THE EFFECTS OF CURRICULA ON STUDENTS' ABILITY TO ANALYZE AND SOLVE PROBLEMS IN ALGEBRA

by

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ABSTRACT

Title of Dissertation: The Effects of Curricula on Students' Ability to Analyze and Solve Problems in Algebra

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The purpose of this study was to compare students in two different curricula on two problem solving abilities, the ability to identify the underlying mathematical structure in a problem and the ability to solve problems. The two curricula were the Algebra with Computers project materials and the algebra 1 curriculum normally taught in the schools where the Algebra with Computers materials, which sought to give an existence proof for a computer-intense algebra curriculum, were being field tested. The experimental and control classes were at two sites in the Middle Atlantic region.

At both sites, control and experimental students were given two researcher-designed tasks. The Problem Solving Test contained ten problems typical of algebra 1 which

students were asked to solve in any way known to them. The Triads Task consisted of 24 groups of three problems each. Students were asked to identify which pair of each triad would be solved in the same manner.

The results of the two tasks were analyzed using analysis of covariance with each site and each task being analyzed separately. The covariates, taken from the eighth grade scores on the California Achievement Test, were total reading, mathematical concepts and mathematical computation. Sex was used as a blocking variable.

The ANCOVA results were significant in favor of the experimental groups for both Site One (at the .05 level) and Site Two (at the .01 level) on the Problem Solving Test and at Site Two (at the .01 level) for the Triads Task. The Site One Triads Task results were not significant. There were no significant differences between the performances of males and females at both sites on the Problem Solving Test or at Site Two on the Triads Task. At Site One there was a significant difference in favor of the females on the Triads Task. There were no significant interactions between sex and treatment.

The students using the Algebra with Computers materials were better problem solvers in three of the four analyses. This study has shown that as measured by the two tasks given, the Algebra with Computers curriculum does produce better problem solvers.

To John

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INTRODUCTION

CHAPTER I

Rationale

In 1980, the National Council of Teachers of Mathematics (NCTM) issued An Agenda for Action: Recommendations for School Mathematics of the 1980s (NCTM, 1980). Its first recommendation was "Problem solving must be the focus of school mathematics in the 1980s." Problem solving was seen as an appropriate curriculum core because, while emerging information technologies may make many traditional skills less useful or even obsolete, the need for problem solvers will not change. This call for more problem solving in the mathematics curriculum has been repeated in many other reports on the future of mathematics (e.g. NACOME, 1975; Conference Board of the Mathematical Sciences, 1983).

Another recommendation of the Agenda for Action argues that "Mathematics programs must take full advantage of the power of calculators and computers at all grade levels." (NCTM, 1980). Proponents of technology in mathematics education contend that calculators and computers make it possible for students to focus on problem solving by freeing them from computational tasks; that by using computers and calculators students can explore problem solving situations that would be too difficult or too lengthy to do without them; and that students can then focus their energies on building mathematical models for situations, on analyzing those models mathematically using calculator and computer tools and then on analyzing their results in light of the model and the problem. The NACOME report states:

> Because computers can easily cope with complex calculations and numerical values, once programmed by a human problem solver, it seems likely that teachers and students will be able to focus more attention on the modeling process-- problem formulation and interpretation of results--and less on practicing manipulative skills and accuracy.(NACOME, 1975, p.37).

Sharing the vision of NCTM and NACOME is a 1983 report from the Conference Board of the Mathematical Sciences which also envisioned a new school mathematics curriculum that makes use of the "...computer as a tool to solve mathematical problems." (Conference Board of the Mathematical Sciences, 1983, p. 7). A group of mathematics educators meeting in a 1982 NSF-sponsored conference at the University of Maryland further elaborated on this vision by calling for this new mathematics curriculum to have a different sequence of skills that require that students do by hand only those things that are done more quickly by hand while doing by calculator and computer those things that are done more quickly by them (Fey, 1984).

Interwoven with the call for a focus on problem solving and the use of the computer as a tool is a call for the use of realistic application problems in mathematics classes (Lesh, 1982, NCTM, 1980). Application problems give students a feel for the uses of mathematics and enable them to practice using their mathematics skills on useful problems. The use of the computer allows the student to try realistic applications that would be too difficult or too long without the use of those tools. "Computer capabilities also open up fascinating possibilities for realistic application of mathematical techniques and simulation of interdisciplinary problem solving situations." (NACOME, 1975, p. 37).

Sharing this vision of using computers and calculators in a curriculum that emphasized problem solving and realistic application problems, researchers at the University of Maryland began in 1984 to write and test curricula that embodied those themes. The Algebra With Computers project was an NSF-sponsored curriculum project based at the University of Maryland that sought to give an existence proof for such a computer-intensive algebra 1 curriculum.

The development of experimental curricula always raises the questions of whether or not the experimental curriculum is better than the established curriculum and in what ways might their differences affect the abilities of students. Because of its importance, problem solving was chosen as the focus of this dissertation study which seeks to compare the Algebra with Computers algebra 1 curriculum to the established algebra 1 curriculum on the ways in which these curricula affect the problem solving abilities of students who are taking them. Two problem solving abilities of students involved in the Algebra with Computers project were measured and compared those abilities to students in companion classes taking the traditional algebra 1 course.

Another area of concern for mathematics educators is the question of sex differences in the study and learning of mathematics. While some studies have found that males are better at mathematics than females, others have found this not to be the case (Fennema & Sherman, 1978). There is a continuing debate as to whether, in fact, males are better in mathematics than females or whether these differences may be due to other factors such as number and content of mathematics courses taken or encouragement and expectation to study mathematics. Given this open question, this dissertation study also looks at the performance of males and females on the two problem solving abilities measured.

Statement of Purpose

The purpose of this study was to compare students enrolled an experimental algebra 1 curriculum with students in a control algebra 1 curriculum on two problem solving related skills: (1) the ability to identify problems that have the same mathematical structure; (2) the ability to solve given applied problems. The experimental curriculum was the Algebra with Computers project which used the computer as a tool allowing the focus of the course to be on mathematical structure, concepts, and problem solving with applied problems. The control groups were classes who were taking the standard algebra 1 course that was offered at the two schools where the experimental curriculum was being tried.

The two abilities were measured using two experimenter-designed tasks each of which were given to both the groups using the Algebra with Computers materials for algebra 1 and control groups. The ability to identify the mathematical structure in an applied problem was measured by a Triads Task. The Triads Task contained twenty-four groups of three problems. In each triad, there was one pair of problems that shared the same mathematical structure. Students were to identify that pair. The ability to solve applied problems was measured by a Problem

Solving Test. The Problem Solving Test consisted of ten applied problems that students were to solve by any method known to them.

Significance of the Study

If it is found that students in the Algebra with Computers curriculum are better at identifying problems that have the same mathematical structure as measured by the Triads Task and/or that they are better able to solve problems as measured by the Problem Solving Test, this may indicate that the Algebra with Computers curriculum teaches problem solving skills better than the curricula used by the control groups. Because the Algebra with Computers curriculum uses the computer to emphasize concepts and problem solving skills, this may then indicate that using computers in mathematics classrooms to study algebra allows students to concentrate on problem solving and become better problem solvers.

Should the Algebra with Computers students do less well on the Triads Task and/or the Problem Solving Test, it may indicate that the Algebra with Computers materials need revision to more emphasize problem solving and mathematical structure, that the Algebra with Computers materials need to be tried in different settings, that further teacher in-service is necessary to insure that the Algebra with Computers materials are taught as designed, that the differences in teaching problem solving between the two curricula are not significant or that the skill drill found in the regular algebra 1 course is better preparation for problem solving.

The performance of males and females on the two tasks may show whether the two curricula differ in the mathematics learning of males and females. Should the females enrolled in the Algebra with Computers curriculum perform as well as or better than the males, this may indicate that the use of the computer can lessen any advantage that the males may have in mathematics. If the females enrolled in Algebra with Computers perform more poorly than the males, this may indicate that males who have has Algebra with Computers are better mathematical problem solvers than females who had the same curriculum or that perhaps there are other factors such as attitude towards mathematics or computers that may be affecting females' achievement in problem solving.

The Research Questions

1. Will students in the Algebra With Computers classes be better able to identify problems with the same mathematical structure as measured by the Triads Task than those in the control classes ?

2. Will the students in Algebra with Computers classes be better able to solve problems as measured by the Problem Solving Test than those in the control classes?

3. Will males and females differ on their abilities to identify problems with the same mathematical structure as measured by the Triads Test?

4. Will males and females differ on their abilities to solve problems as measured by the Problem Solving Test?

The independent variable in the study is algebra 1 curriculum. The dependent variables are the scores on the Triads Task and Problem Solving Test.

Outline of the Study

Chapter II provides a review of the literature related to problem solving, the use of the computer in the classroom, the teaching of concepts before teaching algorithms and the teaching and learning of algebra.

Chapter III describes the schools and subjects involved in the study and includes a discussion of the Algebra with Computers project and the traditional algebra 1 curriculum. Also includes are the design of the study and the design, administration and scoring of the Triads Task and Problem Solving Test.

Chapter IV provides the results and analysis of the Triads Task, and the Problem Solving Test.

Chapter V summarizes the study, analyzes the results and states the conclusions and implications of the study. The chapter concludes with recommendations for further research.

CHAPTER II

REVIEW OF RELATED LITERATURE

The literature relevant to this study falls into four broad categories. These categories are problem solving research, the use of the computer in the mathematics classroom, research that centers on teaching understanding of concepts before teaching algorithms and research on algebra instruction.

Problem Solving Research

The literature on problem solving is extensive and varied. For this study, areas of problem solving research searched included ones concerning the role of problem solving and problem solving research, problem solving instruction, cognitive theories and problem solving, the role of structure in mathematical problem solving and the role of context in mathematical problem solving. This section will concentrate on general mathematical problem solving with material pertaining to problem solving in algebra treated in the algebra section.

The Role of Problem Solving and Research on Problem Solving

What is problem solving and why is it important for students to be able to solve problems? Problem solving has complex and varied meanings. These meanings may even be subject specific. In this paper problem solving refers to mathematical problem solving and means actions taken to answer a question for which the solver has no readily available algorithm (Lester, 1980). Using this defintion, problem solving will not include repetition of routine operations or symbol manipulations but will include using those skills and other learned concepts in the solution of problems.

The role of problem solving in mathematics learning and instruction is a topic of much concern for teachers, parents and educators. Many mathematics educators feel as Lester does that "...the ultimate aim of learning mathematics at every level is to be able to solve problems." (Lester, 1980, p.287). The 1975 NACOME report on the state of school mathematics criticized schools and curricula for an overemphasis on computational skill development and rote memory of facts while deemphasizing concepts and problem solving.(NACOME, 1975). Among its recommendations is a call for a balance between algorithmic skill learning and problem solving with application problems. In 1980, the National Council of Teachers of

Mathematics (NCTM) echoed these concerns its An Agenda for Action: Recommendations for School Mathematics of the 1980s making its first recommendation that problem solving be the focus of school mathematics. (NCTM, 1980). Another recommendation of these groups is that students in eighth grade and above be allowed to use calculators in all mathematics classes.

Researchers in mathematics education have also recognized the importance of problem solving and have called for more research on problem solving. Lesh (1981, Lesh and Akerstrom, 1982) calls for research focusing on the average ability student learning substantive mathematical content using real problems and realistic settings. Lester has echoed that call adding the need for naturalistic inquiry into problem solving (Lester, 1985).

While mathematics educators continue to call for an emphasis on problem solving and the use of calculators for computation, others may not agree. The College Board in its booklet Academic Preparation for College makes no mention of problem solving skills in its section on mathematics (1983). So while mathematics educators may call for an emphasis on problem solving and problem solving skills and the routine use of the calculator, those who will test students for college entrance, do not.

This difference in opinion concerning what kind of skills students need has been one of continuing interest.

The question is far from resolved. If students do, in fact, need to be able to solve problems then the next area of concern is problem solving instruction. How can we plan instruction so that students learn to solve problems?

<u>Problem Solving Instruction</u>

Research on problem solving has been done in two modes. General problem solving skills have been studied in cross-subject projects and mathematical problem solving has been studied in relation to the mathematics classroom. General problem solving projects have met with little success. This lack of success appears to be because problem solving seems to be subject specific with little transfer of processes between subjects (Lesh, 1981).

The second type of problem solving instruction focuses solely on mathematics and this appears to be more successful. Instructional projects in mathematical problem solving fall into two groups--large curriculum projects focusing on entire classrooms and curricula and smaller projects that involve smaller numbers of students and often focus on teaching heuristics. Two large curriculum projects and four small projects will be mentioned here.

The Applied Problem Solving project was a curriculum project done with seventh-grade average ability students (Lesh, 1981). The problem situations were ten to forty-five minutes long and resembled real-life problems that students or their parents might encounter. Such situations took more thought than the typical textbook problem. The researchers found that the processes needed to solve the real-life problems were easily taught to the students and that these processes were quite different and more complex than the ones needed to solve typical textbook word problems (Lesh, 1982).

A second larger curriculum project was the Mathematical Problem Solving program (Charles, 1984). It involved twelve fifth-grade and ten seventh-grade classes whose teachers implemented the program for twenty-three weeks. The program consisted of instructional material for problem solving, guidelines concerning ways to create a classroom atmosphere conducive to problem solving, group instruction guidelines and a teaching strategy for problem solving. Based on Polya's four phase model of problem solving stressing extensive experience with process problems, it incorporated a specific strategy. MPS students and teachers had one problem solving experience for each day of the week in addition to the regular mathematics program. Results indicate that treatment classes scored significantly higher than control clases on three measures of problem solving and that the program was teachable within the normal classroom environment.

The importance of the MPS was that it was a long-term look at problem solving within normal classroom settings and a normal time schedule. Teachers in the program found that the teaching strategy and the problem solving guide were essential to success.

The four smaller projects considered focused on heuristic training. A study by Vos (1976) looked at the effect of three instructional strategies on the acquistion and use of five problem solving strategies. The three strategies were repetition where students were simply given a problem task; list where students were given a problem task, a checklist with various strategies on it and behavior instruction before they attempted the problem task; and behavior instruction where students were given behavior insruction before being given the problem task. Vos was able to conclude after fifteen weeks of instruction that instruction in the specific problem solving techniques increased the use of those techniques.

Kantowski (1977) took a small group of ninth grade students and looked at what happened when they were taught both geometry skills and problem solving skills. She specifically looked at the processes used by the students in solving problems both before and after instruction. Kantowski found that the use of heuristics was consistently more evident in solutions with scores above the median and that successful problem solvers had more regular patterns of analysis and synthesis. In addition, Kantowski found that instruction in heuristics tends to decrease the number of superfluous strategies used and that Polya's looking back strategy did not increase even with instruction or for good problem solvers.

Schoenfeld (1979) also looked at explicit instruction in heuristics and its effect on problem solving. He worked with seven college upperclassmen in a winter term mini-course on problem solving that taught heuristics. Among other things, Schoenfeld found that even in an enriched environment, students seldom intuit heuristics, that just because a student masters a heuristic does not mean he will use it; and that when problem solving strategies are taught and students remember to use them, the impact on the solution of problems is substantial.

Darch (1984) and others looked at a program of explicit instruction and its effect on skill deficit fourth graders. In the posttest there was a significant positive effect for the explicit instruction but no effect for the provision for extra review lessons. In a maintenance test administered two weeks later, only the students in the explicit group who had received the extra review performed significantly better than those in the comparison groups.

These six projects illustrate the complexity of teaching students problem solving. Explicit instruction on problem solving strategies and skills and practice in solving problems can raise scores on problem solving tests. The MPS program has shown that this is possible within the normal classroom environment. Concerns remain, however, on how to get students to use strategies taught them and on the different processes necessary to solve real-life applications problems.

Success in various projects has also been limited by the lack of a theory to guide research on problem solving and the complex nature of the problem solving process (Lester, 1982a). There appears not to be a single set of problem solving skills even within just the area of mathematics (Greeno, 1980). In addition, "The development of problem solving ability is a cumulative process which depends on the history of problem solving experiences of the student." (Kulm, 1984).

