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COHERENT SIGNAL PROCESSING USING ARRAYS OF ARBITRARY GEOMETRY

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ABSTRACT

The existing spatial smoothing (SS) technique, although it is effective in decorrelating coherent signals, can only be applied to uniformly spaced linear arrays which are very sensitive to the directions-of-arrival (DOA's) and can be used to estimate azimuth angles only. To significantly improve the robustness of DOA estimation and of beamforming and to estimate both azimuth and elevation angles, we developed techniques for applying SS to arrays of arbitrary geometry. We found that an array must have an orientational invariance structure with an ambiguity free center array for applying SS. We also study the cause of ambiguities in a multiple signal environment and find the necessary and sufficient conditions for an array manifold to be ambiguity free. If an array is also central symmetric, the forward/backward spatial smoothing can be used to improve the resolution. Finally, we expand the application of our technique not only to MUSIC and adaptive beamforming algorithms but also to ESPRIT algorithm. All the predicted results are verified by simulations.

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I Introduction

Sensor array processing has been a key technology in radar, sonar, communications and biomedical signal processing. Recently, as the cellular communication technology advances, sensor array processing emerges as a potential technology to improve the spectral efficiency [1], [2], [3].

Much of the work in array processing has focused on methods for high-resolution DOA estimation and optimum adaptive beamforming. These methods include the well known MUSIC [4] algorithm and ESPRIT [5] algorithm for DOA estimation and MVDR and LCMV algorithms [6], [7], [8] for beamforming. However, an important drawback of these techniques is the severe degradation of the estimation accuracy in DOA estimation [9] or signal cancelation [10] in adaptive beamforming, in the presense of highly correlated or coherent signals.

To counter the deleterious effects due to some coherent signals, a pre-processing scheme referred to as spatial smoothing (SS) proposed by Evans et al. [11] and further developed by Shan et al. [9], [12] has been shown to be effective in decorrelating coherent signals. However, such a scheme can only be applied to uniformly spaced linear arrays. Linear arrays are known to be limited to estimating azimuth angles within 180^0, and practically effective only for signals from broadside direction. The degree of SS using a uniformly spaced linear array is also sensitive to DOA's [13]. As a result, a linear array is not very effective in radar, sonar, or especially in cellular communications where users can never predict the incoming directions of the moving targets.

In the past decade, researches have been carried out in developing algorithms for coherent interference using arrays of arbitrary geometry. In the area of DOA estimation, multidimensional subspace fitting algorithms such as deterministic maximum likelihood (DML) [14], multidimensional (MD)-MUSIC [15], and recently proposed weighted subspace fitting (WSF) [16], [17], are effective in both coherent and noncoherent environment and can be applied to arrays of arbitrary geometry. However, all these algorithms involve some searching procedures used to solve nonlinear equations. They are computationally intensive and are not practical in real-time applications. In the area of narrow-band adaptive beamforming, The coherent interference suppression using null constraint with an array of arbitrary geometry was addressed in [18]. This approach still requires pre-estimation of arrival angles of coherent interferences. Recently, the SPT-LCMV beamforming algorithm applicable to arrays of arbitrary geometry was considered in [19]. This algorithm requires increased computational complexity compared to LCMV.

In this work, we develop a general SS technique for arrays of arbitrary geometry to make MUSIC, ESPRIT algorithms and optimum adaptive beamforming algorithms operative in a coherent interference environment and meanwhile achieve robustness in performance. Compared with the aforementioned methods for arrays of arbitrary geometry, this SS technique can be easily implemented. It does not increase the computational complexity of either MUSIC, ESPRIT, or adaptive beamforming. It allows us to work on a data domain [20], and thus enables us to incorporate the recently developed URV [21] [22] algorithm to DOA estimation and updating and enables us to implement MVDR beamforming algorithm using systolic arrays. Therefore it has great potential in mobile radio communication where coherent and cochannel multipath interference is a major problem. Also, it can be used in conjunction with MUSIC or ESPRIT algorithm to provide an initialization for the WSF method to get a more accurate DOA estimation [17].

Specifically, we discovered and proved the conditions on an array of arbitrary geometry for applying SS. They are: (1) such an array must have an orientational invariance structure; (2) its center array has an ambiguity free array manifold; and (3) the number of subarrays is larger than or equal to the largest number of mutually coherent signals. By working on a smoothed data matrix obtained from SS, we can use MUSIC and optimum adaptive beamformers effectively in a coherent interference environment. To further increase efficiency and estimation resolution, we found that the forward/backward spatial smoothing [23] (FBSS), when applied to a nonlinear array of central symmetry, can reduce the number of sensors required and improve the estimation resolution for closely spaced incoming signals. Finally we expand the application of our results to ESPRIT.

In all the papers cited above that dealt with DOA estimation with arrays of arbitrary geometry, ambiguity free array manifolds were assumed. In [4] Schmidt discovered and defined the rank-n ambiguity in an array manifold. However, no more studies on such ambiguity had been reported in the open literature [24]. Then in [24], Lo and Marple proved the conditions for a rank-2 ambiguity. In this paper, we report a more thorough study on this issue. We proved the necessary and sufficient conditions for a three-sensor array manifold to be ambiguity free. We then identified several situations, for higher order sensor array manifolds, in which ambiguity may arise. Thus we get corresponding necessary conditions to design ambiguity free center arrays and subarrays.

This paper is divided into six sections. In section II, we prove the necessary and sufficient conditions on an array of arbitrary geometry for applying SS, and consider the FBSS technique for applications in nonlinear arrays. In section III, we study the cause of ambiguities in a multiple signal environment. In section IV, we present some simulation results and show that dense square arrays have a preferred geometry for applying SS. In section V, we expand our results to ESPRIT. Section VI concludes our work.

II SS for Array of Arbitrary Geometry

We first assume that all the sensors in an array discussed in this paper are omnidirectional and identical. Consider an array of p sensors. Let d narrow-band signals with additive white Gaussian noise impinge on the array at incident angles $\theta_1, \cdots, \theta_d$. The array output covariance matrix has the form [20]

$$R = E(\mathbf{r}(t)\mathbf{r}^H(t)) = AR_sA^H + \sigma^2 I. \tag{1}$$

where $\mathbf{r}(t)$ is the received signal by the array at time t, A is a $p \times d$ steering matrix and σ^2 is the variance of the white Gaussian noise. When there are coherent interferences, the signal covariance matrix R_s is no longer full rank. Therefore, all the high resolution DOA estimation methods based on eigendecomposition and all the adaptive beamforming algorithms which assume that interfering signals are not fully correlated with the desired signals fail to operate effectively.

In the case of a uniformly spaced linear array, with a sensor spacing Δ, the SS [9] [12] algorithm can be applied to achieve the nonsingularity of the modified covariance matrix of the signals. This technique begins by dividing a uniformly spaced linear array of L sensors into K overlapping subarrays of size p, with sensors $\{1, \cdots, p\}$ forming the first subarray, and sensors $\{2, \cdots, p+1\}$ forming the second subarray, etc. It was shown that [9]

$$A_k = A_1 E^{(k-1)}, \tag{2}$$

where A_k, $k = 1, \cdots, K$, is a $p \times d$ steering matrix consisting of steering vectors associated with the kth subarray, and $E^{(k)}$ denotes the kth power of a $d \times d$ diagonal matrix E.

The spatially smoothed covariance matrix is defined as the average of the subarray covariances:

$$\bar{R} = \frac{1}{K}\sum_{k=1}^{K} R_k = A_1 \bar{R}_s A_1^H + \sigma^2 I, \tag{3}$$

where R_k is the covariance matrix associated with the kth subarray, $\bar{R}_s$ is the modified covariance matrix of the signals, and has been proved [9] to be full rank when $K \geq d$. The signals are thus progressively decorrelated [13]. However, linear arrays have limitations in the domain of estimable DOA's. It has been shown in ref. [25] that $\bar{R}_s$ can be decomposed as follows:

$$\bar{R}_s = \frac{1}{K} C C^H \tag{4}$$

where $C = PA^T$ with $P = diag(p_1, p_2, \cdots, p_d)$, and

$$A = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ e^{-j2\pi\frac{\Delta}{\lambda}\sin(\theta_1)} & e^{-j2\pi\frac{\Delta}{\lambda}\sin(\theta_2)} & \cdots & e^{-j2\pi\frac{\Delta}{\lambda}\sin(\theta_d)} \\ e^{-j4\pi\frac{\Delta}{\lambda}\sin(\theta_1)} & e^{-j4\pi\frac{\Delta}{\lambda}\sin(\theta_2)} & \cdots & e^{-j4\pi\frac{\Delta}{\lambda}\sin(\theta_d)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j2K\pi\frac{\Delta}{\lambda}\sin(\theta_1)} & e^{-j2K\pi\frac{\Delta}{\lambda}\sin(\theta_2)} & \cdots & e^{-j2K\pi\frac{\Delta}{\lambda}\sin(\theta_d)} \end{bmatrix}. \tag{5}$$

When incoming signals are closely spaced, the columns of both A and A_1 become almost linearly dependent [25]. The dependency increases drastically when some of $sin(\theta_i), i = 1, \cdots, d$ approach 1 for DOA's near 90^0. As a result, the performance of a linear array deteriorates quickly when some of DOA's approach 90^0. The highly directional sensitivity of the linear array causes the lack of performance robustness to the DOA's and limits the domain of estimable angles to azimuth angles from broadside direction of the array. The lack of performance robustness of a linear array is even more severe when SS technique is applied, because in the smoothed covariance matrix, not only the steering matrix A_1, but also A is ill-conditioned in the situation described above. A general SS technique that is robust and can be applied to directionally independent arrays is thus more desirable.

