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Observer Based Nonlinear Quadratic Dynamic Matrix Control for State Space and I/O Models

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Abstract

Observer based nonlinear QDMC algorithm is presented for use with nonlinear state space and input-output models. The proposed algorithm is an extension of Nonlinear Quadratic Dynamic Matrix Control (NLQDMC) by García (1984) and its extension by Gattu and Zafiriou (1992a). García proposed an extension of linear Quadratic Dynamic Matrix Control (QDMC) to nonlinear processes. Although a nonlinear model is used, only a single Quadratic Program (QP) is solved on-line. Gattu and Zafiriou extended this formulation to open-loop unstable systems, by incorporating a Kalman filter. The requirement of solving only one QP on-line at each sampling time makes this algorithm an attractive option for industrial implementation. This extension of NLQDMC to open-loop unstable systems was *ad hoc* and did not address the problem of offset free tracking and disturbance rejection in a general state space setting. Independent white noise was added to the model states to handle unstable processes. The approach can stabilize the system but leads to an offset in the presence of persistent disturbances. To obtain offset free tracking Gattu and Zafiriou added a constant disturbance to the predicted output as done in DMC-type algorithms. This addition is *ad hoc* and does not result from the filtering/prediction theory. The proposed algorithm eliminates the major drawbacks of the algorithm presented by Gattu and Zafiriou and extends that algorithm for nonlinear models identified based on input-output information. An algorithm schematic is presented for measurement delay cases. The algorithm preserves the computational advantages when compared to the other algorithms based on nonlinear programming techniques. The illustrating examples demonstrate the usage of tuning parameters for unstable and stable systems and points out the benefits and short comings of the algorithm.

1 Introduction

Lately, the importance of utilizing nonlinear process models in process control applications has been very well recognized both by academia and industry. A significant number of Model Predictive Control (MPC) algorithms that utilize nonlinear process models in the on-line optimization have been appeared in the literature. In all these algorithms an objective function is minimized to compute the future manipulated variables. The various algorithms based on nonlinear programming techniques (Jang *et al.*, 1987; Brengel and Seider, 1989; Li and Biegler, 1989; Eaton and Rawlings, 1990; Patwardhan *et al.*, 1990; Bequette, 1991) differ in the way how the ordinary differential equations are solved and in the optimization approach utilized. García (1984) proposed an extension of linear Quadratic Dynamic Matrix Control (QDMC) to nonlinear processes (abbreviated to NLQDMC from here onwards). Although a nonlinear model is used, only a single Quadratic Program (QP) is solved on-line. Gattu and Zafiriou (1992a) extended this formulation to open-loop unstable systems, by incorporating a Kalman filter. The requirement of solving only one QP on-line at each sampling time makes this algorithm an attractive option for industrial implementation. This extension of NLQDMC to open-loop unstable systems was *ad hoc* and did not address the problem of offset free tracking and disturbance rejection in a general state space setting. Independent white noise was added to the model states to handle unstable processes. The approach can stabilize the system but leads to an offset in the presence of persistent disturbances. To obtain offset free tracking Gattu and Zafiriou (1992a) added a constant disturbance to the predicted output as done in DMC-type algorithms. This addition is *ad hoc* and does not result from the filtering/prediction theory. This is also pointed out by Lee and Ricker (1993).

There is a recent surge in the use of nonlinear models identified based on input-output information, for control purposes using the Model Predictive Control schemes. Saint-Donat *et al.* (1991) used neural network models in the on-line optimization. They solved the on-line optimization problem utilizing the nonlinear programming techniques. Hernandez and Arkun (1992, 1993) used the neural network models and polynomial ARMA models in the on-line optimization. They used the extended DMC algorithm (Peterson *et al.*, 1992) and the algorithm based on nonlinear programming techniques (Hernandez, 1992) for on-line control.

In this paper, we present an algorithm for use with nonlinear state space and input-output models which addresses the offset free tracking problem and disturbance rejection problem in a general setting. The paper is organized as follows. In section 2, observer design is presented. In section 3, we present the algorithm schematic for filter and predictor formulations and illustrate the use of tuning parameters by application of the algorithm to a simple example. The algorithm is extended to measurement delay cases in section 4. In section 5, we present the algorithm for input-output models and demonstrate the applicability on a simple example. The concluding remarks are presented in section 6.

2 Observer Design

Consider the nonlinear process and measurement models of the form

$$\dot{x} = f_c(x, u) \tag{1}$$

$$y = h(x) \tag{2}$$

where x is the state vector, u is the input vector, y is the output vector. Define

$$\begin{aligned} A_k &= (\frac{\partial f}{\partial x})|_{x=\xi_k, u=\nu_k} \\ B_k &= (\frac{\partial f}{\partial u})|_{x=\xi_k, u=\nu_k} \\ C_k &= (\frac{\partial h}{\partial x})|_{x=\xi_k} \end{aligned}$$

where the subscript k indicates the sampling instant k. ξ_k and ν_k are the values of x and u at sampling instant k, for which the values are set appropriately in the algorithm schematic. Let Φ_k and Γ_k be discrete state space matrices (e.g., Åström and Wittenmark, 1984), obtained from A_k, B_k and the sampling time. y_k be the measurement of the plant at k and $f(x_k, u_k)$ is denoted as the value of the state when the system model $\dot{x} = f_c(x, u)$ is integrated over one sampling time from the initial conditions x_k and u_k.

The idea is to approximate the nonlinear process as a linear model around the sampling instant, augment the linear model with additional linear states to describe the appropriate disturbances, then compute the estimator gains for the augmented system. Once the estimator gains have been computed, we use these estimator gains to update the nonlinear states and the augmented linear states to capture the effect of nonlinearity and disturbances. The material presented in this subsection is very well discussed in the linear system literature in some form or other. For the details the reader is referred to (Bitmead *et al.*,1990; Anderson and Moore, 1979). We consider the two sets of linear discrete models given as

Type A:

$$\begin{aligned} z_{j+1} &= \Phi_k z_j + \Gamma_k u_j + w_{1j} \\ \eta_{j+1} &= \eta_j + w_{2j} \\ y_j &= C_k z_j + \eta_j + v_j \end{aligned} \tag{3}$$

Type B:

$$\begin{aligned} z_{j+1} &= \Phi_k z_j + \Gamma_k u_j + \Gamma_k w_j + w_{1j} \\ w_{j+1} &= w_j + w_{2j} \\ y_j &= C_k z_j + v_j \end{aligned} \tag{4}$$

where w_{1j}, w_{2j} and v_j are the uncorrelated white noise sequences with $[{w_{1j}}^T, {w_{2j}}^T]^T \sim (0, Q)$ and $v \sim (0, R)$, Q and R being covariance matrices associated with process and measurement noises. z is the state vector of the linearized model, y_j is the measurement and η and w represent additional linear states to describe the disturbances.

Type A model, represents the process model augmented with the disturbance model for the disturbances which are step-like at the output. Type B model represents the augmented process and disturbances models for the step-like disturbances at the input. Offset free tracking in the presence of model-plant mismatch can be handled in an effective manner by the use of either type of models. Also, the observer designed based on the description of the either type can stabilize the open-loop unstable processes by putting the closed-loop observer poles inside the unit disk provided that the controller is designed such that the regulator poles are inside the unit disk. So, the disturbance model selection should be based on the knowledge of disturbances i.e. whether the disturbances are like additive steps at the output or slow drifts or like steps at the input or states. The only technical requirement in using these kind of disturbance models is the requirement of detectability. For the existence of stable filter, it is necessary that the augmented system is detectable. In general, it is required that the number of states augmented are less than or equal to the number of outputs for the detectability of the augmented system. Muske and Rawlings (1993a) also state this requirement. This requirement forced us to consider two separate models instead of treating them in a composite setting. For more details on the detectability of the augmented system, the reader is referred to Morari and Stephanopoulos (1980) and Davison and Smith (1972).

In our development, it is assumed that $Q \approx \begin{bmatrix} {\sigma_{w1}}^2 & 0 \\ 0 & {\sigma_{w2}}^2 \end{bmatrix}$ and $R \approx {\sigma_v}^2 I$ where ${\sigma_{w1}}^2, {\sigma_{w2}}^2$ and ${\sigma_v}^2$ are scalar variances. Define $\sigma_1 = \sigma_{w1}/\sigma_v, \sigma_2 = \sigma_{w2}/\sigma_v$ and let ${\sigma_v}^2 = 1$. The parameters σ_1 and σ_2 are used as a tuning parameters which determine the value of estimator gains. A detailed discussion on this kind of tuning parameter can be found in Gattu and Zafiriou (1992a).