Though problem solving has been taught successfully in the mathematics classroom and much research has been done in this area, much remains to be done as there is much which we still do not know about how students learn to solve problems and how to best teach students to solve problems. "...the literature on mathematical problem-solving instruction is still largely based on folklore and the sage advice of master teachers like George Polya." (Lester, 1982a).

Cognitive Theories and Problem Solving

Although there is now no reasonable, complete theory of mathematical problem solving, mathematical problem solving has a theoretical basis in cognitive psychology (Lester, 1982a). The Gestalt psychologist Wertheimer made the distinction between understanding the problem and the rote execution of the problem (Wertheimer, 1959, Mayer, 1985). Cognitive psychologists have since identified four types of knowledge necesary for mathematical problem solving. They are factual knowledge, schematas, algorithmic knowledge and strategic knowledge (Mayer, 1981b, 1982a). Schema are elements of an student's cognitive make-up that allow him to classify problems. (Hergeghahn, 1982, Mayer, 1982b, Silver, 1982). Schema allow students to make a general representation for a class of problems. Strategic knowledge involves the approach to a problem and the heuristics used to solve the problem. (Mayer, 1981a). Mayer believes that current methods of instruction ignore the schema and strategic aspects of problem solving. Mayer also identified four types of training necessary for learning problem solving. They are translation, schemata, strategy training and algorithhn automaticity (Mayer, 1985). These correspond to the four types of knowledge necessary for problem solving. Algorithm automaticity may be seen in the rote drill often done in the mathematics classroom and translation training involves translating situations into mathematical sentences or operations. This also is often now part of mathematics instruction. Mayer suggested that his four types of training would increase problem solving ability if integrated into mathematical instruction. His suggestions include giving students practice in recognizing different types of problems, in mixing the types of problems given in an assignment and giving students practice in changing representations in problems. (Mayer, 1985,1982a).

Cognitive theories have also been part of much of the research on expert and novice problem solvers. Early research in this area centered around the playing of chess. Researchers found that expert players tended to represent data in very efficient and large representations. These representations helped them focus on key features of a problem and were too complicated for novice players to learn (Newell and Simon, 1972, Lockhead, 1981). Referred to as "chunking" information, this means that expert problem solvers can store and access large amounts of information in short amounts of time. This allows them to solve problems more quickly and effectively. This research into experts and novices has been extended into physics (Larkin, 1980) and mathematics (Chi, Glaser and Reese, 1982) with the chunking of information by experts also found in both subjects.

<u>The Role of Structure in Problem Solving</u>

The role of mathematical structure in problem solving has been well documented in recent years. Mathematics by its nature is highly structured. Skills, facts, concepts and procedures are interrelated. Good problem solvers appear to be able to take advantage of this structure in solving problems (Carpenter, 1985). They often use structure to decide on what processes to use or to decide what information is relevent to the problem (Hayes, Waterman and Robinson, 1977).

The role of mathematical structure in problem solving was one of the major findings of Krutetskii (1976) in his studies of children and their problem solving. Working with students partitioned by teacher judgment into groups of mathematically capable, mathematically average and mathematically incapable, Krutetskii had students solve a great number of carefully constructed problems. Krutetskii found that good problem solvers were superior to poor problem solvers in their ability to perceive the underlying mathematical structure of a problem and to generalize to problems of similiar structure. Poorer problem solvers were found to attend more to contextual details than to the mathematical structure of the problems. Krutetskii hypothesized a three stage model for problem solving. The stages were the acquisition of initial facts, the transformation of the initial items to obtain a solution and the retention of information about the problem and the process of solving it (Silver, 1977). Other differences that he found between good and poor problem solvers include that good problem solvers have abilities to distinquish relevant information and to remember the formal structure for a long time after solving the problem. Krutetskii worked primarily with gifted children and his work was non-experimental and based on long term observations (Krutetskii, 1976).

Attempting to see if this ability to see mathematical structure would be present in non-gifted students, Silver (1977, 1979, 1981) designed several experiments testing to see if good problem solvers did indeed see the structure of a problem. This was done by looking at the question of how students perceived problems as related. The principle technique used was a Card Sorting Task where students were asked to sort twenty-four problems presented on cards into groups of problems that are "mathematically alike". The problems in the Card Sorting Task were primarily from Krutetskii's collection of problems and were short problems of the puzzle type or solvable by linear equations. The students involved were 98 eighth grade students who were in junior high mathematics classes taught by Silver in a suburban public school in New York. The day after the students sorted the cards, they were asked to solve the problems and this produced a problem solving ability score. Students were also given three other tasks. In one task they matched 10 new problems to the original 24 problems from the card sorting task again asked to match those that are mathematically related; this was called the Problem Relations Task. The Problem Triads Task involved groups of three problems of which two were structurally alike. Students were asked to identify the ones that were most mathematically alike. Finally, in the Problem Memory Task, the students were asked to remember problems that were familiar and their responses were coded as to the nature of their recall. (Silver, 1977).

Silver had hypothesized that students would sort the cards in the Card Sorting Task along either structure or context lines. The mathematical structure of a problem is the underlying part of the problem that does not change when details such as the statement or numbers in the problem change. Two problems are said to have the same structure "...when algebraic equations into which the problems may be translated are the same..." (Goldin, 1984). Problems have the same context when they have similar story and details. Silver found in the initial task that two other ways of sorting problems began to emerge. One Silver called pseudostructure. This involved problems that were related along lines such as age problems or coin problems or problems that has the same units in them. The fourth line of sorting was named question form. Students might group together all problems with the question beginning "how many" or "how much". Silver then did a second round of Card Sorting and problem solving using another group of eighth graders and a modified set of problems. The second set was designed so that two problems were alike only along one of the four dimensions, structure, context, pseudostructure and question, that Silver now identified as sorting dimensions. Silver called these four dimensions categories of problem relatedness.

In the initial round of tasks, Silver found that students who associated along the structure dimension did not associate along the context or pseudostrucutre dimensions. There were significant positive correlations between scores on the Card Sorting Tasks and the Problem Relations Tasks on all four dimensions and there were significantly positive correlations between sorting along the structure dimension and verbal IQ, nonverbal IQ, mathematics concept knowledge and mathematics computations ability (Silver, 1977). In addition, students' scores on the contextual dimension were significantly negatively correlated with each of the variables mentioned above as positively correlated with the structure score. Using partial correlations, the correlation between the structure score and the students' problem solving ability remained significant even when the effects of the four variables named above were controlled simultaneously. The Problem Triads Task scores were positively correlated to the Card Sorting Task scores along the structure and context dimensions.

In the second round, all of the findings given above were supported and in addition, the tendancy to sort on the basis of pseudostrucutre was found to be negatively correlated to mathematical ability and the tendancy to sort on the basis of question form was found to be unrelated to mathematical ability.

On the whole, Silver was able to confirm Krutetskii's observations that good problem solvers were indeed able to perceive the underlying mathematical structure of problems better than poorer problem solvers. In additional research, Silver found that better problem solvers remembered the structure and used the structure to solve related problems (Silver, 1981).

What is mathematical structure? Silver used expert opinion to determine which of his problems had the same structure but other researchers have sought ways to define structures as the same. Kilpatrick (1975) divided problem solving research into three categories of independent variables and four of dependent variables. The independent variables were subject, task and situation variables.

Structure variables are a subset of the task variables. Kilpatrick used structure to mean the intrinsic mathematical structure of a problem. Problems with the same structure may be understood as problems with the same formula or relation (Kulm, 1984) or it may be characterized by the "state-space" approach of Goldin (1984). The "space-state" approach was developed by Goldin and Luger (1974, 1975) by applying certain artificial intelligence techniques to problem solving and it involves such constructs as the number of moves in problem solution, the number of blind alleys and the length of the shortest path to solution (Goldin, 1984b). The "state-space" approach has been most often used with puzzle type problems such as the Tower of Hanoi or the Missionaries and Cannibals problems. The use of structure as the underlying functions or equations of a problem is more in line with the application problems that are a part of this study and for that reason, that characterization of structure will be used rather than the "space-state" characterization.

The cognitive level of the learner may be a factor in the effect that problem structure has on his ability to solve problems. Days (1977, 1979) studied the processes used by concrete and formal operational students solving problems classified as simple and complex in structure. Days found that problem structure had a greater role in the processes used by formal operational students than the concrete operational students. The two groups did not differ in processes used for simple structure problems but did for complex structure problems suggesting that simple structure problems did not call for high level processes but that complex structure problems did. The formal operational students were better able to use higher processes in order to solve the problems with the more complex structures.

The role of structure in mathematical problem solving has been studied in several different ways. Research by Silver and Krutetskii indicates that students who are aware of the mathematical structure of a problem are better problem solvers and that poorer problem solvers may attend instead to contextual details of a problem. At this point no research has been done on the teaching of mathematical structure to students. If we teach the students to recognize that mathematical structure of a problem, will they then become better problem solvers? Can we teach students to recognize the underlying structure? These remain two questions unanswered by research.

The Role of Context in Mathematical Problem Solving

In most problem solving research the role of context has been to be a variable that is held constant while other variables are studied. Context is defined as the parts of a problem that "...describe surface characteristics that can be observed directly from the problem statement or its immediate surroundings." (Webb, 1984, p. 70). Context variables may be the field from which the problem comes or whether a problem is an application or fantasy problem.

Recent research has included a call for more applied problems to be used as research tasks. Lesh (1981, 1982) has called for research focusing on the average ability student solving problems of substantive mathematical content on real problems in realistic settings. Lesh, has, in addition, offered a model of the interaction between real world situations, models, pictures and written and spoken symbols (Lesh, 1981). This model was used in the Applied Problem Solving project. This project with average ability junior high students involves problems that require ten to forty-five minutes for solution and that resemble problem solving situations that might occur in the everyday lives of the students and their parents. Among the findings of the APS project is that students in applied problem solving tend to develop and use powerful content-dependent processes rather than general processes (Lesh, 1982).

Other research on context includes the work of Christensen (1980) who gave ninth grade algebra students a set of eight pairs of matched word problems. One of each pair was written in terms of a real life context and one was in a fantasy context. Students were told to select one of each pair to solve. Two days later the students completed the regular chapter test based on the current material and this was used as a measure of achievement. Results indicated that female students had no preference for fantasy or real life problems but that males had a strong preference for the fantasy problems. The level of students' achievement was not a significant factor in predicting preferences. Students' success in solving fantasy problems was higher but not practically so.

Blake (1977) used two groups of algebra II students with one given problems using real world setting and one using mathematical setting problems. He found that problem context was not related either to the total number of heuristics used or the number of different heuristics used. Students working on problems in the mathematical setting had a slightly more difficult time understanding the problems but performed as well as the other group.

The question of transfer of information is one of concern and related to the context of a problem. Moskol (1980) in an exploratory study with college students and real-world problems found that the college students had trouble transferring mathematical concepts to real-world problems. Kulm and Days (1979) found that transfer of information does take place but that contexts must be the same for the transfer to take place. Silver (1981) also found significant transfer of information occured but that neither good nor poor problem solvers tended to acknowledge having used information from the target problems when solving the structurally related problems.

The role of context is still unclear in mathematical problem solving. Student preference for one type of problem or another and the role of context in transfer of information has been studied with no definitive answers. The use of applied problems has been called for among the mathematics education community but the question still remains if applied problems enhance problem solving.

The Computer as a Tool for Learning Mathematics

The advent of the computer age has caused much debate in the mathematics education community concerning the role of the computer in the mathematics classroom. Mathematics classrooms were among the first classrooms to have computers in them as mathematics teachers became the computer experts in many schools and as computer programming became part of the mathematics curriculum. These first classroom computers were usually terminals with phone lines to main frame computers. With the reduction of the cost and the increase in abilities of the microcomputer, more and more mathematics classrooms added computers. The focus of these first computer users was programming. Many studies were done focusing on the learning of programming skills as problem solving skills (Robitaille, 1977, Reding, 1982). The next wave of computer use was that of the drill and practice software. Microcomputers became electronic ditto sheets as students practiced skills on them. In recent years with the writing of software that will do symbol manipulation, the micro computer can now be used not only as a drill master but also as a tool for solving problems. Software is available that will allow students to graph equations, produce tables of values for and solve equations (Fey, 1984). Research has begun to look at the use of the computer as a tool and the effect that tool might have on classroom activity and on the skills taught (Heid, 1984, 1988).

Goldstein (1980) suggested four roles for computers in problem solving tasks. A computer coach is a type of computer assisted instruction where the coaching program observes the student engaged in an intellectual game and may intervene occassionally to discuss skills that might improve play. As a personal assistant, a computer can assume part of the problem solving task freeing the student to solve more complex problems. In cognitive programming environments, students implement their own problem solving programs and in cognitive simulation environments students can use simulations to explore the consequences of various strategies. Two of these four modes are present in the Algebra with Computers project. The students have several simulations that they use to explore the relationship between various variables and the students use several plotting and symbol manipulation programs to have the computer perform the personal assistant role.

An NSF sponsored working conference in 1982 spent time conjecturing what changes could take place in the mathematics classroom if the two modes mentioned above were available and used (Fey, 1984). Conference participants envisioned a mathematics classroom where the computer did much of the symbol manipulation and where students learned to use the computer to produce tables of values and graphs to aid in problem solving. Participants envisioned a classroom where students would learn to do by hand those skills that can be done effectively by hand and use the computer to do those things which the computer can do faster or with more accuracy. Students in such classrooms would focus on concept learning and problem solving skills and concepts and problem solving skills would be learned before any skill training.

Heid (1984, 1988) used the computer as a tool to teach a calculus class. She focused much of the first 12 weeks of the semester on concept learning using the computer as a tool to do the symbol manipulations needed to solve the various calculus problems. The last three weeks of the semester she focused on the manipulation skills of calculus. Her two experimental sections then took the uniform departmental final exam of routine skills in calculus where they performed almost as well as the sections that had spent the fifteen weeks primarily on routine skill development. The experimental sections also showed a much deeper and broader understanding of course concepts than the control classes as evidenced in analysis of class assignments, quizzes and interviews.

Other researchers have looked into the use of computers in the classroom but their focus has not been primarily on the computer as a tool (Shavelson, 1985, Signer, 1982). Shavelson looked at the patterns of microcomputer use in classrooms and identified four clusters of uses among teachers who were identified by their peers as excellent users of microcomputers in the classroom. The four clusters are drill and practice, enrichment, adjunct instruction and orchestration. Of the four, orchestration comes closest to the use of the computer as a tool but it, in fact, refers to teachers who use many different types of programs in their classrooms. Using the computer as a tool does involve orchestration of several different types of programs and uses of the computer but it also involves using the computer in place of learned routine skills. Shavelson did not mention finding this in the mathematics and science classrooms in his study. Signer (1982) did an experimental study using a non-equivalent control group model. Using a computer integrated instruction model, teachers used microcomputers to perform simulations and calculations as part of a total class presentation in five algebra II classes. The classrooms each had one 19 inch monitor in them. Signer found a significant difference in mathematics achievement in favor of the control class. Reasons for the difference in favor of the control class may be that the computer was not used as frequently as it could have been, that the use of one monitor in the classroom did not have the impact on students that a computer lab would have and that teachers needed better in-service training in order to know how to use the computer effectively in the classroom.

The use of the computer as a tool in the classroom has not been tested to a great extent at this time. The study by Heid (1984, 1988) shows great promise as to what could be done in terms of teaching concepts and problem solving skills before teaching routine drill skills in a calculus class. The study by Signer showed problems that can arise when teachers are not prepared for using the computer and the limitations of the use of one large monitor in the classroom.

Interaction of Concepts and Skill Development

The questions of what skills and concepts should be taught when and how they should be taught are ones that are under constant debate in the mathematics education community. One debate that rages constantly is whether skill development or understanding of concepts should come first in instruction. Much of the "new mathematics" was a reaction to the overemphasis on skill development of the 1950's and then the Back to Basics movement was a reaction to the overemphasis on concept learning during the "New Mathematics" era. (NACOME, 1975). The NACOME report warned against too much emphasis on one or the other of skill development or concept learning.