A Orientational Invariance Structure

It is apparent that the mapping relation between A_k and A_1 is the key to successful application of the SS technique. In general, we can divide an arbitrary array into K subarrays which may overlap. There is not always a steering matrix A to map all the steering matrix A_k, for $k = 1, \cdots, K$ to A. In this section, we will develop necessary and sufficient conditions on array geometries for implementing the general SS. First, we give the following lemmas.

Lemma 1 *For steering matrices A and B, given by $A = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_d)]$ and $B = [\mathbf{b}(\theta_1), \cdots, \mathbf{b}(\theta_d)]$, there exists a mapping relation $B = AC$ if and only if C is a diagonal matrix.*

Lemma 2 *For K steering matrices $A_1, A_2, \cdots, A_K$, each A_i can be mapped to a steering matrix B if and only if there exists a mapping relation $A_j = A_i C_{ij}$ for any i and j.*

The proofs of both lemmas are given in the Appendix.

Consider an array that is divided into K subarrays. Suppose A_i and A_j are the steering matrices associated with the ith and the jth subarrays, and there are d signals with incoming angles $\theta_1, \cdots, \theta_d$. A_i can be written as

$$A_i = [\mathbf{a}_i(\theta_1), \mathbf{a}_i(\theta_2), \cdots, \mathbf{a}_i(\theta_d)] \tag{6}$$

where $\mathbf{a}_i^T(\theta_k) = [e^{-j\phi_{i1}(\theta_k)}, e^{-j\phi_{i2}(\theta_k)}, \cdots, e^{-j\phi_{ip}(\theta_k)}], k = 1, \cdots, d$, is the steering vector associated with the ith subarray, and $\phi_{il}(\theta_k)$, $l \in \{1, \cdots, p\}$, is the phase delay of the kth signal at the lth sensor of the ith subarray from the first sensor of the first subarray. We refer to the sensor of an array associated with the lth row of a steering matrix of the array as the lth sensor of the array.

Let Δ_{ijl}, $1 \leq l \leq p$, represent the distance between the lth sensor in the ith subarray and the lth sensor in the jth subarray. Let β_{ijl} represent the angle of the line on which these two sensors are located. If the ith and the jth subarrays are identical and have the same orientation, i.e. all Δ_{ijl} for $l = 1, \cdots, p$ are equal and all $\beta_{ijl}, l = 1, \cdots, p$ are equal, then the phase delay of a signal with an incoming angle θ_k from each sensor in the ith subarray to the corresponding sensor in the jth subarray is the same according to the far field assumption. We denote this phase delay by $\Phi_{ij}(\theta_k)$. For any $l \in \{1, \cdots, p\}$, we have

$$\Phi_{ij}(\theta_k) = \phi_{jl}(\theta_k) - \phi_{il}(\theta_k) = 2\pi \frac{\Delta_{ijl}}{\lambda} \sin(\beta_{ijl} - \theta_k + \frac{\pi}{2}), \tag{7}$$

then $A_j = A_i C_{ij}$, where C_{ij} is a diagonal matrix with the mth diagonal element $e^{-j\Phi_{ij}(\theta_m)}$. The identical and orientational invariance properties between two subarrays guarantee a mapping relation between their steering matrices.

On the other hand, if $A_j = A_i C$, by Lemma 1, C should be a diagonal matrix and can be represented by $C = diag\{c_{11}(\theta_1), c_{22}(\theta_2), \cdots, c_{dd}(\theta_d)\}$. It requires that

$$e^{-j\phi_{il}(\theta_k)} c_{kk}(\theta_k) = e^{-j\phi_{jl}(\theta_k)} \quad \text{for} \ \ l = 1, \cdots, p, \tag{8}$$

which can be simplified to

$$\phi_{jl}(\theta_k) - \phi_{il}(\theta_k) = \Phi'_{ij}(\theta_k) + 2\pi n, \quad \text{for} \ \ l = 1, \cdots, p \tag{9}$$

where n can be any integer. The relation in (9) holds for all θ_k in $[0, 360)$ only if $\Delta_{ij1} = \Delta_{ij2} = \cdots = \Delta_{ijp}$ and $\beta_{ij1} = \beta_{ij2} = \cdots = \beta_{ijp}$, i.e. the ith and the jth subarrays must be identical and have the same orientation. Thus, we have Lemma 3:

Lemma 3 *Suppose A_i and A_j are steering matrices associated with the ith and the jth subarrays. The sensors in each subarray are numbered in the same sequence. There exists a mapping relation $A_j = A_i C_{ij}$ if and only if the ith and the jth subarrays are identical and have the same orientation.*

From Lemmas 2 and 3, we have:

Theorem 1 *Suppose an array can be divided into K subarrays, each having a $p \times d$ steering matrix A_i, $i = 1, 2, \cdots, K$. All $A_1, A_2, \cdots, A_K$ can be mapped to a $p \times d$ steering matrix B by $A_i = BD_i$ if and only if all these subarrays are identical and have the same orientation.*

We call the array structure held by an array satisfying conditions in Theorem 1 the orientational invariance structure. A more rigorous definition is given as follows:

Definition 1 (Orientational Invariance Structure) *An array has an orientational invariance structure if it can be divided into subarrays that are identical and have the same orientation.*

For an array with orientational invariance structure, we can consider each subarray as one element located at its first sensor. Then all these elements form a center array. A more rigorous definition for center array is given as follows:

Definition 2 (Center Array) *If an array with orientational invariance structure is divided into subarrays (which can have overlap), then the collection of all the first sensors of these subarrays form a center array.*

B Necessary and Sufficient Conditions

Suppose an array has an orientational invariance structure. Moreover, its center array has an ambiguity free structure and the number of subarrays is larger than or equal to the largest number of mutually coherent signals. The $p \times d$ steering matrices $A_1, A_2, \cdots, A_K$ are associated with the subarrays $1, 2, \cdots, K$, respectively, and d_k is the distance between the first sensor in the first subarray and the first sensor in the kth subarray. The angle β_k represents the direction of the line on which the first sensor in the first subarray and the first sensor in the kth subarray are located (see Fig.1). We have

$$A_k = A_1 D_k, \quad k = 2, \cdots, K \tag{10}$$

where

$$D_k = \begin{bmatrix} e^{-j\frac{2\pi d_k}{\lambda}\sin(\beta_k - \theta_1 + \frac{\pi}{2})} & & & \\ & e^{-j\frac{2\pi d_k}{\lambda}\sin(\beta_k - \theta_2 + \frac{\pi}{2})} & & \\ & & \ddots & \\ & & & e^{-j\frac{2\pi d_k}{\lambda}\sin(\beta_k - \theta_d + \frac{\pi}{2})} \end{bmatrix}. \tag{11}$$

The covariance matrix of the kth subarray is thus given by

$$R_k = A_1 D_k R_s D_k^H A_1^H + \sigma^2 I, \tag{12}$$

where R_s is the covariance matrix of the source. The spatially smoothed covariance matrix is defined as the average of the subarray covariances

$$\bar{R} = \frac{1}{K}\sum_{k=1}^{K} R_k = A_1 \bar{R}_s A_1 + \sigma^2 I, \tag{13}$$

where $\bar{R}_s$ is the modified covariance matrix of the signal given by

$$\bar{R}_s = \frac{1}{K}\sum_{k=1}^{K} D_k R_s D_k^H. \tag{14}$$

We will show in the following that $\bar{R}_s$ is nonsingular. First, $\bar{R}_s$ can be written as

$$\bar{R}_s = [I \ D_2 \ \cdots D_K] \begin{bmatrix} \frac{1}{K}R_s & & & \\ & \frac{1}{K}R_s & & \\ & & \ddots & \\ & & & \frac{1}{K}R_s \end{bmatrix} \begin{bmatrix} I \\ D_2 \\ \vdots \\ D_K \end{bmatrix}. \tag{15}$$

