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Optimal Admission Control of Two Traffic Types at a Circuit-Switched Network Node

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OPTIMAL ADMISSION CONTROL OF TWO TRAFFIC TYPES AT A CIRCUIT-SWITCHED NETWORK NODE

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Abstract

Two communication traffic streams with Poisson statistics arrive at a network node on separate routes. These streams are to be forwarded to their destinations *via* a common trunk. The two links leading to the common trunk have capacities C_1 and C_2 bandwidth units, respectively, while the capacity of the common trunk is C bandwidth units, where $C < C_1 + C_2$. Calls of either traffic type that are not admitted at the node are assumed to be discarded. An admitted call of either type will occupy, for an exponentially distributed random time, one bandwidth unit on its forwarding link as well as on the common trunk. Our objective is to determine a scheme for the optimal dynamic allocation of available bandwidth among the two traffic streams so as to minimize a weighted blocking cost. The problem is formulated as a Markov decision process. By using dynamic programming principles, the optimal admission policy is shown to be of the "bang-bang" type, characterized by appropriate "switching curves." The case of a general circuit-switched network, as well as numerical examples, are also presented.

Index Terms: admission control, Markov decision processes, dynamic programming, linear programming, switching curves.

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1. Introduction

In modern telecommunication networks there is an increasing need to transmit simultaneously heterogeneous traffic types with diverse characteristics, performance requirements, and grades of service. This has resulted in a recent surge of interest in the study of *integrated systems* capable of realizing an efficient sharing of facilities such as transmission and switching. Indeed, it is expected that in communication networks of the future, large *integrated service digital networks* (ISDN's) will be designed to accommodate random demands for bandwidth usage from a population of heterogeneous users.

Various schemes have been proposed to date for multiplexing several types of traffic on the same channel. Much of the work has concentrated on two types of traffic, namely voice and data [5]. Furthermore, the techniques addressed thus far can be classified into three broad categories:

1)*Complete Sharing Scheme:* In this scheme, a call of a particular traffic type is always offered access to the network whenever sufficient bandwidth is available to accomodate it.

2)*Complete Partitioning Scheme:* In this technique, the available channel bandwidth at each node is partitioned and a portion of the bandwidth is dedicated to each traffic type.

3)*Moving Boundary Scheme* [6,8,9]: This scheme applies to two traffic types, namely voice calls and data packets. The total bandwidth is partioned into two compartments. One compartment is typically allocated to voice traffic, while data can use the remaining compartment as well as any unused slots in the voice compartment. On the other hand, voice traffic cannot use any unused data slots, and operates as a *loss* system, i.e., voice calls that are not accepted upon arrival are assumed to be lost.

In the schemes mentioned above, the customary objective is to control the multiplexer so as to maximize channel utilization or minimize blocking probability. Usually, the first call type, viz., voice, operates as a loss system while the second call type, viz., data, is queued. Also, much of the work to date involves *static* schemes for the control of the multiplexer, i.e., the allocation policy is chosen *a priori* for all time, and its performance analyzed. Much less work is available on *dynamic* control schemes, where the *best* allocation policy according to some criterion may vary in time; such a policy may be stationary (deterministic) or nonstationary.

In this paper, we consider the problem of allocating channel bandwidth to two communication traffic streams that arrive at a network node on two different routes and must be forwarded to their destinations *via* a common trunk. Calls of either traffic type that are not given admission at the node are assumed to be discarded. Our objective is to determine a scheme for the optimal dynamic allocation of available bandwidth among the two traffic streams so as to minimize a weighted blocking cost. The problem is formulated as a Markov decision process where the control actions consist of accepting or discarding a call at the instant of its arrival into the system. By using dynamic programming principles, we demonstrate that the optimal admission policy is of the "bang-bang" type characterized by two "switching-curves" in the state space of the system.

The paper is organized as follows. The control problem is formulated in section 2. Section 3 considers the discounted cost case and establishes key properties of the optimal discounted cost function; the associated optimal policy for this case is characterized in section 4. The average cost problem is addressed in section 5. Numerical results for a given link are given in section 6. Finally, in section 7 we consider the optimal admission control problem for a general circuit-switched network.

2. A Description of the Problem

We consider an integrated scheme for providing service to two types of non-queueing traffic (e.g., voice and video) with different statistics and requiring different grades of service. We shall direct our attention to two traffic streams, one of each type, arriving at nodes 1 and 2, respectively (see Figure 1), for transmission to node 3 which subsequently directs the traffic onto a trunk. We assume that nodes 1 and 2 are connected to node 3 by means of two links of capacities C_1 and C_2 frequency slots, respectively, and that the trunk capacity is C slots. It is further supposed that each call (i.e., voice call or video message) occupies exactly one frequency slot on one of the two links and on the trunk.

When a call arrives at node 1 or 2, a decision is made at the node on either accepting it or blocking it. If accepted, the call is granted a slot *simultaneously* on the corresponding forwarding link as well as on the common trunk; if blocked it is assumed to be lost. These decisions are based on minimizing appropriate blocking costs associated with lost calls. We remark that deliberate blocking of a call of one type may be advantageous for the following reason: It may be worthwhile reserving an empty slot on the shared trunk for a call of the other type since blocking the latter (at a later time) would incur a greater cost.

The following statistical assumptions are made on the arrival and service times of the incoming traffic. Calls of type-i arrive in a Poisson stream of rate λ_i; their corresponding service times are i.i.d. exponential random variables with mean μ_i, $i = 1, 2$. We shall assume without any loss of generality that $\mu_1 \leq \mu_2$ and that all arrival and service processes are mutually independent. The *state* of the system describing the distribution of the load at time $t \geq 0$ is defined by the two-dimensional vector $\mathbf{x}_t = (x_t^1, x_t^2)^T$ where x_t^1 and x_t^2 denote the number of calls in service of type-1 and type-2, respectively, such that $x_t^1 \leq C_1$, $x_t^2 \leq C_2$ and $x_t^1 + x_t^2 \leq C$. We further assume that $C < C_1 + C_2$. Then the state space of the system (see Figure 2) is the set:

$$\mathcal{X} = \{\mathbf{x}_t = (x_t^1, x_t^2) : \; x_t^1, x_t^2 \in Z, \; 0 \leq x_t^1 \leq C_1, \; 0 \leq x_t^2 \leq C_2, \; x_t^1 + x_t^2 \leq C\}.$$

Transitions among the states in $\mathcal{X}$ are described in terms of the operators A_i and D_i representing, respectively, an arrival or a departure of a message of type i, $i = 1, 2$. Thus, the operators $A_i : \mathcal{X} \to \mathcal{X}, D_i : \mathcal{X} \to \mathcal{X}, i = 1, 2$, are defined by:

$$A_1(x^1, x^2) = ((x^1 + 1)^*, x^2)$$

$$A_2(x^1, x^2) = (x^1, (x^2 + 1)^*)$$

$$D_1(x^1, x^2) = ((x^1 - 1)^+, x^2)$$

$$D_2(x^1, x^2) = (x^1, (x^2 - 1)^+),$$

where

$$((x^1 + 1)^*, x^2) = \begin{cases} (x^1 + 1, x^2) & \text{if } x^1 < C_1, \; x^1 + x^2 < C \\ x^1 & \text{otherwise,} \end{cases}$$

and $m^+ = \max\{0, m\}$.

At this point, we provide a heuristic motivation of the nature of the control actions at nodes 1 and 2. Denoting by $z_t^i = z_t^i(\mathbf{x}_t)$ the probability of blocking an incoming call of type-i, $i = 1, 2$, arriving in the time interval $[t, t + dt)$, we must suitably select this probability based on a knowledge of $\mathbf{x}_t$. We refer to z_t^i as the *control action* taken at time t. If $a_i > 0$ is a blocking cost associated with the type-i traffic, the total cost incurred during $[t, t + dt)$ is $\sum_{i=1}^{2} \lambda_i a_i z_t^i(\mathbf{x}_t) dt$. We can then write the *normalized cost per unit time* at time t with the system state being $\mathbf{x}_t$, as $c_t(\mathbf{x}_t) = z_t^1(\mathbf{x}_t) + a z_t^2(\mathbf{x}_t)$, where $a > 0$ (assuming, of course, that $\lambda_i > 0, i = 1, 2$).

