ABSTRACT

Title of dissertation:	UNDERSTANDING HOW PRESERVICE TEACHERS USE FOCUSING QUESTIONING STRUCTURES: A MULTIPLE CASE STUDY
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The study explores how five secondary mathematics preservice teachers use questioning structures as they develop understanding of how to teach. Teacher questioning impacts the degree of student thinking during solving problems, specifically selecting focusing over funneling questioning structures (Herbel-Eisenmann & Breyfogle, 2005; Wood, 1998). Questioning structures are investigated as the participants plan a lesson, practice it to their peers, and then teach it to high school students. As these preservice teachers explore this lesson over most of a semester, a rich analysis of questioning is developed through planning, practicing, and teaching the lesson. Investigation includes how participants elicit and interpret student thinking and

how their responses either focused on the thinking of students or funneled students to the thinking of teachers. The research questions of this study are:

- Do preservice teachers use focus and funnel questioning structures as they elicit, interpret, and respond to student thinking and, if so, how do they use them?
- In what ways does preservice teachers' use of focus and funnel questioning structures change through the plan-practice-teach cycle?

Data for the study include an initial peer rehearsal activity; draft and final lesson plans; reflections on experiences with planning, peers, and students; and transcripts of peer rehearsals and interviews with each participant at the end of the study. Analysis of the data explored the types of questions asked and questioning structures used, how the preservice teachers used questioning to privilege or minimize the role of student thinking, and how flexible the preservice teachers were in asking questions, be they planned or extemporaneous. While each of the participants stated the goal of creating student-centered learning environments, they varied widely in their ability to privilege student thinking. Some reasons for the differences in these abilities are explored.

The study demonstrated four potential areas of future research in regard to teacher preparation: preservice teachers need help to learn about and use focusing questioning structures; opportunities may need to allow preservice teachers to address and overcome their current beliefs; preservice teachers need support to effectively elicit, interpret, and respond to student thinking; and peer practice needs specific structures to be effective.

UNDERSTANDING HOW PRESERVICE TEACHERS
USE FOCUSING QUESTIONING STRUCTURES:
A MULTIPLE CASE STUDY

By

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Dedication

This dissertation is dedicated to all of my students and colleagues with whom I have worked that have helped me to better understand the learning and teaching of mathematics.

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Many people have supported me on this journey, which has taken twists and turns that I never expected when I started. First of all, I need to thank my wife, Michele, for all of the support and understanding. My focus was too often on finishing readings and writing reflections when I should have focused on being present with her and others in my family. She is always patient and far too generous with allowing me to develop my interests and helpful in allowing me time and space to accomplish so many different goals. Her support – both direct and indirect – has allowed me to create what you see here.

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Chapter 1: Introduction

My Story

My views on teaching mathematics have changed dramatically since I began teaching twenty-nine years ago. My experiences have taught me the importance of learning about and using student thinking to teach mathematics most effectively. I have learned that I need to use questioning to find out how students are making sense of mathematical concepts. I know that I need to take more of a "back seat" and allow student thinking to guide the direction of lessons and not to let my thinking and my strategies dominate the lesson. I have learned that the more teachers know about their students, the better they will be able to help their students learn.

In looking back on my experiences in education, I find interesting parallels of key transition points in my career and the introduction of important documents in the field of mathematics education. I started teaching in the fall of 1990, shortly after the release of the National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). When I started, I taught the way that I had learned, beginning most lessons by presenting a problem and modeling the desired solution strategy, then providing students with practice problems, observing them and providing feedback as they practiced, and finally having students work on additional examples independently. Such lessons followed the Gradual Release of Responsibility teaching strategy (Pearson & Gallagher, 1983), i.e., I (as the teacher) "do" (solve the problem), we (teachers and students) "do" (solve similar problems), and you (as the student) "do" (solve more problems). This model focuses on the thinking and strategies of the teacher, minimizing approaches that students might consider. As the impact of *Curriculum and Evaluation Standards* (NCTM, 1989) was felt throughout the country, my teaching evolved to privilege the ideas of students, as I realized I needed to question and learn more about how students interpreted a problem rather than telling them how solve it, which would allow connections to form between the students' initial thinking and the strategies they developed to solve them. The importance of using student thinking to guide mathematics instruction is an assumption that underlies both my work in education and this study.

Another important document in the development of the field of mathematics education was *Principles and Standards for School Mathematics* (NCTM, 2000). At this time in my career I was moving into my first non-classroom-based position as the supervisor of mathematics of my school district. While I was only out of the classroom for one year, I began to realize the link between policy at the district level and the effects on teaching in the classroom. As I returned to the classroom, I witnessed the effects of *Principles and Standards* (NCTM, 2000) on the teaching of mathematics. This helped me to continue to evolve in my teaching and I began to take opportunities to work more with fellow teachers in coaching and professional development roles. I next chose to leave the classroom and return to a district level position in 2010, the same year as the release of the Common Core State Standards (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). Now, my position as a district leader provided me the opportunity to help determine the best way to address the needs these new standards identified for both students and teachers.

In my own development as a classroom teacher and education leader, student thinking had come to represent one of the most important aspects, if not the most important, in the learning and teaching of mathematics. My own teaching has changed significantly since I earned my teaching certification many years ago and my coaching now focuses on supporting others engaged in similar shifts in their thinking about how to teach mathematics. My own change was evolutionary, with many different experiences causing me to continually evaluate my own teaching. In viewing recordings of my own teaching, I realized that I was doing too much of the thinking in my lessons. I saw that the teacher I was in the video was not the teacher that I was in my head. I started to do more collaborating with other teachers and working with them to experience the need for change that I experienced. This has led to reconsidering the role of questioning and the development of eliciting, interpreting, and responding to student thinking, and how teachers can best engage in the teaching of mathematics.

As I became more involved in coaching situations with preservice and in-service teachers, I noticed that many lessons focused on the thinking of the teacher rather than that of the students. Many lessons I observed showed students how to solve the problem, then "allowed" them to solve problems using a given strategy. When I noticed that both planning and implementing lessons rarely addressed the thinking of students, I decided to learn more about how to support teachers in eliciting, interpreting, and responding to student thinking to provide the proper foundation for their instruction.

Topic of the Study

The job of teaching is a complex act during which teachers make multiple decisions to help students achieve specific learning goals (Hammerness, Darling-Hammond, with Grossman, Rust, & Shulman, 2005). Preparing preservice teachers for the challenges of the classroom is equally complex. Teacher preparation programs need to provide strategies and guidance on how preservice teachers can learn to make instructional decisions that will help students understand concepts and achieve success. At the same time, there are only so many hours in which to prepare teachers and help them understand everything that makes teaching complex. Preparation programs need to identify which practices are most important by developing a set of "core" practices, which allow teacher educators and preservice teachers to prioritize which practices will enable them to become effective teachers (Ghousseini, 2015; Grossman, Hammerness, & McDonald, 2009). Core teaching practices are those used by teachers on a regular basis, which require them to use professional judgment and support students in learning critical content (McDonald, Kazemi, & Kavanagh, 2013; Windschitl, Thompson, Braaten, & Stroupe, 2012). An additional vital aspect of core practices needed within teacher preparation programs is training preservice teachers to both learn and apply these practices (Hatch & Grossman, 2009).

One of the primary core teaching practices for preservice teachers involves questioning that supports eliciting, interpreting, and responding to student thinking (Ghousseini, 2015; Singer-Gabriella, Stengel, Shahan, & Kim, 2016; Sleep & Boerst, 2012; Stein, Engle, Smith, & Hughes, 2008). Over time, the role of the mathematics teacher in the classroom has shifted from a provider of knowledge and strategies to a collector of student ideas, who interprets them and then figures out how to use them as the class moves through a lesson (Stein et al., 2008). Because of the varying progress of teachers accepting and embodying this shift, preservice teachers have a range of experiences viewing teachers as both provider and collector of ideas. Interestingly, these experiences can sometimes create tensions for preservice teachers, causing tensions around attending to and engaging with student thinking (Singer-Gabriella et al., 2016). With these experiences and tensions in mind, preparation programs need to include how to elicit, interpret, and respond to student thinking as a core practice and determine what practices work best to support preservice teachers developing this core practice. The participants in this study draft and revise their lesson plans over several weeks and practice lesson implementation with their peers, creating a situation that is different than for most in-service teachers. This structure provides opportunities for preservice teachers to work on practices with the support of their peers and teacher educators. There is also a growing body of evidence that preservice teachers can learn and develop these practices (Singer-Gabriella et al., 2016; Sleep & Boerst, 2012; Star, Lynch, & Perova, 2011; Star & Strickland, 2008).

Effective learning and teaching of mathematics requires understanding of the meaning of eliciting, interpreting, and responding to student thinking (see Figure 1). Eliciting student

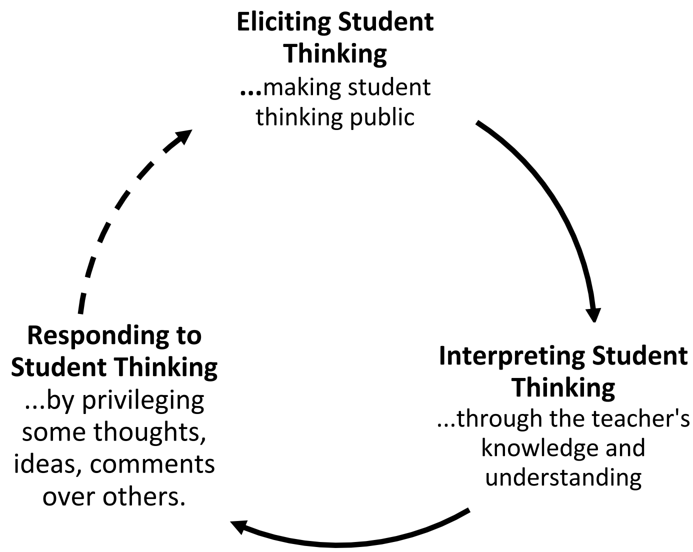


Figure 1. Eliciting, interpreting, and responding to student thinking.

thinking involves making student thinking public in the learning environment, that is, "made visible and elaborated" (Singer-Gabriella et al., 2016, p. 413). Teachers interpret student thinking using their mathematical knowledge and experience through the framework of teacher noticing, described later in this chapter. They then choose how to respond to student thinking, what to privilege and what to bypass (Singer-Gabriella et al., 2016). Teachers may also decide in what manner to respond, i.e., through questioning, displaying student work, or providing more information (Stein et al., 2008).

What preservice teachers respond to during a lesson and how they choose to respond may be influenced by how they have envisioned the lesson during the planning process. Teachers start lessons with a vision of how they see the lesson proceeding, as defined by their lesson plan. Of interest to this study are two ideas: how do preservice teachers' beliefs about the learning of mathematics impact their view of lessons and whether these beliefs influence how preservice teachers listen to and use student thinking during instruction. This study examines how preservice teachers use different questioning structures, specifically focusing and funneling, to teach. Examining questioning structures provides insight into preservice teachers use of student thinking, whether they focus on student thinking to guide the direction of instruction or funnel instruction to match expectations, how they demonstrate their views about the role of the teacher, and whether interpretations of student thinking or those envisioned from the lesson plan guide the lesson.

Significance of the Study

This study explores how preservice teachers view the questioning of students: how they plan questions, what questions they ask, and what they act upon in student responses. When preservice teachers consider the questions they anticipate asking, the student responses they expect, and the actions they plan to take based on those responses, they may end up citing only one expected response, the correct one. This may be evident in their plans, in their practice teaching activities, and in their instruction. When I have observed the preservice teachers practicing and implementing lessons, they sometimes struggled when students gave an incorrect answer or exhibited misconceptions. Later, as they began to work on how to elicit student thinking, interpret student responses, and respond to students, their anticipation and questioning improved once presented with different models of questioning and practicing questioning and responses in the classroom. As I came to research questioning in the literature, the model of funneling and focusing structures emerged as a way to indicate how teachers collect and use student thinking (Wood, 1998). When teachers use their own reasoning to guide students to solution, they are using funneling questioning structures. When student ideas guide the lesson, teachers are using focusing questioning structures. Funneling and focusing questioning structures are an effective way to examine how preservice teachers emphasize thinking – funneling shows that the teacher is using their own thinking during the lesson; focusing shows that the teacher expects students to use their own thinking to learn mathematics (Wood, 1998). In examining how preservice teachers use questioning structures, and specifically when they privilege student thinking in the use of focusing over funneling, this study provides information for teacher educators about how help teachers support a focus on student thinking during planning, rehearsals, and instruction.

We know from the literature that preservice teachers can learn, teach, and reflect on questioning practices and using student thinking (Blanton, Berenson, & Norwood, 2001; Grosser-Clarkson, 2016; Singer-Gabella et al., 2016), especially at the elementary level (Nicol, 1999; Singer-Gabella et al., 2016; van den Kieboom, Magiera, & Moyer, 2014); but there is little in the literature about how secondary mathematics preservice teachers plan and use funneling and focusing questioning structures, especially as a benefit in understanding how to use student thinking. In looking at the development of questioning over time in a course where preservice teachers spend weeks revising and practicing their lesson, this study shows how preservice teachers develop and use questioning in planning, practicing, and teaching at an in-depth level. The research presented here shows that preservice teachers vary widely in their consideration and use of questioning for instruction, and while they intend to focus on the thinking of students, many struggle to implement it. This study builds on areas that have been identified in previous research in using questioning to examine what preservice teachers plan for and how they use student responses during instruction. This links research in questioning and questioning structures with teacher noticing research, showing what preservice teachers notice and how they respond to what they notice through examination of their choice of questioning through the planning, practicing, and teaching. This study demonstrates the need for teacher educators to assist preservice teachers in learning to use student thinking during instruction.

There is evidence that teacher beliefs play a role in how one learns to teach (Philipp, 2007; van Es & Conroy, 2009), but there is little known about how preservice teachers address the potential tensions between their beliefs and their effective use of questioning and student thinking. Focus questioning structures provide instruction better geared to elicit, interpret, and respond to student thinking (Wood, 1998), but preservice teachers can struggle with focusing and often move to funnel questioning structures. While there is research on the positive impact of rehearsal on helping preservice teachers develop effective teaching practices (Ghousseini, Beasley, & Lord, 2015; Grossman et al., 2009; Kazemi et al., 2009), there is a gap in the literature in understanding how preservice teachers use these rehearsals to improve questioning structures and use student thinking through when teaching lessons. This study provided information about how preservice teachers use questioning structures to elicit, interpret, and respond to student thinking. This information in turn suggests strategies to teacher educators to support preservice teachers to use focus questioning structures to help elicit, interpret, and respond to student thinking.

This study investigated how well preservice teachers manage the tension between interpreting and responding to student thinking and their goals for instruction. An overview of the study follows: its purpose, theoretical framework, specific research questions investigated, definition of some key terms, and the significance of this study for the preparation of preservice teachers.

Purpose of the Study

The purpose of this study is to determine how preservice teachers use questioning structures to elicit, interpret, and respond to student thinking through the planning, practicing, and teaching of mathematics lessons (referred to in the study as the plan-practice-teach cycle). Participants in the study:

- planned a high school mathematics lesson, which included the expectation that the preservice teacher would formulate what questions to ask, anticipate how students respond to these questions, and predict which actions to take based on those responses;
- practiced the lesson with their peers in a methods course; and
- taught the lesson in a high school mathematics classroom.

In thinking about this study, two of the key questions became "How do preservice teachers plan what questions to ask and what student responses to expect?" and "Does practicing the lesson with their peers help prepare them to teach in the classroom?" In other words, how does the plan-practice-teach cycle help preservice teachers determine what a useful lesson will look like. Planning which questions to ask and predicting how students will attend to them can affect instructional decisions (Sherin, Jacobs, & Philipp, 2011a; Tienken, Goldberg, & DiRocco, 2009). Effective teachers plan lessons and questions that support students in achieving their learning goals (Shahrill, 2013; TeachingWorks, 2018). This initial step of the plan-practice-teach cycle can provide preservice teachers information about how the lesson will go, and what errors and misconceptions they anticipate in the lesson.

In many teacher preparation courses, preservice teachers move on to practice the lesson with their peers. These practice sessions allow them an opportunity to try out their lessons and check whether their expectations of the lesson may be correct. Preservice teachers can then use the evidence of learning elicited in student (i.e., peer) responses to interpret and inform their in-the-moment teaching decisions (Davis, 1997; Doerr, 2006). Preservice teachers can also pose questions to elicit and interpret student thinking (Di Teodoro, Donders, Kemp-Davidson, Robertson, & Schuyler, 2011; TeachingWorks, 2018). By doing so, teachers may discover what their students already know how to do. Practicing the lesson provides information to preservice teachers about how the lesson may go with high school students, while practice with peers supports the use of questioning to elicit and interpret student thinking: from planning appropriate questions and anticipating possible student responses, noticing student responses in order to gauge what students are thinking during instruction, and asking questions that address student needs during instruction. Preservice teachers receive feedback on their lessons from both the course instructor and peers, then revise their lesson plans in order to reflect this feedback.

At the end of this preparation, preservice teachers teach a mathematics lesson in an actual high school classroom. The questions they plan and ask serve as markers for what preservice teachers are thinking as they elicit student thinking. How they interpret what students say and do guides their actions and responses, which should provide insight into how they interpret student thinking. Do they "funnel" students into predetermined learning pathways, perhaps guided by what was predicted in their lesson plan or what they experienced with peers? Or did they "focus" on student responses to guide their decisions about which questions to ask (Herbel-Eisenmann & Breyfogle, 2005)?

This study investigated the questioning sequences that occur throughout the plan-practice-teach cycle in order to understand how preservice teachers elicit, interpret, and respond to student thinking. One focus consisted of how student thinking is privileged during instruction by the use of different questioning structures, and how the experience of the plan-practice-teach cycle may influence the actions of preservice teachers as they elicit, interpret, and respond to student thinking during the lesson. Data collection occurs throughout the plan-practice-teach cycle as a result of the lesson plan, preservice teacher reflection assignments, and an end-of-semester interview, designed to learn about the reasoning of preservice teachers. Analyzing the questioning sequences derived from the lesson plan, practice with peers, and the lesson teaching high school students provided data about the use of student thinking throughout the plan-practice-teach cycle. Determining what preservice teachers attend to during mathematics instruction and the questions they ask in response to this information informs teacher education programs on how to devise better ways of questioning to elicit, interpret, and respond to student thinking during mathematics instruction.

Theoretical Framework

This study examines how preservice teachers decide what questions to ask and predict the responses students will provide (what they expect will happen); how they practice a lesson (what they observe and how they respond to what they notice); and how they teach a lesson with students (what they observe and how they respond to what they notice). To answer these questions, the study explores the things preservice teachers anticipate before or notice during instruction and how they then act upon what is expected or noticed, and uses *teacher noticing* as a theoretical framework.

Teacher noticing explores what teachers see, how they interpret what they see, and how they act, based on their interpretations (Sherin et al., 2011a). Teachers use what they notice while teaching in order to ask the right questions at the right time and to adjust questions based on perceived student understanding of a specific topic (NCTM, 1991; Tienken et al., 2009; van den Kieboom et al., 2014). As such, teacher noticing should be a component of all teacher education programs (van Es & Sherin, 2002). When teachers notice student thinking during instruction, they can interpret and subsequently select what they will attend and respond to, which may influence how they respond to students (Levin, Hammer, & Coffey, 2009).

This kind of skill is critical for creating effective learning environments (Sherin, Russ, & Colestock, 2011b; van Es & Sherin, 2002). However, as preservice teachers attempt to notice and attend to student thinking, they may struggle in moving from an idealized version of what they envision to the reality of enacting lessons in the classroom (Clift & Brady, 2005; van Es & Conroy, 2009). In this study, teacher noticing serves as a lens for how preservice teachers make sense of what students say and do in response to questions. What do preservice teachers anticipate, as evidenced in their lesson plan? What do they notice as they teach to their peers? What do they notice during instruction with students? What is the relationship between what is expected and what is noticed, and does one (expectation of what students will do or noticing student thinking) take priority over the other during instruction?

Research questions

This study investigates the following research questions:

- Do preservice teachers use focus and funnel questioning structures as they elicit, interpret, and respond to student thinking and, if so, how do they use them?
- In what ways does preservice teachers' use of focus and funnel questioning structures change through the plan-practice-teach cycle?

Definitions

As work on this study has progressed, especially in sharing ideas it has generated with others, it became obvious that some of the terms used needed clarification. Many important terms used in the study, such as *practice, anticipation*, and *expectation*, have multiple meanings both within and outside of mathematics education. Other terms will be defined as they occur in the narrative. Following is the definition of a few general key terms:

Anticipation

This term describes what preservice teachers think students will do during instruction and what actions they will take. During lesson planning, participants in the methods class develop questions to use as well as possible student responses to those questions; thus, they *anticipate* which questions they will ask, as well as how students will respond. They anticipate not only the correct solutions, but also incorrect ones. In fact, they are working to develop questions designed to uncover student misconceptions so that they become part of the instruction. They also provide suggested teacher actions to student responses so that lesson plans include anticipated student responses and teacher actions.

Expectations

In this study, such a term describes how preservice teachers view the responses that students will make or have made in the lesson. What *expectations* do they have for what students are able to do? Throughout the study, collected data and analysis include what preservice teachers expect from students; this analysis links such expectations with teacher actions, including whether they use funneling or focusing questioning structures.

Practice

This is perhaps the most challenging word found in this study, as it has two primary meanings. First, it describes the actions that teachers take during instruction, as in teaching *practice* or *practices*. A major focus of the study is to explore important teaching practices that preservice teachers develop through their preparation program. The study also looks at the opportunity preservice teachers have to *practice* their lessons with their peers. In this context, *practice* refers to the ability of preservice teachers in methods classes to implement their lesson plan with peers. This provides them with information on how the lesson could go in a regular classroom, and what may occur while teaching the lesson with high school students.

Role of the Plan-Practice-Teach Cycle

This study seeks to deepen our understanding of whether preservice teachers change through the plan-practice-teach cycle. In my work with preservice teachers, I have examined how they plan for instruction, practice implementing those plans with their peers, revise their plans based on the practice, and then enact the lessons in classrooms. I have witnessed how their eliciting, interpreting, and responding to student thinking develops through these experiences. This has offered insights into how preservice teachers respond to student thinking and has linked teacher actions to funneling and focusing questioning structures. This study also explores how preservice teachers make choices about questioning through the plan-practice-teach cycle, which may lead to new ways of supporting preservice teachers as they elicit, interpret, and respond to student thinking.

An important element of the study is the influence of practicing with peers in the use of student thinking by preservice teachers. Preservice teachers have the opportunity to "test out" their lessons with peers, providing them with the experience of trying out questioning sequences before enacting them with high school students. The enactment of the lesson takes such ideas and experimentation into the classroom. While research has shown the benefit of incorporating practice-based models of teacher education into preservice teacher preparation programs (Ball & Forzani, 2009; Grossman et al., 2009; McDonald et al., 2013), there are many different models of practice-based teacher education. For this study, the practice-based model focuses on how preservice teachers enact lessons with their peers and adjust lessons by analyzing the responses to student thinking as they support instructional outcomes. Integrating effective teaching strategies by practicing lessons with peers and teaching the same lesson with high school students provided preservice teachers the chance to develop as teachers by practicing new strategies in a safe environment (Kazemi, Franke, & Lampert, 2009; Lampert, 2010) before applying them in the classroom. Practicing with peers allows preservice teachers to think about how lessons may progress in a sheltered environment, without the need to monitor classroom behaviors, address interruptions, and consider other pedagogical distractions. This model of practice-based teacher education allows preservice teachers (and their instructors) to explore strategies with peers before teaching students and connects thinking about instruction (planning) with practicing teaching in coursework (rehearsals) and enacting instruction (teaching) (Ghousseini et al., 2015; Grossman et al., 2009; Kazemi et al., 2009).

This study explores how preservice teachers develop questioning in response to student thinking using the plan-practice-teach cycle. By developing, enacting, and analyzing their lesson plans, preservice teachers learn to think about what they will notice and the actions they will take and act on while teaching, and how they may adjust questions and instruction to elicit and interpret student thinking. The next chapter explores the research areas presented in this chapter in more depth.

Chapter 2: Review of the Literature

Research has shown that preservice teachers understand the key characteristics of questioning and eliciting, interpreting, and responding to student thinking. They can learn about practices associated with questioning and student thinking and apply them in the early stages of their teaching career. What is not so well-known is how these practices evolve through the cycle of planning a lesson, practicing it with fellow preservice teachers, and teaching the lesson to students. Of particular interest is the role of practicing the lesson with peers, which is a unique element in the preservice stage of teachers' careers. This study investigates each phase of the plan-practice-teach cycle that guides how preservice teachers use questions and elicit, interpret, and respond to student thinking. It explores how preservice teachers make choices about what questions to ask, what they anticipate and notice about student thinking, how they respond to student thinking, and how they reflect on the practice of eliciting, interpreting, and responding to student thinking.

This section offers background information upon which this research study is based. The opening section explores the importance of core practices for teacher education and the need to focus limited resources on key instructional practices such as developing effective questioning structures and eliciting, interpreting, and responding to student thinking, with an emphasis on preservice teacher preparation. Continuing the focus of teacher preparation, research demonstrates how practice-based teacher education supports preservice teachers in learning to teach. Next, eliciting, interpreting, and responding to student thinking are identified as core practices for preservice teachers. How preservice teachers use questioning to access and identify student thinking leads to different ways to categorize types of questions and questioning sequences. The section concludes with research on teacher noticing practices and how this applies to preservice teachers and the cycle of plan-practice-teach.

Core Practices

Teaching is a highly complex act that involves both deep and connected conceptual understanding of content as well as the ability to apply the practices of teaching that will best support students at a given time (Ball & Forzani, 2009; Bransford, Darling-Hammond, & LePage, 2005). Currently many teacher education programs are shifting to a balance of content knowledge and teaching practices (practical experiences) in order to prepare candidates for the complexities of the teaching profession (Grossman et al., 2009; McDonald et al., 2013). While some research focuses on mathematics education, a review of research literature for core practices examines all content areas. This allows for a consideration of general teaching practices as well as those specific to mathematics education. A search of the literature identifying core practices includes significant work in the field of math and science education, which will become apparent through the consistent citation of articles published in multiple content areas.

Given the many practices teachers use during instruction, researchers have begun to identify shared core practices that use available resources to maximum effect (Ball & Forzani, 2011; Forzani, 2014; Franke, Kazemi, & Battey, 2007; McDonald et al., 2013; Windschitl et al., 2012). For core practices to be an efficient tool, there must be a determination of what constitutes a core practice. As stated earlier, core practices are common teaching practices that require teachers' professional judgment and support student learning. While teachers use many practices throughout the day, core practices focus on instruction, excluding such activities as taking attendance or collecting papers following a test. The core practices movement assumes three important ideas about teaching and teacher education: 1) all students should develop critical thinking, reasoning, and problem-solving skills; 2) instruction should adjust to the thinking and needs of the students in the classroom; and 3) the role of subject-matter knowledge to support adjustments is critical for effective instruction (Forzani, 2014). These assumptions are necessary in understanding how learning occurs for both students and preservice teachers, given the use of core practices and the practice-based teacher education models described below.

Core practices are essential for creating teacher competency and a set of baseline abilities with which teachers can grow (Ball & Forzani, 2011). In order to determine these practices, one must "deconstruct" teaching to arrive at common understandings and goals (Ball & Forzani, 2011), as well as determining the importance of each teaching practice (Ball & Forzani, 2009). However, lack of a consistent language and the need to identify proper "grain size" of teaching practices between researchers and content areas may challenge the creation of core practices (Ball & Forzani, 2011). Core practices for the preparation of teachers must be applicable to the regular daily work of teachers; be accessible to preservice teachers in order for them to learn, practice, and apply; be part of a coherent and systematic system of teaching; and "reflect priorities of equitable and effective teaching and allow significant time for novices to develop and receive feedback on approximations of each of these practices" (Windschitl et al., 2012, p. 883). These are ambitious goals, reflecting the importance of such practices and how preservice teachers can become capable of learning and applying them.

The creation of a focused set of core practices can lead to a common approach and curriculum across preparation and in-service programs (Ball & Forzani, 2011). So, what practices are core practices? Grossman et al. (2009) identify the following characteristics for core teaching practices:

- Practices that occur with high frequency in teaching;
- Practices that novices can enact in classrooms across different curricula or instructional approaches;
- Practices that novices can actually begin to master;
- Practices that allow novices to learn more about students and about teaching;
- Practices that preserve the integrity and complexity of teaching; and
- Practices that are research-based and have the potential to improve student achievement (Grossman et al., 2009, p. 277).

These criteria distinguish core practices as important instructional activities, separate from categorizing other activities that teachers conduct during the day (McDonald et al., 2013). Using these criteria, McDonald et al. (2013) synthesized the research of other experts, including Lampert, Ball, Forzani, and Grossman, to identify “eliciting and responding to students’ ideas, setting and maintaining expectations, or leading particular types of discussions as they come to life in particular content areas” (p. 380) as examples of core practices. The researchers emphasize the importance of attending to both the conceptual and practical aspects of these practices, an area reinforced in the section on practice-based teacher education which follows.

Ghousseini and Sleep (2011) have identified five categories that insure activities are accessible (“studyable”) for preservice teachers: “(1) engaging the content, (2) providing insight into student thinking, (3) orienting to the instructional context, (4) providing lenses for viewing, and (5) developing a disposition of inquiry” (p. 151). These categories show that teaching practices must be able to be both learned and applied by preservice teachers to be a core practice at this level of teacher preparation.

Windschitl et al. (2012) synthesize previous research to identify important teaching practices in a subject area, including 1) reasoning about important concepts of a subject; 2) participating in conversations about the subject; and 3) solving real world problems within a subject. They detail four core practices for model-based inquiries in science:

1. Selecting big ideas, treating them as models;
2. Eliciting students' ideas, using them to adapt instruction;
3. Choosing activity and framing intellectual work; and
4. Pressing for explanation (Windschitl et al., 2012, p. 899).

The first practice in this set focuses on planning while the latter three focus on classroom discourse, supporting the need for students to reason out important concepts in science. While the researchers provide further details on how this set of core practices may apply to science, they could clearly apply to mathematics instruction as well. Ghousseini et al. (2015) offer "eliciting and responding to student reasoning, representing student thinking, orienting students to one another's ideas, and attending to students' errors" (p. 462) as examples of cross-discipline core practices, with a clear connection to the core practices identified by Windschitl et al. (2012) above.

Core practices are neither intuitive nor spontaneous but require the support of mentors and practice for development within teacher preparation programs. Ghousseini (2015) examined how one preservice teacher worked to enact classroom discourse, specifically key teaching practices such as eliciting student thinking, representing and connecting student ideas, and focusing on important mathematical ideas that in student solutions. Preparation for using these practices begins with learning and practicing them in methods courses, using preservice teachers in the role of students, with teacher educators identifying the instructional practices as they model them. Preservice teachers then engage in a transcript activity, fishbowl activity, and field placement activity to rehearse using these practices. Ghousseini (2015) found that while preservice teachers may learn core practices and use them to support student understanding, they sometimes struggle with competency. Through personal reflection and help from the course instructor, the preservice teacher in this case study improved her ability to support students in making sense of each other's ideas. This study shows the importance of purposeful consideration of methods course experiences, practice with peers, and classroom experiences to provide preservice teachers with the support they need to learn and develop core practices.

While such core practices are an important part of effective teaching, they are not applied consistently, even in the classrooms of experienced teachers. Windschitl et al. (2012) contends that the incorporation of core practices requires a culture shift for all teachers, suggesting that few preservice teachers have seen such practices in their own learning experiences. Teachers may have difficulty supporting active student discourse in mathematics lessons where students provide responses and reasons and the teacher guides the lesson, based on student ideas. Students may raise concepts the teacher has not anticipated, requiring teachers to think on their feet about how to use such ideas to support the learning goal of the lesson (Stein et al., 2008). It is thus critical that teacher education programs focus on important core practices, and link theoretical discussions with practice in the classroom, allowing for feedback and reflection to support preservice teacher training (McDonald et al., 2013). Preservice teachers need guidance that connects their understanding of important strategies with instruction, paving the way for practice-based experiences to link theoretical and practical preparation.

Practice-based Teacher Education

Another important direction currently advocated in preservice teacher education is the use of practice-based activities, or "professional training that attempts to focus novices' learning more directly on the work of teaching rather than on traditional academic or theoretical topics that may have only marginal relevance to the realities of the classroom" (Forzani, 2014). The literature around practice-based teacher education is closely associated with core practices, with many researchers developing ideas in both areas. When exploring important studies in the area of practice-based teacher education, the same experts are often working to develop core practices in teacher education.

The benefit of linking ideas and concepts of teaching and their practical application is not new (Chuickshank & Metcalf, 1990), although practice-based teacher education is only now emerging in research (Grossman, 2009; McDonald et al., 2013). The current focus is on creating clear definitions and common understandings for teacher educators and education programs (Grossman & McDonald, 2008). [There are multiple definitions of practice-based teacher education programs and activities, including variations on preservice teacher field experiences and residency programs (Forzani, 2014); even defining the term *practice* may be challenging (Lampert, 2010). This study focuses on practice-based teacher education as a way of linking theoretical topics traditionally presented in preservice teacher coursework based on practical experiences in classrooms.]. Practicing allows lessons to be "tried out, corrected, refined, and mastered" (Ball & Forzani, 2009, p. 504).

While teacher education programs effectively design activities and discussions to examine instructional practices around student work, lesson planning, and classroom video, there is a gap in helping preservice teachers respond to student thinking and actions in the classroom

(Kazemi et al., 2009). While research indicates the benefits of linking classroom and practical experiences, there is still a need to develop activities and contexts to support preservice teachers in the actual enactment of classroom teaching (Grossman et al., 2009; Kazemi et al., 2009; Matsko & Hammerness, 2014). Preservice teachers who participate in cycles of instruction (planning that uses targeted instructional practices, teaching to apply these practices, and participating in guided reflection around these practices) demonstrated improvement in their teaching practices (Zembal-Saul, Blumenfeld, & Krajcik, 2000). They also tend to connect prior knowledge to relevant aspects of teaching the given content (Lampert et al., 2013). Such research demonstrates a key element of practice-based teacher education, the linking of preservice teacher coursework with practical classroom experiences, and the need for activities that help preservice teachers benefit from the link.

An emphasis on practice-based teacher education relies on particular views about how students learn, focusing on a student-centered learning environment (recall the assumptions of Forzani [2014] above). The practice-based teacher education perspective allows preservice teachers to work on understanding student-centered instruction by considering instruction through multiple formats: thinking about the lesson in planning, practicing the lesson with peers, and teaching the lesson to students. The coordination of such practices within a teacher preparation program allows preservice teachers to identify the benefit of visualizing and enacting lessons within the classroom experience (Ball & Cohen, 1999). Preservice teachers experience two different worlds, one in their collegiate environment and one as a preservice teacher in the classroom. As a result, there is a gap between how preservice teachers consider what to do (from their collegiate experiences) and what they *can* do (while teaching in the classrooms) (Kennedy, 1999).

The goal of practice-based teacher education programs is to tie the conceptual frameworks of theoretical pedagogy and the practical aspect of classroom teaching within teacher preparation programs, helping preservice teachers to understand the *how* and *why* of teaching (Boerst, Sleep, Ball, & Bass, 2009). Providing opportunities for novice teachers to experience teaching through rehearsal is beneficial as it approximates the act of teaching to problems of practice in controlled activities accompanied by coaching by teacher educators (Ghousseini et al., 2015). This allows preservice teachers to explore the complexity of teaching so that they can learn to think systematically "through activities that engage them in deliberating about problems of practice" (Ghousseini & Herbst, 2016, p. 82). In the ability to experience instruction in well-considered and manageable pieces, they can make sense of teaching as a realistic approximation (Grossman et al., 2009).

McDonald et al. (2013) presents a learning cycle for the enactment of core practices grounded in practice-based teaching experiences for preservice teachers (see Figure 2). This cycle explores how preservice teachers experience core practices in their coursework and links that learning with course rehearsals and practical application with students. It is important that activities allow for the teachers to progress from methods course discussions to rehearsals and then to classroom enactment; beginning with abstractions and moving to developmental practical experiences (McDonald et al., 2013). This learning cycle connects the analysis of Grossman et al. (2009) to three primary elements: representation, decomposition, and approximation of practice. These elements support preservice teachers in making sense of teaching through analysis and planning, lesson rehearsal, and experimentation (Lampert, Beasley, Ghousseini, Kazemi, & Franke, 2010). *Representations* include the different ways preservice teachers can view the practice of teaching, including videos, transcripts, and simulations. Representations

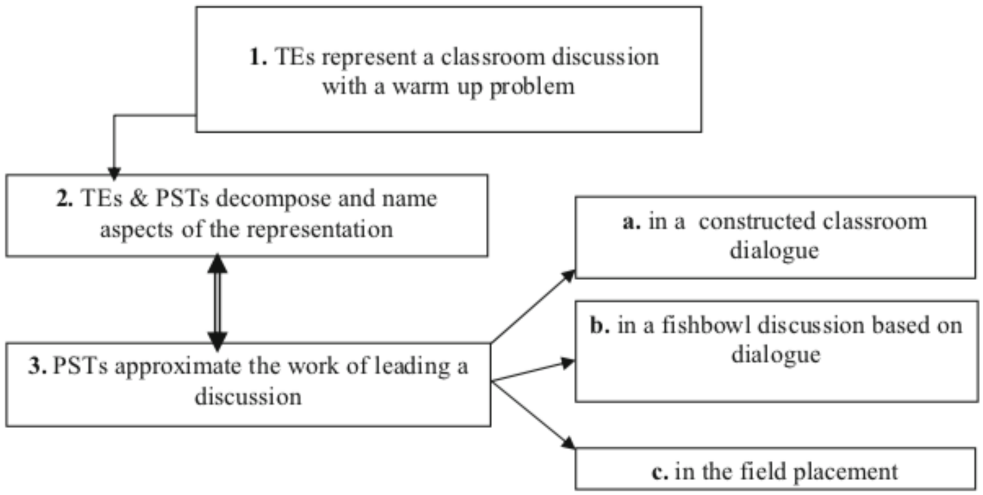
Figure 2. Cycle for collectively learning to engage in an authentic and ambitious instructional activity (from McDonald et al., 2013, p. 382).

correspond to the activities preservice teachers learn in their coursework (the top right quadrant of the learning cycle of McDonald et al., 2013). Activities then move clockwise to the second quadrant of the cycle. Preparation includes both planning what instruction should look like and practicing the lesson with peers. Effective teaching requires planning in order to predict different ways students learn a concept or topic, determine effective ways to help them, and evaluate when students achieve the desired learning goal (Ball & Forzani, 2009). These activities are all part of the representations preservice teachers should experience.

Preparation and rehearsal activities lead to *approximations of practice*. These can provide opportunities for preservice teachers to align coursework experiences with classroom apprenticeship, linking rehearsal to enactment (Lampert, 2010). Rehearsals approximate the actual work of teaching while providing preservice teachers with specific activities and support from teacher educators, so that these practices become routines. These allow preservice teachers to work on developing teaching practices where they can "hold some things constant while [they] work on others" (Lampert et al., 2010, p. 130). While many teacher education programs provide preservice teachers the opportunity to learn about, plan, and enact teaching practices through lessons, what may be missing are "opportunities to practice elements of interactive teaching in settings of reduced complexity" (Grossman & McDonald, 2008, p. 190).

Rehearsals also allow preservice teachers to practice teaching in a moderately controlled environment where they can reflect on their experiences (Ghousseini et al., 2015). Ghousseini and Herbst (2016) found that rehearsals provide preservice teachers the chance to learn about and manage important elements of teaching and "examine the work and develop the perspective of the teacher" (p. 101). Through rehearsals, preservice teachers can share instructional decisions in a safe environment (perhaps while pausing during a lesson in order to receive feedback during an instructional activity). They also improve alignment between instructional goals, planning the lesson, practicing with peers, and teaching the lesson in the classroom (Kazemi, Ghousseini, Cunard, & Turrou, 2016; Lampert et al., 2013). Through rehearsing a lesson, preservice teachers can apply instructional practices, learn from their mistakes, receive support from teacher educators, and prepare for engaging with actual students (Kazemi et al., 2016), while avoiding some of the problems that preservice teachers encounter during internships, such as classroom management issues or external interruptions. Finally, rehearsals allow for guided feedback on specific teaching strategies (Grossman et al., 2009; Lampert et al., 2013).