Let C denote the Hermitian square root of $\frac{1}{K}R_s$, i.e.

$$CC^H = \frac{1}{K}R_s. \tag{16}$$

It follows that

$$\bar{R}_s = GG^H \tag{17}$$

where G is a $d \times Kd$ block matrix given by

$$G = [C \ D_2C \ \cdots \ D_KC]. \tag{18}$$

Clearly, the rank of $\bar{R}_s$ is equal to the rank of G. Suppose there are q groups of signals in d incoming signals, with l_i, $i = 1, \cdots, q$, correlated signals in each group. R_s must be a block diagonal matrix with block size l_i, $i = 1, \cdots, q$. We can thus get a corresponding block diagonal matrix C. Recall that the rank of a matrix is unchanged after a change of its columns. By grouping columns of similar elements, we can verify that

$$\rho(G) = \rho \begin{bmatrix} c_{1,1}\mathbf{b}_1 & \cdots & c_{1,l_1}\mathbf{b}_1 & & & & \\ \vdots & \cdots & \vdots & & & & \\ c_{l_1,1}\mathbf{b}_{l_1} & \cdots & c_{l_1,l_1}\mathbf{b}_{l_1} & & & & \\ & & & \ddots & & & \\ & & & & c_{d-l_q+1,d-l_q+1}\mathbf{b}_{d-l_q+1} & \cdots & c_{d-l_q+1,d}\mathbf{b}_{d-l_q+1} \\ & & & & \vdots & \cdots & \vdots \\ & & & & c_{d,d-l_q+1}\mathbf{b}_d & \cdots & c_{d,d}\mathbf{b}_d \end{bmatrix} \tag{19}$$

where ρ is a rank operator, c_{ij} is the ijth element of matrix C, and $\mathbf{b}_i$ $(i = 1, \cdots, d)$ is the $1 \times K$ row vector given by

$$\mathbf{b}_i = [1 \; e^{-j\frac{2\pi d_2}{\lambda} sin(\beta_2 - \theta_i + \frac{\pi}{2})} \; e^{-j\frac{2\pi d_3}{\lambda} sin(\beta_3 - \theta_i + \frac{\pi}{2})} \; \cdots e^{-j\frac{2\pi d_K}{\lambda} sin(\beta_K - \theta_i + \frac{\pi}{2})} \;]. \tag{20}$$

Each row of matrix C has at least one nonzero element because the energy of each signal is nonzero. It is observed that $\mathbf{b}_i$ is the transpose of the steering vector associated with the center array. Since the center array is assumed to have ambiguity free array manifold, when $K \geq \max\{l_1, l_2, \cdots, l_q\}$, all the $\mathbf{b}$ vectors associated with all the signals within a group of coherent signals are thus linearly independent. Therefore, G is of full row rank and the modified covariance matrix $\bar{R_S}$ is of full rank. Otherwise, if the center array is not ambiguity free, or if $K < \max\{l_1, l_2, \cdots, l_q\}$, G cannot be ensured to be of full row rank, and neither can R_s. From Theorem 1 and the proof above, we get the following theorem.

Theorem 2 *SS can be applied to an array of arbitrary geometry to obtain a full rank smoothed signal covariance matrix if and only if an array has an orientational invariance structure, its center array has an ambiguity free structure, and the number of subarrays is larger than or equal to the size of the largest group of coherent signals.*

C Further Improvement

To get a smoothed nonsingular covariance matrix $\bar{R}_s$ by using the SS technique, we need $K \geq max\{l_1, l_2, \cdots, l_q\}$. We can further reduce the number of subarrays by getting another K backward subarrays similar to the case in a linear array [23]. Although, the Forward-Backward Spatial Smoothing (FBSS) [23] can always be applied in a uniformly spaced linear array. For arrays of arbitrary geometry, there is some requirements on the geometry for successful implementation of the backward method. We first give the definition of central symmetry:

Definition 3 (Central Symmetry) *The array is central symmetric if it is identical before and after rotating* 180^0 *about its center of mass.*

If an array is central symmetric, we can get K additional backward subarrays by reversing the order of the subarrays and the order of the sensors within each subarray.

Let $\mathbf{r}_k^b(t)$ denote the complex conjugate of the output of the kth backward subarray for $k = 1, \cdots, K$. We have

$$\mathbf{r}_k^b(t) = A_1 D_k (D_L \mathbf{s}(t))^* + \tilde{\mathbf{n}}^*(t) \tag{21}$$

where $\tilde{\mathbf{n}}(t)$ is an additive white Gaussian noise vector, D_L is a diagonal matrix with the ith diagonal element given by $e^{-j2\pi \frac{d_{Kp}}{\lambda} \sin(\beta_{Kp} - \theta_i + \frac{\pi}{2})}$ and d_{Kp} is the distance between the first sensor in the first forward subarray and the first sensor in the first backward subarray. The angle β_{Kp} represents the direction of the line on which the two sensors are located.

The covariance matrix of the kth backward subarray is given by

$$R_k^b = A_1 D_k R_s^b D_k^H A_1^H + \sigma^2 I \tag{22}$$

with

$$R_s^b = E(D_L^* \mathbf{s}^*(t) \mathbf{s}^T(t) D_L^T) = D_L^* R_s^* D_L^T. \tag{23}$$

Define the spatially smoothed backward subarray covariance matrix $\bar{R}^b$ as the average of these subarray covariance matrices, i.e.,

$$\bar{R}^b = \frac{1}{K} \sum_{k=1}^{K} R_k^b = A_1 \bar{R}_s^b A_1^H + \sigma^2 I, \tag{24}$$

where

$$\bar{R}_s^b = \frac{1}{K} \sum_{k=1}^{K} D_k R_s^b D_k^H, \tag{25}$$

and define the forward/backward smoothed covariance matrix $\tilde{R}$ as the average of $\bar{R}$ in (13) and $\bar{R}^b$, i.e.,

$$\tilde{R} = \frac{\bar{R} + \bar{R}^b}{2} = A_1 \tilde{R}_s A_1^H + \sigma^2 I. \tag{26}$$

It follows that

$$\tilde{R}_s = \frac{\bar{R}_s + \bar{R}_s^b}{2}. \tag{27}$$

We can show, in a similar way as in the case of a linear array [23], that the modified source covariance matrix $\tilde{R}_s$ is nonsingular as long as $2K \geq \max\{l_1, l_2, \cdots, l_q\}$.

III Ambiguity Free Array Structure

To perform SS, we need an ambiguity free center array manifold. Also, to perform MUSIC, we further require ambiguity free subarray manifolds. Ambiguity arises when a steering vector can be expressed as a linear combination of other steering vectors in an array manifold $\mathcal{A}$ [4]. For a uniformly spaced linear array, rank-1 ambiguity [4] cannot be avoided since the DOA's which are "mirror images" with respect to the array line, have the same steering vector. This limits the range of DOA's estimable by a uniformly spaced linear array to within 180^0. Suppose an array has p elements, then rank-p [4] ambiguities cannot be avoided. In this paper, an ambiguity free array manifold of an array of p sensors refers to rank-(p-1) ambiguity free. Generally, to avoid ambiguity, an array used for high-resolution DOA estimation must have a proper structure. An ambiguity free array manifold has been assumed in several papers [5], [17], [15]. Our attempt is to identify all the situations in which ambiguity may arise. One of our guidelines in designing arrays is to avoid these identified ambiguities.

Theorem 3 *In an azimuth only system, the necessary and sufficient condition for an ambiguity free three-sensor array manifold is that all these three sensors are not on one line and that the distance between any two sensors is less than or equal to* $\frac{\lambda}{2}$.

The proof is given in the appendix.

