ABSTRACT

Title of Dissertation: THE DEVELOPMENT OF SYMBOLIC MAGNITUDE UNDERSTANDING IN EARLY CHILDHOOD

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The path towards mathematics success starts early, as initial numerical knowledge sets the foundation for children's later mathematics learning. In particular, young children's knowledge of numerical magnitudes, like knowing that seven is more than three, is theorized to play an important role in their mathematical development. In support of this perspective, there is consistent evidence that symbolic magnitude skills, or knowledge of how written numerals and number words can be ordered and compared, predict mathematical achievement in childhood and adulthood. Yet less is known about the antecedents and consequents of symbolic magnitude understanding in preschool. The goal of the present study was to understand whether symbolic magnitude knowledge in early childhood relates to later math achievement and identify foundational numerical and general cognitive skills that underlie the development of symbolic magnitude knowledge.

One hundred and forty Head Start preschoolers aged 3 – 5 years old were assessed in the winter and spring of the school year to test a theory-driven conceptual model of symbolic magnitude development. Specifically, children's knowledge of the cardinal value represented by numbers, such as knowing that the number word "four" can be represented with four objects, was hypothesized to predict their symbolic magnitude understanding, with children's symbolic magnitude understanding in turn predicting their symbolic addition skills controlling for children's executive functioning skills, age, and gender.

There was significant evidence in favor of the proposed conceptual model, specifically that children's cardinality skills predicted their concurrent and later symbolic magnitude understanding; children's symbolic magnitude understanding predicted their later addition skills; and children's executive functioning skills predicted each of their numerical skills uniquely. Findings suggest symbolic magnitude understanding fully mediates the relation between children's cardinality and addition skills, and both domain-general executive functioning and domain-specific cardinality and magnitude skills assessed in the winter explain a similar amount of variability in children's spring addition skills. These findings will be used to inform the design of comprehensive early numeracy interventions to help parents, teachers, and researchers best support the mathematical development of young children.

THE DEVELOPMENT OF SYMBOLIC MAGNITUDE UNDERSTANDING IN EARLY CHILDHOOD

by

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# Chapter 1: Introduction

## Statement of the Problem

Mathematics achievement has been linked to a host of critical adult outcomes, including educational attainment, rates of full-time employment, and wages (Geary, 2011; Ritchie & Bates, 2013). The path towards mathematics success starts early, as initial numerical knowledge sets the foundation for children's later mathematics learning. Children's numeracy skills in preschool and the beginning of kindergarten predict their math achievement in later grades (Duncan et al., 2007; Watts, Duncan, Siegler, & Davis-Kean, 2014). Longitudinal data has shown that children's performance on counting and simple arithmetic problems at 54 months old significantly predicts mathematical achievement through age 15, even when controlling for early reading skills, short-term memory, attention, and demographic variables (Watts et al., 2014). Importantly, evidence of the strong and consistent relation between early and later mathematical skills underscores the significance of building strong numeracy skills in early childhood. Furthermore, variability in starting points matters: children who enter school behind in math tend to have a slower rate of improvement throughout early elementary school, effectively widening the achievement gap with their higher performing peers (Jordan, Kaplan, Ramineni, & Locuniak, 2009).

In particular, young children's knowledge of numerical magnitudes, like knowing that seven is more than three, is theorized to play an important role in their mathematical development. Early on in childhood, children learn the numerical symbols of their culture, including number words (e.g., one, two, three) and numerals (e.g., 1, 2, 3). Symbolic magnitude understanding refers to children's understanding of how the number symbols can be compared to one another and ordered. This type of numerical magnitude knowledge is thought to support children's broader mathematical achievement through grounding the higher order manipulations of more advanced math problems, by helping students learn arithmetic, select appropriate strategies when solving math problems, and identify more plausible answers (Mussolin, Nys, Leybaert, & Content, 2016). For example, a child who understands the magnitudes of the numbers 10 and 1 may be less likely to say their sum is equal to 101. The integrated theory of numerical development posits that an understanding of the magnitude of numbers underlies math learning across the lifespan (Siegler, 2016; Siegler & Braithwaite, 2017; Siegler & Lortie-Forgues, 2014; Siegler, Thompson, & Schneider, 2011). In support of this perspective, there is consistent evidence that symbolic magnitude skills predict mathematical achievement in childhood and adulthood (Schneider et al., 2016; 2018). Recent meta-analyses suggest the relation between symbolic magnitude knowledge and broader mathematical competence across samples of children and adults is statistically significant and positive, with a medium effect size (*r*s= .20 – .44; Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider et al., 2016; 2018).

Much of the early mathematical literature has focused on describing how children develop an understanding of number symbols, such as learning to count, map number words to their corresponding quantities, and recognize written numerals (e.g., Gelman & Gallistel, 1978; Ginsburg & Baroody, 2003; Wynn, 1992). However, less is known about the development of symbolic magnitude knowledge. While the integrated theory of numerical development clearly describes children's progressive understanding of magnitudes from 0 to 10, 0 to 100, fractions, and ultimately, all rational numbers, Siegler and colleagues acknowledge the theoretical debates on the antecedents of early symbolic magnitude knowledge – namely, the answer to how children learn what number words and numerals mean (Siegler, 2016; Siegler & Braithwaite, 2017; Siegler & Lortie-Forgues, 2014; Siegler, Thompson, & Schneider, 2011). Furthermore, few empirical studies linking magnitude knowledge to their general mathematical competence in early childhood have involved measures of symbolic magnitude knowledge, raising the question of whether the relation exists for children prior to the start of kindergarten, a period in which children are rapidly developing symbolic magnitude knowledge for numbers 1 through 10 (Siegler, 2016). Understanding the developmental trajectories of younger children throughout their early acquisition of numerical skills may help to address whether symbolic magnitude knowledge in early childhood relates to later math achievement, and if so, what foundational numerical and general cognitive skills underlie improvements in symbolic magnitude knowledge. Answering these questions will ultimately help researchers identify and develop interventions to support children at risk for later mathematics difficulties.

**Study Rationale**

Symbolic magnitude knowledge is thought to play an important role in supporting children's later mathematical understanding (De Smedt, Noel, Gilmore & Ansari, 2013; Fazio et al., 2014; Mussolin et al., 2016; Siegler, 2016). In particular, symbolic magnitude knowledge is thought to help young children learn arithmetic, in part by helping them identify more plausible answers to arithmetic problems. However, the vast majority of research examining the relations between magnitude and broader mathematical performance has relied on studies of school-aged children and adults (e.g., De Smedt, Verschaffel, & Ghesquiere, 2009; Friso van den Bos et al., 2015; Schneider et al., 2016, 2018; Simms, Clayton, Cragg, Gilmore, & Johnson, 2016). It is less clear whether preschool children's symbolic magnitude skills would be related to their early arithmetic understanding. Furthermore, the integrated theory of numerical development is less clear on the precursors of symbolic magnitude knowledge, namely, the early numerical skills that predict preschool-aged children's symbolic magnitude understanding. Identifying the antecedents and consequents of children's symbolic magnitude understanding in preschool would help to answer critical theoretical and practical questions about the best ways to support children's early mathematical learning.

One potential antecedent to children's symbolic magnitude knowledge is their understanding of the cardinal value of numbers, or the connection between number symbols and their corresponding quantities. For example, a child who understands the cardinal value of the number three would be able to give their parent three forks to set the table and understand that counting "one, two, three" forks means they have three in total (Gelman & Gallistel, 1978; Wynn, 1992). The cardinal-knowledge proposal suggests that once children understand cardinality, or the quantities represented by individual number words and numerals, they undergo a significant shift in their numerical knowledge that launches their understanding of symbolic mathematics (Geary et al., 2017). In particular, this proposal states that only after young children understand cardinality can they begin to understand the relations among symbolic numbers (i.e., symbolic magnitude knowledge; Geary et al., 2017; Geary & vanMarle, 2018). Under this proposal, children who acquire cardinality knowledge at a younger age will have more knowledge of symbolic magnitude and arithmetic at school entry than children who acquire cardinality knowledge at an older age, who are left with a shorter timeframe to develop their symbolic number skills.

Beyond foundational numerical skills like cardinality, it is also possible that domain-general executive functioning (EF) skills influence children's symbolic magnitude understanding. Executive functioning (EF) is defined as a set of top-down cognitive processes consisting of three core skills: inhibition, working memory, and cognitive flexibility (Diamond, 2013). Inhibition refers to a person's ability to control their behaviors, thoughts, or emotions to do what is appropriate rather than what is intuitively appealing. For example, a child with strong inhibitory control would be better able to raise a quiet hand rather than shout out their answer to a teacher's question. Working memory refers to a person's ability to hold information in mind and mentally manipulate it without a physical representation. Children with strong working memory skills would be able to remember multiple steps in a series of instructions and implement them in order. Cognitive flexibility, also called attention shifting, is the ability to change one's perspective or solve a problem in a creative way. Cognitive flexibility relies on inhibition and working memory skills and develops later on in childhood (Diamond, 2013). A child who is more cognitively flexible would be better able to adapt to novel situations, like being asked to line up in age order when the typical instruction is to line up in height order.

EF skills relate to children's skill development broadly and predict their math achievement specifically (Blair & Razza, 2007; Duncan et al., 2007; Geary, 2011). EF skills may allow children to concentrate, pay attention to, and learn from symbolic magnitude information in their surroundings. Furthermore, EF skills may directly affect children's ability to inhibit incorrect responses, keep relevant information in mind, and shift their attention to the relative features of the tasks that assess symbolic magnitude understanding. Some empirical evidence supports this link: young children's EF skills positively correlate to their symbolic magnitude knowledge (Geary, 2011; Geary & vanMarle, 2016; Passolunghi, Vercelloni, & Schadee, 2007; Toll & Van Luit, 2014), and predict improvements in symbolic magnitude understanding over time (Geary, Hoard, Nugent, & Byrd-Craven, 2008).

Though both cardinality and executive functioning skills have been implicated in the development of symbolic magnitude understanding, few studies have examined their effects in concert. Furthermore, it remains an open question as to whether children's symbolic magnitude understanding in preschool significantly predicts performance on more advanced mathematical tasks, such as single digit whole number arithmetic, above and beyond other foundational numerical and executive functioning skills. Thus, the goal of the present study is to test a more comprehensive conceptual model of children's symbolic magnitude understanding with a sample of preschoolers (Figure 1). By investigating the developmental trajectory of symbolic magnitude understanding in early childhood, the results of the present study may be used to inform adaptive interventions appropriate for a broad cross-section of children with varying skills. To date, there is little work on the development of young children’s symbolic magnitude understanding, particularly in combining domain-general and domain-specific theoretical perspectives in a unified model. Unpacking the development of children’s symbolic magnitude understanding will pave the way for future comprehensive intervention measures.

**Study Design**

The present study used a short-term longitudinal design to test the proposed conceptual model of preschool children’s symbolic magnitude development across the school year. Previous research suggests that children at these ages are beginning to acquire symbolic magnitude understanding, yet there remains significant variability in their performance on magnitude comparison and number line estimation tasks (Ramani & Scalise, 2018; Scalise, Daubert, & Ramani, 2017, 2019). Furthermore, there is evidence that there are detectable improvements to preschool children’s symbolic magnitude understanding across a span of three to four months (Geary & vanMarle, 2018).

Preschool children aged 3- to 5-years old completed assessments in the winter and spring of the school year. In the winter, children’s cardinality, symbolic numerical magnitude knowledge, symbolic addition skills, and executive functioning skills were assessed. In the spring, children’s cardinality, symbolic numerical magnitude knowledge, and symbolic addition skills were assessed for a second time.

Measured variable path analysis (MVPA) was used to test whether the resulting data supported the hypothesized relations in the conceptual model. MVPA is an analytic technique that falls under the broad methodological umbrella of structural equation modeling, and is used to test theoretically driven hypotheses about structural relations among variables. Unlike traditional linear regression techniques, which require multiple analyses to test the mediating effect (Baron & Kenny, 1986), MVPA allows for the simultaneous estimation of multiple direct and indirect effects within a single unified model.

## Research Aims and Hypotheses

Broadly, the goal of this study is to investigate the development of symbolic magnitude understanding in early childhood, as well as its relation to other numerical and executive functioning skills. Informed by the conceptual model, children's early cardinality knowledge was expected to predict their later symbolic magnitude knowledge, with children's early symbolic magnitude knowledge in turn predicting their later arithmetic skills, operationalized as symbolic addition skills. Further, children's executive functioning skills were expected to predict their performance on all measures of numerical knowledge, in keeping with domain-general models of cognitive development.

**Aim 1. To determine whether children's cardinality knowledge predicts their symbolic magnitude knowledge.** In light of recent empirical findings showing preschoolers' knowledge of the cardinal principle significantly predicts their improvement on symbolic magnitude comparison tasks (Geary & vanMarle, 2018; Le Corre, 2014), it was expected that children's cardinality knowledge would predict their symbolic magnitude knowledge. Unlike previous studies, the present study included more rigorous measures of children's cardinality and symbolic magnitude knowledge with more tasks per construct and more individual trials per task. The measures allowed for additional descriptive analyses of the relations among these skills, including identifying children with partial cardinality knowledge (e.g., knowledge of cardinal values of one, two, three, and so on) and comparing their accuracy on symbolic magnitude trials involving smaller versus larger numbers.

**Aim 2. To determine whether children's symbolic magnitude knowledge predicts their addition skills.** Though research with older children and adults suggests that symbolic magnitude understanding predicts later mathematics achievement, research testing this hypothesis with preschool children is limited. The present study examined whether preschoolers' symbolic magnitude understanding at the beginning of the school year predicted their spring performance on symbolic addition assessments, controlling for other numerical and EF skills. Symbolic magnitude understanding in the winter was expected to predict children's spring addition scores above and beyond other foundational numerical skills, in keeping with the moderate relations observed from studies of school-aged children and adults (Schneider et al., 2016, 2018). While previous research has identified cardinality knowledge as a more substantive predictor of their later mathematics achievement than symbolic magnitude knowledge (Geary & vanMarle, 2016, 2018), the present study included more comprehensive measures of children's symbolic magnitude knowledge that could account for additional variability in their broader math ability.

**Aim 3. To determine whether children's symbolic magnitude knowledge fully mediates the relation between their cardinality and addition skills.** Recent longitudinal work suggests that preschoolers' cardinality knowledge is the most significant predictor of their later math achievement (Geary & vanMarle, 2018), while research with older children implicates symbolic magnitude understanding as a key predictor of math achievement (Schneider et al., 2016, 2018). In the present study, it was expected that children's cardinality skills would significantly predict their symbolic magnitude knowledge, such that symbolic magnitude knowledge would fully mediate the previously documented relations between these numerical skills and math achievement. In the landscape of early numerical skills, symbolic magnitude knowledge is arguably the most complex – integrating knowledge of symbolic numbers (both number words and numerals), an understanding of what quantity each number refers to, and the consecutive and non-consecutive relations among numbers from across the count sequence. Thus, symbolic magnitude knowledge may represent the culmination of the foundational numerical skills and was therefore expected to fully mediate the relation between children's cardinality and later symbolic addition skills.

**Aim 4. To investigate the relative contributions of domain-specific numerical skills and domain-general executive functioning (EF) skills in explaining children's addition skills.** Domain-general theories of cognitive development highlight the role of EF skills in children's mathematical development, yet little research to date has directly compared the relative effects of children's domain-specific numerical and domain-general EF skills on their later math achievement. It was expected that both numerical and EF skills would be significant predictors of children's arithmetic skills, however, the direct and indirect effects of children's numerical skills were expected to have a larger standardized effect on their addition skills than the direct and indirect effects of children's EF skills.

## Contribution to the Field

The present study is one of the first to investigate the development of children's symbolic magnitude understanding during preschool, a period when numerical understanding undergoes rapid improvement and yields wide variability in children's skills. The use of measured variable path analysis (MVPA) allows for the testing of the complete model of structural relations simultaneously, moving beyond traditional linear regression approaches that investigate the predictors of one skill at a time. This research has both theoretical and practical implications, by investigating which numerical and executive functioning skills support symbolic magnitude understanding and providing evidence as to whether symbolic magnitude understanding is as important to general mathematics achievement in early childhood as it is in samples of older children and adults. Results from the current study will be used to inform the design of comprehensive early numeracy interventions for young children, providing parents, teachers, and researchers with insight into key skills to focus on to best early mathematics learning.

## Chapter 2: Review of the Literature

### Overview

This review presents the research to date on the development of symbolic magnitude understanding in early childhood. It begins with an overview of how children learn about symbolic numbers, describing the hierarchical development of their symbolic numerical skills that in turn support more advanced mathematical skills. Next, it provides an overview of numerical magnitude understanding embedded in a theoretical framework of numerical development across the lifespan, followed by a review of the relations between symbolic magnitude understanding in early childhood with concurrent and later mathematics achievement. It subsequently reviews potential predictors of children's symbolic magnitude understanding in preschool, including both numerical and executive functioning skills. The concluding section presents a discussion of remaining gaps in the literature, including the identification of antecedents and consequents of symbolic magnitude understanding in early childhood and the strength of the relation between children's symbolic magnitude understanding and later mathematics achievement in models with rigorous numerical and executive functioning controls.

### Understanding Symbolic Numbers

During early childhood, children learn symbolic number representations in the form of number words and written numerals. This process unfolds over the course of several years, with many interrelated symbolic skills developing in conjunction. One of the first symbolic number skills that children develop is counting. The development of children's counting skills has been heavily debated by cognitive development researchers, who seek to answer questions including when children develop an understanding of symbolic numbers, how researchers can accurately measure their knowledge, and whether their skills relate to a type of innate numerical understanding. Piaget claimed that children do not understand what symbolic numbers mean until age 5 – 6, when they enter the concrete operations stage and can successfully pass the conservation of number task (Piaget, 1952). However, Gelman and Gallistel (1978) argued that children who can count in accordance with a set of principles demonstrate an understanding of numbers. These five counting principles include: 1) the one-one principle, that every item is only counted once; 2) the stable order principle, that sequence of counting words is the same across all situations; 3) the cardinal principle, that the last number word said when counting a set represents the total magnitude of the set; 4) the abstraction principle, that any kinds of objects can be counted; and 5) the order-irrelevance principle, that the objects in a set may be counted in any order as long as the other principles are also upheld (Gelman & Gallistel, 1978). Gelman and Gallistel argued that the five counting principles are implicit to young children, meaning that children tend to count according to these principles without being explicitly aware of or able to describe them. For example, even when young children use idiosyncratic count lists (e.g., "one, two, seven…"), they tend to apply the same inaccurate count lists across counting situations, in keeping with the counting principles (Gelman & Meck, 1983).

In contrast, Carey and colleagues argued against implicit, core representations of the five counting principles, claiming children's understanding of counting relies instead on core representations for identifying and tracking individual objects and comparing sets of objects (Le Corre & Carey, 2007). They further argue that young children first learn the verbal count list, then use that information to derive the counting principles described by Gelman and Gallistel (Le Corre & Carey, 2008).

Despite the theoretical debates on the origins of children's counting knowledge, much is descriptively known about the symbolic number skills of children at different ages. Children demonstrate knowledge of the verbal counting sequence as early as 2 and 3 years old (Gelman & Gallistel, 1978). Though young children may be able to recite the count sequence aloud (e.g., "one, two, three…"), they are not immediately proficient at assigning number words to the correct numerosities (Wynn, 1992). Although children may learn to count and identify canonical sets of objects at a young age (e.g., "two shoes – one, two" at 2 years old; Mix, 2009), their verbal count lists are often still idiosyncratic at 2.5 years (e.g., "one, two, six…"; Gelman & Gallistel, 1978). It may take children until they are 3 years or older to be able to count aloud from 1 to 10 correctly (Fuson, 1988; Siegler & Robinson, 1982), during which time they are also developing the ability to count sets of objects up to 5 (Baroody & Price, 1983; Schaeffer, Eggleston, & Scott, 1974) and correctly label small sets of four or less without counting (Schaeffer et al., 1974).

Though young children may have the ability to recite the counting sequence in order and count small sets of objects correctly does not mean that they understand what the numbers they are using refer to (i.e., that counting "one, two, three" objects means that there are three objects total in the set). A child who understands the cardinal value for a given number would be expected to provide the correct number of objects when asked by an experimenter, for example, giving an experimenter three yellow rubber ducks from their pile of ten ducks when the experimenter asked them for three. This more sophisticated understanding of number is termed cardinality, and is thought to develop for individual numbers up to a year after children can correctly count to the same quantities (Wynn, 1992). Children's understanding of cardinality develops in numerical order, with children progressing from not knowing the meaning of "one" (often referred to as pre-knowers), to knowing the meaning of "one" but not two or more (one-knowers), and so forth (Wynn, 1992). After children successfully demonstrate an understanding of the numbers five and six by being able to reliably provide sets of five and six objects when asked, they are thought to understand the cardinal principle (Sarnecka & Carey, 2008; Wynn, 1992).

The acquisition of the cardinal principle is theorized to be a gateway for children's broader mathematical understanding. In particular, the cardinal-knowledge proposal suggests that once children understand the quantities represented by individual number words and numerals, they undergo a significant shift in their numerical knowledge that launches their understanding of symbolic mathematics (Geary et al., 2017; Spaepen, Gunderson, Gibson, Goldin-Meadow, & Levine, 2018). After children understand the cardinal principle, but not beforehand, they show evidence of understanding that each number can be generated by adding one to the previous number (the successor principle; Sarnecka & Carey, 2008; Spaepen et al., 2018), only sets that are perfectly matched on quantity are numerically equivalent (equinumerosity; Sarnecka & Wright, 2013), and can consistently implement matching rules based on numerical quantity rather than visual similarity of stimuli (crossmapping; Mix, 2008). Moreover, children's cardinality skills in preschool predict children's number set knowledge in kindergarten, including numerical quantity and symbol mapping tasks and symbolic magnitude comparisons (Le Corre, 2014; Moore et al., 2016). Cardinality knowledge in preschool is also predictive of later math but not reading achievement, signaling that it may be a domain-specific gateway skill (Geary et al., 2017). Proponents of the cardinal-knowledge proposal suggest that children who acquire cardinality knowledge at a younger age will demonstrate higher levels of math achievement at school entry than children who acquire cardinality knowledge at an older age, who have a shorter window to develop their more advanced symbolic number skills (Geary et al., 2017; Geary & vanMarle, 2018; Spaepen et al., 2018).

In addition to learning the meanings of symbolic number words, young children must also learn about written number symbols (i.e., numerals). Although knowledge of number words via the verbal count sequence occurs first in development, young children quickly develop knowledge of numerals. By age four, approximately one-quarter of children can correctly label numerals 1 – 9 (Ginsburg & Baroody, 2003). Numeral knowledge represents an important link to children's success in formal mathematical knowledge, such as basic arithmetic, since much of school-based mathematics requires children to correctly translate and produce written numerals (Purpura, Baroody, & Lonigan, 2013). Indeed, preschool children's ability to identify written numerals is a significant predictor of their later mathematics ability (Chard et al., 2005; Clarke & Shinn, 2004; Purpura et al., 2013). Just as children learn both the sequence of verbal number words and how those words map onto different quantities, children must learn to connect the verbal number word to the correct written numeral and how the written numerals map onto different quantities. There is some evidence that 4 – 5 year old children first recognize written numerals by mapping the corresponding number word and then begin mapping between written numerals and their corresponding cardinal values (Knudsen, Fischer, Henning, & Aschersleben, 2015). However, children's cardinal knowledge of number words was significantly greater than their cardinal knowledge of numerals and more accurate than their verbal labels of numerals at age 4 (Knudsen et al., 2015). This suggests that children may be using cardinal knowledge of number words to understand the cardinal values of number symbols, with performance on traditional verbal cardinality tasks yet again serving as a gateway skill to more advanced symbolic numeral knowledge. Moreover, this implies that while children's knowledge of numerals is key for performance on mathematical tasks that require decoding or producing written numerals, it may be less important for tasks that include verbal labels for number symbols (i.e., for problems read aloud by an experimenter).