Let $\delta > 0$ be the interest rate used for discounting future cost, i.e., the present value of a cost a incurred at time t is $ae^{-\delta t}$. Let $J_t^\delta(\mathbf{x})$ be the minimum "expected" total discounted-cost with respect to $z_t^i(\cdot)$, $i = 1, 2$, when the time horizon is $\{t : t \geq 0\}$ and the initial state is $\mathbf{x} = (x^1, x^2)$. Then, dynamic programming considerations lead to the following optimality conditions for $J_t^\delta(\mathbf{x})$:

$$
\begin{aligned}
J_{t+dt}^\delta(\mathbf{x}) = \min_{0 \leq z_0^1, z_0^2 \leq 1} \{ & z_0^1 dt + a z_0^2 dt + \\
& e^{-\delta dt} \Big(\sum_{i=1}^{2} (z_0^i \lambda_i J_t^\delta(\mathbf{x}) + \lambda_i (1 - z_0^i) J_t^\delta(A_i \mathbf{x}) + \\
& + x^i \mu_i J_t^\delta(D_i \mathbf{x})) dt \\
& + (1 - (x^1 \mu_1 + x^2 \mu_2) dt) J_t^\delta(\mathbf{x}) \} + o(dt).
\end{aligned}
$$

It readily follows that:

$$
\begin{aligned}
J_{t+dt}^\delta(\mathbf{x}) = & \min_{0 \leq z_0^1 \leq 1} \{ z_0^1 (1 - e^{-\delta dt} \lambda_1 (J_t^\delta(A_1 \mathbf{x}) - J_t^\delta(\mathbf{x})) \} + \\
& + \min_{0 \leq z_0^2 \leq 1} \{ z_0^2 (a - e^{-\delta dt} \lambda_2 (J_t^\delta(A_2 \mathbf{x}) - J_t^\delta(\mathbf{x})) \} \\
& + \text{ terms not depending on } z_0^1, z_0^2.
\end{aligned}
$$

Consequently $z_0^{1(2)} = 0$ (i.e., a type-1(2) call is accepted) if $J_t^\delta(A_{1(2)}\mathbf{x}) - J_t^\delta(\mathbf{x}) \leq e^{\delta dt}(a)/\lambda_{1(2)}$; otherwise $z_0^{1(2)} = 1$. Thus, we can associate with every state $\mathbf{x}$ in $\mathcal{X}$ a set of admissible actions $\mathcal{D} = \{0, 1\}^2$ with the understanding that an admissible action $\mathbf{z}_t(\mathbf{x})$ at state $\mathbf{x}$ and at time t will have the form:

$$
\mathbf{z}_t(\mathbf{x}) = \left(z_t^1(\mathbf{x}), z_t^2(\mathbf{x}) \right)
$$

where $z_t^i = 1$ or 0 according to whether an arriving call of type-i is rejected or accepted into the system. The *action space* is then defined as the product set D^S, and we represent an *admissible control strategy* (CS) as a D^S-valued stochastic process $(z_t, t \geq 0)$ where $z_t = (z_t(\mathbf{x}), \mathbf{x} \in \mathcal{X})$. Hereafter, we shall use the abbreviated notation z for the CS $(z_t,\ t \geq 0)$. We denote by $\mathcal{P}$ the set of all admissible control strategies. Further, for simplicity, we write z_t^i instead of $z_t^i(\mathbf{x}_t)$. Finally, observe that $((\mathbf{x}_t,\ \mathbf{z}_t), t \geq 0)$ is a Markov decision process with transition rates shown in Figure 2.

At this juncture, it is convenient to relate the continuous time Markov chain $(\mathbf{x}_t, t \geq 0)$ to a "suitable" discrete time chain by following the method of "uniformization" [7,13]. To this end, we first define the *total event rate* by:

$$\rho = \lambda_1 + \lambda_2 + C\mu_2.$$

Then, let $0 = t_0 < t_1 < t_2 \ldots < t_n < \ldots$ be the transition epochs (due to arrivals or departures) of the state process $(\mathbf{x}_t, t \geq 0)$. By suitably introducing "dummy" transitions as in [10,7], it follows that the interepoch intervals are i.i.d random variables with a common distribution determined by $\mathbb{P}(t_{k+1} - t_k > t) = e^{-t\rho},\ t \geq 0,\ k = 1, 2\ldots$ Then it can be easily shown [10,13] that the δ-discounted expected cost accrued up to time t_n upon starting with initial state $\mathbf{x}$ and following a policy z in $\mathcal{P}$, viz.,

$$\mathbb{E}_{\mathbf{x}}^{z}\left(\int_0^{t_n} e^{-\delta t}(z_t^1 + a z_t^2)dt\right),$$

is equal to a cost of the form:

$$\mathbb{E}_{\mathbf{x}}^{z}\left(\sum_{k=0}^{n-1} \beta^k (z_k^1 + a z_k^2)\right),$$

with $z_k^i \triangleq z_{t_k}^i$ and $\beta = \rho/(\delta + \rho) < 1$. The last expectation is taken with respect to the probability distribution associated with a discrete time Markov decision process $(\mathbf{x}_k, k \geq 0)$ with transition probabilities:

$$\mathbb{P}(\mathbf{x}_{k+1}|\mathbf{x}_k, \mathbf{z}_k) \cdot \rho = \begin{cases} \lambda_1 & \text{if } \mathbf{x}_{k+1} = A_1\mathbf{x}_k, z_k^1 = 0 \\ \lambda_1 & \text{if } \mathbf{x}_{k+1} = \mathbf{x}_k, z_k^1 = 1 \\ \lambda_2 & \text{if } \mathbf{x}_{k+1} = A_2\mathbf{x}_k, z_k^2 = 0 \\ \lambda_2 & \text{if } \mathbf{x}_{k+1} = \mathbf{x}_k, z_k^2 = 1 \\ \mu_1 x^1 & \text{if } \mathbf{x}_{k+1} = D_1\mathbf{x}_k \\ \mu_2 x^2 & \text{if } \mathbf{x}_{k+1} = D_2\mathbf{x}_k \\ C\mu_2 - x^1\mu_1 - x^2\mu_2 & \text{if } \mathbf{x}_{k+1} = x_k, \mathbf{z}_k = 0. \end{cases} \tag{2.1}$$

Then for each initial state $\mathbf{x}$ in $\mathcal{X}$, we can define,

$$J_n^\beta(\mathbf{x}) = \min_{z \in \mathcal{P}} \mathbb{E}_{\mathbf{x}}^{z}\left(\sum_{k=0}^{n-1} \beta^k (z_k^1 + a z_k^2)\right), \tag{2.2a}$$

as the n-step optimal β-discounted expected cost. Further, we introduce the infinite horizon optimal β-discounted expected cost:

$$J^{\beta}(\mathbf{x}) = \min_{z\in\mathcal{P}} \mathbb{E}^{z}_{\mathbf{x}} \sum_{k=0}^{\infty} \beta^k (z^1_k + a z^2_k) < +\infty. \tag{2.2b}$$

Since the underlying state space S is finite, it can be shown [11] that $\lim_{n\to\infty} J^{\beta}_n(\mathbf{x}) = J^{\beta}(\mathbf{x})$; moreover for the infinite horizon problem an optimal policy exists and it is stationary, i.e., the minimizing CS $\hat{z}$ satisfies $\hat{\mathbf{z}}_t(\mathbf{x}_t) = \hat{\mathbf{z}}(\mathbf{x}_t)$, $\mathbf{x}_t$ in $\mathcal{X}$, $t \geq 0$.

In terms of the discrete-time formulation, the Dynamic Programming equation can be written as follows (assume for simplicity that $\rho = 1$):

$$\begin{aligned} J^{\beta}_{k+1}(\mathbf{x}) = \min_{\substack{\{z^i_0\in\{0,1\} \\ z^i_0=1 \text{ if } x^i=C_i \\ \text{or } x^1+x^2=C,\ i=1,2\}}} & \{ z^1_k + a z^2_k \\ & + \beta\lambda_1(1-z^1_k)J^{\beta}_k(A_1\mathbf{x}) + \beta\lambda_1 z^1_k J^{\beta}_k(\mathbf{x}) \\ & + \beta\lambda_2(1-z^2_k)J^{\beta}_k(A_2\mathbf{x}) + \beta\lambda_2 z^2_k J^{\beta}_k(\mathbf{x}) \\ & + \beta\mu_1 x^1 J^{\beta}_k(D_1\mathbf{x}) + \beta\mu_2 x^2 J^{\beta}_k(D_2\mathbf{x}) \\ & + \beta(C\mu_2 - x^1\mu_1 - x^2\mu_2)J^{\beta}_k(\mathbf{x})\}, \end{aligned} \tag{2.3}$$

with $\mathbf{x} = (x^1, x^2)$ being the initial state and z^1_0, z^2_0 the corresponding actions at that state. Rearranging terms in (2.3), we get the following optimality criteria:

While at state $\mathbf{x}$ such that $x^1 < C_1$ ($x^2 < C_2$), and $x^1 + x^2 < C$, an incoming type-1(2) call is blocked, i.e., $z_0^{1(2)} = 1$, if

$$J^{\beta}_k(A_{1(2)}(\mathbf{x})) - J^{\beta}_k(\mathbf{x}) \geq 1(a)/\beta\lambda_{1(2)}. \tag{2.4}$$

Furthermore, if $x^{1(2)} = C_{1(2)}$ or $x^1 + x^2 = C$, we set $z_0^{1(2)} = 1$.

3. Properties of the Optimal Discounted Cost Function

We now derive a few properties of the optimal n-step β-discounted cost function $J^{\beta}_n(\cdot)$ which will be employed in the next section to characterize the optimal policy for call acceptance or rejection.

Proposition 3.1: For each $x^1 \geq 0,\ x^2 \geq 0,\ J_n^\beta(\cdot,\cdot)$ is an increasing function of x^1, x^2.

Proof: We will first show that

$$J_n^\beta(x^1+1,x^2) \geq J_n^\beta(x^1,x^2),$$

by using simple coupling arguments. We consider two identical systems starting with initial conditions (x^1+1,x^2) and (x^1,x^2), respectively. We apply the same control strategy to both systems, namely the one that is optimal for the n-step cost problem with initial conditions (x^1+1,x^2). Whenever the system with initial condition (x^1+1,x^2) admits a new call then it is feasible for the system starting at (x^1,x^2) to do so. Let $\mathbf{x}_k$ denote the state trajectory of the first system, and let $\sigma \triangleq \min(k : x_k^1 = 0)$. For $n \geq \sigma$, the states of the two systems coincide. Since we follow the same control strategy for both systems, elementary coupling arguments provide that

$$J_n^\beta(x^1+1,x^2) - J_n^\beta(x^1,x^2) \geq 0.$$

In a similar manner we show that $J_n^\beta(x^1,x^2+1) \geq J_n^\beta(x^1,x^2)$.

