# ABSTRACT


Title of dissertation: USING COMMERCIAL LIST INFORMATION IN SCREENING ELIGIBLE HOUSING UNITS

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When using commercial address lists to sample households, investigators spend considerable time and money on screening households for eligibility as well as locating certain subpopulations (to achieve target sample sizes). Utilizing the demographic information on these lists to target eligible persons and subgroups has the potential to lower costs and field workers workload. Unfortunately, the information attached to the lists is error prone. We propose to evaluate the use of demographic information available on commercial lists in multistage household sampling. Specifically, this research will study how to efficiently design a three-stage sample that involves screening of housing units to determine eligibility. This research will also examine more complex estimators than have been previously studied.

The goals of this study are to (1) estimate the accuracy rates in which commercial lists can correctly identify households with certain

characteristics (e.g., Hispanics, Non-Hispanic Blacks, etc.); (2) Derive a theoretical variance formula, including variance components, for estimated totals; (3) Estimate variance components and evaluate alternative variance component estimators (design-based ANOVA, anticipated variance (model + design)); (4) Determine how to allocate two and three stage samples supplemented with commercial lists accounting for inaccuracy of listings, costs at each stage of sampling, target sample sizes and coefficient of variations (CVs), stratification of SSUs, and stratification of HU's by MSG characteristics (e.g., Race/Ethnicity, ages of persons in HU, etc.).

This research seeks to better understand the quality of demographic data attached to commercial lists and to use this information to increase sampling efficiency in the HRS by recovering more information for lower costs. This research potentially creates an improved sample design for HRS and similar surveys that is less costly and equally or more statistically efficient than the current design. In particular, the proposed design will help sample designers reduce the amount of housing unit screening needed to identify target subpopulations (e.g., Blacks, Hispanics, teenagers, and females). Furthermore, the results of this research will extend to other multistage household surveys that use commercial lists for sampling.

USING COMMERCIAL LIST INFORMATION IN SCREENING ELIGIBLE HOUSING UNITS

by

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To

Amyah, Akyli, Azaio, Arazo, Ajedi, Ajoui, & Baby

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# 1 Introduction

Chapter 1 reviews the general problem of allocating a household survey based on information from lists of addresses that are sold commercially. Section 1.1 sketches the topics covered in this dissertation and describes how it extends earlier research. Section 1.2 discusses the sources of commercial lists, how they are typically constructed in the U.S., and how they have been used in address-based sampling. Information on the lists can be used to target a sample toward certain demographic groups; this involves screening households as discussed in Section 1.3. The fourth section covers auxiliary, demographic information on the lists that can be used for sampling.

## 1.1 Overview

When using commercial address lists to sample households, investigators spend considerable time and money on screening housing units (HUs) for eligibility as well as locating certain subpopulations (to achieve target sample sizes). Utilizing the demographic information (e.g., age, race/ethnicity) on these lists to target eligible persons and subgroups has the potential to lower costs and field workers' workload. Unfortunately, the information attached to the lists is error prone. However, Valliant, Hubbard, et al. (2014) showed that nonlinear programming, using the commercial list information, could be used to screen more efficiently for some demographic groups even when the list information is not entirely accurate. The purpose of this study is to further evaluate the use of demographic information available on commercial lists in multistage household sampling. Specifically, this research will study how to efficiently design a three-stage sample that involves screening of HUs to determine eligibility.

Similar to Valliant, Hubbard, et al. (2014), we use nonlinear programming to find sample allocations subject to a variety of constraints. In determining how to allocate two- and three-stage samples supplemented with commercial lists, we extend variance formulas for two- and three-stage sampling to formulate an optimization problem (Valliant, Dever, & Kreuter, 2013). The theoretical work will be to derive component formulas that account for strata of secondary sampling units (SSUs) and substrata of HUs and are specific to inverse weighted estimators of totals and means. This research will also examine more complex sample designs than have been previously studied. Specifically, the goals of this research are to:

(1) Estimate the accuracy rates in which commercial lists can correctly identify households with certain characteristics (e.g., Hispanics, Non-Hispanic Blacks, Persons 50 and over, etc.).

(2) Study how to efficiently design two and three-stage samples that involve screening of housing units using demographic information on commercial lists to determine eligibility accounting for inaccuracy of listings, costs at each stage of sampling, target sample sizes of demographic subgroups, stratification of SSUs by some area characteristics (e.g., density of Blacks, Hispanics, Others), stratification of HU's by commercial list characteristics (e.g., Race/Ethnicity, ages of persons in HU, etc.), and characteristics of different variables of interest.

(3) Derive a theoretical variance formula, including variance components, for estimated totals and estimate variance components.

(4) Study the use of ANOVA and anticipated variances as alternative variance component estimators.

This research seeks to better understand the quality of demographic data attached to commercial lists and to use this information to increase sampling efficiency in the ABS surveys by recovering more information for lower costs. This research potentially creates an improved sample design for ABS surveys that is less costly and equally or more statistically efficient than designs that do not use list information. In particular, the proposed design could help sample designers reduce the amount of housing unit screening needed to identify and target hard-to-reach subpopulations. Although sample allocation in multistage designs has been studied previously, the combination of topics listed above is unique. Combining a design with multiple goals, a solution via nonlinear programming, inclusion of commercial lists to refine an area sample, together with modern variance component methods will be a new and practically useful contribution to the sample design literature.

## 1.2 Commercial Lists

### 1.2.1 Source of Commercial Lists

The United States Postal Service (USPS®) Address Management System (AMS; USPS 2013b) database serves as the official record of US mailing addresses. A mailing address contains information on street/box number, city, state, ZIP code, carrier route number, delivery sequence number (order in which letter carrier delivers mail) and vacant/seasonal indicator flags (Iannacchione V., 2011). The USPS® Computerized Delivery Sequence (CDS; USPS 2013a) file is built from the information contained within the AMS database. For a monthly subscription fee, the CDS program provides updated delivery sequence information to qualified commercial vendors given they provide their own address lists. Such vendors include marketing database companies like Experian (http://www.experian.com), infoUSA (http://www.infousa.com), Marketing Systems Group (MSG, http://www.m-s-g.com), Valassis (http://www.valassis.com), and Acxiom (http://www.acxiom.com).

As part of the licensing agreements, vendor-supplied address lists must include at least 90 percent (but at most 110 percent) of the possible delivery addresses in the ZIP Code(s) for which they wish to receive updates (USPS 2013b). Vendors can choose to receive updates on a weekly or bimonthly basis and can also request updates from what the USPS calls the CDS-No Stat file (USPS 2013a), which contains around 7 million predominantly rural addresses (Shook-Sa, Currivan, McMichael, & Iannacchione, 2013). Vendors can alternatively maintain a Delivery Sequence File Second Generation (DSF2) license that does not require the same rigorous standards. The DSF2 license allows vendors to update their address lists monthly at a lower cost than the CDS. Consequently, less detailed information is provided, limiting services to checking whether an address is currently represented in the DSF2 as a known address record and recording vacancy information. In this paper, residential address lists maintained by commercial vendors (through updates received from the USPS® CDS or DSF2 license) will be referred to as commercial lists[1].

### 1.2.2 Coverage of Commercial Lists

Although the correspondence between mailing addresses and housing units is not exactly one to one, Iannacchione (2011) provides evidence that the residential mailing addresses contained in the CDS and CDS No-Stat files provide nearly complete coverage of the U.S. household population[2]. The coverage of commercial lists varies by vendor but is generally

[1] Also defined as consumer databases in AAPOR (2015).
[2] Housing units may contain more than one household. However, calculations of this study will not account for that.

of good quality and good coverage (Dohrmann, Han and Mohadjer 2006, 2007; English, O'Muirheartaigh, et al. 2009; English, Bilgen and Fiorio 2012).

### 1.2.3 Use of Commercial Lists in Sampling: Address-Based Sampling (ABS) Frames

Accordingly, survey research organizations purchase these commercial lists to build sampling frames for household surveys. Any set of sampling procedures that use commercial lists as sampling frames for surveys is better known as Address Based Sampling (ABS; Link, Battaglia, et al. 2008; Roth, Han and Montaquila 2013). A relatively new method, ABS first emerged as a popular cost and time saving alternative to the manual field listing of HUs (O'Muircheartaigh, Eckman and Weiss 2003; Iannacchione, Staab and Redden 2003; Dorhmann, Han and Mohadjer 2006) and later as a solution to decreased in response rates of random digit dialing (RDD) due to the rising cell phone only population (Link, Battaglia, et al. 2008; Link, Daily, et al. 2009; Brick, Williams and Montaquila 2011).

Iannacchione, Staab and Redden (2003) report one of the earliest usages of address-based sampling: a metropolitan household survey that estimated the impact of a mass media campaign on the poorest African American adults (aged 18-45) in Houston, TX. Aware that field enumeration was not an option due to the time and nature of the campaign, yet needing to conduct the survey fairly quickly, researchers opted to use a commercial list to develop the sampling frame. Additionally, when using commercial lists in place of field listing for a heart disease prevention study conducted in Dallas County, TX, Iannacchione, Staab and Redden (2003) cut sampling frame creation costs by an estimated 90 percent. The ability to uncover and target hard-to-reach subpopulations in comparably shorter time frames using less resources than in traditional field listings serves as a classic example highlighting important advantages of ABS.

Address based sampling allows smaller organizations the possibility to conduct household surveys targeting populations that were previously reserved for National organizations with large budgets and comprehensive coordination. Today, address-based sampling is used in a multitude of surveys including National Survey of Family Growth (NSFG; Lepkowski, Mosher, et al. 2010), General Social Survey (GSS; O'Muircheartaigh, Eckman and Weiss 2003; O'Muircheartaigh, English, et al. 2009), National Children’s Study (NCS; English, O'Muirheartaigh, et al. 2009), National Survey on Drug Use and Health (NSDUH; Iannacchione, McMichael, et al. 2012), Health and Retirement Study (HRS; Valliant, et al. 2014), National Household Education Surveys (NHES; Brick, Williams and Montaquila 2011), and Nielsen TV Ratings Diary (Link, Daily, et al. 2009).

## 1.3 Standard Screening Practices

The time and cost savings associated with using commercial lists in place of traditional field listing for constructing sampling frames are clearly established and well documented in the past literature, cited above. However, researchers still spend a considerable amount of resources screening HUs for eligibility for relatively small gains in target sample sizes. Moreover, screening becomes exceedingly lengthy and costly when studies target hard to reach subpopulations such as younger age groups or Blacks (Brick, Williams and Montaquila 2011; Bilgen, English and Fiorio 2012).

Traditional methods used to identify subpopulation members include large-scale screening and the use of Census block group data to target sample areas (Waksberg, Judkins, & Massey, 1997). In large-scale screening of HUs, relatively large screening operations (a large pool of interviewers and a large amount of time) are needed to locate enough eligible members to generate an adequate domain sample size, adding substantial costs for conducting the survey (Lepkowski, Davis, et al. 2001; Kalton, Kali and Sigman 2014). Costs often arise from the need to use multiple modes or two-phase designs (e.g., mail and phone, mail and face-to-face, mail and mail) during the screening process (Murphy, Harter and Xia 2010; Brick, Williams and Montaquila 2011).

For decades, the decennial census and, more recently, the American Community Survey (ACS) have provided high quality information to attach to HU sampling frames. This includes longitude, latitude, census tract, census block, and area level demographic information. Vendors use approximate latitude and longitude coordinates from the Census to geocode addresses in order to assign addresses to the correct Census block, block group or tract (Dohrmann, et al. 2014; AAPOR Task Force on Address-based Sampling 2016). This information can be used in multistage sampling to stratify secondary sampling units (SSUs), which are typically groups of census blocks, by their concentrations of demographic groups and then sample SSUs at different rates (Waksberg, Judkins, & Massey, 1997). Within each SSU, HUs are sampled and then screened for eligibility. Oversampling based on Census tract or block information can provide some help in finding subpopulations such as race/ethnicity or socioeconomic status that are concentrated in specific geographic areas. However, this method does not help much in finding groups of persons that are widely dispersed like teenagers, the elderly, or households with children (Brick, Williams and Montaquila 2011; English, Li, et al. 2014).

Since traditional screening methods are not always cost-efficient in identifying eligible HUs, supplemental designs that use more targeted screening at the HU level may prove useful in these cases. Improved designs are dependent on identifying HUs that are likely to contain members of the subpopulation prior to the sampling process. In previous decades, the lack of available information at the HU level made this pre-screening nearly impossible. However, with the available HU level auxiliary data on commercial lists, researchers can potentially utilize this information to more efficiently identify eligible HUs (Chmura & Yancey, 2012).

## 1.4 Auxiliary Information on Commercial Lists

Commercial vendors often enhance the original CDS/DSF2 data by appending auxiliary demographic and/or geographic HU information to mailing addresses. This includes the publicly available census area level information as well as items for individual households from other sources. According to the *Task Force on Address-based Sampling* (AAPOR Task Force on Address-based Sampling, 2016), "[vendors] often amass and compile personal and household information from thousands of sources". These proprietary sources include consumer activity data (e.g., warranty cards and magazine subscriptions) as well as public records (e.g., phone listings, credit records, property records, and voter-registration lists) (Smith and Kim 2009; English, Bilgen and Fiorio 2012; AAPOR Task Force on Address-based Sampling 2016).

In turn, vendors use this information to construct the auxiliary variables that they include on commercial lists. Usually, auxiliary variables from a certain commercial list originate from a combination of different sources. Some variables may be directly extracted from proprietary databases, while other variables are modeled or imputed based on variable(s) in the proprietary databases (AAPOR Task Force on Address-based Sampling, 2016).

Table 1.1 Example Substrata and Definitions from Valliant, Hubbard, et al. (2014)

| MSG Substratum | Label | Definition |
|---|---|---|
| 1 | MBB H | One or more MBB Hispanic persons in the HU |
| 2 | MBB NH | One or more MBB non-Hispanic persons in the HU |
| 3 | EBB H | One or more EBB Hispanic persons in the HU |
| 4 | EBB NH | One or more EBB non-Hispanic persons in the HU |
| 5 | No MBB/EBB | No EBB or MBB persons in the HU (No MBB/EBB) |
| 6 | Unknown | Unknown whether the HU contained an EBB or MBB person based on MSG data. |

Alternatively, some vendors, such as MSG, source auxiliary data from other commercial vendors. The auxiliary information consists of variables at both the household and person level. HU level data may include landline telephone numbers, geographic coordinates, and income. Person level data may provide details specific to the head of the household, such as name, Hispanic surname indicator, marital status, age, sex, race/ethnicity, and email addresses (Link, Daily, et al. 2009; Valliant, Hubbard, et al. 2014). The vendors considered in this thesis provide information on up to six persons in a HU.

In an effort to improve the efficiency at which some target domains are sampled, HUs can be stratified during the third stage of sampling according to the attached demographic information from MSG. As an example, Valliant, Hubbard, et al. (2014) used MSG data on two persons in the household as well as the Hispanic ethnicity of at least one person in the household to classify HUs into one of six MSG substrata used for sampling, shown in

Table 1.1.

Their application was based on the Health and Retirement Study which recruits HUs for different age cohorts based on the ages of persons in the household. The first four substrata contained HUs that MSG expected to be eligible for the Early Baby Boomer (EBB) or Middle Baby Boomer (MBB) cohorts. The fifth substratum contained HUs that MSG did not expect to be eligible for the EBB or MBB cohorts. The sixth substratum contained HUs for which MSG was missing the demographic information to predict eligibility. Note that HUs in the sixth substratum must be given a positive inclusion probability since auxiliary data are not missing at random and HUs in that substratum may actually be eligible for a cohort. The example substrata could be used to oversample Hispanics in the third stage of selection. Alternative groupings of MSG substrata (e.g., income groups, marital status, etc.) can be made depending on the available auxiliary data. As part of this thesis research, I will consider other types of stratification that would be appropriate for targeting different demographic groups.

This research focuses particularly on the use of demographic auxiliary variables to determine eligibility in a subpopulation directly from commercial lists. This approach is attractive for efficiently controlling sample allocations to subpopulations by means of reducing and/or eliminating time-consuming, costly screening practices. As noted earlier, area stratification is not always efficient in identifying eligible HUs. Standard approaches to screening and locating subpopulation members are time consuming and expensive. In these cases, supplemental designs that conduct further stratification at the HU level may prove useful. Despite its potential use, little research has extensively explored utilizing such data. This is partially due to the large number of HUs with unavailable or inaccurate auxiliary data, making it difficult for researchers to correctly identify eligible HUs.

### 1.4.1 Quality of Auxiliary Information

The utility of auxiliary information will depend on several factors that affect the quality of the data including the proportion of HUs for which data are available (i.e., *availability*), the ability of ancillary data to predict the true characteristics of HUs (i.e., *accuracy*), and the vendor chosen to supply the data. The utility will also depend on various characteristics of the survey itself, such as the variables of interest, the subgroups of interest, and the variable level of aggregation (household or person-level).

### 1.4.2 Availability of Auxiliary Information

Availability problems occur when variable information is missing for all or for a portion of HUs on a list. Vendors collect and compile vast amounts of information that they acquire from a variety of sources; each source varies in completeness. For this reason, variables on a single commercial list often differ considerably on their availability. Several studies support this claim. For example, in two samples taken from commercial lists provided by MSG, auxiliary data was missing 20 percent to 43 percent of the time (Roth, Han and Montaquila 2012, 2013) and 5 percent to 27 percent of the time (DiSogra, Dennis, & Fahimi, 2010) for differing variables. While comparing differences in variable availability between vendor lists, Buskirk, Malarek and Bareham (2014) found that missing rates within a single vendors' commercial list ranged from as little as 26 percent to as much as 96 percent.

The availability of a variable generally depends on its source. It is likely that vendors have more complete information on variables sourced from quality sources, such as instances where the data are generally known for each HU (e.g., phone numbers from public telephone directories) or instances where the data are derived from models that include Census/ACS data (e.g., HU income modeled from area level income data). Income has relatively high availability across samples taken from commercial lists (DiSogra, Dennis and Fahimi 2010; Roth, Han and Montaquila 2012, 2013; Buskirk, Malarek and Bareham 2014) while educational attainment and ethnicity have been noted to have relatively high rates of missingness (DiSogra, Dennis and Fahimi 2010; Roth, Han and Montaquila 2012, 2013).

Variables also have varying degrees of availability within subgroups. Some demographic subgroups of HUs have more missing data than others. Such differences in availability within variables are not ignorable. The missingness of subgroups for specific variables can give insight to the characteristics of HUs that are likely to be under-represented with respect to the demographic auxiliary data on commercial lists. This makes it more difficult for researchers to locate these subgroups using auxiliary data. Past research has shown that with respect to the general population (not accounting for inaccuracies in the auxiliary data):

1. HUs with available auxiliary data tend to be composed of older people. Adults in age groups 55+ and 65+ are usually over-represented while younger adults 34 and under are usually under-represented (Link and Burks 2103; Buskirk, Malarek and Bareham 2014).
2. HUs with available auxiliary data are more economically advantaged, with homeowners overrepresented and renters underrepresented (English, Bilgen and Fiorio 2012; Buskirk, Malarek and Bareham 2014). For income subgroups, lower income HUs are under-represented while higher income HUs are over-represented. (English,

Bilgen and Fiorio 2012; English, Li, et al. 2014; Pasek, et al. 2014).

3. Auxiliary data are considerably less available in high poverty areas (Pasek, et al. 2014) and high population density areas (English, Bilgen, & Fiorio, 2012).

These outcomes are expected given that the demographic variables derived from credit agencies, consumer-spending databases, and warranty information most likely pertain to older, wealthier persons (English, Li, Mayfield, & Frasier, 2014). However, the research does not agree across the board for all variables in which subgroups are more likely to be unavailable. In particular, race/ethnicity subgroups are not consistently available. Link and Burks (2103) found that Hispanics were under-represented and Blacks over-represented with respect to their distribution in the population. On the other hand, English, Li, et al. (2014) found that Hispanics were over-represented compared to the population distribution. In addition, for commercial lists that based race/ethnicity on surnames, Blacks were under-represented compared to the population distribution since Blacks generally do not have distinct surnames (English, Bilgen and Fiorio 2012; English, Li, et al. 2014).

Because the availability of variables depends on sources, which vary from vendor to vendor, and because much of the auxiliary data are not missing at random, it is important to account for the difference in availability between subgroups.

### 1.4.3 Accuracy of Auxiliary Information

Given that auxiliary data are available for a HU, the next concern is whether the information is accurate. Inaccuracies in auxiliary data can occur from a variety of reasons. Common sources of inaccuracies include differences in variable definitions between commercial lists and the comparing data (e.g., Census data, respondent screener data), time lapses in when a vendor last updated its database relative to when a researcher collects data in the field, and simple mismatch errors on part of the vendor (Roth, Han and Montaquila 2012; Buskirk, Malarek and Bareham 2014).

Variables on a single commercial list often differ considerably on their accuracy rates. Person level variables are more prone to inaccuracies than housing unit level variables. The reference person identified in the field may not be the same reference person identified on commercial lists, especially for households with more than two people. Roth et al. (2012, 2013) evaluated the data quality of demographic variables provided on ABS frames by matching MSG data to the 2011 National Household Education Surveys (NHES) Field Test mail screener. They found that HU characteristics from NHES matched those of the demographic information found on the MSG commercial lists 26 to 75 percent of the time. When including missingness, education attainment had the lowest accuracy rate and home tenure had the highest. Even when the variable definition varied from MSG to NHES, the MSG data correctly identified households with children 41 percent of the time.

Only a few other studies have explored accuracies of commercial lists further. The results of these studies are limited to the sample from which they were taken (see Table 1.2). DiSogra, Dennis and Fahimi (2010) found that MSG variables correctly predicted home ownership about 94 percent of the time and household income about 41 to 52 percent of the time when compared to a self-reported web survey of housing units. In addition, Chmura and Yancey (2012) found that sample indicators for the age of head of household $\leq 35$ were accurate 79 percent of the time. For race/ethnicity variables, commercial lists were able to correctly identify Blacks 66 to 85 percent of the time and Hispanics 75 to 88 percent of the time in two separate studies (DiSogra, Dennis and Fahimi 2010; Chumra and

Yancey 2012). In all studies, accuracy rates were more variable for person level information like gender and educational attainment than variables that are more likely to be related to HU characteristics like home ownership and surname (DiSogra, Dennis and Fahimi 2010; Roth, Han and Montaquila 2012, 2013; Buskirk, Malarek and Bareham 2014).

Table 1.2 Accuracy Rates of Variables by Reference Paper and Type of List

| **Variable** | **Accuracy Rate (Percent)** | **Reference** | **Type of List** |
|---|---|---|---|
| Home Tenure | 60-70 | Buskirk et al. (2014) | Multiple general vendor lists |
| | 75 | Roth, Han, and Montaquila (2012) | MSG |
| | 94 | DiSogra, Dennis and Fahimi (2010) | MSG |
| HH Income | 41-50 | DiSogra, Dennis and Fahimi (2010) | MSG |
| | 48 | Roth, Han, and Montaquila (2012) | MSG |
| | | | |
| Race/Ethnicity-Black | 66 | DiSogra, Dennis and Fahimi (2010) | MSG |
| | 85 | Chmura and Yancey (2012) | General vendor list |
| Race/Ethnicity-Hispanic | 64 | Roth, Han, and Montaquila (2012) | MSG |
| | 75 | DiSogra, Dennis and Fahimi (2010) | MSG |
| | 88 | Chmura and Yancey (2012) | General vendor list |
| Children Present | 35-39 | English et al. (2014) | Targeted Lists |
| | 40 | Roth, Han, and Montaquila (2012) | MSG |
| No. of Children | 13-15 | Buskirk et al. (2014) | Multiple general vendor lists |
| Hispanic Surname | 92 | Roth, Han, and Montaquila (2012) | MSG |
| Surname Suffix | 4-5 | Buskirk et al. (2014) | Multiple general vendor lists |
| Marital Status | 20-74 | Buskirk et al. (2014) | Multiple general vendor lists |
| Educational Attainment | 26 | Roth, Han, and Montaquila (2012) | MSG |
| HOH Age ≤ 35 | 79 | Chmura and Yancey (2012) | General vendor list |

### 1.4.4 Vendors Role in Auxiliary Information

Vendors will likely differ in the way variables are updated, captured, defined and coded (AAPOR 2015) resulting in discrepancies between vendor lists. These discrepancies between vendors' lists add to the varying quality of auxiliary data. For example, marital status may have good accuracy and availability on the vendor list that has a reliable source, but bad accuracy and availability on the vendor list with no reliable source and had to impute marital status. In addition, as noted previously, vendors may be not be referring to the same reference persons for variables that are captured at the person level. For that reason, HU level characteristics are more consistent across vendors than person level characterizes.

Buskirk, Malarek and Bareham (2014) compared availability across three vendors and found that availability varied between 65-80% for vendor lists for certain key variables (income, age 65+, number of adults, age groups, surname, own/rent and given name). The worst variation was in marital status, which was available 20% of the time on one list and 74% of the time on another. Still some variables were fairly consistent across vendors including number of children (13-15%) and surname suffix (4-5%). Although some variables, such as the number of children in a household, did not suffer from variation among vendors (15%), the variables did not match the prevalence in the population (20%). This serves as a reminder that variables that agree across vendor lists may still be highly inaccurate.

Lastly, English, Li, et al. (2014) found little variation (35-39%) in the accuracy rates between vendors' lists when attempting to predict which HUs have small children. A possible reason for this level of consistency is that the vendors may be acquiring information on children from the same propriety source.

### 1.4.5 Use of Auxiliary Data in Sampling

Auxiliary information can be useful in the screening process even when the list information is not entirely accurate (Valliant R. , Hubbard, Lee, & Chang, 2014). The three methods discussed below employ auxiliary information at different stages of the survey design with the common goal of drawing more efficient samples by reducing screening efforts for target demographic subgroups.

The first method aims at increasing response rates for hard to reach subgroups by using demographic data to tailor incentives for target subgroups. Link, Daily, et al. (2009) first used this technique to reduce the amount of oversample needed to achieve target sample sizes. As a result, the number of completed Nielson diaries for householder's aged 18-34 years old was especially high compared to previous years. However, this technique heavily relies on the cooperation rate of the subgroup. In contrast, the same technique showed no significant improvement in penetrating Black and Hispanic households (Link, Daily, et al. 2009; Chmura and Yancey 2012).

The two remaining methods focus on screening HUs at higher eligibility rates than they occur in the general U.S. population, using targeted lists or HU level stratification. *Targeted lists* are vendor originated lists of HUs likely to contain members of specific demographic subgroups (Bilgen, English and Fiorio 2012; English, Bilgen and Fiorio 2012; English, Li, et al. 2014). These demographic subgroups are often the hardest to reach populations in address-based sampling (e.g., Blacks, Hispanics, 18-34 year olds, and children). The targeted lists are expected to contain the requested subpopulation in higher concentrations compared to the general population and therefore are ideal lists to use as an enhancement to ABS frames.

Alternatively, researchers can stratify HUs by subgroups formed with auxiliary demographic information (Roth, Han and Montaquila 2013; English, Li, et al. 2014; Valliant, Hubbard, et al. 2014). For example, consider a multistage ABS design, where the HUs are the third stage sampling units. The race/ethnicity variable from the auxiliary data can be used to assign the HUs to one of five strata: Non-Hispanic Black, Hispanic, White, Other, or Unknown. The target race/ethnicity group is then oversampled from the respective stratum under the assumption that the stratum contains higher concentrations of the targeted subgroup. Note that because auxiliary data are not missing at random nor is it inaccurate at random, the mentioned methods must allow HUs in the unknown stratum as well as HUs not on the targeted lists to have a positive probability of inclusion.

Valliant, Hubbard, et al. (2014) demonstrated that stratifying using error-prone auxiliary data improved the efficiency of locating members of some subgroups but not all. For some subgroups where the distribution in the HU level strata or on targeted lists is not higher than in the general population, the above methods may not prove more useful than randomly sampling from the general population. This is especially true for those subgroups that are underrepresented in the auxiliary data or where the data are highly inaccurate. Furthermore, there is some evidence that representation of subgroups is not only reliant on the subgroup of interest but also on the sourced commercial list. Some commercial lists may be better at targeting Blacks (Link & Burks, 2103), while other commercial lists better at targeting Hispanics (English, Li, et al. 2014; Valliant, Hubbard, et al. 2014).

# 2 Derivation and Results of a Theoretical Three-Stage Variance Formula with Strata of SSUs and Substrata of HUs

Chapter 2 covers the theory for estimators of totals in three-stage samples, including point estimators and their design-based variances. Sections 2.1 and 2.2 introduce the approach and notation. Sections 2.3 and 2.4 consider an estimator of a population total and its variance. The variance is broken into components associated with each stage of sampling. Analysis of variance estimators of the components are derived along with anticipated variances that use a random effects model.

## 2.1 Introduction

In determining how to allocate two- and three-stage samples, the contributions of each stage of sampling to the variance of an estimator must be accounted for (Valliant, Dever, & Kreuter, 2013). The following chapter details the derivation and results for variance component formulas, for estimators of totals that account for strata of SSUs and substrata of HUs. In Sections 2.2 and 2.3 , we set up the general three-stage sample design, which will serve as the framework for the design specific to the Health and Retirement Study (HRS) data that will be used for empirical illustration. The two-stage sample design is covered inherently. Much of the derivation work for the variance formulas can be found in Appendix A.1. In Section 2.4 , I present direct estimates of the variance components, as well as plug-in estimators for the measures of homogeneity, with the use of ANOVA estimators, and anticipated variances using a random effect model as an alternative variance component estimator.

## 2.2 Summary of Notation

In household surveys, it is common to select PSUs, SSUs within PSUs, and households within SSUs. Consider a three-stage sample design in which the first-stage units are selected with probability proportional to size with replacement, i.e., *ppswr*, second-stage units are stratified within each PSU and selected with *ppswr*, and third-stage units are stratified within each SSU and selected using *srswor*. Although sampling without replacement is more common in practice, the variance component formulas for with-replacement sampling are more useful when determining how to allocate a sample. In this scenario, there are three variance components that need to be considered to allocate a sample among the different stages of sampling. To specify this situation the following notation is needed.

### 2.2.1 Sample Design

$i$ : PSU *index*
$a$ : SSU *stratum index*
$j$ : SSU *index*
$b$ : HU *substratum index*
$k$ : HU *index*

### 2.2.2 Population Values

| | | |
|---|---|---|
| $U$ | : | Set of all PSUs in the universe |
| $U_{ia}$ | : | Set of all SSUs in PSU $i$, SSU stratum $a$ in the universe |
| $U_{iajb}$ | : | Set of all HUs in PSU $i$, SSU stratum $a$, SSU $j$, HU substratum $b$ in the universe |
| $U_{iaj}$ | : | Set of all HUs in PSU $i$, SSU stratum $a$, SSU $j$, across all HU substrata in the universe |
| $M$ | : | Number of PSUs in the population |
| $A$ | : | Number of SSU strata in each PSU $i$ |
| $N_{ia}$ | : | Number of SSUs in the population for PSU $i$, SSU stratum $a$ |
| $B$ | : | Number of HU substrata in each PSU $i$, SSU $j$ |
| $Q$ | : | Number of HUs in the population |
| $Q_i$ | : | Number of HUs in the population for PSU $i$ |
| $Q_{ia}$ | : | Number of HUs in the population for PSU $i$, SSU stratum $a$ |
| $Q_{iaj}$ | : | Number of HUs in the population for PSU $i$, SSU stratum $a$, SSU $j$ |
| $Q_{iajb}$ | : | Number of HUs in the population for PSU $i$, SSU stratum $a$, SSU $j$, HU substratum $b$ |
| $Q_a$ | : | Number of HUs in the population for SSU stratum $a$ across all PSUs |
| $Q_{ab}$ | : | Number of HUs in the population for SSU stratum $a$, HU substratum $b$ across all PSUs |
| $t_{U_i}$ | : | Population total of an analysis variable for PSU $i$ |
| $t_{U_{ia}}$ | : | Population total of an analysis variable for PSU $i$, SSU stratum $a$ |
| $t_{U_{iaj}}$ | : | Population total of an analysis variable for PSU $i$, SSU stratum $a$, SSU $j$ |

### 2.2.3 Sample Values

$s_1$ : Set of sample PSUs

$s_{1,SR}$ : Set of sample Self-Representing (SR) PSUs

$s_{1,NSR}$ : Set of sample Non-Self-Representing (NSR) PSUs

$s_{ia}$ : Set of sample SSUs in PSU *i*, SSU stratum *a*

$s_2$ : Set of all sample SSUs across all PSUs; $s_2 = \{s_{ia};\ i \in s_1, a = 1, ..., A\}$

$s_{iaj}$ : Set of sample HUs in PSU *i*, SSU stratum *a*, SSU *j*

$s_{iajb}$ : Set of sample HUs in PSU *i*, SSU stratum *a*, SSU *j*, HU substratum *b*

$m$ : Number of sample PSUs

$n_{ia}$ : Number of sample SSUs selected from PSU *i*, SSU stratum *a*

$q_{iajb}$ : Number of sample HUs selected from PSU *i*, SSU stratum *a*, SSU *j*, HU substratum *b*

$p_i$ : Single draw probability of PSU *i*

$p_{j|ia}$ : Single draw probability of SSU *j* within PSU *i*, SSU stratum *a*

$\pi_{k|iajb}$ : Probability of selection of HU *k* within PSU *i*, SSU stratum *a*, SSU *j*, HU substratum *b*

## 2.3 Three Stage Sample Design

### 2.3.1 *PWR* Estimators

#### 2.3.1.1 General Design

Using the notation above, consider the following three-stage design in which *m* PSUs are selected with probability proportional to size with replacement (*ppswr*), the SSUs are stratified within each PSU and $n_{ia}$ are selected with *ppswr* within PSU *i*, SSU stratum *a*, and the HUs are stratified within each SSU and $q_{iajb}$ are selected with simple random sampling without replacement (*srswor*) within PSU *i*, SSU *j* in SSU stratum *a*, and HU substratum *b*. We assume that the sampling fraction in the third stage is negligible. Shorthand for this three-stage design is *ppswr/ppswr/srswor*. Although most samples are selected without replacement, modeling the sample selection as being done with-replacement is a practical workaround. The with-replacement formulation avoids complex design-based variance formulas that involve joint selection probabilities and is not useful for determining sample allocations. The *ppswr* variance formulas are simpler and contain sample sizes in a direct way that facilitate theoretical variance calculations.

Let $y_k$ be the value of an analysis variable associated with HU *k*. Then the p-expanded with replacement (*pwr*; Särndal, Swensson and Wretman 1992) estimate of the population total, $t_U = \sum_{i \in U} \sum_{a=1}^{A} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} y_k$, of an analysis variable *Y* is:

$$\hat{t}_{pwr} = \frac{1}{m} \sum_{i \in s_1} \frac{\hat{t}_{U_i}}{p_i} \tag{1.1}$$

where

$$\hat{t}_{U_i} = \sum_{a=1}^{A} \hat{t}_{U_{ia}} \tag{1.2}$$

is the estimated total for PSU *i*,

$$\hat{t}_{U_{ia}} = \frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{\hat{t}_{U_{iaj}}}{p_{j|ia}} \tag{1.3}$$

is the estimated total for SSU stratum *a*, in PSU *i* from a *ppswr* sample of SSUs, and

$$\hat{t}_{U_{iaj}} = \sum_{b=1}^{B} \frac{Q_{iajb}}{q_{iajb}} \sum_{k \in s_{iajb}} y_k \tag{1.4}$$

is the estimated total for SSU *j*, in PSU *i*, SSU stratum *a* across all HU strata from a simple random sample of $q_{iajb}$ HUs in HU stratum *b*. In cases where population values are unknown, $Q_{iajb}$ will need to be estimated from a sample. Alternatively, we can write Equation (1.1) in terms of $w_k = \left(mp_i n_{ia} p_{j|ia} \pi_{k|iajb}\right)^{-1}$, the overall weight for HU *k*, such that

$$\hat{t}_{pwr} = \sum_{i \in s_1} \sum_{a=1}^{A} \sum_{j \in s_{ia}} \sum_{b=1}^{B} \sum_{k \in s_{iajb}} w_k y_k \tag{1.5}$$

Note that $w_k$ is not the inverse of the selection probability of HU *k* since the first two stages of sampling are treated as with-replacement.

The *pwr* estimator, $\hat{t}_{pwr}$, is a design-unbiased estimator of the population total of *y*'s under the *ppswr/ppswr/srswor* design. The overall weight for HU *k* is the product of individual weights at each stage: PSU, SSU segments stratified by SSU level strata, households stratified by HU level strata.

### 2.3.1.2 Non Self-Representing (NSR) PSUs

In the actual HRS sampling design (and many other household survey designs), PSUs are stratified into PSU strata before selection. Self-representing PSUs each constitute a stratum and are selected with certainty (i.e., one draw probability = 1). Non-self-representing PSUs are selected with *ppswr* in their respective strata. Because the HRS data contains both SR and NSR PSUs, the *pwr* estimator must be estimated by two separate parts. Below we formulate $\hat{t}_{pwr}$ for both SR and NSR PSUs. To formulate the *pwr* estimator for SR PSUs, we recognize that SR PSUs are essentially strata where a stratified 2 stage sample of SSUs and HUs is selected. In practice, the NSR PSUs are stratified then picked with *ppswor* inside each stratum. We use the same practical work-around as above and treat the NSR PSUs as being selected with replacement, i.e., *ppswr*. We make an adjustment to the stratified one-draw probability and treat the HRS NSR PSU sample as unstratified. Let

$m_{NSR}$ denote the number of NSR PSUs. Thus, the *pwr* estimate of the population total, $t_U$, of an analysis variable *Y* is:

$$\hat{t}_{pwr} = \hat{t}_{pwr,SR} + \hat{t}_{pwr,NSR} \tag{1.6}$$

with

$$\hat{t}_{pwr,SR} = \sum_{i \in s_{1,SR}} \sum_{a=1}^{A} \frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{1}{p_{j|ia}} \sum_{b=1}^{B} \frac{Q_{iajb}}{q_{iajb}} \sum_{k \in s_{iajb}} y_k \tag{1.7}$$

and

$$\hat{t}_{pwr,NSR} = \frac{1}{m_{NSR}} \sum_{i \in s_{1,NSR}} \frac{1}{p_i} \sum_{a=1}^{A} \frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{1}{p_{j|ia}} \sum_{b=1}^{B} \frac{Q_{iajb}}{q_{iajb}} \sum_{k \in s_{iajb}} y_k \tag{1.8}$$

where $p_i = p_{hi} \frac{Q_h}{Q}$ is the adjusted one-draw probability for PSU *i* assuming an unstratified sample of NSR PSUs was selected. The logic behind this adjustment is explained below.

The HRS data file only contains the stratified one-draw probabilities for NSR PSUs, i.e.,

$$\begin{aligned} p_{hi} = \frac{Q_{hi}}{Q_h} &= \frac{\text{no. of HUs in PSU stratum } h\text{, PSU } i}{\text{no. of HUs in PSU stratum } h} \\ &= \text{probability PSU } i \text{ is chosen from PSU stratum } h \end{aligned}$$

However, $\hat{t}_{pwr}$, requires one-draw PSU probabilities, the probability PSU is chosen from the universe of all NSR PSUs, i.e., $Q_{hi}/Q$. The following adjustment was made to convert the available HRS stratified probabilities, $p_{hi}$, to the one draw probabilities, $p_i = Q_{hi}/Q$, assuming that the HRS PSUs were allocated to the NSR strata in proportion to the population number of HUs in each stratum.

$$p_i = \frac{Q_{hi}}{Q} \bullet \frac{Q_h}{Q_h} = \frac{Q_{hi}}{Q_h} \bullet \frac{Q_h}{Q} = p_{hi} \frac{Q_h}{Q} \qquad (1.9)$$

where $Q_h$ and $Q$ are estimated by

$$\hat{Q}_h = \sum_{i \in s_h} \sum_{a} \sum_{j \in s_{hia}} \sum_{b} \sum_{k \in s_{hiajb}} w_{k,NSR} \qquad (1.10)$$

with $w_{k,NSR} = \left( m_{NSR} p_i n_{ia} p_{j|ia} \pi_{k|iajb} \right)^{-1}$ is the overall sample weight of a HU assuming a stratified selection of PSUs, and $\hat{Q} = \sum_h \hat{Q}_h$ . Also, $s_h$ denotes the set of sample PSUs in PSU stratum *h*, $s_{hia}$ denotes the set of sample SSUs in PSU stratum *h*, PSU *i*, SSU stratum *a*, $s_{hiajb}$ denotes the set of sample HUs in PSU stratum *h*, PSU *i*, SSU stratum *a*, SSU *j*, HU substratum *b* . Note that in the 2010-11 HRS design each $s_h$ has only 1 sample PSU.

### 2.3.2 Components of Variance

As shown in Appendix A.1, the design relvariance of $\hat{t}_{pwr}$ is obtained by extending results in (Hansen, Hurwitz, & Madow, 1953) and is

$$\frac{V\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{1}{t_U^2} \left\{ \begin{array}{l} \dfrac{S_{U1(pwr)}^2}{m} \\ + \dfrac{1}{m} \displaystyle\sum_{i \in U} \frac{1}{p_i} \sum_{a=1}^{A} \frac{S_{U2(pwr)ia}^2}{n_{ia}} \\ + \dfrac{1}{m} \displaystyle\sum_{i \in U} \frac{1}{p_i} \sum_{a=1}^{A} \frac{1}{n_{ia}} \sum_{j \in U_{ia}} \frac{1}{p_{j|ia}} \sum_{b=1}^{B} Q_{iajb}^2 \frac{S_{U3iajb}^2}{q_{iajb}} \end{array} \right\} \qquad (1.11)$$

$$\equiv \frac{1}{t_U^2} \left\{ V_{PSU} + V_{SSU} + V_{HU} \right\}$$

where $V_{PSU}$ , $V_{SSU}$ , and $V_{HU}$ are defined by the last equality,

$$S_{U1(pwr)}^2 = \sum_{i \in U} p_i \left( \frac{t_{U_i}}{p_i} - t_U \right)^2 \tag{1.12}$$

is the population (unit) variance between PSU totals appropriate to the *ppswr* PSU sample,

$$S_{U2(pwr)ia}^2 = \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right)^2 \tag{1.13}$$

is the unit variance among SSU totals in SSU stratum *a*, PSU *i*, appropriate to the *ppswr* SSU sample design,

$$S_{U3iajb}^2 = \frac{1}{Q_{iajb} - 1} \sum_{k \in U_{iajb}} \left( y_k - \bar{y}_{U_{iajb}} \right)^2 \tag{1.14}$$

is the unit variance among HUs, in HU substratum *b*, SSU *j*, SSU stratum *a*, PSU *i*, with $\bar{y}_{U_{iajb}} = \sum_{k \in U_{iajb}} y_k \big/ Q_{iajb}$ . We use the terms substrata or substratum to refer to the HU substrata and the terms strata or stratum when referring to the SSU strata for simplicity. We also assume that every SSU stratum *a* occurs in every PSU *i* and that every HU substratum *b* occurs in every SSU *j*. In practice, this will not always be true in which case some terms in subsequent formulas will drop out.

In order to write Equation (1.11) in a more useful form for sample calculation, assume that the same number of SSUs is selected from SSU stratum *a* across each PSU, that is, $n_{ia} = \bar{n}_a$ , and that the same number of HUs is selected from substratum *b* within stratum *a*, for every PSU/SSU *ij* combination that is, $q_{iajb} = \bar{\bar{q}}_{ab}$. Define

$$t_{U_a} = \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} y_k$$

and

$$t_{U_{ab}} = \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{k \in U_{iajb}} y_k \ . \tag{1.15}$$

As shown in Appendix A.2, the relvariance then can be rewritten as a sum of three components,

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{B^2}{m} + \sum_{a=1}^{A} K_a^2 \frac{W_{2a}^2}{m\bar{n}_a} + \sum_{a=1}^{A}\sum_{b=1}^{B} K_{ab}^2 \frac{W_{3ab}^2}{m\bar{n}_a \bar{\bar{q}}_{ab}} \tag{1.16}$$

where $K_a = t_{U_a} \big/ t_U$ , $K_{ab} = t_{U_{ab}} \big/ t_U$ ,

$$B^2 = \frac{\mathrm{S}_{U1(pwr)}^2}{t_U^2},$$

$W_{2a}^2 = \dfrac{1}{t_{U_a}^2} \sum_{i \in U} \dfrac{\mathrm{S}_{U2(pwr)ia}^2}{p_i}$ is the contribution to the unit relvariance due to the second stage SSUs within SSU stratum *a*, and

$W_{3ab}^2 = \dfrac{1}{t_{U_{ab}}^2} \sum_{i \in U} \dfrac{1}{p_i} \sum_{j \in U_{ia}} \dfrac{Q_{iajb}^2 S_{U3iajb}^2}{p_{j|ia}}$ is the contribution to the relvariance due to the third stage HUs within SSU stratum *a*, HU substrata *b*.

### 2.3.3 Measures of Homogeneity

Define,

$W^2 = \frac{1}{t_U^2}\sum_{i\in U}\frac{Q_i^2 S_{U3i}^2}{p_i}$, a weighted unit relvariance among HUs across all PSUs and SSUs ignoring the $a$ and $b$ strata; $S_{U3i}^2 = \frac{1}{Q_i - 1}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left(y_k - \bar{y}_{U_i}\right)^2$, the element level variance among all elements in PSU $i$; $\bar{y}_{U_i} = \sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{y_k}{Q_i}$

$W_{3a}^2 = \frac{1}{t_{U_a}^2}\sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{Q_{iaj}^2 S_{U3iaj}^2}{p_{j|ia}}$, a weighted unit relvariance among HUs in SSU stratum $a$, across all PSUs, ignoring SSU membership and the $b$ strata;

$$S_{U3iaj}^2 = \frac{\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left(y_k - \bar{y}_{U_{iaj}}\right)^2}{Q_{iaj}-1} = \frac{\sum_{k\in U_{iaj}}\left(y_k - \bar{y}_{U_{iaj}}\right)^2}{Q_{iaj}-1};\ \bar{y}_{U_{iaj}} = \sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{y_k}{Q_{iaj}}$$

As show in Appendix A.2, the relvariance in (1.16) can also be written in terms of two measures of homogeneity:

$$\frac{V\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{\tilde{V}}{m}k_1\delta_1 + \sum_{a=1}^{A}K_a^2\frac{\tilde{V}_a}{m\bar{n}_a}k_{2a}\delta_{2a} + \sum_{a=1}^{A}\sum_{b=1}^{B}K_{ab}^2\frac{W_{3ab}^2}{m\bar{n}_a\bar{\bar{q}}_{ab}} \tag{1.17}$$

where

$\tilde{V} = \frac{1}{\bar{y}_U^2}\frac{1}{Q-1}\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left(y_k - \bar{y}_U\right)^2$ is the unit relvariance of $y$ in the population across all PSUs, SSUs, $a$ strata, and $b$ strata, $\bar{y}_U = \sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{y_k}{Q}$

$\tilde{V}_a = \frac{1}{\bar{y}_{U_a}^2} \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \frac{1}{Q_a - 1} \left( y_k - \bar{y}_{U_a} \right)^2$ is the unit relvariance of $y$ among elements (HUs) in SSU stratum $a$ across all PSUs in the population and all $b$ strata with, $\bar{y}_{U_a} = \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \frac{y_k}{Q_a}$.

Then the measures of homogeneity are defined as

$$\delta_1 = \frac{B^2}{B^2 + W^2} \quad , \quad k_1 = \frac{B^2 + W^2}{\tilde{V}}$$

$$\delta_{2a} = \frac{W_{2a}^2}{W_{2a}^2 + W_{3a}^2} , \quad k_{2a} = \frac{W_{2a}^2 + W_{3a}^2}{\tilde{V}_a} \tag{1.18}$$

where $B^2$, $W_{2a}^2$, $W_{3ab}^2$, $K_a$, and $K_{ab}$ are defined above in Section 2.3.2 and $W^2$ and $W_{3a}^2$ are previously defined above in this section.

Also shown in Appendix A.2, when there are no $b$ strata, the relvariance in Eq. (1.16) deduces to

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{\tilde{V}}{m\bar{n}_+\bar{\bar{q}}_+} \left\{ k_1 \delta_1 \bar{n}_+ \bar{\bar{q}}_+ + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{\tilde{V}} \frac{\bar{n}_+}{\bar{n}_a} \frac{\bar{\bar{q}}_+}{\bar{\bar{q}}_a} k_{2a} \left[ 1 + \delta_{2a} \left( \bar{\bar{q}}_a - 1 \right) \right] \right\} \tag{1.19}$$

where

$\bar{n}_+ = \sum_{a=1}^{A} \bar{n}_a$ is the number of sample SSUs allocated and

$\bar{\bar{q}}_+ = \sum_{a=1}^{A} \bar{n}_a \bar{\bar{q}}_a \Big/ \sum_{a=1}^{A} \bar{n}_a$ is the mean number of sample elements (HUs) per SSU across all SSU strata.

In the special case of no *a* nor *b* strata, we have $\bar{n}_a = \bar{n}_+ \equiv \bar{n}$, $\bar{\bar{q}}_a = \bar{\bar{q}}_+ \equiv \bar{\bar{q}}$, $\delta_{2a} = \delta_2$, and $\tilde{V}_a = \tilde{V}$. The relvariance then reduces to

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{\tilde{V}}{m\bar{n}\,\bar{\bar{q}}}\left\{k_1\delta_1\bar{n}\,\bar{\bar{q}} + k_2\left[1+\delta_2\left(\bar{\bar{q}}-1\right)\right]\right\} \tag{1.20}$$

matching equation (9.21) in Valliant, et. al. (2013). The relvariances written as above in Eqs. (1.17), (1.19), and (1.20) are useful for the sample allocation problem since they include design-effect-like terms. Note that, as in earlier sections, the entire notation above is for a linear estimator of a total that is a weighted summation of *y*'s.

### 2.3.4 Non Self-Representing (NSR) and Self-Representing (SR) PSUs

Because the HRS data used in this analysis contains both SR and NSR PSUs, the relvariance components must be estimated separately.

The relvariance of the *pwr* estimator, $\hat{t}_{pwr}$, is:

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = F_{SR}^2\frac{\mathrm{V}\left(\hat{t}_{pwr,SR}\right)}{t_{SR}^2} + F_{NSR}^2\frac{\mathrm{V}\left(\hat{t}_{pwr,NSR}\right)}{t_{NSR}^2} \tag{1.21}$$

where

$t_U = t_{SR} + t_{NSR}$ (the population total of *y* broken into totals for the SR and NSR parts of the frame), $F_{SR} = \dfrac{t_{SR}}{t_U}$, and $F_{NSR} = \dfrac{t_{NSR}}{t_U} = 1 - F_{SR}$. Part of the general allocation problem would be determining the definition and number of self-representing PSUs. In this thesis, we assume that the split between *SR* and *NSR* PSUs is predetermined. Thus, $F_{SR}$ and $F_{NSR}$ are treated as constants.

*Non Self-Representing (NSR) PSUs*

The relvariance formula for NSR PSUs will be the exact form of Eq. (1.17) with $m =$ no. of sample NSR PSUs and $p_i =$ adjusted one draw probability defined earlier. The sample sizes of SSUs and HUs are within NSR PSUs only; to avoid notational clutter, we do not add *NSR* subscripts to the sample sizes and variance components. The calculations and universe *U* are restricted to the universe of NSR PSUs. Thus, we have

$$\frac{\mathrm{V}\left(\hat{t}_{pwr,NSR}\right)}{t_{NSR}^2} = \frac{B^2}{m} + \sum_{a=1}^{A} K_a^2 \frac{W_{2a}^2}{m\bar{n}_a} + \sum_{a=1}^{A}\sum_{b=1}^{B} K_{ab}^2 \frac{W_{3ab}^2}{m\bar{n}_a \bar{\bar{q}}_{ab}} \tag{1.22}$$

where $K_a = t_{U_a} / t_{NSR}$, $K_{ab} = t_{U_{ab}} / t_{NSR}$, and $t_{U_a}, t_{U_{ab}}$ are defined similarly to those in Eq. (1.15) but use terms specific to the NSR PSUs. $B^2$, $W_{2a}^2$, and $W_{3ab}^2$ are defined in Section 2.3.2. The relvariance in (1.22) can also be written in terms of two measures of homogeneity:

$$\frac{\mathrm{V}\left(\hat{t}_{pwr,NSR}\right)}{t_{NSR}^2} = \frac{\tilde{V}}{m} k_1 \delta_1 + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{m\bar{n}_a} k_{2a}\delta_{2a} + \sum_{a=1}^{A}\sum_{b=1}^{B} K_{ab}^2 \frac{W_{3ab}^2}{m\bar{n}_a \bar{\bar{q}}_{ab}} \tag{1.23}$$

In the case where there are no *b* strata, this relvariance can be written as

$$\frac{\mathrm{V}\left(\hat{t}_{pwr,NSR}\right)}{t_{NSR}^2} = \frac{\tilde{V}}{m\bar{n}_+ \bar{\bar{q}}_+} \left\{ k_1 \delta_1 \bar{n}_+ \bar{\bar{q}}_+ + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{\tilde{V}} \frac{\bar{n}_+}{\bar{n}_a} \frac{\bar{\bar{q}}_+}{\bar{\bar{q}}_a} k_{2a} \left[1 + \delta_{2a}\left(\bar{\bar{q}}_a - 1\right)\right] \right\} \tag{1.24}$$

where $\tilde{V}$, $\tilde{V}_a$, $k_1$, $\delta_1$, $k_{2a}$, and $\delta_{2a}$ are defined similarly to those in Section 2.3.3 but use terms specific to the NSR PSUs.

*Self-Representing (SR) PSUs*

In this section, we use some of the same formulas in Section 2.3.2 and Section 2.3.3, but we restrict the calculations to the set of SR PSUs. As for the variance formula for the NSR PSUs, *SR* subscripts are not added below to simplify the notation. Restrict $U$ (and all alike indices) to the set of all SR PSUs. Here we treat each SR PSUs as a stratum and define:

$\bar{n}_a =$ average number of sample SSUs selected from SSU stratum $a$, across all SR PSUs

$\bar{\bar{q}}_{ab} =$ average number of sample HUs selected from SSU stratum $a$, HU substratum $b$ across all SR PSUs

$Q_{iajb} =$ total number of HUs in SR PSU $i$, SSU stratum $a$, SSU $j$, HU substratum $b$ in the population

$Q_{iaj} =$ total number of HUs in SR PSU $i$, SSU stratum $a$, SSU $j$ across all HU substratum $b$ in the population

$Q_a =$ total number of SR PSUs in SSU stratum $a$ in the population

Then the relvariance formula corresponding to Equation (1.16) for SR PSUs is

$$\frac{\mathrm{V}\left(\hat{t}_{pwr,SR}\right)}{t_{SR}^2} = \sum_{a=1}^{A} K_a^2 \frac{W_{2a}^2}{\bar{n}_a} + \sum_{a=1}^{A}\sum_{b=1}^{B} K_{ab}^2 \frac{W_{3ab}^2}{\bar{n}_a \bar{\bar{q}}_{ab}} \tag{1.25}$$

where $K_a = t_{U_a} / t_{SR}$, $K_{ab} = t_{U_{ab}} / t_{SR}$, and $t_{U_a}, t_{U_{ab}}$ are defined similarly to those in Eq. (1.15) but use terms specific to the SR PSUs. Also,

$$W_{2a}^2 = \frac{1}{t_{U_a}^2} \sum_{i \in U_{SR}} S_{U2(pwr)ia}^2 \text{, and}$$

$$W_{3ab}^2 = \frac{1}{t_{U_{ab}}^2} \sum_{i \in U_{SR}} \sum_{j \in U_{ia}} \frac{Q_{iajb}^2 S_{U3iajb}^2}{p_{j|ia}}$$

where $S_{U2(pwr)ia}^2$ and $S_{U3iajb}^2$ are defined as in Section 2.3.2.

The relvariance in (1.25) can also be written in terms of two measures of homogeneity:

$$\frac{\mathrm{V}\left(\hat{t}_{pwr,SR}\right)}{t_{SR}^2} = \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{\bar{n}_a} k_{2a}\delta_{2a} + \sum_{a=1}^{A}\sum_{b=1}^{B} K_{ab}^2 \frac{W_{3ab}^2}{\bar{n}_a \bar{\bar{q}}_{ab}} \tag{1.26}$$

with

$$\tilde{V}_a = \frac{1}{\bar{y}_{U_a}^2} \sum_{i\in U_{SR}} \sum_{j\in U_{ia}} \sum_{b=1}^{B} \sum_{k\in U_{iajb}} \frac{1}{Q_a - 1}\left(y_k - \bar{y}_{U_a}\right)^2, \text{ and}$$

$$W_{3a}^2 = \frac{1}{t_{U_a}^2} \sum_{i\in U_{SR}} \sum_{j\in U_{ia}} \frac{Q_{iaj}^2 S_{U3iaj}^2}{p_{j|ia}} .$$

The measures of homogeneity for SR PSUs are defined as

$$k_{2a} = \frac{W_{2a}^2 + W_{3a}^2}{\tilde{V}_a} \quad \text{and} \quad \delta_{2a} = \frac{W_{2a}^2}{W_{2a}^2 + W_{3a}^2} .$$

When there are no *b* strata for HUs, the relvariance in Equation (1.25) can also be written in terms of a single measure of homogeneity

$$\frac{\mathrm{V}\left(\hat{t}_{pwr,SR}\right)}{t_{SR}^2} = \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{\bar{n}_a \bar{\bar{q}}_a} k_{2a}\left[1 + \delta_{2a}\left(\bar{\bar{q}}_a - 1\right)\right] \tag{1.27}$$

Note that the summations above are restricted to the *SR* PSUs.

## 2.4 Estimating Variance Components and Measures of Homogeneity

In this section, we present direct estimates of the variance components in Eq.(1.16), Eq. (1.22), and Eq. (1.25), as well as plug-in estimators for the measures of homogeneity in Eq. (1.17) which can be made from the sample. Two alternative estimation methods will be studied: (1) design-based ANOVA and (2) anticipated variance (model + design).

### 2.4.1 Design-based ANOVA Variance Component Estimation

#### 2.4.1.1 General Case

The following design-based variance component estimators are extensions of ones in Hansen et al (1953) for the case of *ppswr/ppswr/srswor*. These are generally referred to as ANOVA estimators because of their similarity to standard analysis of variance estimators. In this general case, we cover a design in which PSUs are not divided into SRs and NSRs. Subsequent to the general case, we discuss a design like the HRS which has SR and NSR PSUs.

First define,

$\bar{y}_{s_{iajb}} = \dfrac{\sum_{k \in s_{iajb}} y_k}{q_{iajb}}$, the sample mean of HUs in HU substratum $b \,|\, iaj$

$\hat{t}_{iajb} = \hat{Q}_{iajb} \bar{y}_{s_{iajb}}$, the estimated total for HU substratum $b \,|\, iaj$; $\hat{Q}_{iajb} = \sum\limits_{k \in s_{iajb}} w_{k|iajb}$

estimated number of HUs in HU substratum $b \,|\, iaj$ in the population;

$w_{k|iajb} = \dfrac{1}{\pi_{k|iajb}}$, the weight for HU $k \mid iajb$

$\hat{t}_{iaj} = \sum_b \hat{t}_{iajb}$, the estimated total for SSU $j \,|\, ia$

$\hat{t}_{ia\,(pwr)} = \dfrac{1}{n_{ia}} \sum_{j \in s_{ia}} \dfrac{\hat{t}_{iaj}}{p_{j|ia}}$, the estimated total for SSU stratum $a \,|\, i$

$\hat{t}_{i(pwr)} = \sum_a \hat{t}_{ia\,(pwr)}$, the estimated total for PSU $i$

$\hat{t}_{a\,(pwr)} = \dfrac{1}{m} \sum_{i \in s_1} \dfrac{\hat{t}_{ia\,(pwr)}}{p_i}$, the estimated total for SSU stratum *a*, across all PSUs,

$\hat{t}_{iab\,(pwr)} = \dfrac{1}{n_{ia}} \sum_{j \in s_{ia}} \dfrac{\hat{t}_{iajb}}{p_{j|ia}}$, the estimated total for HU substratum $b \,|\, ia$ across all SSUs

$\hat{t}_{ab\,(pwr)} = \dfrac{1}{m} \sum_{i \in s_1} \dfrac{\hat{t}_{iab(pwr)}}{p_i}$, the estimated total for SSU stratum *a*, HU substratum *b*, across all PSUs

$\hat{Q}_{iaj} = \sum_{b} \sum_{k \in s_{iajb}} \dfrac{1}{n_{ia} p_{j|ia}} \dfrac{1}{\pi_{k|iajb}}$, the estimated number of HUs in SSU $j|ia$

$\hat{Q}_{ia} = \sum_{j \in s_{ia}} \dfrac{1}{n_{ia} p_{j|ia}} \sum_{b} \sum_{k \in s_{iajb}} \dfrac{1}{\pi_{k|iajb}}$, the estimated number of HUs in SSU stratum $a|i$

$\hat{Q}_{a} = \sum_{i \in s_1} \dfrac{1}{mp_i} \sum_{j \in s_{ia}} \dfrac{1}{n_{ia} p_{j|ia}} \sum_{b} \sum_{k \in s_{iajb}} \dfrac{1}{\pi_{k|iajb}}$, the estimated number of HUs in SSU stratum *a* across all PSUs

$\hat{Q} = \sum_{i \in s_1} \dfrac{1}{mp_i} \sum_{a} \sum_{j \in s_{ia}} \dfrac{1}{n_{ia} p_{j|ia}} \sum_{b} \sum_{k \in s_{iajb}} \dfrac{1}{\pi_{k|iajb}}$, the estimated number of HUs across all PSUs

$\hat{S}^2_{3iajb} = \dfrac{1}{q_{iajb} - 1} \sum_{k \in s_{iajb}} \left( y_k - \bar{y}_{s_{iajb}} \right)^2$, the sample variance among HUs in HU substratum $b \,|\, iaj$

$\hat{V}_{3iajb} = \frac{\hat{Q}_{iajb}^2}{q_{iajb}} \hat{S}_{3iajb}^2$, the estimated variance of the estimated total $\hat{t}_{iajb}$ for HU substratum $b \mid iaj$ assuming that the sampling fraction $q_{iajb} / Q_{iajb}$ in *iajb* is small; $\hat{Q}_{iajb}$ and $q_{iajb}$ are based on all sample HUs while $\hat{S}_{3iajb}^2$ is based on HUs with nonmissing data.

$\hat{V}_{3iaj} = \sum_b \hat{V}_{3iajb} = \sum_b \frac{\hat{Q}_{iajb}^2}{q_{iajb}} \hat{S}_{3iajb}^2$, the estimated variance of the estimated total for SSU $j \mid ia$ in a stratified *srswor* with a negligible sampling fraction in each HU stratum

$\hat{S}_{2\mathbb{A}ia}^2 = \frac{1}{n_{ia} - 1} \sum_{j \in s_{ia}} \left( \frac{\hat{t}_{iaj}}{p_{j|ia}} - \hat{t}_{ia(pwr)} \right)^2$, the sample variance among estimated SSU totals in SSU strata $a \mid i$

$$\hat{S}_{2\mathbb{B}ia}^2 = \frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{\hat{V}_{3iaj}}{p_{j|ia}^2}$$

$$\hat{S}_{2(pwr)ia}^2 = \hat{S}_{2\mathbb{A}ia}^2 - \hat{S}_{2\mathbb{B}ia}^2$$

$\hat{S}_{1(pwr)\mathbb{A}}^2 = \frac{1}{m-1} \sum_{i \in s_1} \left( \frac{\hat{t}_{i(pwr)}}{p_i} - \hat{t}_{pwr} \right)^2$, the sample variance among estimated PSU totals

$$\hat{S}_{1(pwr)\mathbb{B}}^2 = \frac{1}{m} \sum_{i \in s_1} \frac{1}{p_i^2} \sum_a \frac{\hat{S}_{2\mathbb{A}ia}^2}{n_{ia}}$$

$$\hat{S}_{1(pwr)}^2 = \hat{S}_{1(pwr)\mathbb{A}}^2 - \hat{S}_{1(pwr)\mathbb{B}}^2$$

where $\hat{S}_{1(pwr)}^2$ estimates $S_{U1(pwr)}^2$, $\hat{S}_{2(pwr)ia}^2$ estimates $S_{U2(pwr)\text{ia}}^2$, and $\hat{S}_{3iajb}^2$ estimates $S_{U3iajb}^2$.

Then the estimators of $V_{PSU}$, $V_{SSU}$, and $V_{HU}$ in Eq. (1.11) are

$$v_{PSU} = \frac{\hat{S}^2_{1(pwr)}}{m}$$

$$v_{SSU} = \sum_{i \in s_1} \frac{1}{(mp_i)^2} \sum_a \frac{1}{n_{ia}} \hat{S}^2_{2(pwr)ia}$$

$$v_{HU} = \sum_{i \in s_1} \frac{1}{(mp_i)^2} \sum_a \sum_{j \in s_{ia}} \frac{1}{\left(n_{ia} p_{j|ia}\right)^2} \sum_b \hat{V}_{3iajb}$$

(1.28)

and the relvariance of the *pwr* estimator is estimated by

$$\frac{v\left(\hat{t}_{pwr}\right)}{\hat{t}^2_{pwr}} = \frac{1}{\hat{t}^2_{pwr}} \left(v_{PSU} + v_{SSU} + v_{HU}\right).$$

Now assuming that for each PSU $i$, $n_{ia} = \bar{n}_a$, and for every PSU/SSU $ij$ combination, $q_{iajb} = \bar{\bar{q}}_{ab}$, the estimated ANOVA relvariance of $\hat{t}_{pwr}$ can be written as:

$$\frac{v\left(\hat{t}_{pwr}\right)}{\hat{t}^2_{pwr}} = \frac{\hat{B}^2}{m} + \sum_{a=1}^{A} \hat{K}^2_a \frac{\hat{W}^2_{2a}}{m\bar{n}_a} + \sum_{a=1}^{A} \sum_{b=1}^{B} \hat{K}^2_{ab} \frac{\hat{W}^2_{3ab}}{m\bar{n}_a \bar{\bar{q}}_{ab}} \qquad (1.29)$$

where $\hat{K}_a = \hat{t}_{a(pwr)} \big/ \hat{t}_{pwr}$, $\hat{K}_{ab} = \hat{t}_{ab(pwr)} \big/ \hat{t}_{pwr}$,

$$\hat{B}^2 = \frac{\hat{S}^2_{1(pwr)}}{\hat{t}^2_{pwr}}$$

$$\hat{W}^2_{2a} = \frac{1}{\hat{t}^2_{a(pwr)}} \sum_{i \in s_1} \frac{\hat{S}^2_{2(pwr)ia}}{mp_i^2}$$

$$\hat{W}^2_{3ab} = \frac{1}{\hat{t}^2_{ab(pwr)}} \left\{ \sum_{i \in s_1} \frac{1}{mp_i^2} \sum_{j \in s_{ia}} \frac{1}{n_{ia} p^2_{j|ia}} \hat{Q}^2_{iajb} \hat{S}^2_{3iajb} \right\}$$

The above estimators $\hat{B}^2$, $\hat{W}_{2a}^2$, and $\hat{W}_{3ab}^2$ estimate the corresponding components $B^2$, $W_{2a}^2$, and $W_{3ab}^2$ of Eq. (1.16). (Note that the estimator $\hat{W}_{3ab}^2$ applies whether or not $n_{ia} = \bar{n}_a$. We also note that $\hat{B}^2$ and $\hat{W}_{2a}^2$ can be negative because of the subtraction term that occurs in the sample variances. However, using anticipated variances may help correct this problem.) Using plug-in estimators for the measures of homogeneity in Eq. (1.17), the relvariance is estimated by

$$\frac{v\left(\hat{t}_{pwr}\right)}{\hat{t}_{pwr}^2} = \frac{\hat{\tilde{V}}}{m}\hat{k}_1\hat{\delta}_1 + \sum_{a=1}^{A}\hat{K}_a^2\frac{\tilde{V}_a}{m\bar{n}_a}\hat{k}_{2a}\hat{\delta}_{2a} + \sum_{a=1}^{A}\sum_{b=1}^{B}\hat{K}_{ab}^2\frac{\hat{W}_{3ab}^2}{m\bar{n}_a\bar{\bar{q}}_{ab}} \tag{1.30}$$

The plug-in estimators of the measures of homogeneity of Eq. (1.18) are

$$\hat{\delta}_1 = \frac{\hat{B}^2}{\hat{B}^2 + \hat{W}^2}, \quad \hat{k}_1 = \frac{\hat{B}^2 + \hat{W}^2}{\hat{\tilde{V}}}$$

$$\hat{\delta}_{2a} = \frac{\hat{W}_{2a}^2}{\hat{W}_{2a}^2 + \hat{W}_{3a}^2}, \quad \hat{k}_{2a} = \frac{\hat{W}_{2a}^2 + \hat{W}_{3a}^2}{\hat{\tilde{V}}_a} \tag{1.31}$$

where

$$\hat{W}^2 = \frac{1}{\hat{t}_{pwr}^2}\sum_{i\in s_1}\frac{\hat{Q}_i^2\hat{S}_{3i}^2}{mp_i^2}, \qquad \hat{S}_{3i}^2 = \left(\sum_a\sum_{j\in s_{ia}}\sum_b\sum_{k\in s_{iajb}} w_{k|i}\right)^{-1}\sum_a\sum_{j\in s_{ia}}\sum_b\sum_{k\in s_{iajb}} w_{k|i}\left(y_k - \hat{\bar{y}}_i\right)^2,$$

$$\hat{\bar{y}}_i = \frac{\sum_a\sum_{j\in s_{ia}}\sum_b\sum_{k\in s_{iajb}} w_{k|i}y_k}{\sum_a\sum_{j\in s_{ia}}\sum_b\sum_{k\in s_{iajb}} w_{k|i}}, \quad w_{k|i} = \frac{1}{n_{ia}p_{j|ia}}\frac{1}{\pi_{k|iajb}}$$

$$\hat{W}_{3a}^2 = \frac{1}{\hat{t}_{a(pwr)}^2}\left\{\sum_{i \in s_1} \frac{1}{mp_i^2} \sum_{j \in s_{ia}} \frac{1}{n_{ia} p_{j|ia}^2} \hat{Q}_{iaj}^2 \hat{S}_{3iaj}^2\right\},$$

$$\hat{S}_{3iaj}^2 = \left(\sum_b \sum_{k \in s_{iajb}} w_{k|iajb}\right)^{-1} \sum_b \sum_{k \in s_{iajb}} w_{k|iajb}\left(y_k - \hat{\bar{y}}_{s_{iaj}}\right)^2, \quad \text{with } \hat{\bar{y}}_{s_{iaj}} = \frac{\sum_b \sum_{k \in s_{iajb}} w_{k|iajb} y_k}{\sum_b \sum_{k \in s_{iajb}} w_{k|iajb}},$$

$$\hat{\hat{V}} = \frac{q}{q-1}\left(\sum_{i \in s_1} \sum_a \sum_{j \in s_{ia}} \sum_b \sum_{k \in s_{iajb}} w_k - 1\right)^{-1} \sum_{i \in s_1} \sum_a \sum_{j \in s_{ia}} \sum_b \sum_{k \in s_{iajb}} \left\{w_k \left(y_k - \hat{\bar{y}}_{s_1}\right)^2 \Big/ \hat{\bar{y}}_{s_1}^2\right\} \quad \text{with}$$

$$\hat{\bar{y}}_{s_1} = \frac{\sum_{i \in s_1} \sum_a \sum_{j \in s_{ia}} \sum_b \sum_{k \in s_{iajb}} w_k y_k}{\sum_{i \in s_1} \sum_a \sum_{j \in s_{ia}} \sum_b \sum_{k \in s_{iajb}} w_k}$$

, $q$ is the total number of HUs in the sample, and

$$\hat{\hat{V}}_a = \frac{q_a}{q_a - 1}\left(\sum_{i \in s_1} \sum_{j \in s_{ia}} \sum_{k \in s_{iaj}} w_k - 1\right)^{-1} \sum_{i \in s_1} \sum_{j \in s_{ia}} \sum_{k \in s_{iaj}} \left\{w_k \left(y_k - \hat{\bar{y}}_{s_a}\right)^2 \Big/ \hat{\bar{y}}_{s_a}^2\right\} \text{ with}$$

$$\hat{\bar{y}}_{s_a} = \frac{\sum_{i \in s_1} \sum_{j \in s_{ia}} \sum_{k \in s_{iaj}} w_k y_k}{\sum_{i \in s_1} \sum_{j \in s_{ia}} \sum_{k \in s_{iaj}} w_k}$$

, and $q_a$ is the total number of HUs in SSU stratum $a$ in the sample.

When there are no $b$ strata and using plug-in estimators for the measures of homogeneity in Eq. (1.19), the relvariance is estimated by

$$\frac{v\left(\hat{t}_{pwr}\right)}{\hat{t}_{pwr}^2} = \frac{\hat{\hat{V}}}{m\bar{n}_+ \bar{\bar{q}}_+}\left\{\hat{k}_1 \hat{\delta}_1 \bar{n}_+ \bar{\bar{q}}_+ + \sum_{a=1}^{A} \hat{K}_a^2 \frac{\hat{\hat{V}}_a}{\hat{\hat{V}}} \frac{\bar{n}_+}{\bar{n}_a} \frac{\bar{\bar{q}}_+}{\bar{\bar{q}}_a} \hat{k}_{2a}\left[1 + \hat{\delta}_{2a}\left(\bar{\bar{q}}_a - 1\right)\right]\right\} \quad (1.32)$$

where

$\bar{n}_+ = \sum_{a=1}^{A} \bar{n}_a$ is the number of sample SSUs allocated and

$\bar{\bar{q}}_+ = \sum_{a=1}^{A} \bar{n}_a \bar{\bar{q}}_a \Big/ \sum_{a=1}^{A} \bar{n}_a$ is the mean number of sample elements (HUs) per SSU across all SSU strata.

Estimation when there are no *a* or *b* strata is also a special case of Eq. (1.30) and is not shown here.

### 2.4.1.2 Handling Non Self-Representing (NSR) PSUs in the HRS Design

The estimator of the relvariance in Equation (1.29) can be written as a function of the self-representing and non self-representing PSUs:

$$\frac{v\left(\hat{t}_{pwr}\right)}{\hat{t}_{pwr}^2} = \hat{F}_{SR}^2 \frac{v\left(\hat{t}_{pwr,SR}\right)}{\hat{t}_{pwr,SR}^2} + \hat{F}_{NSR}^2 \frac{v\left(\hat{t}_{pwr,NSR}\right)}{\hat{t}_{pwr,NSR}^2} \tag{1.33}$$

where $\hat{F}_{SR}^2 = \left(\frac{\hat{t}_{pwr,SR}}{\hat{t}_{pwr}}\right)^2$ and $\hat{F}_{NSR}^2 = \left(\frac{\hat{t}_{pwr,NSR}}{\hat{t}_{pwr}}\right)^2 = 1 - \hat{F}_{SR}^2$ .

The design-based variance components formulas for NSR PSUs will be the exact form of Eqs. (1.29)-(1.32). The only distinction is that the sample is now restricted to the sample of NSR PSUs and their SSUs and HUs that are within NSR PSUs only, such that $m$, $\bar{n}_a$ , $\bar{\bar{q}}_{ab}$, and $p_i$ are now specific to the NSR PSUs.

Assuming that for each PSU *i*, $n_{ia} = \bar{n}_a$ and for every PSU/SSU *ij* combination, $q_{iajb} = \bar{\bar{q}}_{ab}$, the estimated ANOVA relvariance of $\hat{t}_{pwr,NSR}$ in Eq. (1.16) can be written as:

$$\frac{v\left(\hat{t}_{pwr,NSR}\right)}{\hat{t}_{pwr,NSR}^2} = \frac{\hat{B}^2}{m} + \sum_{a=1}^{A} \hat{K}_a^2 \frac{\hat{W}_{2a}^2}{m\bar{n}_a} + \hat{K}_{ab}^2 \sum_{a=1}^{A} \sum_{b=1}^{B} \frac{\hat{W}_{3ab}^2}{m\bar{n}_a \bar{\bar{q}}_{ab}} \tag{1.34}$$

where $\hat{K}_a = \hat{t}_{a(pwr)} \big/ \hat{t}_{pwr,NSR}$, $\hat{K}_{ab} = \hat{t}_{ab(pwr)} \big/ \hat{t}_{pwr,NSR}$, and $\hat{t}_{a(pwr)}$, $\hat{t}_{ab(pwr)}$ are defined similarly to those in Section 2.4.1.1 but use terms specific to the NSR PSUs. $\hat{B}^2$, $\hat{W}_{2a}^2$, and $\hat{W}_{3ab}^2$, are defined similarly to those in Eq. (1.29) but use terms specific to the NSR PSUs. *NSR* subscripts could be used on $\hat{B}^2$ and other terms, but we omit those to simply the notation.

Using plug-in estimators for the measures of homogeneity in Eq. (1.17), the relvariance is estimated by

$$\frac{v\left(\hat{t}_{pwr,NSR}\right)}{\hat{t}_{pwr,NSR}^2} = \frac{\hat{\tilde{V}}}{m}\hat{k}_1\hat{\delta}_1 + \sum_{a=1}^{A}\frac{\tilde{V}_a}{m\bar{n}_a}\hat{k}_{2a}\hat{\delta}_{2a} + \sum_{a=1}^{A}\sum_{b=1}^{B}\frac{\hat{W}_{3ab}^2}{m\bar{n}_a\bar{\bar{q}}_{ab}} \tag{1.35}$$

where $\hat{\tilde{V}}$, $\hat{\tilde{V}}_a$, $\hat{k}_1$, $\hat{\delta}_1$, $\hat{k}_{2a}$, and $\hat{\delta}_{2a}$ are defined similarly to those in Section 2.4.1.1 but use terms specific to the NSR PSUs.

### 2.4.1.3 Handling Self-Representing (SR) PSUs in the HRS Design

In this section, we again use some of the same formulas in Section 2.3.2 -2.3.3 but we restrict the calculations to the set of SR PSUs. As in the previous section, we omit *SR* subscripts to simply the notation.

Assuming that for each SR PSU *i*, $n_{ia} = \bar{n}_a$ and for every PSU/SSU *ij* combination, $q_{iajb} = \bar{\bar{q}}_{ab}$, the estimated ANOVA relvariance of $\hat{t}_{pwr,SR}$ in Eq. (1.16) can be written as:

$$\frac{v\left(\hat{t}_{pwr,SR}\right)}{\hat{t}_{pwr,SR}^2} = \sum_{a=1}^{A}\hat{K}_a^2\frac{\hat{W}_{2a}^2}{\bar{n}_a} + \sum_{a=1}^{A}\sum_{b=1}^{B}\hat{K}_{ab}^2\frac{\hat{W}_{3ab}^2}{\bar{n}_a\bar{\bar{q}}_{ab}} \tag{1.36}$$

where $\hat{K}_a = \hat{t}_{a(pwr)} \big/ \hat{t}_{pwr,SR}$, $\hat{K}_{ab} = \hat{t}_{ab(pwr)} \big/ \hat{t}_{pwr,SR}$, and $\hat{t}_{a(pwr)}$, $\hat{t}_{ab(pwr)}$ are defined similarly to those in Section 2.4.1.1 but use terms specific to the SR PSUs. Also,

$$\hat{W}_{2a}^{2} = \frac{1}{\hat{t}_{a(pwr)}^{2}} \sum_{i \in s_{1,SR}} \hat{S}_{2(pwr)ia}^{2} \text{ and } \hat{W}_{3ab}^{2} = \frac{1}{\hat{t}_{ab(pwr)}^{2}} \sum_{i \in s_{1,SR}} \sum_{j \in s_{ia}} \frac{1}{n_{ia} p_{j|ia}^{2}} \hat{Q}_{iajb}^{2} \hat{S}_{3iajb}^{2}$$

where $\hat{S}_{2(pwr)ia}^{2}$ , $\hat{S}_{3iajb}^{2}$ are defined as in Section 2.4.1. Note that the SR PSUs are treated as strata in the formulas for $\hat{W}_{2a}^{2}$ and $\hat{W}_{3ab}^{2}$ so that a PSU weight is not included. Using plug-in estimators for the measures of homogeneity in Eq. (1.17), the relvariance is estimated by

$$\frac{v\left(\hat{t}_{pwr,SR}\right)}{\hat{t}_{pwr,SR}^{2}} = \sum_{a=1}^{A} \hat{K}_{a}^{2} \frac{\hat{\tilde{V}}_{a}}{\bar{n}_{a}} \hat{k}_{2a} \hat{\delta}_{2a} + \sum_{a=1}^{A} \sum_{b=1}^{B} \hat{K}_{ab}^{2} \frac{\hat{W}_{3ab}^{2}}{\bar{n}_{a} \bar{\bar{q}}_{ab}} \tag{1.37}$$

And the plug-in estimators of the measures of homogeneity are

$$\hat{\delta}_{2a} = \frac{\hat{W}_{2a}^{2}}{\hat{W}_{2a}^{2} + \hat{W}_{3a}^{2}} \ , \ \hat{k}_{2a} = \frac{\hat{W}_{2a}^{2} + \hat{W}_{3a}^{2}}{\hat{\tilde{V}}_{a}} \tag{1.38}$$

$\hat{W}_{3a}^{2} = \frac{1}{\hat{t}_{a(pwr)}^{2}} \left\{ \sum_{i \in s_{1,SR}} \sum_{j \in s_{ia}} \frac{1}{n_{ia} p_{j|ia}^{2}} \hat{Q}_{iaj}^{2} \hat{S}_{3iaj}^{2} \right\}$ , $\hat{S}_{3iaj}^{2}$ defined as in Section 2.4.1.

$\hat{S}_{3iaj}^{2} = \left( \sum_{b} \sum_{k \in s_{iajb}} w_{k|iajb} \right)^{-1} \sum_{b} \sum_{k \in s_{iajb}} w_{k|iajb} \left( y_k - \hat{\bar{y}}_{s_{iaj}} \right)^2 , \quad \hat{\bar{y}}_{s_{iaj}} = \frac{\sum_{b} \sum_{k \in s_{iajb}} w_{k|iajb} y_k}{\sum_{b} \sum_{k \in s_{iajb}} w_{k|iajb}}$, and

$\hat{\tilde{V}}_{a} = \frac{q_a}{q_a - 1} \left( \sum_{i \in s_{1,SR}} \sum_{j \in s_{ia}} \sum_{k \in s_{iaj}} w_k - 1 \right)^{-1} \sum_{i \in s_{1,SR}} \sum_{j \in s_{ia}} \sum_{k \in s_{iaj}} \left\{ w_k \left( y_k - \hat{\bar{y}}_{s_a} \right)^2 \Big/ \hat{\bar{y}}_{s_a}^{2} \right\}$ using terms specific to the SR PSUs.

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### 2.4.2 Anticipated Variances

In this section, we discuss the super population model used for the HRS data and the computation of anticipated variances.

#### 2.4.2.1 Superpopulation Model

Another way to circumvent the potential negative $\hat{B}^2$ and $\hat{W}_{3ab}^2$ terms from the estimators of design-based variances is to use the anticipated variance, i.e., the variance expected or anticipated under a certain model. If the model holds for the population and a sample is selected from it, existing non-survey software can be used to estimate model variance components to help stabilize the estimates. Being able to make use of available variance estimation software is desirable for the ease of estimation. The anticipated variance (Isaki & Fuller, 1982) is defined as

$$\begin{aligned} AV\left(\hat{t}_{pwr}\right) &= E_M\left[E\left\{\left(\hat{t}_{pwr}-t_U\right)^2\right\}\right]-\left[E_M\left\{E\left(\hat{t}_{pwr}-t_U\right)\right\}\right]^2 \\ &= E_M\left[var\left(\hat{t}_{pwr}-t_U\right)\right] \end{aligned}$$

where $\hat{t}_{pwr}$ is an unbiased *pwr* estimator of the population total $t_U$.

Consider a model for $y_k$ with common mean, $\mu$, and random effects for PSUs, $\alpha_i$, SSUs, $\gamma_{iaj}$, and HUs in SSU/HU substratum *ab*, $\varepsilon_{iajbk}$:

$$y_k = \mu + \alpha_i + \gamma_{iaj} + \varepsilon_{iajbk} \tag{1.39}$$

with $\alpha_i \sim \left(0\,,\,\sigma_\alpha^2\right)$, $\gamma_{iaj} \sim \left(0\,,\,\sigma_\gamma^2\right)$, $\varepsilon_{iajbk} \sim \left(0\,,\,\sigma_{\varepsilon_{ab}}^2\right)$ and the errors being independent, such that

$$Var_M\left(y_k\right) = \sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_{\varepsilon_{ab}}^2 \quad \text{and} \quad E_M\left(y_k\right) = \mu \quad \text{for} \quad k \in U_{iajb}\,.$$

The model in Eq. (1.39) seeks to estimate the variance between HUs based on SSU stratum/HU substratum *ab*. There are other mixed models to consider. Still keeping PSUs, SSUs, and HUs as random effects, we could experiment with which fixed effects (SSU strata, HU substrata) may fit the model better. It is possible to use only the SSU strata or only the HU substrata to model the residuals. We will show in Section 3, the model with both *ab* strata fits the HRS data better than the model with only *a* or *b* strata 60 percent of the time. Hence, we use the model shown in Eq. (1.39) to get variance component estimates.

For other datasets different mixed models may be more appropriate. For example, demographic factors (HH composition—married, single, education level of persons in the HH, and so on) may be predictive of *y*'s collected in a household survey. However, these may not be available in advance to use for sample design. Consequently, aggregate-level covariates for PSUs and SSUs are likely to be the most practical to use when designing a survey.

#### 2.4.2.2 Model Expectations of Design-Based Variance Components

The model expectation of the design-based variance can be computed under the above model, but for the sample allocation we only need the approximate expectations of $B^2$, $W_{2a}^2$, $W_{3ab}^2$. Assume that there are a large number of SSUs, $N_{ia}$, in every PSU/SSU stratum *ia* combination so that, $N_{ia} \approx N_{ia} - 1$. Then the model expectations of $B^2$, $W_{2a}^2$, $W_{3ab}^2$, $W^2$, and $W_{3a}^2$ from Eqs. (1.16) and (1.18) can be found as follows (see Appendices A.4.5 - A.4.9 for derivation):

$$E_M\left(t_U^2 B^2\right) \doteq \sigma_\alpha^2 M^2 \bar{Q}^2 \left( \frac{v_{Q(pwr)}^2}{M^2} + 1 \right) + \sigma_\gamma^2 \sum_{i \in U} \frac{1}{p_i} \left\{ \sum_{a=1}^{A} N_{ia} \bar{Q}_{ia}^2 \left( v_{Q_{ia}}^2 + 1 \right) + \sum_{a=1}^{A} \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 Q_{ia\bullet b} \right\} + \mu^2 \bar{Q}^2 v_{Q(pwr)}^2 \tag{1.40}$$

$$E_M\left(t_{U_a}^2 W_{2a}^2\right) \doteq \sum_{i \in U} \frac{1}{p_i} \left\{ \left( \mu^2 + \sigma_\alpha^2 \right) \bar{Q}_{ia}^2 v_{Q_{ia(pwr)}}^2 + \sigma_\gamma^2 N_{ia}^2 \bar{Q}_{ia}^2 \left( \frac{v_{Q_{ia(pwr)}}^2}{N_{ia}^2} - \frac{v_{Q_{ia}}^2}{N_{ia}} + 1 \right) + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \left[ \sum_{j \in U_{ia}} \frac{Q_{iajb}}{p_{j|ia}} - Q_{ia\bullet b} \right] \right\} \tag{1.41}$$

$$E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right) = \sum_{i \in U} \frac{1}{p_i} \sum_{j \in U_{ia}} \frac{1}{p_{j|ia}} \sigma_{\varepsilon_{ab}}^2 Q_{iajb}^2 \tag{1.42}$$

$$E_M\left(t_U^2 W^2\right) \doteq \sum_{i \in U} \frac{Q_i}{p_i} \sum_{a=1}^{A} N_{ia} \left\{ \sigma_\gamma^2 \bar{Q}_{ia} \left[ 1 - \frac{\bar{Q}_{ia}}{Q_i} \left( v_{Q_{ia}}^2 + 1 \right) \right] + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \bar{Q}_{ia\bullet b} \right\} \tag{1.43}$$

$$E_M\left(t_{U_a}^2 W_{3a}^2\right) = \sum_{i \in U} \frac{1}{p_i} \sum_{j \in U_{ia}} \frac{Q_{iaj}}{p_{j|ia}} \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 Q_{iajb} \tag{1.44}$$

where

$t_U^2 = \bar{y}_U^2 \left( M\bar{N}\bar{\bar{Q}} \right)^2$, $\bar{N} = \sum_{i \in U} \sum_{a} \frac{N_{ia}}{M}$ is the mean number of SSUs in the population per PSU, and $\bar{\bar{Q}} = \sum_{i \in U} \frac{Q_i}{M\bar{N}}$ is the mean number of HUs per SSU in the population;

$t_{U_a} = \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} y_k$ and $t_{U_{ab}} = \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{k \in U_{iajb}} y_k$;

$v_{Q(pwr)}^2 = \frac{S_{Q(pwr)}^2}{\bar{Q}^2}$ is the unit relvariance of PSU sizes $Q_i$. When PSUs are selected using *ppswr*, $S_{Q(pwr)}^2 = \sum_{i \in U} p_i \left( \frac{Q_i}{p_i} - Q \right)^2$, $\bar{Q} = \sum_{i \in U} \frac{Q_i}{M}$ is the mean number of HUs per PSU, and $Q = \sum_{i \in U} Q_i$;

$v_{Q_{ia(pwr)}}^2 = \frac{S_{Q_{ia(pwr)}}^2}{\bar{Q}_{ia}^2}$ is the unit relvariance among SSU counts of HUs within SSU stratum *a*. When SSUs are selected using *ppswr*, $S_{Q_{ia(pwr)}}^2 = \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right)^2$, with $Q_{ia} = \sum_{j \in U_{ia}} Q_{iaj}$;

$v_{Q_{ia}}^2 = \frac{S_{Q_{ia}}^2}{\bar{Q}_{ia}^2}$ is the unit relvariance of SSU sizes $Q_{iaj}$, $S_{Q_{ia}}^2 = \frac{1}{N_{ia} - 1} \sum_{j \in U_{ia}} \left( Q_{iaj} - \bar{Q}_{ia} \right)^2$, with $\bar{Q}_{ia} = \sum_{j \in U_{ia}} \frac{Q_{iaj}}{N_{ia}}$.

### 2.4.2.3 Model Expectations of Measures of Homogeneity

The measures of homogeneity of the design-based variance can also be computed under the model in Eq. (1.39). Assuming $M$, $N_{ia}$, $N_a$, $Q_a$, and $Q$ are large such that $M \approx M-1$, $N_{ia} \approx N_{ia}-1$, $N_a \approx N_a-1$, $Q_a \approx Q_a-1$ and $Q \approx Q-1$, the model expectations of $\tilde{V}$ and $\tilde{V}_a$ from Eq. (1.17) can be found as follows (see Appendices A.4.10 and A.4.11 for derivation):

$$\begin{aligned} E_M\left(\bar{y}_U^2\tilde{V}\right) &\doteq \sigma_\alpha^2\left[1-\frac{1}{M}\left(v_Q^2+1\right)\right]+\sigma_\gamma^2\left[1-\frac{1}{Q^2}\sum_{i\in U}\sum_{a=1}^{A} N_{ia}\bar{Q}_{ia}^2\left(v_{Q_{ia}}^2+1\right)\right] \\ &\quad +\sum_{i\in U}\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\frac{Q_{ia\bullet b}}{Q} \end{aligned} \tag{1.45}$$

$$\begin{aligned} E_M\left(\bar{y}_{U_a}^2\tilde{V}_a\right) &\doteq \sigma_\alpha^2\left\{1-\frac{1}{Q_a^2}M\bar{Q}_{1a}^2\left(v_{Q_{1a}}^2+1\right)\right\}+\sigma_\gamma^2\left\{1-\frac{1}{Q_a^2}N_a\bar{Q}_{2a}^2\left(v_{Q_{2a}}^2+1\right)\right\} \\ &\quad +\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\frac{Q_{ab}}{Q_a} \end{aligned} \tag{1.46}$$

where

$v_Q^2=\dfrac{S_Q^2}{\bar{Q}^2}$ is the unit relvariance of the $Q_i$'s, $S_Q^2=\left(\dfrac{1}{M-1}\right)\sum_{i\in U}\left(Q_i-\bar{Q}\right)^2$, and

$\bar{Q}=\sum_{i\in U}\dfrac{Q_i}{M}$;

$v_{Q_{1a}}^2=S_{Q_{1a}}^2\big/\bar{Q}_{1a}^2$ is the population relvariance of the $Q_{ia}$'s within SSU stratum *a*,

$S_{Q_{1a}}^2=(M-1)^{-1}\sum_{i\in U}\left(Q_{ia}-\bar{Q}_{1a}\right)^2$, and $\bar{Q}_{1a}=M^{-1}\sum_{i\in U}Q_{ia}=Q_a/M$ is the mean number of HUs per PSU in SSU stratum *a*;

$v_{Q_{2a}}^2 = S_{Q_{2a}}^2 / \bar{Q}_{2a}^2$ is the population relvariance of the $Q_{iaj}$ 's within SSU stratum *a*,

$S_{Q_{2a}}^2 = (N_a - 1)^{-1} \sum_{i \in U} \sum_{j \in U_{ia}} \left(Q_{iaj} - \bar{Q}_{2a}\right)^2$, and

$\bar{Q}_{2a} = N_a{}^{-1} \sum_{i \in U} \sum_{j \in U_{ia}} Q_{iaj} = Q_a / N_a$ is the mean number of HUs in SSU stratum *a* across all PSUs;

$Q_{ab} = \sum_{i \in U} \sum_{j \in U_{ia}} Q_{iajb}$ is the number of HUs per SSU stratum/HU substratum *ab* combination across all PSUs.

The model expectations in Eqs. (1.40) - (1.46) can be used to evaluate the model expectations of homogeneity terms, $\delta_1$, $\delta_{2a}$, $k_1$, and $k_{2a}$, below. Assume that the number of PSUs, *M*, is large so that the expectation of a ratio can be approximated as the ratio of expectations.

$$E_M\left(\delta_1\right) \doteq \frac{E_M\left(t_U^2 B^2\right)}{E_M\left(t_U^2 B^2\right) + E_M\left(t_U^2 W^2\right)}, \qquad E_M\left(k_1\right) \doteq \frac{E_M\left(t_U^2 B^2\right) + E_M\left(t_U^2 W^2\right)}{E_M\left(t_U^2 \tilde{V}\right)}$$

$$E_M\left(\delta_{2a}\right) \doteq \frac{E_M\left(t_{U_a}^2 W_{2a}^2\right)}{E_M\left(t_{U_a}^2 W_{2a}^2\right) + E_M\left(t_{U_a}^2 W_{3a}^2\right)}, \quad E_M\left(k_{2a}\right) \doteq \frac{E_M\left(t_{U_a}^2 W_{2a}^2\right) + E_M\left(t_{U_a}^2 W_{3a}^2\right)}{E_M\left(t_{U_a}^2 \tilde{V}_a\right)} \tag{1.47}$$

The above expectations depend on complex variances that involve $Q$, $Q_i$, $Q_{ia}$, $Q_{iaj}$, $Q_{iajb}$, $Q_a$, and $Q_{ab}$. To simplify results as well as make them more comparable to formulas found in earlier sampling literature, we show how the formulas in Eqs. (1.40) - (1.46) reduce under the special conditions that assumptions (*A1*) – (*A5*) in Appendix A.4.2 hold. We repeat those assumptions here for convenience.

(A1) Every SSU stratum *a* occurs in every PSU *i* and that every HU substratum *b* occurs in every SSU *j*.

(A2) Define $p_i = \dfrac{Q_i}{Q}$, $p_{j|ia} = \dfrac{Q_{iaj}}{Q_{ia}}$, and suppose that $Q_{iajb} = \bar{\bar{Q}}_b$, that is, the same number of elements occurs in HU substratum *b* (everywhere) for every PSU/SSU stratum/SSU *iaj* combination. As noted in Lemma 2 of Appendix A.4.2, these restrictions along with assumption (*A1*) imply that $p_i = \dfrac{N_i}{M\bar{N}}$ and $p_{j|ia} = \dfrac{1}{N_{ia}}$, where $N_i = \sum_{a=1}^{A} N_{ia}$ is the number of SSUs in the population for PSU *i*, and $\bar{N} = M^{-1} \sum_{i \in U} N_i$ is the average number of SSUs per PSU in the population.

(A3) $Q_{iajb} = \bar{\bar{Q}}_b$. As noted in Lemma 2 of Appendix A.4.2, this implies $\bar{Q}_{ia \bullet b} = \bar{\bar{Q}}_b$ and $\bar{Q}_{ab} = \bar{\bar{Q}}_b$.[3] It follows that the same number of elements per SSU, $\bar{\bar{Q}} = \sum_{b=1}^{B} \bar{\bar{Q}}_b$, occurs everywhere, that is $Q_{iaj} = \sum\limits_b Q_{iajb} = \sum\limits_b \bar{\bar{Q}}_b \equiv \bar{\bar{Q}} \equiv \bar{Q}_{ia} \equiv \bar{Q}_i \equiv \bar{Q}_{2a}$. As noted in Lemma 3 of Appendix A.4.3, $S^2_{Q_{2a}} = 0.$ We conclude that these restrictions imply that $S^2_{Q_{(pwr)}} = S^2_{Q_{ia(pwr)}} = S^2_{Q_{ia}} = 0$.

(A4) $P_{ia} = N_{ia} / N_i \equiv P_a$, i.e., the proportion of SSUs in SSU stratum *a*, is the same for every PSU *i*.

(A5) $N_{ia} \approx N_{ia} - 1$, i.e., the number of SSUs in the population in every PSU/SSU stratum *ia* combination is large.

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[3] $\bar{\bar{Q}}_b$ indicates that the same number of HUs occurs in each PSU/SSU combination**.**

Using assumptions ($A1$) – ($A5$), we also arrive at the following (see Appendix A.4.2, Lemma 2 for derivation):

$$Q = M\bar{N}\bar{\bar{Q}} \qquad \bar{Q} = \bar{N}\bar{\bar{Q}} \qquad Q_i = N_i\bar{\bar{Q}} \qquad Q_{ia} = N_{ia}\bar{\bar{Q}} \qquad Q_a = N_a\bar{\bar{Q}}$$

$$Q_{ab} = N_a\bar{\bar{Q}}_b \qquad \bar{Q}_{ab} = \bar{\bar{Q}}_b \qquad \bar{Q}_{1a} = \frac{N_a\bar{\bar{Q}}}{M} \qquad \bar{Q}_{2a} = \bar{\bar{Q}}$$

Additionally, $S_Q^2$, $S_{Q_{1a}}^2$, and $S_{Q_a(pwr)}^2$ also reduce to the less complex variance estimations of the HU sizes below (See Appendix A.4.3, Lemma 3):

$$S_Q^2 = \frac{\bar{\bar{Q}}^2}{M-1}\sum_{i\in U}\left(N_i - \bar{N}\right)^2 \equiv \bar{\bar{Q}}^2 S_N^2 \tag{1.48}$$

where $S_N^2$ is the unit variance of the number of SSUs, $N_i$, across PSUs.

$$S_{Q_{1a}}^2 = \frac{\bar{\bar{Q}}^2}{M-1}\sum_{i\in U}\left(N_{ia} - \bar{N}\right)^2 \equiv \bar{\bar{Q}}^2 S_{N_a}^2 \tag{1.49}$$

where $S_{N_a}^2$ is the unit variance of the number of SSUs in SSU stratum $a$, $N_{ia}$, across PSUs;

$$\begin{aligned} S_{Q_a(pwr)}^2 &= \bar{\bar{Q}}^2\sum_{i\in U} p_{2i}\left(\frac{N_{ia}}{p_{2i}} - N_a\right)^2 \quad \text{where } p_{2i} = \frac{N_i}{N} \\ &\equiv \bar{\bar{Q}}^2 S_{N_a(pwr)}^2 \end{aligned} \tag{1.50}$$

where $S_{N_{a(pwr)}}^2$ is the variance if PSUs are selected with probability proportional to $N_{ia}$, the number of SSUs in PSU $i$, SSU stratum $a$. Using the results from Eq. (1.48), we also obtain the following simplification

$$v_Q^2 = \frac{S_Q^2}{\bar{Q}^2} \equiv \frac{\bar{\bar{Q}}^2 S_N^2}{\bar{N}^2\bar{\bar{Q}}^2} = \frac{S_N^2}{\bar{N}^2} = v_N^2\,. \tag{1.51}$$

**When assumptions (*A1*) - (*A5*) hold.** Using assumptions (A1) – (A5), and when $M,\ M\bar{N}$ are large, we arrive at the following (see Appendices A.4.5-A.4.11):

$$E_M\left(t_U^2 B^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma_\alpha^2 + \frac{\sigma_\gamma^2}{\bar{N}} + \sum_{a=1}^{A} P_a \sum_{b=1}^{B} \frac{\sigma_{\varepsilon_{ab}}^2}{\bar{N}} \frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}^2}\right] \tag{1.52}$$

$$E_M\left(t_{U_a}^2 W_{2a}^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2 \left[\sigma_\gamma^2 + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}^2}\right] \tag{1.53}$$

$$t_{U_{ab}}^2 E_M\left(W_{3ab}^2\right) = \left(M\bar{N}\bar{\bar{Q}}_b\right)^2 P_a^2 \sigma_{\varepsilon_{ab}}^2 \tag{1.54}$$

$$E_M\left(t_U^2 W^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 \sum_{a=1}^{A} P_a \left[\sigma_\gamma^2 + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}}\right] \tag{1.55}$$

$$E_M\left(t_{U_a}^2 W_{3a}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2 \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}} \tag{1.56}$$

$$E_M\left(\bar{y}_U^2 \tilde{V}\right) \doteq \sigma_\alpha^2 + \sigma_\gamma^2 + \sum_{a=1}^{A} \sum_{b=1}^{B} P_a \sigma_{\varepsilon_{ab}}^2 \frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}} \tag{1.57}$$

$$E_M\left(\bar{y}_{U_a}^2 \tilde{V}_a\right) \doteq \sigma_\alpha^2 + \sigma_\gamma^2 + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}} \tag{1.58}$$

**The case of no *b* HU substrata.** In the special case when there are no *b* substrata, such that $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectations in Eqs. (1.52) – (1.58) reduce to (see Appendices A.4.5-A.4.11 for derivation):

$$E_M\left(t_U^2 B^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma_\alpha^2 + \frac{\sigma_\gamma^2}{\bar{N}} + \sum_{a=1}^{A} P_a \frac{\sigma_{\varepsilon_a}^2}{\bar{\bar{Q}}\bar{N}}\right] \tag{1.59}$$

$$E_M\left(t_{U_a}^2 W_{2a}^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2 \left[\sigma_\gamma^2 + \frac{\sigma_{\varepsilon_a}^2}{\bar{\bar{Q}}}\right] \tag{1.60}$$

$$E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2 \sigma_{\varepsilon_a}^2 \tag{1.61}$$

$$E_M\left(t_U^2 W^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 \sum_{a=1}^{A} P_a \left[\sigma_\gamma^2 + \sigma_{\varepsilon_a}^2\right] \tag{1.62}$$

$$E_M\left(t_{U_a}^2 W_{3a}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2 \sigma_{\varepsilon_a}^2 \tag{1.63}$$

$$E_M\left(\bar{y}_U^2 \tilde{V}\right) \doteq \sigma_\alpha^2 + \sigma_\gamma^2 + \sum_{a=1}^{A} P_a \sigma_{\varepsilon_a}^2 \tag{1.64}$$

$$E_M\left(\bar{y}_{U_a}^2 \tilde{V}_a\right) \doteq \sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_{\varepsilon_a}^2 \tag{1.65}$$

**The case of no *a* SSU strata and no *b* HU substrata.** In the special case where there are no $a$ strata, such that $\sigma_{\varepsilon_a}^2 = \sigma_\varepsilon^2$ and $P_a = 1$, the model expectations in Eqs. (1.59) -(1.65) reduce to:

$$E_M\left(t_U^2 B^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma_\alpha^2 + \frac{\sigma_\gamma^2}{\bar{N}} + \frac{\sigma_\varepsilon^2}{\bar{N}\bar{\bar{Q}}}\right] \tag{1.66}$$

$$E_M\left(t_{U_a}^2 W_{2a}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma_\gamma^2 + \frac{\sigma_\varepsilon^2}{\bar{\bar{Q}}}\right] \tag{1.67}$$

$$E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 \sigma_\varepsilon^2 \tag{1.68}$$

$$E_M\left(t_U^2 W^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma_\gamma^2 + \sigma_\varepsilon^2\right] \tag{1.69}$$

$$E_M\left(t_{U_a}^2 W_{3a}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 \sigma_\varepsilon^2 \tag{1.70}$$

$$E_M\left(\bar{y}_U^2 \tilde{V}\right) \doteq \sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_\varepsilon^2 \tag{1.71}$$

$$E_M\left(\bar{y}_{U_a}^2 \tilde{V}_a\right) \doteq \sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_\varepsilon^2 \tag{1.72}$$

In that case, the approximate model expectations of Eqs. (1.66) - (1.72) can be used to evaluate the model expectations of $\delta_1$, $k_1$, $\delta_{2a}$, and $k_{2a}$ when there are no *a* or *b* strata. Assuming that $\bar{N}$, $\bar{N}\bar{\bar{Q}}$, and $\bar{\bar{Q}}$ are large, the approximate model expectation of $\delta_1$ reduce to (see Appendix A.4.12 for derivation);

$$E_M(\delta_1) \doteq \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_\varepsilon^2} \qquad E_M(k_1) \doteq \frac{\sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_\varepsilon^2}{\sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_\varepsilon^2} = 1$$

$$E_M(\delta_{2a}) \doteq \frac{\sigma_\gamma^2}{\sigma_\gamma^2 + \sigma_\varepsilon^2} \qquad E_M(k_{2a}) \doteq \frac{\sigma_\lambda^2 + \sigma_\varepsilon^2}{\sigma_\alpha^2 + \sigma_\lambda^2 + \sigma_\varepsilon^2}$$

(1.73)

Using the output from standard (non-survey) variance component estimation software in SAS, we can evaluate Eq. (1.73). However, this will require that all PSUs and all SSUs have the same sizes everywhere. If this is not the case, then for the full anticipated model, we can use the standard variance component estimates from software as inputs to evaluate Eqs. (1.40)-(1.46), which can then be used to evaluate the model expectations of $\delta_1$, $k_1, \delta_{2a}$, and $k_{2a}$.

