INSTITUTE FOR SYSTEMS RESEARCH

TECHNICAL RESEARCH REPORT

Frequency Domain Design of Robustly Stable Constrained Model Predictive Controllers

by H-W. Chiou and E. Zafiriou

T.R. 93-66

The Institute for Systems Research is supported by the National Science Foundation Engineering Research Center Program (NSFD CD 8803012), the University of Maryland, Harvard University, and Industry

Frequency Domain Design of Robustly Stable Constrained Model Predictive Controllers

Hung-Wen Chiou and Evanghelos Zafiriou*

Department of Chemical Engineering and Institute for Systems Research
University of Maryland, College Park, MD 20742

Abstract

The robust stability analysis of Constrained Model Predictive Control (CMPC) for linear time invariant and open-loop stable processes is the main topic of this paper. Based on the CMPC algorithm, the feedback controller is a piece-wise linear operator because of the constraints. This piece-wise linear operator can be thought of as an array of linear feedback controllers in parallel, handling different types of predicted active constraint situations. Each term in the linear operator corresponding to the predicted active constraint situation can be decomposed to have an uncertainty block. Hence, the linear operator can be written as a linear closed-form with uncertainty blocks inside. According to the linear robust stability analysis method, the robust stability of CMPC can be analyzed and the computer aided off-line tuning for the stability of CMPC can also be developed by solving a minimum maximum problem based on the stability analysis method. Some examples are given to show the feasibility of the analysis and tuning methods.

1. Introduction

A framework based on the contraction mapping theory for the robust stability analysis of Constrained Model Predictive Control (CMPC) was developed by Zafiriou (1990). This framework allows for stability analysis in the frequency domain. A framework with an infinite horizon formulation is discussed by Muske and Rawlings (1991,1993). Genceli and Nikolaou (1992) use a formulation with and end constraint in the on-line optimization.

Zafiriou (1990) showed that the CMPC algorithm was shown to be piece-wise linear with each region corresponding to set of active constraints. This "inherent" set of linear feedback controllers was completely characterized and both necessary and sufficient conditions for the closed-loop operator to be a contraction were developed. The necessary condition has been shown to be an excellent indicator of stability for CMPC. In addition, some theoretical results and simulations on the stability of MPC with hard output constraints for SISO processes have been given in Zafiriou and Marchal (1991). The case of soft constraints as well as a mix of soft and hard constraints, has been discussed by Zafiriou (1991) and Zafiriou and Chiou (1992,1993). de Oliveira and Biegler (1992) have suggested an alternative way of constraint softening.

*Author to whom correspondence should be addressed. E-mail: zafiriou@src.umd.edu

To study the robust stability problem of this nonlinear control system, the state space model is used in this paper. This nonlinear control can be thought as a piece-wise linear controller. It contains a sequence of linear operators handling different active constraint situations over the whole control period. On analyzing its stability, a closed-form linear control law with uncertainty blocks corresponding to different types of active constraint situations is constructed. Then, by applying standard linear robust stability techniques in the frequency domain [e.g. Morari and Zafiriou, 1989] we can use the necessary contraction mapping condition to study the robust stability of the CMPC system. Based on the robust stability condition, an off-line computer aided optimization based technique for tuning of CMPC is developed. It is required to solve a min-max problem to obtain a set of CMPC tuning parameters that stabilize the CMPC system.

2. Stability Framework

The preliminaries give the basics of a framework ,based on the contraction mapping, for the stability analysis of Constrained Model Predictive Control (CMPC). For more details and discussion the reader is referred to Zafiriou (1990).