Accepting the need for both skills and concepts, there remains the question of which should be taught first and in what manner. Many educators, parents and students believe that skills must be taught before concepts, that concepts cannot be understood until much skill drill has taken place. It is this idea that was central to the Back to Basics movement. Yet there is little experimental evidence that drill will improve concept learning. Warnke (1969) used elementary education students to see if practice on computational skills would improve concept understanding. Although the experimental group did make significant gains a subtest of the Iowa Test of Basic Skills meant to test concept understanding, the research may be suspect since the experimental group did extra practice over the control group so that the gains may be due to increased work and because the Iowa Test of Basic Skills test on "understanding math concepts" may not be a true test of concept understanding for elementary education majors in college.

There are as many proponents of teaching concepts first as there are of skill development. Cognitive psychologists (Mayer, Stiehl and Greeno, 1975) and mathematics educators (Fey, 1984; Heid, 1984) have called for the teaching first of concepts and following that with appropriate skill development. Again the experimental basis for this view is limited. Webb (1979) looking at the relationship between conceptual knowledge and the use of heuristics found that conceptual knowledge was responsible for 50% of the variance in the solving of eight post test questions in a think aloud interview setting while processes accounted for only 13% of the variance.

The study by Heid (1984, 1988) discussed in the section on the Computer as a Tool remains the pivital study on the concepts first model. Students in the two calculus sections taught by Heid were given concepts of calculus for the first 12 weeks of the semester with the computer being used as a tool to do any calculations and symbol manipulations necessary. The last three weeks of the semester the students were taught the hand techniques for calculations and symbol manipulations. Students did almost as well as students in sections that spent 15 weeks on hand techniques on a test that emphasized the skills over the concepts. Other data consisting of analysis of quizzes, interviews and field notes indicate that the students in the experimental group had a much deeper and broader understanding of the calculus concepts than those in the control group.

With experimental data limited on both models, the controversy over the skill development and concept emphasis models will continue. Research needs to be done on these two models.

Research on Algebra Instruction

Algebra is the central subject in the high school mathematics curriculum. Most of the rest of the high school mathematics is based on or uses algebra. Science courses such as chemistry and physics require algebra skills and frequently have a course in algebra as a prerequisite. Science and mathematics courses in college also depend heavily on algebra. Algebra is, then, an important part of the high school curriculum. Yet, in a review of over 140 studies of school algebra, Dessart found that there is not yet a comprehensive theory for the instruction of algebra (Dessart and Suydam, 1983). Research in the areas of cognitive psychology and algebra has made some contributions to our understanding of the learning of algebra, however.

Algebra carries with it certain expected problems that have been done for years because it is believed that they help students learn to set up equations and to solve problems. These problems include ones about age, coins, mixtures and rates. Algebra students try to figure the meeting of two trains traveling at certain speeds or the proper price for a nut mixture of almonds and peanuts. Mayer has found that students develop schema for each of these types of problems rather than learning a general skill of setting up equations and solving them to solve word problems (Mayer, 1982b). He has identified over 100 different problem types commonly found in algebra, each with a distinctive solution procedure (Mayer, 1981). In an experimental study, Mayer had students read problems and try to recall them. Problems that were common in algebra texts were recalled by students at a higher level than low frequency problems. Students frequently remembered low frequency problems by changing them to a more common version of the problem (Mayer, 1982b). Mayer found that students have a knowledge of problem schema and use that knowledge in the solving of algebra problems. Hayes, Waterman and Robinson (1977) had very similiar findings.

They asked students to judge which parts of a problem were revelant and found that students first decided to which category the problem belonged and then made accurate judgments about relevant information.

Another area of algebra research that is relevant to this study is that concerning the problems that more advanced students have with algebra skills. Moskol (1980) in a exploratory study with college students and real world problems found that students tended to use more aritmetic than algebraic models when modelling real world problems. Students had trouble with transferring their mathematics concepts to real world problems. Clement (1972, Clement, Lockhead and Burton, 1981) found that college students have a hard time transferring English sentences to mathematical sentences and that they had serious difficulty in symbolizing relationships with algeraic equations. These skills are generally considered to be the skills taught in Algebra 1.

The research on algebra shows that students tend to memorize types or categories of problems and how to solve them and then when they are faced in advanced high school mathematics classes or college mathematics or science classes with solving problems for which they have not memorized a solution technique, they are not able to solve the problems.

Summary

There are no comprehensive theories for either the learning and instruction of algebra or the learning and instruction of problem solving. The research has shown several things that deserve further study. Students who understand the mathematical structure of a problem seem to be better problem solvers. Algebra has over 100 different problem types for which students tend to memorize solution schema. Later, when students are faced with new problems that do not fit the memorized schema, they are unable to solve the problems. These two facts lead to the question, can we design instruction so that students will be taught to recognize the underlying structure of a problem and will this then make them better problem solvers? The concepts first model of instruction for calculus seems to hold promise for students to learn a deeper and broader understanding of mathematics. If the computer is used to facilitate the computation and symbol manipulation necessary, could students using the concepts first model to become better problem solvers?

CHAPTER III

METHODOLOGY

This chapter describes the setting of the study, the Algebra with Computers project, the conduct of the control and experimental classes, and the differences between the control and experimental classes. Additionally, it contains descriptions of the design, administration and scoring of the Triads Task and the Problem Solving Test. The final section of this chapter discusess the analysis of the data.

All students in control and experimental classes at two sites were given a Triads Task and a Problem Solving Test. Additional data in the form of achievement and functional mathematics test scores were also collected.

The Setting

Subjects

The students in this study were all freshmen enrolled in Algebra 1 classes in two different high schools. Both high schools were located near major universities in the Middle Atlantic area.

The Site One high school was in a district which included a small university-dominated town and the surrounding rural areas. The students were mostly white and middle class coming from the community and surrounding areas. Many of the students had expectations of attending college. The school district tracked its top students into algebra 1 in the eighth grade, so that those students who were taking algebra 1 as freshman were considered average students for this school district.

The Site Two school was located in a large metropolitan school district which had great numbers of minority and immigrant students. The high school was in a neighborhood of low socio-economic status and few of its students had expectations of college attendance. This district also tracked its top students into algebra 1 as eighth graders so that students taking algebra 1 in their freshman year were considered average students for this school district.

The two school districts may represent two ends of the spectrum of school districts. Site One being in a rural area with a university town at its center. Student scores on standardized test in this district tend to be fairly high. Support of education from the community is also high. The Site Two district is perhaps typical of an urban district with much lower scores on standardized tests and much less community involvement and support. The districts are not comparable.

The students were enrolled in the experimental and control classes by the guidance departments at each school. The only additional criterion for being in either group was that the student be in the ninth grade. After enrollment, parents were sent a letter explaining the project and asking permission for their children to participate. Parents were given the option of removing their children from the experimental group.

At Site One, there were two experimental classes, two control classes and two teachers involved in the study. Each teacher taught one control and one experimental class. Class sizes ranged from 20 to 30 students in both the control and experimental classes.

At Site Two, there were three experimental classes, one control class and three teachers involved. Each teacher taught one experimental class and one teacher taught the control class in addition to her experimental class. Class sizes ranged from 20 to 30 students per class in both the control and experimental classes.

<u>Description of the Control Classes</u>

The control classes were normal algebra 1 classes and were conducted according to the guidelines for algebra 1 within each school district. Each school district had different guidelines for algebra 1. The guidelines followed closely the textbooks used. The only difference between the control classes and any other algebra 1 classes in the schools was their involvement in being observed, inverviewed and tested as controls for the study.

The Site One control classes used Algebra One by Max Sobel, Evan M. Maletsky, Norbert Lerner and Louis S. Cohen published in 1985 by Harper & Row as a textbook while the site control class Algebra Structure and Method Book 1 by Mary P. Dolciani published by Houghton Mifflin Company in 1984 as a textbook.

The control classes were conducted in a mannner that is familiar for most algebra 1 classes. The class typically began with correction and discussion of homework or a short quiz or both. The Site Two class had a short quiz every day. This was followed by a lesson presentation by the teacher which frequently involved showing the students how to do some particular type of problem. The lesson presentation often focused on solving problems mainly by symbol manipulation. Following the answering of students' questions or sending them to the board to try some problems in front of the class, homework was assigned and any remaining time in the period was used for this seatwork. Students usually worked alone on the seat work.

Description of The Algebra with Computers Project

The NSF-funded Algebra With Computers curriculum uses computers and calculators as tools in an algebra 1 sequence that emphasizes concept learning and mathematical structure while teaching some specific problem solving techniques using primarily application problems. In the Algebra with Computers curriculum, many traditional symbol manipulations were done by the computer. During this study, students at the two sites were using the revised year 2 curricula.

The Algebra with Computers curriculum eliminates or reorders many symbol manipulation skills; concepts and problem solving skills are with a heavy reliance on applied problems. The notion of function is the organizing focus of the Algebra with Computers. Students are introduced to functions early in the materials and functions are then used throughout the rest of the materials. This may help students to see the mathematical structure of problems. Students are taught to use functions build mathematical models for applied problems and to look for the underlying mathematical structure in a problem as a part of building that model.

The Algebra with Computers materials consisted of four modules. Module A was an introduction to mathematical modelling, to the use of variables and to using the calculator and computer. The Module B materials introduced the concept of function and used the computer to do the symbol manipulations needed for the modelling of problem situations. Module C concentrated on linear functions while Module D had quadratic and higher order functions.

Throughout the modules, students were taught to do by hand those manipulations that were most quickly done by hand. Students in the Algebra with Computers classes at Site Two were issued calculators along with their text books; at Site One, students used their own calculators. They were taught to use the calculators whenever hand calculations are necessary. Computers were used in the curriculum as tools. Students went to the computer lab with specific problems to solve or discovery exercises to do and used the computer to solve problems, to do manipulations, to produce a graph or a table as needed. Computers were very seldom used for drill and practice.

The conduct of a typical class was not much different (except in the labs) from that of the control classes. The class usually began with the correction and discussion of homework and/or with a short quiz. The lesson presentation by the teacher might include the solving of problems by symbol manipulation as found in the control classes but it might also include the conducting of experiments for the collection of data, the use of a large screen monitored computer to solve problems or the discussion of a mathematical model for a problem solving situation.

Several times a week the students were involved in computer lab situations. A typical computer lab was set up by a teacher-led discussion of the tasks to be accomplished followed by some or all of the students going to the computer lab to complete the assigned tasks. At both sites the computer labs were close to the classrooms and the teachers were able to monitor the labs themselves. Both labs contained enough computers so that students were able to work in groups of no more than three. Most frequently the students were able to work in pairs. Some of the teachers assigned lab partners and some allowed the students to choose their partners. Where students were allowed to choose, there was some changing of partners over the school year.

Typically students went to the lab with a specific task. The task might be exploratory when the students were working on new materials or they might be using the computer to solve particular problems that had been posed in the classroom or in the materials. Students usually took turns at the keyboard with the other person acting in a quality control capacity to check accuracy of his partners' entries. There was much discussion between partners as to how to approach problems.

The four modules did not comprise the whole course for the Algebra with Computers students. Because some of the symbol manipulations done on the computer in the Algebra with Computers classes are on standardized tests, after the completion of the four Algebra with Computers modules, the

Algebra with Computers classes were taught those skills using the same textbook as the control classes.

The major differences between the Algebra with Computers classes and the traditional classes included the use of different materials, different skills in different sequences, the use of the computer and calculator, the deemphasis of symbol manipulation skills in the Algebra with Computers sections, the emphasis on concepts, mathematical structure and problem solving in the Algebra with Computers section.

Differences Between the Two Sites

Differences between the programs at the two sites centered around the amount of time spent on the experimental materials, the lab situation and the administration of the Triads Task and Problem Solving Test.

Amount of Time Spent on Experimental Materials

At Site One, students in the experimental classes only worked on the experimental materials for the first semester of the school year. These experimental classes completed only Modules A and B and about half of Module C. The second semester was then spent in the same textbook as the control classes, covering those topics that the teachers felt needed to be covered for the students to be able to take a departmental final in early June.

At Site Two, the experimental classes completed most of the four modules with only a couple of sections in Module D being skipped by two of the experimental classes. At this site, the experimental classes worked on the materials until the first of May at which time the Triads Task and Problem Solving Test were administered. The rest of the school year was then spent by the experimental classes in the text used by the control class in preparation for their departmental final exam.

Lab Situations

The lab situation differed at the two sites also. At Site One, the lab was down the hall and required that the whole class and teacher go to the lab together and return together. Apple computers were available at Site One. While all necessary programs were available for the Apple, the software was not as elegant or as easy to use as that available at Site Two which had IBM-compatible computers. At Site Two, the computer lab was located just across the hall from the classroom and it was possible for the teacher to monitor the lab and the classroom at the same time. This allowed for students to be sent to the lab in small groups or for short periods of time. Additionally, one experimental class had a half-period study hall in the mathematics classroom with their teacher just prior to lunch each day. This gave this class additional time in the lab and additional access to their teacher for help. The Site Two lab had IBM-compatible computers with software that was easier to use than the Apple.

Administration of the Problem Solving Test and Triads Task

Another area of difference was the administration of the Triads Task and the Problem Solving Test. At Site One both instruments were administered by the researcher to both the control and experimental classes on two consecutive days in late May. The Problem Solving Test was given on the first day. Students were allowed to use calculators on the Problem Solving Test but few were observed doing so. The Triads Task was administered on the second day.

At Site Two, students in the experimental classes were allowed to use the computer on the Problem Solving Test. To facilitate this use, on the first day of testing half of the students in a class went to the lab and did the Problem Solving Test while the other half did the Triads Task. The second day the groups switched. At Site Two, the two instruments were admininstered by the classroom teachers in early May. For one experimental class, due to a misunderstanding of the administration instructions, each student only did one of Problem Solving Test or Triads Task.

These differences between the two sites of the amount of time spent on the experimental materials, the lab situation, the administration of the Triads Task and Problem Solving Test led to the decision to analyze all data from the two sites separately.

Overall Design of the Study

The overall design of the study is a pre- and post-test control group quasi-experimental design. Test scores on the California Achievement test were collected and used as covariates. The post-tests were scores from the Problem Solving Test and the Triads Task. Analysis of data was done using analysis of covariance with the two sites analyzed separately.

Instruments

Two instruments designed by the researcher were given to all students at both sites. The instruments were a Triads Task and a Problem Solving Approach Test.

The Triads Task

The Triads Task consists of 24 groups of three problems. Students are asked to identify the two problems in each triad that, if they were to solve them, would have the same solution procedure. Students were also asked to give a reason for their selection. Students were not asked to solve the problems. A copy of the Triads Task and its answer key is found in Appenidx A.

Design of the Triads Task

The Triads Task was designed using the problems from a previously developed Problem Bank (see Appendix B). The Problem Bank contained applied problems and variations on them. The intent in the design was for the problems to be applied ones with a context and meaning that were fairly typical of algebra 1 and fairly short in reading time. The Triads Task was designed using linear and quadratic problems in a proportion of two-thirds linear and one-third quadratic as this was judged to be the proportion in which those two types of problems are typically found in algebra 1. In addition, each type of problem was divided into those requiring solution and those requiring evaluation of expressions. Half of the problems in the Triads Task are require solutions and half are require evaluation of expressions. Within the triads, problem types of two linear and one quadratic, two quadratic and one linear, three linear and three quadratic were used with variations of using solve and evaluate within those types. With all the variations, there were eight problem types.

An additional consideration for the construction of the Triads Task was having triads that contained pairs that matched according to the four categories of problem relatedness identified by Silver (1977). Those four categories were mathematical structure (S), context (C), question (Q) and pseudostructure (P). In any triad there are three pairs of problems. One pair matches on structure. The other two match on two of context, pseduostructure and question. Thus there were three patterns of triads--S-C-Q (one pair matches on structure, one pair matches on context and one pair matches on question), S-C-P and S-Q-P.