In sum, young children must learn symbolic number representations, beginning with knowledge of the verbal count sequence, followed by an understanding of the quantities or cardinal values associated with number words, the corresponding written numerals, and the cardinal values associated with numerals. In particular, knowledge of the cardinal principle is thought to serve as a gateway for children's broader symbolic mathematical skills. However, a wealth of theoretical and empirical evidence also points to the importance of numerical magnitude understanding, or knowledge of how symbolic numbers can be compared and ordered, in grounding mathematical learning from infancy throughout adulthood.

## Numerical Magnitude Understanding

Mathematics is a domain where knowledge builds over time, requiring mastery of basic numerical skills like arithmetic to lay the foundation for more complex topics like algebra. In the integrated theory of numerical development, Siegler and colleagues propose that an understanding of the magnitude of numbers underlies math learning across the lifespan (Siegler, Thompson, & Schneider, 2011; Siegler & Lortie-Forgues, 2014; Siegler, 2016). An accurate representation of numerical magnitude may ground the higher order manipulations of more advanced math problems, by helping students learn arithmetic, select appropriate strategies when solving math problems, and identify more plausible answers (Mussolin et al., 2016). Indeed, magnitude understanding is correlated with mathematics achievement in children and adults (Schneider et al., 2016, 2018), and the effects of magnitude interventions have generalized to other math skills, including arithmetic (e.g., Honoré & Noël, 2016; Park & Brannon, 2013). Magnitude understanding is hypothesized to follow a developmental trajectory, beginning with the ability to discriminate between non-symbolic magnitudes, followed by magnitudes of small whole numbers (1-10), broadening to include larger whole numbers, and ultimately encompassing all rational numbers (Siegler, 2016). During the early childhood period, the focus of magnitude skills development is on refining non-symbolic representations and assigning meaning to symbolic representations of numbers (Carey, 2009; Halberda &

Feigenson, 2008; Siegler, 2016; Wynn, 1992). Each of these progressions is described in more detail below.

**Non-Symbolic Magnitude**

From infancy, humans have the ability to discriminate between sets of different magnitudes, such as a set of 4 objects and a set of 12 objects (Izard, Sann, Spelke, & Streri, 2009; Cordes & Brannon, 2008; Xu & Spelke, 2000). This imprecise non-symbolic magnitude understanding is frequently referred to as Approximate Number Sense (ANS), shared at a basic level by other species (Brannon, 2006; Dehaene, 2011). It is typically assessed using forced-choice comparison tasks of sets of objects, where children and adults are asked to view two sets of objects and say which has the numerically greater amount (Figure 2). The ANS is characterized by ratio dependence, whereby sets of objects with a larger ratio (e.g., 1 dot versus 10 dots, 1:10) are easier to discriminate than sets of objects with a smaller ratio (e.g., 10 dots versus 11 dots, 10:11). Over time, children and adults improve their magnitude discrimination abilities: infants are able to discriminate between sets of objects when they differ by larger ratios of 1:3 or 1:2, while 4- and 5-year old children can discriminate between ratios of 4:5 or 5:6, and some adults can reliably discriminate between ratios of 9:10 (Halberda & Feigenson, 2008; Izard et al., 2009; Xu & Spelke, 2000). The most challenging ratio that an individual is able to consistently and accurately compare is captured by a Weber fraction (*w*), a common measure of individuals' ANS acuity. Interestingly, children can compare numerical magnitudes in visual and auditory formats and other types of magnitude, including surface area and time, at equivalent ratios (Brannon, Lutz, & Cordes, 2006; Lipton & Spelke, 2003; Siegler, 2016; vanMarle & Wynn, 2006; Xu & Spelke, 2000).

Improvements in ANS acuity across the lifespan are generally attributed to increased experience comparing quantities, as well as general maturation in neural circuitry and related executive functioning skills that support the ANS (Halberda & Feigenson, 2008).

**Symbolic Magnitude**

During early childhood children begin to learn symbolic representations for numbers, namely number words (e.g., one, two, three) and written numerals (e.g., 1, 2, 3). The integrated theory of numerical development describes the second time point of magnitude understanding as the process of linking newly acquired small number symbols (1 – 10) with foundational non-symbolic magnitude knowledge (Siegler, 2016). While symbolic number knowledge refers broadly to skills dependent on number symbols, including counting and identifying written numerals, symbolic magnitude knowledge refers specifically to the understanding of how numbers represented by symbols can be compared and ordered (Fazio et al., 2014).

One method of assessing symbolic magnitude knowledge is through comparisons of number words and written symbols (i.e., "which is more, 2 or 7?"; Figure 2). Children with a strong understanding of symbolic magnitude for small whole numbers would be expected to be highly accurate at comparing pairs of numbers and saying which number is bigger. As with non-symbolic magnitude comparison tasks, children and adults exhibit ratio dependence on symbolic magnitude comparison tasks in their accuracy and reaction times, and are both more accurate and quicker to respond to comparisons of larger ratios than smaller ratios (Halberda & Feigenson, 2008). A second method of measuring symbolic magnitude knowledge is number line estimation tasks, where children mark where a target number would fall on a bounded left-to-right number line (Figure 2). Children who have a strong understanding of symbolic magnitude are expected to make estimates that are fairly close to where the target number would actually appear on a fully partitioned number line. Although young children with knowledge of the counting sequence may correctly order the magnitudes of their estimates, they often compress the space between their estimates of larger numbers (Siegler, 2016). For example, preschoolers' estimates of the numbers 1-10 frequently show logarithmic patterns, with more space between smaller numbers and less space between larger numbers, failing to represent the standard unit of distance between all numbers and their neighbors. As children gain more experience with smaller whole numbers through counting and basic arithmetic their estimates on a 1-10 number line become more linear, while their estimates on 1-100 or 1-1000 number lines show logarithmic curves (Berteletti, Lucangeli, Piazza, Dehaene, & Zorzi, 2010; Laski & Siegler, 2007; Siegler & Booth, 2004).

## Symbolic Magnitude and Mathematics Achievement

Descriptive reviews on the relations between magnitude and achievement report a consistent relation between symbolic magnitude knowledge and math achievement across samples of children and adults (De Smedt et al., 2013; Merkley & Ansari, 2016). Two meta-analyses to date have calculated an estimate of the effect of symbolic magnitude knowledge on math achievement. Schneider and colleagues

(2016) calculated the overall effect size estimate of the relation between symbolic magnitude comparison tasks and mathematics achievement to be *r* = .302, equivalent to a 9 percent overlap between the variance in participants' symbolic magnitude comparison proficiency and the variance in mathematical competence. This effect size was significantly larger than the estimated effect of non-symbolic magnitude comparison tasks and mathematics achievement (*r* = .241). In general, knowledge from magnitude comparison tasks appears to have stronger effects on math achievement measures that rely on magnitude directly, such as mental arithmetic or early math assessments that focus on counting and informal addition (Schneider et al., 2016). A second meta-analysis by Schneider and colleagues (2018) estimated the effect of the relation between symbolic number line estimation tasks and mathematical competence to be *r* = .443, significantly larger than the effect size estimate of symbolic magnitude comparison and mathematics achievement. Although the type of mathematical competence measure did not significantly moderate the estimated effect size, measures of number line estimation that involved fractions were more strongly related to math competence than measures than involved whole numbers (Schneider et al., 2018).

The results of the descriptive reviews and meta-analyses suggest that symbolic magnitude knowledge is related to mathematics achievement across a range of participant samples. However, they do not rule out the possibility that the relations between symbolic magnitude knowledge and math achievement vary by age. Much of the data examining magnitude and math achievement, including the meta-data analyzed by Schneider et al. (2016, 2018), has relied on studies of school-aged children and adults. It is less clear whether preschool children's symbolic magnitude skills would be related to their math achievement. In preschool, children typically develop their symbolic number skills broadly, and it is possible that other symbolic skills such as understanding the cardinal principle are more important to supporting math achievement at this stage than symbolic magnitude knowledge. If preschoolers' symbolic magnitude skills do relate to their math achievement, it would motivate future research on supporting symbolic magnitude skill development in early childhood. Few studies to date have assessed preschoolers' symbolic magnitude skills in relation to their broader math performance; three longitudinal studies and three experimental studies are described in turn.

**Longitudinal Studies**

Longitudinal studies of preschoolers' symbolic magnitude skills suggest they significantly predict later math achievement, more so than non-symbolic magnitude skills but perhaps less than other types of symbolic number knowledge. VanMarle and colleagues (2014) assessed low-income children's symbolic magnitude comparison skills (limited to numerals they could correctly identify) in the fall and spring of the school year. Preschoolers' symbolic magnitude knowledge in the spring was significantly related to their concurrent math achievement ($r = .64$, $p < .001$), and their fall symbolic magnitude knowledge positively correlated with their later math achievement in the spring ($r = .44$, $p < .001$). However, in a multiple mediation analysis that also included verbal counting, cardinality, and numeral recognition, symbolic magnitude comparison skills no longer significantly predicted math achievement. This suggests that symbolic magnitude comparison skills may relate to preschoolers' math achievement, but other related symbolic number skills like counting, numeral identification, and cardinality may be more predictive of young children's math achievement.

Two studies investigated children's symbolic magnitude skills at age 4 and their relation to math achievement at age 6 (Kolkman, Kroesbergen, & Leseman, 2013; Toll & Van Luit, 2014; Toll et al., 2015). In a structural equation model, preschoolers' symbolic number line estimation performance predicted their symbolic magnitude knowledge (number line estimation and magnitude comparison) at age 5, which was the only direct pathway to their math achievement at age 6 (Kolkman et al., 2013). Similarly, Toll and Van Luit (2014) found that typically developing preschoolers' symbolic magnitude comparison skills predicted both the intercept and slope of their mathematical development between ages 4 and 6, assessed by repeated administrations of a standardized math assessment. A reanalysis of the same dataset showed that children's math fluency and reasoning ability at age 6 was mainly predicted by their prior symbolic magnitude skills, both their average level and the development in their skills from preschool (Toll et al., 2015). Taken as a whole, the three longitudinal studies of preschool children suggest that early symbolic magnitude skills predict later math achievement, but may not relate to concurrent preschool math skills as much as other measures of foundational symbolic number knowledge such as counting, numeral identification, and cardinality. In particular, for children who are especially young preschoolers (under 4 years old), from low-income backgrounds, and are at or near chance on measures of symbolic magnitude comparison, basic symbolic number skills may be more informative of future math performance than symbolic magnitude skills.

**Experimental Training Studies**

Three experimental studies randomly assigned preschoolers to a symbolic magnitude training and evaluated the effects on their mathematical skills. Honoré and Noël (2016) randomly assigned some preschoolers to be trained with symbolic magnitude games, involving practice with comparing symbolic numerals and placing numerals on a symbolic number line. Children who played the symbolic magnitude training for ten 30-minute sessions improved their symbolic magnitude skills as well as their arithmetic skills, while children who played a parallel non-symbolic magnitude game did not. This suggests that the symbolic magnitude training was effective at improving children's symbolic magnitude skills, and in turn, their broader mathematical skills.

Two studies focused on training preschoolers' symbolic magnitude skills through a numerical linear board game (Cankaya, LeFevre, & Dunbar, 2014; Siegler & Ramani, 2009). Previous research has shown that practice with a numerical linear board game can successfully improve preschoolers' symbolic magnitude skills, assessed as both number line estimation and magnitude comparison (e.g., Ramani & Siegler, 2008; Siegler & Ramani, 2008; Whyte & Bull, 2008), yet fewer studies have included a broader measure of math achievement. Siegler and Ramani (2009) found that preschoolers who played the linear numerical board game significantly improved their symbolic magnitude representations compared to children who practiced basic number skills (verbal counting, object counting, and numeral identification) or played with a circular numerical board game. Children who played the linear numerical board game also significantly improved their accuracy on solving simple arithmetic problems following a brief training. In a similar design, Cankaya and colleagues (2014) randomly assigned 3 year old children to play either a linear number board game or a color board game for four 15-minute sessions, and assessed their growth on a math achievement measure. However, children in both conditions improved their math achievement over the course of the study, with no selective improvements for children who played the linear numerical board game intended to bolster their symbolic magnitude skills. Moreover, children showed no significant improvement in their symbolic number line estimation skills, providing evidence that the intervention was not a successful test of the effect of improving symbolic magnitude skills on math achievement. The lack of significant improvements may be due to the age of the children; previous studies with a similar dosage had focused on older preschoolers (e.g., Ramani & Siegler, 2008), whereas studies with similarly aged participants found success with greater exposure to the training game (i.e., four 25-minute sessions; Whyte & Bull, 2008). The results of the intervention studies suggest that when preschoolers' symbolic magnitude skills are successfully improved, they also improve their more general mathematics skills such as arithmetic. These findings are compatible with other symbolic magnitude interventions with kindergarten children, which also find improvements in children's arithmetic skills (e.g., Maertens, De Smedt, Sasanguie, Elen, & Reynvoet, 2016).

## Predictors of Symbolic Magnitude Understanding

Across the few studies available, the data suggest that symbolic magnitude skills in preschool do predict children's math achievement, as long as children are proficient with basic numerical skills such as counting and numeral identification. This qualification is itself worthy of investigation; specifically, does children's symbolic magnitude understanding build from particular skills that serve as developmental precursors? The literature on this topic can be broken into two distinct schools of thought: 1) that symbolic magnitude knowledge is built from non-symbolic magnitude knowledge (e.g., Dehaene, 2001), and 2) that symbolic magnitude knowledge is a separate representational system that develops independently of non-symbolic magnitude knowledge but relies on other types of numerical knowledge (e.g., Carey, 2001; Lyons, Ansari, & Beilock, 2012; Reynvoet & Sasanguie, 2016). The broader theoretical and empirical evidence base in favor of each perspective is discussed in turn, followed by the specific findings from the early childhood period. Given that more studies have specifically tested the predictive relations between preschool-aged children's non-symbolic and symbolic magnitude knowledge than other foundational numerical skills, the treatment of each type of predictor is weighted accordingly in the present review.

### Non-Symbolic Magnitude Underlies Symbolic Magnitude

In attempts to answer the question of how humans associate meaning with symbolic numbers (number words and numerals), referred to as the numerical symbol-grounding problem, some researchers have suggested that symbols are grounded in innate, non-symbolic magnitude representations (e.g., Dehaene, 2001;

Feigenson, Dehaene, & Spelke, 2004; Piazza, 2010; Wynn, 1992). This implies that observed individual differences in non-symbolic magnitude knowledge underlie individual differences in symbolic magnitude knowledge (Siegler, 2016).

Several recent reviews have described the numerical symbol-grounding problem and the empirical evidence in favor of underlying non-symbolic representations (Leibovich & Ansari, 2016; Mussolin et al., 2016; Reynvoet & Sasanguie, 2016; Siegler, 2016). Behavioral data have shown that ratio effects characterize performance on both non-symbolic and symbolic magnitude comparison tasks, such that smaller ratios between stimuli (closer to 1) are more difficult to discriminate (e.g., Halberda & Feigenson, 2008; Holloway & Ansari, 2009). In addition, ANS acuity is predictive of symbolic magnitude comparison skills and performance on other symbolic measures, including math achievement (Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Halberda, Mazzocco, & Feigenson, 2008; Libertus, Feigenson, & Halberda, 2013; Moore, vanMarle, & Geary, 2016; Mussolin, Nys, Leybaert, & Content, 2012). Experimental studies have also shown that training non-symbolic magnitude skills can lead to significant improvements in children and adults' symbolic arithmetic skills (Hyde, Khanum, & Spelke, 2014; Park & Brannon, 2013).

Data from neuroimaging studies have implicated the same region of the brain, the Intraparietal Sulcus (IPS), in the processing of both non-symbolic and symbolic magnitude information (see Nieder & Dehaene, 2009). Neural activation patterns during the presentations of non-symbolic and symbolic numbers showed participants responded to similar numerical quantities, regardless of whether presentation format was a set of dots or digits (Piazza, Pinel, Le Bihan, & Dehaene, 2007). Activation patterns within the IPS from the presentation of symbolic numerals were also predictive of activation patterns for the same quantities presented as non-symbolic arrays of dots (Eger et al., 2009).

Grounding culturally constructed numerical symbols in a primary system of non-symbolic representations is intuitive and supported by evidence from behavioral and neuroscience research. It suggests that both diagnostic screening of and potential interventions to support non-symbolic magnitude development may be particularly important in early childhood. However, not all researchers accept the premise that non-symbolic representations underlie symbolic representations, arguing instead that the two systems can be mapped onto one another but develop in parallel.

**Symbolic Magnitude Emerges from Other Skills**

A growing number of researchers have proposed counterarguments to the claim that non-symbolic magnitude representations form the basis of symbolic magnitude knowledge (e.g., Carey, 2009; Lyons, Vogel, & Ansari, 2016; Reynvoet & Sansanguie, 2016; Siegler, 2016). Some researchers propose a solution to the symbol-grounding problem that relies on an object tracking system for small, exact numbers up to four (Carey, 2009; Lyons & Ansari, 2015) or up to ten (Siegler, 2016). Others suggest that symbolic numbers are based on an understanding of ordinal relations gained through knowledge of the count sequence (e.g., Reynvoet & Sansanguie, 2016). Together, proponents of an alternative solution to the symbol-grounding problem cite similar evidence against the claims of Dehaene and others.

Although both non-symbolic and symbolic magnitude comparison tasks elicit ratio dependent performance, ratio effects similarly characterize other non-numerical discrimination tasks, including putting letters in alphabetical order (Van Opstal, Gevers, De Moor, & Verguts, 2008). Thus, ratio effects may be due to cognitive processes of ordering information or making magnitude judgments rather than shared numerical information. Similarly, although behavioral studies that found training non-symbolic skills led to transfer effects on symbolic arithmetic performance provide compelling evidence, several non-symbolic trainings have failed to replicate that pattern of results (e.g., Honoré & Noël, 2016; Park & Brannon, 2014).

While neuroimaging studies have implicated the IPS as a shared region for numerical representation, and recorded activation patterns that suggest similar processing of non-symbolic and symbolic numbers, other studies have described distinct activation patterns and regions. Specifically, several studies found the activation patterns to non-symbolic and symbolic representations of the same number were not the same regardless of the format of presentation (Bulthé, De Smedt, & Op de Beek, 2014; Cohen Kadosh et al., 2011; Lyons, Ansari, & Beilock, 2015). Lyons and colleagues (2015) found that the activation patterns in the IPS from viewing non-symbolic stimuli were broader with more overlap between neighboring quantities than the discrete activation patterns from symbolic stimuli, reinforcing the distinct nature of the approximate non-symbolic numerical representations and the precise symbolic numerical representations. In a similar vein, there is some evidence that representations of symbolic numbers are stored separately, as opposed to directly overlapping, from non-symbolic number representations (Price & Ansari, 2011; Sasanguie, Göbel, & Reynvoet, 2013).

Beyond the contradictory evidence from behavioral and neuroscience studies, some researchers question principle tenants of the non-symbolic grounding of symbolic numbers. For example, Siegler (2016) questions how an approximate non-symbolic representational system could underlie the precise symbolic knowledge of large numbers, such as 100 and 101, given that not even adults reach that level of precision with non-symbolic representations. Leibovich and Ansari (2016) point to the overlap of numerosity and other continuous variables in the measurement of non-symbolic magnitude tasks, arguing that current measures of ANS acuity require significant cognitive control skills that may explain the observed relations between non-symbolic knowledge and math achievement.

In sum, there is mixed evidence that non-symbolic magnitude representations underlie symbolic magnitude representations. Such data from older children and adults that find little overlap between representations are suggestive but not conclusive; indeed, it is possible that symbolic magnitude representations develop originally from non-symbolic representations but separate over time. Rather, investigating whether non-symbolic magnitude representations underlie symbolic magnitude representations should center on young children who are actively learning numerical symbols and refining their non-symbolic representations. The following section reviews evidence of potential links during early childhood.

## Relations Between Non-Symbolic and Symbolic Magnitude in Early Childhood

Although many of the theoretical arguments on the relations between non-symbolic and symbolic magnitude have relied on evidence from school-aged children and adults, similar behavioral patterns are relevant during the early childhood period. If non-symbolic magnitude representations underlie symbolic magnitude representations, one would expect strong correlations both concurrently and longitudinally. Similarly, one would expect improvements in non-symbolic magnitude representations, due to either developmental maturation or targeted intervention, to lead to improvements in symbolic magnitude skills. If different patterns emerged between non-symbolic and symbolic magnitude skills in preschool, it may provide evidence in favor of the two as distinct representations. Studies that related non-symbolic and symbolic magnitude skills in early childhood included both non-experimental, correlational designs and experimental designs, discussed in turn below.

**Non-experimental studies.** Non-experimental studies of preschoolers' non-symbolic magnitude knowledge and concurrent or later symbolic magnitude knowledge generally support the claim that the two are related (Table 1). Three studies asked children to compare sets of objects (non-symbolic magnitude comparison) and compare symbolic numerals (symbolic magnitude comparison; Batchelor, Inglis, & Gilmore, 2015; Moore et al., 2016; Toll, Van Viersen, Kroesbergen, & Van Luit, 2015). All three studies reported that children's accuracy on the non-symbolic and symbolic magnitude comparison tasks were significantly correlated when assessed concurrently, and one study also reported a significant correlation between children's preschool non-symbolic comparison skills and their symbolic magnitude skills 2.5 years later (Toll et al., 2015).

Two studies have investigated the relations between preschoolers' non-symbolic and symbolic magnitude knowledge using number line estimation tasks (Kolkman et al., 2013; Toll et al., 2015). Toll and colleagues found a significant relation between preschoolers' non-symbolic magnitude comparison skills and their performance on a 0-100 symbolic number line estimation task in first grade ($r = .12$, $p < .01$; Toll et al., 2015). However, although the relation was statistically significant, the magnitude of the effect explained less than two percent of the total variance, which raises questions about its practical importance. A second study found no significant concurrent relation between non-symbolic magnitude comparison skills and symbolic number line estimation performance ($r = -.01$, $p > .05$), but a significant relation between non-symbolic and symbolic number line estimation ($r = .35$, $p < .01$; Kolkman et al., 2013). This implies that task features may moderate the relations between non-symbolic and symbolic magnitude skills, at least in single time point analyses. When children are assessed using the same type of task (i.e., magnitude comparison, number line estimation) across non-symbolic and symbolic representations, there appear to be significant concurrent correlations. However, when studies vary the type of task between non-symbolic and symbolic magnitude measures, there may be little or no relation. Schneider, Thompson, and Rittle-Johnson (2017) suggest that low inter-correlations between magnitude comparison and number line estimation tasks may be because the tasks tap into different components of magnitude knowledge (ordinal versus ratio scale) and thus elicit different problem-solving strategies.

The remaining two studies examined children's non-symbolic skills and symbolic number sense skills, operationalized as assessment batteries targeting counting skills, Arabic numeral knowledge, symbolic magnitude comparison, canonical number patterns, and identifying number words (Mussolin et al., 2012, 2014) and symbolic magnitude comparison and number-to-quantity mapping skills (Rittle-Johnson, Fyfe, Hofer, & Farran, 2016). In line with previously described studies, there were significant correlations between concurrent non-symbolic skills and symbolic number sense skills (Mussolin et al., 2012, 2014; Rittle-Johnson et al., 2016), as well as between non-symbolic skills and symbolic number sense skills 7 months later (Mussolin et al., 2014). However, Mussolin and colleagues (2012) noted variability in the relation depending on their participants' ages: while 4-year-olds' non-symbolic and symbolic skills were significantly related ($r = .43$, $p < .01$), symbolic and non-symbolic skills of 3-year-olds were not ($r = .28$, $p > .05$). This discrepancy may be due in part to the limited symbolic number knowledge of the 3-year-old children (average score of 3 out of 63 on the symbolic number battery; Mussolin et al., 2012). Taken together, these results suggest that non-symbolic magnitude skills relate to children's early symbolic number knowledge broadly, as long as there is sufficient variability in the measures of symbolic number knowledge.

