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Kinematic Analysis of Epicyclic-Type Transmission Mechanisms Using the Concept of Fundamental Geared Entities

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ABSTRACT

A new methodology for the speed ratio analysis of epicyclic-type transmission mechanisms is presented. First, the kinematic characteristics associated with various operation modes of fundamental geared entities are investigated. Then, it is shown that the overall speed ratio of an epicyclic gear mechanism can be expressed in terms of its fundamental geared entities. This method leads to an automated derivation of the speed ratio of an epicyclic-type transmission mechanism without the need of a symbolic manipulation software.

NOMENCLATURE

K_i	Velocity parameter of the ith operation mode.
$N_{p,x}$	Gear ratio defined by a planet gear p with respect to a sun or a ring gear x. $N_{p,x} = \pm\, T_p/T_x$, where T_p and T_x denote the numbers of teeth on the planet and the sun or ring gear, respectively, and the positive or negative sign depends on whether x is a ring or sun gear.
$R^z_{x,y}$	Speed ratio between an input link x and an output link y with reference to a ground link z.
ω_i	Angular velocity of link i

1 Introduction

Most automatic transmission mechanisms use epicyclic gear trains (EGTs) to achieve a set of desired speed ratios. The central axis of an EGT is supported by bearings housed in the casing of an automatic transmission. The mechanism formed by an EGT and the casing of a transmission gearbox is a fractionated mechanism called the epicyclic gear mechanism (EGM). An EGM may possess two or three degrees of freedom (DOF) depending on whether the EGT is a one or two-DOF gear train (Tsai et al., 1988). In this paper, we assume that an EGM has two degrees of freedom.

Figure 1 shows a typical epicyclic-type automatic transmission mechanism. In such a transmission mechanism, the *speed ratio* is defined as the ratio of the input shaft speed to the output shaft speed. Different speed ratios are obtained by using clutches to connect various links to the input power source and the casing of a transmission gearbox, respectively. Typically, a rotating clutch is used for connecting two rotating links, and a band clutch is used to connect a link to the casing. In Fig. 1, the rotating and band clutches are denoted as C and B, respectively, and numbered sequentially.

The speed ratios selected for a transmission are tailored for vehicle performance including a first gear for starting, a second or third gear for passing, and a high gear for fuel economy at road speeds. A table of clutching conditions showing a specific order of speed ratios is called the *clutching sequence* of a transmission. The clutching sequence for the transmission shown in Fig. 1 is listed in Table 1, where an X_i indicates that the corresponding clutch is activated on the ith link for that *gear*.

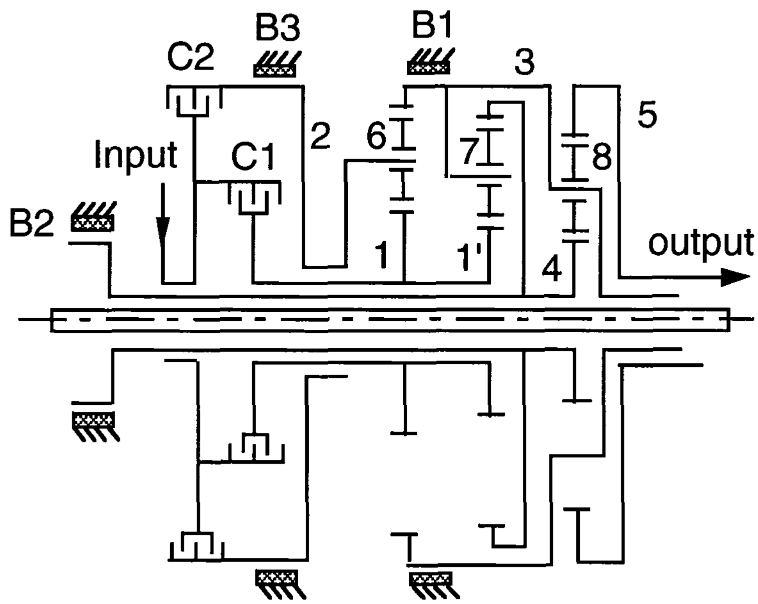


Figure 1: A Typical Transmission Mechanism.

Table 1: Clutching sequence.