We can see in general that (a) rank-1 ambiguity occurs not only in uniformly spaced linear arrays but also in rectangular arrays with sensors having a uniform spacing of $\frac{\lambda}{2}$ along either x-axis or y-axis, (b) rank-2 ambiguity occurs in an array that consists of two parallelly positioned linear arrays with an identical uniform sensor spacing that is larger than $\frac{\lambda}{2}$, (c) rank-3 ambiguity occurs in an array that consists of three parallelly positioned linear arrays with an identical uniform sensors spacing that is larger than $\frac{\lambda}{2}$, and (d) higher order ambiguity occurs if more than $\lceil\frac{k}{2}\rceil$ sensors are on one line in a k sensor array or if an array consists of m parallelly positioned linear arrays with an identical uniform sensor spacing that is larger than $\frac{\lambda}{2}\lfloor\frac{m}{2}\rfloor$. These situations are shown schematically in Fig.12(a)-(d). In Fig.12 (b) and (c), the angles θ and α satisfy the following constraint:

$$2\pi\frac{d}{\lambda}\sin(\alpha) + 2k\pi = 2\pi\frac{d}{\lambda}\sin(\theta), \quad k \in \{1, 2, \cdots\}. \tag{28}$$

In Fig.12 (d), the angles θ, β and α satisfy the following constraint:

$$2\pi\frac{d}{\lambda}\sin(\alpha)+2k_1\pi=2\pi\frac{d}{\lambda}\sin(\beta)+2k_2\pi=2\pi\frac{d}{\lambda}\sin(\theta), \tag{29}$$

where k_1,$k_2 \in \{1,2,\cdots\}$ and $k_1 \neq k_2$.

To get an ambiguity free array manifold, it is necessary to avoid these identified situations. It is our conjecture, although not yet proved at this stage, that if the spacing between two neighboring sensors of a uniformly spaced rectangle array is $< \frac{\lambda}{2}$, this rectangle array is ambiguity free. The rationale is that it is less likely to have higher order ambiguities [24].

IV Implementation and Experimental Results

A Some Practical Considerations

In practice, we can perform FBSS by setting up a special data matrix. Specifically, for the nth snapshot we set up the data matrix

$$A^H(n)=\begin{bmatrix} u(p,1,n) & \cdots & u(p,k,n) & u^*(1,k,n) & \cdots & u^*(1,1,n) \\ u(p-1,1,n) & \cdots & u(p-1,k,n) & u^*(2,k,n) & \cdots & u^*(2,1,n) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ u(1,1,n) & \cdots & u(1,k,n) & u^*(p,k,n) & \cdots & u^*(p,1,n) \end{bmatrix} \tag{30}$$

where $u(i,j,n)$ denotes the sample taken at the ith sensor of the jth subarray. For the totality of N snapshots, we can define the overall data matrix

$$A^H=\left[A^H(1),A^H(2),\cdots,A^H(N)\right]. \tag{31}$$

It follows that the averaged smoothed correlation matrix $\tilde{R}$ (as defined in (26)) is estimated as follows:

$$\tilde{R}=\frac{1}{2NK}A^H A. \tag{32}$$

As we know, more robust results can be obtained from data domain rather than from covariance domain. We can proceed with MUSIC [20] algorithm or MVDR [7] [20] beamforming algorithm based on A instead of $\tilde{R}$.

We also need to choose an array for applying SS. Such an array should satisfy all conditions aforementioned. An omnidirectional circular array has been a conventional choice for mobile communications [1] [2], and there have been active research efforts to find a pre-processing scheme for the circular array to handle the coherent interference [27]. However, we can see clearly from our discussion that a circular array is not an array with orientational invariance structure. Therefore it does not satisfy the necessary condition for applying SS. This implies that the circular array cannot overcome the coherent interference by using the SS technique.

B Experimental Results

In this section, we present some simulation results on MUSIC algorithm to show the effectiveness of our SS and FBSS. We choose a square array, which has an orientational invariance structure, central symmetriy, and a sensor spacing less than $\frac{\lambda}{2}$.

Example 1: we use a nine-sensor linear array and a nine-sensor square array as shown in Fig.2, both having a spacing of 0.45λ between neighboring sensors. We consider two narrow-band coherent signals with DOA's at 70^0 and 85^0. The SNR is 20 dB. A total of 500 samples ("snapshots") are taken from the array. We use SS as a pre-processing scheme for MUSIC. Fig.3 shows that the DOA's of the two coherent signals are not resolved using a linear array, whereas the square array gives a satisfactory result.

Example 2: We use the square array shown in Fig.2 to receive two coherent signals with DOA's at 40^0 and 50^0. The SNR is 20 dB. A total of 500 samples are taken. We apply SS and FBSS separately. Fig.4 shows that the DOA estimation resolution achieved by a central symmetric array is significantly improved by using the FBSS method.

Example 3: We use a dense square array of sixty-four sensors as shown in Fig.7(b). The array contains 4 subarrays each of 49 sensors. The spacing between two neighboring sensors is 0.45λ. The array receives four groups of different signals: four coherent signals at $20^0, 65^0, 150^0$ and 200^0, three coherent signals at $230^0, 250^0$ and 280^0, two other coherent signals at 30^0 and 300^0 and another at 320^0. The SNR is 20 dB. A total of 500 samples are used. First, we apply FBSS and then apply MUSIC. Simulation results are shown in Fig.5.

We found the results obtained for a nonlinear array in an azimuth-only system remain valid in an azimuth-elevation system. The following is an example.

Example 4: We use the square array shown in Fig.2 to receive two coherent signals, one is at an azimuth of 40^0 and an elevation of 30^0, and the other is at an azimuth of 50^0 and an elevation of 60^0. The SNR is 20dB. The number of samples taken is 500. By using FBSS and MUSIC, we obtain the result in Fig.6.

C Choices of Orientational Invariance Structure

In this section, we like to study some guidelines for designing an optimal sensor array for SS. We found that the sensor utilization rate is an important factor for estimating DOA's of coherent signals with SS.

Definition 4 (Sensor Utilization Rate (SUR))

$$SUR = \frac{\sum n_{subarray}}{n_{array}} \tag{33}$$

where $\sum n_{subarray}$ is the sum of the number of sensors in each subarray, and n_{array} is the total number of sensors in the whole array. Obviously $SUR \geq 1$, because of possible overlap of subarrays.

Emample 5: We perform simulations on two 64-element arrays: (1) a dense square array as shown in Fig.7(b), which has a high SUR for a given number of sensors, and (2) a hollow square array, as shown in Fig.7(a), which has a low SUR. The dense square array contains 4 subarrays each having 49 sensors. The spacing between two neighboring sensors is 0.45λ. The SUR of the array is approximately 3. The hollow square array contains 4 subarrays, each having 32 sensors. The spacing between two neighboring sensors is 0.45λ. The SUR is 2. Both structures are used to estimate the DOA's of two coherent signals. The input SNR is 20dB. We use the FBSS method. The simulation results are shown in Fig.8. In case (a) and (b), the two coherent signals are at 40^0 and 50^0. Both arrays can clearly identify the DOA's. In case (c) and (d), the two coherent signals are at 45^0 and 50^0, only the dense square array can identify the DOA's.

Our results show that dense square array structure is better than the hollow square array structure. Since both arrays have the same number of elements, we infer that the SUR is an important factor and needs to be maximized in the array design.

V Spatial Smoothing for ESPRIT

Similar to MUSIC, the ESPRIT algorithm [5] is an approach to signal parameter estimation. It exploits an underlying data model at significant computational savings. The ESPRIT algorithm is also limited to estimating parameters in noncoherent incoming signals. The conventional SS can be incorporated into ESPRIT [28], but it requires the center array to be a uniformly spaced linear array. In this section, we show that our scheme also works for the ESPRIT algorithm to estimate parameters in a coherent interference environment.

In the ESPRIT algorithm, we consider d narrow-band plane waves with incident angles $\theta_1, \cdots \theta_d$, and wavelength λ, impinge on a planar array of m sensors (m is even), arranged in $\frac{m}{2}$ doublet pairs. The displacement vector is the same for each doublet pair, but the location of each pair is arbitrary. The sensor output $x(t)$ is given by

$$\mathbf{x}(t) = \begin{pmatrix} A \\ A\Phi \end{pmatrix} \mathbf{s}(t) + \mathbf{n}(t) \tag{34}$$

where $\mathbf{n}(\mathbf{t})$ is a white Gaussian noise vector. A and $A\Phi$ are the steering matrices corresponding to the first sensors and the second sensors in all pairs, respectively. The matrix Φ is a diagonal $d \times d$ matrix of phase delays between the doublet sensors for the d signals. The sensor output covariance matrix R_x is thus measured by

$$R_x = \begin{pmatrix} A \\ A\Phi \end{pmatrix} R_s \begin{pmatrix} A \\ A\Phi \end{pmatrix}^H + \sigma^2 I. \tag{35}$$

A full rank matrix R_s is assumed when the ESPRIT algorithm is performed. If some of the incoming signals are coherent, R_s will not be a full rank matrix and the ESPRIT will fail. The spatial smoothing technique we introduced in the previous sections can then be applied here to get a modified full rank signal covariance matrix.