The bivariate correlations between measures of non-symbolic magnitude and symbolic number knowledge suggest that they are related when assessed at the same time and up to 2.5 years later (Table 1). However, more sophisticated analyses tell a more nuanced story. Although non-symbolic magnitude comparison skills in preschool were correlated with symbolic magnitude comparison skills, preschool ANS acuity did not significantly predict children's kindergarten performance on number sets, a task involving mapping between symbolic and non-symbolic magnitudes (Moore et al., 2016). Similarly, children's improvement in non-symbolic comparison performance covaried with their improvement in symbolic magnitude comparison performance, but only their symbolic magnitude comparison skills predicted their symbolic number line estimation skills (Toll et al., 2015). Kolkman and colleagues (2013) found that preschoolers' non-symbolic magnitude skills (measured with magnitude comparison and number line estimation tasks) formed a separate factor from their symbolic magnitude skills, and their factor score on non-symbolic magnitude in preschool did not significantly predict their symbolic magnitude skills in kindergarten. With a cross-lagged correlation controlling for participant age, initial skill level at first assessment, and general cognitive skills, Mussolin and colleagues found that early symbolic number sense scores significantly predicted later non-symbolic comparison scores, but not the reverse (Mussolin et al., 2014). Although Rittle-Johnson and colleagues (2016) reported consistent predictive relations between non-symbolic skills in preschool and symbolic skills in first grade, their non-symbolic magnitude comparison and ordering tasks were untimed and children were allowed to count to reach their answer, suggesting their performance may reflect both non-symbolic and symbolic skills. Only one study of preschoolers' non-symbolic comparison skills showed a significant predictive relation with their symbolic comparison skills, controlling for working memory, verbal skills, numerical identification, spontaneous focus on number, and ability to match the correct symbolic numeral to a non-symbolic array of dots (Batchelor et al., 2015).

In sum, across the non-experimental studies that assessed preschoolers' non-symbolic and symbolic magnitude skills, all bivariate correlations between measures of the same task type were positive and statistically significant. However, most studies revealed that when controlling for other variables or conducting a factor analysis, non-symbolic magnitude skills were no longer significantly related to symbolic magnitude skills (c.f. Batchelor et al., 2015). This pattern of results suggests that non-symbolic and symbolic magnitude skills may be related, though the relation appears weak and may be explained by other factors. The next section turns to experimental studies to evaluate whether preschoolers' non-symbolic magnitude skills are causally related to their symbolic magnitude skills.

**Experimental training studies.** If non-symbolic magnitude representations form the basis for symbolic magnitude representations, interventions that target children's non-symbolic skills should also lead to improvements in their symbolic skills. Two experimental studies randomly assigned preschoolers to training conditions, including non-symbolic magnitude training. Whyte and Bull (2008) had children play a non-symbolic magnitude comparison game in small groups across four 25-min sessions. Children were shown two cards with varying quantities of apples, ranging from 1 to 100, and asked to say which card had more apples. After each child made their guess, the experimenter showed the children the symbolic number associated with each quantity, and gave each child accuracy feedback on their non-symbolic magnitude comparison. Children who played the non-symbolic magnitude comparison game significantly improved their symbolic magnitude comparison skills, but showed no improvement on symbolic number line estimation. This study suggests that practice making non-symbolic magnitude comparisons led to improvements on symbolic magnitude comparison skills, however, the results may also be attributed to the symbolic magnitude information provided during the training game. Although children were first asked to make non-symbolic magnitude comparisons, they were also shown symbolic magnitudes and told which symbolic quantity was greater. Thus, it is unclear whether the non-symbolic magnitude, symbolic magnitude, or the linking of non-symbolic and symbolic magnitude information underlies the improvement in symbolic magnitude comparison skills.

More recently, Honoré and Noël (2016) trained preschoolers for ten sessions on one of two non-symbolic games, a non-symbolic magnitude comparison game and a non-symbolic number line estimation game. When playing the non-symbolic magnitude comparison game, children were shown three sets of dots of varying quantities, and asked to identify the sets with the most and least dots. In the non-symbolic number line estimation game, children placed a set of dots in the appropriate place along a number line with endpoints of one dot and 20 dots. Accuracy feedback was provided to children in both games after each trial. Children in the non-symbolic training condition did not significantly improve their performance on symbolic magnitude comparison with either Arabic numerals or verbal number words, nor did they improve their symbolic number line estimation skills. However, the non-symbolic magnitude training did successfully improve children's performance on non-symbolic magnitude comparison and number line estimation tasks, and a separate symbolic magnitude training condition led to improvements in children's symbolic magnitude skills. These results suggest non-symbolic and symbolic magnitude representations are separate in preschool, inconsistent with a single representational system of numerical magnitude (Honoré & Noël, 2016).

**Summary.** Taken as a whole, the evidence from correlational and experimental studies of preschoolers implies that non-symbolic and symbolic magnitude representations may be related, but non-symbolic magnitude representations do not appear to underlie symbolic representations. Preschoolers' non-symbolic skills did not predict their kindergarten symbolic magnitude skills (e.g., Kolkman et al., 2013; Moore et al., 2016). Latent variable techniques further suggested that non-symbolic and symbolic magnitude knowledge form separate but related factors in preschool (Kolkman et al., 2013), similar to patterns observed in kindergarteners (Cirino, 2011). Experimental studies also fail to demonstrate improvement on symbolic magnitude skills after non-symbolic magnitude training, implying that the skills are not causally linked (Honoré & Noël, 2016). In sum, preschoolers' non-symbolic and symbolic magnitude skills may be related, but non-symbolic magnitude may not underlie symbolic magnitude. Rather, it seems likely that children's foundational numerical skills may play a key role in the development of early symbolic magnitude knowledge.

## Relations Between Numerical Skills and Symbolic Magnitude

Children's magnitude knowledge does not develop in isolation, and indeed, other early symbolic numerical skills also play an important role in supporting later math achievement. It is possible that particular numerical skills are similarly related to children's symbolic magnitude skills, and may perhaps lay a necessary foundation for symbolic magnitude knowledge. The relations between symbolic magnitude knowledge and children's verbal counting, numeral identification, and cardinality skills are discussed in turn.

**Verbal counting.** Verbal counting knowledge is often the first symbolic numerical knowledge that children master, occurring well before children are proficient on tasks of symbolic magnitude knowledge (for a review, see Ramani, Daubert, & Scalise, 2019). Nevertheless, the verbal counting string provides early clues to the ordinal nature of numbers, communicating their relative magnitudes from their position in the sequence. As described by Carey (2009), learning to count may allow children to form a placeholder structure that they later apply to understanding the magnitudes of numbers. As children learn their first verbal counting words (i.e., one, two, three, four) they may use an enriched form of parallel individuation, innate working memory models that can represent small quantities, to create long-term memory representations of small sets (Carey, 2009). For example, verbal number words from the count sequence might be stored as canonical representations of each quantity, such as "me" as "one", "Mommy and Daddy" as "two", and "me, Mommy, Daddy" as "three" (Carey, Shusterman, Haward, & Distefano, 2017; Mix, 2002). In this way, verbal counting words and the meanings assigned to them – which are not immediately connected to the ANS – may form the beginnings of children's symbolic magnitude knowledge (Carey et al., 2017; Mix, 2002). This in turn implies that children must have knowledge of the verbal counting sequence from at least numbers 1-9 before they can accurately compare symbolic magnitudes within the same range.

Given the relative lack of research predicting the development of children's symbolic magnitude knowledge, few studies have assessed both children's verbal counting and symbolic magnitude knowledge. Ramani and Siegler (2008) found preschool children's symbolic magnitude skills (assessed by a magnitude comparison task and a number line estimation task) formed a separate factor from children's verbal counting skills, both before and after a symbolic magnitude intervention. This provides some evidence that verbal counting is a unique, separable skill from symbolic magnitude knowledge, yet does not rule out the possibility of a predictive relation. Indeed, Mussolin and colleagues (2014) found that preschoolers' verbal counting significantly predicted their symbolic number skills (a composite of tasks including symbolic magnitude comparison) up to seven months later, controlling for child age. However, verbal counting skills may be necessary but not sufficient for learning about the magnitudes of numbers: cross-sectional data suggest that children can count in a number range for nearly a year prior to understanding of the magnitudes of those numbers (Le Corre & Carey, 2007). In sum, although counting skills predict children's later math performance (e.g., Aunio & Niemivirta 2010; Aunola, Leskinen, Lerkkanen, & Nurmi, 2004), its close relations to other foundational numerical skills like cardinality may make it unlikely to be the strongest predictor of symbolic magnitude knowledge.

**Numeral knowledge.** Beyond serving as the theorized link between informal and formal mathematical knowledge (e.g., Purpura et al., 2013), numeral knowledge may be particularly relevant to measures of children's symbolic magnitude skills. Symbolic magnitude knowledge is typically assessed with tasks that incorporate symbolic numerals, such as magnitude comparison tasks that present two numerals and ask the child to identify the larger number, or number line estimation tasks that display a target symbolic numeral with a number line labeled with 0 and 10 numerals as endpoints. The task formats might imply that being able to identify numerals is a prerequisite to accurately completing symbolic magnitude tasks, yet some presentations involve an experimenter labeling the numbers for children (e.g., Siegler & Ramani, 2009). Other tasks explicitly limit symbolic magnitude comparison to only numerals that children can correctly identify (e.g., vanMarle et al., 2014), raising an important question in the measurement of symbolic magnitude knowledge. Namely, is it possible that children have some verbal knowledge of symbolic magnitude that goes beyond their capacity to correctly identify written number symbols? It is conceivable that a child with knowledge of the verbal counting string would be able to accurately say which of two numbers was larger if an experimenter verbally identified the options, even if they had inaccurately identified one of the numerals previously. For example, a child may know that eight is larger than two from the relative positions in the verbal count sequence, even if they struggle to recognize the numeral 8. This question was empirically tested using data from over 250 preschoolers, with results suggesting that when an experimenter verbally labels the numerals in a symbolic magnitude comparison task, children perform significantly above chance on trials that include a numeral that they could not correctly identify (Appendix A).

However, although there is some evidence that numeral knowledge is not a prerequisite to symbolic magnitude knowledge, children with greater numeral knowledge do tend to have more integrated symbolic magnitude representations. For example, a study of kindergarteners found that children who could correctly write more Arabic numerals also showed a stronger tendency to associate smaller symbolic magnitudes with the left-hand side of space and larger symbolic magnitudes with the right-hand side of space (SNARC effect; Hoffman, Hornung, Martin, & Schiltz, 2013). Similarly, on symbolic number line estimation tasks, being able to correctly identify the numerals presented would significantly reduce the cognitive load of completing the trial compared to a child who needs to retain the experimenter's labels for each numeral in their working memory. In sum, numeral knowledge may be particularly important when symbolic magnitude knowledge is assessed tasks that require children to identify the numerals independently, but less so for predicting performance on magnitude comparison tasks with verbal labels for each numeral, especially if children have strong knowledge of the cardinal values of the corresponding number words.

**Cardinality.** Cardinality refers to children's ability to link numerical symbols (number words and numerals) to their corresponding quantities, in essence translating between symbolic and non-symbolic magnitude representations. As with other numerical skills, it is possible that with the order cues embedded in the verbal counting sequence, young children could have some understanding of symbolic magnitude without cardinality knowledge. Indeed, preschoolers can successfully compare number words larger than four, even before knowing the cardinal principle

(Davidson, Eng, & Barner, 2012). Nonetheless, cardinality knowledge could help children compare and order symbolic magnitudes, allowing for the use of symbolic representations (verbal count sequence, written numerals) or translating back to non-symbolic representations (sets of quantities). Indeed, children's performance on verbal cardinality tasks in preschool predicts their symbolic magnitude comparison skills in kindergarten (Moore et al., 2016).

One of the only known studies to investigate the development of preschool children's symbolic magnitude understanding found that children's knowledge of the cardinal principle was a significant predictor of their improvements in symbolic magnitude comparisons, controlling for child age, intelligence, EF skills, and preliteracy skills (Geary & vanMarle, 2018). Specifically, before children understood the cardinal principle, their performance on a symbolic magnitude comparison task was statistically equivalent to chance, whereas after they acquired knowledge of the cardinal principle, their performance on the same task improved significantly. Similar to theories of numeral knowledge described previously, children's cardinality knowledge is thought of as a translational skill that bridges foundational non-symbolic and symbolic number skills (e.g., non-symbolic magnitude, verbal counting) with more advanced school-based numerical knowledge (Geary & vanMarle, 2018).

However, unlike numeral knowledge, cardinality knowledge provides some numerical magnitude information. It implies that a child knows that "four" means □□□□, whereas symbolic magnitude knowledge implies that a child knows that "four" (□□□□) is less than "six" (□□□□□□), whether or not they refer back to non-symbolic representations to reach that conclusion. Symbolic magnitude knowledge subsumes the magnitude information conveyed by cardinality, adding an understanding of the relations among various magnitudes. In sum, cardinality is likely the best predictor of children's symbolic magnitude understanding.

**Summary.** Taken as a whole, the data suggest that young children's symbolic magnitude understanding relates broadly to other early symbolic numerical skills. However, cardinality is likely the best predictor of children's symbolic magnitude understanding – more so than verbal counting or numeral knowledge. If symbolic magnitude knowledge is particularly important for general numerical development and later mathematical achievement (e.g., Siegler, 2016), it is important to unpack the relations with other developing skills. In a similar vein, the role of children's executive functioning skills in potentially influencing the development of symbolic magnitude knowledge and mathematical learning broadly is important to consider.

## Executive Functioning and Symbolic Magnitude Understanding

Executive functioning (EF) refers to a core set of cognitive processes including inhibition, working memory, and cognitive flexibility (Diamond, 2013). Inhibition refers to the ability to control one's behavior, thoughts, and emotions in order to respond appropriately or as instructed rather than the natural response, such as in the game "Simon Says". Working memory refers to the ability to hold information in mind and manipulate it without a physical representation, such as remembering a short grocery list and ordering it by section of the grocery store. Cognitive flexibility includes both inhibition and working memory skills (Diamond, 2013), and refers to the ability to switch perspectives or solve problems creatively with a new way of thinking.

EF skills are theorized to be central to the information processing required by mathematics, such as ignoring irrelevant information in a word problem (inhibition), keeping different quantities in mind and manipulating them while solving a multi-digit arithmetic problem (WM), and being able to use a variety of strategies to generate a solution (cognitive flexibility; Geary & Hoard, 2005). EF skills may also be key to young children's developing symbolic magnitude understanding, such as suppressing inappropriate strategies like choosing their favorite number instead of the bigger number (inhibition), keeping in mind and mentally comparing multiple quantities to form an understanding of relative magnitudes (WM), and translating various sources of symbolic magnitude information presented verbally and visually as sets and numerals (cognitive flexibility). Although inhibition, WM, and cognitive flexibility are theoretically distinct, there is considerable evidence that young children's EF skills form a single latent factor, emerging as separate constructs later on in middle childhood (Bull & Lee, 2014; Clark, Sheffield, Wiebe, & Epsy, 2013; Shing, Lindenberger, Diamond, Li, & Davidson, 2010; Wiebe, Epsy, & Charak, 2008).

Empirical studies have found consistent positive and significant relations between children's EF skills and their concurrent and later mathematics achievement (Ackerman & Friedman-Krauss, 2017; Bull & Lee, 2014). Preschool children's EF skills (modeled as a single latent factor) significantly predict their concurrent performance on a standardized math achievement measure, controlling for child age and intelligence (Bull, Espy, Wiebe, Sheffield, & Nelson, 2011). Children's growth in EF skills during preschool predicts higher growth in their emerging math skills (McClelland et al., 2007). A longitudinal study that followed preschoolers over a two year period found that children's EF skills at age 3 significantly predicted their numeracy skills at ages 3.75 and 4.5 years as well as their kindergarten math achievement, controlling for socioeconomic status, language, and processing speed (Clark et al., 2013). Children's earlier informal numeracy skills and their later formal math achievement scores were best predicted by unique, independent paths from their EF skills (Clark et al., 2013). There is some evidence that children's EF skills may directly influence their math skills and moderate the relation between early and later math skills: Ribner and colleagues (2017) found that preschoolers' EF skills significantly predicted their 5th grade math achievement, and that relation remained significant after the interaction of preschool EF skills and preschool numeracy skills was added to the model. However, most of the research to date has focused on the direct influence between preschoolers' EF and math skills.

In sum, EF skills are likely an additional source of variability in children's early mathematical skills, above and beyond their foundational numerical skills. EF skills assessed in preschool are significant predictors of concurrent and later math abilities, even when controlling for other numerical, language, and general cognitive abilities. Understanding the role of executive functioning and early numerical skills on later mathematics achievement may be particularly relevant for preschool children, who are setting the foundation for their later mathematics achievement.

## Addressing Gaps in the Literature

In the present review of the research on the development of symbolic magnitude understanding in early childhood, several open questions remain to guide future research. Despite extensive research with school-aged children and adults, there is far less evidence in support of the link between children's symbolic magnitude understanding in early childhood and their math achievement, particularly in models with rigorous numerical and executive functioning controls. In addition, it is unknown whether other foundational numerical skills, such as cardinality, are more predictive of math achievement in early childhood than symbolic magnitude understanding. Finally, although there is substantial evidence linking mathematical achievement and executive functioning skills, little research to date has addressed the relative contributions of domain-specific numerical skills and domain-general executive functioning skills on children's mathematical development. Each of these questions is explored briefly below.

### Symbolic Magnitude Skills and Mathematical Achievement in Early Childhood

Across the few studies available, the data suggest that symbolic magnitude skills in preschool do predict children's math achievement, as long as children are proficient with symbolic numbers. The findings are compatible with descriptive and meta-analytic reviews of older children and adults, which consistently report that symbolic magnitude skills relate to math achievement (e.g., Chen & Li, 2014; Fazio et al., 2014; Schneider et al., 2016). In particular, there is some evidence that magnitude knowledge may predict early math achievement like mental arithmetic (Schneider et al., 2016). One potential mechanism for the relation between symbolic magnitude understanding and early arithmetic skills is that children who understand the magnitudes of different numbers may be better able to check their solutions to addition problems. For example, a child who knows the symbolic magnitudes of 3 and 4 would be unlikely to say that the sum of those numbers was 2, because they understand that both the magnitudes of 3 and 4 are greater than 2. They may also be more sensitive to part-part-whole relationships because they have a stronger understanding of the individual magnitudes of each addend and sum, as well as how their magnitudes can be compared and ordered. The current study tests a more comprehensive model of the relation between symbolic magnitude and math achievement that included domain-general and domain-specific controls. Specifically, children's symbolic magnitude skills were hypothesized to predict their later addition skills, above and beyond the variance explained by executive functioning and cardinality skills.

**Cardinality and Symbolic Magnitude Skills**

The empirical evidence reviewed suggests that preschool children's cardinality skills are likely the strongest predictor of their symbolic magnitude skills, more so than their non-symbolic magnitude, verbal counting, or numeral recognition skills. In a sense, symbolic magnitude understanding is a natural extension of children's cardinality knowledge, as they move from understanding the unique magnitudes of individual small numbers to being able to successfully compare and order the magnitudes of different numbers. One potential mechanism for the relation between cardinality and symbolic magnitude understanding is that once a child understands the cardinal value of a set of small numbers, such as one, two, three, and four, they can then draw from their understanding of cardinal values to relate the values to each other (e.g., think about the value of 1 and the value of 4 and recognize that the former is a smaller quantity). This in turn suggests that children who understand some cardinal values of numbers and not others (i.e., subset knowers) would be expected to have higher accuracy on symbolic magnitude task trials involving numbers with values they know versus larger numbers with cardinal values they do not yet know.

Beyond the relation to symbolic magnitude understanding, cardinal number knowledge is hypothesized as a gateway to broader mathematical achievement (Geary & vanMarle, 2018). It is important to consider whether children's cardinality knowledge is potentially a stronger predictor of mathematics achievement in early childhood than their magnitude knowledge. Although some research has found that preschool children's cardinality skills were better predictors of later math achievement than their symbolic magnitude skills (e.g., Geary & vanMarle, 2016), no study to date has tested whether symbolic magnitude understanding actually mediates the relation between children's cardinality skills and math achievement. The present study examines the evidence for symbolic magnitude as a full or partial mediator of children's cardinality and addition skills. By modeling the predictive relation between cardinality and symbolic magnitude simultaneously with the relation between symbolic magnitude and addition, this study extends previous research focused on investigating the predictors of one skill at a time.

**Contributions of Domain-General and Domain-Specific Skills**

The three core components of EF – inhibition, working memory, and cognitive flexibility – have theoretical links to the learning of mathematics (Geary & Hoard, 2005), as well as learning more broadly. EF skills have similarly been linked to children's concurrent and later mathematics performance (e.g., Blair & Razza, 2007; Duncan et al., 2007; Geary, 2011; Geary & Hoard, 2005; Geary et al., 2008; Kolkman et al., 2013; Peng, Namkung, Barnes, & Sun, 2016). However, few studies have directly compared the amount of variance explained by domain-general EF skills and domain-specific numerical skills on young children's broader mathematical skills. Unpacking the relative contributions of EF and other numerical skills has significant implications for the design of more comprehensive interventions to best support children's emerging mathematical skills. The current study includes both numerical and executive functioning skills as predictors of young children's addition skills, directly comparing the magnitude of the effects to determine whether EF skills are more, less, or equivalently important as numerical skills in explaining variability in early math ability.

## Chapter 3: Method

### Participants

Participants were 140 3 – 5 year old children at the first assessment ($M_{age}$ = 4 years, 5 months; range = 3 years, 2 months – 5 years, 4 months; 49% female). This age range was selected because of previous research suggesting that during this period children are beginning to acquire symbolic magnitude knowledge, yet there remains significant variability in their performance on magnitude comparison and number line estimation tasks (Daubert, 2018; Siegler & Ramani, 2008). An additional four children were recruited but excluded from the study due to: extreme distraction that prevented the experimenter from completing assessments ($n$ = 1); little to no English comprehension and production capacity ($n$ = 2); repeated declinations to participate ($n$ = 1). The number of participants exceeds the recommended ratio of sample size to number of free parameters (5 to 1; Bentler & Chou, 1987), and statisticians suggest that sample sizes lower than 200 participants can be used to effectively test models without latent variables with strong correlations between constructs (Kenny, 2015).

Children were recruited from four local Head Start Centers. Head Start is a federally funded early childhood education program for children living in households at or below the federal poverty line. To be eligible for enrollment in Head Start during the 2018 – 2019 school year, a family of four had to have an annual household income of $25,100 or less. Head Start Center directors were initially contacted by email, followed by a phone call or in-person meeting to answer any questions. Directors who agreed to have their center participate in the study were provided with a letter to send home to parents describing the study, along with a study consent form and parent survey. All materials were available in English and Spanish. Parents who were interested in having their child participate in the study completed the consent form and returned it to their child's school. Some parents also completed and returned the optional parent survey, which asked basic demographic questions about their child's age, gender, race, and ethnicity, and parents' highest level of educational attainment (Appendix B). Seventy-five percent of parents reported their child's race; of those children, 60% were identified as African American or Black, 19% as Asian or Pacific Islander, 9% as Caucasian or White, 1% as Native American or Alaskan, and as 11% Biracial or Multiracial. Eighty-two percent of parents reported their child's ethnicity; of those children, 21% were identified as Hispanic or Latino and 79% were identified as non-Hispanic/Latino. Eighty-seven percent of parents reported the highest level of education they attained or their child's other parent attained; of those children, 2% had parents with less than high school educations, 4% had parents with some high school education, 23% had parents with a high school diploma or GED, 27% had parents with some college, 12% had parents with a 2-year or Associate's degree, 30% had parents with a 4-year or Bachelor's degree, and 2% had parents with a graduate or professional degree.

**Design**

This study used a short-term longitudinal design to test the proposed conceptual model of children's numerical skill development across the school year, namely, whether children's cardinality knowledge predicted their symbolic magnitude understanding, and whether their symbolic magnitude understanding predicted their early symbolic arithmetic skills. These relations were tested controlling for domain-general executive functioning skills. Children's knowledge was assessed in two time points separated by 3 – 4 months, referred to as Time 1 and Time 2 (Figure 3). During Time 1, children's cardinality, symbolic magnitude, symbolic arithmetic, and executive functioning skills were assessed. During Time 2, children's cardinality, symbolic magnitude, and symbolic arithmetic skills were assessed.

