2}, \quad (3.69)$$

resulting from the sum of final photon polarizations. As discussed in the last section, $\mathcal{G}_{\mu\nu}$ reduces to $-g_{\mu\nu}$ only when the Compton tensor is exactly conserved.

The last step is to contract our hadronic part in eq. (3.59) with the leptonic part in eq.

(3.50), and from this we have

$$|\mathcal{T}_{\text{DVCS}}|^2 = \frac{1}{Q^4} \left\{ F_{UU} + (2\Lambda_L)F_{UL} + (2\Lambda_T)(\cos(\phi_S - \phi)F_{UT,\text{in}} + \sin(\phi_S - \phi)F_{UT,\text{out}}) \right. \\ \left. + (2h)[F_{LU} + (2\Lambda_L)F_{LL} + (2\Lambda_T)(\cos(\phi_S - \phi)F_{LT,\text{in}} + \sin(\phi_S - \phi)F_{LT,\text{out}})] \right\}, \quad (3.70)$$

where we define

$$\begin{aligned} F_{UU} &\equiv L_{\rho\sigma}^{\text{DVCS,U}} H_{\text{DVCS,U}}^{\rho\sigma}, & F_{LU} &\equiv iL_{\rho\sigma}^{\text{DVCS,L}} H_{\text{DVCS,U}}^{\rho\sigma}, \\ F_{UL} &\equiv L_{\rho\sigma}^{\text{DVCS,U}} H_{\text{DVCS,L}}^{\rho\sigma}, & F_{LL} &\equiv iL_{\rho\sigma}^{\text{DVCS,L}} H_{\text{DVCS,L}}^{\rho\sigma}, \\ F_{UT,\text{in}} &\equiv L_{\rho\sigma}^{\text{DVCS,U}} H_{\text{DVCS,T,in}}^{\rho\sigma}, & F_{LT,\text{in}} &\equiv iL_{\rho\sigma}^{\text{DVCS,L}} H_{\text{DVCS,T,in}}^{\rho\sigma}, \\ F_{UT,\text{out}} &\equiv L_{\rho\sigma}^{\text{DVCS,U}} H_{\text{DVCS,T,out}}^{\rho\sigma}, & F_{LT,\text{out}} &\equiv iL_{\rho\sigma}^{\text{DVCS,L}} H_{\text{DVCS,T,out}}^{\rho\sigma}, \end{aligned} \quad (3.71)$$

analogous to those in ref. [133]. The two structure functions F_{LU} and $F_{LT,\text{out}}$ vanish with leading twist CFFs, and we list them here for completeness. Since those hadronic tensor $H_{\text{DVCS}}^{\rho\sigma}$ only contain the tensor structures defined in eq. (3.68), it is only those tensor structures contracted with the leptonic tensor that will enter the cross-section. Thus, we define the following four scalar coefficients for the pure DVCS cross-section

$$\begin{aligned} h^{\text{U}} &\equiv L_{\rho\sigma}^{\text{DVCS,U}} \mathcal{H}^{\rho\sigma}, & h^{+, \text{U}} &\equiv L_{\rho\sigma}^{\text{DVCS,U}} \mathcal{H}_+^{\rho\sigma}, \\ \tilde{h}^{\text{U}} &\equiv L_{\rho\sigma}^{\text{DVCS,U}} \widetilde{\mathcal{H}}^{\rho\sigma}, & h^{-, \text{L}} &\equiv L_{\rho\sigma}^{\text{DVCS,L}} \mathcal{H}_-^{\rho\sigma}. \end{aligned} \quad (3.72)$$

Since the four tensor structures are either symmetric/antisymmetric in their ρ, σ indices, they only couple to the unpolarized/polarized leptonic tensor respectively, whereas coefficients such as h^{L} or $\tilde{h}^{\text{L}}$ vanish. Therefore, only four independent coefficients can be defined. Then we can express

the structure functions in terms of them, which are given in Appendix. D.

In the leading twist approximation, it can be easily checked that our F_{UU} and F_{LL} agrees with the unpolarized and double spin process result in ref. [102]. Besides, our F_{UU} , F_{LL} , $F_{LT,\text{in}}$ and $F_{UT,\text{out}}$ agree with the $\mathcal{C}_{\text{unp}}^{\text{DVCS}}$, $\mathcal{C}_{\text{LP}}^{\text{DVCS}}$, $\mathcal{C}_{\text{TP},+}^{\text{DVCS}}$, $\mathcal{C}_{\text{TP},-}^{\text{DVCS}}$ in refs. [105, 136]. In addition, they correspond to the $F_{UU,T}$, F_{LL} , $F_{LT,T}^{\cos(\phi-\phi_S)}$, $F_{UT,T}^{\sin(\phi-\phi_S)}$ terms in ref. [133] respectively. Therefore, our polarized cross-section reduces to the results in ref. [102, 105, 133, 136] at leading twist.

3.2.3 Interference cross-section with polarizations

The interference cross-section can be carried out similarly. Again, we write the interference squared amplitude as a product of their leptonic and hadronic parts as

$$\mathcal{I} = -\frac{e_l}{Q^2 t} L_{\text{INT}}^{\mu\rho\sigma} H_{\mu\rho\sigma}^{\text{INT}} + \text{c.c.} . \quad (3.73)$$

where we have,

$$L_{\text{INT}}^{\mu\rho\sigma} \equiv \sum_{h'} \bar{u}(k, h) \gamma^\rho u(k', h') l_{\text{BH}}^{\mu\sigma} , \quad (3.74)$$

$$H_{\text{INT}}^{\mu\rho\sigma} \equiv \sum_{S', \Lambda'} T^{*\nu\rho} \varepsilon_\nu(q', \Lambda') \varepsilon_\mu^*(q', \Lambda') \bar{u}(P', S') \left[(F_1 + F_2) \gamma^\sigma - \frac{\bar{P}^\sigma}{M} F_2 \right] u(P, S) . \quad (3.75)$$

Then we split the leptonic term into a polarized and unpolarized part as,

$$L_{\mu\rho\sigma}^{\text{INT}} = L_{\mu\rho\sigma}^{\text{INT,U}} + i2h L_{\mu\rho\sigma}^{\text{INT,L}} , \quad (3.76)$$

whose explicit expressions are given in Appendix E. The hadronic tensor can be written as,

$$\begin{aligned}
H_{\text{INT}}^{\mu\rho\sigma} = & \left\{ 4\bar{P}^\sigma \left(F_1 \mathcal{H}^* - \frac{t}{4M^2} F_2 \mathcal{E}^* \right) + [(tn^\sigma + 2\xi\Delta^\sigma) + 2iM\epsilon^{\sigma n\Delta S}] (F_1 + F_2)(\mathcal{H}^* + \mathcal{E}^*) \right. \\
& - \frac{2i}{M} \epsilon^{\sigma\bar{P}\Delta S} (F_1 + F_2) \mathcal{E}^* + \frac{2i}{M} \bar{P}^\sigma \epsilon^{n\bar{P}\Delta S} F_2 (\mathcal{H}^* + \mathcal{E}^*) \left. \right\} \mathcal{G}_\nu^\mu \mathcal{T}_{(2)}^{\nu\rho} \\
& + \left\{ \left[2i\epsilon^{\sigma n\bar{P}\Delta} + 2M \left(n^\sigma + \frac{2\xi}{t} \Delta^\sigma \right) (\Delta \cdot S) \right] (F_1 + F_2) \tilde{\mathcal{H}}^* \right. \\
& + 4M \left(S^\sigma - \frac{(\Delta \cdot S)\Delta^\sigma}{t} \right) \xi (F_1 + F_2) \left(\tilde{\mathcal{H}}^* + \frac{t}{4M^2} \tilde{\mathcal{E}}^* \right) \\
& - 4M \bar{P}^\sigma (n \cdot S) \left(F_1 + \frac{t}{4M^2} F_2 \right) \tilde{\mathcal{H}}^* \\
& \left. + \frac{2}{M} \bar{P}^\sigma (\Delta \cdot S) \left[\xi F_1 \tilde{\mathcal{E}}^* - (1 + \xi) F_2 \tilde{\mathcal{H}}^* \right] \right\} \mathcal{G}_\nu^\mu \widetilde{\mathcal{T}}_{(2)}^{\nu\rho} .
\end{aligned} \tag{3.77}$$

Terms proportional to Δ^σ can be dropped since $\Delta^\sigma L_{\mu\rho\sigma}^{\text{INT}} = 0$, though they are kept in the above expression such that $\Delta^\sigma H_{\mu\rho\sigma}^{\text{INT}} = 0$ is manifest. Then following the same polarization decomposition in eq. (3.58), we can write the hadronic tensor in terms of four different terms as

$$H_{\text{INT}}^{\mu\rho\sigma} = H_{\text{INT,U}}^{\mu\rho\sigma} + (2\Lambda_L) H_{\text{INT,L}}^{\mu\rho\sigma} + 2\Lambda_T \left[H_{\text{INT,T,in}}^{\mu\rho\sigma} \cos(\phi_S - \phi) + H_{\text{INT,T,out}}^{\mu\rho\sigma} \sin(\phi_S - \phi) \right] , \tag{3.78}$$

where we define similarly

$$H_{\text{INT,U}}^{\mu\rho\sigma} \equiv \frac{1}{2} \left[H_{\text{INT}}^{\mu\rho\sigma}(S) + H_{\text{INT}}^{\mu\rho\sigma}(-S) \right], \quad (3.79)$$

$$H_{\text{INT,L}}^{\mu\rho\sigma} \equiv \frac{1}{2} \left[H_{\text{INT}}^{\mu\rho\sigma}(S_{\text{L}}) - H_{\text{INT}}^{\mu\rho\sigma}(-S_{\text{L}}) \right], \quad (3.80)$$

$$H_{\text{INT,T,in}}^{\rho\sigma} \equiv \frac{1}{2} \left[H_{\text{INT}}^{\mu\rho\sigma}(S_{\text{T,in}}) - H_{\text{INT}}^{\mu\rho\sigma}(-S_{\text{T,in}}) \right], \quad (3.81)$$

$$H_{\text{INT,T,out}}^{\rho\sigma} \equiv \frac{1}{2} \left[H_{\text{INT}}^{\mu\rho\sigma}(S_{\text{T,out}}) - H_{\text{INT}}^{\mu\rho\sigma}(-S_{\text{T,out}}) \right]. \quad (3.82)$$

The four hadronic tensor can be solved by inserting our S , and we have for $H_{\text{INT,U}}^{\mu\rho\sigma}$,

$$\begin{aligned} H_{\text{INT,U}}^{\mu\rho\sigma} = & \left[4\bar{P}^\sigma \left(F_1 \mathcal{H}^* - \frac{t}{4M^2} F_2 \mathcal{E}^* \right) + tn^\sigma (F_1 + F_2) (\mathcal{H}^* + \mathcal{E}^*) \right] \mathcal{G}_\nu^\mu \mathcal{T}_{(2)}^{\nu\rho} \\ & + 2i\epsilon^{\sigma n \bar{P} \Delta} (F_1 + F_2) \tilde{\mathcal{H}}^* \mathcal{G}_\nu^\mu \tilde{\mathcal{T}}_{(2)}^{\nu\rho}, \end{aligned} \quad (3.83)$$

and for $H_{\text{INT,L}}^{\mu\rho\sigma}$,

$$\begin{aligned} H_{\text{INT,L}}^{\mu\rho\sigma} = & \left\{ -4\bar{P}^\sigma \left[F_1 \left(\tilde{\mathcal{H}}^* - \frac{\xi^2}{1+\xi} \tilde{\mathcal{E}}^* \right) - F_2 \frac{t}{4M^2} \xi \tilde{\mathcal{E}}^* \right] \right. \\ & \left. - tn^\sigma (F_1 + F_2) \left(\tilde{\mathcal{H}}^* + \frac{\xi}{1+\xi} \tilde{\mathcal{E}}^* \right) \right\} \mathcal{G}_\nu^\mu \tilde{\mathcal{T}}_{(2)}^{\nu\rho} \\ & - 2i\epsilon^{\sigma n \bar{P} \Delta} (F_1 + F_2) \left(\mathcal{H}^* + \frac{\xi}{1+\xi} \mathcal{E}^* \right) \mathcal{G}_\nu^\mu \mathcal{T}_{(2)}^{\nu\rho}, \end{aligned} \quad (3.84)$$

and for $H_{\text{INT,T,in}}^{\mu\rho\sigma}$,

$$\begin{aligned}
H_{\text{INT,T,in}}^{\mu\rho\sigma} = & \frac{1}{N} \left\{ -8\bar{P}^\sigma \left[\xi F_1 \left(\xi \tilde{\mathcal{H}}^* + \left(\frac{\xi^2}{1+\xi} + \frac{t}{4M^2} \right) \tilde{\mathcal{E}}^* \right) + F_2 \frac{t}{4M^2} \left((\xi^2 - 1) \tilde{\mathcal{H}}^* + \xi^2 \tilde{\mathcal{E}}^* \right) \right] \right. \\
& \left. - 2tn^\sigma (F_1 + F_2) \left[\tilde{\mathcal{H}}^* + \left(\frac{t}{4M^2} - \frac{\xi}{1+\xi} \right) \xi \tilde{\mathcal{E}}^* \right] \right\} \mathcal{G}_\nu^\mu \widetilde{\mathcal{T}}_{(2)}^{\nu\rho} \\
& + \frac{4i}{N} \epsilon^{\sigma n \bar{P} \Delta} (F_1 + F_2) \left[\xi \mathcal{H}^* + \left(\frac{\xi^2}{1+\xi} + \frac{t}{4M^2} \right) \mathcal{E}^* \right] \mathcal{G}_\nu^\mu \mathcal{T}_{(2)}^{\nu\rho},
\end{aligned} \tag{3.85}$$

and for $H_{\text{INT,T,out}}^{\mu\rho\sigma}$,

$$\begin{aligned}
H_{\text{INT,T,out}}^{\mu\rho\sigma} = & \frac{1}{N} \left\{ -8i\bar{P}^\sigma \left[F_1 \left(\xi^2 \mathcal{H}^* + \left(\xi^2 + \frac{t}{4M^2} \right) \mathcal{E}^* \right) + \frac{t}{4M^2} F_2 \left((\xi^2 - 1) \mathcal{H}^* + \xi^2 \mathcal{E}^* \right) \right] \right. \\
& \left. - 2itn^\sigma (F_1 + F_2) \left(\mathcal{H}^* + \frac{t}{4M^2} \mathcal{E}^* \right) \right\} \mathcal{G}_\nu^\mu \mathcal{T}_{(2)}^{\nu\rho} \\
& - \frac{4}{N} \epsilon^{\sigma n \bar{P} \Delta} \xi (F_1 + F_2) \left(\tilde{\mathcal{H}}^* + \frac{t}{4M^2} \tilde{\mathcal{E}}^* \right) \mathcal{G}_\nu^\mu \widetilde{\mathcal{T}}_{(2)}^{\nu\rho},
\end{aligned} \tag{3.86}$$

respectively. Again, the last step is to contract the leptonic and hadronic part to get the squared amplitude. We have the same eight polarization configurations as what we get for the DVCS in eq. (3.70), which are defined as,

$$\begin{aligned}
\mathcal{I} = & \frac{-e_l}{Q^2 t} \left\{ F_{UU}^I + 2\Lambda_L F_{UL}^I + 2\Lambda_T \left(F_{UT,\text{in}}^I \cos(\phi_S - \phi) + F_{UT,\text{out}}^I \sin(\phi_S - \phi) \right) \right. \\
& \left. + 2h \left[F_{LU}^I + 2\Lambda_L F_{LL}^I + 2\Lambda_T \left(F_{LT,\text{in}}^I \cos(\phi_S - \phi) + F_{LT,\text{out}}^I \sin(\phi_S - \phi) \right) \right] \right\},
\end{aligned} \tag{3.87}$$

where those eight polarized cross-sections can be expressed as,

$$\begin{aligned}
F_{UU}^I &\equiv L_{\mu\rho\sigma}^{\text{INT},\text{U}} H_{\text{INT},\text{U}}^{\mu\rho\sigma} + \text{c.c.} , & F_{LU}^I &\equiv iL_{\mu\rho\sigma}^{\text{INT},\text{L}} H_{\text{INT},\text{U}}^{\mu\rho\sigma} + \text{c.c.} \\
F_{UL}^I &\equiv L_{\mu\rho\sigma}^{\text{INT},\text{U}} H_{\text{INT},\text{L}}^{\mu\rho\sigma} + \text{c.c.} , & F_{LL}^I &\equiv iL_{\mu\rho\sigma}^{\text{INT},\text{L}} H_{\text{INT},\text{L}}^{\mu\rho\sigma} + \text{c.c.} , \\
F_{UT,\text{in}}^I &\equiv L_{\mu\rho\sigma}^{\text{INT},\text{U}} H_{\text{INT},\text{T},\text{in}}^{\mu\rho\sigma} + \text{c.c.} , & F_{LT,\text{in}}^I &\equiv iL_{\mu\rho\sigma}^{\text{INT},\text{L}} H_{\text{INT},\text{T},\text{in}}^{\mu\rho\sigma} + \text{c.c.} , \\
F_{UT,\text{out}}^I &\equiv L_{\mu\rho\sigma}^{\text{INT},\text{U}} H_{\text{INT},\text{T},\text{out}}^{\mu\rho\sigma} + \text{c.c.} , & F_{LT,\text{out}}^I &\equiv iL_{\mu\rho\sigma}^{\text{INT},\text{L}} H_{\text{INT},\text{T},\text{out}}^{\mu\rho\sigma} + \text{c.c.} .
\end{aligned} \tag{3.88}$$

analogous to those in ref. [133]. In order to express those cross-sections further, we define 6

coefficients $A^I, B^I, C^I, \tilde{A}^I, \tilde{B}^I$ and $\tilde{C}^I$ which are defined as

$$\begin{aligned}
A^I &\equiv 8\bar{P}^\sigma \mathcal{G}_\nu^\mu \mathcal{T}_{(2)}^{\nu\rho} L_{\mu\rho\sigma}^{\text{INT}} , & \tilde{A}^I &\equiv 8i\bar{P}^\sigma \mathcal{G}_\nu^\mu \widetilde{\mathcal{T}}_{(2)}^{\nu\rho} L_{\mu\rho\sigma}^{\text{INT}} , \\
B^I &\equiv 2tn^\sigma \mathcal{G}_\nu^\mu \mathcal{T}_{(2)}^{\nu\rho} L_{\mu\rho\sigma}^{\text{INT}} , & \tilde{B}^I &\equiv 2itn^\sigma \mathcal{G}_\nu^\mu \widetilde{\mathcal{T}}_{(2)}^{\nu\rho} L_{\mu\rho\sigma}^{\text{INT}} , \\
C^I &\equiv 4i\epsilon^{\sigma n\bar{P}\Delta} \mathcal{G}_\nu^\mu \widetilde{\mathcal{T}}_{(2)}^{\nu\rho} L_{\mu\rho\sigma}^{\text{INT}} , & \tilde{C}^I &\equiv 4\epsilon^{\sigma n\bar{P}\Delta} \mathcal{G}_\nu^\mu \mathcal{T}_{(2)}^{\nu\rho} L_{\mu\rho\sigma}^{\text{INT}} ,
\end{aligned} \tag{3.89}$$

where each of them can be written as the sum of their unpolarized and polarized parts,

$$\mathcal{A}^I \equiv \mathcal{A}^{I,\text{U}} + i2h\mathcal{A}^{I,\text{L}} , \tag{3.90}$$

with $\mathcal{A} = \{A, B, C, \tilde{A}, \tilde{B}, \tilde{C}\}$ such that each $\mathcal{A}^{I,\text{U/L}}$ is given by the same definition in eq. (3.89)

but with L^{INT} replaced by $L^{\text{INT},\text{U}}/L^{\text{INT},\text{L}}$ respectively. The $A^{I,\text{U}}, B^{I,\text{U}}$ and $C^{I,\text{U}}$ are the same as

the coefficients defined in ref. [133]. Then the eight polarized cross-sections can be written with

these coefficients, which are given in Appendix E. In the unpolarized case, our expression of F_{UU}^I

agrees with the F_{UU}^I in ref. [133] and the C_{unp}^I in ref. [105, 136]. More detailed comparisons are

left to the next section.

3.3 Comparison of different cross-section formulas

In the last section, we expressed the square amplitude in terms of combinations of hadronic tensors and leptonic ones. Complete analytical expressions of those terms are cumbersome, and in this section we mainly compare them through twist expansion and numerical evaluation with the help of FEYNALC [137–139]. As we discussed before, the BH contribution will be treated as the background and omitted for the comparison.

3.3.1 Comparisons of unpolarized cross-section

To start with, we compare our pure DVCS cross-section with the BKM10 results [136], whereas the BKM01 [105] results lack all the higher-twist kinematics and do not agree with our formula nor the BKM10 one, so we will not have it for comparison. The four-fold cross-section

$$\frac{d^4\sigma_{\text{DVCS}}^{UU}}{dx_B dQ^2 d|t| d\phi} = \int_0^{2\pi} d\phi_S \frac{d^4\sigma_{\text{DVCS}}^{UU}}{dx_B dQ^2 d|t| d\phi d\phi_S}, \quad (3.91)$$

is of interest, since a transverse polarized target will not be involved in this paper.

In the subsection 3.2.2, we worked out the pure DVCS cross-section formulas and argue that they agree with the BKM10 results to the leading order. Here we compare them beyond the leading order. The twist expansion of those coefficients h^U , $\tilde{h}^U$, $h^{+,U}$ and $h^{-,L}$ which are needed for the pure DVCS cross-section are given in Appendix. D, and it can be checked that in the unpolarized case, our leading term, corresponding to $h_{(2)}^U = \tilde{h}_{(2)}^U$, agrees with the $c_{0,\text{unp}}^{\text{DVCS}}$ term in BKM10 [136] and the subleading term, corresponding to $h_{(3)}^U = \tilde{h}_{(3)}^U$, agrees with the $c_{1,\text{unp}}^{\text{DVCS}}$ term in BKM10 if we put in $\beta = 1/2$ which corresponds to the convention chosen there. Therefore, our

result agrees with the BKM10 result up to twist-four terms. However, we do not have twist-four agreement due to the approximation made in BKM10, for which they use the light-cone vectors n_{BKM} and p_{BKM} that are not light-like and drop the twist-four term in $\mathcal{T}_{(2)}^{\Lambda\Lambda' = ++}$ in the helicity amplitude in eq. (3.30). As a result, we do not have exactly the same hadronic matrix element as the $\mathcal{C}_{\text{unp}}^{\text{DVCS}}$, since their twist-four terms are incomplete, and we will focus on the numerical comparison at higher order.

One comment on the hadronic structures such as $\mathcal{C}_{\text{unp}}^{\text{DVCS}}$ in the BKM10 results is that those structures by definition must be functions of the parton momentum fraction x (which will be convoluted with Wilson coefficients and eventually vanish in CFFs), skewness parameter ξ , momentum transfer t and proton mass M only, whereas the Q^2 dependence only comes in through the evolution of GPD. Therefore, the hadronic part must be expressible in terms of ξ, t and M . However, in the BKM10 results those structures are written with x_B and it can not be completely converted into ξ which make their expression unjustified, as x_B are defined from the virtual photon momentum q through $q \cdot P = Q^2/(2x_B)$ which the hadronic part should not depend on. This results from the effectively light-cone vectors $n_{\text{BKM}}^\mu \propto \bar{q}^\mu$ used in their paper.

In Fig. 3.6, we compare those coefficients h^U and $\tilde{h}^\text{U}$ with the BKM10 results [136], which are almost the same with twist-four agreement and numerically very close to each other as we can see from the plot. To do so, we need to convert the BKM10 results into our form, which are given in eq. (D.7). It turns out that our twist expansion converges reasonably fast at twist four for both the JLab 6 GeV and 12 GeV points, where it converges faster for the 12 GeV which has a larger $Q^2 = 4.55 \text{ GeV}^2$. Besides, the all-order results and the twist-four results agree well with the BKM10 result [136], when $\alpha = \beta = 1/2$, corresponding to the choice of light-cone vectors made there.

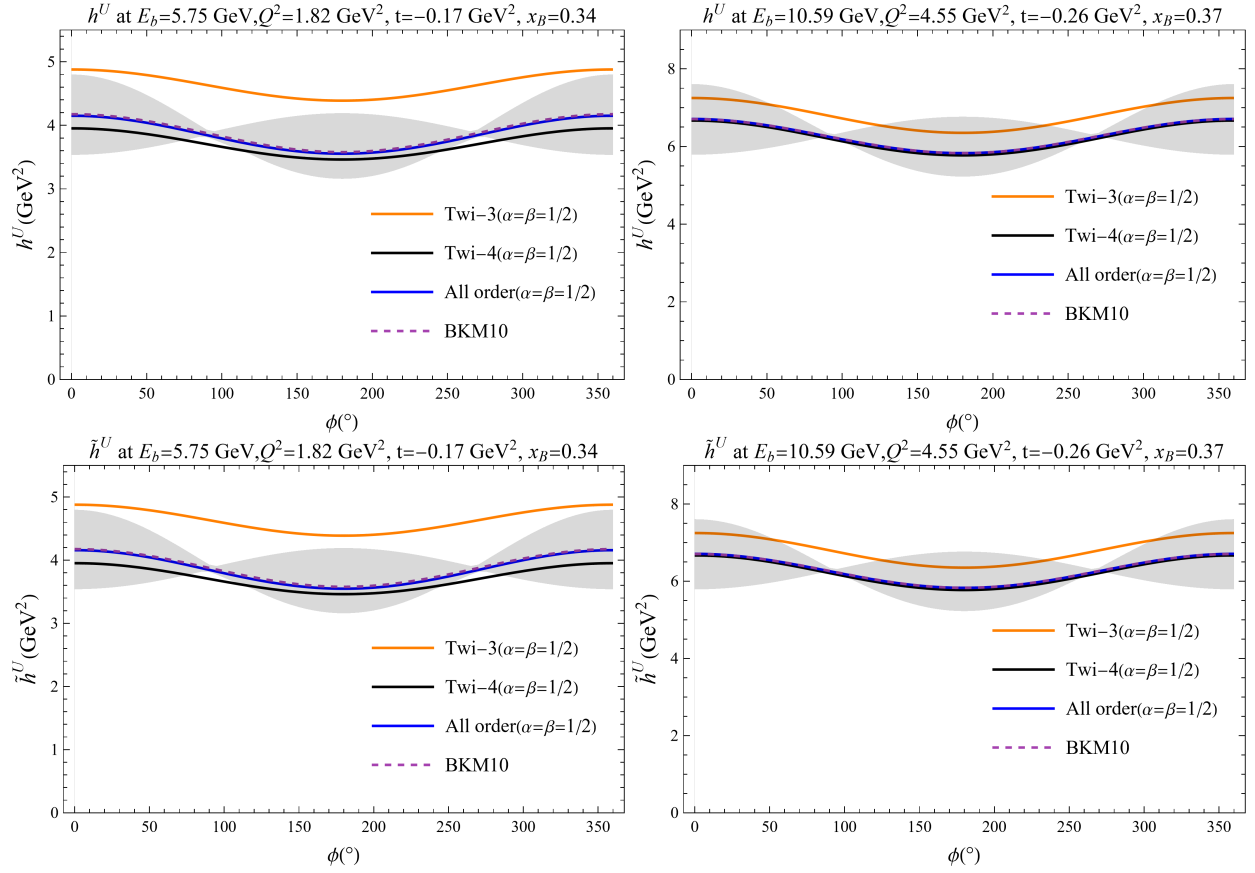


Figure 3.6: Comparisons of the coefficients h^U and $\tilde{h}^U$ from light-cone calculation with the BKM10 result. The orange line stands for the results with only twist-three kinematical accuracy, the black line has twist-four kinematical accuracy, while the blue line contains kinematics of all order. The dashed line is the BKM10 result. The gray band consists of all possible values of those coefficients with $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 2$.

With those coefficients, we can move on to compare the cross-section. In order to perform the comparison of the cross-section, we need input for the CFFs. Here we used the UVa CFFs listed in ref. [140] which are shown in Tab. 3.1. With those CFFs, we can calculate and compare all the different DVCS cross-section formulas, which are plotted in Fig. 3.7. The light-cone result with $\alpha = \beta = 1/2$ has the same twist-three behavior as the BKM10, while the light-cone result at $\alpha = \beta = 1$, corresponding to the UVa [133] formula, will have different twist-three behavior due to its different β , since the cross-section depends on β at twist three as indicated by eq. (D.13).

CFF	x_{Bj}	$ t (\text{GeV}^2)$	$Q^2(\text{GeV}^2)$	$\text{Re}\mathcal{H}$	$\text{Re}\mathcal{E}$	$\text{Re}\tilde{\mathcal{H}}$	$\text{Re}\tilde{\mathcal{E}}$	$\text{Im}\mathcal{H}$	$\text{Im}\mathcal{E}$	$\text{Im}\tilde{\mathcal{H}}$	$\text{Im}\tilde{\mathcal{E}}$
UVa	0.34	0.17	1.82	-0.897	-0.541	2.444	2.207	2.421	0.903	1.131	5.383
UVa	0.37	0.26	4.55	-0.884	-0.424	3.118	2.900	1.851	0.649	0.911	3.915

Table 3.1: CFFs used for numerical comparison of the cross-section calculation, see ref. [140].

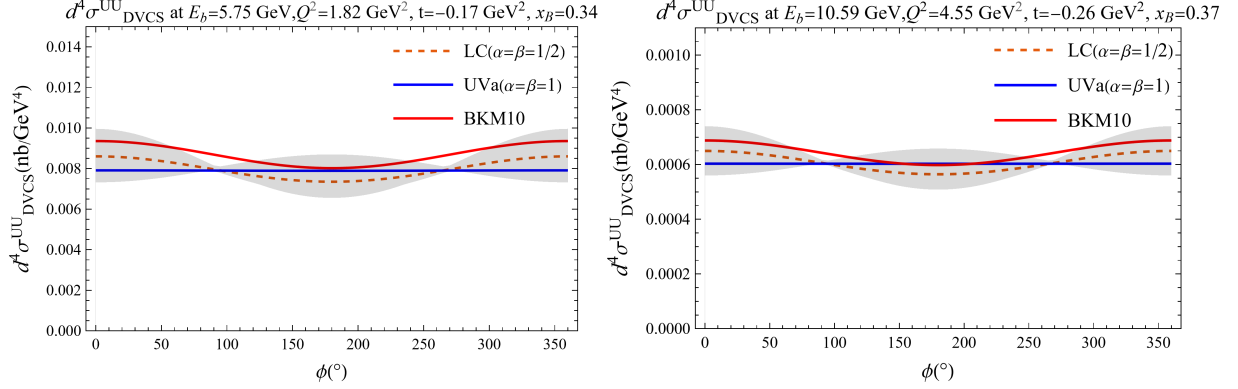


Figure 3.7: Comparison of the unpolarized four-fold pure DVCS cross-section $d^4\sigma_{\text{DVCS}}^{UU}$. The dashed line corresponds to our light-cone calculation with $\alpha = \beta = 1/2$, the blue line corresponds to our reproduced UVa result with $\alpha = \beta = 1$, and the red line is the BKM10 result. Besides, the gray band consists of $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 2$. The three results shown here are reasonably close, while their difference are higher twist effects.

One obvious difference is that the UVa results with $\beta = 1$ actually have no angular dependence, while the BKM10 results with $\beta = 1/2$ DO have. Both features are correctly reproduced using our general set-up. With that said, those results are reasonably close to each other numerically. In Fig. 3.7, we also show a gray band consists of all cross-section predictions with $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 2$. this band serves as an estimation of the intrinsic theoretical uncertainties for the pure DVCS cross-section.

Besides the pure DVCS cross-section, we also compare the four-fold interference cross-section $d^4\sigma_{\text{INT}}^{UU}$ with similar definition as the pure DVCS one with the BKM10 [136] result. Recall that in eq. (3.89), we define 6 coefficients that are independent of the CFFs, we first compare those coefficients.

For unpolarized target, the coefficients contain only the unpolarized part $A^{I,U}$, $B^{I,U}$ and

$C^{I,U}$. The coefficients of BKM10 [136] can be expressed into those coefficients as in Eqs. (E.9) - (E.11). Then similarly to the case of the pure DVCS cross-section, we perform a twist expansion comparison for those coefficients $A^{I,U}$, $B^{I,U}$ and $C^{I,U}$ (see Eqs. (E.35) - (E.37) for the expansion). It can be easily checked that the expansions agree with the BKM10 results when taking $\beta = 1/2$ at twist-three level, but not at twist-four. In Fig. 3.8, we compare both our complete and expanded coefficients $A^{I,U}$, $B^{I,U}$, $C^{I,U}$ with BKM10 ones. As we can see from the plot, the all-order results agree well with the BKM10 result, while the twist-four expansion works fine. It is also evident that for the JLab 12 GeV point with $Q^2 = 4.55\text{GeV}^2$, the twist-expansion converges much faster than the JLab 6 GeV with $Q^2 = 1.82\text{GeV}^2$, due to the larger Q^2 .

Though the all-order light-cone result at $\alpha = \beta = 1/2$ agrees well with the BKM10 result numerically, an exact twist-four agreement is not expected due to the approximation made in BKM10. Given the effective light-cone vectors n_{BKM} and p_{BKM} used in BKM, our light-cone vectors n and p will not reduce to those off-light-cone vectors by setting $\alpha = \beta = 1/2$.

With those coefficients compared, we could move on to the cross-section. In Fig. 3.9, we show the different cross-section prediction with different conventions (α and β) corresponding to the difference choice illustrated in Fig. 3.5. While the BMP convention with $\beta \rightarrow \infty$ differs most from the other conventions, the others are quite close to each other. Again, we vary α from 0 to 1 and β from 0 to 2, and then our predicted cross-sections will form a band as shown in Fig. 3.9. Since the choice of α and β are conventional, those bands serve as an estimation of the theoretical intrinsic uncertainties of the cross-section prediction.

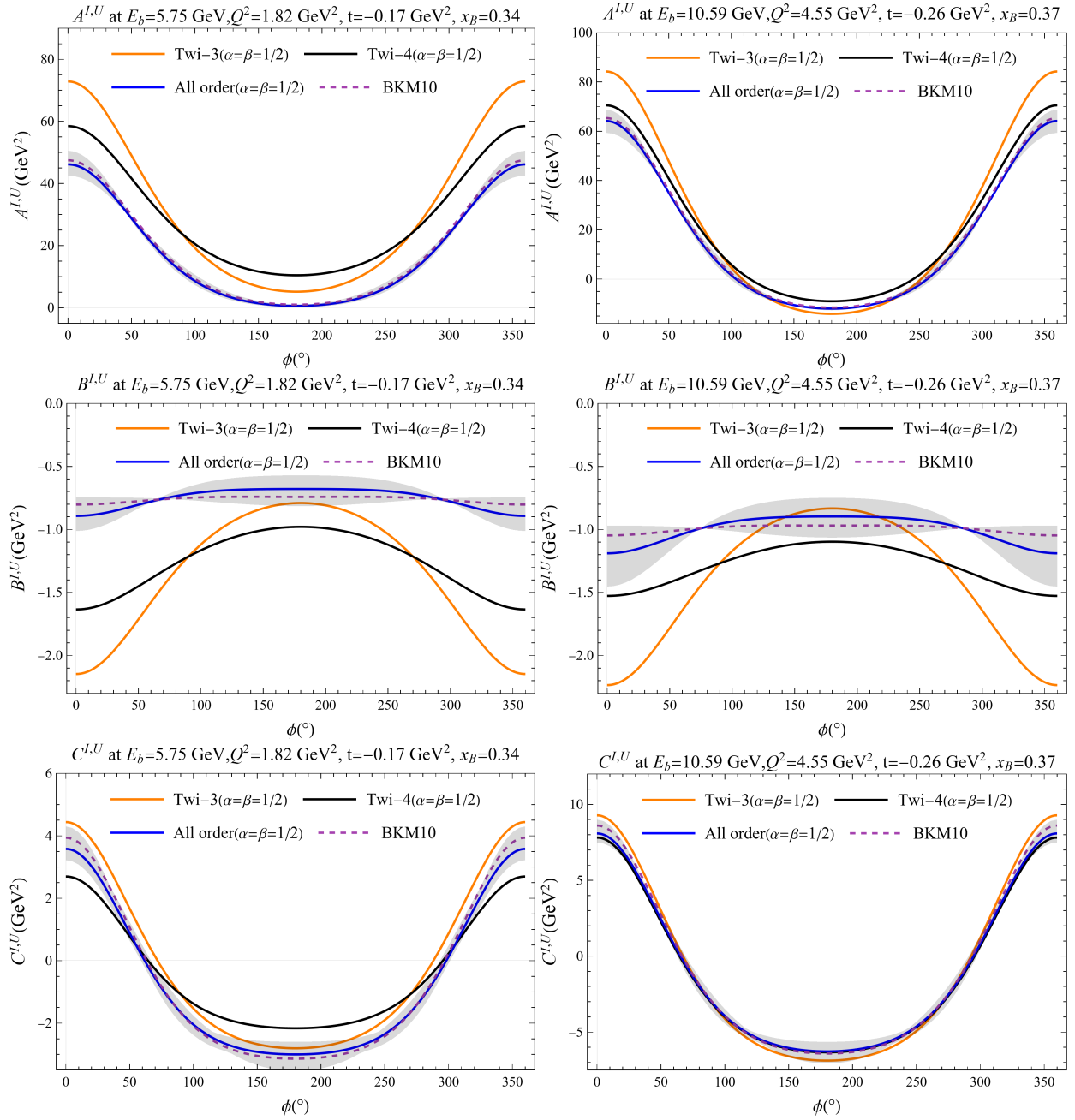


Figure 3.8: Comparison of the coefficients $A^{I,U}$, $B^{I,U}$, $C^{I,U}$ from light-cone calculation with the BKM10 result. The orange line stands for the results with only twist-three kinematical accuracy, the black line has twist-four kinematical accuracy, while the blue line contains kinematics of all order. The dashed line is the BKM10 result. The gray band consists of all possible values of those coefficients with $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 2$.

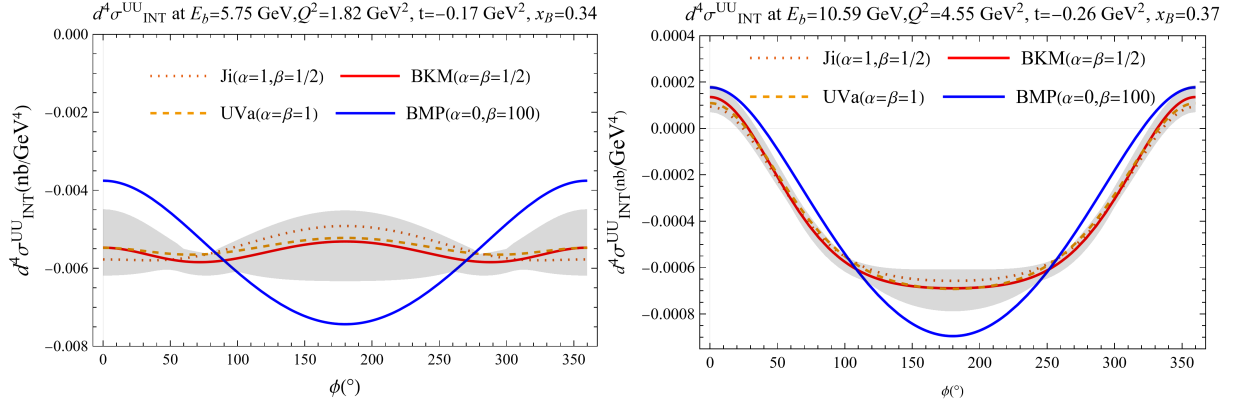


Figure 3.9: Comparison of the four-fold interference cross-section $d^4\sigma_{\text{INT}}^{UU}$ of different choice of α and β . Besides the different lines showing different choice of parameters, the gray band consists of all cross-section results with $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 2$. The UVa results shown here are obtained by taking $\alpha = \beta = 1$ in our set-up and differ from the actual result in ref. [133] with an overall factor of $\cos \phi$.

3.3.2 Comparisons of cross-sections with longitudinally polarized target

In the last subsection, we compared the cross-section in the unpolarized case, and in this subsection we show that our formulas apply to the polarized cross-section which also agrees with the BKM10 results [136]. For simplicity, we consider the cross-section with longitudinally polarized target and unpolarized beam.

The pure DVCS cross-section does not have leading-twist contribution, as we discussed in subsection 3.2.2. Therefore, we focus on comparing the interference cross-section. In eq. (E.3), we show that the cross-section for unpolarized beam and longitudinally polarized target can be expressed in terms of the other three coefficients $\tilde{A}^{I,U}$, $\tilde{B}^{I,U}$ and $\tilde{C}^{I,U}$. In ref. [136], the same hadronic structure shows up, and a comparison of those coefficients would suffice. Again, in order to compare them, we need to convert the BKM result to the form of ours, which are given in Eqs. (E.12) - (E.14). We perform a twist expansion for those three coefficients in Eqs. (E.38) - (E.40) and compare both the expanded formulas and the all-order formulas numerically, as shown

in Fig. 3.10. Note that there is an overall minus sign difference between ours and the BKM10 result which we take away, since the polarization vectors can always be redefined with a minus sign.

