TR-87-16

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Submitted to IEEE Transactions on Communications, October 1986

A DISTRIBUTED RESERVATION-BASED CDMA PROTOCOL THAT DOES NOT REQUIRE FEEDBACK INFORMATION

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ABSTRACT

In some multi-user radio systems the receiver is not allowed to transmit feedback information to the senders. In other applications the transmitters cannot receive feedback information sent by the receiver. For these cases a multiple access protocol is needed that maintains satisfactory performance in the absence of feedback. In this paper, such a protocol is introduced and an exact analysis of it is provided. The protocol, which uses an unconventional reservation mechanism, exploits the capability of interference rejection that Code Division Multiple Access (CDMA) provides.

* Ms. Tarr is an NRL Fellow under ONR Grant No. N00014-85-G-0207

I. INTRODUCTION

The use of spread-spectrum signaling enables the users of a multiple access system to transmit over the channel on a *code-division* basis. Code Division Multiple Access (CDMA) provides new degrees of freedom in the design of multiple access protocols. In this paper we focus on the case where no feedback information is available to the transmitter. Such a situation arises in cases of radio-silent receivers or of strong interference in the neighborhood of the transmitting users. We present a protocol that is suitable for the no-feedback case and that exploits CDMA capabilities. This protocol was first introduced in [1], but without an accurate analysis. Here we present an exact analysis method and a complete performance evaluation.

We consider a system in which a population of bursty users wishes to transmit to a single destination, as shown in Figure 1, without the possibility of feedback from the destination. From the perspective of strict *time domain* multiple access, a destructive *collision* occurs whenever two packets are transmitted simultaneously. Here we depart from the constraints of purely time-domain protocols, and consider a reservation scheme that takes advantage of the quasi-orthogonality property that can be achieved through the use of spread spectrum Code Division Multiple Access (CDMA) signaling. This property permits the successful reception of a packet despite the simultaneous transmission by other users. In addition, this reservation scheme is totally distributed in that it does not require any coordination among the users.

Spread spectrum signaling, normally used because of its antijamming capability, leads naturally to the use of CDMA techniques [2,3]. For example, in a frequency hopping (FH) CDMA system the code corresponds to the FH pattern. *Selective addressing* is achieved by the transmitting node through the use of a code associated with a particular destination. In this paper we are more interested in the *selective reception* capability, which is the ability of a receiver to monitor the code associated with any particular transmitting user, despite the presence of other signals on the channel.

For the purpose of analysis we assume that each hopping pattern is random, and that patterns corresponding to different codes are independent of each other. It is therefore possible for two or more users (using different hopping patterns) to transmit simultaneously in the same frequency bin, resulting in loss of data. The loss of data caused by such *frequency hits** can be handled via the use of error control coding. The number of users that can share a wideband channel by means of CDMA techniques and the resulting performance depend on the modulation/coding scheme, channel characteristics, and receiver implementation. This problem has been investigated by Pursley [4,5], Hajek [6], and Wieselthier and Ephremides [7].

There is an analogy between TDMA and CDMA. In the case of TDMA the channel is shared by assigning a distinct time slot to each user, thus ensuring that the users don't interfere with each other; the users are therefore orthogonal in the time domain. In the case of FH-CDMA a distinct code may be assigned to each user. A number of users can transmit simultaneously on different codes, with no individual user able by himself to cause significant interference to any other user; however, the combined effect of a sufficiently large number of users can disrupt each other's communication. There is thus only a ***quasi-orthogonality*** among users in the code-domain. Fortunately, a large number of quasi-orthogonal codes can be defined. Performance will be acceptable as long as not too many signals (using different codes) are transmitted simultaneously.**

Note that we use the term CDMA to refer to a class of multiple access techniques that use the CDMA multi-user channel property. Although CDMA itself permits a number of signals to share a wideband channel simultaneously, we retain the constraint that a user can correctly receive at most one signal at a time. It is therefore necessary to develop multiple access protocols that operate in the joint *time-code domain* by taking advantage of the CDMA multi-user channel property, while living within the constraints imposed by the use of spread spectrum signaling.

* These hits involve fractions of packets in this case, in contrast to the usual case of time-domain multiple access schemes in which collisions result in the destruction of entire packets.

** In some cases it is possible to have a set of orthogonal codes in a FH-CDMA system. This means that no two users ever transmit in the same frequency bin simultaneously. Clearly, the number of codes in such a system can be no greater than the number of frequency bins. In addition, very accurate timing is required to maintain synchronization among the hopping patterns corresponding to different codes; such timing is often impossible to achieve as a consequence of the inherent propagation delays in radio communication systems. In contrast, most analyses of quasi-orthogonal CDMA systems assume that there is no synchronization among the various codes, although of course the receiver must be synchronized to the particular code that it is monitoring.

The protocol we propose is based on the concept of reservations. In typical reservation schemes (such as Roberts' Reservation scheme [8]), each terminal first transmits reservation minipackets (which are much smaller than the actual information packets) to request the assignment of a time slot for each packet in its queue. Upon the successful transmission of a reservation minipacket, the corresponding information packet(s) will in effect join a common queue from which they may be transmitted, without danger of collision and according to whatever service discipline has been chosen (e.g., first come first served).

Under ideal conditions (i.e., a noiseless channel where all users are within communication range of each other and can thus monitor each other's reservation requests) no central controller would be needed. The Interleaved Frame Flush-Out (IFFO) protocols [9], for example, are a class of schemes in which transmission schedules are generated by the users in a distributed fashion, based upon the reservation requests of the population of users. In realistic situations, however, as a consequence of potential channel errors or equipment malfunction it cannot be assumed that all users have the same information. Inconsistent transmisision schedules can then be generated, resulting in the simultaneous transmission by two or more users and the subsequent destructive collision. Therefore, a central controller is usually needed to allocate slots to the users requesting them to ensure that at most one user transmits in any slot.

In the case we are interested in, namely when no feedback information from the common destination to the users is possible or available, a distributed reservation scheme is needed that can in fact operate in an environment of noisy channels characterized by lack of complete connectivity. It is especially important that the control of a channel access scheme be distributed when multiple users are attempting to communicate with a central station that cannot communicate back to the users, and thus cannot provide schedules.

II. SYSTEM MODEL AND PROTOCOL

A population of users transmits to a single radio-silent destination. We assume fixed length packets and a fixed length slotted frame structure (L slots per frame), as shown in Figure 2. As is typical of slotted systems, the slot duration is equal to the length of a packet, and all packet transmissions start at the beginning of a time slot. Each user is allowed to transmit at most one packet in any frame. The traffic process is assumed to be bursty, so that pure TDMA is ruled out. There is no coordination among users, who are assumed to be incapable of monitoring each others' reservations. Spread spectrum signaling is used, thereby facilitating the use of CDMA techniques, which permit the correct reception of signals despite the simultaneous transmission by other users. However, as we have already noted, the destination is assumed to be able to monitor only one of these signals at a time as in standard multiple access systems. In most of the discussion, we consider a noiseless channel, in which the only source of interference is the transmission by other members of the user population; however, in our discussion of quasi-orthogonal codes we also consider the case in which background channel noise can contribute to packet errors.

A contention-free reservation process is often assumed in the study of reservation-based channel access protocols. Such a mechanism can be implemented if the size of the total user population is not too great by designating the first slot of each frame as a reservation slot and dividing it into TDMA minislots. Alternatively, a separate reservation channel can be implemented. Contention-based reservation procedures have also sometimes been considered. In such cases only the reservation minipackets (which are much smaller in length than message packets) are competing for channel access, and significantly higher data throughput can be maintained than in a purely contention-based channel access scheme. In most of the discussion presented in this paper, it is assumed that the reservation process is contention-free and that it does not result in any overhead or other penalty in system performance. In doing so, we are able to separate the inherent performance of the distributed reservation scheme from the mechanics of the reservation process, which may be specifically related to the particular system implementation.

The reservation procedure that we propose is quite different from conventional schemes. Under the basic version of our protocol, each user with a packet to transmit chooses one of the L

slots in the frame at random. He sends a reservation minipacket that consists not only of a declaration of intent to transmit in the coming frame, but, in addition, of the actual slot number in which he will transmit. Since the users are uncoordinated, it is possible that two or more of them choose the same slot. Since the reservation process is contention-free, the receiver has full knowledge of all of the transmitters' intentions. In conventional *time-domain* schemes such interference would result in collisions that destroy each packet that is involved. However, the use of spread spectrum code division signaling permits the selective reception of one signal. Whenever two or more users declare their intent to transmit in the same slot, it is up to the receiver to decide which of these signals it will in fact monitor. Initially it is assumed that the codes are orthogonal, so that any number of other users can be tolerated in the same slot. In this case there is always a successful reception in any slot chosen by one or more users. In the more realistic case of quasi-orthogonal codes, a successful reception is not always possible. Our analysis establishes the achievable performance in that case, as well.

Three factors can contribute to the failure of a packet to be received successfully: (a) the reservation is not received correctly, (b) the packet cannot be scheduled to be received (because the destination is going to listen to another user's transmission), (c) the level of other-user interference is sufficiently high to cause a packet error. Since the destination does not transmit a listening schedule, the users do not know whether or not their transmissions will be monitored. We sometimes use the term *assigned slot* to refer to the slot in which a user's transmission will be monitored by the destination, although the destination does not transmit a listening schedule. The ability to receive one signal correctly, despite the presence of others, results in considerable performance improvement as compared with conventional time-domain ALOHA-type schemes, as we shall demonstrate.

A sample realization of a frame of protocol operation is illustrated in Figure 3 for the simple case of frame length $L = 5$ slots and $M = 5$ users transmitting in the frame. Only the actual data slots are shown, and not the reservation slot (or subchannel). The slots chosen by the users have been shaded. Users #1, #3, and #5 have chosen slots #2, #1, and #5 respectively; all of these users are successful because they are the only ones to transmit in their respective slots. Users #2 and #4, however, have both chosen slot #4. Only one of these is successful; the decision of whom to monitor is left up to the destination.

There are two basic ways to handle unsuccessful packets. They either may be retransmitted at a later time (e.g., in the next frame) or they may be simply dropped from the users' buffers. The former approach is the one most often taken in contention-based schemes. However, we have assumed that there is no feedback information of any type transmitted by the destination or among the users. The success or failure of individual transmissions cannot be determined, and so there is no information available on which to base a decision to retransmit. We thus assume that unsuccessful packets are dropped from the user's buffer, and therefore lost. The probability of packet loss is easily evaluated in the course of evaluating channel performance.

Exactly because the outcome of the transmission does not become known to the transmitter, it is natural to allow for *packet transmission diversity* by allowing each transmitting user to designate several (say Q) slots in which he will transmit the *same* packet within the frame. The destination node, after receiving all of these reservations, will attempt to generate a monitoring schedule that maximizes the number of distinct packets it will receive correctly. We can consider an extreme case in which each transmitting user transmits in all L slots of the frame. If the spread spectrum codes employed by each user were truly orthogonal, then such a scheme would provide optimum performance. However, only a quasi-orthogonality normally exists among such codes, and errors will result if too many signals attempt to share the channel simultaneously. The optimum value of the packet diversity parameter Q therefore depends on the degree of other-user interference that can be tolerated.

We first consider the orthogonal CDMA code case in which other-user interference is not troublesome (i.e., any number of other simultaneous users can be tolerated). One packet is thus successful in every assigned slot. A simple Markov chain model is sufficient to describe the behavior of this system. To model the more realistic case in which the CDMA codes are not orthogonal, it has been necessary to develop a three dimensional Markov chain model to characterize the level of other-user interference.

III. PROTOCOL ANALYSIS — ORTHOGONAL CODES

We evaluate the conditional probability distribution of the number of successful transmissions in a frame consisting of L slots, *given* that M users transmit packets in the frame, and each of these transmits its packet in Q slots chosen at random in the frame. We do not consider the mechanism used to make reservations; it is assumed in the present discussion that the destination receives error-free reservation information from all users. In Section VIII we address the impact of unsuccessful reservations on system performance. We also do not consider the statistics of the arrival process at this point. It is straightforward to incorporate the arrival statistics into the analysis, as we demonstrate in Section VII. In this section we consider only the case of perfectly orthogonal codes.

No Packet Diversity: $Q = 1$

When there is no packet diversity, each packet is transmitted exactly once in the frame. One way to analyze this case is to use combinatorial techniques to determine the probability distribution for the number of non-empty time slots (see e.g., [10]). We consider, however, the following Markov chain approach, which can be extended to apply directly to the case of $Q > 1$ as well.

