ABSTRACT

Title of dissertation: NOVICE MIDDLE-SCHOOL MATHEMATICS TEACHERS LEARNING TO PROMOTE STUDENT SENSE MAKING THROUGH PRODUCTIVE DISCUSSION

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While mathematics education researchers have long characterized student performance marked by mathematical explanations, arguments, and justifications as evidence of mathematical reasoning and understanding (e.g. Schoenfeld, 1992), current education policy has begun to move in a similar direction, emphasizing sense making and mathematical communication as features of mathematics education (Common Core State Standards Initiative, 2010). However, designing and implementing instruction with these features is challenging (e.g. Lampert, 1990). Furthermore, classroom and instructional norms norms must be carefully developed for this discursive-heavy instruction to be equitable (e.g. Boaler & Staples, 2008). If mathematical discussions are to be a feature of the mathematics classroom, then teachers must learn to learn from their own teaching to enact practices that promote discussion (Hiebert, Morris & Glass, 2003).

This study is a qualitative investigation of how three novice middle-school mathematics teachers learned to promote in-class student discussion, with a focus on the features of and strategies for instruction to which they attended, as well as their

negotiation of challenges that arose during practice. Supported by a mentor, these teachers participated in a reflective teaching cycle that included a continuing teacher seminar, planning sessions, classroom observations, and reflection sessions over the course of 5 months. Through case studies, these teachers' instructional planning, practice and reflection were analyzed. Each case offers a perspective addressing how a teacher approached promoting student sense making through discussion, the challenges faced, and how those challenges were negotiated.

Cross-case analysis yielded five findings. First, the teachers found that building relationships with their students encouraged student participation in discussion. Second, the teachers were able to leverage accountability in the design and implementation of their lessons. Third, school context either supported or impeded the teachers' ability to engage students in discussion. The fourth finding illuminated the ways in which the organizational practice of tracking students impacted teacher perceptions and eventual decision-making. The final finding clarified the effect of mentoring support on teacher efficacy and self-efficacy. These findings have implications for mathematics teacher education, as well as induction mentoring programs.

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Chapter 1: Introduction

Over the course of mathematics education history, there have been many different arguments voiced around how mathematics should be taught. Researchers and mathematics educators have long argued for mathematics education that encourages students to make sense of challenging mathematics so that they can develop the necessary critical thinking skills that will serve them in the future (e.g. Lakatos, 1976; Kitcher, 1984; Schoenfeld, 1992; Stein, Grover & Henningsen, 1996). Often, arguments have been made for students to be able to develop mathematical explanations, arguments, and justifications for their thinking as evidence of mathematical understanding (e.g., Ball, 1993; Lampert, 1990; Schoenfeld, 1992). More recently, education policy has begun to move in a similar direction, emphasizing critical thinking, sense making, and mathematical communication as features of mathematics education (Common Core State Standards Initiative [CCSSI], 2010). In response to suggestions from research and policy, expectations characterizing how students are expected to interact in the classroom have changed, and this has implications for how teachers might design and enact instruction (Foreman & Ansell, 2001; Herrenkohl & Guerra, 1998).

The mathematical practices that are proposed in recent policy documents promote the inclusion of productive mathematical discussion through sense making as a feature of mathematics instruction (CCSSI, 2010). Schoenfeld defined *mathematical sense making* here as "(a) developing a mathematical point of view — valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure" (Schoenfeld, 1994, p. 60). *Productive mathematical* *discussions* are those student-centered discussions that "support student learning of mathematics by helping students learn how to communicate their ideas, making students' thinking public so it can be guided in mathematically sound directions, and encouraging students to evaluate their own and each other's mathematical ideas" (Smith & Stein, 2011, p. 1). Features of productive discussions, for the purpose of this study, include explaining and justifying ones' ideas, revoicing others' ideas in order to verify and internalize them, questioning others' ideas for clarity, and challenging others' ideas when there is a disagreement. But how might the mathematics education community educate and support both prospective and practicing teachers as they attempt to enact instruction that features students making sense of mathematics through participation in productive discussion?

Significance of the Problem

In the United States since World War II, secondary school mathematics has traditionally been taught as a system of skills and procedures. Historically, the perceived student responsibility for learning mathematics focused on following rules presented by the teacher or the textbook, memorizing and applying those rules, and verifying correctness through an authority such as the teacher or the textbook (Cobb & Yackel, 1996; Lampert, 1990; Schoenfeld, 1992). This type of practice can be very formal and, for some students, can limit opportunities for them to develop their mathematical reasoning (Brown, Collins & Duguid, 1989; Stein, Grover & Henningsen, 1996).

An in-depth understanding of mathematics includes not only the knowledge of rules and procedures, but also the ability to think mathematically. While not solely limited to the following understandings, an individual is viewed as exhibiting characteristics associated with thinking mathematically when explaining and justifying mathematical ideas, expressing a mathematical point of view, and using different tools to make sense of mathematical structures (Lakatos, 1976; Kitcher, 1984; Schoenfeld, 1992; Stein, Grover & Henningsen, 1996). Each of these features, especially explaining and justifying mathematical ideas, can be supported through classroom discussion.

Many policy documents addressing mathematics education in K-12 schools posit that classroom discussion addressing the meaning and solution of mathematics problems is an important venue through which schoolchildren may build their mathematical knowledge base. The Common Core State Standards for Mathematics (CCSSI, 2010), the National Council of Teachers of Mathematics (NCTM) through its *Principles and Standards for School Mathematics* (2000), and mathematics education researchers (e.g., Ball, 1993; Lampert, 1990; Schoenfeld, 1992) argue for the importance of students conversing about the meaning and solution of mathematics problems in K-12 schools. For instance, the Common Core State Standards for Mathematics (CCSSI, 2010) include "standards for mathematical practice" (p. 6). These include "make sense of problems and persevere in solving them," "reason abstractly and quantitatively," (p. 6) and "attend to precision" (p. 7). Text describing these standards notes that students, at all levels, should be able to "construct viable arguments and critique the reasoning of others" (p. 6) and to "communicate precisely with others" (p. 7). Similarly, NCTM (2000) states that students should be able to "make and investigate mathematical conjectures" and to "develop and evaluate mathematical arguments and proofs" (p. 57-58). Furthermore, this form of reasoning should be augmented through communication as students

> [o]rganize and consolidate their mathematical thinking through communication; communicate their mathematical thinking coherently and clearly to peers, teachers and others; analyze and evaluate the mathematical thinking and strategies of others; [and] use the language of mathematics to express mathematical ideas precisely (NCTM, 2000, p. 60).

If these understanding, communication, and reasoning goals and standards are to characterize norms of practice as evidenced in schools, there will need to be a fundamental adjustment in participation structures within the mathematics classroom (Foreman & Ansell, 2001; Herrenkohl & Guerra, 1998). Through mathematical communication, students and teachers may work and reason together as they “do mathematics” in a way that augments the mathematical knowledge that students are expected to know (Cobb & Yackel, 1996).

Implementing Discussion-focused Norms of Instruction with Underrepresented Groups. Despite the empirical claims of the benefits of instruction focused on problem solving, some researchers have argued that when instructional schemes in mathematics classrooms focus on problem solving, communication in groups, and more indirect pedagogy, students from the dominant culture may be privileged (Apple, 1992; Bernstein, 1990; Delpit, 2006; Lubienski, 2000, 2002). Lubienski (2000, 2002) contended that the low-income students in her classroom were confused and distracted by divergent explanations in the classroom and were unsure of their or the teachers’ classroom roles. These students’ explanations were more context-dependent than those of students of higher socioeconomic status (SES) who contributed explanations that were general and referred to the underlying mathematics.

However, other researchers have found that classrooms organized around group work, discussion, and sense making can be equitable and beneficial to low-income students of color if supporting norms and socially negotiated “rules” for appropriate behavior in group situations are carefully developed during implementation (Boaler, 1998, 2002a, 2002b; Boaler & Staples, 2008; Gutierrez, 2000; Kitchen, DePree, Celedon-Pattichis & Brinkerhoff, 2007; Martin, 2000; Moses & Cobb, 2001). This conditional clarification is consistent with the recommendations of theorists who suggest that students from outside the dominant culture must be given explicit access to the “culture of power” that will allow them to be successful (Bourdieu & Passeron, 1994; Cobb, 1999; Delpit, 2006; Herrenkohl & Guerra, 1998).

Nevertheless, researchers studying the implementation of reform models of mathematics instruction have noted that the cognitive demands of classrooms may be lowered in schools that serve students of color or from low-income backgrounds (Haberman, 1991; Silver & Stein, 1996). This lowering of cognitive demand during mathematics lessons may occur for many reasons, including low expectations of low-income students of color. Since the cognitive demand expected during instruction could be lowered for minority and low-income students, this raises the question of whether or not these students may have fewer prior experiences with the expectations and norms of a discussion-based classroom that focuses on explaining and justifying divergent solution strategies.

Bourdieu (1977) argued that early experiences have more weight on the ingrained and often tacit ways of thinking and behaving than do experiences later in life. However, Bourdieu also stated that explicit instruction could have an effect on changing those habits. It may be that in order to implement instructional strategies emphasizing communication and reasoning in a secondary mathematics classroom, it is initially important to have explicit instruction that focuses on norms and routines for discursive group-based problem solving and sense making. Lubienski (2000) noted that while she found investigation and discussion-based instruction to be disadvantageous to the low-income students in her study, she also was not explicit with her students about her role as teacher within this instructional approach. She noted that she did not make the students aware of the rationale or intention underlying mathematical discussions nor how these discussions were supposed to benefit the students' learning. It may be that explicit care needs to be taken when implementing new norms of discursive sense-making because, as can be inferred from Lubienski's research, if the development of norms is left up to chance, the results may not be advantageous for underrepresented students.

This is not to say that students should be acculturated into the dominant paradigm without any thought to or acknowledgement of the values of their personal experiences or knowledge. Instead, norms are cultural practices that are conveyed, not with the intention of replacing a tacit or home culture, but rather to allow students the multi-cultural access that they need to be successful in the dominant world (Brown, Collins & Duguid, 1988) without reproducing inequity. Boaler and Staples (2008) allege that not teaching students to code switch, that is to switch back and forth between different linguistic or interpretive forms depending on surrounding, is to perpetuate inequity. In their study, they found that low-income, culturally and linguistically diverse students who participated in classroom activities that relied heavily on small-group discussion outperformed less diverse, higher-SES students receiving more traditional instruction. However, the teachers in Boaler and

Staples' study carefully developed the social norms and socio-mathematical norms of the classrooms with each class and were consistent in their application.

Other studies have found similar results: If the teachers carefully develop the norms of the classroom and do not reduce the cognitive demand of the mathematics tasks, low-income students of color can be successful in discussion-oriented mathematics classes (Herrenkohl & Guerra, 1998; Huffard-Ackles, Fuson & Sherin, 2004; Schoenfeld, 1992; Silver & Stein, 1996; Stein, Grover & Henningsen, 1996). Therefore, it is critical to determine how mathematics teacher educators might prepare and support teachers for consciously developing the classroom culture and norms of mathematical discussion.

Challenges faced by new and experienced teachers while attempting to promote discussion. There have been many researchers who have studied teaching with a focus on student-centered discourse. Chazan (2000) studied teaching using problem-based investigation and student discussion in an urban high school class. He found that there were many difficulties in teaching in this way. First, there was a culturally acceptable way to practice mathematics that pervaded the school experience. The way he was trying to teach was fundamentally different, and therefore, there was resistance. Also, he found it difficult when restructuring the concepts of what is correct and incorrect in mathematics not only to teach the dogmatic mathematics that will be expected of the students in future studies, but also to allow for student investigation and thinking about topics that are still being argued about in the mathematics community. In this setting, constructing what mathematics *was* caused difficulties for Chazan when teaching with a focus on student-centered discourse

Lampert (1990) also studied her own and others' teaching of mathematics when that teaching was marked by efforts to engage students in mathematical discussion. Lampert found that there were many dilemmas that arose in this instructional approach. Certain decisions—such as where to stand in the classroom, how to group students, how to validate all children's learning, and the idea of what is correct and what is incorrect—posed problems to teachers in the classroom. Lampert suggested that there is no correct way to solve these problems and that teachers will need to negotiate with problems daily. These dilemmas can stump even veteran teachers; therefore, those dilemmas, and even simpler challenges, may pose more problems for novice teachers.

Ball (1993) studied her own teaching and found that negotiating a mathematical path when allowing students to discuss and investigate mathematics using discussion about different representations was difficult. In particular, planning for classroom teaching and anticipating different representations of mathematical ideas that students could use in order to convey their thinking or that she as a teacher could use, ultimately, to elicit a mathematical concept posed dilemmas for Ball. The path of students' learning was not certain, and certain changes during the course of the lesson were necessary to keep students thinking about, in her case, the concepts of positive and negative numbers. Being able to stay true to the mathematics was another dilemma that Ball identified in this type of classroom teaching.

Beyond the strictly mathematical challenges that teaching in this way can pose, other difficulties may cause problems for teachers when trying to instruct using problems and discussion. Problems with classroom management can derail what may be successful small- or whole-group discussion. Turner and colleagues (2002) stated that avoidance strategies in mathematics seem to increase as students progress from elementary to secondary school. These avoidance strategies as employed by students are often used to save face when attempting to hide competence or lack of confidence. There can be a type of unspoken contract that teachers and students enter into, one in which the students say: I will not disrupt your class if you do not ask me to think or work too hard (Powell, Farrar & Cohen, 1985). This type of contract becomes pervasive, especially in schools that serve low-income students of color, where expectations for students are often low (Habermann, 1991).

This type of disengagement of students can present problems for teachers who are trying to involve students actively in the learning process. Cooney (1985) found that a new teacher who whole-heartedly believed in problem solving as central to mathematics had difficulties when students were not receptive to his methods. Often this teacher resorted to more directed-teaching activities because students did not seem interested or engaged in his problem-solving activities. He felt that all his students wanted to do was socialize and that he had to structure the classroom strictly in order to control students' participation in mathematics activities. This presented a conflict between the teacher's belief in what mathematics was and what forms engaging with mathematical content would encompass during instruction and his resultant teaching.

Furthermore, when students were tracked in mathematics, not only may the students in the lower tracks have no intrinsic reasons to pursue mathematics, they may not have extrinsic ones either (Gregg, 1995). The external motivation associated with the benefit of earning high grades was not real or important to these students because getting "good grades in mathematics" was not congruent with their perceived futures. These students were simply convinced that they did not need or care to learn mathematics. This presented issues of control in the classroom. When there were issues of control, the teacher limited her leading questions and the opportunities that she provided for students to participate in discussion. She also limited activities with which students could engage. In particular, the teacher viewed activities that were deemed more engaging as only being useful for her honors classes. She felt she could not employ these sorts of lessons with her lower-tracked classes because it would give students more opportunity to get out of control. However, the remaining emphasis on teaching only “boring” rules and procedurally focused lessons yielded instructional sessions that were not engaging for the lower-tracked students. By focusing on control, the teacher put herself in a never-ending loop where the solution to her perceived problem may have been making the situation worse. These types of actions and beliefs are often taken-as-shared among mathematics teachers as part of the school mathematics tradition. This tradition is very difficult to change. Therefore, it is necessary to conduct more research addressing how teachers might change and improve their implementation of mathematics instruction that includes student sense making through discussion.

Rationale for Study

Due to the many challenges that new and experienced teachers face in consistently planning and implementing instruction that promotes students’ sense making through productive discussion, and the uncertainty of how teachers negotiate those challenges, it is critical to understand better how mathematics teacher education programs can support teachers in their efforts to learn how to develop these practices. The next section of this chapter discusses reasons why this research is necessary by considering the challenges that *all* teachers face when implementing this type of instruction, as well as why, in particular, middle school teachers in an alternative-certification program should be studied.

Challenges teachers face and the rationale for those challenges. For many teachers, teaching in a manner that expects students to make sense of mathematics through productive discussion does not come naturally. Sometimes, even when teachers think that they are teaching in a manner that is student centered and focused on discourse, the predominant interaction present between teachers and students still evidences an Initiation-Response-Evaluation (IRE) format (Nuthall, 2005). Furthermore, Nuthall makes the argument that teaching is a culture, and when individuals become teachers, they assimilate to the existing culture, even unintentionally. The argument could be made that teachers move away from more student-centered, discussion-based teaching styles because this type of teaching is difficult to implement, especially in an isolated environment. This shift away from student-centered teaching may be exacerbated by the norms of schooling that are pervasive in schools in urban or urbanized locales that are populated by low-income students of color (Habermann, 1991). Many researchers make the case that even when expert teachers, often the researchers themselves, attempt to implement this type of teaching that there are many challenges (Ball, 1993; Chazan, 2000; Lampert, 1990; Lubienski, 2002; Simon, 1995). Therefore, before expecting teachers to be able to teach in this manner, it may be important to ensure that teachers have adequate supports to learn how to negotiate this type of teaching.

Teaching is a demanding task. As noted by Rowan, Correnti and Miller (2002), “teaching is a form of expert work that requires extensive professional preparation, strong subject-matter knowledge, and a variety of pedagogical skills, all of which are drawn upon in the complex and dynamic environment of classrooms” (p. 1538). However, teachers often teach in the way that they were taught (e.g., Lortie, 1975). Furthermore, many people who choose to become mathematics teachers had been successful in their previous mathematics classes, frequently in classes that were taught in skill- or procedurally-focused ways. Since teachers are likely to have experienced, and been successful in participating in, this type of teaching when learning mathematics themselves, one should not presume that teachers will be able to teach in ways that highlight explanation, justification, and understanding just because they learned about the existence of these instructional approaches in a pre-service program. Mathematics educators are, of yet, unsure as how to address this particular problem in a manner that is consistently successful. Due to this uncertainty, there is a need to study efforts to support teachers’ learning of how to teach in this way.

Middle-school mathematics teachers in an alternative-certification program serving urban school districts. Middle-school mathematics teachers in alternative-certification program serving urban school districts have particular features that identify them as a population worthy of continued study. Consider the rationale for further research on alternatively-certified teachers placed in urban schools, both middle-school teachers in particular and in urban schools in general.

Alternatively certified teachers placed in urban school districts. After the No Child Left Behind (NCLB, 2001) act was passed, it became a legal requirement for schools to staff their faculties with “highly-qualified” teachers. Urban school districts have had a more difficult time than suburban school districts in recruiting and retaining qualified teachers, specifically mathematics teachers. There are many reasons for this, including, but not limited to salary, location of residence as opposed to location of school, and history of school success (Jacob, 2007). However, NCLB forced districts to employ increasing numbers of highly-qualified teachers to respond to growing student populations. This has caused urban school districts to resort to recruiting teachers from, or collaborating with, alternative-certification programs in order to fill vacant positions with qualified teachers (Jacob, 2007). These alternative certification programs are often different from traditional teacher-education programs in that teachers in alternative programs are frequently placed into classrooms as teachers of record with fewer years of education and training than traditional graduates of university teacher-training programs (New York City Teaching Fellows, 2013; Teach for America, 2013). The knowledge that these alternatively certified teachers possess, the challenges they face during teaching, and characterization of their developing instructional strategies, particularly strategies that promote student mathematical sense making through participation in productive discussion, are not addressed by either research on the experiences of pre-service teachers or the experiences of new or veteran teachers who were graduates of traditional teacher-preparation programs. While alternatively certified teachers are novices, they are not graduates of the same post-baccalaureate teacher-preparation program that research on novice teachers typically addresses. They are novice teachers with differing, and possibly more limited, pre-induction classroom experience. Therefore, additional research is needed to investigate and analyze the experiences of these novice, alternatively-certified, teachers.

Middle-school teachers. In addition, middle-school teachers of mathematics are of particular interest. Teachers become certified to teach middle-school mathematics in several ways. They may become certified through a middle-school mathematics program specifically, often in conjunction with a certification in a second middle-school subject area; they may become certified as generalist teachers, teaching all subjects K-8; or they may be certified as secondary, or grades 6-12 or 7-12, specialized-subject mathematics teachers (Tatto & Senk, 2011). Not only do the multiple paths to becoming a middle-school mathematics teacher complicate discussions about middle-grades teachers' pedagogical knowledge, it also may have implications for content knowledge. Many teachers who become middle-school mathematics teachers are required to complete substantially fewer mathematics classes as compared to traditional requirements for secondary teachers (Lutzer, Rody, Kirkman & Maxwell, 2005). This may cause additional difficulties when graduates of these programs are attempting to plan and enact instruction that promotes students' mathematical sense making through participation in productive discussion. Anticipating and responding to student thinking in a student-centered classroom demands knowledge of different content for middle-school teachers as compared to elementary-school teachers. The mathematics education community is not certain what content requirements would be advisable for middle-school teacher preparation, in part because we do not currently know enough about what is necessary to learn how to teach in a student-centered manner. Further research is necessary to characterize the development and enactment of middle-school teachers' instructional strategies promoting student sense making through participation in productive mathematics discussion.

Urban schools. As was aforementioned, there are concerns about teaching in a student-centered, or indirect, manner when the student population is largely low-income students and/or students of color (Apple, 1992; Bernstein, 1990; Delpit, 2006; Lubienski, 2000, 2002). Studies have established that when teachers carefully attend to building classroom norms and to using instructional strategies that allow their students access to rigorous and challenging mathematics, these actions result in equitable results in classrooms populated my low-income students of color as compared to schools with populations from more dominant cultural backgrounds (e.g. Boaler & Staples, 2008). However, studies such as that conducted by Boaler and Staples have accessed veteran teachers and have not indicated whether the teachers were products of alternative-certification programs rather than traditional teacher-preparation programs. Furthermore, this research has typically been conducted in high school settings. What is needed is research that investigates how novice middle-school mathematics teachers who are enrolled in alternative-certification programs develop their ability to promote students' sense making through participation in productive discussion. This research may ascertain the strategies these teachers use and the challenges they face and negotiate in order to provide rigorous, equitable, mathematics instruction to low-income students of color.

This dissertation is a study that investigates the issue of teachers learning how, and learning how to learn how, to teach in ways that highlight mathematical problem solving with student-to-student explanation and questioning. This issue is of particular importance when deciding how to support teachers' efforts to learn to teach in ways that research suggests as positively benefitting student understanding. The study has implications for the preparation of both pre- and in-service teachers of secondary mathematics.

Research Questions

In order for teachers to learn to teach in ways that include sense making and discussion as features of their teaching, which has been established as a difficult practice, more information is needed. Mathematics teacher educators need to understand the challenges that new teachers face as well as the strategies they use to negotiate these challenges, and the strategies they use in the normal course of their teaching to promote discussion and mathematical sense making. This study specifically focused on teachers in an alternative-certification program who were completing the requirements to obtain their teacher certification, and therefore were categorized as novices. To this end, this dissertation addressed the following research questions:

1) What features of and strategies for instruction do novice teachers attend to and implement with regard to promoting sense making through student participation in productive mathematical discussion?
2) What challenges do these novice teachers face in the process of promoting sense making through student participation in mathematical discussion, and how do they negotiate these challenges?

Much of the research investigating the developing knowledge and skills of pre-service or veteran teachers has addressed these individuals' learning about teaching within traditional teacher-preparation programs and/or traditional professional development models. However, it is also important to understand and address the needs of novice teachers graduating from alternative certification programs if we are to inform our methods of preparing and supporting new teachers.

Overview of Conceptual Framework

The conceptual framework of this study focuses on instructional strategies and challenges. In order to enact successful and equitable mathematics instruction that engages students in sense making through productive mathematical discussion, teachers must develop, and use, instructional strategies. The framework through which these strategies and challenges was identified and eventually analyzed is outlined in the following sections.

Instructional strategies. Teachers may use several strategies in order to scaffold students' engagement in productive sense-making mathematical discussions. Teachers may structure lessons around problematic or investigative tasks, set up these tasks in order to facilitate student access to these tasks, and require students to complete the tasks collaboratively in small groups (Boaler, 2002a, 2002b, 2006). Teachers may assert and maintain expectations, and develop norms, for students' production of explanations of mathematical solutions strategies in small- and whole-group discussions, well as require students' active engagement with making sense of, responding to, and attempting to understand student explanations and justifications (Boaler & Staples, 2008; Wood, 1999; Yackel, 2001, Yackel & Cobb, 1996). This means that students must be able to answer questions such as "how" and "why" (Bowers, Cobb, & McClain, 1999; McClain & Cobb, 2001; Yackel, 2001; Yackel & Cobb, 1996), and that teachers may ask probing, leading, or advancing questions to encourage students to develop these answers. Having students pose questions to others is another important component of sense making through discussion (Borasi, 1992; Ciardello, 1998; Zack & Graves, 2001).

In order to promote this behavior, teachers may direct student questions to other students, asking students to revoice a students' explanation or to pose a question if they cannot revoice. In order to structure discussions that promote sense making, teachers may also carefully choose solutions to be presented so that a discussion of reasonableness or correctness of those solutions may result (Smith & Stein, 2011). Teachers may use instructional strategies of their own design, or use strategies that are presented to them during teacher preparation courses or through teacher-support systems, such as teacher mentoring.

Challenges. Teachers may face challenges in designing instruction that promotes mathematical sense making through student participation in productive mathematical discussion (Au, 2007; Ball, 1993; Chazan, 2000; Cooney, 1985; Darling-Hammong & Rustique-Forrester, 2005; Gregg, 1995; Hoy & Spero, 2005; Lampert, 1990; Lubienski, 2002; Oakes, 2005; Powell, Farrar & Cohen, 1985; Simon, 1995; Turner et al., 2002). These challenges may include: time constraints due to curriculum and testing requirements; norms of schooling including student perceptions of mathematical authority, students' desire or reluctance to discuss, teachers' perceptions of student ability, and teachers' desire or reluctance to allow students to struggle; issues of classroom management; lack of a supportive school context; or issues of efficacy and self-efficacy. These challenges may inhibit teachers' effectiveness when attempting to enact instruction that incorporates features emphasizing productive mathematical discussion. Due to these challenges, teachers may develop, adapt, and implement instructional strategies that negotiate or mitigate these challenges. They may implement these strategies as drawn from their own design, or in conjunction with strategies evolving from teacher-preparation courses or other avenues of school-based support, such as mentoring sessions or on-site observation of other teachers.

Overview of Research Design and Study

This study took place within an alternative-certification program. This alternative-certification program was a post-baccalaureate program that recruited candidates with mathematics and science, rather than education, backgrounds who had a commitment to urban school populations and/or the specific community with which the alternative-certification program partnered. The teachers in this program enrolled in teacher-preparation courses during the summer before their initial placement as a half-time teacher of record. They continued courses during their first year of placement. They were also provided with a university-based mentor. For the purposes of this study, I was the mentor as a participant observer, and some of the participants with whom I worked were the sources for subsequent case studies. One of my mentees was removed from this study due to her failure to successfully complete the alternative-certification program. I engaged these teachers in a reflective-teaching cycle (Smith, 2001) that included planning sessions, classroom observations, and reflections, as well as a teacher seminar in which all teachers participating in my study engaged. The intention was that through this process and through active reflection, the participating teachers would begin to learn to learn to teach in a manner that engaged students in sense-making discussions with their peers as well as with their teachers.

Through this process, I studied the strategies, challenges, and negotiation of these challenges that these novice teachers developed, experienced, and enacted during their first year of teaching. I studied these qualitatively by analyzing the discourse of the participating teachers during planning- and reflection-based mentoring sessions and teacher seminars and by analyzing their talk and actions during actual classroom implementation of instructional strategies. These analyses addressed both their efforts to implement instructional strategies that they believed would promote student sense making and engagement in productive mathematical discussion and their efforts to negotiate the challenges that they experienced while attempting to promote these instructional, classroom-based discussions.

Planning-focused mentoring sessions allowed me to access the considerations that teachers verbally offered or made evident while being encouraged to develop student-centered instruction. Reflection sessions provided opportunities for teachers to talk about the challenges that presented themselves during implementation and allowed space for teachers to reason through strategies that could negotiate these challenges. The teacher seminars allowed the participating teachers time to collaborate with me as they planned for and interpreted the occurrences that arose during instruction.

I coded and analyzed both teacher discourse and action through the lenses of strategies and challenges, as described above in the conceptual framework. Through this analytic process, I created a case study for each participating novice teacher that described and analyzed the development of that teacher's design and implementation of instruction that promoted sense making through productive student discussion. These cases also highlighted the instructional strategies that these novice teachers used, the challenges that they faced, and how they negotiated those challenges in order to learn. Looking across cases provided data for analysis that permitted highlighting of the relevant features of this development, providing further insights in terms of the professional advancement of alternatively-certified middle-school mathematics teachers.

Chapter 2: Literature Review and Conceptual Framework

This review highlights major contributions in the literature that are theoretical and practical predecessors to this study. First, I discuss the equitability of discussion-based education for minority populations. Second, I explicate the features of discussion-based pedagogy that are relevant to this study. Third, I present one conception of how teachers learn to learn to teach while focusing on promoting productive mathematical discussion that result in sense making. Finally, I outline the conceptual framework for this study, referencing key literature.

Discussion-based Education in Low-Income Populations of Color

A review of the literature addressing cases of successful and equitable mathematics education for traditionally underrepresented students identified three major themes. The first theme is the presence of either a dedicated teacher or a dedicated activist force assuming responsibility for and advancing the implemented practices. The second theme builds off of the proposition that student identity and students' mathematical dispositions are critical to mathematical learning as research holds that the mathematics curriculum, tasks, and problems used during instruction should align with and support students' mathematical dispositions. The third theme is that classrooms marked by the presence of instructional practice(s) advancing equitable mathematical education involved teachers taking the time to acclimate and acculturate students into the social norms and sociomathematical norms of the intended classroom and community, as well as to the nature and conduct of mathematical discussion. The third theme, and to some extent the second, are particularly salient to this study.

Developing Norms for the Purpose of Equity

Some researchers have claimed that classrooms that focus on problem solving, communication in groups, and more indirect pedagogical features may privilege students from the dominant culture (Apple, 1992; Bernstein, 1990; Delpit, 2006; Lubienski, 2000, 2002). However, other researchers have found that classrooms built around group work, discussion, and sense making can be equitable and beneficial to low-income students of color if necessary supporting norms are carefully developed during implementation (Boaler, 1998, 2002a, 2002b; Boaler & Staples, 2008; Gutierrez, 2000; Kitchen, DePree, Celedon-Pattichis, & Brinkerhoff, 2007; Moses & Cobb, 2001; Martin, 2000).

In settings where students engaged in the work of making sense of mathematics through participation in productive discussions and where their classrooms were also found to be equitable and beneficial, qualitative research reports offered insights as to key classroom norms for instruction privileging mathematical discussion. For example, frequently students find it difficult to interpret meaning and to define goals within the ambiguous contexts that frequently characterize open-ended or open-entry, authentic or applied mathematics problems. However, when teachers use questioning to orchestrate a developmental discussion with the students, prior to expecting the students to solve the problem, students learn how to approach and analyze these sorts of problems (Boaler 2002a, 2002b, Kitchen, et al., 2007). The critical aspect to scaffolding these developmental discussions is modeling the different types of questions that students should ask themselves when they are navigating new problem contexts: “What is it [the problem] asking? How could we rephrase this question? What are the key parts of the problem?” (Boaler, 2006, p. 367). After persistent modeling of these questions by a teacher, the students begin to ask themselves these questions without prompting. This research suggests that teachers should provide many examples of what "counts as good mathematics" work, so that students may clearly understand what is expected of them. Through their pedagogical strategies of group examinations and student-questioning practices, teachers painstakingly develop the norms of collective responsibility for understanding.

However, this is not to say that implementing instruction that focuses on student sense making through productive discussion, even with the aforementioned suggestions from literature, is easy or automatic. The teachers who carefully developed norms and achieved equitable results were often experienced teachers within supportive school or department contexts. Indeed, there are teachers who have not been successful when attempting to implement student-centered mathematics instruction. However, a critical feature present in classrooms where this form of mathematics instruction is productively applied is an emphasis on norms for classroom activity and discussion, norms that may be transferrable to differing mathematical settings. For example, when using the curriculum *Mathematics in Context*, Gutstein (2003) developed the norms of questioning, of explanation, and of justification and then used these same expectations and strategies when incorporating social justice projects into the school mathematics curriculum.

It is not known if it is the presence and specificity of established norms that are essential to subsequent productive, student-generated discussion or if norms may be a vehicle for engaging *all* students in indirect, student-centered teaching focusing on discussion. One study that attempted to engage students in student-centered discussions in mathematics classrooms with a diverse socioeconomic (SES) population found that higher-SES students seemed to benefit more from this type of instruction. However, the teacher-researcher in this study revealed that she had assumed that students understood that her role was one of facilitating discussion instead of developing the norms and shared understanding of what participation in mathematical discussion meant (Lubienski, 2000). Similar assumptions were characteristic of some classrooms in the Algebra Project, a curricular and instructional project that had successes with equitably engaging low-income students of color in mathematical sense making (Moses & Cobb, 2001). Some teachers in the Algebra Project felt that developing the norms associated with scaffolding sense-making discussion was not a priority to their teaching because they found taking the time to develop the norms distracted both teachers and students from attending to the mathematics curriculum. However, these teachers were less successful in engaging students in sense-making discussions (Martin, 2000). This suggests that teachers' development of social and sociomathematical norms may directly affect their success in engaging their students in productive mathematics discussions.

Furthermore, addressing the needs of low-achieving students or students with disabilities related to mathematics is a concern for equity. Research has shown that when low-achievers and students with mathematical disabilities are engaged in mathematics instruction that has student-centered sense making and productive mathematical discussion as a feature of instruction, these students sometimes provide marginal or confusing explanations, and are not fully engaged in the small-group work that leads to a productive whole-group discussion. Other times these students are silent during the verbal work of sense making (Baxter, Woodward & Olson, 2001; Baxter, Woodward, Wong & Voorhies, 2002). However, these researchers do not advocate removing these low-achieving students and students with mathematical disabilities to a different classroom for intensive instruction, as they may not be exposed to the work of sense making and verbal argumentation in these settings. Instead, they advocate for careful social scaffolding in whole-group settings, dedicated individual scaffolding in small groups, and greater opportunities to participate in the work of sense making and argumentation in mathematics.

Classrooms in which norms of productive mathematical discussion were not carefully developed or made explicit yielded uneven results with regards to engagement of students and equitable results (Lubienski, 2000; Martin, 2000). Other studies demonstrate that the careful development of norms is an important feature of discussion-based mathematics classes that yielded equitable results (Boaler, 1998, 2002a, 2002b; Boaler & Staples, 2008; Gutierrez, 2000; Kitchen, DePree, Celedon-Pattichis, & Brinkerhoff, 2007; Moses & Cobb, 2001; Martin, 2000). However, in these latter studies, the teachers were often experienced teachers in supportive contexts. Teachers often collaborated and supported each other throughout an extended period of time to develop practices and norms that would support equitable student learning. Furthermore, research into the results of student-centered mathematics for low-achievers and students with mathematical disabilities suggests a need for greater exposure and scaffolding in order to equitable engage these students in productive mathematical discussion and sense making. Therefore, more research is needed in order to determine how novice teachers might learn to learn how to develop norms for discussion-based mathematics classes, and scaffolding for low-achievers, to support their students, and to attend to equity.

Features of Norms of Student-to-Student Discussion

What instructional strategies might contribute to communicating and developing classroom norms for productive student-to-student discussion that promotes mathematical sense making? A synthesis of this literature identifies both social norms and sociomathematical norms, characterizing features of productive mathematical discussion as carried out by students. These include expectations for explanation and justification, revoicing, and questioning. A final feature, herein referred to as "challenging," refers to participants in a discussion challenging an explanation or justification as being insufficient or invalid.

Social norms and sociomathematical norms. At the start of the school year or at the beginning of a new course, social norms for behavior will quickly become established. Ideally, teachers and students take the time to develop and establish social norms. In classes where what is and what will be expected of students are quite different from students' prior practices, it is arguably more important to develop social norms carefully. But before one can develop norms of productive mathematical discussion in a middle-grades mathematics class, one must know what types of norms are intended and what they look like in practice, as well as how to go about developing them. The following section addresses the definitions of social norms and sociomathematical norms, as well as the difficulties that may arise when developing norms of productive mathematical discussion.

Social norms. Norms are present in every classroom regardless of the tradition of the classroom (Yackel & Cobb, 1996). In order to develop a student-centered classroom culture that promotes sense making, it is important that the structuring of classroom norms advance or support this approach to mathematics teaching (Lampert, 1990). In mathematics classrooms, both social and mathematical norms are necessary in order to structure interactions as well as content (Cobb, 1994). Research suggests that if the norms of the classroom are such that students collaborate and communicate, natural opportunities for them to explain, justify, and ask clarifying questions will arise (Yackel, Cobb & Wood, 1991). This implies a norm of cooperation is present in the classroom. When students collaborate and communicate in classrooms, a focus on agreement on the veracity of solution strategies or ideas breeds opportunities for explanation, justification, questioning and challenging (Yackel, Cobb & Wood, 1991). Listening, especially active listening, is an important norm to establish in a community-based classroom (Wood, 1999).

Sociomathematical norms. However, to make these normative practices specific to mathematics, sociomathematical norms must be included (Yackel & Cobb, 1996). Sociomathematical norms can be emergent and taken-as-shared when coupled with the development of relevant social norms (Cobb, 1999; Yackel, 2001; Yackel & Cobb, 1996). Use of the social norm of questioning can inspire the development of the sociomathematical norm of what counts as an acceptable and clear explanation (Bowers, Cobb & McClain, 1999). Similarly, in order to inspire social norms of questioning and of challenging in mathematics classrooms, what counts as mathematically different must be established in order to motivate questions and challenges about divergent responses (Horn, 2005). When students explain, justify and generalize, students develop understandings of what they are communicating, which is important for individual and social knowledge development (Horn, 2005). Developing these norms is important for student achievement, especially in classrooms serving low-income students of color

(Boaler, 2006; Boaler & Staples, 2008; Horn, 2005; Martin, 2000). Students who are taught mathematics in classrooms that are based on collaborative mathematical discussion may be very successful academically, however reported exemplars of this instructional approach are marked by a "unique" mathematics department focused tirelessly on building the norms of classrooms where every student could have access (Boaler & Staples, 2008). Therefore, teacher implementation of normative development in classrooms is arguably very important.

Discussion-centered norms. Classrooms oriented towards sense making through productive mathematical discussion often are very different from what students have come to expect from schooling, and therefore care has to be taken to renegotiate norms for these classrooms and to build a new type of classroom culture (Cobb & Yackel, 1996). However, these new classroom norms are consistent with calls for with attention to mathematical disposition (Bowers, Cobb & McClain, 1999; Cobb & Yackel, 1996, National Research Council, 2001). Social norms contribute to the development of social autonomy and sociomathematical norms contribute to the development of intellectual autonomy (Cobb, 1999; McClain & Cobb, 2001; Yackel & Cobb, 1996). Students play a big role in the structuring of norms since oftentimes their beliefs, specifically their beliefs about mathematics and mathematics learning, have to be reorganized (Bowers, Cobb & McClain, 1999; Cobb & Yackel, 1996). However, the role of the teacher is equally important (Cobb, Wood, Yackel & McNeal, 1992; Lampert, Rittenhouse & Crumbaugh, 1996; Nathan, Eiliam & Kim, 2007; Yackel & Cobb, 1996).

Since developing social and sociomathematical norms is so critical to the development of a classroom culture oriented towards discussion, it is important to investigate how teachers learn to learn to develop these norms. Most of the research that focuses on the development of social and sociomathematical norms has been conducted in elementary school classrooms, with "normative" populations of students, where whole-group interactions in classrooms are a focus. Since the types of norms that support a discussion-oriented classroom are distinct from many students' prior experiences in schooling and in mathematics instruction, developing these norms later in the students' educational and mathematical career may be more difficult than doing so earlier. Bourdieu (1977) argues that early experiences have the most weight in shaping a person's tacit beliefs, dispositions, and habits. Furthermore, it may be easier for a teacher to model and scaffold norms in whole-group situations because then the teacher may have access to all student interaction at once and can highlight types of privileged types of student contributions in front of the whole class. Yet, recent research has highlighted the importance of developing small-group norms of mathematical activity in secondary classrooms that serve low-income students of color in order to allow diverse students access to mathematical success and persistence (Boaler, 2006; Boaler & Staples, 2008; Horn, 2005).

Explanation. Explanation is a mathematical activity that uses mathematical objects, concepts, procedures, and actions in the description of ideas or solution strategies that answer questions such as, "How?" and "Why?" with regard to those ideas and solutions (Bowers, Cobb, & McClain, 1999; McClain & Cobb, 2001; Yackel, 2001; Yackel & Cobb, 1996). Theorists suggest that participation in both producing and engaging with others' mathematical explanation is an important component of making sense of mathematics. Empirical research suggests that students who participate in this kind of activity have higher achievement (Boaler, 2002a, 2002b; Horn, 2005; Schoenfeld, 2002). However, it is important for the educational community not only to know what forms student explanation may assume in the classroom, but also how teachers can learn to develop norms in the classroom so that students constantly engage in these behaviors.

Revoicing. Revoicing can be defined as a practice in which a speaker rephrases, summarizes, elaborates, or translates what someone else has said in a way that can be evaluated by all participants in a discourse community (Foreman & Ansell, 2002). Furthermore, it can be used to clarify, explicate, or provide support to a speaker's utterance. Revoicing has generally been studied as a way for teachers to facilitate whole class discussions (O'Conner & Michaels, 1993). However, the concept of revoicing can be extended with student-to-student implications. O'Conner and Michaels (1993) suggest that teachers can use revoicing as a tool to socialize students from diverse backgrounds into academic roles and identities by coordinating academics, roles, and responsibilities, which can create opportunities for student learning. These researchers argue that revoicing allows teachers to link student experiences and knowledge to the practices of the wider disciplinary world of mathematics. Through revoicing, the speaker may also verify or reject inferences as made or offered by others.

Revoicing can be used to give status or power to the original speaker and their ideas (O'Connor & Michaels, 1993). Revoicing can also be used to position speakers in opposition, which can promote active reflection and can change the way claims are proposed, justified, and contested, as well as creating opportunity for intellectual authority (Foreman & Ansell, 2002; O'Connor & Michaels, 1993). Wood (1999) contends that teachers' development of normative expectations marked by each student listening to others is important in developing discursive mathematical classrooms. This can be done by encouraging student revoicing of others (Huffard-Ackles, Fuson & Sherin, 2004). Teachers can also ask students to revoice explanations to ensure understanding (Cobb, 1999). Students' own sense making can also benefit from participating in revoicing of ideas. Students can assign legitimacy and give authority to statements offered by other students or offered by the teacher when using revoicing in the same manner that teachers do (Forman & Ansell, 2002).

Bakhtin (1981) states that "the word in language is half someone else's. It becomes one's own only when the speaker populates it with his own intention, his own accent, when he appropriates the word adapting it to his own semantic and expressive intention" (p. 293-294). The road to understanding for students, as opposed to repetition of authoritative ideas without sense making, is supported and developed through appropriating others' ideas and intertwining those voices with one's own (Forman, McCormick & Donato, 1998; Bakhtin, 1981; Wertsch, 1998). This research suggests the importance of designing a classroom environment where knowledge and discussion can be "genuinely appropriated by students, not just mastered and ventriloquated by them" (Forman, McCormick & Donato, 1998, p. 333).

Questioning. Questioning is an act wherein a person asks a question in order to clarify that person's understanding. Questioning serves a powerful role in making sense of mathematics. Borasi (1992) states that "the essential component to critical thinking is to pose questions and evaluate their worthiness" (p. 202, as cited in Zack & Graves, 2001). Questioning requires students to make connections, to develop internal cognitive processes, and to act metacognitively (Ciardello, 1998). Furthermore, the sociomathematical norm of a valid explanation can be developed through student questioning (Bowers, Cobb & McClain, 1999). When students convey a lack of understanding by posing questions, a more clear definition of what counts as an explanation can be developed. Students may ask questions when the justification of a mathematical idea is not taken-as-shared, and the subsequent responses to those questions can build up a community ideas of what is taken-as-shared (Cobb, 1999). For example, Zack and Graves (2001) found that students asked questions to verify their own understanding. According to a hierarchical framework characterizing levels of a math-talk community, communities where students ask questions of each other is a more developed math-talk community (Huffard-Ackles, Fuson & Sherin, 2004).

However, classroom environments that involve questioning others' thinking require a major shift in cultural norms (Lampert, Rittenhouse & Crumbaugh, 1996). Lampert, Rittenhouse and Crumbaugh (1996) suggest that there are strong folk norms against disagreement because most people will avoid disagreement as it may serve as a source of conflict. However, cognitive conflict in the classroom is a powerful tool for learning (Nathan, Eiliam & Kim, 2007). A sociomathematical norm of what counts as mathematically different and norms of valuing this difference must be developed in concert with norms of questioning and challenging to reconcile this difference (Bowers, Cobb & McClain, 1999). Theoretical perspectives imply that questioning and challenging may be fundamental building blocks of learning, and these components are mentioned throughout the research literature as key elements of mathematical practice. Therefore, it may be beneficial for students to participate in these practices in their mathematics classrooms in order to increase their mathematical content knowledge and to deepen their achievement.

Challenging. Lampert (1990) suggests that the practice of challenging, especially by counterexample, leads to the conception that mathematical truth is established by providing evidence and making logical arguments, which may help students become more comfortable with participating in these very mathematical activities. Justifying and challenging are both equal parts of mathematical argumentation and knowledge building (Foreman, et al, 1998; Lakatos, 1976; Lampert, 1990), therefore simply teaching students to justify their response without similarly teaching them to challenge others' ideas may be insufficient. Similarly to Lampert, Wood (1999) suggests that challenging ideas and the subsequent discussion leading to a resolution in a classroom is a "precursor to the development of mathematical argumentation" (p. 189). Also, according to Wood, cognitive conflict is a way to transform through and to assist in knowledge construction. Nathan (2007) also suggests that disagreement is fundamentally important to the development of thinking and knowing. It is important for students to see mistakes as valid starting points for learning, instead of something undesirable (Lakatos, 1976; Lampert, 1990; Lampert, Rittenhouse & Crumbaugh, 1996; Yackel, Cobb & Wood, 1991). This implies that the act of challenging should be an integral part of any mathematics students' learning experience.

However, as noted in the discussion of questioning, classroom environments that involve challenging others' thinking requires a major shift in cultural norms because the strong folk norms against disagreement cause most people to avoid challenging as a source of conflict (Lampert, Rittenhouse & Crumbaugh, 1996). However, conflict in the classroom is a powerful tool for learning (Nathan, Eiliam & Kim, 2007). Just as sociomathematical norm of what counts as mathematically different and norms of valuing this difference must be developed in concert with norms of questioning, these norms must also be established if challenging is to be reconciled as appropriate behavior in a classroom (Bowers, Cobb & McClain, 1999).

These theoretical perspectives imply that questioning and challenging may be fundamental building blocks of learning. They are mentioned throughout the research as key elements of mathematical practice. Therefore, it may be beneficial to students to participate in these practices in their mathematics classrooms in order to increase their mathematical content knowledge and achievement. If it is an important component of a students' mathematical experience, it should also be considered important to investigate how to build those types of norms in a classroom. The studies by King (1992, 1994) articulate how questioning was developed in the science classroom; however, it is not known how this type of implementation translates to mathematics education. Also, the studies by King do not address the development of the norm of challenging. This may be considered as a higher priority than studying the effects of questioning and challenging on achievement, because it is important to see how to enact these types of norms in a classroom before studying the effects of such practices.

Throughout mathematics education literature, explanation, justification, revoicing, questioning and challenging are featured as practices that advance student learning in mathematics. However, in order to develop these practices as social norms within a classroom, as well as elicit and develop the subsequent and related sociomathematical norms, it is important to ask how these social and sociomathematical norms are developed in classrooms. Particularly, it is of interest to consider how novice teachers can learn to develop these norms and practices within their mathematics classrooms, for the benefit of their students' mathematical understandings.

Instructional Challenges

Many researchers have studied teaching with a focus on student-centered discussion. Chazan (2000) found that since this type of mathematics instruction that he was consciously trying to develop was fundamentally different from that which students were familiar, he was met with resistance. Lampert (1985) also found that there many instructional challenges that arise when teaching in a student-centered manner and these included decisions such as where to stand, how to validate students' utterances, and what is correct and incorrect. Lampert herself was a veteran teacher and was still troubled by these "dilemmas," which she suggested had no correct response. Ball (1993) found that planning different representations with which students would interact in order to elucidate a mathematical concept was difficult due to the uncertainty of the future path of students' learning and to the need to stay true to mathematical content.

