

**TECHNICAL  
RESEARCH  
REPORT**

*Institute for  
Systems  
Research*

**Estimation of Achievable Tolerances**

*by S.K. Gupta, D.S. Nau and G.M. Zhang*

*The Institute for Systems  
Research is supported by the  
National Science Foundation  
Engineering Research Center  
Program (NSFD CD 8803012),  
Industry and the University*

TR 93-44

# Estimation of Achievable Tolerances\*

Satyandra K. Gupta<sup>†</sup>      Dana S. Nau<sup>‡</sup>      Guangming Zhang<sup>§</sup>

University of Maryland  
College Park, MD 20742 USA

## Abstract

This report presents a new and systematic approach to assist decision-making in selecting machining operation plans. We present a methodology to estimate achievable tolerances of operations plan. Given an operation plan, we use variety of empirical and mathematical models to evaluate process capabilities of various machining operations and compute achievable tolerances using tolerance charting techniques.

## 1 Introduction

Increasing global competition is challenging the U.S. manufacturing industry to bring competitively-priced, well-designed and well-manufactured products to market in a timely fashion. Since decisions made during the design stage can have significant effects on product cost, quality, and lead time, increasing research attention is being given to integrating engineering design and manufacturing, with a focus on design for manufacturability.

Consider the task of designing and manufacturing machined parts. A machined part is often considered as a collection of machinable features and thus the problem of evaluating the machinability of the part<sup>1</sup> reduces to the problem of evaluating the machinability of the machinable features [2, 9, 6, 10]. One approach for evaluating the machinability of the machinable features is to compare each feature's machining tolerance and surface finish requirements against a list of process capabilities. This approach has been used for process selection in several generative process planning systems [3, 8, 1, 5]. However, this is not the most accurate way of determining machinability, for at least two reasons:

1. Whether or not a given machining process is capable of creating a given machinable feature will depend not only on the feature geometry, tolerance requirements, and surface finish requirements [13], but also on statistical variations in the process capabilities. In particular, random tool motion caused by tool wear or/and variations in basic material properties, such as hardness and ductility, in the material being machined is one of the major factors affecting the surface quality.

---

\*This work was supported in part by NSF Grants NSF DDM-88003012, IRI-8907890, and DDM-9201779.

<sup>†</sup>Institute for Systems Research and Department of Mechanical Engineering. Email: skgupta@src.umd.edu

<sup>‡</sup>Institute for Systems Research and Department of Computer Science. Email: nau@cs.umd.edu

<sup>§</sup>Institute for Systems Research and Mechanical Engineering Department. Email: zhang@src.umd.edu.

<sup>1</sup>By the machinability of a part, we mean how easy it will be to achieve the required machining accuracy. This is somewhat broader than the usual usage of "machinability."

2. Existing approaches for obtaining machinable features from a CAD model will normally produce a single interpretation of the part as a collection of machinable features. However, there can be several different interpretations of the same part as different collections of machinable features—and each interpretation will lead to different sequences of machining operations for creating the same part [12, 7]. To determine the machinability of the part, all of the alternative interpretations should be generated and examined.

This report discusses the development of a new approach whereby, given a proposed design for a machinable part, we can automatically generate the alternative machining operation, and then evaluate each of them to determine the achievable machining accuracies or costs. We anticipate that this information will be useful in several ways: (1) to provide feedback to the designer about possible problems that may arise in trying to meet the specified geometry and tolerances and (2) to provide information to the manufacturing engineer about alternative ways to machine the part, for use in process planning.

## 2 Definitions

A *machined part* (or just a *part*) is the finished component to be produced as a result of a set of machining operations on a piece of *stock*, i.e., the raw material from which the part is to be machined. For example, Fig. 1 shows a part. We assume a cylindrical stock of correct length and diameter. Part is to be made from the stock by machining the surfaces  $S1$  through  $S3$ .

We will represent both the part and the stock as geometric solids. For our purposes, a *solid* is any regular, semi-analytic set. In CAD systems, solids may be represented using any of several techniques, such as constructive solid geometry or boundary representations. If  $R$  is any solid, then  $b(R)$  is the *boundary* of  $R$  and  $i(R)$  is the *interior* of  $R$ . Note that  $R = i(R) \cup b(R)$  and that  $i(R) \cap b(R) = \emptyset$ . Throughout the rest of this paper, we will let  $P$  be a solid representing a part, and  $S$  be a solid representing the stock from which  $P$  is to be made.

A *machining feature* is the portion of the workpiece affected by a machining operation. We consider two types of machining features: *roughing features*, which correspond to roughing operations, and *finishing features*, which correspond to finishing operations. Since each roughing operation removes some volume of material from the workpiece, the corresponding roughing feature represents the volume of material removed. Since each finishing operation changes the quality of one or more surfaces of the workpiece, the corresponding finishing feature represents this set of surfaces. More formally, a roughing feature is a triple

$$f = (rem(f), acc(f), class(f)),$$

and a finishing feature is a triple

$$f = (fin(f), acc(f), class(f)),$$

where  $rem(f)$ ,  $fin(f)$ ,  $acc(f)$ , and  $class(f)$  are as defined below:

- To perform the machining operation, one sweeps the tool volume  $T$  along some trajectory  $t$ , creating a swept volume  $T_{sw}$ . The *accessibility volume*,  $acc(f)$ , is the set of all points in  $T_{sw}$  that are on or above the *partition face*  $\pi(f)$ , in some cases the approach face is planar, and in other cases it is cylindrical).