We consider one frame of L slots and M users, each of which transmits in exactly one slot. The number of successful packets in the frame is equal to the number of slots in which one or more packets are transmitted. We approach this problem by considering the M users, one by one, as they independently place a packet into one of the L time slots. As each user picks a slot, we determine whether this slot has already been chosen by another user. We consider the probability of the number of successes in the frame, as the number of users we have counted is increased from j to $j+1$, for $1 < j < M-1$. Thus, we define

$$P(n \mid i) = Pr(n \text{ successes by first } j+1 \text{ users} \mid \text{given } i \text{ successes by first } j \text{ users}). \quad (1)$$

Clearly, the only possible transitions from i successes when one more user is accounted for are to $n = i$ and $n = i+1$. An unsuccessful transition occurs if user $j+1$ chooses one of the i slots chosen by the first j users; thus we have

$$P(i \mid i) = \frac{i}{L}. \tag{2}$$

A successful transition occurs if user $j+1$ chooses one of the $(L-i)$ slots not chosen by the first j users; thus we have

$$P(i+1 \mid i) = 1 - \frac{i}{L}. \tag{3}$$

We define,

$$p_j(i) = Pr(i \text{ successes in the group of the first } j \text{ users}). \tag{4}$$

This probability can be expressed in terms of the transition probabilities as,

$$p_j(i) = p_{j-1}(i)P(i \mid i) + p_{j-1}(i-1)P(i \mid i-1), \tag{5}$$

with initial condition $p_1(1) = 1$. The distribution for $p_j(i)$ is evaluated recursively until we obtain,

$$\begin{aligned} p_M(i) &= Pr(i \text{ successes in the group of the } M \text{ users}) \\ &= Pr(i \text{ successes in frame}). \end{aligned} \tag{6}$$

Packet Diversity: $Q > 1$

For $Q > 1$ the distribution for the number of successful transmissions depends on the strategy used by the destination to determine whom it will monitor in each slot. In Figure 4 we illustrate the difficulty of slot assignment for $Q > 1$ in extremely simplified form for the case of $Q = 2$, $L = 3$, and $M = 3$. In this example, if the destination decides to monitor user #1 in slot #1 and user #2 in slot #2 then no assignment is possible for user #3. If, however, user #1 is monitored in slot #4, then slot #1 would be available for user #3. For large values of L, M, and Q it is considerably more difficult to create an optimum set of slot assignments (i.e., one that maximizes the number of successful packets), unless an exhaustive search is made amongst all possible listening schedules. We therefore consider a non-optimal scheme for constructing a listening schedule that is, in fact, amenable to exact analysis, which we proceed to describe.

We let the destination assign slots in its listening schedule before it has complete knowledge of the reservations made by all users. As in the case of $Q = 1$, we consider the transition probabilities as we count the users, until all M users have been considered. The first user ($j = 1$) is always successful. The destination chooses to monitor one of that user's Q slots at random; the remaining $Q-1$ slots reserved by the first user are treated as empty slots by the destination. No effort is made to coordinate this assignment choice with those for the remaining users; specifically, the destination does not backtrack to change the earlier entries of his schedule as he reviews the reservation list. He proceeds to make an arbitrary assignment for the second user by choosing randomly one of that user's Q slots, with the only constraint that the already blocked slot for listening to user #1 is no longer considered. The process continues in this fashion until all users' reservation announcements have been looked at, and a listening schedule has been constructed. The order of looking at the reservations of the M users is arbitrary. This procedure will obviously result in some inefficiencies, as discussed above. The analysis thus provides a pessimistic estimate of the system performance as compared with that of a more intelligent decision maker.* We emphasize that the analysis is exact for the case of this non-optimal scheduling process.

Note that the first Q users are always successful, even if they all choose the same set of Q slots. In general, user j will be successful if one or more of his Q slots has not already been assigned to another user. Since the destination randomly assigns one of the (not previously assigned) slots to that user, we have

$$P(i \mid i) = \begin{cases} \dfrac{i}{L}\dfrac{(i-1)}{(L-1)} \cdots \dfrac{(i-Q+1)}{(L-Q+1)}, & i \geq Q \\ 0, & i < Q \end{cases} \tag{7}$$

and,

*A more intelligent decision maker might also be able to avoid scheduling users in slots in which there is an especially high level of other-user interference, as determined by examining the reservations.

$$P(i+1 \mid i) = 1 - P(i \mid i). \tag{8}$$

We can again use the recursion defined by eq. (5) to evaluate the probability distribution for the number of successful transmissions in the frame.

IV. PERFORMANCE EVALUATION — ORTHOGONAL CODES

The throughput is defined as the expected number of successful packets received per slot. For the present case of orthogonal CDMA codes, the throughput is easily expressed as follows.

$$S = \mathit{Throughput} = \frac{\sum_{i=1}^{M} i \; p_M(i)}{L}. \tag{9}$$

Any overhead caused by the reservation process has been neglected. In Figure 5 we illustrate throughput as a function of M, the number of users transmitting in the frame, for packet diversity values of $Q = 1$, 2, 3, and 4. The frame length is $L = 10$ slots. These performance curves were generated under the assumption that the spread spectrum codes are orthogonal, thus permitting other-user interference to be ignored, and that there is no background noise in the communication channel. An upper bound on throughput (corresponding to the case of $Q = L$, i.e., all users transmitting in every slot) is also provided. Throughput of course increases as Q increases, with the most significant increase occurring as Q is increased from 1 to 2. The throughput of an ALOHA-type system (i.e., $Q = 1$, but transmission is unsuccessful if two or more users transmit in the same slot) is also shown to illustrate the considerable improvement that is obtained as a result of the ability to tolerate other-user interference by means of spread spectrum CDMA signaling.

We also consider the probability that a user is unsuccessful in a given frame, which is obtained directly from the throughput calculation:

$$Pr(\text{user unsuccessful}) = 1 - \frac{L\,S}{M}. \tag{10}$$

This expression is valid whether or not the CDMA codes are orthogonal. This performance criterion is especially significant for our protocol, because there is no feedback from the destination, and thus no possibility for the retransmission of unsuccessful packets. One can consider the throughput that is achievable under the constraint that the probability that a user is unsuccessful does not exceed some predetermined value. For the present case of orthogonal CDMA codes, a user is unsuccessful in a frame only if the destination does not schedule to monitor his packet in any of his Q slots in the frame. Figure 6 shows the probability that a user is unsuccessful, as a function of M, for the same set of system parameters used in Figure 5. Note that for orthogonal codes the probability that a user is unsuccessful is zero if M (the total number of users) is less than or equal to Q (the packet diversity).

The probability that a packet is unsuccessful, which we just discussed, represents the average failure probability of all users in the system. The failure probability of a packet of a particular user actually depends on the order in which his reservation is processed by the destination. As discussed earlier, the packets of the first Q users are always scheduled, whereas the packets of subsequent users have a decreasing probability of being scheduled. Thus, this protocol contains an inherent priority mechanism. The probability that the j^{th} user is scheduled is calculated in the course of evaluating the state probabilities. It is evaluated by summing over all states in which the number of successful users, i, increases when the j^{th} user is added to the system. Thus, we have

$$\begin{aligned} Pr\,(j^{th} \text{ user successful}) &= Pr\,(j^{th} \text{ user scheduled}) \\ &= \sum_{i=0}^{j} p_{j-1}(i)\,P\,(i+1 \mid i) \qquad j \geq 2. \end{aligned} \tag{11}$$

Figure 7 shows the probability that the j^{th} user is unsuccessful for the case of orthogonal codes and packet diversity values of $Q = 1$, 2, 3, and 4. Note that the curves intersect at approximately the point where the 11^{th} user is added to the system. This behavior occurs because larger values of Q increase the probability of success for the earlier users, thus leaving fewer slots available for the last few users.

V. AN AUGMENTED STATE MODEL

When the CDMA codes are not orthogonal, the probability that a packet is received correctly depends on the level of interference produced by other users that share the same wideband channel. Frequency hopping systems, unlike direct sequence systems, are relatively insensitive to the relative signal strength of the interfering signals. It is generally reasonable to assume that a particular hop of a FH signal will experience destructive interference only if another signal transmits in the same frequency bin at the same time. Therefore, when random hopping patterns are used it is sufficient to characterize the other-user interference process in terms of the number of other users that share the same wideband channel [4,5].

The Markov chain model described above for the orthogonal code case provides only a characterization of the number of assigned slots in the frame, but not the number of users that transmit in the individual slots. We now present an analysis model which employs a more detailed state description to model accurately the effects of other-user interference. As before, a non-optimal slot assignment scheme is considered, in which the destination assigns slots before he has complete knowledge of the reservations made by all users. As mentioned earlier, he does so by constructing a listening schedule as he reviews the reservation announcements of the users in an arbitrary order, and without backtracking to change earlier entries in his schedule. The approach we take in the more accurate model is an extension of that presented for the orthogonal code case. We again count the users one at a time until all M users have been considered. The augmented state model reflects not only the total number of successes in the frame, but also the detailed events at an arbitrarily chosen time slot, designated as *our slot*. So let us choose such a slot. For each value of j, a state is defined as

$$\{i,t,A\}_j$$

where

$i =$ number of slots assigned when the reservations of the first j users are processed.

$t =$ number of users who have chosen *our slot* from the group of the first j users.

$$A = \begin{cases} 0 & \text{if } \textit{our slot} \text{ is not assigned yet} \\ 1 & \text{if } \textit{our slot} \text{ is already assigned} \end{cases}$$

From any such state, there are at most four states that can be entered, namely

$$\begin{array}{llll} \{i,t,0\} \rightarrow & \{i,t,0\} & \{i,t,1\} \rightarrow & \{i,t,1\} \\ \text{or} & \{i+1,t,0\} & \text{or} & \{i,t+1,1\} \\ \text{or} & \{i+1,t+1,0\} & \text{or} & \{i+1,t,1\} \\ \text{or} & \{i+1,t+1,1\} & \text{or} & \{i+1,t+1,1\} \end{array}$$

The transition probabilities are presented and derived in the Appendix.

The probability of the state $\{i,t,A\}_j$ after the j^{th} user has been considered is now defined as $p_j(\{i,t,A\})$, and it can be expressed as:

$$\begin{aligned} p_j(\{i,t,0\}) = {} & p_{j-1}(\{i,t,0\})P(\{i,t,0\} \mid \{i,t,0\}) + p_{j-1}(\{i\text{-}1,t,0\})P(\{i,t,0\}|\{i\text{-}1,t,0\}) \\ & + p_{j-1}(\{i\text{-}1,t\text{-}1,0\})P(\{i,t,0\}|\{i\text{-}1,t\text{-}1,0\}) \end{aligned} \tag{12}$$

or,

$$\begin{aligned} p_j(\{i,t,1\}) = {} & p_{j-1}(\{i\text{-}1,t\text{-}1,0\})P(\{i,t,1\}|\{i\text{-}1,t\text{-}1,0\}) + p_{j-1}(\{i,t,1\})P(\{i,t,1\}|\{i,t,1\}) \\ & + p_{j-1}(\{i,t\text{-}1,1\})P(\{i,t,1\}|\{i,t\text{-}1,1\}) + p_{j-1}(\{i\text{-}1,t,1\})P(\{i,t,1\}|\{i\text{-}1,t,1\}) \\ & + p_{j-1}(\{i\text{-}1,t\text{-}1,1\})P(\{i,t,1\}|\{i\text{-}1,t\text{-}1,1\}) \end{aligned} \tag{13}$$

with initial condition $p_0(\{0,0,0\}) = 1$. Thus the distribution for $p_j(\{i,t,A\})$ can be evaluated recursively until $p_M(\{i,t,A\}) = Pr(\text{state } \{i,t,A\} \text{ after all } M \text{ users have been accounted for})$ is obtained. The distribution of $\{i,t,A\}_M$ provides the joint distribution of the number of assigned slots in the frame, the number of users assigned to *our slot*, and whether or not *our slot* is assigned after the M^{th} (the final) user has been counted. The probability of success or failure in *our slot* can thus be determined for any model of other-user interference, including threshold-based [11,12] or probabilistic ones [4-6].

VI. PERFORMANCE EVALUATION USING THE AUGMENTED STATE MODEL

System performance can be determined from the distribution of $\{i,t,A\}_M$, in conjunction with a model for the effects of other-user interference on packet error probability. Although the augmented state model is not needed to evaluate the performance for the case of orthogonal codes, a discussion of this case first provides the necessary background for the discussion of the quasi-orthogonal code case.

1) Orthogonal Codes

Channel throughput is defined as the expected number of successful packets per slot. For orthogonal codes the throughput is calculated in terms of the augmented model as

$$S = Throughput = \frac{\sum_{i=1}^{M}\sum_{t=1}^{M}\sum_{A=0}^{1} i \; p_M(\{i,t,A\})}{L}. \tag{14}$$

In this expression, we have simply determined the expected value of i, i.e., the number of assigned slots in the frame, and divided it by the number of slots in the frame. This is sufficient because any assigned slot corresponds to a successful transmission and vice-versa.

There is another equivalent way to determine throughput. We recognize that the statistics of *our slot* are identical to those of any other slot. Thus, the probability that our slot is assigned (i.e., the probability that $A = 1$) is equal to the expected throughput per slot for the present case of orthogonal codes. We therefore have,

$$S = Pr(A = 1) = \sum_{i=1}^{M}\sum_{t=1}^{M} p_M(\{i,t,1\}). \tag{15}$$

This expression is easily modified to accommodate the properties of quasi-orthogonal codes, as we demonstrate next.

2) Quasi-orthogonal Codes

We first consider a threshold-based model for other-user interference in which a packet is never received correctly if the number of users transmitting in the same slot is equal to or greater than some threshold T, but always received correctly if it is less than the threshold. We define I to be the number of *other* users that can be tolerated in a slot. Clearly, $I = T-2$. Thus, a packet is successful as long as there are not more than $I = T-2$ *other* packets transmitted in the same time slot. The throughput of the system for a given threshold value is given by

$$S = \sum_{i=1}^{M} \sum_{t=1}^{T-1} p_M(\{i,t,1\}) \tag{16}$$

which is simply a truncation of eq. (15). Figures 8 and 9 show the throughput performance for threshold values of $I = 0$ and $I = 2$ respectively and $L = 10$. We see that the optimum value of Q, for a given threshold I, depends on the number of users. For the case of $I = 0$, however, in which the presence of one or more other users always causes a packet error (as in ALOHA), a packet diversity value of $Q = 1$ is best for any number of users.