**Procedure**

Children whose parents/guardians agreed to their participation provided verbal assent to participate prior to every session. Participating children completed three 15 – 20 minute sessions of tasks one-on-one with a trained experimenter in a quiet area of their classroom or school. There were four experimenters: two female Caucasian graduate students, one female Caucasian undergraduate student, and one female African American undergraduate student. All experimenters were trained by the author and CITI-approved. The author (female Caucasian graduate student) served as the experimenter for 89% of the sessions ($n$ = 363 individual sessions). Assessment data were recorded by the experimenter by hand on de-identified recording sheets or recorded electronically on a tablet computer, depending on the task administration format.

Time 1 of the study (November 2018 – January 2019) included two sessions, and assessed children's cardinality, symbolic magnitude, and arithmetic skills (Give-N, Point-to-X, How-Many, symbolic magnitude comparison, symbolic number line estimation, forced choice addition, story problem addition; Session 1), and executive functioning skills (inhibitory control, working memory, attention shifting; Session 2). Time 2 of the study included one session, three to four months later (March – May 2019). On average, there was a gap of 3.7 months between participants' Time 1 and Time 2 sessions (*SD* = 0.23 months; range = 3.5 to 5.4 months). During Time 2, children's cardinality, symbolic magnitude, and arithmetic skills were assessed a second time (Give-N, Point-to-X, How-Many, symbolic magnitude comparison, symbolic number line estimation, forced choice addition, story problem addition; Session 3). Each construct (i.e., cardinality, symbolic magnitude, arithmetic, executive functioning) was assessed using multiple measures to reduce the potential for measurement error and in turn increase the precision of statistical estimates. Task scripts are included in Appendix C.

## Measures

**Cardinality skills.** Children's cardinality knowledge was assessed with the Give-N task, the Point-to-X task, and the How-Many task. In the Give-N task (Wynn, 1990), children are given a set of 15 plastic tokens and asked to give some of the tokens back to the experimenter. Children were asked to provide 1, 2, 3, 4, 5, and 6 tokens. If the child provided a correct response, the experimenter next asked them for N + 1 tokens. If the child provided an incorrect response for any trial greater than one, the experimenter then asked for N-1 in the next trial. The task ended when the child reached six tokens correctly, or gave at least two correct responses for N and two incorrect responses for N +1. The dependent measure is children's knower-level, scored as the highest number of tokens that children responded to correctly (Le Corre & Carey, 2007; Sarnecka & Carey, 2008).

In the Point-to-X task, children were shown pieces of paper with two vertically arranged sets of candies on the left and right hand side of the paper, separated by a vertical line (adapted from Levine et al., 2010; Wynn, 1992). On each item, children were asked to point to X (e.g., “two candies”), with quantities ranging from 1 to 6 across a total of 15 trials. Children indicated their response by pointing to the set on the left or right side of the page. The location of the target set was counterbalanced across items. The dependent variable is the number of trials in which the child correctly identified the target set.

In the How-Many task (adapted from Wynn, 1992), children were shown a picture with stars on it and asked to count the stars. After the child counted the stars, the experimenter turned over the picture to “hide” the stars, and asked the child how many stars they were hiding. Children were shown pictures of 2, 3, 4, 5, and 6 stars presented in random order. The dependent variable is the number of trials in which the child correctly stated the number of hidden stars.

**Symbolic magnitude.** Children’s symbolic numerical magnitude knowledge was assessed with a symbolic magnitude comparison task and a 0-10 number line estimation task. In the symbolic magnitude comparison task, children were asked to compare 24 pairs of symbolic numerals ranging from 1-9 presented on a paper flipbook (Ramani & Siegler, 2008). After two practice trials with experimenter feedback, participants were shown 22 test pairs of numbers and asked to indicate which number is larger. The test pairs were read aloud by the experimenter without accuracy feedback. Each number was counterbalanced for side of presentation (i.e., 3|8, 8|3), with the ratio between pairs ranging from 1.1 (e.g., 8|9) to 9.0 (e.g., 9|1). The dependent measure is the percentage of correct comparisons.

In the number line estimation task, children were shown 20 cm lines on a tablet computer, with a 0 labeled at the left end and 10 labeled at the right end, and asked to make a mark on the line where a target number would go (Ramani & Siegler, 2008). After practice making marks on an example trial, children were administered 18 trials with numbers ranging from 1 to 9 presented in random order. On each trial, the experimenter identified the number at the top and then asked, "If this is where 0 goes (pointing) and this is where 10 goes (pointing), where does N go?" The dependent variable is the accuracy of children's estimates compared to the target quantity, measured by percentage of absolute error (PAE = (|estimate - estimated quantity| / scale of estimates) x 100). For example, if a child was asked to make a mark for the number 5, but their corresponding placement was actually where the number 9 would go, their PAE would be equal to 40 percent (e.g., (|9 – 5| / 10) x 100). Lower PAE scores indicate higher accuracy; therefore PAE scores were reversed prior to analyses to aid in interpretability.

**Symbolic addition.** Children's symbolic addition skills were assessed with a forced-choice and story problem addition task. In the forced-choice addition task (adapted from Daubert, 2018; Prather & Alibali, 2011), children were asked to make judgments about which of two imaginary children (names gender-matched to participants) answered arithmetic problems correctly. Children were shown two complete problems (e.g. 2 + 1 = 3, 2 + 1 = 1), one of which was correct. All equations were shown in the form of symbolic numerals and their corresponding non-symbolic quantities (i.e. sets of cookies). The experimenter read both of the problems aloud as they pointed to the problem they were reading. The child was then asked to indicate the problem belonging to the child who they thought *"is right"*. The two equations were repeated by the experimenter as many times as the child needed. Children completed seven addition items, and the dependent variable is the number of items the child answered correctly.

In the story problem task, children were asked to solve addition problems embedded in a story context of a child and their tokens. The experimenter described a child who has a starting number of tokens (e.g., 3), then gets some number of additional tokens (e.g., 2), and asked the participant to determine how many tokens the child in the story has altogether. On each trial, the experimenter set out the number of tokens referenced for the participant to use in problem solving. Each child completed 10 trials, and the dependent variable is the number of trials that the child answered correctly.

**Executive functioning.** Three tasks of children's executive functioning skills were administered and analyzed as a composite score, corresponding to the three core components of executive functioning processes: inhibition, working memory, and cognitive flexibility (Diamond, 2013). Each task was administered on a tablet computer using the National Institutes of Health (NIH) Toolbox Cognition Battery (Weintraub et al., 2013). All tasks have been normed and validated using a nationally representative sample based on gender, race/ethnicity, and socioeconomic status of English-speaking children and adults aged 3 to 85 (Weintraub et al., 2013). Children's performance on each task was scored using the protocols provided by the

NIH Toolbox Cognition Battery for uncorrected standardized scores. Uncorrected standardized scores were used rather than age-corrected standardized scores because the participants in the present study ranged in age from 3 – 5 years old, hence the use of uncorrected scores allowed for comparable scores across the sample.

The Flanker task was used to assess children's inhibitory control skills (Eriksen & Eriksen, 1974). Children were shown displays with a fish in the middle of the screen surrounded by additional fish on either side, and asked to touch a button indicating the direction that the middle fish is facing. On congruent trials, the middle fish faced the same direction as the surrounding fish. On incongruent trials, the middle fish faced the opposite direction of the surrounding fish. After four practice trials with accuracy feedback, children completed twenty test trials without accuracy feedback. Children who missed no more than one congruent and one incongruent test fish trial completed an additional twenty test trials with arrows. For each accurate trial children received 0.125 points, which were summed over all test trials for a total accuracy score (range: 0 to 5 points). Children that proceeded to the arrow test trials also received a reaction time score based on their median reaction time on incongruent trials that they answered correctly (range: 0 to 5 points). The dependent variable is children's uncorrected standardized scores, calculated as the sum of the accuracy and reaction time scores and then converted into a normative score.

Children's working memory skills were assessed with the List Sorting task (Tulsky et al., 2013). Children were shown a series of stimuli (e.g., pictures of animals) while simultaneously hearing verbal labels (e.g., dog, horse) and asked to repeat the stimuli back to the experimenter in size order, from smallest to largest.

Four practice trials with lists of two and three stimuli were administered with accuracy feedback. Assessment trials began with a list of two stimuli; if a child correctly ordered by size, the next trial increased the list length by one (up to seven stimuli). If a child incorrectly ordered a list by size, the next trial repeated the same list length. The task ended if a child incorrectly responded to two trials of the same list length. If a child correctly answered at least one list of three stimuli, they were also given test trials from two lists simultaneously: a list of animals interspersed with a list of foods. Children who received 2-list trials had to sort the items first by category (food or animal) and then by size (smallest to largest) within each category. If a child correctly ordered both lists by size, the next trial increased the list length by one (up to seven stimuli per list). If a child incorrectly ordered either list by size, the next trial repeated the same list length. The task ended if a child incorrectly responded to two trials of the same list length. Children received one point for each list that was correctly sorted. The dependent measure is the uncorrected standardized score, calculated as the normalized sum of scores across all lists presented.

The Dimensional Change Card Sort task was used to assess children's attention shifting or cognitive flexibility (Zelazo, 2006). Children were asked to sort objects by either color or shape. Children were shown four practice trials and asked to sort by color (e.g., white, brown) with accuracy feedback, followed by four practice trials to sort by shape (e.g., rabbit, sailboat). If a child responded accurately to three or more practice trials, they were asked to sort five additional test items by color (e.g., blue, yellow) without accuracy feedback. If children successfully sorted at least four of the test trials by color, they were next asked to sort five additional test cards by shape (e.g., truck, ball) instead of color. If a child correctly sorted at least four test cards by shape, they were next asked to sort a mixed set of cards by either shape or color depending on a verbal prompt. The experimenter demonstrated the new sorting procedure, then the child completed 30 test trials. For each accurate trial children received 0.125 points, which was summed over all test trials for a total accuracy score (range: 0 to 5 points). Children that proceeded to the mixed test trials also received a reaction time score based on their median reaction time on trials that they answered correctly of the less frequently cued dimension (range: 0 to 5 points). The dependent variable is children's uncorrected standardized scores, calculated as the sum of the accuracy and reaction time scores that are then converted into a normative score.

**Parent survey.** Parents/guardians were asked to complete a brief demographic survey asking their child's age, gender, race, ethnicity, as well as the educational attainment of both parents, and the number of people residing in the household (see Appendix B). Children's age at the time of first assessment (in months) was included as a control variable in the statistical models.

## Data Analysis

**Missing data.** A subset of the 140 participants was missing data on one or more of the assessments. Table 2 summarizes the amount missing data by assessment. Less than 3% of scores on individual tasks were missing. Across Times 1 and 2, a total of 11 children were missing scores on at least one task, representing less than 8% of the overall sample. The vast majority of missing data was due to child absence during the assessment time points. The missing data was thought to be missing at random (MAR), meaning the missing scores were unrelated to children's actual abilities on those types of assessments, but were related to children's scores on other observed variables in the model (e.g., the other related numerical and EF skills). There is no statistical test for determining whether missing data qualifies as MAR or missing not at random (MNAR), which can cause severe bias when analysis results are based on complete cases only (Klein, 2016). To address this issue, the treatment of missing data was handled in two ways and the corresponding results were compared in a sensitivity analysis: 1) listwise deletion, and 2) full information maximum likelihood (FIML), conditioned on the model covariates. Listwise deletion reduces the analysis sample to only cases with complete data on all measures, bringing the sample size for the present study to 129 participants. In contrast, FIML is an available data analysis method, meaning it relies on all observed data to estimate the model parameters. FIML estimates a population variance and covariance matrix that maximizes the log likelihood of the observed data occurring. The log likelihood function quantifies the discrepancy between the observed data and the expected population data using a casewise method, and the estimated parameters are based on the maximum of the aggregated casewise log likelihoods. The estimates were conditioned on the covariates of child age and gender in the present study, and the use of all available data with FIML allowed for the retention of the full sample of 140 participants. However, FIML cannot be used with some correction methods for non-normality (i.e., the Satorra-Bentler scaling method, see below), hence in the case of non-normally distributed variables a sensitivity analysis of the same models using listwise deletion versus FIML imputed estimates is presented in the appendices. In general, the amount of missing data in the present study could be considered low compared to missing rates of many educational and psychological studies (15 – 20%; Enders, 2003), and is less likely to bias the resulting estimates than studies with more than 10% of the data missing (Bennett, 2001).

**Preliminary analyses.** Prior to addressing the primary study aims, preliminary analyses were conducted to summarize children's performance on each measure at each time point and test for age- and gender-related differences. Bivariate correlations were conducted between each numerical knowledge task administered during Time 1 and Time 2 to assess the stability of children's numerical knowledge over time. Bivariate correlations between each measure of numerical and executive functioning skills and children's age at the first assessment as well as descriptive statistics children's performance by age (3, 4, or 5 years old) were conducted to determine whether child age should be included in the main analytic models as a covariate. Independent samples *t*-tests were used to test for significant gender differences on the numerical and executive functioning tasks.

**Primary analyses.** Measured variable path analysis (MVPA) was used to address the study aims. MVPA is an analytic technique that falls under the broad methodological umbrella of structural equation modeling, and is used to test theoretically driven hypotheses about structural relations among variables. Although traditional linear regression techniques are also a type of path analysis model, MVPA allows for the estimation of the complete model comprising multiple direct and indirect effects simultaneously, as opposed to conducting a multi-step mediation test with linear regression which requires individual analyses for the direct effect of the predictor on the outcome, the direct effect of the predictor on the mediator, the direct effect of the predictor on the outcome, and the indirect effect of the predictor on the outcome via the mediator (Baron & Kenny, 1986). As a first step in conducting the MVPA, composites were created for each construct (cardinality, symbolic magnitude, symbolic addition, executive functioning) using principle components analysis (PCA). Since the goal of the MVPA was to model children's cardinality, symbolic magnitude, addition, and executing functioning skills as opposed to their specific task performance, PCA provided an overall measure of children's performance across tasks. Unlike using a summed score or averaging standardized task scores, PCA explains the total variation in the observed variables, which can result in unequal weights for each task. Although composites created using PCA do still contain measurement error, PCA was determined to be a more appropriate method for composite creation given the sample size of the present study ($N = 140$), as opposed to using a latent variable approach such as factor analysis to account for measurement error which typically requires larger sample sizes to produce reliable factor scores.

Next, Mplus was used to fit the complete model with structural paths for Time 1 cardinality predicting Time 1 symbolic magnitude, Time 1 symbolic magnitude predicting Time 2 symbolic addition, and EF skills predicting cardinality, symbolic magnitude, and symbolic addition skills, controlling for child age and gender (Figure 4). The conceptual model was over-identified, meaning the parameters of interest could be estimated from the amount of information available in the data and the model fit was testable. Standard model-fit indices were used to evaluate whether the MVPA model based on the conceptual model was able to sufficiently explain the relations in the observed data, including the Standardized Root Mean Squared Residual (SRMR) and Comparative Fit Index (CFI). This two-index presentation strategy was recommended by Hu and Bentler (1998), who further recommended the use of SRMR and at least one supplemental index to interpret the fit of models with maximum likelihood estimation methods. Based on their sensitivity analyses, SRMR was the most sensitive to model misspecification but robust to issues of smaller sample sizes (less than 250 participants). The SRMR is a measure of absolute model fit and captures the difference between the observed and model-predicted correlation, without penalizing more complex models with additional paths. Simulation research suggests that an SRMR of less than .08 indicates good model fit (Hu & Bentler, 1999). CFI was selected as the supplemental index of model fit. It is an incremental measure of model fit compared to the fit of the null model, with a complexity penalty for each additional parameter included in the model. Unlike other similar measures (e.g., TLI, NNFI, RMSEA), the CFI is appropriate for use with smaller samples sizes ($N < 250$; Hu & Bentler, 1998). CFI values greater than .95 are considered to indicate good model fit (Hu & Bentler, 1999). In addition, the magnitude and significance of each parameter estimate was interpreted to determine whether there is support for the proposed structural relations (e.g., cardinality predicting symbolic magnitude understanding). Standardized path coefficients between all constructs are reported to aid interpretability.

In the case of significant age or gender effects on children's performance on the mathematical or executive functioning measures, the demographic variables were included in all models as controls. If there were ceiling or floor effects that led to multivariate non-normal distributions between the set of composite variables, Satorra-Bentler (S-B) scaling methods was used to correct the model fit indices and standard errors using a factor based on the degree of non-normality of the variables (Satorra & Bentler, 2001). Non-normal distributions combined with smaller sample sizes can lead to biased fit indices that over-reject correctly specified models and biased standard errors that underestimate the true variation in parameters (Finney & DiStefano, 2013).

**Aim 1. To determine whether children's cardinality knowledge predicts their symbolic magnitude knowledge.** The present study tested whether children's early cardinality knowledge predicted their symbolic magnitude understanding concurrently and 3 – 4 months later. The magnitude and significance of the structural path for Time 1 cardinality predicting Time 1 symbolic magnitude in the overall MVPA model was used to address this aim. In keeping with previous empirical findings (e.g., Geary & vanMarle, 2018), children's cardinality knowledge was expected to predict their symbolic magnitude knowledge. Unlike previous studies, the present study included more rigorous measures of children's cardinality and symbolic magnitude knowledge, with both more tasks per construct as well as more individual trials within each of the tasks. To further investigate this aim, a separate linear regression was conducted to test if children's Time 1 cardinality skills predicted their Time 2 symbolic magnitude skills, controlling for Time 1 symbolic magnitude skills and child age. This allowed for a more stringent test of the hypothesized causal relation, across longitudinal as opposed to concurrent assessment time points while controlling for children's earlier performance on the same measure.

In addition, descriptive analyses were used to unpack the relations among these skills, including identifying children with partial cardinality knowledge (e.g., knowledge of cardinal values of one, two, three, and so on) and comparing their accuracy on symbolic magnitude trials involving numbers within and beyond their knower-levels.

**Aim 2. To determine whether children's symbolic magnitude knowledge predicts their addition skills.** Theoretical and empirical research with older children and adults suggests that symbolic magnitude understanding predicts later mathematics achievement, however, research testing this hypothesis with preschool children is limited. The present study examined whether preschoolers' symbolic magnitude understanding predicts their later performance on symbolic addition assessments, controlling for other foundational numerical and executive functioning skills. It was expected that symbolic magnitude understanding assessed by Time 1 performance on magnitude comparison and number line estimation tasks would significantly predict children's Time 2 arithmetic scores, in keeping with prior theoretical and empirical trends (Schneider et al., 2016, 2018; Siegler, 2016). Though previous research has identified other numerical skills including verbal counting and cardinality as significant predictors of math achievement (e.g., Aunio & Niemivirta, 2010; Geary & vanMarle, 2016), symbolic magnitude knowledge is arguably the culmination of the other foundational numerical skills. Symbolic magnitude knowledge relies on the integration of symbolic number knowledge (both number words and numerals), a cardinal understanding of what quantity each number refers to, and the knowledge of consecutive and non-consecutive relations between numbers. Children's symbolic magnitude knowledge was therefore expected to significantly predict their later addition skills, above and beyond the variance explained by their cardinality and executive function skills.

Like in Aim 1, the magnitude and significance of the corresponding structural pathway in the MVPA model was used to determine whether Time 1 symbolic magnitude understanding significantly predicted Time 2 addition performance. A separate linear regression was also conducted to determine whether Time 1 symbolic magnitude understanding predicted Time 2 addition scores, controlling for Time 1 addition scores and child age. This provided a more rigorous test of the hypothesized causal relation by controlling for children's previous addition scores. In addition, children's symbolic magnitude knowledge was categorized as at or below chance versus above chance performance on the magnitude comparison task, and their mean accuracy on addition measures were compared using independent samples *t*-tests.

**Aim 3. To determine whether children's symbolic magnitude knowledge fully mediates the relation between their cardinality and addition skills.** Recent longitudinal work suggests that preschoolers' cardinality knowledge is the most significant predictor of their later math achievement (Geary & vanMarle, 2018), while research with older children implicates symbolic magnitude understanding as a key predictor of math achievement (Schneider et al., 2016, 2018). The present study was the first to test whether symbolic magnitude understanding mediates the relation between preschool children's cardinality and addition skills. To this end, second MVPA model was tested with an additional direct path between Time 1 cardinality and Time 2 addition skills, representing a partial mediation.

Next, the Akaike Information Criteria with a second-order bias correction (AICc) was used as a comparative measure of fit when determining the better of two models. The second-order bias correction was developed to adjust for poor performance when there are too many estimated parameters relative to the sample size (Anderson, 2008). The AICc is computed by adding a correction term to the AIC equivalent to $2K(K + 1) / (n – K – 1)$, where K refers to the number of predictors and n refers to the sample size. The difference in model AICcs was computed as $\Delta_i = AICc_i – AICc_{min}$, where $AICc_{min}$ refers to the model with the lower AICc value. To better interpret the $\Delta_i$ values, the following benchmarks were applied: 1) if $\Delta_i < 2$ there is substantial support for the *i*-th model; 2) if $2 < \Delta i < 4$ there is strong support for the *i*-th model; 3) if $4 < \Delta_i < 7$ there is little support for the *i*-th model; and 4) if $\Delta_i > 7$ there is no support for the *i*-th model (Burnham & Anderson, 2004). Given that the partial mediation model was just-identified (fully saturated), the two models could not be compared using absolute fit statistics like a chi-squared difference test. Since symbolic magnitude knowledge of the relations between multiple numbers builds from a cardinal understanding of each individual number, it was expected that preschool children's symbolic magnitude understanding would fully mediate the relation between their cardinality and arithmetic skills.

**Aim 4. To investigate the relative contributions of domain-specific numerical skills and domain-general executive functioning (EF) skills in explaining children's addition skills.** Domain-general theories of cognitive development highlight the role of EF skills in children's mathematical development, yet little research to date has directly compared the relative effects of children's domain-specific numerical and domain-general EF skills on their later math achievement. It was expected that both numerical and EF skills would be significant predictors of children's arithmetic skills, however, children's numerical skills would have a larger standardized effect on their addition skills than children's EF skills. This hypothesis was assessed by comparing the magnitude of the direct and indirect standardized path coefficients of the EF skills composite predicting the addition composite with the path coefficients of the numerical skills composites (i.e., cardinality and symbolic magnitude) predicting the addition composite.

# Chapter 4: Results

## Preliminary Analyses

Descriptive statistics were conducted to summarize children's performance on each measure at each time point and compare their performance on the numerical measures across time points (Table 3). On average, children showed significant improvements on all numerical knowledge measures between Time 1 and Time 2 except for the number line estimation task ($t(132) = 1.11$, $p = .27$). Bivariate correlations of the measures administered within time point are presented in Table 4. Bivariate correlations were conducted between each numerical knowledge task administered during Time 1 and Time 2 to assess the stability of children's numerical knowledge over time (Table 5). All of the numerical knowledge tasks were significantly and positively correlated between Time 1 and Time 2 administration. Performance on the Give-N, Point-to-X, How Many, magnitude comparison, and story problem addition tasks were strongly correlated across Time 1 and Time 2 administrations, while the number line estimation and forced choice addition tasks showed weak to moderate correlations.

**Gender.** A total of 68 girls and 72 boys participated in the study. Means and standard deviations on all of the numerical and executive functioning tasks by child gender are presented in Table 6. To determine if there were differences in numerical or executive functioning skills between genders, independent samples *t*-tests were conducted. There were no differences on children's executive functioning skills, however, there were significant differences on children's Time 1 Point-to-X and story problem addition performance and Time 2 Give-N and Point-to-X performance, favoring girls. There were also significant gender differences on the composite scores for cardinality and addition. To account for these differences in children's performance, gender was controlled for in all primary analyses.

**Age.** The mean age of participants at the first assessment was 4 years and 5 months, with ages ranging from 3 years, 2 months to 5 years, 4 months. To determine whether children's numerical or executive functioning skills varied as a function of age, correlations were conducted between child age and each measure of numerical knowledge and executive functioning (Table 7). All of the measures of numerical and executive functioning skills were significantly correlated to children's age except Time 1 forced choice addition, which was marginally significant ($r(137) = .163$, $p = .056$). Table 8 provides descriptive statistics of children's performance on each numerical and executive functioning task by age (3, 4, or 5 years old). Across tasks, 5 year olds tended to have higher performance than 4 year olds, who in turn tended to have higher performance than 3 year olds. There was not a significant difference in the average age of male and female participants ($M_{\text{boys}} = 53.3$ months, $M_{\text{girls}} = 52.8$ months, $t(138) = .411$, $p = .681$). Given the significant and consistent relations between children's age and performance on numerical and executive functioning tasks, child age during the first assessment session was controlled for in all subsequent analyses.