Several researchers argue that as students progress from elementary to middle school, they may disengage from participation in mathematical discussion (Turner, et al., 2002). In order to avoid this direct resistance, teachers, specifically teachers of students from low-income backgrounds, may decide not to press students to engage in productive mathematical discussion where the students would be expected to make sense autonomously (Haberman, 1991; Powell, Farrar & Cohen, 1985). However this lack of student buy-in, which may have been reinforced by instructional methods and social contracts, can be challenging for teachers who are attempting to engage their students in productive discussion (Cooney, 1985). Further, this lack of student buy-in could be exacerbated by tracking (Gregg, 1995).

Learning how to engage students in sense making through participation in productive discussion is difficult, even for veteran teachers. Veteran teachers are able to draw from a wealth of experience in order to negotiate instructional challenges, although it is not clear that there is any “right way” to address these. This draws more attention to the need for research on how novice teachers may learn to negotiate instructional challenges, so they may not only persist in developing normative practices, but also have successful outcomes with their students.

Learning to Learn to Teach

Presuming that the teacher’s role in structuring a successful classroom that focuses on student discussion and problem solving is critically important, it stands to reason that the preparation of these teachers is also critically important. This may be especially true because of the difficulties that teachers have expressed in trying to teach in this manner. According to Hiebert, Morris, and Glass (2003), mathematics educators and researchers are always interested in how to best prepare teachers in ways that actually have practical results in actual classrooms. But it is very difficult to change the practice of teachers, whether novice or experienced.

Although experienced teachers have spent more time in classrooms teaching in their specific ways, even new teachers come to the classrooms with pre-conceived notions of how they should behave and enact instruction in their particular subject. Lortie (1975) argued that these pre-conceived notions about teaching are a result of the apprenticeship of observation. By this Lortie meant that those who go into teaching may feel as if they know what it means to teach because they have “apprenticed” to this approach to teaching as a student throughout their years in K-12 education and beyond. These beliefs about teaching are hard to change. Most of the time, the observed style of mathematics teaching was a traditional approach to mathematics teaching, centered on lecture and “delivery” of knowledge. This may lead teachers to feel the most comfortable with the concept of “teaching as telling” (Smith, 1996). This appreticeship of observaton can be hard to overcome, therefore teacher professional development must approach teacher learning with the same philosophies that are used when approaching student learning. Teachers need to be put into positions where they experience cognitive dissonance with respect to their current beliefs about teaching if they are to change their practice (Thompson & Zeuli, 1999).

Many studies have investigated the professional development of teachers. Throughout the literature, there are some frequently cited points highlighted in many studies as important features of effective professional development. For example as synthesized from self-report surveys and interviews of teachers, the most promising professional development programs affecting teacher learning and change are those that included the following design characteristics: a focus on content knowledge, opportunities for active learning, coherence with other learning activities, collaboration of teachers who are teaching the same content, and duration of activity (Garet, Porter, Desimone, Birman, & Yoon, 2001). Other studies have concurred, emphasizing direct connection to teachers’ everyday practice (Wayne, Yoon, Zhu, Cronen, & Garet, 2008; Stein, Smith, & Silver, 1999; Heck, Banilower, Weiss, & Rosenberg, 2008). It may be that the richest environments for learning to teach are classrooms (Ball & Cohen, 1999;

Hiebert, Morris & Glass, 2003). It also has been found that focusing on educational artifacts such as student work (Little, 1993) and case studies (Sowder, 2007) are effective strategies for promoting teacher change. Structures such as lesson studies not only allow in-service teachers to conduct case studies, but also allow them to develop a professional community within their school (Hiebert, Morris, & Glass, 2003).

Hiebert, Morris and Glass (2003) argued that it is unlikely that new teachers will gain the "knowledge, competencies, and dispositions that [they] need to become expert mathematics teachers" within a preparation program alone (p. 202). They, instead, argued that prospective teachers need to learn how to learn to teach from reflecting on their own emerging practice rather than expecting to graduate from a preparation program with completely effective competnecies of teaching. Furthermore, they argued, this learning should not take place in a vacuum. In order to professionalize teaching, according to Heibert, Morris and Glass, teachers need to work together to build a collective knowledge base for practice, similar to those held by other professional disciplines. To this end, it can be inferred that the argument is for teachers to work together to learn to learn to teach in order to build this professional knowledge base within a community. Learning to learn to teach means "knowing how to learn from classroom teaching experiences. It means planning these experiences in a way that affords learning and then reflecting on the outcomes in order to maximize the benefits that can be gained from the experiences (Artzt, 1999)" (Hiebert, Morris & Glass, 2003, p. 206). These researchers argue that "the complexity of teaching and the difficulty of mastering all aspects of effective teaching, especially as defined by the new and ambitious learning goal of mathematical proficiency, nearly ensure that prospective teachers cannot become experts, or even accomplished novices, during a relatively brief program" (p. 204).

Professional communities of teachers, sometimes involving other players, such as administrators, facilitators and specicailists, are contexts that provide opportunities for teacher learning (e.g. Campbell, 2009; Cobb, et al., 2003; Cobb, et al., 2009; Grossman, Wineburg & Woolworth, 2001; Kazemi & Franke, 2004; Little, 2002). Although teachers are often considered and profess to be lifelong learners, the structure of the American K-12 school system does not allow many opportunities for teachers to learn within their school day. Therefore all learning that teachers participate in is conducted in their "free" time (Grossman, Wineburg & Woolworth, 2001). However, in professional communities, teachers come together and develop shared purpose or enterprise, shared repertoire of ways of reasoning with tools and artifacts, and norms of mutual engagement (Wenger, 1998) that have a direct impact on their daily practice.

Horn (2005) found that teachers who interacted in successful collegial teacher groups learned how to work together to interpret the artifacts of their own teaching as a group. The teachers' talk in groups helped the teachers address practical problems while working through their own assumptions about students, mathematics, and pedagogy. Teacher learning communities, or seminars, can be a means to build a community, to develop a theory-practice connection, to build a knowledge base about the change, and to provide an interesting context for adult learning (Short, Giorgis & Pritchard, 1993). Professional learning communities may be useful when teachers are attempting to implement instructional change as these address the concerns that teachers express, as communities destroy the teacher isolation that will hinder progress and also address teachers' need for more collaborative work and time to reflect on their practice (Short, Giorgis & Pritchard, 1993). There is an increasing foundation of research that indicates when teachers collaborate and work collegially there are positive results on student achievement (Louis & Marks, 1998; Mitchell, 1989; Rosenholtz, 1989; Stoll, et al., 2006). Professional learning communities can provide a space and an environment for teacher learning about their own practice.

However, it is important for teachers to focus on pedagogical moves and solutions within their pedagogical community, instead of on surface features of children's behaviors (Nickerson & Moriarty, 2005). Specifically in places where teachers are striving to build classroom environments where students' learn collaboratively, it is suggested that teachers benefit from opportunities to learn collaboratively. Teachers, just like students, have different opportunities to learn when their learning environment is structured collaboratively. In addition, in places where there were positive professional teacher communities, improved achievement was most marked for schools enrolling students from areas that were considered "disadvantaged" (Lee & Smith, 1996). Instructional reforms, not only locally within the United States but also internationally, hinge on having both individual and collective capacities for the factors that produce school change. Professional learning communities are sites that encourage both collective and individual learning and contribute to sustainable improvement of schools (Stoll et al., 2006).

Teacher communities may be associated with additional benefits. For example, it has been shown that teacher collaboration supports the retention of teachers in urban schools (Bloland & Selby, 1980; Popkewitz & Myrdal, 1991; Yee, 1990). Teachers' relationships with other teachers, and their relationship with the curriculum, are two factors that affect teacher retention (Shann, 1998). However, it is important to remember that “communities of practice should not be romanticized; they can reproduce counter-productive patterns, injustices, racism, sexism, and abuses of all kinds” (Wenger, 1998, p. 132). Therefore it is important to address the focus and conduct of these professional learning communities carefully.

Hiebert, Morris and Glass (2003) suggest that teachers should collaborate and focus their learning by treating the lessons that they develop as miniature design experiments. In this approach, teachers focus on building opportunities for their students’ learning and then reflect on those students’ learning in their professional learning communities. Design experiments reflect a research methodology that is used by researchers in differing fields. Simon (1995) used a semblance of a design experiment when teaching pre-service teachers in order to better structure their learning experience in a constructivist way. Cobb, Zhao and Dean (2009) used a design experiment to study teachers’ actions within the university classroom as well as teachers’ and students’ resultant actions within that classroom in order to struture and modify instruction. Other researchers have used the concept of a design experiment in order to focus carefully and reflectively on their own teaching practice as they conducted teacher-research studies (e.g., Lampert, 1990). However, Hiebert et al. (2003) suggests that teachers themselves, within a teacher community, should conduct these design experiments and then collaboratively discuss their findings so they can plan future lessons based on what they know and are learning about their students’ learning.

Design research accomplishes the goals inherent within the pespective of teachers learning to learn to teach. The first goal is learning how to learn from lessons as mini-experiments. Design experiments allow for those researchers conducting those experiments to analyze and modify their approaches reciprocally. Design experiments start with thought experiments where experimenters think through possible trajectories of what may happen in their setting, which is, in this case, the mathematics classroom (Cobb, et al., 2001, Gravemeijer, 1994). In these thought experiments, teacher planning is undertaken as an intentional process that includes not only activities but potential teacher moves. This process turns what usually are spontaneous decisions made as a reaction to student output to intentional, planned, and well-thought-out plans of how to react to potential student output (Hiebert, Morris & Glass, 2003). Thse designs are very specific and are able to be evaluated. These evaluations then produce information for both educational theory and practice (Gravemeijer, 1994).

The second goal of learning to learn to teach is the development of a knowledge base for the profession of teaching. Hiebert and others (2003) argue that while many professions have a professional knowledge base and that the practitioners within these professions actively contribute to that knowledge base, teachers are often unable to do so. Collaborative teacher-led design experiments meet this goal because design experiments can be a useful way to develop generalizable theories (Edelson, 2002). Cobb (2001) suggests that design experiments incorporate four steps that test hypothesized theories: developing a theory, derivation of principles of design from theory, the translation of principles into concrete designs, and the assessment of those designs. This formulation of theory through design experiments could be an individually assumed responsibility of a single teacher and that teacher would be able to learn from it. However, the learning to learn to teach model that is suggested by Hiebert, Morris and Glass (2003) posits that individual teacher learning is not sufficient; instead, the learning of teachers must become part of and contribute to a profession. This happens when teachers interact collegially in a community. These researchers argued that “[w]orking with colleagues ensures that the learning goals, lesson designs, and data interpretation become explicit and public so they are accessible to others” (p. 212). This is important because it is not enough simply to treat a lesson as an experiment. Everything involved must be explicit and transparent so that teachers are able to evaluate their work and learn from it.

Usually, traditional teacher preparation programs are only able to engage prosective teachers in the work of reflecting and learning from their teaching during the prospective teachers’ field experience. In this field experience, the prospective teacher is often not responsible for all aspects of the daily work of teaching. After the field experience is completed and the teacher successfully completes the program, the teacher finds a placement often disconnected from any support. However, it has been argued that support is important to helping novice teachers learn to learn from their teaching. Also, collegial support is important to teacher development. Therefore, this study provides this support. The teachers in this study were provided, through an alternative-certification program, a mentor who guided their participation in a reflective teaching cycle (Smith, 2001). Through this cycle, the teachers were supported in learning to learn from their teaching. Furthermore, an aspect of collegiality was provided through an addition of a teacher seminar through this study. In this seminar, the teachers were able to learn *with* other teachers, with the support of their mentor.

Conceptual Framework

This study investigated teachers' discussion of strategies and challenges with regard to developing classroom norms that promoted students' productive sense making through discussion. This study's conceptual framework defines productive mathematical discussion and sense making in order to investigate use of instructional strategies within classrooms with emerging norms, the challenges that arise while planning and implementing these strategies, and how teachers negotiate those challenges through use of new strategies or modification of existing strategies.

Productive Mathematical Discussion. Productive mathematical discussion is student-centered discussion that "support[s] student learning of mathematics by helping students learn how to communicate their ideas, making students' thinking public so it can be guided in mathematically sound directions, and encouraging students to evaluate their own and each other's mathematical ideas" (Smith & Stein, 2011, p. 1). Features of productive discussions, for the purpose of this study, include explaining and justifying ones' ideas, revoicing others' ideas in order to verify and internalize them, questioning others' ideas for clarity, and challenging others ideas when there is a disagreement.

Mathematical Sense Making. Mathematical sense making is defined as "(a) developing a mathematical point of view — valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure" (Schoenfeld, 1994). Recent policy documents support the development of instruction that promotes mathematical sense making (Common Core State Standards for Mathematics, 2013).

Instructional Strategies. In classrooms where norms of explanation and justification have been successfully supported and maintained, teachers' instructional strategies have been highlighted as being particularly important in the development, support, and maintenance of these norms (Boaler & Staples, 2008). Developing a task that allows for multiple entry points and for multiple solution strategies provides students multiple avenues for explaining their work to other students and the teacher. Carefully setting up the task by having students ask themselves and others key questions, such as "What is the question asking?" allows students access to solve problematic tasks. When teachers ask their students questions that require them to explain their work in detail, it introduces the expectation that student work will be explained. Requiring a single student in a group, who is not identified in advance, to explain the work of the group to the teacher conveys the expectation of students clarifying their own understanding by asking their group members questions until each has an explanation that is well developed and suitable for the teacher.

Norms of revoicing may be supported by asking students whether they agree or disagree with a students' argument and why. Also, teachers may redirect students' questions, as directed to the teacher, to other students in the classroom. Teachers may also have students rephrase a students' explanation or justification in their own words.

Norms of questioning may be supported and maintained through the use of question stems (King, 1992, 1994). Questions stems are suggested starting points for questions such as "What do you mean by … ?" or "Why did you … ?" Students who are provided with question stems may interpret the stems as models that show them particular ways to ask questions. Also, during whole class discussions, implementation of expected student roles and question stems can help students to question other students, not only to reinforce their understanding but also to help students who would be responsible for presenting the groups' work clarify their explanations (Herrenkohl & Guerra, 1998). Norms of challenging may be supported by having students present differing solution strategies to the class and by having students compare and contrast these strategies (Smith & Stein, 2001). Teachers may ask students to disagree politely with students in either whole- or small-group discussion.

Certain instructional strategies may maintain and support classroom norms that will promote student sense making through productive whole- and small-group discussions (Boaler & Staples, 2008). Having problematic tasks with room for explanation, allowing students to discuss their solution strategies in small groups, and requiring students to share out in whole class discussions have been suggested as useful instructional strategies for promoting students' explanation and questioning (Cobb, personal communication, 2010; Smith & Stein, 2011). It can be inferred that without careful attention to implementing supportive instructional strategies, norms of explanation and questioning may not be established and maintained (Lubienski, 2002; Martin, 2000).

Instructional Challenges. When teaching, many instructional challenges may arise that are not easily solved since there may not be a single or expected "right" answer (Lampert, 1985). It is not unusual for these challenges to arise in classes where teachers are promoting sense making through productive mathematics discussion (Ball, 1993). These challenges may arise as teachers are considering how to represent content, to respect children as thinkers while providing guidance, to create and use community, and to try to be intellectually honest. Other researchers have noted that teachers have faced challenges that have to do with classroom management (Gregg, 1985; Lampert, 1985; Turner, et al., 2002), mathematical authority (Chazan, 2000; Lampert, 1985), and student buy-in (Cooney, 1985; Lubienski, 2002). These challenges that are inherent in teaching practices may influence teachers' decision-making processes when deciding on instructional strategies.

Chapter 3: Methodology and Research Design

The following sections describe the methodology and research design of this study addressing novice teachers' strategies and challenges, as well as their negotiation of these challenges, when promoting students' efforts to make sense of mathematics through student participation in productive mathematical discussion. The data analyzed in this study were drawn from several settings, including: observations of methods courses pre-placement, course documents remitted pre-placement, initial interviews, mentoring sessions, teacher seminars, and actual classroom observation.

Research Questions

The research questions, as outlined in the previous chapter, are as follows:

1) What features of and strategies for instruction do novice teachers attend to and implement with regard to promoting sense making through student participation in productive mathematical discussion?
2) What challenges do these novice teachers face in the process of promoting sense making through student participation in mathematical discussion, and how do they negotiate these challenges?

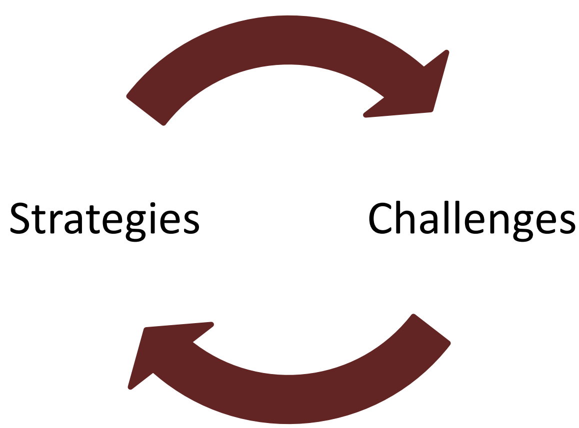


Figure 1: Relationship between strategies and challenges

The relationship between strategies and challenges is considered to be reciprocal: a teacher will devise a strategy, they may face challenges in their instruction, either as a result of, or unrelated to the implementation of that strategy. They will then develop a new strategy or refine a previous strategy in order to address this challenge. By analyzing this process of moving from strategies to challenges, through negotiation, and back to strategies, I address how and why the teacher participants changed in terms of their promotion of sense making through student participation in mathematical discussion.

Design

This study used participant observation as the method of data collection. As a researcher, I worked very closely with the participants in my study. I was their program-provided mentor and supported them during their first year of teaching. I also served as an instructor for some class sessions in some of their teacher preparation courses. Furthermore, I was an active participant in their novice-teacher seminar during their field placement. This meant that during their initial year as a teacher, I influenced what instructional strategies were considered and potentially discussed as mechanisms for supporting and maintaining emerging norms of explanation and questioning within the teachers' classrooms. Since these teachers were novice teachers, many of the instructional strategies that I proposed arose in discussion with them during either individual mentoring sessions or in the seminar. Through the application of the participant-observer model, I assumed two roles with reciprocity. I served not only as a researcher collecting data but also as mentor of these individuals operating with the intent of supporting each of them as they grew to assume the role of teacher throughout their first year of teaching. Therefore, the methodology employed in this study allowed me to describe the instructional strategies considered and co-constructed by these novice teachers and me within the context of mentoring sessions and a teacher seminar. The collected data provided firsthand notations and records describing their experiences and professional change, as evidenced through the talk and action of these three teachers within the context of their mentoring and seminar support as well as their actions within their classrooms and schools.

Program Context

The program for which I served as a mentor and in which the participants enrolled was a collaborative effort affiliated with a large mid-Atlantic university and a large local school district. The National Center of Education Statistics categorizes this school district as being situated in a “Large Suburb,” which is defined as a “territory outside a principal city and inside an urbanized area with population of 250,000 or more” (Institute of Education Sciences, 2013). Locally, it was not unusual for this school district to be described as “hard to staff,” in that many of the schools in the district had a history of low student performance on the state’s high-stakes standardized achievement assessments, had a high rate of teacher turnover, and had a predominant target population of students who were children of color and low-income. By design, this alternative certification program recruited prospective teachers who were committed to the community and/or the population of the student body. Those who enrolled in and successfully completed the requirements of the program were certified as teachers of middle-school mathematics or science and one other core middle-school subject, which could be the remaining field of either science or mathematics. The program’s expressed goal was to bring into the field of mathematics or science education individuals who were committed to educating students who were traditionally underserved and to provide these individuals with the academic background and support necessary to become qualified practitioners. While it encompassed some of the features of alternative resident teacher certification programs, the program was designed so that prospective teachers were slowly introduced to teaching. Cohorts of prospective teachers were enrolled in summer coursework that included an introduction to teaching course that focused on equity and diversity, a methods class in their primary content area, and a content-area reading course. During the summer they also were enrolled in a short field experience. When they entered teaching in the fall, they were partnered with a cooperating teacher for a month-long internship, and then subsequently assumed a half-time (rather than a full-time) teaching load while partnered with another novice teacher from the cohort. In this way, together a pair of novice teachers filled a full-time teaching vacancy. In this way, the paired teachers would be able to gain experience with the work of teaching and to observe other teachers during their non-teaching time. During the fall semester, after they had begun teaching, they were enrolled simultaneously in a second methods course in their primary content area, an adolescent development course, and a seminar course that met every other week. During the spring semester, they were enrolled in their second content-area reading course, an equity and diversity course, and the same every other week seminar course. In the summer following their year of half-time teaching, they were enrolled in a methods course in their secondary content area.

Participants

The participants in this study were three novice teachers enrolled in the alternative certification program described above. During the year in which I collected data, there were 13 teachers enrolled in the alternative certification program which served as the site for this research. Eleven of those teachers' primary certification was in mathematics. Of those 11 teachers, 9 agreed to participate in the study. After conducting preliminary observations of these 9 teachers in their summer methods class, I selected three based on my perception of their initial predispositions for and conceptions of teaching. The first, Jack Davis was selected due to his unabashed commitments to student-centered instruction. The second, Eleanor Scott, was selected because of her significant experience with teaching and the conceptions of teaching that these experiences provided. The third, Michelle Miller, was selected because of my perception of her reticence to teaching in a holistic, conceptual, and student-centered manner. These teachers were my mentees in the alternative certification program and participated in a teacher seminar while simultaneously participating in the requirements of this teacher preparation program. All three teachers were placed in K-8 academies. This was not a required feature of the alternative certification program, but a coincidental placement of the participating teachers in three schools that were organized as K-8 academies.

Michelle Miller. Michelle Miller is an African-American woman. She had earned both a bachelor's degree in physics and a master's degree in mechanical engineering and previously worked as an automotive engineer before deciding to become a teacher. She had no prior teaching experience before entering the program. The licensures that she subsequently earned through enrollment in the program were in middle-grades mathematics and science. Michelle's initial one-month internship was in the same school where she was subsequently assigned as a paired, half-time teacher. This school was a public K-8 academy; Michelle's permanent placement assigned her to teach seventh-grade mathematics and science. She taught one section of each subject.

Eleanor Scott. Eleanor Scott is a Caucasian woman. She earned a bachelor's degree in history and a master's degree from a divinity school. She had completed all requirements but that of a dissertation for a doctoral degree in history. She had previous experience teaching students across all grade levels from pre-kindergarten through graduate school, however she had not previously completed the requirements for teacher certification. The licensures that she earned through enrollment in the program were in middle-grades mathematics and social studies. The program assigned Eleanor to intern in a public middle school (grades 6-8), but then she was transferred to a K-8 public charter school for her paired, permanent placement. Eleanor's placement was housed in a local church. She taught two sections of eighth-grade mathematics.

Jack Davis. Jack Davis is an African-American man. He entered the program immediately after receiving his undergraduate degree in economics. His previous instructional experience encompassed mentoring and providing tutoring for middle school students. The licensures that he earned through enrollment in the program were in middle-grades mathematics and social studies. Jack interned and was permanently placed in the same K-8 school as Michelle. He taught one section of seventh-grade, honors mathematics and one section of seventh-grade social studies.

Researcher

I am a former high school mathematics teacher with 8 years of experience teaching in the same school district where this study was conducted. My teaching experience has been in a public school with similar student demographics and comparable challenges and affordances as the schools wherein these three teachers were placed. However, since I have only taught high school mathematics and statistics to students, and the participants in this study taught middle-grade students in K-8 academies, my experiences did differ from theirs. As such, I had to adapt to new content and curriculum, as well as the needs and background of students of a differing, younger age.

Implementation

This study's subjects were novice teachers who were identified during their teacher-preparation program and were accessed as they taught mathematics half time in local K-8 academies. As originally designed, the teacher preparation program intended each of the teacher candidates to complete a month-long internship in a school with a retired teacher as their cooperating teacher and then to assume half-time teaching responsibilities at that school, which would serve as a permanent placement. However, when the school year began, the school district did not have sufficient full-time vacancies available in which to place all of the candidates in the teacher-preparation program. Due to this fact, each of this study's participating teachers completed their month-long internship in a practicing teacher's classroom and remained there until teaching vacancies became available. Therefore, the program's participating teachers completed a supervised internship with a cooperating mentor teacher that lasted from 1 to 2 months.

Michelle and Jack's internship supervisors created a vacancy at their K-8 public school enabling the hiring of both of them after their completion of a 1.5-month internship. However, this opportunity was not available at the school where Eleanor was completing her internship. Subsequently program administrators learned that a local K-8 charter school had a vacancy that was being temporarily filled with a long-term substitute. When the program discovered this vacancy, Eleanor and a prospective teacher in the alternative certification program who was paired with Eleanor were moved to the charter school after completion of a 2-month internship. After securing their permanent placements, all three of these teachers began participating in two activities related to this research project: individual mentoring sessions and a teacher seminar.

The school in which Jack and Michelle were placed enrolled 660 elementary and middle-school students in 2011. Ninety-five percent of the students enrolled at the school in which they were placed were African-American, and 2 percent of the students enrolled were Hispanic or Latino. Sixty-nine percent of the middle-school students were considered low-income, according to the data on students receiving free or reduced-price meals. Fourteen percent of the middle-school students were receiving special-education services. The school in which Eleanor was placed enrolled 466 students in 2011. Ninety-one percent of the students enrolled at the school in which she was placed were African-American, and 6 percent of the students enrolled were Hispanic or Latino. Fifty-four percent of the middle-school students were considered low-income, according to the data on students receiving free or reduced-price meals. Eleven percent of the middle-school students were receiving special-education services.

Data Sources

I collected data from several sources including observations of a pre-placement methods course, course documents remitted pre-placement, baseline interviews, seven teacher-support reflection cycles, and follow-up interviews. These sources allowed me access to the features of and strategies for instruction to which these novice teachers attended, as well as how they added or modified strategies to negotiate challenges that arose in implementation. The following sections describe each of these sources.

Methods courses. In the summer before the participants began teaching, they enrolled in a mathematics methods course. In this course, teachers worked on mathematics problems, planned and implemented mini-lessons, and reflected with their classmates on those plans and implementations. The observation of the participants in this setting helped me to establish a baseline as to how these teachers approached and enacted teaching before their actual experiences in the classroom.

Course documents. The participants were also enrolled in a second course during the summer prior to their placement. In this course, students submitted several documents which were collected in a final portfolio. This portfolio had three components, one on reflective teaching, one on a field experience and instructional practice, and one on expectations, management, and discipline. The reflective teaching component included a mathematics autobiography, a subsequent reflection on that autobiography, reflections on readings that were originally posted on a course message board, and a later critique of those posts. The field experience and instructional practice component included the teacher's initial impressions of a brief summer field experience held at a local community center with a small group of students, an analysis of the mathematical life history of a student from that community center, and a self-reflection on the participating prospective teacher's instruction during that field experience. The expectation, management, and discipline component of the portfolio included the prospective teacher's management philosophy and a listing/explanation of the planned rules, procedures and expectations that the prospective teacher expected to implement once he or she began teaching. These documents helped me to establish a baseline about how these teachers approached and thought about teaching before entering the classroom.

Baseline interviews. The baseline interview referenced in Appendix A was designed to elicit teachers' initial thoughts regarding the instructional strategies, decisions, and practices that they would use in the classroom to promote productive mathematical discussion and sense making. The interview protocol presented a mathematics problem and asked the teachers to suppose that they were going to ask their students to solve this problem in small groups. The teachers were asked how they would set up the problem and how they encourage their students to explain their thinking to and as questions of each other. This allowed me to acquire an early perspective of what types of instructional strategies, decisions, and practices to which the teachers were attending.

Teacher-support reflection cycles. Several data sources resulted from the teachers' participation, with me, in a teacher-support reflection cycle. This cycle, as represented in Figure 2, is a modification of a reflective teaching cycle as described by Smith (2001). In this study, the reflective teaching cycle included four components: a collaborative teacher seminar that included all participating teachers and the mentor, a planning session with the mentor, implementation of the planned lesson observed by a mentor, and a post-lesson reflection session with a mentor. This cycle was repeated seven times over the course of the study. I describe each component in the following sections.

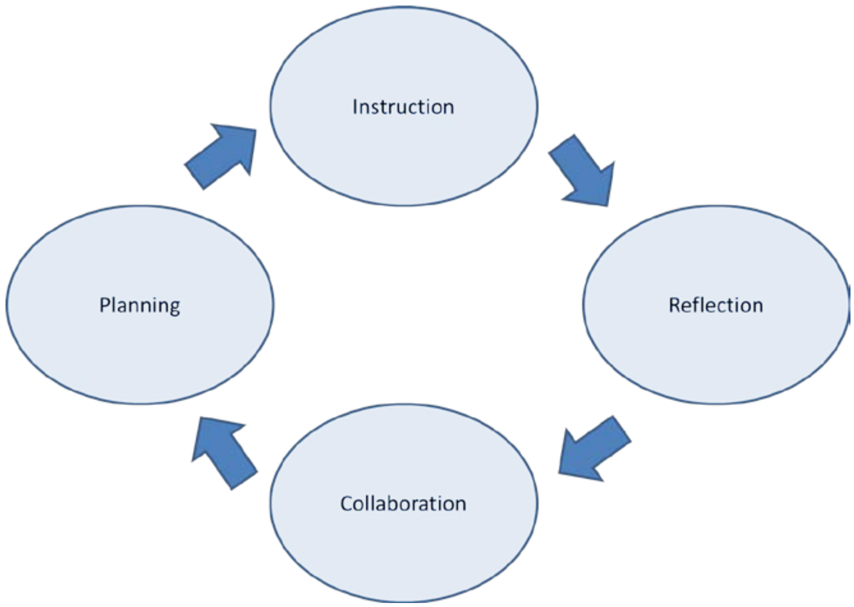


Figure 2: Teacher support-reflection cycle

The intent of these mentoring sessions and the teacher seminar was to assist teachers in developing viable strategies for promoting student discussion in their classrooms.

Collaboration: Teacher seminar. The teacher seminar was conducted for one hour approximately every two weeks after their permanent placement, allowing the three participants and I to meet and discuss their own teaching and the teaching of their colleagues in the seminar. The teacher seminar was structured in two different ways.

The initial structure of the teacher seminar is outlined in Table 1. Novice teachers often have difficulty reflecting on their own lessons. Therefore, initially I guided the teachers to reflect on video-taped episodes of teaching as conducted by others, episodes that exhibited or failed to exhibit viable methods of encouraging student discussion. Watching these videos was an assignment for their fall mathematics methods course. After the teachers were permanently placed, I pulled the three participants out of their fall methods course for an hour to conduct the teacher seminar. During this hour, we spent approximately 20 minutes reflecting on the video. We would then spend approximately 15 minutes reflecting on their teaching experiences with regard to promoting productive discussion and sense making. We would conclude with a discussion of future content and potential instructional strategies, decisions or practices.

Table 1
Initial teacher seminar model

Activity	Time allotment
Reflection on teacher video and lesson plan from database/website	25 minutes
Reflection on own teaching	25 minutes
Discussing future content and instructional strategies	10 minutes

As the teachers gained more experience, and completed their fall methods class, we transitioned to the second model where the primary focus of teacher reflection was on their own teaching (Table 2). Since they had completed their methods class, we scheduled time outside of their teaching and fall and spring coursework to meet and discuss teaching. During these sessions, we spent approximately 45 minutes reflecting on their teaching, with a specific focus on promoting sense making through participation in productive mathematical discussion. After reflecting, we spent approximately 15 minutes discussing future content and instructional strategies, decisions, and practices. These settings allowed me to access what features of and strategies for teaching to which they attended, as well as the challenges they felt they were facing in planning and implementation.

Table 2
Second teacher seminar model

Activity	Time allotment

Reflection on their own teaching	45 minutes
Discussing future content and instructional strategies	15 minutes

Planning: Mentoring sessions. I met with the candidates in their schools seven times over the course of their field placement in order to support their instructional planning through mentoring. Promoting productive discussion and mathematical sense making was a commitment that guided my interactions during these planning and mentoring sessions. I met with Eleanor one-on-one to support her planning. I had two one-on-one planning meetings with Michelle and Jack, however since they were placed in the same school and taught the same grade level, five of the planning mentoring sessions included Michelle, Jack *and* I. While these sessions varied in duration, these sessions not only allowed me access to these novice teachers, but also allowed me to collect data addressing how the teachers were thinking about planning and instruction and the concerns and challenges they faced, while simultaneously supporting their lesson planning for future instruction.

Observations. After the planning and mentoring sessions, I observed the teachers as they implemented instruction. I observed each of the teachers throughout entire, 70-minute, class periods. Observation of their teaching allowed me direct access to the instructional strategies, decisions, and practices that these teachers implemented in their classroom, as well as the responses of their students.

Reflection: Mentoring sessions. After I observed a class period of instruction, I would meet with each teacher one-on-one to reflect on the observed lesson. Promoting productive mathematical discussion and sense making was a guiding feature that influenced my facilitation of this reflection. The teacher and I collaborated to assess the observed lesson. We discussed teacher moves and student reactions during the lesson. We also discussed potential next steps. These reflection and mentoring sessions varied in duration. Reflecting with the teachers allowed me access to the teachers' thoughts regarding instructional strategies, decisions and practices, as well as any challenges that we felt they were facing.

Follow-up interviews. I conducted a single, follow-up, individual interview with each teacher in late April and early May of 2013. During these interviews, the teachers were asked to reflect on their internship and teaching in their first year of teaching and identify what they felt were critical moments in their development, specifically with regard to promoting student discussion. After the teacher had responded to this open-ended question, we engaged in a follow-up discussion to clarify the remarks made by the teacher. This follow-up interview also permitted member checking to verify my preliminary interpretations resulting from my analysis of the collected data. This interview allowed me to clarify and affirm these initial analytic interpretations.

Connections to the research questions. Each of the data sources described above permitted the collection of data addressing one or both of the research questions. The table below identifies the differing data sources that were accessed when addressing the two separate research questions.

Table 3
Data Sources

Research Question	Data Sources
What do novice teachers attend to and implement with regard to promoting sense making through student participation in	Course documents from their summer internship course Field notes of summer methods class

productive mathematical discussion?	observations Transcripts of baseline interviews Transcripts of mentoring sessions Field notes of classroom observation Transcripts of classroom teaching Transcripts of teacher seminars Transcripts of follow-up interviews
What challenges do these novice teachers face in the process of promoting sense making through students participation in productive mathematical discussion, and how do they negotiate these challenges?	Transcripts of baseline interviews Transcripts of mentoring sessions Field notes of classroom observation Transcripts of classroom teaching Transcripts of teacher seminars Transcripts of follow-up interviews

Data Collection

The initial data were collected during the teachers' methods course during the months of June and July 2011. I observed the teachers during their methods course and took field notes about the features of their discussions as it related to planning and enacting mathematics instruction. After the teachers were permanently placed in November 2011, I conducted a baseline interview with each teacher. This interview was audio-recorded and transcribed. Following these interviews, we initiated the teacher-support reflection cycle, a cycle that was repeated seven times over a period ranging from November 2011,through March 2012. Within the teacher-support reflection cycle, all of the planning and reflection mentoring sessions were audio-recorded and transcribed. Each teacher seminar, as conducted between November 2011 and March 2012, was also video-recorded and transcribed.

Although seven classroom observations of each participant were conducted, only three of these were sources for data collection. The teaching observations that served as sources for data collection were conducted in November 2011, during Cycle 1 of the teacher-support reflection cycle, in January 2012, during Cycle 4 of the teacher-support reflection cycle, and in March 2012, during Cycle 7 of the teacher-support reflection cycle. I identified these particular classroom observations as sources for data collection in order to establish whether instructional change had transpired as documented by the beginning, middle, and end of the study. I took field notes, including time indicators, during each of these three observations and made notations that indicated particular features of the lesson that were relevant to analysis pursuant to answering the research questions. These notations consisted of codes that I hypothesized before conducting the study. The teacher also carried an audio-recorder to document their verbal teacher moves as well as responses from students with whom they were interacting. These audio-recordings were transcribed. Although this is a study addressing instruction to promote student discussion, due to restrictions related to approval for conducting research involving human subjects, I will not be offering analyses drawn from transcripts of students' independent discussion as it was carried out in either whole-class or small-group discussion within classrooms. My observational data consisted of field notes and audio transcripts; however, my audio transcripts are limited to statements of teachers and the responses of students in interaction with their teachers.

In May 2013, I acquired a document file consisting of each teacher's final teaching portfolio as developed during from their summer introduction to teaching course that focused on equity and diversity in which they were enrolled in the summer of 2011. During the last week of April or the first week of May 2013, I conducted a follow-up interview with each teacher. These interviews were audio-taped and transcribed. It is important to state that I was serving as the mentor for each of the teachers. Therefore I was an integral part of each teacher seminar, planning mentoring session, and reflection mentoring session, and my goals of promoting mathematical sense making through student participation in productive discussion featured prominently in these interactions. While I cannot quantify the impact of *my* presence as a mentor on these seminars and mentoring sessions, the inclusion of mentoring support was likely a critical variable in the learning and development of these teachers. The timeline of data collection for this study is noted in Table 4.

Table 4
Timeline of data collection

Time Collected	**Data Collected**
Baseline: June – July 2011	Methods class observations
Week 0: November 2011	Baseline interview
Week 1 - 2: November 2011	Teacher seminar Planning mentoring session Recorded classroom observation Reflection mentoring session
Week 3 - 4: November – December 2011	Teacher seminar Planning mentoring session Non-recorded classroom observation Reflection mentoring session
Week 5 – 6: December 2011	Teacher seminar Planning mentoring session Non-recorded classroom observation Reflection mentoring session
Week 7 – 8: January 2012	Teacher seminar Planning mentoring session Recorded classroom observation Reflection mentoring session
Week 9 – 10: February 2012	Teacher seminar Planning mentoring session Non-recorded classroom observation Reflection mentoring session
Week 11 – 12: March 2012	Teacher seminar

	Planning mentoring session Non-recorded classroom observation Reflection mentoring session
Week 13 – 14: March 2012	Teacher seminar Planning mentoring session Recorded classroom observation Reflection mentoring session
Follow-up: April – May 2013	Course documents from summer course Follow-up interviews

Data Analysis

While the method of data collection for this study was participant observation, the over-arching analytic method was that of case study. A case study can be defined as

> An empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident.(Yin, 2009, p. 18)

As noted by Yin (2006), “the strength of the case study method is its ability to examine, in depth, a ‘case’ within its ‘real-life’ context” (p. 111). In this study, teachers are in multiple contexts that affect their practice: university coursework, schools, mentoring sessions, and teacher seminars. The development of these teachers’ instructional strategies, whether based on their previously held beliefs or their negotiation of dilemmas, is inseparable from these contexts. These three participants were treated as cases of a phenomenon in similar, but not identical, contexts.

In order to analyze this data, I coded documents and transcripts with codes defined by the teachers’ instructional strategies, decisions, or practices, as well as the challenges that they faced during the course of the study, as was outlined in the conceptual framework. In the following sections, I operationalize these codes.

Instructional strategies. There were several sources for analyzing instructional strategies. Frequently, during the teacher seminars, the discussion turned to planning. Through these discussions, with me serving as an organizer/participant, the teachers co-constructed definitions of instructional strategies that might support student discussion and then discussed how those strategies could be implemented. In addition, each of the teachers continued developing and reflecting on instructional strategies when meeting with me during mentoring sessions. The following definitions are phrased to characterize occurrences during instruction. For purposes of analysis and coding, these strategies, decisions, and practices were grouped into five overarching categories: nature, set-up, and design of interaction with a task; expectations; questioning/explanation; discussion; and relationship building.

Nature, set-up, and design of interaction with a task. Instructional strategies may promote differing learning goals and convey differing expectations through the nature of an assigned task, its design, and its set-up. This code indicated that the participants were either discussing, planning for, or implementing instructional strategies regarding task design and setting. This code was applied in the following instances: use of problematic, hands on, or investigative task; the teacher setting up the task to facilitate student engagement; students working on this task individually, in pairs, in small groups, or as a whole class; and students use of or discussion regarding use of manipulatives to facilitate mathematical sense making.

Teacher uses a problematic, investigative, or hands-on task. The use of a problematic, investigative, and/or hands-on task identifies a focus for student activity in the classroom. Problematic tasks are those where the solution strategy is not immediately apparent and may induce student struggle and eventual sense making. Investigative tasks are those where students have opportunity to investigate a pattern that may elucidate a particular mathematical property or concept. Hands-on tasks are ones that allow students access to abstract mathematics through the use of concrete manipulatives. A task can be categorized as one, or more than one of these things. A task may be open-ended, meaning there are multiple correct solutions, or open-entry, meaning there are many possible ways to solve the problem to get the single correct answer. Typically, a task is deemed to have a level of rigor if it requires the student to do more than simply retrieve a fact from memory. In addition, a task may allow students opportunities to investigate patterns and to generalize from those patterns to define properties. Finally, a task may allow students concrete ways of thinking about abstract concepts.

Teacher frames task. This characterization references a teacher's use of an instructional practice that included determining and asking questions in order to set up a task for some students or for whole-class investigation. Teachers may ask questions to clarify contextual cues if the problem is set in a real-world context or questions that prompt students to notice what the task is asking them to do or find. Exemplar questions include, "What is the problem asking?" or "What may your answer look like?"

Teacher groups students in small groups. Student groups consisting of six or fewer students were defined as small groups. When teachers assigned students to these groupings, they determined whether the collective of students were were mathematically heterogeneous or homogeneous, as well as determining which students were assigned to the differing student groups. When this coding was applied, a small group of six or fewer students would be working collectively on the same problem, which may or may not be the same problem assigned to other groups of students in the classroom.

Manipulatives. Teachers may provide students with manipulatives, or make manipulatives available to students, in order to facilitate their investigation of mathematical concepts. Manipulatives allow students to use concrete items to represent mathematical problems and contexts and may facilitate student sense making about potential solution strategies.

Expectations. This code reflected the instructional strategies, decisions, or practices communicating expectations that were voiced and reinforced by the teacher in order to promote discussion in the classroom. This code was applied when a teacher communicated the following: teacher expectations for student behavior as they participated in small groups, teacher expectations in terms of requiring students to share their thinking, teacher expectations for student interaction during whole-group discussions, and teacher expectations for individual students during both small- and whole-group activity.

Teacher defines expectations of small-group work. A teacher could use either routines or statements to define expectations for group work. For example, a strategy defining expectations may include a rubric, either developed by the teacher or co-developed by the teacher and students, or a chart that describes what a teacher or outsider should see and hear when students were working in small-group settings. Expectations for students could include providing assistance to one another, making sure that each person in the group understands and can explain the actions or decisions of the group, working together on the same problem, providing explanations of work, seeking help, and/or staying on task. Additionally, the teacher could incorporate techniques to monitor students' work by providing a signature, stamp, or sticker when students have completed one question and are ready to move to the next question. Other instructional techniques associated with this category included the teacher verbalizing when he/she is hearing model statements or questions in the groups and providing examples of groups who are working together well. Another technique is for the teacher to provide scripts for students that convey what a group discussion might look or sound like. The strategy employed by a teacher to communicate expectations did not have to be elaborate. For example, a teacher might simply remind students to remain on task and to ask each other questions rather than to ask questions of the teacher.

Teacher requires student(s) to share solutions to the class. This instructional decision and practice occurred when the teacher identified either one student or all students a single small group of students, to present a solution to the class. The expectation presumed with this characterization is that the selected students should be able to explain their solution strategy to the other students. This also permits students in the class to compare their work, to find inconsistencies, to ask clarifying questions, and to learn of other solution strategies. When a teacher is planning for a lesson, ideally the plan will convey an allotment of time for whole-group student discussion, either in one day or across multiple, contiguous days of instruction.

Teacher defines expectations of whole class discussion. While the prior expectations defined expectations for small-group work, this expectation identified the teacher's decision to engage with the class in order to address and co-define expectations for whole-group discussions as conducted by the students. The teacher may create a rubric, a chart, or statement stems such as: “I disagree with the idea that …;” “I think that maybe instead … because ….” Question stems such as, “Why did you …?” establish the nature of student talk and student-to-student interaction that a teacher expects to hear and see during whole group discussion. In addition to the content of the discussion, the code “expectation” was applied when a teacher conveyed expectations regarding the conduct of the discussion, including standards for respectful questioning and disagreement as well as for student attentiveness.

Teacher defines expectations of individuals. The teacher might ensure or communicated the expectation that students were responsible for their own learning. For example, the teacher may explicitly require students to think individually prior to collaborating. The teacher may also require all students to document their thinking, regardless of whether or not they are collaborating.

Questioning/Explanation. This code identified those instructional strategies, decisions and practices through which the teacher questioned students or elicited explanations from students in order to promote productive small- or whole-group discussion. This code was applied when a teacher used use of question stems, asked probing or clarifying questions of their students, asked whether students disagreed or agreed with an explanation as offered by a student and why, and directed student questions to other students in the class rather than the teacher responding to all student questions.

Teacher provides question stems. A teacher could share teacher-developed question stems with students with the intention that the students should subsequently ask these types of questions of other students during either whole- or small-group interactions. Teachers used these question stems as a basis for discussion of expectations and then co-constructed with the students a more complete list of question stems that students could use to provoke explanation and shared understanding in the class. Question stems may include queries such as: “How did you get …?” “How do you know ...?” and “Why did you …?”

Teacher asks/models open-ended and probing questions. This an instructional practice occurred when a teacher identified or prepared open-ended and probing questions that were then asked of students during small-group and whole-group discussions. An open-ended question is a question that may or may not have differing correct answers, but does have differing possible approaches for addressing the question. A probing question is a question that requires the students to identify or clarify what they know, so that they may begin to think about the implications of that knowledge with regard to the mathematics involved. During lesson planning, the teacher may hypothesize or anticipate different solutions or solution strategies and plan questions to ask when the teacher sees a student or group of students using that particular strategy. During instruction, the teacher might ask these questions to promote student discussion and to model for the students the types of questions that students can ask each other.

Teacher scaffolds with probing and leading questions when students are “stuck.” During a lesson, a teacher may offer open-ended and/or leading questions in response to student errors or misconceptions. When planning a lesson, a teacher should anticipate possible student misconceptions or errors and plan a combination of probing and leading questions for use when these misconceptions arise, particularly if the misconceptions are not pointed out or addressed by other students. During instruction, a teacher asks these questions and prompts other students to ask these questions. While a leading question frequently has a procedural or factual answer, it may be used in conjunction with probing questions in order to develop mathematical explanations and to solicit justifications.

Teacher requires student to explain correct or incorrect answers. Teachers may require their students to explain and justify their work. When responding to either correct or incorrect answers, the teacher's intent was to always request an explanation and justification, regardless of whether the offered answer was correct or incorrect. The teacher then might subsequently engage with other students, in either the small group or across the whole class, with questions asking whether the provided explanation or justification was reasonable or required revision.

Teacher requires single (random) student to explain to the teacher. A teacher could purposefully select a single student in a group (not always the same student) to explain that group's work. The intent of this strategy is to monitor the participation and understanding of every group member by requiring one student and that one student alone to answer the teacher's request for explanation. If that student cannot explain, then, in order to promote student discussion and responsibility for making sense of the mathematics, the expected reaction of the teacher is to walk away, after conveying to the group of students that it is their responsibility as well as the teacher's expectation that the group members will talk and work together in order to help the selected student subsequently explain their work and reasoning to the teacher. This small-group interaction should continue until the selected student can satisfactorily explain the group's work to the teacher.

Teacher asks if students agree or disagree and why. In order to promote questioning and explanation, the teacher may ask either small groups of students or the whole class whether they agreed or disagreed with the efficacy of a particular solution or solution strategy. In doing this, the teacher does not validate or invalidate particular answers as a mathematical authority, but instead asks the students to explain their reasoning and prompts them to ask questions of each other.

Teacher directs students to speak to each other. When the teacher was asked questions by students, a teacher could reply by asking students questions such as “What do you think?” and “Why?” By deflecting a student question back to the class or a small group of students, the teacher promotes and conveys the intention that when questions are raised by one or more students, it is the responsibility of the other students to explain and justify to and with each other in order to produce a response to those student questions, rather than simply seeking teacher explanation. This strategy scaffolds discussion wherein students critique and question others’ work in response to the requirement that they explain why they did certain things and how they knew that they are correct.

Discussion. This code refers to those instructional strategies, decisions, or practices through which a teacher directly facilitates the development of productive whole-group discussion. This code was applied when a teacher was discussing the reasonableness of students’ solutions, choosing students to present their differing or divergent solution strategies and answers, and revoicing or encouraging the students to revoice students’ explanations.

