ABSTRACT

Dissertation Title: EXPLORING THE CLASSROOM NORMS OF AN UNDERGRADUATE PRECALCULUS COURSE AND THEIR RELATIONSHIP WITH STUDENTS' SELF-EFFICACY, ACHIEVEMENT, AND STEM INTENTIONS: A CONVERGENT MIXED-METHODS STUDY

Sean Gruber, Doctor of Philosophy, 2022

Dissertation directed by: Dr. Andrew Brantlinger, Department of Teaching and Learning, Policy and Leadership

The number of students pursuing a science, technology, engineering, or mathematics (STEM) degree in the United States has continued to decline over the last two decades. These trends are alarming considering the national focus on providing accessible and quality STEM education to underrepresented students, as well as the fact that the number of STEM careers is projected to continue growing over the next decade. Following the nationwide push to retain students and workers in STEM fields within the United States, educational researchers have attempted to explain what goes on within undergraduate STEM classrooms to explain these trends. In so doing, researchers answer the call to analyze the teaching practices of college STEM instructors, particularly mathematics teachers, with the goal of improving instruction and student outcomes. Researchers generally agree that findings from research in K-12 classrooms on practices that engage students in the learning process, including student-centered learning, may be beneficial to students in undergraduate STEM classrooms.

This study followed a convergent mixed-methods design that integrated quantitative and qualitative results in the analytic and results stages. The study utilized survey, interview, and observational data from the Precalculus course offered at Blackboard University (pseudonym) to describe the classroom norms of Precalculus and their predictive power of students' achievement, self-efficacy, and STEM intentions. While evidence suggested some variation by dimensions of teaching considered and the Teaching Assistant (TA) for a discussion section, in general, instructors' perceptions of classroom norms in the large lecture and discussion sections aligned with those of the students. Evidence from participants' survey responses and interview comments suggested that both instructors and students perceived a hybrid of instructor- and student-centered norms in the large lecture and discussion sections, with more instructor-centered norms being perceived in the large lecture and more student-centered norms in the discussion sections.

Hierarchical linear modeling was used to explain differences in students' final exam grades, self-efficacy, and STEM intentions, controlling for the discussion sections students were in. Results suggested that students' perceptions of the norms related to the teaching dimension of *variation in instruction* (e.g., having students explore different solution pathways and representations of problems) in the large lecture predicted an increase in students' final exam grades and self-efficacy. However, norms related to the teaching dimension of *instructor-to-student engagement* (e.g., the instructor and students engaging with each other through asking and answering questions) in the large lecture predicted a decrease in students' final exam grades. With respect to the discussion sections, norms related to the teaching dimension of *instructor-to-student engagement* predicted an increase in both students' final exam grades and self-efficacy. None of the variables considered in this study predicted students' STEM intentions.

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by
Sean Gruber

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Advisory Committee:
Dr. Andrew Brantlinger, Chair
Dr. Lawrence Clark
Dr. Daniel Chazan
Dr. Dana Grosser-Clarkson
Dr. Doron Levy

Dedication

To the love of my life, Lydia - you are my unwavering respite that reminds me of my value and blessings found in the Lord.

To my students - past, current, and future - you are my inspiration to never stop exploring and asking questions about the mystery of mathematics and teaching.

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Chapter 1. Introduction and Aims of Research

The number of students pursuing a science, technology, engineering, or mathematics (STEM) degree in the United States has continued to decline over the last two decades (Chen, 2013; Fayer et al., 2017; Langen & Dekkers, 2005; National Center for Education Statistics, 2017; Seymour & Hewitt, 1997). Moreover, students of color and female students make up a small percentage of STEM bachelor's degrees conferred in the United States (Musu-Gillette et al., 2016; National Science Foundation, 2014). These trends are alarming considering the national focus on providing accessible and quality STEM education to underrepresented students (The White House Office of Science and Technology Policy, 2018), as well as the fact that the number of STEM careers are projected to continue to grow over the next decade (Zilberman & Ice, 2021).

Following the nationwide push to retain students and workers in STEM fields within the United States (U.S. Department of Education, 2018), educational researchers have attempted to explain what goes on within undergraduate STEM classrooms to explain these trends (e.g., Stains et al., 2018). In so doing, researchers answer the call to analyze the teaching practices of college STEM instructors, particularly mathematics teachers (Speer et al., 2010), with the goal of improving instruction and student outcomes. Improving the instruction of undergraduate courses can also, as the American Mathematical Society (1999) argues, help math departments attain the resources and support of their university or college. Thus, prioritizing the improvement of undergraduate courses not only addresses issues of retention but also may lead to "support for the department's highest priorities" (p. 28).

Echoing research on mathematics instruction in K-12 settings (e.g., Boaler & Greeno, 2000), research on undergraduate mathematics instruction has highlighted an over-reliance on traditional (i.e., teacher-centered) instruction, resulting in calls for a shift to student-centered or standards-based instruction and learning (American Mathematical Association of Two-Year Colleges, 2006; *The MAA and the New NCTM Standards*, 2000; The Mathematical Association of America, 2018). Advocates of standards-based learning environments view students as "partners in the learning experience" (American Mathematical Association of Two-Year Colleges, 2006, p. 17). Others have described classroom practices that engage students beyond "the traditional lecture where students passively receive information from the instructor" (Prince, 2004, p. 223) as "reform-oriented" learning (Boaler & Greeno, 2000, p. 172), "active learning" (Freeman et al., 2014, pp. 8413–8414), collaborative learning, and cooperative learning, to name a few (see Prince, 2004 for more on the unique differences between each). Relatedly, other researchers have argued for the use of complex problems to drive discussion, such as in Problem-Based Learning, or PBL (Hmelo-Silver, 2004), in actively engaging students during learning. Regardless of the term used, researchers generally agree that findings from research in K-12 classrooms on practices that engage students in the learning process "can be profitably leveraged and adapted to the university setting" (Rasmussen et al., 2006, p. 91).

For the purposes of this study, I use the term "student-centered" instruction or instructional practices to refer to those "practices in which the students are the sole or key actor(s), including interactions among students in class, students' active and constructive engagement with course content, and formative assessment practices" (Walter et al., 2016, p. 5). According to the Mathematical Association of America's Instructional Practices Guide (2018), student-centered classroom in an undergraduate math classroom may be described as a space in which the following practices are implemented.

Students are…

- Critiquing the ideas of their classmates during peer-to-peer discussion.
- Given the time to work through problems, even if the students struggle, and are asked to reflect on what aspects of the problems proved challenging.
- Engaging in group explorations that ask students to apply familiar concepts to new contexts and representations, while considering key assumptions of problems.
- Explain their thinking to two or three other students through, for instance, the use of whiteboards.
- Asking other students and the instructor questions related to difficulties with the problem/concept at hand and any related misconceptions.

Instructors are…

- Identifying what parts of a student's answer to a question are correct even if the final solution is incorrect.
- Having students complete short formative assessments (e.g., exit tickets) to demonstrate their understanding of a concept.
- Using assignments given before class to inform their instruction, including any adjustments that are needed to address misconceptions and use what students already know.
- Providing students time to respond to questions (i.e., wait time).
- Connecting students' ideas and using them to guide instruction.

In contrast, "instructor-centered practices" are "practices in which the instructor is the sole or primary actor" (Walter et al., 2016, p. 5). While both student-centered and instructor-centered practices may occur within the same classroom (Stains et al., 2018; Walter et al., 2016), an ideas I revisit in later sections, the above descriptions suggest there are, indeed, differences between the two sets of practices.

Succinctly, empirical evidence suggests that the implementation of student-centered practices in undergraduate mathematics courses supports students' conceptual understanding (Freeman et al., 2014) and seems to encourage students to persist and remain in a STEM field (Seymour & Hewitt, 1997; Xu, 2016) A second, yet related strand of research finds a positive correlation between STEM students' achievement in courses and their level of self-efficacy as STEM learners (Bandura, 1977; Peters, 2013; Schunk, 1991). This strand also shows that increased self-efficacy may increase students' interest in pursuing a STEM degree/career (Bandura, 1993; Betz & Hackett, 1983), a result that extends to traditionally underrepresented students in STEM fields (MacPhee et al., 2013).

However, it is important to consider the nuances of the above relationships. First, evidence in support of the relationship between the type of instruction (e.g., student-centered) and student achievement may be weak, at best (see Peters, 2013, p. 472). If such a relationship does exist, student-centered instruction may offer great improvement to student achievement when measured using concept inventory assessments (see Libarkin, 2008 for a description of concept inventories) than when using skill-based assessments or traditional exams written by course instructors (Freeman et al., 2014). Additionally, there is potential for students taught using student-centered instruction to perform worse on problems involving computational skills than students taught using teacher-centered (i.e., instructor-centered) instruction (Bookman & Friedman, 1994).

Second, increasing student-centered practices, such as having students explain their thinking, may actually deter students from switching into a STEM field of study, possibly due to the practices going against students' beliefs about mathematics (see Ellis et al., 2014 for a case in mathematics). Third, one relationship not represented above is the direct relationship between student-centered practices and students' self-efficacy. Indeed, questions remain as to if an increase in student-centered practices in an undergraduate classroom predicts an increase students' self-efficacy. For instance, it may be that instructor-centered practices, not student-centered, are positively related to students' self-efficacy, as instructor-centered practices may match what students experienced in their high school math classrooms (see Peters, 2013).

Lastly, it is important to consider the quality of student-to-student and student-to-instructor interactions that occur within a student-centered classroom. As Langer-Osuna (2017) argues, "our field cannot fully understand (and thereby support) how students author, share, and debate mathematical ideas without taking into consideration how they negotiate relationships of power in the collaborative mathematics classroom" (p. 238). In other words, part of building a classroom community that is built on student-centered practices is reminding students of "behavioral norms and guidelines" that give space for their classmates to also share their ideas and questions so discussions are not dominated by one or two students in a group (The Mathematical Association of America, 2018). Additionally, a classroom climate that is not supportive of traditionally underrepresented STEM students may be predictive of declines in the very skills student-centered instruction promotes, such as the ability to work collaboratively (see also Cabrera & La Nasa, 2005; Cabrera et al., 2001). Thus, the needs of a diverse group of students must be considered when implementing student-centered practices.

Combining the historical push to better support STEM students at the undergraduate level with the above-mentioned nuanced findings, more research is needed to better understand what types of practices can support STEM students. The current study offers one way to advance this understanding.

1.1. Practices Framed as Classroom Norms

The context of this study is an undergraduate precalculus course recently redesigned to implement student-centered practices. The American Mathematical Association of Two-Year Colleges describes "standard-based learning environments," that is, student-centered classrooms, as those in which "students are viewed as partners in the learning experience" (American Mathematical Association of Two-Year Colleges, 2006, p. 17). Additionally, in their 2018

Instructional Practices Guide, the Mathematical Association of America noted that if instructors are to implement student-centered practices, they "will need to create a classroom environment where students feel accountable both as individuals and as members of the classroom community of learners" (p. 10). This framing of a classroom as a larger entity in which instructors and students interact suggest that "it is not sufficient to explain regularities of communication by means of individual routines; interactively constituted structures must be considered as well" (Voigt, 1985, p. 71). Even so, "[the teacher] is seen to express [their] authority in action by initiating, guiding, and organizing the [norms] renegotiation process" (Cobb & Yackel, 1996, p. 178). Thus, to capture the teaching and learning practices in a course redesigned to focus on engaging students during instruction, it was necessary to adopt a framework that considers both what the instructors *and* students are doing in the classroom. Classroom norms is one such framework.

Classroom norms may be used to capture those "regularities in communal or collective classroom activity and are considered to be jointly established by the teacher and students as members of the classroom community" (Cobb & Yackel, 1996, p. 178). In other words, the norms[1] of a classroom entail what the teacher *and* students are doing. This perspective is particularly advantageous when looking at student-centered classrooms given the communal nature of such classrooms (i.e., teacher-to-student and student-to-student interactions). While I provide a more concrete definition of what I considered classroom norms for this study in Chapter Three, examples of norms from past research include students offering an explanation for how they reasoned through a problem or considering different explanations for the same problem (Yackel et al., 2000, p. 276).

[1] While conducted in an elementary school setting, see Levenson (2009) for an interesting study on " teachers' endorsed norms, teachers' and students' enacted norms, and students' perceived norms."

The current study focuses on *both* instructor and students' perceptions of what goes on within an undergraduate math classroom. Considering this focus, I use the phrase 'classroom norms' to refer to the agreed-upon practices of students and instructors that make up the classroom. More specifically, the classroom norms of the course in question were derived from instructor and student responses to a set of survey items that described different practices of the instructors and students. Following the results of factor analysis, these norms were grouped into different dimensions of teaching, such as how the students engaged with one another and how the instructor engaged with the students. To be consistent and clear throughout this paper, I use the phrase 'instructional practices' when referencing past work that used this (or similar) terminology. However, when describing the methodology and results of the current study, I use the phrase classroom norms given the instructor and student perspectives were used to answer the research questions I present below.

1.2. Research Questions

The following research questions were answered within the context of a precalculus course offered at a large, mid-Atlantic, public university, hereon referred to as Blackboard University (BU).

1) To what extent are the classroom norms of the large lecture and discussion sections of an undergraduate precalculus course instructor- or student-centered, as perceived by surveyed students?
2) In comparing surveyed instructors to students, to what extent are the classroom norms perceived as instructor- or student-centered?
3) How do the instructors and students describe the classroom norms of the precalculus course in interviews?
4) What perceived classroom norms of an undergraduate precalculus course are predictive of students': (a) academic achievement in the course, (b) mathematical self-efficacy, and (c) STEM intentions?

1.3. Overview of Research Design

The current study used a convergent mixed-methods approach (Creswell, 2015), with cyclical interactions between the quantitative and qualitative components (QUAN-QUAL-QUAN), to answer the research questions. As a broad overview, I conducted a factor analysis of student survey responses to items that asked students to comment on the extent a provided practice, that is, classroom norm was descriptive of their classrooms. The resulting factors were identified as different dimensions of teaching. While the surveys served as a starting point in the quantitative component of this study, a survey may miss "complex interactions between students and teachers" (Mayer, 1999, p. 33). This limitation is more likely to be the case when considering classroom norms that may not necessarily be made explicit to students, yet are implicitly embedded in classroom practices or culture (Code et al., 2016). Thus, I presented the initial dimensions of teaching and their associated classroom norms to instructors and students during semi-structured interviews to refine the list of norms to those most relevant to either the large lecture or discussion sections of the course (QUAN→QUAL). I also conducted a limited number of classroom observations to guide the interviews and provide contextual examples of what instructors and students referenced, as well as developed a vignette of each of the instructors.

Following the interviews, I retained only those norms relevant to either the large lecture or discussion sections of the precalculus course. I then reran the factor analysis of student responses, resulting in a final set of teaching dimensions with associated classroom norms (QUAL→QUAN). Considering the retained norms were written in such a way that they depicted student-centered practices, as described above, instructor and student survey and interview responses were used to comment on the extent the classroom norms were instructor-centered, student-centered, or a hybrid of the two. Lastly, I used students' factor scores for each dimension of teaching to conduct an HLM regression analysis exploring the relationships between the norms associated with a given dimension and the student outcome variables of student achievement, self-efficacy, and STEM intentions.

1.4. Contributions of Study

Following a thorough review of relevant literature, I address a gap in literature by analyzing the classroom norms of an undergraduate math course in relation to students' achievement, self-efficacy, and STEM intentions. From this research, the field may have a better understanding of how to address the negative trends in undergraduate STEM fields discussed at the outset of this chapter. Moreover, the current study offers a unique contribution to the field of STEM education in its focus on an undergraduate precalculus course, in particular.

Several studies over the past two decades have provided valuable information on the teaching practices within an undergraduate STEM course and their relation to various student outcomes (Freeman et al., 2014; Gasiewski et al., 2012; Jimenez-Soffa, 2006; Seymour & Hewitt, 1997; Stains et al., 2018; Watkins & Mazur, 2013, to name a few). Within the field of mathematics, these studies have focused on a range of mathematical courses (Code et al., 2016; Laursen et al., 2011), such as calculus courses (Bookman & Friedman, 1998; Bressoud et al., 2013; Ellis et al., 2014) and more advanced or logic/proof-based courses (Fukawa-Connelly, 2012; Rasmussen et al., 2006; Stylianou & Blanton, 2002; Weber, 2004). However, my review revealed that only recently (Association of Public and Land-Grant Universities, 2018; Mathematical Association of America, 2019.; McGowen, 2006; Rasmussen et al., 2019) has more specific attention been given to exploring student achievement, self-efficacy, and STEM intentions/persistence in the courses directly preceding calculus, namely, precalculus. Moreover, the trends discussed at the outset of this paper suggest more analysis is needed to support students taking “lower-division mathematics courses” (Smith & Funk, 2021, p. 4) considering undergraduates’ persistence in STEM studies may directly be linked with their achievement (Chen, 2013) and experiences in introductory and developmental math courses (Crisp et al., 2009; Gainen, 1995; Hagerty et al., 2010), such as precalculus. As a result, similar to the view of calculus one as a “gateway” to STEM (Bressoud et al., 2013, p. 697), precalculus students may be filtered from STEM fields early on in their coursework.

Having presented an overview of this study, the remainder of this dissertation is organized into the following sections: a review of the literature including relevant terms, methodologies, and instruments used in past research; a detailed description of the research design and methods of analysis in the current study; the results of the analysis; a discussion of the results and limitations; and finally, a call for future research when considering the practical implications of this study. I begin by reviewing relevant literature used to build the conceptual framework used in answering the research questions.

Chapter 2. Review of Literature and Conceptual Framework

Chapter Two presents a review of relevant literature and contextualizes the key terms used throughout this study. Throughout my review, organized by key term, I provide a detailed description of findings across the literature on the predictive relationships between student-centered and instructor-centered practices or norms in undergraduate STEM classrooms, and students' self-efficacy, achievement, and STEM intentions. I also review how past studies have defined these terms while clarifying the definitions used in the current study. Additionally, I review relevant literature that guided how I measured the different variables discussed. I conclude this chapter with a presentation of the conceptual framework.

In my review of the literature, I first considered studies within the context of undergraduate STEM courses. To conduct a thorough review not limited by a search for the same terms used in this study, other criteria for review included studies that focused on the following: practices of the teacher; practices of students; agreed upon practices of a classroom, that is, classroom norms; student achievement; self-efficacy; and STEM intentions or persistence. Additionally, I reviewed theoretical pieces and instructional guides developed by prominent mathematical associations (e.g., the Mathematical Association of America) that comment on the use of student-centered practices at the undergraduate level, as well as the student outcomes of interest in this study. While I did not place a restriction on the date of the literature, all of the pieces I reviewed were authored between 1976 and 2022.

2.1. Overview of Self-efficacy Research

In discussing self-efficacy of STEM students in undergraduate classrooms, I borrow Bandura's (1977) description of "efficacy expectations," or "the conviction that one can successfully execute the behavior required to produce [certain] outcomes" (p. 193). Connections between students' self-efficacy and their subject-specific skills have been an ongoing topic of research across education and psychology (see Multon et al., 1991 for an in-depth review). Most relevant to this proposal, research suggests that the instructional practices implemented in post-secondary mathematics classrooms may be positively associated with students' confidence in their mathematical abilities (American Mathematical Association of Two-Year Colleges, 2006), including how willing students are to engage with a mathematical task (Code et al., 2016). These findings align with similar arguments within K-12 education (National Council of Teachers of Mathematics, 2000, 2014).

As an example of student-centered practices supporting students' confidence, I highlight Laursen et al.'s (2011) exploration of the effects of inquiry-based learning (IBL) versus non-IBL college math courses on student outcome variables. Succinctly, an IBL course implements several student-centered practices, including students giving "presentations, working in small groups, [and] discussing ideas that generally arose from a group problem or student-presented solution" (p. iii). To clarify, the courses analyzed included first-year math courses (e.g., calculus) and advanced math courses (e.g., number theory), collectively referred to as "math-track" courses. All math-tracked courses had IBL sections, some of which also had non-IBL sections. Additionally, pre-service teacher, math content courses (i.e., courses reviewing mathematical material needed to teach K-12), all of which implemented IBL, were considered in the study. The authors reported that not only did math-track IBL students have statistically significantly higher averages in their self-reported cognitive gains[2] than non-IBL students, but also higher affective gains, including confidence. While the authors' analysis of students' "confidence about doing

[2] Laursen et al. (2011) defined cognitive gains as "growth in understanding mathematical concepts and ideas, in thinking and problem-solving, and in applying knowledge outside [the] classroom" (p. 46). Cognitive gains were measured following students' completion of a learning gains survey.

mathematics" is not equivalent to analyzing students' mathematical self-efficacy, or confidence in their ability to complete specific math problems (see Bandura, 2006), these findings demonstrate the positive effects of more student-centered practices that are a part of undergraduate STEM courses implementing student-centered practices, such as IBL math courses.

2.2. Measuring Self-efficacy

The application of Bandura's (1977) definition of self-efficacy has played a pivotal role in exploring self-efficacy at the undergraduate level. For instance, MacPhee et al. (2013) explored the effects of a STEM mentoring program on traditionally underrepresented STEM students' academic performance, academic self-efficacy, and confidence between their junior and senior years of their STEM major. The authors used the What I am Like Scale (WIAL) survey, a more general self-appraisal instrument measuring students' academic self-efficacy. For example, the instrument may be used to capture to what degree students feel smarter than other students (Neemann & Harter, 2012). Still, MacPhee et al. (2013) stated that the WIAL may not have served as the best choice in instruments to measure academic self-efficacy given that self-efficacy most often pertains to specific contexts and domains (see Pajares & Miller, 1994).

Considering the importance of domain and context when measuring undergraduate students' mathematical self-efficacy, a primary instrument used in several studies (see Kranzler & Pajares, 1997 for a review) is the Mathematics Self-Efficacy Scale*,* or MSES, developed by Betz and Hackett (1983). Pajares and Miller (1995) provide a succinct summary of the MSES and its various subscales:

> The instrument has 52 items and three subscales representing three domains of math-related behavior: solution of math problems, completion of math tasks used in everyday life (e.g., balancing a checkbook, computing income taxes), and satisfactory performance

> in college courses that require knowledge and mastery of mathematics (e.g., calculus, statistics, and biochemistry). (p. 194)

Since its development, researchers have continued to use both the MSES and adaptations of the MSES to measure students' self-efficacy within a variety of math course settings, including developmental algebra and calculus classes (for example, Hall & Ponton, 2005; Peters, 2013). Moreover, Pajares and Miller (1995) used a revised version (see Kranzler & Pajares, 1997, p. 218 for specific changes, including changing the Likert scale from 10 to five points) of the MSES, or the MSES-R, to explore relationships between students' self-efficacy and their pursuit of a math major.

Aside from the MSES-R, Zakariya et al. (2019) asked students how confident they were in their current ability to solve calculus problems from a past final exam, resulting in the Calculus Self-Efficacy Inventory (CSEI). In short, the CSEI asks students to rate their confidence in solving a set of 15 questions on a scale of 0 (not confident at all) to 100 (very confident), with problems ranging in topic and difficulty level. Such a domain-specific approach to measuring self-efficacy is useful given that past exams play a critical role in measuring student achievement within the current course of study (more on this in Chapter Three). Moreover, the authors argue that the CSEI has multiple benefits over the MSES and MSES-R, including, but not limited to, its shorter format and the specific nature of the tasks themselves.

While the CSEI offers one means of measuring self-efficacy in a calculus course, the current study focuses on a precalculus course. After a thorough review of the literature, the only self-efficacy instrument specifically designed for an undergraduate precalculus course was that of Carter (2022). With the help of Dr. Michelle Peters (see Peters, 2013) at the University of Houston-Clear Lake, Carter developed the Pre-calculus Self-efficacy Instrument (PCSEI) using learning outcomes for students taking undergraduate precalculus courses in the state of Texas.

Following an expert-panel review and exploratory factor analysis (EFA), Carter found the PCSEI to be a valid and reliable instrument for measuring self-efficacy within the context of an undergraduate precalculus course. The survey consists of 25 items, with 12 algebra and 13 trigonometry questions. Given the PCSEI is directly applicable to measuring self-efficacy specifically in an undergraduate precalculus course, I revisit the use of the PCSEI in Chapter Three.

2.3. Overview of STEM Intentions Research

Calls to better understand what instructional practices encourage students to pursue STEM degrees has continued to grow (Xu, 2016), especially in the United States (The White House Office of Science and Technology Policy, 2018; U.S. Department of Education, 2018) as the number of students leaving STEM fields continues to increase (Fayer et al., 2017, p. 7). Most relevant to the current study, empirical evidence suggests that students' sense of self-efficacy may predict students' academic and career pursuits (Bandura, 1993; Betz & Hackett, 1981; Schunk, 1991; Wheeler, 1983). As Hall and Ponton (2005) argue, "without confidence in mathematical ability, students' choices of majors, and ultimately their futures, may be limited to nonmathematical areas" (p. 30). Put another way, students' attitudes towards mathematics may provide insight into their sense of self-efficacy (Randhawa et al., 1993), and in turn, students' mathematical self-efficacy may provide insight into their pursuit of a STEM degree/career (Pajares & Miller, 1994). Indeed, mathematical dispositions (Royster et al., 1999) and attitudes towards mathematics (see Aiken, 1976 for a review; Sonnert et al., 2015) have been found to relate to students' STEM intentions.

In considering specific instructional practices that might best support students' pursuit of STEM fields of study, I highlight Bressoud et al.'s (2013) follow-up analysis to their exploration of how the attitudes of college students changed from the beginning to end of a calculus I course, considered to be a "gatekeeper" course to STEM fields (Gasiewski et al., 2012). Results suggested that instructors who engaged in practices such as listening to students, checking their understanding, and going over different methods for solving the same problems (i.e., student-centered practices) appeared to encourage students to continue studying mathematics. In general, empirical evidence suggests implementation of student-centered practices, including engaging students in conceptual discussions and peer-to-peer interactions, may help retain students in a STEM degree/course series (for example, see Watkins & Mazur, 2013; Xu, 2016). Moreover, encouraging students to engage in student-centered practices outside of class, such as forming peer study and support groups, may encourage students to remain in a STEM field of study (Seymour & Hewitt, 1997), including students of color (Palmer et al., 2011).

Most notably, one of the largest studies to explore factors related to STEM retention is that of the Mathematical Association of America (MAA) in their analysis of Characteristics of Successful Programs in College Calculus, or CSPCC (MAA, 2009). Among the 212 colleges and universities across the nation that were surveyed in the CSPCC study, 17 were deemed successful in terms of retention, student achievement, and "productive[3] disposition" towards mathematics. In particular, Bressoud and Rasmussen (2015) highlighted that five of these successful institutions implemented "student-centered pedagogies and active-learning strategies" (Bressoud & Rasmussen, 2015, p. 145). The term "active learning" aligns with the definition of student-centered practices used in the current study: "Active learning engages students in the process of learning through activities and/or discussion in class, as opposed to passively listening to an expert. It emphasizes higher-order thinking and often involves group work" (Freeman et

[3] Productive disposition is defined as "habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy" (National Research Council, 2001, p. 5).

al., 2014, pp. 8413–8414). The MAA has further argued for the implementation of evidence-based instructional practices (i.e., student-centered practices) in their recent follow-up study, Progress through Calculus[4] (MAA, 2015-2019), analyzing the progress institutions have made in implementing the characteristics of successful programs highlighted in the CSPCC study (see also Johnson et al., 2022).

Before continuing to the next section on how studies have measured students' STEM intentions, I acknowledge that understanding students' STEM intentions, as is the aim of the current study, and their actual persistence in attaining a STEM degree (i.e., retention) are two different endeavors (see the seminal work of Seymour & Hewitt, 1997 exploring the reasons students do not persist in earning a degree in the sciences). Indeed, many studies have measured students' persistence in attaining a STEM degree (Crisp et al., 2009; Green & Sanderson, 2018; Maltese & Tai, 2011; Mau, 2016) through comparing STEM intentions with STEM degrees earned (see Daempfle, 2003 and; Gainen, 1995 for a larger discussion on STEM attrition). The current study may inform the field by analyzing students' STEM intentions when considering the classroom norms of an undergraduate math course.

2.4. Measuring STEM Intentions

In general, empirical evidence suggests that interest in STEM is often tied to attaining a STEM degree (Green & Sanderson, 2018, p. 81). Thus, I reviewed literature that explored students' interests in attaining a STEM degree as a means of understanding how to measure undergraduates' STEM intentions (e.g., Lent et al., 2001). In particular, the work of Lin et al. (2018) exploring college students' STEM interest was relevant to the current study given their

[4] Part of this analysis included the administration of the Postsecondary Instructional Practices Survey for Mathematics (PIPS-M) that is also used in the current study.

simultaneous focus on self-efficacy. As seen in the following description, the authors asked students to comment on their interest in majoring in a STEM field.

> Participants were asked to indicate their interest in STEM subjects on a scale from 1 (Strongly Dislike) to 5 (Strongly Like). The list of STEM subjects included Statistics, Chemistry, Physics, Basic Math, Computer Science, Biology, Advanced Math, and Engineering. Higher scores indicate that participants have higher interest in STEM subjects. (p. 5)

Still, the work of Apkarian et al. (2019) stood out in my review given the alignment between their focus on instructor and student-centered practices and the focus of these practices in the current study. As a mathematics-specific version of the original Postsecondary Instructional Practices Survey, or PIPS (Walter et al., 2016), Apkarian et al. developed the PIPS-M as a means to characterize instructors' teaching practices. One characterization included considering instructors' teaching practices along the range of being more instructor-centered to student-centered. Additionally, the student version of the survey, or SPIPS-M, includes an item that explicitly asks students about their STEM intentions. Not only does this item offer direct insight into students' STEM intentions, similar to the question presented from Lin et al. (2018), but the PIPS-M/SPIPS-M is currently being used in collecting data around the redesign of the precalculus course for the current study. For this reason, I offer a more detailed description of the PIPS-M/SPIPS-M in Chapter Three.