Ghousseini and Herbst (2016) conducted a study of secondary mathematics preservice teachers in which they explored representation, decomposition, and approximation of practice (see Figure 3). Preservice teachers begin the process by completing mathematics tasks as learners, followed by a reflection about the task by a teacher educator leading the discussion. Next, the preservice teachers analyze the task and its implementation. These activities prepare preservice teachers to use the specified instructional practice in their own teaching. As shown in Figure 3, preservice teachers may employ three different approximations of practice.



[TE refers to the teacher educator; PST refers to the preservice teacher.]

Figure 3. Three pedagogies of practice to teach the work of leading discussions *(Ghousseini & Herbst, 2016, p. 85).*

First, the preservice teachers read a sample script from a mathematics lesson that has some of the lines of discussion missing. They provide their own lines, explaining why they would use such statements. This activity helps the preservice teacher to analyze student contributions and consider their actions based on what the students have contributed.

Next, preservice teachers enact the lesson with peers acting as the students in the lesson. This helps the preservice teacher practice the lesson and consider alternative pathways a lesson might take, protected by a more controlled environment than the actual classroom. The final approximation of practice calls for preservice teachers to teach part of a lesson during their field placements.

This moves to the next quadrant of the learning cycle, enacting the activity with students (McDonald et al., 2013), which includes keeping records of their practice and reflecting on the implementation of effective classroom discourse (Ghousseini & Herbst, 2016). Ghousseini and Herbst (2016) shows how a clear pedagogy that builds from representation and decomposition to multiple approximations of practice can help preservice teachers enact successful classroom teaching practices. The authors cite the importance of teacher educators selecting appropriate and varied activities that involve key instructional practices, including those aligned with core practices, while providing key opportunities and analysis of practices. Cruickshank and Metcalf (1990) found in their review of teacher preparation program research that both preservice and in-service teachers should be able to master these practices. Teacher educators must be intentional in their enactment of opportunities, just as the preservice teachers are intentional in their implementation. The opportunities to practice and apply teaching practices should be occurring in all educational environments.

Throughout the cycle of learning and practicing how to teach, preservice teachers experience a *decomposition* of practices, as they move into the final quadrant of the learning cycle which involves enactment and moving forward (McDonald et al., 2013). Decomposition involves breaking down the practice of teaching into manageable pieces. Referring to the characteristics of core practices as described by Ghousseini and Sleep (2011) and Grossman et al. (2009), it is critical that preservice teachers be able to unpack teaching instruction in a way that makes sense. Decomposition can occur in many different ways, including methods courses that examine multiple solutions to mathematics tasks, the use of transcripts and videos by both expert and preservice teachers, and providing ways to reflect on their practice. However, one must be careful in deconstructing a practice, as it is important to learn practices in small chunks and be able to connect those pieces with the purpose of the lesson (Boerst et al., 2009). These three components—representation, approximation, and decomposition of practice—should build upon and complement one another, allowing preservice teachers to experience, analyze, and implement best teaching practices.

However, preservice teachers rarely follow a single path in building competence in using key teaching practices. Thompson, Windschitl, and Braaten (2013) showed how their participants, 26 secondary science preservice teachers, developed through three different groups: one group readily integrated new practices into their teaching experiences, a second group included new practices at specific times and places in a given lesson, and the third group used language from the new practices to label what they were doing without adapting the practice. Such groups of preservice teachers differed in their development based on three different factors: 1) negotiated membership in and across communities that provided different images of teaching (of both theoretical and practical approaches); 2) prioritizing student thinking in pedagogical discourse; and 3) the use of tools based on the gap between their vision and their practice (Thompson et al., 2013).

What is not complete in the literature is how preservice teacher rehearsals support the development of using questioning to elicit, interpret, and respond to student thinking. Do rehearsals with peers provide appropriate practice that provides an improved understanding of how to teach the lesson with students? This study provides insights on how rehearsals can provide either helpful feedback to improve the lesson or a false sense of teaching the actual lesson. The findings provide suggestions about how to provide rehearsal experiences that support how to use focus questions to elicit, interpret, and respond to student thinking.

Eliciting, Interpreting, and Responding to Student Thinking

One focus of this study is how preservice teachers elicit, interpret, and respond to student thinking in mathematics instruction. With ongoing development of what constitutes a core practice, and the eliciting, interpreting, and responding to student thinking that is considered by many to be a core practice (Ghousseini & Sleep, 2011; McDonald et al., 2013; Windschitl et al., 2012), research in this area has expanded to connect such practices to student success in mathematics (Bransford, Brown, & Cocking, 1999; Stein et al., 2008). In order to consider how preservice teachers implement these practices, the meaning of each instructional strategy should be clearly identified, along with how each of them support student thinking. *Eliciting student thinking* means the ways in which teachers allow student ideas to become visible, while giving the entire class the opportunity to discuss them openly (Singer-Gabella et al., 2016; Stein et al., 2008). The environment created by the teacher is an important element of eliciting student thinking; students should feel free to share creative ideas, and both the teacher and the activity should give students a chance to reason about mathematics.

Singer-Gabella et al. (2016) identify three key types of tasks that provide an opening to a shared investigation: 1) those that ask students to reason about mathematics, not just apply a known procedure; 2) those which "invite diverse solution strategies, reflecting a range of understandings, rather than funneling students toward the same approach" (p. 413); and 3) tasks that use students' prior knowledge and push them "toward new ways of thinking about the mathematics at hand" (p. 413). These features should be part of tasks and the teaching of them (Franke et al., 2009; Stein et al., 2008), as they highlight how eliciting student thinking can provide the genesis of what occurs in the lesson. Teachers may use eliciting student thinking as a guide to learn about what a student does or does not know about a mathematics topic and formulate follow-up questions that will provide detail about the student's thinking, allowing them to clarify their thinking for both the teacher and the class (Singer-Gabella et al., 2016).

Follow-up questions can clarify ambiguous explanations, uncover reasons for student errors, and let students elaborate on their solution strategy (Franke et al., 2009). A sequence of specific probing questions requires teachers to actively listen to student responses and then press students for clearer explanations. It may provide teachers with a deeper understanding of how students understand mathematics as they learn to talk with students about mathematics, understand the meaning of certain responses, and decide what to do with what they have learned (Franke, Carpenter, Levi, & Fennema, 2001). While not all follow-up questions move student thinking forward, those that maintain the focus on student thinking have these characteristics.

In *interpreting student thinking*, teachers use student responses to make sense of what students appear to be thinking. Interpreting student responses requires a deep understanding of content, understanding the multiple ways that students may understand mathematics, and the ability to actively listen to what students are saying without allowing outside factors or their own interpretations get in the way (Davis, 1997; Lambert, 1985; Singer-Gabella et al., 2016; Stein et al., 2008). Such in-the-moment actions can serve as assessments of what students know, and guide teacher actions throughout the lesson, by allowing teachers to 1) notice what is important and meaningful in student work; 2) make sense and connect student thinking and strategies to broader mathematical concepts; and 3) determine where students belong in various learning progressions (Singer-Gabella et al., 2016). How a teacher interprets student thinking is often determined by the actions of the teacher, so it is important to provide a structure for how to plan and pose mathematical tasks, which is why eliciting, interpreting, and responding to student thinking is such an important teaching practice (Singer-Gabella et al., 2016).

When teachers *respond to student thinking*, they act on what they have learned while eliciting student thinking and interpreting what the students have said. However, not every teacher action is based solely on student thinking. The degree to which they are depends on the degree to which teachers carefully respond to that thinking (Pierson, 2008), requiring the teacher to determine an appropriate instructional move to support student development; “the task for the teacher is to challenge students’ present thinking in a constructive way” (Singer-Gabella et al., 2016, p. 414). The choice of instructional move may be determined by how well teachers understand the mathematics involved and the students’ responses, the experience of teachers as they approach a lesson, and how flexible teachers are in listening to and interpreting what students say.

Another question in the research is whether preservice teachers are ready to elicit, interpret, and respond to student thinking as they develop their teaching practices. Researchers agree that preservice teachers vary in their ability to attend to and use student thinking (Levin et al., 2009; Singer-Gabella et al., 2016). Beliefs and knowledge of preservice teachers about the learning of mathematics differs and plays a crucial role in how they learn to teach and how they use questioning (van Es & Conroy, 2009) and beliefs can be changed through teacher preparation opportunities (Philipp, 2007). Philipp (2007) goes on to identify “that the feelings teachers experienced as learners carry forward to their adult lives, and these feelings are important factors in the ways teachers interpret their mathematical worlds” (p. 258). Teacher preparation programs need to address how these feelings impact the views of future teachers. Studies show while leveraging student thinking is not simple (Moyer & Milewicz, 2002), with the proper structure and support, preservice teachers can attend to student thinking while learning how to manage classroom instruction (Freese, 2006; Levin et al., 2009; Singer-Gabella et al., 2016;

Windschitl et al., 2012). These studies contradict the “stage-based” view of teacher preparation, where preservice teachers master classroom management skills and routines before engaging with student thinking (Levin et al., 2009). However, if we wish to position student thinking as the central element in mathematics instruction, emphasis on preparing teachers needs to attend to student thinking, integrated with developing other teaching skills, including classroom management (Davis, 2006; Levin et al., 2009). Teachers can learn about student thinking through the effective use of questioning strategies.

Questioning

An important part of eliciting, interpreting, and responding to student thinking is planning effective questions (Corley & Rauscher, 2013; McCarthy, Sithole, McCarthy, Cho, & Gyan, 2016; Moyer & Milewicz, 2002). Questioning during mathematics instruction has moved away from teacher thinking to a focus on supporting students to build on their own thinking to solve problems (Mason, 2010; Ontario Ministry of Education, 2011). Teachers need to anticipate student thinking and be prepared with productive questions to follow a student's reasoning within a lesson, as well as to address any misconceptions students may have about mathematical concepts. Questioning is an important link to eliciting, interpreting, and responding to student thinking. One of the first steps is to prepare for effective questioning prior to teaching a lesson (McCullough & Findley, 1983). Tienken et al. (2009) have identified the importance of planning and asking questions that will prompt students to think critically in order to solve real-world problems. They found that teachers did not ask enough what they called “productive” questions. Instead, they recommend a style of questioning that's often used in the legal profession, where lawyers prepare questions in advance of trial to develop a line of reasoning aimed at achieving a predetermined goal. Educators might follow a similar strategy, particularly in their use of open-ended questions that encourage critical thinking, as these are difficult to create in the midst of a lesson (Tienken et al., 2009). Teachers could create pathways to possible responses, keeping them in mind as they teach, and prepare questions that help students identify and make sense of these pathways.

Planning questions is an important element of effective instruction, given that “teachers need to plan a route and strategy in order to use questions productively and develop students’ thinking based on the learning objectives of their lessons” (Tienken et al., 2009, p. 42). Questioning can provide teachers a structure for helping students through hints and clues as well as probing student responses to understand their thinking (van den Kieboom et al., 2014). This does not mean, however, that a lesson must follow a script, as teachers should remain flexible and open to student thinking and ideas whenever possible (Shahrill, 2013). Teacher questions need to build on one another, allowing students “to identify thinking processes, to see the connections between ideas and to build new understanding as they work their way to a solution that makes sense to them” (Ontario Ministry of Education, 2011). The development of questions occurs in planning a lesson as well as what teachers notice during the lesson and how they act on what they notice (Kazemi & Stipek, 2001). Teachers should listen carefully to student responses in order to become aware of their needs, as these should guide questions to ask and actions to take during a lesson (Leahy, Lyon, Thompson, & Wiliam, 2005).

Teachers need to be able to ask questions guided not only by the task at hand, but in consideration of students' present abilities, as well as those they need to develop in the future (Thompson & Zeuli, 1999). Developing and using appropriate questioning is important in order to “encourage students to become independent thinkers, develop the ability to reason logically, and communicate their ideas to others” (van den Kieboom et al., 2014, p. 430). Teacher questioning plays a vital role in helping students understand mathematics; the choice and implementation of questions may get students to start thinking about content, see connections to other mathematical concepts, and allow students to make sense of any mathematical foundation they are building (Ontario Ministry of Education, 2011).

In order to tell whether a student is correct, some teachers funnel students to how they would solve the problem. Preservice teachers often express desires to create student-centered learning environment but teach in ways in which the teacher takes over the thinking (Singer-Gabella et al., 2016). Singer-Gabella et al. (2016) identify four pressure points, or tensions, that hold preservice teachers back from focusing on student thinking: how they view the learning of mathematics, what learning mathematics in school should look like, how mathematical understanding develops (such as the importance of effort and perseverance in learning mathematics), and how they encourage student interactions during instruction. This study shows that preservice teachers must have both the desire and the ability to set the stage for leveraging student thinking during instruction. Preservice teachers also struggle finding the time to plan lessons and to find curriculum resources that improve their understanding of key teaching practices (Grosser-Clarkson, 2016).

In their case study, Blanton et al. (2001) observed a preservice middle school mathematics teacher as she initially guided students as they worked on mathematical tasks, instead of allowing them to apply their own reasoning. She did not anticipate student thinking, but rather used student responses to funnel the class toward her way of thinking (a questioning structure described in depth later in this chapter). After listening to suggestions and discussions during her internship, the teacher eventually allowed student thinking to guide problem-solving. By analyzing the discussion in her classroom and considering the role of student ideas in learning mathematics, this preservice teacher was able to listen to student thinking and use it to support her teaching practices. Over time, with appropriate scaffolding from teacher educators, she was comfortable with letting students learn from their mistakes (Blanton et al., 2001). This case study demonstrates how some preservice teachers, with appropriate supports, can learn to attend to student thinking while developing other skills necessary for effective teaching, reinforcing the notion that preservice teachers should learn to use student thinking as they develop their teaching abilities.

Another example of the importance of listening to and incorporating student ideas during instruction is Doerr's (2006) case study. Here, the secondary school teacher focused on listening carefully to student responses and used questions to probe student thinking, rather than just using questions to determine whether a student response was correct. This shift in approach to question selection followed the teacher's progress from telling the class how to solve problems and asking questions simply to clarify understanding (by following a script) to allowing students to explain and make sense of the mathematical task themselves. Teachers need to bear in mind that it is the student's consideration of possible pathways to a solution that makes problem solving productive. As a result, teachers should design questions that help students make sense of such pathways, and support students to build on their own ideas and make connections within mathematical learning progressions (Doerr, 2006).

Effective questioning and probing can support students as they create new ideas and build on previous ones. In their ten-year longitudinal study, Martino and Maher (1999) examined how teachers use questions to learn how students develop and understand mathematical ideas. They recommended that teachers listen carefully to student ideas and be "prepared, whenever possible, to seize an opportunity to stimulate further thought" (p. 54). This comes about through carefully eliciting and interpreting what students are thinking. Teachers need to understand the close relationship between listening to what students say and responding to their thinking by posing timely questions designed to help students develop their own ideas based on the lesson (Martino & Maher, 1999). Questioning thus becomes a critical tool for teachers to elicit, interpret, and respond to student thinking.

While there is little research that describes questioning abilities of preservice teachers or how teacher educators can support preservice teachers to develop effective questioning strategies (Moyer & Milewicz, 2002; van den Kieboom et al., 2014), there are a few studies that provide some insight in this area. In their studies with preservice teachers and middle school-aged students, Coles and Brown (2016) describe the challenge of linking a vision of what teachers expect from their lessons to expressing that vision in planning and enacting lessons. They learned that questioning can provide evidence for how well students understand the learning goal and may determine how to adjust lessons to meet the needs of students. The goal of mathematics learning is "a convergence of teacher intentions and student mathematical activity" (p. 149), using questioning to elicit and interpret student thinking. Proper questioning can help teachers learn what students are thinking and guide their decisions during instruction (Coles & Brown, 2016). Lessons should provide opportunities for students to use their own reasoning in performing mathematical tasks (Lobato, Hohensee, Rhodehamel, & Diamond, 2012).

The TIMSS Video Study of 1999, however, showed that questioning does not always provide guidance for how students understand the learning goal. It investigated mathematics instruction across seven counties to examine eighth-grade mathematics teaching around the world (National Center for Education Statistics, 2003). Results from a nationally representative sample of teachers "suggests a general lack of attention among teachers [in the United States] to the ideas students develop" (Thompson & Saldanha, 2004, p. 96). Instead of fostering a student-centered, problem-solving learning environment where students develop generalizable ideas, the TIMSS Video Study showed that eighth-grade mathematics classrooms in the United States tend to emphasize having students do what they're told to do and working hard to remember what they were told (Thompson & Saldanha, 2004).

The effective use of questions in the mathematics classroom is not a simple task for preservice teachers. In a study conducted during a methods course that included weekly classroom interactions, Nicol (1999) noted that preservice teachers struggled with knowing what questions to ask and how to ask them, as well as how to integrate questioning, listening, and responding to students while teaching. Preservice teachers also had difficulty trying to balance questions that uncover student thinking with those designed to lead students toward a specific answer. Through course activities designed to help use questioning to learn what students were thinking, Nicol (1999) showed that preservice teachers could improve their ability to elicit, interpret, and respond to student thinking. That is, preservice teachers need to learn how to make appropriate decisions about what questions to ask, whether to support students or let them work things out on their own, whether to ask a leading or open-ended question, and when to tell students what to do (Moyer & Milewicz, 2002; NCTM, 1991). Deciding how such questions push students is critical to effective learning. "Learning occurs when assistance is provided at opportune points in the learner's zone of proximal development" (Hufferd-Ackles, Fuson, & Sherin, 2004, p. 83). Van den Kieboom et al. (2014) found that preservice teachers often asked questions one after another, without attempting to probe student thinking or encourage explanations, even when coached to incorporate such strategies. Van Es and Conroy (2009) found that preservice teachers struggled to ask questions that promote student sense-making and reasoning about mathematics while they were teaching. Asking questions and incorporating student responses are important activities, as they can guide classroom discourse and help develop understanding for all students (Hufferd-Ackles et al., 2004).

Preservice teachers can improve their questioning strategies by working closely with peers and revising lessons in a recursive manner. Fernandez and Zilliox (2011) examined how preservice teachers learned the elements of effective lessons through lesson study cycles where they collaborated with peers on tasks that involve unfamiliar mathematics content. The preservice teachers researched the content, planned the lessons, enacted them with peers serving as students, and then reflected on the experience together. They then repeated the planning and enactment cycle using revised lessons covering the same content. This experience showed that early lessons, even those backed by research that support the planning and teaching of content, rarely included appropriate question-and-responses that addressed the needs of students during instruction. Through the iterative process of lesson study, preservice teachers were able to improve their anticipation of which questions to ask students and focus on the needs of learners (Fernandez & Zilliox, 2011). Now, investigation needs to consider the research on questions and questioning structures.

Questioning Types and Structures

How preservice teachers develop different types of questions within structures of questions is an important element for study in the preparation of preservice teachers (van den Kieboom et al., 2014). There are many different ways to categorize and/or organize questions, including whether the stance of the question is open or closed (Ontario Ministry of Education, 2011), the characteristics of questions such as those of lower or higher cognitive demand (Henningsen & Stein, 1997), how cognitive demand is supported by teachers' use of questions

(Stein et al., 2008), teacher's intention and expectation of student responses (Ulleberg & Solem, 2018), or the use of Bloom's taxonomy to determine types of questions (Bloom, Krathwohl, & Masia, 1956). These different ways to categorize questions have allowed researchers to explore how teachers make sense of questioning and how it impacts the learning environment within their studies. It is important for studies to connect their categorization of questioning to the goals of their study. For this research study, questions are grouped together and examined for how they help teachers to elicit, interpret, and respond to student thinking.

When Myhill & Dunkin (2005) examined mathematics education at three elementary schools by analyzing videotaped lessons, they learned that the dominant form of instruction was through teachers transmitting information to students. As they examined lessons, looking for four types of questions: procedural, factual, speculative, and process, they found that most lessons reflected procedural and factual questions. This shows that there is a need to examine patterns of questioning to learn how to support preservice teachers and develop activities that will prepare them to use questioning effectively.

This finding was similar to those of Tienken et al. (2009). The lack of higher-level cognitive questions, designed to invite student opinions and explanations, is "counterproductive to the enterprise of learning" (Myhill & Dunkin, 2005, p. 424). Even when teachers used questions to elicit student thinking and encourage divergent responses and explanations, they often sought specific, pre-planned answers, or offered answers when students did not supply them. A focus on procedural and factual questions where teachers retained control of the discourse lost the opportunity to use student thinking to guide the lesson (Myhill & Dunkin, 2005).

Questioning structures are formed by grouping questions together based on a common meaning or theme. Questions structures are important as they demonstrate how teachers learn about and develop student thinking beyond individual questions. Teachers can organize their questions to (1) *assess* and clarify student understanding of a problem and (2) *advance* student thinking to achieve the learning goal of the lesson (Smith, Bill, & Hughes, 2008). This organization of questions allows teachers to first elicit student thinking, understand where that student is along the learning continuum, and use questions to help the student progress toward the learning goal.

Organizing teacher questions by type can also classify different types of classroom environments created by teachers. In their analysis of the differences between how teachers implement the same curriculum, Boaler and Brodie (2004) cited nine questioning types:

1. Gathering information, leading students through a method;
2. Inserting terminology;
3. Exploring mathematical meanings and/or relationships;
4. Probing, getting students to explain their thinking;
5. Generating discussion;
6. Linking and applying;
7. Extending thinking;
8. Orienting and focusing; and
9. Establishing content (Boaler & Broadie, 2004, p. 777).

The researchers define *questions* as “utterances that had both the form and function of questions, and which were mathematical” (p. 777) in nature. Their findings on questioning demonstrate how teachers use different types of questions to create different levels of cognitive opportunities for students during mathematics instruction, to create the kind of classroom environment needed to support student learning. They also identify the importance of exploring questioning and the questioning structures necessary for understanding how questions help teachers elicit, interpret, and respond to student thinking (Boaler & Brodie, 2004).

Questioning structures group questions together in a sequence and are one way to explore how teachers create and influence student thinking. The IRE (initiation, reply, and evaluation/feedback) is a common method often called the traditional questioning structure (Waring, 2009; Wells & Arauz, 2006) which is prevalent in many classrooms (Stigler & Hiebert, 1999). In this pattern, the teacher initiates interactions by asking a question, seeking replies from one or more students, and then evaluating the reply with feedback to the student. Mehan (1979) also identifies examples of classroom interactions where the IRE pattern can guide students toward correct answers but do not represent student understanding of content. One example involves students providing answers to a teacher's question until the teacher accepts an answer as correct (Mehan, 1979). This is a closed structure that limits how teachers probe student thinking and focus on teachers progressing through multiple phases of a lesson efficiently.

Franke et al. (2009) examined how teachers use questioning to support making student thinking explicit. They observed three elementary classrooms where teachers were participating in professional development experiences to support students' algebraic reasoning. In examining the data from videotaped lessons, they tried to identify *segments* of lessons to support the analysis of questioning. A segment is "an extended interaction or discussion between the teacher and an individual student, in which that student had at least two conversational turns" (p. 383). Within these segments, questioning sequences were coded as "general questions, specific questions, probing sequences of specific questions, and leading questions" (p. 383). In organizing question sequences in this manner, the researchers were able to link the type of question used to students' initial explanations during mathematics instruction. While teachers might initiate a problem by asking students to explain their thinking, follow-up questions varied in their degree of specificity, sometimes leading students to the desired result. The study demonstrated how teachers were able to use questioning structures to improve student explanations, and the degree to which teachers were able to enable deeper understanding as the lesson progressed (Franke et al., 2009).

McCarthy et al. (2016) studied two eighth-grade mathematics teachers to explore the use of questioning strategies while teaching quadratic modeling. After viewing videotaped lessons from these teachers, the researchers developed four categories of questioning strategies: probing and follow-up, leading, check-listing, and student-specific. Probing and follow-up questions assessed student thinking, allowing teachers to deepen student understanding by supporting student investigation. When teachers used questioning sequences to guide students and prompt them in a certain direction, they employed leading questions, which included scaffolding to encourage students to achieve a specific learning goal. Check lists of questioning occurs when teachers proceed with a string of questions without noting student responses and ideas; it involved little responsiveness to student thinking and few opportunities for students to present questions. Targeted questions invited students to contribute to the classroom conversation. The researchers identified "the importance of the awareness of the questioning strategies to facilitate effective interaction in mathematical classroom discourse" (p. 88). Thus, teachers need to be aware of multiple questioning strategies and select those that effectively guide student thinking during instruction.

Wood (1998) described two structures teachers use to support communicating ideas in the classroom, *funneling* and *focusing*. In funneling, teachers were looking for a predetermined pathway for “a solution that is representative of the method they intend the students to use” (p. 167). This type of structure led to a student belief that mathematics is about learning a predetermined solution that teachers have in mind (Wood, 1998). Funneling of questions can occur when teachers guide students through a “series of explicit questions until [the student] provides the correct answer” (Wood, 1998, p. 171). The teacher controls the thinking and the students become bystanders in how the teacher makes sense of the problem. There is, however, a distinction between funneling and the support offered by teachers when using scaffolding, as described by Herbel-Eisenmann and Breyfogle (2005). When teachers provide scaffolded questions to support student understanding, they “discuss these particular questions and the purpose for attending to them” (p. 486); such questions should decrease over time as students internalize them and the teacher eliminates their use. In order to scaffold questions, students must understand the purpose of the questioning so that they may begin to include them as they think about finding solutions to problems.

In contrast, the focusing structure requires students to express their thinking, given an expectation that there will be a variety of pathways to a solution when problem solving (Wood, 1998). In focusing, the teacher listens to and respects student responses and explanations and uses them to guide the lesson by eliciting student thinking, interpreting what students offer as solutions, and responding to their thinking by helping “students notice an idea” (Wood, 1998, p. 168). This differs from funneling, in which the teacher simply identifies the correct strategy to use. In focusing, student thinking is the critical element in implementing a mathematics lesson (Wood, 1998). By eliciting students’ solutions and explanations, focusing allows teachers to understand and interpret student thinking (Herbel-Eisenmann & Breyfogle, 2005). Focusing and funneling questioning structures provide a way to view how teachers elicit, interpret, and respond to student thinking. These structures provide two views on using student thinking, one guided by responses from the students and the other guided by the thinking of the teacher. How the teacher uses questions demonstrates how they privilege student thinking and links to eliciting, interpreting, and responding to student thinking.

Questioning and questioning structures are critical elements of effective instruction and the study of how preservice teachers develop an understanding of questioning types and structures is important within teacher preparation programs. Questioning helps students make sense of other solutions, while the teacher monitors the level of understanding for all students and uses questioning structures – series of questions linked together – to let students clarify and extend their explanations. Teachers notice student thinking when eliciting it and interpret what they notice to determine their questioning as a way to respond to student thinking.

Teacher Noticing

The construct of *teacher noticing* frames what teachers see, how they interpret what they see, and how they react to these interpretations (Sherin et al., 2011a; van Es & Sherin, 2002) and provides links to elements of effective instruction (Linsenmeier, Sherin, Walkoe, & Mulligan, 2014; Sherin et al., 2011a; van Es & Sherin, 2002). Teacher noticing involves what teachers notice as well as "their reflections, reasoning, and decisions based on this noticing" (McDuffie et al., 2014, p. 247). When it is associated with student thinking, teacher noticing can become an integral element in how to interpret student thinking and respond to that thinking. Teachers need to make sense of what they observe in the classroom and how to move understanding mathematics forward (Sherin et al., 2011a). This is the fundamental purpose of teacher noticing. "What makes noticing consequential, of course, is that people act on what they notice" (Schoenfeld, 2011, p. 230), which has an impact on the responses and decisions teachers make in order to move learning forward.

Teacher noticing includes how teachers prioritize what they see and hear, where some things are important to the lesson and others less so. This identification and privileging of observations is critical to effective teaching; teachers must make many observations throughout a lesson and there is never enough time to address everything that arises. This is a critical part of interpreting and responding to student thinking, since what teachers respond to guides the rest of the lesson. In order to succeed, teachers must create a construct of what they see and hear, establishing a hierarchy for what is important to the lesson (Erickson, 2011). This is a critical piece of teacher noticing, with teachers constructing their own view of what requires particular attention.

Teacher noticing may also drive adjustments to the lesson, as the "ability to adapt instruction in the moment requires that teachers be able to notice and interpret aspects of classroom interactions" (van Es & Sherin, 2002, p. 571). This includes the ability to prioritize events and then adapting instruction to meet the needs of learners (van Es & Sherin, 2002). It may also lead to the development of teacher-created reasoning within a lesson (Sherin & Star, 2011). Though not always intentional, this structure allows teachers to filter what they notice and determine whether to respond or not, based on what they observe. The development of this discriminating sense is important for creating pathways to student learning in the classroom. How preservice and in-service teachers develop this discriminating sense and apply it across lessons and topics forms connections to broader principles of mathematics teaching and learning.

Preservice Teacher Noticing

Many teacher preparation programs address the different steps of student thinking. The design of most preparation programs supports preservice teachers, as they develop their own questioning techniques, noticing what occurs, and supporting students in reaching specific goals (Kennedy, 1999; Shahrill, 2013). As teachers develop the ability to teach mathematics, how they view interactions in the classroom will evolve, as well as how they interpret student errors and misconceptions (Erickson, 2011). As preservice teachers learn to become teachers, they may lack well-developed observation skills (Star & Strickland, 2008). In their work in a secondary mathematics methods course designed to improve observation skills, using pre- and post-tests of the ability to notice instructional features in classroom video, Star and Strickland (2008) found preservice teachers were able to significantly improve their ability to observe classroom events within such categories as *classroom environment* (observing the physical arrangement of the classroom, materials, and equipment available to students during the lesson, and demographic information of the class) and *tasks* (activities students are working on). Preservice teachers scored relatively well in noticing features of *classroom management* (how teachers address disruptions, pacing, and calling on students, as well as the "presence" of the teacher) on both pre- and post-tests, but showed smaller gains in noticing *mathematical content* (examining the mathematical representations in the lesson) and *communication* (the different discursive interactions in the lesson, including teacher-to-student and student-to-student). The study demonstrated how preservice teachers learn elements of noticing over the span of a methods course. Preservice teachers were better able to attend to static features of the classroom such as management and environment, but less able when it came to dynamic items such as mathematical content (Star & Strickland, 2008).

Star and colleagues replicated this study a few years later (Star et al., 2011) and found that the ways in which preservice teachers noticed elements of classroom environment and communication were similar in both studies. The results diverged in the areas of tasks, mathematical content, and classroom management (Star et al., 2011). That is, participants showed no improvement when it came to identifying elements of the task in a recorded lesson and features of mathematical content in the lesson. While participants in the second study showed some improvement in identifying elements of classroom management, this change could have been an indication of a greater space for growth, as pre-test scores were lower in the second study compared to the first.

While different results emerged from these studies, they both demonstrated that preservice teachers can improve their ability to notice over the course of a semester-long class (Star et al., 2011), with two important caveats regarding preservice teachers. First, there was no identification within the coursework of the *important* features to which preservice teachers should attend. The goal of both studies was to determine whether noticing skills could improve, not whether what preservice teachers noticed could be prioritized or privileged (Star et al., 2011; Star & Strickland, 2008). Second, both studies were based on videotaped lessons and not live teaching experiences. This allowed participants to observe the instruction without the need to balance those observations against actual teaching of the lesson. How participants might have observed instruction while it was occurring was not a part of the study.

Preservice teachers may notice important elements of student thinking, only to struggle with responding to what they notice (Jacobs, Lamb, Philipp, & Schappelle, 2011; Star & Strickland, 2008; Walkoe, 2015). As noted, Star and Strickland (2008) and Star et al. (2011) got mixed outcomes when preservice teachers noticed and identified questions they felt were likely to lead to student learning. That is, they noticed features of the environment and student communication, but not necessarily elements involving instructional activity and content of a mathematics lesson. Levin et al. (2009) found that preservice teachers begin to identify important features within student thinking given the proper framing of teacher preparation activities. Their findings present a challenge to the perspective that preservice teachers cannot elicit, interpret, nor respond to student thinking until they become competent in other so-called fundamental teaching skills such as creating a teaching identity or managing a classroom.

What Is Missing in the Literature?

While there are connections between some of the research topics described above, such as the links between core practices and practice-based teacher education, there is a gap in the research literature on the connection between how preservice teachers develop the core practice of eliciting, interpreting, and responding to student thinking, use and develop questioning structures, and learn to use these effective practices through the plan-practice-teach cycle. Specifically, there is little available research linking:

- how preservice teachers plan for and use questioning structures;
- what preservice teachers notice in student thinking as they rehearse and instruct lessons; and
- how they respond to that student thinking.

In adding to the research literature, this study examines different questioning structures to understand how preservice teachers use what they notice in student thinking, as focusing questions demonstrate use of student thinking to guide the path of instruction and funneling questions steer instruction toward teacher interpretations or pathways planned for the lesson. How these structures are imagined, developed, and used over the plan-practice-teach cycle adds to the literature and provides understanding that can allow for the development of activities to support preservice teachers to elicit, interpret, and respond to student thinking.

Chapter 3: Methods

Research concerning core practices, practice-based teacher education, eliciting, interpreting, and responding to student thinking, question types and structures, and teacher noticing provided the foundation for this study. Each of these topics demonstrates the importance of effective questioning to engage and support students during the learning of mathematics. Research indicates that preservice teachers can use questions effectively and have shown the ability to learn how to elicit, interpret, and respond to student thinking, but there are variances in these areas across different subjects and studies. Preservice teachers often desire to leverage student thinking in their instruction but struggle to provide the climate in which student thinking guides the development of mathematical understanding. In many cases, even though it might not be their intent, the teacher takes over the thinking in the lesson. Research also indicates that there are elements of effective instruction that preservice teachers can attend to and some areas that they struggle to identify as important. Based on those findings, this study uses questioning structures to investigate how preservice teachers use questions to elicit, interpret, and respond to student thinking and how preservice teachers change through the plan-practice-teach cycle. Questioning is a way to view teacher noticing during instruction – what do they notice about student thinking and how do they interpret and respond to that thinking. They give information about what the teacher is learning about student thinking and what they decide to do about it. Questions will be grouped into sequences to explore how preservice teachers elicit, interpret, and respond to student thinking – grouping questions allows deeper investigation than exploring single questions.

The results of this study provides an understanding of how preservice teachers change over the course of a semester in their use of questioning structures, what characteristics of the plan-practice-teach cycle provided the best opportunities to make changes in questioning, and what were the key elements that supported improving the preservice teachers' ability to elicit, interpret, and respond to student thinking.

Using the cycle of planning, practicing, and teaching high school mathematics lessons, the questions that preservice teachers plan provide information on how they may expect lessons to look. Through anticipating questions, student responses, and how to respond in their lesson plan documents, and practicing with their peers, the degree to which practice teaching matches those expectations of what teachers expected to notice and how they respond demonstrates the alignment of lesson plan documents, peer practice, and teaching. Such an analysis examines the use of questioning structures throughout the plan-practice-teach cycle. Specifically:

- Do preservice teachers use focus and funnel questioning structures as they elicit, interpret, and respond to student thinking and, if so, how do they use them?
- In what ways does preservice teachers' use of focus and funnel questioning structures change through the plan-practice-teach cycle?

Focusing and funneling questioning structures are the focus of this study as they represent how teachers elicit, interpret, and respond to student thinking. As these questioning structures identify how the teacher uses student thinking, they provide insight into how the teacher reasons about using student thinking. When using the focusing questioning structure, teachers use their interpretations of student thinking to allow students to clarify and extend their thinking. When using funneling questioning structure, teachers guide students to a specific interpretation, narrowing the outcomes for student thinking. These questioning structures demonstrate how teachers respond to their interpretations of student thinking.

Choice of Design

The design chosen for this study is the multiple case study. This allows for in-depth analysis of the topic of research and collecting data bounded by specific systems and times to provide greater understanding of how the phenomenon of interest develops (Creswell, 2013), specifically how preservice teachers use focusing and funneling questioning structures and elicit, interpret, and respond to student thinking. The case study design requires research into what happens in a situation and the reasons how and why something happens (Bernard & Ryan, 2010; Yin, 2006). In case study design, multiple lines of evidence combine to provide a rich understanding of the topic under investigation (Yin, 2006). Using multiple participants in a study may allow for different sources of data to address the research question and provide powerful findings, based on commonalities among cases which result in generalizations across some or all of the participants. Using multiple cases increases the power of generalizations beyond findings using an individual case (Borman, Clarke, Cotner, & Lee, 2006). In addition, differences across cases can illuminate potential limitations of the study.

Setting

Participants volunteered from a secondary mathematics methods course offered at a large mid-Atlantic public university. Using a methods course provided an appropriate setting for this research: it is one of the courses where preservice teachers focus on the use of effective teaching practices through secondary mathematics content. The required course includes undergraduate and graduate students seeking certification to teach mathematics at either the middle or high school level. A methods course includes activities that focus on planning and implementing high school mathematics lessons that encourage effective discourse, examine patterns and techniques of questioning, and investigate how mathematics content plays a role in the discursive process.

Course sessions examine instructional strategies and discuss strengths and challenges within lessons for students and teachers. Discussions that arise in the course concern how to collect and use evidence of student thinking to support effective instruction. Activities include in-class noticing and interpretation of student thinking (through completion of mathematics tasks and discussion of possible lines of student thinking), development of lesson plans, reflections on peer teaching episodes (rehearsing lessons with others in the course), and reflections and analysis of implemented lessons.

The graduate students in the course were part of a preparation program designed to provide state certification in secondary mathematics education along with a master's degree in education within a 13-month time frame. During both the fall and spring semesters, graduate students intern full time at local public high schools in nearby school districts. In addition to support offered on campus, each preservice teacher has a mentoring classroom teacher who provides feedback and support.

Participant Pool

In consultation with the course instructor, the choice was to include only graduate students in the study based on the fact that they were in full-time teaching placements, compared to the undergraduate preservice teachers in part-time placements. Since the graduate preservice teachers worked with students on a daily basis, they developed a better understanding of their students than the undergraduate students in the class who only participate in class once per week. While all of the participants share the characteristic of being graduate students, they also had a number of differences. Each of the participants comes with their own unique experiences that have defined the learning and teaching of mathematics; personalities that contribute to their teaching ability; and different students, mentor teachers, and school climates that have differential impact of the goals of their lessons. Each of the participants also bring different classroom teaching experiences, with some having taught during undergraduate study and some not. These differences provide variation in the beginning point for each participant that may lead to variation in their evolution throughout the study.

Participant Selection

The study included all of the voluntary participants who provided data. Of the ten graduate-level preservice teachers enrolled in the course, six initially volunteered to participate in the study. While the graduate students in the program come from diverse educational backgrounds, each of the volunteers has gone directly from undergraduate study into this graduate program. This provides some commonalities among the volunteers in their experiences related to teaching. Of these six, five provided complete data and participated in the end-of-the-semester individual interview. All five of these volunteers are part of the study.

Implementation Design and Data Collection

The research design and data collection occurred throughout the semester. It was bounded by an introduction of the study during the first class and the interview near the end of the semester, after the participants taught the focus lesson in a high school mathematics classroom. During the introduction of the study, all graduate-level preservice teachers heard a brief biography of the researcher, the goals of the study, and a description of the included data and collection process, which were also part of the consent form distributed. During this discussion, there was an emphasis placed on wanting to learn more about how preservice teachers develop and implement instruction during their teacher preparation experience.

The researcher sat in on four of the first six methods classes, taking field notes on the participants' comments and actions. The purpose of these observations was to learn more about the experiences of the participants and the activities of the course.

