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Fast Blind Adaptive Algorithms for Equalization and Diversity Reception in Wireless Communications Using Antenna Arrays

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Fast Blind Adaptive Algorithms for Equalization and Diversity Reception in Wireless Communications using Antenna Arrays*

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Abstract: *To combat the multipath and time-variant fading of wireless communication channels, antenna arrays are usually used to improve the quality and increase the capacity of communication service. This paper investigates the fast blind adaptive algorithms for the equalization and diversity combining in wireless communication systems using antenna arrays. Two second-order statistics based algorithms, SOSA and MSOS, for equalization and diversity combining are proposed and their convergence in noiseless and noisy channels is analyzed. Since the proposed algorithms use only second-order statistics or correlation of the channel outputs, they converge faster than the higher-order statistics based algorithms, which is also confirmed by computer simulation examples.*

Keywords: blind adaptive equalization, fractionally-spaced equalizer, diversity reception, antenna arrays.

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1 Introduction

Due to the multipath phenomena in wireless communication systems, channel equalizers have to be used to remove the intersymbol interference (ISI). Traditionally, training sequences have been applied to estimate the channel parameters or to initialize the equalizers. However, when the training sequence is costly or impracticable, blind identification or equalization algorithms have to be employed.

If the channel output of a digital communication system is sampled at baud-rate, a single-input/single-output (SISO) system is obtained. Since second-order statistics of a system output only carries its amplitude information, higher-order statistics has to be used to identify or equalize the channel. Many higher-order statistics based channel identification or equalization algorithms have been proposed in [7, 9, 10, 18, 19, 20, 22, 23, 24] and the references therein. Godard algorithm [10] or constant-modulus algorithm (CMA)[22, 23] is one of the most popular blind equalization algorithms because of its simplicity and effectiveness. It is proved in [8] that the CMA converges to one of its global minima if implemented with a double-infinite-length equalizer. However, if it is implemented with a finite-length equalizer, as is the case in practical communication systems, an improper initial setting of the equalizer will cause ill-convergence [6, 15]. The investigation has also indicated that for the channels with deep nulls in its spectrum or for the channels with non-constant modulus input, the CMA equalizer converges very slowly.

To improve the quality of wireless communication systems, antenna arrays are used for diversity reception. The digital communication systems with antenna arrays can be modeled as single-input/multiple-output (SIMO) systems if there is only one dominant signal received by the antenna arrays. The SIMO systems can also be view as the over-sampled digital communication systems with single sensor. The SIMO channels satisfying certain conditions can be identified using second-order statistics or correlation function of the channel outputs[1, 12, 14, 17, 21]. Once the channel parameters are estimated, the optimum diversity combiner and equalizer [2, 3, 13] can be designed to remove intersymbol interference caused by channel distortion. The CMA can also be applied in SIMO systems to exploit the channel diversity and equalize systems. In this case, it is called *fractionally-spaced CMA (FS-CMA) equalizer*. Even though it has been proved that FS-CMA equalizer is globally convergent under a *length-and-zero condition* [16], it converges slowly since it is a higher-order statistics based algorithm, and it sometimes suffers from noise amplification, especially when the signal-to-noise ratio of system is low.

As indicated before, the SIMO channels under certain conditions can be identified using second-order statistics. Therefore, there should exist second-order statistics based blind adaptive equalization and diversity combining algorithms for the SIMO systems satisfying some condition. In fact, we will derive blind adaptive algorithms for the diversity reception and equalization of the SIMO systems using second-order statistics of the channel outputs, and analyze the convergence of the proposed algorithms.

The rest of this paper is organized as following. In Section 2, we will describe the mathematical model of wireless communication systems using antenna arrays. Then we will develop two second-order statistics based algorithms for blind equalization and diversity combining in Section 3. Next, in Section 4, we will prove the global convergence of the proposed algorithms for noiseless channels and analyze their performance for noisy channels. Finally, we will demonstrate the fast convergence of the new algorithms by computer simulation examples.

2 Problem Formulation

Figure 1 demonstrates the scenario of a wireless communication system using antenna arrays. Due to the multipath effect of the wireless channel, each antenna will receive a distorted signal. Hence, the communication system shown in Figure 1 can be described as a single-input/multiple-output (SIMO) system shown as in Figure 2. The input sequence $\{s[n]\}$, with zero-mean and variance σ_s^2, is sent through M different linear channels with impulse response $\{h_m[n]\}$ for $m = 1, 2, \cdots, M$. Therefore, channel outputs can be expressed as

$$x_m[n] = h_m[n] * s[n], \text{ for } m = 1, \cdots, M, \tag{2.1}$$

or in vector form as

$$\mathbf{x}[n] = \mathbf{h}[n] * s[n], \tag{2.2}$$

where $*$ denotes the convolution of sequences (or vectors), and $\mathbf{h}[n]$ and $\mathbf{x}[n]$ are respectively defined as

$$\mathbf{h}[n] \triangleq \begin{pmatrix} h_1[n] \\ \vdots \\ h_M[n] \end{pmatrix}, \text{ and } \mathbf{x}[n] \triangleq \begin{pmatrix} x_1[n] \\ \vdots \\ x_M[n] \end{pmatrix}. \tag{2.3}$$

In this paper, we will assume that the SIMO channels are of finite impulse response (FIR), and furthermore, they satisfy the *length-and-zero condition* [16], i.e. the M subchannels satisfy the following two conditions:

1. *$h_{m_1}[0] \neq 0$ and $h_{m_2}[L-1] \neq 0$ for some $1 \leq m_1,\ m_2 \leq M$, where L is the largest length of the M subchannels.*

2. *$\{H_m(z)\}_1^M$ have no common zeros, where $H_m(z)$ is the Z-transform of $\{h_m[n]\}$.*

For the SIMO FIR channels, the channel outputs can be written in matrix form as

$$\mathbf{x}_K[n] = \mathcal{H}_K \mathbf{s}_K[n], \tag{2.4}$$

or

$$\mathcal{X}_K[n, N] = \mathcal{H}_K \mathcal{S}_K[n, N], \tag{2.5}$$

where we have used the notations

$$\mathcal{H}_K \triangleq \begin{pmatrix} \mathbf{h}[L-1] & \cdots & \mathbf{h}[0] & \cdots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{h}[L-1] & \cdots & \mathbf{h}[0] \end{pmatrix}, \tag{2.6}$$

$$\mathbf{x}_K[n] \triangleq \begin{pmatrix} \mathbf{x}[n] \\ \vdots \\ \mathbf{x}[n+K-1] \end{pmatrix}, \quad \mathcal{X}_K[n,N] \triangleq (\mathbf{x}_K[n], \cdots, \mathbf{x}_K[n+N-1]), \tag{2.7}$$

and

$$\mathbf{s}_K[n] \triangleq \begin{pmatrix} \mathbf{s}[n-L+1] \\ \vdots \\ \mathbf{s}[n+K-1] \end{pmatrix}, \quad \mathcal{S}_K[n,N] \triangleq (\mathbf{s}_K[n], \cdots, \mathbf{s}_K[n+N-1]). \tag{2.8}$$