2.5. Overview of Academic Achievement Research

Another aim of this study is to analyze the relationship between the norms of a college math classroom and student achievement in the class. Some evidence suggests that in comparison to teacher-centered practices, student-centered practices may better support students in passing STEM courses (Freeman et al., 2014, p. 8410), helping students attain greater conceptual understanding of material on assessments (Rasmussen et al., 2006), and close achievement gaps between initially low- and high-achieving students (Kogan & Laursen, 2014). For example, consider the work of Kuh et al. (2008) in better understanding how to support college students' success through analyzing institutional practices and conditions, as well as student behaviors. Results suggested that students being asked questions in class, contributing to class discussions, and discussing class content with people outside of the class, all of which are student-centered practices, may help them compensate for initially low achievement levels when entering college. In other words, while having students actively participate in class and engage with self-assessment may lead to increased student performance (American Mathematical Association of Two-Year Colleges, 2006), actively engaging students during instruction may be most effective in helping those students who already are struggling in their courses (Bressoud, 2018, section "Efforts to change").

Building on the self-efficacy literature reviewed in an earlier section, the relationship between students' self-efficacy and their achievement in undergraduate math courses has been a heavily researched topic (Betz & Hackett, 1983; Hall & Ponton, 2005; MacPhee et al., 2013; Pajares & Miller, 1994; Peters, 2013). Empirical evidence and theoretical arguments suggest a positive relationship between students' achievement and self-efficacy (Bandura, 1977; Schunk, 1991), including confidence in their mathematical abilities (Peters, 2013, p. 2). Moreover, college students' ability to self-regulate their learning may predict an increase in students' conceptual understanding of complex ideas (Azevedo & Cromley, 2004) and grades (Wadsworth et al., 2007), although not always (Cho & Heron, 2015).

It is worth noting that the potentially predictive relationship between students' self-efficacy and achievement extends to research beyond STEM[5] courses. In their comprehensive and complex meta-analysis, Multon et al. (1991) analyzed 38 samples from 36 different studies, in which 28.9% of the 4,998 sample was made up of college students, to explore potential relationships between self-efficacy and academic outcomes. The authors found that "the relationship of self-efficacy to performance… may vary across types of students, measures, and study characteristics" (p. 34). However, the authors highlighted the stronger relationship between self-efficacy and performance for low-achieving students. This finding underscores the role that instructional practices supportive of students' self-efficacy may play in helping low-achieving students perform well in their classes.

2.6. Measuring Academic Achievement

Most frequently, researchers use students' grades on exams, final course grades, or GPA as the primary sources for measuring student performance (Chen, 2013; MacPhee et al., 2013; Peters, 2013). Final exams serve as a useful and valid means for measuring student achievement for the current study, a point I return to in more depth in Chapter Three.

2.7. Nuances of Self-efficacy, STEM Intentions, and Academic Achievement

As emphasized in Chapter One, questions persist about *who* benefits from certain teaching and learning practices when considering students' self-efficacy, STEM intentions, and academic achievement. For example, findings from Sonnert et al. (2015) suggested student-centered practices were "more beneficial (in terms of influencing students' mathematics attitudes) for students with initially more positive attitudes than for students with initially more negative attitudes" (p. 19). Similar findings were highlighted by Laursen et al. (2011). Moreover,

[5] One of these studies includes the work of Betz and Hackett (1983) cited earlier in the discussion of self-efficacy research in STEM.

students' confidence in and attitude towards STEM as a subject are influenced by many factors (Aiken, 1976; Cabrera & La Nasa, 2005) aside from whether or not a classroom is, for instance, more student-centered or instructor-centered. Such factors include gender and learning preferences (Middleton et al., 2013), as well as if students are STEM majors to begin with (Ellis et al., 2014). Research also suggests that student engagement in peer study groups and STEM clubs on campus, that is, factors outside the classroom, may positively predict traditionally underrepresented students' self-efficacy and pursuit of a STEM degree (Palmer et al., 2011).

In addition, many students in higher education come to mathematics classrooms with preferred teaching math practices (Hativa & Birenbaum, 2000; Middleton et al., 2013) and pre-formed ideas of what learning mathematics looks like (Bookman & Friedman, 1998; Ellis et al., 2014; Sonnert et al., 2015), agreeing with studies taking place in primary and secondary school settings (Jansen & Herbel-Eisenmann, 2001; Schoenfeld, 1989; Star et al., 2008). For example, in her study examining potential relationships between the classroom climate, self-efficacy, and achievement for undergraduate students taking a college algebra course, Peters (2013) found that "classroom climates leaning more towards teacher-centered teaching styles [tended] to have students with higher mathematics self-efficacy," which may have been explained by these students' familiarity with teacher-centered (i.e., instructor-centered) practices due to high-stakes testing in high school (p. 474). Additionally, students' preferences in particular instructional practices may be more so influenced by a teacher's clarity in instruction, as well as how "interesting" the teacher is, rather than whether or not the teacher makes a classroom more student-centered or not (Gasiewski et al., 2012; Hativa & Birenbaum, 2000).

Despite the above nuanced findings, a review of literature suggested that student-centered practices may provide a potential avenue for supporting students' achievement, self-efficacy, and

STEM intentions. More specifically, researchers[6] across the field of mathematics education have emphasized the need to build on both "the conceptual and cultural knowledge that students bring with them to the classroom" (National Research Council: Commission on Behavioral and Social Sciences and Education, 2004, p. 134). At the undergraduate level, this emphasis may be implemented through student-centered practices that actively engaging students in the learning process (The Mathematical Association of America, 2018). In turn, implementing these practices may address differences in achievement with respect to socioeconomic status, race, and ethnicity (American Mathematical Association of Two-Year Colleges, 2006). Succinctly, the following points represent common themes across STEM literature on student-centered practices.

- To support student learning within STEM classrooms, teachers must consider issues of race (Middleton et al., 2013), gender (Cabrera et al., 2001; Jimenez-Soffa, 2006; Kogan & Laursen, 2014), identity (Boaler & Greeno, 2000), and precollege factors, such as parental education (Terenzini et al., 1995).
- From a practical standpoint, teachers can support a diverse group of students by "setting the classroom norms and engagement expectations to incorporate student-centered learning on the first day of class" (The Mathematical Association of America, 2018, p. 11).
- The call for student-centered practices at the undergraduate level align with arguments at the K-12 level pushing for students to "take on forms of intellectual authority that fuel the collaborative mathematics classroom" (Langer-Osuna, 2017, p. 238).

2.8. Research on Practices and Norms of Undergraduate Classrooms

Characterizing a classroom by the teaching and learning practices, as well as interactions between students and the instructor, has been collectively framed in many ways, including the classroom climate (Franke et al., 2007; Peters, 2013), culture (Çakır & Akkoç, 2020), or environment (Boaler, 1998). For instance, in reflecting on her own experience as a fifth grade teacher, Lampert (2001) described the role of implementing "routines" to help students learn

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[6] See also the National Council of Teachers of Mathematics' *Principles to Actions* (2014, p. 4).

mathematics while considering "students bring interpretations of how to use what they find in the classroom and norms of social interaction from diverse family backgrounds and different school experiences" (p. 95). As another example, the authors of the seminal book, *How People Learn* (National Research Council: Commission on Behavioral and Social Sciences and Education, 2004), describe "learner-centered environments" to "include teachers who are aware that learners construct their own meanings, beginning with the beliefs, understandings, and cultural practices they bring to the classroom" (p. 36). The authors present classrooms as a type of "community"[7] that, depending on the classroom and school, "reflect different sets of norms and expectations" (p. 145).

Regardless of the term used to describe what instructors and students are doing in the classroom, there may be an intrinsic difficulty in distinguishing between the contributions of teachers and students, as Lampert (2001) suggested in her description of roles in the classroom.

> The distinction between the people in the classroom and the physical tools they use, like furniture and books, is not a clear one. The teacher uses herself and her students to get tasks of teaching accomplished, and students use one another and the teacher to study. Depending on how a class is structured, students may also play the role of teacher and use the tools and tasks of teaching. (p. 95)

From this perspective, it may be difficult to identify who initiates a particular practice, especially within a classroom focused on implementing student-centered practices. This difficulty points to a potential benefit in considering the practices implemented in the classroom, whether they be initiated by the instructor or students, in terms of classroom norms. More specifically, classroom norms are "jointly established" (Cobb & Yackel, 1996, p. 178). Thus, the framing of teaching and learning practices as classroom norms considers not only the experience of the initiator of a certain practice, but that of the respondent to the practice.

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[7] While not explicit to classrooms, see also Wenger (2000) description of "communities of practices."

As an example, consider the student-centered practice of an instructor asking students to explain their reasoning. While this practice may communicate an action and expectation of the teacher (Çakır & Akkoç, 2020), it is left to the students to engage with this expectation by explaining their reasoning. Particularly in a student-centered classroom, it may be that explaining one's reasoning is not an explicit expectation spoken by the instructor but is built into assignments, such as group quizzes. Thus, students may still engage in explaining their reasoning without the explicit initiation of the teacher asking students to do this. In addition, students may decide to engage in explaining their thinking through requesting to work alone on a quiz (see Boaler & Greeno, 2000, p. 172 on the inclusion of individual work when considering social practices), thus engaging with this practice but not with others. While not necessarily the only means, it may be advantageous to theoretically frame what goes on within the classroom as classroom norms to account for these more nuanced interactions given norms consider both the role of instructors *and* students. From a practical perspective, the data for this study is primarily drawn from surveys and interviews collected from both instructors and students. This data allowed for an analysis of practices as agreed upon by both instructors and students, that is, the classroom norms of the precalculus course in question.

2.9. Measuring Classroom Norms

Characterizing the norms of a classroom was essential in answering the research questions guiding this study. As argued above, this characterization entails analyzing the practices of both instructors and students in the classroom. Put another way, as Yackel et al. (2000) described in their analysis of social and sociomathematical[8] norms of an undergraduate differential equations class, "while it is the teacher who typically initiates the constitution of

[8] Cobb and Yackel (1996) referred to norms specific to mathematical thinking, explaining, and problem solving as "sociomathematical norms."

norms, all participants in the interaction contribute to their ongoing negotiation" (p. 281). As alluded to above, this characterization can be difficult when considering norms "are seldom taught explicitly but are nonetheless experienced by the students" (Code et al., 2016, p. 917). In my review of the literature, while not[9] all studies framed what instructors and students do in the classroom as "classroom norms," some methodologies and frameworks offered a starting point in considering how to answer the current research questions from this perspective.

As an example, Peters (2013) analyzed the "classroom climate," or "the learning environment that the instructor creates by teaching in a teacher-centered (TC) or learner-centered (LC) manner," of instructors for college algebra courses across the United States (p. 461). More specifically, Peters identified predictive relationships between students' mathematical self-efficacy (measured via the MSES-R), achievement (course final exam), gender, and classroom climate (teacher-reported survey data). She then generated hierarchical linear models (HLM) to predict the relationship between more teacher- or student-centered climates, and students' self-efficacy and achievement. Peters' (2013) focus on self-efficacy and achievement as they relate to the practices of an instructor being more teacher- or learner-centered overlaps somewhat with the aims of the current study in considering the agreed upon instructor and student-centered classroom norms in an undergraduate precalculus course. Thus, Peters' HLM analysis was informative in building the models used to answer the research questions posed here. However, in considering the current study's focus on classroom norms that are "jointly established by the teacher and students as members of the classroom community" (Cobb & Yackel, 1996, p. 178), I also reviewed literature that considered the perspectives of both instructors and students in STEM undergraduate classrooms.

[9] See my criteria for the literature review at the beginning of Chapter Two.

In my review, few studies provided such an explicit framework for measuring both instructors and students' contributions to classroom norms as that of Çakır and Akkoç (2020). Most relevant to the current study, Çakır & Akkoç defined a classroom norm as "a collective notion and refers to what is taken-as-shared by a group" (p. 22). Moreover, the authors' followed the same theoretical stance of this proposal, underscoring the negotiation and renegotiation of norms from the perspective of both instructor and students. The authors considered "both [the] expectations and actions" of instructors and students when suggesting evidence for a particular norm (p. 23). To summarize, Çakır and Akkoç (2020) analyzed 43 video-recorded lessons to identify the presence or absence of a pre-defined set of sociomathematical norms (see p. 22) within a private, fifth-grade, gifted and talented math classroom. More specifically, the authors took "both expectations (together with awareness on students' sides) and actions (e.g., posing problems) as evidence of" a sociomathematical norm (p. 25).

Conceptually, Çakır & Akkoç (2020) considered two dimensions when identifying a norm: "The teacher dimension is concerned with teachers' expectations and actions, while the student dimension refers to actions, an awareness of teachers' expectations and the students' expectations from the teacher" (p. 23). Consider the following example to clarify the application of these dimensions.

> Concerning expectations, an example would be a teacher's or students' expectations and awareness of these expectations about generating problems. An action would be a teacher's assignment of a PP [problem posing] task or students' initiation of generating problems. Therefore, posed problems in a classroom could be considered as evidence of SMNs [sociomathematical norms] related to PP. (p. 23)

In other words, the authors analyzed classroom recordings, a teacher interview, and student-generated problems to check for the following components that served as evidence of a classroom norm: 1) the teacher *enacted* the norm and also *expected* students to practice the norm;

and 2) the students were *aware* of the teacher's expectation surrounding the norm, the students *enacted* the norm, and the students *expected* the teacher to also enact the norm (note that the awareness component is not present in the teacher dimension).

As an example, consider the classroom norm of reformulating problems. Çakır and Akkoç's (2020) framed the "expectation" component of this norm as the teacher expectation[10] that students reformulate problems and the student expectation that the teacher will initiate reformulations. Awareness of this norm, a component specific only to students, came in the form of students suggesting to reformulate a problem. However, only after students reformulated a problem (i.e., giving actual suggestions for how to change the problem) did student awareness of the norm move to an "action" of the norm. Çakır and Akkoç described the difference between actions and awareness when considering if a sociomathematical (or classroom norm, for the purposes of this study) norm was "revealed" through an "act" that was "evident beyond awareness" (p. 26). In other words, students' awareness of a particular classroom norm was represented by a cognizant noticing of a norm that may or may not result in an enactment of the norm. Similarly, the authors saw the teacher engaging with the students' suggestions or offering suggestions himself as evidence of the norm in action from the teacher's perspective. Given the practical framework offered by Çakır & Akkoç (2020) in measuring classroom norms, I return to a discussion on applying their framework to the current study in Chapter Three.

In the above sections, I have reviewed relevant literature to display the complexities in making sense of students' self-efficacy, achievement, and STEM intentions within the context of an undergraduate math classroom. In particular, I have focused on literature analyzing student-centered versus instructor-centered practices, in line with recent foci of undergraduate STEM

[10] Note that other sociomathematical norms identified by the authors were framed using the word "should," which may be interpreted as an expectation.

research. Additionally, I have reviewed empirical studies and theoretical arguments on how to measure students' mathematical self-efficacy, STEM intentions, academic achievement, and classroom norms, while highlighting the complexities of this research. Building on this literature, I now turn to clarifying the key terms used in the overarching conceptual framework of this study.

2.10. Clarifying Key Terms

Given the large number of terms discussed above and the varying definitions used across the literature, in the paragraphs below, I provide a condensed review of definitions of key terms used throughout this study.

2.10.1. Classroom Norms

To begin, thus far I have used the terms instructor- and student-centered practices, and related terms, to refer to who is doing the mathematical problem-solving, thinking, and communicating in the classroom. As mentioned at the outset of this paper, using the terms 'practices' or 'instructional practices' was necessary in describing past studies that have used this language. Still, the current study considered the joint perspectives of instructors and students on these instructional practices as classroom norms (Cobb & Yackel, 1996). In other words, I discuss the results from the current study in terms of the agreed upon practices of the classroom from the perspective of *both* instructors and students that, in particular, may be more instructor and student-centered in nature. Succinctly, I borrow Cobb and Yackel's (1996) definition of "classroom social norms" to define what I mean by classroom norms, that is, "regularities in communal or collective classroom activity and are considered to be jointly established by the teacher and students as members of the classroom community" (p. 178). These classroom norms include what students and teachers *both* come to expect, act on, and are aware of (Çakır & Akkoç, 2020).

To clarify, researchers have made a distinction between classroom[11] social norms and "sociomathematical norms," or "normative aspects of mathematical discussions specific to students' mathematical activity" (Yackel & Cobb, 1996, p. 461). Examples of sociomathematical norms would be what characterizes a mathematical solution to be different from another or what is considered an efficient solution. While warranted for future research, for the purposes the current study, I did not[12] explore sociomathematical norms and instead focused only on classroom social norms. Using the previous examples to distinguish between the two terms, while a sociomathematical norm would involve what defines a "different" solution in a math classroom, a classroom norm involves the extent to which alternative solutions are explored. I provide a more detailed list of the classroom norms that stemmed from the survey given to this study's participants in Chapter Three.

2.10.2. Self-Efficacy

I borrow Bandura's (1977) description of "efficacy expectations," or "the conviction that one can successfully execute the behavior required to produce [certain] outcomes" (p. 193), to define what I mean by students' self-efficacy.

2.10.3. Academic Achievement

I follow the example of past research in using students' performance on the final exam of the precalculus course in question to define what I mean by achievement (see, Peters, 2013, for example).

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[11] To be succinct, I hereon use the term classroom norms as opposed to classroom social norms.
[12] In addition, the specific mathematical content discussed (e.g., how to graph cubic functions) was not considered a classroom norm as such an analysis of class curriculum was beyond the scope of this study.

2.10.4. STEM Intentions

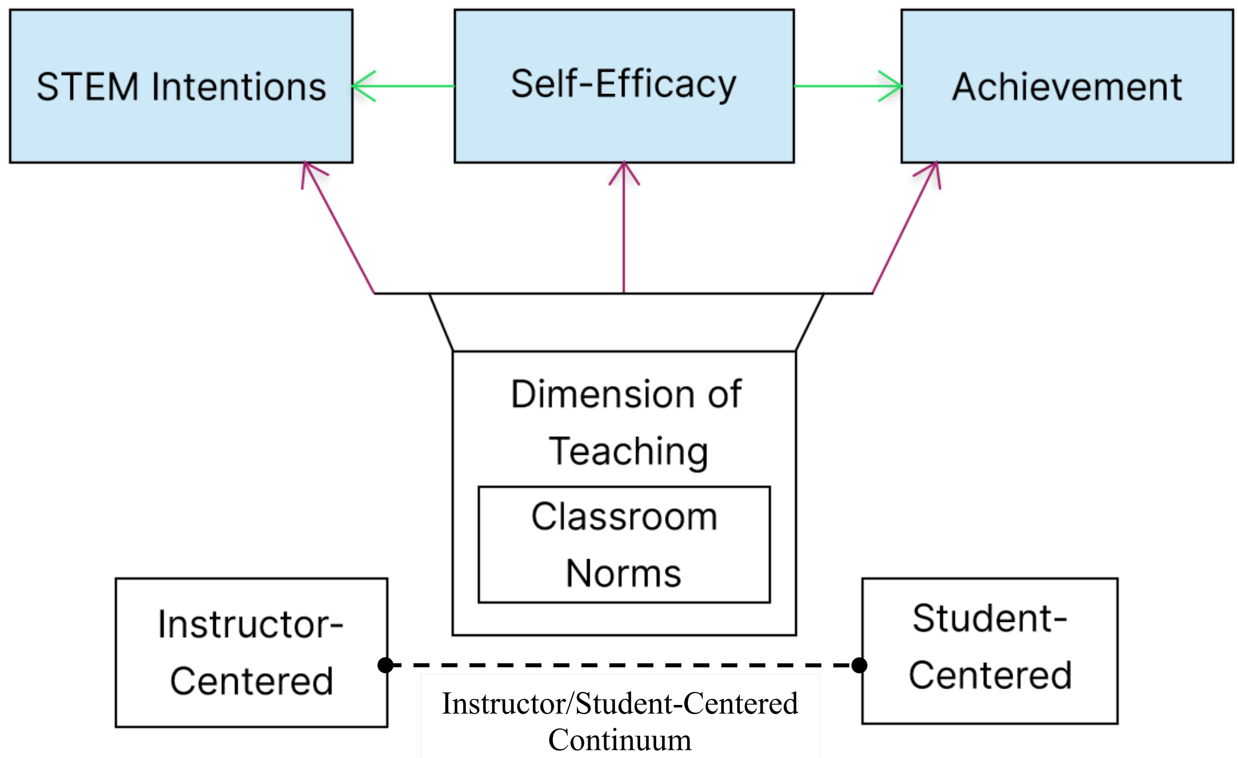
I define students' STEM intentions as students' plans to major in a STEM field at the time of this study.

2.10.5. Dimensions of Teaching

Lastly, a factor analysis was conducted to reduce the survey items representing classroom norms of the precalculus course into a set of 'dimensions of teaching' (or 'teaching dimensions') that served as dependent variables in the regression models. I define these dimensions of teaching as the "underlying dimensions" (Field et al., 2012, p. 751) representative of their corresponding classroom norms from the factor analysis. More specifically, factor analysis suggested the norms of the course reviewed here could be reduced into the different ways instructors and students engaged with one another and the mathematics. To summarize the relationships between the variables of interest in the current study, I offer Figure 1 as the conceptual framework.

Figure 1

Conceptual Framework



Note. Relationships between the classroom norms of a dimension of teaching, and students' self-efficacy, academic achievement in a course, and STEM intentions. The green arrows represent positive relationships supported by the mentioned literature, while the purple arrows are relationships that the current study is meant to explore as it is uncertain whether they are positive or negative relationships. The dimensions of teaching and their associated classroom norms are placed on a continuum ranging between instructor- and student-centered (see Rasmussen & Marrongelle, 2006 for more on this continuum).

To clarify the many parts of Figure 1, research suggests a single classroom can be characterized by both instructor-centered *and* student-centered practices (Stains et al., 2018), or classroom norms, for the purposes of this study. These characterizations are represented in Figure 1 by the instructor- to student-centered continuum, a concept borrowed from Rasmussen and Marrongelle (2006) and their discussion of a "noninterventionist-total responsibility continuum" (p. 415). Moreover, the one-way arrows used in Figure 1 represent the predictive relationship between the classroom norms and student outcomes (i.e., self-efficacy, achievement, and STEM intentions) that follow empirical research and, thus, should not necessarily be interpreted as a causal relationship. In particular, the green arrows in Figure 1 follow empirical evidence suggesting positive-predictive relationships between the noted outcomes. For example, an increase in students' self-efficacy often is predictive of an increase in students' STEM achievement.

The purple arrows represent the relationships being explored in the current study. While research suggests that, for example, student-centered practices positively relate to students' sense of self-efficacy, the current research sees this relationship and the others represented in Figure 1 by purple arrows as needing more exploration for the reasons discussed in the above review of literature. Furthermore, the purple arrows represent relationships between two given variables with no assumption as to whether the relationship is positive or negative (unlike the green arrows). For example, there is conflicting evidence as to whether a more student-centered classroom leads to students' pursuit of a STEM (Ellis et al., 2014; Kogan & Laursen, 2014). Additionally, in reviewing the literature, very few studies have asked questions surrounding self-efficacy, student achievement, and STEM intentions within the current context, namely, an undergraduate precalculus course. Thus, I leave room for unique findings and questions about the relationships presented in Figure 1 that have yet to be discussed within a precalculus course.

Lastly, please note that I do not include arrows representing the mediating role of classroom norms between, for example, self-efficacy and student achievement. While Peters (2013) provided an example of such research, understanding these mediating relationships within the context described below is outside the scope of the current research. Instead, the focus of this research is to explore the direct relationships between the classroom norms of an undergraduate math classroom and students' self-efficacy, academic achievement, and STEM intentions. Given this focus, I also do not include a direct arrow between achievement and STEM intentions (for more on this topic, see Maltese & Tai, 2011). In addition, I leave an analysis of the potential bidirectional relationships (i.e., two-way arrows) between components in the framework for future research, such as whether students coming into a classroom with higher self-efficacy lends the classroom to more student-centered norms (see Peters, 2013, p. 474 for an interesting finding on higher self-efficacy of students in a teacher-centered classrooms). Indeed, while focused more on instructors' perceptions of their students, research at the K-12 level, for example, suggests teachers' willingness to implement more student-centered practices may relate to teachers' perceived abilities of their students (Cross, 2009; Steele, 2001). Lastly, while the diagram presented in Figure 1 does not capture all the factors that may predict the included outcomes, it does provide a useful framework to address the aims of the current study.

Having described the conceptual framework guiding this study, I now turn to describing the quantitative and qualitative components used to answer the research questions posed above.

Chapter 3. Research Design and Methodology

Having presented the aims of the current study in light of past research and current national trends in undergraduate STEM education, I offer the following sections detailing the research design and methodology used in answering the research questions. I begin by describing the context of the precalculus course, including my involvement in the course as both a researcher and instructor. I then provide descriptions of the instruments I used in collecting data. Following these descriptions, I provide a detailed walkthrough of the steps I followed in my data analysis, including a timeline of the data collection and analysis.

3.1. Description of Course

The context of the current study is the Precalculus course offered at a large, mid-Atlantic, public university, hereon referred to as Blackboard University (BU). Precalculus serves as a prerequisite course for STEM students who will subsequently take the mainstream Calculus course. There are three versions of Precalculus offered by the Math Department at BU. First, there is the traditional large-lecture, or Main Precalculus (~200 students), version of the course that meets three times a week for a 50-minute lecture. In addition, students taking Main Precalculus meet multiple times[13] per week in their discussion sections made up of roughly 20 to 30 students. The discussion sections are led by Teaching Assistants (TAs) who are graduate students in the Math Department. At the time of this study, the course coordinator for all three versions of Precalculus was the instructor for the Main Precalculus large lecture. Second, the Math Department offers a version of Precalculus, referred to here as Bridge Precalculus (typically four sections with roughly 25 students in each section), through a fall extension

[13] Discussion sections that are held on Mondays, Wednesdays, and Fridays meet for 50 minutes while those that are held on Tuesdays and Thursdays meet for 75 minutes.

program for incoming freshmen officially admitted for the spring semester who wish to begin earning credit in the fall. Bridge Precalculus sections meet twice a week for 75-minute sessions, with no recitation sections. Lastly, the Math Department offers a special version of Precalculus, or Remedial Precalculus (two sections of ~25 students), that follows a five-week developmental algebra course and is designed for STEM-intending students who have low mathematics placement test scores. Remedial Precalculus sections meet five times a week for 50-minute meetings with no recitation sections.

All three versions of the course have historically had a high failure rate (30-50%) and serve a disproportionate number of underrepresented students in STEM fields of study. To address these trends, certain instructors of Main and Bridge Precalculus have been a part of a recent redesign focused on active learning. Over the past three years, I have worked with the Precalculus course coordinator (for all three versions of the course) and several BU faculty members across multiple departments to support active learning, or student-centered practices, as defined here. In 2017, BU joined the NSF-funded Student Engagement in Mathematics through an Institutional Network for Active Learning, or SEMINAL network (Association of Public and Land-Grant Universities, 2018). The SEMINAL network aims to support universities in their work to adopt active learning practices (recall active learning is comparable to learning involving student-centered practices) in their precalculus to calculus 2 (P2C2) courses. Preliminary findings from this work suggest active learning may be connected to students' successful completion of the Main[14] and Bridge versions of the course (Gruber et al., 2020).

[14] A redesign of Bridge Precalculus is still in its beginning stages.

3.2. Study Participants

A major aim of this study was to characterize the classroom norms within the different Precalculus settings at BU, namely, the Main Precalculus large lecture and discussion sections. The Precalculus course coordinator has played a critical role in the redesign of the course, has shown consistent interest in reflecting on the course, and has encouraged TAs to participate in the pilot studies leading up to the current study (more information on these studies is presented below). Considering the course coordinator is the instructor for the large lecture of Main Precalculus, I chose to focus[15] my efforts in answering the research questions posed here within the context of Main Precalculus, heron referred to as Precalculus (unless otherwise discussed with the other versions of the course). Following IRB approval, I invited all Precalculus instructors, including the course coordinator and discussion section TAs, and their students to participate in the study. To encourage participation, student participants were entered into a lottery upon completion of the student survey for a chance to be one of 100 students awarded a $10 gift card, while instructors who completed an instructor survey were automatically awarded a $10 gift card. Additionally, each student who participated in a student interview was awarded a $30 gift card, while instructors who participated in an interview were awarded a $10 electronic gift card. All the gift cards were for Amazon.com and were electronically sent to participants using the emails they provided at the end of the surveys. In total, the Precalculus course coordinator (i.e., large lecture instructor), four[16] of the five discussion section TAs, and 181 of the 249 (73%) students served as the participants of the current study.

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[15] While I also invited Bridge and Remedial Precalculus instructors, only one of the two Bridge instructors responded, and none of the Remedial instructors responded.
[16] The one TA who did not participate did, however, allow me to observe one of his discussion sections.

3.3. Ethical Considerations and Researcher Bias

Given my considerable investment in the Precalculus course at BU, it is important to note that on the "participant/observer continuum" (Bogdan & Biklen, 2007, p. 91), I solely acted in the role of an observer and interviewer during data collection and did not participate in any classroom instruction or activities. Moreover, while I have taught sections of Precalculus[17] in the past, I did not teach any sections during the semester I collected data. Therefore, there were no apparent conflicts of interest in conducting the current study. Even so, students may have viewed me as an authoritative figure as the Principal Investigator of the study and, in turn, felt pressured to participate in the study. To minimize this pressure, following the approved IRB protocol for this study, students were reminded that their participation in the study had no effect on their grades in Precalculus. In addition, if students chose to participate in the study, they could stop participating at any point.