In Figure 10 we illustrate throughput performance for $Q = 3$ as I is varied from 0 to 4. As I is increased the throughput increases, until it reaches the limiting case for $I \geq M-1$, which is in fact the orthogonal code case in which other user interference can be neglected.

If a threshold-based model for other-user interference is unsatisfactory, it is straightforward to incorporate probabilistic models of other-user interference into the throughput calculations. The throughput is easily expressed as

$$S = \sum_{i=1}^{M} \sum_{t=1}^{M} p_M(\{i,t,1\}) \, (1 - Pr(E \mid t-1)) \tag{17}$$

where $Pr(E \mid t-1)$ is the probability that a packet is received incorrectly, given that there are $t-1$ *other* packets transmitted in the same slot. This probability depends on the error control code that is used to correct errors caused by other-user interference and background noise, and on the number of frequency bins over which the signal is hopped.

We consider an application in which each packet is encoded as a single Reed-Solomon codeword. A Reed-Solomon (n, v) code is capable of correcting any pattern of $\tau = \lfloor (n-v)/2 \rfloor$ n-ary symbol errors.* The packet error probability, given k *other* users, is therefore

$$Pr(E \mid k) = \sum_{i=\tau+1}^{n} \binom{n}{i} \, p_k^{\,i} \, (1-p_k)^{n-i} \tag{18}$$

where p_k is the symbol error probability, given that k *other* users are transmitting in the same time slot (all frequency hits are assumed to result in symbol errors**). This quantity is easily

*The probability of undetected codeword error is less than $1/\tau!$, which is negligible in many applications.

**Considerable performance improvement can be obtained if frequency hits can be detected, and the corresponding symbols erased by the decoder. A codeword can be decoded correctly as long as the number of symbol erasures plus twice the number of symbol errors is not greater than 2τ.

expressed as [4,5]

$$p_k = 1 - (1 - \frac{2}{q})^k (1 - p_0) \tag{19}$$

where p_0 is the symbol error probability resulting from background noise in the absence of other-user interference, and q is the number of orthogonal frequency bins in the wideband channel.

Figure 11 shows the throughput performance for several examples of quasi-orthogonal CDMA codes with Reed-Solomon coding, compared with the performance of orthogonal CDMA codes and a threshold interference model. We have considered RS-(32,16) and RS-(32,24) codes, which have error-correcting capability of $r = 8$ and 4, respectively. The 32-ary symbol error probability in the absence of other-user interference was assumed to be 0.0 and 0.1. The number of frequency bins is $q = 50$ and the packet diversity is $Q = 3$. Note that the RS-(32,16) code (in a noiseless channel) performs nearly as well as the infinite threshold (orthogonal CDMA code) case, except when the number of users transmitting is large. Increasing q would improve performance further. Note also that the threshold model, which is often used in the modeling of spread spectrum systems for reasons of convenience, rather than accuracy, predicts quite different performance from the Reed-Solomon coding model which is based on channel and code properties. **These results indicate that threshold models do not predict performance accurately in spread spectrum multiple access systems.**

Figure 12 shows the effect on throughput performance of varying packet diversity ($Q = 1$, 2, 3, and 4) when the RS-(32,16) code is used, the channel is noiseless except for other-user interference, and $q = 50$ frequency bins. As in the case of orthogonal CDMA codes, the probability that a user is unsuccessful is once again given by $1 - LS/M$. Figure 13 shows the probability that a user is unsuccessful for the case of a RS-(32,16) code, a noiseless channel, and $q = 50$ frequency bins. Note that this probability is zero when $M = 1$, and non-zero whenever $M \geq 2$.

We can determine the probability that the j^{th} user is successful. However, the present case of quasi-orthogonal codes is more complicated than that of orthogonal codes. We have,

$$Pr(\text{user } j \text{ is successful}) = Pr(\text{user } j \text{ is successful} \mid \text{user } j \text{ is scheduled})\, Pr(\text{user } j \text{ is scheduled}). \tag{20}$$

Since the first factor on the right hand side is independent of j (it depends only on the number of users that transmit in the slot), we may express it as:

$$\begin{aligned} Pr(\text{user } j \text{ is successful} \mid \text{scheduled}) &= Pr(\text{a user is successful} \mid \text{scheduled}) \\ &= Pr(\textit{our slot} \text{ is successful} \mid \text{assigned}). \end{aligned} \tag{21}$$

The second equality reflects the fact that all slots have identical statistics, and the fact that the success of a user in an assigned slot depends only on the number of users in the slot. Recall that the event that ***our slot*** is assigned is designated as $A = 1$. We therefore obtain,

$$\begin{aligned} Pr(\text{user } j \text{ is successful} \mid \text{scheduled}) &= \frac{Pr(\textit{our slot} \text{ is successful}, A = 1)}{Pr(A = 1)} \\ &= \frac{Pr(\textit{our slot} \text{ is successful})}{Pr(A = 1)}. \end{aligned} \tag{22}$$

Both the numerator and denominator depend on M, the total number of users that transmit in the frame.

The second factor on the right hand side of eq. (20), namely Pr(user j is scheduled), which depends on j, can be evaluated from the simple one-dimensional Markov chain model used for the orthogonal code case (see eq. (11)).

Figure 14 shows the probability that the j^{th} user is unsuccessful, for an example with RS(32,16) coding, $M = 20$ users, $q = 100$ frequency bins, and $Q = 1, 2, 3,$ and 4. Note that, for a given value of Q, this probability is the same for each of the first Q users, because each has the same probability of being scheduled.

VII. ARRIVAL STATISTICS

In the results presented thus far, performance has been evaluated as a function of the number of users that attempt to transmit in a given frame. Certainly, in a practical system the number of users that transmit in each frame is not known a priori. The results presented thus far, in terms of a fixed number of users, are useful because they do not depend on any particular arrival process. We can now incorporate the statistics of the packet arrivals into our model.

Suppose that we have a fixed population of M users, each of which transmits in a given frame with probability Φ, a Bernoulli trial. The throughput is easily evaluated as

$$S = \sum_{m=1}^{M} \binom{M}{m} \Phi^m (1-\Phi)^{M-m} S_m \tag{23}$$

where S_m is the throughput of a channel in which m users transmit, as calculated earlier. This expression is valid for any model of other-user interference (e.g., threshold or Reed-Solomon coding for interference rejection).

Figure 15 compares the throughput performance of a system with a Bernoulli arrival process with one in which all of a fixed population transmit. For the case of the Bernoulli arrival process, each of the $M = 20$ users transmits with probability Φ. The horizontal axis is thus equal to 20 Φ. For the case in which all users transmit, the horizontal axis is simply the number of users in the system. The throughput achieved in the Bernouilli arrival process case is somewhat lower than that in the fixed population case for much of the region shown on the curves.

VIII. THE EFFECT OF ADDITIONAL INTERFERENCE SOURCES AND LOST RESERVATIONS

The interference model presented thus far takes into account the effects of the population of M users that transmit to the same destination, as well as the effects of background noise, modeled as p_0 (the Reed-Solomon symbol error probability in the absence of other-user interference). It is straightforward to incorporate the effects of additional users that employ the same FH signaling format, but which are not among the population of M users that make reservations. It is important to be able to do so, because the users taking part in the distributed reservation protocol may have to share a wideband channel with additional users that may be either using the distributed reservation protocol to communicate to a different destination, or that may be using a different protocol entirely.

We define N to be the number of additional interfering users. Since reservations for these users are not processed by the destination, they do not interfere with the scheduling of the M users of interest. Thus, each of these N users has the potential to add to the interference process in *our slot*, i.e., to the quantity we have defined as t, but not to add to the successful packet process i. Thus, the transition probabilities corresponding to these N users are

$$P(\{i,t+1,0\} \mid \{i,t,0\}) = \frac{\hat{Q}}{L} \tag{24}$$

$$P(\{i,t,0\} \mid \{i,t,0\}) = 1 - \frac{\hat{Q}}{L} \tag{25}$$

We have used the symbol $\hat{Q}$ to represent the packet diversity for the N additional users to indicate that it can be different from the value Q used by the M users of interest (actually each of the N users can use a different value of $\hat{Q}$). The frame length L must be the same, however. All other transitions have zero probability. The probability distribution of the system state is evaluated by stepping the distribution obtained for the M-user system through these additional transitions. These transitions may be incorporated either before or after those corresponding to the M users that are actually participating in the protocol.

The same approach can be used to model the effects of lost reservations. Since a user does not know whether his reservation is successful, he will transmit in the Q slots he selected, independently of whether or not the reservation is actually received correctly. Thus, his packets will add to the interference process seen by the other users, although they have no chance of correct reception. Since the destination constructs a listening schedule that is based only on the reservations that are received correctly, the users whose reservations are unsuccessful cannot interfere with the scheduling of the users whose reservations are successful. In the problem formulation, M is now the number of users whose reservations are successful. Users whose reservations are unsuccessful now become part of the population of N additional interfering users. It is assumed that virtually all errors in the reservation process are detected through the use of error control coding, so that the only effect of errors in the reservation process is that reservations are lost.

Figure 16 shows throughput performance as a function of M, for $Q = 3$ with N varying between 0 and 10, for the case of an RS-(32,16) code, a noiseless channel, and $q = 50$ frequency bins. Throughput experiences graceful degradation as N increases.

If a characterization of the statistics of N is available, performance can be obtained by averaging over the distribution, as follows,

$$S = \sum_{n=1}^{N} Pr(n \text{ additional users}) \, S_n \qquad (26)$$

where S_n is now the throughput for a system in which there are n additional users and all other system parameters are held fixed (i.e., M, Q, q, and the error control code properties). We have not attempted to characterize this distribution, because it depends highly on specific system parameters. It is, in fact, straightforward to incorporate any model for errors in the reservation process into the system model presented here. In particular, we note that the statistics of the lost-reservation process depend on the number of users that attempt to make reservations, and on the specific mechanism used to implement the reservation process.

IX. SOME FINAL REMARKS

The problems of Code Division Multiple Access represent a relatively new facet of the field of multiple access protocols, especially for the case of mobile users and radio channel applications. Many of the studies so far have focused on the analytical modeling of the spread-spectrum signaling schemes that make CDMA possible [4,5]. Not much attention has been paid to the exploitation of the new degrees of freedom that become available to the protocol designer through the use of CDMA. In this paper we have tried to illustrate the potential of such exploitation by focusing on a distributed reservation scheme that does, in fact, take advantage of such properties. Our analysis is exact, and accommodates a general model for other-user interference in FH-CDMA systems. Our results have demonstrated that a threshold model for other-user interference does not predict performance acceptably, and that a probabilistic model which incorporates the properties of error control codes and channel characteristics is necessary.

A crucial feature of the distributed reservation protocol, relating to robustness and survivability, is that the destination does not have to broadcast schedules, and can thus maintain radio silence. Therefore, to disrupt protocol operation one must disrupt the actual link from user to destination, since there is no (potentially weak) feedback or acknowledgment channel from destination to users. Another feature aiding survivability is the fixed frame length. One does not have to monitor either data traffic or control traffic to know frame boundaries. These are important considerations in assessing the suitability of the scheme in certain military applications.

Although we have considered a single destination to whom all packets are directed, we may also consider an extension to multiple destinations, as shown in Figure 17. We can assume that some of the users communicate with more than one destination, and that not all users are within communication range of all destinations. As in the single destination case, the use of multiple transmissions permits greater flexibility in slot assignment. It is certainly possible for one destination to monitor one transmission of a packet while another destination monitors one of the other redundant transmissions.

In contrast, we can consider a conventional centrally controlled reservation scheme (i.e., one in which the destination prepares and distributes schedules to the users) operating in a multiple destination environment. In such a system multiple transmissions from users would often be required (i.e., one to each destination), unless the destinations were able to coordinate their listening schedules, a task which requires the exchange of information among the destinations. Such coordination would have to be done for every frame because of the assumed bursty nature of the traffic process, and in many cases would not be feasible. Furthermore, to do so would violate the assumption of radio-silence. The distributed reservation scheme, on the other hand, is very well suited for communication from a population of bursty users to a group of geographically separated uncoordinated destinations. This is again an important consideration for mobile, multihop network applications.

In conclusion, we observe that the idea of one-way reservations as the cornerstone of protocol design yields attractive solutions to the multiple access problem in certain applications. In addition, it suggests ways in which to exploit the new degrees of freedom provided by the use of spread-spectrum CDMA techniques.

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APPENDIX

In this appendix we derive the exact expressions for the transition probabilities for the distributed reservation scheme. These transitions, which were enumerated in Section V, reflect the evolution of the number of successes in the frame as the reservations of each user are processed by the destination. We divide these transitions into two groups, i.e., those from state $\{i,t,0\}_j$ (*our slot* is not assigned after the reservations of the first j users have been processed) and those from state $\{i,t,1\}_j$ (*our slot* is assigned after the reservations of the first j users have been processed).

I. TRANSITIONS FROM $\{i,t,0\}_j$

I.1 $\{i,t,0\}_j \rightarrow \{i,t,0\}_{j+1}$

This transition implies that all Q transmissions of the $(j+1)^{st}$ user are in slots that have already been assigned to the first j users. The probability of this transition is:

$$P(\{i,t,0\}_{j+1} \mid \{i,t,0\}_j) = \left\{ \begin{array}{ll} \left(\frac{i}{L}\right)\left(\frac{i-1}{L-1}\right) \cdots \left(\frac{i-Q+1}{L-Q+1}\right) & i \geq Q \\ 0 & i < Q \end{array} \right\} = P(i \mid i) \qquad \text{(I.1)}$$

which is identical to the expression for $P(i \mid i)$ derived in Section III for the one-dimensional Markov chain model. Most of the transition probabilities derived in this appendix will, in fact, be expressed in terms of this quantity.