Teacher chooses divergent strategies or solutions for presentation. During planning sessions, a teacher might hypothesize or anticipate, in advance of instruction, the differing ways through which students might attempt to solve an assigned problem. During instruction, a teacher could circulate about the classroom in order to identify students with different solution strategies for solving a particular problem. If different solution strategies were not noted, the teacher could suggest or model differing strategies in whole-group discussion time, asking for student evaluation and input. If a strategy that the teacher has not anticipated has arisen within student work, the teacher may join the other students in the class in the role of a learner and ask the authoring student to explain the strategy. Student explanation of strategies may offer the opportunity for the teacher to model the asking of clarifying questions.

Teacher models/chooses incorrect strategies for presentation. During either planning or instructional sessions, a teacher might hypothesize or anticipate incorrect strategies that students may use when solving particular problems. Coupled with the prior teacher strategy of responding consistently to incorrect answers by asking for explanation or justification, the intent of this strategy is to allow for the development of classroom norms wherein students realize that asking clarifying questions and revising answers, explanations, or solution strategies are both permissible and encouraged actions. This norm has to be carefully co-constructed by the students and the teacher within the classroom so that students feel safe sharing their answers and solution strategies, whether those approaches are correct or incorrect. In order for students to learn how to respond respectfully to incorrect answers, before expecting students to question each other's errors respectfully as expected during discussion, the teacher may model the presentation of incorrect strategies setting up situations wherein students are put in a position of needing to question that teacher's work.

Teacher encourages discussion of reasonableness of student solutions. Planning to use this instructional practice involves the teacher hypothesizing or anticipating possible student approaches or solutions. During instruction, a teacher might question the reasonableness of responses or ideas by asking students questions such as, “Does that make sense in the problem?” “Is that what we had determined the question was asking for?” or “Is that the kind of solution that we were expecting?” When applying this strategy, the teacher does not immediately affirm or deny correct or incorrect answers to questions.

Teacher revoices or encourages students to revoice a student’s thinking. A teacher might revoice a student’s explanation, either to verify understanding of the student’s thinking or to rephrase the student’s explanation using mathematical terminology. Similarly, a teacher could ask another student to rephrase a prior student’s explanation using their own words, in order to verify a targeted student’s understanding while maintaining the class’ attentiveness to students’ presentations.

Developing relationships with students. This instructional decision and practice is defined by making students feel safe and supported by the teacher. The teacher can set expectations of student-to-student interaction, get to know students personally and academically, and provide praise in appropriate situations.

Challenges. Research Question 2 references the challenges that the novice teachers confronted in the process of promoting sense making through student participation in mathematical discussion. There were two contexts wherein these challenges were made public and addressed, either collaboratively or individually, namely when the teachers reflected on their lessons in mentoring sessions and in the teacher seminars. Reflections addressing challenges that arose out of problems regarding student discussion were of particular focus.

As defined for analysis, these challenges are distinguished by five overarching codes. These codes include time constraints, norms of schooling, classroom management, context, and teacher efficacy and self-efficacy.

Time constraints. This challenge was confronted during discussions addressing the teachers' concerns that there was inadequate time to implement instructional strategies that focused on providing support and opportunity for student explanation and questioning. Since lessons that allow time for student discourse about mathematics may take a longer period of time for consideration of the nature and meaning of the mathematics content then required during direct instruction, participants in the seminar or mentoring sessions may express concerns indicating their opinion that lesson and/or unit pacing was at risk due to the inclusion of these strategies and time allotments. Teachers feel pressures due to the limited amount of instructional time allotted and its relation to the amount of content that they are expected to address in that time. Teachers also feel pressure resulting from the timing of fixed schedules for administration of district- or state-level assessments.

Norms of schooling. In a particular school or grade, many teachers might teach in a similar manner with similar routines and expectations. This then defines a norm or expectation for teaching and schooling and establishes a type of instruction with which many students are familiar. Approaches or assumptions that are contrary to these norms may cause dissonance, as features associated with these norms or expectations are questioned or redefined. The code "norms of schooling identified challenges including perceptions of mathematical authority, teacher and student response to potential or actual struggle, perceptions of student ability, and the students' desire or reticence to participate in discussion.

Mathematical authority. A challenge associated with mathematical authority may be manifested during discussions wherein the participating teachers address the lack of student critique or their difficulty in questioning use of student strategies because of the students' assumption that any student work that was presented or shared with the class would have to be correct, as otherwise the teacher would not have chosen it as an exemplar for class review. This dilemma includes students' perception that the teacher is the only arbiter of mathematical correctness in the classroom, rather than their own or others' reasoning. Initially, because of prior norms for classroom activity, students presumed that since a student's solution strategy was selected for whole-class presentation by the teacher, that strategy must be correct. Therefore, because the strategy was correct, the students presumed there was no need for them to actively and critically listen to student explanations during the presentations. This challenge may cause a roadblock for student questioning and emergent discourse.

Student struggle. This challenge to student discussion was coded when a student did not understand how to access a problematic task or was unwilling to struggle with the mathematical content and to engage in the work of making sense of the mathematics because the solution path was not readily apparent. Since, in the United States, solving problems in mathematics is often characterized as applying a rapid, routinized procedural solution to a relatively straightforward problem, students may not recognize or may resist the need to reason and think critically in order to solve problems. This tension may cause teachers to change tactics and possibly lower the cognitive demand of a problematic task. Teachers may fear student struggle and immediately direct students to a particular procedure or solution strategy.

Perceptions of student ability. Teachers' perception of student ability, or the students' actual ability, may affect teachers' planning and implementation of lessons that promote students' sense-making and engagement in mathematical discussions. In response to this challenge, teachers may choose to increase or reduce the cognitive demand of tasks, either during planning or implementation, in order to adapt to what they perceive as their students' abilities.

Students desire to discuss. Due to familiarity with instruction that does not demand sense making or participation in discussion, students may resist instruction that demands the increased level of participation associated with the expectation of students' active discussion. This may result in a challenge for teachers as they worked to develop new norms of instruction and discussion over a long period. In order to address this challenge, teachers must be willing to be persistent in the face of student resistance.

Classroom management. The challenge code of classroom management refers both to inadequate classroom management, that does not establish a context for productive student interaction, and restrictive classroom management, that inhibits student interaction. Each of these are roadblocks to instructional practices marked by student group work, discussion, and respectful questioning. In either of these forms of classroom management challenges, a teacher may feel that students are not on-task during group assignments, that they are not attentive to questions or explanations as offered by their peers, and/or that they are not respectful when addressing incorrect or differing solutions or solution strategies. Some classroom management factors that may contribute to problems inhibiting student-to-student discussion and student-to-teacher discussion include, but are not limited to, the design and focus of the task, class flow and pacing, management of off-task behavior, and student inattentiveness.

Context. School context may present a challenge to teaching in a manner that promotes student sense making through participation in discussion. This code indicates challenges arising from: inadequate access to resources; the perceived or actual support of teaching endeavors by local school administrators, infrastructure, or other teachers; and the culture of the school and the conduct of instruction in other classrooms. School context may also challenge a teacher to change his or her instructional approaches to those that will promote student sense making through participation in discussion.

Teacher efficacy and self-efficacy. This challenge was coded when the novice teachers admitted or experienced a sense of inadequacy due to their limited experience with and ability to plan, implement, and reflect on instruction and their limited experience with and ability to address, react to, and reflect on student discussion during instruction. This was particularly true when the novice teachers were attempting to implement instruction that was not consistent with their experience as students, such as mathematics instruction that required their students to investigate and make sense of mathematics and to participate in productive mathematical discussion.

Thematic Analysis

Analysis was initiated by coding the field notes, course documents, and transcripts of audio. The analytic codes and which collected data they refer to are noted in Table 4.

Table 4
Summary of Context for the Data Analysis

Source	Data Collection	Analytic Codes
Preliminary data	Field notes, Course documents, Baseline interviews	Nature of Task Expectations Questioning/Explanation Discussion Relationship Building* Time constraints Norms of Schooling Classroom Management Context Efficacy and Self-Efficacy*
Mentoring sessions	Tapes and transcripts	Nature of Task Expectations Questioning/Explanation Discussion Relationship Building* Time constraints Norms of Schooling Classroom Management Context Efficacy and Self-Efficacy*
Teacher seminars	Tapes and transcripts	Nature of Task Expectations Questioning/Explanation Discussion Relationship Building* Time constraints Norms of Schooling Classroom Management Context Efficacy and Self-Efficacy*
Classroom observation	Tapes, notes and transcripts	Nature of Task Expectations Questioning/Explanation Discussion Relationship Building* How strategies implemented

All transcripts were coded referencing the analytic codes highlighted in Table 4. Analytic codes that were emergent during the course of analysis are marked with an asterisk.

Following coding, transcripts were analyzed again using thematic analysis of discourse in order to discern similarities and differences over contexts and cases.

Thematic analysis (Boyatzis, 1998) is a process of coding qualitative data using themes. As Boyatzis describes:

> A theme is a pattern found in the information that at the minimum describes and organizes possible observations or at the maximum interprets aspects of the phenomenon. A theme may be identified at the manifest level (directly observable in the information) or at the latent level (underlying the phenomenon). The themes may be initially generated inductively from the raw information or generated deductively from theory or prior research. (p. vii)

This allowed for flexibility since themes were from both theory and the data.

For example, the following portion of a seminar transcript was coded as “norms of schooling” because Jack was talking about the ways in which his honors students were familiar with participating in mathematics class.

> JD: I have an interesting…when I sit at my desk and watch her teach and I see her students doing it and they’re very…it’s easier for them to work off script but like…I think my kids were technically supposed to be in honors classes, they like much more direct instruction and they don’t like to be asked to do something first, they’re a lot more resistant to it but it’s cool watching when she teaches them and it’s like they’re all about it but I think like maybe if you were in an honors class, you probably would be good doing school and you’ll be good at doing school if you were good at just listening and taking notes, you know.
>
> (EY nods)
>
> JD: Just a different idea. (Seminar Two, November 21, 2011).

After the coding process was completed, I analyzed the code saturation for each participant. The saturation of codes indicated to me the strength of the influence of individual strategies and challenges on each of the participants' practice. Then, I retrieved and read the series of quotes attached to a highly saturated code to uncover underlying themes that motivated the teacher's development. If other quotes were related to that particular theme, regardless of their code assignment, I retrieved those quotes as well. I selected the quotes that were rich in their contribution to the theme for inclusion in the analysis chapters, as well as an array of quotes that illuminated the theme completely, without redundancy.

After I completed the analysis of each participant, I conducted a cross-case analysis. In order to conduct this analysis, I focused on themes that had been very significant to at least two of the participants. I summarized these as common findings across all three cases.

Overview of Subsequent Chapters

Chapters 4, 5, and 6 present descriptions and analyses of the individual cases of Michelle Miller, Eleanor Scott, and Jack Davis respectively. Each chapter begins with a description and analysis of the participating novice teacher and that teacher's initial conception of teaching. Each chapter also includes an analysis of that teacher's changing approach to lesson development as well as that teacher's changing use of small- and whole-group discussion. Chapter 4, focusing on Michelle Miller, includes an analysis of her changing conception of collaboration. Chapter 5, focusing on Eleanor Scott, includes an analysis of her adaptation to her school context. Chapter 6, focusing on Jack Davis, includes an analysis of his changing efficacy with regard to facilitating discussion.

Chapter 7 presents a discussion of findings and implications that arose from a cross-case analysis of the three cases. Chapter 8 discusses this study's contributions to literature, limitations, and future research questions.

Chapter 4: Michelle Miller

Michelle Miller changed her career to become a teacher. Previously she earned an undergraduate degree in physics and a master’s degree in mechanical engineering and then she worked as an automotive engineer for a large American automaker. She entered the alternative-certification program in order to prepare for a position as a teacher of middle-school mathematics.

I observed Michelle during multiple sessions within her program’s initial course addressing methods for teaching mathematics. It was during these sessions that I observed Michelle both planning and implementing lessons in class. She seemed to be concerned with defining isolated individual mathematical skills as targets for instruction and subsequently focusing single lessons on each of those individual skills. Her methods instructor pushed her to think more about overarching mathematical ideas and concepts. When she performed mock-implementations of these lessons, she attempted to integrate a conceptual view of the individual skill that she selected; however, she did not allow for much time for small-group thinking or student investigation of the mathematical content. Rather than probing her classmates’ thinking, she walked them through the process of considering a mathematical feature of a concept by asking leading questions that would direct the whole-group conversation toward the right answer or idea. As soon as one student stated the critical or culminating correct answer, Michelle concluded that the teaching episode was complete and the whole class learned that skill or property. As carried out by Michelle in her methods class, her role as the teacher was to lead instruction directly and to manage all mathematical thinking completely. She did this by posing a series of low-level leading questions for the students to answer, resulting in the recitation of the intended mathematics.

Michelle was paired with a co-teacher for the initial period of her internship (August-October) and received her final placement as teacher of record in November. Early in her final placement, Michelle spoke about her interest in being a teacher who would expect students to integrate many mathematical skills into one activity and to engage in productive small-group discussion addressing how to select and apply the appropriate skill to a real-world situation. However, the reality of Michelle's initial teaching was very different. Michelle continued to implement an instructional approach similar to that evidenced during her methods classes, as she isolated individual procedures from within the overarching mathematical objective and targeted a particular procedure during each class. The district-provided student textbook included activities for lessons intended for delivering as group investigations. Although these were not necessarily problem-based lessons, Michelle modified the lessons and required students to work through them individually. After students had attempted to work through the investigations individually, she would guide them through a mathematical sense-making process by calling on an individual student to make a presentation during which time Michelle would guide the student to clarify his work by asking leading questions. If the presenting student had not been able to complete the investigation successfully, she would guide the student through completion of the problem in the front of the class, either by asking leading questions herself or by having the presenting student call on a classmate who would explain the work. Once an individual student had "discovered" the pattern and stated it to the class, she would move on to applying that individual skill to different exercises.

Michelle's initial approach to planning was to break down a mathematical idea into isolated skills or a hierarchy of skills and to teach them one at a time. She focused her planning time on deciding how to present sequentially these skills in isolation. The textbook she used facilitated this approach. The introduction of each section of the textbook provided investigations through which students could discover mathematical rules and properties. However, the sections themselves often were separated into single skills, concepts, or groups of skills within a concept. For example, one of the introductory activities intended that students work together to find the slope between different points on that line in order to determine the answer to the essential question "How can the slope of a line be used to describe the line?" (Larson & Boswell, 2010, p. 54). Although this activity introduced slope within a chapter on graphing linear equations and linear systems, the activity simply focused on slope and the underlying mathematical features of the slope of a linear equation. Thus, this textbook allowed Michelle to implement what may be termed investigations and, at the same time, left space for her to isolate skills. Although she was intrigued with the potential of teaching for conceptual understanding, or teaching that allowed students to interact and apply multiple skills to real-world data, Michelle did not demonstrate this instructional approach either in her methods classes or in the lessons that she conducted in November during the beginning of her permanent placement.

Analysis of Michelle's Instructional Practice: Challenges and Change

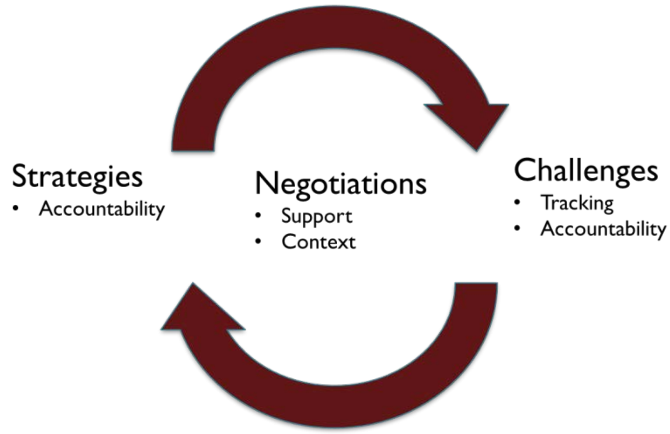


Figure 3: Findings for Michelle

Throughout the 5-month period of this study (November-March), Michelle tried many new instructional strategies, was faced with many challenges, and implemented or modified her instructional approach as a response to those challenges. She faced challenges of self-efficacy, norms of schooling, and individual student ability and success. She negotiated these challenges through application of differing instructional strategies. In so doing, Michelle's perception of teaching, and the planning and implementation of that teaching, changed. This chapter initially presents an analysis of Michelle's early conception of teaching, addressing how she approached and perceived mathematics teaching, during her initial placement as teacher of record in November, as well as nature of mathematics teaching that she characterized as good teaching but did not or could not implement herself. The next section of this chapter analyses changes in her approach to lesson development, as Michelle transitioned from a focus on individual skills and step-by-step instruction addressing those skills to a focus on a combination of overarching concepts along with skill development. This section of the chapter also considers the instructional decision making that facilitated that change. The third section analyses Michelle's change in the amount and type of discussion she chose to use in her classroom, from whole-group recitation with small amounts of small-group collaboration to greater incorporation of small-group collaboration while Michelle checked individuals understanding. Finally, the chapter concludes with an analysis of the change in Michelle's conception of discussion and its use in mathematics teaching, from a focus on individual understanding assessed through whole-group recitation, to productive whole-group and small-group discussions with the teacher serving as facilitator.

Michelle's Initial Conception of Teaching

I accessed three data sources in order to establish Michelle's initial conception of teaching. These were the lessons she planned and implemented in her mathematics methods course prior to her internship, an initial observed lesson in November delivered early in her permanent field placement, and her reflections on commercially available videotaped lessons that she interpreted as model lessons in a teacher seminar in November.

Michelle's mathematics methods class. The instructor of the Michelle's mathematics methods course challenged his students to consider teaching through use of difficult mathematics problems that incorporated mathematical thinking and required sense making in order to solve them. That is, he suggested that a teacher should plan lessons with these problems as a centerpiece, with a period for launching or scaffolding the set-up of a problem-based activity before the students engaged in the activity. After the completion of the activity, he argued that the lesson should transition for a planned period of time to a period when students would present their solution strategies, with the underlying mathematical concepts being summarized through a discussion carried out between the students and the teacher. However, through November, when Michelle approached planning these lessons, she would first isolate individual skills from within the intended mathematical content standard for presentation within separate lessons. She then focused her lessons on the separate procedures that the students would have to know. When she implemented these lessons, she used leading questions to guide students or, in the case of a methods-class demonstration involving her fellow classmates, she managed guided recitation through which the correct answer and mathematical idea would surface. She was accepting of different solution strategies if they arose, but did not engage students in discussing the validity or lack of validity of students' different solution strategies. During this period, Michelle professed an interest in mathematics teaching that did not look like the teaching she exhibited in either her methods classes or her classroom.

Michelle's interest in student small-group work. In our first seminar, which took place in early November directly after Michelle's permanent placement, we reflected on a videotaped lesson in which students were looking at two different graphs of real-world data collected by a calculator-based laboratory. One graph displayed temperature as the dependent variable over the independent variable of time, as students removed a temperature probe from hot water and allowed it to cool to room temperature. The other graph mapped the changing height of the position of a ball over time as the students dropped the ball towards a sensor. The students in the video attempted to fit an equation to those graphs using what they knew about linear and non-linear functions and their translations. This activity did not focus on a single individual skill. Rather, it expected the students to draw on their knowledge of linear, quadratic, exponential, and polynomial functions in order to choose a model that would closely match the graph of the real-world data. The focus was to find an equation that matched the data closely, by first identifying the function type, using the features of that function in conjunction with the data points to write an equation. The teacher also expected the students to explain their understandings and mathematical reasoning as they justified the match between data and the equation.

The students in the video worked in small groups facilitated by a teacher without her direct interference into the students' thinking processes. The presence of autonomous student discussion in the video intrigued Michelle. In the seminar, she said that she drew a connection between the real-world and "messy" nature of the problem that the teacher used in this instruction as being beneficial to what she referred to as "open discussion." Michelle was familiar with problems in which a particular graph or table that mapped directly to a well-defined function rule was presented to students, with the expectation that students could write the associated rule. She felt that the fact that the videotaped lesson presented real-world data that did not lend itself to a perfect fit to a single function rule allowed students to have more conversation about how they were going to approach finding a solution. In the video, the students had to talk to each other in their groups about the characteristics of the distribution of the plotted data and to refer to their notes and their textbooks in order to determine which type of function would have the closest fit to the data. Michelle watched the videotaped students have conversations about which function would be appropriate and attributed the presence of this discussion to the real-world nature of the problem. Michelle stated:

> …the fact that [the teacher in the video] used real-world data. It wasn't giving a picture of a graph and saying "Guess the equation." She actually had data that was

> graphed and her point was really good–that data can be messy, it's not perfect. [The task] left it open for anyone to guess. [The solution] wasn't going to be a perfect guess anyway because the data was messy. But, at least she facilitated the conversation amongst the groups to figure out how we're going to guess it. Is it linear? No, it's not linear. You know, does it look like something else, something familiar? So, I think the task, like Jack said, in itself lended [sic] to open discussion (Michelle, Seminar One, November 7, 2011)

Michelle thought the messiness and the real-world nature of the data forced the students to interact with each other in order to choose the best model to fit the data, out of all the types of functions to which they were exposed. She felt that having a presentation of data that did not obviously fit a single, well-defined function allowed students to work productively together. Importantly, Michelle characterized the discussion that the students were having as *productive*. During a preliminary interview in early November, before this particular seminar, she had expressed concerns about her students' ability to work *productively* in small groups. Instead, Michelle felt that the students primarily focused on who calculated the answer correctly and often dissolved into argument rather than having productive discussion. She said that one of her goals for her students was that they would engage in productive discussion rather than:

> …debating or discounting ideas. Because I've seen instances in my classroom where one person was absolutely right and the whole class tried to shut him down: "Oh, that's not right!" I said, "No, that is!" So, it's like, "Listen." Or because it's not coming out of … [the teacher's mouth] it's not valued. You know, they're not

valuing each other's opinion. (Michelle, Preliminary Interview, November 3, 2011)

Michelle was interested in the small-group interaction in the video because the students were working productively together. This was a contrast to what she had seen during whole-group instruction in her classrooms when students would attempt to discount each other's ideas when they believed an offered answer was not correct. However, in the video, there was not a correct answer about which to argue, since the students were only supposed to use their mathematical ideas to find a model that fit closely.

Furthermore, she thought that it was important that the task incorporated not only real-world data but also the type of mathematical thinking that students would use in the field after they had finished with formal schooling. Her experience as an engineer provided her with an understanding of how mathematics was applied in industry, and she found the lesson that the teacher in the video presented more applicable to the types of situations in which students would use mathematics outside of the classroom. She stated that she "just liked the fact of using technology and graphing real-world data. It's so much more realistic from an engineering perspective, that's what we do every day" (Michelle, Seminar One, November 7, 2011). She had seen that, in her engineering profession, mathematics was less straightforward, not solely skills-based, and often messy. She was intrigued that the students in the video were using their mathematical knowledge of function and working together without much teacher guidance, and that through this interaction with the teacher's task, the students were able to fit a model that matched with the "messy" data.

The real-world nature of the data was important to Michelle, because she thought it would be more interesting to the students, rather than "sitting around talking about the Pythagorean Theorem" (Michelle, Seminar One, November 7, 2011). In principle, Michelle seemed to be interested in mathematics instruction providing students with an environment in which they could apply their individualized skills and procedures. Nevertheless, however interested Michelle was in providing students the aforementioned environment, this was not the instruction that she was planning for and implementing in her classroom.

The difference between Michelle's interest and her classroom reality. Although Michelle thought that the videotaped lesson that she and the other participants reflected on in our first seminar provided students with opportunities for productive discussion as well as interacting in realistic contexts with a wide variety of mathematics, she felt that she was incapable of implementing similar lessons in her classrooms with her students. She believed that her students would not be able to struggle through real-world problems that involved multiple skills or through problems for which a solution strategy or correct solution was not clearly evident without her, as the teacher, directly leading them to that solution. Although Michelle stated that teachers need to create a classroom culture that is conducive to this type of discussion, at this point in her teacher development, she was not deconstructing any of the multiple forms of instructional strategies that teachers might employ when developing or fostering a classroom culture in which student-centered problem solving happened seamlessly.

When Michelle reflected on the video, she was impressed with the results that she saw: students working together productively and using their resources of multiple mathematical skills to find an appropriate solution to a complex problem. However, she did not analyze the strategies and scaffolding that the teacher in the video had implemented over period of time, a time longer than the single videotaped lesson, in order to induce the results that Michelle saw in the video. Although she recognized that she only saw a portion of the class period, she did not seem to recognize that the scaffolding that the teacher put into developing this classroom culture might have gone beyond that single class period. She posited that it may have been the level of the students in the video or perhaps something that the teacher had done to set up the task. She stated, “I don’t know if it was the level of the students, or, as Jack said, the preparation, what happened before they got started. It was only 14 minutes of video, so I’m pretty sure it wasn’t a whole entire class” (Michelle, Seminar One, November 7, 2011)

At this point, Michelle was not considering the strategies that a teacher may use over a long period of time in order to create or foster a classroom culture wherein the students would learn to be able to discuss mathematics productively without direct teacher leading. When I asked the seminar group to reflect on what the video may not have shown that would be necessary to facilitate the students’ discussion in their small groups, Michelle clearly focused on the results of the lesson rather than the teacher moves that may have led to those results. Indeed, she countered, “[w]hen I watched it, I couldn’t see that anything was missing” (Michelle, Seminar One, November 7, 2011). Michelle was impressed with the results depicted in this videotaped lesson and wanted her class to proceed in this manner, however she was not thinking about the work that the teacher may have done outside of the videotaped lesson segment in order to teach her students how to interact and struggle through difficult mathematical problems.

At this point in her teacher development, Michelle seemed to believe that if students were intelligent or disciplined enough, then those students would be able to consider a problem, without prior teacher set-up or scaffolding, and subsequently be able to collaborate productively in small groups so as to solve that problem. However, her perception of her own students led her to believe that she would not be able to accomplish similar results in her classroom. She felt that “the students themselves have to have the discipline to do it,” and she believed that her “students are not there yet–to be able to take a task and run with it successfully like that without scaffolding” (Michelle, Seminar One, November 7, 2011). Michelle was either not aware of or not focusing on actions that a teacher would need to take in order to establish routines and expectations for his form of mathematical behavior in the classroom, steps that would not only set up the task at hand but would also build classroom norms over a longer period of time. As a result, she felt incapable of reproducing these results in her classroom. She seemed to feel that student collaboration on difficult problems was a more immediate classroom result than is realistic in schooling. This made this form of instruction feel out of her reach. She stated, “I wish I could do that in my class” (Michelle, Seminar One, November 7, 2011). Since she felt as if she was unable to duplicate results similar to what she had seen in the video, she relied on her initial conception of teaching when faced with the responsibility of planning lessons for use in her actual classroom.

When I first met with Michelle about planning in November, she planned her mathematics lessons by isolating a single mathematical topic, breaking it up into its constituent skills, and then leading her students through those individual skills one lesson at a time. When I spoke with her about her lesson planning in that first meeting, she was focusing on properties of exponents. She had identified each of the properties of exponents that the high-stakes, standardized state test would assess and had allocated each of those properties to individual lessons. She did this even though the textbook, which she had as a resource, included activities that would allow the students to investigate the underlying patterns that would elucidate an individual property of exponents, complete with instructions for working with another student on this investigation. Michelle controlled the lesson tightly, allowing little room for student investigation and almost no room for collaborative work. Also, after a single student presented the property to the class as Michelle walked him through his explanation using leading questions, she was satisfied that all students had discovered the pattern and internalized understanding of the pattern. This, however, was not true, as students were heard telling the special educator who was also in the classroom that they did not understand subsequent activities requiring application of exponent rules.

However, as Michelle continued to teach and to participate in mentoring and seminars from November to March, she began to attend to different details about her teaching, details that eventually resulted in Michelle successfully implementing lessons that were similar to what she had seen in the video.

Michelle's Changing Approach to Lesson Development

An Overview. This section will address how Michelle's instructional decision-making and approach to lesson development changed. Michelle and I first had the opportunity to discuss her planning for her actual classroom instruction after her permanent placement in November. When we initially spoke, Michelle was planning one lesson at a time. Although she prepared lesson plans that covered an entire week in advance, due to a school policy of submitting an upcoming week's lesson plans, those lessons did not always have an overarching theme, idea, or context with which to tie them together. She mentioned that she was in the process of teaching exponents. She had separated all the constituent parts of what she called "exponents" into individual parts. She mentioned that in the previous 70-minute class period she had covered the definition of exponents as well as ways to convert between exponential and standard form. She further remarked that she intended to isolate each of the properties of exponents needed for the high-stakes standardized test (i.e., multiplying powers with the same base, dividing powers with the same base, and raising a power to a power).

Michelle used what her textbook termed "activities" for her lessons, and these activities sometimes involved investigation, problems to solve, or applications of skills. Initially, she selected her activities directly from her curricular materials. These materials included a district-provided pacing guide, which gave direction as to lesson and unit topics, overarching content questions, suggested textbook resources, and district-written practice problems formatted similarly to the high-stakes standardized test; as well as two textbooks, one of which was a comprehensive mathematics textbook with an accompanying workbook, and another of which was a state-test-preparation book. Her main textbook, *Big Ideas* (Larson & Boswell, 2010), had chapters that were organized around a mathematical idea. Within each chapter, the authors had separated related individual or groups of skills into sections. Each section contained an essential question that foreshadowed the presented mathematical topic, one or more investigative activities, a summary of the key ideas covered by the section in both words and formal notation, and several examples that would show the application of the individual key ideas. Often the textbook would suggest completion of investigative activities in partner groups. Each section would end with a group of exercises intended for practice.

When Michelle described her lesson plan to me, she indicated that she was using the investigative activities as the main tasks in her lesson; however, when she implemented her plan, she had students work through the activities individually as exercises rather than in small groups as investigations. Throughout our work together, from our initial conversations outlined in the previous section to the end of the 5-month (November-March) period of this study, she expressed a desire to incorporate more investigative activities that engaged students in making sense of mathematical ideas; however she often felt that she was incapable of doing so. Through our mentoring sessions, Michelle progressed in her approach to lesson development from her early conception and enactment of teaching, which included mostly planning teacher-led and individual-skills-based lessons, to the development of a unit-encompassing project in which her students could engage in both mathematical concepts *and* skills with far less teacher direction.

The following sections describe that change. First, I analyze how, through negotiation of challenges and use of different instructional strategies, Michelle's lesson planning changed from solely focusing her lessons on individual skills or procedures to the unification of both mathematical concepts and skills. Second, I describe Michelle's negotiation of her self-efficacy through mentoring, leading to her development of a unit-encompassing project in which her students can apply multiple mathematical ideas in order to solve open-ended questions.

Moving from skill-based to concepts and skills. Michelle expressed a desire to incorporate lessons that allowed students to integrate and apply their mathematical knowledge in order to solve difficult mathematical problems, while collaborating with other students in her classroom. She seemed to intend to engage students in mathematical investigations in their small-groups since she used mathematical tasks from her textbook that directed students to investigate mathematical structures and properties. However, when discussing her lesson plans in our first meeting, Michelle had isolated the textbook section covering multiplying powers with the same base as the focus of her 70-minute lesson. Although her plans were part of a larger unit on exponent properties, she made sure to highlight that she would be "getting to the laws of exponents but" she was "only teaching one [law] at a time" (Michelle, Mentoring Session, November 17, 2011). Her insistence on teaching one property or law at a time was consistent with what seemed to be her initial perception of teaching as identifying and organizing individual skills that she needed to separate and present to students in a particular order. This idea of going step-by-step or one at a time surfaced often with her students, however, through her own experience she began to negotiate through her fears and challenges of approaching mathematics conceptually and in an integrated manner.

Michelle's approach to lesson development began to change because of her need and desire to incorporate more manipulatives in her mathematics instruction. In order to incorporate manipulatives we changed how we talked about lessons during our mentoring sessions, and Michelle's participation in those sessions changed as well. Another way that her approach to lesson development changed was how she negotiated her perception of her students' ability to engage with other students to solve difficult problems in a conceptual and integrated way. Finally she began to incorporate students' multiple solution strategies into her instruction, and this allowed her to incorporate more concept-based discussions that had students apply the knowledge and skills that they had been taught. A discussion of each of these changes follows.

Manipulatives. In the beginning of November, when Michelle and I spoke about planning her lessons, she highlighted activities in the textbook that would help her teach the skill that she had selected for the day. One of the first lessons that we spoke about addressed the properties involved when multiplying two powers with the same base. For this lesson, she planned to use paper-and-pencil activities from the book. These activities presented multiplication of two powers with the same base and directed the students to write each power in expanded for, and then to consolidate the resultant string into a single power. At the end of November, I spoke to her about planning a lesson on integers on absolute value. Michelle, again, chose activities from the text. These were also pencil-and-paper activities. The activities directed students to complete tables of values and use the ideas of speed and velocity in order to talk about absolute value and ordering integers. Michelle had asked me for help planning this lesson, and I suggested an open-ended task that also used speed and velocity where students would choose their own speed and velocity, complete a table using the idea of unit rate, and then come up with a story that could be modeled by their data. Although we incorporated multiple skills (i.e. integers, absolute value, unit rate, and numeracy) in the second lesson that we spoke about, it was not until Michelle encountered a site-based dilemma where we truly began speaking about mathematical concepts.

I began integrating talk about mathematical concepts into our planning discussions during our third round of meetings in December. In order to do this, I leveraged a dilemma that had come up in Michelle's teaching. The administration at Michelle's school had completed an observation, and Michelle felt that the administrator was concerned that she and Jack "weren't using a lot of manipulatives in our instruction" (Michelle, Mentoring Session, December 9, 2011). It was important to her to find ways to incorporate manipulatives in her instruction in order to conform to what she felt the administration wanted from her. Therefore, we began to speak about lesson planning in a different way to address her concern.

Since Michelle would often have to make, find, or purchase the manipulatives that she would be using in her teaching, it became useful for us to plan farther than a week ahead, in order to give her the time to secure or locate the manipulatives. Since we were talking about longer stretches of time within the curriculum, I was able to focus the planning discussions around overarching mathematical concepts. The first unit that we began discussing in this manner covered writing and solving algebraic equations and inequalities. We were able to discuss the type of thinking that students should be engaging in when approaching writing and solving equations. We discussed multiple ways to model the algebraic thinking process and incorporate manipulatives so that students were able to access their algebraic thinking in a concrete way.

When we spoke about how to incorporate manipulatives, we talked about using them to model how to solve equations by undoing the operations. I suggested that

Michelle embed solving equations in a context that was relevant to her students, such as going out to the movies, and that she could use fake money to have the students model how to undo each of the operations in a concrete manner.

Michelle related that she began her instruction on equations using algebra tiles to model solving equations. However, Michelle had opted to separate equations with a single operation from those with more than one operation. Her isolation of equations with different numbers of steps reduced the mathematical concepts of equality and undoing to unrelated procedures. Even though Michelle incorporated the manipulatives, the students still struggled with making sense of the idea of solving equations. When Michelle advanced to two-step equations with the algebra tiles, she found that students had difficulty making sense of the mathematical process that the equation, and the algebra tiles, represented. She stated, “the hardest thing for them is to identify the two steps involved and try to isolate the variables” (Michelle, Mentoring Session, December 30, 2012). Michelle found that the students still had difficulty, even when using the manipulatives, after the constituent procedures were broken up into separate pieces. Although she had experienced some success with working with the algebra tiles, she felt that the students were still having problems determining when to use a particular procedure.

However, Michelle also reflected on a situation in which she used a conceptual approach that incorporated manipulatives to make sense of a situation and then to work backwards. In this approach, she seemed to have more success. Michelle had presented the students with a real world problem that was relevant to the students’ lives: going out on a date to the movies and buying food there. Michelle set up the problem by telling the story of the two students who went out to the movies and spent $10 at the concession stand. She told them that the total amount spent was $30 and then she gave them $30 in play money to help them solve the problem. Michelle felt that the students were successful when using the money to help them solve the word problem. While the students worked, she facilitated their transition from manipulating the money to assigning a mathematical procedure to their manner of thinking. She described her students' success accessing and solving two-step equations when using the money as a manipulative.

> I also use money too as another example. …I started out a problem that had the total date cost $30 and $10 dollars was for food and how much were the two movie tickets? …So I gave each group $30 [in play money] and then they showed me they divvied up the money. And I asked them to "You see what you're doing with the money? Translate that to a mathematic[al] operation." So, for example, a lot of them, they started off with 30 and then $30 was for food, so they took out $10 out of the total 30. And I said "What are you doing? What are you actually doing when you're doing that?" And they get it, they say "Oh, I'm taking away." "Oh, what's the mathematic[al] term?" "Oh subtracting 10." Yeah. So, as they were playing with the money, I had them translate that to a mathematic[al] operation and then they connect it to solving for how much that ticket was. (Michelle, Mentoring Session, December 30, 2011)

When she used the manipulatives to allow students to approach the problem from a perspective which they understood, and not as separate procedures that required students to isolate individual operations and apply the particular procedure, Michelle found that the students had success. This may have provided Michelle with the reinforcement she needed to continue to try to incorporate more conceptual thinking into her lesson planning and her implementation.

However, the most important thing that came from our initial discussion of using manipulatives in the classroom was our continued focus on the overarching mathematical ideas in a given unit and the associated skills. Instead of solely focusing on skills, we began focusing on mathematical topics and ordered different topics so that they would make mathematical sense. For example, the unit that followed algebraic expressions and equations was a unit that was a mix of different indicators that the district called measurement. It included measurements of angles of plane figures as well as area and surface area of different figures. Michelle asked what manipulatives or hands-on activities she could use in the unit. This caused us to focus our conversation on certain topics that would be more conducive to the use of manipulatives. Since Michelle was a student of mine in a mathematics content course for middle school teachers in the previous summer, I was able to reference several hands-on activities that we used in that course. In the course, I began with the area of a square and used the square to then derive the area formula of several plane figures, either by dividing the square into pieces, as is the case with the triangle, or by treating the figure as a compound figure constructed with shapes of which the formula was already known. Through this reference, I was also able to connect the area of plane figures to surface area, as I had done in the summer course. Using nets as manipulatives, a student would be able to connect the area of compound figures to surface area. Michelle recalled these activities and planned to incorporate them, however, she intended to “precut and have shapes ready” (Michelle, Mentoring Session, January 20, 2012) in order to streamline her lesson and avoid issues with scissors.

Michelle’s participation also began to change. As we focused on mathematical topics in our planning discussions, Michelle began to realize when her district pacing guide had missing concepts or suggested an order that did not make sense. In February, after teaching constructions, Michelle reflected that in order for students to understand why constructions work they would have to have some background on circles. She had noticed that the instructional period set aside for circles and parts of circles in the pacing guide occurred after the instructional period on geometric figures and constructions. She noted that they “needed it to have [circles] in the curriculum earlier” (Michelle, Mentoring Session, February 28, 2012). Conversations about mathematical concepts and their related skills seemed to have affected Michelle’s ability to analyze curricular materials critically and allowed her to relate mathematical concepts and skills in ways that made sense. Our mentoring sessions were marked by a consistent focus on making sense of mathematics and over time this seemed to allow or encourage her to think about and organize mathematical content more autonomously. For example, when she, Jack and I were discussing plans for the unit-length project, Michelle addressed a rationale for the use of different data displays. It was important to her that there was a meaningful reason conveyed or discussed for these data displays, beyond establishing students’ proficiency in completing or constructing the graphs as a simple procedure for her students. So, she suggested a reason for the use of statistical displays. She contributed, “My thought is showing reasons why we use graphs, the different graphs. Because it’s easier to read [graphs than] … presenting … this whole page of numbers. It’s like okay, what does this mean? … Showing the numbers in that form of a graph, showing the ease of using or accessing the data" (Michelle, Mentoring Session, February 23, 2012). She felt that the accessibility of the distribution of the data in a graph was the fundamental rationale for using data displays; therefore, she felt it was important to include that topic in her project on data analysis.

Michelle's desire to use manipulatives in her instruction caused us to plan for longer periods in her curriculum. During these planning sessions, I continued to push Michelle to consider the goals or objectives of an entire unit over time and to think about mathematical topics across the unit, rather than only individual lessons. Our conversations about manipulatives led to consideration of how to group certain lessons together into the overarching topic in which they belonged. As mentioned previously, Michelle had transitioned from planning for lessons that presented single properties in isolation through using manipulatives to address the overarching algebraic theme of doing and undoing, through grouping together plane figures and connecting them to surface area, and through focusing on a pragmatic rationale for why a particular mathematics topic or concept is useful. As our planning conversations began to focus more and more on bigger mathematical ideas, it was not a large jump for her to begin thinking about intersecting mathematical topics within an overarching project. In this way, Michelle's desire to use hands-on activities and manipulatives in the classroom was leveraged to push her to think about mathematics by topic instead of lesson. This eventually led to Michelle's desire to do a project-based unit, which will be analyzed in a subsequent section.

Student ability. As we discussed mathematical content in a manner that focused more on overarching mathematical ideas and topics rather than presentation of single, isolated skills, Michelle's perception of her students' ability and the successes they would be able to experience when approaching mathematics through both concepts and skills often troubled her. In one mentoring session, I suggested that a teacher could teach equation writing and solving from a conceptual perspective based on generalization from arithmetic and working backwards to undo arithmetic procedures. I posited that then it would be unnecessary to teach solving one-step equations using addition, subtraction, multiplication, and division, and then two-step equations with combinations of those as individual elements. I suggested that initially thinking about an equation as a procedure with a result would allow students to think about what was done to the variable and then what would have to be undone to return back to the value of that variable (see Powell, 2009). Michelle responded by stating that she would separate one-step and two-step equations and then further break down one-step equations to focus on those that involved addition and subtraction as a single skill, and then subsequently address those one-step equations that involved multiplication and division.

> MM: Well, I don't know, I probably would [my class].Maybe I wouldn't take two days for each [operation]… I can combine addition, subtraction and then multiplication, division.
>
> JD: I can see that.
>
> MM: I would put those together but I would do step-by-step. I wouldn't combine them all at once and do several different operations with mine [my students]. So,

> okay, well I’m happy that it gives us a time to teach conceptually in step-by-step with the operations. Okay.

In this explanation, Michelle immediately followed her statement of preference for isolating addition and subtraction one-step equations from one-step equations involving multiplication and division with the statement that she was happy to have the time to teach conceptually. This suggests that although she wanted to use the aforementioned manipulatives to have the students visualize the individual operations, she still felt the need to isolate a single skill into a lesson. When I suggested that she could address solving equations as one concept, she resisted. She stated, “Yeah, you probably could do it in your class” (Michelle, Mentoring Session, December 9, 2012), referring to Jack’s class. This highlights her concern about her students’ ability level, since the school labeled Jack’s class as an honors class and labeled Michelle’s class as comprehensive, ostensibly populated by students with a lower level of ability.

It may be that Michelle was experiencing some dissonance between her perception of what she felt her students were able to accomplish successfully and what they actually were able to do. She had been seeing her students have successes in the classroom on problems that she felt might have been too difficult for her students to approach. She had stated previously, “Sometimes you’re kind of leery in giving them stuff and then run off on their own. But sometimes they surprise you” (Michelle, Seminar Two, November, 21, 2012). The successes that she was seeing her students experience in the classroom challenged her previous ideas regarding the limitations associated with perceptions of her students’ limited ability and allowed her to try more difficult mathematics problems with her students.

Not only did Michelle begin to incorporate more difficult mathematics problems into her teaching, she also began to include more small-group work. She had students work together on real-world problems and allowed students to reason through the problems concretely before she asked questions that would help them think about the mathematical ideas behind them. She found that her students were successful in translating their thinking into mathematical terminology.

> So, for example, a lot of them, they started off with 30 and then $10 was for food, so they took out $10 out of the total 30 and I said, “What are you doing? What are you actually doing when you’re doing that?” And they get it! They say, “Oh, I’m taking away. Oh, what’s the mathematical term [for that]? Oh, subtracting.” (Michelle, Mentoring Session, December 30, 2011)

Michelle was happy to see that her students were “getting it” when they were approaching word problems, as Michelle had initially considered word problems to be very difficult for her students. She saw her students have success working on a problem in their small groups without her intervention and without her leading the students through the whole process of problem solving. These instances of students working together in order to solve difficult mathematics problems allowed her to feel that she could take more risks in her mathematics classroom. She began to change her perception of scaffolding. Originally, she thought of scaffolding as the practice of offering leading questions that directed students through the intended individual procedures and skills in the classroom. Now she was scaffolding by asking guiding questions that facilitated students’ mathematical thinking while they investigated problems using concepts and ideas.

Michelle seemed to use what she knew or learned about student difficulties in a lesson as an insight for influencing her future instructional decisions, instead of abandoning her plans entirely. Michelle taught a mathematics course and a science course to the same students. In an attempt to engage her students in a conceptual discussion about different systems within the body, she had given her science class a homework assignment that required them to read about a certain system in the human body and then come prepared to discuss their knowledge with the class in small groups. However, she was disappointed that her students did not come to class prepared. She assigned blame to the students for not doing the homework assignment and therefore for sabotaging a lesson that would have focused on discussion. She noted that, because of their lack of preparation, the students simply did not have anything to add to the discussion.

> I don't know. I had this nice thing planned, and it's one thing too when you're trying to have group work and facilitate discussion, you have to have something to add. I had a homework assignment, they were supposed to investigate things that they identified that they were going to do. If half the class doesn't do that, it's like I wanted to rotate them so that they would (inaudible) each other and provide information for one another. It's like, when they don't do the task, it just shuts the discussion down. Okay, you've got nothing to add. You know, they'll just be sitting there. (Michelle, Seminar Five, February 1, 2012)

She expressed frustration with her students for their inability to participate in the discussion. In this case, it seemed that the students were not able to participate in the discussion because of the individual, at-home nature of the preparatory assignment. Michelle seemed to recognize this, as she did not give up on having conceptual-themed discussions with this group of students. Instead, she modified her instructional strategies to allow time in class for students to formulate their responses in their small groups before engaging in the discussion. Subsequently, she tried again to organize a class based on the assumption that students would prepare for class in advance. But, this time Michelle did so with her mathematics class. In a unit on data analysis, she provided her students with a list of scenarios and asked the students to decide which data display would be the most appropriate in the given scenario. She also provided them with a reference sheet of key terms, definitions of those terms, and their exemplar applications, in order to allow students access to a discussion where there could be multiple correct responses to a single question. Instead of Michelle's initial scaffolding design, which was to assume control of the conversation and carefully guide the trajectory of student talk in the classroom, she positioned the students centrally in the discussion. She provided the students with a means of facilitating their discussion through their reference sheet and then allowed them to express their thinking to each other, only providing direction and comments when necessary to continue the flow of discussion. She began to place herself in a position of facilitator rather than director of classroom talk. After Michelle made these different instructional adjustments, she saw that her students were could successfully participate in a conceptual discussion.

> And we're at the point [where] we're talking about which data display to choose and why, depending on the circumstance. So I had two scenarios, and I asked each group to talk amongst themselves to determine which data display they'll choose for [each of] the scenario[s]. And so, once they did that I opened it up to the floor. … and I called a group, a table, and they shared their response, and I

> said, "Okay, who agreed or disagreed and want[s] to add to it?" So, that spawned a lot of discussion. "Oh, look, I disagree because this, this and this." And they were able to use the terminology of why … and recommend something else. And so, that kind of went on back and forth for a good 10 minutes, and it was a really good discussion. So I got to hear what they were thinking. So it was really good. Positive. (Michelle, Seminar Six, March 7, 2012)

Although Michelle had experienced earlier failures when expecting the students to prepare for and participate in whole-class discussions that focused on mathematical concepts and had multiple solutions, she had tried again and found success. She negotiated this dilemma by providing not only class time to prepare in their small groups, but a reference sheet to remind students of the mathematical tools that they could use in order to respond to the tasks and to explain and justify their reasoning and solutions. This provided her with another example of student success in her classroom.

Throughout the year, Michelle saw more successes in terms of her students' ability to access different mathematical problems in her classroom as she changed the types of scaffolds she provided. Instead of leading students through systematic procedures that she considered difficult for her students, she began to facilitate their interaction with mathematical concepts and their constituent skills. Where earlier in the year she would stop small-group time to demonstrate solutions to the exercises that the students were working on, later in the year she provided more indirect scaffolding and allowed the students to work together in order to solve problems. She discovered that her students were capable of working with other students on difficult mathematics with less teacher direction. She reflected on her surprise and happiness about her students successes: “They do it and then you’re shocked. … It went very, very well” (Michelle, Seminar, March 14, 2012). She seemed to continue allowing students more autonomy in their small groups as a result of her negotiation of this dilemma, in addition to allowing her students more opportunities for working together.. Furthermore, instead of removing instances in which she thought her students would struggle, she made different instructional decisions to support her students’ efforts to working through not only mathematical skills, but also the meaning of concepts \.