**Correlates of improvement.** Children significantly improved on all numerical knowledge measures between Time 1 and Time 2, except on the number line estimation task (Table 3). Standardized difference scores were calculated for each numerical knowledge measure by subtracting children's Time 1 score from their

Time 2 score and dividing by the standard deviation of Time 1 scores. The standardized difference scores were not significantly correlated with children's age or gender on any measure, but were significantly negatively correlated with children's initial Time 1 scores on every measure (Table 9). This suggests that children with the lowest scores at Time 1 on each measure gained the most between Time 1 and Time 2 on the respective numerical knowledge measures.

### Primary Analyses

**Composite scores.** Composites were created for each construct (cardinality, symbolic magnitude, symbolic addition, executive functioning) during each time point (Time 1 and Time 2), for a total of seven composites. Each composite was created using principle components analysis (PCA) without rotation on its corresponding measures using SPSS Statistics Version 25 (see Table 10 for component loadings). The PCA for Time 1 cardinality was conducted on the Time 1 Give-N, Point-to-X, and How Many measures. It resulted in one component explaining 77 percent of the variance. The PCA for Time 1 symbolic magnitude was conducted on the Time 1 symbolic magnitude comparison and reverse-scored number line estimation tasks. It resulted in one component explaining 71 percent of the variance. The PCA for Time 1 addition was conducted on the Time 1 forced choice addition and story problem addition tasks. It resulted in one component explaining 61 percent of the variance. The PCA for executive functioning was conducted on the Time 1 working memory, inhibitory control, and attention shifting tasks. It resulted in one component explaining 61 percent of the variance. The PCA for Time 2 cardinality was conducted on the Time 2 Give-N, Point-to-X, and How Many measures. It resulted in one component explaining 76 percent of the variance. The PCA for Time 2 symbolic magnitude was conducted on the Time 2 symbolic magnitude comparison and reverse-scored number line estimation tasks. It resulted in one component explaining 77 percent of the variance. The PCA for Time 2 addition was conducted on the Time 2 forced choice addition and story problem addition tasks. It also resulted in one component explaining 61 percent of the variance.

In sum, all of the PCA analyses resulted in a single component per construct, explaining between 61 and 77 percent of the variance. Regression scores for each PCA were generated for participants and used in the measured variable path analysis (MVPA). Descriptive statistics of each composite score are reported in Table 11. There were no outlying composite scores (+/- 3 standard deviations from the mean) on any composite, so all composite scores were retained for use in the primary analysis. The skew and kurtosis statistics suggested that the Time 1 cardinality and Time 2 addition composites were not normally distributed, nor was the multivariate distribution of Time 1 cardinality, Time 1 symbolic magnitude, and Time 2 addition composites. Violations of multivariate non-normality, often signaled by non-normal distributions on individual variables, can lead to biased fit indices when used in a MVPA model (Finney & DiStefano, 2013; Satorra & Bentler, 2001). To adjust for the non-normality of the composite variables, the Satorra-Bentler (S-B) scaling methods was used to correct the model fit indices and standard errors using a factor based on the degree of multivariate non-normality. Since the S-B scaling methods require listwise deletion, an additional MVPA model using Full Information

Maximum Likelihood (FIML) to estimate parameters based on all available data was also conducted (Appendix D).

**Measured variable path analysis (MVPA).** Mplus8 Demo Editor, Version 1.6 was used to test the hypothesized conceptual model using MVPA. The model specified a direct path from Time 1 cardinality predicting Time 1 symbolic magnitude, a direct path from Time 1 symbolic magnitude predicting Time 2 symbolic addition, and three direct paths for EF skills predicting cardinality, symbolic magnitude, and symbolic addition skills, respectively. As is standard in structural equation modeling frameworks, child age and gender were added as control variables with direct paths to each of the constructs (cardinality, magnitude, addition, and executive functioning) and allowed to covary (Figure 4). The chi-square for the model was significant $\chi^2(1) = 4.73$, $p = .03$, and the fit indices met the benchmark criteria for a good fit to the data. The CFI is 0.987, greater than the benchmark value of 0.95, suggesting the model explains a good amount of variance in the data compared to a null model. The SRMR is 0.022, less than the benchmark of 0.08, suggesting there is little variance left unexplained after accounting for the model. These results suggested that there is adequate data-model fit and the proposed model could be retained. All of the hypothesized structural pathways of interest were statistically significant and positive. Table 12 reports the unstandardized and standardized path estimates, standard errors, and significance levels.

**Aim 1. To determine whether children's cardinality knowledge predicts their symbolic magnitude knowledge.** Results from the MVPA model indicated that children's Time 1 cardinality skills significantly predicted their Time 1 symbolic magnitude skills, $\beta = 0.51$, S.E. = 0.07, $p < .001$. Assuming a correct underlying model, the standardized path from cardinality to symbolic magnitude can be interpreted as a one standard deviation increase in cardinality causes, on average, a 0.51 standard deviation increase in symbolic magnitude holding all else constant.

Multiple linear regression analysis was used to test if children's Time 1 cardinality skills significantly predicted their Time 2 symbolic magnitude skills, controlling for Time 1 symbolic magnitude skills, child age, and child gender. The results indicated that the regression was statistically significant and accounted for 58% of the variance in Time 2 symbolic magnitude skills ($R^2 = .58$, $F(4, 127) = 43.42$, $p < .001$). Specifically, Time 1 cardinality skills significantly predicted Time 2 symbolic magnitude skills ($\beta = .50$, $t(127) = 5.85$, $p < .001$), meaning a one standard deviation change in Time 1 cardinality skills was associated with a 0.50 standard deviation change in Time 2 symbolic magnitude skills, holding Time 1 symbolic magnitude skills, age, and gender constant (Table 13).

Descriptive analyses were used to categorize children into their knower-level (based on Time 1 Give-N scores) and summarize their performance on symbolic magnitude tasks during Time 1 and Time 2 (Table 14). As hypothesized, children with more cardinality knowledge at Time 1 generally had more accurate performance on symbolic magnitude tasks. One-way ANOVAs with Give-N scores as the grouping variable and Time 1 and Time 2 symbolic magnitude comparison and number line estimation tasks as the outcomes revealed significant differences between Give-N ability levels on all magnitude measures. Looking at the trial level data, children's performance on symbolic magnitude comparison trials where they understood the cardinal values of both numbers was compared against performance on trials where they did not understand the cardinal value of one or both numbers. For example, if a child was categorized as a 4-knower based on their Time 1 Give-N task performance, their accuracy across all symbolic magnitude comparison trials that involved numbers 5 through 9 were averaged (e.g., 7 vs. 2, 1 vs. 8). These average magnitude comparison accuracy scores for trials with known and unknown cardinal values reported in Table 15. The results suggest that knowledge of individual cardinal values does relate to higher performance on symbolic magnitude trials involving those numbers: across knower-levels, children's average performance on comparison trials involving known cardinal values was greater than their average performance on comparison trials involving at least one unknown cardinal value.

**Aim 2. To determine whether children's symbolic magnitude knowledge predicts their addition skills.** Results from the MVPA model indicated that children's Time 1 symbolic magnitude skills significantly predicted their Time 2 addition skills, $\beta = 0.23$, S.E. = 0.10, $p = .018$. Assuming a correct underlying model, the standardized path from symbolic magnitude to addition can be interpreted as a one standard deviation increase in cardinality causes, on average, a 0.23 standard deviation increase in symbolic magnitude holding all else constant.

Multiple linear regression analysis was used to test if children's Time 1 symbolic magnitude skills significantly predicted their Time 2 addition skills, controlling for Time 1 addition skills, child age, and child gender. The results indicated that the regression was statistically significant and explained 37% of the variance in children's Time 2 addition skills ($R^2 = .37$, $F(4, 126) = 18.14$, $p < .001$).

Specifically, Time 1 symbolic magnitude skills significantly predicted Time 2 addition skills ($\beta = .23$, $t(126) = 2.71$, $p = .008$), meaning a one standard deviation change in Time 1 symbolic magnitude skills was associated with a 0.23 standard deviation change in Time 2 addition skills, holding Time 1 addition skills, age, and gender constant (Table 16).

Descriptive analyses were used to categorize children's Time 1 symbolic magnitude comparison performance as at or below chance versus above chance performance. Next, the mean accuracy on Time 1 and 2 addition measures of children above and below chance on symbolic magnitude were compared using independent samples *t*-tests (Table 17). Children who performed above chance on the Time 1 symbolic magnitude comparison task had significantly higher performance on the Time 1 and 2 forced choice and story problem addition tasks than children who performed at or below chance.

**Aim 3. To determine whether children's symbolic magnitude knowledge fully mediates the relation between their cardinality and addition skills.** The conceptual model tested with the primary MVPA assumes that symbolic magnitude fully mediates the relation between children's cardinality and addition skills. To determine whether a partial mediation model that includes an additional direct path between Time 1 cardinality and Time 2 addition skills better fit the data, a second MVPA model was run (Figure 5). The partial mediation model was just-identified meaning it had zero degrees of freedom, therefore absolute model fit could not be assessed with the CFI and SRMR. However, the full and partial MVPA models could be compared with comparative measures of fit. Model comparison can be carried out in a variety of ways for models subsumed within SEM, including difference of SEM fit indices (*ΔCFI* or *ΔRMSEA*) and the likelihood ratio test for nested models. Methods based on information criteria can be used as well (Anderson, 2008). In particular, the Akaike Information Criteria with a second-order bias correction (AICc) can be used to compare model fit with an additional adjustment to improve estimation when there are a larger number of parameters relative to the sample size. The AICc for the partial mediation model ($AICc_p$ = 1231.05) was slightly smaller than the AIC for the full mediation model ($AICc_f$ = 1233.36), although the absolute value of the difference in AICcs between the two models was less than the benchmark value of 4, indicating strong support for the full mediation model (Burnham & Anderson, 2004). This suggested that the more parsimonious full mediation model was a better fit for the data than the saturated partial mediation model.

**Aim 4. To investigate the relative contributions of domain-specific numerical skills and domain-general executive functioning (EF) skills in explaining children's addition skills.** Results from the full mediation MVPA model indicated that children's symbolic magnitude skills significantly predicted their addition skills (see above). In addition, all of the direct paths between children's Time 1 EF skills and numerical skills were statistically significant, including the direct path between Time 1 EF skills and Time 2 addition skills, $\beta = 0.22$, S.E. = 0.10, $p = .023$. The total standardized direct and indirect effects of children's cardinality, symbolic magnitude, and executive functioning skills on Time 2 addition are shown in Table 18. The total effects of both numerical skills and EF skills were positive and statistically significant. Contrary to the original hypothesis, the magnitude of the total effects of numerical and EF skills were comparable (0.347 and 0.328, respectively), suggesting that both numerical and EF skills played an important role in explaining variability in children's addition skills.

**Summary.** In sum, the majority of the hypothesized relations were supported by the data. The preliminary analyses revealed significant age effects (favoring older children over younger children) and gender effects (favoring girls over boys). The numerical and EF measures were significantly correlated within time points, and the individual numerical measures were correlated across Time 1 and 2 administrations. Principle components analysis yielded seven composite scores, with one-component solutions for each of the hypothesized constructs. Overall, the conceptual model fit the data well, and all hypothesized paths were statistically significant and positive. Additional linear regressions and descriptive task-level analyses provide further evidence for cardinality predicting symbolic magnitude and symbolic magnitude predicting addition. Model comparisons suggest strong evidence in favor of the full mediation model with symbolic magnitude mediating the relation between cardinality and addition rather than the partial mediation model. Finally, both EF and numerical skills explained a significant amount of variability in children's addition skills, with total effects that were comparable.

## Chapter 5: Discussion

Broadly, the goal of the present study was to investigate the development of children's symbolic magnitude understanding in early childhood. Specifically, this study aimed to: 1) Determine whether children's cardinality knowledge predicts their symbolic magnitude knowledge, 2) Determine whether children's symbolic magnitude knowledge predicts their addition skills, 3) Determine whether children's symbolic magnitude knowledge fully mediates the relation between their cardinality and addition skills, and 4) Investigate the relative contributions of domain-specific numerical skills and domain-general executive functioning (EF) skills in explaining children's addition skills. The results of the study provide evidence that symbolic magnitude understanding in preschool relates to children's later addition skills, above and beyond other foundational numerical and executive functioning skills.

The present study extended previous research on children's early numerical understanding in several important ways. Despite significant theoretical and empirical support for the importance of symbolic magnitude understanding for later mathematical achievement, the present study was one of the first to investigate its relation with later addition skills during the preschool period when children are first learning the meaning of symbolic numbers. Further, the present study examined the unique contributions of symbolic numerical and executive functioning skills in predicting children's symbolic magnitude understanding, moving away from previous literature that asked whether non-symbolic magnitude skills predict symbolic magnitude skills or pitted both types of magnitude skills against one another as predictors of math achievement. Finally, by using measured variable path analysis, the present study was able to test a broader conceptual model of predictive longitudinal relations simultaneously as opposed to alternative approaches that investigate the predictors of one skill at a time. In sum, the present study was the first to test a comprehensive longitudinal model of the symbolic magnitude understanding of preschoolers, which in turn has critical implications for the design of comprehensive early numeracy interventions to support school readiness skills.

### Variability in Children's Numerical and Executive Functioning Skills

In the present study, there was substantial variability across children's numerical and executive functioning skills. As in previous studies of preschool children, child age was related to all of the individual numerical and executive functioning tasks. On average, children who were older at the first assessment session tended to have higher performance on the measures than children who were younger. Older children (5 year olds) would be more likely to be attending their second year of preschool than younger children (3 – 4 year olds), and in turn would have likely had significantly more exposure to school-based math instruction. Moreover, the age-related differences in EF likely reflect broader age-related trends in children's general cognitive abilities. During the early childhood period, rapid changes in children's brain development are strongly correlated with improvements in children's EF skills (for reviews, see Best & Miller, 2010; Shanmugan & Satterthwaite, 2016). To account for the significant age-related differences, child age was controlled for in all analyses. An alternative method would have been to model children's skill development separately by their age (3, 4, or 5 years old), however, the sample size of the present study prevented subgroup analyses (see Limitations).

Surprisingly, child gender was also related to performance across time points of the study, specifically on measures of cardinality and addition. Meta-analyses and descriptive reviews show little evidence of gender gaps in overall math performance (Hyde, 2005; Hyde, Fennema, & Lamon, 1990; Lindberg, Hyde, Petersen, & Linn, 2010; Spelke, 2005). A recent empirical study compared 1,391 male and female children aged 6 – 13 years old on a number of foundational numerical tasks, including symbolic and non-symbolic magnitude comparison, number line estimation, and addition and subtraction problems (Hutchison, Lyons, & Ansari, 2019). Their results indicated little evidence for gender difference on measures of basic numerical skills. When using Bayesian analyses, the only task to show a significant gender difference was children's 0 – 1000 number line estimation, favoring boys (Hutchison et al., 2019). Thus, given the trends in the broader research literature, the significant gender differences in children's cardinality and addition skills in the present study were unexpected. Although there was no significant age difference between male and female children in this study, the overall MVPA model revealed a significant relation between gender and EF skills, also favoring girls. It is possible that the gender differences on numerical skills observed in the present study relate to EF or more general attention skills, as young female children tend to have higher performance on executive control tasks than young male children (e.g., Wiebe, Espy, & Charak, 2008). It is unlikely that the gender differences observed here are due to the socioeconomic status of the children that participated in the study, as previous research suggests gender differences on mathematical and spatial measures tend to be observed among higher-SES samples but not among lower-SES samples (e.g., Levine, Vasilyeva, Lourenco, Newcombe, & Huttenlocher, 2005; McGraw, Lubienski, & Strutchens, 2006).

In sum, the present study found significant age and gender differences in children's performance on numerical and executive functioning tasks. Age-related differences were expected based on previous literature, however, gender-related differences on numerical skills were unexpected, potentially due to shared variability with children's executive functioning skills. Additional research is needed to determine whether gender differences in early cardinality and addition skills of preschoolers are replicable, and test for potential mediating variables such as executive functioning or self-regulation skills.

**The Relation Between Cardinality and Symbolic Magnitude Understanding**

Findings from the present study indicate that preschool children's cardinality knowledge significantly predicts their symbolic magnitude understanding. Although much of the previous research on the magnitude understanding of preschool-aged children focused on the relations between non-symbolic and symbolic magnitude, a review of the literature suggested that other symbolic numerical skills likely explain more of the variance in children's symbolic magnitude skills than non-symbolic magnitude skills. The present study focused on children's cardinality knowledge as a predictor of symbolic magnitude skills, due to emerging research and theory suggesting cardinality is a so-called "gateway" skill to children's broader mathematical achievement (e.g., Geary et al., 2017; Geary & vanMarle, 2018; Spaepen et al., 2018). Specifically, the present study tested whether children's cardinality skills predicted their symbolic magnitude skills, controlling for demographic variables and domain-general executive functioning skills.

The relation between cardinality and symbolic magnitude skills was hypothesized because understanding what individual numbers mean (i.e., their cardinal values) could theoretically serve as the foundation for understanding the relative meanings of numbers (i.e., symbolic magnitude). Developmentally, when children acquire an understanding of small whole numbers they may begin to successfully order and compare the numbers that they understand individually. As children begin to understand the cardinal values associated with larger numbers, they may be better able to order and compare those numbers against other known quantities. Crucially, this hypothesis suggests that children who understand the cardinal value of smaller whole numbers may be able to accurately compare and order those numbers in symbolic magnitude tasks, even if they have not yet acquired the cardinal principle and demonstrated an understanding of the cardinal value of all numbers assessed.

The present study found support for the relation between cardinality and symbolic magnitude in several ways. First, the structural path between Time 1 cardinality and symbolic magnitude skills was statistically significant and positive as hypothesized. Given the broad conceptual model tested in the MVPA, the significance and magnitude of this pathway suggests the relation between cardinality and symbolic magnitude understanding 3 – 4 months later is robust, even when controlling for related domain-general executive functioning skills, child age, and child gender. Second, the multiple linear regression of Time 1 cardinality predicting

Time 2 symbolic magnitude skills demonstrated evidence of a longitudinal predictive relation, lending more support to the hypothesized causal direction. This relation across the time between assessments remained significant when controlling for Time 1 symbolic magnitude skills, child age, and child gender. Third, trial-level descriptive analyses showed a pattern of improved symbolic magnitude comparisons for children with greater cardinality knowledge, and higher accuracy for trials relying on numbers with cardinal values familiar to the child. Specifically, across knower-levels, children's average performance on comparison trials involving known cardinal values was greater than their average performance on comparison trials involving at least one unknown cardinal value. This provides support for the hypothesized developmental mechanism between cardinality and symbolic magnitude understanding, such that a child who knows the cardinal value of the numbers 1, 2, and 3 would be more accurate in making magnitude comparisons with those numbers than comparisons with larger numbers beyond their cardinality knower-level. Furthermore, unlike previous research that suggests that children must acquire the cardinal knowledge principle before understanding symbolic numbers (Geary et al., 2017), the present results suggest that children with partial cardinality knowledge demonstrate some understanding of symbolic magnitudes of numbers within their cardinal knower-levels.

In sum, preschool children's cardinality skills predict their concurrent and later symbolic magnitude skills. This finding is in keeping with previous studies of young children's numerical skills, which found the age at which children became cardinal principle knowers significantly predicted their symbolic magnitude skills and performance on a standardized math assessment (Geary et al., 2017; Geary & vanMarle, 2018). It extends previous studies by testing the relation with a more rigorous assessment of cardinality and symbolic magnitude understanding, including multiple measures of each construct with many opportunities for children to demonstrate their knowledge. Importantly, the selected symbolic magnitude measures allow for children to use their knowledge of symbolic magnitudes based on number words, unlike previous studies that required children to make comparisons based on their knowledge of written numerals. Furthermore, the present study is the first to analyze trial-level accuracy data to demonstrate evidence of the incremental developmental improvements for children's symbolic magnitude understanding as they acquire an understanding of the cardinal values of specific numbers.

**The Relation Between Symbolic Magnitude Understanding and Addition**

As hypothesized, the present study found evidence of a significant predictive relation between children's symbolic magnitude understanding and later symbolic addition skills. Previous research with school-aged children and adults has shown significant positive relations between symbolic magnitude understanding and broader mathematical achievement, but few studies have tested the relation with samples of preschool-aged children. Of the research that has examined the links between magnitude understanding and math achievement in younger children, the most consistent links arose between preschool symbolic magnitude skills and math achievement later on in elementary school, with less support for significant relations with math achievement during preschool (e.g., Cankaya et al., 2014; De Smedt et al., 2009; Friso van de Bos et al., 2015; Simms et al., 2016; vanMarle et al., 2014).

Findings from experimental training studies showed some evidence of positive improvements on children's arithmetic skills after a symbolic magnitude intervention (e.g., Maertens et al., 2016; Siegler & Ramani, 2009), however, these studies did not control for other domain-specific numerical skills like cardinality or domain-general skills like executive functioning. The present study expanded on previous research to determine whether symbolic magnitude understanding was significantly related to children's preschool addition skills, above and beyond cardinality and executive functioning.

The integrated theory of numerical development suggests magnitude understanding provides the foundation for later math achievement by grounding the higher order manipulations of more advanced math problems, helping students learn arithmetic, select appropriate strategies when solving math problems, and identify more plausible answers to problems (Mussolin et al., 2016; Siegler, 2016). One potential developmental mechanism between children's symbolic magnitude understanding and addition skills may relate to their understanding of part-whole relations. Specifically, it is possible that understanding the magnitudes of numbers and how they relate to one another could help children to recognize set combinations and the part-whole relations that form the basis of simple addition problems. This in turn could facilitate children's understanding of basic arithmetic facts, particularly in problem types supported by non-symbolic representations of quantities like the forced choice and story problem addition measures used in the present study (Daubert, 2018; Prather & Alibali, 2011). Given the substantial role of arithmetic in children's elementary school mathematics learning, the early numerical skills that predict arithmetic represent critical school readiness skills for preschool children.

The relation between symbolic magnitude understanding and addition skills was supported by the significant and positive path between Time 1 magnitude and Time 2 addition skills, the significant and positive coefficient of the symbolic magnitude composite in the multiple linear regression predicting later addition skills, and the descriptive analyses revealing significantly higher performance on concurrent and later addition tasks for children who were above chance on the symbolic magnitude comparison task. This significant predictive relation between symbolic magnitude understanding and addition skills was robust despite controls for children's cardinality skills, prior addition skills, executive functioning skills, age, and gender.

In sum, preschool children's symbolic magnitude skills predict their addition skills 3 – 4 months later. This finding is similar to previous studies of elementary school children (e.g., De Smedt et al., 2009; Fris van de Bos et al., 2013; Maertens et al., 2016), but contrasts with some previous longitudinal work with preschoolers that found there was no significant relation between symbolic magnitude understanding and later math achievement when controlling for children's cardinality and EF skills (e.g., vanMarle et al., 2014; Geary & vanMarle, 2016). This discrepancy in findings may be due in part to the operationalization of math achievement. While previous work used children's standardized scores on the *Test of Early Mathematical Ability-3* (TEMA-3; Ginsburg & Baroody, 2003), the present study used addition tasks that included non-symbolic representations of quantities. The TEMA-3 for 3 – 4 year old children includes items such as verbal and object counting, making numerical comparisons, simple arithmetic and number line problems, many of which are the types of tasks traditionally used to assess cardinality and symbolic magnitude understanding. By focusing on simple addition problems rather than using a broader math measures with overlapping tasks, the findings from the present study inform a more nuanced skill development model. Furthermore, the present study used more rigorous measures of symbolic magnitude understanding, with both substantially more trials and assessment protocols that allowed for children to access their knowledge of symbolic magnitudes based on number words rather than requiring them to recognize written numerals. By focusing on preschool-aged children, controlling for related numerical and executive functioning skills, and incorporating ample measures of children's symbolic magnitude understanding and addition skills, the present study extends the previous research with strong support for the predictive relation between preschool children's symbolic magnitude understanding and later addition skills.