I.2 $\{i,t,0\}_j \rightarrow \{i+1,t,0\}_{j+1}$

This transition implies that at least one of the Q transmissions by the $(j+1)^{st}$ user is in a new (i.e., as yet unassigned) slot, but none are in *our slot*. The probability of this transition is given by:

$$P(\{i+1,t,0\}_{j+1} \mid \{i,t,0\}_j) = Pr\{\text{at least one in new slot, none in } \textit{our slot}\}$$
$$= Pr\{\text{at least one in new slot} \mid \text{none in } \textit{our slot}\} Pr\{\text{none in } \textit{our slot}\}.$$

We have,

$$Pr\{\text{none in } \textit{our slot}\} = \left(\frac{L-1}{L}\right)\left(\frac{L-2}{L-1}\right) \cdots \left(\frac{L-Q}{L-Q+1}\right) = \frac{L-Q}{L}.$$

$$Pr\{\text{at least one in new slot} \mid \text{none in } \textit{our slot}\} = 1 - Pr\{\text{none in new slot} \mid \text{none in } \textit{our slot}\}.$$

$$Pr\{\text{none in new slot} \mid \text{none in } \textit{our slot}\}$$

$$= \left\{ \begin{array}{ll} \left(\frac{i}{L-1}\right)\left(\frac{i-1}{L-2}\right) \cdots \left(\frac{i-Q+1}{L-Q}\right) & i \geq Q \\ 0 & i < Q \end{array} \right\} = \left(\frac{L}{L-Q}\right) P(i \mid i).$$

Therefore,

$$P(\{i+1,t,0\}_{j+1} \mid \{i,t,0\}_j) = \left[1 - \left(\frac{L}{L-Q}\right) P(i \mid i)\right] \left(\frac{L-Q}{L}\right)$$

$$= \frac{L-Q}{L} - P(i \mid i). \tag{I.2}$$

I.3 $\{i,t,0\}_j \rightarrow \{i+1,t+1,0\}_{j+1}$

This transition implies that at least one of the Q transmissions by the $(j+1)^{st}$ user is in an unassigned slot, and one of them is in *our slot*, but this user is not assigned to our slot. We write this transition probability as follows:

$$P(\{i+1,t+1,0\}_{j+1} \mid \{i,t,0\}_j) = P(\{0\}_{j+1} \mid \{i+1,t+1,\cdot\}_{j+1},\{i,t,0\}_j)P(\{i+1,t+1,\cdot\}_{j+1} \mid \{i,t,0\}_j),$$

where the notation $\{0\}_{j+1}$ refers to $A = 0$ (i.e., *our slot* is not yet assigned) after the reservation of the $(j+1)^{st}$ user has been processed.

Before proceeding with the derivation, we introduce the final transition type from state $\{i,t,0\}_j$, because the derivation of its transition probability is very similar to that which we are now discussing.

I.4 $\{i,t,0\}_j \rightarrow \{i+1,t+1,1\}_{j+1}$

This transition implies that one of the Q transmissions by the $(j+1)^{st}$ user is in *our slot*, and the user is assigned to *our slot*. We write this transition probability as follows:

$$P(\{i+1,t+1,1\}_{j+1} \mid \{i,t,0\}_j) = P(\{1\}_{j+1} \mid \{i+1,t+1,\cdot\}_{j+1},\{i,t,0\}_j)P(\{i+1,t+1,\cdot\}_{j+1} \mid \{i,t,0\}_j).$$

We now proceed with the derivations for cases I.3 and I.4. The second term on the right hand side of the transition probabilities for both of these cases is easily evaluated as follows,

$$P(\{i+1,t+1,\cdot\}_{j+1} \mid \{i,t,0\}_j) = 1 - Pr\{\text{none in } \textit{our slot}\}$$

$$= 1 - \left(\frac{L-1}{L}\right)\left(\frac{L-2}{L-1}\right) \cdots \left(\frac{L-Q}{L-Q+1}\right)$$

$$= 1 - \frac{L-Q}{L} = \frac{Q}{L}.$$

To complete the derivations for cases I.3 and I.4 we must evaluate,

$$P(\{0\}_{j+1} \mid \{i+1,t+1,\cdot\}_{j+1},\{i,t,0\}_j) \text{ and } P(\{1\}_{j+1} \mid \{i+1,t+1,\cdot\}_{j+1},\{i,t,0\}_j).$$

To obtain the probability that *our slot* is assigned, we recall the strategy that is used by the destination to assign slots, which was discussed in Section III. The destination chooses one slot at random from all of the slots of the current user that have not yet been assigned to some other user. Therefore, if $k+1$ of the Q slots chosen by the current user have not previously been assigned (one of which is *our slot*), then the probability that the user is assigned to *our slot* is $1/(k+1)$. Thus, we can write,

$$P(\{0\}_{j+1} \mid \{i+1,t+1,\cdot\}_{j+1},\{i,t,0\}_j) = \sum_{k=0}^{Q-1} Z(k)\left(\frac{k}{k+1}\right)$$

$$P(\{1\}_{j+1} \mid \{i+1,t+1,\cdot\}_{j+1},\{i,t,0\}_j) = \sum_{k=0}^{Q-1} \frac{Z(k)}{k+1}$$

where we have defined,

$$Z(k) = Pr\{k \text{ other packets in unassigned slots}\} \qquad k = 0,1,\cdots Q-1.$$

The transition probabilities are therefore:

$$P(\{i+1,t+1,0\}_{j+1} \mid \{i,t,0\}_j) = \left(\frac{Q}{L}\right) \sum_{k=0}^{Q-1} Z(k)\left(\frac{k}{k+1}\right) \tag{I.3}$$

$$P(\{i+1,t+1,1\}_{j+1} \mid \{i,t,0\}_j) = \left(\frac{Q}{L}\right) \sum_{k=0}^{Q-1} \frac{Z(k)}{k+1}. \tag{I.4}$$

The $Z(k)$'s are evaluated as follows:

$Z(0)$ is simply the probability that none of the $Q-1$ other diversity transmissions of user $(j+1)$ (i.e., not including the transmission in *our slot*) are in slots that are as yet unassigned; thus all $Q-1$ are in assigned slots, and we may consider them as *failures*.

$$Z(0) = \underbrace{\left(\frac{i}{L-1}\right)\left(\frac{i-1}{L-2}\right)\cdots\left(\frac{i-Q+2}{L-Q+1}\right)}_{|<\text{—— } Q-1 \text{ failures ——}>|} \qquad i > Q-2.$$

$Z(1)$ is the probability that one of the $Q-1$ other transmissions is in an unassigned slot (one *success*), and $Q-2$ are in assigned slots (*failures*).

$$Z(1) = (Q-1)\underbrace{\left(\frac{L-i-1}{L-1}\right)}_{|<\text{ 1 success }>|}\underbrace{\left(\frac{i}{L-2}\right)\left(\frac{i-1}{L-3}\right)\cdots\left(\frac{i-Q+3}{L-Q+1}\right)}_{|<\text{—— } Q-2 \text{ failures ——}>|}.$$

The general term is as follows:

$$Z(k) = Pr(k \text{ others in unassigned slots})$$

$$= \binom{Q-1}{k}\left(\frac{L-i-1}{L-1}\right)\left(\frac{L-i-2}{L-2}\right)\cdots\left(\frac{L-i-k}{L-k}\right)\left(\frac{i}{L-k-1}\right)\left(\frac{i-1}{L-k-2}\right)\cdots\left(\frac{i-Q+k+2}{L-Q+1}\right).$$

$|<$———— k successes ————$>|$ $|<$———— $Q-k-1$ failures ————$>|$

II. TRANSITIONS FROM $\{i,t,1\}_j$

II.1 $\{i,t,1\}_j \rightarrow \{i,t,1\}_{j+1}$

This transition implies that all of the Q transmissions by the $(j+1)^{st}$ user are in slots that have already been assigned, and none are in *our slot*.

$$P(\{i,t,1\}_{j+1} \mid \{i,t,1\}_j) = \begin{cases} \left(\frac{i-1}{L}\right)\left(\frac{i-2}{L-1}\right)\cdots\left(\frac{i-Q}{L-Q+1}\right) & i \geq Q \\ 0 & i < Q \end{cases}$$

$$= \left(\frac{i-Q}{i}\right) P(i \mid i). \qquad \text{(II.1)}$$

II.2 $\{i,t,1\}_j \rightarrow \{i,t+1,1\}_{j+1}$

This transition implies that all of the Q transmissions by the $(j+1)^{st}$ user are in slots that have already been assigned, and one is in *our slot*.

$$P(\{i,t+1,1\}_{j+1} \mid \{i,t,1\}_j) = Q\left(\frac{1}{L}\right)\left(\frac{i-1}{L-1}\right)\left(\frac{i-2}{L-2}\right)\cdots\left(\frac{i-Q+1}{L-Q+1}\right)$$

$$= \left(\frac{Q}{i}\right) P(i \mid i). \qquad \text{(II.2)}$$

II.3 $\{i,t,1\}_j \rightarrow \{i+1,t,1\}_{j+1}$

This transition implies that at least one of the Q transmissions by the $(j+1)^{st}$ user is in an unassigned slot, and none of them are in *our slot*.

$P(\{i+1,t,1\}_{j+1} \mid \{i,t,1\}_j) = Pr\{$at least one in new slot, none in *our slot*$\}$

$$= Pr\{\text{at least one in new slot} \mid \text{none in } \mathit{our\ slot}\} Pr\{\text{none in } \mathit{our\ slot}\}.$$

We already have

$$Pr\{\text{none in } \mathit{our\ slot}\} = \frac{L-Q}{L}.$$

Therefore,

$$\begin{aligned} P(\{i{+}1,t,1\}_{j+1} \mid \{i,t,1\}_j) &= \left(1 - Pr\{\text{none in new slot} \mid \text{none in } \mathit{our\ slot}\}\right)\left(\frac{L-Q}{L}\right) \\ &= \left[1 - \left(\frac{i-1}{L-1}\right)\left(\frac{i-2}{L-2}\right)\cdots\left(\frac{i-Q+1}{L-Q+1}\right)\left(\frac{i-Q}{L-Q}\right)\right]\left(\frac{L-Q}{L}\right) \\ &= \left[1 - P(i \mid i)\left(\frac{L}{i}\right)\left(\frac{i-Q}{L-Q}\right)\right]\left(\frac{L-Q}{L}\right). \end{aligned} \qquad \text{(II.3)}$$

II.4 $\{i,t,1\}_j \rightarrow \{i{+}1,t{+}1,1\}_{j+1}$

This transition implies that at least one of the Q transmissions by the $(j+1)^{st}$ user is in an unassigned slot, and one is in ***our slot***.

$$\begin{aligned} &P(\{i{+}1,t{+}1,1\}_{j+1} \mid \{i,t,1\}_j) \\ &\quad = Pr\{\text{one is in } \mathit{our\ slot}\}\left[1 - Pr\{\text{none of } Q-1 \text{ others are in new slot}\}\right]. \end{aligned}$$

$$Pr\{\text{one is in } \mathit{our\ slot}\} = \frac{Q}{L}.$$

$$Pr\{\text{none of } Q-1 \text{ others are in new slot}\} = \left(\frac{i-1}{L-1}\right)\left(\frac{i-2}{L-2}\right)\cdots\left(\frac{i-Q+1}{L-Q+1}\right).$$

Therefore, we obtain

$$\begin{aligned} P(\{i{+}1,t{+}1,1\}_{j+1} \mid \{i,t,1\}_j) &= \frac{Q}{L}\left[1 - \left(\frac{i-1}{L-1}\right)\left(\frac{i-2}{L-2}\right)\cdots\left(\frac{i-Q+1}{L-Q+1}\right)\right] \\ &= \frac{Q}{L}\left[1 - \frac{L}{i}P(i \mid i)\right]. \end{aligned} \qquad \text{(II.4)}$$

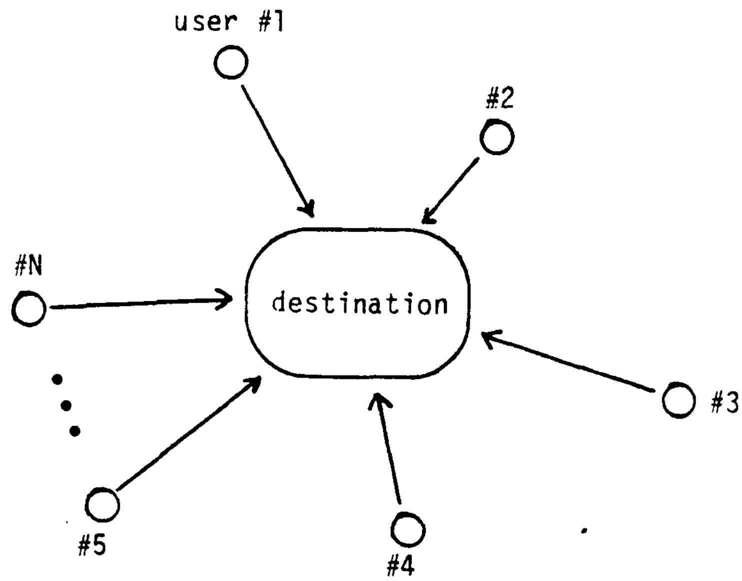

Fig. 1 Population of independent users communicating to a single radio-silent destination over a common radio or satellite channel.

