ABSTRACT

Title of dissertation: STRATEGIC DEFAULT, RENEGOTIATION, AND RECOURSE IN RESIDENTIAL MORTGAGE CONTRACTS

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In this article we model strategic default and renegotiation in residential mortgage contracts. In particular, we study how recourse affects mortgage rates and default. We find that in the presence of recourse, default rates are lower for a given loan-to-value ratio, equilibrium coupon rates are lower, loan-to-value ratios are higher and welfare is improved. We find that higher loan-to-value ratios under recourse, increase welfare but can lead to higher equilibrium default rates. We find that when the bank has monopoly power during renegotiation, contracts with renegotiation are an improvement over contracts without renegotiation. Increase in homeowner renegotiation bargaining power, beyond a threshold, has a negative effect on equity value since the surplus that the homeowner can extract ex-post is priced into the initial mortgage rate. We show that the provision of recourse and the balance of bargaining power during renegotiation alleviates some of the distortions due to moral hazard implicit in debt contracts. Our equilibrium concept is sub-game perfect Nash and we derive closed form solutions in the model.

STRATEGIC DEFAULT, RENEGOTIATION, AND RECOURSE IN RESIDENTIAL MORTGAGE CONTRACTS

by

Anshuman Sinha

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Dedication

This thesis is dedicated to Aysun Alp.

Acknowledgments

It takes a village to raise a child and indeed it has taken a village to get me through this process. The time Ive spent in the Finance Ph.D. program will probably be my defining experience. This grueling process has been the most rewarding experience of my life and the many incredible people that I met here have inspired me to try and become like them.

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Chapter 1: Introduction

A home purchase is the most significant investment made by most households. According to the U.S. Census Bureau up to 66% of residential home ownership is financed by residential mortgages.[1] Residential mortgages are secured debt contracts where the property serves as a collateral for the loan. It is well known that debt contracts induce moral hazard in the contracting environment. In this paper, we study the moral hazard resulting from strategic default, renegotiation of contracts, and the mitigation of these inefficiencies in the presence of recourse. A borrower is said to strategically default on his debt payments if the borrower's decision to default is driven by his unwillingness to pay rather than his inability to pay. Banks rationally price this strategic default into the initial terms of the contract, and the price for the strategic default is paid ex-ante by the borrowers. After a default, the lender repossesses the home using a legal process known as foreclosure. Foreclosure is a complex, time consuming and expensive process for both the bank and the borrower. Dead-weight foreclosure costs can be delayed if the bank and the homeowner can successfully renegotiate the terms of the mortgage. Contract renegotiations, while efficient ex-post, can increase the ex-ante cost to the borrower; it is not credible

[1] `http://www.census.gov/compendia/statab/cats/construction_housing.html`

for banks to commit to not renegotiate ex-post. This dynamic consistency problem further distorts the incentives of the borrowers by making it lucrative for them to sue ex-post for renegotiation. This inefficiency is also priced into the initial terms of the contract and is reflected in the ex-ante coupon rates and loan-to-value ratio. the ability of banks to sue the borrower and get a deficiency judgement against him, alleviates some of this distortion. The pursuit of a deficiency judgement, henceforth "lender recourse", converts the gap between the value of the foreclosed home and the value of the debt into an unsecured loan which enables the bank's access to the borrower's personal assets to pay for this shortfall. Recourse makes default less desirable from the perspective of the borrower and reduces some of the deadweight loss due to strategic default and foreclosure. This feeds back into the ex-ante coupon rate and loan-to-value ratio.

In this paper we model strategic default and renegotiation in residential mortgage contracts and study how the features of recourse and the balance of bargaining power alleviate some of the incentive problems associated with debt and renegotiation. Our model is a continuous time infinite horizon game of endogenous strategic default and renegotiation. We derive closed form solutions. We have two state variables, the house price and the borrower's income. The house price is a geometric Brownian motion while the homeowner's income is a continuous time Markov chain that switches between n states. Homeowner income is assumed to be not contractible. The homeowner derives a psychic utility from ownership over renting. He enters into a mortgage contract because he either has insufficient funds to buy the house with his own funds or chooses not to use all of his funds to do so. We have no savings in the model. The homeowner is risk neutral and consumes all of his cash flows. The debt contract is a perpetuity with a constant coupon rate and some fixed face value. The homeowner endogenously chooses at every instant whether to stay current in the contract or to exit the contract via selling the house and prepaying the loan or by defaulting on his coupon payments and suing for renegotiation. In the event of renegotiation, the homeowner and the lender renegotiate the coupon rate. Renegotiation imposes a fixed cost on the participants. We examine a variety of bargaining power regimes during renegotiation, ranging from the bank having monopoly power to the bank pricing the renegotiation at a competitive rate. We also study contracts without a renegotiation feature, ignoring the ex-post incentives the bank would have to enter into a renegotiation. In contracts without renegotiation or when renegotiations fail, we assume that the debt contract ends and the foreclosure process is initiated. The house is possessed by the bank which sells it to recoup some of its investment back. Foreclosure is assumed to be costly on both the banks and the borrowers. We study foreclosure regimes with and without lender recourse.

Our model is a sequential game in continuous time; our equilibrium concept is a sub-game perfect Nash equilibrium. Our model produces a rich set of results. We find that in the absence of recourse, the homeowner's default decision is independent of his personal wealth. We also find that in regimes with recourse, the homeowners stay in the homes longer before defaulting. We show that under recourse, there are lower default rates, lower coupon rates, higher debt capacity and increased welfare. An implication of higher debt capacity under recourse is that there are potentially higher default rates in these states. When the homeowner's income is stochastic, we find that he defaults at a lower price when his income is high and the lender can garnish more of it. When his income is high, however, he sells his house at a lower price since the option to default is even further out of the money in the high income state. We find that, when the homeowner has high bargaining power during renegotiation, contracts with renegotiation violate the bank's individual rationality constraint and are not offered in equilibrium. On the other hand, when the bank has high bargaining power, contracts with renegotiation are an improvement over contracts without renegotiation. Increased homeowner bargaining power during renegotiation distorts their incentives and turns out to be inefficient as the moral hazard is priced into the initial coupon rate. Bargaining power is clearly an important determination of the feasibility of a mortgage modification program and subsequent default rates. A legal regime that enables lender recourse reduces homeowner bargaining power during a renegotiation. Lender recourse removes the incentives of the homeowner to engage in foreclosure stripping and increases the reservation value of the bank in the renegotiation process.

Our results are consistent with some of the empirical results in Ghent and Kudlyak (2011). Ghent and Kudlyak estimate a probit model using loan level data on a sample of over 3 million U.S. mortgages. Consistent with the predictions of our model, they find that borrowers in recourse states have slightly lower average FICO scores and slightly higher LTVs at origination. Our model predicts that for a given level of leverage, recourse states with have lower default but for a given house our model predicts that recourse states will have higher leverage and thus the equilibrium default rates can be higher than in the no-recourse states. Ghent and Kudlyak find that default probabilities are 1.32 times higher where there is no threat of recourse and there is negative equity in the house but unconditionally Ghent and Kudlyak find no difference between the default rates in the recourse and non-recourse states. Interestingly, they find no evidence that interest rates are lower in recourse states. This is consistent with our model where at optimal leverage levels, the coupon rates are similar in recourse and no-recourse states but controlling for leverage, the recourse states have a lower coupon rate. Ghent and Kudlyak find that for properties appraised lower than 200K there is no difference in impact of recourse. This is also consistent with our model since lower valued homes are bought by people with lower incomes and when there is sufficiently low income to be garnished, there is little impact on the default behavior of the borrower.

In a related paper, Hatchondo, Martinez and Sanchez (2010) look at the influence of recourse rules on a mortgage market in a discrete time, infinite horizon, overlapping generation equilibrium model. They numerically solve their model and calibrate it to fit empirically observed ratios. They find that as the lender is allowed to garnish more from a defaulting households income, the default probability declines while home ownership and welfare increases. Our research agenda, while similar to theirs, differs in three important respects. Firstly, we look at the influence and interaction of contract renegotiation with recourse rules and strategic default incentives. Secondly, we model prepayment jointly with strategic default enabling us to study the interaction in equilibrium of these two aspects of the contract. Thirdly, our results are closed form, enabling us to analyze a rich array of comparative statics results. In other related literature, Corbae and Quintin (2012) calibrate an overlapping generations model with earnings shocks and quantify the influence of non-traditional mortgage contracts with low down payments and delayed amortization on foreclosure rates. They find that lender recourse reduces mortgage rates and foreclosure rates fall substantially. Earlier work by Clauretie (1987), Jones (1993), and Ambrose, Capone, and Deng (2001) look empirically at differences in defaults across states in the U.S.A. Clauretie (1987) estimates a linear regression model of aggregate state default rates and finds that whether or not a state permits a deficiency judgment does not significantly affect the state's default rate. Ambrose, Buttimer, and Capone (1997) find that the probability of default is a decreasing function of the probability of obtaining a deficiency judgment.

Theoretical work on the pricing of mortgages as derivative assets started with Epperson et al. (1985) and Kau et al. (1992, 1995). These papers took the house price and the short rate as the underlying stochastic state variables and derived the value of the mortgage to the borrower and lender, given optimal exercise of the options to default and to prepay. They arrived at the continuous fundamental partial differential equation which usually did not admit analytical solutions and were solved numerically. Our model extends this literature in three main directions. Firstly, we have analytical solutions because we solve a system of linked ordinary differential equations instead of a partial differential equation. This enables us to have explicit analytical solutions, despite having two state variables. Our two variables are the house price and borrower income but our model can be easily extended to include a spot rate which is a continuous time Markov chain. Secondly, we study renegotiation of contracts, which was not studied in these seminal papers. Thirdly, we look at the impact of recourse on strategic default. One shortcoming of our approach is that we do not explicitly study the time dependency, which is an important aspect of a borrowers financial decisions. Consistent with this literature, we model the default and prepayment options simultaneously.

Another strand of literature that relates to mortgage renegotiation and bargaining power is one of competitiveness in the banking industry. Sharpe (1990) and Rajan (1992) argue that relationship lending gives banks a monopoly on information about their borrowers and therefore give banks bargaining power over the firm's profits. The benefits to the firm of the banks having an increased bargaining power are manifested in the firm's lower likelihood of being liquidity constrained as shown in the case of Japanese banks by Hoshi, Kashyap, and Scharfstein (1990,1991). A downside of outside bank competition or the loss of bank bargaining power is the distortion of borrower incentives. If the lenders are arm's length lenders then management can indulge in empire building and risk shifting. Our renegotiation results are similar to this literature as we capture the upside of bank's bargaining power and how it mitigates the moral hazard problem with debt. We find that high ex-post homeowner bargaining power distorts their incentives to strategically default and the surplus extracted ex-post by the homeowner is captured by the bank via higher initial mortgage rates. In our model, when the bank has monopoly renegotiation power or sufficiently high bargaining power, it leads to a welfare improvement over contracts without renegotiation. The housing context does not have the debt overhang problems related with this literature.

The repayment incentives of borrowers and the debt restructuring issues that we study in our paper are also related to similar issues studied in sovereign debt contracts. Strategic default in a sovereign debt is first studied by Eaton and Gersovitz (1981). Sovereign default is legal by definition and lenders do not have any collateral as they cannot confiscate any national assets. Payment incentives in these markets are primarily said to be driven by a threat to the sovereign of loss of access to international capital markets and disruption of international trading opportunities. These effects are similar to what a homeowner faces in the event of a severe loss in their credit score. Kletzer and Wright (2000) show that permanent credit embargoes are not credible threats because the gains from risk-sharing that motivate lending in the first place give incentives for forgiveness and new lending. Sovereign debt renegotiation suffers from co-ordination, free riding and hold out problems between myriad lenders that are absent in residential mortgage contracts.

The rest of the paper is organized as follows. Chapter 2 presents the game and looks at contracts without renegotiation. Chapter 3 looks at contracts with renegotiation. Chapter 4 contains all numerical results and our conclusions. Finally, we have the appendices with some derivations.

Chapter 2: Strategic default in contracts without renegotiation. Constant income and Stochastic income

2.1 The Game

2.1.1 Timeline

The bank and the homeowner play a sequential game. Both the homeowner and the bank are risk neutral. The homeowner has a higher time discount parameter than the bank. Going forward we use the words borrower and homeowner interchangeably. We also use the words bank and lender interchangeably henceforth. The borrower is interested in homeownership because he derives a psychic utility from ownership over renting. The borrower may ask the bank for a loan because he has insufficient funds to purchase the house outright or he might prefer to finance the purchase via some amount of debt because of his high time discount parameter. When homeowner approaches the bank for a loan of a given face value, the bank offers the homeowner a debt contract which is a perpetuity with a fixed coupon rate. The debt contract may have up to one provision for renegotiation. This initial rate is assumed to price debt competitively at par. If this contract meets the homeowner's individual rationality constraint then the game begins. From this time onwards, time is continuous and at each instant the homeowner decides if he wants to stay current on his coupon payments or exit the contract. The contract can be exited in two ways. The first way is to sell the house. The homeowner could sell the house and repay the face value of the loan. The second way to exit the contract is to default on the payments. If the contract did not have a provision for renegotiation then this action would take both the agents straight into the foreclosure process. The foreclosure is settled according to whether the legal regime allows for lender recourse. If the initial debt contract did have a provision for renegotiation then the bank and the borrower renegotiate the coupon rate. The homeowner chooses the sale and default boundary endogenously and optimally. The bank can rationally anticipate the borrower's actions for a given choice of coupon rate. Therefore the coupon that the banks choose and the boundaries that the borrower chooses are best mutual responses to each other and our equilibrium is a sub-game perfect Nash equilibrium. We solve for this equilibrium using dynamic programming. We derive a system of linked ordinary differential equations that the borrower's value function satisfies and then solve for the borrower's value function.

2.1.2 Sale boundary

The borrower can sell the home at any point and pay the lender the face value of the loan. The borrower makes up for any shortfall on his own account. Since the borrower has a high time discount parameter, for high enough house prices the borrower has an incentive to realize the difference between the market price of the house and his lower valuation in terms of current consumption.

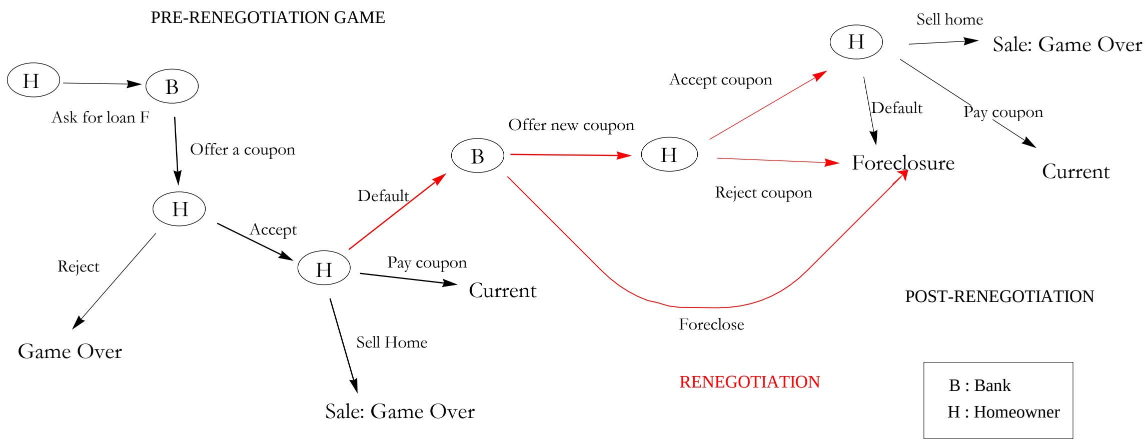
2.1.3 Default boundary

The borrower can also choose to simply default on his at any point in time. If the default leads to foreclosure then both the bank and the borrower suffer fixed dead-weight costs. If the bank and the borrower successfully renegotiate a new coupon then they incur a small renegotiation cost. It is easy to see that when the homeowner has negative equity, he would prefer defaulting over selling and when he has high positive equity that he would prefer to sell the house rather than default.

2.1.4 Recourse

There are two foreclosure regimes that we study in this model. In the first no-recourse regime, the bank has possession of the house and the homeowner is able to walk away from his obligation. In the second, recourse regime, the bank is able garnish some constant amount of the homeowner's future wages. This amount, Ie, is assumed to be independent of the current house price or the homeowner's income. It is an upper limit to the extent to which the bank can recoup its losses. The bank can go after the borrower's wealth to make up for its shortfall, up to this amount Ie. In equilibrium we see that, given Ie, the homeowner picks a default house price such that the garnishment limit is passed and the bank is left with some shortfall.

Figure 2.1: **The Game**



2.1.5 Renegotiation: bargaining power

If the debt contract has a provision for renegotiation then the bank and the homeowner try to arrive at a new coupon so that their individual rationality constraints are met and that they can share the surplus from delaying the dead-weight loss associated with foreclosure. We introduce a parameter γ that captures the homeowner's bargaining power. We study the extreme cases when the bank prices the new loan as a monopolist or when the homeowner has all the bargaining power and the loan is priced as if the bank was operating in a competitive market. Then we study the cases with intermediate bargaining power where the two agents are able to get the renegotiated coupon to move away from the monopolistic and competitive extremes.

2.2 Model

There are two risk neutral agents in our model. The borrower who takes a loan F to buy a house of value H_0 and the lender who offers a loan with face value F and coupon rate c. The borrower has the wealth to make the down payment of $d = H_0 - F$ but needs financing for the remaining amount. The borrower derives a psychic utility from owning over renting a home (see Figure 2.3). The house Price follows a geometric Brownian motion. The model has an infinite horizon and is in continuous time. The debt contract is a perpetuity with the provision for a one-time[1] costly loan renegotiation. At each instant in time, the borrower has the choice of staying current on his loan, selling the home to a third party or defaulting on his loan. To stay current on his loan, the homeowner needs to keep making payments to the bank at the constant coupon rate. If the homeowner decides to sell the home, he will have to pay the bank the face-value, F, of the loan from the proceeds of the sale while the difference will be credited to or debited from his personal wealth. When the homeowner chooses to default for the first time, the bank and the homeowner can renegotiate a new coupon. If the homeowner defaults a second time, the bank forecloses on the house. Renegotiation and foreclosure impose fixed dead-weight costs on both the agents.

The model timeline is as follows(see Figure 2.1): The borrower approaches the bank for a loan to buy a house. The borrower has insufficient funds to buy

[1]The model extends easily to any fixed finite number of renegotiations. We ignore the time inconsistency of the agents pre-committing to any such number ex-ante.

the house out right. The borrower derives a psychic utility from ownership that he does not receive from renting a home. Any tax advantage from home ownership can be thought of as subsumed in this psychic utility. The lender offers a mortgage contract to the borrower. A contract is a perpetuity with a fixed coupon rate c and a face value of debt F. The debt contract has one provision for a borrower initiated renegotiation. The borrower also has the option to terminate the contract by selling the house at any time and paying off the face value of the loan. Finally, the borrower can default on the loan at any time of his choosing. A failed renegotiation or a second default leads to the house being foreclosed. Foreclosure is costly to both the lender and the borrower. The lender loses funds in administrative and legal fees while the borrower's creditworthiness takes a non-trivial hit.

2.2.1 State variables

2.2.1.1 House Price

The house price process follows a geometric Brownian motion, where r is the risk free rate and $\delta > 0$ is the rental rate that the homeowner saves on by living in the house or earns by renting the house out.

$$dH_t = (r - \delta)H_t\,dt + \sigma H_t\,dW_t \tag{2.1}$$

We assume that the risk free rate is deterministic.

$$dB_t = rB_t\,dt \tag{2.2}$$

2.2.1.2 Income Process

The borrower earns an income at one of two possible constant rates. A high rate i_e and a low rate i_u, where the subscripts e and u signify employment and unemployment. We refer to the high income rate state as the employed state and the lower income rate state as the unemployed state. When the borrower is employed, there is chance of arrival of unemployment with a Poisson arrival rate of λ_u. Similarly, when the borrower is unemployed, there is a chance of securing employment with a Poisson arrival rate of λ_e. The income process is independent of the house price. The borrower's income process is a two state continuous time Markov chain(see Figure 2.2). The set up can be extended to an n-state chain. The present value of future income, if the current income state is high (low) is denoted by $I_t = I_e$ (I_u).

Proposition 2.2.1. *The present value of future income, given some current state of employment and a discount rate of ρ, is given as follows. I_e is the present value if the borrower is currently employed and I_u is the present value if the borrower is unemployed.*

$$I_e = \frac{i_e}{\rho} - \frac{(i_e - i_u)\lambda_e}{\rho(\rho + \lambda_e + \lambda_u)} \tag{2.3}$$

$$I_u = \frac{i_u}{\rho} + \frac{(i_e - i_u)\lambda_u}{\rho(\rho + \lambda_e + \lambda_u)} \tag{2.4}$$

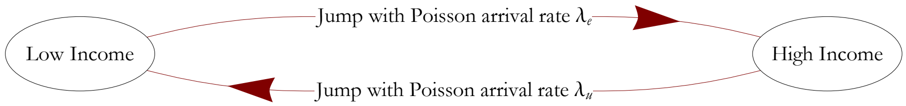


Figure 2.2: **Income Process**

Proof. See Appendix A. □

2.2.2 Borrower characteristic

A key feature of the borrower is that he is risk neutral. This feature takes away any insurance motives from the borrower's actions. Borrower's decisions are purely strategic. The next important feature is that the borrower's time discount parameter is higher than the risk free rate. This higher discount rate creates motives for the borrower to take on debt and to sell the house when the price is high enough. The final important characteristic is the psychic utility that the borrower derives from ownership. This is taken to be a function of the house price. It is zero when the house is worthless and it steadily but sub-linearly increases as the house price increases, indicating a relative satiation in the status or other happiness that the homeowner can derive from having a bigger and better house. In out numerical results, we have taken the instantaneous rate of psychic utility to be $\psi(H) = H^p$ where $p = 0.15 \quad \text{or} \quad 0.25$ were used. The present value of Psychic Utility, if you never defaulted, was derived to be $\Psi(H) = \frac{H^p}{(-p(r+\delta)+\rho+\frac{p(p-1)}{2}\sigma^2)}$.