### 2.4.2.4 Non Self-Representing (NSR) and Self-Representing (SR) PSUs in the HRS Design

*Non Self-Representing (NSR) PSUs*

The same model in Section 2.4.2.1 will be used for the NSR PSUs but now the mean, $\mu$, and random effects $\alpha_i$ are specific to the NSR PSU. The calculations and universe *U* are restricted to the universe of NSR PSUs. *NSR* subscripts are not added below to simplify the notation. The model expectations of $B^2$, $W_{2a}^2$, $W_{3ab}^2$, $W^2$, $W_{3a}^2$, $\tilde{V}$, and $\tilde{V}_a$ for NSR

PSUs will be the exact form of Eqs. (1.40) - (1.46) with $m =$ no. of sample NSR PSUs, $p_i =$ adjusted one draw probability defined earlier. The only distinction is that the sample is now restricted to the sample of NSR PSUs and their SSUs and HUs that are within NSR PSUs only, such that $M$ , $N_{ia}$, $\bar{Q}_{ia}$ and $\bar{Q}_{ia\bullet b}$ are now specific to the NSR PSUs.

Equations in Section 2.4.2.2 are helpful because they show the effects of the sizes of PSUs/SSUs on the components of variances. However, we will use the following equations which are easier to compute for numerical calculations. These equations are earlier forms of the derivations of the model expectations before substituting for the relvariances of the $Q$'s:

$$E_M\left(t_U^2 B^2\right) \doteq \sigma_\alpha^2 \sum_{i\in U} \frac{Q_i^2}{p_i} + \sigma_\gamma^2 \sum_{i\in U} \frac{1}{p_i}\left\{\sum_{a=1}^{A}\sum_{j\in U_{ia}} Q_{iaj}^2 + \sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{ia\bullet b}\right\} + \mu^2 \sum_{i\in U} p_i\left(\frac{Q_i}{p_i} - Q\right)^2 \tag{1.74}$$

(see Eq. (A.68) for derivation)

$$\begin{aligned} E_M\left(t_{U_a}^2 W_{2a}^2\right) \doteq \sum_{i\in U}\frac{1}{p_i}\Bigg\{ & \left(\mu^2 + \sigma_\alpha^2\right)\sum_{j\in U_{ia}} p_{j|ia}\left(\frac{Q_{iaj}}{p_{j|ia}} - Q_{ia}\right)^2 \\ & + \sigma_\gamma^2\left[\sum_{j\in U_{ia}}\frac{Q_{iaj}^2}{p_{j|ia}} - \left(\sum_{j\in U_{ia}} Q_{iaj}^2\right)\right] \\ & + \sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left[\sum_{j\in U_{ia}}\frac{Q_{iajb}}{p_{j|ia}} - Q_{ia\bullet b}\right]\Bigg\} \end{aligned} \tag{1.75}$$

(see Eq. (A.92) for derivation)

$$E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right)=\sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{1}{p_{j|ia}}\sigma_{\varepsilon_{ab}}^2 Q_{iajb}^2 \tag{1.76}$$

(see Eq. (A.101) for derivation)

$$\begin{aligned}
E_M\left(t_U^2 W^2\right) &\doteq \sum_{i\in U}\frac{Q_i}{p_i}\sum_{a=1}^{A}N_{ia}\left\{\sigma_\gamma^2\bar{Q}_{ia}\left[1-\frac{\bar{Q}_{ia}}{Q_i}\left(v_{Q_{ia}}^2+1\right)\right]+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\bar{Q}_{ia\bullet b}\right\}\\
&=\sum_{i\in U}\frac{Q_i}{p_i}\sum_{a=1}^{A}N_{ia}\left\{\sigma_\gamma^2\bar{Q}_{ia}\left[1-\frac{\bar{Q}_{ia}}{Q_i}\frac{1}{N_{ia}\bar{Q}_{ia}^2}\sum_{j\in U_{ia}}Q_{iaj}^2\right]+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\bar{Q}_{ia\bullet b}\right\}\\
&=\sum_{i\in U}\frac{Q_i}{p_i}\sum_{a=1}^{A}N_{ia}\left\{\sigma_\gamma^2\left[\bar{Q}_{ia}-\frac{1}{Q_i N_{ia}}\sum_{j\in U_{ia}}Q_{iaj}^2\right]+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\bar{Q}_{ia\bullet b}\right\}
\end{aligned}$$

(See Eq. (A.116) for derivation) (1.77)

$$E_M\left(t_{U_a}^2 W_{3a}^2\right)=\sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{iajb} \tag{1.78}$$

(see Eq. (A.133) for derivation)

$$\begin{aligned}
E_M\left(\bar{y}_U^2\tilde{V}\right) &\doteq \sigma_\alpha^2\left[1-\frac{1}{Q^2}\sum_{i\in U}Q_i^2\right]+\sigma_\gamma^2\left[1-\frac{1}{Q^2}\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}Q_{iaj}^2\right]\\
&\quad+\frac{1}{Q}\sum_{i\in U}\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{ia\bullet b}
\end{aligned} \tag{1.79}$$

(see Eq. (A.151) for derivation)

$$E_M\left(\bar{y}_{U_a}^2\tilde{V}_a\right)\doteq\sigma_\alpha^2\left\{1-\frac{1}{Q_a^2}\sum_{i\in U}Q_{ia}^2\right\}+\sigma_\gamma^2\left\{1-\frac{1}{Q_a^2}\sum_{i\in U}\sum_{j\in U_{ia}}Q_{iaj}^2\right\}+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\frac{Q_{ab}}{Q_a} \tag{1.80}$$

(see Eq. (A.174) for derivation)

where $t_U^2, t_{U_a}, t_{U_{ab}}$ and $\bar{y}_U^2, \bar{y}_{U_a}^2$ are defined similarly to those in Sections 2.3.2 - 2.3.3 but use terms specific to the NSR PSUs.

*Self-Representing (SR) PSUs*

In this section, we use some of the same formulas in Section 2.4.2.1 through Section 2.4.2.3, but we restrict the calculations to the set of SR PSUs. *SR* subscripts are not added below to simplify the notation. Restrict *U* (and all alike indices) to the set of all SR PSUs. Here we treat each SR PSU as a stratum and let $p_i = 1$.

Consider a model for $y_k$ with common mean, $\mu$, fixed effects for SR PSUs, $\alpha_i$, and random effects for SSUs, $\gamma_{iaj}$, and HUs in SSU/HU substratum a*b*, $\varepsilon_{iajbk}$:

$$y_k = \mu + \alpha_i + \gamma_{iaj} + \varepsilon_{iajbk} \tag{1.81}$$

with $\gamma_{iaj} \sim \left(0\,,\, \sigma_{\gamma}^2\right)$, $\varepsilon_{iajbk} \sim \left(0\,,\, \sigma_{\varepsilon_{ab}}^2\right)$,

and the errors being independent, such that

$$Var_M\left(y_k\right) = \sigma_{\gamma}^2 + \sigma_{\varepsilon_{ab}}^2 \quad \text{and} \quad E_M\left(y_k\right) = \mu + \alpha_i \quad \text{for} \quad k \in U_{iajb}\,.$$

The model differs from the NSR PSUs because the SR PSUs are treated as strata with fixed effects. The variance of fixed effects for SR PSUs is zero, i.e., $Var_M\left(\alpha_i\right) = \sigma_{\alpha}^2 = 0$. Let $t_U$, $t_{U_a}$, $t_{U_{ab}}$, $\bar{y}_U^2$, $\bar{y}_{U_a}^2$ be defined similarly to those in Section 2.3.3 but use terms specific to the SR PSUs. In order to get model expectations for SR PSUs, we substitute $\mu$ with $\mu + \alpha_i$, $\sigma_{\alpha}^2 = 0$, and $p_i = 1$ into equations for NSR PSUs in Section 2.4.2.2 to obtain:

$$\begin{aligned} E_M\left(t_{U_a}^2 W_{2a}^2\right) \doteq \sum_{i \in U} \Bigg\{ & \left(\mu + \alpha_i\right)^2 \bar{Q}_{ia}^2 v_{Q_{ia(pwr)}}^2 + \sigma_{\gamma}^2 N_{ia}^2 \bar{Q}_{ia}^2 \left( \frac{v_{Q_{ia(pwr)}}^2}{N_{ia}^2} - \frac{v_{Q_{ia}}^2}{N_{ia}} + 1 \right) \\ & + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \left[ \sum_{j \in U_{ia}} \frac{Q_{iajb}}{p_{j|ia}} - Q_{ia \bullet b} \right] \Bigg\} \end{aligned}$$

$$E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right)=\sum_{i\in U}\sum_{j\in U_{ia}}\frac{1}{p_{j|ia}}\sigma_{\varepsilon_{ab}}^2 Q_{iajb}^2$$

$$E_M\left(t_{U_a}^2 W_{3a}^2\right)=\sum_{i\in U}\sum_{j\in U_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{iajb}$$

$$E_M\left(\bar{y}_{U_a}^2\tilde{V}_a\right)\doteq\sigma_\gamma^2\left\{1-\frac{1}{Q_a^2}N_a\bar{Q}_{2a}^2\left(v_{2Q_a}^2+1\right)\right\}+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\frac{Q_{ab}}{Q_a}$$

(1.82)

For numerical calculations the following equations are easier to compute (see Appendix A.4):

$$\begin{aligned}E_M\left(t_{U_a}^2 W_{2a}^2\right)\doteq\sum_{i\in U}&\left\{\left(\mu+\alpha_i\right)^2\sum_{j\in U_{ia}}p_{j|ia}\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)^2\right.\\&+\sigma_\gamma^2\left[\sum_{j\in U_{ia}}\frac{Q_{iaj}^2}{p_{j|ia}}-\left(\sum_{j\in U_{ia}}Q_{iaj}^2\right)\right]\\&\left.+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left[\sum_{j\in U_{ia}}\frac{Q_{iajb}}{p_{j|ia}}-Q_{ia\bullet b}\right]\right\}\end{aligned}$$

$$E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right)=\sum_{i\in U}\sum_{j\in U_{ia}}\frac{1}{p_{j|ia}}\sigma_{\varepsilon_{ab}}^2 Q_{iajb}^2$$

$$E_M\left(t_{U_a}^2 W_{3a}^2\right)=\sum_{i\in U}\sum_{j\in U_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{iajb}$$

$$E_M\left(\bar{y}_{U_a}^2\tilde{V}_a\right)\doteq\sigma_\gamma^2\left\{1-\frac{1}{Q_a^2}\sum_{i\in U}\sum_{j\in U_{ia}}Q_{iaj}^2\right\}+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\frac{Q_{ab}}{Q_a}$$

(1.83)

## 2.4.3 Estimators of Anticipated Variances

Estimators of variance are needed to evaluate Eqs. (1.40) - (1.46). These are plug in estimators. Although we do not provide theoretical details, in large PSU, SSU, and HU samples the estimators will be consistent.

### 2.4.3.1 General Case

In addition to the estimators of the unit sizes defined in Section 2.4.1.1, we define additional estimators assuming the 3rd stage is *srs* so that the following hold:

$$\hat{M} = \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}, \quad \hat{N} = \frac{1}{m}\sum_{i\in s_1}\frac{\hat{N}_i}{p_i} = \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{1}{p_{j|ia}}, \quad \hat{\hat{N}} = \frac{\hat{N}}{\hat{M}}$$

$$\hat{N}_i = \sum_a \hat{N}_{ia}, \qquad \hat{N}_{ia} = \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{1}{p_{j|ia}}, \qquad \hat{N}_a = \frac{1}{m}\sum_{i\in s_1}\frac{\hat{N}_{ia}}{p_i}$$

$$\hat{Q}_{iaj} = \sum_b \sum_{k\in s_{iajb}} w_{k|iajb} = \sum_b \sum_{k\in s_{iajb}} \frac{Q_{iajb}}{q_{iajb}} = \sum_b \cancel{q_{iajb}}\frac{Q_{iajb}}{\cancel{q_{iajb}}} = \sum_b Q_{iajb} = Q_{iaj}$$

$$\hat{Q}_{ia\bullet b} = \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iajb}}{p_{j|ia}}, \quad \hat{\hat{Q}}_{ia\bullet b} = \frac{\hat{Q}_{ia\bullet b}}{\hat{N}_{ia}}$$

$$\hat{Q}_{ab} = \sum_{i\in s_1}\frac{1}{m}\frac{1}{p_i}\sum_{j\in s_{ia}}\frac{Q_{iajb}}{n_{ia}p_{j|ia}}$$

$$\hat{Q}_{ia} = \sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}}{n_{ia}p_{j|ia}} \text{ and when } 3^{rd} \text{ stage SRS } \hat{Q}_{ia} = \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}.$$

$$\hat{Q}_a = \sum_{i\in s_1}\frac{\hat{Q}_{ia}}{mp_i}, \quad \hat{\hat{Q}}_{1a} = \frac{\hat{Q}_a}{\hat{M}}, \quad \hat{\hat{Q}}_{2a} = \frac{\hat{Q}_a}{\hat{N}_a}$$

$$\hat{Q} = \sum_{i\in s_1}\frac{1}{mp_i}\hat{Q}_i, \quad \hat{Q}_i = \sum_a \hat{Q}_{ia}, \qquad \hat{\hat{Q}} = \frac{\hat{Q}}{\hat{M}}$$

Then the estimators of the unit relvariances in Sections 2.4.2.2- 2.4.2.3 are (see Appendix A.5 for derivations):

$$\hat{v}^2_{Q(pwr)} = \frac{\hat{S}^2_{Q(pwr)}}{\hat{\hat{Q}}^2} \text{ where}$$

$$\hat{S}^2_{Q(pwr)} = \frac{1}{m-1}\sum_{i\in s_1}\left(\frac{\hat{Q}_i}{p_i}-\hat{Q}\right)^2 - \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a \frac{1}{n_{ia}}\hat{S}^2_{Q_{ia}(pwr)}; \tag{1.84}$$

$$\hat{v}^2_{Q_{ia}(pwr)} = \frac{\hat{S}^2_{Q_{ia}(pwr)}}{\hat{\hat{Q}}^2_{ia}} \text{ where}$$

$$\hat{S}^2_{Q_{ia}(pwr)} = \frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\left(\frac{\hat{Q}_{iaj}}{p_{j|ia}}-\hat{Q}_{ia}\right)^2; \tag{1.85}$$

$$\hat{v}^2_{Q_{ia}} = \frac{\hat{S}^2_{Q_{ia}}}{\hat{\hat{Q}}^2_{ia}} \text{ where } \hat{S}^2_{Q_{ia}} = \hat{S}^2_{\mathrm{AQ}_{ia}} + \hat{S}^2_{\mathrm{B}Q_{ia}}$$

$$\hat{S}^2_{\mathrm{AQ}_{ia}} = \frac{n_{ia}}{n_{ia}-1}\frac{\sum_{j\in s_{ia}} w_{j|ia}\left(\hat{Q}_{iaj}-\hat{\hat{Q}}_{ia}\right)^2}{\sum_{j\in s_{ia}}\left(w_{j|ia}-1\right)} \quad \text{and} \quad \hat{S}^2_{\mathrm{B}Q_{ia}} = \frac{1}{\hat{N}^2_{ia}}\frac{1}{n_{ia}}\hat{S}^2_{Q_{ia}(pwr)}; \tag{1.86}$$

$$\hat{v}^2_Q = \frac{\hat{S}^2_Q}{\hat{\hat{Q}}^2} \text{ where } \hat{S}^2_Q = \hat{S}^2_{\mathrm{A}Q} + \hat{S}^2_{\mathrm{B}Q}$$

$$\hat{S}^2_{\text{AQ}} = \frac{m}{m-1}\frac{\sum_{j\in s_1} w_i\left(\hat{Q}_i - \hat{\bar{Q}}\right)^2}{\sum_{j\in s_1}\left(w_i - 1\right)}$$

$$\hat{S}^2_{\text{B}Q} = \frac{1}{\hat{M}^2}\frac{1}{m}\hat{S}^2_{Q(pwr)} - \sum_{i\in s_1}\frac{1}{mp_i}\sum_{a}\frac{\hat{S}^2_{Q_{ia(pwr)}}}{\hat{M}n_{ia}}\left(1-\frac{1}{\hat{M}mp_i}\right);$$

(1.87)

$$\hat{v}^2_{Q_{1a}} = \frac{\hat{S}^2_{Q_{1a}}}{\hat{\bar{Q}}^2_{1a}} \text{ where } \hat{S}^2_{Q_{1a}} = \hat{S}^2_{\text{A}Q_{1a}} + \hat{S}^2_{\text{B}Q_{1a}}$$

$$\hat{S}^2_{\text{A}Q_{1a}} = \frac{m}{m-1}\frac{\sum_{i\in s_1} w_i\left(\hat{Q}_{ia} - \hat{\bar{Q}}_{1a}\right)^2}{\sum_{i\in s_1}\left(w_i - 1\right)} = \frac{m}{m-1}\frac{1}{\hat{M}}\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}^2_{ia}}{p_i} - \hat{M}\hat{\bar{Q}}^2_{1a}\right)$$

$$\hat{S}^2_{\text{B}Q_{1a}} = \frac{1}{\hat{M}^2}\frac{1}{m}\hat{S}^2_{Q_{a(pwr)}} - \sum_{i\in s_1}\frac{1}{mp_i}\frac{\hat{S}^2_{Q_{ia(pwr)}}}{\hat{M}n_{ia}}\left(1-\frac{1}{\hat{M}mp_i}\right)$$

$$\hat{S}^2_{Q_{a(pwr)}} = \frac{1}{m}\sum_{i\in s_1}\left(\frac{\hat{Q}_{ia}}{p_i} - \hat{Q}_a\right)^2; \qquad (1.88)$$

$$\hat{v}^2_{Q_{2a}} = \frac{\hat{S}^2_{Q_{2a}}}{\hat{\bar{Q}}^2_{2a}} \text{ where } \hat{S}^2_{Q_{2a}} = \hat{S}^2_{\text{A}Q_{2a}} + \hat{S}^2_{\text{B}Q_{2a}}$$

$$\hat{S}^2_{\text{A}Q_{2a}} = \frac{1}{\hat{N}_a - 1}\sum_{i\in s_1}\frac{1}{mp_i}\sum_{j\in s_{ia}}\frac{\hat{Q}^2_{iaj}}{n_{ia}p_{j|ia}} - \hat{N}_a\hat{\bar{Q}}^2_{2a}$$

$$\hat{S}^2_{\text{B}Q_{2a}} = \frac{1}{\hat{N}^2_a}\frac{1}{m}\hat{S}^2_{Q_{a(pwr)}} + \frac{1}{\hat{N}^2_a}\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\frac{\hat{S}^2_{Q_{ia(pwr)}}}{n_{ia}}; \qquad (1.89)$$

Then the estimators of the model expectations in Eqs. (1.40) - (1.46) are

$$\hat{E}_M\left(t_U^2 B^2\right)=\hat{\sigma}_\alpha^2 \hat{M}^2 \hat{\bar{Q}}^2\left(\frac{\hat{v}_{Q(pwr)}^2}{\hat{M}^2}+1\right)+\hat{\sigma}_\gamma^2 \sum_{i\in s_1}\frac{1}{mp_i^2}\left\{\sum_{a=1}^{A}\hat{N}_{ia}\hat{\bar{Q}}_{ia}^2\left(\hat{v}_{Q_{ia}}^2+1\right)\right.$$
$$\left.+\sum_{a=1}^{A}\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2\hat{Q}_{ia\bullet b}\right\}+\hat{\mu}^2\hat{\bar{Q}}^2\hat{v}_{Q(pwr)}^2 \tag{1.90}$$

$$\hat{E}_M\left(t_{U_a}^2 W_{2a}^2\right)=\sum_{i\in s_1}\frac{1}{mp_i^2}\left\{\left(\hat{\mu}^2+\hat{\sigma}_\alpha^2\right)\frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\left(\frac{\hat{Q}_{iaj}}{p_{j|ia}}-\hat{Q}_{ia}\right)^2\right.$$
$$+\hat{\sigma}_\gamma^2\left[\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{n_{ia}p_{j|ia}^2}-\left(\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{n_{ia}p_{j|ia}}\right)\right] \tag{1.91}$$
$$\left.+\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2\left[\left(\sum_{j\in s_{ia}}\frac{\hat{Q}_{iajb}}{n_{ia}p_{j|ia}^2}\right)-\hat{Q}_{ia\bullet b}\right]\right\}$$

$$\hat{E}_M\left(t_{U_{ab}}^2 W_{3ab}^2\right)=\hat{\sigma}_{\varepsilon_{ab}}^2\sum_{i\in s_1}\frac{1}{mp_i^2}\sum_{j\in s_{ia}}\frac{1}{n_{ia}p_{j|ia}^2}\hat{Q}_{iajb}^2 \tag{1.92}$$

$$\hat{E}_M\left(t_U^2 W^2\right)=\sum_{i\in s_1}\frac{\hat{Q}_i}{mp_i^2}\sum_{a=1}^{A}\hat{N}_{ia}\left\{\hat{\sigma}_\gamma^2\hat{\bar{Q}}_{ia}\left[1-\frac{\hat{\bar{Q}}_{ia}}{\hat{Q}_i}\left(\hat{v}_{Q_{ia}}^2+1\right)\right]+\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2\hat{\bar{Q}}_{ia\bullet b}\right\} \tag{1.93}$$

$$\hat{E}_M\left(t_{U_a}^2 W_{3a}^2\right)=\sum_{i\in s_1}\frac{1}{mp_i^2}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}}{n_{ia}p_{j|ia}^2}\sum_{b=1}^{B}\hat{Q}_{iajb}\hat{\sigma}_{\varepsilon_{ab}}^2 \tag{1.94}$$

$$\hat{E}_M\left(\bar{y}_U^2\tilde{V}\right)=\hat{\sigma}_\alpha^2\left[1-\frac{1}{\hat{M}}\left(\hat{v}_Q^2+1\right)\right]+\hat{\sigma}_\gamma^2\left[1-\frac{1}{\hat{\bar{Q}}^2}\sum_{i\in s_1}\frac{1}{mp_i}\sum_{a=1}^{A}\hat{N}_{ia}\hat{\bar{Q}}_{ia}^2\left(\hat{v}_{Q_{ia}}^2+1\right)\right]$$
$$+\sum_{i\in s_1}\frac{1}{mp_i}\sum_{a=1}^{A}\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2\frac{\hat{Q}_{ia\bullet b}}{\hat{Q}} \tag{1.95}$$

$$E_M\left(\bar{y}_{U_a}^2\tilde{V}_a\right)=\hat{\sigma}_\alpha^2\left\{1-\frac{1}{\hat{Q}_a^2}\hat{M}\hat{\bar{Q}}_{1a}^2\left(\hat{v}_{Q_{1a}}^2+1\right)\right\}+\hat{\sigma}_\gamma^2\left\{1-\frac{1}{\hat{Q}_a^2}\hat{N}_a\hat{\bar{Q}}_{2a}^2\left(\hat{v}_{Q_{2a}}^2+1\right)\right\}+\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2\frac{\hat{Q}_{ab}}{\hat{Q}_a} \tag{1.96}$$

### 2.4.3.2 NSR PSUs

*Non Self-Representing (NSR) PSUs*

The anticipated variance formulas for NSR PSUs will be the exact form of Eqs. (1.90) - (1.96) with $m=$ no. of sample NSR PSUs, $p_i=$ adjusted one draw probability defined earlier. The sample is now restricted to the sample of NSR PSUs and their SSUs and HUs that are within NSR PSUs only, such that $m$, $n_{ia}$, and are now specific to the NSR PSUs. The model is also specific to the NSRs when estimating $\mu^2, \sigma_\alpha^2, \sigma_\gamma^2$ ,and $\sigma_{\varepsilon_{ab}}^2$.

$$\begin{aligned}\hat{E}_M\left(t_U^2B^2\right)=&\hat{\sigma}_\alpha^2\sum_{i\in s_{1,NSR}}\frac{\hat{Q}_i^2}{mp_i^2}+\hat{\sigma}_\gamma^2\sum_{i\in s_{1,NSR}}\frac{1}{mp_i^2}\sum_{a=1}^{A}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{n_{ia}p_{j|ia}}\\&+\sum_{i\in s_{1,NSR}}\frac{1}{mp_i^2}\sum_{a=1}^{A}\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2\hat{Q}_{ia\bullet b}+\hat{\mu}^2\hat{\bar{Q}}^2\hat{v}_{Q_{(pwr)}}^2\end{aligned} \tag{1.97}$$

$$\begin{aligned}\hat{E}_M\left(t_{U_a}^2W_{2a}^2\right)=\sum_{i\in s_{1,NSR}}\frac{1}{mp_i^2}&\left\{\left(\hat{\mu}^2+\hat{\sigma}_\alpha^2\right)\frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\left(\frac{\hat{Q}_{iaj}}{p_{j|ia}}-\hat{Q}_{ia}\right)^2\right.\\&+\hat{\sigma}_\gamma^2\left[\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{n_{ia}p_{j|ia}^2}-\left(\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{n_{ia}p_{j|ia}}\right)\right]\\&\left.+\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2\left[\left(\sum_{j\in s_{ia}}\frac{\hat{Q}_{iajb}}{n_{ia}p_{j|ia}^2}\right)-\hat{Q}_{ia\bullet b}\right]\right\}\end{aligned} \tag{1.98}$$

$$\hat{E}_M\left(t_{U_{ab}}^2 W_{3ab}^2\right)=\hat{\sigma}_{\varepsilon_{ab}}^2 \sum_{i\in s_{1,NSR}} \frac{1}{mp_i^2} \sum_{j\in s_{ia}} \frac{1}{n_{ia}\,p_{j|ia}^2} \hat{Q}_{iajb}^2 \tag{1.99}$$

$$\hat{E}_M\left(t_U^2 W^2\right)=\sum_{i\in s_{1,NSR}} \frac{\hat{Q}_i}{mp_i^2} \sum_{a=1}^{A} \hat{N}_{ia}\left\{\hat{\sigma}_\gamma^2\left[\hat{\hat{Q}}_{ia}-\frac{1}{\hat{N}_{ia}\hat{Q}_i}\sum_{j\in s_{ia}} \frac{\hat{Q}_{iaj}^2}{n_{ia}\,p_{j|ia}^2}\right]+\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2 \hat{\hat{Q}}_{ia\bullet b}\right\} \tag{1.100}$$

$$\hat{E}_M\left(t_{U_a}^2 W_{3a}^2\right)=\sum_{i\in s_{1,NSR}} \frac{1}{mp_i^2} \sum_{j\in s_{ia}} \frac{\hat{Q}_{iaj}}{n_{ia}\,p_{j|ia}^2} \sum_{b=1}^{B} \hat{\sigma}_{\varepsilon_{ab}}^2 \hat{Q}_{iajb} \tag{1.101}$$

$$\begin{aligned}\hat{E}_M\left(\bar{y}_U^2 \tilde{V}\right)=&\,\hat{\sigma}_\alpha^2\left(1-\frac{1}{\hat{Q}^2}\sum_{i\in s_{1,NSR}} \frac{\hat{Q}_i^2}{mp_i}\right)+\hat{\sigma}_\gamma^2\left(1-\frac{1}{\hat{Q}^2}\sum_{i\in s_{1,NSR}} \frac{1}{mp_i}\sum_{a=1}^{A}\sum_{j\in s_{ia}} \frac{\hat{Q}_{iaj}^2}{n_{ia}\,p_{j|ia}}\right)\\ &+\frac{1}{\hat{Q}}\sum_{i\in s_{1,NSR}} \frac{1}{mp_i}\sum_{a=1}^{A}\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2 \hat{Q}_{ia\bullet b}\end{aligned} \tag{1.102}$$

$$\begin{aligned}E_M\left(\bar{y}_{U_a}^2 \tilde{V}_a\right)=&\,\hat{\sigma}_\alpha^2\left\{1-\frac{1}{\hat{Q}_a^2}\sum_{i\in s_{1,NSR}} \frac{\hat{Q}_{ia}^2}{mp_i}\right\}+\hat{\sigma}_\gamma^2\left\{1-\frac{1}{\hat{Q}_a^2}\sum_{i\in s_{1,NSR}} \frac{1}{mp_i}\sum_{j\in s_{ia}} \frac{\hat{Q}_{iaj}^2}{n_{ia}\,p_{j|ia}}\right\}\\ &+\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2 \frac{\hat{Q}_{ab}}{\hat{Q}_a}\end{aligned} \tag{1.103}$$

### 2.4.3.3 SR PSUs

In this section, we again use some of the same formulas in Section 2.4.3.2 -2.3.3 but we restrict the calculations to the set of SR PSUs. As in the previous section, we omit *SR* subscripts to simply the notation. For SR PSUs let $p_i = 1$, then

$$\hat{E}_M\left(t_{U_a}^2 W_{2a}^2\right)=\sum_{i\in s_{1,SR}}\left\{\left(\hat{\mu}+\hat{\alpha}_i\right)^2\frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\left(\frac{\hat{Q}_{iaj}}{p_{j|ia}}-\hat{Q}_{ia}\right)^2\right.$$

$$+\hat{\sigma}_\gamma^2\left[\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{n_{ia}p_{j|ia}^2}-\left(\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{n_{ia}p_{j|ia}}\right)\right] \tag{1.104}$$

$$\left.+\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2\left[\left(\sum_{j\in s_{ia}}\frac{\hat{Q}_{iajb}}{n_{ia}p_{j|ia}^2}\right)-\hat{Q}_{ia\bullet b}\right]\right\}$$

$$\hat{E}_M\left(t_{U_{ab}}^2 W_{3ab}^2\right)=\sum_{i\in s_{1,SR}}\sum_{j\in s_{ia}}\frac{1}{n_{ia}p_{j|ia}^2}\hat{\sigma}_{\varepsilon_{ab}}^2\hat{Q}_{iajb}^2 \tag{1.105}$$

$$\hat{E}_M\left(t_{U_a}^2 W_{3a}^2\right)=\sum_{i\in s_{1,SR}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}}{n_{ia}p_{j|ia}^2}\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2\hat{Q}_{iajb} \tag{1.106}$$

$$E_M\left(\bar{y}_{U_a}^2\tilde{V}_a\right)=\hat{\sigma}_\gamma^2\left\{1-\frac{1}{\hat{Q}_a^2}\sum_{i\in s_{1,SR}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{n_{ia}p_{j|ia}}\right\}+\sum_{b=1}^{B}\hat{\sigma}_{\varepsilon_{ab}}^2\frac{\hat{Q}_{ab}}{\hat{Q}_a} \tag{1.107}$$

# 3 Application to the Health and Retirement Survey of Designed Based ANOVA Variance Component Estimation for Sample Allocations

Using the formulas from Chapter 2, we will obtain ANOVA variance estimates and use them for sample size calculations for several variables from the Health and Retirement Study (HRS). The HRS is sponsored by the National Institute of Aging and the Social Security Administration University. For comparison we also calculate estimates using the anticipated variance method covered in Chapter 2.

## 3.1 The Health and Retirement Study (HRS)

In this section, we will describe how the household level file that was used for this thesis was constructed. The household level dataset comprises information on housing units taken from several data files:

(1) HRS Screener Files 2010-2011

(2) HRS Interview File for households interviewed in March 2010 - November 2011 for the Middle Baby Boomer (MBB) cohort

(3) HRS Interview File for households interviewed in March 2010 - November 2011 for the Early Baby Boomer (EBB) cohort

(4) The corresponding data from Marketing Systems Group (MSG) for the HRS Interview/Screener file in 2010-2011

The following sections detail the sample design of the HRS, and the variable matching on MSG-HRS to create the final dataset.

### 3.1.1 Overview

The Health and Retirement Study (HRS, http://hrsonline.isr.umich.edu/) is a longitudinal panel study that surveys a representative sample of approximately 20,000 adults, over the age of 50, living in households in the 48 contiguous states and the District of Columbia. Every two years, from pre-retirement into retirement, the HRS collects information on the changes in income, work, health insurance, disability, physical health, and health care expenditures of aging Americans. The HRS is designed to help understand and address the challenges and opportunities of aging.

### 3.1.2 Sample Design and Procedures

The full HRS sample is composed of several age cohorts, each of which covers six birth years. Every six years, HRS adds a new age cohort to the study. The latest three age cohorts added in 2004, 2010, and 2016 are respectively the Early Baby Boomers (EBBs) born 1948-1953, Middle Baby Boomers (MBBs) born 1954-59, and the Late Baby Boomers (LBBs) born 1960-1965. For this research, the focus is narrowed to the EBB and MBB age cohorts. Specifically, the data used for analysis are the HRS interview data for households interviewed during the period of March 2010 through November 2011 for the EBB and MBB age cohorts, the corresponding HRS screener data, and the corresponding MSG data.

The cohorts are derived from two multistage area probability samples, completed in four stages. In the first stage, a probability proportional to size selection of 75 Primary Sample Units (PSUs), based on U.S. Metropolitan Statistical Areas (MSAs) and non-MSA counties, are chosen.

For the second stage, the Secondary Sampling Units (SSUs) are composed of Census blocks or groups of blocks. Because HRS oversamples Hispanics and Blacks, SSUs are divided into one of four strata according to the Hispanic and Black racial density of its respective block group, as found in the 2000 decennial census. The SSU strata are defined within geographic strata of PSUs shown in Table 3.1. These strata have been found to be generally useful in household surveys that target the Black and Hispanic minority groups and have been used in a number of surveys conducted previously by the University of Michigan. Only those SSUs in SSU strata 2, 3, and 4, that is, those with racial proportions more than 10% Hispanic or more than 10% Black, are sampled in the data that are available for this study.

Table 3.1 SSU Stratum Definitions for 2010-11 HRS Data

| SSU Stratum No. | Label | Definition |
|---|---|---|
| 01 | <10% B, <10% H | < 10% Black, < 10% Hispanic |
| 02 | ≥10% B, <10% H | ≥ 10% Black, < 10% Hispanic |
| 03 | <10% B, ≥10% H | < 10% Black, ≥ 10% Hispanic |
| 04 | ≥10% B, ≥10% H | ≥ 10% Black, ≥ 10% Hispanic |

In the third stage of sampling, a list of all HUs physically located within the bounds of the selected SSUs is enumerated. The list of HUs is sent to the commercial list vendor MSG for matching to the available auxiliary data. During the time of the study, MSG was receiving updates from the USPS CDS as well as compiling information from four commercial vendors: InfoUSA, Targus, Experian and Acxiom[4]. In addition, MSG contained information for addresses on the "Do Not Mail" list but not for those addresses on the No-Stat file. MSG attached a variety of demographic information (e.g., age, gender, Hispanic surname, marital status, income, etc.) on up to two persons for each HU for which there was data (see Appendix B.2 for a full list of MSG variables). HRS corrects any errors found in the enumeration of HUs. The MSG age and race/ethnicity information on addresses in sample segments is then used for sampling housing units. HRS collects the

[4] As of 2014, MSG no longer uses Axciom.

actual demographic information for each responding HU since the MSG data are not always correct. The accuracy rates for the MSG data are presented in Section 3.2 .

The final and fourth stage of sampling involves screening for qualified household members living inside of HUs. During the screening process, HRS collects data on the ages of every household member as well as the marital status and race/ethnicity of certain household members. This information is used to determine qualified household financial units. A household financial unit is a single age-eligible person or a married couple where one or both parties are age-eligible (at the time of the first interview)[5]. In the EBB and MBB cohorts, almost all HUs contain only one financial household. Because the number of HUs with multiple financial households is extremely small, the fourth stage of sampling will be ignored for this research when doing sample size calculations (Valliant R. , Hubbard, Lee, & Chang, 2014).

Table 3.2. MSG Substrata and Definitions for Application to 2010-2011 HRS

| MSG Substratum | Label | Definition |
|---|---|---|
| 1 | 45-62 H | One or more 45-62 Hispanic persons in the HU |
| 2 | 45-62 NH B | One or more 45-62 non-Hispanic, Black persons in the HU |
| 3 | 45-62 NH O | One or more 45-62 non -Hispanic Other persons in the HU |
| 4 | 45-62 No Race/Eth | One or more 45-62 persons with missing race/ethnicity |
| 5 | Not 45-62 | No persons 45-62 in the HU |
| 6 | Unknown | Unknown whether the HU contained persons 45-62 based on MSG data. Age is missing or No Record. |

[5] More information on the HRS can be found at Michigan's Institute for Social Research website http://hrsonline.isr.umich.edu/.

### 3.1.3 Stratification of HUs Using Commercial Lists

In an effort to improve the efficiency at which some target domains are sampled, HUs can be stratified during the third stage of sampling according to the attached demographic information from MSG. In the empirical work here, we use MSG data on two persons in the household as well as the race/ethnicity of the head of household (assuming the race of person 1 and person 2 are the same) to classify HUs into one of six MSG substrata used for sampling, as shown in Table 3.2. (In this paper, the terms HU substrata/MSG substrata are used interchangeably.) The range of ages covered in the HRS data and treated as being eligible here was 45-62. Note that this is different from the ages covered by any HRS cohort; all cases that had an HRS age were included to increase the sample size used for analysis. The first four substrata contained HUs that MSG anticipated as having someone in the 45-62 age range. The fifth substratum contained HUs that had no one 45-62 according to MSG. The sixth substratum contained HUs for which MSG was either missing the demographic information, i.e., age, to predict eligibility or missing the address completely. Note that HUs in the fifth and sixth substratum must be given a positive inclusion probability since some of those HUs may have one or more persons in the eligible age range. The proposed substrata could be used to oversample Blacks or Hispanics in the third stage of selection. Alternative groupings of MSG substrata (e.g., income groups, marital status, etc.) can be made depending on the available auxiliary data. As part of this thesis research, I will only consider stratification in Table 3.2 that would be appropriate for targeting Black and Hispanic persons aged 45-62.

When determining MSG classification for age, we classified HUs into age groups if either the head of household or person 2 was in the target age group. For MSG race/ethnicity classification, we used the head of household race/ethnicity to determine race/ethnicity for the entire HU. Table 3.3, Table 3.4, and Table 3.5 show an unweighted and weighted summary for all addresses based on the classification using MSG data on age, race/ethnicity, and both age crossed with race/ethnicity, respectively. The weights used were HRS base weights that weight the sample up to population counts in SSU strata 2-4, and did not include any nonresponse, or post-stratified adjustments.

Table 3.3. Summary of Classification Results using MSG data Age

| MSG classification | Unweighted | | Weighted | |
|---|---|---|---|---|
| | No. of HUs | Percent | No. of HUs | Percent |
| 45-62 | 6,606 | 23 | 8,909,868 | 22 |
| Not 45-62 | 7,775 | 28 | 12,374,079 | 31 |
| No available data | 13,783 | 49 | 18,763,670 | 47 |
| Total | 28,164 | 100 | 40,047,617 | 100 |

Table 3.4. Summary of Classification Results using MSG data Race/Ethnicity

| MSG classification | Unweighted | | Weighted | |
|---|---|---|---|---|
| | No. of HUs | Percent | No. of HUs | Percent |
| Hispanic | 4,205 | 15 | 6,569,096 | 16.5 |
| Non-Hispanic Black | 3,173 | 11 | 4,578,628 | 11.5 |
| Non-Hispanic Other | 6,273 | 22 | 10,686,189 | 27 |
| No available data | 14,513 | 52 | 18,213,704 | 45 |
| Total | 28,164 | 100 | 40,047,617 | 100 |

Table 3.5. Summary of Classification Results using MSG data Age and Race/Ethnicity

| MSG classification | Unweighted | | Weighted | |
|---|---|---|---|---|
| | No. of HUs | Percent | No. of HUs | Percent |
| 45-62 Hispanic | 1,187 | 4.21 | 1,531,239 | 3.82 |
| 45-62 NH Black | 1,113 | 3.95 | 1,537,944 | 3.84 |
| 45-62 NH Other | 2,468 | 8.76 | 3,825,802 | 9.55 |
| 45-62 No Race-Eth | 1,838 | 6.53 | 2,014,882 | 5.03 |
| Not 45-62 | 7,775 | 27.61 | 12,374,079 | 30.9 |
| Unknown | 13,783 | 48.94 | 18,763,670 | 46.85 |
| Total | 28,164 | 100 | 40,047,617 | 100 |

## 3.2 Availability of MSG and HRS Data

For the period March 2010–November 2011, HRS data on respondents to the screener and interview were compared to information obtained from MSG. From SSU strata 2-4, there were a total of 28,164 sampled addresses (20,887 addresses in 2010 and 7,277 addresses in 2011) selected for screening and sent to MSG for matching.[6] MSG reported whether or not a HU was on the MSG files and if so the individual data for that HU. Table 3.6 shows that at the time of the matching, MSG reported that 14,381 HUs (51 percent) had age information available, while only 13,651 HUs (48 percent) had race/ethnicity information available. Because we often need information on more than one variable to target a specific group of interest, i.e., Hispanic, females ages 18-35, the availability of crossed variables is also a valuable measure. The number of HUs having both age and race/ethnicity data available on the MSG list decreased to only 10,273 HUs, or 36 percent.

Table 3.6. Summary of Information Available on MSG for Age and Race/Ethnicity Variables

| MSG provided information on address | Age | | Race/Ethnicity | | Age/Race/Ethnicity | |
|---|---|---|---|---|---|---|
| | No. of HUs | Percent | No. of HUs | Percent | No. of HUs | Percent |
| Address not sent to MSG[7] | 212 | 0.75 | 220 | 0.78 | 220 | 0.78 |
| No | 13,571 | 48.19 | 14,293 | 50.75 | 17,671 | 62.74 |
| Yes | 14,381 | 51.06 | 13,651 | 48.47 | 10,273 | 36.48 |
| Total | 28,164 | 100.00 | 28,164 | 100.00 | 28,164 | 100.00 |

[6] It was discovered later that some of the 28,164 sampled addresses were actually not sent to MSG.
[7] The difference in the number of addresses not sent to MSG across variables is because addresses were sent to MSG separately for age and race/ethnicity matching.

## 3.3 MSG Accuracy Rates in Classifying HUs by Race/Ethnicity and Age

Given that auxiliary data are available for a HU, the next concern is whether or not the information is accurate. In this section, we estimate the accuracy rates in which commercial lists from MSG can correctly identify households with certain characteristics (e.g., Hispanics, Non-Hispanic Blacks, Persons 45-62, etc.).

To estimate accuracy rates, we used HRS screener data as the measure of true classification. When information on age and race/ethnicity for sampled HUs was not available in the HRS screener, we used the HRS interview responses (when available) to classify HUs into age and race/ethnicity groups. Out of the 28,164 HUs on file, a total of 15,272 HUs had information on age in either the HRS screener or interview data and 4,449 HUs were vacant. This resulted in a total of 19,721 HUs on the HRS files to match to MSG age information. Out of those HUs with HRS age information, all but 64 had race/ethnicity information available on the HRS data. This resulted in a final total of 19,657 HUs available in the HRS data for matching to MSG information to obtain accuracy rates.

Table 3.7. Summary of Classification Results using MSG data by Age group 45-62 and Race/Ethnicity

| MSG classification | Unweighted | | Weighted | |
|---|---|---|---|---|
| | No. of HUs | Percent | No. of HUs | Percent |
| Correctly classified | 6,061 | 31 | 9,817,268 | 34 |
| Incorrectly classified | 2,675 | 14 | 4,385,539 | 15 |
| No available data | 10,921 | 55 | 14,819,672 | 51 |
| Total | 19,657 | 100 | 29,022,480 | 100 |

Table 3.7 shows an aggregated summary based on the classification using MSG data. Out of 19,657 HUs with HRS data, 55 percent of addresses had no available data either because MSG was missing age, race/ethnicity, or there was simply no record. The unweighted analyses show that 45 percent had matched MSG data, while the weighted analyses show 49 percent had matched MSG data. MSG was able to correctly identify HUs with persons aged 45-62 in race categories with 31 percent accuracy unweighted compared to 34 percent weighted. The breakdown of those 8,736 (6,061+2,675) HUs for which MSG and HRS both had age/race/ethnicity is also informative. Given both MSG and HRS had age and race/ethnicity data available, MSG was able to correctly identify HUs into MSG substrata one to three, 69 percent (6,061/8,736) of the time.

Although MSG is not totally accurate, it does give a way to target the sample towards HUs more likely to be eligible. When HUs are stratified into groups of people who may be more likely to be eligible for a survey, those strata can be sampled at higher rates. To illustrate how this may be achieved, the following notation is needed:

*d* = analytic domain in HRS: 1=45-62 Hispanic; 2=45-62 NH Black; 3=45-62 NH Other; in addition, define two other domains: 4=Not 45-62; 5=Unoccupied HU.

*b* = MSG substratum used to sample HU in the third stage of sampling; *b*=1, 2, …, 6;

$p_{ab}(d) =$ proportion of HUs in SSU stratum *a*, MSG sampling substratum *b* that are classified to be in HRS analysis domain *d*.

Table 3.8 shows unweighted estimates of $p_{ab}(d)$ within each of the six MSG sampling substrata based on HRS screener and interview responses. HUs in substrata 1-3, with known race/ethnicity, have a higher proportion of eligible HUs (ranging from 0.743-0.785) than those in substrata 4. The eligibility rate for HUs in substrata 4, with unknown race/ethnicity, fell to 0.581 due to a significant proportion (0.178) of HUs being unoccupied. Among HUs in substrata 5 which were expected to be non-eligible, 74.4 percent were in fact not eligible. Of those HUs in substrata 6, which had unknown eligibility, 77.7 percent were not eligible: 43.8 percent were not 45-62 and 33.9 percent were unoccupied. To use this information on accuracy rates to more efficiently sample HUs, we should sample HUs from substrata 1-4 at higher rates than substrata 5 and 6.

Furthermore, when considering race/ethnicity as a factor, HUs sampled from substrata 1-3 that were expected to be Hispanic, Non-Hispanic Black, and Non-Hispanic, respectively, were confirmed by HRS respondents to be that race/ethnicity 61.7, 54.2, and 64.9 percent of the time. Weighted estimates are given in **Error! Reference source not found.** and show slightly higher eligibility rates. The overall eligibility rate was 35.09 percent, unweighted, and 42.83 percent, weighted (see Table 3.10 and Table 3.11). The weights used were base weights that weight the sample up to population counts in SSU strata 2-4, and did not include any nonresponse, or post-stratified adjustments. We will use the unweighted accuracy rates in Table 3.8 to compute sample allocations later in this thesis.

Table 3.8. Estimated Proportion of HUs, $p_{ab}(d)$ ,in each MSG substratum Classified into HRS Domains, Unweighted

| MSG sampling substratum ($b$) | HRS analysis domains based on responses to screener and interview ($d$) | | | | | |
|---|---|---|---|---|---|---|
| | 1) 45-62 Hispanic | 2) 45-62 NH Black | 3) 45-62 NH Other | 4) Not 45-62 | 5)Unoccupied HU | All eligible* |
| 1 45-62 H | 0.617 | 0.025 | 0.102 | 0.184 | 0.073 | 0.743 |
| 2 45-62 NH Black | 0.010 | 0.542 | 0.222 | 0.147 | 0.079 | 0.774 |
| 3 45-62 NH Other | 0.029 | 0.107 | 0.649 | 0.137 | 0.078 | 0.785 |
| 4 45-62 No Race-Eth | 0.036 | 0.200 | 0.345 | 0.241 | 0.178 | 0.581 |
| 5 Not 45-62 | 0.058 | 0.082 | 0.117 | 0.612 | 0.132 | 0.256 |
| 6 Unknown | 0.073 | 0.069 | 0.081 | 0.438 | 0.339 | 0.223 |

*HRS domains 1-3

Table 3.9. Estimated Proportion of HUs, $p_{ab}(d)$ , in each MSG substratum Classified into HRS Domains, Weighted

| MSG sampling substratum ($b$) | HRS analysis domains based on responses to screener and interview ($d$) | | | | | |
|---|---|---|---|---|---|---|
| | 1) 45-62 Hispanic | 2) 45-62 NH Black | 3) 45-62 NH Other | 4) Not 45-62 | 5)Unoccupied HU | All eligible* |
| 1 45-62 H | 0.674 | 0.017 | 0.133 | 0.131 | 0.046 | 0.823 |
| 2 45-62 NH Black | 0.015 | 0.491 | 0.319 | 0.113 | 0.062 | 0.825 |
| 3 45-62 NH Other | 0.018 | 0.061 | 0.746 | 0.137 | 0.037 | 0.825 |
| 4 45-62 No Race-Eth | 0.017 | 0.144 | 0.529 | 0.200 | 0.110 | 0.690 |
| 5 Not 45-62 | 0.069 | 0.059 | 0.169 | 0.607 | 0.096 | 0.297 |
| 6 Unknown | 0.103 | 0.070 | 0.147 | 0.402 | 0.278 | 0.320 |

*HRS domains 1-3

Table 3.10 Weighted Accuracy Counts of MSG data when compared to 2010-2011 MSG Screener Data

| Age/Race Groups Identified by MSG sampling substratum (*b*) | HRS analysis domains based on responses to screener and interview, *d* | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1) 45-62 Hispanic | 2) 45-62 NH Black | 3) 45-62 NH Other | 4) Not 45-62 | 5)Unoccupied HU | All eligible (HRS domains 1-3) | No. Persons in Age/Race Groups | Percent |
| 1 45-62 H | 757,692 | 19,131 | 149,162 | 146,859 | 51,720 | 925,984 | 1,124,563 | 3.87 |
| 2 45-62 NH Black | 19,236 | 615,750 | 400,510 | 141,240 | 78,306 | 1,035,496 | 1,255,042 | 4.32 |
| 3 45-62 NH Other | 55,233 | 184,343 | 2,249,494 | 413,771 | 112,778 | 2,489,069 | 3,015,618 | 10.39 |
| 4 45-62 No Race-Eth | 28,038 | 244,054 | 897,260 | 338,877 | 187,070 | 1,169,353 | 1,695,299 | 5.84 |
| 5 Not 45-62 | 609,075 | 517,974 | 1,486,203 | 5,348,204 | 846,129 | 2,613,252 | 8,807,585 | 30.35 |
| 6 Unknown | 1,353,516 | 913,985 | 1,929,176 | 5,281,583 | 3,646,113 | 4,196,677 | 13,124,373 | 45.22 |
| Total | 2,822,790 | 2,495,236 | 7,111,805 | 11,670,533 | 4,922,116 | 12,429,831 | 29,022,480 | 100.00 |
| Percent | 9.73 | 8.60 | 24.50 | 40.21 | 16.96 | 42.83 | 100.00 | |

Table 3.11 Unweighted Accuracy Counts of MSG data when compared to 2010-2011 MSG Screener Data

| Age/Race Groups Identified by MSG sampling substratum (*b*) | HRS analysis domains based on responses to screener and interview, *d* | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1) 45-62 Hispanic | 2) 45-62 NH Black | 3) 45-62 NH Other | 4) Not 45-62 | 5)Unoccupied HU | All eligible (HRS domains 1-3) | No. Persons in Age/Race Groups | Percent |
| 1 45-62 H | 526 | 21 | 87 | 157 | 62 | 634 | 853 | 4.34 |
| 2 45-62 NH Black | 8 | 449 | 184 | 122 | 65 | 641 | 828 | 4.21 |
| 3 45-62 NH Other | 49 | 182 | 1,105 | 234 | 132 | 1,336 | 1,702 | 8.66 |
| 4 45-62 No Race-Eth | 48 | 267 | 461 | 322 | 238 | 776 | 1,336 | 6.80 |
| 5 Not 45-62 | 308 | 438 | 626 | 3,277 | 704 | 1,372 | 5,353 | 27.23 |
| 6 Unknown | 699 | 665 | 775 | 4,198 | 3,248 | 2,139 | 9,585 | 48.76 |
| Total | 1,638 | 2,022 | 3,238 | 8,310 | 4,449 | 6,898 | 19,657 | 100.00 |
| Percent | 8.33 | 10.29 | 16.47 | 42.28 | 22.63 | 35.09 | 100.00 | |

## 3.4 Estimating Totals, Means, and Variance Components from the 2010-11 HRS Interview Data

When determining the sample allocation, we would like to set a level of precision for key variables of interest. Recall our goal of utilizing accuracy rates to find an optimal sample allocation with the objective of minimizing the variance of some target estimate(s) subject to a variety of contraints. In this section, we consider the contributions of the different stages to the variance of an estimator in order to allocate a sample among the three stages of sampling. Valliant, Hubbard, et al. (2014) showed how to determine an allocation while achieving target sample sizes and minimizing costs. Here, we advance that work one more step by minimizing the variance for key variables given fixed costs using MSG substrata accuracy rates. In Section 3.4.1, we estimate the population totals for select HRS interview variables (see Appendix B.1). Then, in Section 3.4.2 we describe the imputation techniques used to satisfy the assumption that all *iajb* combinations in the 2010-2011 HRS data are nonempty in the population. Finally in Sections 3.4.3-3.4.4, we estimate the components of variance associated with the different stages of the sample design for those HRS interview variables using the two techniques found in the formulas of Sections 2.4.1-2.4.2, respectively. Variances will be presented in terms of the relvariance to reduce the variance components of differing dimensions and differing types of estimates (totals, means) to the same scale. All variance estimation was performed in R version 3.5.0.

### 3.4.1 *PWR* Estimates

In the following estimations, we used HRS selection probabilities that were proportional to full population housing unit counts (not just HUs that contain persons aged 45-62). The design-based properties of the weights allow us to get approximately unbiased estimates of means, proportions, and variance components for the domain of 45-62 year olds. Note that this analysis does not directly apply to the way HRS designs its samples to obtain HRS cohorts, because our data includes

HRS files from different years that span a broader age range than any HRS cohort. This is rather an illustration of how to design a three-stage sample that involves screening of HUs to determine eligibility using some HRS data as the basis for analysis.

To satisfy Eq. (2.6) and Eq. (2.35), estimates of overall population totals, population totals for analysis variables Y and their corresponding variance must be made separately for HUs contained in self-representing (SR) PSUs versus HUs contained in non-self-representing (NSR) PSUs. Because the data were collected over two years, some PSUs were sampled in both the 2010 sample and the 2011 sample and therefore had two distinct one-draw probabilities $p_i$ for 2010 and 2011. We treated the PSU *i* sampled in both 2010 and 2011 as two distinct PSUs: PSU *i* sampled in year 2010, and PSU *i* sampled in year 2011. There were a total of $m$ = 82 PSUs (54 NSR PSUs + 28 SR PSUs). The number of sample PSUs, sample SSUs, and screened HUs are displayed in Table 3.12. Overall, there were 454 SSUs (277 in NSR PSUs + 177 in SR PSUs). Of the 19,657 sample HUs, 12,933 were in the NSR PSUs and 6,724 were in the SR PSUs.

Table 3.13 shows the breakdown on the average number of SSUs selected from SSU strata $a$=2, 3, 4 across all PSUs, and the average number of HUs selected from MSG substrata $b$=1, …,6 within each stratum $a$, across all PSU/SSU $ij$ combinations, separately for NSR and SR PSU samples. Table 3.14 displays the average number of HUs screened from MSG substratum $b$ ($b$ = 01, 02, 03, 04, 05, 06) within SSU stratum $a$ ($a$ = 02, 03, 04), across all PSU/SSU $ij$ combinations, for SR PSUs and NSR PSUs. The actual HRS design did not include $b$ substrata, so when assigning HUs to the proposed MSG sampling substrata $b$, there were not enough HUs to span every $ab$ combination. The SSU strata/MSG substrata 0303[8] had a very low HU count. In fact, there were

[8] The "less than 10 percent Black, more than 10 percent Hispanic" SSU stratum, and the "one or more 45-62 non-Hispanic other persons" HU substratum

Table 3.12. Number of Sample PSUs, Sample SSUs, and Screened HUs, Overall and Separately for SR and NSR PSUs in HRS 2010-11

| | PSUs ($m$) | SSUs ($n$) | HUs Screened |
|---|---|---|---|
| NSR | 54 | 277 | 12,933 |
| SR | 28 | 177 | 6,724 |
| Total | 82 | 454 | 19,657 |

Table 3.13. Average number of SSUs selected from SSU stratum *a* across all SR and NSR PSUs in HRS 2010-11

| | SSU Stratum | $\bar{n}_{a,SR}$ | $\bar{n}_{a,NSR}$ |
|---|---|---|---|
| 02 | >10% B, <10% H | 2.11 | 2.71 |
| 03 | <10% B, >10% H | 4.00 | 4.26 |
| 04 | >10% B, >10% H | 3.48 | 2.24 |

Table 3.14. Average number of screened HUs within HU substratum *b* (*b* = 01, 02,…, 06) within SSU stratum *a* (*a* = 02, 03, 04) across all PSU/SSU *ij* combinations and population estimates of HUs, $\hat{Q}_{ab,SR}$ and $\hat{Q}_{ab,NSR}$ in HRS 2010-11

| SSU/MSG *ab* | Average No. of HUs in *ab* for SR PSUs | Average No. of HUs in *ab* for NSR PSUs | $\hat{Q}_{ab,SR}$ | $\hat{Q}_{ab,NSR}$ |
|---|---|---|---|---|
| 0201 | 1.40 | 1.80 | 15,515 | 50,441 |
| 0202 | 4.16 | 4.84 | 157,023 | 783,493 |
| 0203 | 6.17 | 7.62 | 171,502 | 963,377 |
| 0204 | 7.81 | 6.36 | 218,646 | 1,246,277 |
| 0205 | 14.79 | 14.75 | 785,866 | 4,295,112 |
| 0206 | 19.67 | 25.39 | 831,323 | 5,448,707 |
| 0301 | 3.56 | 5.43 | 233,359 | 276,933 |
| 0302 | 2.69 | 5.08 | 139,825 | 330,172 |
| 0303 | 0.00 | 1.00 | 0 | 2,248 |
| 0304 | 4.32 | 7.33 | 386,887 | 697,302 |
| 0305 | 7.48 | 14.78 | 774,329 | 1,968,936 |
| 0306 | 13.33 | 22.58 | 1,246,113 | 2,635,977 |
| 0401 | 4.49 | 3.76 | 151,555 | 107,901 |
| 0402 | 4.45 | 4.82 | 142,684 | 195,082 |
| 0403 | 5.06 | 4.73 | 135,176 | 80,117 |
| 0404 | 3.75 | 4.40 | 223,989 | 260,442 |
| 0405 | 12.06 | 13.61 | 699,568 | 787,112 |
| 0406 | 23.13 | 23.31 | 1,292,753 | 1,286,740 |

no HUs in the SR PSUs, and the average of 1.00 in the NSR PSUs represents only one HU in the NSR PSUs, resulting in very low estimated population counts. In practice, we want to sample some minimum number of HUs from each *ab*, but for purposes of our illustration we ignore this mishap and use the values as is. However, as discussed below, we will impute values for some terms in the variance components when sample sizes are inadequate to make direct estimates.

In 2010-2011, the HRS sample goals were to hit specified sample size targets for Black, Hispanic, and Other race-ethnicity groups. Through screening, each sample HU was classified into one of these groups. The HUs were then subsampled at rates designed to achieve the target sample sizes. This led to many HUs not being interviewed, particularly ones classified as Other. A consequence of this is that although all screened HUs can be categorized by which *ab* combination they are in, interview data on income, wealth, etc. were not collected on all HUs. This missing data issue must be dealt with for the analysis in this thesis.

Define the HRS sample data for a key variable of interest as the set of HUs that have HU level HRS interview data available for that specific variable. The sample sizes for selected HRS variables are listed in Table 3.15 separately for SR and NSR PSUs. Since some HUs had missing HU level *k* interview data for specific variables, HRS sample sizes varied slightly between variables.[9] Sample sizes ranged from 893 to 1,565 HUs. These sample sizes were not large enough to have observations that span across all combinations of PSUs, SSU strata, SSUs, and MSG substrata of the screened HUs. In fact, when assigning HUs to the proposed MSG sampling substrata *b*, many *iajb* combinations did not contain any sample HUs. For example, consider the income variable where 12,933 screened HUs in the NSR PSUs, formed 1,060 possible *iajb*

[9] Among HUs that were interviewed, HU level *k* responses were missing less than 8 percent of the time for all variables. Therefore, we did not impute at the HU level *k*.

combinations but only 615 of the combinations (58.02 percent) contained sample HUs that were interviewed. For the 6,724 screened HUs in the SR PSUs which formed 723 *iajb* combinations 406 (56.15 percent) contained interviewed sample HUs and 317 (43.85 percent) contained no sample HUs that were interviewed. Such empty combinations occurred because the entire *iajb* combination had no interviewed HUs.

Table 3.15. Sample counts, $\hat{t}_{pwr.alt}$ estimates of the population total, and $\hat{\bar{y}}_{s_1}$ mean estimates for selected HRS Interview Variables, by SR and NSR PSUs. See Appendix B.1 for explanation of the variables.

| Selected HRS Interview Variables | $q_{SR}$ | $q_{NSR}$ | $\hat{t}_{pwr.alt}$ | $\hat{t}_{pwr.alt,SR}$ | $\hat{t}_{pwr.alt,NSR}$ | $\hat{\bar{y}}_{s_{1,SR}}$ | $\hat{\bar{y}}_{s_{1,NSR}}$ | $\hat{F}_{SR}$ | $\hat{F}_{NSR}$ |
|---|---|---|---|---|---|---|---|---|---|
| income | 946 | 1527 | 1.37E+12 | 4.20E+11 | 9.55E+11 | 55,194 | 44,598 | 0.3053 | 0.6947 |
| wealtha | 946 | 1527 | 3.79E+12 | 1.19E+12 | 2.60E+12 | 156,587 | 121,247 | 0.3144 | 0.6856 |
| wealthb | 946 | 1527 | 4.04E+12 | 1.25E+12 | 2.78E+12 | 164,867 | 129,927 | 0.3107 | 0.6893 |
| other_debts | 893 | 1439 | 1.39E+07 | 3.58E+06 | 1.03E+07 | 0.47 | 0.48 | 0.2581 | 0.7419 |
| charity_donate | 898 | 1443 | 9.00E+06 | 2.46E+06 | 6.54E+06 | 0.32 | 0.31 | 0.2730 | 0.7270 |
| employed | 944 | 1520 | 1.49E+07 | 4.18E+06 | 1.07E+07 | 0.55 | 0.50 | 0.2804 | 0.7196 |
| ownHome | 922 | 1478 | 1.43E+07 | 3.55E+06 | 1.07E+07 | 0.47 | 0.50 | 0.2488 | 0.7512 |
| ownStock | 899 | 1442 | 4.95E+06 | 1.64E+06 | 3.31E+06 | 0.22 | 0.15 | 0.3316 | 0.6684 |
| own_2nd_home | 920 | 1478 | 2.96E+06 | 7.17E+05 | 2.24E+06 | 0.09 | 0.10 | 0.2424 | 0.7576 |
| own_transport | 903 | 1452 | 2.07E+07 | 4.90E+06 | 1.58E+07 | 0.64 | 0.74 | 0.2373 | 0.7627 |
| selfRatedHealth | 968 | 1565 | 1.16E+07 | 2.98E+06 | 8.58E+06 | 0.39 | 0.40 | 0.2580 | 0.7420 |

$q_{SR}$ and $q_{NSR}$ are counts of HUs that had interview data for specific variables

Table 3.16 Proportion of missing combinations needed to estimate totals and means across all *y* variables by design level. Percentages are based on all 19,537 HUs including those HUs not interviewed.

| Design Level | SR PSUs | | NSR PSUs | |
|---|---|---|---|---|
| | Total No. of Combinations in dataset | Range of Percent Missing | Total No. of Combinations in dataset | Range of Percent Missing |
| *iajb* | 723 | (43.5 - 45.8) | 1,060 | (40.9 - 43.0) |
| *iaj* | 177 | (13.6 - 14.7) | 277 | (16.2 -17.0) |
| *ia* | 56 | (1.8 - 3.6) | 94 | (9.6 - 10.6) |

Overall, the frequency of missing PSU, SSU strata, SSU, and MSG substrata totals, $\hat{t}_{iajb}$, and means, $\hat{\bar{y}}_{s_{iajb}}$, ranged from 40.9 percent to 45.8 percent for key variables (see Table 3.16). Totals and means at the PSU, SSU strata, SSU *iaj* level were missing 13.6 percent to 17.0 percent of the time, and at the PSU, SSU strata *ia* level 1.8 to 10.6 percent of the time. However, no imputation was necessary at the *iaj* or *ia* level since lower level imputation at the *iajb* level took care of those missing values.

Having an empty HU sample for an *iajb* combination is problematic because there is no data to estimate means or totals at certain levels of stratification, and thus no data to estimate variances. Having only one HU in an *iajb* combination is also problematic because only one sample HU is insufficient for variance estimation. This also extends to any single PSU *i*, PSU/SSU stratum *ia*, SSU *j* given PSU/SSU stratum *ia*, and SSU stratum/MSG substratum *ab* combination for which there are less than two sampled HUs. In the analyses for this thesis, we can assume that all *iajb* combinations occur in the population, even though some may be empty in the HRS 2010-11 data file. Values are imputed where necessary as described in Section 3.4.2. However, in order to avoid large amounts of imputation at the *iajb* level, we choose an alternative estimation for the *pwr* estimator in Eq. (1.6) as specified below. All screened HUs could be used to estimate the population counts of HUs in the SR and NSR PSUs. The smaller set of HUs that provided interview data could be used to estimate means per HU. Thus, we estimated population totals by multiplying estimated population counts by estimated population means:

$$\hat{t}_{pwr.alt} = \hat{t}_{pwr.alt,SR} + \hat{t}_{pwr.alt,NSR} \tag{2.1}$$

where

$$\hat{t}_{pwr.alt,SR} = \hat{Q}_{SR}\hat{\bar{y}}_{s_{1,SR}} \tag{2.2}$$

and

$$\hat{t}_{pwr.alt,NSR} = \hat{Q}_{NSR}\hat{\bar{y}}_{s_{1,NSR}} \tag{2.3}$$

where

$\hat{Q}_{SR} = 7,606,112$ is the estimated total number of HUs in the population of SR PSUs in SSU strata 2-4,

$\hat{Q}_{NSR} = 21,416,368$ is the estimated total number of HUs in the population of NSR PSUs in SSU strata 2-4 ,

$\hat{Q} = \hat{Q}_{SR} + \hat{Q}_{NSR} = 29,022,480$ is estimated total number of HUs in the population , and

$$\hat{\bar{y}}_{s_{1,SR}} = \frac{\sum_{i \in s_{1,SR}} \sum_{a} \sum_{j \in s_{ia}} \sum_{b} \sum_{k \in s_{iajb}} w_k y_k}{\sum_{i \in s_{1,SR}} \sum_{a} \sum_{j \in s_{ia}} \sum_{b} \sum_{k \in s_{iajb}} w_k} \text{ and } \hat{\bar{y}}_{s_{1,NSR}} = \frac{\sum_{i \in s_{1,NSR}} \sum_{a} \sum_{j \in s_{ia}} \sum_{b} \sum_{k \in s_{iajb}} w_k y_k}{\sum_{i \in s_{1,NSR}} \sum_{a} \sum_{j \in s_{ia}} \sum_{b} \sum_{k \in s_{iajb}} w_k}$$

are estimated means from HUs that reported an item like income for SR and NSR PSUs and $w_k$ is the weight for an HU defined in Section 2.3.1.

The $\hat{t}_{pwr.alt}$ estimate for each analysis variable is displayed in Table 3.15 along with the number of sample HUs selected from the SR and NSR PSUs and the values of $\hat{\bar{y}}_{s_{1,SR}}$ and $\hat{\bar{y}}_{s_{1,NSR}}$. Here we also display $\hat{F}_{SR}$ and $\hat{F}_{NSR}$, which will be estimated as

$$\hat{F}_{SR} = \frac{\hat{t}_{pwr.alt,SR}}{\hat{t}_{pwr.alt}} \qquad \text{and} \qquad \hat{F}_{NSR} = \frac{\hat{t}_{pwr.alt,NSR}}{\hat{t}_{pwr.alt}} \tag{2.4}$$

in Section 4.3.1 in the variance of the optimization problem.

### 3.4.2 Imputation of Missing Data

Empty combinations can occur either because the population itself contained no cases in a particular *iajb* or, by chance the sample contained no such cases even though there may have been some in the population. We impute values for different components in the variance formulas where necessary as described in the following sections.

#### 3.4.2.1 PSU, SSU Strata, SSU, MSG Substrata Level (*iajb*) Imputation

When computing $\hat{t}_{pwr}$ for use in variance formulas, we used Eq. (2.6) along with the imputation methods described below. This is unlike the alternative approach we took in Section 3.4.1 to estimate totals and means found in Table 3.15. The different approaches lead to very similar estimates of $\hat{t}_{pwr}$.

$\boldsymbol{\hat{\bar{y}}_{s_{iajb}}}$: For $\hat{\bar{y}}_{s_{iajb}}$ that were missing we used the mean of the $y_k$'s for SSU strata/MSG substrata *ab* to impute values for all variables. The SSU stratum/MSG substratum *0303* had zero sample HUs in the SR PSUs and only one sample HU in the NSR PSUs. Since there were no HUs in the SR PSUs in 0303, we used the imputed value of SSU stratum/MSG substratum *0403* for SSU stratum/MSG substratum 0303[10]. We used this same substitution method for other statistics when necessary.

$\boldsymbol{\hat{t}_{iajb}}$: For missing $\hat{t}_{iajb} = \hat{Q}_{iajb}\bar{y}_{s_{iajb}}$, we conducted the imputation in two ways. For continuous variables, we calculated $\hat{t}_{iajb}$ for those cases where $y_k$ was non-missing (and $\hat{\bar{y}}_{s_{iajb}}$ was not imputed) then used the median value of $\hat{t}_{iajb}$ in each SSU strata/MSG substrata *ab* for the imputation. We did this because the totals appeared more reasonable imputing at the $\hat{t}_{iajb}$ level than if we had used the imputed values at the $\hat{\bar{y}}_{s_{iajb}}$ level. However, for categorical 0-1 variables, we found that calculating $\hat{t}_{iajb}$ using the imputed values of $\hat{\bar{y}}_{s_{iajb}}$ was sufficient. For the one HU in the NSR PSUs in SSU stratum/MSG substratum 0303, we imputed the median value of $\hat{t}_{iajb}$ in SSU stratum 03.

$\boldsymbol{\hat{S}^2_{3iajb}}$: The variance $\hat{S}^2_{3iajb} = \frac{1}{q_{iajb}-1}\sum_{k \in s_{iajb}}\left(y_k - \bar{y}_{s_{iajb}}\right)^2$ was missing, on average, 68 percent for NSR PSUs and 73 percent of the time for SR PSUs. The variance was missing more often than for $\hat{t}_{iajb}$ because there were PSU/SSU strata/SSU/MSG substrata where there was only 1 HU in *iajb*. In such cases, the denominator of $\hat{S}^2_{3iajb}$ was zero, and thus the value was undefined. We imputed median and mean values of $\hat{S}^2_{3iajb}$ in SSU strata/MSG substrata *ab* for continuous and categorical variables, respectively.

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[10] We chose to use estimates from the SSU stratum/ MSG substratum 0403 as a replacement for those in 0303, because both belong to high Hispanic SSU strata ( ≥ 10% Hispanic) and both are contained in the same MSG substratum 03 “One or more 45-62 non -Hispanic Other persons in the HU.

$\hat{\boldsymbol{V}}_{\boldsymbol{3iajb}}$ **:** $\hat{V}_{3iajb} = q_{iajb}^{-1}\hat{Q}_{iajb}^{2}\hat{S}_{3iajb}^{2}$ was missing 41.9 to 45.8 percent of the time. We calculated $\hat{V}_{3iajb}$ for those cases where $q_{iajb}$ was non-missing and then used the median value of $\hat{V}_{3iajb}$ in each SSU strata/MSG substrata *ab* for the imputation.

### 3.4.2.2 SSU Strata, MSG Substrata Level (ab) Imputation

Since there were little to no HUs in SSU stratum/MSG substratum *0303*, we replaced the value of $\hat{W}_{3ab}^{2}$ for SSU stratum/MSG substratum *0303* with the value of $\hat{W}_{3ab}^{2}$ for SSU stratum/MSG substratum *0403*, for both SR and NSR PSUs,

### 3.4.2.3 PSU, SSU Strata, SSU Level (*iaj*) Imputation

Because we imputed at lower levels, all *iaj* combinations were non-missing when calculated and thus there was no imputation at this level.

### 3.4.2.4 PSU, SSU Strata Level *(ia)* Imputation

$\hat{\boldsymbol{S}}_{\boldsymbol{2\mathrm{A}ia}}^{\boldsymbol{2}}$ : The sample variance among estimated SSU totals in SSU strata $a|i$, $\hat{S}_{2\mathrm{A}ia}^{2}$, was missing 39.3% of the time when there was only one SSU in PSU *i*, SSU strata *a*, i.e., $n_{ia} = 1$, thus resulting in a zero in the denominator of the variance. When $\hat{S}_{2\mathrm{A}ia}^{2}$ was empty or very small (less than .0009), it was imputed using the median value of $\hat{S}_{2\mathrm{A}ia}^{2}$ for SSU stratum *a* across the PSUs with non-missing $\hat{S}_{2\mathrm{A}ia}^{2}$.

$\hat{\boldsymbol{S}}_{\boldsymbol{2(pwr)ia}}^{\boldsymbol{2}}$ : $\hat{S}_{2(pwr)ia}^{2}$ was negative 23 percent of the time for SR PSUs and 34 percent of the time for NSR PSUs, due to the subtraction, $\hat{S}_{2\mathrm{A}ia}^{2} - \hat{S}_{2\mathrm{B}ia}^{2}$. This is a known defect of ANOVA variance component estimators but may be exacerbated in this case by the imputations. The negative values were replaced with the minimum value of the non-negative values of $\hat{S}^2_{2(pwr)ia}$. In every variable but income, the value $\hat{S}^2_{2(pwr)ia} \big/ mp_i$ that makes up $\hat{W}^2_{2a}$ had substantial outliers. For outlier values above the 95 percent quantile, we replaced $\hat{S}^2_{2(pwr)ia} \big/ mp_i$ with the median value of $\hat{S}^2_{2(pwr)ia} \big/ mp_i$ for the non-outliers.

$\boldsymbol{\hat{S}^2_{Q_{ia}}}$ : $\hat{S}^2_{Q_{ia}}$ was missing 39.3 percent of the time when there was only one SSU in PSU *i*, SSU strata *a*, i.e., $n_{ia} = 1$, thus resulting in a zero in the denominator of the variance. When $\hat{S}^2_{Q_{ia}}$ was empty it was imputed using the median value of $\hat{S}^2_{Q_{ia}}$ for SSU stratum *a* across the PSUs with non-missing $\hat{S}^2_{Q_{ia}}$.

### 3.4.2.5 PSU Level (*i*) Imputation

The 2011 HRS sample was a subsample of the full HRS sample and was geared toward obtaining more cases in racial minority groups. Some PSUs were omitted in the subsample leading to some PSU-level missing data in 2011. Two of the NSR PSUs contained no sample HUs. Consider empty-PSU $i \in s_r$, the set of PSUs (both empty and non-empty) contained in Census Region *r*. The imputed value for the empty PSU-level estimate is the median estimate among the non-empty PSUs from the same Census Region *r*. The following estimates in the NSR PSUs were imputed using this method: $\hat{t}_{i(pwr)}$ , $\hat{\bar{y}}_i$, and $\hat{\hat{S}}^2_{3i}$. Outliers were a concern for the PSU level value $\sum_a \hat{S}^2_{2\mathbb{A}ia} \big/ n_{ia}$ that makes up $\hat{S}^2_{1(pwr)\mathbb{B}}$.

We replaced values that fell above the 90 percent quantile with the corresponding Census region median value of $\sum_a \hat{S}^2_{2\mathbb{A}ia} \big/ n_{ia}$ of the non-outliers.

### 3.4.3 Designed Based ANOVA Variance Components from the HRS Data

Since we want to know how we should design a survey prior to selecting the sample, it is customary to use design-based variances, which measure variability across all the samples that could be selected using a particular design. Design-based variance techniques measure the changes in the statistics of interest from different possible PSU, SSU, and element samples selected from the frame. The formulas in Section 2.4.1 were used to estimate the variance components directly from the 2010–11 HRS sample for different variables.

Table 3.17 shows the relvariance component estimates of $B^2$, $W^2$, $B^2+W^2$, $\delta_1$, $k_1$, and $\tilde{V}$ shown in Eqs. (1.29)-(1.34), for NSR PSUs by selected HRS interview variables. The between PSU variance component, $\hat{B}^2$, is small for all variables and negative for some. The variables `wealthb`, `other_debts`, `employed`, `own_2nd_home` and `own_transport` all have negative values of $\hat{B}^2$ and thus also for $\hat{\delta}_1$. As noted in Section 2.4.1, the estimates can be negative due to the subtraction term that occurs in the sample variances. When $\hat{B}^2$ is negative, it is likely this component is small. We will see in the next section how the anticipated variance can give better estimates of $\hat{B}^2$. Values of $\hat{W}^2$ are larger than $\hat{B}^2$ everywhere. In the case of variables `wealtha` (total wealth excluding secondary residence) and `wealthb` (total wealth including secondary residence), the value of $\hat{W}^2$ is much bigger than $\hat{B}^2$ implying that majority of the variance comes from within PSUs, i.e., the variance among HUs within PSUs is larger than the variance among PSU means per HU (or equivalently, among PSU totals).

Values of $\hat{\delta}_1$ range from -0.0448 to 0.0805, except for the variable `own_transport`. Small values of $\hat{\delta}_1$ agree with literature that the effect of clustering should be small in a population like this population for a *pps* design. Many of the values of $\hat{k}_1$ are near 1 which means the values of $\hat{B}^2 + \hat{W}^2$ are close to the unit relvariances of the population, $\hat{\hat{V}}$. `Income` has a $\hat{k}_1$ value of 0.6740 indicating that $\hat{B}^2 + \hat{W}^2$ is smaller than the unit relvariance of the population. `Own_transport` has a $\hat{k}_1$ value of 0.57 indicating that $\hat{B}^2 + \hat{W}^2$ is half the unit relvariance of the population. However, `own_transport` also has a negative value of $\hat{B}^2$ that is contributing to this smaller value of $\hat{k}_1$.

Table 3.17. Relvariance Component Estimates for Selected HRS Interview Variables from the 2010-2011 HRS, NSR PSUs

| HRS Interview Variables | NSR PSUs only | | | | | |
|---|---|---|---|---|---|---|
| | $\hat{B}^2$ | $\hat{W}^2$ | $\hat{B}^2 + \hat{W}^2$ | $\hat{\hat{V}}$ | $\hat{\delta}_1$ | $\hat{k}_1$ |
| `income` | 0.0815 | 0.9311 | 1.0126 | 1.5024 | 0.0805 | 0.6740 |
| `wealtha` | 0.0819 | 40.1710 | 40.2529 | 34.3650 | 0.0020 | 1.1713 |
| `wealthb` | -0.3722 | 35.7472 | 35.3749 | 30.6955 | -0.0150 | 1.1524 |
| `other_debts` | -0.0452 | 1.0561 | 1.0109 | 1.0833 | -0.0448 | 0.9332 |
| `charity_donate` | 0.0462 | 2.3140 | 2.3602 | 2.2765 | 0.0196 | 1.0368 |
| `employed` | -0.0310 | 0.8419 | 0.8109 | 0.9973 | -0.0382 | 0.8131 |
| `ownHome` | 0.0123 | 1.1159 | 1.1282 | 0.9971 | 0.0109 | 1.1315 |
| `ownStock` | 0.0541 | 4.9139 | 4.9680 | 5.4763 | 0.0109 | 0.9072 |
| `own_2nd_home` | -0.0507 | 8.0886 | 8.0379 | 8.5608 | -0.0063 | 0.9389 |
| `own_transport` | -0.1222 | 0.3303 | 0.2081 | 0.3599 | -0.5874 | 0.5781 |
| `selfRatedHealth` | 0.0130 | 1.2411 | 1.2541 | 1.4967 | 0.0104 | 0.8379 |

Other measures of homogeneity and estimates of variance components for both SR and NSR PSUs, such as $\hat{W}_{2a}^2$, $\hat{W}_{3ab}^2$, $\hat{W}_{3a}^2$, $\hat{\delta}_{2a}$, $\hat{k}_{2a}$ and $\tilde{V}_a$, found in Eqs. (1.29)-(1.38), are shown in Appendix B.3. Overall, relvariances are bigger in NSR PSUs than SR PSUs. This could be because of the spareness of the data in SSU strata or from anomalies in HRS data that should be investigated in practice first when redesigning the HRS data. Negative values of $\hat{W}_{2a}^2$ are highlighted in Table 5.3.

For NSR PSUs, sample sizes are smallest in SSU Stratum 04. Small sample sizes in SSU stratum 04 may result in very small sample sizes in PSU/SSU stratum *ia* which may result in bigger variances in SSU Stratum 04 for some variables, especially for those where it is not prevalent in that population. Note also that that SSU Stratum 04 is ≥ 10% Black, ≥ 10% Hispanic population which we would expect to be similar to each other on many of the HRS variables but may be hard to capture due to small sample sizes. In the NRS PSUs, `wealtha` and `wealthb` have fairly large estimates of $\hat{W}_{2a}^2$ indicating large variability among SSUs. However, the value of $\hat{W}_{3a}^2$ for `wealtha` and `wealthb` is much larger than $\hat{W}_{2a}^2$ implying that majority of the variance comes from within SSUs, i.e., the variance among HUs within SSUs is larger than the variance among SSU means per HU (or equivalently, among SSU totals). We expect our allocation to select more HUs inside of SSUs than more SSUs. Estimates of $\hat{\delta}_{2a}$ for `wealtha` and `wealthb` that are smaller than for other variables agree with this.

The above holds for all variables except `income` and `own_transport`. `Income` has the opposite effect with more variability for SSU stratum 02 and 04 (which are both ≥ 10% Black) coming from among SSUs. For income, the opposite was true. $\hat{W}_{2a}^2$ was bigger than $\hat{W}_{3a}^2$ implying that the variance among SSU means is larger than the variance among HUS within SSUs. HUs are more similar within SSUs on income than between SSUs. We may expect an allocation from the ANOVA relvariances for income to look different from other HRS variables, putting more emphasize on selecting more SSUs in those strata and less HUs per SSU. Estimates of $\hat{\delta}_{2a}$ range from (0.28-0.76) for `income` and (0.27-0.57) for `own_transport`. Using a univariate optimization problem would help us distinguish the best allocation to choose given a set of variables.

For SR PSUs, $\hat{W}_{3a}^2$ was bigger than $\hat{W}_{2a}^2$ in most cases. For `own_2nd_home` $\hat{W}_{2a}^2$ was negative everywhere for SR and NSR PSUs. The $\hat{W}_{3a}^2$ estimates for `own_transport` (0.050-0.058) and `other_debts` (0.115-0.164) were similar across SSU strata, while $\hat{W}_{3a}^2$ estimates of `wealtha` and `wealthb` were near 1. Overall, for NSR PSUs $\hat{\delta}_{2a}$ ranged from (-0.04 – 0.649). In Section 3.4.4.4, we compare differences to results found in ANOVA versus anticipated variances.

### 3.4.4 Anticipated Variance Components from the HRS Data

Because $\hat{B}^2$ and $\hat{W}_{2a}^2$ can be negative, alternative variance estimation techniques need to be examined and evaluated. One of those techniques is anticipated variances which uses model-based estimation. For the anticipated model estimates, we did not treat PSUs sampled from different years as distinct PSUs. Consequently, there are 16 SR PSUs defined in the model which is the actual number of SR PSUs in the HRS. The formulas in Section 2.4.3 are used to estimate the variance components directly from the 2010–11 HRS sample for different variables.

### 3.4.4.1 Model Selection

To arrive at the model selection of Eqs. (1.39) and (1.81), we considered three mixed models that had the same exact form with the distinction being that of the variance of residual term $\varepsilon_{iajbk}$. The three models change this variance in the following manner:

$Var_{M_1}(y_k) = \sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_{\varepsilon_{ab}}^2$, error term based on SSU strata/ MSG substrata

$Var_{M_2}(y_k) = \sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_{\varepsilon_a}^2$, error term based on SSU strata only

$Var_{M_3}(y_k) = \sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_{\varepsilon_b}^2$, error term based on MSG substrata only

where $M_1$, $M_2$, and $M_3$ denote Model 1, 2, and 3, respectively. for the three different models. All three models were fit using SAS `proc mixed` separately for SR and NSR PSUs for each HRS variable of interest.

Table 3.18 shows the results for several fit statistics for the continuous variables `income`, `wealtha`, and `wealthb`. The REML, residual (restricted) maximum likelihood, estimation method was used in all cases. For NSR PSUs Model 3, `income, wealtha,` and `wealthb` the estimation stopped because of too many likelihood evaluations. This was corrected by scaling `income`, `wealtha`, and `wealthb` by a factor of $1/1000$. To keep results on the same scale, the SR PSUs model was also scaled. The variance component estimates for Eqs. (1.97) - (1.107) produced by `proc mixed` are also scaled by a factor of $1/1000^2$. In Table 3.18, fit statistics with smaller numbers are better fits and the best fit is highlighted. Model 1, where the error term is based on both SSU stratum/MSG substratum *ab* the HU is in, i.e., $\varepsilon_{iajbk} \sim \left(0\,,\,\sigma_{\varepsilon_{ab}}^2\right)$, fit the data best 92 percent of the time for `income`, `wealtha`, and `wealthb`.

Table 3.19 shows the results for the fit statistics for selected 2010-11 HRS categorical variables. The REML, residual (restricted) maximum likelihood, estimation method was used in all cases. In Model 1 with SR PSUs, the estimation for `own_transport` stopped because of too many likelihood evaluations. This was due to all HUs in SSU strata/MSG substrata *0204* and *0304* having almost all the same value for `own_transport`. To correct this, we removed all HUs in SSU strata/MSG substrata *0204* and *0304* out of the model. In Table **3.19** the model with the best fit is highlighted. For both SR and NSR PSUs, Model 1, where the error term is based on both which SSU stratum/MSG substratum *ab*, was the best fit 47 percent of the time for NSR PSUs. Since Model 1 was the best fit for both SR and NSR PSUs, as well as both continuous and categorical variables, we selected this as our model.

Table 3.18 Fit Statistics for Selected Continuous Variables for 2010-11 HRS Data, SR and NSR PSUs

| Selected Continuous HRS Variables | NSR PSUs | | | SR PSUs | | |
|---|---|---|---|---|---|---|
| | Model 1 | Model 2 | Model 3 | Model 1 | Model 2 | Model 3 |
| | $\sigma^2_{\varepsilon_{ab}}$ | $\sigma^2_{\varepsilon_a}$ | $\sigma^2_{\varepsilon_b}$ | $\sigma^2_{\varepsilon_{ab}}$ | $\sigma^2_{\varepsilon_a}$ | $\sigma^2_{\varepsilon_b}$ |
| *Percent best fit* | *100%* | *0%* | *0%* | *92%* | *0%* | *8%* |
| `income` | | | | | | |
| -2 Res Log Likelihood | 16218.0 | 16405.8 | 16339.5 | 10317.9 | 10571.3 | 10365.7 |
| AIC | 16258.0 | 16415.8 | 16355.5 | 10385.9 | 10611.3 | 10411.7 |
| AICC | 16258.6 | 16415.9 | 16355.6 | 10388.5 | 10612.3 | 10412.9 |
| BIC | 16294.2 | 16424.8 | 16370 | 10493.9 | 10674.9 | 10484.8 |
| `wealtha` | | | | | | |
| -2 Res Log Likelihood | 21814.1 | 24229.4 | 23006.1 | 13341.7 | 13688.1 | 13531.0 |
| AIC | 21854.1 | 24239.4 | 23022.1 | 13377.7 | 13696.1 | 13545.0 |
| AICC | 21854.6 | 24239.5 | 23022.2 | 13378.4 | 13696.1 | 13545.1 |
| BIC | 21890.2 | 24248.4 | 23036.6 | 13434.9 | 13708.8 | 13567.2 |
| `wealthb` | | | | | | |
| -2 Res Log Likelihood | 21943.9 | 24358.6 | 23111.8 | 13487.5 | 13781.8 | 13680.7 |
| AIC | 21983.9 | 24368.6 | 23127.8 | 13523.5 | 13789.8 | 13694.7 |
| AICC | 21984.5 | 24368.7 | 23127.9 | 13524.3 | 13789.8 | 13694.8 |
| BIC | 22020.1 | 24377.7 | 23142.2 | 13580.7 | 13802.5 | 13717.0 |

Table 3.19 Fit Statistics for Selected Categorical Variables for 2010-11 HRS Data, SR and NSR PSUs

| Selected Categorical HRS Variables | NSR PSUs | | | SR PSUs | | |
|---|---|---|---|---|---|---|
| | $\sigma^2_{\varepsilon_{ab}}$ | $\sigma^2_{\varepsilon_a}$ | $\sigma^2_{\varepsilon_b}$ | $\sigma^2_{\varepsilon_{ab}}$ | $\sigma^2_{\varepsilon_a}$ | $\sigma^2_{\varepsilon_b}$ |
| *Percent best fit* | *47%* | *31%* | *22%* | *47%* | *28%* | *25%* |
| other_debts | | | | | | |
| -2 Res Log Likelihood | 2087.8 | 2088.0 | 2088.0 | 1320.4 | 1321.8 | 1321.1 |
| AIC | 2125.8 | 2096.0 | 2102.0 | 1356.4 | 1329.8 | 1335.1 |
| AICC | 2126.4 | 2096.1 | 2102.1 | 1357.2 | 1329.9 | 1335.3 |
| BIC | 2160.2 | 2103.3 | 2114.6 | 1413.5 | 1342.5 | 1357.4 |
| charity_donate | | | | | | |
| -2 Res Log Likelihood | 1777.3 | 1817.3 | 1785.3 | 1119.1 | 1149.4 | 1133.6 |
| AIC | 1817.3 | 1827.3 | 1801.3 | 1155.1 | 1157.4 | 1147.6 |
| AICC | 1817.9 | 1827.4 | 1801.4 | 1155.9 | 1157.4 | 1147.8 |
| BIC | 1853.5 | 1836.4 | 1815.8 | 1212.3 | 1170.1 | 1169.9 |
| employed | | | | | | |
| -2 Res Log Likelihood | 2148.0 | 2150.0 | 2149.2 | 1362.1 | 1364.1 | 1363.0 |
| AIC | 2188.0 | 2160.0 | 2165.2 | 1398.1 | 1372.1 | 1377 |
| AICC | 2188.6 | 2160.1 | 2165.3 | 1398.8 | 1372.2 | 1377.1 |
| BIC | 2224.1 | 2169.0 | 2179.6 | 1455.3 | 1384.8 | 1399.2 |
| ownHome | | | | | | |
| -2 Res Log Likelihood | 1914.8 | 1924.4 | 1922.6 | 1176.8 | 1185.3 | 1186.2 |
| AIC | 1954.8 | 1932.4 | 1936.6 | 1212.8 | 1193.3 | 1200.2 |
| AICC | 1955.4 | 1932.4 | 1936.6 | 1213.5 | 1193.4 | 1200.4 |
| BIC | 1990.9 | 1939.6 | 1949.2 | 1269.9 | 1206.0 | 1222.5 |
| ownStock | | | | | | |
| -2 Res Log Likelihood | 952.1 | 1098.2 | 980.2 | 710.6 | 780.6 | 734.9 |
| AIC | 992.1 | 1108.2 | 996.2 | 746.6 | 788.6 | 748.9 |
| AICC | 992.7 | 1108.3 | 996.3 | 747.4 | 788.7 | 749.0 |
| BIC | 1028.3 | 1117.2 | 1010.6 | 803.8 | 801.3 | 771.1 |
| own_2nd_home | | | | | | |
| -2 Res Log Likelihood | 260.2 | 336.8 | 363.8 | 258.8* | 335.3 | 308.1 |
| AIC | 300.2 | 346.8 | 379.8 | 294.8* | 343.3 | 322.1 |
| AICC | 300.8 | 346.8 | 379.8 | 295.5* | 343.4 | 322.2 |
| BIC | 336.3 | 355.8 | 394.2 | 351.9* | 356.0 | 344.3 |
| own_transport | | | | | | |
| -2 Res Log Likelihood | 1527.8 | 1643.8 | 1553.7 | 1070.4 | 1136.1 | 1116.2 |
| AIC | 1567.8 | 1653.8 | 1569.7 | 1102.4 | 1144.1 | 1130.2 |
| AICC | 1568.4 | 1653.8 | 1569.8 | 1103.1 | 1144.2 | 1130.3 |
| BIC | 1603.9 | 1662.8 | 1584.2 | 1153.2 | 1156.8 | 1152.4 |
| selfRatedHealth | | | | | | |
| -2 Res Log Likelihood | 2125.1 | 2148.9 | 2139.8 | 1339.0 | 1353.7 | 1344.6 |
| AIC | 2165.1 | 2158.9 | 2155.8 | 1375 | 1361.7 | 1358.6 |
| AICC | 2165.6 | 2158.9 | 2155.9 | 1375.7 | 1361.8 | 1358.8 |
| BIC | 2201.2 | 2167.9 | 2170.3 | 1432.2 | 1374.4 | 1380.9 |

*Does not include HUs in SSU strata/ MSG substrata 0204 and 0304

### 3.4.4.2 SAS Code for Fitting Random Effects Models

We use the `proc mixed` procedure in SAS to obtain estimates for the variances $\sigma_\alpha^2$, $\sigma_\gamma^2$, and $\sigma_{\varepsilon_{ab}}^2$, and the fixed effect mean $\mu$ for selected 2010-11 HRS variables for both SR and NSR PSUs. These values will be used in Section 3.4.4.4 to obtain the anticipated variance estimates by plugging them into the estimators of the model expectations from Section 2.4.3, $\hat{E}_M\left(B^2\right)$, $\hat{E}_M\left(W_{2a}^2\right)$, $\hat{E}_M\left(W_{3ab}^2\right)$, $\hat{E}_M\left(W^2\right)$, $\hat{E}_M\left(W_{3a}^2\right)$, $\hat{E}_M\left(\tilde{V}^2\right)$, and $\hat{E}_M\left(\tilde{V}_a^2\right)$, separately for SR and NSR PSUs. Since the SAS code for these estimates is fairly specialized we include it here.

*Non-Self Representing PSUs*

Recall from Section 2.4.2.1 the model for $y_k$ with common mean, $\mu$, and random effects for NSR PSUs, $\alpha_i$, SSUs, $\gamma_{iaj}$, and HUs in SSU/HU substratum *ab*, $\varepsilon_{iajbk}$:

$$y_k = \mu + \alpha_i + \gamma_{iaj} + \varepsilon_{iajbk}$$

with $\alpha_i \sim \left(0, \sigma_\alpha^2\right)$, $\gamma_{iaj} \sim \left(0, \sigma_\gamma^2\right)$, $\varepsilon_{iajbk} \sim \left(0, \sigma_{\varepsilon_{ab}}^2\right)$ and the errors being independent, such that

$$Var_M\left(y_k\right) = \sigma_\alpha^2 + \sigma_\gamma^2 + \sigma_{\varepsilon_{ab}}^2 \quad \text{and} \quad E_M\left(y_k\right) = \mu .$$

The corresponding SAS statement is shown in Figure 1.

**Figure 1** SAS MIXED Statement for NSR PSUs for 2010-11 HRS Data for income

```
proc mixed data=hrs.NSR noclprint noitprint covtest;
   class PSU_ID ssu_str SSU_ID  msg_str ;
   model income = /s;
   random int / subject = PSU_ID ;
   random int / subject = SSU_ID(PSU_ID*ssu_str) ;
   repeated /group =ssu_str*msg_str;
   run;
```

The proc mixed statement selects only the HRS data that belongs to the NSR PSUs "hrs.NSR". To save space in the SAS Output (shown in Appendix B.4) we included the NOCLPRINT, NOITPRINT, and COVTEST options. NOCLPRINT and NOITPRINT suppress the printing of information at the CLASS level and of the iteration history, respectively. COVTEST displays the hypothesis testing of the variance and covariance components. The CLASS statement names the classification variables to be used in the model. For our model the classification variables are PSU_ID, ssu_str, SSU_ID, and msg_str which correspond to PSUs, SSU strata (categorized by Black/Hispanic racial proportions), SSUs, and MSG/HU substratum (Hispanic ethnicity and EBB/MBB age categories), respectively.

The MODEL, RANDOM, and REPEATED statements together specify the model. The MODEL statement specifies the fixed effects and the RANDOM statement specifies the random effects. Since the intercept is our only fixed effect which is included by default, there are no variables after the equal sign in the MODEL statement. Additionally, the s option in the MODEL statement asks SAS to print the estimates for the fixed effects, i.e., $\mu$. The syntax in Figure 1 says that the dependent variable, income, is modeled by a fixed intercept, $\mu$, (by default when fixed-effects are included), a random effect (int in the random statement) for PSUs , $\alpha_i$, ("subject=PSU_ID") , a random effect for SSUs nested within PSUs and SSU strata, $\gamma_{iaj}$, ("subject=SSU_ID(PSU_ID*ssu_str)"), and a random error, $\varepsilon_{iajbk}$, that varies among groups in the REPEATED statement. The REPEATED group statement ("group=ssu_str*msg_str") allows for 18 differing estimates of the variance of the residual term, i.e., $\sigma^2_{\varepsilon_{ab}}$, in each SSU/MSG substrata *ab*.

**Figure 2** SAS MIXED Statement for SR PSUS for 2010-11 HRS Data for income

```
proc mixed data=hrs.SR cl noclprint noitprint covtest ;
   class PSU_ID ssu_str SSU_ID  msg_str HU_ID;
   model income = PSU_ID /s;
   random int / subject = SSU_ID(PSU_ID*ssu_str) ;
   repeated /group =msg_str*ssu_str;
   run;
```

*Self Representing PSUs*

Recall from Section 2.4.2.1 a model for $y_k$ with common mean, $\mu$, fixed effects for SR PSUs, $\alpha_i$, and random effects for SSUs, $\gamma_{iaj}$, and HUs in SSU/HU substratum a*b*, $\varepsilon_{iajbk}$:

$$y_k = \mu + \alpha_i + \gamma_{iaj} + \varepsilon_{iajbk}$$

with $\gamma_{iaj} \sim \left(0\,,\,\sigma_\gamma^2\right)$, $\varepsilon_{iajbk} \sim \left(0\,,\,\sigma_{\varepsilon_{ab}}^2\right)$, and the errors being independent, such that

$$Var_M\left(y_k\right) = \sigma_\gamma^2 + \sigma_{\varepsilon_{ab}}^2 \quad \text{and} \quad E_M\left(y_k\right) = \mu + \alpha_i \quad \text{for} \quad k \in U_{iajb}.$$

The corresponding SAS statement is shown above in Figure 2. The `proc mixed` statement selects only the HRS data that belongs to the SR PSUs "`hrs.SR`". For the SR PSU model the classification variables are the same as in NSR PSUs. The distinction for SR PSUs is the MODEL statement which specifies the fixed effects which are the intercept which is include by default and `PSU_ID`. Now, the `s` option in the MODEL prints the estimates for the fixed effects $\mu$ and $\alpha_i$. The syntax expresses that the dependent variable, `income`, is modeled by a fixed intercept, $\mu$, 16 fixed effects for the SR PSUs, $\alpha_i$, ("`model income2 = PSU_ID`") , a random intercept clustered by SSUs , $\gamma_{iaj}$, ("`subject = SSU_ID(PSU_ID*ssu_str)`"), and a random error, $\varepsilon_{iajbk}$, that varies among 18 groups in the REPEATED statement ("`group =ssu_str*msg_str`").

### 3.4.4.3 Anticipated Model Variance Parameter Estimates

The summary of the variance models are in Table **3.19** and Table 3.20 in this section and in Table 5.17 (see Appendix B.4). In some cases, variance components were not significantly different from zero or the sample data were too sparse to support estimation. In such cases, we used ad hoc values described below. Without assigning nonzero values to all components, the sample allocation algorithm in Section 4 would assign zero units for some HRS variables for some stages of sampling.

In both SR and NSR PSUs, the PSU variance estimates were all non-significant indicating that the random effect for PSUs does not play a large part in predicting the outcome of the dependent variables for a HU. Because the SSU stratum/MSG substratum *0303* had zero sample HUs in the SR PSUs and only one sample HU in the NSR PSUs, the estimates for SSU stratum/MSG substratum *0303* were replaced with the estimates in SSU stratum/MSG substratum *0403* everywhere. The estimate of $\sigma_\alpha^2$ for `other_debts` was 0 for NSR PSUs. To correct for this, the minimum value of $\hat{\sigma}_\alpha^2$ for NSR PSUs (which was the .0001 for `own_2nd_home)` was used as a replacement estimate for `other_debts`. For SR PSUs, for `own_transport`, HUs in SSU stratum/MSG substratum *0204* and *0304* were removed when fitting the model with SR PSUs.

The estimates of Tables Table **3.19**, Table 3.20, and Table 5.17 will be used in the Section 3.4.4.4 to obtain the anticipated variance component estimates by plugging them into the estimators of the model expectations from Section 2.4.3, $\hat{E}_M\left(B^2\right)$ , $\hat{E}_M\left(W_{2a}^2\right)$, $\hat{E}_M\left(W_{3ab}^2\right)$, $\hat{E}_M\left(W^2\right)$, $\hat{E}_M\left(W_{3a}^2\right)$, $\hat{E}_M\left(\tilde{V}^2\right)$, and $\hat{E}_M\left(\tilde{V}_a^2\right)$, separately for SR and NSR PSUs.

Table 3.20 Variance Component Estimates, $\hat{\mu}$, $\hat{\sigma}_{\alpha}^{2}$, $\hat{\sigma}_{\gamma}^{2}$, for NSR PSUs for selected 2010-11 HRS variables.

| Parameter | NSR PSUs only | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | income | wealtha | wealthb | other debts | charity donate | employed | ownHome | ownStock | own 2nd home | own transport | self Rated Health |
| Intercept $\hat{\mu}$ | 44.36 | 85.66 | 88.22 | 0.4818 | 0.2927 | 0.5493 | 0.5303 | 0.1409 | 0.0737 | 0.7787 | 0.3611 |
| PSU $\hat{\sigma}_{\alpha}^{2}$ | 37.14 | 228.57 | 339.05 | 0.0001*† | 0.0031† | 0.0022† | 0.0004† | 0.0016† | 0.0001† | 0.0034† | 0.0043† |
| SSU $\hat{\sigma}_{\gamma}^{2}$ | 555.76 | 5659.90 | 4667.90 | 0.0082 | 0.0231 | 0.0245 | 0.0709 | 0.0187 | 0.0043 | 0.0414 | 0.0168 |

(`income`, `wealtha`, `wealthb` are in thousands of dollars)

*NSR PSU estimate from own 2nd home used as a replacement for other debts

†Not significantly different from 0

Table 3.21 Variance Component Estimates, $\hat{\mu}$, $\hat{\sigma}_{\gamma}^{2}$, for SR PSUs for selected 2010-11 HRS variables.

| Parameter | SR PSUs only | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | income | wealtha | wealthb | other debts | charity donate | employed | ownHome | ownStock | own 2nd home | own transport* | self Rated Health |
| Intercept $\hat{\mu}$ | 66.17 | 187.34 | 205.02 | 0.5382 | 0.3716 | 0.5787 | 0.5419 | 0.3259 | 0.1140 | 0.7349 | 0.2775 |
| SSU $\hat{\sigma}_{\gamma}^{2}$ | 3.44E+02 | 1.10E+04 | 1.24E+04 | 0.0025 | 0.0136 | 0.0181 | 0.0822 | 0.0229 | 0.0029 | 0.0190 | 0.0202 |

(`income`, `wealtha`, `wealthb` are in thousands of dollars)

*Did not include HUs in SSU strata/ MSG substrata 0204 and 0304

### 3.4.4.4 Anticipated Relvariance Component Estimates

Table 3.22 shows the relvariance component estimates of $B^2$, $W^2$, $B^2+W^2$, $\delta_1$, $k_1$, and $\tilde{V}$ shown in Eqs. (1.97) - (1.103) for NSR PSUs by selected HRS interview variables. The between PSU variance component, $\hat{B}^2$, is small for all variables. The negative ANOVA values of $\hat{B}^2$ for the variables `wealthb`, `other_debts`, `employed`, `own_2nd_home` and `own_transport` have been corrected to be positive and thus also for $\hat{\delta}_1$. We noted in Section 2.4.1, that when the ANOVA estimate for $\hat{B}^2$ is negative, it is likely this component is small. The anticipated variance does indeed give better non-negative estimates of $\hat{B}^2$ that are small. Values of $\hat{W}^2$ remain larger than $\hat{B}^2$ everywhere.