Since the study examines how preservice teachers use questions and elicit, interpret, and respond to student thinking in mathematics, it was important to note that throughout their methods course, the preservice teachers explored student thinking through multiple experiences of using questioning. As learners, the preservice teachers first solved mathematical tasks and considered their own learning experiences and how teachers elicit and use their thinking. They explored alternate ways to solve the tasks and the meaning behind these alternate student strategies in order to consider how their students think about mathematics. This led to generating possible questions to ask during the task, anticipating how students might respond, and planning what to do, given different student responses. Finally, the preservice teachers enacted their questioning strategies with peers and in high school classrooms. Throughout the semester they explored activities including reading about questioning, investigating lesson designs, and observing authentic classroom scenarios/videos to gain guidance in planning, orchestrating, and reflecting on how questioning can benefit student thinking. This study also benefits from the context of deep investigation into a single lesson – weeks allowed to develop a draft lesson plan, time in class to practice the lesson with peers acting as students, revision of the lesson plan, teaching the lesson to high school students, and reflecting on each step of the process. The time for examining this one lesson and the allowance to rehearse the lesson and make adjustments to the plan provides a unique opportunity that is not replicable for in-service teachers.

Developing an understanding of the questioning process first occurs as preservice teachers make sense of mathematics tasks as learners. Completing the mathematics task is an integral part of the lesson planning process; follow-up included sharing questioning strategies and possible sequences that might benefit students. These discussions allowed the preservice teachers to plan questions from two perspectives. First, they are able to consider questions that elicit how well students understand the learning goal. Second, once they determine that students have achieved proficiency of the learning goal, they can consider questions that support students building on the learning goal. Incorporated into these activities were videos and transcripts of the questioning processes for mathematics activities, activities that offered preservice teachers the chance to discuss teaching practices removed from the "immediate demands of instruction" (Sherin & Linsenmeier, 2011, p. 39). Through these activities and discussions, preservice teachers could analyze the context of teaching, what was important to notice and interpretations of what they noticed, without worrying about assessing their own instruction (Castro, Clark, Jacobs, & Givvin, 2005).

Peer Interview

Preservice teachers prepared for their high school lesson by using a common task and investigating how to use assessing and advancing questions. Assessing questions allow teachers to learn more about how well students understand the task and learning goal. Advancing questions press students to move beyond their current thinking into a new situation (Smith et al., 2008). Both of these question types fit well with a focus questioning structure, as teachers elicit student thinking, interpret how well the student represents understanding of the learning goal, and use either assessing or advancing questions based on the needs of the student. Each preservice teacher used the same task (see Appendix A) and other preservice teachers acted as students in these practice lessons. The lessons were audio recorded and analyzed by each preservice teacher (see Appendix B), focusing on identifying whether questions were assessing or advancing, what they learned from the student response, and possible teacher actions based on what they learned, including possible revised questions. This analysis assignment was the first data source for the study.

Draft Lesson Plan

The next data source collected for this study was a draft of the participants' lesson plan and accompanying commentary (see Appendix C). This represented preservice teachers' initial thinking of how to develop mathematical understanding of their assigned learning goal. Mentor teachers provide the lesson's learning goal to the preservice teacher from the curriculum for the class. The lesson plan assignment allowed the preservice teacher to apply the class-discussed questioning techniques as they related to the learning goal of their lesson. This also provides another area of variability as each participant had different topics and different levels of students with which to work.

While the specific questioning structures of funneling and focusing were not part of the course content, encouraging student-centered learning environments and working to uncover and support student thinking were high priorities in both the activity and in-course discussions. The expectation was that lesson plans would include questions the preservice teacher would ask in the lesson, anticipated responses from students, and teacher actions based on these responses. Including teacher questions, student responses, and teacher actions provided an overview of what to observe in the lesson while the potential actions provided insight into how the teacher interpreted student thinking. The course instructor provided feedback on lesson plans prior to collection of the second data source for the study: the practice lesson with peers. Coding of the lesson plan focused on different types of anticipated question and potential questioning structures represented in the lesson plan (see explanation of coding below).

In the commentary on the lesson plan, the participants reflected on how they designed their lesson to support mathematical understanding, used knowledge of students to inform teaching and learning, and monitored student learning (see Appendix C). This commentary provided insights into how the preservice teachers were thinking about their lesson and their expectations for student participation and learning.

Peer Practice

Another source of data involved practicing the lesson plan with peers. Peer practice provides a useful source of data as working within the controlled environment of a methods class allows preservice teachers to focus on one particular core practice, such as eliciting, interpreting, and responding to student thinking. During peer practice, preservice teachers gain experience and feedback on implementation of the lessons which they are able to apply to actual teaching episodes. One advantage of peer practicing is the ability to pause the practice and reflect on teacher actions during instruction and make adjustments. The goal of this activity was to practice implementing the lesson in order to learn how a lesson might go when enacted with high school students. Groups of preservice teachers practiced teaching important elements of each of lesson (preservice teachers were each given about 30 minutes to present the important ideas in their lessons). They then taught their lessons to a small group of other preservice teachers, with peers acting as high school students for the purpose of the lesson.

While there were benefits derived from practicing the lesson with peers, secondary mathematics preservice teachers may not encounter the same challenges that high school students would; rather, the roles played by peers often reflect well-behaved high school students focused on problem solving (Kazemi et al., 2016). Limited thinking and responses provided by peers may not match actual thinking and responses presented by high school students.

Therefore, how preservice teachers elicited, interpreted, and responded to peers did not provide a complete picture of how the lesson might go enacted with real high school students.

Each participant made a video recording of their practice lesson. The researcher then transcribed each lesson. Recordings and transcriptions provided data about preservice teachers' decision-making during teaching of the lesson. The process of coding of the transcript matched the coding of the draft lesson plan. Linking these two data sources allowed for an examination of how study participants planned and implemented the questions in their lessons. While these data were not an exact representation of what they might do in a lesson with actual high school students, it did provide insight into how they used questioning to elicit, interpret, and respond to student thinking.

Peer Practice Reflection

Each participant completed an analysis of their peer practice (see Appendix D). This activity allowed for preservice teachers to deepen their knowledge about teaching from the peer practice, as video recording can reinforce learning experiences (McDonald et al., 2013). Preservice teachers analyzed their peer practice video to look specifically for examples of creating a positive learning environment and developing conceptual understanding, procedural fluency, and reasoning and/or problem-solving skills, as well as analyzing their own teaching. In these responses, preservice teachers developed claims about their teaching during the peer practice and supported their claims with evidence from their recording of the lesson. These responses provided data about their views on the use of questioning and its success in eliciting, interpreting, and responding to student thinking.

Revised Lesson Plan

After practicing the lesson and reflecting on the questioning in their video of the practice lesson, participants completed a revision of their lesson plan as a final preparation for the high school lesson. Participants provided these revised lesson plans to the researcher who then coded the questions and questioning structures used in the final lesson plan. Comparisons between the draft and the final lesson plans and practice implementation of the lesson explored whether questions and questioning sequences were consistent or had changed, based on feedback and practice implementation.

Teaching the Lesson

The next element of the study called for each participant to teach a lesson to local high school mathematics students. The mentor teacher and course instructor observed the lesson. As part of their reflection, each preservice teacher videotaped and analyzed the lesson (see Appendix E). While the IRB did not permit use of any video that included high school students, the assignment allowed for sharing of in-class discussion as the preservice teachers excluded student identities in their assignment.

The preservice teachers analyzed their classroom videos and responded to questions on their teaching experience. These responses provide insight into what each preservice teacher noticed and how they interpreted what they noticed, as well as what actions they used to support student thinking. Preservice teachers then shared the assignment with the researcher. The analysis of questions and questioning sequences used the same coding scheme as the draft lesson plan, and the alignment of questions was tracked across draft lesson plan, the peer practice, and the taught lesson. In addition, preservice teachers made annotations on copies of their lesson materials about their teaching as it occurred and provided commentary reflections on their lesson plan changes. The participants in the study provided these materials as data sources to the researcher.

Interview

The final data collection of the study was the individual interview with each participant. This interview was semi-structured, with similar questions for each participant, allowing the collection of common information from each one, with some specific question for each participant to allow the researcher to go deeper where needed (Bernard & Ryan, 2010) (see Appendix F). It included questions addressing data from the entire semester, including questions about the design of questioning in the lesson plans, what changes the preservice teacher made, the impact of practicing with peers, and whether teaching the lesson with high school students matched expectations. These questions linked analysis performed throughout the semester with the thinking of each participant, particularly whether they were able to elicit student thinking through the use of selective questioning, what they expected from students while planning lessons, how they interpreted student thinking during the lesson, and how they responded to student thinking.

Data Sources and the Structure of Data Analysis

To summarize, the data collected and analyzed for the study included lesson plans, transcripts, reflections, and interviews (see Table 1).

Table 1. Data Sources and Types of Data Analysis.

Data Source	**Type of Data Analysis**
Peer interview – audio recording, transcript, and participant reflection/analysis	Coding analysis of the questions asked and the reasoning for questioning provided by the preservice teacher, focusing on how they explore questioning and how they exhibit focusing and funneling question structures.

Draft lesson plan – lesson plan and commentary	Coding analysis of the predicted path of the lesson, with a focus on how the preservice teacher guided student thinking throughout the lesson. Commentary provided some participant reasoning for questions and actions predicted in the lesson plan.
Peer practice – video recording and transcript	Coding analysis of the transcript of the peer practice, looking at how participants elicited, interpreted, and responded to student thinking and the alignment between the lesson plan and the peer practice lesson.
Peer practice reflection – participant reflection/analysis	Linked to the transcripts of the peer practice – provided some participant reasoning for questions and actions during the practice lesson.
Revised lesson plan – lesson plan and commentary	Coding analysis of any revisions to the predicted path of the lesson and some of the reasons for them.
Teaching the lesson – participant reflection/analysis	Provided some participant reasoning for questions/action of the implemented lessons.
Interview – audio recording and transcript	Linked to the questions/actions of the plans and teaching episodes (both peer practice and high school lesson). Provided some participant reasoning for questions and actions throughout the process of planning and teaching.

All of the data sources except the interview were part of the assignments for the secondary mathematics methods course. Each data source provided insights into the thinking of preservice teachers and supported their understanding of the different components of each of the research questions.

Data analysis investigated how preservice teachers used questioning through the plan-practice-teach cycle of lesson development and implementation. For the purpose of the study, we need to define the term *question*. While many phrases can function as a question, in this case, a question is a statement that sought to do one of several tasks in the development of mathematical understanding:

- gather information from students;
- explore mathematical meanings and relationships;

- get students to explain their thinking and reasoning;
- support connecting ideas together; or
- extend student thinking (adapted from Boaler & Brodie, 2004) (see Table 2 for examples).

Table 2. Question Characteristics and Examples.

Statement Type	**Examples**
Gather information from students	• Why did you create a bar graph?
Explore mathematical meanings and relationships	• Why did you connect the points?
Get students to explain their thinking and reasoning	• Tell me more about your thinking.
Support connecting ideas together	• Where do you see elements of the graph in your equation?
Extend student thinking	• What would happen if you changed the *y*-intercept?

This definition meant questions that concerned classroom management ("Would you come to the board?" "Can you repeat that?"), were not part of developing understanding ("Do you have your book today?"), or required choral responses from students ("Did everybody agree with that?") would be omitted. Note that statements do not require a question mark to be a question, as the example for 'Get students to explain their thinking and reasoning' in Table 2 demonstrates. This is an attempt to differentiate between teachers comments and teacher questions. While the researcher determines if a statement meets one of these five characteristics of a question, reaction from students during instruction can support the choice. If students respond to an utterance by explaining their thinking, then the utterance served as a question. This definition allowed for the development of mathematical understanding by either the teacher or student and supported examining how the preservice teacher used questions and student responses to elicit, interpret, and respond to student thinking.

Once an understanding of what constitutes a question was established, the next step was analyzing and coding questioning, i.e., whether to consider questions individually throughout the lesson or determine which group of questions to analyze (see Appendix G). Coding involves "reducing the data into meaningful segments and assigning names for the segments" (Creswell, 2013, p. 180), with codes coming from both inductive (from the data) and deductive (from understanding of the topic studied) approaches (Bernard & Ryan, 2010). Combining or grouping codes into broader categories supports describing of information and themes in the data (Creswell, 2013). As the early research into this study determined the importance of questioning and questioning structures, the choice was to create codes that identify the type of questions asked and that would allow them to be grouped together to "mark instances of themes in a set of data" (Bernard & Ryan, 2010, p. 76). The coding showed that there were themes among the participants, both commonalities and differences, in how each developed their questioning. There of these themes were discrete, such as whether the participant used initial-response-evaluate (IRE), focusing, or funneling questioning structures. Some feel along a continuum, such as how did the pre-service teacher design the rehearsal environment in order to benefit from the practice. As the coding process continued, these themes began to show how to use the patterns to begin the cross-case analysis.

The themes in the data focused around how preservice teachers use sequences of questions to develop mathematical thinking in their students. The patterns that emerged were how they elicited, interpreted, and responded to student thinking. Specifically, did the preservice teacher allow elicited student responses to guide the course of the questioning (focusing) or did they use their thinking to determine the pathway (funneling). In order to determine whether a preservice teacher was using funneling or focusing questioning structures, sequences of questions along with their codings were grouped together to allow for examination of how the preservice teacher used student thinking. This process allows for the researcher to consider the underlying aspect of the thinking of the preservice teacher (Saldaña, 2011), in this case, how they were interpreting and responding to student thinking (see Appendix G for examples and explanation of the coding of questions and question sequences).

For the purpose of this study, groups of coded questions are "chunks." The creation of these chunks helps researchers to "develop a sense of the content of the theme" (Bernard & Ryan, 2010, p. 101) during the coding process. A chunk was an extended interaction or discussion in either small group or whole class environments, between the teacher and students which explored the reasoning behind a mathematical topic or subtopic. An extended interaction or discussion had to include at least two conversational turns by students. This requirement was to eliminate the traditional I-R-E pattern of question coding, where the teacher *initiates* the discussion with a question, a student *responds*, and the teacher *evaluates* (or provides feedback for) the response before moving on to another topic (Mehan, 1979). In this question structure, the student has only one opportunity to offer their thinking on a topic before the teacher moves the class to the next part of the lesson progression. By requiring at least two student interactions in coding, the focus is on investigating student thinking with depth. A chunk, for coding purposes, lasts through the end of a particular exploration of student thinking on the topic or subtopic (based on the work of Franke et al., 2009).

The development of codes for the different types of questions that preservice teachers used in planning and teaching was next. While the unit of analysis was a sequence of questions, analyzing each question within the lesson plans and enacted lessons proved helpful in learning how preservice teachers were planning or guiding the discussion. Looking at both individual questions and sequences of questions helped in understanding how the preservice teacher thought about how to learn more about the current level of student understanding. It also demonstrated how the preservice teacher used questions within the sequence to either facilitate student-centeredness or shifted into more teacher-centered interactions. Looking at the number of questions in the sequence, and when and if a shift occurred to teacher-centered interactions, provided insights into the depth of how the preservice teacher interpreted and responded to student thinking. This analysis of codes at both the individual question and the "chunked" level of question sequences provides a stronger foundation for the analysis (Bernard & Ryan, 2010). Researching the literature on questioning provided an initial identification of question types (Boaler & Brodie, 2004; McCarthy et al., 2016; Moyer-Packenham & Milewicz, 2002; Myhill & Dunkin, 2005). The use of preexisting codes helped to guide the coding process (Bernard & Ryan, 2010; Creswell, 2013). Boaler and Brodie (2004) provided the foundation for draft question types, while other studies augmented or included additional question types.

Development of codes occurred over time, based on the literature and evolution of codes as well as through practice with the data (Bernard & Ryan, 2010; Creswell, 2013). Practice coding revised and clarified question types and structures and their descriptions (Grosser-Clarkson, 2016).

Below are the question types and descriptions, used in this study, categorized with source documentation and notes:

- F – factual question – expectation for students to provide a pre-determined answer (Myhill & Dunkin, 2005). "Requires immediate answer; rehearses known facts/

procedures; enables students to state facts/procedures" (Boaler & Brodie, 2004, p. 777)

- P – procedure question – allows student to identify a procedure or the next step of a procedure or asked to move to the next part of the lesson (Myhill & Dunkin, 2005)
    - PC: Checklisting: teacher uses amount of wait time to gauge the need to move on. "No matter what answer the student gives, the teacher simply moves on to the next item in mind" (McCarthy et al., 2016, p. 81)
    - PL: Leading question that allows only one answer (and answer may be imbedded in the question) to move the lesson forward
- WH – What/how question – asking students to explain what they know or how they know their reasoning
- Q – inquiry question – allow students to articulate understanding of mathematical concept – explain the reasons for their thinking (Myhill & Dunkin, 2005). "Exploring mathematical meanings and/or relationships - Points to underlying mathematical relationships and meanings. Makes links between mathematical ideas and representations" (Boaler & Brodie, 2004, p. 777)
    - QC: extend to clarify prior questions/response
- S – speculative question/why – no pre-determined answer, asking for opinions, hypotheses, ideas, possibilities (Myhill & Dunkin, 2005)
    - SP: Probing and follow-up: "teacher can question students based on incorrect responses, non-specific, or competent questioning of the student's view or answer (Moyer-Packenham & Milewicz, 2002)" (quoted from McCarthy et al., 2016, p. 81). "Probing, getting students to explain their thinking - Asks

student to articulate, elaborate or clarify ideas" (Boaler & Brodie, 2004, p. 777). Additional examples include: "What happened when…"; "Did you…"; "How would you…"

  - SE: evaluating decisions or the work of another
  - ST: "Extending thinking - Extends the situation under discussion to other situations where similar ideas may be used" (Boaler & Brodie, 2004, p. 777). Find another way, compare across situations, create or evaluate generalizations

- G – general question – not related to student response (Franke et al., 2009)
  - May manage classroom behavior or guide conversation

These questioning categories provide a coding scheme for individual questions asked. Coding questions at the individual level provided the evidence needed to consider how the sequences of questions developed either focusing or funneling questioning structures within chunks of questions. Coding took place throughout the data gathering period, with adjustments made to codes based on the data provided and the ongoing coding process (see Appendix G for a representation of the coding of data). As researchers develop a better understanding of the data, the adjustment of codes allows for a better representation of the data (Bernard & Ryan, 2010).

After the coding of questions within each chunk, the next step was to determine whether the chunks consisted of funneling or focusing questioning structures. In funneling, teachers continue questioning until "a student gives a solution that is representative of the method that they [the teacher] intend the student to use" (Wood, 1998, p. 167) or directing student thinking to a predetermined path, based on how teacher views the problem's solution or development of mathematics (Herbel-Eisenmann & Breyfogle, 2005). In this structure the teacher is the primary thinker and the one who is conducting a cognitive activity in the lesson (Herbel-Eisenmann & Breyfogle, 2005; Wood, 1998). A funneling sequence may being with a "probing question but allows little wait time and immediately follows up with questions that become more directed toward one particular answer" (NCTM, 2014, pp. 40-41). In funneling, the teacher tries to get students to view what the teacher sees through a series of indirect but increasingly directive questions (Mason, 2010) and where students are responsible to provide the answers that the teacher is seeking.

In a focusing question structure, the emphasis shifts from the teacher's thinking to that of the student, where the teacher listens to student ideas and guides the conversation based on the ideas provided by students (Herbel-Eisenmann & Breyfogle, 2005). The teacher asks questions to prompt students to share their thinking, and to explain and justify it to the class. Students provide the cognitive activity around the mathematics while the teacher supports that activity by determining which questions to ask in order to keep the focus on student thinking. Instead of funneling students toward the teacher's way of thinking (which encourages students to see the teacher as the sole authority), focusing allows students to think about mathematics and communicate their ideas to one another (Wood, 1998) and supports students in making connections (Herbel-Eisenmann & Breyfogle, 2005).

After coding the questions and question chunks in lesson plans and in practice and implemented lessons, chunks of questions about the same topic or subtopic were connected throughout different aspects of the lesson: the lesson plans (both before and after practice with peers), the transcript and video of the practice lesson with peers, and the reflection provided by the preservice teachers from the lessons employed in high school classrooms. These connections examine the planning and enactment of a topic or subtopic as they occur in each aspect of the lesson and explore how the preservice teachers elicit, interpret, and respond to student thinking. For example, in Beth's data, there are examples of similar questions throughout the plan-practice-teach cycle. In the draft lesson plan, she includes the questioning sequence of "Is Kate's work wrong? What was the mistake that led her to the wrong solution? Did she apply all of the properties of logarithms correctly?" (Beth lesson plan draft, 2018, p. 5). During her peer practice, she asks, "Is that a wrong application of the property, or is that not the best way to solve this equation?" (Beth transcript of peer practice, 2018, lines 275-276). In her final lesson plan, Beth includes the questions, "How does the domain of logarithmic functions impact when we apply the properties of logarithms? I.e. Why can't we always apply the properties when we have variable arguments in the logarithm?" (Beth lesson plan final, 2018, p. 4). And in her lesson with students, she asks, "It seems like he did apply the property correctly, but maybe look back at your property notes and think about, what do we have to know about what's inside the logarithm in order to apply the property?" (Beth analysis #3, 2018, p. 2). While these questions are different in wording, they are similar in that they are asking students to connect the use of properties to the worked-out problem. Beth demonstrates that her questions changed based on how she interprets student thinking from the responses to earlier questions and the work that she sees represented on student papers. From this series of questions, Beth demonstrates how she makes adjustments to the interpretation of student thinking and the context of the discussion, and then shows that she is responding to student thinking.

In addition, preservice teachers' reflections after the draft lesson plan and the practice lesson with peers, as well as responses from the interview following the enactment of the lesson in the classroom, provided reasons for some of the decisions made by the teacher. They also offered understanding for how participants believed they were developing and teaching lessons that elicited, interpreted, and responded to student thinking. For example, as Beth discussed her views on questioning during the end-of-the-semester interview, she shares, “I think that my questions evolved, mainly again because my understanding of the task evolved” (Beth interview transcript, 2018, lines 97-98). She demonstrates that she used the plan-practice-teach cycle to learn more about the development of mathematical understanding in her lesson and connects to the changes in the questions that she plans and asks.

The preservice teacher reflections and interview provide supporting (and non-supporting) data about how preservice teachers view their questioning and actions as they work to uncover student thinking. Examination of each case study connects questions, actions, and decisions across the plan-practice-teach cycle for each participant. Next, commonalities and differences across cases develops the findings of this study. In the next two chapters, learnings are shared based on tracking questions and actions of preservice teachers through the plan-practice-teach cycle. These learnings provide information on what each of the preservice teachers anticipate and expect, what they notice during practice rounds with peers and actual instruction with high school students, and how they respond to what they notice during both sessions. These responses are based on what preservice teachers notice and indicate what they elicit about student thinking, how they interpret such thinking, and what responses they give, based on their interpretations.

Chapter 4: Case Studies

The five participating preservice teachers considered here are all students in a secondary mathematics education graduate program at a large mid-Atlantic university. Each is a full-time intern as a mathematics teacher at public high school and each participant attends graduate school classes in the evening. All entered the graduate program upon completing their undergraduate degrees and have attended classes over the summer and in the evening during the school year to receive teaching certification along with master's degrees in education. While the participants have these experiences in common, they also have unique experiences, characteristics, expectations, and goals for how they approach teaching mathematics, view learning mathematics, and respond to student thinking during instruction. A pseudonym identifies each participant. There is no intention implied in the order by which the cases are presented; they are done randomly.

Each of the following case studies begins with a short biography and an overview of how the participants elicited, interpreted, and responded to student thinking over the course of the semester. The data begins with a description of a one-to-one interview between the preservice teacher and a peer. This instructional experience provides insight into how each participant approaches questioning with little preparation. Next is the analysis of a draft of the lesson plan, which is the beginning of the development of a lesson that occurs over approximately half of the semester. This is the beginning of the plan-practice-teach cycle. Analysis of the lesson plan included how each preservice teacher plans to elicit and respond to student thinking and the questioning sequences they anticipate. Analysis moves to the practice of the lesson with peers, looking at the alignment between questions in the lesson plan and asked during the practice and how the participant enacts what they planned. The final draft of the lesson plan indicates what the preservice teacher feels is most likely to occur, taking into account learnings gained from the peer practice. Finally, an analysis of the teaching of the lesson in a high school classroom links the pieces of the plan-practice-teach cycle and provides an endpoint for the scope of this study on how the participant elicited, interpreted, and responded to student thinking.

Gloria: Things Go Better with a Plan

Gloria decided near the end of high school that she wanted to be a librarian. When she was looking for colleges to attend, she learned there were no undergraduate programs in the field of library sciences. When she learned that many librarians earn undergraduate degrees in education before pursuing graduate study in the library sciences, Gloria decided to complete an undergraduate degree in education. She chose mathematics because “I am good at math” (Gloria interview transcript, 2018, p. 1). She selected an undergraduate program that allows students to teach in the classroom early in their coursework. Once she had experienced classroom teaching during her freshman year, she decided that teaching was a good fit for her. She chose to participate in the graduate program after completing her undergraduate program since it would provide teacher certification along with a financial benefit (teachers with master’s degrees earn a higher salary than those with only bachelor’s degrees), and she could complete it in the same amount of time she had planned for her undergraduate training. She believes the course has better prepared her to become a teacher, especially in terms of creating detailed lesson plans (Gloria interview transcript, 2018, p. 8).

Gloria identifies at the end of the semester that she wanted to find a balance between the questions she plans before a lesson and those she develops during a lesson, based on what her students need at the time. However, she does not get much of a chance to do this, as her experiences make it difficult for her to enact planned questions and she often ends up asking questions extemporaneously. Her in-the-moment questioning often guides students toward making sense of problems in the same manner that she does. She particularly struggles when students are uncertain about their reasoning or do not want to share their thinking.

Initial Peer Practice: Struggling without a Plan

In a teaching activity early in the methods course, peers took the roles of teacher and student in a teacher-to-student interviews that involved student-created graphs (see Appendix A). Gloria served as the teacher while another preservice teacher acted as her student (identified as "student" in the rest of this section). Their task was to practice how a teacher would make sense of graphs created by a student. With little opportunity to plan, Gloria struggles to develop an understanding of how the student is thinking about the task. She is able to elicit student thinking by asking probing questions, but without the planning time to anticipate student reasoning and develop teacher actions, she struggles in responding to the student's thinking.

After the student completed his graphs, Gloria initiates the discussion by asking, "Why don't you go ahead and tell me what you were thinking when you were doing B?" (Gloria analysis #1 transcript, 2018, line 1), a question designed to elicit student thinking. The student's response shows his thinking was based on one particular element of the graph, "I thought that since there are zero ounces, it would be just like zero dollars" (Gloria analysis #1 transcript, 2018, line 3). Gloria' response offers her interpretation of the student's thinking and leads him to make sense of a particular part of his graph: "What you're looking at here is that it tells you that you pay quite a lot for letters weighing up to one ounce. What does up to an ounce mean?" (Gloria analysis #1 transcript, 2018, lines 5-6). She is leading the student to make sense of his answer through her own lens of making sense of his graph. The student answers that an amount less than an ounce is the same as an ounce, except for negative numbers. Gloria guides the student toward her interpretation again when she states, “Okay, that's good. I'm glad that you brought up negative numbers, because we can't have negative weight, right?” (Gloria analysis #1 transcript, 2018, lines 10-11). While the student introduced the idea that negative numbers were not part of the domain of the problem, he did not have the opportunity to identify why or comprehend the impact of this reasoning. The teacher elicits the student’s thinking, makes an interpretation, and then funnels the student toward her interpretation through the use of comments embedded in her questions, which narrow the student’s choices to making connections to her interpretation.

In a later discussion of another graph the student created, Gloria makes a connection to his previous work and asks the student to explain the connection: “You have sort of a really similar graph going on, and I wondered if maybe you could explain why” (Gloria analysis #1 transcript, 2018, lines 55-56). Asking the student to explain why provides an opportunity to elicit student thinking, but having the question begin by making a connection to the previous graph narrows the opportunity to how the teacher is making sense of the graph. During this short interview activity, Gloria initiated a discussion of two student-created graphs. Both discussions began with an attempt to elicit student thinking. In the first segment, Gloria started the sequence with an open-ended question but then narrowed the student’s thinking with her second question. In the second segment, the wording of the opening question narrows the student’s thinking. Both examples represent a teacher funneling student thinking toward how she is making sense of the task.

While the structure of funneling and focusing questioning is not part of the coursework or mentioned in her reflection assignments, Gloria seems to be aware of the impact of her questioning on student thinking. In her reflection assignment, she notes that her questioning often involves getting the student to agree with her interpretation. She identifies that she sometimes asks leading questions, ending with the word, “right?” (linking to the episode in the interview described above). She understands how her response limits student thinking: “I took away an opportunity for him to do more thinking” (Gloria analysis #1, 2018, p. 2). She states in her reflection that she would prefer to use “why” questions during the lesson, to allow the ‘student/peer’ to explain his thinking rather than doing it for him, specifically to give him “a chance to tell me that there is no such thing as negative weight, rather than him doing as most students would do and agree with the teacher” (Gloria analysis #1, 2018, p. 3). While her reflection indicates that she wants to advance student thinking, she leads students to her view of the task. When the student suggests that the first graph may need to be a piecewise function, she asks, “So what would your piecewise function, maybe, look like?” (Gloria analysis #1 transcript, 2018, line 24). Gloria explains in her reflection that this question was meant to advance the student’s thinking but does not elicit the student’s understanding of a piecewise function. She suggests a revision: “If we reread the situation now knowing what you just told me about what *up to an ounce* means, how could you draw a piecewise function that accommodates that definition?” (Gloria analysis #1, 2018, p. 3, emphasis in the original). This still fails to determine the student’s understanding of piecewise functions. What she is really asking is for the student to reflect the approach she is visualizing (how a piecewise function fits the given context) without first determining whether he is familiar with the concept. Instead, Gloria jumps into guiding the student to interpret the graph in the way in which she sees it, assuming the student already knows about piecewise functions.

In her reflection assignment, Gloria proposes a different opening question for the second segment of the interview. Instead of her question making a connection between the two graphs (as described above), she suggests, "Why did you choose a linear function to describe situations C and A? What was similar about them?" (Gloria analysis #1, 2018, pp. 4-5). However, this question assumes that the student understands the meaning of linear functions. In order to best elicit and interpret student thinking, teacher questions need to probe student understanding which can include learning their definitions of key terms. A question that asks about similarities and differences between the graphs, without the teacher setting boundaries such as identifying linear functions in the case above, provides a better opportunity for the teacher to elicit student thinking on which to base interpretations and subsequent responses.

While Gloria believes that questioning is "one of the most important things a teacher can do for a student" (Gloria analysis #1, 2018, p. 1), she identified that it can be a challenge for the teacher. This includes areas where she wants to improve her own questioning, such as using purposeful questions (avoiding ones that give students the answer) and being mindful of carefully sequencing questions for the best effect (Gloria' analysis #1, 2018). Her reflection provides evidence of how Gloria struggles to allow students to explain their own thinking with extemporaneous questioning. Even some of her questions developed after reflection make assumptions about what students understand.

Planning (Draft Lesson Plan): Developing Good Questions

When given the opportunity to plan questioning sequences, Gloria demonstrates the ability to develop questions to help elicit and understand student thinking. She plans teacher questions to deepen understanding of correct thinking and help students find ways to fix incorrect ones. She is able to consider how to interpret student responses and help them to make sense of the mathematics of the task.

Gloria' draft lesson plan includes a mixture of questions, some intended to elicit student thinking, along with potential focused questioning sequences, and some that aim students toward the teacher's perspective (Gloria lesson plan draft, 2018). Her lesson plan includes options for how students might respond, including both correct answers and possible errors or misconceptions. She organizes her thinking about the lesson plan in two ways. During small group work where students investigate relationships between logarithmic and exponential functions, she describes her role as monitoring student thinking. To that end she creates two categories of questions: assessing and advancing. During the whole group phase of a lesson, where students share ideas developed in small groups, she helps students connect different parts of a lesson. During this phase, Gloria categorizes questions as probing, discussion generating, and making mathematics visible. These categories also come from her coursework.

She organizes her monitoring questions during small group activities by the types of strategy she expects students to use for a task (see Appendices H and I). Monitoring student thinking and the structure of assessing and advancing questions come directly from course resources. Assessing questions help teachers learn "what the student did and why he or she did it" (Smith & Stein, 2018, p. 44) while advancing questions move students to think more deeply about a concept. Gloria formulates questions based on three different strategies she anticipates: "logarithmic functions as inverses of exponential functions, logarithm *is* the exponent, and mirroring nature of properties of exponents and logarithms" (Gloria lesson plan draft, 2018, pp. 3-4, emphasis in the original).

For each question, Gloria includes a potential student response as well as how the teacher might address the student's response. She is thinking about how to elicit student thinking, anticipate a student response, interpret the response, and determine a teacher action based on her interpretation. In some cases, she identifies misconceptions students may have that are associated with the content of the question. This planning helps Gloria envision how the lesson may develop and prepares her to respond to student thinking in ways that she wants – keeping thinking with the students.

Gloria's assessing questions help her to identify student errors. This is evident not only from the question but also Gloria' expectation of student and teacher responses. For example, "How did you set this up?" is followed by the student response: "I put these here and here," the teacher question of, "What happens if you try inputting some numbers where you know what the output should be?" and the student response, "That doesn't work . . . maybe I need to switch these" (Gloria lesson plan draft, 2018, p. 3). Modeling anticipated dialogue helps Gloria prepare to question during the instructed lesson. Another example of the sequence would be:

Teacher: How did you find the logarithm?

Student: (explains process) But I don't know how to use it.

Teacher: Think about the inverse relationship, and how you can use it to set up this problem as an exponential (Gloria lesson plan draft, 2018, p. 3).

Such examples are representative of question and response expectations in Gloria' assessing question section of the lesson plan. While her questions help students find errors, they do not elicit or deepen correct student thinking. Creating questions to explore errors in student thinking limits the lesson if these are the only questions used during implementation.

Her advancing questions help to complete her understanding of student thinking by expecting that students will demonstrate correct understanding of the content. Examples include, "What could you model with these? Would they be the same or different?" and "What could you do if the base is not given?" (Gloria lesson plan draft, 2018, p. 3). While other examples of questions allow students to advance their thinking, Gloria' student and teacher responses show that she sees these as helping the students to demonstrate their understanding of the content, as shown by this excerpt from the lesson plan:

> T: How can you use what you know about properties of exponents and apply it to logarithms?
>
> S: You can use the Product Rule to add the insides.
>
> T: Try an example for x and y to see that works (Gloria lesson plan draft, 2018, p. 4).

Variance within actual student responses to these questions may take the questioning in a different direction, so it is difficult to tell how Gloria might use them in the classroom. From her planning, she indicates the goal of trying to elicit, interpret, and respond to student thinking while monitoring small group instruction. This was not evident during the peer interview, even though she expresses that eliciting, interpreting, and responding to student thinking is a desired goal.

Questions during the whole-class section of the lesson provide openings for students to share their thinking, make connections between their thinking and that of their classmates, and generalize about the properties of mathematics embedded in the task. While the lesson plan includes possible student responses, there are no teacher actions/responses indicated, so it is unclear how Gloria might interpret these responses. In addition, such opportunities occur during whole-class instruction, leaving fewer spaces for the teacher to elicit and interpret this thinking in individual students. It would help if Gloria would plan questions that elicit such deep thinking in both phases of the lesson, small group and whole group. Based on the challenges Gloria experiences in trying to encourage student thinking in working with one student during the peer interview, she might find it more challenging to address individual student thinking when she is trying to manage the instruction of the entire class.

Peer Practice of Actual Lesson: No Chance to Use Those Good Questions

For the peer practice of her lesson, Gloria chooses to start at the beginning of her lesson and get as far as she can in the available time. However, she only has 30 minutes to practice her 90-minute lesson. As a result of a shortage of time and her peers asking many questions at the start of the lesson, Gloria is unable to practice the questioning sequences she has planned for the lesson.

During the peer practice, some of Gloria' questioning sequences start by trying to elicit student thinking. For example, she asks, "How did you get x^{13}?" (Gloria transcript of peer practice, 2018, line 47) and "How did you get m^6?" (Gloria transcript of peer practice, 2018, line 206). However, she struggles to gain information about the lesson's learning goal, in part because she does not reach the key activity that addresses her learning goal. Due to time constraints, the practice lesson does not progress to the part of the lesson that contains her planned questions. Gloria is unable to practice the integral elements of the learning goal because of her choice to start at the beginning of the lesson and work through as much as she can, as well as the roles her peers take in working through the lesson. If Gloria had focused on the primary elements of her lesson, she might have used questioning more effectively during the peer practice and noted elements relevant to her learning goal and realized how her questioning could elicit and respond to student thinking.

For their part, Gloria's peers demonstrate a number of misconceptions during the introduction to the main activity, a point where her lesson plan had anticipated the students would better understand the material. Those acting as students lacked clarity about mathematical concepts such as like terms and Laws of Exponents, something Gloria's lesson did not predict in a Pre-Calculus class. Her reflection assignment for the practice lesson states that she slowed the pace to interpret the needs of the students (Gloria analysis #2, 2018). As a result, the questioning used in the peer practice was extemporaneous (since the lesson plan did not include planned questions for this section) and focused more on procedural fluency more than student sense making. This is evident in questions such as, “Why did you add them?” (Gloria transcript of peer practice, 2018, line 51), “Which Law of Exponents?” (Gloria transcript of peer practice, 2018, line 63), and “Are they the same kind of x?” (Gloria transcript of peer practice, 2018, line 122). The goal of these questions is to reinforce students remembering the rules. Questions which ask students to recall the Laws of Exponents do not provide opportunities for students to demonstrate thinking. Had Gloria designed the activity and questioning to teach the Laws instead of review them, similarly to how she designed the main activity described in the previous section, the task and questioning might have allowed her to elicit and respond to student thinking in a deeper manner.

Without the benefit of planning how questions will help unpack student thinking, Gloria is the one who identifies the importance of making connections between elements of the lesson, not the students. Here Gloria is funneling students toward the outcome she desires instead of using a focusing questioning structure.

Final Lesson Plan: Tweaking the Plan

Based on her consideration of how to teach the lesson, Gloria revised some questions and clearly defined when to ask questions that support learning and student thinking. As she did not have an opportunity to practice her questioning during the peer practice, she keeps many of her questions from the draft but changes how she organized her collection of student thinking in the lesson. Gloria moved from a general monitoring tool, organized by student strategy across the whole lesson, to a questioning tool that divided the lesson into four parts and guided questions to problems that students were working on, following problems 2, 4, and 7 of the task and the close of the lesson (see Appendices H and I for the task) (Gloria revised lesson plan explanation, 2018). Some questions remained consistent in both lesson plans and some questions closely paralleled one another. There are also many questions that are new in the final lesson plan. Some differences include asking students about the graph, asking them to make connections between the equation and the graph, and asking them to compare their strategies with those of their peers (Gloria lesson plan final, 2018).

In her commentary on revisions to the lesson plan, and in the interview at the end of the semester, Gloria declared that such a format fit the flow of the lesson and the whole-class format (as opposed to more one-on-one support) (Gloria interview transcript, 2018; Gloria revised lesson plan explanation, 2018). Her final lesson plan included questions that elicited student thinking, anticipated student responses, and included teacher responses that used a majority of "what" and "how" questions. These questions may provide either a focusing or funneling questioning sequence, depending on how the questions respond to student thinking. These questions do show that she values understanding student thinking and how she envisions the progression of the lesson.

Teaching the Lesson: Enacting the Plan

Gloria reflects on how planning can support her instruction in the analysis of her enacted lesson. She was able to ask many of her planned questions which allowed her to interpret and respond to student thinking. While there were still moments where she did some of the thinking for students, she also adjusted the lesson based on the student thinking she collects.