Multiple solutions or multiple solution strategies. One approach that was important to Michelle was allowing students to solve problems using multiple solution strategies. She first mentioned this strategy while reflecting on a video in our seminar. In this particular video, a Japanese teacher had put several problems on the board in which students would have to solve for a particular angle in a pair of parallel lines. The key to solving these problems was to extend different lines or to draw additional lines on the diagram in order to represent the diagram’s properties in ways that supported application of a theorem that would then permit determination of the desired angle. When reflecting on this video, what was important to Michelle was that, in her interpretation of the event, the teacher wanted each of his students to “ ‘Do it in the easiest way, not one way, but the easiest way for you’… instead of pushing them down one lane of solving” (Michelle, Seminar Two, November 21, 2011). Michelle did not force her students to solve problems using one particular procedure. If a student could demonstrate another mathematically correct way to solve a problem, Michelle would validate that student’s work and suggest that he continue to use the strategy with which he was most comfortable when solving the problem.

Initially, Michelle had included multiple solution strategies in her classroom by simply validating the different ways that her students used when they approached a problem or by allowing students to present several different solution strategies. She did not mention, nor did I witness, an instance where she challenged students to say why a student could solve a particular problem with different methods. However, since she was interested in students possibly approaching problems from many ways, we became intentional in our mentoring sessions by discussing different solution strategies for problems. We considered different ways to conceptualize writing and solving equations that allowed students the space to make sense of the mathematical thinking that went into undoing operations. We incorporated this strategy into our talk about the underlying mathematics, as Michelle and I began to discuss inclusion of more conceptual thinking in her planning for lessons that were focusing on individual skills. When Michelle was thinking about how her students could approach writing and solving equations, we not only talked about sense making, but also about the many problem-solving strategies that a student could use to solve an equation in one variable. These strategies included use of manipulatives, mental mathematics, diagrams such as flow-charts, and traditional symbolic ways of solving. Michelle incorporated many of these strategies into her instruction.

> And some people do the negative-positive… cancellations and putting pairs together and some… try to cancel it and subtract it, you did the opposite or inverse operation on both sides. … We … use[d] the algebraic tiles to model that and then … I also showed them your method of drawing the circles and then you connect the circles with the arrow showing one way, which operation you did, and

> then go in the reverse way on the opposite. So I used that representation… (Michelle, Mentoring Session, December 30, 2011).

Michelle began to show her students many different ways to model their thinking and allowed the students to choose which method was the most beneficial to them. She said that these different representations of each student's thinking were beneficial to her as a teacher. She stated that "it gave me a representation on how they solved" (Michelle, Mentoring Session, December 30, 2011). Giving students access to many different representations of the underlying mathematics not only helped the students establish different ways for solving problems, the representations also gave Michelle access to each students' thinking.

Michelle was able to continue incorporating the idea of multiple solutions into her classroom and her facilitation of discussion. She used particular instructional moves in order to cause students to confront, consider, and accept not only multiple solution strategies, but also the possibility of multiple correct solutions. In order to do this, she presented her students with questions that either had multiple ways for answering the question or multiple correct answers. Later in the year, she recounted a whole-class discussion in which students were presenting their information, and agreeing and disagreeing with each other's solutions. Michelle had presented each small group with a situation in which they would have to use a graph to display data in order to answer a specific question. The students had time to talk about these questions in their small groups and then spoke about their conclusions with the whole class. Michelle described her incorporation of multiple solutions: "Sometimes it could be more than one data display for a certain situation" (Michelle, Seminar Six, March 7, 2012). For Michelle, the important criteria that she expected her students to meet was to interpret a mathematical representation, to verbalize their conceptual understanding of the underlying distribution or relationship between the variables in the display, and to explain their thinking, rather than to simply identify the solution. This instructional intent was evident in the design of the questions to which the students were to respond, as these questions intentionally permitted multiple possible solutions. Therefore, the students were expected to use their knowledge of mathematics to justify their solutions. When describing how her students discussed different solutions to a particular question, Michelle reflected that her major contribution to the discussion was to share the judgment that multiple responses could be equally correct. This further reinforced the idea that multiple solutions and multiple solution strategies could be valid in a mathematics classroom. This is evidence that Michelle was no longer only teaching in a strictly procedural style, since she structured her class so that multiple strategies and multiple solutions were valid in a discussion about a particular mathematical topic.

Project-based planning. One of the major turning points for Michelle was her admission of feeling unable to plan a project that encompassed an entire unit and allowed the students autonomy. In November, she had discussed a feeling of inability to incorporate any autonomous productive student discussion in her classroom. However, as she continued teaching, she began to feel more comfortable incorporating different strategies and approaches with her students. She expressed in January that she felt that she had reached a point where she would be able to “really try to implement some things that I’ve been wanting to do, now [that] I’ve got a handle on my students” (Michelle, Mentoring Session, January 20, 2012). The more experience and success she had in the classroom, the more adventurous she became with incorporating difficult mathematics and expecting student discussion in her classroom. During a seminar in February, Michelle mentioned that she was very interested in designing and implementing an overarching project that would encompass all of the mathematics for an entire unit. However, she did not feel that she was capable of designing a project-based unit on her own.

> Right. So is there some project-based type of learning where you could tie [all of the mathematics] into it? I think, it [could] address everything we're saying…You know it's like my mind isn't there yet to be able to do that on my own…But if something exists where it's just a … project. Piece-to-piece where it's building up to something. It would be wonderful. (Michelle, Seminar Five, February 1, 2012)

Michelle was looking for a project-based sequence of lessons that would require students to demonstrate performance-based understandings, but she wanted an already developed sequence of lessons so she could use the lessons, as is, in her classroom because she did not feel that she would be able to design a comprehensive project by herself. She reiterated, stating, "I just don't have the experience" (Michelle, Seminar Five, February 1, 2012). She felt her inexperience with mathematics teaching made her unable to plan a project that would productively address the mathematical content in which she was expected to engage her students. Therefore, I suggested that the mentoring sessions could serve as a setting wherein Jack, Michelle, and I could work together to plan this type of project. Michelle, Jack and I chose an upcoming unit from the curriculum pacing guide. Since units in this particular pacing guide were often supposed to be covered in a matter of days, we chose a unit far enough in the future so we would have time to think through and to develop a project plan that would incorporate all of the mathematical concepts of that particular unit, which included data analysis and data displays.

During this planning, Michelle, Jack, and I went through and read the unit plan from the district curriculum, noting the isolated skills and concepts that the unit contained. I helped direct their efforts to reorganize the content so that it would make more sense mathematically and also would be more accessible to the students. We discussed topics that the students would have to think through and understand, if they were to make sense of mathematical relationships in order to make informed decisions. We also discussed the types of questions that would push the students to do some thinking and use the content knowledge that they were specified as learning objectives in this unit. The curriculum guide stated that, within the period of this unit, the included topics were: central tendency, spread, histograms, bar graphs, line graphs, pictograms, appropriate graphs for a given scenario, and misleading graphs. We decided that the topics of central tendency and range should be addressed after students evaluated data displays so that students would be more likely to have developed accessible knowledge addressing why the mean, median and mode described the center of a data set, as well as why particular measures or descriptive statistics would be appropriate to use in different situations. We also decided that the data set used in this unit would be consistent across lessons, and that it would be contextualized, so that students could see data in multiple representations and could meaningfully discuss the similarities and differences of the graphs. They chose the context of data reflecting past scores from a high-stakes assessment as Michelle and Jack felt that this context would be accessible and relevant to all of the students in their class. Furthermore, we discussed crafting data sets that would illustrate intended relationships and allow consideration of specific mathematical characteristics. For example, these included data sets that introduced situations of skewed distributions or distributions that were difficult to compare point-by-point because of outliers or long-tailed distributions.

After talking through the general design of the project, Michelle worked on her own conception of the project and the sequence of lessons presenting the project (Appendix A). Michelle showed great growth in her instructional decision-making with regard to productive student discourse during the development of this project. Michelle, Jack, and I collaboratively discussed the major mathematical topics that the project would include, and, because Michelle and Jack shared a classroom, they would both be using a project for the same unit during the same time-frame. However, Michelle developed her project independently of Jack and me. While Michelle's project and Jack's project each included the same content and were situated in the same context, their projects focused on two different important features of data analysis. Whereas Jack encouraged his students to focus on data displays that could be used either to represent data correctly or to misrepresent data intentionally, Michelle focused her project on developing her students' understanding of which data display would be appropriate to answer a series of questions and why. Although some of her questions directed students to answer the question using a particular data display, many the questions that she included in her project packet allowed students to choose any display as long as they could use it to answer the question. For example, Michelle asked students to answer questions such as:

- "How well did your class perform on the [high-stakes standardized test]?"

- "Did the students' [high-stakes standardized test] scores improve from year to year?", and
- "Was there a difference in performance between your male and female students?" (Michelle, project document, March 20, 2012).

While Michelle incorporated the mathematical topics that she, Jack, and I had discussed, she created a project that took a unique approach. She developed this project document on her own, incorporating open-ended questions that focused on an important overarching mathematical concept. Furthermore, these questions provided an opportunity for student discussion within groups and multiple solution strategies between groups.

In summary, Michelle changed her approach to lesson development by successfully negotiating her feelings of inexperience as she worked with Jack and I during mentoring sessions and developed an overarching project on which the students worked in their small groups for several days. Due to Michelle's desire to use hands-on activities and eventually to develop a project, Michelle and I were able to discuss the overarching mathematical ideas around data displays and analysis in our mentoring sessions. Instead of working on individual lessons with single skills, Michelle was able to build up to, and incorporate, a broader mathematical idea into a project-based unit, which required students to use both skills and concepts to answer questions. Michelle's approach to lesson development and her instructional strategies had changed a great deal.

Michelle's Changing Use of Small- and Whole-Group Discussion

An Overview. In November, Michelle used a combination of whole-group and small-group talk to achieve certain goals in her lessons. She used whole-group talk in three settings. First, when calling on students to share with the entire class a recitation of the steps used to arrive at a solution to a warm-up question. Second, when she was setting up or presenting a task that she subsequently would expect the students to complete. Finally, when students were explaining to the class how and why they used mathematical procedures to complete the given task. She organized her class instruction in small groups when she provided time for the students to work together on a presented task. In the beginning of her permanent placement, Michelle allotted 5 minutes for each instance of small-group talk, and the mathematical tasks being addressed often did not require nor provide opportunity for discussion. For example, when I observed her classroom in November, the students worked in small groups to evaluate and simplify an exponential formula and to use the calculator to determine a numerical solution to an exponential expression. In this particular lesson, she used the investigations in the textbook as the source for her lesson's main task. However, despite the fact that the students were positioned in small groups, when she assigned these investigations she told the students that they were expected to carry out their work individually and silently and not to interact with each other about the tasks.

During a second observation of Michelle's instruction in January, I noted that she had modified her approach, in order to address what she viewed as students' inadequate response to the cognitive demand of the task. As previously observed, she allotted the students 5 minutes of time to carry out small-group work in order to solve problems that required the application of geometrical definitions, but she quickly co-opted their small-group discussions when she felt that students were struggling. Instead of scaffolding the task or using questioning to foster student analysis of the task as they worked to make sense of and solve the task in their small groups, she interrupted their small-group work, calling the students' attention to her as she demonstrated the procedure for solving all of the assigned problems in front of the class.

However, when I conducted a final in-class observation of Michelle's instruction in March, I noted that she had students working in small groups for 34 minutes of the 70-minute class. Michelle primarily used whole-group instructional time as an opportunity for students to present their findings and to talk about the overarching mathematical concepts of the work that they had done. Furthermore, while in seminar, Michelle spoke of the conduct of some whole-group discussions in her classroom during which the students interacted with each other on the mathematical topic by responding to each other's contributions rather than solely engaging in recitation with Michelle. Throughout the course of this study, Michelle encountered challenges with her conception of discussion and addressing student struggle which she negotiated using different strategies. Through this negotiation, she began to incorporate more instances of productive small- and whole-group discussion in her classroom. Michelle developed instructional strategies that transitioned the student conversations in her classroom from recitation to discussion, helped to negotiate her challenges through assigning roles so that she felt able to assign small-group work, and allowed her to negotiate students' struggle during small-group time rather than through teacher-led demonstrations.

Recitation versus discussion. The type of classroom talk that Michelle orchestrated in her classroom in November in was that of whole-group recitation, rather than whole-group or small-group discussion. She originally characterized this whole-group recitation *as* student discussion. She would allot a small amount of time for the students to work on problems either by themselves, which was initially the most prevalent occurrence, or in small groups defined by who was positioned at a table. She would then call the groups together and use equity-sticks as a method to call on students randomly, expecting them to explain their work at the board. She would prompt the students through their verbal explanations by using leading questions. Once a student had completed this directed recitation, that student would take his or her seat and she would call on the next student in order to explain the next problem or question. As the year progressed, Michelle and I discussed several challenges that seemed to be preventing her from allowing the students to have discussions in the classroom: her own self-efficacy, her need for individual knowledge and contribution, and her perceptions of students' facility with vocabulary. Her negotiation of these challenges contributed to her change in her use of small- and whole-group discussion.

Self-efficacy. Michelle had expressed a desire to have students engaged in discussion in her class and, at the outset of her permanent placement, mentioned that she felt unable to do so. Michelle's lack of efficacy again emerged in a mentoring session conducted after my first observation of her teaching in November. While reflecting on this lesson, I remarked to her that students were not engaging productively in discussion. Furthermore, I reflected that she was doing most of the work in the discussion, as she was asking all of the questions and carefully leading the students' talk in the direction in which she wanted it to go. When students made mistakes in their mathematics or in their procedures, Michelle would be the one to indicate that the student made a mistake. Of greater concern was the fact that when other students tried to suggest corrections to a student's work, Michelle would quiet their interjections. When I mentioned these observations to Michelle in debriefing and mentoring sessions, it became clear that her conception of discussion was different from the one that I maintained. When I referred to the type of interactive discourse that she could have potentially facilitated with her students during implementation of the lesson that I observed, she stated, “I thought that’s what I was doing” (Michelle, Mentoring Session, November 17, 2011). She believed that by requiring students to demonstrate the procedures in front of the classroom and by asking them questions to guide their recitation, she was engaging students in a whole-group mathematical discussion.

Initially, Michelle was not aware that she was dominating the discussion time in her classroom. When we debriefed after my first observation of her classroom in November, I stated that she was doing the majority of the talking during whole-group discussion time, as she asked all of the questions and evaluated all of the responses. I reflected that even though she prompted students to prepare questions for the presenter, she did not allow time or encourage other students to participate in the one-on-one recitation in which she and her selected student engaged. When I described these classroom instances to Michelle, she stated, “I think I’m missing that. So yes, I’ll look at what I did yesterday on the video [I recorded] because I don’t see what you just said” (Michelle, Mentoring Session, November 17, 2011). Not only did she believe that the recitation that she was facilitating was synonymous with guided whole-class discussion, she was unaware that the teacher moves that she made would stymie the whole class’ involvement in discussion. She felt that she had been engaging the other students in what she interpreted to be a whole-class discussion through prompting the students who were not presenting to be listening and to be ready with questions to ask. However, the students did not ask any questions, and the interaction remained a recitation rather than a productive discussion.

This interaction during the mentoring session seemed to cause Michelle to reflect on her own feelings of self-efficacy with respect to leading or facilitating student discussion. Although during our debriefing session directly following her implementation, Michelle seemed to push back on my assessment that her facilitation of discussion in her classroom was lacking, she contacted me via email the following day requesting help with future efforts to support and orchestrate discussion. She intimated her awareness of our difference conceptions of facilitating discussion and requested to see my interpretation of discussion in practice. She wrote:

> I have an idea to run past you. I would like to invite you to teach one of my math classes so you can model facilitating group activities and discussion. This will give Jack and I an opportunity to see it in action so we are better able to implement it on our own. Let me know if and when you are available. (Michelle, Email Communication, November 18, 2011)

Upon reflection of our conversation about the meaning of discussion versus recitation after my first observation of her class, Michelle requested assistance in her classroom so she could observe an experienced teacher's efforts to support and foster student whole- and small-group discussion. She wanted to have an understanding of what I meant by facilitation of discussion. Her request to "see it in action" intimated that she felt she had too little familiarity with what productive discussion looked like in practice and that may have inhibited her ability to enact it on her own. In response to that email request, we co-planned a lesson over the next two weeks and then co-taught that lesson. After the lesson concluded, Michelle reflected on our episode of co-teaching by stating that the "discussion went well" (Michelle, Mentoring Session, December 1, 2011).Subsequently, Michelle expressed more comfort in facilitating discussion with her students during a December seminar. She reflected that after our co-teaching experience, she saw her students more engaged in their small groups. She also mentioned that her students were speaking to each other more respectfully during their discussions.

> MM: … Of course you came into our classroom to help with the discussion piece which was very good as far as the students asking each other questions or correcting each other without having…
>
> HD: Are you having success with that?
>
> MM: Yeah, I mean they're [discussing] in their own way. That's when I see them talking and they are on track and they are engaged. I'm happy. And you know, they're not always polite, but you know, they're getting there. (Michelle, Seminar Three, December 4, 2011)

At this point, instead of feeling that she was not sure what discussion "looked like," she reflected on her students talking about the task and being engaged in their work with their peers. She mentioned that she and the students were making progress on incorporating productive discussion in her mathematics class, and she indicated that this progress was, at least in part, because of our co-teaching episode.

Individual knowledge. One of the challenges that initially inhibited Michelle from using discussion in her classroom was her desire to be able to assess students' individual knowledge. She had expectations for individuals and sometimes felt that she could not accurately assess student knowledge when students were working in groups.

Initially, Michelle did not find small-group discussion important, when compared to her need to set and evaluate individual expectations for students. She felt that when students worked individually she could ensure that each student had internalized the mathematical content covered during the lesson. Furthermore, she felt that if she tightly controlled the whole-group talk and led students toward the expected or desired answer, every student would understand and know the presented content.

In my first classroom observation in November, Michelle was delivering a lesson that I considered to have the potential to be investigative, in which students would write out powers as repeated multiplication problems. They could then follow this up by writing the multiplication of powers as repeated multiplication problems and then simplify the resultant string of multiplied bases in one power. Through this intended lesson in the textbook resource, the students were to discover the property of multiplying two powers with the same base. Michelle had the students do this individually in the warm-up, and when she had students present their findings from that activity, one student noticed, with the help of Michelle's leading questions, the pattern of adding the exponents and presented it to the class. After this single student explained the pattern that he noticed about adding exponents when multiplying two powers with the same base, Michelle prompted another student to revoice what the original presenter had said. When this student did not do so using vocabulary that was satisfactory to Michelle, she called on other students to revoice the explanation until she heard the answer she desired. After this episode of whole-class recitation was completed, she stated "So here, see you all taught the class; I don't have to teach anything. … Add the exponent, keep the same base …." (Michelle, Classroom Observation One, November 17, 2011). Since she had a few students summarize the property while the rest of the class was silent, she assumed that the teaching episode was ended and that the rest of the students knew and understood this information. Since Michelle had tightly controlled the whole-group talk, what actually transpired what that a single student presented the correct mathematical property and other students revoiced what the presenting student had said. Nevertheless, the verbal presentation of the property by the student was evidence to her that she had fulfilled her responsibility to require students to investigate and discuss this property.

After the warm-up, she then had the students individually work on what was supposed to be the further investigation into this pattern. Instead of having them investigate the single student's statement as a conjecture, she assigned them more problems, requiring them to demonstrate use of this pattern as practice. When asked about this particular structure in the lesson, she replied that having individual students record the information on their own was more important than having them work together to investigate the property further. In addition to mentioning that she felt the class had learned the expected information during the short explanation period after the warm up, she offered a few other reasons for having the students work individually. These included both a desire to not lose valuable classroom time on what she through was unnecessary or inefficient reiteration and a desire for students to work on and have their own information. Because Michelle had seen a student present the information that was the goal of her lesson, she felt that there was no pedagogical reason to spend more time investigating this mathematical property. It was, she posited, a property that the student had already heard and internalized. She felt that doing further investigation would "[take] too much more time so [she and the class] moved on" (Michelle, Mentoring Session, November 17, 2011). To Michelle, her students' individual understanding was the most important expectation, and since she believed that her students' learning had already taken place, she poignantly mentioned that it "wasn't a big deal for [her] as far as [the students] working together" (Michelle, Mentoring Session, November 17, 2011). She did not see the benefit of small-group work as a contributor to individual student knowledge, and therefore it was not as important to incorporate opportunities for small-group student work in her teaching practice as compared to whole-group recitation and individual student work.

Furthermore, Michelle seemed to feel that if the students worked in groups, they all would not have the same access to the mathematical knowledge and information as they would have if they worked individually. It was important to Michelle that each student have a copy of the class work for reference in order to be able to complete a practice assignment at home. She explained,

> …although the book said work in partners I wanted … each individual person to have it [a record of the exponent properties or rules] and they fill out themselves because on the flipside of it, it's their homework, at least they will have a reference. … So everybody did it on their own so when it's time for homework …. (Michelle, Mentoring Session, November 17, 2011)

She was afraid that if the students worked in groups and shared their information on a single work product, they would not have access to the necessary reference information when they were expected to complete similar work on their own. Her major concern was for students to come to know the information on their own and to have their own recording of their work to use for reference when completing their homework. Individual student knowledge was clearly important to Michelle, and it limited the type of discussion that Michelle incorporated into her classroom early in her teaching experience.

Michelle clarified how important individual knowledge and understanding was to her when she reflected one of the commercially available videos in our seminar later in November. In this particular video, a Japanese teacher had students create a problem where they would have to solve for a missing angle in a set of parallel lines and transversals by extending an existing transversal or placing an additional transversal in their diagram. The activity in which the Japanese teacher engaged his students was for the students to create problems individually that were solvable using these previously discussed methods. Following the creation of these problems, the teacher asked them to solve their own problems individually to make sure they were solvable. Following this, the teacher asked the students to pair up with a group of three additional students in order to choose the problem that was the most difficult among them to pose to the class. When I asked the group of teachers what the teacher in the video could have done to promote more small-group discussion, Michelle reiterated her desire for individual thinking, specifically differentiating between what she called *individual thinking* and *group thinking.* She privileged individual thinking over thinking that happens in a group, especially in the beginning of the activity. She felt that, like her, this teacher's goal primarily was individual thinking, rather than the type of collaborative thinking that she felt happened in small groups.

> I don't know if [the teacher] wanted to, I mean what was his goal in that lesson? He didn't want to take that time. And then later on, they came together and looked at their problems and choose the hardest one to present. So I value that. I wanted

> them to think individually first, share their ideas and their thinking, not group thinking. I want individual thinking at certain times, especially in the beginning, and then you share your ideas and then you're supposed to pick which one is the hardest like they did in the video (Michelle, Seminar Two, November 21, 2011).

She valued the individual thinking that she thought working together in small groups might mask. She was concerned that working together might cause any individual student to value the thinking of another student over their own, regardless of whether their own thinking was correct. She thought it was important for students to develop their own thinking before the thinking of the group co-opted their process.

Furthermore, Michelle was quite concerned that some students in the group may give another student an answer, rather than explaining their thinking to that person. Therefore, that student may not actually know the information but may just have regurgitated what another student in his group had said. This was a concern to Michelle because she felt it was part of her duty as a teacher to be able to assess a student's knowledge accurately, and she felt that it might not be truly possible to do this when the students could be relying on each other to form an explanation.

> How can you tell who knows, and who really knows because they're in a group and they can just say, "Say this." I love the group thing, but when it comes down to individual, I need to know what that person knows. And it's hard to gauge sometimes in groups because of their dynamics. They want to help each other but just in the wrong way. Just give them the answers. (Michelle, Seminar Five, February 1, 2012)

This statement offered after 3 months of permanent placement indicates that Michelle's perspective of discussion and its use had changed somewhat. Instead of dismissing discussion entirely while pointing out her need for accurate assessment, she intimated at the same time that she felt that group work could be useful if students were able to help each other in the *right way* as opposed to what she categorized as the *wrong way*. If she was going to incorporate small-group discussion in her class, it was important that her students explain the work while they were helping each other instead of simply providing their group-mates with final answers without engaging each other in sense making.

Michelle's priority was to know what every single student in the group actually knew on their own. She also wanted to make sure that students understood and completed assignments instead of simply talking. In her classroom, Michelle always had her students seated in groups of four, so there were opportunities while the students were working on an assigned mathematics task for them to talk to each other as well as the risk that the discussions that her students were having were not achieving the mathematical outcomes that she intended. Her desire was productive discussion that resulted in individual understanding. If the discussion that was happening in her classroom did not further her agenda of individual learning, it was "not her focus." When asked about methods for facilitating discussion, Michelle replied:

> That's not my focus, as far as what can I do to have them discuss. They talk a lot. My thing is, how do I get them to learn from each other? You know, just learn the concept. So, when you ask that, it's like, "That's not at the top of my list." I mean, it kind of plays into it, [but] I'm not looking for ways to make them talk. It's

> mainly understanding things and completing things. I haven't really thought about facilitating discussion. (Michelle, Seminar Five, February 1, 2012).

Michelle acknowledged that her students talked to each other a great deal in her classroom; however, it may not have been yielding the desired results. Getting them to talk, or discuss, more was not a priority for her. Instead, getting them to learn, complete, and understand an assignment while collaborating was more of her focus. However, when Michelle described her priorities, she indicated that her views on discussion were evolving. She clearly stated that in her view it was important for the children to "learn from each other," instead of solely learning individually or from the teacher. Therefore, small-group discussion seemed to be beginning to have more of a value for her in the sense-making process, although her students did not necessarily discuss or carry out discussions in what she conceived as "the right way."

Through discussions in mentoring sessions and in seminar, Michelle began to develop a teaching strategy within which to negotiate her pedagogical conflict between her need for individual understanding and her desire for students to learn with and from each other: She expected the students to think first on their own and then to check their answers with their group members. She did this in several ways. First, Michelle gave students a short amount of thinking time before sharing their work with another person. Second, she gave students an assignment at home in order to prepare for a group discussion. Third, she had the students complete an individual brainstorming session in which they recorded their ideas on sticky notes and affixed them to the group's document before proceeding to negotiation as a small group. Her attempts at implementing this strategy were variable, but by March, when I observed her classroom for the last time, she had honed her strategy of requiring students to think on their own and then collaborate with their group members to discuss and verify their results.

During this observation, each student in a group was expected to complete their own calculations and then to share their solution and solution strategy with the rest of their small group. Michelle facilitated this type of student discussion by consistently asking students to “make sure everybody at [their] table get [sic] the same answer” (Michelle, Classroom Observation Three, March 22, 2012). This approach required the students to do their own work *and* to share their solutions with their group mates. If the solutions did not match, the students would have to share their strategies to find where their work had differed, resulting in differing answers. Using this strategy, Michelle could encourage students to analyze each other’s work and to engage in error checking. She had discovered a strategy that was acceptable to her, as she could have students participating in small-group discussion and be sure students were doing and understanding their work individually.

The second strategy that Michelle attempted in order to negotiate her dilemma of evaluating individual learning versus small-group interaction entailed students completing an individual fact-finding assignment as homework, an assignment that would ostensibly prepare them for engaging in a small-group, fact-sharing discussion during the subsequent class. She realized that expecting students to prepare at home for a small-group discussion might not be the most beneficial of implementation as “half the class [did not] do that” (Michelle, Seminar Five, February 1, 2012). If the students did not complete the home assignment, they would not have anything to contribute to the discussion, and therefore no worthwhile sense making would take place during class.

Instead of discarding this strategy, she modified this strategy to include in-class thinking time prior to collaboration. This way, she could ensure that students completed the activity by holding students accountable for producing a document that represented their individual ideas. She used this modified strategy during her project-based unit in March. She distributed the questions that the students would be required to answer to complete the project and had them think individually about how they would approach each question. The students would them document their thinking on sticky notes which they then affixed to the project paper for reference during their collaboration time.

During February and March, as her conception of use of discussion changed and she began to incorporate different strategies to facilitate discussion, Michelle began to use more small-group discussion in her class. She discovered that these small- and whole-group discussions were beneficial to her pursuit of individual understanding. She found that when she allowed the students to work in small groups, she was no longer required to be solely positioned in front of the classroom leading a recitation of results. Since she had more time during her class period where she was not leading the class, she was able to recognize and address student misconceptions within their work while circulating through the class. She noticed that when she required students to work in small groups on their project, she "was able to work with people one-on-one" (Michelle, Mentoring Session, March 22, 2012), which she realized she had not been able to do when she designed lessons that were more teacher-centered and recitation-based. Instead of stating that small-group work was not a priority for her and was not important, as she had said in November and February, she now in March stated how the use of small-group work benefited her as a teacher. She realized that when she circulated among the small groups she was able to find and correct student mistakes and to help ensure student individual understanding.

> I like the discussion. For the first time, I told Mr. Davis, I said, “I love it because I get to interact with the students I normally don’t get to interact with one-on-one.” So I’m going around touching, or interacting with each one of my students and seeing what they’re doing. (Michelle, Seminar Seven, March 14, 2012).

Michelle utilized the strategies of allowing students individual think time before collaboration in addition to requiring students to verify their work with their group mates. This allowed her to integrate the use of small-group work with her desire to focus on individual understanding. When she began to incorporate more use of small-group organization during her instruction, she realized that having students work in their small group and discuss the work amongst themselves allowed her more time as a teacher to inquire as to what individual students in her classroom understood.

Facility with vocabulary. During my second observation of Michelle’s classroom in January, Michelle spent 45 minutes during her 70-minute lesson having students recite and explain definitions of different structures in plane geometry. She had one student read aloud the definitions that the student had taken in her notes the previous day, and she asked questions of the whole class to clarify each definition. Then she explained some new definitions. Following that, Michelle asked students to spend some time in their small groups to apply these definitions in order to solve problems. However, she only allowed the students to begin to make sense of the problems for 5 minutes before calling back the attention of the class so she could demonstrate for the whole class how to apply the intended procedures in order to solve or complete the problems as expected. After observing this class, I asked Michelle how many days she had allocated for review of vocabulary and she reported that this was one of three days. I suggested that three days might be too much time to dedicate to reciting definitions, with limited room for application of these definitions to problem solving. But she stated that it was important that her class understand the vocabulary since they often struggled with mathematical terminology.

> …so that's my approach so they will always have a reference point. And it's like the [state test]… they'll use those definitions, and I want them to remember what it is and what it represented. So I felt, based on my group of kids… I have to slow it down a little bit. …I mean I don't know, I did what I felt was necessary for my group of kids…because they always, the vocab, it messes them up all the time so that's why I did it that way. (Michelle, Mentoring Session, January 27, 2012).

It was very important to Michelle that her students use the proper mathematical terminology. Often, in her classroom, she asked students to revoice their own explanations using the proper mathematical terminology. She led the students to the proper terminology by using leading questions during their individual recitations or presentations. She had observed her students having difficulty with using proper vocabulary or identifying the features identified by the terminology that they would use to solve different problems, especially those on high-stakes standardized tests. Due to these concerns, she spent a self-reported several days having students recite definitions because she felt that it was "necessary" for her students.

However, subsequently Michelle began to implement a strategy that allowed her to incorporate work on mathematical terminology *and* whole-group discussion in her classroom simultaneously. During the same data analysis unit in which she had her students work on a project, she provided the students with a reference sheet that consisted of a summary of information about each data display. She then posited several questions to her students, requiring them to select an appropriate data display in order to answer a question. Some of the questions had a single correct answer, and some of the questions had multiple possible solutions. She allowed the students time to think and discuss their solutions in their small groups, in addition to accessing the vocabulary reference sheet, before opening up the questions to the whole class as a discussion period. She recounted the productive whole-group discussion that occurred in her classroom when asked about how student discussion was progressing. She cited the fact that she provided the students with the reference sheet as being the key to her students' ability to engage in productive discussion.

> I had a good day today…in my mod class on data analysis. And we're at the point we're talking about which data display to choose and why, depending on the circumstance. So I had two scenarios and I asked each group to talk amongst themselves to determine which data display they'll choose for the scenario… and I called a group…and they shared their response and I said, "Okay, who agreed or disagreed and want to add to it?" So, that spawned a lot of discussion. ... And they were able to use the terminology of why.…I think what helped was that I gave them, well, we went over the list of displays the day before and they had a hand out and then they could refer to the hand out and say, "Oh, because a histogram talks about change over time, or intervals." So they were able to be able to look at their sheet and kind of say why they chose certain data displays. And so, I think

> that kind of made them a little bit more comfortable because they had something to refer to. (Michelle, Seminar Six, March 7, 2012)

In this summary, Michelle notes that she had her students focus on using the proper mathematical terminology *while* participating in small-group and whole-group discussions. She found the discussion to be productive and cited her use of a data-display reference sheet as a way to enable her to facilitate the discussion and, at the same time, address her focus on vocabulary within the discussion format.

Negotiation of challenges by using roles. Michelle also, during her project-based unit, established clearly defined group-member roles so that each student would have an individual responsibility for participation in small-group work. In the project document, she described four roles, which students would assume during completion of the project. These roles included group facilitator, group recorder and folder monitor, group reporter, and timekeeper. The group facilitator was responsible for:

> …moderating all team discussions, keeping the group on task for each assignment, and ensuring that everybody assumes their share of the work involved. They must also be certain that everyone benefits from an optimal learning situation: Everyone should have the opportunity to learn, to participate, and to earn the respect of their teammates (Michelle, project document, March 2012)

This role addressed Michelle's earlier concerns of individual knowledge and participation in productive discussion. Since she had expressed concerns in February about group-thinking and the propensity for students to co-opt final answers from a single member of the group, she addressed this concern by making a single student responsible for making sure everyone was involved and participating in the required group discussion.

The second role that Michelle created for use in her project was a group recorder and folder monitor. The folder monitor portion of the role required the student to collect and organize the materials of the group for storage in a folder in the classroom. However, the group recorder role indicated that the student was required to “keep all necessary records, including attendance and homework check-offs, and record any assigned team activities. They also prepare the group's activities, completing work sheets or written assignments or summarizing discussions for their group's oral reports or for submission to the instructor” (Michelle, project document, March 2012). This role encouraged the group to have autonomy. If they needed to complete additional work for the project outside of class, the recorder would keep track of individual students’ responsibilities and assignments. Furthermore, this student was required to document the group’s conclusions and have them ready for presentation. However, this did not isolate all requirements for completion of work onto this student. Since Michelle separated the recorder, who would summarize the group’s work in writing in preparation for presentation, from the reporter, who “orally summarizes the group's activities or conclusions. They also routinely assist the Group Recorder with the preparation of group reports and worksheets” (Michelle, project document, March 2012), Michelle spread the work of preparing for presentation among group members. This requirement of collaboration addressed Michelle’s need for every student to be individually required to participate and be involved in their small-group discussion, and additionally require them to be prepared to present either directly to the teacher, or to the class in a whole-group discussion.

The final role that Michelle included in her project document was the timekeeper. The timekeeper was responsible for “keeping the group aware of time constraints for any activities. With the facilitator, they help the group remain on task, consulting with other teams when needed” (Michelle, project document, March 2012). Michelle created this role to make sure that students were talking about the project task at all times. Therefore, she could feel that students were talking productively about mathematics, rather than one person completing the work while the others watched. This also ensured that the students completed the mathematical work in a timely fashion as to not waste any needed instructional time, which addressed a concern about time constraints that Michelle had articulated in November. By including the instructional strategy of group roles in the project, Michelle addressed her challenges of individual participation, time constraints, and desire for productive discussion while at the same time allowing her to include substantial amounts of small-group discussion time in her classroom. These roles allowed the students to work autonomously, with Michelle as an outside facilitator, instead of in a tightly controlled, and predominantly teacher-directed, environment. The existence of these roles in the project document structured the lesson so that the small-group discussion could take on this different form.

Negotiating struggle. One of the major uses of the whole-group questioning format that Michelle used in classroom observations in November and February was to negotiate student struggle. When she found that students were struggling on a particular task, she would call the class together as a whole group and either set up the task for them by questioning students in the room or direct questions to different students in the classroom. She did this in order to guide the students into presenting the proper procedure by responding to her step-by-step questions. She seemed to have a fear of students having to endure too much struggle while working on their problems and seemed compelled to correct their misconceptions immediately in a whole-group format. Clear evidence of this occurred during her second observation in February. Michelle had reviewed definitions for an extended period. She then had the students try to complete a worksheet that contained several problems on which application of the definitions of complementary and supplementary angles would give students the information they needed to calculate the missing angle in a diagram. She told them to work in small groups, and she circulated around to many different groups. While she was circulating, she discovered that several groups of students required assistance to begin the different problems. After 5 minutes, she recalled the students' attention to the front of the classroom where she demonstrated procedures to solve the problems on the worksheet. After seeing several mistakes, Michelle stopped the class and said:

> Okay, hold on. This is going to help. This is going to help the entire class on the second section where it says find the value of x. Remember this diagram from yesterday? … Remember this from yesterday? How do we measure angles? Which way do we measure? (Michelle, Classroom Observation Two, January 27, 2012).

Michelle had circulated the class for those 5 minutes and discovered several students with incorrect answers. Students also called on her for general help. Instead of asking students leading questions and allowing the students to struggle in their small groups to make sense of and solve the procedure, she terminated the small-group time and used her leading questions to guide students through a recitation of the expected procedures.

As the year progressed, Michelle seemed to take a different approach to negotiating student struggle. She set up her tasks more consistently before students undertook the assignment, and, when she provided in-class time for the students to work, she allowed them work in small groups for extended periods. Michelle then used this time when she was not personally leading the classroom to circulate among the small groups and to check for understanding in individual groups. She was able to isolate student misconceptions in these small groups instead of addressing the whole class and clarifying the problems for them. During my observation near the end of March, the students spent 35 minutes of time in their small groups calculating percentages or fractions in order to construct a circle diagram of data that they generated themselves. This was in stark contrast to the 5 minutes of small-group time that she allowed her students in February. Not only did the students spend an extended period working in their small groups, but Michelle also circulated the classroom and checked for student understanding. Furthermore, during the observation in March, most of the whole-group discussion time was used for recording data or investigating overarching mathematical concepts through facilitated discussion. In this case, the students came together as a whole class to record data addressing which sports they liked, to record the fractions and percents that each group calculated to contribute to a whole circle graph, and to use principles of percents and fractions to determine the validity of the data. Michelle had changed her approach to negotiating student struggle from a whole-group, teacher-led lecture to enacting the role of facilitator where she checked individual student understanding while the groups worked.

Michelle's Changing Conception of Collaboration and its Use

An Overview. In November, Michelle showed a desire, superficially, for students to work in small-groups in her classroom. She initially addressed that desire by arranging the desks in her classroom in groups of four, assigning students to these groups, and asking them to talk to each other during the completion of short activities in the classroom. Her students struggled initially with simply having polite, on topic, conversations with the other students in their small groups without Michelle's direct intervention and arbitration. Needing to address this, Michelle's efforts with respect to developing a culture of student collaboration were focused on establishing norms for student behavior and verbal interaction within the small groups. However, as evidenced during classroom observations during this month, Michelle did not leverage small-group work as an instructional organization through which students might learn and make sense of mathematics. Although she expected the students to be working on the assignment, which was mathematical, her conversations about her students' small group work were initially more focused on their social behavior and interaction rather than on their mathematical findings, explanations, or mathematical uses for small-group work.

In January, Michelle began to suspect that simply positioning the students in small groups and expecting them talk with each other productively as they worked on assigned mathematical tasks was not sufficient. She feared that her students were using small groups as a setting in which to give each other what they felt were the correct answers and or explanations without engaging each other in mathematical thinking and learning. In February during a teacher seminar, she expressed the concern that she wanted more productivity to be a characteristic of her small-group configurations. At this time, as she began to consider what was missing in her use of this instructional strategy, she began to focus on her intended goal: her students' mathematical thinking. Through this reflection, Michelle came to the realization that she wanted her students to use the small-group time as a setting for *learning from each other*, rather than simply as time for talking or sharing answers. She began to think about her students' mathematical learning in the context of small-group work.

In order to accomplish this end, Michelle began incorporating two different instructional strategies. First, she gave each student in a small group a role that defined how that student would be expected to participate in discussing the assigned talk. Second, she required individual thinking time before the students came together to discuss their mathematical explanations, solution strategies, and justifications. By establishing these expectations and clarifying how they would be applied, Michelle began to scaffold for her students how to engage in collaboration in ways that resulted in mathematical sense making.

Michelle's perception of collaboration and the strategies that she employed to facilitate student collaboration changed over the 5-month period from November to March. First, I describe Michelle's initial conceptualization as expressed in her statements regarding the essential need to develop a classroom culture in which students trusted each other enough to feel comfortable talking to each other and were able to participate positively in polite, on-topic, discussions during small-group work. Second, I analyze the challenges that Michelle faced with regard to the nature and form of her students' discussion in small groups and how she negotiated those challenges while developing a deeper notion of the mathematical purpose for small-group collaboration. Finally, I analyze the development of two instructional strategies that Michelle began to incorporate in order to develop the type of collaboration that encouraged mathematical learning that she desired. First, I analyze Michelle's strategy of assigning students specific roles to play during small-group work on a mathematical talk. Secondly, I analyze her strategy of requiring individual think time.

The need to develop a collaborative culture. In November, at the beginning of Michelle's permanent field placement as teacher of record, students working together autonomously characterized Michelle's idealized image of teaching. In her idealized image, students would work together and rely on each other, rather than looking solely to a teacher for assistance. Students would debate productively, without requiring engagement with the teacher as a means of securing validation of their perspective. Further, she imagined students consistently providing "productive" help and not simply giving answers to each other without explanation or justification. When Michelle reflected on commercially available videos of teaching as shared during her methods course and as reflected upon during our teacher seminar, she lauded the autonomous collaboration of the students depicted in the video. She spoke admirably of their ability to work productively in a setting that the teacher did not directly and tightly control and of their ability to attack difficult problems through sustained struggle while utilizing their own knowledge and resources. However, Michelle was initially disappointed in her attempts to have students collaborate in her own class. She felt that her students argued unproductively, and that they had no mechanism for making sense of or evaluating the reasonableness of a tendered solution. Thus, they had no recourse other than to rely on her as the arbiter. She recounted, with disappointment

> … instances in my classroom where one person was absolutely right and the whole class tried to shut him down. “Oh, that’s not right!” I said, “No, that is!” So, it’s like, “Listen.” Or, because it’s not coming out of … my mouth, it’s not valued. You know, they’re not valuing each other’s opinion.” (Michelle, Preliminary Interview, November 3, 2011)

She felt that her students were mostly concerned about whether a response or solution was right or wrong, as they would unproductively argue instead of making sense of a solution strategy or trusting each other’s thinking.

In order to respond to these concerns, Michelle tried to imitate or use the behaviors or instructional steps that she saw the teachers in the videos employ, as a way of helping her students learn to collaborate effectively. She began to speak about instructional strategies that she could use to help her students become familiar with talking to each other and with valuing each other’s contributions. During the new-teacher seminar, she stated,

> So I’m trying to do that for [the students]. We [as teachers] have to, maybe, have a set of questions for them to ask each other. Get them used to talking and explaining to one another without [there] being a right or wrong [judgment] … . Just let me hear your conversation. That sort of thing. And I talk about a level of trust, building trust amongst them[selves] so that they’re used to working with each other. They’ve had some camaraderie, and they’ve had some successes, and [now] they’re able to trust one another. [They’re] more to where they can listen (Michelle, Seminar One, November 7, 2011).

Michelle felt that providing students with question stems as conversation starters would help the students acclimate to the process of asking and answering each other's questions in a way that would lead to productive discussion. She had seen her students becoming more comfortable with working with and talking to each other on an assigned mathematical task during class. She felt that by positioning her students in groups of four and giving them repeated experiences working with other students in small groups, they would build a level of trust that would result in productive collaboration similar to that which she had observed in the video. In addition, she felt that providing students question stems in the future might help her students practice how to talk and explain productively. However, this use of question stems was only implemented indirectly, through the hanging of posers displaying different types of questions as decoration in the classroom that she shared with Jack.

While reflecting on the commercially available video shared during the first November teacher seminar, Michelle stated that the students in the video seemed to be comfortable with asking questions of other students in their group and truly listened to the responses offered by the student who was asked a question.

> They ask and listen. They feel comfortable with asking this person or that person. So, I think creating that culture … is key. (Michelle, Seminar One, November 7, 2011)

As Michelle had mentioned in a preliminary interview, she felt that although her students were beginning to trust each other, she needed to include additional instructional strategies to support her students' efforts to question, explain to, and listen to each other during small-group interactions. She felt that *she* needed to develop a collaborative culture in *her* classroom so that students could "ask [questions] and listen [to each other]" while in their small groups. In order to develop that culture, we spoke in our mentoring sessions about strategies that Michelle could use to facilitate collaboration in the classroom.

From a fear of regurgitation to a conception of learning from each other. During the mentoring sessions, Michelle and I spoke about different strategies that a teacher could use to promote students' productive collaboration in their groups and how this could help foster a classroom culture. Our conversations regarding productive collaboration continued through conversations addressing use of other strategies. We spoke about facilitating students' discussion, rather than feeling it was necessary to directly guide and control their discussions, and we spoke about procedures or routines that would lead students to be prepared to explain and justify their work. As one technique, I suggested a practice of selecting a single student from a small group and requiring that student to answer a question posed by the teacher, a question that would require a mathematical explanation or justification. If that student could not respond, this technique required the teacher to walk away after tasking the small group with discussing that question until the initially teacher-selected student could answer the question. In this way, the teacher could use questioning to further a particular student's understanding while at the same time requiring the group to collaborate in order to construct knowledge. Michelle had seen other teachers attempting this strategy, but she did not feel that it was a useful strategy for fostering individual student understanding. Michelle noted,

> …We're trying to do a little bit of that now because I know [teacher's name] does that well. [What I noticed is] that instead of [the students in the group who

> understood] really helping [the other students] know, they just tell them to … "Say this. Say that." But, [they're] not really helping them understand. (Michelle, Mentoring Session, January 18, 2012)

During our discussions, Michelle began expressing her concern that student collaboration could mask individual knowledge. She believed that a student who did not come to understand the concept or procedure on their own could possibly regurgitate a procedure as demonstrated or shared by another student member of the group without truly understanding that procedure. She felt that the students truly did not know how to help each other in their small groups. She felt that all that they knew to do was to simply tell each other the correct answer and *possibly* the correct procedure, without helping their tablemates understand the "why" justifying that response.

Michelle began to negotiate this dilemma when she began to speak about discussions in a different way. During a February teacher seminar, when I raised a question addressing how the teachers were developing discussion in their classrooms, it became clear that what I meant by discussion and what Michelle believed I meant by discussion were two different things. Michelle had initially characterized discussion as student talk during class without a clear goal to the nature or purpose of that talk. She had situated her students in table-groups of four, and her students had, at least to some extent, an opportunity to talk to each other every day. But, because the type of talk that was happening in those table groups was not sufficiently productive with regard to promoting mathematical understanding, she was not convinced that there was any need to promote more of this talk. However, as she reflected upon her teaching and her students' resultant behavior, she began to find a deeper purpose for discussion in her classroom. Instead of simply equating student talk with student discussion, she began to characterize intended student discussion in her classroom as having a mathematical purpose that resulted in collaborative student learning of mathematical concepts. Her focus was to develop, in her classroom, a culture where students productively *learned from each other*.

> That's not my focus, as far as what can I do to have them discuss. They talk a lot. My thing is: How do I get them to learn from each other? You know, just learn the concept. So, when you ask that, it's like that's not at the top of my list. I mean, it kind of plays into it, [but] I'm not looking for ways to make them talk. It's mainly understanding things and completing things. I haven't really thought about facilitating discussion… . If we have a group that can get it and a group that doesn't, it's helpful even when they're in close proximity to help. Not [to] tell the answer, [but] to help them find their way to the answer. They're so quick to give each other answers and not show them how to do it. (Michelle, Seminar Five, February 1, 2012)

Michelle's conception of *learning from each othe*r, rather than simply giving each other answers included not only her initial conception of discussion, which she equated with student talk, but also incorporated a mathematical purpose that would be developed through productive, collaborative learning. She wanted her students to talk with a purpose, and that purpose was for students to help each other find "their way to an answer" so they could learn mathematical content with and from each other. This addition of an expectation of mathematical purpose within student discussion and collaboration suggested Michelle's deeper understanding of the pedagogical purpose for collaboration. This, in turn, could result in student discussion similar to that which she idealized in November, where small groups worked “pretty much on their own looking at [the problem] and researching and finding [the solution]. They utilized the resources that they had” (Michelle, Seminar One, November 7, 2011). However, the actions of the students in her classroom did not yet reach her expressed ideal of student collaboration and mutual struggle, since her students were providing each other with answers rather than collaboratively making sense of the mathematics.

