APPROVAL SHEET

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The Application of the Gyrator Concept to Transistors

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Date:

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THE APPLICATION OF THE GYRATOR CONCEPT TO TRANSISTORS

by

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INTRODUCTION

In 1948 the term "gyrator" was first used to define a non-reciprocal passive network element.[1] Prior to that time, passive and reciprocal were occasionally used interchangeably. In other fields comparable progress had been made. Mechanical devices with gyrostatic terms had been used. Electro-mechanical anti-reciprocal terms were also used. In the latter field, it was shown that an electric current setting up a magnetic field could energize a mechanical device that would convert the motion back into electrical oscillations.[2] This was the closest analogue to an electrical four pole, but would only be useful over a limited frequency range. In each of these three fields, two equations represent the action of the device. For all devices, except electrostatic transducers, a constant appears in one equation as a positive coefficient, and as a negative coefficient in the other.[3] Generally it can be expressed as

$$A_1x_1 + A_2x_2 + A_3x_3 = y_1$$
$$B_1x_4 + B_2x_5 + B_3x_6 = y_2,$$

in which x and y are variables, and A and B are constants, with $A_3 = -B_3$.

The gyrator was visualized as being a fifth passive circuit element, in addition to the capacitor, inductor, resistor, and ideal transformer. This new circuit element was to simplify the synthesis of 2-n pole networks mathematically, and physically, by reducing the computation, and the number of elements required.

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1. B.D.H. Tellegen. The Gyrator a New Electric Network Element. Phillips Res. Rep. 3:81-101, 1948.

2. E.M. McMillan. Violation of the Reciprocity Theorem in Linear Passive Electro-mechanical Systems. J. Acous. Soc. Am. 18:344-347.

3. B.D.H. Tellegen, op. cit. p. 91.

SECTION I

VARIATIONS OF THE CONVENTIONAL GYRATOR

The electrical gyrator, as first conceived, was an ideal passive, non-reciprocal, four pole device.[4] This is shown with different conventions as indicated in Fig. 1.

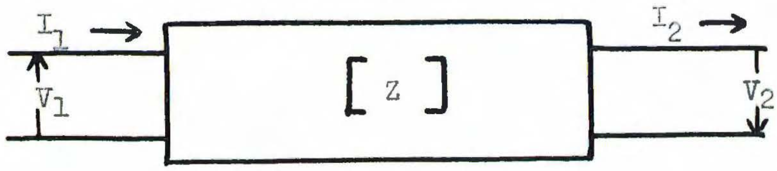

Fig. 1 The Impedance Four-Pole

The associated equations are given by

$$V_1 = Z_{11} I_1 - Z_{12} I_2 \qquad (1)$$
$$V_2 = -Z_{21} I_1 + Z_{22} I_2.$$

As it was passive, with no loss

$$Z_{11} = Z_{22} = 0.$$

The non-reciprocal feature occurs by letting

$$Z_{12} = - s$$
$$Z_{21} = + s$$

This defines (s) as the gyration resistance. Then

$$V_1 = + s I_2 \qquad (2)$$
$$V_2 = - s I_1.$$

Similarly, gyration conductance (G) can be defined as

$$G = - \frac{1}{s}.$$

Using different conventions, this is used in an admittance four pole shown in Fig. 2.[5]

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4. Ibid, p. 87

5. Jacob Shekel. The Gyrator as a Three Terminal Element. Proc. I.R.E. 41:1014

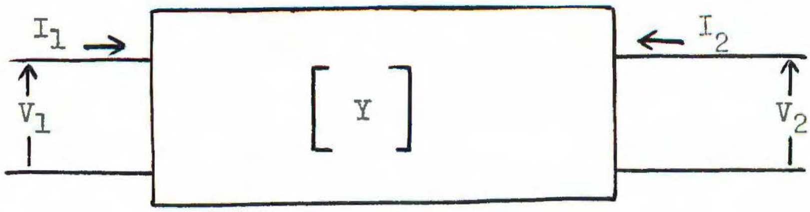

Fig. 2 The Admittance Four-Pole

Then,

$$I_1 = GV_2$$
$$I_2 = -GV_1. \qquad (3)$$

The impedance gyrator, with arrow going from input to output, represents +s. The admittance gyrator, using + G, is also shown in Fig. 3.[6]