As we can see from the plot, our results agree well with the BKM10 result [136] when taking $\alpha = \beta = 1/2$, corresponding to the choice made there. The larger difference in $\tilde{B}^{I,U}$ can be explained for two reasons. For one thing, $\tilde{B}^{I,U}$ does not have a leading twist contribution, so the differences at a lesser-dominant higher twist will be more significant in $\tilde{B}^{I,U}$. Another reason is that the effective light-cone vectors n_{BKM} used in ref. [136] is not light-like, while $\tilde{B}^{I,U}$ is defined as the contraction of n^σ with the leptonic part as shown in eq. (3.89), which depends directly on the light-cone vector. Apart from that, we see similar behavior as the unpolarized case that the twist expansion converges much faster for the JLab 12 GeV point, due to its larger Q^2 . Also, our agreements with the BKM10 results improve as Q^2 gets large, since their difference is a higher twist effect.

With those coefficients and nucleon form factors, the cross-sections can be worked out the same way as the unpolarized case. We compare the polarized cross-section with different choice of parameters as shown in Fig. 3.11. Again, we show that they form a band which estimates the intrinsic theoretical uncertainties of cross-section prediction.

3.3.3 Gauge dependence of the cross-section

In the subsection 3.1.2, we discussed that the cross-section actually depends on the gauge choice due to the breaking of exact current conservation condition of the Compton tensor: $q'_\mu T^{\mu\nu} \sim T^{\mu\nu} q_\nu \sim \mathcal{O}(\Delta_\perp^2)$. Therefore, the cross-section will not be invariant under the gauge transforma-

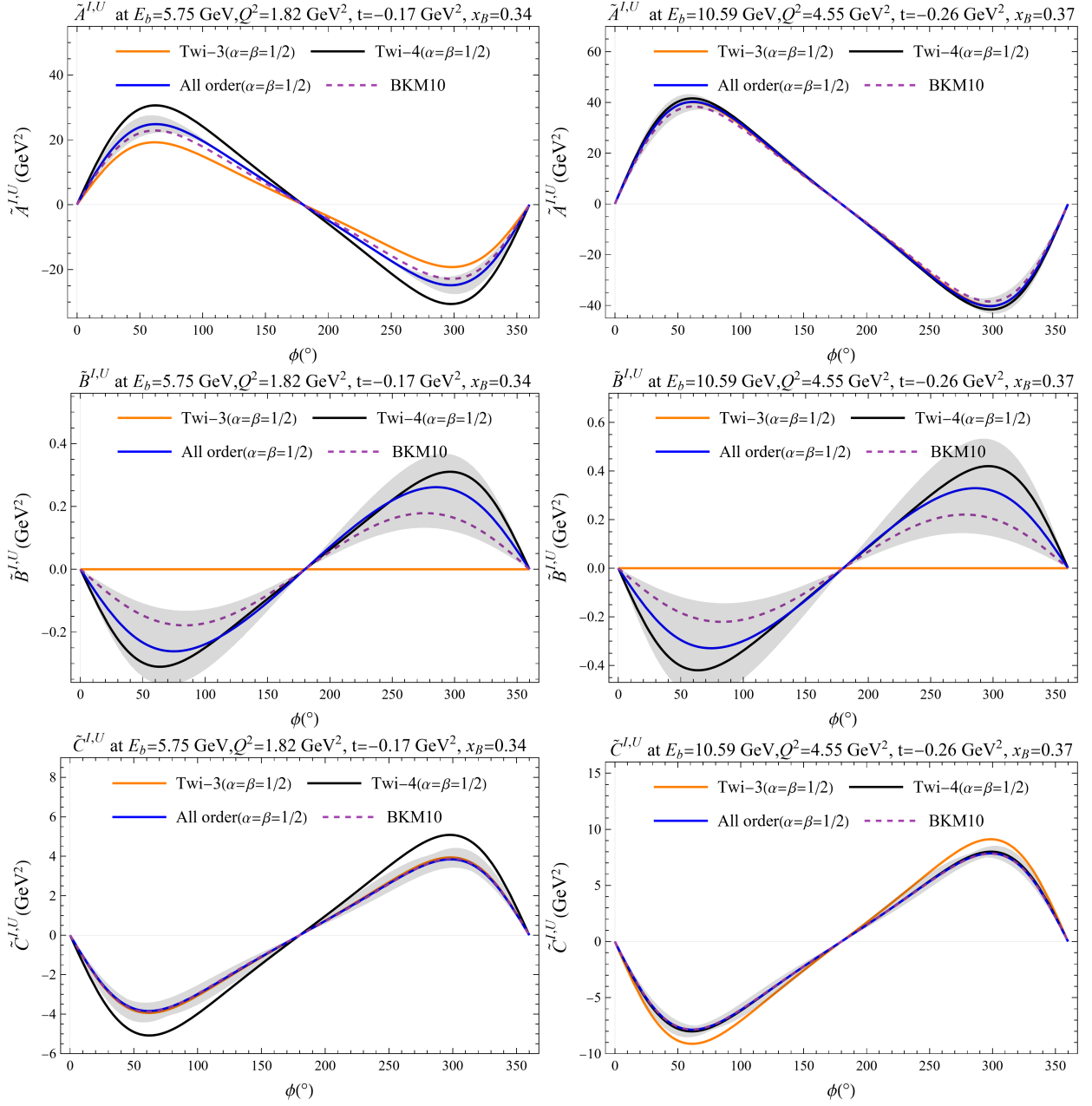


Figure 3.10: Comparisons of the coefficients $\tilde{A}^{I,U}$, $\tilde{B}^{I,U}$, $\tilde{C}^{I,U}$ from light-cone calculation with the BKM10 result. The orange line stands for the results with only twist-three kinematical accuracy, the black line has twist-four kinematical accuracy, while the blue line contains kinematics of all order. The dashed line is the BKM10 result. The gray band consists of all possible values of those coefficients with $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 2$.

tion $\varepsilon^\mu(q', \Lambda') \rightarrow \varepsilon^\mu(q', \Lambda') + \lambda q'^\mu$. In this subsection, we study how the cross-section will vary under the gauge transformation.

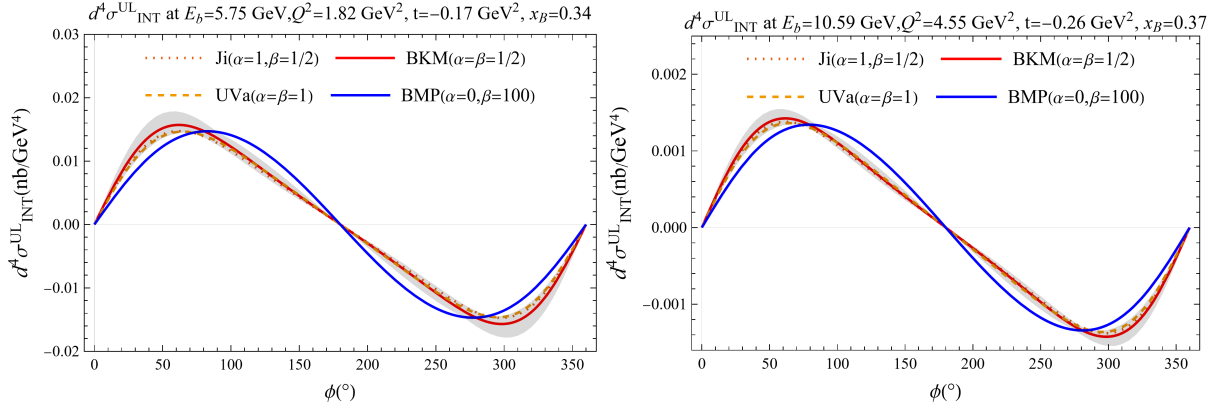


Figure 3.11: Comparisons of the four-fold interference cross-section $d^4\sigma_{\text{INT}}^{UL}$ of different choice of α and β for longitudinally polarized target and unpolarized beam. The gray band consists of all cross-section results with $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 2$. The UVa results shown here are obtained by taking $\alpha = \beta = 1$ in our set-up and might differ from the actual result in ref. [133].

Note that the gauge fixing condition $V \cdot \varepsilon(q') = 0$ actually requires that $V \cdot q' \neq 0$. Otherwise, the condition $V \cdot \varepsilon(q') = 0$ may never be satisfied under the gauge transformation as $V \cdot (\varepsilon(q') + \lambda q') = V \cdot \varepsilon(q')$, leaving a pole at $V \cdot q'$ in the polarization sum in eq. (3.24), which should be avoided when choosing the gauge. For arbitrary gauge choice, the $\mathcal{G}^{\mu\nu}$ will reduce to $-g^{\mu\nu}$ if the current conservation condition is exactly satisfied. Therefore, we define

$$\mathcal{G}_{(0)}^{\mu\nu} = -g^{\mu\nu}, \quad \mathcal{G}_{(1)}^{\mu\nu} = \frac{V^\mu q'^\nu + V^\nu q'^\mu}{V \cdot q'} - \frac{V^2 q'^\mu q'^\nu}{(V \cdot q')^2}, \quad (3.92)$$

such that $\mathcal{G}^{\mu\nu} = \mathcal{G}_{(0)}^{\mu\nu} + \mathcal{G}_{(1)}^{\mu\nu}$. Then the gauge dependence is all contained $\mathcal{G}_{(1)}^{\mu\nu}$ and when the Compton tensor is exactly conserved, it can be set to zero. Since both the pure DVCS and interference cross-sections are linear in $\mathcal{G}^{\mu\nu}$, we can use the ratio

$$r_{\text{DVCS/INT}} = \frac{d\sigma_{\text{DVCS/INT}}(\mathcal{G}_{(1)}^{\mu\nu})}{d\sigma_{\text{DVCS/INT}}(\mathcal{G}^{\mu\nu})}, \quad (3.93)$$

to estimate the dependence of cross-sections on the gauge choices, and the numerical results with

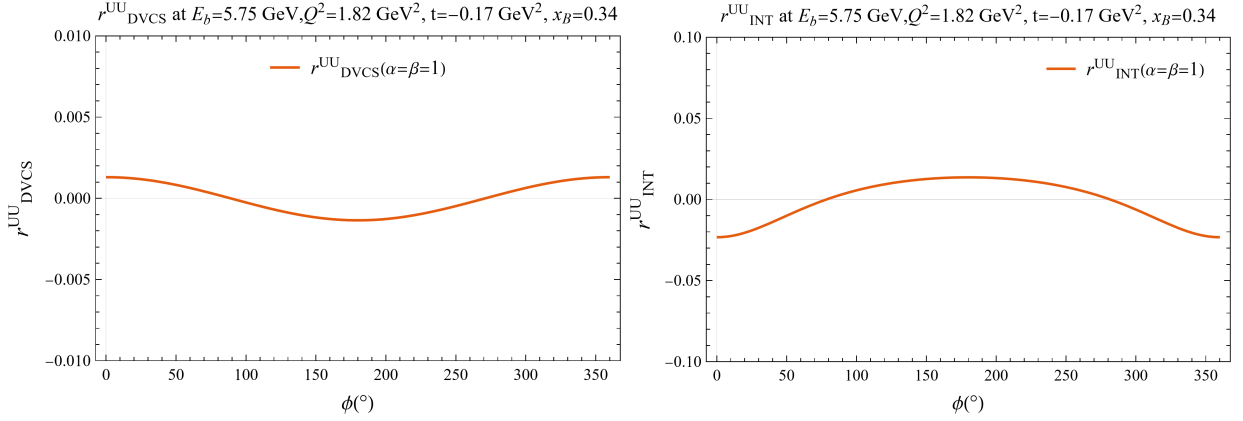


Figure 3.12: The ratio $r_{\text{DVCS/INT}}^{\text{UU}}$ defined above in the unpolarized case, calculated at one given kinematic point and radiation gauge in the lab frame with $V = P$. The gauge dependence is in general not significant, though it is more relevant in the case of interference cross-section.

the radiation gauge in the lab frame, corresponding to $V = P$, are shown in Fig. 3.12.

As we can see from the plot, the gauge dependence is indeed a higher order effect. Even for $Q^2 = 1.82$ GeV², the ratio is already small enough. Therefore, the gauge dependence of the cross-section will not severely affect the results, though they still add to the intrinsic uncertainties in the theoretical prediction of the cross-section. Some numerical studies on the gauge dependence of the cross-section by choosing different V as P , P' and q respectively show that they give almost the same cross-section with difference at the order of 0.1%, and thus the gauge dependence is indeed a less relevant effect than the dependence on the choice of light-cone vectors.

It is also worth noting that the gauge dependence will be more significant if the leading approximation of Compton tensor coefficients $\mathcal{T}_{(2)}^{\mu\nu} \approx -\frac{1}{2}g_{\perp}^{\mu\nu}$, $\widetilde{\mathcal{T}}_{(2)}^{\mu\nu} \approx \frac{i}{2}\epsilon_{\perp}^{\mu\nu}$ is used. The reason is simple - if we have the Compton tensor coefficients to all order, it will be exactly conserved, and the contribution of those gauge-dependent terms vanishes. This also indicates that the gauge dependence shown in Fig. 3.12 can only be eliminated by calculating the Compton tensor coefficients to a higher order, rather than choosing a specific gauge.

3.4 Operator production expansion and covariant coefficients

In previous sections, we show how the cross-section formulas depend on the choice of light-cone vectors in the leading twist GPD approximation, where all the higher twist GPDs are set to be zero. However, the light-cone dependence persists at twist three even when the Compton tensor coefficients are accurate to twist three, indicating the existence of some other kinematical twist-three correction from higher twist GPDs. These relations between GPDs of different twist are the results of Lorentz symmetry and the equations of motion, which are generally known as the Wandzura-Wilczek (WW) relations, stating that the higher-twist GPDs can be split into the WW parts that are expressible in terms of the lower-twist GPDs and the genuine twist-three parts that are related to higher-twist quark-gluon-quark operators [29, 123, 141–143]. In this section, we show that it is necessary to include the kinematical corrections from twist-three GPDs in order for the dependence on light-cone vectors to cancel at twist three.

3.4.1 Covariant Compton tensor coefficients

The best method to study the twist expansion without light-cone vectors is the Operator Production Expansion (OPE), which states that a non-local operator in z^μ can be expanded in terms of a series of local operators when $z^2 \ll 1/\Lambda_{\text{QCD}}^2$

$$\mathcal{J}_1(z)\mathcal{J}_2(0) = \sum_i C_i(z^2)\mathcal{O}_i(0) \quad (3.94)$$

where $\mathcal{O}_i(0)$ are a set of local operators and the corresponding coefficients $C_i(z^2)$ are their Wilson coefficients calculable in perturbative QCD in strong coupling α_s expansion. As the separation of

the non-local operator z approaches light-cone as $z^2 \rightarrow 0$, local operators in the lowest power of z will dominate, which are defined as the leading twist operators. As we discussed before, the twist defined in this case differs from the dynamical twist in light-cone power counting, and is known as the geometric twist, which is defined through $t \equiv d - s$, with d the mass dimension of the operator and s the Lorentz spin. For instance, a rank-2 tensor $O^{\mu\nu}$ can in general be decomposed into symmetric traceless, antisymmetric and traceful parts as

$$O^{\mu\nu} = \left(O^{(\mu\nu)} - \frac{1}{4} g^{\mu\nu} \text{Tr}(O) \right) + O^{[\mu\nu]} + \frac{1}{4} g^{\mu\nu} \text{Tr}(O) \quad (3.95)$$

and each of them will be of spin 2, 1 and 0. Consequently, if the operator has a mass dimension 4, the three parts will be twist-two, twist-three and twist-four respectively. Therefore, equations of a general tensor operator will contain operators of different twist, making it possible to express part of the higher twist operators in terms of the lower twist operators through equations of motion. These relations for the twist-three GPDs in DVCS process has been studied and derived in the literatures [29, 123, 141–143], which can be written as

$$W^{[\gamma^\mu]}(x, \xi, t) = \frac{\Delta^\mu}{-2\xi} n_\nu W^{[\gamma^\nu]}(x, \xi, t) + \dots, \quad (3.96)$$

$$W^{[\gamma^\mu \gamma^5]}(x, \xi, t) = \frac{\Delta^\mu}{-2\xi} n_\nu W^{[\gamma^\nu \gamma^5]}(x, \xi, t) + \dots, \quad (3.97)$$

where a general tensor is projected out with the twist-two part, and the ellipsis stands for twist-two GPDs convoluted with WW kernel and genuine twist-three GPDs. It is worth noting that terms with twist-two GPDs convoluted with WW kernel are in general comparable to the twist-two GPDs themselves, but they are dropped here because they lead to a different set of CFFs which do

not mix with the leading twist CFFs we care about in this paper. Therefore, it is sufficient to study the terms above for our purpose, and a more thorough study of the twist-three CFFs is left to future works. While the above relation leads to the trivial identity $n_\mu W^{[\gamma^\mu]}(x, \xi, t) = n_\nu W^{[\gamma^\nu]}(x, \xi, t)$ if we project out the leading twist contribution, it also involves a twist-three part in the form of $W^{[\gamma_\perp^\mu]}(x, \xi, t) = \Delta_\perp^\mu / (-2\xi) n_\nu W^{[\gamma^\nu]}(x, \xi, t) + \dots$. This kinematical twist-three term will modify our Compton tensor coefficients and cancel the light-cone dependence at twist three, as we will see.

In order to get the modified Compton tensor coefficients, we need to redo the calculation with the above substitution of the leading twist GPDs. The result is surprisingly simple and leads to a modified Compton tensor coefficients as (compared with eq. 3.18),

$$\begin{aligned}\mathcal{T}_{(2)}^{\mu\nu} &= -\frac{1}{2} \left[g^{\mu\nu} - \frac{n^\mu \Delta^\nu + n^\nu \Delta^\mu}{n \cdot \Delta} \right], \\ \widetilde{\mathcal{T}}_{(2)}^{\mu\nu} &= -\frac{i}{2(n \cdot \Delta)} \epsilon^{\mu\nu m \Delta}.\end{aligned}\tag{3.98}$$

The above expressions reduce to $-\frac{1}{2}g_\perp^{\mu\nu}$ and $\frac{i}{2}\epsilon_\perp^{\mu\nu}$ respectively if we choose the light-cone vectors such that $\Delta_\perp^\mu = 0$, which corresponds to the BMP choice [132] that $\beta \rightarrow \infty$. This indicates the kinematical correction to the leading twist CFFs vanishes at twist-three if we choose the light-cone vectors such that $\Delta_\perp^\mu = 0$. On the other hand, the above Compton tensor coefficients can also be promoted with the physical vector q, q' up to twist-four terms as,

$$\begin{aligned}\mathcal{T}_{(2),C}^{\mu\nu} &= -\frac{1}{2} \left[g^{\mu\nu} - \frac{q^\mu q'^\nu + q^\nu q'^\mu}{q \cdot q'} + \frac{q'^\mu q'^\nu q^2}{(q \cdot q')} \right], \\ \widetilde{\mathcal{T}}_{(2),C}^{\mu\nu} &= \frac{i}{2(q \cdot q')} \epsilon^{\mu\nu q q'},\end{aligned}\tag{3.99}$$

which are given in ref. [132] as well. These promoted Compton tensor coefficients are covariant

and apply to a general choice of light-cone vectors. Besides, they are also manifestly gauge-invariant, which frees the scattering amplitude from any specific gauge fixing conditions.

3.4.2 Comparison of the cross-section with covariant coefficients

Now we are in a position to go back to the cross-section formulas, and compare different formulas with the kinematical corrections above. Our results above indicates two ways to calculate the cross-section: either to take $\beta \rightarrow \infty$ with the light-cone Compton tensor coefficients in eq. (3.18) such that all kinematical twist-three corrections vanish, or to use the covariant Compton tensor coefficients defined in eq. (3.99). While the former approach provides a simple connection to our previous results, it is restrained to one specific choice of light-cone vectors and will not be general enough to study the light-cone dependence of the cross-section formulas. Therefore, we focus on the latter approach, for which we use the covariant Compton tensor coefficients and study the remnant light-cone dependence in the cross-section formulas.

Recall that in previous sections, we defined 4 scalar coefficients $h^U, \tilde{h}^U, h^{-,L}$ and $h^{+,U}$ for DVCS amplitude and 6 complex (or 12 real) scalar coefficients $A^I, B^I, C^I, \tilde{A}^I, \tilde{B}^I$ and $\tilde{C}^I$. A comparison of those coefficients would suffice in order to compare different formulas. For the pure DVCS cross-section, it turns out that with the covariant Compton tensor coefficients, those coefficients have a compact form that is independent of the light-cone vectors, for which we have

$$\begin{aligned}
h^U &= \tilde{h}^U = \frac{2(k \cdot q)(k \cdot q')}{q \cdot q'} - \frac{q^2(k \cdot q')^2}{(q \cdot q')^2} - k \cdot q, \\
h^{-,L} &= \frac{-q^2(k \cdot q')}{q \cdot q'} + k \cdot q, \\
h^{+,U} &= 0.
\end{aligned} \tag{3.100}$$

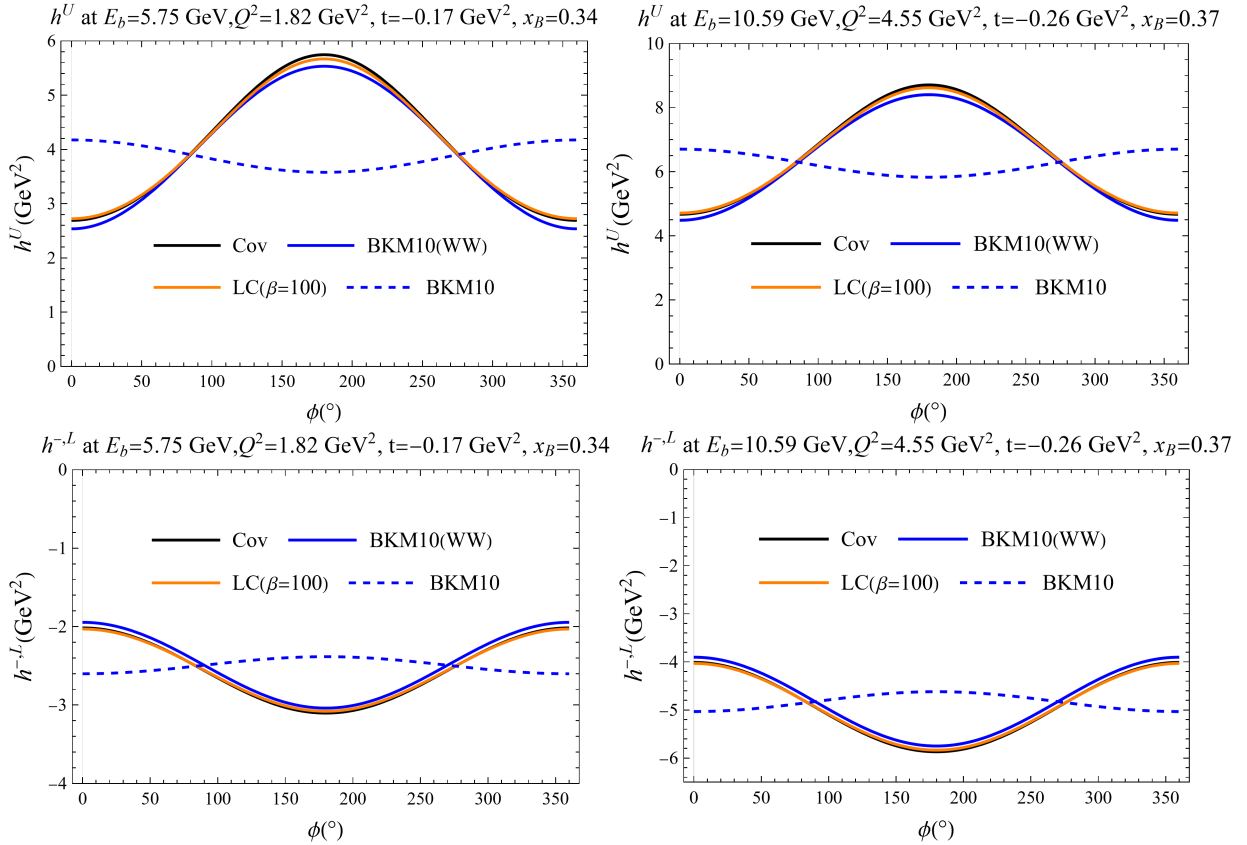


Figure 3.13: Comparisons of the coefficients h^U and $h^{-,L}$ over different formulas. In the plots, we can see that the covariant coefficients above agrees well with the BKM10 results with WW relations and also our light-cone calculation with $\beta \rightarrow \infty$, while they differ a lot from the result without WW relation. Note that in the $\beta \rightarrow \infty$ limit, the value α is actually degenerate and irrelevant.

Using those coefficients, we compare different pure DVCS cross-section formulas numerically, as shown in Fig. 3.13. Apparently, they confirm our observation that the general covariant coefficients agree with the light-cone results with $\beta \rightarrow \infty$ as well as the BKM10 results with the WW relation, whereas their differences to BKM10 results without WW relation which are twist-three effects are shown to be quite significant. Note that in the $\beta \rightarrow \infty$ limit, the value α is actually degenerate and irrelevant. Therefore, we suppress the choice of α in the limit $\beta \rightarrow \infty$ hereafter.

The comparison of the interference cross-section formulas can be done similarly, except

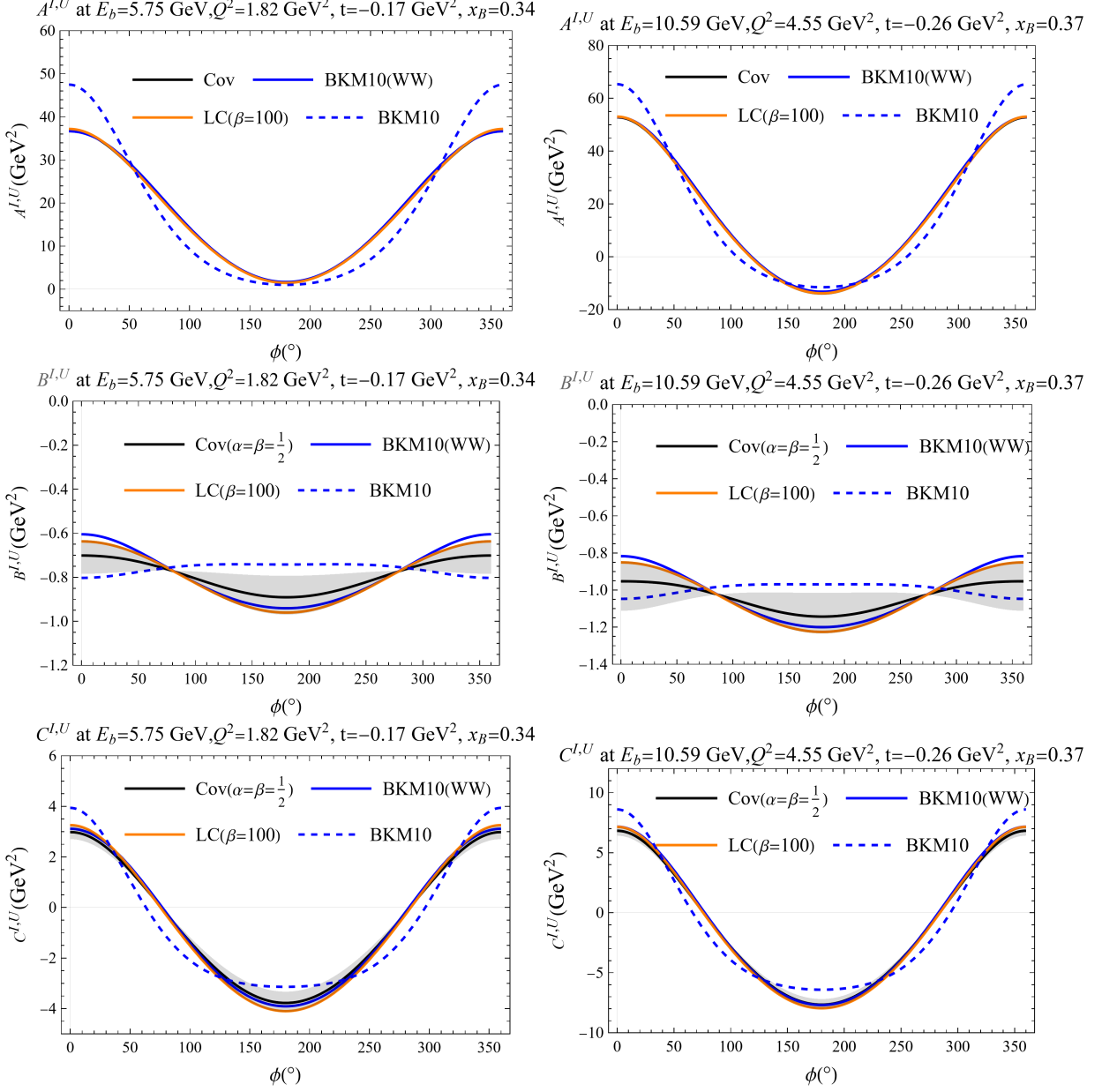


Figure 3.14: Comparisons of the coefficients $A^{I,U}$, $B^{I,U}$ and $C^{I,U}$ over different formulas. The coefficients $B^{I,U}$ and $C^{I,U}$ depend on the light-cone vectors explicitly, and thus we vary the α and β from $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 100$ to get the gray band in the plot. The blue lines in the plot agree well with each other.

that we have more coefficients in the case of interference. The expressions of those scalar coefficients are collected in Appendix E, and here we focus on comparing them numerically.

As shown in Fig. 3.14, the covariant coefficients agree well with the BKM10 formula with

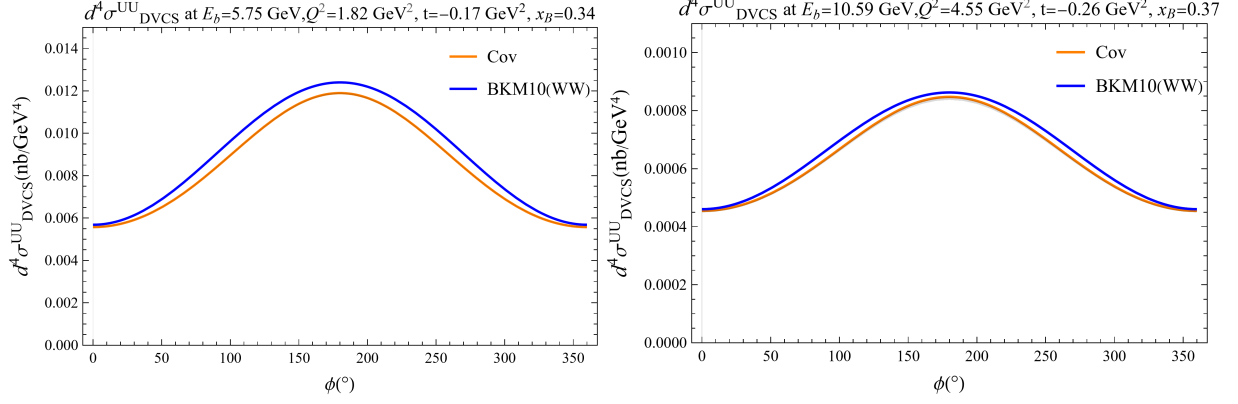


Figure 3.15: Comparisons of the unpolarized four-fold pure DVCS cross-section $d^4\sigma_{\text{DVCS}}^{UU}$. Our pure DVCS cross-sections almost have no light-cone dependence, while they differ a bit from the BKM10 results with WW relations.

WW relation and they both agree well with the previous light-cone result at $\beta \rightarrow \infty$, confirming that the kinematical twist-three corrections vanish when we choose the light-cone vectors such that $\Delta_{\perp} = 0$. However, the coefficients B^I and C^I still depend on the light-cone vectors, which is shown in the plot as the gray band for which we vary the α and β from $0 \leq \alpha \leq 1$ and $0 \leq \beta \leq 100$. We emphasize that after using the covariant Compton tensor coefficients, the remnant light-cone dependence is twist-four suppressed, so it will be less significant at large Q^2 , as indicated by the plots on the right of Fig. 3.14 which has $Q^2 = 4.55\text{GeV}^2$, much larger than the left plots with $Q^2 = 1.82\text{GeV}^2$.

3.4.3 Light-cone dependence of cross-sections

With those scalar coefficients compared, we can now move on to compare the cross-section formulas with our covariant coefficients and find out how the cross-section depends on the choice of light-cone vectors. In Fig. 3.15, we compare the pure DVCS cross-section numerically. It turns out that our covariant results for the pure DVCS cross-sections has almost no light-cone dependence. We get almost identical results by varying α and β in the region $0 \leq \alpha \leq 1, 0 \leq$

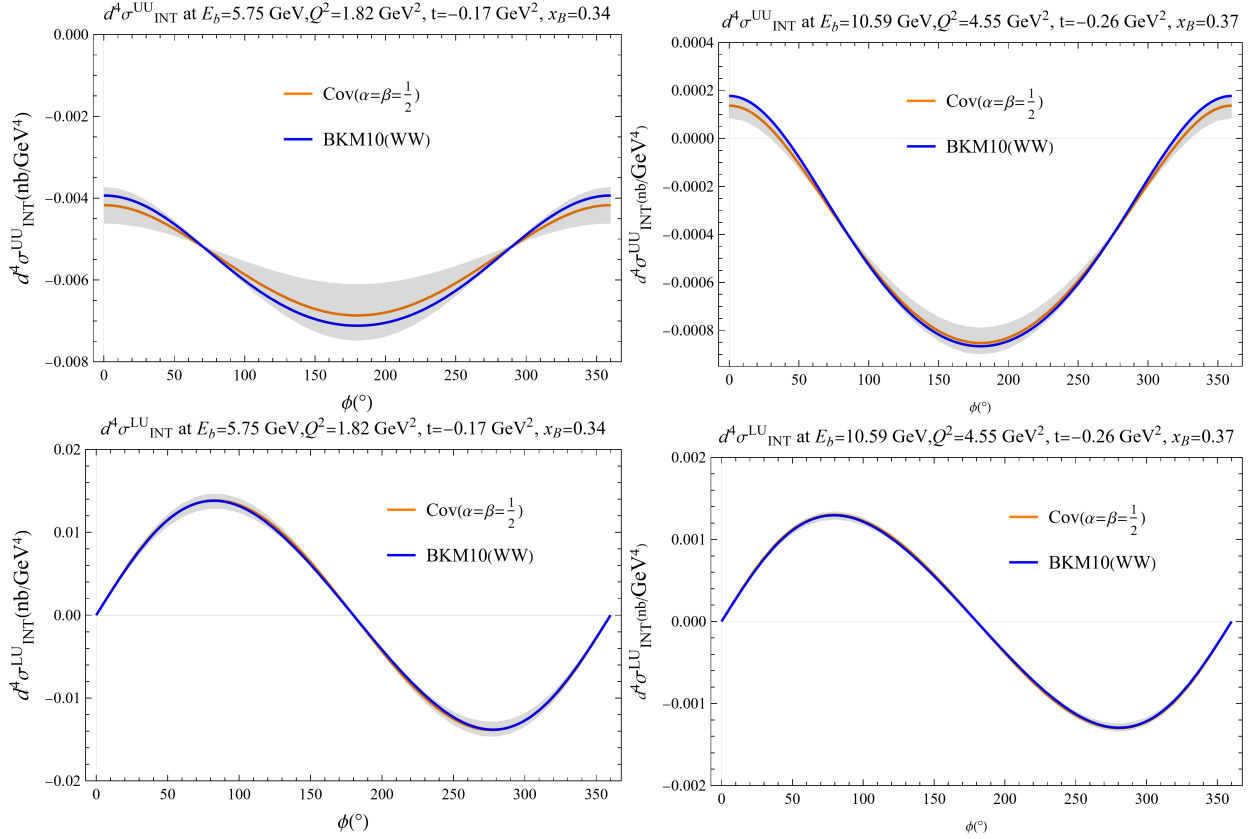


Figure 3.16: Comparisons of the four-fold interference cross-section $d^4\sigma_{\text{INT}}^{UU}$ and $d^4\sigma_{\text{INT}}^{LU}$. Our interference cross-sections agree well with the BKM10 results with WW relations. Different from the case of pure DVCS cross-sections, the interference cross-sections still show dependence on the choice of light-cone vector, which is indicated by the gray band for which we vary α and β in the region $0 \leq \alpha \leq 1, 0 \leq \beta \leq 100$.

$\beta \leq 100$. Therefore, we suppress the α and β when presenting the pure DVCS cross-sections.

On the other hand, our results still differ from the BKM10 results with a twist-four effect. As we discussed in the previous section, the main reason is that BKM10 used the effective light-cone vectors that are not on the light-cone, leading to the differences at twist-four level.

Besides, we also compare the interference cross-sections for both unpolarized and longitudinally polarized beam and unpolarized target as shown in Fig. 3.16. Due to the explicit light-cone dependence in the coefficients $B^{I,U}$ and $C^{I,U}$ (also $\tilde{B}^{I,U}$ and $\tilde{C}^{I,U}$ in the case of a longitudinally polarized beam), the interference cross-sections still show some dependence on the

choice of light-cone vectors. Again, we emphasize that after using the covariant Compton tensor coefficients, the light-cone dependence vanishes at twist three, so the light-cone dependence indicated by the gray bands are twist-four effects.

3.5 Comments on different cross-section formulas

In the end, we comment on the major cross-section formulas in the literature. We denote one of the most popular formulas as the Belitsky-Kirchner-Müller (BKM01) one [105], which receives higher-order kinematical corrections [136] (BKM10) and then gets refreshed in the Belitsky-Müller-Ji (BMJ) formulas [134] to include the transversely polarized target. The latest version of this series used in the fitting of the recent JLab Hall A DVCS measurement [144] is the Braun-Manashov-Müller-Pirnay (BMMP) formulas [132], where kinematical twist-four effects such as target-mass and finite- t corrections are considered. Another recent work in the literature, denoted UVa, is given in ref. [133] which utilizes helicity amplitudes to separate out the different contributions to the cross-section at both leading and sub-leading twist.

Several differences between our formulas and the BKM01, BKM10, BMJ and BMMP formulas should be noted here. In both the BKM01 and BKM10 formulas, the twist-three parts are given in terms of the so-called effective CFFs which are mixtures of twist-two and twist-three CFFs. In the later BMJ formulas [134], the mixing of CFFs gets more complicated due to the introduction of the helicity-dependent CFFs, where each of them might involve CFFs of different twist. In the BMMP formulas [132], such mixing is resolved by choosing a different set of photon polarization vectors, which rotates the helicity-dependent CFFs and disentangles the mixing. This agrees with our argument in ref. [145] that with the BMMP coordinate choice, such

kinematical twist-three effects vanish.

Another difference worth noting is the choice of definitions of twist-three GPDs. Commonly in the literature (as well as here), the definitions in refs. [70, 146] are used, where the GPDs are defined according to their different Dirac structures that do not mix the Dirac tensor (without γ^5) and pseudo-tensor (with γ^5 and Levi-Civita symbol). With this choice, the physical interpretation of GPDs gets clearer. For instance, the GPD $E_{2T}(x, \xi, t)$ is closely related to the orbital angular momentum of partons [61, 73, 74, 143]. On the other hand, in the BKM01, BKM10, BMJ and BMMP formulas, another set of twist-three GPDs $H_{\pm}^3, E_{\pm}^3, \tilde{H}_{\pm}^3, \tilde{E}_{\pm}^3$ (or ones with equivalent definitions) are defined, resembling the definition of $H, E, \tilde{H}, \tilde{E}$. This choice makes the calculation simpler, as it will be similar to the twist-two one, while they make the physical interpretation of GPDs, such as the orbital angular momentum for E_{2T} , less obvious.

At last, we comment on the quantitative comparison of all the formulas. The BKM01 formulas lack kinematical higher order effects, while the UVa formulas differ from the others with an extra $\cos(\phi)$ phase factor in the interference cross-section. The connections between the BMMP and BMJ formulas are discussed with details in ref. [132]. Besides the kinematical twist-four corrections added in the BMMP formula, the major difference between them is the redefinition of light-cone vectors in the BMMP formulas, while the cross-section formulas are in essence the same. Both the BMMP and BMJ formulas are closely related to the BKM10 formulas — the corresponding scalar coefficients are identical to those of the BKM10, which agree well with the scalar coefficients here.

Chapter 4: QCD factorization for threshold production of heavy vector meson with GPDs

In addition to the quark structures that can be probed with DVCS, gluon structures are more mysterious and can only be probed with strongly-interacting particles like mesons, e.g., through meson productions like DVMP. The meson productions typically require higher collision energy compared to that of processes like DVCS. One reason is that the meson produced in the final state can still interact with the nucleon with soft gluon exchange, thus higher energy is required to avoid the dominance of the soft contributions. Also, since the meson could also be produced by the constituent quarks in the nucleon through processes shown in Fig. 4.1, the heavier meson production for which such contributions shall be suppressed will be of more interest for probing the gluon structures, requiring higher energy as well.