Proposition 3.2: For each $x^2 > 0,\ \ J_n^\beta(\cdot,x^2)$ is a convex function, i.e.,

$$J_n^\beta(x^1+1,x^2) - J_n^\beta(x^1,x^2) \geq J_n^\beta(x^1,x^2) - J_n^\beta(x^1-1,x^2). \tag{3.1}$$

A similar statement is true of $J_n^\beta(x^1,\cdot)$ for each $x^1 > 0$.

Proposition 3.2 can be established using linear programming techniques and duality theory in the manner of [10]. To this end, we shall need the following definitions:

A ***sample path*** ω^k (of arrivals and departures) is a sequence of k events, $k = 1,2,\cdots$, defined by

$$\omega^k = \{\omega_1,\omega_2,\ldots\omega_k\},\ \omega_j \in \{A_1,A_2,D_1,D_2\}$$
$$j = 1,\ldots,k,$$

with j representing the j^{th} arrival or departure epoch, and A_i, D_i denoting respectively an arrival or a departure of a type-i call, $i = 1,2$. We define the ***basic sample space***, Ω^k for the MDP to be the set of all sequences ω^k.

A *transition* ξ_k is specified by

$$\xi_k(\omega^k) = \begin{cases} (1,0) & \text{if } \omega_k = A_1 \\ (0,1) & \text{if } \omega_k = A_2 \\ (-1,0) & \text{if } \omega_k = D_1 \\ (0,-1) & \text{if } \omega_k = D_2. \end{cases}$$

We can then express the evolution of the state trajectory corresponding to a policy z in $\mathcal{P}$, through the following recursive equation:

$$\begin{aligned} \mathbf{x}_0 &= \mathbf{x} \\ \mathbf{x}_k(\omega^k) &= \mathbf{x}_{k-1}(\omega^{k-1}) + \xi_k(\omega^k) - \text{ diag } \xi_k(\omega^k)\mathbf{z}_k(\omega^k), \end{aligned} \tag{3.2}$$

where $\mathbf{x}_0$ is the initial state and diag $\xi_k(\omega^k)$ is a 2×2-diagonal matrix with diagonal elements $\xi_k(\omega^k)$. Solving the recursive state evolution equation (3.2), we obtain that

$$\mathbf{x}_k(\omega^k) = \mathbf{x} + \sum_{j=1}^{k} \xi_j(\omega^j) - \sum_{j=1}^{k} \text{ diag } \xi_j(\omega^i)\mathbf{z}_j(\omega^j).$$

The n-step β-discounted cost corresponding to a control policy z in $\mathcal{P}$ and with initial condition $\mathbf{x}$ can be written as follows:

$$\begin{aligned} V_n^\beta(\mathbf{x}, z) &= \mathbb{E}_{\mathbf{x}}^z \left(\sum_{k=1}^{n} \beta^k (z_k^1 \mathbf{1}(\omega_k = A_1) + a z_k^2 \mathbf{1}(\omega_k = A_2) \right) \\ &= \sum_{k=1}^{n} \sum_{\omega^k \in \Omega^k} \gamma_k(\omega^k)\mathbf{z}_k(\omega^k), \end{aligned}$$

where $\mathbf{1}(\cdot)$ denotes the indicator function, and $\gamma_k(\omega^k) = \beta^k\big(\mathbf{1}(\omega_k = A_1), a\mathbf{1}(\omega_k = A_2)\big)\,\mathbb{P}(\omega^k)$, with $\mathbb{P}(\omega^k)$ being the probability of the sample path ω^k.

The optimal n-step discounted cost (2.2a) then becomes

$$J_n^\beta(\mathbf{x}) = \min_{\mathbf{z}_k \in \{0,1\}^2} V_n^\beta(\mathbf{x}, z).$$

In an analogous manner, we define:

$$W_n^\beta(\mathbf{x}) = \min_{\mathbf{z}_k \in [0,1]^2} V_n^\beta(\mathbf{x}, z).$$

Then $W_n^\beta(\mathbf{x})$ is the (optimal) value function of a minimization problem of the form:

$$W_n^\beta(\mathbf{x}) = \min_{\{\mathbf{z}_k(\omega^k)\}_{k=1}^n} \sum_{k=1}^{n} \sum_{\omega^k \in \Omega^k} \gamma_k(\omega^k)\mathbf{z}_k(\omega^k),$$

such that

$$z_k^{1(2)}(\omega^k) = \begin{cases} 0 & \text{if } \omega_k = A_{2(1)} \text{ or } D_{2(1)}, \\ \in [0,1] & \text{otherwise.} \end{cases} \qquad (LP)$$

$$\omega^k \in \Omega^k,$$

$$k = 1, 2, \ldots n,$$

under the constraints

$$0 \leq \mathbf{x} + \sum_{j=1}^{k} \xi_j(\omega^j) - \sum_{j=1}^{k} \text{ diag } \xi_j(\omega^j)\mathbf{z}_j(\omega^j) \leq \binom{C_1}{C_2},$$

and

$$(1,1)(\mathbf{x} + \sum_{j=1}^{k} \xi_j(\omega^j) - \sum_{j=1}^{k} \text{ diag } \xi_j(\omega^j)\mathbf{z}_j(\omega^j)) \leq C.$$

This is a linear program in the finite array of variables $\{\mathbf{z}_k(\omega^k), \omega^k \in \Omega^k, 1 \leq k \leq n\}$. Since $\mathbf{x}$, the initial condition, enters linearly in the constraint equation, it can be shown [13] that $W_n^\beta(\cdot)$ is a convex function of $\mathbf{x}$. Note that, $W_n^\beta(\mathbf{x})$ cannot as yet be associated with $J_n^\beta(\mathbf{x})$, since the variables $z_k^i(\omega^k)$ in (LP) can take values in the interval [0,1]. We now proceed to prove that there exists a solution $\mathbf{z}_k(\omega^k), \omega^k$ in $\Omega^k, 1 \leq k \leq n$, such that $\mathbf{z}_k(\omega^k)$ belongs to $\{0,1\}^2$. We first derive the necessary and sufficient conditions of optimality for the solutions of the linear program (LP). By duality theory [13, p. 50], $z^* = \{\mathbf{z}_k^*(\omega^k), \omega^k \in \Omega^k, 1 \leq k \leq n\}$ is an optimal solution of the (LP) above if and only if there exist nonegative dual variables

$$\lambda_k^*(\omega^k) \in \mathbb{R}_+^2,\ \mu_k^*(\omega^k) \in \mathbb{R}_+^2,\ v_k^*(\omega^k) \in \mathbb{R}_+,\ 1 \leq k \leq n,\ \omega^k \in \Omega^k$$

such that (we drop in our notation the dependence of certain variables on ω^k to make the presentation simpler):

1) z^* is an optimal solution to the following unconstrained problem:

$$\min_{\left\{\substack{z_k(\omega^k)\in[0,1]\\ k=1,2,\ldots n\\ \omega^k\in\Omega^k}\right\}} \sum_{k=1}^{n}\sum_{\omega^k\in\Omega^k}(\gamma_k \mathbf{z}^k - \lambda_k^*(\mathbf{x}+\sum_{j=1}^{k}\xi_j - \sum_{j=1}^{k} \text{diag } \xi_j\mathbf{z}_j)$$

$$+\mu_k^*(\mathbf{x}+\sum_{j=1}^{k}\xi_j - \sum_{j=1}^{k} \text{diag } \xi_j\mathbf{z}_j - (C_1,C_2)^T) \tag{3.3a}$$

$$+v_k^*((1,1)(\mathbf{x}+\sum_{j=1}^{k}\xi_j - \sum_{j=1}^{k} \text{diag } \xi_j\mathbf{z}_j) - C)).$$

2) If by $\{\mathbf{x}_k(\omega^k,z^*)\}_{k=1}^n$ we denote the state trajectory generated by z^* through (3.2), then

$$0 \le \mathbf{x}_k(\omega^k,z^*) \le (C_1,C_2)^T, \ \ \mathbf{x}_k^1(\omega^k,z^*)+\mathbf{x}_k^2(\omega^k,z^*) \le C. \tag{3.3b}$$

3) If $\lambda_k^{i*} > 0$, then $x_k^i(\omega^k,z^*) = 0$, $i = 1,2$.
If $\mu_k^{i*} > 0$, then $x_k^i(\omega^k,z^*) = C_i$, $i = 1,2$. (3.3c)
If $v_k^* > 0$, then $x_k^1(\omega^k,z^*) + x_k^2(\omega^k,z^*) = C$.