We consider each doublet sensor pair in the array used by ESPRIT algorithm as one element. Then the array consists $\frac{m}{2}$ elements. If this array has an orientational invariance structure with K subarrays and the corresponding center array has an ambiguity free structure, the sensor output at the kth subarray is given by

$$\mathbf{x}_k(t) = \begin{pmatrix} A_1 \\ A_1\Phi \end{pmatrix} D_k \mathbf{s}(t) + \mathbf{n}(t). \tag{36}$$

Matrix D_k is a diagonal $d \times d$ matrix of the phase delays in the form given in (11). The corresponding covariance matrix R_{x_k} is given by

$$R_{x_k} = \begin{pmatrix} A_1 \\ A_1\Phi \end{pmatrix} D_k R_s D_k^H \begin{pmatrix} A_1 \\ A_1\Phi \end{pmatrix}^H + \sigma^2 I. \tag{37}$$

A smoothed output covariance matrix $\bar{R_x}$ can thus be defined as

$$\bar{R_x} = \frac{1}{K}\sum_{i=1}^{K} R_{x_k} = \begin{pmatrix} A_1 \\ A_1\Phi \end{pmatrix} \bar{R_s} \begin{pmatrix} A_1 \\ A_1\Phi \end{pmatrix}^H + \sigma^2 I \tag{38}$$

where $\bar{R_s}$ is the modified signal covariance matrix as defined in (14). As proved in Section II, $\bar{R_s}$ is of full rank if K is larger than or equal to the size of the largest group of coherent signals. We can now successfully perform ESPRIT based on $\bar{R_x}$. We can also use FBSS to further reduce the number of sensors required and to improve the estimation resolution if the array of $\frac{m}{2}$ element is central symmetric.

Although SS enables ESPRIT to estimate DOA's in a coherent interference environment, the estimation is still limited to identifying DOA's within 180^0 in an azimuth only system. Hence, in terms of performance robustness to DOA's, our SS is more effective for MUSIC than for ESPRIT.

Example 6: A twelve sensor array shown in Fig.9 is used in this example to receive two coherent signals at 70^0 and 80^0. This array consists of two overlapping nine-sensor square arrays. Each sensor in one square array and its counterpart in another form a doublet pair. These nine doublet pairs form an array which has orientational invariance structure and is central symmetric. The spacing between two neighboring sensors is 0.45λ. The doublet spacing for ESPRIR is 0.45λ. The SNR is 20dB. A total of 2000 trials are run. A histogram of the results is given in Fig.10. We apply FBSS first and then applied the ESPRIT. The two angles are clearly identified.

VI Conclusions

To significantly improve performance robustness in DOA estimation and in adaptive beamforming, we developed techniques for applying SS on arrays of arbitrary geometry, thus making MUSIC, ESPRIT and adaptive beamformers operative in a coherent interference environment. In order to apply SS to an array of arbitrary geometry, this array must have an orientational invariance structure and its center array must be ambiguity free. Also the number of subarrays must be greater than or equal to the largest number of mutually coherent signals. To apply SS in conjunction with MUSIC, all the subarrays must also be ambiguity free, and the number of sensors in each subarrays must be larger than the number of incoming signals. For ESPRIT, two identical arrays (or subarrays) separated by a displacement vector are used each satisfying the conditions for applying SS and MUSIC.

When a nonlinear array is central symmetric, the FBSS can be used and it outperforms the regular SS in terms of improved efficiency and estimation resolution.

We proved the necessary and sufficient conditions for a three-sensor array manifold to be ambiguity free. We identified several situations, for higher order sensor array manifolds, in which ambiguity may arise. It is necessary to avoid the identified ambiguities in designing ambiguity free center arrays and subarrays.

In practice, we found that we can choose a square array with a sensor spacing less than $\frac{\lambda}{2}$ to meet all the conditions required for applying SS. Simulation results also show that for DOA estimation of coherent signals using SS, a square array has a prefered geometry in terms of the DOA estimation resolution and performance robustness.

VII Appendix

Proof of Theorem 1:

If part:

If sensors A, B and C are not on one line and their mutual distance is less than $\frac{\lambda}{2}$, without loss of generality, we let sensor A be the first sensor in the array, B the second and C the third. The steering matrix of the array has the form

$$V = \begin{bmatrix} 1 & 1 & 1 \\ e^{j\phi_1(\theta_1)} & e^{j\phi_1(\theta_2)} & e^{j\phi_1(\theta_3)} \\ e^{j\phi_2(\theta_1)} & e^{j\phi_2(\theta_2)} & e^{j\phi_2(\theta_3)} \end{bmatrix} \tag{39}$$

where ϕ denotes phase delay. If the distance between any two sensors is $< \frac{\lambda}{2}$, the phase delay $\phi_1(\theta_i)$ and $\phi_2(\theta_i), i = 1, 2, 3$, are real numbers from $(-\pi, \pi)$.

Note that the steering matrix of the array corresponding to three incoming signals at different angles is a special case of the general array in Lemma 2 of [24]. By Lemma 2 in [24], V is nonsingular with possible exception in one of the following three situations:

(1) When $\phi_1(\theta_1)=\phi_1(\theta_2)$, i.e. the two incoming signals are symmetric with respect to the line on which sensors A and B are located. Note that

$$\begin{aligned} det(V) &= \begin{vmatrix} 0 & 1 & 0 \\ e^{j\phi_1(\theta_1)} - e^{j\phi_1(\theta_2)} & e^{j\phi_1(\theta_2)} & e^{j\phi_1(\theta_3)} - e^{j\phi_1(\theta_2)} \\ e^{j\phi_2(\theta_1)} - e^{j\phi_2(\theta_2)} & e^{j\phi_2(\theta_2)} & e^{j\phi_2(\theta_3)} - e^{j\phi_2(\theta_2)} \end{vmatrix} && (40) \\ &= - \begin{vmatrix} e^{j\phi_1(\theta_1)} - e^{j\phi_1(\theta_2)} & e^{j\phi_1(\theta_3)} - e^{j\phi_1(\theta_2)} \\ e^{j\phi_2(\theta_1)} - e^{j\phi_2(\theta_2)} & e^{j\phi_2(\theta_3)} - e^{j\phi_2(\theta_2)} \end{vmatrix}. && (41) \end{aligned}$$

When $\phi_1(\theta_1) = \phi_1(\theta_2)$, $\det(V) = 0$ if and only if $\phi_2(\theta_1) = \phi_2(\theta_2)$ or $\phi_1(\theta_3) = \phi_1(\theta_2)$. Since these sensors are not on one line, if $\phi_1(\theta_1) = \phi_1(\theta_2)$, we have $\phi_2(\theta_1) \neq \phi_2(\theta_2)$. Since θ_1, θ_2 and θ_3 are three different angles, when θ_1 and θ_2 are symmetric with respect to the line, θ_3 and θ_2 can not be symmetric to the line, i.e. if $\phi_1(\theta_1) = \phi_1(\theta_2)$, then we get $\phi_1(\theta_3) \neq \phi_1(\theta_2)$. Thus, when $\phi_1(\theta_1) = \phi_1(\theta_2)$, the matrix V is nonsingular.

(2) Similarly, we can prove that when $\phi_2(\theta_1) = \phi_2(\theta_2)$, the matrix V is nonsingular.

(3) When $\phi_1(\theta_1) - \phi_1(\theta_2) = \phi_2(\theta_1) - \phi_2(\theta_2)$, i.e. $\phi_1(\theta_1) - \phi_2(\theta_1) = \phi_1(\theta_2) - \phi_2(\theta_2)$, θ_1 and θ_2

are symmetric with respect to the line connecting sensors B and C. Note that

$$det(V) = \begin{vmatrix} e^{-j\phi_2(\theta_1)} & e^{-j\phi_2(\theta_2)} & e^{-j\phi_2(\theta_3)} \\ e^{j\phi_1(\theta_1)-j\phi_2(\theta_1)} & e^{j\phi_1(\theta_2)-j\phi_2(\theta_2)} & e^{j\phi_1(\theta_3)-j\phi_2(\theta_3)} \\ 1 & 1 & 1 \end{vmatrix} \tag{42}$$

$$= (e^{-j\phi_2(\theta_1)} - e^{-j\phi_2(\theta_2)})(e^{j\phi_1(\theta_2)-j\phi_2(\theta_2)} - e^{j\phi_1(\theta_3)-j\phi_2(\theta_3)}) \tag{43}$$

When $\phi_1(\theta_1) - \phi_2(\theta_1) = \phi_1(\theta_2) - \phi_2(\theta_2)$, $\det(V) = 0$ if and only if $e^{-j\phi_2(\theta_1)} = e^{-j\phi_2(\theta_2)}$ or $e^{j\phi_1(\theta_2)-j\phi_2(\theta_2)} = e^{j\phi_1(\theta_3)-j\phi_2(\theta_3)}$. Since the mutual distance between A, B and C are less than $\frac{\lambda}{2}$, $\phi_2(\theta_1)$, $\phi_2(\theta_2)$, $\phi_1(\theta_2) - \phi_2(\theta_2)$ and $\phi_1(\theta_3) - \phi_2(\theta_3)$ are all real numbers in $(-\pi, \pi)$. $e^{-j\phi_2(\theta_1)} = e^{-j\phi_2(\theta_2)}$ if and only if $\phi_2(\theta_1) = \phi_2(\theta_2)$. $e^{j\phi_1(\theta_2)-j\phi_2(\theta_2)} = e^{j\phi_1(\theta_3)-j\phi_2(\theta_3)}$ if and only if $\phi_1(\theta_2) - \phi_2(\theta_2) = \phi_1(\theta_3) - \phi_2(\theta_3)$.