Therefore, DVMP measurements are mostly accessible with collider experiments which allow more center-of-mass energy compared to the fixed-target ones, and the future EIC [6] will be crucial for such task. On the other hand, at lower energy near the threshold, the exclusive meson productions, especially for heavy mesons like J/ψ , also contain rich gluonic physics and can be measured currently at JLab [147], which has raised lots of interest in the communities [148–154]. An intuitive picture for the gluonic sensitivity of such processes is that when a heavy meson is produced in the nucleon, it can be treated as a color dipole in the non-relativistic limit. Near the

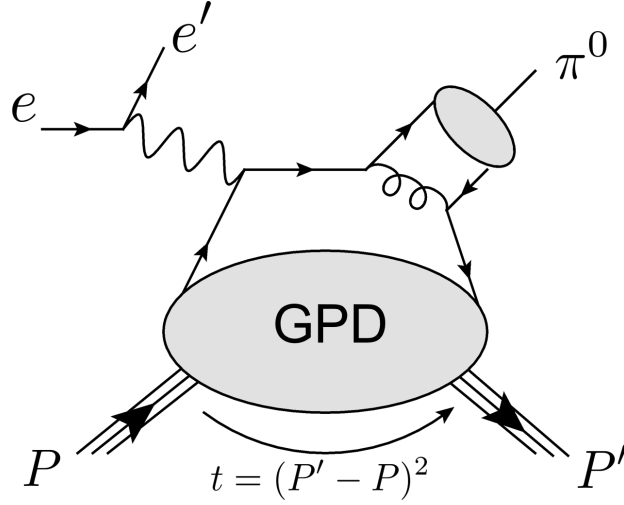


Figure 4.1: An example of meson production from quark GPDs.

threshold where the dipole has almost zero kinetic energy in the center-of-mass frame, it will be coupled to the local color electric field E_a [155, 156], as illustrated in Fig. 4.2.

Of course, we can not use a static heavy meson that decays very shortly to probe the local gluon distributions of the proton in reality. Instead, one could consider the photo- or lepto-production of such heavy meson off the nucleon near the threshold. In this chapter, we will consider such a threshold photoproduction process. We will show that the QCD factorization formula in terms of gluon GPD for the photo- or lepto- production of heavy vector meson remains to be valid in the near threshold region. At a large vector meson mass, the amplitude is dominated by the leading moment of the gluon GPD, the gluon GFFs of the proton, allowing them to be extracted from experiments [147, 157]. We apply the formalism to the case of unpolarized J/ψ production from recent GlueX measurements [147], which shows encouraging results.

In the literature, different methods have been adopted to analyze the process. In the vector dominance model [156, 158, 159], the vector meson photoproduction is related to the forward meson-nucleon scattering where a direct OPE in terms of gluonic matrix elements is applicable [160–163]. Recently, this process has also been approached using dispersive analy-

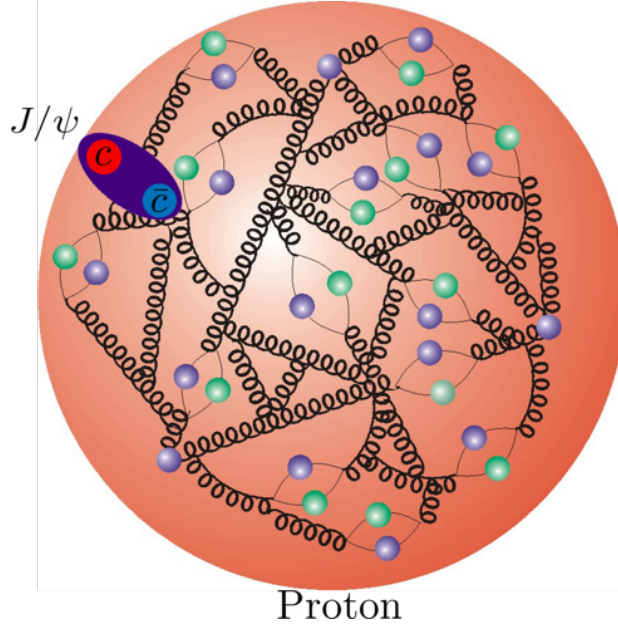


Figure 4.2: An illustration of using heavy meson as the probe of local color field.

sis [164, 165] and holographic QCD [148–151]. However, a comprehensive understanding of the process in perturbative QCD in the threshold region is still lacking except a few early attempts [166, 167]. This is in sharp contrary to the large Q^2 and large W diffractive region where the process has attracted attentions since mid-1990s [168–172] with well-established all order factorization [104] in terms of GPDs [3, 4, 103], and the $Q^2 = 0$, large W and small t region where the factorization has been explicitly shown at next to leading order [173].

In this chapter, we study the near-threshold photoproduction of heavy vector meson with mass M_V in the heavy-meson mass limit $M_V \rightarrow \infty$. In this limit, the process is dominated by the direct photon coupling with the heavy quarks, and the heavy quark production through gluonic subprocesses including possible intrinsic heavy flavor is suppressed by $\alpha_s(M_V) \rightarrow 0$. We show that the QCD factorization in terms of gluon GPDs in ref. [173] remains valid in the threshold region as well. Different from the proposals in refs. [156, 158] and recent calculations in a different limit [174, 175], the leading contribution comes from the tensor part of gluonic

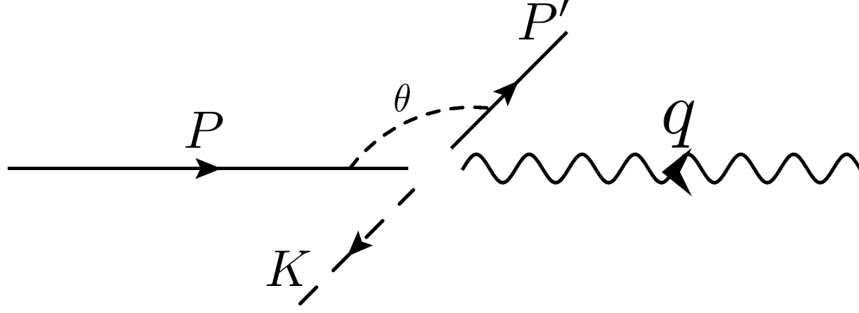


Figure 4.3: The heavy vector-meson photoproduction kinematics in the center-of-mass frame.

EMT and high-dimensional twist-two gluonic operators, due to the emergent light-cone structure in the large M_V limit. The near-threshold region is characterized by large skewness parameter $\xi \sim 1$ regardless of the Q^2 [175], and the gluon EMT dominates over high dimensional operators as well as three gluon exchanges [166, 174]. Therefore, the process can be used to probe the gluonic GFFs of the proton, which provide important information about the gluon contributions to the proton's mass and spin as well as pressure structures [23, 26, 27, 176, 177].

4.1 Threshold production of heavy vector meson

In this section, we show how the near-threshold photoproduction of heavy vector meson can be used to access the gluonic structures through a QCD factorization in terms of the gluon GPDs in the heavy-meson mass limit $M_V \rightarrow \infty$.

4.1.1 Near-threshold kinematics of exclusive production

We first consider the near-threshold kinematics of the process. Without loss of generality, we work in the center-of-mass frame though the final result is frame-independent. The four momenta of the incoming photon, outgoing vector meson, incoming proton, outgoing proton are denoted by q , K , P , and P' , respectively as shown in Fig. 4.3. We restrict ourselves to the real

photon and near-threshold region, although similar analysis can be extended to a finite virtual photon mass Q^2 . We choose the incoming proton to move in the $+z$ direction, as in the lab frame. In the center-of-mass frame, the magnitudes of the three momenta can be expressed in Lorentz scalars as

$$|\vec{K}| = \sqrt{\frac{(W^2 - (M_V + M_N)^2)(W^2 - (M_V - M_N)^2)}{4W^2}}, \quad (4.1)$$

$$|\vec{P}| = \frac{W^2 - M_N^2}{2W}, \quad (4.2)$$

where we label the nucleon mass M_N , the center-of-mass energy squared $W^2 \equiv (P + q)^2 \geq (M_N + M_V)^2$, the average proton four momentum $\bar{P} \equiv (P' + P)/2$, four momentum transfer $\Delta = P' - P$ and associated invariant $t \equiv \Delta^2$. In the heavy-quark limit, the vector-boson mass $M_V \gg M_N$. As a result $W \gg M_N$ and the incoming proton travels almost along the $+$ light-cone direction in terms of the light-front coordinate $x^\pm \equiv (x^0 \pm x^z)/\sqrt{2}$, where x^μ is a four coordinate. One has

$$P^\mu \rightarrow 2p^\mu, \quad (4.3)$$

with $p = \bar{P}^+/\sqrt{2}(1, 1, 0_\perp)$ and $n = 1/(\sqrt{2}\bar{P}^+)(1, -1, 0_\perp)$ being the two opposite light-cone unit vectors. The four momenta of the final-state proton and vector meson are

$$K = (K^0, |\vec{K}|\vec{e}), \quad (4.4)$$

$$P' = (P'^0, -|\vec{K}|\vec{e}), \quad (4.5)$$

where the unit vector $\vec{e}$ can be in any spatial direction.

Here we focus on the threshold region defined by the condition that the heavy meson produced in the final state will be non-relativistic, whose velocity will be suppressed by the heavy meson mass $\beta_V \equiv |\vec{K}|/K^0 \sim \mathcal{O}(M_N/M_V)$ while the final proton can still have a velocity $\beta \equiv |\vec{K}|/P^0$ of order 1. Right at the threshold, the invariant momentum transfer t equals to

$$-t_{\text{th}} = \frac{M_N M_V^2}{M_N + M_V}, \quad (4.6)$$

which is of order $M_V M_N$ in the heavy-quark limit. As the center-of-mass energy squared W increases, the allowed region of $-t$ forms a band $[|t|_{\min}(W), |t|_{\max}(W)]$ between the backward case with $-t = |t|_{\min}$ and forward case with $-t = |t|_{\max}$. Near the threshold, both of them are of order $\mathcal{O}(M_N M_V)$, much larger than M_N^2 . In the standard notation of GPDs [4], this implies that the skewness

$$\xi = -\frac{\Delta \cdot n}{2\bar{P} \cdot n} = \frac{P^+ - P'^+}{P^+ + P'^+} = 1 + \mathcal{O}\left(\frac{M_N}{M_V}\right), \quad (4.7)$$

is close to 1, which has also been observed for the large Q^2 region [175]. In Fig. 4.4, we show the ξ value on the $(W, -t)$ plane for J/ψ and Υ production in the kinematically allowed region. This condition $\xi \rightarrow 1$ near the threshold allows us to study the form factors of the gluon EMT as we discuss later.

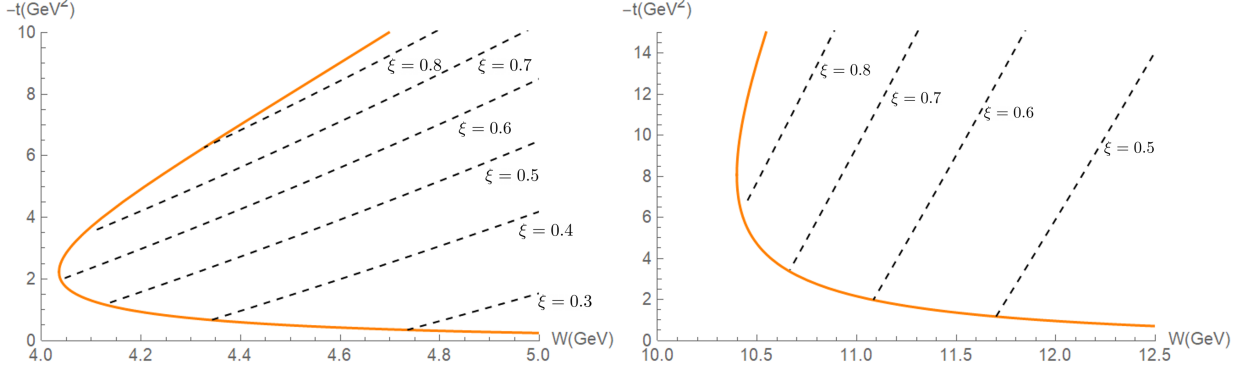


Figure 4.4: Kinematically allowed region on the $(W, -t)$ plane for J/ψ (left) and Υ (right). The masses of the vector mesons are $M_{J/\psi} = 3.097$ GeV and $M_{\Upsilon} = 9.46$ GeV. Dashed lines with equal ξ are shown on the $(W, -t)$ plane in the kinematically allowed region for each meson respectively.

4.1.2 Leading-order QCD factorization

We now proceed to consider the amplitude of the process written as

$$\mathcal{M}(q) = \int d^4z e^{iq \cdot z} \langle P', K(\varepsilon_V) | \varepsilon_\mu J^\mu(z) | P \rangle, \quad (4.8)$$

where J^μ is the electromagnetic current operator, ε^μ is the polarization vector of the photon normalized as $\varepsilon^2 = -1$ and $|K(\varepsilon_V)\rangle$ denotes the vector meson state with momentum K and polarization vector ε_V normalized as $\varepsilon_V^2 = -1$ and satisfying $K \cdot \varepsilon_V = 0$. All states are normalized covariantly. In the heavy-quark limit, the leading-order contribution to $\mathcal{M}$ is given by two-gluon exchange diagrams as illustrated in Fig. 4.5.

The following amplitude can be read from the Feynman diagrams as,

$$\begin{aligned} \varepsilon^\rho M_\rho(q) = (ig)^2 \int \frac{d^4k d^4l}{(2\pi)^8} \langle P' | A^\mu \left(-l - \frac{\Delta}{2} \right) A^\nu \left(l - \frac{\Delta}{2} \right) | P \rangle \\ \times \text{Tr} \left\{ \sum_n I_{\mu\nu}^n(K, k, l, \Delta, \varepsilon) \Psi_V(K, k) \right\}, \end{aligned} \quad (4.9)$$

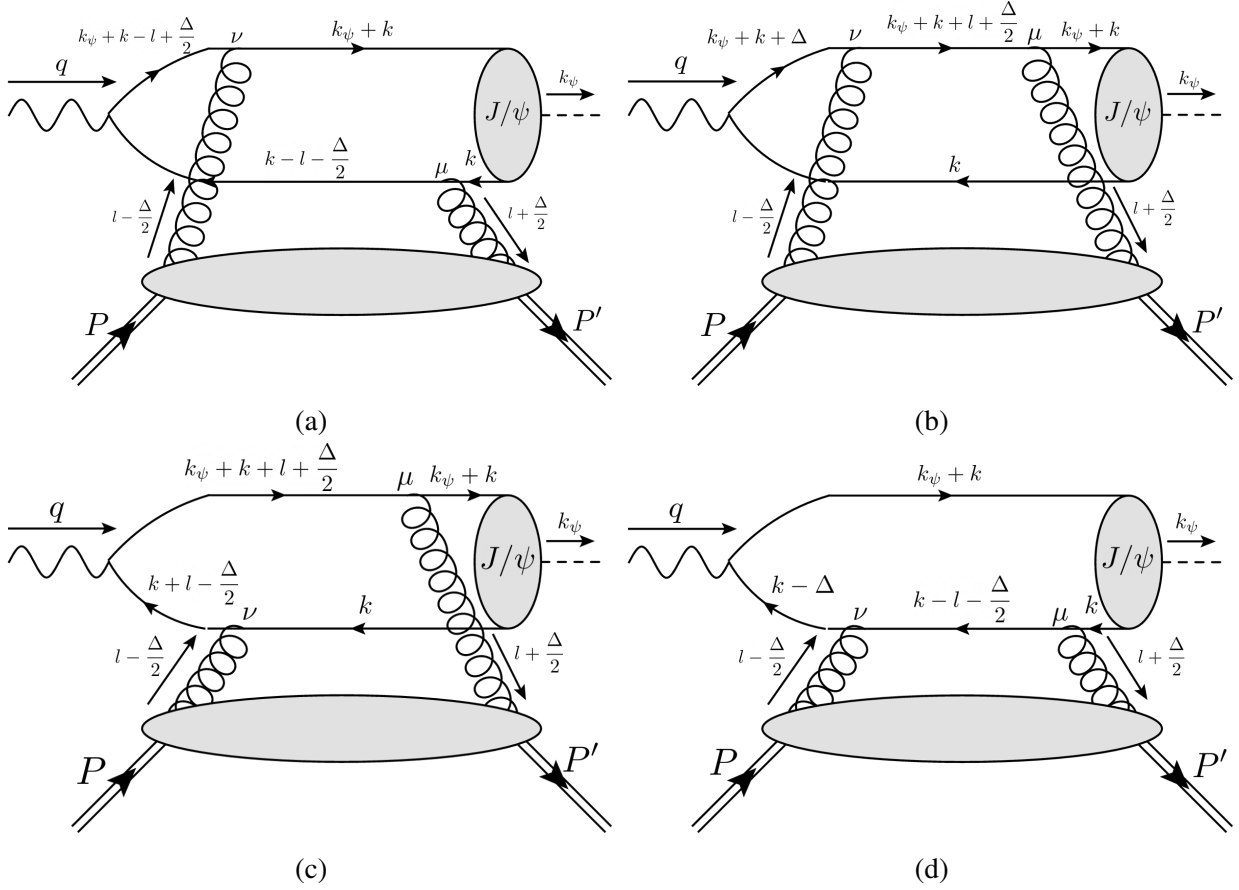


Figure 4.5: Four leading-order Feynman diagrams that contribute to the photoproduction of a heavy vector meson ($V = J/\psi$ and $K = k_\psi$ here).

where the index n sums over the four diagrams in Fig. 4.5 and the $\Psi_V(K, k)$ stands for the DA of the vector meson defined as

$$\Psi_V(K, k) \equiv \langle K(\varepsilon_V) | T \{ \bar{\psi}(-K - k) \psi(k) \} | 0 \rangle, \quad (4.10)$$

The perturbative hard part $I_{\mu\nu}^n$ can be read from the four diagrams as well, which are

$$I_a^{\mu\nu} = \gamma^\nu \frac{i}{\not{K} + \not{k} - \not{l} + \not{\Delta}/2 - m_Q + i\epsilon} \not{\epsilon} \frac{i}{\not{k} - \not{l} - \not{\Delta}/2 - m_Q + i\epsilon} \gamma^\mu, \quad (4.11)$$

$$I_b^{\mu\nu} = \gamma^\mu \frac{i}{\not{K} + \not{k} + \not{l} + \not{\Delta}/2 - m_Q + i\epsilon} \gamma^\nu \frac{i}{\not{K} + \not{k} + \not{\Delta} - m_Q + i\epsilon} \not{\epsilon}, \quad (4.12)$$

$$I_c^{\mu\nu} = \gamma^\mu \frac{i}{\not{K} + \not{k} + \not{l} + \not{\Delta}/2 - m_Q + i\epsilon} \not{\epsilon} \frac{i}{\not{k} + \not{l} - \not{\Delta}/2 - m_Q + i\epsilon} \gamma^\nu, \quad (4.13)$$

$$I_d^{\mu\nu} = \not{\epsilon} \frac{i}{\not{k} - \not{\Delta} - m_Q + i\epsilon} \gamma^\nu \frac{i}{\not{k} - \not{l} - \not{\Delta}/2 - m_Q + i\epsilon} \gamma^\mu. \quad (4.14)$$

To simplify these contributions, we perform the following approximations which are justified in leading order in $\mathcal{O}(M_N/M_V)$ and $\mathcal{O}(\alpha_S)$. First, since the momentum transfer is mostly in the light-cone direction p , only the $+$ components of the loop momenta l and $-\Delta - l$ for the gluons are kept in the heavy-quark loop and can be expressed as

$$-\frac{\Delta}{2} + l = (\xi + x)p + \dots, \quad (4.15)$$

$$-\frac{\Delta}{2} - l = (\xi - x)p + \dots, \quad (4.16)$$

where $-1 < x < 1$ is the momentum fraction for l . In the threshold region, ξ approaches 1 in the heavy-quark limit, though we will keep the ξ dependence in general.

Second, at the large M_V limit, we perform the non-relativistic approximation to the heavy-meson wave function which amounts to the substitution

$$\langle K(\varepsilon_V) | \psi(-k - K) \bar{\psi}(k) | 0 \rangle = \frac{1 - \not{v}_V}{2} \frac{\varepsilon_V^*}{\sqrt{2}} \frac{1 + \not{v}_V}{2} \tilde{\phi}^*(k). \quad (4.17)$$

Here v_V is the four velocity of the vector meson, and the scalar function $\tilde{\phi}(k)$ corresponds to

the distribution amplitude of the vector meson. The relative momentum between the quark and anti-quark is of order $\mathcal{O}(\alpha_S M_V)$ for hydrogen-like systems, and to the leading order in α_s we can further approximate the wave function as

$$\begin{aligned}\phi(k) &= (2\pi)^4 \delta^4 \left(k - \frac{M_V}{2} v_V \right) \int \frac{d^4 k'}{(2\pi)^4} \tilde{\phi}(k') , \\ &= (2\pi)^4 \delta^4 \left(k - \frac{M_V}{2} v_V \right) \phi(0) ,\end{aligned}\tag{4.18}$$

where the velocity v_V of the final state heavy-meson can be approximated as the static one $v = (1, 0, 0_\perp)$ to the leading order in $\frac{M_N}{M_V}$. The $\phi(0) = \sqrt{2M_V/3} \psi_{\text{NR}}(0)$ is proportional to the non-relativistic hydrogen-like wave function ψ_{NR} at the origin. The above meson wave function can be studied more systematically with Non-Relativistic QCD (NRQCD), which shows that the leading correction to the wave function is indeed suppressed by order $\alpha_S(M_V)$ [178–180].

Finally, we note that for collinear gluons in light-cone gauge $A^+ = 0$, the A^- component is suppressed by the large boost factor. Therefore, to the leading order approximation, it is sufficient to keep the transverse components of the gluon gauge potential only.

Given these approximations, we can evaluate the Feynman diagrams in light-cone gauge straightforwardly. The perturbative hard part $I_{\mu\nu}^n$ will be simplified into

$$\mathcal{I}_a^{\mu\nu} = \mathcal{I}_c^{\mu\nu} = 0 ,\tag{4.19}$$

$$\mathcal{I}_b^{\mu\nu} = \mathcal{I}_d^{\mu\nu} = -\frac{1}{\sqrt{2}m_Q^2} \left(g^{\mu\nu} (\varepsilon \cdot \varepsilon_\psi) + \varepsilon^\nu \varepsilon_\psi^\mu - \varepsilon^\mu \varepsilon_\psi^\nu \right)_\perp ,\tag{4.20}$$

where the $\perp$ subscript indicates that only the transverse components are non-zero. Then the

leading amplitude $\mathcal{M}(\varepsilon_V, \varepsilon)$ reads:

$$\mathcal{M}(\varepsilon_V, \varepsilon) = \frac{8\sqrt{2}\pi\alpha_S(M_V)}{M_V^2} \phi^*(0) G(t, \xi) (\varepsilon_V^* \cdot \varepsilon) . \quad (4.21)$$

Here the function $G(t, \xi)$ implicitly depends on the polarization of the initial and final proton which is suppressed and can be expressed in terms of the gluon GPDs as

$$G(t, \xi) = \frac{1}{2\xi} \int_{-1}^1 dx \mathcal{A}(x, \xi) F_g(x, \xi, t) . \quad (4.22)$$

where the hard kernel $\mathcal{A}(x, \xi)$ reads

$$\mathcal{A}(x, \xi) \equiv \frac{1}{x + \xi - i0} - \frac{1}{x - \xi + i0} . \quad (4.23)$$

The standard gluon GPD F_g is defined as [5]

$$F_g(x, \xi, t) \equiv \frac{1}{(\bar{P}^+)^2} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | F^{+i} \left(-\frac{\lambda n}{2} \right) F_i^+ \left(\frac{\lambda n}{2} \right) | P \rangle , \quad (4.24)$$

where the states are normalized as $\langle P | P \rangle = 2E_P (2\pi)^3 \delta^{(3)}(\vec{0})$ and the renormalization scale has been omitted. Similar results have been derived in the literature both in the high-energy limit [181, 182] and in the heavy-quark limit [173], and here we show equivalent ones in the near-threshold region that we are interested in.

While the hadronic matrix element $G(t, \xi)$ can be evaluated at a generic ξ , near the threshold and in the heavy-quark limit, we will have t approaching infinity and ξ approaching 1. However, there are two reasons to keep the generic ξ dependency. First, the expression with

generic ξ agrees with the leading order result in the kinematic region at large W and small $|t|$ [173, 181, 182]. What we have shown is that in the heavy-quark limit, the validity region of the leading order factorization formula in terms of GPDs can be smoothly extended to the threshold region along the $|t|_{\min}$ line. It turns out that the large M_V is sufficient to generate light-cone structure in the threshold region. Therefore, our result can be viewed as a generalization of ref. [173]. Second, although ξ approaches 1 in the heavy meson limit near the threshold, in reality, especially the production of J/ψ , the kinematic finite meson mass corrections are important. By using the physical ξ , one can expect to take part of these kinematic corrections into account.

Given the amplitude above, the cross-section can then be calculated accordingly, which reads

$$\begin{aligned} \frac{d\sigma}{dt} &= \frac{e^2 e_Q^2}{16\pi (W^2 - M_N^2)^2} \frac{1}{2} \sum_{\text{polarization}} |\mathcal{M}(\varepsilon_V, \varepsilon)|^2, \\ &= \frac{\alpha_{\text{EM}} e_Q^2}{4 (W^2 - M_N^2)^2} \frac{(16\pi\alpha_S)^2}{3M_V^3} |\psi_{\text{NR}}(0)|^2 |G(t, \xi)|^2, \end{aligned} \quad (4.25)$$

with e_Q the charge of the quark in the unit of proton charge, and the photon and meson polarization are summed over. The kinematic pre-factor is the same as that in refs. [148, 149] and in the second line it is shown that the cross-section of near-threshold meson production is related to the non-relativistic wave function at origin and the gluon GPDs. Again, the renormalization scales in GPDs and wave function have been omitted and shall be on the order of M_V .

4.1.3 Expansion in moments of GPDs

As shown in the previous subsection, the entire gluon GPD enters the amplitude for the threshold region heavy quarkonium production because of the light-cone dominance. However,

in the heavy vector meson limit where ξ approaches one, one can Taylor-expand the $\mathcal{A}(x, \xi)$ and express the $G(t, \xi)$ in terms of the even moments of the GPD

$$G(t, \xi) = \sum_{n=0}^{\infty} \frac{1}{\xi^{2n+2}} \int_{-1}^1 dx x^{2n} F_g(x, \xi, t) . \quad (4.26)$$

The moments can be related to the matrix element of the gluonic twist-two operator $O_g^{\mu\mu_1\cdots\mu_{n-2}\nu}$ [5]

$$\begin{aligned} & \langle P' | O_g^{\mu\mu_1\cdots\mu_{n-2}\nu} | P \rangle \\ &= \mathcal{S} \bar{u}(P') \gamma^\mu u(P) \sum_{i, \text{even}} A_{n,i}^g(t) \Delta^{\mu_1} \cdots \Delta^{\mu_i} P^{\mu_{i+1}} \cdots P^\nu \\ &+ \mathcal{S} \bar{u}(P') \frac{i\sigma^{\mu\alpha} \Delta_\alpha}{2m} u(P) \sum_{i, \text{even}} B_{n,i}^g(t) \Delta^{\mu_1} \cdots \Delta^{\mu_i} P^{\mu_{i+1}} \cdots P^\nu \\ &+ \mathcal{S} \frac{\Delta^\mu}{m} \bar{u}(P') u(P) \bmod(n, 2) C_n(t) \Delta^{\mu_1} \cdots \Delta^\nu . \end{aligned} \quad (4.27)$$

By parametrizing the gluon GPDs as

$$F_g(x, \xi, t) = \frac{1}{2\bar{P}^+} \left[H_g(x, \xi, t) \bar{u}(P') \gamma^+ u(P) + E_g(x, \xi, t) \bar{u}(P') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2m} u(P) \right] , \quad (4.28)$$

the moments of the scalar functions H and E can then be written as [5]

$$\int_0^1 dx x^{2n} H_g(x, \xi, t) = \sum_{i=0}^n (2\xi)^{2i} A_{2n+2,2i}^g + (2\xi)^{2n+2} C_{2n+2}^g , \quad (4.29)$$

$$\int_0^1 dx x^{2n} E_g(x, \xi, t) = \sum_{i=0}^n (2\xi)^{2i} B_{2n+2,2i}^g - (2\xi)^{2n+2} C_{2n+2}^g . \quad (4.30)$$

where C -terms represent the highest power in ξ . In particular, for $n = 0$ we have

$$\int_0^1 dx H_g(x, \xi, t) = A_{2,0}^g(t) + (2\xi)^2 C_2^g \equiv H_2(t, \xi), \quad (4.31)$$

$$\int_0^1 dx E_g(x, \xi, t) = B_{2,0}^g(t) - (2\xi)^2 C_2^g \equiv E_2(t, \xi). \quad (4.32)$$

for the leading moments of the GPDs.

As we will argue, the higher moment contributions shall be suppressed in the threshold region. If one only keep the leading moment corresponding to $n = 0$, the $G(t, \xi)$ becomes

$$\begin{aligned} G(t, \xi) &= \frac{1}{\xi^2 (\bar{P}^+)^2} \langle P' | \sum_{a,i} F^{a,+i}(0) F^{a,+}_i(0) | P \rangle \\ &= \frac{1}{\xi^2 (\bar{P}^+)^2} \langle P' | T_g^{++} | P \rangle, \end{aligned} \quad (4.33)$$

in terms of the matrix elements of gluon EMT. Then with the gluon GFFs defined as [4]

$$\begin{aligned} \langle P' | T_{q,g}^{\mu\nu} | P \rangle &= \bar{u}(P') \left[A_{q,g}(t) \gamma^{(\mu} \bar{P}^{\nu)} + B_{q,g}(t) \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M_N} \right. \\ &\quad \left. + C_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M_N} + \bar{C}_{q,g}(t) M_N g^{\mu\nu} \right] u(P). \end{aligned} \quad (4.34)$$

where $A = A_{2,0}$, $B = B_{2,0}$ and $C = C_{2,0}$ corresponding to the moments of gluon GPDs defined before, the hadronic matrix element squared $|G(t, \xi)|^2$ can then be written as

$$|G(t, \xi)|^2 = \frac{4}{\xi^4} \left\{ \left(1 - \frac{t}{4M_N^2} \right) E_2^2 - 2E_2(H_2 + E_2) + (1 - \xi^2) (H_2 + E_2)^2 \right\}, \quad (4.35)$$

after summing/averaging over the final and initial proton spin, where $H_2 \equiv H_2(t, \xi)$, $E_2 \equiv E_2(t, \xi)$ are defined before. Combining with eq. (4.25), the cross-section of heavy vector meson

photoproduction can be expressed in terms of those GFFs. This result agrees with the holographic QCD predictions [150, 151] that the leading contribution to the cross-section is due to exchange of 2^{++} excitations, or the spin-2 twist-two operators, instead of the 0^{++} F^2 operators suggested in refs. [156, 159]. However, in the holographic approach the twist-two part of the GFFs is dual to the graviton exchange and is free from the C_g contribution. This differs from the generic QCD parametrization in eq. (4.33). Because of the EMT conservation, the quantum anomalous energy F^2 form factor can be related to the twist-two ones here. If the further limit $\xi \rightarrow 0$ is taken in eq. (4.35), only the A form factors are leading and all the results agrees. However, this is inconsistent with our approximation here.

Here we return to the validity of the moment expansion and the leading moment approximation. Note that for generic $\xi < 1$, the $G(t)$ has imaginary part

$$\text{Im } G(t, \xi) \propto F_g(\xi, \xi, t) + F_g(-\xi, \xi, t) , \quad (4.36)$$

which implies that the moment expansion in eq. (4.26) has some limitations. However, as ξ gets closer to 1 in the heavy-quark limit, the above imaginary part vanishes in power of $1 - \xi$. The renormalization group evolution will also help to improve the convergence of the expansion at large renormalization scale $\mu \sim M_V$. Indeed, it has been shown that the asymptotic form of gluon GPD reads [64, 183]

$$F_g^{\text{asym}}(x, \xi, t) \propto \left(1 - \frac{x^2}{\xi^2}\right)^2 \theta\left(1 - \frac{x}{\xi}\right) , \quad (4.37)$$

which vanishes quadratically at $x = \xi$. With this form of asymptotic behavior, one can show that

the expansion in eq. (4.26) is convergent. To summarize, eq. (4.26) should be a good approximation for $G(t, \xi)$ in the threshold region where ξ is close to 1.

Regarding the validity of the leading moment approximation in eq. (4.33), one can read from eq. (4.26) that due to the ξ^{2n} in the denominator, eq. (4.26) is simultaneously a moment expansion and a ξ^{-1} expansion. The closer the ξ to 1, the larger the contribution from the leading term $n = 0$. Using the asymptotic form in eq. (4.37), in the exact $\mu \sim M_V \rightarrow \infty$ limit the ratio between the contribution from higher ($n \geq 1$) moments and the leading moment is

$$\frac{\sum_{n \geq 1}^{\infty} \int_{-1}^1 dx \frac{x^{2n}}{\xi^{2n}} F_g^{\text{asym}}(x, \xi, t)}{\int_{-1}^1 dx F_g^{\text{asym}}(x, \xi, t)} = \frac{1}{4}, \quad (4.38)$$

the total higher-moment contribution is 25 percent. Notice that in the large Q^2 region, a similar estimation in favor of the leading moment dominance has first been performed in ref. [175].

Realistically, when $\mu \sim M_V$ is not very large and ξ is not exactly at 1, we can use the ratio between the second and the leading terms to estimate the effect of the high-order terms. Using the explicit expressions for GPD moments in eq.(4.29), the ration between second and leading moments can be calculated as

$$\frac{A_{4,0}^g + 4\xi^2 A_{4,2}^g + 16\xi^4 C_4^g}{\xi^2 (A_{2,0}^g + 4\xi^2 C_2^g)}, \quad (4.39)$$

for the moments of H and

$$\frac{B_{4,0}^g + 4\xi^2 B_{4,2}^g - 16\xi^4 C_4^g}{\xi^2 (B_{2,0}^g - 4\xi^2 C_2^g)}, \quad (4.40)$$

for the moments of E . For a quick estimation, one can keep only the A_0 form factors which equal

to moments of gluon-PDFs at $t = 0$. Away from $t = 0$, one can use the following dipole model

$$A_{2,0}^g = \left(1 - \frac{t}{m_A^2}\right)^{-2} \int_0^1 dx x f_g(x) , \quad (4.41)$$

$$A_{4,0}^g = \left(1 - \frac{t}{m_{A4}^2}\right)^{-2} \int_0^1 dx x^3 f_g(x) . \quad (4.42)$$

Here f_g is the gluon PDF at zero momentum transfer and m_A, m_{A4} are unknown dipole masses.

Neglecting the difference in dipole masses and using the recent CTEQ global analysis [34], at renormalization scale $\mu = 1.3$ GeV one has

$$\frac{\int_0^1 dx x^3 f_g(x)}{\xi^2 \int_0^1 dx x f_g(x)} = \frac{0.038}{\xi^2} . \quad (4.43)$$

For J/ψ production, one has $\xi = 0.6$ right at the threshold, and the ratio is around 0.1, consistent with the dominance of the leading moment. Away from the threshold, the ratio increases as ξ drops, and the dominance of leading moment become less pronounced. As one increases the renormalization scale, more momentum fraction will be carried by small x gluons and the ratio becomes smaller. However, since the form factor C is also important for the amplitude, a more realistic estimation needs inputs from lattice calculations.

4.2 Gravitational form factors from threshold production

As we have discussed in the previous section, threshold quarkonium photoproduction in the heavy-quark limit is dominated by the gluonic GFFs in the leading moment approximation. In this section, we study the phenomenology of determining these GFFs from realistic heavy quarkonium production data, neglecting various higher-order corrections which we will take into

account in the future. We consider the J/ψ photoproduction where more data are available near the threshold, and predict the near-threshold Υ photoproduction cross-section where the heavy-quark expansion works better.

4.2.1 Threshold J/ψ photoproduction

To start with, we consider the J/ψ photoproduction total cross-sections from Stanford Linear Accelerator Center (SLAC) [184], Cornell [185] and the most recent GlueX experiments [147]. Given that the GlueX data seem to deviate from the SLAC and Cornell data, we will focus on comparing with the GlueX result which includes more data as well as measurements of differential cross-section. Besides the physical constants such as the proton mass $M_N = 0.938$ GeV, the J/ψ mass $M_{J/\psi} = 3.097$ GeV etc., we use the same α_S as in ref. [149]

$$\alpha_S(\mu = 2 \text{ GeV}) = 0.3 . \quad (4.44)$$

For the non-relativistic wave function $|\psi_{\text{NR}}(0)|^2$, it can be measured from the leptonic decay rate of J/ψ [180, 186, 187]

$$\Gamma(V \rightarrow e^+e^-) = \frac{16\pi\alpha_{\text{EM}}^2 e_Q^2 |\psi_{\text{NR}}(0)|^2}{M_V^2} \left(1 - \frac{16\alpha_s}{3\pi}\right) . \quad (4.45)$$

Then one has [187, 188]

$$|\psi_{\text{NR}}(0)|^2 = 1.0952/(4\pi) (\text{GeV})^3 . \quad (4.46)$$

As for the hadronic matrix element $G(t, \xi)$, inputs of GFFs are required. At large momentum transfer it has been argued that the form factors decays polynomially [189–191] based on power-

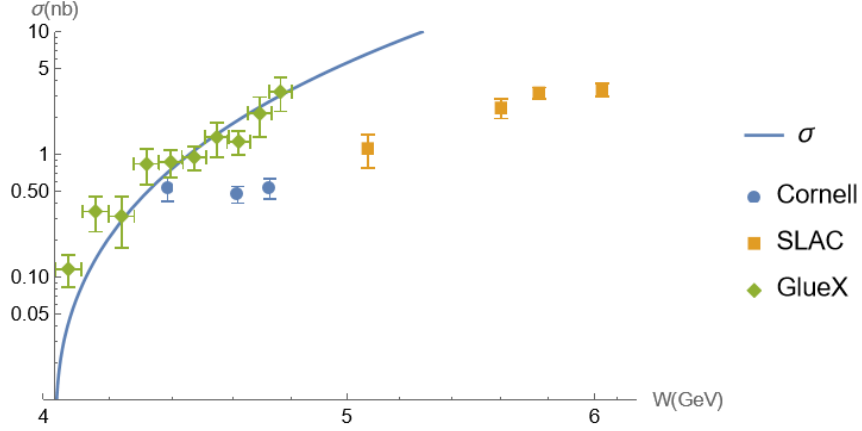


Figure 4.6: Comparison of the cross-section with GFFs given by lattice [192] with experiments data from SLAC [185], Cornell [184] and GlueX [147].

counting methods. Here we use the GFFs from lattice QCD calculations, which model those form factors with dipole expansion

$$A_g(t) = \frac{A_g(0)}{\left(1 - \frac{t}{m_A^2}\right)^2}, \quad (4.47)$$

$$C_g(t) = \frac{C_g(0)}{\left(1 - \frac{t}{m_C^2}\right)^2}. \quad (4.48)$$

In a recent lattice calculation of gluon GFFs [192], those parameters are found approximately, $m_A = 1.13$ GeV, $m_C = 0.48$ GeV, $A_g(0) = 0.58$ and $C_g(0) = -1.0$, while the $B_g(t)$ form factor is numerically small. These inputs without any fitting parameters yield the curve shown in Fig. 4.6.

On the other hand, our theoretical formulas allow us to extract the GFFs from the J/ψ photoproduction data. Since the data are quite limited at this stage, We choose to fit the GlueX total cross-section and differential cross-section combined. If we have both $A_g(0)$ and m_C fixed to be the lattice values in order to avoid overfitting, we get $m_A = 1.18 \pm 0.07$ GeV, $C_g(0) = -2.4 \pm 0.7$. In Fig. 4.7, we compare the fit results with the data from GlueX. The uncertainties

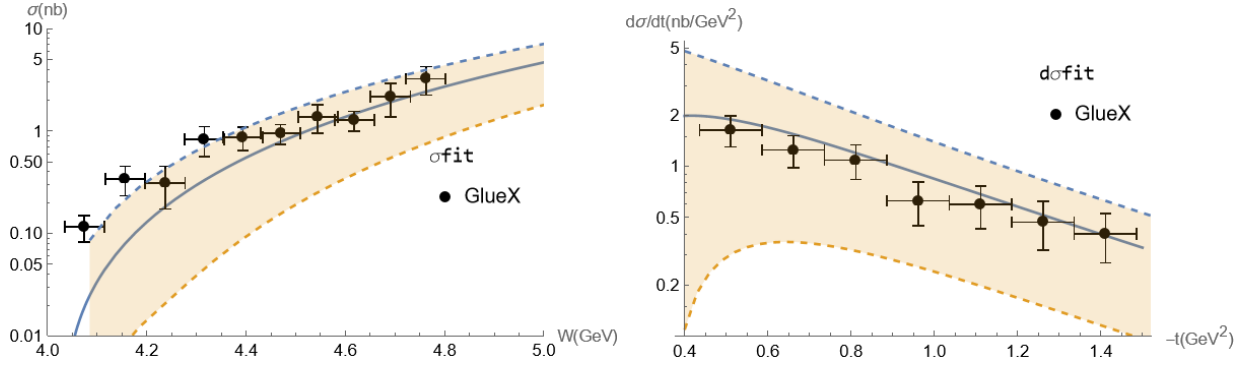


Figure 4.7: Fitted total cross-section (left) and differential cross-section (right) of J/ψ production compared with the total cross-section measured at GlueX [147]. The 95% confidence band is shown as the shaded region hereafter.

of m_A and $C_g(0)$ indicate that the data are more sensitive to m_A rather than $C_g(0)$ in the region of GlueX data. One can of course explore other fitting scenarios as well.