- In the case of a roughing operation, the set of all points in  $T_{sw}$  on or below  $\pi$  is a volume, which we call the *removal volume*,  $rem(f)$ . In the case of a finishing operation, the set of all points in  $T_{sw}$  on or below  $\pi$  is a surface or set of surfaces, which we call the *finishing surface*  $fin(f)$ .
- The feature  $f$  will be an *instance* of some *feature class*  $\phi$ , which is a parameterized set of machining features characterized by the shape and trajectory of the cutting tool. If  $f$  is a feature in  $\phi$ , then the *class* of  $f$  is  $class(f) = \phi$ . If  $f$  is an instance of  $\phi$ , then the  $f$ 's *parameters* in  $\phi$  are the specific set of parameter values for  $\phi$  that result in  $f$ .

Normally we will have some fixed finite set of feature classes  $\Phi = \{\phi_1, \phi_2, \dots, \phi_n\}$ , and for each part that we want to machine, we will be interested in describing the part in terms of features from  $\Phi$ . Each set of features from  $\Phi$  that describes the part is a feature-based model of the part. Suppose we are given a part  $P$  and stock  $S$ . A *feature-based model* (or FBM) of  $P$  and  $S$  is any set of features  $F$  having the following properties:

1. Each  $f \in F$  is an instance of some feature class in  $\Phi$ .
2. If we subtract the roughing features in  $F$  from  $S$ , we get  $P$ ; i.e.,

$$S - \bigcup_{f \in F} rem(f) = P.$$

3. No roughing feature in  $F$  is redundant, i.e., for every feature  $f \in F$ ,

$$S - \bigcup_{g \in F - \{f\}} rem(g) \neq P.$$

4. For every finishing feature  $f \in F$ , some portion of  $fin(f)$  is part of the surface of  $P$ .

### 3 Estimating Machining Accuracy

Each machining operation creates a feature which has certain geometric variations compared to its nominal geometry. Designers normally give tolerance specifications on the nominal geometry, to specify how large these variations are allowed to be. Given a candidate operation sequence, the machining data for that sequence, the feature's dimensions, and the material from which the part is to be made, we want to evaluate whether or not it can satisfactorily achieve the tolerance specifications.

To get the most accurate results, the best technique is to construct a mathematical model of the machining process. To date, we have done this for turning and boring—and our methodology can easily be extended to model all machining processes involving single-point cutting tools. By modeling the relative motion of the workpiece and the cutting tool, we produce models of topography resulting from the machining process—and from these models, we calculate the achievable tolerances and surface finishes produced by the machining process. Our models take into account the following factors:

1. The machining system parameters, such as the feed rate, cutting speed, depth of cut, and structural dynamics. Their effects on the process capabilities can be modeled and evaluated deterministically.

Table 1: Estimation of Incremental Increases during Drilling

Drill Diameter (mm)	Hardness of Workpiece Material (BHN)	Enlargement Coefficient
1.0 to 5.0	100	0.05 - 0.10
1.0 to 5.0	300	0.05 - 0.20
5.0 to 20	100	0.08 - 0.15
5.0 to 20	300	0.15 - 0.25
20 to 60	100	0.12 - 0.30
20 to 60	300	0.20 - 0.35

2. The natural and external variations in the machining process. For example, variations in hardness in the material being machined cause random vibration, which is one of the major factors affecting the surface quality. We model these variations statistically.

Machining processes that do not involve single-point cutting tools are complex enough that we have not yet succeeded in constructing accurate mathematical models of them. For these machining processes, we are developing empirical models.

### 3.1 Example of Empirical Modeling: Achievable Tolerance for Drilling

**Estimating the Incremental Increase in Hole Diameter.** Drilling creates a hole whose diameter is larger than the diameter of the drilling tool. The difference between the hole’s diameter and the drill’s diameter is defined to be the *incremental increase* associated with the drilling process. Empirical data are often used to estimate the incremental increase associated with a specific drilling process. Table 1 lists some of the typical data used on the shop floor. As an example, the incremental increase would be approximately 0.10 mm when a drill having 5 mm in diameter is used to make a hole on aluminum-based materials. Therefore, the diameter after drilling would be approximately 5.10 mm.

**Estimating Geometrical Variations.** In addition to the incremental increase in diameter, the holes produced by drilling have roundness, straightness, cylindricity, and other errors. Such errors are usually induced by the runout in attaching a drill to the machine tool, and the drill whirling motion due to the drill deflection during machining. As illustrated in Fig. 2(a), these machining errors vary along the axial direction. The error has its minimum value at one end of the hole where the drilling process starts, marked as  $\Delta d_{min}$  in Fig. 2(b). It reaches its maximum value at other end of the hole, marked as  $\Delta d_{max}$  in Fig. 2(b). The difference between  $\Delta d_{max}$  and  $\Delta d_{min}$  increases as the ratio of the length to diameter of a drill and the hardness of the workpiece material increase. In our work, we assume  $\Delta d_{min} = 0$ , and a database is constructed to determine  $\Delta d_{max}$  for a given drilling condition. Fig. 3 outlines the basic structure of the database.