During the lesson Gloria made annotations on a copy of the lesson plan. She noted elements of the lesson she did not need (such as reviewing the Laws of Exponents, which could have been included in the lesson based on her peer practice experience), parts she wanted to improve (i.e., when to collect the whiteboards), unexpected misconceptions (applications of the product rule and understanding the representation of logarithms), and which parts of the planned lesson she did not get a chance to teach (sentence synthesis summary) (Gloria lesson plan annotations, 2018). She identified which starting questions (those that began a sequence of teacher question, student response, teacher response) were asked during the lesson. She modified one question ("What did you notice about the use of properties in this task?"), as well as one that did not get many responses but might be worth trying again ("What did we learn from looking at the way logarithmic graphs change vertically or horizontally?"), and one that she did not get to ask ("How could we have used the product/quotient rule to help us with the problem from the video?") (Gloria lesson plan annotations, 2018).

In her interview at the end of the semester, Gloria explained that she modified the question about properties because she saw students making connections between the Laws of Exponents and the properties of logarithms. As a result, she changed the question to respond to how students were thinking at the time (Gloria interview transcript, 2018), and modified the lesson to match her interpretation of student thinking. This demonstrates how her planning provided a foundation on which she could adapt questions based on student responses. This was not evident in her teaching when she did not have the opportunity to plan questions. She asked most of her "starter questions" in each section of the lesson plan, indicating that the lesson closely reflected the way that she had envisioned it (this close alignment between lesson plan and the delivered lesson makes one wonder whether the planned lesson guided the direction of the lesson more than the student thinking collected during the lesson).

In her reflection analysis, Gloria states that students shared their thinking when asked to break down explanations of their thinking. She also shared an example that demonstrated that she sometimes provided thinking for the students. “I replied that the order mattered less for the example we were currently working with, but I cautioned him to think about the polarity of the terms involved, prompting him by saying, “All of the terms here are… positive or negative?”” (Gloria analysis #3, 2018, p. 3). She later describes how *she* makes links for the students between types of rules within the lesson – the power rule, the product rule, and the quotient rule (Gloria analysis #3, 2018). While these statements provide examples of effectively sharing student thinking and ideas, Gloria indicates it was a challenge to have students communicate throughout the lesson, as she was concerned there was not enough discourse about analyzing logarithmic functions graphically. To facilitate student discourse, she hoped the lesson would provide structures to help each group share their work on the activity, partly due to their lack of experience communicating in the classroom.

Summary

In her interview at the end of the semester, Gloria refers to working to find her “sweet spot of questioning techniques” (Gloria interview transcript, 2018, line 53). She seeks a balance between the questions she plans and adjusting the questions while teaching. During the semester, finding this balance was challenging. During both the peer interview and the peer practice, there were very limited opportunities to use planned questioning. This represents one of the challenges in the peer rehearsals during this study. Gloria was not able to benefit fully from her peer practice due to the way that she set up the experience. In her extemporaneous questioning, while she often opens with a question to elicit student thinking, she usually funnels students toward her interpretation of their response. In the high school lesson, she used many of her planned questions and made adjustments based on the student thinking she collected. What is difficult to determine is how she uses a balance between planned questions, which are predominant in her high school lesson, and in-the-moment questions, which are exclusive in her peer practices. While her reflections indicate a desire to use student thinking to guide the instruction, it is unclear if the reasoning of the students was used as the focus of the high school lesson.

Beth: Develop Active Listening

Beth entered college as a music major (flute performance) and added mathematics as a second major with the desire to become a teacher (Beth interview transcript, 2018). With a double major, along with the requirements for teaching certification, Beth realized that she would need a fifth year of study to meet all the requirements. When she learned that she needed the same amount of time to complete the double major and earn a master's degree in teaching, she decided to pursue a master's degree upon finishing her degrees in music and mathematics.

Beth develops her collection and use of student reasoning over the course of the semester. She elicits student thinking during the peer interview but there are also times when she guides students to her view of the mathematics. She is patient with the development of her lesson plan, comfortable with its progress and the gaps where she needs further development as she creates drafts and practices with her peers. She is mindful to listen actively in order to elicit and interpret student thinking in her peer practice and high school lesson. She is consistent with the goals of her questioning in the lesson plan and how she teaches using those questioning structures. She is careful not to put too much of her own thinking into her questions and clearly wants to let students guide the thinking of the lesson.

Initial Peer Practice: Starting with a Blend

In her peer practice teaching interview (see Appendix A for the task that accompanies this interview), Beth offered both funneling and focusing questioning structures. She used her interpretation of student responses in different ways, sometimes probing the student's thinking, sometimes guiding the student to her interpretation. She varies in how she acts on what she hears from the student.

Her initial question asks the 'student' (peer participant), "I'm wondering why you decided not to connect these points [B], but you connected them in this graph [A], this graph [C], and this graph [D]" (Beth analysis #1, 2018, p. 1). Beth identifies this as an assessing question whose goal is to learn what aspects of scenario B led to a different representation compared to the other three scenarios (Beth analysis #1, 2018). She is asking an inquiry question to elicit the student's thinking regarding the difference between discrete and continuous real-world situations.

Her interpretation of this student's response is that the student understands how to break time into smaller segments, leaving it continuous while still having a constant rate of change. Beth reasons he does not have the same interpretation for breaking down weight, i.e. ounces into smaller units. Reflecting on the interview, she says she wished she had connected the student's thinking to an example of breaking time into smaller increments (Beth analysis #1, 2018). However, the task was what created the difference in the graphs, as the second scenario included fixed pricing for each ounce or part of an ounce (see Appendix A for the precise wording of the task). Beth's response to the student's thinking was based on whether weight is a continuous variable, not learning how the student made sense of a connection between the graph and the context of the scenario. Here Beth shifts to a funneling questioning structure, working to help the student understand both the task and how she is making sense of his work (the graph). She was asking the student to connect to her thinking rather than carefully listening to his reasoning.

For the second questioning sequence, Beth starts the sequence with a desire to elicit information on how the student understands the concept of continuity, as stated in her reflection. She asks an inquiry question of the student so he can compare his discontinuous graph from the first questioning sequence to a new graph about people renting a bus. While this scenario is discrete (the number of people renting the bus can be represented only in whole numbers), the student has created a continuous graph. Beth poses an inquiry question designed to learn more about the student's thinking. However, in her reflection analysis Beth explains she changed the direction of her inquiry based on how the student responded to her question. Instead of asking follow-up questions that would clarify the student's understanding of continuous variables, she asks about a different property of the student's graph, noting the arrow drawn at the end of his graph which indicates that the values on the graph will continue to decrease. She asks, "If this arrow is going down, what happens when your graph intercepts the x-axis?" (Beth analysis #1, 2018, p. 2). In her reflection, Beth explains she wanted the student to realize that the graph cannot cross the x-axis, as the scenario does not make sense for the rental cost of the bus to be a negative value. She is working to elicit student thinking and help the student deepen his understanding of how to represent this context. The student responds that as more people pay to rent the bus, the cost will go down to $0, commenting, "It will be so many people so you could just take over the bus company" (Beth analysis #1, 2018, p. 3). Beth is unsure how to respond and decides that the student has a misperception about the scenario and decides to move to a different questioning sequence.

Her next questioning sequence asks the student to interpret the x-intercept of another graph. The scenario involves the value of a car, for which the student indicates on his graph that zero and negative values are possible. In her analysis Beth says she was "trying to push the student to critically evaluate his graph and its reflection of the scenario" (Beth analysis #1, 2018, p. 3). She was trying to elicit student thinking and help him to see that some values in his graph did not correctly represent the scenario. She asks questions about the slope of a graph after seeing that the student is confusing a linear graph with a slope of ½ with the exponential function represented in this scenario, as the value of the car loses about ½ of its value each year. The student explains: "it says half of its value and I know half is one over two, so the rise is going down one and it's going over two" (Beth analysis #1, 2018, p. 4).

At this point Beth changes the goal of her questioning back to exploring the student's understanding of continuity, ignoring the previous response of the student. While there had been discussion about connected points on the graph, neither has mentioned the term "continuous." The teacher asks, "Are all of your graphs continuous or are some of them not continuous?" (Beth's analysis #1, 2018, p. 4). This is a point in the lesson where the teacher is funneling the student toward her interpretation of his graphs, bringing him back to the topic she wants to discuss. After the student response, which indicates that he is unclear about the meaning of "continuous," she defines the term "discontinuous." She asks, "Based on the different scenarios, how were you able to decide if your graph was gonna be continuous or discontinuous?" (Beth analysis #1, 2018, p. 4). Here she is clearly funneling him to the outcome she wants, pushing him to connect his thinking after he has demonstrated a lack of understanding of the concept of continuity. She is not listening carefully and not interpreting the student's thinking relative to the multiple graphs he has created. The student links the idea of fixed amounts to discontinuity, working to make sense of his graph and the discussions of the peer interview. At the end of the interview, Beth provides evaluative feedback that does not represent the level of student understanding of the concepts in the task. She ends the interview by saying, "That makes sense" (Beth analysis #1, p. 6), but gaps of understanding in the interview that makes this a questionable evaluation of the interview.

In her interview reflection, Beth states what she learned from the student and alternate questions she could have asked. Her reflection demonstrates she is trying to focus on student thinking, but some of the questions funnel the student toward what the teacher is seeing. Beth does point out times when she does not probe student thinking deeply enough to truly understand what the student is thinking. She identifies questions she would ask in a revised interview, some aligned to listening to the student's response and some funneling the student to see aspects of his work in the same way she does. During some of these question structures, she attempts to reengage a focusing structure by asking probing questions of the student. In those questions she uses terms such as "slope" and "continuous" without checking whether the student understands them or how they connect to his work.

Planning (Draft Lesson Plan): Still a Work-in-Progress

Beth acknowledges that parts of the lesson still need development. She includes ideas she has for the warm-up and anticipated student responses and indicates that these areas of the lesson need more development. She includes both assessing and advancing questions in the segment of her lesson where students are making sense of the task in small groups, organized by possible student strategies, plugging in possible solutions and logarithm power property for the first task (The Lost Solution – see Appendix J) and plugging in possible solutions, absolute value, and logarithm power property, exponential form, and square root for the second task (The Wrong Solution – see Appendix K).

Possible questioning sequences in small group discussions start with an inquiry or speculative question and then move to additional speculation or what/how questions, as Beth anticipates learning how the student is making sense of the task through careful listening to student thinking. Her questions focus on reasons for student thinking and making connections within activities in the lesson. For example, in the assessing questions for the absolute value strategy for The Wrong Solution task (see Appendix K), her planned questions are:

- Why did you put absolute value signs around that expression? I don't see any in the original problem.
- Which solution did your method lead you to? Did it lead you to both solutions? Then, why is one wrong and one correct?
- How would you explain your method to Kate so that she doesn't make this mistake again? (Beth lesson plan draft, 2018, pp. 6-7)

From these planned questions, Beth demonstrates ideas about how she predicts students will approach the task and how she will ask questions to elicit student thinking.

During the whole class discussion that follows small group explorations, Beth offers questions such as, "How is the logarithm power property used similarly or differently in these two tasks?" and "How can the errors you found and corrected help you when solving logarithmic equations in the future?" (Beth lesson plan draft, 2018, p. 9). These questions show that she anticipates helping students extend their thinking to solving similar problems in the future based on their current thinking. Thus, she anticipates different ways that students will solve the tasks and any errors or misconceptions they may make along the way.

While she cannot necessarily anticipate all student responses, she predicts that some students will not find any errors in the given student work (Beth lesson plan draft, 2018). In planning for multiple students' responses, she plans to support students who struggle to start the assignment, including asking students to plug in the provided values to see what happens. She wants students to identify any misconceptions in the provided work, deciding that she will scaffold questions to encourage students to make connections to their previous lessons. Such assessing questions will allow her to elicit student thinking, and by listening carefully to student thinking, she plans to have students present their thinking to her and other students (Beth lesson plan draft, 2018).

Peer Practice of Actual Lesson: A True Desire to Learn

In the peer practice, Beth identifies that her goal is to understand how learners will interact with the lesson. She knows that there are places in her draft lesson plan that she needs to develop further and wants to use the peer practice to help her develop those sections. She begins the peer practice by offering a background for the lesson to set the stage for her peers and provides a handout that shows what students have learned up to now. Her practice benefits from focusing on an important part of the lesson. Rather than starting at the beginning, she begins with the part she feels matters the most. She provides an overview for how the lesson will go and asks the group to provide feedback on areas for which she is interested, including her prepared questions. She wants feedback on her questions because she feels that sometimes her questions confuse students more than help them (Beth transcript of peer practice, 2018).

There is a mixture of questions asked during the practice, some directly from the lesson plan and some adapted to help develop clarity in student thinking. While these are similar to those developed previously, Beth does not read directly from a script. She asks, "So, when you plugged in, you got the solution, when you plugged in -2, you find that it did work?" (Beth transcript of peer practice, 2018, lines 93-94). This question echoes, "Why did you plug their solutions back in to the original equation?" (Beth lesson plan draft, 2018, p. 3). "Is there a way to find that one solution without finding both, I guess is the question?" (Beth transcript of peer practice, 2018, lines 221-222) ties back to her planned question, "Did it lead you to both solutions? (Beth lesson plan draft, 2018, p. 6). Beth understands her lesson well enough to ask questions similar to those she had planned, without having to rely on reading questions from her lesson plan.

She also wants to elicit more depth from students' reasoning, demonstrating that she is basing her actions on her interpretation of student thinking. Listening to how one group is trying to determine whether a student answer is correct, she asks, "Is there a way you can find out if her solution works in the equation?" (Beth transcript of peer practice, 2018, line 82). Similar questions suggest substituting values into equations, but none ask how to determine if a given solution would work in the equation. She adjusts her questions to address what she interprets of the students' thinking, based on what she has heard. Beth also asks questions that elicit clarity in student thinking. When a student discusses redoing the work represented in the task, she asks, "How did you redo it? What was different?" (Beth transcript of peer practice, 2018, line 159). This question provides a better understanding of student thinking, thus allowing for a better interpretation and therefore a better teacher response. There are multiple examples of this type of questioning during her peer practice, demonstrating her active listening.

During the peer practice, some students identified valid mathematical processes as inappropriate for this task, such as when one student remarked, "I don't think that you are allowed to cancel these 2's out, for some reason" (Beth transcript of peer practice, 2018, line

320). Here the student has correctly applied the division property of equality but is seeking ways to explain how to achieve answers they know to be correct. Beth explains that canceling both sides by 2 (dividing both sides of an equation by the same value) is mathematically appropriate in other instances, and lets the students ponder why it would not be appropriate in this case. She listens carefully and interprets the student as trying to use a mathematical property incorrectly. Rather than tell the student, she points out the inconsistency and walks away. Throughout the lesson Beth asks questions that allow students to make sense of their thinking, either individually or in small groups, and provides space for students to focus on responses, not necessarily the thinking of the teacher. These questions link to a focusing questioning structure.

During her interview, Beth shared that her goal during the practice was not so much about the structure of the lesson but rather to learn about how students might think about the tasks and methods they might use to make sense of them (Beth interview transcript, 2018). She carefully listened to student thinking to learn how others make sense of the learning goal of the lesson. Her questioning in the small group discussions demonstrates a desire for students to explain their thinking and interact during the questioning period. There were numerous examples of Beth asking an initial question and students asking and answering each other's questions during small group discussions before the teacher interjected again. She promoted student-to-student interactions that allowed the students to make sense of the task themselves. Her follow-up questions sought clarity or worked to extend the thinking of students in a small group environment. She did not attempt to funnel students to her own way of thinking or a particular answer but asked open-ended questions that allowed them to provide and discuss their way of thinking. This demonstrates a focusing questioning structure, actively listening in order to connect her questions to student responses.

Final Lesson Plan: Focusing on the Goal

In her final lesson plan Beth provides more detail about her thinking and completed elements of her lesson that were missing in the earlier draft. She takes what she learned from the peer practice and completes sections of the lesson plan that were still forming in the draft. While many of the questions remain the same as the draft, she does change some questions in the group discussion to reflect her better understanding of the goal of the lesson. Beth is aware that her questions need to match student responses collected during the lesson, and only so much planning will prepare her for that. She knows that she will need to adjust to what she hears from students.

She has learned more about the goal of her lesson from her course instructor which helped her create "a more clear and focused objective for the lesson" (Beth lesson plan change reasons, 2018, p. 1). She includes a warm-up that asks students to find an error in solving quadratic and rational equations and a homework assignment regarding logarithmic equations with two potential solutions, linking the beginning and end of the lesson to tasks experienced in the peer practice.

Beth created a monitoring tool with her planned questions during small group work, so that she would have a resource to use while teaching to the class. There were no changes in the questions she anticipated needing during the small group portion of the lesson. Again, she asked questions guided by a desire to clarify and deepen understanding of student work (anticipating the need for revision due to the extemporaneous nature of small group interactions). This was an interesting factor, as she stated in the interview at the end of the semester: "All I needed for the rehearsal was to get more, potential student thinking and methods" (Beth interview transcript, 2018, line 63). Beth explained that she did not change her questions because she wanted to focus on how students were making sense of the student work rather than the methods she anticipated students would use when solving it on their own (Beth interview transcript, 2018). It was almost as if she was worried that over-preparation would affect how she listened to student thinking. She provided evidence of her active listening in the way she responded to student thinking during the peer practice. Even though her goal for the peer practice was to see how learners approach the tasks, what she learned made her realize that did not change the questioning planned for the small group segment of the lesson.

Beth did make changes to the questions for the whole group discussion to better focus on the learning goal of the lesson (Beth lesson plan change reasons, 2018). While her original questions allowed students to extend their thinking, the questions in her final lesson plan reflect how she envisions the lesson proceeding based on her experience with the peer practice and conversations with her course instructor and mentor teacher:

> What is similar or different about what happened to Gil and Kate when solving their respective equations?
>
> What advice could help both of them in the future when solving logarithmic equations?
>
> What similarities and differences do you see between methods for solving the two different equations, and the mistakes that both Gil and Kate made?
>
> Both students ran into issues with finding the correct solutions because of how they applied the logarithm properties. What "rule" can you come up with to avoid losing a solution in the future?
>
> How does the domain of logarithmic functions impact when we apply the properties of logarithms? I.e. Why can't we always apply the properties when we have variable arguments in the logarithm? (Beth lesson plan final, 2018, pp. 3-4)

These questions demonstrate where Beth anticipates student challenge and discovery: that students will think about how they determined their answers and the impact of domain restrictions on applying properties. Revisions of the whole group questions indicate what Beth learned from the development of the lesson plan and practice with peers in narrowing down key elements of the learning goal and highlighting them at the end of the lesson.

Teaching the Lesson: Active Listening in Action

For her enacted lesson, Beth puts all of her thinking and learning into action. She asks questions to help elicit student thinking. She listens to the responses of students and asks follow-up questions that allow students to build on their own thinking.

In analysis #3 Beth described how she used questions during the lesson to give students an opportunity to provide reasons for their answers, as well as redirect them when their thinking was incorrect or incomplete.

> I re-directed them to examine other steps in the equation. I would ask, "It seems like he did apply the property correctly, but maybe look back at your property notes and think about, what do we have to know about what's inside the logarithm in order to apply the property?" (Beth analysis #3, p. 2)

Her questioning prompts students to examine the meaning of rules and procedures involving logarithms. She shares that she "pushed them [students] to consider other possible errors by saying, "Do you think she ever saw anything being squared?"" (Beth analysis #3, 2018, p. 2). She also asked students to revisit answers where she felt they had not considered the situation deeply or needed to provide richer explanations, asking them to keep thinking about their reasoning and giving them ideas when they needed them. She comments,

> Some of the most interesting mathematical conversations in the lesson came from students' own questions. One student asked, "If the solution is extraneous, is it still a correct answer?" This question was an indicator that this student was thinking deeply about mathematics and working hard to solve the problem at hand. I built on this by asking, "Is it really a solution to the equation?" to which he replied no, since when you plug it in it does not solve the equation. (Analysis #3, p. 3)

Here Beth demonstrated the desire and ability to actively listen in order to understand and interpret student thinking. This is a key component in a focused questioning strategy. She also described how discussion and questioning led students to a better understanding of the rules and properties of logarithms, as well as procedural fluency.

Summary

Overall, Beth strategically used opportunities in the peer practice and feedback from both her course instructor and mentor teacher to improve her understanding of eliciting, interpreting, and responding to student thinking. Over the course of the semester, she showed how the role of purposeful questioning increased in her planning and teaching, and how that paralleled her use of student thinking during instruction. She used more focusing questioning than funneling questioning in all but her initial experience, and her belief about the role of planning and lesson development allowed her to learn to use questioning to learn about how students made sense of her tasks. She effectively used the plan-practice-teach cycle to benefit the learning of students. In her interview at the end of the semester, Beth explained that both the practice and the actual lesson represented a need to be flexible and adjust her questioning (Beth interview transcript, 2018), indicating how active listening supported student understanding during instruction.

Throughout the process of planning, practicing, and teaching, she showed the importance of deepening her understanding of student thinking to achieve the goal of a lesson.

Clara: Step In to Help Students Out

Clara grew up wanting to be a teacher. During her end-of-semester interview, she told of the importance of learning in her family and the unstated expectation of studying a STEM field, as all of the members in her family are involved in them. In high school she identified math as "more of a fallback" subject (Clara interview transcript, 2018, line 20). Clara pursued a master's degree in teaching because she wanted to learn more about teaching before she was in charge of her own classroom (Clara interview transcript, 2018).

Clara appreciates the importance of learning about what students are thinking, and the development of her lesson provides insight into how she tries to elicit, interpret, and respond to student thinking. Over the course of the semester, she provides more support and guidance for students to help them understand what she wants them to know. Her challenge is to learn what students are thinking without revealing her own interpretation of that thinking. While she wants to have student thinking guide the lesson, she struggles with how to help them when they are unsure, which often leads to her providing an answer.

Initial Peer Practice: Disconnected Interpretations

In her peer interview, Clara asked questions intended to have the student share her thinking about the choice of graph, but the student interprets these questions differently from Clara's intention. From her reflection, Clara is seeking broad perspectives on how the student is making sense of the situations, but the student provides more specific responses and Clara is unsure how to respond. The two participants in the discussion ae not able to connect their different views on the task before the end of the interview.

Clara opened the peer interview graph activity by asking, “How did you come up with the form you wanted to put this graph in?” (Clara analysis #1, 2018, p. 1). The goal of this question was to determine if a student could distinguish between forms of graphs and explain their reasoning for a particular form. The “student” responded by identifying one point on her graph and providing an explanation for its context (the second graph on the assignment, see Appendix A), “For zero letters, I have zero ounces. For one letter I have a bit more ounces and for each letter I increase by the same amount of ounces…or it is cost?” (Clara analysis #1, 2018, p. 1). Clara interprets this student’s thinking as reasoning behind why she graphed distinct points rather than a continuous function. She interprets the student response as the graph should start at (0, 0) and that the cost should increase for each ounce. Her interpretation assumes certain elements of the student’s understanding. That said, the student’s response does not identify the form of the graph, does not clearly identify an understanding of discrete versus continuous, nor why the graph should start at (0, 0). Clara provides possible alternate questions in her reflection of, “What is the graph you made called? Why did you pick this form? What does this graph tell me about the situation?” (Clara analysis #1, 2018, p. 1) which Clara identifies as guiding the student her responses (It is not clear what responses Clara is looking for the student to provide.) nor how these alternate initial questions will obtain those student responses.

Next, she rereads the question, explaining that there was a different rate for one ounce compared to each ounce thereafter. She indirectly acknowledges the student’s response by asking, “So are yours going up in an equal amount each ounce?” (Clara analysis #1, 2018, p. 1). Here, Clara leads the student to reason the way she wants. The student explains why there are equal increases across the graph, “With each additional ounce you pay the same amount of money” (Clara analysis #1, 2018, p. 1). Clara repeats the student’s answer to confirm her thinking, the student agrees, and the teacher replies, “Awesome” (Clara analysis #1, 2018, p. 2). In this way Clara funnels the student toward the answer she desires; and when the student fails to see the same thing the teacher does, Clara is unsure how to support the student.

Asking about the third graph on the assignment, Clara begins with a question from the previous example: “How did you come up with this form of the graph?” (Clara analysis #1, 2018, p. 3), even though she was not satisfied with the student’s previous response to this question. Again, the student explains how she represents the problem on her graph. Clara’s interpretation shows she believes the student understood that increasing the number of students decreases the cost per student but did not know which operation she should link to her answer. Clara expects the student to provide a response to an unasked question, that of the operation involved in the problem. Clara feels she is allowing the student to explain her graph but the student does not possess the vocabulary to describe her thinking (Clara analysis #1, 2018). Clara is looking for particular responses from the student and not adjusting her questioning to learn about the construction of the graph.

She next asks if the line should be continuous. The student responds it should, “because you have to connect the dots and so I drew a bunch of dots and then I had to connect them, because that is what you do” (Clara analysis #1, 2018, p. 3). The teacher asks if it is possible to have a cost based on half of a person, and the student replies you cannot have such a value, but you have to connect all the dots. This sequence is another example of Clara interpreting student thinking, using an initial question to help the student recognize a misconception, but then not being sure how to support the student beyond that. Though she attempts to use funneling question structures to get the student to identify differences between her understanding and that of the teacher, her attempts here are unsuccessful.

Planning (Draft Lesson Plan): Variance in Expectation

In her draft lesson plan, Clara varies in the types of questioning structures she anticipates needing and the depth of the vision she has for different sections of the lesson. Some of her questioning provides opportunity for students to extend their thinking and some guides students toward her thinking of the solutions. She has provided some possible student responses in some places of the plan, but others lack a clear view of what she expects to occur. As this is a draft of her lesson plan, it is appropriate that some elements are more developed than others.

Clara's lesson plan extends over two consecutive days. The goal is for students to "use knowledge of exponential and logarithm rules to solve logarithmic expressions" (Clara lesson plan 1 draft, 2018). In the opening activity, students make sense of the work of two hypothetical students which include typical misconceptions that students have with logarithms. The first problem misrepresents of the meaning of negative exponents $\left(log_{\frac{1}{2}}\left(\frac{1}{4}\right)=-2\right)$ and the second misrepresents square roots $\left(log_{25}\left(\frac{1}{5}\right)=\frac{1}{2}\right)$ (Clara's lesson plan 1 PowerPoint, 2018). These problems provide an opportunity for students to demonstrate their understanding of logarithmic functions leading into the lesson.

The questions Clara creates in anticipation of the opening activity include multiple directions for student thinking to go. Possible questions and student responses for the first problem include different ways to read a logarithm, follow-up questions about whether the student work is correct or not, and questions about the meaning of a negative exponent. There is a mixture of speculative questions that ask for student reasoning and factual/procedural questions to determine if the students can read and re-express logarithmic equations (Clara lesson plan 1 draft, 2018). Overall, the questions should determine students' understanding of negative exponents. The teacher appears to funnel students toward this result, though the implementation of this problem may differ based on student responses. The second problem includes a fractional exponent which should result in a square root that is a whole number, while a misconception would maintain that a fractional exponent provides a fractional result. The structure of the questions is similar to the first problem, including whether the questioning should be open-ended or closed, depending upon how the teacher uses the planned questions.

The major activity of the first day has students encountering examples of exponential rules and creating conjectures for those rules and participate in a gallery walk to share their groups' thinking with the class (Class lesson plan 1 draft, 2018). Questions for this activity focus on procedures more than concepts ("What were your steps when trying to figure out [the] rule?" "If I let you use a calculator, I want you to check your answer and see if you get the same") as well as "what"/"how" questions to ask students to provide their reasoning ("What is the relationships between exponents and logarithms?" "Does your rule make sense for any number?" "Are there any numbers that I cannot take the log of?") (Clara lesson plan 1 draft, pp. 4-5). Such questions lead to either focusing or funneling questioning, depending on how responsive the teacher is to student thinking and their interpretation of that thinking. While Clara states that she wants students to use their own mathematical reasoning for how and why solutions of problems work, and to be able to create rules on their own (Clara lesson plan draft commentary, 2018), there is little explanation for how students will make sense of the examples that will lead to development of the rules. Her peer interview experience demonstrates that she struggles with helping students develop new understandings, but her planning does not provide options that will help her. Some of the examples provide structures that students can use to make sense of the task (determining a solution to $log_2(2 \cdot 4)$ that may lead to the solution to $log_2 8$ and the connection between those solutions), but others seem to go beyond the ability of students to make a connection to the rules they are intended to create (such as how to use $log_8(7 \cdot 10)$ to create a rules about multiplying logarithms) (Clara lesson plan 1 draft PowerPoint, 2018).

After going over the homework problems, Clara opens the second day with an activity where students make sense of logarithmic equations that involve variable quantities, the first use of variables in this particular lesson. Questioning planned for this activity focuses on the procedural steps needed to complete the problems ("Now can you explain the steps you took to figure out your answer?" "How did you pick the order in which you solved the problem?" "Did you plug your value back in and check your answer? Did it work?") and one that probes student understanding ("Is there more than one answer? Why or why not?") (Clara lesson plan 2 draft, 2018, pp. 2-3).

The main activity for this second part of the lesson explores a mathematical contradiction (starting with the inequality $\frac{1}{4} > \frac{1}{8}$ and using properties of logarithms to get to 2 > 3) (see Appendix L). Students work individually and then in small groups to make sense of the process provided. Clara prepares the following possible questions:

- How did you approach this problem?
- What can you do to try to explain why this happened?
- What part of your work/steps did we not do correctly? Why are they incorrect?
- How does using absolute value signs reflect the properties of the logarithmic function?
- How can you check if your solutions are correct? (Clara lesson plan 2 draft, 2018, p. 4)

These questions support student speculation as well as funnel them toward considering absolute value properties. Clara does not anticipate student responses in this part of her draft lesson plan, so it is not possible to determine if her planned questions are responsive to student thinking or intended to guide students to her way of making sense of the activity. Her plan during the whole group explanation of the activity includes generic summation questions which apply to many tasks:

- What were some ways people approached this problem? What was your explanation for the problem?
- What can we use from our previous knowledge to make sense of this problem?
- How can we use what we just did moving forward? (Clara lesson plan 2 draft, 2018, pp. 4-5)

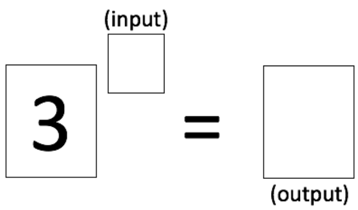
These questions do not reflect how she envisions the discussion to proceed, and she does not include any anticipation of student responses for these questions. In her commentary about her lesson plan draft, Clara identifies that, "During the lesson students will learn logarithm rules which they will then use on day 2 to figure out why a certain task/activity works" (Clara lesson plan draft commentary, 2018, p. 2). However, the activity on Day 2 relies on an understanding of the properties of logarithms not included in the Day 1 lesson, so the connection between the two lessons is more superficial than described.

Peer Practice of Actual Lesson: Allowing A Chance for Exploration

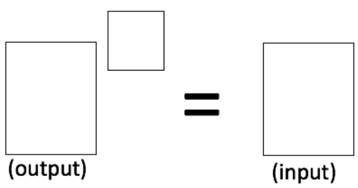
In the peer practice, Clara provides two different activities from her lesson that allow students to explore mathematical relationship. The first activity explores the connection between exponential and logarithmic expressions and the second helps students conjecture rules involving logarithms. While both of these activities begin with student exploration, one ends without closure for the students and the second ends with the teacher guiding the students to a desired result.

Clara begins the peer practice by stating that she is seeking ways to make learning rules more interesting to her students (Clara transcript of peer practice, 2018). She offers background on what students have done and gives her peers an opening activity she needs help on. Like Beth, she begins the practice with a part of the lesson she wants to learn more about. This activity is not part of the draft lesson plan but designed to help students determine the relationship between exponential and logarithmic functions. She provides the following:

When looking at an exponential function,



When looking at the inverse of an exponential function,

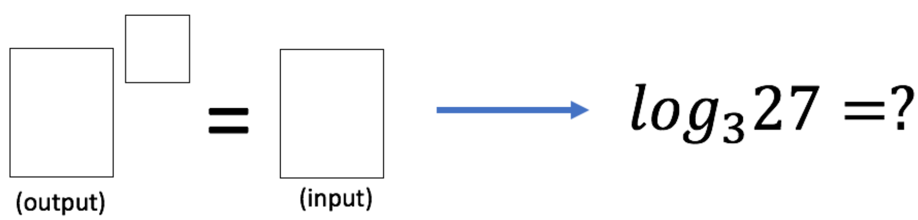


(Clara practice lesson plan feedback, 2018, p. 4)

She allows time for "students" (peers in the roles of students) to work on the activity while she circulates, asking what the students are thinking, i.e., "Is there a reason you all did that?" and "Where did you get that from?" (Clara transcript of peer practice, 2018, lines 48 and 52). She asks students to explain where they are putting key values (often concerning the placement of 1/3 or negative 3). She suggests such strategies as checking to see if the answer works and if students have worked the problems out completely (Clara transcript of peer practice, 2018). In her reflection on this section, Clara explains she "tries to push students to think about what would they be undoing in this case" (Clara analysis #2, 2018, p. 2). She is trying to balance helping the students look at their own thinking and funneling them to understand the role of inverse functions.

After combined individual and small group work time, Clara calls the class to order. She asks about the meaning of the given information, “How did you interpret this statement? What does it mean for me to find an inverse?” (Clara transcript of peer practice, 2018, lines 161-162). She asks students to explain their thinking to the whole group (“Can you explain to the group why you thought negative 3 was originally the exponent?” “Can you lead us through what the three options are?”) (Clara transcript of peer practice, 2018, lines 186-187 and 193-194). After allowing students to share their ideas, she moves on to the next part of the exploration,

When looking at the inverse of an exponential function,



(Clara practice lesson plan feedback, 2018, p. 4)

Students are asked to work in groups to fill in the missing information, based on what they have done in the first part of the activity (Clara transcript of peer practice, 2018). Clara circulates and asks questions to monitor their understanding of logarithmic functions. When she asks students what l-o-g stands for, most state they do not know. Clara tells students to continue exploring but does not pose any questions that help students make the desired connections. She calls the class back together and explains she wants them to gain an understanding of natural logarithms and inverses, and has the groups discuss how to implement the activity (Clara transcript of peer practice, 2018). There is no further discussion about potential connections between exponential and logarithmic expressions.

After a discussion about teaching of this part of the lesson, Clara moves on to how to create rules from examples, part of her draft lesson plan for Day 1. The first part asks students to make sense of the product rule. Once again, Clara circulates among the groups, asking questions that encourage students to continue their explorations and explain why they are doing what they are doing. Examples include, “Now what’s the next step that you want to do?” “How’d you get that?” “What would be an easier way to figure out what that is?” (Clara transcript of peer practice, 2018, lines 423, 430, and 438). She often opens a questioning sequence with a speculative question that leads to guiding or procedural questioning such as:

Teacher: Is there any way for us to know what those unknown values are?

Student 1: Maybe, like I know that 7^2 is 49, I know 7^3 is – I don’t know but I can figure it out with a calculator. And then from there I can’t take the square root of 7, that’s where I’m end up getting stuck in.

Teacher: Ok. So if I look at this rule (*points to product rule*), is there some way I can split it up, so I can at least try to make sense of something helpful?

Student 1: Yeah. So I need to do something, I mean…

Teacher: Your first steps here were to multiply 2 and 8. So if I look at this next one, what’s 8 times 9.

Student 8: 72.

Teacher: And that’s what … *garbled* … so how is there a way to make that smaller?

(Clara transcript of peer practice, 2018, lines 485-501)

Evident in this sequence is Clara’s role in working out how to create a rule. Instead of waiting for the student to identify previous work and connect it to the example, Clara steps in and identifies what the student did and how that should connect to the example. She presents her thinking and funnels the student toward analyzing the problem in the same way that she does.

At this point, the peer practice lesson ends and the class provides feedback to Clara.

Final Lesson Plan: Expectations Remain Unclear

In the final lesson plan, Clara adjusts the lesson based on learning more about students background in mathematics and what they should have already learned. However, those changes occur only in the opening section; there are not corresponding changes to the latter parts of the lesson. In addition, the elements of the lesson plan that were unclear in the draft have not been clarified in the final version of the plan.

Clara cuts the opening activity she used in her peer practice once she realized the students had prior experience with connections between exponential and logarithmic functions. In designing a new entry point, she shifts to having students examine common misconceptions about evaluating logarithms. Students must consider the possibility of negative exponents, negative outcomes, and fractional exponents and their impact on a logarithmic function. These are places where students commonly make errors; by setting up the warm-up as an evaluation of another student's work, the class has a chance to make sense of their reasoning–first to see if the work is correct; then to determine if there are any errors.

However, she makes no changes to the questions or anticipated student responses and teacher actions in the final lesson plan. One challenge is how different the warm-up activity is from the one delivered during the peer practice. Also, the key investigation of the final lesson plan was not part of the peer practice. There is no added consideration for how students will respond to the questioning during the investigative activity (which was new to the lesson). The revised lesson plan does not reflect how Clara is using her experiences developed from the peer practice.

Teaching the Lesson: Moving Away from Exploration

Throughout her development from draft lesson plan to implementation, Clara narrows the scope of the exploration by students. She initiated an open exploration of the relationship between exponential and logarithmic expressions in the peer practice but eliminated it when aware that this would not be new for the students. The replacement activity allows for students to investigate common errors, but she adjusts the questioning within her expectation of the lesson to narrow student opportunity to guide the direction of the lesson with their thinking. Overall, the role of student thinking from the draft lesson plan to the enactment of the lesson has declined and more thinking is provided by the teacher.

As she prepares to teach the lesson, Clara makes adjustments based on the time available. Instead of having two days to teach the lesson, there was a snow day and Clara has only one class period for the lesson (Class lesson plan annotations commentary, 2018). The taught lesson includes the warm-ups from the original two days of lessons and the investigative activity where the students need to find an error in the steps of a mathematical solution (see Appendix L)).

She creates a note sheet version of the lesson, using a hard copy of the PowerPoint on which she writes the questions she plans to ask, alternate questions she considers during the lesson based on student responses, and notes taken during instructions to help her later reflections (Clara interview transcript, 2018). Some of these questions match her final lesson plan, she adds some and omits others. For the warm-up problem where students identify an error, she continues to ask them to compare their work with the given student's, but asks them also to interpret the basic meaning of the task ("What does the negative 2 rep [represent]? What does ½ rep [represent]?") (Clara lesson plan annotations, 2018, p. 2).

In the second warm-up problem, her questions are another mix of new questions and those taken from the lesson plan. Of the five questions on her note sheet, two are from the lesson plan and three are new. The new ones ("Why did you do the reverse operations?" "What does the x = rep [represent]? Where do I plug it in to check my work?" (Clara lesson plan annotations, 2018, p. 3) concentrate on guiding students to what they need to obtain the correct answer.

Moving to the task where students must identify an error in the problem that ends with 2 > 3, Clara's notes vary slightly from her final lesson plan. In the lesson plan, she asks:

- How did you approach this problem?
- What can you do to try to explain why this happened?
- What part of our work/steps did we not do correctly?
- Why are they incorrect?
- How does using absolute value signs reflect the properties of the logarithmic function?
- How can you check if your solutions are correct?
- What were some ways people approached this problem?
    - What was your explanation for the problem?
- What can we use from our previous knowledge to make sense of this problem?
- How can we use what we just did moving forward? (Clara lesson plan final, 2018, pp. 5-6)

Speculative questions like these allow students to see different ways of being correct, as they are based in reasoning and elicit student thinking for the teacher to interpret. On her annotation sheet, Clara has listed

- What did they do incorrectly? Why?
- Maybe try to plug in a # [number] for a and see what happens. Can we generalize it now?
- How did we use our previous knowledge? What does this tell you moving forward?