I also acknowledge that, following my involvement in the redesign of Precalculus to incorporate student-centered instructional practices, I came to this study with certain experiences and beliefs as to how to successfully support Precalculus students. These experiences and beliefs were helpful in formulating the research questions guiding this study in that the topic of student-centered classroom norms are of interest to me. This interest was important for me to persist in the completion of the current study that took multiple years, between becoming familiar with the literature, designing the study, and conducting the study. Even so, I acknowledge that my time as a student, instructor, and now someone familiar with the literature reviewed above may have influenced the current study at different points. To be candid, I believe that a combination of both student-centered and instructor-centered norms, co-constructed by instructors and students,

[17] Bridge Precalculus, to be more specific.

would best support Precalculus students. To minimize the influence and bias due to my own beliefs, I shared both qualitative and quantitative results with fellow colleagues in the field throughout my analysis.

3.4. Prior Analyses and Pilot Study of Precalculus

Beyond my involvement in the SEMINAL project, I conducted two smaller analyses of students' experiences in Precalculus within the past five years that provided me entry into the Precalculus course and initial insights into the classroom norms of Precalculus. In the first small-scale study, as a part of a qualitative methods course in the fall of 2017, I observed the Precalculus large lecture and interviewed a student about his experiences in the course. In the second small-scale study, as a part of an independent study with Dr. Daniel Chazan, I conducted IRB-approved research of the Bridge and Main Precalculus courses in the fall of 2018. Succinctly, I analyzed students' experiences in, expectations of, and engagement in these two versions of Precalculus. The study included a single observation of the large lecture for Main Precalculus and one Bridge Precalculus section using the Classroom Observation Protocol for Undergraduate STEM (COPUS) developed by Smith et al. (2013). In addition, I interviewed the instructors of the sections I observed, as well one student from Main Precalculus and one from Bridge Precalculus. Finally, I obtained survey data from 29% (n=67) of the Main Precalculus students and 33% (n=11) of the participating Bridge Precalculus section. I wrote the survey items using the coding scheme that Star et al. (2008) developed for their analysis of differences that students notice between reform (or student-centered) and traditional math programs. In short, the survey asked students about their experience in past math classes in comparison to their Precalculus course at BU, as well as their plans to take future math courses.

Findings from both small-scale studies suggest two insights into the instructional practices of the Main Precalculus (i.e., Precalculus in the current study) large lecture instructor, an active member of the SEMINAL group. First, from my observations, the Main Precalculus instructor resorted to more instructor-centered practices, such as lecturing, when not implementing more student-centered practices, further supporting the continuum presented in Figure 1. Second, the instructor did not exclusively utilize student-centered or instructor-centered practices but rather used both at different times throughout a lecture. Stains et al. (2018) found similar results from their observations of 2008 STEM classrooms across North America. Thus, it may be that the classroom practices implemented in Main Precalculus are representative of those practices implemented in other STEM courses outside of BU. In turn, answering the research questions posed here within the context of BU may provide findings that may be generalized to other introductory math courses. In addition, such insights would be valuable to the Math Department at BU given their current efforts to reform Precalculus and improve undergraduate math instruction, more generally.

In addition to these smaller analyses, I conducted a pilot study of the current study in the spring of 2021. The pilot study was implemented during the COVID-19 pandemic in which all classes were taught virtually via Zoom. To account for these unique circumstances, and following insights from the SEMINAL group (Smith, personal communication, October 1, 2020), students had the added option to indicate if the ongoing COVID-19 pandemic was a barrier to their regular participation in Precalculus. In addition, instructors were given the option to comment on how effectively they felt they had incorporated student-centered practices compared to before the pandemic. While the final data collection for the current study was completed in the fall of 2021 when courses resumed completely in-person, the above questions were kept in the PIPS-M for the purposes of future research related to pre- and post-pandemic[18] instruction.

To summarize, the pilot study, which included[19] 75 student survey responses, resulted in the validation of principal component analysis (PCA), or exploratory factor[20] analysis, as a means to measure the classroom norms in the large lecture of Main Precalculus (see Cabrera et al., 2001, p. 337 for another example applying principal component analysis to instructional practices). More specifically, the current study utilized Postsecondary Instructional Practices Survey for Mathematics, or PIPS-M (Apkarian et al., 2019), an adaption of the subject-general PIPS developed by Walter et al. (2016).

Having provided an overview of the context of the current study, I now turn to describing each data source used in answering the research questions, beginning with the PIPS/PIPS-M surveys. This set of surveys served as the source for characterizing the Precalculus classroom norms as perceived by instructors and students. More specifically, the survey data was used to answer the first, second, and fourth research questions.

3.5. Quantitative Data

The first, third, and fourth research questions guiding this study were as follows.

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[18] While instruction for Precalculus during the semester data collection took place was conducted in-person (what I refer to here as "post-pandemic") the worldwide COVID-19 pandemic was technically still ongoing.

[19] In total, the pilot study included survey responses from 75 Main Precalculus students, 9 Remedial Precalculus students, the Main Precalculus course coordinator, and the one Remedial Precalculus instructor at that time. The Kaiser–Meyer–Olkin (KMO) measure of 0.86 suggested the sample of 75 students was a sufficient sample size for factor analysis. In addition, according to Bartlett's test for sphericity, $\chi^2(91) = 959.82, p < 0.001$, the individual SPIPS-M items were statistically significantly correlated to justify factor analysis. Without any type of rotation, four factors with eigenvalues above 1 explained 64.59% of the variance in students' responses with respect to the large lecture.

[20] See Fields et al. (2012) for comments and sources that speak to the nuanced differences between principal component analysis (PCA) and factor analysis (p. 760). For the purposes of this study, PCA was a sufficient statistical analysis method.

- To what extent are the classroom norms of the large lecture and discussion sections of an undergraduate precalculus course instructor or student-centered, as perceived by surveyed students?
- In comparing surveyed instructors to students, to what extent are the classroom norms perceived as instructor or student-centered?
- What perceived classroom norms of an undergraduate precalculus course are predictive of students': (a) academic achievement in the course, (b) mathematical self-efficacy, and (c) STEM intentions?

These questions were included to build off past research analyzing student outcomes in relation to instructor- or student-centered norms in an undergraduate math course. To answer these questions, I used a survey to collect data on the classroom norms of both the large lecture and discussion sections from the perspective of the large lecture instructor, the discussion section TAs, and the students. I describe the survey I used in the sections that follow. As mentioned above, when describing the development of the survey, I use the term 'practices' to follow the language used by the authors of the survey. However, the analysis used in the current study considered the agreed upon practices, or classroom norms, by analyzing the survey responses and interview comments of both instructors and students.

3.5.1. Postsecondary Instructional Practices Survey (PIPS)

To measure the self-reported instructional practices of interdisciplinary, postsecondary instructors, Walter et al. (2016) developed the Postsecondary Instructional Practices Survey (PIPS) using empirical evidence and writing on theories related to practices of a classroom. As an overview, beginning with 153 items from other surveys and observational protocols, the authors generated, revised, and removed statements that they conceptualized into four categories[21] to assist in making sense of the items: "instructor-student interactions, student-content interactions, student-student interactions, and assessment" (p. 3). The final version of the

[21] These categories, or conceptualizations, closely align with the dimensions of teaching that resulted from my own factor analysis of survey responses in the current study.

PIPS contains 24 instructional practice statements. For each of these items, instructors are asked to indicate, on a scale of (0) “Not at all descriptive of my teaching” to (4) “Very descriptive of my teaching,” how descriptive the provided statements were of their instructional practices. As an example, one of the statements is, “I structure class so that students discuss the difficulties they have with this subject with other students.”

In testing the PIPS for reliability and validity, Walter et al. (2016) collected 891 survey responses of instructors (including instructors from STEM and non-STEM departments) at four universities across the United States. Using exploratory and confirmatory factor analysis, the authors found that the items could be grouped into either five or two factors. Most relevant to the current study, the two-factor model included one factor representing instructor-centered practices (including nine items) and a second factor representing student-centered practices (15 items). The authors found that “the PIPS has an overall instrument reliability of $\alpha = 0.800$,” with factor one and two explaining roughly 24% and 14% of the variance, respectively. Additionally, the instrument was refined for validity purposes with the help of a small group of instructors and educational researchers, none of whom were a part of the 891 participants mentioned above.

3.5.2. The PIPS for Mathematics (PIPS-M)

The Postsecondary Instructional Practices Survey for Mathematics (PIPS-M) used in the current study is an adapted version of the PIPS specifically designed for college-level mathematics courses. The PIPS-M was used as a part of the Progress through Calculus (PtC) study (MAA, 2015-2019), and has been used over the past several years at BU as a part of the SEMINAL project (Apkarian et al., 2019). Beginning with the PIPS, the PIPS-M contains 17 additional items to address the goals of the SEMINAL project described above. These items include: one item that states, “I structure class so that students work on problems individually during class"; eight items focusing on practices that are specific to active learning, as well as mathematical knowledge for teaching; and eight items focusing on inclusive instructional practices (see Apkarian et al., 2019, pp. 36-38, for an item-by-item comparison between the PIPS, PIPS-M, and SPIPS-M). To clarify, while Apkarian et al. (2019) recoded the instructional practice items to be from a scale of (1) Not at all descriptive to (5) Very descriptive, the current study used the following scale of 0 to 4 as in the work of Walter et al. (2016): (0) Not at all descriptive of my experience; (1) Minimally descriptive; (2) Somewhat descriptive; (3) Mostly descriptive; (4) Very descriptive. The prompt for the instructor questions read as follows.

> Please indicate the degree to which the following statements are descriptive of your teaching in Precalculus. Select "Not at all descriptive" if the statement is not descriptive of your teaching. Please note that you are given space following the questions below for any additional comments/clarifications you wish to make.

Outside of the instructional practice items, the instructor PIPS-M asks about participant demographics and course coordination, among other topics. In total, the PIPS-M includes 41 instructional practice statements that are directed towards undergraduate math instructors.

The PIPS-M is accompanied by a student version, or the SPIPS-M[22], in which students are presented with 26 of the 41 PIPS-M instructional practice statements reframed from the perspective of a student. Using the above example on the instructor PIPS-M, the student version of the item on students working individually read, "I work on problems individually during class time." Students are asked to answer these items considering the context of a regular course meeting (i.e., lecture) and discussion section, separately, and respond using the same Likert-scale as that of the instructors. Below are the prompts students are presented with, once when

[22] To be succinct, I heron refer to the instructor and student versions of the Postsecondary Instructional Practices Survey for Mathematics as the PIPS-M. When necessary, I make the distinction between the instructor and student survey by referring to the student version as the SPIPS-M. Please also note that there is a student-instructor (e.g., undergraduate teaching assistants) version (UGPIPS-M) of the PIPS-M that was not relevant considering the participants of this study (see Apkarian et al., 2019).

responding to the items with respect to the large lecture, and once with respect to the discussion sections.

- Student Large-Lecture Prompt: Indicate the degree to which the following statements describe your experience in the large lecture with Dr. Riemann. Select "Not at all descriptive" if the statement does not describe your experience
- Student Discussion Section Prompt: Indicate the degree to which the following statements describe your experience in your discussion/lab section. Select "Not at all descriptive" if the statement does not describe your experience.

In addition, the SPIPS-M asks students for demographic information, as well as other topics including attendance and resources students use (e.g., online tutorials and graphing calculators) throughout the semester.

Given that the view of the current study suggesting classroom norms are negotiated by both the instructor and students, the core statistical analysis of this study was of the PIPS-M instructional practice items that appeared on both the instructor and student survey. Thus, there were a total of 22 potential Precalculus classroom norms listed on the PIPS-M and SPIPS-M. Following the work of Walter et al. (2016) and Apkarian et al. (2019), each[23] of these norms, as defined here, could be described as being instructor or student-centered in nature. Thus, in the current study, an instructor noting that, for instance, a student-centered norm was very descriptive of their classroom could be interpreted as the instructor's perceived implementation of that norm being more student-centered. As a reference, Table 1 lists the PIPS-M instructional

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[23] Please note that in developing the PIPS-M, Apkarian et al. (2019) added multiple items to the PIPS "with the intention of measuring instructional practices that support an inclusive experience for students" (p. 6). In other words, the following items were not explicitly characterized as being either instructor or student-centered: *participation strategies*, *wide participation*, *wide student response to questions*, *sense of community*, and *student names*. Considering the overlap in these practices with those of student-centered practices listed at the outset of this paper and highlighted throughout the literature reviewed here (e.g., engaging students by asking questions and having students work with others), for the purposes of this study, these items were categorized as student-centered in nature.

practice items, or classroom norms in the current study, listed from the perspective of both an instructor and student (i.e., the SPIPS-M items) and categorized as instructor or student-centered.

Table 1

PIPS-M/SPIIPS-M Items

Instructional Practice (IC: Instructor-Centered, or SC: Student-Centered)	Survey Statement
Connecting Content (SC)	(Instructor) I provide activities that connect course content to my students' lives and future work. / (Student) The class activities connect course content to my life and future work.
Constructive Criticism (SC)	(Instructor) I structure class so that students constructively criticize one another's ideas. / (Student) I constructively criticize other student's ideas during class.
Immediate Feedback (SC)	(Instructor) I provide students with immediate feedback on their work during class (e.g., student response systems, short quizzes). / (Student) I receive immediate feedback on my work during class (e.g., student response systems such as clickers or voting systems, short quizzes).
Instructor Feedback on Assignments (SC)	(Instructor) I give feedback on homework, exams, quizzes, etc. / (Student) I receive feedback from my instructor on homework, exams, quizzes, etc.
Participation Strategies (SC)	(Instructor) I use strategies to encourage participation from a wide range of students. / (Student) My instructor uses strategies to encourage participation from a wide range of students.

Student Response to Questions (SC)	(Instructor) I ask students to respond to questions during class time. / (Student) I am asked to respond to questions during class time.

In interpreting the results of the PIPS-M, I highlight Walter et al.'s (2016) two means of summarizing the data collected via the PIPS. First, the "actual factor sum, "or sum of all responses under factor, and the "maximum possible factor sum" can be used to compute a "factor score" for the instructor. To avoid confusion with how students' factor scores were computed using loadings from factor analysis, I hereon refer to this score as an "awareness score" for a factor, using the terminology presented in my conceptual framework borrowed from Çakır and Akkoç (2020). As an example (see below), if an instructor answered (3) Mostly descriptive to all 9 items for a given factor, then this instructor's awareness score for that factor would be calculated as follows (see Walter et al, 2016, p. 6, for more details).

> Actual factor sum for survey items loading onto the given factor = 3*9=27
> Maximum possible factor sum = 4*9=36
> Instructor awareness score for given factor = (27/36)*100=75%

Second, instructors' responses can be used to generate a scatterplot that places their awareness scores (or PIPS scores in Figure 2) along axes, where each axis represents a different factor. As an example, I include Walter et al.'s scatterplot in Figure 2, comparing the PIPS instructor scores across different departments at a public research university.

Figure 2

Example Comparison of Instructors PIPS Scores

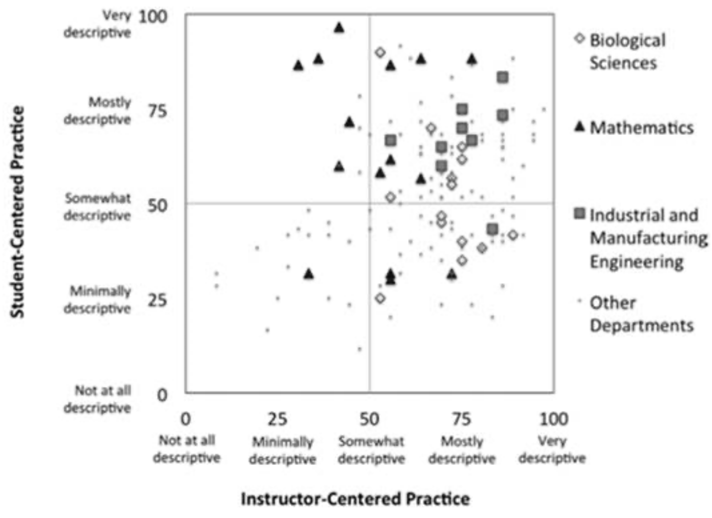

FIGURE 3. PIPS scores for instructors in the 19 sampled departments at Institution A ($N = 152$). Case study departments are identified.

Note. Example comparison of instructors' PIPS scores along the factors of instructor- and student-centered practices. Borrowed from Walter et al. (2016, p. 7).

In considering the presentation of instructor awareness scores in Figure 2, I highlight that the two-dimensional graph suggests instructors' awareness scores may 'range' along a scale of the factors of interest, such as an instructor or student-centered practices. Thus, the PIPS-M provides a useful means of measuring the implementation of classroom norms, as defined in the current study, that range between instructor- and student-centered (see conceptual framework). Such a measurement is beneficial to answering the current research questions within the context of the Precalculus course given that, following to results from the pilot study, the large-lecture instructor implemented instructional practices that were more instructor-centered when not[24] implementing practices that were more student-centered. This 'range' of instructor and student-

[24] As opposed to another type of practice not categorized as instructor- or student-centered that was outside the scope of the current study.

centered practices, or norms in the current study, is supported by literature suggesting instructors may move along a "noninterventionist-total responsibility continuum" ranging from "pure discovery to pure telling" (see Rasmussen & Marrongelle, 2006, pp. 391, 415), that is, instructor-centered to student-centered.

For the current study, the factors of interest were the various dimensions of teaching that resulted from a factor analysis of classroom norms. Moreover, the classroom norms that remained following a removal of those not relevant to the Precalculus class were all student-centered[25] in nature. As a result, an instructor with a lower awareness score along each dimension of teaching may be interpreted as the instructor's perceiving their classroom as more instructor-centered when considering that dimension's classroom norms. That is, a lower instructor awareness score for a certain dimension of teaching suggests the instructor generally perceived instructor-centered norms on the PIPS-M as being a part of their classroom (e.g., ranging[26] between not at all and minimally descriptive). Conversely, a higher instructor awareness score suggested the instructor perceived more student-centered norms under a given teaching dimension. Finally, an instructor awareness score closer to 50% (i.e., being 'pulled' to the middle of the continuum) suggests a hybrid of instructor- and student-centered classroom norms under a given teaching dimension. A similar analysis was conducted when calculating the mean student awareness score for each dimension of teaching (not to be confused with student factor scores used in the regression models).

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[25] That is, an instructor or student noting a norm was, for example, mostly descriptive of their classroom aligned with the definition of implementing student-centered norms in the current study.
[26] More specific cutoffs are presented in the presentation of results in Chapter Four to consistently interpret instructors' awareness scores.

3.5.3. Measuring Academic Achievement

Following the past work of the SEMINAL team at BU in analyzing Precalculus student achievement, I chose to use students' final exam grades at the end of the semester as the measure of achievement. This measure of achievement is useful considering the SEMINAL team's interest to compare student achievement across the different versions of Precalculus (i.e., Main, Bridge, and Remedial). More specifically, there are inconsistencies in the syllabi, including grading schemes, between Main, Bridge, and Remedial Precalculus. In addition, there are variations on exams and quizzes given throughout the semester due to the different time constraints and contextual differences between the different versions of the course. However, the same final exam is given to all Precalculus students, regardless of the version. Thus, while the current study does not include data from Bridge and Remedial Precalculus, using the final exam data from Main Precalculus sets up future research in comparing student achievement across the different versions of the course. Moreover, the use of the final exams in this way aligns with the current course coordinator's interest in analyzing student success on specific final exam questions. While a question-by-question analysis of the final exam is beyond the scope of the current[27] study, the use of final exam grades to measure achievement offers initial steps in this direction.

3.5.4. Precalculus Self-Efficacy PIPS-M Items

Returning to the definition of self-efficacy, Bandura (1977) argues that students' self-efficacy is specific to both the context and tasks in question (see also Pajares & Miller, 1994). Applying this definition to such a broad domain as undergraduate STEM courses creates

[27] Future projects may also compare responses on the PCSEI with similar questions presented on the final exam (see Pajares & Miller, 1995, p. 194 for an example of such an analysis).

challenges in measuring students' self-efficacy, considering both context and content would vary across different courses. Therefore, to answer the current research questions specifically within the context of an undergraduate precalculus course, I will use the Precalculus Self-efficacy Instrument, or PCSEI, developed by Carter (2022).

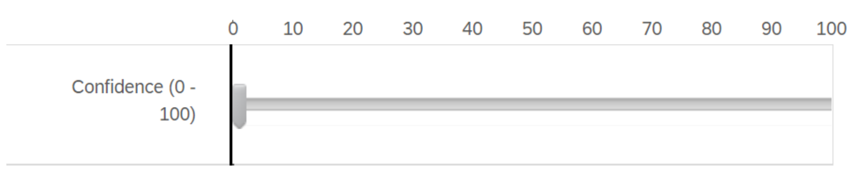
Following a review of the problems with the Precalculus course coordinator, as well as my own experience in teaching the course, I selected 20 of the 25 questions on the PCSEI that were representative of the content historically covered on Precalculus final exams at BU. As described above, the PCSEI follows the format of the Calculus Self-efficacy Scale from Zakariya (2019) in asking students to rate their confidence in solving the included problems on a scale of 0 (not confident at all) to 100 (very confident), with problems ranging in topic and difficulty level. Below is the prompt for the PCSEI and a sample item.

Figure 3

Sample Item from Precalculus Self-efficacy Instrument

> Below you will find a set of different tasks representative of the content covered in Precalculus at Blackboard University. For each task, use the slider tool to indicate how much confidence you have that you could manage the given task at this point in time, ranging from 0 (No possibility I could solve this) to 100 (Totally confident that I could solve this). You are not asked to solve the tasks.

Find the domain of the function $f(x)=\dfrac{x}{\sqrt{x+1}}$.



After taking the PCSEI, students were assigned a self-efficacy score by summing their responses to the 20 items, resulting in a self-efficacy score between 0 and 2000 (the upper-rating for a given item was 100, giving 100*20=2000 as a maximum self-efficacy score), following the work of Zakariya et al. (2019, p. 8). To efficiently capture students' responses, students were presented with the PCSEI items on the SPIPS-M.

3.5.5. STEM Intentions PIPS-M Items

Students were asked the following questions on the SPIPS-M related to their intentions to major in a STEM field.

- STEM Intentions Question A:
Have you declared, or do you intend to declare, a STEM (science, technology, engineering, or mathematics) major?
Answer Options: Yes, No, Unsure, Prefer not to disclose

- STEM Intentions Question B:
Which major(s) have you declared, or do you intend to declare? (open-ended question)

While Question A directly asks students about their STEM intentions, there were several students who incorrectly categorize their major as a STEM major (e.g., dietetics, economics, immersive media design) or not a STEM major (e.g., civil engineering, computer science, neuroscience). Thus, to have a consistent categorization of students' STEM intentions, I recoded[28] students' responses to Question A using their free responses to Question B. To consistently and accurately recode students' STEM intentions, I followed the STEM classification guidelines and definitions of the U.S. Department of Education (2021) and the National Science Foundation (Fiegener, 2015).

Some students shared comments that indicated they had declared a major but were intending on switching it. Consider the following sample responses to Question B.

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[28] To clarify, any majors related to health sciences was categorized as non-STEM majors. Following Fiegener's (2015) categorization of "health" degrees, examples of these non-STEM majors include kinesiology, dietetics, hearing and speech sciences, family science, nursing, pre-med, and pre-pharmacy.

- I have declared a mathematics major but I am looking to switch to business analytics
- Biology (or public health, I'm on the fence)
- Biological science but I am switching to a psychology major
- I have declared chem and intend to declare undecided
- Engineering but I will change it to something else

For these students, I followed their original categorization of STEM intentions to avoid misinterpreting their intentions. Finally, for students who categorized their STEM intentions for Question A but left Question B blank, I used their original categorization of their major intentions (i.e., STEM or non-STEM).

Such questions of intention may be different from merely being interested in a STEM field. Students may initially be interested (Lin et al., 2018), or even begin majoring in a STEM field, yet not complete the degree for various reasons (Crisp et al., 2009; Ellis et al., 2014). However, exploring students' persistence in a STEM field would require longitudinal data collection that is beyond the scope of this study. Thus, asking students about their major intentions sufficed for the purposes of this study.

3.6. Qualitative Data

The third research question guiding this study was stated as follows: How do the instructors and students describe the classroom norms of the precalculus course in interviews? This question was included to provide capture more detailed and nuanced descriptions of the classroom norms listed on the PIPS-M. To answer this question, I conducted semi-structured interviews with the Precalculus large-lecture instructor, four participating TAs, and 10 Precalculus students. In addition, I conducted a limited number of classroom observations to provide context to participants' comments. I describe each of these data sources below.

3.6.1 Instructor and Student Interviews

Of the 53 students indicating interest in being interviewed (students indicated interest at the end of the SPIPS-M), I interviewed one student from each of the ten discussion sections. Students with the same TA (but from different sections) were interviewed together. These interviews were essential in characterizing the classroom norms of Precalculus from the student perspective using the PIPS-M items. More specifically, these interviews allowed space for instructors and students to comment on specific classroom norms from the PIPS-M that were or were not a part of the large lecture and discussion sections, separately. In turn, I was presented with details as to was normative in the Precalculus large lecture and discussion sections that may have otherwise been less salient and, thus, more difficult to capture through the survey responses or my observations of class meetings alone. In other words, the interviews served to capture the classroom norms as perceived by those who were immersed in day-to-day activities and interactions of the classroom (see Cobb & Yackel, 1996, p. 176, on the importance of considering “students’ mathematical development as it occurs in the social context of the classroom").

Aside from providing greater insight into the classroom norms of Precalculus, student interviews, in particular, align with calls for using interviews in understanding the unique experiences of a diverse group of students, including those students historically underrepresented in STEM fields (Palmer et al., 2011, p. 495). Moreover, the norms of a classroom may be perceived differently by instructors and students (see Levenson et al., 2009 for a primary school example). Thus, to capture an accurate perspective of a diverse and representative group of Precalculus students, interviewees were invited using the following criteria as a guide.

- The student indicated interest in and consented to being interviewed, as noted at the end of the SPIPS-M.
- The student consented to audio/video recording of the student interview for research purposes.
- The student completed the entire SPIPS-M.
- The student did not select "other" as a response to attendance SPIPS-M item, ranging from (0) I've never missed a class to (3) I've missed at least one class per week the attendance scale for either, with preference shown to those students with higher attendance in hopes of receiving a better day-to-day account of classroom activities and interactions.
- Both STEM and non-STEM intending majors were considered.
- The pool of potential interviewees was racially and ethnically diverse, as well as diverse in their gender identities and sexual orientations.

In addition, I implemented extreme/divergent case sampling techniques (Kemper et al., 2003) to invite students who were diverse in terms of their self-efficacy scores, as well as what they final exam and final course grade was. For example, one student interviewee completed Precalculus with a final grade of 97% while another failed the course and was retaking it during the interview process. Another interview consisted of a student whose self-efficacy responses summed to 960 out of a possible 2000 points on the self-efficacy scale while the other student's responses summed to 1727. Still another interview included one of the five students whose self-efficacy responses summed to 2000 (i.e., a 'perfect' score).

3.6.2. Classroom Observations

After administering the PIPS-M, I conducted a single observation of the Precalculus large lecture. In addition, while one of the TAs did not participate in the survey or interview process, all five of the TAs agreed to my observing one of their two discussion sections. Thus, in total I observed six Precalculus classroom meetings. During each observation, I took descriptive fieldnotes (Bogdan & Biklen, 2007) to capture the words of students and the instructors, as well as what activities they were engaging with (e.g., a group quiz). These fieldnotes helped me ask questions and make references to the classes during the interviews, as well as provide context during my analysis and writeup of results. Following the consent of instructors and students, I audio and video recorded four of the six meetings to revisit what happened in each meeting and transcribe moments throughout the classes. As mentioned above, one of the TAs did not complete a survey but still agreed to my observing one of his discussion sections and taking notes of his instruction. However, most of his students did not consent to being audio or video recorded. In addition, a second TA did not consent to a video and audio recording of my observation of one of his sections. Thus, for these two discussion sections, I relied primarily on taking fieldnotes (in addition to taking photos of the board following the consent of both TAs) to describe what happened during my observation. To be clear, direct quotes of any student who did not consent to being audio or video recorded during my observations, as indicated on the SPIPS-M, were not included in any reference I made (including the vignettes provided in Appendices E-J) to observations of the Precalculus classrooms.

3.7. Mixed Methodology

Using the above quantitative and qualitative components, this study followed a convergent mixed-methods approach in which each component described above informed the other (Creswell, 2015). To begin, an assumption of this study was that the norms of a classroom consist of interactions and negotiations between instructors and their students, components a classroom a survey alone may not be able to fully measure (Mayer, 1999). Moreover, both in conversations with experts in the field and my reading of the literature, the idea of 'measuring' classroom norms is not an easy process, especially considering norms may be something "experienced" by not necessarily "taught" (Code et al., 2016, p. 917). To address this limitation, I used the qualitative and quantitative data as a validity check in measuring the classroom norms of Precalculus.

To review, the quantitative component of the study provided insight into potential predictive relationship between classroom norms, grouped into different dimensions of teaching as determined by factor analysis, and the student outcomes in question (achievement, self-efficacy, and STEM intentions). The qualitative component consisted of using instructor and student interviews to refine the quantitative models. More specifically, following an initial factor analysis of student SPIPS-M responses to generate an initial list of Precalculus classroom norms, I used instructor and student interviews to eliminate classroom norms not relevant to either the large lecture or discussion sections, followed by rerunning the factor analysis on the remaining classroom norms and generating the final predictive models.