	Actuated clutches				
Gear	C_1	C_2	B_1	B_2	B_3
First	X_1		X_3		
Second	X_1			X_4	
Third		X_2		X_4	
Fourth	X_1	X_2			
Reverse	X_1				X_2

Various approaches such as the relative velocity method (Glover, 1965; Levai, 1968), energy method (Wilkinson, 1960), bond graph method (Allen, 1979), the vector-loop method (Gibson and Kramer, 1984; Smith, 1979; Willis, 1982), and the signal-flow graphs (Ma and Gupta, 1994) have been proposed for the kinematic analysis of EGTs. Conventionally, such analysis is performed by manual formulation. Recently, there has been an increasing interest in the development of computer-aided analysis programs (Tsai, 1985; Hedman, 1989a; Hedman, 1989b; Hedman, 1993). Freudenstein and Yang (1972) introduced the concept of fundamental circuits. The concept was further extended by other researchers (Belfiore and Pennestri, 1989; Saggere and Olson, 1992; Pennestri and Freudenstein, 1993; Pennestri et al., 1993).

The concept of fundamental circuit is a powerful tool for the kinematic analysis of EGTs. However, the analysis involves the solution of a set of linear equations for all the kinematic variables. It does not provide much insight into the mechanics of an EGT. This paper applies the concept of fundamental geared entities (FGEs) for the kinematic analysis of EGMs. First, the kinematic characteristics associated with various operation modes of FGEs are investigated. Then, it is shown that, by decomposing an EGM into two subsystems and each subsystem into two sub-subsystems until each lowest level subsystem contains only one FGE, the overall speed ratio of an EGM can be symbolically expressed in terms of the FGEs. The main advantage of this method is that an algebraic expression for the overall speed ratio of an EGM can be easily obtained by observation without mathematically manipulating the fundamental circuit equations. It leads to an automated derivation of speed ratio without the need of a symbolic manipulation software. In addition, it also provides a better insight of the effects of the FGEs and their gear sizes on the overall speed ratio of an EGM. Thus, the speed ratio of an EGM can be estimated without specifying the gear sizes. This can be very helpful in the identification of a clutching sequence during the design phase of an epicyclic-type automatic transmission.

In what follows, the definitions of canonical graph and fundamental geared entity are reviewed.

1.1 Canonical Graph Representation

In a graph representation, links are represented by vertices and joints are represented by edges. The gear pairs are represented by thick edges, revolute joints are represented by

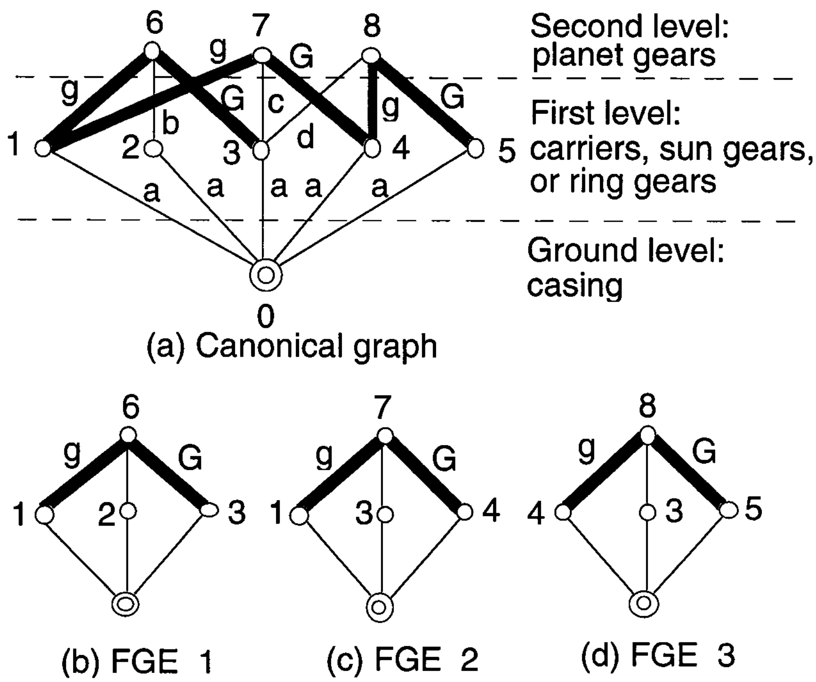


Figure 2: (a) Canonical graph of Fig. 1, (b) through (d) Single planet FGEs.

thin edges, and the thin edges are labeled according to their axis locations in space. In order to avoid the problem of pseudo-isomorphism due to the existence of coaxial links, a canonical graph representation was suggested (Tsai, 1988, Chatterjee and Tsai, 1994a). In a canonical graph representation, the joints among some coaxial links are rearranged such that all the thin-edged paths originated from the root (grounded link) have distinct edge labels. Further, the vertices can be divided into several levels. The ground-level vertex denotes the casing. The first-level vertices denote the coaxial sun gears, ring gears, and carriers. The second-level vertices denote the planet gears. The canonical graph representation of the *EGM* shown in Fig. 1 is sketched in Fig. 2(a). A literature survey reveals that all existing automatic transmission mechanisms have their links distributed up to the second level (Tsai et al., 1988; Gott, 1991). In this paper, only those *EGM*s with their vertex distribution up to the second level are considered.