For ordinary holes, the above computation is sufficient to determine the maximum error value,  $\Delta d_{max}$ . However,  $h_{12}$  is not an ordinary hole, because part of it will later be removed when  $h_{22}$  is machined. Thus, as illustrated in Fig. 2(c), the maximum error value  $\Delta d_{max}$  for  $h_{12}$  has to be modified, or reduced to  $\frac{L_1}{L} \times \Delta d_{max}$ . Note that the low limit of the achievable tolerance should be modified when the right portion is being removed, as illustrated in Fig. 2(d). The modified low

limit is given by  $[\frac{L-L_1}{L}\Delta d_{max}]$ . In both cases,  $L_1$  represents the length of the remaining part of  $h_{12}$ .

**Precision of the Machine Tool Being Used.** The achievable accuracy for a given machining operation is also related to the precision of the machine tool being used. If we have a high-precision machine tool (e.g., one with high spindle accuracy), we can significantly reduce machining errors such as  $\Delta d_{max}$  and  $\Delta d_{min}$ , as shown in Fig. 3. To account for machine tool precision, we multiply  $\Delta d_{max}$  and  $\Delta d_{min}$  by a modification factor, MF1. For example, consider a process plan in which all processes are to be performed in a single setup, on an engine lathe. Since the machine tool accuracy of lathes is better than the accuracy of drill presses, a value of 0.8 may be assigned to MF1.

**Effects of Machining Parameters.** The achievable accuracy for a given machining operation is also related to machining parameters such as spindle speed and feed rate. For example, a high spindle speed increases the runout error, and drilling at a large feed rate leaves large feed marks on the drilled surface. To account for these effects, we multiply  $\Delta d_{max}$  and  $\Delta d_{min}$  by a second modification factor, MF2.

**Determining the Achievable Tolerance.** Let the hole diameter be expressed as:

$$\text{Drill Size} \begin{array}{l} +\text{upper limit} \\ +\text{lower limit} \end{array} \quad (1)$$

Combining all of the above factors, we get the following formulas for the upper and lower limits of the achievable tolerances. Suppose the hole is drilled from right to left, as shown in Fig. 2. Then there are three cases.

For a complete hole, the tolerances are

$$\text{upper limit} = \text{Incremental Increase} + \Delta d_{max} \times \text{MF1} \times \text{MF2} \quad (2)$$

$$\text{lower Limit} = 0.0. \quad (3)$$

If the right side is later removed, then the tolerances are

$$\text{upper limit} = \text{incremental increase} + \Delta d_{max} \times \text{MF1} \times \text{MF2} \quad (4)$$

$$\text{lower limit} = \frac{L - L_1}{L} \Delta d_{max}. \quad (5)$$

If the left side is later removed, then the tolerances are

$$\text{upper limit} = \text{incremental increase} + \left[ \frac{L_1}{L} \Delta d_{max} \right] \times \text{MF1} \times \text{MF2} \quad (6)$$

$$\text{lower Limit} = 0.0. \quad (7)$$

**Estimation the Concentricity Error.** In practice, possible errors in the concentricity mainly come from the errors in the machine tool movement, the drill runouts, and the fixturing error when the workpiece is being machined on different setups or orientations. We use the following formula to calculate the concentricity error between two machined holes.

$$\begin{aligned} \text{concentricity error} &= \frac{\frac{1}{2}(\Delta d_{max} + \Delta d_{min})_{H1} + \frac{1}{2}(\Delta d_{max} + \Delta d_{min})_{H2}}{2} \times \text{MF1} \times \text{MF2} \\ &+ \text{errors due to multiple setups} \end{aligned} \quad (8)$$

### 3.2 Example of Mathematical Modeling: Achievable Tolerance for Boring

**Estimating the Number of Machining Passes.** Boring operations are widely used on the shop floor as finishing or semi-finishing operations to enlarge an existing hole. However, the inherent properties of the long, slender boring bar make it susceptible to severe vibration during machining. To maintain a stable and satisfactory operation, the material removal rate during machining has to be limited. This may require more than a single pass to complete the machining operation.

In general, a single pass during the machining operation is sufficient if specifications on dimensional accuracy and finish quality are low. The enlargement of H3 from the existing H1 is a typical example, since the surface quality of a recess is of little relevance to its function of storing lubricant. For surfaces where dimensional accuracy and finish quality are major concerns, two passes during the machining operation are usually needed. The first pass removes the major surface irregularities resulting from drilling, and the second pass produces a smooth surface and assures the dimensional accuracy. To achieve higher machining accuracy, grinding, reaming, or other finish machining operations are often recommended. In our work, we assume that, depending on the required specification, either a single-pass operation or a two-pass operation is needed during boring. During a two-pass operation, the first pass removes 2/3 of the excessive material, and the second pass removes the remaining 1/3.