The QDMC-type algorithms [Garcia and Morshedi, 1986; Garcia and Morari, 1985b] use a quadratic objective function that includes the square of the weighted norm of the predicted error (setpoint minus predicted output) over a finite horizon in the future (sample points $k+1,\ldots,k+P$, where k is the current sample point) as well as penalty terms on u or Δu:

$$\min_{\Delta u(k),\cdots,\Delta u(k+M-1)} \sum_{l=1}^{P} [e(k+l)^T \Gamma^2 e(k+l) + u(k+l-1)^T B^2 u(k+l-1) + \Delta u(k+l-1)^T D^2 \Delta u(k+l-1)] \quad (1)$$

The minimization of the objective function is carried out over the values of $\Delta u(k)$, $\Delta u(k+1),\ldots,$ $\Delta u(k+M-1)$, where M is a specified parameter. The minimization is subject to possible hard constraints on the inputs u, their rate of change Δu, the outputs y and other process variables usually referred to as associated variables. The details on the formulation of the optimization problem can be found in Prett and Garcia (1988). We can soften the hard constraints on the predicted outputs by allowing violation by an amount ϵ. In the formulation in this paper, the same violation variable $\epsilon \geq 0$ is used for all the points in the constraint window. Hence the output constraints are softened to be:

$$y_L - \epsilon \leq y(k+l) \leq y_U + \epsilon, \quad w_b \leq l \leq w_e \qquad (2)$$

where y_L, y_U are the lower and upper limits respectively; w_b, w_e are the beginning and ending points of output constraint window. The term $\epsilon^T W^2 \epsilon$ is added to the objective function, where W is the weight that determines the extent of softening. For $W = \infty$ we get hard constraints. $W = 0$ corresponds to completely removing the constraints. This formulation covers any mix of hard and soft constraints.

After the problem is solved on-line at k, only the optimal value for the first input vector $\Delta u(k)$ is implemented and the problem is solved again at $k+1$. The optimization problem of the QDMC algorithm can be written as a standard Quadratic Programming problem:

$$\min_v q(v) = \frac{1}{2} v^T G v + g^T v \qquad (3)$$

subject to

$$A^T v \geq b \qquad (4)$$

where $v = [\ \Delta u(k)\ \cdots\ \Delta u(k+M-1)\ ]^T$ and the matrices $G\ (>\ 0\ )$, A, and vectors g, b are functions of the tuning parameters (weights, horizon P, M, some of the hard constraints). The vectors g, b are also linear functions of $y(k)$, $u(k-1), \cdots, u(k-N)$. When some of the constraints have been softened, v is augmented to include all the corresponding ϵs. Inequality (4) includes both the soft and hard constraints. For the optimal solution v^* with respect to a certain active constraint situation, we have [Fletcher, 1981]:

$$\begin{bmatrix} G & -\hat{A} \\ -\hat{A}^T & 0 \end{bmatrix} \begin{bmatrix} v^* \\ \lambda^* \end{bmatrix} = - \begin{bmatrix} g \\ \hat{b} \end{bmatrix} \qquad (5)$$

where $\hat{A}^T$, $\hat{b}$ consist of the rows of A^T, b that correspond to the constraints that are active at the optimum and λ^* is the vector of the Lagrange multipliers corresponding to these constraints. The optimal $\Delta u(k)$ corresponds to the first m elements of the v^* that solves (5), where m is the dimension of u.

The special form of the LHS matrix in (5) allows the numerically efficient computation of its inverse in a partitioned form [Fletcher, 1981]:

$$\begin{bmatrix} G & -\hat{A} \\ -\hat{A}^T & 0 \end{bmatrix}^{-1} = \begin{bmatrix} H & -T \\ -T^T & U_L \end{bmatrix} \qquad (6)$$

Then

$$\begin{aligned} v^* &= -Hg + T\hat{b} \\ \lambda^* &= T^T g - U_L \hat{b} \end{aligned} \qquad (7)$$

and

$$\begin{aligned} u(k) &= [\, I\, 0 \cdots 0\,](-Hg + T\hat{b}) \qquad (8) \\ &\stackrel{\text{def}}{=} f_{Ji}(y(k), u(k-1), \cdots, u(k-N); r_P(k), d(k)) \end{aligned}$$

where $r_P(k)$ includes all the values of reference signal (setpoint) during the prediction horizon from $k+1$ to $k+P$ and $d(k)$ is the disturbance effect at the output at k. J_i is an index set that defines the active constraints. The function f_{J_i} is linear in y and u. The corresponding linear feedback controller, describing the behavior of the system at this operating region can be computed from expression involving the derivatives of f_{J_i} w.r.t the y and u variables [Zafiriou, 1990,1991].