Since A, B and C are not on one line, if θ_1 and θ_2 are symmetric to the line connecting B and C, they can not be symmetric to the line connecting A and B or A and C. i.e., if $\phi_1(\theta_1) - \phi_2(\theta_1) = \phi_1(\theta_2) - \phi_2(\theta_2)$, we have $\phi_2(\theta_1) \neq \phi_2(\theta_2)$. Since θ_1, θ_2 and θ_3 are three different incoming angles, if θ_1 and θ_2 are symmetric to the line connecting B and C, θ_2 and θ_3 can not be symmetric to the line. i.e., if $\phi_1(\theta_1) - \phi_2(\theta_1) = \phi_1(\theta_2) - \phi_2(\theta_2)$, we have $\phi_1(\theta_2) - \phi_2(\theta_2) \neq \phi_1(\theta_3) - \phi_2(\theta_3)$. Thus, when $\phi_1(\theta_1) - \phi_2(\theta_1) = \phi_1(\theta_2) - \phi_2(\theta_2)$, the matrix V is nonsingular.

Therefore, we conclude that all the three situations which cause the singularity of the matrix in Lemma 2 of [24] will not cause the singularity of three-sensor steering matrix if three sensors are not on one line and their mutual distance is less than $\frac{\lambda}{2}$. Therefore the matrix V is full rank.

If the spacing between any two of the three sensors is not larger than $\frac{\lambda}{2}$, and there is at least one pair in these three sensors with a spacing of $\frac{\lambda}{2}$, then the only situation that the phase delay $\phi_1(\theta_i)$ and $\phi_2(\theta_i), i = 1, 2, 3$, are not all in $(-\pi, \pi)$ is when one of the incoming signals is from the direction parallel to a line on which the two sensors with spacing $\frac{\lambda}{2}$ are located. The other two signals can be either from the opposite direction or from other directions. If one of the other two signals is from the opposite direction, it can be easily proved that the corresponding steering matrix is full rank. If the other two signals are from the two other different directions, then one of $\phi_n(\theta_i)$, $n = 1, 2$, $i = 1, 2, 3$ is equal to π and the rest are real numbers from $(-\pi, \pi)$. Similarly, we can prove that the matrix V is of full rank.

Only if part:

If the conditions in Theorem 1 are not satisfied, rank-1 or rank-2 ambiguity occurs for some incoming signals. These situations are shown schematically in Fig.11(a)(b). In Fig.11(a), the relation between θ and α is

$$2\pi\frac{d}{\lambda}sin(\theta-\alpha)+k2\pi=2\pi\frac{d}{\lambda}sin(\theta+\alpha) \quad k\in\{1,2,\cdots\}. \tag{44}$$

In Fig.11(b), the relation between θ and α is

$$2\pi\frac{d}{\lambda}sin(\alpha)+k2\pi=2\pi\frac{d}{\lambda}sin(\frac{\pi}{2}-\theta), \quad k\in\{1,2,\cdots\}. \tag{45}$$

Proof of Lemma 1:

If part:

The proof is obvious and is omited.

Only If part:

If $B=AC$, $A=[a(\theta_1),\cdots,a(\theta_d)]$ and $B=[b(\theta_1),\cdots,b(\theta_d)]$ and also assume C is not a diagonal matrix, i.e. it has non-zero element c_{lm} for $l\neq m$, then the steering vector $b(\theta_m)$ is

$$b(\theta_m)=\sum_{i=1}^{d}c_{im}a(\theta_i)=c_{lm}a(\theta_l)+\sum_{i=1,i\neq l}^{d}c_{im}a(\theta_i). \tag{46}$$

This means that $b(\theta_m)$ is a function of variable θ_l, which contradicts to the definition that $b(\theta_m)$ is only a function of θ_m. Thus the assumption that C is a non-diagonal matrix is false. C has to be a diagonal matrix with $c_{ii}=\frac{b(\theta_i)}{a(\theta_i)}$.

Proof of Lemma 2:

If part:

Obviously, B can be any of $\{A_1,A_2,\cdots,A_K\}$.

Only if part:

If each A_i can be mapped to a steering matrix B, by definition there exist C_i, C_j such that $A_i=BC_i$, $A_j=BC_j$. By Lemma 1, C_i is a diagonal matrix. So C_i^{-1} exists and is also a diagonal matrix. We have $A_j=A_iC_i^{-1}C_j$. Let $C_{ij}=C_i^{-1}C_j$, C_{ij} is the product of two diagonal matrices. So C_{ij} is also an diagonal matrix. $A_j=A_iC_{ij}$.

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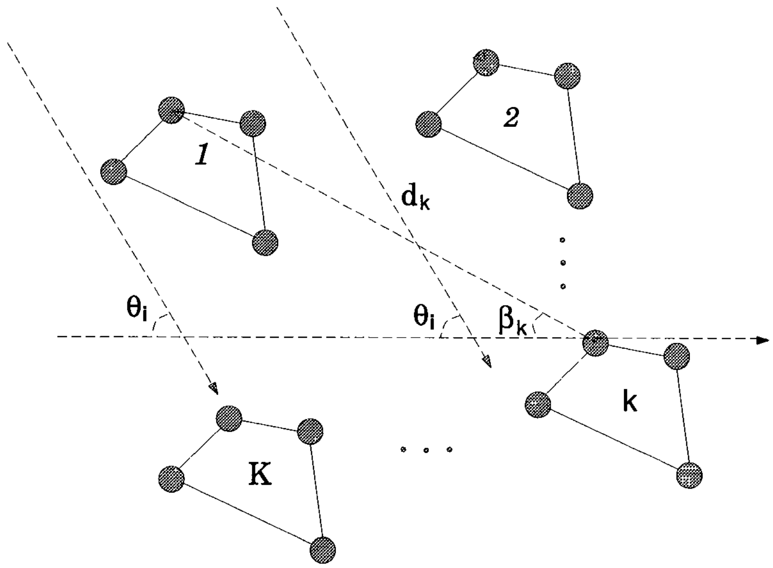


Figure 1: Orientational invariance sensor array geometry

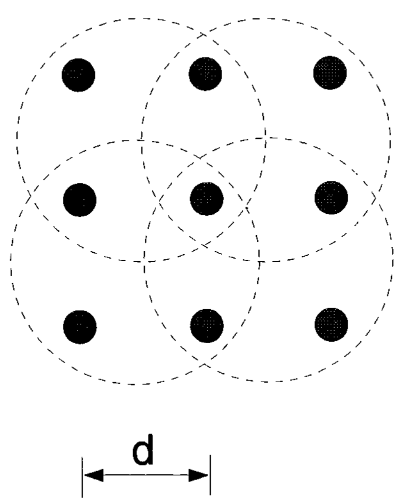


Figure 2: A nine-sensor square array with spacing d

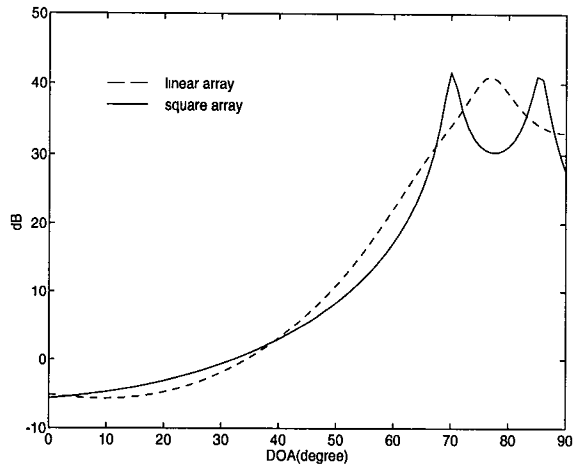


Figure 3: SS and MUSIC for DOA estimation of two coherent signals at 70^0 and 85^0