**Estimating the Achievable Machining Accuracy for Each Pass.** Fig. 4 illustrates the error generation due to the boring bar vibration during machining. The vibration magnitude,  $\Delta A$ , has its direct influence on dimensional accuracy, such as  $d_{max}$  and  $d_{min}$  shown in Fig. 4(b), and finish quality. Major factors contributing to the error generation can be the hardness of the workpiece material, the total machining load, the slenderness ratio of the boring bar, the slenderness ratio of the workpiece, and the progress of tool wear during machining. Similar to the method used to determine the achievable tolerance during drilling, we calculate the achievable tolerance during boring based on the following formulas:

Let the hole diameter be expressed as:

$$\text{Boring Dia} \begin{array}{l} +\text{upper limit} \\ +\text{lower limit} \end{array} \quad (9)$$

For the first pass:

$$\text{upper limit} = \text{incremental increase} + \Delta B \times \text{MF1} \times \text{MF2} \times \text{MF3} \quad (10)$$

$$\text{lower limit} = \text{incremental increase.} \quad (11)$$

For the second pass:

$$\text{upper limit} = \text{incremental increase} + [\Delta B \times \text{MF1} \times \text{MF2} \times \text{MF3}] \times 0.5 \quad (12)$$

$$\text{lower limit} = \text{incremental increase.} \quad (13)$$

In Eqs. 10 and 12,  $\Delta B$  represents a nominal value of the machining error. Fig. 5 presents the basic structure of a database prepared for selecting the value  $\Delta B$  under various machining conditions. In Eqs. 10 and 12, the MF's are modification factors to account for the machine tool accuracy, rigidity of the workpiece-boring bar combination, and machining parameters. The lower limit of the achievable tolerance is determined by the incremental increase necessary for completing a single pass during machining. In our work, the numerical value of the lower limit is set to equal the achievable tolerance of its preceding operation. In Eq. 12 a proportionality coefficient of 0.5 is used in evaluating the upper limit of the achievable tolerance for the second pass. This indicates that the tolerance achievable during a finish cut is higher than the tolerance achievable during a rough or semi-finish cut.

**Estimating the Concentricity Error.** As a semi-finishing or finishing operation, the boring process significantly reduces the concentricity error resulting from the drilling operation. It is evident that the achievable concentricity tolerance between the two enlarged holes is also related to the achievable machining accuracy for each of them. In our work, the following formula is used to calculate the concentricity error between the two machined holes after the boring operation.

For the first pass:

$$\begin{aligned} \text{concentricity error} &= \frac{1}{2}(\Delta B_{H1} + \Delta B_{H2}) \times \text{MF1} \times \text{MF2} \\ &+ \text{errors due multiple setups} \end{aligned} \quad (14)$$

For the second pass:

$$\begin{aligned} \text{concentricity error} &= \left[ \frac{1}{2}(\Delta B_{H1} + \Delta B_{H2}) \times \text{MF1} \times \text{MF2} \times \text{MF3} \right] \times 0.5 \\ &+ \text{errors due to multiple setups} \end{aligned} \quad (15)$$

To demonstrate the basic mechanism used to estimate achievable machining accuracy for drilling operations, we present the steps in evaluating the achievable machining accuracy for drilling  $h_{12}$  and drilling  $h_{22}$ . In the operation sequence listed above, if we use drill diameters of 29 mm and 49 mm to make  $h_{12}$  and  $h_{22}$ , respectively, then the corresponding incremental increases, as listed in Table 1, would be approximately 0.20 mm. Based on the constructed database, the values for  $\Delta d_{max}$  and  $\Delta d_{min}$  are 0.15 mm and 0.00 mm, respectively.

As shown in Fig 6,  $h_{11}$ ,  $h_{12}$ , and  $h_{13}$  will be made in one setup, offering an opportunity to achieve high machining accuracy. Thus, this operation sequence will be preferable when the specifications of the tolerances of the two holes and the concentricity between them are tight. Since all three of the features are rotational, the essential machining operations involved will be drilling and boring operations.

Fig. 6 shows a graphical representation of the dynamic process to derive the achievable machining accuracy. Fig. 6 illustrates the transition of the achievable tolerance from a drilling operation to a boring operation. The tolerance range narrows as the boring operation progresses. Fig. 6 also indicates the dynamic generation of concentricity error at the end of the second operation. Note the symbol to represent the achievable concentricity between H1 and H2 after drilling H2. It means that that the two holes of H1 and H2 may not be aligned along a unified axis due to the machining errors resulted in the first and second operations of the selected operation sequence. Fig. 6 also

illustrates the transition of the concentricity error from a high value to a low value during the boring operation.

In the current case study, the low limit of the achievable tolerance of the remaining part of  $h_{11}$  would be 0.10 mm after the

## 4 Conclusions

We have described a new approach for use in the concurrent engineering of products and manufacturing processes. Its uniqueness lies in the generation and evaluation of machining alternatives. For a given part design, the approach generates a variety of alternative process sequences based on feature interpretations, and then evaluates these sequences to determine their cost and machining accuracy. The work presented in this paper clearly demonstrates the benefits and importance of an early integration of the manufacturing knowledge and design specifications in the product development cycle. The integration offers an effective information flow between the product designer and process planner. It will give the production engineer knowledge about what processes and process parameters are most desirable over the various ways in which the part might be machined; and will give the product designer a better understanding of about the robustness of the design, and how the design might be changed to improve its manufacturability.