Let ${K_k}^F \triangleq \begin{bmatrix} {K_k}^{F1} \\ {K_k}^{F2} \end{bmatrix}$ and ${K_k}^P \triangleq \begin{bmatrix} {K_k}^{P1} \\ {K_k}^{P2} \end{bmatrix}$ be the estimator gains computed using the Kalman filter formulations and predictor formulations respectively. The superscripts F and P stands for filter and predictor, 1 stands for the gain for the subsystem consisting of original states and 2 stands for the gain for the subsystem consisting of augmented states. These estimator gains are computed by solving Algebraic Riccati Equation (ARE) using the augmented system matrices and tuning parameters σ_1 and σ_2. For the literature on the solution of ARE and expressions for the estimator gains, the reader is referred to Åström and Wittenmark (1984), Bitmead *et al.*,(1990), Anderson and Moore (1979).

In this subsection, we have summarized the essential elements of observer design useful in the context of the paper and for details, we referred to the relevant literature appropriately.

2.1 Prediction equations for nonlinear system

In this subsection, we present the prediction equations for the nonlinear process model states and the augmented linear disturbance model states based on the estimator gains computed in the previous subsection.

2.1.1 Predictor formulation

Type A augmented system:

$$\begin{aligned}
\hat{x}_{k+1|k} &= f(\hat{x}_{k|k-1}, u_k) + K_k{}^{P1}[y_k - h(\hat{x}_{k|k-1}) - \hat{\eta}_{k|k-1}] && (5)\\
\hat{\eta}_{k+1|k} &= \hat{\eta}_{k|k-1} + K_k{}^{P2}[y_k - h(\hat{x}_{k|k-1}) - \hat{\eta}_{k|k-1}] && (6)\\
\hat{y}_{k+1|k} &= h(\hat{x}_{k+1|k}) + \hat{\eta}_{k+1|k} && (7)
\end{aligned}$$

By taking the conditional mean (Anderson and Moore, 1979), the P-step ahead predictions are

$$\begin{aligned}
\hat{x}_{k+i|k} &= f(\hat{x}_{k+i-1|k}, u_{k+i-1}) \quad i = 2, \ldots, P\\
\hat{\eta}_{k+i|k} &= \hat{\eta}_{k+i-1|k} \quad i = 2, \ldots, P\\
\hat{y}_{k+i|k} &= h(\hat{x}_{k+i|k}) + \hat{\eta}_{k+i|k} \quad i = 2, \ldots, P && (8)
\end{aligned}$$

Type B augmented system:

$$\begin{aligned}
\hat{x}_{k+1|k} &= f(\hat{x}_{k|k-1}, u_k) + \Gamma_k \hat{w}_{k|k-1} + K_k{}^{P1}[y_k - h(\hat{x}_{k|k-1})] && (9)\\
\hat{w}_{k+1|k} &= \hat{w}_{k|k-1} + K_k{}^{P2}[y_k - h(\hat{x}_{k|k-1})] && (10)\\
\hat{y}_{k+1|k} &= h(\hat{x}_{k+1|k}) && (11)
\end{aligned}$$

The P-step ahead prediction equations are

$$\begin{aligned}
\hat{x}_{k+i|k} &= f(\hat{x}_{k+i-1|k}, u_{k+i-1}) + \Gamma_k \hat{w}_{k+i-1|k} \quad i = 2, \ldots, P\\
\hat{w}_{k+i|k} &= \hat{w}_{k+i-1|k} \quad i = 2, \ldots, P\\
\hat{y}_{k+i|k} &= h(\hat{x}_{k+i|k}) \quad i = 2, \ldots, P && (12)
\end{aligned}$$

with $\hat{x}_{0|-1} = x_0$, $\hat{\eta}_{0|-1} = 0$ and $\hat{w}_{0|-1} = 0$. The notation $\hat{x}_{k|k-1}$ represents the estimate of x at k based on the information at $k-1$. P is the prediction horizon. To avoid complexity in notation, we used the same notation for estimator gains for both Type A and Type B systems. But in actuality they come from solving ARE's with different system matrices.

2.1.2 Filter formulation

Type A augmented system:

$$\begin{aligned}
\hat{x}_{k|k} &= \hat{x}_{k|k-1} + K_k{}^{F1}[y_k - h(\hat{x}_{k|k-1}) - \hat{\eta}_{k|k-1}] && (13)\\
\hat{\eta}_{k|k} &= \hat{\eta}_{k|k-1} + K_k{}^{F2}[y_k - h(\hat{x}_{k|k-1}) - \hat{\eta}_{k|k-1}] && (14)
\end{aligned}$$

with

$$\begin{aligned}\hat{x}_{k|k-1} &= f(\hat{x}_{k-1|k-1}, u_{k-1}) \\ \hat{\eta}_{k|k-1} &= \hat{\eta}_{k-1|k-1}\end{aligned}$$

The P-step ahead predictions are

$$\hat{x}_{k+i|k} = f(\hat{x}_{k+i-1|k}, u_{k+i-1}) \quad i = 1, \dots, P \tag{15}$$
$$\hat{\eta}_{k+i|k} = \hat{\eta}_{k+i-1|k} \quad i = 1, \dots, P \tag{16}$$
$$\hat{y}_{k+i|k} = h(\hat{x}_{k+i|k}) + \hat{\eta}_{k+i|k} \quad i = 1, \dots, P \tag{17}$$

Type B augmented system:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + \Gamma_k \hat{w}_{k|k-1} + K_k{}^{F1}[y_k - h(\hat{x}_{k|k-1} + \Gamma_k \hat{w}_{k|k-1})] \tag{18}$$
$$\hat{w}_{k|k} = \hat{w}_{k|k-1} + K_k{}^{F2}[y_k - h(\hat{x}_{k|k-1} + \Gamma_k \hat{w}_{k|k-1})] \tag{19}$$

with

$$\begin{aligned}\hat{x}_{k|k-1} &= f(\hat{x}_{k-1|k-1}, u_{k-1}) \\ \hat{w}_{k|k-1} &= \hat{w}_{k-1|k-1}\end{aligned}$$

The P-step ahead predictions are

$$\hat{x}_{k+i|k} = f(\hat{x}_{k+i-1|k}, u_{k+i-1}) + \Gamma_k \hat{w}_{k+i-1|k} \quad i = 1, \dots, P \tag{20}$$
$$\hat{w}_{k+i|k} = \hat{w}_{k+i-1|k} \quad i = 1, \dots, P \tag{21}$$
$$\hat{y}_{k+i|k} = h(\hat{x}_{k+i|k}) \quad i = 1, \dots, P \tag{22}$$

with the Initial conditions $\hat{x}_{0|-1} = x_0$, $\hat{\eta}_{0|-1} = 0$ and $\hat{w}_{0|-1} = 0$.

3 Algorithm

The predicted output is divided into two parts as the effect of past and the effect of future manipulated variables as done in the original version of NLQDMC (García, 1984).

$$\hat{y}_{k+i|k} = \hat{y}^{\star}_{k+i|k} + \hat{y}^{f}_{k+i|k} \quad i = 1, \dots, P \tag{23}$$

where $\hat{y}^{\star}$ is the effect of past and $\hat{y}^{f}$ is the effect of future.

The effect of future control moves

The contribution of the effect of future manipulated variables to the predicted output is given as $\sum_{i=1}^{l} S_{l,k} \Delta u_{k+i-l}$ $(i = 1, 2, \dots, P)$, where Δu is the change in manipulated variables, defined as $\Delta u_k \triangleq u_k - u_{k-1}$ and $S_{l,k}$ are the step response coefficient matrices obtained by

$$S_{l,k} = \sum_{j=1}^{l} C_k \Phi_k^{j-1} \Gamma_k \quad (l = 1, 2, \dots, P) \tag{24}$$

For more details on computation of step response coefficients, the reader is referred to Gattu and Zafiriou (1992a).