Figure 4

Study Components

QUAN

- **Procedures/Instruments**
  - Administered of PIPS-M (instructor survey) and SPIPS-M (student survey) with PCSEI (self-efficacy) and STEM Intention items.
  - Performed factor analysis to group survey items into different dimensions of teaching and guide semi-structured interviews.
- **Products**
  - Initial teaching dimensions (i.e., the factors resulting from factor analysis) and their associated classroom norms (i.e., the individual items nested under a teaching dimension).
  - Descriptive statistics of instructor and student survey data.

QUAL

- **Procedures/Instruments**
  - Used student survey results to invite a representative sample (including one from each discussion section) to semi-structured interviews.
  - Invited all instructors to semi-structured interviews.
  - Coded recorded interviews for expectations and examples of the initial list of classroom norms.
  - Observed the large lecture and one discussion section of each TA to provide more context of classroom norms interviewees commented on.
- **Products**
  - Final list of relevant Precalculus classroom norms framed in terms of what instructors and students come to "expect" and are "aware" of.
  - Examples, that is, "actions" of relevant classroom norms.
  - Pool of instructor and student interviewees.
  - Observational fieldnotes to provide a vignette of the discussion sections and the large lecture.

QUAN

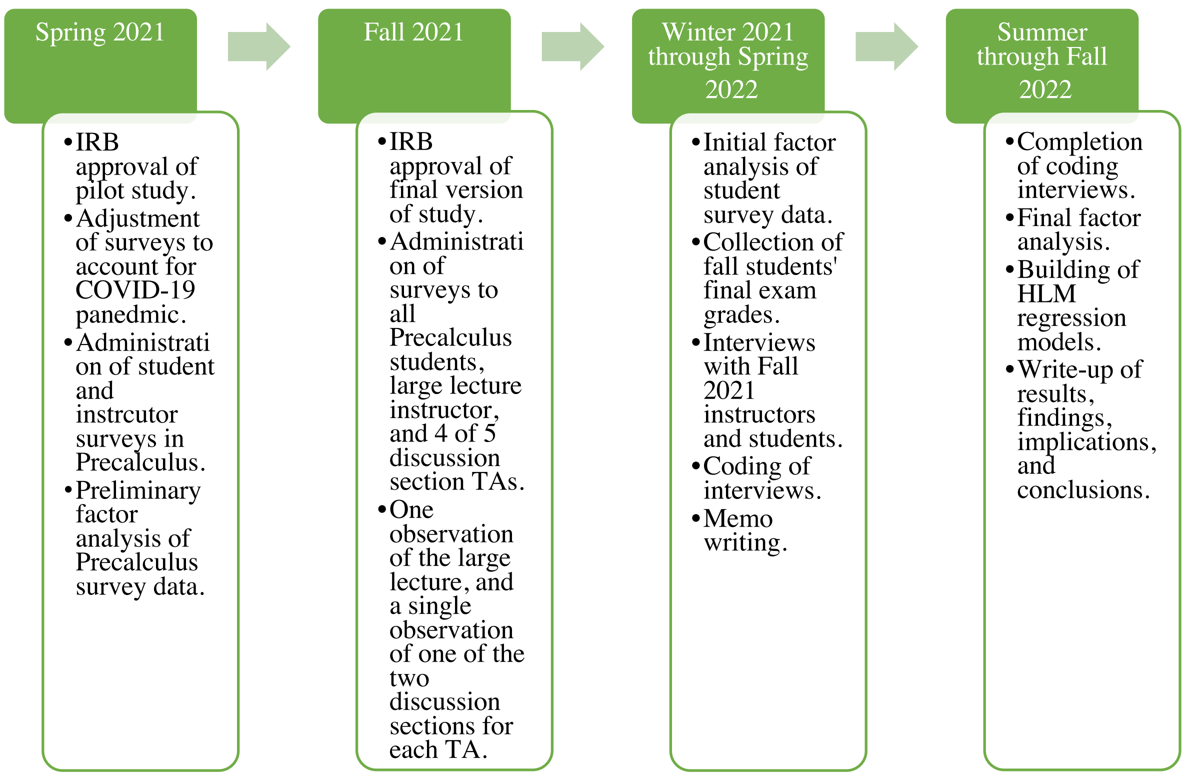
- **Procedures/Instruments**
  - Reran factor analysis of relevant classroom norms (i.e., survey items interviewees said were representative of Precalculus) to form the final set of classroom norms for Precalculus from the perspective of instructors *and* students.
  - Collected final exam grades.
  - Built HLM models with students nested within discusion sections.
- **Products**
  - Awarenes scores for instructors and students for each dimensions of teaching.
  - Factor scores for each dimension of teaching that were used in predicting student outcomes in the regression models.
  - Hierarchical linear regression models for self-efficacy and achievement outcomes.
  - Logistic regression model for STEM intentions outcome.

Note. The above is a visual representation of the quantitative and qualitative components of the mixed-methods study. Products highlighted in green in Figure 4 represent components of the study that were used in answering the research questions.

Having presented each component of the study, I now present a detailed description of the methods of analysis to answer the research questions. Additionally, I comment on why each component is useful in answering the research questions, as well as any limitations of the component. I offer Figure 5 as a visual guide in understanding the procedures, instruments, and products of each part of the study, including where each component falls in its placement of the QUAN-QUAL-QUAN design. I also provide an overview of the study timeline in Figure 5.

Figure 5

Study Timeline



3.7.1. Preliminary Norms and Dimensions of Teaching (QUAN→QUAL)

To begin, I conducted a factor analysis of student responses to the 22 SPIPS-M items, or preliminary Precalculus classroom norms, resulting in an initial set of factors. Initial analysis of students'[29] responses with regards to the discussion sections suggested a three-factor solution with eigenvalues greater than 1, satisfying Kaiser's criterion (as cited in Field et al., 2012). This criterion also resulted in an initial three-factor solution for the large lecture. These factors captured different dimensions of teaching for the large lecture and discussion sections, separately. More specifically, the factors that resulted from a factor analysis of student responses to the individual PIPS-M survey items[30] with respect to either the large lecture or discussions sections described different aspects of teaching and learning in Precalculus. For instance, a factor analysis of students' responses with respect to the large lecture resulted in survey items that described having students explore varied representations, solutions, and explanations (among other activities) loading onto a single factor. In turn, I labeled this factor as "Variation in Instruction" to summarize this factor and its associated survey items.

Second, using the above set of six factors (three for the responses about the large lecture and three for the discussion sections), I conducted semi-structured interviews to capture the experiences of Precalculus instructors and students. I invited all instructors and students who had completed the PIPS-M to semi-structured interviews, including two students of each TA (one from each of their sections). During the interviews, I asked students and instructors to comment on those classroom norms listed on the PIPS-M survey that were expectations of students/instructors in the Precalculus class. More specifically, the classroom norms were

[29] Listwise deletion was implemented in the preliminary factor analysis. No more than two of the 181 responses (1.1%) were missing from any one of the 22 items for students' responses with respect to the large lecture, while no more than 8 (4.4%) were missing for any one of the 22 items with respect to the discussion sections.
[30] Or eventual classroom norms, once I considered student and instructor perspectives.

presented in their different dimensions of teaching (i.e., the resulting factors) specific to the large lecture or discussion sections. I summarized these factors into a set of short statements to capture the norms represented in each dimension of teaching and provide some structure to the interview. For example, Factor 1 of the preliminary factor analysis of the large lecture resulted in the following classroom norms from the PIPS-M being grouped together.

1) Multiple approaches to solving a problem are discussed in class.
2) The instructor explains concepts in this class in a variety of ways.
3) The instructor adjusts teaching based upon what the class understands and does not understand.
4) I have enough time during class to reflect about the processes I use to solve problems.
5) Class is structured to encourage peer-to-peer support among students (e.g., ask peer before you ask instructor, having group roles, developing a group solution to share).
6) I receive feedback from my instructor on homework, exams, quizzes, etc.

The summarizing statement for these classroom norms read as follows: The instructor is expected to provide student feedback and explain concepts in a variety of ways to address students' misunderstandings; Students are expected to use class time and peers to reflect on solution processes. For each dimension of teaching, participants were asked the following questions, in addition to follow-up questions to receive clarity and ensure my understanding of interviewees' comments: 1) To what extent would you say these practices[31] are an expectation you have of the instructor/students, or they have of you; 2) Are there any practices that stand out to you that are/aren't a part of your class?

The framing of items in terms of expectations and what is or is not a part of the Precalculus class follows Çakır and Akkoç's (2020) framing of "expectations and actions as evidence of a norm" (p. 23). However, given the focus of this study on those classroom norms that are present in the Precalculus class, the identification of actions (i.e., the implementation) of

[31] Note that while I used the term practices in the interviews, these practices served as the classroom norms of Precalculus given I eventually considered instructor *and* student responses.

the various classroom norms took precedence over what participants expected. In other words, if instructors or students did not expect a norm yet commented that the norm was a part of the class, the norm remained as a part of the final data set analyzed in answering the research questions. Framing the potential norms as expectations primarily served as a conversation starter for students and instructors to share, from their perspectives, what went on within the discussion sections and large lecture.

As an example of how I identified the actions or expectations of a preliminary classroom norm, the large lecture instructor commented during her interview on the use of graphs and algebraic structures to help students approach problems using different strategies when discussing how to find the equation of a given sinusoidal graph. I marked this timestamp with the concept code of *varied representation*. As Saldaña (2015) suggested, concept coding uses "a word or short phrase that symbolically represents a suggested meaning broader than a single item or action, a 'bigger picture' that suggests an idea rather than an object or observable behavior" (p. 387). In other words, timestamps marked with the code *varied representation* did not provide insight into the extent this norm was a part of Precalculus by, for instance, counting the number of moments in interviews this norm was discussed. Instead, I used the concept code of *varied representation* to revisit these moments and use the words of the interviewees to express, from their perspective, the extent the classroom norm of *varied representation* was a part of the class. In addition, I synthesized interviewees' comments to collect examples of the norm in action. In the end, I used the words of the interviewees and my synthesis of examples as a measure of a norm being present in either the large lecture or discussion sections. When appropriate, I also used my observational fieldnotes to provide more context to these examples, including transcriptions of interactions between students and instructors.

In addition to presenting the preliminary classroom norms to interviewees, I shared a summary of what occurred during the large lecture and discussion section specific to the TA and students I was interviewing. Upon sharing the summaries, I asked participants to comment on how representative the class I observed was of other class meetings throughout the semester. I present these observational summaries as vignettes (see Appendices E-J) to provide the reader with a brief description of what a 'typical' day in the large lecture and each of the discussion sections looked like in terms of what instructors and students did. In general, all participants shared that the vignettes were representative of a typical day in their classroom.

3.7.2. Removal of Classroom Norms (QUAL→QUAN)

Following the interviews, I marked classroom norms on the PIPS-M/SPIPS-M that students and instructors noted as not relevant to either the Precalculus large lecture or discussion sections. More specifically, classroom norms were not included in the identification of the final set of classroom norms describing Precalculus if the following criteria were met.

- For the large lecture, the instructor and at least two of the students commented that a norm was not a part of the class, using language such as 'that didn't really happen' or 'we didn't have that in class.'
- For the discussion sections, at least two of the TAs and two of the students commented that a norm was not a part of the class.

Given the purposeful selection described above of how students were selected to be interviewed, as well as the number of instructors participating in this study, the criteria above seemed reasonable for the purposes of this study. As an example of a removal of a norm, the large lecture instructor shared she did not know most students' names and multiple students confirmed this fact. Thus, student and instructor responses to this survey norm was removed before further analysis of the large lecture PIPS-M responses.

In addition to the above criteria, single norms that made up an entire factor (i.e., dimensions of teaching) were removed from further analysis, followed by removing of norms that loaded with only one other norm and were also deemed[32] as generic in wording. For instance, *guiding students*, or “I guide students through major topics as they listen,” was removed from further analysis of the large lecture responses given that this norm loaded onto a factor with only one other norm, and may be interpreted by students and instructors differently depending on their perspective of what “guiding” entails (indeed, students shared that guiding involved a back-and-forth between TAs and students in the discussion sections). To clarify, other norms that still loaded by themselves or with one other norm, loaded onto more than one factor, and were, according to participant interview data, relevant to the respective classroom spaces, were not removed. Other norms deemed generic in wording that had low loadings on their respective factor were then removed (e.g., *student reflections* in the discussion responses). Finally, of the removed norms, those whose average response from *either* the instructor or student perspective were above a score of 2 (i.e., “Somewhat descriptive”) were considered for being put back into the set of classroom norms used in the final analysis. However, if the return of any of the norms meeting this criterion caused any of the existing norms to load onto more than one factor or led to the creation of an additional factor with two or fewer norms, these norms were not reintroduced into the respective data pool (i.e., data analyzed for the large lecture or discussion sections).

To clarify, I did not expect instructors and students to comment on all classroom norms from the PIPS-M during the interviews for two main reasons. First, time constraints may prevent

[32] A colleague not associated with the current study assisted in deeming whether the items were generic in wording. In addition, this colleague helped in conducting the final factor analysis, building the hierarchical models described in later sections, and ensuring the proper statistical assumptions were met in the analysis.

instructors and students from being able to fully comment on all the norms. Second, some classroom norms may simply not be at the forefront of instructors' and students' experiences within the classrooms. As demonstrated by Çakır and Akkoç (2020), not all norms initially considered to possibly be present in a classroom are observed in the "negotiation among teacher and students" (p. 30). Even so, a classroom norm on the PIPS-M not being mentioned during the interviews did *not* disqualify the norm from future analysis as some norms are "experienced" by not necessarily "taught" (Code et al., 2016, p. 917). That is, some norms are implicitly[33] a part of the classroom and thus may not be highlighted by instructors and students. Thus, only if a classroom norm on the PIPS-M met the above criteria was it be removed from future analysis.

Following the removal of any classroom norms, I reran the factor analysis of student responses to develop a finalized set of relevant classroom norms specific to either the large lecture or discussion sections. The resulting factor scores (with each factors represented a different dimension of teaching) for students were regressed onto the student outcomes of achievement, self-efficacy, and STEM intentions in the HLM analysis.

3.7.3. Awareness Scores

To clarify, in Çakır and Akkoç's (2020) analysis, "awareness" referred to students' awareness of the their teacher's expectations. However, I more broadly use the term awareness to refer to what students expect of the teacher, what the teacher expects of them, and what classroom norms are a part of the Precalculus classroom. In other words, students' awareness in the current study is measured by students' experience (i.e., the extent a norm is present, either in the large lecture or discussion sections) and encompasses the teacher and student actions of that norm. This perspective matches the wording of the PIPS-M items that ask students to "indicate

[33] For example, students may work with one another because the tables in a classroom are grouped together, and not necessarily because the instructor explicitly asked for this.

the degree to which the following statements describe your experience" for each classroom norm. Moreover, my adaptation[34] of the Çakır and Akkoç's framework follows my presentation of the initial set of relevant classroom norms during student interviews that included all norms (i.e., items) from the survey, whether they were more focused on what the teacher does or what the students do.

3.7.4. Final List of Classroom Norms

To summarize, factor analysis of the student responses to SPIPS-M norms about the large lecture resulted in a Kaiser–Meyer–Olkin (KMO) measure of 0.84, suggesting 181 students was a sufficient sample size for factor analysis. In addition, according to Bartlett's test for sphericity, $\chi^2(78) = 852.95, p < 0.001$, the individual SPIPS-M norms were statistically significantly correlated with each other to justify factor analysis. An analysis of the scree plot suggested three factors with Eigenvalues of at least 0.90 could be used to explain the variance of student responses to the large lecture survey items describing the classroom norms. The three factors, or dimensions of teaching, were retained to explain 59.47% of the variance for 13 of the original 22 SPIPS-M norms following the above removal process. Oblique rotation was used to account for correlation between factors (0.30 between Factors 1 and 2, 0.47 between Factors 1 and 3, and 0.17 between Factors 2 and 3). The following norms were removed with regards to large lecture responses.

- SPIPS-M Norms Removed from Final Large Lecture Factor Analysis:
    - Connecting Content, Student Names, Immediate Feedback, Sense of Community, Peer to Peer Support, Participation Strategies, Student Reflection, Guiding Students, Individual Work

[34] To clarify, I include both instructor and student awareness scores, deviating slightly from Çakır and Akkoç's (2020) framework that only defines students' awareness of a norm. In the end, having an awareness score for both instructors and students proved helpful in having a measure of how instructor- or student-centered their classroom was when considering a specific norm.

A factor analysis of the discussion section responses resulted in a KMO of 0.93 and $\chi^2(105) = 1974.71, p < 0.001$ for Bartlett's test of sphericity. An analysis of the scree plot suggested three factors with Eigenvalues of at least 0.90 could be used to explain the variance of student responses to the discussion section survey items describing the classroom norms. Three factors (dimensions of teaching) were retained to explain 71.33% of the variance for 15 of the original 22 SPIPS-M norms following the above removal process. Oblique rotation was used to account for correlation between factors (0.39 between Factors 1 and 2, 0.55 between Factors 1 and 3, and 0.50 between Factors 2 and 3). The following norms were removed with regards to discussion responses.

- SPIPS-M Norms Removed from Final Discussion Section Factor Analysis:
  - Connecting Content, Immediate Feedback, Student Reflection, Sense of Community, Guiding Students, Individual Work, Wide Student Response to Questions

Using the perspectives of both instructors and students, the final set of SPIPS-M norms used in answering the research questions guiding this study are included in Table 2, along the following information: a description of the different dimensions of teaching represented by the factors, specific to either the large lecture or discussion sections; Cronbach's alpha as a measure of reliability for the overall student responses with respect to the large lecture and discussion sections; Cronbach's alpha for each of the underlying dimensions of teaching; and each norm's factor loading. Following suite with practices in the literature (Field et al., 2012), factor loadings above 0.30 were considered significant. As the results suggested, each dimension of teaching was a reliable measure of the underlying classroom norms. The Cronbach alpha for each dimension of teaching was at least .70, indicating the classroom norms loading onto each dimension reliably measured the same aspect of teaching and learning. With the exception of *instructor feedback on* assignments and *constructive* criticism, all norms that remained in the pool of final classroom norms loaded onto equivalent dimensions of teaching for both the large lecture and discussion sections.

Table 2

Final Classroom Norms by Classroom Space and Dimension of Teaching

Classroom Space (Cronbach's α)	Dimension of Teaching (Cronbach's α)	Description[35]	Classroom Norms from PIPS-M Items (item loadings)
Large Lecture (.85)	Factor 1: Variation in Instruction (.81)	Instructors are expected to adjust their instruction based upon what students currently do or do not understand, as we as provide explanations and feedback on assignments (e.g., homework, quizzes, and exams). Students and instructors are expected to engage with multiple solutions to problems and representations of course topics/problems.	Varied Representation (.84) Multiple Solutions (.74) Varied Explanations (.67) Instructional Adjustments (.58) Instructor Feedback on Assignments (.43)
	Factor 2: Student-to-Student Collaboration (.76)	Students are expected to work both individually and in small groups where they provide constructive criticism to one another, as well as discuss course topics and their mathematical difficulties.	Mathematical Difficulties (.88) Small Groups (.72) Students Talking About Topics (.77) Constructive Criticism (.60)

[35] As with the preliminary factors presented to participants during the interviews, each description provided in Table 2 acts as a summarizing statement to reference the expectations and actions related to the given classroom norms. However, as mentioned in overview of research of classroom norms (see chapter three), assigning expectations and actions in this way does not place the responsibility of the enactment of a norm on the instructor or students separately given that classroom norms are "jointly established" (Cobb & Yackel, 1996, p. 178). Instead, these descriptions are meant to capture the general characteristics of the classroom norms associated with a dimension of teaching, while providing language to guide discussions around what a dimension of teaching entails.

	Factor 3: Instructor-to-Student Engagement (.76)	The instructor is expected to ask students questions, while a wide range of students are expected to participate through responding to these questions and sharing their ideas during whole class discussions.	Wide Participation (.87) Whole-Class Sharing (.76) Wide Student Response to Questions (.68) Student Response to Questions (.60)
Discussion Sections (.94)	Factor 3: Variation in Instruction (.87)	Instructors are expected to guide students through major topics as students listen and engage with multiple solutions to problems and representations of course topics/problems. Instructors are expected to offer varied explanations and adjust their instruction based upon what students currently do or do not understand.	Varied Representation (.48) Varied Explanations (.83) Instructional Adjustments (.85) Multiple Solutions (.59)
	Factor 2: Student-to-Student Collaboration (.90)	Students are expected to work individually on problems while also supporting each other during small group work as they provide constructive criticism to other's ideas, discuss course topics, and talk about their mathematical difficulties.	Mathematical Difficulties (.92) Small Groups (.88) Students Talking About Topics (.89) Peer-to-Peer Support (.60)
	Factor 1: Instructor-to-Student Engagement (.90)	The instructor is expected to ask students questions while implementing participation strategies and using students' names. The instructor is also expected to provide feedback on assignments (e.g., homework, quizzes, and exams). A wide range of students are expected to participate through responding to their instructor's questions, sharing their ideas during whole class discussions, and constructively criticizing other's ideas.	Wide Participation (.71) Whole-Class Sharing (.89) Student Response to Questions (.62) Participation Strategies (.62) Student Names (.34) Instructor Feedback on Assignments (.51) Constructive Criticism (.79)

3.7.5. Missing Data

Before running the regression models described in the next section, a review of the student SPIPS-M survey data revealed that 26% (n = 47) of the 181 participating Precalculus students did not consent to sharing their coursework, which included final exam grades. In addition, each of the 20 self-efficacy items were missing no more than five responses (2.8%, n = 181). Finally, nine students left multiple SPIPS-M items blank. As a result, I used multiple imputation to replace the final exam grades of those students who did not consent to sharing their coursework, as well as those missing self-efficacy or SPIPS-M responses, with the average outcome generated from five imputations of student demographic and attendance data. More specifically, I used the following variables in the multiple imputation.

- Attendance in large lecture
- Attendance in discussion
- Gender
- Race & Ethnicity
- Sexual Orientation
- Special Population (e.g., commuter, veteran, etc.)
- Existing final exam grades, self-efficacy responses, and SPIPS-M responses

The imputation resulted in a complete set of 181 cases used in conducting the regression analysis described below.

3.7.6. Classroom Norms Regression Analysis

After identifying relevant classroom norms in both the large lecture and discussion sections of Precalculus, as well as imputing missing data, I generated hierarchical regression models to comment on the extent the classroom norms, grouped by dimension of teaching, predicted students' academic achievement, self-efficacy, and STEM intentions. While the intraclass[36] correlation (ICC) suggested that variation in students' final exam scores, self-efficacy, and STEM intentions was not explained by students being nested within the sections of different TAs, the use of hierarchical models in answering the current research questions agrees with the theoretical assumption that, even if minimally, there may be variations between the different discussion sections for each of the student outcomes in question (see Peters, 2013 for a similar argument). Given that the self-efficacy and achievement outcomes are continuous variables, I used hierarchical linear modeling (HLM) to explore the relationships between the Precalculus classroom norms and students' self-efficacy and achievement. As for STEM intentions, because this outcome is categorical, I used hierarchical logistic regression to model the likelihood of participants' intentions to major in a STEM field. Level one of each model consisted of individual student data while level two consisted of classroom data (i.e., variations in the outcome variable accounted for by the TA a student had). At the student level, I included the factor scores for each of the dimensions of teaching in the large lecture and discussion sections. I also included variables for race and gender at the student level given empirical evidence (see Chapters One and Two) that historically underrepresented student in STEM may have different experiences in STEM courses. In addition, I included a binary variable for low[37] attendance following comments from the large lecture instructor and one of the TAs during the

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[36] A multilevel model with no predictors was first used to calculus the ICC for each student outcome. Results suggested an ICC of approximately 0% ($1.7\ x\ 10^{-12}$) for final exam scores (i.e., academic achievement), and 1.5% for students' standardized self-efficacy scores. Given that the outcome variable in logistic regression is binary (in this case, STEM or non-STEM intending), a pseudo-ICC for was calculated using the variance of a logistic distribution ($\frac{\pi^2}{3} \approx 3.29$) and the estimated level-2 variance for students' STEM intentions (Goldstein et al., 2002, p. 7), resulting in a pseudo-ICC of approximately 0%.

[37] Students were coded as having low attendance if they missed at least one class per week in the large lecture (as indicated on the SPIPS-M survey). Alternatively, students were coded as having low attendance if they missed 4-6 discussion sections or at least one discussion section a week (some students had discussion twice a week for longer meeting times while others met three times per week for shorter meeting times).

interviews suggesting attendance was influential on students' performance in the class. Below I provide a list of the dependent and independent variables included in the models.

- Dependent Variables:
    - Student achievement - final exam grades as a percentage.
    - Student self-efficacy – standardized variable of the sum of students' total self-efficacy responses out of 2000.
    - Students' STEM intentions - categorical response to SPIPS-M STEM intentions item (i.e., 0 = non-STEM intending, 1 = STEM intending).
- Independent Variables:
    - Dimension of teaching representing classroom norms – factors (separate for large lecture and discussion sections) from analysis of student responses to the PIPS-M items remaining after analysis of the instructor and student interviews.
    - Student demographics[38] variables from the SPIPS-M, including gender, race, and attendance in both the large lecture and discussion.

The following multilevel model was used to predict students' final exam grades, followed by a list summarizing the meaning of each of the variables when considering the reference group of White, male students not identified as having low attendance.

- Level-1 (Student Level) Achievement Model:

$$\begin{aligned} A_{ij} = \beta_{0\mathrm{j}} &+ \beta_{1\mathrm{j}} Female_{\mathrm{ij}} + \beta_{2\mathrm{j}} AfricanAmerican_{\mathrm{ij}} + \beta_{3\mathrm{j}} HispanicLatinx_{\mathrm{ij}} + \beta_{4\mathrm{j}} Asian_{\mathrm{ij}} \\ &+ \beta_{5\mathrm{j}} MultiRace_{\mathrm{ij}} + \beta_{6\mathrm{j}} LowAttendance_{\mathrm{ij}} + \beta_{7\mathrm{j}} LVariation_{\mathrm{ij}} \\ &+ \beta_{8\mathrm{j}} LCollaboration_{\mathrm{ij}} + \beta_{9\mathrm{j}} LEngage_{\mathrm{ij}} + \beta_{10\mathrm{j}} DVariation_{\mathrm{ij}} \\ &+ \beta_{11\mathrm{j}} DCollaboration_{\mathrm{ij}} + \beta_{12\mathrm{j}} DEngage_{\mathrm{ij}} + \varepsilon_{ij} \end{aligned}$$

- $\boldsymbol{A_{ij}}$ - the predicted final exam grade (as a percentage) for student i nested within classroom j taught by one of the five TAs ($j = 0\ to\ 4$)
- $\boldsymbol{\beta_{0j}}$ - the intercept term representing the within-TA average final exam grade for the sections led by TA j
- $\boldsymbol{\beta_{nj}}$ $(for\ n = 1\ to\ 12)$ - the regression coefficient for a given predictor for the sections led by TA j
- $\boldsymbol{\varepsilon_{ij}}$ - the difference in the predicted (A_{ij}) and observed final exam grade of student i nested within TA j (i.e., the level-1 error term)

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[38] Please note the following: 1) one student identified as gender fluid and male, but was placed into the reference group of male (all other students identified as male or female); 2) five students who preferred not to answer and one who did not provide their race/ethnicity were placed in the reference group of White; 3) three students identified as Middle Eastern or North African but were recoded as Asian; and 4) Asian consisted of students who identified as Central Asian, East Asian, Southeast Asian, and South Asian;

- Level-2 (Classroom Level) Achievement Model:

$$\beta_{0j} = \gamma_{00} + u_{0j}$$

$$\beta_{nj} = \gamma_{\mathrm{n}0} + u_{nj}\ (for\ n = 1\ to\ 12)$$

- $\boldsymbol{\gamma_{00}}$ - the grand mean final exam grade for all students, irrespective of TA
- $\boldsymbol{u_{0j}}$ - the difference between the grand mean final exam grade and the average final exam grade of students nested within TA j (i.e., the level-2 error term)
- $\boldsymbol{\gamma_{\mathrm{n}0}}$ - the grand mean value for predictor β_n (for each of the 12 predictors) across all TAs
- $\boldsymbol{u_{nj}}$ - the difference between the grand mean for predictor β_n and the average value of predictor β_n of students nested within TA j (that is, β_{nj})

The self-efficacy model exactly matched that of students' final exam grades, with the exception of the outcome variable being the standardized self-efficacy score of student i nested in TA j, or SE_{ij}. Lastly, the STEM[39] Intentions model was an adjusted version of the above models for final exam grades and self-efficacy. The variables in the equation below correspond to those of the above equations but for the natural logarithm of a logistic function. More specifically, the outcome of the following function evaluates the probability, or $P(M_{ij})$, that student i nested in TA j said they intended to major in a STEM field based on the included predictors.

[39] Note that five students selected "Unsure" to the question of if they had declared or intended to declare a STEM major, while once student said they "Preferred not to disclose." These students were recoded as non-STEM intending majors.