The term being minimized in (3.3a) can be written as:

$$\sum_{k=1}^{n}\sum_{\omega^k\in\Omega^k}(\gamma_k(\omega^k) - \mathbf{c}_k(\omega^k))z_k(\omega^k) + K,$$

where K does not depend on z, and

$$\mathbf{c}_k(\omega^k) = \left(\sum_{j=k}^{n}(\lambda_j^*(\omega^j) - \mu_j^*(\omega^j) - v_j^*(1,1))\right) \text{diag } \xi_k(\omega^k).$$

Hence, condition 1 above can be written more conveniently as

$$z_k^{*1(2)}(\omega^k) = 0 \text{ if } \omega_k = A_{2(1)} \text{ or } D_{2(1)};$$

otherwise,

$$z_k^i(\omega^k) = \begin{cases} 1 & \text{if } \gamma_k^i(\omega^k) - c_k^1(\omega^k) < 0 \\ 0 & \text{if } \gamma_k^i(\omega^k) - c_k^i(\omega^h) > 0 \\ \in [0,1] & \text{if } \gamma_k^i(\omega^k) - c_k^i(\omega^h) = 0 \end{cases} \tag{3.4}$$

for $i = 1, 2$.

Lemma 3.1: Let $X = \{\mathbf{x} : \mathbf{x} = p_1\xi(A_1) + p_2\xi(A_2),\ p_1 \in (-\frac{1}{2}, \frac{1}{2}],\ p_2 \in [-\frac{1}{2}, \frac{1}{2})\}$. Then

$$X - \{\ \mathrm{diag}\xi(\omega)\ \mathbf{z} \mid \mathbf{z} \in [0,1]^2\} \subset X \cup \{X - \xi(\omega)\},$$
$$\omega \in \{A_1, A_2, D_1, D_2\}.$$

Proof. The proof of the lemma is straightforward (see Figure 3).

Lemma 3.2: There is an integer-valued policy $z = \{\mathbf{z}_k(\omega^k) | \omega^k \in \Omega^k, 1 \le k \le n\}$ such that $\mathbf{z}_k(\omega^k) = \mathbf{z}_k^*(\omega^k)$ whenever the latter is integer-valued, and

$$\Delta_k \triangleq (\mathbf{x}_k(\omega^k, z^*) - \mathbf{x}_k(\omega^k, z)) \in X,$$

for all ω^k in $\Omega^k, 1 \le k \le n$.

Proof. We use induction. Assume that Δ_k lies in X. Since:

$$\Delta_{k+1} = \Delta_k - \ \mathrm{diag}\ \xi_{k+1}(\omega^{k+1})\mathbf{z}_{k-1}^*(\omega^{k+1}(\omega^{k+1}) + \ \mathrm{diag}\ \xi_{k+1}(\omega^{k+1})\mathbf{z}_{k+1}(\omega^{k+1}),$$

if $\Delta_{k+1} = \Delta_k -$ diag $\xi_{k+1}(\omega^{k+1})\mathbf{z}_{k+1}^*(\omega^{k+1})$ lies in X, we choose $z_{k+1}^i(\omega^{k+1}) = 0,\ i = 1, 2$. If $\Delta_k -$ diag $\xi_{k+1}(\omega^{k+1})\mathbf{z}_{k+1}^*(\omega^{k+1})$ lies in $X - \xi(\omega^{k+1})$, and if $\omega^{k+1} = A_{1(2)}$ (or $D_{1(2)}$), we choose $z_{k+1}^{1(2)}(\omega^{k+1}) = 1$ and $z_{k+1}^{2(1)}(\omega^{k+1}) = 0$, and in either case Δ_{k+1} belongs to X.

Remark: That $z_k^{1(2)}(\omega^k) = 1$ for $\omega_k = D_{1(2)}$ is not surprising. In this case we "disable" dummy departures so that $\mathbf{x}_k(\omega^k) \ge 0$ in order that the linear program (LP) may have a "feasible" solution.

Proposition 3.3: The integer-valued policy $\{\mathbf{z}_k(\omega^k)\}_{k=1}^n$ in Lemma 2 is an optimal solution for the linear program (LP).

Proof: We show that the necessary conditions for optimality in (LP) are satisfied by the integer-valued policy $\{\mathbf{z}_k(\omega^k)\}_{k=1}^n$ in Lemma 3.2. Since $\mathbf{z}_k(\omega^k)$ is integer-valued when $\mathbf{z}_k^*(\omega^k)$ is, relation (3.4) and hence the minimization of (3.3a), automatically hold.

We now check the remaining two conditions of optimality, namely (3.3b) and (3.3c). We first show that $\mathbf{x}_k(\omega^k, z) \ge 0$. Suppose the opposite, and let $x_k^1(\omega^k, z) < 0$. Since

$x_n^1(\omega^k, z)$ is integer-valued, $x_k^1(\omega^k, k) \leq -1$ and from Lemma 3.2 we have $x_k^1(\omega^k, z^*) = x_n^1(\omega^k, z) + p_1$ for a suitable p_1 in $(-\frac{1}{2}, \frac{1}{2}]$. Then, $x_k^1(\omega^k, z^*) \leq -1 + \frac{1}{2} = -\frac{1}{2} < 0$, a fact contradicting the feasibility of $x_k^1(\omega^k, z^*)$, and hence the optimality of z^*.

In a similar manner, we show that $\mathbf{x}_k(\omega^k, z) \leq (C_1, C_2)^T$. Assume that $x_k^1(\omega_1^k, z) > C_1$, whence $x_k^1(\omega^k, z) \geq C_1 + 1$; we then get $x_k^1(\omega^k, z^*) \geq C_1 + 1 - \frac{1}{2} = C_1 + \frac{1}{2} > C_1$, which is again a contradiction. Further, $x_k^1(\omega^k, z) + x_k^2(\omega^k, z) \leq C$ since in the opposite case $x_k^1(\omega^k, z^*) + x_k^2(\omega^k, z^*) > C + 1 - \frac{1}{2} - \frac{1}{2} = C$. Notice, however, that the last argument relies heavily on the fact that p_1 lies in $(-\frac{1}{2}, \frac{1}{2}]$ and p_2 lies in $[-\frac{1}{2}, \frac{1}{2})$, i.e., p_1 and p_2 cannot equal $\frac{1}{2}$ (or $-\frac{1}{2}$) simultaneously. Finally, we check the conditions (3.3c). We prove first that $v_k^* > 0$ implies $x_k^1(\omega^k, z) + x_k^2(\omega^j, z) = C$. Since z^* is an optimal solution of the linear program (LP), it is enough to show that $x_k^1(\omega^k, z^*) + x_k^2(\omega^k, z^*) = C$ implies $x_k^1(\omega^k, z) + x_k^2(\omega^k, z) = C$. To this end, we observe by Lemma 3.2 that for a suitable choice of p_1, p_2,

$$x_k^1(\omega^k, z) + x_k^2(\omega^k, z) = x_k^1(\omega^k, z^*) + x_k^2(\omega^k, z^*) - p_1 - p_2 = C - p_1 - p_2 \in (C - \frac{1}{2}, C + \frac{1}{2}),$$

and $x_k^1(\omega^k, z) + x_k^2(\omega^k, z) = C$, since the sum is integer-valued. Similarly it can be shown that $\lambda_k^{i*} > 0$ implies $x_k^i(\omega^k, 2) = 0$, and $\mu_k^{i*} > 0$ implies $x_k^i(\omega^k, z^*) = C_i, i = 1, 2$. Since the necessary and sufficient conditions of optimality for (LP) are satisfied, the optimality of $z = \{\mathbf{z}_k(\omega^k), 1 \leq k \leq n, \omega^k \in \Omega^k\}$ is now evident.

Proof of Proposition 3.2: Since there exists an integer-valued solution $z = \{\mathbf{z}_k(\omega^k), 1 \leq k \leq n, \omega^k \in \Omega^k\}$ for (LP), if the initial condition $\mathbf{x}$ is integer-valued, it follows that $J_n^\beta(\cdot) = W_n^\beta(\mathbf{x})$. Therefore, the convexity of $W_n^\beta(\cdot)$ with respect to $\mathbf{x}$ implies the same for $J_n^\beta(\cdot)$.

Further, since $W_n^\beta(\cdot)$ is the value function of a linear program, it is a piecewise linear function of $\mathbf{x}$ [13, page 56], so that the following corollary holds:

Corollary 3.1: $J_n^\beta(\cdot)$ is a piecewise linear function of $\mathbf{x}$.

Proposition 3.4: For every $\mathbf{x} \geq 0$, $J_n^\beta(\cdot)$ is a "supermodular" function of $\mathbf{x}$, i.e.,

$$J_n^\beta(x^1 + 1, x^2 + 1) - J_n^\beta(x^1 + 1, x^2) \geq J_n^\beta(x^1, x^2 + 1) - J_n^\beta(x^1, x^2).$$

Proof: The proposition is a direct consequence of Corrollary 3.1 and Proposition 3.1. The proof is provided in the appendix.

4. Determination of the Optimal Strategy

We now show that the optimal policy for call allocation is of the "bang-bang" type. Specifically we prove that for type-1 calls, there is a monotone *switching curve* which partitions the state space into two regions. One of them is a blocking region (i.e., blocking is optimal for all states belonging to the region) while the other is nonblocking. Analogous results hold for type-2 calls also.

We begin by making the following assertions:

Assertion 1: Assuming that state (x^1, x^2) is a blocking state for type-1 calls, all states $(\overline{x}^1, x^2)$ with $\overline{x}^1 > x^1$ are also blocking states for type-1 calls.