1.2 Fundamental Geared Entity

Chatterjee and Tsai (1994b) defined the *FGE* as a subgraph of an *EGM* formed either by a single second-level vertex or a chain of heavy-edge connected second-level vertices together with all the lower level vertices connecting them to the root. Figures 2(b) through 2(d) show

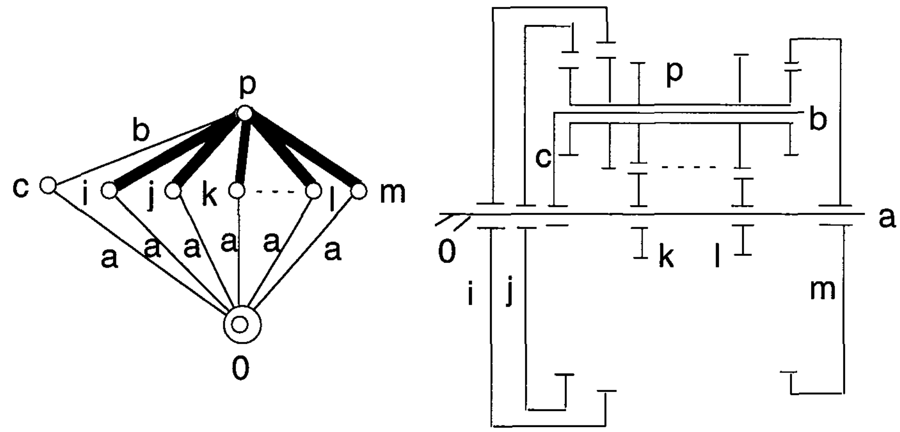


Figure 3: A typical single-planet FGE.

the graphs of three *FGE*s identified from Fig. 2(a). Fundamental geared entities having more than two meshing planet gears are not practical and will be excluded from further consideration. Thus, only two general types of *FGE*s will be considered. They are the *single-planet FGE* and the *double-planet FGE* as shown in Figs. 3 and 4.

2 Kinematic Characteristics

An *FGE* is said to be *active* if it takes power from and gives power to its external environment. The external environment includes the input power source, output shaft, and its adjacent *FGE*s. It has been shown (Chatterjee and Tsai, 1994b) that an active *FGE* must contain at least three ports of communication: one local input, one local output, and one local reaction link. Since only the first-level coaxial links are used as the input, output, or reaction link, the velocity relationship among these coaxial links should be established in order to effectively compute the various speed ratios of a transmission mechanism. This can be accomplished by eliminating the velocities of planet gears and inactive coaxial links (i.e., non-torque carrying links) from the fundamental circuit equations.

2.1 Kinematics of a Single-Planet FGE

In a single-planet *FGE*, the coaxial links include a carrier and several first-level gears meshing with one planet gear. An active single-planet *FGE* can have two different modes of operation. Each operation mode contains three ports of communication. The first operation mode

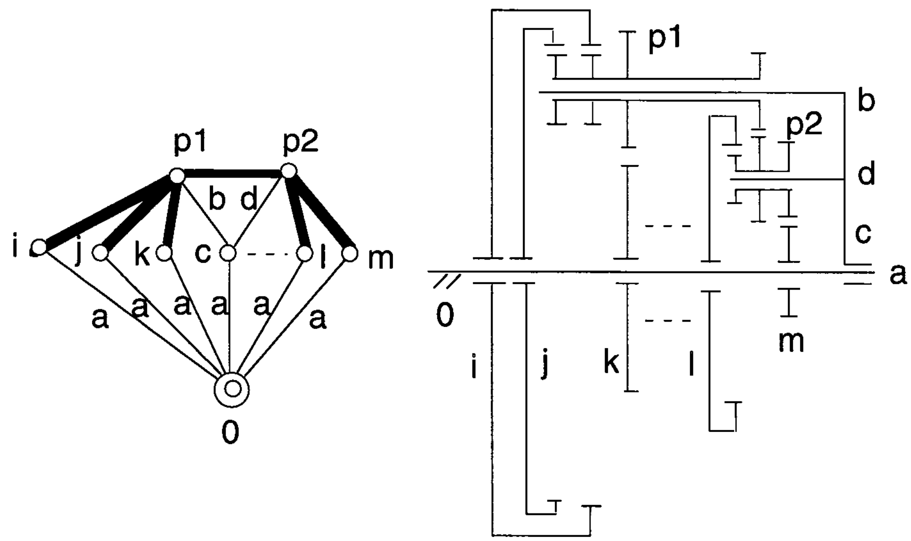
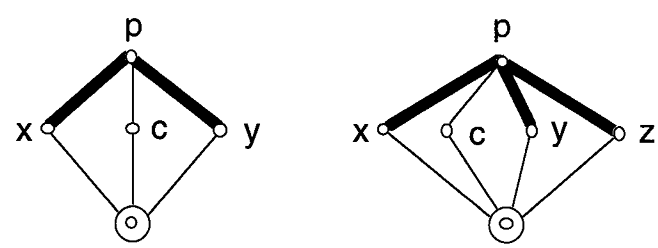