It is proved in [12] that if an FIR SIMO channels satisfy the length-and-zero condition, then $\mathcal{H}_K$ for any $K \geq L-1$ is of full column rank, which is also the identifiable condition [21] of the SIMO FIR channels using second-order statistics of the channel outputs. Hence, from (2.4), there exists a $KM \times (K+L-1)$ matrix $\mathcal{F}$ (not necessarily unique), such that

$$\mathbf{s}_K[n] = \mathcal{F}^H \mathbf{x}_K[n]. \tag{2.9}$$

Let

$$\mathcal{F} = (\mathbf{f}_1,\ \mathbf{f}_2, \cdots,\ \mathbf{f}_{K+L-1}), \tag{2.10}$$

where $\mathbf{f}_k$ for $k = 1, \cdots, K+L-1$ are column vectors with KM elements. From the above identities, we can implement the channel equalizer and diversity combiner as given in Figure 3. The task of blind adaptive equalization and diversity combining is to find algorithms to adjust the parameters $\mathbf{f}_k$ of the filters in Figure 3 so that the system output

$$y[n] = cs[n-n_d] \tag{2.11}$$

for some unit norm constant c and delay n_d.

3 Algorithm Development

Having described the system model and summarized some properties of SIMO FIR channels, we now develop blind adaptive equalization and diversity algorithms for SIMO FIR channels using second-order statistics of the channel outputs.

3.1 Basic Principle

From the definitions of (2.8) and (2.9), we have

$$\mathbf{f}_k^H \mathbf{x}_K[n] = \mathbf{f}_{k+1}^H \mathbf{x}_K[n-1], \tag{3.1}$$

for $k = 1, \cdots, K+L-2$, and every integer n. Hence, using the above identity, we may obtain $\mathbf{f}_k$. If the channel noise is considered, $\mathbf{f}_k$ for $k = 1, \cdots, K+L-1$ can be estimated by minimizing the cost function

$$\mathcal{C}(\mathbf{f}) = \sum_{k=1}^{K+L-2} E|\mathbf{f}_k^H \mathbf{x}_K[n] - \mathbf{f}_{k+1}^H \mathbf{x}_K[n-1]|^2, \tag{3.2}$$

subject to

$$\sum_{k=1}^{K+L-1} |\mathbf{f}_k^H \mathbf{x}_K[n]|^2 = c_o, \tag{3.3}$$

where c_o is a positive constant. The constraint is added here to prevent from the trivial solution $\mathbf{f}_k = \mathbf{0}$ for all $k = 1, \cdots, K+L-1$.

Based on the principle presented here, we are able to develop a second-order statistics based algorithm (SOSA) and a modified second-order statistics based algorithm (MSOSA).

3.2 Second-Order Statistics based Algorithm

From (3.2), direct calculation yields that

$$\mathcal{C}(\mathbf{f}) = \sum_{k=1}^{K+L-2} (\mathbf{f}_k^H R_x[0]\mathbf{f}_k - \mathbf{f}_k^H R_x[1]\mathbf{f}_{k+1} - \mathbf{f}_{k+1}^H R_x^H[1]\mathbf{f}_k + \mathbf{f}_{k+1}^H R_x[0]\mathbf{f}_{k+1}), \tag{3.4}$$

where we have used the definition

$$R_x[m] \triangleq E\{\mathbf{x}_K[n]\mathbf{x}_K^H[n-m]\}, \tag{3.5}$$

and the identity

$$R_x[-m] = R_x^H[m]. \tag{3.6}$$

Hence,

$$\frac{\partial \mathcal{C}(\mathbf{f})}{\partial \mathbf{f}_k} = \begin{cases} R_x[0]\mathbf{f}_1 - R_x[1]\mathbf{f}_2, & k=1, \\ 2R_x[0]\mathbf{f}_k - R_x[1]\mathbf{f}_{k+1} - R_x^H[1]\mathbf{f}_{k-1}, & k=2,\cdots,K+L-2, \\ R_x[0]\mathbf{f}_{K+L-1} - R_x^H[1]\mathbf{f}_{K+L-2}, & k=K+L-1. \end{cases} \tag{3.7}$$

Let

$$\mathbf{f} = \begin{pmatrix} \mathbf{f}_1 \\ \vdots \\ \mathbf{f}_{K+L-1} \end{pmatrix}, \tag{3.8}$$

then, (3.7) can also be written as

$$\frac{\partial \mathcal{C}(\mathbf{f})}{\partial \mathbf{f}} = R\mathbf{f}, \tag{3.9}$$

where R is a $KM(K+L-1) \times KM(K+L-1)$ matrix defined as

$$R \triangleq \begin{pmatrix} R_x[0] & -R_x[1] & \mathbf{0} & \cdots & \mathbf{0} \\ -R_x^H[1] & 2R_x[0] & -R_x[1] & \ddots & \mathbf{0} \\ \mathbf{0} & -R_x^H[1] & 2R_x[0] & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \cdots & R_x[0] \end{pmatrix}. \tag{3.10}$$

Using gradient-based algorithm, we can obtain a set of iteration formula to estimate $\mathbf{f}$ as following:

$$\widehat{\mathbf{f}}^{(n+1)} = \mathbf{f}^{(n)} - \mu R\mathbf{f}^{(n)}, \tag{3.11}$$

$$\mathbf{f}^{(n+1)} = \frac{c_o}{p^{(n+1)}}\widehat{\mathbf{f}}^{(n+1)}, \quad p^{(n+1)} = \Big(\sum_{k=1}^{K+L-1} (\widehat{\mathbf{f}}_k^{(n+1)})^H R_x^{(n+1)}[0]\widehat{\mathbf{f}}_k^{(n+1)}\Big)^{1/2}, \tag{3.12}$$

and $R_x[m]$ in R can be estimated using

$$R_x^{(n+1)}[m] = \lambda R_x^{(n)}[m] + (1-\lambda)\mathbf{x}_K[n]\mathbf{x}_K^H[n-m], \tag{3.13}$$

where μ is a small step-size and λ is a forgetting factor close to 1.

The blind adaptive algorithm for equalization and diversity combining defined by (3.11)-(3.13) is called second-order statistics based algorithm (SOSA).