4.2.2 Threshold Υ photoproduction

The above result can be used to predict the Υ photoproduction rate near-threshold with $M_\Upsilon = 9.46$ GeV and the wave function at origin for Υ [187, 188]

$$|\psi_{\text{NR}}(0)|^2 = 5.8588/(4\pi) (\text{GeV})^3. \quad (4.49)$$

Considering the effect of running coupling constant, $\alpha_S = 0.2$ is used for the Υ production. Then we have the predicted total cross-section and differential cross-section in Fig. 4.8. The large mass of Υ implies that our calculation in the heavy meson limit works better for Υ production, as there are less theoretical uncertainties from higher order correction. However, the production rate is suppressed by the heavy meson mass and thus the cross-section is much lower than J/ψ .

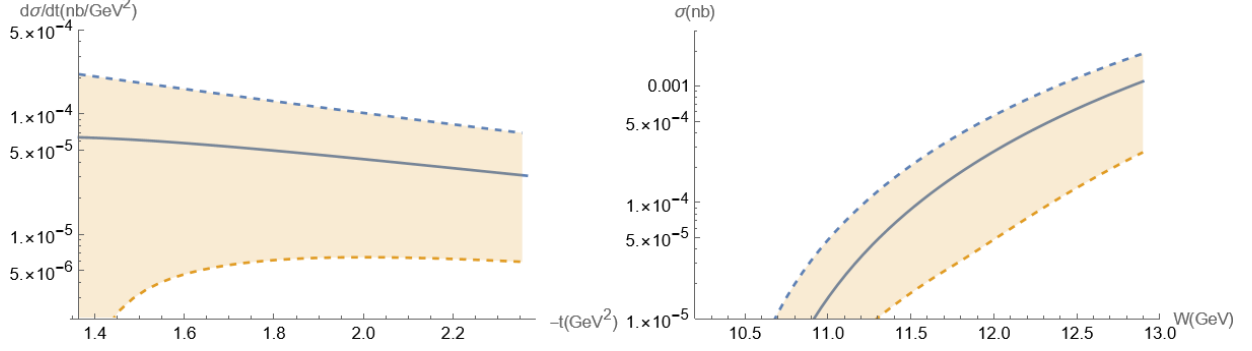


Figure 4.8: Solid line shows the predicted total cross-section (left) and differential cross-section (right) of Υ photoproduction near the threshold. $W = 11.5$ GeV is chosen for the differential cross-section.

4.2.3 Quantum anomalous energy and mass radius

The GFFs measured from J/ψ photoproduction can be used to study mass, spin, and pressure properties of the nucleon [4, 26, 27, 159, 177]. There are two methods to access the quantum anomalous energy through the matrix elements of the EMT. The first is through the matrix element of F^2 suggested in ref. [158]. In our factorization, this is a sub-leading contribution. Alternatively, one can also access the same quantity by considering the trace of the full EMT,

$$H_a = \langle P | T^\mu_\mu | P \rangle \quad (4.50)$$

$$= \frac{1}{4} (A_q(0) + A_g(0)) M_N, \quad (4.51)$$

where $A_q(0)$ and $A_g(0)$ are traditionally interpreted as the momentum fractions carried by quarks and gluons, respectively. The above relation comes from the conservation of the EMT. Thus fitting $A_g(0)$ through heavy quarkonium production data, one obtains the gluonic contribution to the quantum anomalous energy, and the result can be compared with the vector-dominance model analysis [158, 193]. Of course, it also provides an alternative determination of the gluonic

momentum fraction in the nucleon. It shall be clear that our formula for the cross-section works for the large t . To extrapolate to $t = 0$, one has to learn the t dependence through lattice QCD calculations.

The scalar and mass radii of the proton are defined as [177],

$$\begin{aligned}\langle r^2 \rangle_s &= 6 \frac{dA(t)}{dt} - 18 \frac{C(0)}{M_N^2}, \\ \langle r^2 \rangle_m &= 6 \frac{dA(t)}{dt} - 6 \frac{C(0)}{M_N^2}.\end{aligned}\tag{4.52}$$

The two different mass radii are related with the $C(0)$ term as

$$\langle r^2 \rangle_s - \langle r^2 \rangle_m = -12 \frac{C(0)}{M_N^2}.\tag{4.53}$$

The GFFs here are for the total EMT, so one has $A(t) = A_q(t) + A_g(t)$ and $C(t) = C_q(t) + C_g(t)$. For the quark contribution we use the lattice data $C_{u+d}(0) = -0.267$ or $C_{u+d}(0) = -0.421$ depending on the extrapolation method [194]. As for $A_{u+d}(t)$, one could also fit the form factor with above dipole expansion as

$$A_{u+d}(t) = \frac{A_{u+d}(0)}{\left(1 - \frac{t}{m_{\text{Di},u+d}^2}\right)^2},\tag{4.54}$$

for which one has the dipole mass $m_{\text{Di},u+d} \approx 1.8$ GeV and $A_{u+d}(0) \approx 0.5$ [195]. Then the quark

form factors combined with above gluon form factors from fitting gives

$$\langle r^2 \rangle_A \equiv 6 \frac{dA(t)}{dt} \approx (0.52 \text{ fm})^2, \quad (4.55)$$

$$\langle r^2 \rangle_C \equiv -6 \frac{C(0)}{M_N^2} \approx (0.84 \text{ fm})^2. \quad (4.56)$$

and then we have

$$\langle r^2 \rangle_s \approx (1.55 \text{ fm})^2, \quad (4.57)$$

$$\langle r^2 \rangle_m \approx (0.98 \text{ fm})^2. \quad (4.58)$$

One should be aware that those results are associated with large uncertainties resulting from the lack of precision for the fitted value $C_g(0) = -2.4 \pm 0.7$.

The above results serve as an example of measuring the GFFs from the threshold photoproduction of heavy vector mesons. Although the results rely on the input from lattice calculations as well as the models for the form factors, it can be a useful tool when more data are available and higher-order corrections are taken into account. One can also make a full GPD-based analysis directly using the unexpanded formula.

4.3 Cross-section with polarization

Up to now we have only considered the unpolarized cross-section. In this section we study the polarization effect. By measuring the polarization of the photon and the vector meson, we are able to verify the specific structure $\varepsilon_V^* \cdot \varepsilon$ of the factorization formula eq. (4.21) which will serve as a crucial test of the factorization formula itself. Since eq. (4.21) also predicts that the dependence

on the proton polarization is included in the matrix element $G(t, \xi)$, therefore by measuring the polarization of the proton also allow us to obtain more information to extract the GFFs. Notice that the measurement of the polarization of the final state proton can be non-trivial [196, 197]. In this section we shall study the polarized cross-section from two different point of view, the vector meson polarization and the proton polarization.

Polarization observables allow us to separate out the contributions from different form factors. In particular, even though the $B_{q,g}(t)$ is small, the polarized cross-sections can be sensitive to the $B_g(t)$ form factor which is related to the gluon angular momentum [4]. If given sufficient data with high precision, it might be possible to shed some light on the gluon angular momentum contribution to the proton spin.

4.3.1 Vector meson polarization

The total cross-section is proportional to $|\mathcal{M}(\varepsilon_V, \varepsilon)|^2$ and thus one has from eq. (4.21),

$$\frac{d\sigma}{dt} \propto \varepsilon_\mu^*(q, \lambda) \varepsilon_\nu(q, \lambda) \varepsilon_V^{*\mu}(K, \lambda') \varepsilon_V^\nu(K, \lambda') . \quad (4.59)$$

In general one could define the photon polarization tensor and meson polarization as $\rho_{\mu\nu}(q)$ and $\rho_{V\mu\nu}(K)$ so $\frac{d\sigma}{dt} \propto \rho_{\mu\nu}(q) \rho_V^{\mu\nu}(K)$. For physical photon or meson, the polarization tensor is simply

$$\rho_{\mu\nu}(q) = \varepsilon_\mu^*(q, \lambda) \varepsilon_\nu(q, \lambda) , \quad (4.60)$$

$$\rho_{V\mu\nu}(K) = \varepsilon_{V,\mu}^*(K, \lambda') \varepsilon_{V,\nu}(K, \lambda') , \quad (4.61)$$

while in the case there are virtual particles coupled with leptonic current, the polarization tensor can be written with those leptonic current as

$$\rho_{\mu\nu}(q) = \bar{u}(l_1, S_1) \gamma_\mu u(l_2, S_2) \bar{u}(l_2, S_2) \gamma_\nu u(l_1, S_1) , \quad (4.62)$$

with l_i and S_i the momenta and the polarization vectors of the lepton and anti-lepton pair. They satisfy $S_i \cdot l_i = 0$, $S_i^2 = -1$ and similarly for the vector meson. If we consider the photoproduction process when the photon is on-shell and unpolarized, the polarization tensor can be written as

$$\rho_{\mu\nu}(q) = \frac{1}{2} g_{\mu i} g_{\nu j} (\delta^{ij} - \hat{q}^i \hat{q}^j) , \quad (4.63)$$

where $\hat{q}^i$ is the unit vector in the direction of photon three momentum. As for the vector meson, assuming the final leptons are unpolarized and averaging over the lepton polarization we have

$$\rho_{V\mu\nu}(K) = -\frac{1}{2} K^2 g_{\mu\nu} + l_{1,\mu} l_{2,\nu} + l_{2,\mu} l_{1,\nu} . \quad (4.64)$$

with l_1, l_2 the momenta of the two leptons satisfying $l_2 + l_1 = K$, and thus we have

$$\frac{d\sigma}{dt} \propto \frac{K^2}{2} + \vec{l}_1 \cdot \vec{l}_2 - \hat{q} \cdot \vec{l}_1 \hat{q} \cdot \vec{l}_2 . \quad (4.65)$$

The last term $\hat{q} \cdot \vec{l}_1 \hat{q} \cdot \vec{l}_2$ indicates that the cross-section depends on the angle between the momentum of outgoing lepton and the momentum of initial photon and can be examined by experiments.

4.3.2 Proton polarization

Here we consider the polarization of the initial and final proton. For a demonstration, let us consider the asymmetric-difference $\delta|G(t, \xi, S', S)|^2$ as

$$\delta|G(t, \xi, S', S)|^2 \equiv |G(t, \xi, S', S)|^2 - |G(t, \xi, S', -S)|^2, \quad (4.66)$$

which measures the difference in the cross-section when the spin S^μ of the initial proton has been flipped. Notice the polarization vector satisfies $S \cdot P = 0$ and $S^2 = -1$ and similarly for S' . Then one has

$$\begin{aligned} \delta|G(t, \xi, S', S)|^2 = \frac{4}{\xi^4} \Bigg\{ & -S \cdot S' \left[\left(1 - \frac{t}{4M_N^2}\right) E_2^2 - 2(E_2 + H_2)E_2 + (1 - \xi^2)(E_2 + H_2)^2 \right] \\ & + \frac{S'^+(P' \cdot S)}{\bar{P}^+} [H_2 + E_2] [(1 + \xi)H_2 + \xi E_2] + \frac{(P \cdot S')(P' \cdot S)}{2M_N^2} E_2^2 \\ & + \frac{S^+(P \cdot S')}{\bar{P}^+} [H_2 + E_2] [(1 - \xi)H_2 - \xi E_2] + \frac{S^+ S'^+ t}{2(\bar{P}^+)^2} (H_2 + E_2)^2 \Bigg\}, \end{aligned} \quad (4.67)$$

where $E_2 \equiv E_2(t, \xi)$ and $H_2 \equiv H_2(t, \xi)$ are leading moments of GPDs defined in eq. (4.31).

When the initial proton is transversely polarized, one can show that

$$\begin{aligned} |G(t, \xi, S', S)|^2 = & \frac{1 - S \cdot S'}{4} \sum_{S, S'} |G(t, \xi, S, S')|^2 \\ & + 2 \frac{S'^+(P' \cdot S)}{\xi^4 \bar{P}^+} [H_2 + E_2] [(1 + \xi)H_2 + \xi E_2] + \frac{(P \cdot S')(P' \cdot S)}{\xi^4 M_N^2} E_2^2. \end{aligned} \quad (4.68)$$

In particular, if both S and S' are perpendicular to the scattering plane, then only the first

term in eq. (4.68) survives and the cross-section is non-zero only when S, S' are parallel. For generic transverse spin S, S' the second term is also non-zero. By measuring the cross-section at different S, S' , one is allowed to extract all the three form factors.

4.4 Comments

Before ending this chapter, we would like to make several comments. First, our calculation is performed only at leading order in $\mathcal{O}(M_N/M_V)$ and $\mathcal{O}(\alpha_S)$. For application to J/ψ production, since the J/ψ mass is not heavy enough, one expects large mass correction and high-twist effect. A comprehensive study of the mass correction is definitely very important but is beyond the current scope. Beyond leading order in $\mathcal{O}(\alpha_S)$, there are quantum corrections both from the emission of virtual gluon and from the internal motion of the vector meson [173]. For production of J/ψ , the order $\mathcal{O}(\alpha_s)$ corrections can be significant [173].

Second, we should emphasize again that our result should be viewed as a generalization of the leading order factorization in ref. [173]. The kinematic region considered therein is large W and small momentum transfer t , corresponding to the large W part of the $t_{\min}$ line. The heavy-quark limit is performed after taking the large W limit. Here we have extended the factorization along the $t_{\min}$ line into the entire threshold region where W is of the same order of M_V and is a single ultraviolet scale. While the factorization seems to work in leading and next leading order, the validity of the factorization theorem to all orders in perturbation theory remains to be established.

Third, although we considered only the small $Q^2 \sim 0$ region, at large Q^2 the factorization formula in terms of GPD should remain valid. In fact, as Q^2 increase, the incoming proton

becomes faster and the momentum transfer becomes larger. The light-cone structure shall be more relevant. This picture differs qualitatively from the Euclidean OPE approach in ref. [174].

Forth, we comment on the scalar contributions. In eq. (4.21), only the $F^{+i}F^{+i}$ component has been kept due to the standard power-counting rule of collinear gluon. If one keep all the terms, then the $F^{+-}F^{+-}$ contribution, corresponding to the F^2 contribution is proportional to $1 - \xi^2$ and is suppressed in the heavy-quark limit.

Finally, one should also notice that although in the current work only two-gluon exchange has been considered. It has been argued in ref. [166] that in the near-threshold region the three-gluon contributions might be important. However, as pointed out in ref. [153], the three-gluon exchange contribution is forbidden by the C-parity for such an exclusive production process.

Chapter 5: GUMP program for GPD Global analysis

In the previous chapters, we discussed various experimental efforts in accessing the nucleon 3D structures with GPDs. In addition, lattice QCD provides another opportunity to study the parton physics in the nucleon from first-principles calculations. The moments of GPDs are calculated on the lattice for the quark [194, 195] and the gluon [198]. In recent years, the large momentum effective theory (LaMET) [8–10] was proposed, which expands lattice technology further to allow the direct calculations of the x -dependence of the parton distributions including GPDs [199–202].

Despite the efforts put into investigating GPDs, limited knowledge of them has been obtained at this point due to their higher-dimension nature. Due to the lack of knowledge, we are faced with the so-called inverse problem when reconstructing GPDs, which states that GPDs can not be uniquely determined with, for instance, the combination of PDFs and the CFFs [203]. Therefore, we are in need of a global analysis program that can combine different constraints on GPDs to solve the inverse problem. One possible way is to parameterize the GPDs with finite (and extendable) numbers of free parameters, such that one can match the number of free parameters to the number of constraints on the GPDs till they are completely determined with controlled uncertainties. This is in the same spirit of determining proton collinear PDFs from various experimental data [34, 204].

With the above motivation, lots of GPD parameterization methods are proposed in the literature [37–47]. They generally fall into three categories: the double distribution representations, the moment space representations and other dynamic GPD models. The Radyushkin double distribution ansatz (DDA) is one of the earliest GPD parameterization method [37, 38]. Under the double distribution framework, the Vanderhaeghen-Guichon-Guidal (VGG) model [40, 183, 205] and the Goloskokov-Kroll (GK) model [41, 206] were developed for phenomenological applications. In general, the DDA expresses the GPD in terms of an integral expression of the double distribution. Then by constructing proper double distributions, the GPD will be reduced to the corresponding PDF in the forward limit and satisfies the polynomiality conditions. However, since the original double distributions have only one free parameter to control the overall ξ dependence, such GPDs parameterization methods lack the flexibility when fitting to measurements with different ξ [207].

The limitation of DDA leads to explorations of other possibilities. Another approach to constructing GPDs is to consider the polynomial expansion of GPDs. Two major GPD parameterization methods involving the polynomial expansion are the dual parameterization based on the Shuvaev-Noritzsch transform [39, 208, 209] and the Kumerički-Müller (KM) model [43, 44, 210, 211] based on conformal moment approach [42] commonly used in DVCS analysis, which are shown to be mathematically equivalent in ref. [212]. The basic idea of the polynomial expansion or the moment expansion is to expand the GPD in terms of a complete set of polynomials where the expansion coefficients are the corresponding moments. Then the GPD can be expressed as the sum of these polynomials without loss of generality. The polynomial expansion requires the polynomials to properly imitate the behavior of the GPD and extra care will be needed for practical uses.

In addition, there are various dynamic models of GPDs [45–47, 213, 214]. These models generally approximate the hadron states as expansions in the Fock space of quarks and gluons, usually with the help of light front wave functions. Since the parton distributions are the matrix elements of certain light-cone operators between hadrons, they are calculable when the hadron states are replaced with the bound states of quarks and gluons with some effective interactions. In this sense, the parameterization of GPDs is turned into the task of determining the free parameters in the wave functions and the effective interactions.

However, a general parameterization strategy that combines various physical constraints on GPDs with the lattice calculations still seems to be lacking in the literature. Therefore, in this chapter we present a program to parameterize GPDs through Universal Moment Parameterization (GUMP), which can be fitted to lattice calculations, PDFs and experimental measurements of CFFs altogether and help solve the inverse problem. We will continue exploring the moment expansion, especially the conformal moment expansion, for reasons to be explained.

5.1 Conformal moment expansion of GPDs

In this section, we will introduce the general technique and review salient features of conformal moment expansion for GPDs [42], which will be our basis for the GUMP program. We start with the physical constraints on GPDs which are important guides to finding proper parameterization strategies.

5.1.1 Physical constraints on GPDs

GPDs are simply functions F with three arguments (x, ξ, t) , which can be defined as the off-forward matrix element of light-cone operators as,

$$F(x, \xi, t) \equiv \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | O(\lambda n) | P \rangle , \quad (5.1)$$

with n the light-cone vector ($n^2 = 0$), $O(\lambda n)$ the light-cone operators of the quark/gluon and P/P' the initial/final nucleon momenta. We define the average nucleon momentum $\bar{P} \equiv (P + P')/2$, the momentum transfer $\Delta \equiv P' - P$ and the momentum transfer square $t \equiv \Delta^2$. Besides, the skewness parameter ξ is defined as $\xi \equiv -n \cdot \Delta / (2n \cdot \bar{P})$. In the forward limit, GPDs are reduced to the corresponding PDF $f(x)$, so we have

$$\lim_{\xi, t \rightarrow 0} F(x, \xi, t) = f(x) , \quad (5.2)$$

which is the most important and stringent constraint on GPDs.

The two arguments x and ξ of GPDs have support $x, \xi \in [-1, 1]$, whereas the momentum transfer squared t could take any negative value $t < 0$. Since GPDs are typically defined symmetric in ξ , we will consider only positive $\xi > 0$ without loss of generality. GPDs do not exist everywhere in the above 3D subspace. For instance, since the skewness parameter ξ corresponds to the longitudinal projection of the momentum transfer, there exists a minimum $|t|_{\min}$ for each ξ as

$$|t| \geq |t|_{\min} = \frac{4\xi^2 M^2}{1 - \xi^2} , \quad (5.3)$$

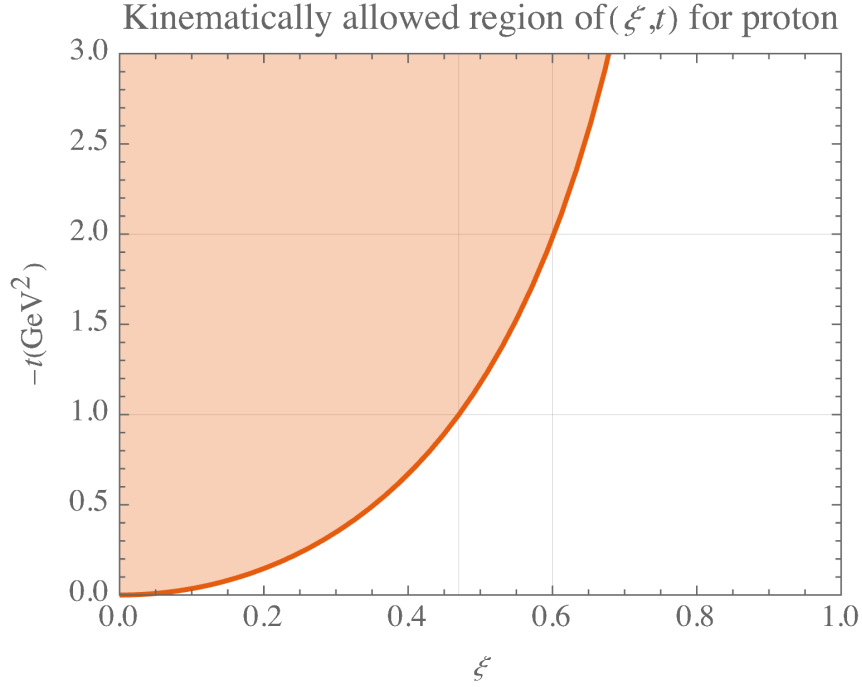


Figure 5.1: A plot showing the kinematically allowed region on the (ξ, t) plane for proton scattering with proton mass $M = 0.938$ GeV. Typically, larger momentum transfer $|t|$ is needed to reach the region with larger skewness ξ .

with M the nucleon mass such that it takes infinite momentum transfer $|t| \rightarrow \infty$ to reach $|\xi| \rightarrow 1$, as shown in Fig. 5.1. Accordingly, we restrain ourselves to the kinematically allowed region.

Furthermore, there exists non-analyticity on the (x, ξ) plane, specifically on the crossover line $|x| = \xi$. Since GPDs are essentially parton distributions with non-zero momentum transfer Δ , their physical interpretation varies accordingly [5]: with small momentum transfer in the near forward region $\xi < |x|$, GPDs can still be interpreted as the amplitudes of emitting and reabsorbing a parton in the nucleon and behave like the PDFs, whereas with large momentum transfer in the region $\xi > |x|$, they are interpreted as the amplitudes of emitting/absorbing a pair of partons and behave like the DAs. The two regions will be referred to as the PDF-like region ($\xi < |x|$) and the DA-like region ($\xi > |x|$) respectively, and they are joined together at the crossover line $\xi = |x|$ where one of the partons has zero momentum fraction. The collinear

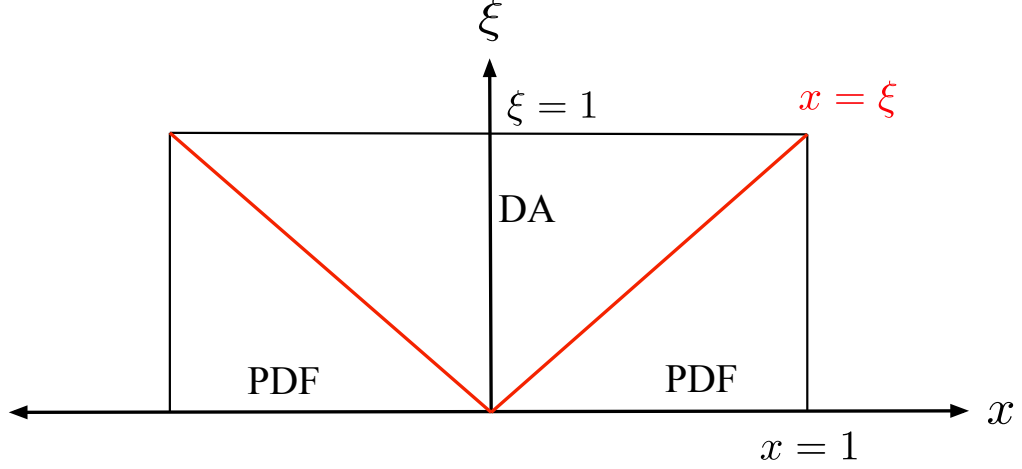


Figure 5.2: Graphical representation of the kinematical regions of GPDs on the (x, ξ) plane. The GPDs need to be continuous at the crossover line $|x| = \xi$ in order for factorization to work, while their derivatives might not exist there.

factorization requires the GPDs to be continuous at the crossover lines, but their derivatives might not be. Consequently, these crossover lines can be extremely subtle for the parameterization of GPDs on the (x, ξ) plane.

In addition, GPDs are also constrained by the polynomiality condition, which states that the Mellin moments of GPDs

$$F_n(\xi, t) \equiv \int_{-1}^1 dx x^{n-1} F(x, \xi, t) , \quad (5.4)$$

must be finite order polynomials of ξ such that [5],

$$F_n(\xi, t) = \sum_{k=0, \text{even}}^n \xi^k F_{n,k}(t) . \quad (5.5)$$

The polynomiality condition is the result of Lorentz invariance, and it holds even though the

integral in eq. (5.4) involves both the PDF-like region ($|x| > \xi$) and the DA-like region ($|x| < \xi$) in spite of the non-analyticity at the crossover lines.

5.1.2 Conformal moment expansion and scale evolution of GPDs

With the physical constraints discussed above, we now introduce the conformal moment expansion and show how the constraints can be imposed with such expansion. Consider if we expand the GPD in terms of a complete set of polynomials, denoted as $C_i(x)$, such that

$$F(x, \xi, t) = \sum_i \rho_C(x) C_i(x) F_i^C(\xi, t) , \quad (5.6)$$

with $F_i^C(\xi, t)$ the corresponding coefficient for fixed ξ and t . Since $C_i(x)$ form a complete set of basis, we have the completeness condition

$$\sum_i C_i(x) C_i(y) = \rho_C^{-1}(x) \delta(x - y) , \quad (5.7)$$

and the orthonormal condition

$$\int_{-1}^1 dx \rho_C(x) C_i(x) C_j(x) = \delta_{ij} , \quad (5.8)$$

with $\rho_C(x)$ the weight function of the polynomials $C_i(x)$. Then the $F_i^C(\xi, t)$ is simply the moment of $F(x, \xi, t)$:

$$F_i^C(\xi, t) = \int_{-1}^1 dx C_i(x) F(x, \xi, t) . \quad (5.9)$$

The above is generally true for any complete set of polynomials $C_i(x)$ and there are infinite possible choices. Therefore, one naturally would ask: which choice is the most suitable one for GPDs? An answer is motivated by the renormalization group equations of GPDs. But before getting into that, we start with a simpler case — the scale evolution of PDFs, or the Dokshitzer–Gribov–Lipatov–Altarelli–Parisi (DGLAP) equations [215–217].

The leading-order DGLAP equations for non-singlet quark distributions reads,

$$\frac{d}{d \ln Q^2} q(x, Q) = \frac{\alpha_s(Q)}{2\pi} \int_x^1 dy \frac{q(y, Q)}{y} \cdot P\left(\frac{x}{y}\right) + \mathcal{O}(\alpha_s(Q)^2), \quad (5.10)$$

with $P(z)$ the splitting function. Generally, one needs to solve the integro-differential equation in order to perform the scale evolution of PDFs. However, if one performs the Mellin transform of the DGLAP equation according to

$$f_n = \int_{-1}^1 dx x^{n-1} f(x), \quad (5.11)$$

the DGLAP equation will read,

$$\frac{d}{d \ln Q^2} q_n(Q) = \frac{\alpha_s(Q)}{2\pi} q_n(Q) \cdot P_n + \mathcal{O}(\alpha_s(Q)^2), \quad (5.12)$$

in Mellin space, where $q_n(Q)$ and P_n are the Mellin transformations of $q(x, Q)$ and $P(x)$ according to eq. (5.11) and the DGLAP equation becomes multiplicative in the Mellin moment space which holds true to all order in perturbation theory. Therefore, if one parameterizes the PDFs in the Mellin moment space, the scale evolution equations can be easily solved.

The above moment representation also applies to the construction of GPDs. Consider the

scale evolution equations of non-singlet quark GPDs in the form of [3, 102, 212, 218–225],

$$\frac{d}{d \ln Q^2} F(x, \xi, t, Q^2) = \frac{\alpha_s(Q)}{2\pi} \int_{-1}^1 \frac{dx'}{|\xi|} \left[V\left(\frac{x}{\xi}, \frac{x'}{\xi}\right) \right]_+ F(x', \xi, t, Q^2) \quad (5.13)$$

with $\left[V\left(\frac{x}{\xi}, \frac{x'}{\xi}\right) \right]_+$ the evolution kernel. It can be shown that in the forward limit, the evolution kernel is reduced to the splitting function of DGLAP equation:

$$\lim_{\xi \rightarrow 0} \frac{1}{|\xi|} \left[V\left(\frac{x}{\xi}, \frac{1}{\xi}\right) \right]_+ = P(x), \quad (5.14)$$

while for $|x|, |x'| < 1$ it is related to the Efremov-Radyushkin-Brodsky-Lepage (ERBL) kernel [226–229] for the DA scale evolution:

$$V(2x - 1, 2y - 1)_{0 < x, y < 1} = V_{\text{ERBL}}(x, y). \quad (5.15)$$

For this reason, the PDF- and DA-like regions of GPDs have also been called DGLAP and ERBL regions respectively.

Analogous to the analysis of PDFs and their Mellin moments, the GPD moments that diagonalize the right-hand side (RHS) of eq. (5.13) will renormalize multiplicatively. To the leading order, such a solution is found to be the Gegenbauer polynomials $C_j^{(\lambda)}(x)$, which satisfy [219]

$$\int_{-1}^1 \frac{dx'}{|\xi|} \left[V^{(0)}\left(\frac{x}{\xi}, \frac{x'}{\xi}\right) \right]_+ C_j^{\frac{3}{2}}\left(\frac{x}{\xi}\right) = \gamma_j C_j^{\frac{3}{2}}\left(\frac{x'}{\xi}\right), \quad (5.16)$$

with $V^{(0)}$ the leading-order evolution kernel and γ_j the corresponding anomalous dimension (eigenvalue) of each Gegenbauer polynomial. For the quark we have $\lambda = 3/2$, and for the

gluon $\lambda = 5/2$ should be used. Accordingly, the conformal moments defined as

$$\mathcal{F}_j(\xi, t) \equiv \xi^j \frac{\Gamma(\frac{3}{2})\Gamma(j+1)}{2^j \Gamma(\frac{3}{2}+j)} \int_{-1}^1 dx C_j^{\frac{3}{2}}\left(\frac{x}{\xi}\right) F(x, \xi, t), \quad (5.17)$$

are multiplicatively renormalizable, which will not hold if the higher order evolution effects are taken into account and more care will be needed then [43]. Here an extra normalization constant is multiplied to ensure that in the forward limit, the conformal moments are reduced to the Mellin moments,

$$\lim_{\xi \rightarrow 0} \mathcal{F}_j(\xi, t) = \int_{-1}^1 dx x^j F(x, \xi, t), \quad (5.18)$$

which shall be expected as the evolution equation itself will be reduced to the DGLAP equation. Also note that there exists a mismatch of conventions in the literature that the j th conformal moment $\mathcal{F}_j(\xi, t)$ actually corresponds to the n th Mellin moment with $n = j + 1$ instead of j , according to eq. (5.11). Here the two will be distinguished by having j for the conformal moments and n or s for the Mellin moments respectively, and they are related by $n = s = j + 1$.

Inspired by the leading-order evolution and with eq. (5.6), in general GPDs can be written in terms of a formal summation as [212],

$$F(x, \xi, t) = \sum_{j=0}^{\infty} \xi^{-j-1} \frac{2^j \Gamma(\frac{5}{2}+j)}{\Gamma(\frac{3}{2})\Gamma(j+3)} \left[1 - \left(\frac{x}{\xi}\right)^2\right] C_j^{\frac{3}{2}}\left(\frac{x}{\xi}\right) \mathcal{F}_j(\xi, t) \quad \text{for } |x| < \xi, \quad (5.19)$$

where we have the weight function of Gegenbauer polynomials $\rho_C^{\frac{3}{2}}(x) = 1 - x^2$ and some extra normalization constants to match normalization of the conformal moments $\mathcal{F}_j(\xi, t)$ in eq. (5.17).

The whole prefactors of the conformal moments can be collected to define

$$(-1)^j p_j(x, \xi) \equiv \xi^{-j-1} \frac{2^j \Gamma(\frac{5}{2} + j)}{\Gamma(\frac{3}{2}) \Gamma(j+3)} \left[1 - \left(\frac{x}{\xi} \right)^2 \right] C_j^{\frac{3}{2}} \left(\frac{x}{\xi} \right) \quad \text{for } |x| < \xi, \quad (5.20)$$

where an extra $(-1)^j$ factor is extracted for future convenience. Analogous to the partial wave expansion, the $p_j(x, \xi)$ s are called the conformal wave functions and $\mathcal{F}_j(\xi, t)$ the conformal partial wave amplitude respectively [42].

Also note that with the above conformal partial wave expansion, the non-analyticity at $|x| = \xi$ can be taken care of — GPDs might not have well-defined derivatives there, but the moments still exist such that the polynomiality condition can be imposed.

5.1.3 Resummation of conformal partial wave expansion

In moment expansion, the reconstruction of PDFs or GPDs requires summing over the contributions of all moments. Such summation does not always converge, and analytic methods shall be developed to perform the formal summation.

Consider the Mellin moment representation of the PDF $f(x)$ for example. In order to express the PDF in terms of its Mellin moments, one can write $f(x)$ in terms of a formal summation as

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \delta^{(n)}(x) f_{n+1}, \quad (5.21)$$

which is obviously not summable, since the $\delta(x)$ function and its n th derivatives $\delta^{(n)}(x)$ are singular. Even though the RHS of eq. (5.21) can be shown to be equivalent to the PDF $f(x)$ formally, the expression is of no practical use. Therefore, the concept of formal summation has to

be introduced, which implies that we consider the RHS of eq. (5.21) as an abstract mathematical entity regardless of the convergence of the series.

To actually reconstruct the PDF $f(x)$, the inverse Mellin transformation can be employed:

$$f(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-s} f_s ds, \quad (5.22)$$

where the Mellin moments f_s are analytically continued in the sense that s can take complex values rather than just positive integers n . This method has been adopted for the numerical analysis of PDFs in the literatures [230, 231]. Note that the two equations (5.21) and (5.22) define the same PDF $f(x)$, though they look totally different.

To resum eq. (5.19) for GPDs, one obstacle is that the Gegenbauer polynomials are only complete on the interval $[-1, 1]$, while the argument of the Gegenbauer polynomials x/ξ in the above expansion can take any real value. For this reason, we restrict the expression eq. (5.19) to the region $|x| < \xi$, while extension to the PDF-like region $|x| > \xi$ has to be defined through analytic continuation. This can be done with the Schl\"afli integral, as discussed in ref. [42, 212],

$$p_j(x, \xi) = -\frac{\Gamma(5/2 + j)}{\Gamma(1/2)\Gamma(2 + j)} \frac{1}{2i\pi} \oint_{-1}^1 du \frac{(u^2 - 1)^{j+1}}{(x + u\xi)^{j+1}}, \quad (5.23)$$

where the integration contour encircles the line segment $u \in [-1, 1]$. The analytic continued wave functions can then be derived with this integral representation and one gets [212],

$$p_j(|x| \leq \xi, \xi) = \frac{2^{j+1}\Gamma(5/2 + j)\xi^{-j-1}}{\Gamma(1/2)\Gamma(1 + j)} (1 + x/\xi) {}_2F_1 \left(-1 - j, j + 2, 2 \mid \frac{\xi + x}{2\xi} \right), \quad (5.24)$$

and

$$p_j(x > \xi, \xi) = \frac{\sin(\pi[j+1])}{\pi} x^{-j-1} {}_2F_1 \left(\begin{matrix} (j+1)/2, (j+2)/2 \\ 5/2+j \end{matrix} \middle| \frac{\xi^2}{x^2} \right). \quad (5.25)$$

These wave functions are equivalent to the Gegenbauer polynomials definition of conformal wave functions when j is integer, and they can be used for more general propose, for instance, the resummation of moments. Note that the above conformal wave functions are continuous at $|x| = \xi$ but their derivatives are not, which is consistent with the behavior of GPDs at the crossover lines.

Two techniques, the Mellin-Barnes integral [43, 44] and the Shuvaev-Noritzsch transform [208, 209] have been used in the literature to reconstruct the GPDs with moments. Both methods are shown equivalent mathematically [212], and we will focus on the former one here. The Mellin-Barnes integral is based on the Sommerfeld-Watson transformation,

$$\sum_{n=-\infty}^{\infty} (-1)^n f_n = \frac{1}{2i} \oint_C \frac{f(z)}{\sin \pi z} dz, \quad (5.26)$$

assuming $f(z)$ is analytic. The proof is straightforward, as the $\sin(\pi z)$ in the denominator of RHS leads to poles at all integers z with the residue of $\pi(-1)^n f(z)$ and the equation is simply the residual theorem applied to the RHS. Then, the formal summation in eq. (5.19) can be written in terms of the following Mellin-Barnes integral in the same manner [43, 44]

$$F(x, \xi, t) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \frac{p_j(x, \xi)}{\sin(\pi[j+1])} \mathcal{F}_j(\xi, t), \quad (5.27)$$

where the constant c should be carefully chosen, see for instance ref. [44, 212].

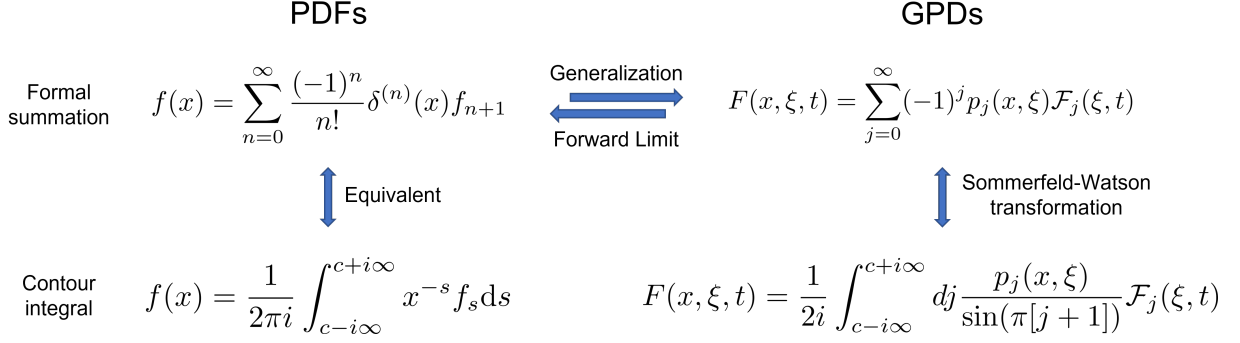


Figure 5.3: A comparison of the inverse Mellin transformation expression of the PDFs and the Mellin-Barnes integral expression of GPDs together with the corresponding formal summation expressions.

The comparison of the different methods used for PDFs and GPDs are shown in Fig. 5.3.

In principle, the Mellin-Barnes integral should reproduce the PDFs in the forward limit $\xi = 0$.

Indeed, since the conformal wave functions $p_j(x, \xi)$ s have the limit behavior

$$\lim_{\xi \rightarrow 0} p_j(x, \xi) = \frac{1}{j!} \delta^{(j)}(x), \quad (5.28)$$

the formal summation in eq. (5.19) will be reduced to eq. (5.21). However, due to the singular behavior of δ functions, the Mellin-Barnes integral will not converge in the $\xi \rightarrow 0$ limit. Instead, one can use a finite but small ξ as the regulator for the δ function, and the Mellin-Barnes integral will reproduce the PDF when $x \gg \xi$, as shown in Fig. 5.4.