**Symbolic Magnitude as a Mediator of Cardinality and Addition Skills**

Given the parallel research findings of the importance of cardinality and symbolic magnitude skills for predicting children's more advanced math skills, one goal of the present study was to test whether symbolic magnitude partially or fully mediated the relation between children's cardinality and addition skills. Although the results strongly suggest that symbolic magnitude is important to preschoolers' addition skills, the implications of partial versus full mediation differ. For example, if symbolic magnitude fully mediated the relation between cardinality and addition, it would suggest that symbolic magnitude acts as a type of gateway skill between foundational and more advanced numerical knowledge. If symbolic magnitude partially mediated the relation between cardinality and addition, it would underscore the significance of children's cardinality skills for broader math achievement.

The results of the present study suggest that there was strong evidence in favor of retaining the more simplistic full mediation model over the partial mediation model with an additional direct path between cardinality and addition skills. However, children's cardinality skills were a statistically significant predictor of their symbolic magnitude skills and the indirect effect of cardinality on addition was also significant, suggesting that both cardinality and magnitude skills are important to laying the foundations in early childhood for later mathematical success. Specifically, this finding suggests that cardinality and symbolic magnitude understanding should be a focal point of early childhood mathematics instruction prior to the introduction of simple arithmetic (see Implications and Applications section below). Furthermore, it is quite likely that the evidence in favor of a full versus partial mediation depends on the age of the children sampled. For example, the influence of cardinality skills on addition may be more important for samples of younger children (e.g., 2 and 3 year olds). The present study assessed mainly 4 and 5 year olds, which may have bolstered the relative importance of symbolic magnitude skills during the brief developmental window observed.

**Role of Domain-General and Domain-Specific Skills**

A wealth of research suggests that children's executive functioning skills are domain-general predictors of their mathematics achievement (e.g., Ackerman & Friedman-Krauss, 2017; Bull et al., 2011; 2014; Clark et al., 2013; McClelland et al., 2007). EF skills may be particularly important for young children's developing symbolic magnitude understanding in enabling children to suppress inappropriate strategies like choosing their favorite number instead of the bigger number (inhibition), mentally compare multiple quantities to form an understanding of relative magnitudes (working memory), and translate various sources of symbolic magnitude information presented verbally and visually as sets and numerals (attention shifting). Both EF skills and domain-specific numerical skills were hypothesized to significantly predict children's addition skills, however, the total effect of children's cardinality and symbolic magnitude skills was expected to be greater than the total effect of children's domain-general EF skills.

In keeping with the hypothesized relation, children's EF skills had a significant and positive total effect on their addition skills. Indeed, EF skills significantly predicted all three numerical skills (cardinality, symbolic magnitude, addition) in the MVPA model. However, contrary to the original hypothesis, the total standardized effects of children's numerical and EF skills on their addition skills were comparable in magnitude. This suggests that EF skills in early childhood may be equivalently important to explaining variability later addition skills as other numerical skills, particularly with samples of children from low-income households. Previous research has similarly highlighted the importance of EF skills for later math achievement for children from low-income households (e.g., Blair & Razza, 2007; Nayfeld et al., 2013), with some researchers hypothesizing that income-related gaps in children's EF skills may partially explain the observed gaps in children's math achievement (Ramani et al., 2017). Although some prior research has included measures of executive functioning as controls in models of children's mathematical development, the present study is one of the first to directly compare the unique contributions of EF and numerical skills on preschoolers' developing addition skills. Given that executive functioning and numerical skills explained a similar amount of variability in children's spring addition scores, researchers of numerical cognition should work towards incorporating more domain-general skills as predictors in broader models of numerical development rather than relegating them to be control variables.

**A Conceptual Model of Symbolic Magnitude in Early Childhood**

The overarching goal of the present study was to test the proposed conceptual model of symbolic magnitude understanding, complete with antecedents (cardinality and executive functioning) and a consequent (symbolic addition). The results provide support for each of the hypothesized structural relations, as described above. As one of the first studies to directly investigate children's symbolic magnitude skills during the preschool period, the findings provide evidence that symbolic magnitude understanding is as important to general mathematics achievement in early childhood as it is in samples of older children and adults. Previous research with elementary school aged children has shown that symbolic magnitude understanding predicts later broader math achievement (De Smedt et al., 2009; Friso van den Bos et al., 2015; Simms et al., 2016). By using developmentally appropriate symbolic addition tasks with pictures and physical manipulatives, it was possible to show a significant relation between children's symbolic magnitude skills and their earliest addition skills – some of the first skills that relate to numeracy but are more computationally complex. Finding support for the conceptual model of symbolic magnitude understanding provides a blueprint for the numerical skill development of children aged 3 to 5 years old; start by providing support for cardinality skills, followed by symbolic magnitude skills, and then begin to practice supported arithmetic problems.

The conceptual model tested by the present study was informed by and extends previous research on children's early mathematical development. Early in childhood, children begin to learn symbolic number representations in the form of number words and numerals, typically starting with the verbal count sequence (Gelman & Gallistel, 1978). There is a significant delay between knowledge of the count sequence and being able to connect known number words to their corresponding quantities, and understand the broader cardinal principle that the last word in the counting sequence also refers to the total set size (Wynn, 1992). However, as the present study demonstrates even children with partial cardinality knowledge show a beginning understanding of symbolic magnitude, or the relationships between symbolic numbers. In keeping with previous literature, the present study found that children who have acquired the cardinal principle tend to have the highest average performance on symbolic magnitude tasks. Furthermore, once children become more accurate in comparing and ordering symbolic numbers, the present study found that they also begin to improve their simple addition skills. This connection between symbolic magnitude and arithmetic skills had been shown previously in a longitudinal sample of elementary school children, but the present study is among the first to demonstrate the relation among preschool children. Moreover, children's simple arithmetic skills in the preschool period have been found to significantly predict their mathematical achievement through age 15 (Watts et al., 2014). The results of the present study describe a sequence of numerical skill development, beginning with cardinality, followed by symbolic magnitude understanding, leading to simple addition. Coupled with findings from prior cross-sectional and longitudinal studies, the results of the present study inform a piece of a broader developmental trajectory beginning with children's earliest verbal counting skills and continuing through their high school math achievement.

## Limitations

There are several limitations of the present study. First, although the sample size was adequate for assessing the proposed conceptual model with measured variable path analysis, it was a relatively small sample for a structural equation model. This limited the types of model fit indices that could be used to interpret the results, as well as the ability to assess model fit across subgroups of the sample. For example, the present study found significant relations between children's age at first assessment and performance on all numerical and executive functioning measures. It is possible that the specific relations between individual skills may differ as a function of child age, such that with a sample of only 3-year old participants with less cardinality knowledge, there would be less evidence for the direct path between magnitude understanding and addition skills. Although the present study assessed children ranging from 3 to 5 years old, there were too few children in each age group to justify fitting separate MVPA models for 3-, 4-, and 5-year olds. The MVPA model was corrected to account for non-normality in the measures from any age-related floor or ceiling effects and age at first assessment was included as a control variable in all analyses, however, future work should enroll more participants in order to better investigate age-related differences.

Second, the proposed conceptual model and the modeling of structural paths presume a causal influence between the numerical and executive functioning skills however the study employed a longitudinal design. Although the longitudinal nature of the assessments and the additional descriptive analyses provide evidence in favor of the causal hypotheses, an experimental design would be more conclusive. Specifically, a study that randomly assigned children to receive a cardinality or symbolic magnitude training followed by assessments of their magnitude and addition skills would be better equipped to assess a causal relation. The present study takes an important first step with significant longitudinal findings, but further experimental research is needed in this area.

Finally, although the present study established some support for the proposed conceptual model of symbolic magnitude understanding, it did not account for many related domain-general and domain-specific skills. It remains an open question whether children's general ability (i.e., intelligence or processing speed) would help to explain variability in children's numerical skills over time, above and beyond demographic and executive functioning controls. Furthermore, it is likely that children's other core symbolic number skills relate to their development of symbolic magnitude understanding and addition performance, including verbal counting, ordinality, and numeral identification skills (Carey, 2009; Purpura et al., 2013; Reynvoet & Sasanguie, 2016). Additional longitudinal or microgenetic work may be needed to better capture incremental changes in children's symbolic number knowledge, bolstering the evidence from landmark cross-sectional studies of children's early math skills (e.g., Gelman & Gallistel, 1978; Schaeffer et al., 1974; Wynn, 1992).

**Future Directions**

There are a number of future directions for research related to these constructs. One important contribution of the present study was the use of multiple measures for each construct, with more trials per measure to assess children's knowledge. However, future research should look beyond the traditional measures from the numerical cognition research literature to identify more sensitive measures of children's knowledge. For example, children's performance on traditional magnitude comparison tasks with close-ended, pre-specified comparisons may be masking their partial understanding of symbolic magnitudes. It is possible that a child may struggle to correctly identify the larger number between 5 and 6, but could easily share other examples of numbers with magnitudes larger than 5 if asked with an open-ended prompt (e.g., "what is a number that is bigger than 5?" rather than "which is more, 5 or 6?"). The use of such measures could ultimately lead to a more accurate representation of children's numerical knowledge that in turn reveals more nuanced patterns of relations among the constructs of interest. Relatedly, the present study focused on children's performance at individual time points, but looking at their growth in skills across time may be more informative to understanding the relations among numerical skills. Future research should collect the same scalable measures across multiple time points spanning the preschool and early elementary school years. Indeed, some longitudinal analyses suggest children's growth in early mathematical skills is more predictive of their later math achievement than their individual starting points (e.g., Watts et al., 2014).

Another direction for future research relates to unpacking the role of executive functioning skills in the development of children's symbolic magnitude understanding. Research with older children and adults has suggested that some components of EF such as working memory are more important to math learning than others (Bull & Lee, 2014; Kolkman, Hoijtink, Kroesbergen, & Leseman, 2013; Van der Ven, Kroesbergen, Boom, & Leseman, 2012). However, that hypothesis may be less applicable for samples of preschool-aged children, whose EF skills are better explained by a single factor (Bull & Lee, 2014; Clark et al., 2013; Shing et al., 2010; Wiebe et al., 2008). In a similar vein, the components of preschoolers' EF skills are thought to fall under the conceptual umbrella of self-regulation skills, closely related to attention (Blair & Razza, 2007). Given the methodological challenges in assessing young children's cognitive skills accurately, it becomes conceptually unclear whether the relations between EF and children's mathematics performance is truly unique or indicative of relations with more general self-regulation abilities. Furthermore, recent research has identified EF skills as both significant mediators and moderators of the relations between numerical skills (e.g., Ribner et al., 2017). It is also possible that children need a set amount or threshold of executive functioning skill to perform optimally on mathematics tasks. Future studies should compare alternative models of EF skills on children's numerical skill development to determine whether mediation, moderation, or a minimum threshold best capture the relations observed in the data.

Moreover, given the evidence in the present study suggesting the importance of symbolic magnitude understanding for early addition skills, there are several open questions about how to best support young children's magnitude learning. A number of studies have found significant effects on children's symbolic magnitude and broader numerical skills following practice with intervention materials (e.g., Maertens et al., 2016; Ramani et al., 2017; Ramani & Siegler, 2008; Scalise et al., 2017, 2019; Siegler & Ramani, 2009; Whyte & Bull, 2008), yet it is not clear what would be the best method of symbolic magnitude practice that would lead to the largest improvements. For example, Ramani and Siegler (2008; 2009) randomly assigned Head Start preschoolers to play numerical linear board games, and saw significant improvements on their magnitude comparison and number line estimation skills relative to control conditions. More recently, researchers have demonstrated the effectiveness of playing numerical tablet computer games (Ramani et al., 2017), and making magnitude comparisons between pairs of numbered cards at improving young children's symbolic magnitude understanding (Scalise et al., 2017; 2019). It is possible that some forms of magnitude training like exposure to linear number representations while playing board games or exposure to non-symbolic and symbolic quantity representations while playing card games may be more or less effective at promoting children's magnitude knowledge. Future research could contrast the effectiveness of these types of trainings by randomly assigning children to play magnitude board games or card games, or test the effectiveness of interleaved training with exposure to both linear and non-symbolic magnitude representations.

Finally, it remains an open question as to the role of social influences, particularly interactions with parents and educators, on children's symbolic magnitude understanding. Observational studies of parents and educators have found infrequent occurrences of numerical magnitude talk in naturalistic situations (Klibanoff, Levine, Huttenlocher, Vasilyeva, & Hedges, 2006; Ramani, Rowe, Eason, & Leech, 2015; Scalise, Ramani, & Pak, 2019), although in general, parents' and educators' talk about numerical concepts significantly predicts their young children's mathematical achievement (see Scalise, Gladstone, & Ramani, 2019 for a review). The experimental research literature to date is less clear on the types of guidance or scaffolding that is key to improving children's magnitude understanding. For example, it is possible that the relative success of experimenter-led magnitude trainings on promoting children's magnitude understanding relates to adults guiding children's attention to particular magnitude features, or asking children open-ended questions rather than telling them the correct solution. Indeed, a recent study found significant improvements to preschool children's addition skills following a brief magnitude training incorporating open-ended questions posed to children (Daubert, 2018), underscoring the importance of one aspect of adult guidance during magnitude-related interactions with young children.

**Implications and Applications**

The results of the present study suggest several implications for parents and educators of young children. First, it provides evidence that symbolic magnitude understanding during the preschool period is important for the development of addition skills. Addition skills are a hallmark of early elementary school mathematics instruction, and in turn, the results of the present study suggest that magnitude understanding should be considered a key school readiness skill. Although a number of intervention studies have demonstrated that children's symbolic magnitude skills are malleable with brief trainings (e.g., Ramani & Siegler, 2008; Scalise et al., 2017; Whyte & Bull, 2008), observational research suggests many parents and educators refer far less frequently to symbolic magnitude than other more basic numerical concepts in their conversations with young children (Klibanoff et al., 2006; Ramani et al., 2015). In general, it is helpful to ask children to compare or order the magnitudes of different numbers in everyday conversations, such as whether a 7-year old or a 10-year old is older, how to place the numbers in order for a hopscotch game, and whether one sibling received more, less, or the same number of apple slices. This contrasts with typical counting instruction, where parents and educators often count out loud with children or prompt them to count up from 1. In particular, emphasizing symbolic magnitude skills with preschoolers is likely more effective at supporting short-term math achievement than emphasizing non-symbolic magnitude skills as the evidence for preschoolers' non-symbolic magnitude skills relating to their math achievement is mixed, with some studies suggesting that shared variation in non-symbolic magnitude and math performance can be explained by EF skills or symbolic number knowledge (Fuhs & McNeil, 2013; Kolkman et al., 2013; Purpura et al., 2013; vanMarle et al., 2014). Boosting symbolic magnitude skills may also be especially important for children from low-income households who show significant gaps on their symbolic magnitude skills compared to children from middle-income households (Scalise et al., 2017; Siegler & Ramani, 2008).

Educators could consider incorporating simple magnitude screeners to identify children who are in the most need of additional support. For example, researchers have designed a 2 – 4 minute paper-and-pencil magnitude assessment for kindergarten through 3rd grade students, where children see pairs of written numerals and circle the side with the larger number (Nosworthy, Bugden, Archibald, Evans, & Ansari, 2013). This assessment could be adapted for individual administration with preschool children with a teacher or researcher reading the numerals to the child, or translated into a tablet or computer app with verbal instructions that state the numerals. Some preschool-aged children may also need additional practice on their cardinality skills. On the other hand, screening measures may identify other children who are proficient on cardinality and symbolic magnitude tasks and instead are ready for practice with symbolic arithmetic problems. The use of adaptive or broader interventions that target multiple skills may help classroom teachers support children with varying needs more effectively. For example, some existing symbolic magnitude interventions with numerical card games (e.g., Scalise et al., 2017, 2019) could be modified to include more explicit cardinality scaffolding, such as strategically pairing verbal counting and set labeling to underscore the cardinal principle (Mix, Sandhofer, Moore, & Russell, 2012).

Finally, given the comparable influence of numerical and EF skills on children's addition, parents and educators should consider incorporating practice with executive functioning to help boost children's math skills. Indeed, EF skills are related to mathematical achievement across development (Blair & Razza, 2007; Bull & Lee, 2014; Geary, 2011), and relate to math more so than literacy or language outcomes (Blair et al., 2015). Previous research has tested the effectiveness of EF trainings, such as computerized games requiring children to remember a series of objects in appearing order (Jaeggi, Buschkuehl, Jonides, & Shah, 2011; Ramani et al., 2017). In general, there is mixed evidence on the malleability of EF skills, with some studies showing positive training effects and others failing to replicate any improvements (Diamond & Lee, 2011; Jaeggi et al., 2011; Thorell, Lindquist, Bergman, Bohlin, & Klingberg, 2009; Wass, Scerif, & Johnson, 2012). Several research groups have used EF and numerical trainings to support children's skill development (e.g., Kroesbergen, van 't Noordende, & Kolkman, 2014; Ramani et al., 2017). For example, Kroesbergen and colleagues (2014) found that children who completed four weeks of working memory training games where they were taught to remember a series of objects and their orientations, with and without additional numerical information (e.g., "two pairs of socks"), significantly improved both their working memory and numerical skills. Furthermore, engaging in some typical everyday activities with parents, teachers, or peers may also help to support children's EF skills, such as waiting to take a turn while playing a game. Other researchers have suggested using "X-ray vision" or memory games, which require children to remember the value of one or more numerical cards that are turned face down, as a way to promote EF and numerical skills simultaneously (Joswick, Clements, Sarama, Banse, & Day-Hess, 2019). When possible, making small adaptations to existing mathematics instruction, such as increasing the number of steps to complete a task or asking open-ended questions that require flexible thinking (e.g., "what are all the numbers less than 6?") can also provide support for children's executive functioning skills (Joswick et al., 2019).

**Conclusion**

Symbolic magnitude understanding is thought to play an important role in mathematical achievement across the lifespan. However, this is one of few studies to have directly examined the role of symbolic magnitude understanding during the preschool period. The findings provide evidence that symbolic magnitude understanding predicts concurrent and later math skills, above and beyond the effects of cardinality, executive functioning, child age, and child gender. The current study helps to explain the developmental trajectory of preschool children's numerical skills across the academic year, providing key insights for parents, teachers, and researchers seeking to support early mathematical development.

## Tables and Figures

Table 1

*Correlations between Non-Symbolic and Symbolic Magnitude Skills*

| Study | Non-symbolic task | Symbolic task | Concurrent correlation | Longitudinal correlation | Time elapsed |
|---|---|---|---|---|---|
| Batchelor et al. (2015) | MC | MC | +0.55*** | ---- | ---- |
| Kolkman et al. (2013) | MC | NLE | -0.01 | ---- | ---- |
| | NLE | | +0.35** | | |
| Moore et al. (2016) | MC | MC | +0.28* | ---- | ---- |
| Mussolin (2012, 2014) | MC | SNS | +0.67*** | +0.59*** | 7 months |
| Rittle-Johnson (2016) | NNS | SNS | +0.56* | ---- | ---- |
| Toll et al. (2015) | MC | MC | +0.45** | +0.15** | 2.5 years |
| | | NLE | | +0.12** | |

Note: MC = magnitude comparison task; NLE = number line estimation task; SNS = symbolic number sense tasks; NNS = non-symbolic number sense tasks; *$p < .05$, **$p < .01$, ***$p < .001$.

Table 2

*Missing Data by Measure*

| Measure | N | N Missing |
|---|---|---|
| Time 1 Give-N | 139 | 1 |
| Time 1 Point to X | 139 | 1 |
| Time 1 How Many? | 138 | 2 |
| Time 1 Symbolic Magnitude Comparison | 138 | 2 |
| Time 1 Number Line Estimation | 138 | 2 |
| Time 1 Forced Choice Addition | 138 | 2 |
| Time 1 Story Problem Addition | 136 | 4 |
| Time 1 Working Memory | 136 | 4 |
| Time 1 Inhibitory Control | 136 | 4 |
| Time 1 Attention Shifting | 136 | 4 |
| Time 2 Give-N | 135 | 5 |
| Time 2 Point to X | 135 | 5 |
| Time 2 How Many? | 133 | 2 |
| Time 2 Symbolic Magnitude Comparison | 135 | 5 |
| Time 2 Number Line Estimation | 135 | 5 |
| Time 2 Forced Choice Addition | 134 | 6 |
| Time 2 Story Problem Addition | 132 | 8 |
| Total number of scores across measures | 2313 | 62 |
| Percentage of total scores across measures | 97.4% | 2.6% |

Table 3

*Descriptive Statistics of Participants' Performance on Numerical and Executive Functioning Measures by Session*

| Measure | N | M | SD | Min | Max | Skew (*S.E.*) | Kurtosis (*S.E.*) | *t* | *p* |
|---|---|---|---|---|---|---|---|---|---|
| T1 Give-N | 139 | 3.74 | 2.08 | 0.00 | 6.00 | -0.13 (.21) | -1.59 (.41) | 4.16 | <.001 |
| T2 Give-N | 135 | 4.24 | 1.97 | 0.00 | 6.00 | -0.51 (.21) | -1.35 (.41) | | |
| T1 Point to X | 139 | 81.8 | 16.0 | 33.3 | 100.0 | -0.78 (.21) | -0.30 (.41) | 4.32 | <.001 |
| T2 Point to X | 135 | 86.6 | 13.9 | 33.3 | 100.0 | -1.35 (.21) | 1.66 (.41) | | |
| T1 How Many? | 138 | 55.2 | 41.4 | 0.0 | 100.0 | -0.21 (.21) | -1.64 (.41) | 3.18 | .002 |
| T2 How Many? | 133 | 63.5 | 38.9 | 0.0 | 100.0 | -0.56 (.21) | -1.33 (.42) | | |
| T1 Magnitude | 138 | 67.0 | 20.9 | 22.7 | 100.0 | 0.06 (.21) | -1.25 (.41) | 4.61 | <.001 |
| T2 Magnitude | 135 | 73.4 | 21.1 | 31.8 | 100.0 | -0.19 (.21) | -1.42 (.41) | | |
| T1 NLE | 138 | 70.3 | 11.3 | 34.0 | 96.0 | -0.20 (.21) | -0.19 (.41) | 1.11 | .271 |
| T2 NLE | 135 | 71.2 | 11.2 | 41.0 | 93.0 | -0.52 (.21) | -0.30 (.41) | | |

| Measure | N | M | SD | Min | Max | Skew (*S.E.*) | Kurtosis (*S.E.*) | *t* | *p* |
|---|---|---|---|---|---|---|---|---|---|
| T1 Forced Choice | 138 | 67.6 | 21.4 | 14.3 | 100.0 | -0.14 (.21) | -0.63 (.41) | 2.18 | .031 |
| T2 Forced Choice | 134 | 72.3 | 21.7 | 14.3 | 100.0 | -0.35 (.21) | -0.76 (.42) | | |
| T1 Story Problem | 136 | 51.6 | 31.3 | 0.0 | 100.0 | 0.11 (.21) | -1.43 (.41) | 2.23 | .028 |
| T2 Story Problem | 132 | 57.2 | 30.3 | 0.0 | 100.0 | -0.18 (.21) | -1.20 (.42) | | |
| T1 Working Memory | 136 | 22.8 | 26.3 | 0.0 | 78.0 | 0.44 (.21) | -1.50 (.41) | ---- | ---- |
| T1 Inhib Control | 136 | 37.1 | 17.6 | 0.0 | 83.0 | 0.26 (.21) | 0.41 (.41) | ---- | ---- |
| T1 Attention Shifting | 136 | 40.7 | 19.2 | 0.0 | 92.0 | -0.32 (.21) | 0.74 (.41) | ---- | ---- |

Note: T1 = Time 1, T2 = Time 2, NLE = number line estimation. Paired sample *t*-tests of Time 1 and Time 2 numerical measures assess the change in children's performance over time.