(Clara lesson plan annotations, 2018, p. 4)

These questions are specific to the task and more teacher-directed than those in the lesson plan. Instead of letting students determine the strategy, Clara is providing the strategy and the students are justifying it. Here, Clara creates a more open-ended presentation for the lesson plan and a funneling question approach in her immediate preparation and implementation of the lesson. It appears she has less confidence that students will make sense of the task on their own. In the interview at the end of the semester, Clara states there was little conversation in class on this part of the lesson, since students had not considered inequalities recently and were having trouble with the base of the logarithm being a variable (Clara interview transcript, 2018). As more and more scaffolding is provided to the lesson, it is difficult to know the trouble stems from a lack of student understanding or what is expected of them.

She adds that her students struggled with parts of the lesson where they had to find errors in other's work. In the first part of the warm-up, "they thought that this statement was true but based on their knowledge did not make that much sense" (Class lesson plan annotations commentary, 2018, p. 1). During the lesson, she rewrote the problem to include a variable, changing $log_{\frac{1}{2}}\left(-\frac{1}{4}\right) = -2$ to $log_{\frac{1}{2}}x = -2$ and asking students to determine the answer of

$x = -\frac{1}{4}$ (Clara lesson plan annotations, 2018). Her annotations show her thinking to review exponential rules before to the warm-up question and her reflection states a need to rewrite the problems where students struggle. She wants to help students before they have a chance to struggle.

To Clara, the investigative problem needed an easier entry point, one that would allow students to start with a specific value for the base of the logarithm (giving it a value of 2 or 3 instead of having an unknown "a"). She states, "A lot of my students still struggle with thinking abstractly and therefore had trouble finding a starting place in the activity" (Class lesson plan annotations commentary, 2018, p. 1). This presents two challenges for the lesson. First, by eliminating the abstract level of the problem, the challenge of the problem changed, as well as its solution. The abstraction is what leads to the contradiction in the first place. Second, by providing scaffolding for all students, Clara would eliminate the challenge for students who could adapt to the situation. Rather, some students should be able to work on a higher-order-thinking-skill task while others get the support they need when provided access to the problem.

In her reflection on the lesson, Clara identifies ways to improve her questioning. In one part of her lesson, she comments that her questioning took the student through a sequence of steps to solve a problem. While this supported development of procedural fluency, there was no extension question connecting the procedure to the reasoning behind the steps (Clara analysis #3, 2018). During a second part of the lesson, Clara saw herself doing most of the thinking, "Rather than having the students lead the discussion, the teacher candidate seems to rush them to the conclusion" (Clara analysis #3, 2018, p. 4). Clara identifies the importance of using student thinking to guide the questioning in the lesson. In addition, Clara's expectations of student thinking helping to guide the lesson changed, especially when she realized that the questions in the lesson plan were more speculative and inquiry based than those she ended up asking.

Summary

In her course reflection at the end of the semester, Clara stated, "it is important to plan for misconceptions, questions, and potential struggles" (Clara final reflection, 2018, p. 2). This seems to contradict some of the revisions made and challenges identified in her lesson planning, peer practice, and teaching experience. Her statements appear to demonstrate that she may believe that she is providing opportunity for students to struggle but is not clear on what that looks like during instruction. While she planned for misconceptions, she uses the plan-practice-teach cycle to make revisions to decrease the role of student thinking and instead provide additional review material, limiting student interaction with their misconceptions. She refers to the importance of planning for any struggles students may encounter in a lesson, but it was not clear that she could plan for the supports students needed when struggling. Her initial peer interview, the peer practice, and the instructed lessons reveal examples of student struggle that went unattended. She was more likely to step in and funnel students to the correct solution than to allow students to work through their thinking. Throughout the semester, she decreases the expectations of the students to provide thinking to guide the lesson. At the end of her reflection, she admitted that she sometimes had to stop herself from guiding students through a task and continues to "struggle with giving students the opportunity to productively struggle" (Clara final reflection, 2018, p. 5). Clara identifies areas she can improve and shows that she is interested in making changes in her teaching that will help elicit, interpret, and respond to student thinking.

Michael: Here's What to Do

Michael decided relatively late that he wanted to be a teacher. After experiencing tutoring in college, he took an introductory teaching class midway through his junior year and enjoyed the experience. He chose to finish his undergraduate degree in mathematics before entering the graduate program for teacher certification (Michael interview transcript, 2018).

Michael is very confident in his positioning himself as a teacher and in his teaching abilities, stating in this end-of-course reflection that "I am surer than ever before that teaching is the right career for me" (Michael course reflection, 2018, p. 5). In his lesson planning and instruction, he works hard to provide everything that he thinks students need to be successful. While it is unclear what this means from his lesson plans, his teaching demonstrates how he views instruction as a teacher-centered activity. He states that student thinking is an important part of the learning process and he appreciates being able to help students solve problems, but his lessons often provide thinking to the students rather than collect it from the students.

Initial Peer Practice: Using an IRE Approach

In the initial peer practice, Michael demonstrates clear use of the IRE questioning approach with the teacher initiating discourse with a question, obtaining a response from one student, and providing evaluative feedback to the student and moving on to the next question (see earlier discussion on types of questioning structures). There is no probing of student thinking and teacher actions indicate that he understands the student's response without probing student thinking to ensure that this is true.

In his assignment Michael's goal was to discover why the student created their selected graphs (see Appendix A) that they did (Michael analysis #1, 2018; Michael interview transcript, 2018). He starts with the question, "Why did you decide to do this curvy shape?" (Michael analysis #1, 2018, p. 1). Rather than asking the student to describe her thinking about the graph and allow her to identify what characteristics of the graph she values, Michael limits the student response to a specific area of interest. The problem states, "Each hour a candle burns down the same amount." The student responds that in her experience, candles burn more quickly at the beginning and more slowly at the end. This is in direct contradiction to what the problem in the task presents (see Appendix A). Instead of asking the student to explain the difference between the problem and her interpretation, Michael provides positive feedback: "Okay, that makes sense. You're using like a real-world approach" (Michael analysis #1, 2018, p. 1). In his reflection, he admits that the student missed the point that the candle should burn the same amount each hour, but it is unclear how this exchange might look during instruction if he were to alter his questioning.

Discussions between the teacher and the peer acting as student for the second and fourth graphs occurred in a similar manner to the first, with Michael asking a question, the student responding, and the teacher evaluating the student's thinking, the IRE questioning structure. Michael reflected that his alternate questioning would be to ask for more detail about the graph (such as labels on the axes) or examples of specific values that the student considered appropriate. The student drew one graph correctly and one graph incorrectly. In the incorrect graph, the situation modeled an exponential function showing the value of a car decreasing by one-half each year at a non-constant rate of change; the student created a linear graph with the slope of negative one-half. Michael did not probe the student's thinking about the incorrect graph or ask questions that would help the student correct her misconceptions.

The only change in the questioning structure occurred during the third graph. In addition to a general question (why the third graph is the only one with disconnected dots), Michael follows up with another question about the student-created graph. The student responds by explaining about discrete situations and the need to include integers (as the student notes, you cannot have one and a half people). Michael does not interpret this thinking from the student. He next asks about a second characteristic of the student graph: “It looks like here at the end it kind of starts to taper off. Why is that?” (Michael analysis #1, 2018, p. 1). The student responds and the teacher states he follows that thinking and moves on to the next graph.

While Michael expresses a desire to understand why the student developed specific graphs, in the interview at the end of the semester he states his questions did help him meet the goal of understanding the student’s graphs (Michael interview transcript, 2018). Still, he asks only one follow-up question about the four graphs the student created, and this question is not in response to student thinking. Even his reflection on the peer interview which includes possible alternative pathways the interview could have taken, he does not identify a need to probe student thinking more deeply or ask questions that would help the student identify misconceptions. His alternatives are asking the student to consider more detailed versions of her graphs or ask about the situation and then have the student redraw her graphs, in the hopes that she will discover her errors. He also focuses on the graphs rather than the learning goal of the activity. The primary goal of the assignment was to “Think about the shape of the graph and whether it should be a continuous line or not” (see Appendix A). While a few of his questions mention the shape of the graph, none probe deeply as to how and why the student chose to create the graphs in the way she has. Only one question directly addresses continuity, and it is unclear how the teacher interprets the student’s thinking since his actions are not a result of student responses.

Planning (Draft Lesson Plan): Follow the Teacher's Steps

Michael's draft lesson plan is a step-by-step process of how to use the problems on the worksheet to help students learn how to compute iterated and double integrals (Michael draft lesson plan, 2018). There is an emphasis on the teaching of the teacher and how students will need to follow his thinking. There are multiple places where he refers to his actions, such as:

- "I will briefly explain why" (Michael draft lesson plan, 2018, p. 1).
- "I will walk them through how the bounds were made" (p. 3).
- "If a student questions whether this holds true for *all* functions, I will tell them that that is a good question, and one that will be addressed in a future lesson" (p. 3, emphasis in the original).

There are other places in the lesson plan where the teacher asks questions of students, but in the draft, these are identified only as places where questions will be asked. For example:

> I will ask students what the integral of $\frac{1}{\sqrt{1-x^2}}$ is. Someone will likely answer this right away, and I will write the answer on the board. Doing this should help prepare the class for integrals that involve inverse trigonometric functions (arctrig).
>
> (Michael lesson plan draft, 2018, p. 2)

Instead of generating questions, possible student responses, and potential teacher actions, Michael shows where he feels students may struggle and the general topic of the questions he will ask. This is not a complete preparation of the lesson plan, in part because of not having completed the problems prior to the peer practice, which he shared during the practice (Michael transcript of peer practice, 2018). His questioning preparation only gives the topic for questions and what the student should do correctly, based on their answer to the question. There are no examples of alternative solutions, incorrect responses, or possible sequences of questions to support struggling students. Since the lesson plan provides only short exchanges of teacher-guided discussions, there is little evidence to show how Michael will support student perseverance, even though it is one of the Standards for Mathematical Practice identified as a goal for this lesson (Michael lesson plan draft, 2018).

The main part of the lesson is a worksheet that students complete while working in small groups (see Appendix M for the final version of the worksheet). The lesson plan outlines the anticipated student successes organized by question. The teacher responds to students with either single question-answer examples (as above) or teacher-guided actions that guide the student on how to correct an error or misconception. After 40 minutes of working collaboratively, the lesson plan calls for the teacher to bring the class together for a brief, 5-minute discussion. Selected students present their work on two of the problems from the worksheet, identified as important topics. The lesson plan and accompanying commentary identify misconceptions and errors that Michael anticipates students may have during the lesson. Many of the instructional strategies designed to address these needs are teacher-guided, as seen above. Once the teacher has elicited and interpreted student thinking, his responses provide the directions that students are encouraged to follow.

Peer Practice of Actual Lesson: The Teacher Knows Best

For Michael, the peer practice becomes a guided teaching experience as many of his peers struggle with the content of the lesson. The level of mathematics in the lesson involves higher level mathematics and his peers are less familiar with the content than he expects of his students. Michael's response to their needs is to provide directions and immediate feedback to move them along in the lesson. The role of the teacher in the peer practice aligns with the description of the teacher from the draft lesson plan.

In the peer practice lesson, Michael sets the stage by giving his peers a brief explanation of what students will know coming into the lesson (what a double integral is and how it can represent the area of a region) and that the lesson will include giving the class a definition that connects double integrals and iterated integration. He distributes the worksheet to the "students" (his peers from the class). Michael circulates around the room as they work, offering hints at the individual, small group, and whole group level. For example, when he notices that many groups are struggling, he asks, "Does anyone remember, if you have a function, like $f(x, y)$, and we're taking a partial derivative of that function with respect to x, then what do we do with all of the y terms?" (Michael transcript of peer practice, 2018, lines 64-66). A student responds that they are considered constants in the function, and Michael says "Yes, with double integration, it's a lot the same way. Whatever your inner *dy* or *dx* is, you are integrating as if all the other variables are kept constant" (Michael transcript of peer practice, 2018, lines 70-71). The questioning structure is again teacher-initiate, student respond, teacher respond with information that connects the students' struggles with support the teacher wants to provide. This is consistent with how he approaches questioning in the draft lesson plan. While the student provides the answer to the original question, there is little interpretation offered by the teacher (since it was a response to a procedural question) and no opportunity for student thinking, as the teacher provides the connection.

The next nine minutes involve students working in small groups and the teacher monitoring progress and answering questions as they arise. He uses the e^{2x} pattern question he identifies in his lesson plan, and the student who asked for help is able to determine how the pattern may solve the problem. Asked if the solutions are correct, Michael provides direct feedback (yes or no) without asking further questions about why students believe their answer is correct. He offers hints for how students can progress which follow how his lesson plan is structured. Instead of asking questions that would allow a student to consider what to do, he provides guidance, just in case students might need it. A number of hints include statements for what to do next or what to be cautious of, as the students work. There are also elements of encouraging students to support each other as they work. When he is working with a pair of students at different levels of understanding, he asks one student to check with his partner with future questions. In this way, he is encouraging students to see each other as a resource to solve problems (Michael transcript of peer practice, 2018). Still, he more often offers support when students ask him, not allowing them to turn to their peers as a resource or ask a student for some guidance without telling them what to do.

When one student asks about one of the problems on the worksheet, Michael brings the group together to discuss how to identify boundaries. This is one of the problems he anticipated students would struggle with, so he decides to present this to the whole class and tell them how to solve it. This is how he described what he will do in his draft lesson plan. Afterwards, he asks the student who raised the problem question if she sees her mistake (Michael transcript of peer practice, 2018). Instead of allowing students to determine the boundary, he solves the problem for the class and asks if they understand.

The remainder of the lesson follows the same structure as earlier, with Michael circulating while students are working. There is a mix of questions and statements, with most of the information coming from the teacher. Sometimes Michael comments on what he observes on how to solve the problem., pausing while a student is working to ask about their work:

Teacher: How did you get $6x$?

Student5: Because you get $2xy$, oh wait, it shouldn't be 6, it should be $9x$, right?

Teacher: Yeah, yeah, that's it. *Teacher continues to observe student working.*

Student5: So, you plug in the 2?

Teacher: So, you don't just plug it in, you actually have to integrate that now, and you're doing it with respect to *dx*. Nice move, I'm glad you included the *dx*.

(Michael transcript of peer practice, 2018, lines 494-504)

The questioning structure follows the same pattern throughout the lesson, with the teacher asking a question about the student's work, the student responding, and the teacher evaluating the response. Often these responses embed what the teacher is thinking, which may be what the student is thinking, but it is difficult to tell without hearing more from the students. In his reflection on the lesson, Michael refers to the actions of the teacher, but rarely to the actions of the students (Michael analysis #2, 2018). It is a challenge to tell what he knows about student thinking based on the peer practice. In the interview at the end of the semester, Michael states that the peer practice was not of particular value, as the "students" had not solved similar problems in a while and the approaches they took were not consistent with the strategies that students in his class would use. While there were students who struggled during the peer practice, Michael felt he would not have any students like that in his high school classroom. In this peer practice experience, the content of the lesson became the learning goal for the peers, and the preservice teacher did not take advantage of this to deepen his understanding of possible questioning structures to privilege student thinking.

Final Lesson Plan: Provide Up-front Support

In terms of changes to the draft lesson plan, it was clear that the peer practice actually did have an impact on how Michael envisioned the lesson. The final lesson plan includes more direct support and guidance than the draft. The worksheet of problems provides more hints and scaffolding for students than the original, which was under development when the peer practice took place. Michael includes hints on the worksheet to support the places where he anticipates students may experience difficulty and gives students the answers to all problems so they will be able to check their own work (Michael lesson plan final, 2018). He also includes an additional problem as a warm-up before the worksheet to help set the students up for success, including a problem that can act as a model for the work they will be doing (Michael lesson plan revision summary, 2018). There are still very few actual "scripted" questions and student responses in the lesson plan which serve to summarizes Michael's expectations. With the inclusion of hints on the worksheet and an answer key, "students do not have to depend on me (the teacher) for a hint that could just as easily be included on the worksheet itself" (Michael lesson plan revision summary, 2018, p. 1). This is how Michael views his role in the classroom, which was evident during peer practice. As he circulated, guiding students based on how he interpreted their work and the solution of the problems, the hints support this view of instruction. Based on the challenge his peers experienced during the practice, Michael provides more support and structure for all students, whether they need it or not.

Teaching the Lesson: What is the Best Support?

Michael indicates that he adjusted the lesson during the multiple times he taught to better meet the needs of students. He discusses how he changed some parts of the lesson throughout the day to best meet the needs of his students and how the provided hints on the handouts had various impacts on his students.

Some students solved the opening challenge problem shortly after starting the worksheet activity (Michael lesson plan annotations, 2018). Since Michael assumed this problem would not be easy to solve, he amended the problem later to increase the challenge and forge a deeper link to future lessons.

His notes reflect his goal to ensure that all students showed all work correctly (Michael lesson plan annotations, 2018). He recommends additional problem review so that students would not have to struggle with each problem on the worksheet. Interestingly, he identified that some students did not like having hints on the worksheet and decided that in the future he would either move hints to a separate page or another part of the worksheet (Michael lesson plan annotations, 2018). In consideration of revisions to future implementations of the lesson, Michael determined how students could access hints and supports when they needed them, allowing them some challenges based on their incoming and developing understanding of the topic.

Some students experienced unexpected misconceptions, which Michael stated he would discuss when going over the worksheet with the class, but in his view not enough of the students erred for him to add a hint on the worksheet. The hints offered varied degrees of support for students (Michael lesson plan annotations, 2018). This may indicate the need for supports to connect to student thinking derived from the teacher's interpretation of what the student needs. This should be an active process involving student and teacher, not simply included on the worksheet for students to interpret.

In his reflection on teaching the lesson to high school students, Michael provides an example of how students created a new technique that replicated the iterated integral strategy and did so by using the problems on the worksheet to build an understanding of the strategy (Michael analysis #3, 2018). His description includes how students originally "reversed" the order of the integration but did so in a way that made it difficult to solve the integral. The teacher suggests shifting to "an idea that approaches this problem without switching our bounds of integration" (Michael analysis #3, 2018, p. 1). One student comes up with an idea and the teacher asks her to elaborate on it, to elicit more information about the strategy as well as help another student understand her thinking. This demonstrates the importance of teacher interpretation of student thinking and that taking an appropriate action is better than providing hints on the worksheet.

Summary

Although he states in his end-of-semester interview that planning is a critical element of effective questioning to get what you want from student responses, Michael states that his questioning style became more extemporaneous than he had planned. This may show that his beliefs about what learning in a mathematics classroom should look conflict with how he describes the learning environment he wishes to create. This is evident throughout the semester. He struggled with trying to make the lesson plan match the vision he had for the lesson and to bring student thinking into his planning and his teaching. "How I anticipated dealing with students was completely different than how I would actually deal with student responses" (Michael interview transcript, 2018, lines 191-192). One wonders whether the disconnect involves his goals for lesson planning and his teaching style. While his planning of questions was based more on sequencing of how he expected students to think, as evidenced by including hints to guide students toward his view of the problem, his planning and teaching throughout the plan-practice-teach cycle was teacher-centered and guided students to his way of viewing problems. His questioning focused more on sharing his thinking than eliciting and interpreting student thinking.

Elise: Explain It for Them

Like Michael, Elise came to the idea of being a teacher during her undergraduate years. As one of her extracurricular activities, she tutored high school students and found she enjoyed it. Upon learning there was a program that would allow her to complete her undergraduate degree in mathematics as well as earn a master's degree in education, she chose to work toward both.

Elise struggles with balance between providing support and allowing students to persevere. While she plans lessons that include questions designed to elicit student thinking and encourage students to create multiple pathways to solutions, she also wants students to be successful and get the right answers. She collects student thinking but is not always sure how to use it to guide students to form their own conjectures and correct errors and misconceptions. In her teaching, the desire for students to be successful can lead her to providing answers to students.

Initial Peer Practice: Superficial Probing of Student Thinking

In the peer interview assignment, Elise opens questioning sequences with probing questions to elicit student thinking. However, she is sometimes unsure what to do with that thinking, especially when the thinking of the student is unclear.

While one of her peers acts the role of a student, Elise begins by asking him to explain his first graph (see Appendix A for the activity). The student responds saying, "You have a candle and it's burning so you start with some candle and then it goes down over time until you have no candle left" (Elise analysis #1, 2018, p. 4). Elise asks the student to explain his answer, asking what "no candle left" refers to and why the student drew a straight line. When the student explains that the straight line represents the candle burning down the same amount each hour, the teacher introduces the concept of a constant slope. Elise also reflected that this is a place where she could alter her questioning to provide the student with the type of information she wanted to hear; to word questions in a way that would allow the student to provide the terms (Elise analysis #1, 2018).

For the step function graph (the cost of sending a letter, see Appendix A), the student includes only whole values of weight. Elise asks the student what the point (0, 0) represents on his graph, and he answers, "That is the cost if you don't have a letter you don't pay anything…so zero ounces zero letters" (Elise analysis #1, p. 4). The teacher explains this means you would not pay anything for not sending a letter and asks the student why he includes only points on his graph. The student answers that you would need to round up, which Elise accepts (Elise analysis #1, 2018). In her reflection, she admits that she should have asked about the student's views on rounding, but the focus was on whether rounding is the correct term to use for what the student is doing (Elise analysis #1, 2018).

For the third and fourth graph, Elise asks the student about details, such as the arrow at the end of the line. The third graph includes an arrow and the fourth graph does not. The student provides thinking for each graph and Elise asks a follow-up question to help the student think beyond his current representation, but once the student provides his thinking, Elise moves on to another question or ends the interview (Elise analysis #1, 2018). In her reflection she reveals that she wants to learn more about the student's thinking and help him link his graphs back to the context, but it shows her lack of focus on the learning goal (discussing the shape of the graph and continuity).

In this activity Elise's questioning, even with possible revisions, falls short of understanding student thinking. She starts by eliciting thoughts from the student but struggles to determine what she needs to learn and how to ask questions that will reveal the student's thinking. When the student offers erroneous statements, such as the inappropriate use of rounding or a linear graph for an exponential context, Elise is unsure how to address them. In the reflection after the peer interview, some of her revised questions fall short of helping elicit student thinking. In her instruction and reflection, Elise is funneling the student to provide confirmation of how she views the graphs, not the other way around.

Planning (Draft Lesson Plan): Conceptual Understanding and Focusing is Possible

In her draft lesson plan, Elise provides a lesson that supports students developing conceptual understanding and practicing procedures with transformations. Included questions may lead to focused questioning structures if the teacher uses them to provide opportunities for student thinking to guide the lesson. The lesson is an activity in which students apply their recent work on transformations to a series of real-world situations (Elise lesson plan draft, 2018). Students work in groups to use transformations in order to move around a map of a university campus (see Appendix N for the handout of this activity).

For each of the three parts of the lesson–the warm-up, the main activity, and the closing activity–Elise has developed questions to ask students as they work. Some questions preview the work that students will be doing during the main activity, allowing Elise to learn what knowledge students bring to the lesson and what potential misconceptions they may have. Her first question checks for understanding of map reading and others get students thinking about important characteristics of rotation and reflection, specifically the direction of rotation and what is important when doing a reflection.

For the initial part of the lesson, where students complete transformations on a map, Elise creates questions to enrich student thinking:

- Can you explain how you are working through these transformations?
    - If I reflected over Line 13 instead of Line 14 would I land in a different spot? Why?
- Why does it not specify if the 180-degree rotation is clockwise or counter-clockwise?
- Does order matter for translation?
- Why does my center of rotation matter? (Elise lesson plan draft, 2018, p. 3)

These questions elicit student thinking on the meaning of transformations. In her reflection on the lesson plan, she states that, "Students will have to engage with their conceptual understanding of transformations in order to complete the task and to successfully map each transformation on the coordinate plane" (Elise lesson plan draft commentary, 2018, p. 1). These questions help the teacher determine how well students are meeting that goal.

Elise has also developed questions for each of the challenge problems that are the second part of the activity. A sampling of these questions demonstrates a focus on conceptual understanding:

- What if I asked you to only use translation or reflection or rotation? Is it possible?
- Are you trying to use the least amount of transformations as possible? To get from this class to this class is there an easier way to do it?

(Elise lesson plan draft, 2018, p. 3)

She has also included possible student errors and misconceptions in this part of the lesson. She identifies each student error and possible teacher actions, for example:

- Students may have trouble understanding rotation
    - Mark center of rotation on map and draw a line extending to the coordinate point. Tell students to "swing around" this "pivot point" 90 degrees

- Students may struggle choosing a center of rotation
    - Have students analyze what happens to the point if different centers of rotations are chosen (Elise lesson plan draft, 2018, p. 4).

It is interesting to note that while the questions focus on conceptual understanding, the potential errors and teacher actions focus more on procedure. Elise's questions for the closing technology activity ask students to compare different types of transformation, consider whether different combinations are possible, and whether they can determine the transformation based solely on the pre-image and image (Elise lesson plan draft, 2018). These questions support Elise's focus on conceptual understanding in the lesson. As in all lesson plans, these questions may lead to either focusing or funneling question sequences, depending how the teacher incorporates student thinking in the selection and use of questions, as some questions provide multiple strategies for students to use and others are more guided by the teacher's way of thinking. Success will depend on how the teacher asks questions, what other questions the teacher includes, and how the teacher uses student thinking during instruction.

Peer Practice of Actual Lesson: A Focus on Procedures

The peer practice provides Elise with the opportunity to explore how to support student thinking while they are practicing procedures, specifically those dealing with transformations. However, the practice evolves to focus more on the procedures than reinforcing concepts, with the teacher guiding the discussion around how to properly construct transformations. When the opportunity arises to allow for student thinking to guide the development of a conceptual idea, the teacher funnels students to her way of thinking.

During practice with her peers, Elise explains that the students have already worked with transformations in previous lessons and that this lesson is an extension of that work. When her peers ask if there are particular roles that she would like them to enact, Elise states that she just wants them to be regular high school students (Elise transcript of peer practice, 2018). In this section, the term "students" refers to Elise's peers during the lesson.

The lesson begins with students working in small groups to complete the activity (see Appendix N for the activity handout). Elise circulates and checks on student progress. Some groups struggle with the activity (Elise transcript of peer practice, 2018). One student is trying to make sense of reflections. Working through the example, the student says, "Because it's two away from this point, it's two away from here and I pick it up and lay it down, it's still going to be two away" (Elise transcript of peer practice, 2018, lines 151-152). Elise responds, "Yeah, and that's the line of symmetry we were talking about, right?" (Elise transcript of peer practice, 2018, line 154). Like the example from the peer interview where she introduced the terms constant and slope, she introduces a concept without having a corresponding introduction by the student. Rather than using questioning to elicit the idea, or at least the beginning of the idea from the student, Elise has inserted the term here to move the student closer to her goal for the problem, funneling the student to where she sees a solution and wants the learning to go. This is similar to what occurred at the beginning of the peer interview.

Next, she works on another goal of the lesson, understanding that rotations of 180 degrees clockwise and counter-clockwise are equivalent. This question comes directly from her lesson plan: "Why does it not specify if the 180-degree rotation is clockwise or counterclockwise?" (Elise lesson plan draft, 2018, p. 3). This question has the potential to elicit student thinking while focusing on how the student is making sense of a problem. Here Elise scaffolds the discussion with questions that provide the steps a student should consider. She asks the student to rotate a point counterclockwise, then identify the center of rotation, the location of the image of the point, and finally do the same rotation clockwise. The student states that the two images are in the same place. Elise asks, "Why is it in the same place? Do you know?" (Elise transcript of peer practice, 2018, line 195). When the student responds, "Because a clock has 36 hours on it?" (Elise transcript of peer practice, 2018, lines 197), the teacher moves on to another student. To end the sequence, the teacher should respond by helping students make connections and complete their thinking. Since Elise is apparently unsure how to deal with the student response, she moves to help another student.

She completes a similar questioning sequence with another student, again providing the steps for constructing the two rotations and asking, "Does it matter if it's clockwise or counterclockwise?" (Elise transcript of peer practice, 2018, line 263). The student responds, "Because if I'm going clockwise, I could end up in the wrong, in a different place" (Elise transcript of peer practice, 2018, lines 271-272). The student has not seen that the two rotations are equivalent. Elise asks the student to show her, and he realizes it does not matter. She checks to be sure the student understands that this relationship is only true for 180-degree rotations and asks why it is important that the rotation is 180 degrees. The student responds that it is a straight line, and Elise says, "So, it's also, if we talk about it in terms of a circle, it's also half of a circle, right?" (Elise transcript of peer practice, 2018, lines 290-291). Again, Elise provides the interpretation she desires instead of findings ways to guide the student to find it.

Later in the lesson, she checks in with a pair of students who are debating the proper location of an image of a point. She determines they both started in the same place and checks the steps they did in order to determine their images. She focuses on procedure and has the students repeat the process with her support to find the error (Elise transcript of peer practice, 2018). There is no mention of a conceptual meaning of the constructions (such as drawing an arc creates a set of points that are equidistant from a given point). Instead, the focus is on how to use the procedure to determine the correct image of a given point. Near the end of the practice lesson, over 28 minutes into the 30-minute lesson, she asks the second question from her lesson plan, asking students to evaluate whether their solution is the most efficient way to travel (Elise transcript of peer practice, 2018). In the last two minutes of the lesson, she asks more questions designed to help students think about the links between transformations and evaluate them relative to one another:

- Is there a different transformation we could use to get to the same destination? (Elise transcript of peer practice, 2018, lines 607-608)
- Can you use reflection? (Elise transcript of peer practice, 2018, line 641)
- Is there something you can do just once? Try to think of a transformation you can do just once (Elise transcript of peer practice, 2018, lines 650-651).

While she has developed many possible questions in the lesson plan, very few were asked during the peer practice. Instead, most involve which procedure to use for the different transformations, not the conceptual understanding she identifies as important in her reflections. In her reflection on the peer lesson, Elise identifies that the peer lesson was successful because

> Students were able to engage in problem-solving skills because they engaged in a task for which the solution method was not known in advance; the task allowed them to struggle. Students were asked challenging questions that pushed them to use transformations in a new way. (Elise analysis #2, 2018, p. 3)

While students were able to reach an understanding, students should determine ideas through their own thinking without significant guidance from the teacher. Elise does point out that, “On multiple occasions I was too quick to provide students with an explanation, instead of questioning them further" (Elise analysis #2, 2018, p. 4) as something she needs to address in her teaching.

Final Lesson Plan: Stay the Course

Based on her reflections after the peer practice, Elise does not make significant changes to her planning. There are only minor edits to one of the challenge questions for the activity. Elise defines which closure sequence to use for the lesson, which in the draft has options based on time limits. She includes additional questions for the students to consider, including "Does the order of transformations matter?" and "What are some important features?" (Elise lesson plan final, 2018, p. 5), which offer a better balance of conceptual understanding and knowledge of procedures.

Teaching the Lesson: I Can Explain

In her teaching of the lesson, Elise allows students to explore the constructions and transformations in a similar manner as in the peer practice and finds that she should have segmented the activity into multiple sections to allow for more support during the activity. She also notes that her questioning could be better and there were times when she was doing the explaining instead of the students.

Elise identifies that the lesson with her high school students went well, that the students enjoyed the lesson and had the opportunity to create their own transformations in response to the challenge questions (Elise lesson plan commentary, 2018). From notes taken during the lesson, Elise indicates she would like to extend the warm-up so that students are prepared for the next step (Elise lesson plan final annotation, 2018). She feels that the students needed more support before taking on the challenge of the main activity. However, since the students had covered this topic in middle school and this was an application lesson of transformations they had seen in previous lessons, this may provide more support than students actually need.

Elise identifies that some students struggled on the main activity based on the length of the exploration, as an early mistake has ramifications throughout the activity (Elise lesson plan final annotations, 2018). She decided that in the future, she would break the exploration into multiple sections, to let students "know if they are doing it right or if they need to check their work" (Elise lesson plan final annotation, 2018, p. 3). She also notes that parts of the activity need greater emphasis, such as the order of transformations in Challenge Question 2 and how the figure moves while it is rotating (Elise lesson plan final annotation, 2018). She notes that some questions were highly effective with students, specifically:

- Why did you choose this dorm and this restaurant? Could I make more effective choices that would make me cut down the number of transformations?
- Which transformations did you use?
- Which transformation allows you to move the most distance?

(Elise lesson plan final annotation, 2018, p. 3)

Her comments show there were successes during the lesson and places where she sees revisions may lead to an improved experience for her students.

She also notes areas of improvement needed for the development of her questioning. "Instead of pushing her with more questioning, I simply began to explain. I also did this when working with students individually" (Elise analysis #3, 2018, p. 4). She reinforced this view during the interview at the end of the semester: "I noticed that I had the tendency to like, explain instead of question, and even when I did question, I didn't really give them a lot of time to think about it" (Elise interview transcript, 2018, lines 145-146). She includes a similar sentiment in her reflection on the lesson:

> I planned a lot of questioning, questions specifically to probe student thinking more, just deepen their understanding, and just to make sure they have concepts . . . and like all those questions. And when the lesson actually came down to it, I feel like I was just so, I just wanted them to get through the first part that they could get more like the second part. (Elise interview transcript, 2018, lines 148-151)

The desire to move to another part of the lesson caused her to push students ahead and not spend time eliciting, interpreting, and responding to student thinking, an observation Elise made watching a video of the lesson and reflection. This demonstrates that she is examining her own teaching and considering other ways to develop her vision of how she wants the instruction to go.

Summary

Elise plans lessons that have the potential to elicit student thinking. In enacting those lessons, she evolves from more directive questioning of the peer interview to increased use of focused questioning structures in her high school lesson. However, sometimes she does not deliver planned questions in ways that support the development of student thinking. Instead, her questions tend to guide students to answers though the use of funneling questioning rather than letting the students persevere to find their own solutions by the use of focusing questioning structures. Elise uses the plan-practice-teach cycle to identify the need to be more specific in her questioning (such as the open-ended question which starts off the peer interview) and provide support for students before they encounter challenges, by using a warm-up to the instructed lesson. She limits student interactions with errors by teaching them how to uncover mathematical relationships through those struggles and using funneling to guide them to her responses. With a desire to create a smooth pathway to learning, Elise explains the mathematics to her students and limits the ways that students may think about the mathematics.

Chapter 5: Cross-case Analysis

With these detailed descriptions of each case study, it is now time to look across cases to look for commonalities and differences. Examination of each case looked for examples of topics that are key to this study, based on the research questions:

- Do preservice teachers use focus and funnel questioning structures as they elicit, interpret, and respond to student thinking and, if so, how do they use them?
- In what ways does preservice teachers' use of focus and funnel questioning structures change through the plan-practice-teach cycle?

Color-coded highlighting of a previous draft of case study summaries indicated when one of these topics occurred (see Appendix O for a sample of this coding). Notes made during this coding process detailed how the topic connected across the case, looking to see either similarity or differences across individual cases. Next, commonalities and differences across the cases showed that there were specific ways the research questions were answered by the data.

In connecting to the first research question, each of the preservice teachers used focusing and funneling questioning structures differently to elicit, interpret, and respond to student thinking, advancing learning through how they use student thinking to support understanding of the learning goal. While they often use probing questions to open questioning sequences, they differ in how they respond to student answers to these questions. Teacher responses range from questions that allow students to provide clarity to their own thinking or explore an error, embed guidance to direct students to a particular way of thinking, inform students of how the teacher interprets their thinking, or give evaluative feedback to the student. These different responses to student thinking create ways to use student thinking to support sense making of the learning goal.

In exploring the plan-practice-teach cycle of the second research question, the participants use of focusing and funneling questioning structures develops in different ways. In the initial draft lesson plan, each of the preservice teachers provides insight into how they envision how their lesson will develop student understanding of the learning goal. While the lesson plan does not always represent what a teacher will do when teaching a lesson, it does provide insight into how they anticipate student thinking. During the peer practice, the participants vary in what they are able to learn about their lesson. Some focus on practicing key features of their lesson while others start their lesson at the beginning and see how much of the lesson they can complete. Those that focus on the key features learn more about how to support learners to make sense of the learning goal; those that simply start at the beginning often do not complete key features that would help them to consider key moments in questioning.

The peer practice has an impact on the revisions each preservice teacher makes to their lesson plan. These revisions identify that these teachers have adjusted how they feel the lesson will develop, and some of the teachers make changes that allow student thinking a greater role in making sense of the learning goal while others make changes that limit the vision of how they plan to use student thinking in the lesson. Finally, the participants teach their lesson in high school mathematics classrooms. They vary in how student thinking guides the lesson, with some of the participants using student thinking to a greater extent than they did at the start of the semester, one preservice teacher changing little over the course of the study, and one participant narrowing the use of student thinking in her lesson.

Each of the preservice teachers in this study experiences different degrees of change in their questioning and eliciting, interpreting, and responding to student thinking through the plan-practice-teach cycle over the semester. One of the teachers demonstrates significant growth in her ability to elicit, interpret, and respond to student thinking through focusing questioning structures, two of the preservice teachers show some growth in these areas, one of them stagnates in using student thinking as a foundation for his lesson, and the final preservice teacher regresses in how she uses focusing questions and student thinking within her teaching. While there is a limited amount of data to determine how deeply change occurs across these participants, there are indications that the experiences provided within the course do have some influence on some of the participants in the study.

This organization of the evidence provides the structure to present the results of the cross-case analysis. The first section explores how each of the preservice teachers uses questioning to elicit, interpret, and respond to student thinking. The next section examines how the preservice teachers develop and use questioning structures through the plan-practice-teach cycle. The final section examines how much participants change over the course of the semester. The analysis concludes with a summary of what learnings emerge from the study.

Questioning Structures: Use by the Five Teachers

Just like veteran teachers, preservice teachers vary in their ability to use questioning to elicit, interpret, and respond to student thinking. Throughout the semester, each of the preservice teachers in this study identified that they strove to use questions to learn about student thinking during mathematics instruction, through both the visions of learning represented by their lesson plans and their actions taken while teaching, both in practice and in the classroom (see Table 3). Each believed that they provided opportunities for students to showcase their thinking and diverse ideas, connecting to the meaning of eliciting student thinking (Singer-Gabella et al., 2016). These preservice teachers varied greatly in their use of questions and questioning structures, just like in-service teachers (Herbel-Eisenmann & Breyfogle, 2005; Ontario Ministry of Education, 2011). The difference in this study is that the preservice teachers have the opportunity to plan, practice, revise, teach, and reflect on their lessons with depth.

Questioning Structures, Table 3. Typical Questioning Techniques for Each Preservice Teacher

Participant	**Typical Questioning**	**Example**
Gloria	More Funneling than Focusing, providing students with her interpretations of their responses	She sometimes supplies the thinking to the students instead of collecting it from the students, “I replied that the order mattered less for the example we were currently working with, but I cautioned him to think about the polarity of the terms involved” (Gloria analysis #3, 2018, p. 3).
Beth	More Focusing than Funneling	She works to help students think together about how to properly use mathematics to solve problems. There were multiple occurrences during the peer practice where she would ask an initial question and then allow students to ask questions back and forth to one another before the teacher interjected again to ask another question to spark student thinking (Beth transcript of peer practice, 2018).
Clara	More Funneling than Focusing, guiding students to the answer for which she sought	Clara provides more structure to the lesson through revisions to the questions in the lesson plan and as a result of her analysis of her peer practice. During the peer practice, her questions sometimes guided students to the answer she was looking for, “So if I look at this rule [points to product rule], is there some way I can split it up, so I can at least try to make sense of something helpful?” (Clara transcript of peer practice, 2018, lines 491-492). Here, she guides the student to see the problem the same way she does.
Michael	Mostly Initiate-Respond-Evaluate (IRE)	Michael uses short questioning sequences of teacher question, student response, and teacher evaluation of response, then begins the next question. During the peer practice, when he notices that many students are not completing the double integral boundary problem correctly (which he had anticipated), he models how to solve the problem, taking over the thinking on how to solve the problem.
Elise	More Funneling than Focusing, providing answers to students	During the peer practice, when a student provides a response that is a correct way of explaining a relationship about 180-degree rotations, but not what Elise is looking for, she tells the student the explanation she has in mind.