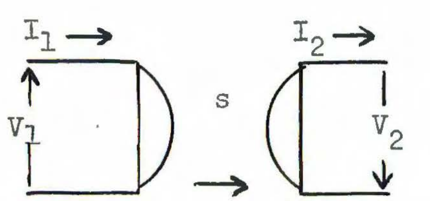

Fig. 3a The Impedance Gyrator

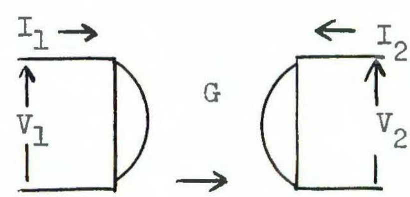

Fig. 3b The Admittance Gyrator

The main property of the impedance gyrator is that the input impedance is the inverse of the termination with a constant multiplier (s^2). This implies that a capacitive reactance ($\frac{1}{jwc}$) in the output will look like an inductive reactance, with an inductance, $L = s^2C$ in the input. Also this causes two impedances, Z_1 and Z_2 in the output to appear as two impedances, $\frac{s^2}{Z_1}, \frac{s^2}{Z_2}$ in parallel at the input. In addition, if the output is shorted, the input appears open, and with no load, the input appears shorted. This can be shown for the impedance basis from the equations below.

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6. H. J. Carlin. On the Physical Realizibility of Linear Non-Reciprocal Networks. Proc. I.R.E. 43:608

$$V_1 = + sI_2$$

$$V_2 = - sI_1$$

$$Z\ in = \frac{V_1}{I_1} = -s^2 \frac{I_2}{V_2} = \frac{-s^2}{Z_{Load}}, \tag{4}$$

where the minus sign denotes a reversal of current in the output from the conventional direction. The admittance gyrator will give a similar result.

Other properties can be found similarly from Fig. 4.

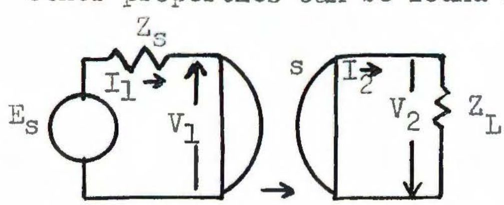

Fig. 4a The Terminated Impedance Gyrator

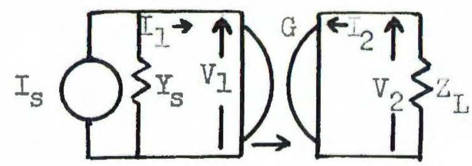

Fig. 4b The Terminated Admittance Gyrator

The output impedance of the gyrator would be the impedance presented to the load by the source through gyrator action, i.e.,

$$Z_{out} = \frac{s^2}{Z_{source}}.$$

The voltage amplification (Av) can be found by using Eq. (2),

$$V_1 = sI_2 = s(\frac{V_2}{Z_L})$$

$$A_v = - \frac{V_2}{V_1} = - \frac{Z_L}{s}. \tag{5}$$

Similarly, the current amplification (Ai) can be found from Eq. (3),

$$I_1 = GV_2 = G(-I_2 \cdot R_L)$$

$$Ai = - \frac{I_2}{I_1} = \frac{1}{GZ_L} = - \frac{s}{Z_L} \tag{6}$$

The Transducer Power gain, (G_t), for a matched resistance input and matched resistance output can also be found,[7]

$$G_t = (\frac{I_2}{I_1})^2 \cdot \frac{R_L}{R_{in}} = \frac{I_2}{I_1} \cdot \frac{I_2 R_L}{I_1 R_{in}} = \frac{I_2}{I_1} \cdot \frac{V_2}{V_1} = Ai \cdot Av$$