3.3 Modified Second-Order based Algorithm

If we examine identities (2.8) and (2.9) carefully, we will find that

$$\mathbf{f}_{k_1}^H \mathbf{x}_K[n-k_1] = \mathbf{f}_{k_2}^H \mathbf{x}_K[n-k_2] \tag{3.14}$$

for $k_1,\ k_2 = 1, 2, \cdots, K+L-1$ and all integer n. Hence, we can modify the cost function (3.2) as

$$\tilde{C}(\mathbf{f}) = \sum_{k_1,k_2=1}^{K+L-1} E|\mathbf{f}_{k_1}^H \mathbf{x}_K[n-k_1] - \mathbf{f}_{k_2}^H \mathbf{x}_K[n-k_2]|^2. \tag{3.15}$$

From this cost function, we are able to derive a modified second-order statistics based algorithm (MSOSA), which is similar to (3.11)-(3.13), except that R there is substituted by

$$\tilde{R} = \begin{pmatrix} (K+L-2)R_x[0] & -R_x[1] & \cdots & -R_x[K+L-2] \\ -R_x^H[1] & (K+L-2)R_x[0] & \ddots & -R_x[K+L-3] \\ \vdots & \ddots & \ddots & \vdots \\ -R_x^H[K+L-2] & \cdots & \cdots & (K+L-2)R_x[0] \end{pmatrix}. \tag{3.16}$$

Since the MSOSA exploits more information about the structure of $\mathbf{s}_K[n]$, as comfirmed by computer simulations, it converges faster than the SOSA.

4 Performance of the New Algorithms

Having developed the SOSA and the MSOSA in the previous section, we will investigate their convergence in noiseless and noisy channels in this section. First, we study the global convergence of the proposed algorithms under noiseless situation.

4.1 Global Convergence for Noiseless Channels

Before investigating the convergence, we give a relevant definition. A sequence $\{s[n]\}$ is said to be *K-th order persistently exciting* [26] if and only if $\mathcal{S}_K(1, N)$ defined in (2.8) is of full row rank for some N. Most of the existing blind equalization algorithms assume that the channel input is with independent identical distribution (i.i.d.). Since the channel input sequence in communication systems is often coded data, this assumption is not true in these cases. Fortunately, the proposed algorithms only require that the channel input is persistently exciting upto certain order, which is easily satisfied by almost all digital signals in communication systems.

The convergence performance of the new algorithms can be expressed by the following theorem.

Theorem 1: *For digital communication systems, assume that the channel input sequence $\{s[n]\}$ is $K+1$-th order persistently exciting for some $K \geq L-1$ and N in (2.8), and the noiseless channel satisfies the length-and-zero condition. If for some k and n*

$$\mathbf{f}_k^H \mathbf{x}_K[n] \neq 0, \tag{4.1}$$

and for all $k = 1, 2, \cdots, K+L-2$ and $n = 1, 2, \cdots, N$

$$\mathbf{f}_k^H \mathbf{x}_K[n+1] = \mathbf{f}_{k+1}^H \mathbf{x}_K[n], \tag{4.2}$$

then,

$$\mathcal{F}^H \mathcal{H}_K = c I_{K+L-1} \tag{4.3}$$

for some non-zero c, where I_{K+L-1} is a $(K+L-1) \times (K+L-1)$ identity matrix.

Proof: Let

$$\mathbf{g}_k = \mathcal{H}_K^H \mathbf{f}_k, \tag{4.4}$$

for $k = 1, 2, \cdots, K+L-1$, then, from (4.1) and (4.2), we have

$$\mathcal{S}_K^H(2, N)\mathbf{g}_k = \mathcal{S}_K^H(1, N)\mathbf{g}_{k+1}, \tag{4.5}$$

for $k = 1, 2, \cdots, K+L-2$. Since $s[n]$ is $K+1$-th order persistently exciting, the different columns in $\mathcal{S}_K^H(1,N)$ and $\mathcal{S}_K^H(2,N)$ ($K+1$ different columns) are linearly independent. Therefore, we have

$$g_{k,K+L-1} = g_{k+1,1} = 0, \;\; g_{k,m} = g_{k+1,m+1}, \tag{4.6}$$

for $k,\; m = 1, \cdots, K+L-2$, where $g_{k,m}$ is the m-th element of $\mathbf{g}_k$. (4.6) implies

$$g_{k,m} = \begin{cases} c & \text{if } \; k = m, \\ 0 & \text{otherwise,} \end{cases}, \tag{4.7}$$

it follows that

$$\mathcal{F}^H \mathcal{H}_K = \begin{pmatrix} \mathbf{g}_1^H \\ \vdots \\ \mathbf{g}_{K+L-1}^H \end{pmatrix} = cI_{K+L-1} \tag{4.8}$$

and c is non-zero since $\mathbf{f}_k^H \mathbf{x}_k[n] \neq 0$ for some k and n.

$\square$

Theorem 1 indicates the global convergence of the SOS and MSOS algorithms used in noiseless channel regardless of the initial setting

4.2 Performance for Noisy Channels

In practical communication systems, channel noise always exists besides the channel distortion. For SIMO channels with additive white noise, the channel output $\mathbf{x}_K[n]$ can be written as

$$\mathbf{x}_K[n] = \mathcal{H}_K \mathbf{s}_K[n] + \mathbf{w}_K[n], \tag{4.9}$$

where

$$\mathbf{w}_K[n] \triangleq \begin{pmatrix} \mathbf{w}[n] \\ \vdots \\ \mathbf{w}[n+K-1] \end{pmatrix}, \;\; \text{and} \;\; \mathbf{w}[n] \triangleq \begin{pmatrix} w_1[n] \\ \vdots \\ w_M[n] \end{pmatrix}, \tag{4.10}$$

with $w_m[n]$ being the additive noise of m-th sensor at time n, which is assumed to be uncorrelated for all m and n and with zero-mean and variance σ_w^2. In this case, the correlation function of the channel output can be expressed as

$$
\begin{aligned}
R_x[m] &\triangleq E\{\mathbf{x}_K[n]\mathbf{x}_K^H[n-m]\} \\
&= \mathcal{H}_K E\{\mathbf{s}_K[n]\mathbf{s}_K^H[n-m]\}\mathcal{H}_K^H + E\{\mathbf{w}_K[n]\mathbf{w}_K^H[n-m]\} \\
&= \sigma_s^2 \mathcal{H}_K J_1^m \mathcal{H}_K^H + \sigma_w^2 J_2^m,
\end{aligned} \tag{4.11}
$$