Questioning Structures: Focusing and Funneling

In this study, four of the five preservice teachers demonstrated varying degrees of focus and funnel questioning structures (Herbel-Eisenmann & Breyfogle, 2005; Wood, 1998). A focusing question structure allows students to share their thinking and looks for a variety of responses and strategies to solve a given problem. The teacher selects questions in terms of what they notice, what students share, and how the lesson follows student ideas and thinking (Herbel-Eisenmann & Breyfogle, 2005). In funneling, teachers choose questions based on a predetermined pathway and their own thinking rather than the thinking of their students (Herbel-Eisenmann & Breyfogle, 2005; Wood, 1998).

Gloria, Beth, Clara, and Elise all used differing balances of focusing and funneling question structures in their teaching. When they used focusing question structures, the questioning sequence began with an open-ended, speculative question designed to elicit student thinking. The next action was to ask a question to demonstrate that the teachers had listened to the response and interpreted the student thinking. However, it is important to let the student provide their thinking without directing them in a particular direction or toward a particular strategy. As described below, during her peer practice, Beth asked questions that allowed students to provide their thinking.

Beth actively listened to the student responses before asking another inquiry question, providing space and options for students to continue explaining their thinking. She did not guide the students to a particular answer nor does she include her own interpretation in her question. She simply allowed the students to continue to explain their thinking or clarify their response as indicated by the teacher's question. She identified the importance of students being able to explain their thinking and her role was to redirect thinking, not provide it, when they were in error. She also used questions to allow students to deepen their understanding of the learning goal, allowing students to make sense of errors, even ones that may not have come up during student responses. In this way, Beth is focusing students to make sense of their own thinking, not hers. While each of the four preservice teachers included some examples of focusing in their teaching, Beth demonstrated this consistently, as opposed to Gloria, Clara, and Elise, who often use the funnel questioning structure and include their own thinking during questioning.

The funneling sequences employed by Gloria, Clara, and Elise usually began in the same manner as the focusing structure, i.e., with an open-ended or inquiry question. While this was not a requirement of funneling question sequences, each of the preservice teachers would usually initiate questioning sequences in ways designed to elicit student thinking in an open manner, and each of these preservice teachers state they were eliciting student thinking. The questioning structure used, however, is determined not just by the first question, but also by how the teacher responds based on what they have noticed and what their interpret of what the students have said and done. This is the importance of examining sequences of questions, not individual ones. These teachers' actions would often guide students to where they had planned the lesson to go or would include their thinking about the students' response. At this point they are funneling students toward their goals for the lesson. There were four basic funneling responses used: leading the student to the teacher's view of the problem, getting students to agree, providing their answer to the question, or trying to move the lesson forward without students answering questions.

When teachers lead a student to their view of the problem, they often included their own interpretation of the student's thinking in their question. When Gloria heard a student mention negative numbers in her peer interview, she responded by saying, "Okay, that's good. I'm glad that you mentioned negative numbers, because we can't have negative numbers, right?" (Gloria analysis #1 transcript, 2018, lines 10-11). Gloria identifies that this is an example of her leading the student to the answer, and an area that she wants to improve. Here, Gloria interpreted what the student said by offering an assumption about what he was thinking. She then shared that assumption with the student and asked the student to agree with her assumption. In this way she led the student to see the choice of negative numbers in the problem the same way she did.

There were also times when a teacher was unsure of how to lead a student from his or her thinking to where they wanted the student to go. Working with her peers on her practice lesson, Elise asked a student to explain why the 180-degree rotation in a clockwise direction was the same as a 180-degree rotation in the counter-clockwise direction. The student responded by connecting the 180-degree rotation with a straight line. Seeking a different response, Elise answered with a description that it is also half of a circle (Elise transcript of peer practice, 2018). In using funneling, teachers sometimes deprive students of the depth of understanding to make connections across learning experiences (NCTM, 2014). This may result in teachers providing the connections, since they are not eliciting them from students. Here, Elise cannot see a way to help the student move from his idea to her way of thinking, so she funnels him toward it by simply stating what she wants. This can occur when the teacher is unsure how to bridge the gap between what the student is thinking and the goal of the lesson.

There are also times when teachers are concerned that a lesson is not going well or when they feel a need to quicken the pace. This may occur when the class period is about to end. After teaching her lesson to high school students, Clara reflected that she did most of the thinking. Rather than ask the students to explain the steps in the process, she pushed them ahead and provided the thinking and answers for them. This is one reason why teachers use funneling questioning structures; when they feel that students are not communicating well, they begin making the connections for the students. Clara felt that she rushed them to the conclusion of the activity rather than allowing the students to reach their own conclusions (Clara analysis #3, 2018).

Questioning Structures: Another Style

In contrast, Michael did not use clear focusing or funneling question structures. Instead, his questioning relied on initiate-respond-evaluate (IRE) questioning (Mehan, 1979). Similar to focusing and funneling questioning structures, IRE questioning has the teacher initiate discussion by asking a question and allowing the student to respond to the question. What makes the IRE questioning different is that the teacher does not ask a subsequent question to either guide or allow for the development of student thinking but rather the teacher evaluates the response and moves on to another topic. Rather than deepening understanding with focusing or guiding the student like funneling, IRE questioning provides the teacher's evaluation of the student response and continues to the next part of the lesson. There is no chunk of questions and responses, but rather individual cycles of teacher questions, student responses, and teacher evaluations.

While Michael's lesson plans did not specify the type of questioning structure he envisioned, during his teaching he provided no interpretation of student responses or open-ended responses to student thinking. Since his lesson plans simply indicate the content of the lesson, not how he envisions implementing questioning practices, it was impossible to determine the type of questioning structure he anticipated. Michael's teaching and reflections demonstrate a preference for teacher-dominated lessons, with few opportunities for him to interpret student thinking or continue the discourse by asking questions to explore and expand student responses. Therefore, most of Michael's discourse structures are brief as the teacher provides evaluative feedback to initial student responses. There are times when Michael embeds what he wants the student to do in his statement or question. In his reflections on both the practice and instructed lessons, Michael identifies the need to provide hints and supports to learners in order to help them to achieve the learning goal. He demonstrates a teacher-centered classroom that provides models for students to replicate. When the lesson would benefit from a connection, it is teacher who makes it.

While Michael's questioning structure relies on IRE, he does demonstrate some aspects of the goals of a funneling question structure in offering support to students even when they do not need it. This was evident in the handout used in his lesson with high school students. In addition to the hints offered on the handout, Michael considered adding review problems when students struggled with the lesson, to prevent them from making errors on the handout. This was a key component of his teaching, how to prevent students from making mistakes in the learning process.

Each of these preservice teachers show variations in how they use and consistently adopt questioning structures designed to elicit, interpret, and respond to student thinking. While each expressed a vision of teaching that included a prominent role for student thinking in guiding student learning, their various teaching approaches demonstrate how they consider and use student thinking differently to promote student understanding.

Questioning Structures: Variation of Technique

This study demonstrates how preservice teachers within the same methods class, who possess similar beliefs about the importance of using student thinking, may develop different questioning techniques. How these preservice teachers used student thinking and developed questions to ask while they were teaching differed widely. While each of the participants experienced the plan-practice-teach cycle, collaborating and sharing ideas with each other and had multiple opportunities to revise their vision of their lesson, their questioning styles remained unique. The impact of the semester-long methods class varied, with three of the preservice teachers choosing to make greater use of teacher actions based on student thinking and working to incorporate more focused-based question structures. Of the remaining preservice teachers, one teacher chose to narrow opportunities to use student thinking, and another employed the IRE questioning structure which resulted in fewer opportunities to incorporate student thinking into his lessons.

The Plan-Practice-Teach Cycle: Questioning

Overall, each of these teachers expressed a desire to use student thinking in the learning process, in different ways and with varying degrees of success (see Table 4). Michael guided his students the most, using student thinking to confirm how he would solve the problem. This was less evident in his lesson plan, but obvious in his practice and teach cycles. Over the semester Gloria, Elise, and Beth all improved their eliciting, interpreting, and responding skills through the plan-practice-teach cycle. The more they planned, the more likely they were to provide responses to student thinking which encouraged the expansion of student ideas. By comparison, Clara's lessons offered direct questions with fewer opportunities to elicit, interpret, and respond to student thinking, decreasing the role of student thinking as her lesson evolved over the semester. This is how the questions that preservice teachers plan and use indicate what they notice in students' responses and how they act based on those responses.

The Plan-Practice-Teach Cycle, Table 4. Development of Questioning Through the Plan-Practice-Teach Cycle for Each Preservice Teacher

Participant	**Plan**	**Practice as Link to Planning and Teaching**	**Teach**
Gloria	Some gaps in what lessons might look like	Matches the script when available but struggles without one	Leads students to her way of thinking
Beth	Clarifies and adds detail to questions to better learn what students are thinking	Provides opportunities for students to explain thinking	Focuses on using student thinking to move forward in the lesson
Clara	Unclear what to do with student errors	Increasingly provides more and more guidance to students	Guides students to her view of the task
Michael	Only provides a vague idea of what will happen	Guide students to answers and give hints to students to avoid questions	Few responses – little evidence of interpreting student thinking
Elise	Some gaps in what lessons might look like	Unsure how to unpack student thinking and how to help when students struggle	A mix of deepening student thinking and guiding students to her thinking

The Plan-Practice-Teach Cycle: Developing the Plan

Lesson plans are only guides into what teachers are thinking about how a lesson may proceed and generally provide options based on how students react to the problems; as such, they describe multiple questioning pathways a teacher may take. It is not always clear what type of questioning structure teachers plan to use based on a review of their lesson plan. Each of the preservice teachers in the study revealed different aspects of how they plan to elicit, interpret, and respond to student thinking. Gloria, Beth, and Elise created initial questions to elicit student thinking along with possible correct and incorrect student responses to those questions. They also included possible teacher actions to address those responses in their planning, showing that they had considered what the task would look like when they taught and how they envisioned the instruction of the lesson. In this way, they were modeling lessons that could lead to focusing questioning structures. These teachers created plans that value each element of eliciting, interpreting, and responding to student thinking and how the lesson would focus on that thinking.

Still, since parts of Gloria's and Elise's lessons did not include student misconceptions as possible responses, one wonders if they felt students would not need support in these particular sections, how they plan to focus on student responses, and whether they considered the errors that students might make at this point in the lesson. Beth proved to be the clearest in her understanding of the lesson, identifying gaps in her development and incorporating multiple sources (curriculum materials, mentor teacher, and course instructor) to improve the design of her lesson through the semester. This allowed her to prepare for using focused questioning structures during her lesson. For the draft lesson plan, she acknowledged that she did not know how some parts would go and that she was still working to design the structure of the lesson. Her questions in both the draft and final lesson plans offered ways for students to do their own thinking and make sense of both correct and incorrect ways to solve problems.

Clara's lesson plan included both questions and possible student responses, but her vision involved only correct responses, guiding the lesson to the outcome she desired. Again, while lesson plans do not necessarily indicate the direction of lesson, not including incorrect student responses does not provide the flexibility necessary for teachers to adjust instruction and questioning based on their interpretation of student thinking. Clara showed little consideration of student struggles, such as how to address a student who failed to see that the initial problem, based on a hypothetical student, was incorrect. In fact, as the lesson develops through the semester, she decreases the level of cognitive demand she expects of her students and provides more scaffolding in anticipation of student need.

Michael's lesson planning differed from his peers. Rather than consider possible student responses and potential teacher actions, he constructed summaries about what he expected would occur. These narratives included topics of questions he might ask but not the actual questions, and errors and misconceptions he anticipated his students would make, providing a step-by-step outline for how he envisioned the lesson. This provided less information about how he envisioned the questioning within his lesson. His attention rests on the actions of the teacher, as represented in the structure and writing of the lesson plan. While he identified possible student struggles and the topics of questions he might ask, he included no questions designed to learn about and respond to student thinking. Michael's approach differed in format from his peers but did demonstrate a vision of what a lesson might look like and considered places where he could support student thinking. Left unclear in how to address student errors and what types of questions he would use to elicit, interpret, and respond to student thinking. The edits that Michael made between the draft and final lesson plans mostly involved offering hints used during the peer practice of the lesson. Where he felt students might need help, Michael provided assistance, whether they needed it or not.

The Plan-Practice-Teach Cycle: Using Practice to Link Planning and Instruction

How these preservice teachers linked the use of questioning in their lesson plans to their instruction varied. Some attempted to use the questions in their lesson plans as a script, aligning their teaching closely to a vision described in their lesson plans. For others, the lesson plan offered an outline of what they wanted to happen in the lesson, while they remained open to taking alternative paths and using extemporaneous questions during instruction. This was evident in the three teaching episodes during the semester–the peer interview of the graph activity; the peer practice of developing a lesson; and the implemented lesson in a high school mathematics classroom.

All of the preservice teachers struggled with their questioning during the peer interview activity. Gloria's and Elise's peer interview questioning sequences began with inquiry questions that elicited thinking from the peer "student," their responses tended to lead the student toward the teacher's way of thinking, without deepening the teacher's understanding of the student's approach. Beth and Clara struggled to interpret and respond to student thinking and often made unconfirmed assumptions about what students were thinking in order to funnel them to their interpretation of their work. Michael asked questions, received the students' responses, and then moved on to his next question. For each, there was a gap between what they envisioned in the lesson plan and knowing how to help students achieve their vision, which often led to the use of funneling questioning structures.

During the peer practice, the preservice teachers differed in what they learned about questioning and the development of their lesson. Gloria and Clara learned little from the peer practice to support how they wanted to elicit, interpret, and respond to student thinking for their lesson. Gloria encountered an obstacle as the opening review activity becomes a challenge for her peers. She spends most of the practice explaining the laws of exponents. Gloria did not anticipate this area of struggle in her planning for the lesson and required her to adjust the lesson to address this need exhibited by the students. Using extemporaneous questions (there were no planned questions as this was not the primary learning experience of the lesson), Gloria concentrated more on the procedures that students needed to follow in order to complete the exercises, and less on their thinking. She funneled students to the correct answer rather than focusing on whether student thinking was correct. Clara also learns little, but for her, it is because of the changes that she makes to her lesson after the peer practice. She learns that students already know the connection that she hopes they will discover in her lesson, so she revises the lesson and changes her perspective on both how to use student thinking and how she will question students. That change becomes evident when she teaches the lesson to high school students, as described in the next section.

There are also similarities between Michael's and Elise's peer practice. Both explore important parts of their lessons during the practice and both demonstrate limiting student thinking. As Michael circulated around the classroom during the practice, offering hints to students while they worked, there was little teacher-student interaction. He included episodes from the lesson plan in his teaching, such as the discussion of the bounds of a double integral, but instead of allowing students to lead the discussion and offering responses based on his interpretation of student thinking, Michael provided the explanation himself. At the end of the presentation, he asked the students if they understood. During Elise's peer practice, she placed a greater focus on how to complete various transformations than the more conceptual goal identified in her draft lesson plan. Some of the questions she used during the peer practice came directly from the draft lesson plan, such as helping students see that a 180-degree clockwise rotation produces the same image as a 180-degree counter-clockwise rotation, and why this was so. But when implemented, the discussion ended with Elise providing the answer that she wanted instead of building on the response given by the students. Both Michael and Elise limited their use of student responses and thinking and guided students to their interpretation of the learning goal.

Beth's peer practice experience was different from the other participants. Beth balanced the choice of questions, some from her lesson plan, some responding to student thinking, to clarify her understanding without providing her own interpretation. As she was able to make use of both questions she anticipated and those she could develop in the moment based on student thinking, her questioning structure in the final lesson plan was similar to her draft. This experience allowed her to be flexible in her questioning and focus more on student thinking, both in the peer practice lesson and the high school lesson. None of the rehearsals made the most of the controlled environment created by practicing a lesson in a methods course. There was no pausing of the lesson to discuss the effectiveness of chosen pedagogical strategies nor to discuss the success of questioning at uncovering student thinking. This gap lost an opportunity to support preservice teachers in the development of their questioning structures.

The participants' reflections on the instructed lessons show some commonalities and some differences. Each of the preservice teachers shares that they were pleased with various aspects of the lesson and students benefited from the experience. Beth described how she offered opportunities for students to provide their own thinking. This allowed her to obtain a better understanding of how students thought as well as how to interpret their thinking. A few also provide insight that they are still working on how to elicit, interpret, and respond to student thinking effectively. For example, Clara adjusted the questioning in her lesson as she prepared to teach. She lists the questions she wants to use right before the lesson. These questions, a combination of questions from the lesson plan and those created immediately before the lesson, were more directive, led to more funneling questioning structures, and reflect her desire to give students a better understanding of the task. She eliminated questions that asked students to compare their approach with those of other students and added some to guide students toward the correct answer, reflecting her goal to include more review of prior learning in order to better prepare students (Clara lesson plan annotations commentary, 2018).

Overall, the preservice teachers who participated in this study demonstrated multiple questioning methods during instruction. Throughout the plan-practice-teach cycle, each exhibited benefits and challenges of using planned and extemporaneous questioning. There were times when they were able to follow questions from their plan and times when they struggled to develop questions in the moment. In addition, some tried to balance between planned and unplanned questioning. In the interview at the end of the semester, Gloria explained that she wanted to find the 'sweet spot' between using questions planned for the lesson and ones that responded to student thinking (Gloria interview transcript, 2018). Each of the participants struggle with this balance at some point of the semester, in part from not always being able to use prepared questions and sometimes not knowing how to respond to student thinking.

The Plan-Practice-Teach Cycle: Responding to Student Thinking During Teaching

Each of the preservice teachers responded differently to the student thinking they elicited, particularly when the students struggled to understand the goals of the lesson. During the peer interview (the initial practice teaching episode of the semester), each teacher had different levels of success with eliciting, interpreting, and responding to student thinking. For example, Michael offered little interaction with his student during the interview. Although he managed to elicit student thinking, there was no interpretation of that thinking nor was there a teacher response linked to the student's ideas. Each question was limited to a specific characteristic of the student's graph, and once the student had responded to his question, Michael would move on to another question without building on the student's thinking.

While the other four participants did provide some response to student thinking, each of them used mostly funneling questions to guide students to their way of thinking. Elise provides a model of this process. Elise opened with a speculative question, listened to the student's response, and then asked him to explain his response in greater depth. However, other follow-up questions would guide the student to Elise's interpretation. Throughout the peer interview, Elise would offer her view of the student's thinking before the student had a chance to fully explain his response. Her questions reflected a desire to learn from the student, but if the student's thinking was incorrect or incomplete, Elise seemed unsure how to address it. Although she made some interpretations to certain responses, there were gaps when she did not respond to the student's thinking.

During the teaching episodes, each preservice teacher demonstrated that they valued eliciting, interpreting, and responding to student thinking in the actions they took and the questions they asked. At one end of the spectrum, Michael focused on the thinking of the teacher rather than that of the students. He gave a presentation during the peer practice, based on witnessing many of his students in need of guidance on a particular problem. Michael would embed his interpretation of student work by offering a hint or asking a question that funneled students to his strategy or solution. During the peer practice, he would circulate around the room, observing what students were doing and suggesting what students could do next, without asking questions of students. Michael stated that the peer practice did not have particular value for him, in part because some of his peers struggled to understand the content of the lesson. Yet, the experience caused him to place multiple hints in the final version of the activity so there would be less need for the teacher to help individual students working on the problems (Michael lesson plan revision summary, 2018). Some students were confused by the hints and others did not want such help. Michael overlooked the importance of questioning and the teacher's interpretation of student thinking and subsequent responses. In placing hints directly in the handout, he presumes to know what students will need and provides it, whether they ask for help or not. This eliminates the need for teacher questioning.

Three of the preservice teachers demonstrate during the cycle that they are still working on how to receive and respond to student thinking. Gloria wants to use questioning and student thinking to help students learn, but sometimes offers an explanation of the problem or funnels students to her view of the problem. While her high school students were able to contribute to the lesson, Gloria states that *she* is the one who makes the connections between the rules at the end of the lesson, not the students (Gloria analysis #3, 2018). This was evident in her peer interview and peer practice as well. Gloria allowed students opportunity to provide their thinking, but sometimes led them to the outcome she had planned. This was not clear in the lesson plan in terms of how Gloria implements the thinking in her lesson plan. Like Gloria, Clara wants to use student thinking in ways that support the learning process, but sometimes uses questioning in ways that guide students to the thinking that she envisions.

This range of teacher responses was also evident in the peer practice and the high school implementation of the lesson for Elise. There were elements in her questioning that demonstrated she listened carefully to student thinking and asked questions to learn more about his or her thinking. There were also examples of Elise guiding the student to a particular outcome as well as examples of moving to another student when she was unsure how to respond to a certain answer. When students struggled to provide the reasoning she was looking for in the discussion involving 180-degree rotation, Elise moved away from the student's answer ("because it's just a straight line") (Elise transcript of peer practice, 2018, line 288) to offer another answer ("If we talk about it in terms of a circle, it's also half of a circle, right?") (Elise transcript of peer practice, 2018, lines 290-291). A wide range of teacher responses were evident in each of these three approaches.

Beth's questioning consistently examined student thinking and allowed her students to provide it without interruptions or guidance from the teacher. After the peer interview, in which Beth occasionally led the student to her thinking, she helped students make sense of problems in their own way and encouraged them to examine the meaning of rules and procedures in both the peer practices and high school classes. Her peer practice transcript and the analysis of her implemented high school lesson demonstrates that she focuses on active listening while teaching (Beth analysis #3, 2018; Beth transcript of peer practice, 2018), as well as the ability to balance what she has planned with what she needs to do in the moment to respond to student thinking. Beth focused on asking questions of students, not giving answers. She demonstrates the most growth in questioning and eliciting, interpreting, and responding to student thinking in the plan-practice-teach cycle.

The Plan-Practice-Teach Cycle: Change

This study supports both the view that preservice teachers who participate in plan-practice-teach cycles may experience improvement (Zembal-Saul et al., 2000) and that it can be difficult to help preservice teachers enact effective practices in the classroom (Kazemi et al., 2009). However, the ability to expand these findings to a wider audience of preservice teachers is limited by the number of participants involved in the study and the differences experienced by the participants during their peer practices. With that said, four of the candidates showed some change over the semester regarding their views of and practices using eliciting, interpreting, and responding to student thinking. Gloria, Beth, and Elise increased the roles that student thinking played in their instruction, with Beth demonstrating the ability to focus on student thinking on a consistent basis. Gloria and Elise increased their ability to respond to student thinking, but still had moments where they relied on their interpretation of student thinking and/or attempted to guide the student to how to view a problem.

Clara had a different experience than Gloria, Beth, and Elise. She started with an open-ended, discovery activity which asked students to determine a connection between exponential and logarithmic functions and focused on student thinking. However, she later learned that students already knew this connection coming into the lesson. When she changed the activity, its goals were not as explicit as the one she delivered during peer practice. She provided fewer speculative and inquiry-based questions and used more directive and procedural ones as she developed her thinking about the lesson, moving from many open-ended questions to only a few. She took over the thinking for students and felt the need to adjust the challenge of the lesson to offer them an easier path to the learning goal for her students.

The fifth preservice teacher, Michael, showed little change in how he viewed the role of and drew on student thinking throughout the semester. From the beginning, he saw questioning as a tool to move the lesson forward, with his views guiding development of the lesson. This was evident in both the design of his lessons and their implementation. Research has shown that preservice teachers need to develop how to attend to student thinking (Davis, 2006; Levin et al., 2009) and understand that use of student thinking is a core teaching practice (Ghousseini & Sleep, 2011; McDonald et al., 2013; Windschitl et al., 2012).

Summary

The outcomes from the study show how five preservice teachers plan and use questioning during a secondary mathematics methods course. In reference to the first research question, the teachers differed in how they used focusing and funneling questions to elicit, interpret, and respond to student thinking. Four of the five participants used different balances of focusing and funneling questioning structures, with the fifth one using mostly an IRE questioning technique. The three participants who used more funneling questioning structures each used them in different ways that limited student thinking. This may be impacted by the variation within each case: different students, different mentor teachers, different beliefs and values held by the preservice teachers, and different topics of the lessons. In reference to the second research question, how the participants use questioning through the plan-practice-teach cycle differed. This was in part due to different experiences in the practice element of the cycle and in part due to different actions taken by the preservice teachers both in revising the lesson plan and during instruction. Also, each of the preservice teachers falls along a continuum of how they privilege student thinking during instruction. One rarely used student thinking and the other four used student thinking to different degrees to move the learning forward. Also, the preservice teachers exhibited different types of change in their use of questioning and eliciting, interpreting, and responding to student thinking over the semester. This study shows that the experiences of each of the participants, both those that they have prior to their methods course as well as those that are contained within it, has varied impact on their questioning and actions, and confirms the fact that preservice teachers can develop their questioning skills while learning to maintain positive classroom environments in their preparation to provide mathematics instruction on their own. While the total amount of data is not sufficient to make broad claims about the significance of the change across all preservice teachers, there are some important commonalities. These are discussed in the following chapter.

Chapter 6: Discussion

This final chapter begins with a descriptive overview of the study. The experiences of the participants and analysis of the cases lead to a deeper understanding of the findings. The individual and cross-case analyses lead to four implications important to teacher educators: encouraging preservice teachers to use focusing questioning structures is not a simple task for teacher educators; preservice teachers enter teacher preparation programs with beliefs that need to be addressed in developing and implementing student-centered lessons; activities need to be flexible with the understanding that preservice teachers elicit, interpret, and respond to student thinking differently through the plan-practice-teach cycle; and practice with peers needs to be structured in ways that will benefit how preservice teachers develop lessons. Lastly, recommendations for possible future research and limitations of the study are considered.

Summary of the Study

The preservice teachers in this study spent more than half of the semester developing a lesson to use in their high school mathematics classrooms. In the journey to developing the best lesson they could on a selected topic, they experienced both success and challenges. They endeavored to learn how to use questioning structures to elicit, interpret, and respond to student thinking. Some experienced various degrees of success in using student thinking to lead instruction, while others concentrated more on working with students in ways that felt comfortable and matched their beliefs about the learning of mathematics. The participants privileged student thinking in different ways based on what they tended to notice and how they interpreted what they noticed. As they presented information to students and tried to interact with them in ways that would promote learning, the transcripts and reflections demonstrate a wide range between how they wanted to interact and their actual classroom interactions. A key outcome was in how the preservice teachers responded to student thinking, starting with whether they responded to it at all.

This study explored the development of five preservice teachers over the course of one secondary mathematics methods class within a 13-month graduate program in teacher preparation. Each of the preservice teachers was teaching fulltime in their internships and taking multiple classes in the evening to complete their requirements for certification. The focus of this study was how these preservice teachers use questioning structures to elicit, interpret, and respond to student thinking. Also examined was how preservice teachers developed their use of questioning through the plan-practice-teach cycle.

This study sought answers to the following research questions:

- Do preservice teachers use focus and funnel questioning structures as they elicit, interpret, and respond to student thinking and, if so, how do they use them?
- In what ways does preservice teachers' use of focus and funnel questioning structures change through the plan-practice-teach cycle?

Data sources included three analytical activities (one from the peer interview, one from peer practice of the lesson, and one from teaching the lesson with high school students), copies of the draft and final lesson plans, commentary on composition and revisions of the lesson plans, resources that accompanied the lesson plans, transcript of peer practices, notes taken on the teaching of the lesson to high school students, and the transcripts of interviews conducted by the researcher with the preservice teachers at the end of the semester.

Analysis examined how questioning developed through the plan-practice-teach cycle and looked for themes of how the preservice teachers changed over the semester. The coding of transcripts examined the types of questions and questioning structures used by each participant and all data sources were explored for change within cases and themes across them. Exploration included how preservice teachers used different questioning structures, with emphasis on focusing and funneling questioning, to elicit, interpret, and respond to student thinking.

The chapters on the case studies and the cross-case analysis demonstrate that these preservice teachers vary in terms of eliciting, interpreting, and responding to student thinking and in the development of questioning structures used in their teaching. Gloria sought a balance between questions that she planned to use and questions that she developed as a respond to student thinking, but she struggled to keep the focus on how the students were making sense of the problem. Beth was able to develop student-centered learning environment through an increased use of focusing questioning structures. In her desire to support students in solving problems, Clara found it difficult to withhold her thinking and allow opportunities for students to make sense of problems on their own. Michael created learning environments that maintained the focus on the teacher and used more directive instruction practices that guided students to solve problems in predictive ways. Elise found it challenging to help students when they responded in ways that she did not anticipate, and sometimes resorted to providing answers in order to guide the flow of the lesson in predicted ways.

In comparing these cases, Gloria and Elise provided opportunities to elicit student thinking but often funneled students to the strategies and answers they desired. Michael used an initiate-respond-evaluate (IRE) questioning structures (Mehan, 1979) to retain control of thinking about solving problems. Clara, in worrying about the success of her students, provided additional support and limited the range of student thinking. She funneled thinking toward the outcomes she wanted. Beth encouraged her students to think about their responses and focused the lesson to develop student thinking. The variation of teaching practices of these preservice teachers was not unexpected (Thompson et al., 2013). In addition, the difference between what they believe they are doing and what students are actually experiencing may reflect a gap between what preservice teachers think they can do and what they demonstrate in the classroom (Kennedy, 1999).

Meaning of Participant Experiences

The preservice teachers in this study were able to explore the development of a single lesson. This in-depth study of a single topic provided data unique to the preservice learning environment. They created a lesson plan, practiced the lesson, revised their plan, and implemented their lesson in classrooms over the course of the semester. Throughout the plan-practice-teach cycle, they were able to make adjustments to their lessons and improve how to elicit, interpret, and respond to student thinking, showing that it is possible to use practice-based teacher education successfully (Grossman et al., 2009). They benefitted from feedback from their course instructor, their mentor teacher, and their peers.

However, while the participants shared some common experiences as part of the course, each of them varied on what they learned during the plan-practice-teach cycle. This recalls the research of Philipp (2007) that teachers have differing beliefs about how mathematics is learned and how teachers should position themselves during instruction, as well as the work of Thompson, Windschitl, and Braaten (2013) on the different paths that preservice teachers take to build competence. They each brought different backgrounds and resources into their teaching experiences and each taught different lessons in different classrooms with different classroom expectations and different mentor teachers. All of these variances account for the different paths for each of the preservice teachers in this study. Each of the participants started at a different place in their perspective about using student thinking during instruction, impacted by tensions such as those described by Singer-Gabella et al. (2016), even while describing similar goals in their writing and their discussions. What was not directly addressed was how their views varied on what is effective mathematics instruction and how to achieve it. This is evident in the variance between how Beth left lesson development open until after the peer practice so that she could learn more about the topic to Michael stating that he learned nothing from his rehearsal. While each preservice teacher changed during the semester, the variance of starting points and backgrounds meant that the participants were widely varied in their approaches, what they learned, and how much they changed. This study showed that preservice teachers can start to learn about the use of focusing questioning structures and each of them will continue to learn more about questioning as they develop through their entire careers.

Implications

This analysis leads to implications for teacher educators and teacher preparation programs. Noting where the preservice teachers involved in this study met success and where they struggled provides important information for what best supports the development of core instructional practices. There are four implications from this study that have significant impact on the work of teacher educators:

- how to encourage preservice teachers to use focusing questioning structures over funneling questioning structures;
- how to understand and, when needed, overcome the beliefs that preservice teachers bring into their initial practice;
- how to effectively use the plan-practice-teach cycle to support preservice teachers eliciting, interpreting, and responding to student thinking; and

- how peer rehearsals need to be specifically designed in order to effectively address teaching strategies.

Teacher Educators Must Address How Preservice Teachers Implement Questioning Structures and Encourage the Use of Focusing

Research shows that focusing questioning structures allow students to create meanings of the mathematics they learn and to better understand mathematics through explanation (Wood, 1998), while funneling questioning structures give "the illusion that learning is actually happening" (Wood, 1998, p. 172). This study demonstrates that preservice teachers can learn to develop and use focusing questioning structures, as evidenced by how Beth develops and uses questioning through the plan-practice-teach cycle. However, the other participants struggled to move from planning to use questioning to create student-centered learning environments. They were more likely to provide students with guidance directed toward how the teacher was making sense of the problem rather than allow the students to guide the thinking. Emphasis in teacher preparation programs needs to consider how to foster focusing questioning structures and create the conditions to allow preservice teachers to benefit from activities that uncover and confront their beliefs and facilitate multiple ways to use questioning to better learn the thinking of students (building on the ideas of Philipp, 2007).

For example, Beth is able to plan questioning that allows for students to provide their own thinking. During instruction, she circulated around the room, using questions to elicit further thinking from her students. She would either ask the student for more explanation or extends their thinking to a broader environment. With either choice, Beth used the students' thinking to enrich her own understanding of what was taking place in the class. This also allowed students to discover their own mistakes and move toward solving problems independently rather than depending on the teacher for answers, similar to research of what is possible in growth in teacher questioning (Blanton et al., 2001; Doerr, 2006). The lesson develops through Beth's interpretation of student responses and her actions that privilege the use of student thinking to achieve learning goals.

However, while Beth was able to develop her use of focusing questioning, the other preservice teachers struggled to try to use focusing questioning structures while teaching. While these preservice teachers included in their reflections that they desired to engage students to think during instruction, when they moved from planning to teaching, they mixed in funneling questioning approaches; they moved to guide students to a particular outcome or provide interpretation of student thinking quickly in students' development of understanding. Teacher educators need to consider how to support preservice teachers to enact instruction to use student thinking throughout the questioning sequence, not just as an initial, open-ended question. Practice and coursework needs to include experiences that allow preservice teachers to consider multiple actions that respond to student thinking and learn how to choose actions that maintain student-centered learning environments throughout the questioning sequence. More discussion during planning should address incorrect and incomplete responses along with actions that preservice teachers should take. This is especially true for preservice teachers who may not have experienced these types of instructional environments in their own learning. Many of the preservice teachers in this study exhibited beliefs about the teaching and learning of mathematics that funneled students to their ideas. These beliefs presented challenges to eliciting, interpreting, and responding to student thinking.

Teacher Educators Must Provide Ways for Preservice Teachers to Address and Overcome

Their Current Beliefs and Work to Relax the Tensions these Beliefs Cause

The preservice teachers involved in this study created different roles for themselves during questioning that represented their views of what it means to teach mathematics. Those who used the funneling or IRE questioning structures saw themselves as the expert in the room, providing evaluative feedback or guiding students to the correct way of solving a problem. The lesson plan gave them a view of the lesson, and there were times when they steered the teaching of the lesson toward that vision. For instance, when Michael saw that a few students in the peer practice were identifying the boundaries of the double integral incorrectly, as predicted in his lesson plan, he immediately moved to present his explanation to the class. He saw his role as providing answers when students struggle. This belief in the role of the teacher of taking over the class contrasts with using student thinking to support development in a constructive manner, helping students make sense of problems and eventually solve the problems themselves (Singer-Gabella et al., 2016).

While research demonstrates that preservice teachers can leverage student thinking, this study confirmed that there are pressure points that can hold them back from success, as described in Singer-Gabella et al. (2016). The reservice teachers in this study sometimes struggled with how to move between their vision of what they expected in their lesson and how they taught the lesson, with some of their own views and beliefs about the learning of mathematics impacting how they created the learning environment in the classroom. When students struggled or provided an incorrect answer, many of the participants moved quickly to share their own thinking (as Clara did during her lesson with high school students) or to tell the students the correct answer (as Elise did during the peer practice). They struggled with the balance of allowing student thinking to guide instruction with managing the lesson and helping students to move toward achievement of the learning goal. This demonstrates the tension between the vision they expressed in their lesson plan and how mathematical learning looked during instruction. Michael, who expressed goals of having students make sense of problems in the reflection on his lesson plan, taught using more direct methods and with I-R-E questioning structures.

How preservice teachers move from planning a lesson to practicing the lesson with peers to implementing the lesson with students is a critical progression. Preservice teachers need to learn more about their own beliefs and how those beliefs impact their views on teaching. Learning more deeply about their own thinking can help preservice teachers to both envision and implement instruction that addresses student thinking in a consistent manner. Teacher educators need to support preservice teachers by helping them learn what to notice and how to react to what they notice during instruction. By eliciting what students are thinking and interpreting those thoughts, teachers notice what they view as important and adapt instruction based on what they notice (van Es & Sherin, 2002). Teacher preparation programs need to provide preservice teachers with experiences that allow them to practice questioning that maintains students being in control of their own thinking. The preservice teachers in this study often responded in ways that did not allow students to build on their thinking but instead provided teacher interpretations on how to solve the problem. The preservice teachers were able to elicit student thinking, but it is unclear if they were able to notice what was important, and their actions took the thinking away from students. In controlling their thinking, the preservice teachers guided students to construct understanding in the ways they felt were correct. This control limited their ability to adapt instruction to meet the needs of the learners, an important element of teaching that results from teacher noticing (van Es & Sherin, 2002). Teacher educators need to provide experiences for preservice teachers that practice identifying what is important during instruction, discuss why it is important, and discuss strategies on how to respond to what is important. Most of the participants in this study did not demonstrate progress in terms of learning how to encourage student thinking by attending to the mathematics content of the lesson, which is consistent with several earlier studies regarding teacher noticing (Star et al., 2011; Star & Strickland, 2008).

Teacher Educators Need to Support Preservice Teachers to Effectively Elicit, Interpret, and Respond to Student Thinking

Eliciting, interpreting, and responding to student thinking is a core practice in effective mathematics teaching (Ghousseini et al., 2015; McDonald et al., 2013; Windschitl et al., 2012). Each of the preservice teachers showed unique ways in how they use this practice. While four of the five teachers used opening questions to prompt student thinking, they still struggled with knowing what to do next. Each of them, including the preservice teacher who predominantly used the IRE questioning structure, were able to interpret student thinking. However, with the exception of Beth (who used focusing questions more than the others), all of the teachers viewed that interpretation as guidance for how to proceed in the lesson. Only Beth focused her follow-up questions to allow students to confirm her interpretations or change her interpretation based on additional elicited student thinking. Like Beth, teachers need to allow for student thinking to diverge from their expectations, provide a space for students to offer a range of solutions, and not have their ideas funneled into a common approach (Singer-Gabella et al., 2016). While each of these preservice teachers was able to initially elicit student thinking, they needed additional support in order to consider what to do with that information in the context of their lesson. Some of their plans included possible student responses and potential teacher actions, but little of that made its way into the instructed lesson. Teacher educators need to plan on how to aid preservice teachers to move from vision to enactment that elicits student thinking, interprets that thinking, and then determine appropriate ways to respond to that thinking. This can be done within the plan-practice-teach cycle, with peer practice allowing for a safe space for preservice teachers to consider how to respond to different student answers. Teacher educators can use these practice lessons to instruct preservice teachers on options that can be available to maintain the thinking with students during instruction (Lampert et al., 2010).

Perhaps due to the complexity of the classroom interactions or a failure to refer to the lesson plan during instruction, most of the response sequences during this study either provided the teacher's interpretation during questioning or did not become part of the instructed lesson. As Franke et al. (2009) showed, follow-up questions can clarify ambiguities, help teachers learn more about the reasoning behind student errors, and allow students to elaborate on their strategies. In this study, preservice teachers rarely used follow-up questions in these ways. Teacher educators need to support how to move preservice teachers from automatic responses that take over the thinking in a lesson and provide opportunities for preservice teachers to unpack student reasoning and encourage students to understand and elaborate. Too often, the follow-up questions by the preservice teachers in this study guided students in a direction the preservice teacher directed or provided the answer the teacher was seeking, prompting the notion that preservice teachers need more experiences that allow them to unpack classroom interactions and practice longer questioning sequences that go beyond identifying an individual response and teacher action sequence spelled out in a lesson plan. Instead, they need developmental experiences that would allow them to decompose practice in ways that make sense to them (Grossman et al., 2009), perhaps combining lesson planning, role playing, and peer practice in ways that allow for preservice teachers to respond to student thinking in meaningful ways that do not disconnect them from practice (Boerst et al., 2011).