Figure 4: A typical double-planet FGE.



(a) First mode (x, y, c) (b) Second mode (x, y, z)

Figure 5: Two operation modes.

contains two gears x and y, and one carrier c as the three ports of communication as shown in Fig. 5(a). The second operation mode contains three gears x, y, and z as the three ports of communication while the carrier does not communicate with the external environment as shown in Fig. 5(b).

The fundamental circuit equations for the first operation mode shown in Fig. 5(a) can be written as

$$\frac{\omega_x - \omega_c}{\omega_p - \omega_c} = N_{p,x} \tag{1}$$

and

$$\frac{\omega_y - \omega_c}{\omega_p - \omega_c} = N_{p,y} \tag{2}$$

Dividing Eq. (2) by (1) yields a kinematic equation relating the angular velocities of the three ports of communication as

$$R^c_{y,x} \equiv \frac{\omega_y - \omega_c}{\omega_x - \omega_c} = K_1 \tag{3}$$

where $K_1 = N_{p,y}/N_{p,x}$ is called the *velocity parameter* of the first operation mode.

Similarly, three fundamental circuit equations can be written for the second operation mode shown in Fig. 5(b). Eliminating ω_p and ω_c from these fundamental circuit equations, yields

$$R^z_{y,x} \equiv \frac{\omega_y - \omega_z}{\omega_x - \omega_z} = K_2 \tag{4}$$

where $K_2 = (N_{p,y} - N_{p,z})/(N_{p,x} - N_{p,z})$ is called the *velocity parameter* of the second operation mode.

2.2 Kinematics of a Double-Planet FGE

In a double-planet FGE, there are two meshing planet gears, one carrier, and several coaxial gears meshed with either one of the two planet gears. There are four possible modes of operation. The three ports of communication in the first and second modes involve only one planet gear. They function like a single-planet FGE. Hence, their kinematic equations are given by Eqs. (3) and (4).

The third operation mode contains a coaxial gear x meshing with the first planet $p1$, another coaxial gear y meshing with the second planet $p2$, and the carrier c as the three ports of communication as shown in Fig. 6(a). Three fundamental circuit equations can be written for the three gear pairs. Eliminating the variables associated with the two planets, yields

$$R^c_{y,x} \equiv \frac{\omega_y - \omega_c}{\omega_x - \omega_c} = K_3 \tag{5}$$

where $K_3 = N_{p2,y}N_{p1,p2}/N_{p1,x}$ is called the *velocity parameter* of the third operation mode.

The fourth operation mode contains three coaxial gears x, y, and z as the three ports of communication while the carrier does not communicate with the external environment as shown in Fig. 6(b). Four fundamental circuit equations can be written. Eliminating the variables associated with the two planets and the carrier, yields

$$R^z_{y,x} \equiv \frac{\omega_y - \omega_z}{\omega_x - \omega_z} = K_4 \tag{6}$$

where $K_4 = (N_{p2,z} - N_{p1,y}N_{p2,p1})/(N_{p2,z} - N_{p1,x}N_{p2,p1})$ is called the *velocity parameter* of the fourth operation mode.

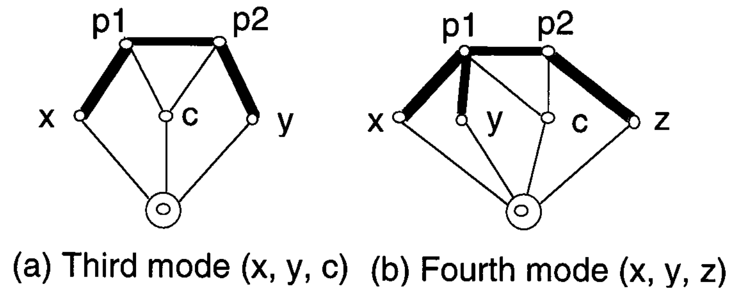


Figure 6: Two additional operation modes.