This framework can be used with any type of linear models. In this paper we use state space descriptions. Consider the discrete state space model with disturbance directly added to the output for a process given by

$$\begin{aligned} x(k+1) &= \phi x(k) + \theta u(k) \\ y(k) &= c x(k) + d(k) \end{aligned} \qquad (9)$$

where $x(k)$, $x(k+1)$ are the state vectors of the model; $u(k)$ and $y(k)$ are the input and output vectors of the model respectively; $d(k)$ is the disturbance; ϕ, θ, c are the coefficient matrices of the model. Use the state space model (9) to predict the plant outputs over the prediction horizon (P) and assume that the predictive plant output is equal to the summation of model output and disturbance $d(k)$ and that $d(k)$ is constant over the whole prediction horizon ($\Delta d(k+i) = d(k+i) - d(k+i-1) = 0$, $i = 1, \cdots, P$).

3. Robust Stability Analysis Method

For each set of active constraints, the corresponding feedback control law can be written in a form that contains a block that depends on the constraints. The necessary condition for contraction that is used as an indicator of stability, requires that all such controllers stabilize the plant. One way to address this problem is by treating the block that depends on J_i, as "uncertainty" whose value varies with the constraint set. A bound can then be calculated and robustness for all "uncertainty" be required. This uncertainty is real-valued and one-sided for which the results of Lee and Tits (1992) can be used. True plant "uncertainty" can then be added in a straight forward manner.

We consider here the case where a mix of u, Δu, and y constraints are used. The case where no y constraints exist results in simpler conditions but these special cases are omitted from this paper for lack of space. For the general case, let $\hat{A}^T = \varpi^T \bar{s}$ where $\bar{s}$ is a full rank submatrix of A^T, and ϖ is an extraction matrix to extract the rows of $\hat{A}^T$ from $\bar{s}$. The matrix $\bar{s}$ has to be such that the following condition holds: (a) $(\bar{s}\bar{s}^T)^{-1}\bar{s}G\bar{s}^T(\bar{s}\bar{s}^T)^{-1} \geq \varpi(\hat{A}^T G^{-1}\hat{A})^{-1}\varpi^T$. The selection of $\bar{s}$ is not always simple, but in many cases, as in the two examples in the paper, it is obvious. The following decompositions can be obtained [Horn and Johnson, 1990]: (b) $(\bar{s}\bar{s}^T)^{-1}\bar{s}G\bar{s}^T(\bar{s}\bar{s}^T)^{-1} = E^{-1}U^T U E^{-T}$, where E is a nonsingular matrix and U is an unitary matrix variably dependent of the active constraint situation. (c) $\varpi(\hat{A}^T G^{-1}\hat{A})^{-1}\varpi^T = E^{-1}U^T \Delta_c U E^{-T}$, where Δ_c is a diagonal matrix with the value of each diagonal entry element between 0 an 1. The following lemma allows the simplification of the control law for stability purposes.

Lemma 1 *If the active constraint set includes some inputs reaching their active constraints of $\Delta u(k)$, then the term $u(k-1)$ arising from the bound ($\hat{b}$) does not affect the equivalent linear control law for stability (contraction mapping) analysis.*

Proof: see appendix A.

From equations (6) – (9), (b), (c), lemma 1, we can obtain after matrix operations and taking the z-transform:

$$u(z) = (I + \bar{\varphi}_3 z^{-1})^{-1}(\bar{\varphi}_1 x(z) + \bar{\varphi}_2 d(z)) \qquad (10)$$