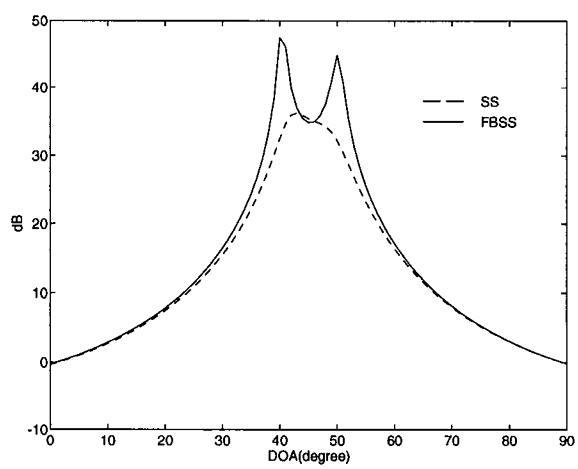


Figure 4: DOA estimation of two coherent signals at 40^0 and 50^0 by using a nine sensor square array

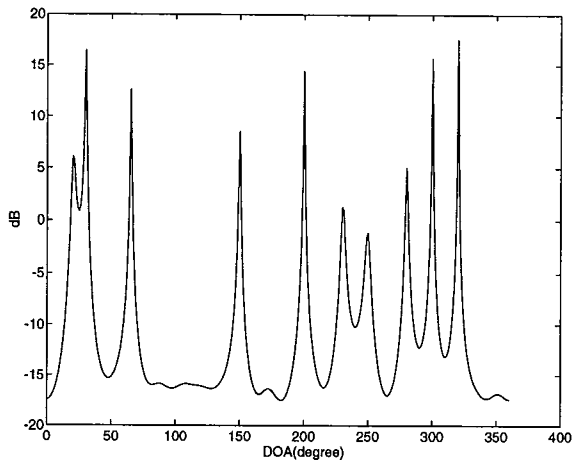


Figure 5: DOA estimation of four groups of coherent signals at (20^0,65^0,150^0,200^0), (230,250^0,280^0), (30^0,300^0) and 320^0 based on a sixty four-sensor square array

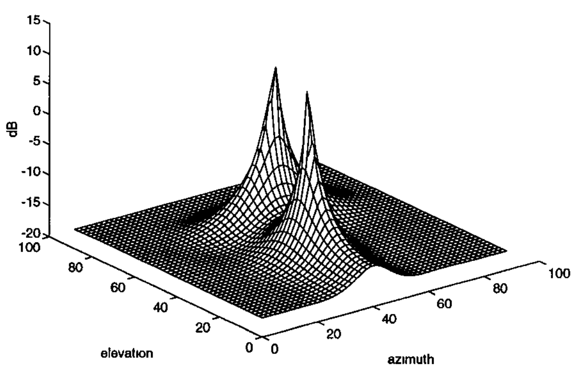


Figure 6: DOA estimation of two coherent signals at an azimuth of 40^0 and an elevation of 30^0, and at an azimuth of 50^0 and an elevation of 60^0, respectively

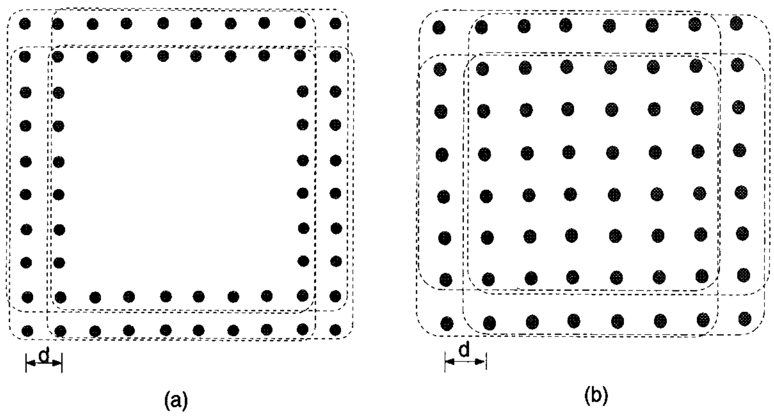


Figure 7: (a) A sixty four-sensor dense square array with four overlapping dense square subarrays of forty nine-sensors (b)A sixty four-sensor hollow square array with four overlapping hollow square subarrays of thirty two-sensors

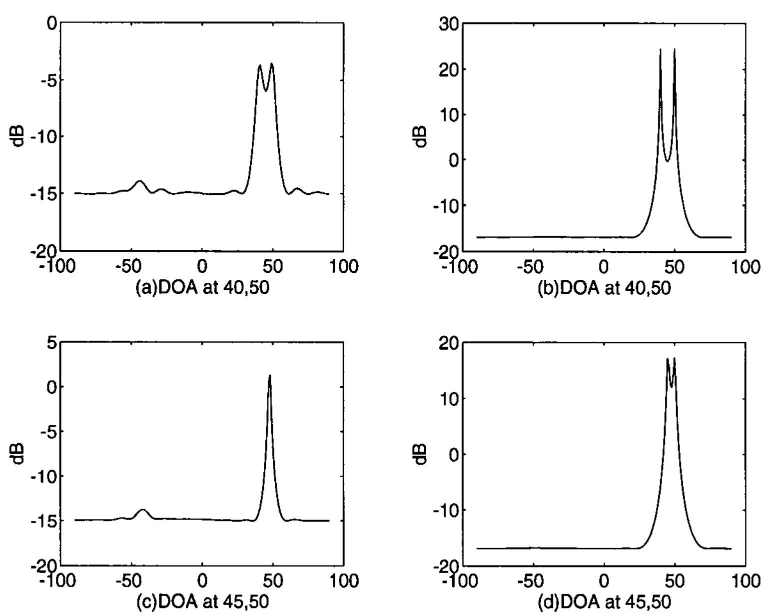


Figure 8: A sixty four-sensor hollow square array is used in (a) and (c), A sixty four-sensor dense square array is used in (b) and (d)