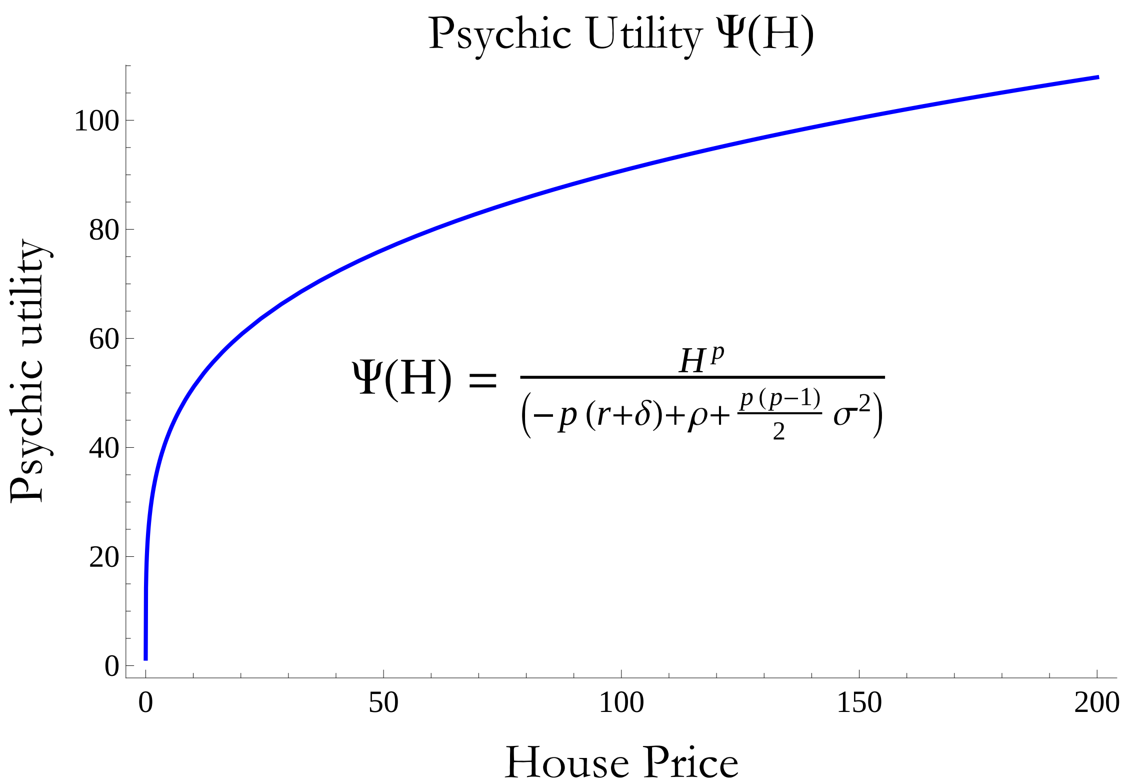


Figure 2.3: **Psychic Utility from home ownership**

2.2.3 Homeowner Utility Function

Let $U(\cdot)$ be the instantaneous utility function, which is of class C^2, and let ρ be the discount rate. The homeowner's optimization problem is to pick the optimal times to default, τ_1, or to sell the house, τ_2, so as to maximize his expected utility.

$$J(H_0, I_0) =$$

$$max_{\tau_1,\tau_2} E_{H_t,I_t}\left[\int_0^{\tau_1\wedge\tau_2} e^{-\rho t} U(i_t - cF + \psi(H_t) + \delta H_t)dt \right.$$
$$\left. + e^{-\rho(\tau_1\wedge\tau_2)}\left(f_1(H_{\tau_1}, I_{\tau_1})\mathbb{1}_{\tau_1<\tau_2} + f_2(H_{\tau_2}, I_{\tau_2})\mathbb{1}_{\tau_1>\tau_2}\right)\right]$$

where $i_t - cF + \psi(H_t) + \delta H_t$ is the instantaneous cashflow the homeowner derives from owning a mortgaged home. We assume that the homeowner is risk neutral, i.e. $U(x) \equiv x$. $f_1(\cdot,\cdot)$ is the boundary payoff in the event of default, while $f_2(\cdot,\cdot)$ is the boundary payoff in the event of prepayment.

$$f_1(H_t, I_t) = \begin{cases} -CF + I_t + max[H_t - F, 0] & \text{in a No-Recourse regime} \\ -CF + max[H_t + I_t - F, 0] & \text{in a Recourse regime} \\ J_0(H_t, I_t) & \text{in the event of a successfuly renegotiation} \end{cases}$$

$$f_2(H_t, I_t) = I_t + H_t - F \quad \text{Sale payoff is independent of the legal regime}$$

2.2.4 Debt Valuation

For a given contract, and given optimal behavior of the homeowner, we calculate the present value of debt, $V(H_0, I_0)$ as given below:

$$\begin{aligned} V(H_0, I_0) = & \\ & E_{H_t,I_t}\Bigg[\int_0^{\tau_1\wedge\tau_2} e^{-rt} cF dt \\ & + e^{-r(\tau_1\wedge\tau_2)}\Big(g_1(H_{\tau_1}, I_{\tau_1})\mathbb{1}_{\tau_1<\tau_2} + g_2(H_{\tau_2}, I_{\tau_2})\mathbb{1}_{\tau_1>\tau_2}\Big)\Bigg] \end{aligned}$$

where cF is the instantaneous cashflow the bank derives from mortgage payments and τ_1 and τ_2 come from the homeowner's optimization. $g_1(\cdot,\cdot)$ is the boundary payoff in the event of default, while $g_2(\cdot,\cdot)$ is the boundary payoff in the event of prepayment.

$$g_1(H_t, I_t) = \begin{cases} -CFL + min[H_t, F] & \text{in a No-Recourse regime} \\ -CFL + min[H_t + I_t, F] & \text{in a Recourse regime} \\ V_0(H_t, I_t) & \text{in the event of a successfuly renegotiation} \end{cases}$$

$$g_2(H_t, I_t) = F \quad \text{Sale payoff is independent of the legal regime}$$

2.3 Constant Income: Contracts without renegotiation

2.3.1 No Recourse regime

In our continuous time setting, the homeowner has to decide at each instant if he should stay current on his debt or exit the contract via sale or default(see Figure 2.4). The Bellman principle optimizes over the trade-off between exiting at a point or staying on and acting optimally an instant later. Figure 2.5 shows the state space and the possible movement of the borrowers value function. The region of the state space where the homeowner chooses to stay current is known as the continuation region. The dotted arrows show the movement of the high income value function

$J^e(H)$ depending on how the state variables evolve. The lateral movement shows the case where no income shock arrives in the next instant and only the house price changes. The vertical move indicates the possibility of an income shock moving us into a region of the state space with a potentially different low income value function $J^u(H)$. In the absence of recourse the borrowers boundary choices are not influenced by his income. As the choice of boundaries and boundary payoffs are identical in the absence of recourse, we find that $J^e(H) = J^u(H) + a$, were a is a constant that accounts for the differences in income levels. Since income does not influence any incentives, we take income to be zero in the no-recourse regime. The homeowner winds up with a single value function $J^e(H) \equiv J^u(H) = J(H)$.

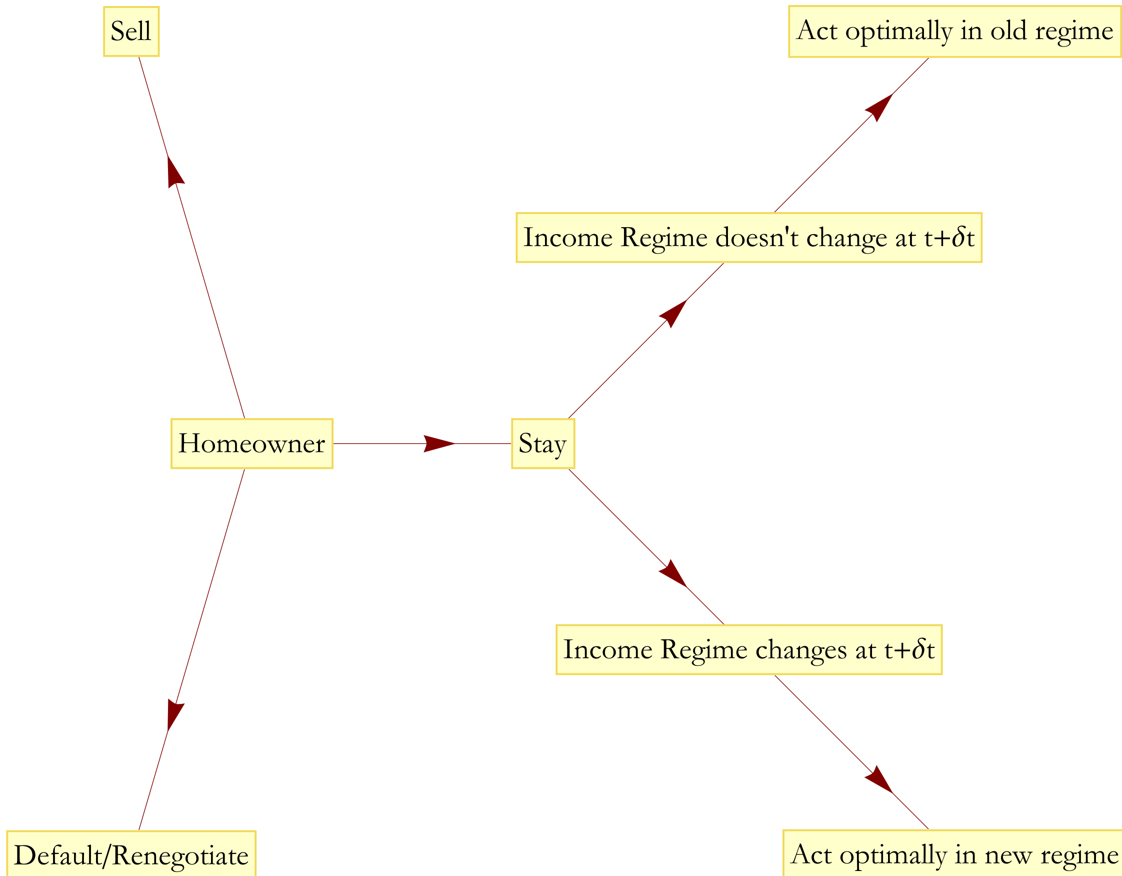


Figure 2.4: **Homeowner's problem**

For a given contract (c_1, F), the borrower faces a free boundary problem and Bellman equations in equation (2.5) describe his value functions. $J_k^i(H_t)$ is the homeowner's value function when the house price is H_t and his income flows at the rate $i \in \{i_u, i_e\}$ and $k \in \{0, 1\}$ indicates whether the borrower is in a pre-renegotiation or post-renegotiation state. The homeowner derives psychic utility from ownership at the rate of $\psi(H)$.

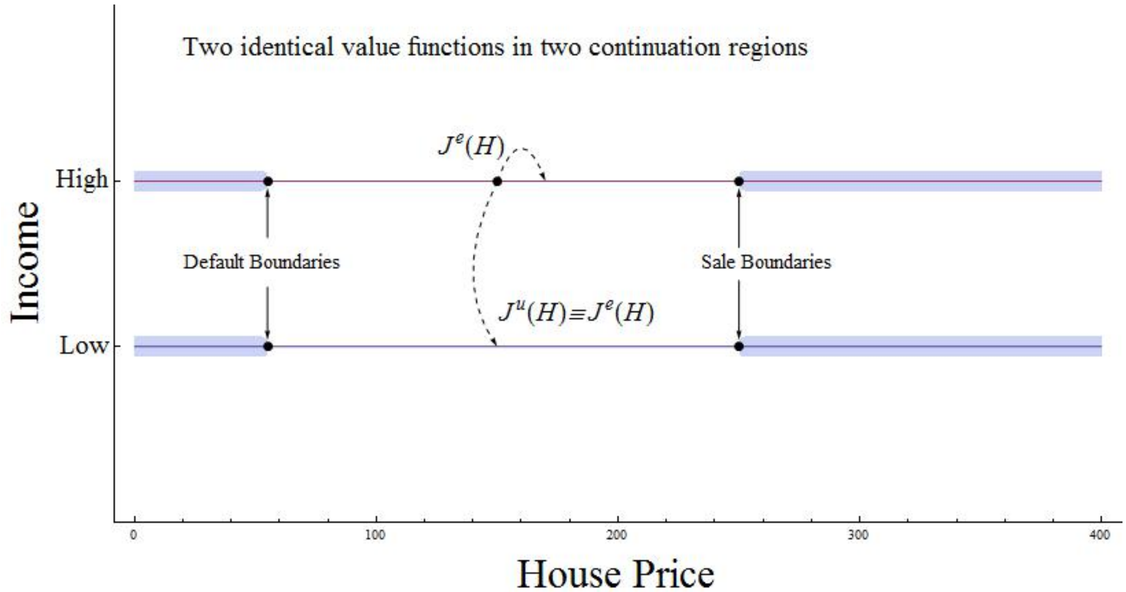


Figure 2.5: **Movement of value function across the state space under no-recourse**

$$J^e(H_t) = \max\left\{ \underbrace{B(H_t)}_{\text{exit contract}}, E_t\left[\underbrace{e^{-\lambda_e h}\left(\int_0^h e^{-\rho s}(i_e + \psi(H) + \delta H_{t+s} - c_1 F)ds + e^{-\rho h} J^e(H_{t+h})\right)}_{\text{stay current and income doesn't switch}} \right.\right.$$
$$\left.\left. + \underbrace{\int_0^h \left(\int_0^\tau e^{-\rho s}(i_e + \psi(H) + \delta H_{t+s} - c_1 F)ds + e^{-\rho\tau} J^u(H_{t+\tau}) \right) \lambda_e e^{-\lambda_e \tau} d\tau}_{\text{stay current and income switches}} \right] \right\}$$
$$J^u(H_t) = \max\left\{ \underbrace{B(H_t)}_{\text{exit contract}}, E_t\left[\underbrace{e^{-\lambda_u h}\left(\int_0^h e^{-\rho s}(i_u + \psi(H) + \delta H_{t+s} - c_1 F)ds + e^{-\rho h} J^u(H_{t+h})\right)}_{\text{stay current and income doesn't switch}} \right.\right.$$
$$\left.\left. + \underbrace{\int_0^h \left(\int_0^\tau e^{-\rho s}(i_u + \psi(H) + \delta H_{t+s} - c_1 F)ds + e^{-\rho\tau} J^e(H_{t+\tau}) \right) \lambda_u e^{-\lambda_u \tau} d\tau}_{\text{stay current and income switches}} \right] \right\} \tag{2.5}$$

where $B(H)$ is the payoff if the homeowner either defaults or sells the home.

$$B(H_t) = \max \begin{cases} H_t - F & \text{At the Sale boundary} \\ \begin{cases} I_t - C_f + \max\{H_t - F, 0\} & \text{Foreclosure: No Recourse} \\ -C_f + \max\{H_t + I_t - F, 0\} & \text{Foreclosure: Recourse} \\ J_0^{\{e,u\}} - C_r & \text{Post Renegotiation Value Function} \end{cases} \end{cases}$$

and

$$\begin{aligned} C_f &= \text{dead-weight cost to the homeowner associated with Foreclosure} \\ C_r &= \text{dead-weight cost to the homeowner incurred during Renegotiation} \end{aligned}$$

Equation (2.5) is the borrowers Bellman equation and it describes the choices of the borrower in the continuation region. At any instant the borrower chooses between exiting the contract and accepting the boundary payoff or staying current. If he stays current and the income state is high, in the next instant he may receive an income shock and be moved to the low income continuation region or he may simply move around in the high income continuation region. Whichever portion of the continuation region he winds up in, the homeowner acts optimally going forward from that point on. We derive that the value functions represented by the Bellman equation satisfy a system of linked ordinary differential equations described in Equation (2.6). These second-order non-homogeneous ordinary differential equations are known as Euler differential equations.

Proposition 2.3.1. *The home owner's value functions satisfy the following system of linked second order, second degree ordinary differential equations.*

$$J^e_{HH} + \frac{2(r-\delta)}{\sigma^2 H} J^e_H + \frac{2(\lambda_e - \rho)}{\sigma^2 H^2} J^e + \frac{2\lambda_e}{\sigma^2 H^2} J^u = -\frac{2(i_e + \psi(H) - cF)}{\sigma^2 H^2} - \frac{2\delta}{\sigma^2 H} \tag{2.6}$$

$$J^u_{HH} + \frac{2(r-\delta)}{\sigma^2 H} J^u_H + \frac{2(\lambda_u - \rho)}{\sigma^2 H^2} J^u + \frac{2\lambda_u}{\sigma^2 H^2} J^e = -\frac{2(i_u + \psi(H) - cF)}{\sigma^2 H^2} - \frac{2\delta}{\sigma^2 H} \tag{2.7}$$

Proof. See Appendix. □

Proposition 2.3.2. *When there is no recourse during default, the borrower's sale and default decision are not influenced by his income state. The lender's value function thus solves the following ordinary differential equation.*

$$J_{HH} + \frac{2(r-\delta)}{\sigma^2 H} J_H - \frac{2\rho}{\sigma^2 H^2} J = -\frac{2(\psi(H) - cF)}{\sigma^2 H^2} - \frac{2\delta}{\sigma^2 H} \tag{2.8}$$

Proof. See Appendix. □

Since the borrower's income does not factor into any of the borrower's decision, we will take income to be zero for the remainder of this section. We use the method of variation of parameters to solve these equations. Equation (2.9) represents the homeowner value function. The first two terms, $AH^{k_1} + BH^{k_2}$, represent the value of the option to default on the contract and the value of the option to sell the house. The next terms, $\frac{\delta H}{\delta+\rho-r} + \Psi(H)$, represent the homeowners subjective valuation of the home and his psychic utility from homeownership. The final term, $\frac{-cF}{\rho}$, is his valuation of the debt burden were he to hold his debt to perpetuity. The constants A and B are determined by the boundary payoffs. We use value matching and smooth pasting conditions on the value function to jointly determine the default boundaries, $H0S$ and $H0D$, and also the coefficients A and B of the homeowners value function.

Proposition 2.3.3. *The borrower's value function is as follows:*

$$J(H) = AH^{k_1} + BH^{k_2} + \frac{\delta H}{\delta + \rho - r} + \Psi(H) - \frac{cF}{\rho} \tag{2.9}$$

Where k_1 and k_2 satisfy the following quadratic

$$k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2} = 0$$

The coefficients A and B, for each level, are determined by the borrower's choice of renegotiation, default and sale boundaries.

Proof. See Appendix. □

Equation (2.10) represents the valuation of debt from the perspective of the debt holder. The lender can anticipate borrower default behavior given a contract and calculate the value of debt. The first two terms, $LH^{k_1} + MH^{k_2}$, represent the short position the bank has on the borrowers options to exit the contract and the third term, $\frac{cF}{r}$, is the present value of receiving the debt coupons in perpetuity. The coefficients are determined by value matching at the default and sale boundaries.

Proposition 2.3.4. *The valuation of debt, given a certain coupon and face value* *of debt, is as follows:*

$$V(H) = LH^{k_1} + MH^{k_2} + \frac{cF}{r} \tag{2.10}$$

The coefficients L and M, for each level, are determined by the borrower's choice of renegotiation, default and sale boundaries.

Proof. See Appendix. □

2.3.2 Recourse regime: constant income

The portion of the borrower's income that is not subject to recourse does not influence any of his decisions. We lose no generality by ignoring the portion of his income not subject to recourse. Going forward, we use the term income as the portion of borrower income or wealth that can be garnished by the lender. So constant income can be interpreted as a limit on the amount that the bank can garnish from the current wealth of the borrower to cover any deficiency after the foreclosure process.

In the continuation region, the equity value function and debt valuation follow the same laws of motion as they did in the no-recourse regime. The only difference comes in the default boundary payoffs. The option positions of the bank and the homeowner have different valuation in this regime and the difference shows up in the different coefficient values that we get for A, B, L and M.

2.4 Stochastic Labor Income

The borrower's income process is a two state continuous time Markov chain. The set up can be extended to an n-state chain. The borrower earns an income at one of two possible constant rates. A high rate i_e and a low rate i_u, where the subscripts e and u signify employment and unemployment. In the high income state, there is chance of arrival of a negative income shock with a Poisson arrival rate of λ_u. Similarly, in the low income state, there is a chance of a positive income shock with a Poisson arrival rate of λ_e. The income process is independent of the house price. The present value of future income, if the current income state is high (low) is denoted by $I_t = I_e$ (I_u).

Proposition 2.4.1. *The present value of future income, given some current state of employment and a discount rate of ρ, is given as follows. I_e is the present value if the borrower is currently employed and I_u is the present value if the borrower is unemployed.*

$$I_e = \frac{i_e}{\rho} - \frac{(i_e - i_u)\lambda_e}{\rho(\rho + \lambda_e + \lambda_u)} \tag{2.11}$$

$$I_u = \frac{i_u}{\rho} + \frac{(i_e - i_u)\lambda_u}{\rho(\rho + \lambda_e + \lambda_u)} \tag{2.12}$$

Theorem 2.4.2. *In a No-Recourse regime, the homeowner's stochastic labor Income has no impact on his default and sale decisions.*

Observation 2.4.3. *In a Recourse regime, the homeowner's Income level impacts his decision to default and to sell. When the homeowner's income is high, he defaults* *at a lower house price. When the homeowner's income is low, he sells at a higher house price.*

In the presence of recourse, the homeowner stands to lose more when he has a higher income and that incentivizes him to default later. At the sale point, income does not factor in and the only factor that separates the high income state from the low income state is the value of the default options. Since the default option has a higher strike for the low income state, it is more valuable and hence the homeowner waits longer before giving up that option and selling the house.

The continuation region for the high income and the low income states are depicted in figure 2.6. In the high income state, the continuation region is the set of house prices H such that $H \in [HDD, H0S]$. The homeowner defaults at the price HDD and sells at the price $H0S$ and stays current for intermediate prices. In the low income state, the continuation region is the set of house prices H such that $H \in [H0D, HSS]$. The homeowner defaults at the price $H0D$ and sells at the price HSS and stays current for intermediate prices. Note that, $HDD < H0D < H0S < HSS$.

2.4.1 Splitting the continuation region

We split both the continuation regions into two parts. The high income continuation region ranges from HDD to $H0S$. We split it into the regions $[HDD, H0D]$ and $[H0D, H0S]$. The low income continuation region ranges from $H0D$ to HSS.

Table 2.1: **This table lists how, when income is stochastic, the value function splits in different portions of the domain**

	High Income		Low Income	
Range	Equity	Debt	Equity	Debt
$[HDD, H0D]$	J^D	V^D	None	None
$[H0D, H0S]$	J^e	V^e	J^u	V^u
$[H0S, HSS]$	None	None	J^S	V^S

We split this into the regions $[H0D, H0S]$ and $[H0S, HSS]$.See figure 2.6. We define Equity and Debt Value functions as follows:

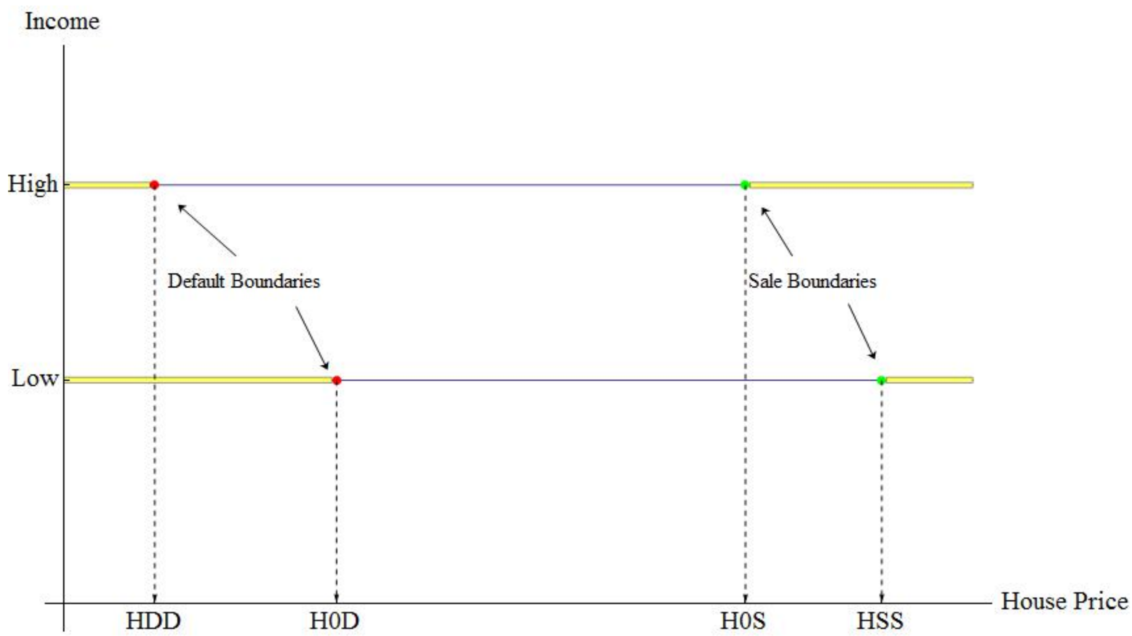


Figure 2.6: **Continuation Region with Labor Income uncertainty**

2.4.1.1 Smooth Pasting and Value Matching

We have 4 smooth pasting and 4 value matching conditions for Equity at the boundaries of the continuation regions. These conditions arrive out of No-Arbitrage and continuity. In addition we have 4 more conditions from the continuity and

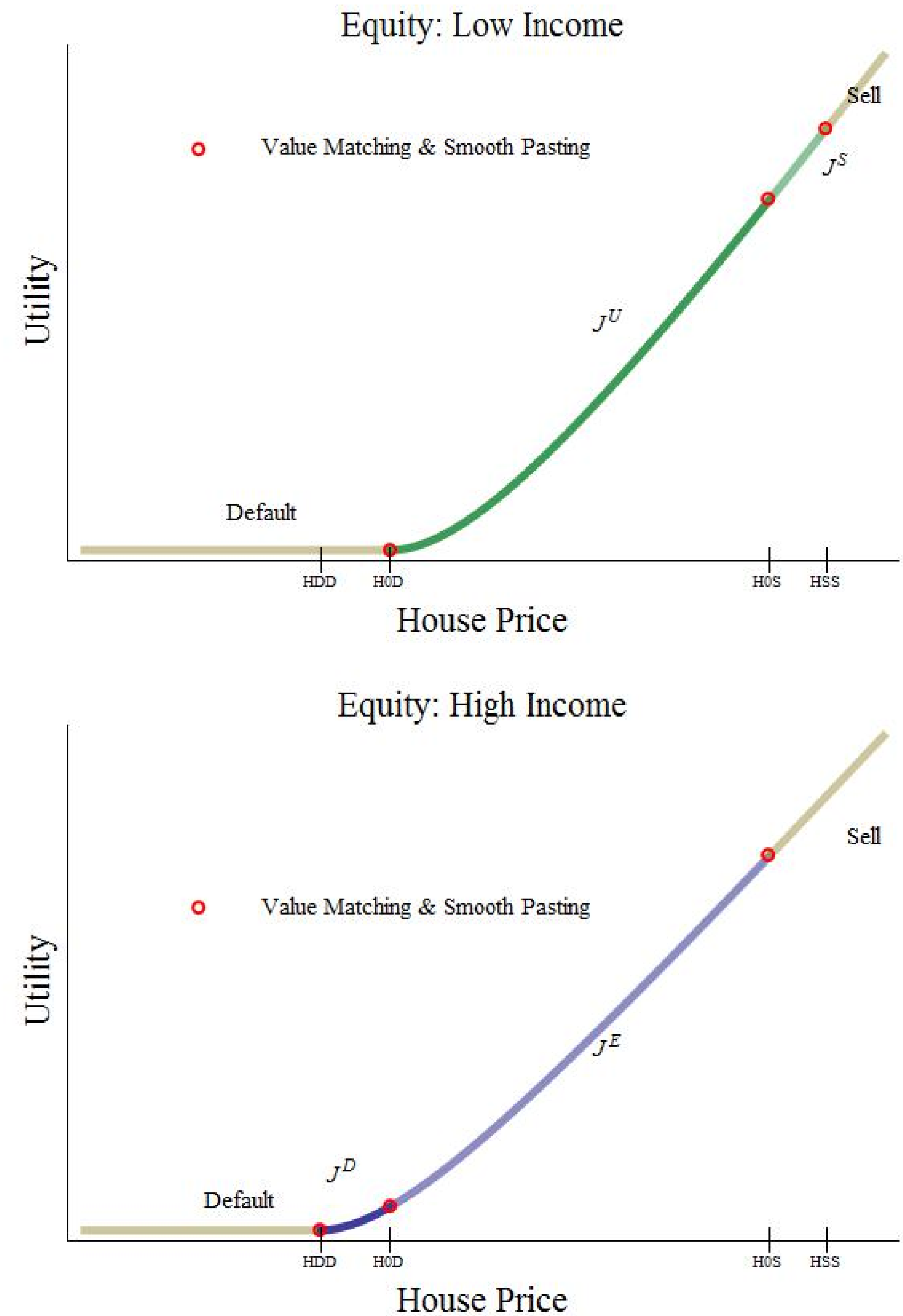


Figure 2.7: **Value Matching and Smooth Pasting conditions for Equity**

differentiability of the value function in the interior of the continuation region. See figure 2.7.