where

$$
\begin{aligned}
\bar{\varphi}_1 &= -\tau G^{-1}S^T\Gamma^T\Gamma\eta + \\
&\quad \tau G^{-1}\bar{s}^T E^{-1}U^T\Delta_c U E^{-T}(\bar{s}G^{-1}S^T\Gamma^T\Gamma\eta + [0\ \bar{\eta}]^T) \\
\bar{\varphi}_2 &= -\tau G^{-1}S^T\Gamma^T\Gamma\alpha + \\
&\quad \tau G^{-1}\bar{s}^T E^{-1}U^T\Delta_c U E^{-T}(\bar{s}G^{-1}S^T\Gamma^T\Gamma\eta + [0\ \bar{\alpha}]^T) \\
\bar{\varphi}_3 &= -\tau G^{-1}[d^2\ 0\cdots 0]^T + \\
&\quad \tau G^{-1}\bar{s}^T E^{-1}U^T\Delta_c U E^{-T}\bar{s}G^{-1}[d^2\ 0\cdots 0]^T \\
\tau &= [\,I\,0\ \cdots\ 0\,]^T \qquad (11)
\end{aligned}
$$

where I is an identity matrix with dimension corresponding to the dimension of the input vector u; d is the penalty of $\Delta u(k)$.

$$\eta^T = [\,c\phi\ \cdots\ c\phi^P\,]^T,\ \alpha = [\,I_1\ \cdots\ I_P\,]^T \qquad (12)$$

where I_i ($i = 1\cdots P$) are identity matrices with dimension corresponding to the dimension of the output vector. $\bar{\eta}$, $\bar{\alpha}$ contain the elements of η, α, corresponding to the output constraint window, respectively. The control block diagram with repeated uncertainty blocks is shown in figure 1, and it can be rearranged as the control block diagram in figure 2, where M_u contains the fixed part (independent of $\bar{\Delta}_c$) of the control block diagram of figure 1;

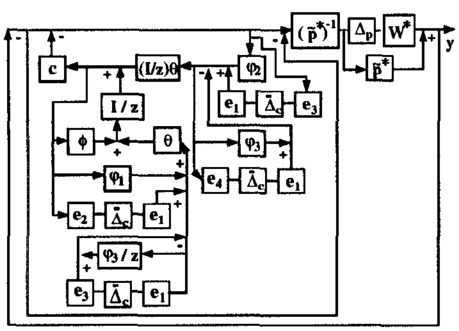

Figure 1: Control block diagram for CMPC robust stability analysis (W^* is a weighting matrix determined by the plant uncertainty type and bounds and $\bar{p}^*$ is a discrete model.)

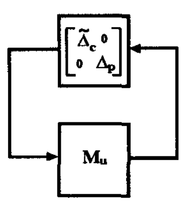

Figure 2: Control block diagram for robust stability analysis

$$
\begin{aligned}
\varphi_1 &= -\tau G^{-1}S^T\Gamma^T\Gamma\eta,\ \varphi_2 = -\tau G^{-1}S^T\Gamma^T\Gamma\alpha \\
\varphi_3 &= -\tau G^{-1}[d^2\ 0\cdots 0]^T \\
e_1 &= \tau G^{-1}\bar{s}^T E^{-1},\ e_2 = E^{-T}(\bar{s}G^{-1}S^T\Gamma^T\Gamma\eta + [0\ \bar{\eta}]^T) \\
e_3 &= E^{-T}(\bar{s}G^{-1}S^T\Gamma^T\Gamma\alpha + [0\ \bar{\alpha}]^T) \\
e_4 &= E^{-T}\bar{s}G^{-1}[d^2\ 0\cdots 0]^T,\ \bar{\Delta}_c = U^T\Delta_c U
\end{aligned}
$$

Then, from robust control theory, we can obtain the following robust stability condition:

$$\bar{\sigma}(\begin{bmatrix} \tilde{\Delta}_c & 0 \\ 0 & \Delta_p \end{bmatrix})\mu(M_u(\omega)) < 1\ ,\ \ 0 \le \omega \le \frac{\pi}{T} \qquad (13)$$

where Δ_p is the uncertainty from the plant, which may be a complex matrix. $\tilde{\Delta}_c = diag(\bar{\Delta}_c,\ \bar{\Delta}_c, \bar{\Delta}_c, \bar{\Delta}_c)$. μ denotes the structured singular value. This condition guarantees satisfaction of the necessary contraction condition, that is used as an indicator of stability, for all possible plants, described by the Δ_p uncertainty, assuming that the unconstrained control law has been designed to be stable for the nominal plant.