However, Michelle’s practical conception of productive whole-group discussion as evidenced in praxis and subsequent reflection on praxis seemed to evolve. In November, she believed that she was facilitating discussion while simply participating in a pattern of talk, a pattern that consisted only of questions and responses with single students in front of the whole class. However, after negotiating challenges in January and February, Michelle began to characterize discussion as having the mathematical purpose of students collaborating to learn from each other. As a result, in March, Michelle categorized an episode of teaching as a “good day” because it contained “a lot of discussion” (Michelle, Seminar Six, March 7, 2012). During that day, she had students think about data displays that were appropriate for different situations in their table groups and then engage in a conversation about their ideas. Michelle recounted that discussion on this day was marked by back-and-forth verbalizations between table groups, as the students presented their findings and reasoning, and other groups disagreed, or agreed and presented their reasoning. Michelle reported that she facilitated cross-group interaction by allowing students to negotiate conceptions of either mathematical correctness or the possibility of multiple solutions. Through her reflection on her implementation, she seemed to begin to conceptualize collaboration as productive discussion in a group where each student had an equal role, rather than the recitation she had displayed in November. As such, this event and its recounting provided evidence of Michelle's emerging conceptualization of collaboration,

Strategies to promote collaboration. By February, Michelle had begun to conceptualize collaboration as students *learning from each other*. However, she realized that her students were not effectively collaborating in their mathematical sense making process. Therefore, in March, she began to modify her prior instructional approach to incorporate new strategies that would facilitate her students' collaboration in their small groups. Two instructional strategies that she developed were assigning students roles and requiring students to take individual think-time.

Student roles. In November, Michelle said that it would be helpful, as a scaffold, to assign students roles in order to acculturate students into working together in small groups. She stated in a preliminary interview that she wanted to

> …assign [the students] roles before getting started, so they understand the expectations going in. Roles, like someone is actually going to record, someone to do the calculations, someone to build something or make a model or something, and then maybe present questions in the beginning that they could ask each other (Michelle, Preliminary Interview, November 3, 2011).

She thought that giving the students a clear definition of how they would participate in a small-group situation would help the students work together productively. She continued to talk about assigning students roles throughout our interactions over the 5-month period from November to March.

During a seminar in November, Michelle referred to assignation of student roles when the group of teachers discussed the reticence of their students with respect to providing detailed explanations in either small- or whole-group discussions. The other teachers reflected that their students often deferred to a student they identified as being smart or to the authority of the teacher's utterances in the classroom as their justifications of their solutions or solution strategies. Michelle asserted that she needed to develop a culture of discussion and mathematical explanation and justification in the classroom in order for productive discussion to commence. She argued,

> I think it's a culture that we have to create in our classroom[s] because some kids are not used to working in groups. They don't know how. So, I know I need to facilitate my group discussions and activities more by assigning roles so [the students] have a purpose (Michelle, Seminar One, November 7, 2011).

Other teachers in this seminar, after Michelle's argument, began furiously taking notes on her ideas. It resonated with all the teachers that the students did not know how to work in groups efficiently, and Michelle's argument for the assignation of group roles seemed to be an effective instructional strategy to achieve those goals. Michelle seemed to be committed to assigning roles to students in their small groups. When I asked her what she wanted to work on concerning the development of small-group discussion, she clearly stated that she wanted to work on trying to "to come up with roles" (Michelle, Seminar One, November 7, 2011).

Since Michelle had consistently positioned her students in groups of four in her classroom, she continued to brainstorm about the type of roles she should develop. In her scholarly endeavors in her teacher preparation program, she had read about and heard descriptions of standard roles that she could assign to students in order to facilitate small-group work. However, she was not satisfied with those standard roles that she, herself, had suggested earlier. She stated that she would like to create tailored roles to a particular activity. After I asserted that an assignation of roles should not excuse any individual students from participating in the mathematical sense making during small-group work, Michelle suggested that she should create "some roles to a different problem. Oh, you're the engineer, I need you to talk about this. You are the verifier…. Connect to the problem or the task at hand. You're the counter…" (Michelle, Seminar One, November 7, 2011). It was important to her to give her students' a purpose for collaboration that was reasonable in the real-world context with which she was familiar through her previous work experience. However, this constant re-creation of roles did not seem feasible to Michelle in practice.

Immediately after this discussion in seminar, Michelle attempted to assign roles in her classroom. When I asked if she had followed up on her statement that she was going to attempt role assignments, Michelle said that she assigned three roles to the group of students. She explained

> …one [was] the recorder. They actually wrote it in and determine[d] the value, and the other person had the calculator to where they would calculate the value, and the other one was a verifier … to verify in their calculator make sure it's correct. So, they liked that. I had to remind them [of] that, though (Michelle, Mentoring Session, November 17, 2011).

She assigned the roles so the students would, hopefully, be more autonomous in their groups and not constantly ask for validation from the teacher or from the student that the group had identified as being smart or successful. Since Michelle was concerned about productive discussion dissolving into non-productive arguments with no viable solution, she found that by assigning one student as a calculator and one as a verifier, she was able to redirect the arbitration of the arguments to the students' jurisdiction rather than her own. She commented that when an argument arose, she would ask the group a question about their roles. She commented that reminding them of the roles to which she had assigned them supported the goal of having their discussions remain productive. She stated,

> …then they had an argument so I walked over and I said, "Who's the verifier? If you all don't agree, all of you all do it yourselves and see what you come up with and then do it on the calculator and see." So I had to tell them and to actually do that to remind them (Michelle, Mentoring Session, November 17, 2011).

Michelle felt, when she reminded students of their assigned roles, that this kept their discussion from dissolving into argument. Keeping discussions polite and on task, at this point, was Michelle's major focus. Her role, in facilitating discussion, became one of reminding students of the roles that they had to assume.

Although Michelle reported that she was assigning roles, it seemed that the role of the "verifier" became a norm of behavior in her classroom. When I asked Michelle if she was still assigning the verifying role in her classroom, she responded, "It depends on the task, I always say, 'Verify. …Check it yourself. Does that make sense?'" (Michelle, Seminar Four, January 12, 2012). Although she stated she was still occasionally assigning roles in her classroom, the idea of students verifying, or checking, their group's work was pervasive in the classroom. What had initially started as assigning roles had developed into a normative practice. This was important because Michelle, at the same time, was negotiating another dilemma: Students were not individually learning through their collaboration. In November, assigning the role of the “verifier” was specifically created to prevent argument, encourage politeness, and address issues of mathematical authority. The intent of this role was to encourage students to agree on answer. However, in January, Michelle began speaking about verifying in a different way. Instead of specifically getting students to agree on an answer, now Michelle specifically mentioned that, in her classroom, the norm of verification was intended to encourage the students to engage in collaborative sense making about the reasonableness of the solution. Michelle had begun to engage her students in talk that had a specific mathematical purpose, that of answering the question “Does that make sense?” with regard to a solution or solution strategy.

Although in January verifying had become a norm in Michelle’s classroom, Michelle had subsequently become concerned about her application of norms. She had decided in November that it would be best to create different, appropriate roles for every task she assigned, but she had discovered that the creation of these task-specific norms was not feasible for her with the time required of her as a teacher and a student. Furthermore, since she was not developing task-specific roles, she found that her small-group classroom management was suffering. Since her expectations of small-group participation were not clear, she felt that she spent too much time reminding students to remain on task rather than effectively facilitating discussion. She first divulged this discovery in February, during a teacher seminar. She felt that, if she was to be effective in facilitating small-group work using norms, she had to create general student norms that would be applicable to any task. She confided that she would

> …have to come up with general roles that will fit with, like, every task. Because sometimes you just want them [the students] to discuss and you don't have to have roles for discussing, just like comparing … . Their off-task behavior is veering [them] off from their discussions, They're talking about personal things. And so, with 25 people … you're just monitoring. You're walking around and telling them to stop and focus. That's pretty much all I've been doing (Michelle, Seminar Five, February 1, 2012).

Michelle felt that, because she had not created general norms for each task, the students' small-group discussions were suffering for lack of productivity. She felt that not only were the students becoming increasingly off task because of the lack of consistent roles, but also that she was unable to do what she wanted to do in the classroom. She was not able to engage students productively with mathematics because she was monitoring student behavior.

In March, when Michelle developed her unit-long project, she incorporated more general student roles that would promote collaboration. Within the project packet, she included the definitions of four roles that she would ask her students to assume during the completion of the project. These roles were group facilitator, group recorder and folder monitor, group reporter, and timekeeper. The most important characteristic of the definitions of these roles was the fact she defined these roles in order to direct and encourage students to collaborate. The role of facilitator included the responsibility to ensure "that everybody assumes their share of the work involved" (Michelle, project document, March 2012). This required the student who assumed the role of facilitator to encourage each group member to share in the thinking process and in the work required to respond to the questions that Michelle had posed in the project.

The group recorder was required to “complet[e] work sheets or written assignments or summariz[e] discussions for their group’s oral reports or for submission to the instructor” (Michelle, project document, March 2012). However, the reporter was the student who was responsible for “summariz[ing] the group’s activities or conclusions. They also routinely assist the Group Recorder with the preparation of group reports and worksheets” (Michelle, project document, March 2012). These overlapping definitions of student responsibilities facilitated collaboration in several ways. First, both the recorder and the reporter were required to synthesize the thinking and the conclusions of the group, one on paper, and one orally. This required both students to be actively involved in the small group’s discussions and to be prepared for any questions from Michelle. Second, these definitions specifically required the reporter and the recorder to “routinely assist” each other. This would promote the reporter’s and the recorder’s collaboration to create written summaries and solutions as well as oral explanations and justifications of their work.

In a similar fashion, the role of the facilitator and the role of the timekeeper included instructions to keep the group’s conversations “on task” (Michelle, project document, March 2012). Specifically, the instructions for the timekeeper required the student who assumed that role to work “with the facilitator” (Michelle, project document, March 2012). Similar to the way the definitions of the group roles required the recorder and the reporter to work collaboratively with their group to document their progress, the facilitator and the timekeeper were required to collaborate to maintain the focus of group work and discussion on the task at hand. Michelle, in this way, used the strategy of assigning group roles to promote collaboration within small groups.

These roles were particularly important for three reasons. They were sufficiently general so they could be applied to any task, they addressed Michelle's desire for more autonomy so she would not have to serve as a monitor; and they focused students' small-group discussion on making sense of the mathematical concepts in the task. By having general tasks that were used consistently, the students could become familiar with the roles which they had been assigned. Furthermore, the definitions of the roles put the onus of behavior monitoring onto the students. Both of these features would allow Michelle the opportunity to focus more on the mathematics with which the students were engaged instead of simply "walking around and telling [the students] to stop and focus." Also, Michelle had defined the roles of the reporter and recorder so that the group would have to collaborate to interpret the question, to form a solution, to summarize their ideas, and to be prepared to present or submit their mathematical thinking to the class or Michelle. The definition of the assigned roles encouraged students to collaborate, work autonomously, and focus on making sense of not only the task, but also of the thinking of the students in their group in order to formulate a final solution.

Individual think time. Another strategy that Michelle used to promote collaboration was the use of individual think time before a student engaged in discussion with their group members. Michelle implemented this strategy to insure that the students would collaborate by beginning with each student's individual ideas and then working toward a common solution strategy. This strategy potentially addressed Michelle's fear of one student co-opting the small-group talk and the other students simply copying and regurgitating that single student's work. Michelle mentioned that she was afraid that individuals might not be equally involved in the thinking process that resulted in a completed explanation and justification. She initially expressed this concern, stating that students may, "instead of really helping them know, they just tell them to regurgitate … It's like, 'Say this. Say that'" (Michelle, Mentoring Session, January 18, 2012). She repeated her concern later in February when she expressed that when students were "in a group … they can just say, 'Say this.'" (Michelle, Seminar Five, February 1, 2012). However, when Michelle developed her project-based unit, she began using a strategy of providing students with an opportunity for individual think time before talking to their group.

When students were completing procedures when working in groups, even though these procedures were embedded in student-centered activities, Michelle encouraged a type of collaboration that incorporated her own ideas. She had students do their own calculations and/or brainstorm their own ideas before engaging in small-group discussions and then used the expectation of verification in order to create the culture wherein students talked to each other and helped each other to find solution strategies and not simply answers. She first implemented this strategy when the students received the packet for the project-based unit. Her initial instructions to the groups required them to read the questions individually and to develop their own initial approach to answering each of the questions. She stated,

> I had them initially kind of brainstorm what they are going to do in the beginning. And, to get everybody's input, I handed out stickies. I didn't want anybody to say

> anything because the [students with the role of] recorder could be the one[s] trying to be in control and only record the ideas that they think is viable. But, in order to bypass that or avoid that, I had everybody have their own little stickies and write their ideas before they even got started (Michelle, Seminar Seven, March 14, 2012).

In order to scaffold the groups into acknowledging each student's contribution to the questions in the project, Michelle had the students record their thinking and affix the "stickies" to the project paper. That way, each student would be able to read the thinking of each of the students in the small group before deciding on a solution that would be sufficient as a representative of the whole group. By allowing think time, Michelle encouraged the students to collaborate and incorporate all of the individual ideas into the final product. Furthermore, since the students would have to read or listen to other students' thought processes before developing their final product for submission, Michelle was requiring them to make sense of and evaluate different students' thinking instead of relying on either their own or that of the student or teacher who they characterized as having the greatest mathematical authority. In this way, the students were being expected to think mathematically and to make decisions about an approach or a solution.

Michelle continued this strategy later in March. During the final observation of her classroom, Michelle assigned students the task of working in groups to create a circle graph that recorded data that the class collected. The students remained situated in their table groups; however, Michelle encouraged them to "individually see if you can come up with the same answer" (Michelle, Classroom Observation Three, March 22, 2012).

She asked them to attempt a solution strategy on their own and then to share their results within their small group in order to verify or modify their strategy and to create a solution strategy that would suffice for the entire small group. She was consistent in her requirement of individual think time while circulating. She prompted groups to collaborate in this way by saying to students, "… after this make sure everybody at your table get[s] the same answer … not just you. Make sure you see it from everybody else" (Michelle, Classroom Observation Three, March 22, 2012). By continuing to have students think for themselves and then commiserate with their group members, she encouraged students to collaborate in their small groups.

From November to March, Michelle's conception of collaboration and discussion, and her implementation of these things, began to emerge and develop. In November, she recognized the need for creating a culture where collaboration was the norm. She struggled in January and in February with the actual performance of her students in their small groups and her fears of small groups leaving individuals behind in the mathematical sense making process. However, by March, her conception of collaboration had grown from simple student talk to *learning from each other,* and she began to include strategies, such as student roles and individual think time, to encourage students to collaborate in their learning process.

Conclusion

During Michelle's journey through teaching, mentoring, and seminar sessions, Michelle changed a great deal. In order to achieve these changes she posited strategies, faced challenges, and negotiated some of the challenges with different teacher moves that served both her concerns and the promotion of student discussion. Michelle entered her teaching focusing on individual skills and presenting lessons in a teacher centered manner, and left the study with an emerging perspective on what mathematics classroom discussion was, how it might appear in practice, and the practical uses of student discussion with regard to mathematical sense making in the classroom.

Michelle changed the way she approached her lesson development and the instructional decisions she was making, the amount and type of student discussion that she incorporated in her lessons, and her conceptualization of collaboration and its use. She changed her approach to lesson development and the instructional decisions she was making in several ways. Instead of focusing on breaking down mathematical content into individual skills and focusing single lessons on isolated skills, one skill at a time, she began thinking about mathematical concepts and skills simultaneously, and finding ways to integrate them. Instead of only thinking about a single lesson at a time, she began to think about overarching mathematical ideas, or curricular units before she began to plan individual lessons. With regard to the amount and type of student discussion that she incorporated in her lessons, Michelle began to use small-group work in conjunction with student-centered lessons, as well as allowing students to spend more class time interacting in small groups. She also began using whole-group time to set up lessons or record results rather than walking through exercises through individual recitation to the whole class. She initially found individual work to be more important than having students work together, however, as she used different strategies and negotiated her challenges, she began to define collaboration differently and found collaboration and small-group work to be useful for students in making sense of mathematics.

The following is a summary of the findings for Michelle Miller that positions my analysis of Michelle's data in light of the conceptual lens addressing challenges, negotiations, and strategies. Accountability was something to which Michelle initially attended and strategized. However, it was also a challenge that she faced with regard to being able to adequately assess student performance. Michelle leveraged her desire for individual accountability by incorporating strategies for individual thinking time before group work, and for small-group thinking time before participation in whole-class discussion. She also found that when she incorporated more time for small group discussion in her classroom, she was able to better assess students' individual thinking and correct misconceptions that students have. This was due to her mobility in the classroom during group work, rather than being required to remain in front of the classroom giving whole-group instruction. Second, negotiation of the local school context forced her to change and develop her teaching practice. Her administrator, and the intended school culture and instructional strategy of the administration, required her to change her instruction to include more hands-on, student-centered, investigative instruction. Without the prompting of her administration, she may not have been motivated to change her instructional strategies. Third, the way her students were tracked was a challenge that Michelle had to negotiate. Since her students were labeled as the "comprehensive" class, she perceived their ability levels to be low, and therefore she initially lowered the cognitive demand of the instruction. However, she found that when she provided appropriate scaffolds, her students were able to participate in mathematical sense making and productive discussions. Finally, the support that Michelle was provided helped her negotiate multiple challenges in the classroom. I modeled the facilitation of discussion in the classroom so that she had a clearer picture of what discussion in mathematics classrooms looked like. I also assisted her in planning for overarching mathematical conceptual understanding rather than individual skills. Finally, I helped her plan for a project-based unit when she felt she was too inexperienced to do so herself.

Chapter 5: Eleanor Scott

Before Eleanor Scott joined the alternative certification program, she had experience as a short- and long-term substitute teacher. She said that she had taught every age "from 3 to 43" (Eleanor, Follow-Up interview, April 23, 2013) and had worked in pre-school, elementary, middle-school classrooms and in graduate-level classrooms over the course of her prior graduate studies. However, she was not licensed as a teacher in the state. Previously she earned an undergraduate degree in history and a master's degree from a Divinity school. Also, she had completed her coursework and comprehensive examinations before withdrawing from her doctoral studies in history prior to completing her dissertation. She entered the alternative certification program in order to become certified to teach middle-school mathematics and eventually to become employed full-time as a teacher.

I observed Eleanor during multiple sessions within her program's initial course addressing methods for teaching mathematics. It was during these sessions that I observed Eleanor planning for her own micro-teaching sessions, as well as responding to others' implementation of an episode of micro-teaching while she assumed the role of student. Her comments during this course led me to infer that she felt that it was necessary to provide students with a simpler example of how to approach a problem before engaging them in a problem-based lesson and that she felt teachers should be able to respond supportively to a child's thinking even if that thinking was different from the classroom's consensus. Eleanor would use the problems that her methods teacher provided as the focus of her lesson, however, her lesson plans would call for the teacher to set up the problem for the students by demonstrating a solution strategy for a similar, but simpler, problem. She also offered ways in which to respond to a child's thinking that were different from the consensus expressed by peers. For example, she pressured the other prospective teachers in her class to manage divergent student responses; however, she did not initially address ways through which teachers might manage those student responses that were not correct or were not within the domain of the problem's context. I also reviewed some of the course materials that Eleanor submitted in her summer internship course. Eleanor made many references to the importance of building relationships with students. She stated that teachers needed to build rapport and trust with a classroom in order to be able to invite their students to take mathematical risks and to participate in mathematics learning.

Eleanor was paired with a co-teacher for the initial period of her internship (August – October) and received her final placement as teacher of record in November. However, the school that was established as her final placement was a completely different educational setting in terms of context such as size, discipline procedures, administrative management, instructional routines, and resources. Although she had been successful in enacting and maintaining the pattern of facilitating student-centered lessons promoting student discussion in the classroom with her co-teacher, she was subsequently permanently placed at a school where norms to support student-centered lessons were not established. The students in her new placement were not familiar with collaborative learning, and she seemed to be the only teacher in the school interested in teaching in ways that promoted student discussion. In light of this, Eleanor almost had to begin refining her assumptions about teaching anew, as now she was faced with initiating and defending intended instructional practices that were viewed as atypical at her teaching placement rather than maintaining presumed approaches. This required Eleanor to consider whether and how to adapt her teaching practices to fit in her new and very different school context.

Eleanor's initial approach to planning at her final placement was to access resource activities and investigations from websites or curricula that she had used previously, since the textbook that her charter school had purchased for the eighth-grade students was not aligned with the content that the district's curriculum demanded. She had a strong desire to engage her students in hands-on investigations, however she had to introduce adaptations to these practices slowly in order to be successful within the context of her school. At the same time, because Eleanor had previous experience in public schools at the elementary and middle-school level, she was very aware of the culture and pressures around high-stakes standardized assessments. These prior experiences caused her to have a commitment focused on adequately preparing students for success on these exams, and this commitment influenced her planning.

Analysis of Eleanor's Instructional Practice: Challenges and Change

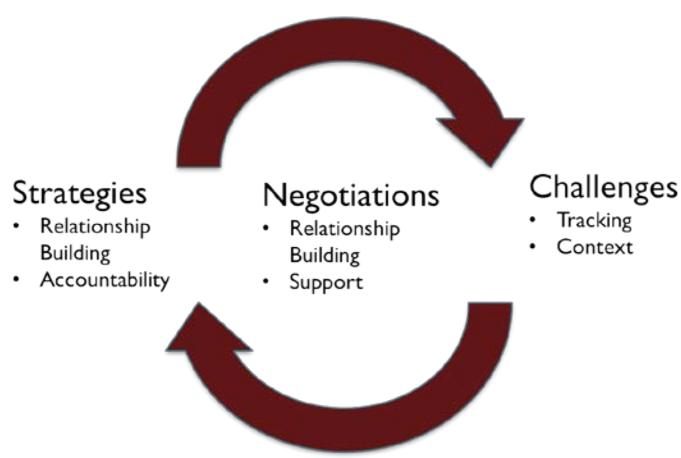


Figure 4: Findings for Eleanor

Throughout the 5-month period of data collection (November – March), as Eleanor implemented her instructional strategies that included hands-on investigative activities with many opportunities for small-group work, she was faced with many challenges, and she implemented or modified her instructional approach as a response to those challenges. She faced challenges related to context, such as classroom management, time constraints, and challenges related to her interpretation of her students' needs, such as testing, student ability, and her and her students' self-efficacy. As she negotiated these challenges, Eleanor's planning for and implementation of teaching changed. This chapter initially presents an analysis of Eleanor's early conception of the role of teacher, addressing how she approached and perceived mathematics teaching during her summer coursework and her initial placement as teacher of record in November. The next section of this chapter analyses the manner in which Eleanor adapted to her new school context after reaching her permanent placement, as she attempted to create a safe space for student participation, as well as to find ways to manage behavior that encouraged students to engage mathematically. The third section analyzes changes Eleanor made during the middle of her observed teaching, changes that affected the types of activities upon which her instructional plans became centered, and the challenges that induced that change. This third section of the chapter includes an analysis of how Eleanor began using worksheets, contrary to her beliefs as to what defined exemplars of good teaching, in order to cover content before the a high-stakes state examination, and then her return to use of investigative, hands-on activities. Finally, the chapter concludes with an analysis of changes in the type of discussion that Eleanor implemented in her classroom. This analysis portrays a shift not only in how she set-up a activity to give students access, but also a related shift from small-group discussions, that dominated a whole class period and that varied in their effectiveness, to guided whole-group sense-making discussions that included short periods of time for individual or small-group work.

Eleanor's Initial Conception of Teaching

I accessed seven data sources in order to establish Eleanor's initial conception of teaching over time. These included her actions during her mathematics methods class prior to her internship, available course documents from her summer internship course, and audio-taped transcripts recording a preliminary interview regarding her initial perspective on instructional strategies that might promote students discussion. These data sources also include the first of three observed lessons in November, mentoring sessions that took place before and after that observed lesson, and the first of seven teacher seminars in which she participated.

Eleanor was not a true novice teacher when she entered the alternative certification program at the University. She had experience as a graduate assistant during her doctoral program, as a teaching assistant in a pre-school, and as a short-term and long-term substitute in elementary and middle schools. However, in her K-12 educational settings, she never functioned as a certified teacher, and so she entered the alternative certification program. However, because of her prior experiences, Eleanor was able to engage with the education course content and her responsibilities as a teacher drawing on the wealth of knowledge that she already possessed. This suggests that Eleanor was soon ready to consider how her instruction should address or adapt to meet the needs of the students, rather than focusing her efforts on developing her identity and practice as a teacher. This was in contrast to the focus of the other, novice teachers in her teacher preparation program.

In this section, I first analyze how Eleanor's initial conception of teaching included recognition of the potential presence of divergent or unexpected student thinking in classrooms without pragmatic or concrete knowledge of how to manage those responses. Second, I analyze a conception of student-centered teaching as hands-on and investigative as opposed to problem-based. Finally, I discuss her desire, as a teacher, to have knowledge of and relationships with her students, as this affected the way she managed her classroom in her permanent placement. In addition, this analysis of her initial conceptualization of teaching includes a discussion of her school-based context, as her permanent placement was very different from her prior teaching experiences and from the setting of her internship period; therefore, she had to adjust her thinking considerably when faced with challenges associated with this placement.

Prior teaching experience. Eleanor's prior teaching experience provided her with a modicum of insight into the experience of teaching. However, since her teaching experiences were in a very different environment, she would have to adapt her approach to lesson planning and enactment in order to address the needs of the students in the school in which she was permanently placed. Furthermore, her experiences were those of a substitute teacher, which often does not allow a person the experience of a permanent classroom teacher. Eleanor's prior experiences allowed her some insight into teaching, which led to some affordances and constraints in her ability to adapt to her new circumstances. The salient points which had an effect on her future teaching were her perception of access to materials and her knowledge of different, or divergent, solutions or solution strategies. She would have to address both of these in her teaching and planning for teaching.

Access to materials. Eleanor's prior teaching was in relatively well-resourced contexts. In both the graduate program in which she was appointed as a graduate assistant and the school district in which she served as a teacher's assistant and as a short-term and long-term substitute teacher, she had access to any materials that she needed to enact her planned lessons. When she entered the alternative certification program, she participated in a short summer internship and encountered her first difficulty with materials. She stated,

> There were a few problems I did not foresee. One was the availability of materials. Because we were not sure how many students to expect, I had brought only four or five copies of each handout. When we realized we had more students than that, I found blank paper and wrote the problems on a white board I had brought. We ran into more problems fairly quickly, however. Some students had forgotten to bring a pencil, and we had no working calculators. A second problem was related to the first. Because we didn't want to use up too much of the big [chart] paper and because he [one of the other pre-service teachers] wanted to keep the lesson moving, he relied on the three teachers in the room to check answers rather than reviewing problems together on the board. While I am sure he and [a third pre-service teacher] had gone over the steps in adding fractions, the girls with whom I was working probably would have benefited from seeing a few more problems and hearing the thought process spelled out, maybe with some

> visual presentations. The paper problem seemed to create a cascade of issues, at least in that first week (Eleanor, Field Experience Paper, July 6, 2011).

The lack of access to materials was not a problem with which she was familiar, since she had not had problems of this nature in the past. Therefore, she needed to develop techniques to address these concerns. After her permanent placement, concerns about the availability of materials contributed to her modification of planned lessons. But at this point, she had her first experience with needing to adapt to circumstances or a placement where materials were not as readily available as those locations in which she was used to working.

Knowledge of the possibility of multiple solutions: both correct and incorrect. As observed during sessions of the mathematics methods course, Eleanor was tasked with developing a lesson that used a mathematical problem as a centerpiece, as well as to interact and reflect with peer teachers who were presenting their planned problem-based lesson to their classmates. Eleanor seemed to push the thinking of her classmates to include or anticipate alternative and mathematically acceptable solutions to potential open-ended questions. During one session of the methods course, Michelle was presenting a problem to her peers that asked the participants to draw a qualitative graph (i.e., a graph on Cartesian plane without numeric labels). The intent was to model the relationship between the degree to which people liked to drink milk, ranging from not at all to a great deal, versus the temperature of the milk, ranging from cold to warm. There were two graphs proposed. One graph was in the shape of a U as the graph modeled the degree to which people would “like” milk as its temperature ranged from cold to warm; this graph mimicked the graph presented in the intended problem solution. However, Eleanor proposed another graph that was approximately in the shape of an M, modeling the degree to which people would "like" milk as its temperature ranged from frozen to "too hot." Although the original task did not indicate an inclusion of the situations of frozen and of "too hot" milk, Eleanor felt that it was important to consider and acknowledge the modeling of these contexts as a possible solution. This illustrated that she was entering this teacher preparation program cognizant of different ways that students could think about a situation, even when those ways of thinking were outside the bounds of the mathematics or the conditions presented in the problem. However, during the class discussion involving the teacher candidates in the course, the conversation remained focused on the participants attempting to determine the validity of Eleanor's solution and not on the instructional challenge or decisions of teachers when faced with unexpected solutions. Eleanor was aware of the possibility of divergent student responses; however, when she raised one of these responses in her methods class, the teacher candidates did not address what a teacher should do in these situations.

Although Eleanor was aware of the different solutions that students may offer during the course of a class period, she would eventually have to address ways not only to solicit multiple solution strategies, but ways to manage different or divergent solution strategies. This was specifically relevant with regard to the context of her placement, since she would have to diligently develop a classroom culture in which students felt comfortable expressing their thinking out loud during discussions. Being cognizant of potentially different and correct, or different and incorrect, solutions that may arise during instruction afforded her an enhanced ability to think critically about how to address these solutions when they were uttered.

Conception of student-centered teaching. Eleanor's summer internship course included a short field experience that included four days of interacting with students at a local community center. When writing about that field experience, Eleanor indicated that she was interested in using the field experience to "practice student-centered (or, at least, more student-centered) teaching" (Eleanor, Course Paper, June 27, 2011). I initially interpreted this statement to mean that Eleanor was interested in learning to develop engaging, student-centered activities and to begin to implement them. However, further discussion with Eleanor in January regarding her use of student-centered activities in her prior teaching experience clarified that this interpretation was incorrect. After becoming aware of this, I then interpreted this statement to mean that she desired to use her summer internship as an opportunity to improve by making her teaching *more* student centered before her permanent placement began.

Eleanor initially had a very optimistic perspective of using hands-on, investigative, student-centered activities in her instruction, activities that she presumed would promote student discussion. This is in contrast to using problem-based instruction, with which she expressed discomfort. In the school in which her initial internship was positioned, student-centered learning with an emphasis on discussion was supported, and she engaged with and observed other teachers beginning to develop and establish norms of productive mathematical discussion immediately upon the school's opening. When she taught independently during her internship period, she implemented lessons that were very student-centered and required students to engage in both small- and whole-group discussions as a feature of the class. However, when she was permanently placed in her independent placement, the context for her teaching changed. Since this context was so very different, her conception of teaching when she was initially placed is interpreted as a beginning point. In the following sections, I describe the school context of her permanent placement, her classroom context, and analyze her conceptions of hands-on investigative instruction as opposed to problem-based instruction.

School context and culture. Eleanor's permanent placement was at a K-8 public charter school. It was the first year the school had an eighth grade, and Eleanor was the only on-level, eighth-grade, mathematics teacher. The other mathematics teacher who taught eighth-grade students taught algebra for students who were labeled as advanced. Eleanor's students did not have a consistent qualified teacher before Eleanor arrived at her placement in November. As a result, Eleanor's students had not begun to study the intended mathematics content until 1.5 months into the school year. Her classes were very small, one enrolling only 9 students and the other consisting of 12 students. At the same time, students were frequently absent, often due to suspension, so Eleanor felt that it was difficult simply to keep up with the curriculum pacing and even more difficult to be able to have the students work in groups.

One of the challenges Eleanor faced with regard to incorporating any type of small-group discussion in her classroom was that of addressing classroom logistics. Eleanor wanted to rearrange the classroom so that the desks were situated in groups of four in order to facilitate small-group work. Her first obstacle with regards to room arrangement was due in part to the lack of communication with the teacher from the cohort with whom she shared a teaching vacancy. Since they did not share any planning time in the central location of their shared classroom, there was very little time available during the day in which they could discuss room arrangement.

> [The other teacher] and I are still trying to figure out how to rearrange the desks and have all the desks that we need in the classroom, and maybe have a projector in a better place and have people to be, like, close… But we'd have to rearrange the desks. And had [the other teacher] and I be able to sit down and agree on something …. It's been a logistics issue … I don't want [the other teacher] to come in one morning, and oh, I've changed the room. So, we sort of have to have time to work that out together. You know we don't have planning time in our room, so it's a little tricky (Eleanor, Preliminary Interview, November 3, 2011).

Since Eleanor and her partner teacher did not share planning time in their room, it was difficult for them to have time during the school day to discuss the rearranging of the room. This made it difficult for Eleanor to incorporate the types of small-group work around hands-on, investigative, and student-centered lessons that she had used before and practiced in the summer internship.

Eleanor and her partner teacher did eventually find time to discuss rearranging the room, and they grouped their desks in fours. However, they were not able to keep the room arranged in groups of four due to the protestations of a third teacher who floated into their classroom for the last period of the day.

> We've got the initial problem that [the other teacher] and I rearranged our room, [the students have] been sitting in pairs…and we wanted to rearrange it, so we rearranged it in mostly group of fours. There's one group of five. And the art teacher who's in our room during our planning fourth period basically had a meltdown. And so today she made [the students] rearrange the chairs, so they're back in pairs…. She's like "Well, I can't work this way." …, I'm like, "I'm sorry,

> you're in here for 40 minutes at the end of the day. We're in here all day."… So, [the art teacher says] "Well, eighth graders won't work this way" (Eleanor, Seminar One, November 7, 2011).

Since the art teacher did not feel comfortable with a classroom arrangement of desks whereby students were organized in small groups and felt that "eighth graders won't work this way," Eleanor felt she was not able to arrange her desks in a way to support the formation of small groups. Although the art teacher only worked in the classroom for 40 minutes, Eleanor believed that she was the only teacher in her school who wanted to utilize small-group instruction. Therefore, she felt that she had to acquiesce to the desires of the art teacher and keep her desks situated in rows where pairs of students could possibly have discussions. This shows a conflict between the pervasive context of the school norms and the type of instruction that Eleanor was trying to implement.

According to Eleanor, this resistance to student small-group work and discussion was endemic of the school's culture. Eleanor found that when she tried to have small- or whole-group discussions in her classroom, she was met by resistance from the school's other faculty. For example, during one lesson she had her students engage in an activity wherein they were given box-and-whisker plots that had the same minimum and maximum values and they had to go around the room and match data sets to the different plots. During this activity, while the students were discussing amongst themselves, another teacher came in to the classroom assuming that the students were misbehaving. Eleanor described,

> … In my school they don't work in groups in any of their other classes. …they sit by themselves; they work quietly. There is no small group work. So, I mean, the

> best day we had they were working in groups, they were up out of their seats, they were doing stuff, and I got in trouble because the class was making noise. Well. But, they're making the right noise. Right? They're arguing over where the lower quartile should be; they're fine. …To the other teacher I said, "I'm sorry they're near your door. I'm sorry it's bothering you, but they're actually completely on task. Thank you for your concern." … Um, but I think it's the culture. If [the students are] not used to doing that … (Eleanor, Seminar One, November 7, 2011).

As described by Eleanor, the culture of the school in which she was placed was one that she felt emphasized focused, individual, silent student work during class time. When Eleanor tried to have group discussions, she was met with resistance from the school staff. This caused Eleanor difficulty with other staff members while she was trying to develop a classroom culture of collaboration.

Classroom culture. Recall that Eleanor was the eighth-grade mathematics teacher in a K-8 school. The students in her classroom had been enrolled in the school and experiencing mathematics teaching and learning in that school since it opened when they were in the third grade. Therefore, Eleanor felt that the students had been familiarized to a particular type of instruction, and that her students were resistant to different ways of learning and experiencing mathematics. She felt that since the students were not familiar with investigative, problem-based, or even collaborative learning, it was difficult for her to get her students to participate in the type of classroom that she was trying to create. She stated:

> I think the biggest thing is just having kids so used to doing that kind of thing. Because I don't do a problem every day, the days we do do problems they're really resistant. They're not used to doing it. They're not used to asking each other questions. They're used to looking and saying, "I don't know. I don't get this." And they want me to provide all the answers and they get really mad at me because I won't. … And I feel like I'm the only one trying to say, "You've got to, you know, struggle with it. Figure it out." But they think I'm just trying to be mean. …It's not what they're used to. They want a worksheet with the same [type] of problems (Eleanor, Preliminary Interview, November 3, 2011).

The difficulties that Eleanor faced with regard to school norms challenged Eleanor's ability both to set-up mathematical tasks and then to scaffold resultant classroom discussions so that her students might be able to make sense of mathematics. Eleanor would have to adapt her approach to address both of these issues in order to engage the students in her classroom within the context of this school.

Eleanor also struggled with group dynamics. Since the school was so small, the students had been in class with the same students for all 6 years that the school had been open, and they were very familiar with each other. This caused problems with small-group interactions in her classroom. She stated that

> They've been with each other, most of them, since third grade with exactly the same group of kids, and they're kind of sick of each other by eighth grade. … But they know each other so well. So, they'll just start harassing each other about something that happened years ago. I'm like, "Can we get over that? Really?" (Eleanor, Follow-Up Interview, April 23, 2013).

The small population of the school and the way that the students advanced together year to year in all the same classes caused disruption in the class when Eleanor asked her students to work collaboratively.

Also, the close-knit nature of the school contributed to certain students being ostracized from the group. One of the students in Eleanor's class of 12 students had just recently enrolled at the school, so she was the only new student in a class of children who had been in all the same classes since they were in the third grade. This also disrupted the group dynamics in certain ways, as some students refused to work with this new student. One student in particular would be very insulting to the student who had recently enrolled. She stated, "[One girl] just refuses [to work with the new girl] and gets really insulting. I don't bring it up because it descends into bullying especially [the new girl]. [This girl] repeatedly referred to [the new girl] as 'that.' 'I won't work with that.'" (Eleanor, Mentoring Session, January 25, 2012). This suggests because of her concern with creating a safe space in the classroom for all of her students, Eleanor did not force students who would be insulting to each other to work together.

Furthermore, when she did have students working on making sense of rigorous mathematics tasks, she felt that the students would immediately disengage when they met with something that was difficult. If they could not immediately ask a question about how to solve a particular problem of Eleanor, they would sit in their seats and wait to be told what to do. Eleanor felt that the students were resistant to any type of struggle. She described

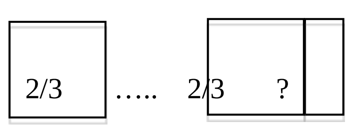
> …When I tell my kids that you can't come and chase me around the class and ask me questions,…they just sit there. And wait for me to come to them. That's not

how it works either. You keep working. (Eleanor, Seminar One, November 7, 2011).

She wanted her students to learn that struggle was part of making sense of and learning mathematics. However, she felt that the presumed culture as established by the students in her classroom, in November, was not conducive to this type of instruction, and that it would be difficult for her to try and develop the classroom norms that would allow her students to participate more fully.

Setting up tasks in student-centered lessons. Eleanor seemed committed and idealistic about implementing student-centered instruction. However, it seemed that her conception of student-centered instruction was different from that which was held by the teacher of her methods class. Her methods instructor encouraged the prospective teachers to design their lessons with a problem as the centerpiece. They were asked to use the "Before-During-After" format. In this format, the Before portion of the lesson was intended to be a period of time during which the teacher would launch or set-up the problem-based activity. The During segment of the lesson would include both the presentation of the problem to students and the time during which the students would work to solve the problem with teacher serving the role of questioning, monitoring, or scaffolding in order to support and challenge the students' work. The After portion of the lesson was a time period permitting discussion of solutions and strategies, as well as a summary of the mathematical concepts presented. When Eleanor was observed planning her lesson, the Before segment in her lesson focused on teacher modeling, with the teacher demonstrating a solution to a simpler problem than the one that the students subsequently would be asked to solve. Instead of helping students understand what the problem asked and providing them with supports that they could use to solve the intended problem, she planned to provide an exemplar for their assigned problem. Although she stated that she wanted her students to investigate and engage in hands-on activities, when she attempted to design a problem-based lesson, she limited the amount of student investigation by modeling a similar problem.

This conception of using a simpler example when attempting to plan for problem-based instruction continued when she reached her permanent placement. Indeed, it became stronger. After she had been switched to the context of her permanent placement, I presented her with a problem around which to conceptualize a lesson about dividing fractions during a November preliminary interview. The problem I presented to her was:

> Delonte has a summer job helping the manager at his apartment complex. The manager has asked Delonte to help him build a concrete patio at the back of one of the apartments. The patio will be laid down in square sections marked by wooden dividers, with each section holding 2/3 of a cubic yard of concrete. The concrete truck holds 2 ¼ cubic yards of concrete and the manager has to pay for the entire truckload.
>
> The manager told Delonte that he knows there will not be enough concrete for a full section at the very end of the patio, so the manager will use the wooden dividers to fill a smaller area at the end of the patio.
>
> 
>
>
> How many sections can Delonte and the manager fill if they use all of the concrete in the truck? (Your answer should state how many complete sections and a fraction to indicate what part [how much] of a section is on the end.)

After considering the problem, Eleanor confessed that she had "a hard time teaching dividing fractions this way. I mean, I understand why you do it, but … ." (Eleanor, Preliminary Interview, November 3, 2011). She stated that she had difficulty with planning and teaching lessons around mathematical problems, although felt she knew why problems were used in instruction. Even though she felt that she had difficulties teaching using problems such as this one, she was able to think about ways that she could provide scaffolding to support her students as they worked through solving the problem that I had presented. To do this, she recalled what she had learned during her methods class. She explained:

> …We did something like this over the summer in [methods teacher's] methods class. We had a … bar that looked like a whole but it was like four fifths… . So, I mean I guess I probably would use -- we don't really have Cuisinaire rods, but I would probably use some kind of rectangular manipulative, you know, that was just out of paper … and have the kids try and figure it out from that. Like, how do you figure out how much that last section is going to be after you take away all the two-thirds that you can … . I mean, you would definitely have to use some kind of manipulatives (Eleanor, Preliminary Interview, November 3, 2011).

Eleanor was able to reference what she had learned in her mathematics methods course the previous summer to think of a way to scaffold the problem so her students would be able to begin to think about a solution strategy. However, in addition to giving her students manipulatives in order to help them address the problem, she added that she would, again, have to model a simpler problem before letting her students begin to solve the problem on her own. In this case, however, she clarified her rationale for using this strategy. She said that she would set up the problem by

> …Making sure they understand, like, how to model division to begin with. Right. When you have … whole numbers, how do you do it? How do you take … a set of something? You'd, maybe 2 yards, and you try and divide that evenly into five different pieces, or you're going to divide that 2, you can divide it evenly. I guess I would start with something like that.
>
> Um, probably with just whole numbers to begin with. Like, you've got 15 cookies and you have to divide them among three people. The idea that you sort of give everybody their piece, where you figure out what to do with that last piece. I can't think of another way to do it (Eleanor, Preliminary Interview, November 3, 2011).

Eleanor explained that the reason that she would model a simpler problem was not necessarily to show students how to solve the problem, but instead to demonstrate to students how they would use the provided manipulatives to model an operation such as division. However, the need to show simpler examples before allowing students to engage in problem-based activities was still present.

Although Eleanor was able to think through a way to scaffold problem-based instruction in her class, she became less likely to incorporate it in her instruction because of the complexity of her context. Immediately when presented with the aforementioned problem, she mentioned that her students would be resistant to engaging with this type of problem, saying, "I just know my kids would already be like 'Ahhhhhh!'" (Eleanor, Preliminary Interview, November 3, 2011). She felt that her students would immediately reject the possibility of engaging in work on this type of problem with any level of autonomy.

Yet, Eleanor believed in including problem-based or student-centered investigations in her instruction. However, she initially had a particular conception of what was required in the set-up, or Before, phase of the lesson. She was concerned about her students' ability to engage with the problem or investigation, and she had an evolving conception of how best to allow students enough access to begin conceptualizing solution strategies to that particular situation. Since she struggled with her own efficacy of planning for this type of instruction, she also was struggling with the type of set-up or scaffolding she would provide for students. At this point, she heavily included modeling in her approach to initiating instruction. This would influence the way her lessons changed over the course of the study.

Eleanor's tension between meaningful mathematics and test preparation. During her summer courses, Eleanor expressed an internal conflict between acknowledging and rewarding effort and persistence in mathematics work and the need to make sure that students were able to be successful on unit and high-stakes standardized tests. She stated,

> I'm not sure how to balance [acknowledging effort as well as test results] at this point because, whether we like it or not, getting the right answers on certain tests matters. At a certain point, we have to work with students on test-taking skills along with all those other student skills (taking notes, participating in discussions, defending an argument), but this needs to be balanced with an emphasis on process. I want my students to know that taking risks is valued, and I want them

> to know that the effort that goes into solving problems will somehow be reflected in their grade (if not in their [high-stakes standardized test] score). At some point, I guess I hope those things will come together, but I don't think we get those students across the [high-stakes standardized test]-proficient line unless they have some confidence in their abilities. They need to have taken risks and [to] come out on top if we want them to put the effort into the tests other people deem important (Eleanor, Discussion Board Post, July 24, 2011).

Eleanor understood that acknowledging the students' effort may help them be confident enough to take risks and "come out on top," but she also indicated that results on the tests were potentially important for their future mathematics trajectory. However, she was not yet sure how to balance a focus on test-taking skills and procedures with the other behaviors that she felt were important in a mathematics classroom.

Eleanor expressed, in the first teacher seminar and in a mentoring session in November, a tension between the need to make mathematics meaningful and engaging and her desire to make sure that her students knew the content and were able to pass the state's high-stakes standardized test. In the first teaching seminar in November, the topic of task context and its interest to students arose after the teachers reflected on a commercially available video and began relating what they saw in that video to their daily lives as teachers. The video had depicted students modeling equations to match real life scientific situations, such as the phenomenon of temperature cooling, and the acceleration of an object due to gravity. Into this discussion, Eleanor suggested that a teacher could use a "variety of tasks" with different contexts in order to engage the students' interest in the practice of doing mathematics. She offered

> Maybe you could list of a variety of tasks. Like this time it was more an engineering/physics type of task and the next time it's going to have to be … maybe you can have these different kinds of problems…a range of problems and have them be that engaging … . (Eleanor, Seminar One, November 7, 2011)

However, the tension that she was feeling between providing real-world engaging contexts became evident when she responded to a comment that Michelle made about the same topic.

> Jack: And I saw it and said this should be in a science classroom, not in a math classroom. And maybe that's like
>
> Michelle: But the two worlds meet
>
> Jack: I know, but
>
> Michelle: and that's more realistic
>
> Jack: It is
>
> Michelle: than us sitting around talking about the Pythagorean theorem.
>
> Eleanor: I want them to learn the Pythagorean Theorem (Jack, Michelle & Eleanor, Seminar One, November 7, 2011).

Although, initially, Eleanor was stating the importance of using problem contexts that would engage all of the students, she made clear that teaching using these contexts could not be *at the expense* of learning the necessary content in her mathematics class. It was important for her both to engage the students into thinking about mathematics problems and to ensure that the students learned the necessary material with and without real-world contexts.

Eleanor also felt that it was important for the students to understand the uses for the mathematics content they were learning. When Eleanor spoke about having her students learn scientific notation, she spoke about incorporating a project that would have them select real-world items or concepts that would require scientific notation to denote the size or amount of the quantity accurately. Eleanor described:

> I was going to have them, but it was much more of a research project … . I wanted them to find … five big things and five small things and sort of come up with, I mean I want them to write it in scientific notation … I mean like a planet … . Like how far Jupiter is from … . And the size of a blood cell (Eleanor, Seminar One, November 7, 2011).