Note that while the GPDs calculated above are ready for all kinds of applications, there exist some shortcuts, such as the evaluation of CFFs and scale evolution, as these calculations are easier to perform in moment space. The detailed discussion of this topic is in ref. [43].

Following the above framework, for any given set of (well-behaved) moments $\mathcal{F}_j(\xi, t)$, the corresponding GPDs can be calculated and vice versa. Practically, a finite set of parameters must be chosen to parameterize the infinite set of moments. Therefore, all the moments have to be

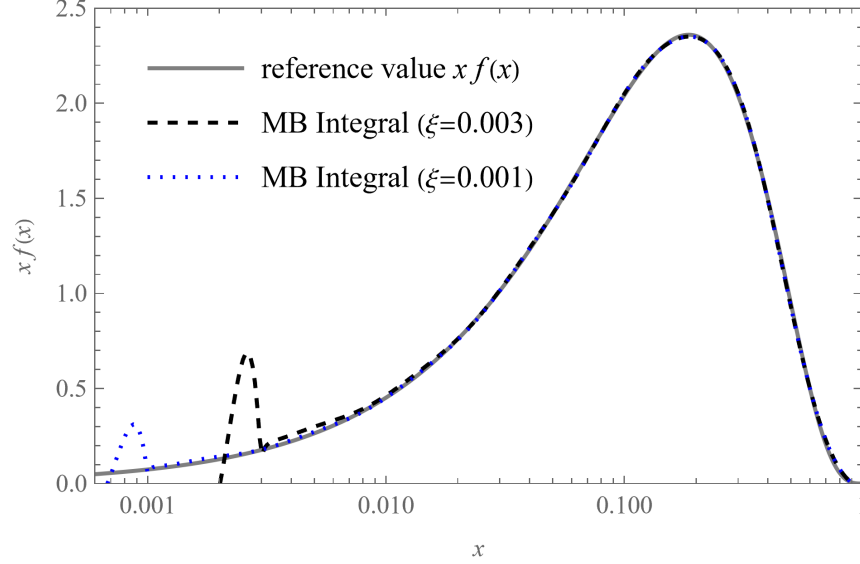


Figure 5.4: A plot showing that the Mellin-Barnes (MB) integral can indeed reproduce the reference PDF $f(x) = 18.6x^{-0.2}(1-x)^{3.5}$ when $x \gg \xi$, where ξ serves as the regulator. On the other hand, in the $x \lesssim \xi$ region, the PDFs calculated this way are strong affected by the none-zero skewness ξ and deviate from the reference value.

parameterized universally, whereas the flexibility of such parameterization can be maintained by the extensive parameter set. The GUMP program for global analysis follows this approach by parametrizing the general forms of all the moments or *universal moment parametrization*. More care will surely be needed concerning the actual parameterization of moments, which will be discussed in the next section.

5.2 Universal parameterization of GPD moments

In the last section, we show that the GPDs $F(x, \xi, t)$ can be equivalently expressed in terms of their moments $\mathcal{F}_j(\xi, t)$. While the equivalence results from the nature of the moment space representation and requires no model assumption, practically one has to choose a set of parameters to construct the GPDs. In this section, we will discuss the parameterization of moments.

5.2.1 Convention and decomposition of GPDs

We start by considering the four leading-twist GPDs H , E , $\tilde{H}$ and $\tilde{E}$, where each of them has different flavors u , d and g corresponding to the up and down quarks as well as the gluon. Although the strange quark might have sizable effects that should be considered for a more careful treatment, the flavor separation of GPDs is typically challenging due to the lack of flavor-sensitive probe [232]. Strange quark distributions will not be well constrained with just DVCS measurements, and thus we will leave this for the future work with more flavor-sensitive measurements.

For simplicity, the four different species of GPDs will be treated almost equally here. Except that the vector GPDs H and E satisfy slightly different scale evolution equations from the axial-vector ones $\tilde{H}$ and $\tilde{E}$ [43], the same parameterization will be used for all four GPDs. Although GPDs or combinations of GPDs of different species H , E , $\tilde{H}$ and $\tilde{E}$ correspond to different helicity amplitudes and might have different behaviors accordingly, knowledge of different GPDs species is limited and thus implementing different parameterization for them individually may not lead to any significant difference. Therefore, we collectively denote all GPDs as $F_{q,g}(x, \xi, t)$ where $F = \{H, E, \tilde{H}, \tilde{E}\}$ and $q = \{u, d\}$, with x the parton momentum fraction, ξ the skewness parameter and t the total momentum transfer squared and parameterize them in the same manner. We note that the skewness parameter ξ generally takes the value from -1 to 1 , but it will be considered to be positive hereafter, as GPDs are commonly defined to be symmetric in ξ , subject to their parity symmetry.

An intriguing feature of the GPDs is that they consist of two regions with totally different physical interpretation. In the PDF-like region where $x > \xi$ or $x < -\xi$, GPDs resemble the PDFs which are interpreted as the amplitudes of emitting and reabsorbing a parton (quark,

antiquark or gluon). On the other hand, in the distribution amplitude (DA)-like region where $-\xi < x < \xi$, they resemble the DAs instead and are interpreted as the amplitudes of emitting/absorbing a parton-antiparton pair. *Therefore, GPDs do not naturally have distinguishable quark and antiquark components, especially in the DA-like region.* This feature indicates that the parameterization of GPDs should be flexible in both the PDF and DA-like regions, since different physics are involved.

Motivated by such property of GPDs, one can write them as,

$$F_q(x, \xi, t) \equiv F_{\hat{q}}(x, \xi, t) \mp F_{\bar{q}}(-x, \xi, t) + F_{q\bar{q}}(x, \xi, t), \quad (5.29)$$

where the $\mp$ depends on the parity of the GPDs which takes $-$ for vector GPDs H and E and $+$ for axial vector GPDs $\tilde{H}$ and $\tilde{E}$. Here the subscript $\hat{q}$ stands for the quark distributions excluding antiquark, to be distinguished from the subscript q for the quark flavor. The three terms above describe the amplitudes corresponding to the quark, antiquark, and quark-antiquark pair respectively. Accordingly, $F_{\hat{q}}(x, \xi, t)$ and $F_{\bar{q}}(x, \xi, t)$ have support $x > -\xi$, while the $F_{q\bar{q}}(x, \xi, t)$ has support $\xi > x > -\xi$. In the semi-forward limit where $\xi = 0$ and t generally non-zero, $F_{\hat{q}}(x, \xi, t)$ and $F_{\bar{q}}(x, \xi, t)$ reduce to the quark and antiquark t -dependent PDFs with support $x > 0$, whereas $F_{q\bar{q}}(x, \xi, t)$ vanishes with its vanishing support. We note that GPDs can always be decomposed into such three parts that each term represents the behavior of GPDs in different regions respectively. The extra DA terms $F_{q\bar{q}}(x, \xi, t)$ allow extra flexibility for the parameterization of GPDs particularly in the DA-like region, as we will discuss with more details in the following section.

While the quark and antiquark GPDs $F_{\hat{q}}(x, \xi, t)$ and $F_{\bar{q}}(x, \xi, t)$ are the natural generalization of the quark and antiquark PDFs, the DA terms $F_{q\bar{q}}(x, \xi, t)$, also known as the D -terms in

some other contexts [233], only exists in the off-forward case. Such terms live in the DA-like region only, and they will be the dominant effects in the large ξ limit $\xi \rightarrow 1$ or the asymptotic limit where renormalization scale $\mu \rightarrow \infty$ [64, 183]. Since the DA terms vanish at the crossover line $x = \pm\xi$, they can always be expressed with a set of complete polynomials like Gegenbauer polynomials, which is exactly the conformal moment expansion. Therefore, each of the three terms can be expressed in terms of their conformal moments and satisfies the polynomiality condition.

Here we will focus on the quark and antiquark GPDs $F_q(x, \xi, t)$ and $F_{\bar{q}}(x, \xi, t)$, whereas the extra DA terms $F_{q\bar{q}}(x, \xi, t)$ will not be put into the global analysis for two reasons. First, as discussed in the previous work, we will consider the high energy limit where $\xi \rightarrow 0$ ideally and the DA-like region disappears in such limit. Second, since the DA terms vanish at the crossover line $x = \pm\xi$, they are hard to extract with measurements of DVCS or similar processes which provide effectively just CFFs — the DA terms contribute to the real part of the CFFs only, corresponding to the subtraction terms in the dispersion relations of CFFs [44]. Consequently, the behaviors of GPDs in the DA-like region given by the quark and antiquark GPDs $F_q(x, \xi, t)$ and $F_{\bar{q}}(-x, \xi, t)$ will be ambiguous, since one can add extra DA terms without affecting the CFFs too much in the small ξ limit. This will be discussed with more details in the next section.

We note that another notation of quark and antiquark GPDs has been used in the literatures, see for instance ref. [212]

$$\begin{aligned} F_{\hat{q}}(x > \xi, \xi, t) &= F_q(x, \xi, t) , \\ F_{\hat{\bar{q}}}(x > \xi, \xi, t) &= \mp F_q(-x, \xi, t) , \end{aligned} \tag{5.30}$$

with the same convention for the $\mp$ sign. Since the quark and antiquark GPDs are defined with

the same function $F_q(x, \xi, t)$ partitioned into two part, the full quark GPD $F_q(x, \xi, t)$ can not be written as the sum of them $F_q(x, \xi, t) \neq F_{\hat{q}}(x, \xi, t) \mp F_{\bar{\hat{q}}}(-x, \xi, t)$. Due to the lost of linearity in such GPD decomposition which will also break the polynomiality condition for quark and antiquark GPDs respectively, this definition will not be used here.

To summary, we will consider quark and antiquark GPDs $F_{\hat{q}}(x, \xi, t)$, $F_{\bar{\hat{q}}}(x, \xi, t)$ as well as the gluon GPDs $F_g(x, \xi, t)$ here with four twist-two GPDs $H, E, \tilde{H}$ and $\tilde{E}$, forming a basis with 20 GPDs. Conventionally, we also rewrite the basis by defining the valence combination: $F_{qv}(x, \xi, t) \equiv F_{\hat{q}}(x, \xi, t) - F_{\bar{\hat{q}}}(x, \xi, t)$, and thus the basis can be written with $F_{qv}(x, \xi, t)$, $F_{\bar{q}}(x, \xi, t)$ and $F_g(x, \xi, t)$ equivalently, analogous to the basis commonly chosen for PDFs global analysis in the literatures [34, 35, 234].

5.2.2 Polynomiality condition and small ξ expansion of GPD moments

To impose the physical constraints on GPDs, we start by noting that the polynomiality condition, originally proved for the Mellin moments of the GPDs, can be equivalently applied to the conformal moments. Since each Gegenbauer polynomial $C_j^{(\lambda)}(x)$ can be expressed in terms of linear combination of x^k ,

$$C_j^{(\lambda)}(x) = \sum_{k=0}^j c_{j,k}^{(\lambda)} x^k, \quad (5.31)$$

and vice versa:

$$x^j = \sum_{k=0}^j c_{j,k}^{-1,(\lambda)} C_k^{(\lambda)}(x), \quad (5.32)$$

where $c_{j,k}^{(\lambda)}$ and $c_{j,k}^{-1,(\lambda)}$ are some constant coefficients, according to eq. (5.17) we have

$$\begin{aligned}
\mathcal{F}_j(\xi, t) &\propto \int_{-1}^1 dx \, \xi^j C_j^{\frac{3}{2}}\left(\frac{x}{\xi}\right) F(x, \xi, t) , \\
&= \int_{-1}^1 dx \sum_{k=0}^j c_{j,k}^{\frac{3}{2}} \xi^{j-k} x^k F(x, \xi, t) , \\
&= \sum_{k=0}^j c_{j,k}^{\frac{3}{2}} \xi^{j-k} \int_{-1}^1 dx x^k F(x, \xi, t) .
\end{aligned} \tag{5.33}$$

Thus the polynomiality condition of Mellin moments immediately leads to the polynomiality condition of conformal moments. The reverse can be proved in the same way using eq (5.32), so the two polynomiality conditions are equivalent. Therefore, we can write the conformal moments of GPDs as

$$\mathcal{F}_j(\xi, t) = \sum_{k=0, \text{even}}^{j+1} \xi^k \mathcal{F}_{j,k}(t) , \tag{5.34}$$

where the form factors $\mathcal{F}_{j,k}(t)$ with double subscripts do not depend on ξ . While the GPDs $F(x, \xi, t)$ are completely determined by $\mathcal{F}_{j,k}(t)$, limited information on them is known. Two limit behaviors, the PDF limit $\xi = 0$ and the DA limit $\xi = 1$, can be considered separately. In the semi-forward limit $\xi = 0$, the conformal moments $\mathcal{F}_j(\xi, t)$ are given by $\mathcal{F}_{j,k}(t)$ with $k = 0$:

$$\mathcal{F}_j(\xi = 0, t) = \mathcal{F}_{j,0}(t) , \tag{5.35}$$

where we know that the GPD will be reduced to the PDF and the conformal moments correspond to the Mellin moments of the PDF if we further take $t = 0$. Thus, the $\mathcal{F}_{j,0}(t)$ at zero momentum transfer $t = 0$ is entirely determined by the forward PDF $f(x)$, which can always be determined

well from global fitting, and we have the following requirements

$$\mathcal{F}_{j,0}(t=0) = \int dx x^j f(x) . \quad (5.36)$$

Commonly in the PDF analysis, for instance in the CTEQ global analysis [34], the ansatz of the PDF $f(x)$

$$f(x) = \sum_{i=1}^{i_{\max}} N_i x^{-\alpha_i} (1-x)^{\beta_i} , \quad (5.37)$$

is chosen, whereas further improvements can be done based on this. This ansatz can be immediately transformed to the moment space, and we have the following ansatz for the Mellin moments of the PDF

$$\mathcal{F}_{j,0}(t=0) = \int dx x^j f(x) = \sum_{i=1}^{i_{\max}} N_i B(j+1-\alpha_i, 1+\beta_i) , \quad (5.38)$$

which will be our building blocks for the GPD at $\xi = 0$ or small ξ .

On the other hand, in the DA limit $\xi = 1$, we have

$$\mathcal{F}_j(\xi=1, t) = \sum_{k=0, \text{even}}^{j+1} \mathcal{F}_{j,k}(t) , \quad (5.39)$$

which could have completely different behavior than the forward case. Actually, they must behave differently. The reason is that the formal summation in eq. (5.19) with the moments in eq. (5.38) does not converge, and thus it can only be resummed by the Mellin-Barnes integral method or other equivalent methods. Consequently, GPDs corresponding to such moments do not generally vanish at $|x| = \xi$. While it is expected as GPDs do not generally vanish at the crossover line, for large ξ , especially for ξ close to 1, such moments lead to GPDs that do not vanish at the end

point $|x| = 1$. Therefore, the moments at large ξ must converge faster than the moments in eq. (5.38) in order to have proper end-point behavior.

Due to the lack of constraints from physically observable to the large ξ behavior of GPDs, we will restrain ourselves to the region with small ξ to avoid such complexity, whereas the large ξ behavior (which also corresponds to the large t behavior as shown in Fig. 5.1) of GPDs requires separate studies. Then, though the moments of the GPDs are generally unknown, in the small ξ region, the first few terms will dominate:

$$\mathcal{F}_j(\xi, t) = \mathcal{F}_{j,0}(t) + \xi^2 \mathcal{F}_{j,2}(t) + \mathcal{O}(\xi^4) . \quad (5.40)$$

In the region with small ξ , it is assumed that the ansatz used for the PDF in eq. (5.38) applies not only to the leading moment with $k = 0$, but also the first few moments with $k = 2, 4, \dots$ which can certainly break down as k gets large.

Note that the small ξ expansion does not apply to the GPDs themselves, since GPDs are not analytic at the crossover line $|x| = \xi$. However, the moments of GPDs are always polynomials of ξ , and they are always regular and expandable in the small ξ region, which, on the other hand, need proper resummation in order to recover the GPDs.

5.2.3 Regge trajectory and t -dependence of GPDs

The last piece to complete the universal moment parameterization is the t -dependence of GPDs. In the framework of moment expansion, the t -dependence is encoded in all the form factors $\mathcal{F}_{j,k}(t)$ and thus the t -dependence of the form factors $\mathcal{F}_{j,k}(t)$ are crucial. While each term can be in principle fitted to the experimental measurements or lattice calculations, it is not

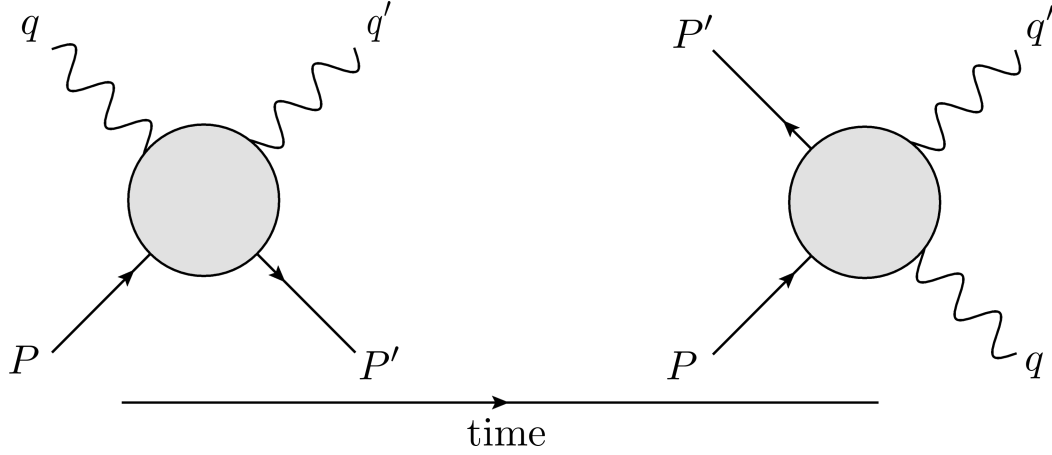


Figure 5.5: Crossing symmetry shown in terms of Feynman diagrams. The two processes are related to the same set of Feynman diagrams by simply exchanging the initial and final particles. The right plot corresponds to the t -channel process of the left one.

practical to do so, because the resummation procedure requires a set of $\mathcal{F}_{j,k}(t)$ with j goes to infinity. Neither experimental measurements nor lattice calculations allow us to access all the moments, especially the higher order ones, and thus some extrapolation methods that take the known moments into account and allow us to extrapolate the higher moments will be necessary. Motivated by that, we introduce the Regge trajectory in this subsection and discuss how it can help build the t -dependence of GPDs.

The starting point is the crossing symmetry, with which processes of different channels but with the same Feynman diagrams can be connected, as shown in Fig. 5.5. The scattering amplitude $\mathcal{T}$ can be written with these Feynman diagrams regardless of which channel one chooses as a function of the Mandelstam variables s, t, u , ignoring the spin degrees of freedom for simplicity. The Mandelstam variables s, t, u for $2 \rightarrow 2$ processes are defined as $s \equiv (P + q)^2$, $t \equiv (P' - P)^2$ and $u \equiv (P - q')^2$. The 3 Mandelstam variables s, t, u satisfy the condition $s + t + u = P^2 + P'^2 + q^2 + q'^2$, with which we can eliminate the variable u for simplicity.

One key hypothetical property in Regge theory is the duality [235], which states that the scattering amplitude is given by the sum of all t -channel resonances and also equals to the sum of all s -channel resonances. Therefore, the t -dependence of the scattering amplitude can be tentatively written as

$$\mathcal{T}(s, t, u) = \sum_{\text{res.}} \frac{g_t(s, u)}{t - m_{\text{res}}^2}, \quad (5.41)$$

with $g_t(s, u)$ the residuals of the resonance poles and the RHS sums over all the t -channel resonances. In principle, the scattering amplitude can then be written directly with all the resonances. However, there are potentially infinite resonances and summing over their contributions are highly non-trivial. Thus, one will need to collect the poles together in order to sum over them, which leads to the Regge theory [236, 237].

Regge theory is, in short, a theory about complex angular momentum J (different from the conformal spin j). The key observation is that the mass of the resonances increases with increasing angular momentum J on the hadron spectrum. Thus, a linear Regge trajectory usually written as $\alpha(t) \equiv \alpha + \alpha' t$ with $\alpha(t)$ the angular momentum and t the mass squared, also known as the Reggeon. The major assumption in Regge theory is that the resonances and their analytic continuation to complex J are the only singularities in the scattering amplitudes. Then the scattering amplitude can be expressed in terms of all the Regge trajectories or Reggeons.

Note that the scattering amplitude $\mathcal{T}(s, t, u)$ is not explicitly a function of J . To separate the contributions of different J , it can be written in terms of the partial wave expansion of a t -channel scattering as

$$\mathcal{T}(s, t, u) = 16\pi \sum_{J=0}^{\infty} (2J+1) \mathcal{T}_J(t, u) P_J(\cos \theta_t). \quad (5.42)$$

Then the partial wave amplitudes $\mathcal{T}_J(t, u)$ will have poles at $J = \alpha(t)$ for each Regge trajectory according to the assumption made for Regge theory, and they can be written with these poles as

$$\mathcal{T}_J(t, u) = \sum_{\text{Reg.}} \frac{r(t, u)}{J - \alpha(t)} , \quad (5.43)$$

which sums over all the Regge trajectory and $r(t, u)$ is the corresponding residual. Note that eq. (5.41) and eq. (5.43) are equivalent descriptions of those resonance poles. Their relation is intuitive — eq. (5.41) approaches those poles with a fixed integer J in the M^2 direction, whereas eq. (5.43) approaches those poles with fixed M^2 in the J direction.

Then, we can discuss the t -dependence of the GPDs or equivalently the form factors $\mathcal{F}_{j,k}(t)$. Since they are essentially the nucleon matrix elements of certain quark (gluon) operators which are closely related to the quark (gluon)-nucleon scattering amplitude, the above arguments also apply to them, at least qualitatively. Consider the partial wave expansion in eq. (5.42). In the higher energy limit $s \rightarrow \infty$ and $\xi \rightarrow 0$, the $\cos(\theta_t)$ can be expressed in terms of ξ as

$$\cos(\theta_t) \propto -\frac{1}{\xi} \rightarrow \infty . \quad (5.44)$$

Then with the asymptotic behavior of the Legendre polynomials,

$$\lim_{z \rightarrow \infty} P_J(z) \propto z^J , \quad (5.45)$$

one finds that the partial wave amplitude with angular momentum J has the asymptotic behavior of ξ^{-J} in the high energy limit. Then it can be quickly identified that the term $\mathcal{F}_{j,k}(t)$ corresponds

to the angular momentum $J = j + 1 - k$, which leads to

$$\mathcal{F}_{j,k}(t) = \sum_{\text{Reg.}} \frac{r_{j,k}(t)}{j + 1 - k - \alpha(t)}, \quad (5.46)$$

with $r(t)$ the corresponding residuals of the Regge poles. Again, if we stay in the region where momentum transfer $|t|$ is small, the leading lightest Regge trajectory is sufficiently good to describe the t -dependence of the GPDs, while more terms can always be added to make it more flexible. Therefore, we reach the final proposed ansatz for the moments of GPDs as

$$\mathcal{F}_{j,k}(t) = \sum_{i=1}^{i_{\max}} N_{i,k} B(j + 1 - \alpha_{i,k}, 1 + \beta_{i,k}) \frac{j + 1 - k - \alpha_{i,k}(0)}{j + 1 - k - \alpha_{i,k}(t)} \beta_{i,j,k}(t), \quad (5.47)$$

with $k = 0, 2$ which breaks down at large k . Note that the residuals $\beta_{i,j,k}(t)$ are smooth functions of t which satisfies $\beta_{i,j,k}(t = 0) = 1$ to restore the corresponding forward limit in eq. (5.38). For the valence quark distributions we simply set the residual function $\beta_{i,j,k}(t) = 1$, since their algebraic decaying behaviors in $|t|$ can be parameterized well by the Regge trajectory. However, for the sea distributions, it has been observed that in high energy processes, the different cross-sections drop exponentially as the momentum transfer $|t|$ increase e.g., for DVCS [108, 109], which differs from the power-law behavior indicated by the Regge theory. Therefore, we incorporate the extra exponential behavior into the $\beta(t)$ to set $\beta_{i,j,k}(t) = \exp(bt)$ with b a constant slope parameter.

In the end, note that the proposed form in eq. (5.47) is equivalent to the ansatz in KM model [44] and the similar proposed forward-like function in the dual parameterization [212] except for the $SO(3)$ partial wave expansion that we dropped for simplicity. However, the main goal of this

work is to focus on both sea and valence distributions and the corresponding constraints from lattice calculation in for instance [194, 195, 198–202] besides the experimental measurements which only partially constrain the GPDs.

5.3 A demonstrative example: t -dependent PDF and proton tomography

Following the above global analysis strategy for GPDs, in this section we show an example to apply it to the $\xi = 0$, t -dependent PDF and proton tomography from various constraints. Note that once the parameterization is specified, in principle one just needs to put all the constraints together, and try fitting to them with for instance, χ^2 minimization. However, the amount of parameters needed for a global fitting is huge — we needed at least 4 parameters for each different GPDs (H , E , $\tilde{H}$ and $\tilde{E}$) and each flavors (u , d , g and even more) at $\xi = 0$ and even more for non-zero skewness ξ . Therefore, as a proof of principle we start by considering the H and E GPDs of u and d quarks at $\xi = 0$, which are the so-called t -dependent PDF $H_{u/d}(x, t)$ and $E_{u/d}(x, t)$. These two quantities are necessary to understand the angular momentum densities of u and d quarks [4]

$$J_{u/d}(x, t) = \frac{1}{2}x \left(H_{u/d}(x, t) + E_{u/d}(x, t) \right) , \quad (5.48)$$

which will be our main focus in this section.

We note that in the zero skewness limit the evolution of GPDs will be reduced to that of PDFs, which is easy to perform. All the observables used are either calculated at the factorization scale $\mu_f = 2 \text{ GeV}$ or evolved to the same scale in order to be consistent. Therefore, we suppress all the factorization scale when presenting the results. Leading order evolution of GPDs in moment space can be done within our program, which is, however, not used for this analysis,

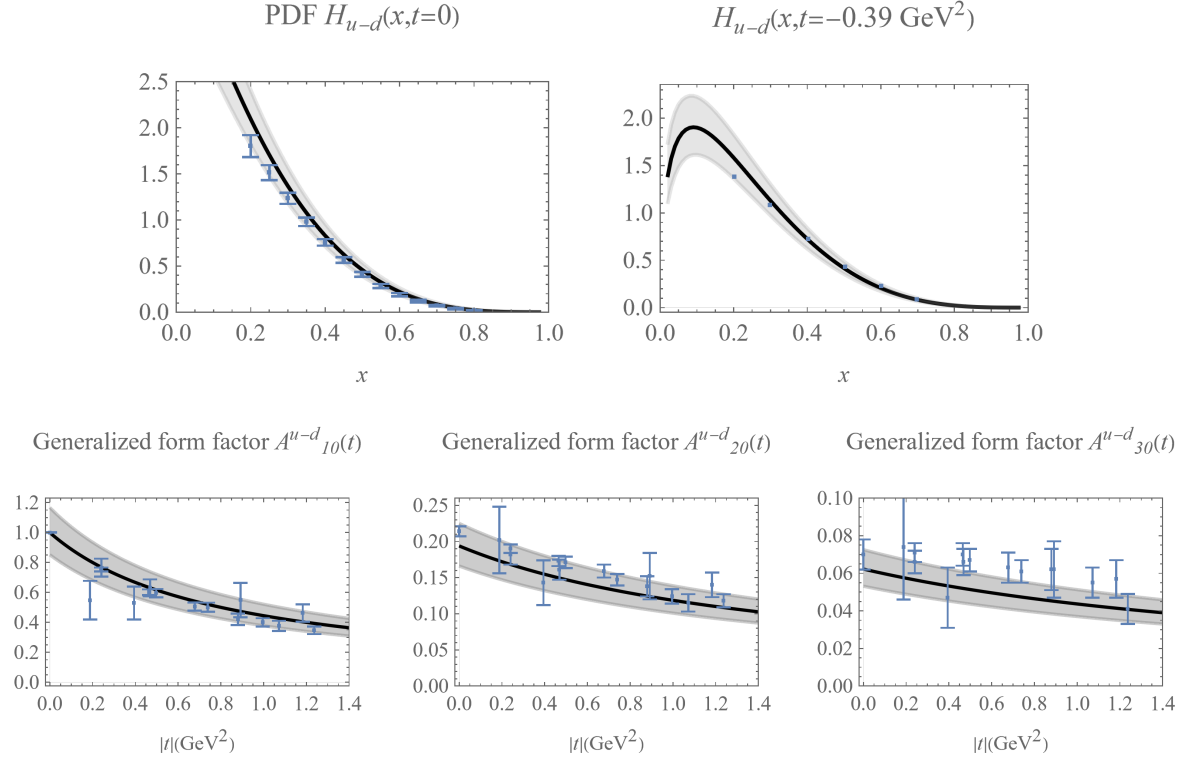


Figure 5.6: A simultaneous fit to the forward PDF $H_{u-d}(x)$ (corresponding to the PDF $q_{u-d}(x)$) [34, 204], lattice calculated PDF $H_{u-d}(x, t = -0.39 \text{ GeV}^2)$ [201] and generalized form factors $A_{10}^{u-d}(t)$, $A_{20}^{u-d}(t)$ and $A_{30}^{u-d}(t)$ [194]. The solid lines are the central value of $H_{u-d}(x, t)$, the gray bands are the regions within 1σ uncertainty and the dots are measurements that are fitted to. The χ^2 per d.o.f. is 3.0 for our 4 parameters fitting. The parameters from the fitting read $N_{H_{\text{vec}}} = 4.518$ (327), $\alpha_{H_{\text{vec}}} = -0.027$ (26), $\beta_{H_{\text{vec}}} = 3.269$ (76) and $\alpha'_{H_{\text{vec}}} = 1.286$ (51).

as those results are taken at the same scale already.

5.3.1 Fitting the valence t -dependent PDF

In this subsection, we use the proposed form in eq. (5.47) for the t -dependent PDF $H_{u/d}(x, t)$ and $E_{u/d}(x, t)$. We take $k = 0$ in the semi-forward $\xi = 0$ limit, and set $i_{\text{max}} = 1$ for the simplest ansatz. Since in lattice QCD the flavor singlet $u + d$ and non-singlet $u - d$ combinations are commonly separately calculated, we apply such ansatz for the $H/E_{u+d}(x, t)$ and $H/E_{u-d}(x, t)$ respectively.

We will neglect the sea contributions in the fitting to lattice results here since the sea contributions to GFFs are suppressed compared to the valence contributions. According to the CTEQ18 PDFs [34], the moments of sea distributions are about one order of magnitude smaller than the valence ones, where similar suppression is expected for the GFFs. Therefore, one should not expect strong constraints to the sea distributions from lattice results, and the experimental measurements are crucial to get the sea distributions.

For the t -dependent PDF $H_{u/d}^{\text{val}}(x, t)$, we can simultaneously fit to the forward PDF $q(x)$, for which we choose the recommended CTEQ18qed results [34, 204], the quark GFFs [194] and lattice calculated isovector t -dependent PDF $H_{u-d}^{\text{val}}(x, t)$ [201]. We used the ManeParse [238] Mathematica package to read the CTEQ18qed PDF and the MINUIT2 minimizer [239] for χ^2 analysis, and the results are given in Fig. 5.6, where the generalized form factors are defined by [4, 5]

$$\begin{aligned} \int_{-1}^{+1} dx x^{n-1} H(x, \xi, t) &= \sum_{i=0, \text{even}}^{n-1} (-2\xi)^i A_{ni}(t) + (-2\xi)^n C_{n0}(t) \big|_{n \text{ even}} , \\ \int_{-1}^{+1} dx x^{n-1} E(x, \xi, t) &= \sum_{i=0, \text{even}}^{n-1} (-2\xi)^i B_{ni}(t) - (-2\xi)^n C_{n0}(t) \big|_{n \text{ even}} , \end{aligned} \quad (5.49)$$

thus only $A_{n0}(t)$ and $B_{n0}(t)$ survive in the semi-forward limit $\xi = 0$. Note that the χ^2 per degree of freedom (d.o.f.) is 3.0 for our 4 parameters fitting. The major source of χ^2 is the discrepancy between lattice calculated form factors and the global fitted PDFs — the moments of PDF from global fitting are significantly lower than the lattice calculations. Therefore, the χ^2 per d.o.f. indicates the systematic uncertainties including the effects of unphysical pion mass of $m_\pi = 496$ MeV, omitted disconnected diagrams and other effects in lattice calculation that are not taken into account. Also, it will be helpful to consider fitting to the experiment measurements

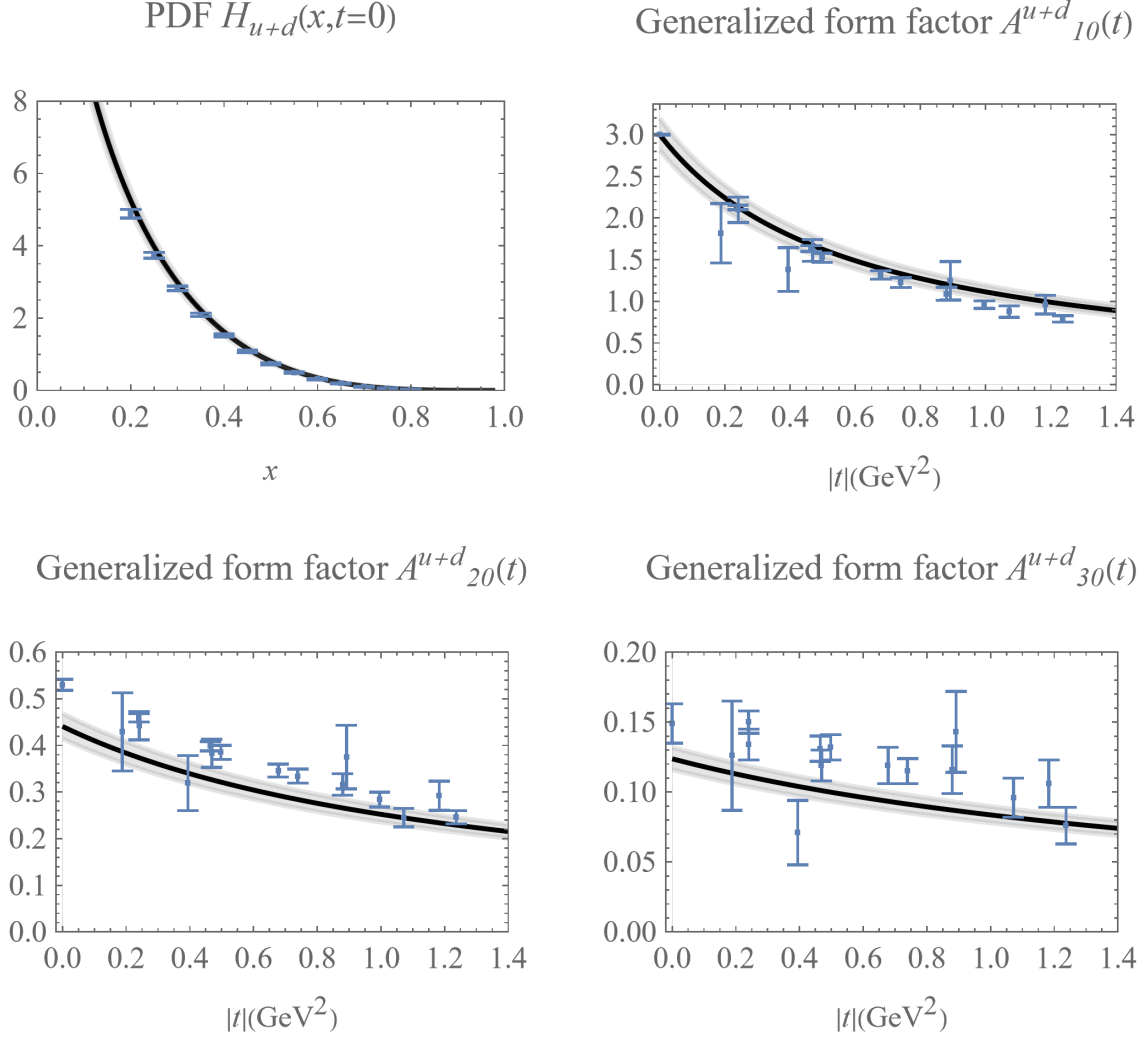


Figure 5.7: A simultaneous fit to the forward PDF $H_{u+d}(x)$ (corresponding to the PDF $q_{u+d}(x)$) [34, 204] and generalized form factors $A_{10}^{u+d}(t)$, $A_{20}^{u+d}(t)$ and $A_{30}^{u+d}(t)$ [194]. The solid lines are the central value of $H_{u+d}(x, t)$, the gray bands are the regions within 1σ uncertainty and the dots are measurements that are fitted to. The χ^2 per d.o.f. is 8.47 for our 4 parameters fitting. The parameters from the fitting read $N_{H_{\text{scal}}} = 8.340$ (263), $\alpha_{H_{\text{scal}}} = 0.210$ (10), $\beta_{H_{\text{scal}}} = 3.584$ (43) and $\alpha'_{H_{\text{scal}}} = 1.338$ (27).

of elastic form factors as described in ref. [240].

Similar fitting can be done for the isoscalar valence distributions $H_{u+d}^{\text{val}}(x, t)$ as shown in Fig. 5.7. However, since the isoscalar t -dependent PDF $H_{u+d}^{\text{val}}(x, t)$ is not calculated in ref. [201], we fit to the forward PDF $H_{u+d}^{\text{val}}(x, t=0)$ and the lattice isoscalar form factors $A_{10}^{u+d}(t)$, $A_{20}^{u+d}(t)$

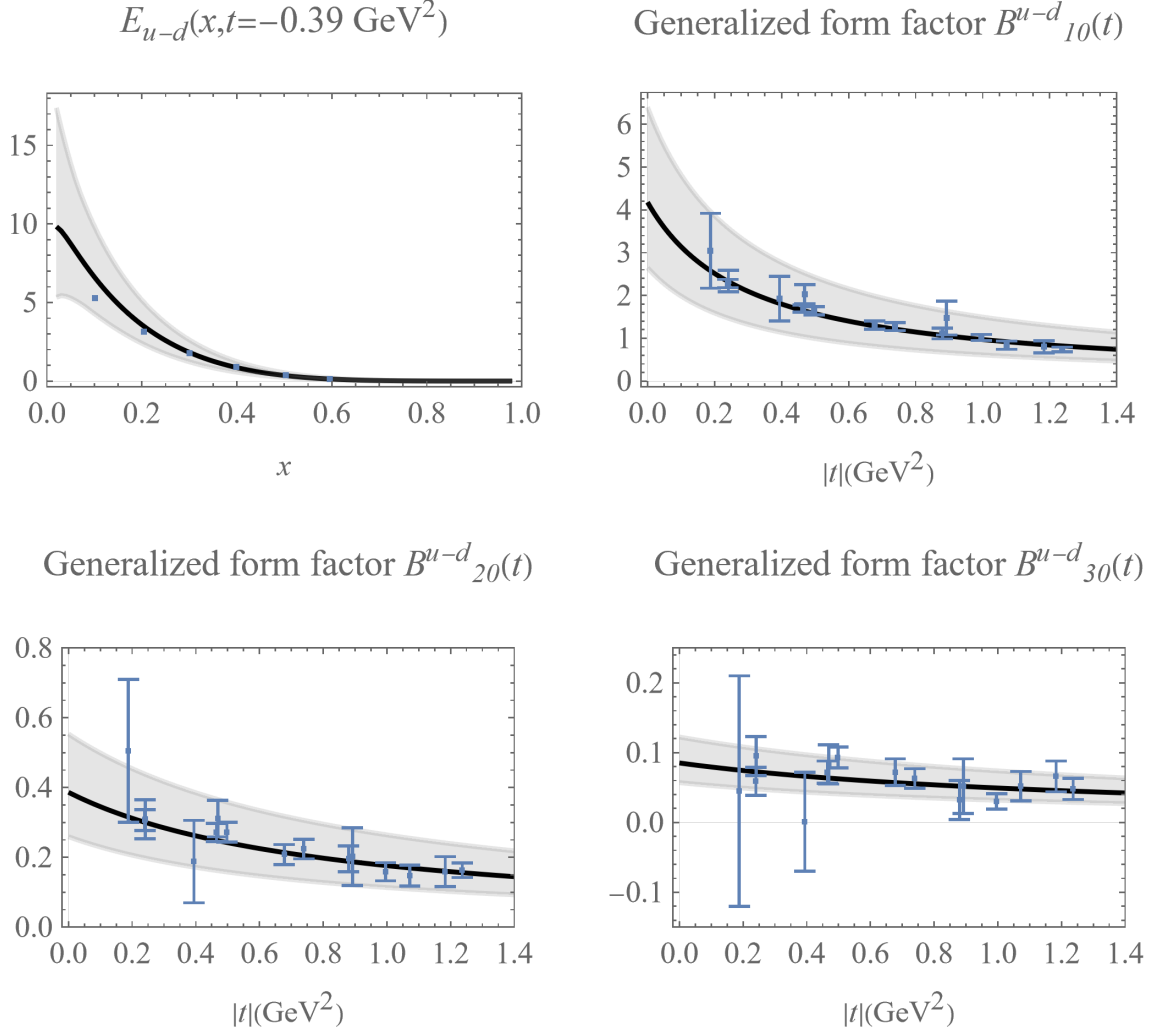


Figure 5.8: A simultaneous fit to the t -dependent PDF $E_{u-d}(x)$ [201] and generalized form factors $B_{10}^{u-d}(t)$, $B_{20}^{u-d}(t)$ and $B_{30}^{u-d}(t)$ [194]. The solid lines are the central value of $E_{u-d}(x, t)$, the gray bands are the regions within 1σ uncertainty and the dots are measurements that are fitted to. The χ^2 per d.o.f. is 0.8 for our 4 parameters fitting. The parameters from the fitting read $N_{Evec} = 6.770$ (1.354), $\alpha_{Evec} = 0.437$ (62), $\beta_{Evec} = 4.527$ (240) and $\alpha'_{Evec} = 1.864$ (222).

and $A_{30}^{u+d}(t)$ [194]. Again, the major source of χ^2 is the discrepancy between lattice calculated form factors and the global fitted PDFs which do not indicate the failure of the parameterization form used and can be mitigated if the systematical uncertainties are under control.