Table 3.22. Anticipated Relvariance Component Estimates for Selected HRS Interview Variables from the 2010-2011 HRS, NSR PSUs

| HRS Interview Variables | NSR PSUs only | | | | | |
|---|---|---|---|---|---|---|
| | $\hat{B}^2$ | $\hat{W}^2$ | $\hat{B}^2+\hat{W}^2$ | $\hat{\tilde{V}}$ | $\hat{\delta}_1$ | $\hat{k}_1$ |
| `income` | 0.0186 | 1.051 | 1.0701 | 1.4800 | 0.0174 | 0.7231 |
| `wealtha` | 0.0528 | 55.968 | 56.0212 | 27.0051 | 0.0009 | 2.0745 |
| `wealthb` | 0.0538 | 48.643 | 48.6965 | 24.6216 | 0.0011 | 1.9778 |
| `other_debts` | 0.0019 | 1.173 | 1.174 | 1.0800 | 0.0016 | 1.0875 |
| `charity_donate` | 0.0460 | 2.410 | 2.456 | 2.1590 | 0.0187 | 1.1378 |
| `employed` | 0.0115 | 0.986 | 0.997 | 1.0010 | 0.0115 | 0.9964 |
| `ownHome` | 0.0124 | 1.356 | 1.368 | 1.0280 | 0.0090 | 1.3310 |
| `ownStock` | 0.0861 | 4.917 | 5.004 | 5.0820 | 0.0172 | 0.9846 |
| `own_2nd_home` | 0.0080 | 7.878 | 7.886 | 6.7840 | 0.0010 | 1.1624 |
| `own_transport` | 0.0092 | 0.450 | 0.459 | 0.4180 | 0.0202 | 1.0975 |
| `selfRatedHealth` | 0.0303 | 1.557 | 1.587 | 1.5500 | 0.0191 | 1.0239 |

Values highlighted were negative in ANOVA but now corrected to non-zero values through anticipated variances

Values of $\hat{\delta}_1$ for anticipated estimates range from about 0.001 to 0.020. Small values of $\hat{\delta}_1$ agree with the ANOVA estimates. Many of the values of $\hat{k}_1$ are near 1 which means the values of $\hat{B}^2 + \hat{W}^2$ are close to the unit relvariances of the population, $\hat{\hat{V}}$. Table 3.23 compares the anticipated variance estimates to the earlier ANOVA estimates. We notice that values of $\hat{k}_1$ for `employed`, `own_transport` and `selfRatedHealth` which were far from 1 in ANOVA are now nearing 1, indicating that anticipated variances did a good job of correcting the negative values of ANOVA. `Own_transport` which has a $\hat{k}_1$ value of 0.57 for ANOVA now has a value of 1.0975. This change happened because the negative value of $\hat{B}^2$ that was contributing to a smaller value of $\hat{k}_1$ in ANOVA is now positive. The ANOVA value of $\hat{k}_1$ 0.8131 for `employed` is now 0.9964 when using anticipated variances. `Income` has a $\hat{k}_1$ value of 0.723 indicating that $\hat{B}^2 + \hat{W}^2$ is smaller than the unit relvariance of the population. For `wealtha` and `wealthb`, $\hat{k}_1$ increased nearly to 2.

Table 3.23. Comparison of ANOVA and Anticipated Relvariance Component Estimates, $\hat{B}^2$, $\hat{W}^2$, $\hat{\delta}_1$, and $\hat{k}_1$ for Selected HRS Interview Variables from the 2010-11 HRS, NSR PSUs

| HRS Interview Variables | ANOVA | | | | ANTICIPATED | | | |
|---|---|---|---|---|---|---|---|---|
| | $\hat{B}^2$ | $\hat{W}^2$ | $\hat{\delta}_1$ | $\hat{k}_1$ | $\hat{B}^2$ | $\hat{W}^2$ | $\hat{\delta}_1$ | $\hat{k}_1$ |
| `income` | 0.082 | 0.93 | 0.081 | 0.674 | 0.019 | 1.052 | 0.017 | 0.723 |
| `wealtha` | 0.082 | 40.17 | 0.002 | 1.171 | 0.053 | 55.97 | 0.001 | 2.075 |
| `wealthb` | -0.372 | 35.75 | -0.015 | 1.152 | 0.054 | 48.64 | 0.001 | 1.978 |
| `other_debts` | -0.045 | 1.06 | -0.045 | 0.933 | 0.002 | 1.17 | 0.002 | 1.088 |
| `charity_donate` | 0.046 | 2.31 | 0.020 | 1.037 | 0.046 | 2.41 | 0.019 | 1.138 |
| `employed` | -0.031 | 0.84 | -0.038 | 0.813 | 0.012 | 0.99 | 0.012 | 0.996 |
| `ownHome` | 0.012 | 1.12 | 0.011 | 1.132 | 0.012 | 1.36 | 0.010 | 1.331 |
| `ownStock` | 0.054 | 4.91 | 0.011 | 0.907 | 0.086 | 4.92 | 0.017 | 0.985 |
| `own_2nd_home` | -0.051 | 8.09 | -0.006 | 0.939 | 0.008 | 7.88 | 0.001 | 1.162 |
| `own_transport` | -0.122 | 0.33 | -0.587 | 0.578 | 0.009 | 0.45 | 0.020 | 1.098 |
| `selfRatedHealth` | 0.013 | 1.24 | 0.010 | 0.838 | 0.030 | 1.56 | 0.019 | 1.024 |

Other measures of homogeneity and estimates of variance components for both SR and NSR PSUs, such as $\hat{W}_{2a}^2$, $\hat{W}_{3ab}^2$, $\hat{W}_{3a}^2$, $\hat{\delta}_{2a}$, $\hat{k}_{2a}$ and $\tilde{V}_a$, found in Eqs. (1.29) - (1.38), are shown in Appendix B.5.

The anticipated variance method corrected negative values of $\hat{W}_{2a}^2$. We can now see for `own_2nd_home`, $\hat{W}_{2a}^2$ ranges from 0.3474 to 0.4755 for SR PSUs, while in the NSR PSUs show more variability in `own_2nd_home` with values of 1.4 in SSU stratum *03* and 3.17 in SSU stratum 04. Overall, the SR and NSR estimates obtained using anticipated variances were smaller more similar to each other for the categorical variables than they were using ANOVA. However, for `wealtha`, and `wealthb`, both methods had much higher values for $\hat{W}_{2a}^2$ in NSR PSUs than in SR PSUs. In addition, there are some discrepancies in the variance estimation for $\hat{W}_{2a}^2$ across the ANOVA and anticipated methods. This should be studied further when designing a survey.

For NSR PSUs, $\hat{W}_{3a}^2$ for `income` and categorical variables were similar to ANOVA in all SSU strata. For `wealtha` and `wealthb`, SSU stratum *03* still had smaller variance in comparison to the other SSU strata; however, estimates for $\hat{W}_{3a}^2$ increased significantly in size (e.g., for `wealtha` in NSR PSUs in SSU stratum *04* increased from 82.9 for ANOVA to 1573.9 for anticipated). This is probably due to outliers in SSU stratum *04.* Results for $\hat{W}_{3ab}^2$ were similar for ANOVA and anticipated variances in Table 5.9 and Table 5.23. In NSR PSUs, $\hat{W}_{3ab}^2$ estimates were very large in some cases. Some variables such as `wealtha`, `wealthb`, `ownStock`, `own_2nd_home`, with large values of $\hat{W}_{3ab}^2$ (e.g., `wealtha` has a value of 11426.92 for $\hat{W}^2_{3ab,NSR}$ in Table 5.24). These large values may be due to a variety of reasons such as (i) instability due to a very low sample size in an *ab* substratum (for HRS this was sometimes less than 3), (ii) outliers, or a combination of both of these factors. As noted earlier, these matters should be studied further in practice when redesigning any multistage sample.

# 4 The Optimization Problem

Chapter 4 describes how to optimally allocate a three-stage sample using mathematical programming methods. Certainty and non-certainty PSUs are handled separately subject to a cost constraint.

## 4.1 Introduction

When designing the sample described in Section 2.3 it is necessary to determine how many PSUs to select and how many sample units to allocate to each SSU stratum and HU substratum subject to a total fixed cost. We can determine this allocation based on a single estimate (univariate), as we will demonstrate in Section 4.3, or for a set of estimates (multivariate). Generally, a national survey, such as HRS, wishes to make estimates for many analysis variables $l$ (e.g., income, employment status, etc.) and subgroups (or domain $d$) (e.g., Blacks, age groups, etc.). This type of allocation problem is less straightforward than if only a single estimate is desired, considering that compromises must be made to find an allocation that will give acceptable levels of precision for all target estimates.

In Section 4.2, we develop a cost function associated with collecting data for the three-stage sample design in the HRS data. In Section 4.3, an optimization problem is formulated that finds an optimal sample allocation with the objective of minimizing the approximate relvariance of a single target estimate for a total fixed cost subject to a variety of constraints. A more complex approach is to minimize a weighted sum of the relvariances of estimates of target variables, where the variables are weighted according to their degree of importance to the goals of the survey. We formulate the multivariate optimization problem in Section 4.3.2. The relvariance is used to reduce the variance components of differing dimensions and differing types of estimates (totals, means) to the same scale.

The Solver tool in Microsoft Excel uses nonlinear programming to find solutions to problems with “up to 200 decision variables and constraints on up to 100 cells in the spreadsheet” (Valliant, Dever, & Kreuter, 2013). All solutions to the optimal allocation problems are found using Excel Solver.

## 4.2 Cost Functions

### 4.2.1 General Cost Function

Consider the costs associated with collecting data in the three-stage sample design of Section 2.3. Assume there are costs per sample PSU, sample SSU in stratum *a*, and sample HU in substratum *ab*, denoted as $C_1$, $C_{2a}$, and $C_{3ab}$ respectively. Then a simple cost function is

$$C = C_0 + C_1 m + \sum_{a=1}^{A} C_{2a} m \bar{n}_a + \sum_{a=1}^{A} \sum_{b=1}^{B} C_{3ab} m \bar{n}_a \bar{\bar{q}}_{ab} \qquad (3.1)$$

where $C_0$ denotes fixed costs that do not depend on the number of sample PSUs, SSUs, or HUs, $\bar{n}_a$ is the mean number of sample SSUs allocated to SSU stratum *a*, and $\bar{\bar{q}}_{ab}$ is the mean number of HUs allocated to substratum *ab*.

We seek to find the optimal allocation that minimizes the relvariance in Eq. (1.16) , subject to a total cost $C - C_0$ and to a list of constraints described in Section 4.3 . This allocation problem does not have a closed form solution but can be solved using nonlinear programming methods.

### 4.2.2 Self-Representing (SR) and Non-Self-Representing (NSR) PSUs

Because the HRS data used in this analysis contains both SR and NSR PSUs, the optimal solution components must be calculated separately. The cost function that considers both SR and NSR PSUs is

$$\begin{aligned} C - C_0 &= C_1(m_{SR} + m_{NSR}) + \sum_{a=1}^{A} C_2\left(m_{SR}\bar{n}_{a,SR} + m_{NSR}\bar{n}_{a,NSR}\right) \\ &+ \sum_{a=1}^{A}\sum_{b=1}^{B} C_3\left(m_{SR}\bar{n}_{a,SR}\bar{\bar{q}}_{ab,SR} + m_{NSR}\bar{n}_{a,NSR}\bar{\bar{q}}_{ab,NSR}\right) \end{aligned} \tag{3.2}$$

*Non Self-Representing (NSR) PSUs*

The optimal allocations for NSR PSUs will be found using nonlinear programming with the decision variables being with $m_{opt} = m_{opt,NSR} =$ the optimal number of NSR PSUs to select, and the optimal sample sizes of SSUs to select per SSU stratum *a*, $\bar{n}_{opt,a} = \bar{n}_{opt,a,NSR}$, and the optimal number of HUs to select per SSU stratum *a*, HU substratum *b*, $\bar{\bar{q}}_{opt,ab} = \bar{\bar{q}}_{opt,ab,NSR}$, are within NSR PSUs only.

*Self-Representing (SR) PSUs*

When there are not a fixed number of certainty PSUs, the optimal allocation problem is also complicated because there is no closed form solution. One could partially resolve this complication by allocating a fixed number of SR PSUs. Even with this simplification, the numbers of sample SSUs and HUs must still be determined. In our demonstration, we utilize a nonlinear programming algorithm in Excel Solver to complete this complex allocation problem.

## 4.3 Optimization Problem

To write down the mathematical formalization of the optimization problem, first define:

$S_E$ = a set of $L$ estimates for which target levels of precision are desired

$\theta_l$ = the population value (e.g., total or mean) of analysis variable $l$

$\hat{\theta}_l$ = the estimate of $\theta_l$ (which will be estimated using Eq. (1.6) for totals)

$\hat{\theta}_{ld}$ = the estimate of analysis variable $l$ in domain $d$

$\omega_l$ = the importance weight for analysis variable $l$

$CV^2(\hat{\theta}_l)$ = the relvariance of $\hat{\theta}_l$ (which is defined by Eq. (1.16) for totals)

$CV(\hat{\theta}_l) = \sqrt{CV^2(\hat{\theta}_l)}$ = the *coefficient of variation* of $\hat{\theta}_l$

$CV(\hat{\theta}_{ld}) = \sqrt{CV^2(\hat{\theta}_{ld})}$ = the *coefficient of variation* of $\hat{\theta}_{ld}$

The importance weights are based on the subjective judgment of the survey designer about how important particular estimates are to a survey.

### 4.3.1 Univariate Optimization Problem

We formed the univariate optimization problem as follows:

Find $\left\{m_{NSR},\ \bar{n}_{a,SR},\ \bar{n}_{a,NSR},\ \bar{\bar{q}}_{ab,SR},\ \bar{\bar{q}}_{ab,NSR};\ a=1,\ldots,A,\ b=1,\ldots,B\right\}$ that

minimizes the approximate relvariance which is defined by Eq. (1.33) (i.e., the objective function),

$$\begin{aligned}\phi &= \hat{CV}^2\left(\hat{\theta}_l\right)\\ &= \hat{F}_{SR}^2 \frac{v\left(\hat{t}_{pwr,SR}\right)}{\hat{t}_{pwr,SR}^2} + \hat{F}_{NSR}^2 \frac{v\left(\hat{t}_{pwr,NSR}\right)}{\hat{t}_{pwr,NSR}^2}\end{aligned} \tag{3.3}$$

where in order to solve the optimization problem, relvariances, $\hat{CV}^2\left(\hat{\theta}_l\right)$, were evaluated based on estimated variance components, $\hat{F}_{SR}^2 = \left(\frac{\hat{t}_{pwr.alt,SR}}{\hat{t}_{pwr.alt}}\right)^2$, and $\hat{F}_{NSR}^2 = \left(\frac{\hat{t}_{pwr.alt,NSR}}{\hat{t}_{pwr.alt}}\right)^2$, subject to the constraints:

(1) **Fixed SR PSU sample size:** $m_{SR} = 16$, We choose to fix the total number of sample SR PSUs, mimicking the idea that an allocation is being made to an existing PSU sample. The number of SR PSUs in the HRS is actually 16.

(2) **Minimum NSR PSU sample size:** $m_{NSR} \geq 25$ , a lower bound on the total number of sample NSR PSUs

(3) **Maximum SSU strata sample size**: $\bar{n}_{a,SR}, \leq \min\{ N_{ia} \mid i = 1, \dots, m_{SR}\}$ $and$ $\bar{n}_{a,NSR}, \leq \min\{ N_{ia} \mid i = 1, \dots, m_{NSR}\}$ for all $a$, i.e., $\bar{n}_a$ is bounded above by the value for $N_{ia}$ of the SR/NSR PSU that has the minimum number of SSUs in the population for SSU stratum $a$

(4) **Minimum SSU strata sample size:** $\bar{n}_{a,SR}$, $\bar{n}_{a,NSR} \geq 2$ for all $a$, i.e., a minimum number of SSUs sampled per SR/NSR PSU from SSU stratum $a$ (in general $\bar{n}_{a,\min} \geq 2$)

(5) **Maximum HU substrata sample size**: $\bar{\bar{q}}_{ab,SR} \leq \min\{Q_{iajb} \mid i = 1, \dots, m_{SR}, j = 1, \dots, n\}$ and $\bar{\bar{q}}_{ab,NSR} \leq \min\{Q_{iajb} \mid i = 1, \dots, m_{NSR}, j = 1, \dots, n\}$ for all $ab$, i.e., $\bar{\bar{q}}_{ab}$ is bounded above by the value for $Q_{iajb}$ of the SR/NSR PSU/SSU $ij$ that has the minimum number of HUs in the population for SSU stratum/HU substratum $ab$

(6) **Minimum HU substrata sample size**: $\bar{\bar{q}}_{ab,SR}$, $\bar{\bar{q}}_{ab,NSR} \geq 2$ for all $ab$, i.e., a minimum number of HUs sampled per SR/NSR PSU/SSU $ij$ from SSU strata/ HU substrata $ab$ $\left(\text{in general } \bar{\bar{q}}_{ab,min} \geq 2\right)$

(7) **Minimum and Maximum sample size of HUs per PSU:**

$50 \leq \sum_a \sum_b \bar{n}_{a,SR} \bar{\bar{q}}_{ab,SR}, \ \sum_a \sum_b \bar{n}_{a,NSR} \bar{\bar{q}}_{ab,NSR} \leq 100$, i.e., a minimum and maximum number of HUs sampled per SR/NSR PSU $i$

(8) **Maximum HU sample size:** $q_{SR} \leq Q_{SR} = 7,606,112$ and $q_{NSR} \leq Q_{NSR} = 21,416,368$, i.e., the number of sample HUs for the SR and NSR PSU sample cannot be more than the number of HUs in the population for SR and NSR PSUs, respectively

(9) **Fixed costs:** Assume that the cost per sample SSU is the same in every substratum *a* and that the cost per sample HU is the same for every substratum *ab*. Define the costs at each stage of sampling as

$$\bar{C} = \left( C_1 = \$35,000, \quad C_2 = \$2,600, C_3 = \begin{cases} \$850, & \text{Occupied HU} \\ \$150, & \text{Unoccupied HU} \end{cases} \right) \tag{3.4}$$

such that

$$\begin{aligned} C - C_0 = {} & C_1(m_{SR} + m_{NSR}) + \sum_{a=1}^{A} C_2 \left( m_{SR}\bar{n}_{a,SR} + m_{NSR}\bar{n}_{a,NSR} \right) \\ & + \sum_{a=1}^{A}\sum_{b=1}^{B} C_3 \left( m_{SR}\bar{n}_{a,SR}\bar{\bar{q}}_{ab,SR} + m_{NSR}\bar{n}_{a,NSR}\bar{\bar{q}}_{ab,NSR} \right) \\ & \leq C_{tot} \end{aligned}$$

where $C_{tot}$ is the total budget for costs that vary with sample sizes. For this optimization problem we set $C_{tot} = \$10\ million$.

(10) **Target sample sizes for analytical domains** $d$=(1=45-62 Hispanic; 2=45-62 NH Black; 3=45-62 NH Other) **that account for inaccuracy of listings due to commercial list data**: The expected number of sample HUs found to be eligible by being in HRS analytical domain $d$ | $d$=1,..,3 (1=45-62 Hispanic; 2=45-62 NH Black; 3=45-62 NH Other; and two other domains: 4=Not 45-62; 5=Unoccupied HU) is

$$q(d) = \sum_{a=1}^{A} \sum_{b=1}^{B} q_{ab} p_{ab}(d)$$

where

$q_{ab} = m\bar{n}_a \bar{\bar{q}}_{ab}$ , the number of HUs allocated to SSU /HU substratum $ab$

$p_{ab}(d)$ = the proportion of HUs in SSU stratum/HU substratum $ab$ that are correctly identified by the commercial list data as being in domain $d$ (i.e., the accuracy rate of the commercial list data for domain $d$ in SSU strata/HU substratum $ab$). For this analysis, we used the unweighted accuracy rates, $p_{ab}(d)$, from Table 3.8.

The constraint is to set $q(d) = q_0(d)$, the target number of sample HUs for each domain $d$. This will allocate $q_{ab}$ to SSU stratum/HU substratum $ab$ while accounting for inaccuracy of commercial list data.

Instead of expected target sample sizes for domain $d$, we used the expected proportion of sample HUs allocated to domain $d$

$$\frac{q(d)}{\sum_{d=1}^{3} q(d)} \tag{3.5}$$

The constraint is to set $\frac{q(d)}{\sum_{d=1}^{3} q(d)} \geq .30$, such that the sample size of HUs for each domain $d$ is spread more or less evenly throughout the total sample.

(11) **Maximum design effects for weights:**

$deff_k = 1 + relvar(base\ sampling\ weights) \leq D_0 = 1.75$, a bound on weighting design effects.

Constraining the variability of the weights is a standard technique in sampling and helps reduce the variance of full population estimates (Kish 1965).

Although not used here, additional constraints that might be used in some problems are the following:

(12) **Maximum design effects for weights of domains:**

$deff_{kd} = 1 + relvar_d(base\ sampling\ weights) \leq D_{0d} = 1.75$, a bound on weighting design effects for each domain $d$.

(13) **Target coefficient of variations for estimates of domains:**

$CV(\hat{\theta}_{ld}) \leq CV_0(\hat{\theta}_{ld})$ for all $\hat{\theta}_{ld} \in S_E$, i.e., the *coefficient of variation* of an estimate $\hat{\theta}_{ld}$ is bounded above for all $\hat{\theta}_{ld}$ in some set of estimates that have desired precision targets.

### 4.3.2 Multivariate Optimization Problem

The optimal allocation will be different for different variables and a compromise needs to be made. To accomplish this, we minimize the weighted average of the *CV*'s for different variables. The multivariate optimization problem is as follows:

Find $\left\{m_{NSR},\ \bar{n}_{a,SR},\ \bar{n}_{a,NSR},\ \bar{\bar{q}}_{ab,SR},\ \bar{\bar{q}}_{ab,NSR};\ a=1,\ldots,A,\ b=1,\ldots,B\right\}$ that minimizes the weighted sum of the relvariances (i.e., the objective function),

$$\phi=\sum_{l=1}^{L}\omega_l\cdot CV^2\left(\hat{\theta}_l\right) \tag{3.6}$$

The constraints will be the same as in the univariate case (1) – (11). An importance weight, $\omega_l$, will be assigned for each desired of analysis variable $l$ we want to include in the optimization. Importance weights are often assigned depending on the objective of the survey. In some surveys it may be possible to identify variables that are the main outcomes of interest, giving them more weight in the optimization. For example, in HRS data important variables such as income and wealth may be given more weight in the objective function above.

## 4.4 Optimal Allocations for a Three Stage Sample Using Accuracy Rates from an HRS Survey

In this section, we share the results of the optimal allocation we computed using Excel Solver based on Eqs. (4.2), (4.3), and (4.4). Section 4.3.2 illustrates how a multivariate optimization problem can be solved using equal importance weights when applied to HRS data. Appendix C displays table of the optimization results for each variable of interest by the different methods of variance estimation.

### 4.4.1 Univariate Allocation Using Design-Based ANOVA and Model Based Anticipated Relvariances

The set up for the Excel Solver allocation using design-based ANOVA relvariances and model based anticipated relvariances, as well as their solutions for each selected HRS variable are displayed in Appendix C.1 and Appendix C.2, respectively. We give an overview of the Excel Solver table below.

Recall in Section 3.3 we estimated the accuracy rates in which commercial lists from MSG can correctly identify households for each analytic domain *d* of interested in HRS: 1=45-62 Hispanic; 2=45-62 NH Black; 3=45-62 NH Other and two other domains: 4=Not 45-62; 5=Unoccupied HU. The unweighted accuracy rates, $p_{ab}(d)$, from Table 3.8 are displayed at the top left of the Excel Solver table. (Note that the accuracy rates do not depend on PSU classification (SR or NSR)). These are needed to find out how many HUs would be eligible for the HRS in each SSU/MSG substratum *ab*. The overall proportion of eligible HUs for each SSU/MSG substratum *ab* is displayed under the column heading "All Eligibles". This information allows us to determine how many HUs need to be sampled and screened, overall and in each SSU/MSG substratum *ab*, to achieve our actual allocation.

Highlighted in blue in the figures in Appendices C.1 and C.2 are the ANOVA variance components estimates for both SR and NSR PSUs that were estimated in Section 3.4.3 for ANOVA and Section 3.4.4 for anticipated variances. The sample allocations for both SR and NSR PSUs are heighted in grey and are summarized in Tables 4.4 – 4.8. The bottom left corner of each Excel table in the Appendices holds the constraints of the optimization problem as detailed in Section 4.3.1. A column displays either TRUE or FALSE indicating whether or not a constraint has been satisfied. In the adjacent columns is a summary of the total number of HUs needed to be screened separately for SR and NSR PSUs as well as the actual allocation achieved or the number of HUs we expect to be eligible for the HRS given the screening of HUs.

A summary of the Excel Solver solutions is displayed in Table 4.1 for ANOVA. For most variables the weighting design effect, $deff_k$ , was 1.75, which is the maximum bound on design effects for the optimization problem. `Other_debts`, `Own_2`$^{nd}$`_home` and `selfRatedHealth` had design effects of 1.67, 1.54 and 1.56, respectively. The total relvariances estimated using Eq. (4.7) range from (0.001-0.015). The relvariances for SR and NSR PSUs estimated using Eq. (2.19) range from (0.008-0.035) and (0.001-0.024), respectively. `Wealthb` had the maximum coefficient of variation 0.121 while `own_transport` had the lowest at 0.037.

Table 4.1 Summary of Excel Solver Solutions Using Design-Based ANOVA Variances

| Summary of Solution ANOVA | | | | | | | |
|---|---|---|---|---|---|---|---|
| Selected HRS Variables | $deff_k$ | Total RelVar | $CV$ | $\hat{F}_{SR}$ | $\hat{F}_{NSR}$ | RelVar ($t_{pwr,SR}$) | RelVar ($t_{pwr,NSR}$) |
| income | 1.75 | 0.006 | 0.075 | 0.093 | 0.483 | 0.016 | 0.009 |
| wealtha | 1.75 | 0.014 | 0.117 | 0.099 | 0.470 | 0.035 | 0.022 |
| wealthb | 1.75 | 0.015 | 0.121 | 0.097 | 0.475 | 0.034 | 0.024 |
| other_debts | 1.67 | 0.002 | 0.043 | 0.067 | 0.550 | 0.012 | 0.002 |
| charity_donate | 1.74 | 0.004 | 0.066 | 0.075 | 0.529 | 0.018 | 0.006 |
| employed | 1.75 | 0.002 | 0.046 | 0.079 | 0.518 | 0.011 | 0.002 |
| ownHome | 1.75 | 0.003 | 0.054 | 0.062 | 0.564 | 0.015 | 0.003 |
| ownStock | 1.75 | 0.009 | 0.094 | 0.110 | 0.447 | 0.031 | 0.012 |
| own_2nd_home | 1.54 | 0.008 | 0.091 | 0.059 | 0.574 | 0.018 | 0.013 |
| own_transport | 1.75 | 0.001 | 0.037 | 0.056 | 0.582 | 0.013 | 0.001 |
| selfRatedHealth | 1.56 | 0.002 | 0.042 | 0.067 | 0.551 | 0.008 | 0.002 |

Table 4.2 Summary of Excel Solver Solutions Using Anticipated Variance

| Summary of Solution Anticipated | | | | | | | |
|---|---|---|---|---|---|---|---|
| Selected HRS Variables | $deff_k$ | Total RelVar | $CV$ | $\hat{F}_{SR}$ | $\hat{F}_{NSR}$ | RelVar ($t_{pwr,SR}$) | RelVar ($t_{pwr,NSR}$) |
| income | 1.67 | 0.001 | 0.036 | 0.305 | 0.695 | 0.005 | 0.002 |
| wealtha | 1.75 | 0.026 | 0.162 | 0.314 | 0.686 | 0.049 | 0.045 |
| wealthb | 1.75 | 0.022 | 0.149 | 0.311 | 0.689 | 0.046 | 0.038 |
| other_debts | 1.69 | 0.002 | 0.044 | 0.258 | 0.742 | 0.013 | 0.002 |
| charity_donate | 1.69 | 0.004 | 0.062 | 0.273 | 0.727 | 0.021 | 0.004 |
| employed | 1.74 | 0.002 | 0.041 | 0.280 | 0.720 | 0.010 | 0.002 |
| ownHome | 1.75 | 0.002 | 0.047 | 0.249 | 0.751 | 0.018 | 0.002 |
| ownStock | 1.75 | 0.007 | 0.083 | 0.332 | 0.668 | 0.029 | 0.008 |
| own_2nd_home | 1.62 | 0.014 | 0.118 | 0.242 | 0.758 | 0.092 | 0.015 |
| own_transport | 1.75 | 0.001 | 0.029 | 0.237 | 0.763 | 0.007 | 0.001 |
| selfRatedHealth | 1.70 | 0.003 | 0.053 | 0.258 | 0.742 | 0.018 | 0.003 |

Table 4.3 PSU Allocation for Selected Variables using ANOVA and Anticipated Variances for Self-Representing and Non-Representing PSUs

| Selected HRS Variables | ANOVA | | ANTICIPATED | |
|---|---|---|---|---|
| | $m_{SR}$ | $m_{NSR}$ | $m_{SR}$ | $m_{NSR}$ |
| income | 16.0 | 88.1 | 16.0 | 80.9 |
| wealtha | 16.0 | 85.4 | 16.0 | 84.3 |
| wealthb | 16.0 | 85.9 | 16.0 | 83.8 |
| other_debts | 16.0 | 81.1 | 16.0 | 80.7 |
| charity_donate | 16.0 | 85.4 | 16.0 | 80.9 |
| employed | 16.0 | 83.5 | 16.0 | 80.9 |
| ownHome | 16.0 | 85.4 | 16.0 | 81.1 |
| ownStock | 16.0 | 80.2 | 16.0 | 81.0 |
| own_2nd_home | 16.0 | 80.2 | 16.0 | 80.6 |
| own_transport | 16.0 | 85.6 | 16.0 | 80.9 |
| selfRatedHealth | 16.0 | 80.1 | 16.0 | 80.8 |

The results in Tables 4.2 using anticipated variances are similar to results from ANOVA. Here the design effects range from 1.62 - 1.75. `Own_2nd_home` has the smallest design effect of 1.62. The total relvariances estimated using Eq. (4.7) range from (0.001-0.026). The relvariances for SR and NSR PSUs estimated using Eq. (2.19) range from (0.007-

0.092) and (0.001-0.045), respectively. The variable with the largest coefficient of variation was `wealtha` at 0.162. `Own_transport` had the lowest coefficient of variation at 0.029.

The number of sample SR PSUs was held constant at $m_{SR} = 16$. Across all variables the optimal number of NSR PSUs to select in the allocations were around 80 to 84 NSR PSUs when using ANOVA, and 80 to 88 NSR PSUs when using anticipated variances. For ANOVA, the allocation for `income` stood out by allocating 88 NSR PSUs more than any other variable. The optimal number of SSUs allocated to each SSU stratum is displayed in Table 4.4. When using both ANOVA and anticipated variances, the number of SSUs allocated to the NSR PSUs was $\bar{n}_{a,NSR} = 2$ (the minimum SSU strata size in the constraints) in SSU stratum *03* and *04* and for all variables. This agrees with the estimates $\hat{W}_{3a}^2$ being bigger than $\hat{W}_{2a}^2$ in NSR PSUs, which allocates more HUs in SSUs than more SSUs. Slightly more SSUs were allocated to SSU stratum *02* for almost all variables when using the ANOVA, and similarly for Anticipated variance allocations slightly more SSUs were allocated to SSU stratum *02* but only for `income`, `wealtha`, and `wealthb`.

The optimization problem using anticipated variances allocates slightly more SSUs to SSU stratum *03* across all variables except for `wealtha` and `wealthb`, which remain constant at 2.0 SSUs per SSU stratum. The overall range for $\bar{n}_{a,SR}$ is 2.0 – 4.2 SSUs. ANOVA had only a slightly bigger range with $\bar{n}_{a,SR}$ spanning from 2.0 to 4.5 SSUs. Design-based ANOVA allocates around twice the number of SSUs (4.0 and 3.6) to SSU stratum *03* for `wealtha` and `wealthb` than other variables. We also note a larger amount of SSUs being allocated in SSU stratum *02* for `ownHome`. For the variables `own_2nd_home` and `selfRatedHealth`, the allocation under ANOVA for SR PSUs was the minimum number of SSUs, 2.0, in all SSU strata.

Allocations results for HUs, displayed in Table 4.5, Table 4.6, and Table 4.7, were similar across variables. For SR PSUs, across continuous variables, a large amount of the HUs were allocated to SSU/MSG substratum *0302* (< 10% Black, ≥ 10% Hispanic / 45-62 NH Black) when using both the ANOVA and anticipated variances. In particular, SSU/MSG substrata *0202*, *0301*, *0401*, and *0406* had the highest numbers of HUs allocated and expected to be eligible. The allocation of HUs to MSG substrata *03* and *04* (45-62 NH Other and 45-62 No Race/Eth) was minimal. In these substrata, the allocation was always $\bar{\bar{q}}_{ab,SR} = 2.0$ HUs per SSU/MSG substratum, the minimum number of HUs as constrained by the optimization. The allocation seemed to favor allocating HUs to MSG substrata *05* and *06*.

Using both ANOVA and anticipated variances to obtain an allocation for HUs in NSR PSUs, more HUs were allocated to SSU strata *02* than other SSU strata *a* across all variables. In particular, SSU/MSG substrata *0201* and *0202* had the highest numbers of HUs allocated and expected to be eligible. Overall, the optimization problem using ANOVA and anticipated variances allocated $m_{NSR}$ , $\bar{n}_{a,SR}$, $\bar{n}_{a,NSR}$, $\bar{\bar{q}}_{ab,SR}$ and $\bar{\bar{q}}_{ab,NSR}$, slightly differently across variables. For example, in SSU/MSG substrata *0301* for the SR HU allocation in Table 4.5, $\bar{\bar{q}}_{ab,SR} = 4.5$ for `income`, $\bar{\bar{q}}_{ab,SR} = 10.6$ for `wealtha`, and $\bar{\bar{q}}_{ab,SR} = 16.2$ for `wealthb`. In this demonstration, every variable behaves differently in the allocation and a compromise needs to be made. This is when a multivariate allocation is appropriate that minimizes the weighted average of relvariances depending on what variables we are more interested in.

The allocations for analysis variables that had negative values for ANOVA relvariance estimates were somewhat different when the anticipated relvariance estimates were used. For example, for `own_2nd_home` was negative for all three SSU strata in SR PSUs, and the minimum of 2 SSUs were allocated. But, when these relvariance estimates are corrected to be positive with the anticipated relvariances, the allocations of SSUs are 2.5, 3.1, and 2.0 in Table 4.4. For `selfRatedHealth`, ANOVA relvariances for SR PSUs led to the minimum of 2 SSUs in the allocation while anticipated relvariances allocated 3.3, 3.5, and 2.9. Because the anticipated relvariances correct the problem of negative component estimates, the allocations using them seem more reliable.

Table 4.4 SSU Allocation for Selected HRS Variables using ANOVA and Anticipated Variances for Self-Representing and Non-Representing PSUs

| SSU stratum $a$ | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd home | own trans port | self Rated Health |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **ANOVA** | | | | | | | | | | | |
| | $\bar{n}_{a,SR}$ | | | | | | | | | | |
| 02 | 2.6 | 4.0 | 3.6 | 2.0 | 2.1 | 2.0 | 2.1 | 2.5 | 2.0 | 2.0 | 2.0 |
| 03 | 2.0 | 2.0 | 2.0 | 2.8 | 2.0 | 3.3 | 4.5 | 3.5 | 2.0 | 3.6 | 2.0 |
| 04 | 2.3 | 2.0 | 2.0 | 3.2 | 3.8 | 3.7 | 2.0 | 2.8 | 2.0 | 2.9 | 2.0 |
| | $\bar{n}_{a,NSR}$ | | | | | | | | | | |
| 02 | 2.9 | 2.6 | 2.7 | 2.1 | 2.5 | 2.3 | 2.5 | 2.2 | 2.0 | 2.5 | 2.0 |
| 03 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 04 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| **ANTICIPATED** | | | | | | | | | | | |
| SSU stratum $a$ | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd home | own trans port | self Rated Health |
| | $\bar{n}_{a,SR}$ | | | | | | | | | | |
| 02 | 2.1 | 2.0 | 2.0 | 3.1 | 3.0 | 3.3 | 3.3 | 2.8 | 2.5 | 3.3 | 3.3 |
| 03 | 3.8 | 2.0 | 2.0 | 3.4 | 3.6 | 3.9 | 4.1 | 4.2 | 3.1 | 3.9 | 3.5 |
| 04 | 2.6 | 2.0 | 2.0 | 2.7 | 2.9 | 3.1 | 3.2 | 3.2 | 2.2 | 3.0 | 2.9 |
| | $\bar{n}_{a,NSR}$ | | | | | | | | | | |
| 02 | 2.0 | 2.5 | 2.4 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 03 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 04 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |

Table 4.5 HU Allocation for Continuous Variables using ANOVA and Anticipated Variances for Self-Representing and Non-Representing PSUs

| | ANOVA | | | | | |
|---|---|---|---|---|---|---|
| SSU/MSG Substratum *ab* | `income` | | `wealtha` | | `wealthb` | |
| | $\bar{\bar{q}}_{ab,SR}$ | $\bar{\bar{q}}_{ab,NSR}$ | $\bar{\bar{q}}_{ab,SR}$ | $\bar{\bar{q}}_{ab,NSR}$ | $\bar{\bar{q}}_{ab,SR}$ | $\bar{\bar{q}}_{ab,NSR}$ |
| 0201 | 2.0 | 13.0 | 2.0 | 14.8 | 2.0 | 14.2 |
| 0202 | 2.0 | 11.6 | 2.0 | 10.3 | 2.0 | 8.6 |
| 0203 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0204 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0205 | 4.7 | 5.1 | 4.3 | 4.9 | 4.5 | 4.9 |
| 0206 | 6.5 | 7.7 | 8.5 | 7.2 | 9.4 | 7.0 |
| 0301 | 10.8 | 2.0 | 3.5 | 3.0 | 5.4 | 2.6 |
| 0302 | 13.2 | 2.0 | 19.0 | 4.5 | 18.9 | 6.2 |
| 0303 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0304 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0305 | 7.1 | 3.6 | 4.7 | 3.8 | 4.6 | 3.7 |
| 0306 | 9.3 | 6.4 | 11.1 | 6.7 | 11.3 | 6.3 |
| 0401 | 3.7 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0402 | 5.0 | 2.0 | 2.3 | 2.0 | 2.4 | 2.0 |
| 0403 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0404 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0405 | 3.3 | 2.0 | 3.8 | 2.0 | 3.8 | 2.0 |
| 0406 | 11.0 | 2.9 | 7.9 | 6.3 | 8.2 | 5.7 |
| | **ANTICIPATED** | | | | | |
| SSU/MSG Substratum *ab* | `income` | | `wealtha` | | `wealthb` | |
| | $\bar{\bar{q}}_{ab,SR}$ | $\bar{\bar{q}}_{ab,NSR}$ | $\bar{\bar{q}}_{ab,SR}$ | $\bar{\bar{q}}_{ab,NSR}$ | $\bar{\bar{q}}_{ab,SR}$ | $\bar{\bar{q}}_{ab,NSR}$ |
| 0201 | 2.0 | 11.8 | 6.5 | 11.9 | 2.0 | 11.9 |
| 0202 | 6.2 | 7.7 | 2.0 | 12.8 | 2.0 | 12.5 |
| 0203 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0204 | 2.9 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0205 | 5.8 | 8.8 | 9.1 | 5.3 | 8.0 | 5.5 |
| 0206 | 8.9 | 9.8 | 11.0 | 9.3 | 10.4 | 9.9 |
| 0301 | 4.5 | 7.0 | 10.6 | 3.3 | 16.2 | 3.2 |
| 0302 | 3.7 | 9.7 | 9.7 | 3.2 | 8.7 | 3.9 |
| 0303 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0304 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0305 | 6.0 | 5.3 | 6.3 | 4.2 | 6.1 | 4.4 |
| 0306 | 6.0 | 7.2 | 12.3 | 5.5 | 11.3 | 5.6 |
| 0401 | 2.9 | 4.0 | 8.3 | 2.0 | 8.8 | 2.0 |
| 0402 | 2.4 | 3.5 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0403 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0404 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0405 | 5.7 | 2.0 | 4.1 | 2.0 | 4.3 | 2.0 |
| 0406 | 10.6 | 3.4 | 9.7 | 13.5 | 10.3 | 14.1 |

Table 4.6 HU Allocation for Categorical Variables using ANOVA Variances for Self-Representing and Non-Representing PSUs

| | $\bar{\bar{q}}_{ab,SR}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| SSU/MSG Substrata *ab* | other debts | charity donate | employed | own Home | own Stock | own 2nd home | own transport | self Rated Health |
| 0201 | 2.0 | 3.5 | 2.9 | 4.0 | 2.0 | 4.5 | 3.1 | 2.3 |
| 0202 | 4.2 | 4.8 | 3.8 | 3.5 | 4.3 | 6.3 | 3.3 | 5.7 |
| 0203 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0204 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0205 | 5.4 | 6.6 | 4.6 | 5.9 | 3.9 | 7.3 | 6.5 | 6.6 |
| 0206 | 9.4 | 8.9 | 7.8 | 6.9 | 7.0 | 12.4 | 7.9 | 9.6 |
| 0301 | 7.4 | 6.9 | 6.2 | 4.7 | 3.8 | 8.6 | 5.7 | 9.5 |
| 0302 | 4.7 | 4.9 | 3.0 | 4.0 | 4.2 | 3.6 | 4.1 | 4.3 |
| 0303 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0304 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0305 | 5.3 | 9.6 | 3.3 | 3.5 | 6.4 | 5.6 | 3.3 | 9.8 |
| 0306 | 13.0 | 13.0 | 9.9 | 4.3 | 10.8 | 20.6 | 6.6 | 21.8 |
| 0401 | 3.2 | 4.3 | 3.7 | 4.8 | 2.7 | 6.3 | 4.9 | 6.8 |
| 0402 | 2.4 | 2.5 | 2.3 | 3.5 | 2.7 | 4.3 | 3.4 | 3.8 |
| 0403 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0404 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0405 | 3.8 | 2.7 | 3.0 | 3.9 | 3.6 | 3.6 | 2.6 | 6.5 |
| 0406 | 10.4 | 7.2 | 7.9 | 13.2 | 13.8 | 21.3 | 8.4 | 19.7 |
| | $\bar{\bar{q}}_{ab,NSR}$ | | | | | | | |
| SSU/MSG Substrata *ab* | other debts | charity donate | employed | own Home | own Stock | own 2nd home | own transport | self Rated Health |
| 0201 | 13.7 | 14.6 | 15.0 | 14.5 | 16.2 | 12.1 | 13.2 | 10.9 |
| 0202 | 12.6 | 14.1 | 15.1 | 14.2 | 13.9 | 8.0 | 13.6 | 10.5 |
| 0203 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0204 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0205 | 6.6 | 5.2 | 5.7 | 5.6 | 6.0 | 10.5 | 5.5 | 8.4 |
| 0206 | 12.4 | 8.3 | 10.1 | 8.8 | 8.9 | 14.8 | 9.1 | 14.8 |
| 0301 | 4.2 | 2.0 | 2.8 | 2.4 | 3.2 | 6.4 | 3.4 | 6.2 |
| 0302 | 5.3 | 2.1 | 2.7 | 2.0 | 2.9 | 7.3 | 2.5 | 5.6 |
| 0303 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0304 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0305 | 5.0 | 4.6 | 4.2 | 4.3 | 4.8 | 5.6 | 3.7 | 4.8 |
| 0306 | 9.6 | 7.3 | 7.8 | 7.2 | 8.5 | 10.3 | 7.0 | 11.0 |
| 0401 | 3.2 | 2.0 | 2.0 | 2.0 | 2.1 | 2.0 | 2.0 | 3.5 |
| 0402 | 2.6 | 2.0 | 2.0 | 2.0 | 2.9 | 5.0 | 2.0 | 4.2 |
| 0403 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0404 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0405 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.2 |
| 0406 | 4.3 | 3.2 | 3.6 | 3.1 | 2.4 | 2.2 | 3.3 | 4.6 |

Table 4.7 HU Allocation for Categorical Variables using Anticipated Variances for Self-Representing and Non-Representing PSUs

| $\bar{\bar{q}}_{ab,SR}$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| SSU/MSG Substrata *ab* | other debts | charity donate | employed | own Home | own Stock | own 2nd home | own transport | self Rated Health |
| 0201 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 3.4 | 2.0 | 2.0 |
| 0202 | 2.0 | 3.3 | 2.7 | 2.7 | 2.8 | 4.8 | 2.8 | 3.1 |
| 0203 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0204 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0205 | 3.5 | 4.2 | 2.9 | 2.9 | 3.3 | 5.4 | 3.0 | 3.1 |
| 0206 | 4.9 | 5.5 | 4.5 | 4.4 | 6.0 | 5.0 | 4.5 | 5.0 |
| 0301 | 6.3 | 4.3 | 4.9 | 4.1 | 4.2 | 5.0 | 4.5 | 4.3 |
| 0302 | 2.7 | 3.7 | 2.3 | 3.0 | 3.2 | 4.6 | 2.4 | 3.3 |
| 0303 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0304 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0305 | 5.0 | 5.0 | 3.7 | 3.4 | 4.3 | 4.7 | 3.1 | 4.3 |
| 0306 | 9.8 | 6.4 | 8.2 | 5.6 | 8.0 | 11.2 | 8.2 | 9.3 |
| 0401 | 4.8 | 3.1 | 3.2 | 3.8 | 2.6 | 5.1 | 3.8 | 3.5 |
| 0402 | 2.4 | 2.9 | 2.7 | 2.6 | 2.5 | 4.3 | 2.8 | 3.0 |
| 0403 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0404 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0405 | 3.9 | 3.8 | 3.2 | 2.5 | 3.0 | 3.8 | 2.9 | 3.5 |
| 0406 | 11.7 | 10.6 | 9.7 | 8.0 | 8.0 | 10.2 | 9.3 | 10.7 |
| $\bar{\bar{q}}_{ab,NSR}$ | | | | | | | | |
| SSU/MSG Substrata *ab* | other debts | charity donate | employed | own Home | own Stock | own 2nd home | own transport | self Rated Health |
| 0201 | 9.3 | 14.5 | 14.2 | 11.3 | 12.6 | 12.9 | 12.9 | 14.5 |
| 0202 | 10.7 | 10.8 | 10.6 | 11.0 | 9.6 | 8.8 | 11.7 | 10.9 |
| 0203 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0204 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0205 | 6.8 | 7.3 | 6.5 | 6.8 | 7.2 | 8.2 | 6.2 | 6.9 |
| 0206 | 12.2 | 11.2 | 11.3 | 10.9 | 9.8 | 13.8 | 12.2 | 12.1 |
| 0301 | 7.6 | 4.7 | 4.5 | 6.7 | 6.1 | 5.3 | 5.5 | 4.4 |
| 0302 | 6.7 | 5.0 | 6.0 | 5.8 | 6.8 | 6.1 | 5.1 | 4.9 |
| 0303 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0304 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0305 | 4.4 | 4.9 | 4.1 | 4.5 | 5.0 | 5.6 | 3.6 | 4.0 |
| 0306 | 8.8 | 7.5 | 8.1 | 8.0 | 7.1 | 10.3 | 8.6 | 8.5 |
| 0401 | 4.4 | 3.4 | 3.6 | 4.4 | 4.2 | 3.3 | 3.7 | 3.4 |
| 0402 | 4.5 | 5.4 | 5.3 | 4.9 | 5.3 | 5.3 | 4.8 | 5.5 |
| 0403 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0404 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0405 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.0 | 2.2 | 2.0 |
| 0406 | 3.8 | 3.4 | 3.6 | 3.2 | 2.7 | 3.4 | 3.9 | 3.8 |

### 4.4.2 Multivariate Allocation Equal Weights

When not every variable behaves in the same way a compromise must be made. Multivariate allocation allows for equal or more importance to be given to certain variables when allocation. In this section, we provide a general look into an allocation that takes into consideration all the selected HRS variables with equal importance. This type of weighting may not be true for the HRS but we demonstrate the technique here.

For the purposes of this demonstration, we use Excel Solver to find an allocation which uses equal importance weights $\omega_l = 1/11$, for each of the selected HRS analysis variables $l$. The multivariate optimization objective function is that of Eq. (3.6) has the same constraints as the univariate allocation from Section 4.3.1. The set up for the multivariate Excel Solver allocations are displayed in Appendices C.3 and C.4. The results for ANOVA and anticipated variances are show in Table 4.8. The number of SR PSUs was fixed at $m_{SR} = 16$. The ANOVA method allocated 88.1 PSUs to the NSR PSUs while the anticipated variance method allocated 83.1 PSUs to the NSR PSUs.

Table 4.8 Summary of Solution for Multivariate Allocation Equal Weights, ANOVA and Anticipated Variances

| Selected HRS Variables | ANOVA | | | | ANTICIPATED | | | |
|---|---|---|---|---|---|---|---|---|
| | Total RelVar | CV | RelVar ($t_{pwr,SR}$) | RelVar ($t_{pwr,NSR}$) | Total RelVar | CV | RelVar ($t_{pwr,SR}$) | RelVar ($t_{pwr,NSR}$) |
| income | 0.006 | 0.075 | 0.016 | 0.009 | 0.001 | 0.038 | 0.007 | 0.002 |
| wealtha | 0.015 | 0.123 | 0.047 | 0.022 | 0.027 | 0.164 | 0.043 | 0.048 |
| wealthb | 0.016 | 0.126 | 0.042 | 0.025 | 0.023 | 0.151 | 0.041 | 0.040 |
| other debts | 0.002 | 0.046 | 0.015 | 0.002 | 0.002 | 0.046 | 0.017 | 0.002 |
| charity donate | 0.005 | 0.068 | 0.024 | 0.005 | 0.004 | 0.065 | 0.028 | 0.004 |
| employed | 0.002 | 0.048 | 0.015 | 0.002 | 0.002 | 0.044 | 0.013 | 0.002 |
| own Home | 0.003 | 0.058 | 0.024 | 0.003 | 0.003 | 0.051 | 0.024 | 0.002 |
| own Stock | 0.010 | 0.099 | 0.044 | 0.011 | 0.008 | 0.089 | 0.039 | 0.008 |
| own 2nd home | 0.012 | 0.109 | 0.040 | 0.016 | 0.015 | 0.124 | 0.101 | 0.017 |
| own transport | 0.001 | 0.039 | 0.017 | 0.001 | 0.001 | 0.031 | 0.010 | 0.001 |
| selfRatedHealth | 0.002 | 0.046 | 0.011 | 0.003 | 0.003 | 0.056 | 0.024 | 0.003 |

Table 4.9 Excel Solver Multivariate Optimization Results for PSUs, ANOVA and Anticipated Variance

| ANOVA | | Anticipated | |
|---|---|---|---|
| $m_{SR}$ | $m_{NSR}$ | $m_{SR}$ | $m_{NSR}$ |
| 16.0 | 88.1 | 16.0 | 83.1 |

Table 4.10 Excel Solver Multivariate Optimization Results for SSUs, ANOVA and Anticipated Variance

| SSU stratum *a* | $\bar{n}_{a,SR}$ | |
|---|---|---|
| | ANOVA | Anticipated |
| 02 | 2.6 | 2.2 |
| 03 | 2.0 | 2.8 |
| 04 | 2.3 | 2.1 |
| SSU stratum *a* | $\bar{n}_{a,NSR}$ | |
| | ANOVA | Anticipated |
| 02 | 2.9 | 2.3 |
| 03 | 2.0 | 2.0 |
| 04 | 2.0 | 2.0 |

Table 4.11 Excel Solver Multivariate Optimization Results for HUs, ANOVA and Anticipated Variance

| SSU/MSG substratum *ab* | ANOVA | | Anticipated | |
|---|---|---|---|---|
| | $\bar{\bar{q}}_{ab,SR}$ | $\bar{\bar{q}}_{ab,NSR}$ | $\bar{\bar{q}}_{ab,SR}$ | $\bar{\bar{q}}_{ab,NSR}$ |
| 0201 | 2.0 | 13.1 | 2.0 | 13.2 |
| 0202 | 2.7 | 11.6 | 2.7 | 13.3 |
| 0203 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0204 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0205 | 4.8 | 5.1 | 5.7 | 5.7 |
| 0206 | 6.6 | 7.7 | 7.9 | 9.9 |
| 0301 | 9.1 | 2.0 | 9.4 | 3.1 |
| 0302 | 12.4 | 2.0 | 6.3 | 3.7 |
| 0303 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0304 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0305 | 7.1 | 3.6 | 4.9 | 4.4 |
| 0306 | 9.4 | 6.4 | 9.0 | 6.6 |
| 0401 | 4.6 | 2.0 | 8.2 | 2.0 |
| 0402 | 5.3 | 2.0 | 2.3 | 2.0 |
| 0403 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0404 | 2.0 | 2.0 | 2.0 | 2.0 |
| 0405 | 3.3 | 2.0 | 4.2 | 2.0 |
| 0406 | 11.5 | 2.8 | 11.5 | 12.2 |

Table 4.10 displays the SSU allocation for the multivariate optimization. For SR PSUs, ANOVA and anticipated variance methods allocated between 2.0 - 2.6 SSUs and 2.1 - 2.8 SSUs, respectively, in each SSU strata. ANOVA allocated slightly more SSUs to SSU stratum *02* $\bar{n}_{a,SR} = 2.6$, while the anticipated variance method allocated more SSUs to SSU stratum *03,* $\bar{n}_{a,SR} = 2.8$. For NSR PSUs, both variance methods allocation was 2.0 SSUs to SSU stratum *03* and *04* (as in the univariate allocation) and slightly more SSUs to SSU stratum *02* (2.9 for ANOVA, 2.3 for Anticipated variance). Table 4.11 displays the HU allocation for the multivariate optimization. Results are fairly similar across both methods demonstrating this method is useful to the HRS data.

# 5 Discussion

Commercial address lists have been used to sample households, but investigators spend considerable time and money on screening households for eligibility as well as locating certain subpopulations (to achieve target sample sizes). Commercial lists have errors in the auxiliary data they include. We explored the accuracies of commercial lists further by estimating the accuracy rates in which commercial lists from MSG can correctly identify households with certain characteristics (e.g., Hispanics, Non-Hispanic Blacks, Persons 45-62, etc.). Even with inaccuracies, we demonstrated that utilizing the demographic information on commercial lists can be used to better identify eligible households with certain characteristics and subgroups than equal probability sampling as applied to the 2010-11 HRS data. We found that some characteristics like age and race-ethnicity are more accurately specified on commercial lists than others.

In Chapter 2 and in the Appendix to Chapter 2, a theoretical variance formula, including variance components for estimated totals and estimators of variance components was derived for a 3-stage sample design with stratification of both second-stage and third-stage units. Design-based ANOVA estimators of relvariance components were derived as a method of variance estimation. Model-assisted (anticipated variance) estimators of relvariance components were derived using a random effects model that reflects the complexity of the sample design and the underlying population. In Chapter 3, both variance estimation methods were studied and evaluated for use as alternative variance component estimators. The empirical work used the 2010-11 HRS data to apply and illustrate the variance component theory.

In Chapter 4, we demonstrated how math programming can be used with inaccurate information on commercial lists to find a sample allocation in a complex survey design. We determined how to allocate a three-stage sample supplemented with auxiliary information from commercial lists while accounting for errors in that information. Nonlinear programming (NLP) can be used to efficiently allocate a sample that has a variety of estimation goals and constraints. NLP can find a solution that accounts for (i) sizes of contributions of different stages of sampling to relvariances of estimates, (ii) CV goals of a survey, (iii) cost constraints, (iv) other constraints like required minimum sample sizes in demographic subgroups, maximum design effect due to weighting, minimum and maximum sample sizes for each stage of sampling, and (iv) error rates in commercial lists.

Future work could be done to apply the work in this thesis to other populations that use three stage survey samples that stratify SSUs and final stage sampling units. To apply the methods here, one needs preliminary sample data to estimate variance components, list error rates, and other parameters that affect sample allocations. A critical step is to evaluate the quality of the input data used for estimation. In particular, attention must be paid to how to impute for missing values and how to deal with sparse samples in order to get acceptable variance component estimates at all stages.

# A Appendix Supplement to Chapter 2

Appendix A gives the details of derivations for the design-based relvariance of an estimated total, the optimal allocation to stages for a single variable, ANOVA estimators of variance components, and the anticipated variances of relvariance components. All notation was defined earlier in Section 2.3.2.

## A.1 Derivation of the Design Relvariance of $\hat{t}_{pwr}$

**Theorem 1** *Let* $y_k$ *be the* $k^{th}$ *value of the unit drawn on the* $k^{th}$ *draw and* $p_k$ *be the corresponding one draw selection probability, k=1,2,...,n. Then an unbiased pwr estimator,* $\hat{t}_{pwr}$ *, for the population total is of the form* $\frac{1}{n}\sum_{k\in s}\frac{y_k}{p_k}$*. Its expected value is*

$$E\left[\frac{1}{n}\sum_{k\in s}\frac{y_k}{p_k}\right]=\sum_{k\in U}y_k$$

*and its variance in single-stage sampling is*

$$Var\left[\frac{1}{n}\sum_{k\in s}\frac{y_k}{p_k}\right]=\frac{1}{n}\sum_{k\in U}p_k\left(\frac{y_k}{p_k}-y_U\right)^2.$$

The proof can be found in Särndal (1992, Result 2.9.1).

**Theorem 2** *Assume that a three-stage sample is selected using ppswr/ppswr/srswr, that SSUs in each PSU are stratified into a=1, …, A strata, and that HUs in each PSU/SSU are substratified into b=1, …, B substrata. Sampling of SSUs is done independently from one stratum to another. Sampling of HUs is done independently from one substratum to another. The design relative variance (relvariance) of* $\hat{t}_{pwr}$ *is*

$$\frac{V\left(\hat{t}_{pwr}\right)}{t_U^2}=\frac{1}{t_U^2}\left\{\begin{array}{l}\frac{S_{U1(pwr)}^2}{m} \\ +\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_{a=1}^{A}\frac{S_{U2(pwr)ia}^2}{n_{ia}} \\ +\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_{a=1}^{A}\frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{1}{p_{j|ia}}\sum_{b=1}^{B}Q_{iajb}^2\frac{S_{U3iajb}^2}{q_{iajb}}\end{array}\right\}$$

*Proof*. In the remainder of this appendix, the subscripts "1", "2", and "3", denote stages of sampling. For example, $E_1$ denotes expectation over PSU sampling; $E_{2,3}$ denotes expectation over SSU and HU sampling. Other notation is defined in Section 2.2 . Using the law of total variance, $V(Y)=V\left[E(Y|X)\right]+E\left[V(Y|X)\right]$, and the law of total expectation, $E(Y)=E\left[E(Y|X)\right]$, we have

$$\begin{aligned}V\left(\hat{t}_{pwr}\right)&=V_1\left[E_{2,3}\left(\hat{t}_{pwr}\mid s_1,s_2\right)\right]+E_1\left[V_{2,3}\left(\hat{t}_{pwr}\mid s_1,s_2\right)\right]\\&=V_1\left\{E_2\left[E_3\left(\hat{t}_{pwr}\mid s_1,s_2\right)\mid s_1\right]\right\}\\&\quad+E_1\left\{V_2\left[E_3\left(\hat{t}_{pwr}\mid s_1,s_2\right)\mid s_1\right]\right\}\\&\quad+E_1\left\{E_2\left[V_3\left(\hat{t}_{pwr}\mid s_1,s_2\right)\mid s_1\right]\right\}\end{aligned} \tag{A.1}$$

1. Consider $V_1\left\{E_2\left[E_3\left(\hat{t}_{pwr}\mid s_1,s_2\right)\mid s_1\right]\right\}$. We start by taking the expected value with respect to the third stage and obtain

$$
\begin{aligned}
E_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) &= E_3\left(\sum_{i\in s_1}\sum_{a=1}^{A}\sum_{j\in s_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}} \frac{y_k}{mp_i n_{ia} p_{j|ia}\pi_{k|iajb}}\delta_{\mathrm{k}|iajb} \mid s_1, s_2\right) \\
&= \sum_{i\in s_1}\sum_{a=1}^{A}\sum_{j\in s_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}} \frac{y_k}{mp_i n_{ia} p_{j|ia}\pi_{k|iajb}} E_3\left(\delta_{\mathrm{k}|iajb} \mid s_1, s_2\right) \\
&= \sum_{i\in s_1}\sum_{a=1}^{A}\sum_{j\in s_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}} \frac{y_k}{mp_i n_{ia} p_{j|ia}\cancel{\pi_{k|iajb}}}\cancel{\pi_{k|iajb}} \\
&= \sum_{i\in s_1}\frac{1}{mp_i}\sum_{a=1}^{A}\sum_{j\in s_{ia}}\frac{\sum_{b=1}^{B}\sum_{k\in U_{iajb}} y_k}{n_{ia} p_{j|ia}} \\
&= \sum_{i\in s_1}\frac{1}{mp_i}\sum_{a=1}^{A}\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}
\end{aligned} \tag{A.2}
$$

Next, note that $\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}$ is the *pwr* estimator of $t_{U_{ia}}$ and is therefore unbiased in *pwr* sampling. Then, taking the expected value with respect to the second stage

$$
\begin{aligned}
E_2\left[E_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) \mid s_1\right] &= E_2\left[\sum_{i\in s}\frac{1}{mp_i}\sum_{a=1}^{A}\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}} \mid s_1\right] \\
&= \sum_{i\in s_1}\frac{1}{mp_i}\sum_{a=1}^{A}E_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}} \mid s_1\right) \\
&= \sum_{i\in s_1}\frac{1}{mp_i}\sum_{a=1}^{A}t_{U_{ia}} \\
&= \frac{1}{m}\sum_{i\in s_1}\frac{t_{U_i}}{p_i}
\end{aligned} \tag{A.3}
$$

Finally taking the variance with respect to the first stage yields the PSU variance component

$$V_1\left\{E_2\left[E_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) \mid s_1\right]\right\} = V_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{t_{U_i}}{p_i} \mid s_1\right)$$

$$= \frac{1}{m}\sum_{i\in U} p_i\left(\frac{t_{U_i}}{p_i} - t_U\right)^2 \tag{A.4}$$

$$= \frac{S^2_{U1(pwr)}}{m}.$$

2. Consider $E_1\left\{V_2\left[E_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) \mid s_1\right]\right\}$. Using the result from above that,

$$E_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) = \sum_{i\in s_1}\frac{1}{mp_i}\sum_{a=1}^{A}\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}$$

and taking the variance with respect to second stage which is *ppswr*, we obtain

$$V_2\left[E_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) \mid s_1\right] = V_2\left(\sum_{i\in s_1}\frac{1}{mp_i}\sum_{a=1}^{A}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}} \mid s_1\right)$$

$$= \sum_{i\in s_1}\frac{1}{(mp_i)^2}\sum_{a=1}^{A} V_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}} \mid s_1\right)$$

$$= \sum_{i\in s_1}\frac{1}{(mp_i)^2}\sum_{a=1}^{A}\frac{1}{n_{ia}}\sum_{j\in U_{ia}} p_{j|ia}\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right)^2$$

(A.5)

This follows from the fact that

$$\frac{1}{n_{ia}}\sum_{j\in U_{ia}} p_{j|ia}\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right)^2$$

is the variance of the estimated *pwr* total from a with-replacement sample of SSUs, $\frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{t_{U_{iaj}}}{p_{j|ia}}$. Finally, taking the expected value with respect to the first stage, and using the fact that

$$\frac{1}{m} \sum_{i \in s_1} \frac{1}{p_i} \left[ \frac{1}{p_i} \sum_{a=1}^{A} \frac{1}{n_{ia}} \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right)^2 \right]$$

is a *pwr* estimator of a population total, yields the SSU variance component,

$$\begin{aligned}
E_1\left\{V_2\left[E_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) \mid s_1\right]\right\} &= E_1\left( \sum_{i \in s_1} \frac{1}{(mp_i)^2} \sum_{a=1}^{A} \frac{1}{n_{ia}} \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right)^2 \mid \mathrm{s}_1 \right) \\
&= \frac{1}{m} E_1\left( \frac{1}{m} \sum_{i \in s_1} \frac{\frac{1}{p_i} \sum_{a=1}^{A} \frac{1}{n_{ia}} \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right)^2}{p_i} \mid \mathrm{s}_1 \right) \\
&= \frac{1}{m} \sum_{i \in U} \frac{1}{p_i} \sum_{a=1}^{A} \frac{1}{n_{ia}} \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right)^2 \\
&= \frac{1}{m} \sum_{i \in U} \frac{1}{p_i} \sum_{a=1}^{A} \frac{S_{U2(pwr)ia}^2}{n_{ia}}.
\end{aligned}$$

(A.6)

3. Consider $E_1\left\{E_2\left[V_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) \mid s_1\right]\right\}$. We start by taking the variance with respect to the third stage, which is a simple random sample selected with replacement:

$$V_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) = V_3\left(\sum_{i\in s_1}\sum_{a=1}^{A}\sum_{j\in s_{ia}}\sum_{b=1}^{B}\sum_{k\in s_{iajb}} \frac{y_k}{mp_i n_{ia} p_{j|ia}\pi_{k|iajb}} \mid s_1, s_2\right)$$

$$= \sum_{i\in s_1}\sum_{a=1}^{A}\sum_{j\in s_{ia}} \frac{1}{\left(mp_i n_{ia} p_{j|ia}\right)^2} V_3\left(\sum_{b=1}^{B} Q_{iajb} \sum_{k\in s_{iajb}} \frac{y_k}{q_{iajb}} \mid s_1, s_2\right)$$

$$= \sum_{i\in s_1}\sum_{a=1}^{A}\sum_{j\in s_{ia}} \frac{1}{\left(mp_i n_{ia} p_{j|ia}\right)^2} \sum_{b=1}^{B} Q_{iajb}^2 \frac{S_{U3iajb}^2}{q_{iajb}}$$

(A.7)

This follows from the fact that $\sum_{b=1}^{B} Q_{iajb}^2 \frac{S_{U3iajb}^2}{q_{iajb}}$ is the variance of the estimated total of a stratified *srswor* sample of HUs, $\sum_{b=1}^{B} Q_{iajb} \sum_{k\in s_{iajb}} \frac{y_k}{q_{iajb}}$, when the sampling fraction is small.

Next, using the fact that

$$\frac{1}{n_{ia}} \sum_{j\in s_{ia}} \frac{\sum_b Q_{iajb}^2\, S_{U3iajb}^2 \big/ q_{iajb}}{p_{j|ia}^2}$$

is the *pwr* estimator of $\sum_{j\in U_{ia}} \frac{1}{p_{j|ia}} \sum_{b=1}^{B} Q_{iajb}^2 S_{U3iajb}^2 \big/ q_{iajb}$ , we have

$$E_2\left[V_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) \mid s_1\right] = E_2\left(\sum_{i\in s_1}\sum_{a=1}^{A}\sum_{j\in s_{ia}} \frac{1}{\left(mp_i n_{ia} p_{j|ia}\right)^2} \sum_{b=1}^{B} Q_{iajb}^2 \frac{S_{U3iajb}^2}{q_{iajb}} \mid s_1\right)$$

$$= \sum_{i\in s_1} \frac{1}{\left(mp_i\right)^2} \sum_{a=1}^{A} \frac{1}{n_{ia}} E_2\left(\frac{1}{n_{ia}} \sum_{j\in s_{ia}} \frac{\frac{1}{p_{j|ia}} \sum_{b=1}^{B} Q_{iajb}^2 \frac{S_{U3iajb}^2}{q_{iajb}}}{p_{j|ia}} \mid s_1\right) \quad \text{(A.8)}$$

$$= \sum_{i\in s_1} \frac{1}{\left(mp_i\right)^2} \sum_{a=1}^{A} \sum_{j\in U_{ia}} \frac{1}{n_{ia} p_{j|ia}} \sum_{b=1}^{B} Q_{iajb}^2 \frac{S_{U3iajb}^2}{q_{iajb}}.$$

Lastly, taking the expected value due to the first stage with respect to with-replacement sampling yields the HU variance component

$$E_1\left\{E_2\left[V_3\left(\hat{t}_{pwr} \mid s_1, s_2\right) \mid s_1\right]\right\} = E_1\left(\sum_{i\in s_1}\frac{1}{\left(mp_i\right)^2}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\frac{1}{n_{ia}\,p_{j|ia}}\sum_{b=1}^{B}Q_{iajb}^2\,\frac{S_{U3iajb}^2}{q_{iajb}}\right)$$

$$= \frac{1}{m}E_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{\frac{1}{p_i}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\frac{1}{n_{ia}\,p_{j|ia}}\sum_{b=1}^{B}Q_{iajb}^2\,\frac{S_{U3iajb}^2}{q_{iajb}}}{p_i}\right)$$

$$= \frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\frac{1}{n_{ia}\,p_{j|ia}}\sum_{b=1}^{B}Q_{iajb}^2\,\frac{S_{U3iajb}^2}{q_{iajb}}. \tag{A.9}$$

Substituting Eq. (A.4), (A.6), and (A.9) into Eq. (A.1), we obtain

$$\mathrm{V}\left(\hat{t}_{pwr}\right) = \frac{S_{U1(pwr)}^2}{m} + \frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_{a=1}^{A}\frac{S_{U2(pwr)ia}^2}{n_{ia}} + \frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_{a=1}^{A}\frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{1}{p_{j|ia}}\sum_{b=1}^{B}Q_{iajb}^2\,\frac{S_{U3iajb}^2}{q_{iajb}}$$

and dividing both sides by $t_U^2$,

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{1}{t_U^2}\left\{\frac{S_{U1(pwr)}^2}{m} + \frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_{a=1}^{A}\frac{S_{U2(pwr)ia}^2}{n_{ia}} + \frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_{a=1}^{A}\frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{1}{p_{j|ia}}\sum_{b=1}^{B}Q_{iajb}^2\,\frac{S_{U3iajb}^2}{q_{iajb}}\right\} \square \tag{A.10}$$

## A.2 Alternative Expressions for the Relvariance

In the following section, we assume that the same number of SSUs is selected from SSU stratum *a* across each PSU, that is, $n_{ia} = \bar{n}_a$, and that the same number of HUs is selected from substratum *b* within stratum *a*, for every PSU/SSU *ij* combination, that is, $q_{iajb} = \bar{\bar{q}}_{ab}$. We also assume that every SSU stratum *a* occurs in every PSU *i* and that every HU substratum *b* occurs in every SSU *j*.

**Proposition 1** *The relvariance can be rewritten as a sum of three components,*

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{B^2}{m} + \sum_{a=1}^{A} K_a^2 \frac{W_{2a}^2}{m\bar{n}_a} + \sum_{a=1}^{A}\sum_{b=1}^{B} K_{ab}^2 \frac{W_{3ab}^2}{m\bar{n}_a\bar{\bar{q}}_{ab}}$$

*where* $K_a = t_{U_a}/t_U$ *,* $K_{ab} = t_{U_{ab}}/t_U$ *; and* $B^2$ *,* $W_{2a}^2$ *, and* $W_{3ab}^2$ *are defined below.*

*Proof*. From Eq. (A.10), the relvariance of $\hat{t}_{pwr}$ is

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{1}{t_U^2}\left\{\begin{array}{l} \dfrac{S_{U1(pwr)}^2}{m} + \dfrac{1}{m}\sum_{i\in U}\dfrac{1}{p_i}\sum_{a=1}^{A}\dfrac{S_{U2(pwr)ia}^2}{n_{ia}} \\ + \dfrac{1}{m}\sum_{i\in U}\dfrac{1}{p_i}\sum_{a=1}^{A}\dfrac{1}{n_{ia}}\sum_{j\in U_{ia}}\dfrac{1}{p_{j|ia}}\sum_{b=1}^{B}Q_{iajb}^2\dfrac{S_{U3iajb}^2}{q_{iajb}} \end{array}\right\}$$

Substituting $n_{ia} = \bar{n}_a$ and $q_{iajb} = \bar{\bar{q}}_{ab}$ then rearranging terms, we obtain

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{1}{t_U^2}\left\{\begin{array}{l} \dfrac{S_{U1(pwr)}^2}{m} \\ + \dfrac{1}{m}\sum_{i\in U}\dfrac{1}{p_i}\sum_{a=1}^{A}\dfrac{S_{U2(pwr)ia}^2}{\bar{n}_a} \\ + \dfrac{1}{m}\sum_{i\in U}\dfrac{1}{p_i}\sum_{a=1}^{A}\dfrac{1}{\bar{n}_a}\sum_{j\in U_{ia}}\dfrac{1}{p_{j|ia}}\sum_{b=1}^{B}Q_{iajb}^2\dfrac{S_{U3iajb}^2}{\bar{\bar{q}}_{ab}} \end{array}\right\}$$

Multiplying through by $t_{U_a}^2 \big/ t_{U_a}^2$ in the second term and $t_{U_{ab}}^2 \big/ t_{U_{ab}}^2$ in the third term, we obtain

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{1}{m}\frac{S_{U1(pwr)}^2}{t_U^2} + \frac{1}{m}\sum_{a=1}^{A}\frac{1}{\overline{n}_a}\frac{t_{U_a}^2}{t_U^2}\frac{1}{t_{U_a}^2}\sum_{i\in U}\frac{S_{U2(pwr)ia}^2}{p_i} + \frac{1}{m}\sum_{a=1}^{A}\sum_{b=1}^{B}\frac{1}{\overline{n}_a}\frac{1}{\overline{\overline{q}}_{ab}}\frac{t_{U_{ab}}^2}{t_U^2}\frac{1}{t_{U_{ab}}^2}\sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{Q_{iajb}^2 S_{U3iajb}^2}{p_{j|ia}}. \tag{A.11}$$

Define

$$B^2 = \frac{\mathrm{S}_{U1(pwr)}^2}{t_U^2},$$

$$W_{2a}^2 = \frac{1}{t_{U_a}^2}\sum_{i\in U}\frac{\mathrm{S}_{U2(pwr)ia}^2}{p_i}, \text{ and}$$

$$W_{3ab}^2 = \frac{1}{t_{U_{ab}}^2}\sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{Q_{iajb}^2 S_{U3iajb}^2}{p_{j|ia}}.$$

Substituting $K_a$, $K_{ab}$, $B^2$, $W_{2a}^2$ and $W_{3ab}^2$ back into Eq. (A.11), we have

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{1}{m}B^2 + \frac{1}{m}\sum_{a=1}^{A}\frac{t_{U_a}^2}{t_U^2}\frac{1}{\overline{n}_a}W_{2a}^2 + \frac{1}{m}\sum_{a=1}^{A}\sum_{b=1}^{B}\frac{t_{U_{ab}}^2}{t_U^2}\frac{1}{\overline{n}_a}\frac{1}{\overline{\overline{q}}_{ab}}W_{3ab}^2$$

$$= \frac{B^2}{m} + \sum_{a=1}^{A}K_a^2\frac{W_{2a}^2}{m\overline{n}_a} + \sum_{a=1}^{A}\sum_{b=1}^{B}K_{ab}^2\frac{W_{3ab}^2}{m\overline{n}_a\overline{\overline{q}}_{ab}} \quad \square \tag{A.12}$$

**Proposition 2** *The relvariance in Eq.(1.16) can also be written in terms of measures of homogeneity*

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2}=\frac{\tilde{V}}{m}k_1\delta_1+\sum_{a=1}^{A}K_a^2\frac{\tilde{V}_a}{m\bar{n}_a}k_{2a}\delta_{2a}+\sum_{a=1}^{A}\sum_{b=1}^{B}K_{ab}^2\frac{W_{3ab}^2}{m\bar{n}_a\bar{\bar{q}}_{ab}}$$

*where* $\tilde{V}$, $\tilde{V}_a$, $K_a^2$, $K_{ab}^2$, $\delta_1$, $k_1$, $\delta_{2a}$, *and* $k_{2a}$ *are defined in Section 2.3.3. Furthermore, when there are no B strata the relvariance in Eq.*(1.16) *reduces to*

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2}=\frac{\tilde{V}}{m\bar{n}_+\bar{\bar{q}}_+}\left\{k_1\delta_1\bar{n}_+\bar{\bar{q}}_++\sum_{a=1}^{A}K_a^2\frac{\tilde{V}_a}{\tilde{V}}\frac{\bar{n}_+}{\bar{n}_a}\frac{\bar{\bar{q}}_+}{\bar{\bar{q}}_a}k_{2a}\left[1+\delta_{2a}\left(\bar{\bar{q}}_a-1\right)\right]\right\}$$

where $\bar{n}_+=\sum_{a=1}^{A}\bar{n}_a$ is the number of sample SSUs allocated and

$\bar{\bar{q}}_+=\sum_{a=1}^{A}\bar{n}_a\bar{\bar{q}}_a\Big/\sum_{a=1}^{A}\bar{n}_a$ is the mean number of sample elements (HUs) per SSU across all SSU strata.

*Proof*. From Eq.(1.16), the relvariance of $\hat{t}_{pwr}$ is

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2}=\frac{B^2}{m}+\sum_{a=1}^{A}K_a^2\frac{W_{2a}^2}{m\bar{n}_a}+\sum_{a=1}^{A}\sum_{b=1}^{B}K_{ab}^2\frac{W_{3ab}^2}{m\bar{n}_a\bar{\bar{q}}_{ab}}$$

Multiplying and dividing through by 1,

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2}=\frac{B^2}{m}\frac{B^2+W^2}{B^2+W^2}\frac{\tilde{V}}{\tilde{V}}+\sum_{a=1}^{A}K_a^2\frac{W_{2a}^2}{m\bar{n}_a}\frac{W_{2a}^2+W_{3a}^2}{W_{2a}^2+W_{3a}^2}\frac{\tilde{V}_a}{\tilde{V}_a}+\sum_{a=1}^{A}\sum_{b=1}^{B}K_{ab}^2\frac{W_{3ab}^2}{m\bar{n}_a\bar{\bar{q}}_{ab}}$$

(A.13)

Recall $B^2$, $W_{2a}^2$, and $W_{3ab}^2$ are defined in Section 2.3.2. Note that $B$ and $B^2$ are not used for the same notation, as $B$ represents the number of SSU strata and $B^2$ represents part of the PSU variance component. Define

$$\delta_1 = \frac{B^2}{B^2 + W^2}, \qquad k_1 = \frac{B^2 + W^2}{\tilde{V}}$$

$$\delta_{2a} = \frac{W_{2a}^2}{W_{2a}^2 + W_{3a}^2}, \quad k_{2a} = \frac{W_{2a}^2 + W_{3a}^2}{\tilde{V}_a}$$

$\tilde{V} = \dfrac{1}{\bar{y}_U^2} \dfrac{1}{Q-1} \sum_{i \in U} \sum_{a=1}^{A} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \left( y_k - \bar{y}_U \right)^2$ is the unit relvariance of $y$ in the population across all PSUs, SSUs, $a$ strata, and $b$ strata

$\tilde{V}_a = \dfrac{1}{\bar{y}_{U_a}^2} \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \dfrac{1}{Q_a - 1} \left( y_k - \bar{y}_{U_a} \right)^2$ is the unit relvariance of $y$ among elements (HUs) in SSU stratum $a$ across all PSUs in the population and all $b$ strata. Substituting the above terms into Eq. (A.13) and rearranging we obtain,

$$\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} = \frac{B^2}{m} \frac{B^2 + W^2}{B^2 + W^2} \frac{\tilde{V}}{\tilde{V}} + \sum_{a=1}^{A} K_a^2 \frac{W_{2a}^2}{m\bar{n}_a} \frac{W_{2a}^2 + W_{3a}^2}{W_{2a}^2 + W_{3a}^2} \frac{\tilde{V}_a}{\tilde{V}_a} + \sum_{a=1}^{A} \sum_{b=1}^{B} K_{ab}^2 \frac{W_{3ab}^2}{m\bar{n}_a \bar{\bar{q}}_{ab}}$$

$$= \frac{\tilde{V}}{m} \frac{B^2}{B^2 + W^2} \frac{B^2 + W^2}{\tilde{V}} + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{m\bar{n}_a} \frac{W_{2a}^2}{W_{2a}^2 + W_{3a}^2} \frac{W_{2a}^2 + W_{3a}^2}{\tilde{V}_a} + \sum_{a=1}^{A} \sum_{b=1}^{B} K_{ab}^2 \frac{W_{3ab}^2}{m\bar{n}_a \bar{\bar{q}}_{ab}}$$

$$= \frac{\tilde{V}}{m} k_1 \delta_1 + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{m\bar{n}_a} k_{2a} \delta_{2a} + \sum_{a=1}^{A} \sum_{b=1}^{B} K_{ab}^2 \frac{W_{3ab}^2}{m\bar{n}_a \bar{\bar{q}}_{ab}} \tag{A.14}$$

Now assume there are no $b$ strata. Substituting the above terms back into Eq. (A.14) and rearranging terms, we obtain

$$
\begin{aligned}
\frac{\mathrm{V}\left(\hat{t}_{pwr}\right)}{t_U^2} &= \frac{\tilde{V}}{m} k_1 \delta_1 + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{m\bar{n}_a} k_{2a} \delta_{2a} + \sum_{a=1}^{A} K_a^2 \frac{W_{3a}^2}{m\bar{n}_a \bar{\bar{q}}_a} \\
&= \frac{\tilde{V}}{m} k_1 \delta_1 + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{m\bar{n}_a} \left( k_{2a} \delta_{2a} + \frac{W_{3a}^2}{\bar{\bar{q}}_a} \frac{W_{2a}^2 + W_{3a}^2}{W_{2a}^2 + W_{3a}^2} \frac{1}{\tilde{V}_a} \right) \\
&= \frac{\tilde{V}}{m} k_1 \delta_1 + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{m\bar{n}_a} \left( k_{2a} \delta_{2a} + \frac{1}{\bar{\bar{q}}_a} \frac{W_{3a}^2}{W_{2a}^2 + W_{3a}^2} \frac{W_{2a}^2 + W_{3a}^2}{\tilde{V}_a} \right) \\
&= \frac{\tilde{V}}{m} k_1 \delta_1 + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{m\bar{n}_a \bar{\bar{q}}_a} \left[ \bar{\bar{q}}_a k_{2a} \delta_{2a} + \left(1 - \delta_{2a}\right) k_{2a} \right] \\
&= \frac{\tilde{V}}{m} k_1 \delta_1 + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{m\bar{n}_a \bar{\bar{q}}_a} k_{2a} \left[ 1 + \delta_{2a} \left( \bar{\bar{q}}_a - 1 \right) \right] \\
&= \frac{\tilde{V}}{m\bar{n}_+ \bar{\bar{q}}_+} \left\{ k_1 \delta_1 \bar{n}_+ \bar{\bar{q}}_+ + \sum_{a=1}^{A} K_a^2 \frac{\tilde{V}_a}{\tilde{V}} \frac{\bar{n}_+}{\bar{n}_a} \frac{\bar{\bar{q}}_+}{\bar{\bar{q}}_a} k_{2a} \left[ 1 + \delta_{2a} \left( \bar{\bar{q}}_a - 1 \right) \right] \right\} \square
\end{aligned}
\tag{A.15}
$$

## A.3 Derivation of the ANOVA estimates of the components of the relvariance

**Theorem 3.** *An unbiased estimate of the HU component of relvariance, $V_{HU}$, is*

$$v_{HU} = \sum_{i \in s_1} \frac{1}{(mp_i)^2} \sum_a \sum_{j \in s_{ia}} \frac{1}{\left(n_{ia} p_{j|ia}\right)^2} \sum_b \hat{V}_{3iajb}$$

*where* $\sum_b \hat{V}_{3iajb} = \sum_b \frac{Q_{iajb}^2}{q_{iajb}} \hat{S}_{3iajb}^2$ is the estimated variance of the estimated total $\hat{t}_{iaj}$ for SSU $j|ia$ and $\hat{S}_{3iajb}^2 = \frac{1}{q_{iajb}-1} \sum_{k \in s_{iajb}} \left(y_k - \bar{y}_{s_{iajb}}\right)^2$ is the sample variance among HUs in HU substratum $b|iaj$.

*Proof.*

$$\begin{aligned}
E\left[v_{HU}\right] &= E_1\left\{E_2\left[E_3\left(\sum_{i \in s} \frac{1}{(mp_i)^2} \sum_a \sum_{j \in s_{ia}} \frac{1}{\left(n_{ia} p_{j|ia}\right)^2} \sum_b \frac{Q_{iajb}^2}{q_{iajb}} \hat{S}_{3iajb}^2 \;\middle|\; s_1, s_2\right) \;\middle|\; s_1\right]\right\} \\
&= E_1\left\{\sum_{i \in s_1} \frac{1}{(mp_i)^2} \sum_a E_2\left[\sum_{j \in s_{ia}} \frac{1}{\left(n_{ia} p_{j|ia}\right)^2} \sum_b \frac{Q_{iajb}^2}{q_{iajb}} E_3\left(\hat{S}_{3iajb}^2 \;\middle|\; s_1, s_2\right) \;\middle|\; s_1\right]\right\} \\
&= E_1\left\{\sum_{i \in s_1} \frac{1}{(mp_i)^2} \sum_a E_2\left[\frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{1}{p_{j|ia}}\left(\frac{1}{n_{ia} p_{j|ia}} \sum_b \frac{Q_{iajb}^2}{q_{iajb}} S_{U3iajb}^2\right) \;\middle|\; s_1\right]\right\} \\
&= E_1\left\{\frac{1}{m} \sum_{i \in s_1} \frac{1}{p_i}\left[\frac{1}{mp_i} \sum_a \sum_{j \in U_{ia}}\left(\frac{1}{n_{ia} p_{j|ia}} \sum_b \frac{Q_{iajb}^2}{q_{iajb}} S_{U3iajb}^2\right)\right]\right\} \\
&= \sum_{i \in U} \frac{1}{mp_i} \sum_a \sum_{j \in U_{ia}} \frac{1}{n_{ia} p_{j|ia}} \sum_b \frac{Q_{iajb}^2}{q_{iajb}} S_{U3iajb}^2 \\
&= \frac{1}{m} \sum_{i \in U} \frac{1}{p_i} \sum_{a=1}^{A} \frac{1}{n_{ia}} \sum_{j \in U_{ia}} \frac{1}{p_{j|ia}} \sum_{b=1}^{B} Q_{iajb}^2 \frac{S_{U3iajb}^2}{q_{iajb}} \\
&= V_{HU} \;\square
\end{aligned}$$

**Theorem 4.** *An unbiased estimate of the SSU component of relvariance,* $V_{SSU}$ *, is*

$$v_{SSU} = \sum_{i \in s} \frac{1}{(mp_i)^2} \sum_a \frac{1}{n_{ia}} \hat{S}^2_{2(pwr)ia}$$

*where*

$$\hat{S}^2_{2(pwr)ia} = \hat{S}^2_{2\mathbb{A}ia} - \hat{S}^2_{2\mathbb{B}ia}$$

*with*

$$\hat{S}^2_{2\mathbb{A}ia} = \frac{1}{n_{ia}-1} \sum_{j \in s_{ia}} \left( \frac{\hat{t}_{iaj}}{p_{j|ia}} - \hat{t}_{ia(pwr)} \right)^2, \qquad \hat{S}^2_{2\mathbb{B}ia} = \frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{\hat{V}_{3iaj} / p_{j|ia}}{p_{j|ia}},$$

$$\hat{V}_{3iaj} = \sum_b \frac{Q^2_{iajb}}{q_{iajb}} \hat{S}^2_{3iajb}, \qquad \hat{S}^2_{3iajb} = \frac{1}{q_{iajb}-1} \sum_{k \in s_{iajb}} \left( y_k - \bar{y}_{s_{iajb}} \right)^2.$$

*Proof*. First, we show that $\hat{S}^2_{2(pwr)ia} = \hat{S}^2_{2\mathbb{A}ia} - \hat{S}^2_{2\mathbb{B}ia}$ is an unbiased estimator of $S^2_{U2(pwr)ia} = \sum_{j \in U_{ia}} p_{j|ia} \left( t_{U_{iaj+}} \big/ p_{j|ia} - t_{U_{ia}} \right)^2$. A biased estimator of $S^2_{U2(pwr)ia}$ is obtained by writing what would be the estimator of $S^2_{U2(pwr)ia}$ in a single-stage sample:

$$\hat{S}^2_{2\mathbb{A}ia} = \frac{1}{n_{ia}-1} \sum_{j \in s_{ia}} \left( \frac{\hat{t}_{iaj+}}{p_{j|ia}} - \hat{t}_{ia(pwr)} \right)^2 \tag{A.16}$$

where

$\hat{t}_{iaj} = \sum_b \frac{Q_{iajb}}{q_{iajb}} \sum_{k \in s_{iajb}} y_k$ is the stratified *srs* estimated total for SSU *j*, within PSU *i*, SSU stratum *a* and

$\hat{t}_{ia(pwr)} = \frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{\hat{t}_{iaj}}{p_{j|ia}}$ is the *ppswr* estimated total for PSU *i*, SSU stratum *a*.

By expanding and simplifying Eq. (A.16), and using the fact that $n_{ia}\hat{t}_{ia(pwr)} = \sum_{j \in s_{ia}} \frac{\hat{t}_{iaj}}{p_{j|ia}}$,

we obtain an alternative expression for $\hat{S}^2_{2\mathrm{A}ia}$

$$
\begin{aligned}
\hat{S}^2_{2\mathbb{A}ia} &= \frac{1}{n_{ia}-1} \sum_{j \in s_{ia}} \left( \frac{\hat{t}_{iaj+}}{p_{j|ia}} - \hat{t}_{ia(pwr)} \right)^2 \\
&= \frac{1}{n_{ia}-1} \sum_{j \in s_{ia}} \left( \frac{\hat{t}^2_{iaj}}{p^2_{j|ia}} - 2\frac{\hat{t}_{iaj}}{p_{j|ia}} \hat{t}_{ia(pwr)} + \hat{t}^2_{ia(pwr)} \right) \\
&= \frac{1}{n_{ia}-1} \left( \sum_{j \in s_{ia}} \frac{\hat{t}^2_{iaj}}{p^2_{j|ia}} - 2\hat{t}_{ia(pwr)} \sum_{j \in s_{ia}} \frac{\hat{t}_{iaj}}{p_{j|ia}} + \hat{t}^2_{ia(pwr)} \sum_{j \in s_{ia}} 1 \right) \\
&= \frac{1}{n_{ia}-1} \left( \sum_{j \in s_{ia}} \frac{\hat{t}^2_{iaj}}{p^2_{j|ia}} - 2\hat{t}_{ia(pwr)} n_{ia} \hat{t}_{ia(pwr)} + n_{ia}\hat{t}^2_{ia(pwr)} \right) \\
&= \frac{1}{n_{ia}-1} \left( \sum_{j \in s_{ia}} \frac{\hat{t}^2_{iaj}}{p^2_{j|ia}} - n_{ia}\hat{t}^2_{ia(pwr)} \right)
\end{aligned}
\tag{A.17}
$$

Using the form of $\hat{S}^2_{2\mathbb{A}ia}$ in Eq. (A.35), we take the expected value of $\hat{S}^2_{2\mathbb{A}ia}$

$$
\begin{aligned}
E_2 E_3 \left[ \hat{S}^2_{2\mathbb{A}ia} \mid s_2 \right] &= E_2 \left[ E_3 \left( \frac{1}{n_{ia}-1} \left[ \sum_{j \in s_{ia}} \frac{\hat{t}^2_{iaj}}{p^2_{j|ia}} - n_{ia}\hat{t}^2_{ia(pwr)} \right] \mid s_2 \right) \right] \\
&= E_2 \left[ E_3 \left( \frac{1}{n_{ia}-1} \sum_{j \in s_{ia}} \frac{\hat{t}^2_{iaj}}{p^2_{j|ia}} \middle| s_2 \right) \right] - E_2 \left[ E_3 \left( \frac{n_{ia}}{n_{ia}-1} \hat{t}^2_{ia(pwr)} \middle| s_2 \right) \right]
\end{aligned}
\tag{A.18}
$$

Continuing from (A.18),

$$E_2 E_3\left[\hat{S}^2_{2\mathbb{A}ia} \mid s_2\right] = E_2\left[\frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\frac{1}{p^2_{j|ia}} E_3\left(\hat{t}^2_{iaj} \mid s_2\right)\right] - E_2\left[\frac{n_{ia}}{n_{ia}-1} E_3\left(\hat{t}^2_{ia(pwr)} \mid s_2\right)\right]$$

$$= E_2\left[\frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\frac{1}{p^2_{j|ia}}\left\{Var_3\left(\hat{t}_{iaj} \mid s_2\right) + \left[E_3\left(\hat{t}_{iaj} \mid s_2\right)\right]^2\right\}\right]$$

$$-E_2\left[\frac{n_{ia}}{n_{ia}-1}\left\{Var_3\left(\hat{t}_{ia(pwr)} \mid s_2\right) + \left[E_3\left(\hat{t}_{ia(pwr)} \mid s_2\right)\right]^2\right\}\right] \tag{A.19}$$

Now,

$$Var_3\left(\hat{t}_{iaj} \mid s_2\right) = \sum_b \frac{Q^2_{iajb}}{q_{iajb}} S^2_{U3iajb} = V_{3iaj} \qquad \text{(A.20)} \quad \text{and} \quad E_3\left(\hat{t}_{iaj} \mid s_2\right) = t_{U_{iaj}} \qquad \text{(A.21)}$$

This follows from the form of the variance of an estimated total for a stratified *srswor* design with a small sampling fraction in each stratum

$$Var_3\left(\hat{t}_{ia(pwr)} \mid s_2\right) = Var_3\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{t}_{iaj}}{p_{j|ia}} \mid s_2\right) = \frac{1}{n^2_{ia}}\sum_{j\in s_{ia}}\frac{Var_3\left(\hat{t}_{iaj} \mid s_2\right)}{p^2_{j|ia}} = \frac{1}{n^2_{ia}}\sum_{j\in s_{ia}}\frac{V_{3iaj}}{p^2_{j|ia}} \tag{A.22}$$

$$\left[E_3\left(\hat{t}_{ia(pwr)} \mid s_2\right)\right]^2 = \left[E_3\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{t}_{iaj}}{p_{j|ia}} \mid s_2\right)\right]^2 = \left[\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{E_3\left(\hat{t}_{iaj} \mid s_2\right)}{p_{j|ia}}\right]^2 = \left[\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}\right]^2 \tag{A.23}$$

Substituting Eqs. (A.38)- (A.41) into Eq. (A.37) and pulling out $n_{ia}/(n_{ia}-1)$ in the second term, we obtain

$$
\begin{aligned}
E_2 E_3\left[\hat{S}^2_{2\mathbb{A}ia} \mid s_2\right] &= E_2\left[\frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\frac{1}{p^2_{j|ia}}\left\{V_{3iaj}+t^2_{U_{iaj}}\right\}\right] \\
&\quad - E_2\left[\frac{n_{ia}}{n_{ia}-1}\left\{\frac{1}{n^2_{ia}}\sum_{j\in s_{ia}}\frac{V_{3iaj}}{p^2_{j|ia}}+\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}\right)^2\right\}\right] \\
&= E_2\left[\frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\frac{V_{3iaj}}{p^2_{j|ia}}\right]+E_2\left[\frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\frac{t^2_{U_{iaj}}/p_{j|ia}}{p_{j|ia}}\right] \\
&\quad - E_2\left[\frac{n_{ia}}{n_{ia}-1}\left\{\frac{1}{n^2_{ia}}\sum_{j\in s_{ia}}\frac{V_{3iaj}}{p^2_{j|ia}}\right\}\right]-E_2\left[\frac{n_{ia}}{n_{ia}-1}\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}\right)^2\right] \\
&= \frac{n_{ia}}{n_{ia}-1}E_2\left[\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{V_{3iaj}}{p^2_{j|ia}}\right]+\frac{n_{ia}}{n_{ia}-1}E_2\left[\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t^2_{U_{iaj}}/p_{j|ia}}{p_{j|ia}}\right] \\
&\quad - \frac{n_{ia}}{n_{ia}-1}\frac{1}{n_{ia}}E_2\left[\left\{\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{V_{3iaj}}{p^2_{j|ia}}\right\}\right]-\frac{n_{ia}}{n_{ia}-1}E_2\left[\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}\right)^2\right]
\end{aligned}
$$

(A.24)

Now applying Theorem 2,

$$
\begin{aligned}
E_2\left[\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}\right)^2\right] &= Var_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}\right)+\left[E_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}\right)\right]^2 \\
&= \frac{1}{n_{ia}}\sum_{j\in U_{ia}}p_{j|ia}\left(\frac{t_{U_{iaj+}}}{p_{j|ia}}-t_{U_{ia}}\right)^2+\left(\sum_{j\in U_{ia}}t_{U_{iaj}}\right)^2 \qquad \text{(A.25)} \\
&= \frac{S^2_{U2(pwr)ia}}{n_{ia}}+t^2_{U_{ia}}
\end{aligned}
$$

Substituting Eq. (A.43) into Eq. (A.42), and applying Theorem 2, we obtain

$$
\begin{aligned}
E_2\left[\hat{S}^2_{2\mathbb{A}ia}\right] &= \frac{n_{ia}}{n_{ia}-1}\left(\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}\right)+\frac{n_{ia}}{n_{ia}-1}\left(\sum_{j\in U_{ia}}\frac{t^2_{U_{iaj}}}{p_{j|ia}}\right) \\
&\quad -\frac{n_{ia}}{n_{ia}-1}\frac{1}{n_{ia}}\left(\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}\right)-\frac{n_{ia}}{n_{ia}-1}\frac{S^2_{U2(pwr)ia}}{n_{ia}}-\frac{n_{ia}}{n_{ia}-1}t^2_{U_{ia}} \\
&= \frac{n_{ia}}{n_{ia}-1}\left(\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}-\frac{1}{n_{ia}}\left(\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}\right)\right) \\
&\quad +\frac{n_{ia}}{n_{ia}-1}\left(\sum_{j\in U_{ia}}\frac{t^2_{U_{iaj}}}{p_{j|ia}}-t^2_{U_{ia}}\right)-\frac{n_{ia}}{n_{ia}-1}\frac{S^2_{U2(pwr)ia}}{n_{ia}} \\
&= \cancel{\frac{n_{ia}}{n_{ia}-1}}\left(\cancel{\frac{n_{ia}-1}{n_{ia}}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}\right) \\
&\quad +\frac{n_{ia}}{n_{ia}-1}\left\{\sum_{j\in U_{ia}}p_{j|ia}\left(\frac{t_{U_{iaj}}}{p_{j|ia}}-t_{U_{ia}}\right)^2\right\}-\frac{n_{ia}}{n_{ia}-1}\frac{S^2_{U2(pwr)ia}}{n_{ia}} \\
&= \sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}+\frac{n_{ia}}{n_{ia}-1}\left\{S^2_{U2(pwr)ia}-\frac{S^2_{U2(pwr)ia}}{n_{ia}}\right\} \\
&= \sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}+\cancel{\frac{n_{ia}}{n_{ia}-1}}\left\{\cancel{\frac{n_{ia}-1}{n_{ia}}}S^2_{U2(pwr)ia}\right\} \\
&= S^2_{U2(pwr)ia}+\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}
\end{aligned}
$$

(A.26)

Define

$$S^2_{2\mathbb{B}ia}=\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}} \tag{A.27}$$

Then substituting Eq. (A.27) into Eq. (A.44), we obtain

$$E\left[\hat{S}^2_{2\mathrm{A}ia}\right] = S^2_{U2(pwr)ia} + S^2_{2\mathrm{B}ia} \quad \text{(A.28)}$$

The bias of $\hat{S}^2_{2\mathrm{A}ia}$ is

$$\begin{aligned} Bias\left[\hat{S}^2_{2\mathrm{A}ia}\right] &= E\left[\hat{S}^2_{2\mathrm{A}ia}\right] - S^2_{U2(pwr)ia} \\ &= S^2_{U2(pwr)ia} + S^2_{2\mathrm{B}ia} - S^2_{U2(pwr)ia} \\ &= S^2_{2\mathrm{B}ia} \end{aligned}$$

and an unbiased estimator of the bias, $\hat{S}^2_{2\mathrm{B}ia}$, is

$$\hat{S}^2_{2\mathrm{B}ia} = \frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{\hat{V}_{3iaj} / p_{j|ia}}{p_{j|ia}}$$

To form an unbiased estimator of $S^2_{U2(pwr)ia}$, we subtract an unbiased estimator of the

bias, $\hat{S}^2_{2\mathrm{B}ia}$, from the biased estimator of $S^2_{U2(pwr)ia}$, $\hat{S}^2_{2\mathrm{A}ia}$, and obtain

$$\hat{S}^2_{U2(pwr)ia} = \hat{S}^2_{2\mathrm{A}ia} - \hat{S}^2_{2\mathrm{B}ia} \quad \text{(A.29)}$$

such that

$$\begin{aligned} E\left[\hat{S}^2_{U2(pwr)ia}\right] &= E\left[\hat{S}^2_{2\mathrm{A}ia} - \hat{S}^2_{2\mathrm{B}ia}\right] \\ &= E\left[\hat{S}^2_{2\mathrm{A}ia}\right] - E\left[\hat{S}^2_{2\mathrm{B}ia}\right] \\ &= \left(S^2_{U2(pwr)ia} + S^2_{2\mathrm{B}ia}\right) - S^2_{2\mathrm{B}ia} \\ &= S^2_{U2(pwr)ia} \square \end{aligned}$$

(A.30)

Finally, we show that $E\left[v_{SSU}\right] = V_{SSU}$ .

$$
\begin{aligned}
E\left[v_{SSU}\right] &= E_1\left\{E_2\left[E_3\left(\sum_{i\in s}\frac{1}{\left(mp_i\right)^2}\sum_{a=1}^{A}\frac{1}{n_{ia}}\hat{S}^2_{2(pwr)ia} \quad | \; s_1, s_2\right) \quad | \; s_1\right]\right\} \\
&= E_1\left\{\sum_{i\in s}\frac{1}{\left(mp_i\right)^2}\sum_{a=1}^{A}\frac{1}{n_{ia}}E_2\left[E_3\left(\hat{S}^2_{2(pwr)ia} \mid s_1, s_2\right) \mid s_1\right]\right\} \\
&= E_1\left[\sum_{i\in s}\frac{1}{\left(mp_i\right)^2}\sum_{a=1}^{A}\frac{S^2_{U2(pwr)ia}}{n_{ia}}\right] = E_1\left[\frac{1}{m}\sum_{i\in s}\frac{1}{p_i}\left(\frac{1}{mp_i}\sum_{a=1}^{A}\frac{S^2_{U2(pwr)ia}}{n_{ia}}\right)\right] \\
&= \sum_{i\in U}\left(\frac{1}{mp_i}\sum_{a=1}^{A}\frac{S^2_{U2(pwr)ia}}{n_{ia}}\right) = \frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_{a=1}^{A}\frac{S^2_{U2(pwr)ia}}{n_{ia}} \\
&= V_{SSU} \; \square
\end{aligned}
$$

(A.31)

**Theorem 5.** *An unbiased estimate of the PSU component of relvariance,*

$V_{PSU} = S^2_{U1(pwr)}\big/m$, *is*

$$v_{PSU} = \frac{\hat{S}^2_{1(pwr)}}{m}$$

*where*

$$\hat{S}^2_{1(pwr)} = \hat{S}^2_{1(pwr)\mathbb{A}} - \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_{a}\frac{\hat{S}^2_{2\mathbb{A}ia}}{n_{ia}}$$

*with*

$$\hat{S}^2_{1(pwr)\mathbb{A}} = \frac{1}{m-1}\sum_{i\in s_1}\left(\frac{\hat{t}_{i(pwr)}}{p_i} - \hat{t}_{pwr}\right)^2$$

and $\hat{S}^2_{2\mathbb{A}ia}$ as defined in Theorem 4.

