ABSTRACT

Title of dissertation: UNMAPPED HOLDINGS
AND THE PERFORMANCE MEASUREMENT
OF U.S. EQUITY MUTUAL FUNDS

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This paper investigates a dataset that provides information about assets held by U.S. equity mutual funds, but are not U.S. equities ('unmapped holdings'). I show the widespread presence of these assets and investigate how they are used within mutual fund portfolios. I find that their effects are statistically significant upon both portfolio risk and return. They can either hedge or complement mapped asset returns. I show that predictability of mutual fund returns are reduced when unmapped holdings returns are controlled. Since unmapped holdings returns are not observable, I define an econometric technique that in chapter two that can control for their effect. This technique uses an average return (an 'endogenous benchmark') to control for common but immeasurable or unobservable characteristics in a group of funds. I find that an 'endogenous benchmark' alone produces estimates nearly as good as those using common risk factor regression models. By combining an endogenous bechmark with other risk factors in regression models, I find that estimates are improved.

UNMAPPED HOLDINGS AND THE PERFORMANCE MEASUREMENT OF U.S. EQUITY MUTUAL FUNDS

by

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Dissertation submitted to the Faculty of the Graduate School of the
University of Maryland, College Park in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2009

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Acknowledgments

I owe my gratitude to all the people who have made this thesis possible and because of whom my graduate experience has been one that I will cherish forever. First and foremost I'd like to thank my advisor, Professor Russell Wermers for giving me an invaluable opportunity to work on challenging and extremely interesting projects over the past few years. He has always made himself available for help and advice and there has never been an occasion when I've knocked on his door and he hasn't shared with me his time. It has been a pleasure to work with and learn from such an extraordinary individual.

I would also like to thank Professor Pete Kyle. His insight and guidance helped me refine my research techniques and taught me how to thoroughly investigate an idea. He always made opportunities to meet with me a priority, and offered insightful direction and advice. Thanks are due to Professor Lemma Senbet, Professor Steve Heston and Professor John Chao for agreeing to serve on my thesis committee and for sparing their invaluable time reviewing the manuscript. Thanks are also due to Soeren Hvidkjaer, Mark Loewenstein, Greg Willard, Gerard Hoeberg, and Anna Obizhaeva for their helpful guidance and advice during my studies.

I owe my deepest thanks to my family: my wife and children have always stood by me and supported my efforts throughout my career. Words cannot express the gratitude I owe them. I also thank my father, John Hunter, whose love for finance and research formed my inspiration to pursue similar dreams.

I would like to acknowledge financial support from the Graduate School of

Business. Their committment to the PhD program and its students has created a standard of excellence in learning and research. I discovered a faculty, department, program, and facility that exceeded my greatest expectations.

This paper represents the culmination of many years of effort. During this time I have had the opportunity to interact with many great individuals, each of whom deepened my knowledge and understanding in some way, great or small. Thus I believe that it is impossible to mention all that may have influenced this work, and I sincerely apologize to those that I might have inadvertently omitted.

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Chapter 1

Mind the Gap: Unmapped Holdings and the Performance of U.S. Equity Mutual Funds

1.1 Introduction

A majority of U.S. equity mutual funds do not restrict their investments exclusively to U.S. equities. In fact, in the period from July 2003 to September 2006, 33% of domestic equity mutual funds held over 10% of their assets in assets that were not U.S. equities, and over 20% held at least 10% throughout the entire period. I refer to these assets as 'unmapped holdings' and include assets such as cash, international equities, bonds, convertibles and preferreds, and even derivatives. Despite this widespread practice, researchers have typically performed their analysis of these funds as if all funds were fully invested in U.S. equities.

In this paper, I group funds with common unmapped holdings properties and show that unmapped holdings have a statistically significant effect upon a fund's return and volatility. Using these fund groups, I investigate the value of active management while controlling for the unmapped holdings effects.

I focus this study upon U.S. domestic equity mutual funds, and I group them by the type of unmapped assets they hold. Between 2003 and 2007, CRSP temporarily distributed CRSP temporarily distributed mutual fund holdings data that included non-equity holdings, including derivatives, money markets, bonds, and foreign assets. This information has not been available to past studies of mutual funds. By using this data, I demonstrate that there are strong performance differences between groups of funds that hold different types of unmapped holdings.

While unmapped holdings legally are permitted to exist within U.S. equity mutual fund portfolios[1], the question of whether they should exist is more complex. The presence of unmapped holdings can induce both positive and negative effects both in the evaluation of portfolio performance and in the discretion of portfolio managers. For example, unmapped holdings differences can obscure performance comparisons between funds. Unmapped holdings might introduce undefined risk factors into a portfolio, such that regression measures like Jensen's α become subject to omitted variables bias. Although the directional impact of the bias is unclear, an omitted risk factor can induce the illusion of significance and persistence in parameter estimates. If exposure to undefined risk factors are commonly held across sub-groups of funds, this can produce an illusion of winning and losing groups. Non-regression comparisons, such as performance-to-benchmark return comparisons, performance vs. characteristic based benchmarks (Daniel et. al. (1997[5])) would also be subject to similar bias.

The presence of unmapped holdings may simultaneously create desirable effects in actively managed portfolios. Returns of unmapped holdings might either complement (or leverage) mapped holdings returns due to strong correlations, or they may hedge mapped holdings returns through zero or negative correlation. Un-

[1]See Appendix A for an overview of regulations governing unmapped holdings

mapped holdings provide a portfolio manager with a greater degree of freedom to maximize returns.

In this paper, mutual funds are categorized directly by the unmapped holdings they possess. This categorization identifies groups of funds with differences both in unmapped assets and in portfolio effects of those assets. I focus upon risk and return properties of unmapped holdings and, more specifically, how unmapped holdings returns complement or hedge a portfolio's equity only return. I segregate mutual funds into these groups and then apply the endogenous benchmarks technique of Hunter, Kandel, Kandel, and Wermers[13] to model fund performance.

By nature, unmapped holdings are neither well understood within mutual fund portfolios, nor are they easily measurable due to a lack of available data for these assets. The endogenous benchmarks regression technique is perfectly suited to analyze the effects of these assets because of their inherent lack of definition and data. In this technique, my regressions use the average returns of groups of funds that share similar unmapped holdings to proxy for the unknown unmapped holdings effects. By doing so, I show that common unmapped holdings can better model fund returns and then better explain out-of-sample performance.

Most studies have assumed unmapped assets to be insignificant. One study by Koski and Pontiff[16] seems to confirm this notion. They investigate the risk and return characteristics of funds that use derivatives and argue that those funds are indistinguishable from funds that do not use derivatives.

Other studies imply that unmapped holdings have a substantial effect upon mutual fund returns. Wermers[22] indirectly calculates the average return of unmapped holdings using portfolio returns and mapped holdings. He infers that the average mutual fund loses 70 basis points per year due to investment in unmapped holdings, after controlling for stock characteristics, asset selection, transaction costs, and fees. This results suggests that if managers had exclusively held U.S. stock holdings, then their return would very nearly offset the expenses and fees they incur.

I use unmapped holdings and revisit the question of if mutual fund managers have skill. This question has been investigated as early as Jensen[14]. Recent research has argued that active fund managers are successful at finding good investments. In addition to Wermers[22], Baks et al.[1] find that mean-variance investors who are skeptical about active management skills can identify mutual funds that generate ex-ante positive alphas. More recently, Kacperczyk, Sialm, and Zheng[15] also produce a measure called the return gap that they show has strong ability to predict fund returns due to skilled manager's unobserved trades.

My investigation of mutual fund performance predictability is inspired by the work by Kacperczyk, Sialm, and Zheng[15]. Kacperczyk et al. used domestic equity mutual funds to investigate the return gap, a measure of the difference between reported fund returns and the return on a hypothetical portfolio that invests in previously disclosed portfolio holdings. They find that the return gap strongly predicts fund performance and they attribute the strength of this measure to evidence of manager skill. The Kacperczyk et al. paper is particularly relevant to my study of unmapped holdings because the size of the return gap is directly related to the portfolio weight that a fund invests in unmapped holdings. Since unmapped holdings give a manager latitude to alter portfolio risk, the effect of these assets must be controlled to accurately assess manager skill.

This paper will proceed as follows: section 1.2 describes the data and the endogenous benchmarks regression technique; section 1.3 groups funds by their unmapped holdings with supporting data; section 1.4 presents results that showing the statistical significance of unmapped holdings, correlation data within and between different unmapped holdings groups of fund, in-sample regression results, and out-of-sample return prediction tests; and section 1.5 concludes.

1.2 Data

The results of this paper are constructed using holdings data that was temporarily provided in the CRSP mutual funds database. Between 2003 and late 2007, CRSP was updating their database with mutual fund holdings provided to them by S&P. This data was voluntarily provided by participating mutual funds to Standard & Poor's, and afterwards acquired by Morningstar. CRSP obtained this data from Standard & Poor's and proceeded to map equity holdings into their U.S. stock database.

One particular advantage to this dataset is that it includes information on bond, international stock, and derivative holdings for a large number of funds. Such data has always been absent from the Thompson/CDA dataset and is again absent from the present CRSP database. This is one reason for the omission of unmapped holdings from much of the research in the past. One shortcoming of this dataset is that the data was voluntarily offered by each fund company and thus is not verified to be 100% complete and accurate. In a random sample of funds, I have compared the CRSP holdings data against the Certified Shareholder Report for Investment Companies (SEC Edgar form N-CSR) and found perfect matches in assets, share amounts, and market values. I have also randomly sampled mutual fund prospectus statements (presented in appendix B) and found that they appear consistent with the holdings in my sample. In cases when assets were not the same between the CRSP dataset and the N-CSR filing, I found the CRSP holdings data to contain greater detail than what was contained in the N-CSR. Also, the statistical properties of fund returns seem generally consistent with the types of unmapped holdings reported by the funds, such as correlations and variances.

In a few instances CRSP incorrectly populated its dataset without identifying any mapped holdings. In these cases, mapped and unmapped holdings are indistinguishable. For this reason, all holdings data showing a 100% investment in unmapped assets were omitted from the dataset. In other cases, CRSP may have partially identified mapped holdings within a portfolio. In such cases this would make some mapped holdings appear to be unmapped. These portfolios are included within my sample. Such holdings can overstate the market value of unmapped holdings and inflate the correlation between unmapped and mapped assets. The influence of such assets is relatively small because over a time series the mis-classification is temporary by construction.

I study quarterly holdings of domestic equity mutual funds. Domestic funds are selected following criteria applied in other studies such as the return gap paper by Kacperczyk et al[15]. One criteria that is intentionally omitted is a filter to restrict the sample only to funds where the aggregate market value of mapped assets is within a percentage of a fund's total net asset value. This percentage is typically set to 10% or 20%. In this study, the convention arbitrarily eliminates an important set of funds from the sample since these funds are likely to hold greater exposure to unmapped holdings. Though not shown, I also have found that unmapped holdings exist and influence portfolios regardless of this restriction. In fact, a large majority of mutual funds have substantial unmapped holdings below 10% of net asset value.

Table 1.1 presents summary statistics for funds with different percentage allocations to unmapped holdings. The first group of data, labeled "Extent of Unmapped Assets" presents the percentage of the fund population that holds different market weights of unmapped assets. 16.6% of all mutual funds had no exposure to unmapped assets at least once during the measurement period, and the majority of funds (62.7%) held between 2 and 5% of their market value in unmapped assets at least once during the measurement period. On average, most funds (34.5%) held between 2 and 5% of their assets in unmapped holdings, but a substantial 18.12% (11.28% + 6.33% + 0.51%) of the fund population held over 10% of their assets in unmapped holdings on average.

One does not find substantial variation in fund characteristics as unmapped asset allocation varies. The second group of data in table 1.1 presents the characteristics of funds that have different allocations to unmapped assets. The smallest funds (avg. \$1.43 mil) tend to hold between a 2 and 5% allocation to unmapped holdings, and the largest funds (avg. \$54.07 mil.) tend to have the greatest allocation to unmapped assets. This relationship is not monotonic, as slightly larger funds (avg. \$6.94 mil) also tend to hold no unmapped assets. The number of issues held by a fund does not appear to correlate with a fund's unmapped holdings allocation.

There is also little distinction in the asset investment across different allocations to unmapped assets. The third group of data in table 1.1 shows the average percentage market value allocation that funds invest in different types of unmapped assets. One observes generally increasing values in all of the asset types as the allocation to unmapped assets increases.

This analysis spans mutual fund holdings report dates from 9/30/2003 through 6/30/2007. Table 1.1 reports that 1,931 out of 1,959 funds average a non-zero exposure to unmapped holdings, representing 98.6% of the sample population. As the sample is restricted to a greater allocation in unmapped holdings, the percentage expectedly declines. Even so, a substantial number of funds hold an allocation of 10% or more of unmapped assets in their portfolio. Differences between funds are not well distinguished across fund characteristics and styles.

1.3 Methodology

1.3.1 Portfolio Effects of Unmapped Holdings

Unmapped holdings are closely related to the return gap, a measure defined as the difference between reported fund returns and the return on a hypothetical portfolio that invests in previously disclosed portfolio holdings. The return gap is a useful measure that captures the combined effect of both unmapped holdings and unobserved trading actions by fund managers.[2] It represents a joint measure of both unmapped holdings and unobserved trading. If fund returns are cross-sectionally averaged across funds that are invested in unmapped assets that have similar return characteristics, the influence of unmapped holdings should persist in the average while uncorrelated trading actions of funds would be reduced.

I use portfolio measures that are derived from the return gap to investigate if they significantly predict the type of unmapped assets held by a group of funds. The three measures that I use are the weight of investment in unmapped holdings (w_{unmap}), the volatility of unmapped holdings (σ^2_{unmap}), and the correlation between mapped and unmapped holdings ($\rho_{map,unmap}$). The weight of investment in unmapped holdings is directly observed from reported quarterly fund holdings, and the volatility and correlation are implied measures that are constructed from the return gap and the return of mapped portfolio assets.

Fund groups include those holding bonds and cash, derivatives, and foreign assets. Within these three broad classes, I further segment funds into more specialized subgroups such as funds with index or futures assets, and funds with foreign assets and derivatives, among others. By using a binary variable g_i to represent a fund's ownership of unmapped holdings common to group i, I then run a logit regression to find which portfolio parameters significantly correlate to asset ownership in each

[2]Recent studies[15] have used the return gap to make inferences about the unobserved actions of fund managers. Their return gap measure applies a t-bill rate of return to proxy for the return effects of unmapped holdings. Since derivatives and foreign securities can produce returns that are substantially different from t-bills, I do not do so in this study.

group.

$$g_i = \eta_1 w^+_{unmap} + \eta_2 w^-_{unmap} + \eta_3 \sigma^2_{unmap} + \eta_4 \rho^+_{map,unmap} + \eta_5 \rho^-_{map,unmap} + \epsilon \qquad (1.1)$$

The results of this test are presented in table 1.2. They show that indeed funds with particular types of unmapped assets tend to share common unmapped portfolio characteristics. The first result set shows the probability of group membership relative to all other U.S. equity mutual funds. The majority of funds (82%) hold some combination of bond and cash assets. Furthermore, the probability of funds holdings bonds and cash significantly increases with the proportion a fund invests in unmapped holdings. Perhaps more surprising, the greater the correlation between mapped and unmapped holdings, the higher the probability that a fund holds bonds and cash. This outcome is better understood when one separates the funds that exclusively use bonds and cash from funds that use bonds, cash, and derivative instruments.

Funds that exclusively use bonds and cash represent 67% of the fund population. These funds are best fit by a small allocation to unmapped holdings (w^+_{unmap}) and a correlation ($\rho_{map,unmap}$) close to zero. Funds holdings derivatives in addition to bonds and cash are quite distinct from the exclusive cash and bond group. Funds with index derivatives only represent 1.9% of mutual funds. These funds typically have very small unmapped holdings weight, and their correlation between mapped and unmapped holdings is either a significant positive value (complement) or a significant negative value (hedge). Similar results appear for funds holding bonds, cash, and non-index derivatives, except funds with non-index derivatives tend to have significantly larger allocations to unmapped holdings.

Derivatives use appears to have an important role in the portfolio effects of unmapped holdings. Derivative holdings in general, and more particularly options and swaps correlate with large allocations to unmapped holdings while index and bond derivative positions tend to be more prevalent in small allocations. All derivative holdings show strong correlations between mapped and unmapped assets, both positive (complement) and negative (hedge).

Funds with foreign assets represent a large proportion of equity funds. Funds with these assets show significant unmapped allocations, decreased unmapped asset volatility, and limited uncorrelation between mapped and unmapped assets. Funds holding currency assets are similar.

By narrowing the focus to only funds with derivatives, I find that futures and index asset holders appear quite similar, swap holders show even stronger positive and negative correlation effects, and option holders show negative correlation with the greatest significance. Among foreign asset holders, funds with derivatives also show some evidence of both negative and positive correlation portfolio effects.

Next, I apply the results from table 1.2 to construct economically significant groups of funds using their investment in unmapped holdings.

1.3.2 Mutual Fund Group Construction

Since funds with foreign assets and funds with derivative assets both showed strong correlation to mapped holdings, and only funds invested exclusively in bonds and cash showed little correlation, I grouped funds using both asset holdings and correlation effects. Table 1.3 shows the resulting groups and criteria. Among funds invested exclusively in foreign assets, I created one group that only holds foreign assets, another group that holds foreign assets and derivatives with positive correlation to mapped holdings, and another with negative correlation. I do likewise with funds that hold derivatives and with funds that hold index or futures assets. I did not create additional groups of funds that exclusively invest in bonds and cash due to a lack of correlation differences.

Panel A of table 1.3 presents evidence that indeed these groups show significant and distinct portfolio effects. Each group slightly differs from other groups by its particular portfolio properties.

1.3.3 Predictability of Mutual Fund Returns and Endogenous Benchmarks

In this section, I briefly outline how the 'endogenous benchmarks' technique by Hunter, Kandel, Kandel, and Wermers[13] can be applied to control for unmapped holdings in a test of mutual fund performance predictability. In particular, a fund's use of unmapped holdings introduces unmodeled return effects into portfolio returns. Consider first if the unmodeled portfolio returns are due to an omitted priced risk factor.

Suppose that the excess gross return of a fund i at time t is spanned by two priced risk factors $f_{1,t}$ and $f_{2,t}$. Then the return $r^e_{i,t}$ of this fund is defined as:

$$r^e_{i,t} = \alpha_i + \beta_{1,i} f_{1,t} + \beta_{2,i} f_{2,t} + \epsilon_{i,t}. \tag{1.2}$$

If the factor included by unmapped holdings ($f_{2,t}$) is omitted from the regression then this construct is a classical omitted variables problem, and the resulting estimates are biased.

$$r^e_{i,t} = \gamma_i + b_{1,i} f_{1,t} + \epsilon_{i,t} \tag{1.3}$$

$$E_t \widehat{b}_{1,i} = \beta_{1,i} + P_{1,2} \beta_{2,i} \tag{1.4}$$

$$E_t \gamma_i = \alpha_i + \beta_{2,i} \tag{1.5}$$

$$(E_t f_{2,t} - P_{1,2} E_t f_{1,t}) \tag{1.6}$$

$P_{1,2}$ is the slope of the regression of $f_{2,t}$ on $f_{1,t}$.

The endogenous benchmarks technique of Hunter, Kandel, Kandel, and Wermers uses the fact that the average return of a group of funds at time t contains an average loading on the unmodeled factor $f_{2,t}$ if those funds share an exposure to that factor. The average group return can thus be used as a proxy for the omitted factors.