for $m \geq 0$, and

$$R_x[m] = R_x[-m]^H \tag{4.12}$$

for $m < 0$, where J_1 is a $(K+L-1)\times(K+L-1)$ backward shift matrix defined as

$$J_1 \triangleq \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \ddots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}, \tag{4.13}$$

and J_2 is a $KM \times KM$ block backward shift matrix defined as

$$J_2 \triangleq \begin{pmatrix} \mathbf{0} & I & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I & \cdots & \mathbf{0} \\ \cdots & \ddots & \ddots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \ddots & I \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \end{pmatrix} \tag{4.14}$$

with $\mathbf{0}$ and I being $M \times M$ zero matrix and identity matrix respectively. Because of the channel noise, $R_x[0]$ is positive-definite and can be decomposed as

$$R_x[0] = U\Sigma^2 U^H, \tag{4.15}$$

where U is a unitary matrix and

$$\Sigma^2 = diag\{\sigma_1^2, \cdots, \sigma_{KM}^2\} \tag{4.16}$$

with σ_k^2 for $k = 1, \cdots, KM$ being the eigenvalues of $R_x[0]$.

The impulse response of the channel and equalized system and mean-square-error (MSE) of the channel output can be expressed in terms of the $R_x[m]$ as given in the following theorem.

Theorem 2: *Let*

$$\Sigma \triangleq diag\{\sigma_1, \cdots, \sigma_{KM}\}, \tag{4.17}$$

and

$$\mathcal{Q} \triangleq \begin{pmatrix} \Sigma U^H & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \Sigma U^H \end{pmatrix}. \tag{4.18}$$

Then the impulse response of the channel and equalized system is

$$t_n = \begin{cases} \frac{1}{K+L-1} \sum_{i=\max\{1,n+1\}}^{\min\{K+L-1+n, K+L-1\}} g_{i,i-n}, & -(K+L-2) \le n \le K+L-2, \\ 0 & otherwise. \end{cases}, \tag{4.19}$$

where

$$[g_{i,j}] = [\mathbf{f}_1^{(o)}, \cdots, \mathbf{f}_{K+L-1}^{(o)}]^H \mathcal{H}_K, \tag{4.20}$$

and

$$\mathbf{f}^{(o)} = \frac{1}{\sigma_s (K+L-1)^{1/2}} \mathcal{Q}^{-1} \mathbf{r}, \tag{4.21}$$

with $\mathbf{r}$ *being the eigenvector of* ${\mathcal{Q}^{-1}}^H \mathcal{R} \mathcal{Q}^{-1}$ *corresponding to its smallest eigenvalue.* $\mathcal{R}$ *in the above expression is either* R *defined as (3.10) or* $\tilde{R}$ *defined as (3.16), depending on the discussed algorithm.*

The mean-square-error (MSE) of the equalizer output is

$$MSE = \sigma_s^2 (|1 - t_0|^2 + \sum_{n \ne 0} |t_n|^2) + \sigma_w^2 \sum_{n,k} |u_{n,k}|^2, \tag{4.22}$$

where

$$u_{n,k} = \begin{cases} \frac{1}{K+L-1} \sum_{i=\max\{1,n-K+1\}}^{\min\{n,K+L-1\}} f_{(n-i)M+k,i}^{(o)} & 1 \le n \le 2K+L-2 \text{ and } k = 1, \cdots, M, \\ 0 & otherwise. \end{cases} \tag{4.23}$$

Proof : Let $\mathcal{R}$ be either R defined as (3.10) or $\tilde{R}$ defined as (3.16), then SOS algorithm or MSOS algorithm is to find $\mathbf{f}$ minimizing $\mathbf{f}^H\mathcal{R}\mathbf{f}$ subject to $\sum_{i=1}^{K+L-1}\mathbf{f}_i^H R_x[0]\mathbf{f}_i = (K+L-1)\sigma_s^2$. Let

$$\Sigma = diag\{\sigma_1, \cdots, \sigma_{KM}\}, \tag{4.24}$$

$$\mathbf{d}_k = \Sigma U^H \mathbf{f}_k \quad \text{or} \quad \mathbf{d} = \mathcal{Q}\mathbf{f}. \tag{4.25}$$

Then, the SOS or MSOS algorithm can be expressed as to find $\mathbf{d}$ minimizing $\mathbf{d}^H {\mathcal{Q}^{-1}}^H \mathcal{R}\mathcal{Q}^{-1}\mathbf{d}$ subject to $\| \mathbf{d} \|^2 = (K+L-1)\sigma_s^2$.

Let $\mathbf{r}$ be the eigenvector of ${\mathcal{Q}^{-1}}^H\mathcal{R}\mathcal{Q}^{-1}$ corresponding to its smallest eigenvalues, then

$$\mathbf{d}^{(o)} = \frac{1}{\sigma_s(K+L-1)^{1/2}}\mathbf{r}, \tag{4.26}$$

and hence

$$\mathbf{f}^{(o)} = \mathcal{Q}^{-1}\mathbf{d}^{(o)}, \quad \text{or} \quad \mathbf{f}_k^{(o)} = U\Sigma^{-1}\mathbf{d}_k^{(o)} \tag{4.27}$$

is the solution of SOS algorithm or MSOS algorithm, depending upon the choice of $\mathcal{R}$. Therefore, if $\alpha = \frac{1}{K+L-1}$ in Figure 2, then the equalizer output will be

$$y[n] = \frac{1}{K+L-1}\sum_{i=1}^{K+L-1} {\mathbf{f}_i^{(o)}}^H \mathbf{x}_K[n+L-i]. \tag{4.28}$$

From (4.28), the impulse response of the equalized system is

$$t_n = \begin{cases} \frac{1}{K+L-1}\sum_{i=\max\{1,n+1\}}^{\min\{K+L-1+n, K+L-1} g_{i,i-n}\}, & -(K+L-2) \le n \le K+L-2, \\ 0 & \text{otherwise.} \end{cases} \tag{4.29}$$

Hence, the MSE of the equalizer output is

$$\begin{aligned} MSE &\triangleq E\{|y[n]-s[n]|^2\} \\ &= \sigma_s^2(|1-t_0|^2 + \sum_{n\neq 0}|t_n|^2) + \sigma_w^2\sum_{n,k}|u_{n,k}|^2, \end{aligned} \tag{4.30}$$

with $u_{n,k}$ defined as (4.23).

□

From the above theorem, the convergence of the system can be uniquely determined once the parameters of the channel is known, which is different from the higher-order statistics based algorithms whose convergence depends on both the channel parameters and equalizer initial setting.