*Proof*. First, we show that $\hat{S}^2_{1(pwr)}$ is an unbiased estimator of $S^2_{U1(pwr)} = \sum_{i\in U} p_i \left(\frac{t_i}{p_i} - t_U\right)^2$

A biased estimator of $S^2_{U1(pwr)}$ is obtained by writing what would be the estimator of $S^2_{U1(pwr)}$ in a single-stage sample:

$$\hat{S}^2_{1(pwr)\mathrm{A}} = \frac{1}{m-1}\sum_{i\in s_1}\left(\frac{\hat{t}_{i(pwr)}}{p_i} - \hat{t}_{pwr}\right)^2 \tag{A.32}$$

where $\hat{t}_{i(pwr)} = \sum_a \frac{1}{n_{ia}} \sum_{j\in s_{ia}} \frac{\hat{t}_{iaj}}{p_{j|ia}}$ is the estimated total for PSU $i$ with $\hat{t}_{iaj} = \sum_b \frac{\hat{Q}_{iajb}}{q_{iajb}} \sum_{k\in s_{iajb}} y_k$

and $\hat{t}_{pwr} = \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\sum_a \frac{1}{n_{ia}} \sum_{j\in s_{ia}} \frac{\hat{t}_{iaj}}{p_{j|ia}}\right)$ is the estimate of the population total.

By expanding and simplifying Eq. (A.32), and using the fact that $m\hat{t}_{pwr} = \sum_{i\in s_1}\frac{1}{p_i}\hat{t}_{i(pwr)}$,

$$\begin{aligned}
\hat{S}^2_{1(pwr)\mathrm{A}} &= \frac{1}{m-1}\sum_{i\in s_1}\left(\frac{\hat{t}_{i(pwr)}}{p_i} - \hat{t}_{pwr}\right)^2 \\
&= \frac{1}{m-1}\sum_{i\in s_1}\left(\frac{\hat{t}^2_{i(pwr)}}{p_i^2} - 2\frac{\hat{t}_{i(pwr)}}{p_i}\hat{t}_{pwr} + \hat{t}^2_{pwr}\right) \\
&= \frac{1}{m-1}\left[\sum_{i\in s_1}\frac{\hat{t}^2_{i(pwr)}}{p_i^2} - 2\hat{t}_{pwr}\sum_{i\in s_1}\frac{\hat{t}_{i(pwr)}}{p_i} + \sum_{i\in s_1}\hat{t}^2_{pwr}\right] \qquad \text{(A.33)} \\
&= \frac{1}{m-1}\left[\sum_{i\in s_1}\frac{\hat{t}^2_{i(pwr)}}{p_i^2} - 2\hat{t}^2_{pwr}m + \hat{t}^2_{pwr}m\right] \\
&= \frac{1}{m-1}\left[\sum_{i\in s_1}\frac{\hat{t}^2_{i(pwr)}}{p_i^2} - m\hat{t}^2_{pwr}\right]
\end{aligned}$$

Using the alternative form of $\hat{S}^2_{1(pwr)\mathrm{A}}$ in Eq. (A.36) and taking its expected value with respect to the third stage of sampling we obtain,

$$
\begin{aligned}
E_3\left[\hat{S}^2_{1(pwr)\mathrm{A}} \mid s_1, s_2\right] &= E_3\left[\frac{1}{m-1}\left(\sum_{i\in s_1}\frac{\hat{t}^2_{i(pwr)}}{p_i^2} - m\hat{t}^2_{pwr}\right) \mid s_1, s_2\right] \\
&= \frac{1}{m-1}\sum_{i\in s_1}\frac{1}{p_i^2}E_3\left[\hat{t}^2_{i(pwr)} \mid s_1, s_2\right] - \frac{m}{m-1}E_3\left[\hat{t}^2_{pwr} \mid s_1, s_2\right] \\
&= \frac{1}{m-1}\sum_{i\in s_1}\frac{1}{p_i^2}\left\{Var_3\left[\hat{t}_{i(pwr)} \mid s_1, s_2\right] + \left[E_3\left(\hat{t}_{i(pwr)} \mid s_1, s_2\right)\right]^2\right\} \\
&\quad - \frac{m}{m-1}\left\{Var_3\left[\hat{t}_{pwr} \mid s_1, s_2\right] + \left[E_3\left(\hat{t}_{pwr} \mid s_1, s_2\right)\right]^2\right\}
\end{aligned}
\tag{A.34}
$$

Now,

$$
\begin{aligned}
Var_3\left[\hat{t}_{i(pwr)} \mid s_1, s_2\right] &= Var_3\left[\sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{t}_{iaj}}{p_{j|ia}} \mid s_1, s_2\right] = \sum_a \frac{1}{n_{ia}^2}\sum_{j\in s_{ia}}\frac{Var_3\left[\hat{t}_{iaj} \mid s_1, s_2\right]}{p^2_{j|ia}} \\
&= \sum_a \frac{1}{n_{ia}^2}\sum_{j\in s_{ia}}\frac{V_{3iaj}}{p^2_{j|ia}} \\
&= \sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{1}{p_{j|ia}}\left(\frac{V_{3iaj}}{n_{ia}p_{j|ia}}\right)
\end{aligned}
\tag{A.35}
$$

$$
\begin{aligned}
E_3\left[\hat{t}_{i(pwr)} \mid s_1, s_2\right] &= E_3\left[\sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{t}_{iaj}}{p_{j|ia}} \mid s_1, s_2\right] = \sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{E_3\left(\hat{t}_{iaj} \mid s_1, s_2\right)}{p_{j|ia}} \\
&= \sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}} = \sum_a \hat{\hat{t}}_{ia}
\end{aligned}
\tag{A.36}
$$

where $\hat{\hat{t}}_{ia}$ is defined by the last equality, that is $\hat{\hat{t}}_{ia} = \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{1}{p_{j|ia}}t_{U_{iaj}}$ .

Also,

$$Var_3\left[\hat{t}_{pwr} \mid s_1, s_2\right] = Var_3\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{t}_{iaj}}{p_{j|ia}}\right) \mid s_1, s_2\right]$$

$$= \frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\left[\sum_a \frac{1}{n_{ia}^2}\sum_{j\in s_{ia}}\frac{1}{p_{j|ia}^2} Var_3\left[\hat{t}_{iaj} \mid s_1, s_2\right]\right]$$

$$= \frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\left[\sum_a \frac{1}{n_{ia}^2}\sum_{j\in s_{ia}}\frac{1}{p_{j|ia}^2} V_{3iaj}\right]$$

$$= \frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\left[\sum_a \frac{1}{n_{ia}^2}\sum_{j\in s_{ia}}\frac{V_{3iaj}/p_{j|ia}}{p_{j|ia}}\right]$$

(A.37)

and

$$E_3\left[\hat{t}_{pwr} \mid s_1, s_2\right] = E_3\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{t}_{iaj}}{p_{j|ia}}\right) \mid s_1, s_2\right]$$

$$= \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{1}{p_{j|ia}} E_3\left[\hat{t}_{iaj} \mid s_1, s_2\right]\right)$$

$$= \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{t_{U_{iaj}}}{p_{j|ia}}\right) = \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a \hat{\hat{t}}_{ia}$$

(A.38)

Substituting Eqs. (A.53) – (A.56) back into Eq. (A.52), we obtain

$$E_3\left[\hat{S}_{1(pwr)\mathrm{A}}^2 \mid s_1, s_2\right] = \frac{1}{m-1}\sum_{i\in s_1}\frac{1}{p_i^2}\left\{\sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{1}{p_{j|ia}}\left[\frac{V_{3iaj}}{n_{ia}p_{j|ia}}\right] + \left[\sum_a \hat{\hat{t}}_{ia}\right]^2\right\}$$

$$-\frac{m}{m-1}\left\{\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\left[\sum_a \frac{1}{n_{ia}^2}\sum_{j\in s_{ia}}\frac{V_{3iaj}/p_{j|ia}}{p_{j|ia}}\right] + \left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a \hat{\hat{t}}_{ia}\right]^2\right\}$$

(A.39)

Next taking the expected value of Eq. (A.57) with respect to the second stage sample, we obtain

$$E_2\left[E_3\left(\hat{S}^2_{1(pwr)\mathrm{A}} \mid s_1, s_2\right) \mid s_1\right] = \frac{1}{m-1}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a E_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{1}{p_{j|ia}}\left[\frac{V_{3iaj}}{n_{ia}p_{j|ia}}\right] \mid s_1\right)$$
$$+\frac{1}{m-1}\sum_{i\in s_1}E_2\left[\left(\sum_a \hat{\hat{t}}_{ia}\right)^2 \mid s_1\right]$$
$$-\frac{m}{m-1}\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\left[\sum_a\frac{1}{n_{ia}}E_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{V_{3iaj}/p_{j|ia}}{p_{j|ia}} \mid s_1\right)\right]$$
$$-\frac{m}{m-1}E_2\left[\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a\hat{\hat{t}}_{ia}\right)^2 \mid s_1\right]$$

$$=\frac{1}{m-1}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}$$
$$+\frac{1}{m-1}\sum_{i\in s_1}\frac{1}{p_i^2}E_2\left[\left(\sum_a\hat{\hat{t}}_{ia}\right)^2 \mid s_1\right]$$
$$-\frac{m}{m-1}\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\left[\sum_a\frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}\right]$$
$$-\frac{m}{m-1}E_2\left[\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a\hat{\hat{t}}_{ia}\right)^2 \mid s_1\right]$$

(A.40)

Now using basic properties of expectation, Theorem 2, and the definition of $S^2_{U2(pwr)ia}$,

we have

$$
\begin{aligned}
E_2\left[\left(\sum_a \hat{\hat{t}}_{ia}\right)^2 \mid s_1\right] &= Var_2\left(\sum_a \hat{\hat{t}}_{ia} \mid s_1\right) + \left[E_2\left(\sum_a \hat{\hat{t}}_{ia} \mid s_1\right)\right]^2 \\
&= Var_2\left(\sum_a \frac{1}{n_{ia}} \sum_{j\in s_{ia}} \frac{t_{U_{iaj}}}{p_{j|ia}} \mid s_1\right) + \left[E_2\left(\sum_a \frac{1}{n_{ia}} \sum_{j\in s_{ia}} \frac{t_{U_{iaj}}}{p_{j|ia}} \mid s_1\right)\right]^2 \\
&= \sum_a \frac{1}{n_{ia}} \sum_{j\in U_{ia}} p_{j|ia}\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right)^2 + \left[\sum_a \sum_{j\in U_{ia}} t_{U_{iaj}}\right]^2 \\
&= \sum_a \frac{S^2_{U2(pwr)ia}}{n_{ia}} + t^2_{U_{i+++}}
\end{aligned}
\tag{A.41}
$$

and

$$
\begin{aligned}
E_2\left[\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a \hat{\hat{t}}_{ia}\right)^2 \mid s_1\right] &= Var_2\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a \hat{\hat{t}}_{ia} \mid s_1\right] + \left[E_2\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a \hat{\hat{t}}_{ia} \mid s_1\right)\right]^2 \\
&= \frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2} Var_2\left[\sum_a \hat{\hat{t}}_{ia} \mid s_1\right] + \left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i} E_2\left(\sum_a \hat{\hat{t}}_{ia} \mid s_1\right)\right]^2 \\
&= \frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a \frac{S^2_{U2(pwr)ia}}{n_{ia}} + \left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i} t_{U_{i+++}}\right]^2
\end{aligned}
\tag{A.42}
$$

Substituting Eq. (A.59-A.60) back into Eq. (A.58) and simplifying, we obtain

$$
\begin{aligned}
E_2\left[E_3\left(\hat{S}^2_{1(pwr)\mathrm{A}} \mid s_1, s_2\right) \mid s_1\right] &= \frac{1}{m-1}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a \frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}} \\
&+\frac{1}{m-1}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{S^2_{U2(pwr)ia}}{n_{ia}}+\frac{1}{m-1}\sum_{i\in s_1}\frac{t^2_{U_{i+++}}}{p_i^2} \\
&-\frac{m}{m-1}\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\left[\sum_a\frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}\right] \\
&-\frac{m}{m-1}\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{S^2_{U2(pwr)ia}}{n_{ia}}-\frac{m}{m-1}\left[\frac{1}{m}\sum_{i\in s_1}\frac{t_{U_{i+++}}}{p_i}\right]^2 \\
&=\left[\frac{1}{m-1}-\frac{m}{m-1}\frac{1}{m^2}\right]\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}} \\
&+\left[\frac{1}{m-1}-\frac{m}{m-1}\frac{1}{m^2}\right]\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{S^2_{U2(pwr)ia}}{n_{ia}} \\
&+\frac{m}{m-1}\left[\frac{1}{m}\sum_{i\in s_1}\frac{t^2_{U_{i+++}}}{p_i^2}-\left(\frac{1}{m}\sum_{i\in s_1}\frac{t_{U_{i+++}}}{p_i}\right)^2\right] \\
&=\left[\frac{m-1}{m(m-1)}\right]\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}} \\
&+\left[\frac{m-1}{m(m-1)}\right]\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{S^2_{U2(pwr)ia}}{n_{ia}} \\
&+\frac{m}{m-1}\left[\frac{1}{m}\sum_{i\in s_1}\frac{t^2_{U_{i+++}}}{p_i^2}-\left(\frac{1}{m}\sum_{i\in s_1}\frac{t_{U_{i+++}}}{p_i}\right)^2\right]
\end{aligned}
\tag{A.43}
$$

Continuing from Eq. (A.61),

$$E_2\left[E_3\left(\hat{S}^2_{1(pwr)\mathrm{A}} \mid s_1, s_2\right) \mid s_1\right] = \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a \frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}$$

$$+\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a \frac{S^2_{U2(pwr)ia}}{n_{ia}} \tag{A.44}$$

$$+\frac{m}{m-1}\left[\frac{1}{m}\sum_{i\in s_1}\frac{t^2_{U_{i+++}}}{p_i^2} - \left(\frac{1}{m}\sum_{i\in s_1}\frac{t_{U_{i+++}}}{p_i}\right)^2\right]$$

Next taking the expected value of Eq. (A.62) with respect to the first stage sample, we obtain

$$E_1\left\{E_2\left[E_3\left(\hat{S}^2_{1(pwr)\mathrm{A}} \mid s_1, s_2\right) \mid s_1\right]\right\} = E_1\left\{\begin{array}{l} \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a \frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}} \\ +\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a \frac{S^2_{U2(pwr)ia}}{n_{ia}} \\ +\frac{m}{m-1}\left[\frac{1}{m}\sum_{i\in s_1}\frac{t^2_{U_{i+++}}}{p_i^2} - \left(\frac{1}{m}\sum_{i\in s_1}\frac{t_{U_{i+++}}}{p_i}\right)^2\right] \end{array}\right\}$$

$$= E_1\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\frac{1}{p_i}\sum_a \frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}\right)\right]$$

$$+E_1\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\frac{1}{p_i}\sum_a \frac{S^2_{U2(pwr)ia}}{n_{ia}}\right)\right]$$

$$+\frac{m}{m-1}\left\{E_1\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\frac{t^2_{U_{i+++}}}{p_i}\right)\right] - E_1\left[\left(\frac{1}{m}\sum_{i\in s_1}\frac{t_{U_{i+++}}}{p_i}\right)^2\right]\right\}$$

(A.45)

Continuing from Eq. (A.45),

$$
\begin{aligned}
E_1\left\{E_2\left[E_3\left(\hat{S}^2_{1(pwr)\mathrm{A}} \mid s_1, s_2\right) \mid s_1\right]\right\} &= E_1\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\frac{1}{p_i}\sum_{a}\frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}\right)\right] \\
&+E_1\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\frac{1}{p_i}\sum_{a}\frac{S^2_{U2(pwr)ia}}{n_{ia}}\right)\right] \\
&+\frac{m}{m-1}\left\{E_{11}\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\frac{t^2_{U_{i+++}}}{p_i}\right)\right]\right. \\
&\left.-Var_1\left[\frac{1}{m}\sum_{i\in s_1}\frac{t_{U_{i+++}}}{p_i}\right]-\left[E_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{t_{U_{i+++}}}{p_i}\right)\right]^2\right\} \\
&=\sum_{i\in U}\frac{1}{p_i}\sum_{a}\frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}}+\sum_{i\in U}\frac{1}{p_i}\sum_{a}\frac{S^2_{U2(pwr)ia}}{n_{ia}} \\
&+\frac{m}{m-1}\sum_{i\in U}\frac{t^2_{U_{i+++}}}{p_i}-\frac{\cancel{m}}{m-1}\frac{1}{\cancel{m}}\sum_{i\in U}p_i\left(\frac{t^2_{U_{i+++}}}{p_i}-t_U\right)^2 \\
&-\frac{m}{m-1}\left(\sum_{i\in U}t_{U_{i+++}}\right)^2
\end{aligned}
\tag{A.46}
$$

Now simplifying the third, fourth, and fifth terms of Eq.(A.46)

$$
\begin{aligned}
\frac{m}{m-1}\sum_{i\in U}\frac{t^2_{U_i}}{p_i}-\frac{m}{m-1}\left(\sum_{i\in U}t_{U_i}\right)^2-\frac{1}{m-1}\sum_{i\in U}p_i\left(\frac{t_{U_i}}{p_i}-t_U\right)^2 \\
&=\frac{m}{m-1}\left(\sum_{i\in U}\frac{t^2_{U_i}}{p_i}-t^2_U\right)-\frac{1}{m-1}\sum_{i\in U}p_i\left(\frac{t_{U_i}}{p_i}-t_U\right)^2 \\
&=\frac{m}{m-1}\sum_{i\in U}p_i\left(\frac{t_{U_i}}{p_i}-t_U\right)^2-\frac{1}{m-1}\sum_{i\in U}p_i\left(\frac{t_{U_i}}{p_i}-t_U\right)^2 \qquad \text{(A.47)} \\
&=\frac{m-1}{m-1}S^2_{U1(pwr)} \\
&=S^2_{U1(pwr)}
\end{aligned}
$$

Substituting Eq. (A.64) into Eq. (A.63) and rearranging terms, we obtain

$$
\begin{aligned}
E\left[\hat{S}^2_{1(pwr)\mathrm{A}}\right] &= E_1\left\{E_2\left[E_3\left(\hat{S}^2_{1(pwr)\mathrm{A}} \mid s_1, s_2\right) \mid s_1\right]\right\} \\
&= S^2_{U1(pwr)} + \sum_{i\in U}\frac{1}{p_i}\sum_a \frac{S^2_{U2(pwr)ia}}{n_{ia}} + \sum_{i\in U}\frac{1}{p_i}\sum_a \frac{1}{n_{ia}}\sum_{j\in U_{ia}}\frac{V_{3iaj}}{p_{j|ia}} \\
&= S^2_{U1(pwr)} + \sum_{i\in U}\frac{1}{p_i}\sum_a \frac{S^2_{U2(pwr)ia}}{n_{ia}} + \sum_{i\in U}\frac{1}{p_i}\sum_a \frac{S^2_{2\mathrm{B}ia}}{n_{ia}} \\
&= S^2_{U1(pwr)} + \sum_{i\in U}\frac{1}{p_i}\sum_a \frac{S^2_{U2(pwr)ia} + S^2_{2\mathrm{B}ia}}{n_{ia}}
\end{aligned}
$$

(A.48)

The bias of $\hat{S}^2_{1(pwr)\mathrm{A}}$ is

$$
\begin{aligned}
Bias\left[\hat{S}^2_{1(pwr)\mathrm{A}}\right] &= E\left[\hat{S}^2_{1(pwr)\mathrm{A}}\right] - S^2_{U1(pwr)} \\
&= S^2_{U1(pwr)} + \sum_{i\in U}\frac{1}{p_i}\sum_a \frac{S^2_{U2(pwr)ia} + S^2_{2\mathrm{B}ia}}{n_{ia}} - S^2_{U1(pwr)} \\
&= \sum_{i\in U}\frac{1}{p_i}\sum_a \frac{S^2_{U2(pwr)ia} + S^2_{2\mathrm{B}ia}}{n_{ia}}
\end{aligned}
$$

and an unbiased estimator of the bias is

$$
\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\frac{1}{p_i}\sum_a \frac{\hat{S}^2_{U2(pwr)ia} + \hat{S}^2_{2\mathrm{B}ia}}{n_{ia}}\right) \tag{A.49}
$$

Recall from Eq. (A.47) $\hat{S}^2_{U2(pwr)ia} + \hat{S}^2_{2\mathrm{B}ia} = \hat{S}^2_{2\mathbb{A}ia}$, we can rewrite Eq. (A.67) as

$$
\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a \frac{\hat{S}^2_{2\mathbb{A}ia}}{n_{ia}} \tag{A.50}
$$

To form an unbiased estimator of $S^2_{U1(pwr)}$, we subtract an unbiased estimator of the bias,

$\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{\hat{S}^2_{2\mathbb{A}ia}}{n_{ia}}$, from the biased estimator, $\hat{S}^2_{1(pwr)\mathbb{A}}$, and obtain

$$\hat{S}^2_{1(pwr)} = \hat{S}^2_{1(pwr)\mathbb{A}} - \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{\hat{S}^2_{2\mathbb{A}ia}}{n_{ia}} \tag{A.51}$$

such that

$$\begin{aligned}
E\left[\hat{S}^2_{1(pwr)}\right] &= E\left[\hat{S}^2_{1(pwr)\mathbb{A}} - \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{\hat{S}^2_{2\mathbb{A}ia}}{n_{ia}}\right] \\
&= E\left[\hat{S}^2_{1(pwr)\mathbb{A}}\right] - E\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left(\frac{1}{p_i}\sum_a\frac{\hat{S}^2_{2\mathbb{A}ia}}{n_{ia}}\right)\right] \\
&= \left(S^2_{U1(pwr)} + \sum_{i\in U}\frac{1}{p_i}\sum_a\frac{S^2_{U2(pwr)ia}+S^2_{2\mathrm{B}ia}}{n_{ia}}\right) - \sum_{i\in U}\frac{1}{p_i}\sum_a\frac{S^2_{U2(pwr)ia}+S^2_{2\mathrm{B}ia}}{n_{ia}} \\
&= S^2_{U1(pwr)} \square
\end{aligned}$$

Finally, we show that $E\left[v_{PSU}\right] = V_{PSU}$.

$$\begin{aligned}
E\left[v_{PSU}\right] &= E_1\left\{E_2\left[E_3\left(\hat{S}^2_{1(pwr)}/m \mid s_1, s_2\right) \mid s_1\right]\right\} \\
&= \frac{1}{m}E_1\left\{E_2\left[E_3\left(\hat{S}^2_{1(pwr)} \mid s_1, s_2\right) \mid s_1\right]\right\} \\
&= \frac{S^2_{U1(pwr)}}{m} \\
&= V_{PSU} \square
\end{aligned}$$

**Corollary to Theorem 5.** Assume that for each PSU $i$, $n_{ia} = \overline{n}_a$ and for every PSU/SSU $ij$ combination, $q_{iajb} = \overline{\overline{q}}_{ab}$. Then the estimated ANOVA relvariance $\hat{t}_{pwr}$ can be written as:

$$\frac{v\left(\hat{t}_{pwr}\right)}{\hat{t}_{pwr}^2}=\frac{\hat{B}^2}{m}+\sum_{a=1}^{A}\frac{\hat{W}_{2a}^2}{m\bar{n}_a}+\sum_{a=1}^{A}\sum_{b=1}^{B}\frac{\hat{W}_{3ab}^2}{m\bar{n}_a\bar{\bar{q}}_{ab}}$$

where

$$\hat{B}^2=\frac{\hat{S}_{1(pwr)}^2}{\hat{t}_{pwr}^2}$$

$$\hat{W}_{2a}^2=\frac{1}{\hat{t}_{pwr}^2}\sum_{i\in s_1}\frac{\hat{S}_{2(pwr)ia}^2}{mp_i^2}$$

$$\hat{W}_{3ab}^2=\frac{1}{\hat{t}_{pwr}^2}\left\{\sum_{i\in s_1}\frac{1}{mp_i^2}\sum_{j\in s_{ia}}\frac{1}{n_{ia}p_{j|ia}^2}\hat{Q}_{iajb}^2\hat{S}_{3iajb}^2\right\}$$

Proof.

$$\frac{v\left(\hat{t}_{pwr}\right)}{\hat{t}_{pwr}^2}=\frac{1}{\hat{t}_{pwr}^2}\left(v_{PSU}+v_{SSU}+v_{HU}\right)$$

$$=\frac{1}{\hat{t}_{pwr}^2}\left\{\frac{\hat{S}_{1(pwr)}^2}{m}+\sum_{i\in s_1}\frac{1}{\left(mp_i\right)^2}\sum_{a=1}^{A}\frac{\hat{S}_{2(pwr)ia}^2}{n_{ia}}+\sum_{i\in s_1}\frac{1}{\left(mp_i\right)^2}\sum_{a=1}^{A}\sum_{j\in s_{ia}}\frac{\sum_{b=1}^{B}\hat{V}_{3iajb}}{\left(n_{ia}p_{j|ia}\right)^2}\right\}$$

$$=\frac{1}{\hat{t}_{pwr}^2}\left\{\frac{\hat{S}_{1(pwr)}^2}{m}+\sum_{i\in s_1}\frac{1}{\left(mp_i\right)^2}\sum_{a=1}^{A}\frac{\hat{S}_{2(pwr)ia}^2}{\bar{n}_a}+\sum_{i\in s_1}\frac{1}{\left(mp_i\right)^2}\sum_{a=1}^{A}\sum_{j\in s_{ia}}\frac{\sum_{b=1}^{B}\frac{Q_{iajb}^2}{\bar{\bar{q}}_{ab}}\hat{S}_{3iajb}^2}{\left(\bar{n}_a p_{j|ia}\right)^2}\right\}$$

$$=\frac{1}{m}\frac{\hat{S}_{1(pwr)}^2}{\hat{t}_{pwr}^2}+\sum_{a=1}^{A}\frac{1}{m\bar{n}_a}\left(\frac{1}{\hat{t}_{pwr}^2}\sum_{i\in s_1}\frac{\hat{S}_{2(pwr)ia}^2}{mp_i^2}\right)$$

$$+\sum_{a=1}^{A}\sum_{b=1}^{B}\frac{1}{m\bar{n}_a\bar{\bar{q}}_{ab}}\left(\frac{1}{\hat{t}_{pwr}^2}\sum_{i\in s_1}\frac{}{mp_i^2}\sum_{j\in s_{ia}}\frac{Q_{iajb}^2\hat{S}_{3iajb}^2}{\bar{n}_a p_{j|ia}^2}\right)$$

$$=\frac{\hat{B}^2}{m}+\sum_{a=1}^{A}\frac{\hat{W}_{2a}^2}{m\bar{n}_a}+\sum_{a=1}^{A}\sum_{b=1}^{B}\frac{\hat{W}_{3ab}^2}{m\bar{n}_a\bar{\bar{q}}_{ab}}\square$$

## A.4 Derivation of Anticipated Variances

For the following section, consider a model for $y_k$ with common mean, $\mu$, and random effects for PSUs, $\alpha_i$, SSUs within PSU/SSU stratum $ia$, $\gamma_{iaj}$, HUs within PSU *i*/ SSU stratum *a*/ HU substratum *b*, $\lambda_{iajb}$, and elements, $\varepsilon_{iajbk}$:

$$y_k = \mu + \alpha_i + \gamma_{iaj} + \lambda_{iajb} + \varepsilon_{iajbk}$$

with $\alpha_i \sim \left(0\,,\,\sigma_\alpha^2\right)$, $\gamma_{iaj} \sim \left(0\,,\,\sigma_{\gamma_a}^2\right)$, $\lambda_{iajb} \sim \left(0\,,\,\sigma_{\lambda_b}^2\right)$, $\varepsilon_{iajbk} \sim \left(0\,,\,\sigma_{\varepsilon_{ab}}^2\right)$ and the errors being independent.

We first establish some preliminary results in Lemmas 1-4 that will be used in the proofs of the theorems and corollaries that follow.

Recall the following notation:

$M$ = the number of PSUs

$\bar{N} = \sum_{i \in U} \frac{N_i}{M}$, the average number of SSUs per PSU , $N_i = \sum_{a=1}^{A} N_{ia}$

$P_{ia} = \frac{N_{ia}}{N_i}$, the proportion of SSUs in strata *a*, PSU *i*

$Q = \sum_{i \in U} Q_i$, the total number of HUs in the population , $Q_i = \sum_{a=1}^{A} \sum_{j \in U_{ia}} Q_{iaj}$ ,

$Q_{iaj} = \sum_b Q_{iajb}$

$\bar{Q} = \sum_{i \in U} \frac{Q_i}{M} = \frac{Q}{M}$, the average number of HUs per PSU,

$\bar{Q}_{ia} = \sum_{j \in U_{ia}} \frac{Q_{iaj}}{N_{ia}}$, $Q_{ia} = \sum_{j \in U_{ia}} Q_{iaj}$ , $\bar{Q}_{ia\bullet b} = \sum_{j \in U_{iab}} \frac{Q_{iajb}}{N_{ia}}$, $Q_{ia\bullet b} = \sum_{j \in U_{ia}} Q_{iajb}$

$v^2_{Q,pwr} = \dfrac{S^2_{Q,pwr}}{\bar{Q}^2}$ is the unit relvariance of PSU sizes $Q_i$, when the PSUs are selected

using *ppswr*, $S^2_{Q,pwr} = \sum_{i \in U} p_i \left( \dfrac{Q_i}{p_i} - Q \right)^2$

$v^2_{Q_{ia}} = \dfrac{S^2_{Q_{ia}}}{\bar{Q}^2_{ia}}$ is the unit relvariance of SSU sizes $Q_{iaj}$, $S^2_{Q_{ia}} = \left( \dfrac{1}{N_{ia} - 1} \right) \sum_{j \in U_{ia}} \left( Q_{iaj} - \bar{Q}_{ia} \right)^2$

$v^2_{Q_{ia\bullet b}} = \dfrac{S^2_{Q_{ia\bullet b}}}{\bar{Q}^2_{ia\bullet b}}$ is the unit relvariance of HU substratum sizes $Q_{iajb}$, within *iab*

$$S^2_{Q_{ia\bullet b}} = \frac{1}{N_{ia} - 1} \sum_{j \in U_{iab}} \left( Q_{iajb} - \bar{Q}_{ia\bullet b} \right)^2$$

$v^2_{Q_{ia,pwr}} = \dfrac{S^2_{Q_{ia,pwr}}}{\bar{Q}^2_{ia}}$ is the unit relvariance among SSU counts of HUs within SSU stratum

*a* when SSUs are selected using *ppswr*, $S^2_{Q_{ia,pwr}} = \sum_{j \in U_{ia}} p_{j|ia} \left( \dfrac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right)^2$

$v^2_{Q_{ia\bullet b,pwr}} = \dfrac{S^2_{Q_{ia\bullet b,pwr}}}{\bar{Q}^2_{ia\bullet b}}$ is the unit relvariance among SSU counts of HUs within SSU

stratum *a* and HU substratum *b* when SSUs are selected using *ppswr*,

$$S^2_{Q_{ia\bullet b,pwr}} = \sum_{j \in U_{iab}} p_{j|ia} \left( \frac{Q_{iajb}}{p_{j|ia}} - Q_{ia\bullet b} \right)^2$$

### A.4.1 Lemma 1

**Lemma 1.** Assume a large number of SSUs in every PSU/SSU stratum *ia* combination is large so that $N_{ia} \approx N_{ia} - 1$. Then the following equalities hold:

(a) $\sum_{j \in U_{ia}} Q_{iajb}^2 = N_{ia}\left(S_{Q_{ia \bullet b}}^2 + \bar{Q}_{ia \bullet b}^2\right)$

(b) $\sum_{j \in U_{ia}} Q_{iajb}^2 = N_{ia}\bar{Q}_{ia \bullet b}^2\left(v_{Q_{ia \bullet b}}^2 + 1\right)$

(c) $\sum_{j \in U_{ia}} \frac{Q_{iaj}^2}{p_{j|ia}} = S_{Q_{ia,pwr}}^2 + Q_{ia}^2 = S_{Q_{ia,pwr}}^2 + N_{ia}^2\bar{Q}_{ia}^2 = N_{ia}^2\bar{Q}_{ia}^2\left(\frac{v_{Q_{ia,pwr}}^2}{N_{ia}^2} + 1\right)$

(d) $\sum_{j \in U_{ia}} Q_{iaj}^2 = N_{ia}\bar{Q}_{ia}^2\left(v_{Q_{ia}}^2 + 1\right)$

(e) $\sum_{j \in U_{ia}} \frac{Q_{iajb}^2}{p_{j|ia}} = S_{Q_{ia \bullet b,pwr}}^2 + Q_{ia \bullet b}^2 = S_{Q_{ia \bullet b,pwr}}^2 + N_{ia}^2\bar{Q}_{ia \bullet b}^2 = N_{ia}^2\bar{Q}_{ia \bullet b}^2\left(\frac{v_{Q_{ia \bullet b,pwr}}^2}{N_{ia}^2} + 1\right)$

(f) $\sum_{j \in U_{ia}} \frac{Q_{iaj}^2}{p_{j|ia}} - \left(\sum_{j \in U_{ia}} Q_{iaj}^2\right) = N_{ia}^2\bar{Q}_{ia}^2\left(\frac{v_{Q_{ia},pwr}^2}{N_{ia}^2} - \frac{v_{Q_{ia}}^2}{N_{ia}} + 1\right)$

(g) $\sum_{j \in U_{ia}} \frac{Q_{iajb}^2}{p_{j|ia}} - \left(\sum_{j \in U_{ia}} Q_{iajb}^2\right) = N_{ia}^2\bar{Q}_{ia \bullet b}^2\left(\frac{v_{Q_{ia \bullet b},pwr}^2}{N_{ia}^2} - \frac{v_{Q_{ia \bullet b}}^2}{N_{ia}} + 1\right)$

(h) $\sum_{i \in U} \frac{Q_i^2}{p_i} = M^2\bar{Q}^2\left(\frac{v_{Q_{(pwr)}}^2}{M^2} + 1\right)$

(i) $\sum_{i \in U} Q_i^2 = M\bar{Q}^2\left(v_Q^2 + 1\right)$, assuming $M \approx M - 1$.

Proof.

(a) Let $S^2_{Q_{ia\bullet b}} = \dfrac{1}{N_{ia}-1} \sum_{j \in U_{iab}} \left( Q_{iajb} - \bar{Q}_{ia\bullet b} \right)^2$ be the unit variance of the number of HUs for SSU $j$ in PSU $i$, SSU stratum $a$, HU substratum $b$ where $\bar{Q}_{ia\bullet b} = \dfrac{Q_{ia\bullet b}}{N_{ia}}$. Suppose a large number of SSUs so $N_{ia} \approx N_{ia} - 1$. Expanding and simplifying we obtain

$$\begin{aligned} N_{ia} S^2_{Q_{ia\bullet b}} &\doteq \sum_{j \in U_{ia}} \left( Q_{iajb} - \bar{Q}_{ia\bullet b} \right)^2 \\ &= \left( \sum_{j \in U_{ia}} Q^2_{iajb} \right) - N_{ia} \bar{Q}^2_{ia\bullet b} \end{aligned}$$

Rearranging terms leads to

$$N_{ia} S^2_{Q_{ia\bullet b}} = \left( \sum_{j \in U_{ia}} Q^2_{iajb} \right) - N_{ia} \bar{Q}^2_{ia\bullet b}$$

$$\left( \sum_{j \in U_{ia}} Q^2_{iajb} \right) = N_{ia} \left( S^2_{Q_{ia\bullet b}} + \bar{Q}^2_{ia\bullet b} \right) \quad \square$$

(b) Define the unit relvariance of the count of HUs across all SSUs in PSU $i$, SSU stratum $a$, HU substratum $b$ as $v^2_{Q_{ia\bullet b}} = S^2_{Q_{ia\bullet b}} \big/ \bar{Q}^2_{ia\bullet b}$. Suppose a large number of SSUs such that $N_{ia} \approx N_{ia} - 1$. Then

$$\begin{aligned} \sum_{j \in U_{ia}} Q^2_{iajb} &= \left( N_{ia} - 1 \right) S^2_{Q_{iab}} + N_{ia} \bar{Q}^2_{ia\bullet b} \\ &\doteq N_{ia} \bar{Q}^2_{ia\bullet b} v^2_{Q_{ia}} + N_{ia} \bar{Q}^2_{ia\bullet b} \\ &= N_{ia} \bar{Q}^2_{ia\bullet b} \left( v^2_{Q_{ia}} + 1 \right) \end{aligned}$$

(c) Let $S^2_{Q_{ia,pwr}} = \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right)^2$ be the unit variance when SSUs are selected *ppswr* of the number of HUs for SSU *j* in PSU *i*, SSU stratum *a*, HU substratum *b*, when SSUs are selected *ppswr*. Expanding and simplifying we obtain:

$$
\begin{aligned}
S^2_{Q_{ia,pwr}} &= \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right)^2 \\
&= \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{Q^2_{iaj}}{p^2_{j|ia}} - 2Q_{ia} \frac{Q_{iaj}}{p_{j|ia}} + Q^2_{ia} \right) \\
&= \sum_{j \in U_{ia}} \left( \frac{Q^2_{iaj}}{p_{j|ia}} \right) - 2Q_{ia} \left( \sum_{j \in U_{ia}} Q_{iaj} \right) + Q^2_{ia} \sum_{j \in U_{ia}} p_{j|ia} \\
&= \sum_{j \in U_{ia}} \left( \frac{Q^2_{iaj}}{p_{j|ia}} \right) - 2Q_{ia}Q_{ia} + Q^2_{ia} \qquad \text{since } \sum_{j \in U_{ia}} p_{j|ia} = 1 \\
&= \sum_{j \in U_{ia}} \left( \frac{Q^2_{iaj}}{p_{j|ia}} \right) - Q^2_{ia}
\end{aligned}
$$

Rearranging terms leads to

$$\sum_{j \in U_{ia}} \left( \frac{Q_{iaj}^2}{p_{j|ia}} \right) = S_{Q_{ia},pwr}^2 + Q_{ia}^2$$

$$\sum_{j \in U_{ia}} \left( \frac{Q_{iaj}^2}{p_{j|ia}} \right) = S_{Q_{ia},pwr}^2 + N_{ia}^2 \bar{Q}_{ia}^2 \qquad \text{since } Q_{ia} = N_{ia} \bar{Q}_{ia}$$

(d) Let $N_{ia} S_{Q_{ia}}^2 = \sum_{j \in U_{ia}} \left( Q_{iaj} - \bar{Q}_{ia} \right)^2$ be the unit variance of the number of HUs for SSU *j* in PSU *i*, SSU stratum *a*. Then

$$\begin{aligned} N_{ia} S_{Q_{ia}}^2 &= \sum_{j \in U_{ia}} \left( Q_{iaj} - \bar{Q}_{ia} \right)^2 \\ &= \left( \sum_{j \in U_{ia}} Q_{iaj}^2 \right) - N_{ia} \bar{Q}_{ia}^2 \end{aligned}$$

Rearranging terms leads to

$$N_{ia} S_{Q_{ia}}^2 = \left( \sum_{j \in U_{ia}} Q_{iaj}^2 \right) - N_{ia} \bar{Q}_{ia}^2$$

$$\begin{aligned} \left( \sum_{j \in U_{ia}} Q_{iaj}^2 \right) &= N_{ia} S_{Q_{ia}}^2 + N_{ia} \bar{Q}_{ia}^2 \\ &= N_{ia} \bar{Q}_{ia}^2 \left( v_{Q_{ia}}^2 + 1 \right) \text{ by definition of } v_{Q_{ia}}^2 \end{aligned}$$

(e) Let $S_{Q_{ia \bullet b},pwr}^2 = \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{Q_{iajb}}{p_{j|ia}} - Q_{ia \bullet b} \right)^2$ be the unit variance when SSUs are selected *ppswr* of the number of HUs for SSU *j* in PSU *i*, SSU stratum *a*, HU substratum *b*. Expanding and simplifying we obtain:

$$
\begin{aligned}
S^2_{Q_{ia\bullet b,\,pwr}} &= \sum_{j\in U_{ia}} p_{j|ia}\left(\frac{Q_{iajb}}{p_{j|ia}} - Q_{ia\bullet b}\right)^2 \\
&= \sum_{j\in U_{ia}} p_{j|ia}\left(\frac{Q^2_{iajb}}{p^2_{j|ia}} - 2Q_{ia\bullet b}\frac{Q_{iajb}}{p_{j|ia}} - Q^2_{ia\bullet b}\right) \\
&= \sum_{j\in U_{ia}}\left(\frac{Q^2_{iajb}}{p_{j|ia}}\right) - 2Q_{ia\bullet b}\left(\sum_{j\in U_{ia}} Q_{iajb}\right) - Q^2_{ia\bullet b}\sum_{j\in U_{ia}} p_{j|ia} \\
&= \sum_{j\in U_{ia}}\left(\frac{Q^2_{iajb}}{p_{j|ia}}\right) - 2Q_{ia\bullet b}Q_{ia\bullet b} - Q^2_{ia\bullet b} \qquad \text{since} \sum_{j\in U_{ia}} p_{j|ia} = 1 \\
&= \sum_{j\in U_{ia}}\left(\frac{Q^2_{iajb}}{p_{j|ia}}\right) - Q^2_{ia\bullet b}
\end{aligned}
$$

Rearranging terms leads to

$$
S^2_{Q_{ia\bullet b,\,pwr}} = \sum_{j\in U_{ia}}\left(\frac{Q^2_{iajb}}{p_{j|ia}}\right) - Q^2_{ia\bullet b}
$$

$$
\sum_{j\in U_{ia}}\left(\frac{Q^2_{iajb}}{p_{j|ia}}\right) = S^2_{Q_{ia\bullet b,\,pwr}} + Q^2_{ia\bullet b}
$$

$$
\sum_{j\in U_{ia}}\left(\frac{Q^2_{iajb}}{p_{j|ia}}\right) = S^2_{Q_{ia\bullet b,\,pwr}} + N^2_{ia}\bar{Q}^2_{ia\bullet b} \square
$$

(f) Using results from Lemma 1(c)-1(d) and assuming $N_{ia} \approx N_{ia} - 1$, we obtain

$$\sum_{j \in U_{ia}} \frac{Q_{iaj}^2}{p_{j|ia}} - \left( \sum_{j \in U_{ia}} Q_{iaj}^2 \right) = S_{Q_{ia},pwr}^2 + N_{ia}^2 \bar{Q}_{ia}^2 - N_{ia} S_{Q_{ia}}^2 - N_{ia} \bar{Q}_{ia}^2 \quad \text{by Lemma 1(c)-1(d)}$$

$$= S_{Q_{ia},pwr}^2 - N_{ia} S_{Q_{ia}}^2 + N_{ia}^2 \bar{Q}_{ia}^2 - N_{ia} \bar{Q}_{ia}^2$$

$$= S_{Q_{ia},pwr}^2 - N_{ia} S_{Q_{ia}}^2 + \bar{Q}_{ia}^2 N_{ia} \left( N_{ia} - 1 \right)$$

$$\doteq S_{Q_{ia},pwr}^2 - N_{ia} S_{Q_{ia}}^2 + \bar{Q}_{ia}^2 N_{ia}^2 \qquad \text{since } N_{ia} \approx \left( N_{ia} - 1 \right)$$

$$= N_{ia}^2 \bar{Q}_{ia}^2 \left( \frac{S_{Q_{ia},pwr}^2}{N_{ia}^2 \bar{Q}_{ia}^2} - \frac{S_{Q_{ia}}^2}{N_{ia} \bar{Q}_{ia}^2} + 1 \right)$$

$$= N_{ia}^2 \bar{Q}_{ia}^2 \left( \frac{v_{Q_{ia},pwr}^2}{N_{ia}^2} - \frac{v_{Q_{ia}}^2}{N_{ia}} + 1 \right) \qquad \text{by definition of relvariance}$$

(g) Using results from Lemma 1 and assuming $N_{ia} \approx N_{ia} - 1$, we obtain

$$\sum_{j \in U_{ia}} \frac{Q_{iajb}^2}{p_{j|ia}} - \left( \sum_{j \in U_{ia}} Q_{iajb}^2 \right) = S_{Q_{ia\bullet b},pwr}^2 + N_{ia}^2 \bar{Q}_{ia\bullet b}^2 - N_{ia} S_{Q_{ia\bullet b}}^2 - N_{ia} \bar{Q}_{ia\bullet b}^2 \quad \text{by Lemma 1(a),1(e)}$$

$$= S_{Q_{ia\bullet b},pwr}^2 - N_{ia} S_{Q_{ia\bullet b}}^2 + N_{ia}^2 \bar{Q}_{ia\bullet b}^2 - N_{ia} \bar{Q}_{ia\bullet b}^2$$

$$= S_{Q_{ia\bullet b},pwr}^2 - N_{ia} S_{Q_{ia\bullet b}}^2 + \bar{Q}_{ia\bullet b}^2 N_{ia} \left( N_{ia} - 1 \right)$$

$$\doteq S_{Q_{ia\bullet b},pwr}^2 - N_{ia} S_{Q_{ia\bullet b}}^2 + \bar{Q}_{ia\bullet b}^2 N_{ia}^2 \qquad \text{since } N_{ia} \approx \left( N_{ia} - 1 \right)$$

$$= N_{ia}^2 \bar{Q}_{ia\bullet b}^2 \left( \frac{S_{Q_{ia\bullet b},pwr}^2}{N_{ia}^2 \bar{Q}_{ia}^2} - \frac{S_{Q_{ia\bullet b}}^2}{N_{ia} \bar{Q}_{ia}^2} + 1 \right)$$

$$= N_{ia}^2 \bar{Q}_{ia\bullet b}^2 \left( \frac{v_{Q_{ia\bullet b},pwr}^2}{N_{ia}^2} - \frac{v_{Q_{ia\bullet b}}^2}{N_{ia}} + 1 \right) \qquad \text{by definition of relvariance}$$

(h) Let $S^2_{Q_{(pwr)}}$ be the unit variance of PSU sizes when a *ppswr* sample is selected. Then

$$S^2_{Q_{(pwr)}} = \sum_{i\in U} p_i \left( \frac{Q_i}{p_i} - Q \right)^2 = \sum_{i\in U} \frac{Q_i^2}{p_i} - Q^2$$

Rearranging terms leads to

$$\begin{aligned} \sum_{i\in U} \frac{Q_i^2}{p_i} &= S^2_{Q_{(pwr)}} + Q^2 \\ &= S^2_{Q_{(pwr)}} + M^2 \bar{Q}^2 \\ &= M^2 \bar{Q}^2 \left( \frac{v^2_{Q_{(pwr)}}}{M^2} + 1 \right) \quad \text{by definition of } v^2_{Q_{(pwr)}} \end{aligned}$$

(i) Let $S^2_Q = \left( \frac{1}{M-1} \right) \sum_{i\in U} \left( Q_i - \bar{Q} \right)^2$ be the unit variance of PSU sizes $Q_i$ where

$\bar{Q} = \sum_{i\in U} \frac{Q_i}{M}$. Additionally, suppose a large number of PSUs such that $M \approx M-1$.

Expanding and simplifying we obtain:

$$\begin{aligned} M\, S^2_Q &\doteq \sum_{i\in U} \left( Q_i - \bar{Q} \right)^2 \\ &= \left( \sum_{i\in U} Q_i^2 \right) - M\bar{Q}^2 \end{aligned}$$

Rearranging terms leads to

$$\begin{aligned} \sum_{i\in U} Q_i^2 &\doteq M \left( S^2_Q + \bar{Q}^2 \right) \\ &= M \left( \bar{Q}^2 v^2_Q + \bar{Q}^2 \right) \\ &= M\bar{Q}^2 \left( v^2_Q + 1 \right) \quad \square \end{aligned}$$

### A.4.2 Lemma 2

In Lemmas 2 - 3, we assume following:

(*A1*) Every SSU stratum/HU substratum *ab* combination occurs in every SSU in the population.

(*A2*) $p_i = \dfrac{Q_i}{Q}$ and $p_{j|ia} = \dfrac{Q_{iaj}}{Q_{ia}}$. That is, PSUs and SSUs are sampled with probabilities proportional to the number of HUs they contain.

(*A3*) $Q_{iajb} = \bar{\bar{Q}}_b$. That is, every PSU/SSU stratum *a*/SSU *iaj* combination, has the same number of HUs in HU substratum *b*.

(*A4*) $P_{ia} = \dfrac{N_{ia}}{N_i} \equiv P_a$, i.e., the proportion of SSUs in stratum *a* is the same in every PSU *i*.

(*A5*) The number of SSUs in every PSU/SSU stratum *ia* combination is large, i.e., $N_{ia} \approx N_{ia} - 1$.

**Lemma 2.** Assume that (*A1*) - (*A3*) hold. Assumptions (*A2*) and (*A3*) imply that all SSUs have the same number of elements, $\bar{\bar{Q}} = \sum_b \bar{\bar{Q}}_b$. Then the following hold:

(a) $Q_{iaj} = \bar{\bar{Q}}$

(b) $Q_{ia\bullet b} = N_{ia}\bar{\bar{Q}}_b$

(c) $\bar{Q}_{ia\bullet b} = \bar{\bar{Q}}_b$

(d) $Q_{ia} = N_{ia}\bar{\bar{Q}}$

(e) $\bar{Q}_{ia} = \bar{\bar{Q}}$

(f) $\bar{Q}_i = \bar{\bar{Q}}$

(g) $Q_i = N_i \bar{\bar{Q}}$

(h) $Q = M\bar{N}\bar{\bar{Q}}$

(i) $\bar{Q} = \bar{N}\bar{\bar{Q}}$ and $M\bar{Q} = M\bar{N}\bar{\bar{Q}}$

(j) $Q_a = N_a \bar{\bar{Q}}$

(k) $Q_{ab} = N_a \bar{\bar{Q}}_b$

(l) $\bar{Q}_{ab} = \bar{\bar{Q}}_b$

(m) $\bar{Q}_{1a} = \dfrac{N_a \bar{\bar{Q}}}{M}$

(n) $\bar{Q}_{2a} = \bar{\bar{Q}}$

(o) $p_{j|ia} = \dfrac{1}{N_{ia}}$

(p) $p_i = \dfrac{N_i}{M\bar{N}}$

Proof.

(a) $Q_{iaj} = \sum_b Q_{iajb} = \sum_b \bar{\bar{Q}}_b = \bar{\bar{Q}}$

(b) $Q_{ia\bullet b} = \sum_{j \in U_{ia}} Q_{iajb} = \sum_{j \in U_{ia}} \bar{\bar{Q}}_b = N_{ia}\bar{\bar{Q}}_b$

(c) $\bar{Q}_{ia\bullet b} = \dfrac{Q_{ia\bullet b}}{N_{ia}} = \dfrac{\cancel{N_{ia}}\bar{\bar{Q}}_b}{\cancel{N_{ia}}} = \bar{\bar{Q}}_b$

(d) $Q_{ia} = \sum_{j \in U_{ia}} \sum_b Q_{iajb} = \sum_{j \in U_{ia}} \bar{\bar{Q}} = N_{ia}\bar{\bar{Q}}$

(e) $\bar{Q}_{ia} = \dfrac{Q_{ia}}{N_{ia}} = \dfrac{N_{ia}\bar{\bar{Q}}}{N_{ia}} = \bar{\bar{Q}}$

(f) $\bar{Q}_i = \dfrac{Q_i}{N_i} = \dfrac{1}{N_i}\sum_a Q_{ia} = \dfrac{1}{N_i}\sum_a N_{ia}\bar{\bar{Q}} = \dfrac{1}{\cancel{N_i}}\cancel{N_i}\bar{\bar{Q}} = \bar{\bar{Q}}$

(g) $Q_i = N_i\bar{Q}_i = N_i\bar{\bar{Q}}$

(h) $Q = \sum_{i\in U} Q_i = \left(\sum_{i\in U} N_i\right)\bar{\bar{Q}} = M\bar{N}\bar{\bar{Q}}$

(i) $\bar{Q} = \dfrac{Q}{M} = \dfrac{M\bar{N}\bar{\bar{Q}}}{M} = \bar{N}\bar{\bar{Q}}$; multiplying both sides by $M$ we obtain $M\bar{Q} = M\bar{N}\bar{\bar{Q}}$

(j) $Q_a = \sum_{i\in U} Q_{ia} = \sum_{i\in U} N_{ia}\bar{\bar{Q}} = N_a\bar{\bar{Q}}$

(k) $Q_{ab} = \sum_{i\in U}\sum_{j\in U_{ia}} Q_{iajb} = \sum_{i\in U}\sum_{j\in U_{ia}} \bar{\bar{Q}}_b = N_a\bar{\bar{Q}}_b$

(l) $\bar{Q}_{ab} = \dfrac{Q_{ab}}{N_a} = \dfrac{N_a\bar{\bar{Q}}_b}{N_a} = \bar{\bar{Q}}_b$

(m) $\bar{Q}_{1a} = \dfrac{Q_a}{M} = \dfrac{N_a\bar{\bar{Q}}}{M}$

(n) $\bar{Q}_{2a} = \dfrac{Q_a}{N_a} = \dfrac{N_a\bar{\bar{Q}}}{N_a} = \bar{\bar{Q}}$

(o) $p_{j|ia} = \dfrac{Q_{iaj}}{Q_{ia}} = \dfrac{\sum_b \bar{\bar{Q}}_b}{N_{ia}\sum_b \bar{\bar{Q}}_b} = \dfrac{1}{N_{ia}}$

(p) $p_i = \dfrac{Q_i}{Q} = \dfrac{N_i\bar{\bar{Q}}}{M\bar{N}\bar{\bar{Q}}} = \dfrac{N_i}{M\bar{N}}$

### A.4.3 Lemma 3

**Lemma 3.** Assume that (*A1*) - (*A3*) hold. Then

(a) $S^2_{Q_{(pwr)}} = v^2_{Q_{(pwr)}} = 0$ if only (*A1*) holds

(b) $S^2_{Q_{ia\bullet b}} = v^2_{Q_{ia\bullet b}} = 0$

(c) $S^2_{Q_{ia}} = v^2_{Q_{ia}} = 0$

(d) $S^2_{Q_{ia(pwr)}} = v^2_{Q_{ia(pwr)}} = 0$ if only (*A1*) holds

(e) $S^2_{Q_{2a}} = v^2_{Q_{2a}} = 0$

(f) $S^2_Q = \dfrac{\bar{\bar{Q}}^2}{M-1}\sum_{i\in U}\left(N_i - \bar{N}\right)^2 \equiv \bar{\bar{Q}}^2 S^2_N$, assuming $M \approx M-1$

(g) $v^2_Q \equiv \dfrac{S^2_N}{\bar{N}^2} = v^2_N$, assuming $M \approx M$ -1

(h) $S^2_{Q_{1a}} = \dfrac{\bar{\bar{Q}}^2}{M-1}\sum_{i\in U}\left(N_i - \bar{N}\right)^2 \equiv \bar{\bar{Q}}^2 S^2_{N_a}$

(i) $S^2_{Q_a(pwr)} = \bar{\bar{Q}}^2 \sum_{i\in U} p_{2i}\left(\dfrac{N_{ia}}{p_{2i}} - N_a\right)^2 \equiv \bar{\bar{Q}}^2 S^2_{N_a(pwr)}$ where $p_{2i} = \dfrac{N_i}{N}$

Proof.

(a) $S^2_{Q_{(pwr)}} = \sum_{i\in U} p_i\left(\dfrac{Q_i}{p_i} - Q\right)^2 = \sum_{i\in U} p_i\left(\dfrac{\cancel{Q_i}}{\cancel{Q_i}}Q - Q\right)^2 = 0$; then $v^2_{Q_{(pwr)}} = \dfrac{S^2_{Q_{(pwr)}}}{\bar{Q}^2} = 0$

which holds when only (*A2*) is true.

(b) $S^2_{Q_{ia\bullet b}} = \dfrac{1}{N_{ia}-1}\sum_{j\in U_{ia}}\left(Q_{iajb} - \bar{Q}_{ia\bullet b}\right)^2 = \dfrac{1}{N_{ia}-1}\sum_{j\in U_{ia}}\left(\bar{\bar{Q}}_b - \bar{\bar{Q}}_b\right)^2 = 0$; then

$v^2_{Q_{ia\bullet b}} = \dfrac{S^2_{Q_{ia\bullet b}}}{\bar{Q}^2_{ia\bullet b}} = 0$

(c) $S^2_{Q_{ia}} = \left(\dfrac{1}{N_{ia}-1}\right)\sum_{j\in U_{ia}}\left(Q_{iaj}-\bar{Q}_{ia}\right)^2 = \left(\dfrac{1}{N_{ia}-1}\right)\sum_{j\in U_{ia}}\left(\sum_b \bar{\bar{Q}}_b - \sum_b \bar{\bar{Q}}_b\right)^2 = 0$ ; then

$$v^2_{Q_{ia}} = \frac{S^2_{Q_{ia}}}{\bar{Q}^2_{ia}} = 0$$

(d) $S^2_{Q_{ia(pwr)}} = \sum_{j\in U_{ia}} p_{j|ia}\left(\dfrac{Q_{iaj}}{p_{j|ia}} - Q_{ia}\right)^2 = \sum_{j\in U_{ia}} \dfrac{1}{N_{ia}}\left(\dfrac{\cancel{Q_{iaj}}\,Q_{ia}}{\cancel{Q_{iaj}}} - Q_{ia}\right)^2 = 0$ ; then

$v^2_{Q_{ia(pwr)}} = \dfrac{S^2_{Q_{ia(pwr)}}}{\bar{Q}^2_{ia}} = 0$ which holds when only ($A2$) is true

(e) We know that $Q_{iaj} = \bar{\bar{Q}} = \bar{Q}_{1a}$ by Lemma 2(a) and 2(n). Expanding and substituting we obtain,

$$S^2_{Q_{2a}} = \left(N_a - 1\right)^{-1}\sum_{i\in U}\sum_{j\in U_{ia}}\left(Q_{iaj} - \bar{Q}_{2a}\right) = \frac{1}{N_a-1}\left(\sum_{i\in U}\sum_{j\in U_{ia}} Q^2_{iaj} - \bar{Q}^2_{2a}N_a\right)$$

$$= \frac{1}{N_a-1}\left(\sum_{i\in U}\sum_{j\in U_{ia}} \bar{\bar{Q}}^2 - \bar{\bar{Q}}^2 N_a\right) = \frac{1}{N_a-1}\left(\bar{\bar{Q}}^2 N_a - \bar{\bar{Q}}^2 N_a\right) = 0$$

then $v^2_{Q_{2a}} = \dfrac{S^2_{Q_{2a}}}{\bar{Q}^2_{2a}} = 0$

(f) $S^2_Q = \dfrac{1}{M-1}\sum_{i\in U}\left(Q_i - \bar{Q}\right)^2 = \dfrac{1}{M-1}\left(\sum_{i\in U} N_i^2 \bar{\bar{Q}}^2 - M\bar{N}^2\bar{\bar{Q}}^2\right)$

$$= \frac{1}{M-1}\bar{\bar{Q}}^2\left(\sum_{i\in U} N_i^2 - M\bar{N}^2\right) = \frac{\bar{\bar{Q}}^2}{M-1}\sum_{i\in U}\left(N_i - \bar{N}\right)^2$$

$\equiv \bar{\bar{Q}}^2 S^2_N$ assuming $M \approx M$ -1

(g) $v_Q^2 = \frac{S_Q^2}{\bar{Q}^2} \equiv \frac{\bar{\bar{Q}}^2 S_N^2}{\left(\bar{N}\bar{\bar{Q}}\right)^2} = \frac{S_N^2}{\bar{N}^2} = v_N^2$ assuming $M \approx M$ -1

(h) $$S_{Q_{1a}}^2 = \frac{1}{M-1}\sum_{i\in U}\left(Q_{ia} - \bar{Q}_{1a}\right)^2 = \frac{1}{M-1}\left(\sum_{i\in U} Q_{ia}^2 - M\bar{Q}_{1a}^2\right)$$

$$= \frac{1}{M-1}\left[\sum_{i\in U}\left(N_{ia}\sum_b \bar{\bar{Q}}_b\right)^2 - M\left(M^{-1}\sum\nolimits_{i\in U} N_{ia}\sum_b \bar{\bar{Q}}_b\right)^2\right]$$

$$= \frac{1}{M-1}\left[\sum_{i\in U} N_{ia}^2\bar{\bar{Q}}^2 - M\left(M^{-2}N_a^2\bar{\bar{Q}}^2\right)\right]$$

$$= \frac{1}{M-1}\bar{\bar{Q}}^2\left(\sum_{i\in U} N_{ia}^2 - \frac{N_a^2}{M}\right) = \frac{1}{M-1}\bar{\bar{Q}}^2\sum_{i\in U}\left(N_{ia} - \bar{N}_a\right)^2$$

$$\equiv \bar{\bar{Q}}^2 S_{N_a}^2$$

(i) $$S_{Q_a(pwr)}^2 = \sum_{i\in U} p_i\left(\frac{Q_{ia}}{p_i} - Q_a\right)^2 = \sum_{i\in U}\frac{Q_{ia}^2}{p_i} - Q_a^2$$

$$= \sum_{i\in U}\frac{N_{ia}^2\bar{\bar{Q}}^2}{Q_i Q^{-1}} - N_a^2\bar{\bar{Q}}^2 = \sum_{i\in U}\frac{N_{ia}^2\bar{\bar{Q}}^2 M\bar{N}\cancel{\bar{\bar{Q}}}}{N_i\cancel{\bar{\bar{Q}}}} - N_a^2\bar{\bar{Q}}^2$$

$$= M\bar{N}\bar{\bar{Q}}^2\sum_{i\in U}\frac{N_{ia}^2}{N_i} - N_a^2\bar{\bar{Q}}^2 = \bar{\bar{Q}}^2\left(M\bar{N}\sum_{i\in U}\frac{N_{ia}^2}{N_i} - N_a^2\right)$$

$$= \bar{\bar{Q}}^2\sum_{i\in U}\frac{N_i}{N}\left(\frac{N_{ia}}{N_i/N} - N_a\right)^2$$

$$= \bar{\bar{Q}}^2\sum_{i\in U} p_{2i}\left(\frac{N_{ia}}{p_{2i}} - N_a\right)^2$$

$$\equiv \bar{\bar{Q}}^2 S_{N_a(pwr)}^2$$

### A.4.4 Lemma 4

**Lemma 4.** For any *p*,

$$\left(\frac{1}{p}-1\right)^2-1=\left(\frac{1-2p}{p^2}\right)$$

Proof.

$$\begin{aligned}\left(\frac{1}{p}-1\right)^2-1&=\frac{1}{p^2}-\frac{2}{p}+1-1\\&=\left(\frac{1}{p^2}-\frac{2p}{p^2}\right)\\&=\left(\frac{1-2p}{p^2}\right)\end{aligned}$$

### A.4.5 Model Expectation of B$^2$

**Theorem 6.** *The approximate model expectation of* $B^2$ *is*

$$E_M\left(t_U^2 B^2\right) \doteq \sigma_\alpha^2 M^2 \bar{Q}^2\left(\frac{v_{Q(pwr)}^2}{M^2}+1\right)+\sigma_\gamma^2 \sum_{i\in U}\frac{1}{p_i}\left\{\sum_{a=1}^{A} N_{ia}\bar{Q}_{ia}^2\left(v_{Q_{ia}}^2+1\right)\right.$$

$$\left.+\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{ia\bullet b}\right\}+\mu^2\bar{Q}^2 v_{Q(pwr)}^2 .$$

*Proof*.

Recall that

$$\begin{aligned} t_U^2\, B^2 &= \mathrm{S}_{U1(pwr)}^2 \\ &= \sum_{i\in U} p_i\left(\frac{t_{U_i}}{p_i}-t_U\right)^2 \end{aligned} \tag{A.52}$$

where $t_U = \sum_{i\in U} t_{U_i}$ and $t_{U_i} = \sum_{a=1}^{A} t_{U_{ia}} = \sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}} y_k$ . Substituting the model form of $y_k = \mu+\alpha_i+\gamma_{iaj}+\varepsilon_{iajbk}$ into Equation (A.52), we have

$$\begin{aligned} t_{U_i} &= \sum_{a=1}^{A} t_{U_{ia}} \\ &= Q_i\left(\mu+\alpha_i\right)+\sum_{a=1}^{A}\sum_{j\in U_{ia}} Q_{iaj}\gamma_{iaj}+\varepsilon_i \end{aligned} \tag{A.53}$$

$$\begin{aligned} t_U &= \sum_{i\in U} t_{U_i} \\ &= Q\mu+\sum_{i\in U} Q_i\alpha_i+\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}} Q_{iaj}\gamma_{iaj}+\varepsilon \end{aligned} \tag{A.54}$$

where $\varepsilon_i = \sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}} \varepsilon_{iajbk}$ and $\varepsilon = \sum_{i\in U}\varepsilon_i$ .

Taking the expected value of Eqs. (A.53) and (A.54) we obtain

$$E_M\left(t_{U_i}\right)=E_M\left(Q_i\mu\right)=Q_i\mu \tag{A.55}$$

$$E_M\left(t_U\right)=E_M\left(Q\mu\right)=Q\mu \tag{A.56}$$

(All other terms have expected value with respect to the model equal to zero.)

Taking the expected value of Equation (A.52) we obtain,

$$\begin{aligned} E_M\left(t_U^2B^2\right) &= \sum_{i\in U} p_i E_M\left(\frac{t_{U_i}}{p_i}-t_U\right)^2 \\ &= \sum_{i\in U} p_i\left[E_M\left(\frac{t_{U_i}}{p_i}-t_U\right)\right]^2+\sum_{i\in U} p_i Var_M\left(\frac{t_{U_i}}{p_i}-t_U\right) \end{aligned} \tag{A.57}$$

So we need to find

$$1.\ \sum_{i\in U} p_i\left[E_M\left(\frac{t_{U_i}}{p_i}-t_U\right)\right]^2 \tag{A.58}$$

and

$$2.\ \sum_{i\in U} p_i\, Var_M\left(\frac{t_{U_i}}{p_i}-t_U\right) \tag{A.59}$$

1. Taking the expectation in Equation (A.58), we obtain

$$\left[E_M\left(\frac{t_{U_i}}{p_i}-t_U\right)\right]^2=\left(\frac{Q_i\mu}{p_i}-Q\mu\right)^2=\left(\frac{Q_i}{p_i}-Q\right)^2\mu^2 \tag{A.60}$$

Multiplying Equation (A.60) through by $p_i$ and summing over the all PSUs $i$ in the population, we obtain

$$\begin{aligned}\sum_{i\in U} p_i\left[E_M\left(\frac{t_{U_i}}{p_i}-t_U\right)\right]^2 &= \sum_{i\in U} p_i\left(\frac{Q_i}{p_i}-Q\right)^2\mu^2 \\ &= \mu^2 S^2_{Q_{pwr}} \\ &= \mu^2\bar{Q}^2 v^2_{Q_{pwr}} \quad \square\end{aligned} \tag{A.61}$$

Still solving Equation (A.58), but now assuming $p_i = \dfrac{Q_i}{Q}$, we obtain

$$\begin{aligned}\sum_{i\in U} p_i\left[E_M\left(\frac{t_{U_i}}{p_i}-t_U\right)\right]^2 &= \sum_{i\in U} p_i\left(\cancel{Q_i}\frac{Q}{\cancel{Q_i}}-Q\right)^2\mu^2 \\ &= 0 \quad \square\end{aligned} \tag{A.62}$$

2. To evaluate $\sum_{i\in U} p_i\, Var_M\left(\dfrac{t_{U_i}}{p_i}-t_U\right)$, first find

$$\begin{aligned}\left(\frac{t_{U_i}}{p_i}-t_U\right) &= \mu\left(\frac{Q_i}{p_i}-Q\right)+\alpha_i\frac{Q_i}{p_i}-\sum_{i\in U}\alpha_i Q_i+\sum_{a=1}^{A}\left[\sum_{j\in U_{ia}}\gamma_{iaj}\frac{Q_{iaj}}{p_i}-\sum_{i\in U}\sum_{j\in U_{ia}}\gamma_{iaj}Q_{iaj}\right] \\ &\quad+\frac{\varepsilon_i}{p_i}-\sum_{i\in U}\varepsilon_i \\ &= \mu\left(\frac{Q_{i'}}{p_{i'}}-Q\right)+\alpha_i Q_i\left(\frac{1}{p_i}-1\right)-\sum_{i'\neq i\in U}\alpha_{i'}Q_{i'} \\ &\quad+\sum_{a=1}^{A}\left[\sum_{j\in U_{ia}}\gamma_{iaj}Q_{iaj}\left(\frac{1}{p_i}-1\right)-\sum_{i'j'\neq(ij)}^{U,U_{i'a}}\gamma_{i'aj'}Q_{i'aj'}\right] \\ &\quad+\varepsilon_i\left(\frac{1}{p_i}-1\right)-\sum_{i'\neq i\in U}\varepsilon_{i'}\end{aligned} \tag{A.63}$$

Now taking the variance of Equation (A.63) with respect to the model, and using the assumption that the random effects are uncorrelated, we have

$$
\begin{aligned}
Var_M\left(\frac{t_{U_i}}{p_i}-t_U\right) &= Var_M\left[\mu\left(\frac{Q_{i'}}{p_{i'}}-Q\right)\right]+Var_M\left[\alpha_i Q_i\left(\frac{1}{p_i}-1\right)-\sum_{i'\neq i\in U}\alpha_{i'}Q_{i'}\right] \\
&+Var_M\left[\sum_{a=1}^{A}\left(\sum_{j\in U_{ia}}\gamma_{iaj}Q_{iaj}\left(\frac{1}{p_i}-1\right)-\sum_{i'j'\neq(ij)}^{U,U_{i'a}}\gamma_{i'aj'}Q_{i'aj'}\right)\right] \\
&+Var_M\left[\varepsilon_i\left(\frac{1}{p_i}-1\right)-\sum_{i'\neq i\in U}\varepsilon_{i'}\right]
\end{aligned}
\tag{A.64}
$$

Next, we show the details of evaluating the terms of Equation (A.64) :

A. $$Var_M\left[\mu\left(\frac{Q_{i'}}{p_{i'}}-Q\right)\right]=\left(\frac{Q_{i'}}{p_{i'}}-Q\right)Var_M\left(\mu\right)$$

$$=0.$$

B. $$
\begin{aligned}
Var_M\left[\alpha_i Q_i\left(\frac{1}{p_i}-1\right)-\sum_{i'\neq i\in U}\alpha_i Q_i\right] &= Q_i^2\left(\frac{1}{p_i}-1\right)^2\sigma_\alpha^2+\sum_{i'\neq i\in U}Q_{i'}^2\sigma_\alpha^2 \\
&= Q_i^2\left[\left(\frac{1}{p_i}-1\right)^2-1\right]\sigma_\alpha^2+\sum_{i'\in U}Q_{i'}^2\sigma_\alpha^2 \\
&= \sigma_\alpha^2\left[\frac{Q_i^2}{p_i^2}-\frac{2Q_i^2}{p_i}+\sum_{i'\in U}Q_{i'}^2\right]\text{by Lemma 4.}
\end{aligned}
$$

$$\text{C. } Var_M\left[\sum_{a=1}^{A}\left(\sum_{j\in U_{ia}}\gamma_{iaj}Q_{iaj}\left(\frac{1}{p_i}-1\right)-\sum_{i'j'\neq(ij)}^{U,U_{i'a}}\gamma_{i'aj'}Q_{i'aj'}\right)\right]$$

$$=\sigma_\gamma^2\sum_{a=1}^{A}\left(\sum_{j\in U_{ia}}Q_{iaj}^2\left(\frac{1}{p_i}-1\right)^2+\sum_{i'j'\neq(ij)}^{U,U_{i'a}}Q_{i'aj'}^2\right)$$

$$=\sigma_\gamma^2\sum_{a=1}^{A}\left(\sum_{j\in U_{ia}}Q_{iaj}^2\left[\left(\frac{1}{p_i}-1\right)^2-1\right]+\sum_{i'j'\neq(ij)}^{U,U_{i'a}}Q_{i'aj'}^2\right)$$

$$=\sigma_\gamma^2\sum_{a=1}^{A}\left(\sum_{j\in U_{ia}}Q_{iaj}^2\left[\frac{1}{p_i^2}-\frac{2}{p_i}\right]+\sum_{i'j'\neq(ij)}^{U,U_{i'a}}Q_{i'aj'}^2\right)\text{ by Lemma 4.}$$

$$D.\ Var_M\left[\varepsilon_i\left(\frac{1}{p_i}-1\right)-\sum_{i'\neq i\in U}\varepsilon_{i'}\right]$$

$$=\left(Var_M\left(\varepsilon_i\right)\left(\frac{1}{p_i}-1\right)^2+\sum_{i'\neq i\in U}Var_M\left(\varepsilon_i\right)\right)$$

$$=Var_M\left(\varepsilon_i\right)\left[\left(\frac{1}{p_i}-1\right)^2-1\right]+\sum_{i'\in U}Var_M\left(\varepsilon_i\right)$$

$$=Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\left[\frac{1}{p_i^2}-\frac{2}{p_i}\right]+\sum_{i'\in U}Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\quad\text{by Lemma 4.}$$

since $Var_M\left(\varepsilon_i\right)=\sum_a\sum_{j\in U_{ia}}\sum_b\sum_{k\in U_{iajb}}Var_M\left(\varepsilon_{iajbk}\right)=\sum_a\sum_b\sum_j\sum_k\sigma_{\varepsilon_{ab}}^2=Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2$ .

Substituting A, B, C, and D back into Equation (A.64) we obtain,

$$Var_M\left(\frac{t_{U_i}}{p_i}-t_U\right)=0+\sigma_\alpha^2\left[\frac{Q_i^2}{p_i^2}-\frac{2Q_i^2}{p_i}+\sum_{i\in U}Q_i^2\right]$$
$$+\sigma_\gamma^2\sum_{a=1}^{A}\sum_{j\in U_{ia}}\left[\frac{Q_{iaj}^2}{p_i^2}-\frac{2Q_{iaj}^2}{p_i}\right]+\sigma_\gamma^2\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}Q_{iaj}^2$$
$$+\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left[\frac{Q_{ia\bullet b}}{p_i^2}-\frac{2Q_{ia\bullet b}}{p_i}+\sum_{i\in U}Q_{ia\bullet b}\right]$$

Multiplying through by $p_i$, summing over all PSUs $i$ in the population, and reversing the sum over $i$ and the other sums, we obtain

$$\begin{aligned}\sum_{i\in U} p_i Var_M\left(\frac{t_{U_i}}{p_i}-t_U\right) &= \sigma_\alpha^2\left[\sum_{i\in U}\frac{Q_i^2}{p_i}-\sum_{i\in U}2Q_i^2+\left(\sum_{i\in U}p_i\right)\sum_{i\in U}Q_i^2\right]\\ &+\sigma_\gamma^2\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\left[\frac{Q_{iaj}^2}{p_i}-2Q_{iaj}^2\right]+\sigma_\gamma^2\left(\sum_{i\in U}p_i\right)\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}Q_{iaj}^2\\ &+\sum_{i\in U}\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left[\frac{Q_{ia\bullet b}}{p_i}-2Q_{ia\bullet b}\right]+\left(\sum_{i\in U}p_i\right)\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\sum_{i\in U}Q_{ia\bullet b}\end{aligned}\tag{A.65}$$

Since $\sum_{i\in U} p_i = 1$, Equation (A.65) reduces to

$$\begin{aligned}\sum_{i\in U} p_i Var_M\left(\frac{t_{U_i}}{p_i}-t_U\right) &= \sigma_\alpha^2\left[\sum_{i\in U}\frac{Q_i^2}{p_i}-\sum_{i\in U}Q_i^2\right]\\ &+\sigma_\gamma^2\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\left[\frac{Q_{iaj}^2}{p_i}-Q_{iaj}^2\right]\\ &+\sum_{i\in U}\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left[\frac{Q_{ia\bullet b}}{p_i}-Q_{ia\bullet b}\right]\end{aligned}\tag{A.66}$$

Assuming $\frac{1}{p_i}$ is large so $\frac{1}{p_i}-1\approx\frac{1}{p_i}$, we obtain

$$\begin{aligned}\sum_{i\in U} p_i Var_M\left(\frac{t_{U_i}}{p_i}-t_U\right) &\doteq \sigma_\alpha^2\sum_{i\in U}\frac{Q_i^2}{p_i}\\ &+\sigma_\gamma^2\sum_{i\in U}\frac{1}{p_i}\sum_{a=1}^{A}\sum_{j\in U_{ia}}Q_{iaj}^2\\ &+\sum_{i\in U}\frac{1}{p_i}\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{ia\bullet b}\end{aligned}\tag{A.67}$$

Substituting the results from Eqs. (A.61) and (A.67) back into Equation (A.57),

$$E_M\left(t_U^2 B^2\right) \doteq \sigma_\alpha^2 \sum_{i\in U} \frac{Q_i^2}{p_i} + \sigma_\gamma^2 \sum_{i\in U} \frac{1}{p_i} \sum_{a=1}^{A} \sum_{j\in U_{ia}} Q_{iaj}^2$$
$$+\sum_{i\in U} \frac{1}{p_i} \sum_{a=1}^{A} \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 Q_{ia\bullet b} + \sum_{i\in U} p_i \left(\frac{Q_i}{p_i} - Q\right)^2 \mu^2$$

$$= \sigma_\alpha^2 \sum_{i\in U} \frac{Q_i^2}{p_i}$$
$$+\sigma_\gamma^2 \sum_{i\in U} \frac{1}{p_i} \sum_{a=1}^{A} \sum_{j\in U_{ia}} Q_{iaj}^2$$
$$+\sum_{i\in U} \frac{1}{p_i} \sum_{a=1}^{A} \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 Q_{ia\bullet b}$$
$$+\mu^2 \bar{Q}^2 v_{Q_{(pwr)}}^2$$

(A.68)

Assuming $N_{ia}$ is large so, $N_{ia} \approx N_{ia} - 1$

$$E_M\left(t_U^2 B^2\right) \doteq \sigma_\alpha^2 M^2 \bar{Q}^2 \left(\frac{v_{Q_{(pwr)}}^2}{M^2} + 1\right) \text{ by Lemma 1(h)}$$
$$+\sigma_\gamma^2 \sum_{i\in U} \frac{1}{p_i} \sum_{a=1}^{A} N_{ia} \bar{Q}_{ia}^2 \left(v_{Q_{ia}}^2 + 1\right) \text{ by Lemma 1(d)}$$
$$+\sum_{i\in U} \frac{1}{p_i} \sum_{a=1}^{A} \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 Q_{ia\bullet b}$$
$$+\mu^2 \bar{Q}^2 v_{Q_{(pwr)}}^2 \ \square$$

(A.69)

**Corollary to Theorem 6.** Assume that conditions (*A1*) – (*A5*) in Appendix A.4.2 hold. Then in the special case of no *b* strata so that $\sigma^2_{\varepsilon_{ab}} = \sigma^2_{\varepsilon_a}$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectation of $B^2$ can be simplified to

$$E_M\left(t_U^2 B^2\right) \doteq M^2 \bar{N}^2 \bar{\bar{Q}}^2 \left[\sigma^2_\alpha + \frac{\sigma^2_\gamma}{\bar{N}} + \sum_{a=1}^{A} P_a \frac{\sigma^2_{\varepsilon_a}}{\bar{N}\bar{\bar{Q}}}\right]$$

and when there are no *a* strata so that $P_a = 1$ and $\sigma^2_{\varepsilon_a} = \sigma^2_\varepsilon$,

$$E_M\left(t_U^2 B^2\right) \doteq M^2 \bar{N}^2 \bar{\bar{Q}}^2 \left[\sigma^2_\alpha + \frac{\sigma^2_\gamma}{\bar{N}} + \frac{\sigma^2_\varepsilon}{\bar{N}\bar{\bar{Q}}}\right].$$

*Proof.*

When (*A1*) - (*A3*) hold, $v^2_{Q(pwr)} = v^2_{Q_{ia}} = 0$. Substituting this result back into Eq. (A.69),

$$\begin{aligned} E_M\left(t_U^2 B^2\right) &\doteq \sigma^2_\alpha M^2 \bar{Q}^2 \\ &+ \sigma^2_\gamma \sum_{i \in U} \frac{1}{p_i} \sum_{a=1}^{A} N_{ia} \bar{Q}^2_{ia} \\ &+ \sum_{i \in U} \frac{1}{p_i} \sum_{a=1}^{A} \sum_{b=1}^{B} \sigma^2_{\varepsilon_{ab}} Q_{ia \bullet b} \end{aligned} \tag{A.70}$$

When (*A3*) holds, we can use the results from Lemma 2 to obtain

$$\begin{aligned} E_M\left(t_U^2 B^2\right) &\doteq \sigma^2_\alpha M^2 \bar{N}^2 \bar{\bar{Q}}^2 && \text{by Lemma 2(i)} \\ &+ \sigma^2_\gamma \sum_{i \in U} \frac{1}{p_i} \sum_{a=1}^{A} N_{ia} \bar{\bar{Q}}^2 && \text{since } \bar{Q}^2_{ia} = \bar{\bar{Q}}^2 \text{ Lemma 2(e)} \\ &+ \sum_{i \in U} \frac{1}{p_i} \sum_{a=1}^{A} \sum_{b=1}^{B} \sigma^2_{\varepsilon_{ab}} N_{ia} \bar{\bar{Q}}_b && \text{since } Q_{ia \bullet b} = N_{ia} \bar{\bar{Q}}^2_b \text{ Lemma 2(b)} \end{aligned} \tag{A.71}$$

By Lemma 2(p), $p_i = \dfrac{N_i}{M\bar{N}}$. Rearranging terms in $P_{ia} = \dfrac{N_{ia}}{N_i} \equiv P_a$ from (*A4*), we have

$N_i P_{ia} = N_{ia} \equiv N_i P_a$. And so, $\dfrac{N_{ia}}{p_i} = \cancel{N_i} P_a \dfrac{M\bar{N}}{\cancel{N_i}} = P_a M\bar{N}$. Substituting $\dfrac{N_{ia}}{p_i} = P_a M\bar{N}$,

$$
\begin{aligned}
E_M\left(t_U^2 B^2\right) &\doteq \sigma_\alpha^2 M^2 \bar{N}^2 \bar{\bar{Q}}^2 + \sigma_\gamma^2 \sum_{i\in U} \sum_{a=1}^{A} P_a M\bar{N}\bar{\bar{Q}}^2 + \sum_{i\in U} \sum_{a=1}^{A} \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 P_a M\bar{N}\bar{\bar{Q}}_b \\
&= M^2 \bar{N}^2 \bar{\bar{Q}}^2 \left[\sigma_\alpha^2 + \frac{\sigma_\gamma^2}{\bar{N}}\right] + M^2 \bar{N}^2 \sum_{a=1}^{A} P_a \sum_{b=1}^{B} \bar{\bar{Q}}_b \frac{\sigma_{\varepsilon_{ab}}^2}{\bar{N}} \\
&= \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma_\alpha^2 + \frac{\sigma_\gamma^2}{\bar{N}} + \sum_{a=1}^{A} P_a \sum_{b=1}^{B} \frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}^2} \frac{\sigma_{\varepsilon_{ab}}^2}{\bar{N}} \right]
\end{aligned}
\tag{A.72}
$$

Note that reversing the *a* and *b* summations in Eq. (A.72) assumes that every SSU stratum *a* contains all HU *b* substrata. In the special case of no *b* strata so that $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectation of $B^2$ can be simplified to

$$
E_M\left(t_U^2 B^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma_\alpha^2 + \frac{\sigma_\gamma^2}{\bar{N}} + \sum_{a=1}^{A}\left( P_a \frac{\sigma_{\varepsilon_a}^2}{\bar{N}\bar{\bar{Q}}}\right)\right]
$$

and when there are no *a* strata so that $P_a = 1$ and $\sigma_{\varepsilon_a}^2 = \sigma_\varepsilon^2$,

$$
E_M\left(t_U^2 B^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma_\alpha^2 + \frac{\sigma_\gamma^2}{\bar{N}} + \frac{\sigma_\varepsilon^2}{\bar{N}\bar{\bar{Q}}}\right] \square
$$

## A.4.6 Model Expectation $W_{2a}^2$

**Theorem 7.** *The approximate model expectation of $W_{2a}^2$ is*

$$E_M\left(t_{U_a}^2 W_{2a}^2\right) \doteq \sum_{i\in U}\frac{1}{p_i}\left\{\left(\mu^2+\sigma_\alpha^2\right)\bar{Q}_{ia}^2 v_{Q_{ia(pwr)}}^2 + \sigma_\gamma^2 N_{ia}^2 \bar{Q}_{ia}^2\left(\frac{v_{Q_{ia(pwr)}}^2}{N_{ia}^2}-\frac{v_{Q_{ia}}^2}{N_{ia}}+1\right)\right.$$
$$\left. + \sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left[\sum_{j\in U_{ia}}\frac{Q_{iajb}}{p_{j|ia}} - Q_{ia\bullet b}\right]\right\}.$$

*Proof.*

Recall that

$$t_{U_a}^2 W_{2a}^2 = \frac{\sum_{i\in U} S_{U2(pwr)ia}^2}{p_i} = \sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}} p_{j|ia}\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right)^2 \tag{A.73}$$

where $t_{U_{iaj}} = \sum_{b=1}^{B}\sum_{k\in U_{iajb}} y_k$ and $t_{U_{ia}} = \sum_{j\in U_{ia}} t_{U_{iaj}}$. Substituting the model form of $y_k$ into Eq. (A.73), we have

$$\begin{aligned} t_{U_{iaj}} &= \sum_{b=1}^{B}\sum_{k\in U_{iajb}} \mu + \alpha_i + \gamma_{iaj} + \varepsilon_{iajbk} \\ &= \sum_{b=1}^{B} Q_{iajb}\left(\mu+\alpha_i+\gamma_{iaj}\right) + \sum_{b=1}^{B}\sum_{k\in U_{iajb}}\varepsilon_{iajbk} \\ &= Q_{iaj}\left(\mu+\alpha_i+\gamma_{iaj}\right)+\varepsilon_{iaj} \end{aligned} \tag{A.74}$$

and

$$\begin{aligned} t_{U_{ia}} &= \sum_{j\in U_{ia}} Q_{iaj}\left(\mu+\alpha_i+\gamma_{iaj}\right) + \sum_{j\in U_{ia}}\varepsilon_{iaj} \\ &= Q_{ia}\left(\mu+\alpha_i\right) + \sum_{j\in U_{ia}} Q_{iaj}\gamma_{iaj} + \varepsilon_{ia} \end{aligned} \tag{A.75}$$

Taking the expected value of Eqs. (A.74) and (A.75) we obtain

$$E_M\left(t_{U_{iaj}}\right) = E_M\left(Q_{iaj}\mu\right) = Q_{iaj}\mu \tag{A.76}$$

$$E_M\left(t_{U_{ia}}\right) = E_M\left(Q_{ia}\mu\right) = Q_{ia}\mu \tag{A.77}$$

(All other terms have expected value with respect to the model equal to zero.)

Taking the expected value of Equation (A.73) we obtain,

$$\begin{aligned}
E_M\left(t_{U_a}^2 W_{2a}^2\right) &= E_M\left[\sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}} p_{j|ia}\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right)^2\right] \\
&= \sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}} p_{j|ia} E_M\left[\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right)^2\right] \\
&= \sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}} p_{j|ia}\left\{\left[E_M\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right)\right]^2 + Var_M\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right)\right\}
\end{aligned} \tag{A.78}$$

So we need to find

$$1. \quad \sum_{j\in U_{ia}} p_{j|ia}\left[E_M\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right)\right]^2 \tag{A.79}$$

and

$$2. \quad \sum_{j\in U_{ia}} p_{j|ia} Var_M\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right) \tag{A.80}$$

1. Solving Equation (A.79), we obtain

$$\left[E_M\left(\frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}}\right)\right]^2 = \left(\frac{Q_{iaj}\mu}{p_{j|ia}} - Q_{ia}\mu\right)^2 = \left(\frac{Q_{iaj}}{p_{j|ia}} - Q_{ia}\right)^2\mu^2 \tag{A.81}$$

Multiplying Equation (A.81) through by $p_{j|ia}$ and summing over all SSUs $j|ia$ in the population, we obtain

$$\sum_{j \in U_{ia}} p_{j|ia} \left[ E_M \left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right) \right]^2 = \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right)^2 \mu^2$$

$$= \mu^2 S_{Q_{ia(pwr)}} \tag{A.82}$$

$$= \mu^2 \bar{Q}_{ia}^2 v_{Q_{ia(pwr)}}^2 \quad \square$$

4. To evaluate $\sum_{j \in U_{ia}} p_{j|ia} Var_M \left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right)$, first find

$$\left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right) = \frac{Q_{iaj}}{p_{j|ia}} \left( \mu + \alpha_i + \gamma_{iaj} \right) + \frac{\varepsilon_{iaj}}{p_{j|ia}} - Q_{ia} \left( \mu + \alpha_i \right) - \sum_{j \in U_{ia}} Q_{iaj} \gamma_{iaj} - \varepsilon_{iaj}$$

$$= \left( \frac{Q_{iaj}}{p_{j|ia}} \mu - Q_{ia} \mu \right) + \left( \frac{Q_{iaj}}{p_{j|ia}} \alpha_i - Q_{ia} \alpha_i \right)$$
$$+ \left( \frac{Q_{iaj}}{p_{j|ia}} \gamma_{iaj} - \sum_{j \in U_{ia}} Q_{iaj} \gamma_{iaj} \right) + \left( \frac{\varepsilon_{iaj}}{p_{j|ia}} - \varepsilon_{iaj} \right)$$

$$= \mu \left( \frac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right) + \alpha_i \left( \frac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right)$$
$$+ \left[ \gamma_{iaj} Q_{iaj} \left( \frac{1}{p_{j|ia}} - 1 \right) - \sum_{j' \neq j \in U_{ia}} \gamma_{iaj'} Q_{iaj'} \right]$$
$$+ \left[ \varepsilon_{iaj} \left( \frac{1}{p_{j|ia}} - 1 \right) - \sum_{j' \neq j \in U_{ia}} \varepsilon_{iaj'} \right] \quad \square \tag{A.83}$$

Now taking the variance of Equation (A.83) with respect to the model, we have

$$Var_M\left(\frac{t_{U_{iaj}}}{p_{j|ia}}-t_{U_{ia}}\right)=Var_M\left[\mu\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)\right]$$
$$+Var_M\left[\alpha_i\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)\right]$$
$$+Var_M\left[\gamma_{iaj}Q_{iaj}\left(\frac{1}{p_{j|ia}}-1\right)-\sum_{j'\neq j\in U_{ia}}\gamma_{iaj'}Q_{iaj'}\right]$$
$$+Var_M\left[\varepsilon_{iaj}\left(\frac{1}{p_{j|ia}}-1\right)-\sum_{j'\neq j\in U_{ia}}\varepsilon_{iaj'}\right]$$

(A.84)

Evaluating each term of Equation (A.84) separately, we obtain

A. $$Var_M\left[\mu\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)\right]=\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)^2 Var_M\left(\mu\right)=0 \qquad \text{(A.85)}$$

B. $$Var_M\left[\alpha_i\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)\right]=\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)^2 Var_M\left(\alpha_i\right)=\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)^2\sigma_\alpha^2 \qquad \text{(A.86)}$$

C. $$Var_M\left[\gamma_{iaj}Q_{iaj}\left(\frac{1}{p_{j|ia}}-1\right)-\sum_{j'\neq j\in U_{ia}}\gamma_{iaj'}Q_{iaj'}\right]$$
$$=Q_{iaj}^2\left(\frac{1}{p_{j|ia}}-1\right)^2 Var_M\left(\gamma_{iaj}\right)+\sum_{j'\neq j\in U_{ia}}Q_{iaj'}^2 Var_M\left(\gamma_{iaj'}\right)$$
$$=Q_{iaj}^2\left(\frac{1}{p_{j|ia}}-1\right)^2\sigma_\gamma^2+\sum_{j'\neq j\in U_{ia}}Q_{iaj'}^2\sigma_\gamma^2$$
$$=\sigma_\gamma^2\left[Q_{iaj}^2\left(\frac{1}{p_{j|ia}}-1\right)^2+\sum_{j'\neq j\in U_{ia}}Q_{iaj'}^2+Q_{iaj}^2-Q_{iaj}^2\right]$$

Continuing from C,

$$
\begin{aligned}
&Var_M\left[\gamma_{iaj}Q_{iaj}\left(\frac{1}{p_{j|ia}}-1\right)-\sum_{j'\neq j\in U_{ia}}\gamma_{iaj'}Q_{iaj'}\right]\\
&\qquad=\sigma_\gamma^2\left[Q_{iaj}^2\left[\left(\frac{1}{p_{j|ia}}-1\right)^2-1\right]+\sum_{j\in U_{ia}}Q_{iaj}^2\right]\\
&\qquad=\sigma_\gamma^2\left[Q_{iaj}^2\left(\frac{1-2p_{j|ia}}{p_{j|ia}^2}\right)+\sum_{j\in U_{ia}}Q_{iaj}^2\right]\text{ by Lemma 4}\\
&\qquad=\sigma_\gamma^2\left[\frac{Q_{iaj}^2}{p_{j|ia}^2}-\frac{2Q_{iaj}^2}{p_{j|ia}}+\sum_{j\in U_{ia}}Q_{iaj}^2\right]
\end{aligned}
\tag{A.87}
$$

D. To evaluate $Var_M\left[\varepsilon_{iaj}\left(\frac{1}{p_{j|ia}}-1\right)-\sum_{j'\neq j\in U_{ia}}\varepsilon_{iaj'}\right]$, first we find

$$
Var_M\left(\varepsilon_{iaj}\right)=\sum_{b=1}^{B}\sum_{k\in U_{iajb}}Var_M\left(\varepsilon_{iajbk}\right)=\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\sigma_{\varepsilon_{ab}}^2=\sum_{b=1}^{B}Q_{iajb}\sigma_{\varepsilon_{ab}}^2
$$

Then

$$
\begin{aligned}
Var_M\left[\varepsilon_{iaj}\left(\frac{1}{p_{j|ia}}-1\right)-\sum_{j'\neq j\in U_{ia}}\varepsilon_{iaj'}\right]&=\left(\frac{1}{p_{j|ia}}-1\right)^2\sum_{b=1}^{B}Q_{iajb}\sigma_{\varepsilon_{ab}}^2+\sum_{j'\neq j\in U_{ia}}\sum_{b=1}^{B}Q_{iaj'b}\sigma_{\varepsilon_{ab}}^2\\
&=\left[\left(\frac{1}{p_{j|ia}}-1\right)^2-1\right]\sum_{b=1}^{B}Q_{iajb}\sigma_{\varepsilon_{ab}}^2+\sum_{j\in U_{ia}}\sum_{b=1}^{B}Q_{iajb}\sigma_{\varepsilon_{ab}}^2\\
&=\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left[\frac{Q_{iajb}}{p_{j|ia}^2}-\frac{2Q_{iajb}}{p_{j|ia}}+\sum_{j\in U_{ia}}Q_{iajb}\right]\text{by Lemma 4}\\
&=\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left[\frac{Q_{iajb}}{p_{j|ia}^2}-\frac{2Q_{iajb}}{p_{j|ia}}+Q_{ia\bullet b}\right]
\end{aligned}
\tag{A.88}
$$

Substituting A - D back into Equation (A.84) we obtain,

$$
\begin{aligned}
Var_M\left(\frac{t_{U_{iaj}}}{p_{j|ia}}-t_{U_{ia}}\right) &= 0+\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)^2\sigma_\alpha^2 \\
&+\sigma_\gamma^2\left[\frac{Q_{iaj}^2}{p_{j|ia}^2}-\frac{2Q_{iaj}^2}{p_{j|ia}}+\sum_{j\in U_{ia}}Q_{iaj}^2\right] \\
&+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left[\frac{Q_{iajb}}{p_{j|ia}^2}-\frac{2Q_{iajb}}{p_{j|ia}}+Q_{ia\bullet b}\right]
\end{aligned}
\tag{A.89}
$$

Multiplying through by $p_{j|ia}$ and summing over the all SSUs $j|ia$ in the population, we obtain

$$
\begin{aligned}
\sum_{j\in U_{ia}}p_{j|ia}Var_M\left(\frac{t_{U_{iaj}}}{p_{j|ia}}-t_{U_{ia}}\right) &= \sigma_\alpha^2\sum_{j\in U_{ia}}p_{j|ia}\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)^2 \\
&+\sigma_\gamma^2\sum_{j\in U_{ia}}p_{j|ia}\left(\frac{Q_{iaj}^2}{p_{j|ia}^2}-\frac{2Q_{iaj}^2}{p_{j|ia}}+\sum_{j\in U_{ia}}Q_{iaj}^2\right) \\
&+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\sum_{j\in U_{ia}}p_{j|ia}\left(\frac{Q_{iajb}}{p_{j|ia}^2}-\frac{2Q_{iajb}}{p_{j|ia}}+Q_{ia\bullet b}\right) \\
&= \sigma_\alpha^2\sum_{j\in U_{ia}}p_{j|ia}\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)^2 \\
&+\sigma_\gamma^2\left(\sum_{j\in U_{ia}}\frac{Q_{iaj}^2}{p_{j|ia}}-2\sum_{j\in U_{ia}}Q_{iaj}^2+\sum_{j\in U_{ia}}p_{j|ia}\sum_{j\in U_{ia}}Q_{iaj}^2\right) \\
&+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left(\sum_{j\in U_{ia}}\frac{Q_{iajb}}{p_{j|ia}}-2\sum_{j\in U_{ia}}Q_{iajb}+\sum_{j\in U_{ia}}p_{j|ia}Q_{ia\bullet b}\right)
\end{aligned}
\tag{A.90}
$$

Since $\sum_{j \in U_{ia}} p_{j|ia} = 1$ , Equation (A.90) reduces to

$$\sum_{j \in U_{ia}} p_{j|ia} Var_M \left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right) = \sigma_\alpha^2 \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right)^2$$

$$+\sigma_\gamma^2 \left( \sum_{j \in U_{ia}} \frac{Q_{iaj}^2}{p_{j|ia}} - 2 \sum_{j \in U_{ia}} Q_{iaj}^2 + \sum_{j \in U_{ia}} Q_{iaj}^2 \right)$$

$$+\sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \left( \sum_{j \in U_{ia}} \frac{Q_{iajb}}{p_{j|ia}} - 2Q_{ia\bullet b} + Q_{ia\bullet b} \right)$$

$$= \sigma_\alpha^2 \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right)^2$$

$$+\sigma_\gamma^2 \left[ \sum_{j \in U_{ia}} \frac{Q_{iaj}^2}{p_{j|ia}} - \left( \sum_{j \in U_{ia}} Q_{iaj}^2 \right) \right]$$

$$+\sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \left[ \left( \sum_{j \in U_{ia}} \frac{Q_{iajb}}{p_{j|ia}} \right) - Q_{ia\bullet b} \right]$$

(A.91)

Substituting the results from Eqs. (A.82) and (A.91) back into Eq. (A.78), we have

$$E_M\left(t_{U_a}^2 W_{2a}^2\right) = \sum_{i\in U} \frac{1}{p_i} \sum_{j\in U_{ia}} p_{j|ia} \left\{ \left[ E_M\left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right) \right]^2 + Var_M\left( \frac{t_{U_{iaj}}}{p_{j|ia}} - t_{U_{ia}} \right) \right\}$$

$$\doteq \sum_{i\in U} \frac{1}{p_i} \left\{ \left(\mu^2 + \sigma_\alpha^2\right) \sum_{j\in U_{ia}} p_{j|ia} \left( \frac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right)^2 \right.$$

$$+\sigma_\gamma^2 \left[ \sum_{j\in U_{ia}} \frac{Q_{iaj}^2}{p_{j|ia}} - \left( \sum_{j\in U_{ia}} Q_{iaj}^2 \right) \right]$$

$$\left. +\sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \left[ \left( \sum_{j\in U_{ia}} \frac{Q_{iajb}}{p_{j|ia}} \right) - Q_{ia\bullet b} \right] \right\}$$

$$\doteq \sum_{i\in U} \frac{1}{p_i} \left\{ \left(\mu^2 + \sigma_\alpha^2\right) \bar{Q}_{ia}^2 v_{Q_{ia(pwr)}}^2 \right. \qquad \text{by definition of relvariance}$$

$$+\sigma_\gamma^2 N_{ia}^2 \bar{Q}_{ia}^2 \left( \frac{v_{Q_{ia(pwr)}}^2}{N_{ia}^2} - \frac{v_{Q_{ia}}^2}{N_{ia}} + 1 \right) \qquad \text{by Lemma 1(f)}$$

$$\left. +\sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \left[ \left( \sum_{j\in U_{ia}} \frac{Q_{iajb}}{p_{j|ia}} \right) - Q_{ia\bullet b} \right] \right\}$$

$$= \sum_{i\in U} \frac{1}{p_i} \left\{ \left(\mu^2 + \sigma_\alpha^2\right) \bar{Q}_{ia}^2 v_{Q_{ia(pwr)}}^2 + \sigma_\gamma^2 N_{ia}^2 \bar{Q}_{ia}^2 \left( \frac{v_{Q_{ia(pwr)}}^2}{N_{ia}^2} - \frac{v_{Q_{ia}}^2}{N_{ia}} + 1 \right) \right.$$

$$\left. +\sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \left[ \left( \sum_{j\in U_{ia}} \frac{Q_{iajb}}{p_{j|ia}} \right) - Q_{ia\bullet b} \right] \right\} \square \qquad \text{(A.92)}$$

**Corollary to Theorem 7.** Assume that conditions (*A1*) – (*A5*) in Appendix A.4.2 hold. Then in the special case of no $b$ strata so that $\sigma^2_{\varepsilon_{ab}} = \sigma^2_{\varepsilon_a}$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectation of $W^2_{2a}$ can be simplified to

$$E_M\left(t^2_{U_a} W^2_{2a}\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2 \left[\sigma^2_\gamma + \frac{\sigma^2_{\varepsilon_a}}{\bar{\bar{Q}}}\right]$$

and when there are no $a$ strata so that $P_a = 1 \textit{ and } \sigma^2_{\varepsilon_a} = \sigma^2_\varepsilon$,

$$E_M\left(t^2_{U_a} W^2_{2a}\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma^2_\gamma + \frac{\sigma^2_{\varepsilon}}{\bar{\bar{Q}}}\right].$$

*Proof.*

We rewrite Equation (A.92) as

$$\begin{aligned} E_M\left(t^2_{U_a} W^2_{2a}\right) = \sum_{i \in U} \frac{1}{p_i} \Bigg\{ & \left(\mu^2 + \sigma^2_\alpha\right) N^2_{ia} \bar{Q}^2_{ia} \frac{v^2_{Q_{ia(pwr)}}}{N^2_{ia}} \\ & + \sigma^2_\gamma N^2_{ia} \bar{Q}^2_{ia} \left( \frac{v^2_{Q_{ia(pwr)}}}{N^2_{ia}} - \frac{v^2_{Q_{ia}}}{N_{ia}} + 1 \right) \\ & + \sum_{b=1}^{B} \sigma^2_{\varepsilon_{ab}} \left[ \left( \sum_{j \in U_{ia}} \frac{Q_{iajb}}{p_{j|ia}} \right) - Q_{ia \bullet b} \right] \Bigg\} \end{aligned} \tag{A.93}$$

When (*A2*) holds, we use the result from Lemma 2 that $v^2_{Q_{ia(pwr)}} = v^2_{Q_{ia}} = 0$, and obtain

$$E_M\left(t^2_{U_a} W^2_{2a}\right) = \sum_{i \in U} \frac{1}{p_i} \left\{ \sigma^2_\gamma N^2_{ia} \bar{Q}^2_{ia} + \sum_{b=1}^{B} \sigma^2_{\varepsilon_{ab}} \left[ \left( \sum_{j \in U_{ia}} \frac{Q_{iajb}}{p_{j|ia}} \right) - Q_{ia \bullet b} \right] \right\} \tag{A.94}$$

Assume that $Q_{iajb} = \bar{\bar{Q}}_b$ and $p_{j|ia} = \frac{Q_{iaj}}{Q_{ia}}$ as in (*A2*) and (*A3*).