For the t -dependent PDF $E_{u/d}^{val}(x, t)$, the forward constraints from PDFs no longer exist, and we have to rely on the lattice calculated t -dependent GPDs $E_{u/d}^{val}(x, t)$ [201]. Though the isoscalar

GPD $E_{u+d}(x, t)$ is not calculated there, there is strong evidence showing that the isoscalar form factors $B_{u+d}(t)$ s are consistent with zero [194, 198]. Therefore, we will assume $E_{u+d}^{\text{val}}(x, t) = 0$ with which the flavor separation becomes trivial. The results for the $E_{u-d}^{\text{val}}(x, t)$ are shown in Fig. 5.8. The χ^2 per d.o.f. is about 0.8, which is really due to the lack of precision in the lattice results, since there are no constraints from better determined forward PDFs. Therefore, the uncertainties of the GPD $E_{u-d}^{\text{val}}(x, t)$ are significantly larger.

5.3.2 The angular momentum sum rule and proton tomography

With the fitted t -dependent PDFs $H_{u/d}(x, t)$ and $E_{u/d}(x, t)$, interesting information of the nucleon 3D structure can be revealed. On the one hand, as mentioned at the beginning of the section, these GPDs carry information about the transverse angular momentum densities of quarks. On the other hand, as first pointed out by M. Burkhardt in refs. [28, 241], the GPDs in the $\xi \rightarrow 0$ limit can be Fourier transformed in the transverse space to get the impact parameter space parton distribution which describe the 3D structure of the nucleon. In this section, we will explore the proton 3D structure with the above fitted GPDs.

In Fig. 5.9, we plot the transverse angular momentum densities of quark with eq. (5.48). Note again that, this plot needs to be further improved with the sea distributions, whose contributions to the angular momentum are, however, mostly in the small x region and one order of magnitude smaller than the valence ones. The contributions from the d quark are always negative according to the fitted results, which are essentially the consequence of the assumption $E_{u+d}(0) = 0$ supported by the lattice calculations of quark and gluon generalized form factors.

In addition to the parton transverse angular momentum densities which only make use of

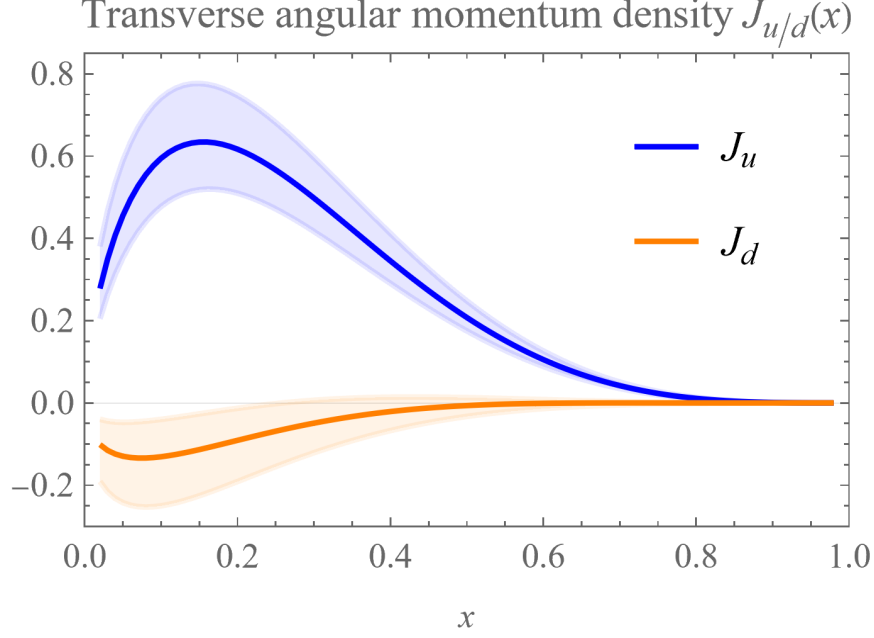


Figure 5.9: A plot of the quark transverse angular momentum densities $J_{u/d}(x)$ with the fitted GPDs $H_{u/d}(x, t)$ and $E_{u/d}(x, t)$. Most of the uncertainties are from the GPDs $E_{u/d}(x, t)$ which are less constrained without corresponding forward PDFs.

the GPD in the forward limit $\xi = t = 0$, we can also take advantage of the t -dependent of the GPDs and study the impact parameter space distribution of the quarks. For instance, the quark densities in unpolarized protons are given by the Fourier transform of the GPDs $H_{u/d}(x, t)$:

$$\rho_q(x, \mathbf{b}) = \int \frac{d^2\Delta}{(2\pi)^2} e^{-i\Delta \cdot \mathbf{b}} H_q(x, -\Delta^2) = \mathcal{H}_q(x, \mathbf{b}) , \quad (5.50)$$

where $\mathcal{H}_q(x, \mathbf{b})$ is defined as the 2-dimensional Fourier transformation of $H_q(x, -\Delta^2)$. Then we plot the quark distribution with the above expression and the fitted GPDs in Fig. 5.10. The quark densities typically get more concentrated but suppressed for increasing x , since the number densities should decrease for quarks with larger momentum due to momentum conservation, and the less deflected they will be.

Another set of densities that is of interest is the quark densities in a polarized proton. As

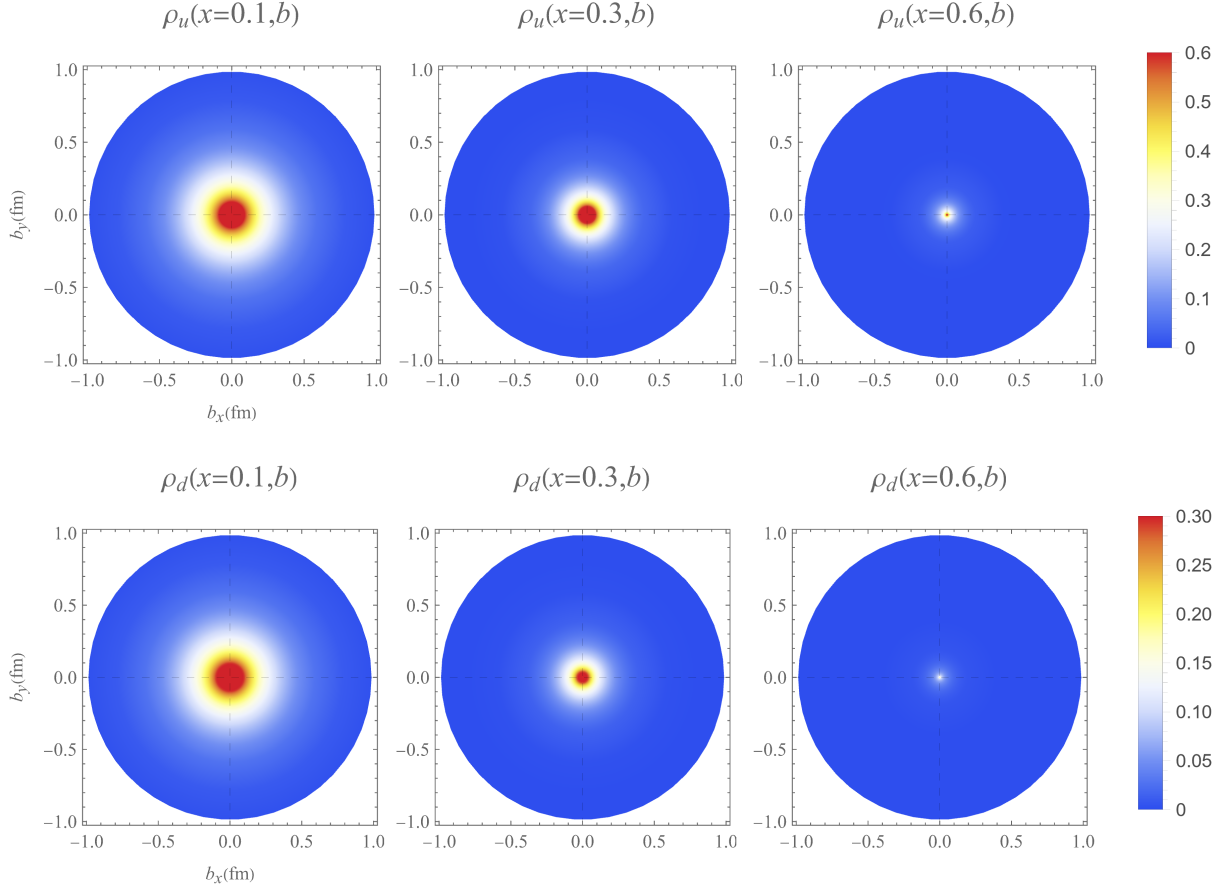


Figure 5.10: Plots of quark densities $\rho_q(x, \mathbf{b})$ for both u (the upper row) and d (the lower row) quarks in unpolarized proton with increasing momentum fraction x . The densities typically get more concentrated but suppressed for increasing x .

we discussed before, it is crucial to introduce the intrinsic transverse distributions to remove the contributions from center-of-mass motion, for which we define the intrinsic quark densities as

$$\begin{aligned}
\rho_{q,\text{In}}^X(x, \mathbf{b}) &= \int \frac{d^2\Delta}{(2\pi)^2} e^{-i\Delta \cdot \mathbf{b}} \left[H_q(x, -\Delta^2) + \frac{i\Delta_y}{2M} (H_q(x, -\Delta^2) + E_q(x, -\Delta^2)) \right], \\
&= \mathcal{H}_q(x, \mathbf{b}) - \frac{1}{2M} \frac{\partial}{\partial b_y} (\mathcal{H}_q(x, \mathbf{b}) + \mathcal{E}_q(x, \mathbf{b})), \\
&= \mathcal{H}_q(x, \mathbf{b}) - \frac{1}{2M} \frac{b_y}{|\mathbf{b}|} \frac{\partial}{\partial |\mathbf{b}|} (\mathcal{H}_q(x, \mathbf{b}) + \mathcal{E}_q(x, \mathbf{b})),
\end{aligned} \tag{5.51}$$

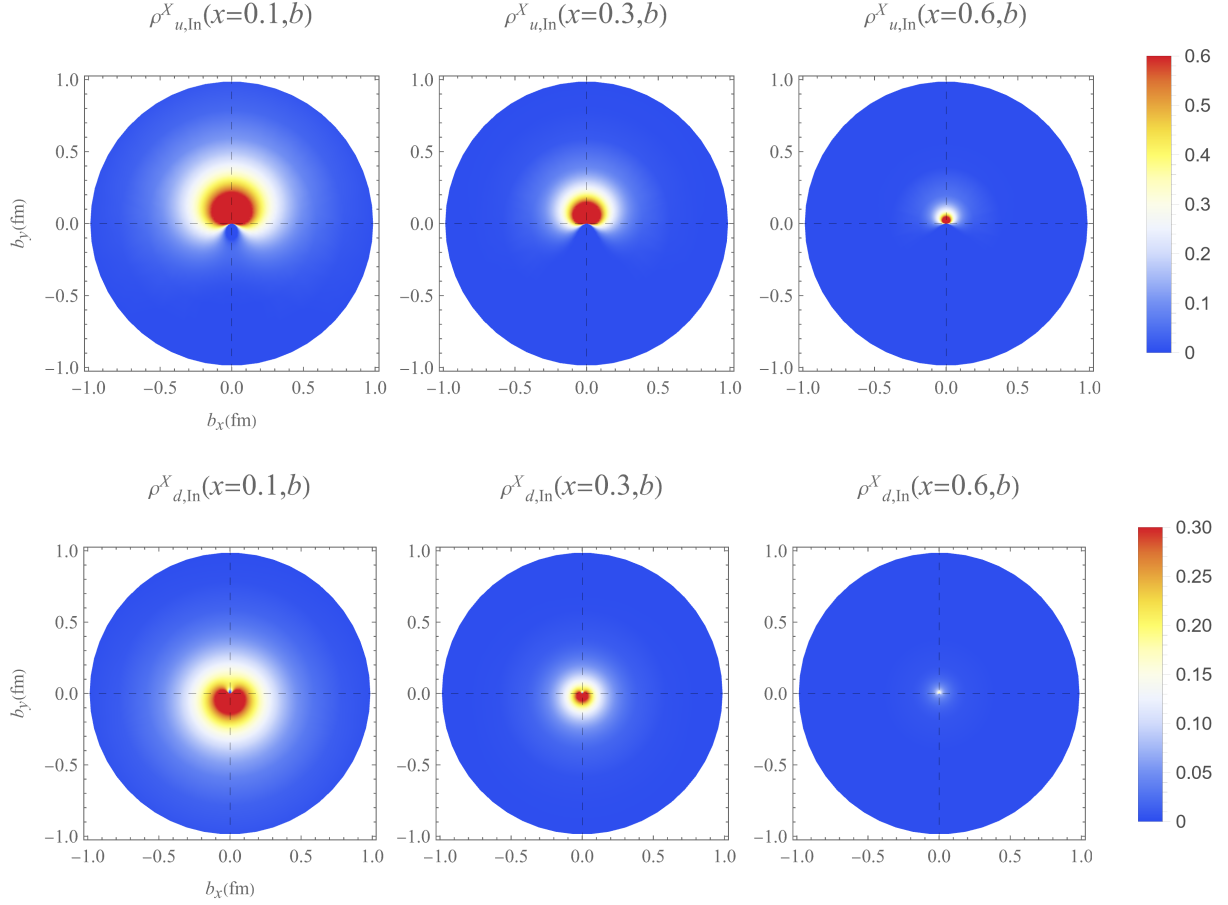


Figure 5.11: Plots of the intrinsic quark densities $\rho_{q,\text{ln}}^X(x, \mathbf{b})$ for both u (the upper row) and d (the lower row) quarks in transversely polarized proton (in the X direction) with increasing momentum fraction x . Both the u and d quark densities are shifted in the y direction which contribute to the transverse angular momentum J^X , while the u contributions are positive ($+y$ direction) and the d contributions are negative ($-y$ direction).

with which one has

$$J_q^X(x) = \int d^2\mathbf{b} (b^y \times xP^+) \rho_{q,\text{ln}}^X(x, \mathbf{b}) . \quad (5.52)$$

With the above fitted GPDs $H_{u/d}(x, t)$ and $E_{u/d}(x, t)$, the intrinsic quark densities in transversely polarized nucleon can be studied as well as shown in Fig. 5.11. While the quark densities in a polarized proton get more concentrated but suppressed for increasing x just like in the unpolarized

case, we find that clearly the quarks have non-zero transverse displacement, which contributes to the transverse angular momentum of the nucleon along with their large longitudinal momenta.

5.3.3 Sea distributions and experimental observables

Here we discuss the effects of the sea distributions, which are important sources of systematic uncertainties in the above analysis, since in principle both sea and valence distributions contribute to the generalized form factors. As mentioned above, the sea distributions can be dropped because they are mostly in the small x regions and their contributions to the form factors are suppressed compared to that of the valence distributions. On the other hand, this indicates that even if we consider the sea distributions in the above analysis, they will not be constrained as much. Therefore, it is crucial to also include the experimental observables, especially the ones with small ξ (since it can not be zero) in order to constrain the sea distributions. Then, we are inevitably faced with the effects of both non-zero skewness ξ and the sea distributions simultaneously.

Fortunately, in the KM model [44], they show that the sea distributions with such parameterization method can successfully fit to the experimental data which proves the flexibility of this parameterization when extending to non-zero ξ . In this sense, our results here with the valence distributions are complementary to the sea distributions obtained in the KM models. On the other hand, GPDs at non-zero ξ are faced with the inverse problem [203], as the experimental measurements of CFFs alone cannot determine the x -dependence at non-zero ξ . The solution (compromise) in the KM model is to model the x -dependence of the GPDs at non-zero ξ with the same parameters as the PDFs, except for a different overall normalization constant, which

might be too constraining. However, there have been results of lattice calculated x -dependence of GPDs with LaMET [199] at non-zero ξ . These results, even just qualitatively, can be very helpful in mitigating or resolving the inverse problem at non-zero ξ .

Therefore, to obtain the GPDs at non-zero ξ reliably, we need to extend our present results with sea distributions and fit to both experimental observables and lattice results at non-zero ξ as well. A global fitting program is then necessary for this purpose, which will be studied in the next section.

5.4 Global analysis with DVCS and extracted GPDs

In the previous sections, we discussed how GPDs can be parameterized in terms of their moments based on the technique that has been systematically developed in ref. [42] and used in for instance ref. [44]. In this section, we will discuss how we apply such framework to the GPDs at non-zero skewness, including GPDs of various different species and flavors. To find the set of parameters that fit to the measurements best, we employ the *iminuit* interface of *Minuit2* [242, 243] as the minimizer.

5.4.1 Empirical constraints and inputs of the global analysis

The challenge in the global analysis is due to the lack of off-forward constraints. Therefore, extra empirical constraints are still needed for the extraction of GPDs. For the ξ -dependent terms, the simplest choice is to set them to be proportional to the leading terms, as has been done in ref. [44]:

$$\mathcal{F}_{j,k}(t) = R_k \mathcal{F}_{j-k,0}(t) , \quad (5.53)$$

with R_k the ratio between them. Here we have just one parameter R_2 for the ξ^2 terms for each GPD $F(x, \xi, t)$ that accounts for its extra ξ -dependence. As for the residual function $\beta(t)$, two $\beta(t)$ s of different b slope are embedded in the parameterization of the vector GPDs $H_{\bar{q},g}$ and the axial vector GPDs $\tilde{H}_{\bar{q},g}$ respectively, with $q = \{u, d\}$ for sea quarks. We note that due to the lack of flavor-sensitive probe, we assume the same b slope for the gluon and sea quarks of different flavor and also the same Regge slope α' for them which is fixed to be $\alpha'_{\bar{q},g} = 0.15$ from the pomeron trajectory [244].

Another difficulty in GPD parameterization is about the flavors and species of GPDs. While the extraction of one 3D function is already challenging, the simultaneous extraction of GPDs with multiple flavors and species is just harder. Two of the four different species of GPDs, E and $\tilde{E}$, do not have corresponding forward PDFs, unlike H and $\tilde{H}$ which reduce to the PDF $f(x)$ and helicity PDF $\Delta f(x)$ respectively, adding extra difficulties to their extraction. Consequently, we have to reduce the number of parameters associated to these two GPDs due to the lack of constraints. To do so, we set the two valence quark distributions to be proportional ($E_{u_V} \propto E_{d_V}$ and $\tilde{E}_{u_V} \propto \tilde{E}_{d_V}$) and the sea quark as well as the gluon distributions to be proportional to the corresponding H and $\tilde{H}$ GPDs.

We note that these extra empirical constraints, both for the ξ -dependent terms and for the $E/\tilde{E}$ GPDs, are added for pure practical purposes. As mentioned above, they simply represent the lack of information of GPDs in the off-forward region, which can and should be improved in the future with more inputs from both lattice calculations and experiment measurements. In table 5.1, we collect whether GPDs of different species and flavors are either parameterized independently with eq. (5.47) or linked to the other GPDs with some empirical assumptions here.

Finally, we comment on the comparison of the GUMP program to the other GPD global

GPDs species and flavors	Fully parameterized	GPDs linked to	Proportional constants
H_{u_V} and $\tilde{H}_{u_V}$	✓	-	-
E_{u_V} and $\tilde{E}_{u_V}$	✓	-	-
H_{d_V} and $\tilde{H}_{d_V}$	✓	-	-
E_{d_V} and $\tilde{E}_{d_V}$	✗	E_{u_V} and $\tilde{E}_{u_V}$	$R_{d_V}^{E/\tilde{E}}$
$H_{\bar{u}}$ and $\tilde{H}_{\bar{u}}$	✓	-	-
$E_{\bar{u}}$ and $\tilde{E}_{\bar{u}}$	✗	$H_{\bar{u}}$ and $\tilde{H}_{\bar{u}}$	$R_{\text{sea}}^{E/\tilde{E}}$
$H_{\bar{d}}$ and $\tilde{H}_{\bar{d}}$	✓	-	-
$E_{\bar{d}}$ and $\tilde{E}_{\bar{d}}$	✗	$H_{\bar{d}}$ and $\tilde{H}_{\bar{d}}$	$R_{\text{sea}}^{E/\tilde{E}}$
H_g and $\tilde{H}_g$	✓	-	-
E_g and $\tilde{E}_g$	✗	H_g and $\tilde{H}_g$	$R_{\text{sea}}^{E/\tilde{E}}$

Table 5.1: A summary of how each GPDs with different species and flavors are parameterized respectively. Fully parameterized GPDs are expressed in terms of eq. (5.47), whereas the other GPDs are linked to the fully parameterized GPDs with proportional constants.

analysis programs with conform moments framework, specifically the Kumerički-Müller (KM) model and its extensions [43, 44, 210, 211]. As clearly stated before, the GUMP program follows the general conformal moment parameterization of GPDs, and adopts many useful observations there to make the GUMP parameterization practical. On the other hand, this program focuses more on the extraction of all four leading-twist GPDs including both valence and sea quarks of different flavors with the combined inputs from both lattice calculations and experiments. Therefore, though in a similar spirit, GUMP program allows a more comprehensive study of the GPDs, especially the valence parts which are effectively constrained by lattice calculations while less accessible from experiments.

With the above parameterization, the global analysis can be performed with adequate inputs

to pin down all the parameters. Therefore, we continue discussing how we select inputs from all the available results for the global analysis.

We start with the forward inputs where GPDs reduce to simply PDFs. While we could have various inclusive measurements as the inputs rather than the global PDFs extracted from a specific work to avoid the bias, there are several reasons we choose the extracted PDFs for the GPD global analysis. First, the extraction of the PDFs itself is considerably involving which requires dedicated works, especially for the simultaneous extraction of both unpolarized and polarized PDFs [245, 246]. Second, much less is known for the off-forward behaviors of GPDs compared to the forward ones. Consequently, a fine forward analysis can not be matched with an equivalent off-forward analysis, making it less urgent to improve the forward part of the analysis. Therefore, we avoid repeating the forward fitting that have been studied extensively by the PDF global analysis community and take their globally extracted PDFs as the inputs. More specially, we consider one of the recent analysis by the JAM collaboration [245], where both the unpolarized and polarized proton PDFs are extracted simultaneously.

The off-forward inputs, on the other hand, consist of various constraints including globally extracted FFs [247], lattice calculations [199, 248] and exclusive measurements, or the DVCS measurements [109, 144, 249–251] more specifically for this work. For the form factors, we take the globally extracted charge FFs [247] rather than fit to the elastic scattering data directly [240] for the same reasons as for the PDFs. Since the charge form factors are averaged over all flavors, we combine the charge form factors of both proton and neutron to obtain the charge form factors of up and down quarks respectively, assuming isospin symmetry and ignoring the contributions of strange and other heavy flavors.

The lattice inputs include both calculation of generalized form factors and x -dependence of

GPDs [199, 248]. Putting the lattice QCD results together with other experiment measurements can be quite subtle, due to the systematic uncertainties of the lattice results that are hard to fully understand. Progresses have been made in getting the systematic uncertainties under control and calculations from different group are converging for certain quantities like the (axial) charge form factors, see for instance the review in ref. [200], but more efforts are still in demand to get the other results consistent. Therefore, here we consider the lattice results from a single group only [199, 248]¹ to avoid the potential tension among lattice results from different groups. On the other hand, we adjust their weights in the global analysis by increasing the uncertainties of the results to account for the unknown systematic uncertainties. We note that the calculation of the x -dependence of GPDs at non-zero skewness has been done in the literature [199], although the reliable regions with controlled systematic uncertainties get even more subtle due to the irregular behavior of GPDs at $x = \xi$. Thus, the inclusion of the x -dependence of GPDs at non-zero skewness in the global analysis will be left to the future work.

Last but not the least, we have the experimental exclusive measurements. In principle, the exclusive measurements can and should include as many processes as possible to get better constraints on the GPDs as well as to test the universality of GPDs. However, the main challenge for putting different processes together is the mismatch in the amount of data available. Two types of Compton scattering processes, DVCS and Time-like Compton Scattering (TCS) [252], have been measured at JLab. Much more DVCS data have been obtained than that of TCS, of which the first measurement was made just recently [253]. On the other hand, DVMP typically requires much larger virtuality than DVCS to suppress the unwanted contributions of transverse

¹Here the choice is made by considering only the variety of GPD related calculations and does not reflect a preference over the other lattice calculations.

polarization, which are mostly accessible with colliders such as HERA [244, 254] and the future EIC [6, 7]. Therefore, we start by considering the DVCS measurements only, which has the broadest global kinematical coverage among these process that includes both the low x_B region covered by H1 [109] at HERA and the medium x_B region covered by various experiments at JLab [144, 249–251]. It is worth noting that DVCS process is mostly sensitive to the quark distributions whereas as the gluon distributions must be obtained from meson productions or other gluon-sensitive processes. Obtaining the gluonic distributions from them is indeed of high interest and importance, which will be carried out in a separate work.

With all the inputs above, it seems straightforward to simply put them together and perform the global analysis with the parameterization described before. However, it will not be quite practical with the large number of free parameters, which typically means extremely slow convergence in the search of best-fit parameters. Besides, the evaluation of GPDs, especially with scale evolution, is much more computationally intensive, compared to that of the PDFs for instance, making the global analysis even less handleable. Therefore, we split the global analysis into two steps by first performing the semi-forward fit at zero skewness $\xi = 0$ (while the momentum transfer squared t can still be non-zero) and then fit to the off-forward inputs with non-zero skewness $\xi \neq 0$. Apparently, the two-step fitting introduces bias that favors the semi-forward constraints over the off-forward constraints. However, given the fact that much more semi-forward constraints can be obtained from lattice and experiments than the off-forwards ones, it is a reasonable and practical assumption for such large system of parameters.



Figure 5.12: A plot showing the fitting procedure of the GUMP program. The semi-forward ($\xi = 0$) fit that consists of four individual fits for each t -dependent PDFs is performed before the off-forward ($\xi \neq 0$) fit. The results of the semi-forward fit are then passed to the off-forward fit as fixed parameters, whereas the off-forward parameters will be determined with the off-forward constraints.

5.4.2 Basics of the fit

As discussed before, the whole fit will be split into the semi-forward ($\xi = 0$) part and the off-forward ($\xi \neq 0$) part to avoid dealing with the huge parameter set in a single fit. Furthermore, in the semi-forward case constraints on different species H , E , $\tilde{H}$, and $\tilde{E}$ decouple, unlike the off-forward case where all four of them are involved. This allows one to further decompose the semi-forward fit into four separate fits that do not interfere for each of the four t -dependent PDFs. Therefore, the whole fitting procedure eventually consists of five individual fits as shown in Fig. 5.12. Correspondingly, the total χ^2 can be decomposed into five parts:

$$\chi_{\text{tot}}^2 = \chi_{\text{fwd}}^2 + \chi_{\text{off-fwd}}^2 = \chi_H^2 + \chi_E^2 + \chi_{\tilde{H}}^2 + \chi_{\tilde{E}}^2 + \chi_{\text{off-fwd}}^2, \quad (5.54)$$

where each term χ_F^2 with $F = \{H, E, \tilde{H}, \tilde{E}\}$ stands for the χ^2 for the semi-forward fits of the corresponding t -dependent PDFs. Then one just need to minimize these χ^2 s separately in order to perform the fit and the results are summarized in table 5.2. More details of the fitted parameters are presented in Appendix F.

Several comments are as follows: first, we take the globally fitted unpolarized and polar-

Sub-fits	χ^2	N_{data}	$\chi^2_\nu \equiv \chi^2/\nu$
Semi-forward			
$t\text{PDF } H$	281.7	217	1.41
$t\text{PDF } E$	59.7	50	1.36
$t\text{PDF } \tilde{H}$	159.3	206	0.84
$t\text{PDF } \tilde{E}$	63.8	58	1.23
Off-forward			
JLab DVCS	1413.7	926	~ 1.53
H1 DVCS	19.7	24	~ 0.82
Off-forward total	1433	950	1.53
Total	2042	1481	1.40

Table 5.2: A table summarizing the total χ^2 versus the number of data points N_{data} as well as the χ^2 per degree of freedom χ^2_ν . Since the JLab and H1 DVCS measurements are fitted simultaneously in the off-forward fit, the χ^2/ν is replaced with χ^2/N_{data} instead to estimate their separate contributions.

ized PDFs [245] with 31 points sampled in the region $x \in [0.005, 0.6]$ for each flavor, indicating 155 points for the t -dependent PDFs H and $\tilde{H}$ respectively. Note that since the parameterization we used has only 3 parameters in the forward limit for each PDF, much less than the PDF global analysis [34], the sample size can not be too large correspondingly. The extra constraints from PDFs on H and $\tilde{H}$ account for their larger N_{data} as shown in table 5.2, while the other around 50 data for each t -dependent PDF are taken from the globally fitted FFs [247] and lattice calculations [199, 248].

Second, in the interest of this work, we select the JLab DVCS data with larger Q^2 ($Q > 1.8$ GeV) and smaller x_B ($x_B < 0.5$). The larger Q^2 is required by the factorization theorem and also to suppress the higher twist effects, while the low x_B region is selected in accord with the small $\xi(x_B)$ expansion such that we have the expansion parameter $\xi^2 \lesssim 0.1$. Even with such selection, it still leaves much more JLab DVCS data than the H1 data (which do not need

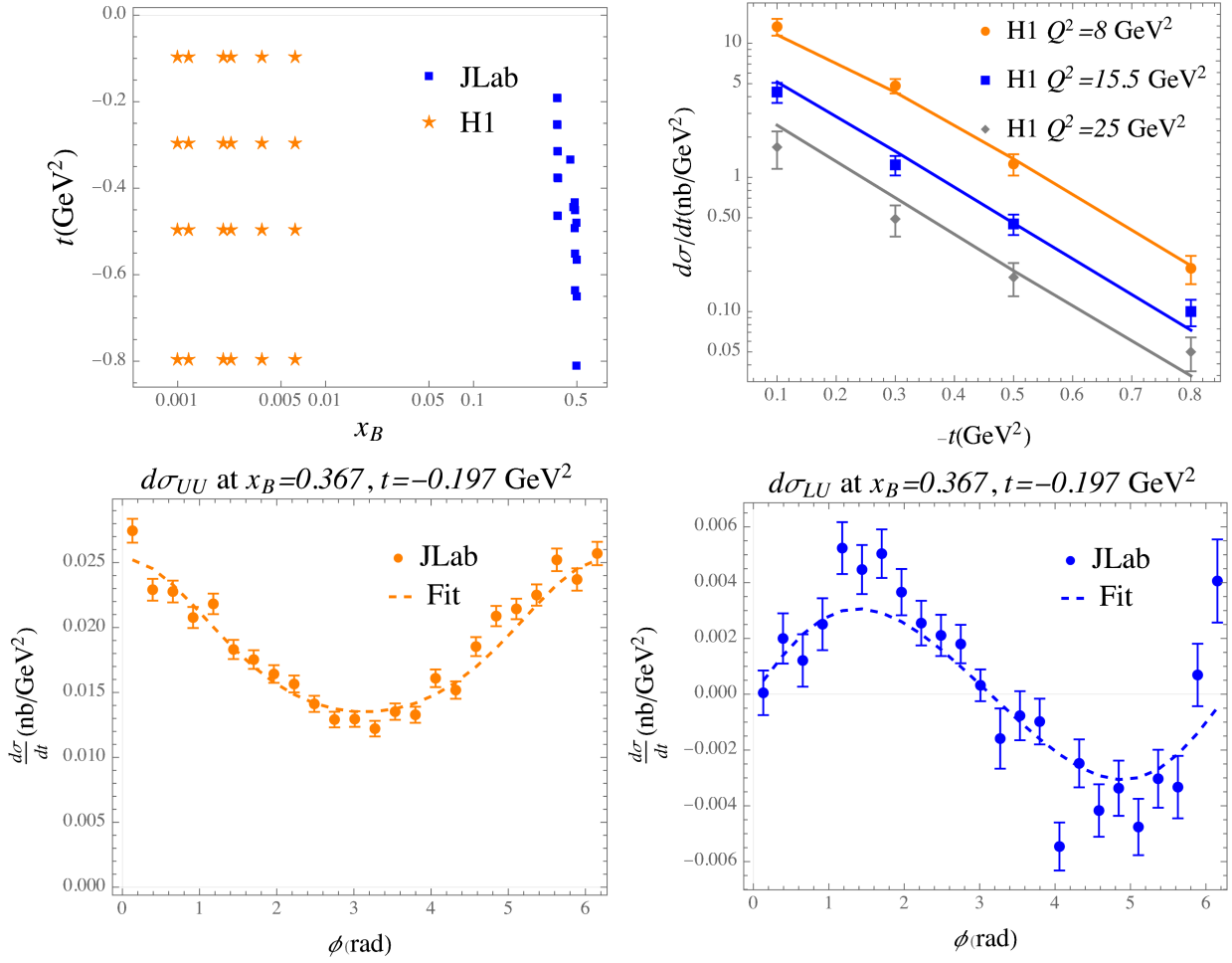


Figure 5.13: The top left panel shows the kinematical coverage of the DVCS measurements including both JLab and H1. The top-right panel shows the fit to the DVCS measurements at H1 with the azimuthal angle ϕ integrated, whereas the lower two figures show examples of the fit to the JLab DVCS cross-section measurements at $Q^2 = 3.65 \text{ GeV}^2$ in the azimuthal angle ϕ for both polarized and unpolarized beam. Contributions of Bethe-Heitler process are calculated with the electromagnetic form factors in ref. [247]. The DVCS and interference cross-sections are calculated with the formulas in ref. [145] including the contributions of twist-two CFFs only.

selection as they are already in the large Q^2 and small x_B region), since both the azimuthal ϕ dependence and beam-polarized cross-sections are measured at JLab. Therefore, the JLab data will be the dominant effects in the off-forward fit even though they cover about the same amount of kinematical points on the (x_B, t) space. In figures 5.13, we show the kinematical coverage of the DVCS measurements at both JLab and H1 and present some typical examples of the fit to the

DVCS measurements.

To summary, the reduced χ^2 s are all around 1 which indicates generally good fits. For the forward fit, the main issue is that the 3 forward parameters for the H PDF cannot perfect describe the x -shape ranging from $x \in [0.005, 0.6]$. Although this could be improved with more sets of ansatz, it would also require more off-forward inputs accordingly. As for the off-forward fit, the main challenge seems to be the potential higher-twist effects in the JLab DVCS measurements given that the typical Q^2 of them is around 4 GeV^2 , which still allow sizable twist-three contributions or even twist-four contributions especially at large momentum transfer [132, 144, 255]. For instance, the LU cross-section plot at the lower-right of Fig. 5.13 still deviates from a perfect sine shape as predicted in the $Q^2 \rightarrow \infty$ limit, which could be due to the higher-twist effects.

5.4.3 The extracted CFFs and GPDs

As discussed before, since the off-forward inputs contain DVCS measurements only which are effectively combinations of CFFs, they do not constrain the x shape nor the flavor structures of GPDs well. Thus, the flavor-dependent x shape of GPDs extracted under the empirical constraints in subsection 5.4.1 will be obviously model-dependent. Therefore, before moving on to the extracted GPDs which relies on the choice of ansatz, we first discuss the extracted CFFs of which the comparison is less model-dependent. More specifically, we will compare the CFFs extracted here with the locally extracted CFFs in ref. [144] and the CFFs predicted by the KM15 model [256] based on the global analysis of CFFs. The comparison of the CFFs are shown in Fig. 5.14.

Before commenting on the comparison of the extracted CFFs, we first note that even the

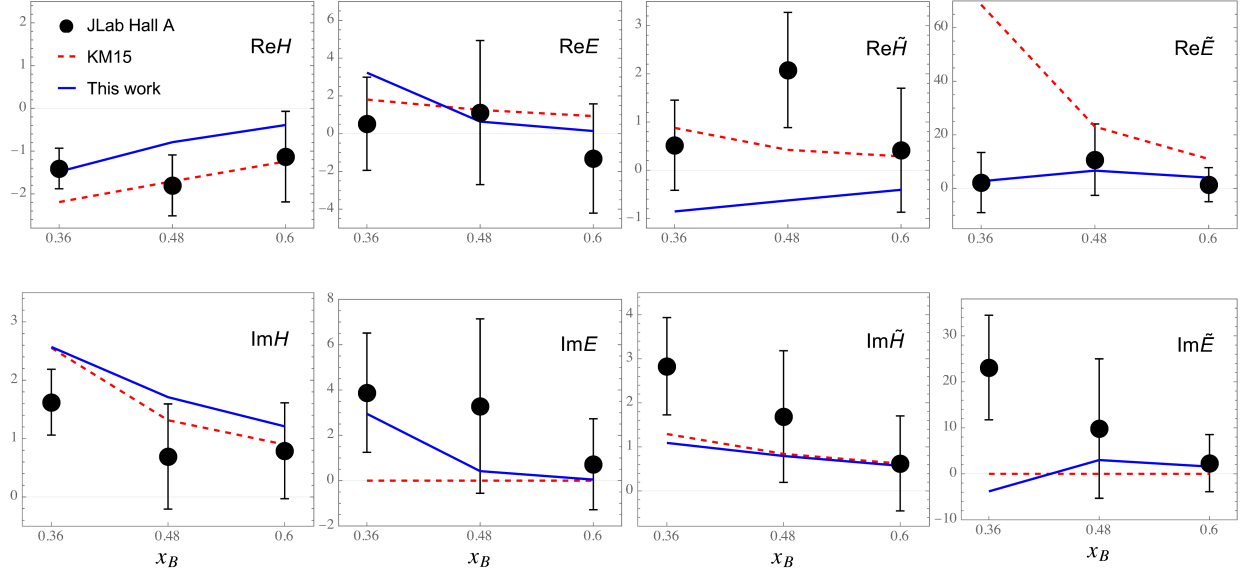


Figure 5.14: A comparison of the CFFs extracted here with the locally extracted CFFs in ref. [144] as well as the CFFs predicted by the KM15 model [256]. The three different kinematical points with $x_B = 0.36, 0.48, 0.60$ have momentum transfer $t = -0.345, -0.702, -1.050$ GeV^2 respectively. The CFFs in ref. [144] are extracted from data with various different Q^2 , whereas the theoretical values are calculated at the reference scale $Q^2 = 4 \text{ GeV}^2$.

local extraction of CFFs suffers from the degeneracy issue. The total DVCS cross-sections are generally quadratic equations of all the 8 CFFs:

$$d\sigma^{P_b P_t} = d\sigma_{\text{DVCS}}^{P_b P_t} + d\sigma_{\text{INT}}^{P_b P_t} + d\sigma_{\text{BH}}^{P_b P_t} = \sum_{i,j} A_{ij}^{P_b P_t} \mathcal{F}^i \mathcal{F}^j + \sum_i B_i^{P_b P_t} \mathcal{F}^i + C^{P_b P_t}, \quad (5.55)$$

with $\mathcal{F}_i = \{\text{Re}\mathcal{H}, \text{Im}\mathcal{H}, \text{Re}\mathcal{E}, \text{Im}\mathcal{E}, \text{Re}\tilde{\mathcal{H}}, \text{Im}\tilde{\mathcal{H}}, \text{Re}\tilde{\mathcal{E}}, \text{Im}\tilde{\mathcal{E}}\}$ which are the real or imaginary parts of the CFFs corresponding to the GPDs H , E , $\tilde{H}$ and $\tilde{E}$. For each combination of beam polarization P_b and target polarization P_t , the pure DVCS (DVCS), interference (INT) and Bethe-Heitler (BH) contributions are quadratic, linear, and constant in the CFFs respectively. Ideally, one would need all 8 possible combinations of P_b and P_t in order to disentangle the 8 CFFs, see for instance ref. [257], but there could still be degeneracy in the solutions as the nature of quadratic equations.