- STEM Intentions Model:

$$P(M_{ij}) = \frac{1}{1 + e^{-(var)}}$$

- $\boldsymbol{P}(\boldsymbol{M_{ij}})$ – the probability that student i nested in TA j said they intended to major in a STEM field
- $\boldsymbol{var} = \beta_{0\mathrm{j}} + \beta_{1\mathrm{j}} Female_{\mathrm{ij}} + \beta_{2\mathrm{j}} AfricanAmerican_{\mathrm{ij}} + \beta_{3\mathrm{j}} HispanicLatinx_{\mathrm{ij}} + \beta_{4\mathrm{j}} Asian_{\mathrm{ij}} + \beta_{5\mathrm{j}} MultiRace_{\mathrm{ij}} + \beta_{6\mathrm{j}} LowAttendance_{\mathrm{ij}} + \beta_{7\mathrm{j}} LVariation_{\mathrm{ij}} + \beta_{8\mathrm{j}} LCollaboration_{\mathrm{ij}} + \beta_{9\mathrm{j}} LEngage_{\mathrm{ij}} + \beta_{10\mathrm{j}} DVariation_{\mathrm{ij}} + \beta_{11\mathrm{j}} DCollaboration_{\mathrm{ij}} + \beta_{12\mathrm{j}} DEngage_{\mathrm{ij}} + \varepsilon_{ij}$

Chapter 4. Results

This chapter presents my findings with respect to the four research questions presented in Chapter One. In answering research question one, I used descriptive statistics of students' PIPS-M survey responses to measure the extent students perceived the classroom norms of Precalculus as being instructor- or student-centered along the teaching dimensions of *student-to-student collaboration, instructor-to-student engagement,* and *variation in instruction*, separately for the large lecture and discussion sections. Results suggested that, in general, students perceived a hybrid of instructor- and student-centered norms in the large lecture and discussion sections, with more instructor-centered norms tending to be implemented in the large lecture and more student-centered in the discussion sections. In answering research question two, I used instructor and student awareness scores from the PIPS-M to compare the extent Precalculus instructors and students perceived the classroom norms as instructor- or student-centered along each teaching dimension. For the most part, instructors' perceptions of classroom norms in the large lecture and discussion sections aligned with students, with some variation depending on the dimension of teaching and, for the discussion sections, the TA in question. In answering question three, I used data from the instructor and student interviews to extend the survey results by providing examples of classroom norms in the large lecture and discussion sections. Finally, in answering research question four, I used students' factor scores for each of the six-total teaching dimensions (three for the large lecture and three for the discussion sections; see Table 2) to check for predictive relationships between the perceived norms of each dimension and students' academic achievement, self-efficacy, and STEM intentions. The hierarchical models showed significant relationships between students' perceptions of norms under two of the large lecture dimensions of teaching, one of the discussion section dimensions of teaching, and two of the three outcomes of interest.

I begin this chapter by answering the first research question by providing descriptive statistics of students' responses to the individual PIPS-M items. These responses were also used in the hierarchical models predicting the different student outcomes.

4.1. Results Part I: Surveyed Students' Perceptions of Classroom Norms

The first research question guiding this study was as follows: To what extent are the classroom norms of the large lecture and discussion sections of an undergraduate precalculus course instructor- or student-centered, as perceived by surveyed students? In answering the first research question, I used students' PIPS-M survey results. As an overview, Table 3 and Table 5 present the mean student response of each of the classroom norms listed on the PIPS-M, organized by classroom space and teaching dimension.

Table 3

Descriptive Statistics of Student Survey Responses for Large Lecture

Dimension of Teaching	Classroom Norm from SPIPS-M Survey	Mean Student Response (SD)
Variation in Instruction	Varied Representation – In my class a variety of means (models, drawings, graphs, symbols, simulations, tables, etc.) are used to represent course topics and/or solve problems.	2.19 (1.19)
	Multiple Solutions – Multiple approaches to solving a problem are discussed in class.	2.00 (1.13)
	Varied Explanations – The instructor explains concepts in this class in a variety of ways.	1.42 (1.21)
	Instructional Adjustments –	1.19 (1.21)

	The instructor adjusts teaching based upon what the class understands and does not understand.	
	Instructor Feedback on Assignments – I receive feedback from my instructor on homework, exams, quizzes, etc.	1.59 (1.17)
Student-to-Student Collaboration	Mathematical Difficulties – I discuss the difficulties I have with math with other students during class.	2.51 (1.28)
	Small Groups – I work with other students in small groups during class.	2.22 (1.27)
	Students Talking About Topics – I talk with other students about course topics during class.	2.40 (1.21)
	Constructive Criticism – I constructively criticize other student's ideas during class.	1.03 (1.12)
Instructor-to-Student Engagement	Wide Participation – A wide range of students participate in class.	1.19 (1.05)
	Whole-Class Sharing – I share my ideas (or my group's ideas) during whole class discussions.	1.44 (1.28)
	Wide Student Response to Questions – A wide range of students respond to the instructor's questions in class.	1.15 (0.92)
	Student Response to Questions – I am asked to respond to questions during class time.	1.07 (1.04)

Note. The above classroom norms were taken from students' responses to the individual SPIPS-M items. The items are organized by the dimension of teaching, that is, the factor an item loaded onto, with respect to either the large lecture or discussion sections. The survey scale was the following: (0) Not at all descriptive of my experience; (1) Minimally descriptive; (2) Somewhat descriptive; (3) Mostly descriptive; (4) Very descriptive.

As a general guide, I used the following cutoffs (see Table 4) to consistently interpret the mean student responses for each norm. Recall that the norms that remained after removal of those that were not relevant to either the large lecture or discussion sections, according to instructor and student interviews, were student-centered in nature (see Table 1 in Chapter Three). Thus, a higher response value for a given norm (e.g., a response of 4 indicated a student perceived a given norm as "Very descriptive" of the large lecture or their discussion section) was interpreted as students perceiving a student-centered implementation of that norm.

Table 4

Interpreting Mean Student Survey Responses

Mean Student Response	Interpretation
$0 \leq Mean < 1.5$	Instructor-Centered
$1.5 \leq Mean < 2.5$	A Hybrid of Instructor and Student-Centered
$2.5 \leq Mean < 4$	Student-Centered

In considering the dimension of *variation in instruction*, students perceived classroom norms of the large lecture as ranging from instructor-centered to a hybrid of instructor- and student-centered. As an example of an instructor-centered norm, the average student reported the large lecture instructor, Dr. Riemann (pseudonym), making minimal *instructional adjustments* ($M = 1.19,\ SD = 1.21$). This suggests that Dr. Riemann guided students through a fixed set of problems as opposed to adjusting the problems, or some related aspect of instruction, to address questions in the large lecture. As for student-centered norms, the average student reported *varied representations* ($M = 2.19, SD = 1.19$) and exploring *multiple solutions* ($M = 2.00, SD = 1.13$) as being at least somewhat descriptive of the large lecture.

Survey responses also suggested that students perceived a hybrid of instructor- and student-centered norms related to the large-lecture teaching dimension of *student-to-student* *collaboration*. For instance, the average student noted that *students talking about topics* ($M = 2.40, SD = 1.21$) and working in *small groups* ($M = 2.22, SD = 1.27$) ranged between somewhat and mostly descriptive of the large lecture. Still, student responses suggested there was little time in the large lecture for students to *constructively criticize* ($M = 1.03,\ SD = 1.12$) classmates' ideas during class, or more broadly, work with others.

Of all the large-lecture dimensions of teaching, responses related to *instructor-to-student engagement* suggested students perceived the associated norms as the most instructor-centered. More specifically, the average student perceived these norms as being minimally descriptive of the large lecture and none of them being somewhat descriptive. For example, the average student reported minimal time for *student response to questions* ($M = 1.07, SD = 1.04$). For those students who did respond to questions in the large lecture, this may have only been true for a select few of the students given *wide student response to questions* ($M = 1.15,\ SD = 0.92$) was perceived by students as being minimally descriptive of the large lecture.

Table 5

Descriptive Statistics of Student Survey Responses for Discussion Sections

Teaching Dimension	Classroom Norm from SPIPS-M Survey	Mean Student Response (SD)
Variation in Instruction	Varied Representation – In my class a variety of means (models, drawings, graphs, symbols, simulations, tables, etc.) are used to represent course topics and/or solve problems.	2.22 (1.26)
	Varied Explanations – The instructor explains concepts in this class in a variety of ways.	2.04 (1.33)

	Instructional Adjustments – The instructor adjusts teaching based upon what the class understands and does not understand.	2.09 (1.38)
	Multiple Solutions – Multiple approaches to solving a problem are discussed in class.	2.17 (1.27)
Student-to-Student Collaboration	Mathematical Difficulties – I discuss the difficulties I have with math with other students during class.	2.71 (1.16)
	Small Groups – I work with other students in small groups during class.	2.81 (1.12)
	Students Talking About Topics – I talk with other students about course topics during class.	2.81 (1.12)
	Peer-to-Peer Support – Class is structured to encourage peer-to-peer support among students (e.g., ask a peer before you ask the instructor, having group roles, developing a group solution to share).	2.56 (1.25)
Instructor-to-Student Engagement	Wide Participation – A wide range of students participate in class.	1.92 (1.27)
	Whole-Class Sharing – I share my ideas (or my group's ideas) during whole class discussions.	2.00 (1.28)
	Student Response to Questions – I am asked to respond to questions during class time.	1.78 (1.32)
	Participation Strategies – My instructor uses strategies to encourage participation from a wide range of students.	1.84 (1.34)
	Student Names – The instructor knows my name.	1.89 (1.52)

Instructor Feedback on Assignments – I receive feedback from my instructor on homework, exams, quizzes, etc.	2.20 (1.21)
Constructive Criticism – I constructively criticize other student's ideas during class.	1.81 (1.33)

Note. The above classroom norms were taken from students' responses to the individual SPIPS-M items. The items are organized by the dimension of teaching, that is, the factor an item loaded onto, with respect to either the large lecture or discussion sections. The survey scale was the following: (0) Not at all descriptive of my experience; (1) Minimally descriptive; (2) Somewhat descriptive; (3) Mostly descriptive; (4) Very descriptive.

Survey responses related to the norms under the discussion teaching dimension of *variation in instruction* suggested students perceived a hybrid of instructor- and student-centered norms and, in comparison to the large lecture, slightly more student-centered norms. For example, the average student reported *instructional adjustments* ($M = 2.09,\ SD = 1.38$) and *varied explanations* ($M = 2.04, \mathrm{SD} = 1.33$) as at least somewhat descriptive of the discussion sections while students reported these norms as minimally descriptive in the large lecture. Put another way, students perceived the TAs and students in the discussion sections as spending some time during class exploring alternative explanations to problems. Moreover, according to students, the TAs sometimes adjusted their instruction based upon what students did or did not understand.

Similarly, the average student perceived the norms in discussion sections as being more student-centered than the large lecture when considering the discussion teaching dimension of *student-to-student collaboration*. For example, having students work in *small groups* ($M = 2.81,$ $\mathrm{SD} = 1.12$) and *talk about topics* ($M = 2.81,\ \mathrm{SD} = 1.12$) were reported by students as ranging between somewhat and mostly descriptive of the discussion sections. These results suggest students perceived the discussion sections as focused on students working with one another.

Finally, students reported classroom norms under the discussion teaching dimension of *instructor-to-student engagement* as ranging from student-centered to a hybrid of instructor- and student-centered. For example, the average student reported *whole-class sharing* ($M = 2.00$, $\mathrm{SD} = 1.28$) and receiving *instructor feedback on assignments* ($M = 2.20,\ \mathrm{SD} = 1.21$) as being at least somewhat descriptive of their discussion section, while the norms of having students engage in *constructive criticism* ($M = 1.81,\ \mathrm{SD} = 1.33$) and *respond to questions* ($M = 1.78$, $\mathrm{SD} = 1.32$) ranged between being minimally and somewhat descriptive of the discussion sections. These results suggest students perceived the TAs as dedicating at least some of the time in the discussion sections to having students engage in asking and answering questions, in addition to providing feedback to students.

4.2. Results Part II: Surveyed Instructors vs. Students' Perceptions of Norms

The previous section reported the extent the classroom norms of the large lecture and discussion sections of Precalculus were instructor- or student-centered, as perceived by surveyed students. In this section, I turn to a comparison of instructor and student survey responses to answer the second research question guiding this study: In comparing surveyed instructors to students, to what extent are the classroom norms perceived as instructor- or student-centered? To answer this question, I used the instructor and mean student awareness[40] scores on the PIPS-M survey for each of the large lecture and discussion section teaching dimensions and their corresponding classroom norms. Use of the awareness scores allowed for an efficient means to compare the classroom norms for each dimension of teaching as perceived by instructors and students. That is, awareness scores helped compare the classroom norms in the large lecture as

[40] Recall that participants' awareness scores (a percentage calculated as the sum of a participant's responses to norms associated with a teaching dimension divided by the maximum possible sum for that dimension) may be interpreted as the extent instructors or students agreed that the norms associated with a given teaching dimension were descriptive of either the large lecture or discussion sections.

perceived by instructors and students, as well as the TAs and students across the different discussion sections. Similar to the scale presented in Table 4, I used the following scale (see Table 6) to interpret awareness scores, with the main difference being that awareness scores were calculated as a percentage.

Table 6

Interpreting Awareness Scores

Awareness Score for Teaching Dimension	Interpretation
$0\% \leq Score < 40\%$	Instructor-Centered
$40\% \leq Score < 60\%$	A Hybrid of Instructor- and Student-Centered
$60\% \leq Score < 100\%$	Student-Centered

Continuing from the results of the previous section, in general, both the large-lecture instructor and her students reported instructor-centered to a hybrid of instructor- and student-centered norms under each large-lecture dimension of teaching. The main difference between the large-lecture instructor and students' perceptions of classroom norms related to the dimension of *variation in instruction*. As for the discussion sections, the TAs and their students reported classroom norms ranging from a hybrid of instructor- and student-centered, to student-centered. In the sections that follow, I present the instructor and student awareness scores under each dimension of teaching.

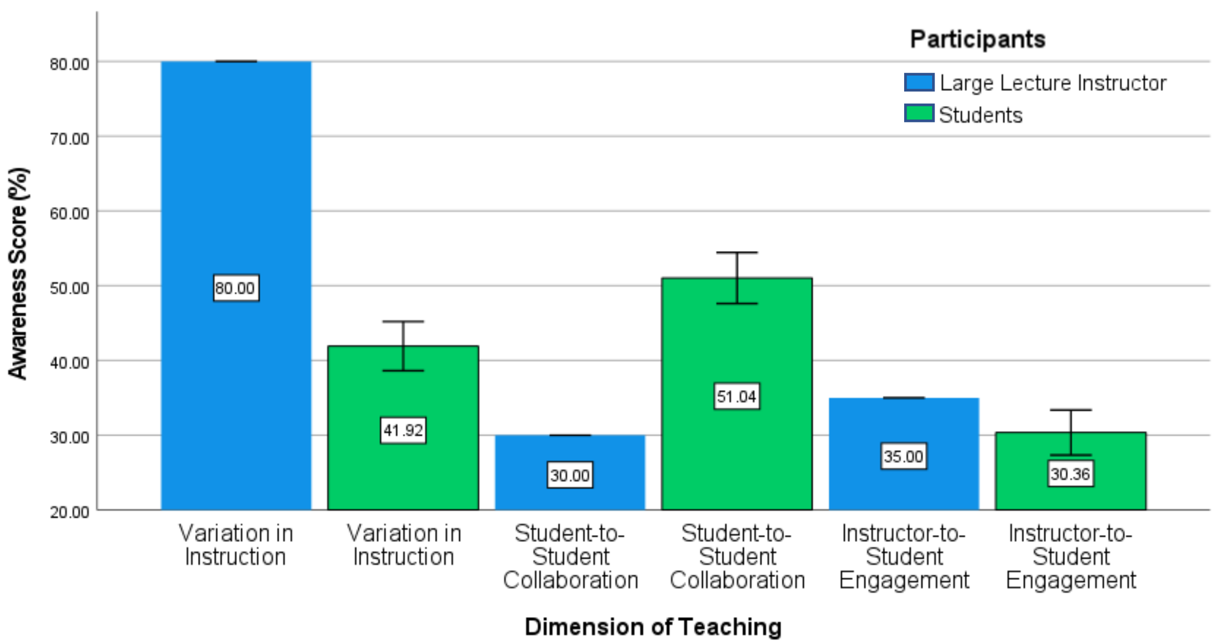
4.2.1. Large-Lecture Awareness Scores

Dr. Riemann's awareness score suggested that she perceived a relatively significant focus on student-centered norms (awareness score of 80%) under the teaching dimension of *variation in instruction* in the large lecture (see Figure 6). In contrast, as reflected in the mean student awareness score, students perceived (41.92%) a hybrid of instructor- and student-centered norms under this dimension. For the teaching dimension of *student-to-student collaboration*, Dr.

Riemann reported more instructor-centered norms (30%) compared to students' perceptions (51.04%) of a hybrid of instructor- and student-centered norms. That is, the large lecture was at times more student-centered and other times more instructor-centered when focusing on opportunities for students to collaborate, with students recalling more student-centered norms than Dr. Riemann under this dimension of teaching. For the teaching dimension of *instructor-to-student engagement*, both Dr. Riemann (35%) and students (30.36%) reported more instructor-centered norms under the dimension of *instructor-to-student engagement*.

Figure 6

Lecture Instructor vs. Mean Student Awareness Score by Dimension of Teaching



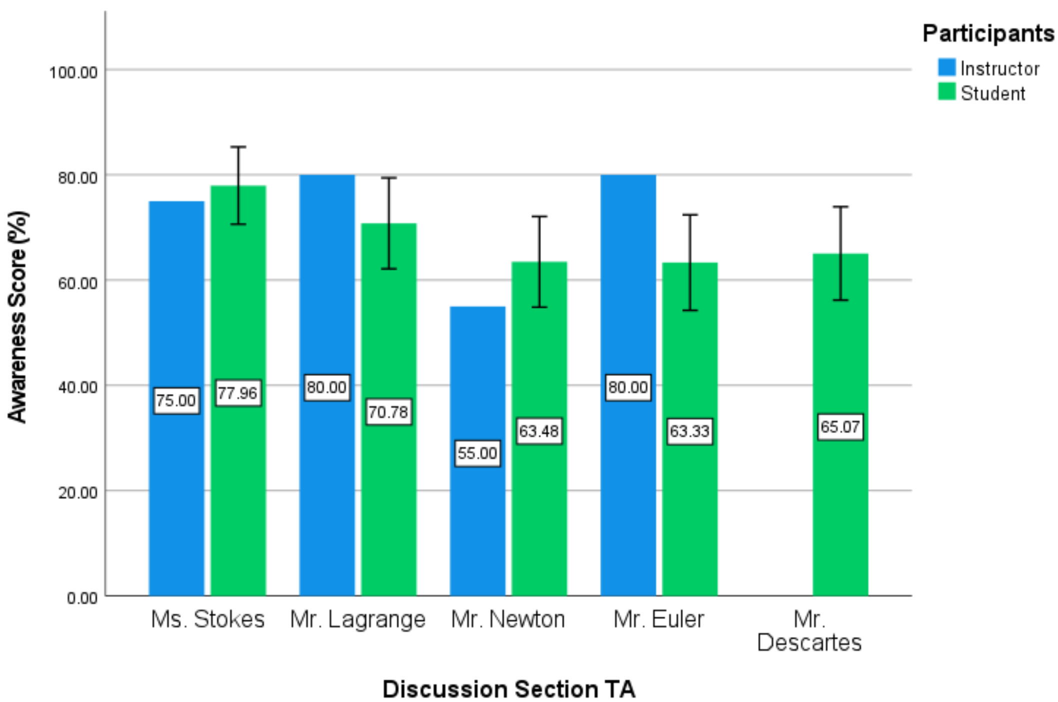
Note. Error bars are 95% confidence intervals for the mean student awareness score of students with respect to the teaching dimension indicated.

4.2.2. Discussion Sections Awareness Scores

Within the context of the discussion sections, instructors and students generally commented that classroom norms were more student-centered than the large lecture when considering the norms related to the dimension of *student-to-student collaboration*. As Figure 7 suggests, the mean student awareness score for the teaching dimension of *student-to-student* collaboration was fairly consistent across the discussion sections of different TAs. More specifically, students perceived student-centered norms related to the dimension of *student-to-student* collaboration, with awareness scores ranging from roughly 63% to 78%. Likewise, the awareness scores of most of the participating TAs depicted a more student-centered classroom when considering norms under *student-to-student collaboration*.

Figure 7

TA vs. Mean Student Awareness Score: Student-to-Student Collaboration

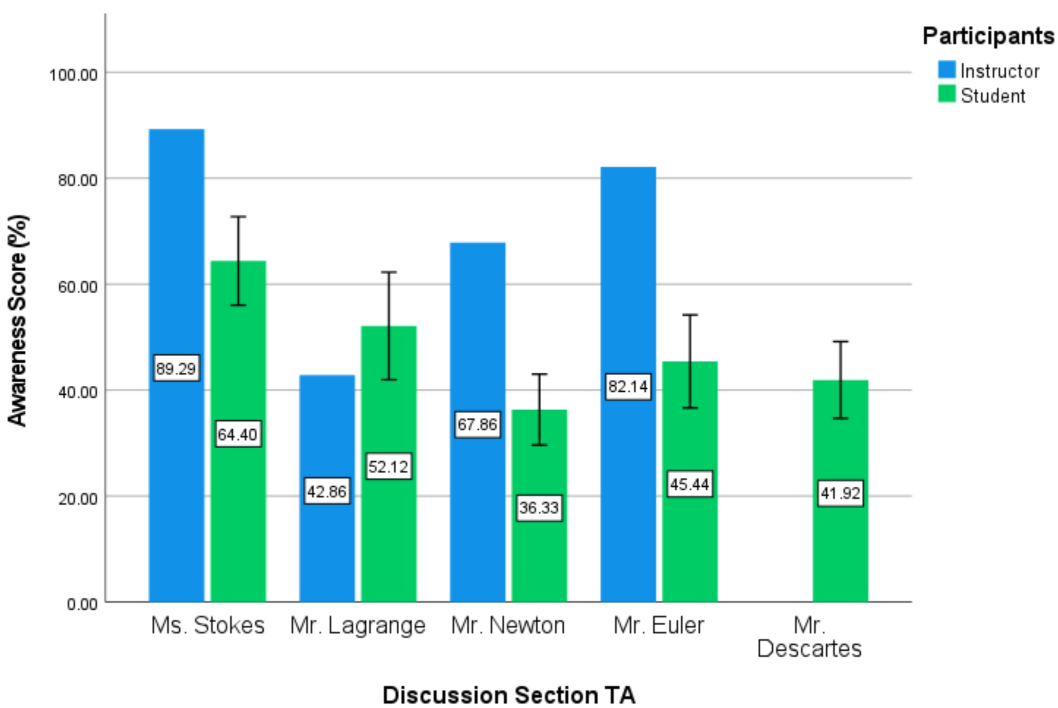


Note. Error bars are 95% confidence intervals for the mean student awareness score of students under the teaching dimension of *student-to-student collaboration*.

As for the dimension of *instructor-to-student* engagement within the discussion sections, with the exception of Ms. Stokes, the mean student awareness score suggested students perceived a hybrid or instructor- and student-centered norms (see Figure 8). Of all the TAs, Ms. Stokes' self-reported awareness score of the classroom norms related to *instructor-to-student engagement* (89.29%) was the highest, in addition to having the highest student awareness score (64.40%). In other words, Ms. Stokes and her students perceived the most student-centered norms related to *instructor-to-student engagement* in their discussion sections. In contrast, Mr. Newton's mean student awareness score (36.33%) suggested students perceived instructor-centered norms related to *instructor-to-student engagement*.

Figure 8

TA vs. Mean Student Awareness Score: Instructor-to-Student Engagement

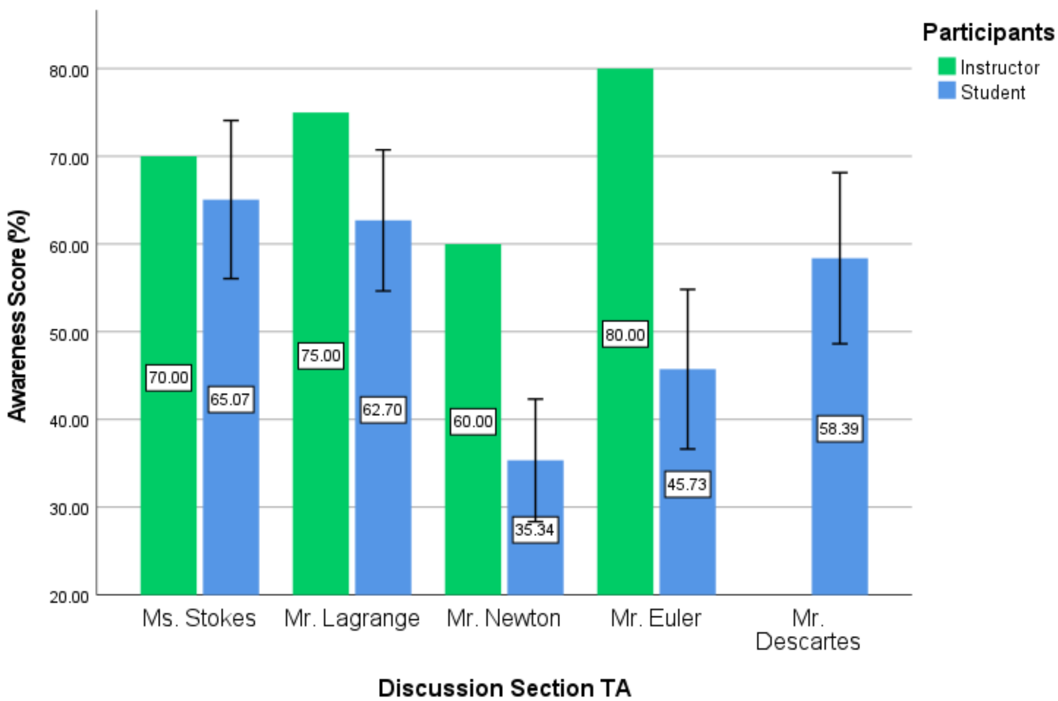


Note. Error bars are 95% confidence intervals for the mean student awareness score of students under the teaching dimension of *instructor-to-student engagement.*

Finally, students mostly perceived a hybrid of instructor- and student-centered norms under the teaching dimension of *variation in instruction*, while all TAs self-reported student-centered norms (see Figure 9). The significant different between Mr. Euler's awareness score and his mean student awareness score (a gap of roughly 35%) suggests Mr. Euler may have overestimated his implementation of student-centered norms under the teaching dimension of *variation in instruction*. Additionally, Ms. Stokes and Mr. Lagrange's similar awareness scores and mean student awareness scores point to both the instructors and students perceiving a similar level of student-centered norms in their discussion sections under the teaching dimension of *variation in instruction*.

Figure 9

TA vs. Mean Student Awareness Score: Variation in Instruction



Note. Error bars are 95% confidence intervals for the mean student awareness score of students under the teaching dimension of *variation in instruction*.

4.3. Results Part III: Interviewed Participants' Descriptions of Norms

The third research question guiding this study was the following: How do the instructors and students describe the classroom norms of the Precalculus course in interviews? In answering the third research question, I used instructor and student interview data to extend the survey results reported above. More specifically, I synthesized representative quotes to provide contextual examples of the classroom norms under each dimension of teaching. The following sections are organized by teaching dimension with respect to either the large lecture or discussions sections. I begin each section with a succinct summary of the awareness scores reviewed above to place the interview comments within the context of the instructor and student survey results.

4.3.1. Variation in Instruction: Descriptions of the Large Lecture

The instructor and mean student awareness score with respect to *variation in instruction* suggested Dr. Riemann perceived student-centered classroom norms within the large lecture while her students perceived a hybrid of instructor- and student-centered classroom norms. For Dr. Riemann, the "skill" of solving a problem in more than one way and considering alternative explanations was an important part of the class. One of the interviewed students made comments related to this skill in sharing that Dr. Riemann made "an environment where it was open to ask questions, and if someone had a question and they were confused in the way that she did it, she would try to explain it in a different way to offer a different perspective." As an example of this skill, Dr. Riemann shared that she had students consider the connection between algebraic and graphical representations of functions. Dr. Riemann's comments below express the importance she places on varied explanations, solutions, and representations.

> The fact that we can solve a problem more than one way, and we try to check things, not necessarily through the method we've worked through, I put a pretty strong emphasis on

> this. Like, how do I know that the result I got is correct? Does it make sense physically? Does it agree with where I started with, and so on and so forth? Because I think this like going forward, this is a useful skill for them. … It's an important skill to have.

In particular, Dr. Riemann made use of the online graphing tool, Desmos, to have students consider different representations of problems during, for example, the quizzes given in lecture (see in Appendix B for a description of the different assignments of the course). One of the interviewed students described this activity in the excerpt below.

> She [Dr. Riemann] did go through a bunch of different ways of solving and use a different bunch of different like, ways to show. Like she would use you know, Desmos for some things. And then she'd like solve using paper like, write out all her steps or whatever.

While Dr. Riemann's emphasis on alternative solutions, representations, and explanations suggested she perceived student-centered norms under the dimension of *variation in instruction* in the large lecture, students reported a much more hybrid implementation set of norms under this dimension of teaching. Of the norms loading onto this dimension of teaching, Dr. Riemann's comments with regards to *instructor feedback on assignments*[41], in particular, proved helpful in understanding why students may have perceived a more instructor-centered implementation of classroom norms related to *variation in instruction*. Consider the following comments from Dr. Riemann.

> And I'm sending them home with the instruction of reviewing their notes and re-reading the chapter. And if they haven't done already, re-work through the textbook examples. And if they find that it takes them long, or it's hard, to also practice a problem at the end of each example. And those problems come with a video [solution] in the electronic textbooks. So they have a way of checking that. Alright. But that's, you know … good students do this. Please do this. But I'm not checking on that. What I'm checking is a post-lecture quiz. … Some of them are doing the good thing of trying to do the post-lecture WebAssign thing and coming with questions to their TA, but not all of them.