The effect of past control moves

The effect of past $\hat{y}^{\star}_{k+i|k}$ is defined as the predicted output if there are no input changes in the future. Therefore, $\hat{y}^{\star}_{k+i|k}$ is computed by using (7) and (8) or (11) and (12) or (17) or (22) by setting $u_{k+i} = u_{k-1}$ for $i = 0, 1, \ldots, P-1$.

Optimization

$$\min_{\Delta u_k, \ldots, \Delta u_{k+M-1}} \sum_{l=1}^{P} (\hat{y}_{k+l|k} - r_{k+l})^T D^2 (\hat{y}_{k+l|k} - r_{k+l}) + \Delta u^T_{k+l-1} \Lambda^2 \Delta u_{k+l-1} \tag{25}$$

where P is the prediction horizon and M is the number of future moves to be optimized. It is assumed that $u_{k+M-1} = u_{k+M} = \ldots = u_{k+P-1}$. D and Λ are diagonal weight matrices and r is the reference setpoint.

The above optimization problem with constraints can be written as a standard Quadratic Programming problem, as shown in García and Morshedi (1986):

$$\min_X \Phi(X) = \frac{1}{2} X^T G X + g^T X \tag{26}$$

subject to:

$$H^T X \geq b \tag{27}$$

where

$$X = [\Delta u_k \ldots \Delta u_{k+M-1}]^T \tag{28}$$

and H and b depend on the constraints on manipulated variables, change in manipulated variables and outputs.

The M future manipulated variables are computed, but only the first move is implemented (García and Morshedi, 1986).

3.1 Algorithm schematic

Depending on whether the predictor formulation is used or the filter formulation is used, the sequence of computations is slightly different. We provide two schematics and make appropriate comments. The sampling instant is k in both the schematics.

Schematic I:

This schematic is based on the predictor formulation.

(a) Set $\xi_k = \hat{x}_{k|k-1}$ and $\nu_k = u_{k-1}$.

(b) Linearize (1) and (2) to get A_k, B_k and C_k and discretize to obtain Φ_k and Γ_k.

(c) Compute the step response coefficients.

(d) Compute the estimator gain K^P

(e) Compute $\hat{y}^{*}_{k+i|k}$ for $i = 1, \ldots, P$ using (7) and (8) or (11) and (12) by setting $u_{k+i-1} = u_{k-1}$ for $i = 1, \ldots, P-1$

(f) Solve QP and implement u_k

(g) Obtain $\hat{x}_{k+1|k}$ using (5) or (9).

Schematic II:

This schematic is based on the filter formulation.

(a) Set $\xi_k = \hat{x}_{k|k-1}$ and $\nu_k = u_{k-1}$.

(b) Linearize (1) and (2) to get A_k, B_k and C_k and discretize to obtain Φ_k and Γ_k.

(c) Compute the estimator gain K^F

(d) Obtain $\hat{x}_{k|k}$ using (13) or (18)

(e) Set $\xi_k = \hat{x}_{k|k}$ and $\nu_k = u_{k-1}$.

(f) Linearize (1) and (2) to get A_k, B_k and C_k.

(g) Compute the step response coefficients.

(h) Compute $\hat{y}^{*}_{k+i|k}$ for $i = 1, \ldots, P$ using (17) or (22) by setting $u_{k+i-1} = u_{k-1}$ for $i = 1, \ldots, P-1$

(i) Solve QP and implement u_k

(j) Obtain $\hat{x}_{k+1|k}$ using (15) or (20).

Remark 1: In the schematic based on filter formulation, linearization is done twice. The additional benefit we are getting by two linearizations is that the linear model obtained by linearing the nonlinear model at $\hat{x}_{k|k}$ and u_{k-1} is used to compute the effect of future inputs. Whereas in the predictor formulation, the linearized model obtained by linearizing the nonlinear model at $\hat{x}_{k|k-1}$ and u_{k-1} is used to compute the effect of the future inputs.

Remark 2: For the Type A augmented system, if there is no process noise i.e., if $\sigma_1 = 0$, the filter and predictor formulations are identical. For a choice of $\sigma_1 = 0$, the disturbances are viewed as independent random steps affecting each of the plant outputs with measurement noise being white, i.e., type 1 disturbances as discussed by Morari and Lee (1991). For this choice of tuning parameter, the gains ${K_k}^{F1}$ and ${K_k}^{P1}$ result in a value of zero. The gains ${K_k}^{F2}$ and ${K_k}^{P2}$ will be equal and result in diagonal matrices with each diagonal element $K_{k,i}$ given by an expression (Ricker, 1991)

$$K_{k,i} = \frac{2}{1+\sqrt{1+\frac{4}{\sigma_2{}^2}}} \qquad for \quad i = 1, 2, \ldots, p \tag{29}$$

where p is the number of outputs. As the value of σ_2 varies from zero to infinity with $\sigma_1 = 0$, ${K_k}^{F2}$ and ${K_k}^{P2}$ will take a value from zero to I. For a value of I, the formulation is identical to García's (1984) nonlinear version of QDMC.

Remark 3: For type 2 disturbances i.e. independent double integrated white noise affecting each of the outputs (Morari and Lee, 1991), the augmented system can be described as

$$\begin{aligned} z_{j+1} &= \Phi_k z_j + \Gamma_k u_j \\ \eta_{j+1} &= \eta_j + \eta_j{}' \\ \eta_{j+1}{}' &= \eta_j{}' + w_{2j} \\ y_j &= C_k z_j + \eta_j + v_j \end{aligned}$$

For a choice of $\sigma_1 = 0.0$ and for a nonzero value of σ_2, the the expression for the estimator gain, $K_{k,i} \triangleq \begin{bmatrix} K_{k,i}{}^1 \\ K_{k,i}{}^2 \end{bmatrix}$ is given by (Ricker, 1991)

$$\begin{aligned} K_{k,i}{}^1 &= \frac{\sqrt{2}}{2}\sqrt{\sigma_2\sqrt{\sigma_2{}^2+16}-\sigma_2{}^2} \qquad for \qquad i = 1, \ldots, p \\ K_{k,i}{}^2 &= \frac{(K_{k,i}{}^1)^2}{K_{k,i}{}^1+2} \qquad for \qquad i = 1, \ldots, p \end{aligned}$$

where $K_{k,i}$ has the same meaning as in remark 2.

Remark 4: In the begining of the simulation, the estimator (Type A or Type B) is initialized by solving equations $5-7$ or $9-11$ open-loop. It is suggested to do an open-loop simulation keeping the input constant at the steady state value to obtain the steady state values for the estimator states.

Remark 5: Throughout the formulation, persistent disturbances entering the state equations are considered as input disturbances by adding $\Gamma_k w$ to the state equations. However, Γ_k in $\Gamma_k w$ of (7) can be different provided that the augmented system satisfies the detectability requirement.

3.2 Tuning guidelines for σ_1 and σ_2

3.2.1 Type A model:

For open-loop unstable systems, nonzero values of σ_1 and σ_2 are recommended. A nonzero value of σ_1 puts the closed-loop poles of the observer inside the unit disk and a nonzero value of σ_2 rejects the disturbances or tracks the nonzero setpoints without offset. For stable systems, σ_1 is not required.

3.2.2 Type B model:

A nonzero value of σ_2 is recommended for both open-loop unstable and stable system for offset free disturbance rejection and tracking.

In both the cases, smaller values for σ_1 and σ_2 are recommended in the presence of measurement noise.

3.3 Illustration

In this section, the proposed algorithm is applied to control a chemical reactor at unstable steady state point. Predictor formulation is used in all the simulations. The example problem is taken from Sistu and Bequette (1991). The process is an exothermic, first order, irreversible reaction in an adiabatic CSTR with dynamic equations given by

$$\begin{aligned}
\frac{dx_1}{dt} &= -\phi x_1 exp(\frac{x_2}{1+x_2/\gamma}) + q(x_{1f} - x_1) \\
\frac{dx_2}{dt} &= \beta\phi x_1 exp(\frac{x_2}{1+x_2/\gamma}) - (q+\delta)x_2 + \delta u + q x_{2f} \\
y &= x_2
\end{aligned}$$

where x_1 is the dimensionless concentration, x_2 is the dimensionless reactor temperature and u is the dimensionless cooling jacket temperature. The controlled variable is the dimensionless reactor temperature and the manipulated variable is the dimensionless cooling jacket temperature. For the manipulated variable value of zero, the plant has three stable steady states, with the unstable steady state at $[0.5528, 2.7517]$ and two stable steady states at $[0.8560, 0.8859]$ and $[.2354, 4.7050]$. The values of the parameters used are $\beta = 8.0, \delta = 0.3, x_{1f} = 1.0, q = 1.0, \phi = 0.072, \gamma = 20.0, x_{2f} = 0.0$ and $u = 0.0$. The objective of the control is to control the reactor temperature at the unstable steady state point in the presence of disturbances and parametric uncertainty. In the simulations, the parametric uncertainty is simulated by giving the values for heat transfer coefficient δ as $\delta_{plant} = 0.3$ and $\delta_{model} = 0.2$. In all the simulations, we assume that there is a parametric uncertainty, a values of $P = 5$, $M = 2$ and a sampling time of 0.25 is used and the input is constrained between -1.0 and 2.0. The steady state equilibrium curve is given in figure 1. Note that the model has only one steady state for the manipulated variable value of zero. However, for a small negative value of input, the model has three steady states.