For Debt, we have 4 value matching conditions at the boundaries of the con-

Table 2.2: **This table lists the smooth pasting and value matching conditions for the homeowner's value function**

$J^D(HDD) = -CF$	Value Matching
$J^D_H(HDD) = 0$	Smooth Pasting
$J^e(H0D) = J^D(H0D)$	Continuity of Value Function
$J^e_H(H0D) = J^D_H(H0D)$	Differentiability of Value Function
$J^e(H0S) = H0S - F + Ie$	Value Matching
$J^e_H(H0S) = 1$	Smooth Pasting
$J^u(H0D) = -CF$	Value Matching
$J^u_H(H0D) = 0$	Smooth Pasting
$J^u(H0S) = J^S(H0S)$	Continuity of Value Function
$J^u_H(H0S) = J^S_H(H0S)$	Differentiability of Value Function
$J^u(HSS) = HSS + Iu - F$	Value Matching
$J^u_H(H0S) = 1$	Smooth Pasting

Table 2.3: **This table lists the smooth pasting and value matching conditions for the bank's value function**

$V^D(HDD) = HDD - CFL$	Value Matching
$V^D(H0D) = V^e(H0D)$	Continuity of Value Function
$V^D_H(H0D) = V^e_H(H0D)$	Differentiability of Value Function
$V^e(H0S) = F$	Value Matching
$V^u(H0D) = H0D - CFL$	Value Matching
$V^u(H0S) = V^S(H0S)$	Continuity of Value Function
$V^u_H(H0S) = V^S_H(H0S)$	Differentiability of Value Function
$V^S(HSS) = F$	Value Matching
$V^e(H0) = F$	Individual Rationality constraint

tinuation region. In addition we have 4 more conditions from the continuity and differentiability of the value function in the interior of the continuation region. In addition, we have an Individual Rationality constraint for the initial pricing of Debt. We have a total of 21 constraints. See figure 2.8.

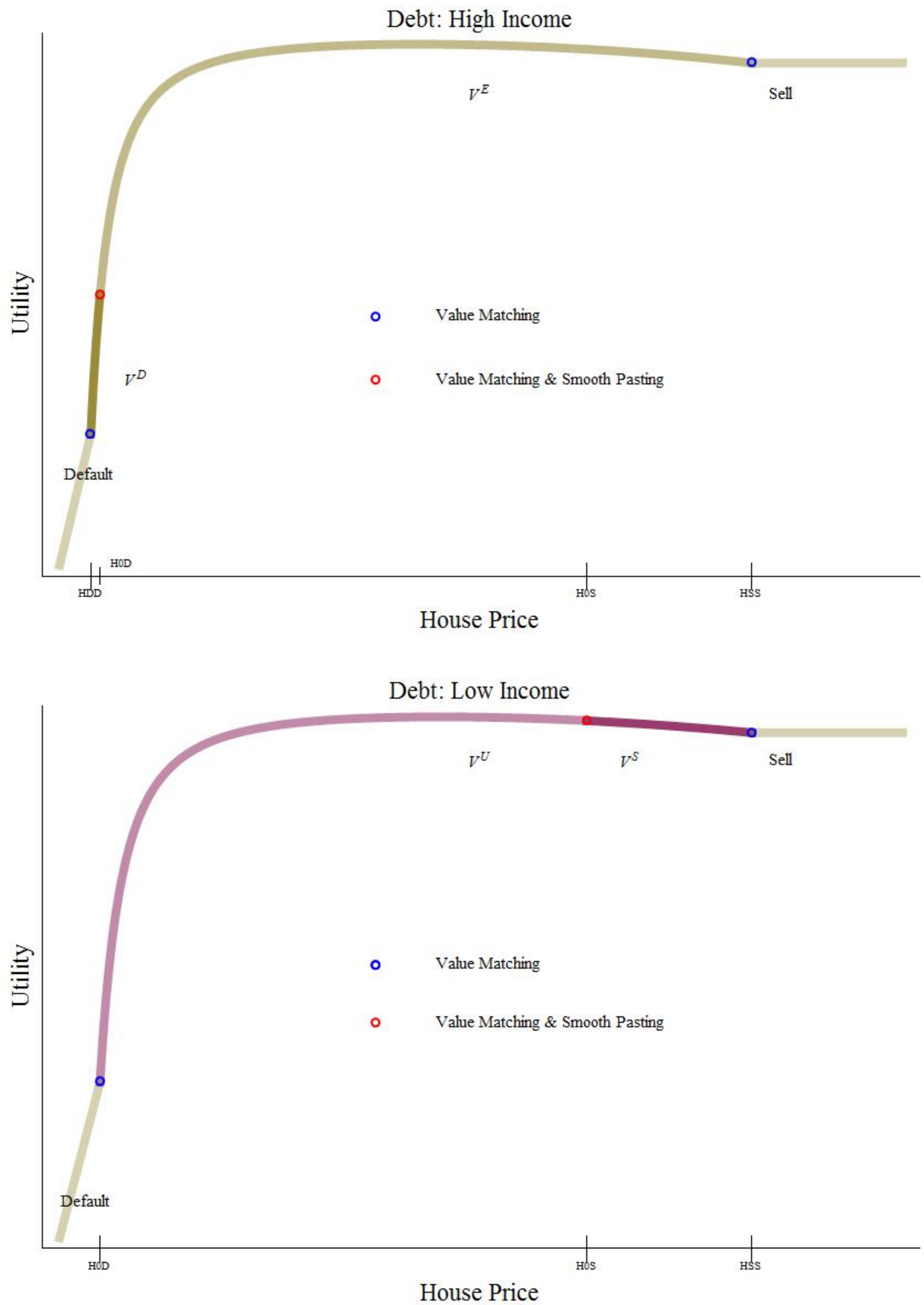


Figure 2.8: **Value Matching and Smooth Pasting conditions for Debt**

Proposition 2.4.4. *When income is stochastic, the domain of the value functions splits into four distinct regions. Equity and Debt value functions take the following functional form. The H^k terms represent the options to sell and default. In addition to the option, Equity valuation consists of Income, Psychic utility of ownership,* *private valuation of the house and the private valuation of the debt burden. Equity value function is denoted by J and debt value function is denoted by V.*

$$J^e = \sum_{j=1}^{4} A_j H^{k_j} + I_e - \frac{cF}{\rho} + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H}{(\rho + \delta - r)}$$

$$J^u = \sum_{j=1}^{2} A_j H^{k_j} - \frac{\lambda_u}{\lambda_e} \sum_{j=3}^{4} A_j H^{k_j} + I_u - \frac{cF}{\rho} + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H}{(\rho + \delta - r)}$$

$$J^S = A_{S1} H^{m_1^*} + A_{S2} H^{m_2^*} + \frac{(i_u + \lambda_u I_e - (c + \lambda_u)F)}{(\lambda_u + \rho)} + \frac{H^p}{((\lambda_u + \rho) - \frac{1}{2}p(p-1)\sigma^2 - (r - \delta)p)}$$
$$+ \frac{(\delta + \lambda_u)H}{((\lambda_u + \rho) - r + \delta)}$$

$$J^D = A_{D1} H^{m_1} + A_{D2} H^{m_2} + \frac{(i_e - \lambda_e CF - cF)}{(\lambda_e + \rho)} + \frac{H^p}{((\lambda_e + \rho) - \frac{1}{2}p(p-1)\sigma^2 - (r - \delta)p)}$$
$$+ \frac{\delta H}{((\lambda_e + \rho) - r + \delta)}$$

$$V^e = \sum_{j=1}^{4} L_j H^{k_j} + \frac{cF}{r}$$

$$V^u = \sum_{j=1}^{2} L_j H^{k_j} - \frac{\lambda_u}{\lambda_e} \sum_{j=3}^{4} L_j H^{k_j} + \frac{cF}{r}$$

$$V^S = L_{S1} H^{m_1^*} + L_{S2} H^{m_2^*} + \frac{(c + \lambda_u)F}{(\lambda_u + r)}$$

$$V^D = L_{D1} H^{m_1} + L_{D2} H^{m_2} + \frac{cF - \lambda_e CFL + \lambda_e I_u}{(\lambda_e + r)} + \frac{\lambda_e H}{\lambda_e + \delta}$$

where k_1, k_2 solve the first equation and k_3, k_4 solve the second equation

$$k(k-1) + \frac{2(r-\delta)}{\sigma^2} k - \frac{2\rho}{\sigma^2} = 0$$
$$k(k-1) + \frac{2(r-\delta)}{\sigma^2} k - \frac{2\rho}{\sigma^2} = \frac{2(\lambda_e + \lambda_u)}{\sigma^2}$$

m_1, m_2 satisfy the following characteristic equation

$$m(m-1) + \frac{2(r-\delta)}{\sigma^2}m - \frac{2(\lambda_e+\rho)}{\sigma^2} = 0$$

and m_1^*, m_2^* satisfy the following characteristic equations

$$m(m-1) + \frac{2(r-\delta)}{\sigma^2}m - \frac{2(\lambda_u+\rho)}{\sigma^2} = 0$$

Chapter 3: Contracts with Renegotiation

3.1 Introduction

In this section we study the impact of the possibility of subsequent renegotiation of mortgage terms on the initial equilibrium mortgage contract. Depending on how costly renegotiation is, there will always be ex-post efficiency gains to be made by prolonging the dead-weight costs arising from foreclosure. We hardwire the number of possible renegotiations to be one though our model can be extended to a finite number of renegotiations agreed upon in advance. We do not study the time inconsistency issue that the bank faces when it comes to pre-committing to a single renegotiation. We limit the renegotiations to be over mortgage rates. Our main results are that when debt has all the bargaining power then renegotiations are welfare improving. Contracts with a renegotiation feature is an improvement on contracts without one. Renegotiation and recourse act like substitutes and contracts with recourse and renegotiations are optimal. These contracts allow the homeowner the highest debt capacity. On the other hand when the homeowner has all the bargaining power, it is optimal for him to default immediately. In the event of an immediate default, the homeowner is able to extract the surplus from avoiding foreclosure and the bank does not meet his reservation value. Therefore such contracts will not be offered by banks. When debt has all the bargaining power, the bank's advantageous position post-renegotiation benefits the homeowner ex-ante in terms of more favorable coupons. As the homeowner bargaining power increases, the homeowner position post-renegotiation improves but any losses the bank faces is recaptured by the bank via increased initial mortgage rates. Increased Homeowner bargaining power leads to earlier renegotiation and higher initial mortgage rates. Interestingly, we find that there exists an intermediate level of bargaining power that leads to the highest welfare.

3.2 Model

In contracts with a renegotiation feature, the homeowner faces a choice, at every instant, between staying current on his loan or exiting the contract. The homeowner can exit the contract via selling the house or defaulting on his contract in an effort to renegotiate the contract terms. After the homeowner defaults on his coupon payments, the homeowner and the bank then attempt to renegotiate a new coupon rate. In the event that they fail to renegotiate, the house will go into foreclosure and both parties will receive their foreclosure payoffs. The foreclosure payoffs are determined by the legal regime. In a No-Recourse regime, the bank gets the house, if it's foreclosure sale price is less than the face value of debt, while the homeowner gets to keep his income and any remaining proceeds from the foreclosure sale after debt has been pad off. Both parties incur foreclosure costs, which are higher for the bank. In a Recourse regime, in addition to taking the house, the bank will garnish the homeowner's wages if the foreclosure sale was insufficient to pay the bank the face-value of debt. It is in the interest of the bank and the homeowner to delay these foreclosure costs and split the savings between them as dictated by their bargaining power.

3.3 The Homeowner's Problem

The homeowner's problem in a contract with renegotiation is similar to the problem studied in contracts without renegotiation. The homeowner has to choose between staying current on his payments or exiting via selling or defaulting and then renegotiating. In the continuation region, the homeowner's value functions, described in equations (3.1) and (3.2), take familiar form. The subscript "1" denotes pre-renegotiation, while the subscript "0" denotes a post-renegotiation contract.

$$J^1(H) = A_1 H^{k_1} + B_1 H^{k_2} + I_e - \frac{c_1 F}{\rho} + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H}{(\rho + \delta - r)} \tag{3.1}$$

$$J^0(H) = A_0 H^{k_1} + B_0 H^{k_2} + I_e - \frac{c_0 F}{\rho} + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H}{(\rho + \delta - r)} \tag{3.2}$$

Let $H1R$ be the price at which the homeowner chooses to renegotiate the contract. $H1S$ is the price at which the homeowner chooses to sell his home when he is under the original contract. $H0D$ and $H0S$ are the default and sale boundaries chosen by the homeowner under the renegotiated contract. The homeowner's value function and boundary choices leave us with 8 unknowns: $A_1, B_1, A_0, B_0, H1R, H1S, H0D, H0S$.

Table 3.1: **Equity smooth pasting and value matching conditions in contracts with renegotiation**

Equation	Type of Contact	Description of Evaluation point	Notation of point
$J^1(H) = H - F + I_e$	Value Matching	Original contract sale point	H1S
$J^1_H(H) = 1$	Smooth Pasting	Original contract sale point	H1S
$J^1(H) = J^0(H) - CR$	Value Matching	Original contract renegotiation point	H1R
$J^1_H(H) = J^0_H(H)$	Smooth Pasting	Original contract renegotiation point	H1R
$J^0(H) = H - F + I_e$	Value Matching	Renegotiated contract sale point	H0S
$J^0_H(H) = 1$	Smooth Pasting	Renegotiated contract sale point	H0S
$J^0(H) = -CF$	Value Matching	Renegotiated contract default point	H0D
$J^0_H(H) = 0$	Smooth Pasting	Renegotiated contract default point	H0D

From no-arbitrage, we have both smooth pasting and value matching at each of the four boundaries, giving us 8 equations that determine our unknowns. Table 3.2 lists all the equations. Since every aspect of the renegotiated contract is a function of the price at which the homeowner chooses to renegotiated and since our post-renegotiation free boundaries are determined implicitly, the homeowner's smooth pasting condition, described in (3.3), at the renegotiation point H1R is evaluated using implicit function theorem.

$$
\begin{aligned}
&A1k_1H1R^{k_1-1} + B1k_2H1R^{k_2-1} + \frac{pH1R^{p-1}}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta}{(\rho + \delta - r)} = \\
&\left(\frac{\partial A0}{\partial H0S}\frac{dH0S}{dH1R} + \frac{\partial A0}{\partial H0D}\frac{dH0D}{dH1R} + \frac{\partial A0}{\partial c_0}\frac{dc_0}{dH1R} + \frac{\partial A0}{\partial H1R}\right) H1R^{k_1} + A0k_1H1R^{k_1-1} \\
&+ \left(\frac{\partial B0}{\partial H0S}\frac{dH0S}{dH1R} + \frac{\partial B0}{\partial H0D}\frac{dH0D}{dH1R} + \frac{\partial B0}{\partial c_0}\frac{dc_0}{dH1R} + \frac{\partial B0}{\partial H1R}\right) H1R^{k_2} + B0k_2H1R^{k_2-1} \\
&- \frac{F}{\rho}\frac{dc_0}{dH1R} + \frac{pH1R^{p-1}}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta}{(\rho + \delta - r)} \qquad (3.3)
\end{aligned}
$$

3.4 Bank's problem

The bank's problem in a contract with renegotiation is also similar to the problem studied in contracts without renegotiation. The bank has to choose an initial coupon anticipating the homeowner's response to such a coupon choice while also pricing the coupon so the bank gets its reservation value. When the homeowner defaults and attempts a renegotiation, the bank will offer a new coupon based on the relative bargaining powers of the parties involved. The bank will collect its boundary payments in accordance with the nature of the boundary and the nature of the legal regime. In the continuation region, the valuation of debt, described in equations (3.4) and (3.5), takes familiar form. The subscript "1" denotes pre-renegotiation, while the subscript "0" denotes a post-renegotiation contract.

$$V^1(H) = L_1H^{k_1} + M_1H^{k_2} + \frac{c_1F}{r} \qquad (3.4)$$

$$V^0(H) = L_0H^{k_1} + M_0H^{k_2} - \frac{c_0F}{r} \qquad (3.5)$$

$H1R$ is the price at which the homeowner chooses to renegotiate the contract. $H1S$ is the price at which the homeowner chooses to sell his home when he is under the original contract. $H0D$ and $H0S$ are the default and sale boundaries chosen by the homeowner under the renegotiated contract. The homeowner's value function and boundary choices leave us with 6 unknowns: $L_1, M_1, L_0, M_0, c_0, c_1$.

The initial coupon, c_1, is determined by the bank's individual rationality constraint

Table 3.2: **Debt value matching conditions in contracts with renegotiation**

Equation	Type of Contact	Description of Evaluation point	Notation of point
$V^1(H) = F$	Value Matching	Original contract sale point	H1S
$V^1(H) = V^0(H) - CRL$	Value Matching	Original contract renegotiation point	H1R
$V^0(H) = F$	Value Matching	Renegotiated contract sale point	H0S
$V^0(H) = H + I_e - CFL$	Value Matching	Renegotiated contract default point	H0D

described in (3.6). The value of debt at the inception of loan is equal to the value of the loan made by the bank.

$$L_1 H0^{k_1} + M_1 H0^{k_2} + \frac{c_1 F}{r} = F \tag{3.6}$$

The renegotiated coupon rate c_0 is determined by the relative bargaining power of the two agents and is discussed in the following section.

3.5 Bargaining Power

Bargaining power in renegotiation is defined as the ability of the agents to influence the renegotiated coupon value. In the first extreme case, when the bank has all the bargaining power, the bank picks the coupon that maximizes his valuation. Equation (3.7) describes the bank's valuation of post-renegotiation debt, where $H1R$ is the house price at renegotiation and CRL is the fixed cost of renegotiation.

$$L0H1R^{k_1} + M0H1R^{k_2} + \frac{c_0F}{r} - CRL \tag{3.7}$$

The bank's first order condition is described in (3.8). In keeping with the terminology used earlier in the paper, $H0S$ is the sale boundary, $H0D$ is the default boundary, $H1R$ is the price at which the contract was renegotiated, F is the face value of debt and c_0 is the renegotiated coupon.

$$\begin{aligned}&\left(\frac{\partial L0}{\partial H0S}\frac{dH0S}{dc_0} + \frac{\partial L0}{\partial H0D}\frac{dH0D}{dc_0} + \frac{\partial L0}{\partial c_0}\right) H1R^{k_1}\\ &+ \left(\frac{\partial M0}{\partial H0S}\frac{dH0S}{dc_0} + \frac{\partial M0}{\partial H0D}\frac{dH0D}{dc_0} + \frac{\partial M0}{\partial c_0}\right) H1R^{k_2} + \frac{F}{r} = 0\end{aligned} \tag{3.8}$$

Since the post-renegotiation default and sale boundaries are implicit functions of the renegotiated coupon, we use the Implicit function theorem to arrive at expressions for $\frac{dH0S}{dc_0}$ and $\frac{dH0S}{dc_0}$. Recall that the post-renegotiation sale and default boundary are determined by a system of simultaneous equations:

$$F_1(H0S, H0D, c_0, H1R) = 0 \tag{3.9}$$

$$F_2(H0S, H0D, c_0, H1R) = 0 \tag{3.10}$$

Let $(c_d, H0S_d, H0D_d)$ solve (3.8), (3.9) and (3.10). c_d is the renegotiated coupon rate when the bank has all the bargaining power. On the other hand, when the homeowner has all the bargaining power, since his valuation decreases monotonically in the coupon rate, he picks the lowest coupon that satisfies the bank's individual rationality constraint and gives the bank its foreclosure payoff. Equation (3.11) describes the bank's post-renegotiation value function equaling its payoff has the renegotiations failed.

$$L0H1R^{k_1} + M0H1R^{k_2} + \frac{c_0 F}{r} - CRL = H1R - CFL \tag{3.11}$$

Let $(c_e, H0S_e, H0D_e)$ solve (3.11), (3.9) and (3.10). c_e is the renegotiated coupon rate when the homeowner has all the bargaining power. Now for any $\gamma \in (0, 1)$,

$$c_\gamma = \gamma c_d + (1 - \gamma) c_e$$

defines the renegotiated coupon that the homeowner and bank would arrive at when the bargaining power was mixed.

Chapter 4: Results

4.1 Results

4.1.1 Contracts without Renegotiation

In contracts without renegotiation, our results are driven by the homeowner's optimal choice of option exercise boundaries and his subjective time discount parameter. We find that in the presence of recourse, default becomes a lot less lucrative for the borrower. This incentivizes him to stay longer in the contract. This feeds back into the initial pricing of the debt contract. As a consequence, the homeowner enjoys a lower mortgage rate for a given face value of the loan. The lowered debt burden is reflected in the homeowner taking longer to shed that burden via selling the house. As a result, under recourse, we see lower mortgage rates, lower default rates and higher equity valuation. We show that default will never occur when there is negative equity and the homeowner sells the home only when he has positive equity. We show that in the presence of recourse the game has a larger continuation region.

Observation 4.1.1. *Lets $H0S_R$, $H0S_{NR}$, $H0D_R$ and $H0D_{NR}$ be the sale and default boundaries under recourse and no-recourse respectively. For a given coupon, the* *default boundary under a regime without recourse is higher than the default boundary under a regime with recourse. The sale boundary in the absence of recourse is higher than in the presence of recourse. The difference in the boundaries is higher at the default boundary than at the sale boundary.*

$$H0D_{Rec} < H0D_{No-Rec}$$

$$H0S_{Rec} < H0S_{No-Rec}$$

$$H0D_{Rec} - H0D_{No-Rec} < H0S_{No-Rec} - H0S_{Rec}$$

Let contract-R be a mortgage contract under renegotiation and let contract-NR be a contract under a no-renegotiation regime. Given a fixed mortgage rate, both contracts are identical in their cash flows in the continuation region and also identical in their payoffs to equity at the sale boundary. The only difference between them is that the default boundary is uniformly more lucrative in the absence of recourse. This makes default in contract-NR more attractive as compared to in contract-R. The more favorable default terms makes the default option more valuable in contract-NR making continuation at the sale boundary more valuable. This results in the sale boundary of contract-NR being higher than the sale boundary of contract-R. The increase in the value of the default put is much higher closer to its exercise value than away at the sale boundary and that is why the shift in the default boundary is larger than the shift in the sale boundary.

The changes in the boundary choices of the homeowner under recourse has a feedback effect on the valuation of debt at the inception of the loan. Debt now has a higher value at the beginning of the loan if the legal regime allows for recourse. Since the initial debt is priced at par, for a given face-value of debt, homeowners now face a lower mortgage rate. The lower mortgage rate leads to a higher valuation of equity.

Observation 4.1.2. *For a given face-value of debt, the equilibrium mortgage rates are lower in a regime with recourse as compared to the mortgage rate in a regime without recourse. The equity values in recourse regimes are higher.*

Our next result shows that the homeowner would prefer to take on a mortgage even if he was able to finance the purchase of the home from his own funds. This result is driven by the homeowner's subjective time discount factor being higher than the risk free rate. For sufficiently low face-value of debt, the homeowner can get a loan arbitrarily close to the risk free rate. At these low rates, it is lucrative for the homeowner to increase his current consumption and finance the home purchase via debt that he values below par. As he loads up on debt, the coupon rate on the entire debt increases. This reduces his incremental gain from trading intertemporally and at some point he reaches his optimal debt capacity. If the homeowner were to have 100% debt, the equilibrium mortgage rate is above his subjective discount rate. This demonstrates that there is an interior optimal debt level. With increasing debt, the default put that the homeowner owns also increases in value but we show that this increase is insufficient to compensate the homeowner for the increasing burden of an increasing mortgage rate that accompanies high levels of leverage. The uniformly lower mortgage rates under recourse imply that the homeowner has a higher debt capacity under recourse.