Then by Lemma 2, $p_{j|ia} = \dfrac{1}{N_{ia}}$ and $Q_{ia\bullet b} = N_{ia}\bar{\bar{Q}}_b$. When $(A5)$ holds , $N_{ia} \approx N_{ia} - 1$ and

$$E_M\left(t_{U_a}^2 W_{2a}^2\right) = \sum_{i\in U}\frac{1}{p_i}\left\{\sigma_\gamma^2 N_{ia}^2 \bar{Q}_{ia}^2 + \sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left[\left(\sum_{j\in U_{ia}} N_{ia}\bar{\bar{Q}}_b\right) - N_{ia}\bar{\bar{Q}}_b\right]\right\}$$

$$= \sum_{i\in U}\frac{1}{p_i}\left(\sigma_\gamma^2 N_{ia}^2 \bar{Q}_{ia}^2 + \sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 N_{ia}\bar{\bar{Q}}_b\left(N_{ia}-1\right)\right) \tag{A.95}$$

$$\doteq \sum_{i\in U}\frac{1}{p_i}N_{ia}^2\left(\sigma_\gamma^2 \bar{Q}_{ia}^2 + \sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 \bar{\bar{Q}}_b\right)$$

When (*A2*) holds, we know that from Lemma 3, $\bar{Q}_{ia} = \bar{\bar{Q}}$. Using this together with the fact that $p_i = \dfrac{N_i}{M\bar{N}}$ by Lemma 2 and $P_{ia} = \dfrac{N_{ia}}{N_i} \equiv P_a$ by (*A4*), we obtain

$$E_M\left(t_{U_a}^2 W_{2a}^2\right) = \sum_{i\in U}\frac{M\bar{N}}{N_i}N_i^2 P_a^2\left[\sigma_\gamma^2\bar{\bar{Q}}^2 + \sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\bar{\bar{Q}}_b\right]$$

$$= M^2\bar{N}^2 P_a^2\left[\sigma_\gamma^2\bar{\bar{Q}}^2 + \sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\bar{\bar{Q}}_b\right] \quad \text{since } \sum_{i\in U} N_i = M\bar{N} \tag{A.96}$$

$$= \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2\left[\sigma_\gamma^2 + \sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}^2}\right]$$

In the special case of no *b* strata so that $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectation of $W_{2a}^2$ can be simplified to

$$E_M\left(t_{U_a}^2 W_{2a}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2\left[\sigma_\gamma^2 + \frac{\sigma_{\varepsilon_a}^2}{\bar{\bar{Q}}}\right] \tag{A.97}$$

and when there are no *a* strata so that $P_a = 1$ and $\sigma_{\varepsilon_{ab}}^2 = \sigma_\varepsilon^2$,

$$E_M\left(t_{U_a}^2 W_{2a}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2\left[\sigma_\gamma^2 + \frac{\sigma_\varepsilon^2}{\bar{\bar{Q}}}\right]\square \tag{A.98}$$

## A.4.7 Model Expectation of $W_{3ab}^2$

**Theorem 8.** *The model expectation of* $W_{3ab}^2$ *is*

$$E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right) = \sigma_{\varepsilon_{ab}}^2 \sum_{i\in U} \frac{1}{p_i} \sum_{j\in U_{ia}} \frac{1}{p_{j|ia}} Q_{iajb}^2 .$$

*Proof.*

Recall that

$$t_{U_{ab}}^2 W_{3ab}^2 = \sum_{i\in U} \frac{1}{p_i} \sum_{j\in U_{ia}} \frac{Q_{iajb}^2}{p_{j|ia}} S_{U3iajb}^2$$

with

$$S_{U3iajb}^2 = \left(Q_{iajb} - 1\right)^{-1} \sum_{k\in U_{iajb}} \left(y_k - \bar{y}_{U_{iajb}}\right)^2$$

and

$$\bar{y}_{U_{iajb}} = \sum_{k\in U_{iajb}} \frac{y_k}{Q_{iajb}} .$$

Substituting the model form of $y_k$ into the above, we have

$$\begin{aligned} y_k - \bar{y}_{U_{iajb}} &= \left(\mu + \alpha_i + \gamma_{iaj} + \varepsilon_{iajbk}\right) - \frac{1}{Q_{iajb}} \sum_{k\in U_{iajb}} \left(\mu + \alpha_i + \gamma_{iaj} + \varepsilon_{iajbk}\right) \\ &= \left(\mu + \alpha_i + \gamma_{iaj} + \varepsilon_{iajbk}\right) - \frac{Q_{iajb}}{Q_{iajb}}\left(\mu + \alpha_i + \gamma_{iaj}\right) - \frac{1}{Q_{iajb}} \sum_{k\in U_{iajb}} \varepsilon_{iajbk} \\ &= \varepsilon_{iajbk} - \bar{\varepsilon}_{iajbk} \end{aligned}$$

(A.99)

and

$$E_M\left[S^2_{U3iajb}\right] = E_M\left[\left(Q_{iajb}-1\right)^{-1}\sum_{k\in U_{iajb}}\left(y_k-\bar{y}_{U_{iajb}}\right)^2\right]$$
$$=\left(Q_{iajb}-1\right)^{-1}E_M\left[\sum_{k\in U_{iajb}}\left(\varepsilon_{iajbk}-\bar{\varepsilon}_{iajb}\right)^2\right]=\frac{\cancel{\left(Q_{iajb}-1\right)}}{\cancel{\left(Q_{iajb}-1\right)}}\sigma^2_{\varepsilon_{ab}}$$
$$=\sigma^2_{\varepsilon_{ab}}$$

(A.100)

Then

$$E_M\left[t^2_{U_{ab}}W^2_{3ab}\right]=E_M\left[\sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{Q^2_{iajb}S^2_{U3iajb}}{p_{j|ia}}\right]$$
$$=\sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{1}{p_{j|ia}}Q^2_{iajb}E_M\left[S^2_{U3iajb}\right]$$
$$=\sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{1}{p_{j|ia}}Q^2_{iajb}\sigma^2_{\varepsilon_{ab}}\ \square$$

(A.101)

**Corollary to Theorem 8** Assume that conditions (*A1*) – (*A5*) in Appendix A.4.2 hold. Then in the special case of no *b* strata so that $\sigma^2_{\varepsilon_{ab}}=\sigma^2_{\varepsilon_a}$ and $\bar{\bar{Q}}_b=\bar{\bar{Q}}$, the model expectation of $W^2_{3ab}$ can be simplified to

$$E_M\left(t^2_{U_{ab}}W^2_{3ab}\right)=\left(M\bar{N}\bar{\bar{Q}}\right)^2P^2_a\sigma^2_{\varepsilon_a}$$

and when there are no *a* strata so that $P_a=1$ and $\sigma^2_{\varepsilon_a}=\sigma^2_{\varepsilon}$,

$$E_M\left(t^2_{U_{ab}}W^2_{3ab}\right)=\left(M\bar{N}\bar{\bar{Q}}\right)^2\sigma^2_{\varepsilon}.$$

*Proof.*

When (*A1*)- (*A3*) holds, we obtain

$$E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right) = \sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{1}{p_{j|ia}}\bar{\bar{Q}}_b^2\sigma_{\varepsilon_{ab}}^2 \qquad \text{since } Q_{iajb} = \bar{\bar{Q}}_b$$

$$= \sigma_{\varepsilon_{ab}}^2\sum_{i\in U}\frac{1}{p_i}N_{ia}\sum_{j\in U_{ia}}\bar{\bar{Q}}_b^2 \qquad \text{since } p_{j|ia} = \frac{1}{N_{ia}} \quad \text{Lemma 2}(o)$$

$$= \sigma_{\varepsilon_{ab}}^2 M\bar{N}\sum_{i\in U}\frac{1}{N_i}N_{ia}^2\bar{\bar{Q}}_b^2 \text{ since} \qquad p_i = \frac{N_i}{M\bar{N}} \quad \text{Lemma 2}(p)$$

(A.102)

When (*A4*) holds, we obtain

$$E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right) = \sigma_{\varepsilon_{ab}}^2 M\bar{N}\sum_{i\in U}\frac{1}{N_i}N_i^2P_a^2\bar{\bar{Q}}_b^2 \qquad \text{since } N_iP_a = N_{ia}$$

$$= \sigma_{\varepsilon_{ab}}^2 M\bar{N}\sum_{i\in U}N_iP_a^2\bar{\bar{Q}}_b^2$$

$$= \sigma_{\varepsilon_{ab}}^2 M^2\bar{N}^2\bar{\bar{Q}}_b^2P_a^2 \qquad \text{since } \sum_{i\in U}N_i = M\bar{N}$$

(A.103)

In the special case of no *b* strata, so that $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$,

$$\begin{aligned} E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right) &= \sigma_{\varepsilon_{ab}}^2 M^2\bar{N}^2P_a^2\bar{\bar{Q}}_b^2 \\ &= \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2\sigma_{\varepsilon_a}^2 \end{aligned} \qquad \text{(A.104)}$$

and when there are no *a* strata so that $P_a = 1$ and $\sigma_{\varepsilon_a}^2 = \sigma_\varepsilon^2$,

$$E_M\left(t_{U_{ab}}^2 W_{3ab}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2\sigma_\varepsilon^2 \square \qquad \text{(A.105)}$$

### A.4.8 Model Expectation of $W^2$

**Theorem 9.** *The approximate model expectation of $W^2$ is*

$$E_M\left(t_U^2 W^2\right) \doteq \sum_{i\in U}\frac{Q_i}{p_i}\sum_{a=1}^{A} N_{ia}\left\{\sigma_\gamma^2 \bar{Q}_{ia}\left[1-\frac{\bar{Q}_{ia}}{Q_i}\left(v_{Q_{ia}}^2+1\right)\right]+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 \bar{Q}_{ia\bullet b}\right\}.$$

*Proof.*

Recall that

$$t_U^2 W^2 = \sum_{i\in U}\frac{Q_i^2 S_{U3i}^2}{p_i} \tag{A.106}$$

where

$$S_{U3i}^2 = \frac{1}{Q_i-1}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left(y_k-\bar{y}_{U_i}\right)^2$$

and

$$\bar{y}_{U_i} = \sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{y_k}{Q_i}$$

Substituting the model form of $y_k$ into the above, we have

$$\begin{aligned}\bar{y}_{U_i} &= \sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{y_k}{Q_i}\\ &= \sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{\left(\mu+\alpha_i+\gamma_{iaj}+\varepsilon_{iajbk}\right)}{Q_i}\\ &= \left(\mu+\alpha_i\right)+\sum_{a=1}^{A}\sum_{j\in U_{ia}}\gamma_{iaj}\frac{Q_{iaj}}{Q_i}+\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\varepsilon_{iajbk}\frac{1}{Q_i}\end{aligned} \tag{A.107}$$

Then,

$$y_k - \bar{y}_{U_i} = \gamma_{iaj} - \sum_{a} \sum_{j \in U_{ia}} \frac{Q_{iaj}}{Q_i} \gamma_{iaj} + \varepsilon_{iajbk} - \frac{\varepsilon_{i++++}}{Q_i}$$

$$= \gamma_{iaj} \left( 1 - \frac{Q_{iaj}}{Q_i} \right) - \sum_{a} \sum_{j' \neq j \in U_{ia}} \frac{Q_{iaj'}}{Q_i} \gamma_{iaj'}$$

$$+ \varepsilon_{iajbk} \left( 1 - \frac{1}{Q_i} \right) - \sum_{a} \sum_{j' \in U_{ia}} \sum_{b' \neq b} \sum_{k' \neq k \in U_{iajb}} \varepsilon_{iaj'b'k'} \frac{1}{Q_i} \qquad \text{(A.108)}$$

Taking the expected value of Equation (A.106), we obtain

$$\begin{aligned}
E_M \left( t_U^2 W^2 \right) &= E_M \left( \sum_{i \in U} \frac{Q_i^2 S_{U3i}^2}{p_i} \right) \\
&= \sum_{i \in U} \frac{Q_i^2 E_M \left( S_{U3i}^2 \right)}{p_i} \\
&= \sum_{i \in U} \frac{Q_i^2}{p_i} \frac{1}{Q_i - 1} \sum_{a=1}^{A} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} E_M \left( y_k - \bar{y}_{U_i} \right)^2 \\
&= \sum_{i \in U} \frac{Q_i^2}{p_i} \frac{1}{Q_i - 1} \sum_{a=1}^{A} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \left\{ \left[ E_M \left( y_k - \bar{y}_{U_i} \right) \right]^2 + Var_M \left( y_k - \bar{y}_{U_i} \right) \right\} \square
\end{aligned} \qquad \text{(A.109)}$$

We need to find

1. $\left[ E_M \left( y_k - \bar{y}_{U_i} \right) \right]^2$ (A.110)

2. $Var_M \left( y_k - \bar{y}_{U_i} \right)$ (A.111)

1. Solving Eq. (A.110), we obtain

$$E_M\left(y_k - \bar{y}_{U_i}\right) = 0$$

$$\left[E_M\left(y_k - \bar{y}_{U_i}\right)\right]^2 = 0 \tag{A.112}$$

2. Taking the variance of Eq. (A.111) with respect to the model, we have

$$\begin{aligned}
Var_M\left(y_k - \bar{y}_{U_i}\right) &= Var_M\left[\gamma_{iaj}\left(1-\frac{Q_{iaj}}{Q_i}\right)\right] + \sum_a \sum_{j'\neq j\in U_{ia}} Var_M\left[\frac{Q_{iaj'}}{Q_i}\gamma_{iaj'}\right] \\
&\quad + Var_M\left[\varepsilon_{iajbk}\left(1-\frac{1}{Q_i}\right)\right] + \sum_a \sum_{(j'b'k')\neq jbk}^{U_{ia},B,U_{iaj'b'}} Var_M\left[\varepsilon_{iaj'b'k'}\frac{1}{Q_i}\right] \\
&= \sigma_\gamma^2\left(1-\frac{Q_{iaj}}{Q_i}\right)^2 + \sum_a \sum_{j'\neq j\in U_{ia}} \sigma_\gamma^2 \frac{Q_{iaj'}^2}{Q_i^2} \\
&\quad + \sigma_{\varepsilon_{ab}}^2\left(1-\frac{1}{Q_i}\right)^2 + \sum_a \sum_{j'b'k'\neq(jbk)}^{U_{ia},B,U_{iaj'b'}} \sigma_{\varepsilon_{ab'}}^2 \frac{1}{Q_i^2} \\
&= \sigma_\gamma^2\left(1-\frac{Q_{iaj}}{Q_i}\right)^2 - \sigma_\gamma^2\frac{Q_{iaj}^2}{Q_i^2} + \sum_a \sum_{j'\neq j\in U_{ia}} \sigma_\gamma^2 \frac{Q_{iaj'}^2}{Q_i^2} + \sigma_\gamma^2\frac{Q_{iaj}^2}{Q_i^2} \\
&\quad + \sigma_{\varepsilon_{ab}}^2\left(1-\frac{1}{Q_i}\right)^2 - \sigma_{\varepsilon_{ab}}^2\frac{1}{Q_i^2} + \sum_a \sum_{j'b'k'\neq(jbk)}^{U_{ia},B,U_{iaj'b'}} \sigma_{\varepsilon_{ab'}}^2 \frac{1}{Q_i^2} + \sigma_{\varepsilon_{ab'}}^2\frac{1}{Q_i^2}
\end{aligned}$$

Let $(jbk)$ denote a HU $k$, in a specific SSU $j$, HU substratum $b$. Then

$\sum_{j'b'k'\neq(jbk)}^{U_{ia},B,U_{iaj'b'}}$ denotes the sum of all SSUs *j* in $U_{ia}$, over all HU substratum *b*, for all HUs $k$ in $U_{iajb}$ in a given PSU *i*, SSU stratum *a*, except the single $(jbk)$ term.

And so,

$$
\begin{aligned}
Var_M\left(y_k-\bar{y}_{U_i}\right) &= \sigma_\gamma^2\left(1-\frac{2Q_{iaj}}{Q_i}+\frac{Q_{iaj}^2}{Q_i^2}-\frac{Q_{iaj}^2}{Q_i^2}\right)+\sum_a\sum_{j\in U_{ia}}\sigma_\gamma^2\,\frac{Q_{iaj}^2}{Q_i^2} \\
&\quad +\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q_i}+\frac{1}{Q_i^2}-\frac{1}{Q_i^2}\right)+\sum_a\sum_{j\in U_{ia}}\sum_b\sum_{k\in U_{iajb}}\sigma_{\varepsilon_{ab}}^2\,\frac{1}{Q_i^2} \\
&= \sigma_\gamma^2\left(1-\frac{2Q_{iaj}}{Q_i}\right)+\sum_a\sum_{j\in U_{ia}}\sigma_\gamma^2\,\frac{Q_{iaj}^2}{Q_i^2} \\
&\quad +\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q_i}\right)+\sum_a\sum_{j\in U_{ia}}\sum_b\sum_{k\in U_{iajb}}\sigma_{\varepsilon_{ab}}^2\,\frac{1}{Q_i^2}\square
\end{aligned}
\tag{A.113}
$$

Substituting Eqs. (A.112) and (A.113) back into the $S_{U3i}^2$ portion of Equation (A.109),

$$
\begin{aligned}
E_M\left(S_{U3i}^2\right) &= \frac{1}{Q_i-1}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left\{\left[E_M\left(y_k-\bar{y}_{U_i}\right)\right]^2+Var_M\left(y_k-\bar{y}_{U_i}\right)\right\} \\
&= \frac{1}{Q_i-1}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}} Var_M\left(y_k-\bar{y}_{U_i}\right) \\
&= \frac{1}{Q_i-1}\sum_a\sum_{j\in U_{ia}}\sum_b\sum_{k\in U_{iajb}}\sigma_\gamma^2\left(1-\frac{2Q_{iaj}}{Q_i}\right)+\frac{Q_i}{Q_i-1}\sum_a\sum_{j\in U_{ia}}\sigma_\gamma^2\,\frac{Q_{iaj}^2}{Q_i^2} \\
&\quad +\frac{1}{Q_i-1}\sum_a\sum_{j\in U_{ia}}\sum_b\sum_{k\in U_{iajb}}\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q_i}\right)+\frac{Q_i}{Q_i-1}\sum_a\sum_{j\in U_{ia}}\sum_b\sum_{k\in U_{iajb}}\sigma_{\varepsilon_{ab}}^2\,\frac{1}{Q_i^2} \\
&= \frac{1}{Q_i-1}\sum_a\sum_{j\in U_{ia}}Q_{iaj}\sigma_\gamma^2\left(1-\frac{2Q_{iaj}}{Q_i}\right)+\frac{\cancel{Q_i}}{Q_i-1}\sum_a\sum_{j\in U_{ia}}\sigma_\gamma^2\,\frac{Q_{iaj}^2}{Q_i^{\cancel{2}}} \\
&\quad +\frac{1}{Q_i-1}\sum_a\sum_b Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q_i}\right)+\frac{\cancel{Q_i}}{Q_i-1}\sum_a\sum_{j\in U_{ia}}\sum_b\sum_{k\in U_{iajb}}\sigma_{\varepsilon_{ab}}^2\,\frac{1}{Q_i^{\cancel{2}}}
\end{aligned}
\tag{A.114}
$$

Continuing from Eq. (A.114),

$$E_M\left(S_{U3i}^2\right)=\frac{1}{Q_i-1}\sum_a\sigma_\gamma^2\left(\sum_{j\in U_{ia}}Q_{iaj}-2\sum_{j\in U_{ia}}\frac{Q_{iaj}^2}{Q_i}\right)+\frac{1}{Q_i-1}\sum_a\sum_{j\in U_{ia}}\sigma_\gamma^2\frac{Q_{iaj}^2}{Q_i}$$

$$+\frac{1}{Q_i-1}\sum_a\sum_b Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q_i}\right)+\frac{1}{Q_i-1}\sum_a\sum_b Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\frac{1}{Q_i}$$

$$=\frac{1}{Q_i-1}\sum_a\sigma_\gamma^2\left(Q_{ia}-\sum_{j\in U_{ia}}\frac{Q_{iaj}^2}{Q_i}\right)+\frac{1}{Q_i-1}\sum_a\sum_b Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\left(1-\frac{1}{Q_i}\right)$$

Using the definition of $\bar{Q}_{ia}=\sum_{j\in U_{ia}}\frac{Q_{iaj}}{N_{ia}}$, then $N_{ia}\bar{Q}_{ia}=\sum_{j\in U_{ia}}Q_{iaj}$. And by definition $\bar{Q}_{ia\bullet b}=\sum_{j\in U_{iab}}\frac{Q_{iajb}}{N_{ia}}$ then $N_{ia}\bar{Q}_{ia\bullet b}=\sum_{j\in U_{iab}}Q_{iajb}$. Assuming $N_{ia}\approx N_{ia}-1$, we have

$$E_M\left(S_{U3i}^2\right)\doteq\frac{1}{Q_i-1}\sum_a\sigma_\gamma^2\left(N_{ia}\bar{Q}_{ia}-\frac{1}{Q_i}N_{ia}\bar{Q}_{ia}^2\left(v_{Q_{ia}}^2+1\right)\right)\text{ by Lemma 1(d)}$$

$$+\frac{1}{Q_i-1}\sum_a\sum_b Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\left(1-\frac{1}{Q_i}\right)$$

$$=\frac{1}{Q_i-1}\sum_a\sigma_\gamma^2 N_{ia}\bar{Q}_{ia}\left(1-\frac{\bar{Q}_{ia}}{Q_i}\left(v_{Q_{ia}}^2+1\right)\right)$$

$$+\frac{1}{\cancel{Q_i-1}}\sum_a\sum_b N_{ia}\bar{Q}_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\left(\frac{\cancel{Q_i-1}}{Q_i}\right)$$

(A.115)

Substituting Eq. (A.115) back into Eq.(A.109) and assuming $Q_i \approx Q_i - 1$,

$$E_M\left(t_U^2 W^2\right) \doteq \sum_{i\in U} \frac{Q_i^2 E_M\left(S_{U3i}^2\right)}{p_i}$$
$$= \sum_{i\in U} \frac{Q_i}{p_i} \sum_a \sigma_\gamma^2 N_{ia} \bar{Q}_{ia} \left(1 - \frac{\bar{Q}_{ia}}{Q_i}\left(v_{Q_{ia}}^2 + 1\right)\right) \tag{A.116}$$
$$+ \sum_{i\in U} \frac{Q_i}{p_i} \sum_a \sum_b N_{ia} \bar{Q}_{ia\bullet b} \sigma_{\varepsilon_{ab}}^2$$

Rearranging terms in Eq. (A.116),

$$E_M\left(t_U^2 W^2\right) \doteq \sum_{i\in U} \frac{Q_i}{p_i} \sum_{a=1}^{A} N_{ia} \left\{ \sigma_\gamma^2 \bar{Q}_{ia} \left[1 - \frac{\bar{Q}_{ia}}{Q_i}\left(v_{Q_{ia}}^2 + 1\right)\right] + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \bar{Q}_{ia\bullet b} \right\} \square$$

**Corollary to Theorem 9.** Assume that conditions (*A1*) – (*A5*) in Appendix A.4.2 hold.

In particular assume that (*A2*) $p_i = \frac{Q_i}{Q}$; $p_{j|ia} = \frac{Q_{iaj}}{Q_{ia}}$, (*A3*) $Q_{iajb} = \bar{\bar{Q}}_b$, $(A4)$ $P_{ia} = \frac{N_{ia}}{N_i} \equiv P_a$

, and (*A5*) $N_{ia} \approx N_{ia} - 1$ hold. Then in the special case of no *b* strata so that $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$ and

$\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectation of $W^2$ can be simplified to

$$E_M\left(t_U^2 W^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 \sum_{a=1}^{A} P_a \left[\sigma_\gamma^2 + \sigma_{\varepsilon_a}^2\right]$$

and when there are no *a* strata so that $P_a = 1$ and $\sigma_{\varepsilon_a}^2 = \sigma_\varepsilon^2$,

$$E_M\left(t_U^2 W^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma_\gamma^2 + \sigma_\varepsilon^2\right].$$

*Proof.*

When (*A2*) holds, by Lemma 3, $v_{Q_{ia}}^{2} = v_{Q_{ia\bullet b}}^{2} = 0$. Substituting this result back into Eq. (A.116),

$$E_M\left(t_U^2 W^2\right) = \sum_{i\in U} \frac{Q_i}{p_i} \sum_{a=1}^{A} N_{ia} \left\{\sigma_\gamma^2 \bar{Q}_{ia}\left[1 - \frac{\bar{Q}_{ia}}{Q_i}\right] + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \bar{Q}_{ia\bullet b}\right\}$$

(A.117)

When (*A3*) holds, we use the results from Lemma 2(g) that $Q_i = N_i \bar{\bar{Q}}$ and obtain

$$\begin{aligned} E_M\left(t_U^2 W^2\right) &= \sum_{i\in U} \frac{N_i \bar{\bar{Q}}}{p_i} \sum_{a=1}^{A} N_{ia} \left\{\sigma_\gamma^2 \bar{\bar{Q}}\left[1 - \frac{\bar{\bar{Q}}}{N_i \bar{\bar{Q}}}\right]\right. && \text{since } \bar{Q}_{ia} = \bar{\bar{Q}} \quad \text{Lemma 2(e)} \\ &\quad \left. + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \bar{\bar{Q}}_b \right\} && \text{since } \bar{Q}_{ia\bullet b} = \bar{\bar{Q}}_b \quad \text{Lemma 2(c)} \end{aligned}$$

(A.118)

Assuming (*A4*) $P_{ia} = \dfrac{N_{ia}}{N_i} \equiv P_a$, and substituting $\dfrac{N_{ia}}{p_i} = P_a M\bar{N}$,

$$E_M\left(t_U^2 W^2\right) = M\bar{N} \sum_{i\in U} N_i \bar{\bar{Q}} \sum_{a=1}^{A} P_a \left\{\sigma_\gamma^2 \bar{\bar{Q}}\left[1 - \frac{1}{N_i}\right] + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \bar{\bar{Q}}_b\right\}$$

(A.119)

Using the fact that $M\bar{N} = \sum_{i\in U} N_i$ and $N_i \approx N_i - 1$ by (*A5*),

$$\begin{aligned} E_M\left(t_U^2 W^2\right) &= M\bar{N} \sum_{i\in U} N_i \bar{\bar{Q}} \sum_{a=1}^{A} P_a \sigma_\gamma^2 \bar{\bar{Q}}\left[\cancel{\frac{N_i - 1}{N_i}}\right] + M\bar{N} \sum_{i\in U} N_i \bar{\bar{Q}} \sum_{a=1}^{A} P_a \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \bar{\bar{Q}}_b \\ &\doteq M^2 \bar{N}^2 \bar{\bar{Q}}^2 \sigma_\gamma^2 \sum_{a=1}^{A} P_a + M^2 \bar{N}^2 \bar{\bar{Q}} \sum_{a=1}^{A} P_a \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \bar{\bar{Q}}_b \end{aligned}$$

(A.120)

And so

$$t_U^2 E_M\left(W^2\right) \doteq M^2 \bar{N}^2 \bar{\bar{Q}}^2 \sigma_\gamma^2 \sum_{a=1}^{A} P_a + M^2 \bar{N}^2 \bar{\bar{Q}} \sum_{a=1}^{A} P_a \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \bar{\bar{Q}}_b$$

(A.121)

In the special case of no *b* strata so that $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectation of $W^2$ can be simplified to

$$E_M\left(t_U^2 W^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 \sum_{a=1}^{A} P_a \left[\sigma_\gamma^2 + \sigma_{\varepsilon_a}^2\right]$$

(A.122)

and when there are no *a* strata so that $P_a = 1$ and $\sigma_{\varepsilon_a}^2 = \sigma_\varepsilon^2$,

$$E_M\left(t_U^2 W^2\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2 \left[\sigma_\gamma^2 + \sigma_\varepsilon^2\right]\square$$

(A.123)

### A.4.9 Model Expectation of $W_{3a}^2$

**Theorem 10.** *The model expectation of $W_{3a}^2$ is*

$$E_M\left(t_{U_a}^2 W_{3a}^2\right) = \sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{iajb}.$$

*Proof.*

Recall from Section 2.3.3 that

$$t_{U_a}^2 W_{3a}^2 = \sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{Q_{iaj}^2}{p_{j|ia}} S_{U3iaj}^2 \tag{A.124}$$

where

$$S_{U3iaj}^2 = \frac{1}{Q_{iaj}-1}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left(y_k - \bar{y}_{U_{iaj}}\right)^2$$

and

$$\bar{y}_{U_{iaj}} = \sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{y_k}{Q_{iaj}}.$$

Substituting the model form of $y_k$ into the above, we have

$$\begin{aligned}
\bar{y}_{U_{iaj}} &= \sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{y_k}{Q_{iaj}} \\
&= \sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{\left(\mu+\alpha_i+\gamma_{iaj}+\varepsilon_{iajbk}\right)}{Q_{iaj}} \\
&= \frac{1}{Q_{iaj}}Q_{iaj}\left(\mu+\alpha_i+\gamma_{iaj}\right)+\frac{1}{Q_{iaj}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\varepsilon_{iajbk} \\
&= \left(\mu+\alpha_i+\gamma_{iaj}\right)+\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\varepsilon_{iajbk}\frac{1}{Q_{iaj}}
\end{aligned} \tag{A.125}$$

Then,

$$y_k - \bar{y}_{U_{iaj}} = \varepsilon_{iajbk} - \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \varepsilon_{iajbk} \frac{1}{Q_{iaj}}$$
$$= \varepsilon_{iajbk}\left(1 - \frac{1}{Q_{iaj}}\right) - \sum_{b'k' \neq (bk)}^{B, U_{iajb'}} \varepsilon_{iajb'k'} \frac{1}{Q_{iaj}}$$

(A.126)

Taking the expected value of Equation (A.124), we obtain

$$E_M\left(t_{U_a}^2 W_{3a}^2\right) = \sum_{i \in U} \frac{1}{p_i} \sum_{j \in U_{ia}} \frac{Q_{iaj}^2}{p_{j|ia}} E_M\left(S_{U_{3iaj}}^2\right)$$
$$= \sum_{i \in U} \frac{1}{p_i} \sum_{j \in U_{ia}} \frac{Q_{iaj}^2}{p_{j|ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \frac{1}{Q_{iaj}-1} E_M\left(y_k - \bar{y}_{U_{iaj}}\right)^2$$
$$= \sum_{i \in U} \frac{1}{p_i} \sum_{j \in U_{ia}} \frac{Q_{iaj}^2}{p_{j|ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \frac{1}{Q_{iaj}-1} \left\{\left[E_M\left(y_k - \bar{y}_{U_{iaj}}\right)\right]^2 + Var_M\left(y_k - \bar{y}_{U_{iaj}}\right)\right\}.$$

(A.127)

Need to find

1. $$\left[E_M\left(y_k - \bar{y}_{U_{iaj}}\right)\right]^2 \quad \text{(A.128)}$$

2. $$Var_M\left(y_k - \bar{y}_{U_{iaj}}\right) \quad \text{(A.129)}$$

1. Since all terms in Eq. (A.126) have expected value with respect to the model equal to zero, solving Eq. (A.128) we obtain

$$E_M\left(y_k - \bar{y}_{U_{iaj}}\right) = 0$$

$$\left[E_M\left(y_k - \bar{y}_{U_{iaj}}\right)\right]^2 = 0 \quad \text{(A.130)}$$

2. Now taking the variance of Equation (A.126) with respect to the model, we have

$$Var_M\left(y_k - \bar{y}_{U_{iaj}}\right) = Var_M\left[\varepsilon_{iajbk}\left(1-\frac{1}{Q_{iaj}}\right)\right] + \sum_{b'k' \neq (bk)}^{B,U_{iajb'}} Var_M\left[\varepsilon_{iajb'k'}\frac{1}{Q_{iaj}}\right]$$

$$= \sigma_{\varepsilon_{ab}}^2\left(1-\frac{1}{Q_{iaj}}\right)^2 + \sum_{b'k' \neq (bk)}^{B,U_{iajb'}} \sigma_{\varepsilon_{ab'}}^2 \frac{1}{Q_{iaj}^2} + \sigma_{\varepsilon_{ab}}^2 \frac{1}{Q_{iaj}^2} - \sigma_{\varepsilon_{ab}}^2 \frac{1}{Q_{iaj}^2}$$

$$= \sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q_{iaj}} + \cancel{\frac{1}{Q_{iaj}^2}} - \cancel{\frac{1}{Q_{iaj}^2}}\right) + \sum_{b'k'}^{B,U_{iajb'}} \sigma_{\varepsilon_{ab'}}^2 \frac{1}{Q_{iaj}^2}$$

$$= \sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q_{iaj}}\right) + \sum_{b'=1}^{B} \sigma_{\varepsilon_{ab'}}^2 \frac{Q_{iajb'}}{Q_{iaj}^2} \tag{A.131}$$

Substituting Eqs. (A.130) and (A.131) back into the formula for $S_{U3iaj}^2$, we have

$$E_M\left(S_{U_{3iaj}}^2\right) = \sum_{b=1}^{B}\sum_{k\in U_{iajb}} \frac{1}{Q_{iaj}-1}\left\{0 + Var_M\left(y_k - \bar{y}_{U_{iaj}}\right)\right\}$$

$$= \sum_{b=1}^{B}\sum_{k\in U_{iajb}} \frac{1}{Q_{iaj}-1}\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q_{iaj}}\right) + \left(\sum_{b=1}^{B}\sum_{k\in U_{iajb}} \frac{1}{Q_{iaj}-1}\right)\left(\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 \frac{Q_{iajb}}{Q_{iaj}^2}\right)$$

$$= \sum_{b=1}^{B}\frac{Q_{iajb}}{Q_{iaj}-1}\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q_{iaj}}\right) + \frac{\cancel{Q_{iaj}}}{Q_{iaj}-1}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 \frac{Q_{iajb}}{Q_{iaj}^{\cancel{2}}}$$

$$= \frac{1}{Q_{iaj}-1}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left(Q_{iajb} - \frac{2Q_{iajb}}{Q_{iaj}} + \frac{Q_{iajb}}{Q_{iaj}}\right) = \frac{1}{Q_{iaj}-1}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\left(Q_{iajb} - \frac{Q_{iajb}}{Q_{iaj}}\right)$$

$$= \frac{1}{\cancel{Q_{iaj}-1}}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{iajb}\left(\frac{\cancel{Q_{iaj}-1}}{Q_{iaj}}\right) = \sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 \frac{Q_{iajb}}{Q_{iaj}} \square \tag{A.132}$$

Substituting Eq. (A.85) back into Eq. (A.80),

$$
\begin{aligned}
E_M\left(t_{U_a}^2 W_{3a}^2\right) &= \sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{Q_{iaj}^2}{p_{j|ia}}E_M\left(S_{U_{3iaj}}^2\right)\\
&= \sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{Q_{iaj}^{\cancel{2}}}{p_{j|ia}}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\frac{Q_{iajb}}{\cancel{Q_{iaj}}}\\
&= \sum_{i\in U}\frac{1}{p_i}\sum_{j\in U_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{iajb}\square
\end{aligned}
\tag{A.133}
$$

**Corollary to Theorem 10.** Assume that conditions (*A1*) – (*A5*) in Appendix A.4.2 hold.

In particular assume that (*A2*) $p_i = \frac{Q_i}{Q}$; $p_{j|ia} = \frac{Q_{iaj}}{Q_{ia}}$, (*A3*) $Q_{iajb} = \bar{\bar{Q}}_b$,

$(A4)\ P_{ia} = \frac{N_{ia}}{N_i} \equiv P_a$, and $(A5)\, N_{ia} \approx N_{ia} - 1$ hold. Then in the special case of no *b* strata so that $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$. Then in the special case of no *b* strata so that $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectation of $W_{3a}^2$ can be simplified to

$$
E_M\left(t_{U_a}^2 W_{3a}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2 \sigma_{\varepsilon_a}^2
$$

and when there are no *a* strata so that $P_a = 1$ and $\sigma_{\varepsilon_a}^2 = \sigma_{\varepsilon}^2$,

$$
E_M\left(t_{U_a}^2 W_{3a}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 \sigma_{\varepsilon}^2
$$

*Proof*.

When ($A2$) and ($A3$) hold $p_i = \dfrac{Q_i}{Q}$, $p_{j|ia} = \dfrac{Q_{iaj}}{Q_{ia}}$, and $Q_{iajb} = \bar{\bar{Q}}_b$. Then we have

$$
\begin{aligned}
E_M\left(t_{U_a}^2 W_{3a}^2\right) &= \sum_{i \in U} \frac{1}{p_i} \sum_{j \in U_{ia}} \frac{Q_{iaj}}{p_{j|ia}} \sum_{b=1}^{B} Q_{iajb} \sigma_{\varepsilon_{ab}}^2 \\
&= \sum_{i \in U} \frac{Q}{Q_i} \sum_{j \in U_{ia}} \frac{Q_{ia}}{\cancel{Q_{iaj}}} \cancel{Q_{iaj}} \sum_{b=1}^{B} \bar{\bar{Q}}_b \sigma_{\varepsilon_{ab}}^2 \\
&= Q \sum_{i \in U} \frac{Q_{ia}}{Q_i} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \bar{\bar{Q}}_b \sigma_{\varepsilon_{ab}}^2 \\
&= M\bar{N}\bar{\bar{Q}} \sum_{i \in U} \frac{Q_{ia}}{Q_i} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \bar{\bar{Q}}_b \sigma_{\varepsilon_{ab}}^2
\end{aligned}
\tag{A.134}
$$

and when ($A4$) holds such that $P_{ia} = \dfrac{N_{ia}}{N_i} \equiv P_a$, we have $N_{ia} \equiv N_i P_a$ and

$$
\begin{aligned}
\sum_{i \in U} \sum_{j \in U_{ia}} 1 &= \sum_{i \in U} N_{ia} \\
\sum_{i \in U} N_{ia} &= \sum_{i \in U} N_i P_a \\
N_a &= P_a \sum_{i \in U} N_i \\
N_a &= P_a M \bar{N}
\end{aligned}
\tag{A.135}
$$

Also when ($A3$) holds, we know that from Lemma 2, $\bar{Q}_{ia} = \bar{\bar{Q}}$, $\bar{Q}_i = \bar{\bar{Q}}$, and $\bar{Q}_{ia \bullet b} = \bar{\bar{Q}}_b$.

Using this together with the fact that $N_{ia} \equiv N_i P_a$ and $\sum_{i \in U} \sum_{j \in U_{ia}} 1 = N_a = P_a M \bar{N}$, we obtain

$$
\begin{aligned}
E_M\left(t_{U_a}^2 W_{3a}^2\right) &= M\bar{N}\bar{\bar{Q}} \sum_{i \in U} \frac{N_{ia}\bar{Q}_{ia}}{N_i \bar{Q}_i} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \bar{\bar{Q}}_b \sigma_{\varepsilon_{ab}}^2 \\
&= M\bar{N}\bar{\bar{Q}} \sum_{i \in U} P_a \frac{\cancel{\bar{\bar{Q}}}}{\cancel{\bar{\bar{Q}}}} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \bar{\bar{Q}}_b \sigma_{\varepsilon_{ab}}^2 \\
&= M\bar{N}\bar{\bar{Q}} P_a \sum_{b=1}^{B} \bar{\bar{Q}}_b \sigma_{\varepsilon_{ab}}^2 \sum_{i \in U} \sum_{j \in U_{ia}} 1 \\
&= M\bar{N}\bar{\bar{Q}} P_a^2 \sum_{b=1}^{B} \bar{\bar{Q}}_b \sigma_{\varepsilon_{ab}}^2 \left(M\bar{N}\right) \\
&= \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2 \sum_{b=1}^{B} \frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}} \sigma_{\varepsilon_{ab}}^2
\end{aligned}
$$

(A.136)

In the special case where there are no $b$ strata so that, $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectation of $W_{3a}^2$ can be simplified to

$$
\begin{aligned}
E_M\left(t_{U_a}^2 W_{3a}^2\right) &= \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2 \frac{\cancel{\bar{\bar{Q}}}}{\cancel{\bar{\bar{Q}}}} \sigma_{\varepsilon_a}^2 \\
&= \left(M\bar{N}\bar{\bar{Q}}\right)^2 P_a^2 \sigma_{\varepsilon_a}^2
\end{aligned}
$$

(A.137)

And when there are no $a$ strata so that, $P_a = 1$ and $\sigma_{\varepsilon_a}^2 = \sigma_\varepsilon^2$,

$$
E_M\left(t_{U_a}^2 W_{3a}^2\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2 \sigma_\varepsilon^2 \square
$$

(A.138)

## A.4.10 Model Expectation of $\tilde{V}$

**Theorem 11.** *The approximate model expectation of* $\tilde{V}$ *is*

$$
\begin{aligned}
E_M\left(\bar{y}_U^2\tilde{V}\right) \doteq \sigma_\alpha^2\left[1-\frac{1}{M}\left(v_Q^2+1\right)\right] \\
+\sigma_\gamma^2\left[1-\frac{1}{Q^2}\sum_{i\in U}\sum_{a=1}^{A} N_{ia}\bar{Q}_{ia}^2\left(v_{Q_{ia}}^2+1\right)\right] \\
+\sum_{i\in U}\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\frac{Q_{ia\bullet b}}{Q}.
\end{aligned}
$$

*Proof.*

Recall that

$$
\bar{y}_U^2\tilde{V} = \frac{1}{Q-1}\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left(y_k-\bar{y}_U\right)^2 \tag{A.139}
$$

where

$$
\bar{y}_U = \sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{y_k}{Q}.
$$

Substituting the model form of $y_k$ into the above, we have

$$
\begin{aligned}
\bar{y}_U &= \sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\frac{y_k}{Q} \\
&= \mu+\sum_{i\in U}\alpha_i\frac{Q_i}{Q}+\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\gamma_{iaj}\frac{Q_{iaj}}{Q} \\
&+\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\varepsilon_{iajbk}\frac{1}{Q}
\end{aligned}
\tag{A.140}
$$

Then,

$$
\begin{aligned}
y_k - \bar{y}_U &= \alpha_i - \sum_{i \in U} \alpha_i \frac{Q_i}{Q} \\
&\quad + \gamma_{iaj} - \sum_{i \in U} \sum_{a=1}^{A} \sum_{j \in U_{ia}} \gamma_{iaj} \frac{Q_{iaj}}{Q} \\
&\quad + \varepsilon_{iajbk} - \sum_{i \in U} \sum_{a=1}^{A} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \varepsilon_{iajbk} \frac{1}{Q}
\end{aligned}
$$

$$
\begin{aligned}
&= \alpha_i \left( 1 - \frac{Q_i}{Q} \right) - \sum_{i' \neq i \in U} \alpha_{i'} \frac{Q_{i'}}{Q} \\
&\quad + \gamma_{iaj} \left( 1 - \frac{Q_{iaj}}{Q} \right) - \sum_{i'a'j' \neq (iaj)}^{U, A, U_{i'a'}} \gamma_{i'a'j'} \frac{Q_{i'a'j'}}{Q} \\
&\quad + \varepsilon_{iajbk} \left( 1 - \frac{1}{Q} \right) - \sum_{i'a'j'b'k' \neq (iajbk)}^{U, A, U_{i'a'}, B, U_{i'a'j'b'}} \varepsilon_{i'a'j'b'k'} \frac{1}{Q}
\end{aligned}
\tag{A.141}
$$

Taking the expected value of Eq. (A.139) we obtain

$$
E_M \left( \bar{y}_U^2 \tilde{V} \right) = E_M \left( \frac{1}{Q-1} \sum_{i \in U} \sum_{a=1}^{A} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \left( y_k - \bar{y}_U \right)^2 \right)
$$

$$
\begin{aligned}
&= \frac{1}{Q-1} \sum_{i \in U} \sum_{a=1}^{A} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} E_M \left[ \left( y_k - \bar{y}_U \right)^2 \right] \\
&= \frac{1}{Q-1} \sum_{i \in U} \sum_{a=1}^{A} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \left\{ \left[ E_M \left( y_k - \bar{y}_U \right) \right]^2 + Var_M \left( y_k - \bar{y}_U \right) \right\}
\end{aligned}
\tag{A.142}
$$

Need to find

1. $\left[ E_M \left( y_k - \bar{y}_U \right) \right]^2$ (A.143)

2. $Var_M \left( y_k - \bar{y}_U \right)$ (A.144)

The expected value of all terms in Eq.(A.141) is zero. Consequently $E_M\left(y_k-\bar{y}_U\right)=0$ and hence

$$\left[E_M\left(y_k-\bar{y}_U\right)\right]^2=0. \tag{A.145}$$

Solving Eq. (A.144) we obtain,

$$\begin{aligned}
Var_M\left(y_k-\bar{y}_U\right)&=Var_M\left[\alpha_i\left(1-\frac{Q_i}{Q}\right)\right]+\sum_{i'\neq i\in U}Var_M\left[\alpha_{i'}\frac{Q_{i'}}{Q}\right]\\
&+Var_M\left[\gamma_{iaj}\left(1-\frac{Q_{iaj}}{Q}\right)\right]+\sum_{i'a'j'\neq(iaj)}^{U,A,U_{i'a'}}Var_M\left[\gamma_{i'a'j'}\frac{Q_{i'a'j'}}{Q}\right]\\
&+Var_M\left[\varepsilon_{iajbk}\left(1-\frac{1}{Q}\right)\right]+\sum_{i'a'j'b'k'\neq(iajbk)}^{U,A,U_{i'a'},B,U_{i'a'j'b'}}Var_M\left[\frac{\varepsilon_{i'a'j'b'k'}}{Q}\right]\\
&=\sigma_\alpha^2\left(1-\frac{Q_i}{Q}\right)^2+\sum_{i'\neq i\in U}\sigma_\alpha^2\frac{Q_{i'}^2}{Q^2}\\
&\sigma_\gamma^2\left(1-\frac{Q_{iaj}}{Q}\right)^2+\sum_{i'a'j'\neq(iaj)}^{U,A,U_{i'a'}}\sigma_\gamma^2\frac{Q_{i'a'j'}^2}{Q^2}\\
&+\sigma_{\varepsilon_{ab}}^2\left(1-\frac{1}{Q}\right)^2+\sum_{i'aj'b'k'\neq(iajbk)}^{U,A,U_{i'a},B,U_{i'aj'b}}\sigma_{\varepsilon_{a'b'}}^2\frac{1}{Q^2}
\end{aligned} \tag{A.146}$$

Continuing on from Eq. (A.146) ,

$$Var_M\left(y_k - \bar{y}_U\right) = \sigma_\alpha^2\left(1-\frac{Q_i}{Q}\right)^2 - \sigma_\alpha^2\frac{Q_i^2}{Q^2} + \left(\sum_{i'\neq i\in U}\sigma_\alpha^2\frac{Q_{i'}^2}{Q^2}\right) + \sigma_\alpha^2\frac{Q_i^2}{Q^2}$$

$$+\sigma_\gamma^2\left(1-\frac{Q_{iaj}}{Q}\right)^2 - \sigma_\gamma^2\frac{Q_{iaj}^2}{Q^2} + \left(\sum_{i'a'j'\neq(iaj)}^{U,A,U_{i'a'}}\sigma_\gamma^2\frac{Q_{i'a'j'}^2}{Q^2}\right) + \sigma_\gamma^2\frac{Q_{iaj}^2}{Q^2}$$

$$+\sigma_{\varepsilon_{ab}}^2\left(1-\frac{1}{Q}\right)^2 - \sigma_{\varepsilon_{ab}}^2\frac{1}{Q^2}$$

$$+\left(\sum_{i'aj'b'k'\neq(iajbk)}^{U,A,U_{i'a},B,U_{i'aj'b}}\sigma_{\varepsilon_{a'b'}}^2\frac{1}{Q^2}\right) + \sigma_{\varepsilon_{ab}}^2\frac{1}{Q^2}$$

(A.147)

And so,

$$Var_M\left(y_k - \bar{y}_{U_i}\right) = \sigma_\alpha^2\left(1-\frac{2Q_i}{Q}+\cancel{\frac{Q_i^2}{Q^2}}-\cancel{\frac{Q_i^2}{Q^2}}\right) + \sum_{i\in U}\sigma_\alpha^2\frac{Q_i^2}{Q^2}$$

$$+\sigma_\gamma^2\left(1-\frac{2Q_{iaj}}{Q}+\cancel{\frac{Q_{iaj}^2}{Q^2}}-\cancel{\frac{Q_{iaj}^2}{Q^2}}\right) + \sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sigma_\gamma^2\frac{Q_{iaj}^2}{Q^2}$$

$$+\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q}+\cancel{\frac{1}{Q_i^2}}-\cancel{\frac{1}{Q^2}}\right) + \sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sum_{b}\sum_{k\in U_{iajb}}\sigma_{\varepsilon_{ab}}^2\frac{1}{Q^2}$$

$$= \sigma_\alpha^2\left(1-\frac{2Q_i}{Q}\right) + \sum_{i\in U}\sigma_\alpha^2\frac{Q_i^2}{Q^2}$$

$$+\sigma_\gamma^2\left(1-\frac{2Q_{iaj}}{Q}\right) + \sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sigma_\gamma^2\frac{Q_{iaj}^2}{Q^2}$$

$$+\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q}\right) + \sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sum_{b}\sum_{k\in U_{iajb}}\sigma_{\varepsilon_{ab}}^2\frac{1}{Q^2} \quad \text{(A.148)}$$

Substituting Eqs. (A.145) and (A.148) back into Eq. (A.142), we have

$$E_M\left(\bar{y}_U^2\tilde{V}\right)=\frac{1}{Q-1}\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left\{\left[E_M\left(y_k-\bar{y}_U\right)\right]^2+Var_M\left(y_k-\bar{y}_U\right)\right\}$$

$$=\frac{1}{Q-1}\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left\{0+Var_M\left(y_k-\bar{y}_U\right)\right\}$$

$$=\frac{1}{Q-1}\sum_{i\in U}\sum_{a=1}^{A}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}Var_M\left(y_k-\bar{y}_U\right)$$

$$=\frac{1}{Q-1}\sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sum_{b}\sum_{k\in U_{iajb}}\sigma_\alpha^2\left(1-\frac{2Q_i}{Q}\right)+\frac{\cancel{Q}}{Q-1}\sum_{i\in U}\sigma_\alpha^2\frac{Q_i^2}{Q^{\cancel{2}}}$$
$$+\frac{1}{Q-1}\sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sum_{b}\sum_{k\in U_{iajb}}\sigma_\gamma^2\left(1-\frac{2Q_{iaj}}{Q}\right)+\frac{\cancel{Q}}{Q-1}\sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sigma_\gamma^2\frac{Q_{iaj}^2}{Q^{\cancel{2}}}$$
$$+\frac{1}{Q-1}\sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sum_{b}\sum_{k\in U_{iajb}}\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q}\right)+\frac{\cancel{Q}}{Q-1}\sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sum_{b}\sum_{k\in U_{iajb}}\sigma_{\varepsilon_{ab}}^2\frac{1}{Q^{\cancel{2}}}$$

$$=\frac{1}{Q-1}\sum_{i\in U}Q_i\sigma_\alpha^2\left(1-\frac{2Q_i}{Q}\right)+\frac{1}{Q-1}\sum_{i\in U}\sigma_\alpha^2\frac{Q_i^2}{Q}$$
$$+\frac{1}{Q-1}\sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}Q_{iaj}\sigma_\gamma^2\left(1-\frac{2Q_{iaj}}{Q}\right)+\frac{1}{Q-1}\sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sigma_\gamma^2\frac{Q_{iaj}^2}{Q}$$
$$+\frac{1}{Q-1}\sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sum_{b}Q_{iajb}\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q}\right)+\frac{1}{Q-1}\sum_{i\in U}\sum_{a}\sum_{j\in U_{ia}}\sum_{b}Q_{iajb}\sigma_{\varepsilon_{ab}}^2\frac{1}{Q}$$

(A.149)

Continuing from Eq.(A.149),

$$
\begin{aligned}
E_M\left(\bar{y}_U^2\tilde{V}\right) &= \frac{1}{Q-1}\sigma_\alpha^2\left(\sum_{i\in U}Q_i-\frac{2}{Q}\sum_{i\in U}Q_i^2\right)+\frac{1}{Q-1}\sigma_\alpha^2\sum_{i\in U}\frac{Q_i^2}{Q} \\
&\quad +\frac{1}{Q-1}\sigma_\gamma^2\sum_{i\in U}\sum_a\left(\sum_{j\in U_{ia}}Q_{iaj}-\frac{2}{Q}\sum_{j\in U_{ia}}Q_{iaj}^2\right)+\frac{1}{Q-1}\sigma_\gamma^2\sum_{i\in U}\sum_a\sum_{j\in U_{ia}}\frac{Q_{iaj}^2}{Q} \\
&\quad +\frac{1}{Q-1}\sum_{i\in U}\sum_a\sum_b Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\left(1-\frac{2}{Q}\right)+\frac{1}{Q-1}\sum_{i\in U}\sum_a\sum_b Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\frac{1}{Q} \\
&= \frac{1}{Q-1}\sigma_\alpha^2\left(\sum_{i\in U}Q_i-\frac{1}{Q}\sum_{i\in U}Q_i^2\right)+\frac{1}{Q-1}\sigma_\gamma^2\sum_{i\in U}\sum_a\left(\sum_{j\in U_{ia}}Q_{iaj}-\frac{1}{Q}\sum_{j\in U_{ia}}Q_{iaj}^2\right) \\
&\quad +\frac{1}{\cancel{Q-1}}\sum_{i\in U}\sum_a\sum_b Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2\left(\frac{\cancel{Q-1}}{Q}\right)
\end{aligned}
$$

(A.150)

Rearranging terms in Eq. (A.150) and assuming $Q\approx Q-1$,

$$
\begin{aligned}
E_M\left(\bar{y}_U^2\tilde{V}\right) &= \frac{1}{Q-1}\sigma_\alpha^2\left(Q-\frac{1}{Q}\sum_{i\in U}Q_i^2\right) \\
&\quad +\frac{1}{Q-1}\sigma_\gamma^2\left(\sum_{i\in U}\sum_a Q_{ia}-\frac{1}{Q}\sum_{i\in U}\sum_a\sum_{j\in U_{ia}}Q_{iaj}^2\right) \\
&\quad +\frac{1}{Q}\sum_{i\in U}\sum_a\sum_b Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2 \\
&\doteq \frac{\cancel{Q}}{\cancel{Q}-1}\sigma_\alpha^2\left(1-\frac{1}{Q^2}\sum_{i\in U}Q_i^2\right) \\
&\quad +\frac{\cancel{Q}}{\cancel{Q}-1}\sigma_\gamma^2\left(1-\frac{1}{Q^2}\sum_{i\in U}\sum_a\sum_{j\in U_{ia}}Q_{iaj}^2\right) \\
&\quad +\frac{1}{Q}\sum_{i\in U}\sum_a\sum_b Q_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2
\end{aligned}
$$

(A.151)

Using the definition of $M\bar{Q} = Q = \sum_{i\in U} Q_i$ and $N_{ia}\bar{Q}_{ia\bullet b} = \sum_{j\in U_{ia}} Q_{iajb}$,

$$
\begin{aligned}
E_M\left(\bar{y}_U^2\tilde{V}\right) &\doteq \sigma_\alpha^2\left(1-\frac{1}{M^{\cancel{2}}\cancel{\bar{Q}^2}}\cancel{M}\,\cancel{\bar{Q}^2}\left(v_Q^2+1\right)\right) \quad \text{by Lemma 1(i)} \\
&\quad +\sigma_\gamma^2\left(1-\frac{1}{Q^2}\sum_{i\in U}\sum_{a} N_{ia}\bar{Q}_{ia}^2\left(v_{Q_{ia}}^2+1\right)\right) \quad \text{by Lemma 1(d)} \\
&\quad +\frac{1}{Q}\sum_{i\in U}\sum_{a}\sum_{b} N_{ia}\bar{Q}_{ia\bullet b}\sigma_{\varepsilon_{ab}}^2 \\
&= \sigma_\alpha^2\left[1-\frac{1}{M}\left(v_Q^2+1\right)\right]+\sigma_\gamma^2\left[1-\frac{1}{Q^2}\sum_{i\in U}\sum_{a=1}^{A} N_{ia}\bar{Q}_{ia}^2\left(v_{Q_{ia}}^2+1\right)\right] \\
&\quad +\sum_{i\in U}\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\,\frac{Q_{ia\bullet b}}{Q}\square
\end{aligned}
\tag{A.152}
$$

**Corollary to Theorem 11.** Assume that conditions (*A1*) – (*A5*) in Appendix A.4.2 hold. In particular assume that (*A2*) $p_i = \frac{Q_i}{Q}$; $p_{j|ia} = \frac{Q_{iaj}}{Q_{ia}}$, (*A3*) $Q_{iajb} = \bar{\bar{Q}}_b$, $(A4)\ P_{ia} = \frac{N_{ia}}{N_i} \equiv P_a$, and (*A5*) $N_{ia} \approx N_{ia}-1$ hold. Furthermore, assume $M \approx M-1$ and $M\bar{N} \approx M\bar{N}-1$. Then in the special case of no b strata so that $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectation of $\tilde{V}$ can be simplified to

$$E_M\left(t_U^2\tilde{V}\right) = \left(M\bar{N}\bar{\bar{Q}}\right)^2\left[\sigma_\alpha^2+\sigma_\gamma^2+\sum_{a=1}^{A} P_a\sigma_{\varepsilon_a}^2\right]$$

and when there are no *a* strata so that $P_a = 1$ and $\sigma_{\varepsilon_{ab}}^2 = \sigma_{\varepsilon_a}^2$,

$$E_M\left(t_U^2\tilde{V}\right) = M^2\bar{N}^2\bar{\bar{Q}}^2\left[\sigma_\alpha^2+\sigma_\gamma^2+\sigma_\varepsilon^2\right].$$

*Proof.*

When (*A2*) holds, we know from Lemma 3 that $v_{Q_{ia(pwr)}}^2 = v_{Q_{ia}}^2 = 0$, and from Section 2.4.2.3 that $v_Q^2 \equiv v_N^2$. Substituting this result back into Eq. (A.152), we obtain

$$E_M\left(\bar{y}_U^2\tilde{V}\right) = \sigma_\alpha^2\left[1 - \frac{1}{M}\left(v_N^2 + 1\right)\right] + \sigma_\gamma^2\left[1 - \frac{1}{Q^2}\sum_{i\in U}\sum_{a=1}^{A} N_{ia}\bar{Q}_{ia}^2\right] + \frac{1}{Q}\sum_{i\in U}\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 N_{ia}\bar{Q}_{ia\bullet b} \tag{A.153}$$

Assuming $Q_{iajb} = \bar{\bar{Q}}_b$, we know that from Lemma 2, $\bar{Q}_{ia} = \bar{Q}_i = \bar{\bar{Q}}$. When (*A4*) holds, we have $P_{ia} = \frac{N_{ia}}{N_i} = \frac{N_{ia}\bar{Q}_{ia}}{N_i\bar{Q}_i} = \frac{N_{ia}\bar{\bar{Q}}}{N_i\bar{\bar{Q}}} = \frac{Q_{ia}}{Q_i} \equiv P_a$. Then assuming $P_{ia} \equiv P_a$ implies $N_{ia} \equiv N_i P_a$. Substituting these results back into Eq. (A.153) , we obtain

$$\begin{aligned} E_M\left(\bar{y}_U^2\tilde{V}\right) &= \sigma_\alpha^2\left[1 - \frac{1}{M}\left(v_N^2 + 1\right)\right] \\ &\quad + \sigma_\gamma^2\left[1 - \frac{1}{Q^2}\sum_{i\in U} N_i\sum_{a=1}^{A} P_a\bar{\bar{Q}}^2\right] \qquad \text{since } \bar{Q}_{ia} = \bar{\bar{Q}} \\ &\quad + \frac{1}{Q}\sum_{i\in U} N_i\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 P_a\bar{\bar{Q}}_b \qquad \text{since } \bar{Q}_{ia\bullet b} = \bar{\bar{Q}}_b \end{aligned} \tag{A.154}$$

Substituting $Q = M\bar{N}\bar{\bar{Q}}$ and using the fact that $M\bar{N} = \sum_{i\in U} N_i$

$$\begin{aligned} E_M\left(\bar{y}_U^2\tilde{V}\right) &= \sigma_\alpha^2\left[1 - \frac{1}{M}\left(v_N^2 + 1\right)\right] + \sigma_\gamma^2\left[1 - \frac{\cancel{M}\,\cancel{\bar{N}}\,\cancel{\bar{\bar{Q}}^2}}{M^{\cancel{2}}\bar{N}^{\cancel{2}}\cancel{\bar{\bar{Q}}^2}}\sum_{a=1}^{A} P_a\right] + \frac{\cancel{M\bar{N}}}{\cancel{M\bar{N}}\,\bar{\bar{Q}}}\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 P_a\bar{\bar{Q}}_b \\ &= \sigma_\alpha^2\left[1 - \frac{1}{M}\left(v_N^2 + 1\right)\right] + \sigma_\gamma^2\left[1 - \frac{1}{M\bar{N}}\right] + \sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 P_a\frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}} \end{aligned} \tag{A.155}$$

Continuing from Eq. (A.155), if $M$ is large, we assume $\frac{1}{M}\left(v_N^2+1\right)\approx 0$, and

$$
\begin{aligned}
E_M\left(\bar{y}_U^2\tilde{V}\right) &\doteq \sigma_\alpha^2\ [1-0] \\
&\quad + \sigma_\gamma^2\left[\cancel{\frac{M\bar{N}-1}{M\bar{N}}}\right] \qquad \text{since } M\bar{N}\approx M\bar{N}-1 \\
&\quad + \frac{1}{\bar{\bar{Q}}}\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 P_a\bar{\bar{Q}}_b
\end{aligned}
\tag{A.156}
$$

$$
= \sigma_\alpha^2+\sigma_\gamma^2+\sum_{a=1}^{A}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 P_a\frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}}
$$

In the special case of no $b$ strata when $\sigma_{\varepsilon_{ab}}^2=\sigma_{\varepsilon_a}^2$ and $\bar{\bar{Q}}_b=\bar{\bar{Q}}$ the model expectation of

$\tilde{V}$ can be simplified to

$$
\begin{aligned}
E_M\left(\bar{y}_U^2\tilde{V}\right) &= \sigma_\alpha^2+\sigma_\gamma^2+\sum_{a=1}^{A}\sigma_{\varepsilon_a}^2 P_a\frac{\cancel{\bar{\bar{Q}}}}{\cancel{\bar{\bar{Q}}}} \\
&= \sigma_\alpha^2+\sigma_\gamma^2+\sum_{a=1}^{A}P_a\sigma_{\varepsilon_a}^2
\end{aligned}
\tag{A.157}
$$

and when there are no $a$ strata so that $P_a=1$ and $\sigma_{\varepsilon_a}^2=\sigma_\varepsilon^2$,

$$
E_M\left(\bar{y}_U^2\tilde{V}\right)=\sigma_\alpha^2+\sigma_\gamma^2+\sigma_\varepsilon^2 \tag{A.158}
$$

Using the definition of $\frac{t_U^2}{\left(M\bar{N}\bar{\bar{Q}}\right)^2}=\bar{y}_U^2$, we can rewrite Eq. (A.157) as

$$
E_M\left(t_U^2\tilde{V}\right)=\left(M\bar{N}\bar{\bar{Q}}\right)^2\left[\sigma_\alpha^2+\sigma_\gamma^2+\sum_{a=1}^{A}P_a\sigma_{\varepsilon_a}^2\right] \tag{A.159}
$$

and Eq.(A.158) as

$$
E_M\left(t_U^2\tilde{V}\right)=\left(M\bar{N}\bar{\bar{Q}}\right)^2\sigma_\alpha^2+\sigma_\gamma^2+\sigma_\varepsilon^2\square \tag{A.160}
$$

## A.4.11 Model Expectation of $\tilde{V}_a$

**Theorem 12.** *Assuming* $Q_a$, $M$, $N_a$ *are large such that,* $Q_a \approx Q_a - 1$, $M \approx M - 1$, *and* $N_a \approx N_a - 1$, *the approximate model expectation of* $\tilde{V}_a$ *is*

$$E_M\left(\bar{y}_{U_a}^2 \tilde{V}_a\right) \doteq \sigma_\alpha^2 \left\{1 - \frac{1}{Q_a^2} M \bar{Q}_{1a}^2 \left(v_{Q_{1a}}^2 + 1\right)\right\} + \sigma_\gamma^2 \left\{1 - \frac{1}{Q_a^2} N_a \bar{Q}_{2a}^2 \left(v_{Q_{2a}}^2 + 1\right)\right\}$$
$$+ \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \frac{Q_{ab}}{Q_a}.$$

*Proof.*

We use the following equalities to get the result. The derivation is straightforward and are similar to those for Lemma 1 in Section A.4.1.

(i) $\sum_{i \in U} Q_{ia}^2 = (M-1) S_{1Qa}^2 + M\bar{Q}_{1a}^2 \approx M\bar{Q}_{1a}^2 \left(v_{Q_{1a}}^2 + 1\right)$ where

$S_{Q_{1a}}^2 = (M-1)^{-1} \sum_{i \in U} \left(Q_{ia} - \bar{Q}_{1a}\right)^2$, $v_{Q_{1a}}^2 = S_{Q_{1a}}^2 \big/ \bar{Q}_{1a}^2$, and $\bar{Q}_{1a} = M^{-1} \sum_{i \in U} Q_{ia}$.

(ii) $\sum_{i \in U} \sum_{j \in U_{ia}} Q_{iaj}^2 = (N_a - 1) S_{Q_{2a}}^2 + N_a \bar{Q}_{2a}^2 \approx N_a \bar{Q}_{2a}^2 \left(v_{Q_{2a}}^2 + 1\right)$ where

$S_{Q_{2a}}^2 = (N_a - 1)^{-1} \sum_{i \in U} \sum_{j \in U_{ia}} \left(Q_{iaj} - \bar{Q}_{2a}\right)^2$, $v_{Q_{2a}}^2 = S_{Q_{2a}}^2 \big/ \bar{Q}_{2a}^2$, and

$\bar{Q}_{2a} = N_a^{-1} \sum_{i \in U} \sum_{j \in U_{ia}} Q_{iaj}$.

Recall that

$$\bar{y}_{U_a}^2 \tilde{V}_a = \frac{1}{Q_a - 1} \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \left(y_k - \bar{y}_{U_a}\right)^2 \tag{A.161}$$

where $\bar{y}_{U_a} = \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \frac{y_k}{Q_a}$.

Substituting the model form of $y_k$ into the above, we have

$$\bar{y}_{U_a} = \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \frac{y_k}{Q_a}$$
$$= \mu + \sum_{i \in U} \alpha_i \frac{Q_{ia}}{Q_a} + \sum_{i \in U} \sum_{j \in U_{ia}} \gamma_{iaj} \frac{Q_{iaj}}{Q_a} + \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \varepsilon_{iajbk} \frac{1}{Q_a} \tag{A.162}$$

Then,

$$y_k - \bar{y}_{U_a} = \alpha_i - \sum_{i \in U} \alpha_i \frac{Q_{ia}}{Q_a}$$
$$+\gamma_{iaj} - \sum_{i \in U} \sum_{j \in U_{ia}} \gamma_{iaj} \frac{Q_{iaj}}{Q_a}$$
$$+\varepsilon_{iajbk} - \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \varepsilon_{iajbk} \frac{1}{Q_a}$$

$$= \alpha_i \left( 1 - \frac{Q_{ia}}{Q_a} \right) - \sum_{i' \neq i \in U} \alpha_{i'} \frac{Q_{i'a}}{Q_a}$$
$$+\gamma_{iaj} \left( 1 - \frac{Q_{iaj}}{Q_a} \right) - \sum_{i'j' \neq (ij)}^{U, U_{i'a}} \gamma_{i'aj'} \frac{Q_{i'aj'}}{Q_a}$$
$$+\varepsilon_{iajbk} \left( 1 - \frac{1}{Q_a} \right) - \sum_{i'j'b'k' \neq (ijbk)}^{U, U_{i'a}, B, U_{i'aj'b'}} \varepsilon_{i'aj'b'k'} \frac{1}{Q_a}$$

(A.163)

Taking the expected value of Eq. (A.161) we obtain

$$E_M \left( \bar{y}_{U_a}^2 \tilde{V}_a \right) = E_M \left( \frac{1}{Q_a - 1} \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \left( y_k - \bar{y}_{U_a} \right)^2 \right)$$
$$= \frac{1}{Q_a - 1} \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} \left\{ \left[ E_M \left( y_k - \bar{y}_{U_a} \right) \right]^2 + Var_M \left( y_k - \bar{y}_{U_a} \right) \right\}$$

(A.164)

We need to find

$$1.\quad \left[E_M\left(y_k-\bar{y}_{U_a}\right)\right]^2 \tag{A.165}$$

$$2.\quad Var_M\left(y_k-\bar{y}_{U_a}\right) \tag{A.166}$$

The expected value of all terms in Eq.(A.163) is zero. Consequently $E_M\left(y_k-\bar{y}_{U_a}\right)=0$ and hence

$$\left[E_M\left(y_k-\bar{y}_{U_a}\right)\right]^2=0. \tag{A.167}$$

Solving Eq. (A.166) we obtain,

$$\begin{aligned} Var_M\left(y_k-\bar{y}_{U_a}\right)=Var_M&\left[\alpha_i\left(1-\frac{Q_{ia}}{Q_a}\right)\right]+\sum_{i'\neq i\in U}Var_M\left[\alpha_{i'}\frac{Q_{i'a}}{Q_a}\right]\\ &+Var_M\left[\gamma_{iaj}\left(1-\frac{Q_{iaj}}{Q_a}\right)\right]+\sum_{i'j'\neq(ij)}^{U,U_{i'a}}Var_M\left[\gamma_{i'aj'}\frac{Q_{i'aj'}}{Q_a}\right]\\ &+Var_M\left[\varepsilon_{iajbk}\left(1-\frac{1}{Q_a}\right)\right]+\sum_{i'j'b'k'\neq(ijbk)}^{U,U_{i'a},B,U_{i'aj'b'}}Var_M\left[\frac{\varepsilon_{i'aj'b'k'}}{Q_a}\right] \end{aligned} \tag{A.168}$$

Evaluating the variances and adding and subtracting terms to complete various sums of squares gives

$$\begin{aligned} Var_M\left(y_k-\bar{y}_{U_a}\right)=\sigma_\alpha^2&\left(1-\frac{Q_{ia}}{Q_a}\right)^2+\sigma_\alpha^2\left\{\sum_{i'\neq i\in U}\left[\frac{Q_{i'a}}{Q_a}\right]^2+\left(\frac{Q_{ia}}{Q_a}\right)^2-\left(\frac{Q_{ia}}{Q_a}\right)^2\right\}\\ &+\sigma_\gamma^2\left(1-\frac{Q_{iaj}}{Q_a}\right)^2+\sigma_\gamma^2\left\{\sum_{i'j'\neq(ij)}^{U,U_{i'a}}\left[\frac{Q_{i'aj'}}{Q_a}\right]^2+\left[\frac{Q_{iaj}}{Q_a}\right]^2-\left[\frac{Q_{iaj}}{Q_a}\right]^2\right\}\\ &+\sigma_{\varepsilon_{ab}}^2\left(1-\frac{1}{Q_a}\right)^2+\sum_{i'j'b'k'\neq(ijbk)}^{U,U_{i'a},B,U_{i'aj'b'}}\sigma_{\varepsilon_{ab'}}^2\frac{1}{Q_a^2}+\sigma_{\varepsilon_{ab}}^2\left\{\frac{1}{Q_a^2}-\frac{1}{Q_a^2}\right\} \end{aligned} \tag{A.169}$$

Collecting terms leads to

$$\begin{aligned} Var_M\left(y_k-\bar{y}_{U_a}\right) &= \sigma_\alpha^2\left(1-2\frac{Q_{ia}}{Q_a}\right)+\sigma_\alpha^2\sum_{i'\in U}\left[\frac{Q_{i'a}}{Q_a}\right]^2 \\ &+\sigma_\gamma^2\left(1-2\frac{Q_{iaj}}{Q_a}\right)+\sigma_\gamma^2\sum_{i'j'}^{U,U_{i'a}}\left[\frac{Q_{i'aj'}}{Q_a}\right]^2 \\ &+\sigma_{\varepsilon_{ab}}^2\left(1-2\frac{1}{Q_a}\right)+\frac{1}{Q_a^2}\sum_{i'j'b'}^{U,U_{i'a},B}\sigma_{\varepsilon_{ab'}}^2 Q_{i'aj'b'} \end{aligned} \tag{A.170}$$

Summing over PSUs, SSUs, *b* strata, and elements gives

$$\begin{aligned} &\sum_{i\in U}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}} Var_M\left(y_k-\bar{y}_{U_a}\right)= \\ &\quad\sigma_\alpha^2\sum_{i\in U}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left\{\left(1-2\frac{Q_{ia}}{Q_a}\right)+\sum_{i'\in U}\left[\frac{Q_{i'a}}{Q_a}\right]^2\right\} \\ &\quad+\sigma_\gamma^2\sum_{i\in U}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left\{\left(1-2\frac{Q_{iaj}}{Q_a}\right)+\sum_{i'j'}^{U,U_{i'a}}\left[\frac{Q_{i'aj'}}{Q_a}\right]^2\right\} \\ &\quad+\sum_{i\in U}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}}\left\{\sigma_{\varepsilon_{ab}}^2\left(1-2\frac{1}{Q_a}\right)+\frac{1}{Q_a^2}\sum_{i'j'b'}^{U,U_{i'a},B}\sigma_{\varepsilon_{ab'}}^2 Q_{i'aj'b'}\right\} \end{aligned} \tag{A.171}$$

Continuing from above we have,

$$\begin{aligned} \sum_{i\in U}\sum_{j\in U_{ia}}\sum_{b=1}^{B}\sum_{k\in U_{iajb}} Var_M\left(y_k-\bar{y}_{U_a}\right) &= \sigma_\alpha^2\left\{Q_a-\frac{2}{Q_a}\sum_{i\in U}Q_{ia}^2+\frac{1}{Q_a}\sum_{i\in U}Q_{ia}^2\right\} \\ &+\sigma_\gamma^2\left\{Q_a-\frac{2}{Q_a}\sum_{i\in U}\sum_{j\in U_{ia}}Q_{iaj}^2+\frac{1}{Q_a}\sum_{i\in U}\sum_{j\in U_{ia}}Q_{iaj}^2\right\} \\ &+\left(1-\frac{2}{Q_a}\right)\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{ab}+\frac{1}{Q_a}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 Q_{ab} \end{aligned} \tag{A.172}$$

Note that to obtain the preceding expression, we used the assumption that all $b$ substrata of HUs occur in every PSU and SSU to move the sum over $b$ outside the other sums.

$$\left(Q_a - 1\right) E_M\left(\bar{y}_{U_a}^2 \tilde{V}_a\right) = \sum_{i \in U} \sum_{j \in U_{ia}} \sum_{b=1}^{B} \sum_{k \in U_{iajb}} Var_M\left(y_k - \bar{y}_{U_a}\right)$$

$$= \sigma_\alpha^2 Q_a \left\{1 - \frac{1}{Q_a^2} \sum_{i \in U} Q_{ia}^2\right\} + \sigma_\gamma^2 Q_a \left\{1 - \frac{1}{Q_a^2} \sum_{i \in U} \sum_{j \in U_{ia}} Q_{iaj}^2\right\}$$

$$+ \left(1 - \frac{1}{Q_a}\right) \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 Q_{ab}$$

(A.173)

Dividing through by $Q_a - 1$ and assuming that $Q_a$ is large so that $Q_a \approx Q_a - 1$ gives

$$E_M\left(\bar{y}_{U_a}^2 \tilde{V}_a\right) \doteq \sigma_\alpha^2 \left\{1 - \frac{1}{Q_a^2} \sum_{i \in U} Q_{ia}^2\right\} + \sigma_\gamma^2 \left\{1 - \frac{1}{Q_a^2} \sum_{i \in U} \sum_{j \in U_{ia}} Q_{iaj}^2\right\} + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \frac{Q_{ab}}{Q_a}$$

(A.174)

Next, we use (i) – (ii) above to obtain

$$E_M\left(\bar{y}_{U_a}^2 \tilde{V}_a\right) \doteq \sigma_\alpha^2 \left\{1 - \frac{M\bar{Q}_{1a}^2}{Q_a^2}\left(v_{Q_{1a}}^2 + 1\right)\right\} + \sigma_\gamma^2 \left\{1 - \frac{N_a \bar{Q}_{2a}^2}{Q_a^2}\left(v_{Q_{2a}}^2 + 1\right)\right\} + \sum_{b=1}^{B} \sigma_{\varepsilon_{ab}}^2 \frac{Q_{ab}}{Q_a} \square$$

(A.175)

**Corollary to Theorem 12.** Assume that conditions (*A1*) – (*A5*) in Appendix A.4.2 hold. Furthermore, assume $M \approx M-1$, and $M\bar{N} \approx M\bar{N}-1$. Then in the special case of no b strata so that $\sigma^2_{\varepsilon_{ab}} = \sigma^2_{\varepsilon_a}$ and $\bar{\bar{Q}}_b = \bar{\bar{Q}}$, the model expectation of $\tilde{V}_a$ can be simplified to

$$E_M\left(\bar{y}^2_{U_a}\tilde{V}_a\right) \doteq \sigma^2_\alpha + \sigma^2_\gamma + \sigma^2_{\varepsilon_a} \text{ and } E_M\left(k_{2a}\right) \doteq \frac{P_a^2\left[\sigma^2_\gamma + \sigma^2_{\varepsilon_a}\right]}{\sigma^2_\alpha + \sigma^2_\gamma + \sigma^2_{\varepsilon_a}}$$

and when there are no *a* strata so that $P_a = 1$ and $\sigma^2_{\varepsilon_{ab}} = \sigma^2_{\varepsilon_a}$,

$$E_M\left(\bar{y}^2_{U_a}\tilde{V}_a\right) \doteq \sigma^2_\alpha + \sigma^2_\gamma + \sigma^2_\varepsilon \text{ and } E_M\left(k_{2a}\right) \doteq \frac{\sigma^2_\gamma + \sigma^2_\varepsilon}{\sigma^2_\alpha + \sigma^2_\gamma + \sigma^2_\varepsilon}, \text{ when } \bar{\bar{Q}} \approx \bar{\bar{Q}} + 1.$$

*Proof.*

By Lemma 3, we know that $v^2_{Q_{1a}} = v^2_N$ and $v^2_{Q_{2a}} = 0$, substituting this back into Eq. (A.175),

$$E_M\left(\bar{y}^2_{U_a}\tilde{V}_a\right) \doteq \sigma^2_\alpha\left\{1 - \frac{1}{Q_a^2} M\bar{Q}^2_{1a}\left(v^2_N + 1\right)\right\} + \sigma^2_\gamma\left\{1 - \frac{1}{Q_a^2} N_a\bar{Q}^2_{2a}\right\} + \frac{1}{Q_a}\sum_{b=1}^{B}\sigma^2_{\varepsilon_{ab}}Q_{ab}$$

(A.176)

Also assuming $Q_{iajb} = \bar{\bar{Q}}_b$. and $N_{ia} \equiv N_i P_a$, we know $N_a = M\bar{N}P_a$. Using this with the results from Lemma 3, it also follows that:

$$\bar{Q}_{1a} = \frac{N_a\bar{\bar{Q}}}{M} = \frac{\cancel{M}\bar{N}\bar{\bar{Q}}P_a}{\cancel{M}} = \bar{N}\bar{\bar{Q}}P_a, \qquad \bar{Q}_{2a} = \bar{\bar{Q}},$$

$$Q_a = N_a\bar{\bar{Q}} = M\bar{N}\bar{\bar{Q}}P_a, \qquad Q_{ab} = N_a\bar{\bar{Q}}_b = M\bar{N}\bar{\bar{Q}}_b P_a.$$

Expression (A.176) then reduces to,

$$E_M\left(\bar{y}_{U_a}^2\tilde{V}_a\right) \doteq \sigma_\alpha^2\left\{1-\frac{M\left(\bar{N}\bar{\bar{Q}}P_a\right)^2}{\left(M\bar{N}\bar{\bar{Q}}P_a\right)^2}\left(v_N^2+1\right)\right\}+\sigma_\gamma^2\left\{1-\frac{M\bar{N}P_a\bar{\bar{Q}}^2}{\left(M\bar{N}\bar{\bar{Q}}P_a\right)^2}\right\}$$
$$+\frac{1}{M\bar{N}\bar{\bar{Q}}P_a}\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2 M\bar{N}\bar{\bar{Q}}_b P_a \tag{A.177}$$
$$=\sigma_\alpha^2\left\{1-\frac{1}{M}\left(v_N^2+1\right)\right\}+\sigma_\gamma^2\left\{1-\frac{1}{M\bar{N}P_a}\right\}+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}}$$

When $M$, $M\bar{N}$ is large,

$$E_M\left(\bar{y}_{U_a}^2\tilde{V}_a\right) \doteq \sigma_\alpha^2+\sigma_\gamma^2+\sum_{b=1}^{B}\sigma_{\varepsilon_{ab}}^2\frac{\bar{\bar{Q}}_b}{\bar{\bar{Q}}} \tag{A.178}$$

If there are no $b$ strata, then $\sigma_{\varepsilon_{ab}}^2=\sigma_{\varepsilon_a}^2$, $\bar{\bar{Q}}_b=\bar{\bar{Q}}$, and

$$E_M\left(\bar{y}_{U_a}^2\tilde{V}_a\right) \doteq \sigma_\alpha^2+\sigma_\gamma^2+\sigma_{\varepsilon_a}^2 \tag{A.179}$$

If there are no $a$ strata so that $\sigma_{\varepsilon_a}^2=\sigma_\varepsilon^2$, and

$$E_M\left(\bar{y}_{U_a}^2\tilde{V}_a\right) \doteq \sigma_\alpha^2+\sigma_\gamma^2+\sigma_\varepsilon^2 \quad \square \tag{A.180}$$

Also note that Eq. (A.180) is the same as Eq. (A.158) for $E_M\left(\bar{y}_U^2\tilde{V}\right)$ as it should be.

Using the definition of $\frac{t_{U_a}^2}{\left(M\bar{N}\bar{\bar{Q}}\right)^2}=\bar{y}_{U_a}^2$, we can rewrite Eq. (A.179) as

$$E_M\left(t_{U_a}^2\tilde{V}_a\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2\left[\sigma_\alpha^2+\sigma_\gamma^2+\sigma_{\varepsilon_a}^2\right] \tag{A.181}$$

and Eq. (A.180) as

$$E_M\left(t_{U_a}^2\tilde{V}_a\right) \doteq \left(M\bar{N}\bar{\bar{Q}}\right)^2\left[\sigma_\alpha^2+\sigma_\gamma^2+\sigma_\varepsilon^2\right]\square \tag{A.182}$$

## A.4.12 Model Expectations of Measures of Homogeneity

When there are no *a* or *b* substrata, assuming $\bar{N}$ and $\bar{N}\bar{\bar{Q}}$ are large and using the results in Appendices A.5.6 - A.5.9, gives the approximate model expectations of $\delta_1$ and $k_1$ :

$$E_M\left(\delta_1\right)=\frac{E_M\left(t_U^2 B^2\right)}{E_M\left(t_U^2 B^2\right)+E_M\left(t_U^2 W^2\right)} = \frac{\sigma_\alpha^2+\dfrac{\sigma_\gamma^2}{\bar{N}}+\dfrac{\sigma_\varepsilon^2}{\bar{N}\bar{\bar{Q}}}}{\sigma_\alpha^2+\dfrac{\sigma_\gamma^2}{\bar{N}}+\dfrac{\sigma_\varepsilon^2}{\bar{N}\bar{\bar{Q}}}+\sigma_\gamma^2+\sigma_\varepsilon^2} \doteq \frac{\sigma_\alpha^2}{\sigma_\alpha^2+\sigma_\gamma^2+\sigma_\varepsilon^2}$$

$$E_M\left(k_1\right)=\frac{E_M\left(t_U^2 B^2\right)+E_M\left(t_U^2 W^2\right)}{E_M\left(t_U^2 \tilde{V}\right)} = \frac{\sigma_\alpha^2+\dfrac{\sigma_\gamma^2}{\bar{N}}+\dfrac{\sigma_\varepsilon^2}{\bar{N}\bar{\bar{Q}}}+\sigma_\gamma^2+\sigma_\varepsilon^2}{\sigma_\alpha^2+\sigma_\gamma^2+\sigma_\varepsilon^2} \doteq \frac{\sigma_\alpha^2+\sigma_\gamma^2+\sigma_\varepsilon^2}{\sigma_\alpha^2+\sigma_\gamma^2+\sigma_\varepsilon^2}=1 \tag{A.183}$$

Likewise, assuming $\bar{\bar{Q}}$ is large, the approximate model expectations of $\delta_{2a}$ and $k_{2a}$ are

$$E_M\left(\delta_{2a}\right)=\frac{E_M\left(t_{U_a}^2 W_{2a}^2\right)}{E_M\left(t_{U_a}^2 W_{2a}^2\right)+E_M\left(t_{U_a}^2 W_{3a}^2\right)} = \frac{\sigma_\gamma^2+\dfrac{\sigma_\varepsilon^2}{\bar{\bar{Q}}}}{\sigma_\gamma^2+\dfrac{\sigma_\varepsilon^2}{\bar{\bar{Q}}}+\sigma_\varepsilon^2} \doteq \frac{\sigma_\gamma^2}{\sigma_\gamma^2+\sigma_\varepsilon^2}$$

$$E_M\left(k_{2a}\right)=\frac{E_M\left(t_{U_a}^2 W_{2a}^2\right)+E_M\left(t_{U_a}^2 W_{3a}^2\right)}{E_M\left(t_{U_a}^2 \tilde{V}_a\right)} = \frac{\sigma_\gamma^2+\dfrac{\sigma_{\varepsilon_a}^2}{\bar{\bar{Q}}}+\sigma_{\varepsilon_a}^2}{\sigma_\alpha^2+\sigma_\gamma^2+\sigma_{\varepsilon_a}^2} \doteq \frac{\sigma_\gamma^2+\sigma_{\varepsilon_a}^2}{\sigma_\alpha^2+\sigma_\gamma^2+\sigma_{\varepsilon_a}^2} \tag{A.184}$$

where $t_U^2=\bar{y}_U^2\left(M\bar{N}\bar{\bar{Q}}\right)^2$ and $t_{U_a}^2=\left(M\bar{N}\bar{\bar{Q}}\right)^2\bar{y}_{U_a}^2$ .

## A.5 Derivation of Estimators Needed For Anticipated Variances

### A.5.1 Estimator of $S^2_{Q(pwr)}$ needed for Anticipated Variances

**Theorem 13.** *An unbiased estimator of* $S^2_{Q(pwr)}$ *is*

$$\hat{S}^2_{Q_{(pwr)}} = \frac{1}{m-1}\sum_{i\in s_1}\left(\frac{\hat{Q}_i}{p_i}-\hat{Q}\right)^2 - \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a \frac{1}{n_{ia}\left(n_{ia}-1\right)}\left(\sum_{j\in s_{ia}}\frac{\hat{Q}^2_{iaj}}{p^2_{j|ia}} - n_{ia}\hat{Q}^2_{ia}\right)$$

*where if the third stage is SRS*

$\hat{Q}_{iaj} = Q_{iaj}$, $\hat{Q}_{ia} = \dfrac{1}{n_{ia}}\sum_{j\in s_{ia}}\dfrac{Q_{iaj}}{p_{j|ia}}$, $\hat{Q}_i = \sum_a \hat{Q}_{ia}$, and $\hat{Q} = \dfrac{1}{m}\sum_{i\in s_1}\dfrac{\hat{Q}_i}{p_i}$.

*Proof*.

The variance $S^2_{Q_{(pwr)}}$ is the special case of Equation (1.12) with $y_k = 1$. The estimator of $S^2_{Q_{(pwr)}}$ is the estimator from Section 2.4.1.1, $\hat{S}^2_{1(pwr)} = \hat{S}^2_{1(pwr)\mathbb{A}} - \hat{S}^2_{1(pwr)\mathbb{B}}$, with $y_k = 1$. Letting $y_k = 1$ in Eq. (1.12) we have,

$$S^2_{U1(pwr)} = \sum_{i\in U} p_i\left(\frac{t_{U_i}}{p_i} - t_U\right)^2 = \sum_{i\in U} p_i\left(\frac{Q_i}{p_i} - Q\right)^2 = S^2_{Q(pwr)}. \tag{A.185}$$

We know that $\hat{S}^2_{1(pwr)}$ is an unbiased estimator of $S^2_{U1(pwr)}$ for all $y_k$ including $y_k = 1$. So plugging in $y_k = 1$ into $\hat{S}^2_{1(pwr)}$ will give an unbiased estimator of $S^2_{Q(pwr)}$, since $S^2_{U1(pwr)} = S^2_{Q(pwr)}$ when $y_k = 1$. Letting $y_k = 1$ in the components of $\hat{S}^2_{1(pwr)}$ in Section 2.4.1.1 gives:

$$\hat{t}_{iajb} = \hat{Q}_{iajb}, \qquad \hat{t}_{iaj} = \hat{Q}_{iaj}, \qquad \hat{t}_{ia(pwr)} = \hat{Q}_{ia},$$

$$\hat{t}_{i(pwr)} = \hat{Q}_{i}, \qquad \hat{t}_{iab(pwr)} = \hat{Q}_{ia\bullet b}, \qquad \hat{t}_{pwr} = \hat{Q}$$

Making these substitutions in the formula for $\hat{S}^2_{1(pwr)\mathbb{A}}$ gives

$$\hat{S}^2_{1(pwr)\mathbb{A}} = \frac{1}{m-1}\sum_{i\in s_1}\left(\frac{\hat{Q}_i}{p_i} - \hat{Q}\right)^2. \tag{A.186}$$

Since, $\hat{S}^2_{1(pwr)\mathbb{B}} = \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a \frac{\hat{S}^2_{2\mathbb{A}ia}}{n_{ia}}$ we need

$$\begin{aligned}
\hat{S}^2_{2\mathbb{A}ia} &= \frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\left(\frac{\hat{t}_{iaj}}{p_{j|ia}} - \hat{t}_{ia(pwr)}\right)^2 = \frac{1}{n_{ia}-1}\sum_{j\in s_{ia}}\left(\frac{\hat{Q}_{iaj}}{p_{j|ia}} - \hat{Q}_{ia}\right)^2 \\
&= \frac{1}{n_{ia}-1}\left(\sum_{j\in s_{ia}}\frac{\hat{Q}^2_{iaj}}{p^2_{j|ia}} - 2\hat{Q}_{ia}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}}{p_{j|ia}} + \hat{Q}^2_{ia}\sum_{j\in s_{ia}} 1\right) \\
&= \frac{1}{n_{ia}-1}\left(\sum_{j\in s_{ia}}\frac{\hat{Q}^2_{iaj}}{p^2_{j|ia}} - 2n_{ia}\hat{Q}_{ia}\hat{Q}_{ia} + n_{ia}\hat{Q}^2_{ia}\right) \\
&= \frac{1}{n_{ia}-1}\left(\sum_{j\in s_{ia}}\frac{\hat{Q}^2_{iaj}}{p^2_{j|ia}} - n_{ia}\hat{Q}^2_{ia}\right)
\end{aligned} \tag{A.187}$$

So,

$$\begin{aligned}
\hat{S}^2_{1(pwr)} &= \frac{1}{m-1}\sum_{i\in s_1}\left(\frac{\hat{t}_{i(pwr)}}{p_i} - \hat{t}_{pwr}\right)^2 - \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{\hat{S}^2_{2\mathbb{A}ia}}{n_{ia}} \\
&= \frac{1}{m-1}\sum_{i\in s_1}\left(\frac{\hat{Q}_i}{p_i} - \hat{Q}\right)^2 - \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{1}{n_{ia}(n_{ia}-1)}\left(\sum_{j\in s_{ia}}\frac{\hat{Q}^2_{iaj}}{p^2_{j|ia}} - n_{ia}\hat{Q}^2_{ia}\right)\square
\end{aligned} \tag{A.188}$$

## A.5.2 Estimator of $S^2_{Q_{ia}(pwr)}$ needed for Anticipated Variances

**Theorem 14.** *An unbiased estimator of* $S^2_{Q_{ia}(pwr)} = \sum_{j \in U_{ia}} p_{j|ia} \left( \frac{Q_{iaj}}{p_{j|ia}} - Q_{ia} \right)^2$ is

$$\hat{S}^2_{Q_{ia}(pwr)} = \frac{1}{n_{ia} - 1} \sum_{j \in s_{ia}} \left( \frac{\hat{Q}_{iaj}}{p_{j|ia}} - \hat{Q}_{ia} \right)^2$$

*where if the third stage is SRS*

$\hat{Q}_{iaj} = Q_{iaj}$, and $\hat{Q}_{ia} = \frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{Q_{iaj}}{p_{j|ia}}$.

*Proof*. Expand

$$\begin{aligned}
\hat{S}^2_{Q_{ia}(pwr)} &= \frac{1}{n_{ia} - 1} \sum_{j \in s_{ia}} \left( \frac{\hat{Q}_{iaj}}{p_{j|ia}} - \hat{Q}_{ia} \right)^2 \\
&= \frac{1}{n_{ia} - 1} \sum_{j \in s_{ia}} \left( \frac{\hat{Q}^2_{iaj}}{p^2_{j|ia}} - 2 \frac{\hat{Q}_{iaj}}{p_{j|ia}} \hat{Q}_{ia} + \hat{Q}^2_{ia} \right) \\
&= \frac{1}{n_{ia} - 1} \left( \sum_{j \in s_{ia}} \frac{\hat{Q}^2_{iaj}}{p^2_{j|ia}} - 2\hat{Q}_{ia} \sum_{j \in s_{ia}} \frac{\hat{Q}_{iaj}}{p_{j|ia}} + n_{ia} \hat{Q}^2_{ia} \right) \\
&= \frac{1}{n_{ia} - 1} \sum_{j \in s_{ia}} \frac{\hat{Q}^2_{iaj}}{p^2_{j|ia}} - \frac{n_{ia}}{n_{ia} - 1} \hat{Q}^2_{ia}
\end{aligned} \tag{A.189}$$

Using $E_2$ and $E_3$ as in earlier sections to denote expectations with respect to the second and third stages of sampling,

$$\begin{aligned}
E_2 E_3 \left( \hat{S}^2_{Q_{ia}(pwr)} \right) &= \frac{1}{n_{ia} - 1} E_2 \sum_{j \in s_{ia}} E_3 \left( \frac{\hat{Q}^2_{iaj}}{p^2_{j|ia}} \right) - \frac{n_{ia}}{n_{ia} - 1} E_2 E_3 \left( \hat{Q}^2_{ia} \right) \\
&= \frac{1}{n_{ia} - 1} E_2 \left( \sum_{j \in s_{ia}} \frac{Q^2_{iaj}}{p^2_{j|ia}} \right) - \frac{n_{ia}}{n_{ia} - 1} E_2 E_3 \left( \hat{Q}^2_{ia} \right) \\
&= C - D
\end{aligned} \tag{A.190}$$

Solving for C and assuming $n_{ia}/(n_{ia}-1)\approx 1$,

$$\frac{1}{n_{ia}-1}E_2\left(\sum_{j\in s_{ia}}\frac{Q_{iaj}^2}{p_{j|ia}^2}\right)=\frac{n_{ia}}{n_{ia}-1}\sum_{j\in U_{ia}}\frac{\cancel{p_{j|ia}}\,Q_{iaj}^2}{p_{j|ia}^{\cancel{2}}}\doteq\sum_{j\in U_{ia}}\frac{Q_{iaj}^2}{p_{j|ia}}. \tag{A.191}$$

Solving for D and assuming $n_{ia}/(n_{ia}-1)\approx 1$,

$$\frac{n_{ia}}{n_{ia}-1}E_2E_3\left(\hat{Q}_{ia}^2\right)\doteq E_2E_3\left[\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)^2\right] \tag{A.192}$$

with

$$E_3\left[\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)^2\right]=Var_3\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)+\left[E_3\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)\right]^2=\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)^2 \tag{A.193}$$

and

$$\begin{aligned}E_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)^2&=Var_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)+\left[E_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)\right]^2\\&=\frac{1}{n_{ia}}\sum_{j\in U_{ia}}p_{j|ia}\left(\frac{Q_{iaj}}{p_{j|ia}}-Q_{ia}\right)^2+Q_{ia}^2\\&=\frac{S_{Q_{ia}(pwr)}^2}{n_{ia}}+Q_{ia}^2\end{aligned} \tag{A.194}$$

Taking the difference of C and D,

$$\begin{aligned}E_2E_3\left(\hat{S}_{Q_{ia}(pwr)}^2\right)&\doteq\left(\sum_{j\in U_{ia}}\frac{Q_{iaj}^2}{p_{j|ia}}-Q_{ia}^2\right)-\frac{S_{Q_{ia}(pwr)}^2}{n_{ia}}\\&=S_{Q_{ia}(pwr)}^2-\frac{S_{Q_{ia}(pwr)}^2}{n_{ia}}\\&=\frac{n_{ia}-1}{n_{ia}}S_{Q_{ia}(pwr)}^2\\&\doteq S_{Q_{ia}(pwr)}^2\square\end{aligned} \tag{A.195}$$

### A.5.3 Estimator of $S^2_{Q_a(pwr)}$ needed for Anticipated Variances

**Theorem 17.** *An unbiased estimator of* $S^2_{Q_a(pwr)} = \sum_{i \in U} p_i \left( \frac{Q_{ia}}{p_i} - Q_a \right)^2$ is

$$\hat{S}^2_{Q_a(pwr)} = \frac{1}{m-1} \sum_{i \in s_1} \left( \frac{\hat{Q}_{ia}}{p_i} - \hat{Q}_a \right)^2$$

*where if the third stage is SRS*

$\hat{Q}_{ia} = \frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{Q_{iaj}}{p_{j|ia}}$ and $\hat{Q}_a = \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_{ia}}{p_i}$.

*Proof*. Expand

$$\begin{aligned}
\hat{S}^2_{Q_a(pwr)} &= \frac{1}{m-1} \sum_{i \in s_1} \left( \frac{\hat{Q}_{ia}}{p_i} - \hat{Q}_a \right)^2 \\
&= \frac{1}{m-1} \sum_{i \in s_1} \left( \frac{\hat{Q}^2_{ia}}{p_i^2} - 2 \frac{\hat{Q}_{ia}}{p_i} \hat{Q}_a + \hat{Q}^2_a \right) \\
&= \frac{1}{m-1} \left( \sum_{i \in s_1} \frac{\hat{Q}^2_{ia}}{p_i^2} - 2\hat{Q}_{ia} \sum_{j \in s_{ia}} \frac{\hat{Q}_{ia}}{p_i} + m\hat{Q}^2_a \right) \\
&= \frac{1}{m-1} \sum_{i \in s_1} \frac{\hat{Q}^2_{ia}}{p_i^2} - \frac{m}{m-1} \hat{Q}^2_a
\end{aligned} \tag{A.196}$$

Using $E_2$ and $E_3$ as in earlier sections to denote expectations with respect to the second and third stages of sampling,

$$\begin{aligned}
E_1 E_2 \left( \hat{S}^2_{Q_a(pwr)} \right) &= \frac{1}{m-1} E_1 \sum_{i \in s_1} E_2 \left( \frac{\hat{Q}^2_{ia}}{p_i^2} \right) - \frac{m}{m-1} E_1 E_2 \left( \hat{Q}^2_a \right) \\
&= \frac{1}{m-1} E_1 \left( \sum_{i \in s_1} \frac{\hat{Q}^2_{ia}}{p_i^2} \right) - \frac{m}{m-1} E_1 E_2 \left( \hat{Q}^2_a \right) \\
&= C - D
\end{aligned} \tag{A.197}$$

Solving for C and assuming $m/(m-1)\approx 1$,

$$\frac{1}{m-1}E_1\left(\sum_{i\in s_1}\frac{\hat{Q}_{ia}^2}{p_i^2}\right)=\frac{m}{m-1}\sum_{i\in U}\frac{\cancel{p_i}\,Q_{ia}^2}{p_i^{\cancel{2}}}\doteq\sum_{i\in U}\frac{Q_{ia}^2}{p_i}. \tag{A.198}$$

Solving for D,

$$\frac{m}{m-1}E_1E_2\left(\hat{Q}_a^2\right)=\frac{m}{m-1}E_1E_2\left[\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right)^2\right]\doteq E_1E_2\left[\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right)^2\right] \tag{A.199}$$

with

$$E_2\left[\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right)^2\right]=Var_2\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right)+\left[E_2\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right)\right]^2=0+\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right)^2 \tag{A.200}$$

and

$$\begin{aligned}E_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right)^2&=Var_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right)+\left[E_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right)\right]^2\\&=\frac{1}{m}\sum_{i\in U}p_i\left(\frac{Q_{ia}}{p_i}-Q_a\right)^2+Q_a^2\\&=\frac{S_{Q_a(pwr)}^2}{m}+Q_a^2\end{aligned} \tag{A.201}$$

Taking the difference of C and D,

$$\begin{aligned}E_2E_3\left(\hat{S}_{Q_a(pwr)}^2\right)&\doteq\left(\sum_{i\in U}\frac{Q_{ia}^2}{p_i}-Q_a^2\right)-\frac{S_{Q_a(pwr)}^2}{m}\\&=S_{Q_a(pwr)}^2-\frac{S_{Q_a(pwr)}^2}{m}\\&=\frac{m-1}{m}S_{Q_a(pwr)}^2\\&\doteq S_{Q_a(pwr)}^2\,\square\end{aligned} \tag{A.202}$$

### A.5.4 Estimator of $S^2_{Q_{ia}}$ needed for Anticipated Variances

**Theorem 18.** *An approximately unbiased estimator of*

$$S^2_{Q_{ia}} = \frac{1}{N_{ia}-1}\sum_{j\in U_{ia}}\left(Q_{iaj}-\bar{Q}_{ia}\right)^2$$

is

$$\hat{S}^2_{Q_{ia}} = \hat{S}^2_{\mathrm{AQ}_{ia}} + \hat{S}^2_{\mathrm{B}Q_{ia}}$$

*where*

$$\hat{S}^2_{\mathrm{AQ}_{ia}} = \frac{n_{ia}}{n_{ia}-1}\frac{\sum_{j\in s_{ia}} w_{j|ia}\left(\hat{Q}_{iaj}-\hat{\bar{Q}}_{ia}\right)^2}{\sum_{j\in s_{ia}}\left(w_{j|ia}-1\right)} \quad \textit{and} \quad \hat{S}^2_{\mathrm{B}Q_{ia}} = \frac{1}{\hat{N}^2_{ia}}\frac{1}{n_{ia}}\hat{S}^2_{Q_{ia(pwr)}}.$$

*Proof.*

A biased estimator of $S^2_{Q_{ia}}$ is obtained by writing what would be the estimator of $S^2_{Q_{ia}}$ in a single-stage sample:

$$\hat{S}^2_{\mathrm{AQ}_{ia}} = \frac{n_{ia}}{n_{ia}-1}\frac{\sum_{j\in s_{ia}} w_{j|ia}\left(\hat{Q}_{iaj}-\hat{\bar{Q}}_{ia}\right)^2}{\sum_{j\in s_{ia}}\left(w_{j|ia}-1\right)} \tag{A.203}$$

where if third stage is SRS

$w_{j|ia} = \frac{1}{n_{ia}p_{j|ia}}$, $\hat{Q}_{iaj} = Q_{iaj}$, and $\hat{\bar{Q}}_{ia} = \frac{\hat{Q}_{ia}}{\hat{N}_{ia}}$ with $\hat{Q}_{ia} = \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}$, and $\hat{N}_{ia} = \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{1}{p_{j|ia}}$.

By expanding and simplifying Eq. (A.203) and using the fact that $w_{j|ia} = \left(n_{ia}p_{j|ia}\right)^{-1}$, we obtain an alternative expression for $\hat{S}^2_{\mathrm{AQ}_{ia}}$.