Thus the average group excess return, $r^e_{g,t}$ is defined as follows:

$$r^e_{g,t} = \alpha_g + \beta_{1,g} f_{1,t} + \beta_{2,g} f_{2,t} + \epsilon_{g,t}. \tag{1.7}$$

Thus in the endogenous model regression one estimates and obtains the following:

$$r^e_{i,t} = \mu_i + c_i f_{1,t} + d_i r^e_{g,t} + \epsilon_{i,t} \tag{1.8}$$

$$E_t \mu_i = \alpha_i - z_i \alpha_g \tag{1.9}$$

In this paper, I test for predictability in mutual fund returns by constructing a trading strategy that is based upon past mutual fund performance. The two measures evaluated here are the the 4-factor model alpha, and the 4-factor plus endogenous benchmark alpha.

1.4 Results

1.4.1 Statistical Significance

The groups of funds defined in section 1.3.2 were distinguished by their unmapped holdings. In this section, these groups are analyzed to demonstrate their economic significance and empirical implications.

Table 1.4 shows each of the groups that were defined in section 1.3.2. It presents group size (number of funds), annual turnover, average return gap, average 4-factor model alpha estimates (using the Fama-French 3 factor model and the

4th Carhart momentum factor), average endogenous factor model alpha estimates (using the same 4 factors previously mentioned, and the endogenous factor defined in section 1.3.3), and the average weight of investment in unmapped holdings (w_{unmap}). Within each group section of the table, statistics for the entire group are presented on the first line, and statistics for the top third and bottom third of the group with respect to turnover are presented on lines two and three. First note that the groups of funds that hold unmapped assets with low or negative correlation to mapped holdings (groups 1c, 2c, 3a, and 4c) also have the lowest return gaps relative to all of the other groups of funds. This result is consistent with our hypothesis that unmapped holdings have an important effect upon the return gap. Also note that funds within each group have relatively consistent allocations to unmapped holdings since w_{unmap} shows little variation between high and low turnover funds within each group.

Perhaps most interesting relationships in table 1.4 are found by comparing estimated returns in the return gap estimates, the Carhart 4-factor model alpha, and the 4-factor plus endogenous benchmark model (hereafter called the endogenous model) alpha. For example, consider the return gap. As a baseline, I compare each of the group return gaps against the average return gap across all funds. The largest return gaps are observed in assets with positive correlation to mapped portfolio exposure. The biggest observation is found in positively correlated foreign assets, next derivatives, then futures. The lowest return gaps are negatively or near-zero correlation asset groups with derivatives appearing at the bottom, followed by futures, bonds, then foreign groups. The data show that the return gap tends to reward unmapped asset exposures that positively correlate with U.S. equities and penalize negative correlation. One would expect this relationship to reverse in a net downward trending market.

Consider instead the alpha estimates of the 4-factor equity risk model. Variation in 4-factor alpha estimates are much smaller between groups, but a predictable pattern still appears. Alpha estimates are closest to the average of all funds in funds that invest in futures and index assets. Derivatives and are the next closest, followed by bonds and cash, with foreign assets bearing the greatest distinction from the average. This outcome demonstrates that unmapped holdings best fit by U.S. equity risk factors are estimated with precision, while assets with distinct sensitivities will vary more from the mean. This demonstrates the effects of omitted variables bias, but this fact in combination with modern portfolio theory implies that fund managers have an incentive to pursue and invest in unmapped holdings.

Consider two U.S. equity portfolio managers, one who exclusively holds U.S. equities and another that deviates into unmapped holdings. The first manager can only earn alpha through stock selection while the second can do likewise, but also inflate alpha through omitted variables bias. Modern portfolio theory suggests that manager two has greater diversification, is more likely to earn a greater Sharpe ratio, and will appear superior against risk factor regressions such as the 4-factor model. In fact, a manager extracts the greatest benefit by selecting assets with the least correlation. This hypothesis is validated in the data, since the overwhelming majority of mutual funds (85%) hold bonds and cash, and the next most popular unmapped holding is foreign assets (27%). The least popular unmapped holdings, futures and index assets, are also those best modeled under U.S. equity risk factors. They only represent 7% of the U.S. equity fund population. The data also show that the 4-factor alpha tends to place the greatest alpha on funds with negatively correlated unmapped assets.

Finally, consider the estimated alphas when the effects unmapped holdings are controlled by proxy, as they are in the endogenous model regressions. One first should note that the magnitude of alpha estimates are considerably smaller under the endogenous model. By construction, group average alpha estimates are not significantly different from zero. The endogenous benchmark directly adjusts for common unmapped holdings within each group and places the group average alpha at zero. No strong pattern appears in the alpha estimates when comparing between groups or correlations. The most visible pattern in endogenous model estimates is that low turnover funds tend to have a higher alpha than high turnover funds within the same group. This pattern appears in every unmapped holdings group except among funds holding futures or index assets and where those assets have a positive correlation with their mapped holdings.

Residual correlations provide additional evidence of the economic significance of unmapped holdings. Table 1.5 presents the percentage of funds with statistically significant pairwise correlations with other funds in the same group. It shows that a substantial percentage of funds retain high residual correlations with other funds in their group after the common risk factors have been removed. On average, 46.9% of funds retained a statistically significant residual correlation with other in their group. When the endogenous benchmark is included as an additional factor, the percentage of funds with statistically significant residual correlations drops substantially.

1.4.2 Group Consistency

Table 1.7 presents statistics representing factor model estimates of funds within each unmapped holdings group. It shows that in all of the unmapped holdings groups, the 4-factor model recorded a high statistically significant positive alpha. Nearly 80% of all unmapped holdings funds within these groups recorded a statistically significant positive intercept. Almost all of the funds also had a statistically significant exposure to the market as measured by RMRF. Perhaps more surprising, however is the distribution of funds around the remaining risk coefficients. No real pattern appears to distinguish any particular risk factor exposure in one group or another. From this outcome we conclude that the styles presently defined are not related to any of the usual investment styles typically claimed by fund managers.

Table 1.7 also permits us to compare how adjusted R^2 increases or decreases when the endogenous factor is or is not used in regression estimates. It shows that the average R^2 is between 89 and 95% in all of the unmapped groups. When the endogenous factor is added, the average R^2 improves to between 90 and and 95%. Interestingly, when the endogenous factor is used alone in a single factor regression, it is inferior to the standard 4 factor model, but still explains a relatively large percentage of variation. The single endogenous factor model attains an average R^2 between 78 and 90%.

Table 1.6 shows how closely the unmapped holdings groups correlate with others. Bonds, futures, and foreign asset unmapped holdings groups show very little correlation to each other. The strongest correlation appears between bonds and the other main categories: foreign, futures, and derivatives. It is not surprising that we find that futures and derivatives are quite highly correlated, since the two groups are not mutually exclusive.

1.4.3 Predictability of Fund Returns

A test for predictability of mutual fund performance is this paper's final test. Having established the relevance and economic significance of unmapped holdings, I now test for predictable mutual fund performance when unmapped holdings are controlled. In this paper, predictability due to the 4 factor regression model, and the 4 plus endogenous factor model are compared. These factors are ranked and a long-short portfolio of top and bottom quintile mutual funds is constructed. The performance results of these portfolios are detailed in table **??**.

The first column of table **??** presents the raw performance results of this test. In nearly all groups of funds, the most predictable returns are obtained using the endogenous model alpha estimates. The only exceptions to this are observed in funds that use derivatives or futures to hedge their other assets, and in funds that use derivatives to make foreign investments more correlated with other assets. On a risk adjusted basis, similar results are also visible.

1.5 Conclusion

This paper has explored a temporarily available dataset containing rich information about unmapped holdings in mutual fund portfolios. Mutual fund managers benefit from the use of unmapped holdings in their portfolios, partly because this introduces an omitted variables bias into analysis of their performance, and also because this leads to greater diversification benefits. Fund manager overwhelmingly invest in unmapped holdings with low correlation to U.S. equity assets. This is primarily done in bonds, and next in foreign assets. Assets with the greatest correlation to U.S. equity markets (U.S. futures and other derivatives) are the least utilized unmapped asset.

Mutual funds can be grouped by their common unmapped assets. By doing so, the resulting groups share common unmapped holdings, and the group average return successfully controls for return effects missed in standard U.S. equity risk factor models.

Correlation and unmapped asset allocations are distinct when compared between groups. Funds with bonds tend to have the greatest correlation with other groups, but this relationship is likely due to the widespread bond assets held in unmapped holdings. The unmapped groups for foreign assets and derivatives have a nearly zero correlation.

Unmapped holdings groups are dramatically different from standard mutual fund groups. They show virtually no common sensitivities to mutual fund styles, such as capitalization or value/growth effects.

Out-of-sample performance tests show that in general, funds show greater performance predictability when unmapped holding effects are controlled. Risk adjusted performance and return predictability is worse among funds that appear to use derivative assets as a form of hedge in their portfolio.

To fairly assess skills of mutual fund managers, the effects of unmapped holdings should not be ignored. To do so increases the incentive for a manager to hold unmapped assets, and the omitted variables bias will induce an illusion of manager alpha. Further, the principal of diversification implies that such alpha estimates will have a positive bias.

Unmapped Holdings Summary Statistics

	Percentage Unmapped Holdings							
	0%	(0%, 1%]	(1%, 2%]	(2%, 5%]	(5%, 10%]	(10%, 20%]	(20%, 50%]	(50%, 100%)
Extent of Unmapped Assets								
Pct Funds	16.64%	2.96%	40.58%	62.69%	48.85%	27.05%	13.17%	1.79%
(1 or more obs)								
Pct Funds (Avg Wt)	1.43%	8.01%	15.01%	34.51%	22.92%	11.28%	6.33%	0.51%
Nbr Funds	28	157	294	676	449	221	124	10
Fund Characteristics by Unmapped Allocation								
Avg MV (mil)	6.94	5.48	3.32	1.43	2.18	4.58	16.02	54.07
Avg Issues Held	66.1	195.2	227.9	145.4	138.9	140.0	209.0	257.7
Avg Unmapped MV (mil)	0.000	0.035	0.052	0.047	0.152	0.657	4.519	34.604
Avg Unmapped Issues	0.0	4.1	5.0	4.7	7.9	15.3	49.0	124.7
Allocation								
Cash	0.00%	0.39%	1.05%	2.26%	4.06%	6.22%	6.97%	23.37%
Derivatives	0.00%	0.00%	0.02%	0.02%	0.14%	0.28%	0.99%	0.53%
Bonds	0.00%	0.08%	0.12%	0.33%	1.03%	2.35%	8.57%	19.75%
Foreign	0.00%	0.01%	0.01%	0.04%	0.13%	0.62%	1.08%	1.76%
Currency	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.08%
Warrant/Pfd	0.00%	0.00%	0.03%	0.03%	0.12%	0.17%	0.62%	0.64%
No ID	0.00%	0.16%	0.34%	0.63%	1.49%	4.73%	9.99%	17.87%

Table 1.1: This table presents summary statistics for funds with various allocation weights to unmapped holdings. The first lines labeled "Extent of Unmapped Assets" presents the percentage of funds at each unmapped holdings allocation level. The.second group of data labeled "Fund Characteristics by Unmapped Allocation" presents market value and holdings characteristics for funds at each level of unmapped holdings. The third group of data, labeled "Allocation" presents the average allocation of funds to different types of unmapped assets.

Appendix A

Appendix: Overview of Regulations Governing Unmapped Holdings

The presence of derivatives, leverage, and short positions in mutual funds has been a growing trend. Among many, it has been believed that mutual funds were prohibited by law from short positions and leverage, including the implicit leverage in derivatives contracts. In fact, section 12 of the Investment Company Act of 1940 specifically prohibits, under rules administered by the SEC, investment companies from engaging in short positions or leverage. However, the present legal interpretation of this section is that the SEC has not administered any rules per section 12 of the 1940 act. Therefore, mutual funds are permitted to do short sales and leverage, and section 18 of the 1940 act instead governs such positions in these funds([**?**]).

By law, it is legal for U.S. equity mutual funds to invest in unmapped holdings. According to section 8 of the Investment Company Act of 1940, mutual funds are required to disclose their investment policy to both the SEC and their shareholders, but some terms by which they define themselves are not regulated and lack precise definition. Many funds define their strategy as 'U.S. equity' or 'domestic equity', but simultaneously maintain positions in unmapped holdings which could be represented as inconsistent with such a claim. U.S. equity funds with positions in foreign stocks, U.S. bonds, or foreign bonds are simple examples of this. On the other hand, other

Proportion of Funds Allocated to an Asset
as a Function of Unmapped Properties

		Coefficient Estimates				
	P(x=1)	$w_{unmap} > 0$	$w_{unmap} < 0$	σ^2_{unmap}	$\rho_{map,unmap} > 0$	$\rho_{map,unmap} < 0$
P(x = 1) vs. All Funds						
Bond & Cash Holders	81.98%	0.204*	0.278	-0.054*	0.064*	-0.077*
		(7.26)	(0.76)	(-3.2)	(2.39)	(-2.14)
Bond & Cash Only	67.30%	-0.505*	0.075	-0.017	-0.056*	-0.077*
		(-23.880)	(0.40)	(-1.01)	(-2.26)	(-1.82)
Bonds, Cash & Non-Index Derivatives	2.36%	0.474*	0.257	-0.355*	0.102*	0.213*
		(15.93)	(1.05)	(-1.52)	(1.83)	(2.75)
Bonds, Cash & Index Derivatives	1.89%	-0.088*	-2.647	-1.396*	0.996*	0.2*
		(-2.07)	(-1.21)	(-2.29)	(18.16)	(2.29)
Derivative Holders	5.63%	0.277*	-1.240	-0.034	0.542*	0.223*
		(11.87)	(-1.27)	(-0.8)	(14.52)	(4.13)
Options Holders	1.25%	0.391*	-1.322	0.134*	0.161*	0.692*
		(11.37)	(-0.81)	(3.61)	(2.17)	(9.42)
Swap Holders	0.00%	0.802*	-2.771	-1.112	3.653*	2.72*
		(10.62)	(-0.19)	(-0.22)	(9.13)	(8.21)
Futures Holders	2.41%	-0.019	-4.728*	-1.346*	0.873*	0.243*
		(-0.5)	(-2.32)	(-2.57)	(17.35)	(3.21)
Index Derivative Holders	2.21%	-0.101*	-5.079*	-0.578*	1.003*	0.206*
		(-2.39)	(-2.88)	(-2.26)	(18.83)	(2.47)
Bond Derivative Holders	0.00%	-1.277*	0.738*	-0.080	3.452*	0.036*
		(-2.34)	(2.67)	(-0.22)	(5.06)	(0.01)
Foreign Holders	10.98%	0.775*	-0.069	-0.173*	-0.231*	-0.115*
		(31.73)	(-0.21)	(-2.74)	(-7.34)	(-2.42)
Currency Holders	2.98%	0.313*	-0.175	-0.160	-0.313*	-0.106
		(9.93)	(-0.22)	(-1.56)	(-5.49)	(-1.25)
P(x = 1) vs. Derivative Holders						
Bond & Cash Holders	89.96%	0.342*	2.516*	-0.695*	0.195	-0.013
		(2.00)	(1.74)	(-2.68)	(1.55)	(-0.06)
Bonds, Cash & No Index Derivatives	44.03%	0.915*	2.928*	-0.025	-0.671*	-0.268*
		(9.25)	(2.89)	(-0.16)	(-7.88)	(-1.86)
Bonds, Cash & Index Derivatives	41.53%	-0.869*	-2.158*	-1.296*	0.749*	0.285*
		(-8.53)	(-2.62)	(-3.47)	(8.71)	(2.02)
Options Holders	24.29%	0.413*	4.287*	2.021*	-0.421*	0.971*
		(5.12)	(2.94)	(4.66)	(-4.60)	(5.71)
Swap Holders	0.01%	1.27*	-2.887	-3.361	4.431*	2.844*
		(8.18)	(-0.46)	(-0.61)	(7.19)	(6.01)
Futures Holders	50.97%	-0.818*	-3.7*	-1.471*	0.661*	0.37*
		(-8.57)	(-2.70)	(-3.91)	(7.85)	(2.56)
Index Derivative Holders	46.01%	-0.945*	-2.473*	-0.773*	0.791*	0.293*
		(-9.19)	(-2.88)	(-3.56)	(9.11)	(2.07)
Bond Derivative Holders	0.19%	-0.431	5.499*	0.377	1.050	0.082
		(-0.51)	(3.84)	(1.50)	(1.21)	(0.06)
Foreign Holders	23.18%	0.402*	-0.864	-0.584	-0.915*	-0.223
		(4.62)	(-0.74)	(-1.50)	(-9.74)	(-1.38)
Currency Holders	27.41%	0.16*	-0.630	-1.924*	-0.969*	-0.220
		(2.09)	(-0.64)	(-1.76)	(-10.83)	(-1.48)
P(x = 1) vs. Foreign Holders						
Bond & Cash Holders	39.39%	5.499*	NA	0.377*	1.05*	0.082*
		(3.84)	(0.00)	(1.50)	(1.21)	(0.06)
Derivative Holders	13.46%	0.048	NA	-0.110	-0.244*	0.208*
		(0.74)	(0.00)	(-0.75)	(-3.22)	(1.68)
Currency Holders	8.20%	0.255*	NA	-4.260	-0.556*	0.019
		(3.81)	(0.00)	(-1.19)	(-6.58)	(0.13)
Options Holders	0.70%	0.365*	NA	-16.077	-0.543*	1.079*
		(3.36)	(0.00)	(-0.85)	(-3.27)	(6.61)
Swap Holders	0.00%					
Futures Holders	4.36%	-0.095	NA	-0.002	0.296*	0.752*
		(-0.91)	(0.00)	(-0.02)	(2.42)	(5.03)
Index Derivative Holders	2.35%	-0.253*	NA	0.028	0.76*	0.814*
		(-1.85)	(0.00)	(0.28)	(5.05)	(4.43)

Table 1.2: This table presents the logit regression results of the probability that a fund has holdings x as a function of weight of investment in unmapped holdings w_{unmap}, volatility of unmapped holdings σ^2_{unmap}, and correlation between mapped and unmapped holdings $\rho_{map,unmap}$.

Unmapped Group Evidence

Panel A: T-Statistics of Estimated Parameters

group		N funds	factors			
			w^+_{unmap}	$-w^-_{unmap}$	$\rho^+_{map,unmap}$	$-\rho^-_{map,unmap}$
	a	114	-0.80	-3.07	17.31	3.57
1	b	103	-3.32	-2.33	19.03	-0.08
	c	33	5.52	-2.80	-0.69	7.42
	a	278	19.64	-0.12	-2.56	-2.91
2	b	45	19.62	-1.55	10.44	-2.11
	c	18	8.52	-0.05	-1.00	5.32
3	a	1318	-16.10	1.28	-5.03	-1.63
	a	206	10.19	-3.59	14.68	4.93
4	b	177	6.16	-3.06	17.50	-0.58
	c	80	9.08	-1.90	-1.63	9.92

Panel B: Group Definition

Group	Effect	Assets Held
1a	Mix	Futures or Index Assets
1b	Complement	Futures or Index Assets ($\rho_{map,unmap} > 0$)
1c	Hedge	Futures or Index Assets ($\rho_{map,unmap} < 0$)
2a	Mix	Foreign Assets
2b	Complement	Foreign Assets and Derivative Assets ($\rho_{map,unmap} > 0$)
2c	Hedge	Foreign Assets and Derivative Assets ($\rho_{map,unmap} < 0$)
3a	Hedge	Bonds and/or cash and nothing else
4a	Mix	Derivatives
4b	Complement	Derivatives ($\rho_{map,unmap} > 0$)
4c	Hedge	Derivatives ($\rho_{map,unmap} < 0$)

Table 1.3: This table presents the criteria used to form groups of mutual funds using their unmapped holdings. Panel A shows t-statistics of portfolio measures that predict group membership. Panel B presents the criteria that was used to assemble each group. Within each type of unmapped holding group, there was evidence that unmapped holdings could both complement or hedge portfolio assets. Several groups were divided using correlation between mapped and unmapped assets to distinguish these effects.