4. A Computer-Aided Off-Line Tuning Method for Robust Stability

The results of the previous section can be used for tuning the parameters of the CMPC algorithm, including the softening weights W, for robust stability. Experience and general known trends can help make this task easier. However, to a large extent, it would remain a trial-and-error procedures. To address this issue, we have experimented with the use of a sophisticated optimization package for designing the CMPC algorithm by tuning its parameters through off-line optimization.

The design problem can be defined as a minimum-maximum (Min Max) optimization problem. The objective is to choose a set of CMPC tuning parameters stabilizing (or minimizing) the maximum structured singular value. The objective function can be given as:

$$\min_{P,\,M,\,B,\,D,\,\Gamma,\,CW,\,W}\ \ \max_{\omega}\ \mu \qquad (14)$$

where CW is a vector containing the constraint window of the predicted outputs. This constraint can in some cases be also considered as a tuning parameter. For cases characterized by the following lemma, the corresponding controller is independent of the tuning parameters and it may destabilize the control system. By readjusting the constraint window, the controller would become tunable.

Lemma 2 *If the number of the inputs is equal to the number of the outputs, and the first impulse response coefficient matrix (after the time delay) is invertible, and the constraints of the first predicted outputs after the time delay are active in the active constraint set, then the control law is tuning parameter independent.*

The approach followed in the proof is similar to that in lemma 1.

5. Illustrations

Example 1. A 2 × 2 process model from the Shell Standard Control Problem is [Prett and Garcia, 1988] used:

$$\tilde{p}(s) = \begin{bmatrix} \frac{4.05e^{-27s}}{50s+1} & \frac{1.77e^{-28s}}{60s+1} \\ \frac{5.39e^{-18s}}{50s+1} & \frac{5.72e^{-14s}}{60s+1} \end{bmatrix}$$

The sampling time is 6 minutes. The plant can be described as:

$$p(s) = \begin{bmatrix} \frac{(4.05+2.11\Delta_1)e^{-27s}}{50s+1} & \frac{(1.77+0.39\Delta_2)e^{-28s}}{60s+1} \\ \frac{(5.39+3.29\Delta_1)e^{-18s}}{50s+1} & \frac{(5.72+0.57\Delta_2)e^{-14s}}{60s+1} \end{bmatrix}$$

where $-1 \le \Delta_i \le 1$ ($i = 1, 2$). Initially, select the tuning parameters as $P = 7$, $M = 1$, $\Gamma = I$, $B = 0$, $D = 0$, $CW1 = 5$, $CW2 = 3$, where the $k + CW1$, $k + CW2$ are the points in the future (k being the current point) for which constraints on outputs 1 and 2 respectively, are placed in the on-line optimization. Here for simplicity we assume only one point in each constraint window. The hard constraints are set on the first point (after delay time) of both predictive outputs. From the off-line analysis or the simulation results shown in figures 3 and 5, we know that the control system is nominally unstable and tuning parameter independent according to lemma 2. Hence, we let the constraint window be tunable during the computer aided tuning procedure. After solving the MinMax problem (14) by CONSOLE (please see reference [10]) over P, M, CW and keeping B, D, Γ constant: during the tuning procedure, we obtain a new set of tuning parameters and constraint points: $P = 7$, $M = 2$, $CW1 = 6$, $CW2 = 4$. The closed-loop structured singular value for this new set of tuning parameters and constraint windows is shown in figure 4. We see that this new set of tuning parameters and constraint windows can stabilize the system. Comparing the on-line simulation results (figures 5, 6), we also can see that the control system is stabilized under the same disturbances and constraint bounds. In the simulations, the lower and upper bounds of the predictive outputs are -0.3 and 0.3, the plant is chosen as $\Delta_i = 1$, $i = 1, 2$, and the disturbance is $[\,3/s,\ 0.1/s\,]^T$.