Setpoint tracking

Figure 2 illustrates the response of the reactor temperature for a setpoint change from the lower steady state of $y_0 = 0.8859$ to the unstable steady state point corresponds to $y = 2.7517$. With type B model, tuning parameter values of $\Lambda = 0.2$,$\sigma_1 = 0.0$ and $\sigma_2 = 100.0$ are used in the simulations. With type A model, tuning parameter values of $\Lambda = 0.2$,$\sigma_1 = 1.0$ and $\sigma_2 = 10.0$ are used. The results are comparable with those of Sistu and Bequette who use both state and parameter estimation. In the presence of parametric uncertainty, Sitsu and Bequette observed that the use of state estimation alone results in a poor performance.

The response by using type A model is much sluggish than the response by using type B model. This is because of the location of the observer poles of the linearized models. The observer poles are not affected much by tuning parameters and therefore the performance cannot be improved by tuning when type A model is used.

Output disturbances

Figure 3 demonstrates the response of the reactor temperature for a step output disturbance in a reactor running at unstable steady state setpoint. A value of 0.3 is used for the output disturbance. The output disturbance value of 0.3 pushes the plant steady state away from the model and increases the model-plant mismatch. With Type A model, tuning parameter values of $\Lambda = 0.05$, $\sigma_1 = 2$ and $\sigma_2 = 10.0$ and with the Type B model, tuning parameter values of $\Lambda = 0.5$, $\sigma_1 = 0.0$ and $\sigma_2 = 100.0$ are used. Since the system is open-loop unstable a nonzero value of σ_1 is must to stabilize the system and a nonzero value of σ_2 is used to get the offset free response. Also, as observed in the case of setpoint tracking, by using Type A model, due to the location of the poles of the observers of the linearized models, the integral action is slow.

Input disturbances

Figure 4 demonstrates the response for an input disturbance of 0.3 in a reactor running at open-loop unstable steady state setpoint. With Type A model, tuning parameter values of $\Lambda = 0.05$, $\sigma_1 = 0.1$ and $\sigma_2 = 10.0$ and with the Type B model, tuning parameter values of $\Lambda = 0.1$, $\sigma_1 = 0.0$ and $\sigma_2 = 100.0$ are used. As observed in the case of output disturbances, the performance using Type B model is better than the performance using Type A model.

To summarize, for open-loop unstable processes use of augmented models of Type B gives a better performance. Similar observation is made by Muske and Rawlings (1993b) for control of linear unstable systems.

4 Measurement delay systems

So far it is assumed that, there is no delay in the measurements. In quite a few process control applications, there is always a delay from the sensors. So, it is important to consider the measurement delay information. In this section we present the algorithm for the measurement delay processes. It is assumed that the sum of distributed delays is represented by a lumped delay of d sampling intervals at the plant output.

For simplicity, we present the prediction equations and the algorithm only for Type A augmented system with the predictor formulation. It is easy to write the other cases based on this formulation.

4.1 Prediction equations

$$\hat{x}_{k-d+1|k-d} = f(\hat{x}_{k-d|k-d-1}, u_{k-d}) + {K_{k-d}}^{P1}[y_{k-d} - h(\hat{x}_{k-d|k-d-1}) - \hat{\eta}_{k-d|k-d-1}] \quad (30)$$

$$
\begin{aligned}
\hat{\eta}_{k-d+1|k-d} &= \hat{\eta}_{k-d|k-d-1} + K_{k-d}{}^{P2}[y_{k-d} - h(\hat{x}_{k-d|k-d-1}) - \hat{\eta}_{k-d|k-d-1}] && (31)\\
\hat{x}_{k-d+2|k-d} &= f(\hat{x}_{k-d+1|k-d}, u_{k-d+1}) && (32)\\
\hat{\eta}_{k-d+2|k-d} &= \hat{\eta}_{k-d+1|k-d} && (33)\\
\vdots &\quad \vdots && (34)\\
\hat{x}_{k|k-d} &= f(\hat{x}_{k-1|k-d}, u_{k-1}) && (35)\\
\hat{\eta}_{k|k-d} &= \hat{\eta}_{k-1|k-d} && (36)\\
& && (37)
\end{aligned}
$$

The P-step ahead predictions are

$$
\begin{aligned}
\hat{x}_{k+i|k-d} &= f(\hat{x}_{k+i-1|k-d}, u_{k+i-1}) \quad i = 1, \ldots, P\\
\hat{\eta}_{k+i|k-d} &= \hat{\eta}_{k+i-1|k-d} \quad i = 1, \ldots, P\\
\hat{y}_{k+i|k-d} &= h(\hat{x}_{k+i|k-d}) + \hat{\eta}_{k+i|k-d} \quad i = 1, \ldots, P
\end{aligned} \tag{38}
$$

4.2 Algorithm schematic

(a) Set $\xi_k = \hat{x}_{k-d|k-d-1}$ and $\nu_k = u_{k-d-1}$.

(b) Linearize (1) and (2) to get A_k, B_k and C_k and discretize to obtain Φ_k and Γ_k.

(c) Compute the estimator gain K^P

(d) Obtain $\hat{x}_{k|k-d}$ using (35)

(e) Set $\xi_k = \hat{x}_{k|k-d}$ and $\nu_k = u_{k-1}$.

(f) Linearize (1) and (2) to get A_k, B_k and C_k.

(g) Compute the step response coefficients.

(h) Compute $\hat{y}^{\star}_{k+i|k-d}$ for $i = 1, \ldots, P$ using (38) by setting $u_{k+i-1} = u_{k-1}$ for $i = 1, \ldots, P-1$

(i) Solve QP and implement u_k

(j) Obtain $\hat{x}_{k-d+1|k-d}$ using (30).

5 Input-Output models

In this section, we present the observer based NLQDMC algorithm for the control of nonlinear processes based on the models identified from input-output data. The models of the form

$$
y_k = f(y_{k-1}, y_{k-2}, \ldots, y_{k-n_y}, u_{k-1}, u_{k-2}, \ldots, u_{k-n_u}) \tag{39}
$$

where n_y is the number of past outputs, n_u is the number of past inputs, are considered whether they are identified using neural networks or polynomial ARMA structure or by some other input-output identification method. For simplicity sake, only the predictor formulation is presented here. The filter formulation can be formulated as in the case of state space formulation. For better understanding, first the procedure for one-step ahead prediction is presented for the linear models and later the nonlinear implementation is outlined. Consider the linear model given by

$$y_j = -A_1 y_{j-1} - A_2 y_{j-2} \ldots - A_{n_y} y_{j-n_y} + B_1 u_{j-1} + B_2 u_{j-2} + \ldots + B_{n_u} u_{j-n_u} \tag{40}$$

Then a minimal state space realization is constructed using the above model. Software from the package CONSYD is used to construct minimal state space model. The software is based on the algorithm developed by Economou (1982) which utilizes Rosenbrock's (1968) algorithm. Let the minimal state space model is given by

$$\begin{aligned} x_{j+1} &= \Phi x_j + \Gamma u_j \\ y_j &= C x_j \end{aligned} \tag{41}$$

Then the observer is designed by augmenting the disturbance models to the above model equations as in the case of state space formulation. The augmented models are given as

Type A:

$$\begin{aligned} x_{j+1} &= \Phi x_j + \Gamma u_j + w_{1j} \\ \eta_{j+1} &= \eta_j + w_{2j} \\ y_j &= C x_j + \eta_j + v_j \end{aligned} \tag{42}$$