Observation 4.1.3. *There exists an optimal debt level when the legal regime does not allow for lender recourse.*

Observation 4.1.4. *Contracts with recourse allow for a higher optimal debt level than contracts without recourse.*

The homeowner and the bank find it ex-post efficient to postpone dead-weight loss from incurring foreclosure costs by modifying the terms of the contract. Any ex-post modification is priced into the contract ex-ante. The balance of bargaining power during renegotiation becomes relevant in determining the ex-ante efficiency of mortgage renegotiation. We show that when the homeowner has all the bargaining power, he instantly defaults on his contract and extracts the value of the dead-weight foreclosure costs from the bank. We also show that contracts without renegotiation dominate contracts with renegotiation if the borrower has all the bargaining power. On the other hand, contracts with costless renegotiation are an improvement over contracts without renegotiation when the bank acts like a monopolist during renegotiation. Homeowner bargaining power is an important determinant of the feasibility of a mortgage modification program and subsequent default rates. Lender recourse reduces homeowner bargaining power during a renegotiation. Lender recourse removes the incentives of the homeowner to engage in foreclosure stripping and increases the reservation value of the bank in the renegotiation process.

Observation 4.1.5. *When the homeowner has all the bargaining power, he defaults immediately and sues for renegotiation. This enables him to instantly extract from* *the bank the dead-weight costs it would incur during foreclosure*

Observation 4.1.6. *When the homeowner has high bargaining power, the banks are strictly better off offering contracts without renegotiation or not offering any contract at all.*

Observation 4.1.7. *When the bank has monopoly power during renegotiation and when renegotiation is costless then contracts with renegotiation are a welfare improvement over contracts without renegotiation.*

The base parameter values that we use are listed in Table 4.1. When we don't vary the face value of the loan, it is assumed to be 80%.

Table 4.1: **Base case parameter values for the numerical results when income is not stochastic**
This table reports the parameter values used in the base case for our numerical results for the examples comparing recourse and no-recourse regimes with constant income and no renegotiation.

Parameter	Symbol	Value
House Price	H_0	100
Recoursable income	I	10
Face value of loan	F	80
Foreclosure cost (lender)	CFL	9
Foreclosure cost (borrower)	CF	3
Renegotiation cost (lender)	CRL	2
Renegotiation cost (borrower)	CR	1
Psychic utility at H_0	$\Psi(H)$	33
risk free rate	r	0.047
Borrower time discount rate	ρ	0.057
House dividend rate	δ	0.05
House price volatility	σ	0.15

4.1.2 Recourse vs No-Recourse

This example illustrates the differences in outcomes with the introduction of lender recourse by comparing two contracts that do not feature renegotiation. In the recourse regime, we fix the extent of recourse to be 10. In Figure 4.1, we see that in a regime with recourse we get lower equilibrium coupon rates and higher equity values. In regimes with recourse, for a given coupon rate, the homeowner delays default and stays longer in the contract. This is viewed favorably by the lenders and gives a higher valuation of the debt. Therefore, for a given valuation of debt, the recourse regime has a lower coupon for the borrower. As we vary the face value of debt, we see that an optimal level of debt emerges in both the regimes. This is due to the fact that the homeowner has a higher time discount parameter and is willing to give up more of his future earnings for current wealth. As the borrower levers up, the coupon rate on the entirety of his debt increases and at some point the marginal gain from borrowing an additional dollar at a low rate is less than the marginal loss in paying a higher coupon on the entire debt burden. In regimes with recourse, for any level of debt, the coupon charged is lower, therefore it takes an increased amount of levering up to hit a satiation point. Figure 4.2 shows how the sale and default boundaries are affected by recourse. The lower debt burden for a given face value of debt incentivizes the homeowner to stay longer in the house. In Figure 4.3, we look at the valuation of the default and sale options at the inception of the loan. We see that the in the no-recourse regime the option to default is considerably more valuable. We also see that the option to default increases in leverage. The option

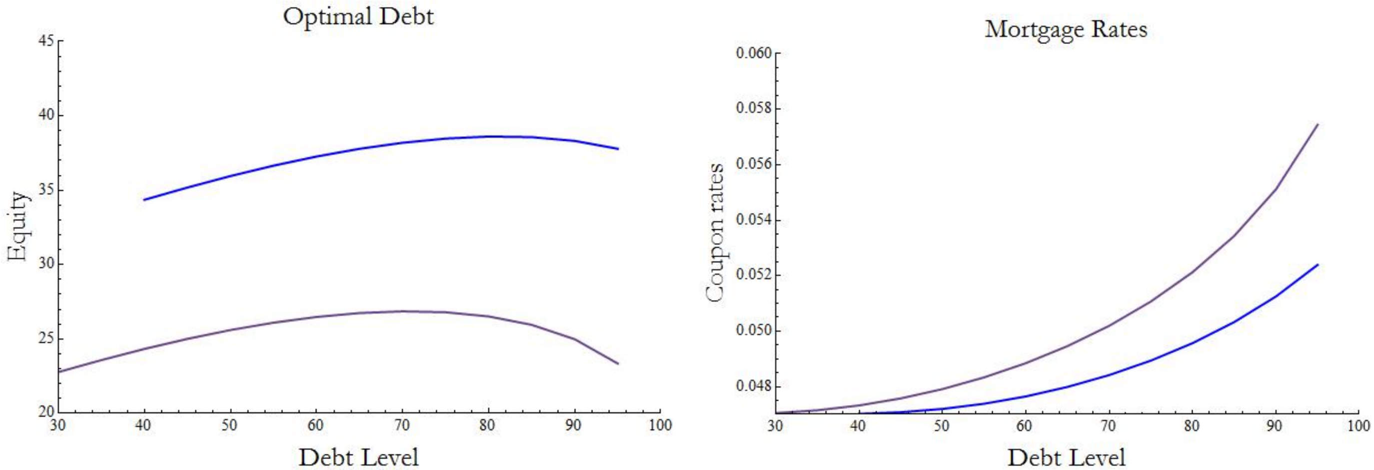


Figure 4.1: **Comparison of Equity value and mortgage rates across Recourse and No-Recourse legal regimes when income is constant**
This figure reports the value of equity less down-payment and equilibrium mortgage rates across contracts in recourse and no-recourse regimes as we vary the face value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%.

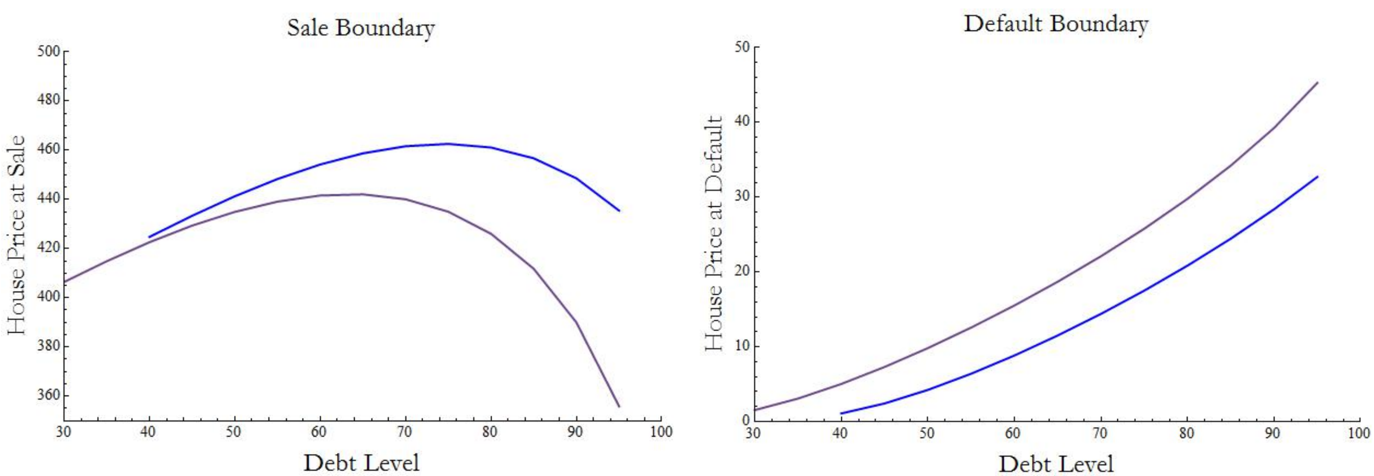


Figure 4.2: **Comparison of sale and default boundaries across Recourse and No-Recourse legal regimes when income is constant**
This figure reports the value of sale and default boundaries across contracts in recourse and no-recourse regimes as we vary the face value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%.

to sell the house is a lot less valuable and a lot less sensitive to leverage.

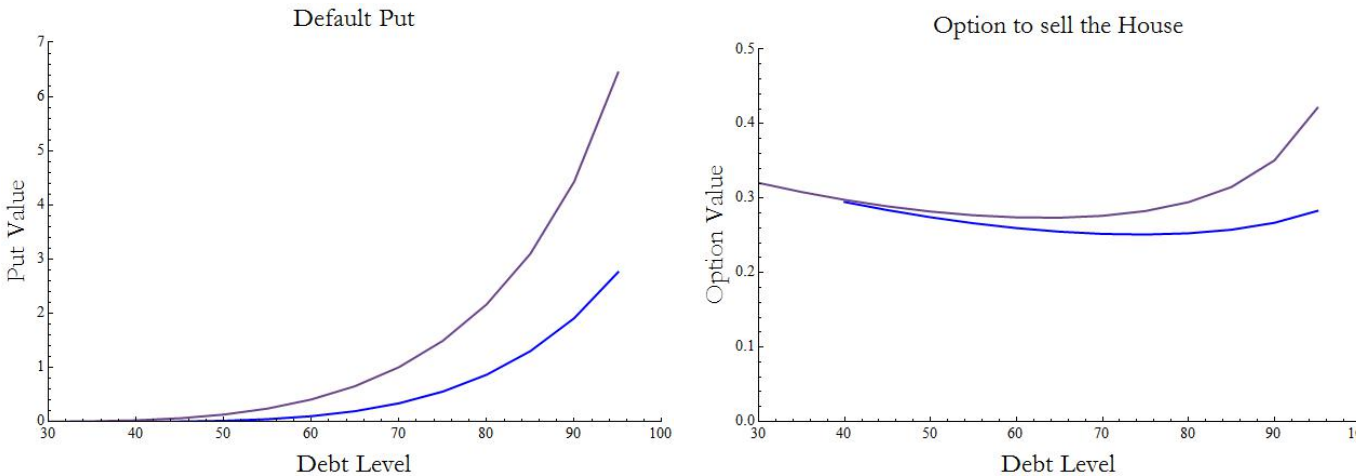


Figure 4.3: **Comparison of default and sale options across Recourse and No-Recourse legal regimes when income is constant**
This figure reports the value of default and sale options across contracts in recourse and no-recourse regimes as we vary the face value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%.

4.1.3 Stochastic Income

We use the base parameter values listed in table **??**. We look at the impact of stochastic income on default decision, sale decision, equilibrium mortgage rates and equilibrium equity values. We find that in the absence of recourse, income fluctuation has no impact on any of the homeowner's decisions. In a Recourse regime, we see that the homeowner defaults at a lower price when he has more income to lose via recourse. We also find that the homeowner sells at a higher price when his income is lower. For the purposes of selling, the income level does not impact things directly. The only difference between the high and low income states is that between the two

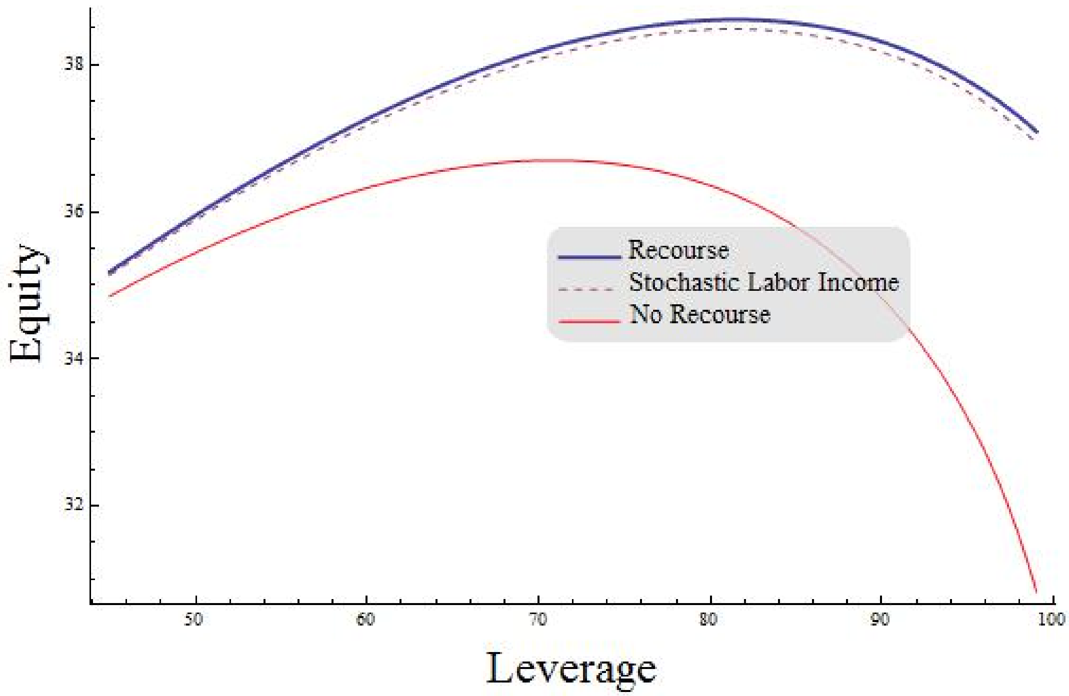


Figure 4.4: **Comparison of Equity value in Stochastic income setting vs constant income setting as debt varies**
This figure reports the value of Equity value in Stochastic income setting vs constant income setting contracts in recourse and no-recourse regimes as we vary the face value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Income in the high state is $I_e = 9.84$, while income in the low state is I_u=2.05

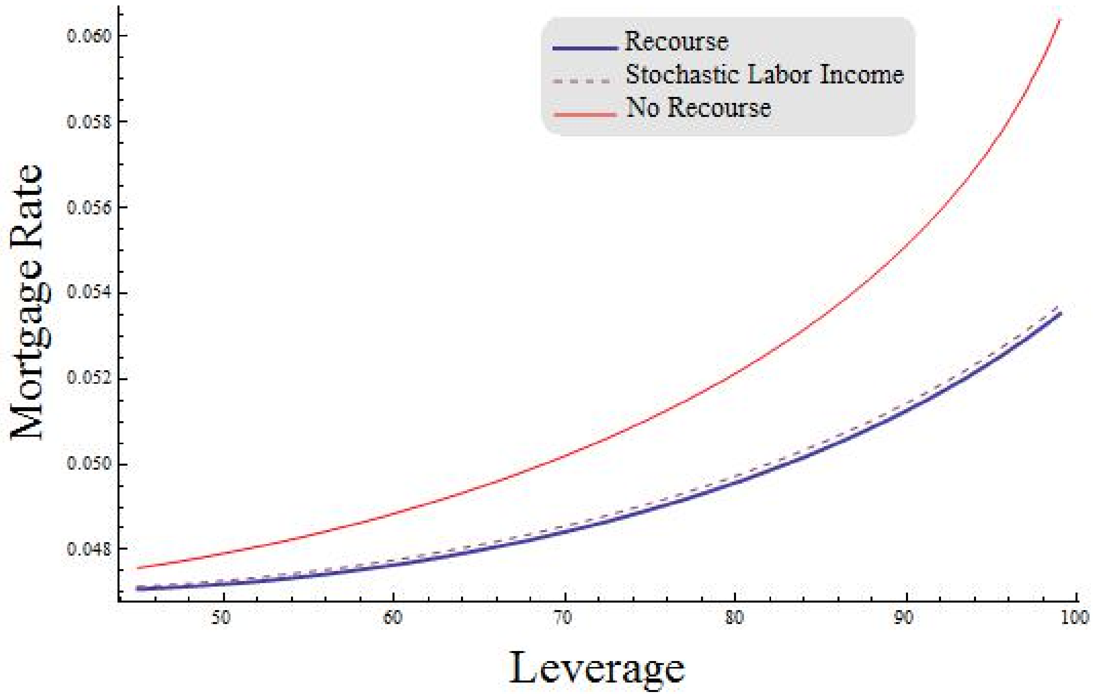


Figure 4.5: **Comparison of equilibrium mortgage rate in Stochastic income setting vs constant income setting as debt varies**

This figure reports the value of equilibrium mortgage rate in Stochastic income setting vs constant income setting contracts in recourse and no-recourse regimes as we vary the face value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Income in the high state is $I_e = 9.84$, while income in the low state is I_u=2.05

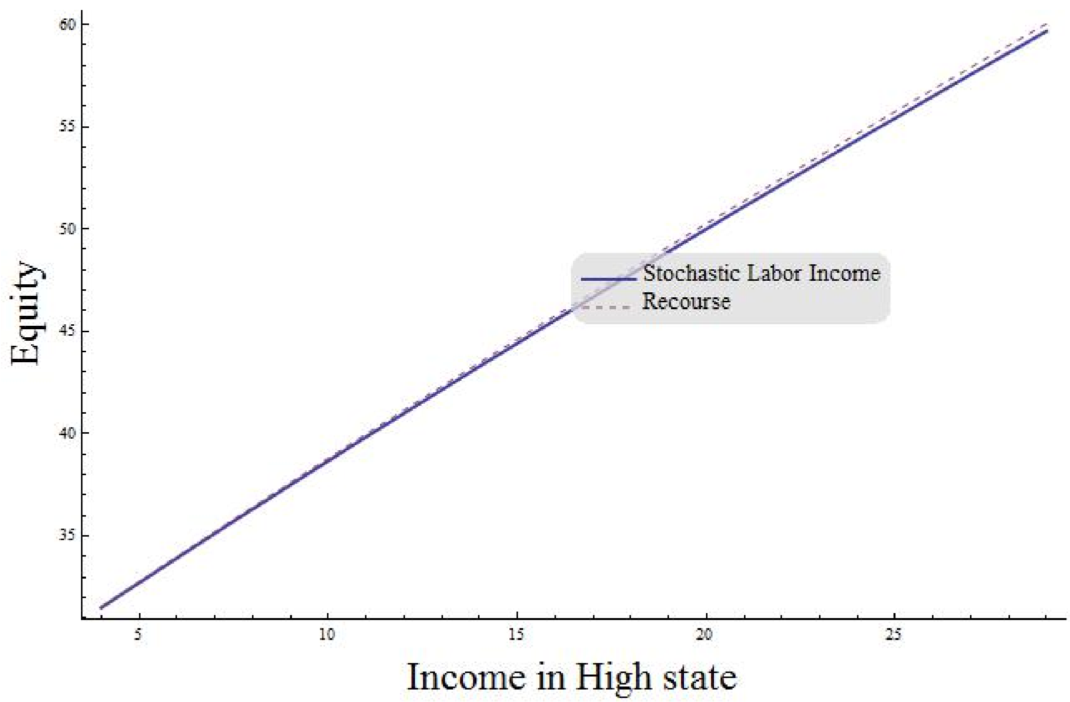


Figure 4.6: **Comparison of Equity value in Stochastic income setting vs constant income setting as income varies**
This figure reports the value of Equity value in Stochastic income setting vs constant income setting contracts in recourse and no-recourse regimes as we vary the face value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate $\rho :$ 5.7%, house price volatility $\sigma :$ 15% and house price dividend rate $\delta :$ 5%. Debt level F is 80.

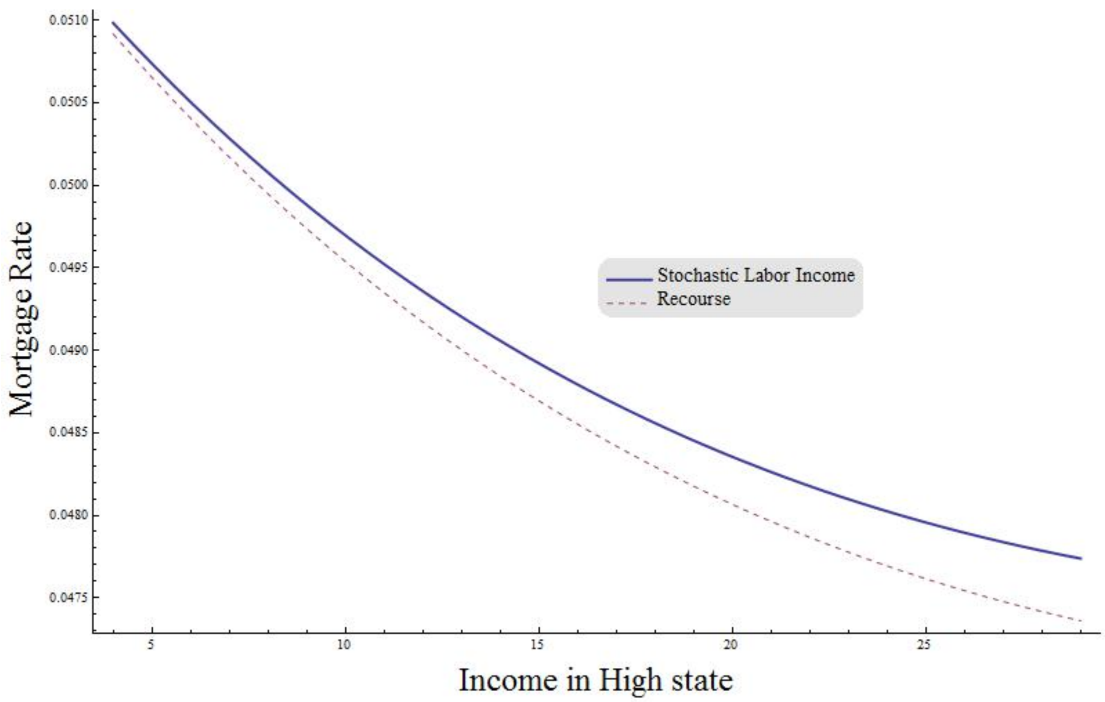


Figure 4.7: **Comparison of equilibrium mortgage rate in Stochastic income setting vs constant income setting as income varies**
This figure reports the value of equilibrium mortgage rate in Stochastic income setting vs constant income setting contracts in recourse and no-recourse regimes as we vary the face value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Debt level F is 80.

options to default. Since the option to default has a lower strike, it's price is lower near the sale point and the homeowner is willing to part with it earlier. See table 4.3

We see that, predictably, the overall mortgage rate when income is stochastic is in between the no-recourse mortgage rate and the recourse mortgage rate if the income was higher, see figure 4.5. We find that the lower mortgage rate in the recourse-stochastic-income state leads the homeowners to have a higher optimal leverage level than in the case without recourse. The optimal leverage level is in between the no-recourse leverage and the recourse high constant level. See figure 4.4. We also look at the value of Equity and the coupon rate as we increased the recoursable income, see figure 4.7 and figure 4.7 and we find that mortgage rates reduce in recoursable income and the value of Equity increases in recoursable income. Interestingly, in 4.10 we see that at optimal leverage, in regimes with recourse default occurs at a higher house price. This result is counter to the intuition that recourse would lead to fewer defaults and foreclosures.

Observation 4.1.8. *In the presence of Labor Income uncertainty, the homeowner defaults later in the state in which his income is higher but sells later in the state in which his income is lower.*

Observation 4.1.9. *There exists an optimal debt level.*

Observation 4.1.10. *The optimal debt level is higher in the presence of labor income uncertainty.*

4.1.4 Renegotiation

Both the bank and the homeowner suffer costly dead-weight loss associated with the foreclosure process. It is in their interest to stave off these dead-weight costs by renegotiating the terms of the contract. Bargaining power during renegotiations becomes a crucial variable in determining if the banks and the homeowners are able to gain the surplus from postponing dead-weight losses associated with the foreclosure process. We see that in the extreme case when bank's have all the bargaining power, we see a welfare gain relative to contracts without renegotiation. As bargaining power shifts from debt to equity, any of the surplus that equity extracts ex-post from debt is re-captured by debt via an increasing initial pre-renegotiation mortgage rate. As equity bargaining power increases, we see the homeowner sue for renegotiation sooner. In the extreme case where equity has all the bargaining power, the homeowner renegotiates immediately and is able to extract the surplus from the bank. These contracts will not be offered by the bank since they do not meet the bank's individual rationality constraint. We find a level of equity bargaining power at which is becomes feasible for the bank to offer contracts with renegotiation. There also exists an optimal level of bargaining power split.