$$= (- \frac{s}{R_L}) (- \frac{R_L}{s}) = 1,$$

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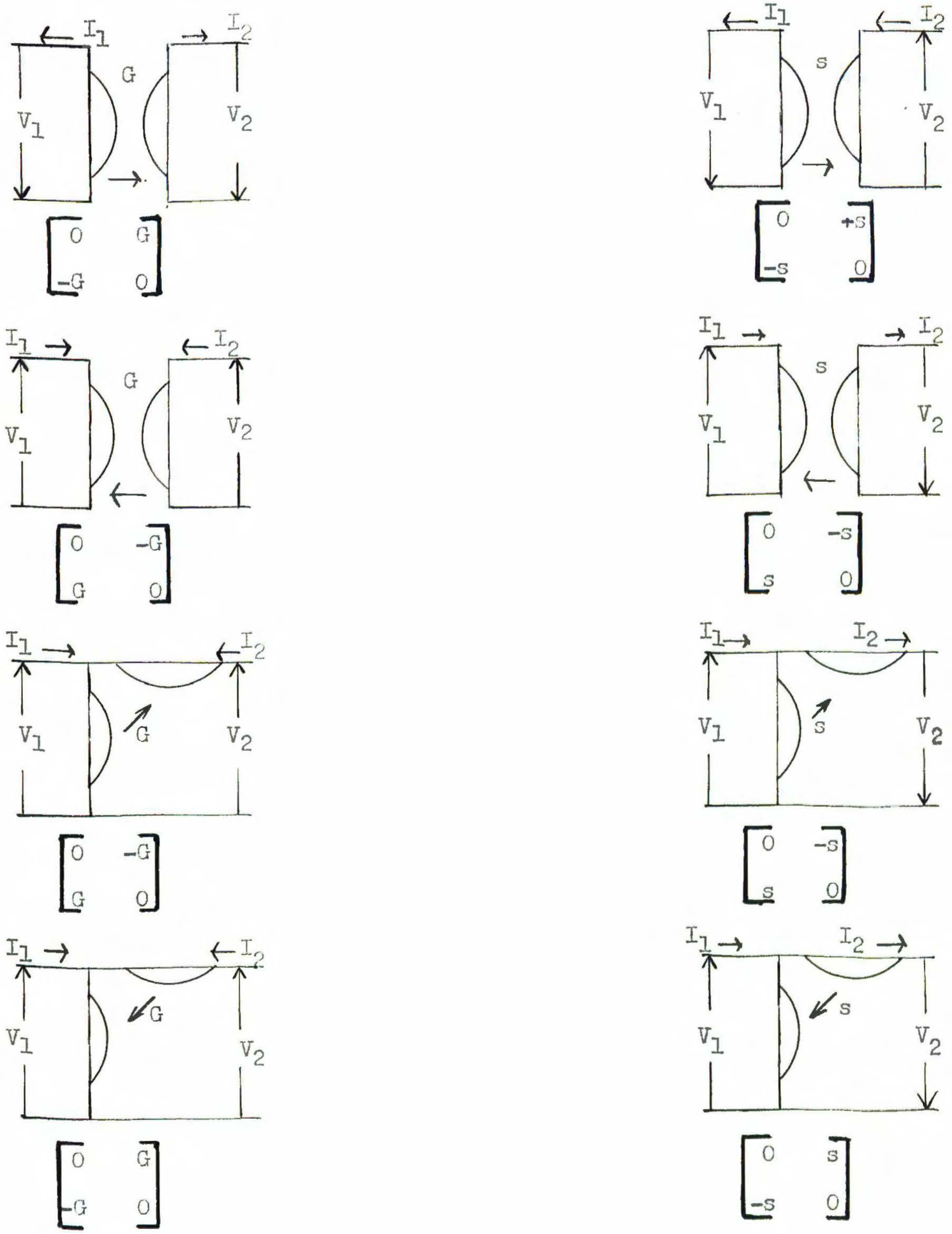
7. I.R.E. Standards on Audio Measurements. Proc. I.R.E. 44:671

which is logical, as the gyrator is assumed to be passive with no loss. In addition to the conventional gyrator, other configurations could be used with changes in their associated matrices, as in Table 1.

TABLE I

IDEAL GYRATORS



SECTION II

THE INDEFINITE MATRIX

The indefinite matrix on the admittance basis can be proved as follows:[8]

For an n-node network let I_{ik} be the current from node (i) to node (k) then,

$$I_{ii} = 0 \text{ and } \quad I_{ik} = -I_{ki}.$$

Let I_i be the current entering node (i). Then, as the current entering a node must equal the current leaving it,

$$I_i + \sum_{k=1}^{n} I_{ki} = 0$$

$$I_i = \sum_{k=1}^{n} I_{ik}.$$

Summing the equations for all nodes,

$$\sum_i I_i + \sum_i \sum_k I_{ki} = 0 \qquad i = 1, 2 \ldots n, \quad k = 1, 2 \ldots n$$

$$I_{ii} = 0, \quad I_{ik} = -I_{ki},$$

$$\sum_i \sum_k I_{ki} = 0$$

$$\therefore \qquad \sum_i I_i = 0. \qquad (5)$$

This shows that the current entering the network equals the current leaving it.