Type B:

$$\begin{aligned} x_{j+1} &= \Phi x_j + \Gamma u_j + \Gamma w_j + w_{1j} \\ w_{j+1} &= w_j + w_{2j} \\ y_j &= C x_j + v_j \end{aligned} \tag{43}$$

Let $K^P \triangleq \begin{bmatrix} K^{P1} \\ K^{P2} \end{bmatrix}$ be the estimator gain computed using either (42) or (43). The one-step ahead prediction equations are given as

Type A augmented system:

$$\hat{x}_{j+1|j} = \Phi \hat{x}_{j|j-1} + \Gamma u_j + K^{P1}[y_j - C\hat{x}_{j|j-1} - \hat{\eta}_{j|j-1}] \tag{44}$$

$$\hat{\eta}_{j+1|j} = \hat{\eta}_{j|j-1} + K^{P2}[y_j - C\hat{x}_{j|j-1} - \hat{\eta}_{j|j-1}] \tag{45}$$

$$\hat{y}_{j+1|j} = C\hat{x}_{j+1|j} + \hat{\eta}_{j+1|j} \tag{46}$$

by taking the z-transform, $\hat{y}(z)$ is given by

$$\hat{y}(z) = C(zI-\Phi)^{-1}\Gamma u(z) + [C(zI-\Phi)^{-1}K^{P1} + (zI-I)^{-1}K^{P2}](y(z) - C\hat{x}(z) - \hat{\eta}(z)) \quad (47)$$

Since by construction $C(zI-\Phi)^{-1}\Gamma = (I+\sum_{i=1}^{n_y} A_i z^{-i})^{-1}(\sum_{i=1}^{n_u} B_i z^{-i})$

$$\begin{aligned} \hat{y}(z) &= (I+\sum_{i=1}^{n_y} A_i z^{-i})^{-1}(\sum_{i=1}^{n_u} B_i z^{-i})u(z) + \\ &\quad [C(zI-\Phi)^{-1}K^{P1} + (zI-I)^{-1}K^{P2}](y(z)-\hat{y}(z)) \end{aligned} \quad (48)$$

$$\begin{aligned} \hat{y}(z) &= -\sum_{i=1}^{n_y} A_i z^{-i}\hat{y}(z) + (\sum_{i=1}^{n_u} B_i z^{-i})u(z) + \\ &\quad (I+\sum_{i=1}^{n_y} A_i z^{-i})[C(zI-\Phi)^{-1}K^{P1} + (zI-I)^{-1}K^{P2}](y(z)-\hat{y}(z)) \end{aligned} \quad (49)$$

The first two terms on the right hand side of the above prediction equation represent the contribution from the original process model and the third term is due to the correction for the disturbance model assumed. Therefore, in the time domain the predicted output at j can be represented as

$$\hat{y}_{j|j-1} = \hat{y}^d_{j|j-1} + \hat{y}^c_{j|j-1} \quad (50)$$

where $\hat{y}^d_{j|j-1}$ represents the deterministic contribution and $\hat{y}^c_{j|j-1}$ represents the correction due to stochastic disturbance assumptions. In the time domain representation the deterministic contribution is given by

$$\hat{y}^d_{j|j-1} = -A_1\hat{y}_{j-1|j-2} - A_2\hat{y}_{j-2|j-3} \ldots - A_{n_y}\hat{y}_{j-n_y|j-n_y-1} + B_1 u_{j-1} + B_2 u_{j-1} + \ldots + B_{n_u} u_{j-n_u} \quad (51)$$

Stochastic contribution

Define $A_0 \triangleq I, \hat{v}(z) \triangleq y(z) - \hat{y}(z)$ and denote the third term on the right hand side of (49) as $\hat{y}^c(z)$.

$$\hat{y}^c(z) = (\sum_{i=0}^{n_y} A_i z^{-i})[C(zI-\Phi)^{-1}K^{P1} + (zI-I)^{-1}K^{P2}]\hat{v}(z) \quad (52)$$

$$\hat{y}^c(z) = (\sum_{i=0}^{n_y} A_i z^{-i})C(zI-\Phi)^{-1}K^{P1}\hat{v}(z) + (\sum_{i=0}^{n_y} A_i z^{-i})(zI-I)^{-1}K^{P2}\hat{v}(z) \quad (53)$$

Denoting the first term and the second term of the right hand side of the above expression as $\hat{y}^{c1}(z)$ and $\hat{y}^{c2}(z)$ respectively,

$$\hat{y}^c(z) = \hat{y}^{c1}(z) + \hat{y}^{c2}(z) \quad (54)$$

The corresponding time domain expression is

$$\hat{y}^c_{j|j-1} = \hat{y}^{c1}_{j|j-1} + \hat{y}^{c2}_{j|j-1} \quad (55)$$

Now, $\hat{y}^{c1}(z)$ can be represented in the time domain using the state space representation as

$$\begin{aligned} x_{l+1} &= \Phi x_l + K^{P1}\hat{v}_l \\ y_{i,l} &= A_i C x_{l-i} \\ \hat{y}^{c1}_{j|j-1} &= \sum_{i=0}^{n_y} y_{i,j} \end{aligned} \tag{56}$$

with initial conditions $x_0 = 0$ and $\hat{v}_l = y_l - \hat{y}_{l|l-1}$. Since the identified input-output model assumes the use of only past n_y outputs information, in the computation of $\hat{y}^{c1}_{j|j-1}$, the values of $\hat{v}_l$ are set to zero for $l < j - n_y$. Similarly $\hat{y}^{c2}_{j|j-1}$ is represented in the state space form by replacing Φ by I of appropriate dimension and A_iC by A_i and K^{P1} by K^{P2} in (56).

Type B augmented system:

$$\hat{x}_{j+1|j} = \Phi\hat{x}_{j|j-1} + \Gamma u_j + \Gamma\hat{w}_{j|j-1} + K^{P1}[y_j - C\hat{x}_{j|j-1}] \tag{57}$$

$$\hat{w}_{j+1|j} = \hat{w}_{j|j-1} + K^{P2}[y_j - C\hat{x}_{j|j-1}] \tag{58}$$

$$\hat{y}_{j+1|j} = C\hat{x}_{j+1|j} \tag{59}$$

By taking the z-transform and on simplification

$$\begin{aligned} \hat{y}(z) &= -\sum_{i=1}^{n_y} A_i z^{-i}\hat{y}(z) + (\sum_{i=1}^{n_u} B_i z^{-i})u(z) + (\sum_{i=1}^{n_u} B_i z^{-i})w(z) + \\ & \quad (I + \sum_{i=1}^{n_y} A_i z^{-i})[C(zI - \Phi)^{-1}K^{P1}](y(z) - \hat{y}(z)) \end{aligned} \tag{60}$$

To represent the predicted output in the time domain similar procedure is used as that for type A augmented system.

5.1 Nonlinear implementation

At sampling instant k, linear model is obtained by linearizing (39) at $y_{k|k-1}, \ldots, y_{k-n_y+1|k-n_y}$ and $u_{k-1}, \ldots, u_{k-n_u}$ and is given by

$$y_j = -A_{1,k}y_{j-1} - A_{2,k}y_{j-2} - \ldots A_{n_y,k}y_{j-n_y} + B_{1,k}u_{j-1} + B_{2,k}u_{j-2} + \ldots + B_{n_u,k}u_{j-n_u} \tag{61}$$

The corresponding minimal state space realization is represented as

$$\begin{aligned} x_{j+1} &= \Phi_k x_j + \Gamma_k u_j \\ y_j &= C_k x_j \end{aligned} \tag{62}$$

Then the additional states to represent the disturbances are augmented to obtain type A or type B augmented model and the estimator gain $K_k{}^P \triangleq \begin{bmatrix} K_k{}^{P1} \\ K_k{}^{P2} \end{bmatrix}$ is computed.

Prediction

The predicted output is expressed as the sum of deterministic contribution and the correction due to stochastic disturbance assumptions.

$$\hat{y}_{k+i|k} = \hat{y}^d_{k+i|k} + \hat{y}^c_{k+i|k} \qquad for \quad i = 1, \ldots, P \tag{63}$$

where P is the prediction horizon.