Within Group Unmapped Holdings Statistics

Fund-Date Observations

		N	τ mean	T-stat (mean)	Return Gap mean	T-stat (mean)	4-factor mean	T-stat (mean)	5-factor mean	T-stat (mean)	w_{unmap} mean	T-stat (mean)
All Funds												
All	1	2965	1.02		-0.0009%		0.0980%		-0.0016%		0.07	
High τ	2	989	2.18	20.44	-0.0004%	0.01	0.1121%	0.16	-0.0240%	-0.17	0.08	0.45
Low τ	3	988	0.21	-17.35	-0.0010%	0	0.0847%	-0.15	0.0181%	0.15	0.08	0.78
1a: Futures or Index Assets												
All	1	214	1.33		0.0007%		0.0817%		-0.0285%		0.09	
High τ	2	72	3.08	7.23	0.0049%	0.03	0.0967%	0.04	-0.0062%	0.05	0.12	0.73
Low τ	3	71	0.12	-6.09	-0.0003%	-0.01	0.0628%	-0.06	-0.0334%	-0.01	0.06	-0.55
1b: Futures or Index Assets ($\rho_{map,unmap} > 0$)												
All	1	192	1.23		0.0008%		0.0778%		-0.0212%		0.09	
High τ	2	64	2.94	6.55	0.0067%	0.04	0.0911%	0.04	-0.0092%	0.03	0.12	0.63
Low τ	3	64	0.1	-5.3	-0.0011%	-0.01	0.0549%	-0.07	-0.0493%	-0.06	0.06	-0.55
1c: Futures or Index Assets ($\rho_{map,unmap} < 0$)												
All	1	55	1.31		-0.0043%		0.1127%		0.0036%		0.07	
High τ	2	19	2.59	4.11	-0.0016%	0.01	0.1053%	-0.01	0.0252%	0.02	0.08	0.21
Low τ	3	18	0.29	-3.67	-0.0092%	-0.01	0.1475%	0.05	0.0822%	0.09	0.08	0.18
2a: Foreign Assets												
All	1	631	0.88		0.0005%		0.0950%		-0.0003%		0.13	
High τ	2	211	1.74	11.35	0.0012%	0.01	0.1036%	0.04	-0.0241%	-0.09	0.13	-0.23
Low τ	3	210	0.22	-10.03	-0.0005%	-0.01	0.0853%	-0.05	0.0181%	0.07	0.15	0.61
2b: Foreign Assets ($\rho_{map,unmap} > 0$)												
All	1	118	0.91		0.0037%		0.0898%		-0.0211%		0.13	
High τ	2	40	1.55	4.56	0.0045%	0	0.0975%	0.02	-0.0035%	0.03	0.2	1.04
Low τ	3	39	0.26	-4.9	0.0025%	-0.01	0.0881%	0	0.0146%	0.06	0.12	-0.09
2c: Foreign Assets ($\rho_{map,unmap} < 0$)												
All	1	38	1.14		-0.0037%		0.1152%		-0.0200%		0.11	
High τ	2	13	2.55	3.97	-0.0143%	-0.02	0.1365%	0.03	-0.0471%	-0.03	0.1	-0.11
Low τ	3	12	0.21	-2.86	-0.0042%	0	0.1082%	-0.01	-0.0019%	0.02	0.14	0.24
3a: Bonds and Cash Alone												
All	1	2528	0.97		-0.0013%		0.0994%		0.0011%		0.06	
High τ	2	843	2.03	17.77	-0.0019%	-0.01	0.1147%	0.16	-0.0213%	-0.16	0.06	0.03
Low τ	3	842	0.23	-15.35	-0.0009%	0.01	0.0853%	-0.15	0.0223%	0.14	0.07	0.67
4a: Derivatives												
All	1	399	1.56		0.0009%		0.1008%		0.0009%		0.11	
High τ	2	133	3.6	11.17	0.0034%	0.02	0.1181%	0.06	0.0028%	0	0.18	1.67
Low τ	3	133	0.19	-9.06	0.0000%	-0.01	0.0837%	-0.07	0.0338%	0.09	0.10	-0.37
4b: Derivatives ($\rho_{map,unmap} > 0$)												
All	1	353	1.49		0.0009%		0.0946%		0.0077%		0.12	
High τ	2	118	3.47	10.07	0.0036%	0.02	0.1088%	0.05	0.0073%	0.00	0.19	1.61
Low τ	3	117	0.17	-8.1	0.0006%	0	0.0799%	-0.05	0.0314%	0.06	0.1	-0.48
4c: Derivatives ($\rho_{map,unmap} < 0$)												
All	1	149	1.37		-0.0022%		0.1220%		-0.0046%		0.1	
High τ	2	50	2.89	6.34	-0.0043%	-0.01	0.1279%	0.01	0.0011%	0.01	0.15	0.69
Low τ	3	49	0.28	-5.37	-0.0023%	0	0.1239%	0.00	0.0568%	0.1	0.11	0.15

Table 1.4: This table presents group size, turnover, return gap, 4-factor (Carhart) model alpha estimates, 4+endogenous factor model alpha estimates, and weight of investment in unmapped holdings values. T-statistics presented are t-statistics of a difference in means from the group average. High turnover and low turnover funds in each group were ranked and the top third were labeled high τ and the bottom third were labeled low τ. The return gap and all model estimates of alpha represent daily values.

U.S. Equity Funds: Residual Correlation

Correlation Coefficients within Group across fund Returns Residuals (percent of funds with significant residual correlations)

Model				**Period**				
	12-2003	06-2004	12-2004	06-2005	12-2005	06-2006	12-2006	2003-2007
1a: Futures & Index Derivatives								
4 Factor	39.1%	42.7%	40.3%	39.5%	38.6%	43.9%	46.5%	53.8%
5 Factor	29.3%	34.0%	32.0%	28.5%	27.4%	33.3%	26.2%	35.9%
1b: Futures & Index Derivatives, $\rho_{map,unmap} > 0$								
4 Factor	44.8%	43.5%	41.9%	41.2%	42.0%	47.6%	46.8%	57.2%
5 Factor	32.7%	35.4%	32.5%	29.8%	28.0%	35.4%	26.9%	36.4%
1c: Futures & Index Derivatives, $\rho_{map,unmap} < 0$								
4 Factor	40.7%	35.7%	33.7%	34.6%	29.0%	32.9%	35.2%	42.5%
5 Factor	21.6%	18.3%	22.7%	19.8%	21.5%	23.2%	23.8%	24.7%
2a: Foreign Assets								
4 Factor	37.1%	38.6%	37.6%	33.5%	31.1%	30.9%	35.6%	44.1%
5 Factor	19.7%	25.7%	23.9%	21.5%	20.9%	24.5%	26.5%	30.3%
2b: Foreign Assets and Derivative Assets, $\rho_{map,unmap} > 0$								
4 Factor	43.8%	37.3%	37.1%	38.0%	35.0%	39.7%	45.1%	49.8%
5 Factor	25.0%	27.7%	27.7%	23.6%	25.7%	28.0%	29.4%	33.1%
2c: Foreign Assets and Derivative Assets, $\rho_{map,unmap} < 0$								
4 Factor	40.0%	49.2%	42.0%	32.2%	34.7%	33.8%	42.4%	42.4%
5 Factor	24.0%	23.4%	23.4%	21.1%	23.8%	24.1%	20.4%	26.0%
3a: Bonds & Cash exclusively								
4 Factor	31.0%	36.9%	32.6%	30.7%	30.2%	33.9%	37.8%	44.3%
5 Factor	20.6%	27.7%	23.4%	22.9%	21.6%	24.1%	24.8%	30.1%
4a: Derivatives								
4 Factor	33.0%	38.1%	32.7%	32.6%	31.7%	36.3%	40.6%	46.2%
5 Factor	25.3%	31.1%	25.9%	23.8%	23.1%	28.8%	26.4%	32.3%
4b: Derivatives, $\rho_{map,unmap} > 0$								
4 Factor	38.8%	39.6%	34.9%	33.8%	33.5%	38.7%	42.4%	48.2%
5 Factor	29.6%	32.7%	28.8%	25.7%	24.8%	30.8%	26.7%	34.1%
4c: Derivatives, $\rho_{map,unmap} < 0$								
4 Factor	27.3%	38.3%	29.6%	26.0%	28.1%	28.5%	34.8%	40.2%
5 Factor	21.8%	22.7%	19.2%	19.0%	18.6%	22.5%	21.5%	24.8%
Average								
4 Factor	37.6%	40.0%	36.2%	34.2%	33.4%	36.6%	40.7%	46.9%
5 Factor	25.0%	27.9%	26.0%	23.6%	23.5%	27.5%	25.3%	30.8%

Table 1.5: This table presents the percentage of funds within each unmapped holdings group that have significant correlation coefficients in their residuals after their returns are regressed on the Carhart 4-factor model and the 4-factor model plus the endogenous factor (5 factor). Due to use of daily returns, all regressions are augmented using the Scholes-Williams technique to control for non-synchronous data.

U.S. Equity Funds

Correlation Coefficients across Group Return Residuals

(first row: correlation, second row: p-value)

Correlation Pairs	**Period**								
	12/2003	06/2004	12/2004	06/2005	12/2005	06/2006	12/2006	06/2007	03-07
Futures Positive Rho	0.16	0.24	0.62	0.69	0.60	1.00	0.66	0.37	0.97
Futures Negative Rho	0.11	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Foreign Positive Rho	0.60	0.42	0.60	0.73	0.83	0.91	0.82	0.76	0.71
Foreign Negative Rho	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Derivatives Positive Rho	0.11	0.19	0.53	0.71	0.80	0.99	0.83	0.71	0.91
Derivatives Negative Rho	0.20	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bonds	0.77	0.77	0.68	0.82	0.80	0.69	0.86	0.85	0.49
Futures	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bonds	0.65	0.65	0.80	0.71	0.71	0.75	0.75	0.76	0.68
Foreign	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Bonds	0.26	0.03	0.34	0.50	0.41	0.33	0.38	0.33	0.67
Derivatives	0.02	0.35	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Futures Positive Rho	0.42	0.34	0.27	0.47	0.56	-0.04	0.37	0.49	0.07
Foreign Positive Rho	0.00	0.00	0.00	0.00	0.00	0.69	0.00	0.00	0.29
Futures Positive Rho	0.90	0.93	0.89	0.93	0.90	0.99	0.89	0.91	0.96
Derivatives Positive Rho	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Foreign Positive Rho	0.38	0.47	0.55	0.66	0.70	0.05	0.65	0.70	0.25
Derivatives Positive Rho	0.00	0.00	0.00	0.00	0.00	0.27	0.00	0.00	0.02
Futures Negative Rho	0.50	0.41	0.22	0.45	0.51	-0.08	0.47	0.54	0.03
Foreign Negative Rho	0.00	0.00	0.01	0.00	0.00	0.81	0.00	0.00	0.41
Futures Negative Rho	0.18	0.15	0.53	0.63	0.61	0.98	0.69	0.47	0.98
Derivatives Negative Rho	0.08	0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Foreign Negative Rho	0.06	-0.07	0.30	0.58	0.54	0.07	0.50	0.38	0.13
Derivatives Negative Rho	0.32	0.79	0.00	0.00	0.00	0.23	0.00	0.00	0.15

Table 1.6: This table presents estimated correlation coefficients between unmapped holdings groups of funds. Returns are calculated as equally weighted averages of fund returns.

U.S. Equity Mutual Funds: In Sample Estimates

	4 Factor Model Alpha	5 Factor Model Alpha (no α_g)	Alpha	RMRF	HML	SMB	UMD	End Factor	Endogenous Alpha
1a: Futures and Index Assets									
Positive Significant	3.3%	4.0%	3.0%	99.9%	17.0%	28.4%	15.5%	57.8%	5.1%
Positive Not Significant	42.2%	41.4%	43.5%	0.1%	21.8%	13.5%	31.3%	32.5%	46.6%
Negative Not Significant	49.4%	48.5%	47.2%	0.0%	29.3%	14.4%	34.9%	8.9%	43.7%
Negative Significant	5.1%	6.1%	6.3%	0.0%	31.9%	43.7%	18.3%	0.7%	4.6%
R-Square	94%	95%							90%
1b: Futures and Index Assets, Positive Correlation									
Positive Significant	2.8%	3.3%	2.8%	100.0%	18.1%	27.2%	14.7%	62.4%	4.8%
Positive Not Significant	41.5%	40.9%	43.8%	0.0%	22.2%	12.3%	31.2%	29.7%	46.7%
Negative Not Significant	50.3%	49.0%	47.1%	0.0%	28.0%	13.5%	34.2%	7.6%	44.1%
Negative Significant	5.4%	6.8%	6.3%	0.0%	31.8%	47.0%	19.9%	0.3%	4.4%
R-Square	95%	96%							92%
1c: Futures and Index Assets, Negative Correlation									
Positive Significant	5.0%	6.2%	4.4%	99.6%	31.8%	30.1%	15.2%	54.9%	6.0%
Positive Not Significant	48.4%	46.4%	45.8%	0.5%	22.1%	16.1%	33.4%	35.1%	46.5%
Negative Not Significant	43.5%	43.9%	45.6%	0.0%	22.5%	13.8%	38.2%	8.7%	41.8%
Negative Significant	3.2%	3.5%	4.3%	0.0%	23.6%	39.9%	13.2%	1.3%	5.6%
R-Square	90%	91%							85%
2a: Foreign Assets									
Positive Significant	4.4%	5.5%	2.6%	100.0%	19.6%	30.1%	22.7%	46.4%	5.1%
Positive Not Significant	47.5%	46.3%	46.0%	0.1%	30.1%	18.6%	31.0%	36.1%	45.3%
Negative Not Significant	45.1%	44.6%	48.5%	0.0%	27.8%	26.5%	31.2%	15.4%	45.3%
Negative Significant	2.9%	3.6%	2.9%	0.0%	22.5%	24.8%	15.1%	2.1%	4.3%
R-Square	93%	93%							89%
2b: Foreign Assets, Positive Correlation									
Positive Significant	4.8%	6.4%	2.0%	100.0%	20.7%	30.9%	18.0%	57.4%	4.9%
Positive Not Significant	46.4%	45.5%	46.3%	0.0%	26.1%	17.5%	30.4%	30.7%	45.8%
Negative Not Significant	46.3%	44.0%	49.1%	0.0%	27.4%	21.3%	38.3%	10.2%	44.6%
Negative Significant	2.5%	4.2%	2.7%	0.0%	25.7%	30.3%	13.3%	1.7%	4.7%
R-Square	93%	94%							90%
2c: Foreign Assets, Negative Correlation									
Positive Significant	4.1%	6.5%	1.7%	99.6%	28.8%	35.4%	23.4%	62.5%	4.5%
Positive Not Significant	48.1%	45.6%	49.2%	0.4%	26.3%	16.2%	27.7%	31.9%	46.8%
Negative Not Significant	45.7%	44.1%	46.3%	0.0%	24.1%	15.7%	35.4%	5.5%	42.5%
Negative Significant	2.1%	3.9%	2.8%	0.0%	20.8%	32.7%	13.5%	0.2%	6.2%
R-Square	90%	92%							87%

Table 1.7: This table presents in-sample statistical significance and R^2 estimates using 4 factor regression models, 4 plus endogenous benchmark regression models, and endogenous benchmark alone regression models. All models were run using daily returns with Scholes-Williams correction factors for non-synchronous data. The final column of the table presents the statistical significance of estimated coefficients on the endogenous benchmark.

U.S. Equity Mutual Funds: In Sample Estimates

	4 Factor Model Alpha	5 Factor Model Alpha (no α_g)	Alpha	RMRF	HML	SMB	UMD	End Factor	Endogenous Alpha
3a: Bond and Cash Assets									
Positive Significant	3.2%	4.0%	3.1%	99.7%	20.1%	42.9%	24.4%	45.1%	4.7%
Positive Not Significant	46.0%	45.0%	45.5%	0.1%	25.2%	12.7%	30.8%	37.1%	46.5%
Negative Not Significant	47.2%	46.8%	47.7%	0.1%	27.0%	18.0%	29.9%	15.7%	44.8%
Negative Significant	3.6%	4.2%	3.7%	0.1%	27.6%	26.4%	14.9%	2.1%	4.0%
R-Square	93%	93%							88%
4a: Derivative Assets									
Positive Significant	3.3%	4.1%	3.1%	99.1%	17.7%	35.2%	18.0%	48.8%	5.7%
Positive Not Significant	42.7%	41.8%	45.0%	0.5%	24.4%	13.6%	30.4%	36.3%	46.8%
Negative Not Significant	49.6%	48.8%	47.6%	0.4%	27.9%	18.4%	35.7%	13.3%	43.4%
Negative Significant	4.5%	5.4%	4.3%	0.0%	30.1%	32.8%	15.9%	1.7%	4.1%
R-Square	93%	93%							89%
4b: Derivative Assets, Positive Correlation									
Positive Significant	3.6%	4.4%	3.0%	99.5%	17.1%	35.2%	18.6%	49.8%	5.1%
Positive Not Significant	43.8%	43.0%	44.6%	0.4%	23.1%	12.4%	32.2%	35.3%	46.2%
Negative Not Significant	47.7%	46.9%	47.2%	0.2%	28.2%	16.1%	33.8%	13.4%	44.4%
Negative Significant	4.9%	5.7%	5.2%	0.0%	31.6%	36.3%	15.4%	1.6%	4.3%
R-Square	94%	94%							90%
4c: Derivative Assets, Negative Correlation									
Positive Significant	4.7%	5.5%	3.5%	98.4%	25.0%	38.8%	16.3%	44.5%	5.2%
Positive Not Significant	46.2%	45.0%	45.7%	1.1%	29.6%	16.8%	35.3%	41.7%	46.5%
Negative Not Significant	46.4%	46.1%	47.9%	0.6%	24.6%	17.2%	34.1%	12.4%	43.7%
Negative Significant	2.8%	3.4%	2.9%	0.0%	20.8%	27.1%	14.2%	1.4%	4.6%
R-Square	89%	90%							84%

Out of Sample Performance Tests

Group	SubGroup	Rank Model	Binary	Tercile	Quartile	Quintile
Futures	All (1a)	4 factors	0.14	0.16	0.21	0.32
And Index			*(0.73)*	*(0.67)*	*(0.82)*	*(1.19)*
Holdings		5 factors	0.12	0.16	0.22	0.31
		(no α_g)	*(0.65)*	*(0.70)*	*(0.85)*	*(1.11)*
		5 factors	0.02	0.04	0.07	-0.02
			(0.10)	*(0.17)*	*(0.29)*	*(-0.06)*
	Positive ρ	4 factors	0.13	0.2	0.27	0.37
	(1b)		*(0.71)*	*(0.85)*	*(1.17)*	*(1.56)*
		5 factors	0.12	0.21	0.31	0.35
		(no α_g)	*(0.66)*	*(0.90)*	*(1.34)*	*(1.38)*
		5 factors	0.06	0.12	0.18	0.2
			(0.30)	*(0.54)*	*(0.72)*	*(0.71)*
	Negative ρ	4 factors	0.14	0.27	0.28	0.14
	(1c)		*(0.63)*	*(0.91)*	*(0.83)*	*(0.34)*
		5 factors	0.12	0.27	0.42	0.33
		(no α_g)	*(0.49)*	*(0.90)*	*(1.20)*	*(0.80)*
		5 factors	0.08	0.23	0.22	0.13
			(0.32)	*(0.82)*	*(0.69)*	*(0.35)*
Foreign	All (2a)	4 factors	-0.13	-0.28	-0.28	-0.25
Asset			*(-0.52)*	*(-0.84)*	*(-0.80)*	*(-0.71)*
Holders		5 factors	-0.14	-0.25	-0.29	-0.33
		(no α_g)	*(-0.57)*	*(-0.74)*	*(-0.82)*	*(-0.91)*
		5 factors	-0.03	-0.01	-0.01	0.02
			(-0.15)	*(-0.03)*	*(-0.03)*	*(0.06)*
	Positive ρ	4 factors	**0.82**	**1.03**	**1.12**	**1.24**
	and Derivatives		*(4.39)*	*(4.36)*	*(4.24)*	*(3.83)*
		5 factors	**0.87**	**1.13**	**1.25**	**1.33**
		(no α_g)	*(4.79)*	*(4.61)*	*(3.85)*	*(3.65)*
		5 factors	**0.6**	**0.79**	**0.74**	**0.87**
			(3.45)	*(5.08)*	*(4.49)*	*(5.47)*
	Negative ρ	4 factors	-0.36	-0.49	-0.59	-0.66
	and Derivatives		*(-1.20)*	*(-1.37)*	*(-1.49)*	*(-1.45)*
		5 factors	-0.29	-0.38	-0.49	-0.57
		(no α_g)	*(-0.94)*	*(-0.97)*	*(-1.13)*	*(-1.18)*
		5 factors	-0.29	-0.4	-0.45	-0.5
			(-1.05)	*(-1.06)*	*(-1.02)*	*(-1.00)*

Table 1.8: This table presents time-series out-of-sample alpha estimates from January 2004 through June 2007 for the equal-weighted portfolio of funds that are constructed from the in-sample ranked alpha from (1) the 4-factor model ($r^{\epsilon}_{i,t} = \alpha_{1,i} + \gamma_{mkt,i} r_{mkt,t} + \gamma_{hml,i} r_{hml,t} + \gamma_{smb,i} r_{smb,t} + \gamma_{umd,i} r_{umd,t} + \epsilon_{i,t}$), from (2) the 4+E factor model when the endogenous factor excludes the group's average alpha (correlated errors or time varying factors model), and from (3) the 4+E factor model when the endogenous factor includes the group's average alpha (omitted variable model). In each category, funds are ranked during each three-month time-period on their in-sample alpha, then binary, tercile, quartile, and quintile portfolios are formed. Out-of-sample alpha estimates are then calculated as the realized performance less predicted performance (intercept omitted) over the subsequent 12 months. Predicted performance numbers are projected using in-sample endogenous model estimates. Means and t-statistics were for the overlapping data samples and were calculated following Jegadeesh (1990). The data represent a long-short investment (an investment in the top n-tile funds less investment in bottom n-tile funds). Statistically significant alpha estimates are shown in bold and t-statistics are shown in a smaller font beneath each estimate.