## References

- [1] S.L. Brooks and K.E. Hummel. Xcut: A rule-based expert system for the automated process planning of machined parts. Technical Report Technical Report BDX-613-3768, Bendix Kansas City Division, 1987.
- [2] Butterfield, Green, Scott, and Stoker. Part features for process planning. Technical Report R-86-PPP-01, Computer Aided Manufacturing International, November 1986.
- [3] T. C. Chang and R. A. Wysk. *An Introduction to Automated Process Planning Systems*. Prentice-Hall, Englewood Cliffs, NJ, 1985.
- [4] K. Dehnad. *Quality Control, Robust Design, and the Taguchi Method*. Wadsworth & Brooks/Cole, Pacific Grove, CA, 1989.
- [5] I. Ham and S. Lu. Computer aided process planning, the present and the future. *Annals of CIRP*, 37(2), 1988.
- [6] K. E. Hummel. The role of features in computer-aided process planning. In *Proc. CAMI Features Symposium*, number P-90-PM-02, pages 285–320, August 9–10 1990.
- [7] R. Karinthi and D. Nau. An algebraic approach to feature interactions. *IEEE Trans. Pattern Analysis and Machine Intelligence*, 1991. To appear.
- [8] D. S. Nau. Automated process planning using hierarchical abstraction. *TI Technical Journal*, pages 39–46, Winter 1987. Award winner, Texas Instruments 1987 Call for Papers on AI for Industrial Automation.

- [9] J. Shah, P. Sreevalsan, M. Rogers, R. Billo, and A. Mathew. Current status of feature technology. Technical Report R-88-GM-04.1, CAM-I Inc., 1988.
- [10] Jami Shah. Philosophical development of form feature concept. In *Proceedings of Feature Symposium*, number P-90-PM-02, Woburn, Boston, MA, August 1990.
- [11] G. Taguchi, E. Elsayed, and T. Hsiang. *Quality Engineering in Production Systems*. McGraw Hill, 1989.
- [12] Jan H. Vandenbrande. *Automatic Recognition of Machinable Features in Solid Models*. PhD thesis, Computer Science Department, UCLA, 1990.
- [13] S.M. Wu. Dynamic data system: A new modeling approach. *Transactions of the ASME, Journal of Engineering for Industry*, 1977.