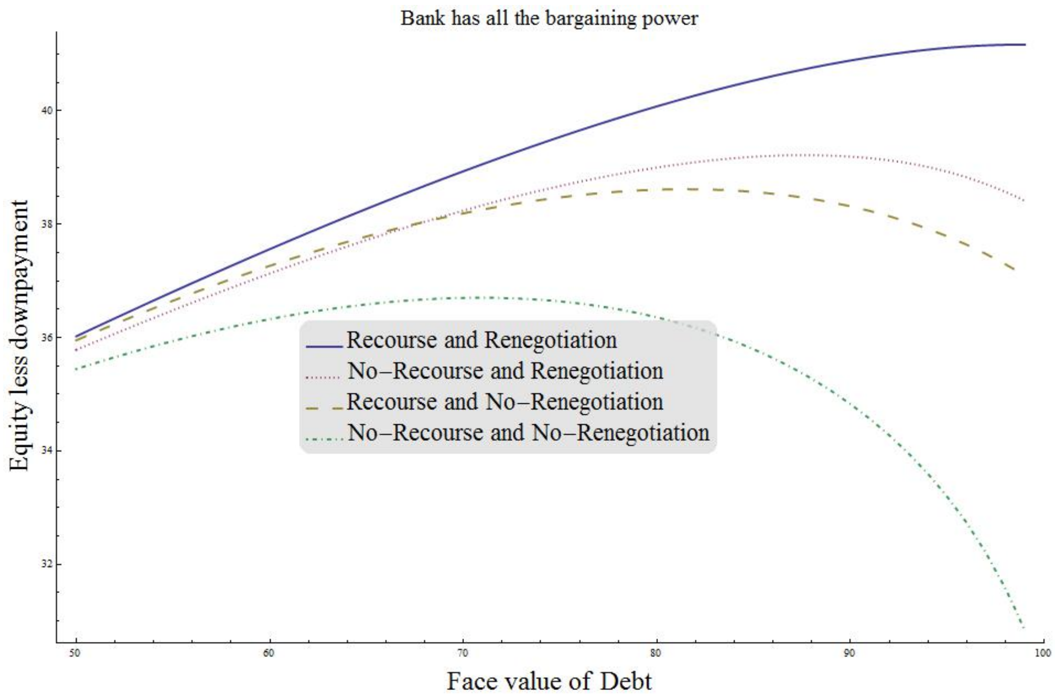
Observation 4.1.11. *When debt has all the bargaining power, contracts with one renegotiation are a welfare improvement on contracts with renegotiation for both recourse and non-recourse legal regimes. In contracts with renegotiation, the homeowner has a higher optimal debt capacity.*

In figure 4.8, we see an example which compares the value equity gains above the down-payment made by entering into a mortgage contract. We see that contracts without renegotiation and without recourse are at the bottom, while contracts with renegotiation and recourse have the highest equity valuation and also suggest the highest optimal debt level. We can see from 4.9 that the renegotiation feature leads to lower ex-ante mortgage rates and lower renegotiated coupons. Table 4.9 lists the equity valuation and mortgage rates in an example in the No-Recourse regime when bank has all the bargaining power. We find that equity is maximized at a debt level of around 95%. Table 4.8 reports the renegotiation price and other boundaries for the same contract. Table 4.9 on the other hand lists the equity values and equilibrium mortgage rates for the same set of paramters but in a recourse legal regime while table 4.8 looks at the boundaries for the same example. We see that contracts with renegotiation in a recourse regime lead to the highest equity valuation and higher debt capacity.

Observation 4.1.12. *When Equity has all the bargaining power, the homeowner defaults immediately to capture the surplus. These contracts violate the bank's individual rationality constraint and will not be offered in equilibrium. There exists a level of equity bargaining power above which contracts with renegotiation will not be offered.*

Figure 4.10 illustrates how extremely high ex-post bargaining power for equity is detrimental for the homeowner. In this example, if equity has bargaining power in excess of 85% he defaults immediately because he is better off doing that than

Figure 4.8: **Comparison of Equity across Recourse and Renegotiation when the bank has all the bargaining power**
This figure reports the value of equity less down-payment across contracts with or without renegotiation in recourse and no-recourse regimes as we vary the face value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Cost of Renegtiation to the Bank is 2 and to the Borrower is 1.

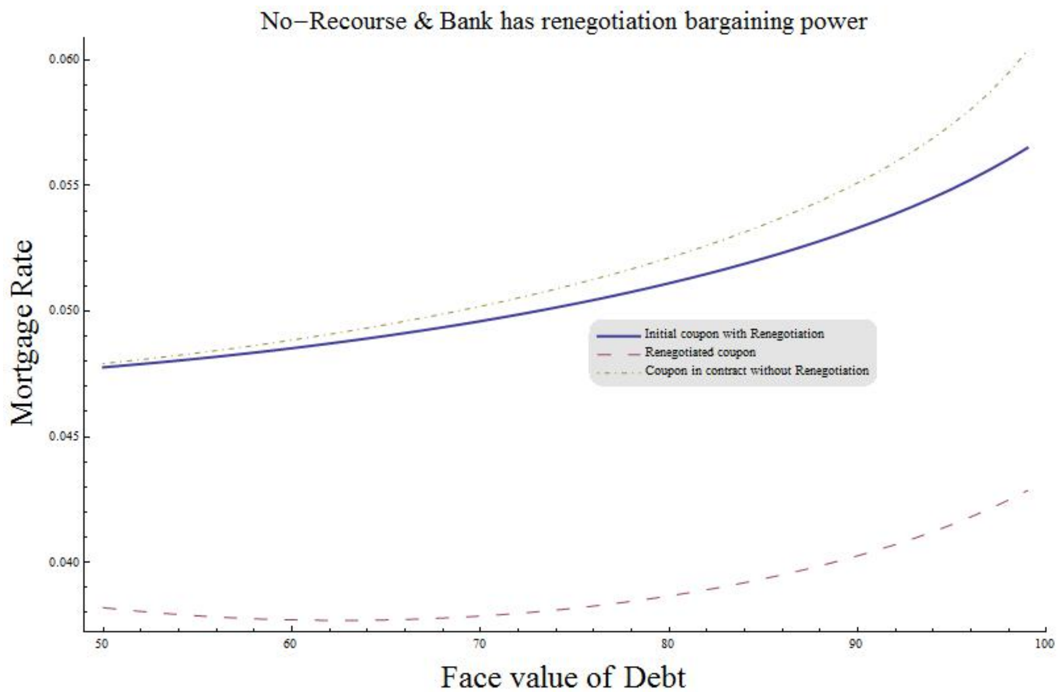


staying current on his loan. Such contracts violate the bank's individual rationality constraint. For slightly lower equity bargaining power, we find that most of the post-renegotiation surplus extracted by the homeonwer is given back to the bank via an increasing ex-ante mortgage rate, see table 4.5.

Observation 4.1.13. *There exists an optimal level of bargaining power. As equity's bargaining power increases from zero, his value increases but beyond a point, any further increase in bargaining power lower equity's valuation.*

We see that the higher the equity's bargaining power, the sooner he defaults.

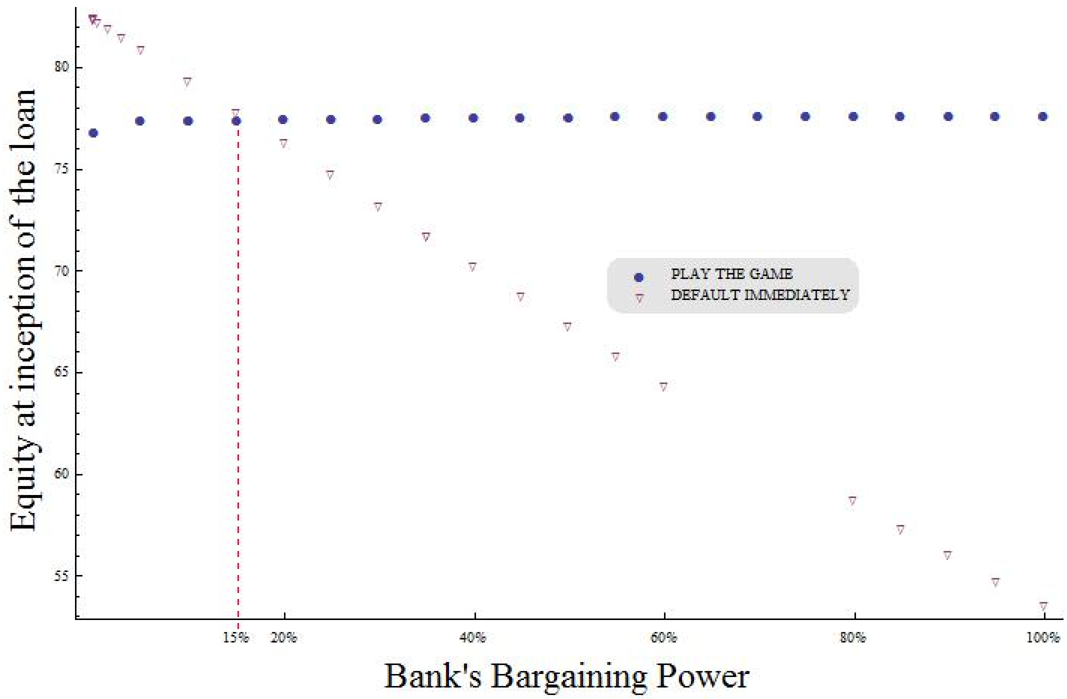
Figure 4.9: **Comparison of Mortgage rates in contracts with or without renegotiation when the bank has all the bargaining power in a No-Recourse regime**
This figure reports the initial and renegotiated mortgage rates in contracts with renegotiation, in comparison to mortgage rates in contracts without renegotiation, as we vary the face value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Cost of Renegtiation to the Bank is 2 and to the Borrower is 1.



Also in 4.5, we see that when equity has bargaining power of 20%, equity gets the highest valuation at 77.5809. Any further increase in equity's bargaining power is detrimental as it lowers his valuation. Any surplus extracted by equity is recaputred by debt via an increasing initial mortgage rate. Table 4.4 shows that with increasing bargaining power, the homeowner renegotiates sooner. The table also shows that the higher the homeowner bargaining power, the longer the homeowner stays in the renegotiated contract.

Figure 4.10: **Comparison of Equity value of immediate renegotiation versus staying current and renegotiating later as the bargaining power varies**

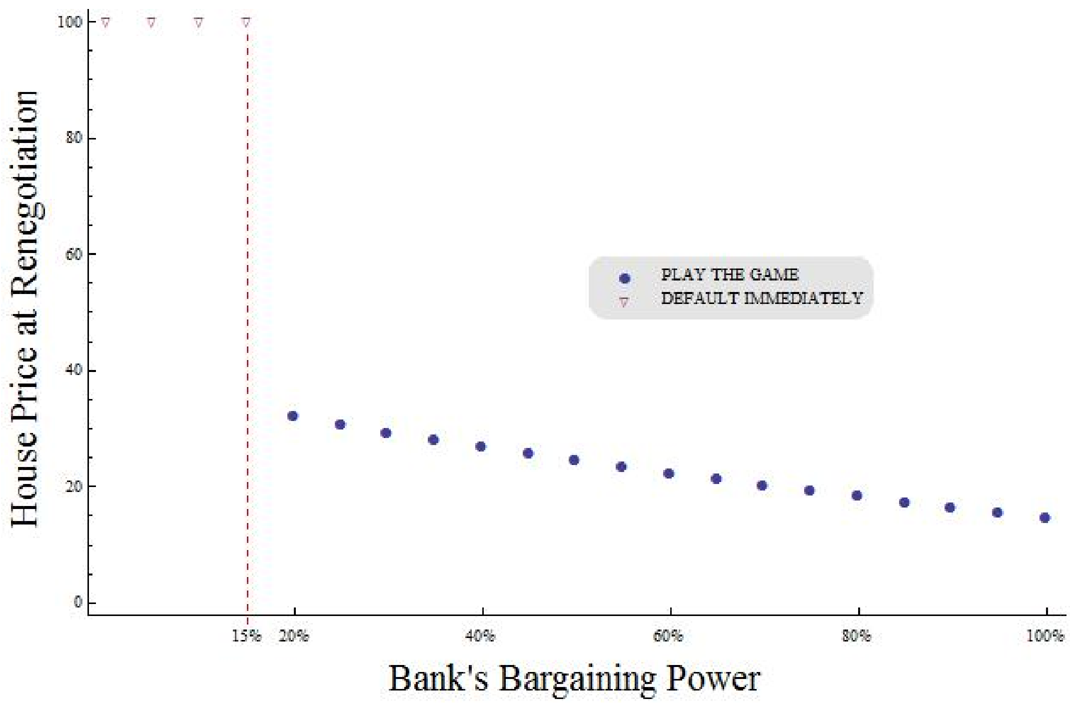
This figure reports the value of Equity for varying levels of bargaining power during renegotiation. The base parameters used here are as follows: Initial House price 100, face value of Debt f 60, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Cost of Renegtiation to the Bank is 2 and to the Borrower is 1.



4.2 Conclusion

In this paper, we model strategic default and renegotiation in residential mortgage contracts and study how the feature of recourse alleviates some of the incentive problems associated with debt and renegotiation. Our model is a sequential game in continuous time and our equilibrium concept is a sub-game perfect Nash equilibrium. We find that in the absence of recourse, the homeowner's default decision is independent of his personal wealth. In the absence of recourse, stochastic income

Figure 4.11: **Renegotiation price as the bargaining power varies**
This figure reports how the optimal renegotiation price varying with bargaining power during renegotiation. The base parameters used here are as follows: Initial House price 100, face value of Debt f 60, risk free rate 4.7%, subjective discount rate $\rho:$ 5.7%, house price volatility $\sigma:$ 15% and house price dividend rate $\delta:$ 5%. Cost of Renegtiation to the Bank is 2 and to the Borrower is 1.



does not influence the homeowner's default and sale decisions. In the presence of recourse, when the homeowner has stochastic income, we find that the homeowner defaults later when his income is higher. Less intuitively, we see that the homeowner sells later when his income is lower. Income uncertainty reduces the payoff that the banks would receive in the presence of recourse and leads to lower welfare relative to a constant income recourse case but higher welfare relative to that in the no-recourse regime. We also find that in regimes with the provision of recourse the homeowners stay longer in the homes before defaulting. We show that under recourse there are lower default rates, lower coupon rates, higher debt capacity and increased welfare.

Interestingly, we see that since recourse allows for a higher level of optimal leverage, it also winds up having a higher probability of default and foreclosure. We also show that contracts without renegotiation offer a welfare improvement if the homeowner does not have excessive ex-post bargaining power during renegotiation. Both the homeowner and the bank have an incentive to save on the dead-weight loss associated with foreclosure, however all the surplus that the homeowner extracts ex-post is priced into the ex-ante mortgage rate further propelling the homeowner to default sooner. In our model, both recourse and renegotiation with a balance of bargaining power, serve to reduce the moral hazard associated with strategic default. The two provisions are complimentary and work to increase ex-ante welfare, lower mortgage rates and increase debt capacity. Homeowner bargaining power is an important determinant of the feasibility of a mortgage modification program and subsequent default rates. A legal regime that enables lender recourse reduces homeowner bargaining power during a renegotiation. Lender recourse removes the incentives of the homeowner to engage in foreclosure stripping and increases the reservation value of the bank in the renegotiation process. Our paper provides a parsimonious framework for analyzing the interaction of lender recourse with strategic default and mortgage renegotiation. This model can be extended to include re-financing of mortgages and stochastic interest rates that follow a continuous time Markov chain.

Table 4.2: **Base case parameter values for the numerical results when income is stochastic**
This table reports the parameter values used in the base case for our numerical results when the homeowner's income is stochastic.

Name	Symbol	Value
House Price	H_0	100
Face value of Debt	F	80
Equity's Foreclosure Cost	CF	3
Debt's Foreclosure cost	CFL	9
Equity's Renegotiation Cost	CR	1
Debt's Renegotiation cost	CRL	3
Risk Free rate	r	0.047
Equity Subjective Discount rate	ρ	0.057
Home dividend rate	δ	0.05
House Price volatility	σ	0.15
High Income accrual rate	i_e	0.6
High Income present value	I_e	9.84279
Low Income accrual rate	i_u	0
Low Income present value	I_u	2.05058
High Income intensity rate	λ_e	0.005
Low Income intensity rate	λ_u	0.015
First characteristic root (jump into interior)	k_1	-1.70499
Second characteristic root (jump into interior)	k_2	2.97166
Third characteristic root (jump into interior)	k_3	-2.05842
Fourth characteristic root (jump into interior)	k_4	3.32509
First characteristic root (jump into default)	m_1	-1.79817
Second characteristic root (jump into default)	m_2	3.06484
First characteristic root (jump into sale)	m_1^*	-1.97456
Second characteristic root (jump into sale)	m_2^*	3.24123
Psychic Utility parameter	p	0.15

Table 4.3: **Free boundaries under Recourse as initial loan-to-value varies when income is stochastic**

This table reports the renegotiation and sale boundaries at the high and low income levels for varying degrees of initial Loan-to-value. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate $\rho:$ 5.7%, house price volatility $\sigma:$ 15% and house price dividend rate $\delta:$ 5%.

Debt	Default boundary High Income	Default boundary Low Income	Sale boundary High Income	Sale boundary Low Income
99.	36.5035	42.9219	416.9	418.032
95.	32.6204	38.9868	432.598	433.449
90.	28.2622	34.5552	446.016	446.629
85.	24.3127	30.5209	454.421	454.868
82.	22.1005	28.251	457.595	457.966
80.	20.683	26.7917	459.055	459.382
75.	17.32	23.31	460.731	460.968
70.	14.1926	20.0382	460.022	460.189
65.	11.2863	16.9518	457.35	457.464
55.	6.15955	11.2879	447.37	447.415
50.	4.00434	8.71122	440.565	440.589
45.	2.22164	6.32781	432.852	432.864

Table 4.4: **Boundaries in Contracts with Renegotiation in a Recourse regime as the bargaining power varies**

This table reports the renegotiation and sale boundaries of the initial contract and the default and sale boundaries of the renegotiated contract for varying degrees of bargaining power. The base parameters used here are as follows: Initial House price 100, face value of debt F is 60, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Cost of Renegtiation to the Bank is 2 and to the Borrower is 1.

Bank Bargaining Power	Initial Contract Sale Price	Renegotiation Price	Renegotiated Contract Sale Price	Renegotiated Contract Default Price
100.%	455.621	14.7663	535.783	4.59363
95.%	455.243	15.6349	541.648	4.31696
90.%	454.775	16.5252	547.345	4.05327
85.%	454.214	17.4372	552.879	3.80221
80.%	453.556	18.3714	558.252	3.56355
75.%	452.796	19.3289	563.463	3.33719
70.%	451.93	20.3112	568.508	3.12305
65.%	450.95	21.3202	573.383	2.9211
60.%	449.851	22.3582	578.082	2.73133
55.%	448.623	23.4278	582.596	2.55372
50.%	447.257	24.5322	586.917	2.38824
45.%	445.742	25.6745	591.036	2.23481
40.%	444.065	26.8585	594.944	2.09329
35.%	442.212	28.0875	598.634	1.96343
30.%	440.166	29.3644	602.106	1.84466
25.%	437.908	30.691	605.372	1.7361
20.%	435.419	32.0668	608.457	1.63642
15.%	N/A	100	469.846	7.98444
10.%	N/A	100	485.83	7.12378
5.%	N/A	100	501.764	6.28751
0.%	N/A	100	517.649	5.47923

Table 4.5: **Renegotiation price and equilibrium coupon rates in Contracts with Renegotiation in a Recourse regime as the bargaining power varies** This table reports the renegotiation price and the equilibrium coupon rates of the initial contract and the renegotiated contract. The base parameters used here are as follows: Initial House price 100, face value of debt F is 60, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Cost of Renegtiation to the Bank is 2 and to the Borrower is 1.

Debt Bargaining Power	Equity Value	Debt Value	Renegotiation Price	Initial Coupon	Renegotiated Coupon
100%	77.5624	60.	14.7663	0.0475407	0.0400334
95%	77.5705	60.	15.6349	0.0475814	0.0394812
90%	77.5763	60.	16.5252	0.0476312	0.0389443
85%	77.5797	60.	17.4372	0.0476905	0.0384223
80%	77.5809	60.	18.3714	0.0477598	0.037915
75%	77.5799	60.	19.3289	0.0478394	0.0374227
70%	77.5767	60.	20.3112	0.0479301	0.0369455
65%	77.5715	60.	21.3202	0.0480325	0.0364841
60%	77.5643	60.	22.3582	0.0481472	0.0360391
55%	77.5549	60.	23.4278	0.0482754	0.0356113
50%	77.5435	60.	24.5322	0.0484179	0.0352014
45%	77.5299	60.	25.6745	0.0485761	0.0348105
40%	77.5142	60.	26.8585	0.0487512	0.0344394
35%	77.4963	60.	28.0875	0.048945	0.0340888
30%	77.4761	60.	29.3644	0.0491591	0.0337587
25%	77.4535	60.	30.691	0.0493958	0.033448
20%	77.4286	60.	32.0668	0.0496573	0.0331545
15%	77.7526	56.3156	100	N/A	0.0462
10%	79.2844	54.5632	100	N/A	0.0447123
5%	80.8212	52.7909	100	N/A	0.0432246
0%	82.3623	51.	100	N/A	0.0417369

Table 4.6: **Boundaries in Contracts with Renegotiation in a Recourse regime when the Bank has all the bargaining power**

This table reports the renegotiation and sale boundaries of the initial contract and the default and sale boundaries of the renegotiated contract for varying levels of face-value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Cost of Renegtiation to the Bank is 2 and to the Borrower is 1.

Debt	Initial Sale Price	Renegotiation Price	Renegotiated Contract Sale Price	Renegotiated Contract Default Price
50.	441.643	8.34434	493.957	2.09705
55.	449.074	11.5576	514.888	3.25618
60.	455.621	14.7662	535.784	4.59359
65.	461.11	17.9659	556.065	6.09436
70.	465.342	21.1805	575.287	7.75608
75.	468.077	24.4427	593.07	9.58479
80.	469.019	27.7889	609.037	11.5937
85.	467.781	31.2601	622.769	13.8037
90.	463.847	34.9055	633.75	16.2457
95.	456.488	38.789	641.277	18.9659
99.	447.426	42.1273	644.131	21.3896

Table 4.7: **Equity values and mortgage rates in Contracts with Renegotiation in a Recourse regime when the Bank has all the bargaining power** This table reports the Equity value, Debt value and coupon rates for the initial and the renegotiated contracts as we vary the levels of face-value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Cost of Renegtiation to the Bank is 2 and to the Borrower is 1.

Debt	Equity	Equity less down-payment	Equity at Renegotiation	Debt at Renegotiation	Initial Coupon	Renegotiated Coupon
50.	86.0237	36.0237	3.13563	38.0825	0.0471666	0.0413392
55.	81.8106	36.8106	4.17491	40.534	0.0473237	0.0406293
60.	77.5624	37.5624	5.004	42.8692	0.0475407	0.0400333
65.	73.2734	38.2734	5.6585	45.1673	0.0478201	0.039576
70.	68.9366	38.9366	6.1717	47.4841	0.0481659	0.0392648
75.	64.543	39.543	6.56998	49.8632	0.0485845	0.0391008
80.	60.0814	40.0814	6.8734	52.3434	0.0490852	0.0390846
85.	55.5371	40.5371	7.09705	54.9637	0.0496817	0.0392195
90.	50.8903	40.8903	7.25215	57.7688	0.0503943	0.0395149
95.	46.1129	41.1129	7.34679	60.816	0.0512537	0.0399893
99.	42.1691	41.1691	7.38263	63.4816	0.0520787	0.0405202

Table 4.8: **Boundaries in Contracts with Renegotiation in a No-Recourse regime when the Bank has all the bargaining power**
This table reports the renegotiation and sale boundaries of the initial contract and the default and sale boundaries of the renegotiated contract for varying levels of face-value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate ρ : 5.7%, house price volatility σ : 15% and house price dividend rate δ : 5%. Cost of Renegtiation to the Bank is 2 and to the Borrower is 1.

Debt	Initial Sale Price	Renegotiation Price	Renegotiated Contract Sale Price	Renegotiated Contract Default Price
45.	430.292	12.5153	501.365	3.77238
50.	436.607	15.6707	522.049	5.18189
55.	441.824	18.8306	541.919	6.75349
60.	445.738	22.0216	560.571	8.48787
65.	448.099	25.2763	577.64	10.3932
70.	448.593	28.6309	592.753	12.4846
75.	446.811	32.1268	605.48	14.7851
80.	442.201	35.8152	615.273	17.3287
85.	433.975	39.7641	621.367	20.1669
90.	420.944	44.0747	622.6	23.3809
95.	401.117	48.917	616.969	27.1137
99.	377.875	53.3949	604.895	30.6626

Table 4.9: **Equity values and mortgage rates in Contracts with Renegotiation in a No-Recourse regime when the Bank has all the bargaining power**

This table reports the Equity value, Debt value and coupon rates for the initial and the renegotiated contracts as we vary the levels of face-value of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate $\rho:\ 5.7\%$, house price volatility $\sigma:\ 15\%$ and house price dividend rate $\delta:\ 5\%$. Cost of Renegtiation to the Bank is 2 and to the Borrower is 1.

Debt	Equity	Equity less down-payment	Equity at Renegotiation	Debt at Renegotiation	Initial Coupon	Renegotiated Coupon
45.	90.0453	35.0453	14.021	29.5635	0.0474861	0.0386785
50.	85.7855	35.7855	14.7376	31.8907	0.0477634	0.0381888
55.	81.4838	36.4838	15.299	34.2058	0.0481078	0.0378639
60.	77.1328	37.1328	15.735	36.5582	0.0485236	0.0377025
65.	72.723	37.723	16.0686	38.9884	0.049018	0.0377002
70.	68.2426	38.2426	16.3174	41.5337	0.0496021	0.0378551
75.	63.6761	38.6761	16.4946	44.2335	0.0502922	0.0381702
80.	59.0021	39.0021	16.6099	47.1344	0.0511125	0.0386564
85.	54.1902	39.1902	16.6703	50.298	0.0521002	0.0393369
90.	49.1929	39.1929	16.6801	53.8155	0.0533156	0.0402558
95.	43.9276	38.9276	16.6402	57.84	0.0548682	0.0414996
99.	39.4123	38.4123	16.5698	61.6239	0.0565035	0.0428516

Table 4.10: **Comparison of default boundaries across regimes for optimal debt**

This table reports the default values across recourse and no-recourse regimes and for constant and stochastic income for optimal level of debt. The base parameters used here are as follows: Initial House price 100, risk free rate 4.7%, subjective discount rate $\rho:\ 5.7\%$, house price volatility $\sigma:\ 15\%$ and house price dividend rate $\delta:\ 5\%$. High income is $I_e = 9.84$ and low income is I_u=2.05.