Let V_i be the voltage of the (i)th node with respect to an arbitrary reference voltage.

Then,

$$[I] = [Y][V].$$

Let all V in the column matrix be zero except V_j then,

$$\sum_i I_i = \sum_i (Y_{ij} V_j) = V_j \sum_i Y_{ij}.$$

As $\sum_i I_i = 0$ was shown above, then

$$0 = V_j \sum_i Y_{ij} \qquad \text{with } V_j \neq 0$$

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8. Jacob Shekel. Voltage Reference Node. Wireless Engineer 31:6-10

As this is true for any V_j, then

$$\sum_i Y_{ij} = 0 \tag{6}$$

or $Y_{1j} + Y_{2j} + \dots Y_{nj} = 0,$

which proves that the sum of the rows = 0.

As the voltages only depend on differences, an arbitrary voltage, V_o, can be added to each V_i. Then the current into the (i)th node will be

$$I_i = \sum_k Y_{ik} (V_k + V_o) = V_o \sum_k Y_{ik} + \sum_k Y_{ik} V_k$$

and, as the currents do not change with the addition of V_o, this means that

$$I_i = \sum_k Y_{ik} V_k$$

so $V_o \sum_k Y_{ik} = 0.$

As this is true for every I_i, this shows that V_o is not zero, so

$$\sum_k Y_{ik} = 0 \tag{7}$$

or $Y_{11} + Y_{12} + Y_{13} + \dots Y_{1n}$

which proves that the sum of the columns = 0.

To change from one common grounded terminal to another, a transformation must be used.

The first step is to define a new voltage matrix V' in terms of the old voltage V.

$$[V] = [A][V']$$

where A is a rectangular matrix. New currents I' are then defined in terms of the old ones, I.

$$I' = A_t I$$

where A_t is the transpose of A. This is necessary to keep the power invariant. Then, to keep the new admittance matrix Y' invariant, using Y as the new one,

$$[Y'] = [A_t][Y][A] \tag{8}$$

When this is applied to a 4 x 4 indefinite matrix it can reduce it to a 3 x 3 by eliminating a row and column. For example if $V_4 = 0$, then

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

This would eliminate the fourth row and column. Similarly if a 3 x 3 were used and

$$V_1 = V_1' - V_4' \quad \text{for } i = 1, 2, 3,$$

then a new row and column is formed, each row and column totalling zero as shown before.

This idea can be applied to a four terminal gyrator. As a real gyrator will have to have some losses in it, the losses can be taken into consideration in a general four terminal matrix.[9] Let

$$G = \tfrac{1}{2}(Y_{12} - Y_{21}),$$

then

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12}-G \\ Y_{21}+G & Y_{22} \end{bmatrix} + \begin{bmatrix} 0 & G \\ -G & 0 \end{bmatrix}, \tag{9}$$

in which the second part is the symmetrical part, and the third part is the unsymmetrical part.

This can also be done on the impedance basis. The general admittance matrix can be changed into the indefinite admittance matrix by completing each row and column to add up to zero. Then the result can be broken down in symmetric and skew-symmetric portions, i.e.,

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12}-G & Y_{13}+G \\ Y_{21}+G & Y_{22} & Y_{23}-G \\ Y_{31}-G & Y_{32}+G & Y_{33} \end{bmatrix} + \begin{bmatrix} 0 & G & -G \\ -G & 0 & G \\ G & -G & 0 \end{bmatrix}.$$

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9. L.M. Vallese. Understanding the Gyrator. Proc. I.R.E. 43, Part 1:483

Similarly, the impedance matrix can be brought to the indefinite form, and then be broken down,

$$\begin{bmatrix} Z_{11} & -Z_{12} & -Z_{13} \\ -Z_{21} & Z_{22} & -Z_{23} \\ -Z_{31} & -Z_{32} & Z_{33} \end{bmatrix} = \begin{bmatrix} Z_{11} & -Z_{12}-s & -Z_{13}+s \\ -Z_{21}+s & Z_{22} & -Z_{23}-s \\ -Z_{31}-s & -Z_{32}+s & Z_{33} \end{bmatrix} + \begin{bmatrix} 0 & s & -s \\ -s & 0 & +s \\ +s & -s & 0 \end{bmatrix}.$$

Generalized gyrators can be used in the general indefinite form. If the input is actually 1, 3 and the output 2, 3, then disregard gyrator between 1 and 2. The combinations of the indefinite impedances and admittances with gyrators are shown in Fig. 5.