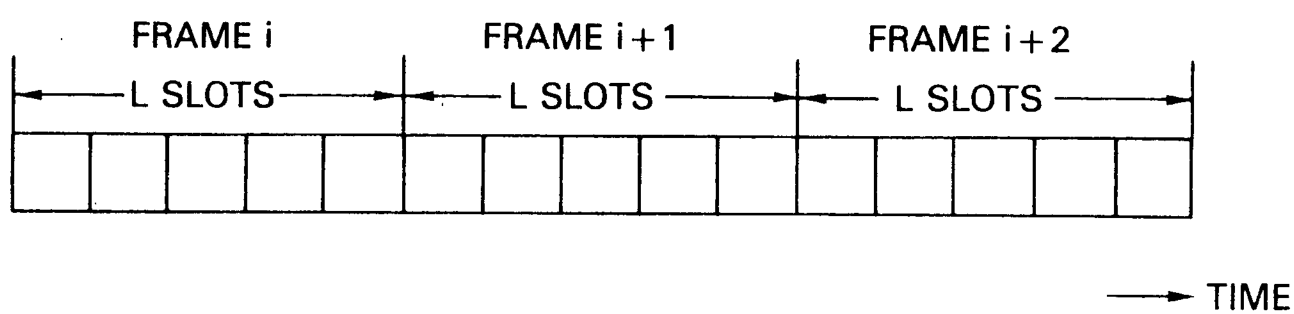

Fig. 2 Slotted frame structure.

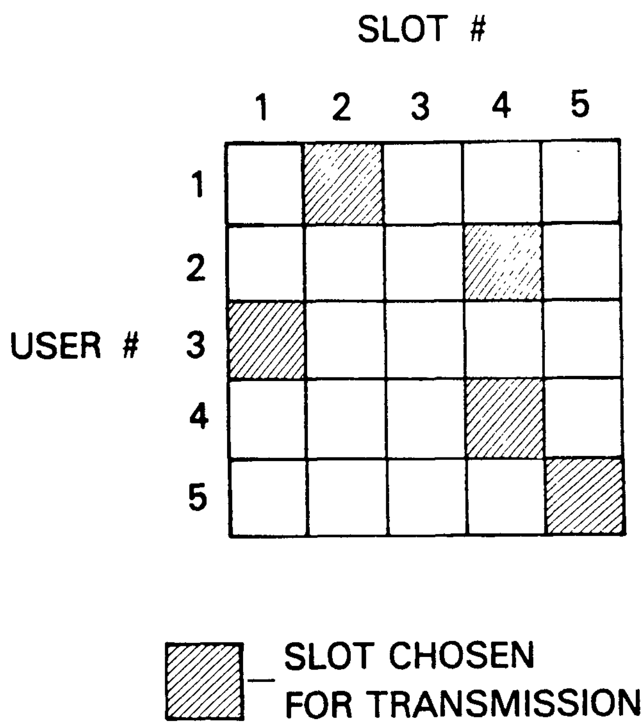

Fig. 3 Sample realization of a frame of protocol operation for $L = 5$, $M = 5$.

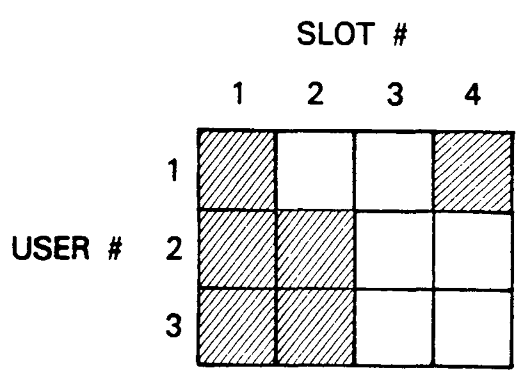

Fig. 4 Simplified illustration of the difficulty of slot assignment.

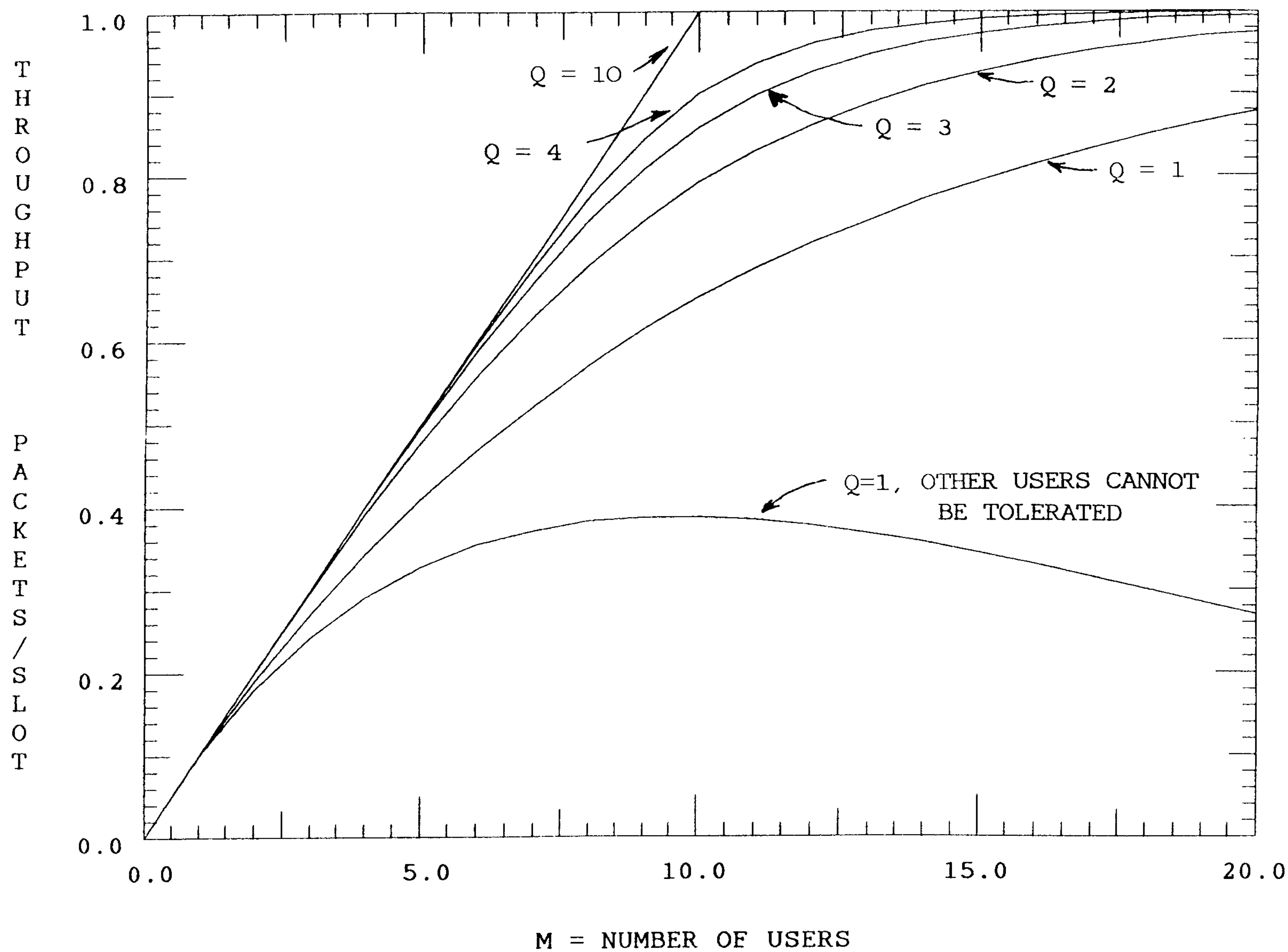

Fig. 5 Throughput performance of distributed reservation scheme; orthogonal CDMA codes.

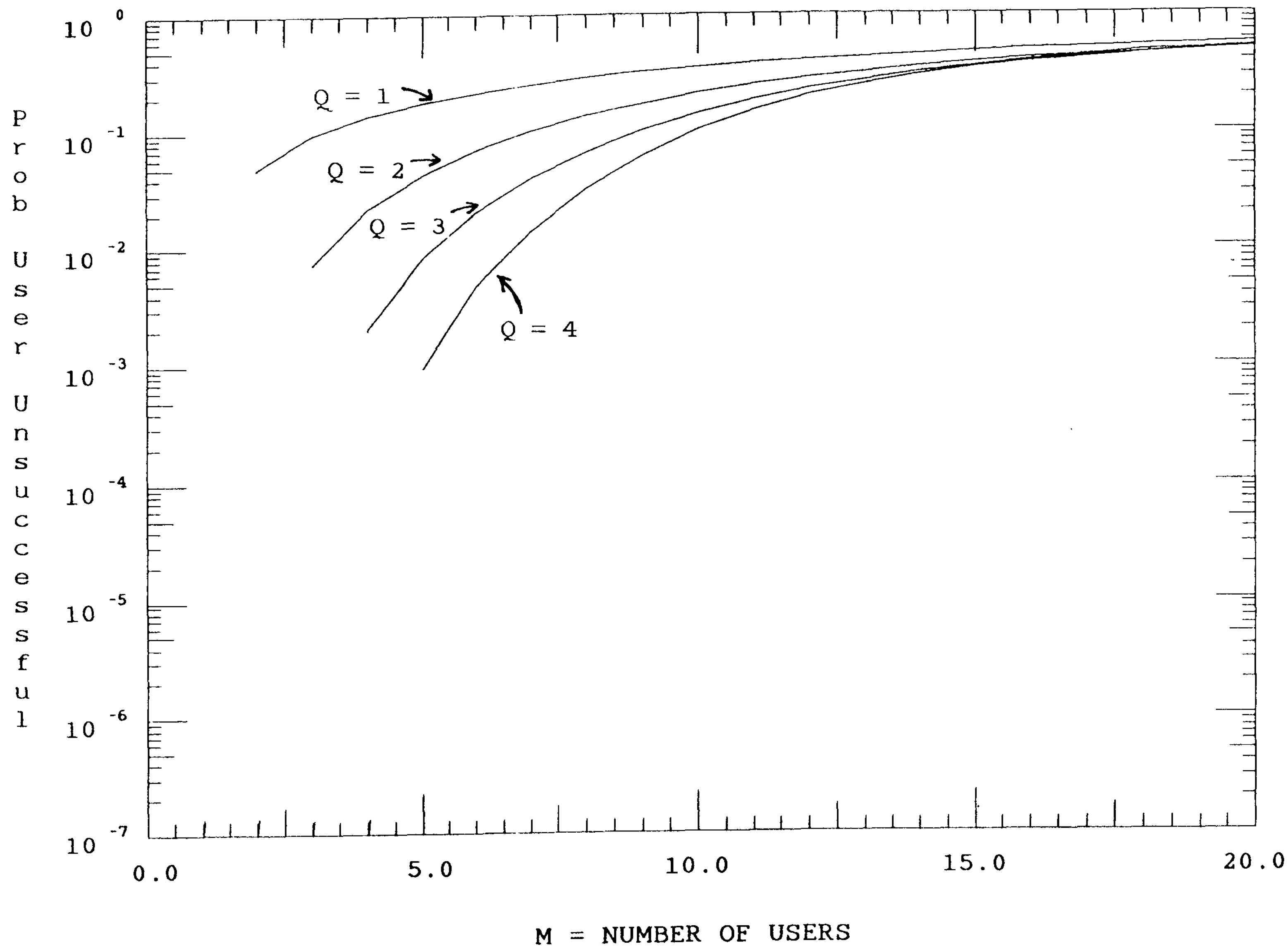

Fig. 6 Probability a user is unsuccessful under distributed reservation scheme; orthogonal CDMA codes.