Deterministic contribution

Define, $m(l) \triangleq min(k, l)$

$$\hat{y}^d_{k+i|k} = f(y_{k+i-1|m(k+i-2)}, \ldots, y_{k+i-n_y|m(k+i-n_y-1)}, u_{k+i-1}, \ldots, u_{k+i-n_u}) \tag{64}$$

for $i = 1, \ldots, P$. The above expression is a nonlinear function in the future manipulated variables. Therefore, to formulate the optimization problem as a single quadratic program the deterministic contribution is subdivided into the effect of past and future.

$$\hat{y}^d_{k+i|k} = \hat{y}^{dp}_{k+i|k} + \hat{y}^{df}_{k+i|k} \qquad for \qquad i = 1, \ldots, P \tag{65}$$

where $\hat{y}^{dp}_{k+i|k}$ is the effect of past and $\hat{y}^{df}_{k+i|k}$ is the effect of future. The effect of past is computed by setting $u_{k+i} = u_{k-1}$ for $i = 0, 1, \ldots, P-1$. The computation of effect of future is same as that described in section 3 for state space formulation of the algorithm.

Stochastic contribution

The computation of the linear correction $\hat{y}^c_{k+i|k}$ is exactly same as that described in the previous subsection with the only modification that the system matrices A_i, Φ, K^P, Γ and C are replaced by $A_{i,k}, \Phi_k, {K_k}^P, \Gamma_k$ and C_k respectively. As in the case of state space formulation of the algorithm, in the absence of measurement information in the future, by taking the conditional mean, it is assumed that $y_{k+i} - \hat{y}_{k+i|k} = 0$ for $i = 1, \ldots, P$.

Once the predicted output is computed, the future manipulated variables are obtained by solving the optimization problem (25).

5.2 Algorithm schematic

(a) Linearize (39) at $y_{k|k-1}, \ldots, y_{k-n_y+1|k-n_y}$ and $u_{k-1}, \ldots, u_{k-n_u}$ to obtain $A_{i,k}$ for $i = 1, \ldots, n_y$ and $B_{i,k}$ for $i = 1, \ldots, n_u$

(b) Obtain the minimal state realization of the linearized input-output model

(c) Compute the step response coefficients

(d) Compute the estimator gain ${K_k}^P$

(e) Compute $\hat{y}^{dp}_{k+i|k}$ for $i = 1, \ldots, P$ using (64) by setting $u_{k+i} = u_{k-1}$ for $i = 0, 1, \ldots, P-1$.

(f) Compute the linear correction based on the expressions (52) – (56)

(g) Solve QP and implement u_k

(h) Obtain $\hat{y}_{k+1|k}$ using (63) with deterministic part computed using nonlinear input-output model

5.3 Illustration

In this section, the algorithm is applied to control the reactions in series ($A \rightarrow B \rightarrow C$) in a CSTR. The desired product is the intermediate product B. The differential equations describing the system are given by (Hernandez, 1992)

$$
\begin{aligned}
\frac{dx_1}{dt} &= 1 - x_1 - E_3 exp(\frac{-E_1}{x_3})x_1 + E_4 exp(\frac{-E_2}{x_3})x_2 \\
\frac{dx_2}{dt} &= -x_2 + E_3 exp(\frac{-E_1}{x_3})x_1 - E_4 exp(\frac{-E_2}{x_3})x_2 \\
\frac{dx_3}{dt} &= u - x_3 + 0.05(E_3 exp(\frac{-E_1}{x_3})x_1 - E_4 exp(\frac{-E_2}{x_3})x_2) \\
y &= x_2
\end{aligned}
$$

with $E_1 = 50$, $E_2 = 70$, $E_3 = 300000$ and $E_4 = 60000000$; where x_1 and x_2 are the dimensionless concentrations of A and B in the reaction mixture, x_3 is the dimensionless reactor temperature and u is the dimensionless temperature of the jacket surrounding the reactor vessel. The reactor equilibrium curve of concentration of B as a function of jacket temperature has a well defined maximum. The control objective is to operate the reactor at the maximum concentration of product B. The concentration has a maximum value of 0.314. The output maximum of this process makes this example a challenging problem for control due to the change in the sign of the gain around this steady state. Gattu and Zafiriou (1992b) demonstrated the successful application of state estimation NLQDMC algorithm to this process for setpoint changes and input disturbances without model-plant mismatch. In this paper, we use the plant described by the above differential equations and the model as the input-output description taken from Hernandez (1992). The model-plant mismatch introduced here makes this example more interesting and challenging.

The input-output model is a sigmoid polynomial model and is given as (Hernandez,1992)

$$
y(k+1) = \sigma(\theta_0 + \theta_1 y(k) + \theta_2 u_s(k-1) + \theta_3 y^3(k-2) + \theta_4 u_s(k) u_s(k-1) u_s(k-3))
$$

where, the map $\sigma(.)$ is the sigmoid function defined as

$$\sigma(x) = \frac{1}{1 + exp(-x)}$$

which is bounded between 0 and 1. The input u_s is the scaled input and is defined as

$$u_s = \frac{u-2}{4}$$

The parameter values are $\theta_0 = -2.751$, $\theta_1 = 6.791$, $\theta_2 = .241$, $\theta_3 = -8.693$ and $\theta_4 = -.226$. The steady state equilibrium curves for the model and plant are given in figure (5). In all the simulations tuning parameter values of $P = 10$ and $M = 1$ are used and a sampling time value of 0.5 is used. The input is constrained between -2 and 2.

Figure 6 demonstrates the response for a step change in setpoint starting from the left of the peak. A step setpoint change is made from the steady state value of $y = 0.1461$ and $u = 3.5$ to a value of $y = 0.314$. The tuning parameter values of $\sigma_1 = 0.0$, $\sigma_2 = 1000.0$ and $\Lambda = 1.0$ are used using both Type A and Type B models. In the entire left side of the peak, the model and plant gains have the same sign. So, it does not a pose a challenging problem and the responses using both Type A and Type B models are excellent.

Figure 7 demonstrates the response for a step change in setpoint from the right of the peak. A step setpoint change is made from the steady state value of $y = 0.1581$ and $u = 5.5$ to a value of $y = 0.314$. The tuning parameter values of $\sigma_1 = 0.0.\sigma_2 = 1000.0$ and $\Lambda = 0.05$ are used with Type A model and the tuning parameter values of $\sigma_1 = 0.0.\sigma_2 = 1000.0$ and $\Lambda = 0.3$ are used with Type B model. Note that the gain of the model equilibrium curve changes its sign on the right side of the plant peak. The use of Type B model resulted in the steady state offset. The control action got "stuck" at the zero gain area of the model and plant settled at an output value corresponding the input value at the zero gain area. Whereas by using Type A model, the control and observer parameters can be tuned such a way that it does not get stuck at the zero gain area. The oscillations are due to the aggressive control around the zero gain area. Figure 8 demonstrates the response for the setpoint trajectory tracking from the same steady state. With Type B model, tuning parameter values $\sigma_1 = 0.0$, $\sigma_2 = 1000.0$ and $\Lambda = 0.2$ are used. With Type A model, tuning parameter values $\sigma_1 = 0.0$, $\sigma_2 = 1000.0$ and $\Lambda = 0.01$ are used and the change in the manipulated variable is constrained between -0.5 and 0.5. Similar responses are observed as in the case of step setpoint change. The oscillation in the response when using the Type A model is due to the aggressive control action in the zero gain area. Figure 9 shows that, in the zero gain area, the change in the manipulated variable hits the constraints. The aggressive control action around the zero gain area is due to the result of using a linear model for future prediction. This situation here points out the shortcoming of the linearization based nonlinear MPC algorithms. By choosing a large value of P in an algorithm utilizing nonlinear programming techniques, the oscillations could be reduced.

Figure 10 demonstrates the response for an input disturbance of -0.2 in a system running at peak. The introduction of negative input disturbance moves the plant curve closer to the zero gain area of the model. The tuning parameter values of $\sigma_2 = 1000.0$ and $\Lambda = 0.5$

are used with Type A model and the tuning parameter values of $\sigma_2 = 1000.0$ and $\Lambda = 0.9$ are used with Type B model. Even in the case of input disturbance, use of Type A model results in a better performance. This is because, using a smaller value of Λ with Type B model moves the estimator states into zero gain area of the model and results in steady state offset. It can be seen from figure 11 that the slower response using Type B model is due to the slower input action.