During this activity, Eleanor hoped her students would understand why they would be using scientific notation, since the numbers that they would be using to describe these sizes and distances would be very large or very small. She wanted this task to be open ended so that students would have a wide variety of different things for which to use scientific notation. However, when I asked Eleanor about the mathematics that would be required during this task, Eleanor stated, “The rigor isn’t there” (Eleanor, Seminar One, November 7, 2011). She felt that while the task would give meaning to the use of scientific notation, it might not be sufficiently rigorous mathematically when it came to the students using and learning the mathematics involved in scientific notation. She suggested that the students might be distracted with finding these large and small objects and not focus on the mathematical concepts with regard to place value and the procedures of changing the form of the number to scientific notation.

Furthermore, with regards to scientific notation, Eleanor was concerned that the students did not have the requisite knowledge of place value that they would need to make sense of the conversions from standard notation to scientific notation.

> I know… I'm trying to decide now after what happened today, because today was the first day we were doing the exponents. I'm trying to decide if tomorrow we go back and do place value or we actually get into exponents (Eleanor, Seminar One, November 7, 2011).

Eleanor was concerned that she might need to teach her students more about place value and the ramifications of multiplying and dividing by 10, so that her students would be able to understand scientific notation. However, it also concerned her that going backwards to discussing place value would take too much time from her curriculum, and that if her coverage of the curriculum was delayed in this way then perhaps her students might not understand the requisite information that they would need to pass the high-stakes standardized test later in the year. She expressed, "They just have to pass this test. …They need to understand that [the significand is] a number … more than [or equal to] one and less than ten" (Eleanor, Seminar One, November 7, 2011). It was important to her that by the end of the unit that her students would know the procedural algorithmic rules defining scientific notation that they would have to demonstrate in order to pass the test, namely, that in scientific notation the significand had to be greater than or equal to one and less than ten. She felt that her students' might not learn the procedural facts through the activity that she had previously described and that she may have to change the activity to focus the students' attention more on the underlying mathematics. Eleanor seemed to be trying to balance her desire to have her students develop mathematical understanding through problem solving and applying mathematics in meaningful situations and context and her need for the students to be able to perform on the test.

Eleanor's prior knowledge of the importance of standardized tests to schools and future student trajectories affected her lesson planning. When she was permanently placed, she began instruction over a month behind in the district curriculum. Her focus on students' ability to pass these tests as well as her consciousness of the need for careful pacing and coverage of content would affect her lesson planning. She would eventually make modifications to her planning in order to address concerns over preparation for testing. She would compromise her own beliefs with regard to student-centered thinking in order to respond to these concerns and cover the necessary content at the pace necessary for test preparation.

Teaching as relationships with and knowledge of students. Eleanor came into teaching with a great deal of attentiveness to students, as well as a sense of responsibility for getting to know about her students, both academically and personally, in ways that would allow her to reach them mathematically. Early on in her coursework, she expressed this clearly when discussing good teaching practices and equitable teaching, saying,

> Is the student not proficient in English, have a learning disability, come from a different cultural perspective? All those children need to be met where they are so that they can meet the same high standards. …Their success in our classroom is dependent upon our ability to get to know them, find out what they know, and search out meaningful ways to help them achieve (Eleanor, Discussion Board Post, June 17, 2011).

She believed that teachers needed to know a student in many different ways in order to be helpful to their mathematics achievement. She needed to know about their linguistic background, their cultural background, *and* their mathematical ability. She specifically mentioned but *getting to know them* and *find out what they know* were two aspects of a teacher's responsibility that would affect a teacher's ability to help the students succeed.

Eleanor also highlighted that she wanted students to be respected, welcomed, and acknowledged in her class. She said,

> The teachers I have seen use a "teacher voice" can come across as cold. Some aspects of it are what I try (but don't always succeed) at doing: I don't want to talk over anyone; I don't want to turn into the Charlie Brown … I'm big into making eye contact, but I think it's important to acknowledge several people in the class (Eleanor, Discussion Board Post, June 25, 2011).

It was important to her that her students not perceive her as "cold" so that her students would feel able to approach her and to ask her questions. She also wanted to use eye contact in the classroom to let the students feel involved and engaged in the classroom when she was speaking. Furthermore, she was clear that she did not want to silence her students by speaking when they were speaking; she wanted her students' voices heard and respected in the classroom.

She reiterated her desire for wanting her students' voices to be heard more than once in her teacher preparation courses. She explained,

> I want to keep the momentum [of the class] going, but I also believe that some students deserve to have the time to express their thought (Eleanor, Discussion Board Post, July 3, 2011).

This clarified that she knew that pacing and momentum were important to managing a class and covering the content, however she wanted her students to have space to communicate their own thinking during a class session.

However, Eleanor was confronted with difficulties when she entered her new context as teacher of record. Although she came into her teaching not wanting to be authoritarian or minimize students' voices, when she entered her new placement she began having problems with classroom management. She maintained her commitment, but then was confronted with the conflict of trying to find ways to manage her classroom while knowing, respecting, and honoring her students' voices. Eleanor, in order to build relationships with students, had to find ways to make her students feel safe to express their thinking in her classroom. Since, as was aforementioned, she was cognizant of the possibility of students offering up different answers that may or may not be supported by correct mathematics, and as she began to realize that her students might be uncomfortable with the possibility of being wrong, she would have to carefully craft a culture of community and collaboration in her classroom.

Eleanor's Adaptation to Her School Context

An Overview. When Eleanor reached her permanent placement, the school context, including the culture of the school, and the classroom context were quite different from what was the norm at the school where she completed her internship. Due to this, Eleanor had some difficulties with the context of her placement and the culture of her classroom. It was very important to Eleanor to persist in the face of this adversity in order to maintain her commitment to teaching in a relational way and to provide student-centered instruction to his students. She incorporated several activities in her classroom that promoted productive discourse. She had students draw squares on each side of a right triangle on centimeter dot paper so they could investigate the relationship between the squared areas of the legs of the triangle as they relate to the hypotenuse. She also had students cut out the angles that were made by a transversal crossing parallel lines and match up the angles that were the same size. She had students construct cylinders made by rolling a sheet of copy paper both length-wise and width-wise and subsequently have a discussion about the difference in volume and the relationship between the impact of height and radius on the volume. She had students collect data on their height and the number of steps they would need to take to transverse a certain distance and then to talk about the linear relationship between those quantities as a negative correlation. She continued to do those things regardless of how successful she felt her she and her students were in completing these activities. When I asked her what motivated her to continue to teach in a student-centered manner that attempted to promote productive mathematics discussion, she responded,

> Because I believe that's what's most worthwhile for them. I mean I really do think that they need to understand the concepts behind it and not just memorize a couple of rules and a formula. Because that just doesn't help you out in the real world. … But it's trying to get them to imagine where they really might need this stuff in the real world (Eleanor, Follow-up interview, April 23, 2013).

Eleanor was committed to the idea that the students should develop the habits of mind to engage productively in the mathematics in a way that would result in true understanding of the mathematical concepts. This belief pushed her to continue to teach in a way that promoted student sense making in the face of pushback by both the students and the other teachers in the school. In order to do this, she needed to adapt her instruction and her way of relating to students within the school context. In this section, I will analyze how she created a safe space and a collaborative culture, as well as how she adapted her classroom management to support her instruction in keeping with her teaching philosophy.

Creating a safe space and collaborative culture. One of the challenges with which Eleanor struggled was getting her students to learn how to collaborate. Eleanor felt that they were not familiar with academic collaboration since her observations of other classrooms in the school revealed that they were not permitted to discuss with or talk to their classmates in their other classes during instruction. She stated,

> Well, frankly, in other classes, they're not allowed to talk at all. … I mean when I've observed other classes, they sit there and they'll sort of whisper to each other but they're not supposed to be talking (Eleanor, Mentoring Session, January 24, 2012).

She felt that since her students were not familiar with discussing content with peers in their class, it was difficult for them to know how to use discussion time productively in her class. She felt that since the instructional style that she used in her classroom was so unfamiliar to her students, then her students needed to be taught how to discuss mathematics productively.

Furthermore, she felt that since other teachers in her school taught in a recitative manner, her students were afraid to take risks and to engage in conversations of multiple solution strategies or in divergent ways of thinking. Instead she felt that students were trying to guess the particular answer that the teacher wanted to hear. She felt that she had to combat this preconception and, at the same time, make her classroom a safe space in which students felt comfortable taking risks. Her initial conception of the importance of building relationships with children affected the manner in which she attempted to create a safe space. She wanted her students to feel comfortable with her as a teacher, rather than simply an evaluator, and believe that they had a safe space in which to voice their thoughts. She reflected,

> I really did want them to take risks and think about what might be going on, and I wasn't going to put them down if they said something wrong. That I wanted them to just think about it. That it wasn't one obvious answer that I was keeping hidden from them. Um, because I think that is just what they're used to. They're used to trying to guess what the right answer is, and that's not really how I teach (Eleanor, Follow Up Interview, April 23, 2013).

Eleanor felt that her students might be apprehensive about sharing their ideas to peers and, even more so, to the teacher in a whole-class discussion. She felt that her students were having to adapt to a completely different learning environment, and this was initially difficult for them.

However, by February and March, Eleanor felt that her students were beginning to understand what she expected when she asked the students to collaborate while working on mathematics. In a teacher seminar, she stated,

> In all their other classes the seats are always separated, and they don't work in groups. And this was the interesting thing, [a student], one day, this was about a month ago and he was trying to work really hard because he wants to go to a particular high school. "So I'm going to work with Greg." I said, "Ok, and what I want here is you guys talk about this, this, and this." He said, "Oh, so … that's

> what you meant when you said that we could talk while we work?" That's what I've been saying all year. A month ago he got it, … "Oh, that's what you mean" (Eleanor, Seminar Seven, March 14, 2012).

Since Eleanor had consistently reinforced her expectations for collaborating in her mathematics classroom, her students were beginning to understand what she meant by working together. However, since she had been teaching in a way that she felt conflicted with the expectations in the school's other classrooms, her students took a long time to be able to understand collaboration.

In order to develop this culture of collaboration, she often used whole-class discussion as a way to create a safe space. She was very conscious of the language she used when speaking to her students and evaluating their responses as she facilitated that whole group discussion. In November, after students filled out a table with *a* squared, *b* squared, and *c* squared, she asked them to think about a relationship between the three values. One student noted that all the numbers in the table were perfect squares. Though this was a true statement, this was not the relationship that she was attempting to elicit. Instead of discounting the student's observation outright, she said "OK, but give me another relationship. Like one, four and five, what's the relationship there?" (Eleanor, Classroom Observation One, November 15, 2011). She did not say "No," outright, instead, she encouraged the student to continue thinking about the relationships that existed. She was trying to help the students feel safe in their struggle to think mathematically.

She was consistent in this type of behavior for the majority of the time, asking leading questions when students were faltering in whole class discussions and moving to a different topic when students would be completely stumped. When she asked a particular student, “So [student name], what does it mean if there’s a negative relationship?” (Eleanor, Classroom Observation Three, March 29, 2012), and the student she questioned faltered, she gave him a sentence starter , “The taller you are …” (Eleanor, Classroom Observation Three, March 29, 2012) instead of discounting the student’s initial answer. She wanted her students to feel comfortable participating in small- and whole-group discussion.

When I asked Eleanor what strategies she used to create a safe environment to encourage student discussion, she stated,

> I think there was just a lot of reassuring them and I know I was very conscious of not saying “Yes, good job.” Because I didn’t want somebody to hear that, me saying that to somebody else, and not saying that to them. So, I tended to do a lot more of “Ok, ok, give me more on that,” and not say, “Yes” or “No.” (Eleanor, Follow-Up Interview, April 23, 2013).

She felt that it was important, in order to build a safe culture that helped promote productive discussion, for her students to feel that their responses would not be dismissed immediately. Instead, she allowed space for students to elaborate on their thinking and work toward a defensible mathematical solution. She did not want to be evaluative, or praise certain students over others. It was important to Eleanor to acknowledge the differences in students’ approaches to different problems without privileging one over the other. Also, she felt that it was important to support students’ efforts toward elaborating and correcting their own answers without feeling judged. Since she had known prior to entering her placement that, certainly, students would have different approaches and solutions to problem solving which may be correct or incorrect, she felt that she had to find a way to address these while *simultaneously* building positive relationships with students and supporting their positive participation in discussions. In this way, she helped to develop a culture of student discussion where students would feel safe to participate.

Classroom management in conjunction with promoting discussion. Eleanor felt that there was a negative culture with regard to discipline in this particular school. She felt that the officials in the school were pressuring her to document every student infraction so that the students could be punitively disciplined. She felt that this sometimes affected her ability to introduce new strategies that would effectively promote student discussion through the use of student-centered investigative instructional strategies. She stated,

> I haven't totally given up on it and I've been trying to introduce some pieces of things. But then I get the pressure that every little thing has to be written up on a [discipline referral] now. So that makes it, it just makes it a little trickier (Eleanor, Preliminary Interview, November 3, 2011).

She felt that the school was trying to document infractions and punish students in order to give them leverage because, in Eleanor view, the administration wanted "[the students they felt were problems] out of the school" (Eleanor, Mentoring Session, November 14, 2011). Eleanor believed that these types of discipline practices were disruptive to her instruction in a number of ways. First, it caused increased amount of absences to suspension. Second, those children who wanted to go to a school other than this charter school would be disruptive on purpose in order to get removed from the school. Third, she felt the excessive documentation put a burden on the amount of lesson planning time she had. Finally, fourth, Eleanor felt the negative discipline detracted from the safe environment that she was trying to develop in her classroom.

Through conversations during mentoring sessions, Eleanor and I devised a positive behavior strategy that would reward positive participation in class and in both small-group and whole-class discussions. Eleanor had initially begun trying to craft a reward strategy that would be used across the entire grade level and reward entire classes at a time based on their behavior. However, there were two reasons that she found that to be difficult. She stated that the other teachers on her grade level team had not responded or been resistant to implementing this strategy. I expressed concern that a single misbehaving student may keep the entire class from being rewarded and that could undermine the usefulness of this strategy. Instead, I suggested a ticket system for individual rewards that I had seen Michelle and Jack use in their classroom. Michelle and Jack had created green and red construction paper squares: green to indicate positive participation and red to indicate that a particular student or group of students were acting outside of classroom expectations. Eleanor modified the strategy for her particular class. She purchased lottery tickets that would reward students for:

> Being on task, asking a good questions of me or just in conversation or somebody else or explaining to somebody else so those kind of accountable talk kinds of things and being able to explain your answers when I come over. …OK so explain to me what you're doing here, what do those units need to be and why? Explain that to me. Like you do that, and you get a ticket (Eleanor, Mentoring Session, January 17, 2012).

The "accountable talk" that Eleanor referenced was the district's phrase that referred to students engaging in productive mathematical discussion. Therefore, she was using the tickets not only to manage student behavior, but also to encourage the students to contribute productively to whole-class and small-group discussions about mathematics. She gave tickets to students who were participating positively, had them write their name on the back, and then kept all the tickets for the week in a jar. She would choose one ticket at random at the end of the week and reward students for their productive participation in mathematics. The students responded to this reward system and began participating more positively in class.

Since Eleanor had entered teaching with a commitment to building relationships and rapport with students as a way to support instruction, it was important to Eleanor to be supportive of students' positive participation in class, rather than being a disciplinarian. As evidenced by the pressure for documentation of every infraction on a discipline referral and the numerous suspensions her students received, Eleanor perceived that the pervasive culture of the school was one of strict discipline rather than one of positive behavior intervention. In order to build relationships with her students *while* maintaining order and productive participation in class, she felt the need to institute a positive system of reward for participation in mathematical activity.

Eleanor's Changing Approach to Lesson Development

Overview. Eleanor clearly bought into and was committed to implementing student-centered investigative instruction that would promote students' explanation and discussion in her classroom. She stated that she believed instruction that would allow students to make sense of mathematics in a meaningful way was "what's most worthwhile for [my students]" (Eleanor, Follow Up Interview, April 23, 2013). However, this became difficult at times during the year for several different reasons. Eleanor started the year having students engage in an investigation supporting their discovery of the Pythagorean Theorem. Students would construct squares on centimeter dot paper off of the edges of triangles of different sizes. They would then measure, or estimate, the area of the squares and enter those values into a table. Eleanor then asked the students to identify the relationship between these squared dimensions.

However, in January, Eleanor had begun using worksheets as her main activity instead of student-centered investigations. She attempted to have students work in pairs or in small groups in order to develop answers to the multiple problems on the worksheet. However, soon after January, she reverted to having students work with hands-on investigations. There were several challenges that Eleanor faced that contributed to the momentary shift to worksheets, as well as affecting the efficacy of her negotiation of student sense making and facilitation of productive discussion. In this section I will analyze her challenges that affected the use of student-centered activities in her class: the availability of materials and the pressures she felt with regard to time, testing and students' individual knowledge.

Availability of materials. Eleanor's prior teaching experiences were in school contexts that were comparatively well resourced. Therefore, when she encountered a school context in which materials were scarce, her lesson planning was effected. Initially, successful engagement of her students in mathematics was affected due to their need to construct their own diagrams. Secondly, Eleanor's lack of experience with having to acquire her own materials because of lack of materials in the school proper caused her to change her lessons to include only materials that were readily available rather than finding her own. However, Eleanor adapted to the dearth of materials by purchasing the materials that she desired to use in her lessons, and this affected the student-centered nature of her lesson planning and enactment.

When I first observed Eleanor's teaching in November, she was engaging students in an investigation of the Pythagorean Theorem. However, this had uneven results because the students were required to draw the triangles and squares on the paper themselves. Much of the students' working time in class was spent on struggling to draw the squares properly, and Eleanor did not get much time to engage students in thinking about the resultant pattern and what it could mean for the equation. When I spoke to Eleanor, she also recognized the concern about what the students were actually spending time on in class: the drawing of the squares as opposed to making sense of the desired mathematics. Furthermore, Michelle spoke to Eleanor about possibly having the models pre-drawn so the construction of the diagrams did not detract from the mathematics.

> MM: And to that point, do you think it would have helped if we had some models already done so, triangles already cut up and you could just piece them together themselves.
>
> ES: That would have helped some of the kids; I mean in terms of some of them they find lower skills to start with but they're
>
> MM: For that reason.
>
> ES: And then they can't figure it out, "It doesn't look right." Yeah, because it's not a square (giggling). So I think there were a lot of layers [where] they were getting stuck (Eleanor and Michelle, Seminar One, November 21, 2011).

Eleanor realized that time spent on drawing their own diagrams was detracting from the mathematical sense making in her lessons, and she knew that this might be a problem going forward. She decided to use peg-boards in a future lesson on transformations instead of having the students negotiate the construction of transformed graphs on their own paper. The reason for these concerns was that the school at which she worked was "virtually out of paper … and I'm trying to save paper for the test" (Eleanor, Mentoring Session, November 15, 2011). Her fear of having to limit the amount of photocopies she was making was affecting her instructional decision-making and having an effect on the amount of mathematical sense making that her students were engaging with in class. In response to this, Eleanor had to find different materials that her students could use rather than making several photocopies every day. Some of the adaptations she made included using aforementioned peg-boards, making materials herself out of her own personal supplies, or having students create "a lot of little mini-posters" (Eleanor, Follow Up Interview, April 24, 2013) so that students would only have to use one sheet of paper in a small group.

The availability of materials also contributed to Eleanor changing her plans in January from having students create scale drawings from maps to investigate the concept of similarity and scaling to a one-page worksheet that had several similarity problems where students would use proportions to find a missing side. When I initially entered her class, Eleanor prefaced her instruction by explaining to me why she had changed her plan. She mentioned that she was not able to acquire maps from the social studies teacher, and that she only had one available tape measure for the students to use. She realized that the instructional goals would be compromised by her inability to access appropriate materials, so she changed her plan to a worksheet.

Furthermore, the only mathematics teacher in the school that had a class set of graphing calculators was the algebra teacher; in addition, many calculators were not working. Initially, Eleanor was apprehensive of borrowing the calculators often since her students had written inappropriate things on the calculators, and she felt that the “algebra teacher got tired of turning on her calculators and seeing that” (Eleanor, Mentoring Sessions, March 22, 2012). If she had access to a set of calculators on her own, she might have been less fearful of her students using the calculators. However, in March, during the last lesson I observed, she did borrow the calculators to allow her students to interact with functional data and the relationship between two variables. She stated, “I want them to be able to have the chance to predict … What kind of relationship do you think this going to be?” (Eleanor, Mentoring Session, March 22, 2012). She did overcome her apprehension and request to borrow the classroom set of calculators so her students could make sense of the relationship between their height and the number of steps they would have to take to traverse a particular distance. It seemed that availability of materials affected Eleanor’s instructional decision-making, however this restriction was less as the school year progressed.

Time constraints, testing, and student knowledge. Eleanor had always been concerned about the need to prepare her students for testing. From the outset of data collection, she mentioned that she perceived a tension between the type of student-centered instruction that she felt committed to providing her students and the need to cover content in that would allow her students to know and retain procedures for their later performance on district and state tests. This tension caused her to briefly modify her lesson plan to include worksheets that would allow her students to practice necessary procedures as well as allow *her* to progress through the curriculum at a rate that would provide her the opportunity to cover the most content possible before the required high-stakes standardized tests. Although she returned to her original planning and implementation of student-centered lessons, she felt the need to change her approach during December and January in order to address these concerns.

Throughout the year, Eleanor expressed concern at her and her students' ability to prepare for both the district's test and the upcoming, high-stakes, standardized assessment required by the state. These pressures influenced her instructional decision-making. In November, during our first teacher seminar, we were discussing how to approach place-value, scientific notation, and exponents in an investigative way that would allow students access to the concepts behind the procedures. However, one of the thoughts that arose during the discussion was that what the students really needed to know about scientific notation to pass the test was simply the rule behind it. Eleanor stated, specifically, "They just have to pass this test" (Eleanor, Seminar One, November 7, 2011). She clearly was experiencing a tension between having her students acquire conceptual understanding and the need to prepare them for the examination.

Eleanor was under pressure for both testing and time. Since she had arrived at her placement in November and there had been no consistent teacher before that time, she felt obligated to start at the beginning of the curriculum. This put her behind according to the district's pacing guide. However, she knew, "[My students] all have to take the [high-stakes standardized assessment] and they're all going to algebra next year. … So, how do

I get them there?" (Eleanor, Mentoring Session, December 20, 2011). She felt obligated to prepare her students in the best way she could so that when they were required to take Algebra I in the ninth grade, they would be prepared. She internalized the responsibility of preparing the students for the content on which they would have to demonstrate proficiency.

These tensions clearly affected her instructional decision-making. Although she had demonstrated commitment to student-centered instruction and the promotion of student discussion in class, during January she reverted to giving her students worksheets. She stated, "No, today was just a worksheet; I'm not crazy about doing that, but you need to see what they remember" (Eleanor, Mentoring Session, January 17, 2012). This statement demonstrates that although she did not like using worksheets and presenting tasks that were less hands-on, investigative, and open for student discussion, she felt compelled to use those tasks and worksheets when evaluating her students' skills.

However, due to her commitment to student-centered instruction that provided opportunities for discussion, she very quickly reverted to using more investigative activities during her instruction. She clarified her rationale in a follow-up interview during the following school year.

> A lot of that is the pressure of getting ready for [the high-stakes standardized assessment] and trying to catch up [in the curriculum]. Because, the hands-on activities, while I think they're worthwhile, the kids don't always see the point. …While they'll remember the activity they don't always remember the math involved. And it tends to slow us down and leave me behind … . We just never got to where we were supposed to be [as defined] by [the high-stakes standardized

> assessment]. So …with the worksheets I was able to do that but, I mean the thing that I like about doing hands-on activities is I just think there are more multiple points of entry, I guess. I mean, it's just more accessible to different skill sets, and … so when I move away from those, I just get very concerned about my really low-level kids (Eleanor, Follow-Up Interview, April 24, 2013).

She believed that investigative activities that allowed space for discussion were worthwhile; however, she felt that sometimes the time required to have the students really engage in the mathematics of that investigation was not worth the time. She felt that the class time required for these more involved activities prevented her from adequately covering curriculum in time for the students to prepare for the high-stakes standardized assessment. This caused her to switch temporarily to using worksheets because they were more straightforward and required less class time to assess and assist her students in their skill development. However, she very quickly resumed sourcing and developing student-centered investigative lessons that allowed space for student discussion because she was concerned that worksheets and more closed lessons would not benefit those students who she perceived were low-level. She wanted to provide support for the students who she felt needed different ways to access the mathematics, and therefore, after testing, she returned to the type of instructional decisions she was making at the beginning of the year.

Teacher self-efficacy. Although Eleanor had several years of teaching experience on many different grade levels, she still had some questions about her own efficacy. Specifically, when speaking with me in a preliminary interview in November about problem-based instruction, she stated, "I have a hard time teaching dividing fractions this way" (Eleanor, Preliminary Interview, November 3, 2011). Although she had discussed a commitment to student-centered instruction that promoted discussion, she did not feel comfortable with problem-based instruction. This might have contributed to the fact that she did not seem to use problems to motivate discussion; rather she used investigations.

Furthermore, as was aforementioned, she spent some time in January using worksheets instead of the investigative activities in which she initially, and later, engaged her students. Her own efficacy may have contributed to this shift in activity use. She stated,

> I was trying to think of one over the weekend then I got sucked into doing planning stuff, Bat Mitzvah planning stuff, but it's probably like that for the next two months. I'm not straight at home so I'm trying to come up with things … While I love this stuff, but I'm having a hard time coming up with projects until we get to things like similar triangles and then there's a map task I used to do in [my old district] … I'm trying to find it (Eleanor, Mentoring Session, January 17, 2012).

Due to the constraints on her time, Eleanor found she was less able to design project and investigative activities that she had not previously used in other teaching assignments. This seemed to contribute to her practice of using worksheets as the central feature of her lesson, rather than the hands-on activities that promoted discussion and sense making.

Eleanor's classes would engage in small-group and whole-group discussion throughout the year, however participation of *all* students in *every* activity was not consistent. Due to the uneven nature of students' participation and buy-in, Eleanor stated, "I'm not sure I ever got really, had too many successes last year with promoting student discussion. There were a few times" (Eleanor, Follow-Up Interview, April 24, 2013). She recognized that some lessons were more successful than others with regard to promoting student discussion, however, she downplayed her own efficacy about enticing and expecting students to explain their work either to her or other students. Students engaged in more productive discussion as the year progressed.

Eleanor's Changing Use of Small- and Whole-Group Discussion

Overview. Eleanor began the year focusing on investigative student-centered activities where students would work together in small groups, ostensibly, to have productive sense making mathematical discussions with their peers. She would assign students different roles and have them collaborate to produce mini-posters and deliverables that explained their group's thinking to the class. She consistently asked students, while in their small groups, to explain to her their thinking and their process. Even as she began to move towards the use of more worksheets, she consistently had her students work with others to complete the assignments. However, in February through March, Eleanor seemed to design opportunities for discussion that involved more whole-group discussion. These discussions involved a great deal of mathematical sense making by the students, however, these discussions were organized differently than her initial use of small-group discussion. She would ask a question and allow several students to voice their thinking. Then she would direct them either to work on something or to write something individually or in groups, and then she would bring the attention of the class back to a whole-group discussion to share what they had discovered or calculated. Her use of whole-group discussion addressed some of her initial concerns about building rapport and relationships with her students by providing them with scaffolding through investigative activities. Her scaffolding allowed her to address issues of her context, in that she maintained student engagement in a mode of instruction with which her students were not familiar. She built a safe space for her students in order to engage them consistently in mathematical thinking without allowing them to disengage because of their perceived lack of ability. Her focus on building relationships with students caused her to modify her instructional tactics to include more whole-group instruction in order to support her struggling students as well as maintain her "high-ability" students' engagement in the lesson. She used several different strategies that to mediate challenges that she faced in engaging her students in investigative activities that supported mathematics learning. First I analyze Eleanor's use of roles, and then I analyze the challenges associated with student ability, student self-efficacy, and teacher-self efficacy that contributed to her change in organization of opportunities for student discussion.

Assigning student roles. From the first time I spoke to Eleanor, she highlighted assigning student roles as a way she would facilitate students' peer-to-peer explanations. Furthermore, Michelle and Eleanor discussed different types of roles that would be useful in the first teacher seminar in November. After discussions in mentoring sessions, Eleanor implemented the roles in her classroom. She instructed her students as follows:

> I'm gonna need one person who's the recorder who makes sure that the table on the other side of the dot paper gets filled out as completely as you can. … One person's gonna be the facilitator. Make sure everybody's doing their job (Eleanor, First Classroom Observation, November 15, 2011).

During this lesson, students worked in groups of four. Eleanor documented which student held which role and circulated the room reminding students of their responsibilities. A certain student who was often off-task chose the role of facilitator. When Eleanor reminded this student of his role, he became serious and began leading his group to remain on task and to discuss the pattern in the Pythagorean Theorem assignment.

When reflecting on the lesson directly after this lesson, Eleanor felt that "[the students] still don't get the roles" (Eleanor, Mentoring Session, November 15, 2011). She believed that her students did not immediately adopt the roles and perform to her expectations, and that her students would need more reinforcement. However, this was something she understood before she attempted to implement the roles. When she first discussed the need to use roles, she stated, "You'd need to build it up over time" (Eleanor, Preliminary Interview, November 3, 2011). Therefore, she understood she would need to reinforce consistently the expectations of her defined roles in order to promote and facilitate discussion. She would have to develop student interactions in groups using roles as a classroom norm. This belief allowed her to persist in role use even though she felt her students had not been initially as successful as she had hoped.

By the next year, when she reflected on what had helped her to promote productive discussions in class, she highlighted her use of student roles and a critical teacher move. She mentioned,

> [Roles] any time they were in the small groups … And [there was] some kind of product that I wanted from them. Whether it was a poster or, almost any time, I guess we did a lot of little mini-posters. So, I used roles similar to that almost every time. … I think that was one of the big things, [it] was the roles (Eleanor, Follow-Up Interview, April 24, 2013).

She used roles, and she used them selectively. She made sure there was a purpose for students to have a role, and the students would have to produce a deliverable by the end of the activity. This helped her to facilitate productive student discussion. However, she persisted in requiring individual students to explain when they were engaged in their small-group activities. She explained,

> I mean a lot of it, it[‘s] just going over, just checking in with the group and asking questions of all the people. They’ll all be, “Ask so-and-so that’s their job.” “No, everybody has to be able to answer my question.” And, you know, “Well, can you explain that to this person? Because she doesn’t seem to understand what you’re saying. So, I’m going to stand here while you explain it to her” (Eleanor, Follow-Up Interview, April 24, 2013).

She did not allow the students to abdicate their responsibility to explain the mathematics to her and to other students because “Reporter” was not their role. She required every student to be an equal participant in mathematical sense making through discussion by asking students to explain and by requiring them to explain their thinking to others.

Student ability and self-efficacy. Eleanor’s perception of her students’ ability, and her students’ self-efficacy, affected the way she designed and implemented lessons that promoted discussion in her class as well as her perception of how successful they were at promoting discussion. Her initial conception of the importance of building relationships with students while teaching caused her to change her tactics with regard to engagement of students in discussion. She began to incorporate more whole-group discussion in order to maintain her safe space for student participation as well as support her students that she perceived here of low-ability.

Student ability. Eleanor’s perception of her students’ ability seemed to have an effect on the type of student discussion she promoted in her class and how she engaged her students in this discussion. During many of our mentoring sessions, Eleanor's conversations tended to address her concerns for her students' ability both in general and in specific. She often made comments such as, "[This student] is very concrete and doing anything abstract is really hard for her" (Eleanor, Mentoring Session, December 20, 2012). She was very concerned about the ability level of her students and about whether they learning the material.

Eleanor's perception of her students' ability was a challenge that became connected to the type of discussion Eleanor promoted in her class. In a follow-up interview the following year, when I mentioned to Eleanor that I noticed she seemed to move from promoting long periods of small-group work to guided periods of whole-group sense making that would be broken up by small periods of individual, paired, or small-group work, she cited student ability as a rationale. She said,

> And I think that was partly a way of trying to hold some of [the students] accountable, and trying to support some of the kids who were lower level. There were some kids in that class who were actually pretty good, if they weren't totally distracted and off-topic… And then there were some kids in that class who … aren't officially special ed but were very, very low. … And I think in part what I was thinking was just that I was trying to support the kids who were low and trying to keep everybody focused so that they knew they were going to be put on the spot a bit more (Eleanor, Follow-Up Interview, April 24, 2013).

Eleanor felt that structuring her classes around guided whole-group sense making would support the students whom she felt were of a lower-ability level. Furthermore, she felt that by structuring her class in that way she was able to keep everyone on task and to hold the students she felt were higher-achievers accountable for continuing participation. She was able to engage the students who would normally struggle by asking them leading questions and scaffolding their thinking and at the same time require explanations and contributions from all students in the class.

Student self-efficacy. Eleanor paid attention to her students' self-efficacy and self-concept regarding learning mathematics. She noted that "there's some kids with real confidence issues" (Eleanor, Mentoring Session, January 17, 2012), and she would work diligently with these students to help them improve their skills. Upon entering the alternative certification program, she believed in the importance of building a rapport with students. Her concern about her students' confidence, and the resultant alteration of the type of discussions she facilitated in her class, were directly related to her conception of the need to provide a safe and supportive space in which trust was shared between student and teacher.

Eleanor explained why she felt her students had feelings of low self-concept: "These are the lower-level eighth graders you know. I mean they know that too" (Eleanor, Mentoring Session, December 5, 2011). There were two sections of eighth graders at this particular school, one contained 24 students and the other contained 14. They traveled together to every other class during the day, but were separated for mathematics. There were three eighth-grade math classes, two "on-level" mathematics classes and one Algebra I class. Eleanor taught the two "on-level" mathematics classes, which contained 9 students and 12 students respectively. All of the other students who were not with Eleanor were in Algebra I. So, Eleanor felt that the students thought of themselves as low-ability students, since the school had placed their classmates into a higher-level mathematics class.

Due to Eleanor's perception of her students' self-concept, it was very important for her to support them continually in their participation in mathematics discussion and sense making. When speaking about what motivated her to promote, scaffold, and support *all* students' participation in these types of discussions, she stated:

> And partly what I knew of their own conceptions of themselves as math learners… . And I hadn't seen that as much, to be honest, in middle schoolers before last year… . Usually it's with algebra that I had seen kids run into a wall and be like, "Oh, you know, I used to be good at math, but obviously I really can't do what needs to be done." I just, I hadn't run into that before. Among my kids at [my school], they were really hitting that wall at 7th or 8th grade as opposed to with algebra or geometry. So … trying to, I don't want to say build up their self-esteem, but, to see that, to sort of encourage them to see themselves as math learners. That played into a lot of what I did (Eleanor, Follow-Up Interview, April 24, 2013).

Eleanor felt that continuing to keep the students engaged in conversations about mathematics would not only help students learn the content, but would also help in developing students' mathematics identity. She felt that having a positive mathematics identity would help them later on in their mathematics learning in high school. This contributed to her using more scaffolded, whole-class discussions rather than letting student work in small groups and giving them the opportunity to disengage when they became frustrated.

Changes in set-up of activities. In November, Eleanor set-up her Pythagorean Theorem activity by assigning roles and putting an example of what the squares should look like as constructed around the sides of the triangle on the document camera. She had the students construct all of the triangles and squares on their paper, then measure their area, and fill in their table so they could have a discussion later about the relationship between the squares. However, the majority of the class period was consumed by the physical construction of the triangles, and there was little time for students to think critically about the relationship. The whole-group discussion was limited to the reporting out of results of the area calculations, and then students were released to work individually or in a group to discover the pattern. The students worked in small groups, but discussions that were mathematical in nature were limited, and result of understanding the Pythagorean Theorem was limited to a single student. When I reflected with Eleanor afterward, we discussed the set-up to the activity and what was provided for them and what was not. I suggested that since the students struggled with drawing the squares correctly, the activity might have been set-up better by providing the students with already drawn diagrams, so the focus would be on mathematics and not drawing. Over the course of the following months (December – March), Eleanor modified the way she set-up the activities, so that students would spend more time talking and thinking about mathematics.

By March, Eleanor had begun to set-up her activities in smaller bursts. Instead of having one extended time of small-group work, she would have them complete their small-group work in shorter timeframes. Each small-group section of the class would be individually set-up, as well as the whole-group sections, so that students had a great deal of support to engage in the productive discussion necessary to result in mathematical sense making. In an activity in March, Eleanor had the students use quick paper folding techniques to create parallel lines that would then be cut by a transversal. The resultant angles were cut out, and the students were then directed to match up the angles that were the same measure. After the angles were matched, Eleanor was able to lead the class in a whole-group discussion about which angles are congruent and the names of those angles.

In an activity at the end of March, Eleanor had the students measure their heights and record how many steps it would take for a person to walk from one chalk marking to another. She designed this activity to engage students in thinking about linear correlations of the data and interpretations of a slope, in this case, a negative slope. She allowed the students to work in small groups to collect their data; first, inside their classroom, students were expected to measure each other's height with a tape measure. The next step was taking the students outside and having them count their steps three separate times. When they returned into the classroom, she directed them to calculate the average of their steps and collect everyone's data in a table of values. Each component was set-up separately. In this case, as a contrast to November, she was able to facilitate a 22-minute productive whole-group discussion about interpretations of slope in relation to that activity.

The struggles of Eleanor's students caused her to find different ways to set-up activities in mini-small-group interactions followed up by long class discussions. Her changes in set-up allowed her students more access to not only discussion, but also the mathematics of the activity. Deconstructing the activity into smaller pieces supported students' on-task behavior and allowed them a greater ability to participate in mathematical sense making through a whole-group discussion.

Conclusion

There were some affordances and constraints that arose from Eleanor's prior experience in teaching. She had little experience in designing instruction where she was not supported with materials and resources. This inexperience caused her development to slow, as she had to become familiar with sourcing her own resources that would support her instruction. However, her knowledge of the possibility of different and/or divergent student responses to mathematical questions caused her to consider how to consciously and deliberately create a safe and supportive space in which students might feel comfortable participating in mathematical discussion.

However, Eleanor's feelings of tension between what she felt was appropriate mathematical instruction and the demands of high-stakes, standardized testing altered her lesson design during the middle of the school year. She changed her initial approach of designing and implementing student-centered instruction in order to serve the need of curriculum coverage. However, her commitment to the benefit of student-centered investigation and productive student discussion overrode her fear of being unable to cover content, and she quickly returned to designing and implementing instruction that was student centered and provided opportunities for discussion. She concentrated on her efforts, and as a result, developed and structured activities that would more directly and efficiently engage students in mathematical sense making.

Eleanor's desire to build relationships with her students and to create safe spaces where they could engage in mathematics affected her design of the amount and type of discussion that her students would engage in during class. Furthermore, her perception of her students' ability in conjunction with her desire to support those students she perceived as having less ability, caused her to change her lesson design. Instead of having long stretches of time where students engaged in small-group activities as a primary feature of instruction, she would have students collaborate in small bursts, and she would guide the investigation through posing questions and engaging students in whole-group discussions in which students would collaboratively make sense of the mathematics involved in the task. These short whole-group discussions would allow *all* students in her classroom access to the mathematics that they then completed either individually or with their classmates in small-group discussions.

The following is a summary of the findings for Eleanor Scott that positions my analysis of Eleanor's data in light of the conceptual lens addressing challenges, negotiations, and strategies. Her ability to build relationships was a strategy to which she originally attended, as well as a way to negotiate challenges she faced in the classroom. In order to motivate students with little confidence to participate in making sense and discussing challenging mathematics, she built relationships with them and supported their participation by praising effort and contributing and leading them to a mathematically sound contribution, rather than praising correct answers and denying the effort of students who contributed incorrect answers. She also developed a strategy to maintain the students accountability to participate in the sense making and productive discussions. She scaffolded whole-group discussions with short opportunities for individual or group thinking in order to require students to consistently be present and participatory in the overall sense making that was required by the mathematics task at hand. Her context was a challenge that she had to negotiate, since her students did not work collaboratively or have productive discussions in any of the classes that they were taking or had previously taken in this school. Therefore, she had to face the disapproval of other teachers, while at the same time teaching her students what it meant to work collaboratively. Also, the tracking of her students was a challenge, as her students knew that they were considered the low-ability students. She had to build relationships with these students to build their confidence and encourage their participation in mathematical discussions. Finally, through the support of myself and her peers in the teacher seminar, she was able to develop roles so her students would know what was expected of them in small groups. Also, Eleanor and I developed a classroom management strategy that would reward positive participation in mathematical discussions.

Chapter 6: Jack Davis

Jack Davis entered the alternative certification program directly after graduating a highly selective university with a degree in economics. He was motivation to enter teaching was influenced by his experience working with middle-school aged students in an after-school program. He began as a mentor for the student participants in this program and noticed quickly that in order to support the students' positive self-esteem, as a component of the mentoring effort, the students in the program needed support for their mathematics learning. Thorough his involvement with this program, Jack developed a passion for both building relationships with middle-school-aged students and for teaching mathematics. Jack's own experiences living and attending school in the same district in which he would be teaching bolstered his passion and dedication to teaching students in this district.

I observed Jack planning for his own microteaching lesson during one session within the program's initial course addressing methods for teaching mathematics. His comments during this session led me to infer that he had a strong dedication to engaging students in student-centered approaches to mathematics; however he was not certain how to set-up a task so that students would be able to have success accessing the task. Initially, Jack would design lessons where the teachers would simply offer or distribute a problematic task to students without any introduction or set-up. During this session of the methods course, I was involved in a group discussion with Jack and his group-mates as to how to set-up a problematic task in a way that would provide students with guidance and allow them to access the task without revealing a particular solution strategy for the task. In addition, I also reviewed all of the written materials that Jack submitted in his summer internship course. Within these submissions, Jack made many references to excellence and equity in teaching, building relationships with students, and providing access to student-centered instruction. He felt that providing students with excellent and equitable student-centered teaching was his duty and that doing so would be instrumental in closing what could be perceived as an opportunity gap.

Jack was paired with a co-teacher for the initial period of his internship (August-October). This co-teacher, who was also the department chair, appreciated his work and created another vacancy in the school in order to retain Jack (and Michelle) in the same school for their permanent placement. Jack immediately began attempting to implement student-centered instruction that included problematic tasks and encouraged students to make sense of mathematics, implementing approaches that he had both learned in his teacher-preparation classes as well as from his own experience as a sixth-grader in an elementary school located approximately 10 miles from where he was interning. Over the course of the year, Jack made use of the school district curriculum resources, professional development from his co-teacher and his district, lessons from his summer content course for which I was the instructor, and mentoring sessions in order to refine and attempt to perfect his implementation of student-centered instruction that promoted discussion.

Jack's initial approach to teaching included a focus on excellence and equity with regard to designing and implementing instruction. He felt it was important to develop relationships with students, which later affected his ability to engage students in productive discussion. He understood the diversity of the district and wanted to learn how to engage students from many different backgrounds. He also wanted to develop his enactment of student-centered instruction, specifically with regard to allocating appropriate and sufficient time for activities in order to engage students in making sense of mathematics while at the same time covering the required curriculum. He also was concerned about his own content knowledge and the affect that any missing content knowledge on his part would contribute to an inability to engage students in sense making. These factors influenced his planning and implantation of instruction throughout the year.

Analysis of Jack's Instructional Practice: Challenges and Change

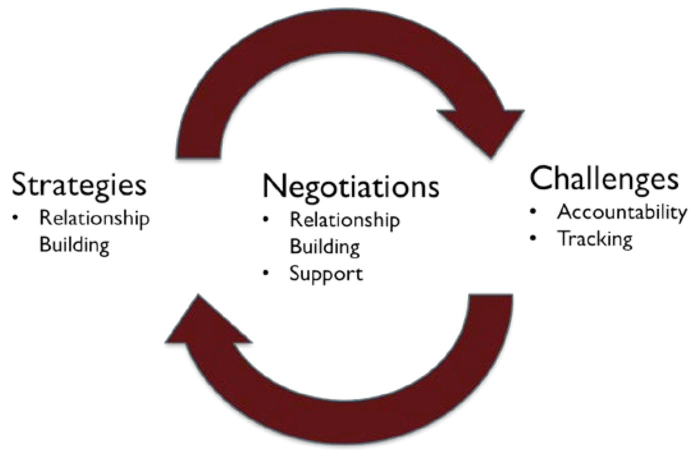


Figure 5: Findings for Jack

Throughout the 5-month period of data collection (November – March), Jack implemented instructional strategies that included applied and investigative tasks that encouraged students to make sense of mathematics in small- and whole-groups. During this time, he was faced with challenges, and he implemented or modified her instructional approach in response to those challenges. These challenges related to self-efficacy, time constraints, testing, and familiarity with mathematical content. As he negotiated these challenges, Jack's planning for teaching and teaching practice shifted. Since he immediately began teaching in ways that promoted sense making and student discussion, these shifts were smaller and more nuanced than what might be expected of a beginning teacher. This chapter initially presents an analysis of Jack's early conception of the role of teaching, addressing how he approached planning for mathematics teaching during his summer coursework. The next section of this chapter analyses the way Jack modified both his planning and his role as facilitator in the classroom, as he simultaneously developed relationships with students and increased their comfort in participating in discussion. This section focuses on how Jack began to think about student misconceptions and potential questions to respond to these misconceptions during planning. Furthermore, It addresses how, as student participation in discussion increased, Jack attributed it to his and his students' increasing comfort with instruction that proceeded in a student-centered, discursive manner. Finally, the chapter concludes with an analysis of the changes in the type of discussion that Jack implemented in his classroom. This analysis portrays a shift toward instruction that included centers, as well as instruction that focused more on sense-making discussions as carried out by small groups of students.

Jack's Initial Conception of Teaching

I accessed three data sources in order to establish Jack's initial conception of teaching. These included his actions during his mathematics methods class prior to his internship, posts on a university-managed, electronic bulletin board where Jack reflected on course readings in his summer internship course, and Jack's final portfolio as submitted for his summer internship course, including every paper that Jack wrote for the course.

Drive for excellence and equity in teaching. Jack's commitment to excellence and equity was apparent early in his teacher preparation program. He stated,

> For me, good teaching is characterized by many traits including content knowledge, prudent lesson planning, flawless lesson delivery, and fair assessment … . It is noteworthy that the modifier “for equity” introduces an additional purpose to teaching that “good” doesn’t fully describe … . I want to become an excellent teacher. I expect it from myself, and our children deserve it. While I accept that it will come over years of professional development, “good” should not be enough for me (Jack, Discussion Board Post, June 19, 2011).

Jack believed that the students in the district in which he would be teaching deserved nothing less than perfection from their teachers with regard to every aspect of their educational experience. He was motivated to work hard to provide well-designed opportunities through which his students could learn. His early experience of learning mathematics in the same district familiarized him with teachers and mathematics specialists who had provided him with excellent opportunities to learn mathematics in a way that forced him to make sense of mathematics through group discussions of mathematical problems.

Jack understood the diversity of the communities that were a part of this school district. Since he had lived and gone to school in this district, his understanding of this diversity supported his commitment to equity. He wrote,

> I don’t have many assumptions of the people in the area because I’ve witnessed the amount of diversity first hand. I have friends from the area in law school and friends in jail. I have friends that didn’t finish high school and I was blessed enough to go on to [a highly selective university] and am now pursuing a graduate degree. So for me it is quite difficult to have any preconceived notions about

> ability or motivation or any other excuse or label commonly placed on Black children. I think I am pretty good at seeing each child as their [sic] own person which may seem idealistic, but it is my reality (Jack, Course Paper, July 6, 2011).

Jack had personal knowledge of the district and the diversity within it. Due to this, Jack was committed to treating children as individuals. Furthermore, he felt he was able to do so without assumptions about his students' potential based on racial stereotypes. He was committed to treating and educating his students in an equitable manner. Jack was aware of what researchers and policymakers called "the achievement gap." Due to this, Jack dedicated himself to improving students' mathematics learning throughout his college years. He then joined this alternative certification program in order to continue that work.

However, Jack immediately realized that he would have to work hard to become an excellent teacher and recognized that there were going to be challenges that he would have to address in order to become excellent. When reflecting upon his summer field experience with middle-school-aged children at a local community center, Jack reflected that,

> It's important for me to also practice responding to student learning while in front of the room. I think it will be difficult to hear, comprehend, and record thinking on the board while standing there and still identify any flaws in their logic (Jack, Course Paper, July 6, 2011).

Jack noticed that a key feature of engaging students in discussion and mathematical sense making would require a teacher to be able to "hear, comprehend, and record" student thinking quickly. Also he would be required to interpret the students' thinking and evaluate the validity of a student's argument in the moment. He recognized that this would be initially difficult and that he would have to work hard to develop this skill. This understanding facilitated future mentoring opportunities that occurred after Jack's permanent placement, as he tried to improve his interactions with students and their thinking.