Since there were eight problem types and three patterns of pairs, twenty-four triads were constructed. Table 1, Construction of the Triad Task shows the construction of the triads, the number of the problems in the Problem Bank in each triad and the number of the group in the Triad Task itself. Groups were ordered in the Triads Task in a random manner with no pattern or problem type being given two times in a row and with a mix of correct answer letters.

Table 1

Construction of Triads Task

Problem Type	Patterns of Pairs		
	S-C-Q	S-C-P	S-Q-P
2 linear/1 quad			
EESo	29/42/23 (1)	6/25/16 (21)	4/28/27 (12)
EESo	30/6/16 (10)	29/30/32 (15)	25/28/27 (24)
SoSE	13/33/8 (23)	5/2/26 (9)	43/34/8 (14)
2 quad/1 linear			
ESoSo	35/16/10 (16)	6/16/31 (2)	45/44/23 (8)
3 linear			
EESo	22/25/36 (11)	4/1/7 (17)	25/6/34 (3)
SoSoE	37/13/22 (7)	36/5/22 (13)	34/37/22 (22)
SoSoE	7/13/39 (18)	7/13/28 (5)	24/38/29 (20)
3 quad			
EESo	15/41/23 (4)	15/40/23 (19)	26/17/32 (6)

KEY

E = evaluate
So = solve
S = structure

C = context
Q = question
P = pseudostructure

The numbers separated by / refer to the Problem Bank Number. The numbers in parentheses are the Group numbers is the Triads Task.

Scoring

The Triads Task was scored giving one point for each correct choice of a pair that had the same mathematical structure. The maximum score is 24.

The Problem Solving Test

The Problem Solving Test contained ten problems judged to be typical of algebra 1. Students were asked to solve the problems showing all of their work on the test booklet. A copy of the Problem Solving Test is found in Appendix C.

Design of the Problem Solving Test

The ten problems on the Problem Solving Test were chosen because they were judged to be typical of algebra 1. One of the problems in testing the problems solving abilities of students involved in these two different curricula is that the students in the different groups have very different tools available to them and they are used to different formats in their tests and problems. The experimental materials involved applied problems that often required some reading and the setting up of a mathematical model. The experimental students were used to using the calculator and computer as tools. The control classes were used to shorter and more traditional problems and to solving most of their problems by symbol manipulation. The Problem Solving Test was an attempt at a middle ground between the two groups in that half of the problems were choosen from ones typical of the control texts and half were items typical of the experimental materials. The problems had more reading than normally found in the control class problems but less than that typical of the experimental materials.

The issue of whether or not students could use calculators or computers was solved by allowing all students to use a calculator if they choose. Students in the experimental sections at Site One had not used the computer since January and the computer room was not available to them any longer so no computers were used at Site One. At Site Two the experimental students were allowed to use computers if they chose.

Scoring

The Problem Solving Tests were scored by giving five points for each problem and scoring using a protocol. A copy of the protocol used is in Appendix D. The Problem Solving Test were scored by the researcher and another mathematics education graduate student using the protocol. The two raters trained on the use of the protocol and then divided the tests for scoring. Twenty per cent of the tests were scored by both raters to check for reliablity.

The raters met at the completion of each problem in the Problem Solving Test to check for reliability. Reliability for the different problems ranged from 100% to 86%; those rates are listed in Table 2. Scores that were in dispute were then agreed upon before scoring the next problem in the test.

Table 2

Reliability on Scoring of the Problem Solving Test

Problem Number	Number of Agreements
1	29 (100)
2	29 (100)
3	29 (100)
4	27 (93.1)
5	28 (96.5)
6	25 (86.2)
7	26 (89.7)
8	26 (89.7)
9	27 (93.1)
10	25 (86.2)

Number of papers in reliability check = 29

Numbers in parentheses are per cents

Administration of the Triads Task and Problem Solving Test

The Triads Task and Problem Solving Test were administered at Site One by the experimenter late in May on two sucessive days. Students in the experimental classes at this site had completed their involvement with the experimental materials at the end of January and had been working in the traditional textbook since that time. The Problem Solving Test was administered to both the control and experimental classes on the first day and the Triads Task was administered on the second day. Both were administered in the regular classroom during the normal class time with the classroom teacher in the room.

At Site Two the tasks were administered by the classroom teachers at the beginning of May when their experimental classes had completed Module D. The control section was given the Problem Solving Test one day and the Triads Task the next day. The experimental classes were split in half for the administration of the two tasks. Half of experimental students took the Triads Task on day one while the other half took the Problem Solving Test. The second day those who had taken the Triads Task the day before took the Problem Solving Test and vise versa. This was done to facilitate computer use. A copy of the Administration Instructions for Site Two teachers is found in Appendix E.

For the Triads Task, students were given directions to put their names and class section on their test papers and then given directions on completing the Triads Task task. The directions emphasized that the students were to choose a pair from the three problems in each group that if they were to solve the problems, would be solved in the same way. Students were told that they were not being asked to solve the problems.

In taking the Problem Solving Test, students were given a test booklet consisting of an instruction sheet and the ten problems each printed on a separated page. Students were instructed to solve each problem in whatever manner they wished and to show all of their work on the test booklet. Experimental students at Site Two who were using the computer were also asked to indicate by a check mark the tool that they used in solving the problem.

Analysis of Data

The Problem Solving Test and the Triads Task

Analysis of data was done using analysis of covariance. Due to the differences at the two sites given above, the two sites were analyzed separately. The analysis of covariance used sex as a blocking variable and had as covariates the California Achievement Test scores for total reading, mathematical computation and mathematical concepts with the Problem Solving Test and the Triads Task as the dependent variables in separate analyses.

CHAPTER 4

Results and Analysis of Data

Introduction

This chapter gives the results and analysis of data for the two tasks, the Problem Solving Test and the Triads Task, administered to the control and experimental students involved in the Algebra with Computers project.

Table 3 gives the frequency of students by sex for each of the control and experimental groups at the time of the administration of the Problem Solving Test and Triads Task. However, complete data was not available for all students. Some students, due to absence or task administration error, completed only one of the two tasks. Some students, due to school transfers, had missing covariate data. The data analysis did not compensate for missing data; when complete data for a student was not available for a particular procedure, the data associated with that student was deleted from that analysis. This accounts for the differences in the number of cases in different analyses.

TABLE 3

Frequency of Students at Each Site by Sex and Treatment

Location	Females	Males
<u>Site One</u>		
Experimental	31	22
Control	24	24
<u>Site Two</u>		
Experimental	38	42
Control	16	10

A number of covariates were available for the students at both sites. Pearson correlations with the Problem Solving Test and the Triads Task were run on all of the covariates available at each site. None of the covariates had a Pearson correlation of 0.6 or greater with either the Problem Solving Test or the Triads Task, this level of correlation being the one usually used for covariates in analysis of covariance (Barcikowski, 1983). However, three covariates chosen for use--the California Achievement test scores taken in eighth grade for total reading, mathematical computation and mathematical concepts-- are standardized scores, they were available for both sites and they were identified by the teachers involved in the

Algebra with Computers project as ones that seemed most likely to influence students' achievement in the Algebra with Computers sections. The California Achievement test score for total reading was chosen because of the great amount of reading expected not only in the Algebra with Computers materials but also in the Triads Task task itself. The Triads Task involved reading sets of paragraph long word problems and then deciding as to which pair in each set would be solved in the same manner. It presented 24 sets of 3 problems each in one class period. The California Achievement Test score for mathematical computation was chosen as a covariate because the control group at Site Two and both groups at Site One completed the Problem Solving Test without using the computer, so computational errors would have affected the scores of those students. The California Achievement Test score for mathematical concepts was used as a covariate because conceptual knowledge is important in problem solving. Table 4 specifies the covariates used and their abbreviations. Table 5 gives the Pearson correlations of the covariates and the Problem Solving Test and Triads Task at both sites.

TABLE 4

Covariates

Abbreviation	Covariate
READ	Total reading score on 8th grade California Achievement Test (CAT)
COMP	Mathematics computation score on 8th grade CAT
CONCPT	Mathematics concepts score on 8th grade CAT

TABLE 5

Pearson Correlations Between the Triads Task
and Problem Solving Test
and the Covariates at Both Sites

Covariate	Correlation with Problem Solving Test	Triads Task
Site One		
READ	.3996	.1564
COMP	.2281	.0777
CONCPT	.5317	.1412
Site Two		
READ	.5041	.2236
COMP	.1382	.1182
CONCPT	.4933	.1498

Because the students in the experimental groups at the two different sites had studied different amounts of the Algebra with Computers materials, giving in effect two different experimental treatments, the results from the two sites were analyzed separately. The results of the two instruments are presented below with the results from each site being presented and discussed separately.

The Problem Solving Test

Results at Site One

Table 6 gives the means, standard deviations and number of students taking the Problem Solving Test at Site One. The Problem Solving Test consisted of ten applied problems on it with each problem given a score of 0 to 5 points making a total of 50 possible points.

TABLE 6

Means and Standard Deviations
on Problem Solving Test at Site One

	Mean	Standard Deviation	Number of Students
Experimental	34.78	8.83	49
Females	35.46		28
Males	33.86		21
Control	33.02	8.89	44
Females	32.96		23
Males	33.10		21

The difference in mean scores between the two treatment groups was only 1.76 points (out of 50 possible points), favoring the experimental group. Standard deviations within the groups were comparable.

The means for males and females in the control and experimental groups show that the mean of the experimental group females is 1.6 points higher than the mean of the males in the experimental group and that the mean of the control group males is 0.14 points higher than that of the females in the control group. In each case, the difference between the means of the males and the females was small.

The results of the Problem Solving Test were analyzed using analysis of covariance. With sex as a blocking variable, the analysis of covariance used as covariates the California Achievement test scores on total reading (READ),mathematical computation (COMP) and mathematical concepts (CONCPT). The analysis of covariance was computed using SPSSX with the alpha level of significance of .05. The results are given in Table 7.

TABLE 7

Analysis of Covariance Results

on Problem Solving Test at Site One

Source of Variation	Sum of Squares	DF	Mean Square	F
Covariates	2225.670	3	741.890	13.388**
READ	290.146	1	290.146	5.236 *
COMP	161.560	1	161.560	2.916
CONCPT	807.937	1	807.937	14.580**
Main Effects	351.328	2	175.664	3.170 *
Sex	2.507	1	2.507	0.832
TRTMNT	350.975	1	350.975	6.334 *
Interactions	1.032	1	1.032	0.892
Sex x TRTMNT	1.032	1	1.032	0.892
Error	3878.956	70	55.414	

* p < .05 ** p < .01

Discussion

The mean scores were lower than anticipated in both groups. The mean for the experimental group was 69.6 % and the mean for the control group was 66.0 %. At Site One the difference between the control and experimental student scores on the Problem Solving Test was significant at the .05 level with the mean of the experimental students being greater than the mean of the control students. The overall test for significance of the covariates was significant at the .01 level with the covariates of READ and CONCPT also significant. The significance of the CONCPT covariate was at the .01 level and the significance of the READ was at the .05 level. These significances were expected given the nature of the Algebra with Computers materials and the Problem Solving Test itself.

Table 7 also shows that the differences between the mean scores of the males and the females were not significant and that there was no interaction effect between sex and the treatment.

Further Data Explorations

In addition to performing an analysis of covariance on the results of the Problem Solving Test at site one, frequencies were also collected of the number and per cent of items completely correct on each item of the Problem Solving Test for the control and experimental groups. This was done to further explore the possible meaning of the statistically analyzed results but is not part of the hypothesis testing done with the analysis of covariance. This item analysis data is given in Table 8.

TABLE 8

Number Correct on Problem Solving Test

by Problem Number at Site One

Problem No.	Experimental	Control
1	48 (97.9)	44 (100)
2	41 (83.7)	42 (95.5)
3	37 (75.5)	31 (70.5)
4	21 (42.9)	11 (25.0)
5	44 (89.8)	41 (93.2)
6	24 (49.0)	25 (56.8)
7	5 (10.2)	7 (15.9)
8	26 (53.1)	22 (50.0)
9	11 (22.4)	8 (18.2)
10	27 (55.1)	28 (63.6)
	N = 49	N = 44

Notes:

1. Numbers is Parentheses are per cents.
2. Problems were counted correct if given 5 points or if given 4 points and the only error was omission of units.

Discussion

It is interesting to note that on items 1, 2, 5, 6, 7 and 10, the control group actually had a greater percentage of students with completely correct solutions. Since the experimental group had a greater mean total score than the control group, apparently among students who received partial credit the experimental group received higher partial scores than the control group, thus indicating that more experimental students attempted the problems.

On problem four, the experimental group outscored the control group by almost 18 percentage points, the largest difference for any of the problems. Problem four involved setting two expressions equal and solving the resulting equation. The solution was fairly simple so that it could have been easily solved by guess-and-test, solved by graphing the two expressions and finding their intersections or by setting the two expressions equal to each other and manipulating symbols. The experimental students may have outscored the control students on this problem because the experimental students were taught more than one way to approach problems. This problem might have been difficult for all students to conceptualize in terms of setting the two expressions equal and solving but would have been easily solved using the guess-and-test strategy that was taught in the Algebra with Computers materials. It is likely that the control students who could not solve this by symbol manipulation had been taught no alternative way to approach this problem while the experimental students had been taught guess-and-test as part of their instruction in several methods for solving linear problems.

Scores on problem seven were very low for both groups-- only 10% of the experimental group and 15% of the control group received full credit. Problem seven was a typical area-of-a-garden problem. Review of the papers of both the control and experimental groups showed that both groups had trouble with writing equations to solve this problem. Students were apparently having problem understanding the area model. In both groups, students had trouble with representing the dimensions with variables and with applying the area formula.

Results at Site Two

The results on the Problem Solving Test at Site Two are given in Table 9.

TABLE 9

Means and Standard Deviations on Problem Solving Test at Site Two

	Mean	Standard Deviation	Number of Students
Experimental	28.98	8.57	48
Females	27.63		27
Males	30.29		28
Control	26.00	9.28	19
Females	24.21		14
Males	29.57		7

The results of the Problem Solving Test were analyzed with analysis of covariance. As at site one, the covariates from the California achievement test of reading comprehension (READ), mathematical computation (COMP) and mathematical concepts (CONCPT) were used and sex was a blocking variable. The results are given in Table 10.

TABLE 10

Analysis of Covariance Results on Problem Solving Test at Site Two

Source of Variation	Sum of Squares	DF	Mean Square	F
Covariates	1819.618	3	606.539	11.469**
READ	388.200	1	388.200	7.340**
COMP	84.701	1	84.701	1.602
CONCPT	433.010	1	433.010	8.187**
Main Effects	452.867	2	226.434	4.281 *
Sex	5.299	1	5.299	0.100
TRTMNT	424.007	1	424.007	8.017**
Interactions	9.259	1	9.259	0.175
Sex x TRTMNT	9.259	1	9.259	0.175
Error	3173.240	60	52.887	

* p < .05 ** p < .01

Discussion

The mean of the experimental group was higher than the mean of the control group with the experimental group scoring at the 57.8% level and the control group scoring at the 52% level. The level of performance of the experimental group is disappointing. However, it is possible that the mean of about 58% is deceiving. The experimental group at Site Two worked using the computers if they wanted. One of the problems encountered by the graders of the Problem Solving Test was that the experimental group at Site Two consistently forgot to write down the steps that they had used in their solutions. They were asked to write down programs and functions used in the workspace on their tests. Since the Problem Solving Test was graded using a protocol that gave partial credit for partial solutions, students who did not show their work were at a disadvantage. Students working on a computer could easily misread a graph or table or misenter a function and get a wrong answer. If they only wrote down their answer, there was no way for them to receive partial credit for the work that they could do. Students at Site One and in the control group at Site Two had only their testbooks to work on, so they were forced to show their work and, as a consequence, they were likely to receive more partial credit and thus higher scores than students in the experimental group at Site Two. Thus, it is likely that the experimental group at Site Two actually performed at a higher level than shown by their scores.