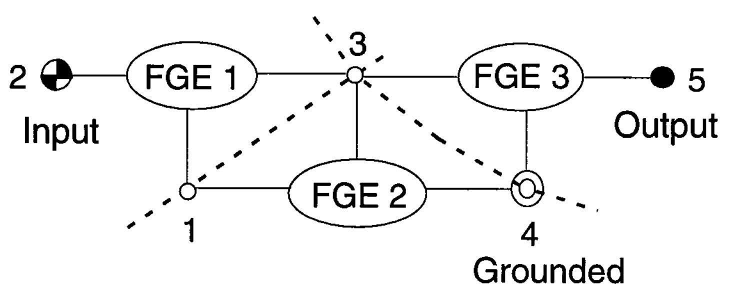


Figure 7: FGE diagram of the mechanism shown in Fig. 1.

3 FGE Diagram

To simplify the speed ratio analysis, an *EGM* is decomposed into several *FGE*s, and a diagram is drawn to demonstrate the interactions between these *FGE*s. We call such a diagram an *FGE* diagram. In an *FGE* diagram, each port of communication is called a *node*. The line connecting a node to an *FGE* is called a *chord*. A node is said to be a *unitary*, *binary*, or *ternary* node depending on whether there is one, two, or three chords incident to it. Figure 7 shows the FGE diagram for the mechanism shown in Fig. 1, where 2 and 5 are unitary nodes, 1 and 4 are binary nodes, 3 is a ternary node.

A node is said to be *activated*, if it is connected to the input power source, the casing, or the output link. A chord is said to be *active* if it carries torque. An *FGE* is said to be active if it contains at least one active chord. Under steady-state operation, the sum of all torques acting on an *FGE* must be equal to zero. Further, the sum of all torques acting on a node must also be equal to zero. Using these two conditions, the following rules are established for the identification of active *FGE*s.

Rule 1. When all the activated nodes belong to one *FGE*, this *FGE* is the only active entity.

Rule 2. If a unitary node is not activated, then its incident chord is inactive. On the other hand, if a unitary node is activated, then its incident chord must be active and so is the associated FGE. Except for the direct drive for which the EGT is locked up as a single body, there are at least three active chords associated with an active FGE.

Rule 3. If a binary node is not activated and one of its incident chord is inactive, then the other incident chord is also inactive. On the other hand, if a binary node is not activated and one of its incident chord is active, then the other incident chord must be active.

Rule 4. When an FGE contains only three chords and one of its incident chords is inactive, then the remaining two chords are also inactive. Thus, this FGE is inactive.

Since only active FGEs are essential for the speed ratio analysis, we apply the above rules to eliminate those inactive chords and inactive FGEs from the FGE diagram. The resulting diagram is called an *active* FGE diagram. For example, when the mechanism shown in Fig. 1 is in the third gear, link 4 is grounded, link 2 is connected to the power source, while link 5 serves as the output link. All the nodes and chords are active. The diagram shown in Fig. 7 is an active FGE diagram.

4 Speed Ratio Relation

When three coaxial links x, y and z of an EGM are selected as the candidates for the input, output and reaction links, there are $3! = 6$ possible clutching conditions. The following basic characteristics relate the speed ratios of these six clutching conditions.

Characteristic 1. If the output and ground links are exchanged, then the two speed ratios are related by

$$R^z_{y,x} + R^x_{y,z} = \frac{\omega_y - \omega_z}{\omega_x - \omega_z} + \frac{\omega_y - \omega_x}{\omega_z - \omega_x} = 1 \tag{7}$$

Characteristic 2. If the output and input links are exchanged, then the two speed ratios are related by

$$R^z_{x,y} = \frac{\omega_x - \omega_z}{\omega_y - \omega_z} = 1/(\frac{\omega_y - \omega_z}{\omega_x - \omega_z}) = \frac{1}{R^z_{y,x}} \tag{8}$$

Characteristic 3. If the input and ground links are exchanged, then the two speed ratios are related by

$$R^y_{z,x} = \frac{1}{R^y_{x,z}} = \frac{1}{1 - R^z_{x,y}} = \frac{1}{1 - \dfrac{1}{R^z_{y,x}}} = \frac{R^z_{y,x}}{R^z_{y,x} - 1} \tag{9}$$