Here the degeneracy will be more severe, since only two polarization configurations, unpolarized or polarized beam with unpolarized target (UU and LU), are considered. For instance, one can show that with UU and LU cross-section, the CFF $\tilde{\mathcal{E}}$ only shows in the quadratic terms, multiplied to either itself or $\tilde{\mathcal{H}}$, implying that the quadratic terms are invariant under the transformation

$$\text{Re}\tilde{\mathcal{H}} \rightarrow -\text{Re}\tilde{\mathcal{H}} \quad , \quad \text{Re}\tilde{\mathcal{E}} \rightarrow -\text{Re}\tilde{\mathcal{E}} \quad , \quad (5.56)$$

and the same for the imaginary part. This degeneracy of CFFs in the DVCS cross-sections leads to the ambiguity in the sign of the extracted $\text{Re}\tilde{\mathcal{H}}$ and $\text{Re}\tilde{\mathcal{H}}$ as well as the $\text{Im}\tilde{\mathcal{E}}$ and $\text{Im}\tilde{\mathcal{E}}$ shown on the right of Fig. 5.14, where the extracted CFFs here seem to take opposite sign compared to that of the local extraction.² Besides this explicit example, there might be other implicit degeneracy in the extracted CFFs which could affect the reliability of such extraction.

Although such degeneracy makes the comparison of the extracted CFFs more subtle and less intuitive, it can certainly be improved in the future with more polarization configurations taken into consideration. On the other hand, many of the CFFs extracted here agree well with the local extraction as shown in Fig. 5.14, adding more confidence to the extraction of those CFFs.

Compared to the CFFs extraction discussed in the previous subsection, the extraction of GPDs will involve more model dependence. The lack of off-forward constraints except the CFFs is the main challenge in the GPD extraction, and the extracted x -shape of GPDs will depend on the ansatz chosen correspondingly. In addition, as the CFFs are averaged over different flavors, the extracted flavor structures could be ambiguous as well. Though suffering from these ambiguities, we still present the extracted GPDs here as an illustration of how GPDs are constrained by

²We note that when testing the fitting program with slightly different set-up, the extracted CFFs indeed turn out to have different signs.

the inputs, with the caveat that the GPDs, especially their off-forward behaviors, are not uniquely determined at this point. These results can certainly be improved with more lattice calculation at non-zero skewness as well as more flavor-sensitive data in the future.

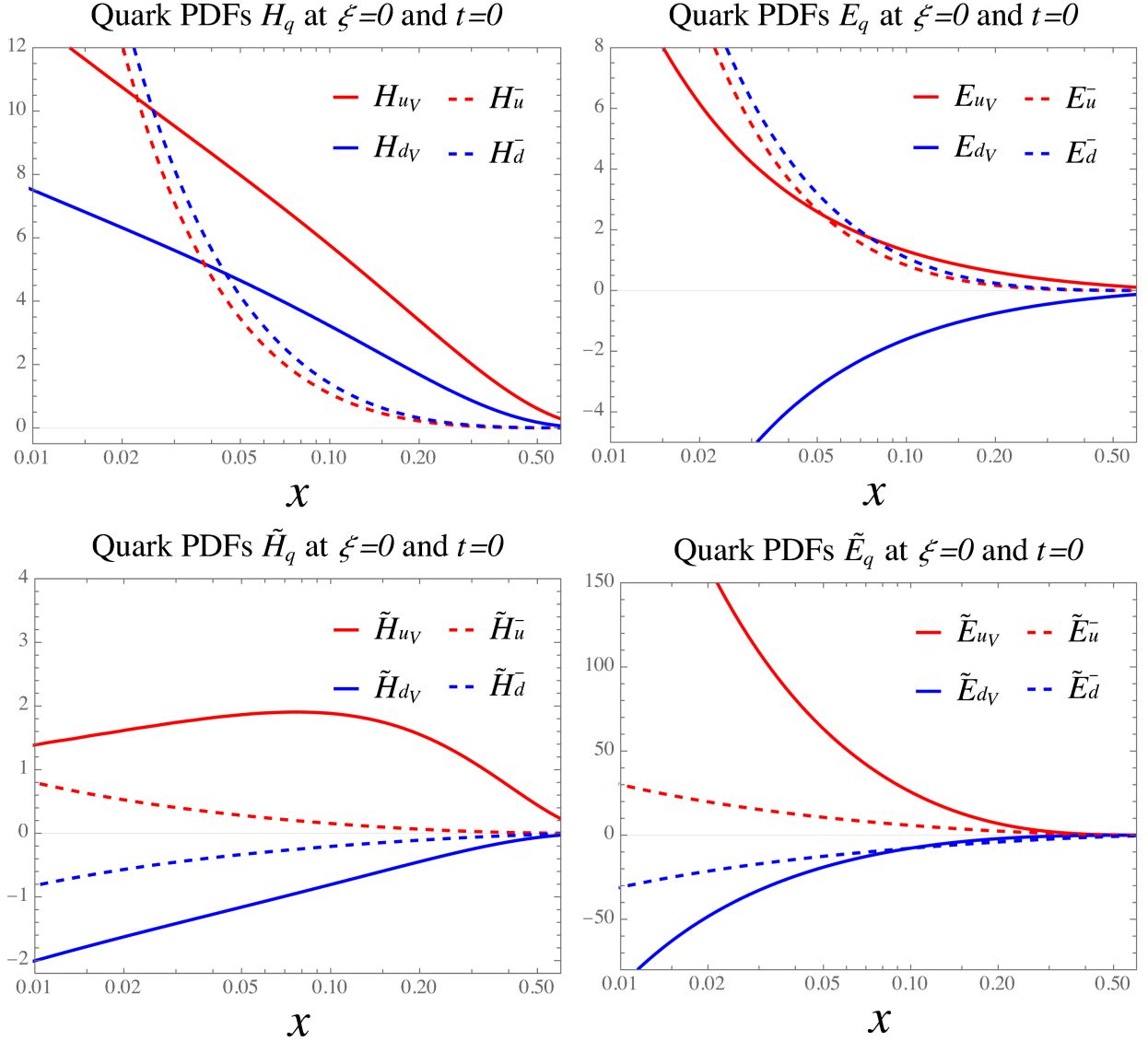


Figure 5.15: Plots present some examples of the extracted quark PDFs at the reference scale $\mu_0 = 2$ GeV. The H and $\tilde{H}$ are obtained from the polarized and unpolarized PDFs whereas the E and $\tilde{E}$ are constrained by the form factors and lattice calculation of form factors as well as GPDs. The gluon PDFs are also implemented here, and they enter the scale evolutions and mix with the quark GPDs. However, since they are not as constrained by the DVCS measurements, they are not presented here.

In Fig. 5.15, we present the extracted PDFs H , E and $\tilde{H}$, and $\tilde{E}$ at the reference scale

$\mu_0 = 2$ GeV. The two PDFs H and $\tilde{H}$ are fully parameterized and fitted to the globally extracted unpolarized and polarized PDFs in ref. [245], and therefore they agree well with the reference values there, which are not shown in the plots though. On the other hand, due to the lack of the information, the PDFs E and $\tilde{E}$ are extracted with the empirical constraints summarized in table 5.1 with globally extracted FFs and lattice calculation of form factors as well as GPDs. The $\tilde{E}$ PDFs turn out to be quite significant due to the contributions of pion pole, according to the lattice calculated form factors of $\tilde{E}$ [248] that we fit the $\tilde{E}$ PDF to. As for the E PDFs, the shape of the E PDFs are obtained combining the lattice calculation of E GPDs and other relevant form factors from both experiments and lattice results. The u and d quark E GPDs are almost the opposite of each other, in accord with the observation that the flavor isoscalar form factors B_{u+d} and gluonic gravitational form factors B_g are consistent with zero according to the lattice results [195, 248, 258]. We note that the valence part of the above PDFs are obtained in the semi-forward fit in the last section as well. However, they are quantitatively slightly different because of the different constraints used here as well as the effects of the sea quark distributions that were not considered in the last section.

In Fig. 5.16, we present quark GPDs at $\xi = 1/3$ and $t = -0.69$ GeV² and reference scale $\mu_0 = 2$ GeV. In general, the extracted GPDs oscillate in the DA-like region ($-\xi < x < \xi$) with a damping tail in the PDF-like region ($x > \xi$ or $x < -\xi$), as the result of the conformal partial wave expansion which expand the GPDs in the DA-like region in terms of Gegenbauer polynomials that oscillate. We note that due to the DA terms of GPDs, namely the $F_{q\bar{q}}$ terms in eq. (5.29), the behavior of GPDs in the DA-like region could look very different from here. With off-forward inputs mainly just CFFs, only the GPDs at the crossover line $x = \pm\xi$ are effectively constrained, which are then extrapolated with decaying tails to the PDF-like region, whereas the

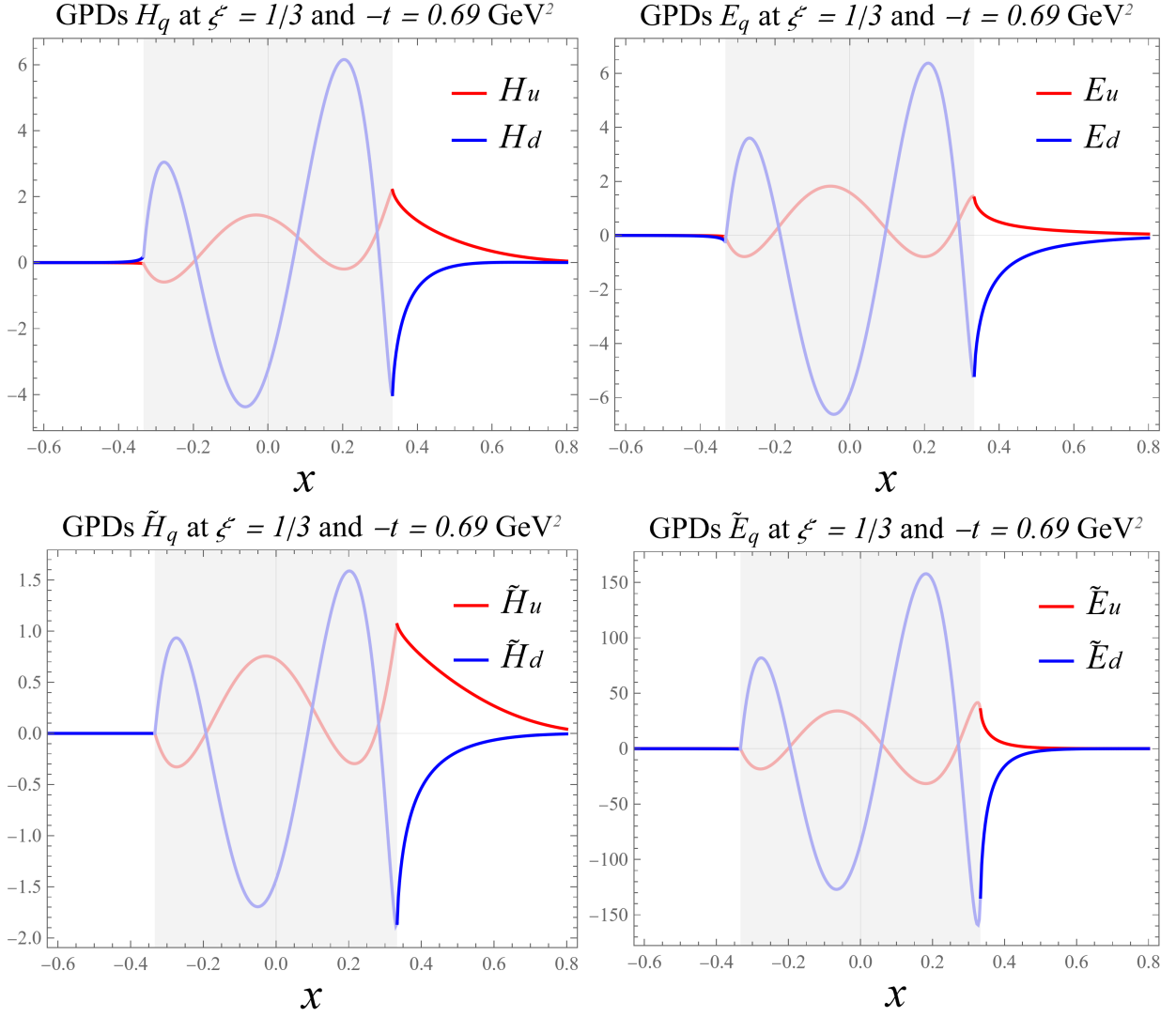


Figure 5.16: Plots of quark GPDs at $\xi = 1/3$, $t = -0.69 \text{ GeV}^2$ and reference scale $\mu_0 = 2 \text{ GeV}$. The DA-like regions of the GPDs ($-\xi < x < \xi$) are shaded, and the curves there are plotted in lighter color for distinction. The oscillating behaviors of the GPDs in the DA-like region are the results of the conformal partial wave expansion, since the GPDs here are expanded in terms of the Gegenbauer polynomials there. We note that the shape of GPDs are determined by the model choice of the moments and the CFFs effectively constrain the GPDs at the crossover line $x = \pm\xi$ only to the leading order.

DA-like regions are not uniquely determined.

The above extracted PDFs and GPDs are generated with the open-source codes of this program [259] which could be used to generate other observables including DVCS cross-section measurements as well, although we should note that only the central values of them are available

at present. While the error propagation is a crucial part of a global analysis, the process is extremely computationally intensive. With 20 functions of three variables where each point must be calculated through a numerical contour integral, sampling them over more than 50 parameters for the statistical uncertainties would be very challenging though not as useful since the main uncertainties are still from the systematics. Therefore, we will leave the error estimation to the future works once the task will be more practical.

5.4.4 GPDs in the DA-like region and DA terms

In the end of this section, we discuss more about the GPDs in the DA-like region. Recall that in the previous section, we discussed that in the small x_B expansion the DA-like region gets less relevant, and so we did not consider the DA-terms $F_{q\bar{q}}$ in the global analysis. However, the behavior of GPDs in this region is of genuine interest as well, which is less known compared to our knowledge of GPDs in the PDF-like region e.g., from PDFs. This region of GPDs is equally important and can be accessed directly from lattice calculations at non-zero skewness [199]. We note that these constraints on GPDs in the DA-like regions were not imposed in the global analysis since only very few of them are available at present. However, in this subsection, we will discuss how the current framework under small x_B expansion can be extended to accommodate these constraints with the extra DA-terms $F_{q\bar{q}}$ and produce smoother GPDs in the DA-like region.

In eq. (5.19), we showed that GPDs can be expressed as the sum of their conformal partial wave, where each term is given by a rescaled Gegenbauer polynomial [42]:

$$(-1)^j p_j(x, \xi) \equiv \xi^{-j-1} \frac{2^j \Gamma(\frac{5}{2} + j)}{\Gamma(\frac{3}{2}) \Gamma(j+3)} \left[1 - \left(\frac{x}{\xi} \right)^2 \right] C_j^{\frac{3}{2}} \left(\frac{x}{\xi} \right) \quad \text{for } |x| < \xi, \quad (5.57)$$

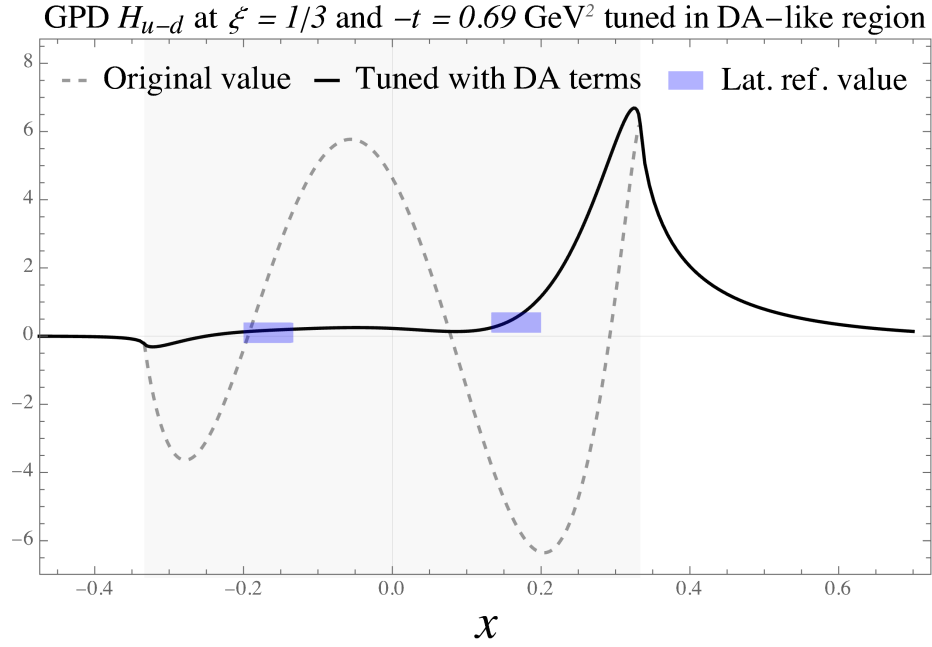


Figure 5.17: The isovector GPD H_{u-d} tuned to fit the reference value calculated on lattice [199] at reference scale $\mu_0 = 2$ GeV. The dashed line is the original extracted shape of GPDs. By adding extra terms in the DA-like region, we obtain the tuned GPDs as the red curve which approach the reference value shown as blue blocks.

that is non-zero only in the DA-like region. Therefore, one can always add finite terms like this to the GPDs freely without changing the GPDs in the PDF-like region. Such terms are called the DA terms here. Since the Wilson coefficients are antisymmetric in x for the CFFs $\mathcal{H}$ and $\mathcal{E}$, adding terms with even j which are symmetric in x will keep both the CFFs $\mathcal{H}$ and $\mathcal{E}$ invariant, which correspond to the so-called shadow GPDs [203]. On the other hand, adding terms with odd j which are antisymmetric in x will modify the real part of the CFFs $\mathcal{H}$ and $\mathcal{E}$ while keeping their imaginary parts the same, which are also known as the subtraction terms [44]. Similar arguments apply to the $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$ CFFs too, except that their Wilson coefficients are symmetric in x .

These terms are in principle hard to extract from experiments, which, however, do affect the generalized form factors and can be constrained by the FFs as well as lattice calculations. Therefore, one should parameterize these extra DA terms, at least part of them, and fit them to

The isovector GPD H_{u-d} at $-t = 0.69 \text{ GeV}^2$ tuned with DA terms

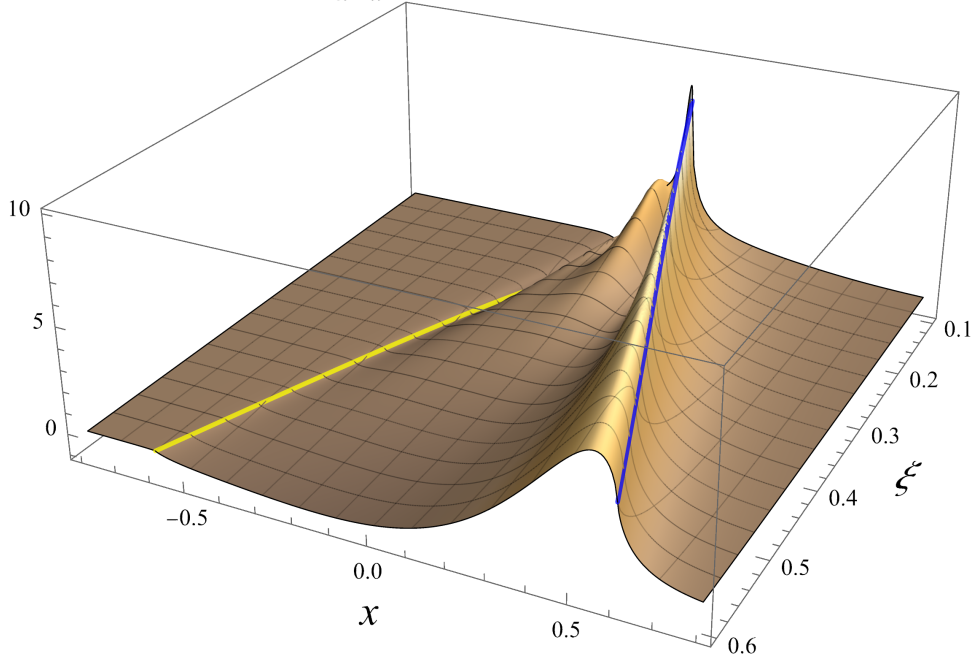


Figure 5.18: The isovector GPD H_{u-d} on the (x, ξ) plane tuned to fit the reference value calculated on lattice [199] at $t = -0.69 \text{ GeV}^2$ and reference scale $\mu_0 = 2 \text{ GeV}$. The blue ($x = \xi$) and yellow ($x = -\xi$) curves correspond to GPDs on the two crossover lines respectively. A cut at $\xi = 0.1$ is made since GPDs in the DA-like region $-\xi < x < \xi$ get singular when ξ approaches 0.

these constraints in order to obtain the shape of GPDs in the DA-like region. There have been lattice calculation of the GPD shape [199], which could be used to constrain such terms and determine the shape of GPDs in the DA-regions. However, since the results contain only the isovector $u - d$ combination of H and $\tilde{H}$ GPDs and such calculation will break down at $x = \pm\xi$, they do not pose enough constraints on the GPDs for global analysis. Therefore, we will leave the extra fitting of the DA terms to those constraints in the future work with more information in the DA-like region. In Fig. 5.17, we show an example of how GPDs can be tuned with the extra DA terms to fit to the constraints in the DA-like regions, where we take the isovector GPDs H_{u-d} calculated on lattice [199] as the reference value.

In Fig. 5.18, we also show the tuned isovector GPD H_{u-d} on the (x, ξ) plane at $t = -0.69$

GeV^2 and reference scale $\mu_0 = 2 \text{ GeV}$. We again note that such results are obtained under the ansatz and empirical constraints used here.

Chapter 6: Summary and outlook

In this dissertation, we studied various topics on the nucleon 3D structures. We discussed how the nucleon 3D structures, especially the spin structures, can be accessed with the so-called GPDs, encoded in which are the coordinate space distributions of the nucleon. We reviewed the longitudinal spin sum rule which is derived by carefully decomposing the QCD AM operator, and then apply to the transverse spin sum rules, noting that extra care must be taken to remove the extrinsic contributions from the center-of-mass motion. With such treatment, we show that a novel twist-three sum rule can be derived for a transversely polarized nucleon, in addition to the well-known twist-two one.

Then we considered several processes that can probe the GPDs, including DVCS and near-threshold heavy meson production that have sensitivities to the quark and gluon structures, respectively. We studied the high order kinematical effects in DVCS related to the choice of light cone vectors and gauge fixing conditions, and showed that the light cone dependence gets canceled at twist three with the kinematical corrections of twist-three GPDs. We also studied the heavy vector meson production near the threshold in the heavy meson limit, and showed that such processes can be factorized in terms of the gluon GPDs, which can be further expanded to get the gluon GFFs. These processes are crucial in accessing the nucleon 3D structures from experiments.

In the end, we discussed the GUMP program for the global analysis of GPDs that employs the moment expansion and can fit to the combination of forward PDFs, the lattice calculation and the experimental measurements. We showed an illustrative example at zero skewness to extract the t -dependent PDFs with various constraints and then moved forward to the global analysis at non-zero skewness considering the global DVCS measurements together with other inputs. With certain empirical constraints that could be removed with more inputs, we extracted PDFs, CFFs and GPDs with the combined inputs from both experiments and lattice calculation for the first time, with the caveat that more inputs, including more polarization configurations for the DVCS measurements and more lattice results of GPDs at non-zero skewness, are still needed to improve the reliability of such an extraction.

The above researches show exciting results, while more still need to be done to improve our knowledge of the nucleon 3D structures. For the near-threshold heavy meson production, important questions have not been answered concerning the high-order corrections in the QCD coupling constant α_S as well as the heavy mass parameter M_V/M_N . In addition, more justifications are in need for the large ξ expansion that expressed the hadronic matrix elements with the gluon GFFs. As for the global analysis program, we consider:

- Including more lattice input, especially more constraints at non-zero skewness.
- Other processes in addition to the DVCS measurements.
- The higher-order correction as well as more quark flavors.

These are essential to improve the reliability and quality of the obtained GPDs.

Appendix A: Parameterization for GPDs

In this appendix, we discuss how to find a complete basis of the Dirac structures to parameterize the GPDs, see also ref. [70]. The general strategy is to eliminate as many Dirac structures as possible, for which the following Gordon identities

$$2M\bar{u}(P', S') \gamma^\alpha u(P, S) = \bar{u}(P', S') [2\bar{P}^\alpha + i\sigma^{\alpha\beta} \Delta_\beta] u(P, S) , \quad (\text{A.1})$$

$$2M\bar{u}(P', S') \gamma^\alpha \gamma_5 u(P, S) = \bar{u}(P', S') [\gamma_5 \Delta^\alpha + 2i\sigma^{\alpha\beta} \gamma_5 \bar{P}_\beta] u(P, S) , \quad (\text{A.2})$$

and

$$\bar{u}(P', S') [\Delta^\alpha + 2i\sigma^{\alpha\beta} \bar{P}_\beta] u(P, S) = 0 , \quad (\text{A.3})$$

$$\bar{u}(P', S') [2\bar{P}^\alpha \gamma_5 + i\sigma^{\alpha\beta} \gamma_5 \Delta_\beta] u(P, S) = 0 , \quad (\text{A.4})$$

are helpful. Eqs. (A.1) and (A.2) indicate that all the Dirac vectors γ^α and Dirac pseudo-vectors $\gamma^\alpha \gamma_5$ can be replaced with a Dirac scalar and a Dirac tensor $\sigma^{\alpha\beta}$ term. Moreover, another identity for the Dirac pseudo-tensor

$$\sigma^{\alpha\beta} \gamma_5 = -\frac{i}{2} \epsilon^{\alpha\beta\mu\nu} \sigma_{\mu\nu} , \quad (\text{A.5})$$

and the projection of eq. (A.4) to n_α which gives

$$\bar{u}(P', S') \left[\gamma_5 + \frac{1}{4} \epsilon^{\alpha\beta\mu\nu} n_\alpha \Delta_\beta \sigma_{\mu\nu} \right] u(P, S) = 0, \quad (\text{A.6})$$

will be helpful as well. These identities indicate that all Dirac pseudo-scalars and pseudo-tensors can be expressed with Dirac scalars and Dirac tensors, while Dirac tensors in the form of $i\sigma^{\alpha\beta} \bar{P}_\beta$ will reduce to Dirac scalars. With this in mind, parameterization for different Dirac structures can be done.

We start by considering the vectors without Dirac matrices, which includes

$$V_S^\mu = \{ \bar{P}^\mu, \Delta^\mu, n^\mu \}, \quad (\text{A.7})$$

where the S (scalar) subscript indicates Dirac scalars. Then, more complicated Dirac structure can be constructed by combining V_S^μ with Dirac matrices. The decomposition must satisfy all the symmetries the origin structure has, including both Lorentz symmetry and the discrete symmetry. An important constraint from the parity is that since all the Dirac pseudo-scalars, pseudo-vectors and pseudo-tensors are expressed with the other Dirac structures, there should be no $\epsilon^{\mu\nu\rho\sigma}$ terms.

Then for the structures that have only one Lorentz index which can be denoted as

$\bar{u}(P', S') \Gamma^\mu u(P, S)$, the possible terms with Dirac scalars will be

$$\Gamma^\mu \text{ with Dirac scalar } \sim V_S^\mu = \{ P^\mu, \Delta^\mu, n^\mu \}. \quad (\text{A.8})$$

In addition, there could be terms with Dirac tensor which include,

$$\Gamma^\mu \text{ with Dirac tensor} \sim \{\sigma^{\mu\nu} V_{S\nu}, V_S^\mu \sigma^{\alpha\beta} V_{S\alpha} V_{S\beta}\} . \quad (\text{A.9})$$

For future convenience, it is helpful to define $V_T^\mu = \{\sigma^{\mu\nu} \Delta_\nu, \sigma^{\mu\nu} n_\nu\}$ which stands for all possible vector combination with Dirac tensors, and $S_T = \sigma^{\alpha\beta} n_\alpha \Delta_\beta$ for all possible scalar combinations with Dirac tensors, where the $\sigma^{\alpha\beta} \bar{P}_\beta$ terms are neglected as they reduce to scalars as discussed above. Then all possible vector structures can be expressed as

$$\begin{aligned} \Gamma^\mu &\sim \{V_S^\mu, V_T^\mu, V_S^\mu S_T\} \\ &= \{\bar{P}^\mu, \Delta^\mu, n^\mu, \sigma^{\mu\nu} \Delta_\nu, \sigma^{\mu\nu} n_\nu, \bar{P}^\mu \sigma^{\alpha\beta} n_\alpha \Delta_\beta, \Delta^\mu \sigma^{\alpha\beta} n_\alpha \Delta_\beta, n^\mu \sigma^{\alpha\beta} n_\alpha \Delta_\beta\} , \end{aligned} \quad (\text{A.10})$$

where each term will be associated with an independent scalar function coefficient when we use this basis to parameterize GPDs.

Though more complicated, the tensor structure $\bar{u}(P', S') \Gamma^{\mu\nu} u(P, S)$ can be parameterized similarly. For the Dirac scalar terms, one has

$$\Gamma^{\mu\nu} \text{ with Dirac scalar} \sim \{V_S^\mu V_S^\nu, g^{\mu\nu}\} , \quad (\text{A.11})$$

and the Dirac tensor terms are

$$\Gamma^{\mu\nu} \text{ with Dirac tensor} \sim \{g^{\mu\nu} S_T, \sigma^{\mu\nu}, V_S^\mu V_S^\nu S_T, V_S^\mu V_T^\nu, V_T^\mu V_S^\nu\} . \quad (\text{A.12})$$

Consequently, $\Gamma^{\mu\nu}$ can in general be expressed in the basis

$$\Gamma^{\mu\nu} \sim \{g^{\mu\nu}, g^{\mu\nu} S_T, \sigma^{\mu\nu}, V_S^\mu V_S^\nu, V_S^\mu V_S^\nu S_T, V_S^\mu V_T^\nu, V_T^\mu V_S^\nu\} . \quad (\text{A.13})$$

It can be checked that there are 33 independent combinations above and one of them needs to be eliminated in order to be consistent with the form factors counting. This can be shown explicitly with a rearrangement of the basis. Since one has in general the expansion

$$V^\mu = p^\mu (V \cdot n) + n^\mu (V \cdot p) + V_\perp^\mu , \quad (\text{A.14})$$

where $V_\perp^\mu$ is non-zero only for μ in transverse direction, one can always choose a new basis with $V_S^\mu = \{\bar{P}_+^\mu, \Delta_\perp^\mu, n_-^\mu\}$ and $V_T^\mu = \{\sigma_\perp^{\mu\nu} \Delta_\nu, \sigma_\perp^{\mu\nu} n_\nu\}$ where again $\perp$ means they are non-zero only for μ in transverse direction. Another useful relation is the Gordon identity eq. (A.1) projected to n_α which gives

$$2M \bar{u}(P', S') \not{n}_\alpha u(P, S) = \bar{u}(P', S') [2 + i\sigma^{\alpha\beta} n_\alpha \Delta_\beta] u(P, S) , \quad (\text{A.15})$$

and this indicates that one could use $\{I, \not{n}\}$ to replace $\{I, \sigma^{\alpha\beta} n_\alpha \Delta_\beta\}$. Then the following parameterization for Γ^μ as

$$\begin{aligned} \Gamma^\mu = & \gamma_+^\mu H(x) + \frac{i\sigma_+^{\mu\alpha} \Delta_\alpha}{2M} E(x) \\ & + \frac{i\Delta_\perp^\mu}{M} G_1(x) + i\Delta_\perp^\mu \not{n} G_2(x) + \frac{i\sigma_\perp^{\mu\alpha} \Delta_\alpha}{2M} G_3(x) \\ & + \sigma_\perp^{\mu\nu} n_\nu G_4(x) \\ & + n_-^\mu F_1(x) + n_-^\mu \not{n} F_2(x) , \end{aligned} \quad (\text{A.16})$$

can be shown to be equivalent, where x here is short for all possible scalars in the system. It should be clear that the first line can be expressed by P_+^μ and $P_+^\mu \not{n}_-$ with the help of the Gordon identities, but it is rewritten to be consistent with the other parameterization. Similarly, for the tensor combination $\bar{u}(P')\Gamma^{\mu\nu}u(P)$, one has

$$\begin{aligned}\Gamma^{\mu\nu} = & A_1(x)g_\perp^{\mu\nu} + A_2(x)g_\perp^{\mu\nu}\not{n}_- + A_3(x)\sigma_\perp^{\mu\nu} + A_4(x)P_+^\mu P_+^\nu + A_5(x)n_-^\mu n_-^\nu \\ & + A_6(x)\Delta_\perp^\mu \Delta_\perp^\nu + A_7(x)P_+^\mu n_-^\nu + A_8(x)n_-^\mu P_+^\nu + A_9(x)P_+^\mu \Delta_\perp^\nu + A_{10}(x)\Delta_\perp^\mu P_+^\nu \\ & + A_{11}(x)n_-^\mu \Delta_\perp^\nu + A_{12}(x)\Delta_\perp^\mu n_-^\nu + A_{13}(x)P_+^\mu P_+^\nu \not{n}_- + \cdots .\end{aligned}\quad (\text{A.17})$$

By counting the independent combinations in eq. (A.13), there should be $33 = 3 + 3 \times 3 + 3 \times 3 + 2 \times 3 + 3 \times 2$ independent parameters with 3 elements in V_S and 2 elements in V_T . However, it can also be shown that

$$\Delta_\perp^{[\mu} \sigma_\perp^{\nu]\rho} \Delta_\rho \in \left\{ \sigma_\perp^{\mu\nu}, \Delta_\perp^{[\mu} \sigma_\perp^{\nu]\rho} n_\rho \right\} , \quad (\text{A.18})$$

where $[\mu\nu]$ in the indices means totally anti-symmetrization, so one of the three terms above can be eliminated and only 32 independent parameters left.

Appendix B: Lab frame kinematics of DVCS

Here we define all the physical four momenta in the lab frame, which can be written in terms of their components $V^\mu = (V^0, V^x, V^y, V^z)$ as

$$\begin{aligned}
P &= (M, 0, 0, 0) , \\
q &= \frac{Q}{\gamma} (1, 0, 0, -\sqrt{1 + \gamma^2}) , \\
k &= k_0 (1, \sin \theta_l, 0, \cos \theta_l) , \\
k' &= k'_0 (1, \sin \theta'_l, 0, \cos \theta'_l) , \\
P' &= \left(M - \frac{t}{2M}, |\vec{P}'| \sin \theta' \cos \phi, |\vec{P}'| \sin \theta' \sin \phi, |\vec{P}'| \cos \theta' \right) , \\
q' &= q'_0 (1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) .
\end{aligned} \tag{B.1}$$

The relevant constants can be solved with the kinematical invariants such as $Q, t, x_B, \dots$ which give [133]

$$\begin{aligned}
|\vec{P}'| &= \sqrt{-t \left(1 - \frac{t}{4M^2} \right)} , \quad q'_0 = \frac{Q^2 + x_B t}{2M x_B} , \quad k_0 = \frac{Q}{\gamma y} , \quad k'_0 = \frac{Q(1 - y)}{\gamma y} , \\
\cos \theta &= -\frac{1 + \frac{\gamma^2}{2} \frac{Q^2 + t}{Q^2 + x_B t}}{\sqrt{1 + \gamma^2}} , \quad \cos \theta' = -\frac{\gamma^2(Q^2 - t) - 2x_B t}{4x_B M \sqrt{1 + \gamma^2} \sqrt{-t(1 - \frac{t}{4M^2})}} , \\
\cos \theta_l &= -\frac{1 + y \frac{\gamma^2}{2}}{\sqrt{1 + \gamma^2}} , \quad \cos \theta'_l = -\frac{1 - y - \gamma^2 y/2}{(1 - y) \sqrt{1 + \gamma^2}} .
\end{aligned} \tag{B.2}$$

Note that those angles are not complete fixed by their cosines, and we have the extra conventional requirements that $\sin \theta_l > 0$, $\sin \theta > 0$ and consequently $\sin \theta'_l < 0$, $\sin \theta' < 0$ from momentum conservation.

Appendix C: Bether-Heitler cross-section

The BH squared amplitude can be written into the leptonic and hadronic part as,

$$|\mathcal{T}_{\text{BH}}|^2 = \frac{1}{t^2} L_{\text{BH}}^{\rho\sigma} H_{\rho\sigma}^{\text{BH}}, \quad (\text{C.1})$$

where we define

$$L_{\text{BH}}^{\rho\sigma} \equiv \sum_{h', \Lambda'} l_{\text{BH}}^{*\mu\rho} l_{\text{BH}}^{\nu\sigma} \varepsilon_\mu(q', \Lambda') \varepsilon_\nu^*(q', \Lambda'), \quad (\text{C.2})$$

$$\begin{aligned} H_{\text{BH}}^{\rho\sigma} \equiv & \sum_{S'} \bar{u}(P, S) \left[(F_1 + F_2) \gamma^\rho - \frac{\bar{P}^\rho}{M} F_2 \right] u(P', S') \\ & \times \bar{u}(P', S') \left[(F_1 + F_2) \gamma^\sigma - \frac{\bar{P}^\sigma}{M} F_2 \right] u(P, S), \end{aligned} \quad (\text{C.3})$$

Then we can write the hadronic part $H_{\text{BH}}^{\rho\sigma}$ as,

$$\begin{aligned} H_{\text{BH}}^{\rho\sigma} = & 4 \left(F_1^2 - \frac{t}{4M^2} F_2^2 \right) \bar{P}^\rho \bar{P}^\sigma + (F_1 + F_2)^2 (\Delta^2 g^{\rho\sigma} - \Delta^\rho \Delta^\sigma) \\ & + 2i (F_1^2 + F_2^2) M \epsilon^{\rho\sigma\alpha\beta} S_\alpha \Delta_\beta + \frac{4i(F_1 F_2 + F_2^2)}{M} \bar{P}^{[\rho} \epsilon^{\sigma]\mu\nu\alpha} \bar{P}_\mu S_\nu \Delta_\alpha, \end{aligned} \quad (\text{C.4})$$

with $[\dots]$ in the indices indicating antisymmetrizing the indices inside, for instance $A^{[\mu} B^{\nu]} = 1/2(A^\mu B^\nu - A^\nu B^\mu)$, and similarly we use $(\dots)$ for symmetrizing the indices inside. The first line corresponds to the unpolarized hadronic part, while the second line corresponds to the polarized

case, which are symmetric/ antisymmetric in the ρ, σ indices respectively. Then, as in Ref. [133],

we can define

$$\begin{aligned} A^{\text{BH}} &\equiv \frac{4\bar{P}^\rho \bar{P}^\sigma}{\Delta^2} L_{(\rho\sigma)}^{\text{BH}} , & B^{\text{BH}} &\equiv \left(g^{\rho\sigma} - \frac{\Delta^\rho \Delta^\sigma}{\Delta^2} \right) L_{(\rho\sigma)}^{\text{BH}} , \\ \tilde{A}^{\text{BH}} &\equiv \frac{4i}{Mt} \bar{P}^{[\rho} \epsilon^{\sigma]\mu\nu\alpha} \bar{P}_\mu S_\nu \Delta_\alpha L_{[\rho\sigma]}^{\text{BH}} , & \tilde{B}^{\text{BH}} &\equiv 2i \frac{M}{t} \epsilon^{\rho\sigma\alpha\beta} S_\alpha \Delta_\beta L_{[\rho\sigma]}^{\text{BH}} , \end{aligned} \quad (\text{C.5})$$

such that the amplitude square can be written as,

$$\begin{aligned} |\mathcal{T}_{\text{BH}}|^2 &= \frac{1}{t} \left[A^{\text{BH}} \left(F_1^2 - \frac{t}{4M^2} F_2^2 \right) + B^{\text{BH}} (F_1 + F_2)^2 \right. \\ &\quad \left. + \tilde{A}^{\text{BH}} (F_1 F_2 + F_2^2) + \tilde{B}^{\text{BH}} (F_1 + F_2)^2 \right] . \end{aligned} \quad (\text{C.6})$$

Those coefficients A^{BH} , B^{BH} , $\tilde{A}^{\text{BH}}$ and $\tilde{B}^{\text{BH}}$ can be solved with spinor algebra, and we get

$$A^{\text{BH}} = \frac{-8}{t(k \cdot q')(k' \cdot q')} \left\{ \bar{P}^2 [(k \cdot \Delta)^2 + (k' \cdot \Delta)^2] + t [(k \cdot \bar{P})^2 + (k' \cdot \bar{P})^2] \right\} , \quad (\text{C.7})$$

$$B^{\text{BH}} = \frac{-4}{(k \cdot q')(k' \cdot q')} [(k \cdot \Delta)^2 + (k' \cdot \Delta)^2] , \quad (\text{C.8})$$

$$\begin{aligned} \tilde{A}^{\text{BH}} &= \frac{16h}{Mt(k \cdot q')(k' \cdot q')} \left\{ t \bar{P}^2 [(k' \cdot \Delta)(k' \cdot S) - (k \cdot \Delta)(k \cdot S)] \right. \\ &\quad \left. + t(S \cdot \bar{P}) [(k \cdot \Delta)(k \cdot \bar{P}) - (k' \cdot \Delta)(k' \cdot \bar{P})] + \bar{P}^2 (S \cdot \Delta) [(k \cdot \Delta)^2 - (k' \cdot \Delta)^2] \right\} , \end{aligned} \quad (\text{C.9})$$

$$\begin{aligned} \tilde{B}^{\text{BH}} &= \frac{16hM}{t(k \cdot q')(k' \cdot q')} \left\{ (S \cdot \Delta) [(k' \cdot \Delta)^2 - (k \cdot \Delta)^2] \right. \\ &\quad \left. + t [(k \cdot \Delta)(k \cdot S) - (k' \cdot \Delta)(k' \cdot S)] \right\} . \end{aligned} \quad (\text{C.10})$$

which agrees the result in [133]. Then the cross-section can be expressed in terms of the combination of inner products of physical four momenta and kinematical scalars.