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[41] To clarify, *instructor feedback on assignments* described the extent students received and instructors gave on "homework, exams, quizzes, etc." Considering the process of receiving feedback from the instructor involves reconsidering one's solutions and explanations to the problems in question, *instructor feedback on assignments* conceptually aligned with other norms under the teaching dimension of *variation in instruction*.

The above quote suggests that some of the responsibility for students' receiving feedback on different assignments was placed directly on the students. As another example, during my observation of the large lecture, Dr. Riemann told students to "treat the WebAssign homework as your own sort of self-quiz" given that it was automatically graded. However, as a broader reflection on the class, students shared that even the feedback they did receive directly from Dr. Riemann and the TAs seemed to be limited. As one interviewed student shared, "I don't think we ever received feedback on exams, homework, or quizzes, other than maybe like, oh, like I got a point off for this and like there's like a tiny little note." Another student similarly commented, "it wasn't really a feedback-heavy class as a whole, regardless of if it was discussion or lecture." This lack of feedback directly from the instructors, in turn, may have contributed to students' perceptions of instructor-centered (with some student-centered) norms in the large lecture, as seen in the mean student awareness scores for *variation in instruction*.

4.3.2. Variation in Instruction: Descriptions of the Discussion Sections

The instructor and mean student awareness score with respect to *variation in instruction* suggested all the TAs perceived student-centered classroom norms within the discussion sections while most students perceived a hybrid or instructor- and student-centered classroom norms. For instance, consider the norms of *varied explanations*, *solutions*, and *representations* of concepts and problems. Both Dr. Riemann and the TAs emphasized that these student-centered norms were implemented through the use of the three to four exploration problems and exam-prep activities assigned in the discussion sections. More specifically, these ungraded problems were designed to have students notice common mistakes, check their group-quiz solutions, and work on more difficult, yet related problems. In general, however, students did not comment on the exploration problems and exam-prep activities, despite these assignments being the only problems handed out during discussion aside from the group quizzes. Instead, when asked if such norms as *varied explanations* and *varied representations* were a part of the discussion sections, most students commented on the ways in which the TAs explained the problems. For instance, one of his Mr. Descartes' students described Mr. Descartes' teaching style as very "standard," at least when guiding the whole class. As this student shared, "[Mr. Descartes] would write the question on the board and then go through it very uniformly if there weren't really any graphs, unless we were graphing. But no like tables or anything. And so it was all pretty standard I think."

Like Mr. Descartes, Mr. Newton did not often provide *varied representations* of problems. As Mr. Newton shared, "You know, I don't think I've actually gone [to] that level of using various representations." Considering these interview students' perceptions in comparison to his relatively high (i.e., student-centered) self-reported awareness score of 60%, Mr. Newton may have overestimated his student-centered implementation of norms such as *varied explanations* and *varied representations*. Similarly, Mr. Euler's self-reported awareness score, in addition to comments from his students during the interview process, suggest he and the interviewed students perceived a relatively high number of student-centered norms under the dimension of *variation in instruction*. However, the significantly lower mean student awareness score of 45.73% suggests this perception may not have been true for all students in his sections. To check this hypothesis, an independent[42] sample t-test with respect to students' achievement in the class, self-efficacy, and STEM intentions (i.e., the student outcomes of interest in this study)

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[42] More specifically, the student awareness scores of the following subgroups of Mr. Euler's students were compared: students intending to major in STEM; students earning a final course grade of an A, B, or C versus a final grade of D, F, or Incomplete; students earning a grade of 60% or higher on the final exam; and students whose standardized self-efficacy score was above the mean score of 0.

was conducted to test for significant difference in the student awareness scores for these subgroups along the teaching dimension of *variation in instruction*. Results suggested a non-significant difference. Thus, other factors not considered here may explain the difference in Mr. Euler's self-reported awareness score and the mean student awareness score for norms under the teaching dimension of *variation in instruction*.

As for Mr. Lagrange and Ms. Stokes, their instructor and the mean student awareness scores depicted a classroom implementing norms that were more student-centered when considering the teaching dimension of *variation in instruction*. For example, interviewed students commented on Ms. Stokes' use of *whole-class discussion* and *peer-to-peer support* to explore *varied solutions*. Indeed, during my observation, Ms. Stokes had students share their work on the front whiteboard and explicitly mentioned to the whole class that they should consider alternative solutions.

4.3.3. Student-to-Student Collaboration: Descriptions of the Large Lecture

The instructor and mean student awareness scores suggested that, when it came to the dimension of *student-to-student collaboration* in the large lecture, Dr. Riemann and her students perceived a hybrid of instructor- and student-centered classroom norms, with students perceiving more instructor-centered norms. Interview comments from Dr. Riemann and her students were consistent with the survey results. While students reported working together during some parts of the large lecture, other parts consisted of students listening to Dr. Riemann or working on problems individually. For example, students were asked to complete "two-part quizzes" in which students first worked on a given problem individually and then on the same problem a second time, but with other students. During these two-part quizzes, Dr. Riemann shared that students were "discussing with the group, moving chairs, turning to neighbors," and submitting a solution that was “as close to a 10 [out of 10] as possible.” Aside from the two-part quizzes, interviewed students did not report collaborating on problems during the large lecture very often, consistent with Dr. Riemann’s interview comments. Dr. Riemann specifically shared that she “rarely [used] the whiteboards” mounted around the lecture hall (see Appendix C for a description of the large-lecture space). As for “turning to [a] neighbor and talking about stuff,” Dr. Riemann shared this happened “two to three times a class.”

In interviews, multiple students agreed that, in the large lecture, *peer-to-peer support* usually only occurred when the two-part quizzes were assigned. Aside from the two-part quizzes, as one student shared during her interview, “because it’s such large groups [in the large lecture] … class wasn’t really structured to ask either peers or her [Dr. Riemann]. It was more structured to sit there and listen to her lecture." In other words, interviewed students reported listening to Dr. Riemann teach in the large lecture with “occasional peer-to-peer support,” as one student described it. These comments aligned with Dr. Riemann’s descriptions and the mean student awareness score suggesting a perceived implementation of instructor-centered norms under the dimension of *student-to-student collaboration*. Still, in reflecting on changes she wanted to see made to the course, Dr. Riemann shared that she wanted the large lecture to be more student-centered when considering norms under this dimension of teaching; however, the size of the lecture made this difficult. As Dr. Riemann explained, “in the discussion, you are more visible if you don't participate …so there is more of an expectation of participation” than in the large lecture.

4.3.4. Student-to-Student Collaboration: Descriptions of the Discussion Sections

The instructor and mean student awareness scores for items under the dimension of *student-to-student collaboration* suggested the TAs and their students perceived the classroom norms for *student-to-student* collaboration to be student-centered in the discussion sections. Similar to the two-part quizzes students took in the large lecture, students and TAs commented on group quizzes given in the discussion sections. Student and instructor comments suggested these quizzes, in addition to a set of exploration (extension) and problems meant to prepare students for upcoming exams, were the main catalyst for student-centered norms in the discussion sections. As one student shared, the "whole point of the quiz … [was] just to encourage working in a group." In further reflecting on these opportunities to work with others in the discussion sections, other interviewed students shared that being able to work both individually and in groups made it so students could "easily work through" the problems and check whether students got the same or different answers.

4.3.5. Instructor-to-Student Engagement. Descriptions of the Large Lecture

The awareness scores of Dr. Riemann and her students suggested they perceived the norms in the large lecture as more instructor-centered under the dimension of *instructor-to-student engagement*. For instance, while Dr. Riemann did ask students questions in the large lecture, many of these questions were rhetorical. Dr. Riemann described her questioning strategies in the excerpt below.

> I say that I make a distinction between kind of rhetorical questions and questions where I'm really, you know, really expecting an answer. So, I do a fair number of rhetorical questions, like, we are solving an equation with substitution, or something like this, and we get to a point where solve the simplified equation, and I'm asking, are we done yet? Are we done now? … But then there are the other things where it's more like, let's make sure we understand this definition, this process, this whatever. So, let's work a little. And that would be kind of a short thing where they need to … provide an answer. And I'm expecting that the answer might be divergent, so I'll collect a few answers.

While Dr. Riemann distinguished between rhetorical and non-rhetorical questions, some students may not have been able to tell the difference between the two. As one interviewed student shared, Dr. Riemann "asked question sometimes, but again, it's not like very clear if it's supposed to be answered or not.” As for non-rhetorical questions, Dr. Riemann shared that she did not do “cold calling,” or call on students who did not voluntarily offer a question or an answer to one of her questions. Another student confirmed that Dr. Riemann did not cold call students.

> She never selected students to ask questions. It was like, raise your hand if you have a question. Very few people participated in class. … We never, ever had a whole-class discussion. We had very limited participation in class. It was mostly just lecture.

A third student similarly shared, “I don’t think she really had any [participation] strategies. The only thing she did was like ask if you had any questions.” Indeed, while Dr. Riemann expected a wide range of students to participate in class, in reflecting on the large lecture she shared, “that’s not what’s happening.”

Although few students participated in answering questions during the large lecture, one student shared that a lack of participation may have been due to not necessarily a lack of trying on the students’ part, but due to the size of the class.

> And with the questions, I feel like it was more like she’d asked questions of [the] whole class and the whole class would answer, if that makes sense. … I do remember, like, a wide range of students would try to participate in class, but it was kind of hard, because there’s so many of us. Yeah, it's kind of like everyone would try. And then it was kind of like everyone's talking. So it's kind of like, if she asked a question to the entire class, like everyone would answer instead of like one person answering.

Still, one student shared Dr. Riemann would sometimes “take questions, but the other times it was not really a thing.” Additionally, the time Dr. Riemann did give for asking and answering questions may have catered to only a select group of students, as suggested in the following comment from one of the interviewed students.

> There was not really, I guess you could say that there was an encouragement of participation from a wider group of students by asking for students to ask questions, but in a way, I feel like that just only caters to the, I guess, most outgoing or like boldest students who feel confident in their question, whereas if someone who has a more foundational question [they] might not feel comfortable sharing.

As another student put it, "only the people in the front or the people that were right there raising their hands" were the students who "ever really got called on for anything."

4.3.6. Instructor-to-Student Engagement: Descriptions of the Discussion Sections

The instructor and mean student awareness score with respect to the teaching dimension of *instructor-to-student engagement* suggested that most of the TAs and their students perceived a hybrid or instructor- and student-centered classroom norms within the discussion sections. In reviewing data related to the norms of *instructor-to-student engagement*, many of the differences seen in the awareness scores across the different sections may have been attributed to the way in which a TA initiated helping students during the discussion sections. More specifically, while all of the TAs worked with students one-on-one, some of the TAs also focused more time on addressing the class as a whole. For instance, comments from students and instructors suggested Mr. Newton and Mr. Descartes' interactions with students took place more one-on-one. As one student shared in reflecting on Mr. Descartes' strategies for explaining concepts to students, "once we went into the practice problems, if people had more questions, then he would maybe explain things differently. But it wasn't necessarily in a group setting. … More like individually he would discuss things more." In other words, students in these discussion sections may have had less time to engage with the instructors along with the rest of the class, limiting opportunities for *constructive criticism* of classmate's ideas. As a results, under the teaching dimension of *instructor-to-student engagement*, such experiences may have contributed to students perceiving more instructor-centered norms in Mr. Newton and Mr. Descartes' discussion sections than the discussion sections of the other TAs.

In contrast to the above TAs, Ms. Stokes focused a great deal of time on engaging students in *whole-class discussions* and encouraging *wide participation*. For example, Ms.

Stokes was the only TA on the day of my observation to have students share their work on the front whiteboard for the class to see. Ms. Stokes was also unique on the day of my observation in that she also held whole-class discussions that involved back-and-forth interactions between her and the students. Ms. Stokes' students confirmed that she regularly addressed the class as a whole. As one of her students described, Ms. Stokes "always looked out for what we were doing right and wrong, and if everyone was doing something wrong, she would make sure to go over that in [the] discussions."

To be clear, the other TAs also addressed the class at different times to comment on important ideas to keep in mind while completing the assignments in the discussion sections. For instance, Mr. Euler shared that while on the day of my observation he mostly worked with students one-on-one, he usually would go over questions with the whole class. Even so, evidence from the student surveys and interviews suggested Ms. Stokes held more whole-class discussions that involved students engaging with her and the other students in the class when compared to the other TAs, a result that may have contributed to her classroom norms being perceived by students as student-centered under the dimension of *instructor-to-student engagement*.

The above sections answer the first, second, and third research questions guiding this study by using survey and interview data to describe where on the instructor- to student-centered continuum the classroom norms under each dimension of teaching fell, as perceived by instructors and students. To summarize, perceived classroom norms ranged, in general, between instructor-centered and a hybrid of instructor- and student-centered along the teaching dimensions of *student-to-student collaboration, instructor-to-student engagement,* and *variation in instruction* in the large lecture. For the discussion sections, perceived classroom norms under each dimension mostly ranged from student-centered to a hybrid of instructor and student-centered, with some variations by TA. I now turn to answering the fourth research question guiding this study to explore what perceived classroom norms were predictive of student achievement, self-efficacy, and STEM intentions within the contexts of both the large lecture and discussion sections.

4.4. Part IV Results: Classroom Norms Associated with Student Outcomes

In this final results section, I use the hierarchical models described at the end of Chapter Three to answer the fourth research question guiding this study: What perceived classroom norms of an undergraduate precalculus course are predictive of students' (a) academic achievement in the course, (b) mathematical self-efficacy, and (c) STEM intentions? As indicated in Chapter Three, hierarchical modeling was used to account for the variation in each of the student outcome variables explained by students nested within discussion sections led by different TAs. Results suggested that perceived norms under two of the dimensions of teaching related to the large lecture were significantly associated with students' academic achievement and self-efficacy. In addition, perceived norms under one of the dimensions of teaching related to the discussion sections was significantly associated with students' academic achievement and self-efficacy. However, none of the variables included in the logistical model were predictive of students' intentions to major in a STEM field.

As an overview of the various student outcomes, the average final exam grade was 68.27% ($SD = 17.56$), with 80.7% (n=146) of the 181 students earning a final exam grade of 60% or above. Before standardization, the average self-efficacy score for participants in this study was 1439.06 ($SD = 398.53$) out of a maximum score of 2000. Out of the 181 participants in this study, approximately 72% (n=131) of students intended on majoring in a STEM field at the time of taking the SPIPS-M.

4.4.1. Academic Achievement

When considering the perceived classroom norms under the large-lecture teaching dimension of *variation in instruction*, an increase in one[43] standard deviation of a student's factor score was associated with an increase in a final exam grade of 5.52% ($p < .001$) when compared to the reference group of White, male students not identified as having low attendance (see Table 7). In contrast, perceived norms under the large-lecture teaching dimension of *instructor-to-student* engagement were associated with a decrease of 6.73% ($p < .001$) on students' final exam grades. In addition, the perceived norms under the discussion teaching dimension of *instructor-to-student engagement* predicted an increase in the average final exam grade of 4.47% ($p < .05$). Moreover, the average final exam grade of a student who identified as African American[44] was 7.52% lower ($p < .05$) than that of the reference group, 9.43% lower for students identifying as Hispanic/Latinx ($p<.05$), and 12.23% lower ($p < .01$) for students who identified as multiracial.

[43] To be succinct, all results related to dimensions of teaching may be interpreted as the change in a student outcome variable (in comparison to the reference group) following an increase in one standard deviation of a student's factor score for a given dimension, holding all other variables constant.
[44] See Appendix A for a frequency table of the attendance and demographic variables included in the models.

Table 7

Estimates of Fixed Effects on Student Final Exam Grades

Parameter	Estimate	Std. Error	t	Sig.
Intercept	72.93	2.50	29.14	<.001
Large Lecture - Variation in Instruction	5.52***	1.41	3.92	<.001
Large Lecture - Student-to-Student Collaboration	.49	1.33	.37	.713
Large Lecture - Instructor-to-Student Engagement	-6.73***	1.51	-4.46	<.001
Discussion - Variation in Instruction	-.59	1.53	-.39	.699
Discussion - Student-to-Student Collaboration	-1.22	1.53	-.80	.427
Discussion - Instructor-to-Student Engagement	4.47*	1.72	2.60	.010
Low Attendance	-3.16	2.74	-1.15	.251
Female	2.13	2.48	.86	.392
African American	-7.52*	3.20	-2.35	.020
Hispanic/Latinx	-9.43*	4.07	-2.32	.022
Asian	-4.80	3.68	-1.30	.194
Multiracial	-12.23**	3.82	-3.20	.002

Note. The reference group is a White-male student who neither missed at least one class per week in the large lecture, 4-6 discussion sections, nor at least one discussion section a week.
$*\, p < .05.$ $**\, p < .01.$ $***\, p < .001.$

4.4.2. Self-Efficacy

For the perceived classroom norms related to the large-lecture teaching dimension of *variation in instruction*, results suggested that an increase in a single standard deviation in students' factor scores was associated with an increase of 0.18 ($p < .05$) standard deviations in students' self-efficacy scores (see Table 8). In addition, the perceived classroom norms related to the discussion teaching dimension of *instructor-to-student engagement* in the discussion sections were associated with an increase of 0.30 ($p < 0.01$) standard deviations in students' self-efficacy scores.

Table 8

Estimates of Fixed Effects on Student Self-efficacy

Parameter	Estimate	Std. Error	t	Sig.
Intercept	-.05	.15	-.35	.730
Large Lecture - Variation in Instruction	.18*	.08	2.11	.036
Large Lecture - Student-to-Student Collaboration	-.06	.08	-.71	.480
Large Lecture - Instructor-to-Student Engagement	-.06	.09	-.67	.504
Discussion - Variation in Instruction	.05	.09	.53	.599
Discussion - Student-to-Student Collaboration	.00	.09	.05	.962
Discussion - Instructor-to-Student Engagement	.30**	.10	2.95	.004
Low Attendance	-.02	.16	-.14	.889
Female	.00	.15	.00	1.000
African American	.19	.19	1.01	.315
Hispanic/Latinx	-.12	.24	-.50	.617
Asian	-.09	.22	-.39	.700
Multiracial	.27	.23	1.18	.240

Note. The reference group is a White-male student who neither missed at least one class per week in the large lecture, 4-6 discussion sections, nor at least one discussion section a week.
$* p < .05$. $** p < .01$. $*** p < .001$.

4.4.3. STEM Intentions

In the end, none of the predictors included in the hierarchical logistical model statistically significantly predicted that a given student intended to major in a STEM field (see Table 9). However, while not statistically significant, a strong association (p=.056) resulted between a student identifying as African American and intending on majoring in a STEM field.

Table 9

Estimates of Probability of Majoring in STEM

Model Term	Coefficient	Std. Error	t	Sig.	Exp (Coefficient)
Intercept	.49	.35	1.40	.162	1.63
Large Lecture - Variation in Instruction	.16	.21	.77	.443	1.17
Large Lecture - Student-to-Student Collaboration	-.08	.20	-.42	.672	.92
Large Lecture - Instructor-to-Student Engagement	-.15	.22	-.66	.510	.86
Discussion - Variation in Instruction	-.15	.23	-.68	.498	.86
Discussion - Student-to-Student Collaboration	.20	.22	.89	.375	1.22
Discussion - Instructor-to-Student Engagement	.20	.26	.77	.444	1.22
Low Attendance	.34	.42	.80	.427	1.40
Female	.06	.36	.17	.864	1.06
African American	.95	.49	1.92	.056	2.58
Hispanic/Latinx	.20	.57	.35	.727	1.22
Asian	.75	.57	1.30	.195	2.11
Multiracial	.44	.55	.79	.429	1.55

Note. The reference group is a White-male student who neither missed at least one class per week in the large lecture, 4-6 discussion sections, nor at least one discussion section a week. In reading the above table, the column titled "Exp (Coefficient)" may be interpreted as the probability that the student group of interest intends on majoring in STEM in comparison to the reference group. For instance, while not significant, a student identifying as Hispanic/Latinx was 1.22 times more likely to major in STEM compared to a student in the reference group.

More specifically, results suggested a student identifying as African American was roughly 2.5 times more likely to have intentions to major in a STEM field when compared to a White-male student not identified as having low attendance.

Chapter 5. Discussion and Conclusion

This dissertation adds to the growing body of research on teaching and learning in undergraduate mathematics courses. More specifically, the quantitative component offers insights into the potentially predictive relationships between students' perceptions of classroom norms, academic achievement, and self-efficacy. Qualitatively, this study provides insights about how course instructors, including graduate students serving as Teaching Assistants (TAs), and students perceived the co-constructed norms of a classroom. Attention to classroom norms, and the extent to which the teaching and learning of undergraduate mathematics is student-centered, has grown in importance as math departments, in particular, look to support undergraduate students taking introductory math courses.

The study followed a convergent mixed-methods design that integrated quantitative and qualitative results in the analytic and results stages. The study utilized survey, interview, and observational data from the Precalculus course offered at Blackboard University (BU) to describe the perceived classroom norms of Precalculus and their predictive power of students' achievement on the final exam, self-efficacy, and STEM intentions. Following an exploratory factor analysis of students' survey responses to items serving as potential Precalculus classroom norms, as well as a qualitative analysis of interviews with students and instructors, norms perceived by instructors and students in Precalculus were categorized into three dimensions of teaching that were considered separately for the large lecture and discussion sections: 1) *student-to-student collaboration*, that is, norms related to the activity of students working together; 2) *instructor-to-student* engagement, or norms involving instructor facilitation and student participation in asking and answering questions; and 3) *variation in instruction*, or norms that highlight alternative explanations, solutions, representations, etc. in the classroom.

While evidence suggested some variation by dimension of teaching and the TA for a discussion section, in general, instructors' perceptions of classroom norms in the large lecture and discussion sections aligned with students. Evidence from participants' survey responses and interview comments suggested that both instructors and students perceived a hybrid of instructor- and student-centered norms in the large lecture and discussion sections, with more instructor-centered norms perceived in the large lecture and more student-centered norms in the discussion sections. Hierarchical modeling was used to explain differences in students' final exam grades, self-efficacy, and STEM intentions, controlling for the discussion sections students were in. Results suggested that students' perceptions of the norms under the teaching dimension of *variation in instruction* in the large lecture predicted an increase in students' final exam grades and students' self-efficacy in the course. However, students' perceived norms under the teaching dimension of *instructor-to-student engagement* in the large lecture predicted a decrease in students' final exam grades. With respect to the discussion sections, students' perceived norms under the teaching dimension of *instructor-to-student engagement* predicted an increase in both students' final exam grades and self-efficacy. None of the predictors considered in this study were associated with students' STEM intentions.

5.1. Conversing with Existing Literature

In considering the current study in light of existing literature, I highlight four findings about the predictive, and non-predictive power in some cases, of the hierarchical models of undergraduate students' course achievement, self-efficacy, and STEM intentions after taking Precalculus at Blackboard University. I begin with comments related the role of a hybrid of instructor- and student-centered norms in Precalculus. Additionally, I comment on the non-significant relationship between the perceived discussion section classroom norms under the dimension of *variation in instruction* with any of the student outcomes. I then comment on results related to race within the context of the recent redesign of Precalculus, followed by a discussion on the non-predictive power of the model for STEM intentions.

5.1.1. Possible Benefits of Hybrid and Instructor-Centered Norms

For the large lecture, the quantitative results suggested that students' perceptions of more student-centered norms under with the teaching dimension of *instructor-to-student engagement* were negatively associated with students' final exam grades. More specifically, an increase in one standard deviation of a students' factor scores for the dimension of *instructor-to-student* engagement corresponded, on average, with a drop in students' final exam grades by 6.7%. Put another way, the more a student perceived the classroom norms of wide participation, whole-class sharing, and a wide range of students responding to an instructor's questions in the large lecture, the lower the student's final exam grade. However, students' increased perceptions of norms related to *variation in instruction* in the large lecture were, on average, associated with an increase in students' final exam grades. Considering the final list of classroom norms related to both the large lecture and discussion sections were student-centered in nature, one interpretation of these results may be that students' perceptions of student-centered norms may support an increase in final exam grades when considering particular dimensions of teaching, but may be detrimental when considering others.

While the focus of this study was on students' perceptions of classroom norms as opposed to the actual[45] implementation of, for instance, student-centered norms, the positive association between students' perceptions of student-centered norms under the dimension of *variation in instruction* in the large lecture speak to previous literature that finds that, in general,

[45] See comments in the sections that follow on future research to address this nuance.

student-centered practices may be positively associated with students' academic achievement in STEM coursework (Freeman et al., 2014). However, the negative association between students perceiving student-centered norms related to *instructor-to-student engagement* in the large lecture sits in contrast to arguments for student-centered instructional practices in undergraduate mathematics (e.g., The Mathematical Association of America, 2018). More specifically, it may be that only particular student-centered norms, or more broadly, practices are beneficial to students. Instead, students may benefit from instructors, such as those of Precalculus, having students explore multiple solutions and varied representations (a more student-centered approach to teaching) while not focusing on engaging a wide-range of students in a large lecture (a more instructor-centered approach).

Indeed, multiple students in interviews commented that the size of the large lecture made it very difficult to participate, despite trying to. In addition, when considering the teaching dimension of *variation in instruction* in the large lecture, while the quantitative results suggested a positive relationship with students' final exam grades, some students voiced not understanding the need for exploring alternative solutions during the interviews. Thus, it may be that instructors of courses like Precalculus may better support students in terms of their academic achievement and self-efficacy by implementing a hybrid of instructor- and student-centered norms along the different dimensions of teaching identified in this study.

5.1.2. Reconsidering Norms Related to Variation in Instruction

The results presented here suggested that the perceived norms under the dimension of *variation in instruction* were not predictive of students' final exam scores, self-efficacy, or intentions to major in a STEM field when considering the discussion sections. As this dimension of teaching comprised classroom norms such as *varied representations* of topics/problems and *multiple solutions* and *explanations*, this finding sits in contrast to arguments in the field that support having students explore varied explanations (The Mathematical Association of America, 2018). In other words, the findings of this study raise questions for future exploration related to the benefit, in terms of the student outcomes discussed here, of having students explore different solutions and explanations for problems. In fact, one of the most prominent critiques students and TAs had of the Precalculus courses related to exploring alternative solutions in the discussion sections. More specifically, two students during the interview process shared their preference for reviewing one solution method. In addition, multiple TAs found the Exploration Problems and time spent considering different solutions either not helpful to students in retaining information or targeted more towards those students "motivated" enough do these problems that were ungraded to begin with. Put another way, while the quantitative results suggested that having students consider varied representations, explanations, and solutions in the discussion sections did no harm when considering students' final exam grades, self-efficacy, and STEM intentions, there also was no evidence that the perceptions of these and related norms under the teaching dimension of *variation in instruction* benefited students in any of the three student outcomes.

5.1.3. Equity in Precalculus

While none of the research questions posed in the current study explicitly addressed issues of equity in Precalculus, a major aim in the redesign of Precalculus at Blackboard University has been to better support traditionally underrepresented students in STEM fields. Thus, it is worth, even if briefly, reviewing the quantitative findings related to race, academic achievement, self-efficacy, and STEM intentions.

As highlighted in the description of the SEMINAL team's redesign of Precalculus at BU, Precalculus serves a disproportionate number of underrepresented students in STEM fields of study. This fact, coupled with the historically high failure rate of Precalculus students, was one of the main issues that motivated the Precalculus team at Blackboard University to join the SEMINAL network roughly five years ago. The results presented in this study suggest there is still work to be done in supporting a diverse group of students taking Precalculus. First, the final exam grades for students who identified as African American (37% of participants), Hispanic/Latinx (11%), or multiracial (13.8%) were predicted to be statistically significantly lower than those of White-male students not identified as having low attendance when considering the different variables of interest in this study. One potential explanation for these results is a lack of support for students of color outside of the Precalculus classroom that have been found to support these students' achievement in STEM classes. For instance, empirical evidence suggests peer-to-peer networks and mentoring relationships with faculty outside of the classroom can support students' of color in their academic achievement in STEM courses (Ellington & Frederick, 2010; Griffin et al., 2010). Aside from academic achievement, peer support groups and mentoring programs have also been shown to predict the self-efficacy and STEM persistence of students of color (Palmer et al., 2011), and more generally, academic[46] self-efficacy (MacPhee et al., 2013). Thus, it may be that students of color in the current study were not receiving the support they needed outside of the classroom to be successful in Precalculus at BU, regardless of whether they perceived classroom norms as being more instructor-centered, student-centered, or a hybrid of the two.

[46] MacPhee et al. (2013) define academic self-efficacy as "confidence in one's ability to accomplish academic tasks" (p. 348).

Second, race and gender were not predictive of Precalculus students' self-efficacy and STEM intentions. While certainly this result may simply be limited to the context of Precalculus at BU and the data collected for the purposes of this study, it is still worth noting when considering the small percentage of STEM bachelor's degrees conferred in the United States (Musu-Gillette et al., 2016; National Science Foundation, 2014). One, albeit generous, interpretation of this result may be that the mathematical experiences offered to students in the Precalculus large lecture and discussion sections at the least do no harm to the self-efficacy and STEM intentions of students traditionally underrepresented in STEM fields. Furthermore, a review of the interviews with students did not reveal[47] any unique experiences in Precalculus with respect to students' race or gender. Thus, even a non-significant relationship between race and students' self-efficacy and STEM intentions within the context of this study may be seen as a potential point of success for the Precalculus course at BU. More explicitly, it may be that the recent redesign to implement more student-centered norms within Precalculus has at least had no negative affect on students traditionally underrepresented in STEM fields in terms of their self-efficacy and STEM intentions.