The difference between the mean scores of the two groups was significant at the .01 level. As at site one, the overall test for significance of all of the covariates was significant at the .01 level. The covariates of READ and CONCPT were also significant when tested separately, showing that students' beginning skills in reading and mathematical concepts influenced their problem solving ability as measured by the Problem Solving Test.

At Site Two on the Problem Solving Test, the mean score of the males was higher than the mean score of the females in both the control and experimental groups. However, the differences of 2.64 points for the experimental group and 5.36 points for the control group were tested using analysis of covariance as given in Table 10. This analysis revealed no significant difference in the mean scores of the males and females nor was there an interaction effect between sex and the treatment.

Further Data Explorations

Tool Use

Students in the experimental group at Site Two were asked to indicate by check marks the tools that they used on a each problem. This task was not done at Site One since the experimental students there did not use computers on the Problem Solving Test. This task was added for the computer users to get a sense of how computer use affects problem solving. It was not scored as a part of the Problem Solving Test nor is this exploratory section part of the statistical hypothesis testing. A summary of the reported tool use is given in Table 11 for those students who got a problem correct and those who got it incorrect.

TABLE 11

Tools Use Reported By Site Two Experimental Students

Tool	Students with Correct Answers	Students with Incorrect Answers
	%	%
Calculator	146 (44.7)	113 (50.4)
Computer graph	18 (5.5)	24 (10.7)
Computer table	69 (21.2)	35 (15.6)
muMath	40 (12.3)	17 (7.6)
Guess-n-test	41 (12.5)	40 (17.9)
Other	21 (6.4)	8 (3.5)
None given	9 (2.8)	46 (20.5)
Number	326 correct answers	224 incorrect answers

Notes:

1. Multiple tools could be reported.
2. Answers were considered correct if they received 5 points or if they received 4 points with the only error being omission of the correct units.

Discussion

Both the students with correct answers and those with incorrect answers displayed a similar pattern of tool use. The preferred single tool was the calculator followed by a computer table. The set of students with incorrect answers gave a greater percentage of guess-and-test indications or gave no tool indication than the students with correct answers. The students with correct answers frequently used muMath and computer graphs. Since the group with incorrect answers showed more calculator use and more use of guess-n-test while those students who got the problem correct used more muMath, it is possible that those students who got the correct answer were those who were better computer users. MuMath was a little more difficult to learn to use than the calculator or the guess-and-test methods but muMath gives more precise answers and muMath and the function graph program make it less likely that students will make errors like missing the second root for a quadratic (a frequent mistake on the Problem Solving Test). Even if this relationship whereby better computer users get more correct answers exists, the direction of the relationship is unclear. That is, it is not clear if being a better computer user allows students to get more correct answers or if the better mathematics students have simply become the better computer users.

Item Analysis

In addition to performing an analysis of covariance on the results of the Problem Solving Test at Site Two and examining the frequency of tool use of the experimental group, an examination of frequency and per cent of items completely correct on each item of the Problem Solving Test for the control and experimental groups was also made (see Table 12).

TABLE 12

Number Correct on Problem Solving Test

by Problem Number at Site Two

Problem No.	Experimental	Control
1	53 (96.4)	18 (85.7)
2	36 (65.5)	17 (81.0)
3	52 (94.5)	12 (57.1)
4	27 (49.1)	1 (4.8)
5	40 (72.7)	18 (85.7)
6	18 (32.7)	6 (28.6)
7	8 (14.5)	0 (0)
8	42 (76.4)	8 (38.1)
9	8 (14.5)	3 (14.3)
10	37 (67.2)	5 (23.8)
	N = 55	N = 21

Notes.

1. Numbers is Parentheses are per cents.
2. Problems were counted as correct is given 5 points or if given 4 points and the only error was omission of units.

Discussion

At Site Two, the percent of correct answers given by the control group was higher than that of the experimental group on problems number two and five. Correct completion of problems two and five involves solving linear equations, so it appears that a greater percentage of the control group members are able to do these correctly than the experimental group members. However, since the experimental group mean was greater that the control group mean in total scores, this means that more of the experimental group members were able to make at least some progress on the problems.

Problems four and seven were, as at Site One, ones in which both groups scored poorly with the experimental group receiving more full credit than the control. Problem four, involving setting two expressions equal to each other and solving, had half of the experimental students receiving full credit while only one student (5%) got it correct in the control group. Problem seven involving a traditional area-of-the-garden quadratic had no control students receiving full credit, while 14% of the experimental students answered correctly.

The Triads Task

The Triads Task was given within a day of the Problem Solving Test at both sites. Consisting of 24 groups of three problems, for each triad, the student was to identify the pair from the three problems that would be solved with the same strategy if the students were required to solve the problems. The 24 triads were scored with one point for each triad making the maximum score for the Triads Task 24 points.

Results at Site One

The results of the Triads Task at Site One are given in Table 13.

TABLE 13

Scores on the Triads Task at Site One

	Mean	Standard Deviation	Number of Students
Experimental	7.55	2.89	53
Females	8.40		30
Males	6.43		23
Control	6.82	3.43	44
Females	7.39		23
Males	6.19		21

The results of the Triads Task were analyzed using an analysis of covariance. The California Achievement test scores for total reading (READ), mathematical computation (COMP) and mathematical concepts (CONCPT) were used as the covariates and sex was used as a blocking variable. The results are shown in Table 14.

TABLE 14

Analysis of Covariance Results on Triads Task at Site One

Source of Variation	Sum of Squares	DF	Mean Square	F
Covariates	24.655	3	8.218	1.010
READ	9.050	1	9.050	1.112
COMP	3.128	1	3.128	0.537
CONCPT	3.041	1	3.041	0.374
Main Effects	40.560	2	20.280	2.492
Sex	34.385	1	34 385	4.225 *
TRTMNT	5.112	1	5.112	0.628
Interactions	18.423	1	18.423	2.264
Sex x TRTMNT	18.423	1	18.423	2.264
Error	602.238	74	8.138	

* p < .05   ** p < .01

Discussion

The results of the Triads Task in both groups were disappointing. The means of the two groups were 7.55 for the experimental group and 6.82 for the control group. Out of twenty-four points, this is low achievement. This task proved to be more difficult and time-consuming for students than anticipated or than indicated by the piloting of the task at Site Two. Since the task involved three problems for each triad and since there were 24 triads, students had to read and understand 72 problems in a class period. An attempt to lighten the reading load was made by reusing problems in different triads so that student had only to glance over them again as they made their decisions on which they would solve the same way. Students at Site One disliked the repeated problems and several spent time discussing this with the researcher after the Triads Task was given. The students felt that using the problem again was boring and repetitious. They felt that they would have rather had different problems throughout the Triads Task. These students also felt that the task was too long.

The difference between the means of the experimental and the control students was not significant. The effects of the covariates together or singly were also not significant.

The Algebra with Computers students had finished their work with the experimental materials at the end of the first semester having only completed Modules A and B and about half of Module C and had been working since then in the same book as the control students. It is quite likely that the experimental students at Site One had been away for too long from the Algebra with Computers materials or had not completed enough of the materials for the curriculum to make a difference in this task. It is also possible that there is simply no difference in the ability to identify the mathematical structure in a problem between students using the two curricula.

There was a significant difference in favor of the females in the overall scores on the Triads Task. The reason for this is unclear. Since this was a long and difficult task, it is possible that more females just finished it or more females took it seriously and tried harder. These reasons are suggested from observations made on the day of administration. At site one, one teacher had some trouble with some of the class members in both the control and experimental classes taking the Triads Task seriously. Specifically there were groups of predominately males in the back corner of the classroom who whispered a lot during the task. The other teacher had a substitute on the day of the administration of the Triads Task and the substitute also had some control problems. It is possible, therefore, that the members of the classes who were not involved or influenced by the groups in the back may have done better on the Triads Task simply as a function of completing the task or of taking the task seriously. It is also possible that the females were better able to see the underlying structure of the problems given.

Further Data Explorations

Students taking the Triads Task were asked to give the reasons why they had chosen a particular pair of problems as being ones that they would solve in the same way. This task proved to be a difficult one for some students. Some gave no answers; some chose one answer and gave it each time; some gave generic reasons such as "because". To get a sense of how students were answering the reason part of the Triads Task, all of the reasons for those students in one class who got a problem correct were listed. Since it appeared that those reasons would fit reasonably into Silver's four categories of structure, context, question and pseudostructure when a category of "other" was added to cover no response. Frequencies were made of all students who got a problem correct and which category his reason fell into. Students who gave no answer were classified into the "other" category. It was decided to classify only those students who got the question correct because a great number of students who got the problems wrong simply did not put any reason down. Table 15 gives the number and percentage of students who gave reasons in the five different categories. Appendix F gives some sample reasons for each of the five categories.

TABLE 15

Reasons Given on Triads Task by Site One Students Who Got the Problem Correct

Dimension	Experimental	Control
Structure	271 (65.9)	184 (58.9)
Context	0 (0)	0 (0)
Question	23 (5.6)	25 (8.0)
Pseudostructure	45 (10.9)	39 (12.5)
Other	57 (13.9)	75 (24.0)
Number of Correct Answers	411	312
Number of Students	52	48

Discussion

From Table 15 it can be seen that the experimental students were more likely than control students to give a structure reason. The two groups gave about the same percentage of question and pseudostructure answers with no one giving context reasons. The experimental group gave fewer other answers than the control group. Since this data were not tested for significance, the only conclusion that can be made about it is that in both groups, students who got the answer correct gave no context reasons and few question and pseudostructure reasons.

Results at Site Two

Mean and standard deviation scores on the Triads Task at Site Two are given in Table 16.

TABLE 16

Scores on the Triads Task at Site Two

	Mean	Standard Deviation	Number of Students
Experimental	7.44	3.61	36
Females	7.97		31
Males	6.94		32
Control	4.23	2.69	16
Females	4.29		14
Males	4.13		8

The differences between the two scores were analyzed with analysis of covariance again using the California Achievement Test scores of total reading (READ), mathematical concepts (CONCPT) and mathematical computation

(COMP) as covariates and using sex as a blocking variable. The results are given in Table 17.

TABLE 17

Analysis of Covariance Results

on Triads Task at Site Two

Source of Variation	Sum of Squares	DF	Mean Square	F
Covariates	54.310	3	18.103	1.537
READ	29.396	1	29.396	2.496
COMP	1.725	1	1.725	0.147
CONCPT	0.202	1	0.202	0.017
Main Effects	190.335	2	95.168	8.082**
Sex	27.156	1	27.156	2.306
TRTMNT	171.443	1	171.443	14.559**
Interactions	0.080	1	0.080	0.007
Sex x TRTMNT	0.080	1	0.080	0.007
Error	777.192	66	11.776	

* p < .05   ** p < .01

Discussion

At Site Two, there was a difference of 3.21 points between the control and experimental means. The overall achievement on the Triads Task was, with the control group at Site Two having an especially low mean. The mean of the experimental group was 31% while the mean of the control group was 17.6%.

As at Site One, students complained about the length and difficulty of the Triads Task task. Many students did not finish the task and many did not give reasons for their choices on the Triads Task.

As indicated in Table 17, the scores of the control and experimental groups were significantly different on the Triads Task task at Site Two. Since the Site Two experimental students had had the Algebra with Computers materials until just a few days before the administration of the Triads Task, this difference is likely due to the differences in the two curricula in teaching identification of structure. The Algebra with Computers students at Site Two were better able to analyze problems as measured by the Triads Task than the control students. As at Site One none of the covariates were significant together or singly in the Triads Task analysis.

The females in both the experimental and control groups had greater mean scores than the males. But there is no significant difference between the Triads Task scores of males and females at Site Two, as measured by the analysis of covariance presented in Table 17. There is also no interaction effect between the treatment and sex variables. This means that there was no difference between the scores of males and females on the Triads Task.

Further Data Explorations

As at Site One, the reasons given by the students in both groups who correctly identified structurally similiar problems were tabulated into five groups: structure, context, pseudostructure and question, and other. The results are given in Table 18.

TABLE 18

Reasons Given on Triads Task by Site Two Students Who Got the Problem Correct

Dimension	Experimental	Control
Structure	312 (68.7)	69 (71.1)
Context	0 (0)	0 (0)
Question	15 (3.3)	4 (4.1)
Pseudostructure	23 (5.1)	4 (4.1)
Other	117(25.7)	20 (20.6)
Number of Correct Answers	454	97
Number of Students	64	23

The percentage of control students who correctly identified the structure of a problem and who gave a structure explanation is higher than the percentage of experimental students giving a structure answer but only by 2.4 percentage points. Since the number of control correct answers was so small, it did not take very many correct answers in the control group to influence the percentage. The experimental group gave a lot of "Other" answers at this site. Most often no reason given was. Since the task was long and the students were required by their teachers to complete it, many appeared to deal with its length and difficulty by not giving a reason. Since the experimental students got the correct answers but simply gave no reasons, it is possible that they would have given more structure reasons had they given reasons at all.

Relationship Between the Triads Task and the Problem Solving Test

Pearson correlation coefficients were calculated for the Triads Task and the Problem Solving Test. Results are given in Table 19.

TABLE 19

Pearson Correlation Coefficients

Between the Triads Task and the Problem Solving Test

Site	Correlation
One	.2721**
Two	.2177*

* $p < .05$ ** $p < .01$

Discussion

One of the reasons for interest in an instrument such as the Triads Task is to attempt development of an instrument that would measure problem solving ability without actually asking students to solve problems. Such an instrument holds the possibility of testing and comparing students from widely differing curricula or with different problem solving skills. The correlation between the Problem Solving Test and the Triads Task was significant at both sites raising the possibility that the Triads Task might be used as a measure of problem solving ability where a single test for two very different programs was not possible. The correlation is low enough, however, to indicate that the Triads Task would need revision and further testing before it possibble use as a single test.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This study focused on the problem solving abilities of the students who were involved in the Algebra with Computers project and of students who were in companion traditional algebra 1 classes. Done in the second year of field tests of the Algebra with Computers materials, this study gave two researcher-designed instruments to students in both experimental and control classes at two sites where the materials were being tested. The two instruments were a Problem Solving Test which consisted of ten problems typical of algebra 1 that students were to solve by any method known to them and a Triads Task which has 24 groups of three problems that students were to read and idenitfy which pair of the three problems would be solved in the same manner.

Summary of Findings Related to Problem Solving Test

At both sites, mean scores for students in the Algebra with Computers classes were greater than the mean scores of students in the control classes on the Problem Solving Test. The differences in the class means were analyzed by Analysis of Covariance using the California Achievement test scores for total reading, mathematical computation and mathematical concepts as covariates and sex as a blocking variable. The differences were significant at both sites. Thus as measured by the Problem Solving Test, the Algebra with Computers students were better problem solvers than the control students.

The covariates as a group and the covariates of total reading and mathematical concepts were also significant at both sites. This indicates that students' abilities to read and to understand mathematical concepts affect their problem solving ability as measured on the Problem Solving Test.

At both sites, the ANCOVAs showed no significant sex differences in the Problem Solving Test scores nor was there any interaction effect between the treatment and sex.

Further data exploration unrelated to the Analysis of Covariance was done on the Problem Solving Test. Analysis of the items on the Problem Solving Test from both sites showed that the control students received more full credit on two problems involving solution of simple linear equations. The Algebra with Computers students received more full credit on a problem that involved setting two linear equations equal and solving. Since the Algebra with Computers students had a greater overall mean than the control students but the control students received more full credit on several problems, it appears that more Algebra with Computers students were able to attempt a wider range of problems.

Summary of Findings Related to Triads Task

At both sites, the mean of the experimental students was greater than the mean of the control students. As analyzed by Analysis of Covariance using the California Achievement test scores of total reading, mathematical computation and mathematical concepts as covariates and sex as a blocking variable, the difference between the treatment groups at Site One was not significant but it was significant at Site Two with the Algebra with Computers students scoring higher than the control students. The Site One experimental students had not completed the Algebra with Computers materials which emphasized functions and thus taught the underlying mathematical structure, while the Site Two students had studied all four of the modules. These results on the Triads Task suggest that the Algebra with Computers materials produced students at one site who are better able to analyze problems but that the partial treatment at Site One consisting of Modules A and B and half of C followed by a semester of traditional algebra was not enough to produce this effect.