Since the state (x^1, x^2) is a blocking state for type-1 calls, from the switching conditions (2.4) we have $J_n^\beta(x^1+1, x^2) - J_n^\beta(x^1+1, x^2) \geq J_n^\beta(x^1+1, x^2) - J_n^\beta(x^1, x^2) \geq 1/\beta\lambda_1$, and as a consequence the state (x^1+1, x^2) is blocking thereby validating the assertion. An analogous result for type-2 calls can be similarly proved.

Assertion 2: Assuming that state (x^1, x^2) is a blocking state for type-1 calls, all states $(x_1, \overline{x}_2)$ with $\overline{x}_2 > x_2$ are also blocking states for type-1 calls. The assertion is proved in a similar manner as assertion 1 by using the supermolularity property (Proposition 2.4) of $J_n^\beta(\cdot)$.

By combining assertions 1 and 2, we conclude that the optimal strategy minimizing the β-discounted n-step blocking cost (2.2a) is characterized by two monotone switching curves, one for each traffic type. Furthermore, since all the previous arguments are valid in the limit as $n \to \infty$, we can assert:

Proposition 4.1: The optimal policy for the blocking system under study with respect to an infinite horizon β-discounted blocking cost is characterized by two monotone (decreasing) switching curves, one for each traffic type (Figure 4).

5. The Average Cost Case

In this section, we determine the structure of the optimal stationary policy with respect to an average cost criterion. To this end, we define the long-run average cost associated with a policy z in $\mathcal{P}$ and starting with initial state $\mathbf{x}$, as:

$$V(\mathbf{x}, z) = \limsup_{n\to\infty} \frac{1}{n} \mathrm{E}^z_{\mathbf{x}} \sum_{k=0}^{n} (z^1_k + a z^2_k), \quad a > 0.$$

The minimum long-run average cost then is:

$$J_{av}(\mathbf{x}) = \min_{z\in\mathcal{P}} V(\mathbf{x}, z),$$

and the policy that achieves the minimum is an average cost optimal strategy.

From [11, Thm. 2.1 and 2.2] we conclude the following: Since the state space of our problem is finite for all discount factors $0 < \beta < 1$, the difference $|J^\beta(x^1, x^2) - J^\beta(0,0)|$ is bounded. It follows that the average cost $J_{av}(\mathbf{x})$ is independent of the initial state $\mathbf{x}$ and $J_{av} = \lim_{\beta\to 1}(1-\beta)J^\beta(0,0)$. Furthermore, there exists a bounded function $h(x^1, x^2)$ and a sequence of discount factors $\beta_n \to 1$ with $h(x^1, x^2) = \lim_{n\to\infty}(h^{\beta_n}(x^1, x^2) - h^{\beta_n}(0,0))$, and satisfying the following DP-equation for the average cost:

$$\begin{aligned} J_{av} + h(\mathbf{x}) = \min_{\substack{\{z^i_k\in\{0,1\} \\ z^i_k=1 \text{ if } x^i=C_1 \\ \text{or } x^1+x^2=C\}}} & \{z^1_k + a z^2_k \\ & + \lambda_1(1-z^1_k)h(A_1\mathbf{x}) + \lambda_1 z^1_k h(\mathbf{x}) \\ & + \lambda_2(1-z^2_k)h(A_2\mathbf{x}) + \lambda_2 z^2_k h(\mathbf{x}) \\ & + \mu_1 x^1 h(D_1, \mathbf{x}) + \mu_2 x^2 h(D_2\mathbf{x}) \\ & + (C\mu_2 - x^1\mu_1 - x^2\mu_2)h(\mathbf{x})\}. \end{aligned}$$

Furthermore, there exists a stationary policy z that is average cost optimal and is the minimizer of the right side of the equation above. Obviously, $h(\cdot)$ has the same properties as $J^\beta(\cdot)$ for $0 < \beta < 1$, i.e., it is increasing, convex and supermodular. Switching conditions similar to (2.4) may be derived for $h(\cdot)$, and using the same arguments as for the discounted cost case, it can be shown that the average cost optimal strategy has the form of two monotone switching curves.

6. The Case of a Single Link

Similar results as before hold true for the problem associated with the optimal admission of two traffic types arriving at a common link having capacity C frequency slots (i.e., when $C_1 > C$ and $C_2 > C$ in the model studied previously). Figure 5a illustrates the optimal admission policy for the β-discounted cost with $\lambda_1 = 10$, $\lambda_2 = 100$, $\mu_1 = 5$, $\mu_2 = 5$, $C = 10$, $\beta = 0.99$. In this example, the dynamic programming recursion (2.3) was iterated 200 times till the β-discounted cost converged. Then the optimal policy was evaluated through relations (2.4). In this particular case, and in many other similar ones, we observed from the computations that the traffic with the highest cost was never blocked (except on the boundaries of the state space). We were not able to provide a formal proof of this rather intuitively evident fact through dynamic programming equations.

We can gain some insight into the optimal admission policy for the simple link problem by examining the associated linear program (LP). Assuming that $\lambda_1 < \alpha\lambda_2$, and by the symmetry with respect to the variables $z_k^1(\omega_k)$, $z_k^2(\omega_k)$ of the constraint associated with the capacity C of the link, we conclude that if $z_1^1(A_1) = 0$ then $z_1^2(A_2) = 0$ since, in the opposite case, i.e. if $z_1^2(A_2) = 0$, we can interchange the values of z^1, z^2 while still satisfying the constraint and simultaneously achieving a smaller increase in the cost function of the linear program. In a similar manner we can show that $z_1^2(A_2) = 1$ (call type-2 is blocked) implies $z_1^1(A_1) = 1$ (call type-1 is blocked) also. We have been yet unable to relate the optimal policy derived from the switching conditions (2.4) to the solution of the linear program (LP). Therefore, no valid conclusions about the the optimal policy can be derived by simply examining the behavior of the solution of (LP).

Another case of interest is the optimal admission policy when the traffic streams have different bandwidth requirements. Computations were performed for two streams arriving at a common link. In this case it is computationally demonstrated that the optimal policy for the β-discounted cost (Figure 5b) is not necessarilly characterized by two monotone switching curves. As a consequence, the β-discounted cost may not be convex.

7. The Optimal Admission Control Problem for a General Circuit Switched Network

We consider a circuit-switched network providing service to different traffic (call) types. The links between the nodes are labeled $j = 1, 2, \ldots J$, and each link j comprises C_j

circuits (channels). Each call upon admission to the network is forwarded to its destination through a prespecified set of interconnected links which constitute a route. Let $\mathcal{R}$ be the set of all routes in the network. We define the matrix $A = (a_{jr}, j = 1, 2, \dots J,\ r \in \mathcal{R})$, where

$$a_{jr} = \begin{cases} 1 & \text{if a message on route } r \text{ uses a circuit of link } j \\ 0 & \text{otherwise.} \end{cases}$$

Assume that the calls requesting route r arrive according to a Poisson process with intensity λ_r. Moreover, the service time of each call (i.e., the time during which it is forwarded through route r) is exponentially distributed with parameter μ_r. A call requesting admission on route r is discarded if at least one link on the route r is saturated, i.e., has no free slots. We denote by $\mathbf{C}$ the capacity vector, i.e., $\mathbf{C} = (C_1, \dots C_J)^T$. Observe that the problem formulated in section 2 is a special case of the general problem with matrix A and capacity vector $\mathbf{C}$ of the form:

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} C_1 \\ C_2 \\ C \end{pmatrix}.$$

We study the equivalent discrete-time problem. As before, we define the state of the system at time instant k to be $\mathbf{x}_k = (x_k^r, r \in \mathcal{R})$ where x_k^r denotes the number of calls forwarded on route r at that time instant. Obviously, $\mathbf{A}\mathbf{x}_k \le \mathbf{C}$, and we define the state space of the system to be $\mathcal{X} = \{\mathbf{x} : \mathbf{A}\mathbf{x} \le \mathbf{C}, \mathbf{x} \ge 0\}$. Recall that the time instants at which the system is observed correspond to state transition epochs (i.e., arrivals or departures). Given that a cost a_r is incurred for each call that is not given access to the network on route r, we seek an optimal admission strategy (in the same spirit as for the simple problem of section 2) minimizing an infinite horizon (n step) β-discounted cost of the form:

$$\mathbb{E}_{\mathbf{x}}^{z} \left(\sum_{k=1}^{\infty(n)} \beta^k \mathbf{a}\mathbf{z}_k \right),$$

where $\mathbf{a} = (a_r, r \in \mathcal{R})$, $a_k > 0$, $\mathbf{z}_k = (z_k^r, r \in \mathcal{R})$ and $z_k^r = 1(0)$ if an incoming call on route r is blocked (accepted).