Peer Practice Must be Carefully Tailored to be Effective

While preservice teachers may benefit from practice teaching with peers (Ghousseini et al., 2015; McDonald et al., 2013) and rehearsing in a safe, controlled environment (Ghousseini et al., 2015; Ghousseini & Herbst, 2016), they also need to experience deliberate and well-planned examples of core practices (Grossman et al., 2009). An interesting element of the study was the different experiences preservice teachers had regarding the peer practice. Part of this stemmed from how each of the teachers set the stage for the peer practice. Three chose to begin their peer practice at a certain section of their lesson, while the other two started at the very beginning of the lesson. Those that chose to emphasize the development of the learning goal of their lesson learned more from the rehearsal than those that merely started at the beginning of the lesson. Since the time allotted for peer practice was only 30 minutes, and each of their lessons took either 90 minutes or 45 minute per day for two days, focusing on a particular part of the lesson proved to be beneficial for those who selected a key feature to teach. In identifying specific elements of the lesson that were critical to the learning goal, the preservice teachers were able to focus their practice rather than hoping to get far enough to learn something of significance. Teacher educators need to set up the peer practices to focus on critical pieces in developing understanding of a lesson.

Another key element of peer practice is clearly defining roles for the peer students. Four of the preservice teachers identified what they expected students knew going into the given lesson. The one who did not, Gloria, struggled as her peers played the role of students at an earlier stage of mathematical development than her actual students. In her peer practice, Gloria spent most of her time supporting the warm-up instead of learning about important development promoted in her lesson. This was in part because she did not clearly assign her peers appropriate roles and expectations; as a result, she did not gain as much from her peer practice as she might have. As the different peer practices showed, when peers were better informed about the knowledge level of the students they were modeling, it provided a better experience for the teachers. The greatest benefit from peer practice for preservice teachers comes from an approximation of practice accompanied by coaching by teacher educators (Ghousseini et al., 2015). The more realistic that the approximation can be, the better the sense making is for the practicing teacher (Grossman et al., 2009).

All of the preservice teachers demonstrated the impact of the peer practice as they made changes to their lessons based on it. Even Michael, who stated that he did not learn much from the peer practice (Michael interview transcript, 2018), made significant changes to the handout in his lesson after the practice. Each of the preservice teachers adjusted the activities in their lesson plans, based on the feedback from peers in their roles as students. However, it is not clear that these changes were necessary for the targeted students in Michael's lesson. When it came to questioning structures, there was not as much change among the participants; some of the preservice teachers did not change their questioning at all, once they realized that their peers did not respond as their students might. This was especially true for Michael and Elise, where the challenge of the mathematics being taught was more of an issue than learning how to instruct the mathematics to students. For Michael, who taught a lesson on double integrals and iterated integration, a topic in multi-variable calculus that was not familiar to his peers, most of the lesson involved teaching his peers how to complete the activity. There was less emphasis on pedagogy as his peers were struggling to engage properly with the content. For Elise, the struggle came from getting her peers to complete geometric transformations using the appropriate tools, with her peers trying to remember the rules for completing transformations. Again, most of the interactions were instructional rather than pedagogical. These preservice teachers did not gain as much from their peer practice as others, due to the readiness levels of their peers compared with actual students.

When considering how to maximize the value of peer practice, teacher educators need to remember to coach both the preservice teachers acting as teachers and those acting as students to participate in ways that will benefit the implementation of the lesson in the actual classroom. In addition, teacher educators need to be active in their support of preservice teachers during these rehearsals of lessons and offer feedback that will support the development of targeted instructional practices. Course instructors need to supervise rehearsals and know when to pause the lesson in order to discuss the use and meaning of instructional strategies. Rehearsals need to be set up to target specific learning goals and teaching strategies that preservice teachers know are they are working to improve. There also needs to be consideration of what mathematical topics make effective rehearsal space. Some of the struggles for the peer practices was based on the peers not recalling mathematics well enough to act as the appropriate students. This may indicate that not all mathematical lessons are appropriate for peer practice; we need to investigate which mathematical topics are best to unpack student thinking (Singer-Gabella et al., 2016). Overall, teacher educators need to provide clear learning goals for practicing with peers, allow preservice teachers to identify critical elements of the lesson to practice, define roles for the peers acting as students in the practice, and focus on and actively support specific teaching strategies during the practices, building on the ideas of others (Grossman et al., 2009; Kazemi et al., 2016; Lampert et al., 2013). Each of these factors will increase the value of the peer practice and allow preservice teachers to investigate important teaching practices within the relative safety of their methods courses.

Generalizability

While the research developed in this case study provides valuable understanding regarding preservice teachers, a few constraining factors limit the generalizability of the findings. All participants were volunteers taken from members of a specific secondary mathematics methods class. There was no attempt to determine or assume that they were representative of the wide diversity of beliefs and experiences of those studying to become secondary mathematics educators or current members of the secondary mathematics teaching profession, especially as the entire population of teachers varies greatly. The scope of the study was one semester in a single secondary mathematics methods course focused on classroom interactions. While not enough time to reach a degree of proficiency on questioning structures, each studied in depth how their own questioning used either student or their own thinking most effectively.

All participants shared a common characteristic of starting graduate school immediately after their undergraduate degree. The main focus of the study was a single lesson taught by each of the participants, which may narrow the scope of the findings from this study. While commonalities of questions were connected across the plan-practice-teach cycle, full transcripts of the teach part of the cycle were not analyzed, so there may be connections that are missing from the analysis. There was no attempt to randomize nor control the participants, the topics of the lessons, or the schools and students with which they worked. Future research may consider selecting participants based on the mathematical topics of the lessons or the beliefs and expectations of the mentor teachers involved.

With that in mind, important understandings still emerge from this study. Preservice teachers can design lessons that help them elicit, interpret, and respond to student thinking. While they may initially elicit student thinking, some may require more experience in order to allow students to develop understandings on their own. Peer practice is beneficial in the preparation of teachers and offer certain characteristics that may benefit some preservice teachers more than others or may be of benefit in particular types of lessons more than others. While preservice teachers are able to identify the places where students are or are not making sense of the mathematics within a lesson, they need support in order to determine what to do with what they notice.

Future Research

This study is based on research in the fields of practice-based teacher education, questioning, and eliciting, interpreting, and responding to student thinking. The research looked at how preservice teachers develop questioning on a single lesson through the plan-practice-teach cycle within a semester course. How preservice teachers make sense of teaching is critical to understand in helping them become effective. Based on the findings and implications in this study, several potential questions arise. While there are many possible future research questions, these represent the questions closest to the research questions and themes of this study.

How do we help those learning to be teachers to use focusing questioning structures? More needs to be understood of how preservice teachers' beliefs impact their thinking about questioning and how to use student thinking. We need to provide vignettes and create activities that provide the opportunity for preservice teachers to share their thinking about initial questions, student responses, and follow-up questions. We need to learn more about teacher decision-making during the process of questioning and how to support effective decisions by teachers.

How do beliefs held by preservice teachers impact how they position themselves during questioning? Many preservice teachers have experience of the teacher being an authority in their own learning. Support will be needed for some preservice teachers to demonstrate how to elicit, interpret, and respond to student thinking, with a focus on making sense of mathematics while teaching, not merely in terms of the right answers, but examining multiple ways that students may think about a problem and how to develop their thinking as they attain different learning goals. Providing these experiences will impact their beliefs about the learning and teaching of mathematics (Philipp, 2007). They need to participate in mathematical discourse in different roles if the goal is for students to do the thinking within a lesson. To this end, research should develop learning experiences for preservice teachers to support them as they address their beliefs and learn about the roles available to them.

What is the most effective design for teaching preservice teachers how to best respond to student thinking during the plan-practice-teach cycle? Eliciting, interpreting, and responding to student thinking is a core practice of teaching mathematics. While each of the preservice teachers elicited student thinking during instruction, how they noticed, interpreted, and responded to that thinking varied, as did the degree of power which students had to think about the problems. How preservice teachers learn to respond to student thinking is the next step in providing speculative questions during instruction. As this is an important factor in how students develop their own thinking about mathematics, it is a critical element in teacher preparation.

What is the most effective structure for peer practice? This study demonstrates very different outcomes for each of the preservice teachers in terms of their peer practice. The cause of the variation in this study links to specific factors in each of the peer practices. Further studies need to clarify the best characteristics of effective peer practice.

Conclusion

The focus of this study was to investigate focusing and funneling questioning structures in lessons of preservice teachers. The study examined how preservice teachers elicit, interpret, and respond to student thinking; the process for development was the plan-practice-teach cycle; and the theoretical framework was teacher noticing, as how teachers use student responses to questions provides insights into what they notice as important and how they react to what they notice. Each of these helped to provide structure for analyzing the data and generating the findings and implications of the study.

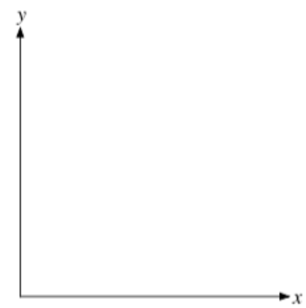
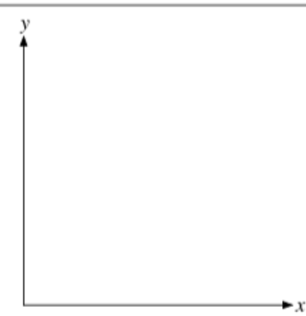
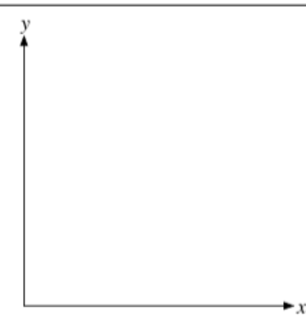
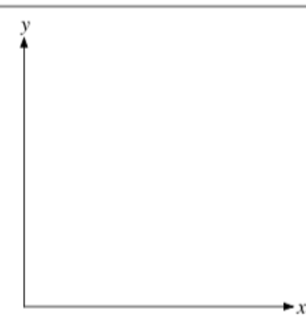
Preservice teachers varied significantly in their implementation of questioning during their lessons. They formed a continuum of questioning structures, with one focused on IRE questioning, three using funneling questioning in different ways, and one teaching consistently with focusing questioning. There were different experiences and benefits of learning from the peer practice activity and the variation of backgrounds and characteristics demonstrated the importance of individualizing support for preservice teacher development.

This study provided me clarity on the need for more research into how to help preservice teachers use questioning and student thinking most effectively, both in my own classroom as well as a profession. We need to provide vignettes that allow preservice teachers to consider multiple pathways within lessons based on correct answers, incorrect answers, and what to do when no students provide answers. Too many of the representations on lesson plans in this study considered a single pathway for questioning. Also, the structure of peer practices and rehearsals needs to be designed in ways that allow for the greatest learning by preservice teachers. Supervision needs to provide opportunities for learning within the practice so that preservice teachers know ways that they can change their practice in order to better elicit, interpret, and respond to student thinking.

Appendix A.Peer Interview Task

Four Situations

1. Sketch a graph to model each of the following situations.
Think about the shape of the graph and whether it should be a continuous line or not.

A: Candle Each hour a candle burns down the same amount. x = the number of hours that have elapsed. y = the height of the candle in inches.	
B: Letter When sending a letter, you pay quite a lot for letters weighing up to an ounce. You then pay a smaller, fixed amount for each additional ounce (or part of an ounce.) x = the weight of the letter in ounces. y = the cost of sending the letter in cents.	
C: Bus A group of people rent a bus for a day. The total cost of the bus is shared equally among the passengers. x = the number of passengers. y = the cost for each passenger in dollars.	
D: Car value My car loses about half of its value each year. x = the time that has elapsed in years. y = the value of my car in dollars.	

Appendix B.Analysis Assignment #1

Audio Analysis #1 (12 points)
Due: Saturday by midnight

The goal of this assignment is to provide an opportunity for you to practice *monitoring* student work. More specifically, you will practice *posing purposeful questions* that both assess and advance student thinking. Last class you completed the Four Situations Task. This week during class you were asked to anticipate student thinking/strategies (see Smith & Stein Chapter 4) and how to respond to their work. Then, you were asked participate in a role-playing activity that was audio recorded.

For this assignment you will use the audio recording during which you played the role of the teacher. Please transcribe the entire audio and then address the following prompts:

- What types of questions did you ask? (e.g., Were they *assessing* or *advancing* questions (Smith & Stein, 2015, p. 44)?)
- Explain what your goal was for each of the questions that you posed
- Reflect back on your questions and articulate what you learned about <u>student thinking</u>
- Suggest refined questions and explain how/why they are an improvement

There is no required format or length for your response. You are required to submit both your response and the audio file. If you have difficulties submitting, you can email the word document and upload the audio file.

<u>Criteria</u>	<u>1 points</u>	<u>2 points</u>	<u>3 points</u>	<u>4 points</u>
List each of your questions. Identify the question type and explain what your goal was or what you had hoped to elicit	Did not list or identify all questions. Need to make significant improvements to be clearer as to why you asked each of the questions.	Questions not all appropriately identified. Need to make very small improvements to be clearer as to why you asked each of the questions.	Clearly identified and articulated why you asked each of the questions – describe what you hope to get from asking each of the questions.	Exceptional articulation of why you asked each of the questions – describe what you hope to get from asking each of the questions.
Explain (in your own words) what you learned about the student's thinking	Did not identify correctly what the student was thinking and their incorrect ideas.	Needs to improve on identifying what the student was thinking and their incorrect ideas.	Satisfactory job identifying what the student was thinking and their incorrect ideas.	Exceptional job identifying what the student was thinking and their incorrect ideas.
Reflect back on your questions suggest refined questions	Need major improvements on your questioning practices, sharing what you learned, no refinements to questions.	Need minor improvements on reflecting on your questioning practices, sharing what you learned, and suggesting refinements to questions.	Satisfactory job reflecting on your questioning practices, sharing what you learned, and suggesting refinements to questions.	Excellent job reflecting on your questioning practices, sharing what you learned, and suggesting exceptional refinements to questions.

Appendix C.Lesson Plan and Commentary Assignment

Lesson Plan Draft & Commentary
(Due Oct. 13th - 16 pts.)

A draft of your lesson plan should be submitted by Saturday, October 13th. Although this draft is work in progress, I expect a thoughtful and thorough lesson plan to be submitted. There is no required format for the lesson plan (e.g., 5 E's, 3 column), however, there is a maximum length of 4 pages per lesson/day. In addition to a lesson plan, you will also submit a lesson planning commentary (maximum of 3 pages). Many of the prompts in the tables below are taken directly from the edTPA handbook [Some information summarized to maintain confidentiality of materials].

Criteria for Lesson Plan (max of 4 pages single spaced per lesson--you can link to materials)	**Pts.**
Standards [include Common Core State Standards for both content and practice]	1
Learning objectives [what mathematics will be learned and how students will learn it]	1
Instructional strategies and learning tasks [what students and teachers are doing, anticipate student responses, include formal and informal assessment]	5

Lesson Planning Commentary (max of 3 single spaced pages)	**Pts**
Planning for Mathematical Understanding (edTPA Rubric 1) [Place the content of the lesson in the progression of learning for the content, describe how connections are built, and include all sources]	3
Using Knowledge of Students to Inform Teaching and Learning (edTPA Rubric 3) [Include instructional strategies, supports, and how they will be used. Include mathematical errors and misconceptions that students may exhibit and how you will address them, identify level of cognitive demand of the task]	3
Monitoring Student Learning (edTPA Rubric 5) [How will you collect evidence of student learning]	3

Appendix D.Analysis Assignment #2

Analysis Assignment #2--Video of Rehearsal (12 pts.)
Due one week after rehearsal
(Monday, October 22nd/29th)

For this assignment you will need to analyze the video of your rehearsed lesson. From this video you will need to "provide 1-2 video clips (together totaling no more than 15 minutes, but not less than 3 minutes) that demonstrate how you interact with students in a positive learning environment to develop conceptual understanding, procedural fluency, AND mathematical reasoning and/or problem solving skills" (edTPA secondary handbook, 2018, p. 19). You will need to submit a written response to the following prompts in no more than 5 single-spaced pages (including prompts). Note that these prompts are taken directly from the secondary mathematics edTPA handbook [Some information summarized to maintain confidentiality of materials], however, not all bullets and criteria for each rubric are expected for this assignment.

Promoting a Positive Learning Environment (Rubric 6)
[Focusing on respect, rapport, and responsiveness]

Engaging Students in Learning (Rubric 7)
[Conceptual understanding, procedural fluency, and reasoning and/or problem-solving skills]

Deepening Student Learning during Instruction (Rubric 8)
[How did you elicit and build on student responses?]

Analyzing Teaching (Rubric 10)
[What changes do you make to improve the learning environment? Justify the changes.]

Appendix E.Analysis Assignment #3

Analysis Assignment #3--Video of Enactment Due two week after rehearsal (12 pts.)

For this assignment you will need to analyze the video of your rehearsed lesson. From this video you will need to "provide 1-2 video clips (together totaling no more than 15 minutes, but not less than 3 minutes) that demonstrate how you interact with students in a positive learning environment to develop conceptual understanding, procedural fluency, AND mathematical reasoning and/or problem solving skills" (edTPA secondary handbook, 2018, p. 19). You will need to submit a written response to the following prompts in no more than 5 single-spaced pages (including prompts). Some of these prompts below are taken from the edTPA handbook [Some information summarized to maintain confidentiality of materials].

Promoting a Positive Learning Environment (Rubric 6) [Focusing on respect, rapport, and responsiveness]

Engaging Students in Learning (Rubric 7)
[Conceptual understanding, procedural fluency, and reasoning and/or problem-solving skills and how the lesson connects students' backgrounds with new learning]

Deepening Student Learning during Instruction (Rubric 8) [How did you elicit and build on student responses?]

Analyzing Teaching (Rubric 10)
[What changes do you make to improve the learning environment? Justify the changes.]

Appendix F.Interview Questions

Interview Questions: Gloria

1. What has been your background leading you here [undergraduate → graduate, etc.]?
2. From you Analysis #1, you talk about how you felt about your questioning. How did you feel that your questioning changed over the development of your lesson?
3. <u>Overall</u>, how did you feel about your lesson with the students?
4. What was the impact of <u>practicing with your peers</u> on your lesson?
    a. What did you learn from that practice?
    b. Did you adjust your questioning based on the practice lesson? How?
5. Did you <u>follow your plan</u> closely or adjust the lesson as you taught?
    a. You changed your organization of your questioning in your lesson plan from assessing-advancing to probing-discussion gathering-making mathematics visible. Can you explain your decision to change?
    b. How did it compare to what you anticipated?
    c. Did anything surprise you? If so, what did you do about it?
    d. If you adjusted the lesson, what was the reason for doing so?
6. I see in your lesson plan annotations that you made some adjustments as well as used many questions that you planned. Can we go through some of those questions and you tell me your specific reflections on them?
7. What is your <u>goal for questions</u> that you ask during a lesson?
    a. What were some of the key questions from your lesson?
        i. Why did you feel that these questions are key/important?
        ii. What responses did you expect from the students?
        iii. How did students actually respond to these questions?
        iv. How did you interpret student responses?
        v. How did you act based on the student responses?
    b. How did your questioning evolve through the development of your lesson plan?

Appendix F.Interview Questions

Interview Questions: Beth

1. What has been your background leading you here [undergraduate → graduate, etc.]?

2. Overall, how did you feel about your lesson with the students?

3. What was the impact of practicing with your peers on your lesson?

    a. What did you learn from that practice?

    b. Did you adjust your questioning based on the practice lesson? How?

    c. During your peer practice, can you describe what your goals were when you approach a small group? Did this change in any way when you taught the lesson with high school students?

4. Did you follow your plan closely or adjust the lesson as you taught?

    a. How did it compare to what you anticipated?

    b. Did anything surprise you? If so, what did you do about it?

    c. If you adjusted the lesson, what was the reason for doing so?

5. What is your goal for questions that you ask during a lesson?

    a. What were some of the key questions from your lesson?

        i. Why did you feel that these questions are key/important?

        ii. What responses did you expect from the students?

        iii. How did students actually respond to these questions?

        iv. How did you interpret student responses?

        v. How did you act based on the student responses?

    b. How did your questioning evolve through the development of your lesson plan?

Appendix F.Interview Questions

Interview Questions: Clara

1. What has been your background leading you here [undergraduate → graduate, etc.]?
2. <u>Overall</u>, how did you feel about your lesson with the students?
3. Your lesson change significantly after the peer practice. What did you learn and why did you change?
    a. What was the impact of <u>practicing with your peers</u> on your lesson?
    b. What did you learn from that practice?
    c. Did you adjust your questioning based on the practice lesson? How?
4. Did you <u>follow your plan</u> closely or adjust the lesson as you taught?
    a. How did it compare to what you anticipated?
    b. Did anything surprise you? If so, what did you do about it?
    c. If you adjusted the lesson, what was the reason for doing so?
5. What is your <u>goal for questions</u> that you ask during a lesson?
    a. What were some of the key questions from your lesson?
        i. Why did you feel that these questions are key/important?
        ii. What responses did you expect from the students?
        iii. How did students actually respond to these questions?
        iv. How did you interpret student responses?
        v. How did you act based on the student responses?
    b. How did your questioning evolve through the development of your lesson plan?

Appendix F.Interview Questions

Interview Questions: Michael

1. What has been your background leading you here [undergraduate → graduate, etc.]?
2. From you Analysis #1, what was your goal for your questioning? Did you meet your goal?
3. <u>Overall</u>, how did you feel about your lesson with the students?
4. What was the impact of <u>practicing with your peers</u> on your lesson?
    a. What did you learn from that practice?
    b. Did you adjust your questioning based on the practice lesson? How?
5. Did you <u>follow your plan</u> closely or adjust the lesson as you taught?
    a. How did it compare to what you anticipated?
    b. Did anything surprise you? If so, what did you do about it?
    c. If you adjusted the lesson, what was the reason for doing so?
6. What is your <u>goal for questions</u> that you ask during a lesson?
    a. What were some of the key questions from your lesson?
        i. Why did you feel that these questions are key/important?
        ii. What responses did you expect from the students?
        iii. How did students actually respond to these questions?
        iv. How did you interpret student responses?
        v. How did you act based on the student responses?
    b. How did your questioning evolve through the development of your lesson plan?

Appendix F.Interview Questions

Interview Questions: Elise

1. What has been your background leading you here [undergraduate → graduate, etc.]?

2. From you Analysis #1, what was your goal for your questioning? Did you meet your goal?

3. <u>Overall</u>, how did you feel about your lesson with the students?

4. You identify in your second analysis assignment that "On multiple occasions I was too quick to provide students with an explanation, instead of questioning them further." What do you mean by this?

5. What was the impact of <u>practicing with your peers</u> on your lesson?

    a. What did you learn from that practice?

    b. Did you adjust your questioning based on the practice lesson? How?

6. Did you <u>follow your plan</u> closely or adjust the lesson as you taught?

    a. How did it compare to what you anticipated?

    b. Did anything surprise you? If so, what did you do about it?

    c. If you adjusted the lesson, what was the reason for doing so?

7. What is your <u>goal for questions</u> that you ask during a lesson?

    a. What were some of the key questions from your lesson?

        i. Why did you feel that these questions are key/important?

        ii. What responses did you expect from the students?

        iii. How did students actually respond to these questions?

        iv. How did you interpret student responses?

        v. How did you act based on the student responses?

    b. How did your questioning evolve through the development of your lesson plan?

Appendix G.Samples of Transcript Analysis

The following two transcript samples provide completed examples of the analysis of questioning and questioning structures. The first transcript (the next three pages) from Gloria' peer interview demonstrates two examples of the funneling questioning structure. The second transcript (the subsequent five pages) from Beth's peer practice provides two examples of the focusing questioning structure as well as an incomplete questioning chunk.

The coding of transcripts used the following process:

1. Questions within the transcript were identified using the definition of "question" (p. 59).
2. These questions were next categorized using question types and descriptions in red (pp. 63-64).
3. Questions were chunked using the definition of chunks (pp. 61-62).
4. The analysis of the chunks allowed for classification as focus, funnel, or IRE questioning structures in green (pp. 61-66).
5. As evidenced in the second example, the linking of questions across lesson plan, peer practice, and high school lesson implementation was done to consider change of the preservice teacher through the plan-practice-teach cycle.

Appendix G.Samples of Transcript Analysis

Appendix G.Samples of Transcript Analysis

T: Okay, why don't you go ahead and tell me what you were thinking when you were doing B?

Commented [MOU1]: WH

Opening probing question – start of a "chunk"

S: On B, well I thought that since there are 0 ounces, it would just be like 0 dollars. You know?

T: Okay, so what you're looking at here is that it tells you that you pay quite a lot for letters weighing up to 1 ounces. What does up to an ounce mean?

Commented [MOU2]: QC

S: I think it means...any amount less than an ounce, excluding negative numbers.

T: Okay, that's good. I'm glad that you brought up negative numbers, because we can't have negative weight? Right?

Commented [MOU3]: PL

Here, the teacher is providing her thinking to the student. She has moved into funneling.

S: It's true, it's true.

T: Yeah. So we're thinking of up to an ounce meaning any amount that's less than one. But what you have here is that you've just got a point on (0,0) and then a point at x=1 and I'm wondering about all of the numbers between 0 and 1. Because you said that up to ounce means anything less than 1.

Commented [MOU4]: QC

S: Well, I was just thinking that...I guess I was thinking that at 0 ounces it's at this price, and when you're at 1 ounce it's at this price. But I guess I forgot about all the numbers in between. So maybe I should make it a piecewise function...

T: Cool! So what would your piecewise function, maybe, look like?

Commented [MOU5]: QC

S: I could make it a line between the one at 0...well, wait, okay so, y is the cost of sending the letter in cents.... and then the weight is x... I don't know. I have no clue what it would look like.

T: Well, I'm really glad that you brought up piecewise functions. Cause this is probably going to work out really well for this function. Because we really, we can't forget about all of those numbers. And the way that the situation is presented it kind of makes you think that you're going to pay the same amount for all of the numbers between 0 and 1.

S: Oh! So would I just draw a horizontal line?

T: Yeah, do you remember what a horizontal line is called? What kind of function that is?

Commented [MOU6]: WH

S: No I don't.

T: It starts with a "c"...

S: Oh, a constant function?

T: Constant function, yeah. So that would probably be one way that you could approach this. It seems like it fits with the problem, right?

Commented [MOU7]: PL

This question reinforces the funneling structure as the teacher is guiding the student to her way of thinking.

Completion of a "chunk"

Appendix G.Samples of Transcript Analysis

Appendix G.Samples of Transcript Analysis

S: Okay.

T: Cool, and then the only other thing I want to talk about is, well I really like the way that you presented situation A with the candle. That looks really good. But for situation B, you have kind of a similar thing going on.

S: You mean C?

T: For C, yes, sorry. Situation C, having to do with the bus. You have sort of a really similar graph going on, and I wondered if maybe you could explain why.

Commented [MOU8]: Q

Start of a new "chunk". Here, the teacher sees a connection and asks the student to see it as well. Funneling – sees a similarity and a difference and wants the student to see it too

S: Well… so I think that I needed to label it more.

T: Okay.

S: So, a group of people rent a bus for a day. The total cost is shared equally, so if this were like 20 people and then this were like 10 people, here and here… then…this were $0 and then this were $20 then this were $40, so it just went down for every time someone else is added. Like, more people.

T: So, can I ask what does the x-axis represent?

Commented [MOU9]: WH

S: The x-axis is the number of passengers.

T: Okay.

S: And then the cost is here. Whoops, I mean this is the people. So, no, this—these are the dollars, so let's make that $200 and that $100, and then these are the people

T: Okay.

S: So, it goes down and...people...wait, it goes down in cost for every time a passenger is added. And then this arrow means that it's just going to keep going down.

T: Okay, so the arrow tells us that it's going to keep going down.

Commented [MOU10]: WH

S: Like you know how when you have this type of function where it goes like that and then you draw an arrow?

T: Yeah.

S: It's just like that, but for this.

T: I'm seeing a little bit more of a difference between this function and that one. Can you maybe spot a difference too?

Commented [MOU11]: ST

Appendix G.Samples of Transcript Analysis

Appendix G.Samples of Transcript Analysis

S: This one is maybe like a logarithmic or exponential and this one is linear.

T: Yeah, and what's a special thing about an exponential function?

Commented [MOU12]: P

S: It never touches the axis, or its asymptote or anything.

T: And that might be something to think about, because no matter how many how many people you add to the bus, the cost will never end up being 0, will it?

Commented [MOU13]: PL.

End of second "chunk" of questions.

S: Nah, you're right.

T: So, that's my only concern. I like your logic. Your logic is really sound for everything else—I could follow what you were talking about, and that was really good. You had a lot of good words in there, and you gave a good example for it. And that was great.

Appendix G.Samples of Transcript Analysis

Appendix G.Samples of Transcript Analysis

Transcript of Peer Practice
Removed some "like"'s if did not impact understanding

Part 1

Course Instructor (0:03): So we are starting our second presentation.

Teacher (0:09): So, I'm interning at [school name] and I'm teaching on level Pre-Calc and so this has been our logarithmic and exponential functions unit. So, this is a front and back of what my students have already learned prior to this lesson, which is an activity for them to further explore the properties of the logarithms and solving logarithmic equations. So, the front is specifically things that they will, they'll have a notes sheet with all of the properties of the logarithmic function and properties of logarithms and they will have the tables worked out, and the back is stuff they should have learned in Algebra 2 but this is also something that they already have in their notebook from the very beginning of the semester. We went over the different families of functions. So, this is just (1:00) in case you, I just want you guys to have the same things to refer to that they will be able to refer to during this activity.

Commented [MOU1]: Provides background.

Student1 (1:07): She's saying we don't know it.

Teacher (1:11): No, no. You never know.

Student1 (1:14): She's right, I want you to know.

Teacher (1:14): It actually helped me when I went through the activity 2. I got the table from the textbook and I was looking at it and it helped me a lot. So, this lesson, I'll have a warm-up, but I am still working on that, so I'm not just going to do a warm-up with you guys, I want to do the main part of the lesson, which is, half the class is going to be working on one activity, and the other half is working on another. They're similar but they're different, but they kind of ask some of the same ideas. So, I'll give about half of you one activity, and half of you the other, and, the way the lesson will go, is they'll work on it individually, which I will give you guys five minutes to work on it individually, and then work in groups. And then they'll be some group culminating discussion or I might have them present their methods or their ideas in groups (2:00), I haven't really totally decided, but, for the purpose of this time, I just want to use it to see how you guys go about solving the tasks, and the problems, and discussing it, so that I can feedback about the tasks themselves and what they're getting at. And, as far as the things that I would want feedback on, are my, I prepared a lot of questions, but if you have any feedback on the way that I ask questions, are they confusing? Sometimes I ask questions and sometimes I think I confuse them more. So if you have feedback on that. Or if you have any suggestions for how students should go about presenting this to their classmates, or not even presenting, but sharing it to the class. So, I'm going to give you guys all one of the tasks. And I am going to set a timer for five minutes to work on it individually. And please, do reference the handout if you do, if you get stuck, it helps to look at some of the things on the handout I already gave you to jog some ideas. *Distributes handouts.*

Appendix G.Samples of Transcript Analysis

Appendix G.Samples of Transcript Analysis

Student2 (3:06): Do you want us to write our names?

Teacher (3:07): That would be great. Yeah, and if you could do work on the paper, or on the back of the paper, so I can look at it.

Students quietly work. Teacher circulates.

Teacher (8:25): Ok, so it's been five minutes. So, if you could share with your groups what you've done so far. What ideas you have and begin working together to see if you can come up with some ideas...

Class splits into three groups. Teacher goes to group A (Student3, Student4, Student5).

Student3 (9:10): That's what I was trying to figure out.

Teacher (9:13): Does it say that Gilberto is wrong? Or, just that hers is right?

Commented [MOU2]: F – modified from lesson plan

Focusing

Start of a "chunk"

Student4 (9:20): I think where the solution gets lost is where she develop track...

Student3 (9:25): Yeah, I got here and

Student4 (9:27): ...where did her solution get lost?

Student5 (9:28): Where is this procedure ...

Teacher (9:33): So if there you guys see two solutions...

Student4 (9:35): Gilberto's right.

Student5 (9:36): We both (*points to Student3*) it as ok, Gilberto is wrong. Find out why he's wrong.

Student3 (9:40): No, no. I read it as he's right. I think he's right. But I also think that's assuming that she's right as well.

Teacher (9:50): Is there a way you can find out if her solution works in the equation?

Commented [MOU3]: SP – not in lesson plan (more of making sense of task, not of strategies)

Student4 (9:54): Yes.

Student3 (9:56): I did plug it in. I plugged the negative in...

Teacher (9:59): You plugged it in and you got...

Appendix G.Samples of Transcript Analysis

Appendix G.Samples of Transcript Analysis

Student3 (10:02): I got I got that it did make sense. But I also got that since it is a squared term, I get a positive and a negative, maybe.

Teacher (10:10): That's an idea to keep in mind. So, when you plugged in, you got the solution, when you plugged in -2, you find that it did work? Did everyone try plugging it in? Maybe try plugging it in and...

Student5 (10:25): Well, wait a minute. I thought it said, I thought it was saying, Ok, Gilberto Is right, or, yeah, Gilberto is wrong. So I didn't even think, I wasn't able to work on it that way at all.

Teacher (10:36): It's actually, at the start, was her solution correct? And then you can try to figure out what is going on.

Teacher moves to group B (Student1, Student6, Student7)

Student1 (10:53): So, then, what happens for that student who doesn't see that piece? Because you're right, mathematically, everything looks valid. And it's that idea of, instead of a false solution, you lost one of the solutions, you lost a correct solution, and why doesn't that work? I think that second one may be more powerful than the first one. Where did the correct solution get lost seems more pertinent than that mathematical insight, what do you do for the kid who doesn't see that? Because I am now starting to look at this term, and figure out, well that has to be positive, what happens when, when I first thought was dividing by 0, something like that. I didn't think about the fact that those two terms being the same – that law, that I can put them back together. Yep, yep.

Teacher (11:45): How can students find, come to some sort of understanding? How is there a way that they can avoid losing a solution? Of not getting a solution? Since you guys are saying that this all is valid. How can they go about the procedure of solving the equation where its valid, but they are not losing a potentially viable solution?

Student6 (12:11): They need to know where the function's defined, so it comes down to domain and range. So it wasn't until I plugged 7 back in, and it is an answer...

Student1 (12:18): Right, right.

Student 6 (12:19): But it's not a solution.

Student7 (12:22): Knowing this is part of the problem. The question is, how do we make them do that?

Student1 (12:33): And then for me, its...

Teacher moves to group C (Student2, Student8)

Commented [MOU4]: Q – modified from lesson plan

Commented [MOU5]: F – not in lesson plan (more of making sense of task, not of strategies)

End of a "chunk"

Commented [MOU6]: WH

Incomplete sequence – multiple questions but students not able to respond appropriately.

Commented [MOU7]: WH–in lesson plan

Commented [MOU8]: WH– not in lesson plan (more of making sense of task, not of strategies)

Commented [MOU9]: WH

Sequencing many questions in a row

Similar to lesson plan sequence in absolute value strategy

Appendix G.Samples of Transcript Analysis

Appendix G.Samples of Transcript Analysis

Student8 (12:35): Like an extra "not real" solution.

Student2 (12:37): It's not necessarily a solution.

Student8 (12:41): It's still a,,,

Student2 (12:43): It's not a number in the domain.

Student8 (12:45): it's still an answer, but not a solution.

Teacher (12:47): Ok, so what do you mean by, what's the difference between an answer and a solution?

Commented [MOU10]: WH

Focusing

Start of a "chuck"

Not directly in lesson plan – but similar to contrast between Kate is wrong and Jason is right

Student8 (12:52): So when I did it out, I did got both 7 and -13.

Teacher (12:56): Ok. Ok. When you did...

Student8 (12:59): So when I, I started from this part, because I agreed with how she used the rule to get to here.

Teacher (13:05): So you agreed with her first step.

Student8 (13:06): So I started from here, and then I redid the whole thing.

Teacher (13:09): Ok, and how did you redo it? What was different?

Commented [MOU11]: WH – not in lesson plan, making sense of student thinking

Student8 (13:12): So what I did was, instead of like, make this, what she did here was log $(x + 3)^2$ equals 2, because she was there was two of them. But instead of doing that, I decided to go maybe the longer route, because I went backwards from this rule, and I just combined them by multiplying in the middle. So that means I got a quadratic. So then I did the quadratic formula.

Teacher (13:34): I see

Student8 (13:35): So then I got both answers. So I agree that this is an answer, it's just not a real answer. So that's what makes it not a solution.

Teacher (13:43): Do you agree with the way they word it as a false solution?

Commented [MOU12]: Q – similar to question in lesson plan

Student8 (13:46): Yeah. So I feel like it's a false solution. I don't think that there is anything's wrong with what she did. I feel like her method is not correct.

Appendix G.Samples of Transcript Analysis

Appendix G.Samples of Transcript Analysis

Teacher (13:52): Well, if her method is not incorrect, how can she make sure that her procedure includes both solutions? So, if her work's correct, but she really did find both solutions, it might be viable solutions? What can she do differently?

Student8 (14:10): I feel like what she didn't do is when you take 10 squared, if I do the opposite of 10 squared, that would be the square root of 10. And when I do the square root of 10, I do the square root of 10, I get plus or minus a number. I feel like she forgot to do that.

Teacher (14:24): Ok.

Student8 (14:25): I personally I agree with his thinking.

Student2 (14:29): What I saw, I tried I kind of like noticed, I ignored she did and started out for myself. So this came, and I said, ok cool, those are gone. Anod those two add, and I made sure the signs are right, and I got the same thing she had, $2 \log_{10}(x + 3) = 2$. And I was, oh, this equals this, or I can cancel those out, and this equals 1. But then I was going to get 7 again. So, then I went back, and I was, ok, I'm going to apply the power rule backwards, and then put the 2 up here, and then multiply it out and I was going to get that...

Student8 (15:01): He was going to what I did.

Student2 (15:02): ...and use the quadratic formula and I was going to get -13, which I saw is what Jason got.

Student8 (15:08): Could you talk about, he said that maybe you weren't allowed to cancel the 2.

...

Commented [MOU13]: WH – in lesson plan

Commented [MOU14]: WH – similar to question in lesson plan

End of a "chunk"

Appendix H.Gloria.Task Part 1

Chopping Logs Part 1

A Solidify Understanding Task

Abe and Mary are working on their math homework together when Abe has a brilliant idea!

Abe: I was just looking at this log function:

$$y = \log_2(x + b).$$

I started to think that maybe I could just "distribute" the log so that I get:

$$y = \log_2 x + \log_2 b.$$

I guess I'm saying that I think these are equivalent expressions, so I could write it this way:

$$\log_2(x + b) = \log_2 x + \log_2 b$$

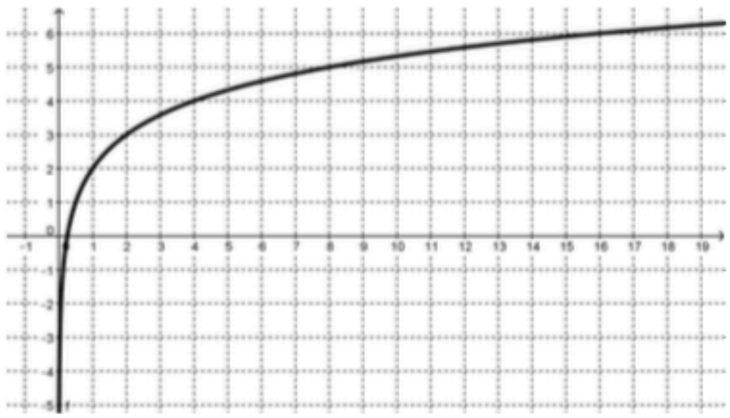
Mary: I don't know about that. I don't think that you're really doing the same thing here as when you distribute a number.

1. What do you think? How can you verify if Abe's idea works?

2. If Abe's idea works, give some examples that illustrate why it works. If Abe's idea doesn't work, give a counter-example.

Appendix H.Gloria.Task Part 1

Abe: I just know that there is something going on with these logs. I just graphed $f(x) = \log_2(4x)$. Here it is:



It's weird because I think that this graph is just a translation of $y = \log_2 x$. Is it possible that the equation of this graph could be written more than one way?