Out of Sample Performance Tests

Group	SubGroup	Rank Model	Binary	Tercile	Quartile	Quintile
Bond and Cash Holders	All (3a)	4 factors	0.17	0.21	0.25	0.29
			(0.76)	*(0.68)*	*(0.73)*	*(0.87)*
		5 factors (no α_g)	0.15	0.2	0.21	0.24
			(0.65)	*(0.63)*	*(0.59)*	*(0.64)*
		5 factors	0.05	0.07	0.05	0.07
			(0.26)	*(0.26)*	*(0.17)*	*(0.22)*
Derivative Holders	All (4a)	4 factors	-0.44	-0.56	-0.66	-0.65
			(-1.30)	*(-1.50)*	*(-1.76)*	*(-1.52)*
		5 factors (no α_g)	-0.52	-0.64	-0.79	**-0.88**
			(-1.53)	*(-1.68)*	*(-1.93)*	*(-1.99)*
		5 factors	-0.34	-0.44	-0.66	-0.74
			(-1.31)	*(-1.40)*	*(-1.68)*	*(-1.74)*
	Positive ρ (4b)	4 factors	**0.38**	0.44	**0.54**	**0.68**
			(2.09)	*(1.86)*	*(1.97)*	*(2.35)*
		5 factors (no α_g)	**0.38**	0.46	0.53	**0.64**
			(2.09)	*(1.90)*	*(1.78)*	*(2.00)*
		5 factors	0.07	0.14	0.09	0.05
			(0.32)	*(0.54)*	*(0.29)*	*(0.16)*
	Negative ρ (4c)	4 factors	-0.3	-0.35	-0.38	-0.41
			(-1.50)	*(-1.47)*	*(-1.33)*	*(-1.15)*
		5 factors (no α_g)	-0.23	-0.33	-0.39	-0.49
			(-1.15)	*(-1.39)*	*(-1.27)*	*(-1.27)*
		5 factors	-0.05	-0.12	-0.2	-0.18
			(-0.26)	*(-0.50)*	*(-0.70)*	*(-0.55)*

T-Stat Difference Test

Panel A: Weighted T-Statistic Difference

	Out-of-Sample α by Style									
	Futures and Index Holdings			Foreign Holdings			Cash and Bonds	Derivative Holdings		
	All (1a)	Pos ρ (1b)	Neg ρ (1c)	All (2a)	Pos ρ (2b)	Neg ρ (2c)	All (3a)	All (4a)	Pos ρ (4b)	Neg ρ (4c)
Long Minus Short	0.44	0.30	-0.04	-0.05	**1.63**	0.07	0.02	**-1.94**	0.71	-0.19
	(1.60)	*(1.32)*	*(-0.08)*	*(-0.08)*	*(4.22)*	*(0.14)*	*(0.04)*	*(-2.34)*	*(1.88)*	*(-0.33)*
Long Only	**0.88**	**0.65**	0.97	-0.11	0.79	0.24	0.30	0.36	**1.15**	0.22
	(2.57)	*(1.99)*	*(1.49)*	*(-0.12)*	*(0.97)*	*(0.29)*	*(0.41)*	*(0.43)*	*(2.46)*	*(0.31)*
Short Only	0.45	0.36	1.02	-0.07	-0.84	0.17	0.31	**2.32**	0.45	0.44
	(1.09)	*(0.96)*	*(1.77)*	*(-0.09)*	*(-0.98)*	*(0.23)*	*(0.36)*	*(2.91)*	*(0.82)*	*(0.52)*
	Out-of-Sample Returns by Style									
	Futures and Index Holdings			Foreign Holdings			Cash and Bonds	Derivative Holdings		
	All (1a)	Pos ρ (1b)	Neg ρ (1c)	All (2a)	Pos ρ (2b)	Neg ρ (2c)	All (3a)	All (4a)	Pos ρ (4b)	Neg ρ (4c)
Long Minus Short	-0.02	0.09	-0.62	-0.10	0.31	0.63	0.02	0.05	0.17	0.00
	(-0.11)	*(0.45)*	*(-1.68)*	*(-0.51)*	*(1.51)*	*(1.42)*	*(0.11)*	*(0.14)*	*(0.60)*	*(-0.01)*
Long Only	**1.33**	**1.33**	**1.10**	**1.34**	**1.51**	**1.68**	**1.26**	**1.24**	**1.45**	**1.44**
	(1.75)	*(1.81)*	*(1.46)*	*(1.71)*	*(1.94)*	*(1.93)*	*(1.57)*	*(1.31)*	*(1.85)*	*(2.11)*
Short Only	**1.36**	**1.25**	**1.73**	**1.45**	**1.20**	**1.04**	**1.26**	**1.20**	**1.29**	**1.45**
	(1.68)	*(1.58)*	*(2.27)*	*(1.75)*	*(1.49)*	*(1.25)*	*(1.39)*	*(1.09)*	*(1.50)*	*(1.80)*

Panel B: Weighted T-Statistic Rank Difference

	Out-of-Sample α by Style									
	Futures and Index Holdings			Foreign Holdings			Cash and Bonds	Derivative Holdings		
	All (1a)	Pos ρ (1b)	Neg ρ (1c)	All (2a)	Pos ρ (2b)	Neg ρ (2c)	All (3a)	All (4a)	Pos ρ (4b)	Neg ρ (4c)
Long Minus Short	0.18	0.11	0.43	-0.43	**1.19**	-0.03	-0.19	**-1.72**	0.44	**-0.18**
	(0.78)	*(0.64)*	*(0.88)*	*(-0.82)*	*(2.98)*	*(-0.07)*	*(-0.37)*	*(-2.51)*	*(1.09)*	*(-0.29)*
Long Only	0.52	0.41	**1.06**	-0.26	0.76	0.16	0.09	0.18	**0.72**	0.03
	(1.75)	*(1.62)*	*(2.71)*	*(-0.39)*	*(1.06)*	*(0.27)*	*(0.15)*	*(0.25)*	*(2.05)*	*(0.06)*
Short Only	0.34	0.30	0.65	0.17	-0.43	0.19	0.31	**1.92**	0.29	0.25
	(1.03)	*(0.98)*	*(1.20)*	*(0.24)*	*(-0.59)*	*(0.28)*	*(0.40)*	*(2.41)*	*(0.56)*	*(0.28)*
	Out-of-Sample Returns by Style									
	Futures and Index Holdings			Foreign Holdings			Cash and Bonds	Derivative Holdings		
	All (1a)	Pos ρ (1b)	Neg ρ (1c)	All (2a)	Pos ρ (2b)	Neg ρ (2c)	All (3a)	All (4a)	Pos ρ (4b)	Neg ρ (4c)
Long Minus Short	**-0.20**	-0.12	-0.49	-0.15	0.20	0.36	-0.19	-0.18	-0.06	0.08
	(-2.02)	*(-1.49)*	*(-1.67)*	*(-1.22)*	*(1.16)*	*(0.89)*	*(-1.26)*	*(-0.51)*	*(-0.25)*	*(0.31)*
Long Only	1.12	1.14	1.12	**1.29**	1.39	1.48	1.16	0.93	1.19	**1.27**
	(1.54)	*(1.61)*	*(1.58)*	*(1.67)*	*(1.86)*	*(1.90)*	*(1.47)*	*(0.95)*	*(1.54)*	*(1.83)*
Short Only	1.34	1.26	**1.61**	**1.44**	1.20	1.12	1.36	1.12	1.26	1.20
	(1.73)	*(1.69)*	*(2.34)*	*(1.81)*	*(1.54)*	*(1.27)*	*(1.59)*	*(0.98)*	*(1.43)*	*(1.41)*

Table 1.9: This table presents the out-of-sample predicted performance from weighted t-statistic differences between model estimates that include the endogenous factor and model estimates when the endogenous factor is excluded. Panel A shows results for weighted t-statistic differences, and panel B presents results for rank weighted t-statistic differences. The t-statistic of intercept estimates from the four factor model are subtracted from corresponding estimates using the four factor model plus an endogenous (no group alpha) factor. Difference weighted long (short) positions are taken in funds with positive (negative) t-statistic differences. Out-of-sample alpha (using actual returns less predicted returns) and return are calculated over the subsequent 3 months for each long, short, and long minus short position. The top half of each table shows results using alpha estimates while the lower half presents the same result using raw returns.

unmapped holdings may be viewed as mere extensions of a fund's 'U.S. equity' claim, such as funds with positions in U.S. equity futures or options or funds with short U.S. equity positions. Each investment company is required to file a registration statement that discloses policy with respect to diversification, leverage, issuance of other senior securities (including short sales), industry concentration, real estate or other commodity purchases and sales, and any other policies which are changeable only if authorized by shareholder vote. Investment companies are also required to disclose any other matters the registrant deems a matter of "fundamental policy". Funds are prohibited from deviating from such policy without majority shareholder approval.

In general, the regulation of mutual funds has been defined to ensure that funds are not deceptive or misleading to both investors and regulators. According to the 1940 act, funds must produce regular shareholder reports that are not "misleading in any material respect". Within these reports, there must be "a list showing the amounts and values of securities owned". Such a list would therefore contain the unmapped holdings within each fund. Thus by law, unmapped holdings are disclosed and a fund's reporting of its investment strategy is, at least to any material extent, not misleading. However, the degree that unmapped holdings may have a significant effect upon mutual fund portfolio returns has been ignored, both in conventional evaluation of equity mutual funds and in academic study.

Although funds that invest in unmapped holdings generally state this fact in their prospectuses, access to unmapped holdings data has been difficult in the past to obtain and their significance has been hard to interpret. Other holdings data such as Thompson/CDA was limited exclusively to long-only positions of U.S. equity assets. Works such as Grinblatt and Titman[12][11], Daniel et al.[5], and Wermers[22] were all constrained in their analysis to this filtered dataset. This paper uses the newly available mutual fund holdings database in CRSP that provides more extensive information about unmapped holdings in mutual funds. Holdings such as derivatives, bonds, international stocks, convertibles, and preferreds are present in this database, but remain unmapped into any other securities datasets.

Appendix B

Appendix: Sample Prospectus Statements

This appendix contains a random sampling of prospectus statements that describe investment strategy and policy pertaining to unmapped holdings. Bold type has been added to emphasize the distinguishing characteristics of each group. In general, we observe that funds that fall in all three subsets a, b, and c tend to have prospectus language that grants wide freedom to shift positions in unmapped holdings, consistent with the fund's designation in all three subsets. Prospectus statements from funds only in subsets a and b tend to have words that convey more constrained use of unmapped holdings. They define their actions in unmapped holdings with greater restraint. Prospectus statements from funds only in subsets a and c tend to place stronger emphasis upon hedging practices.

Since our data only cover a few years, divisions between subsets can be vague. In one case, with the Allegiant Mid Cap Value Fund (in groups 4a, 4b, and 4c) and with the Allegiant Small Cap Value Fund (in groups 4a and 4b), the prospectus language is nearly identical for both funds. The prospectus statements for these funds reflect constraint by permitting derivatives use only to hedge against anticipated security purchases or against anticipated changes to portfolio exposure. This suggests that both funds should be classified in only subsets 4a and 4b. If the Allegiant Mid Cap Value Fund is also classified in subset 4c, then some time during the sample the fund must have held an unmapped holdings position that correlated negatively with its mapped holdings. Upon review of the holdings in this fund, we find the following: 1) Unmapped holdings in this fund is indeed generally constrained to very few investments, usually money market holdings and index futures; 2) During our sample period, the fund was invested in money market cash and futures contracts which resulted in a net positive correlation between its mapped and unmapped holdings; 3) At other times in the sample period, the fund continued to hold cash but no longer held futures contracts, which resulted in a negative correlation between mapped and unmapped assets because of negative equity market returns during a few measurement periods.

B.1 Group 1: Mutual Funds Holding Futures and Index Assets

Group 1a contains funds in both groups 1b and 1c. It represents a broad group of funds. Group 1b represents funds with unmapped holdings that are positively correlated with their mapped equity holdings. Group 1c represents funds with unmapped holdings that are negatively correlated with their mapped equity holdings.

B.1.1 Funds in Groups 1a, 1b, and 1c (All Three)

Group 1 funds that appear in subsets 1a, 1b, and 1c hold futures and index assets, but the correlation between their mapped and unmapped assets vary through time. Their prospectus statements declare direct management of futures and index assets that can produce such an outcome.

Activa Value Fund: The Sub-Adviser may enter into derivative positions for the Fund **for either hedging or non-hedging purposes**... The Portfolio stays as close to fully invested as possible through **cash equitization methods by entering into stock index futures contracts**. The Portfolios are permitted to enter into financial futures contracts, stock index futures contracts and related options (”future contracts”) in accordance with their investment objectives. . . Each of the **Funds may trade in derivative contracts to hedge portfolio holdings and for investment purposes**. Hedging activities are intended to reduce various kinds of risks.

DWS Dreman High Return Equity Fund: The fund is permitted, but not required, to use various types of derivatives (contracts whose value is based on, for example, indices, currencies or securities). **Derivatives may be used for hedging and for risk management or for non-hedging purposes to seek to enhance potential gains.** The fund may use derivatives in circumstances where portfolio management believes they offer an economical means of gaining exposure to a particular asset class or to keep cash on hand to meet shareholder redemptions or other needs while maintaining exposure to the market. In particular, the fund may use futures, currency options and forward currency transactions.

B.1.2 Funds in Groups 1a and 1b

American Century Equity Index Fund: The Equity Index fund seeks to match, as closely as possible, the investment characteristics and results of the S&P 500 Index. **The funds may enter into stock index futures contracts in order to manage each fund's exposure to changes in market conditions.** By investing its cash assets in index futures, the fund can stay fully invested in stocks while having easy access to the money.

Calamos Growth Fund: Although **not the principal investments or strategies of the Fund,** the Fund may utilize other investments and investment techniques that may impact performance, including options, futures and other strategic transactions. Each Fund may use interest rate futures contracts, index futures contracts, volatility index futures contracts and foreign currency futures contracts. Each Fund may purchase and write call and put futures options. (these revisions to the prospectus were implemented in 2005, prior to this the fund held no futures)

Munder Index 500 Fund: The Fund may, but is not required to, use derivative instruments for hedging (attempting to reduce risk by offsetting one investment position with another), for cash management (attempting to remain fully invested while maintaining liquidity) or to gain exposure to an investment in a manner other than investing in the asset directly. **The Fund will not use derivatives for speculative purposes (taking a position to possibly increase return).**

B.1.3 Funds in Groups 1a and 1c

MFS New Discovery Fund: The Funds trade financial instruments with off-balance sheet risk in the normal course of their investing activities in order to **manage exposure to market risks** such as interest rates and foreign currency exchange rates. These financial instruments include written options, forward foreign currency exchange contracts, and futures contracts. Futures contracts, options, and options on futures contracts listed on commodities exchanges are reported at market value using closing settlement prices. The objective of the Fund is capital appreciation. The Fund invests, under normal market conditions, at least 65% of its total assets in equity securities of companies of any size that the Fund's manager believes offer superior prospects for growth. The Fund emphasizes companies in the developing stages of their life cycle that offer the potential for accelerated earnings or revenue growth (emerging growth companies) and may invest up to 35% of its total assets in other securities offering an opportunity for capital appreciation. The Fund may engage in short sales.

Hancock John Large Cap Equity Fund: The fund may attempt to take advantage of short-term market volatility by investing in corporate restructurings or pending acquisitions. The fund may invest up to 20% of its assets in bonds of any maturity, with up to 15% of net assets in junk bonds rated as low as CC by S&P or Ca by Moody's and their unrated equivalents. In selecting bonds, the subadviser looks for the most favorable risk/return ratios. The fund may invest up to 35% of assets in foreign securities. The fund may also make limited use of certain derivatives (investments whose value is based on indexes, securities or currencies). In abnormal circumstances, the fund may temporarily invest extensively in investment-grade short-term securities. In these and other cases, the fund might not achieve its goal. The fund may trade securities actively, which could increase its transaction costs (thus lowering performance) and increase your taxable distributions. A fund may invest in derivatives, which are financial contracts with a value that depends on, or is derived from, the value of underlying assets, reference rates or indexes. Derivatives may relate to stocks, bonds interest rates and related indexes. **Funds may use derivatives for many purposes, including for hedging, and as a substitute for direct investment in securities or other assets. Funds also may use derivatives as a way to adjust efficiently the exposure of the funds to various securities and and markets and currencies without the funds actually having to sell existing investments and make new investments.** This generally will be done when the adjustment is expected to be relatively temporary or in anticipation of effecting the sale of fund assets and making new investments over time.

B.2 Group 2: Mutual Funds Holding Foreign Investments

Group 2a represents a broad group (inclusive of 2b and 2c) of funds. Funds in group 2a alone are funds with foreign holdings and no derivative holdings. Their prospectus statements generally permit limited use of derivative assets, but management appears to not actively do so. Funds in subgroups 2a and 2b hold both foreign assets and derivative assets that are positively correlated with their mapped equity holdings. Funds in groups 2a and 2c hold both foreign assets and derivative assets that are negatively correlated with their mapped equity holdings.