6 Conclusions

Observer based NLQDMC algorithm is presented for use with nonlinear state space and input-output models. The proposed algorithm eliminates the major drawbacks of the algorithm presented in Gattu and Zafiriou (1992a) for nonlinear state space models. The proposed algorithm handles the unstable processes and disturbance rejection in a general setting using linear filtering theory. In addition, the algorithm is presented for nonlinear models identified based on the input-output information. The modifications still preserve the major advantage of the original algorithm: *the computational simplicity by solving only a single quadratic program at each sampling time*

For open-loop unstable processes, it is demonstrated in the example that the process and disturbance augmented models of Type B performs better when compared to augmented models of Type A. Similar observation is made by Muske and Rawlings (1993b) for control of linear unstable systems.

The open-loop stable example with the sign change in the gain, it was demonstrated that the use of augmented models of Type A performs better than augmented models of Type B. The example also demonstrated the shortcomings of the algorithm. It is seen clearly that the performance loss around the zero gain area is due to the use of linear model for the future prediction.

The drawback of this algorithm or any other Model Predictive Control algorithms is that the tuning can be tedious sometimes. Use of automatic off-line tuning scheme proposed by Ali and Zafiriou (1993) would be advantageous to get the desired response. Work in this direction to implement automatic tuning scheme with observer based NLQDMC algorithm is currently under way in our research group.

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Notation

b	inequality constraint equation vector
A, B, C	continuous state space matrices
A_i, B_i	coefficients in input-output model description
D	diagonal weight matrix
E_1, E_2, E_3, E_4	constants
g	the gradient vector
G	hessian matrix
H	inequality constraint equation matrix
k	current sampling time index
$k+1\|k$	estimate at k+1 based on information at k
K	estimator gain
M	no. of future moves
P	prediction horizon
Q, R	covariance matrices
r	set point
$S_{i,k}$	step response coefficient matrix
t	time
T	sampling time
w, v	white noise processes
x	state
X	vector of change in manipulated variables
y	output

Greek letters

Γ, Λ	diagonal weight matrices
Γ_k, Φ_k	discrete state space matrices
Δ	change in the associated variable
θ_i	constants
σ	ratio σ_w/σ_v
σ_w^2, σ_v^2	scalar variances
η_k, ν_k	values of x and u at k

Subscripts

0	initial or nominal value
k	current sampling time index
$k0$	initial or nominal value at k
s	scaled variable

Superscripts

$\hat{}$	estimated value
T	transpose
$\star$	represents the effect of past
f	represents the effect of future
d	represents deterministic contribution
c	correction due to stochastic contribution
F	represens filter
$P1, P2$	represents predictor

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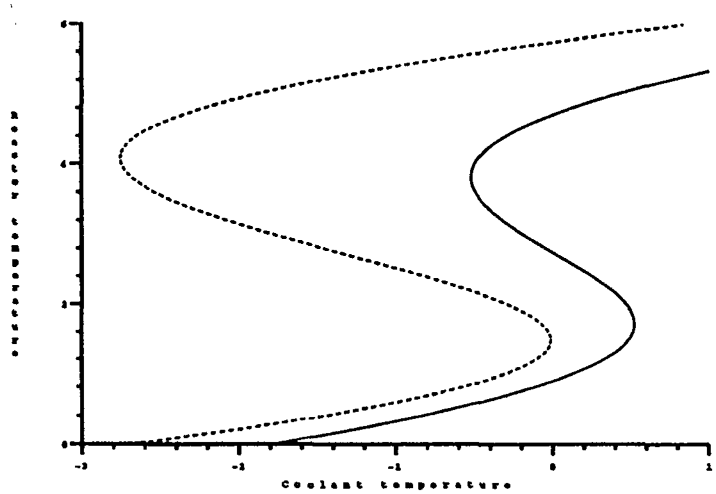

Figure 1: Dimensionless reactor temperature vs. Dimensionless cooling jacket temperature; equilibrium curve

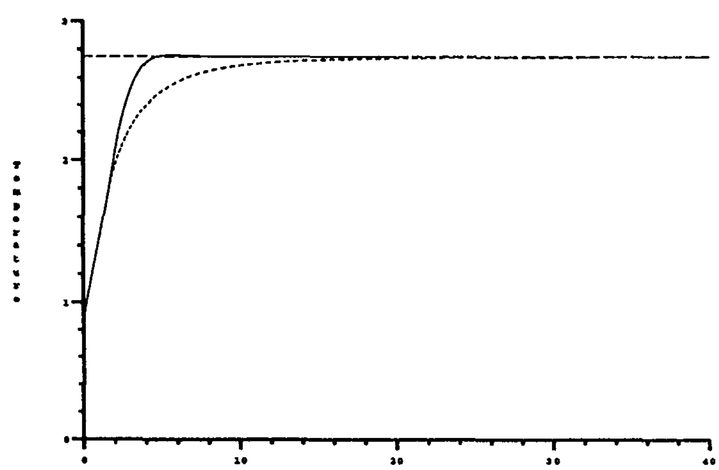

Figure 2: Dimensionless temperature vs. Dimensionless time. Setpoint tracking; Solid line –Type B model; Dotted line –Type A model

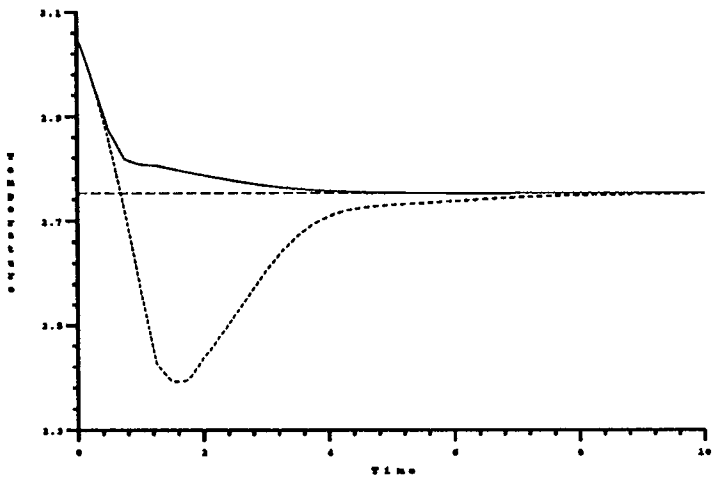

Figure 3: Dimensionless temperature vs. Dimensionless time. Output disturbances; Solid line -Type B model; Dotted line - Type A model

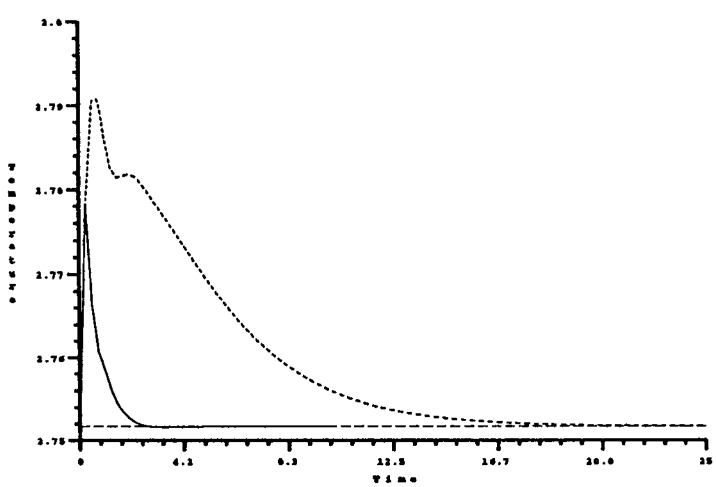

Figure 4: Dimensionless temperature vs. Dimensionless time. Input disturbances; Solid line –Type B model; Dotted line – Type A model