	Optimal Debt	Default boundary
Constant Income No Recourse	70	22.1136
Constant Income Recourse	82	22.27
Stochastic Income Recourse (High)	80	20.683
Stochastic Income Recourse (Low)	80	26.7917

Chapter A: Stochastic Labor Income

A.1 Stochastic Labor Income

A.1.1 Derivation of the Value Functions

We now derive the functional forms that the value of Debt and Equity will take. We'll use the dynamic programming recursive relation from the principal of optimality to derive the Ordinary Differential Equations that debt and equity satisfy. Once we have the differential equations, we'll solve them using the method of variation of parameters to arrive at the functional forms up to some constants. Then we will use the various constraints that we had listed earlier to solve for the unknown coefficients and also the unknown boundary points.

Theorem A.1.1. *The Value functions J^e and J^u satisfy the following system of linked ODEs.*

$$0 = i_e - c_1 F + \psi(H_t) + \delta H_t + (r-\delta)H_t \frac{\partial J^e}{\partial H} + \frac{1}{2}\sigma^2 H_t^2 \frac{\partial^2 J^e}{\partial H^2} - (\lambda_e + \rho)J^e(H_t) + J^u(H_t)\lambda_e$$
$$0 = i_u - c_1 F + \psi(H_t) + \delta H_t + (r-\delta)H_t \frac{\partial J^u}{\partial H} + \frac{1}{2}\sigma^2 H_t^2 \frac{\partial^2 J^u}{\partial H^2} - (\lambda_u + \rho)J^u(H_t) + J^e(H_t)\lambda_u$$

While J^S and J^D satisfy the following ODEs.

$$J^D_{HH} + \frac{2(r-\delta)}{\sigma^2 H_t} J^D_H - \frac{2(\lambda_e + \rho)}{\sigma^2 H^2} J^D = -\frac{2(i_e - c_1 F - CF\lambda_e)}{\sigma^2 H^2} - \frac{2\psi(H_t)}{\sigma^2 H^2} - \frac{2\delta}{\sigma^2 H_t}$$

$$J^S_{HH} + \frac{2(r-\delta)}{\sigma^2 H} J^S_H - \frac{2(\lambda_u + \rho)}{\sigma^2 H^2} J^S = -\frac{2(i_u + \lambda_u I_e - (c_1 + \lambda_u)F)}{\sigma^2 H^2} - \frac{2\psi(H_t)}{\sigma^2 H^2} - \frac{2(\delta + \lambda_u)}{\sigma^2 H_t}$$

Proof.

$$\begin{aligned} J^e(H_t) = \max\Bigg\{ & \underbrace{B(H_t)}_{\text{exit contract}}, E_t\Bigg[\underbrace{e^{-\lambda_e h}\left(\int_0^h e^{-\rho s}(i_e + \psi(H) + \delta H_{t+s} - c_1 F)ds + e^{-\rho h} J^e(H_{t+h}) \right)}_{\text{stay current and income doesn't switch}} \\ & + \underbrace{\int_0^h \left(\int_0^\tau e^{-\rho s}(i_e + \psi(H) + \delta H_{t+s} - c_1 F)ds + e^{-\rho\tau} J^u(H_{t+\tau}) \right) \lambda_e e^{-\lambda_e \tau} d\tau}_{\text{stay current and income switches}} \Bigg] \Bigg\} \end{aligned}$$

$$\begin{aligned} J^u(H_t) = \max\Bigg\{ & \underbrace{B(H_t)}_{\text{exit contract}}, E_t\Bigg[\underbrace{e^{-\lambda_u h}\left(\int_0^h e^{-\rho s}(i_u + \psi(H) + \delta H_{t+s} - c_1 F)ds + e^{-\rho h} J^u(H_{t+h}) \right)}_{\text{stay current and income doesn't switch}} \\ & + \underbrace{\int_0^h \left(\int_0^\tau e^{-\rho s}(i_u + \psi(H) + \delta H_{t+s} - c_1 F)ds + e^{-\rho\tau} J^e(H_{t+\tau}) \right) \lambda_u e^{-\lambda_u \tau} d\tau}_{\text{stay current and income switches}} \Bigg] \Bigg\} \end{aligned}$$

$$
\begin{aligned}
&J^e(H_t)\\
=&E_t\Bigg[e^{-\lambda_e h}\Bigg(\int_0^h e^{-\rho s}(i_e+\psi(H)+\delta H_{t+s}-c_1F)ds+e^{-\rho h}J^e(H_{t+h})\Bigg)\\
&+\int_0^h\Bigg(\int_0^\tau e^{-\rho s}(i_e+\psi(H)+\delta H_{t+s}-c_1F)ds+e^{-\rho\tau}J^u(H_{t+\tau})\Bigg)\lambda_e e^{-\lambda_e\tau}d\tau\Bigg]\\
=&E_t\Bigg[e^{-\lambda_e h}\Bigg(\int_0^h e^{-\rho s}(i_e+\psi(H)+\delta H_{t+s}-c_1F)ds\Bigg)\Bigg]\\
&+E_t\Bigg[e^{-\lambda_e h}\Bigg(e^{-\rho h}J^e(H_{t+h})\Bigg)\Bigg]\\
&+E_t\Bigg[\int_0^h\Bigg(\int_0^\tau e^{-\rho s}(i_e+\psi(H)+\delta H_{t+s}-c_1F)ds\Bigg)\lambda_e e^{-\lambda_e\tau}d\tau\Bigg]\\
&+E_t\Bigg[\int_0^h e^{-\rho\tau}J^u(H_{t+\tau})\lambda_e e^{-\lambda_e\tau}d\tau\Bigg]\\
=&E_t\Bigg[e^{-\lambda_e h}e^{-\rho\theta_1 h}\Bigg(i_e+\psi(H_{t+\theta_1 h})+\delta H_{t+\theta_1 h}-c_1F\Bigg)h\Bigg]\\
&+E_t\Bigg[e^{-(\lambda_e+\rho)h}J^e(H_{t+h})\Bigg]\\
&+E_t\Bigg[\int_0^h e^{-\rho\theta_2\tau}\Bigg(i_e+\psi(H_{t+\theta_2\tau})+\delta H_{t+\theta_2\tau}-c_1F\Bigg)\tau\lambda_e e^{-\lambda_e\tau}d\tau\Bigg]\\
&+E_t\Bigg[\int_0^h e^{-(\rho+\lambda_e)\tau}J^u(H_{t+\tau})\lambda_e d\tau\Bigg]\\
=&E_t\Bigg[\Bigg(i_e+\psi(H_{t+\theta_1 h})+\delta H_{t+\theta_1 h}-c_1F\Bigg)h\Bigg]+O(h^2)\\
&+E_t\Bigg[\Bigg(1-(\lambda_e+\rho)h\Bigg)J^e(H_{t+h})\Bigg]+O(h^2)\\
&+E_t\Bigg[e^{-\rho\theta_2\theta_3 h}\Bigg(i_e+\psi(H_{t+\theta_2\theta_3 h})+\delta H_{t+\theta_2\theta_3 h}-c_1F\Bigg)\theta_3 h\lambda_e e^{-\lambda_e\theta_3 h}h\Bigg]\\
&+E_t\Bigg[e^{-(\rho+\lambda_e)\theta_4 h}J^u(H_{t+\theta_4 h})\lambda_e h\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
=&\Bigg(i_e - c_1F + E_t\Bigg[\psi(H_{t+\theta_1 h})\Bigg] + E_t\Bigg[\delta H_{t+\theta_1 h}\Bigg]\Bigg)h + O(h^2)\\
&+ E_t\Bigg[\Bigg(1-(\lambda_e+\rho)h\Bigg)J^e(H_{t+h})\Bigg] + O(h^2)\\
&+ O(h^2)\\
&+ E_t\Bigg[J^u(H_{t+\theta_4 h})\lambda_e h\Bigg] + O(h^2)\\
=&\Bigg(i_e - c_1F + \psi(H_t) + \left((r-\delta)H_t\frac{\partial\psi}{\partial H} + \frac{1}{2}\sigma^2H_t^2\frac{\partial^2\psi}{\partial H^2}\right)h\\
&+ \delta H_t + \delta(((r-\delta))\,h)\Bigg)h + O(h^2)\\
&+ E_t\Bigg[\Bigg(1-(\lambda_e+\rho)h\Bigg)J^e(H_{t+h})\Bigg] + O(h^2)\\
&+ O(h^2)\\
&+ \Bigg(J^u(H_t) + \left((r-\delta)H_t\frac{\partial J^u}{\partial H} + \frac{1}{2}\sigma^2H_t^2\frac{\partial^2 J^u}{\partial H^2}\right)h\Bigg)\lambda_e h + O(h^2)
\end{aligned}
$$

$$J^e(H_t) = \left(i_e - c_1F + \psi(H_t) + \delta H_t\right)h + E_t\left[\left(1 - (\lambda_e + \rho)h\right)J^e(H_{t+h})\right]$$

$$+ J^u(H_t)\lambda_e h + O(h^2)$$

$$0 = \left(i_e - c_1F + \psi(H_t) + \delta H_t\right)h + E_t\left[J^e(H_{t+h}) - J^e(H_t)\right] - (\lambda_e + \rho)hE_t\left[J^e(H_{t+h})\right]$$

$$+ J^u(H_t)\lambda_e h + O(h^2)$$

$$0 = \left(i_e - c_1F + \psi(H_t) + \delta H_t\right)h + \left((r - \delta)H_t\frac{\partial J^e}{\partial H} + \frac{1}{2}\sigma^2 H_t^2\frac{\partial^2 J^e}{\partial H^2}\right)h$$

$$- (\lambda_e + \rho)h\left(J^e(H_t) + \left((r - \delta)H_t\frac{\partial J^e}{\partial H} + \frac{1}{2}\sigma^2 H_t^2\frac{\partial^2 J^e}{\partial H^2}\right)h\right)$$

$$+ J^u(H_t)\lambda_e h + O(h^2)$$

We divide by h and then take $lim_{h\to 0}$ to get:

$$0 =$$

$$i_e - c_1F + \psi(H_t) + \delta H_t + (r - \delta)H_t\frac{\partial J^e}{\partial H} + \frac{1}{2}\sigma^2 H_t^2\frac{\partial^2 J^e}{\partial H^2} - (\lambda_e + \rho)J^e(H_t) + J^u(H_t)\lambda_e$$

Now all that changes in the regions where only one income level exists is the $J^u(H_t)$ becomes a different function, a boundary payoff.

So when you jump into default:

$$0 = i_e - c_1 F + \psi(H_t) + \delta H_t + (r-\delta) H_t \frac{\partial J^D}{\partial H} + \frac{1}{2}\sigma^2 H_t^2 \frac{\partial^2 J^D}{\partial H^2} - (\lambda_e + \rho) J^D(H_t) - CF\lambda_e$$

$$0 = i_e - c_1 F - CF\lambda_e + \psi(H_t) + \delta H_t + (r-\delta) H_t J_H^D + \frac{1}{2}\sigma^2 H_t^2 J_{HH}^D - (\lambda_e + \rho) J^D$$

$$J_{HH}^D + \frac{2(r-\delta)}{\sigma^2 H_t} J_H^D - \frac{2(\lambda_e + \rho)}{\sigma^2 H^2} J^D = -\frac{2(i_e - c_1 F - CF\lambda_e)}{\sigma^2 H^2} - \frac{2\psi(H_t)}{\sigma^2 H^2} - \frac{2\delta}{\sigma^2 H_t}$$

So when you jump into Sale:

$$0 = i_u - c_1 F + \psi(H_t) + \delta H_t + (r-\delta) H_t \frac{\partial J^S}{\partial H} + \frac{1}{2}\sigma^2 H_t^2 \frac{\partial^2 J^S}{\partial H^2} - (\lambda_u + \rho) J^S + (H_t + I_e - F)\lambda_u$$

$$0 = i_u + \lambda_u I_e - (c_1 + \lambda_u) F + \psi(H_t) + (\delta + \lambda_u) H_t + (r-\delta) H_t J_H^S + \frac{1}{2}\sigma^2 H_t^2 J_{HH}^S - (\lambda_u + \rho) J^S$$

$$J_{HH}^S + \frac{2(r-\delta)}{\sigma^2 H} J_H^S - \frac{2(\lambda_u + \rho)}{\sigma^2 H^2} J^S = -\frac{2(i_u + \lambda_u I_e - (c_1 + \lambda_u) F)}{\sigma^2 H^2} - \frac{2\psi(H_t)}{\sigma^2 H^2} - \frac{2(\delta + \lambda_u)}{\sigma^2 H_t}$$

□

Similarly, debt satisfies the following ordinary differential equations.

Theorem A.1.2. *The Value functions V^e andV^u satisfy the following system of linked ODEs.*

$$0 = cF + (r-\delta) H_t \frac{\partial V^e}{\partial H} + \frac{1}{2}\sigma^2 H_t^2 \frac{\partial^2 V^e}{\partial H^2} - (\lambda_e + \rho) V^e(H_t) + V^u(H_t)\lambda_e$$

$$0 = cF + (r-\delta) H_t \frac{\partial V^u}{\partial H} + \frac{1}{2}\sigma^2 H_t^2 \frac{\partial^2 V^u}{\partial H^2} - (\lambda_u + \rho) V^u(H_t) + V^e(H_t)\lambda_u$$

While V^S and V^D satisfy the following ODEs.

$$V^D_{HH} + \frac{2(r-\delta)}{\sigma^2 H_t} V^D_H - \frac{2(\lambda_e + \rho)}{\sigma^2 H^2} V^D = -\frac{\lambda_e(H + Iu - CFL) + cF}{\sigma^2 H^2}$$
$$J^S_{HH} + \frac{2(r-\delta)}{\sigma^2 H} J^S_H - \frac{2(\lambda_u + \rho)}{\sigma^2 H^2} J^S = -\frac{\lambda_u F + cF}{\sigma^2 H^2}$$

Using the method of variation of parameters we solve these ODEs. For Equity, the value functions consists of five separate components. The components are the value of the house, the present value of gaining a lifetime of psychic utility from ownership, the present value of future income, the present value of the debt if paid in perpetuity and finally the option to default on the debt and the option to sell the house. Debt value function has three components. The first is the present value of receiving debt payments in perpetuity and then second and third are the short positions in the two options that the homeowner holds.

Theorem A.1.3. *Equity has the following functional form. The H^k terms represent the options to sell and default. In addition to the option, Equity valuation consists of Income, Psychic utility of ownership, private valuation of the house and the private* *valuation of the debt burden.*

$$J^e = \sum_{j=1}^{4} A_j H^{k_j} + I_e - \frac{cF}{\rho} + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H}{(\rho + \delta - r)}$$

$$J^u = \sum_{j=1}^{2} A_j H^{k_j} - \frac{\lambda_u}{\lambda_e} \sum_{j=3}^{4} A_j H^{k_j} + I_u - \frac{cF}{\rho} + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H}{(\rho + \delta - r)}$$

$$J^S = A_{S1} H^{m_1^*} + A_{S2} H^{m_2^*} + \frac{(i_u + \lambda_u I_e - (c + \lambda_u)F)}{(\lambda_u + \rho)} + \frac{H^p}{((\lambda_u + \rho) - \frac{1}{2}p(p-1)\sigma^2 - (r - \delta)p)}$$
$$+ \frac{(\delta + \lambda_u)H}{((\lambda_u + \rho) - r + \delta)}$$

$$J^D = A_{D1} H^{m_1} + A_{D2} H^{m_2} + \frac{(i_e - \lambda_e CF - cF)}{(\lambda_e + \rho)} + \frac{H^p}{((\lambda_e + \rho) - \frac{1}{2}p(p-1)\sigma^2 - (r - \delta)p)}$$
$$+ \frac{\delta H}{((\lambda_e + \rho) - r + \delta)}$$

$$V^e = \sum_{j=1}^{4} L_j H^{k_j} + \frac{cF}{r}$$

$$V^u = \sum_{j=1}^{2} L_j H^{k_j} - \frac{\lambda_u}{\lambda_e} \sum_{j=3}^{4} L_j H^{k_j} + \frac{cF}{r}$$

$$V^S = L_{S1} H^{m_1^*} + L_{S2} H^{m_2^*} + \frac{(c + \lambda_u)F}{(\lambda_u + r)}$$

$$V^D = L_{D1} H^{m_1} + L_{D2} H^{m_2} + \frac{cF - \lambda_e CFL + \lambda_e I_u}{(\lambda_e + r)} + \frac{\lambda_e H}{\lambda_e + \delta}$$

where k_1, k_2 *solve the first equation and* k_3, k_4 *solve the second equation*

$$k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2} = 0$$
$$k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2} = \frac{2(\lambda_e + \lambda_u)}{\sigma^2}$$

m_1, m_2 satisfy the following characteristic equation

$$m(m-1) + \frac{2(r-\delta)}{\sigma^2}m - \frac{2(\lambda_e+\rho)}{\sigma^2} = 0$$

and m_1^*, m_2^* satisfy the following characteristic equations

$$m(m-1) + \frac{2(r-\delta)}{\sigma^2}m - \frac{2(\lambda_u+\rho)}{\sigma^2} = 0$$

Proof. We'll first solve the single ODEs and we'll use the method of variation of parameters to solve our ODEs. We'll solve in detail for the "Jump to Sale" ODE and then apply the results to the "Jump to Default" ODE which has identical homogeneous part and differs only in the particular solution.

The homogeneous equation is

$$J^S_{HH} + \frac{2(r-\delta)}{\sigma^2 H}J^S_H - \frac{2(\lambda_e+\rho)}{\sigma^2 H^2}J^S = 0$$

This is a second order Euler equation and since $\left(1 - \frac{2(r-\delta)}{\sigma^2}\right)^2 > -\frac{2(\lambda_e+\rho)}{\sigma^2}$, the general solution to this ODE looks like

$$J_h = A_1 H^{m_1} + A_2 H^{m_2}$$

Where m_1, m_2 satisfy the following characteristic equation

$$m(m-1) + \frac{2(r-\delta)}{\sigma^2}m - \frac{2(\lambda_e+\rho)}{\sigma^2} = 0$$

Let $J_p = C_1(H)H^{m_1} + C_2(H)H^{m_2}$ be the particular solution to our ODE. Then J_p satisfies the following equations:

$$\begin{aligned}
C_1' H^{m_1} + C_2' H^{m_2} &= 0 \\
m_1 C_1' H^{m_1-1} + m_2 C_2' H^{m_2-1} &= -\frac{2(i_e + \lambda_e I_u - (c+\lambda_e)F)}{\sigma^2 H^2} - \frac{2\psi(H_t)}{\sigma^2 H^2} - \frac{2(\delta+\lambda_e)}{\sigma^2 H_t}
\end{aligned}$$

$$\begin{aligned}
C_1' H^{m_1-1} &= -C_2' H^{m_2-1} \\
(m_2 - m_1) C_2' H^{m_2-1} &= -\frac{2(i_e + \lambda_e I_u - (c+\lambda_e)F)}{\sigma^2 H^2} - \frac{2\psi(H_t)}{\sigma^2 H^2} - \frac{2(\delta+\lambda_e)}{\sigma^2 H_t} \\
C_2' &= -\frac{2(i_e + \lambda_e I_u - (c+\lambda_e)F)H^{-1-m_2}}{(m_2-m_1)\sigma^2} - \frac{2\psi(H_t)H^{-1-m_2}}{(m_2-m_1)\sigma^2} - \frac{2(\delta+\lambda_e)H^{-m_2}}{(m_2-m_1)\sigma^2} \\
C_1' &= +\frac{2(i_e + \lambda_e I_u - (c+\lambda_e)F)H^{-1-m_1}}{(m_2-m_1)\sigma^2} + \frac{2\psi(H_t)H^{-1-m_1}}{(m_2-m_1)\sigma^2} + \frac{2(\delta+\lambda_e)H^{-m_1}}{(m_2-m_1)\sigma^2}
\end{aligned}$$

Taking $\psi(H) = H^p$ and integrating, we get:

$$\begin{aligned}
C_2 &= -\frac{2(i_e + \lambda_e I_u - (c+\lambda_e)F)H^{-m_2}}{(-m_2)(m_2-m_1)\sigma^2} - \frac{2H^{p-m_2}}{(p-m_2)(m_2-m_1)\sigma^2} - \frac{2(\delta+\lambda_e)H^{-m_2+1}}{(-m_2+1)(m_2-m_1)\sigma^2} \\
C_1 &= +\frac{2(i_e + \lambda_e I_u - (c+\lambda_e)F)H^{-m_1}}{(-m_1)(m_2-m_1)\sigma^2} + \frac{2H^{p-m_1}}{(p-m_1)(m_2-m_1)\sigma^2} + \frac{2(\delta+\lambda_e)H^{-m_1+1}}{(-m_1+1)(m_2-m_1)\sigma^2}
\end{aligned}$$

So

$$
\begin{aligned}
J_p &= C_1(H)H^{m_1} + C_2(H)H^{m_2} \\
&= + \frac{2(i_e + \lambda_e I_u - (c+\lambda_e)F)}{(-m_1)(m_2-m_1)\sigma^2} + \frac{2H^p}{(p-m_1)(m_2-m_1)\sigma^2} + \frac{2(\delta+\lambda_e)H}{(-m_1+1)(m_2-m_1)\sigma^2} \\
&- \frac{2(i_e + \lambda_e I_u - (c+\lambda_e)F)}{(-m_2)(m_2-m_1)\sigma^2} - \frac{2H^p}{(p-m_2)(m_2-m_1)\sigma^2} - \frac{2(\delta+\lambda_e)H}{(-m_2+1)(m_2-m_1)\sigma^2} \\
&= + \frac{2(i_e + \lambda_e I_u - (c+\lambda_e)F)}{(m_2-m_1)\sigma^2}\left(\frac{1}{-m_1} - \frac{1}{-m_2}\right) + \frac{2H^p}{(m_2-m_1)\sigma^2}\left(\frac{1}{p-m_1} - \frac{1}{p-m_2}\right) \\
&+ \frac{2(\delta+\lambda_e)H}{\sigma^2(m_2-m_1)}\left(\frac{1}{-m_1+1} - \frac{1}{-m_2+1}\right) \\
&= - \frac{2(i_e + \lambda_e I_u - (c+\lambda_e)F)}{\sigma^2}\frac{1}{m_1 m_2} - \frac{2H^p}{\sigma^2}\frac{1}{(p-m_1)(p-m_2)} \\
&- \frac{2(\delta+\lambda_e)H}{\sigma^2}\frac{1}{(1-m_1)(1-m_2)}
\end{aligned}
$$

As m_1, m_2 solve $m(m-1) + \frac{2(r-\delta)}{\sigma^2}m - \frac{2(\lambda_e+\rho)}{\sigma^2}$, we know that

$$
\begin{aligned}
m_1 m_2 &= -\frac{2(\lambda_e+\rho)}{\sigma^2} \\
(1-m_1)(1-m_2) &= \frac{2(r-\delta-\lambda_e-\rho)}{\sigma^2} \\
(p-m_1)(p-m_2) &= \frac{2(\frac{1}{2}p(p-1)\sigma^2 + (r-\delta)p - (\lambda_e+\rho))}{\sigma^2}
\end{aligned}
$$

So

$$
J_p = \frac{(i_e + \lambda_e I_u - (c+\lambda_e)F)}{(\lambda_e+\rho)} - \frac{H^p}{(\frac{1}{2}p(p-1)\sigma^2 + (r-\delta)p - (\lambda_e+\rho))} - \frac{(\delta+\lambda_e)H}{(r-\delta-(\lambda_e+\rho))}
$$

So

$$
\begin{aligned}
J^S &= J_h + J_p \\
&= A_{S1}H^{m_1} + A_{S2}H^{m_2} + \frac{(i_e + \lambda_e I_u - (c+\lambda_e)F)}{(\lambda_e+\rho)} \\
&+ \frac{H^p}{((\lambda_e+\rho) - \frac{1}{2}p(p-1)\sigma^2 - (r-\delta)p)} + \frac{(\delta+\lambda_e)H}{((\lambda_e+\rho) - r + \delta)}
\end{aligned}
$$

Similarly

$$
\begin{aligned}
J^D &= A_{D1}H^{m_1} + A_{D2}H^{m_2} \\
&+ \frac{(i_e - \lambda_e CF - cF)}{(\lambda_e+\rho)} \\
&+ \frac{H^p}{((\lambda_e+\rho) - \frac{1}{2}p(p-1)\sigma^2 - (r-\delta)p)} \\
&+ \frac{\delta H}{((\lambda_e+\rho) - r + \delta)}
\end{aligned}
$$

Now we'll solve the linked ODEs, again using the method of variation in parameters to find general and particular solutions to our system of two Second Order Ordinary Differential Equations .

$$
\begin{aligned}
J^e_{HH} + \frac{2(r-\delta)}{\sigma^2 H_t}J^e_H - \frac{2(\lambda_e+\rho)}{\sigma^2 H_t^2}J^e(H_t) + \frac{2\lambda_e}{\sigma^2 H_t^2}J^u(H_t) &= -\frac{2(i_e + \psi(H_t) + \delta H_t - cF)}{\sigma^2 H_t^2} \\
J^u_{HH} + \frac{2(r-\delta)}{\sigma^2 H_t}J^u_H - \frac{2(\lambda_u+\rho)}{\sigma^2 H_t^2}J^u(H_t) + \frac{2\lambda_u}{\sigma^2 H_t^2}J^e(H_t) &= -\frac{2(i_u + \psi(H_t) + \delta H_t - cF)}{\sigma^2 H_t^2}
\end{aligned}
$$

The ODEs have a homogeneous solution and a particular solution.

$$J^e = J_h^e + J_p^e$$

$$J^u = J_h^u + J_p^u$$

We'll first determine the general solution to the corresponding homogeneous system of differential equations. We hypothesize that the general solution is a linear combinations of linearly independent combination of power functions which take the following form:

$$J_h^e(H) = A^e H^k \quad , J_h^u(H) = A^u H^k$$

Substituting these expressions into the original system and collecting coefficients of A^e and A^u, we obtain the following system for A and B:

$$\begin{pmatrix} k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2(\lambda_e+\rho)}{\sigma^2} & \frac{2\lambda_e}{\sigma^2} \\ \frac{2\lambda_u}{\sigma^2} & k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2(\lambda_u+\rho)}{\sigma^2} \end{pmatrix} \begin{pmatrix} A^e \\ A^u \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{A.1}$$

The determinant of this system must vanish for nontrivial solutions to exist. Hence follows the characteristic equation for the exponent k:

$$\left(k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2(\lambda_e+\rho)}{\sigma^2}\right)\left(k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2(\lambda_u+\rho)}{\sigma^2}\right) - \frac{2\lambda_e}{\sigma^2}\frac{2\lambda_u}{\sigma^2} = 0$$

Let $Q_2 = k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2}$. The above equation can be re-written as:

$$\begin{aligned}
\left(Q_2 - \frac{2\lambda_e}{\sigma^2}\right)\left(Q_2 - \frac{2\lambda_u}{\sigma^2}\right) - \frac{2\lambda_e}{\sigma^2}\frac{2\lambda_u}{\sigma^2} &= 0 \\
Q_2^2 - \frac{2(\lambda_e + \lambda_u)}{\sigma^2}Q_2 &= 0 \\
Q_2\left(Q_2 - \frac{2(\lambda_e + \lambda_u)}{\sigma^2}\right) &= 0
\end{aligned}$$

So our four roots are solutions to the following two quadratics:

$$k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2} = 0 \tag{A.2}$$

$$k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2} = \frac{2(\lambda_e + \lambda_u)}{\sigma^2} \tag{A.3}$$

In contrast, the characteristic equation in the single ODE case was:

$$k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2} = \frac{2\lambda_e}{\sigma^2} \tag{A.4}$$

Let, $k_1 > k_2$ solve (A.2), $k_3 > k_4$ solve (A.3) and $m_1 > m_2$ solve (A.4).