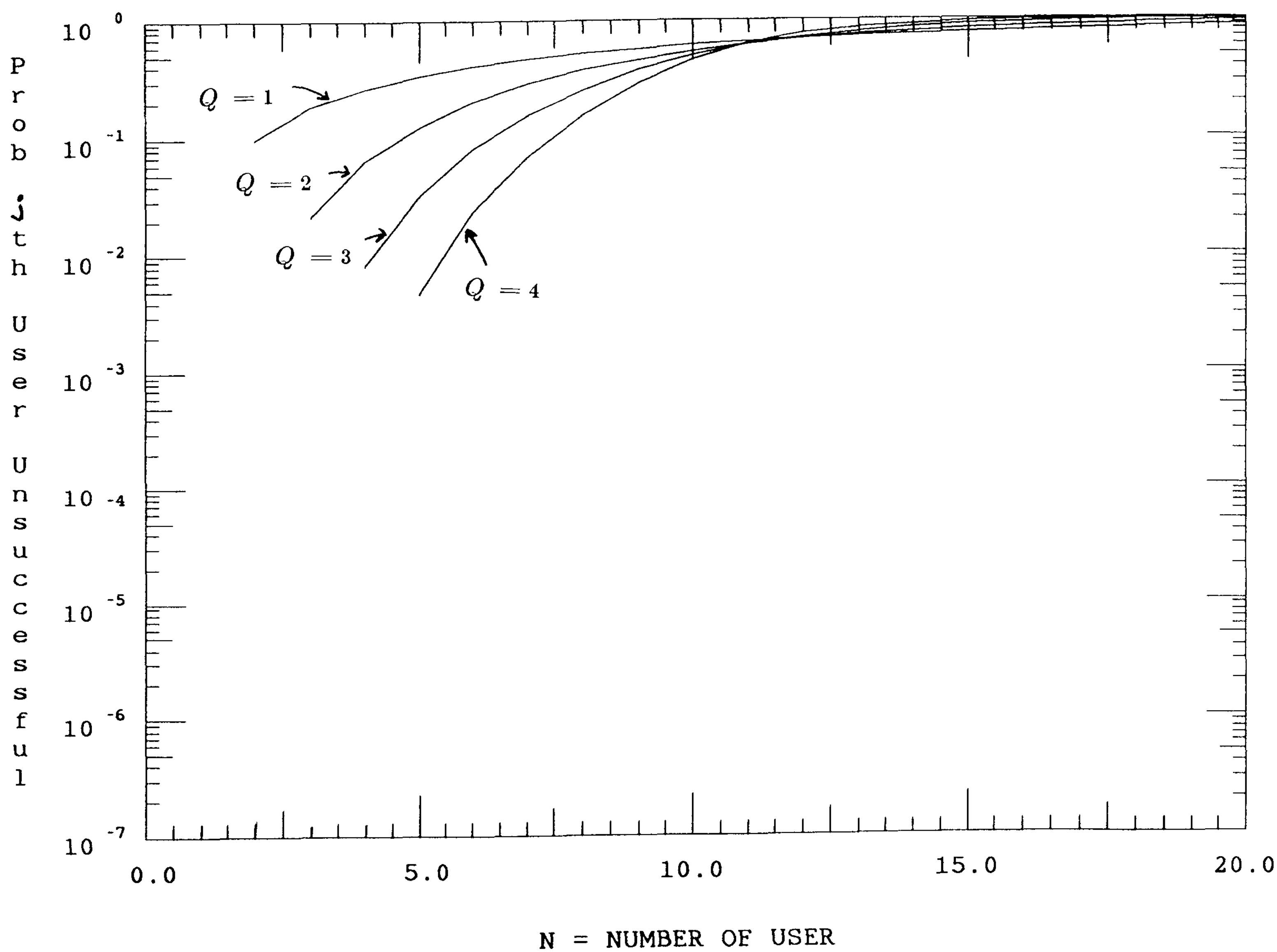

Fig. 7 Probability j^{th} user is unsuccessful under distributed reservation scheme; orthogonal CDMA codes.

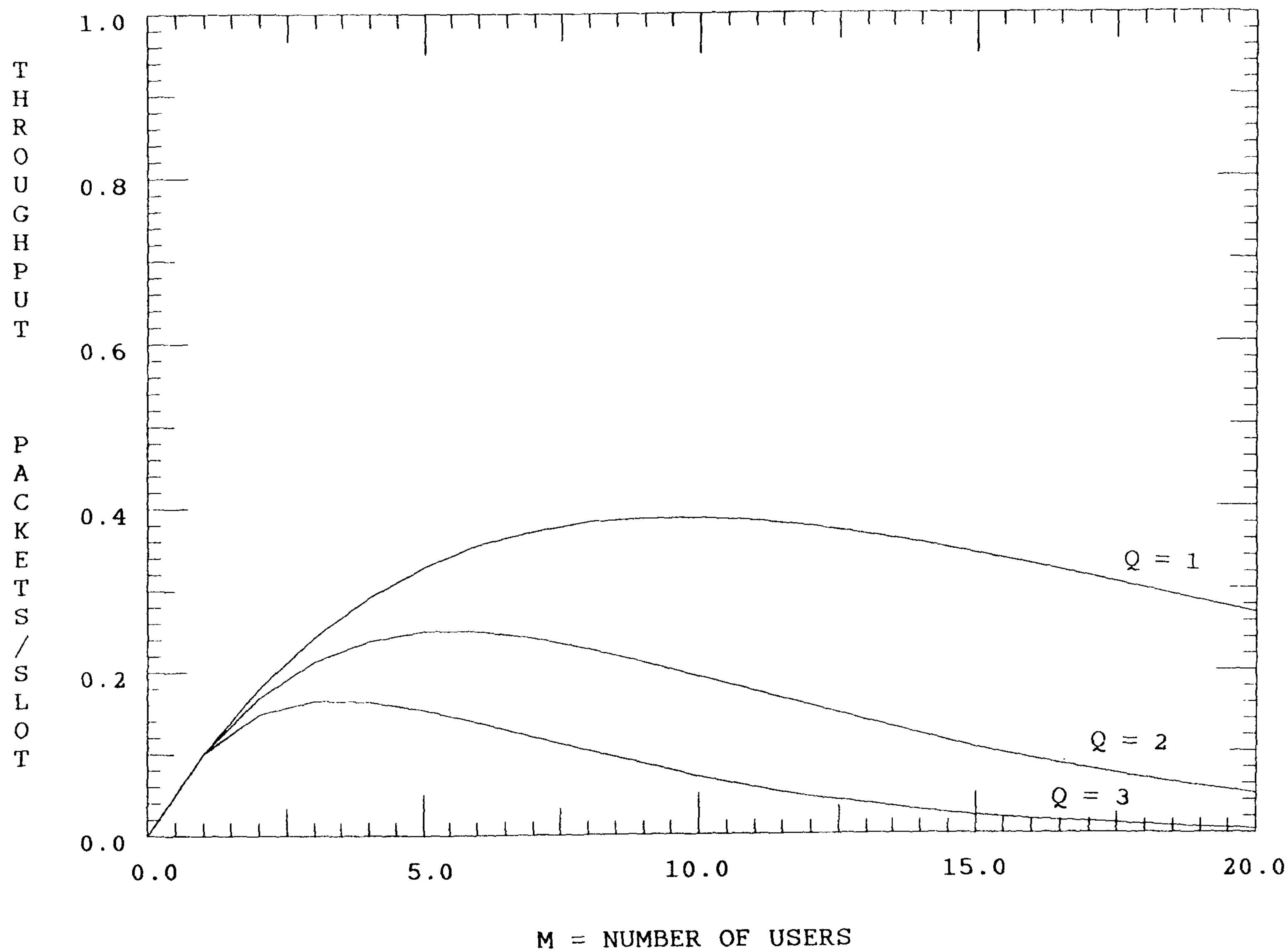

Fig. 8 Throughput performance of distributed reservation scheme; no other users can be tolerated in same slot.

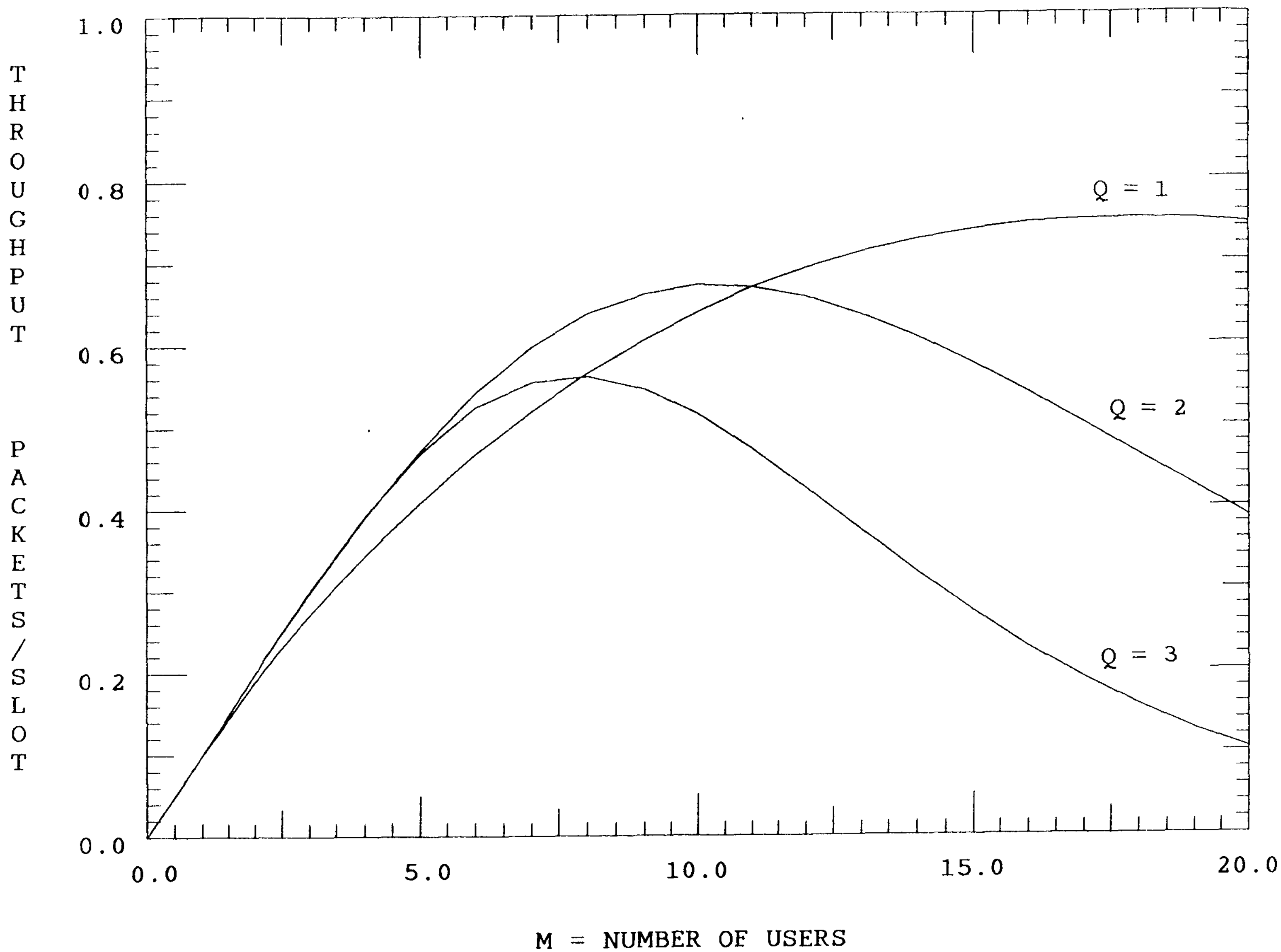

Fig. 9 Throughput performance of distributed reservation scheme; threshold model for effects of other-user interference: 2 other users can be tolerated in same slot.

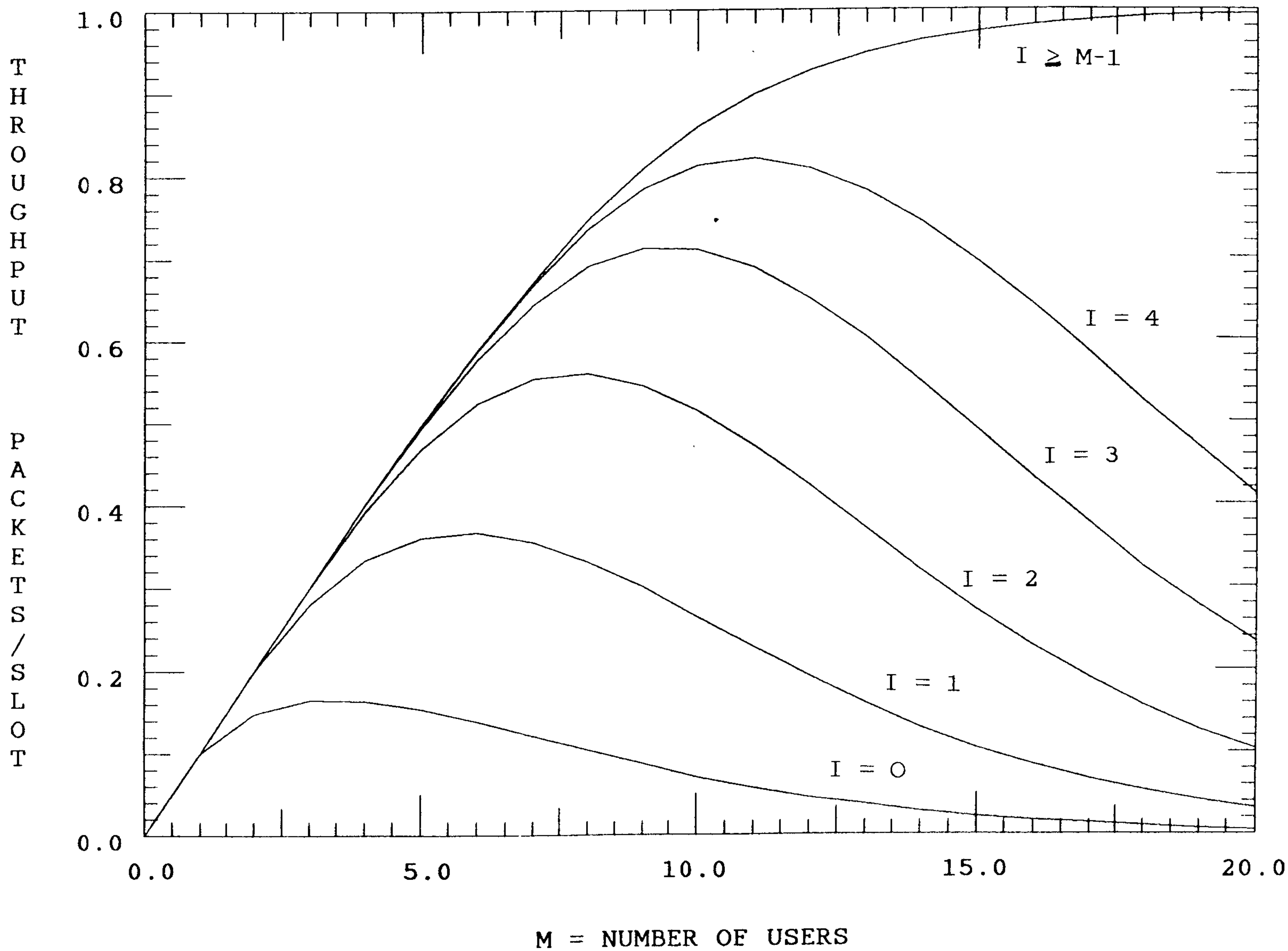

Fig. 10 Throughput performance of distributed reservation scheme; threshold model for effects of other-user interference: several values of I are shown; $Q = 3$.