Table 2: Six Basic Speed Ratios and Their Relations

Type of operation	$R^z_{y,x}$	$R^z_{x,y}$	$R^x_{y,z}$	$R^x_{z,y}$	$R^y_{x,z}$	$R^y_{z,x}$
Speed ratio	r	$\frac{1}{r}$	$1-r$	$\frac{1}{1-r}$	$\frac{r-1}{r}$	$\frac{r}{r-1}$

Applying Characteristics 1 through 3, we can use one speed ratio to determine the remaining five as shown in Table 2. Note that if x, y, and z are the activated nodes of an FGE, then $r = K_i$, i=1, 2, 3, and 4, and Table 2 represents the six basic speed ratios of an FGE.

5 Overall Speed Ratio Analysis

To perform the overall speed ratio analysis, first we sketch the active FGE diagram with respect to a given clutching condition. Second, we decompose the mechanism into two subsystems which are connected to each other by two common nodes as shown in Fig. 8. The two subsystems must share two and only two common nodes so that the overall speed ratio of an EGM can be expressed in terms of the speed ratios of the two subsystems. The common nodes, called the *bridges*, transmit torques from one subsystem to another. Third, we further decompose each first-level subsystem into two second-level subsystems. The process continues until each lowest level subsystem becomes an FGE. This way, the overall speed ratio of an EGM can be expressed in terms of the fundamental geared entities. Since the number of active chords in an EGM depends on the location of activated nodes and the connection among the FGEs, we can classify the clutching conditions into the following two cases.

5.1 Two Subsystems Share One Activated Node

Let x, y, and z denote three activated notes of an EGM. Whenever it is possible, we decompose a system into two subsystems such that the two subsystems share one common activated node, say z, as shown in Fig. 9.

Case 1a: Common Grounded Link. When the common activated node is grounded, we simply express the overall speed ratio as a product of two speed ratios as

$$R^z_{y,x} = R^z_{y,b} R^z_{b,x} \tag{10}$$

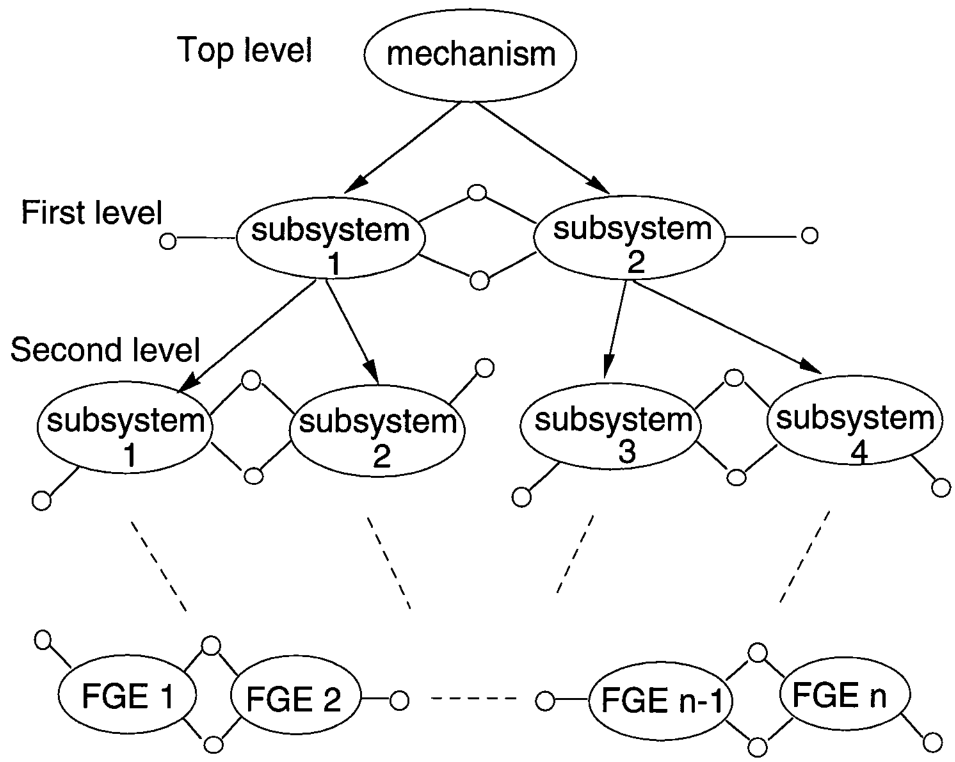


Figure 8: Decomposition of a system.