At Site One there was a significant result in the analysis of covariance on sex differences with the females scoring higher than the males. There was no significant sex difference at Site Two and there was no interaction effect at either site. Since the Site One students had not had the Algebra with Computers materials for a semester and since there was no significant difference at Site Two, this difference may be due to perseverence by females in completing the task or it may mean that females at Site One were better able to analyze problems than were males at Site One.

Summary of the Findings

The Algebra with Computers students at both sites were better able to solve problems and at Site Two the Algebra with Computers students were better able to analyze problems. This indicates that the Algebra with Computers students were better problem solvers than the control students. A major goal of the Algebra with Computers project was to give an existence proof of a computer-intensive environment that produces students who can solve problems. This study has shown that as measured by the Problem Solving Test and the Triads Task, the Algebra with Computers curriculum does produce somewhat better problem solvers.

The differences between the performances of males and females were only significant on the Triads Task at Site One. Here the females outperformed the males. This is an exciting finding since previous research on sex differences in algebra 1 students had found that the males were significantly better solvers of applied problems than females (Swafford, 1980).

Assumptions in this Study

It is assumed that that it is important for students be able to solve algebra problems and that it is important for students to understand the mathematics that they are taught. These two assumptions include the idea that the ability to calculate or manipulate symbols is not sufficient for problem solving. The ability to identify the underlying mathematical structure of a problem is assumed to be an important contributing factor in problem solving. It is also assumed that the Triads Task measures the students' ability to see the underlying mathematical structure of problems. The Problem Solving Test is assumed to measure the students' ability to solve problems.

Limitations

Most of the limitations of this study were due to the differences in the two curricula being studied. The Algebra with Computers and traditional Algebra 1 classes were experiencing different materials with different emphases. The differences in the symbol manipulations taught, the extensive use of application problems and the use of computers and calculators in Algebra with Computers made it difficult to design a problem solving test that was fair to both groups. This study was limited by the use of researcher-constructed materials in the Triads Task and the Problem Solving Test. Further, it is possible that the teachers in the Algebra with Computers did not emphasize or even teach the mathematical structure parts of the Algebra with Computers materials. It is also possible that the teachers of the traditional algebra classes, because they were also teaching Algebra with Computers, classes may have contaminated the traditional classes with Algebra with Computers materials or emphases. Since no true problem solving pre-test was given, it is also possible that the two groups were not equivalent in those skills when they began the programs.

Implications for Practice

This study has shown that the Algebra with Computers program was able to produce better problem solvers. The aspects of the Algebra with Computers curriculum which are different from the traditonal curriculum deserve wider consideration for adoption in classrooms. Those aspects include the use of calculators and computers to provide an emphasis on concept learning and problem solving, the use of extensive applied problems and the use of function as the organizing idea in algebra.

Those who are interested in increasing problem solving in their students but who cannot achieve a computer intensive environment may want to incorporate into their classrooms aspects of the Algebra with Computers project. The use of the function as an organizer of algebra 1 could be achieved using a standard algebra text by doing the last chapter on functions and relations near the beginning of the year. The reduced emphasis on symbol manipulation and arithmetic skills could be achieved by the use of calculators in the classroom. Textbooks that emphasize application problems could be used or teachers could bring to class these problems. The recent reduction in cost of graphic calculators may make them feisible for use in classrooms where computers are not available. Teachers may wish to further explore their use in a problem-rich classroom.

Recommendations for Further Research

While this study has shown that the Algebra with Computers students were better problem solvers than the control students, the level of achievement on both the Triads Task and the Problem Solving Test was not as high as expected. The results of the item analysis on the Problem

Solving Test should perhaps be useful in the revision of the Algebra with Computers materials in showing the relative strengths and weaknesses of the Algebra with Computers students on various types of fairly standard problems. In particular, attention should probably be paid to area problems; the achievement on them was very low in both groups. Further research needs to be done on algebra students' concept of area.

The connection between the ability of students to analyze mathematical problems by recognizing the underlying mathematical structure of a problem and the ability of students to solve problems is more firmly made by this study. The fact that significant differences were found at one site shows that a task such as the Triads Task can measure differences in students' abilities to analyze problems. Silver (1977) had originally had a triads task in his first round of his dissertation study but dropped it from the second round because he did not get a significant result in the first round. Silver hypothesized that his eight item triads task was too short to give differences. This twenty-four item Triads Task did give significant results. And, so, the use of an instrument such as the Triads Task in situations where giving the same problem solving task to two groups of students is difficult, due to different mathematical curricula, now seems reasonable.

However, the low achievement level on the Triads Task combined with the feedback from students make it seem reasonable to revise the Triads Task to contain a smaller number of problems with no repeated problems before using it again. The Triads Task task might also be better done in an interview where the students would have to give a reason and where the interviewer could ask follow-up questions. This use of a modified Triads Task in an interview has the possibility of giving insights into what students are thinking as they try to identify problems that have the same mathematical solution.

Another area for further research would be to try and sort out which of the aspects of the Algebra with Computers materials were responsible for the differences in problem solving ability. The Algebra with Computers materials have many differences from the traditional materials used by the control classes. It may be that one or two of those differences are responsible for the differences in the problem solving abilities. Trying to isolate those significant differences would involve further curriculum development and testing.

The emergence of graphical calculators in recent months brings forth the question of whether they could be used instead of the computer to provide a enriched environment of tables and graphs to facilitate problem solving. Since this is a less costly option than equipping a computer lab, a comparison study of classes using graphical calculators and computers on materials like the Algebra with Computers materials seems an important one.

The issue of teacher training is also an important one that needs to be addressed. The teachers at Site Two in the Algebra with Computers project had been involved with the project from its start, had worked parts of two summers on the project and had quite a bit of input into the materials as they were developed. The teachers at Site Two had had extensive work with the project staff the summer preceding the implementation of the Algebra with Computers materials at their school. The question of what kinds and what levels of teacher in-service are necessary for teachers to successfully teach with Algebra with Computers materials and in a computer-intense environment needs to be explored.

APPENDIX A

TRIADS TASK AND KEY

Name______________________

Period____________________

DIRECTIONS

On the each page that follows is a group of three problems. For each group of three, you are to identify which two problems in the group would be solved in the same way. To do this you circle the choice which tells the two problems that you would solve in the same way:

I & II II & III I & III NONE

After you have chosen the two that would be solved in the same way, write an explanation of why you think they would be solved in the same way under REASON.

Read each problem in each group carefully. Decide which two problems would be solved in the same way. Circle the choice that tells which two would be solved the same way. Then give an explanation for your choice under REASON.

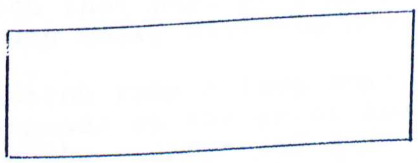
YOU DO NOT HAVE TO SOLVE THE PROBLEMS. YOU ONLY HAVE TO TELL WHICH TWO WOULD BE SOLVED IN THE SAME WAY AND GIVE A REASON FOR YOUR CHOICE.

GROUP 1

I. Sherman works at the Rock-n-Roller roller rink where he earns $4 an hour. Each week he must pay $7 union dues. If Sherman works 12 hours this week, calculate the amount of his paycheck.

II. The tacos at the Taco Shack have 300 calories in them and a small soda has 120 calories. What is the number of calories Bill consumes if he has 3 tacos and a soda?

III. In planning to build a new roller rink, the Rock-n-Roller company needs at least 5000 square meters of land. If the land must be twice as long as it is wide, what length(s) and width(s) would work?



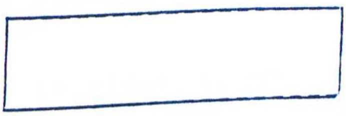
THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II   II & III   I & III   NONE

REASON:

GROUP 2

I. Frieda is going to plant corn in a rectangular patch of her garden. The area of the patch is 72 square meters and it is 6 meters longer than it is wide. Find the length and width of Frieda's garden.



II. Frieda is planning her garden. She needs money for seed and fertilizer. Freida has saved \$12 and has a job that pays \$7 a week. If Freida works 4 weeks, how much money will she have for seed and fertilizer?

III. Sharon runs a lawn mower repair service. Her revenue depends on the price she charges for a tune-up. If y is the revenue and x is the tune up price, Sharon's revenue is $y = -10x^2 + 500x$. What should Sharon charge in order to have \$6000 in revenue?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II II & III I & III NONE

REASON:

GROUP 3

I. John's committee is in charge of ordering material to be made into new band uniforms. Each uniform requires 3.5 meters of material. In order to make matching banners and flags, 18 meters more are needed. How many meters should the committee order if there are 30 band members?

II. Frieda is planning her garden. She needs money for seed and fertilizer. Freida has saved $12 and has a job that pays $7 a week. If Freida works 4 weeks, how much money will she have for seed and fertilizer?

III. The Stanley Snow Disk Company makes and sells the round snow disks that kids use for sledding. For each snow disk, the selling price brings in $.75 more than the cost of manufacturing. However, the company has fixed costs of $185 to advertise their snow disk each week.

How many snow disks must the Stanley Company make and sell each week in order to have at least $200 profit?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

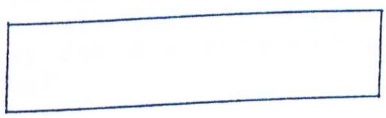
I & II II & III I & III NONE

REASON:

GROUP 4

I. The Rock-n-Roller Roller Rink has found that its profit depends on its admission price. If x is the admission price and y is the profit, then they calculate the profit using $y = -10x^2 + 120x - 200$. Calculate the profit if the admission price is less than $4.00.

II. In planning to build a new roller rink, the Rock-n-Roller company needs at least 5000 square meters of land. If the land must be twice as long as it is wide, what lenght(s) and width(s) would work?



III. The bacteria that causes Hawaiian flu multiplies by the formula $y = 1000 - 50x - 5x^2$ where x is the time in hours and y is the number of bacteria in a blood sample. If it has been 4 hours since Sandy caught the Hawaiian flu, what is the number of bacteria that would be found in his blood sample?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II II & III I & III NONE

REASON:

GROUP 5

I. Paul knows that Mel's Pizza Parlor pays $3.50 an hour and that they charge their employees $5.00 a week for uniform rental. If Paul gets a job at Mel's, figure out his paycheck if he works 15 hours a week.

II. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink, find the number of slices of pizza she can eat and still take in less than 800 calories.

III. Fred is saving to buy a motorcycle. He has saved $630. If the sales tax is 5%, what is the highest price Fred can pay for his motorcycle and still have enough for the tax?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II　　II & III　　I & III　　NONE

REASON:

GROUP 6

I. Mel's Pizza Parlor bases its prices on the diameter of the pizza. If y is the price of the pizza and x is its diameter in inches, Mel charges $y = .02x^2 + 2.00$. What size pizza can Audrey buy for $6?

II. In planning to build a new roller rink, the Rock-n-Roller company has land that is twice as long as it is wide. If the land is 150 meters wide, what is its area?

III. The Stanley Snow Disk Company makes round disks for sledding. If it costs $.001 per square centimeter for plastic plus $1.25 for other costs for each disk, how much does a disk measuring 45 centimeters in radius cost?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II　　　II & III　　　I & III　　　NONE

REASON:

GROUP 7

I. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink, find the number of slices of pizza she can eat and still take in less than 800 calories.

II. Paul knows that Mel's Pizza Parlor pays $3.50 an hour and that they charge their employees $5.00 a week for uniform rental. If Paul gets a job at Mel's, how much will he earn if he works 15 hours a week?

III. Fred's motorcycle travels at least 16 kilometers per liter of gasoline. How much gas must Fred put into his tank to go 128 km?

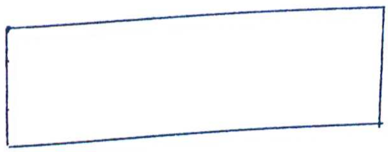
THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II   II & III   I & III   NONE

REASON:

GROUP 8

I. In planning to build a new roller rink, the Rock-n-Roller company needs at least 5000 square meters of land. If the land must be twice as long as it is wide, what lenght(s) and width(s) would work?



II. John's committee is in charge of ordering material to be made into new band uniforms. Each uniform requires 3.5 meters of material. In order to make matching banners and flags, 18 meters more are needed. Calculate the number of meters the committee should order if there are 30 band members.

III. Sharon runs a lawn mower repair service. Her revenue depends on the price she charges for a tune-up. If y is the revenue and $\underline{x}$ is the tune up price, Sharon's revenue is $y = -10x^2 + 500x$. Calculate the price Sharon should charge if she wants a revenue of $6000.

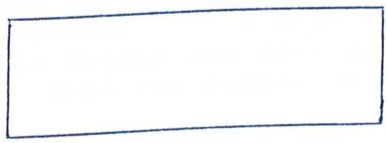
THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II II & III I & III NONE

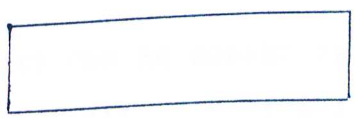
REASON:

GROUP 9

I. In planning to build a new roller rink, the Rock-n-Roller company has land that is twice as long as it is wide. If the land is 150 meters wide, what is its area?



II. The kids on Elm Street have made up a game called Whacko which they play on a vacant lot. The lot's owner has given them permission to put down chalk lines around the rectangular playing field. If they have enough chalk for 200 meters of lines and the end lines must be 30 meters each, how many meters can the side lines be?



III. The Rock-n-Roller roller rink pays its employees $3.50 an hour plus a $5.00 bonus if they work the Saturday night shift. Bill, who always works Saturday night, got a check this week for $75.00. Calculate the number of hours he worked this week.

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II　　　II & III　　　I & III　　　NONE

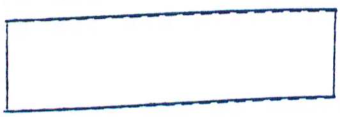
REASON:

GROUP 10

I. Frieda is planning her garden. She needs money for seed and fertilizer. Freida has saved $12 and has a job that pays $7 a week. If Freida works 4 weeks, how much money will she have for seed and fertilizer?

II. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink and has 3 slices of pizza, find the number of calories she has consumed.

III. Frieda is going to plant corn in a rectangular patch of her garden. The patch has area 72 square meters and it is 6 meters longer than it is wide. Find the length and width of Frieda's garden.



THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II   II & III   I & III   NONE

REASON:

GROUP 11

I. Paul knows that Mel's Pizza Parlor pays $3.50 an hour and that they charge their employees $5.00 a week for uniform rental. If Paul gets a job at Mel's, how much will he earn if he works 15 hours a week?

II. John's committee is in charge of ordering material to be made into new band uniforms. Each uniform requires 3.5 meters of material. On order to make matching banners and flags, 18 meters more are needed. How many meters should the committee order if there are 30 band members?

III. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink, how many slices of pizza she can eat and still take in less than 800 calories.

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

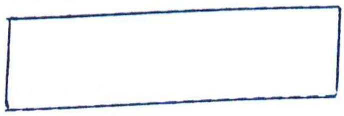
I & II II & III I & III NONE

REASON:

GROUP 12

I. A movie theater's attendance depends on its ticket price. If y is the attendance and x is the ticket price, then y = 300 - 40x. Calculate the number of people who will attend if the ticket price is $2.50.

II. Frieda is going to plant corn in a rectangular patch of her garden. The patch has area 72 square meters and it is 6 meters longer than it is wide. Figure out the length and width of Frieda's garden.



III. Paul knows that Mel's Pizza Parlor pays $3.50 an hour and that they charge their employees $5.00 a week for uniform rental. If Paul gets a job at Mel's, figure out his paycheck if he works 15 hours a week.

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II II & III I & III NONE

REASON:

GROUP 13

I. The Rock-n-Roller roller rink pays its employees $3.50 an hour plus a $5.00 bonus if they work the Saturday night shift. Bill, who always works Saturday night, got a check this week for $75.00. Calculate the number of hours he worked this week.