3. How would you answer Abe's question? Are there conditions that could allow the same graph to have different equations?

Mary: When you say, "a translation of $y = \log_2 x$" do you mean that it is just a vertical or horizontal shift? What could that equation be?

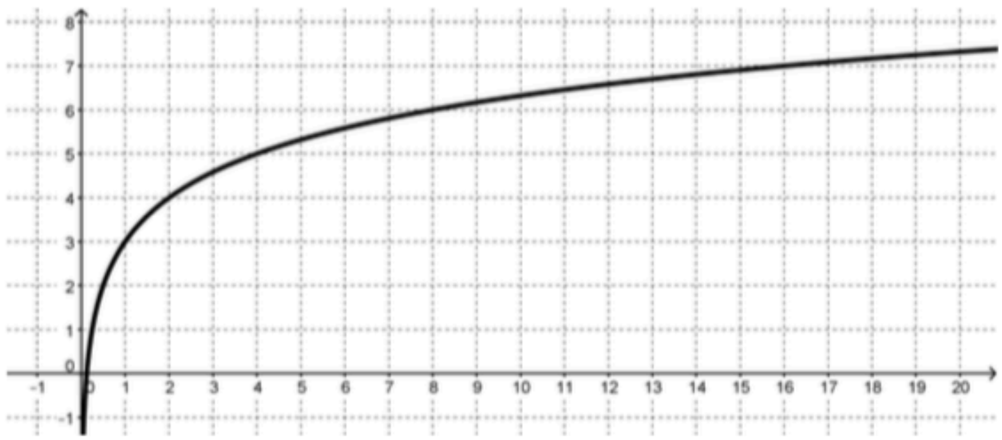
4. Find an equation for $f(x)$ that shows it to be a horizontal or vertical shift of $y = \log_2 x$.

Appendix H.Gloria.Task Part 1

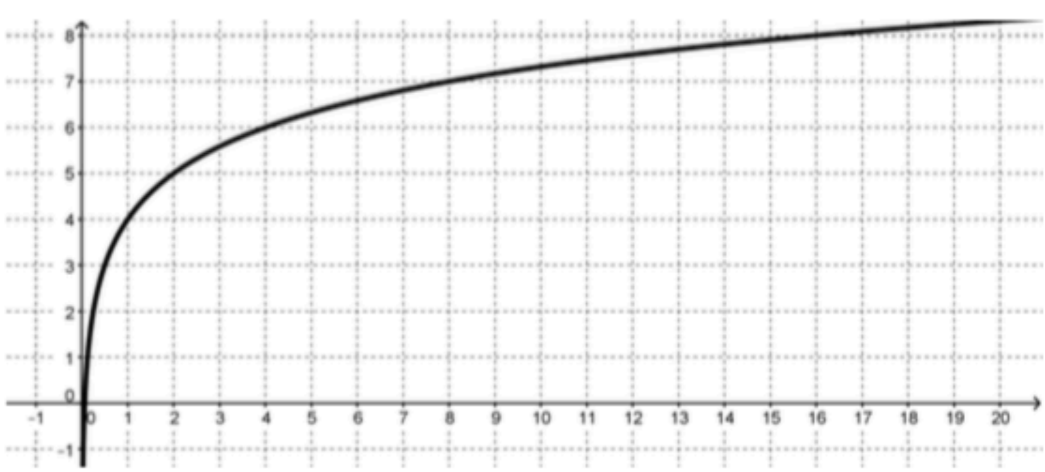
Mary: I wonder why the vertical shift turned out to be up 2 when the x was multiplied by 4. I wonder if it has something to do with the power that the base is raised to, since this is a log function. Let's try to see what happens with $y = \log_2(8x)$ and $y = \log_2(16x)$.

5. Try to write an equivalent equation for each of these graphs that is a vertical shift of $y = \log_2 x$.

a) $y = \log_2(8x)$ Equivalent equation: ________________



b. $y = \log_2(16x)$ Equivalent equation: ________________



Appendix I.Gloria.Task Part 2

Chopping Logs Part 2
A Solidify Understanding Task

Mary: Oh my gosh! I think I know what is happening here! Here's what we see from the graphs:

$$\log_2(4x) = 2 + \log_2 x$$

$$\log_2(8x) = 3 + \log_2 x$$

$$\log_2(16x) = 4 + \log_2 x$$

Here's the brilliant part: We know that $\log_2 4 = 2, \log_2 8 = 3,$ and $\log_2 16 = 4$. So:

$$\log_2(4x) = \log_2 4 + \log_2 x$$

$$\log_2(8x) = \log_2 8 + \log_2 x$$

$$\log_2(16x) = \log_2 16 + \log_2 x$$

I think it looks like the "distributive" thing that you were trying to do, but since you can't really distribute a function, it's really just a log multiplication rule. I guess my rule would be:

$$\log_2(ab) = \log_2 a + \log_2 b$$

6. How can you express Mary's rule in words?

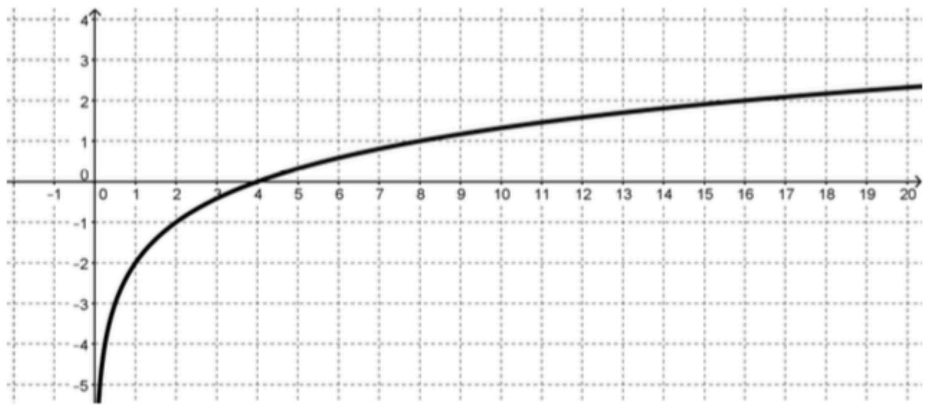
7. Is this statement true? If it is, give some examples that illustrate why it works. If it is not true provide a counter example.

Appendix I.Gloria.Task Part 2

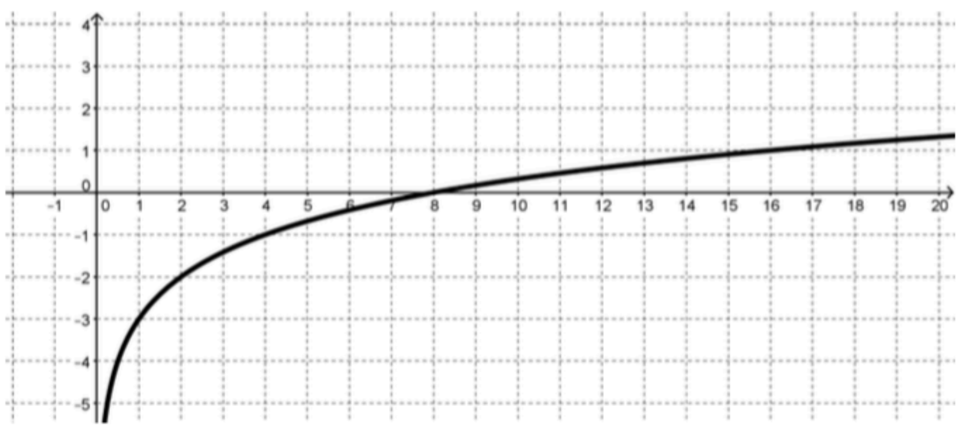
Mary: So, I wonder if a similar thing happens if you have division inside the argument of a log function. I'm going to try some examples. If my theory works, then all of these graphs will just be vertical shifts of $y = \log_2 x$.

8. Here are Abe's examples and their graphs. Test Abe's theory by trying to write an equivalent equation for each of these graphs that is a vertical shift of $y = \log_2 x$.

a) $y = \log_2\left(\frac{x}{4}\right)$ Equivalent equation: ________________________________



b) $y = \log_2\left(\frac{x}{8}\right)$ Equivalent equation: ________________________________



9. Use these examples to write a rule for division inside the argument of a log that is like the rule that Mary wrote for multiplication inside a log.

Appendix I.Gloria.Task Part 2

10. Is this statement true? If it is, give some examples that illustrate why it works. If it is not true provide a counter example.

Abe: You're definitely brilliant for thinking of that multiplication rule. But I'm a genius because I've used your multiplication rule to come up with a power rule. Let's say that you start with:

$$\log_2(x^3)$$

Really that's the same as having: $\log_2(x \cdot x \cdot x)$

So, I could use your multiplying rule and write: $\log_2 x + \log_2 x + \log_2 x$

I notice that there are 3 terms that are all the same. That makes it: $3 \log_2 x$

So my rule is: $\log_2(x^3) = 3 \log_2 x$

If your rule is true, then I have proven my power rule.

Mary: I don't think it's really a power rule unless it works for any power. You only showed how it might work for 3.

Abe: Oh good grief! Ok, I'm going to say that it can be any positive number x, raised to any power, k. My power rule is:

$$\log_2(x^k) = k \log_2 x$$

Are you satisfied?

11. Use similar logic as Abe does at the top of this page that will support his generalized power rule.

Appendix I.Gloria.Task Part 2

Abe: Before we win the Fields Medal for mathematics I suppose that we need to think about whether or not these rules work for any base.

12. The three rules, written for any base $b > 1$ are:

    Log of a Product Rule: $\log_b(xy) =$

    Log of a Quotient Rule: $\log_b\left(\frac{x}{y}\right) =$

    Log of a Power Rule: $\log_b(x^k) =$

    Make an argument for why these rules will work in any base $b > 1$ if they work for base 2.

13. How are these rules similar to the rules for exponents? Why might exponents and logs have similar rules?

Appendix J.Beth.Activity 10_The Lost Solution

10 The Lost Solution

Gilberto solved an equation involving the log function:

$$\log(x-1)^2 = 2\log 3$$
$$2\log(x-1) = 2\log 3$$
$$\log(x-1) = \log 3$$
$$x-1 = 3$$
$$x = 4$$

He checked his solution by substituting it into the original equation and found that it worked. However, his friend Reatha pointed out to him that her solution was −2. Was Reatha's solution correct? At what stage in Gilberto's argument did this solution get lost?

Appendix K.Beth.Activity 12_The Wrong Solution

12 The Wrong Solution

Kate solved the equation

$$\log[(x+3)(x-8)] + \log\left(\frac{x+3}{x-8}\right) = 2 \qquad (1)$$

as follows (logarithms are expressed in base 10):

She used the basic log laws first:

$$\log(x+3) + \log(x-8) + \log(x+3) - \log(x-8) = 2 \qquad (2)$$

She then simplified to

$$2\log(x+3) = 2, \qquad (3)$$

giving $x+3=10$, and hence $x=7$.

However, when she substituted her value into the original equation, she found that neither term on the left-hand side of the equation was defined, so her answer was certainly wrong. Moreover, her friend Jason had used a different method to solve the equation and had found the solution $x=-13$, which worked. What was wrong with Kate's method? Where did the false solution creep in, and where did the correct solution get lost?

Appendix L.Clara.Lesson plan 2 draft classwork

13 2 > 3 by Logarithms

We know that $\frac{1}{4} > \frac{1}{8}$.

Therefore, $\left(\frac{1}{2}\right)^2 > \left(\frac{1}{2}\right)^3$.

Find the logarithms of both sides of the equation to base a for $a > 0$ and $a \neq 1$. Then,

$$\log_a\left(\frac{1}{2}\right)^2 > \log_a\left(\frac{1}{2}\right)^3,$$

giving

$$2\ \log_a\left(\frac{1}{2}\right) > 3\ \log_a\left(\frac{1}{2}\right)$$

or

$$2 > 3.$$

How can that be?

Appendix M.Michael.Class Worksheet

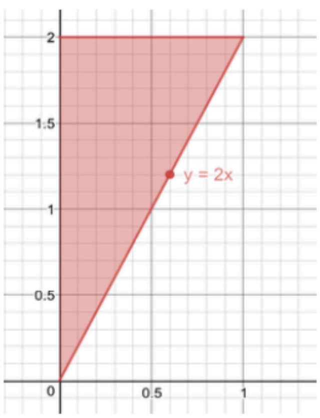
1) $\int_0^2 \int_0^3 (2xy + 3e^{2x})dydx$

2) $\int_0^1 \int_0^1 \frac{1}{\sqrt{1-y^2}}dydx$

Hint: arctrig

3) $\iint_R (x+y)^2 dA$

R is the region depicted below:



Hint: Check your bounds of integration! Do they match up with the diagram?

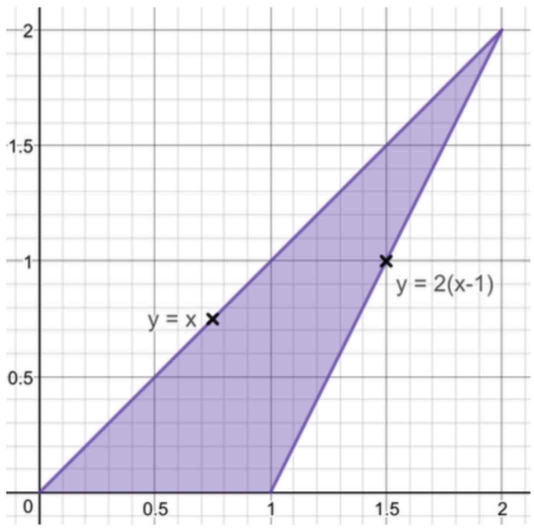
4) $\iint_R \frac{1}{1+y^2} dA$

R is the region in the first quadrant under y = tan(x), and to the left of x = pi/4.

Hint: arctrig
Hint 2: sketch the region being described. Do your bounds of integration match up with the diagram you sketched?

5) $\iint_R \frac{y}{x} dA$

R is the region depicted below:



Hint: Sketch your bounds of integration! Do they match up with the diagram?

6) $\int_1^2 \int_0^{\pi/2} ysin(xy)dydx$

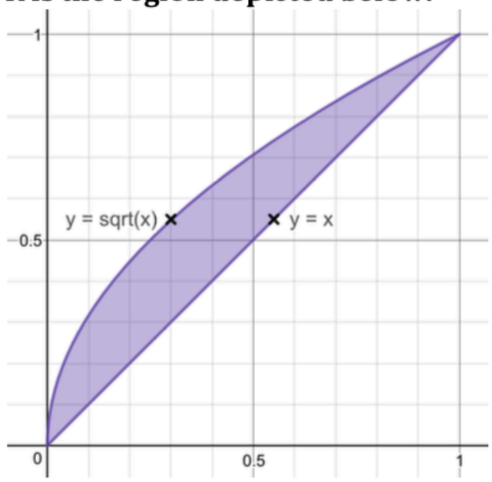
Hint: How did we get dydx from dA? Is there something else we can get from dA that may make this function easier to integrate? Once you have an idea, try applying it to a couple of the problems we have already solved and see what results you get!

7) $\int_0^2 \int_{x^2}^4 \frac{x}{e^{(y^2)}} dydx$

Hint: Did you try doing something similar to what you did for question 6? If so, think about how that change may affect your bounds of integration.

8) $$\iint_R \frac{e^y}{y}\, dA$$

R is the region depicted below:



Hint: If you ever get stuck, try differentiating $f(x) = e^x - xe^x$ a few times. Notice a pattern? If not, try factoring the function and its derivatives.

Appendix N.Elise.Activity Sheet

Mapping Transformations: *Exploration*

Start at the turtle at point **J20 (Visitor parking lot)** on the [redacted] Find the next point on the map by following the steps in the table below to reach your final destination. Record each new point in the table as well as the turtle's direction (Is its nose pointing left, right, upward, or downward?). Record the building name until you reach the final location.

Movement	**Finish point**	**Direction of** turtle's nose	**Building Name**
Translate 3 units left and 1 unit down			
Reflect over Line 15			
Translate 6 units left			
Rotate 180° clockwise centered at **I7**			
Translate 2 units up and 4 units right			
Reflect over Line 14			
Rotate 90° counterclockwise centered at E20			
Translate 1 unit right and 5 units down			
Reflect over Line 13			

With your group, write a small story that goes along with the turtle/your journey. You can reference the ***information booklet*** to find out more about each location.

For example, if I started in the physics building, then the library, and then the [redacted]; I could have the following story:

> *I went to my [redacted] class in the physics building to take my test. I took the test and realized I knew absolutely nothing about [redacted] so I headed over to the library to study. My studying lasted only 20 minutes because I was hungry so I went to the [redacted] to grab some food.*

Try to be creative (and funny)!

Appendix N.Elise.Activity Sheet

Rules for Challenge Questions:

- ***A Translation is a walk.*** For example, if I translate 2 units left, I am walking 2 units.
- ***Reflections and Rotations are "jumps."*** If I reflect or rotate over a building, I am "jumping" over the building.
- ***The center of the turtle is your point.*** However, when starting these problems your turtle may start in any direction. You just have to make sure that the turtle's center lands on the required point.

1. You're hanging out and throwing a frisbee with your friend on [redacted] (**9K**). You check the time and realize you have 2 minutes before your class begins. Your class is all the way in the [redacted] (**15E**). Find the quickest way to get there! Use as little transformations as possible.

2. Your class in [redacted] (**13E**) ended early and you have some time to kill but your next class is right next door in the [redacted] (**11E**). To waste some time, use the 3 transformations (translation, rotation, reflection) at least once. Record the transformations below.

3. You realize you can't cook, so you need to find a place to eat. You start from your dorm and must make your way to some food. Choose your dorm and the restaurant you want to go to (can be found in booklet). Try using the least amount of transformations as possible. Record the transformation below. (If you chose a different dorm or a different restaurant, could you reduce your transformations even more?)

4. You need 16 credits to graduate, which is 4 or more classes. Use the instruction booklet to pick your classes. Record the class and its location in the table below. Once you have scheduled the required 4 classes, you may use the free spaces to take another class or select a different event. For example, you might have practice for a sport you play, a club meeting, a group study session, etc.

   Determine which transformations you will use to get to each location.

Appendix N.Elise.Activity Sheet

Schedule			
Time	**Building or Class Name**	**Building location**	**Transformations used to move from previous location**
8:00 - 9:00 AM	Wake up Dorm: ________________		
9:00 - 10:30 AM			
10:00 - 11:30 AM			
12:00 - 1:30 PM	Lunch Restaurant:______________		
2:00 - 3: 30 PM			
4:00 - 5:30 PM			
6:00 - 7:30 PM			
8:00 - 9:30 PM			
10:00PM - 8:00AM	Sleep Dorm:_________________		

Appendix O.Case Study Highlighting

Case study coding

Planned versus extemporaneous questioning

Focusing questioning structure

Funneling questioning structure

Eliciting student thinking

Interpreting student thinking

Responding to student thinking

Gloria: Follow the Plan

…

Initial peer practice

In a teaching activity early in the methods course, peers took on roles of teacher and student in one-on-one interviews that involved an activity about student-created graphs (see appendix x). Gloria acted as a teacher and another preservice teacher acted as her student (this preservice teacher is identified as the "student" for the remainder of this section) to practice how the teacher would make sense of graphs created by the student. After the student completed his graph, Gloria initiated the discussion about one of the graphs by asking, "Why don't you go ahead and tell me what you were thinking when you were doing B?" (Gloria analysis #1 transcript, 2018, line 1). This question allows Gloria to elicit student thinking. The student's response shows his thinking based on one particular element of his graph, "I thought that since there are 0 ounces, it would be just like 0 dollars" (Gloria analysis #1 transcript, 2018, line 3). In her response, Gloria provides the student her interpretation of his thinking and leads him to make sense of a particular part of his graph, "What you're looking at here is that it tells you that you pay quite a lot for letters weighing up to 1 ounce. What does up to an ounce mean?" (Gloria analysis #1 transcript, 2018, lines 5-6). She is funneling the student to make sense of his answer through the lens of how she is making sense of his graph. The student answers this question with the fact that an amount less than an ounce is the same as an ounce, with the exception of negative numbers. Gloria leads the student to understand her interpretation again when she states, "Okay, that's good. I'm glad that you brought up negative numbers, because we can't have negative weight? Right?" (Gloria analysis #1 transcript, 2018, lines 10-11). While the student introduced the idea that negative numbers were not part of the domain of the problem, he has not had the opportunity to identify why or the impact of this reasoning. The teacher elicits the student thinking, makes an interpretation, and then funnels the student to her interpretation through the use of comments embedded in her questions that narrow the student to make connections to her interpretation.

In a later segment of the interview, moving to start the discussion of another graph that the student has created, Gloria makes a connection for the student to his previous work and asks the student to explain the connection, "You have sort of a really similar graph going on, and I wondered if maybe you could explain why" (Gloria analysis #1 transcript, lines 55-56). While asking the student to explain why provides the opportunity to elicit student thinking, the fact that the question begins with the teacher making a connection to the previous graph narrows that opportunity to how the teacher is making sense of the graph. During this short interview activity, Gloria initiated the discussion of two student-created graphs. Both of these discussions began with an attempt to elicit student thinking. In the first segment, Gloria starts the sequence with an open-ended question but then narrows the student's thinking in her second question. In the second segment, she narrows the student's thinking with the wording of the opening question. These examples represent the teacher funneling student thinking into how she is making sense of the mathematics of the task.

While the structure of funneling and focusing questioning is not part of the coursework or mentioned in her reflection assignments, Gloria demonstrates an awareness of the impact of her questioning on eliciting student thinking. In her reflection assignment on the interview, she notes that her questioning often involves getting the student to agree with her interpretation. She identifies that she sometimes asks leading questions, ending sentences with the word "right?" (linking to the episode from the interview described above). She demonstrates that she understands how her response limited student thinking, saying, "I took away an opportunity for him to do more thinking" (Gloria analysis #1, 2018, p. 2). She states in her reflection that she would prefer to use "why" questions during this lesson to allow the 'student/peer' to explain his thinking rather than doing it for him…

References

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond and G. Sykes (Eds.), *Teaching as the learning profession* (pp. 3-31). San Francisco, CA: Jossey-Bass.

Ball, D. L., & Forzani, F. M. (2009). The work of teaching and the challenge for teacher education. *Journal of Teacher Education*, *60*(5), 497-511.

Ball, D. L., & Forzani, F. M. (2011). Building a common core for learning to teach, and connecting professional learning to practice. *American Educator*, *35*(2), 17–21, 38–39.

Bernard, H. R., & Ryan, G. W. (2010). *Analyzing qualitative data: Systematic approaches*. Los Angeles: Sage Publications, Inc.

Blanton, M. L., Berenson, S. B., & Norwood, K. S. (2001). Using classroom discourse to understand a prospective mathematics teacher's developing practice. *Teaching and Teacher Education, 17*, 227-242.

Bloom, B. S., Krathwohl, D. R., & Masia, B. B. (1956). *Taxonomy of educational objectives: the classification of educational goals*. New York: David McKay Company.

Boaler, J., & Brodie, K. (2004). The importance, nature and impact of teacher questions. In D. E. McDougall, & J. A. Ross (Eds.), Proceedings of the 26th Conference of the Psychology of Mathematics Education (North America, pp. 773-781). Toronto: Ontario Institute for Students in Education of the University of Toronto.

Boerst, T., Sleep, L., Ball, D. L., & Bass, H. (2011). Preparing teachers to lead mathematics discussions. *Teachers College Record, 113*(12), 2844–2877.

Borman, K. M., Clarke, C., Cotner, B., & Lee, R. (2006). Cross-case analysis. In J. L. Green, G. Camilli, & P. B. Elmore (Eds.), *Handbook of complementary methods in education* *research.* New York: Routledge.

Bransford, J., Brown, A. L., & Cocking, R. R. (1999). *How people learn: Brain, mind, experience, and school*. Washington, D.C: National Academy Press.

Bransford, J., Darling-Hammond, L., & LePage, P. (2005). Introduction. In L. Darling-Hammond, & J. Bransford (Eds.), *Preparing Teachers for a Changing World* (pp. 1-39). San Francisco, CA: John Wiley and Sons, Inc.

Castro, A., Clark, K., Jacobs, J., & Givvin, K. (2005). Response to theory & practice question: Using video to support teacher learning. *Association of Mathematics Teacher Educators Connections, 14*(3), 8-12.

Clift, R. T., & Brady, P. (2005). Research on methods courses and field experiences. In M. Cochran-Smith & K. M. Zeichner (Eds.), *Studying teacher education: The report of the AERA panel on research and teacher education* (pp. 309–424). Mahwah, NJ: Lawrence Erlbaum.

Coles, A., & Brown, L. (2016). Task design for ways of working: Making distinctions in teaching and learning mathematics. *Journal of Mathematics Teacher Education, 19*(2-3), 149-168.

Corley, M. A., & Rauscher, W. C. (2013). The Teaching Excellence in Adult Literacy (TEAL) Center fact sheet no. 12: Deeper learning through questioning. Washington, DC: United States Department of Education, Office of Vocational and Adult Education.

Creswell, J. W. (2013). *Qualitative inquiry and research design: Choosing among five approaches* (3rd ed.). Los Angeles: Sage Publications, Inc.

Cruickshank, D. R., & Metcalf, K. K. (1990). Training within teacher preparation. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 469-497). New York:

Macmillan Publishing Co.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education, 28*(3), 355-376.

Davis, E. A. (2006). Characterizing productive reflection among preservice elementary teachers: Seeing what matters. *Teacher and Teacher Education, 22*(3), 281-301.

Di Teodoro, S. D., Donders, S., Kemp-Davidson, J., Robertson, P., & Schuyler, L. (2011). Asking good questions: Promoting greater understanding of mathematics through purposeful teacher and student questioning. *Canadian Journal of Action Research, 12*(2), 18-29.

Doerr, H. M. (2006). Examining the tasks of teaching when using students' mathematical thinking. *Educational Studies in Mathematics, 62*(1), 3-24.

Erickson, F. (2011). On noticing teacher noticing. On noticing teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 17-34). New York: Routledge.

Fernandez, M. L., & Zilliox, J. (2011). Investigating approaches to lesson study in prospective mathematics teacher education. In L. C. Hart, A. Alston, & A. Murata (Eds.) *Lesson study research and practice in mathematics education, learning together* (pp. 85-102). New York, NY: Springer.

Forzani, F. M. (2014). Understanding "core practices" and "practice-based" teacher education: Learning from the past. *Journal of Teacher Education, 65*(4), 357-368.

Franke, M., Carpenter, T. P., Levi, L., & Fennema, E. (2001). Capturing teachers' generative change: A follow-up study of professional development in mathematics. *American Educational Research Journal, 38*(3), 653-689.

Franke, M. L., Kazemi, E., & Battey, D. (2007). Understanding teaching and classroom practice in mathematics. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). New York, NY: Macmillan.

Franke, M. L., Webb, N. M., Chan, A. G., Ing, M., Freund, D., & Battey, D., (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(4), 380-392.

Freese, A. R. (2006). Reframing one's teaching: Discovering our teacher selves through reflection and inquiry. *Teacher and Teacher Education, 22*(1), 100-119.

Ghousseini, H. (2015). Core practices and problems of practice in learning to lead classroom discussions. *The Elementary School Journal, 115*(3), 334-357.

Ghousseini, H., Beasley, H., & Lord, S. (2015). Investigating the potential of guided practice with an enactment tool for supporting adaptive performance. *Journal of the Learning Sciences, 24*(3), 461-497.

Ghousseini, H. & Herbst, P. (2016). Pedagogies of practice and opportunities to learn about classroom mathematics discussions. *Journal of Mathematics Teacher Education, 19*(1), 79-103.

Ghousseini, H., & Sleep, L. (2011). Making practice studyable. *ZDM—The International Journal on Mathematics Education*, *43*(1), 147-160.

Grosser-Clarkson, D. L. (2016). *Examining secondary mathematics teacher candidates' learning and enactment of mathematics teaching practices: A multiple case study*. (Unpublished doctoral dissertation). University of Maryland, College Park.

Grossman, P. (2009). Research on pedagogical approaches in teacher education. In M. Cochran-Smith & K. Zeichner (Eds.), *Studying teacher education* (pp. 425-476). Washington, D.C.: American Educational Research Association.

Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching: theory and practice, 15*(2), 273-289.

Grossman, P., & McDonald, M. (2008). Back to the future: Directions for research in teaching and teacher education. *American Educational Research Journal*, *45*, 184-205.

Hammerness, K., Darling-Hammond, L., with Grossman, P., Rust, F., & Shulman L. (2005). The design of teacher education programs. In L. Darling-Hammond, & J. Bransford (Eds.), *Preparing Teachers for a Changing World* (pp. 390-441). San Francisco, CA: John Wiley and Sons, Inc.

Hatch, T., & Grossman, P. (2009). Learning to look beyond the boundaries of representation: Using technology to examine teaching (overview for a digital exhibition: Learning from the practice of teaching). *Journal of Teacher Education, 60*(1), 70-85.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524–549.

Herbel-Eisenmann, B. A., & Breyfogle, M. L. (2005). Questioning our "patterns" of questioning. *Mathematics Teaching in the Middle School, 10*(9), 484-489.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35*(2), 81-116.

Jacobs, V. R., Lamb, L. L. C., Philipp, R. A., & Schappelle, B. P. (2011). Deciding how to respond on the basis of children's understandings. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics Teacher Noticing* (pp. 97-116). New York: Routledge.

Kazemi, E., Franke, M. F., & Lampert, M. (2009). Developing pedagogies in teacher education to support preservice teachers' ability to enact ambitious instruction. In R. Hunter, B. Bicknell & T. Burgess (Eds.), *Crossing divides: Proceedings of the 32nd annual conference of the mathematics education research group of Australasia* (Vol. 1, pp. 12-29). Palmerston North, New Zealand: MERGA.

Kazemi, E., Ghousseini, H., Cunard, A., & Turrou, A. C. (2016). Getting inside rehearsals: Insights from teacher educators to support work on complex practice. *Journal of Teacher Education, 67*(1), 18-31.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The elementary School Journal, 102*(2), 59-80.

Kennedy, M. (1999). The role of preservice teacher education. In Darling-Hammond, L. and Sykes, G. *Teaching as the learning profession: Handbook of teaching and policy* (pp. 54-86). San Francisco: Jossey Bass.

Lampert, M. (1985). How do teachers manage to teacher? Perspectives on problems in practice. *Harvard Educational Review, 55*(2),

Lampert, M. (2010). Learning teaching in, from, and for practice: What do we mean? *Journal of Teacher Education, 61*(1-2), 21-34.

Lampert, M., Beasley, H., Ghousseini, H., Kazemi, E., & Franke, M. (2010). Using designed instructional activities to enable novices to manage ambitious mathematics teaching. In M. K. Stein & L. Kucan (Eds.), *Instructional explanations in the disciplines* (pp. 129-141). New York: Springer Science+Business Media.

Lampert, M., Franke, M., Kazemi, E., Ghousseini, H., Turrou, A., Beasley, H., . . . Crowe, K. (2013). Keeping it complex: Using rehearsals to support novice teacher learning of ambitious teaching in mathematics. *Journal of Teacher Education*, *64*(3), 226–243.

Leahy, S., Lyon, C., Thompson, M., & Wiliam, D. (2005). Classroom assessment: Minute by minute, day by day. *Educational Leadership, 63*(3), 19-24.

Levin, D. M., Hammer, D., & Coffey, J. E. (2009). Novice teachers' attention to student thinking. *Journal of Teacher Education, 60*(2), 142-154.

Linsenmeier, K. A., Sherin, M., Walkoe, J., & Mulligan, M. (2014). Lenses for examining student mathematical thinking. *Mathematics Teacher, 108*(2), 142-146.

Lobato, J., Hohensee, C., Rhodehamel, B., & Diamond, J. (2012). Using student reasoning to inform the development of conceptual learning goals: The case of quadratic functions. *Mathematical Thinking and Learning, 14*(2), 85-119.

Martino, A. M., & Maher, C. A. (1999). Teacher questioning to promote justification and generalization in mathematics: What research practice has taught us. *Journal of Mathematical Behavior, 18*(1), 53-78.

Mason, J. (2010). Effective questioning and responding in the mathematics classroom. Retrieved from http://mcs.open.ac.uk/jhm3/Selected%20Publications/Effective%20Questioning%20&%20Responding.pdf.

Matsko, K. K., & Hammerness, K. (2014). Unpacking the "urban" in urban teacher education: Making a case for context-specific preparation. *Journal of Teacher Education, 65*(2), 128-144.

McCarthy, P., Sithole, A., McCarthy, P., Cho, J., & Gyan, E. (2016). Teacher questioning strategies in mathematical classroom discourse: A case study of two grade eight teachers in Tennessee, USA. *Journal of Education and Practice, 7*(21), 80-89.

McCullough, D., & Findley, E. (1983). Teaching: How to ask effective questions. *The Arithmetic Teacher, 30*(7), 8-9.

McDonald, M., Kazemi, E., & Kavanagh, S. S. (2013). Core practices and pedagogies of teacher education: A call for a common language and collective activity. *Journal of Teacher Education*, *64*(5), 378-386.

McDuffie, A. R., Foote, M. Q., Bolson, C., Turner, E. E., Aguirre, J. M., Bartell, T. G., Drake, C., & Land, T. (2014). Using video analysis to support prospective K-8 teachers' noticing of students' multiple mathematical knowledge bases. *Journal of Mathematics Teacher Education, 17*(3), 245-270.

Mehan, H. (1979). "What time is it, Denise?": Asking known information questions in classroom discourse. *Theory into Practice, 28*(4), 285-294.

Moyer-Packenham, P. S., & Milewicz, E. (2002). Learning to question: Categories of questioning used by preservice teachers during diagnostic interviews. TEaL Faculty Publications, paper 1624. Logan. UT: Utah State University.

Myhill, D., & Dunkin, F. (2005). Questioning learning. *Language and Education, 19*(5), 415-427.

National Center for Education Statistics. (2003). *Teaching mathematics in seven countries: Results from the TIMSS 1999 video study.* Washington, DC: Author.

National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (NCTM). (1991). *Professional standards for teaching mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for* *school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics (NCTM). (2014). *Principles to actions: ensuring mathematical success for all.* Reston, VA: NCTM.

National Governors Association Center for Best Practices, Council of Chief State School Officers (NGA.CCSSO). (2010). *Common core state standards mathematics*. Washington, DC: NGA/CCSSO.

Nicol, C. (1999). Learning to teach mathematics: Questioning, listening, and responding. *Educational Studies in Mathematics, 37*(1), 45-66.

Ontario Ministry of Education. (2011). Capacity building series: Asking effective questions. Author: Toronto.

Pearson, P. D., & Gallagher, M. (1983). The instruction of reading comprehension. *Contemporary Educational Psychology, 8*(3), 317-344.

Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In Lester, F. K. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257-314). Reston, VA: National Council of Teachers of Mathematics.

Pierson, J. (2008). *The relationship between patterns of classroom discourse and mathematics learning*. (Unpublished doctoral dissertation). University of Texas at Austin.

Saldaña, J. (2011). *Fundamentals of qualitative research*. New York: Oxford University Press, Inc.

Schoenfeld, A. H. (2011). Noticing matters. A lot. Now What? In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 223-238). New York: Routledge.

Shahrill, M. (2013). Review of effective teacher questioning in mathematics classrooms.

International Journal of Humanities and Social Science, 3(17), 224-231.

Sherin, M. G., & Linsenmeier, K. A. (2011). Pause, rewind, reflect. *Journal of Staff Development 32*(5), 38-41.

Sherin, B., & Star, J. R. (2011). Reflections on the study of teacher noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 66-78). New York: Routledge.

Sherin, M. G., Jacobs, V. R., & Philipp, R. A. (Eds.). (2011a). *Mathematics teacher noticing: Seeing through teachers' eyes*. New York: Routledge.

Sherin, M. G., Russ, R. S., & Colestock, A. A. (2011b). Accessing mathematics teachers' in-the-moment noticing. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 79-94). New York: Routledge.

Singer-Gabella, M., Stengel, B., Shahan, E., & Kim, M. (2016). Learning to leverage student thinking: What novice approximations teach us about ambitious practice. *The Elementary School Journal, 116*(3), 411-436.

Sleep, L., & Boerst, T. (2012). Preparing beginning teachers to elicit and interpret students' mathematical thinking. *Teaching and Teacher Education, 28*(7), 1038-1048.

Smith, M. S., Bill, V., & Hughes, E. K. (2008). Thinking through a lesson: Successfully implementing high-level tasks. *Mathematics Teaching in the Middle School, 14*(3), 132-138.

Star, J. R., Lynch, K., & Perova, N. (2011). Using video to improve preservice mathematics teachers' abilities to attend to classroom features. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (117-133). New York: Routledge.

Star, J. R., & Strickland, S. K. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education, 11*(2), 107-125.

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10*(4), 313-340.

Stigler, J.W., & Hiebert, J. (1999). *The teaching gap.* New York: The Free Press.

TeachingWorks. (2018). High-leverage practices. Retrieved from http://www.teachingworks.org/work-of-teaching/high-leverage-practices.

Thompson, C. L., & Zeuli, J. S. (1999). The frame and the tapestry. In L. Darling-Hammond & G. Sykes (Eds)., *Teaching as a learning profession: Handbook of policy and practice* (pp. 341-375). San Francisco: Jossey Bass.

Thompson, J., Windschitl, M., & Braaten, M. (2013). Developing a theory of ambitious earl-career teacher practice. *American Educational Research Journal, 50*(3), 574-615.

Thompson, P. W., & Saldanha, L. A. (2004). Fractions and multiplicative reasoning. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.). *Research companion to the "Principles and standards for school mathematics"* (pp. 95–113). Reston, VA: National Council of Teachers of Mathematics.

Tienken, C. H., Goldberg, S., & DiRocco D. (2009). Questioning the questions. *Kappa Delta Pi Record, 46*(1), 39-43.

Ulleberg, I., & Solem, I. H. (2018). Which questions should be asked in classroom talk in mathematics? *Acta Didactica Norge, 12*(1), 1-21.

van den Kieboom, L. A., Magiera, M. T., & Moyer, J. C. (2014). Exploring the relationship between K-8 prospective teachers' algebraic thinking proficiency and the questions they pose during diagnostic algebraic thinking interviews. *Journal of Mathematics Teachers Education, 17*(5), 429-461.

van Es, E. A., & Conroy, J. (2009). Using the performance assessment for California teachers to examine pre-service teachers' conceptions of teaching mathematics for understanding. *Issues in Teacher Education, 18*(1), 83–100.

van Es, E.A., & Sherin, M.G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education, 10*(4), 571-596.

Walkoe, J. (2015). Exploring teacher noticing of student algebraic thinking in a video club. *Journal of Mathematics Teachers Education, 18*(6), 523-550.

Waring, H. Z. (2009). Moving out of IRF (Initiation-Response-Feedback): A single case analysis. *Language Learning, 59*(4), 796-824.

Wells, G., & Arauz, R. M. (2006). Dialogue in the classroom. *The Journal of the Learning Sciences, 15*(3), 379-428.

Windschitl, M., Thompson, J., Braaten, M., & Stroupe, D. (2012). Proposing a core set of instructional practices and tools for teachers of science. *Science Education, 96*(5), 878-903.

Wood, T. (1998). Alternate patterns of communication in mathematics classes: Funneling or focusing? In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom* (pp. 166-178). Reston, VA: National Council of Teachers of Mathematics.

Yin, R. K. (2006). Case Study Methods. In J. L. Green, G. Camilli, & P. B. Elmore (Eds.), *Handbook of complementary methods in education research* (pp. 111-122). New York: Routledge.

Zembal-Saul, C., Blumenfeld, P., & Krajcik, J. (2000). Influence of guided cycles of planning, teaching, and reflection on prospective elementary teachers' science content representations. *Journal of research in science teaching, 37*(4), 318–339.