Relational teaching. Jack's initial introduction to something similar to teaching was through an after-school mentoring program that he organized and participated in during college. This experience influenced Jack as he perceived both knowledge of and relationships with students as critically important in his conception of teaching. He continued to hone this perspective during his summer field experience where he functioned as one of a few mathematics teachers with a small group of students. When reflecting on this summer practicum experience, he stated,

> With a background in mentoring and small-group tutoring, my style is going to be decidedly relational. My favorite part of being at [the local community center] was working with individual students because I think that making students respect you as a person convinces them to value your interests (like math) and gives them the motivation to push through it on their own (Jack, Discussion Board Post, July 3, 2011)

Jack felt that he would be better able to engage students in mathematics if he developed a favorable relationship with those students. Therefore, he felt that his teaching style would include aspects of developing a professional relationship with the students in his class.

As Jack progressed through the summer field experience, his belief in the importance of relationship building became stronger. As part of his summer internship course, he was required to interview one of the students who was participating in the field experience. After the field experience, he felt that the students whom he interviewed became more responsive to his directions and would consistently engage in the mathematics when asked to do so by Jack. He reflected,

> It is interesting how much more familiar he [a student] has become in the two weeks that followed the interview which reaffirms to me how important it is to build relationships with students so that they see you as human and so that you can do the same … . [G]etting to know students is the most important thing a teacher can do to have an orderly classroom" (Jack, Course Paper, July 15, 2011).

Not only did the student he interviewed re-dedicate himself to mathematics learning in the summer, Jack found that he was more responsive to Jack's reminders to remain on task. Jack found that building relationships with students not only had an effect on students' desire to learn mathematics, but also on the managing of a classroom, and that this management could positively affect the ability of a teacher to implement student-centered mathematics instruction marked by expectations that students would engage in productive mathematical discussion.

Jack's desire to develop relationships and rapport with his students assisted him in his future teaching. Making the students feel comfortable with him and his classroom norms affected the students' willingness and desire to participate in discussion. Jack's efforts to build relationships with his students allowed him to quickly develop routines and procedures, as well as to counter the norms of teacher-directed schooling that may have hampered his ability to develop students' engagement in mathematical discussions.

Dedication to student-centered teaching. From the outset, Jack was determined to teach in a student-centered manner that included discussion. His own mathematical experiences fostered that commitment. Jack's sixth-grade education was in a self-contained classroom, but the students were tracked for mathematics instruction. Jack was in the highest mathematics track. His school also had a mathematics resource teacher who would provide mathematics instruction for the high-tracked student groups and also conduct pullout lessons for selected students that delved deeper into mathematics content. Jack's earliest lasting memory about mathematics was during one of the mathematics workshops. His teacher posed a problem to the class that required the students to make a choice between two payment arrangements for a 28-day summer job, one that would pay $2,000 a week, or one that would require one week of unpaid training and then would provide a salary starting at one cent a day, with the salary doubling every day. The teacher then opened the floor up for discussion. Jack was the sole member of the class to choose and defend the second option, after he worked through the problem using exponents. This problem-based lesson, that provided Jack an opportunity to make sense of mathematics and defend his thinking through discussion, inspired in Jack a love for mathematics. He wrote,

> [The teacher] modeled the type of student-centered approach to teaching that we all strive to achieve by first posing a question to the class. He gave us time to think about the problem and then opened up the floor to discussion. … At that moment, I realized how interesting math could be. … I learned to love that mathematics had a "right" answer, even if there were often different paths to that answer, as I would learn later. But I began to love math and the logical reasoning aspect [of mathematics] (Jack, Course Paper, June 13, 2011).

Since the student-centered approach to teaching had such a profound impact on Jack's perception of mathematics**,** Jack was dedicated to using student-centered teaching methods in order to inspire or provoke his students to appreciate mathematics in the same way.

Jack's experiences in university mathematics and economics courses demonstrated the importance of making sense of mathematics. He found that simply having a set procedure to complete a problem was not sufficient in either upper-level theoretical mathematics courses or applied mathematics courses. This caused him to believe that providing his students opportunities for making sense of problems and developing their own strategies was critical. He wrote,

> [Calculus 3] taught me the importance of teaching students how to think for themselves. Some other math courses applied to economics forced me to realize it is not enough to spoon feed algorithms identifying a singular approach to a problem because real world applications do not come with such algorithms. As an educator, I must teach [the students] how to direct their own learning; how to approach a problem when the methodology is not as clear as one would like. This experience humbled me and helped me to understand why many other students were not as fascinated by math as I. Many do not see mathematics as making sense at its core—[they believe] that it is based on a set of rules (Jack, Course Paper, June 13, 2011).

Jack noted that any sort of problem that a person might encounter in the real world would not necessarily have a set rote procedure in place to solve them. He felt the need to allow students to experience situations in which they would have to make sense of the mathematics in order to prepare them for their future applications of mathematics, whether in school or in life. He wanted students to be able to think for themselves. In a electronic discussion with his fellow prospective teaches, he repeated this perspective when he wrote, “Academically, I want my students to be thinkers, so my teaching style wouldn’t be geared to vain repetitions and procedural tasks but learning to think critically—independently and corporately—in everything they do” (Jack, Discussion Board Post, July 3, 2011). These previously held perceptions caused Jack to enter the classroom with the desire to teach in a way that included student-centered discussions about the mathematics included in a problematic task.

However, through experience both with the field experience at the local community center and through conversations with teachers from the previous year’s cohort in the alternative certification program, Jack discovered that teaching in such a student-centered manner is not simple. He noted,

> Students are incredibly resistant to student-centered teaching. Much of what we have learned in terms of methods of instruction seemed like a struggle to implement. Eventually I learned (with help from the panel with the first cohort) that it was a balancing act between full student direction and full teacher direction and things moved better (Jack, Course Paper, August 1, 2011).

After his experiences with teaching a small group of students during the summer, he anticipated that there might be resistance to student-centered instruction. Nevertheless, he was dedicated and had gleaned some possible strategies for managing that challenge when entering the classroom from the previous cohort. He learned that he would have to balance teacher- and student-directed instruction in his classroom.

Pacing and momentum. One of the things that Jack became aware during his teacher preparation program was the potential challenge of timing and the time constraints that would arise when teaching in a student-centered manner that promoted discussion. Initially he admitted that he was not certain how to address that particular challenge. He wrote,

> The other thing I'd like to work on is time management. A lot of what we have learned in terms of the methods for teaching has been targeted toward inquiry-based or student-centered learning but that takes so much time. … For whatever reasons this approach to teaching takes so long, I felt unprepared with how to speed it up. The reality is that we will have a certain amount of time for the class and a certain amount of material that must be covered in a year (Jack, Course Paper, July 6, 2011).

Jack understood that there would be pressure for him to cover a certain amount of mathematics content in a particular amount of time. He also was aware that teaching in an inquiry-based or student-centered manner would be more time consuming than direct instruction. Initially, Jack had not developed a strategy to address the tension between covering content and engaging students in productive mathematical discussion. However, Jack began to thinking purposefully about the tension between covering content and engaging students in inquiry. He decided that he would provide some structure for his students and introduce time constraints for activities. He said,

> I learned to work off the assumption that students need more structure than just free-range, 100%, student-directed learning. I set time limits, gave them some specific information to work from, and still was able to allow them to put some

> pieces together in the hope that they will retain the information longer (Jack, Course Paper, August 1, 2011).

Jack began to think about and develop ways of addressing the tension between covering content in a pre-determined period and engaging students in inquiry. Due to his reflection on this subject before he became teacher of record, he was able to prepare to meet this challenge as he considered and developed strategies to address this challenge in his own classroom.

Content. One of the challenges that Jack foresaw was his own content knowledge. He was afraid that he would not be able to remember, access, or "unpack" all of the middle-school mathematics content in a way that might allow him to teach the content in an inquiry-based manner. He reflected,

> A secondary hope is that I can recall my middle school mathematics from nine years ago. While it is true that math is sequential and builds on prior knowledge, it is also true that some things simply are not used later. Some statistics like box-and-whisker plots, area formulas like that of a trapezoid, etc. are concepts one doesn't need after graduation unless they teach. It will be interesting to refresh myself on all the things I know but have forgotten. Of course the real task is learning how to communicate all the material to others. I never learned how/why some math (i.e.- formulas and algorithms) is the way it is and it will be interesting trying to make sense of it myself first (Jack, Course Paper, July 6, 2011).

Jack felt that he might have incomplete content knowledge due to the fact that he had not utilized some of the material after learning it in school. Also, he was afraid that the knowledge he retained could be superficial, leaving him either unable or struggling to develop lessons in which students would be able to investigate mathematics conceptually. However, as part of his summer internship class, I was the instructor of a component that addressed middle-school mathematics content in an inquiry-based way. When I developed lessons for this class, I designed them to not only refresh the students' content knowledge, but also to model ways of teaching the content that would inspire sense making and mathematical discussion. Although Jack faced challenges with his content knowledge in some areas, in others he was able to refer to things that he was exposed to in the content class in order to supplement his knowledge.

Background and familiarity with small-groups. The prior experience that Jack had with teaching was in an after-school program that provided one-on-one mentoring and tutoring of students, as well as occasional work with small groups of students. Since Jack was already familiar with individual and small-group instruction, he was more comfortable interacting with students who were positioned in this organizational structure. He said,

> The lessons were structured with periods of full-class discussion as well as periods of group and individual work. It was easy to work with students on an individual level and to some extent at the group level, but I was reluctant to get in front of the class. As time went on and I knew the students better, it became easier to transition to the front of the class (Jack, Course Paper, August 1, 2011).

Through his summer field experience, Jack learned that building relationships with students was one strategy to overcome the challenge of his own unfamiliarity with whole-group instruction. This seemed to affect how Jack initially structured his class, as well as his own self-efficacy in terms of dealing with whole-group instruction. However, both of

Jack's prior experiences with both small-group and whole-group instruction were with a small number of children; there were typically 14 students, sometimes fewer, present during the summer field experience. Jack eventually would have to face challenges of his own efficacy in classrooms with a larger number of students.

Jack's Changing Efficacy in Engaging Students in Discussion

An Overview. This section addresses how Jack's efficacy in engaging students in discussion developed over time. Jack and I first had the opportunity to discuss his approaches to engaging students in discussion after his permanent placement in November. When we initially spoke, Jack was having difficulty encouraging students to work collaboratively. Although he planned lessons that included both opportunities for small- and whole-group discussion, he found students to be reticent to participate in these discussions.

Throughout the school year, Jack seated his students in groups of four every day. However, in November, his students would often work individually on the mathematics even though they were encouraged to work collaboratively. In addition, during whole-group discussions, students would often interact solely with Jack instead of with each other. Throughout our work together, he expressed a desire to get students to work collaboratively and discuss mathematics productively. Jack felt that it was difficult to get his students to participate in these discussions. Through consistently reinforcing the expectations of collaboration and through developing relationships with his students, Jack became progressively more able to facilitate conversations and collaboration between his students.

The following sections describe that change. First, I analyze how Jack consistently reinforced an expectation for collaboration through different verbal directions and instructional strategies. As a result, Jack's students increasingly began to collaborate autonomously. Second, I analyze Jack's ability to facilitate the presentation of multiple solutions strategies in a whole-class setting, as well as his interpretation that developing relationships with students assisted in bolstering his ability to do so.

Expectation for collaboration. Jack consistently required his students to discuss mathematics. During this process, Jack had to combat what he believed were norms of schooling that his students had adopted. He felt that his students, who were labeled "honors," were acculturated to following directed instruction. The students seemed initially to resist his attempts to get them to collaborate. First, I will analyze Jack's statements regarding the challenge that he faced in terms of prior norms of schooling. Second, I will analyze the teacher moves that Jack made in order to promote a norm of collaboration in small groups.

Fighting against norms of schooling. Jack and Michelle taught in the same classroom and split a single teacher vacancy. Jack was assigned the students who were labeled "honors," and Michelle was assigned the students who were labeled "comprehensive." However, these labels were not related to mathematics ability because all of the students were tracked based on their prior performance on a reading assessment. However, Jack perceived that there were differences in his students' and Michelle's students' willingness to participate in student-centered instruction that included discussion. He stated,

> When I sit at my desk and watch [Michelle] teach and I see her students doing … it's easier for them to work off script … . I think my kids were technically supposed to be in honors classes, they like much more direct instruction and they don't like to be asked to do something first. They're a lot more resistant to it … . I think, like maybe, if you were in an honors class, you probably would be good at "doing school" and you'll be good at "doing school" if you were good at just listening and taking notes. (Jack, Seminar Two, November 21, 2011).

Jack felt that his students were acclimated to instructional techniques that directed them as to what to do and what to recall. Furthermore, he felt that his "honors" students were familiar with "doing school" in a particular way, a way that required them simply to sit quietly, listen, and take notes. He felt that Michelle's students might not have been as completely acculturated to direct instruction in the way that the "honors" students were, since they were not considered to be "good at doing school." Therefore, he felt this caused his students to be more resistant to participation in student-centered lessons and mathematical discussion, as compared to Michelle's students.

Jack also believed that his students' familiarity with particular norms of schooling prevented them from productively collaborating. He felt that his students were more comfortable with working individually rather than in small groups. He remarked,

> I hope they drag [the students who do not understand] with them and not just leave them alone … It's hard to make them cooperate you know … Sometimes they just like to do it on their own (Jack, Mentoring Session, January 18, 2012).

He wanted his students to work together so that a student who had a greater understanding of a particular topic could assist the other students who were not as secure.

However, the students' comfort with working individually, rather than collaboratively, made it difficult for Jack to get them to assist each other when addressing a mathematics problem in small groups.

Jack attributed his students' resistance to student-centered mathematics and participation in discussion to the type of instruction with which his "honors" students were familiar. This was not what he had expected as he entered into teaching believing that students who were labeled "honors" would find it easier, as compared to the "comprehensive" students, to participate in discussion and student-centered instruction. However, since the students had not been required to discuss and collaborate in the past, they were not as willing as Jack had expected. He reflected during a follow-interview during the following school year,

> If anyone, I would expect them to be able to handle it more … like I said … it was harder for them to make the jump to doing something extra. I guess what they saw as something extra. As long as they could write it down, I think they thought that was sufficient. Um, but I guess I had to spend time explaining like, "You learn more, you learn more by teaching" (Jack, Follow-Up Interview, May 7, 2013).

He felt that since his students previously had solely been required to complete problems individually and to record their answers in written form, they were resistant to doing something they felt was unnecessary, or "extra." Therefore, Jack had to consistently reinforce his expectations and to provide a rationale as to why students would benefit from discussing mathematics in order to entice his students into participating fully.

Reinforcing expectations and results. Jack's early experiences with his students suggested that his students were not familiar with collaborating and persisting autonomously with problems or problematic tasks. During our first teacher seminar in November, we were reflecting on a teacher video where students collaborated on a mathematics problem with little direction. In the video on which they were reflecting, the teacher had students collect data and then attempt to find an equation that would closely model the data collected. In this video, students worked in groups of approximately four students and collaborated autonomously to find an equation that would approximate the trend of the data. Jack noticed the collaboration and attributed the autonomous nature of the collaboration to the teacher's clear expectations. He noted that his students did not work in this way; instead, they asked Jack questions rather than relying on their knowledge and the knowledge of their group members. He stated,

> It was obvious that her expectations were clear, because if I, when I try to do an assignment like that, it's like 10 seconds in kids are running "Mr. Davis, Mr. Davis" [and] asking me questions, but these students were sitting at their desks trying to work through the problem cooperatively (Jack, Seminar One, November 7, 2011).

Although Jack's students were not collaborating autonomously currently, he understood that clear and consistent expectations for collaboration would be necessary if his students were to learn how to work together autonomously.

Initially, Jack attempted to implement a procedure whereby those students who finished a particular problem first would serve as experts and assist other students in completing this problem. During an early seminar, Jack explained the problem with this particular strategy.

> So, one thing I tried … , in terms of getting students to appreciate each other, [or] what each other has to… say … when they work on something, … Whoever finishes first and gets it correct, they get to go around and explain it to the rest of the class and like check off the papers and stuff. But what I noticed was that … they weren't explaining (Jack, Seminar One, November 7, 2011).

Jack wanted his students to discuss and explain their solution strategies to others in order for each of the students to develop an appreciation of other students' thinking. However, the students who were serving as experts were simply telling the others the answer and having them change their approaches in order to earn a check on their work. This was not the desired outcome; Jack wanted students to learn to collaborate and explain their thinking to others. Jack realized that the strategy of using student experts was not yielding the result of collaboration.

In November, Jack taught a lesson where students investigated patterns in order to discover how to calculate numbers raised to the zero power and negative exponents. In order to investigate negative exponents, Jack had the students complete a table of positive exponents and extend the pattern backwards to a zero exponent. Jack circulated around the classroom and encouraged students to explain their thinking about why the result of raising a number to a zero exponent would be one.

> JD: Can you explain why you think it may be 1? Do think it may be 1? Can you explain, [student name]?
>
> Student: No.
>
> JD: So you just think it'll be 1 but don't know why.
>
> Student: Oh, I used the calculator (laughing)

JD: (To another student) OK, you think you can [explain it]?

Student: Yeah.

JD: Go ahead and explain it (Jack, Classroom Observation One, November 17, 2011)

When one student said that she used the calculator to find her answer and could not explain her thinking further, another student in that group suggested that she could explain it and then proceeded to explain her thinking. Jack persisted in pressing students to explain their thinking rather than just finding a solution. He asked for explanation from individual students as well as encouraging them to speak to each other, saying, "I need you to make sure that everybody at this table [understands]" (Jack, Classroom Observation One, November 17, 2011). Jack was not only eliciting explanations from students so that they could relay their ideas to him, but also encouraging them to explain to each other when he was not present.

Later on, in January, Jack began to incorporate center rotations with his students. In this particular lesson, half of the class was completing a review "scavenger hunt," where folded papers were arranged around the classroom with a multiple-choice problem on the inside, and the answer to a different problem on the outside. Students would move about the classroom searching for the answers to their problems, and, in this way, complete each of the problems. The other half of the class was split into two groups, one that was directed by Jack to take notes on the definitions related to congruence, and the other was attempting to discover the surface area of a rectangular prism through the use of nets. Most of Jack's focus was on the group to which he was providing direct instruction, so the students in the other group had to work collaboratively and mostly autonomously, as Jack would check back with them only intermittently. Jack previously challenged a small group of students to refine their formula by double-checking if the area of each face of the rectangular prism was the same. While Jack was working with the other group, the following conversation transpired,

> Student: Mr. Davis, we got it.
>
> JD: Do [all the sides] go together?
>
> Student: No, because [different side lengths of the different pieces].
>
> JD: Check each other, [student name] and [different student name] check each other's (Jack, Classroom Observation Two, January 27, 2012).

Two things are salient about this conversation. This shows that Jack was still, consistently, not only requiring students to work together, but also asking them to collaborate through validating their answers with other students. Also, there is a shift in the language that the students were using. In the initial observation, the students used the work "I" when explaining their thinking. Here, the student used the word "we." Jack's consistent, high-level expectations were changing the norms of student discussion in the classroom.

In March, Jack reflected in seminar about a lesson in which he engaged students in error correction of a past assessment. By then, Jack was aware of students engaging autonomously in their small groups, in discussions that included both procedural and conceptual talk. Jack was also aware of occasional incidents when students would give answers to each other and then would quickly redirect their conversation to explanation. He reflected,

> I think there was a mix [of procedural and conceptual conversation among students]. Of course, there's going to be some people just saying, "Oh, the answer was A," and so I did hear that. And so for those people I had to say, "But why is it A?" (Jack, Seminar Six, March 7, 2012).

Although there had been a marked shift in both the occurrence and the manner through which students were discussing in the classroom, there still was an occasional deviation from that pattern. While Jack was cognizant of these deviations, he persisted in enforcing his high expectations for these students by reminding them quickly to explain their thinking to each other. Jack's commitment to excellence in teaching and education motivated him to work diligently to change the culture of discussion in the class through teacher moves that reminded students of his persistent expectation of explanation and discussion.

Building relationships. Jack came into teaching believing in the power of relationship building and its influence on a culture of a classroom. To him, knowledge of and rapport with students would be instrumental in encouraging students to discuss and make sense of mathematics. When he entered the classroom, he became immediately aware of the characteristics of students in the classroom, and he worked to remedy any problems that arose with particular students. He stated,

> And the other thing is when I have a whole group discussion … I just feel like it's the same people that are contributing, and some people just don't have a voice. They just, one of the girls that tests the highest in the class, she doesn't … she's ... mute. She won't say anything. You, like, come to her and she'll explain it but she won't talk to the entire class. She just sits there … So having a whole group

> discussion is hard if everyone is not going to be focused and contribute because … I don't know if they don't want to be wrong (Jack, Baseline Interview, November 2, 2011).

First, Jack was aware of the feelings of a particular student in his class, namely that she was reticent to speak up in a whole-group setting. He also noticed that the same students were dominating the whole-group conversation. This directed him to look for a teaching strategy that would address these two issues simultaneously. In response to these concerns, he began to use equity sticks. This way, a student would not feel "picked on" when his or her stick was chosen because it was random. Also, this strategy would make sure that the same students were not the only voices that were heard in a whole-class discussion. Every student would know that everyone in the class might be expected to participate in the discussion at any time. Second, Jack inferred from his observations in the classroom that students may be reticent to participate in whole-class discussions because of a fear of being incorrect and of being embarrassed by either the teacher or other students in the classroom. This reinforced his commitment to building a rapport with his students and an atmosphere of trust in order to allow them to feel comfortable contributing to a conversation without fear of embarrassment.

Jack reflected on relationship building and its connection to developing productive mathematical discussion in his classroom. In a follow-up interview during the May of the following school year, he reflected,

> I think over time, once they just realized it was more of an expectation, they just kind of were more comfortable participating. I think, you know, maybe it was that they lacked some confidence before because they weren't used to doing it, but as

> they tried it out I think it's not as bad. No one's going to laugh at you. No one should be laughing at you. And maybe that makes them more comfortable. Maybe there was like an adjustment period where they had to kind of get comfortable with me and with the classroom in general. … So, I think after you built relationships with students and they kind of get a feel for how comfortable they can be in your room, and I think as time went on they became more comfortable (Jack, Follow-Up Interview, May 7, 2013).

Jack felt that as students became more comfortable with him, his teaching style, and his enforcement of a positive, safe environment for student participation, they became more willing to participate in whole-group discussions. He credited this to the trust that he built with students through relationship building as well as to his ability to entice more students into participating in whole-class discussions.

As Jack built a positive culture through building relationships with students, the characteristics of his whole-class discussions changed. Instead of calling on students to explain their thinking to him from their seats, he was able to encourage students to come to the front of the class to demonstrate their work and to explain their thinking. In an observed lesson in March, Jack was asking students to construct a circle graph from data that the students in the class themselves had generated. While students worked in their small groups, Jack noticed several different methods to find the percentages that each category represented. When he conducted a whole-group discussion, he asked several students to present the different methods of making these particular calculations. After one student came to the front of the classroom and presented their calculations, Jack asked,

> [W]as there another way of doing this? Did anybody do it a different way? In terms of instead of setting up a proportion, I saw it a different way when I was coming around from some people, who did it a different way? (Jack, Classroom Observation Three, March 23, 2012).

After this question, another student came to the front of the class and explained a different way to calculate the percentages. This shows that the students were comfortable presenting their solutions even when they were different from the initial presentation. This was possible because there was a reduction in students' fear of being incorrect, a fear which Jack had noticed earlier in the November. Initially, when a solution is presented by a student in front of the class, other students assumed that this was the sole correct calculation or approach and assumed that their solution was incorrect. However, Jack had built relationships with students, which gave the students the courage to present different solution strategies in front of the whole class. Jack's commitment to developing a rapport with his students so that they felt safe participating helped Jack develop an ability to facilitate productive mathematical discussions in his class.

Jack's Changing Approach to Lesson Development

An Overview. This section addresses how Jack's instructional decision-making and approach to lesson development changed. Jack and I first had the opportunity to discuss his planning for independent instruction after his permanent placement in November. When we initially spoke, Jack was comfortable using the textbook that his district provided, since the lessons either were sufficiently investigative in nature or could be modified slightly to support student inquiry. For example, as suggested by the textbook, Jack had his students develop their understanding of exponents by expecting them to use expanded notation to expand powers and then to simplify the resulting expanded notation into a single power. He did this in order to have the students investigate properties of exponents through observation of patterns. However, Jack then extended his students' investigation beyond the indicators that would be assessed on the high-stakes standardized test, and that were the limits of the district-provided pacing guide, by including investigations of zero and negative exponents.

Jack and I discussed how students might make the connection between two different approaches to negative exponents: by using expanding of powers and then reducing those by simplifying the resultant fraction and by using the algorithm of subtracting exponents. We talked through how students would make that connection and what he could do as a teacher to assist the students while they worked in groups to make sense of negative exponents. Throughout our work together, from our initial conversation to the end of the 5-month (November-March) period of this study, Jack continued to have a desire to have students investigate and make sense of mathematics. In some cases, Jack was unsure about how to design a lesson that would more fully engage students in making sense of mathematics. Furthermore, Jack had to address a need to cover an expected amount of mathematics content in a specified time while at the same time allowing students space to investigate. Over time, he developed strategies in order to address both of these desires.

The following section describes the development of Jack's instructional planning and decision-making. First, I analyze how Jack began to use the lesson activities that he had experienced during his summer content courses as well as to reach out during mentoring to think through lesson planning in order to make sure it was sufficiently investigative. Second, I analyze his instructional decision making as he addressed the challenge of covering content within a particular time period while at the same time engaging his students in mathematical sense making.

Making sense of mathematics. It was important to Jack that his planned lessons would provide opportunities for his students to make sense of mathematics. In addition, it was important to Jack that his lessons should be investigations of pure theoretical mathematics or connected somehow to the real world. However, Jack initially had difficulty planning these lessons when the lessons in the textbook were not sufficiently investigative. In order to address these issues, Jack used lessons from his summer course or reached out to me during lesson planning to assist him in planning for a lesson or a unit. The following section will analyze how Jack addressed the challenges of making the mathematics content engaging or relevant to middle schoolers, as well as how he drew on his mentoring sessions and the ideas addressed in his summer content and methods course to help him plan lessons that allowed students to make sense of mathematics.

Content and connections. Initially, Jack was more comfortable teaching the mathematics content that he found interesting. This became evident after Jack had finished a unit on equations and inequalities and was transitioning into a unit on geometric measurement. When speaking about this unit, Jack stated,

> [Geometry is] just exciting. It's fun math … I mean it's just like inequalities never been my favorite subject. I get it. I can understand how to do it, but it's not fun for me (Jack, Mentoring Session, January 18, 2012).

Since it was important to Jack that mathematics be engaging to students, he was more comfortable with planning for mathematics lessons when those lessons addressed content that was interesting to *him*, as he felt that he would be able to convey that interest to his students.

Jack's novice status as a teacher as well as his own struggles with content initially affected his facility for providing his students opportunities to make sense of all different kinds of mathematics. For instance, after Jack completed the unit on geometric measurement, he transitioned into a unit on geometry that included measurements of angles and constructions. When he was teaching constructions, he only walked students through the process of constructions without pressing students to make sense of why constructions work. During a reflection session following that lesson, I spoke to Jack about the relationship between circles, radii, and constructions. He reflected,

> OK, that's … why [constructions] work. It's not something that I'm not comfortable with; I don't understand that part so that's why it's hard for me to do that … . Yeah, I need to work on that, [because if] someone asked me why do they [need] to know this and I was like, "To me it's important because in math, and in life there are some times [when] you have to be precise about what you're doing and all those steps in order to do things." But, I don't know, it's hard to connect that [explanation or rationale] to real life (Jack, Mentoring Session, February 28, 2012).

Jack had difficulties engaging students in making sense of constructions because his content knowledge was weaker in this area, as compared to other mathematical topics. For Jack, it was important that students were either able to make sense of the theoretical mathematics or to make some connection to real life. In this way, he felt, his students would be either interested in engaging in the lesson or they or he could find some reason that justified learning the mathematics. In this case, Jack felt neither of those situations applied and so he relied on a thorough, step-by-step presentation. Both Jack's insufficient content knowledge and his inexperience with teaching this content contributed to his focus on a procedural approach to geometric constructions.

However, it was still important for Jack that his students be engaged in making sense of mathematics, whether through interesting activities around theoretical mathematics or through a real-world connection. This influenced his lesson planning, and eventually led Jack to ask for assistance with planning lessons in mentoring sessions. He also was able to source activities from his summer content course. In a follow-up interview, he reflected,

> I think that some math lends itself toward doing activities that the kids are more interested in. Or that I can, that I can see a connection for. Maybe I'm just not developed enough to know--I'm still trying to come up with activities to do. And so some things are easier for me to kind of figure out something that kind of goes with it [and] that's more engaging, and other things are more difficult for me to connect (Jack, Follow-Up Interview, May 7, 2013).

Jack understood that there were gaps in his knowledge and experience. He felt that some mathematics topics were by their nature easier topics through which he could develop activities that would be interesting to students, or that he could connect to real life. Therefore, he would need to reach out to find other resources for the mathematics content with which he was less comfortable, as he did during his first year.

Using the content course and mentoring to plan lessons. Many first-year teachers rely on district-provided curricular materials, either lesson plans or textbooks, in order to guide their lesson planning. Jack's school district provided him with a pacing guide with references to the provided textbook. In the beginning, Jack felt the lessons as presented in the textbook provided students with sufficient opportunities to investigate and make sense of mathematics. However in late November, when he looked ahead to a future unit on concepts and operations with integers, he did not feel that the lessons in the textbook were sufficiently investigative, and he was forced to attempt to plan something alone. Since he realized that he was having difficulty, he reached out to me and asked for help planning something problematic that would engage students. He emailed,

> There was actually something I wanted to talk to you about. The next unit is on Integers (comparing and ordering integers is first). I'm having trouble planning something problematic for students to work on. I'm going to get into it tomorrow and I know Michelle told me you might be working with her on delivering that lesson on Thursday. I was wondering if you could help me plan for Wed[nesday] as you plan with her for Thurs[day] so I can introduce the unit well (Jack, email conversation, November 29, 2011).

It was very important to Jack that his students engage with problematic tasks or investigations in order to make sense of mathematics. Due to Jack's inexperience, when his textbook did not meet his expectations, Jack did not have the resources to develop this type of lesson. Since he had a mentor, he emailed me in order to get help planning a lesson. In this way, Jack acknowledged his own inexperience and addressed this by contacting me (his mentor) and working with me to develop a lesson that he felt was a better way to engage students in making sense of mathematics. As a result, Michelle, Jack, and I worked together over email to plan a lesson that would engage students in making sense of the mathematics. Michelle suggested some activities from the textbook. Some of these were investigative and problematic, but others were not. To address this, I suggested an open-ended activity that would incorporate velocity and speed in order to engage students in making sense of absolute value and the ordering of integers.

This first occurrence of Jack asking me for explicit assistance to plan investigative lessons was followed by subsequent sessions where we worked together to think about how to address different concepts in teaching units. The next unit that Jack had to teach covered the topics of equations and inequalities. We met before the unit began and spoke about how to lead students to thinking about the algebra of doing and undoing. I introduced several ways of modeling and thinking about algebra, including: algebra tiles, thinking about number games, flow charts, guess-test-generalize, and contextualized problems. Algebra tiles would model both variables and constants and would allow students to move the tiles around in order to have a concrete way to think about solving equations. A number game would permit the students to hear an equation such as $2x + 5 = 17$ expressed as “I am thinking of a number. I multiplied it by 2. I added 5. My result was 17. What is my number?” This would allow students to use mental mathematics and sense making to undo algebraic operations. Flow charts would allow students to document the arithmetic operations applied to the variable that led to the result, as well as the inverse operations that could be used to work backwards and to determine an answer. These two strategies were presented at a Culturally Relevant Pedagogy conference (Powell, 2009). Guess-test-generalize is a method where students take a contextualized problem and guess possible solutions. They then check the solutions using arithmetic operations as they are stated in the word problem. After repeatedly checking guesses, the student either realizes or is directed to noticing the repeated pattern of operations that they used to check the guesses. These repeated operations can then be used to generalize their method of testing guesses to an equation, an equation that students could then, in turn, solve.

When Jack introduced writing and solving equations to students, he provided them with multiple ways to think about how to solve the problem before introducing the symbolic method. He provided them with algebra tiles, explained the number games and flow charts, and had students solve word problems. He found that students were engaged with certain methods over others. He related,

> Well, it was slow at first because they weren't too crazy about the manipulative part, but they really liked the, they really liked the way with the circles [the flow charts] and the … number games. They really liked that one [the number games], but they understood what to do for the word problem … they could solve it (Jack, Mentoring Session, December 29, 2011).

Jack found that students were successful when he exposed students to different ways of investigating and making sense of solving equations. He found his students were engaged when their minds were engaged in making sense of operations and the undoing of arithmetic procedures mentally and in contexts that made sense to them. Making a connection to a real-world scenario allowed students to think through mathematics in a way in which they were familiar. In addition, playing number games was engaging because it was "fun." Jack saw the resulting student engagement and mathematical understanding and it reaffirmed his commitment to using all resources to develop these types of lessons.

In a subsequent unit on geometric measurement, Jack used another resource other than our mentoring conversations and his textbook. Finding the surface area of a rectangular prism was one of the topics in this particular unit. In order to engage students in an investigation of the formula for calculating the surface area, he sourced an activity from his summer content course. I was the instructor for this course and had used this activity for the prospective teachers in the summer through which they could derive the formula for surface area. In this activity the students have nets of a rectangular prism. They calculate the area of the prism, but then attempt to assign variables to different measures so they can derive the formula. Jack used this lesson in his class. Since he was inexperienced in teaching, and unsure of his own content knowledge. Jack used both his mentoring sessions and his summer experiences as resources for lesson ideas that would address mathematics in a student-centered way as well as resources for developing student-centered lessons where students would have opportunities to discuss and make sense of mathematics. In a follow-up interview, he reflected,

> I kind of realized [that] I didn't know why things were what they were. And so, content knowledge is more than just knowing how to get an answer, but it's about explaining why the way that you took is the proper way to get an answer … I think I learned more [of] that through content sessions that we had [in the teacher preparation course] versus when I was in middle school (Jack, Follow-Up Interview, May 7, 2013),

The summer content course was an important resource for Jack with regard to understanding the why behind the mathematics so that he could then develop student-centered lessons. He used this course, and myself as a mentor, in order to make sure that his students had a seventh-grade experience where they would learn the “why” of mathematics, rather than just the “how.”

Addressing needs of curriculum and testing. Jack was aware that there were pressures directed towards teachers both from the district and from the state to cover a certain amount of topics before the district and state assessments were administered. However, he was committed to having his students develop mathematics through conceptual investigations and discussions, although he knew that instruction designed with these features could be time consuming. He stated,

> [C]onceptual takes longer and [there’s the] the benefit to it, if it’s seen in your classroom and if it’s [then] seen in later classrooms. [When] it’s seen in later classrooms and then it’s like, then the teacher would be like, “Oh you guys completely understand this.” But it’s because they [the students] have a stronger foundation and they can understand more to it. I understand it’s like a challenge trying to do, trying to prepare them for the test (Jack, Seminar Two, November 21, 2011).

Although he knew that this type of instruction would take longer, Jack was committed to including it in his teaching. He believed that students who engaged in the “doing of mathematics in a conceptual way” would develop deeper understandings, which would benefit both these students and their teachers in future mathematics classes.

However, there remained a conflict between the time constraints of scheduled testing and the lengthy duration required by student-centered teaching. However, Jack devised a strategy to address this conflict. He said,“[O]ne of the other teachers on the team introduced me to kind of doing center rotations in class” (Jack, Follow-up

Interview, May 7, 2013). These centers allowed Jack to have students investigate several different mathematics topics at the same time. During January, when I observed Jack for the second time, he had his students working in a center rotation. One group of students were developing definitions in their own words for the terms that were relevant to congruent and similar figures, under close direction from Jack. Half of the class was completing a scavenger hunt in groups of four, where students would collaborate so solve review problems and progress through a set of problems in the expected order by finding the correct answer. Jack specifically required students to show all of their work on their scavenger hunt, so that he could ensure that they were truly thinking about the mathematics and explaining solution strategies to each other, instead of simply copying an answer and moving on. The third group of students was attempting to discover the formula for the surface area of a rectangular prism using nets. In our debriefing after I observed this lesson, Jack specifically mentioned that he was incorporating a center rotation because of the need to cover content in a certain period. He stated, “I want them reviewing [for the exam] on Monday and Tuesday, that’s why I was trying to finish everything today” (Jack, Mentoring Sessions, January 27, 2012). It was important to Jack that his students would engage in mathematics discussions in a meaningful way, but it was also important that Jack orchestrate the progression of his students through the curriculum in a timely fashion. By designing centers, Jack felt that would not have to sacrifice the use of student-centered mathematics investigations and discussions, and yet he would still have time to prepare his students for the examinations.

Furthermore, designing centers allowed Jack to have students investigate new content *at the same time* that they were thinking through and discussing content that should be review topics. In these centers, while the discussions about surface area of a rectangular prism and about congruency and similarity were new topics, the scavenger hunt questions were designed to be a review. Here, the students would have all three experiences in a class period: two new topics and time to review. Therefore, through the use of centers, Jack felt that the students could be constantly learning new things in an investigative manner and simultaneously be preparing for the test. Jack reflected in a follow-up interview,

> Discussion is great, it just … takes more class time to get through the same amount of material. And when you're rushing against [the high-stakes standardized assessment] it kind of feels like there's a lot that you… {need} to at least touch on … . I'm always … trying to, like, bring stuff back up and try to cover a lot of ground. And so, the way I design centers is always kind of multiple indicators a day so they [the students] never, so they don't forget (Jack, Follow-Up Interview, May 7, 2013).

Jack was able to incorporate a strategy of center rotations into his instruction. In this way he could address the needs and the time constraints of upcoming assessments without sacrificing his commitment to investigative instruction with discussion.

Jack's Changing Use of Small- and Whole-Group Discussion.

An Overview. Initially, in November, Jack allowed his students several 5-minute opportunities to work in groups. He questioned his allocation of time for those groups, since he did not feel like he was getting to every group. This was due to the fact that he was spending a great deal of time leading and instructing a single group rather than quickly assessing where the students were, asking a pertinent question that would aid in the development of their thinking, and moving on to another group. As he developed his ability to anticipate misconceptions and plan questions ahead of time, he was able to move more efficiently between groups and to ask questions within the groups.

In November, Jack spent over 50 minutes of his 70-minute class period having students work individually or engage in a discussion as a whole class. This was due to the fact that Jack felt that the students would be off-task and that he could not adequately assess the types of discussion that students were having in their small-groups. He felt the need to control and be aware of students' thinking. Over time, he developed his "teacher ear" and was better able to hear what was transpiring in differing students' small-group discussions. This allowed Jack to provide his students with anywhere from the initial allocation of 20 minutes of small-group work to devoting the majority of the class period to students working in centers. Jack's changing feelings of self-efficacy allowed him to provide his students time in small groups and in whole -group discussion where they were attempting to make sense of mathematics. In the following section, I analyzed how Jack's developing self-efficacy changed his use of small- and whole-group discussions.

Self-efficacy. It is not unusual for a novice teacher to be challenged with feelings of negative self-efficacy. Jack initially felt challenged with his ability to assess where students were in their thinking and to be able to lead or progress students' thinking quickly with a question or a statement before moving on to another group. Initially, Jack felt unequipped to assess the quality and topic of conversations of students in their small groups, and therefore he limited the number of opportunities for students to discuss with other students. In the following sections, I analyze how Jack's questioning and task design changed throughout the school year, including his developing ability to assess students' conversations and thinking in small groups.

Questioning and task design. When I first observed Jack in November, I noticed that when his students were working in small groups, he would spend long periods at a single group, walking them through the task, rather than circulating and questioning several groups. In a reflection session, I mentioned this thought to Jack and suggested that he think about potential difficulties that students might have when working on a particular task, and to think ahead of time about potential questions that he could ask of the students in order to progress their thinking, but at the same time allow them to make sense of the math somewhat autonomously. This conversation had an effect on Jack. He stated,

> One of my biggest take-aways from the last time we visited, observed … you mentioned … to be prepared with the questions and have time to ask the questions, instead of spending time talking to them (Jack, Seminar Two, November 21, 2011).

Jack internalized my comments about how he was interacting with groups. He began thinking ahead about what problems based on students' understandings or confusion could arise, so he was prepared to interpret students' thinking more readily, and so he was more able to ask a question or two that would advance their thinking processes rather than coaching them through the entire task. This pre-planning addressed some of his negative feelings of self-efficacy, as he spent less time trying to determine what students were thinking during the actual lesson, since he had spent time thinking about it during planning. He confirmed this analysis in a follow up interview, saying,

> I think, it was something you said about just thinking about the questions you want to ask before you get there. Think about the misconceptions, like anticipating the misconceptions that students might have makes it easier to kind of ask the right questions [at just the right point] so you don't spend too much time. I think [before] that I spent a lot of time trying to [learn of their thinking without prior reflection]: "Well now, what was [it that] you [were] doing?" (Jack, Follow-Up Interview, May 7, 2013).

Jack believed that thinking about potential student misconceptions in planning helped him to facilitate group work adequately across all small groups, rather than concentrating most of his time on a single group.

Another thing that assisted Jack in the facilitation of small-group work so that he felt more comfortable incorporating it in his lessons was the idea of walking away. When we discussed questioning small groups instead of directing their work, as was aforementioned, I mentioned a critical feature of questioning. I told Jack that it was important to ask a question and then walk away from the group so that the group could discuss their thinking around that question and answer it without being led by the teacher. Jack mentioned that this strategy was important to his facilitation of small-group discussion as early as late November, as he said to his cohort in a seminar that, "'Walk away' is key" (Jack, Seminar Two, November 21, 2011). However, he realized that he also needed to develop norms of student discussion around this strategy. When he began using centers in January as ways to engage students in many investigative or discursive tasks during a single class period, he would use this "walk away" strategy more often, since he was facilitating several groups. However, he set expectations for students during this class period, saying, “When I walk away, that doesn’t mean you stop working” (Jack, Classroom Observation Two, January 27, 2012). He understood that he would have to establish an expectation for student discussion and work around the strategy of “walk away.” He reinforced this expectation by stating to small groups that the students in those groups were expected to keep thinking, working, and making sense of the mathematics even when he was not standing with the group.

“Teacher ear.” One of the reasons that Jack did not make use of much small-group discussion in the beginning of the year was because he felt unable to determine whether students were talking about mathematics while they worked in small groups. His ability to hear what students were talking about through the din of multiple voices in a classroom, or “teacher ear,” was underdeveloped. Therefore, he felt the need to take more teacher-directed control on the classroom initially. He stated,

> It’s difficult when they’re in small groups to make them talk to each other when you’re not there … It’s, I guess, that I don’t know what they’re talking about, and I know that I should let go; it’s that control thing. I just don’t, I feel like, when I hear too much laughter, there’s no way that they’re talking about the problem (Jack, Baseline Interview, November 2, 2011).

His inability to determine if students were on-task made Jack uncomfortable with using small groups, so he limited small-group discussion opportunities to several short bursts of time. Since Jack’s prior experience before becoming a teacher was mostly one-on-one tutoring and mentoring, or small-group mentoring, Jack persisted with his work with small-groups; however, these experiences did not prepare him to comprehend the speech of multiple small groups that were talking at the same time.

When I observed Jack in January, he was using center rotations and students were often working autonomously. When I listened to students who were working autonomously in small groups, even those moving around the classroom completing a scavenger hunt independently, I heard students having conversations that were about the mathematics they were working on. During a reflection session, I mentioned to Jack that I observed his students talking about mathematics, and he was surprised. He replied,

> They were? (laugh) That's the thing, I never really know it, what I hear is a bunch of noise … because when, I guess, when I hear them when they're not talking about math, I do say something each time (Jack, Mentoring Session, January 27, 2012).

Although he still mentioned that he often could not tell if students were on task, he did state that he sometimes could identify when they were off task and address it. The fact that he addressed students' violations of the expectations of discussion made them more likely to discuss mathematics productively rather than to talk about other things. This reinforcement of the expectation of productive discussion of a task facilitated students' autonomous mathematical discussion in small groups.

However, by March, Jack was able to hear what was being discussed in other groups even while he was speaking with a different group. During a teacher seminar, Jack described a task where students were doing an error analysis of a recent assessment in small groups. The students were expected to review their exam results and correct their answers. When Jack was asked if students were just sharing correct answers rather than discussing solution strategies that would lead them to a correct answer, Jack responded,

> So, if I heard [students giving answers without explanation], I heard that a couple of times when I was talking to another group and I was just, say, “Well why is it that?” and then turn back around (Jack, Seminar Six, March 7, 2012).

He stated that when he was working with one group he could overhear a student in another group giving an answer without explanation. Since he had developed his “teacher ear,” he was able to hear that through his conversation with one group and through the noise produced by discussions in the other groups. This helped him to reinforce expectations of productive discussion in small groups and require students to explain their thinking to each other.

By the final teacher seminar, Jack seemed to be more comfortable in facilitating discussion among multiple small groups simultaneously and had incorporated more small-group discussion in his classroom. He had grown from his experience working with one student individually or with one small group of students to facilitating the work and mathematical discussion of several small groups. He reflected,

> I noticed that it starts to get loud after a while, but then I’m hearing more good stuff as they’re talking. So … I think [my teacher ear is] improved. I think so. The level of noise is just different also. When they’re just wildly screaming, that’s obvious that they’re not on task. But it seemed like it was a healthy chatter, I guess. And what they were saying seemed like it was good. And then when I got to them I can tell they had been working based on how far they had gotten (Jack, Seminar Seven, March 14, 2012).

Jack used his “teacher ear” to determine whether students were on task or not and determined that most of the discussion in his classroom was related to mathematics and productive. He also was able to use other context clues, such as the group's progression on the mathematical task they were assigned in conjunction with what Jack could glean from the multiple conversations in the room. As Jack became more comfortable with a "healthy chatter," he released the control he had exerted over his classroom in November. Students were able to discuss mathematics autonomously in small groups with Jack's careful facilitation.

Conclusion

Jack developed his efficacy and facility with engaging students in whole- and small-group discussion throughout his first year. His commitment to high expectations for his students gave him the motivation to combat his students' pre-conceived notions of classroom behavior and to develop norms of participation in any time of discussion. His prior experiences with building rapport with individual students helped him to create a safe environment in which his students trusted both him and the other students and felt comfortable contributing to a discussion without fear of embarrassment. Jack developed lessons that were investigative or contextualized in order to engage his students in discussions that made sense of mathematics by overcoming his fears of deficiencies in his content knowledge by asking for help from me, or by sourcing his activities from his coursework. He restructured his lesson delivery to include center rotations so that he could engage students in sense-making discussion while still meeting the demands of school-district pacing guidelines and of the high-stakes standardized assessment. Also, he managed his feelings of self-efficacy and developed a questioning strategy and his "teacher ear" to manage many groups of students successfully discussing content rather than his monitoring or scaffolding the work of single student or a single group of students, techniques with which he was more familiar initially. Through careful decision-making and development of strategies, Jack's teaching became more accomplished, and he engaged students in productive whole- and small-group discussions in his mathematics class.

The following is a summary of the findings for Jack Davis that positions my analysis of Jack's data in light of the conceptual lens addressing challenges, negotiations, and strategies. Relationship building was a strategy to which Jack attended from the very beginning. He also used relationship building to negotiate the challenges he faced due to the norms of schooling to which his honors students were accustomed. He faced a challenge related to the pressures of accountability that were imposed by the yearly high-stakes standardized assessment. He negotiated this challenge by organizing his instruction into center rotation in which he was able to have students investigate and learn new content, while also having time to review previous content in order to maintain knowledge for the assessment. Also, his students tracking initially proposed a challenge to Jack, since he felt his honors students would be more able to engage in making sense and discussing challenging mathematics, however they were not accustomed to participating in this way in math class. Therefore, he had to maintain expectations and build relationships with his students to encourage their participation in small- and whole-group discussions. Finally, he negotiated challenges through the support of his mentor, me. I assisted him with lesson planning in order to introduce new concepts in a conceptual and investigative way. We also had discussions related to how to facilitate small-group sense making through discussion: instead of spending a large amount of time uncovering student thinking and leading a small group through the entire task, he thought about potential solution strategies prior to instruction, and formulated questions that would advance student thinking so that he could ask a question to a group and walk away, instead of teaching the group through the activity.