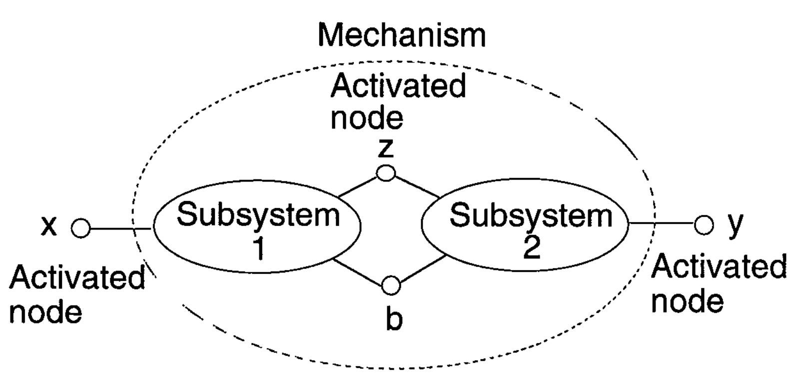


Figure 9: Two subsystems sharing one activated node.

where $R^z_{y,b}$ is associated with subsystem 2 and $R^z_{b,x}$ is associated with subsystem 1.

Case 1b: Common Output Link. When the common activated node is the output link, we apply Table 2 and Eq. (10) to obtain the overall speed ratio as

$$R^x_{y,z} = 1 - R^z_{y,x} = 1 - R^z_{y,b}R^z_{b,x} \tag{11}$$

Case 1c: Common Input Link. When the common activated node is the input link, we apply Table 2 and Eq. (10) to obtain the overall speed ratio as

$$R^x_{z,y} = \frac{1}{1 - R^z_{y,x}} = \frac{1}{1 - R^z_{y,b}R^z_{b,x}} \tag{12}$$

Example 1. Consider the third gear of the mechanism shown in Fig. 1. The active *FGE* diagram is shown in Fig. 7. First we decompose the mechanism into two subsystems: the first contains *FGE* 3 and the second contains *FGE*s 1 and 2. Since the grounded node 4 is shared by the two subsystems, applying Eq. (10) yields

$$R^4_{2,5} = R^4_{2,3}R^4_{3,5} \tag{13}$$

Because the speed ratio $R^4_{2,3}$ in Eq. (13) is associated with a subsystem which contains two *FGE*s, we need to further decompose it into two second-level subsystems by cutting through nodes 1 and 3. Since the common node 3 is now treated as an output node for the speed ratio $R^4_{2,3}$, Eq. (11) yields

$$R^4_{2,3} = 1 - R^3_{2,4} = 1 - R^3_{2,1}R^3_{1,4} \tag{14}$$

Since the speed ratios $R^3_{2,1}$ and $R^3_{1,4}$ in Eq. (14) are associated with *FGE*s 1 and 2, respectively, the speed ratio analysis is now completed. Combining Eqs. (13) and (14) yields the overall speed ratio as

$$R^4_{2,5} = (1 - R^3_{2,1}R^3_{1,4})R^4_{3,5} \tag{15}$$

5.2 Two Subsystems Share No Activated Node

Figure 10 shows a situation when it is impossible to decompose a mechanism into two subsystems with one common activated node. In this case, one subsystem will contain one activated node, say node k, and the other contains two activated nodes, say nodes i and j. Again, we classify the speed ratio analysis into three subcases as follows.

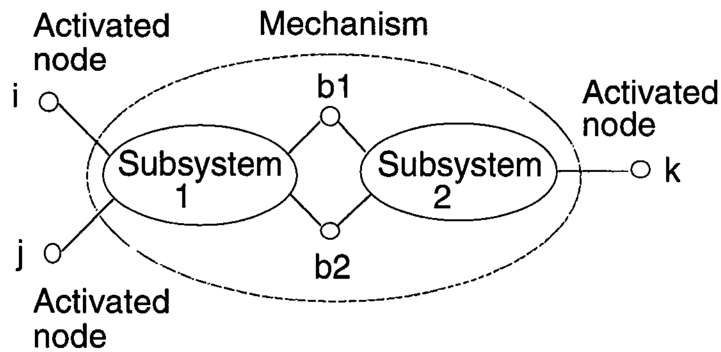


Figure 10: Two subsystems sharing no activated node

Case 2a: Output and Grounded Nodes Belong to One Subsystem. We use the node b_1 as the bridge to obtain the speed ratio as