Since the coefficient of squared term in $Q_2 \equiv k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2}$ is positive, we know that all three of the quadratics are convex. Since $Q_2(0) = -\frac{2\rho}{\sigma^2} < 0$ and $Q_2(1) = \frac{2(r-\rho-\delta)}{\sigma^2} < 0$ we know that $k_1 > 1 > 0 > k_2$. Since (A.3) and (A.4) are downward translations of (A.2), we get that $k_3 > m_1 > k_1 > 1 > 0 > k_2 > m_2 > k_4$

The general solution to the homogeneous system of equations is expressed as follows:

$$J_h^e(H) = \sum_{i=1}^{4} A_i^e H^{k_i}$$
$$J_h^u(H) = \sum_{i=1}^{4} A_i^u H^{k_i}$$

Where $A_i^u = -\frac{(k(k-1)+\frac{2(r-\delta)}{\sigma^2}k-\frac{2(\lambda_e+\rho)}{\sigma^2})}{\frac{2\lambda_e}{\sigma^2}} A_i^e \quad \forall i.$

In particular,

$$A_1^u = A_1^e$$
$$A_2^u = A_2^e$$
$$A_3^u = -\frac{\lambda_u}{\lambda_e} A_3^e$$
$$A_4^u = -\frac{\lambda_u}{\lambda_e} A_4^e$$

Now, following the method of variation of parameters, we'll assume that a particular solution to our inhomogeneous system of ordinary equations takes the following form:

$$J_p^e(H) = \sum_{i=1}^{4} v_i(H) A_i^e H^{k_i}$$
$$J_p^u(H) = \sum_{i=1}^{4} v_i(H) A_i^u H^{k_i}$$

Differentiating the above equations, we get:

$$J_p^{e'}(H) = \sum_{i=1}^{4} v_i^{'}(H) A_i^e H^{k_i} + \sum_{i=1}^{4} v_i(H) A_i^e k_i H^{k_i-1}$$
$$J_p^{u'}(H) = \sum_{i=1}^{4} v_i^{'}(H) A_i^u H^{k_i} + \sum_{i=1}^{4} v_i(H) A_i^u k_i H^{k_i-1}$$

We place the following restrictions on $v^i(H)$:

$$\sum_{i=1}^{4} v_i^{'}(H) A_i^e H^{k_i} = 0 \tag{A.5}$$
$$\sum_{i=1}^{4} v_i^{'}(H) A_i^u H^{k_i} = 0 \tag{A.6}$$

Differentiating $J_p^{e'}(H)$ and $J_p^{u'}(H)$ we get:

$$J_p^{e''}(H) = \sum_{i=1}^{4} v_i^{'}(H) A_i^e k_i H^{k_i-1} + \sum_{i=1}^{4} v_i(H) A_i^e k_i (k_i - 1) H^{k_i-2}$$
$$J_p^{u''}(H) = \sum_{i=1}^{4} v_i^{'}(H) A_i^u k_i H^{k_i-1} + \sum_{i=1}^{4} v_i(H) A_i^u k_i (k_i - 1) H^{k_i-1}$$

We place further restrictions on $v^i(H)$:

$$\sum_{i=1}^{4} v_i^{'}(H) A_i^e k_i H^{k_i-1} = -\frac{2(i_e + \psi(H) + \delta H - cF)}{\sigma^2 H^2} \tag{A.7}$$

$$\sum_{i=1}^{4} v_i^{'}(H) A_i^u k_i H^{k_i-1} = -\frac{2(i_u + \psi(H) + \delta H - cF)}{\sigma^2 H^2} \tag{A.8}$$

Solving this algebraic system below will give us closed from solutions for $v_i^{'}(H) \quad \forall i$, which we can then integrate on a suitable interval to arrive at expressions for $v_i(t) \quad \forall i$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ k_1 & k_2 & k_3 & k_4 \\ 1 & 1 & -\frac{\lambda_u}{\lambda_e} & -\frac{\lambda_u}{\lambda_e} \\ k_1 & k_2 & -\frac{\lambda_u}{\lambda_e}k_3 & -\frac{\lambda_u}{\lambda_e}k_4 \end{bmatrix} \begin{bmatrix} A_1^e H^{k_1} v_1^{'}(H) \\ A_2^e H^{k_2} v_2^{'}(H) \\ A_3^e H^{k_3} v_3^{'}(H) \\ A_4^e H^{k_4} v_4^{'}(H) \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{2(i_e+\psi(H)+\delta H - cF)}{\sigma^2 H} \\ 0 \\ -\frac{2(i_u+\psi(H)+\delta H - cF)}{\sigma^2 H} \end{bmatrix} \tag{A.9}$$

Solving, we get:

$$A_1^e H^{k_1} v_1^{'}(H) = \frac{2(i_e \frac{\lambda_u}{\lambda_e} + i_u)}{\sigma^2 H(-\frac{\lambda_u}{\lambda_e} - 1)(k_1 - k_2)} - \frac{2(\psi(H) + \delta H - cF)}{\sigma^2 H(k_1 - k_2)}$$

$$A_2^e H^{k_2} v_2^{'}(H) = -\frac{2(i_e \frac{\lambda_u}{\lambda_e} + i_u)}{\sigma^2 H(-\frac{\lambda_u}{\lambda_e} - 1)(k_1 - k_2)} + \frac{2(\psi(H) + \delta H - cF)}{\sigma^2 H(k_1 - k_2)}$$

$$A_3^e H^{k_3} v_3^{'}(H) = -\frac{-2i_e + 2i_u}{\sigma^2 H(-\frac{\lambda_u}{\lambda_e} - 1)(k_3 - k_4)}$$

$$A_4^e H^{k_4} v_4^{'}(H) = \frac{-2i_e + 2i_u}{\sigma^2 H(-\frac{\lambda_u}{\lambda_e} - 1)(k_3 - k_4)}$$

Integrating, we get:

$$A_1^e v_1(H) = \frac{2(i_e\frac{\lambda_u}{\lambda_e} + i_u)H^{-k_1}}{\sigma^2(-k_1)(-\frac{\lambda_u}{\lambda_e} - 1)(k_1 - k_2)} - \frac{2H^{p-k_1}}{\sigma^2(p - k_1)(k_1 - k_2)} - \frac{2(\delta H^{-k_1+1})}{\sigma^2(-k_1 + 1)(k_1 - k_2)}$$
$$- \frac{2(-cFH^{-k_1})}{\sigma^2(-k_1)(k_1 - k_2)}$$
$$A_2^e v_2(H) = -\frac{2(i_e\frac{\lambda_u}{\lambda_e} + i_u)H^{-k_2}}{\sigma^2(-k_2)(-\frac{\lambda_u}{\lambda_e} - 1)(k_1 - k_2)} + \frac{2H^{p-k_2}}{\sigma^2(p - k_2)(k_1 - k_2)} + \frac{2(\delta H^{-k_2+1})}{\sigma^2(-k_2 + 1)(k_1 - k_2)}$$
$$+ \frac{2(-cFH^{-k_2})}{\sigma^2(-k_2)(k_1 - k_2)}$$
$$A_3^e v_3(H) = -\frac{(-2i_e + 2i_u)H^{-k_3}}{\sigma^2(-k_3)(-\frac{\lambda_u}{\lambda_e} - 1)(k_3 - k_4)}$$
$$A_4^e v_4(H) = \frac{(-2i_e + 2i_u)H^{-k_4}}{\sigma^2(-k_4)(-\frac{\lambda_u}{\lambda_e} - 1)(k_3 - k_4)}$$

So

$$\begin{aligned} J_p^e &= \frac{2(i_e\frac{\lambda_u}{\lambda_e} + i_u)}{\sigma^2(-\frac{\lambda_u}{\lambda_e} - 1)(k_1 - k_2)}\left(\frac{1}{k_2} - \frac{1}{k_1}\right) \\ &+ \frac{2H^p}{\sigma^2(k_1 - k_2)}\left(\frac{1}{p - k_2} - \frac{1}{p - k_1}\right) \\ &+ \frac{2(\delta H)}{\sigma^2(k_1 - k_2)}\left(\frac{1}{1 - k_2} - \frac{1}{1 - k_1}\right) \\ &+ \frac{2(-cF)}{\sigma^2(k_1 - k_2)}\left(\frac{1}{k_1} - \frac{1}{k_2}\right) \\ &+ \frac{(-2i_e + 2i_u)}{\sigma^2(-\frac{\lambda_u}{\lambda_e} - 1)(k_3 - k_4)}\left(\frac{1}{k_3} - \frac{1}{k_4}\right) \end{aligned}$$

$$
\begin{aligned}
J_p^e = &\frac{2(i_e\frac{\lambda_u}{\lambda_e} + i_u)}{\sigma^2(-\frac{\lambda_u}{\lambda_e} - 1)}\left(\frac{1}{k_1k_2}\right)\\
&+ \frac{2H^p}{\sigma^2}\left(\frac{-1}{(p-k_2)(p-k_1)}\right)\\
&+ \frac{2(\delta H)}{\sigma^2}\left(\frac{-1}{(1-k_2)(1-k_1)}\right)\\
&+ \frac{2(-cF)}{\sigma^2}\left(\frac{-1}{k_1k_2}\right)\\
&+ \frac{(-2i_e + 2i_u)}{\sigma^2(-\frac{\lambda_u}{\lambda_e} - 1)}\left(\frac{-1}{k_3k_4}\right)
\end{aligned}
$$

Recall that k_1, k_2 solve the first equation and k_3, k_4 solve the second equation

$$
\begin{aligned}
k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2} &= 0\\
k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2} &= \frac{2(\lambda_e + \lambda_u)}{\sigma^2}
\end{aligned}
$$

So

$$
\begin{aligned}
k_1k_2 &= -\frac{2\rho}{\sigma^2}\\
k_3k_4 &= -\frac{2(\rho + \lambda_e + \lambda_u)}{\sigma^2}\\
(1-k_2)(1-k_1) &= -\frac{2(\rho + \delta - r)}{\sigma^2}\\
(p-k_2)(p-k_1) &= -\frac{2(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})}{\sigma^2}
\end{aligned}
$$

Substituting

$$
\begin{aligned}
J_p^e = & \frac{(i_e \frac{\lambda_u}{\lambda_e} + i_u)}{(\frac{\lambda_u}{\lambda_e} + 1)\rho} \\
& + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} \\
& + \frac{\delta H}{(\rho + \delta - r)} \\
& - \frac{cF}{\rho} \\
& + \frac{(-i_e + i_u)}{(-\frac{\lambda_u}{\lambda_e} - 1)(\rho + \lambda_e + \lambda_u)}
\end{aligned}
$$

Note that

$$
\begin{aligned}
& \frac{(i_e \frac{\lambda_u}{\lambda_e} + i_u)}{(\frac{\lambda_u}{\lambda_e} + 1)\rho} + \frac{(-i_e + i_u)}{(-\frac{\lambda_u}{\lambda_e} - 1)(\rho + \lambda_e + \lambda_u)} \\
& = \frac{(i_e \frac{\lambda_u}{\lambda_e} + i_u)}{(\frac{\lambda_u}{\lambda_e} + 1)\rho} + \frac{(-i_e + i_u)}{(-\frac{\lambda_u}{\lambda_e} - 1)(\rho + \lambda_e + \lambda_u)} + \frac{i_e}{\rho} - \frac{i_e}{\rho} \\
& = \frac{i_e}{\rho} + \frac{(i_e\lambda_u + i_u\lambda_e)}{(\lambda_u + \lambda_e)\rho} + \frac{(\lambda_e i_e - \lambda_e i_u)}{(\lambda_u + \lambda_e)(\rho + \lambda_e + \lambda_u)} - \frac{i_e}{\rho} \\
& = \frac{i_e}{\rho} + \frac{i_e\lambda_u}{(\lambda_u + \lambda_e)\rho} - \frac{i_e}{\rho} + \frac{\lambda_e i_e}{(\lambda_u + \lambda_e)(\rho + \lambda_e + \lambda_u)} - \frac{\lambda_e i_u}{(\lambda_u + \lambda_e)(\rho + \lambda_e + \lambda_u)} + \frac{i_u\lambda_e}{(\lambda_u + \lambda_e)\rho} \\
& = \frac{i_e}{\rho} - \frac{i_e\lambda_e}{(\lambda_u + \lambda_e)\rho} + \frac{\lambda_e i_e}{(\lambda_u + \lambda_e)(\rho + \lambda_e + \lambda_u)} - \frac{\lambda_e i_u}{(\lambda_u + \lambda_e)(\rho + \lambda_e + \lambda_u)} + \frac{i_u\lambda_e}{(\lambda_u + \lambda_e)\rho} \\
& = \frac{i_e}{\rho} - \frac{i_e\lambda_e}{(\rho + \lambda_u + \lambda_e)\rho} + \frac{\lambda_e i_u}{\rho(\rho + \lambda_e + \lambda_u)} \\
& = \frac{i_e}{\rho} - \frac{(i_e - i_u)\lambda_e}{(\rho + \lambda_u + \lambda_e)\rho} \\
& = I_e
\end{aligned}
$$

Therefore

$$J_p^e = I_e - \frac{cF}{\rho} + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H}{(\rho + \delta - r)}$$

Similarly, we get

$$J_p^u = I_u - \frac{cF}{\rho} + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H}{(\rho + \delta - r)}$$

Therefore

$$J^e(H) = \sum_{i=1}^{4} A_i^e H^{k_i} + I_e - \frac{cF}{\rho} + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H}{(\rho + \delta - r)}$$
$$J^u(H) = \sum_{i=1}^{4} A_i^u H^{k_i} + I_u - \frac{cF}{\rho} + \frac{H^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H}{(\rho + \delta - r)}$$

The value functions for debt, namely, V^e, V^u, V^S and V^D can be derived similarly.

□

We now have twenty one unknowns, namely eight equity coefficients, A_1, A_2, A_3, A_4, A_{D1}, A_{D2}, A_{S1}, A_{S2}, eight debt coefficients, L_1, L_2, L_3, L_4, L_{D1}, L_{D2}, L_{S1}, L_{S2}, four boundaries $HDD, H0D, H0S, HSS$ and the equilibrium mortgage rate c. We solve the system of 21 constraints, listed earlier, for the 21 unknowns.

A.2 Transversality Condition

We assume that the homeowner's value function satisfies the transversality conditions below.

$$lim_{t \to \infty} E_{H_t, I_t} \left[e^{-\rho t} J(H_t, I_t) \right] = 0$$

Chapter B: ODEs

B.1 ODEs

B.1.1 Linked ODEs

We will use the method of variation in parameters to find general and particular solutions to our system of two second order ordinary differential equations .

$$\begin{aligned} J^e_{HH} + \frac{2(r-\delta)}{\sigma^2 H_t} J^e_H - \frac{2(\lambda_e+\rho)}{\sigma^2 H_t^2} J^e(H_t) + \frac{2\lambda_e}{\sigma^2 H_t^2} J^u(H_t) &= -\frac{2(i_e + \psi(H_t) + \delta H_t - cF)}{\sigma^2 H_t^2} \\ J^u_{HH} + \frac{2(r-\delta)}{\sigma^2 H_t} J^u_H - \frac{2(\lambda_u+\rho)}{\sigma^2 H_t^2} J^u(H_t) + \frac{2\lambda_u}{\sigma^2 H_t^2} J^e(H_t) &= -\frac{2(i_u + \psi(H_t) + \delta H_t - cF)}{\sigma^2 H_t^2} \end{aligned}$$

The ODEs have a homogeneous solution and a particular solution.

$$J^e = J^e_h + J^e_p$$

$$J^u = J^u_h + J^u_p$$

We will first determine the general solution to the corresponding homogeneous system of differential equations. We hypothesize that the general solution is a linear combinations of linearly independent combination of power functions which take the following form:

$$J_h^e(H) = A^e H^k \quad , J_h^u(H) = A^u H^k$$

Substituting these expressions into the original system and collecting coefficients of A^e and A^u, we obtain the following system for A and B:

$$\begin{pmatrix} k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2(\lambda_e+\rho)}{\sigma^2} & \frac{2\lambda_e}{\sigma^2} \\ \frac{2\lambda_u}{\sigma^2} & k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2(\lambda_u+\rho)}{\sigma^2} \end{pmatrix} \begin{pmatrix} A^e \\ A^u \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{B.1}$$

The determinant of this system must vanish for nontrivial solutions to exist. Hence follows the characteristic equation for the exponent k:

$$\left(k(k-1)+\frac{2(r-\delta)}{\sigma^2}k-\frac{2(\lambda_e+\rho)}{\sigma^2}\right)\left(k(k-1)+\frac{2(r-\delta)}{\sigma^2}k-\frac{2(\lambda_u+\rho)}{\sigma^2}\right)-\frac{2\lambda_e}{\sigma^2}\frac{2\lambda_u}{\sigma^2}=0 \tag{B.2}$$

Eyeballing the above equation, we see that the quartic will admit solutions at the zeros of the following two quadratic equations:

$$Q_{1,2}(x) = k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2} = \frac{2(\lambda_e + \lambda_u)}{\sigma^2} \tag{B.3}$$

$$Q_{3,4}(x) = k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2\rho}{\sigma^2} = 0 \tag{B.4}$$

Let k_1, k_2 be the zeros of $Q_{1,2}$ and k_3, k_4 be the zeros of $Q_{3,4}$. Since the product of the roots $-\frac{2\rho}{\sigma^2}$, is negative in $Q_{3,4}$ we know that the roots k_3, k_4 have opposite signs. Also since the coefficient of k^2 is positive, we know that $Q_{3,4}$ is convex. Now $Q_{1,2}(x) = Q_{3,4} - \frac{2(\lambda_e+\lambda_u)}{\sigma^2}$ where $\frac{2(\lambda_e+\lambda_u)}{\sigma^2} > 0$. So we know that the roots of $Q_{3,4}$ are also of opposite signs and given a particular sign, are of a larger magnitude. Without loss of generality, we label the roots such that $k_1 > k_3 > 0 > k_4 > k_2$.

When the roots of the above equation, say k_1, k_2, k_3, k_4, are distinct then the general solution to the homogeneous system of equations is expressed as follows:

$$J_h^e(H) = \sum_{i=1}^{4} A_i^e H^{k_i}$$

$$J_h^u(H) = \sum_{i=1}^{4} A_i^u H^{k_i}$$

Where $A_i^u = -\frac{(k(k-1)+\frac{2(r-\delta)}{\sigma^2}k - \frac{2(\lambda_e+\rho)}{\sigma^2})}{\frac{2\lambda_e}{\sigma^2}} A_i^e \quad \forall i.$

Now, following the method of variation of parameters, we'll assume that a particular solution to our inhomogeneous system of ordinary equations takes the following form:

$$J_p^e(H) = \sum_{i=1}^{4} v_i(H) A_i^e H^{k_i}$$
$$J_p^u(H) = \sum_{i=1}^{4} v_i(H) A_i^u H^{k_i}$$

Differentiating the above equations, we get:

$$J_p^{e'}(H) = \sum_{i=1}^{4} v_i^{'}(H) A_i^e H^{k_i} + \sum_{i=1}^{4} v_i(H) A_i^e k_i H^{k_i - 1}$$
$$J_p^{u'}(H) = \sum_{i=1}^{4} v_i^{'}(H) A_i^u H^{k_i} + \sum_{i=1}^{4} v_i(H) A_i^u k_i H^{k_i - 1}$$

We place the following restrictions on $v^i(H)$:

$$\sum_{i=1}^{4} v_i^{'}(H) A_i^e H^{k_i} = 0 \tag{B.5}$$
$$\sum_{i=1}^{4} v_i^{'}(H) A_i^u H^{k_i} = 0 \tag{B.6}$$

Differentiating $J_p^{e'}(H)$ and $J_p^{u'}(H)$ we get:

$$J_p^{e''}(H) = \sum_{i=1}^{4} v_i^{'}(H) A_i^e k_i H^{k_i - 1} + \sum_{i=1}^{4} v_i(H) A_i^e k_i (k_i - 1) H^{k_i - 2}$$
$$J_p^{u''}(H) = \sum_{i=1}^{4} v_i^{'}(H) A_i^u k_i H^{k_i - 1} + \sum_{i=1}^{4} v_i(H) A_i^u k_i (k_i - 1) H^{k_i - 1}$$

We place further restrictions on $v^i(H)$:

$$\sum_{i=1}^{4} v_i^{'}(H) A_i^e k_i H^{k_i - 1} = -\frac{2(i_e + \psi(H) + \delta H - cF)}{\sigma^2 H^2} \tag{B.7}$$

$$\sum_{i=1}^{4} v_i^{'}(H) A_i^u k_i H^{k_i - 1} = -\frac{2(i_u + \psi(H) + \delta H - cF)}{\sigma^2 H^2} \tag{B.8}$$

With the above restrictions in place, we can verify that J_p^e and J_p^u satisfy our original system of inhomogeneous differential equations.

Now equations (B.5),(B.6),(B.7) and (B.8) form a system of linear equations for the functions $v_i^{'}(H) \quad i = 1, ..4$. Solving this algebraic system below will give us closed from solutions for $v_i^{'}(H) \quad \forall i$, which we can then integrate on a suitable interval to arrive at expressions for $v_i(t) \quad \forall i$.