Example 2. We consider the design for robust stability for the same model, plant, and sampling time used as the previous example we use, but this time we allow constraint softening. We start by selecting a set of tuning parameters as:$P = 7$, $M = 1$, $W = 90$, $B = 0$, $D = 0$, $\Gamma = I$. We use the constraints on $\Delta u_1(k)$, $u_1(k)$, $\hat{y}_1(k+5)$. The structured singular value and simulation are shown in figures 7 and 9. During the off-line optimization based tuning, we fix the M, B, D, Γ and constraint point. After several steps of computations in CONSOLE, a new set of parameters for satisfying the robust stability condition ($\mu < 1$) can be found. The new P and W are 73 and 10.0835 respectively. The structured singular value for this new set of tuning parameters is shown in figure 8. Also, the simulation is given in figure 10. The constraints for the on-line simulation are chosen as: $-1 \le u_1(k) \le 1$, $-0.3 \le \Delta u_1(k) \le 0.3$, $-0.1 \le \hat{y}_1(k+5) \le 0.1$, the disturbance is $[\,2/s,\ 1/s\,]^T$ and the plant is chosen as $\Delta_i = 1$, $i = 1, 2$. By using the new set of tuning parameters, we can see that the control system is stable with the softened constraint of the predicted output at $k+5$ without having to slide the constraint point forward as in example 1.

6. Concluding Remarks

Model predictive control with hard constraints is a nonlinear control system even if plant and model are assumed to be linear and time invariant. To analyze the robust stability of this type of control system, the contraction mapping theory can be used. Based on a necessary contraction mapping condition, a stability analysis framework can be set up. This nonlinear control system can be thought of as a sequence of linear feedback controllers handling different active constraint combination situations. A closed-form control law can be constructed that contains "uncertainty" blocks corresponding to different active constraint combination sets. The uncertainty description of the plant can also be added and the structured singular value which servers as a robust stability indicator can then be used to compute this necessary contraction condition. An off-line computer aided tuning method is also discussed in this paper, and the design method requires solving a minimum-maximum problem. Two examples are given to illustrate the effectiveness of the method.

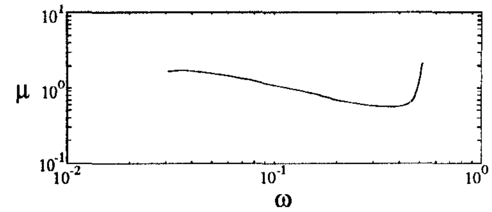

Figure 3: The structured singular value for the initial tuning parameters of example 1

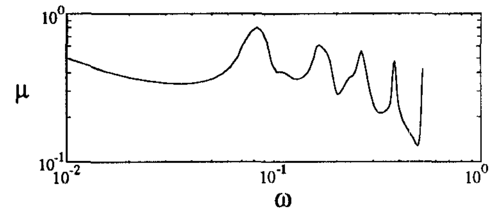

Figure 4: The structured singular value for the final tuning parameters of example 1

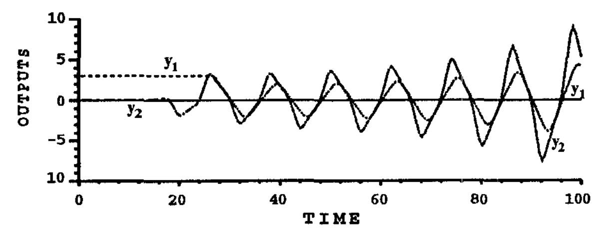

Figure 5: Simulation for example 1 with initial tuning parameters

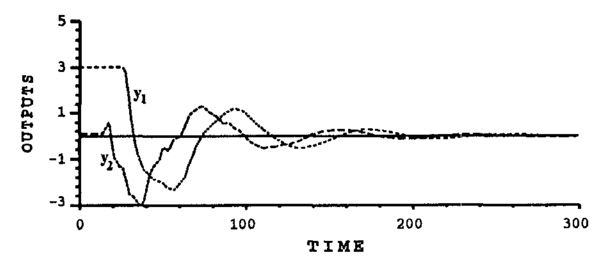

Figure 6: Simulation for example 1 with final tuning parameters

Acknowledgments. The authors are grateful to Dr. Lee Li and Prof. André Tits for making the software for computing the structured singular value available. Support for this project was provided by the National Science Foundation (Presidential Young Investigator grant CTS - 9057292), the Institute for Systems Research and a grant form Shell.