Expand,

$$\begin{aligned}
\sum_{j\in s_{ia}} w_{j|ia}\left(\hat{Q}_{iaj}-\hat{\bar{Q}}_{ia}\right)^2 &= \sum_{j\in s_{ia}} w_{j|ia}\left(\hat{Q}_{iaj}^2-2\hat{Q}_{iaj}\hat{\bar{Q}}_{ia}+\hat{\bar{Q}}_{ia}^2\right)\\
&=\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{p_{j|ia}}-2\hat{\bar{Q}}_{ia}\underbrace{\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}}{p_{j|ia}}\right)}_{pwr\text{ estimator of }Q_{ia}}+\hat{\bar{Q}}_{ia}^2\sum_{j\in s_{ia}}\frac{1}{n_{ia}p_{j|ia}}\\
&=\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{p_{j|ia}}-2\hat{\bar{Q}}_{ia}\hat{N}_{ia}\hat{\bar{Q}}_{ia}+\hat{\bar{Q}}_{ia}^2\hat{N}_{ia}\\
&=\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{p_{j|ia}}-\hat{N}_{ia}\hat{\bar{Q}}_{ia}^2\\
&=\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{p_{j|ia}}-\frac{\hat{Q}_{ia}^2}{\hat{N}_{ia}}.
\end{aligned}
\tag{A.204}$$

Then

$$\begin{aligned}
\hat{S}_{\mathrm{AQ}_{ia}}^2 &= \frac{n_{ia}}{n_{ia}-1}\frac{\sum_{j\in s_{ia}} w_{j|ia}\left(\hat{Q}_{iaj}-\hat{\bar{Q}}_{ia}\right)^2}{\sum_{j\in s_{iab}}\left(w_{j|ia}-1\right)}\\
&=\frac{n_{ia}}{n_{ia}-1}\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{p_{j|ia}}-\frac{\hat{Q}_{ia}^2}{\hat{N}_{ia}}\right)\frac{1}{\hat{N}_{ia}-n_{ia}}\\
&=\frac{n_{ia}}{n_{ia}-1}\frac{1}{\hat{N}_{ia}\left(1-\frac{n_{ia}}{\hat{N}_{ia}}\right)}\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{p_{j|ia}}-\frac{\hat{Q}_{ia}^2}{\hat{N}_{ia}}\right)\\
&\doteq\frac{n_{ia}}{n_{ia}-1}\frac{1}{\hat{N}_{ia}}\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}^2}{p_{j|ia}}-\frac{\hat{Q}_{ia}^2}{\hat{N}_{ia}}\right)\quad\text{if sampling }\frac{n_{ia}}{\hat{N}_{ia}}\text{ fraction is small}
\end{aligned}
\tag{A.205}$$

Rearranging and taking the expectation with respect to the sample design,

$$E_2E_3\left(\frac{n_{ia}-1}{n_{ia}}\hat{S}^2_{\mathrm{AQ}_{ia}}\right)=E_2E_3\left[\frac{1}{\hat{N}_{ia}}\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}^2_{iaj}}{p_{j|ia}}-\frac{\hat{Q}^2_{ia}}{\hat{N}_{ia}}\right)\right]$$

$$=E_2E_3\left(\frac{1}{\hat{N}_{ia}}\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}^2_{iaj}}{p_{j|ia}}\right)\right)-E_2E_3\left(\frac{1}{\hat{N}_{ia}}\frac{\hat{Q}^2_{ia}}{\hat{N}_{ia}}\right)$$

$$=F-G$$

(A.206)

Assuming that $n_{ia}$ is large and using the fact that $\hat{Q}_{iaj}=Q_{iaj}$, we solve for F as

$$F=E_2E_3\left(\frac{1}{\hat{N}_{ia}}\underbrace{\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}^2_{iaj}}{p_{j|ia}}\right)}_{pwr\text{ estimator of }\sum_{j\in U_{ia}}Q^2_{iaj}}\right)\doteq\frac{1}{N_{ia}}\sum_{j\in U_{ia}}Q^2_{iaj} \qquad \text{(A.207)}$$

Now solving for G,

$$G=E_2E_3\left(\frac{\hat{Q}^2_{ia}}{\hat{N}^2_{ia}}\right)\doteq E_2E_3\left(\frac{1}{\hat{N}^2_{ia}}\right)E_2E_3\left(\hat{Q}^2_{ia}\right)\ \doteq\frac{1}{N^2_{ia}}E_2E_3\left[\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}}{p_{j|ia}}\right)^2\right]$$

$$=\frac{1}{N^2_{ia}}E_2\left[\cancel{\underbrace{Var_3\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)}_{0}}+\left[E_3\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)\right]^2\right]$$

$$=\frac{1}{N^2_{ia}}\left[Var_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)+\left[E_2\underbrace{\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)}_{pwr\text{ estimator of }Q_{ia}}\right]^2\right]$$

$$=\frac{1}{N^2_{ia}}\left[\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}+Q^2_{ia}\right].$$

(A.208)

Substituting F and G back in Eq.(A.206),

$$
\begin{aligned}
E_2E_3\left(\frac{n_{ia}-1}{n_{ia}}\hat{S}^2_{\mathrm{AQ}_{ia}}\right) &\doteq \frac{1}{N_{ia}}\sum_{j\in U_{ia}} Q^2_{iaj} - \frac{1}{N^2_{ia}}\left(\frac{1}{n_{ia}}S^2_{\mathrm{Q}_{ia(pwr)}} + Q^2_{ia}\right) \\
&= \frac{1}{N_{ia}}\left(\sum_{j\in U_{ia}} Q^2_{iaj} - \frac{Q^2_{ia}}{N_{ia}}\right) - \frac{1}{N^2_{ia}}\frac{1}{n_{ia}}S^2_{\mathrm{Q}_{ia(pwr)}} \qquad \text{(A.209)} \\
&= S^2_{Q_{ia}} - \frac{1}{N^2_{ia}}\frac{1}{n_{ia}}S^2_{\mathrm{Q}_{ia(pwr)}}
\end{aligned}
$$

And when $n_{ia}/(n_{ia}-1)\approx 1$

$$E\left(\hat{S}^2_{\mathrm{AQ}_{ia}}\right) \doteq S^2_{Q_{ia}} - \frac{1}{N^2_{ia}}\frac{1}{n_{ia}}S^2_{Q_{ia}(pwr)} \qquad \text{(A.210)}$$

Define

$$S^2_{\mathrm{B}Q_{ia}} = \frac{1}{N^2_{ia}}\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}} \qquad \text{(A.211)}$$

Then substituting Eq. (A.211) into Eq. (A.210), we obtain

$$E\left(\hat{S}^2_{\mathrm{AQ}_{ia}}\right) = S^2_{Q_{ia}} - S^2_{\mathrm{B}Q_{ia}} \qquad \text{(A.212)}$$

The bias of $\hat{S}^2_{\mathrm{AQ}_{ia}}$ is $-S^2_{\mathrm{B}Q_{ia}}$ and an unbiased estimator of the bias, $-S^2_{2\mathrm{B}ia}$, is

$$-\hat{S}^2_{\mathrm{B}Q_{ia}} = -\frac{1}{\hat{N}^2_{ia}}\frac{1}{n_{ia}}\hat{S}^2_{Q_{ia(pwr)}} \qquad \text{(A.213)}$$

To form an unbiased estimator of $S^2_{Q_{ia}}$, we subtract an unbiased estimator of the bias,

$-\hat{S}^2_{\mathrm{B}Q_{ia}}$, from the biased estimator of $S^2_{Q_{ia}}$, $\hat{S}^2_{\mathrm{AQ}_{ia}}$, and obtain

$$\hat{S}^2_{Q_{ia}} = \hat{S}^2_{\mathrm{AQ}_{ia}} - \left(-\hat{S}^2_{\mathrm{B}Q_{ia}}\right) = \hat{S}^2_{\mathrm{AQ}_{ia}} + \hat{S}^2_{\mathrm{B}Q_{ia}} \qquad \text{(A.214)}$$

such that

$$E\left(\hat{S}^2_{Q_{ia}}\right) = E\left(\hat{S}^2_{\mathrm{AQ}_{ia}} + \hat{S}^2_{\mathrm{B}Q_{ia}}\right) = \left(S^2_{Q_{ia}} - S^2_{\mathrm{B}Q_{ia}}\right) + S^2_{\mathrm{B}Q_{ia}} = S^2_{Q_{ia}} \ \square \qquad \text{(A.215)}$$

### A.5.5 Estimator of $S_Q^2$ needed for Anticipated Variances

**Theorem 20.** *An approximately unbiased estimator of*

$$S_Q^2 = \frac{1}{M-1}\sum_{i\in U}\left(Q_i - \bar{Q}\right)^2$$

*is*

$$\hat{S}_Q^2 = \hat{S}_{AQ}^2 + \hat{S}_{BQ}^2$$

*where*

$$\hat{S}_{AQ}^2 = \frac{m}{m-1}\frac{\sum_{j\in s_1} w_i\left(\hat{Q}_i - \hat{\bar{Q}}\right)^2}{\sum_{j\in s_1}\left(w_i - 1\right)} \quad \textit{and}$$

$$\hat{S}_{BQ}^2 = \frac{1}{\hat{M}^2}\frac{1}{m}\hat{S}_{Q(pwr)}^2 - \sum_{i\in s_1}\frac{1}{mp_i}\sum_a \frac{\hat{S}_{Q_{ia(pwr)}}^2}{\hat{M}n_{ia}}\left(1 - \frac{1}{\hat{M}mp_i}\right)$$

*with*

$$w_i = \frac{1}{mp_i}\ ,\ \hat{Q}_i = \sum_a \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}} = \sum_a \hat{Q}_{ia}\ ,\ \ \hat{\bar{Q}} = \frac{\hat{Q}}{\hat{M}}\ ,\ \ \hat{Q} = \frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_i}{p_i}\ ,\ \ \hat{M} = \frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}.$$

Proof.

We will show that $\hat{S}_Q^2 = \hat{S}_{AQ}^2 + \hat{S}_{BQ}^2$ is an unbiased estimator of

$S_Q^2 = \frac{1}{M-1}\sum_{i\in U}\left(Q_i - \bar{Q}\right)^2$. A biased estimator of $S_Q^2$ is obtained by writing what would be

the estimator of $S_Q^2$ in a single-stage sample:

$$\hat{S}^2_{\mathrm{AQ}} = \frac{m}{m-1} \frac{\sum_{i \in s_1} w_i \left( \hat{Q}_i - \hat{\bar{Q}} \right)^2}{\sum_{i \in s_1} \left( w_i - 1 \right)} \tag{A.216}$$

By expanding and simplifying Eq. (A.216) and using the fact that $w_i = \left( mp_i \right)^{-1}$, we obtain an alternative expression for $\hat{S}^2_{\mathrm{AQ}}$. First expand,

$$\begin{aligned}
\sum_{i \in s_1} w_i \left( \hat{Q}_i - \hat{\bar{Q}} \right)^2 &= \sum_{i \in s_1} w_i \left( \hat{Q}_i^2 - 2\hat{Q}_i \hat{\bar{Q}} + \hat{\bar{Q}}^2 \right) \\
&= \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_i^2}{p_i} - 2\hat{\bar{Q}} \underbrace{\left( \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_i}{p_i} \right)}_{pwr \text{ estimator of } Q} + \hat{\bar{Q}}^2 \sum_{i \in s_1} \frac{1}{mp_i} \\
&= \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_i^2}{p_i} - \hat{M} \hat{\bar{Q}}^2 \\
&= \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_i^2}{p_i} - \frac{\hat{Q}^2}{\hat{M}}
\end{aligned}$$

Then

$$\hat{S}^2_{\mathrm{AQ}} = \frac{m}{m-1} \frac{\sum_{i \in s_1} w_i \left( \hat{Q}_i - \hat{\bar{Q}} \right)^2}{\sum_{j \in s_{iab}} \left( w_{j|ia} - 1 \right)}$$

$$= \frac{m}{m-1} \left( \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_i^2}{p_i} - \frac{\hat{Q}^2}{\hat{M}} \right) \frac{1}{\hat{M} - m}$$

$$= \frac{m}{m-1} \frac{1}{\hat{M} \left( 1 - \frac{m}{\hat{M}} \right)} \left( \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_i^2}{p_i} - \frac{\hat{Q}^2}{\hat{M}} \right)$$

$$= \frac{m}{m-1} \frac{1}{\hat{M}} \left( \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_i^2}{p_i} - \frac{\hat{Q}^2}{\hat{M}} \right) \quad \text{if sampling } \frac{m}{\hat{M}} \text{ fraction is small}$$

Rearranging and taking the expectation with respect to the sample design,

$$E_1E_2\left(\frac{m-1}{m}\hat{S}^2_{\text{AQ}}\right)=E_1E_2\left[\frac{1}{\hat{M}}\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_i^2}{p_i}-\frac{\hat{Q}^2}{\hat{M}}\right)\right]$$

$$=E_1E_2\left(\frac{1}{\hat{M}}\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_i^2}{p_i}\right)\right)-E_1E_2\left(\frac{1}{\hat{M}}\frac{\hat{Q}^2}{\hat{M}}\right) \qquad \text{(A.217)}$$

$$=F-G$$

Assuming that $m$ is large and using the fact that $\hat{Q}_i=\sum_a \hat{Q}_{ia}$, we solve for F as

$$F=E_1E_2\left[\frac{1}{\hat{M}}\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\hat{Q}_i^2\right)\right]$$

$$=E_1\left[\frac{1}{\hat{M}}\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}E_2\left(\hat{Q}_i^2\right)\right]$$

$$=E_1\left[\frac{1}{\hat{M}}\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left\{Var_2\left(\hat{Q}_i\right)+\left[E_2\left(\hat{Q}_i\right)\right]^2\right\}\right]\hat{Q}_i=$$

$$=E_1\left[\frac{1}{\hat{M}}\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left\{Var_2\left(\sum_a\hat{Q}_{ia}\right)+\left[E_2\left(\sum_a\hat{Q}_{ia}\right)\right]^2\right\}\right]$$

$$=E_1\left[\frac{1}{\hat{M}}\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left\{\sum_a Var_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)+\left[\sum_a E_2\left(\underbrace{\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}}_{pwr\text{ estimate of }Q_{ia}}\right)\right]^2\right\}\right]$$

$$=E_1\left[\frac{1}{\hat{M}}\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\left\{\sum_a\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}+\left[\sum_a Q_{ia}\right]^2\right\}\right]$$

$$=E_1\left[\frac{1}{\hat{M}}\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}+\frac{1}{\hat{M}}\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}Q_i^2\right]$$

$$
\begin{aligned}
F &= E_1\left[\frac{1}{\hat{M}}\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}+\frac{1}{\hat{M}}\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}Q_i^2\right] \\
&\doteq \frac{1}{M}\sum_{i\in U}\sum_a\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}+\frac{1}{M}\sum_{i\in U}Q_i^2 \\
&\doteq \frac{1}{M}\sum_{i\in U}\sum_a\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}+\frac{1}{M}M\left(S_Q^2+\bar{Q}^2\right) \quad \textit{by}\text{ Lemma 1i and M} \approx \text{M-1} \\
&= S_Q^2+\bar{Q}^2+\frac{1}{M}\sum_{i\in U}\sum_a\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}
\end{aligned}
$$

(A.218)

Now solving for G,

$$
\begin{aligned}
G &= E_1E_2\left(\frac{1}{\hat{M}}\frac{\hat{Q}^2}{\hat{M}}\right)=E_1E_2\left(\frac{1}{\hat{M}^2}\right)E_2E_3\left(\hat{Q}^2\right) \\
&\doteq \frac{1}{M^2}E_1E_2\left[\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)^2\right] \\
&= \frac{1}{M^2}E_1\left[\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a Var_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)+\left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_a E_2\left(\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q_{iaj}}{p_{j|ia}}\right)\right]^2\right] \\
&= \frac{1}{M^2}E_1\left[\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\sum_a\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}+\left(\frac{1}{m}\sum_{i\in s_1}\frac{Q_i}{p_i}\right)^2\right] \\
&= \frac{1}{M^2}\left[\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_a\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}+Var_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{Q_i}{p_i}\right)+\left\{E_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{Q_i}{p_i}\right)\right\}^2\right] \\
&= \frac{1}{M^2}\left[\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_a\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}+\frac{1}{m}\sum_{i\in U}p_i\left(\frac{Q_i}{p_i}-Q\right)^2+Q^2\right] \\
&= \frac{1}{M^2}\left[\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_a\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}+\frac{1}{m}S^2_{Q(pwr)}+Q^2\right]
\end{aligned}
$$

(A.219)

Substituting $F$ and $G$ back into Eq.(A.217),

$$
\begin{aligned}
E_1E_2\left(\frac{m-1}{m}\hat{S}^2_{\mathrm{AQ}}\right) &= F-G \\
&\doteq S^2_Q+\bar{Q}^2+\frac{1}{M}\sum_{i\in U}\sum_{a}\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}} \\
&\quad -\frac{1}{M^2}\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\sum_{a}\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}-\frac{1}{M^2}\frac{1}{m}S^2_{Q_{(pwr)}}-\frac{Q^2}{M^2} \\
&= S^2_Q-\frac{1}{M^2}\frac{1}{m}S^2_{Q_{(pwr)}}+\cancel{\bar{Q}^2-\bar{Q}^2}+\frac{1}{M}\sum_{i\in U}\sum_{a}\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}\left(1-\frac{1}{Mmp_i}\right) \\
&= S^2_Q-\frac{1}{M^2}\frac{1}{m}S^2_{Q_{(pwr)}}+\frac{1}{M}\sum_{i\in U}\sum_{a}\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}\left(1-\frac{1}{Mmp_i}\right)
\end{aligned}
$$

(A.220)

And when $m/(m-1)\approx 1$

$$E\left(\hat{S}^2_{\mathrm{A}Q}\right)\doteq S^2_Q-\left[\frac{1}{M^2}\frac{1}{m}S^2_{Q_{(pwr)}}-\frac{1}{M}\sum_{i\in U}\sum_{a}\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}\left(1-\frac{1}{Mmp_i}\right)\right] \qquad \text{(A.221)}$$

Define

$$S^2_{\mathrm{B}Q}=\frac{1}{M^2}\frac{1}{m}S^2_{Q_{(pwr)}}-\frac{1}{M}\sum_{i\in U}\sum_{a}\frac{1}{n_{ia}}S^2_{Q_{ia(pwr)}}\left(1-\frac{1}{Mmp_i}\right) \qquad \text{(A.222)}$$

Then substituting Eq. (A.222) into Eq. (A.221) we obtain

$$E\left(\hat{S}^2_{\mathrm{A}Q}\right)\doteq S^2_Q-S^2_{\mathrm{B}Q} \qquad \text{(A.223)}$$

Since the bias of $\hat{S}^2_{\mathrm{A}Q_{1a}}$ is $-S^2_{\mathrm{B}Q}$ and an unbiased estimator of the bias, $-S^2_{\mathrm{B}Q}$, is

$$-\hat{S}^2_{\mathrm{B}Q}=-\left[\frac{1}{\hat{M}^2}\frac{1}{m}\hat{S}^2_{Q_{(pwr)}}-\sum_{i\in s_1}\frac{1}{mp_i}\sum_{a}\frac{\hat{S}^2_{Q_{ia(pwr)}}}{\hat{M}n_{ia}}\left(1-\frac{1}{\hat{M}mp_i}\right)\right]$$

To form an approximately unbiased estimator of $S_Q^2$ , we subtract an unbiased estimator of the bias, $-\hat{S}_{\mathrm{B}Q}^2$, from the biased estimator, $\hat{S}_{\mathrm{A}Q}^2$, and obtain

$$
\begin{aligned}
\hat{S}_Q^2 &= \hat{S}_{\mathrm{A}Q}^2 - \left(-\hat{S}_{\mathrm{B}Q}^2\right) \\
&= \hat{S}_{\mathrm{A}Q}^2 + \hat{S}_{\mathrm{B}Q}^2
\end{aligned}
\tag{A.224}
$$

such that $E\left(\hat{S}_Q^2\right) \doteq E\left(\hat{S}_{\mathrm{A}Q}^2\right) + E\left(\hat{S}_{\mathrm{B}Q}^2\right) = \left(S_Q^2 - \hat{S}_{\mathrm{B}Q}^2\right) + \hat{S}_{\mathrm{B}Q}^2 = S_Q^2 \square$

## A.5.6 Estimator of $S^2_{Q_{1a}}$ needed for Anticipated Variances

**Theorem 21.** *An approximately unbiased estimator of*

$$S^2_{Q_{1a}} = \frac{1}{M-1}\sum_{i\in U}\left(Q_{ia} - \bar{Q}_{1a}\right)^2$$

is

$$\hat{S}^2_{Q_{1a}} = \hat{S}^2_{\mathrm{A}Q_{1a}} + \hat{S}^2_{\mathrm{B}Q_{1a}}$$

*where*

$$\hat{S}^2_{\mathrm{A}Q_{1a}} = \frac{m}{m-1}\frac{\sum_{i\in s_1} w_i\left(\hat{Q}_{ia} - \hat{\bar{Q}}_{1a}\right)^2}{\sum_{i\in s_1}\left(w_i - 1\right)}$$

*and*

$$\hat{S}^2_{\mathrm{B}Q_{1a}} = \frac{1}{\hat{M}^2}\frac{1}{m}\hat{S}^2_{Q_{a(pwr)}} - \sum_{i\in U}\frac{\hat{S}^2_{Q_{ia(pwr)}}}{\hat{M}n_{ia}}\left(1 - \frac{1}{\hat{M}mp_i}\right).$$

$$\hat{S}^2_{\mathrm{B}Q_{1a}} = \frac{1}{\hat{M}^2}\frac{1}{m}\hat{S}^2_{Q_{a(pwr)}} - \sum_{i\in s_1}\frac{1}{mp_i}\frac{\hat{S}^2_{Q_{ia(pwr)}}}{\hat{M}n_{ia}}\left(1 - \frac{1}{\hat{M}mp_i}\right)$$

*Proof*.

A biased estimator of $S^2_{Q_{1a}}$ can be obtained by writing down an estimator in a single stage sample.

$$\hat{S}^2_{\mathrm{A}Q_{1a}} = \frac{m}{m-1}\frac{\sum_{i\in s_1} w_i\left(\hat{Q}_{ia} - \hat{\bar{Q}}_{1a}\right)^2}{\sum_{i\in s_1}\left(w_i - 1\right)} \tag{A.225}$$

where,

$$w_i = \frac{1}{mp_i}, \quad \hat{Q}_{ia} = \frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{\hat{Q}_{iaj}}{p_{j|ia}}, \quad \hat{\bar{Q}}_{1a} = \frac{\hat{Q}_a}{\hat{M}}, \quad \hat{Q}_a = \frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}.$$

Expanding,

$$
\begin{aligned}
\sum_{i \in s_1} w_i \left( \hat{Q}_{ia} - \hat{\hat{Q}}_{1a} \right)^2 &= \sum_{i \in s_1} w_i \left( \hat{Q}_{ia} - \hat{\hat{Q}}_{1a} \right)^2 \\
&= \sum_{i \in s_1} \frac{1}{mp_i} \left( \hat{Q}_{ia}^2 - 2\hat{Q}_{ia}\hat{\hat{Q}}_{1a} + \hat{\hat{Q}}_{1a}^2 \right) \\
&= \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_{ia}^2}{p_i} - 2\frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_{ia}}{p_i} \hat{\hat{Q}}_{1a} + \frac{1}{m} \sum_{i \in s_1} \frac{\hat{\hat{Q}}_{1a}^2}{p_i} \\
&= \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_{ia}^2}{p_i} - 2\hat{Q}_a \hat{\hat{Q}}_{1a} + \hat{M}\hat{\hat{Q}}_{1a}^2 \\
&= \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_{ia}^2}{p_i} - 2\hat{M}\hat{\hat{Q}}_{1a}^2 + \hat{M}\hat{\hat{Q}}_{1a}^2 \\
&= \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_{ia}^2}{p_i} - \hat{M}\hat{\hat{Q}}_{1a}^2
\end{aligned}
\tag{A.226}
$$

And

$$
\begin{aligned}
\sum_{i \in s_1} (w_i - 1) &= \sum_{i \in s_1} \left( \frac{1}{mp_i} - 1 \right) \\
&= \sum_{i \in s_1} \frac{1}{mp_i} - \sum_{i \in s_1} 1 \\
&= \hat{M} - m \\
&= \hat{M} \left( 1 - \cancel{\frac{m}{\hat{M}}} \right) \quad \text{if } m/\hat{M} \text{ is small} \\
&\doteq \hat{M}
\end{aligned}
\tag{A.227}
$$

Substituting Eq. (A.226) and Eq. (A.227) back into Eq. (A.225) gives us,

$$
\begin{aligned}
\hat{S}^2_{\mathbb{A}Q_{1a}} &= \frac{m}{m-1} \frac{\sum_{i \in s_1} w_i \left( \hat{Q}_{ia} - \hat{\hat{Q}}_{1a} \right)^2}{\sum_{i \in s_1} (w_i - 1)} \\
&\doteq \frac{m}{m-1} \frac{1}{\hat{M}} \left( \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}_{ia}^2}{p_i} - \hat{M}\hat{\hat{Q}}_{1a}^2 \right)
\end{aligned}
\tag{A.228}
$$

Rearranging and taking the expectation with respect to the sample design,

$$E_1 E_2 \left( \frac{m-1}{m} \hat{S}^2_{\mathbb{A}Q_{1a}} \right) = E_1 E_2 \left[ \frac{1}{\hat{M}} \left( \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}^2_{ia}}{p_i} - \hat{M} \hat{\bar{Q}}^2_{1a} \right) \right]$$

$$= E_1 E_2 \left( \frac{1}{\hat{M}} \left( \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}^2_{ia}}{p_i} \right) \right) - E_1 E_2 \left( \hat{\bar{Q}}^2_{1a} \right) \qquad \text{(A.229)}$$

$$= H - J$$

Assuming that $m$ is large and using the fact that $\hat{Q}_{iajb} = Q_{iajb}$ when the third stage sample is SRS, we solve for $H$ as

$$H = E_1 E_2 \left( \frac{1}{\hat{M}} \underbrace{\left( \frac{1}{m} \sum_{i \in s_1} \frac{\hat{Q}^2_{ia}}{p_i} \right)}_{pwr \text{ estimator of } \sum_{i \in U} Q^2_{ia}} \right)$$

$$= E_1 \left( \frac{1}{\hat{M}} \frac{1}{m} \sum_{i \in s_1} \frac{1}{p_i} E_2 \left( \hat{Q}^2_{ia} \right) \right)$$

$$= E_1 \left( \frac{1}{\hat{M}} \frac{1}{m} \sum_{i \in s_1} \frac{1}{p_i} \left[ Var_2 \left( \hat{Q}_{ia} \right) + \left[ E_2 \left( \hat{Q}_{ia} \right)^2 \right] \right] \right)$$

$$= E_1 \left( \frac{1}{\hat{M}} \frac{1}{m} \sum_{i \in s_1} \frac{1}{p_i} \left[ Var_2 \left( \frac{1}{n_{ia}} \sum_{j \in s_{ia}} \frac{Q_{iaj}}{p_{j|ia}} \right) + Q^2_{ia} \right] \right)$$

$$= E_1 \left( \frac{1}{\hat{M}} \frac{1}{m} \sum_{i \in s_1} \frac{1}{p_i} \left[ \frac{1}{n_{ia}} S^2_{Q_{ia(pwr)}} + Q^2_{ia} \right] \right) \qquad \text{(A.230)}$$

$$\doteq \frac{1}{M} \left( \sum_{i \in U} \frac{1}{n_{ia}} S^2_{Q_{ia(pwr)}} + \sum_{i \in U} Q^2_{ia} \right)$$

Solving for J,

$$
\begin{aligned}
J &= E_1 E_2\left(\hat{\bar{Q}}_{1a}^2\right) \doteq E_1\left(E_2\left(\frac{1}{\hat{M}^2}\right)\right) E_1\left(E_2\left(\hat{Q}_a^2\right)\right) \\
&\doteq \frac{1}{M^2} E_1\left(Var_2\left(\hat{Q}_a\right) + E_2\left(\hat{Q}_a\right)^2\right) \\
&= \frac{1}{M^2} E_1\left(Var_2\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right) + \left[E_2\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}_{ia}}{p_i}\right)\right]^2\right) \\
&= \frac{1}{M^2} E_1\left(\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2} Var_2\left(\hat{Q}_{ia}\right) + \left[\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i} E_2\left(\hat{Q}_{ia}\right)\right]^2\right) \\
&= \frac{1}{M^2} E_1\left(\left(\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\ \frac{1}{n_{ia}} S^2_{Q_{ia,\ pwr}}\right) + \left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i} Q_{ia}\right)^2\right) \textit{by}\ \text{pg } 235 \\
&= \frac{1}{M^2}\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia\,(pwr)}}}{n_{ia}} + \frac{1}{M^2} Var_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i} Q_{ia}\right) + \frac{1}{M^2}\left[E_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i} Q_{ia}\right)\right]^2 \\
&= \frac{1}{M^2}\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia\,(pwr)}}}{n_{ia}} + \frac{1}{M^2} Var_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i} Q_{ia}\right) + \frac{1}{M^2} Q_a^2 \\
&= \frac{1}{M^2}\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia\,(pwr)}}}{n_{ia}} + \frac{1}{M^2}\frac{1}{m}\sum_{i\in U} p_i\left(\frac{Q_{ia}}{p_i} - Q_a\right)^2 + \frac{1}{M^2} Q_a^2 \\
&= \frac{1}{M^2}\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia\,(pwr)}}}{n_{ia}} + \frac{1}{M^2}\frac{1}{m} S^2_{Q_{a\,(pwr)}} + \bar{Q}_{1a}^2
\end{aligned}
\tag{A.231}
$$

Substituting $H$ and $J$ back into Eq. (A.229),

$$E_1E_2\left(\frac{m-1}{m}\hat{S}^2_{\mathbb{A}Q_{1a}}\right)=E_1E_2\left(\frac{1}{m}\sum_{i\in s_1}\frac{\hat{Q}^2_{ia}}{p_i}\right)-E_1E_2\left(\hat{\bar{Q}}^2_{1a}\right)$$

$$=\frac{1}{M}\sum_{i\in U}\frac{S^2_{Q_{ia(pwr)}}}{n_{ia}}+\frac{1}{M}\sum_{i\in U}Q^2_{ia}-\frac{1}{M^2}\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia(pwr)}}}{n_{ia}}-\frac{1}{M^2}\frac{1}{m}S^2_{Q_{a(pwr)}}-\bar{Q}^2_{1a}$$

$$=\left[\frac{1}{M}\sum_{i\in U}Q^2_{ia}-\bar{Q}^2_{1a}\right]+\frac{1}{M}\sum_{i\in U}\frac{S^2_{Q_{ia(pwr)}}}{n_{ia}}-\frac{1}{M^2}\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia(pwr)}}}{n_{ia}}-\frac{1}{M^2}\frac{1}{m}S^2_{Q_{a(pwr)}}$$

$$=S^2_{Q_{1a}}+\sum_{i\in U}\frac{S^2_{Q_{ia(pwr)}}}{Mn_{ia}}\left(1-\frac{1}{Mmp_i}\right)-\frac{1}{M^2}\frac{1}{m}S^2_{Q_{a(pwr)}}$$

$$=S^2_{Q_{1a}}-\left[\frac{1}{M^2}\frac{1}{m}S^2_{Q_{a(pwr)}}-\sum_{i\in U}\frac{S^2_{Q_{ia(pwr)}}}{Mn_{ia}}\left(1-\frac{1}{Mmp_i}\right)\right]$$

(A.232)

And when $m/(m-1)\approx 1$

$$E\left(\hat{S}^2_{\mathbb{A}Q_{1a}}\right)\doteq S^2_{Q_{1a}}-\left[\frac{1}{M^2}\frac{1}{m}S^2_{Q_{a(pwr)}}-\sum_{i\in U}\frac{S^2_{Q_{ia(pwr)}}}{Mn_{ia}}\left(1-\frac{1}{Mmp_i}\right)\right] \quad \text{(A.233)}$$

Define

$$S^2_{\mathbb{B}Q_{1a}}=\frac{1}{M^2}\frac{1}{m}S^2_{Q_{a(pwr)}}-\sum_{i\in U}\frac{S^2_{Q_{ia(pwr)}}}{Mn_{ia}}\left(1-\frac{1}{Mmp_i}\right) \quad \text{(A.234)}$$

Then substituting Eq. (A.234) into Eq. (A.233), we obtain

$$E\left(\hat{S}^2_{\mathbb{A}Q_{1a}}\right)\doteq S^2_{Q_{1a}}-S^2_{\mathbb{B}Q_{1a}} \quad \text{(A.235)}$$

The bias of $\hat{S}^2_{\mathrm{A}Q_{1a}}$ is $-S^2_{\mathrm{B}Q_{1a}}$ and an unbiased estimator of the bias, $-S^2_{\mathrm{B}Q_{1a}}$, is

$$-\hat{S}^2_{\mathrm{B}Q_{1a}} = -\left[\frac{1}{\hat{M}^2}\frac{1}{m}\hat{S}^2_{Q_{a(pwr)}} - \sum_{i\in U}\frac{\hat{S}^2_{Q_{ia(pwr)}}}{\hat{M}n_{ia}}\left(1-\frac{1}{\hat{M}mp_i}\right)\right] \qquad \text{(A.236)}$$

To form an approximately unbiased estimator of $S^2_{Q_{1a}}$, we subtract an unbiased estimator of the bias, $-\hat{S}^2_{\mathrm{B}1Q_a}$, from the biased estimator of $S^2_{1Q_a}$, $\hat{S}^2_{\mathrm{A}1Q_a}$, and obtain

$$\begin{aligned}\hat{S}^2_{Q_{1a}} &= \hat{S}^2_{\mathrm{A}Q_{1a}} - \left(-\hat{S}^2_{\mathrm{B}Q_{1a}}\right) \\ &= \hat{S}^2_{\mathrm{A}Q_{1a}} + \hat{S}^2_{\mathrm{B}Q_{1a}}\end{aligned} \qquad \text{(A.237)}$$

such that $E\left(\hat{S}^2_{Q_{1a}}\right) \doteq E\left(\hat{S}^2_{\mathrm{A}Q_{1a}}\right) + E\left(\hat{S}^2_{\mathrm{B}Q_{1a}}\right) = \left(S^2_{Q_{1a}} - \hat{S}^2_{\mathrm{B}Q_{1a}}\right) + \hat{S}^2_{\mathrm{B}Q_{1a}} = S^2_{Q_{1a}}$ □

## A.5.7 Estimator of $S^2_{Q_{2a}}$ needed for Anticipated Variances

**Theorem 22.** *An approximately unbiased estimator of*

$$S^2_{Q_{2a}} = \frac{1}{N_a - 1} \sum_{i \in U} \sum_{j \in U_{ia}} \left( Q_{iaj} - \bar{Q}_{2a} \right)^2 \qquad \textit{where } \bar{Q}_{2a} = \frac{Q_a}{N_a}$$

$$= \frac{1}{N_a - 1} \left[ \sum_{i \in U} \sum_{j \in U_{ia}} Q^2_{iaj} - \frac{Q^2_a}{N_a} \right]$$

is

$$\hat{S}^2_{Q_{2a}} = \hat{S}^2_{\mathbb{A}Q_{2a}} + \hat{S}^2_{\mathbb{B}Q_{2a}}$$

*where*

$$\hat{S}^2_{\mathbb{A}Q_{2a}} = \frac{1}{\hat{N}_a - 1} \sum_{i \in s_1} \frac{1}{mp_i} \sum_{j \in s_{ia}} \frac{\hat{Q}^2_{iaj}}{n_{ia} p_{j|ia}} - \hat{N}_a \hat{\bar{Q}}^2_{2a},$$

$$\hat{S}^2_{\mathbb{B}Q_{2a}} = \frac{1}{\hat{N}^2_a} \frac{1}{m} \hat{S}^2_{Q_{a(pwr)}} + \frac{1}{\hat{N}^2_a} \frac{1}{m^2} \sum_{i \in s_1} \frac{1}{p^2_i} \frac{\hat{S}^2_{Q_{ia,pwr}}}{n_{ia}},$$

*and*

$$\hat{N}_a = \sum_{i \in s_1} \frac{1}{mp_i} \sum_{j \in s_{ia}} \frac{1}{n_{ia} p_{j|ia}}.$$

*Proof.*

A biased estimator of $S^2_{Q_{2a}}$ can be obtained by writing down an estimator in a single stage sample.

$$\hat{S}^2_{\mathbb{A}Q_{2a}} = \frac{1}{\hat{N}_a - 1} \left[ \sum_{i \in s_1} \sum_{j \in s_{ia}} w_i w_{j|ia} \hat{Q}^2_{iaj} - \hat{N}_a \hat{\bar{Q}}^2_{2a} \right] \tag{A.238}$$

where $w_i = \dfrac{1}{mp_i}$ $\quad w_{j|ia} = \dfrac{1}{n_{ia} p_{j|ia}}$ $\quad \hat{Q}_{iaj} = \sum_b \hat{Q}_{iajb} = Q_{iaj}$, $\quad \hat{\bar{Q}}_{2a} = \dfrac{\hat{Q}_a}{\hat{N}_a}$, , and

$$\hat{Q}_{ia} = \sum_{j \in s_{ia}} \frac{Q_{iaj}}{n_{ia} p_{j|ia}}.$$

Taking the expectation with respect to the sample design,

$$E_1 E_2\left(\hat{S}^2_{\text{A}Q_{2a}}\right) = E_1 E_2\left(\frac{1}{\hat{N}_a - 1}\sum_{i\in s_1}\sum_{j\in s_{ia}} w_i w_{j|ia}\hat{Q}^2_{iaj}\right) - E_1 E_2\left(\frac{\hat{N}_a}{\hat{N}_a - 1}\hat{\bar{Q}}^2_{2a}\right) \tag{A.239}$$

$$= K - L$$

Assuming that $m$ is large, $E_1 E_2\left[\left(\hat{N}_a - 1\right)^{-1}\right] \doteq \left(N_a - 1\right)^{-1}$, and using the fact that $\hat{Q}_{iajb} = Q_{iajb}$ when the third stage sample is SRS, we solve for *K* as

$$K \doteq \frac{1}{N_a - 1} E_1 E_2\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\frac{1}{n_{ia}}\sum_{j\in s_{ia}}\frac{Q^2_{iaj}}{p_{j|ia}}\right)$$

$$= \frac{1}{N_a - 1} E_1\left(\frac{1}{m}\sum_{i\in s_1}\frac{1}{p_i}\sum_{j\in U_{ia}} Q^2_{iaj}\right) \tag{A.240}$$

$$= \frac{1}{N_a - 1}\sum_{i\in U}\sum_{j\in U_{ia}} Q^2_{iaj}$$

Using results from Eq.(A.231), we solve for *L* as

$$L = E_1 E_2\left(\frac{\hat{N}_a}{\hat{N}_a - 1}\hat{\bar{Q}}^2_{2a}\right) = E_1 E_2\left(\frac{\hat{N}_a}{\hat{N}_a - 1}\frac{\hat{Q}^2_a}{\hat{N}^2_a}\right)$$

$$\doteq E_1 E_2\left(\frac{1}{\hat{N}_a\left(\hat{N}_a - 1\right)}\right) E_1\left(E_2\left(\hat{Q}^2_a\right)\right)$$

$$\doteq \frac{1}{N_a\left(N_a - 1\right)} E_1\left(Var_2\left(\hat{Q}_a\right) + E_2\left(\hat{Q}_a\right)^2\right)$$

$$= \frac{1}{N_a\left(N_a - 1\right)}\left(\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia(pwr)}}}{n_{ia}} + \frac{1}{m}S^2_{Q_{a(pwr)}} + Q^2_a\right) \tag{A.241}$$

Substituting $K$ and $L$ back into Eq. (A.239)

$$E_1E_2\left(\hat{S}^2_{\text{A}Q_{2a}}\right)=\frac{1}{N_a-1}\sum_{i\in U}\sum_{j\in U_{ia}}Q^2_{iaj}-\frac{1}{N_a\left(N_a-1\right)}\left(\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia(pwr)}}}{n_{ia}}+\frac{1}{m}S^2_{Q_{a(pwr)}}+Q^2_a\right)$$

$$=\frac{1}{N_a-1}\left(\sum_{i\in U}\sum_{j\in U_{ia}}Q^2_{iaj}-\frac{Q^2_a}{N_a}\right)-\frac{1}{N_a\left(N_a-1\right)}\left(\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia(pwr)}}}{n_{ia}}+\frac{1}{m}S^2_{Q_{a(pwr)}}\right)$$

$$=S^2_{Q_{2a}}-\frac{1}{N_a\left(N_a-1\right)}\left(\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia(pwr)}}}{n_{ia}}+\frac{1}{m}S^2_{Q_{a(pwr)}}\right)$$

(A.242)

And when $N_a\approx N_a-1$

$$E_1E_2\left(\hat{S}^2_{\text{A}Q_{2a}}\right)\doteq S^2_{Q_{2a}}-\left(\frac{1}{N_a^2}\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia(pwr)}}}{n_{ia}}+\frac{1}{N_a^2}\frac{1}{m}S^2_{Q_{a(pwr)}}\right) \quad \text{(A.243)}$$

Define

$$S^2_{\text{B}Q_{2a}}=\frac{1}{N_a^2}\frac{1}{m}S^2_{Q_{a(pwr)}}+\frac{1}{N_a^2}\frac{1}{m}\sum_{i\in U}\frac{1}{p_i}\frac{S^2_{Q_{ia(pwr)}}}{n_{ia}} \quad \text{(A.244)}$$

Then substituting Eq .(A.244) into Eq. (A.243) we obtain

$$E\left(\hat{S}^2_{\text{A}Q_{2a}}\right)\doteq S^2_{Q_{2a}}-S^2_{\text{B}Q_{2a}} \quad \text{(A.245)}$$

The bias of $\hat{S}^2_{\text{A}Q_{2a}}$ is $Bias\left(\hat{S}^2_{\text{A}Q_{2a}}\right)=-S^2_{\text{B}Q_{2a}}$

and an unbiased estimator of the bias, $-S^2_{\text{B}Q_{2a}}$, is

$$-\hat{S}^2_{\text{B}Q_{2a}}=-\left(\frac{1}{\hat{N}_a^2}\frac{1}{m}\hat{S}^2_{Q_{a(pwr)}}+\frac{1}{\hat{N}_a^2}\frac{1}{m^2}\sum_{i\in s_1}\frac{1}{p_i^2}\frac{\hat{S}^2_{Q_{ia(pwr)}}}{n_{ia}}\right)$$

To form an approximately unbiased estimator of $S_{Q_{2a}}^2$, we subtract an approximately unbiased estimator of the bias, $-\hat{S}_{\mathbb{B}Q_{2a}}^2$, from the biased estimator of $S_{Q_{2a}}^2$, $\hat{S}_{\mathbb{A}Q_{2a}}^2$, and obtain

$$
\begin{aligned}
\hat{S}_{Q_{2a}}^2 &= \hat{S}_{\mathbb{A}Q_{2a}}^2 - \left(-\hat{S}_{\mathbb{B}Q_{2a}}^2\right) \\
&= \hat{S}_{\mathbb{A}Q_{2a}}^2 + \hat{S}_{\mathbb{B}Q_{2a}}^2 \\
&= \frac{1}{\hat{N}_a - 1} \sum_{i \in s_1} \sum_{j \in s_{ia}} w_i w_{j|ia} \hat{Q}_{iaj}^2 + \hat{N}_a \hat{\bar{Q}}_{2a}^2 + \frac{1}{\hat{N}_a^2} \frac{1}{m} \hat{S}_{Q_{a(pwr)}}^2 + \frac{1}{\hat{N}_a^2} \frac{1}{m^2} \sum_{i \in s_1} \frac{1}{p_i^2} \frac{\hat{S}_{Q_{ia(pwr)}}^2}{n_{ia}}
\end{aligned}
\tag{A.246}
$$

such that $E\left(\hat{S}_{Q_{2a}}^2\right) \doteq E\left(\hat{S}_{\mathbb{A}Q_{2a}}^2\right) + E\left(\hat{S}_{\mathbb{B}Q_{2a}}^2\right) = \left(S_{Q_{2a}}^2 - S_{\mathbb{B}Q_{2a}}^2\right) + S_{\mathbb{B}Q_{2a}}^2 = S_{Q_{2a}}^2$ □

# B Appendix Supplement to Chapter 3

Appendix B lists the fields in the Health and Retirement Study dataset used in calculations and the fields available from the MSG file. Detailed tables are shown of point estimates associated with the ANOVA relvariance component estimates in section B.3. The model variance component estimates used as inputs to anticipated relvariances are shown in B.4 and the estimated anticipated relvariance components themselves in B.5.

## B.1 Partial List of HRS Variables

<u>HH Level Data Set</u>

**Continuous:**

- Income - HRS 2010 Total HH Income (imputed)
- Wealtha - HRS 2010 Total Wealth excluding secondary residence (imputed)
- Wealthb - HRS 2010 Total Wealth including secondary residence (imputed)

**Categorical:**

- Sex (1=Male, 0=Female)
- Currently Employed – HH Level (1=Yes; 0=No)
- Self-Rated Health – HH Level (Low/Poor vs. Other)
- Own Primary Residence – (1=Yes; 0=No)
- Own Stock (1=Yes; 0=No)
- Other Debts - Any Debts Not Asked About Before HH Level (1=Yes; 0=No)
- Own Second Home -HH Level (1=Yes; 0=No)
- Own Transportation - HH Level (1=Yes; 0=No)
- Whether Donate to Charity - HH Level (1=Yes; 0=No)

## B.2 List of MSG Variables and HRS Screener Variables

Table 5.1. Selected MSG Variables

| MSG Variables | Description |
|---|---|
| Age for Person 1 | Head of Household (HoH) age in 2010 provided by MSG. A '.' value means no information provided by MSG |
| Age for Person 2 | The age of a second person in the HH in 2010 provided by MSG. A '.' value means no match |
| Head of household race/ethnicity | Head of Household (HoH) race/ethnicity matched to the address by MSG. (1. Hispanic; 2. non-Hispanic Black; 3. non-Hispanic non-Black; -99 if not sent to MSG to match race/ethnicity; -98: if sent to MSG but no race/ethnicity information provided |
| Gender | M = Male, F = Female, U=Unknown |
| Hispanic_surname | MSG indicator about whether the HoH has a Hispanic surname. 1=Yes, '' = Not sent or No MSG Match |
| Asian surname | MSG indicator about whether the HoH has an Asian surname. 1=Yes, '' = Not sent or No MSG Match |
| Own/Rent | MSG Own Rent HH Status. O=Own, R=Rent, U=Unknown |
| Income | MSG HU Yearly Income |
| Marital status | MSG Marital status of HoH for HRS 2011 address selection. M=Married, S=Single |
| Number of adults | MSG count of number of adults in HH for HRS 2011 address selection |
| Number of children | MSG count of number of children in HH for HRS 2011 address selection. Note that MSG does not provide a value of 0 so unable to tell if HH has zero kids or MSG had no child age data |
| Education | 1 =HS Diploma, 2=Some College -Extremely Likely, 3=Bachelor's degree, 4=Graduate Degree, 5= Less than HS Diploma-Extremely Likely |
| Status of Dwelling Unit (SDU) | S= Single Family Dwelling Unit (SFDU), M= Multiple Family Dwelling Unit (MFDU) |

Table 5.2. Selected HRS Screener Variables

| HRS Screener Variables | Description |
|---|---|
| Ages for Person 1-8 | Ages of all persons 18 and above from HRS HH listing |
| Coupleness status | Coupleness status of persons from HRS screener. Only asked if Informant indicates that persons' YoB was between 1948-1965 (~ age 45-62). 1=Married, 3=Partnered, 6=Not married or partnered) |
| Hispanic ethnicity | Hispanic ethnicity for selected respondent only |
| Race for selected respondent only | Values are 1. Hispanic; 2. non-Hispanic Black; 3. non-Hispanic White; 4. non-Hispanic Other; -98. Don't know/Refusal; -99. if HRS Age Eligibility not in (1,2) |
| HRS Age Eligibility | Known age eligibility status based on data collection outcome. ( '0' = 2004 selected address, no HH roster ages provided; '1'=2010 or 2011 selected address, completed HH listing, age eligible; '2'=2010 selected address, completed HH Listing, age ineligible; '3'=2011 selected address, answered short screening question to more quickly identify age eligible HHs than full rostering, age ineligible; '7'=2010 or 2011 selected address, HU nonresponse to screening questions, age eligibility undetermined; '8' = Unoccupied HU; '9' = Address not selected for data collection). NOTE: In 2011, HRS used a short screening question to more quickly identify age eligible HHs. HHs that indicated that no one in the HH was age 40-64 did not complete a HH Listing. |
| Asian surname | Number of people in HU aged 45-62 |

## B.3 Design-Based ANOVA Variance Component and Measures of Homogeneity Estimates from 2010-11 HRS Data

Table 5.3 Relvariance Component Estimate, $\hat{W}_{2a}^2$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs. Negative values are highlighted.

| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $\hat{W}_{2a}^2$ | | | | | | |
| | | | | | Self-Representing | | | | | | |
| 02 | 0.1195 | 0.3899 | 0.3170 | 0.02991 | 0.0603 | 0.0505 | 0.0597 | 0.2003 | -0.0986 | 0.0574 | 0.0433 |
| 03 | 0.0275 | 0.0200 | 0.0135 | 0.04377 | -0.0337 | 0.0606 | 0.1753 | 0.1194 | -0.3423 | 0.0972 | -0.0038 |
| 04 | 0.1316 | 0.1490 | 0.1353 | 0.08816 | 0.1940 | 0.0971 | 0.0255 | 0.2237 | -0.1203 | 0.1073 | 0.0097 |
| | | | | | Non Self-Representing | | | | | | |
| 02 | 4.5861 | 6.1577 | 8.5233 | 0.2626 | 1.0379 | 0.4567 | 0.7290 | 1.3021 | -0.1911 | 0.8236 | 0.1426 |
| 03 | 1.0776 | 6.7750 | 7.1520 | 0.6789 | -0.3909 | 0.6246 | 0.8896 | 2.7831 | -4.7891 | 0.7307 | -0.4539 |
| 04 | 6.2956 | 18.2309 | 6.0885 | -0.8055 | 1.0917 | 0.1224 | -0.5640 | -2.4624 | -0.6247 | 0.9904 | -1.4984 |

Table 5.4 Relvariance Component Estimate, $\hat{W}_{3a}^2$ , for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs

| | | | | | | $\hat{W}_{3a}^2$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
| | | | | | | Self-Representing | | | | | |
| 02 | 0.0651 | 1.0989 | 0.9212 | 0.11560 | 0.1648 | 0.0890 | 0.0840 | 0.3287 | 0.9081 | 0.0558 | 0.1762 |
| 03 | 0.1622 | 1.6534 | 1.5516 | 0.16427 | 0.5787 | 0.1139 | 0.1346 | 0.3068 | 4.0933 | 0.0503 | 0.2361 |
| 04 | 0.1414 | 1.0849 | 0.9833 | 0.16206 | 0.2570 | 0.0788 | 0.2112 | 0.7617 | 2.6670 | 0.0579 | 0.1674 |
| | | | | | | Non Self-Representing | | | | | |
| 02 | 1.8526 | 73.8981 | 66.4174 | 1.78012 | 4.1237 | 1.4398 | 1.7302 | 5.5844 | 7.7686 | 0.5512 | 2.0482 |
| 03 | 2.7470 | 28.9609 | 34.3562 | 3.81277 | 9.4482 | 3.2523 | 4.3606 | 6.7858 | 7.3561 | 1.2173 | 6.0074 |
| 04 | 1.9326 | 82.9359 | 68.5960 | 7.23227 | 16.7853 | 5.0263 | 8.7629 | 21.2107 | 15.4013 | 2.5609 | 6.1113 |

Table 5.5 Measure of Homogeneity Estimate, $\hat{\delta}_{2a}$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs

| $\hat{\delta}_{2a}$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
| Self-Representing | | | | | | | | | | | |
| 02 | 0.6473 | 0.2619 | 0.2560 | 0.2055 | 0.2679 | 0.3623 | 0.4153 | 0.3787 | -0.1218 | 0.5070 | 0.1974 |
| 03 | 0.1451 | 0.0119 | 0.0086 | 0.2104 | -0.0618 | 0.3474 | 0.5657 | 0.2802 | -0.0913 | 0.6589 | -0.0164 |
| 04 | 0.4820 | 0.1207 | 0.1210 | 0.3523 | 0.4302 | 0.5521 | 0.1079 | 0.2270 | -0.0472 | 0.6494 | 0.0545 |
| Non Self-Representing | | | | | | | | | | | |
| 02 | 0.7123 | 0.0769 | 0.1137 | 0.12857 | 0.2011 | 0.2408 | 0.2964 | 0.1891 | 0.1612 | 0.5991 | 0.0651 |
| 03 | 0.2818 | 0.1896 | 0.1723 | 0.15115 | -0.0432 | 0.1611 | 0.1694 | 0.2908 | 0.1969 | 0.3751 | -0.0817 |
| 04 | 0.7651 | 0.1802 | 0.0815 | -0.12533 | 0.0611 | 0.0238 | -0.0688 | -0.1313 | 0.1157 | 0.2789 | -0.3248 |

Table 5.6 Estimates of the factor, $\hat{k}_{2a}$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs

| | | | | | | $\hat{k}_{2a}$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
| | | | | | | Self-Representing | | | | | |
| 02 | 0.1108 | 0.2050 | 0.1788 | 0.1366 | 0.1683 | 0.1306 | 0.1893 | 0.1295 | 0.1157 | 0.1883 | 0.0985 |
| 03 | 0.1675 | 0.2638 | 0.2594 | 0.1880 | 0.1922 | 0.2812 | 0.2600 | 0.2006 | 0.4247 | 0.3558 | 0.1446 |
| 04 | 0.1782 | 0.1910 | 0.1824 | 0.2064 | 0.1712 | 0.2137 | 0.1484 | 0.1687 | 0.1635 | 0.2564 | 0.1549 |
| | | | | | | Non Self-Representing | | | | | |
| 02 | 3.5886 | 1.8031 | 1.8841 | 1.85172 | 2.3332 | 1.7252 | 2.3505 | 1.1136 | 1.2250 | 3.4996 | 1.5932 |
| 03 | 3.2026 | 5.0420 | 6.0290 | 4.59189 | 3.7721 | 5.3918 | 6.4221 | 2.3014 | 1.0041 | 6.7396 | 2.6731 |
| 04 | 8.9707 | 1.4772 | 1.1719 | 5.31251 | 7.5008 | 4.5194 | 7.1442 | 3.3666 | 1.0672 | 10.1645 | 3.6392 |

Table 5.7 Relvariance Component Estimate, $\hat{\hat{V}}_a$ , for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs

| $\hat{\hat{V}}_a$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
| Self-Representing | | | | | | | | | | | |
| 02 | 1.6665 | 7.2613 | 6.9262 | 1.0651 | 1.3379 | 1.0685 | 0.7587 | 4.0847 | 6.9977 | 0.6015 | 2.2292 |
| 03 | 1.1328 | 6.3423 | 6.0344 | 1.1068 | 2.8360 | 0.6206 | 1.1919 | 2.1252 | 8.8325 | 0.4148 | 1.6068 |
| 04 | 1.5319 | 6.4604 | 6.1325 | 1.2125 | 2.6347 | 0.8227 | 1.5954 | 5.8399 | 15.5717 | 0.6443 | 1.1425 |
| Non Self-Representing | | | | | | | | | | | |
| 02 | 1.7942 | 44.4002 | 39.7743 | 1.1032 | 2.2122 | 1.0993 | 1.0462 | 6.1840 | 7.5603 | 0.3929 | 1.3751 |
| 03 | 1.1942 | 7.0877 | 6.8848 | 0.9782 | 2.4012 | 0.7190 | 0.8175 | 4.1579 | 9.1218 | 0.2890 | 2.0775 |
| 04 | 0.9172 | 68.4846 | 63.7281 | 1.2098 | 2.3833 | 1.1393 | 1.1476 | 5.5688 | 16.3205 | 0.3494 | 1.2675 |

Table 5.8 Proportion, $\hat{K}_a$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs

| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | $\hat{K}_a$ | | | | | | |
| | | | | | Self-Representing | | | | | | |
| 02 | 0.3552 | 0.4066 | 0.4238 | 0.29078 | 0.3503 | 0.2588 | 0.3202 | 0.2522 | 0.3561 | 0.2550 | 0.2566 |
| 03 | 0.3390 | 0.3626 | 0.3485 | 0.38437 | 0.2880 | 0.3851 | 0.4221 | 0.4695 | 0.3709 | 0.4167 | 0.3774 |
| 04 | 0.3058 | 0.2308 | 0.2277 | 0.32485 | 0.3617 | 0.3561 | 0.2577 | 0.2783 | 0.2730 | 0.3283 | 0.3660 |
| | | | | | Non Self-Representing | | | | | | |
| 02 | 0.5513 | 0.5368 | 0.5243 | 0.6092 | 0.6035 | 0.5941 | 0.6027 | 0.5429 | 0.6464 | 0.5990 | 0.6294 |
| 03 | 0.3122 | 0.3808 | 0.3903 | 0.2745 | 0.2843 | 0.2862 | 0.2939 | 0.3864 | 0.2700 | 0.2925 | 0.2372 |
| 04 | 0.1366 | 0.0824 | 0.0854 | 0.1163 | 0.1123 | 0.1197 | 0.1033 | 0.0708 | 0.0836 | 0.1084 | 0.1334 |

Table 5.9 Relvariance Component Estimate, $\hat{W}^2_{3ab,SR}$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR PSUs

| | | | | $\hat{W}^2_{3ab,SR}$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own home | own stock | own 2nd home | own transport | self-rated health |
| 0201 | 0.225 | 4.096 | 0.125 | 0.676 | 0.378 | 0.139 | 0.050 | 0.317 | 0.044 | 0.363 | 0.030 |
| 0202 | 0.020 | 0.113 | 0.106 | 0.178 | 0.250 | 0.145 | 0.047 | 1.044 | 2.902 | 0.112 | 1.664 |
| 0203 | 0.373 | 0.609 | 0.647 | 0.567 | 0.502 | 0.326 | 0.176 | 4.746 | 6.536 | 0.213 | 1.856 |
| 0204 | 0.167 | 1.097 | 1.133 | 0.244 | 0.359 | 0.212 | 0.142 | 0.464 | 6.809 | 0.054 | 1.579 |
| 0205 | 0.027 | 0.171 | 0.140 | 0.146 | 0.191 | 0.079 | 0.095 | 0.183 | 0.996 | 0.099 | 0.255 |
| 0206 | 0.105 | 1.496 | 1.527 | 0.216 | 0.598 | 0.226 | 0.453 | 1.237 | 3.425 | 0.160 | 0.1601 |
| 0301 | 0.239 | 1.702 | 1.685 | 1.549 | 0.745 | 0.451 | 0.298 | 2.606 | 65.151 | 0.247 | 1.399 |
| 0302 | 4.408 | 5.560 | 5.461 | 1.227 | 0.623 | 0.408 | 0.850 | 2.028 | 5.621 | 0.175 | 0.942 |
| 0303* | 0.695 | 3.914 | 2.790 | 1.074 | 0.664 | 0.313 | 0.302 | 3.241 | 1.598 | 0.266 | 2.571 |
| 0304 | 0.198 | 0.515 | 0.504 | 0.345 | 0.249 | 0.249 | 0.047 | 0.092 | 1.514 | 0.052 | 0.597 |
| 0305 | 0.212 | 0.305 | 0.265 | 0.115 | 1.015 | 0.099 | 0.195 | 1.486 | 2.696 | 0.057 | 0.611 |
| 0306 | 0.089 | 0.974 | 0.948 | 0.504 | 4.025 | 0.192 | 0.363 | 0.712 | 2.586 | 0.127 | 0.297 |
| 0401 | 0.146 | 0.450 | 0.347 | 1.440 | 7.058 | 0.646 | 0.306 | 50.887 | 3.873 | 1.117 | 1.090 |
| 0402 | 0.237 | 2.172 | 2.138 | 0.236 | 0.558 | 0.177 | 0.478 | 1.247 | 10.171 | 0.279 | 0.564 |
| 0403 | 0.695 | 3.914 | 2.790 | 1.074 | 0.664 | 0.313 | 0.302 | 3.241 | 1.598 | 0.266 | 2.571 |
| 0404 | 0.346 | 1.910 | 1.880 | 0.278 | 0.619 | 0.075 | 0.153 | 0.189 | 4.781 | 0.093 | 0.973 |
| 0405 | 0.077 | 0.226 | 0.243 | 0.245 | 0.311 | 0.170 | 0.154 | 1.301 | 1.910 | 0.071 | 0.285 |
| 0406 | 0.096 | 0.988 | 1.077 | 0.280 | 0.293 | 0.162 | 1.034 | 1.576 | 8.080 | 0.100 | 0.227 |

* Estimates from the SSU stratum/ MSG substratum 0403 used as a replacement for those in 0303

Table 5.10 Relvariance Component Estimate, $\hat{W}^2_{3ab,NSR}$, for Selected HRS Interview Variables from the 2010-2011 HRS for NSR PSUs

| | $\hat{W}^2_{3ab,NSR}$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own home | own stock | own 2nd home | own transport | self-rated health |
| 0201 | 9.97 | 132.91 | 101.72 | 38.69 | 7.53 | 3.68 | 15.46 | 25.99 | 160.90 | 2.85 | 5.25 |
| 0202 | 1.58 | 1.97 | 1.93 | 3.27 | 9.59 | 1.99 | 1.63 | 11.31 | 27.96 | 0.87 | 8.88 |
| 0203 | 4.23 | 22.01 | 21.71 | 3.98 | 6.32 | 3.85 | 1.65 | 32.38 | 87.45 | 1.65 | 6.36 |
| 0204 | 2.48 | 13.20 | 13.86 | 3.88 | 4.16 | 2.19 | 1.09 | 10.95 | 85.87 | 0.13 | 12.71 |
| 0205 | 0.69 | 5.37 | 5.74 | 2.82 | 4.36 | 2.19 | 2.11 | 8.86 | 45.21 | 0.88 | 3.61 |
| 0206 | 1.50 | 45.55 | 37.68 | 2.88 | 9.50 | 3.19 | 8.60 | 34.91 | 18.26 | 1.37 | 2.88 |
| 0301 | 3.74 | 30.84 | 28.76 | 5.83 | 30.30 | 4.56 | 4.12 | 93.51 | 214.20 | 3.37 | 12.42 |
| 0302 | 5.90 | 92.25 | 169.43 | 7.10 | 8.86 | 3.25 | 3.65 | 11.02 | 63.78 | 0.56 | 26.50 |
| 0303* | 19.72 | 5.52 | 4.13 | 24.53 | 22.851 | 47.49 | 8.67 | 144.46 | 59.63 | 2.79 | 57.84 |
| 0304 | 3.24 | 9.89 | 12.79 | 8.32 | 6.10 | 3.39 | 1.47 | 5.07 | 64.41 | 0.79 | 139.83 |
| 0305 | 4.42 | 11.48 | 9.81 | 9.22 | 10.65 | 5.70 | 5.60 | 16.55 | 55.28 | 1.58 | 13.39 |
| 0306 | 4.91 | 37.96 | 33.65 | 6.69 | 25.08 | 6.85 | 12.25 | 29.05 | 180.85 | 3.27 | 6.63 |
| 0401 | 6.46 | 71.62 | 66.19 | 62.18 | 108.83 | 21.17 | 38.20 | 1156.84 | 47.11 | 2.78 | 47.57 |
| 0402 | 6.93 | 206.31 | 205.99 | 5.16 | 44.24 | 8.07 | 23.93 | 76.10 | 267.45 | 2.62 | 6.30 |
| 0403 | 19.72 | 5.52 | 4.13 | 24.53 | 22.85 | 47.49 | 8.67 | 144.46 | 59.63 | 2.79 | 57.84 |
| 0404 | 5.19 | 25.23 | 32.26 | 10.36 | 13.33 | 5.33 | 4.19 | 54.86 | 827.26 | 1.34 | 22.79 |
| 0405 | 1.31 | 2.79 | 2.79 | 12.40 | 13.62 | 7.20 | 8.06 | 36.84 | 726.95 | 5.91 | 8.88 |
| 0406 | 1.69 | 805.81 | 447.34 | 5.82 | 37.08 | 6.18 | 18.76 | 70.82 | 9.95 | 3.51 | 6.61 |

*Estimates for categorical variables only from the SSU stratum/ MSG substratum 0403 used as a replacement for those in 0303

Table 5.11 Proportion, $\hat{K}_{ab,SR}$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR PSUs

| | $\hat{K}_{ab,SR}$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own home | own stock | own 2nd home | own transport | self-rated health |
| 0201 | 0.004 | 0.006 | 0.030 | 0.003 | 0.006 | 0.002 | 0.004 | 0.003 | 0.012 | 0.001 | 0.005 |
| 0202 | 0.061 | 0.032 | 0.031 | 0.030 | 0.038 | 0.027 | 0.035 | 0.030 | 0.038 | 0.021 | 0.010 |
| 0203 | 0.020 | 0.017 | 0.018 | 0.021 | 0.036 | 0.023 | 0.035 | 0.011 | 0.027 | 0.024 | 0.012 |
| 0204 | 0.065 | 0.099 | 0.092 | 0.039 | 0.051 | 0.032 | 0.049 | 0.056 | 0.026 | 0.042 | 0.015 |
| 0205 | 0.124 | 0.159 | 0.159 | 0.096 | 0.151 | 0.096 | 0.141 | 0.108 | 0.150 | 0.088 | 0.095 |
| 0206 | 0.076 | 0.091 | 0.089 | 0.099 | 0.083 | 0.072 | 0.055 | 0.054 | 0.094 | 0.076 | 0.121 |
| 0301 | 0.021 | 0.022 | 0.020 | 0.022 | 0.030 | 0.036 | 0.047 | 0.022 | 0.014 | 0.035 | 0.036 |
| 0302 | 0.016 | 0.075 | 0.070 | 0.018 | 0.018 | 0.014 | 0.023 | 0.029 | 0.010 | 0.022 | 0.015 |
| 0303* | 0.016 | 0.009 | 0.010 | 0.016 | 0.025 | 0.023 | 0.026 | 0.016 | 0.025 | 0.020 | 0.013 |
| 0304 | 0.087 | 0.085 | 0.080 | 0.063 | 0.093 | 0.048 | 0.102 | 0.145 | 0.083 | 0.074 | 0.025 |
| 0305 | 0.085 | 0.082 | 0.081 | 0.148 | 0.090 | 0.105 | 0.143 | 0.095 | 0.071 | 0.130 | 0.090 |
| 0306 | 0.123 | 0.095 | 0.094 | 0.126 | 0.044 | 0.170 | 0.093 | 0.171 | 0.187 | 0.147 | 0.201 |
| 0401 | 0.018 | 0.012 | 0.014 | 0.013 | 0.009 | 0.019 | 0.028 | 0.003 | 0.033 | 0.014 | 0.023 |
| 0402 | 0.020 | 0.011 | 0.011 | 0.025 | 0.013 | 0.021 | 0.011 | 0.020 | 0.009 | 0.016 | 0.018 |
| 0403 | 0.016 | 0.009 | 0.010 | 0.016 | 0.025 | 0.023 | 0.026 | 0.016 | 0.025 | 0.020 | 0.013 |
| 0404 | 0.045 | 0.024 | 0.022 | 0.030 | 0.033 | 0.038 | 0.033 | 0.064 | 0.036 | 0.032 | 0.014 |
| 0405 | 0.080 | 0.089 | 0.084 | 0.084 | 0.086 | 0.085 | 0.078 | 0.044 | 0.053 | 0.081 | 0.089 |
| 0406 | 0.121 | 0.084 | 0.084 | 0.152 | 0.170 | 0.163 | 0.072 | 0.114 | 0.109 | 0.158 | 0.206 |

* Estimates from the SSU stratum/ MSG substratum 0403 used as a replacement for those in 0303

Table 5.12 Total, $\hat{t}_{ab,SR}$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR PSUs

| | | | | | $\hat{t}_{ab,SR}$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own home | own stock | own 2nd home | own transport | self-rated health |
| 0201 | 1.89E+09 | 6.14E+09 | 3.51E+10 | 9.74E+03 | 1.30E+04 | 1.07E+04 | 1.31E+04 | 4.97E+03 | 8.09E+03 | 7.43E+03 | 1.55E+04 |
| 0202 | 2.96E+10 | 3.49E+10 | 3.67E+10 | 1.09E+05 | 8.94E+04 | 1.20E+05 | 1.18E+05 | 5.23E+04 | 2.62E+04 | 1.09E+05 | 3.05E+04 |
| 0203 | 9.55E+09 | 1.81E+10 | 2.12E+10 | 7.56E+04 | 8.30E+04 | 1.02E+05 | 1.19E+05 | 1.86E+04 | 1.84E+04 | 1.26E+05 | 3.72E+04 |
| 0204 | 3.15E+10 | 1.08E+11 | 1.09E+11 | 1.41E+05 | 1.18E+05 | 1.43E+05 | 1.66E+05 | 9.69E+04 | 1.78E+04 | 2.17E+05 | 4.62E+04 |
| 0205 | 6.00E+10 | 1.73E+11 | 1.87E+11 | 3.42E+05 | 3.53E+05 | 4.27E+05 | 4.78E+05 | 1.86E+05 | 1.04E+05 | 4.57E+05 | 2.87E+05 |
| 0206 | 3.66E+10 | 9.93E+10 | 1.05E+11 | 3.55E+05 | 1.93E+05 | 3.21E+05 | 1.86E+05 | 9.38E+04 | 6.51E+04 | 3.93E+05 | 3.66E+05 |
| 0301 | 1.03E+10 | 2.37E+10 | 2.40E+10 | 7.69E+04 | 7.07E+04 | 1.60E+05 | 1.58E+05 | 3.78E+04 | 9.94E+03 | 1.83E+05 | 1.08E+05 |
| 0302 | 7.86E+09 | 8.22E+10 | 8.29E+10 | 6.50E+04 | 4.10E+04 | 6.42E+04 | 7.81E+04 | 5.06E+04 | 7.22E+03 | 1.13E+05 | 4.62E+04 |
| 0303* | 7.92E+09 | 1.00E+10 | 1.19E+10 | 5.61E+04 | 5.86E+04 | 1.04E+05 | 8.65E+04 | 2.68E+04 | 1.71E+04 | 1.05E+05 | 3.88E+04 |
| 0304 | 4.23E+10 | 9.30E+10 | 9.43E+10 | 2.25E+05 | 2.18E+05 | 2.15E+05 | 3.44E+05 | 2.51E+05 | 5.76E+04 | 3.87E+05 | 7.50E+04 |
| 0305 | 4.13E+10 | 8.91E+10 | 9.51E+10 | 5.27E+05 | 2.09E+05 | 4.65E+05 | 4.83E+05 | 1.64E+05 | 4.91E+04 | 6.74E+05 | 2.73E+05 |
| 0306 | 5.96E+10 | 1.03E+11 | 1.10E+11 | 4.50E+05 | 1.02E+05 | 7.58E+05 | 3.14E+05 | 2.95E+05 | 1.30E+05 | 7.62E+05 | 6.08E+05 |
| 0401 | 8.50E+09 | 1.25E+10 | 1.63E+10 | 4.52E+04 | 2.21E+04 | 8.58E+04 | 9.58E+04 | 4.78E+03 | 2.26E+04 | 7.14E+04 | 7.02E+04 |
| 0402 | 9.72E+09 | 1.24E+10 | 1.25E+10 | 8.79E+04 | 3.10E+04 | 9.50E+04 | 3.82E+04 | 3.45E+04 | 6.22E+03 | 8.12E+04 | 5.31E+04 |
| 0403 | 7.92E+09 | 1.00E+10 | 1.19E+10 | 5.61E+04 | 5.86E+04 | 1.04E+05 | 8.65E+04 | 2.68E+04 | 1.71E+04 | 1.05E+05 | 3.88E+04 |
| 0404 | 2.20E+10 | 2.59E+10 | 2.62E+10 | 1.07E+05 | 7.73E+04 | 1.69E+05 | 1.11E+05 | 1.10E+05 | 2.53E+04 | 1.66E+05 | 4.20E+04 |
| 0405 | 3.90E+10 | 9.69E+10 | 9.96E+10 | 2.99E+05 | 2.01E+05 | 3.76E+05 | 2.62E+05 | 7.63E+04 | 3.69E+04 | 4.19E+05 | 2.68E+05 |
| 0406 | 5.85E+10 | 9.13E+10 | 9.94E+10 | 5.42E+05 | 3.96E+05 | 7.25E+05 | 2.42E+05 | 1.96E+05 | 7.53E+04 | 8.22E+05 | 6.22E+05 |

*Estimates for categorical variables only from the SSU stratum/ MSG substratum 0403 used as a replacement for those in 0303

Table 5.13 Proportion, $\hat{K}_{ab,NSR}$, for Selected HRS Interview Variables from the 2010-2011 HRS for NSR PSUs

| | | | | | $\hat{K}_{ab,NSR}$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own home | own stock | own 2nd home | own transport | self-rated health |
| 0201 | 0.002 | 0.001 | 0.001 | 0.002 | 0.005 | 0.003 | 0.002 | 0.002 | 0.005 | 0.003 | 0.004 |
| 0202 | 0.042 | 0.171 | 0.157 | 0.044 | 0.037 | 0.049 | 0.049 | 0.049 | 0.035 | 0.043 | 0.024 |
| 0203 | 0.047 | 0.055 | 0.052 | 0.055 | 0.070 | 0.052 | 0.079 | 0.048 | 0.041 | 0.051 | 0.049 |
| 0204 | 0.075 | 0.081 | 0.077 | 0.069 | 0.109 | 0.079 | 0.113 | 0.122 | 0.049 | 0.077 | 0.040 |
| 0205 | 0.211 | 0.160 | 0.159 | 0.188 | 0.224 | 0.196 | 0.231 | 0.208 | 0.197 | 0.199 | 0.216 |
| 0206 | 0.174 | 0.069 | 0.078 | 0.249 | 0.172 | 0.215 | 0.127 | 0.108 | 0.311 | 0.224 | 0.296 |
| 0301 | 0.015 | 0.024 | 0.024 | 0.016 | 0.013 | 0.018 | 0.022 | 0.009 | 0.011 | 0.015 | 0.013 |
| 0302 | 0.031 | 0.035 | 0.041 | 0.020 | 0.025 | 0.023 | 0.023 | 0.028 | 0.029 | 0.020 | 0.010 |
| 0303* | 0.001 | 0.000 | 0.000 | 0.003 | 0.008 | 0.003 | 0.007 | 0.003 | 0.003 | 0.005 | 0.003 |
| 0304 | 0.070 | 0.143 | 0.148 | 0.038 | 0.060 | 0.048 | 0.066 | 0.109 | 0.064 | 0.043 | 0.007 |
| 0305 | 0.091 | 0.095 | 0.097 | 0.077 | 0.105 | 0.087 | 0.103 | 0.125 | 0.095 | 0.101 | 0.064 |
| 0306 | 0.106 | 0.083 | 0.080 | 0.123 | 0.077 | 0.108 | 0.081 | 0.118 | 0.069 | 0.113 | 0.143 |
| 0401 | 0.004 | 0.004 | 0.004 | 0.003 | 0.004 | 0.006 | 0.005 | 0.001 | 0.007 | 0.005 | 0.005 |
| 0402 | 0.009 | 0.003 | 0.003 | 0.012 | 0.007 | 0.007 | 0.006 | 0.011 | 0.008 | 0.010 | 0.013 |
| 0403 | 0.004 | 0.006 | 0.006 | 0.003 | 0.008 | 0.003 | 0.007 | 0.003 | 0.003 | 0.005 | 0.003 |
| 0404 | 0.014 | 0.017 | 0.017 | 0.012 | 0.018 | 0.018 | 0.023 | 0.014 | 0.004 | 0.015 | 0.013 |
| 0405 | 0.047 | 0.031 | 0.028 | 0.026 | 0.032 | 0.035 | 0.030 | 0.021 | 0.006 | 0.026 | 0.036 |
| 0406 | 0.059 | 0.022 | 0.027 | 0.059 | 0.028 | 0.050 | 0.027 | 0.021 | 0.062 | 0.047 | 0.062 |

*Estimates for categorical variables only from the SSU stratum/ MSG substratum 0403 used as a replacement for those in 0303

Table 5.14 Total, $\hat{t}_{ab,NSR}$, for Selected HRS Interview Variables from the 2010-2011 HRS for NSR PSUs

| | $\hat{t}_{ab,NSR}$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own home | own stock | own 2nd home | own transport | self-rated health |
| 0201 | 2.02E+09 | 1.23E+09 | 1.50E+09 | 1.69E+04 | 2.72E+04 | 3.42E+04 | 2.03E+04 | 7.25E+03 | 8.28E+03 | 4.42E+04 | 3.03E+04 |
| 0202 | 4.70E+10 | 3.08E+11 | 3.11E+11 | 4.38E+05 | 2.19E+05 | 5.23E+05 | 4.44E+05 | 1.58E+05 | 6.29E+04 | 6.42E+05 | 2.01E+05 |
| 0203 | 5.30E+10 | 9.98E+10 | 1.03E+11 | 5.45E+05 | 4.21E+05 | 5.60E+05 | 7.09E+05 | 1.54E+05 | 7.25E+04 | 7.63E+05 | 4.17E+05 |
| 0204 | 8.50E+10 | 1.46E+11 | 1.53E+11 | 6.86E+05 | 6.52E+05 | 8.51E+05 | 1.02E+06 | 3.91E+05 | 8.83E+04 | 1.17E+06 | 3.40E+05 |
| 0205 | 2.38E+11 | 2.89E+11 | 3.15E+11 | 1.86E+06 | 1.35E+06 | 2.10E+06 | 2.08E+06 | 6.67E+05 | 3.52E+05 | 3.00E+06 | 1.83E+06 |
| 0206 | 1.96E+11 | 1.25E+11 | 1.55E+11 | 2.46E+06 | 1.03E+06 | 2.31E+06 | 1.14E+06 | 3.47E+05 | 5.57E+05 | 3.38E+06 | 2.52E+06 |
| 0301 | 1.64E+10 | 4.42E+10 | 4.68E+10 | 1.54E+05 | 7.85E+04 | 1.97E+05 | 1.96E+05 | 2.74E+04 | 2.05E+04 | 2.20E+05 | 1.13E+05 |
| 0302 | 3.45E+10 | 6.24E+10 | 8.12E+10 | 1.93E+05 | 1.53E+05 | 2.51E+05 | 2.07E+05 | 9.10E+04 | 5.20E+04 | 2.95E+05 | 8.33E+04 |
| 0303* | 1.06E+09 | 5.43E+08 | 6.72E+08 | 3.38E+04 | 4.57E+04 | 3.13E+04 | 6.27E+04 | 1.07E+04 | 5.79E+03 | 7.63E+04 | 2.51E+04 |
| 0304 | 7.87E+10 | 2.58E+11 | 2.92E+11 | 3.72E+05 | 3.57E+05 | 5.12E+05 | 5.95E+05 | 3.49E+05 | 1.14E+05 | 6.47E+05 | 5.73E+04 |
| 0305 | 1.02E+11 | 1.71E+11 | 1.92E+11 | 7.59E+05 | 6.29E+05 | 9.41E+05 | 9.27E+05 | 4.00E+05 | 1.70E+05 | 1.53E+06 | 5.44E+05 |
| 0306 | 1.19E+11 | 1.50E+11 | 1.59E+11 | 1.22E+06 | 4.61E+05 | 1.17E+06 | 7.31E+05 | 3.80E+05 | 1.24E+05 | 1.71E+06 | 1.21E+06 |
| 0401 | 4.83E+09 | 8.10E+09 | 8.43E+09 | 3.20E+04 | 2.50E+04 | 6.46E+04 | 4.53E+04 | 4.08E+03 | 1.24E+04 | 7.08E+04 | 4.32E+04 |
| 0402 | 9.81E+09 | 4.57E+09 | 4.96E+09 | 1.14E+05 | 4.01E+04 | 7.54E+04 | 5.44E+04 | 3.47E+04 | 1.50E+04 | 1.45E+05 | 1.15E+05 |
| 0403 | 4.76E+09 | 1.08E+10 | 1.25E+10 | 3.38E+04 | 4.57E+04 | 3.13E+04 | 6.27E+04 | 1.07E+04 | 5.79E+03 | 7.63E+04 | 2.51E+04 |
| 0404 | 1.58E+10 | 2.99E+10 | 3.44E+10 | 1.23E+05 | 1.10E+05 | 1.90E+05 | 2.09E+05 | 4.61E+04 | 7.66E+03 | 2.30E+05 | 1.14E+05 |
| 0405 | 5.24E+10 | 5.52E+10 | 5.53E+10 | 2.54E+05 | 1.93E+05 | 3.78E+05 | 2.71E+05 | 6.75E+04 | 9.99E+03 | 3.97E+05 | 3.06E+05 |
| 0406 | 6.62E+10 | 3.99E+10 | 5.35E+10 | 5.86E+05 | 1.69E+05 | 5.41E+05 | 2.46E+05 | 6.59E+04 | 1.10E+05 | 7.10E+05 | 5.23E+05 |

*Estimates for categorical variables only from the SSU stratum/ MSG substratum 0403 used as a replacement for those in 0303

## B.4 Variance Component Estimates for SR and NSR PSUs from PROC MIXED SAS

Table 5.15 Variance Component Estimates, $\alpha_i$, for SR PSUs for selected 2010-11 HRS variables.

| Parameter | SR PSUs only | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | income | wealtha | wealthb | other debts | charity donate | employed | ownHome | ownStock | own 2nd home | own transport | self Rated Health |
| Intercept | 66.168 | 187.340 | 205.020 | 0.5382 | 0.3716 | 0.5787 | 0.5419 | 0.3259 | 0.1140 | 0.7349 | 0.2775 |
| PSU 4 | -23.391 | -142.440 | -145.880 | -0.1850 | -0.0978 | -0.1527 | -0.1276 | -0.2116 | 0.0929 | -0.2649 | 0.0407 |
| PSU 6 | -15.357 | -46.920 | -61.355 | -0.1000 | -0.1806 | 0.0634 | -0.1234 | -0.1391 | -0.0428 | -0.0022 | 0.1346 |
| PSU 7 | -27.275 | -80.570 | -82.980 | -0.1097 | -0.1453 | 0.0515 | -0.0334 | -0.0440 | -0.0383 | 0.0387 | 0.0509 |
| PSU 10 | -44.552 | -161.740 | -190.980 | -0.2134 | -0.0785 | -0.2680 | 0.1358 | -0.2944 | -0.0879 | -0.4003 | 0.3281 |
| PSU 17 | -34.357 | -146.290 | -156.230 | 0.0096 | -0.2057 | -0.0059 | 0.0254 | -0.2230 | -0.0845 | 0.1274 | 0.1575 |
| PSU 23 | -29.757 | -68.044 | -86.582 | -0.3430 | -0.1617 | -0.2920 | -0.1997 | -0.2481 | -0.1140 | -0.1940 | 0.1445 |
| PSU 30 | 2.503 | -80.732 | -94.990 | -0.1089 | 0.2389 | 0.0832 | -0.1460 | -0.2383 | 0.0135 | -0.0527 | 0.0694 |
| PSU 35 | -3.558 | 108.530 | 101.800 | -0.1922 | 0.0744 | -0.0316 | 0.1498 | -0.1626 | 0.0222 | -0.1499 | -0.0452 |
| PSU 40 | -14.994 | -55.161 | -63.256 | -0.0003 | -0.0241 | 0.1227 | 0.0837 | -0.0886 | -0.0636 | 0.0700 | -0.0955 |
| PSU 41 | -33.415 | -180.950 | -198.380 | -0.1977 | -0.3716 | -0.0743 | -0.0486 | 0.0590 | -0.1140 | -0.1888 | 0.5998 |
| PSU 47 | -26.918 | -112.910 | -103.920 | -0.0598 | -0.1203 | -0.0962 | -0.1524 | -0.2370 | -0.0717 | -0.1701 | 0.0631 |
| PSU 48 | -21.609 | -106.290 | -103.860 | -0.0793 | -0.1184 | -0.1582 | -0.0143 | -0.1871 | 0.0331 | -0.3463 | 0.2031 |
| PSU 53 | -52.613 | -183.260 | -200.160 | -0.0477 | -0.2876 | -0.1787 | -0.4503 | -0.2477 | -0.0650 | -0.5445 | 0.2917 |
| PSU 54 | -24.832 | -26.719 | -29.477 | -0.1328 | -0.1280 | -0.0687 | -0.2252 | -0.1661 | -0.0587 | -0.4299 | 0.1846 |
| PSU 59 | -6.404 | -127.760 | -113.230 | -0.1746 | -0.0398 | 0.1151 | 0.1126 | -0.2021 | 0.0370 | -0.1136 | 0.0963 |
| PSU 60 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

PSU 60 is the reference category

Table 5.16 Variance Component Estimates for Residual term, $\sigma^2_{\varepsilon_{ab}}$ , for NSR PSUs for selected 2010-11 HRS variables.

| SSU stratum/ MSG substratum $ab$ | NSR PSUs only | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | income | wealtha | wealthb | other debts | charity donate | employed | ownHome | ownStock | own 2nd home | own transport | self Rated Health |
| 0201 | 1.30E+03 | 5.24E+05** | 7.94E+03 | 0.2482 | 0.2858 | 0.1934 | 0.1485 | 0.0982 | 0.2003 | 0.0801 | 0.3156 |
| 0202 | 2.01E+03 | 5.06E+06 | 5.10E+06 | 0.2482 | 0.1839 | 0.2283 | 0.1640 | 0.1130 | 0.0677 | 0.1129 | 0.1910 |
| 0203 | 2.20E+03 | 3.11E+04 | 3.62E+04 | 0.2425 | 0.2419 | 0.2244 | 0.1729 | 0.1209 | 0.0525 | 0.1479 | 0.2216 |
| 0204 | 3.11E+03 | 3.78E+04 | 4.66E+04 | 0.2489 | 0.2911 | 0.2156 | 0.1898 | 0.2139 | 0.0560 | 0.0331 | 0.1782 |
| 0205 | 3.32E+03 | 2.38E+04 | 3.27E+04 | 0.2362 | 0.1892 | 0.2313 | 0.2042 | 0.1042 | 0.0771 | 0.1714 | 0.2525 |
| 0206 | 1.32E+03 | 4.98E+04 | 6.72E+04 | 0.2432 | 0.1398 | 0.2218 | 0.1687 | 0.0568 | 0.0663 | 0.2268 | 0.2429 |
| 0301 | 1.42E+03 | 1.95E+05 | 2.07E+05 | 0.2459 | 0.1795 | 0.2210 | 0.1899 | 0.0758 | 0.0754 | 0.1513 | 0.2247 |
| 0302 | 1.02E+04 | 7.10E+05 | 1.13E+06 | 0.2485 | 0.2361 | 0.2150 | 0.1781 | 0.2044 | 0.1568 | 0.0978 | 0.1615 |
| 0303* | 4.00E+03 | 8.04E+03 | 1.61E+04 | 0.2349 | 0.3394 | 0.2706 | 0.2559 | 0.1232 | 0.0918 | 0.0707 | 0.1645 |
| 0304 | 7.11E+03 | 3.57E+05 | 4.48E+05 | 0.2452 | 0.2302 | 0.1939 | 0.1459 | 0.2756 | 0.1434 | 0.0738 | 0.1078 |
| 0305 | 2.90E+03 | 7.38E+04 | 8.71E+04 | 0.2408 | 0.2059 | 0.2290 | 0.2421 | 0.1379 | 0.0847 | 0.1490 | 0.2002 |
| 0306 | 1.38E+03 | 1.68E+04 | 1.98E+04 | 0.2392 | 0.1237 | 0.2185 | 0.1994 | 0.0700 | 0.0696 | 0.2330 | 0.2451 |
| 0401 | 7.04E+02 | 2.78E+03 | 4.90E+03 | 0.2473 | 0.1920 | 0.2246 | 0.1872 | 0.0558 | 0.0548 | 0.0919 | 0.2231 |
| 0402 | 1.87E+03 | 1.33E+04 | 2.03E+04 | 0.2335 | 0.1755 | 0.2083 | 0.1976 | 0.1999 | 0.0810 | 0.0892 | 0.2256 |
| 0403 | 4.00E+03 | 8.04E+03 | 1.61E+04 | 0.2349 | 0.3394 | 0.2706 | 0.2559 | 0.1232 | 0.0918 | 0.0707 | 0.1645 |
| 0404 | 1.25E+03 | 1.32E+05 | 1.57E+05 | 0.2369 | 0.2054 | 0.2098 | 0.2076 | 0.1510 | 0.0262 | 0.1069 | 0.2461 |
| 0405 | 7.31E+02 | 2.75E+04 | 2.99E+04 | 0.2296 | 0.1396 | 0.2612 | 0.1590 | 0.0512 | 0.0190 | 0.3018 | 0.2446 |
| 0406 | 1.52E+03 | 2.45E+06 | 2.46E+06 | 0.2377 | 0.1233 | 0.2261 | 0.1485 | 0.0476 | 0.0378 | 0.2365 | 0.2368 |

* Estimates from the SSU stratum/ MSG substratum 0403 used as a replacement for those in 0303

** Standard Error (SE) was 0

Table 5.17 Variance Component Estimates, $\sigma^2_{\varepsilon_{ab}}$, for SR PSUs for selected 2010-11 HRS variables.

| SSU stratum/ MSG substratum $ab$ | SR PSUs only | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | income | wealtha | wealthb | other debts | charity donate | employed | ownHome | ownStock | own 2nd home | own transport | self Rated Health |
| 0201 | 3.53E+03 | 5.15E+05 | 5.25E+05 | 0.2423 | 0.3769 | 0.2470 | 0.0478 | 0.2666 | 0.3181 | 0.2363 | 0.0635 |
| 0202 | 1.17E+04 | 5.89E+04 | 6.02E+04 | 0.2550 | 0.2243 | 0.2018 | 0.1373 | 0.1230 | 0.1388 | 0.1824 | 0.1694 |
| 0203 | 2.56E+03 | 2.85E+04 | 6.84E+04 | 0.2255 | 0.2560 | 0.2265 | 0.2063 | 0.0941 | 0.1316 | 0.1919 | 0.1646 |
| 0204 | 1.90E+04 | 6.43E+05 | 7.34E+05 | 0.2903 | 0.2456 | 0.2007 | 0.1956 | 0.2225 | 0.0977 | 0.1229ǂ | 0.1218 |
| 0205 | 2.34E+03 | 1.29E+05 | 1.36E+05 | 0.2485 | 0.2298 | 0.2264 | 0.2091 | 0.0811 | 0.1082 | 0.2093 | 0.2151 |
| 0206 | 2.59E+03 | 1.73E+05 | 1.95E+05 | 0.2333 | 0.1664 | 0.2348 | 0.2122 | 0.1099 | 0.0421 | 0.2283 | 0.2638 |
| 0301 | 2.76E+03 | 3.14E+04 | 5.03E+04 | 0.2442 | 0.1732 | 0.2365 | 0.1162 | 0.1050 | 0.0645 | 0.1382 | 0.2370 |
| 0302 | 6.90E+03 | 1.09E+06 | 1.13E+06 | 0.2387 | 0.2359 | 0.2258 | 0.2204 | 0.2332 | 0.0710 | 0.1229 | 0.2263 |
| 0303* | 2.26E+03 | 2.39E+04 | 2.26E+04 | 0.2439 | 0.2346 | 0.2259 | 0.1949 | 0.1271 | 0.0608 | 0.1855 | 0.2262 |
| 0304 | 9.14E+03 | 2.29E+05 | 2.27E+05 | 0.2425 | 0.3494 | 0.2301 | 0.1341 | 0.3866 | 0.1488 | 0.1229ǂ | 0.1530 |
| 0305 | 3.77E+03 | 7.33E+04 | 8.15E+04 | 0.2927 | 0.2277 | 0.2239 | 0.2157 | 0.1544 | 0.0629 | 0.1503 | 0.2288 |
| 0306 | 8.18E+02 | 3.19E+04 | 3.41E+04 | 0.2384 | 0.0738 | 0.2416 | 0.1298 | 0.1082 | 0.0760 | 0.2408 | 0.2276 |
| 0401 | 1.37E+03 | 5.34E+04 | 9.12E+04 | 0.2304 | 0.1343 | 0.2123 | 0.1774 | 0.0581 | 0.1148 | 0.1765 | 0.2519 |
| 0402 | 3.41E+03 | 3.72E+04 | 3.97E+04 | 0.2554 | 0.1903 | 0.2122 | 0.1533 | 0.1499 | 0.0519 | 0.1908 | 0.2139 |
| 0403 | 2.26E+03 | 2.39E+04 | 2.26E+04 | 0.2439 | 0.2346 | 0.2259 | 0.1949 | 0.1271 | 0.0608 | 0.1855 | 0.2262 |
| 0404 | 1.18E+04 | 2.34E+05 | 2.35E+05 | 0.2510 | 0.2143 | 0.1796 | 0.1607 | 0.2449 | 0.1183 | 0.1809 | 0.1584 |
| 0405 | 3.81E+03 | 7.30E+04 | 8.32E+04 | 0.2501 | 0.1890 | 0.2427 | 0.1546 | 0.0961 | 0.0490 | 0.1872 | 0.2313 |
| 0406 | 1.43E+03 | 1.12E+05 | 1.15E+05 | 0.2376 | 0.1524 | 0.2432 | 0.1637 | 0.0671 | 0.0392 | 0.2086 | 0.2450 |

* Estimates from the SSU stratum/ MSG substratum 0403 used as a replacement for those in 0303

ǂ The minimum estimate of own transport from the SSU stratum/ MSG substratum 0204 used as a replacement

**Not Significant

## B.5 Anticipated Variance Component and Measures of Homogeneity Estimates from 2010-11 HRS Data

Table 5.18 Anticipated Variance Component Estimate, $\hat{E}_M\left(W_{2a}^2\right)$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs.

| | $\hat{E}_M\left(W_{2a}^2\right)$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
| | | | | | Self-Representing | | | | | | |
| 02 | 0.0092 | 0.0402 | 0.0351 | 0.1075 | 0.1088 | 0.1119 | 0.1315 | 0.2432 | 0.3474 | 0.0888 | 0.2169 |
| 03 | 0.0339 | 0.1721 | 0.1789 | 0.0711 | 0.2160 | 0.0666 | 0.1181 | 0.1340 | 0.4526 | 0.0447 | 0.1093 |
| 04 | 0.0227 | 0.2420 | 0.2381 | 0.0621 | 0.0955 | 0.0500 | 0.2024 | 0.2595 | 0.4755 | 0.0438 | 0.0778 |
| | | | | | Non Self-Representing | | | | | | |
| 02 | 0.0882 | 12.0790 | 8.8319 | 0.0850 | 0.1821 | 0.0699 | 0.0736 | 0.2913 | 0.5909 | 0.0240 | 0.0961 |
| 03 | 0.1113 | 6.9880 | 5.6369 | 0.1543 | 0.3069 | 0.1105 | 0.1189 | 0.1805 | 1.4353 | 0.0369 | 0.2544 |
| 04 | 0.1387 | 17.6053 | 8.5748 | 0.2189 | 0.5077 | 0.1625 | 0.2426 | 1.3172 | 3.1735 | 0.0660 | 0.2009 |

Values highlighted were negative in ANOVA but now corrected to non-zero values through Anticipated Variances

Table 5.19 Anticipated Variance Component Estimate, $\hat{E}_M\left(W_{3a}^2\right)$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs.

| | | | | | $\hat{E}_M\left(W_{3a}^2\right)$ | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
| | | | | | Self-Representing | | | | | | |
| 02 | 0.0804 | 0.4471 | 0.3948 | 0.1043 | 0.1324 | 0.0791 | 0.0767 | 0.2429 | 0.6522 | 0.0533 | 0.1629 |
| 03 | 0.1555 | 0.9671 | 0.9423 | 0.1739 | 0.4964 | 0.1059 | 0.0961 | 0.3144 | 1.5480 | 0.0522 | 0.2194 |
| 04 | 0.1239 | 1.5202 | 1.4181 | 0.1691 | 0.2407 | 0.0900 | 0.1971 | 0.4073 | 1.3416 | 0.0648 | 0.1814 |
| | | | | | Non Self-Representing | | | | | | |
| 02 | 1.8038 | 114.82 | 102.93 | 2.0261 | 4.0501 | 1.6710 | 1.7659 | 5.5884 | 13.212 | 0.6513 | 2.4943 |
| 03 | 3.4334 | 35.930 | 38.015 | 4.5249 | 7.8095 | 3.1848 | 3.7912 | 5.4051 | 44.529 | 1.2454 | 7.0669 |
| 04 | 1.7496 | 1573.96 | 1216.25 | 5.7452 | 10.201 | 4.5490 | 5.8179 | 21.599 | 44.466 | 2.6355 | 5.9465 |

Table 5.20 Anticipated Variance Component Estimate, $\hat{\delta}_{2a}$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs

| | | | | | | $\hat{\delta}_{2a}$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
| | | | | | | Self-Representing | | | | | |
| 02 | 0.1028 | 0.0825 | 0.0816 | 0.5076 | 0.4510 | 0.5859 | 0.6317 | 0.5003 | 0.3475 | 0.6250 | 0.5711 |
| 03 | 0.1787 | 0.1511 | 0.1596 | 0.2902 | 0.3032 | 0.3863 | 0.5514 | 0.2989 | 0.2262 | 0.4612 | 0.3324 |
| 04 | 0.1547 | 0.1373 | 0.1438 | 0.2686 | 0.2839 | 0.3571 | 0.5067 | 0.3892 | 0.2617 | 0.4034 | 0.3002 |
| | | | | | | Non Self-Representing | | | | | |
| 02 | 0.0467 | 0.0952 | 0.0791 | 0.0402 | 0.0430 | 0.0401 | 0.0400 | 0.0493 | 0.0426 | 0.0354 | 0.0371 |
| 03 | 0.0315 | 0.1629 | 0.1293 | 0.0329 | 0.0377 | 0.0334 | 0.0303 | 0.0322 | 0.0311 | 0.0287 | 0.0346 |
| 04 | 0.0738 | 0.0111 | 0.0070 | 0.0365 | 0.0471 | 0.0343 | 0.0397 | 0.0571 | 0.0662 | 0.0242 | 0.0325 |

Table 5.21 Anticipated Variance Component Estimates of the factor, $\hat{k}_{2a}$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs

| $\hat{k}_{2a}$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
| Self-Representing | | | | | | | | | | | |
| 02 | 0.1047 | 0.0997 | 0.0995 | 0.1913 | 0.1637 | 0.2109 | 0.1821 | 0.1590 | 0.1444 | 0.2290 | 0.1991 |
| 03 | 0.1707 | 0.1658 | 0.1669 | 0.2192 | 0.2013 | 0.2401 | 0.2283 | 0.1977 | 0.1999 | 0.2697 | 0.2168 |
| 04 | 0.1302 | 0.1407 | 0.1406 | 0.1788 | 0.1679 | 0.1924 | 0.1784 | 0.1672 | 0.1646 | 0.2043 | 0.1758 |
| Non Self-Representing | | | | | | | | | | | |
| 02 | 1.9196 | 2.5331 | 2.4960 | 2.3340 | 2.1069 | 2.1285 | 1.6626 | 2.0200 | 2.2735 | 1.8134 | 2.1810 |
| 03 | 2.6189 | 3.2444 | 3.1993 | 2.8645 | 2.6562 | 2.7146 | 2.3956 | 2.6544 | 2.8342 | 2.5257 | 2.7602 |
| 04 | 3.8647 | 3.2154 | 3.2063 | 3.9004 | 3.9934 | 3.8651 | 3.9861 | 4.2419 | 4.0257 | 3.6987 | 3.8335 |

Table 5.22 Anticipated Variance Component Estimate, $\hat{\hat{V}}_a$ , for Selected HRS Interview Variables from the 2010-2011 HRS for SR and NSR PSUs

| $\hat{\hat{V}}_a$ | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own Home | own Stock | own 2nd Home | own Transport | self-rated health |
| Self-Representing | | | | | | | | | | | |
| 02 | 0.8557 | 4.8884 | 4.3215 | 1.1069 | 1.4733 | 0.9056 | 1.1433 | 3.0580 | 6.9239 | 0.6205 | 1.9078 |
| 03 | 1.1095 | 6.8727 | 6.7176 | 1.1177 | 3.5382 | 0.7185 | 0.9382 | 2.2684 | 10.0093 | 0.3593 | 1.5162 |
| 04 | 1.1261 | 12.5258 | 11.7786 | 1.2928 | 2.0018 | 0.7277 | 2.2391 | 3.9883 | 11.0390 | 0.5315 | 1.4748 |
| Non Self-Representing | | | | | | | | | | | |
| 02 | 0.9856 | 50.0966 | 44.7761 | 0.9045 | 2.0087 | 0.8179 | 1.1064 | 2.9107 | 6.0713 | 0.3724 | 1.1877 |
| 03 | 1.3535 | 13.2284 | 13.6442 | 1.6335 | 3.0556 | 1.2139 | 1.6322 | 2.1043 | 16.2179 | 0.5077 | 2.6525 |
| 04 | 0.4886 | 494.9887 | 382.0067 | 1.5291 | 2.6816 | 1.2190 | 1.5204 | 5.4023 | 11.8338 | 0.7304 | 1.6036 |

Table 5.23 Anticipated Variance Component Estimate, $\hat{E}_M\left(W^2_{3ab,SR}\right)$, for Selected HRS Interview Variables from the 2010-2011 HRS for SR PSUs

| | $\hat{E}_M\left(W^2_{3ab,SR}\right)$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own home | own stock | own 2nd home | own transport | self-rated health |
| 0201 | 0.253 | 1.569 | 0.764 | 0.355 | 0.289 | 0.245 | 0.044 | 1.275 | 0.636 | 0.468 | 0.037 |
| 0202 | 0.133 | 0.311 | 0.297 | 0.148 | 0.201 | 0.099 | 0.071 | 0.334 | 1.425 | 0.094 | 1.298 |
| 0203 | 0.411 | 1.280 | 2.000 | 0.541 | 0.494 | 0.287 | 0.196 | 3.560 | 5.094 | 0.137 | 1.597 |
| 0204 | 0.603 | 1.340 | 1.474 | 0.214 | 0.296 | 0.166 | 0.115 | 0.396 | 5.360 | 0.009 | 1.003 |
| 0205 | 0.054 | 0.273 | 0.244 | 0.149 | 0.128 | 0.085 | 0.064 | 0.173 | 0.691 | 0.068 | 0.178 |
| 0206 | 0.170 | 1.140 | 1.132 | 0.150 | 0.349 | 0.179 | 0.471 | 0.958 | 0.844 | 0.127 | 0.151 |
| 0301 | 0.973 | 1.857 | 2.748 | 1.691 | 1.466 | 0.392 | 0.202 | 3.351 | 29.079 | 0.198 | 0.856 |
| 0302 | 1.297 | 1.907 | 1.891 | 0.772 | 1.807 | 0.736 | 0.478 | 1.121 | 16.141 | 0.141 | 1.333 |
| 0303* | 0.574 | 2.881 | 1.915 | 0.999 | 0.878 | 0.269 | 0.348 | 2.263 | 2.911 | 0.194 | 1.953 |
| 0304 | 0.304 | 1.222 | 1.168 | 0.253 | 0.357 | 0.246 | 0.057 | 0.346 | 2.187 | 0.055 | 1.377 |
| 0305 | 0.285 | 0.734 | 0.643 | 0.142 | 0.763 | 0.159 | 0.135 | 0.870 | 3.956 | 0.036 | 0.465 |
| 0306 | 0.158 | 0.950 | 0.946 | 0.420 | 2.642 | 0.150 | 0.468 | 0.411 | 1.608 | 0.144 | 0.214 |
| 0401 | 0.379 | 4.804 | 5.343 | 1.893 | 4.862 | 0.518 | 0.336 | 44.742 | 3.951 | 0.701 | 0.875 |
| 0402 | 0.250 | 1.088 | 1.121 | 0.181 | 1.083 | 0.131 | 0.582 | 0.702 | 7.241 | 0.157 | 0.414 |
| 0403 | 0.574 | 2.881 | 1.915 | 0.999 | 0.878 | 0.269 | 0.348 | 2.263 | 2.911 | 0.194† | 1.953 |
| 0404 | 0.207 | 2.358 | 2.337 | 0.244 | 0.407 | 0.074 | 0.156 | 0.236 | 2.085 | 0.068 | 1.058 |
| 0405 | 0.161 | 0.339 | 0.349 | 0.176 | 0.307 | 0.110 | 0.144 | 1.036 | 2.427 | 0.074 | 0.204 |
| 0406 | 0.145 | 3.418 | 2.983 | 0.250 | 0.300 | 0.144 | 0.859 | 0.564 | 2.078 | 0.096 | 0.200 |

* Estimates from the SSU stratum/ MSG substratum 0403 used as a replacement for those in 0303
† Estimate uses SSU stratum/ MSG substratum 0404 as a replacement in estimating results in 0303

Table 5.24 Anticipated Variance Component Estimate, $\hat{E}_M\left(W^2_{3ab,NSR}\right)$, for Selected HRS Interview Variables from the 2010-2011 HRS for NSR PSUs

| | $\hat{E}_M\left(W^2_{3ab,NSR}\right)$ | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG Stratum No. | income | wealtha | wealthb | other debts | charity donate | employed | own home | own stock | own 2nd home | own transport | self-rated health |
| 0201 | 10.53 | 11426.92 | 116.29 | 28.81 | 12.82 | 5.47 | 11.91 | 61.85 | 96.69 | 1.36 | 11.36 |
| 0202 | 1.79 | 104.89 | 103.72 | 2.55 | 7.50 | 1.64 | 1.64 | 8.89 | 33.60 | 0.54 | 9.27 |
| 0203 | 3.56 | 14.18 | 15.54 | 3.70 | 6.20 | 3.25 | 1.56 | 23.18 | 45.33 | 1.16 | 5.78 |
| 0204 | 2.88 | 11.92 | 13.40 | 3.54 | 4.59 | 1.99 | 1.23 | 9.36 | 48.07 | 0.16 | 10.31 |
| 0205 | 2.42 | 11.83 | 13.67 | 2.82 | 4.32 | 2.16 | 1.95 | 9.70 | 25.76 | 0.79 | 3.11 |
| 0206 | 2.26 | 209.41 | 182.88 | 2.63 | 8.59 | 2.72 | 8.47 | 30.98 | 14.02 | 1.30 | 2.51 |
| 0301 | 3.95 | 74.36 | 70.40 | 7.68 | 21.69 | 4.22 | 3.68 | 75.17 | 133.77 | 2.32 | 13.18 |
| 0302 | 7.53 | 159.87 | 150.26 | 5.85 | 8.86 | 2.98 | 3.64 | 21.65 | 50.81 | 0.98 | 20.42 |
| 0303* | 22.24 | 8.69 | 13.05 | 25.92 | 20.49 | 34.85 | 8.19 | 135.57 | 344.93 | 1.53 | 32.77 |
| 0304 | 4.23 | 19.79 | 19.30 | 6.53 | 6.64 | 2.73 | 1.52 | 8.36 | 40.52 | 0.65 | 120.81 |
| 0305 | 4.84 | 43.61 | 40.94 | 7.26 | 9.05 | 4.49 | 4.89 | 14.99 | 51.12 | 1.11 | 11.74 |
| 0306 | 3.34 | 25.46 | 26.80 | 5.53 | 19.96 | 5.50 | 12.78 | 16.61 | 155.53 | 2.74 | 5.68 |
| 0401 | 6.64 | 9.31 | 15.16 | 52.98 | 67.78 | 11.84 | 20.09 | 736.10 | 79.06 | 4.03 | 26.30 |
| 0402 | 6.35 | 208.05 | 269.90 | 5.88 | 35.63 | 11.96 | 21.78 | 54.33 | 117.48 | 1.39 | 5.60 |
| 0403 | 22.24 | 8.69 | 13.05 | 25.92 | 20.49 | 34.85 | 8.19 | 135.57 | 344.93 | 1.53 | 32.77 |
| 0404 | 3.77 | 111.21 | 100.29 | 11.87 | 12.70 | 4.38 | 3.59 | 53.56 | 336.13 | 1.52 | 14.38 |
| 0405 | 0.85 | 28.86 | 31.35 | 11.44 | 12.02 | 5.86 | 6.92 | 36.07 | 610.27 | 6.15 | 8.35 |
| 0406 | 2.32 | 10292.88 | 5737.41 | 4.62 | 28.97 | 5.15 | 16.36 | 73.08 | 20.83 | 3.13 | 5.77 |