II. Paul knows that Mel's Pizza Parlor pays $3.50 an hour and that they charge their employees $5.00 a week for uniform rental. If Paul gets a job at Mel's, how much will he earn if he works 15 hours a week?

III. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink, how many slices of pizza she can eat and still take in less than 800 calories.

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

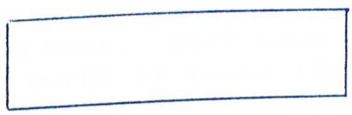
I & II　　II & III　　I & III　　NONE

REASON:

GROUP 14

I. Mel's Pizza Parlor bases its prices on the diameter of the pizza. If y is the price of the pizza and x is its diameter in inches, Mel charges $y = .02x^2 + 2.00$. What is the price of a 14 inch pizza?

II. The kids on Elm Street have made up a game called Whacko which they play on a vacant lot. The lot's owner has given them permission to put down chalk lines around the rectangular playing field. If they have enough chalk for 200 meters of lines and the end lines must be 30 meters each, what lengths will the side lines be?



III. The Stanley Disk Company makes and sells the round snow disks that kids use for sledding. For each snow disk, the selling price brings in $.75 more than the cost of manufacturing. However, the company has fixed costs of $185 to advertise their snow disk each week.

How many snow disks must the Stanley Company make and sell each week in order to have at least $200 profit?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II   II & III   I & III   NONE

REASON:

GROUP 15

I. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink and has 3 slices of pizza, find the number of calories she has consumed.

II. Mel's Pizza Parlor bases its prices on the diameter of the pizza. If y is the price of the pizza and x is its diameter in inches, Mel charges $y = .02x^2 + 2.00$. What size pizza can Audrey buy for $6?

III. Sherman works at the Rock-n-Roller roller rink where he earns $4 an hour. Each week he must pay $7 union dues. If Sherman works 12 hours this week, calculate the amount of his paycheck.

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

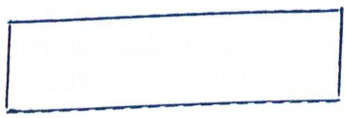
I & II   II & III   I & III   NONE

REASON:

GROUP 16

I. The tacos at the Taco Shack have 300 calories in them and a small soda has 120 calories. Find the number of calories Bill consumes if he has 3 tacos and a soda.

II. Frieda is going to plant corn in a rectangular patch of her garden. The patch has area 72 square meters and it is 6 meters longer than it is wide. Find the length and width of Frieda's garden.



III. The daily profit for the Taco Shack is given by $y = -250x^2 + 500x$ where x is the price of each taco. What price(s) of tacos will give a daily profit of $200?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II　　　II & III　　　I & III　　　NONE

REASON:

GROUP 17

I. A movie theater's attendance depends on its ticket price. If y is the attendance and x is the ticket price, then $y = 300 - 40x$. Calculate the number of people who will attend if the ticket price is $2.50.

II. Fred's motorcycle travels at least 16 kilometers per liter of gasoline. How far can Fred travel on an 8 liter tank of gas?

III. Fred is saving to buy a motorcycle. He has saved $630. If the sales tax is 5%, what is the highest price Fred can pay for his motorcycle and still have enough for the tax?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II   II & III   I & III   NONE

REASON:

GROUP 18

I. Fred's motorcycle travels at least 16 kilometers per liter of gasoline. Find the distance that Fred can travel on an 8 liter tank of gas.

II. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink, find the number of slices of pizza she can eat and still take in less than 800 calories.

III. Fred is saving to buy a motorcycle. He has saved $630. If the sales tax is 5%, what is the highest price Fred can pay for his motorcycle and still have enough for the tax?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

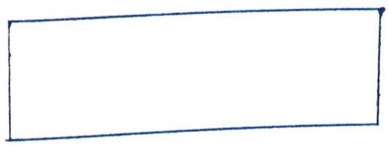
I & II   II & III   I & III   NONE

REASON:

GROUP 19

I. Frieda's garden is 3 times longer than it is wide. If her garden is 15 meters wide, find its area.

II. In planning to build a new roller rink, the Rock-n-Roller company needs at least 5000 square meters of land. If the land must be twice as long as it is wide, what length(s) and width(s) would work?



III. The Rock-n-Roller Roller Rink has found that its profit depends on its admission price. If x is the admission price and y is the profit, then they calculate the profit using $y = -10x^2 + 120x - 200$. Calculate the profit if the admission price is less than $4.00.

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II   II & III   I & III   NONE

REASON:

GROUP 20

I. Sherman works at the Rock-n-Roller roller rink where he earns $4 an hour. Each week he must pay $7 union dues. If Sherman works 12 hours this week, calculate the amount of his paycheck.

II. David belongs to a long distance bike riding club. In order to ride each week, David pays $2 an hour for bike rental plus $5 for dues. If David has $17 per week to spend on bike riding, how many hours can he ride?

III. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink, calculate the number of slices of pizza she can eat and still ta 120 calories. If Valerie drinks one soft drink, calculate the number of slices of pizza she can eat and still take in less than 800 calories.

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

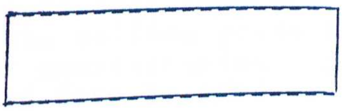
I & II　　II & III　　I & III　　NONE

REASON:

GROUP 21

I. Frieda is planning her garden. She needs money for seed and fertilizer. Freida has saved \$12 and has a job that pays \$7 a week. If Freida works 4 weeks, how much money will she have for seed and fertilizer?

II. Frieda is going to plant corn in a rectangular patch of her garden. The patch has area 72 square meters and it is 6 meters longer than it is wide. Find the length and width of Frieda's garden.



III. John's committee is in charge of ordering material to be made into new band uniforms. Each uniform requires 3.5 meters of material. On order to make matching banners and flags, 18 meters more are needed. How many meters should the committee order if there are 30 band members?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II II & III I & III NONE

REASON:

GROUP 22

I. Fred's motorcycle travels at least 16 kilometers per liter of gasoline. How much gas must Fred put into his tank to go 128 km?

II. Paul knows that Mel's Pizza Parlor pays $3.50 an hour and that they charge their employees $5.00 a week for uniform rental. If Paul gets a job at Mel's, how much will he earn if he works 15 hours a week?

III. The Stanley Disk Company makes and sells the round snow disks that kids use for sledding. For each snow disk, the selling price brings in $.75 more than the cost of manufacturing. However, the company has fixed costs of $185 to advertise their snow disk each week.

How many snow disks must the Stanley Company make and sell each week in order to have at least $200 profit?

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II   II & III   I & III   NONE

REASON:

GROUP 23

I. Mel's Pizza Parlor bases its prices on the diameter of the pizza. If y is the price of the pizza and x is its diameter in inches, Mel charges $y = .02x^2 + 2.00$. What is the price of a 14 inch pizza?

II. Fred's motorcycle travels at least 16 kilometers per liter of gasoline. What is the capacity of his gas tank if Fred can travel 128 kilometers on a tank of gas?

III. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink, find the number of slices of pizza she can eat and still take in less than 800 calories.

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II　　II & III　　I & III　　NONE

REASON:

GROUP 24

I. Paul knows that Mel's Pizza Parlor pays $3.50 an hour and that they charge their employees $5.00 a week for uniform rental. If Paul gets a job at Mel's, figure out his paycheck if he works 15 hours a week.

II. John's committee is in charge of ordering material to be made into new band uniforms. Each uniform requires 3.5 meters of material. In order to make matching banners and flags, 18 meters more are needed. How many meters should the committee order if there are 30 band members?

III. Frieda is going to plant corn in a rectangular patch of her garden. The patch has area 72 square meters and it is 6 meters longer than it is wide. Figure out the length and width of Frieda's garden.

THE PROBLEMS THAT CAN BE SOLVED IN THE SAME WAY ARE:

I & II　　　II & III　　　I & III　　　NONE

REASON:

Answer Key to Triads Task

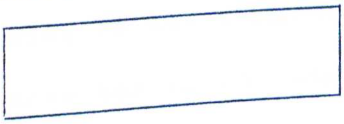
Group	Correct Answer
1	A
2	C
3	A
4	C
5	B
6	B
7	C
8	C
9	B
10	A
11	A
12	C
13	C
14	B
15	C
16	B
17	A
18	B
19	C
20	B
21	C
22	C
23	B
24	A

APPENDIX B

PROBLEM BANK FOR TRIADS TASK

1. Fred's motorcycle travels at least 16 kilometers per liter of gasoline. How far can Fred travel on an 8 liter tank of gas?

2. The kids on Elm Street have made up a game called Whacko which they play on a vacant lot. The lot's owner has given them permission to put down chalk lines around the rectangular playing field. If they have enough chalk for 200 meters of lines and the end lines must be 30 meters each, how many meters can the side lines be?



3. The Stanley Snow Disk Company makes and sells the round snow disks that kids use for sledding. For each snow disk, the selling price brings in $.75 more than the cost of manufacturing. However, the company has fixed costs of $185 to advertise their snow disk each week.

Figure out the number of snow disks the Stanley Company must make and sell each week in order to have at least $200 profit?

4. A movie theater's attendance depends on its ticket price. If y is the attendance and x is the ticket price, then $y = 300 - 40x$. Calculate the number of people who will attend if the ticket price is $2.50.

5. The Rock-n-Roller roller rink pays its employees \$3.50 an hour plus a \$5.00 bonus if they work the Saturday night shift. Bill, who always works Saturday night, got a check this week for \$75.00. Calculate the number of hours he worked this week.

6. Frieda is planning her garden. She needs money for seed and fertilizer. Freida has saved \$12 and has a job that pays \$7 a week. If Freida works 4 weeks, how much money will she have for seed and fertilizer?

7. Fred is saving to buy a motorcycle. He has saved \$630. If the sales tax is 5%, what is the highest price Fred can pay for his motorcycle and still have enough for the tax?

8. Mel's Pizza Parlor bases its prices on the diameter of the pizza. If y is the price of the pizza and x is its diameter in inches, Mel charges $y = .02x^2 + 2.00$. What is the price of a 14 inch pizza?

9. Tickets to a movie theater cost \$4.50 for adults and \$2.50 for students. In order to make a profit each evening, the theater must take in at least \$2000. How many tickets of each type will allow the theater to make a profit?

10. The daily profit for the Taco Shack is given by $y = -3x \quad + 215$ where x is the price of each taco. What price(s) of tacos will give a daily profit of at least $200?

11. In scoring Whacko, a game they invented, the kids on Elm Street count two points for a dribbler and three points for a whammy. Tim figures his team needs to score at least 35 points to beat Earl's team on Saturday. What combination(s) of dribblers and whammys will Tim's team need?

12. Sharon runs a lawn mower repair service. Her revenue depends on the price she charges for a tune-up. If y is the revenue and x is the tune up price, Sharon's revenue is $y = -10x^2 + 500x$. Calculate the revenue if the price is $35.

13. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink, find the number of slices of pizza she can eat and still take in less than 800 calories.

14. David and his friend are packing for a long distance bike ride. David packs 6 containers of drink and 12 packs of food and the total weight of his food and drink is 228 ounces. His friend packs 8 containers of drink and 10 packs of food and his food drink weigh 220 ounces. Figure out the weight of each pack of food and each container of drink.

15. The Rock-n-Roller Roller Rink has found that its profit depends on its admission price. If x is the admission price and y is the profit, then they calculate the profit using $y = -10x^2 + 120x - 200$. Calculate the profit if the admission price is less than \$4.00.

16. Frieda is going to plant corn in a rectangular patch of her garden. The patch has area 72 square meters and it is 6 meters longer than it is wide. Find the length and width of Frieda's garden.

17. The Stanley Snow Disk Company makes round disks for sledding. If it costs \$.001 per square centimeter for plastic plus \$1.25 for other costs for each disk, how much does a disk measuring 45 centimeters in radius cost?

18. David is planning a cross country bicycle ride. He will carry some high nutrition/low weight food bars and drink similar to those used by astronauts. Each gram of food bar supplies 7 calories of energy; each gram of drink gives 4 calories. David's trainer says he needs at least 6000 calories a day. What choice(s) of food and drink could he take for each day?

19. The profit a movie house makes on its sales of a candy bar depends on the price for each bar. If x is the price of the candy, then the profit is $y = -750x^2 + 1200x$. What profit will be made if the price is $.45?

20. Sharon's lawn mower repair service took in $269 in one week. Sharon's bills for parts and rent were $59 and $75. If Sharon worked 15 hours this week, what was her profit per hour?

21. At the Taco Shack Hal bought two sodas and three tacos for $ 3.75. Ernie bought two tacos and three sodas for $ 3.50. Find the cost of each taco and soda.

22. Paul knows that Mel's Pizza Parlor pays $3.50 an hour and that they charge their employees $5.00 a week for uniform rental. If Paul gets a job at Mel's, how much will he earn if he works 15 hours a week?

23. In planning to build a new roller rink, the Rock-n-Roller company needs at least 5000 square meters of land. If the land must be twice as long as it is wide, what lenght(s) and width(s) would work?

24. David belongs to a long distance bike riding club. In order to ride each week, David pays $2 an hour for bike rental plus $5 for dues. If David has $17 per week to spend on bike riding, how many hours can he ride?

25. John's committee is in charge of ordering material to be made into new band uniforms. Each uniform requires 3.5 meters of material. On order to make matching banners and flags, 18 meters more are needed. Hoe many meters should the committee order if there are 30 band members?

26. In planning to build a new roller rink, the Rock-n-Roller company has land that is twice as long as it is wide. If the land is 150 meters wide, what is its area?

27. Frieda is going to plant corn in a rectangular patch of her garden. The patch has area 72 square meters and it is 6 meters longer than it is wide. Figure out the length and width of Frieda's garden.

28. Paul knows that Mel's Pizza Parlor pays $3.50 an hour and that they charge their employees $5.00 a week for uniform rental. If Paul gets a job at Mel's, figure out his paycheck if he works 15 hours a week.

29. Sherman works at the Rock-n-Roller roller rink where he earns $4 an hour. Each week he must pay $7 union dues. If Sherman works 12 hours this week, calculate the amount of his paycheck.

30. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink and has 3 slices of pizza, find the number of calories she has consumed.

31. Sharon runs a lawn mower repair service. Her revenue depends on the price she charges for a tune-up. If y is the revenue and x is the tune up price, Sharon's revenue is $y = -10x^2 + 500x$. What should Sharon charge in order to have $6000 in revenue?

32. Mel's Pizza Parlor bases its prices on the diameter of the pizza. If y is the price of the pizza and x is its diameter in inches, Mel charges $y = .02x^2 + 2.00$. What size pizza can Audrey buy for $6?

33. Fred's motorcycle travels at least 16 kilometers per liter of gasoline. What is the capacity of his gas tank if Fred can travel 128 kilometers on a tank of gas?

34. The Stanley Snow Disk Company makes and sells the round snow disks that kids use for sledding. For each snow disk, the selling price brings in $.75 more than the cost of manyfacturing. However, the company has fixed costs of $185 to advertise their snow disk each week.

How many snow disks must the Stanley Company make and sell each week in order to have at least $200 profit?

35. The tacos at the Taco Shack have 300 calories in them and a small soda has 120 calories. Find the number of calories Bill consumes if he has 3 tacos and a soda.

36. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink, how many slices of pizza she can eat and still take in less than 800 calories.

37. Fred's motorcycle travels at least 16 kilometers per liter of gasoline. How much gas must Fred put into his tank to go 128 km?

38. A slice of pepperoni pizza at Mel's Pizza Parlor has 225 calories. A soft drink has 120 calories. If Valerie drinks one soft drink, calculate the number of slices of pizza she can eat and still take in less than 800 calories.

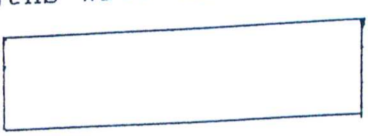
39. Fred's motorcycle travels at least 16 kilometers per liter of gasoline. Find the distance that Fred can travel on an 8 liter tank of gas.