With the formulas given by Theorem 2, we are now able to investigate the effects of the length-and-zero condition on the convergence of our algorithms. To study how the length-and-zero condition influences the equalizer performance, we consider a single-input/two-output channel with impulse responses given by

$$\{1,\ 0.8+\varepsilon\},\quad \text{and}\quad \{1,\ 0.8-\varepsilon\}. \tag{4.31}$$

When there is no channel noise, from Theorem 1, both SOS and MSOS algorithms will converge to the desired parameters if $\varepsilon \neq 0$. However, if the channel noise exists, the performance of the both algorithms will degenerate as ε tends to zero, as demonstrated in Figure 4 which is calculated from (4.23) for $SNR = 20dB$.

We can also calculate the MSE's of SOS and MSOS algorithms in Rayleigh fading channel using Theorem 2. Figure 5 illustrates the MSE's of SOS and MSOS equalizers applied in a single-input/two-output two-ray Rayleigh fading channels.

5 Computer Simulation Examples

Two Monte Carlo simulation examples have been conducted to demonstrate the performance of the new algorithms in digital communication systems.

5.1 The SOS and MSOS Equalizers in Time-Invariant Channel

This example will demonstrate that SOSA and MSOSA equalizers have better convergence performance than FS-CMA equalizer. The channel used in this example is a single-input/two-output time-invariant linear channel with impulse response responses

$\{0.6662 - \jmath 0.8427,\ 1.6323 - \jmath 0.2503,\ -0.6617 - \jmath 0.4102\}$ and $\{0.4607 + \jmath 0.5789,\ 0.5855 - \jmath 2.6912,\ 1.3273 - \jmath 0.4184\}$. The channel noise is additive complex white Gaussian noise with zero-mean and variance determined by SNR. In our simulation, the step-size μ and the forgetting factor λ are chosen to optimize the performance of each equalization algorithm.

If the channel input sequence $\{s[n]\}$ is independent and identically distributed uniformly over $\{\pm 1 \pm \jmath\}$, $\{\pm 3 \pm \jmath\}$, $\{\pm 1 \pm 3\jmath\}$ and $\{\pm 3 \pm 3\jmath\}$, Figure 6 (a), (b) and (c) show the eye patterns of the SOSA, the MSOSA and the FS-CMA equalizers respectively after 200 iterations, and Figure 6 (d) illustrates the convergence of the ISI of the three algorithms with respect to the number of iterations. From Figure 6, it only takes about 250 symbols for the SOSA equalizer or about 150 symbols for the MSOSA equalizer to converge. Both of them converge much faster than the FS-CMA equalizer.

If the CCITT trellis-coded modulated (TCM) scheme in [4, page 81] is employed in a communication system, then the channel input sequence $\{s[n]\}$ is not *i.i.d.*. In this environment, the CMA equalizer based on higher-order statistics will not converge to the channel inverse. However, since our second-order based equalizers do not require the i.i.d assumption, the SOSA equalizer and the MSOSA equalizer are still able to converge to the desired equalizer parameters as illustrated in Figure 7.

5.2 The SOS and MSOS Equalizers in Time-Variant Channel

In section 4, we have already analyzed the performance of the SOS equalizer and the MSOS equalizer in Rayleigh fading channel where we assume that the channel is *time-invariant* or slowly varying compared with the convergence speed of the equalizers. In this example, we are going to study the performance of the SOS and MSOS equalizers used in a single-input/two-output time-variant Rayleigh fading channel. Each subchannel is a two-ray time-variant multipath channel described as in Model 3 of [5]. Figure 8 demonstrates the probability of symbol error of the SOS equalizer and the MSOS equalizer used in 16-QAM digital communication system operated at 4800 bauds, corresponding to different *channel fading bandwidth*[5]. From Figure 8(a) and 8(b), the performance of the MSOS equalizer is better than that of the SOS equalizer for time-variant channels since the former one has faster convergence speed than the latter one.

6 Conclusion

In this paper, we have proposed two second-order statistics based algorithms for adaptive diversity combining and equalization. The proposed algorithms have faster convergence than higher-order statistics based algorithms. They can also be used in systems with non i.i.d. inputs, such as coded data in communication systems. The new algorithms can be used in wireless communication systems with antenna arrays to cancel the ISI and improve the system performance. We are currently studying how to use the proposed algorithms in IS-54 systems to the tracking of the fast fading mobile radio channels.

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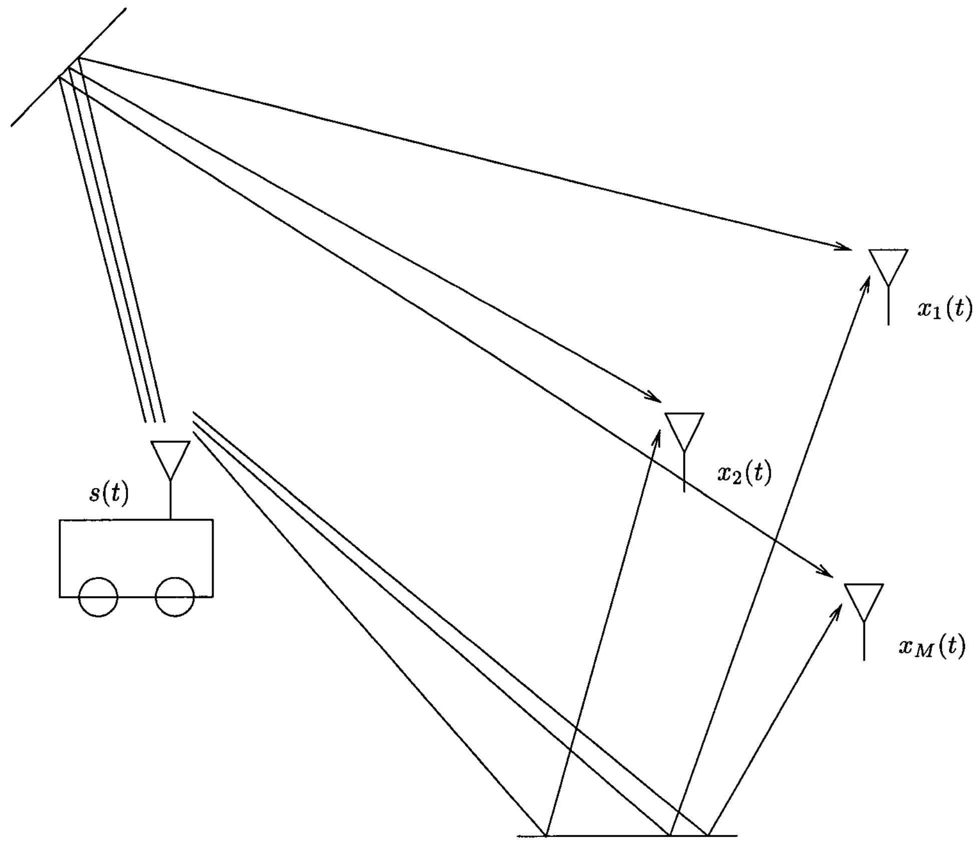

Figure 1: Wireless communication system with antenna arrays.