$$R^i_{k,j} = R^i_{k,b1} R^i_{b1,j} \tag{16}$$

Since the speed ratio $R^i_{b1,j}$ in Eq. (16) belongs to only one subsystem, we only need to analyze the speed ratio $R^i_{k,b1}$. Since the common node b_1 is treated as the output node for $R^i_{k,b1}$, applying Eq. (11) yields

$$R^i_{k,b1} = 1 - R^{b1}_{k,i} = 1 - R^{b1}_{k,b2} R^{b1}_{b2,i} \tag{17}$$

Combining Eqs. (16) and (17) yields the overall speed ratio as

$$R^i_{k,j} = (1 - R^{b1}_{k,b2} R^{b1}_{b2,i}) R^i_{b1,j} \tag{18}$$

Case 2b: Input and Grounded Nodes Belong to One Subsystem. We first obtain the speed ratio of an inverted mechanism for which the input and output links are interchanged. Then, we apply Eq. (18) to derive the overall speed ratio as

$$R^i_{j,k} = \frac{1}{R^i_{k,j}} = \frac{1}{(1 - R^{b1}_{k,b2} R^{b1}_{b2,i}) R^i_{b1,j}} \tag{19}$$

Case 2c: Input and Output Nodes Belong to One Subsystem. We first obtain the speed ratio of an inverted mechanism for which the output and grounded links are interchanged. Then, we apply Eq. (19) to derive the overall speed ratio as

$$R^k_{j,i} = 1 - R^i_{j,k} = 1 - \frac{1}{(1 - R^{b1}_{k,b2} R^{b1}_{b2,i}) R^i_{b1,j}} \tag{20}$$

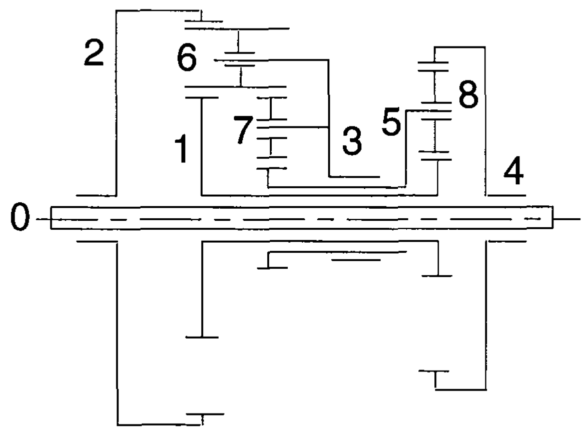


Figure 11: Schematic diagram of an *EGM*

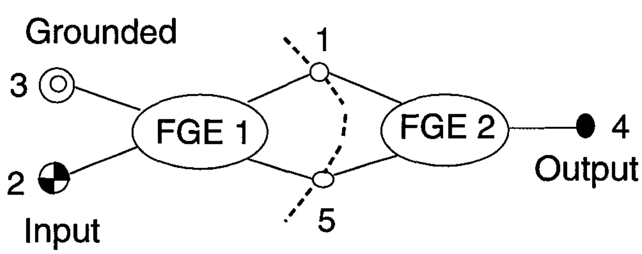


Figure 12: Active FGE diagram with no commonly activated node.

Example 2. Figure 11 shows the schematic diagram of a compound EGM. Assume that links 2, 3, and 4 are connected to the power source, the casing, and the output shaft, respectively. The corresponding active FGE diagram is shown in Fig. 12.

First, we decompose the system by cutting through nodes 1 and 5. These two subsystems share no common activated note. Since the input and grounded nodes belong to FGE 1, applying Eq. (19) yields the overall speed ratio as

$$R^3_{2,4} = \frac{1}{(1 - R^1_{4,5} R^1_{5,3}) R^3_{1,2}} \tag{21}$$

6 Conclusion

We have investigated the kinematic characteristics associated with the various operation modes of FGEs. These kinematic characteristics are then applied to the analysis of the overall speed ratio of an EGM. It is shown that, by decomposing a mechanism into two subsystems and each subsystem into two second-level subsystems, and so on until each lowest level subsystem contains only one FGE, the overall speed ratio of an EGM can be symbolically expressed in terms of its FGEs. The main advantage of this method is that an algebraic expression for the overall speed ratio can be derived without mathematically manipulating the fundamental circuit equations. The method leads to an automated derivation of the speed ratio of an EGM without the need of a symbolic manipulation software. Future work will apply these characteristics and the method of analysis to the development of an automated methodology for the enumeration of various clutching sequences associated with an epicyclic gear mechanism.

7 Acknowledgment

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