5.1.4. Non-Predictive Power of STEM-Intentions Model

The logistic regression model revealed that the variables considered in the current study did not significantly explain the probability of Precalculus students' intentions to major in a STEM field. This finding potentially points to other factors outside of Precalculus that may better explain students' intentions to major in a STEM field. More specifically, students' experiences in high school math classrooms and their mathematical understanding prior to attending college may be better predictors of undergraduate students' interest and persistence in

[47] Admittedly, students may not have been comfortable sharing these experiences during the interview process.

majoring in a STEM field (see Crisp et al., 2009 on factors related to students switching into STEM; see also Green & Sanderson, 2018; Maltese & Tai, 2011). Research also suggests that traditionally underrepresented students in STEM may be more likely to pursue STEM majors as a result of participating in peer-support groups outside of class (Palmer et al., 2011). Returning to the participants in the current study, all but one of the student interviewees commented that their experience in Precalculus did not inform their choice of major or shared that they had not changed their major since finishing[48] the course. Only Turing (pseudonym), a biology major at the time of taking the survey and the only student interviewee to fail Precalculus, decided to switch his major to psychology (i.e., STEM to non-STEM) after the semester ended. Turing was retaking Precalculus at the time of the interview process and, when asked if his experience in Precalculus affected this switch to a non-STEM major, he shared, "I guess a little" when he realized he would have to take Calculus[49] I for STEM majors at BU if he remained a biology major. While the current study lacks data to see if all participants switched from or to a STEM major following Precalculus, Turing's experience prompted a question as to the relationship between students' STEM intentions and their performance in the class. A post-hoc analysis suggested that neither a student failing[50] the final exam (19.34%, or n=35) nor earning a grade of D, F, or Incomplete (25.97%, or n=47) in the course were associated with students' STEM intentions. Thus, factors aside from academic achievement on the final exam or end-of-semester grades should also be considered in making sense of why students may switch from a STEM to non-STEM major following taking courses like Precalculus.

[48] Recall the interviews took place after the semester had ended.
[49] As a psychology major, Turing would take an alternative form of Calculus I for non-STEM majors known as Elementary Calculus in the Math Department at Blackboard University. In reflecting on this alternative course to Calculus I for STEM majors, Turing shared, "I think it's an easier math."
[50] For the purposes of this post-hoc analysis, 60% was used as a cutoff for passing the final exam. The following are the Chi-square test results: with respect to a student failing the final exam, $\chi^2(1) = 1.97,\ p = .161$; with respect to students earning a grade of D, F, or Incomplete in the course, $\chi^2(1) = 2.32,\ p = .128$.

5.2. Limitations of Study and Future Research

In the following five sections, I highlight the limitations of the current study. For each limitation, I discuss a need for future research that may better be able to inform the research questions posed in this study. I begin with limitations related to the framework implemented here, namely, the framing of the classroom norms of Precalculus on a continuum of instructor- to student-centered. I end this subsection with a discussion of the limitations in the data used to answer the research questions.

5.2.1. Further Defining Instructor- vs. Student-Centered Norms

The perceived classroom norms identified in this study stemmed from the 22 overlapping PIPS-M items on both instructor and student surveys. A review of these items reveals that the norms considered in the current study were inherently student-centered to begin with. For instance, classrooms implementing *whole-class sharing* and *peer-to-peer support* may be characterized as implementing student-centered norms. Indeed, the descriptions provided in Table 3 and Table 5 of each of these norms use language that communicates student-centered pedagogy, as described in the introduction of this paper.

While such descriptions were helpful in assigning language to each of the norms for reference purposes, questions may remain as to what instructor-centered 'versions' of these norms may entail. In other words, while an instructor-centered implementation of the *peer-to-peer support* may be easier to envision (e.g., students working individually on problems for the entirety of a class), it may be more difficult to envision a more instructor-centered implementation of *whole-class sharing*. For instance, an instructor-centered implementation of *whole-class sharing* may be interpreted as students *not* sharing their work with the whole class;

however, students may still work with one another in smaller groups, thus implementing a different norm in a student-centered fashion.

Considering the above difficulty, future research should be conducted to define the classroom norms of undergraduate mathematics courses such as Precalculus along the continuum of instructor- and student-centeredness. For instance, a future iteration of this study may entail explicitly asking interviewees to define the classroom norms resulting from a factor analysis of survey items as opposed to presenting the norms in predetermined dimensions of teaching (i.e., the factors) during the interviews. Giving such explicit attention to how the instructors and students of a course define perceived norms may reveal more nuanced insights into what makes up a norm and to what extent (if any) it aligns more with instructor- or student-centered tenets of instruction. This presentation may also assist in further refining a factor analysis, similar to that conducted in the current study, by allotting more freedom to instructors and students in defining perceived classroom norms not captured by the predefined PIPS-M survey items. Indeed, while beyond the scope of the current study, there may certainly be norms not captured by the PIPS-M that would be worth considering in the development of a survey that more accurately characterizes the potential norms undergraduate students and instructors perceive in a class like Precalculus.

5.2.2. Research on Students' Perceptions of Classroom Norms

The research questions guiding this study were framed to focus on instructors' and students' *perceptions* of classroom norms. In turn, I carefully presented the findings in terms of the perceived classroom norms from the perspectives of instructors and students. This presentation followed from a consideration that the use of the SPIPS-M survey and student interviews, methodologically speaking, may have separated how students perceived (i.e., reported) Precalculus classroom norms from those norms actually making up the classroom. As a result, the findings presented here may not seem directly applicable to the work of Cobb and Yackel in their identification of classroom norms as opposed to perceptions of classroom norms. However, I highlight Yackel and Rasmussen's (2002) comments below to suggest that the analysis of instructors' and students' perceptions of norms may be considered alongside past research on, more directly, classroom norms.

> Methodologically, both general social norms and sociomathematical norms are inferred by identifying regularities in patterns of social interaction. Thus social norms are identified from the perspective of the observer and indicate an aspect of the social reality of the classroom. (p. 316).

Put another way, identifying the social norms of a classroom necessitates an outside observer taking some leap of faith in interpreting what is 'normal' in a classroom. Moreover, in the case of the current study, I relied on participants' perceptions to identify classroom norms. In either case, identifying the classroom norms contains an inherent risk in the researcher (i.e., an outsider to the class) misidentifying what is normal for a classroom. This risk only increases when considering "what becomes normative in a classroom is constrained by the current goals, beliefs, suppositions, and assumptions of the classroom participants" (p. 316), of which an outside researcher acting in the role of an observer may have no clear understanding of or influence on. In turn, despite differences with the past work of Cobb and Yackel, the current study follows suit with their work and the work of others in identifying the norms of a classroom and, thus, may inform such research. More specifically, the current study offers a unique means of utilizing qualitative and quantitative methods to measure instructors' and students' perceptions of classroom norms of an undergraduate math course.

Still, to strengthen the measurement of classroom norms, I note that while the perceptions of norms varied, the data collected here is not sufficient in answering questions related to whether the norms, themselves, varied by classroom. More specifically, students within the same classroom are arguably[51] subject to the same classroom norms, whether they are instructor-centered, student-centered, or a hybrid of the two. Therefore, future research should consider including data across multiple large lectures, for instance, to capture variance in the norms used to build the models generated in this study. Including data from multiple large lectures may also serve as a source of validation of the norms that make up Precalculus that, in turn, may allow for stronger claims about the norms of Precalculus beyond how students and an instructor perceive them in a single classroom space.

Aside from the above consideration for future analysis of classroom norms, the standard deviations presented in Table 3 and Table 5 demonstrate variance in students' perceptions of classroom norms, whether in the large lecture or discussion sections, even within the same classroom. For example, the average student perceived students responding to questions as ranging between "Not at all descriptive" to "Somewhat descriptive" in the large lecture ($M = 1.07, SD = 1.04$). Moreover, while students' perceptions were roughly consistent in terms of perceiving more student-centered norms in the discussion sections when compared to the large lecture, students' perceptions of norms varied both within the large lecture and within a single discussion section. This variance agrees with empirical evidence suggesting that students within the same math classroom can perceive different levels of student-centered practices (Ellis et al., 2014). Considering students' perceptions of different classroom norms were found to be significantly associated with Precalculus students' final exam grades and sense of self-efficacy in the course, future research should consider other factors that may result in students' perceptions of classroom norms being different. For example, Star et al. (2008) found that students notice

[51] Students may experience what goes on within the same classroom differently from their peers (Ellis et al., 2014).

differences in the aspects of their college math course (e.g., the expectation to clearly explain one's work, or the time dedicated to group work) based on whether they were coming from more traditional curricula in high school to standards-based curricula in college (i.e., instructor- to student-centered), or vice versa. Thus, students' experiences in math classrooms *prior* to college most likely would shed light on how students perceive the classroom norms of their college math classrooms and any differences in these perceptions across students.

5.2.3. Alignment Between Norms and the Measure of Achievement

During the semester of data collection, Dr. Riemann gave an instructor-written final exam that was similar in topics and format to those given in past iterations of Precalculus, that is, iterations of the course that were not a part of the SEMINAL redesign to focus more on student-centered learning. However, it was beyond the scope of this study to consider what changes, if any, Dr. Riemann had made to the final exam to reflect the tenets of student-centered learning (e.g., focusing on questions that tested students' conceptual understanding as opposed to their ability to perform mathematical procedures). Thus, future research should consider the extent this final exam aligned with the tenets promoted by the student-centered norms identified in the current study. More specifically, while empirical evidence suggests student-centered instruction in STEM courses may offer an advantage over instructor-centered when considering student achievement on assessments testing for conceptual understanding (Rasmussen et al., 2006), this advantage may be less noticeable when considering instructor-written exams (Freeman et al., 2014). In other words, the final exam given by Dr. Riemann may have been more instructor-centered in nature and, in turn, misaligned with the increased focus of student-centered instruction in the course. Thus, the results presented here with regards to students' final exam grades may be limited in that the final exam given during the semester of data collection may not have evaluated, for instance, students' ability to provide multiple solution pathways and representations of problems (i.e., components of student-centered learning). Ensuring an alignment between the norms of Precalculus and student achievement may reveal a stronger relationship when answering the research questions posed here.

5.2.4. Differences in Findings Between Large Lectures and Discussion Sections

While the same dimensions of teaching were considered for both the large lecture and discussion sections, the quantitative results differed somewhat for each classroom space. For instance, while norms under the teaching dimension of *variation in instruction* were associated with an increase in students' final exam grades and self-efficacy, the norms associated with this dimension when considering the discussion sections were not significant predictors of the student outcomes. While this finding may seem unsurprising given the drastically different format of the large lecture when compared to the smaller discussion sections, it also raises questions when considering the overlap in norms under this dimension for both the large lecture and discussion sections (see Table 2). More specifically, are the dimensions of teaching and their associated norms more different between the large lecture and discussion sections than what was captured in the current study? Are there other norms that make up the discussion sections that are not present in the large lecture, and vice versa? As students referenced multiple times in the interview, the sheer size of the large lecture often restricted their ability to, for instance, ask and answer questions. Future research should consider more finely identifying the unique norms making up a large lecture and discussion sections.

5.2.5. Considering Factors Outside of Precalculus: Future Versions of Models

As highlighted above, a post-hoc analysis was conducted to check for relationships between students' overall performance in the course and on the final exam, as well as students'

STEM intentions. These results are limited in that students (like Turing) may have switched their major after failing the course (i.e., after the survey was administered). Moreover, it may be that factors outside of Precalculus were more predictive of students' STEM intentions (see literature highlighted in the above section on the non-predictive power of STEM-intentions model). Thus, data collection specific to this cohort (and future cohorts) of Precalculus students after taking the course would assist in analyzing their STEM trajectories, similar to the work conducted by the National Center for Education Statistics in their Beginning Postsecondary Students Longitudinal Study (as cited in Green & Sanderson, 2018). In addition, when considering the student outcome of STEM intentions, in particular, researchers often include a measure of pre-college achievement (e.g., SAT score or high school GPA) given such a measure has been shown to be potentially predictive of undergraduates' persistence in earning a STEM degree (Mau, 2016; see also Oakes, 1990, for a literature review), although not always (Maltese & Tai, 2011). Thus, a future iteration of this study should include a measure of prior achievement when answering questions related to STEM intentions. As a starting point, I highlight that Dr. Riemann assigns an algebra diagnostic test at the beginning of the semester. Results from such a diagnostic tool may serve as one source of prior achievement specific to students' fluency with prerequisite algebraic skills and concepts coming into Precalculus.

Considering the differences in instructors' perceptions of norms (see Figures 7, 8, and 9), future versions of the quantitative models should also include variables at level two (i.e., the classroom level) that measures instructors' perceptions of the classroom norms for each dimension of teaching. However, considering Mr. Descartes did not complete the PIPS-M, adding such a variable would require giving careful consideration as to whether to remove his students' survey responses from analysis or impute survey scores for Mr. Descartes using the existing data (e.g., the patterns in differences between TA and students' survey responses). Including such a variable in the models would strengthen alignment between the models and the theoretical framing of classroom norms as practices that are informed by both students and instructors. Lastly, at the time of this study, the Math Department at BU was interested in the role of students' Math Placement Exam scores, as well as students' prior math classes, when considering students' academic success and persistence in attaining a STEM degree. Thus, variables for these two measurements should be considered in future versions of the models.

While the above additions to the models are warranted, I predict that instructors' perceptions of the classroom norms would play an insignificant role in predicting Precalculus students' academic achievement, self-efficacy, and STEM intentions considering how a student perceives the classroom norms can be different[52] from that of the instructor. Likewise, from my experience as a Precalculus instructor, even if students have taken calculus courses in high school, there are often algebra skills that students struggle with that make doing well in the course more difficult. Thus, of all the variables considered above, I believe the algebra diagnostic test may be predictive of Precalculus students' academic achievement in the course, in particular. Indeed, concern over the algebra skills of students taking introductory math courses in college, as well calculus courses, has been documented elsewhere (see Kornelson, Moore-Russo, & Reeder, 2020) and has been a common point of conversation with my colleagues in explaining poor precalculus and calculus grades during my time teaching at the undergraduate level.

5.2.6. Limitations in Data Collected

There are three limitations related to data collection in the current study that, if addressed, may better answer the research questions posed here. First, I conducted a limited number of

[52] See Levenson et al. (2009) for a case on sociomathematical norms in an elementary school setting.

observations of each of the instructors for the course. While these observations were only meant to provide context to the comments that instructors and students made during the interviews rather than contributing to the identification of Precalculus classroom norms, observing each instructor more than once may have provided insight into better understanding a hybrid implementation of a given norm, as perceived by instructors and students. For instance, using classrooms observations of 548 STEM instructors across the United States, Stains et al. (2018) generated "instructor profiles" to describe the practices of each instructor, including "didactic," "interactive lecture," and "student-centered" profiles (p. 1469). In particular, instructors classified in the interactive-lecture profile "[supplemented] lecture with more student-centered strategies" (p. 1469). Such a description, generated from observational data of the classroom, may help clarify what is meant by a Precalculus classroom in the current study being characterized by a hybrid implementation (i.e., instructor- and student-centered) of a classroom norm. Moreover, an increased number of observations may help explain the, in some cases, significant differences in instructor self-reported awareness scores and the mean student awareness scores.

Second, students completed the SPIPS-M survey after the drop and withdrawal dates for BU for the fall 2021 semester. Thus, this study did not consider the perspectives of students who withdrew from Precalculus. The perspectives of these students may offer further insight into the classroom norms that were either supportive or hindering to their successful completion of Precalculus. A future analysis of the data collected here should consider inviting these students to share their experiences in the course. Relatedly, it is not uncommon for students to retake Precalculus in the spring semester, as in the case of Turing. Thus, it may be helpful to consider the unique experiences of those students who retake Precalculus. More specifically, there may be differences in the perceived classroom norms in the spring semester that help these students successfully complete Precalculus.

Third, the lack of survey data from Mr. Descartes limits the results discussed here for his discussion sections considering classroom norms entail the perspectives of both the instructor and students. Without the survey data from Mr. Descartes, there is some uncertainty as to the extent his discussion sections were perceived as instructor-centered, student-centered, or hybrid of the two. Still, the claims made here are supported by the culmination of data collected across all the discussion sections and do not rely on the perception of a single discussion section TA. While certainly having the perspective of Mr. Descartes would have assisted in answering the research questions guiding this study, the findings of this study are supported by the reasonably considerable number of instructor and student participants.

5.2.7. Other Considerations for Future Research

The current study focused on the predictive relationships between the perceived Precalculus classroom norms and student outcomes of academic achievement, self-efficacy, and STEM intentions. While beyond the scope of the current study, future analysis of the data collected here should include exploring the relationships between the student outcome variables, themselves. For instance, Peters (2013) found a positive-predictive relationship between students' self-efficacy and achievement. Indeed, a simple correlation between students' final exam grades and standardized self-efficacy scores in the current study revealed a significantly positive relationship ($r(179) = 0.43, p < .001$). Future research should consider the question of how the relationship between academic achievement, self-efficacy, and STEM intentions of Precalculus students differs, if at all, between the different discussion sections included here. Additionally, analysis should be conducted to understand how these differences may be attributed to unique classroom norms of each discussion section as perceived by the students and the TAs.

Related to differences in the implementation of classroom norms among the TAs, future research should consider the role the explicit versus implicit attention to these norms in the classroom play in predicting student outcomes. Similar analyses have been conducted in comparing instructors who have given explicit attention to different social and sociomathematical norms in an undergraduate math classroom (Yackel et al., 2000). As an example of such an analysis, I highlight that the mean student awareness scores for Ms. Stokes and Mr. Newton when considering the teaching dimensions of *student-to-student collaboration* were not statistically significantly different. However, interview and observational data revealed that Ms. Stokes explicitly pointed students to each other for help throughout the entire semester. In addition, Ms. Stokes made direct comments during the class I observed that reflected a student-centered implementation of norms related to *student-to-student collaboration*. For instance, during my observation, Ms. Stokes made comments such as, “Guys, the whole purpose of the class is to think and work together and learn,” and, “if there is a part that doesn’t make sense, ask your teammates, guys. You might learn better from your peer students than me.” In contrast, Mr. Newton commented that while he did at first explicitly ask students to work together, he eventually stopped once this became a regular routine students adopted. Indeed, on the day of my observation, students worked together without the explicit instruction of Mr. Newton.

Considering Ms. Stokes’ students had significantly higher self-efficacy scores than Mr. Newton’s, future exploration should be conducted to see if, for instance, Ms. Stokes’ explicit and continuous attention to the norms under *student-to-student collaboration* played a role in these results. In other words, it may be that Ms. Stokes' explicit attention to having students use each other as resources over the whole semester may be a contributing factor in supporting students' self-efficacy in the course. This result should also be considered alongside the fact that there was not a significant difference between the final exam grades of Ms. Stokes and Mr. Newton's students. Put another way, analysis should be conducted to see if the positive association between Ms. Stokes' explicit attention to norms under the teaching dimension of *student-to-student collaboration* and students' sense of self-efficacy was limited only to this outcome (assuming the explicit attention to norms related to *student-to-student collaboration* was, in fact, a contributing factor).

5.3. Implications

Considering the context of the current study, the findings presented here provide potential implications for the math departments of colleges and universities offering introductory math courses similar to that of Precalculus. More specifically, this study offers implications for the ways in which instructors, math departments, and teaching and learning centers can work together in the implementation of the perceived classroom norms identified in this study as being supportive of undergraduate students.

5.3.1. Undergraduate Math Instructors

In general, this study found that students perceiving student-centered, as well as a hybrid of instructor- and student-centered norms, may support undergraduate students' academic performance and self-efficacy. For example, the results presented here suggest that students perceiving student-centered norms related to the teaching dimensions of *variation in instruction* in the Precalculus large lecture were associated with increased final exam grades and students' sense of self-efficacy, while perceiving student-centered norms under the dimension of *instructor-to-student engagement* was negatively associated with final exam grades. From these results, there appears to be some benefit to students perceiving an instructor-centered implementations of norms related to engaging a wide range of students in whole-class discussions within the context of a large lecture.

Examples of such an implementation may include the instructor focusing on reviewing strategies to solve problems as the class listens and takes notes, as opposed to the instructor attempting to engage students in a whole-class discussion around how to solve the problems. Indeed, interviewees in this study commented on how very few people answered questions from the instructor in the large-lecture setting, and those who did were usually the same students. Instead, most students reported engaging in asking and answering questions within the discussion sections, in which student-centered norms related to *instructor-to-student engagement* were positively related to students' final exam grades and sense of self-efficacy. Thus, lead instructors of large undergraduate math courses such as Precalculus should give careful consideration as to what student-centered norms they choose to implement in a large lecture and those they support TAs in adopting within discussion sections.

5.3.2. Math Departments

The Precalculus course discussed here has historically had a high failure rate while also serving a disproportionate number of students traditionally underrepresented in STEM courses. The findings presented here can support math departments wishing to address these patterns. More specifically, math departments should support instructors' implementation of the classroom norms identified here as, when perceived by instructors and students, being supportive of a diverse group of undergraduate students taking a course such as Precalculus. In turn, these classroom norms may increase the retention of students majoring in a STEM field, including students of color. Math departments' support of faculty and graduate student TAs teaching these undergraduate courses is of paramount importance given the achievement and experiences of students in these introductory courses has been found to be related to students' persistence in majoring in STEM (e.g., Chen, 2013; Gainen, 1995). As a result of improving students' successful completion of courses such as Precalculus, math departments may also find an increase in the resources provided by their universities, as John (1999) argues.

> Inadequate concern for a department's undergraduate instructional program is sure to bring increased criticism. On the other hand, a department that earns a reputation for excellence in teaching undergraduates generally finds that this pays clear benefits in terms of the resources that are allocated to the department. (p. 20)

Put another way, math departments focusing on teaching that implements supportive classroom norms, as perceived by the participants in this study, may be rewarding both for students and the departments, themselves.

5.3.3. Teaching and Learning Centers

On a more practical note, math departments can structure their support in the implementation of the supportive norms identified in this study by joining forces with existing teaching and learning centers dedicated to providing faculty of different courses on campus with learning communities aimed at reflecting on one's teaching. As Bressoud (2018) argues, "faculty need both departmental encouragement and a supportive network if they are to make the transition to more effective teaching" (section "Efforts to change"). Some universities offer such a network through a teaching and learning center (TLC). For instance, the TLC at Blackboard University offers professional development for faculty seeking to develop their instructional skills supported by evidence-based teaching practices. Moreover, the TLC at BU currently offers communities of practice that focus on student-centered learning, as well as how to engage students during a lecture. Such existing structures may guide math departments as they support instructors in their implementation of the beneficial classroom norms identified in this study, without having to 'reinvent the wheel' for such structures. In addition, instructors may benefit from these communities by demonstrating their commitment to student success and, in turn, sharing evidence of this commitment (e.g., a teaching portfolio or a teacher training certificate) when applying for different department roles and future jobs.

5.4. Conclusion

This study sought to address the gap in literature that considers the role of classroom norms in undergraduate math courses directly preceding calculus, and their association with students' academic achievement, self-efficacy, and STEM intentions. Using instructor and student interviews and surveys, results suggested that instructors and students perceived the classroom norms of the Precalculus course offered at Blackboard University to include a hybrid of instructor- and student-centered norms, with more instructor-centered norms perceived in the large lecture of the course and more student-centered norms in the discussion sections. Furthermore, students' perceived norms related to the teaching dimensions of *variation in instruction* in the large lecture, as well as *instructor-to-student engagement* in both the large lecture and discussion sections, were found to be associated with students' achievement on the Precalculus final exam and sense of self-efficacy.

These findings point to the importance of students' perceptions of classroom norms and highlight next steps for undergraduate math instructors, math departments, and teaching and learning centers. More specifically, undergraduate math instructors, math departments, and teaching and learning centers must work together to support the implementation of student-centered, and in some cases, a hybrid of instructor- and student-centered norms, if they wish to see increases in the academic achievement and self-efficacy of undergraduate students' taking introductory math courses such as Precalculus. Additionally, this study suggests that great care should be taken to follow-up with students and gauge their perception of these norms as opposed to assuming certain norms are experienced by all students. Still, future research is needed in considering factors outside of the classroom that may support students' STEM intentions. Faced with declining trends of students earning a STEM degree in the United States, the classroom norms highlighted in the current study may at the very least offer a starting point for math departments wishing to support the academic achievement and self-efficacy of their undergraduate students.

Appendices

Appendix A: Frequency Data of Student Demographics

		Low Attendance	Female	White	African American	Hispanic/ Latinx	Asian	Multiracial
Yes	Count	43	105	74	42	20	25	25
	% (n=181)	23.8%	58.0%	40.9%	23.2%	11.0%	13.8%	13.8%
No	Count	138	76	107	139	161	156	156
	%	76.2%	42.0%	59.1%	76.8%	89.0%	86.2%	86.2%

Note. The reference group for all models was a White-male student who neither missed at least one class per week in the large lecture, 4-6 discussion sections, nor at least one discussion section a week (i.e., not identified as having “Low Attendance”).

Appendix B: Summary of Precalculus Assignments

Assignment	Due Date	Access	Grader
Pre-Lecture Quiz	Before lecture	Via learning management software (LMS)	Automatically by LMS
Post-Lecture WebAssign	After lecture or before next lecture	WebAssign	Automatically by WebAssign
Review of Lecture Content	Not officially assigned, but recommended by Dr. Riemann	Textbook readings, solved problems from textbook	Not graded
Two-Part Quiz	During lecture	Paper	Group – Dr. Riemann Individual - TA
Group Quiz	During discussion	Each person gets a copy of the problem but turns in a group answer	TA
Discussion Exam-Prep	During discussion	Each person gets a copy of the problem	Not graded
WebAssign Exam-Prep	Sunday	WebAssign	Automatically by WebAssign
Exploration Problems	During discussion	Each person gets a copy of the problem	Not graded

Note. The large lecture instructor and Precalculus course coordinator, Dr. Riemann, reviewed the above table to ensure an accurate portrayal of the different assignments.

Appendix C: Description of Large Lecture Space

The Precalculus large lecture takes place in a stadium-style hall in the Blackboard University. Students and instructors can enter and exit the large lecture hall from either side of the top (i.e., back) or bottom (front) of the room. The lecture hall consists of six rows of long tables, each populated by swiveling chairs attached to the underside of the tables and plenty of outlets for students to charge their electronics. There are wood panels that scale the sides and back of the hall with speakers mounted at symmetrical points throughout the room for optimal acoustics.

The tables cascade down to the front of the room where the instructor teaches from. There are whiteboards on both ends of the rows of desks for students to write on, as well as two nearer to the back wall of each side of the room. There is a (rarely used) recording camera that sits in the middle of the back wall above the recycling and trash bins. The front of the room consists of three large projector screens that can be used independently, mobile whiteboards, a single table with five rolling chairs, and three TV monitors. The instructor lectures from a separate desk tall enough for her stand behind while teaching. This desk has a computer monitor, a handheld microphone (although the instructor uses a clip-on mic that wirelessly connects to the room's speaker system), and two document cameras.

Appendix D: Description of Discussion Section Space

All Precalculus discussion sections take place on the top floor of the 4-level physics building on campus. The room stays busy with no more than a 15-minute gap between each class and each section taking place one right after another, each day of the week. While there are two doors in the classroom, only the front nearest a rolling whiteboard opens from the outside for entry. The classroom is filled with six sections of two tables each and four moveable chairs. Each section of tables has a set of outlets accessible at the two ends and middle of a given table, and four of the six sections have a small whiteboard mounted on the wall nearest them. The other two sections on the left side of the room do not have these whiteboards due to the windows taking up wall space.

There is a black cabinet next to the entrance (i.e., the front of the room) that sits behind the rollable whiteboard and next to a larger whiteboard. A larger whiteboard is also mounted on the back wall but is rarely used. There is no projector screen in this room. In the corner opposite to the entry doorway, the TAs have a table where they place their materials and papers to be passed out, as well as a smaller whiteboard mounted on the wall that is identical to those seen around the room for students.

Appendix E: Dr. Riemann's Large Lecture Vignette

Dr. Riemann (pseudonym) is 49 years old, identifies as White, and noted that English is not her primary language. She shared that she is an international instructor. Her primary area of research is in the scholarship of teaching and learning.

Approximately 90 - 100 students were present for the large lecture I observed. Dr. Riemann primarily lectured during the class. Two students (one from the first row of tables and one from the fourth row) responded when Dr. Riemann asked for "burning" or "non-burning" questions from the class. The focus of the class was reviewing how to write a trigonometric function for a given graph, as well as how to use trigonometric identities. Given most of the class was spent on the former topic, I focus on this part of the class in the descriptions below.

Multiple times during Dr. Riemann's lecture she focused on presenting students with strategies for answering different types of questions and emphasized her thought process for each problem. In the following excerpts, Dr. Riemann walks students through identifying the parent function to get started in answering a review question that she mentions is resemblant of problems from WebAssign and past exams.

> So for those types of problems, what we want to do is first of all identify the parent function. Right? … I [am] looking at this graph, and does it look like a sine/cosine graph? Does it look like a tangent-cotangent? Or does it look like a secant-cosecant? And the shapes of those functions should be, if not in the internal memory, then at least on our formula sheet. Right? So when I'm talking about the sine-cosine, what shape am I looking at? Up and down. Right? … So things that have this wavey shape I'm trying to fit them to sine-cosine. What about things that are more like, hmmm, an s-like? … Those would be functions that I can model with a tangent-cotangent. And now we are looking at this last one. Oh, well, if it's none of those, then maybe it's a secant-cosecant. And if elimination doesn't work for you, then remember the definition of those functions. … So they will not be defined where the denominator is 0. Those functions have asymptotes.