B.2.1 Funds in Groups 2a, 2b, and 2c

Acorn Fund: Acorn Fund invests the majority of its assets in U.S. companies, but also may invest up to 33% of its assets in companies outside the United States in developed markets (for example, Japan, Canada and the United Kingdom) and emerging markets (for example, Mexico, Brazil and Korea). **The Fund may enter into a number of derivative strategies, including those that employ futures, options, straddles or similar transactions, to gain or reduce exposure to particular securities or markets.**

CMG Strategic Equity Fund: The Fund's investment objective is to provide long-term growth of capital by investing at least 80% of its total assets in common stocks. Most of the Fund's assets will be invested in U.S. common stocks; however, the Fund may invest up to 33% of its total assets in equity securities, including American Depositary Receipts and Global Depositary Receipts, of foreign issuers when consistent with the Fund's investment objective. The Fund may also invest in real estate investment trusts and securities convertible into or exercisable for stock (including preferred stocks, warrants and debentures). **The Fund may purchase derivative instruments, such as futures, options, swap contracts, and options on futures, to gain or reduce exposure to particular securities or segments of the equity markets. . . . The Fund may use derivatives for both hedging and non-hedging purposes, such as to adjust the Fund's sensitivity to changes in the prices of certain securities held by the Fund, or to offset a potential loss in one position by establishing an opposite position.** The Fund typically uses derivatives in an effort to achieve more efficiently economic exposures similar to those it could have achieved through the purchase and sale of equity securities.

B.2.2 Funds in Group 2a (only)

American Century Vista Fund: Although the portfolio managers intend to invest the fund's assets primarily in U.S. stocks, **the fund may invest in securities of foreign companies.** Most of the fund's foreign investments are in companies located and doing business in developed countries. . . . **The portfolio managers do not attempt to time the market. Instead, under normal market conditions, they intend to keep the fund essentially fully invested in stocks regardless of the movement of stock prices generally.** When the portfolio managers believe it is prudent, the fund may invest a portion of its assets in debt securities, options, preferred stock and equity-equivalent securities, such as convertible securities, stock futures contracts or stock index futures contracts. The fund generally limits its purchase of debt securities to investment-grade obligations. Futures contracts, a type of derivative security, can help the fund's cash assets remain liquid while performing more like stocks. The fund has a policy governing futures contracts and similar derivative securities to help manage the risk of these types of investments. A complete description of the derivatives policy is included in the statement of additional information.

Allianz OCC Renaissance Fund: The Fund seeks to achieve its investment objective by normally investing at least 65% of its assets in common stocks of companies that the portfolio managers believe are trading at prices below their intrinsic values and whose business fundamentals are expected to improve. The Fund may also invest in other kinds of equity securities, including preferred stocks and convertible securities. **The Fund may invest up to 25% of its assets in non-U.S. securities, except that it may invest without limit in American Depository Receipts (ADRs).** The Fund may utilize non-U.S. currency exchange contracts, stock index futures contracts options and other derivative instruments. In response to unfavorable market and other conditions, the Fund may make temporary investments of some or all of its assets in high-quality fixed income securities, cash and cash equivalents. This would be inconsistent with the Fund's investment objective and principal strategies.

BlackRock Aurora Portfolio: Under normal market conditions, the fund invests at least 80% of its total assets in small- and mid-capitalization common and preferred stocks and securities convertible into common and preferred stocks. Although a universal definition of small- and mid-capitalization companies does not exist, the fund generally defines these companies as those with market capitalizations comparable in size to the companies in the Russell 2500 Value Index (between approximately $38 million and $10.8 billion as of December 31, 2005) or a similar index. The fund reserves the right to invest up to 20% of total assets in other securities. These may include other types of stocks, such as large-capitalization stocks, growth stocks, and bonds. The fund may invest up to 5% of total assets in bonds that are below Standard & Poor's BBB or Moody's Baa rating categories, or their unrated equivalents (junk bonds). Split rated bonds will be considered to have the higher credit rating. From time to time the fund may invest without limit in shares of companies through initial public offerings (IPOs). It is possible that in extreme market conditions the fund temporarily may invest some or all of its assets in high quality money market securities. Such a temporary defensive strategy would be inconsistent with the fund's primary investment strategies. The reason for acquiring money market securities would be to avoid market losses. **The management team may, when consistent with the fund's investment goal, buy or sell options or futures on a security or an index of securities (collectively, commonly known as derivatives). The primary purpose of using derivatives is to attempt to reduce risk to the fund as a whole (hedge) but they may also be used to maintain liquidity and commit cash pending investment. The management team also may, but under normal market conditions generally does not intend to, use derivatives for speculation to increase**

returns.

B.2.3 Funds in Groups 2a and 2b

DWS Growth & Income Fund: The fund invests at least 65% of total assets in equities, mainly common stocks. Although the fund can invest in companies of any size and from any country, it invests primarily in large US companies. The fund is permitted, but not required, to use various types of derivatives (contracts whose value is based on, for example, indexes, currencies or securities). **The fund may use derivatives in circumstances where the managers believe they offer an economical means of gaining exposure to a particular asset class or to keep cash on hand to meet shareholder redemptions or other needs while maintaining exposure to the market.**

Fidelity Spartan 500 Index Fund: Geode Capital Management LLC (Geode) normally invests at least 80% of the fund's assets in common stocks included in the S&P 500. The S&P 500 is a widely recognized, unmanaged index of common stock prices. The fund may not always hold all of the same securities as the S&P 500. In addition to the principal investment strategies discussed above, **Geode may use various techniques, such as buying and selling futures contracts, swaps, and exchange traded funds, to increase or decrease the fund's exposure to changing security prices or other factors that affect security values.** (There is no explicit allowance for foreign securities, but the risks section has a complete section regarding risks of the fund's foreign investments.) Geode also intends to follow certain other limitations on the fund's futures and option activities. The fund will not purchase any option if, as a result, more than 5% of its total assets would be invested in option premiums. Under normal conditions, the fund will not enter into any futures contract, option, or swap agreement if, as a result, the sum of (i) the current value of assets hedged in the case of strategies involving the sale of securities, and (ii) the current value of the indices or other instruments underlying the fund's other futures, options, or swaps positions, would exceed 35% of the fund's total assets. These limitations do not apply to options attached to, or acquired or traded together with their underlying securities, and do not apply to securities that incorporate features similar to futures, options, or swaps.

B.2.4 Funds in Groups 2a and 2c

Merger Fund: Under normal market conditions, the Fund invests at least 80% of its assets principally in the equity securities of companies which are involved in publicly announced mergers, takeovers, tender offers, leveraged buyouts, spin-offs, liquidations and other corporate reorganizations. Merger arbitrage is a highly specialized investment approach generally designed to profit from the successful completion of such transactions. **The Fund may employ various hedging techniques, such as short selling and the selective use of put and call options, in an effort to reduce the risks associated with certain of its investments.** For example, when the terms of a proposed acquisition call for the exchange of stock, the shares of the company to be acquired may be purchased and, at approximately the same time, an equivalent amount of the acquiring company's shares may be sold short. Any such short sale will be made with the intention of later closing out ("covering") the short position with the shares of the acquiring company received upon consummation of the acquisition. The purpose of the short sale is to protect against a decline in the market value of the acquiring company's shares prior to the acquisition's completion. The purchase of put options may be similarly used for hedging purposes. The sale of covered call options may also be used by the Fund to reduce the risks associated with individual investments and to increase total investment return. The Fund is permitted to hold both long and short positions in foreign securities.

Tilson Dividend Fund: In seeking to achieve its objective, the Dividend Fund invests in common stocks of companies that the Advisor and the Dividend Fund's investment sub-advisor, Centaur Capital Partners, L.P. (collectively, "Advisors"), believe are undervalued in the securities markets, but which also offer high dividend yields relative to the yield of the broad market averages such as the S&P 500 Total Return Index ("S&P 500"). The Dividend Fund typically invests in common stocks and other equity securities including real estate investment trusts (REITs), publicly traded master limited partnerships (MLPs), royalty trusts, preferred stocks, convertible bonds, convertible preferred stocks, and warrants. **The Advisors also anticipate the moderate and prudent use of covered call option strategies to further the Dividend Fund's goal of current income.** In addition, at the discretion of the Advisors, the Dividend Fund may allocate its capital to bonds and short-term instruments.

B.3 Group 3: Mutual Funds Holding Bonds and/or Cash

American Eagle Twenty Fund: TWENTY FUND is a non-diversified fund that, in normal market conditions, maintains a more concentrated portfolio of approximately, but not less than, 20 securities of primarily American growth companies, without regard to their size. **In normal market conditions, at least 65% of the Fund's total assets must be invested in equity investments,** such as common and preferred stocks, convertible debt securities and options and futures contracts with respect to these securities. The Fund may enter into options and futures transactions to attempt to protect against adverse market price changes when the Fund's investment adviser believes that market conditions make it advisable to do so. In addition, the Fund may employ leverage, sell securities short and buy and sell futures and options contracts on an opportunistic basis to attempt to generate additional investment returns.

Aquila Rocky Mountain Equity Fund: The Fund seeks to achieve its objective by investing primarily in equity securities of companies (”Rocky Mountain Companies”) having a significant business presence in the Rocky Mountain Region. These are companies (i) whose principal executive offices are located in the Rocky Mountain Region, (ii) which have more than 50% of their assets located in the Rocky Mountain Region or (iii) which derive more than 50% of their revenues or profits from the Rocky Mountain Region. Under normal circumstances **the Fund will invest at least 80% of its assets in securities issued by such companies.** In addition to common stocks, equity securities can include preferred stock and convertible fixed-income securities.

GJMB Growth Fund: The investment objective of the GJMB Growth Fund is long term capital appreciation. The Fund intends to remain substantially invested in large cap equity securities. If, however, the advisor believes that adequate investment opportunities that meet the Fund's investment criteria are not currently available, **the Fund may invest up to 50% of its total assets in money market funds, investment grade short-term money market instruments including U.S. Government and agency securities, commercial paper, certificates of deposit, repurchase agreements and other cash equivalents.** The Fund will incur duplicate management and other fees from investments in other mutual funds, primarily money market funds. The Fund may not achieve its investment objective when holding a substantial cash position.

B.4 Group 4: Mutual Funds Holding Derivatives

B.4.1 Funds in Group 4a, 4b, and 4c

Allegiant Mid Cap Value Fund: The Fund invests in value-oriented common stocks of U.S. mid cap companies. The Adviser uses a value-oriented approach and focuses on securities of companies that offer good value and good news. The Adviser generally seeks to invest in companies trading at a discount to intrinsic value; traditionally these companies have price-to-sales, price-to-book and price-to-cash flow ratios that are lower than market averages. Under normal circumstances, at least 80% of the Fund's net assets plus any borrowings for investment purposes will be invested in securities issued by mid cap companies. The Fund may invest up to 20% of its net assets in foreign securities. . . . Each of the Equity Funds . . . may invest in stock index futures contracts and options on futures contracts **in attempting to hedge against changes in the value of securities that they hold or intend to purchase.** Futures contracts may also be based on financial instruments such as stock index option prices. Each of the Equity **Funds may invest in stock index futures contracts in attempting to hedge against changes in the value of securities that it holds or intends to purchase or to maintain liquidity.** Each of these Funds may invest in the instruments described either to hedge the value of their respective portfolio securities as a whole, or to protect against declines occurring prior to sales of securities in the value of the securities to be sold. Conversely, a Fund may purchase a futures contract in anticipation of purchases of securities. In addition, each of these Funds may utilize futures contracts in anticipation of changes in the composition of its holdings for hedging purposes or to maintain liquidity.

Putnam Vista Fund: The fund seeks capital appreciation. Any investment carries with it some level of risk that generally reflects its potential for reward. We pursue the fund's goal by investing mainly in growth stocks. We may invest in foreign investments. **We may engage in a variety of transactions involving derivatives, such as futures, options, warrants and swap contracts. We may make use of "short" derivatives positions,** the values of which move in the opposite direction from the price of the underlying investment, pool of investments, index or currency. **We may use derivatives both for hedging and**

non-hedging purposes, including as a substitute for a direct investment in the securities of one or more issuers. However, we may also choose not to use derivatives, based on our evaluation of market conditions or the availability of suitable derivatives. Investments in derivatives may be applied toward meeting a requirement to invest in a particular kind of investment if the derivatives have economic characteristics similar to that investment.

B.4.2 Funds in Groups 4a and 4b

Allegiant Multi-Factor Small Cap Value Fund: The Fund invests in common stocks of U.S. companies with small stock market capitalizations that are believed to be conservatively valued. Using an analytical process together with fundamental research methods to implement a "value" approach, the Adviser rates the performance potential of companies and buys those securities it considers to be conservatively valued relative to the securities of comparable companies. Under normal circumstances, at least 80% of the Fund's net assets plus any borrowings for investment purposes will be invested in securities of small cap companies (i.e., companies with market capitalizations approximately equivalent to those that fall in the lowest 15% of publicly traded companies represented in the Russell 2000 Value Index.) ... Each of the Equity Funds ... may invest in stock index futures contracts and options on futures contracts **in attempting to hedge against changes in the value of securities that they hold or intend to purchase.** Futures contracts may also be based on financial instruments such as stock index option prices. Each of the Equity **Funds may invest in stock index futures contracts in attempting to hedge against changes in the value of securities that it holds or intends to purchase or to maintain liquidity.** Each of these Funds may invest in the instruments described either to hedge the value of their respective portfolio securities as a whole, or to protect against declines occurring prior to sales of securities in the value of the securities to be sold. Conversely, a Fund may purchase a futures contract in anticipation of purchases of securities. In addition, each of these Funds may utilize futures contracts in anticipation of changes in the composition of its holdings for hedging purposes or to maintain liquidity.

Wasatch Micro Cap Fund: Under normal market conditions, we will invest at least 80% of the Fund's net assets in the equity securities of micro cap companies with market capitalizations of less than $1 billion at the time of purchase. **To a limited extent, the Equity Funds may use derivatives such as options and futures contracts to hedge against certain risks like adverse movements in securities prices.** The Equity Funds may also use options and futures contracts for non-hedging purposes such as seeking to enhance returns. The goal of using options and futures contracts will be to benefit the Equity Funds. However, using options and futures contracts could hurt the Funds' performance if the Advisor incorrectly judges the direction of securities prices.

B.4.3 Funds in Groups 4a and 4c

Allianz OCC Value Fund: The Fund seeks to achieve its investment objective by investing under normal conditions at least 65% of its total assets in common stocks of companies with market capitalizations of more than $5 billion at the time of investment and prices below the Subadviser's estimate of intrinsic value. Intrinsic value refers to the value placed on a company by the Subadviser consistent with its expectation of longer term economic earnings and cash flows. To achieve income, the Fund invests a portion of its assets in income-producing (e.g., dividend-paying) common stocks. The Fund may also invest to a limited degree in other kinds of equity securities, including preferred stocks and convertible securities. The Fund may invest up to 15% of its total assets in foreign securities, except that it may invest without limit in American Depositary Receipts (ADRs). **The Fund may utilize foreign currency exchange contracts, options, or other derivative instruments.** For temporary defensive purposes, or when cash is temporarily available, the Fund may invest in investment grade, short-term debt instruments, including government, corporate and money market securities. If the Fund invests substantially in such instruments, it may not be pursuing its principal investment strategies and may not achieve its investment objective.

Weitz Partners Value Fund: The investment objective of each of the Weitz Equity Funds is capital appreciation. The Weitz Equity Funds seek to achieve their objective by investing primarily in common stocks and a variety of securities convertible into common stocks such as rights, warrants, convertible preferred stock and convertible bonds. **The Fund may buy and sell stock index futures contracts.** A stock index fluctuates generally with changes in the market values of the stocks included in the index. **The Fund's primary purpose in entering into such contracts is to protect it from fluctuations in the value of securities without actually buying or selling the underlying security.** Futures transactions involve brokerage costs and require the Fund to segregate liquid assets to cover its performance under such contracts. The Fund's overall performance could be adversely affected by entering into such contracts if the Adviser's judgment with respect to the investment proves incorrect.

Bibliography

[1] Klaas P. Baks, Andrew Metrick, and Jessica Wachter, *Should investors avoid all actively managed mutual funds? a study in bayesian performance evaluation*, Journal of Finance **56** (20010201), no. 1, p45 – 85.

[2] Christopher R. Blake, Edwin J. Elton, and Martin J. Gruber, *The performance of bond mutual funds*, Journal of Business **66** (1993), no. 3, 370–403.

[3] Stephen J. Brown and William N. Goetzmann, *Mutual fund styles*, Journal of Financial Economics **43** (1997), no. 3, 373–99.

[4] Judith Chevalier and Glenn Ellison, *Risk taking by mutual funds as a response to incentives*, Journal of Political Economy **105** (1997), no. 6, 1167–1200.

[5] Kent Daniel, Mark Grinblatt, Sheridan Titman, and Russ Wermers, *Measuring mutual fund performance with characteristic-based benchmarks*, Journal of Finance **52** (1997), no. 3, Papers and Proceedings Fifty-Seventh Annual Meeting, American Finance Association, New Orleans, Louisiana January 4-6, 1997, 1035–1058.

[6] Roger M. Edelen, *Investor flows and the assessed performance of open-end mutual funds*, Journal of Financial Economics **53** (1999), no. 3, 439–66.

[7] Edwin J. Elton and Martin J. Gruber, *The rationality of asset allocation recommendations*, Journal of Financial and Quantitative Analysis **35** (2000), no. 1, 27–41.

[8] Edwin J. Elton, Martin J. Gruber, Yoel Krasny, and Sadi Ozelge, *The effect of the frequency of holding data on conclusions about mutual fund mangement behavior*, Elton 2006, July 2006.

[9] Eric G. Falkenstein, *Preferences for stock characteristics as revealed by mutual fund portfolio holdings*, Journal of Finance **51** (1996), no. 1, 111–35.

[10] Jason T. Greene and Charles W. Hodges, *The dilution impact of daily fund flows on open-end mutual funds*, Journal of Financial Economics **65** (2002), no. 1, 131–58.

[11] Mark Grinblatt and Sheridan Titman, *Performance measurement without benchmarks: An examination of mutual fund returns*, Journal of Business **66** (1993), no. 1, 47–68.

[12] Mark Grinblatt and Sheridan D. Titman, *Mutual fund performance: An analysis of quarterly portfolio holdings*, Journal of Business **62** (1989), no. 3, 393–416.

[13] David L. Hunter, Eugene Kandel, Russell R. Wermers, and Shmuel Kandel, *Endogenous Benchmarks*, SSRN eLibrary (2009) (English).

[14] Michael C. Jensen, *The performance of mutual funds in the period 1945-1964*, Journal of Finance **23** (1968), 389–416.

[15] Marcin Kacperczyk, Clemens Sialm, and Lu Zheng, *Unobserved actions of mutual funds*, Review of Financial Studies (2006), hhl041.

[16] Jennifer Lynch Koski and Jeffrey Pontiff, *How are derivatives used? evidence from the mutual fund industry*, Journal of Finance **54** (19990401), no. 2, p791 – 816.

[17] Lubos Pastor and Robert F. Stambaugh, *Investing in equity mutual funds*, Journal of Financial Economics **63** (2002), no. 3, 351–80.

[18] Myron Scholes and Joseph T. Williams, *Estimating betas from nonsynchronous data*, Journal of Financial Economics **5** (19771201), no. 3, p309 – 327.

[19] Erik R. Sirri and Peter Tufano, *Costly search and mutual fund flows*, Journal of Finance **53** (1998), no. 5, 1589–1622.

[20] Dee Anne Sjgren, Brian R. Wahlquist, and Seth B. Chertok, *Limits on mutual funds' use of hedging strategies and derivatives. (cover story)*, Investment Lawyer **12** (20051201), no. 12, p1 – 19.