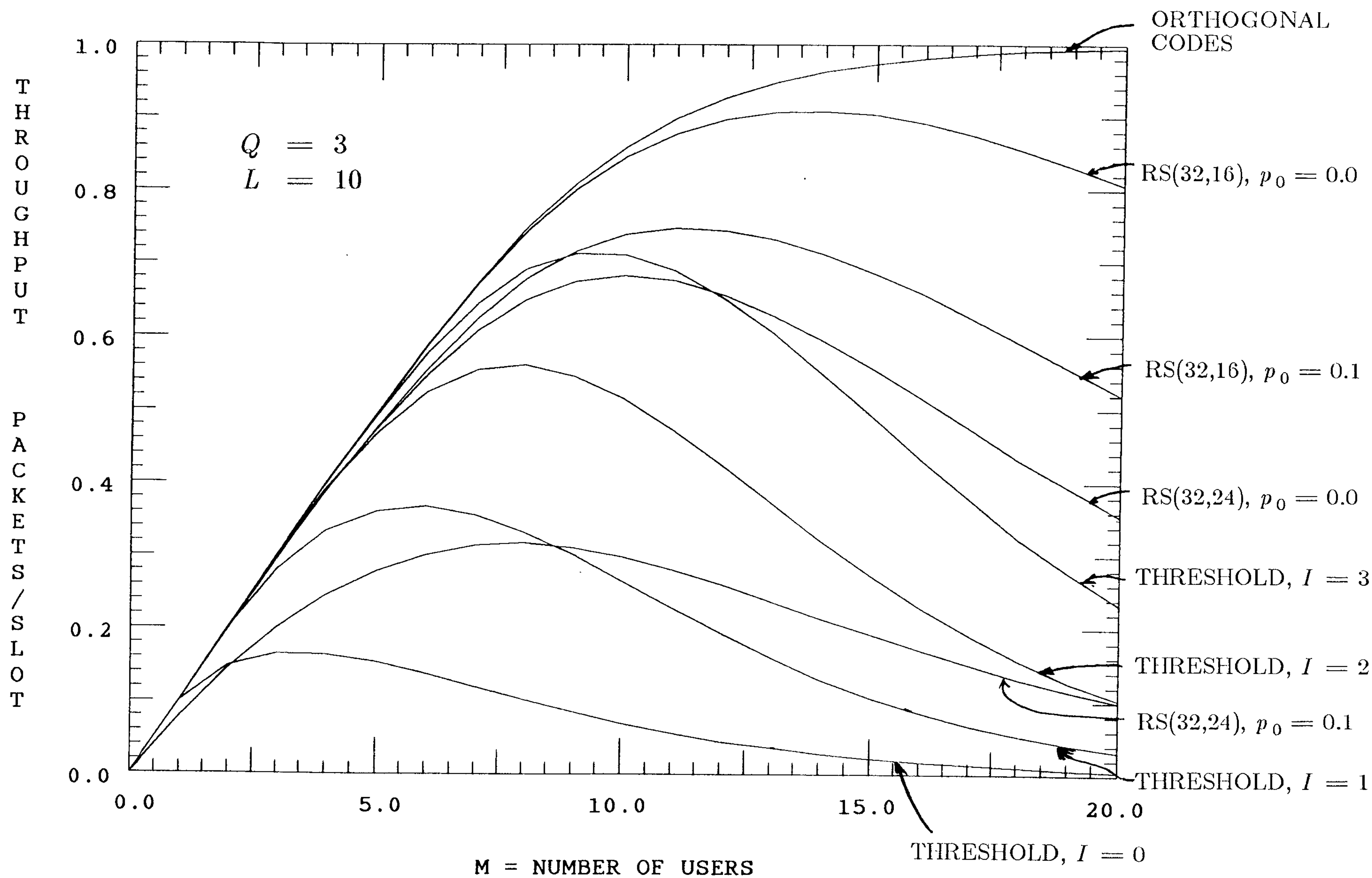

Fig. 11 Throughput performance of distributed reservation scheme; probabilistic (Reed-Solomon coding with $q = 50$ frequency bins) model for effects of other-user interference compared with threshold model

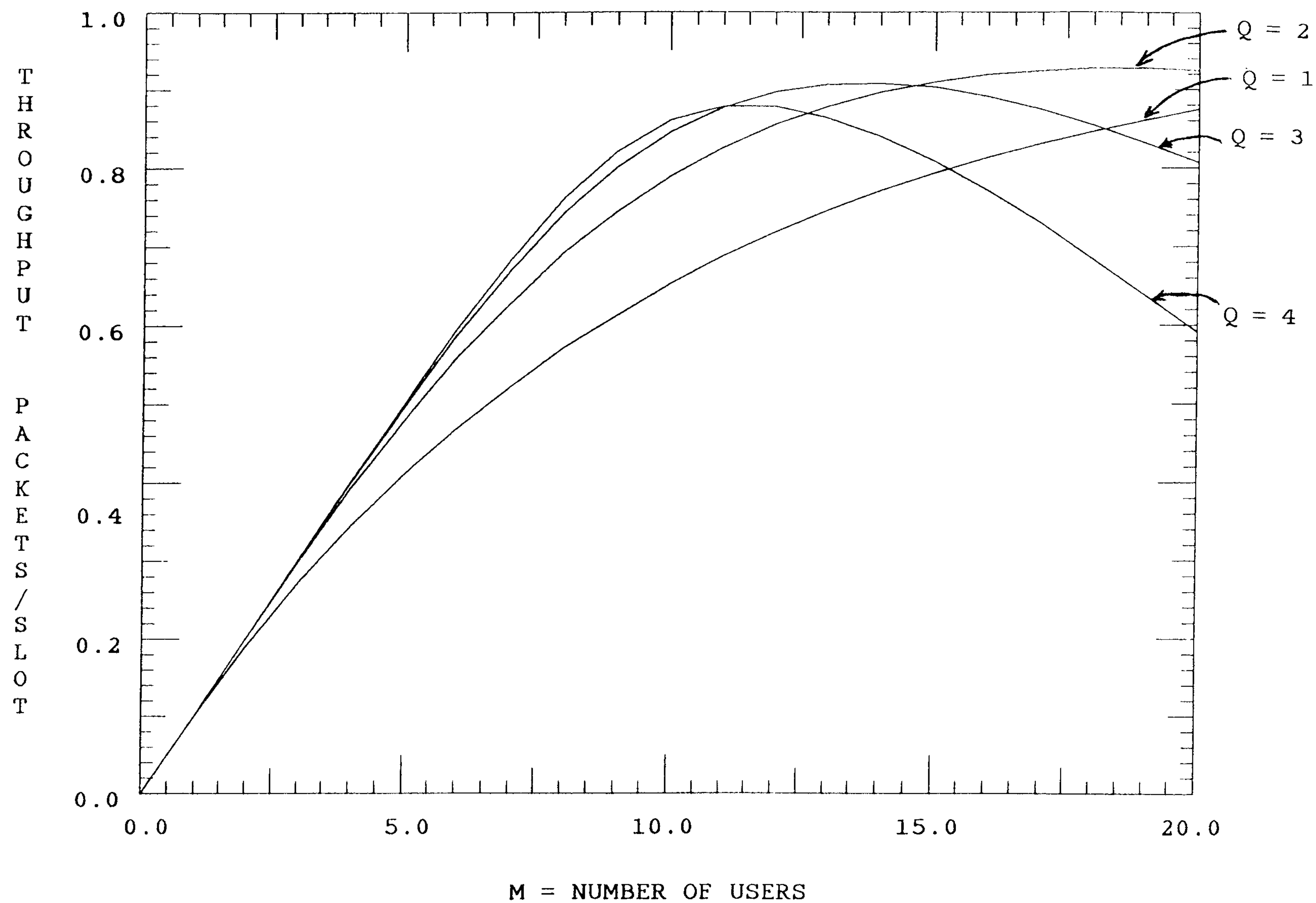

Fig. 12 Throughput performance of distributed reservation scheme; RS(32,16) coding; noiseless channel; $q = 50$ frequency bins; $Q = 1, 2, 3, 4$.

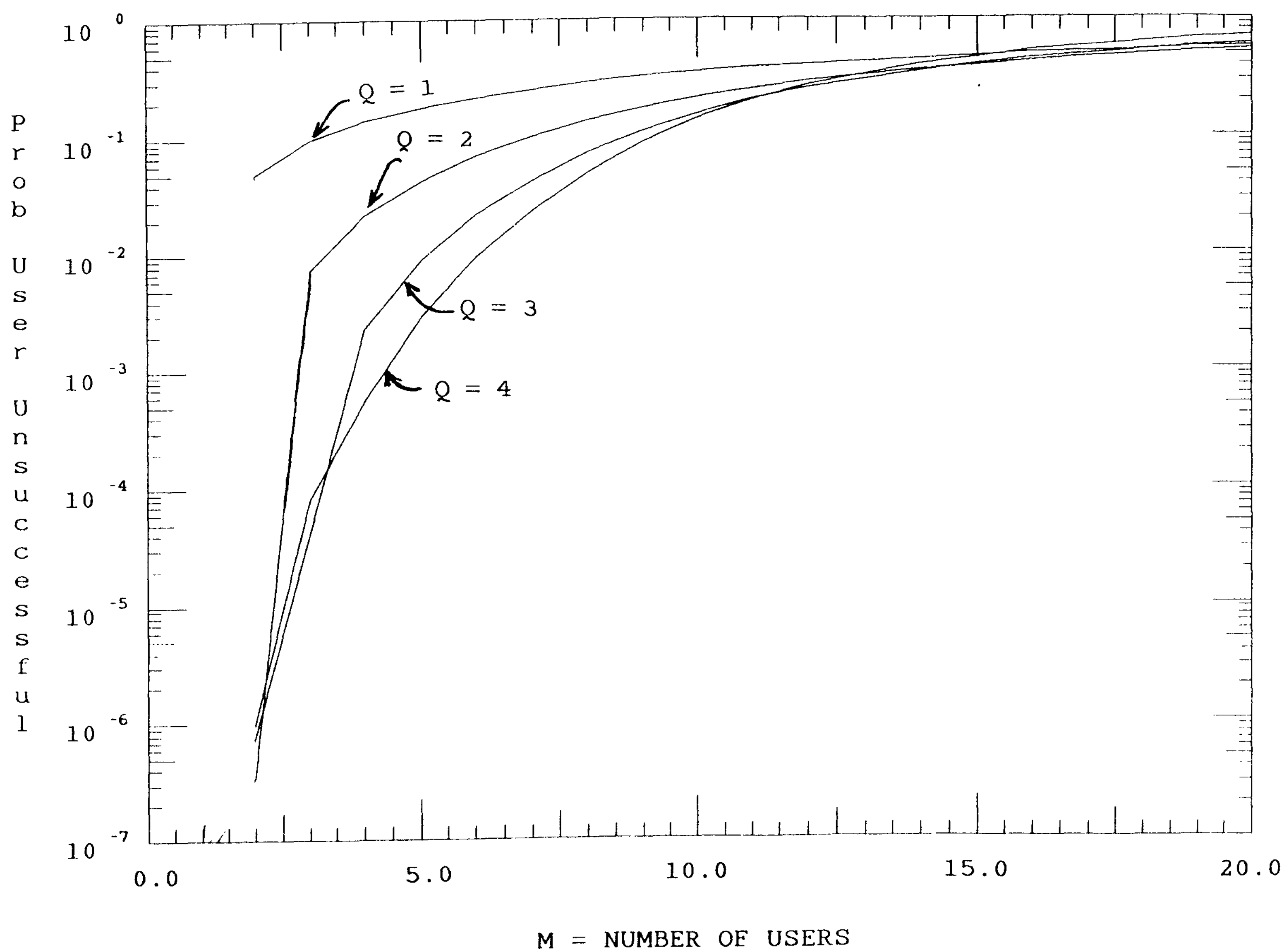

Fig. 13 Probability a user is unsuccessful for distributed reservation scheme; RS(32,16) coding; noiseless channel; $q = 50$ frequency bins; $Q = 1, 2, 3, 4$.