Table 4

*Correlations Between Numerical and Executive Functioning Skills Within Time Points*

| Time 1 Measures | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| 1. Give-N | 1 | | | | | | | | | |
| 2. Point-to-X | .74[a] | 1 | | | | | | | | |
| 3. How Many? | .62[b] | .61[b] | 1 | | | | | | | |
| 4. Magnitude comp. | .68[b] | .63[b] | .65[c] | 1 | | | | | | |
| 5. NLE | .34[b] | .33[b] | .36[c] | .41[b] | 1 | | | | | |
| 6. Forced choice | .18[b] | .06[b] | .20[c] | .23[b] | .11[b] | 1 | | | | |
| 7. Story problem | .66[d] | .64[d] | .58[d] | .58[d] | .28[d] | .21[d] | 1 | | | |
| 8. Working memory | .43[d] | .44[d] | .49[e] | .53[e] | .16[e] | .19[e] | .53[g] | 1 | | |
| 9. Inhibitory control | .46[d] | .35[d] | .46[e] | .45[e] | .28[e] | .19[e] | .54[g] | .36[d] | 1 | |
| 10. Attention shift | .41[d] | .41[d] | .44[e] | .48[e] | .31[e] | .02[e] | .45[g] | .38[d] | .50[d] | 1 |
| **Time 2 Measures** | | | | | | | | | | |
| 1. Give-N | 1 | | | | | | | | | |
| 2. Point-to-X | .67[e] | 1 | | | | | | | | |
| 3. How Many? | .65[g] | .60[g] | 1 | | | | | | | |
| 4. Magnitude comp. | .67[e] | .53[e] | .63[g] | 1 | | | | | | |
| 5. NLE | .43[e] | .35[e] | .43[g] | .54[e] | 1 | | | | | |
| 6. Forced choice | .16[f] | .21[f] | .03[h] | .21[f] | .17[f] | 1 | | | | |
| 7. Story problem | .71[h] | .58[h] | .56[h] | .64[h] | .49[h] | .22[h] | 1 | | | |

Note: NLE = number line estimation. If $r > .3$, then $p < .001$. If $.22 < r < .29$, then $p < .01$. If $.19 < r < .22$, then $p < .05$. If $r < .19$, then $p > .05$.

[a] N = 139

[b] N = 138

[c] N = 137

[d] N = 136

[e] N = 135

[f] N = 134

[g] N = 133

[h] N = 132

Table 5

*Correlations Between Numerical Skills Across Times*

| Measure | N | *r* | *p* |
|---|---|---|---|
| Give-N | 134 | .761 | <.001 |
| Point-to-X | 134 | .600 | <.001 |
| How Many? | 132 | .676 | <.001 |
| Magnitude comparison | 133 | .697 | <.001 |
| Number line estimation | 133 | .302 | <.001 |
| Forced choice addition | 133 | .273 | .001 |
| Story problem addition | 131 | .596 | <.001 |

Table 6

*Numerical and Executive Functioning Skills by Child Gender*

| Measure | Female | | | Male | | | Difference statistics | |
|---|---|---|---|---|---|---|---|---|
| | N | *M* | *SD* | N | *M* | *SD* | *t* | *p* |
| T1 Give-N | 68 | 4.06 | 2.03 | 71 | 3.44 | 2.10 | 1.77 | .078 |
| T1 Point-to-X | 68 | 84.7 | 13.5 | 71 | 79.0 | 17.7 | 2.15 | .033 |
| T1 How Many? | 68 | 59.7 | 40.1 | 70 | 50.9 | 42.5 | 1.26 | .211 |
| T1 Magnitude comparison | 68 | 68.6 | 20.4 | 70 | 65.5 | 21.4 | 0.90 | .371 |
| T1 Number line estimation | 68 | 70.2 | 11.6 | 70 | 70.3 | 11.2 | -0.02 | .988 |
| T1 Forced choice addition | 68 | 68.3 | 23.0 | 70 | 66.9 | 19.8 | 0.37 | .715 |
| T1 Story problem addition | 68 | 57.2 | 31.4 | 68 | 46.0 | 30.4 | 2.11 | .037 |
| T1 Working memory | 67 | 25.9 | 27.5 | 69 | 20.1 | 25.1 | 1.22 | .224 |
| T1 Inhibitory control | 67 | 37.3 | 17.0 | 69 | 37.0 | 18.4 | 0.09 | .930 |
| T1 Attention shifting | 67 | 43.1 | 17.9 | 69 | 38.5 | 20.2 | 1.42 | .157 |
| T2 Give-N | 67 | 4.64 | 1.74 | 68 | 3.84 | 2.12 | 2.41 | .017 |
| T2 Point-to-X | 67 | 89.4 | 11.3 | 68 | 83.8 | 15.7 | 2.35 | .020 |
| T2 How Many? | 67 | 68.7 | 36.4 | 66 | 58.2 | 40.9 | 1.56 | .121 |
| T2 Magnitude comparison | 67 | 75.8 | 20.9 | 68 | 71.1 | 21.3 | 1.33 | .190 |
| T2 Number line estimation | 67 | 72.5 | 11.0 | 68 | 71.0 | 11.2 | 0.27 | .788 |
| T2 Forced choice addition | 67 | 75.7 | 19.7 | 67 | 68.9 | 23.1 | 1.84 | .068 |
| T2 Story problem addition | 67 | 61.2 | 30.2 | 65 | 53.1 | 30.2 | 1.55 | .125 |

Note: T1 = Time 1, T2 = Time 2.

Table 7

*Correlations Between Child Age at First Assessment and Numerical and Executive Functioning Skills*

| Measure | N | *r* | *p* |
|---|---|---|---|
| T1 Give-N | 139 | .51 | <.001 |
| T1 Point-to-X | 139 | .46 | <.001 |
| T1 How Many? | 138 | .46 | <.001 |
| T1 Magnitude comparison | 138 | .53 | <.001 |
| T1 Number line estimation | 138 | .21 | .015 |
| T1 Forced choice addition | 138 | .16 | .056 |
| T1 Story problem addition | 136 | .51 | <.001 |
| T1 Working memory | 136 | .49 | <.001 |
| T1 Inhibitory control | 136 | .37 | <.001 |
| T1 Attention shifting | 136 | .40 | <.001 |
| T2 Give-N | 135 | .48 | <.001 |
| T2 Point-to-X | 135 | .41 | <.001 |
| T2 How Many? | 133 | .45 | <.001 |
| T2 Magnitude comparison | 135 | .52 | <.001 |
| T2 Number line estimation | 135 | .37 | <.001 |
| T2 Forced choice addition | 134 | .18 | .039 |
| T2 Story problem addition | 132 | .49 | <.001 |

Note: T1 = Time 1, T2 = Time 2.

Table 8

*Numerical and Executive Functioning Skills by Child Age at First Assessment*

| Measure | 3 year olds | | | 4 year olds | | | 5 year olds | | |
|---|---|---|---|---|---|---|---|---|---|
| | *N* | *M* | *SD* | *N* | *M* | *SD* | *N* | *M* | *SD* |
| T1 Give-N | 38 | 2.13 | 1.30 | 63 | 4.05 | 2.03 | 38 | 4.84 | 1.88 |
| T1 Point-to-X | 38 | 68.9 | 13.9 | 63 | 85.8 | 14.5 | 38 | 87.9 | 13.1 |
| T1 How Many? | 37 | 25.9 | 28.2 | 63 | 59.7 | 41.5 | 38 | 76.3 | 36.6 |
| T1 Magnitude comparison | 37 | 50.7 | 11.4 | 63 | 68.5 | 21.1 | 38 | 80.5 | 1.17 |
| T1 Number line estimation | 37 | 68.7 | 9.4 | 63 | 68.0 | 11.8 | 38 | 75.6 | 10.7 |
| T1 Forced choice addition | 37 | 61.8 | 14.7 | 63 | 67.3 | 23.0 | 38 | 73.7 | 22.9 |
| T1 Story problem addition | 36 | 28.3 | 19.6 | 62 | 56.6 | 32.4 | 38 | 65.5 | 26.8 |
| T1 Working memory | 37 | 6.4 | 16.9 | 62 | 22.4 | 25.9 | 37 | 39.8 | 25.4 |
| T1 Inhibitory control | 37 | 29.3 | 12.0 | 62 | 37.5 | 18.5 | 37 | 44.3 | 18.2 |
| T1 Attention shifting | 37 | 31.8 | 18.3 | 62 | 39.5 | 17.6 | 37 | 51.9 | 17.5 |
| T2 Give-N | 37 | 2.76 | 1.64 | 61 | 4.54 | 1.92 | 37 | 5.22 | 1.51 |
| T2 Point-to-X | 37 | 75.7 | 15.0 | 61 | 91.0 | 10.4 | 37 | 90.1 | 12.1 |
| T2 How Many? | 36 | 38.3 | 36.5 | 60 | 66.3 | 38.2 | 37 | 83.2 | 28.5 |
| T2 Magnitude comparison | 37 | 58.0 | 16.9 | 61 | 74.3 | 20.5 | 37 | 87.5 | 15.2 |
| T2 Number line estimation | 37 | 33.2 | 9.4 | 61 | 29.3 | 12.4 | 37 | 23.5 | 7.8 |
| T2 Forced choice addition | 36 | 63.1 | 20.9 | 61 | 78.0 | 19.6 | 37 | 71.8 | 23.0 |
| T2 Story problem addition | 35 | 34.3 | 26.2 | 60 | 61.7 | 27.4 | 37 | 71.6 | 26.7 |

Note: T1 = Time 1, T2 = Time 2.

Table 9

*Correlations between Standardized Difference Scores and Child Age, Gender, and Time 1 Performance*

| Standardized Difference | Age | | Gender | | Time 1 Performance | |
|---|---|---|---|---|---|---|
| | *r* | *p* | *r* | *p* | *r* | *p* |
| Give-N | -.09 | .29 | .04 | .63 | -.42 | <.001 |
| Point-to-X | -.14 | .11 | -.04 | .67 | -.57 | <.001 |
| How Many? | -.06 | .52 | .00 | .99 | -.46 | <.001 |
| Magnitude comparison | -.02 | .84 | .02 | .82 | -.37 | <.001 |
| Number line estimation | .145 | .10 | .01 | .96 | -.60 | <.001 |
| Forced choice addition | .02 | .84 | .10 | .27 | -.61 | <.001 |
| Story problem addition | -.07 | .45 | -.06 | .48 | -.49 | <.001 |

Note: Standardized difference scores calculated as: [(Time 2 – Time 1) / *SD*(Time 1)]. $N$ = 134 for correlations with Give-N, Point-to-X; $N$ = 133 for correlations with magnitude comparison, number line estimation, forced choice addition; $N$ = 132 for correlations with How Many?; $N$ = 131 for correlations with story problem addition.

Table 10

*Factor Loadings by Construct*

| Measure | Factor Loading |
|---|---|
| Time 1 Cardinality | |
| T1 Give-N | .899 |
| T1 Point to X | .895 |
| T1 How Many? | .839 |
| Time 1 Magnitude | |
| T1 Magnitude comparison | .841 |
| T1 Number line estimation | .841 |
| Time 1 Addition | |
| T1 Forced choice | .778 |
| T1 Story problem | .778 |
| Time 1 Executive functioning | |
| T1 Working memory | .723 |
| T1 Inhibitory control | .804 |
| T1 Attention shifting | .812 |
| Time 2 Cardinality | |
| T1 Give-N | .891 |
| T2 Point to X | .872 |
| T2 How Many? | .855 |
| Time 2 Magnitude | |
| T2 Magnitude comparison | .877 |
| T2 Number line estimation | .877 |
| Time 2 Addition | |
| T2 Forced choice | .779 |
| T2 Story problem | .779 |

Table 11

*Descriptive Statistics of Composite Scores*

| Measure | N | *Min.* | *Max.* | *Skew (S.E.)* | *Kurtosis (S.E.)* |
|---|---|---|---|---|---|
| Time 1 Cardinality | 138 | -2.36 | 1.25 | -0.34 (.21) | 1.14 (.41) |
| Time 1 Magnitude | 138 | -2.65 | 2.15 | 0.13 (.21) | -0.62 (.41) |
| Time 1 Addition | 136 | -2.27 | 1.96 | 0.19 (.21) | -0.81 (.41) |
| Time 1 Executive functioning | 136 | -2.21 | 2.65 | 0.14 (.21) | -0.10 (.41) |
| Time 2 Cardinality | 133 | -2.73 | 1.06 | -0.68 (.21) | -0.67 (.42) |
| Time 2 Magnitude | 135 | -2.19 | 1.85 | -0.21 (.21) | -1.00 (.42) |
| Time 2 Addition | 132 | -2.10 | 1.74 | -0.15 (.21) | -0.86 (.42) |

Note: Each composite score is standardized with $M = 0$ and $SD = 1$.

Table 12

*Measured Variable Path Analysis Model of Numerical and Executive Functioning Skills*

| | Dependent variable | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Cardinality | | | | Symbolic Magnitude | | | | Addition | | | | Executive Functioning | | | |
| Source | *B* | *SE* | *p* | β | *B* | *SE* | *p* | β | *B* | *SE* | *p* | β | *B* | *SE* | *p* | β |
| Control variables | | | | | | | | | | | | | | | | |
| Gender | .04 | .01 | .001 | .31 | .00 | .01 | .654 | .03 | .03 | .01 | .028 | .22 | .07 | .01 | .001 | .52 |
| Age | .22 | .12 | .071 | .11 | -.09 | .13 | .486 | -.04 | .28 | .15 | .066 | .14 | .27 | .15 | .069 | .14 |
| Domain-general and domain-specific predictors | | | | | | | | | | | | | | | | |
| EF | .47 | .06 | .001 | .48 | .22 | .07 | .003 | .22 | .22 | .10 | .025 | .22 | | | | |
| Cardinality | | | | | .52 | .08 | .001 | .51 | | | | | | | | |
| Magnitude | | | | | | | | | .23 | .10 | .024 | .23 | | | | |

Note: N = 129 with Satorra-Bentler correction for non-normality.

Table 13

*Summary of Multiple Regression Analysis Predicting Time 2 Symbolic Magnitude (N = 132)*

| | *B* | *SE* | β |
|---|---|---|---|
| Age | .02 | .01 | .12 |
| Gender | -.06 | .12 | -.03 |
| Time 1 symbolic magnitude | .25 | .08 | .25** |
| Time 1 cardinality | .51 | .09 | .50*** |
| $R^2$ | | .58 | |
| *F* | | 43.42*** | |

Note: ** $p < .01$; *** $p < .001$.

Table 14

*Means and Standard Deviations on Time 1 and 2 Symbolic Magnitude Measures by Time 1 Give-N Score*

| | | Time 1 Magnitude | | | | Time 2 Magnitude | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Comparison | | NLE | | Comparison | | NLE | |
| T1 Give-N | N | *M* | *SD* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| Pre-knower | 3 | 47.7 | 9.6 | 59.0 | 5.0 | 48.5 | 2.6 | 62.2 | 8.8 |
| 1-knower | 25 | 52.0 | 11.4 | 65.0 | 11.0 | 58.9 | 16.6 | 62.8 | 12.0 |
| 2-knower | 23 | 49.6 | 11.7 | 68.1 | 7.4 | 58.3 | 16.3 | 67.4 | 8.7 |
| 3-knower | 18 | 58.1 | 17.3 | 68.8 | 10.1 | 61.1 | 20.3 | 71.3 | 9.0 |
| 4-knower | 10 | 72.3 | 19.2 | 68.3 | 10.7 | 75.8 | 15.1 | 70.3 | 7.4 |
| 5-knower | 5 | 78.2 | 18.9 | 75.1 | 2.7 | 83.6 | 20.5 | 72.3 | 16.0 |
| CP-knower | 55 | 82.8 | 16.3 | 74.4 | 12.5 | 90.1 | 11.6 | 77.4 | 8.9 |

Note: NLE = number line estimation. Pre-knowers scored 0 on the Time 1 Give-N measure; Cardinal Principle (CP)-knowers scored 6. One-way analysis of variance (ANOVA) revealed significant differences between Give-N ability levels on all magnitude measures.

Table 15

*Average Magnitude Comparison Accuracy on Trials with Known and Unknown Cardinal Values by Knower-Level*

| | | Time 1 | | | | Time 2 | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Known Values | | Unknown Values | | Known Values | | Unknown Values | |
| T1 Give-N | N | *M* | *SD* | *M* | *SD* | *M* | *SD* | *M* | *SD* |
| Pre-knower | 2 | -- | -- | 47.7 | 9.6 | -- | -- | 47.7 | 3.2 |
| 1-knower | 25 | -- | -- | 52.0 | 11.4 | -- | -- | 58.9 | 16.6 |
| 2-knower | 23 | 56.5 | 50.7 | 49.3 | 12.2 | 95.7 | 39.7 | 57.8 | 18.2 |
| 3-knower | 17 | 67.6 | 39.3 | 57.9 | 16.2 | 61.8 | 41.6 | 54.7 | 27.2 |
| 4-knower | 10 | 77.5 | 32.2 | 71.1 | 19.2 | 77.5 | 34.3 | 66.1 | 27.3 |
| 5-knower | 5 | 77.5 | 20.5 | 78.6 | 20.8 | 90.0 | 16.3 | 80.0 | 23.4 |
| CP-knower | 55 | 82.8 | 16.3 | -- | -- | 86.9 | 20.5 | -- | -- |

Note: Pre-knowers scored 0 on the Time 1 Give-N measure; Cardinal Principle (CP)-knowers scored 6.

Table 16

*Summary of Multiple Regression Analysis Predicting Time 2 Addition (N = 131)*

| | *B* | *SE* | β |
|---|---|---|---|
| Age | .03 | .01 | .18* |
| Gender | .27 | .14 | .14 |
| Time 1 addition | .31 | .08 | .32*** |
| Time 1 symbolic magnitude | .23 | .08 | .23* |
| $R^2$ | | .37 | |
| *F* | | 18.14*** | |

Note: * $p < .05$; ** $p < .01$; *** $p < .001$.

Table 17

*Means and Standard Deviations on Time 1 and 2 Addition Measures by Time 1 Magnitude Comparison Accuracy*

| | At or below chance (N = 42) | | Above chance (N = 96) | | *t* | *p* |
|---|---|---|---|---|---|---|
| | *M* | *SD* | *M* | *SD* | | |
| T1 forced choice addition | 62.6 | 16.1 | 70.0 | 23.1 | 2.12 | .037 |
| T1 story problem addition | 34.0 | 23.2 | 59.0 | 31.4 | 98.04 | <.001 |
| T2 forced choice addition | 63.2 | 21.7 | 76.3 | 20.6 | 3.32 | .001 |
| T2 story problem addition | 41.1 | 32.2 | 63.3 | 27.0 | 4.05 | <.001 |

Note: T1 = Time 1; T2 = Time 2.

Table 18

*Total Standardized Direct and Indirect Effects on Time 2 Addition (N = 129)*

| | Total Direct Effects | | | Total Indirect Effects | | | Total Effects | | |
|---|---|---|---|---|---|---|---|---|---|
| | β | *SE* | *p* | β | *SE* | *p* | β | *SE* | *p* |
| Numerical effects | .229 | .097 | .018 | .118 | .058 | .043 | .347 | --- | --- |
| Cardinality | --- | --- | --- | .118 | .058 | .043 | .118 | .058 | .043 |
| Magnitude | .229 | .097 | .018 | --- | --- | --- | .229 | .097 | .018 |
| EF effects | .220 | .097 | .023 | .108 | .053 | .041 | .328 | .087 | <.001 |

Note: EF = executive functioning.

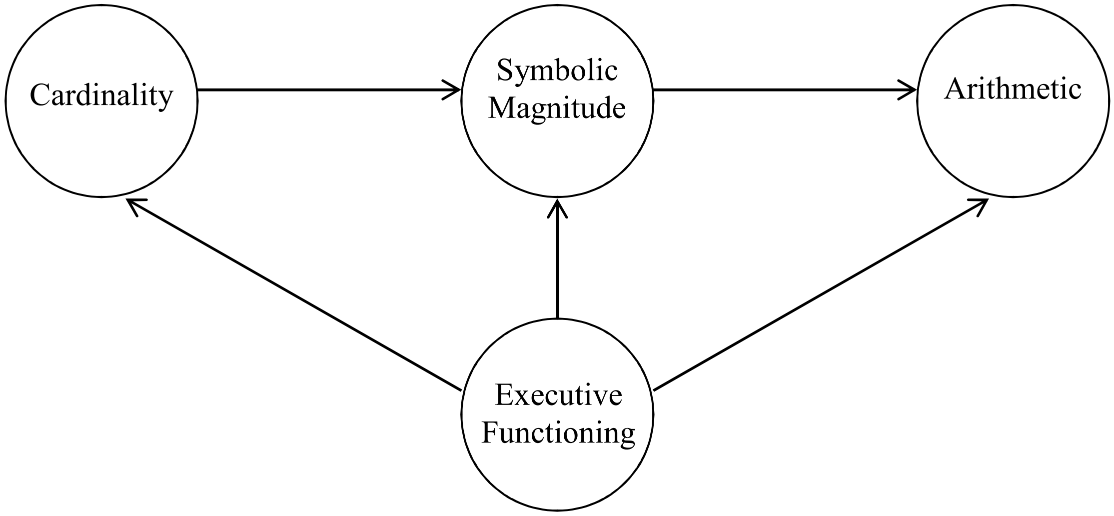


*Figure 1*. Conceptual model of the development of symbolic magnitude understanding