40. Frieda's garden is 3 times longer than it is wide. If her garden is 15 meters wide, find its area.

41. The bacteria that causes Hawaiian flu multiplies by the formula $y = 1000 - 50x - 5x^2$ where x is the time in hours and y is the number of bacteria in a blood sample. If it has been 4 hours since Sandy caught the Hawaiian flu, what is the number of bacteria that would be found in his blood sample?

42. The tacos at the Taco Shack have 300 calories in them and a small soda has 120 calories. What is the number of calories Bill consumes if he has 3 tacos and a soda?

43. The kids on Elm Street have made up a game called Whacko which they play on a vacant lot. The lot's owner has given them permission to put down chalk lines around the rectangular playing field. If they have enough chalk for 200 meters of lines and the end lines must be 30 meters each, what lengths will the side lines be?



44. Sharon runs a lawn mower repair service. Her revenue depends on the price she charges for a tune-up. If y is the revenue and x is the tune up price, Sharon's revenue is $y = -10x^2 + 500x$. Calculate the price Sharon should charge if she wants a revenue of $6000.

45. John's committee is in charge of ordering material to be made into new band uniforms. Each uniform requires 3.5 meters of material. In order to make matching banners and flags, 18 meters more are needed. Calculate the number of meters the committee should order if there are 30 band members.

APPENDIX C

THE PROBLEM SOLVING TEST

NAME_____________________

CLASS____________________

PROBLEM SOLVING TEST

(COMPUTER VERSION)

On the next few pages are ten problems for you to solve. You may use whatever method you prefer to solve the problems. Record your work in the work space and enter your answer in the space given. After you have finished the problem, indicate all of the tools that you used in solving the problem.

1. Sherman works at the Rock-n-Roller roller rink where he earns $4 an hour. Each week $7 is taken from his paycheck for union dues. If Sherman works 12 hours this week, how much money will he have to spend?

Workspace:

Answer:____________________

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

2. The kids on Elm Street have made up a game called Whacko which they play on a vacant lot. They are putting down chalk lines around the rectangular playing field. If they have enough chalk for 200 meters of lines and the end lines must be 30 meters each, how many meters long can each side line be?

Workspace:

Answer: ______________________

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

3. A movie theater's attendance depends on its ticket price. If y is the attendance and x is the ticket price, then y = 300 - 40x. Calculate the number of people who will attend if the ticket price is $2.50.

Workspace:

Answer:________________________

Check any of the following that you used:

______ calculator

______ computer graph

______ computer table

______ muMath

______ guess-n-test

4. David wants to join a long distance bike riding club. There are two clubs in his area that interest him so he made a chart of their costs. Each charges membership dues and an hourly bike rental.

Club	Dues	Hourly Bike Rental
Wildcat	$5.00	$2.00
Turtle	$8.00	$1.50

When (in hours of bike rental) are the two clubs' charges equal?

Workspace:

Answer:____________________

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

5. The Rock-n-Roller roller rink pays its employees $3.50 an hour plus a $5.00 bonus if they work the Saturday night shift. Bill, who always works Saturday night, got a check this week for $75.00. Calculate the number of hours he worked this week.

Workspace:

Answer:____________________

Check any of the following that you used:

______ calculator

______ computer graph

______ computer table

______ muMath

______ guess-n-test

6. The next stop for a family traveling on vacation is 765 kilometers away. When they are on the road, the family drives 90 km/hr. If they make rest stops totaling 2 hours in length, how long will it take them to get to their next stop?

Workspace:

Answer: ____________________

Check any of the following that you used:

_____ calculator

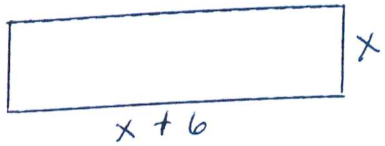
_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

7. Frieda is going to plant corn in a rectangular patch of her garden. The patch has an area of 72 square meters and it is 6 meters longer than it is wide. Find the length and width of Frieda's corn patch.



Workspace:

Answer: ____________________

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

8. An archer shoots an arrow straight up into the air. The rule for finding h the height (in feet) of the arrow after a certain time in seconds t is $h = -16t^2 + 40t + 2$. Find the height of the arrow after 2 seconds.

Workspace:

Answer:____________________

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

9. A rectangular piece of cloth has an area of 12 square meters. If the piece of cloth is 3 times long as it is wide, find the dimensions of the cloth.

Workspace:

Answer:____________________

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

10. The Rock-n-Roller Roller Rink has found that its profit depends on its admission price. If x is the admission price and y is the profit, then they calculate the profit using $y = -10x^2 + 120x - 200$. Calculate the profit if the admission price is $4.00.

Workspace:

Answer:____________________

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

NAME________________________

CLASS________________________

PROBLEM SOLVING TEST

(NON-COMPUTER VERSION)

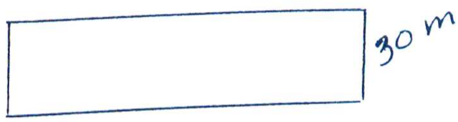
On the next few pages are ten problems for you to solve. You may use whatever method you prefer to solve the problems. Record your work in the work space and enter your answer in the space given.

1. Sherman works at the Rock-n-Roller roller rink where he earns $4 an hour. Each week $7 is taken from his paycheck for union dues. If Sherman works 12 hours this week, how much money will he have to spend?

Workspace:

Answer:____________________

2. The kids on Elm Street have made up a game called Whacko which they play on a vacant lot. They are putting down chalk lines around the rectangular playing field. If they have enough chalk for 200 meters of lines and the end lines must be 30 meters each, how many meters long can each side line be?



Workspace:

Answer:____________________

3. A movie theater's attendance depends on its ticket price. If y is the attendance and x is the ticket price, then $y = 300 - 40x$. Calculate the number of people who will attend if the ticket price is \$2.50.

Workspace:

Answer:____________________

4. David wants to join a long distance bike riding club. There are two clubs in his area that interest him so he made a chart of their costs. Each charges membership dies and an hourly bike rental.

Club	Dues	Hourly Bike Rental
Wildcat	\$5.00	\$2.00
Turtle	\$8.00	\$1.50

When (in hours of bike rental) are the two clubs' charges equal?

Workspace:

Answer: ______________________

5. The Rock-n-Roller roller rink pays its employees \$3.50 an hour plus a \$5.00 bonus if they work the Saturday night shift. Bill, who always works Saturday night, got a check this week for \$75.00. Calculate the number of hours he worked this week.

Workspace:

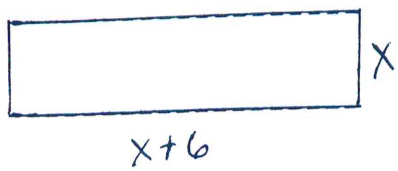
Answer: ____________________

6. The next stop for a family traveling on vacation is 765 kilometers away. When they are on the road, the family drives 90 km/hr. IF they make rest stops totaling 2 hours in length, how long will it take them to get to their next stop?

Workspace:

Answer:________________________

7. Frieda is going to plant corn in a rectangular patch of her garden. The patch has an area of 72 square meters and it is 6 meters longer than it is wide. Find the length and width of Frieda's corn patch.



Workspace:

Answer:____________________

8. An archer shoots an arrow straight up into the air. The rule for finding h the height (in feet) of the arrow after a certain time in seconds t is $h = -16t^2 + 40t + 2$. Find the height of the arrow after 3 seconds.

Workspace:

Answer:____________________

9. A rectangular piece of cloth has an area of 12 square meters. If the piece of cloth is 3 times long as it is wide, find the dimensions of the cloth.

Workspace:

Answer:______________________

10. The Rock-n-Roller Roller Rink has found that its profit depends on its admission price. If x is the admission price and y is the profit, then they calculate the profit using $y = -10x^2 + 120x - 200$. Calculate the profit if the admission price is $4.00.

Workspace:

Answer:____________________

APPENDIX D

PROTOCOL FOR SCORING THE PROBLEM SOLVING TEST

NAME______________________

CLASS______________________

PROBLEM SOLVING TEST

On the next few pages are ten problems for you to solve. You may use whatever method you prefer to solve the problems. Record your work in the work space and enter your answer in the space given. After you have finished the problem, indicate all of the tools that you used in solving the problem.

SCORING

SITUATION	POINTS
1. CORRECT NO. AND UNITS WHERE NEEDED	5
2. MISSING, NEEDED UNITS BUT CORRECT NUMBER	-1
3. COMPUTATION OR SYMBOL MANIPULATION ERROR	-1
4. FUNCTION OR FORMULA CORRECT ALL ELSE WRONG	3
5. INCORRECT FUNCTION OR FORMULA	2
6. VARIABLES ASSIGNED CORRECTLY	1
7. WRONG SUBSTITUTION	-1
8. INCORRECT ANSWER -- NO WORK SHOWN	0

1. Sherman works at the Rock-n-Roller roller rink where he earns $4 an hour. Each week $7 is taken from his paycheck for union dues. If Sherman works 12 hours this week, how much money will he have to spend?

Workspace:

$y = 4x - 7$

$x = 12$

$y = 4 \cdot 12 - 7$

$= 48 - 7$

$= 41$

Answer: \$41

Check any of the following that you used:

_____ calculator

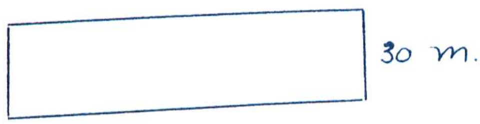
_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

2. The kids on Elm Street have made up a game called Whacko which they play on a vacant lot. They are putting down chalk lines around the rectangular playing field. If they have enough chalk for 200 meters of lines and the end lines must be 30 meters each, how many meters long can each side line be?



Workspace:

$$200 = 30 \cdot 2 + 2x$$
$$200 = 60 + 2x$$
$$140 = 2x$$
$$70 = x$$

Answer: 70

Check any of the following that you used:

______ calculator

______ computer graph

______ computer table

______ muMath

______ guess-n-test

3. A movie theater's attendance depends on its ticket price. If y is the attendance and x is the ticket price, then y = 300 - 40x. Calculate the number of people who will attend if the ticket price is $2.50.

Workspace:

$$y = 300 - 40x \qquad x = 2.50$$

$$= 300 - 40(2.5)$$

$$= 300 - 100$$

$$= 200$$

Answer: 200

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

4. David wants to join a long distance bike riding club. There are two clubs in his area that interest him so he made a chart of their costs. Each charges membership dues and an hourly bike rental.

Club	Dues	Hourly Bike Rental
Wildcat	\$5.00	\$2.00
Turtle	\$8.00	\$1.50

When (in hours of bike rental) are the two clubs' charges equal?

Workspace:

$x = \#$ of hours

$2x + 5 = 1.5x + 8$

$.5x + 5 = 8$

$.5x = 3$

$x = 6$

Answer: 6 hours

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

5. The Rock-n-Roller roller rink pays its employees $3.50 an hour plus a $5.00 bonus if they work the Saturday night shift. Bill, who always works Saturday night, got a check this week for $75.00. Calculate the number of hours he worked this week.

Workspace:

$X = \#$ of hours

$75 = 3.5x + 5$

$70 = 3.5X$

$\frac{70}{3.5} = X$

$20 = X$

Answer: 20

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

6. The next stop for a family traveling on vacation is 765 kilometers away. When they are on the road, the family drives 90 km/hr. If they make rest stops totaling 2 hours in length, how long will it take them to get to their next stop?

Workspace:

t = time in hours

$765 = 90(t-2)$

$765 = 90t - 180$

$945 = 90t$

$10.5 = t$

Answer: 10.5 hours

Check any of the following that you used:

_____ calculator

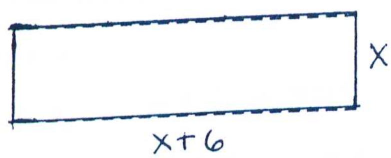
_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

7. Frieda is going to plant corn in a rectangular patch of her garden. The patch has an area of 72 square meters and it is 6 meters longer than it is wide. Find the length and width of Frieda's corn patch.



Workspace:

$x(x+6)=72$

$x^2+6x-72=0$

$(x+12)(x-6)=0$

$x=6, -12$

$x=6$

$x+6=12$

Answer: 6 m, 12 m

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

1 answer correct = 3 points

8. An archer shoots an arrow straight up into the air. The rule for finding h the height (in feet) of the arrow after a certain time in seconds t is $h = -16t^2 + 40t + 2$. Find the height of the arrow after 2 seconds.

Workspace:

$$h = -16t^2 + 40t + 2 \quad t = 2$$
$$= -16(4) + 40(2) + 2$$
$$= -64 + 80 + 2$$
$$= 18$$

Answer: 18 feet

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

$h = -16t^2 \Rightarrow h = -64$ 2 points

9. A rectangular piece of cloth has an area of 12 square meters. If the piece of cloth is 3 times long as it is wide, find the dimensions of the cloth.

Workspace:

$x = \text{width}$

$3x = \text{length}$

$x(3x) = 12$

$3x^2 = 12$

$x^2 = 4$

$x = 2$

$3x = 6$

Answer: 2m, 6m

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

1 correct answer 3pts

Correct ratio with units 2pts

Correct ratio without units 1pt.

10. The Rock-n-Roller Roller Rink has found that its profit depends on its admission price. If x is the admission price and y is the profit, then they calculate the profit using y = -10x + 120x -200. Calculate the profit if the admission price is $4.00.

Workspace:

$y = -10x^2 + 120x - 200 \qquad x=4$

$= -10(4)^2 + 120(4) - 200$

$= 120$

Answer: $120

Check any of the following that you used:

_____ calculator

_____ computer graph

_____ computer table

_____ muMath

_____ guess-n-test

APPENDIX E

ADMINISTRATION INSTRUCTIONS SITE TWO

INSTRUCTIONS FOR ADMINISTERING THE TRIADS TASK AND THE PROBLEM SOLVING TEST

During two periods, students in the experimental classes will be doing two instruments. The Triads Task consists of twenty-four groups of three problems. Students are asked to read the three problems in each triad and to indicate by circling given letters which of two of the three they would solve in the same way and to give a reason fo their choice. The Problem Solving Tes has ten problems which the students are to solve using any method of their choice. Students are to show their work and to enter their answers in a given space.

So that those students who are doing the Problem Solving Test may use the computer if they choose, these tasks will be given much in the same way as we have given the tests in the past. Half of the students should be given the Triad Task the first day while the other half are doing the Problem Solving Test. On the second day, student would switch.

On each day, please ask each student to PRINT his/her name and class name (GREEN 6 or FISCHER 1) in the spaces provided. Then read the directions aloud with each group. It is important to emphasize to the Triads group that they must give a reason and to the Probelm Solving test group that they should show all their work.

Other than that, please answer questions only concerning procedures. Specifically, do not indicate in any way to the Triads group what "solved in the same way" may mean or give any clues to the Problem Solving Test Group as to what methods may be used. If students ask questions of this nature, tell them that we are interested in what they think.

I will try to be there when these are given to help out but this being May, I am not sure I will be able to do this as I would like. If I am not there when these are given, please hold onto them and I will make arangements to pick them up from you. Thanks for your help in this.

The piloting indicated that each instrument takes more than half of the period but less than the whole period. Since students should not start a second instrument on the first day, you may want to provide an assignment for students to work on when they have completed their task for that day.

APPENDIX F

SAMPLE REASONS GIVEN ON THE TRIADS TASK

Sample Structure Reasons

lenght and width

two numbers are multiplied

same equation

same steps

multiply and then divide

a(b + c)

table for solving

y = x + x ^2

subtraction

same formula

division

add and then multiply

Sample Content Reasons

pizza

fuel

food

Mel's

motorcycles

income

Sample Pseudostructure Reasons

amount

money

meters

profit

number per week

diameter and radius

round things

cost per square unit

measurement

salaries

Sample Question Reasons

how much

similiar question

how many

Sample Other Reasons

solved the same way

calculation

no answer given

sound the same

look good

because

look the same

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