Chapter 7: Discussion and Implications

The individual cases considered in Chapters 4, 5, and 6 provide unique and individualized perspectives on how first-year teachers change their practices over time, and the challenges and strategies that motivate those changes. In order to synthesize the information drawn from these cases into findings, I analyzed features of teacher change across each of the cases seeking to identify specific challenges or commitments that were key to motivating change across all three teachers or across at least two of the cases. When there was a particular challenge or commitment that was a common characteristic, I compared how this challenge or commitment was negotiated across two or more teachers. In these situations, I highlighted similarities and differences in the cases to develop findings. I then interpreted these findings in relationship to previous research and for future implications.

My influence on these findings, as a participant-observer, must be acknowledged. Firstly, since I was motivated by commitments to teaching that promoted sense making through participation in productive mathematical discussion, those topics were often a focus of mentoring sessions and teacher seminars. This focus may have influenced the foci of the individual teacher participants. Secondly, each teacher clearly persisted in their efforts to promote student sense making through productive discussion although they were faced with several challenges. Their persistence toward this goal may have been impacted by my support, focus, and commitments as a mentor.

In this chapter, I discuss five findings that were relevant across all three of the individual cases. These findings include: (1) the interaction between relationship building and student discussion, (2) the effect of context on teacher development, (3) the effects of tracking on teacher perceptions and decision-making, (4) support and its influence on teacher efficacy and self-efficacy, and (5) accountability and its relationship to lesson design. I discuss details of each of the cases to explain each finding and to address the different ways that the teachers planned for and actively facilitated student discussion in their classrooms. In addition, I explain how each finding contributes to the literature and practice in teacher preparation, mentoring, and professional development, in particular with regard to novice teachers in “hard to staff” areas. I conclude with a discussion connecting the findings to the research questions.

The Interaction between Relationship-Building and Student Discussion

Building relationships with students was a critical focus in the development of two out of the three participants in the study. Jack and Eleanor used the goal of building relationships with their students as a strategy to promote productive mathematical discussions in class. Building relationships and having consistent high-expectations for students are important, especially for minority or low-income students in middle school. Murdock (1999) noted that,

> Teachers are quite proximally related to students’ academic lives; as such, it is not surprising that students’ perceptions of their teachers’ appraisals and support were the most consistent and largest predictors of students’ behavior. The best correlate of engagement and disciplinary problems was the perceptions students had of the long-term expectations held by their teachers. (p. 71)

Building relationships and having high-expectations for students are critical strategies to promote students’ engagement in middle-school classrooms.

It was not the case that Michelle did not develop any relationships with the students in her classroom. There were several observed occasions where Michelle used humor or positive feedback to build a comfortable relationship between herself and her class. Furthermore, she consistently held her students to high-expectations by using equity sticks to require all students to share their thinking in front of the class. She also used green or red cards to let students know if they were participating positively in their small groups, or if they were violating normed expectations. However, Michelle did not speak, either before or during her teaching experience, about building relationships with her students. She did not indicate that relationship building could serve as a useful strategy when working with her students, nor did she indicate it as an important feature of teaching. Instead, she spoke about managing behavior and requiring student participation. In this, she attended to maintaining high expectations for students.

In contrast, both before and during his first year of teaching, Jack indicated that he felt that building relationships with students should be of high importance to a teacher. His background was in mentoring and tutoring, and he expected to be able to apply those types of rapport-building strategies in a classroom. During his summer field experience, his belief that building relationships with students would help with classroom order and student engagement was reinforced when he realized that the student whom he interviewed became more likely to follow his directions and to remain on task after the interview. During his time as teacher as record, Jack indicated that his ability to build relationships with students and to establish a safe classroom culture was an important instructional strategy that was key as he worked to promote small- and whole-group discussion. Jack felt that once students knew that other students would not ridicule them when they were presenting their work or explaining their ideas, they would be more likely to participate actively and verbally. By the end of the year, his students were comfortable not only with talking to each other within small groups, but also with sharing multiple solution strategies and talking about their validity as a whole class.

From the beginning of her teacher preparation work, Eleanor believed that building relationships with students was important. She posited the need to let students' voices be heard in the classroom because she felt this would provide students with a feeling of ownership and respect. Furthermore, she felt that it was important to be warm and welcoming as a teacher, so that students would feel comfortable in a classroom. This became increasingly important in Eleanor's permanent placement, since her students were not familiar with student-centered learning experiences and had negative self-concepts with regard to mathematics. Eleanor felt it necessary to build a safe and comfortable environment in her classroom through building relationships with students so they would feel comfortable sharing their thinking in front of their peers. Eleanor also provided scaffolding for students in guided, whole-group discussions to support her students' efforts to make sense of challenging mathematics, doing so in order that they would not disengage. She was able to provide targeted scaffolding because she was careful to get to know her students not only academically, but also in terms of their dispositions and experiences at home. Her students became more persistent in participating in discussion because of these relationships.

Implications. Research has documented the importance of teachers developing relationships with and expectations for their students (Wentzel, 2010). Therefore, a feature of any teacher education program should be fostering pre-service and in-service teachers' attentiveness to building relationships with students, to maintaining high expectations, and to building a positive and supportive classroom culture. During two separate courses in this alternative certification program, teachers were required to interview a student: first during their summer field experience before their permanent placement and again during their permanent placement. This was intended to encourage these novice teachers to get to know students in different ways and to reflect on how that knowledge influenced their teaching. Pre-service and in-service teachers should be engaged in discussions about the benefits to building relationships with and maintaining high expectations for students as it relates to classroom management, student engagement, and promoting discussion about mathematics. This is particularly important when teaching in diverse settings (Murdock, 1999).

Accountability and its Relationship to Lesson Design

All three of these novice teachers were in some way motivated by their school districts' clear focus on assessing students and on holding teachers accountable for student learning. The ultimate outcome of teaching is student learning. The current climate of high-stakes standardized assessments and the tying of teacher evaluations to student performance on those assessments have increased the pressure on teachers not only to advance students' learning but also to assess student understanding constantly. Since these teachers wanted their students to learn, and were held accountable for their students' achievement on district- and state-level assessments, it was important for these teachers to hold their students accountable for learning the requisite mathematics content. Due to these pressures, it is not unusual for teachers to change their teaching practice to model the content and format of these assessments directly, rather than to teach in ways that foster conceptual understanding and reasoning skills. However, both the alternative certification program's goals and my goals as a mentor were aligned with an instructional model that encouraged students' mathematical sense making, rather than rote learning of assessment indicators. As a result, assessment and accountability were challenges that all three teachers faced when attempting to develop methods of instruction that included student-centered discussion.

Michelle was very concerned that her students individually develop knowledge. Due to her desire to have every student be proficient in the mathematics content, she initially resisted small-group work, and she tightly controlled whole-group discussions so that she could evaluate whether and how fluently individual students could recite desired mathematics content. She was afraid that allowing students to work collaboratively, or orchestrating time-consuming techniques such as student-centered, whole-group discussions, would limit an individual student's ability to internalize and understand the information being studied. Her desire for individual student knowledge and understanding did not change, nor should it. However, she began to appreciate using small-group discussion when she realized that when students were working in small groups, she was able to directly assess the knowledge of every student in the classroom by circulating and communicating with group members. This was in contrast to the tightly controlled discussions that she led early in her teaching placement, as she did not initially realize that this only allowed her to assess the few students with whom she interacted during whole-group recitation. Her commitment to using small-group discussion for longer periods in her classroom was bolstered by the access to student assessment that small-group work provided.

Although Jack was immediately interested in using small student groups during instruction, he was not sure that he was or would be able to assess the content of students' discussions adequately. Due to this concern, he limited small-group conversations to short bursts, and he expected students to spend the remaining time during the instructional period either working individually or presenting their thinking to the whole class. However, as he became more comfortable with the sounds of students discussing their work in their small groups, he became more comfortable with using small groups as a feature of his lessons. Furthermore, the timing of the high-stakes standardized assessments and their associated benchmark testing motivated him to include center rotations in his classroom. By doing this, Jack was able to engage students in investigative, sense-making discussions while simultaneously addressing the necessary mathematics content that would allow students to perform well on the assessment.

Eleanor was deeply concerned about her students' potential performance on the annual, high-stakes, standardized assessment and its associated benchmark tests, as well as her students' performance on unit assessments that were standardized within the district. She wanted her students to perform well not only on the state tests, but also on the classroom tests so that they would earn a high grade in her class. This apprehension made her temporarily change her lesson planning to include fewer hands-on investigative activities and more skill-based worksheets for a short time, after which she returned to lesson planning that evidenced her commitments to student-centered instruction. She felt the need to have students practice those skills that would be tested so she could directly evaluate her students' current understandings. However, she felt that her students understood the information more deeply when given a chance to investigate.

Furthermore, her students would often disengage from challenging mathematics problems when left to solve them autonomously in their small groups. Therefore, Eleanor changed her use of discussion in her class from small-group discussions to guided whole-group discussions where she would scaffold her class through making sense of relevant mathematics. By scaffolding students through a whole-group discussion, and offering short opportunities for individual and small-group think-time, she was able to hold her students accountable for participation in mathematical thinking.

Implications. Teacher preparation programs must acknowledge the pressures of high-stakes standardized assessments and seriously address teachers' concerns about the test. Recognizing and orchestrating open dialogues about these pressures with prospective or practicing teachers may motivate the teachers' thinking about ways to address these pressures. It cannot be assumed that teachers will simply utilize the teaching strategies learned in their teacher preparation or professional development programs when faced with the constraints of limited time in the school year and the potential implications of using linked student assessment data as the basis for evaluating teacher performance. Instructional strategies such as whole- and small-group discussion, center rotations, and guided whole-group discussions should be examined and discussed in methods classes. A single strategy does not always address the concerns of widely different classrooms and contexts, as is evidenced in these case studies. Furthermore, prospective teachers should learn how to carry out long-term planning with the high-stakes assessments in mind. This too should be a component of a teacher-preparation program.

However, we cannot assume that teachers will learn everything they need to know about teaching through a teacher preparation program that is bounded by time (Hiebert, Morris & Glass, 2003). This leads to two implications. First, teachers must learn to be consistently reflective about their lesson planning, their implementation of those lessons, and their students' learning. If novice and early career teachers learn to treat teaching as a constant experiment, they can learn from their experiences and modify their teaching to address the needs of the situation. In this way, they can learn from their own teaching. Second, if sustained mentoring, as a form of externship, can be incorporated either in the local schools or as a feature in an induction component within teacher-preparation-transition programs, early career teachers could have more of the support that they need as they come to reflect and learn from *real* teaching experiences as a teacher of record. In this type of setting, if a novice teacher faced a challenge that was perceived to be constricting, such as the pressures of high-stakes assessment, a more experienced mentor would be available to offer support and guidance.

The Effects of Context on Teacher Development

While the school contexts in which they were placed differed, each of those settings did influence how the novice teacher who was placed in that school considered, defined, and developed instructional strategies to support productive mathematical discussion in their classrooms. Each of these teachers cited features of their school's culture as relevant to their development as teachers who implemented student-centered instruction that included student small- and whole-group discussion. It has been theorized that the lack of a consistent and agreed-upon infrastructure regarding what good teaching is, what the desired result of teaching is, and defining responsibility for these results can undermine the instructional practice of a teacher, and vice-versa (Cohen, 2011). In this case, all of the teachers cited the context, or infrastructure, of their K-8 schools as relevant to their development. For two of these teachers, the school context served as a positive asset advancing their efforts to orchestrate, scaffold and foster students' mathematical discussion, while for the third teacher, the established norms, assumptions, and routines impacted her efforts negatively.

Michelle began the year teaching in a manner that, as compared to the other participants in this study, was the least likely to include features of student-centered teaching and opportunities for productive mathematical discussion. However, the administration in her school not only strongly encouraged, but *expected* her to teach in a student-centered manner that allowed students to investigate and make sense of mathematics. Michelle only began to change her professional practice after her principal evaluated her teaching, and she interpreted the results of that evaluation to mean that she needed to incorporate more hands-on student investigation as a feature of her teaching. In this way, the context of Michelle's placement not only supported the culture of learning introduced to Michelle in her summer methods course and cultivated by her supervisors, but also encouraged her to undertake instructional changes and to seek my support as she tried to make these changes with respect to the nature of her teaching. As a result, I was able to leverage the expectations of her administration to encourage Michelle to think about planning and teaching in a different way. Since this observation and subsequent evaluation by her school administrator occurred early in the school year, Michelle realized that she needed to make substantive changes while there was still sufficient time in the school year for us to work together.

Jack taught in the same school in which Michelle taught, and therefore he was under the same expectations. However, in addition, Jack had further support for his teacher development. During his first year of teaching, he was able to observe other teachers in his school, as was the design of the program. He cited an observation of one particular teacher as instrumental in his changing approach to incorporating opportunities for student-centered instruction and small-group discussion. The cooperating teacher in whose classroom Jack was placed for his internship period prior to his permanent placement was the department chair for mathematics. Jack observed this teacher conducting center rotations with his students. This gave Jack the idea and the warrant to conduct his own center rotations. Due to the alternative certification program's design, this opportunity to observe another teacher's instruction provided Jack support as it allowed him to see a planned center rotation enacted and allowed him to observe how another, more experienced, teacher facilitated a center rotation. After this, Jack began to plan center rotations for his class and to incorporate more small-group discussion in his classroom. In this case, the school context and the practices of other teachers in the same school supported Jack's changing perspectives on lesson development and his skill in facilitating discussion.

However, Eleanor had a far different experience. Eleanor was moved to a different school context after her internship period. Contrary to Michelle's experience, Eleanor did not feel that the administration at her permanent placement was concerned about how she planned or implemented her mathematics lessons. Contrary to Jack's experience, Eleanor felt that she was the only teacher in the school who was attempting to teach in a student-centered manner that allowed students opportunities for discussion.

Therefore, it was more difficult for Eleanor to teach students how to interact in this way and to encourage their buy-in to struggling through difficult mathematics. Although Eleanor had extensive experience in teaching, specifically in a student-centered way that promoted discussion, her students were not familiar with this form of learning. Thus, Eleanor had to modify her approach several times in order to adapt to her context. She had to create a classroom culture where students felt that it was safe to share their thinking. Also, she had to carefully scaffold whole-class discussions in order to maintain student engagement while promoting sense-making.

Implications. The context of a school placement is an important consideration as it may strongly influence the development of first-year teachers. For the most part, Eleanor persisted in her attempts to develop and implement student-centered lessons that promoted productive discussion because of her extensive experience and commitment to this manner of teaching. However, if a true novice who was not as committed to student-centered instruction or who was not as confident about her own potential for orchestrating this form of instruction was placed in Eleanor's context, what result might occur? Would Michelle have had the same trajectory of change as she did in her supportive context if she were placed in Eleanor's school? Often, teacher education programs place pre-service teachers in locally convenient sites, with available and willing teachers who volunteer to be cooperating teachers. Similarly, school districts may find that the only option is to position new teachers in a limited collection of schools due to staffing vacancies. However, if we expect teachers to teach in the way in which they are instructed in their teacher preparation program, teacher educators must concern themselves with making sure that the context of the teacher candidate's placement will ideally be supportive, or at least not opposed to the types of teaching being cultivated or encouraged. Research indicates that although teachers' efficacy may improve during their student teaching, there are significant declines in efficacy during the first year of teaching (Hoy & Spero, 2005). Therefore, it is important to consider how supportive a permanent placement will be for a first-year and early-career teacher.

Consideration of both administrative policies and support, and the practices of other teachers and professionals in the school, is prudent. Administrative support might help to motivate a new teacher's desire to change or develop their practice. However, the support of other, more experienced professionals in the school who would be able to mentor the new teacher or to provide a site for observation of intended teacher practice is critical. In this way, new teachers would not have to navigate teaching on their own without assistance; rather, they could have the opportunity to learn of and attempt effective strategies with guidance from experienced teachers, as well as benefiting from observing these strategies in practice as carried out by experienced teachers within the reality of schooling. Furthermore, if a new teacher's colleagues were supportive and implemented instructional practices aligned with those discussed within the teacher-preparation program, the novice teacher would be less likely to be discouraged from teaching in the manner espoused by the teacher-preparation program, as was the case with Eleanor.

Since the context of school placement affects teacher development, teacher-education programs need to consider this component in their design of field components and in their efforts to advance an induction year. In this particular alternative certification program, there was a partnership between the university and a particular school district.

In this partnership, the program director was permitted to attempt to influence the novice teachers' permanent placement. In traditional teacher preparation programs, a prospective teacher must independently find a place of employment after graduation. Unless prospective teachers have guidance from someone with knowledge of particular school contexts, or are taught how to ascertain how the context of a school might characterize intended instructional models, they may find themselves positioned to teach in schools that are not supportive of the manner of teaching that they learned in their preparation program. If our goal as teacher educators is sustained teaching that promotes student sense making about mathematics, we must think about the role of school context, recognizing that difficult contexts might cause novice teachers to return to a teaching method that is less challenging or to instructional models that they learned through the apprenticeship of observation (Lortie, 1975).

The Effects of Tracking on Teacher Perceptions and Decision-Making

For Michelle, Jack, and Eleanor, the fact that their students were tracked had at least an initial effect on their instructional decision making. Research indicates that tracking has an influence on teachers' perceptions of students, especially their assumptions about their lower-tracked students (Oakes, 2005). In this case, tracking affected the perceptions that these teachers had regarding their students in both the classes labeled comprehensive *and* in those labeled honors.

Michelle taught a class that was labeled as comprehensive, even though this tracking was based on her students' prior reading scores and not on any prior assessments of their mathematics proficiency. Nevertheless, Michelle often used her pre-conceived notions of her students' low ability as an excuse for not engaging them in challenging, student-centered mathematics lessons that would require problem solving and provide the students with opportunities for sense-making discussions. In addition, Michelle did not recognize the differing strengths or needs of her students. She spoke of her students as an aggregate, as a whole class only composed of students with low levels of ability. Although she made attempts at incorporating opportunities for discussion in her class, her perception of her students' ability caused her to fear student struggle, to provide direct instruction of procedures, and to focus a great deal of attention on rote memorization of vocabulary terms. However, she never indicated that she had any evidence that her students were performing any differently on the mathematics assessments that Jack's "honors" students were. Her instructional decision making, particularly her decision to slow down her pacing and to include more directed instruction seemed to be based solely on her preconceived notion of what the "limited" ability level of students placed in a lower-tracked class meant *to her*. However, she was able to negotiate this challenge and engage her students in productive mathematical discussion in whole- and small-groups that allowed students opportunities to make sense of mathematics.

Michelle used strategies to mitigate her students' perceived struggle in order to encourage their participation in discussion. She gave students both individual and small-group thinking time before requiring them to share their thinking to the class. She also provided reference sheets with mathematical terminology so that students could use the correct mathematical vocabulary in their discussions. She required them to verify individually calculated solutions with other small-group members. Also, Michelle began to appreciate using small-group work because it provided her with more contact time with individual students. During this time, she could correct any misconceptions held by individuals. She may not have changed her mind about her students' initial ability, but she was able to see her students' successes in mathematical sense making as the year progressed.

Jack taught the class that was labeled "honors," however; these students were placed in this class by the same prior reading assessment that determined the placement of Michelle's students. Although Jack stated that he had automatically assumed that his honors students would be more capable of engaging in student-centered mathematics investigations and discussions, he found his students resistant to this type of instruction. He attributed their resistance to the students' assignment to tracked classes. He felt that since his students had been tracked into honors courses, a feature of scheduling that had been in place for more than one year, his students had become acclimated to learning in a particular manner: through listening, note taking during directed instruction, and completion of written solutions to mathematics problems reflecting their individual and independent efforts. He felt that his students' prior experiences with tracked classes, and their prior teachers' mode of instruction in these tracked classes, made his students appreciate and expect specific types of instruction over others. They were not only familiar with these instructional routines, they had learned how to be "good at school" in environments marked by these practices. Jack felt that he had to struggle harder than Michelle to teach students how to participate in student-centered learning environments where the expectation was for students to explain their thinking to others, and he attributed that struggle to the fact that his students were tracked into the honors section.

However, Jack used his experience with building relationships with middle-school students to establish a safe environment in his classroom. He built a rapport with his students that caused them to trust that he would not embarrass them publicly. Furthermore, he established norms in his classroom whereby students would be respectful of others during small- and whole-group discussions and would not devalue the thinking of others. Through these techniques, he began to develop a culture of participation in small- and whole-group discussion in his classrooms. His students became familiar with the practices of struggling with, and making sense of, mathematics collaboratively.

Like Michelle, Eleanor taught a class that was tracked as comprehensive. Eleanor’s students were in the on-level eighth-grade mathematics class, while the other eighth-grade students were accelerated into Algebra I. However, Eleanor’s concerns about tracking were more related to her students’ own limited if not derogatory perceptions of their ability due to tracking. Since the school was very small and the students’ had moved together over the years as a cohort from third to eighth grade, Eleanor felt that the acceleration of some students and the placement of the remainder of the students into the on-level class made the on-level students feel insecure about their mathematical ability. She felt that this insecurity led her students to disengage in mathematical discussions, especially if they were unsure of how to proceed and began to struggle. Therefore, she found it difficult to engage her students in instruction that was student-centered and investigative. Also, she found that her students were initially reticent to participate in discussions for fear they would be incorrect and shamed in front of their peers. In order to combat this fear, Eleanor carefully developed a culture in her classroom that clearly supported attempts at mathematical sense making through discussion. Her perception of her students’ own feelings of self-efficacy based on tracking influenced how she would question and respond to student thinking during small- and whole-group discussions.

In order to address this, Eleanor established norms of respect in her classroom. She privileged students' sharing of their thinking rather than praising correct answers. She facilitated whole-group discussions where she guided students through the practice of sense making and interspersed moments for individual or small-group think time. In this way, she was able to scaffold her insecure students, while holding them accountable for their participation in productive mathematical discussions.

Implications. Tracking has long been targeted for discussion in the mathematics education community. Many mathematics researchers have advocated for heterogeneous classrooms, claiming that they are better sites to support all students' learning across and within all achievement levels, advancing both equity and mathematics achievement (e.g. Boaler & Staples, 2008; Burris, Hubert & Levin, 2006). Tracking not only affects teachers' preconceptions of students' potential to achieve, but it might also affect students' development of a productive disposition about mathematics (e.g. Oakes, 2005). However, there has been much resistance to grouping students' heterogeneously both in public discourse and in policy documents (e.g. Loveless, 1998). Education policy is not likely to eliminate tracking. Thus it is important to prepare teachers for the possibility of teaching in a tracked class.

This study highlights two points. First, teacher education courses must include an opportunity for both prospective and practicing teachers to think about and discuss tracking critically. Teacher educators must provide counter-narratives to the pre-conceptions that tracked former students bring to their classrooms as teachers. Through literature, hands-on experiences, and classroom discussions, we may be able to temper the pre-conceptions that new teachers might have about students who have been tracked. This will help to address situations similar to that evidenced by Michelle and Jack, as their assumptions about their students' abilities contributed to the challenges that they faced when developing student-centered instruction, instruction that included opportunities for students to discuss and make sense of mathematics collaboratively. Furthermore, teacher educators need to prepare teachers to have discussions with their students that include counter-narratives to ability grouping. Teachers must be equipped with ways to talk to students about their ability so that students placed in low-track classes are able to build the same types of productive dispositions that students in high-track mathematics classes might develop. In this way, teacher educators can help to reduce the challenges that novice teachers face. However, we cannot expect that teachers learn everything they need to know about teaching during their teacher preparation program (Hiebert, Morris & Glass, 2003). Therefore, it is important that we prepare teachers to analyze their students' abilities in ways that enable the teachers to recognize student strengths as well as weaknesses and to learn for themselves that students of any track are capable of being successful.

Support and its Influence on Teacher Efficacy and Self-Efficacy

A key feature of the design of this alternative certification program was the degree of support that the novice teachers received during their first year of teaching. This alternative certification program was marked by a year of half-time teaching, so that teachers could have both more time to plan and prepare and to observe other teachers, as well as the opportunity to access a mentor for the entire first year. In addition to this, the participants in this study met bi-weekly with me as a seminar group. Support for a teacher has positive effects on their efficacy and self-efficacy (Bruce & Ross, 2008; Cantrell & Hughes, 2008; Hoy & Spero, 2005). Furthermore, a decrease in efficacy in a teacher's first year, as compared to the student-teaching experience, can be mediated by the amount of support that a novice teacher has during the first year of teaching (Hoy & Spero, 2005). Bi-weekly seminars as well as weekly, individual mentoring sessions were intended to provide increased support for the teachers in this alternative certification program as they would have the support of their coursework, a mentor, and other teachers in their cohort. Each of the teachers who participated in this study increased both their efficacy with respect to having their students discuss mathematics productively, as evidenced by classroom observations, as well as their feelings of self-efficacy, as evidenced by their statements.

During her initial permanent placement, Michelle struggled to embrace teaching in ways that would promote autonomous student discussion and sense making. Therefore, Michelle reached out to me for help in understanding and implementing this type of teaching. During one of our sessions, I modeled with Michelle how to plan a lesson incorporating both small- and whole-group discussions and then subsequently demonstrated how to orchestrate this lesson, facilitating both small- and whole-group discussions in Michelle's classroom. Michelle expressed that this modeling/demonstration was beneficial both to her understanding of what I meant by facilitating discussion and to her ability to enact the practice herself. Subsequently, Michelle and I worked together diligently to plan lessons that included hands-on investigations and small-group work. Near the end of the year, Michelle mentioned during a seminar that she was interested in developing a unit-long project. She mentioned that she did not feel capable of doing so on her own. In response to that feeling, Michelle, Jack, and I participated in a mentoring session where we talked through the planning and structuring of this project. After this conversation, Michelle developed her project, and it included both open-ended questions and questions that could be addressed via multiple solution strategies. These types of questions provided many opportunities for productive student discussion about mathematics. Furthermore, she was able to facilitate both whole- and small-group discussions during the course of the project's implementation. The level of support provided to Michelle increased both her efficacy and self-efficacy with regard to promoting student discussion.

Jack initially had difficulty planning lessons that were investigative and offered opportunities for collaboration if such a lesson was not readily available for his use or required modification within his district-provided textbook. Jack reached out to me, asking for assistance with planning a lesson that would introduce a new topic to students in a way that allowed them to make sense of the new concept through small- and whole-group discussions. In order to support him, we collaboratively planned the lesson. Through this, Jack was able to see ways of thinking about mathematics. In addition, during our mentoring sessions we spoke about different problem-solving strategies that students could use to learn how to think algebraically, which Jack then used in his class to foster students' understanding of their own reasoning about algebra. Jack also had the continuing support of his department chair, who was his original cooperating teacher during his internship period. Since Jack was placed in the same school in which he completed his internship, he was able to observe his cooperating teacher and to learn a variety of instructional strategies from those observations. It was from these experiences that Jack developed his use of center rotations. With regard to facilitating discussions, Jack and I had many conversations where I encouraged him to ask questions of the students in their small groups and to allow them to think about an answer to the question autonomously rather than leading a small group through an activity. I also suggested to him that he think about the misconceptions that students may have before the lesson, so that he could better manage his time with students, instead of spending a great deal of time trying to understand the students' thinking. He reflected that those mentoring sessions were helpful for his developing ability to facilitate small-group discussions, discussions that resulted in students engaged in more autonomous sense making.

Eleanor was a veteran teacher, but she also benefitted from the support provided to her through mentoring and the teacher seminars. This was critical because she was the only teacher who taught the eighth-grade curriculum at her school, and she was placed at a different school from where she completed her internship. Furthermore, she felt that no other teacher in her permanent placement was interested in teaching in a hands-on, investigative way. Therefore, she had very little school-based support. During the year, Eleanor assigned her students roles when working in small groups in order to facilitate their collaboration. This idea was taken from a discussion that we had during an early teacher seminar. Eleanor had been struggling with getting her students to collaborate in small groups, and she used this strategy to help her small-group discussions become more affective. In addition, Eleanor had little previous experience with the type of classroom management and disengagement issues that she was experiencing in her new placement. These issues were affecting her ability to engage students in on-topic conversations about mathematical tasks. During mentoring, Eleanor and I developed a classroom management strategy that would reward students for positively participating in small- and whole-group discussions. These roles and rewards helped her feel that her students were learning and also made their subsequent mathematical discussions more productive.

Implications. The supportive benefits of mentoring and coaching on the development of these teachers' practice implies that sustained mentoring during an induction year or two might be an important addition to teacher preparation programs. Would these teachers have developed their ability to plan for and implement student-centered lessons and facilitate small- and whole-group discussions without mentoring that focused on these features? Would first-year teachers without mentoring support simply revert to an instructional style that was similar to their own experiences or to teaching strategies that were simpler? What would have happened if a novice teacher was placed in Eleanor's context without mentoring? Could that teacher be independently successful in developing productive student discussions?

It has been argued that in order for professional development to be more effective, it must be sustained and directly related to the daily work of the teacher. That is, it must be content related and embedded in the work that teachers do, and it must be sustained in terms of contact hours (Cohen & Hill, 2001; Heck et al., 2009; Sowder, 2007; Stein, Smith & Silver, 1999; Wayne et al., 2009). Mentoring can be considered a form of individual, or small-group professional development that meets all of these criteria. Mentoring has the potential to increase teacher efficacy, self-efficacy, and as a result, both retention and positive student achievement.

Conclusion

Prior to entering teaching, Michelle, Jack and Eleanor attended to the importance of setting and maintaining high expectations and building relationships and rapport with their students. As a result, each of these teachers implemented strategies to this end. Michelle developed an expectation of all students participating positively. Jack treated his students with respect and developed norms of respect among the students. Eleanor rewarded student thinking and carefully guided that thinking toward an understanding of mathematics, while not solely privileging correct student answers. Due to the attention each of these teachers paid to developing relationships and maintaining high expectations, each of the teachers was able to build a classroom culture that was supportive of students' participation in productive mathematical discussion.

All three teachers in this study recognized the potential challenge posed by the pressures of high-stakes testing. During their summer teacher-preparation courses, they voiced an understanding of the time constraints imposed by these tests and the limitations that would place on teaching "time-consuming" lessons. They also attended to the need to assess students adequately and to hold students accountable for learning. When they entered the classroom, this potential challenge became reality, and therefore they needed to develop strategies to negotiate this challenge. Desire for individual accountability initially inhibited Michelle's desire and ability to promote student discussion. However, as she began to use small groups as a feature of her lesson design, she realized it allowed her space to assess individual knowledge and correct misconceptions. Jack developed abilities to anticipate student thinking and misconceptions, as well as to isolate student talk while students were working in small groups. He also began including center rotations in order to include review as well as new content in his teaching and to speed a lesson's pacing. Eleanor almost modified her lessons to include skill-based worksheets in order to negotiate the challenges of accountability. However, she instead used a strategy of guided whole-class discussions where students could make sense of mathematics as a group, or in short periods of small-group or individual work. In this way, she could hold her students accountable for learning while engaging them in conceptual mathematical thought and discussion.

While Michelle and Jack's school context was supportive of their desire to engage students in student-centered investigations and sense-making discussions, Eleanor's school context was different. This means that while the culture and the expectations of Michelle and Jack's school was something to which they attended, in Eleanor's case context was a challenge she had to negotiate. Michelle attended to the desires of her administrators for her to incorporate more hands-on activities, so this motivated a change in her approach to teaching. Jack attended to the strategies that his cooperating teacher used in order to promote student discussion in his class and began to implement the same strategies in his class. In contrast, Eleanor faced the challenge that her students were not familiar with student-centered instruction that included discussion, as the other teachers in the school did not include these features in their lessons. Therefore, Eleanor had to develop a safe and supportive culture in her class, and use her knowledge of individual students to scaffold whole-group investigative discussion to encourage the participation of all students. In this way, school culture and context was either supportive of teacher change, or a challenge that needed negotiation.

Teacher perception of student ability due to tracking was a challenge that each teacher faced, regardless of the track of the students. Michelle initially resisted providing opportunities for autonomous discussion and investigation due to her perception of her students' ability to be successful in those situations. However, she began to scaffold indirectly by providing students access to vocabulary supports during discussions and by monitoring students' misconceptions while using small groups. Jack initially thought that since his students were labeled as honors students, their tracked status would not present a challenge. However, he realized that his honors students were not only familiar with directed instruction and individual recording of solution strategies, but also comfortably expected those approaches. He focused on developing norms of respect in his classroom and developing trusting relationships with his students. Due to this, his students began to participate in discussion without reticence. Eleanor also developed relationships with her students so that they would feel safe participating in discussion. She also provided scaffolds within whole-class discussions to mitigate the frustrations felt by students who considered themselves deficient in mathematics as indicated by tracking.

All three teachers faced the challenges of efficacy and self-efficacy and had to negotiate these challenges to be effective facilitators of student discussion. Michelle, Jack, and Eleanor each utilized the additional supports they had in place to negotiate this challenge. Michelle sought support through mentoring and worked with me to develop student-centered lesson plans that included student discussion. Jack utilized mentoring and the expertise of other teachers in his school to learn new strategies. Through mentoring, he learned how to think through a mathematical concept in order to lesson plan, how to ask advancing or leading questions of students, and how to prepare for potential misconceptions. He learned the teaching strategy of rotating his students through different centers from his cooperating teacher. Mentoring and participation in the teacher seminar supported Eleanor. Through mentoring, she developed a classroom-management strategy that supported her desire to engage students in small- and whole-group discussions. From other teachers in the seminar, she learned about using roles to motivate student participation in small groups.

Each of these findings and features of the experiences of these first-year teachers have implications. The facets that these teachers attended to and that were supportive of their positive development should be included and broadened in teacher-preparation programs. Those features that provide challenges should also be addressed before and during field placements. Teachers should also learn how to be reflective and learn from their practice while actively teaching. Furthermore, mentoring support beyond a student-teaching experience should be considered as a feature of the first 1 to 3 years of teaching in order to assist teachers in developing and implementing student-centered lessons that include opportunities for discussion, as these practices are often difficult for veteran teachers, lessening the potential for on-site support.

Chapter 8: Contributions, Limitations, and Future Research

This study has several contributions to the existing literature about novice teachers, specifically those novice, middle-grades mathematics teachers who are situated in “hard-to-staff” school contexts, and their ability to promote their students’ mathematical sense making through participation in discussion. This chapter will discuss those contributions, as well as the limitations of this study, offering possible directions for future research.

Contributions

The contributions of this study are directly linked to the study’s findings and implications as stated in the previous chapter. These include contributions with regard to novice teachers’, and their pre-service or alternative, teacher education programs’, attentiveness to the potential impact of building relationships, providing supportive contexts for teachers, addressing the existence of tracking, considering the availability of support, and attending to the pervasiveness and unavoidable nature of accountability.

Equity. These teachers, with support from their mentor and occasionally other teachers or administrators, were able to learn to promote students’ sense making and participation in productive mathematical discussion. This is particularly important due to the context of their placement: “hard-to-staff” schools that served largely low-income students of color. These teachers also found success engaging students in sense making and productive discussion in situations where their classrooms included students they perceived as low achieving. Although some researchers have concerns about a student-centered and indirect approach being successful with this particular population (Baxter, Woodward, & Olson, 2001; Baxter, Woodward, Wong & Voorhies, 2002; Lubienski, 2000, 2002), these teachers used several methods to equitably provide access for all the students in their classroom. In order to accomplish this, these teachers built relationships with their students, planned conceptual and accessible tasks, provided appropriate scaffolding (Baxter, Woodward & Olson, 2001), and carefully developed classroom expectations and norms (Boaler & Staples, 2008; Yackel & Cobb, 1996).

Relationships. Research argues that building relationships with students can affect student engagement, and transitively, student achievement (e.g. Klem & Connell, 2004). Many of the current recommendations that prescribe the practical knowledge that effective teachers must have and apply include relationship building (Learning In, From, & For Teaching in Practice [LTP], 2013b; Teachingworks, 2013). Proponents of high-leverage practices name this practice as “engaging in strategic relationship-building conversations with students” (Teachingworks, 2013, para. 13), and proponents of ambitious teaching state “teachers must know their students as individuals and as learners” (LTP, 2013, para. 4). However, documentation through research does not make clear whether or not explicit and pervasive conversations about relationship building are commonplace throughout existing teacher education programs. In this alternative certification program, I am aware that these conversations took place in at least one course, but I have no evidence that these conversations reoccurred in other courses in which the candidates were enrolled.

When analyzing the results of this study, although all of the participants built relationships with students in different ways, only two of the three participants appeared to enter the program with strong commitments to relationship building and then *directly leveraged* those commitments to develop classroom cultures that promoted students’ sense making through discussion. These two participants used their relationships with students and the resultant classroom environment defined by those relationships to promote discussion. The third participant, while she did build relationships with her students, did not leverage those relationships to the end of promoting discussion. This study affirms the current prescriptions for teacher knowledge, in that explicit understanding of relationship building facilitated teachers' ability to promote mathematical sense making through student discussion. This study also suggests that many different teacher commitments and practices are related and interdependent.

Contexts. In traditional pre-service, teacher education programs, a great deal of attention has been paid to the schools in which prospective teachers are placed and to the identification of the cooperating teacher with whom the prospective teacher is placed (e.g. Beck & Kosnick, 2002; Goodlad, 1994). However, in traditional teacher preparation programs, the program has little control over the eventual permanent placement of the graduating, just-certified teacher. Alternative certification programs may have more control, since those programs permanently place or have influence over the permanent placement of many of the candidates. This is potentially important because regardless of the positive progress in prospective teachers' self-efficacy during their practicum period, when a teacher reaches their first year of teaching as the teacher of record, their self-efficacy declines, although the amount of support that a teacher has during the first year of teaching mitigates decreases in teacher self-efficacy (Hoy & Spero, 2005). The implication of this research is that a teacher who receives more support, or is placed in a supportive environment during the first year of teaching, may experience more successes. Furthermore, cohesive infrastructure in the context of a teacher's placement may support teacher efficacy, whereas the opposite may undermine teacher efficacy (Cohen, 2011). However, these studies are largely based on novice-teacher self-reporting. This suggests that classroom-level data may be necessary to confirm or deny these claims.

This study extends the aforementioned research addressing concerns of the quality of teacher placements by providing classroom-level data as well self-reports from teachers. One teacher who initially seemed somewhat established in her teaching routines did improve her teaching practice in part because she was encouraged and required to do so by her school's administrators, who supported innovative teaching practices congruent with those expressed by the university. This alignment, as reflected in an evaluative interaction, helped to leverage and motivate her teaching practice. Whereas, when another teacher was placed in a school context where some other teachers in other content disciplines discouraged teaching that featured investigation and sense making through discussion, this teachers' improvement was mostly leveraged by her own prior teaching experience and professional commitments. If these teachers' placements were reversed, the first teacher may not have made such dramatic changes in her teaching, and the second teacher may have made changes in her teaching that were more dramatic. Teacher preparation programs should consider their options with regard to influencing the eventual teacher placements, possibly through partnerships with districts and/or with schools within districts.

Tracking. Oakes (2005) argues that tracking influences teachers' perceptions of students, and this perpetuates inequity in education. This is specifically relevant in urban or urbanized school districts due to the overrepresentation of minority and low-income students in lower tracked classes. The perceptions that teachers hold of low-tracked students influences the teachers' perceptions of appropriate activities in which to engage students, as well as students' perceptions of their teachers' actions to and perceptions of them. This study confirms those findings, in that the teachers who taught classes labeled as comprehensive cited either their perception of their students' ability or their perception of how their students perceived their own ability as relevant to their instructional decision making. However, this study extends this literature as well. Oakes argues that teachers of high-tracked classes automatically assume that these students will agree with or buy into and engage in rigorous instruction. However, a teacher in this study found his students' placement in high-tracked classes to be a challenge, rather than an asset. He believed that norms of schooling perpetuated by teachers with particular perceptions of what teaching high-tracked students entailed acculturated students to teaching in which student participation was defined by listening, taking notes, and independently providing written solutions to problems. He had to negotiate the challenge of student buy-in to student-centered instruction where students were required to make sense of mathematics collaboratively.

Support. Research findings note that the amount of support that a new teacher experiences can mitigate declines in self-efficacy between new teachers' internship period and their permanent placement (Hoy & Spero, 2005). Other studies have documented increases in teacher efficacy in practice when teachers are provided intensive mentoring support (Stanulis & Floden, 2009). However, there are concerns that these increases in efficacy may not include improving efficacy with regard to "standards-based" mathematics instruction (Wang & Odell, 2011), instruction marked by features such as students communicating about mathematics with their peers in order to develop their mathematical understanding. Even studies that have established that mentoring supporting "standards-based" instruction across multiple disciplines is largely beneficial and is related to increases in teacher efficacy were inconclusive when advancing mentoring in a large urban district with a population of low-income students of color (Ingersoll & Strong, 2011). Such contradictions invite more research. This study established that novice middle-school mathematics teachers may benefit from mentoring. Advice, guidance, and facilitation of discussion as fostered in mentoring sessions and teacher seminars assisted these new teachers' development of instructional strategies, as well as their ability to negotiate challenges in order to promote students' participation in productive mathematical discussion. Mentoring support should be provided to all novice middle-school mathematics teachers that enter teaching through alternative routes.

Accountability. High stakes testing often compromises, or threatens to compromise, high-quality instruction (Darling-Hammond & Rustique-Forrester, 2005). Teachers often narrow their vision of delivered curriculum content and skills to what is tested on high-stakes standardized assessments, frequently teaching in a manner that is limited to drilling procedural skills rather than promoting understanding through student-centered instruction (Au, 2007). Since high-stakes standardized testing is unlikely to disappear, teachers must be taught to negotiate this challenge. This study recommends that teachers, even novice teachers, can *leverage, instead of being limited by,* the demands of accountability and testing. The teachers in this study found teaching in a student-centered way could allow them to assess students more closely. Also, one teacher changed her practice in a way that engaged students to engage in collaborative sense making *in order to* hold her students accountable for taking responsibility of their own understanding.

Limitations

I designed and enacted this study with interpretivist and pragmatist epistemological commitments. I believe that research that investigates the experiences and actions of teachers in context is best addressed through listening to, observing, and interpreting teachers' words and actions while taking into account the teachers' experiences. I address the limitations of this study with respect to these commitments.

This study took place during participants' pre-service coursework and a single year of half-time resident teaching. I did perform a follow-up interview in the subsequent school year when the participants had assumed full-time teaching responsibilities. In this interview, the teachers described a continuation of the implementation of instructional strategies intended to engage students in productive sense-making discussions. However, I was unable to confirm the validity of these assertions, since I did not perform follow-up classroom observations. Therefore, I cannot confirm with any certainty that these teachers were continuing to implement, nor that they were continuing their development as teachers who implement, student-centered instruction. Furthermore, I cannot be certain that these teachers, if required to assume full-time rather than half-time teaching responsibilities during their first year, would have had the sufficient time to reflect in a way that would facilitate the development I witnessed during the time that I observed them.

Although this study was motivated, in part, by educational policies such as those reflected in the standards espoused by the CCSSI, the content standards are required by the Common Core State Standards for Mathematics (CCSSM) had not yet been adopted by the district in which the study took place. I did assess the teachers' implementation of certain mathematical practices as outlined by the CCSSM, however I was not able to ascertain the challenges that may present themselves once the CCSSM content standards become policy. Since mathematical content knowledge is essential for teachers' efficacy in developing student-centered instruction with sense-making discussion as a feature of mathematics lessons (e.g. Ball, Thames & Phelps, 2008; Shulman, 1987), a change in the level of content taught and content knowledge required may pose challenges not investigated in this study.

Finally, the development exhibited by these teachers cannot be ascribed to any middle-school teacher in alternative-certification programs who are provided with mentors. Other teachers in this particular program, even in this particular year, may not have shown the same progression, or even spoken about or implemented the same strategies or experienced the same challenges. I was their mentor, but also a researcher. My commitments were both to what I believed was good teaching, but also to helping teachers learn to learn to promote productive mathematics discussion that engages students in sense making. If my research agenda was different, perhaps these teachers would have developed in different ways, talked about and implemented different strategies, or expressed or attempted to negotiate different challenges.

Future Research

This study was conducted in the context of an alternative-certification program that selected prospective teachers with community commitments. Future studies could focus on alternative-certification programs with different selection criteria to ascertain teachers' development in these contexts. In addition, studies could focus on the development of teachers that were products of traditional teacher-preparation programs. However, since mentoring was such a critical component of this study, these studies should include a mentoring component, or else compare programs that do include mentoring to those who do not.

Secondly, this study was conducted within the first year of half-time teaching. There are several differing study designs that could extend the findings in this study. A researcher could replicate this study while the participants are teaching full time. A differing study design could examine the implications of providing teachers with mentoring lasting for more than one year. It would also be interesting to determine the long-term effect of mentoring on instructional practice and on teachers' understanding and professional engagement by studying teachers after mentoring support is withdrawn. Do mentored teachers assume responsibility for seeking continued professional interaction and growth after formal mentoring ceases?

Thirdly, a study similar to this one should be undertaken once the requirements of the CCSSM are adopted, in order to ascertain whether and how the added pressures and requirements of those standards influence the challenges that novice teachers face. There will be more rigorous content knowledge requirements for teachers and different ways that districts introduce these requirements to their staff and their students, and there will be associated pressures of the new high-stakes standardized assessments for students. These may present different challenges for novice teachers in "hard-to-staff" school contexts.

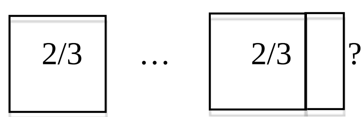
Appendix A: Exemplar Baseline Interview Questions

A mathematical topic that the research subject is familiar with will be selected. For example, the interview may address division of fractions. Then the subject will be asked to explain how they would use a particular mathematics problem on that topic to promote student-to-student explanation and questioning. The script below illustrates exemplar interview questions for a particular problem.

1) Suppose you and your students are working on division of fractions. Suppose for one lesson, you are going to give your students this problem and you are going to ask the students to work together in small groups to solve this problem.

   Delonte has a summer job helping the manager at his apartment complex. The manager has asked Delonte to help him build a concrete patio at the back of one of the apartments. The patio will be laid down in square sections marked by wooden dividers, with each section holding 2/3 of a cubic yard of concrete. The concrete truck holds 2 ¼ cubic yards of concrete, and the manager has to pay for the entire truckload.

   The manager told Delonte that he knows there will not be enough concrete for a full section at the very end of the patio, so the manager will use the wooden dividers to fill a smaller area at the end of the patio.



   How many sections can Delonte and the manager fill if they use all of the concrete in the truck? (Your answer should state how many complete sections and a fraction to indicate what part [how much] of a section is on the end.)

   What might you say or do to set up this lesson so that your students are able to successfully work in small groups to solve this problem?

2) Suppose you not only wanted your students to solve this problem in their small groups, but you were interested in having students explain their work to each other in their small group. What would you say or do so that would happen? (Think about both what you would say or do and when in the lesson you would say or do it.)

3) Suppose you not only wanted your students to solve this problem in their small groups, but you also wanted students to ask each other questions about their work and/or their solution process. What would you say or do so that would happen? (Think about both what you would say or do and when in the lesson you would say or do it.)

Glossary

Challenges: Any voiced or observed difficulty that is viewed as an impediment to developing and implementing instruction**.**

Instructional strategy: Any teacher plan, decision, practice, technique, or move that he or she discusses before or after, or utilizes during instruction.

Mathematical sense making: "(a) developing a mathematical point of view — valuing the processes of mathematization and abstraction and having the predilection to apply them, and (b) developing competence with the tools of the trade, and using those tools in the service of the goal of understanding structure" (Schoenfeld, 1994, p. 60).

Mentoring Session: A mentoring session is when a teacher meets with their mentor either before (planning) or after (reflection) instruction.

Productive mathematical discussion: Student-centered discussions that "support student learning of mathematics by helping students learn how to communicate their ideas, making students' thinking public so it can be guided in mathematically sound directions, and encouraging students to evaluate their own and each other's mathematical ideas" (Smith & Stein, 2011, p. 1).

Reform models of mathematics instruction: Instruction that promotes and facilitates students' productive mathematical discussion that results in student sense making.

Teacher seminar: A teacher seminar is when a group of teachers meet with a mentor and collaborate to reflect on and plan for teaching.

Teacher-support reflection cycle: A modified reflective teaching cycle (Smith, 2001) that included (a) a teacher seminar wherein all teachers and their mentor met, reflected, and planned (b) a planning mentoring session (c) a classroom observation and (d) a reflection mentoring session.

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