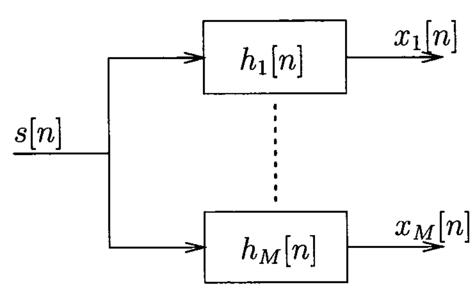

Figure 2: Single-input/multiple-output channel model.

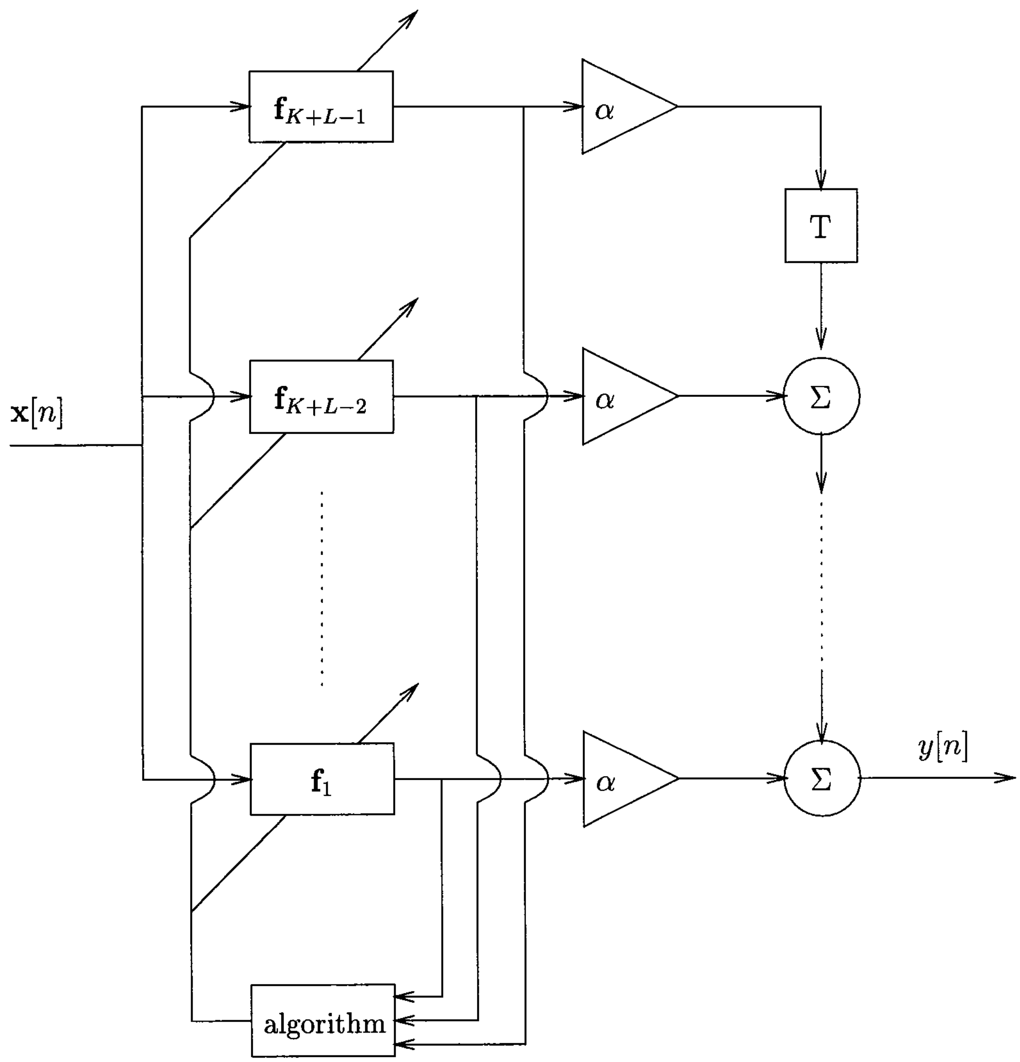

Figure 3: Blind adaptive equalizer with diversity combining.

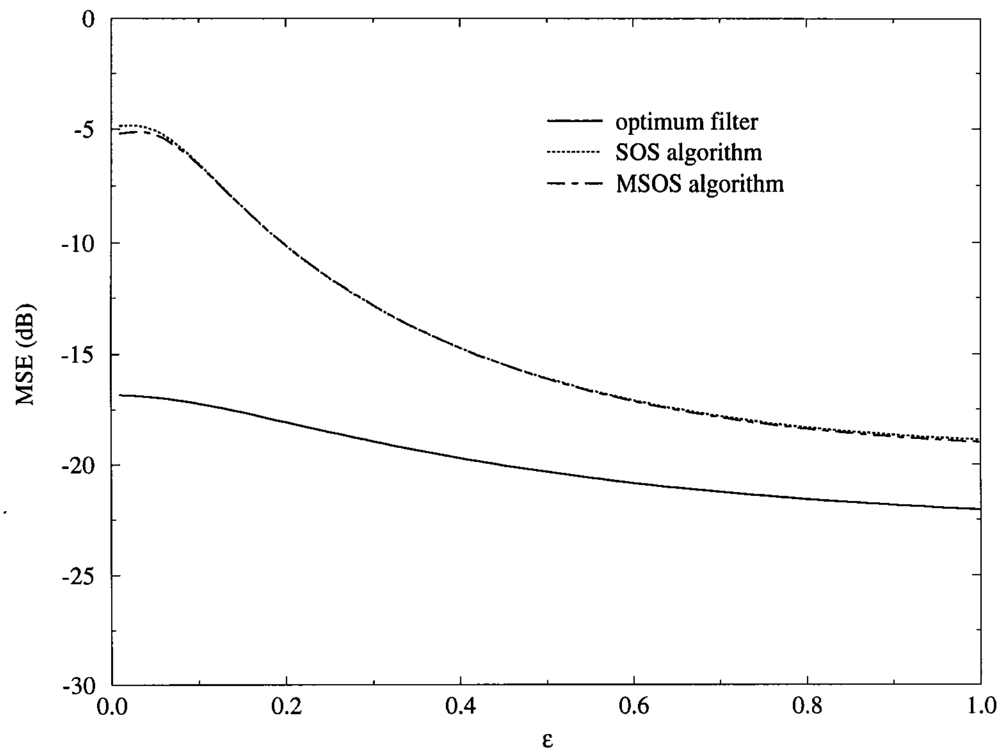

Figure 4: The MSE's of the SOS algorithm, the MSOS algorithm and optimum filter (Wiener filter) vis the diversity of the channels.