We define the total event rate (i.e., the "uniformization" rate) out of a state to be:

$$\rho = \sum_{r \in \mathcal{R}} \lambda_r + \overline{\mathbf{x}} \cdot \mu,$$

where $\mu = (\mu_i, i \in \mathcal{R})$ and $\overline{\mathbf{x}}$ is a solution to the following Linear Program:

$$\max_{x} \mu \mathbf{x},$$

such that:

$$\mathbf{A}\mathbf{x} \leq \mathbf{C}, \mathbf{x} \geq 0.$$

After normalizing all rates with respect to ρ (equivalently assuming $\rho = 1$), we can write the dynamic programming equation associated with the optimal n-step β-discounted cost $J_n^\beta(\mathbf{x})$. To this end we denote by $\mathbf{e_r}$ the column vector $(e_i, i \in \mathcal{R})$, with $e_i = 0$ for $i \neq r$ and $e_r = 1$. Moreover, we define the arrival and departure operators $A_r, D_r : \mathcal{X} \to \mathcal{X}$ as follows:

$$A_r(\mathbf{x}) = (\mathbf{x} + \mathbf{e}_r)^*, \ D_r(\mathbf{x}) = (\mathbf{x} - \mathbf{e}_r)^*,$$

where

$$(\mathbf{x} + \mathbf{e}_r)^* = \begin{cases} \mathbf{x} + \mathbf{e}_r & \text{if } A(\mathbf{x} + \mathbf{e}_r) \leq \mathbf{C} \\ \mathbf{x} & \text{otherwise} \end{cases}, \ (\mathbf{x} - \mathbf{e}_r)^+ = \begin{cases} \mathbf{x} - \mathbf{e}_r & \text{if } x_r \geq 1 \\ \mathbf{x} & \text{otherwise} \end{cases},$$

for r in $\mathcal{R}$. Then we can write:

$$J_{k+1}^\beta(\mathbf{x}) = \min_{\substack{\{z_k^i \in [0,1], r \in \mathcal{R} \\ z_k^r = 1 \text{ if } A(\mathbf{x}+\mathbf{e}_r) > \mathbf{C}\}}} (\mathbf{a}\mathbf{z}_k + \beta \sum_{r \in \mathcal{R}} ((1 - z_k^r)\lambda_r J_k^\beta(A_r\mathbf{x}) + \lambda_r z_k^\beta(\mathbf{x})$$

$$+ x_r \mu_r J_k^\beta(D_r\mathbf{x}) + (\overline{\mathbf{x}}_r - \mathbf{x}_r)\mu_r J_k^\beta(\mathbf{x}))),$$

and the following criterion for admission can be obtained:

If $A(\mathbf{x} + \mathbf{e}_r) \leq \mathbf{C}$ then an incoming call to route r is blocked ($z_k^r = 1$) if:

$$J_n^\beta(A_r\mathbf{x}) - J_n^\beta(\mathbf{x}) \geq \frac{a_r}{\beta \lambda_r}.$$

Unfortunately the convexity and supermodularity properties (cf. Propositions 2.2 and 2.4) for $J_n^\beta(\cdot)$ cannot be derived for an arbitrary network topology. This is due to the fact that an integer-valued solution to an associated linear program similar to (LP) cannot be found. For example, Proposition 3.3 fails to hold if we have a constraint of the form $x_k^1(\omega^k) + x_k^2(\omega^k) + x_k^3(\omega^k) \leq C$ for a system with 3 routes. As a consequence, the optimal strategy for the general problem cannot be shown to have a "switching" surface structure. Nevertheless, for the case where the matrix A has the simple form,

$$\mathbf{A} = \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & \cdots & \\ & & & & 1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 11 & & & & \\ & 11 & & & \\ & & 11 & & \\ & & & \cdots & \\ & & & & 11 \end{pmatrix},$$

it can be easily verified that all the proofs of convexity and supermodularity of the optimal β-discounted cost hold true. In this case, the optimal admission strategy has a structure of a "monotone" switching surface, i.e., if $z^r(\mathbf{x}) = 1$ then $z^r(\mathbf{x} + \mathbf{e}_i) = 1$ for i in $\mathcal{R}$ and $\mathbf{x} + \mathbf{e}_i$ in $\mathcal{X}$. Furthermore, similar results are also true for the long-run average cost problem.

A network with matrix A having the simplified form, as well as the associated switching surface for type-1 calls is shown in Figure 6.

Appendix

Proof of Proposition 3.4: In this section we prove that $J_n^\beta(\cdot)$ is a "supermodular" function. Specifically, we prove that a piecewise linear increasing function $W(\cdot)$ is supermodular, i.e.,

$$W(x^1+1, x^2+1) - W(x^1+1, x^2) \geq W(x^1, x^2+1) - W(x^1, x^2) \qquad (A.1)$$

for $x^1 \geq 0,\ x^2 \geq 0$.

The proof consists of the following cases:

Case 1: Refer to Figure 7. Since $W(\cdot)$ is piecewise linear, its graph consists of intersecting planes. Assume that the points $(x^1, x^2, W(x^1, x^2))$ and $(x^1, x^2+1, W(x^1, x^2+1))$ belong to plane (1), while the other two points, i.e., $(x^1+1, x^2, W(x^1+1, x^2))$ and $(x^1+1, x^2+1, W(x^1+1, x^2+1))$ belong to plane (2). We now make a projection on the (x^1, x^2)-plane. The line l is the projection of the intersection of the planes (1) and (2) (Figure 7). Since $W(\cdot)$ is piecewise linear, we set $W(x^1, x^2) = A_1x^1 + B_1x^2 + C_1$, $W(x^1+1, x^2) = A_2(x^1+1) + B_2x^2 + C_2$, $W(x^1, x^2+1) = A_1x^1 + B_1(x^2+1) + C_1$, and $W(x^1+1, x^2+1) = A_2(x^1+1) + B_2(x^2+1) + C_2$, where the subscript i in the group of coefficients (A_i, B_i, C_i) refers to plane $i = 1, 2$. Direct substitution in (A.1) gives $A_2 \geq A_1$, a valid fact due to the increasing nature of $W(\cdot)$.

For all subsequent cases, the reader is refered to Figure 8.

Case 2: By direct substitution in (A.1) we get:

$$A_1x^1 + B_1x^2 + C_1 \geq A_2x^1 + B_2x^2 + C_2.$$

This inequality is true due to the increasing property of $W(\cdot)$.

Case 3: By substitution in (A.1) we get :

$$A_2(x^1+1) + B_2(x^2+1) + C_2 \geq A_1(x^1+1) + B_1(x^2+1) + C_1,$$

in a manner similar to case 2.

Case 4: Inequality (A.1) is established since $B_2 \geq B_1$, similar to case 1.

In cases (5)-(9) we consider the intersection of 3 planes.

Case 5: By direct substitution in (A.1) we get,

$$A_2(x^1+1)+B_2(x^2+1)+C_2-A_3(x^1+1)-B_3(x^2+1)-C_3+A_3 \geq A_1,$$

since $A_3 \geq A_1$ and $A_2(x^1+1)+B_2(x^2+1)+C_2 \geq A_3(x^1+1)+B_3(x^2+1)+C_3$.

Cases $(6)-(9)$ (see Figure 8) can be established in a similar manner. The case of four intersecting hyperplanes can easily be reduced to any of the previous cases.

Remark: For the proof of "supermodularity ", only the increasing nature and the piecewise linearity of $W(\cdot)$ are used; the convexity property is not needed.

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FIGURES

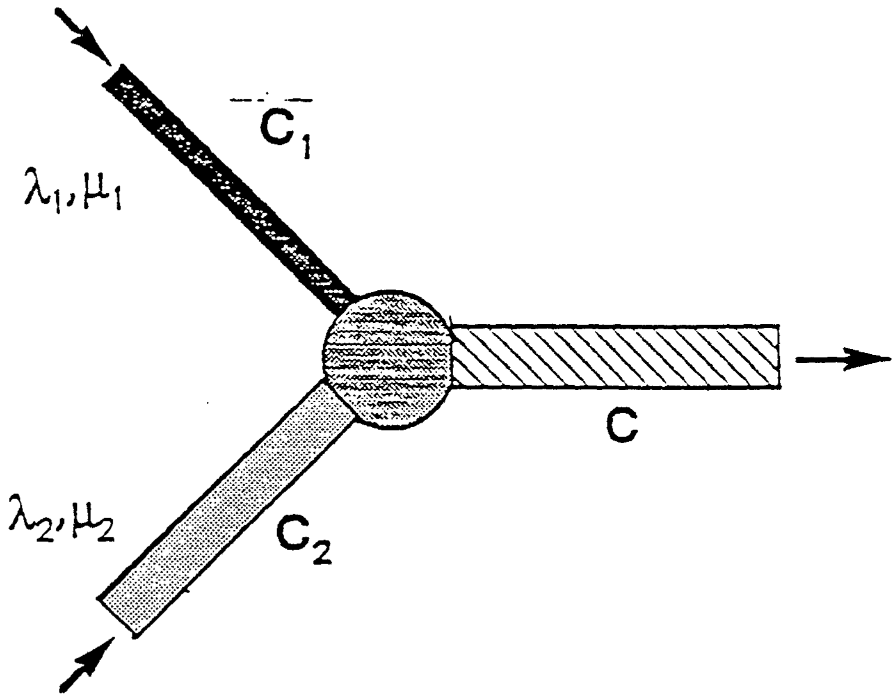
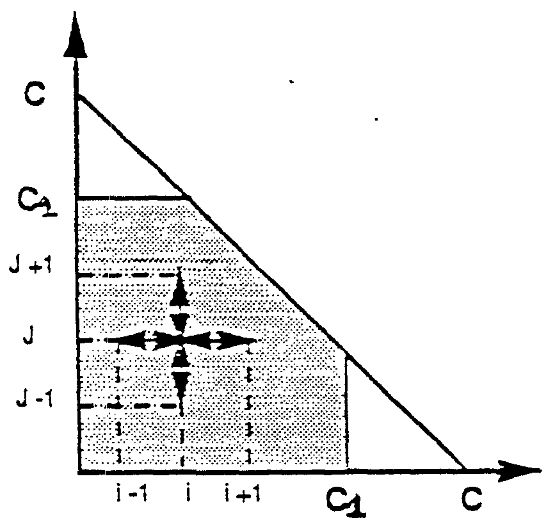