[21] Russ Wermers, *Mutual fund herding and the impact on stock prices*, Journal of Finance **54** (1999), no. 2, 581–622.

[22] ———, *Mutual fund performance: An empirical decomposition into stock-picking talent, style, transactions costs, and expenses*, Journal of Finance **55** (2000), no. 4, 1655–95.

Bibliography

[1] Klaas P. Baks, Andrew Metrick, and Jessica Wachter, *Should investors avoid all actively managed mutual funds? a study in bayesian performance evaluation*, Journal of Finance **56** (20010201), no. 1, p45 – 85.

[2] Christopher R. Blake, Edwin J. Elton, and Martin J. Gruber, *The performance of bond mutual funds*, Journal of Business **66** (1993), no. 3, 370–403.

[3] Stephen J. Brown and William N. Goetzmann, *Mutual fund styles*, Journal of Financial Economics **43** (1997), no. 3, 373–99.

[4] Judith Chevalier and Glenn Ellison, *Risk taking by mutual funds as a response to incentives*, Journal of Political Economy **105** (1997), no. 6, 1167–1200.

[5] Kent Daniel, Mark Grinblatt, Sheridan Titman, and Russ Wermers, *Measuring mutual fund performance with characteristic-based benchmarks*, Journal of Finance **52** (1997), no. 3, Papers and Proceedings Fifty-Seventh Annual Meeting, American Finance Association, New Orleans, Louisiana January 4-6, 1997, 1035–1058.

[6] Roger M. Edelen, *Investor flows and the assessed performance of open-end mutual funds*, Journal of Financial Economics **53** (1999), no. 3, 439–66.

[7] Edwin J. Elton and Martin J. Gruber, *The rationality of asset allocation recommendations*, Journal of Financial and Quantitative Analysis **35** (2000), no. 1, 27–41.

[8] Edwin J. Elton, Martin J. Gruber, Yoel Krasny, and Sadi Ozelge, *The effect of the frequency of holding data on conclusions about mutual fund mangement behavior*, Elton 2006, July 2006.

[9] Eric G. Falkenstein, *Preferences for stock characteristics as revealed by mutual fund portfolio holdings*, Journal of Finance **51** (1996), no. 1, 111–35.

[10] Jason T. Greene and Charles W. Hodges, *The dilution impact of daily fund flows on open-end mutual funds*, Journal of Financial Economics **65** (2002), no. 1, 131–58.

[11] Mark Grinblatt and Sheridan Titman, *Performance measurement without benchmarks: An examination of mutual fund returns*, Journal of Business **66** (1993), no. 1, 47–68.

[12] Mark Grinblatt and Sheridan D. Titman, *Mutual fund performance: An analysis of quarterly portfolio holdings*, Journal of Business **62** (1989), no. 3, 393–416.

[13] David L. Hunter, Eugene Kandel, Russell R. Wermers, and Shmuel Kandel, *Endogenous Benchmarks*, SSRN eLibrary (2009) (English).

[14] Michael C. Jensen, *The performance of mutual funds in the period 1945-1964*, Journal of Finance **23** (1968), 389–416.

[15] Marcin Kacperczyk, Clemens Sialm, and Lu Zheng, *Unobserved actions of mutual funds*, Review of Financial Studies (2006), hhl041.

[16] Jennifer Lynch Koski and Jeffrey Pontiff, *How are derivatives used? evidence from the mutual fund industry*, Journal of Finance **54** (19990401), no. 2, p791 – 816.

[17] Lubos Pastor and Robert F. Stambaugh, *Investing in equity mutual funds*, Journal of Financial Economics **63** (2002), no. 3, 351–80.

[18] Myron Scholes and Joseph T. Williams, *Estimating betas from nonsynchronous data*, Journal of Financial Economics **5** (19771201), no. 3, p309 – 327.

[19] Erik R. Sirri and Peter Tufano, *Costly search and mutual fund flows*, Journal of Finance **53** (1998), no. 5, 1589–1622.

[20] Dee Anne Sjgren, Brian R. Wahlquist, and Seth B. Chertok, *Limits on mutual funds' use of hedging strategies and derivatives. (cover story)*, Investment Lawyer **12** (20051201), no. 12, p1 – 19.

[21] Russ Wermers, *Mutual fund herding and the impact on stock prices*, Journal of Finance **54** (1999), no. 2, 581–622.

[22] ———, *Mutual fund performance: An empirical decomposition into stock-picking talent, style, transactions costs, and expenses*, Journal of Finance **55** (2000), no. 4, 1655–95.

Appendix 2

Endogenous Benchmarks

2.1 Introduction and Motivation

The open-end mutual fund industry is the main venue through which retail investors invest in traded securities. The industry has been growing rapidly during the last few decades – in fact, the total number of equity funds in the U.S. now exceeds the number of stocks traded on the New York Stock Exchange.

A growing number of fund managers follow passive strategies, linking their investments to a particular index. The majority, however, still claim that they can add value to investors by actively managing their portfolios. The basic question facing academics, regulators, and investors alike is whether the active fund managers deliver superior performance to investors, as they claim, or just aggressively solicit additional funds when they are lucky, and downplay their poor performance when they are not. Consequently, the literature on active fund management has been expanding rapidly. It attempts to answer the basic question: are we able to detect whether active management produces superior investment performance that persists over time?

The literature evolved from simple Sharpe ratio comparisons to Jensen's alpha using a single risk factor, to Fama and French (1993) three-factor model, to which Carhart (1997) added the momentum as the fourth factor. Subsequently, the literature tried to account for time-varying factor loadings using conditional β's (e.g., using macroeconomic variables, as in Ferson and Schadt (1996), Ferson and Siegel (2003), and Avramov and Wermers (2006), or with Kalman filters, as in Mamaysky, Spiegel and Zhang (2003). This branch of literature uses exogenously-determined risk factors that are imposed by the researcher on all funds. Most research in this area is limited to US domestic equity funds, for which there exist accepted benchmarks. Extensive literature reviews can be found in Wermers (2000), and Carhart et al. (2000).[1]

An ongoing problem with performance evaluation is the presence of similar strategies among funds, which produce correlated residuals from commonly used models and, therefore, reduce the power of such models to identify skilled fund managers. Jones and Shanken (2005) and Cohen, Coval, and Pastor (2005) recognize this issue, and develop approaches to exploit commonalities in fund returns to improve performance evaluation. However, these papers require fund portfolio holdings data or knowledge about the commonalities that may not be available in practice.

In this paper, we propose a simple approach to account for the commonalities in fund strategies that only uses information on fund returns and the investment objective of the fund. Our approach is to form an additional "factor" from the return on the group of funds to which a given fund belongs, which we call the "endogenous

[1]Another branch tries to attribute the performance to various types of decisions made by the manager: asset allocation, security selection, and high frequency market timing. Such analyses require data on fund holdings, which became available and widely used in the last decade. Examples are Daniel, Grinblatt, Titman, and Wermers (1997), Wermers (2000), and Kosowski et al (2006).

benchmark," since each fund manager chooses the group within which it intends to compete.[2] We note that it is much simpler to account for commonalities using the reference group return, than trying to identify exogenous factors for all of their complex strategies. As such, we postulate several reasons for using this variable as an additional factor. We must stress, that while we call it an additional factor for convenience, we do not imply that it is necessarily priced (this question is left for future research). In this paper, we use it only to improve the estimation of the parameters of interest.

First, let us take the point of view of the investor, who already made the asset allocation decision in terms of choosing the type of funds in which she would like to invest (e.g., Growth vs. Value), but needs help in choosing the best funds within the reference group. Even the least sophisticated investor always has a fallback strategy of equally-weighting (or value-weighting) all funds in the group every period; this tradeable strategy is quite simple.[3] To deserve a higher (than the proportional) share of an investor's portfolio, the fund manager must convince the investor that the fund can deliver superior performance, relative to this naive strategy of investing in the entire group. Superior performance is calculated, of course, by controlling for the risk of the fund relative to the same strategy. Consequently, we can use the group investment as an endogenously-determined benchmark for each fund that belongs to that group. We claim that, by choosing the strategy and advertising herself as managing a growth fund, the manager implicitly chooses the benchmark of growth

[2] In principle, group classification can be based on self-proclaimed goals of the fund manager, or determined ex post from her actual investment strategy. We use the former method.

[3] We only consider no-load funds, thus the cost of rebalancing is low.

funds, thus it is only natural to evaluate her performance using the portfolio of all growth funds during the same time period. In effect, using the entire reference group to benchmark the individual funds for risk has an alternative investment interpretation: it focuses on identifying the best strategies within that group.

Second, suppose we have identified several common risk factors for all funds that belong to a particular group, such as the market return, size, and book-to-market among a group of equity funds. It is likely that these funds expose their investors to additional priced factors that are not observed by the evaluator. If many funds in the group are exposed to these factors, then the group as a whole is exposed to them as well. We then show that the group return can be used to control for the average exposure to these factors, yielding a potentially better estimate of fund α.

Third, even if there are no hidden priced factors, it may well be that many fund managers in a group make similar bets. They may use similar models, have similar behavioral biases, or locate in the same geographical area. This would make the error terms in the individual funds returns (after controlling for the priced factors) correlated across funds in the same group. In such a case the group return also captures these commonalities. Then using group returns in the performance regression improves the estimation of α (see Pastor and Stambaugh (2002) for a detailed econometric argument).

Finally, if the exposure of mutual funds to the known risk factors varies over time, and there is a common component in this variation across funds, then the group return captures this comovement as well. Following Ferson and Schadt (1996), it is easy to see that including group return as an explanatory variable also improves the estimation of α.

Apart from the above advantages, as well as its simplicity, our approach offers additional advantages: it allows evaluation of the performance of any fund, and is not limited to equities. For instance, while risk models are well-developed for most of the mutual funds that we consider (domestic equity, fixed-income), there are many asset classes where this is not so, such as hedge funds. Moreover, the benchmark is a tradeable asset, unlike most of the risk factors in the conventional models–one can easily invest equal amounts of one's wealth in many mutual funds.

To demonstrate the effect of using an "endogenous benchmark," we use data on U.S. mutual funds specializing in equities and in fixed income. We compare the performance of the standard models: the four-factor Carhart (1997) model for equity funds, and the six-factor Elton, Gruber, and Blake (1996) model for fixed-income funds with the same models augmented by the endogenous (group) return as a "factor", and models that use the endogenous return as the only factor.

We begin our analysis by documenting two results from applying the four-factor model to the average group returns in equities. First, the group α can be rather sizeable (positive or negative) and significant during certain periods, even though it is indistinguishable from zero over the long sample period. This suggests that periodic measurements of fund α contain a large group component. Second, we find large and significant correlations between the four-factor model residuals across group portfolios (e.g. Growth and Aggressive Growth). Similar results are found for residuals from the six-factor model for fixed-income funds. This suggests that there are significant unexplained commonalities across groups, which may represent omitted risk factors, which should be accounted for when estimating performance.

Then we show that after controlling for the four- or six-factor models for each fund within a group of equity or debt funds, respectively, the residuals are highly correlated between individual funds within groups, up to 90% in some cases. This suggests strong commonalities in the fund managers behavior, which are not captured by the traditional models. Together, the above results indicate a strong need to account for these commonalities, and we argue that using the group return that captures those is the most parsimonious way. The first indication of the power of this approach becomes evident when we add the orthogonalized endogenous (group) factor to the standard model specifications, the within-group residual correlations drop to half or a third of their former levels, depending on the group. This clearly indicates that the group return is a simple yet powerful tool to control for the commonalities in behavior.

Further, among several equity and fixed-income fund categories, we show that the endogenous factor alone accounts for over 90% of the explanatory power of the more complicated four-factor or six-factor models. Moreover, the endogenous factor loadings are significant for over half of the funds in the traditional equity categories, even after accounting for the standard factors; which is higher than the degree of significance of all the traditional factor apart from the market return.[4]

[4] If one adds to the single factor model the returns of the adjacent groups (well defined in equities), then the explanatory power in the commonly studied groups reaches the levels of the standard models. The main advantage of our approach in this case is that all these factors are tradeable.

Even more noteworthy is that in the less studied equity and fixed-income strategies the endogenous factor alone has a higher explanatory power relative to the standard models. For instance, among Technology funds, the endogenous factor model has $R^2 = 88\%$, while the four-factor model has $R^2 = 81\%$. Similarly, in the Long Term Municipal group, the single factor model yields $R^2 = 94\%$, which dwarfs the 74% of the six factor model.

While this is not the main point of the paper, we also test whether the augmented models estimate the ability of the managers to generate excess returns improves relative to the standard models. We have no clear prediction on that, as better estimation of Jensen's α may actually imply lower predictability of future returns, if these superior abilities are non-existent, and are correctly removed by our procedure. On the other hand, if they do exist, our procedure should improve the predictability. The results indicate that both models seem to predict future performance in some cases, but there is no clear dominance of one over the other.

The paper is organized as follows. Section 2 presents the intuition behind the choice of the group return as an explanatory variable and presents simple econometric arguments for doing so. Section 3 describes the data, while section 4 presents the empirical methodology. Section 5 presents our results, while section 6 concludes.

2.2 Motivating Endogenous Benchmarks

Imagine a group of unsophisticated individuals interested in investing in actively managed mutual funds. They have already obtained expert advice on asset allocation, which means that they have already determined the amounts they would like to invest in each asset class. For simplicity, let us constraint ourselves to equities, and define broad asset classes and corresponding groups of funds: Aggressive Growth, Growth, and Growth and Income. They hire an advisor to suggest the allocation within each group of funds. It is clear to both parties that investors can always save the advisor's fees by using a simple strategy of periodically rebalanced equally weighted portfolio of all the funds in the group. To justify the fee, the advisor must present an evaluation procedure that adds value over their default strategy.

The advisor can force exogenous benchmarks on these investors, but it is unlikely that these benchmarks represent their alternative investment. Instead, we propose treating the investors own default strategy as the benchmark. We refer to this as an "endogenous benchmark", since it is determined by the investor's choice of the asset group, and by the fund's choice to belong to this group. This endogenous benchmark is the cornerstone of the proposed performance evaluation strategy. The investors are advised to modify their naive strategy and invest more in funds that generate positive excess risk-adjusted return, while less in those that generate a negative one.

The basic procedure we propose is to estimate Jensen's α (intercept of the OLS), however instead of the four standard factors (see e.g. Carhart (1997), or Wermers (2000)), we use the excess return on the equally weighted portfolio of all the funds in the group. In computing the periodic return of the default strategy we propose to use all the funds that were available for investment in that group at the beginning of each period. To make this alternative more realistic, we include only the no-load funds in our analyses.

The heterogeneity of the group in terms of investment strategies represents a problem. If all funds in the group invest roughly in the same assets, and vary only in the degree of exposure to the group benchmark, then the estimated Jensen's α indeed captures the excess ability of the manager. However, if funds make significant investments in other assets, deviating from the group's policy in several dimensions, then in addition to the manager's ability the estimated Jensen's α would capture the risk premia associated with these deviations. To control for these, one must use standard risk factors in addition to the group performance. In the context of the US domestic equity this would amount to using a five-factor model of mutual funds' performance evaluation. This approach allows for easy comparison with the existing literature; and, as we show below, represents a solution to econometric problems associated with omitted risk factors and with time varying factor loadings.[5]

2.2.1 Econometric models

In this section we outline the econometric advantages of using the group return in addition to the traditional risk factors. But let us first define the main variables. We denote by $R_{i,t}$ the actual reported return of fund i during month t. This return is net of the management fee, as is customary in fund reporting. Let $m_{i,t}$ be the

[5]The shortcoming of this approach is that it deviates from our basic premise and again imposes external benchmarks on investors. This is especially problematic since many external benchmarks are not easily tradeable.An alternative is to extend the basic premise of our approach across asset classes and treat all group benchmarks as risk factors. This makes the comparison with the existing literature difficult, but is more consistent with our overall approach.

periodic percentage management fee that fund i charged at period t, and $r_{f,t}$ be the risk-free rate for the same period. Together we can use these variables to define the gross excess return of fund i at time t:

$$r_{i,t}^{e} \equiv R_{i,t} + m_{i,t} - r_{f} \tag{2.1}$$

We define by $r_{g,t}^{e}$ the average gross access return of the group of funds to which fund i belongs:[6]

$$r_{g,t}^{e} \equiv \sum_{i=1}^{I} r_{i,t}^{e} \tag{2.2}$$

Next we present three simple specifications of this model to illustrate the potential advantages of adding the endogenous benchmark to the traditional estimation of Jensen's α.

2.2.1.1 Omitted factor model

Suppose that the excess gross return of a fund i at time t is spanned by two priced risk factors $f_{1,t}$ and $f_{2,t}$:

$$r_{i,t}^{e} = \alpha_i + \beta_{1,i} f_{1,t} + \beta_{2,i} f_{2,t} + \epsilon_{i,t}. \tag{2.3}$$

[6] We proceed with identifying all the parameters/variables with subscript g with the group averages of the corresponding fund-specific parameters/variables.

The problem is that an econometrician interested in estimating α_i can only observe the first factor realizations, thus can only run a regression on the observable factor:

$$r^e_{i,t} = \gamma_i + b_{1,i} f_{1,t} + \epsilon_{i,t}. \tag{2.4}$$

We know that in this case the coefficient estimate is biased:

$$E_t \widehat{b}_{1,i} = \beta_{1,i} + P_{1,2}\beta_{2,i}, \tag{2.5}$$

where $P_{1,2}$ is the slope of the regression of $f_{2,t}$ on $f_{1,t}$. Then the expected intercept value is:

$$E_t \gamma_i = \alpha_i + \beta_{2,i} \left(E_t f_{2,t} - P_{1,2} E_t f_{1,t} \right) \tag{2.6}$$

Notice that the estimation error declines in the correlation between $f_{2,t}$ and $f_{1,t}$, assuming $E_t f_{1,t} > 0$. If the two factors are independent, then the error, $\beta_{2,i} E_t f_{2,t}$, is positive and could be quite significant in some periods. Moreover, if the $E_t f_{2,t}$ is large, small variations in the fund exposure to the unobserved risk factor, $\beta_{2,i}$, may change the estimated relative performance of the funds, which is usually of interest.

The proposed approach utilizes the fact that we observe returns of many funds. The average return in the fund population at time t contains the average loading of the funds in the group on the unobserved factor, thus can be used as a proxy. Formally,

$$r^e_{g,t} = \alpha_g + \beta_{1,g} f_{1,t} + \beta_{2,g} f_{2,t} + \epsilon_{g,t}. \tag{2.7}$$

where $\beta_{1,g}$ and $\beta_{2,g}$ are the average loadings on these factors in the group.

Let us first run the following regression:

$$r^e_{g,t} = \gamma_g + b_{1,g} f_{1,t} + \epsilon_{g,t}. \tag{2.8}$$

We know that in this case the coefficient estimate is biased:

$$E_t \widehat{b}_{1,g} = \beta_{1,g} + P_{1,2}\beta_{2,g}, \tag{2.9}$$

where $P_{1,2}$ is the slope of the regression of $f_{2,t}$ on $f_{1,t}$.