Appendix D: Pure DVCS structure functions

Here we express all the pure DVCS structure functions explicitly. With their definition in Eq. (3.71), we can perform the contraction and express them in terms of the scalar amplitude defined in Eq. (3.72) as,

$$F_{UU} = 4 \left[(1 - \xi^2) \left(h^U \mathcal{H}^* \mathcal{H} + \tilde{h}^U \tilde{\mathcal{H}}^* \tilde{\mathcal{H}} \right) - \frac{t}{4M^2} \left(h^U \mathcal{E}^* \mathcal{E} + \xi^2 \tilde{h}^U \tilde{\mathcal{E}}^* \tilde{\mathcal{E}} \right) - \xi^2 \left(h^U \mathcal{E}^* \mathcal{E} + h^U (\mathcal{E}^* \mathcal{H} + \mathcal{H}^* \mathcal{E}) + \tilde{h}^U (\tilde{\mathcal{E}}^* \tilde{\mathcal{H}} + \tilde{\mathcal{H}}^* \tilde{\mathcal{E}}) \right) \right], \quad (\text{D.1})$$

$$F_{UT,\text{out}} = N 4 \text{Im} \left[h^U \mathcal{H}^* \mathcal{E} - \tilde{h}^U \xi \tilde{\mathcal{H}}^* \tilde{\mathcal{E}} \right], \quad (\text{D.2})$$

$$F_{UL} = 8h^{+,U} \text{Im} \left\{ (1 - \xi^2) \tilde{\mathcal{H}}^* \mathcal{H} - \xi^2 \left(\tilde{\mathcal{H}}^* \mathcal{E} + \tilde{\mathcal{E}}^* \mathcal{H} \right) - \left(\frac{\xi^2}{1 + \xi} + \frac{t}{4M^2} \right) \xi \tilde{\mathcal{E}}^* \mathcal{E} \right\}, \quad (\text{D.3})$$

$$F_{LL} = 8h^{-,L} \text{Re} \left\{ (1 - \xi^2) \tilde{\mathcal{H}}^* \mathcal{H} - \xi^2 \left(\tilde{\mathcal{H}}^* \mathcal{E} + \tilde{\mathcal{E}}^* \mathcal{H} \right) - \left(\frac{\xi^2}{1 + \xi} + \frac{t}{4M^2} \right) \xi \tilde{\mathcal{E}}^* \mathcal{E} \right\}, \quad (\text{D.4})$$

$$F_{UT,\text{in}} = 4N h^{+,U} \text{Im} \left\{ \tilde{\mathcal{H}}^* \mathcal{E} - \xi \tilde{\mathcal{E}}^* \mathcal{H} - \frac{\xi^2}{1 + \xi} \tilde{\mathcal{E}}^* \mathcal{E} \right\}, \quad (\text{D.5})$$

$$F_{LT,\text{in}} = 4N h^{-,L} \text{Re} \left\{ \tilde{\mathcal{H}}^* \mathcal{E} - \xi \tilde{\mathcal{E}}^* \mathcal{H} - \frac{\xi^2}{1 + \xi} \tilde{\mathcal{E}}^* \mathcal{E} \right\}, \quad (\text{D.6})$$

Therefore, it will be sufficient to understand those four coefficients h^U , $\tilde{h}^U$, $h^{+,U}$ and $h^{-,L}$ in order to get the pure DVCS cross-section besides the CFFs. Similar structures show up in the

BKM10 [136] and the conversion relation can be written as

$$h_{\text{BKM}}^{\text{U}} = \tilde{h}_{\text{BKM}}^{\text{U}} = \frac{Q^2}{4y^2} \left[2 \frac{2 - 2y + y^2 + \gamma^2 y^2 / 2}{1 + \gamma^2} + \frac{16K^2}{(2 - x_B^2)(1 + \gamma^2)} \left(\frac{-2\xi_{\text{BKM}}}{1 + \xi_{\text{BKM}}} \right)^2 + \frac{8K(2 - y)}{(2 - x_B)(1 + \gamma^2)} \frac{-2\xi_{\text{BKM}}}{1 + \xi_{\text{BKM}}} \cos \phi \right], \quad (\text{D.7})$$

where the ξ_{BKM} differs from our ξ and is defined as

$$\xi_{\text{BKM}} \equiv \frac{x_B \left(1 + \frac{t}{2Q^2} \right)}{2 - x_B + x_B \frac{t}{Q^2}}. \quad (\text{D.8})$$

and K are defined from

$$K^2 = \frac{(t_0 - t)}{Q^2} (1 - x_B) \left(1 - y - \frac{1}{4} \gamma^2 y^2 \right) \left[\sqrt{\gamma^2 + 1} + \left(\frac{\gamma^2}{4(1 - x_B)} + x_B \right) \frac{(t - t_0)}{Q^2} \right], \quad (\text{D.9})$$

with t_0 the minimum momentum transfer defined as,

$$t_0 = -Q^2 \frac{2(1 - x_B)(1 - \sqrt{1 + \gamma^2}) + \gamma^2}{4x_B(1 - x_B) + \gamma^2}. \quad (\text{D.10})$$

Note that there are contribution from those so-called effective twist three Compton form factors in BKM which are defined as [105]

$$\mathcal{F}_{\text{eff}} \equiv -2\xi_{\text{BKM}} \left(\frac{1}{1 + \xi_{\text{BKM}}} \mathcal{F} + \mathcal{F}_+^3 - \mathcal{F}_-^3 \right), \quad (\text{D.11})$$

and they reduce to the $-2\xi_{\text{BKM}}/(1 + \xi_{\text{BKM}})\mathcal{F}$ terms in the coefficients when setting the twist-three CFFs $\mathcal{F}_+^3$ and $\mathcal{F}_-^3$ to zero. We also need to mention that when the WW relations are taken into account, one have $\mathcal{F}_+^3 = -1/\xi_{\text{BKM}}\mathcal{F} + \dots$, where ellipsis stands for twist-two GPDs with WW kernel and genuine twist-three GPDs. Then we have $F_{\text{eff}} \rightarrow 2/(1 + \xi_{\text{BKM}})\mathcal{F}$ with WW relations.

While our covariant scalar coefficients are already presented in Eq. (3.100), of which the code is available in [260] as well, we also present our scalar coefficients calculated from light cone vectors with twist expansion, for which we define

$$\mathcal{H} = \sum_{t=2}^{\infty} Q^{4-t} \mathcal{H}_{(t)} , \quad (\text{D.12})$$

with $\mathcal{H} = \{h^{\text{U}}, \tilde{h}^{\text{U}}, h^{-,\text{L}}, h^{+,\text{U}}\}$. A twist expansion will then yield the following result

$$\begin{aligned} h_{(2)}^{\text{U}} &= \tilde{h}_{(2)}^{\text{U}} = \frac{2 - 2y + y^2}{2y^2} , \\ h_{(3)}^{\text{U}} &= \tilde{h}_{(3)}^{\text{U}} = \frac{2x_B(y-2)\sqrt{[t(x_B-1) - M^2x_B^2](1-y)(\beta-1)\cos\phi}}{[1 + (\beta-1)x_B]y^2} , \\ h_{(4)}^{\text{U}} &= \tilde{h}_{(4)}^{\text{U}} = \frac{1}{2[1 + (\beta-1)x_B]^2y^2} \\ &\quad \times \left[8(\beta-1)^2x_B^2(y-1)\cos^2\phi (M^2x_B^2 - tx_B + t) \right. \\ &\quad \left. - x_B^2(y-2)^2 (M^2 ([2(\beta-1)x_B + 1]^2 + 1) - 2(\beta-1)^2t(x_B-1)) \right] , \end{aligned} \quad (\text{D.13})$$

and for $h^{-,\text{L}}$,

$$\begin{aligned}
h_{(2)}^{-,\text{L}} &= \frac{y-2}{2y} , \\
h_{(3)}^{-,\text{L}} &= \frac{2x_B \sqrt{[t(x_B-1) - M^2 x_B^2](1-y)(\beta-1) \cos \phi}}{[1 + (\beta-1)x_B]y} , \\
h_{(4)}^{-,\text{L}} &= \frac{x_B^2(y-2) (M^2 (2(\beta-1)^2 x_B^2 + 2(\beta-1)x_B + 1) - (\beta-1)^2 t(x_B-1))}{y[(\beta-1)x_B + 1]^2} ,
\end{aligned} \tag{D.14}$$

and for $h^{+,\text{U}}$,

$$h_{(2)}^{+,\text{U}} = h_{(3)}^{+,\text{U}} = h_{(4)}^{+,\text{U}} = 0 . \tag{D.15}$$

Note that with twist-four accuracy, we have the relation $h^{\text{U}} \approx \tilde{h}^{\text{U}}$ and $h^{+,\text{U}} \approx 0$, which can lead to some simplification to the pure DVCS cross-section formula.

Appendix E: Interference structure functions

In the Subsection 3.2.3, we write down the interference cross-section for eight different polarization configurations. Here we put down their expressions explicitly. By simply contracting according to the definitions in Eq. (3.88), we have the following expression for all the F^I s

$$F_{UU}^I = \text{Re} \left[A^{I,U} \left(F_1 \mathcal{H}^* - \frac{t}{4M^2} F_2 \mathcal{E}^* \right) + B^{I,U} (F_1 + F_2) (\mathcal{H}^* + \mathcal{E}^*) \right. \\ \left. + C^{I,U} (F_1 + F_2) \tilde{\mathcal{H}}^* \right], \quad (\text{E.1})$$

$$F_{LU}^I = -\text{Im} \left[A^{I,L} \left(F_1 \mathcal{H}^* - \frac{t}{4M^2} F_2 \mathcal{E}^* \right) + B^{I,L} (F_1 + F_2) (\mathcal{H}^* + \mathcal{E}^*) \right. \\ \left. + C^{I,L} (F_1 + F_2) \tilde{\mathcal{H}}^* \right], \quad (\text{E.2})$$

$$F_{UL}^I = -\text{Im} \left\{ \tilde{A}^{I,U} \left[F_1 \left(\tilde{\mathcal{H}}^* - \frac{\xi^2}{1+\xi} \tilde{\mathcal{E}}^* \right) - F_2 \frac{t}{4M^2} \xi \tilde{\mathcal{E}}^* \right] \right. \\ \left. + \tilde{B}^{I,U} (F_1 + F_2) \left(\tilde{\mathcal{H}}^* + \frac{\xi}{1+\xi} \tilde{\mathcal{E}}^* \right) - \tilde{C}^{I,U} (F_1 + F_2) \left(\mathcal{H}^* + \frac{\xi}{1+\xi} \mathcal{E}^* \right) \right\}, \quad (\text{E.3})$$

$$\begin{aligned}
F_{LL}^I = -\text{Re} \Bigg\{ & \tilde{A}^{I,\text{L}} \left[F_1 \left(\tilde{\mathcal{H}}^* - \frac{\xi^2}{1+\xi} \tilde{\mathcal{E}}^* \right) - F_2 \frac{t}{4M^2} \xi \tilde{\mathcal{E}}^* \right] \\
& + \tilde{B}^{I,\text{L}} (F_1 + F_2) \left(\tilde{\mathcal{H}}^* + \frac{\xi}{1+\xi} \tilde{\mathcal{E}}^* \right) - \tilde{C}^{I,\text{L}} (F_1 + F_2) \left(\mathcal{H}^* + \frac{\xi}{1+\xi} \mathcal{E}^* \right) \Bigg\}, \tag{E.4}
\end{aligned}$$

$$\begin{aligned}
F_{UT,\text{in}}^I = -\frac{2}{N} \text{Im} \Bigg\{ & \tilde{A}^{I,\text{U}} \left[\xi F_1 \left(\xi \tilde{\mathcal{H}}^* + \left(\frac{\xi^2}{1+\xi} + \frac{t}{4M^2} \right) \tilde{\mathcal{E}}^* \right) + F_2 \frac{t}{4M^2} \left((\xi^2 - 1) \tilde{\mathcal{H}}^* + \xi^2 \tilde{\mathcal{E}}^* \right) \right] \\
& + \tilde{B}^{I,\text{U}} (F_1 + F_2) \left[\tilde{\mathcal{H}}^* + \left(\frac{t}{4M^2} - \frac{\xi}{1+\xi} \right) \xi \tilde{\mathcal{E}}^* \right] \\
& + \tilde{C}^{I,\text{U}} (F_1 + F_2) \left[\xi \mathcal{H}^* + \left(\frac{\xi^2}{1+\xi} + \frac{t}{4M^2} \right) \mathcal{E}^* \right] \Bigg\}, \tag{E.5}
\end{aligned}$$

$$\begin{aligned}
F_{LT,\text{in}}^I = -\frac{2}{N} \text{Re} \Bigg\{ & \tilde{A}^{I,\text{L}} \left[\xi F_1 \left(\xi \tilde{\mathcal{H}}^* + \left(\frac{\xi^2}{1+\xi} + \frac{t}{4M^2} \right) \tilde{\mathcal{E}}^* \right) + F_2 \frac{t}{4M^2} \left((\xi^2 - 1) \tilde{\mathcal{H}}^* + \xi^2 \tilde{\mathcal{E}}^* \right) \right] \\
& + \tilde{B}^{I,\text{L}} (F_1 + F_2) \left[\tilde{\mathcal{H}}^* + \left(\frac{t}{4M^2} - \frac{\xi}{1+\xi} \right) \xi \tilde{\mathcal{E}}^* \right] \\
& + \tilde{C}^{I,\text{L}} (F_1 + F_2) \left[\xi \mathcal{H}^* + \left(\frac{\xi^2}{1+\xi} + \frac{t}{4M^2} \right) \mathcal{E}^* \right] \Bigg\}, \tag{E.6}
\end{aligned}$$

$$\begin{aligned}
F_{UT,\text{out}}^I = \frac{2}{N} \text{Im} \Bigg\{ & A^{I,\text{U}} \left[F_1 \left(\xi^2 \mathcal{H}^* + \left(\xi^2 + \frac{t}{4M^2} \right) \mathcal{E}^* \right) + \frac{t}{4M^2} F_2 \left((\xi^2 - 1) \mathcal{H}^* + \xi^2 \mathcal{E}^* \right) \right] \\
& + B^{I,\text{U}} (F_1 + F_2) \left(\mathcal{H}^* + \frac{t}{4M^2} \mathcal{E}^* \right) - C^{I,\text{U}} \xi (F_1 + F_2) \left(\tilde{\mathcal{H}}^* + \frac{t}{4M^2} \tilde{\mathcal{E}}^* \right) \Bigg\}, \tag{E.7}
\end{aligned}$$

$$\begin{aligned}
F_{LT,\text{out}}^I = \frac{2}{N} \text{Re} \left\{ A^{I,\text{L}} \left[F_1 \left(\xi^2 \mathcal{H}^* + \left(\xi^2 + \frac{t}{4M^2} \right) \mathcal{E}^* \right) + \frac{t}{4M^2} F_2 \left((\xi^2 - 1) \mathcal{H}^* + \xi^2 \mathcal{E}^* \right) \right] \right. \\
\left. + B^{I,\text{L}} (F_1 + F_2) \left(\mathcal{H}^* + \frac{t}{4M^2} \mathcal{E}^* \right) - C^{I,\text{L}} \xi (F_1 + F_2) \left(\tilde{\mathcal{H}}^* + \frac{t}{4M^2} \tilde{\mathcal{E}}^* \right) \right\}.
\end{aligned} \tag{E.8}$$

In [136], similar structures show up, and the BKM10 results can be re-expressed in terms of those

coefficients the conversion relations for $A^{I,\text{U}}$, $B^{I,\text{U}}$ and $C^{I,\text{U}}$ are

$$A_{\text{BKM}}^{I,\text{U}} = \frac{-Q^2}{x_B y^3 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ \sum_{n=0}^3 \left[C_{++}^{\text{unp}}(n) + \frac{\sqrt{2}\tilde{K}}{(2-x_B)Q} \frac{-2\xi_{\text{BKM}}}{1+\xi_{\text{BKM}}} C_{0+}^{\text{unp}}(n) \right] \cos(n\phi) \right\}, \tag{E.9}$$

$$\begin{aligned}
B_{\text{BKM}}^{I,\text{U}} = \frac{-Q^2 \xi_{\text{BKM}}}{x_B y^3 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi) \left(1 + \frac{t}{2Q^2} \right)} \left\{ \sum_{n=0}^3 \left[C_{++}^{\text{unp},V}(n) \right. \right. \\
\left. \left. + \frac{\sqrt{2}\tilde{K}}{(2-x_B)Q} \frac{-2\xi_{\text{BKM}}}{1+\xi_{\text{BKM}}} C_{0+}^{\text{unp},V}(n) \right] \cos(n\phi) \right\},
\end{aligned} \tag{E.10}$$

$$\begin{aligned}
C_{\text{BKM}}^{I,\text{U}} = \frac{-Q^2 \xi_{\text{BKM}}}{x_B y^3 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi) \left(1 + \frac{t}{2Q^2} \right)} \left\{ \sum_{n=0}^3 \left[C_{++}^{\text{unp},\text{A}}(n) + C_{++}^{\text{unp}}(n) \right. \right. \\
\left. \left. + \frac{\sqrt{2}\tilde{K}}{(2-x_B)Q} \frac{-2\xi_{\text{BKM}}}{1+\xi_{\text{BKM}}} \left(C_{0+}^{\text{unp},\text{A}}(n) + C_{0+}^{\text{unp}}(n) \right) \right] \cos(n\phi) \right\},
\end{aligned} \tag{E.11}$$

while for $\tilde{A}^{I,U}$, $\tilde{B}^{I,U}$ and $\tilde{C}^{I,U}$, the conversion relations are

$$\tilde{A}_{\text{BKM}}^{I,U} = \frac{-Q^2}{x_B y^3 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \left\{ \sum_{n=1}^3 \left[S_{++}^{\text{LP}}(n) + \frac{\sqrt{2}\tilde{K}}{(2-x_B)Q} \frac{-2\xi_{\text{BKM}}}{1+\xi_{\text{BKM}}} S_{0+}^{\text{LP}}(n) \right] \sin(n\phi) \right\}, \quad (\text{E.12})$$

$$\begin{aligned} \tilde{B}_{\text{BKM}}^{I,U} = \frac{-Q^2 \xi_{\text{BKM}}}{x_B y^3 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi) \left(1 + \frac{t}{2Q^2}\right)} & \left\{ \sum_{n=1}^3 \left[S_{++}^{\text{LP},A}(n) \right. \right. \\ & \left. \left. + \frac{\sqrt{2}\tilde{K}}{(2-x_B)Q} \frac{-2\xi_{\text{BKM}}}{1+\xi_{\text{BKM}}} S_{0+}^{\text{LP},A}(n) \right] \sin(n\phi) \right\}, \end{aligned} \quad (\text{E.13})$$

$$\begin{aligned} \tilde{C}_{\text{BKM}}^{I,U} = -\frac{-Q^2 \xi_{\text{BKM}}}{x_B y^3 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi) \left(1 + \frac{t}{2Q^2}\right)} & \left\{ \sum_{n=1}^3 \left[S_{++}^{\text{LP},V}(n) + S_{++}^{\text{LP}}(n) \right. \right. \\ & \left. \left. + \frac{\sqrt{2}\tilde{K}}{(2-x_B)Q} \frac{-2\xi_{\text{BKM}}}{1+\xi_{\text{BKM}}} (S_{0+}^{\text{LP},V}(n) + S_{0+}^{\text{LP}}(n)) \right] \sin(n\phi) \right\}. \end{aligned} \quad (\text{E.14})$$

$\mathcal{P}_1(\phi)$ and $\mathcal{P}_2(\phi)$ are defined from the scalar products in the denominators of the BH lepton amplitude as,

$$\mathcal{P}_1(\phi) \equiv \frac{(k-q')^2}{Q^2}, \quad \mathcal{P}_2(\phi) \equiv \frac{(k'+q')^2}{Q^2}, \quad (\text{E.15})$$

and $\tilde{K}$ are defined from

$$\tilde{K}^2 = \frac{Q^2 K^2}{1-y+\gamma^2 y^2/4}, \quad (\text{E.16})$$

with K given in Eq. (D.9) and ξ_{BKM} given in Eq. (D.8). Again, there are contributions from those so-called effective twist three Compton form factors corresponding to the $-2\xi_{\text{BKM}}/(1 + \xi_{\text{BKM}})\mathcal{F}$ terms in the coefficients, or $2/(1 + \xi_{\text{BKM}})\mathcal{F}$ with WW relations taken into account.

In our case, we can derive those 6 coefficients right from their definitions in Eq. (3.89), for which the expression of the leptonic tensor $L_{\text{INT}}^{\mu\rho\sigma}$ is needed. We have split the leptonic tensor into the polarized and unpolarized part, and it will be helpful to further split them into the symmetric and antisymmetric parts such that

$$L_{\text{INT,U}}^{\mu\rho\sigma} = L_{\text{INT,U}}^{(\mu\rho)\sigma} + L_{\text{INT,U}}^{[\mu\rho]\sigma}, \quad L_{\text{INT,L}}^{\mu\rho\sigma} = L_{\text{INT,L}}^{(\mu\rho)\sigma} + L_{\text{INT,L}}^{[\mu\rho]\sigma}. \quad (\text{E.17})$$

Those tensors can be written as [102]

$$\begin{aligned} L_{\text{INT,U}}^{(\mu\rho)\sigma} = & \frac{2}{(k - \Delta)^2} \left[[k'^\mu(2k - \Delta)^\rho + k'^\rho(2k - \Delta)^\mu - g^{\mu\rho}k' \cdot (2k - \Delta)] k^\sigma \right. \\ & \left. - (k'^\mu k^\rho + k'^\rho k^\mu - g^{\mu\rho}k \cdot k')\Delta^\sigma + (k'^\mu g^{\rho\sigma} + k'^\rho g^{\mu\sigma} - g^{\mu\rho}k'^\sigma)(k \cdot \Delta) \right] \\ & + \frac{2}{(k' + \Delta)^2} \left[[(2k' + \Delta)^\mu k^\rho + (2k' + \Delta)^\rho k^\mu - g^{\mu\rho}k \cdot (2k' + \Delta)] k'^\sigma \right. \\ & \left. + (k'^\mu k^\rho + k'^\rho k^\mu - g^{\mu\rho}k \cdot k')\Delta^\sigma - (k'^\mu g^{\rho\sigma} + k'^\rho g^{\mu\sigma} - g^{\mu\rho}k'^\sigma)(k' \cdot \Delta) \right], \end{aligned} \quad (\text{E.18})$$

the antisymmetric unpolarized tensor

$$\begin{aligned} L_{\text{INT,U}}^{[\mu\rho]\sigma} = & \frac{2[-(k \cdot k')(\Delta^\mu g^{\rho\sigma} - \Delta^\rho g^{\mu\sigma}) + (k' \cdot \Delta)(k^\mu g^{\rho\sigma} - k^\rho g^{\mu\sigma}) - (k^\mu \Delta^\rho - k^\rho \Delta^\mu)k'^\sigma]}{(k - \Delta)^2} \\ & + \frac{2[(k \cdot k')(\Delta^\mu g^{\rho\sigma} - \Delta^\rho g^{\mu\sigma}) - (k \cdot \Delta)(k'^\mu g^{\rho\sigma} - k'^\rho g^{\mu\sigma}) + (k'^\mu \Delta^\rho - k'^\rho \Delta^\mu)k^\sigma]}{(k' + \Delta)^2}, \end{aligned} \quad (\text{E.19})$$

the symmetric polarized tensor

$$L_{\text{INT,L}}^{(\mu\rho)\sigma} = \frac{2}{(k - \Delta)^2} \left[g^{\mu\rho} \epsilon^{\sigma k k' \Delta} - k'^{\mu} \epsilon^{\rho \sigma k \Delta} - k'^{\rho} \epsilon^{\mu \sigma k \Delta} \right] \\ + \frac{2}{(k' + \Delta)^2} \left[-g^{\mu\rho} \epsilon^{\sigma k k' \Delta} - k^{\mu} \epsilon^{\rho \sigma k' \Delta} - k^{\rho} \epsilon^{\mu \sigma k' \Delta} \right], \quad (\text{E.20})$$

and the antisymmetric polarized tensor

$$L_{\text{INT,L}}^{[\mu\rho]\sigma} = \frac{2}{(k - \Delta)^2} \left[\epsilon^{\mu\rho k k'} (2k - \Delta)^{\sigma} + \epsilon^{\mu\rho k' \Delta} k^{\sigma} + \epsilon^{\mu\rho \sigma k'} (k \cdot \Delta) \right] \\ + \frac{2}{(k' + \Delta)^2} \left[\epsilon^{\mu\rho k k'} (2k' + \Delta)^{\sigma} + \epsilon^{\mu\rho k \Delta} k'^{\sigma} + \epsilon^{\mu\rho \sigma k} (k' \cdot \Delta) \right]. \quad (\text{E.21})$$

Then all the scalar coefficients can be worked out by contracting all the indices with $L_{\mu\rho\sigma}^{\text{INT,U}}$ and

$L_{\mu\rho\sigma}^{\text{INT,L}}$ and written as scalar products. Here we write down all 12 real scalar coefficients:

$$A^{I,U} = \frac{8}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} \left\{ 2(\bar{P} \cdot q)(k \cdot q')^3 \right. \\ - (k \cdot q')^2 \left[2(k \cdot q)(\bar{P} \cdot q') - 2(k \cdot \bar{P})(q^2 - q \cdot q') + 2(\bar{P} \cdot q)(q \cdot q') + q^2 \bar{P} \cdot (q - q') \right] \\ + (k \cdot q') \left[2(k \cdot \bar{P})(q \cdot q')^2 - (\bar{P} \cdot q')(q \cdot q')q^2 \right. \\ \left. + k \cdot q \left((q \cdot q') \bar{P} \cdot (q - 4k) + \bar{P} \cdot q'(q^2 + 2q \cdot q') \right) \right] \\ \left. + (k \cdot q)(q \cdot q') \left[(\bar{P} \cdot k)(q \cdot q') - (\bar{P} \cdot q')(k \cdot q) \right] \right\}, \quad (\text{E.22})$$

$$\begin{aligned}
B^{I,U} = & \frac{2t}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} \left\{ 2(n \cdot q)(k \cdot q')^3 \right. \\
& - (k \cdot q')^2 \left[2(k \cdot q)(n \cdot q') - 2(k \cdot n)(q^2 - q \cdot q') + 2(n \cdot q)(q \cdot q') + q^2 n \cdot (q - q') \right] \\
& + (k \cdot q') \left[2(k \cdot n)(q \cdot q')^2 - (n \cdot q')(q \cdot q')q^2 \right. \\
& \quad \left. + k \cdot q \left((q \cdot q')n \cdot (q - 4k) + n \cdot q'(q^2 + 2q \cdot q') \right) \right] \\
& \left. + (k \cdot q)(q \cdot q') \left[(n \cdot k)(q \cdot q') - (n \cdot q')(k \cdot q) \right] \right\}.
\end{aligned}
\tag{E.23}$$

$$\begin{aligned}
C^{I,U} = & \frac{-4 \left[2(k \cdot q')(k - q) \cdot q' + (k \cdot q)(q \cdot q') \right]}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} \left\{ (k \cdot P') \left[(n \cdot q')(P \cdot q) - (n \cdot q)(P \cdot q') \right] \right. \\
& \left. + (k \cdot P) \left[(n \cdot q)(P' \cdot q') - (n \cdot q')(P' \cdot q) \right] + (k \cdot n) \left[(P \cdot q')(P' \cdot q) - (P \cdot q)(P' \cdot q') \right] \right\}.
\end{aligned}
\tag{E.24}$$

$$\tilde{A}^{I,U} = \frac{8 \left[2(k \cdot q')(k - q) \cdot q' + (k \cdot q)(q \cdot q') \right]}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} \epsilon^{k \bar{P} q q'}, \tag{E.25}$$

$$\tilde{B}^{I,U} = \frac{2t \left[2(k \cdot q')(k - q) \cdot q' + (k \cdot q)(q \cdot q') \right]}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} \epsilon^{k n q q'}, \tag{E.26}$$

$$\tilde{C}^{I,U} = \frac{4 \left[2(q^2 - q \cdot q')(k \cdot q')^2 + (k \cdot q)(q \cdot q')^2 + 2(k \cdot q')(q \cdot q')(q' - 2k) \cdot q \right]}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} \epsilon^{k n P P'}, \tag{E.27}$$

$$A^{I,L} = \frac{8 \left[(k \cdot q)(q \cdot q') - q^2(k \cdot q') \right]}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} \epsilon^{k\bar{P}qq'} , \quad (\text{E.28})$$

$$B^{I,L} = \frac{2t \left[(k \cdot q)(q \cdot q') - q^2(k \cdot q') \right]}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} \epsilon^{knqq'} , \quad (\text{E.29})$$

$$C^{I,L} = \frac{-4(q \cdot q') \left[(k \cdot q)(q \cdot q') - q^2(k \cdot q') \right]}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} \epsilon^{knPP'} , \quad (\text{E.30})$$

$$\begin{aligned} \tilde{A}^{I,L} = \frac{8}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} & \left\{ q^2(k \cdot q')^2(\bar{P} \cdot q') \right. \\ & - (k \cdot q')(q \cdot q') \left[q^2(q' - k) \cdot \bar{P} + (k \cdot q)(\bar{P} \cdot q) \right] \\ & \left. - (k \cdot q)(q \cdot q') \left[(\bar{P} \cdot k)(q \cdot q') - (k \cdot q)(\bar{P} \cdot q') \right] \right\} , \quad (\text{E.31}) \end{aligned}$$

$$\begin{aligned} \tilde{B}^{I,L} = \frac{2t}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} & \left\{ q^2(k \cdot q')^2(n \cdot q') \right. \\ & - (k \cdot q')(q \cdot q') \left[q^2(q' - k) \cdot n + (k \cdot q)(n \cdot q) \right] \\ & \left. - (k \cdot q)(q \cdot q') \left[(n \cdot k)(q \cdot q') - (k \cdot q)(n \cdot q') \right] \right\} , \quad (\text{E.32}) \end{aligned}$$

$$\begin{aligned}
\tilde{C}^{I,L} = \frac{4[q^2(k \cdot q') - (k \cdot q)(q \cdot q')]}{(k \cdot q')(q \cdot q')(k - q) \cdot q'} & \left\{ (k \cdot P') [(n \cdot q')(P \cdot q) - (n \cdot q)(P \cdot q')] \right. \\
& + (k \cdot P) [(n \cdot q)(P' \cdot q') - (n \cdot q')(P' \cdot q)] \\
& \left. + (k \cdot n) [(P \cdot q')(P' \cdot q) - (P \cdot q)(P' \cdot q')] \right\}. \quad (\text{E.33})
\end{aligned}$$

Those coefficients are all we need for the interference cross-section formula for any polarization configurations, of which the code is also available in [260]. Note that all the B^I coefficients are related to the A^I coefficients by $4\bar{P}^\mu \rightarrow tn^\mu$. We also mention that for a given kinematics where twist-four effects are negligible, one can use the substitution $n^\mu \rightarrow q'^\mu/(\bar{P} \cdot q')$ for an approximate light-cone-independent expression, corresponding to the limit $\beta \rightarrow \infty$.

Besides, we also present our scalar coefficients calculated from light cone vectors with twist expansion, for which we define

$$\mathcal{A}^{I,U/L} = \frac{1}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)} \sum_{t=2}^{\infty} Q^{3-t} \mathcal{A}_{(t)}^{I,U/L}, \quad (\text{E.34})$$

with the twist t from 2 to ∞ for each coefficient. Then we have the following results

$$\begin{aligned}
A_{(2)}^{I,U} &= \frac{-8(2 - 2y + y^2)}{x_B y^3} \sqrt{[t(x_B - 1) - M^2 x_B^2](1 - y) \cos \phi}, \\
A_{(3)}^{I,U} &= \frac{-8}{x_B y^3} \left\{ (y - 2) \left\{ -M^2 x_B^2 (y - 2)^2 + t \left[-2 + 2y - y^2 + x_B (3 - 3y + y^2) \right] \right\} \right. \\
&\quad \left. + \frac{2x_B [t(x_B - 1) - M^2 x_B^2] (y - 2)(1 - y)(\beta - 1) \cos 2\phi}{1 + (\beta - 1)x_B} \right\}, \quad (\text{E.35})
\end{aligned}$$

and for $B^{I,U}$,

$$\begin{aligned}
B_{(2)}^{I,U} &= 0, \\
B_{(3)}^{I,U} &= \frac{8tx_B(2-3y+y^2)}{(x_B-2)y^3}, \\
B_{(4)}^{I,U} &= \frac{-8tx_B\sqrt{[t(x_B-1)-M^2x_B^2](1-y)}}{(x_B-2)(1+(\beta-1)x_B)y^3} \\
&\quad \times \left[2(y-2)^2 - (2-2y+y^2)\alpha + 2x_B(6-6y+y^2)(\beta-1) \right] \cos \phi,
\end{aligned} \tag{E.36}$$

and for $C^{I,U}$,

$$\begin{aligned}
C_{(2)}^{I,U} &= \frac{8(2-2y+y^2)}{(x_B-2)y^3} \sqrt{[t(x_B-1)-M^2x_B^2](1-y)} \cos \phi, \\
C_{(3)}^{I,U} &= \frac{8(y-2)[t(x_B-1)-M^2x_B^2]}{(x_B-2)y^3} \left[(y-2)^2 + \frac{2x_B(1-y)(\beta-1) \cos 2\phi}{1+(\beta-1)x_B} \right].
\end{aligned} \tag{E.37}$$

In the case of longitudinally polarized cross-section, the other three coefficients $\tilde{A}^{I,U}$, $\tilde{B}^{I,U}$ and $\tilde{C}^{I,U}$ are needed and their expansions read

$$\begin{aligned}
\tilde{A}_{(2)}^{I,U} &= \frac{-8(2-2y+y^2)}{x_By^3} \sqrt{[t(x_B-1)-M^2x_B^2](1-y)} \sin \phi, \\
\tilde{A}_{(3)}^{I,U} &= \frac{16[t(x_B-1)-M^2x_B^2](1-y)(2-y)(\beta-1) \sin 2\phi}{y^3[1+(\beta-1)x_B]},
\end{aligned} \tag{E.38}$$

and for $\tilde{B}^{I,U}$,

$$\begin{aligned}
\tilde{B}_{(2)}^{I,U} &= 0, \\
\tilde{B}_{(3)}^{I,U} &= 0, \\
\tilde{B}_{(4)}^{I,U} &= \frac{8x_Bt\sqrt{[t(x_B-1)-M^2x_B^2](1-y)}[4(1-y)+(2-2y+y^2)\alpha] \sin \phi}{(x_B-2)y^3[1+(\beta-1)x_B]},
\end{aligned} \tag{E.39}$$

and for $\tilde{C}^{I,U}$,

$$\begin{aligned}\tilde{C}_{(2)}^{I,U} &= -\frac{8(2-2y+y^2)}{(x_B-2)y^3} \sqrt{[t(x_B-1) - M^2 x_B^2](1-y) \sin \phi}, \\ \tilde{C}_{(3)}^{I,U} &= \frac{16x_B [t(x_B-1) - M^2 x_B^2] (1-y)(2-y)(\beta-1) \sin 2\phi}{(x_B-2)y^3 [1 + (\beta-1)x_B]},\end{aligned}\tag{E.40}$$

where some twist-four terms are not shown as the expressions get complicated. According to the previous comparison, those expansions converge to the all-order results at large Q^2 .

Appendix F: GUMP parameters and their best-fit values

In this appendix we present more details of the fit, especially the best-fit parameters obtained from the global analysis. In table [F.1](#), we show all the independent parameters with their statistic uncertainties estimated by the Hessian matrix with the *Minuit2* package.

We note that the $N, \alpha, \beta, \alpha', b$ are the parameters in the moments of GPDs according to eq. (5.47) where $\alpha(t) \equiv \alpha + \alpha' t$ corresponds to the linear Regge trajectory and $\beta(t) = \exp(bt)$ corresponds to the extra exponential term. Each of these parameters have a superscript representing its GPD species ($H, E, \tilde{H}$ or $\tilde{E}$) and a subscript representing its flavor ($u_V, \bar{u}, d_V, \bar{d}$ or g). The R_{sea}^E and $R_{\text{sea}}^{\tilde{E}}$ are the ratio of the $E(\tilde{E})$ GPDs to the $H(\tilde{H})$ for the sea quarks. Besides them, there are also parameters like $R_{u,2}^H$ that are the ratio of the ξ^2 terms to the forward terms as defined in eq. (5.53) for GPDs with different species and flavors. We again note that more details of them and the GUMP program that generate the GUMP GPDs, CFFs and cross-sections are available online [\[259\]](#).

Vector GPDs H and E		Axial-vector GPDs $\tilde{H}$ and $\tilde{E}$	
Parameter	Value (uncertainty)	Parameter	Value (uncertainty)
N_{uV}^H	4.923 (89)	$N_{uV}^{\tilde{H}}$	4.833 (429)
α_{uV}^H	0.216 (7)	$\alpha_{uV}^{\tilde{H}}$	-0.264 (34)
β_{uV}^H	3.229 (23)	$\beta_{uV}^{\tilde{H}}$	3.186 (122)
α'_{uV}^H	2.347 (51)	$\alpha'_{uV}^{\tilde{H}}$	2.182 (175)
$N_{\bar{u}}^H$	0.163 (8)	$N_{\bar{u}}^{\tilde{H}}$	0.070 (33)
$\alpha_{\bar{u}}^H$	1.136 (10)	$\alpha_{\bar{u}}^{\tilde{H}}$	0.538 (112)
$\beta_{\bar{u}}^H$	6.894 (207)	$\beta_{\bar{u}}^{\tilde{H}}$	4.229 (1320)
N_{dV}^H	3.359 (170)	$N_{dV}^{\tilde{H}}$	-0.664 (170)
α_{dV}^H	0.184 (18)	$\alpha_{dV}^{\tilde{H}}$	0.248 (76)
β_{dV}^H	4.418 (77)	$\beta_{dV}^{\tilde{H}}$	3.572 (477)
α'_{dV}^H	3.482 (171)	$\alpha'_{dV}^{\tilde{H}}$	0.542 (103)
$N_{\bar{d}}^H$	0.249 (12)	$N_{\bar{d}}^{\tilde{H}}$	-0.086 (42)
$\alpha_{\bar{d}}^H$	1.052 (10)	$\alpha_{\bar{d}}^{\tilde{H}}$	0.495 (137)
$\beta_{\bar{d}}^H$	6.554 (216)	$\beta_{\bar{d}}^{\tilde{H}}$	2.554 (897)
N_g^H	2.864 (108)	$N_g^{\tilde{H}}$	0.243 (304)
α_g^H	1.052 (8)	$\alpha_g^{\tilde{H}}$	0.631 (330)
β_g^H	7.413 (165)	$\beta_g^{\tilde{H}}$	2.717 (2865)
N_{uV}^E	0.181 (38)	N_{uV}^E	7.993 (3480)
α_{uV}^E	0.907 (17)	α_{uV}^E	0.800 (116)
β_{uV}^E	1.102 (245)	β_{uV}^E	6.415 (1577)
α'_{uV}^E	0.461 (86)	α'_{uV}^E	2.076 (933)
N_{dV}^E	-0.223 (47)	N_{dV}^E	-2.407 (1239)
R_{sea}^E	0.768 (169)	R_{sea}^E	38 (8)
$R_{u,2}^H$	0.229 (0.032)	$R_{u,2}^H$	0.246 (81)
$R_{d,2}^H$	-2.639 (202)	$R_{d,2}^H$	1.656 (375)
$R_{u,2}^E$	0.799 (285)	$R_{u,2}^E$	2.684 (171)
$R_{d,2}^E$	3.404 (1157)	$R_{d,2}^E$	38 (2)
b_{sea}^H	3.448 (133)	b_{sea}^H	9.852 (1330)

Table F.1: A summary of the obtained independent GUMP parameters.

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