Figure 1


C
C2
J +1
J
J -1
i -1
i
i +1
C1
C

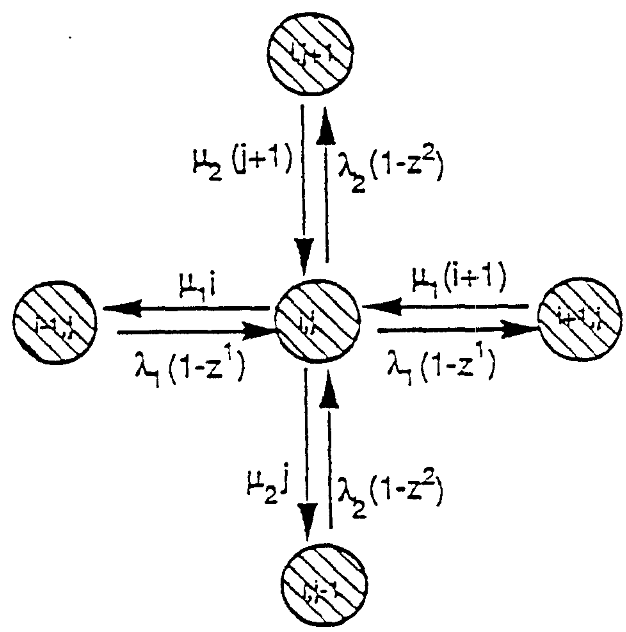

i,j+1
μ2 (j+1)
λ2(1-z2)
μ1 i
μ1 (i+1)
i,j
λ1 (1-z1)
λ1 (1-z1)
μ2 j
λ2(1-z2)

Figure 2

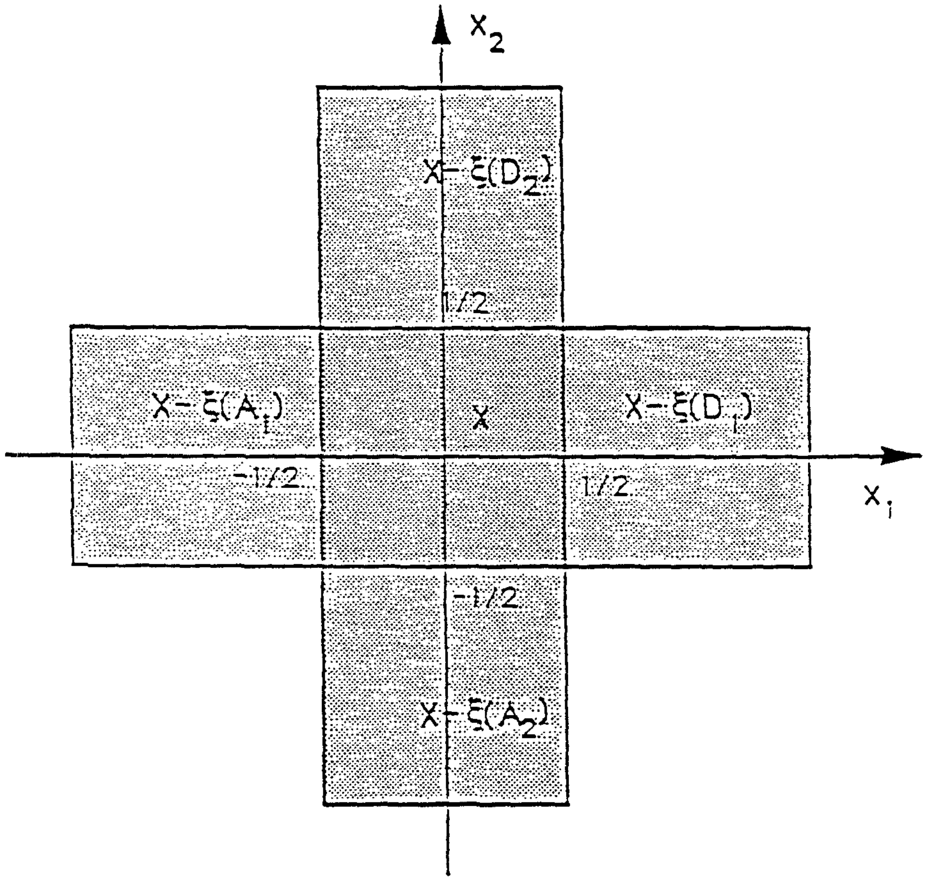


Figure 3

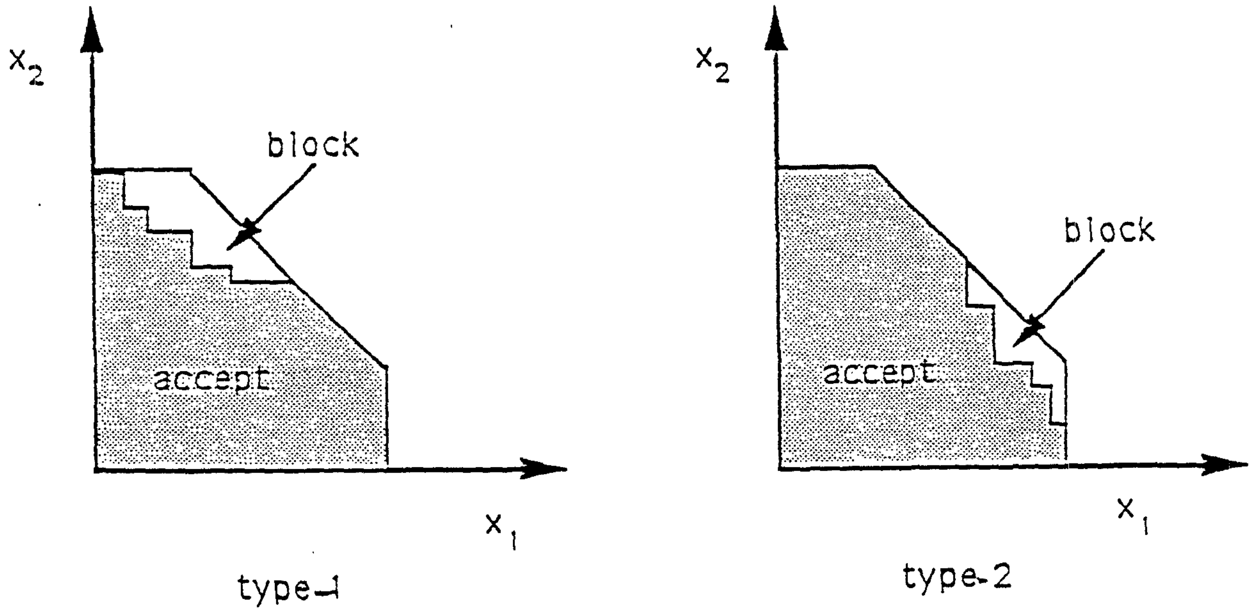


Figure 4

x_1 (horizontal axis), x_2 (vertical axis)

```
nnnnnnnbbbb
nnnnnnbbbb
nnnnnbbbb
nnnnbbbb
nnnbbbb
nnbbbb
nbbbb
bbbb
bbb
bb
b
```

x_1 (horizontal axis), x_2 (vertical axis)

```
nnnnnnnnnnb
nnnnnnnnnb
nnnnnnnnb
nnnnnnnb
nnnnnnb
nnnnnb
nnnnb
nnnb
nnb
nb
b
```

Figure 5a

b: block a call, n: accept a call

$\lambda_1 = 10,\ \lambda_2 = 100,\ \mu 1 = \mu 2 = 5,\ a = 5,\ C = 10$

x_1 (horizontal axis), x_2 (vertical axis)

```
nnnnnnnnnnnnnnnnnnnnnnnnnnnnnnnnnnbbb
nnnnnnnnnnnnnnnnnnnnnnnnnnnnnnnbbb
nnnnnnnnnnnnnnnnnnnnnnnnnnnnbbb
nnnnnnnnnnnnnnnnnnnnnnnnnbbb
nnnnnnnnnnnnnnnnnnnnnnbbb
nnnnnnnnnnnnnnnnnnnbbb
nnnnnnnnnnnnnnnnbbb
nnnnnnnnnnnnnbbb
nnnnnnnnnnbbb
nnnnnnnbbb
nnnnbbb
nbbb
b
```

x_1 (horizontal axis), x_2 (vertical axis)

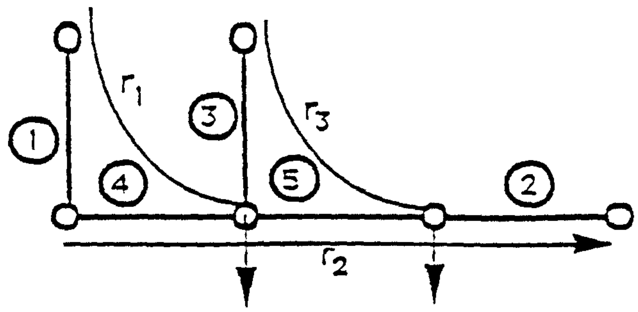
```
nnnnnnnnnnnnnnnbbbbbbbbbbbbbbbbbbbbbb
nnnnnnnnnnnnbbnbbbbbbbbbbbbbbbbbbb
nnnnnnnnnnnnbbbbbbbbbbbbbbbbbbb
nnnnnnnnnnnnbbbbbbbbbbbbbbbb
nnnnnnnnnbbnbbbbbbbbbbbbb
nnnnnnnnnbbbbbbbbbbbbb
nnnnnnbnnbbbbbbbbbb
nnnbnnbbnbbbbbbb
nnnbnnbbbbbbb
bnnbbnbbbb
bnnbbNb
bbNb
b
```