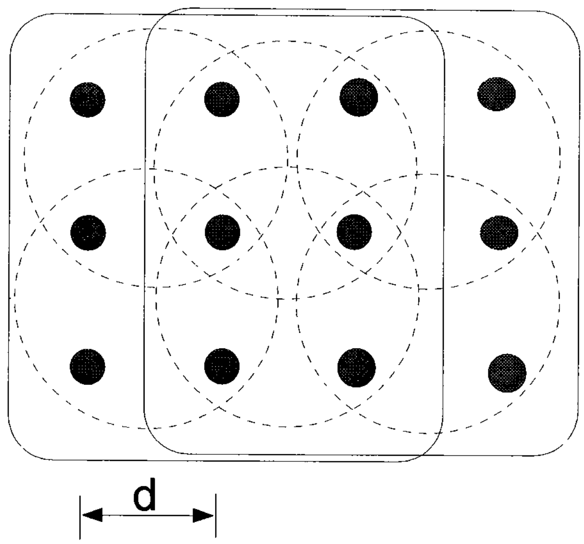


Figure 9: A twelve-sensor rectangle array with spacing d

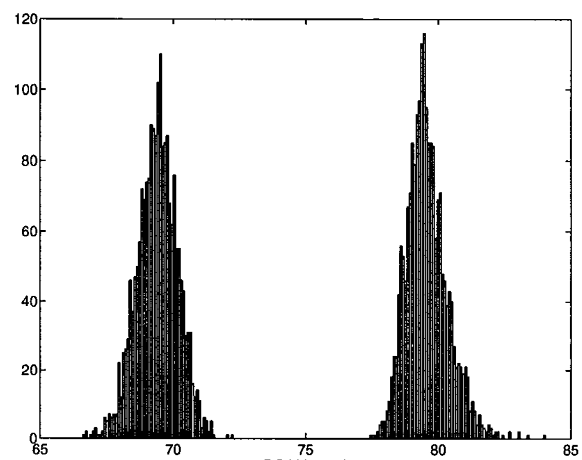
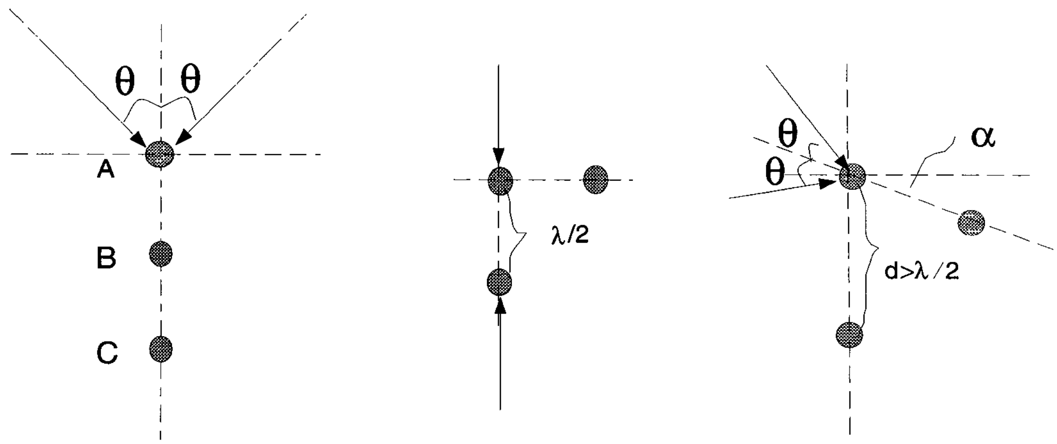
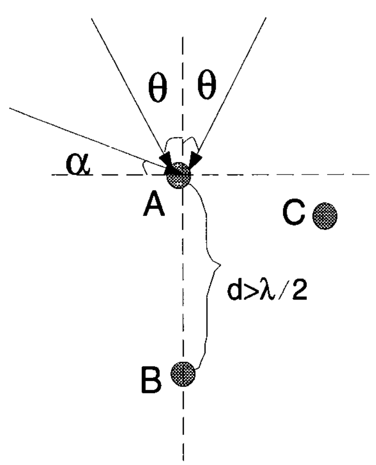


Figure 10: FBSS and ESPRIT for DOA estimation of two coherent signals at 70^0 and 80^0

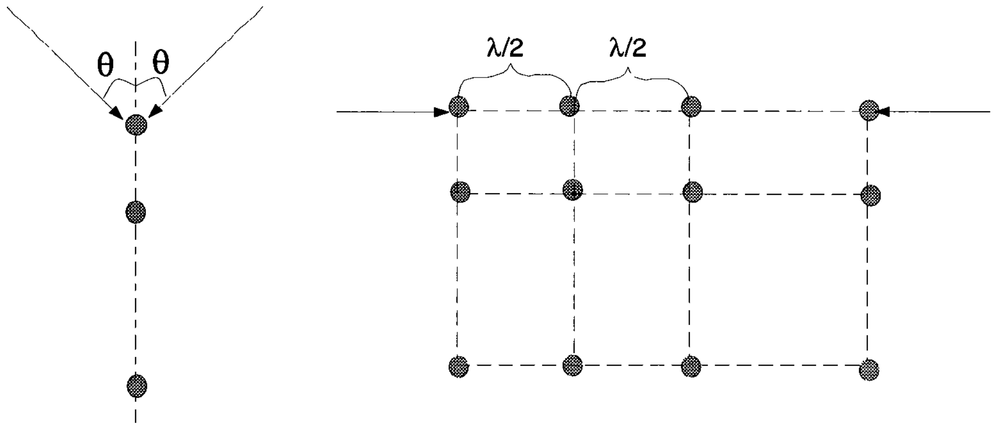


(a) rank-1 ambiguity

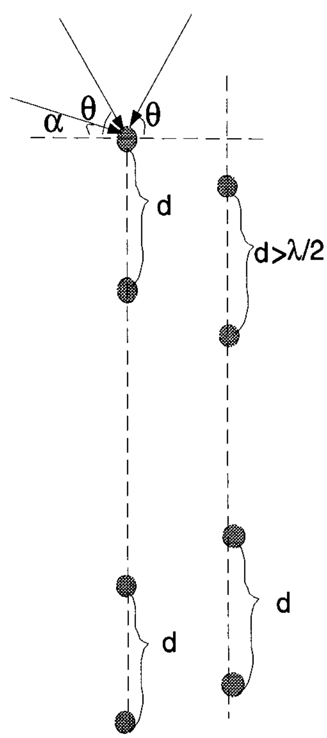


(b) rank-2 ambiguity

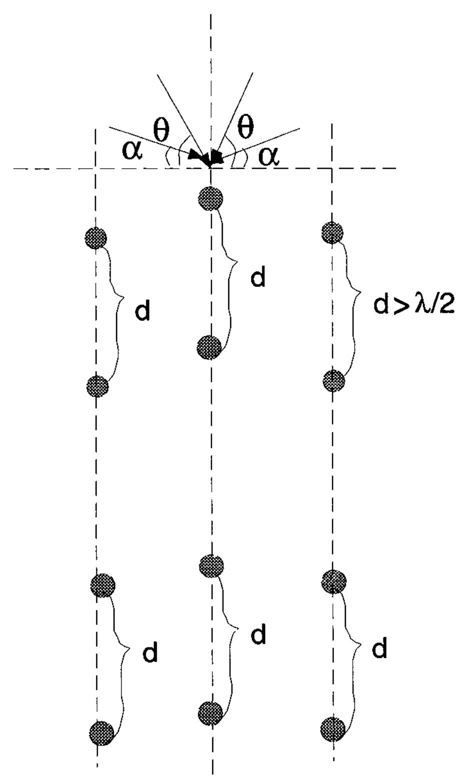
Figure 11: Three-sensor array structures that can cause ambiguities



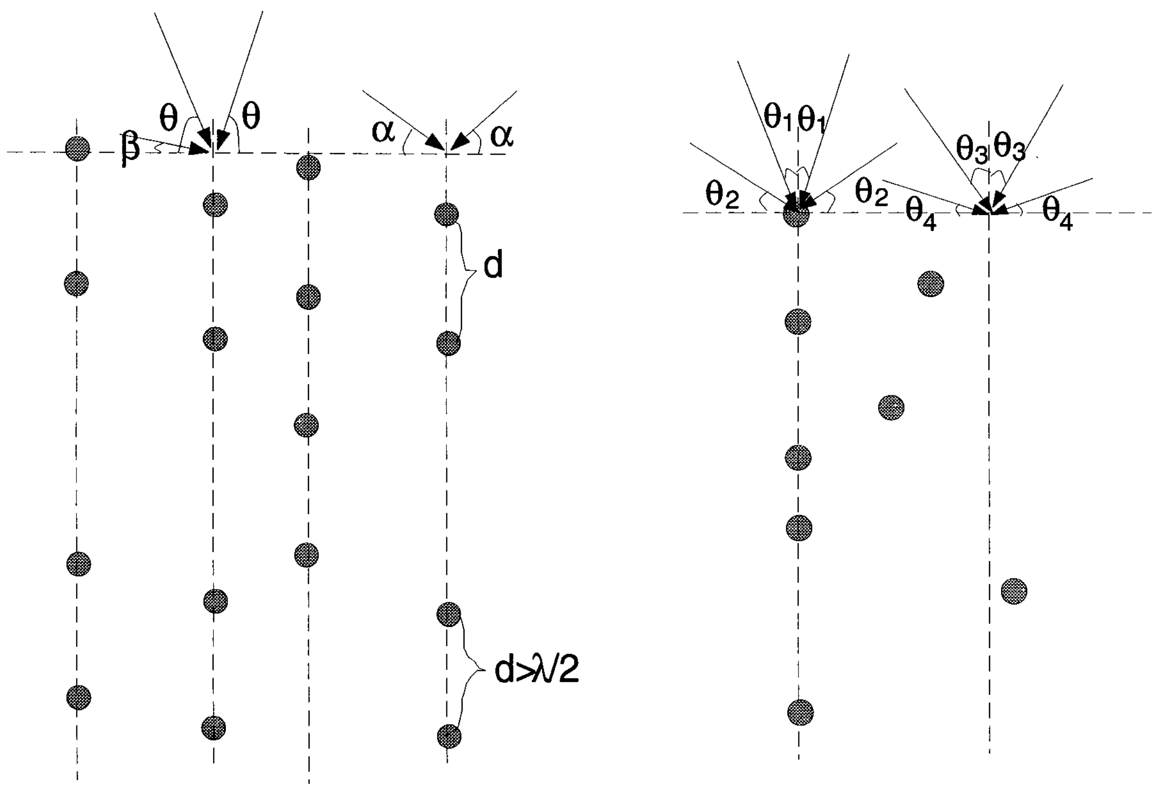
(a) rank-1 ambiguity



(b) rank-2 ambiguity



(c) rank-3 ambiguity



(d) high order ambiguity

Figure 12: high order array structures that can cause ambiguities