Recall that:

$$\begin{aligned}
A_1^u &= -\frac{(k_1(k_1-1) + \frac{2(r-\delta)}{\sigma^2}k_1 - \frac{2(\lambda_e+\rho)}{\sigma^2})}{\frac{2\lambda_e}{\sigma^2}} A_1^e \\
A_2^u &= -\frac{(k_2(k_2-1) + \frac{2(r-\delta)}{\sigma^2}k_2 - \frac{2(\lambda_e+\rho)}{\sigma^2})}{\frac{2\lambda_e}{\sigma^2}} A_2^e \\
A_3^u &= -\frac{(k_3(k_3-1) + \frac{2(r-\delta)}{\sigma^2}k_3 - \frac{2(\lambda_e+\rho)}{\sigma^2})}{\frac{2\lambda_e}{\sigma^2}} A_3^e \\
A_4^u &= -\frac{(k_4(k_4-1) + \frac{2(r-\delta)}{\sigma^2}k_4 - \frac{2(\lambda_e+\rho)}{\sigma^2})}{\frac{2\lambda_e}{\sigma^2}} A_4^e
\end{aligned}$$

Let

$$
\begin{aligned}
m_1 &= -\frac{(k_1(k_1-1)+\frac{2(r-\delta)}{\sigma^2}k_1-\frac{2(\lambda_e+\rho)}{\sigma^2})}{\frac{2\lambda_e}{\sigma^2}} = -\frac{\lambda_u}{\lambda_e} \\
m_2 &= -\frac{(k_2(k_2-1)+\frac{2(r-\delta)}{\sigma^2}k_2-\frac{2(\lambda_e+\rho)}{\sigma^2})}{\frac{2\lambda_e}{\sigma^2}} = -\frac{\lambda_u}{\lambda_e} \\
m_3 &= -\frac{(k_3(k_3-1)+\frac{2(r-\delta)}{\sigma^2}k_3-\frac{2(\lambda_e+\rho)}{\sigma^2})}{\frac{2\lambda_e}{\sigma^2}} = 1 \\
m_4 &= -\frac{(k_4(k_4-1)+\frac{2(r-\delta)}{\sigma^2}k_4-\frac{2(\lambda_e+\rho)}{\sigma^2})}{\frac{2\lambda_e}{\sigma^2}} = 1
\end{aligned}
$$

$$
\begin{bmatrix}
H^{k_1} & H^{k_2} & H^{k_3} & H^{k_4} \\
k_1H^{k_1-1} & k_2H^{k_2-1} & k_3H^{k_3-1} & k_4H^{k_4-1} \\
m_1H^{k_1} & m_2H^{k_2} & m_3H^{k_3} & m_4H^{k_4} \\
m_1k_1H^{k_1-1} & m_2k_2H^{k_2-1} & m_3k_3H^{k_3-1} & m_4k_4H^{k_4-1}
\end{bmatrix}
\begin{bmatrix}
A_1^e v_1'(H) \\ A_2^e v_2'(H) \\ A_3^e v_3'(H) \\ A_4^e v_4'(H)
\end{bmatrix}
=
\begin{bmatrix}
0 \\ -\frac{2(i_e+\psi(H)+\delta H-cF)}{\sigma^2H^2} \\ 0 \\ -\frac{2(i_u+\psi(H)+\delta H-cF)}{\sigma^2H^2}
\end{bmatrix}
\tag{B.9}
$$

We invert this matrix, solve for $v_i'(H)$ and integrate the expression to finally get:

$$
\begin{aligned}
J_p^e(H) &= \sum_{i=1}^{4} v_i(H)A_i^eH^{k_i} = -\frac{cF}{\rho} + \frac{\delta H}{\delta+\rho-r} + I_e + \Psi(H) \\
J_p^u(H) &= \sum_{i=1}^{4} v_i(H)A_i^uH^{k_i} = -\frac{cF}{\rho} + \frac{\delta H}{\delta+\rho-r} + I_u + \Psi(H)
\end{aligned}
$$

Therefore, combining the particular and the homogeneous solutions, we have our value functions:

$$
\begin{aligned}
J^e(H) &= J_h^e(H) + J_p^e(H) = \sum_{i=1}^{4} A_i^e H^{k_i} - \frac{cF}{\rho} + \frac{\delta H}{\delta + \rho - r} + I_e + \Psi(H) \\
J^u(H) &= J_h^u(H) + J_p^u(H) = \sum_{i=1}^{4} m_i A_i^e H^{k_i} - \frac{cF}{\rho} + \frac{\delta H}{\delta + \rho - r} + I_u + \Psi(H)
\end{aligned}
$$

We take $\psi(H) = H^{0.25}$ then it turns out that $\Psi(H) = \frac{H^{0.25}}{-0.25(r+\delta)+\rho+0.09375\sigma^2}$.

B.1.2 Ordinary differential equations

B.1.3 No recourse equity

We use the method of variation in parameters to find general and particular solutions to the second order ordinary differential equations that characterizes equity:

$$
J_{HH} + \frac{2(r-\delta)}{\sigma^2 H} J_H - \frac{2\rho}{\sigma^2 H^2} J = -\frac{2(\psi(H) - cF + \delta H)}{\sigma^2 H^2}
$$

The solution will be the sum of the homogeneous solution and the particular solution.

$$
J(H) = J_h(H) + J_p(H)
$$

We first determine the general solution to the corresponding homogeneous part of the equation. We hypothesize that the general solution is a linear combination of linearly independent power functions which take the following form:

$J_h(H) = AH^k$

Substituting these expressions into the homogeneous part of the original equation, we obtain the following relation

$$\left(k(k-1) + \frac{2(r-\delta)k}{\sigma^2} - \frac{2\rho}{\sigma^2}\right)AH^{k-2} = 0$$

The coefficient of this equation must vanish for nontrivial solutions to exist. Hence follows the characteristic equation for the exponent k:

$$k(k-1) + \frac{2(r-\delta)k}{\sigma^2} - \frac{2\rho}{\sigma^2} = 0$$

When the roots of the above equation, say k_1, k_2, are distinct then the general solution to the homogeneous system of equations is expressed as follows:

$$J_h(H) = \sum_{i=1}^{2} A_i H^{k_i} \tag{B.10}$$

Now, following the method of variation of parameters, we assume that the particular solution to the inhomogeneous part of our ordinary differential equation takes the following form:

$$J_p(H) = \sum_{i=1}^{2} v_i(H) A_i H^{k_i} \tag{B.11}$$

Differentiating, we get:

$$J_p^{'}(H) = \sum_{i=1}^{2} v_i^{'}(H) A_i H^{k_i} + \sum_{i=1}^{2} v_i(H) A_i k_i H^{k_i - 1}$$

We place the following restrictions on $v^i(H)$:

$$\sum_{i=1}^{2} v_i^{'}(H) A_i H^{k_i} = 0 \tag{B.12}$$

Differentiating $J_p^{'}(H)$ we get:

$$J_p^{''}(H) = \sum_{i=1}^{2} v_i^{'}(H) A_i k_i H^{k_i - 1} + \sum_{i=1}^{2} v_i(H) A_i k_i (k_i - 1) H^{k_i - 2}$$

We place further restrictions on $v^i(H)$:

$$\sum_{i=1}^{2} v_i^{'}(H) A_i k_i H^{k_i - 1} = -\frac{2(\psi(H) - cF + \delta H)}{\sigma^2 H^2} \tag{B.13}$$

With the above restrictions in place, we can verify that J_p satisfies our original in-homogeneous differential equation:

$$
\begin{aligned}
& J_p^{''} + \frac{2(r-\delta)}{\sigma^2 H} J_p^{'} - \frac{2\rho}{\sigma^2 H^2} J_p \\
= & \sum_{i=1}^{2} v_i^{'}(H) A_i k_i H^{k_i - 1} + \sum_{i=1}^{2} v_i(H) A_i k_i (k_i - 1) H^{k_i - 2} \\
& + \frac{2(r-\delta)}{\sigma^2 H} \sum_{i=1}^{2} v_i(H) A_i k_i H^{k_i - 1} - \frac{2\rho}{\sigma^2 H^2} \sum_{i=1}^{2} v_i(H) A_i H^{k_i} \\
= & -\frac{2(\psi(H) - cF + \delta H)}{\sigma^2 H^2} + \sum_{i=1}^{2} v_i(H) H^{k_i - 2} A_i \left\{ k(k-1) + \frac{2(r-\delta)k}{\sigma^2} - \frac{2\rho}{\sigma^2} \right\} \\
= & -\frac{2(\psi(H) - cF + \delta H)}{\sigma^2 H^2}
\end{aligned}
$$

Now equations (B.12),(B.13) form a system of linear algebraic equations for the functions $v_i^{'}(H) \quad i = 1, 2$. Solving this algebraic system below will give us closed from solutions for $v_i^{'}(H) \quad \forall i$, which we can then integrate on a suitable interval to arrive at expressions for $v_i(H) \quad \forall i$.

$$
\begin{bmatrix} A_1 H^{k_1} & A_2 H^{k_2} \\ A_1 k_1 H^{k_1 - 1} & A_2 k_2 H^{k_2 - 1} \end{bmatrix} \begin{bmatrix} v_1^{'}(H) \\ v_2^{'}(H) \end{bmatrix} = \begin{bmatrix} 0 \\ -\frac{2(\psi(H) - cF + \delta H)}{\sigma^2 H^2} \end{bmatrix}, \tag{B.14}
$$

Solving, we get:

$$
\begin{aligned}
A_1 v_1^{'}(H) &= -\frac{2H^{-1-k_1}(cF - \delta H - \psi(H))}{(k_2 - k_1)\sigma^2} \\
A_2 v_2^{'}(H) &= \frac{2H^{-1-k_2}(cF - \delta H - \psi(H))}{(k_2 - k_1)\sigma^2}
\end{aligned}
$$

Integrating

$$\begin{aligned} A_1 v_1(H) &= \int_{H*}^{H} A_1 v_1^{'}(H) = -\frac{2cFH^{-k_1}}{k_1(k_1-k_2)\sigma^2} + \frac{2H^{1-k_1}\delta}{(1-k_1)(-k_1+k_2)\sigma^2} + \Psi^*(H) \\ A_2 v_2(H) &= \int_{H*}^{H} A_2 v_2^{'}(H) = \frac{2cFH^{-k_2}}{k_2(k_1-k_2)\sigma^2} - \frac{2H^{1-k_2}\delta}{(1-k_2)(-k_1+k_2)\sigma^2} + \Psi^{**}(H) \end{aligned}$$

Therefore

$$\begin{aligned} J(H) &= J_h(H) + J_p(H) \\ &= \sum_{i=1}^{2} A_i H^{k_i} + v_1(H)A_1 H^{k_1} + v_2(H)A_2 H^{k_2} \\ &= \sum_{i=1}^{2} A_i H^{k_i} - \frac{2cF}{(k_1-k_2)\sigma^2}\left[\frac{1}{k_2} - \frac{1}{k_1}\right] + \frac{2\delta H}{\sigma^2(k_2-k_1)}\left[\frac{1}{1-k_1} - \frac{1}{1-k_2}\right] + \Psi(H) \\ &= \sum_{i=1}^{2} A_i H^{k_i} - \frac{2cF}{\sigma^2}\left[\frac{1}{k_1 k_2}\right] - \frac{2\delta H}{\sigma^2(k_2-k_1)}\left[\frac{1}{(1-k_1)(1-k_2)}\right] + \Psi(H) \\ &= \sum_{i=1}^{2} A_i H^{k_i} - \frac{cF}{\rho} + \frac{\delta H}{(\delta+\rho-r)} + \Psi(H) \end{aligned}$$

Since

$$\begin{aligned} k_1 k_2 &= Q^2(0) = -\frac{2\rho}{\sigma^2} \\ (1-K_1)(1-k_2) &= Q^2(1) = \frac{2(r-\delta)}{\sigma^2} - \frac{2\rho}{\sigma^2} \end{aligned}$$

because k_1 and k_2 solve

$$Q^2(k) = k(k-1) + \frac{2(r-\delta)k}{\sigma^2} - \frac{2\rho}{\sigma^2} = 0$$

The level-1 and level-0 equity value functions are as follows:

$$
\begin{aligned}
J_1(H) &= A_1 H^{k_1} + B_1 H^{k_2} - \frac{c_1 F}{\rho} + \frac{\delta H}{(\delta+\rho-r)} + \Psi(H) && \text{(B.15)} \\
J_0(H) &= A_0 H^{k_1} + B_0 H^{k_2} - \frac{c_0 F}{\rho} + \frac{\delta H}{(\delta+\rho-r)} + \Psi(H) && \text{(B.16)}
\end{aligned}
$$

We take $\psi(H) = H^{0.25}$ then it turns out that $\Psi(H) = \frac{H^{0.25}}{-0.25(r+\delta)+\rho+0.09375\sigma^2}$. The same as what we had derived in the recourse linked equity case.

B.2 Recourse : Single ODE

B.2.1 Equity

$$
{}^S J^e_{HH} + \frac{2(r-\delta)}{\sigma^2 H} {}^S J^e_H - \frac{2(\lambda_e+\rho)}{\sigma^2 H^2} {}^S J^e + \frac{2\lambda_e}{\sigma^2 H^2} B^u(H_t) = -\frac{2(i_e+\psi(H)-cF)}{\sigma^2 H^2} - \frac{2\delta}{\sigma^2 H}
$$

$$
{}^D J^e_{HH} + \frac{2(r-\delta)}{\sigma^2 H} {}^D J^e_H - \frac{2(\lambda_e+\rho)}{\sigma^2 H^2} {}^D J^e + \frac{2\lambda_e}{\sigma^2 H^2} B^u(H_t) = -\frac{2(i_e+\psi(H)-cF)}{\sigma^2 H^2} - \frac{2\delta}{\sigma^2 H}
$$

The solutions to the above ODEs are as follows:

$$
{}^S J^e = {}^S A H^{l_1} + {}^S B H^{l_2} + \frac{i_e - cF + (I_u - F)\lambda_e}{\lambda_e+\rho} + \frac{(\delta+\lambda_e)H}{\delta+\rho+\lambda_e-r} + \hat{\Psi}(H)
$$

$$
{}^D J^e = \begin{cases} {}^D A H^{l_1} + {}^D B H^{l_2} + \frac{i_e - cF - C_f\lambda_e}{\lambda_e+\rho} + \frac{\delta H}{\delta+\rho+\lambda_e-r} + \hat{\Psi}(H) & \text{if } H < F - I_u \\ {}^D A H^{l_1} + {}^D B H^{l_2} + \frac{i_e - cF - (C_f+F-I_u)\lambda_e}{\lambda_e+\rho} + \frac{(\delta+\lambda_e)H}{\delta+\rho+\lambda_e-r} + \hat{\Psi}(H) & \text{if } H > F - I_u \end{cases}
$$

Where l_1 and l_2 are solutions to the following quadratic:

$$k(k-1) + \frac{2(r-\delta)}{\sigma^2}k - \frac{2(\lambda_e+\rho)}{\sigma^2} = 0$$

The coefficients ${}^{S}A, {}^{S}B, {}^{D}A$ and ${}^{D}B$ are determined by boundary conditions.

When we take $\psi(H) = H^{0.25}$ then we get $\hat{\Psi}(H) = \frac{H^{0.25}}{-0.25(r+\delta)+\rho+\lambda_e+0.09375\sigma^2}$

B.3 Markov Chains

Proposition B.3.1. *The present value of future income, given some current state of employment, is given as follows. I_e is the present value when the current state is of employment while I_u is the present value in the low income state of the world.*

$$I_e = \frac{i_e}{r} - \frac{(i_e - i_u)\lambda_e}{r(r+\lambda_e+\lambda_u)} \tag{B.17}$$

$$I_u = \frac{i_u}{r} + \frac{(i_e - i_u)\lambda_u}{r(r+\lambda_e+\lambda_u)} \tag{B.18}$$

Proof.

$$
\begin{aligned}
& I^e \\
= \quad & \int_0^\infty \int_0^\infty \left(\int_0^{\tau_1} e^{-rt} i_e \, dt + e^{-r\tau_1} \int_0^{\tau_2} e^{-rt} i_u dt + e^{-r\tau_1} e^{-r\tau_2} I^e \right) \lambda_e \lambda_u e^{-\lambda_e \tau_1} e^{-\lambda_u \tau_2} \, d\tau_1 \, d\tau_2 \\
= \quad & \int_0^\infty \int_0^\infty \left(\frac{i_e(1 - e^{-r\tau_1})}{r} + \frac{i_u(1 - e^{-r\tau_2}) e^{-r\tau_1}}{r} + e^{-r\tau_1} e^{-r\tau_2} I^e \right) \lambda_e \lambda_u e^{-\lambda_e \tau_1} e^{-\lambda_u \tau_2} \, d\tau_1 \, d\tau_2 \\
= \quad & \int_0^\infty \left(\frac{i_e}{r} + e^{-r\tau_1} \left(\frac{-i_e}{r} + \frac{i_u}{r} + \left(I_e - \frac{i_u}{r} \right) \frac{\lambda_u}{\lambda_u + r} \right) \right) \lambda_e e^{-\lambda_e \tau_1} \, d\tau_1 \\
= \quad & \frac{i_e}{r} + \frac{\lambda_e}{\lambda_e + r} \left(\frac{-i_e}{r} + \frac{i_u}{r} + \left(I_e - \frac{i_u}{r} \right) \frac{\lambda_u}{\lambda_u + r} \right) \\
= \quad & \frac{i_e}{r + \lambda_e} + \frac{i_u \lambda_e}{(r + \lambda_u)(r + \lambda_e)} + I_e \frac{\lambda_u \lambda_e}{(r + \lambda_u)(r + \lambda_e)} \\
= \quad & \frac{i_e(r + \lambda_u)}{r(r + \lambda_e + \lambda_u)} + \frac{i_u(\lambda_e)}{r(r + \lambda_e + \lambda_u)} \\
= \quad & \frac{i_e}{r} - \frac{(i_e - i_u)\lambda_e}{r(r + \lambda_e + \lambda_u)}
\end{aligned}
$$

I^u can be calculated by symmetry.

$$
I^u = \frac{i_u}{r} + \frac{(i_e - i_u)\lambda_u}{r(r + \lambda_e + \lambda_u)}
$$

□

B.4 Implicit Differentiation

B.4.1 Post Renegotiation

Post renegotiation, for a given H1R, the Bank, given some bargaining power, chooses a coupon rate. The two extreme cases here are:

Homeowner has all the power and hence the Bank gets his reservation value.

$$L0H1R^{k_1} + M0H1R^{k_2} + \frac{c_0F}{r} - CR = H1R - CFL \tag{B.19}$$

The other coefficients are found by solving:

$$A0H0S^{k_1} + B0H0S^{k_2} + I_e - \frac{c_0F}{\rho} + \frac{H0S^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H0S}{(\rho + \delta - r)}$$

$$= H0S + Ie - F$$

$$A0k_1H0S^{k_1-1} + B0k_2H0S^{k_2-1} + \frac{pH0S^{p-1}}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta}{(\rho + \delta - r)} = 1$$

$$A0H0D^{k_1} + B0H0D^{k_2} + I_e - \frac{c_0F}{\rho} + \frac{H0D^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H0D}{(\rho + \delta - r)}$$

$$= -CF$$

$$A0k_1H0D^{k_1-1} + B0k_2H0D^{k_2-1} + \frac{pH0D^{p-1}}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta}{(\rho + \delta - r)} = 0$$

$$L0H0S^{k_1} + M0H0S^{k_2} + \frac{c_0F}{r} = F$$

$$L0H0D^{k_1} + M0H0D^{k_2} + \frac{c_0F}{r} = H0D + Ie - CFL$$

The above six equations are linear in four variables, A0, B0, L0 and M0 and non-linear in $H0D, H0S$. We can reduce the system to two equations in $H0S, H0D$ and c_0. Lets call them

$$F_1(H0S, H0D, c_0) = 0 \tag{B.20}$$

$$F_2(H0S, H0D, c_0) = 0 \tag{B.21}$$

Equations (B.20) and (B.21) implicitly define H0S and H0D as functions of c_0 and we will use these equations to compute $\frac{dH0S}{dc_0}$ and $\frac{dH0D}{dc_0}$. We have solved L0 and M0 as functions of H0S,H0D and c_0 so we use those expressions to calculate $\frac{\partial L0}{\partial H0S}$, $\frac{\partial L0}{\partial H0D}$,$\frac{\partial M0}{\partial H0S}$, $\frac{\partial M0}{\partial H0D}$, $\frac{dM0}{dc_0}$ and $\frac{dL0}{dc_0}$.

Differentiating equations (B.20) and (B.21), we get:

$$\frac{\partial F_1(H0S, H0D, c_0)}{\partial H0S}\frac{dH0S}{dc_0} + \frac{\partial F_1(H0S, H0D, c_0)}{\partial H0D}\frac{dH0D}{dc_0} + \frac{\partial F_1}{\partial c_0} = 0 \tag{B.22}$$

$$\frac{\partial F_2(H0S, H0D, c_0)}{\partial H0S}\frac{dH0S}{dc_0} + \frac{\partial F_2(H0S, H0D, c_0)}{\partial H0D}\frac{dH0D}{dc_0} + \frac{\partial F_2}{\partial c_0} = 0 \tag{B.23}$$

Equations (B.22) and (B.23) are linear in $\frac{dH0S}{dc_0}$ and $\frac{dH0D}{dc_0}$ and we'll solve them to evaluate $\frac{dH0S}{dc_0}$ and $\frac{dH0D}{dc_0}$.

Ultimately, we have three equations (B.20), (B.21) and (3.8) which we will use to solve for H0S, H0D and c_0

B.4.2 Pre-renegotiation

The Bank sets a coupon using a competitive contract which satisfies his individual rationality constraint.

$$L1H0^{k_1} + M1H0^{k_2} + \frac{c_1 F}{r} = F \tag{B.24}$$

For a given contract, the homeonwer decides where to sell and where to renegotiate. From No-Arbitrage conditions, we get smooth pasting and value matching. The debt is a martingale, so we get value matching as well. First we write down the value matching conditions for debt

$$L1H1S^{k_1} + M1H1S^{k_2} + \frac{c_1 F}{r} = F \tag{B.25}$$

$$L1H1R^{k_1} + M1H1R^{k_2} + \frac{c_1 F}{r} = L0H1R^{k_1} + M0H1R^{k_2} + \frac{c_0 F}{r} - CRL \tag{B.26}$$

Now we write the Equity value matching and smooth pasting conditions at the sale boundary.

$$A1H1S^{k_1} + B1H1S^{k_2} + I_e - \frac{c_1 F}{\rho} + \frac{H1S^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H1S}{(\rho + \delta - r)}$$

$$= H1S + Ie - F \tag{B.27}$$

$$A1k_1H1S^{k_1 - 1} + B1k_2H1S^{k_2 - 1} + \frac{pH1S^{p-1}}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta}{(\rho + \delta - r)} = 1 \tag{B.28}$$

We use equations (B.27), (B.28), (B.25), (B.26) and (B.24) to solve for L1, M1, A1, B1 and c_1. That leaves us with the value matching and smooth pasting of equity to solve for H1S and H1R. These value matching equation is

$$A1H1R^{k_1} + B1H1R^{k_2} + I_e - \frac{c_1 F}{\rho} + \frac{H1R^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H1R}{(\rho + \delta - r)} =$$
$$A0H1R^{k_1} + B0H1R^{k_2} + I_e - \frac{c_0 F}{\rho} + \frac{H1R^p}{(\rho + p(\delta - r) - p(p-1)\frac{\sigma^2}{2})} + \frac{\delta H1R}{(\rho + \delta - r)} - CR \tag{B.29}$$

The smooth pasting equation is c_0, H0S and H0D are implicit functions of H1R. They are implicitly defined by these three equations:

$$F_1(H0S, H0D, c_0, H1R) = 0 \tag{B.30}$$

$$F_2(H0S, H0D, c_0, H1R) = 0 \tag{B.31}$$

$$F_3(H0S, H0D, c_0, H1R) = 0 \tag{B.32}$$

Where

$$F_3(H0S, H0D, c_0) \equiv$$
$$\left(\frac{\partial L0}{\partial H0S}\frac{dH0S}{dc_0} + \frac{\partial L0}{\partial H0D}\frac{dH0D}{dc_0} + \frac{\partial L0}{\partial c_0}\right) H1R^{k_1}$$
$$+ \left(\frac{\partial M0}{\partial H0S}\frac{dH0S}{dc_0} + \frac{\partial M0}{\partial H0D}\frac{dH0D}{dc_0} + \frac{\partial M0}{\partial c_0}\right) H1R^{k_2} + \frac{F}{r} = 0 \tag{B.33}$$

Differentiating (B.30), (B.31) and (B.32), we get

$$\frac{\partial F_1}{\partial H0S}\frac{dH0S}{dH1R} + \frac{\partial F_1}{\partial H0D}\frac{dH0D}{dH1R} + \frac{\partial F_1}{\partial c_0}\frac{dc_0}{dH1R} + \frac{\partial F_1}{\partial H1R} = 0 \tag{B.34}$$

$$\frac{\partial F_2}{\partial H0S}\frac{dH0S}{dH1R} + \frac{\partial F_2}{\partial H0D}\frac{dH0D}{dH1R} + \frac{\partial F_2}{\partial c_0}\frac{dc_0}{dH1R} + \frac{\partial F_2}{\partial H1R} = 0 \tag{B.35}$$

$$\frac{\partial F_3}{\partial H0S}\frac{dH0S}{dH1R} + \frac{\partial F_3}{\partial H0D}\frac{dH0D}{dH1R} + \frac{\partial F_3}{\partial c_0}\frac{dc_0}{dH1R} + \frac{\partial F_3}{\partial H1R} = 0 \tag{B.36}$$

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