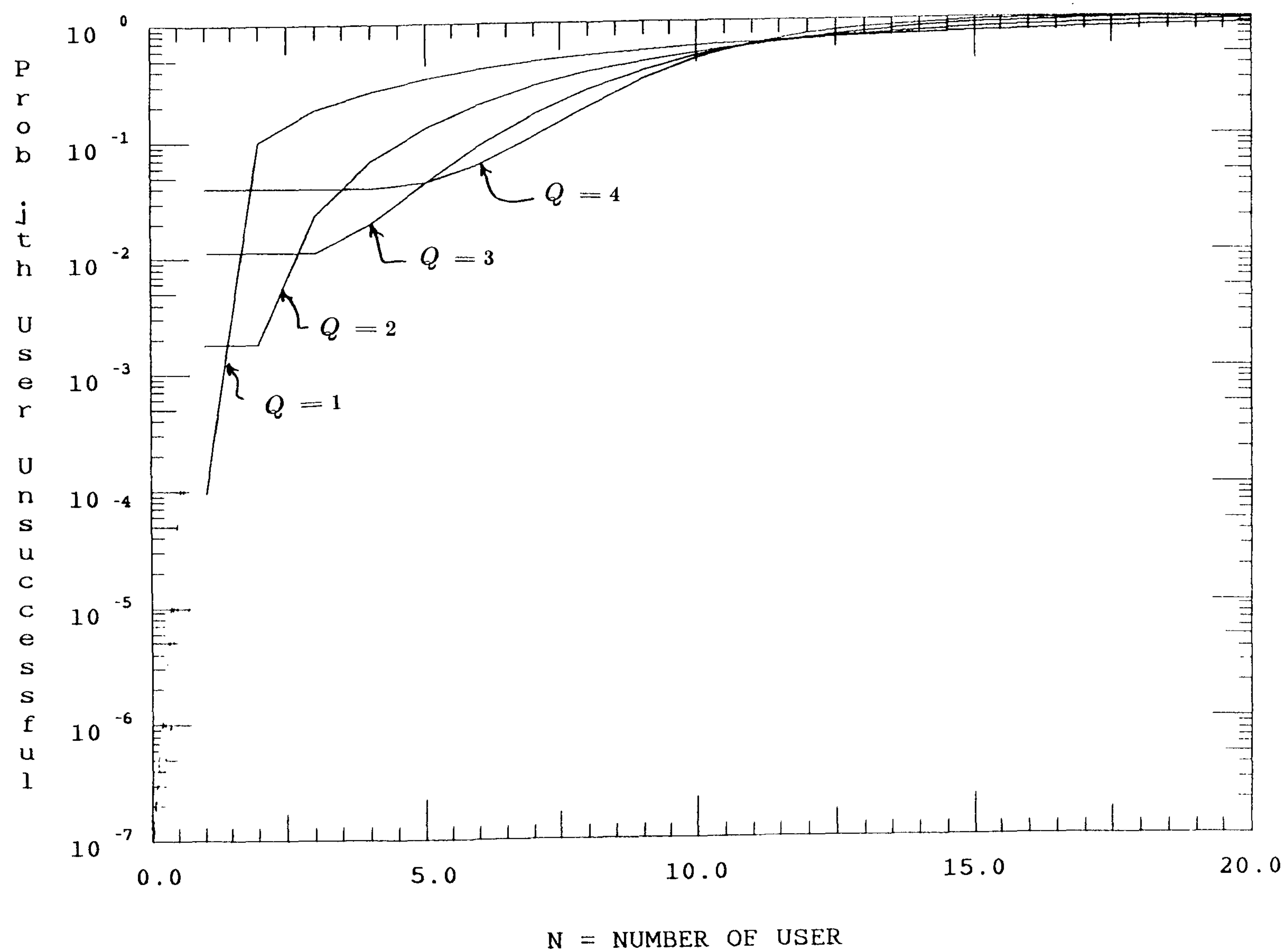

Fig. 14 Probability j^{th} user is unsuccessful for distributed reservation scheme; RS(32,16) coding; noiseless channel; $M = 20$ users; $q = 100$ frequency bins; $Q = 1, 2, 3, 4$.

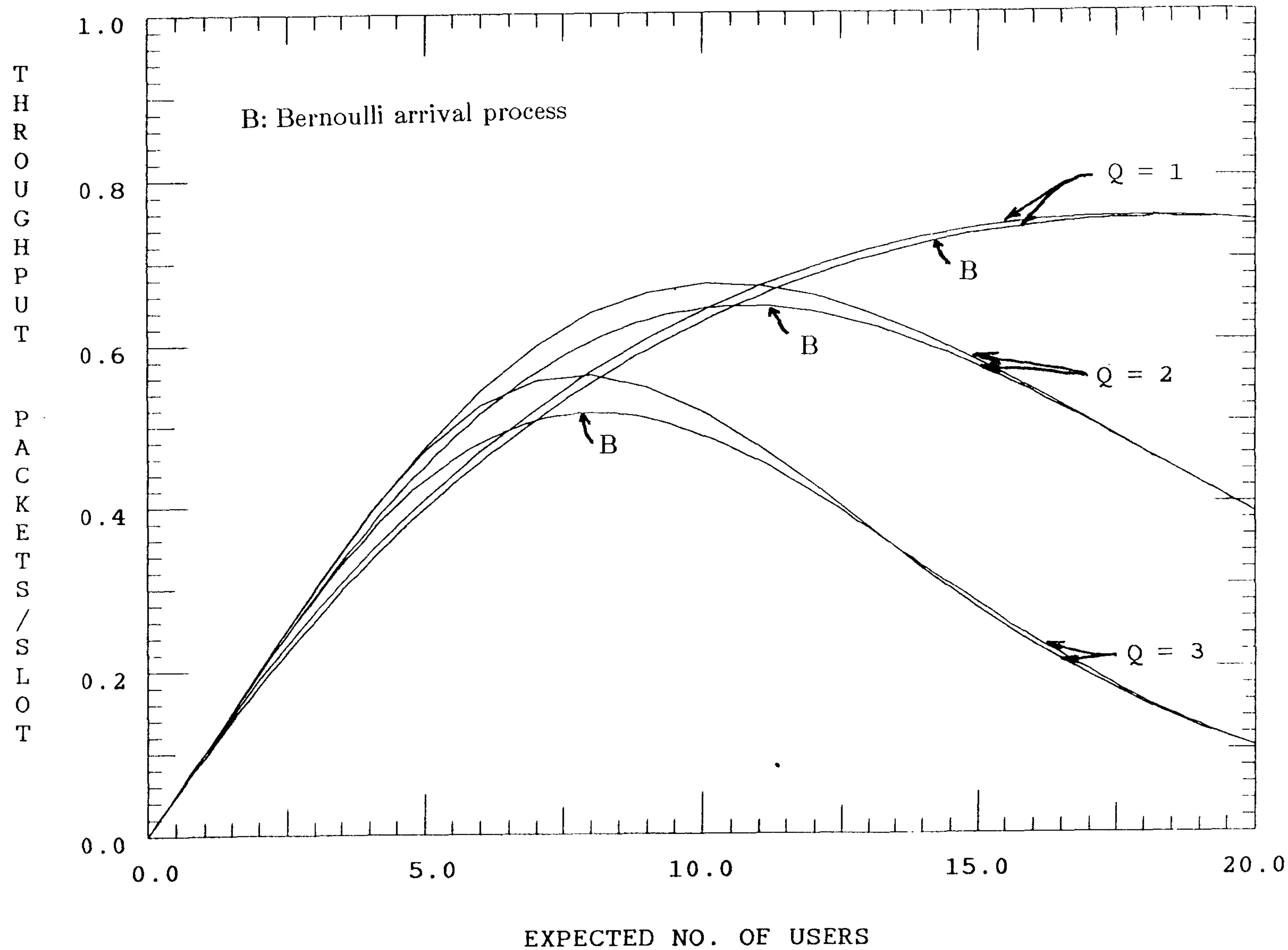

Fig. 15 Throughput performance of distributed reservation scheme; Bernoulli arrival process (M = 20 users) compared with system in which all users transmit; threshold model for effects of other-user interference: 2 other users can be tolerated; $Q = 1, 2, 3$.

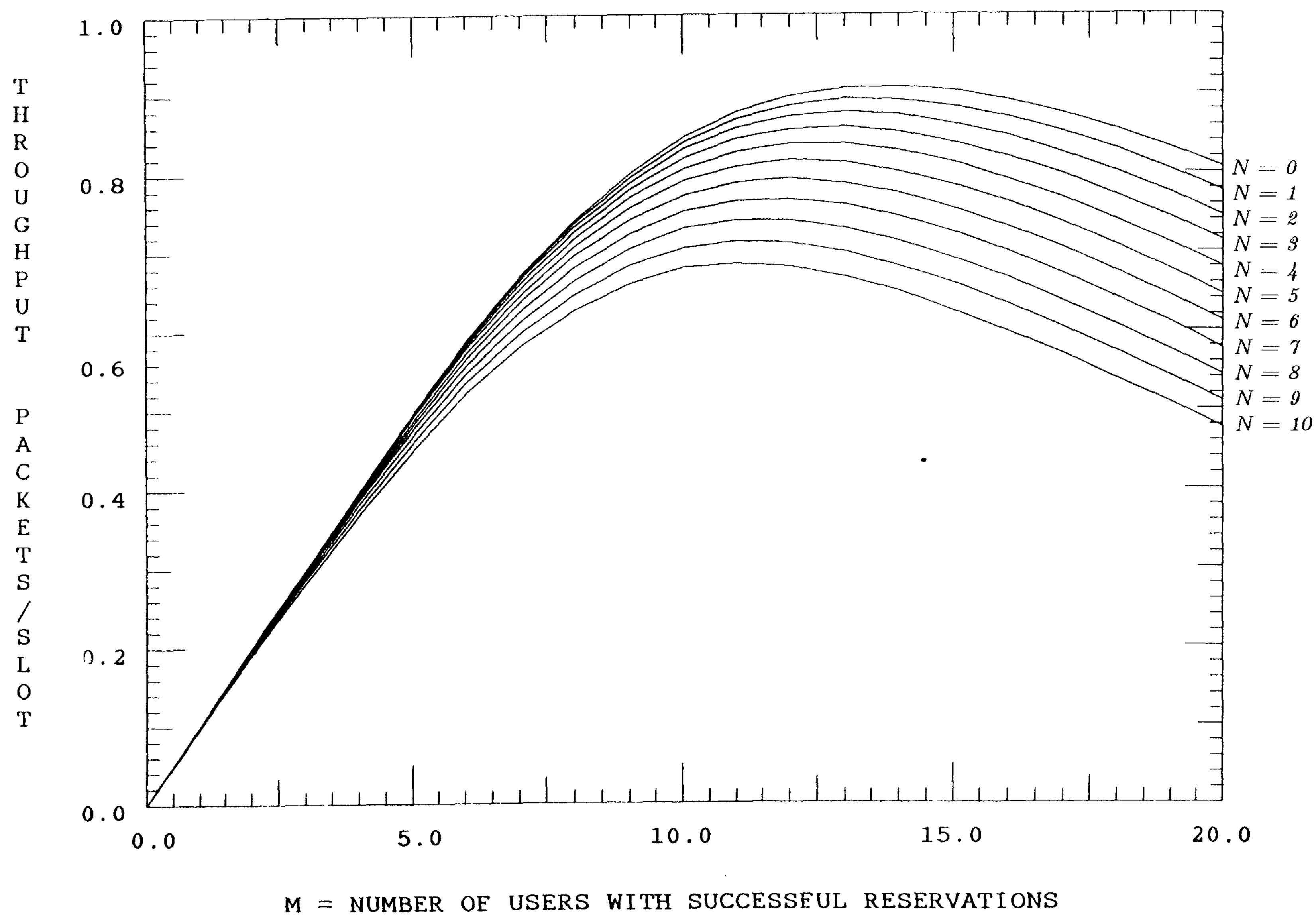

Fig. 16 Throughput performance of distributed reservation scheme; M is the number of users that make reservations successfully; N is the number of additional FH users that share channel; RS(32,16) coding; noiseless channel; $q = 50$ frequency bins; $Q = 3$

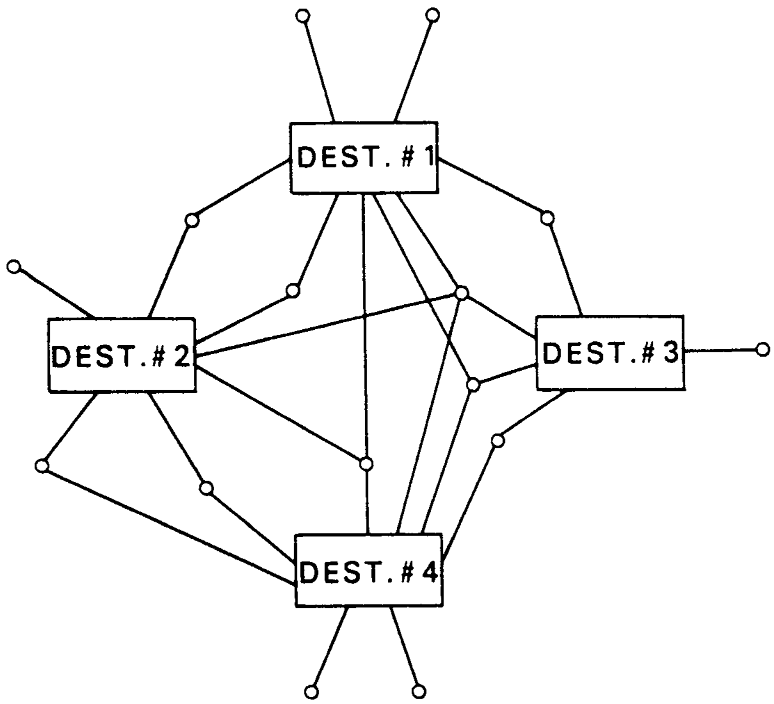

Fig. 17 Sample multiple destination geometry.