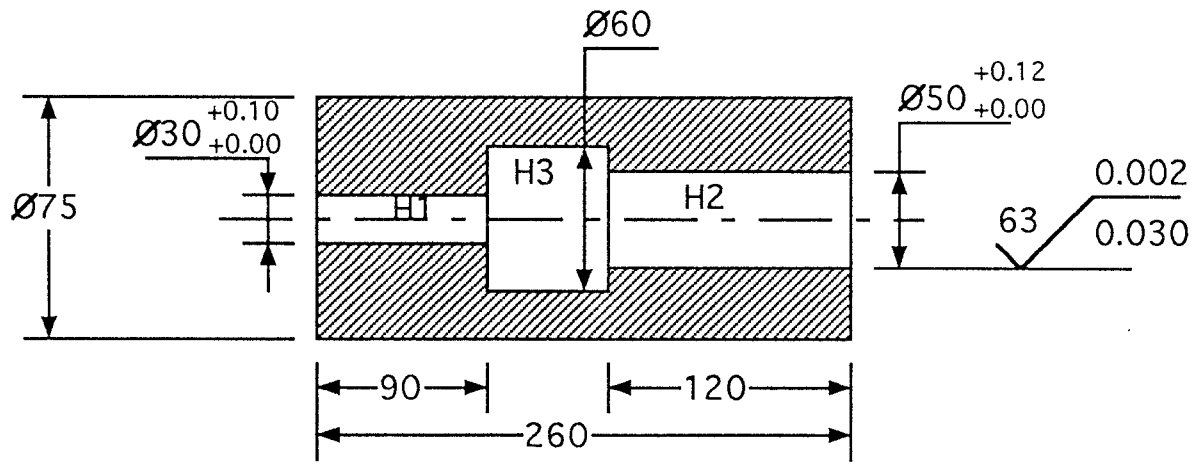


Fig. 1: Part to be machined

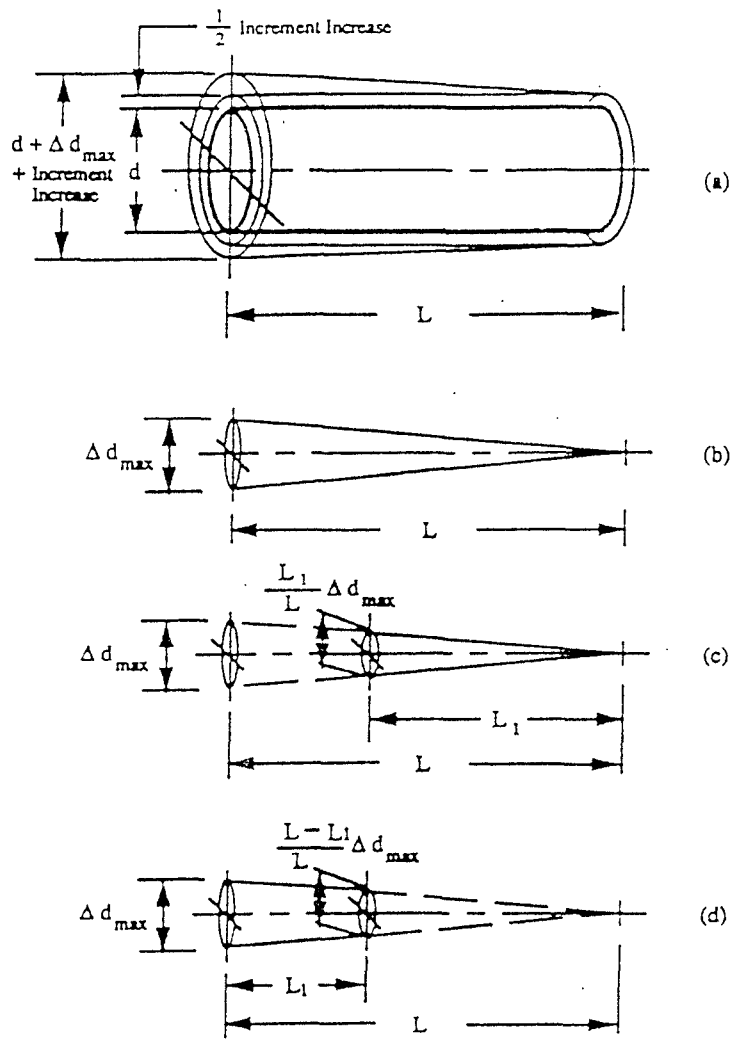
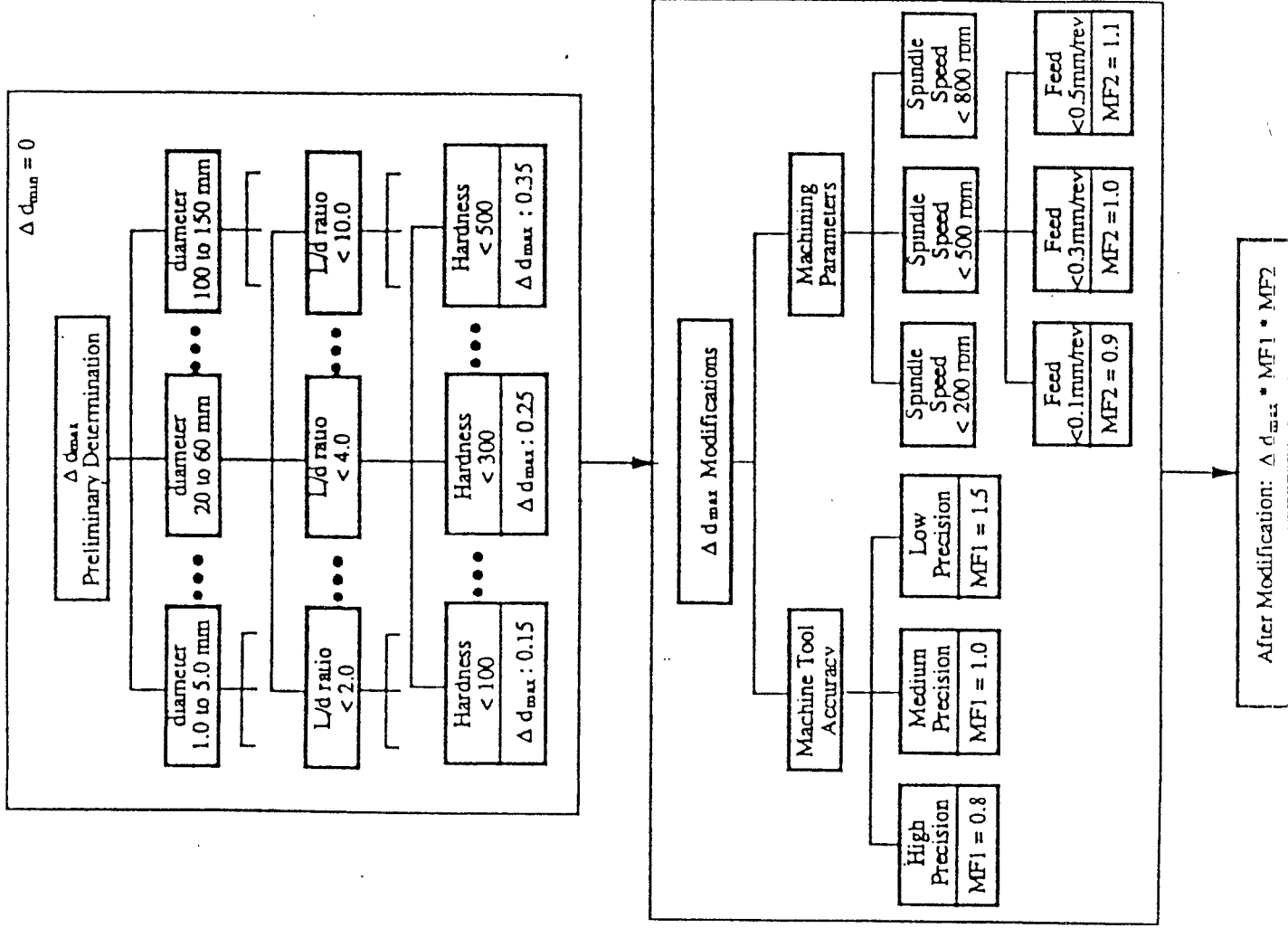