*Estimates from the SSU stratum/ MSG substratum 0403 used as a replacement for those in 0303

# C Appendix Supplement to Chapter 4

## C.1 Design-Based ANOVA Optimization Results

Figure 3. ANOVA Excel Solver Set Up and Results for `income`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 51 | 2 | 8 | 15 | 6 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 45 | 18 | 12 | 7 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 9 | 54 | 11 | 6 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 17 | 29 | 20 | 15 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 11 | 16 | 23 | 118 | 25 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 19 | 19 | 22 | 117 | 91 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 212 | 8 | 35 | 63 | 25 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 4 | 229 | 94 | 62 | 33 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 13 | 19 | 26 | 139 | 30 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 22 | 21 | 24 | 131 | 101 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 82 | 3 | 14 | 25 | 10 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 98 | 40 | 27 | 14 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 8 | 47 | 10 | 6 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 14 | 25 | 17 | 13 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 7 | 10 | 14 | 74 | 16 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 29 | 28 | 32 | 174 | 135 |
| | | | | | | Totals | *468* | *564* | *568* | *1,040* | *548* |

| | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
| 0201 | 83 | 61 | 3% | $ 1,315 | 16 | 2.6 | 2.0 | 1.49 | $ 657.44 | 0.12 | 0.23 | 0.00 | 0.0058 | 9.19E-07 | 15,515 |
| 0202 | 83 | 64 | 3% | $ 1,366 | | 2.0 | 2.0 | 1.55 | $ 680.62 | 0.03 | 0.02 | 0.06 | | 1.96E-05 | 157,023 |
| 0203 | 83 | 65 | 3% | $ 1,377 | | 2.3 | 2.0 | 1.57 | $ 688.72 | 0.13 | 0.37 | 0.02 | | 3.70E-05 | 171,502 |
| 0204 | 83 | 48 | 3% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.17 | 0.07 | | 2.43E-04 | 218,646 |
| 0205 | 193 | 49 | 6% | $ 1,364 | | | 4.7 | 1.20 | $ 292.23 | | 0.03 | 0.13 | | 1.40E-04 | 785,866 |
| 0206 | 267 | 60 | 8% | $ 1,728 | | | 6.5 | 1.44 | $ 267.37 | | 0.11 | 0.08 | | 1.67E-04 | 831,323 |
| 0301 | 344 | 256 | 11% | $ 7,068 | | | 10.8 | 7.99 | | | 0.24 | 0.02 | 0.0016 | 7.03E-06 | 233,359 |
| 0302 | 422 | 327 | 13% | $ 8,978 | | | 13.2 | 10.21 | | $K_{a,SR}$ | 4.41 | 0.02 | | 5.88E-05 | 139,825 |
| 0303 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.36 | 0.69 | 0.02 | | 6.13E-05 | 0 |
| 0304 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.34 | 0.20 | 0.09 | | 6.73E-04 | 386,887 |
| 0305 | 226 | 58 | 7% | $ 2,068 | | | 7.1 | 1.81 | | 0.31 | 0.21 | 0.09 | | 4.41E-04 | 774,329 |
| 0306 | 299 | 67 | 9% | $ 2,499 | | | 9.3 | 2.09 | | | 0.09 | 0.13 | | 3.34E-04 | 1,246,113 |
| 0401 | 134 | 99 | 4% | $ 2,425 | | | 3.7 | 2.74 | | | 0.15 | 0.02 | 0.0054 | 7.51E-06 | 151,555 |
| 0402 | 180 | 140 | 6% | $ 3,388 | | | 5.0 | 3.85 | | | 0.24 | 0.02 | | 1.13E-05 | 142,684 |
| 0403 | 72 | 57 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 0.69 | 0.02 | | 5.42E-05 | 135,176 |
| 0404 | 72 | 42 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 0.35 | 0.05 | | 2.81E-04 | 223,989 |
| 0405 | 121 | 31 | 4% | $ 973 | | | 3.3 | 0.85 | | | 0.08 | 0.08 | | 2.67E-04 | 699,568 |
| 0406 | 398 | 89 | 12% | $ 2,937 | | | 11.0 | 2.45 | | | 0.10 | 0.12 | | 2.62E-04 | 1,292,753 |
| | *3,188* | *1,600* | *100%* | *$1.52E+06* | | | | | | | | Totals | **0.0129** | **0.0031** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0056** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,188 | 15,902 | 19,090 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,811 | 10,411 |
| % NH B | TRUE | 0.3 | Percent | 0.15 | 0.85 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.3 | Count | 3,436 | 3,436 | 3,540 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 3,084,005 |
| | 32767 | | *SSU* | $ 2,600 | $ 17,806.92 | $ 1,571,624 |
| | 0 | | *OCC HU* | $ 850 | $ 1,518,836 | $ 3,247,729 |
| | | | *UNOCC HU* | $ 100 | $ 2,096,643 | $ 7,903,357 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
| 2027 | 81 | 335 | 605 | 239 | 3288 | 2444 | 21% | $ 8,577 | 88 | 2.9 | 13.0 | 9.70 | 0.081 | 4.586 | 9.975 | 0.002 | 0.0055 | 1.32E-08 | 50,441 |
| 28 | 1586 | 650 | 431 | 230 | 2924 | 2264 | 18% | $ 7,897 | | 2.0 | 11.6 | 8.98 | | 1.078 | 1.584 | 0.042 | | 1.22E-06 | 783,493 |
| 15 | 54 | 327 | 69 | 39 | 504 | 396 | 3% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 6.296 | 4.226 | 0.047 | | 2.37E-05 | 963,377 |
| 18 | 101 | 174 | 121 | 90 | 504 | 293 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 2.479 | 0.075 | | 4.83E-05 | 1,246,277 |
| 74 | 105 | 150 | 783 | 168 | 1279 | 328 | 8% | $ 1,483 | | | 5.1 | 1.30 | | | 0.692 | 0.211 | | 9.43E-05 | 4,295,112 |
| 142 | 135 | 158 | 854 | 661 | 1951 | 435 | 12% | $ 2,070 | | | 7.7 | 1.73 | | | 1.503 | 0.174 | | 1.04E-04 | 5,448,707 |
| 217 | 9 | 36 | 65 | 26 | 352 | 262 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 3.738 | 0.015 | 0.0006 | 3.01E-06 | 276,933 |
| 3 | 191 | 78 | 52 | 28 | 352 | 273 | 2% | $ 1,361 | | | 2.0 | 1.55 | | $K_{a,NSR}$ | 5.904 | 0.031 | | 2.03E-05 | 330,172 |
| 10 | 38 | 229 | 48 | 27 | 352 | 277 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.55 | 1.541 | 0.001 | | 4.95E-09 | 2,248 |
| 13 | 70 | 122 | 85 | 63 | 352 | 205 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.31 | 3.240 | 0.070 | | 7.72E-05 | 697,302 |
| 36 | 52 | 74 | 387 | 83 | 631 | 162 | 4% | $ 1,047 | | | 3.6 | 0.92 | | 0.14 | 4.422 | 0.091 | | 2.24E-04 | 1,968,936 |
| 83 | 79 | 92 | 497 | 384 | 1135 | 253 | 7% | $ 1,721 | | | 6.4 | 1.44 | | | 4.910 | 0.106 | | 2.16E-04 | 2,635,977 |
| 217 | 9 | 36 | 65 | 26 | 352 | 262 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 6.458 | 0.004 | 0.0007 | 4.53E-07 | 107,901 |
| 3 | 191 | 78 | 52 | 28 | 352 | 273 | 2% | $ 1,361 | | | 2.0 | 1.55 | | | 6.934 | 0.009 | | 1.93E-06 | 195,082 |
| 10 | 38 | 229 | 48 | 27 | 352 | 277 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 19.722 | 0.004 | | 1.27E-06 | 80,117 |
| 13 | 70 | 122 | 85 | 63 | 352 | 205 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 5.188 | 0.014 | | 4.99E-06 | 260,442 |
| 20 | 29 | 41 | 216 | 46 | 352 | 90 | 2% | $ 584 | | | 2.0 | 0.51 | | | 1.309 | 0.047 | | 3.14E-05 | 787,112 |
| 37 | 36 | 42 | 225 | 174 | 514 | 115 | 3% | $ 780 | | | 2.9 | 0.65 | | | 1.693 | 0.059 | | 5.10E-05 | 1,286,740 |
| 2,968 | 2,872 | 2,971 | 4,689 | 2,402 | *15,902* | *8,811* | *100%* | *$3.25E+06* | | | | | | | | Totals | **0.0068** | **0.0009** | **2.14E+07** |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.75 | 0.0056 | 0.0751 | 0.093 | 0.483 | 0.0159 | 0.0086 |

Figure 4. ANOVA Excel Solver Set Up and Results for `wealtha`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 79 | 3 | 13 | 24 | 9 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 69 | 28 | 19 | 10 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 4 | 14 | 83 | 18 | 10 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 5 | 26 | 44 | 31 | 23 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 16 | 22 | 32 | 167 | 36 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 40 | 38 | 44 | 239 | 185 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 69 | 3 | 11 | 21 | 8 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 6 | 330 | 135 | 90 | 48 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 12 | 17 | 91 | 20 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 26 | 25 | 29 | 156 | 120 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 39 | 2 | 7 | 12 | 5 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 40 | 16 | 11 | 6 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 7 | 10 | 14 | 75 | 16 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 19 | 18 | 21 | 111 | 86 |
| | | | | | | Totals | *327* | *650* | *623* | *1,112* | *614* |

| | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_a q_{ab}$ | $Q_{ab}$ |
| 0201 | 128 | 95 | 4% | $ 1,315 | 16 | 4.0 | 2.0 | 1.49 | $ 657.44 | 0.39 | 4.10 | 0.01 | 0.0161 | 2.18E-05 | 15,515 |
| 0202 | 128 | 99 | 4% | $ 1,361 | | 2.0 | 2.0 | 1.55 | $ 680.62 | 0.02 | 0.11 | 0.03 | | 1.88E-05 | 157,023 |
| 0203 | 128 | 101 | 4% | $ 1,377 | | 2.0 | 2.0 | 1.57 | $ 688.72 | 0.15 | 0.61 | 0.02 | | 2.68E-05 | 171,502 |
| 0204 | 128 | 74 | 4% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 1.10 | 0.10 | | 2.30E-03 | 218,646 |
| 0205 | 273 | 70 | 8% | $ 1,247 | | | 4.3 | 1.09 | $ 292.23 | | 0.17 | 0.16 | | 9.80E-04 | 785,866 |
| 0206 | 545 | 122 | 16% | $ 2,275 | | | 8.5 | 1.90 | $ 267.37 | | 1.50 | 0.09 | | 1.63E-03 | 831,323 |
| 0301 | 112 | 83 | 3% | $ 2,297 | | | 3.5 | 2.60 | | | 1.70 | 0.02 | 0.0013 | 1.55E-04 | 233,359 |
| 0302 | 608 | 471 | 18% | $ 12,932 | | | 19.0 | 14.71 | | $K_{a,SR}$ | 5.56 | 0.08 | | 1.08E-03 | 139,825 |
| 0303 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.41 | 3.91 | 0.01 | | 1.05E-04 | 0 |
| 0304 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.36 | 0.52 | 0.09 | | 1.62E-03 | 386,887 |
| 0305 | 149 | 38 | 4% | $ 1,361 | | | 4.7 | 1.19 | | 0.23 | 0.30 | 0.08 | | 8.54E-04 | 774,329 |
| 0306 | 355 | 79 | 11% | $ 2,969 | | | 11.1 | 2.48 | | | 0.97 | 0.09 | | 1.77E-03 | 1,246,113 |
| 0401 | 64 | 48 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 0.45 | 0.01 | 0.0040 | 2.00E-05 | 151,555 |
| 0402 | 73 | 57 | 2% | $ 1,561 | | | 2.3 | 1.78 | | | 2.17 | 0.01 | | 7.92E-05 | 142,684 |
| 0403 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 3.91 | 0.01 | | 1.05E-04 | 135,176 |
| 0404 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 1.91 | 0.02 | | 4.66E-04 | 223,989 |
| 0405 | 123 | 32 | 4% | $ 1,125 | | | 3.8 | 0.99 | | | 0.23 | 0.09 | | 9.07E-04 | 699,568 |
| 0406 | 254 | 57 | 8% | $ 2,125 | | | 7.9 | 1.77 | | | 0.99 | 0.08 | | 1.96E-03 | 1,292,753 |
| | *3,326* | *1,600* | *100%* | *$1.53E+06* | | | | | | | | Totals | **0.0214** | **0.0141** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0138** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,326 | 15,543 | 18,869 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,544 | 10,144 |
| % NH B | TRUE | 0.3 | Percent | 0.16 | 0.84 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.3 | Count | 3,347 | 3,347 | 3,449 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,990,300 |
| | 32767 | | *SSU* | $ 2,600 | $ 20,811.09 | $ 1,458,326 |
| | 0 | | *OCC HU* | $ 850 | $ 1,532,618 | $ 3,437,945 |
| | | | *UNOCC HU* | $ 100 | $ 2,113,429 | $ 7,886,571 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_a q_{ab}$ | $Q_{ab}$ |
| 1994 | 80 | 330 | 595 | 235 | 3233 | 2403 | 21% | $ 9,700 | 85 | 2.6 | 14.8 | 10.97 | 0.082 | 6.158 | 132.910 | 0.001 | 0.0081 | 2.58E-08 | 50,441 |
| 22 | 1222 | 501 | 332 | 177 | 2254 | 1745 | 15% | $ 7,000 | | 2.0 | 10.3 | 7.96 | | 6.775 | 1.966 | 0.171 | | 3.28E-05 | 783,493 |
| 13 | 47 | 285 | 60 | 34 | 438 | 344 | 3% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 18.23 | 22.014 | 0.055 | | 1.96E-04 | 963,377 |
| 16 | 88 | 151 | 106 | 78 | 438 | 255 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 13.202 | 0.081 | | 3.39E-04 | 1,246,277 |
| 62 | 88 | 126 | 657 | 141 | 1074 | 275 | 7% | $ 1,432 | | | 4.9 | 1.26 | | | 5.370 | 0.160 | | 5.00E-04 | 4,295,112 |
| 115 | 109 | 127 | 690 | 534 | 1576 | 352 | 10% | $ 1,922 | | | 7.2 | 1.60 | | | 45.555 | 0.069 | | 6.21E-04 | 5,448,707 |
| 315 | 13 | 52 | 94 | 37 | 511 | 380 | 3% | $ 1,966 | | | 3.0 | 2.22 | | | 30.844 | 0.024 | 0.0057 | 4.87E-05 | 276,933 |
| 7 | 414 | 169 | 112 | 60 | 763 | 590 | 5% | $ 3,037 | | | 4.5 | 3.45 | | $K_{a,NSR}$ | 92.253 | 0.035 | | 1.87E-04 | 330,172 |
| 10 | 37 | 222 | 47 | 27 | 342 | 268 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.54 | 6.968 | 0.000 | | 2.36E-09 | 2,248 |
| 12 | 68 | 118 | 82 | 61 | 342 | 199 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.38 | 9.889 | 0.143 | | 1.02E-03 | 697,302 |
| 37 | 53 | 75 | 394 | 85 | 643 | 165 | 4% | $ 1,100 | | | 3.8 | 0.96 | | 0.08 | 11.483 | 0.095 | | 6.30E-04 | 1,968,936 |
| 83 | 79 | 92 | 500 | 387 | 1141 | 255 | 7% | $ 1,786 | | | 6.7 | 1.49 | | | 37.958 | 0.083 | | 1.03E-03 | 2,635,977 |
| 211 | 8 | 35 | 63 | 25 | 342 | 254 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 71.617 | 0.004 | 0.0007 | 5.70E-06 | 107,901 |
| 3 | 185 | 76 | 50 | 27 | 342 | 265 | 2% | $ 1,361 | | | 2.0 | 1.55 | | | 206.311 | 0.003 | | 5.01E-06 | 195,082 |
| 10 | 37 | 222 | 47 | 27 | 342 | 268 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 5.518 | 0.006 | | 7.37E-07 | 80,117 |
| 12 | 68 | 118 | 82 | 61 | 342 | 199 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 25.227 | 0.017 | | 3.50E-05 | 260,442 |
| 20 | 28 | 40 | 209 | 45 | 342 | 88 | 2% | $ 584 | | | 2.0 | 0.51 | | | 2.793 | 0.031 | | 2.99E-05 | 787,112 |
| 79 | 75 | 87 | 473 | 366 | 1080 | 241 | 7% | $ 1,690 | | | 6.3 | 1.41 | | | 805.812 | 0.022 | | 1.63E-03 | 1,286,740 |
| 3,020 | 2,698 | 2,826 | 4,595 | 2,405 | *15,543* | *8,544* | *100%* | *$3.44E+06* | | | | | | | | Totals | **0.0146** | **0.0063** | **2.14E+07** |

| | | **Summary of Solution** | | | | | | |
|---|---|---|---|---|---|---|---|---|
| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
| 100 | 100 | 1.75 | 0.0138 | 0.1174 | 0.099 | 0.470 | 0.0355 | 0.0218 |

Figure 5. ANOVA Excel Solver Set Up and Results for `wealthb`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d* *(SR PSUs)* | | | | | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 70 | 3 | 12 | 21 | 8 | 114 | 85 | 3% | $ 1,315 | 16 | 3.6 | 2.0 | 1.49 | $ 657.44 | 0.32 | 0.13 | 0.030 | 0.0160 | 2.10E-05 | 15,515 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 62 | 25 | 17 | 9 | 114 | 88 | 3% | $ 1,361 | | 2.0 | 2.0 | 1.55 | $ 680.62 | 0.01 | 0.11 | 0.031 | | 1.86E-05 | 157,023 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 12 | 74 | 16 | 9 | 114 | 89 | 3% | $ 1,377 | | 2.0 | 2.0 | 1.57 | $ 688.72 | 0.14 | 0.65 | 0.018 | | 3.76E-05 | 171,502 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 23 | 39 | 27 | 20 | 114 | 66 | 3% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 1.13 | 0.092 | | 2.34E-03 | 218,646 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 15 | 21 | 30 | 158 | 34 | 258 | 66 | 8% | $ 1,325 | | | 4.5 | 1.16 | $ 292.23 | | 0.14 | 0.159 | | 8.53E-04 | 785,866 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 39 | 37 | 43 | 235 | 182 | 537 | 120 | 16% | $ 2,521 | | | 9.4 | 2.10 | $ 267.37 | | 1.53 | 0.089 | | 1.63E-03 | 831,323 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 106 | 4 | 18 | 32 | 13 | 172 | 128 | 5% | $ 3,536 | | | 5.4 | 4.00 | | | 1.69 | 0.020 | 0.0008 | 8.71E-05 | 233,359 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 6 | 328 | 134 | 89 | 47 | 605 | 468 | 18% | $ 12,863 | | | 18.9 | 14.63 | | $K_{a,SR}$ | 5.46 | 0.070 | | 9.23E-04 | 139,825 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.42 | 2.79 | 0.010 | | 8.99E-05 | 0 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.35 | 0.50 | 0.080 | | 1.39E-03 | 386,887 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 12 | 17 | 91 | 19 | 148 | 38 | 4% | $ 1,352 | | | 4.6 | 1.19 | | 0.23 | 0.27 | 0.081 | | 7.28E-04 | 774,329 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 26 | 25 | 29 | 158 | 122 | 360 | 80 | 11% | $ 3,012 | | | 11.3 | 2.51 | | | 0.95 | 0.094 | | 1.66E-03 | 1,246,113 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 39 | 2 | 7 | 12 | 5 | 64 | 48 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 0.35 | 0.014 | 0.0035 | 2.22E-05 | 151,555 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 41 | 17 | 11 | 6 | 76 | 59 | 2% | $ 1,621 | | | 2.4 | 1.84 | | | 2.14 | 0.011 | | 6.55E-05 | 142,684 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 2.79 | 0.010 | | 8.99E-05 | 135,176 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 1.88 | 0.022 | | 3.99E-04 | 223,989 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 7 | 10 | 14 | 75 | 16 | 122 | 31 | 4% | $ 1,115 | | | 3.8 | 0.98 | | | 0.24 | 0.084 | | 8.88E-04 | 699,568 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 19 | 18 | 21 | 115 | 89 | 262 | 58 | 8% | $ 2,190 | | | 8.2 | 1.83 | | | 1.08 | 0.084 | | 2.09E-03 | 1,292,753 |
| | | | | | | Totals | *354* | *638* | *608* | *1,104* | *612* | *3,316* | *1,600* | *100%* | *$1.53E+06* | | | | | | | | Totals | **0.0203** | **0.0133** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | |
|---|---|---|
| **Total RelVariance** | **0.0147** | |
| # of Parameters | 43 | Contraints |
| SR PSU | Constant | 16 |
| NSR PSU min | TRUE | 25 |
| NSR SSU per ab min | TRUE | 2 |
| NSR HU per ab min | TRUE | 2 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 |
| MAX Budget | TRUE | 10000000 |
| % H | TRUE | 0.3 |
| SR MAX HUs per PSU | TRUE | 100.0 |
| SR MIN HUs per PSU | TRUE | 50.0 |
| % NH B | TRUE | 0.3 |
| NSR MAX HUs per PSU | TRUE | 100.0 |
| NSR MIN HUs per PSU | TRUE | 50.0 |
| % NH O | TRUE | 0.3 |
| deff | TRUE | 1.75 |
| SR SSU per ab min | TRUE | 2.0 |
| SR HU per ab min | TRUE | 2.0 |
| | 32767 | |
| | 0 | |

| Population Size | | | |
|---|---|---|---|
| *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| Percent | 0.26 | 0.74 | 1.00 |
| **Total Needed To Be Screened** | | | |
| | SR | NSR | Total |
| | 3,316 | 15,565 | 18,882 |
| **Actual Achieved Allocation** | | | |
| *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| Count | 1,600 | 8,594 | 10,194 |
| Percent | 0.16 | 0.84 | 1 |
| **Sample Size By Demographic Domain *d*** | | | |
| | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| Count | 3,364 | 3,364 | 3,466 |
| Percent | 0.33 | 0.33 | 0.34 |
| **COST** | Unit Cost | SR COST | NSR COST |
| *PSU* | $ 35,000 | $ 560,000 | $ 3,007,791 |
| *SSU* | $ 2,600 | $ 19,653.35 | $ 1,492,253 |
| *OCC HU* | $ 850 | $ 1,531,632 | $ 3,388,670 |
| *UNOCC HU* | $ 100 | $ 2,111,285 | $ 7,888,714 |
| | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d* *(NSR PSUs)* | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
| 2022 | 81 | 334 | 604 | 238 | 3280 | 2438 | 21% | $ 9,366 | 86 | 2.7 | 14.2 | 10.59 | 0.082 | 8.523 | 101.722 | 0.001 | 0.0102 | 2.41E-08 | 50,441 |
| 19 | 1078 | 442 | 293 | 156 | 1988 | 1539 | 13% | $ 5,879 | | 2.0 | 8.6 | 6.69 | | 7.152 | 1.929 | 0.157 | | 3.09E-05 | 783,493 |
| 13 | 49 | 299 | 63 | 36 | 460 | 361 | 3% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 6.089 | 21.712 | 0.052 | | 1.62E-04 | 963,377 |
| 17 | 92 | 159 | 111 | 82 | 460 | 267 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 13.862 | 0.077 | | 3.08E-04 | 1,246,277 |
| 65 | 92 | 131 | 686 | 147 | 1121 | 287 | 7% | $ 1,423 | | | 4.9 | 1.25 | | | 5.741 | 0.159 | | 5.05E-04 | 4,295,112 |
| 118 | 112 | 131 | 708 | 548 | 1617 | 361 | 10% | $ 1,878 | | | 7.0 | 1.57 | | | 37.677 | 0.078 | | 6.42E-04 | 5,448,707 |
| 278 | 11 | 46 | 83 | 33 | 451 | 336 | 3% | $ 1,727 | | | 2.6 | 1.95 | | | 28.757 | 0.024 | 0.0063 | 4.79E-05 | 276,933 |
| 10 | 581 | 238 | 158 | 84 | 1072 | 830 | 7% | $ 4,245 | | | 6.2 | 4.83 | | $K_{a,NSR}$ | 169.427 | 0.041 | | 3.44E-04 | 330,172 |
| 10 | 37 | 223 | 47 | 27 | 344 | 270 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.52 | 4.549 | 0.000 | | 1.95E-09 | 2,248 |
| 12 | 69 | 119 | 83 | 61 | 344 | 200 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.39 | 12.792 | 0.148 | | 1.40E-03 | 697,302 |
| 37 | 52 | 74 | 390 | 84 | 637 | 163 | 4% | $ 1,083 | | | 3.7 | 0.95 | | 0.09 | 9.812 | 0.097 | | 5.68E-04 | 1,968,936 |
| 79 | 76 | 88 | 477 | 369 | 1088 | 243 | 7% | $ 1,693 | | | 6.3 | 1.41 | | | 33.647 | 0.080 | | 8.95E-04 | 2,635,977 |
| 212 | 8 | 35 | 63 | 25 | 344 | 255 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 66.191 | 0.004 | 0.0003 | 4.70E-06 | 107,901 |
| 3 | 186 | 76 | 51 | 27 | 344 | 266 | 2% | $ 1,361 | | | 2.0 | 1.55 | | | 205.991 | 0.003 | | 4.86E-06 | 195,082 |
| 10 | 37 | 223 | 47 | 27 | 344 | 270 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 4.134 | 0.006 | | 6.08E-07 | 80,117 |
| 12 | 69 | 119 | 83 | 61 | 344 | 200 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 32.256 | 0.017 | | 4.87E-05 | 260,442 |
| 20 | 28 | 40 | 210 | 45 | 344 | 88 | 2% | $ 584 | | | 2.0 | 0.51 | | | 2.794 | 0.028 | | 2.47E-05 | 787,112 |
| 72 | 68 | 80 | 431 | 334 | 984 | 220 | 6% | $ 1,531 | | | 5.7 | 1.28 | | | 447.339 | 0.027 | | 1.49E-03 | 1,286,740 |
| 3,010 | 2,726 | 2,857 | 4,588 | 2,383 | *15,565* | *8,594* | *100%* | *$3.39E+06* | | | | | | | | Totals | **0.0168** | **0.0065** | **2.14E+07** |

| Summary of Solution | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
| 100 | 100 | 1.75 | 0.0147 | 0.1214 | 0.097 | 0.475 | 0.0337 | 0.0242 |

Figure 6. ANOVA Excel Solver Set Up and Results for `other_debts`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d* (SR PSUs) | | | | | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 39 | 2 | 7 | 12 | 5 | 64 | 48 | 2% | $ 1,315 | 16 | 2.0 | 2.0 | 1.49 | $ 657.44 | 0.030 | 0.676 | 0.003 | 0.0013 | 1.69E-06 | 15,515 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 72 | 30 | 20 | 10 | 134 | 103 | 4% | $ 2,843 | | 2.8 | 4.2 | 3.23 | $ 680.62 | 0.044 | 0.178 | 0.030 | | 2.55E-05 | 157,023 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 | 64 | 50 | 2% | $ 1,377 | | 3.2 | 2.0 | 1.57 | $ 688.72 | 0.088 | 0.567 | 0.021 | | 8.10E-05 | 171,502 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.244 | 0.039 | | 1.63E-04 | 218,646 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 10 | 14 | 20 | 105 | 23 | 172 | 44 | 5% | $ 1,568 | | | 5.4 | 1.38 | $ 292.23 | | 0.146 | 0.096 | | 4.86E-04 | 785,866 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 22 | 21 | 24 | 132 | 102 | 302 | 67 | 8% | $ 2,523 | | | 9.4 | 2.11 | $ 267.37 | | 0.216 | 0.099 | | 5.07E-04 | 831,323 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 207 | 8 | 34 | 62 | 24 | 336 | 250 | 9% | $ 4,891 | | | 7.4 | 5.53 | | | 1.549 | 0.022 | 0.0023 | 4.60E-05 | 233,359 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 116 | 47 | 31 | 17 | 214 | 165 | 6% | $ 3,219 | | | 4.7 | 3.66 | | $K_{a,SR}$ | 1.227 | 0.018 | | 3.94E-05 | 139,825 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 10 | 59 | 12 | 7 | 90 | 71 | 3% | $ 1,377 | | | 2.0 | 1.57 | | 0.29 | 1.074 | 0.016 | | 5.98E-05 | 0 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 18 | 31 | 22 | 16 | 90 | 52 | 3% | $ 1,071 | | | 2.0 | 1.16 | | 0.38 | 0.345 | 0.063 | | 4.18E-04 | 386,887 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 14 | 19 | 28 | 145 | 31 | 237 | 61 | 7% | $ 1,536 | | | 5.3 | 1.35 | | 0.32 | 0.115 | 0.148 | | 6.57E-04 | 774,329 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 43 | 41 | 48 | 258 | 199 | 589 | 131 | 17% | $ 3,483 | | | 13.0 | 2.91 | | | 0.504 | 0.126 | | 9.76E-04 | 1,246,113 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 99 | 4 | 16 | 30 | 12 | 161 | 120 | 5% | $ 2,089 | | | 3.2 | 2.36 | | | 1.440 | 0.013 | 0.0029 | 3.09E-05 | 151,555 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 66 | 27 | 18 | 10 | 121 | 94 | 3% | $ 1,631 | | | 2.4 | 1.85 | | | 0.236 | 0.025 | | 2.44E-05 | 142,684 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 11 | 66 | 14 | 8 | 101 | 80 | 3% | $ 1,377 | | | 2.0 | 1.57 | | | 1.074 | 0.016 | | 5.33E-05 | 135,176 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 20 | 35 | 24 | 18 | 101 | 59 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 0.278 | 0.030 | | 6.83E-05 | 223,989 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 11 | 16 | 23 | 119 | 26 | 195 | 50 | 5% | $ 1,123 | | | 3.8 | 0.99 | | | 0.245 | 0.084 | | 5.50E-04 | 699,568 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 38 | 37 | 43 | 231 | 178 | 526 | 117 | 15% | $ 2,779 | | | 10.4 | 2.32 | | | 0.280 | 0.152 | | 8.80E-04 | 1,292,753 |
| | | | | | | Totals | *505* | *494* | *601* | *1,259* | *702* | *3,562* | *1,600* | *100%* | *$1.56E+06* | | | | | | | | Totals | **0.0065** | **0.0051** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | | No. of Eligible HUs in SSU/MSG *ab*, Domain *d* (NSR PSUs) | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0019** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 | 1417 | 57 | 234 | 423 | 167 | 2298 | 1708 | 15% | $ 9,025 | 81 | 2.1 | 13.7 | 10.20 | -0.045 | 0.263 | 38.688 | 0.002 | 0.0006 | 6.62E-08 | 50,441 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 | 20 | 1148 | 471 | 312 | 166 | 2117 | 1639 | 14% | $ 8,609 | | 2.0 | 12.6 | 9.79 | | 0.679 | 3.273 | 0.044 | | 3.92E-06 | 783,493 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 | 10 | 36 | 217 | 46 | 26 | 335 | 263 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | -0.805 | 3.977 | 0.055 | | 4.61E-05 | 963,377 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 | 12 | 67 | 116 | 81 | 60 | 335 | 194 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 3.885 | 0.069 | | 9.64E-05 | 1,246,277 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 | 63 | 90 | 128 | 672 | 144 | 1098 | 281 | 7% | $ 1,916 | | | 6.6 | 1.68 | | | 2.817 | 0.188 | | 3.56E-04 | 4,295,112 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | | 152 | 144 | 168 | 912 | 706 | 2082 | 465 | 13% | $ 3,326 | | | 12.4 | 2.78 | | | 2.884 | 0.249 | | 3.85E-04 | 5,448,707 |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total | 422 | 17 | 70 | 126 | 50 | 685 | 509 | 4% | $ 2,777 | | | 4.2 | 3.14 | | | 5.833 | 0.016 | 0.0003 | 2.80E-06 | 276,933 |
| MAX Budget | TRUE | 10000000 | | 3,562 | 15,434 | 18,996 | 8 | 464 | 190 | 126 | 67 | 855 | 662 | 6% | $ 3,589 | | | 5.3 | 4.08 | | $K_{a,NSR}$ | 7.104 | 0.020 | | 4.10E-06 | 330,172 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | | 9 | 35 | 211 | 45 | 25 | 324 | 255 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.61 | 24.534 | 0.003 | | 1.13E-06 | 2,248 |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* | 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.27 | 8.318 | 0.038 | | 6.25E-05 | 697,302 |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,109 | 9,709 | 46 | 66 | 94 | 495 | 106 | 808 | 207 | 5% | $ 1,456 | | | 5.0 | 1.28 | | 0.12 | 9.222 | 0.077 | | 2.63E-04 | 1,968,936 |
| % NH B | TRUE | 0.3 | Percent | 0.16 | 0.84 | 1 | 114 | 109 | 127 | 685 | 530 | 1565 | 349 | 10% | $ 2,580 | | | 9.6 | 2.15 | | | 6.689 | 0.123 | | 2.91E-04 | 2,635,977 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | | 317 | 13 | 52 | 95 | 37 | 515 | 383 | 3% | $ 2,087 | | | 3.2 | 2.36 | | | 62.178 | 0.003 | -0.0001 | 1.71E-06 | 107,901 |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* | 4 | 226 | 93 | 61 | 33 | 416 | 322 | 3% | $ 1,748 | | | 2.6 | 1.99 | | | 5.162 | 0.012 | | 2.13E-06 | 195,082 |
| % NH O | TRUE | 0.3 | Count | 3,204 | 3,204 | 3,301 | 9 | 35 | 211 | 45 | 25 | 324 | 255 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 24.534 | 0.003 | | 1.13E-06 | 80,117 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 | 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 10.359 | 0.012 | | 8.48E-06 | 260,442 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST | 19 | 27 | 38 | 199 | 43 | 324 | 83 | 2% | $ 584 | | | 2.0 | 0.51 | | | 12.397 | 0.026 | | 9.82E-05 | 787,112 |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,838,076 | 51 | 49 | 57 | 308 | 238 | 704 | 157 | 5% | $ 1,160 | | | 4.3 | 0.97 | | | 5.824 | 0.059 | | 1.30E-04 | 1,286,740 |
| | 32767 | | *SSU* | $ 2,600 | $ 20,772.67 | $ 1,278,534 | 2,699 | 2,710 | 2,700 | 4,786 | 2,539 | *15,434* | *8,109* | *100%* | *$3.75E+06* | | | | | | | | Totals | **0.0008** | **0.0018** | **2.14E+07** |
| | 0 | | *OCC HU* | $ 850 | $ 1,556,177 | $ 3,746,439 | | | | | | #Hus/PSU | #Hus/PSU | **Summary of Solution** | | | | | | | | | | | | |
| | | | *UNOCC HU* | $ 100 | $ 2,136,949 | $ 7,863,050 | | | | | | SR | NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) | | | | | | |
| | | | | | Total Cost | $1.00E+07 | | | | | | 100 | 100 | 1.67 | 0.0019 | 0.0434 | 0.067 | 0.550 | 0.0116 | 0.0020 | | | | | | |

Figure 7. ANOVA Excel Solver Set Up and Results for `charity_donate`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 74 | 3 | 12 | 22 | 9 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 89 | 37 | 24 | 13 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 45 | 9 | 5 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 14 | 24 | 17 | 12 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 13 | 19 | 26 | 139 | 30 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 22 | 21 | 25 | 134 | 104 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 136 | 5 | 23 | 41 | 16 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 86 | 35 | 23 | 12 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 18 | 25 | 36 | 188 | 40 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 30 | 29 | 34 | 182 | 141 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 161 | 6 | 27 | 48 | 19 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 83 | 34 | 23 | 12 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 13 | 79 | 17 | 9 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 24 | 42 | 29 | 22 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 13 | 19 | 100 | 21 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 32 | 30 | 35 | 191 | 148 |
| | | | | | | Totals | *518* | *488* | *595* | *1,211* | *630* |

| | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
| 0201 | 121 | 90 | 4% | $ 2,312 | 16 | 2.1 | 3.5 | 2.61 | $ 657.44 | 0.060 | 0.378 | 0.006 | 0.0034 | 2.10E-06 | 15,515 |
| 0202 | 164 | 127 | 5% | $ 3,259 | | 2.0 | 4.8 | 3.71 | $ 680.62 | -0.034 | 0.250 | 0.038 | | 4.60E-05 | 157,023 |
| 0203 | 69 | 54 | 2% | $ 1,377 | | 3.8 | 2.0 | 1.57 | $ 688.72 | 0.194 | 0.502 | 0.036 | | 1.88E-04 | 171,502 |
| 0204 | 69 | 40 | 2% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.359 | 0.051 | | 3.67E-04 | 218,646 |
| 0205 | 226 | 58 | 7% | $ 1,926 | | | 6.6 | 1.69 | $ 292.23 | | 0.191 | 0.151 | | 1.20E-03 | 785,866 |
| 0206 | 306 | 68 | 9% | $ 2,385 | | | 8.9 | 1.99 | $ 267.37 | | 0.598 | 0.083 | | 9.53E-04 | 831,323 |
| 0301 | 221 | 164 | 6% | $ 4,542 | | | 6.9 | 5.14 | | | 0.745 | 0.030 | -0.0014 | 6.66E-05 | 233,359 |
| 0302 | 158 | 122 | 5% | $ 3,354 | | | 4.9 | 3.81 | | $K_{a,SR}$ | 0.623 | 0.018 | | 2.51E-05 | 139,825 |
| 0303 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.35 | 0.664 | 0.025 | | 1.33E-04 | 0 |
| 0304 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.29 | 0.249 | 0.093 | | 9.32E-04 | 386,887 |
| 0305 | 308 | 79 | 9% | $ 2,811 | | | 9.6 | 2.47 | | 0.36 | 1.015 | 0.090 | | 1.65E-03 | 774,329 |
| 0306 | 415 | 93 | 12% | $ 3,470 | | | 13.0 | 2.90 | | | 4.025 | 0.044 | | 1.34E-03 | 1,246,113 |
| 0401 | 262 | 195 | 8% | $ 2,845 | | | 4.3 | 3.22 | | | 7.058 | 0.009 | 0.0067 | 5.20E-05 | 151,555 |
| 0402 | 153 | 118 | 4% | $ 1,719 | | | 2.5 | 1.96 | | | 0.558 | 0.013 | | 1.33E-05 | 142,684 |
| 0403 | 121 | 95 | 4% | $ 1,377 | | | 2.0 | 1.57 | | | 0.664 | 0.025 | | 7.04E-05 | 135,176 |
| 0404 | 121 | 70 | 4% | $ 1,071 | | | 2.0 | 1.16 | | | 0.619 | 0.033 | | 1.55E-04 | 223,989 |
| 0405 | 163 | 42 | 5% | $ 789 | | | 2.7 | 0.69 | | | 0.311 | 0.086 | | 8.77E-04 | 699,568 |
| 0406 | 435 | 97 | 13% | $ 1,924 | | | 7.2 | 1.61 | | | 0.293 | 0.170 | | 1.39E-03 | 1,292,753 |
| | *3,441* | *1,600* | *100%* | *$1.54E+06* | | | | | | | | Totals | **0.0088** | **0.0095** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0044** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,441 | 15,451 | 18,892 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,537 | 10,137 |
| % NH B | TRUE | 0.3 | Percent | 0.16 | 0.84 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.3 | Count | 3,345 | 3,345 | 3,447 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,988,104 |
| | 32767 | | *SSU* | $ 2,600 | $ 20,615.17 | $ 1,439,915 |
| | 0 | | *OCC HU* | $ 850 | $ 1,544,098 | $ 3,447,282 |
| | | | *UNOCC HU* | $ 100 | $ 2,124,713 | $ 7,875,301 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
| 1911 | 76 | 316 | 570 | 225 | 3098 | 2303 | 20% | $ 9,594 | 85 | 2.5 | 14.6 | 10.85 | 0.046 | 1.038 | 7.532 | 0.005 | 0.0018 | 6.69E-08 | 50,441 |
| 29 | 1622 | 665 | 441 | 235 | 2991 | 2315 | 19% | $ 9,588 | | 2.0 | 14.1 | 10.91 | | -0.391 | 9.589 | 0.037 | | 5.53E-06 | 783,493 |
| 12 | 45 | 276 | 58 | 33 | 425 | 333 | 3% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 1.092 | 6.321 | 0.070 | | 9.32E-05 | 963,377 |
| 15 | 85 | 147 | 102 | 76 | 425 | 247 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 4.164 | 0.109 | | 1.99E-04 | 1,246,277 |
| 63 | 90 | 128 | 670 | 144 | 1094 | 280 | 7% | $ 1,506 | | | 5.2 | 1.32 | | | 4.361 | 0.224 | | 7.82E-04 | 4,295,112 |
| 128 | 122 | 142 | 769 | 595 | 1755 | 392 | 11% | $ 2,210 | | | 8.3 | 1.84 | | | 9.505 | 0.172 | | 7.17E-04 | 5,448,707 |
| 213 | 8 | 35 | 63 | 25 | 345 | 256 | 2% | $ 1,327 | | | 2.0 | 1.50 | | | 30.301 | 0.013 | -0.0002 | 2.02E-05 | 276,933 |
| 3 | 190 | 78 | 52 | 27 | 350 | 271 | 2% | $ 1,396 | | | 2.1 | 1.59 | | $K_{a,NSR}$ | 8.864 | 0.025 | | 2.12E-05 | 330,172 |
| 10 | 37 | 222 | 47 | 26 | 341 | 268 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.60 | 22.851 | 0.008 | | 4.93E-06 | 2,248 |
| 12 | 68 | 118 | 82 | 61 | 341 | 198 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.28 | 6.101 | 0.060 | | 1.09E-04 | 697,302 |
| 45 | 64 | 92 | 480 | 103 | 785 | 201 | 5% | $ 1,343 | | | 4.6 | 1.18 | | 0.11 | 10.647 | 0.105 | | 5.80E-04 | 1,968,936 |
| 91 | 86 | 100 | 544 | 421 | 1241 | 277 | 8% | $ 1,944 | | | 7.3 | 1.62 | | | 25.078 | 0.077 | | 5.32E-04 | 2,635,977 |
| 211 | 8 | 35 | 63 | 25 | 341 | 254 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 108.827 | 0.004 | 0.0001 | 7.41E-06 | 107,901 |
| 3 | 185 | 76 | 50 | 27 | 341 | 264 | 2% | $ 1,361 | | | 2.0 | 1.55 | | | 44.244 | 0.007 | | 7.47E-06 | 195,082 |
| 10 | 37 | 222 | 47 | 26 | 341 | 268 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 22.851 | 0.008 | | 4.93E-06 | 80,117 |
| 12 | 68 | 118 | 82 | 61 | 341 | 198 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 13.329 | 0.018 | | 2.27E-05 | 260,442 |
| 20 | 28 | 40 | 209 | 45 | 341 | 88 | 2% | $ 584 | | | 2.0 | 0.51 | | | 13.619 | 0.032 | | 1.60E-04 | 787,112 |
| 40 | 38 | 45 | 242 | 187 | 552 | 123 | 4% | $ 864 | | | 3.2 | 0.72 | | | 37.076 | 0.028 | | 2.37E-04 | 1,286,740 |
| 2,828 | 2,858 | 2,852 | 4,572 | 2,342 | *15,451* | *8,537* | *100%* | *$3.45E+06* | | | | | | | | Totals | **0.0017** | **0.0035** | **2.14E+07** |

| | | **Summary of Solution** | | | | | | |
|---|---|---|---|---|---|---|---|---|
| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
| 100 | 100 | 1.74 | 0.0044 | 0.0662 | 0.075 | 0.529 | 0.0182 | 0.0057 |

# Figure 8. ANOVA Excel Solver Set Up and Results for `employed`

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$: *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d* *(SR PSUs)*: 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 59 | 2 | 10 | 18 | 7 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 68 | 28 | 18 | 10 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 43 | 9 | 5 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 23 | 16 | 12 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 12 | 18 | 92 | 20 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 19 | 18 | 21 | 111 | 86 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 201 | 8 | 33 | 60 | 24 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 84 | 34 | 23 | 12 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 11 | 68 | 14 | 8 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 21 | 36 | 25 | 19 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 10 | 14 | 20 | 105 | 23 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 38 | 36 | 42 | 228 | 176 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 135 | 5 | 22 | 40 | 16 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 74 | 30 | 20 | 11 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 13 | 77 | 16 | 9 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 24 | 41 | 29 | 21 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 10 | 15 | 21 | 110 | 24 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 34 | 33 | 38 | 206 | 159 |
| | | | | | | Totals | *538* | *458* | *605* | *1,141* | *641* |

| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation: $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | ANOVA Variance Components Estimates, SR: Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 96 | 71 | 3% | $ 1,929 | 16 | 2.0 | 2.9 | 2.18 | $ 657.44 | 0.051 | 0.139 | 0.002 | 0.0017 | 1.82E-07 | 15,515 |
| 0202 | 125 | 97 | 4% | $ 2,601 | | 3.3 | 3.8 | 2.96 | $ 680.62 | 0.061 | 0.145 | 0.027 | | 1.75E-05 | 157,023 |
| 0203 | 66 | 51 | 2% | $ 1,377 | | 3.7 | 2.0 | 1.57 | $ 688.72 | 0.097 | 0.326 | 0.023 | | 5.29E-05 | 171,502 |
| 0204 | 66 | 38 | 2% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.212 | 0.032 | | 9.21E-05 | 218,646 |
| 0205 | 151 | 39 | 4% | $ 1,347 | | | 4.6 | 1.18 | $ 292.23 | | 0.079 | 0.096 | | 3.00E-04 | 785,866 |
| 0206 | 254 | 57 | 8% | $ 2,075 | | | 7.8 | 1.73 | $ 267.37 | | 0.226 | 0.072 | | 3.32E-04 | 831,323 |
| 0301 | 326 | 242 | 10% | $ 4,095 | | | 6.2 | 4.63 | | | 0.451 | 0.036 | 0.0027 | 3.87E-05 | 233,359 |
| 0302 | 155 | 120 | 5% | $ 2,011 | | | 3.0 | 2.29 | | $K_{a,SR}$ | 0.408 | 0.014 | | 1.14E-05 | 139,825 |
| 0303 | 105 | 82 | 3% | $ 1,377 | | | 2.0 | 1.57 | | 0.26 | 0.313 | 0.023 | | 3.36E-05 | 0 |
| 0304 | 105 | 61 | 3% | $ 1,071 | | | 2.0 | 1.16 | | 0.39 | 0.249 | 0.048 | | 1.53E-04 | 386,887 |
| 0305 | 171 | 44 | 5% | $ 956 | | | 3.3 | 0.84 | | 0.36 | 0.099 | 0.105 | | 3.97E-04 | 774,329 |
| 0306 | 521 | 116 | 15% | $ 2,660 | | | 9.9 | 2.22 | | | 0.192 | 0.170 | | 7.69E-04 | 1,246,113 |
| 0401 | 219 | 163 | 6% | $ 2,427 | | | 3.7 | 2.74 | | | 0.646 | 0.019 | 0.0033 | 2.37E-05 | 151,555 |
| 0402 | 137 | 106 | 4% | $ 1,570 | | | 2.3 | 1.79 | | | 0.177 | 0.021 | | 1.22E-05 | 142,684 |
| 0403 | 119 | 93 | 4% | $ 1,377 | | | 2.0 | 1.57 | | | 0.313 | 0.023 | | 2.97E-05 | 135,176 |
| 0404 | 119 | 69 | 4% | $ 1,071 | | | 2.0 | 1.16 | | | 0.075 | 0.038 | | 2.52E-05 | 223,989 |
| 0405 | 179 | 46 | 5% | $ 882 | | | 3.0 | 0.77 | | | 0.170 | 0.085 | | 4.24E-04 | 699,568 |
| 0406 | 470 | 105 | 14% | $ 2,116 | | | 7.9 | 1.77 | | | 0.162 | 0.163 | | 6.58E-04 | 1,292,753 |
| | *3,382* | *1,600* | *100%* | *$1.54E+06* | | | | | | | | Totals | **0.0077** | **0.0034** | **7.61E+06** |

| **EXCEL SOLVER RESULTS** | | | **Population Size** | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0021** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,382 | 15,312 | 18,693 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,349 | 9,949 |
| % NH B | TRUE | 0.3 | Percent | 0.16 | 0.84 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.3 | Count | 3,283 | 3,283 | 3,382 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,921,983 |
| | 32767 | | *SSU* | $ 2,600 | $ 23,472.87 | $ 1,360,236 |
| | 0 | | *OCC HU* | $ 850 | $ 1,538,159 | $ 3,596,150 |
| | | | *UNOCC HU* | $ 100 | $ 2,121,631 | $ 7,878,368 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d* *(NSR PSUs)*: 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab |
|---|---|---|---|---|---|---|---|---|
| 1751 | 70 | 290 | 523 | 206 | 2839 | 2110 | 19% | $ 9,864 |
| 28 | 1549 | 635 | 421 | 224 | 2857 | 2212 | 19% | $ 10,277 |
| 11 | 40 | 246 | 52 | 29 | 378 | 297 | 2% | $ 1,377 |
| 14 | 76 | 131 | 91 | 67 | 378 | 220 | 2% | $ 1,071 |
| 62 | 89 | 127 | 663 | 142 | 1083 | 278 | 7% | $ 1,673 |
| 140 | 133 | 155 | 840 | 650 | 1917 | 428 | 13% | $ 2,708 |
| 285 | 11 | 47 | 85 | 34 | 463 | 344 | 3% | $ 1,822 |
| 4 | 246 | 101 | 67 | 36 | 453 | 351 | 3% | $ 1,847 |
| 10 | 36 | 217 | 46 | 26 | 334 | 262 | 2% | $ 1,377 |
| 12 | 67 | 115 | 80 | 59 | 334 | 194 | 2% | $ 1,071 |
| 41 | 58 | 83 | 434 | 93 | 709 | 182 | 5% | $ 1,241 |
| 95 | 90 | 105 | 570 | 441 | 1302 | 291 | 9% | $ 2,085 |
| 206 | 8 | 34 | 61 | 24 | 334 | 248 | 2% | $ 1,315 |
| 3 | 181 | 74 | 49 | 26 | 334 | 259 | 2% | $ 1,361 |
| 10 | 36 | 217 | 46 | 26 | 334 | 262 | 2% | $ 1,377 |
| 12 | 67 | 115 | 80 | 59 | 334 | 194 | 2% | $ 1,071 |
| 19 | 27 | 39 | 204 | 44 | 334 | 86 | 2% | $ 584 |
| 43 | 41 | 48 | 260 | 201 | 594 | 133 | 4% | $ 951 |
| 2,745 | 2,825 | 2,778 | 4,574 | 2,390 | *15,312* | *8,349* | *100%* | *$3.60E+06* |

| Non Self-Representing Optimum Allocation: $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | ANOVA Variance Components Estimates, NSR: $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 83 | 2.3 | 15.0 | 11.15 | -0.031 | 0.457 | 3.683 | 0.003 | 0.0009 | 1.76E-08 | 50,441 |
| | 2.0 | 15.1 | 11.69 | | 0.625 | 1.994 | 0.049 | | 2.13E-06 | 783,493 |
| | 2.0 | 2.0 | 1.57 | | 0.122 | 3.846 | 0.052 | | 3.50E-05 | 963,377 |
| | | 2.0 | 1.16 | | | 2.185 | 0.079 | | 6.22E-05 | 1,246,277 |
| | | 5.7 | 1.47 | | | 2.192 | 0.196 | | 3.02E-04 | 4,295,112 |
| | | 10.1 | 2.26 | | | 3.186 | 0.215 | | 3.43E-04 | 5,448,707 |
| | | 2.8 | 2.06 | | | 4.563 | 0.018 | 0.0003 | 4.46E-06 | 276,933 |
| | | 2.7 | 2.10 | | $K_{a,NSR}$ | 3.252 | 0.023 | | 5.06E-06 | 330,172 |
| | | 2.0 | 1.57 | | 0.59 | 47.487 | 0.003 | | 1.53E-06 | 2,248 |
| | | 2.0 | 1.16 | | 0.29 | 3.391 | 0.048 | | 3.95E-05 | 697,302 |
| | | 4.2 | 1.09 | | 0.12 | 5.698 | 0.087 | | 2.40E-04 | 1,968,936 |
| | | 7.8 | 1.74 | | | 6.846 | 0.108 | | 2.77E-04 | 2,635,977 |
| | | 2.0 | 1.49 | | | 21.174 | 0.006 | 0.0000 | 3.07E-06 | 107,901 |
| | | 2.0 | 1.55 | | | 8.073 | 0.007 | | 1.53E-06 | 195,082 |
| | | 2.0 | 1.57 | | | 47.487 | 0.003 | | 1.53E-06 | 80,117 |
| | | 2.0 | 1.16 | | | 5.331 | 0.018 | | 8.58E-06 | 260,442 |
| | | 2.0 | 0.51 | | | 7.197 | 0.035 | | 1.04E-04 | 787,112 |
| | | 3.6 | 0.79 | | | 6.178 | 0.050 | | 1.18E-04 | 1,286,740 |
| | | | | | | | Totals | **0.0012** | **0.0015** | **2.14E+07** |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.75 | 0.0021 | 0.0457 | 0.079 | 0.518 | 0.0111 | 0.0023 |

Figure 9. ANOVA Excel Solver Set Up and Results for `ownHome`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* | | | | | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 82 | 3 | 14 | 25 | 10 | 133 | 99 | 4% | $ 2,644 | 16 | 2.1 | 4.0 | 2.99 | $ 657.44 | 0.060 | 0.050 | 0.004 | 0.0030 | 1.22E-07 | 15,515 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 62 | 26 | 17 | 9 | 115 | 89 | 4% | $ 2,369 | | 4.5 | 3.5 | 2.69 | $ 680.62 | 0.175 | 0.047 | 0.035 | | 1.03E-05 | 157,023 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 43 | 9 | 5 | 66 | 52 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | $ 688.72 | 0.026 | 0.176 | 0.035 | | 6.69E-05 | 171,502 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 23 | 16 | 12 | 66 | 38 | 2% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.142 | 0.049 | | 1.42E-04 | 218,646 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 11 | 16 | 23 | 119 | 25 | 194 | 50 | 6% | $ 1,710 | | | 5.9 | 1.50 | $ 292.23 | | 0.095 | 0.141 | | 6.10E-04 | 785,866 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 17 | 16 | 18 | 99 | 77 | 227 | 51 | 7% | $ 1,835 | | | 6.9 | 1.53 | $ 267.37 | | 0.453 | 0.055 | | 4.35E-04 | 831,323 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 209 | 8 | 35 | 62 | 25 | 338 | 252 | 10% | $ 3,084 | | | 4.7 | 3.49 | | | 0.298 | 0.047 | 0.0069 | 4.15E-05 | 233,359 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 3 | 157 | 64 | 43 | 23 | 289 | 224 | 9% | $ 2,724 | | | 4.0 | 3.10 | | $K_{a,SR}$ | 0.850 | 0.023 | | 3.25E-05 | 139,825 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 4 | 15 | 94 | 20 | 11 | 144 | 113 | 4% | $ 1,377 | | | 2.0 | 1.57 | | 0.32 | 0.302 | 0.026 | | 2.79E-05 | 0 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 5 | 29 | 50 | 35 | 26 | 144 | 84 | 4% | $ 1,071 | | | 2.0 | 1.16 | | 0.42 | 0.047 | 0.102 | | 9.28E-05 | 386,887 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 14 | 20 | 29 | 153 | 33 | 250 | 64 | 8% | $ 1,012 | | | 3.5 | 0.89 | | 0.26 | 0.195 | 0.143 | | 9.92E-04 | 774,329 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 23 | 22 | 25 | 137 | 106 | 313 | 70 | 10% | $ 1,159 | | | 4.3 | 0.97 | | | 0.363 | 0.093 | | 7.18E-04 | 1,246,113 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 95 | 4 | 16 | 28 | 11 | 153 | 114 | 5% | $ 3,150 | | | 4.8 | 3.56 | | | 0.306 | 0.028 | 0.0008 | 3.45E-05 | 151,555 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 61 | 25 | 17 | 9 | 112 | 87 | 3% | $ 2,391 | | | 3.5 | 2.72 | | | 0.478 | 0.011 | | 1.12E-05 | 142,684 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 0.302 | 0.026 | | 6.30E-05 | 135,176 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 0.153 | 0.033 | | 7.08E-05 | 223,989 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 7 | 10 | 15 | 77 | 17 | 126 | 32 | 4% | $ 1,149 | | | 3.9 | 1.01 | | | 0.154 | 0.078 | | 4.61E-04 | 699,568 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 31 | 29 | 34 | 186 | 144 | 424 | 95 | 13% | $ 3,541 | | | 13.2 | 2.96 | | | 1.034 | 0.072 | | 9.00E-04 | 1,292,753 |
| | | | | | | Totals | *511* | *493* | *596* | *1,066* | *558* | *3,223* | *1,600* | *100%* | *$1.52E+06* | | | | | | | | Totals | **0.0107** | **0.0047** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | | No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0029** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 | 1876 | 75 | 310 | 560 | 221 | 3043 | 2262 | 20% | $ 9,535 | 85 | 2.5 | 14.5 | 10.78 | 0.012 | 0.729 | 15.464 | 0.002 | 0.0013 | 3.46E-08 | 50,441 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 | 29 | 1617 | 663 | 439 | 234 | 2981 | 2308 | 19% | $ 9,671 | | 2.0 | 14.2 | 11.00 | | 0.890 | 1.633 | 0.049 | | 1.71E-06 | 783,493 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 | 12 | 45 | 272 | 58 | 33 | 420 | 329 | 3% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | -0.564 | 1.652 | 0.079 | | 3.10E-05 | 963,377 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 | 15 | 84 | 145 | 101 | 75 | 420 | 244 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 1.087 | 0.113 | | 5.68E-05 | 1,246,277 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 | 67 | 96 | 137 | 715 | 154 | 1167 | 299 | 8% | $ 1,626 | | | 5.6 | 1.43 | | | 2.110 | 0.231 | | 3.76E-04 | 4,295,112 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | | 134 | 128 | 149 | 806 | 623 | 1839 | 410 | 12% | $ 2,344 | | | 8.8 | 1.96 | | | 8.598 | 0.127 | | 3.36E-04 | 5,448,707 |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total | 248 | 10 | 41 | 74 | 29 | 403 | 299 | 3% | $ 1,551 | | | 2.4 | 1.75 | | | 4.120 | 0.022 | 0.0005 | 6.48E-06 | 276,933 |
| MAX Budget | TRUE | 10000000 | | 3,223 | 15,499 | 18,722 | 3 | 185 | 76 | 50 | 27 | 341 | 264 | 2% | $ 1,361 | | | 2.0 | 1.55 | | $K_{a,NSR}$ | 3.652 | 0.023 | | 7.28E-06 | 330,172 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | | 10 | 37 | 222 | 47 | 26 | 341 | 268 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.60 | 8.671 | 0.007 | | 1.56E-06 | 2,248 |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* | 12 | 68 | 118 | 82 | 61 | 341 | 198 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.29 | 1.470 | 0.066 | | 3.22E-05 | 697,302 |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,536 | 10,136 | 42 | 60 | 86 | 452 | 97 | 738 | 189 | 5% | $ 1,263 | | | 4.3 | 1.11 | | 0.10 | 5.602 | 0.103 | | 3.13E-04 | 1,968,936 |
| % NH B | TRUE | 0.3 | Percent | 0.16 | 0.84 | 1 | 90 | 86 | 100 | 540 | 418 | 1233 | 275 | 8% | $ 1,931 | | | 7.2 | 1.61 | | | 12.247 | 0.081 | | 2.92E-04 | 2,635,977 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | | 211 | 8 | 35 | 63 | 25 | 341 | 254 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 38.201 | 0.005 | 0.0000 | 3.79E-06 | 107,901 |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* | 3 | 185 | 76 | 50 | 27 | 341 | 264 | 2% | $ 1,361 | | | 2.0 | 1.55 | | | 23.926 | 0.006 | | 3.29E-06 | 195,082 |
| % NH O | TRUE | 0.3 | Count | 3,345 | 3,345 | 3,446 | 10 | 37 | 222 | 47 | 26 | 341 | 268 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 8.671 | 0.007 | | 1.56E-06 | 80,117 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 | 12 | 68 | 118 | 82 | 61 | 341 | 198 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 4.188 | 0.023 | | 1.13E-05 | 260,442 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST | 20 | 28 | 40 | 209 | 45 | 341 | 88 | 2% | $ 584 | | | 2.0 | 0.51 | | | 8.064 | 0.030 | | 8.33E-05 | 787,112 |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,987,452 | 38 | 36 | 42 | 229 | 177 | 524 | 117 | 3% | $ 820 | | | 3.1 | 0.68 | | | 18.764 | 0.027 | | 1.19E-04 | 1,286,740 |
| | 32767 | | *SSU* | $ 2,600 | $ 22,301.39 | $ 1,433,240 | 2,834 | 2,852 | 2,850 | 4,605 | 2,359 | *15,499* | *8,536* | *100%* | *$3.47E+06* | | | | | | | | Totals | **0.0017** | **0.0017** | **2.14E+07** |
| | 0 | | *OCC HU* | $ 850 | $ 1,522,309 | $ 3,474,697 | | | | | | #Hus/PSU SR | #Hus/PSU NSR | | **Summary of Solution** | | | | | | | | | | | |
| | | | *UNOCC HU* | $ 100 | $ 2,104,610 | $ 7,895,390 | | | | | | | | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) | | | | | | |
| | | | | | Total Cost | $1.00E+07 | | | | | | 100 | 100 | 1.75 | 0.0029 | 0.0541 | 0.062 | 0.564 | 0.0154 | 0.0035 | | | | | | |

Figure 10. ANOVA Excel Solver Set Up and Results for `ownStock`

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$ (d=1) 45-62 H | (d=2) 45-62 NH B | (d=3) 45-62 NH O | (d=4) NOT 45-62 | (d=5) UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d* *(SR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 49 | 2 | 8 | 15 | 6 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 92 | 38 | 25 | 13 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 9 | 52 | 11 | 6 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 16 | 27 | 19 | 14 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 13 | 18 | 94 | 20 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 20 | 19 | 23 | 122 | 95 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 130 | 5 | 22 | 39 | 15 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 128 | 52 | 35 | 18 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 12 | 72 | 15 | 9 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 22 | 38 | 27 | 20 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 20 | 29 | 42 | 217 | 47 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 43 | 41 | 48 | 261 | 202 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 75 | 3 | 12 | 22 | 9 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 66 | 27 | 18 | 10 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 10 | 58 | 12 | 7 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 18 | 31 | 22 | 16 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 13 | 19 | 98 | 21 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 45 | 43 | 50 | 270 | 209 |
| | | | | | | Totals | *425* | *539* | *636* | *1,322* | *736* |

| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | ANOVA Variance Components Estimates, SR Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 80 | 59 | 2% | $ 1,315 | 16 | 2.5 | 2.0 | 1.49 | $ 657.44 | 0.200 | 0.317 | 0.003 | 0.0051 | 7.12E-07 | 15,515 |
| 0202 | 169 | 131 | 5% | $ 2,897 | | 3.5 | 4.3 | 3.30 | $ 680.62 | 0.119 | 1.044 | 0.030 | | 1.17E-04 | 157,023 |
| 0203 | 80 | 62 | 2% | $ 1,377 | | 2.8 | 2.0 | 1.57 | $ 688.72 | 0.224 | 4.746 | 0.011 | | 1.42E-04 | 171,502 |
| 0204 | 80 | 46 | 2% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.464 | 0.056 | | 5.07E-04 | 218,646 |
| 0205 | 154 | 40 | 4% | $ 1,134 | | | 3.9 | 0.99 | $ 292.23 | | 0.183 | 0.108 | | 8.55E-04 | 785,866 |
| 0206 | 279 | 62 | 8% | $ 1,875 | | | 7.0 | 1.57 | $ 267.37 | | 1.237 | 0.054 | | 9.39E-04 | 831,323 |
| 0301 | 211 | 157 | 6% | $ 2,506 | | | 3.8 | 2.83 | | | 2.606 | 0.022 | 0.0076 | 1.28E-04 | 233,359 |
| 0302 | 235 | 182 | 6% | $ 2,892 | | | 4.2 | 3.29 | | $K_{a,SR}$ | 2.028 | 0.029 | | 1.53E-04 | 139,825 |
| 0303 | 111 | 87 | 3% | $ 1,377 | | | 2.0 | 1.57 | | 0.25 | 3.241 | 0.016 | | 1.44E-04 | 0 |
| 0304 | 111 | 64 | 3% | $ 1,071 | | | 2.0 | 1.16 | | 0.47 | 0.092 | 0.145 | | 4.86E-04 | 386,887 |
| 0305 | 355 | 91 | 10% | $ 1,874 | | | 6.4 | 1.64 | | 0.28 | 1.486 | 0.095 | | 2.35E-03 | 774,329 |
| 0306 | 596 | 133 | 16% | $ 2,876 | | | 10.8 | 2.40 | | | 0.712 | 0.171 | | 2.51E-03 | 1,246,113 |
| 0401 | 122 | 91 | 3% | $ 1,795 | | | 2.7 | 2.03 | | | 50.887 | 0.003 | 0.0062 | 6.89E-05 | 151,555 |
| 0402 | 121 | 94 | 3% | $ 1,853 | | | 2.7 | 2.11 | | | 1.247 | 0.020 | | 8.48E-05 | 142,684 |
| 0403 | 89 | 70 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 3.241 | 0.016 | | 1.79E-04 | 135,176 |
| 0404 | 89 | 52 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 0.189 | 0.064 | | 2.36E-04 | 223,989 |
| 0405 | 160 | 41 | 4% | $ 1,049 | | | 3.6 | 0.92 | | | 1.301 | 0.044 | | 9.90E-04 | 699,568 |
| 0406 | 617 | 138 | 17% | $ 3,700 | | | 13.8 | 3.09 | | | 1.576 | 0.114 | | 2.36E-03 | 1,292,753 |
| | *3,659* | *1,600* | *100%* | *$1.57E+06* | | | | | | | | Totals | **0.0189** | **0.0123** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0089** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,659 | 14,979 | 18,638 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,260 | 9,860 |
| % NH B | TRUE | 0.3 | Percent | 0.16 | 0.84 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.3 | Count | 3,254 | 3,254 | 3,352 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,890,988 |
| | 32767 | | *SSU* | $ 2,600 | $ 22,709.73 | $ 1,330,823 |
| | 0 | | *OCC HU* | $ 850 | $ 1,565,891 | $ 3,629,595 |
| | | | *UNOCC HU* | $ 100 | $ 2,148,600 | $ 7,851,407 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d* *(NSR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Non Self-Representing Optimum Allocation $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | ANOVA Variance Components Estimates, NSR $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1811 | 72 | 299 | 540 | 213 | 2936 | 2182 | 20% | $ 10,638 | 83 | 2.2 | 16.2 | 12.03 | 0.054 | 1.302 | 25.991 | 0.002 | 0.0021 | 6.07E-08 | 50,441 |
| 24 | 1367 | 560 | 372 | 198 | 2522 | 1952 | 17% | $ 9,459 | | 2.0 | 13.9 | 10.76 | | 2.783 | 11.312 | 0.049 | | 1.40E-05 | 783,493 |
| 10 | 39 | 236 | 50 | 28 | 363 | 285 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | -2.462 | 32.380 | 0.048 | | 2.61E-04 | 963,377 |
| 13 | 73 | 125 | 87 | 65 | 363 | 211 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 10.953 | 0.122 | | 7.72E-04 | 1,246,277 |
| 63 | 90 | 128 | 671 | 144 | 1096 | 281 | 7% | $ 1,765 | | | 6.0 | 1.55 | | | 8.860 | 0.208 | | 1.36E-03 | 4,295,112 |
| 117 | 112 | 130 | 704 | 545 | 1607 | 359 | 11% | $ 2,368 | | | 8.9 | 1.98 | | | 34.913 | 0.108 | | 1.14E-03 | 5,448,707 |
| 325 | 13 | 54 | 97 | 38 | 527 | 392 | 4% | $ 2,098 | | | 3.2 | 2.37 | | | 93.510 | 0.009 | 0.0025 | 1.74E-05 | 276,933 |
| 5 | 263 | 108 | 72 | 38 | 486 | 376 | 3% | $ 2,000 | | | 2.9 | 2.28 | | $K_{a,NSR}$ | 11.025 | 0.028 | | 2.36E-05 | 330,172 |
| 10 | 35 | 215 | 45 | 26 | 330 | 259 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.54 | 144.460 | 0.003 | | 6.19E-06 | 2,248 |
| 12 | 66 | 114 | 80 | 59 | 330 | 192 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.39 | 5.066 | 0.109 | | 3.11E-04 | 697,302 |
| 46 | 66 | 94 | 490 | 105 | 801 | 205 | 5% | $ 1,417 | | | 4.8 | 1.24 | | 0.07 | 16.546 | 0.125 | | 1.25E-03 | 1,968,936 |
| 102 | 97 | 113 | 615 | 476 | 1404 | 313 | 9% | $ 2,272 | | | 8.5 | 1.90 | | | 29.053 | 0.118 | | 1.30E-03 | 2,635,977 |
| 217 | 9 | 36 | 65 | 26 | 351 | 261 | 2% | $ 1,398 | | | 2.1 | 1.58 | | | 1156.835 | 0.001 | -0.0001 | 7.17E-06 | 107,901 |
| 5 | 257 | 105 | 70 | 37 | 474 | 367 | 3% | $ 1,952 | | | 2.9 | 2.22 | | | 76.104 | 0.011 | | 2.42E-05 | 195,082 |
| 10 | 35 | 215 | 45 | 26 | 330 | 259 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 144.460 | 0.003 | | 6.19E-06 | 80,117 |
| 12 | 66 | 114 | 80 | 59 | 330 | 192 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 54.857 | 0.014 | | 5.90E-05 | 260,442 |
| 19 | 27 | 39 | 202 | 43 | 330 | 85 | 2% | $ 584 | | | 2.0 | 0.51 | | | 36.835 | 0.021 | | 1.92E-04 | 787,112 |
| 29 | 28 | 32 | 174 | 135 | 398 | 89 | 3% | $ 644 | | | 2.4 | 0.54 | | | 70.816 | 0.021 | | 3.36E-04 | 1,286,740 |
| 2,829 | 2,714 | 2,717 | 4,459 | 2,260 | *14,979* | *8,260* | *100%* | *$3.63E+06* | | | | | | | | Totals | **0.0046** | **0.0071** | **2.14E+07** |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.75 | 0.0089 | 0.0944 | 0.110 | 0.447 | 0.0312 | 0.0123 |

Figure 11. ANOVA Excel Solver Set Up and Results for `own_2nd_home`

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$ (d=1) 45-62 H | (d=2) 45-62 NH B | (d=3) 45-62 NH O | (d=4) NOT 45-62 | (d=5) UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 89 | 4 | 15 | 27 | 11 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 109 | 44 | 29 | 16 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 13 | 19 | 27 | 143 | 31 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 29 | 28 | 32 | 174 | 135 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 171 | 7 | 28 | 51 | 20 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 62 | 26 | 17 | 9 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 10 | 15 | 21 | 110 | 24 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 48 | 46 | 53 | 289 | 224 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 125 | 5 | 21 | 37 | 15 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 74 | 30 | 20 | 11 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 7 | 9 | 13 | 70 | 15 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 50 | 47 | 55 | 299 | 231 |
| | | | | | | Totals | *559* | *483* | *558* | *1,341* | *790* |

| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | ANOVA Variance Components Estimates, SR Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_a q_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 145 | 107 | 4% | $ 2,971 | 16 | 2.0 | 4.5 | 3.36 | $ 657.44 | -0.099 | 0.044 | 0.012 | -0.0063 | 8.89E-07 | 15,515 |
| 0202 | 200 | 155 | 5% | $ 4,257 | | 2.0 | 6.3 | 4.84 | $ 680.62 | -0.342 | 2.902 | 0.038 | | 4.28E-04 | 157,023 |
| 0203 | 64 | 50 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | $ 688.72 | -0.120 | 6.536 | 0.027 | | 1.46E-03 | 171,502 |
| 0204 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 6.809 | 0.026 | | 1.93E-03 | 218,646 |
| 0205 | 234 | 60 | 6% | $ 2,138 | | | 7.3 | 1.88 | $ 292.23 | | 0.996 | 0.150 | | 5.94E-03 | 785,866 |
| 0206 | 398 | 89 | 11% | $ 3,326 | | | 12.4 | 2.78 | $ 267.37 | | 3.425 | 0.094 | | 5.44E-03 | 831,323 |
| 0301 | 277 | 206 | 7% | $ 5,685 | | | 8.6 | 6.43 | | | 65.151 | 0.014 | -0.0235 | 1.04E-03 | 233,359 |
| 0302 | 115 | 89 | 3% | $ 2,448 | | | 3.6 | 2.78 | | $K_{a,SR}$ | 5.621 | 0.010 | | 1.09E-04 | 139,825 |
| 0303 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.36 | 1.598 | 0.025 | | 3.11E-04 | 0 |
| 0304 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.37 | 1.514 | 0.083 | | 4.50E-03 | 386,887 |
| 0305 | 180 | 46 | 5% | $ 1,647 | | | 5.6 | 1.44 | | 0.27 | 2.696 | 0.071 | | 4.67E-03 | 774,329 |
| 0306 | 661 | 147 | 18% | $ 5,520 | | | 20.6 | 4.61 | | | 2.586 | 0.187 | | 9.83E-03 | 1,246,113 |
| 0401 | 202 | 150 | 5% | $ 4,160 | | | 6.3 | 4.70 | | | 3.873 | 0.033 | -0.0045 | 4.36E-04 | 151,555 |
| 0402 | 137 | 106 | 4% | $ 2,909 | | | 4.3 | 3.31 | | | 10.171 | 0.009 | | 1.24E-04 | 142,684 |
| 0403 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 1.598 | 0.025 | | 3.11E-04 | 135,176 |
| 0404 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 4.781 | 0.036 | | 2.73E-03 | 223,989 |
| 0405 | 114 | 29 | 3% | $ 1,044 | | | 3.6 | 0.92 | | | 1.910 | 0.053 | | 2.95E-03 | 699,568 |
| 0406 | 683 | 152 | 18% | $ 5,705 | | | 21.3 | 4.76 | | | 8.080 | 0.109 | | 1.00E-02 | 1,292,753 |
| | *3,730* | *1,600* | *100%* | *$1.57E+06* | | | | | | | | Totals | **-0.0343** | **0.0522** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0084** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,730 | 15,762 | 19,492 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,017 | 9,617 |
| % NH B | TRUE | 0.3 | Percent | 0.17 | 0.83 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.3 | Count | 3,174 | 3,174 | 3,270 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,806,068 |
| | 32767 | | *SSU* | $ 2,600 | $ 15,600.00 | $ 1,250,704 |
| | 0 | | *OCC HU* | $ 850 | $ 1,573,026 | $ 3,794,610 |
| | | | *UNOCC HU* | $ 100 | $ 2,148,626 | $ 7,851,382 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Non Self-Representing Optimum Allocation $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | ANOVA Variance Components Estimates, NSR $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_a q_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1200 | 48 | 198 | 358 | 141 | 1946 | 1446 | 12% | $ 7,979 | 80 | 2.0 | 12.1 | 9.02 | -0.051 | -0.191 | 160.904 | 0.005 | -0.0005 | 2.39E-06 | 50,441 |
| 12 | 700 | 287 | 190 | 101 | 1290 | 999 | 8% | $ 5,477 | | 2.0 | 8.0 | 6.23 | | -4.789 | 27.958 | 0.035 | | 3.46E-05 | 783,493 |
| 9 | 34 | 208 | 44 | 25 | 321 | 252 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | -0.625 | 87.454 | 0.041 | | 5.72E-04 | 963,377 |
| 12 | 64 | 111 | 77 | 57 | 321 | 186 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 85.868 | 0.049 | | 1.13E-03 | 1,246,277 |
| 97 | 138 | 197 | 1033 | 222 | 1687 | 432 | 11% | $ 3,074 | | | 10.5 | 2.70 | | | 45.211 | 0.197 | | 4.05E-03 | 4,295,112 |
| 173 | 165 | 192 | 1041 | 805 | 2376 | 530 | 15% | $ 3,962 | | | 14.8 | 3.31 | | | 18.258 | 0.311 | | 3.34E-03 | 5,448,707 |
| 633 | 25 | 105 | 189 | 75 | 1027 | 763 | 7% | $ 4,209 | | | 6.4 | 4.76 | | | 214.196 | 0.011 | -0.0022 | 3.68E-05 | 276,933 |
| 11 | 638 | 261 | 173 | 92 | 1176 | 910 | 7% | $ 4,990 | | | 7.3 | 5.68 | | $K_{a,NSR}$ | 63.777 | 0.029 | | 5.94E-05 | 330,172 |
| 9 | 34 | 208 | 44 | 25 | 321 | 252 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.65 | 59.634 | 0.003 | | 2.49E-06 | 2,248 |
| 12 | 64 | 111 | 77 | 57 | 321 | 186 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.27 | 64.414 | 0.064 | | 1.41E-03 | 697,302 |
| 51 | 73 | 104 | 546 | 117 | 892 | 229 | 6% | $ 1,626 | | | 5.6 | 1.43 | | 0.08 | 55.283 | 0.095 | | 2.18E-03 | 1,968,936 |
| 120 | 114 | 133 | 720 | 557 | 1645 | 367 | 10% | $ 2,743 | | | 10.3 | 2.29 | | | 180.850 | 0.069 | | 2.36E-03 | 2,635,977 |
| 203 | 8 | 34 | 60 | 24 | 329 | 244 | 2% | $ 1,347 | | | 2.0 | 1.52 | | | 47.107 | 0.007 | 0.0000 | 9.21E-06 | 107,901 |
| 8 | 436 | 179 | 119 | 63 | 805 | 623 | 5% | $ 3,415 | | | 5.0 | 3.88 | | | 267.445 | 0.008 | | 3.03E-05 | 195,082 |
| 9 | 34 | 208 | 44 | 25 | 321 | 252 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 59.634 | 0.003 | | 2.49E-06 | 80,117 |
| 12 | 64 | 111 | 77 | 57 | 321 | 186 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 827.259 | 0.004 | | 8.16E-05 | 260,442 |
| 18 | 26 | 38 | 196 | 42 | 321 | 82 | 2% | $ 584 | | | 2.0 | 0.51 | | | 726.949 | 0.006 | | 2.76E-04 | 787,112 |
| 25 | 24 | 28 | 151 | 117 | 345 | 77 | 2% | $ 576 | | | 2.2 | 0.48 | | | 9.947 | 0.062 | | 4.89E-04 | 1,286,740 |
| 2,615 | 2,690 | 2,712 | 5,141 | 2,604 | *15,762* | *8,017* | *100%* | *$3.79E+06* | | | | | | | | Totals | **-0.0027** | **0.0161** | **2.14E+07** |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.54 | 0.0084 | 0.0914 | 0.059 | 0.574 | 0.0179 | 0.0127 |

Figure 12. ANOVA Excel Solver Set Up and Results for `own_transport`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* | | | | | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 60 | 2 | 10 | 18 | 7 | 98 | 73 | 3% | $ 2,012 | 16 | 2.0 | 3.1 | 2.27 | $ 657.44 | 0.057 | 0.363 | 0.001 | 0.0019 | 1.63E-07 | 15,515 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 56 | 23 | 15 | 8 | 104 | 81 | 3% | $ 2,213 | | 3.6 | 3.3 | 2.52 | $ 680.62 | 0.097 | 0.112 | 0.021 | | 9.71E-06 | 157,023 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 | 64 | 50 | 2% | $ 1,377 | | 2.9 | 2.0 | 1.57 | $ 688.72 | 0.107 | 0.213 | 0.024 | | 3.96E-05 | 171,502 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.054 | 0.042 | | 4.03E-05 | 218,646 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 12 | 17 | 24 | 128 | 27 | 208 | 53 | 6% | $ 1,904 | | | 6.5 | 1.67 | $ 292.23 | | 0.099 | 0.088 | | 2.31E-04 | 785,866 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 19 | 18 | 21 | 111 | 86 | 254 | 57 | 8% | $ 2,125 | | | 7.9 | 1.77 | $ 267.37 | | 0.160 | 0.076 | | 2.57E-04 | 831,323 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 201 | 8 | 33 | 60 | 24 | 325 | 242 | 10% | $ 3,749 | | | 5.7 | 4.24 | | | 0.247 | 0.035 | 0.0047 | 2.02E-05 | 233,359 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 128 | 52 | 35 | 19 | 236 | 183 | 7% | $ 2,814 | | | 4.1 | 3.20 | | $K_{a,SR}$ | 0.175 | 0.022 | | 7.25E-06 | 139,825 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 12 | 74 | 16 | 9 | 114 | 90 | 4% | $ 1,377 | | | 2.0 | 1.57 | | 0.26 | 0.266 | 0.020 | | 1.93E-05 | 0 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 23 | 39 | 28 | 20 | 114 | 66 | 4% | $ 1,071 | | | 2.0 | 1.16 | | 0.42 | 0.052 | 0.074 | | 6.98E-05 | 386,887 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 11 | 15 | 22 | 115 | 25 | 188 | 48 | 6% | $ 962 | | | 3.3 | 0.84 | | 0.33 | 0.057 | 0.130 | | 3.21E-04 | 774,329 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 27 | 26 | 30 | 164 | 127 | 375 | 84 | 12% | $ 1,754 | | | 6.6 | 1.46 | | | 0.127 | 0.147 | | 5.21E-04 | 1,246,113 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 139 | 6 | 23 | 42 | 16 | 226 | 168 | 7% | $ 3,193 | | | 4.9 | 3.61 | | | 1.117 | 0.014 | 0.0040 | 2.01E-05 | 151,555 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 87 | 35 | 24 | 13 | 160 | 124 | 5% | $ 2,340 | | | 3.4 | 2.66 | | | 0.279 | 0.016 | | 8.80E-06 | 142,684 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 10 | 60 | 13 | 7 | 93 | 73 | 3% | $ 1,377 | | | 2.0 | 1.57 | | | 0.266 | 0.020 | | 2.38E-05 | 135,176 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 19 | 32 | 22 | 17 | 93 | 54 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 0.093 | 0.032 | | 2.80E-05 | 223,989 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 7 | 10 | 14 | 74 | 16 | 122 | 31 | 4% | $ 765 | | | 2.6 | 0.67 | | | 0.071 | 0.081 | | 2.37E-04 | 699,568 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 29 | 27 | 32 | 172 | 133 | 392 | 88 | 12% | $ 2,257 | | | 8.4 | 1.88 | | | 0.100 | 0.158 | | 4.57E-04 | 1,292,753 |
| | | | | | | Totals | *527* | *483* | *590* | *1,060* | *570* | *3,230* | *1,600* | *100%* | *$1.52E+06* | | | | | | | | Totals | **0.0106** | **0.0023** | **7.61E+06** |
| **EXCEL SOLVER RESULTS** | | | **Population Size** | | | | **No. of Eligible HUs in SSU/MSG** *ab*, **Domain** *d (NSR PSUs)* | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
| **Total RelVariance** | **0.0013** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 | 1756 | 70 | 290 | 524 | 207 | 2848 | 2117 | 18% | $ 8,703 | 86 | 2.5 | 13.2 | 9.84 | -0.122 | 0.824 | 2.847 | 0.003 | 0.0014 | 1.15E-08 | 50,441 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 | 28 | 1584 | 649 | 430 | 229 | 2921 | 2261 | 19% | $ 9,242 | | 2.0 | 13.6 | 10.51 | | 0.731 | 0.873 | 0.043 | | 6.98E-07 | 783,493 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 | 12 | 46 | 279 | 59 | 33 | 430 | 338 | 3% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 0.990 | 1.649 | 0.051 | | 1.25E-05 | 963,377 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 | 15 | 86 | 148 | 104 | 77 | 430 | 250 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 0.133 | 0.077 | | 3.18E-06 | 1,246,277 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 | 67 | 96 | 137 | 718 | 154 | 1173 | 301 | 8% | $ 1,593 | | | 5.5 | 1.40 | | | 0.879 | 0.199 | | 1.16E-04 | 4,295,112 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | | 142 | 135 | 158 | 855 | 661 | 1952 | 436 | 13% | $ 2,426 | | | 9.1 | 2.02 | | | 1.367 | 0.224 | | 1.57E-04 | 5,448,707 |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total | 356 | 14 | 59 | 106 | 42 | 577 | 429 | 4% | $ 2,217 | | | 3.4 | 2.51 | | | 3.375 | 0.015 | 0.0004 | 1.67E-06 | 276,933 |
| MAX Budget | TRUE | 10000000 | | 3,230 | 15,548 | 18,777 | 4 | 231 | 95 | 63 | 33 | 426 | 330 | 3% | $ 1,695 | | | 2.5 | 1.93 | | $K_{a,NSR}$ | 0.565 | 0.020 | | 6.54E-07 | 330,172 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | | 10 | 37 | 222 | 47 | 27 | 342 | 269 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.60 | 2.790 | 0.005 | | 2.65E-07 | 2,248 |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* | 12 | 68 | 118 | 83 | 61 | 342 | 199 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.29 | 0.787 | 0.043 | | 7.26E-06 | 697,302 |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,558 | 10,158 | 36 | 51 | 73 | 384 | 82 | 627 | 161 | 4% | $ 1,071 | | | 3.7 | 0.94 | | 0.11 | 1.576 | 0.101 | | 1.00E-04 | 1,968,936 |
| % NH B | TRUE | 0.3 | Percent | 0.16 | 0.84 | 1 | 87 | 83 | 97 | 523 | 405 | 1194 | 266 | 8% | $ 1,865 | | | 7.0 | 1.56 | | | 3.268 | 0.113 | | 1.56E-04 | 2,635,977 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | | 211 | 8 | 35 | 63 | 25 | 342 | 254 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 2.777 | 0.005 | 0.0001 | 2.40E-07 | 107,901 |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* | 3 | 186 | 76 | 50 | 27 | 342 | 265 | 2% | $ 1,361 | | | 2.0 | 1.55 | | | 2.624 | 0.010 | | 9.11E-07 | 195,082 |
| % NH O | TRUE | 0.3 | Count | 3,352 | 3,352 | 3,454 | 10 | 37 | 222 | 47 | 27 | 342 | 269 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 2.790 | 0.005 | | 2.65E-07 | 80,117 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 | 12 | 68 | 118 | 83 | 61 | 342 | 199 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 1.341 | 0.015 | | 1.57E-06 | 260,442 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST | 20 | 28 | 40 | 210 | 45 | 342 | 88 | 2% | $ 584 | | | 2.0 | 0.51 | | | 5.907 | 0.026 | | 4.64E-05 | 787,112 |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,995,472 | 42 | 40 | 46 | 250 | 194 | 572 | 128 | 4% | $ 893 | | | 3.3 | 0.75 | | | 3.510 | 0.047 | | 6.09E-05 | 1,286,740 |
| | 32767 | | *SSU* | $ 2,600 | $ 22,023.75 | $ 1,449,449 | 2,826 | 2,869 | 2,864 | 4,599 | 2,390 | *15,548* | *8,558* | *100%* | *$3.45E+06* | | | | | | | | Totals | **0.0018** | **0.0007** | **2.14E+07** |
| | 0 | | *OCC HU* | $ 850 | $ 1,522,964 | $ 3,450,091 | | | | | | #Hus/PSU | #Hus/PSU | **Summary of Solution** | | | | | | | | | | | | |
| | | | *UNOCC HU* | $ 100 | $ 2,104,988 | $ 7,895,012 | | | | | | SR | NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) | | | | | | |
| | | | | | Total Cost | $1.00E+07 | | | | | | 100 | 100 | 1.75 | 0.0013 | 0.0365 | 0.056 | 0.582 | 0.0129 | 0.0010 | | | | | | |

Figure 13. ANOVA Excel Solver Set Up and Results for `selfRatedHealth`

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$ (d=1) 45-62 H | (d=2) 45-62 NH B | (d=3) 45-62 NH O | (d=4) NOT 45-62 | (d=5) UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 46 | 2 | 8 | 14 | 5 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 98 | 40 | 27 | 14 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 12 | 17 | 25 | 130 | 28 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 22 | 21 | 25 | 135 | 104 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 187 | 7 | 31 | 56 | 22 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 75 | 31 | 20 | 11 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 18 | 26 | 37 | 192 | 41 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 51 | 48 | 56 | 306 | 237 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 134 | 5 | 22 | 40 | 16 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 67 | 27 | 18 | 10 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 12 | 17 | 25 | 128 | 28 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 46 | 44 | 51 | 276 | 213 |
| | | | | | | Totals | *545* | *487* | *568* | *1,413* | *778* |

| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | ANOVA Variance Components Estimates, SR Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 74 | 55 | 2% | $ 1,525 | 16 | 2.0 | 2.3 | 1.72 | $ 657.44 | 0.043 | 0.029 | 0.005 | 0.0014 | 2.25E-07 | 15,515 |
| 0202 | 181 | 140 | 5% | $ 3,857 | | 2.0 | 5.7 | 4.39 | $ 680.62 | -0.004 | 1.664 | 0.010 | | 1.92E-05 | 157,023 |
| 0203 | 64 | 50 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | $ 688.72 | 0.010 | 1.856 | 0.012 | | 8.96E-05 | 171,502 |
| 0204 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 1.579 | 0.015 | | 1.59E-04 | 218,646 |
| 0205 | 213 | 54 | 6% | $ 1,941 | | | 6.6 | 1.70 | $ 292.23 | | 0.255 | 0.095 | | 6.75E-04 | 785,866 |
| 0206 | 308 | 69 | 8% | $ 2,572 | | | 9.6 | 2.15 | $ 267.37 | | 0.161 | 0.121 | | 5.47E-04 | 831,323 |
| 0301 | 303 | 225 | 8% | $ 6,216 | | | 9.5 | 7.03 | | | 1.399 | 0.036 | -0.0003 | 1.28E-04 | 233,359 |
| 0302 | 138 | 107 | 4% | $ 2,933 | | | 4.3 | 3.34 | | $K_{a,SR}$ | 0.942 | 0.015 | | 3.30E-05 | 139,825 |
| 0303 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.26 | 2.571 | 0.013 | | 1.34E-04 | 0 |
| 0304 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.38 | 0.597 | 0.025 | | 1.58E-04 | 386,887 |
| 0305 | 313 | 80 | 8% | $ 2,857 | | | 9.8 | 2.51 | | 0.37 | 0.611 | 0.090 | | 9.91E-04 | 774,329 |
| 0306 | 698 | 156 | 18% | $ 5,833 | | | 21.8 | 4.87 | | | 0.296 | 0.201 | | 1.23E-03 | 1,246,113 |
| 0401 | 218 | 162 | 6% | $ 4,481 | | | 6.8 | 5.07 | | | 1.090 | 0.023 | 0.0006 | 5.80E-05 | 151,555 |
| 0402 | 123 | 95 | 3% | $ 2,615 | | | 3.8 | 2.97 | | | 0.564 | 0.018 | | 2.92E-05 | 142,684 |
| 0403 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 2.571 | 0.013 | | 1.34E-04 | 135,176 |
| 0404 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 0.973 | 0.014 | | 8.08E-05 | 223,989 |
| 0405 | 210 | 54 | 6% | $ 1,914 | | | 6.5 | 1.68 | | | 0.285 | 0.089 | | 6.65E-04 | 699,568 |
| 0406 | 629 | 140 | 17% | $ 5,259 | | | 19.7 | 4.39 | | | 0.227 | 0.206 | | 1.09E-03 | 1,292,753 |
| | *3,791* | *1,600* | *100%* | *$1.58E+06* | | | | | | | | Totals | **0.0018** | **0.0062** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0018** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,791 | 15,816 | 19,607 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,007 | 9,607 |
| % NH B | TRUE | 0.3 | Percent | 0.17 | 0.83 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.3 | Count | 3,170 | 3,170 | 3,267 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,802,573 |
| | 32767 | | *SSU* | $ 2,600 | $ 15,600.00 | $ 1,249,147 |
| | 0 | | *OCC HU* | $ 850 | $ 1,579,135 | $ 3,793,547 |
| | | | *UNOCC HU* | $ 100 | $ 2,154,735 | $ 7,845,267 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Non Self-Representing Optimum Allocation $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | ANOVA Variance Components Estimates, NSR $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1077 | 43 | 178 | 321 | 127 | 1747 | 1298 | 11% | $ 7,170 | 80 | 2.0 | 10.9 | 8.11 | 0.013 | 0.143 | 5.252 | 0.004 | 0.0004 | 5.14E-08 | 50,441 |
| 16 | 913 | 374 | 248 | 132 | 1684 | 1304 | 11% | $ 7,157 | | 2.0 | 10.5 | 8.14 | | -0.454 | 8.875 | 0.024 | | 3.81E-06 | 783,493 |
| 9 | 34 | 208 | 44 | 25 | 320 | 251 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | -1.498 | 6.364 | 0.049 | | 6.10E-05 | 963,377 |
| 12 | 64 | 111 | 77 | 57 | 320 | 186 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 12.706 | 0.040 | | 1.09E-04 | 1,246,277 |
| 77 | 110 | 158 | 825 | 177 | 1347 | 345 | 9% | $ 2,458 | | | 8.4 | 2.16 | | | 3.606 | 0.216 | | 4.85E-04 | 4,295,112 |
| 173 | 164 | 192 | 1038 | 803 | 2369 | 529 | 15% | $ 3,956 | | | 14.8 | 3.30 | | | 2.884 | 0.296 | | 4.78E-04 | 5,448,707 |
| 609 | 24 | 101 | 182 | 72 | 988 | 734 | 6% | $ 4,056 | | | 6.2 | 4.59 | | | 12.417 | 0.013 | -0.0002 | 2.96E-06 | 276,933 |
| 9 | 487 | 200 | 132 | 71 | 898 | 696 | 6% | $ 3,818 | | | 5.6 | 4.34 | | $K_{a,NSR}$ | 26.498 | 0.010 | | 3.66E-06 | 330,172 |
| 9 | 34 | 208 | 44 | 25 | 320 | 251 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.63 | 57.836 | 0.003 | | 2.01E-06 | 2,248 |
| 12 | 64 | 111 | 77 | 57 | 320 | 186 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.24 | 139.830 | 0.007 | | 3.42E-05 | 697,302 |
| 44 | 63 | 90 | 470 | 101 | 768 | 197 | 5% | $ 1,401 | | | 4.8 | 1.23 | | 0.13 | 13.387 | 0.064 | | 2.79E-04 | 1,968,936 |
| 128 | 122 | 142 | 771 | 597 | 1761 | 393 | 11% | $ 2,940 | | | 11.0 | 2.45 | | | 6.631 | 0.143 | | 3.44E-04 | 2,635,977 |
| 348 | 14 | 58 | 104 | 41 | 565 | 420 | 4% | $ 2,320 | | | 3.5 | 2.62 | | | 47.568 | 0.005 | -0.0002 | 2.92E-06 | 107,901 |
| 7 | 368 | 151 | 100 | 53 | 678 | 525 | 4% | $ 2,881 | | | 4.2 | 3.28 | | | 6.305 | 0.013 | | 2.19E-06 | 195,082 |
| 9 | 34 | 208 | 44 | 25 | 320 | 251 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 57.836 | 0.003 | | 2.01E-06 | 80,117 |
| 12 | 64 | 111 | 77 | 57 | 320 | 186 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 22.790 | 0.013 | | 2.19E-05 | 260,442 |
| 20 | 29 | 41 | 216 | 46 | 352 | 90 | 2% | $ 643 | | | 2.2 | 0.56 | | | 8.880 | 0.036 | | 1.28E-04 | 787,112 |
| 54 | 51 | 60 | 323 | 250 | 737 | 164 | 5% | $ 1,230 | | | 4.6 | 1.03 | | | 6.608 | 0.062 | | 1.52E-04 | 1,286,740 |
| 2,625 | 2,684 | 2,698 | 5,093 | 2,715 | *15,816* | *8,007* | *100%* | *$3.79E+06* | | | | | | | | Totals | **0.0000** | **0.0021** | **2.14E+07** |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.56 | 0.0018 | 0.0424 | 0.067 | 0.551 | 0.0080 | 0.0023 |

## C.2 Anticipated Variance Optimization Results

Figure 14. Anticipated Variance Excel Solver Set Up and Results for `income`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d* (*SR PSUs*) | | | | | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | (*d=1*) 45-62 H | (*d=2*) 45-62 NH B | (*d=3*) 45-62 NH O | (*d=4*) NOT 45-62 | (*d=5*) UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_a q_{ab}$ | $Q_{ab}$ |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 42 | 2 | 7 | 12 | 5 | 68 | 50 | 2% | $ 1,315 | 16 | 2.1 | 2.0 | 1.49 | $ 657.44 | 0.01 | 0.13 | 0.00 | 0.0006 | 6.38E-07 | 15,515 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 114 | 47 | 31 | 16 | 210 | 162 | 6% | $ 4,225 | | 3.8 | 6.2 | 4.81 | $ 680.62 | 0.03 | 0.09 | 0.06 | | 3.49E-05 | 157,023 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 44 | 9 | 5 | 68 | 53 | 2% | $ 1,377 | | 2.6 | 2.0 | 1.57 | $ 688.72 | 0.02 | 0.37 | 0.02 | | 4.50E-05 | 171,502 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 20 | 34 | 24 | 18 | 99 | 58 | 3% | $ 1,577 | | | 2.9 | 1.71 | $ 535.63 | | 0.31 | 0.07 | | 3.76E-04 | 218,646 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 11 | 16 | 23 | 119 | 26 | 195 | 50 | 6% | $ 1,688 | | | 5.8 | 1.48 | $ 292.23 | | 0.05 | 0.13 | | 2.31E-04 | 785,866 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 22 | 21 | 24 | 132 | 102 | 301 | 67 | 9% | $ 2,380 | | | 8.9 | 1.99 | $ 267.37 | | 0.15 | 0.08 | | 2.16E-04 | 831,323 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 169 | 7 | 28 | 50 | 20 | 274 | 203 | 8% | $ 2,969 | | | 4.5 | 3.36 | | | 1.11 | 0.02 | 0.0010 | 4.09E-05 | 233,359 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 121 | 50 | 33 | 18 | 223 | 173 | 6% | $ 2,507 | | | 3.7 | 2.85 | | $K_{a,SR}$ | 1.46 | 0.02 | | 3.69E-05 | 139,825 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 13 | 79 | 17 | 9 | 121 | 95 | 3% | $ 1,377 | | | 2.0 | 1.57 | | 0.36 | 0.47 | 0.02 | | 2.19E-05 | 0 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 24 | 42 | 29 | 22 | 122 | 71 | 3% | $ 1,075 | | | 2.0 | 1.17 | | 0.34 | 0.26 | 0.09 | | 4.57E-04 | 386,887 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 21 | 30 | 42 | 222 | 48 | 363 | 93 | 10% | $ 1,749 | | | 6.0 | 1.53 | | 0.31 | 0.33 | 0.09 | | 4.29E-04 | 774,329 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 26 | 25 | 29 | 158 | 122 | 361 | 81 | 10% | $ 1,595 | | | 6.0 | 1.33 | | | 0.08 | 0.13 | | 2.57E-04 | 1,246,113 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 76 | 3 | 13 | 23 | 9 | 123 | 91 | 4% | $ 1,930 | | | 2.9 | 2.18 | | | 0.33 | 0.02 | 0.0008 | 1.83E-05 | 151,555 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 55 | 23 | 15 | 8 | 101 | 79 | 3% | $ 1,649 | | | 2.4 | 1.88 | | | 0.20 | 0.02 | | 1.69E-05 | 142,684 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 9 | 54 | 12 | 6 | 84 | 66 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 0.47 | 0.02 | | 3.17E-05 | 135,176 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 17 | 29 | 20 | 15 | 84 | 49 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 0.28 | 0.05 | | 1.96E-04 | 223,989 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 14 | 20 | 28 | 146 | 31 | 239 | 61 | 7% | $ 1,669 | | | 5.7 | 1.46 | | | 0.16 | 0.08 | | 2.81E-04 | 699,568 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 32 | 31 | 36 | 194 | 150 | 443 | 99 | 13% | $ 2,831 | | | 10.6 | 2.36 | | | 0.13 | 0.12 | | 3.21E-04 | 1,292,753 |
| | | | | | | Totals | *437* | *533* | *631* | *1,247* | *630* | *3,477* | *1,600* | *100%* | *$1.55E+06* | | | | | | | | Totals | **0.0024** | **0.0030** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0013** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,477 | 14,907 | 18,384 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,087 | 9,687 |
| % NH B | TRUE | 0.33 | Percent | 0.17 | 0.83 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.30 | Count | 3,197 | 3,197 | 3,294 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,830,519 |
| | 32767 | | *SSU* | $ 2,600 | $ 22,132.81 | $ 1,261,603 |
| | 0 | | *OCC HU* | $ 850 | $ 1,547,717 | $ 3,778,028 |
| | | | *UNOCC HU* | $ 100 | $ 2,129,850 | $ 7,870,150 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d* (*NSR PSUs*) | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_a q_{ab}$ | $Q_{ab}$ |
| 1172 | 47 | 194 | 350 | 138 | 1901 | 1413 | 13% | $ 7,726 | 81 | 2.0 | 11.8 | 8.73 | 0.019 | 0.088 | 10.530 | 0.002 | 0.0002 | 2.40E-08 | 50,441 |
| 12 | 679 | 278 | 184 | 98 | 1252 | 969 | 8% | $ 5,267 | | 2.0 | 7.7 | 5.99 | | 0.112 | 1.790 | 0.042 | | 3.21E-06 | 783,493 |
| 9 | 35 | 210 | 44 | 25 | 323 | 254 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 0.139 | 3.560 | 0.047 | | 3.10E-05 | 963,377 |
| 12 | 65 | 112 | 78 | 58 | 323 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 2.885 | 0.075 | | 8.75E-05 | 1,246,277 |
| 81 | 116 | 166 | 866 | 186 | 1415 | 363 | 9% | $ 2,557 | | | 8.8 | 2.24 | | | 2.424 | 0.211 | | 2.99E-04 | 4,295,112 |
| 115 | 110 | 128 | 693 | 536 | 1583 | 353 | 11% | $ 2,616 | | | 9.8 | 2.18 | | | 2.258 | 0.174 | | 1.93E-04 | 5,448,707 |
| 700 | 28 | 116 | 209 | 82 | 1135 | 843 | 8% | $ 4,612 | | | 7.0 | 5.21 | | | 3.949 | 0.015 | 0.0001 | 9.88E-07 | 276,933 |
| 15 | 851 | 349 | 231 | 123 | 1569 | 1214 | 11% | $ 6,600 | | | 9.7 | 7.51 | | $K_{a,NSR}$ | 7.532 | 0.031 | | 5.81E-06 | 330,172 |
| 9 | 35 | 210 | 44 | 25 | 323 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.55 | 22.237 | 0.001 | | 7.78E-08 | 2,248 |
| 12 | 65 | 112 | 78 | 58 | 323 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.31 | 4.233 | 0.070 | | 1.10E-04 | 697,302 |
| 49 | 70 | 100 | 526 | 113 | 858 | 220 | 6% | $ 1,551 | | | 5.3 | 1.36 | | 0.14 | 4.843 | 0.091 | | 1.81E-04 | 1,968,936 |
| 85 | 81 | 95 | 512 | 396 | 1169 | 261 | 8% | $ 1,933 | | | 7.2 | 1.61 | | | 3.340 | 0.106 | | 1.43E-04 | 2,635,977 |
| 403 | 16 | 67 | 120 | 48 | 654 | 486 | 4% | $ 2,658 | | | 4.0 | 3.00 | | | 6.644 | 0.004 | 0.0000 | 2.51E-07 | 107,901 |
| 5 | 305 | 125 | 83 | 44 | 562 | 435 | 4% | $ 2,365 | | | 3.5 | 2.69 | | | 6.352 | 0.009 | | 1.11E-06 | 195,082 |
| 9 | 35 | 210 | 44 | 25 | 323 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 22.237 | 0.004 | | 1.57E-06 | 80,117 |
| 12 | 65 | 112 | 78 | 58 | 323 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 3.773 | 0.014 | | 3.96E-06 | 260,442 |
| 19 | 26 | 38 | 198 | 43 | 323 | 83 | 2% | $ 584 | | | 2.0 | 0.51 | | | 0.853 | 0.047 | | 2.23E-05 | 787,112 |
| 40 | 38 | 44 | 239 | 185 | 545 | 122 | 4% | $ 901 | | | 3.4 | 0.75 | | | 2.315 | 0.059 | | 6.58E-05 | 1,286,740 |
| 2,760 | 2,664 | 2,663 | 4,579 | 2,241 | *14,907* | *8,087* | *100%* | *$3.78E+06* | | | | | | | | Totals | **0.0002** | **0.0011** | **2.14E+07** |

| Summary of Solution | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
| 100 | 100 | 1.67002 | 0.0013 | 0.0359 | 0.093 | 0.483 | 0.0054 | 0.0016 |