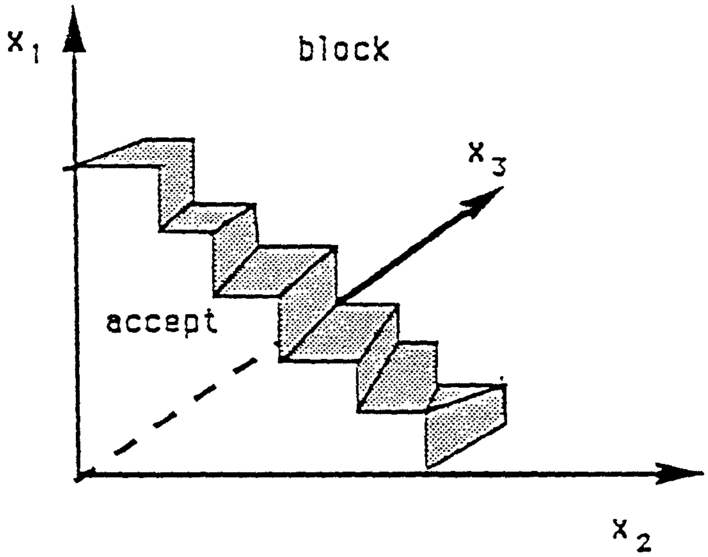
Figure 5b

b: block a call, n: accept a call

Bandwidth ratio 1:3

$\lambda_1 = 16,\ \lambda_2 = 55,\ \mu 1 = 0.5,\ \mu 2 = 8,\ a = 15,\ C = 36$

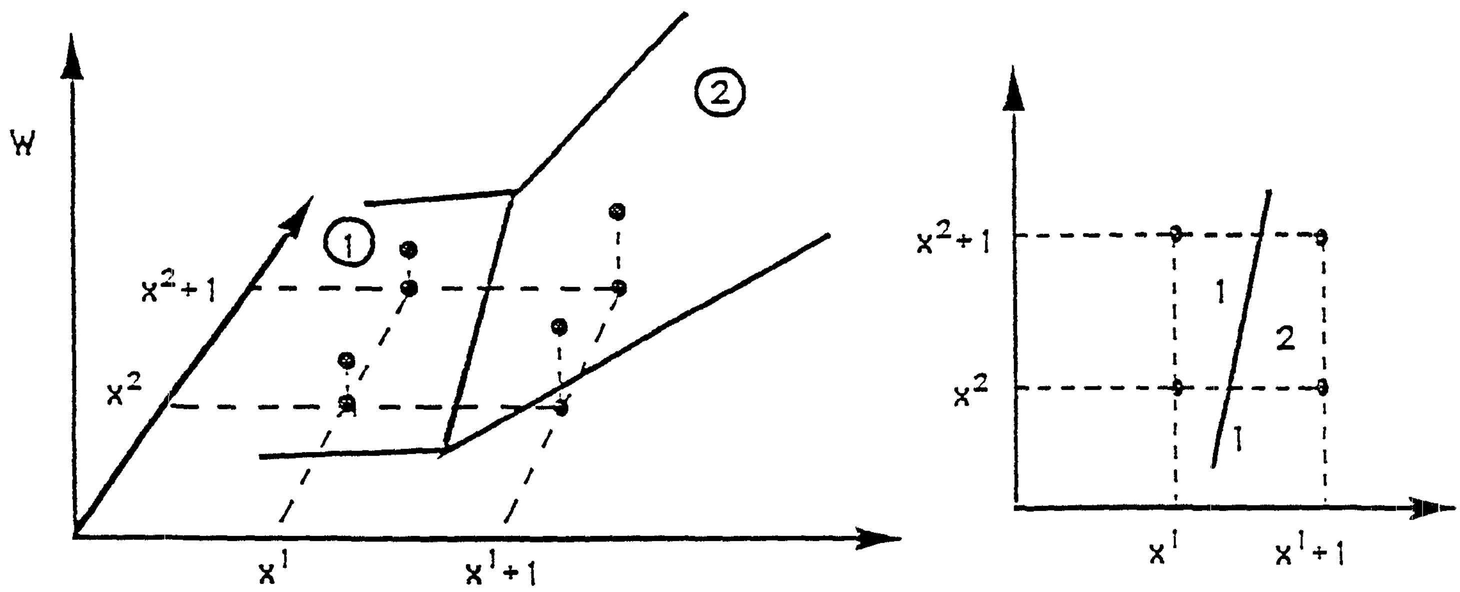




Admission Policy for Traffic Type-1

$$A = \begin{array}{c|c} r_1 \; r_2 \; r_3 \leftarrow \text{routes} & \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} & \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \\ & \uparrow \\ & \text{links} \end{array}$$

Figure 6



Case 1

Figure 7

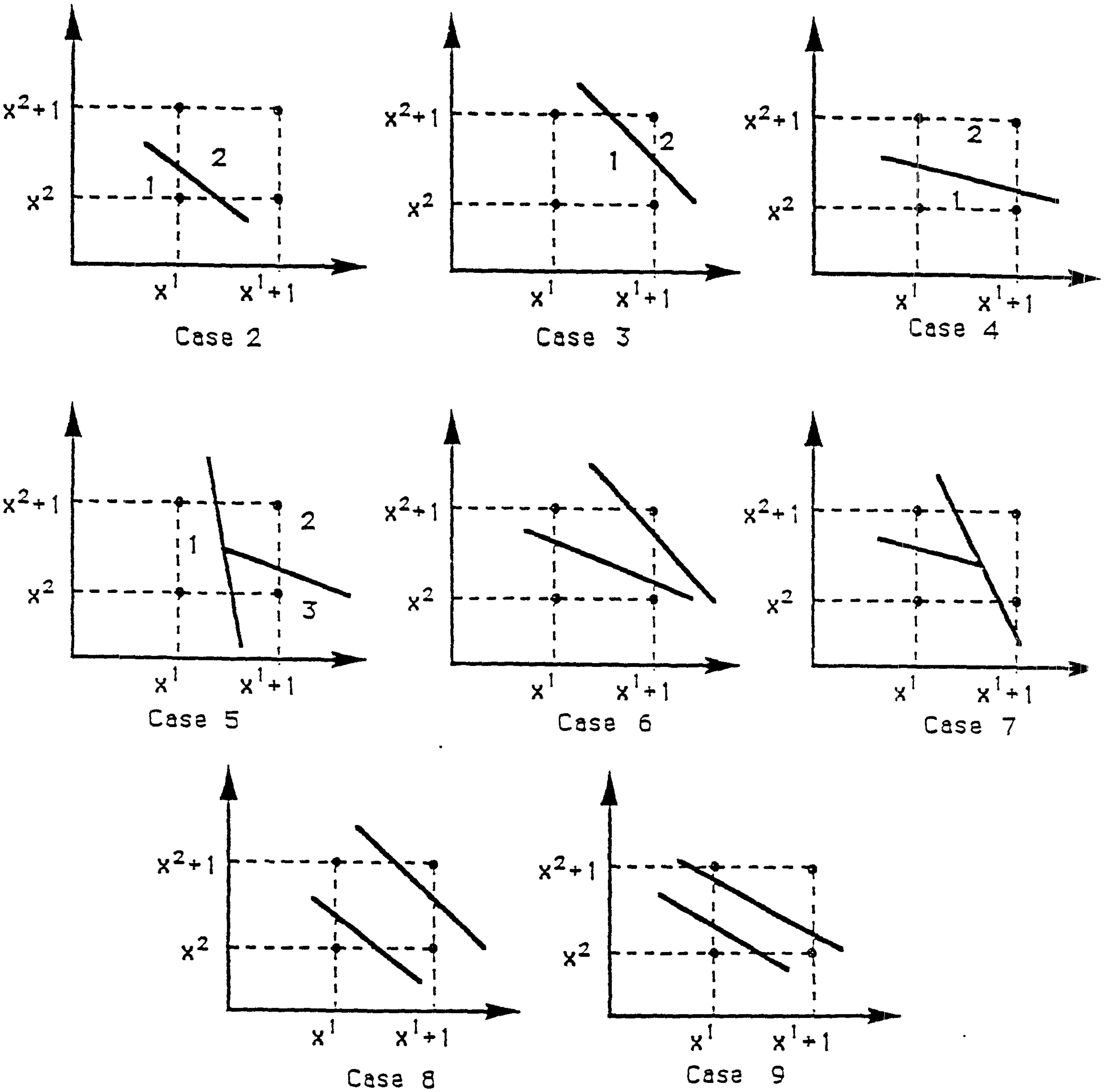

x²+1
x²
x¹
x¹+1
1
2
Case 2
1
2
Case 3
1
2
Case 4
1
2
3
Case 5
Case 6
Case 7
Case 8
Case 9

Figure 8