Fig. 2 Analysis of Machining Errors during the Drilling Operation

# Decision Tree for estimating the values of $\Delta d_{max}$ , MF1 and MF2



Value of  $\Delta d_{max}$  depends on:

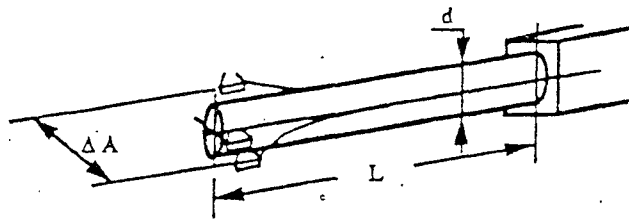
- hole diameter
- L/d ratio
- hardness

Modification Factors:

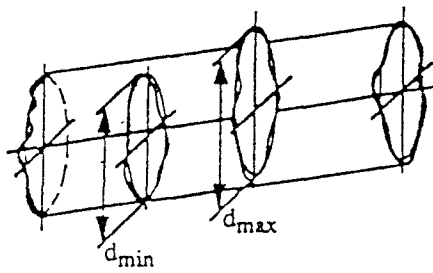
MF1: machine tool accuracy

MF2: machining parameters

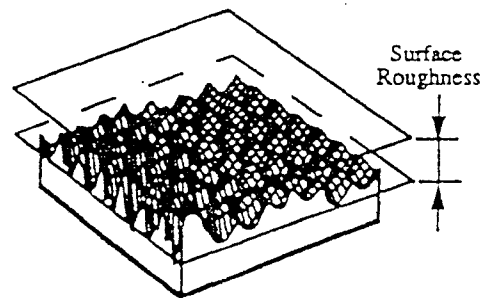
Fig. 3



(a) Vibration of Boring Bar Causes Machining Errors