Appendix A

For a system with m inputs system, assume that ℓ constraints of Δu are active with $\ell \le m$. Without loss of generality assume these are the first ℓ elements of u. Then, the corresponding control law can be written as:

$$\begin{aligned} u(k) \;=\; & -\tau[G^{-1} - G^{-1}\hat{A}(\hat{A}^T G^{-1}\hat{A})^{-1}\hat{A}^T G^{-1}]S^T\Gamma^T\Gamma\eta x(k) - \\ & \tau[G^{-1} - G^{-1}\hat{A}(\hat{A}^T G^{-1}\hat{A})^{-1}\hat{A}^T G^{-1}]S^T\Gamma^T\Gamma\alpha d(k) + \\ & \tau G^{-1}\hat{A}(\hat{A}^T G^{-1}\hat{A})^{-1}\hat{b} + \tau[G^{-1} - G^{-1}\hat{A}(\hat{A}^T G^{-1}\hat{A})^{-1} \\ & \hat{A}^T G^{-1}][\, d^2 u(k-1)\ 0\ \cdots\ 0\,]^T \end{aligned}$$

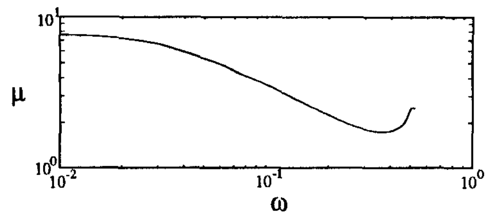

Figure 7: The structured singular value for the initial tuning parameters of example 2

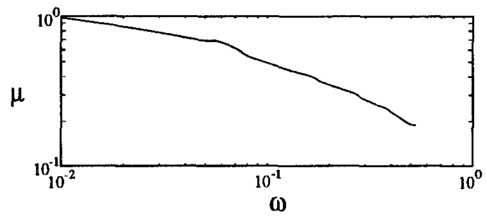

Figure 8: The structured singular value for the final tuning parameters of example 2

where τ, η, α are defined in (11),(12). To examine the contribution of $\hat{b}$ to the state feedback law, we can write $\hat{b}$ as:

$$\hat{b} = [\, u_1(k-1) \; \cdots \; u_\ell(k-1)\; 0 \; \cdots \; 0 \;\; \hat{\eta}x(k) + \hat{\alpha}d(k)\,],$$

where $\hat{\eta}, \hat{\alpha}$ are the matrices consisting of the rows from the η, α corresponding to the active constraint situation. Hence

$$\tau G^{-1}\hat{A}(\hat{A}^T G^{-1}\hat{A})^{-1}U(k-1) = \begin{bmatrix} I_\ell & 0 \\ \bar{x}_5 & 0 \end{bmatrix} U(k-1)$$

where $U(k-1) = [\, u_1(k-1) \; \cdots \; u_\ell(k-1)\; 0 \; \cdots \; 0\,]^T$. By matrix operations and z-transform, the control law can be rewritten as:

$$\begin{aligned} u(z) &= -\begin{bmatrix} I_\ell - z^{-1}I_\ell & 0 \\ -(\bar{x}_5 + x_6)z^{-1} & I - x_7 z^{-1} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} 0 & 0 \\ \bar{x}_1 & \bar{x}_2 \end{bmatrix} x(z) + \begin{bmatrix} 0 & 0 \\ \bar{x}_3 & \bar{x}_4 \end{bmatrix} d(z) \right\} \\ &= -\begin{bmatrix} 0 & 0 \\ (I - x_7 z^{-1})^{-1}\bar{x}_1 & (I - x_7 z^{-1})^{-1}\bar{x}_2 \end{bmatrix} x(z) - \begin{bmatrix} 0 & 0 \\ (I - x_7 z^{-1})^{-1}\bar{x}_3 & (I - x_7 z^{-1})^{-1}\bar{x}_4 \end{bmatrix} d(z) \end{aligned}$$

where x_i, $\bar{x}_i$ correspond to terms which may not be zero in the matrix operations. From the above equation, since I_ℓ and $\bar{x}_5$ have been eliminated from the final expression, the statement of lemma follows □