The above excerpt shows Dr. Riemann helping students identify the parent function using multiple approaches, including remembering what the different parent function graphs look like from “internal memory,” as well as considering what trigonometric functions have vertical asymptotes.

Following the above moment, a student asked Dr. Riemann to “explain how you got 0 for the horizontal shift?” This question served as a segue for Dr. Riemann to continue working on the problem described in the above excerpt while also addressing the use of the secant function as opposed to the cosecant function she just found to represent the graph.

> But what if I wasn’t paying attention and… I started with a secant function? What if I was trying to fit this same graph with a secant function? How can I find the horizontal shift then? … How does the secant look? My memory is not what it used to be, so I don’t remember this. But what I do remember? I remember the definition of secant; sec(x) is 1/cos(x), and I remember the graph of cosine [goes on to describe graph of cosine in terms of domain and range, if it starts at a maximum or minimum, etc.] … And now I’m doing 1/cos(x).

The above excerpt shows Dr. Riemann provided students with a way to think through the same problem using a different approach, or rather, a different function. Additionally, she highlighted strategies for how to work through the case in which students can’t remember key characteristics of a parent function by using what they know about other parent functions. Dr. Riemann’s initial use of a cosecant function (which in this case did not require a horizontal shift) also confirms a comment she made earlier that “some expressions are simpler than others.” As she shared, “I usually try to do a fit that doesn’t have a phase shift, that doesn’t have a horizontal shift. Why? Because I usually get confused by horizontal/phase shifts.”

Other comments provided further evidence of Dr. Riemann’s focus on helping students use different information they had covered in class to check their progress in answering questions related to trigonometric functions and their graphs. As she reminded students, “Alright, so always algebra and the graph, algebra and graphical representation need to fit, and I can use one to check the other.” Furthermore, Dr. Riemann concluded this portion of the class focused on trigonometric functions and their graphs with a comment on how to utilize the WebAssign assignments as a check for understanding.

> Alright, I'll let this stay with you. There are plenty of WebAssign [assignments] of variations of this. … Please treat the WebAssign homework as your own sort of self-quiz. Try to get it. Try to do the work as best you can. Try to get that WebAssign done in one trial. If you get it in one trial, then you really know the material. If it takes you 5 trials before you ask a question, not only do you not know the material, you are not using your time well. You should probably ask a question after the second wrong trial. Right?

Appendix F: Ms. Stokes' Vignette

Ms. Stokes (pseudonym) is a 30-year-old, first-year, biostatistics graduate student who also attended BU as an undergraduate. She identifies as Middle Eastern or North African and noted that English is not her primary language. She is open to many career options following graduation, including an academic position with a focus on teaching and research, an academic position at a two-year college, or a non-academic position (e.g., industry, government). This was her first semester teaching a Precalculus discussion section.

The focus of this class was on working algebraically with inverse trigonometric functions. The following exploration problem is an example of such a problem that was given during the class I observed.

Exploration 23.4.

(a) Write $\sin(\cos^{-1}(x))$ without trigonometric functions. (If you haven't seen a problem like this yet, it's Example 8 in Section 6.4 of the textbook.)
(b) Write $\sin(2\cos^{-1}(x))$ without trigonometric functions.
(c) Write $\sin(\cos^{-1}(x) - \pi/3)$ without trigonometric functions. `https://www.desmos.com/calculator/xnywlbhrit`

During my observation, Ms. Stokes's used student progress on assigned problems, as well as student participation, to drive her instruction. Throughout the class I observed, Ms. Stokes asked for students to communicate if they were making progress with the problems, as seen in the following interchange with multiple students.

> Ms. Stokes: So, what do you think I need to do first? Any suggestions? How about I take this part as my theta? Then you have arc something. You should read it like this, sine inverse of ⅗ is equal to…
> Students: Three fifths.
> Ms. Stokes: Do you remember our popular triangle. So, guys what was the ratio of sine of something? [two other students' responses are inaudible] Use Pythagorean Theorem and tell me, what is this length?
> Students: 4.
> Ms. Stokes: Are you all following? … Guys, be confident, and say it out loud. So I use this information and that is my answer. Questions? If you have a question, that most likely is someone else's question. So say it out loud. … So that was the warm-up.

A few minutes later, Ms. Stokes highlighted a question brought up by a student she was working with one-on-one and asked the class for input on how to approach the problem.

> Ms. Stokes: Guys, your classmate had a good question. She was wondering why do I even need to know this part [points to whiteboard]. I'm looking for sine of theta, right? And I claim this is my theta. I do need this hypotenuse to find sine. Does it make Progress? Eh, eh, eh? [tilts hand back and forth to indicate somewhat] Okay. Okay. Good. Progress, for this part?

While Mr. Lagrange had used the mobile whiteboard for students in the first row of tables, Ms. Stokes was the only TA to utilize the whiteboards mounted on the walls around the room above different group's tables. Furthermore, out of all five TAs I had observed, Ms. Stokes was the only one who had asked a student to come to the board to write out her work and share her thoughts.

> Ms. Stokes: That's when I can use the double-angle identity, right? [finishing a problem involving the double-angle identity] Comments? Feedback? Different idea?
> [student in front row of tables asks a question that is inaudible]
> Ms. Stokes: That was the problem, I mean the mistake that I did. How would that assist us? [student makes an inaudible response]
> Ms. Stokes: So wouldn't that be easier that I tackled it that way because I already know the double-angle identity?
> [student makes an inaudible response]
> Ms. Stokes: Would you like to show me how?
> Student: Unless that's wrong.
> Ms. Stokes: We're gonna figure it out together. Just write it down.
> [student comes to the front to write on the main whiteboard]
> Ms. Stokes: Okay. Thank you. Guys, the whole purpose of the class is to think and work together and learn. So thank you for sharing your thought.

Ms. Stokes made other, similar comments that had students reflect on how they were utilizing the other students in the classroom in understanding the material. For instance, at one point she shared, "I want you to be fast and ask questions from each other," and "if there is a part that doesn't make sense, ask your teammates, guys. You might learn better from your peer students than me."

As a second example of Ms. Stokes emphasizing the participation of students in the learning process, she had another student (from the back row of tables) come up to share an alternative approach to a problem. As she shared with the class, “Guys, remember that math is not written on the stone, and you can write it different ways. As long as that is logical, and you’re following the right rules, you’re fine.”

Ms. Stokes also made two comments that were associated with students’ wellbeing. For example, she started the class with the following statement.

> Good afternoon, everyone. What did you learn in lecture? It sounds like you all need to stand up again. I don’t like the energy of the room. You need to clap for yourself and wake-up. Okay, are you satisfied? You can sit down. Now tell me what you learned during your lecture.

She also wrapped up the class by saying, “Good luck guys [referring to the upcoming final exam]. It’s not the best time of the semester so make sure you do something for your mental health.” Such comments were unique in that, while other TAs had made comments about studying strategies and concepts covered on the final exam (e.g., utilizing past exams for practice, emphasizing certain topics being important to review), none had made comments referencing students’ physical and mental wellbeing.

Appendix G: Mr. Newton's Vignette

Mr. Newton (pseudonym) is a 23-year-old, first-year, mathematics Ph.D. student. He identifies as White. His research focus is homological algebra and plans to pursue an academic position with a focus on teaching and research. This was his first semester teaching a Precalculus discussion section. While Mr. Newton completed an instructor survey, he did not consent to me audio and video recording my observation. As a result, I rely solely on the field notes I took during the observation to describe the classroom on the day I visited.

The focus of the class was on working with inverse functions. The entire class consisted of students working individually or in groups. Almost all of the students had their laptops open (one of which was watching a math instructional video on YouTube). Mr. Newton only addressed the whole class at the beginning of the session I observed.

> Mr. Newton: Since this is the last week of discussion sections, I'll pass back the discussion quizzes. You guys can decide if you want to keep them, or if you don't, that will also work out.
> Student: Does this mean that they got entered into [the learning management system]?
> Mr. Newton: That's really a question for Dr. Riemann because I just grade them.

Mr. Newton did not ask students any questions. Most of the student questions Mr. Newton addressed were related to finding the domain of the inverse of a function, to which Mr. Newton explained multiple times that "The domain of the inverse is the range of f(x) [the original function]." Mr. Newton also used one of the whiteboards at the front of the room to work with a group on an average rate of change (a final topic covered in Precalculus at BU) problem that involved calculating f(a+h) given a function f(x).

In the excerpts that follow, I highlight interactions that took place at the table closest to me given their conversations were easiest to hear. This group was also the loudest in the class. In particular, the excerpt below is an example an interaction between Mr. Newton and a student in this group, whom I will refer to as Donald (pseudonym), who was working with several other students sitting at his table. To provide context, the student was working on a problem that involved simplifying trigonometric functions using various identities.

> Donald: If I could get feedback [from the WebAssign program] in between each step of the problem, that'd be great [laughs]. … I'm not sure what the purpose of trigonometry is if everything we do boils down to one thing [laughs]. … Hey [Mr. Newton], how does this look? Are we on the right track?
> Mr. Newton: Uh, yep.
> Donald: Alright cool. … I got a 100 on a math assignment. I'm so good at math [laughs].

Given that I conducted my observation during the second-to-last week of classes, other points of discussion in Donald's group (as in other discussion sections I observed) centered around what courses they were taking in the spring semester. For example, Donald shared a conversation he had with another student (referenced as Future Student in the interaction below) not currently taking Precalculus but interested in registering for it in the upcoming spring semester.

> Donald: And I was like, okay, what math class are you taking?
> Future Student: PreCalc.
> Donald: Oh, okay. A lot of my friends have failed that.

Another student in Donald's group also commented, "I'm really loving my Fridays. I have like 5 hours between classes. … I'm not having a class before then. 10 am is the earliest class." A few minutes following this conversation, Donald also made comments regarding his high school experience and comfort with trigonometry. As he shared, "The trig's not hard. I can get trig. Trig sub [substitution] was my nickname in high school. It sounded funnier in my head, but it wasn't."

Finally, at the end of the session, a student from a different group asked Mr. Newton if there were going to be review sessions in preparation for the final exam that was to be given the following week.

> Student: Are there going to be reviews for these?
> Mr. Newton: I think … and [says an instructor's name I can't make out] is going to be running these.

Student: And when will those [the review sessions] be occurring?
Mr. Newton: That I don't know. … [inaudible instructor's name] is going to hold a two-hour review session, and he expects you to fill out when you are available to meet.

Appendix H: Mr. Lagrange's Vignette

Mr. Lagrange (pseudonym) is a 23-year-old, second-year, applied mathematics Ph.D. student. He identifies as East Asian and noted that English is not his primary language. He is the first in his family to pursue an advanced degree. He is undecided in what his primary area of research will be but plans to pursue a non-academic position following graduation. Mr. Lagrange has previously served as a TA for other courses at BU.

During my observation of one of his discussion sections, while there were two main moments in the class when Mr. Lagrange lectured (described below), the majority of class time was filled by students working individually or with students at their table (most worked in pairs). He began the class by reminding students about when and where the final exam was to take place. He then proceeded to pass back quizzes, during which he demonstrated that he knew his students' names. On the mobile whiteboard at the front of the room, Mr. Lagrange had written the definition of an inverse of a function given that the discussion quiz for the day focused on this topic. Additionally, he had written on the main whiteboard the algebraic steps for how to find the inverse of a function, which he noted were taken from the textbook. While Mr. Lagrange did not explicitly mention which students should be working on, he referenced the discussion quiz exploration problems when addressing the whole class. In addition, three students were on their computers working on the WebAssign homework for the week.

As mentioned above, aside from beginning the class with providing information about the final exam location and time, there were two times Mr. Lagrange addressed the whole class. The first time was roughly 18 minutes into the 50-minute class when Mr. Lagrange reviewed the algebraic steps to finding the inverse of a function (referencing the main whiteboard with these notes) to help students working on the discussion quiz problems (see below).

MATH 115 Lesson 22: Inverse Functions

Instructions: This assignment is open everything. Open book, open notes, open laptop, open conversation. Write your work for this problem directly on this sheet. You will be graded only on the work that you show.

Group Quiz 22.1. For this problem, let $f(x) = \sqrt{x+1} - 2$.

(a) Find the domain and range of f.
(b) Find f^{-1}. What is the domain of f^{-1}?
(c) Graph $f^{-1}(x)$. (Be careful; part (b) is relevant.)

As Mr. Lagrange shared, "For the discussion quiz problem, so first of all, if you are struggling with part b, 'find the inverse function,' I copied this [references definition of an inverse function] from the textbook." After walking students through this process, he went on to part b of the problem, which asked students to find the domain of the inverse. Mr. Lagrange cautioned students to "be extra careful here because it might not be what it looks like," and suggested that "instead of looking at the result and looking for the domain, try to look at the definition and think about the domain." Mr. Lagrange then instructed students who had finished the discussion quiz problems to work on the exploration problems he described as "very good questions" (see below).

The problems on this side of the sheet are designed to provoke discussion and exploration. A lot of them won't have a single right answer; that's intentional. You may put any written work for them directly in your notebook; it will not be collected. ***You are not expected to complete every problem.***

Exploration 22.2. Prove the following theorem:

$$\text{If } g(x) = \frac{x-1}{3x-2}, \text{ then } (g \circ g)(x) = g^{-1}(x).$$

(This theorem is not true for most functions; g is special. For a major challenge, try to find a different function with the same property! These functions and their properties were a significant component of a doctoral thesis at the University of Maryland.)

Our group spent   *minutes on this, and I am* [not | somewhat | completely] *happy with my answer.*
Right now, this problem feels [not | somewhat | completely] *worth the time we spent on it.*

Exploration 22.3. Let $h(x) = \dfrac{e^x}{e^x+1}$. (This function is called the standard logistic function or the Verhulst model, and it's the single most useful function in biostatistics. This curve is often a shockingly accurate model of population growth – of cells in a Petri dish, of fish in a pond, of the size of a tumor...)

(a) Find $h^{-1}(x)$.
(b) By making a table of signs, find the domain of $h^{-1}(x)$.
(c) Using (b), find the range of $h(x)$.

Our group spent   *minutes on this, and I am* [not | somewhat | completely] *happy with my answer.*
Right now, this problem feels [not | somewhat | completely] *worth the time we spent on it.*

About 13 minutes later (34 minutes into the class), Mr. Lagrange addressed the whole class to review the relationship between the graph of a function and its inverse, referencing part b of the group quiz problems. Below is a partial transcription of this moment.

> Okay, so, some explanation. In the end, your graph looks like this [referencing the answer to Group Quiz 22.1]. … I don't know if you talked about this in class, but there is a relationship between the original graph and the graph of the inverse function [goes on to explains the symmetry of the original graph and the graph of its inverse across the y=x axis]. … I don't know if it's obvious to you or not… but these are reflecting across this line. … I hope you understand the concept. Basically, the thing to take away is, after I solve the inverse, your result doesn't tell you the domain. But rather, you should look for the domain according to the original function.

He then went on to give students guidance as to how to complete the exploration problem 22.2.

> I don't know if you guys looked at the exploration problems, but some of the sample exams ask you to verify two functions, f and g, are inverses. I don't know if you talked about this in class. How do I verify this? So there are two ways, right? [explains using above algebraic steps to find the inverse function and show it is the same as the inverse given, as well as composing the function and its inverse to verify if the result is x] … You still have some time, so if you finish the first problem [group quiz problems], try to do the exploration problems. Also, inverse functions are kind of important, and might be on the final.

The remainder of the class was dedicated to students working individually or with those at their table on the group quiz problems, exploration problems, or WebAssign assignments.

Appendix I: Mr. Euler's Vignette

Mr. Euler (pseudonym) is a 23-year-old, first-year, mathematics Ph.D. student. He identifies as Hispanic or Latinx and noted that English is not his primary language. His primary area of research is algebra and plans to pursue an academic position with a focus on teaching and research following graduation. This was his first semester teaching a Precalculus discussion section.

The entire class was allotted to students working together on the same group quiz and exploration problems, as in Mr. Lagrange's class. In addition, Mr. Euler provided the following additional exam prep problems below after going around to each group of students and explaining that inverse functions have not been a part of final exam since 2008 (as indicated on the handout).

MATH 115 Lesson 22: Inverse Functions Date:____________

Exam Prep Activity

Pb. 1: Use the inverse function property to show that $f(x) = \frac{1}{x} + 2$ is the inverse of $g(x) = \frac{1}{x-2}$. Also find and compare the domain and range of the two functions. (**Hint**: Graphing might be useful).

Pb. 2: For the function $f(x) = \sqrt{3-x}$

(i) Find the inverse function $f^{-1}(x)$.

(ii) Sketch the graphs of f and f^{-1} on the same coordinate axes.

Pb3: Repeat the requirements of pb 2 for $l(x) = \log(2x+5)$.

Looking forward: Inverse functions have not been tested on recent final exams, but they will be tested on our final exam. Look on the Mathematics Testbank (https://www-math.umd.edu/testbank.html) and from there MATH115 for an exam before 2008 that contains an inverse problem. Then solve the problem.

He also shared that at the beginning of the semester, there were typically 19 of 22 students here, but that number had decreased to about 14 since some students had dropped the class. He also said that there had been a lot of people absent during the week of my observation being that it was the second to last week of the semester.

During my observation of one of his discussion sections, Mr. Euler was very attentive to students. There was very little idle time. During the entire session, Mr. Euler was talking with students to help move their groups forward on the problems. As an example of Mr. Euler's conversational teaching style during my observation, consider the following interchange with regards to Group Quiz 22.1.

MATH 115 Lesson 22: Inverse Functions

Instructions: This assignment is open everything. Open book, open notes, open laptop, open conversation. Write your work for this problem directly on this sheet. You will be graded only on the work that you show.

Group Quiz 22.1. For this problem, let $f(x) = \sqrt{x+1} - 2$.

(a) Find the domain and range of f.
(b) Find f^{-1}. What is the domain of f^{-1}?
(c) Graph $f^{-1}(x)$. (Be careful; part (b) is relevant.)

Student: We were trying to look at videos. … The domain I understand to be, like…
Mr. Euler: You're just trying to knock out a negative, right? [explaining the radicand cannot be negative]
Student: Yeah, exactly, so it would be negative 1?
Mr. Euler: So you're like, x+1, if you want to write the work, the thing inside has to [have] no negatives, so it's [the domain] greater than or equal to -1. The interesting part, like the thing you are asking, like how to get the range in general, yeah, the idea is what you're saying. You plug in negative 1, right?
Student: Yeah.
Mr. Euler: It's 0.
Student: Yeah.
Mr. Euler: Then it's, the lowest is negative 2. Then you plug in infinity.
Student: Yeah. I was thinking of it as like, you plug in infinity to x, and if you plug in infinity this is going to be infinitely big.
Mr. Euler: So square root of infinity.
Student: Yeah. So that's still infinity. And then the minus 2 doesn't really do anything, so I know the right-hand side would be to infinity. But then the other, like where it restricts it, I was confused on that end because I was thinking like…
Mr. Euler: Yeah, like maybe include bounds…

Student: Yeah, like negative infinity plus 1 is still negative infinity and square root of negative infinity… so would it just be negative 2 because it's just, you can't do…
Mr. Euler: Yeah, cause you just start plugging in negative 1, like, the lowest you can plug in is negative 1, right?
Student: Yeah.
Mr. Euler: So maybe if you want some really decent work, you can just plug in negative 1, say I get 0 minus 2, it's negative 2, then I plug in infinity, I get infinity, right?
Student: Yeah.
Mr. Euler: And you're like, okay, I know it's negative…
Student: You know it's negative.
Mr. Euler: And I'll draw it to infinity. That's a good idea. Something else I could say is, I know my square roots. It takes anything between 0 and infinity, so square root minus 2, instead of from 0 to infinity, it takes negative 2 to infinity, and just say that [student nods yes]. And just say that right?
Student: Yeah.
Mr. Euler: And just say, because square roots take 0 to infinity, I take it down by 2, right?
Student: Yeah, that makes sense.
Mr. Euler: And in general, you kind of do the same thing, you look at your domain, you plug in like both sides or something…
Student: Yeah.

The excerpt above is representative of other interactions I saw Mr. Euler have with students during my observation. His use of phrases such as, "You're just trying to knock out a negative, right?" and "So you're like, I know my square roots" utilized more informal language (e.g., "knock out") that seemed to help him interact with students on their level of understanding. Mr. Euler was able to effectively identify and communicate student misunderstandings (there was a note on one of the whiteboards, thanking Mr. Euler and saying he was the "best TA ever"), as seen in the following interchange 19 minutes into the class.

Student: So, Mr. Euler, how is the inverse of a square root function a quadratic? They only have one point in common.
Mr. Euler: [rolls mobile whiteboard over to the group] I think you're confusing what an inverse function is.
Student: When I think of inverse, I think of opposite.
Mr. Euler: Okay, so, geometrically… [draws y=x axis] this diagonal is like, 45 degrees, okay? ... You just flip the [over the] axis. So, your inverse looks like this [indicates symmetry of graphs across y=x axis]. So that's your inverse, right?
Student: Okay. That's what we think of.
Mr. Euler: So first of all, [what] does the square root look like? … So, how does it look after reflecting [referencing square root function draw on whiteboard]? So this definitely

> looks like a quadratic [student shakes head up and down]. … So what's happening? … So, the inverse function is not x squared, itself, it's a piece of x squared, the left side of x squared.

Following my observation, Mr. Euler briefly shared about his experience as an instructor in the U.S. compared to teaching outside the United States. He pointed out that having a large class like that of the Precalculus large lecture was very different from his experience. When asked on the survey about any aspects of his identity that have impacted his experience at the BU, he shared, "I just moved to the US so the city and university are very different from what I was used to."

Appendix J: Mr. Descartes' Vignette

Mr. Descartes (pseudonym) did not complete an instructor survey. Therefore, I do not have demographic information about him. However, he did consent to me observing his class. Most of his class did not consent to me video and audio recording my observation. As a result, I chose not to record the class I observed and rely solely on the field notes I took during the observation to describe the session.

Mr. Descartes began the class reviewing when and where the final exam was going to take place, followed by students working on the provided quiz and exploration problems for the day, including the following.

Group Quiz 23.1.

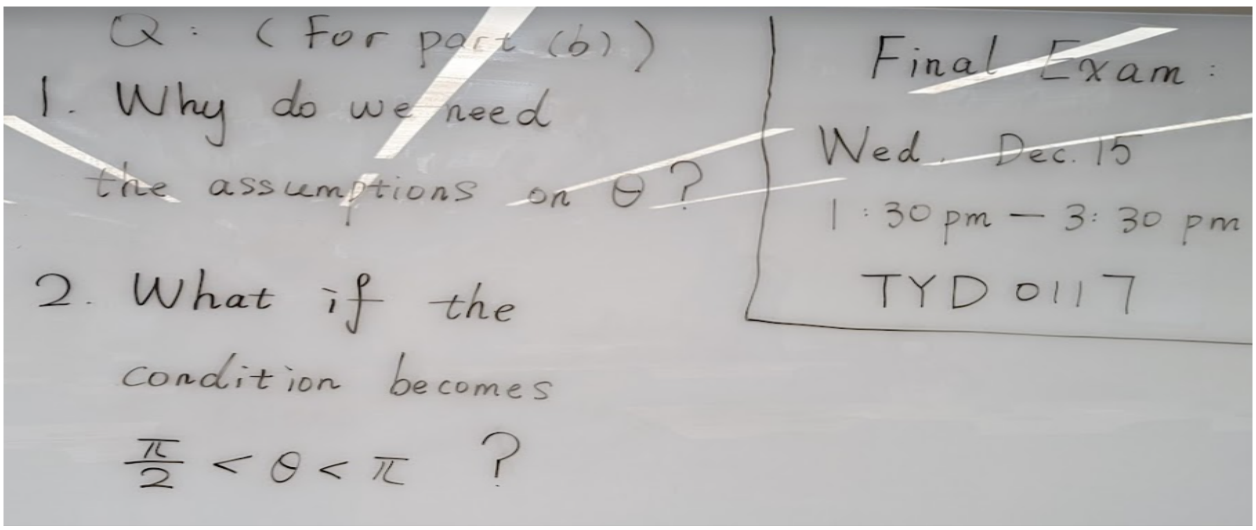
(a) Substitute $x = 2\tan(\theta)$ into $f(x) = \dfrac{x}{\sqrt{4+x^2}}$. (Don't simplify yet.)

(b) Simplify, assuming $0 < \theta < \pi/2$.

The entirety of the class was dedicated to students working on the provided set of problems. Three students had their laptops open, but it was not clear whether they were working on the WebAssign assignment for the week. As with many of the other TAs, Mr. Descartes did not give explicit instructions as to whether students should have been working in groups or individually; in the end, there was a mix of students working individually and in pairs or larger groups.

Throughout the class, Mr. Descartes walked around the room and interacted with students. Some of these interactions consisted of him checking student work after a student had asked if their answer was correct, while others included him checking in with students who had not explicitly asked for help. For example, at one point Mr. Descartes corrected a student's work saying, "This is not correct. So, for part (a)... what did you get? … For a fraction, you can multiply the numerator and denominator. But you cannot square the numerator and denominator." As further examples, at one point Mr. Descartes walked over to a student and asked, “Do you need help, or did you figure it out,” while to another student he asked, “Do you have any results or any questions?” Most student-to-student interactions consisted of students asking each other whether they had gotten the same answer on a given problem.

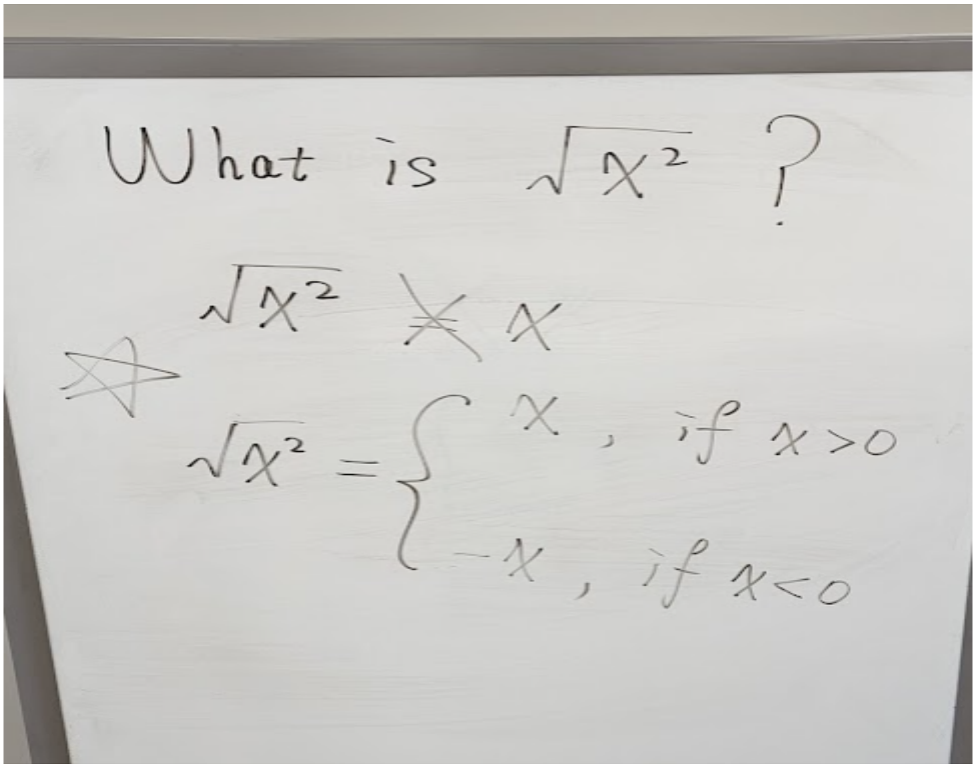
Mr. Descartes’s teaching on the day of my observation allowed a lot of space for students to make sense of the mathematics using prompting questions that he believed addressed common student misunderstandings. Most of these prompting questions stemmed from two questions he had written on the main front whiteboard.



Mr. Descartes referenced these questions when working with multiple students, asking, “At which step [of Group Quiz question b] will you need this assumption?” Consider the following interaction between Mr. Descartes and another student.

> Mr. Descartes: Did you use your assumptions?
> Student: What does that mean, assumptions?
> Mr. Descartes: Here your theta is between 0 and pi/2. So, at which step do you need this assumption?
> Student: Oooohhh!
> …
> Mr. Descartes: So at this is step is where you need this assumption.
> Student: Right. Okay. … Got it. Okay. So if it were in this quadrant, then sine would still be positive?

Additionally, Mr. Descartes referenced another set of questions throughout class that he had posed on the mobile whiteboard at the front of the room.



After speaking with Mr. Descartes after class, he told me that he used the question of "what is the square root of x-squared" to address a common mistake students make in losing the sign (i.e., positive or negative sign) when taking the square root of an equation. Together, the above questions Mr. Descartes posed were meant to guide students in understanding that theta being between 0 and 2pi was a necessary piece of information to answer question b on Group Quiz 23.1. This meaning was made explicit in the following instructions Mr. Descartes gave to the class at the end of the discussion meeting.

> So, uh, let me remind you about something about this question. So actually, this [referencing prompting questions 1 and 2 on the main front whiteboard] question can be generalized to this question [referencing prompting question on what the square root of x-squared is]. So you can think of this general question. What is the square root of x-squared? Someone may think you can take the square root and get x. But in general, you can't get this result. In general, you have to check if x is positive or negative. So you have to consider whether your theta is greater than 0… this is very important. Especially

on your final exam if you want to take the square root, you have to be careful. Do you have any questions for this part?

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