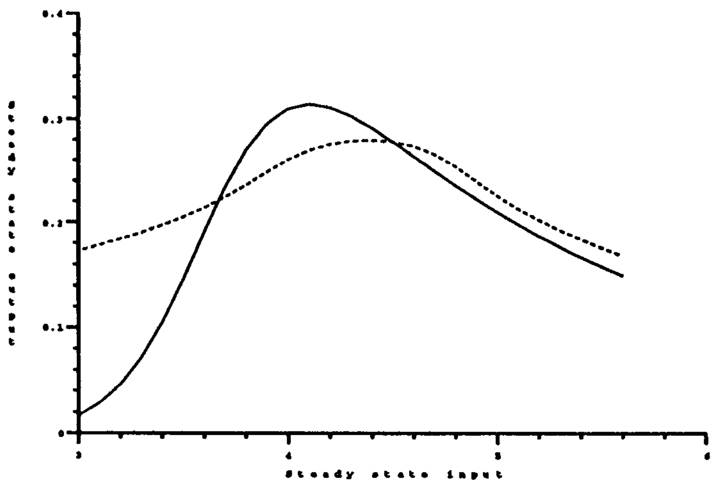

Figure 5: Steady state output vs. Steady state input. Solid line –Plant; Dotted line – Model

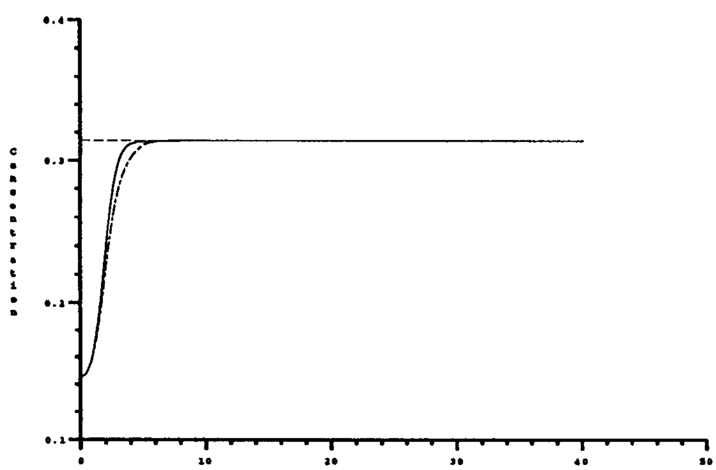

Figure 6: Concentration vs. time. Step setpoint change from left of the peak; Solid line –Type A model; Dotted line – Type B model

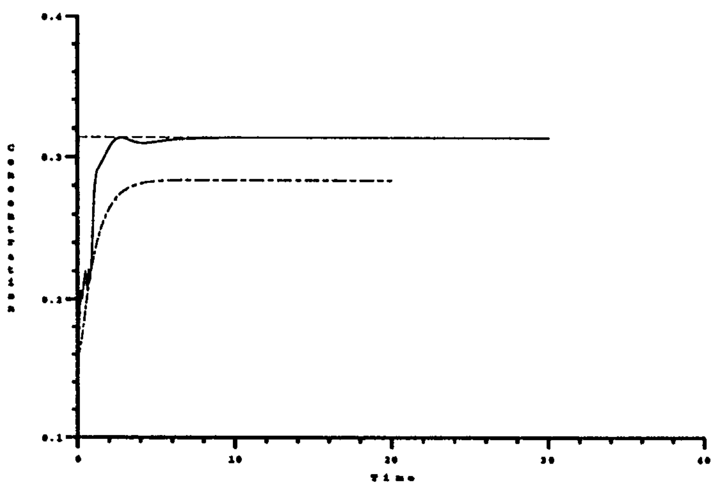

Figure 7: Concentration vs. time. Step setpoint change from right of the peak; Solid line –Type A model; Dotted line – Type B model

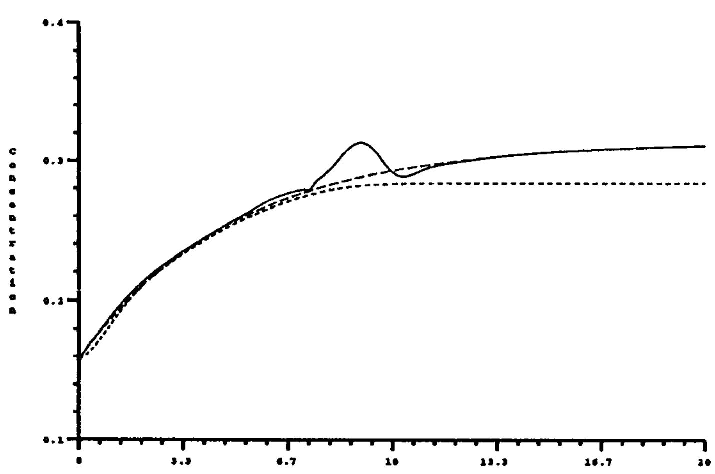

Figure 8: Concentration vs. time. Setpoint trajectory tracking from right of the peak; Solid line –Type A model; Dotted line – Type B model; Dashed line – Setpoint trajectory

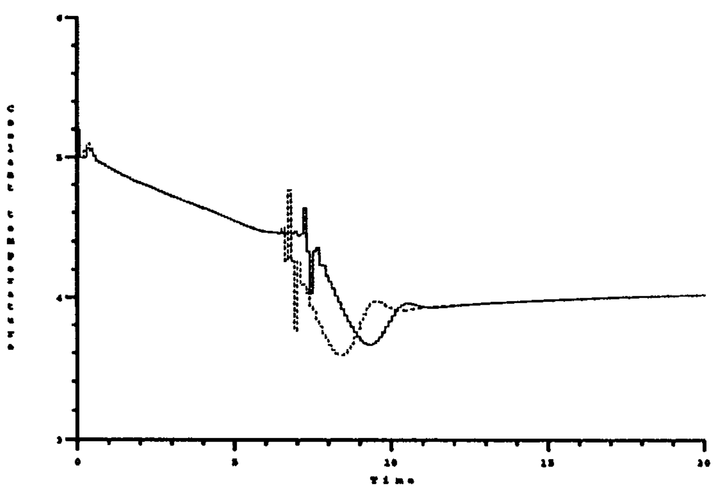

Figure 9: Cooling jacket temperature vs. time. Setpoint trajectory tracking from right of the peak; Solid line –Type A model; Dotted line – Type B model;

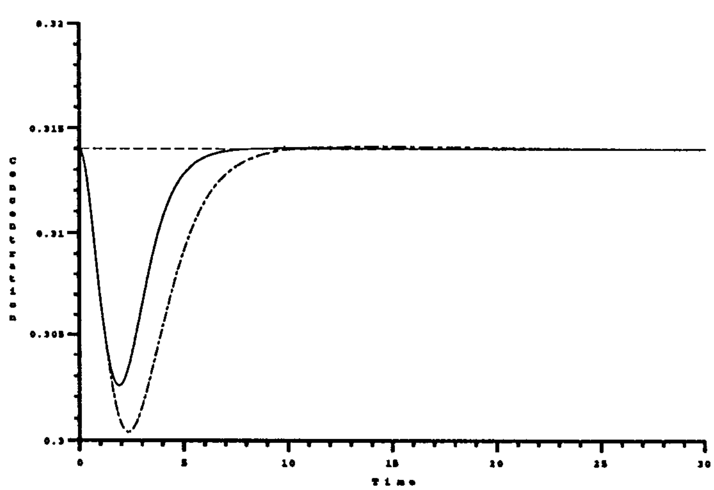

Figure 10: Concentration vs. time. Input disturbances; Solid line –Type A model; Dotted line – Type B model;

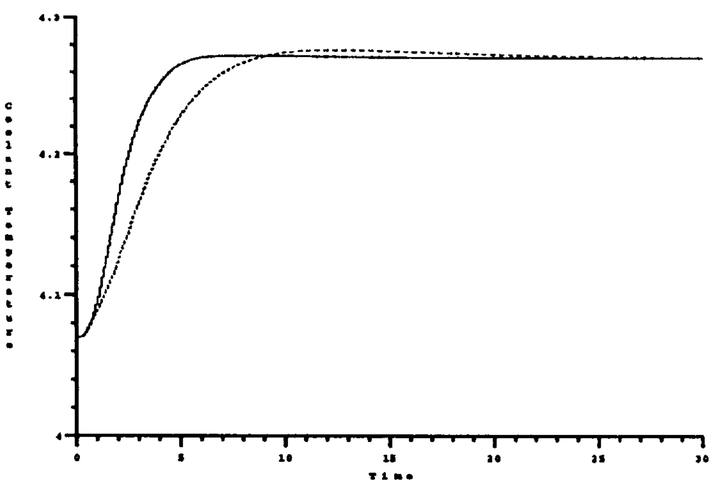

Figure 11: Cooling jacket temperature vs. time. Input disturbances; Solid line –Type A model; Dotted line – Type B model;