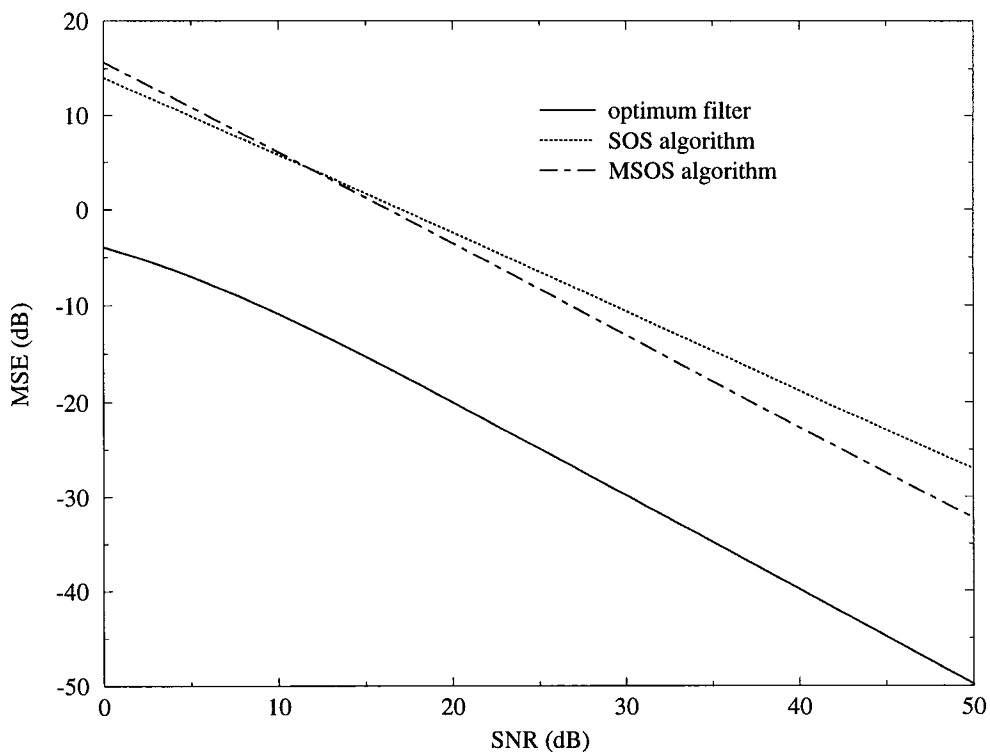

Figure 5: The MSE's of the SOS algorithm, the MSOS algorithm and optimum filter for two-ray Rayleigh fading channel.

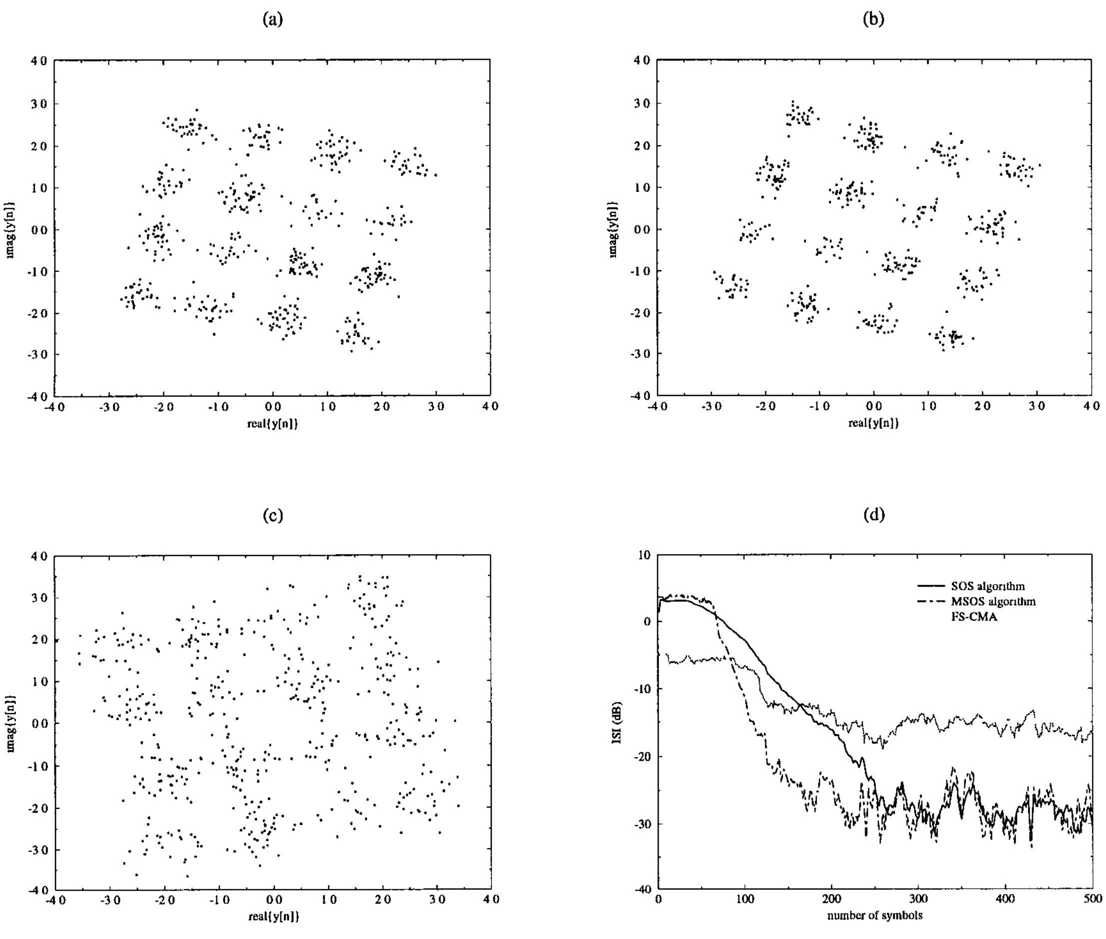

Figure 6: 500 symbol eye patterns of (a) the SOSA, (b) the MSOSA, and (c) the FS-CMA after 200 iterations, and (d) the convergence of the ISI for the three algorithms, when $SNR = 20dB$.