We can then use the sum of the intercept and the residual to construct a new variable, which is the group return net of its exposure to the observed factor:

$$r^{net}_{g,t} \equiv r^e_{g,t} - b_{1,g} f_{1,t} = \alpha_g + \beta_{2,g}\left(f_{2,t} - P_{1,2} f_{1,t}\right) + \epsilon_{g,t} \tag{2.10}$$

Rewriting we obtain:

$$\beta_{2,i} f_{2,t} = -\lambda_i \alpha_g + \lambda_i r^{net}_{g,t} + \beta_{2,i} P_{1,2} f_{1,t} - \lambda_i \epsilon_{g,t}, \tag{2.11}$$

where

$$\lambda_i \equiv \frac{\beta_{2,i}}{\beta_{2,g}}. \tag{2.12}$$

Wooldridge (2002) (pp. 63-64) indicates that in case of an omitted variable, one can use a proxy to consistently estimate the relevant coefficients. There are two sufficient conditions for a "perfect" proxy: first, it has to be redundant, i.e. does not add explanatory power to a fully specified model; and second, that the omitted variable is uncorrelated with the observed explanatory variables, after the proxy is partitioned out of it. In the context of Equation (2.11) and the assumptions of this model the first condition is satisfied: group return does not add by itself to the explanatory power, when both factors are observable, i.e.:

$$E[r_{i,t}^{e}|f_{1,t}, f_{2,t}, r_{g,t}^{net}] = E[r_{i,t}^{e}|f_{1,t}, f_{2,t}] \tag{2.13}$$

Substituting (2.11) into (2.3) we obtain:

$$r_{i,t}^{e} = [\alpha_i - \lambda_i\alpha_g] + [\beta_{1,i} + P_{1,2}\beta_{2,i}]f_{1,t} + \lambda_i r_{g,t}^{net} + [\epsilon_{i,t} - \lambda_i\epsilon_{g,t}], \tag{2.14}$$

Unfortunately, the proxy, $r_{g,t}^{net}$ is correlated with $\epsilon_{g,t}$. This satisfies Wooldridge's (2002) definition of an "imperfect" proxy. Wooldridge (2002, p.64) suggests that unless the proxy is highly correlated with the other regressors, it is usually worthwhile to introduce it, even though it also generates an inconsistent estimate, but it reduces the error. In our case, $r_{g,t}^{net}$ is uncorrelated with $f_{1,t}$ by construction, thus it makes sense to introduce it. We have performed numerous simulations estimating the bias in the estimate of α_i with and without the net group return. In all cases the bias without the group return was significantly larger. This leads us to believe that the introduction of $r_{g,t}^{net}$ improves the estimation of α_i.

2.2.1.2 Correlated Errors

An alternative scenario is when there are no unobserved priced factors, but the errors, $\epsilon_{i,t}$, are correlated across funds in the group due to some commonality in behavior of fund managers. Pastor and Stambaugh (2002) point out that under such scenario "...the estimates of the performance measurements can be improved by using the returns on assets not used to define these measures." In other words the α of the fund can be estimated more precisely by including the returns of the non-benchmark assets in the regression, regardless of whether these assets are priced by the benchmarks. The increased precision comes from the correlation between the random components of the passive assets returns and the fund returns. The noise component of the group return and the individual fund return is likely to be positively correlated, and the excess return of the group is likely to be close to zero. Consequently, the estimation of the fund excess return should be improved by adding the group return to the standard set of benchmarks.

Formally let us assume that the individual funds' errors have the following structure (just one priced risk factor assumed for brevity):

$$r_{i,t}^{e} = \alpha_i + \beta_i f_t + \epsilon_{i,t}, \tag{2.15}$$

where

$$\epsilon_{i,t} = \rho_i L_t + \omega_{i,t}, \tag{2.16}$$

and L_t is a zero-mean random variable (not a priced risk factor). Notice, that unlike the case of omitted priced factors, we can obtain unbiased estimates of α_i directly, thus the exercise is supposed to only increase their precision.

The group return is:

$$r^e_{g,t} = \alpha_g + \beta_g f_t + \epsilon_{g,t}, \tag{2.17}$$

where

$$\epsilon_{g,t} \equiv \rho_g L_t + \omega_{g,t}, \tag{2.18}$$

or

$$L_t = \frac{\widehat{\epsilon}_{g,t}}{\rho_g} - \frac{\omega_{g,t}}{\rho_g}. \tag{2.19}$$

Rearranging (2.17) and substituting the expression for L_t into the model of a single fund,we get:

$$r^e_{i,t} = \alpha_i + \beta_i f_t + \frac{\rho_i}{\rho_g}\widehat{\epsilon}_{g,t} + [\omega_{i,t} - \frac{\rho_i}{\rho_g}\omega_{g,t}]. \tag{2.20}$$

By estimating (2.20) together with (2.17) we can obtain more precise estimates of α_i and β_i for every fund.

2.2.1.3 Time-Varying Exposure

Assume again a single risk factor model, but now the funds change their exposure to this factor over time.

Following Ferson and Schadt (1996) we represent the excess return of a fund

i at time t as:

$$r^e_{i,t} = \alpha_i + \beta_{i,t} f_t + \epsilon_{i,t}, \tag{2.21}$$

and

$$\beta_{i,t} = \beta^0_i + k_i Z_t, \tag{2.22}$$

where f_t is again the single risk factor, β^0_i is the average exposure of the fund to this risk factor, and $k_i Z_t$ is the time varying component of this exposure by fund i; $EZ_t = 0$. Again, we are interested in estimating the value of α_i.

Ferson and Schadt (1996) show that by ignoring Z_t one obtains a biased estimate of α_i(if Z_t is correlated with f_t). Usually researchers do not observe Z_t; to estimate it, Ferson and Schadt (1996) propose various macro factors that may affect the loadings (Avramov and Wermers (2006) also use similar factors in estimating the excess performance of mutual funds). But if these factors affect the loadings of every fund, then they must also affect the whole group. A parsimonious way to capture the commonalities in loadings across funds is to use the average return of the whole group. The logic is the same as in the omitted factor case, and the net group return serves as an imperfect proxy for the $Z_t f_t$ term.

2.2.1.4 Predictions

All the econometric models considered above suggest that group return should be used alongside the standard factors to improve the estimates of Jensen's α. In this section we generate predictions that are consistent with the assumptions of the above models.

First, all three models suggest that if one uses the standard risk factors to estimate the α, the estimated residuals should be correlated across groups if the model is applied to group returns, and across funds within a group, if the model is applied to individual funds. If this is the case, then all three models imply that the introduction of the endogenous factor should significantly reduce the correlation between the estimated residuals within group, as it captures common movements in most funds.

Second, all three econometric models suggest that if the average fund in each group has zero excess ability ($\alpha_g = 0$), then by including the endogenous benchmark, we can obtain better estimates of the absolute α_i for each fund. Even if this condition is not satisfied (which happens to be the case in some groups for some periods), then we estimate the excess ability relative to the group, which is indeed the relevant metric to determine the excess performance of a single fund manager. This is because the group α cannot be and is not persistent over longer periods of time.

Third, while we cannot ascertain which of the three models applies, the predictions from all three is that the inclusion of the group return should bring a significant improvement in the explanatory power, and its loading should be significant for a high proportion of funds in each group.

Finally, the percentage of the significant estimates of α, both positive and negative, should decline following the introduction of the endogenous factor, as the resulting estimate is relative to the overall performance of the group. At the same time, controlling for unobserved factors may work in either direction.

In the next sections we test these predictions.

2.3 Data and Empirical Models

We obtain monthly NAV returns for the universe of U.S. mutual funds from the CRSP Mutual Fund Database for the period 1983-2005. We augment the CRSP investment-objective information with similar information from the mutual fund holdings files obtained from Thomson Financial (since objective information from CRSP is more detailed, but often incomplete), using the MFLINKS of Wharton Research Data Services (WRDS) to link the two datasets. Note that this data is free of survivorship bias, as we include all mutual funds that appear in the linked CRSP/Thomson database during a given period, regardless of whether they survive beyond that period.[7]

For brevity of the exposition we only examine five categories of equity funds: Aggressive Growth (AG), Growth (G), Growth and Income (GI), Technology (T), and Small Cap Growth (SCG). Table 1 presents the sample size of mutual funds within each self-declared investment-objective category during each three-year estimation period. There are many funds in all groups except Technology, which reaches a usable size by 1994.

We also examine six categories of pure bond funds: Intermediate and Long-Term Government (ITG and LTG), Intermediate and Long-Term Corporate (ITC and LTC), and Intermediate and Long-Term Municipals (ITM and LTM). We form these categories based on asset allocations specified in the CRSP Mutual Fund Database, with the requirement that a fund belongs to a given category only if it

[7]For instance, many of our tests require funds to have 36 months of consecutive return data, but do not require survival beyond that period.

invested 70% or more of its assets in that asset category (on average over time). The maturity classifications are as follows: 3 to 7 years is intermediate term, and greater than 7 years is long term.

Table 1 also shows counts for the bond fund groups. Except for the ITC group, which only gathered steam by the late 1980's, all groups have many funds throughout the period. This ensures that our average group return estimates are precise.

2.3.1 Equity Fund Models

2.3.1.1 Baseline Model

We use the Carhart four-factor model as our baseline (reference) model, against which we test our alternative specifications that use endogenous benchmarks. The four-factor model applied to fund or group portfolio i is

$$r_{i,t} = \alpha_i + \beta_{i,rmrf} r_{rmrf,t} + \beta_{i,smb} r_{smb,t} + \beta_{i,hml} r_{hml,t} + \beta_{i,umd} r_{umd,t} + \epsilon_{i,t}, \qquad (2.23)$$

where $r_{i,t}$ is the fund i NAV return, plus 1/12 times the annual expense ratio minus T-bills and $r_{rmrf,t}$, $r_{smb,t}$, $r_{hml,t}$, and $r_{umd,t}$ are the return on the CRSP value-weighted portfolio (NYSE/AMEX/Nasdaq) minus T-bills, and the size, book-to-market, and momentum factor returns (available via Ken French's website). We run this regression each three years, including only funds with a complete record of NAV returns and expense ratios during this period.

We also run the same regression on the equal-weighted group return, which we denote by $r_{g,t}$. This regression yields estimates of α_g, $\epsilon_{g,t}$, and $r_{g,t}^{net} = \alpha_g + \epsilon_{g,t}$.

2.3.1.2 Endogenous Models

Our alternative specification adds the estimate of $r_{g,t}^{net}$ from the four-factor model on the group return (hereafter, orthogonalized group return) to the estimation equation for every fund in the group.[8] In this second stage, we apply the following model to each individual mutual fund within group g:

$$r_{i,t} = \alpha_i + \beta_{i,rmrf} r_{rmrf,t} + \beta_{i,smb} r_{smb,t} + \beta_{i,hml} r_{hml,t} + \beta_{i,umd} r_{umd,t} + \beta_{i,y_{g,t}} r_{g,t}^{net} + \epsilon_{i,t}, \tag{2.24}$$

As stated above, the regression of Equation (2.24) helps to control for misspecification in the form of omitted factors or dynamic factor loadings, or for problems with cross-sectionally (across funds) correlated four-factor model residuals.[9]

However, we gain further insight by applying a third model, which relies solely on the endogenous benchmark in a single-factor model:

$$r_{i,t} = \alpha_i + \beta_{i,r_{g,t}} r_{g,t} + \epsilon_{i,t}. \tag{2.25}$$

This model has the advantage of conserving regression degrees-of-freedom, but will

[8] To ensure that funds are reasonably assigned to groups, we first omit all funds with an $R^2 < 0.35$ in the simple regression of the fund return on the group average return during a particular three-year period. The results without this omission are qualitatively the same, but this procedure eliminates "imposter" funds.

[9] In some specifications we replace the $r_{g,t}^{net}$ by the residual from the same equation, $\epsilon_{g,t}$. This way we can compare the estimates gross and net of the group $\alpha_{g,t}$

not capture different relative factor loadings among funds. For instance, if some funds hold larger capitalization, low momentum stocks, while others hold smaller capitalization high momentum stocks within the Growth category, then the regression of Equation (2.25) will perform much worse than the regression of Equation (2.24). However, it is instructive to determine how well the single endogenous factor model performs in a scenario (i.e., domestic equity funds) where the benchmarks are "tried and true," to gain insights into how it may perform when the proper benchmarks are not known (e.g., among more exotic fund groups, as well as in more complex investments: pension funds, hedge funds).

2.3.2 Fixed-Income Fund Models

2.3.2.1 Baseline Model

Our reference model for bond funds is based on the Blake, Elton, and Gruber six factor model, plus an added equity factor, $RMRF$, which was described in the last section:

$$r_{i,t} = \alpha_i + \beta_{i,IG} r_{IG,t} + \beta_{i,LG} r_{LG,t} + \beta_{i,IC} r_{IC,t} + \beta_{i,LC} r_{LC,t} + \beta_{i,MBS} r_{MBS,t} \quad (2.26)$$

$$+\beta_{i,HY} r_{HY,t} + \beta_{i,RMRF} r_{RMRF,t} + \epsilon_{i,t}, \quad (2.27)$$

where the factors capture risk premia from (1) Intermediate-Term Governments (IG), (2) Long-Term Governments (LG), (3) Intermediate-Term Corporates (IC), (4) Long-Term Corporates (LC), (5) Mortgage-Backed Securities (MBS), High-

Yield corporate bonds (HY), and the excess return on the CRSP NYSE/AMEX/Nasdaq portfolio ($RMRF$). Again, we run this regression every three years, including only funds with a complete record of NAV returns and expense ratios during this period. Funds are reassigned to groups, based on their self-declared investment objectives at the beginning of each three-year period.

2.3.2.2 Endogenous Models

Our alternative specification uses the first-stage intercept plus residual of the group return from the six-factor model of Equation (2.27), $y_{g,t} = \alpha_g + \epsilon_{g,t}$, in a second-stage regression.[10] As with the equity funds above, we apply the following second-stage model to each individual mutual fund within group g:

$$r_{i,t} = \alpha_i + \beta_{i,IG} r_{IG,t} + \beta_{i,LG} r_{LG,t} + \beta_{i,IC} r_{IC,t} + \beta_{i,LC} r_{LC,t} + \beta_{i,MBS} r_{MBS,t} \quad (2.28)$$

$$+\beta_{i,HY} r_{HY,t} + \beta_{i,RMRF} r_{RMRF,t} + \beta_{i,y_{g,t}} y_{g,t} + \epsilon_{i,t}, \quad (2.29)$$

Our third fixed-income model is:

$$r_{i,t} = \alpha_i + \beta_{i,r_{g,t}} r_{g,t} + \epsilon_{i,t}. \quad (2.30)$$

[10] Again, to ensure that funds are reasonably assigned to groups, we first omit all funds with an $R^2 < 0.35$ in the regression of the fund return on the group average return during a particular three-year period.

2.4 Results

2.4.1 Fund Groups

2.4.1.1 Equity

Most of the extant literature on mutual fund performance has focused on equity funds. A priori, we know that the explanatory power of the standard four-factor model is very high, thus we would expect that adding an additional factor will only make a small contribution to the explanatory power of the model. Nevertheless, since rankings by α can change dramatically with only small model changes, the addition of our endogenous factor may change the relative performance of various funds by removing the additional risk to which these funds are exposed in varying degrees.

We first ask a very simple question: do fund groups on average exhibit excess returns after the standard four risk factors are controlled? We run regression (2.23) individually for each of the five equity group returns and estimate the intercept. Table 2 Panel A shows that except for the Growth and Income group, all other equity groups exhibit significant α's during some periods. Moreover, the estimates can be quite sizably positive or negative, depending upon the period. At the same time, none of the groups exhibit a consistent excess performance over the entire sample period, which indicates that these estimates represent temporary risk exposures which should be considered when estimating performance.

We continue with the estimation of the residual correlations across groups.

If the four-factor model of Equation (2.23) adequately explains returns, then the residuals of the first-stage regressions are purely random. In such cases our model has no hope of improving the estimation. To test this we compute the across-group correlations between equal-weighted group return residuals of the above regressions. Under the null we expect to find correlations that are not significantly different from zero. Panel A of Table 3 shows that this is hardly the case, as we can reject the null of no correlation between group residuals quite frequently: 44 out of 74 possible correlation pairs are positive and significant at the 5% confidence level. Indeed, some of the correlations are extremely high.

For instance, Aggressive Growth funds are highly correlated with Growth, Small Cap Growth, and Technology funds over all three-year periods. Growth is correlated with the Growth and Income, but the latter is not correlated with the Aggressive Growth category. Notice, that 7 out of 10 possible correlations over the entire 24-year period are positive and significant, which indicates that this is not period-specific phenomenon. These findings indicate that some important unmodeled common factors are present, but these omitted factors are not common to all groups.[11] It seems that there are at least two independent factors that account for the structure of across-group correlations that we observe.

The above findings suggest that to improve the estimates of the fund's excess returns, we should control for additional, yet unidentified factors. As these factors are likely to be common to many funds in the group, using the group return is an

[11]Although there may be some ambiguity with self-identifying as an "aggressive-growth" fund vs. a "growth fund," we would, nevertheless, not expect widespread similarities between residuals from the four-factor model.

obvious choice.

2.4.1.2 Fixed-Income

Panel B of Table 2 presents the estimates of Jensen's α using a standard six-factor model. While the estimates are occasionally large relative to the fixed income funds expected returns, most are not significant. Yet, we still observe the same pattern of period-related sign of the α's across various groups.

Panel B of Table 3 presents the estimates of the cross-group residuals correlations for the eight fixed-income categories. There are several high correlations between group four-factor residuals. For instance, Intermediate-Term and Long-Term Corporates are highly correlated, and IT and LT Munis are correlated as well. These high correlations indicate that the six-factor model does not capture all of the commonalities in bond fund returns, although the intensity is smaller than in the equities. Similarly, fewer fixed income groups' residuals exhibit significant correlations over the entire sample period, but those that do are very large.

The high correlations in equity and bond groups are likely driven by the heavy loadings on the same securities across groups. We conjecture that these loadings can be controlled by using the groups returns.

2.4.2 Individual Equity and Fixed-Income Funds

Next we turn to evaluating the performance of the standard models at the individual fund level. Panel A of Table 4 presents the percentage of positive and significant (at the 5% confidence level) pairwise correlations between the individual funds' residuals from the four-factor regressions (Equation 2.23) out of all possible pairwise correlations in the group (see rows labeled 4-Factor). First, notice that the percentage of significant correlations ranges between 20% to 85%, averaging 43% over all time periods (random investments would predict 5%). This clearly suggests significant commonalities in the investment strategies across funds that are not captured by the four-factor model. Clearly, some groups are better modeled by the standard four-factor model than others. For instance, the Growth category is much better modeled than the Technology category, where the average percentage of significant positive correlations is 67%.

Panel B shows a very similar picture for the bond funds using six-factor model. The average proportion of significant correlations is much higher; 61%, and for some groups goes as high as 85%. (e.g., LT Munis). Some categories fit the standard model much better than others.

To illustrate the impact of our approach, we present the same correlations after including the orthogonalized group return ($r_{g,t}^{net}$) in the regression. For equities, we run the five-factor regression in Equation (2.24) (rows labeled "5 Factor"), while for bond funds, we run the seven-factor model in Equation (2.29) (rows labeled "7 Factor"). All groups exhibit substantially reduced magnitude of significant positive pairwise correlations when the group return is added to the standard model. This is true for equity funds (Panel A), where the average proportion of positive significant correlations goes down from 43% to 21%, and for fixed-income funds (Panel B), where the same proportion goes down even further from 61% to 23%. This clearly indicates that including the group return captures much of the common variation in the individual funds returns. As one would expect, the effect on the traditional groups, where the extant model works well, is lower than the effect on the groups where the standard model does not fit well, such as Technology funds or LT Munis funds. In the latter groups adding the endogenous factor makes the biggest difference in reducing the individual fund pairwise correlations.

We interpret the results of Tables 3-4 as evidence that (1) standard factors leave a significant degree of unexplained common covariation among funds within a group and across groups, and (2) a significant part of this covariation can be explained by including an endogenous benchmark (group return) on the individual fund level.[12] This provides strong indication that one should include the group return in the performance regressions. Below we show the effect of this inclusion on Jensen's α estimation.