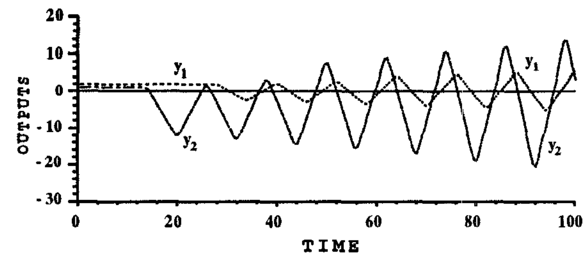

Figure 9: Simulation for example 2 with initial tuning parameters

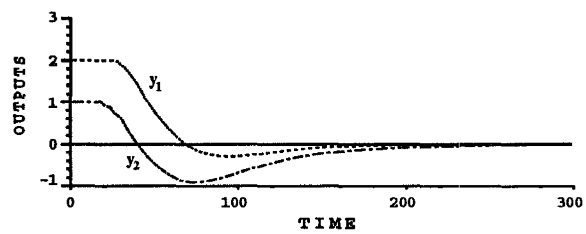

Figure 10: Simulation for example 2 with final tuning parameters

References

[1] E. Zafiriou, "Robust predictive control of processes with hard constraints," *Comp. Chem. Eng.*, vol. 14, pp. 359–371, 1990.

[2] K. R. Muske and J. B. Rawlings, "Linear model predictive control of unstable processes," *Sumitted for publication in the Journal of Process Control*, 1993.

[3] J. B. Rawlings and K. R. Muske, "The stability of constrained receding horizon control," *Sumitted to IEEE Transactions on Automatic Control*, 1991.

[4] H. Genceli and M. Nikolaou, "Robust stability analysis of constrained model predictive control," in *Ann. AIChE mtg. Paper 123d*, (Miami Beach, FL), Nov. 1992.

[5] E. Zafiriou and A. L. Marchal, "'stability of siso quadratic dynamic matrix control with hard output constraints," *AIChE*, vol. 37, no. 10, pp. 1550–1560, 1991.

[6] E. Zafiriou, "On the closed-loop stability of constrained qdmc," in *American Control Conference*, (Boston, MA), pp. 2367–2372, June 1991.

[7] E. Zafiriou and H.-W. Chiou, "Output constraint softening for siso model predictive control," in *American Control Conference*, (San Francisco, CA), pp. 372–376, 1993.

[8] E. Zafiriou and H.-W. Chiou, "On the effect of constraint softening on the stability and performance of model predictive controllers," in *Ann. AIChE mtg. Paper 123f*, (Miami Beach, FL), Nov. 1992.

[9] N. M. de Oliveira and L. T. Biegler, "Algorithms for constrained nonlinear process control," in *Ann. AIChE mtg. Paper 123g*, (Miami Beach, FL), Nov. 1992.

[10] M. Morari and E. Zafiriou, *Robust Process Control*. Prentice-Hall, Inc., 1 ed., 1989.

[11] C. E. Garcia and A. M. Morshedi, "Quadratic programming solution of dynamic matrix control (qdmc)," *Chem. Eng. Commun.*, vol. 46, pp. 73–87, 1986.

[12] C. E. Garcia and M. Morari, "Internal model control. 3. multivariable control law computation and tuning guidelines," *Ind. Eng. Chem. Proc. Des. Dev.*, vol. 24, pp. 484–494, 1985b.

[13] R. Fletcher, *Practical Methods of Optimization: Constrained Optimization*, vol. 2. New York: J. Wiley, 1980.

[14] L. Lee and A. L. Tits, "Mixed one- and two-sided real uncertainty. a vertex result," in *American Control Conference*, (Chicago, IL), 1992.

[15] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1990.

[16] D. M. Prett and C. E. Garcia, *Fundamental Process Control*. Butterworth Publishers, Stoneham, MA, 1988.

[17] M. K. H. Fan, A. L. Tits, and *et al.*, "Console user's manual," in *Systems Research Center Technical Report, TR 90-60r1*, 1990.