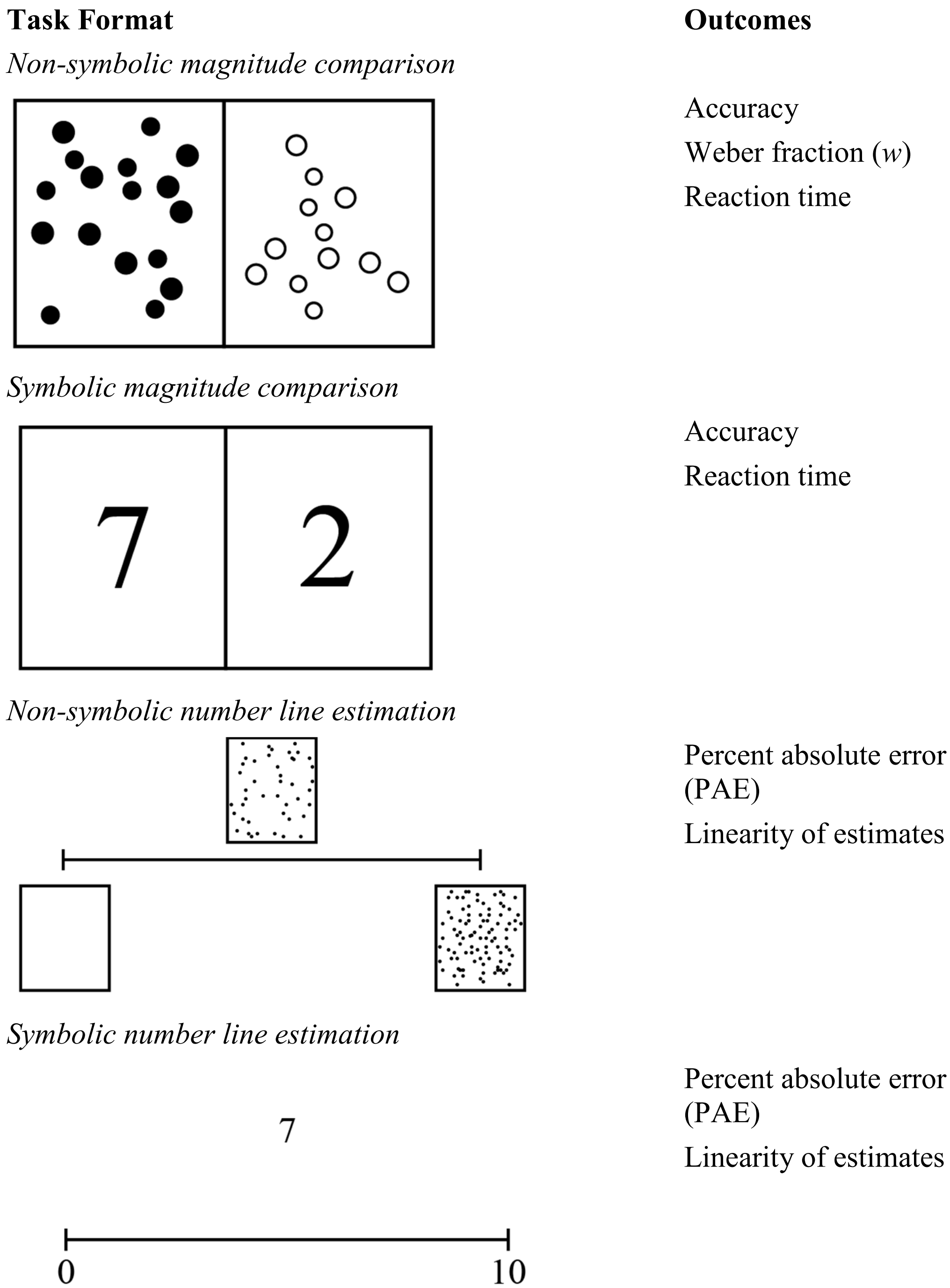


*Figure 2*. Measures of non-symbolic and symbolic magnitude knowledge

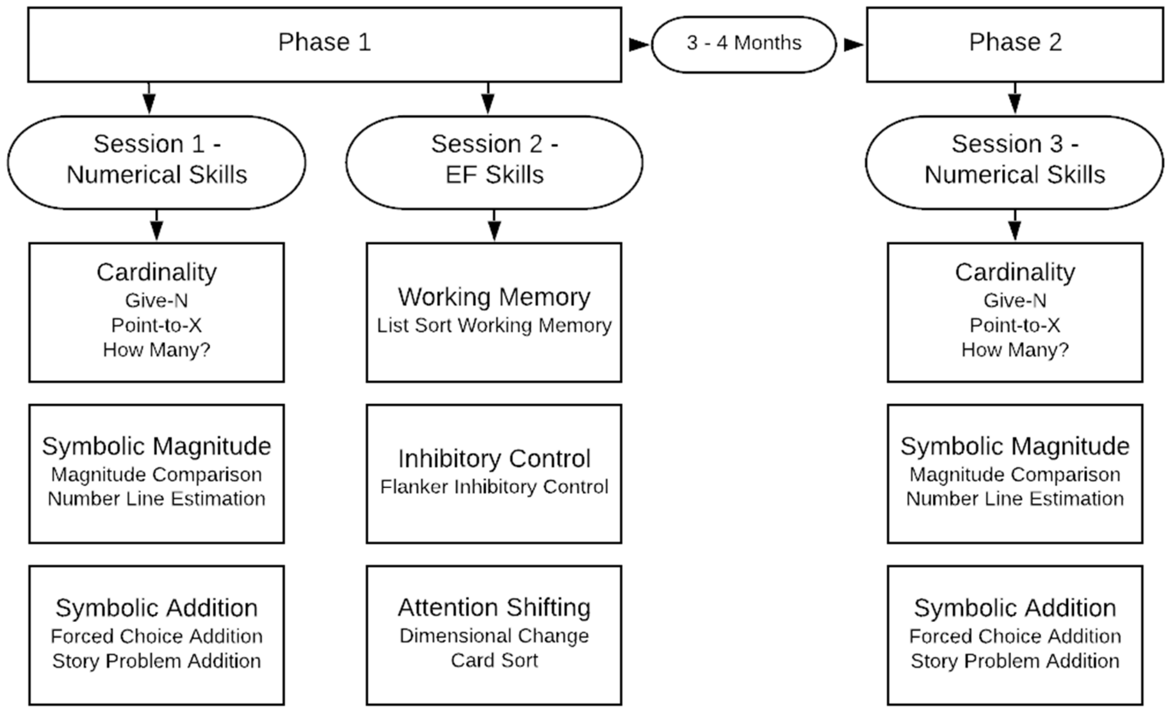


*Figure 3.* Study procedure

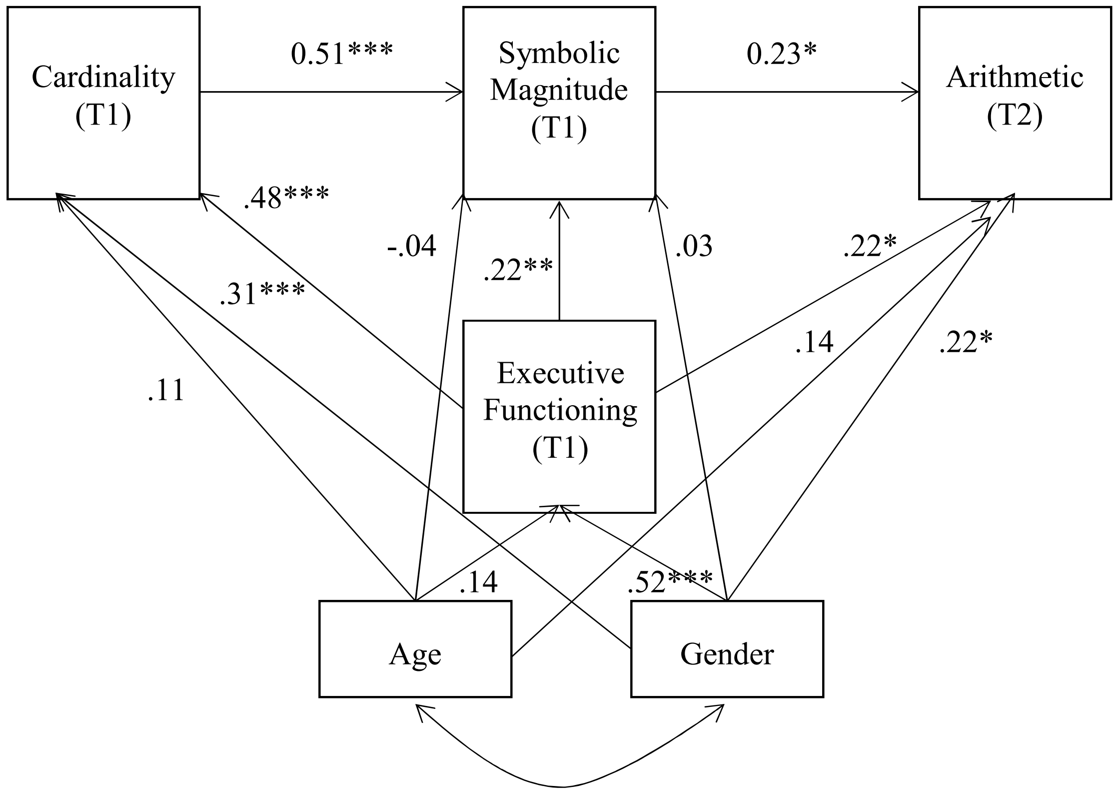


*Figure 4.* Measured Variable Path Analysis – Full Mediation Model

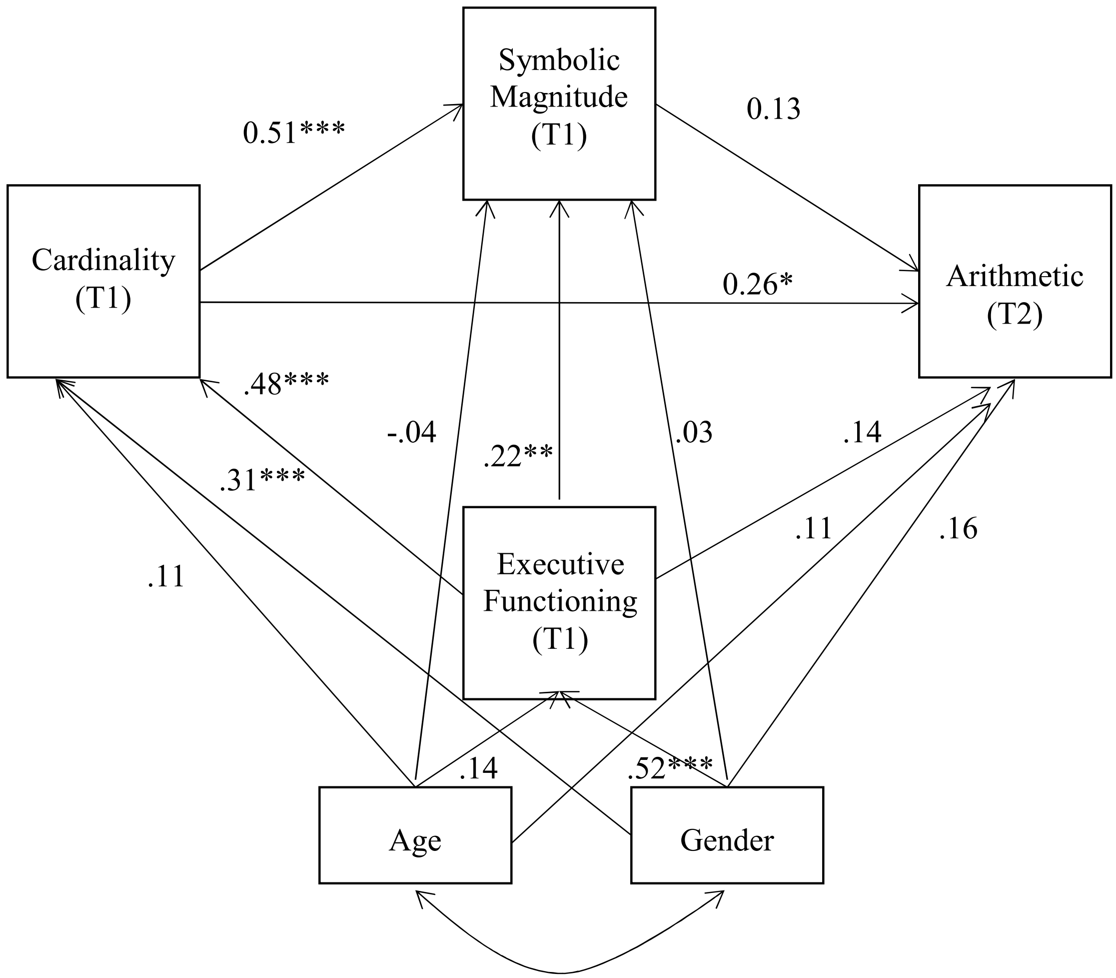


*Figure 5.* Measured Variable Path Analysis – Partial Mediation Model

## Appendices

Appendix A: Preliminary Analysis

To differentiate the present study from recent work examining cardinality as a key predictor of improvements in children's symbolic magnitude comparison skills over the school year (Geary & vanMarle, 2018), a preliminary analysis was conducted on existing data on children's numeral identification and symbolic magnitude comparison skills. The goal of the preliminary analysis was to test the hypothesis that children have some knowledge of the symbolic magnitude of numbers that are presented both verbally and visually even if they are unable to correctly identify the numerals. While Geary and colleagues limited children's symbolic magnitude comparison test trials to only the numbers that they could correctly identify during a numeral identification task, other versions of symbolic magnitude comparison tasks ask all children to compare pairs of numerals ranging from 1 to 9, with an experimenter providing a verbal label to each numeral (e.g., "Which is more, seven or two?"; Ramani & Siegler, 2008). Use of the latter task format implies children may have some magnitude knowledge based on verbal number words, and may be able to make accurate magnitude comparison decisions on trials with numbers that they themselves cannot name.

To test this hypothesis, data from 252 Head Start preschool children collected as pretest measures in multiple numeracy intervention studies were used (Ramani & Scalise, 2018; Scalise, Daubert, & Ramani, 2017, 2019; Scalise, DePascale, Tavassolie, McCown, & Ramani, in preparation). As a part of their pretest assessments, children completed the measures of numeral identification (1-10) and symbolic magnitude comparison described above. In total, 137 children made at least one error identifying a numeral 1-9. Next, their performance on symbolic magnitude comparison trials that involved the number(s) that they incorrectly identified was averaged. For example, if a child inaccurately identified the numeral 9, their accuracy across all symbolic magnitude comparison trials that included nine were averaged (e.g., 4 vs. 9, 1 vs. 9, 9 vs. 6, and 8 vs. 9). These average accuracy scores were then compared against the benchmark of 50 percent, which represents chance level accuracy between the two choices in each comparison trial.

Overall, children's average accuracy on symbolic magnitude comparison trials involving one or more incorrectly identified numerals was 59.7 percent. A one-sample two-tailed *t*-test comparing the accuracy scores to the set point of 50 percent was statistically significant, $t(136) = 5.66$, $p < .001$, $d = 0.48$, indicating the accuracy scores were significantly greater than chance. However, the Shapiro-Wilks test was significant, $W = 0.972$, $p = .007$, suggesting the distribution deviated from normality. The Wilcoxon signed-rank test offers a non-parametric alternative to a one-sample *t*-test that does not assume a normal distribution. The Wilcoxon signed-rank test was also statistically significant, $W = 5045$, $p < .001$, $r(136) = .067$, providing further evidence that the average accuracy scores were greater than chance. These results reveal that children performed above chance at answering symbolic magnitude comparison trials where they could not identify one or both of the numerals, suggesting that children have some knowledge of the magnitude of numbers that does not rely on their numeral identification knowledge. This in turn implies that differences in symbolic magnitude comparison task format, such as limiting comparisons to correctly identified numerals versus testing all numerals with verbal labels by an experimenter, may lead to differences in our estimates of children's symbolic magnitude knowledge.

Appendix B: Parent Survey

Please tell us a little about you and your child. This information will be kept confidential.

1. Child Age/Birthday:

   Age (in years):

   - ❑ 3
   - ❑ 4
   - ❑ 5

   Date of birth (mm/dd/yy):

   _____/ _____/ _____

2. Child gender:
   - ❑ Girl
   - ❑ Boy

3. Child Race: (Check the one that best describes your child.)
   - ❑ African American or Black
   - ❑ Caucasian/White
   - ❑ Asian or Pacific Islander
   - ❑ American Indian or Alaska Native
   - ❑ Biracial/Mixed Race

4. Is your child Hispanic or Latino?
   - ❑ Yes
   - ❑ No

5. Please indicate your relationship to the child (e.g., mother, father, grandparent):

____________________________

6. Parents' highest level of education:

   Parent 1: (Circle One) mother / father / other __________
   - ❑ Less than High School
   - ❑ Some High School Coursework
   - ❑ High School Diploma/GED
   - ❑ Some College Coursework/Vocational Training
   - ❑ 2-year College Degree (Associates)
   - ❑ 4-year College Degree (BA/BS)
   - ❑ Postgraduate or Professional degree (MA, PhD, MD, JD)

   Parent 2: (Circle One) mother / father / other __________
   - ❑ Less than High School

- ❑ Some High School Coursework
- ❑ High School Diploma/GED
- ❑ Some College Coursework/Vocational Training
- ❑ 2-year College Degree (Associates)
- ❑ 4-year College Degree (BA/BS)
- ❑ Postgraduate or Professional degree (MA, PhD, MD, JD)

7. How many people typically reside in your household? _______________

Appendix C: Task Scripts

## Give-N

Place 15 black poker chips in a pile in front of the child.

**For this game, we're going to use these tokens. Can you give me 1 token?** If child does not respond: **Can you show me 1 token? Pick out 1 token for me.**

After the child gives tokens: **Is that N?**

If child says no: **Well, I'd really like N tokens. Can you give me N tokens?** [After child adjusts] **Is that N?**

After child says yes: **Okay!** Put all of the tokens back in front of the child in a pile.

**Now we're going to do another one. Can you give me two tokens?**

Continue same instructions for 2, 3, 4, 5, and 6. If a child gives the wrong number of tokens, ask them for N-1 tokens next. If a child gives the right number of tokens, ask them for N+1 tokens next. Stop the task after either a child reaches 6 tokens correctly, or a child gives N tokens twice correctly but N+1 tokens incorrectly at least twice.

## Point-to-X

**For this game, we're going to look at some pictures. Here are pictures of some candies. Can you show me the two candies?** Guide the child to point to their answer.

**Now we're going to do another one. Can you show me three candies?** Continue for 15 trials.

## How Many?

**Now we're going to look at some pictures of stars. Can you count the stars on this page?** After the child is done counting, say: **Great! Now I'm going to flip it over and hide the stars.** [Turn page over so stars are no longer showing.] **How many stars did we just see?**

If child does not answer: **How many stars are hiding under here? How many stars did we see?**

Repeat for 2, 3, 4, 5, and 6 stars, presented in random order.

**Symbolic Magnitude Comparison**

(Boys: John and Jordan) (Girls: Jaelyn and Sarah)

**For this game, I am going to ask you to tell me which is more. Let's try a couple of easy ones.**

**Ok, if (name) has one 1 cookie and (name) has 6 cookies. Tell me which is more: 1 cookie or 6 cookies?**

If correct, **Great job! 6 cookies is more than 1 cookie. Let's try another one.**
If incorrect, **Actually, 6 cookies is more than 1 cookies. Tell me which is more or higher: 1 cookie or 6 cookies. Let's try another one.**

**Ok, (name) has 5 cookies and (name) has 4 cookies. Tell me which is more: 5 cookies or 4 cookies?**

If correct, **Great job! 5 cookies is more than 4 cookies.**
If incorrect, **Actually, 5 cookies is more than 4 cookies. Tell me which is more or higher: 5 cookies or 4 cookies.**

**Ok, now we are going to do a bunch of other numbers.**

Present the rest of the 22 pairs in the specified order. **Tell me which is more: X cookies or Y cookies?** (Always cookies after each number).

**0-10 Number Line Task Script**

Open the Number Line app on the tablet.

**Now we're going to play a game with number lines. I want you think really hard for this game.**

**Have you ever seen a number line before?**

For all children explain and show 0-100 number line:
**A number line looks just like a line** (point) **with numbers across it. It shows us numbers in order. The numbers go from the smallest to the largest.**

Show child blank 0-10 number line
**The number line we are going to be playing with today goes from 0-10. It starts with zero at this end** (point) **and ends with 10 at this end** (point).

**Your job is to show me where some numbers go on the number line.**

**When you decide where the right place for a number is, I want you to make a mark on the number line. Like this.** Demonstrate for the child on a blank number line. Make the line off-center.

**Now would you like to try?**

**Great job! You can make a mark anywhere on the line where you think the right place for that number is.** Make about 5 marks in random places across the number line.

**Okay? Now I am going to ask you to show me where some numbers go on the number line. Remember, make your mark where you think the right place for the number is.**

Present first number line.

**This is a #. If this is where zero goes** (point), **and this is where 10 goes** (point), **where does N go?**

If child looks confused or does not make a mark immediately, **Make your mark where you think the right place for this number is.**

Go through all of the numbers (1-9), in random order (the tablet will present the numbers in random order).

After 1st set:
**You did such a great job that we are going to go all of the numbers again. Remember, you can make a mark anywhere on the line where you think the right place for that number is.**

Present second set of numbers.

**If this is zero goes (point), and this is where 10 goes (point), where does N go?**

Remind the child periodically through the session, **I want you to think really hard for this game." Or "I want you to think really hard about this next one.**

Praise the children periodically, **I can tell you are really thinking about these.**

**<u>Forced Choice Addition</u>**
**Now we're going to play another game with numbers. For this game, I am going to show you some math problem that two kids from another school did, and I want you to tell me who is right.**

**In the first problem, Jaelyn/Jayden said 2 cookies plus 1 cookie equals 2 cookies** [point to picture]**, and Kaitlyn/Kaleb said 2 cookies plus 1 cookie equals 3 cookies** [point to picture]**. Who do you think is right? Can you point to who you think is right?**

For each problem, say: **[Name] said x cookies + y cookies = z cookies and [Name] said x cookies + y cookies = z cookies. Who do you think is right?** Circle the child's response on the score sheet.

**<u>Story Problem Addition</u>**

**Next I'm going to tell you some stories about Jenny/Johnny and her/his tokens.**

**Jenny/Johnny has 1 token** [lay out 1 token]**, and s/he gets 1 more** [lay out 1 additional token]**. How many does s/he have altogether? If you want, you can use these tokens to help you find the answer.**

For each problem, say: **Jenny/Johnny has X tokens** [lay out X tokens]**, and s/he gets Y more** [lay out Y additional tokens]**. How many does s/he have altogether?**

**<u>Working Memory – Sample Item</u>**

An audio recording with an accompanying animation says: **This is a RABBIT** (point to RABBIT); **this is a SHEEP** (point); **and this is an ELEPHANT** (point).

Question A: The audio recording continues: **Say the smallest animal.**

Correct Response: If child says RABBIT, examiner presses 1 and then the spacebar. The audio recording says: **Good job**. The iPad automatically moves to the next question.

Incorrect Response: If child does not say RABBIT, examiner presses 0 and then the spacebar. The item plays again and the audio recording says: **Let's try that again. The RABBIT is smaller than the SHEEP (point), and the RABBIT is smaller than the ELEPHANT (point). The RABBIT is the smallest animal. Now you say the smallest animal.** If child says RABBIT, examiner presses 1 and then the spacebar. The audio recording says: **Good job**. The iPad automatically moves to the next question.
If child does not know smallest animal after two trials, examiner presses 0 and then the spacebar. Test will be discontinued.

**<u>Inhibitory Control – Sample Item</u>**

**Here is a fish! This is the TAIL [point] and this is the MOUTH [point]. The fish is pointing this way, the same way the fish is swimming.**

**Here is the MIDDLE fish is circled. Can you point to the MIDDLE fish?**

**Look at all the fish! The fish in the MIDDLE is hungry. To feed the MIDDLE fish, choose the button that matches the way the MIDDLE fish is pointing.**

**Let's practice some. If the MIDDLE fish is pointing this way, choose this button [point]. If the MIDDLE fish is pointing this way, choose this button [point].**

**Now you try.**

**<u>Attention-shifting – Sample Item</u>**

**We're going to play a matching game with colors and shapes. We'll play the SHAPE game first. In the SHAPE game, choose the picture that's the same SHAPE as the picture in the middle of the screen. If it's a BOAT, choose this picture [point].**

**If it's a RABBIT, choose that picture [point].**

**Now you try. What SHAPE is this?**

## Appendix D: Sensitivity Analysis

The present study had both missing data and some composite scores with non-normal distributions. In the main analyses presented in text, the Satorra-Bentler scaling method was used to correct for non-normality, however, to do so required listwise deletion of all missing data, bringing the total analytic sample size from N = 140 down to N = 129. To confirm that the listwise deletion did not substantially affect the measured variable path analysis model estimates, a sensitivity analysis was conducted for the MVPA full mediation model using Full Information Maximum Likelihood (FIML) to generate parameter estimates based on all available data.

The chi-square for the model FIML estimation was significant $\chi^2(1) = 6.17$, $p = .01$, the CFI is 0.981, greater than the benchmark value of 0.95, and the SRMR is 0.024, less than the benchmark of 0.08. This suggests acceptable data-model fit, like the main MVPA model presented in the text. Table A1 below shows the unstandardized path estimates, standardized path estimates, and significance levels for the model. The standardized path estimates for the structural relations of interest (e.g., cardinality predicting symbolic magnitude, magnitude predicting arithmetic, and EF skills predicting numerical skills) were all statistically significant and comparable effect sizes to the main analyses presented in text. In sum, the sensitivity analysis suggests the use of the S-B scaling method that led to a slightly reduced sample size did not significantly change the results of the MVPA model.

Table A1

*Measured Variable Path Analysis Model of Numerical and Executive Functioning Skills With Full Information Maximum Likelihood*

| | Dependent variable | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Cardinality | | | | Symbolic Magnitude | | | | Addition | | | | Executive Functioning | | | |
| Source | *B* | *SE* | *p* | β | *B* | *SE* | *p* | β | *B* | *SE* | *p* | β | *B* | *SE* | *p* | β |
| Untransformed Composites | | | | | | | | | | | | | | | | |
| Control variables | | | | | | | | | | | | | | | | |
| Gender | .04 | .01 | .001 | .30 | .01 | .01 | .563 | -.05 | .03 | .01 | .019 | .20 | .07 | .01 | .001 | .53 |
| Age | .27 | .12 | .032 | .13 | -.11 | .12 | .391 | .04 | .30 | .15 | .039 | .15 | .20 | .15 | .159 | .10 |
| Domain-general and domain-specific predictors | | | | | | | | | | | | | | | | |
| EF | .46 | .07 | .001 | .46 | .21 | .08 | .013 | .21 | .23 | .10 | .017 | .23 | | | | |
| Cardinality | | | | | .53 | ,09 | .001 | .53 | | | | | | | | |
| Magnitude | | | | | | | | | .23 | .09 | .011 | .23 | | | | |

Note: N = 140 with Full Information Maximum Likelihood estimation.

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