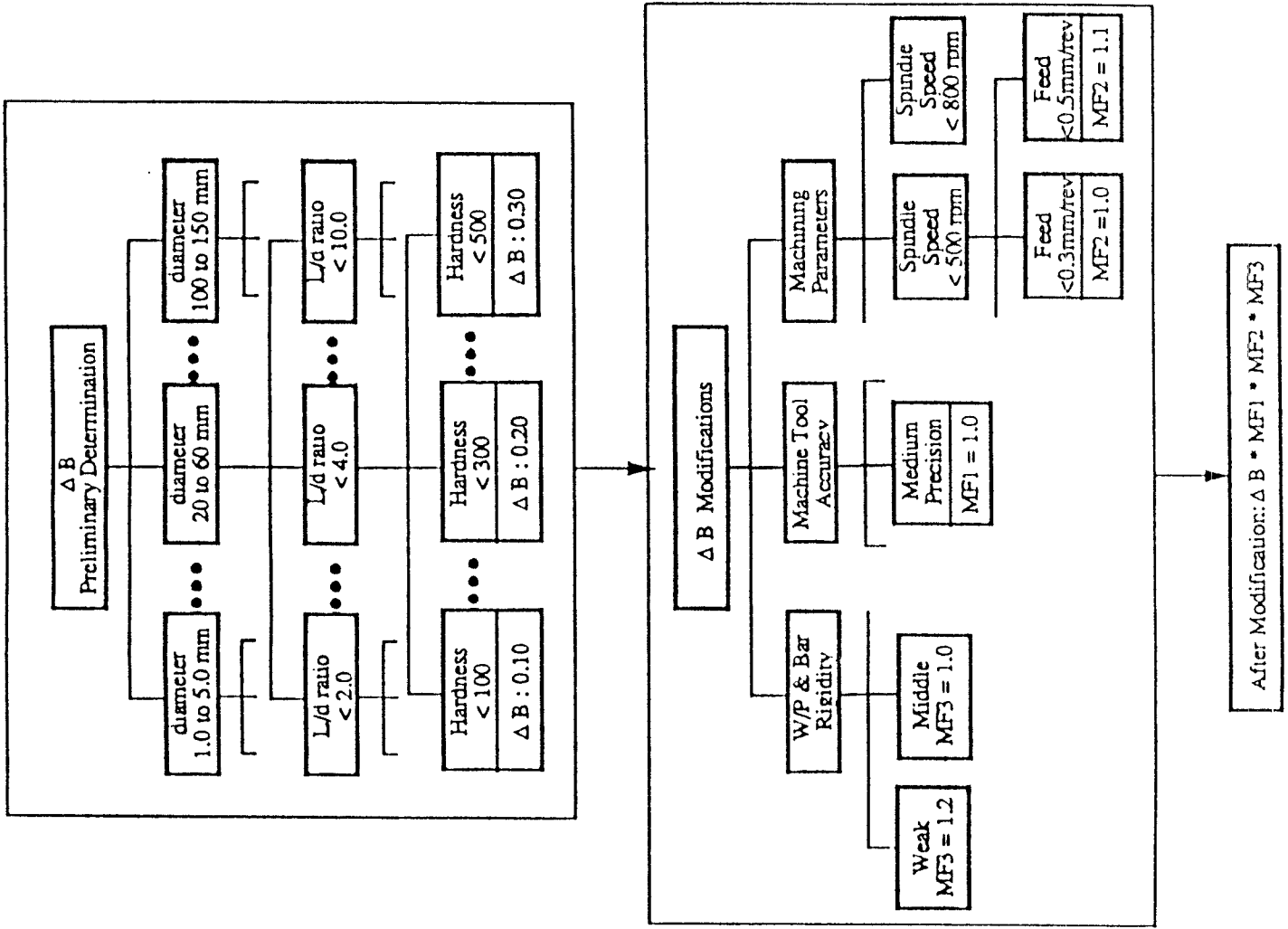
(b) Dimensional Errors



(c) Finish Quality

Fig. 4 Analysis of Machining Errors during the Boring Operation

# Decision Tree for estimating the values of $\Delta B$ , MF1, MF2 and MF3



Value of  $\Delta B$  depends on:

- hole diameter
- L/d ratio
- hardness

Modification Factors:

- MF1: rigidity
- MF2: machine tool accuracy
- MF3: machining parameters

Fig. 5

# Example : Estimation of Machining Accuracy

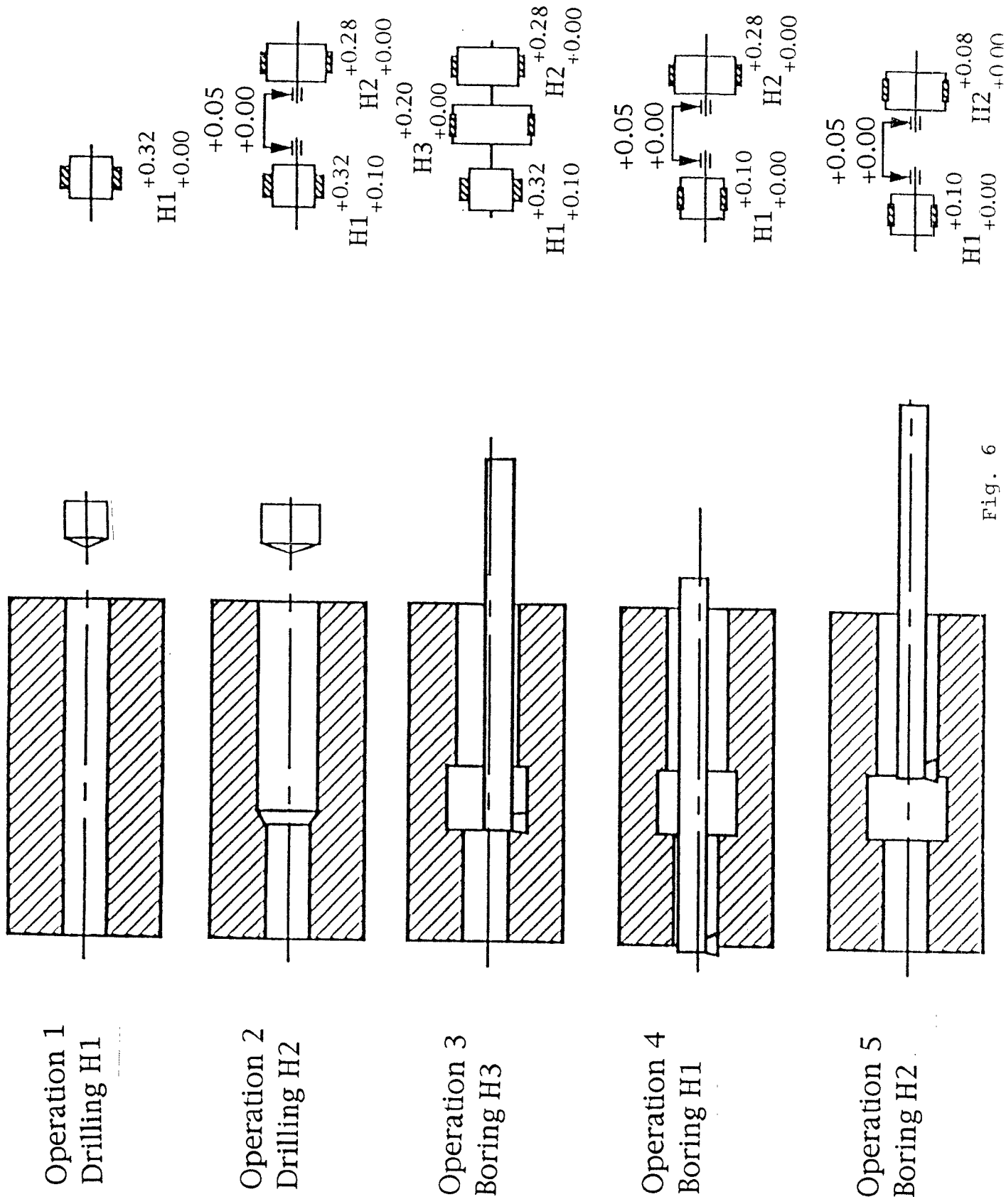


Fig. 6