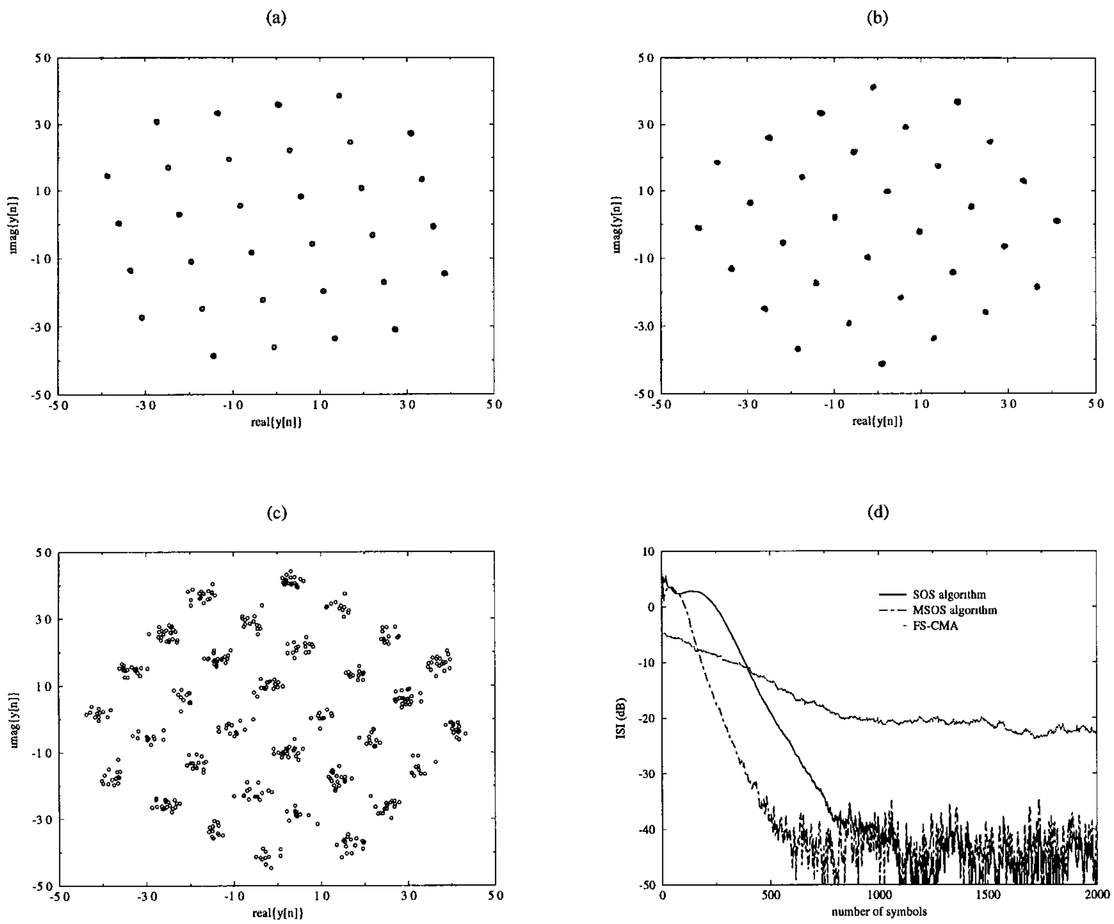

Figure 7: 500 symbol eye patterns of (a) the SOSA, (b) the MSOSA, and (c) the FS-CMA after 2000 iterations, and (d) the convergence of the ISI for the three algorithms, for a TCM signal, when $SNR = \infty$.

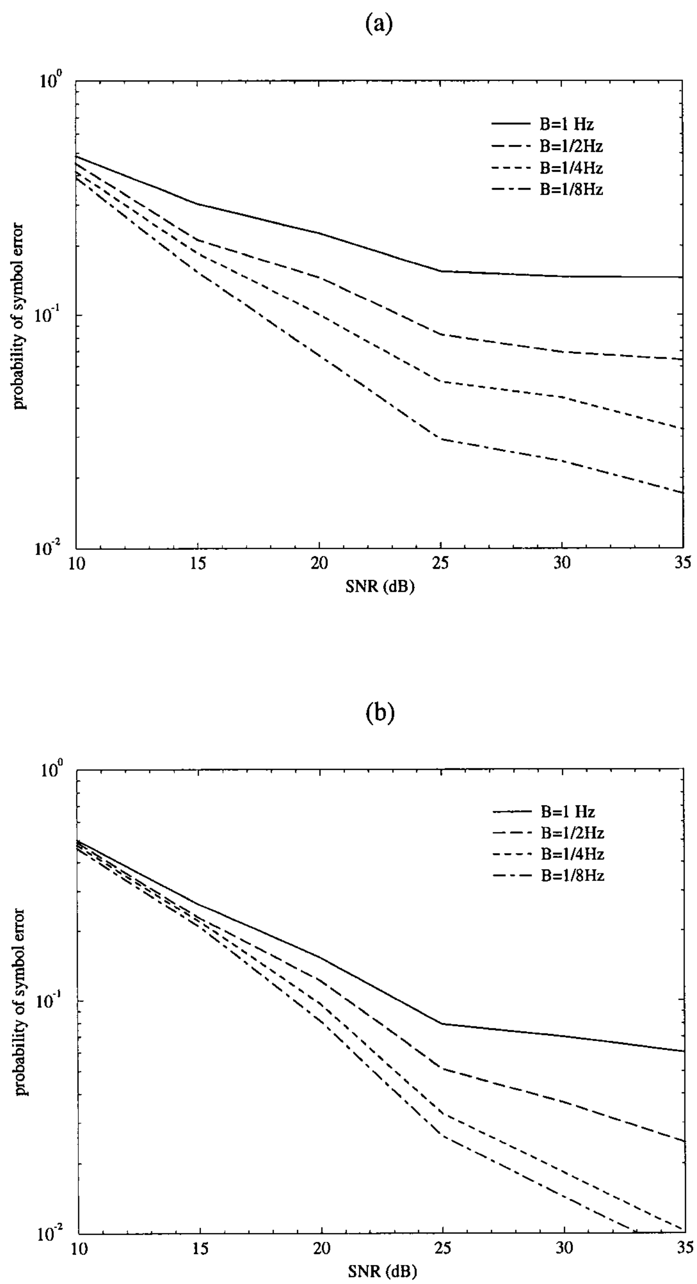

Figure 8: The probability of symbol error of (a) the SOS equalizer, and (b) the MSOS equalizer used in time-variant fading channels with different fading bandwidth B.