2.4.3 Jensen's α Estimation

In this section, we present a comparison between different model in terms of their explanatory power and Jensen's α estimation. First, we compare three models of equities: the traditional four-factor model (Equation (2.23)), the five-factor model (Equation (2.24)), and the Endogenous-factor only model (Equation (2.25)). Panel A of Table 5 presents the results.

[12] We also note that the above results are obtained using fund classification based on the self-declared investment objectives, which could be manipulated. We believe that further gains could likely be made by grouping funds based on commonalities in beginning-of-period holdings or factor loadings.

The four-factor model performs very well in terms of explanatory power, which is not surprising, since much effort was spent to identify factors that explain equity fund returns. In the standard categories - AG, G, GI, and SCG, the adjusted R^2 are in the 83-89% range. In these categories, the marginal addition of the group return cannot be high – it adds at most 3% to the adjusted R^2. However, we would like to point out a different set of statistics: the endogenous factor alone (which is tradeable) has an adjusted R^2 between 74% and 84%, which is very high for a model with only a single factor. Moreover, even though the common four factors had been already taken into account, for between 41% to 69% of funds in these four equity categories the coefficient on the net group return is positive and significant at the 5% level. This is much higher in most cases than the proportion of funds with significant loadings on the traditional factors, except for the market return. The results speak loudly that including the group return into the estimation regression even in the standard equity categories is at least as important as the three non-market factors in the four-factor model.

The Technology group results are different: the returns in this group are less well explained by the four-factors, as indicated by our earlier correlation results, and the R^2 of the group return alone is much higher than the R^2 from the four-factor model. For over 85% of funds, the net group return coefficient is positive and significant. These results indicate that including the group return as a control for unobserved commonalities is useful in all equity funds, but especially in groups that are not well explained by the standard four-factor model, i.e. groups with more specialized investment strategies.

The first column represents the traditional models. The estimation of α in the expanded regression is presented in two ways that differ only by their intercept. The estimate labeled α_1 corresponds to the modification of equation (2.24), where only the residual of the group return equation is used as a regressor. It represents the gross estimate of the individual fund's α.The estimate labeled α_2 corresponds to the equation (2.24), and represents the individual fund's α_i net of the group α_g. Notice that the first estimate, α_1, is more likely to be significant positive or negative than the four-factor model α. However, when we control for the level of the group's α, the percentage of funds with significant positive or negative excess performance estimates drops to the levels almost indistinguishable from random draws (5%). There are marginally more funds that do significantly worse, than those doing significantly better.

The Technology group again exhibits a different picture: the effects are highly non-symmetrical for poorly and well performing funds. First, the proportion of funds with positive significant estimates of α decline quite substantially due to the introduction of the group return from 11.3% to 8.4%. At the same time the proportion of negative significant estimates (funds with rather bad abilities, who may have jumped on the Technology bandwagon) goes up from 2.6% to 8.4%. Many poorly managed Technology funds showed average or even superior returns relative to the standard factors.

As we are mostly interested in the funds that exhibit α's that are significantly different from zero (to pursue and to avoid), we are interested to know whether our procedure improves this allocation of funds into these categories. The test in presented in Table 6, Panel A. We partition all funds (aggregated across groups) into six buckets based on the size of their t-statistic in the four-factor regression. The buckets contain funds with positive and negative α's within one standard deviation from zero (in each direction separately), between one and two standard deviations, and more than two standard deviations. For each bucket, we then compute the percentage of funds that fall in each of the similarly defined buckets using the five-factor model (with and without controlling for the group α). If the two models correctly identify the excess ability of fund managers, then we expect to observe high percentages on the diagonal of the matrix and very low off the diagonal. Without controlling for the group α, we predict that the percentage of funds with significant α's identified by the four-factor model that is also significant in the five-factor model is going to be high, and indeed it is: over 99%. However, after controlling for the group α the picture changes dramatically: only 49.9% of the badly performing funds and 58.8% well-performing funds under the four-factor model remain identified as such. The percentages in the opposite direction are much higher. Almost none of the remaining funds switch sign, but many become insignificant. The least agreement is within the category of funds that are identified to have an estimate that is between one and two standard deviations away from zero.

Next we compute the rank correlation between the estimates from the two models in a cross-section of funds. The idea is to test whether the effect of the group factor inclusion is in the change of the relative magnitude of the coefficients obtained in the two models, or in the changes in their standard errors. Table 7 Panel A shows that the rank correlations are very high (except for some periods in the

Technology group), indicating that the main impact of the group return introduction is on the precision of the estimates, rather on their relative magnitude (rankings).

Finally, we perform an additional test aimed to see whether the group factor inclusion improves the identification of the significantly better and worse run funds. As the main difference between the two models is in the precision of the estimation, we base this test on the differences between the t-statistics of the α estimates under the two models. If a fund's α estimate t-statistic under the five-factor model is higher than the t-stat under the four-factor model, then this fund is included in the long portfolio, and is assigned the weight equal to the differences in the t-statistics. If the relation is reversed, then the fund is included in the short portfolio with similar weight assignment (absolute value of the difference). The top left part of Panel A of Table 9 presents the excess returns (over the predicted from the five-factor model using the prior period loadings) over 12 months for the long only, short only and long-short portfolios. The top right part presents the same, but the actual returns. The bottom repeat it, but instead of the t-stat differences, uses the t-stat ranking differences. The results clearly indicate that while the raw returns of these portfolios are not distinguishable, the excess returns relative to the expectations are significant in four out of five categories when the actual t-statistics are used to form the portfolios. This indicates that when the five-factor model reduces the significance of the fund's excess performance relative to the four-factor model, such excess performance indeed does not show persistence.

Panels B of the same tables present the similar evidence for bond funds, using the six-factor model (Equation (2.27)), the seven-factor model (Equation (2.29)), and the Endogenous-factor only model (Equation (2.30)). Panel B of Table 5 shows that while the results for the Corporate and the Government Groups are well-explained by Elton and Gruber's six factors, the group return is still positive and significant in a large proportion of funds. Indeed, since the managers of the Government groups have a rather limited leeway in their investment choices, their behavior is well explained by the loadings on the indices of appropriate duration. In these groups the group return is significant only in about a third of funds, probably due to common variation in loadings over time. In the Corporate groups the percentage of the funds with significant positive loadings on the group return rises to over 60%, and exceeds the percentage of significant loadings for any other explanatory variable. The Munis groups are not well explained by the standard factors, thus the group return alone yields a much higher explanatory power than the six factors. Similarly, the estimates of α change little following the introduction of the group return in the Government and the LT Corporate groups, but very significantly in the IT Corporate and especially in the Munis groups. The percentage of the funds with significant α's is higher than in the equity category.

Panel B of Table 6 indicates a stronger agreement between the two models in fixed income funds - but it is far from perfect. Over 20% of all funds identified by the six-factor model as having a significant excess performance (positive and negative separately) are no longer identified as such after the group return is included. Finally, the rank correlations are very close to one, indicating again that the main effect of the group return introduction is not on the relative magnitude of the coefficients, but rather on their standard errors.

Based on these and earlier results we conclude that introducing the group return improves the estimation of the manager's ability in equity and fixed income funds, especially in determining its significance.

2.4.4 Out-of-Sample Performance

Our last tests use the endogenous models to form portfolios based on the previous three-year estimates of α, to determine whether the endogenous group factor improves the identification of funds with true skills. We cannot predict a priori whether the introduction of the group return should improve the predictive ability or reduce it, as it has two opposing theoretical effects. On the one hand, the introduction of the group return improves the estimation of the fund's α, thus, if the ability is persistent, the predictive power of the expanded model should improve. On the other hand, as we saw in many cases, the introduction of the group return controls for the risk factors that are not captured by the standard models. If the returns of these unobserved factors are autocorrelated, the introduction of the group return should reduce the predictive power (and correctly so).[13] Thus we can only estimate the net impact of the two effects and report it.

We start by ranking all funds in a group on their base-model α's (gross of 1/12 times the expense ratio) over a given 36-month period, and forming equally-weighted portfolios. For equity funds, the portfolios are formed separately using the four-factor model and the five-factor model. For fixed-income funds, we use the

[13]Related, if funds in a group all have skills, then the group benchmark will extract this from each individual fund in the endogenous regressions. However, in this case, our model correctly concludes that we are just as well off investing in the group as in the individual fund.

six-factor model, and the seven factor model. The first portfolio is binary - long position in the funds with significant positive α, and short position in the funds with significant negative α. The next three portfolios are based on the ranking of the estimated α's: long position on the highest tercile (quartile and quintile), and short position on the lowest tercile (quartile, quintile), respectively.

We then compute the excess returns on these equally-weighted portfolios over the following 12 months using the five-factor model on all of them. Table 8 presents the results of the long-short strategy. We aggregate over all periods, and present the average excess returns and their corresponding standard errors by groups.

Panel A shows the results for equity funds. In every category, except for Aggressive Growth, the traditional four-factor model performs reasonably well, but in practically all cases, the results for one of the five-factor models are stronger in magnitude and many times in significance. A similar conclusion emerges from the fixed income fund (Panel B), but the magnitudes are much smaller.

We conclude that while we can better estimate the excess return in the fund's historical performance, the improved estimation improves the predictive ability only marginally. Recall, that the prediction on this question is ambiguous, thus it is not an evidence against our model.

2.5 Conclusion

The contribution of this paper is to propose a conceptually simple and easily implementable way to control for economy-wide, and asset-group-wide fluctuations in the markets that affect fund returns. We propose adding the group return, which is endogenous, in addition to the exogenously determined factors in the standard regressions estimating the fund loadings and Jensen's α. This approach has intuitive support, since it represents the investment strategy that is always feasible for investors. We also show that the group fund is an imperfect proxy (Wooldridge 2002) for the unobserved risk factors, or common loadings on individual securities, that should be added to the regression to improve the estimation. We show that this addition improves the estimation of the Jensen's α under several plausible scenarios found in the literature. We also demonstrate that the effect of this introduction on the estimates of the excess return in equity and fixed income funds is non-trivial and changes the classification of various managers.

Bibliography

[1] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, CA, 2001), Chap. 1.

[2] N. Bloembergen, *Nonlinear Optics* (Benjamin, Reading, MA, 1977).

[3] Y.R. Shen, *Principles of Nonlinear Optics* (Wiley, New York, 1984).

[4] P.N. Butcher and D.N. Cotter, *The Elements of Nonlinear Optics* (Cambridge University Press, Cambridge, UK, 1990).

[5] R.W. Boyd, *Nonlinear Optics* (Academic Press, San Diego, CA, 1992).

[6] A.C. Newell and J.V. Moloney *Nonlinear Optics (Advanced Topics in the Interdisciplinary Mathematical Sciences)* (Westview Press, Boulder, CO, April 1992).

[7] D. Marcuse,*Light Transmission Optics* (Van Nostrand Reinhold, New York, 1982), Chaps. 8 and 12.

[8] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, CA, 2001), Chap. 2.

[9] P. Diament, *Wave Transmission and Fiber Optics* (Macmillan, New York, 1990).

[10] V.E. Zakharov and A.Shabat, Sov. Phys. JETP **34**, 62 (1972)

[11] R.H. Hardin and F.D. Tappert, SIAM Rev. Chronicle **15**, 423 (1973).

[12] R.A.Fisher and W.K. Bischel, Appl. Phys. Lett. **23**, 661 (1973); J. Appl. Phys **46**, 4921 (1975).

[13] J.W. Cooley and J.W. Tukey, Math. Comput. **19**, 297 (1965).

[14] R. Trebino, D.J. Kane, "Using phase retrieval to measure the intensity and phase of ultrashort pulses: frequency resolved optical gating," J. Opt. Soc. Am. B **10**, 1101 (1993).

[15] D.J. Kane, R. Trebino, "Characterization of Arbitrary Femtosecond Pulses Using Frequency-Resolved Optical Gating," IEEE J. Quant. Elect. **29**, 571 (1993).

[16] D.J. Kane, R. Trebino, "Single-shot measurement of the intensity and phase of an arbitrary ultrashort pulse by using frequency-resolved optical gating," Opt. Lett. **10**, 1101 (1993).

[17] P. O'Shea, M. Kimmel, X. Gu, R. Trebino, "Highly simplified device for ultrashort-pulse measurement," Opt. Lett. **26**, 932 (2001).

[18] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, CA, 2001), Chap. 3.

[19] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, CA, 2001), Chap. 4.

[20] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, CA, 2001), Chap. 10.

[21] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, CA, 2001), Chap. 7.

[22] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, CA, 2001), Chap. 6.

[23] R.H. Stolen, E.P. Ippen, and A.R. Tynes, Appl. Phys. Lett. **20**, 62 (1972).

[24] E.P. Ippen and R.H. Stolen, Appl. Phys. Lett. **21**, 539 (1972).

[25] R.G. Smith, Appl. Opt. **11**, 2489 (1972).

[26] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, CA, 2001), Chap. 9.

[27] G.P. Agrawal, *Nonlinear Fiber Optics* (Academic Press, San Diego, CA, 2001), Chap. 8.

[28] D.L. Hart, Arthur F. Judy, Rajarshi Roy and James W. Beletic, Phys. Rev. E **57**, 4757 (1998); D.L. Hart, Arthur F. Judy, T.A.B. Kennedy, Rajarshi Roy and K. Stoev, Phys. Rev. A **50**, 1807 (1994).

[29] K. Ito, *Lectures on Stochastic Processes* (Tata Institute of Fundamental Research, Bombay, 1960).

[30] R.L. Stratanovich, *Topics in the Theory of Random Noise*, Vols I. and II. (Gordon & Breach, New York, 1963).

[31] H. Risken, *The Fokker-Planck Equation* (Springer-Verlag, Berlin, 1989).

[32] M.J. Werner and P.D. Drummond, J. Comput. Phys. **132**, 312 (1997).

[33] P.D. Drummond and I.K. Mortimer, J. Comput. Phys. **93**, 144 (1991).

[34] S.J. Carter, Phys. Rev. A. **51**, 3274 (1995).

[35] J.R. Thompson and Rajarshi Roy, Phys. Rev. A **43**, 4987 (1991).

[36] W.H. Press, S.A. Teukolsky, W.T. Vetterling and B.P. Flannery, *Numerical Recipes in Fortran: The Art of Scientific Computing* (Cambridge University Press, Cambridge, 1992).

[37] C. Headley, G.P. Agrawal, IEEE J. Quantum Electron. **QE-31**, 2058 (1995), C. Headley, G.P. Agrawal J. Opt. Soc. Am. B. **13**, 2170 (1995).

[38] S.H. Perlmutter, M.D. Levenson, R.M. Shelby and M.B. Weisman, Phys. Rev. Lett. **61** 1388, 1988.

[39] F. Kh. Abdullaev, J.H. Hensen, S. Bischoff and M.P. Sorensen, J. Opt. Soc. Am. B. **15**, 2424 (1998); F. Kh. Abdullaev, J.G. Caputo, and Nikos Flytzanis, Phys. Rev E. **50**, 1552 (1994).

[40] William H. Glenn, IEEE J. Quantum Electron. **QE-25**, 1218 (1989).

[41] R. Trebino. *Frequency-Resolved Optical Gating: The Measurement of Ultrashort Laser Pulses* (Kluwer Academic 2002).

[42] G.P. Agrawal *Nonlinear Fiber Optics* (Academic, San Diego, 2001).

[43] J.M. Dudley, X. Gu, L. Xu, M. Kimmel, E. Zeek, P. O'Shea, R. Trebino, S. Coen, R.S. Windeler, "Cross-correlation frequency resolved optical gating analysis of broadband continuum generation in photonic crystal fiber: simulations and experiments," Opt. Express **10**, 1215 (2002).

[44] Q.D. Liu, J.T. Chen, Q.Z. Wang, P.P. Ho, and R.R. Alfano, "Single pulse degenerate-cross-phase modulation in a single-mode optical fiber," Opt. Lett. **20**, 542 (1995).

[45] T. Sylvestre, H. Maillotte, E. Lantz, and D. Gindre "Combined spectral effects of pulse walk-off and degenerate cross-phase modulation in birefringent fibers", Journal of Nonlinear Optical Physics and Materials 6, 313-320 (1997).

[46] Q.D. Liu, L. Shi, P.P. Ho, R.R. Alfano, R.J. Essiambre, and G.P. Agrawal, "Degenerate cross-phase modulation of femtosecond laser pulses in a birefringent single-mode fiber," IEEE Photon. Tech. Lett. **9**, 1107 (1997).

[47] F.G. Omenetto, B.P. Luce, D. Yarotski and A.J. Taylor, "Observation of chirped soliton dynamics at l= 1.55 mm in a single-mode optical fiber with frequency-resolved optical gating," Opt. Lett. **24**, 1392 (1999).

[48] F.G. Omenetto, Y. Chung, D. Yarotski, T. Shaefer, I. Gabitov and A.J. Taylor, "Phase analysis of nonlinear femtosecond pulse propagation and self-frequency shift in optical fibers," Opt. Commun. **208**, 191 (2002).

[49] F.G. Omenetto, J.W. Nicholson, B.P. Luce, D. Yarotski, A.J. Taylor, "Shaping, propagation and characterization of ultrafast pulses in optical fibers," Appl. Phys. B **70**[Suppl.], S143 (2000).

[50] N. Nishizawa and T. Goto, "Experimental analysis of ultrashort pulse propagation in optical fibers around zero-dispersion region using cross-correlation frequency resolved optical gating," Opt. Express **8**, 328 (2001).

[51] N. Nishizawa and T. Goto, "Trapped pulse generation by femtosecond soliton pulse in birefringent optical fibers," Opt. Express **10**, 256 (2002).

[52] N. Nishizawa and T. Goto, "Characteristics of pulse trapping by use of ultrashort soliton pulses in optical fibers across the zero-dispersion wavelength," Opt. Express **10**, 1151 (2002).

[53] N. Nishizawa and T. Goto, "Ultrafast all optical switching by use of pulse trapping across zero-dispersion wavelength," Opt. Express **11**, 359 (2003).

[54] , K. Ogawa, M.D. Pelusi, "Characterization of ultrashort optical pulses in a dispersion-managed fiber link using two-photon absorption frequency-resolved optical gating," Opt. Commun. **198**, 83-87 (2001).

[55] R.A. Altes, "Detection, estimation, and classification with spectrograms," J. Acoust. Soc. Am. **67**(4), 1232 (1980).

[56] A. Christian Silva, "GRENOUILLE - Practical Issues," unpublished.

[57] J. Garduno-Mejia, A.H. Greenaway, and D.T. Reid, "Designer femtosecond pulses using adaptive optics," Opt. Express **11** 2030 (2003).

[58] P. O'Shea, M. Kimmel, X. Gu, R. Trebino, "Increased-bandwidth in ultrashort-pulse measurement using an angle-dithered nonlinear-optical crystal," Opt. Express **7**, 342 (2000).

[59] P. O'Shea, M. Kimmel, R. Trebino, "Increased phase-matching bandwidth in simple ultrashort-laser-pulse measurements," J. Opt. B **4**, 44 (2002).

[60] S. Akturk, M. Kimmel, P. O'Shea, R. Trebino, "Measuring pulse-front tilt in ultrashort pulses using GRENOUILLE", Opt. Express **11**, 491 (2003).

[61] K. J. Blow, D. Wood, "Theoretical Description of Transient Stimulated Raman Scattering in Optical Fibers," IEEE J. Quant. Elect. **25**, 2665 (1989).

[62] R.H. Stolen, J.P. Gordon, W.J. Tomlinson, J. Opt. Soc. Am. B **6**, 1159 (1989).

[63] P.V. Mamyshev and S.V. Chernikov, Sov. Lightwave Commun. **2**, 97 (1992).

[64] C. Headley III, *Ultrafast Stimulated Raman Scattering in Optical Fibers, Ph.D. Thesis, University of Rochester, NY (1995).*