Figure 15. Anticipated Variance Excel Solver Set Up and Results for `wealtha`

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$ (d=1) 45-62 H | (d=2) 45-62 NH B | (d=3) 45-62 NH O | (d=4) NOT 45-62 | (d=5) UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 127 | 5 | 21 | 38 | 15 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 35 | 14 | 10 | 5 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 17 | 24 | 34 | 178 | 38 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 26 | 24 | 28 | 154 | 119 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 209 | 8 | 35 | 62 | 25 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 3 | 168 | 69 | 46 | 24 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 12 | 17 | 24 | 124 | 27 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 29 | 27 | 32 | 173 | 134 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 163 | 7 | 27 | 49 | 19 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 35 | 14 | 9 | 5 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 7 | 11 | 15 | 80 | 17 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 23 | 22 | 25 | 137 | 106 |
| | | | | | | Totals | *629* | *441* | *529* | *1,131* | *583* |

| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | ANOVA Variance Components Estimates, SR: Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 207 | 154 | 6% | $ 4,247 | 16 | 2.0 | 6.5 | 4.80 | $ 657.44 | 0.04 | 1.75 | 0.01 | 0.0033 | 5.79E-06 | 15,515 |
| 0202 | 65 | 50 | 2% | $ 1,387 | | 2.0 | 2.0 | 1.58 | $ 680.62 | 0.17 | 0.33 | 0.03 | | 1.08E-04 | 157,023 |
| 0203 | 64 | 50 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | $ 688.72 | 0.24 | 1.15 | 0.02 | | 1.01E-04 | 171,502 |
| 0204 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.90 | 0.10 | | 3.79E-03 | 218,646 |
| 0205 | 291 | 75 | 9% | $ 2,659 | | | 9.1 | 2.33 | $ 292.23 | | 0.30 | 0.16 | | 1.63E-03 | 785,866 |
| 0206 | 350 | 78 | 11% | $ 2,928 | | | 11.0 | 2.44 | $ 267.37 | | 1.39 | 0.09 | | 2.37E-03 | 831,323 |
| 0301 | 339 | 252 | 10% | $ 6,966 | | | 10.6 | 7.88 | | | 2.40 | 0.02 | 0.0113 | 7.19E-05 | 233,359 |
| 0302 | 310 | 240 | 9% | $ 6,591 | | | 9.7 | 7.50 | | $K_{a,SR}$ | 2.11 | 0.08 | | 7.99E-04 | 139,825 |
| 0303 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.41 | 3.12 | 0.01 | | 8.37E-05 | 0 |
| 0304 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.36 | 1.32 | 0.09 | | 4.16E-03 | 386,887 |
| 0305 | 202 | 52 | 6% | $ 1,845 | | | 6.3 | 1.62 | | 0.23 | 1.38 | 0.08 | | 2.86E-03 | 774,329 |
| 0306 | 394 | 88 | 12% | $ 3,294 | | | 12.3 | 2.75 | | | 1.07 | 0.09 | | 1.76E-03 | 1,246,113 |
| 0401 | 264 | 197 | 8% | $ 5,434 | | | 8.3 | 6.14 | | | 5.90 | 0.01 | 0.0064 | 6.35E-05 | 151,555 |
| 0402 | 64 | 50 | 2% | $ 1,361 | | | 2.0 | 1.55 | | | 1.34 | 0.01 | | 5.59E-05 | 142,684 |
| 0403 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 3.12 | 0.01 | | 8.37E-05 | 135,176 |
| 0404 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 3.96 | 0.02 | | 9.67E-04 | 223,989 |
| 0405 | 130 | 33 | 4% | $ 1,189 | | | 4.1 | 1.04 | | | 0.50 | 0.09 | | 1.89E-03 | 699,568 |
| 0406 | 312 | 70 | 9% | $ 2,607 | | | 9.7 | 2.18 | | | 4.22 | 0.08 | | 6.81E-03 | 1,292,753 |
| | *3,314* | *1,600* | *100%* | *$1.53E+06* | | | | | | | | Totals | 0.0211 | 0.0276 | 7.61E+06 |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0262** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,314 | 16,372 | 19,685 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,434 | 10,034 |
| % NH B | TRUE | 0.33 | Percent | 0.16 | 0.84 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.30 | Count | 3,311 | 3,311 | 3,411 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,951,801 |
| | 32767 | | *SSU* | $ 2,600 | $ 15,600.00 | $ 1,419,711 |
| | 0 | | *OCC HU* | $ 850 | $ 1,531,351 | $ 3,521,536 |
| | | | *UNOCC HU* | $ 100 | $ 2,106,951 | $ 7,893,048 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Non Self-Representing Optimum Allocation $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | ANOVA Variance Components Estimates, NSR: $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1529 | 61 | 253 | 456 | 180 | 2480 | 1843 | 15% | $ 7,813 | 84 | 2.5 | 11.9 | 8.83 | 0.053 | 12.08 | 11426.9 | 0.001 | 0.0167 | 2.89E-06 | 50,441 |
| 26 | 1446 | 593 | 393 | 209 | 2666 | 2064 | 16% | $ 8,696 | | 2.0 | 12.8 | 9.89 | | 6.991 | 104.887 | 0.171 | | 1.48E-03 | 783,493 |
| 12 | 45 | 271 | 57 | 32 | 417 | 328 | 3% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 17.62 | 14.178 | 0.055 | | 1.33E-04 | 963,377 |
| 15 | 83 | 144 | 101 | 74 | 417 | 242 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 11.916 | 0.081 | | 3.21E-04 | 1,246,277 |
| 63 | 90 | 128 | 671 | 144 | 1096 | 281 | 7% | $ 1,535 | | | 5.3 | 1.35 | | | 11.826 | 0.160 | | 1.08E-03 | 4,295,112 |
| 141 | 134 | 157 | 848 | 656 | 1936 | 432 | 12% | $ 2,480 | | | 9.3 | 2.07 | | | 209.407 | 0.069 | | 2.32E-03 | 5,448,707 |
| 342 | 14 | 57 | 102 | 40 | 555 | 412 | 3% | $ 2,162 | | | 3.3 | 2.44 | | | 74.356 | 0.024 | 0.0060 | 1.08E-04 | 276,933 |
| 5 | 292 | 120 | 79 | 42 | 538 | 416 | 3% | $ 2,171 | | | 3.2 | 2.47 | | $K_{a,NSR}$ | 159.865 | 0.035 | | 4.60E-04 | 330,172 |
| 10 | 36 | 219 | 46 | 26 | 337 | 265 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.54 | 8.695 | 0.000 | | 2.98E-09 | 2,248 |
| 12 | 67 | 116 | 81 | 60 | 337 | 196 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.38 | 19.793 | 0.143 | | 2.06E-03 | 697,302 |
| 40 | 57 | 82 | 429 | 92 | 701 | 180 | 4% | $ 1,214 | | | 4.2 | 1.06 | | 0.08 | 43.613 | 0.095 | | 2.20E-03 | 1,968,936 |
| 68 | 65 | 75 | 409 | 316 | 934 | 208 | 6% | $ 1,480 | | | 5.5 | 1.24 | | | 25.460 | 0.083 | | 8.49E-04 | 2,635,977 |
| 208 | 8 | 34 | 62 | 25 | 337 | 251 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 9.312 | 0.004 | 0.0007 | 7.50E-07 | 107,901 |
| 3 | 183 | 75 | 50 | 26 | 337 | 261 | 2% | $ 1,361 | | | 2.0 | 1.55 | | | 208.049 | 0.003 | | 5.12E-06 | 195,082 |
| 10 | 36 | 219 | 46 | 26 | 337 | 265 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 8.695 | 0.006 | | 1.18E-06 | 80,117 |
| 12 | 67 | 116 | 81 | 60 | 337 | 196 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 111.207 | 0.017 | | 1.56E-04 | 260,442 |
| 19 | 28 | 39 | 207 | 44 | 337 | 86 | 2% | $ 584 | | | 2.0 | 0.51 | | | 28.856 | 0.031 | | 3.13E-04 | 787,112 |
| 166 | 157 | 184 | 994 | 769 | 2270 | 507 | 14% | $ 3,598 | | | 13.5 | 3.00 | | | 10292.9 | 0.022 | | 9.93E-03 | 1,286,740 |
| 2,682 | 2,870 | 2,882 | 5,113 | 2,825 | *16,372* | *8,434* | *100%* | *$3.52E+06* | | | | | | | | Totals | 0.0234 | 0.0214 | 2.14E+07 |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.75 | 0.0262 | 0.1618 | 0.099 | 0.470 | 0.0487 | 0.0455 |

# Figure 16. Anticipated Variance Excel Solver Set Up and Results for `wealthb`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d* (SR PSUs) | | | | | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | (d=1) 45-62 H | (d=2) 45-62 NH B | (d=3) 45-62 NH O | (d=4) NOT 45-62 | (d=5) UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 39 | 2 | 7 | 12 | 5 | 64 | 48 | 2% | $ 1,315 | 16 | 2.0 | 2.0 | 1.49 | $ 657.44 | 0.04 | 0.05 | 0.030 | 0.0031 | 1.63E-05 | 15,515 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 35 | 14 | 9 | 5 | 64 | 50 | 2% | $ 1,361 | | 2.0 | 2.0 | 1.55 | $ 680.62 | 0.18 | 0.31 | 0.031 | | 9.61E-05 | 157,023 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 | 64 | 50 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | $ 688.72 | 0.24 | 2.00 | 0.018 | | 2.07E-04 | 171,502 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 1.00 | 0.092 | | 3.69E-03 | 218,646 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 15 | 21 | 30 | 157 | 34 | 257 | 66 | 8% | $ 2,347 | | | 8.0 | 2.06 | $ 292.23 | | 0.27 | 0.159 | | 1.65E-03 | 785,866 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 24 | 23 | 27 | 145 | 112 | 331 | 74 | 10% | $ 2,768 | | | 10.4 | 2.31 | $ 267.37 | | 1.39 | 0.089 | | 2.41E-03 | 831,323 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 319 | 13 | 53 | 95 | 38 | 518 | 385 | 16% | $ 10,645 | | | 16.2 | 12.03 | | | 3.75 | 0.020 | 0.0109 | 6.43E-05 | 233,359 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 3 | 152 | 62 | 41 | 22 | 280 | 217 | 9% | $ 5,955 | | | 8.7 | 6.77 | | $K_{a,SR}$ | 2.15 | 0.070 | | 7.85E-04 | 139,825 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.42 | 2.10 | 0.010 | | 6.75E-05 | 0 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.35 | 1.28 | 0.080 | | 3.51E-03 | 386,887 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 11 | 16 | 23 | 119 | 26 | 195 | 50 | 6% | $ 1,782 | | | 6.1 | 1.56 | | 0.23 | 1.35 | 0.081 | | 2.80E-03 | 774,329 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 26 | 25 | 29 | 159 | 123 | 363 | 81 | 11% | $ 3,029 | | | 11.3 | 2.53 | | | 1.00 | 0.094 | | 1.74E-03 | 1,246,113 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 174 | 7 | 29 | 52 | 21 | 282 | 210 | 9% | $ 5,799 | | | 8.8 | 6.56 | | | 5.99 | 0.014 | 0.0062 | 8.69E-05 | 151,555 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 35 | 14 | 9 | 5 | 64 | 50 | 2% | $ 1,361 | | | 2.0 | 1.55 | | | 1.39 | 0.011 | | 5.09E-05 | 142,684 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 2.10 | 0.010 | | 6.75E-05 | 135,176 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 3.91 | 0.022 | | 8.28E-04 | 223,989 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 8 | 11 | 16 | 84 | 18 | 137 | 35 | 4% | $ 1,256 | | | 4.3 | 1.10 | | | 0.54 | 0.084 | | 1.74E-03 | 699,568 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 24 | 23 | 27 | 144 | 112 | 330 | 74 | 10% | $ 2,753 | | | 10.3 | 2.30 | | | 3.67 | 0.084 | | 5.67E-03 | 1,292,753 |
| | | | | | | Totals | *658* | *421* | *521* | *1,101* | *568* | *3,269* | *1,600* | *100%* | *$1.53E+06* | | | | | | | | Totals | **0.0202** | **0.0255** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | | No. of Eligible HUs in SSU/MSG *ab*, Domain *d* (NSR PSUs) | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0223** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 | 1486 | 59 | 246 | 444 | 175 | 2410 | 1791 | 15% | $ 7,829 | 84 | 2.4 | 11.9 | 8.85 | 0.053 | 8.840 | 116.290 | 0.001 | 0.0120 | 3.74E-08 | 50,441 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 | 25 | 1376 | 564 | 374 | 199 | 2538 | 1965 | 15% | $ 8,535 | | 2.0 | 12.5 | 9.71 | | 5.643 | 103.719 | 0.157 | | 1.30E-03 | 783,493 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 | 12 | 43 | 263 | 56 | 31 | 405 | 318 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 8.588 | 15.540 | 0.052 | | 1.32E-04 | 963,377 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 | 15 | 81 | 140 | 98 | 72 | 405 | 235 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 13.402 | 0.077 | | 3.39E-04 | 1,246,277 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 | 64 | 91 | 130 | 679 | 146 | 1109 | 284 | 7% | $ 1,601 | | | 5.5 | 1.40 | | | 13.675 | 0.159 | | 1.22E-03 | 4,295,112 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | | 146 | 139 | 162 | 878 | 679 | 2004 | 447 | 12% | $ 2,648 | | | 9.9 | 2.21 | | | 182.876 | 0.078 | | 2.51E-03 | 5,448,707 |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total | 327 | 13 | 54 | 98 | 39 | 530 | 394 | 3% | $ 2,080 | | | 3.2 | 2.35 | | | 70.398 | 0.024 | 0.0051 | 9.97E-05 | 276,933 |
| MAX Budget | TRUE | 10000000 | | 3,269 | 16,449 | 19,718 | 6 | 358 | 147 | 97 | 52 | 661 | 512 | 4% | $ 2,683 | | | 3.9 | 3.05 | | $K_{a,NSR}$ | 150.260 | 0.041 | | 4.95E-04 | 330,172 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | | 10 | 36 | 218 | 46 | 26 | 335 | 263 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.52 | 13.046 | 0.000 | | 5.72E-09 | 2,248 |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* | 12 | 67 | 116 | 81 | 60 | 335 | 195 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.39 | 19.298 | 0.148 | | 2.16E-03 | 697,302 |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,383 | 9,983 | 42 | 60 | 86 | 448 | 96 | 732 | 188 | 4% | $ 1,275 | | | 4.4 | 1.12 | | 0.09 | 40.938 | 0.097 | | 2.06E-03 | 1,968,936 |
| % NH B | TRUE | 0.33 | Percent | 0.16 | 0.84 | 1 | 69 | 66 | 77 | 415 | 321 | 947 | 211 | 6% | $ 1,510 | | | 5.6 | 1.26 | | | 26.798 | 0.080 | | 8.19E-04 | 2,635,977 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | | 207 | 8 | 34 | 62 | 24 | 335 | 249 | 2% | $ 1,315 | | | 2.0 | 1.49 | | | 15.164 | 0.004 | 0.0004 | 1.10E-06 | 107,901 |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* | 3 | 182 | 75 | 49 | 26 | 335 | 260 | 2% | $ 1,361 | | | 2.0 | 1.55 | | | 269.898 | 0.003 | | 6.52E-06 | 195,082 |
| % NH O | TRUE | 0.30 | Count | 3,294 | 3,294 | 3,394 | 10 | 36 | 218 | 46 | 26 | 335 | 263 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 13.046 | 0.006 | | 1.97E-06 | 80,117 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 | 12 | 67 | 116 | 81 | 60 | 335 | 195 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 100.290 | 0.017 | | 1.55E-04 | 260,442 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST | 19 | 27 | 39 | 205 | 44 | 335 | 86 | 2% | $ 584 | | | 2.0 | 0.51 | | | 31.355 | 0.028 | | 2.85E-04 | 787,112 |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,934,013 | 172 | 164 | 191 | 1034 | 800 | 2360 | 527 | 14% | $ 3,764 | | | 14.1 | 3.14 | | | 5737.41 | 0.027 | | 7.96E-03 | 1,286,740 |
| | 32767 | | *SSU* | $ 2,600 | $ 15,600.00 | $ 1,398,066 | 2,636 | 2,874 | 2,873 | 5,189 | 2,877 | *16,449* | *8,383* | *100%* | *$3.57E+06* | | | | | | | | Totals | **0.0175** | **0.0196** | **2.14E+07** |
| | 0 | | *OCC HU* | $ 850 | $ 1,526,930 | $ 3,565,391 | | | | | | #Hus/PSU | #Hus/PSU | **Summary of Solution** | | | | | | | | | | | | |
| | | | *UNOCC HU* | $ 100 | $ 2,102,530 | $ 7,897,470 | | | | | | SR | NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) | | | | | | |
| | | | | | Total Cost | $1.00E+07 | | | | | | 100 | 100.0 | 1.75 | 0.0223 | 0.1494 | 0.097 | 0.475 | 0.0457 | 0.0377 | | | | | | |

Figure 17. Anticipated Variance Excel Solver Set Up and Results for `other_debts`

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$ (d=1) 45-62 H | (d=2) 45-62 NH B | (d=3) 45-62 NH O | (d=4) NOT 45-62 | (d=5) UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 62 | 2 | 10 | 18 | 7 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 55 | 22 | 15 | 8 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 11 | 65 | 14 | 8 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 20 | 35 | 24 | 18 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 10 | 14 | 20 | 106 | 23 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 18 | 17 | 20 | 108 | 84 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 212 | 8 | 35 | 63 | 25 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 81 | 33 | 22 | 12 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 12 | 71 | 15 | 8 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 22 | 38 | 26 | 19 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 16 | 22 | 32 | 168 | 36 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 39 | 37 | 43 | 234 | 181 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 126 | 5 | 21 | 38 | 15 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 55 | 22 | 15 | 8 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 9 | 55 | 12 | 7 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 17 | 29 | 21 | 15 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 10 | 14 | 20 | 102 | 22 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 36 | 35 | 40 | 219 | 169 |
| | | | | | | Totals | *552* | *436* | *613* | *1,220* | *665* |

| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | ANOVA Variance Components Estimates, SR: Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 100 | 75 | 3% | $ 1,315 | 16 | 3.1 | 2.0 | 1.49 | $ 657.44 | 0.108 | 0.328 | 0.003 | 0.0029 | 5.24E-07 | 15,515 |
| 0202 | 101 | 78 | 3% | $ 1,367 | | 3.4 | 2.0 | 1.55 | $ 680.62 | 0.071 | 0.149 | 0.030 | | 2.82E-05 | 157,023 |
| 0203 | 100 | 79 | 3% | $ 1,377 | | 2.7 | 2.0 | 1.57 | $ 688.72 | 0.062 | 0.521 | 0.021 | | 4.75E-05 | 171,502 |
| 0204 | 100 | 58 | 3% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.237 | 0.039 | | 1.01E-04 | 218,646 |
| 0205 | 173 | 44 | 5% | $ 1,009 | | | 3.5 | 0.89 | $ 292.23 | | 0.148 | 0.096 | | 4.91E-04 | 785,866 |
| 0206 | 247 | 55 | 7% | $ 1,318 | | | 4.9 | 1.10 | $ 267.37 | | 0.146 | 0.099 | | 4.21E-04 | 831,323 |
| 0301 | 344 | 256 | 10% | $ 4,135 | | | 6.3 | 4.67 | | | 1.768 | 0.022 | 0.0031 | 5.13E-05 | 233,359 |
| 0302 | 149 | 115 | 4% | $ 1,847 | | | 2.7 | 2.10 | | $K_{a,SR}$ | 0.740 | 0.018 | | 3.41E-05 | 139,825 |
| 0303 | 109 | 86 | 3% | $ 1,377 | | | 2.0 | 1.57 | | 0.29 | 1.012 | 0.016 | | 4.65E-05 | 0 |
| 0304 | 109 | 64 | 3% | $ 1,071 | | | 2.0 | 1.16 | | 0.38 | 0.239 | 0.063 | | 2.40E-04 | 386,887 |
| 0305 | 274 | 70 | 8% | $ 1,464 | | | 5.0 | 1.28 | | 0.32 | 0.158 | 0.148 | | 7.82E-04 | 774,329 |
| 0306 | 535 | 119 | 15% | $ 2,616 | | | 9.8 | 2.18 | | | 0.423 | 0.126 | | 9.00E-04 | 1,246,113 |
| 0401 | 205 | 152 | 6% | $ 3,153 | | | 4.8 | 3.56 | | | 1.959 | 0.013 | 0.0025 | 3.30E-05 | 151,555 |
| 0402 | 101 | 78 | 3% | $ 1,606 | | | 2.4 | 1.83 | | | 0.182 | 0.025 | | 2.27E-05 | 142,684 |
| 0403 | 85 | 67 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 1.012 | 0.016 | | 5.97E-05 | 135,176 |
| 0404 | 85 | 50 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 0.249 | 0.030 | | 7.24E-05 | 223,989 |
| 0405 | 167 | 43 | 5% | $ 1,144 | | | 3.9 | 1.00 | | | 0.180 | 0.084 | | 4.70E-04 | 699,568 |
| 0406 | 500 | 112 | 14% | $ 3,133 | | | 11.7 | 2.61 | | | 0.254 | 0.152 | | 8.41E-04 | 1,292,753 |
| | *3,486* | *1,600* | *100%* | *$1.55E+06* | | | | | | | | Totals | **0.0084** | **0.0046** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0019** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,486 | 15,073 | 18,559 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,075 | 9,675 |
| % NH B | TRUE | 0.33 | Percent | 0.17 | 0.83 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.30 | Count | 3,193 | 3,193 | 3,289 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,826,134 |
| | 32767 | | *SSU* | $ 2,600 | $ 23,974.02 | $ 1,259,648 |
| | 0 | | *OCC HU* | $ 850 | $ 1,548,577 | $ 3,781,655 |
| | | | *UNOCC HU* | $ 100 | $ 2,132,551 | $ 7,867,438 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab |
|---|---|---|---|---|---|---|---|---|
| 928 | 37 | 153 | 277 | 109 | 1504 | 1118 | 10% | $ 6,124 |
| 17 | 941 | 386 | 256 | 136 | 1736 | 1344 | 12% | $ 7,316 |
| 9 | 35 | 210 | 44 | 25 | 323 | 254 | 2% | $ 1,377 |
| 12 | 65 | 111 | 78 | 58 | 323 | 188 | 2% | $ 1,071 |
| 63 | 89 | 128 | 670 | 144 | 1094 | 280 | 7% | $ 1,979 |
| 144 | 137 | 159 | 864 | 668 | 1972 | 440 | 13% | $ 3,265 |
| 761 | 30 | 126 | 227 | 90 | 1234 | 917 | 8% | $ 5,024 |
| 10 | 587 | 241 | 160 | 85 | 1083 | 839 | 7% | $ 4,566 |
| 9 | 35 | 210 | 44 | 25 | 323 | 254 | 2% | $ 1,377 |
| 12 | 65 | 111 | 78 | 58 | 323 | 188 | 2% | $ 1,071 |
| 41 | 58 | 83 | 436 | 94 | 712 | 182 | 5% | $ 1,288 |
| 103 | 98 | 114 | 619 | 479 | 1414 | 316 | 9% | $ 2,342 |
| 441 | 18 | 73 | 132 | 52 | 715 | 531 | 5% | $ 2,910 |
| 7 | 394 | 161 | 107 | 57 | 726 | 562 | 5% | $ 3,061 |
| 9 | 35 | 210 | 44 | 25 | 323 | 254 | 2% | $ 1,377 |
| 12 | 65 | 111 | 78 | 58 | 323 | 188 | 2% | $ 1,071 |
| 19 | 26 | 38 | 198 | 42 | 323 | 83 | 2% | $ 584 |
| 45 | 43 | 50 | 272 | 211 | 621 | 139 | 4% | $ 1,029 |
| 2,641 | 2,757 | 2,677 | 4,583 | 2,415 | *15,073* | *8,075* | *100%* | *$3.78E+06* |

| Non Self-Representing Optimum Allocation $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | ANOVA Variance Components Estimates, NSR: $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 81 | 2.0 | 9.3 | 6.92 | 0.002 | 0.085 | 28.807 | 0.002 | 0.0002 | 7.53E-08 | 50,441 |
| | 2.0 | 10.7 | 8.32 | | 0.154 | 2.546 | 0.044 | | 3.72E-06 | 783,493 |
| | 2.0 | 2.0 | 1.57 | | 0.217 | 3.704 | 0.055 | | 4.45E-05 | 963,377 |
| | | 2.0 | 1.16 | | | 3.541 | 0.069 | | 9.11E-05 | 1,246,277 |
| | | 6.8 | 1.74 | | | 2.820 | 0.188 | | 3.57E-04 | 4,295,112 |
| | | 12.2 | 2.73 | | | 2.631 | 0.249 | | 3.71E-04 | 5,448,707 |
| | | 7.6 | 5.68 | | | 7.681 | 0.016 | 0.0001 | 2.04E-06 | 276,933 |
| | | 6.7 | 5.19 | | $K_{a,NSR}$ | 5.848 | 0.020 | | 2.66E-06 | 330,172 |
| | | 2.0 | 1.57 | | 0.61 | 25.922 | 0.003 | | 1.20E-06 | 2,248 |
| | | 2.0 | 1.16 | | 0.27 | 6.535 | 0.038 | | 4.93E-05 | 697,302 |
| | | 4.4 | 1.13 | | 0.12 | 7.263 | 0.077 | | 2.35E-04 | 1,968,936 |
| | | 8.8 | 1.95 | | | 5.530 | 0.123 | | 2.66E-04 | 2,635,977 |
| | | 4.4 | 3.29 | | | 52.978 | 0.003 | 0.0000 | 1.05E-06 | 107,901 |
| | | 4.5 | 3.48 | | | 5.882 | 0.012 | | 1.39E-06 | 195,082 |
| | | 2.0 | 1.57 | | | 25.922 | 0.003 | | 1.20E-06 | 80,117 |
| | | 2.0 | 1.16 | | | 11.873 | 0.012 | | 9.76E-06 | 260,442 |
| | | 2.0 | 0.51 | | | 11.442 | 0.026 | | 9.10E-05 | 787,112 |
| | | 3.8 | 0.86 | | | 4.625 | 0.059 | | 1.17E-04 | 1,286,740 |
| | | | | | | | Totals | **0.0003** | **0.0016** | **2.14E+07** |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.69 | 0.0019 | 0.0441 | 0.067 | 0.550 | 0.0131 | 0.0020 |

Figure 18. Anticipated Variance Excel Solver Set Up and Results for `charity_donate`

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$ (d=1) 45-62 H | (d=2) 45-62 NH B | (d=3) 45-62 NH O | (d=4) NOT 45-62 | (d=5) UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d* (SR PSUs) 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 59 | 2 | 10 | 18 | 7 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 86 | 35 | 23 | 12 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 10 | 63 | 13 | 7 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 19 | 33 | 23 | 17 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 12 | 16 | 23 | 123 | 26 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 19 | 18 | 21 | 115 | 89 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 154 | 6 | 25 | 46 | 18 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 118 | 48 | 32 | 17 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 12 | 76 | 16 | 9 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 23 | 40 | 28 | 21 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 17 | 24 | 34 | 177 | 38 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 27 | 26 | 30 | 163 | 126 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 88 | 4 | 15 | 26 | 10 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 74 | 30 | 20 | 11 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 10 | 60 | 13 | 7 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 19 | 32 | 22 | 17 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 10 | 14 | 20 | 107 | 23 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 36 | 34 | 40 | 215 | 166 |
| | | | | | | Totals | *447* | *516* | *637* | *1,181* | *623* |

| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | ANOVA Variance Components Estimates, SR: Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_a q_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 96 | 72 | 3% | $ 1,315 | 16 | 3.0 | 2.0 | 1.49 | $ 657.44 | 0.109 | 0.285 | 0.006 | 0.0044 | 1.98E-06 | 15,515 |
| 0202 | 159 | 123 | 5% | $ 2,245 | | 3.6 | 3.3 | 2.55 | $ 680.62 | 0.216 | 0.193 | 0.038 | | 3.68E-05 | 157,023 |
| 0203 | 96 | 76 | 3% | $ 1,377 | | 2.9 | 2.0 | 1.57 | $ 688.72 | 0.095 | 0.491 | 0.036 | | 1.31E-04 | 171,502 |
| 0204 | 96 | 56 | 3% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.286 | 0.051 | | 2.09E-04 | 218,646 |
| 0205 | 200 | 51 | 6% | $ 1,215 | | | 4.2 | 1.07 | $ 292.23 | | 0.129 | 0.151 | | 9.17E-04 | 785,866 |
| 0206 | 263 | 59 | 8% | $ 1,458 | | | 5.5 | 1.22 | $ 267.37 | | 0.355 | 0.083 | | 6.60E-04 | 831,323 |
| 0301 | 250 | 186 | 7% | $ 2,818 | | | 4.3 | 3.19 | | | 1.481 | 0.030 | 0.0049 | 1.17E-04 | 233,359 |
| 0302 | 218 | 168 | 6% | $ 2,540 | | | 3.7 | 2.89 | | $K_{a,SR}$ | 1.840 | 0.018 | | 5.38E-05 | 139,825 |
| 0303 | 117 | 92 | 3% | $ 1,377 | | | 2.0 | 1.57 | | 0.35 | 0.893 | 0.025 | | 9.82E-05 | 0 |
| 0304 | 117 | 68 | 3% | $ 1,071 | | | 2.0 | 1.16 | | 0.29 | 0.368 | 0.093 | | 7.57E-04 | 386,887 |
| 0305 | 289 | 74 | 8% | $ 1,448 | | | 5.0 | 1.27 | | 0.36 | 0.777 | 0.090 | | 1.35E-03 | 774,329 |
| 0306 | 372 | 83 | 11% | $ 1,706 | | | 6.4 | 1.42 | | | 2.522 | 0.044 | | 9.37E-04 | 1,246,113 |
| 0401 | 143 | 107 | 4% | $ 2,032 | | | 3.1 | 2.30 | | | 4.773 | 0.009 | 0.0043 | 6.42E-05 | 151,555 |
| 0402 | 137 | 106 | 4% | $ 2,002 | | | 2.9 | 2.28 | | | 1.093 | 0.013 | | 2.92E-05 | 142,684 |
| 0403 | 93 | 73 | 3% | $ 1,377 | | | 2.0 | 1.57 | | | 0.893 | 0.025 | | 1.23E-04 | 135,176 |
| 0404 | 93 | 54 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 0.408 | 0.033 | | 1.33E-04 | 223,989 |
| 0405 | 175 | 45 | 5% | $ 1,100 | | | 3.8 | 0.96 | | | 0.301 | 0.086 | | 7.94E-04 | 699,568 |
| 0406 | 491 | 110 | 14% | $ 2,830 | | | 10.6 | 2.36 | | | 0.305 | 0.170 | | 1.28E-03 | 1,292,753 |
| | *3,404* | *1,600* | *100%* | *$1.54E+06* | | | | | | | | Totals | **0.0137** | **0.0077** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0039** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,404 | 14,910 | 18,314 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,092 | 9,692 |
| % NH B | TRUE | 0.33 | Percent | 0.17 | 0.83 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.30 | Count | 3,198 | 3,198 | 3,295 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,832,292 |
| | 32767 | | *SSU* | $ 2,600 | $ 24,840.89 | $ 1,262,393 |
| | 0 | | *OCC HU* | $ 850 | $ 1,540,373 | $ 3,780,101 |
| | | | *UNOCC HU* | $ 100 | $ 2,125,214 | $ 7,874,786 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d* (NSR PSUs) 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Non Self-Representing Optimum Allocation $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | ANOVA Variance Components Estimates, NSR: $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_a q_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1450 | 58 | 240 | 433 | 171 | 2351 | 1748 | 16% | $ 9,552 | 81 | 2.0 | 14.5 | 10.80 | 0.046 | 0.182 | 12.820 | 0.005 | 0.0004 | 1.50E-07 | 50,441 |
| 17 | 949 | 389 | 258 | 137 | 1750 | 1355 | 12% | $ 7,358 | | 2.0 | 10.8 | 8.37 | | 0.306 | 7.504 | 0.037 | | 7.39E-06 | 783,493 |
| 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 0.504 | 6.197 | 0.070 | | 1.20E-04 | 963,377 |
| 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 4.587 | 0.109 | | 2.88E-04 | 1,246,277 |
| 68 | 96 | 137 | 720 | 155 | 1175 | 301 | 8% | $ 2,122 | | | 7.3 | 1.86 | | | 4.315 | 0.224 | | 7.21E-04 | 4,295,112 |
| 132 | 125 | 146 | 791 | 612 | 1806 | 403 | 12% | $ 2,983 | | | 11.2 | 2.49 | | | 8.589 | 0.172 | | 6.30E-04 | 5,448,707 |
| 471 | 19 | 78 | 141 | 56 | 764 | 568 | 5% | $ 3,104 | | | 4.7 | 3.51 | | | 21.693 | 0.013 | 0.0002 | 6.52E-06 | 276,933 |
| 8 | 436 | 179 | 118 | 63 | 804 | 623 | 5% | $ 3,382 | | | 5.0 | 3.85 | | $K_{a,NSR}$ | 8.858 | 0.025 | | 9.23E-06 | 330,172 |
| 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.60 | 20.494 | 0.008 | | 4.67E-06 | 2,248 |
| 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.28 | 6.641 | 0.060 | | 1.25E-04 | 697,302 |
| 46 | 65 | 93 | 488 | 105 | 797 | 204 | 5% | $ 1,440 | | | 4.9 | 1.26 | | 0.11 | 9.049 | 0.105 | | 4.85E-04 | 1,968,936 |
| 89 | 85 | 99 | 534 | 413 | 1220 | 272 | 8% | $ 2,016 | | | 7.5 | 1.68 | | | 19.962 | 0.077 | | 4.31E-04 | 2,635,977 |
| 341 | 14 | 56 | 102 | 40 | 553 | 411 | 4% | $ 2,248 | | | 3.4 | 2.54 | | | 67.783 | 0.004 | 0.0000 | 2.85E-06 | 107,901 |
| 8 | 473 | 194 | 128 | 68 | 872 | 675 | 6% | $ 3,667 | | | 5.4 | 4.17 | | | 35.626 | 0.007 | | 2.35E-06 | 195,082 |
| 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 20.494 | 0.008 | | 4.67E-06 | 80,117 |
| 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 12.696 | 0.018 | | 2.28E-05 | 260,442 |
| 19 | 26 | 38 | 198 | 43 | 324 | 83 | 2% | $ 584 | | | 2.0 | 0.51 | | | 12.021 | 0.032 | | 1.49E-04 | 787,112 |
| 40 | 38 | 45 | 241 | 187 | 551 | 123 | 4% | $ 910 | | | 3.4 | 0.76 | | | 28.973 | 0.028 | | 1.86E-04 | 1,286,740 |
| 2,751 | 2,682 | 2,659 | 4,520 | 2,298 | *14,910* | *8,092* | *100%* | *$3.78E+06* | | | | | | | | Totals | **0.0006** | **0.0032** | **2.14E+07** |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.69 | 0.0039 | 0.0624 | 0.075 | 0.529 | 0.0214 | 0.0044 |

# Figure 19. Anticipated Variance Excel Solver Set Up and Results for `employed`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* | | | | | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 65 | 3 | 11 | 19 | 8 | 106 | 79 | 3% | $ 1,315 | 16 | 3.3 | 2.0 | 1.49 | $ 657.44 | 0.112 | 0.276 | 0.002 | 0.0023 | 3.26E-07 | 15,515 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 78 | 32 | 21 | 11 | 143 | 111 | 4% | $ 1,845 | | 3.9 | 2.7 | 2.10 | $ 680.62 | 0.067 | 0.096 | 0.027 | | 1.01E-05 | 157,023 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 11 | 69 | 15 | 8 | 106 | 83 | 3% | $ 1,377 | | 3.1 | 2.0 | 1.57 | $ 688.72 | 0.050 | 0.290 | 0.023 | | 2.92E-05 | 171,502 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 21 | 36 | 25 | 19 | 106 | 61 | 3% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.159 | 0.032 | | 4.29E-05 | 218,646 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 13 | 18 | 95 | 20 | 155 | 40 | 5% | $ 859 | | | 2.9 | 0.75 | $ 292.23 | | 0.087 | 0.096 | | 3.21E-04 | 785,866 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 17 | 16 | 19 | 104 | 81 | 238 | 53 | 7% | $ 1,203 | | | 4.5 | 1.00 | $ 267.37 | | 0.180 | 0.072 | | 2.84E-04 | 831,323 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 186 | 7 | 31 | 55 | 22 | 302 | 224 | 9% | $ 3,203 | | | 4.9 | 3.62 | | | 0.394 | 0.036 | 0.0026 | 3.65E-05 | 233,359 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 77 | 31 | 21 | 11 | 141 | 109 | 4% | $ 1,552 | | | 2.3 | 1.76 | | $K_{a,SR}$ | 0.719 | 0.014 | | 2.19E-05 | 139,825 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 4 | 13 | 80 | 17 | 10 | 124 | 97 | 4% | $ 1,377 | | | 2.0 | 1.57 | | 0.26 | 0.270 | 0.023 | | 2.46E-05 | 0 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 25 | 43 | 30 | 22 | 124 | 72 | 4% | $ 1,071 | | | 2.0 | 1.16 | | 0.39 | 0.249 | 0.048 | | 1.30E-04 | 386,887 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 13 | 19 | 27 | 139 | 30 | 227 | 58 | 7% | $ 1,072 | | | 3.7 | 0.94 | | 0.36 | 0.155 | 0.105 | | 4.66E-04 | 774,329 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 37 | 35 | 41 | 223 | 172 | 508 | 113 | 15% | $ 2,197 | | | 8.2 | 1.83 | | | 0.151 | 0.170 | | 6.19E-04 | 1,246,113 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 98 | 4 | 16 | 29 | 12 | 159 | 118 | 5% | $ 2,136 | | | 3.2 | 2.42 | | | 0.500 | 0.019 | 0.0021 | 2.52E-05 | 151,555 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 71 | 29 | 19 | 10 | 131 | 101 | 4% | $ 1,819 | | | 2.7 | 2.07 | | | 0.130 | 0.021 | | 9.37E-06 | 142,684 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 10 | 64 | 13 | 8 | 98 | 77 | 3% | $ 1,377 | | | 2.0 | 1.57 | | | 0.270 | 0.023 | | 3.11E-05 | 135,176 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 20 | 34 | 24 | 17 | 98 | 57 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 0.072 | 0.038 | | 2.91E-05 | 223,989 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 13 | 18 | 96 | 21 | 157 | 40 | 5% | $ 935 | | | 3.2 | 0.82 | | | 0.110 | 0.085 | | 3.14E-04 | 699,568 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 35 | 33 | 38 | 208 | 161 | 474 | 106 | 14% | $ 2,587 | | | 9.7 | 2.16 | | | 0.145 | 0.163 | | 5.85E-04 | 1,292,753 |
| | | | | | | Totals | *494* | *468* | *637* | *1,154* | *642* | *3,395* | *1,600* | *100%* | *$1.54E+06* | | | | | | | | Totals | **0.0069** | **0.0030** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | | No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0017** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 | 1414 | 56 | 234 | 422 | 167 | 2292 | 1704 | 15% | $ 9,309 | 81 | 2.0 | 14.2 | 10.52 | 0.011 | 0.070 | 5.470 | 0.003 | 0.0002 | 3.24E-08 | 50,441 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 | 17 | 930 | 381 | 253 | 135 | 1715 | 1328 | 12% | $ 7,210 | | 2.0 | 10.6 | 8.20 | | 0.110 | 1.641 | 0.049 | | 2.91E-06 | 783,493 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 | 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 0.161 | 3.251 | 0.052 | | 3.46E-05 | 963,377 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 | 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 1.994 | 0.079 | | 6.63E-05 | 1,246,277 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 | 60 | 86 | 122 | 641 | 138 | 1046 | 268 | 7% | $ 1,889 | | | 6.5 | 1.66 | | | 2.161 | 0.196 | | 3.08E-04 | 4,295,112 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | | 133 | 127 | 148 | 799 | 619 | 1825 | 407 | 12% | $ 3,015 | | | 11.3 | 2.52 | | | 2.724 | 0.215 | | 3.08E-04 | 5,448,707 |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total | 445 | 18 | 74 | 133 | 52 | 722 | 537 | 5% | $ 2,931 | | | 4.5 | 3.31 | | | 4.217 | 0.018 | 0.0001 | 2.65E-06 | 276,933 |
| MAX Budget | TRUE | 10000000 | | 3,395 | 14,844 | 18,240 | 9 | 526 | 216 | 143 | 76 | 970 | 751 | 7% | $ 4,077 | | | 6.0 | 4.64 | | $K_{a,NSR}$ | 2.984 | 0.023 | | 2.17E-06 | 330,172 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | | 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.59 | 34.855 | 0.003 | | 1.16E-06 | 2,248 |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* | 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.29 | 2.728 | 0.048 | | 3.28E-05 | 697,302 |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,095 | 9,695 | 39 | 55 | 78 | 410 | 88 | 670 | 172 | 5% | $ 1,210 | | | 4.1 | 1.06 | | 0.12 | 4.492 | 0.087 | | 2.00E-04 | 1,968,936 |
| % NH B | TRUE | 0.33 | Percent | 0.17 | 0.83 | 1 | 96 | 91 | 106 | 574 | 444 | 1311 | 292 | 9% | $ 2,164 | | | 8.1 | 1.81 | | | 5.496 | 0.108 | | 2.21E-04 | 2,635,977 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | | 360 | 14 | 60 | 108 | 42 | 584 | 434 | 4% | $ 2,372 | | | 3.6 | 2.68 | | | 11.842 | 0.006 | 0.0000 | 9.82E-07 | 107,901 |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* | 8 | 463 | 190 | 126 | 67 | 854 | 661 | 6% | $ 3,591 | | | 5.3 | 4.08 | | | 11.956 | 0.007 | | 8.88E-07 | 195,082 |
| % NH O | TRUE | 0.30 | Count | 3,199 | 3,199 | 3,296 | 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 34.855 | 0.003 | | 1.16E-06 | 80,117 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 | 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 4.375 | 0.018 | | 7.26E-06 | 260,442 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST | 19 | 26 | 38 | 198 | 43 | 324 | 83 | 2% | $ 584 | | | 2.0 | 0.51 | | | 5.859 | 0.035 | | 8.70E-05 | 787,112 |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,833,226 | 43 | 41 | 48 | 257 | 199 | 588 | 131 | 4% | $ 970 | | | 3.6 | 0.81 | | | 5.150 | 0.050 | | 9.94E-05 | 1,286,740 |
| | 32767 | | *SSU* | $ 2,600 | $ 26,602.21 | $ 1,262,809 | 2,705 | 2,731 | 2,659 | 4,431 | 2,318 | *14,844* | *8,095* | *100%* | *$3.78E+06* | | | | | | | | Totals | **0.0002** | **0.0014** | **2.14E+07** |
| | 0 | | *OCC HU* | $ 850 | $ 1,539,546 | $ 3,777,810 | | | | | | | | | **Summary of Solution** | | | | | | | | | | | |
| | | | *UNOCC HU* | $ 100 | $ 2,126,148 | $ 7,873,845 | | | | | | #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) | | | | | | |
| | | | | | Total Cost | $1.00E+07 | | | | | | 100 | 100 | 1.74 | 0.0017 | 0.0410 | 0.079 | 0.518 | 0.0099 | 0.0017 | | | | | | |

Figure 20. Anticipated Variance Excel Solver Set Up and Results for `ownHome`

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$ (d=1) 45-62 H | (d=2) 45-62 NH B | (d=3) 45-62 NH O | (d=4) NOT 45-62 | (d=5) UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 65 | 3 | 11 | 19 | 8 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 77 | 31 | 21 | 11 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 11 | 68 | 14 | 8 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 21 | 36 | 25 | 19 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 13 | 18 | 94 | 20 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 17 | 16 | 19 | 101 | 78 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 164 | 7 | 27 | 49 | 19 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 107 | 44 | 29 | 16 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 4 | 14 | 85 | 18 | 10 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 5 | 26 | 45 | 32 | 23 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 13 | 18 | 26 | 135 | 29 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 27 | 26 | 30 | 162 | 125 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 118 | 5 | 20 | 35 | 14 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 71 | 29 | 19 | 10 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 11 | 66 | 14 | 8 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 20 | 35 | 25 | 18 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 8 | 11 | 15 | 80 | 17 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 30 | 29 | 33 | 180 | 139 |
| | | | | | | Totals | *477* | *484* | *639* | *1,053* | *574* |

| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | ANOVA Variance Components Estimates, SR $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 105 | 78 | 3% | $ 1,315 | 16 | 3.3 | 2.0 | 1.49 | $ 657.44 | 0.131 | 0.036 | 0.004 | 0.0041 | 1.10E-07 | 15,515 |
| 0202 | 141 | 109 | 4% | $ 1,831 | | 4.1 | 2.7 | 2.08 | $ 680.62 | 0.118 | 0.068 | 0.035 | | 1.21E-05 | 157,023 |
| 0203 | 105 | 83 | 3% | $ 1,377 | | 3.2 | 2.0 | 1.57 | $ 688.72 | 0.202 | 0.194 | 0.035 | | 4.63E-05 | 171,502 |
| 0204 | 105 | 61 | 3% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.115 | 0.049 | | 7.28E-05 | 218,646 |
| 0205 | 154 | 39 | 5% | $ 856 | | | 2.9 | 0.75 | $ 292.23 | | 0.064 | 0.141 | | 5.18E-04 | 785,866 |
| 0206 | 231 | 51 | 7% | $ 1,174 | | | 4.4 | 0.98 | $ 267.37 | | 0.484 | 0.055 | | 4.58E-04 | 831,323 |
| 0301 | 266 | 198 | 8% | $ 2,675 | | | 4.1 | 3.02 | | | 0.199 | 0.047 | 0.0051 | 3.52E-05 | 233,359 |
| 0302 | 198 | 153 | 6% | $ 2,057 | | | 3.0 | 2.34 | | $K_{a,SR}$ | 0.473 | 0.023 | | 2.64E-05 | 139,825 |
| 0303 | 131 | 103 | 4% | $ 1,377 | | | 2.0 | 1.57 | | 0.32 | 0.340 | 0.026 | | 3.47E-05 | 0 |
| 0304 | 131 | 76 | 4% | $ 1,071 | | | 2.0 | 1.16 | | 0.42 | 0.057 | 0.102 | | 1.24E-04 | 386,887 |
| 0305 | 221 | 57 | 7% | $ 985 | | | 3.4 | 0.86 | | 0.26 | 0.138 | 0.143 | | 7.99E-04 | 774,329 |
| 0306 | 369 | 82 | 11% | $ 1,509 | | | 5.6 | 1.26 | | | 0.473 | 0.093 | | 7.93E-04 | 1,246,113 |
| 0401 | 192 | 143 | 6% | $ 2,467 | | | 3.8 | 2.79 | | | 0.336 | 0.028 | 0.0042 | 3.02E-05 | 151,555 |
| 0402 | 131 | 101 | 4% | $ 1,739 | | | 2.6 | 1.98 | | | 0.581 | 0.011 | | 1.17E-05 | 142,684 |
| 0403 | 102 | 80 | 3% | $ 1,377 | | | 2.0 | 1.57 | | | 0.340 | 0.026 | | 4.44E-05 | 135,176 |
| 0404 | 102 | 59 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 0.149 | 0.033 | | 4.32E-05 | 223,989 |
| 0405 | 130 | 33 | 4% | $ 745 | | | 2.5 | 0.65 | | | 0.144 | 0.078 | | 4.15E-04 | 699,568 |
| 0406 | 412 | 92 | 13% | $ 2,150 | | | 8.0 | 1.79 | | | 0.876 | 0.072 | | 7.86E-04 | 1,292,753 |
| | *3,227* | *1,600* | *100%* | *$1.52E+06* | | | | | | | | Totals | **0.0135** | **0.0042** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0022** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,227 | 14,851 | 18,078 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,113 | 9,713 |
| % NH B | TRUE | 0.33 | Percent | 0.16 | 0.84 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.30 | Count | 3,205 | 3,205 | 3,302 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,839,431 |
| | 32767 | | *SSU* | $ 2,600 | $ 27,491.84 | $ 1,265,575 |
| | 0 | | *OCC HU* | $ 850 | $ 1,522,695 | $ 3,784,810 |
| | | | *UNOCC HU* | $ 100 | $ 2,110,187 | $ 7,889,816 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Non Self-Representing Optimum Allocation $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | ANOVA Variance Components Estimates, NSR $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1135 | 45 | 188 | 339 | 134 | 1841 | 1368 | 12% | $ 7,460 | 81 | 2.0 | 11.3 | 8.43 | 0.012 | 0.074 | 11.914 | 0.002 | 0.0002 | 4.41E-08 | 50,441 |
| 17 | 964 | 395 | 262 | 140 | 1777 | 1376 | 12% | $ 7,455 | | 2.0 | 11.0 | 8.48 | | 0.118 | 1.636 | 0.049 | | 2.87E-06 | 783,493 |
| 9 | 35 | 211 | 45 | 25 | 325 | 255 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 0.241 | 1.563 | 0.079 | | 3.79E-05 | 963,377 |
| 12 | 65 | 112 | 78 | 58 | 325 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 1.227 | 0.113 | | 8.29E-05 | 1,246,277 |
| 63 | 90 | 129 | 674 | 145 | 1100 | 282 | 7% | $ 1,982 | | | 6.8 | 1.74 | | | 1.948 | 0.231 | | 3.68E-04 | 4,295,112 |
| 129 | 123 | 143 | 774 | 599 | 1767 | 394 | 12% | $ 2,911 | | | 10.9 | 2.43 | | | 8.467 | 0.127 | | 3.44E-04 | 5,448,707 |
| 671 | 27 | 111 | 200 | 79 | 1087 | 808 | 7% | $ 4,406 | | | 6.7 | 4.98 | | | 3.684 | 0.022 | 0.0001 | 2.15E-06 | 276,933 |
| 9 | 511 | 209 | 139 | 74 | 942 | 729 | 6% | $ 3,950 | | | 5.8 | 4.49 | | $K_{a,NSR}$ | 3.640 | 0.023 | | 2.63E-06 | 330,172 |
| 9 | 35 | 211 | 45 | 25 | 325 | 255 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.60 | 8.191 | 0.007 | | 1.55E-06 | 2,248 |
| 12 | 65 | 112 | 78 | 58 | 325 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.29 | 1.519 | 0.066 | | 3.50E-05 | 697,302 |
| 42 | 59 | 85 | 442 | 95 | 723 | 185 | 5% | $ 1,301 | | | 4.5 | 1.14 | | 0.10 | 4.892 | 0.103 | | 2.79E-04 | 1,968,936 |
| 95 | 90 | 105 | 571 | 442 | 1304 | 291 | 9% | $ 2,149 | | | 8.0 | 1.79 | | | 12.776 | 0.081 | | 2.88E-04 | 2,635,977 |
| 440 | 18 | 73 | 131 | 52 | 713 | 530 | 5% | $ 2,889 | | | 4.4 | 3.27 | | | 20.092 | 0.005 | 0.0000 | 9.54E-07 | 107,901 |
| 8 | 433 | 177 | 118 | 63 | 799 | 618 | 5% | $ 3,350 | | | 4.9 | 3.81 | | | 21.784 | 0.006 | | 1.28E-06 | 195,082 |
| 9 | 35 | 211 | 45 | 25 | 325 | 255 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 8.191 | 0.007 | | 1.55E-06 | 80,117 |
| 12 | 65 | 112 | 78 | 58 | 325 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 3.594 | 0.023 | | 1.02E-05 | 260,442 |
| 19 | 27 | 38 | 199 | 43 | 325 | 83 | 2% | $ 584 | | | 2.0 | 0.51 | | | 6.920 | 0.030 | | 7.52E-05 | 787,112 |
| 38 | 37 | 43 | 231 | 178 | 526 | 117 | 4% | $ 867 | | | 3.2 | 0.72 | | | 16.364 | 0.027 | | 1.04E-04 | 1,286,740 |
| 2,728 | 2,721 | 2,663 | 4,447 | 2,291 | *14,851* | *8,113* | *100%* | *$3.78E+06* | | | | | | | | Totals | **0.0002** | **0.0016** | **2.14E+07** |

| #Hus/PSU SR | #Hus/PSU NSR | Summary of Solution deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.75 | 0.0022 | 0.0474 | 0.062 | 0.564 | 0.0177 | 0.0020 |

Figure 21. Anticipated Variance Excel Solver Set Up and Results for `ownStock`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d* (SR PSUs) | | | | | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}$ $/n_a$ | $K^2ab*W^2_{3ab}$ $/n_aq_{ab}$ | $Q_{ab}$ |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 56 | 2 | 9 | 17 | 7 | 91 | 68 | 3% | $ 1,315 | 16 | 2.8 | 2.0 | 1.49 | $ 657.44 | 0.243 | 1.384 | 0.003 | 0.0054 | 2.72E-06 | 15,515 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 69 | 28 | 19 | 10 | 128 | 99 | 4% | $ 1,914 | | 4.2 | 2.8 | 2.18 | $ 680.62 | 0.134 | 0.309 | 0.030 | | 4.59E-05 | 157,023 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 10 | 59 | 13 | 7 | 91 | 71 | 3% | $ 1,377 | | 3.2 | 2.0 | 1.57 | $ 688.72 | 0.260 | 3.587 | 0.011 | | 9.36E-05 | 171,502 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 18 | 31 | 22 | 16 | 91 | 53 | 3% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.384 | 0.056 | | 3.67E-04 | 218,646 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 12 | 18 | 93 | 20 | 151 | 39 | 4% | $ 972 | | | 3.3 | 0.85 | $ 292.23 | | 0.164 | 0.108 | | 7.85E-04 | 785,866 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 20 | 19 | 22 | 120 | 93 | 275 | 61 | 8% | $ 1,616 | | | 6.0 | 1.35 | $ 267.37 | | 0.990 | 0.054 | | 7.62E-04 | 831,323 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 170 | 7 | 28 | 51 | 20 | 276 | 205 | 8% | $ 2,729 | | | 4.2 | 3.09 | | | 3.139 | 0.022 | 0.0071 | 1.18E-04 | 233,359 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 116 | 48 | 32 | 17 | 215 | 166 | 6% | $ 2,195 | | | 3.2 | 2.50 | | $K_{a,SR}$ | 1.195 | 0.029 | | 9.88E-05 | 139,825 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 4 | 14 | 86 | 18 | 10 | 133 | 104 | 4% | $ 1,377 | | | 2.0 | 1.57 | | 0.25 | 2.306 | 0.016 | | 8.54E-05 | 0 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 5 | 27 | 46 | 32 | 24 | 133 | 77 | 4% | $ 1,071 | | | 2.0 | 1.16 | | 0.47 | 0.307 | 0.145 | | 1.34E-03 | 386,887 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 16 | 23 | 34 | 175 | 38 | 287 | 73 | 8% | $ 1,259 | | | 4.3 | 1.10 | | 0.28 | 0.860 | 0.095 | | 1.69E-03 | 774,329 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 39 | 37 | 43 | 232 | 179 | 529 | 118 | 15% | $ 2,126 | | | 8.0 | 1.77 | | | 0.445 | 0.171 | | 1.77E-03 | 1,246,113 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 81 | 3 | 13 | 24 | 10 | 132 | 98 | 4% | $ 1,700 | | | 2.6 | 1.92 | | | 44.209 | 0.003 | 0.0063 | 5.53E-05 | 151,555 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 68 | 28 | 18 | 10 | 126 | 97 | 4% | $ 1,675 | | | 2.5 | 1.91 | | | 0.696 | 0.020 | | 4.58E-05 | 142,684 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 11 | 66 | 14 | 8 | 102 | 80 | 3% | $ 1,377 | | | 2.0 | 1.57 | | | 2.306 | 0.016 | | 1.11E-04 | 135,176 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 20 | 35 | 25 | 18 | 102 | 59 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 0.232 | 0.064 | | 2.53E-04 | 223,989 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 13 | 18 | 94 | 20 | 154 | 40 | 5% | $ 884 | | | 3.0 | 0.78 | | | 1.059 | 0.044 | | 8.37E-04 | 699,568 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 30 | 28 | 33 | 178 | 138 | 406 | 91 | 12% | $ 2,130 | | | 8.0 | 1.78 | | | 0.549 | 0.114 | | 1.25E-03 | 1,292,753 |
| | | | | | | Totals | *456* | *498* | *646* | *1,176* | *644* | *3,421* | *1,600* | *100%* | *$1.54E+06* | | | | | | | | Totals | **0.0189** | **0.0097** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0069** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,421 | 14,658 | 18,079 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,103 | 9,703 |
| % NH B | TRUE | 0.33 | Percent | 0.16 | 0.84 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.30 | Count | 3,202 | 3,202 | 3,299 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,835,966 |
| | 32767 | | *SSU* | $ 2,600 | $ 26,484.85 | $ 1,264,031 |
| | 0 | | *OCC HU* | $ 850 | $ 1,542,066 | $ 3,771,452 |
| | | | *UNOCC HU* | $ 100 | $ 2,128,551 | $ 7,871,449 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d* (NSR PSUs) | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}$ $/mn_a$ | $K^2ab*W^2_{3ab}$ $/mn_aq_{ab}$ | $Q_{ab}$ |
| 1255 | 50 | 208 | 374 | 148 | 2035 | 1512 | 14% | $ 8,254 | 81 | 2.0 | 12.6 | 9.33 | 0.086 | 0.290 | 61.851 | 0.002 | 0.0005 | 2.09E-07 | 50,441 |
| 15 | 842 | 345 | 229 | 122 | 1552 | 1201 | 11% | $ 6,518 | | 2.0 | 9.6 | 7.41 | | 0.180 | 8.888 | 0.049 | | 1.79E-05 | 783,493 |
| 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 1.309 | 23.176 | 0.048 | | 2.09E-04 | 963,377 |
| 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 9.357 | 0.122 | | 7.39E-04 | 1,246,277 |
| 68 | 96 | 137 | 719 | 154 | 1174 | 301 | 8% | $ 2,117 | | | 7.2 | 1.86 | | | 9.702 | 0.208 | | 1.39E-03 | 4,295,112 |
| 115 | 110 | 128 | 693 | 536 | 1581 | 353 | 11% | $ 2,609 | | | 9.8 | 2.18 | | | 30.983 | 0.108 | | 1.02E-03 | 5,448,707 |
| 614 | 25 | 102 | 183 | 72 | 995 | 740 | 7% | $ 4,039 | | | 6.1 | 4.57 | | | 75.174 | 0.009 | 0.0002 | 7.39E-06 | 276,933 |
| 11 | 596 | 244 | 162 | 86 | 1100 | 851 | 8% | $ 4,619 | | | 6.8 | 5.25 | | $K_{a,NSR}$ | 21.648 | 0.028 | | 2.04E-05 | 330,172 |
| 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.54 | 135.570 | 0.003 | | 5.92E-06 | 2,248 |
| 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.39 | 8.356 | 0.109 | | 5.23E-04 | 697,302 |
| 47 | 67 | 96 | 501 | 108 | 818 | 210 | 6% | $ 1,475 | | | 5.0 | 1.29 | | 0.07 | 14.993 | 0.125 | | 1.11E-03 | 1,968,936 |
| 84 | 80 | 94 | 507 | 392 | 1158 | 258 | 8% | $ 1,910 | | | 7.1 | 1.59 | | | 16.609 | 0.118 | | 8.99E-04 | 2,635,977 |
| 416 | 17 | 69 | 124 | 49 | 674 | 501 | 5% | $ 2,734 | | | 4.2 | 3.09 | | | 736.096 | 0.001 | 0.0000 | 2.38E-06 | 107,901 |
| 8 | 466 | 191 | 127 | 67 | 859 | 665 | 6% | $ 3,607 | | | 5.3 | 4.10 | | | 54.335 | 0.011 | | 9.53E-06 | 195,082 |
| 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 135.570 | 0.003 | | 5.92E-06 | 80,117 |
| 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 53.560 | 0.014 | | 5.87E-05 | 260,442 |
| 19 | 27 | 38 | 198 | 43 | 324 | 83 | 2% | $ 584 | | | 2.0 | 0.51 | | | 36.071 | 0.021 | | 1.92E-04 | 787,112 |
| 32 | 31 | 36 | 194 | 150 | 444 | 99 | 3% | $ 732 | | | 2.7 | 0.61 | | | 73.085 | 0.021 | | 3.11E-04 | 1,286,740 |
| 2,746 | 2,704 | 2,653 | 4,379 | 2,177 | *14,658* | *8,103* | *100%* | *$3.77E+06* | | | | | | | | Totals | **0.0007** | **0.0065** | **2.14E+07** |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.75 | 0.0069 | 0.0828 | 0.110 | 0.447 | 0.0286 | 0.0083 |

Figure 22. Anticipated Variance Excel Solver Set Up and Results for `own_2nd_home`

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$ (d=1) 45-62 H | (d=2) 45-62 NH B | (d=3) 45-62 NH O | (d=4) NOT 45-62 | (d=5) UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d* (SR PSUs) 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 83 | 3 | 14 | 25 | 10 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 103 | 42 | 28 | 15 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 8 | 51 | 11 | 6 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 16 | 27 | 19 | 14 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 12 | 17 | 25 | 129 | 28 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 14 | 14 | 16 | 87 | 67 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 153 | 6 | 25 | 46 | 18 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 124 | 51 | 34 | 18 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 11 | 64 | 14 | 8 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 20 | 34 | 24 | 18 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 13 | 19 | 27 | 142 | 30 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 40 | 39 | 45 | 243 | 188 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 113 | 5 | 19 | 34 | 13 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 83 | 34 | 23 | 12 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 8 | 46 | 10 | 6 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 14 | 25 | 17 | 13 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 8 | 11 | 16 | 82 | 18 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 26 | 25 | 29 | 159 | 123 |
| | | | | | | Totals | *485* | *525* | *590* | *1,124* | *604* |

| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR |
|---|---|---|---|---|---|---|---|---|
| 0201 | 135 | 100 | 4% | $ 2,260 | 16 | 2.5 | 3.4 | 2.55 |
| 0202 | 189 | 146 | 6% | $ 3,278 | | 3.1 | 4.8 | 3.73 |
| 0203 | 78 | 62 | 2% | $ 1,377 | | 2.2 | 2.0 | 1.57 |
| 0204 | 78 | 46 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 0205 | 211 | 54 | 6% | $ 1,573 | | | 5.4 | 1.38 |
| 0206 | 198 | 44 | 6% | $ 1,349 | | | 5.0 | 1.13 |
| 0301 | 248 | 185 | 7% | $ 3,297 | | | 5.0 | 3.73 |
| 0302 | 229 | 178 | 7% | $ 3,151 | | | 4.6 | 3.58 |
| 0303 | 99 | 78 | 3% | $ 1,377 | | | 2.0 | 1.57 |
| 0304 | 99 | 58 | 3% | $ 1,071 | | | 2.0 | 1.16 |
| 0305 | 231 | 59 | 7% | $ 1,364 | | | 4.7 | 1.20 |
| 0306 | 555 | 124 | 17% | $ 2,997 | | | 11.2 | 2.50 |
| 0401 | 183 | 136 | 5% | $ 3,373 | | | 5.1 | 3.81 |
| 0402 | 153 | 119 | 5% | $ 2,925 | | | 4.3 | 3.33 |
| 0403 | 71 | 56 | 2% | $ 1,377 | | | 2.0 | 1.57 |
| 0404 | 71 | 41 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 0405 | 134 | 34 | 4% | $ 1,097 | | | 3.8 | 0.96 |
| 0406 | 363 | 81 | 11% | $ 2,722 | | | 10.2 | 2.27 |
| Totals | *3,328* | *1,600* | *100%* | *$1.53E+06* | | | | |

| SSU/MSG strata | ANOVA Variance Components Estimates, SR: Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|
| 0201 | $ 657.44 | 0.347 | 0.624 | 0.012 | 0.0180 | 1.35E-05 | 15,515 |
| 0202 | $ 680.62 | 0.453 | 1.387 | 0.038 | | 2.17E-04 | 157,023 |
| 0203 | $ 688.72 | 0.475 | 5.141 | 0.027 | | 9.39E-04 | 171,502 |
| 0204 | $ 535.63 | | 5.006 | 0.026 | | 1.16E-03 | 218,646 |
| 0205 | $ 292.23 | | 0.702 | 0.150 | | 4.64E-03 | 785,866 |
| 0206 | $ 267.37 | | 0.786 | 0.094 | | 2.51E-03 | 831,323 |
| 0301 | | | 27.911 | 0.014 | 0.0201 | 4.97E-04 | 233,359 |
| 0302 | | $K_{a,SR}$ | 17.823 | 0.010 | | 1.74E-04 | 139,825 |
| 0303 | | 0.36 | 2.698 | 0.025 | | 3.39E-04 | 0 |
| 0304 | | 0.37 | 2.239 | 0.083 | | 4.30E-03 | 386,887 |
| 0305 | | 0.27 | 3.907 | 0.071 | | 5.28E-03 | 774,329 |
| 0306 | | | 1.620 | 0.187 | | 7.33E-03 | 1,246,113 |
| 0401 | | | 3.910 | 0.033 | 0.0159 | 4.88E-04 | 151,555 |
| 0402 | | | 7.412 | 0.009 | | 8.04E-05 | 142,684 |
| 0403 | | | 2.698 | 0.025 | | 4.72E-04 | 135,176 |
| 0404 | | | 2.110 | 0.036 | | 1.08E-03 | 223,989 |
| 0405 | | | 2.310 | 0.053 | | 3.05E-03 | 699,568 |
| 0406 | | | 2.173 | 0.109 | | 5.06E-03 | 1,292,753 |
| | | | | Totals | **0.0540** | **0.0376** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0140** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,328 | 15,637 | 18,965 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,065 | 9,665 |
| % NH B | TRUE | 0.33 | Percent | 0.17 | 0.83 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.30 | Count | 3,189 | 3,189 | 3,286 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,822,693 |
| | 32767 | | *SSU* | $ 2,600 | $ 20,219.96 | $ 1,258,115 |
| | 0 | | *OCC HU* | $ 850 | $ 1,532,799 | $ 3,806,175 |
| | | | *UNOCC HU* | $ 100 | $ 2,113,019 | $ 7,886,982 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d* (NSR PSUs) 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab |
|---|---|---|---|---|---|---|---|---|
| 1286 | 51 | 213 | 384 | 152 | 2085 | 1549 | 13% | $ 8,497 |
| 14 | 772 | 316 | 210 | 112 | 1423 | 1102 | 9% | $ 6,005 |
| 9 | 34 | 209 | 44 | 25 | 323 | 253 | 2% | $ 1,377 |
| 12 | 64 | 111 | 78 | 57 | 323 | 187 | 2% | $ 1,071 |
| 77 | 109 | 156 | 814 | 175 | 1330 | 341 | 9% | $ 2,410 |
| 162 | 154 | 179 | 972 | 752 | 2220 | 495 | 14% | $ 3,680 |
| 528 | 21 | 87 | 158 | 62 | 856 | 636 | 5% | $ 3,489 |
| 9 | 530 | 217 | 144 | 77 | 978 | 757 | 6% | $ 4,128 |
| 9 | 34 | 209 | 44 | 25 | 323 | 253 | 2% | $ 1,377 |
| 12 | 64 | 111 | 78 | 57 | 323 | 187 | 2% | $ 1,071 |
| 52 | 74 | 105 | 551 | 118 | 900 | 231 | 6% | $ 1,631 |
| 121 | 115 | 134 | 724 | 560 | 1653 | 369 | 11% | $ 2,741 |
| 327 | 13 | 54 | 98 | 39 | 531 | 395 | 3% | $ 2,164 |
| 8 | 465 | 190 | 126 | 67 | 857 | 663 | 5% | $ 3,616 |
| 9 | 34 | 209 | 44 | 25 | 323 | 253 | 2% | $ 1,377 |
| 12 | 64 | 111 | 78 | 57 | 323 | 187 | 2% | $ 1,071 |
| 19 | 26 | 38 | 197 | 42 | 323 | 83 | 2% | $ 584 |
| 40 | 38 | 44 | 239 | 185 | 546 | 122 | 3% | $ 905 |
| 2,704 | 2,665 | 2,696 | 4,984 | 2,589 | *15,637* | *8,065* | *100%* | *$3.81E+06* |

| Non Self-Representing Optimum Allocation $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | ANOVA Variance Components Estimates, NSR: $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 81 | 2.0 | 12.9 | 9.61 | 0.008 | 0.589 | 96.687 | 0.005 | 0.0015 | 1.34E-06 | 50,441 |
| | 2.0 | 8.8 | 6.83 | | 1.431 | 33.604 | 0.035 | | 3.78E-05 | 783,493 |
| | 2.0 | 2.0 | 1.57 | | 3.154 | 45.335 | 0.041 | | 2.95E-04 | 963,377 |
| | | 2.0 | 1.16 | | | 48.070 | 0.049 | | 6.26E-04 | 1,246,277 |
| | | 8.2 | 2.11 | | | 25.763 | 0.197 | | 2.93E-03 | 4,295,112 |
| | | 13.8 | 3.07 | | | 14.016 | 0.311 | | 2.74E-03 | 5,448,707 |
| | | 5.3 | 3.94 | | | 133.774 | 0.011 | 0.0006 | 2.76E-05 | 276,933 |
| | | 6.1 | 4.69 | | $K_{a,NSR}$ | 50.814 | 0.029 | | 5.69E-05 | 330,172 |
| | | 2.0 | 1.57 | | 0.65 | 344.927 | 0.003 | | 1.43E-05 | 2,248 |
| | | 2.0 | 1.16 | | 0.27 | 40.517 | 0.064 | | 8.83E-04 | 697,302 |
| | | 5.6 | 1.43 | | 0.08 | 51.120 | 0.095 | | 2.00E-03 | 1,968,936 |
| | | 10.3 | 2.29 | | | 155.528 | 0.069 | | 2.02E-03 | 2,635,977 |
| | | 3.3 | 2.45 | | | 79.061 | 0.007 | 0.0001 | 9.57E-06 | 107,901 |
| | | 5.3 | 4.11 | | | 117.477 | 0.008 | | 1.25E-05 | 195,082 |
| | | 2.0 | 1.57 | | | 344.927 | 0.003 | | 1.43E-05 | 80,117 |
| | | 2.0 | 1.16 | | | 336.127 | 0.004 | | 3.29E-05 | 260,442 |
| | | 2.0 | 0.51 | | | 610.265 | 0.006 | | 2.30E-04 | 787,112 |
| | | 3.4 | 0.76 | | | 20.828 | 0.062 | | 6.48E-04 | 1,286,740 |
| | | | | | | | Totals | **0.0023** | **0.0126** | **2.14E+07** |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.62 | 0.0140 | 0.1183 | 0.059 | 0.574 | 0.0916 | 0.0150 |

Figure 23. Anticipated Variance Excel Solver Set Up and Results for `own_transport`

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$: *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)*: 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 66 | 3 | 11 | 20 | 8 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 81 | 33 | 22 | 12 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 11 | 69 | 15 | 8 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 21 | 37 | 26 | 19 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 13 | 19 | 99 | 21 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 18 | 17 | 20 | 106 | 82 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 171 | 7 | 28 | 51 | 20 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 82 | 34 | 22 | 12 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 4 | 13 | 81 | 17 | 10 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 25 | 43 | 30 | 22 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 11 | 16 | 23 | 118 | 25 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 37 | 35 | 41 | 222 | 172 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 114 | 5 | 19 | 34 | 13 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 73 | 30 | 20 | 11 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 10 | 63 | 13 | 7 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 19 | 33 | 23 | 17 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 8 | 11 | 16 | 84 | 18 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 33 | 31 | 36 | 196 | 152 |
| | | | | | | Totals | *492* | *474* | *634* | *1,119* | *629* |

| SSU/MSG strata | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation: $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | ANOVA Variance Components Estimates, SR: Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/nq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 107 | 79 | 3% | $ 1,315 | 16 | 3.3 | 2.0 | 1.49 | $ 657.44 | 0.089 | 0.549 | 0.001 | 0.0017 | 2.26E-07 | 15,515 |
| 0202 | 149 | 115 | 4% | $ 1,897 | | 3.9 | 2.8 | 2.16 | $ 680.62 | 0.045 | 0.106 | 0.021 | | 6.45E-06 | 157,023 |
| 0203 | 107 | 84 | 3% | $ 1,377 | | 3.0 | 2.0 | 1.57 | $ 688.72 | 0.044 | 0.161 | 0.024 | | 1.79E-05 | 171,502 |
| 0204 | 107 | 62 | 3% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 0.042 | 0.042 | | 1.90E-05 | 218,646 |
| 0205 | 162 | 42 | 5% | $ 890 | | | 3.0 | 0.78 | $ 292.23 | | 0.070 | 0.088 | | 2.08E-04 | 785,866 |
| 0206 | 242 | 54 | 7% | $ 1,212 | | | 4.5 | 1.01 | $ 267.37 | | 0.117 | 0.076 | | 1.98E-04 | 831,323 |
| 0301 | 278 | 206 | 8% | $ 2,941 | | | 4.5 | 3.32 | | | 0.177 | 0.035 | 0.0020 | 1.70E-05 | 233,359 |
| 0302 | 152 | 117 | 5% | $ 1,664 | | | 2.4 | 1.89 | | $K_{a,SR}$ | 0.126 | 0.022 | | 8.11E-06 | 139,825 |
| 0303 | 124 | 97 | 4% | $ 1,377 | | | 2.0 | 1.57 | | 0.26 | 0.220 | 0.020 | | 1.47E-05 | 0 |
| 0304 | 124 | 72 | 4% | $ 1,071 | | | 2.0 | 1.16 | | 0.42 | 0.041 | 0.074 | | 5.05E-05 | 386,887 |
| 0305 | 193 | 50 | 6% | $ 910 | | | 3.1 | 0.80 | | 0.33 | 0.049 | 0.130 | | 2.68E-04 | 774,329 |
| 0306 | 507 | 113 | 15% | $ 2,183 | | | 8.2 | 1.82 | | | 0.149 | 0.147 | | 4.53E-04 | 1,246,113 |
| 0401 | 185 | 137 | 6% | $ 2,519 | | | 3.8 | 2.85 | | | 0.601 | 0.014 | 0.0016 | 1.32E-05 | 151,555 |
| 0402 | 135 | 104 | 4% | $ 1,903 | | | 2.8 | 2.16 | | | 0.160 | 0.016 | | 5.98E-06 | 142,684 |
| 0403 | 96 | 76 | 3% | $ 1,377 | | | 2.0 | 1.57 | | | 0.220 | 0.020 | | 1.89E-05 | 135,176 |
| 0404 | 96 | 56 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 0.075 | 0.032 | | 2.18E-05 | 223,989 |
| 0405 | 138 | 35 | 4% | $ 836 | | | 2.9 | 0.73 | | | 0.068 | 0.081 | | 2.01E-04 | 699,568 |
| 0406 | 448 | 100 | 13% | $ 2,484 | | | 9.3 | 2.07 | | | 0.097 | 0.158 | | 3.89E-04 | 1,292,753 |
| | *3,348* | *1,600* | *100%* | *$1.53E+06* | | | | | | | | Totals | **0.0053** | **0.0019** | **7.61E+06** |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0009** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,348 | 14,973 | 18,322 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,093 | 9,693 |
| % NH B | TRUE | 0.33 | Percent | 0.17 | 0.83 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.30 | Count | 3,199 | 3,199 | 3,296 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,832,550 |
| | 32767 | | *SSU* | $ 2,600 | $ 26,583.47 | $ 1,262,508 |
| | 0 | | *OCC HU* | $ 850 | $ 1,534,806 | $ 3,783,548 |
| | | | *UNOCC HU* | $ 100 | $ 2,121,390 | $ 7,878,605 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)*: 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Non Self-Representing Optimum Allocation: $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | ANOVA Variance Components Estimates, NSR: $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1290 | 52 | 213 | 385 | 152 | 2093 | 1555 | 14% | $ 8,500 | 81 | 2.0 | 12.9 | 9.61 | 0.009 | 0.024 | 1.358 | 0.003 | 0.0001 | 7.48E-09 | 50,441 |
| 18 | 1031 | 423 | 280 | 149 | 1902 | 1472 | 13% | $ 7,996 | | 2.0 | 11.7 | 9.10 | | 0.037 | 0.537 | 0.043 | | 6.61E-07 | 783,493 |
| 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | 0.065 | 1.155 | 0.051 | | 1.16E-05 | 963,377 |
| 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 0.163 | 0.077 | | 5.17E-06 | 1,246,277 |
| 58 | 83 | 118 | 618 | 133 | 1010 | 259 | 7% | $ 1,823 | | | 6.2 | 1.60 | | | 0.785 | 0.199 | | 1.20E-04 | 4,295,112 |
| 144 | 137 | 159 | 864 | 668 | 1972 | 440 | 13% | $ 3,258 | | | 12.2 | 2.72 | | | 1.302 | 0.224 | | 1.48E-04 | 5,448,707 |
| 551 | 22 | 91 | 165 | 65 | 894 | 665 | 6% | $ 3,632 | | | 5.5 | 4.11 | | | 2.323 | 0.015 | 0.0000 | 7.43E-07 | 276,933 |
| 8 | 446 | 183 | 121 | 65 | 823 | 637 | 5% | $ 3,461 | | | 5.1 | 3.94 | | $K_{a,NSR}$ | 0.984 | 0.020 | | 5.91E-07 | 330,172 |
| 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.60 | 1.528 | 0.005 | | 1.54E-07 | 2,248 |
| 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.29 | 0.649 | 0.043 | | 6.34E-06 | 697,302 |
| 34 | 48 | 69 | 360 | 77 | 589 | 151 | 4% | $ 1,063 | | | 3.6 | 0.93 | | 0.11 | 1.113 | 0.101 | | 7.53E-05 | 1,968,936 |
| 102 | 97 | 113 | 610 | 472 | 1393 | 311 | 9% | $ 2,301 | | | 8.6 | 1.92 | | | 2.741 | 0.113 | | 1.13E-04 | 2,635,977 |
| 365 | 15 | 60 | 109 | 43 | 592 | 440 | 4% | $ 2,403 | | | 3.7 | 2.72 | | | 4.031 | 0.005 | 0.0000 | 2.02E-07 | 107,901 |
| 8 | 425 | 174 | 115 | 61 | 783 | 606 | 5% | $ 3,294 | | | 4.8 | 3.75 | | | 1.390 | 0.010 | | 2.11E-07 | 195,082 |
| 9 | 35 | 210 | 45 | 25 | 324 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 1.528 | 0.005 | | 1.54E-07 | 80,117 |
| 12 | 65 | 112 | 78 | 58 | 324 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 1.521 | 0.015 | | 1.88E-06 | 260,442 |
| 20 | 29 | 41 | 215 | 46 | 351 | 90 | 2% | $ 634 | | | 2.2 | 0.56 | | | 6.146 | 0.026 | | 4.71E-05 | 787,112 |
| 46 | 44 | 51 | 276 | 214 | 631 | 141 | 4% | $ 1,042 | | | 3.9 | 0.87 | | | 3.130 | 0.047 | | 4.92E-05 | 1,286,740 |
| 2,707 | 2,725 | 2,661 | 4,486 | 2,394 | *14,973* | *8,093* | *100%* | *$3.78E+06* | | | | | | | | Totals | **0.0001** | **0.0006** | **2.14E+07** |

**Summary of Solution**

| #Hus/PSU SR | #Hus/PSU NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) |
|---|---|---|---|---|---|---|---|---|
| 100 | 100 | 1.75 | 0.0009 | 0.0292 | 0.056 | 0.582 | 0.0072 | 0.0008 |

Figure 24. Anticipated Variance Excel Solver Set Up and Results for `selfRatedHealth`

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* | | | | | | | | | Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, SR | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR | Average Cost Per HU in *b* | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/n_a$ | $K^2ab*W^2_{3ab}/n_aq_{ab}$ | $Q_{ab}$ |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 65 | 3 | 11 | 19 | 8 | 105 | 78 | 3% | $ 1,315 | 16 | 3.3 | 2.0 | 1.49 | $ 657.44 | 0.043 | 0.029 | 0.005 | 0.0009 | 1.60E-07 | 15,515 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 87 | 36 | 24 | 13 | 160 | 124 | 5% | $ 2,080 | | 3.5 | 3.1 | 2.37 | $ 680.62 | -0.004 | 1.664 | 0.010 | | 2.18E-05 | 157,023 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 11 | 68 | 14 | 8 | 105 | 82 | 3% | $ 1,377 | | 2.9 | 2.0 | 1.57 | $ 688.72 | 0.010 | 1.856 | 0.012 | | 5.48E-05 | 171,502 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 21 | 36 | 25 | 19 | 105 | 61 | 3% | $ 1,071 | | | 2.0 | 1.16 | $ 535.63 | | 1.579 | 0.015 | | 9.71E-05 | 218,646 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 13 | 19 | 101 | 22 | 164 | 42 | 5% | $ 919 | | | 3.1 | 0.81 | $ 292.23 | | 0.255 | 0.095 | | 8.72E-04 | 785,866 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 19 | 18 | 21 | 115 | 89 | 263 | 59 | 8% | $ 1,346 | | | 5.0 | 1.12 | $ 267.37 | | 0.161 | 0.121 | | 6.40E-04 | 831,323 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 147 | 6 | 24 | 44 | 17 | 239 | 178 | 7% | $ 2,810 | | | 4.3 | 3.18 | | | 1.399 | 0.036 | -0.0002 | 1.62E-04 | 233,359 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 100 | 41 | 27 | 14 | 184 | 142 | 5% | $ 2,235 | | | 3.3 | 2.54 | | $K_{a,SR}$ | 0.942 | 0.015 | | 2.47E-05 | 139,825 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 12 | 73 | 15 | 9 | 112 | 88 | 3% | $ 1,377 | | | 2.0 | 1.57 | | 0.26 | 2.571 | 0.013 | | 7.68E-05 | 0 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 4 | 22 | 39 | 27 | 20 | 112 | 65 | 3% | $ 1,071 | | | 2.0 | 1.16 | | 0.38 | 0.597 | 0.025 | | 9.02E-05 | 386,887 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 14 | 20 | 28 | 148 | 32 | 242 | 62 | 7% | $ 1,262 | | | 4.3 | 1.11 | | 0.37 | 0.611 | 0.090 | | 1.28E-03 | 774,329 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 38 | 36 | 42 | 227 | 176 | 519 | 116 | 15% | $ 2,482 | | | 9.3 | 2.07 | | | 0.296 | 0.201 | | 1.65E-03 | 1,246,113 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 100 | 4 | 16 | 30 | 12 | 162 | 120 | 5% | $ 2,314 | | | 3.5 | 2.62 | | | 1.090 | 0.023 | 0.0005 | 7.83E-05 | 151,555 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 76 | 31 | 21 | 11 | 140 | 108 | 4% | $ 2,073 | | | 3.0 | 2.36 | | | 0.564 | 0.018 | | 2.57E-05 | 142,684 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 10 | 60 | 13 | 7 | 92 | 72 | 3% | $ 1,377 | | | 2.0 | 1.57 | | | 2.571 | 0.013 | | 9.37E-05 | 135,176 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 18 | 32 | 22 | 16 | 92 | 53 | 3% | $ 1,071 | | | 2.0 | 1.16 | | | 0.973 | 0.014 | | 5.64E-05 | 223,989 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 9 | 13 | 19 | 97 | 21 | 158 | 41 | 5% | $ 1,008 | | | 3.5 | 0.88 | | | 0.285 | 0.089 | | 8.80E-04 | 699,568 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 36 | 34 | 40 | 216 | 167 | 493 | 110 | 14% | $ 2,871 | | | 10.7 | 2.40 | | | 0.227 | 0.206 | | 1.40E-03 | 1,292,753 |
| | | | | | | Totals | *462* | *504* | *635* | *1,185* | *660* | *3,445* | *1,600* | *100%* | *$1.54E+06* | | | | | | | | Totals | **0.0012** | **0.0075** | **7.61E+06** |
| **EXCEL SOLVER RESULTS** | | | **Population Size** | | | | **No. of Eligible HUs in SSU/MSG *ab*, Domain *d* (NSR PSUs)** | | | | | | | | | Non Self-Representing Optimum Allocation | | | | ANOVA Variance Components Estimates, NSR | | | | | | |
| **Total RelVariance** | **0.0021** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR | $B^2$ | $W^2_{2a}$ | $W^2_{3ab}$ | $K_{ab}$ | $K^2a*W^2_{2a}/mn_a$ | $K^2ab*W^2_{3ab}/mn_aq_{ab}$ | $Q_{ab}$ |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 | 1448 | 58 | 240 | 432 | 171 | 2349 | 1746 | 16% | $ 9,553 | 81 | 2.0 | 14.5 | 10.80 | 0.013 | 0.143 | 5.252 | 0.004 | 0.0003 | 3.82E-08 | 50,441 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 | 17 | 955 | 391 | 259 | 138 | 1761 | 1363 | 12% | $ 7,416 | | 2.0 | 10.9 | 8.43 | | -0.454 | 8.875 | 0.024 | | 3.64E-06 | 783,493 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 | 9 | 35 | 210 | 44 | 25 | 323 | 254 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 | | -1.498 | 6.364 | 0.049 | | 6.04E-05 | 963,377 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 | 12 | 65 | 112 | 78 | 58 | 323 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 12.706 | 0.040 | | 1.08E-04 | 1,246,277 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 | 64 | 91 | 130 | 680 | 146 | 1110 | 285 | 7% | $ 2,007 | | | 6.9 | 1.76 | | | 3.606 | 0.216 | | 5.89E-04 | 4,295,112 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | | 143 | 136 | 158 | 857 | 663 | 1957 | 437 | 13% | $ 3,237 | | | 12.1 | 2.70 | | | 2.884 | 0.296 | | 5.79E-04 | 5,448,707 |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total | 443 | 18 | 73 | 132 | 52 | 718 | 534 | 5% | $ 2,920 | | | 4.4 | 3.30 | | | 12.417 | 0.013 | -0.0002 | 4.08E-06 | 276,933 |
| MAX Budget | TRUE | 10000000 | | 3,445 | 15,006 | 18,451 | 8 | 427 | 175 | 116 | 62 | 787 | 609 | 5% | $ 3,314 | | | 4.9 | 3.77 | | $K_{a,NSR}$ | 26.498 | 0.010 | | 4.17E-06 | 330,172 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | | 9 | 35 | 210 | 44 | 25 | 323 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | 0.63 | 57.836 | 0.003 | | 1.99E-06 | 2,248 |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* | 12 | 65 | 112 | 78 | 58 | 323 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | 0.24 | 139.830 | 0.007 | | 3.39E-05 | 697,302 |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,082 | 9,682 | 37 | 52 | 75 | 391 | 84 | 639 | 164 | 4% | $ 1,155 | | | 4.0 | 1.01 | | 0.13 | 13.387 | 0.064 | | 3.35E-04 | 1,968,936 |
| % NH B | TRUE | 0.33 | Percent | 0.17 | 0.83 | 1 | 100 | 95 | 111 | 601 | 465 | 1371 | 306 | 9% | $ 2,268 | | | 8.5 | 1.89 | | | 6.631 | 0.143 | | 4.42E-04 | 2,635,977 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | | 340 | 14 | 56 | 101 | 40 | 551 | 409 | 4% | $ 2,240 | | | 3.4 | 2.53 | | | 47.568 | 0.005 | -0.0002 | 3.00E-06 | 107,901 |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* | 9 | 480 | 197 | 130 | 69 | 885 | 685 | 6% | $ 3,725 | | | 5.5 | 4.24 | | | 6.305 | 0.013 | | 1.67E-06 | 195,082 |
| % NH O | TRUE | 0.30 | Count | 3,195 | 3,195 | 3,292 | 9 | 35 | 210 | 44 | 25 | 323 | 254 | 2% | $ 1,377 | | | 2.0 | 1.57 | | | 57.836 | 0.003 | | 1.99E-06 | 80,117 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 | 12 | 65 | 112 | 78 | 58 | 323 | 188 | 2% | $ 1,071 | | | 2.0 | 1.16 | | | 22.790 | 0.013 | | 2.17E-05 | 260,442 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST | 19 | 26 | 38 | 198 | 43 | 323 | 83 | 2% | $ 584 | | | 2.0 | 0.51 | | | 8.880 | 0.036 | | 1.39E-04 | 787,112 |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,828,678 | 45 | 43 | 50 | 270 | 209 | 616 | 137 | 4% | $ 1,019 | | | 3.8 | 0.85 | | | 6.608 | 0.062 | | 1.82E-04 | 1,286,740 |
| | 32767 | | *SSU* | $ 2,600 | $ 25,047.03 | $ 1,260,782 | 2,733 | 2,691 | 2,657 | 4,534 | 2,389 | *15,006* | *8,082* | *100%* | *$3.78E+06* | | | | | | | | Totals | **0.0000** | **0.0025** | **2.14E+07** |
| | 0 | | *OCC HU* | $ 850 | $ 1,544,477 | $ 3,781,014 | | | | | | #Hus/PSU | #Hus/PSU | | **Summary of Solution** | | | | | | | | | | | |
| | | | *UNOCC HU* | $ 100 | $ 2,129,524 | $ 7,870,474 | | | | | | SR | NSR | deff_kish | TotalRelVar | CV | $F^2_{SR}$ | $F^2_{NSR}$ | relvar($t_{pwr,SR}$) | relvar($t_{pwr,NSR}$) | | | | | | |
| | | | | | Total Cost | $1.00E+07 | | | | | | 100 | 100 | 1.70 | 0.0021 | 0.0454 | 0.067 | 0.551 | 0.0087 | 0.0027 | | | | | | |

# C.3 ANOVA Multivariate Optimization Results

Figure 25. ANOVA Variance Excel Solver Set Up and Results, Multivariate Optimization with Equal Importance Weights

| | Unweighted Accuracy Rates $p_{ab}(d)$ | | | | | | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* | | | | | | | | | Self-Representing Optimum Allocation | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| SSU/MSG strata | *(d=1)* 45-62 H | *(d=2)* 45-62 NH B | *(d=3)* 45-62 NH O | *(d=4)* NOT 45-62 | *(d=5)* UNOCC | All Eligibles | 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR |
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 51 | 2 | 8 | 15 | 6 | 82 | 61 | 3% | $ 1,315 | 16 | 2.6 | 2.0 | 1.49 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 60 | 25 | 16 | 9 | 111 | 86 | 3% | $ 1,840 | | 2.0 | 2.7 | 2.09 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 9 | 53 | 11 | 6 | 82 | 65 | 3% | $ 1,377 | | 2.3 | 2.0 | 1.57 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 16 | 28 | 20 | 15 | 82 | 48 | 3% | $ 1,071 | | | 2.0 | 1.16 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 11 | 16 | 23 | 120 | 26 | 196 | 50 | 6% | $ 1,389 | | | 4.8 | 1.22 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 20 | 19 | 22 | 120 | 92 | 273 | 61 | 9% | $ 1,771 | | | 6.6 | 1.48 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 179 | 7 | 30 | 54 | 21 | 291 | 216 | 9% | $ 5,976 | | | 9.1 | 6.76 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 4 | 214 | 88 | 58 | 31 | 395 | 306 | 12% | $ 8,410 | | | 12.4 | 9.57 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 42 | 9 | 5 | 64 | 50 | 2% | $ 1,377 | | | 2.0 | 1.57 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 22 | 15 | 11 | 64 | 37 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 13 | 19 | 26 | 139 | 30 | 226 | 58 | 7% | $ 2,068 | | | 7.1 | 1.81 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 22 | 21 | 24 | 132 | 102 | 302 | 67 | 9% | $ 2,526 | | | 9.4 | 2.11 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 102 | 4 | 17 | 30 | 12 | 165 | 123 | 5% | $ 3,000 | | | 4.6 | 3.39 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 2 | 103 | 42 | 28 | 15 | 191 | 148 | 6% | $ 3,579 | | | 5.3 | 4.07 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 8 | 47 | 10 | 6 | 73 | 57 | 2% | $ 1,377 | | | 2.0 | 1.57 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 14 | 25 | 17 | 13 | 73 | 42 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 7 | 10 | 14 | 74 | 16 | 121 | 31 | 4% | $ 974 | | | 3.3 | 0.85 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 30 | 29 | 34 | 183 | 141 | 417 | 93 | 13% | $ 3,076 | | | 11.5 | 2.57 |
| | | | | | | Totals | *457* | *572* | *571* | *1,052* | *558* | *3,210* | *1,600* | *100%* | *$1.52E+06* | | | | |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0280** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,210 | 15,883 | 19,093 |
| % H | TRUE | 0.3 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,810 | 10,410 |
| % NH B | TRUE | 0.3 | Percent | 0.15 | 0.85 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.3 | Count | 3,435 | 3,435 | 3,539 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 3,083,426 |
| | 32767 | | *SSU* | $ 2,600 | $ 17,786.03 | $ 1,571,383 |
| | 0 | | *OCC HU* | $ 850 | $ 1,520,958 | $ 3,246,447 |
| | | | *UNOCC HU* | $ 100 | $ 2,098,744 | $ 7,901,256 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* | | | | | | | | | Non Self-Representing Optimum Allocation | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR |
| 2035 | 81 | 337 | 607 | 240 | 3300 | 2453 | 21% | $ 8,609 | 88 | 2.9 | 13.1 | 9.73 |
| 28 | 1578 | 647 | 429 | 229 | 2911 | 2253 | 18% | $ 7,862 | | 2.0 | 11.6 | 8.94 |
| 15 | 54 | 327 | 69 | 39 | 504 | 396 | 3% | $ 1,377 | | 2.0 | 2.0 | 1.57 |
| 18 | 101 | 174 | 121 | 90 | 504 | 293 | 3% | $ 1,071 | | | 2.0 | 1.16 |
| 73 | 104 | 149 | 780 | 168 | 1274 | 327 | 8% | $ 1,478 | | | 5.1 | 1.30 |
| 142 | 135 | 158 | 854 | 661 | 1950 | 435 | 12% | $ 2,069 | | | 7.7 | 1.73 |
| 222 | 9 | 37 | 66 | 26 | 360 | 268 | 2% | $ 1,344 | | | 2.0 | 1.52 |
| 3 | 191 | 78 | 52 | 28 | 352 | 273 | 2% | $ 1,361 | | | 2.0 | 1.55 |
| 10 | 38 | 229 | 48 | 27 | 352 | 277 | 2% | $ 1,377 | | | 2.0 | 1.57 |
| 13 | 70 | 122 | 85 | 63 | 352 | 205 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 36 | 51 | 73 | 385 | 83 | 628 | 161 | 4% | $ 1,042 | | | 3.6 | 0.91 |
| 83 | 79 | 92 | 497 | 384 | 1134 | 253 | 7% | $ 1,720 | | | 6.4 | 1.44 |
| 217 | 9 | 36 | 65 | 26 | 352 | 262 | 2% | $ 1,315 | | | 2.0 | 1.49 |
| 3 | 191 | 78 | 52 | 28 | 352 | 273 | 2% | $ 1,361 | | | 2.0 | 1.55 |
| 10 | 38 | 229 | 48 | 27 | 352 | 277 | 2% | $ 1,377 | | | 2.0 | 1.57 |
| 13 | 70 | 122 | 85 | 63 | 352 | 205 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 20 | 29 | 41 | 216 | 46 | 352 | 90 | 2% | $ 584 | | | 2.0 | 0.51 |
| 36 | 35 | 40 | 219 | 169 | 499 | 111 | 3% | $ 757 | | | 2.8 | 0.63 |
| 2,978 | 2,863 | 2,968 | 4,678 | 2,395 | *15,883* | *8,810* | *100%* | *$3.25E+06* | | | | |

## C.4 Anticipated Multivariate Optimization Results

Figure 26. Anticipated Variance Excel Solver Set Up and Results, Multivariate Optimization with Equal Importance Weights

| SSU/MSG strata | Unweighted Accuracy Rates $p_{ab}(d)$ (d=1) 45-62 H | (d=2) 45-62 NH B | (d=3) 45-62 NH O | (d=4) NOT 45-62 | (d=5) UNOCC | All Eligibles | No. of Excepted HUs *Actually in* Domain *d (SR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Self-Representing Optimum Allocation $m_{SR}$ | $\bar{n}_{a,SR}$ | $\bar{\bar{q}}_{ab,SR}$ | Expected No. Eligibles, SR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0201 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 43 | 2 | 7 | 13 | 5 | 69 | 52 | 2% | $ 1,315 | 16 | 2.2 | 2.0 | 1.49 |
| 0202 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 51 | 21 | 14 | 7 | 94 | 72 | 3% | $ 1,839 | | 2.8 | 2.7 | 2.09 |
| 0203 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 45 | 10 | 5 | 69 | 54 | 2% | $ 1,377 | | 2.1 | 2.0 | 1.57 |
| 0204 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 14 | 24 | 17 | 12 | 69 | 40 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 0205 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 11 | 16 | 23 | 120 | 26 | 196 | 50 | 6% | $ 1,652 | | | 5.7 | 1.45 |
| 0206 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 20 | 19 | 22 | 120 | 92 | 273 | 61 | 8% | $ 2,106 | | | 7.9 | 1.76 |
| 0301 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 259 | 10 | 43 | 77 | 31 | 420 | 312 | 13% | $ 6,174 | | | 9.4 | 6.98 |
| 0302 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 3 | 152 | 62 | 41 | 22 | 281 | 218 | 9% | $ 4,280 | | | 6.3 | 4.87 |
| 0303 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 3 | 10 | 58 | 12 | 7 | 89 | 70 | 3% | $ 1,377 | | | 2.0 | 1.57 |
| 0304 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 3 | 18 | 31 | 22 | 16 | 89 | 52 | 3% | $ 1,071 | | | 2.0 | 1.16 |
| 0305 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 13 | 18 | 26 | 135 | 29 | 221 | 57 | 7% | $ 1,442 | | | 4.9 | 1.26 |
| 0306 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 29 | 28 | 33 | 177 | 137 | 404 | 90 | 12% | $ 2,418 | | | 9.0 | 2.02 |
| 0401 | 0.62 | 0.02 | 0.10 | 0.18 | 0.07 | 0.74 | 167 | 7 | 28 | 50 | 20 | 271 | 201 | 8% | $ 5,393 | | | 8.2 | 6.10 |
| 0402 | 0.01 | 0.54 | 0.22 | 0.15 | 0.08 | 0.77 | 1 | 42 | 17 | 11 | 6 | 77 | 60 | 2% | $ 1,592 | | | 2.3 | 1.81 |
| 0403 | 0.03 | 0.11 | 0.65 | 0.14 | 0.08 | 0.78 | 2 | 7 | 43 | 9 | 5 | 66 | 52 | 2% | $ 1,377 | | | 2.0 | 1.57 |
| 0404 | 0.04 | 0.20 | 0.35 | 0.24 | 0.18 | 0.58 | 2 | 13 | 23 | 16 | 12 | 66 | 38 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 0405 | 0.06 | 0.08 | 0.12 | 0.61 | 0.13 | 0.26 | 8 | 11 | 16 | 85 | 18 | 138 | 35 | 4% | $ 1,226 | | | 4.2 | 1.07 |
| 0406 | 0.07 | 0.07 | 0.08 | 0.44 | 0.34 | 0.22 | 28 | 26 | 31 | 166 | 129 | 380 | 85 | 12% | $ 3,080 | | | 11.5 | 2.57 |
| | | | | | | Totals | *597* | *452* | *552* | *1,094* | *580* | *3,274* | *1,600* | *100%* | *$1.53E+06* | | | | |

| EXCEL SOLVER RESULTS | | | Population Size | | | |
|---|---|---|---|---|---|---|
| **Total RelVariance** | **0.0082** | | *ssu.str* | *Qa_max_SR* | *Qa_max_NSR* | *Q total* |
| # of Parameters | 43 | Contraints | 2 | 2.18E+06 | 1.28E+07 | 14,967,281 |
| SR PSU | Constant | 16 | 3 | 2.78E+06 | 5.91E+06 | 8,692,080 |
| NSR PSU min | TRUE | 25 | 4 | 2.65E+06 | 2.72E+06 | 5,363,119 |
| NSR SSU per ab min | TRUE | 2 | Total | 7.61E+06 | 2.14E+07 | 29,022,480 |
| NSR HU per ab min | TRUE | 2 | Percent | 0.26 | 0.74 | 1.00 |
| #SR HU Sample <= #HU Pop | TRUE | 7606112 | **Total Needed To Be Screened** | | | |
| #NSR HU Sample <= # HU Pop | TRUE | 21416368 | | SR | NSR | Total |
| MAX Budget | TRUE | 10000000 | | 3,274 | 16,139 | 19,413 |
| % H | TRUE | 0.33 | **Actual Achieved Allocation** | | | |
| SR MAX HUs per PSU | TRUE | 100.0 | *Sample Size* | *q_SR* | *qNSR* | *q_total* |
| SR MIN HUs per PSU | TRUE | 50.0 | Count | 1,600 | 8,314 | 9,914 |
| % NH B | TRUE | 0.33 | Percent | 0.16 | 0.84 | 1 |
| NSR MAX HUs per PSU | TRUE | 100.0 | **Sample Size By Demographic Domain *d*** | | | |
| NSR MIN HUs per PSU | TRUE | 50.0 | | *45-62 Hisp* | *45-62 Black* | *45-62 Other* |
| % NH O | TRUE | 0.30 | Count | 3,271 | 3,271 | 3,371 |
| deff | TRUE | 1.75 | Percent | 0.33 | 0.33 | 0.34 |
| SR SSU per ab min | TRUE | 2.0 | **COST** | Unit Cost | SR COST | NSR COST |
| SR HU per ab min | TRUE | 2.0 | *PSU* | $ 35,000 | $ 560,000 | $ 2,909,769 |
| | 32767 | | *SSU* | $ 2,600 | $ 18,260.97 | $ 1,360,713 |
| | 0 | | *OCC HU* | $ 850 | $ 1,527,391 | $ 3,623,866 |
| | | | *UNOCC HU* | $ 100 | $ 2,105,652 | $ 7,894,348 |
| | | | | | Total Cost | $1.00E+07 |

| No. of Eligible HUs in SSU/MSG *ab*, Domain *d (NSR PSUs)* 45-62 H | 45-62 NH B | 45-62 NH O | NOT 45-62 | UNOCC | No. HUs Screened | Total Elig HUs 45-62 | % | Total HU Cost in ab | Non Self-Representing Optimum Allocation $m_{NSR}$ | $\bar{n}_{a,NSR}$ | $\bar{\bar{q}}_{ab,NSR}$ | Expected No. Eligibles, NSR |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1556 | 62 | 257 | 464 | 183 | 2523 | 1875 | 16% | $ 8,694 | 83 | 2.3 | 13.2 | 9.83 |
| 24 | 1375 | 563 | 374 | 199 | 2535 | 1963 | 16% | $ 9,044 | | 2.0 | 13.3 | 10.29 |
| 11 | 41 | 248 | 52 | 30 | 382 | 300 | 2% | $ 1,377 | | 2.0 | 2.0 | 1.57 |
| 14 | 76 | 132 | 92 | 68 | 382 | 222 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 63 | 90 | 128 | 671 | 144 | 1096 | 281 | 7% | $ 1,679 | | | 5.7 | 1.47 |
| 138 | 131 | 153 | 827 | 640 | 1889 | 422 | 12% | $ 2,647 | | | 9.9 | 2.21 |
| 322 | 13 | 53 | 96 | 38 | 522 | 388 | 3% | $ 2,065 | | | 3.1 | 2.33 |
| 6 | 335 | 137 | 91 | 49 | 618 | 479 | 4% | $ 2,531 | | | 3.7 | 2.88 |
| 10 | 36 | 216 | 46 | 26 | 333 | 261 | 2% | $ 1,377 | | | 2.0 | 1.57 |
| 12 | 66 | 115 | 80 | 59 | 333 | 193 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 42 | 60 | 86 | 451 | 97 | 737 | 189 | 5% | $ 1,296 | | | 4.4 | 1.14 |
| 80 | 76 | 89 | 481 | 372 | 1097 | 245 | 7% | $ 1,764 | | | 6.6 | 1.47 |
| 205 | 8 | 34 | 61 | 24 | 333 | 247 | 2% | $ 1,315 | | | 2.0 | 1.49 |
| 3 | 180 | 74 | 49 | 26 | 333 | 257 | 2% | $ 1,361 | | | 2.0 | 1.55 |
| 10 | 36 | 216 | 46 | 26 | 333 | 261 | 2% | $ 1,377 | | | 2.0 | 1.57 |
| 12 | 66 | 115 | 80 | 59 | 333 | 193 | 2% | $ 1,071 | | | 2.0 | 1.16 |
| 19 | 27 | 39 | 204 | 44 | 333 | 85 | 2% | $ 584 | | | 2.0 | 0.51 |
| 148 | 141 | 164 | 889 | 688 | 2029 | 453 | 13% | $ 3,263 | | | 12.2 | 2.72 |
| 2,675 | 2,820 | 2,819 | 5,054 | 2,771 | *16,139* | *8,314* | *100%* | *$3.62E+06* | | | | |

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