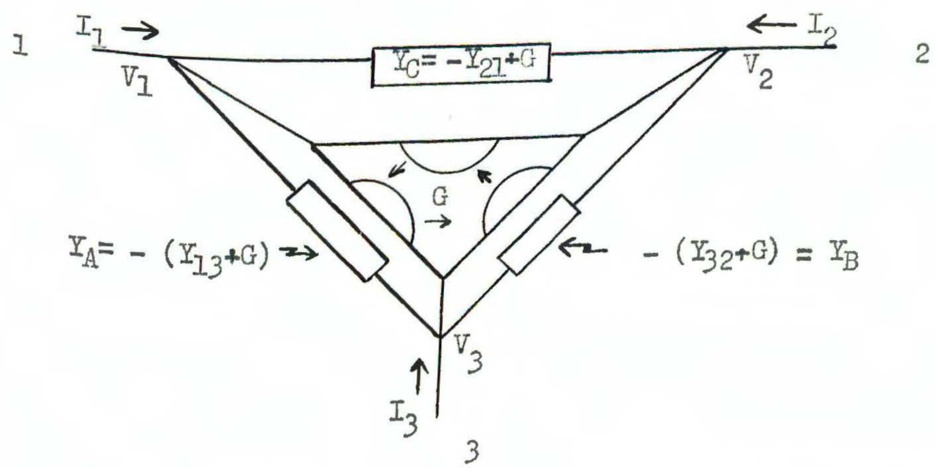

Fig. 5a The General 3 Terminal Admittance Network

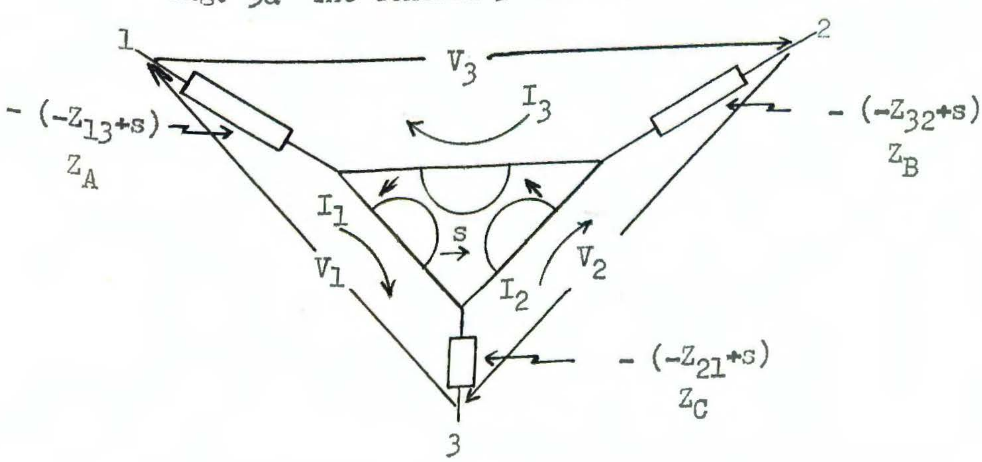

Fig 5b The General 3 Terminal Impedance Network

The three terminal general matrix can be converted into a conventional four pole by crossing out the row and column corresponding to the terminal chosen as common to input and output. The remaining elements in the matrix must then be re-numbered to maintain 1-1 as input and 2-2 as output.

SECTION III

APPLICATION TO THE TRANSISTOR

The values used for the transistor are the low frequency values for it. The elements corresponding to those for the indefinite matrices and figures can be substituted.

Let $\Delta = r_e\,(r_b + r_c) + r_b\ r_c\,(1-a)$

Then,

$$Y_{11} = (r_b + r_c)\,\frac{1}{\Delta} \qquad Y_{12} = -\frac{r_b}{\Delta} \qquad Y_{13} = -\frac{r_c}{\Delta}$$

$$Y_{21} = \frac{-(r_b+ar_c)}{\Delta} \qquad Y_{22} = \frac{r_e+r_b}{\Delta} \qquad Y_{23} = \frac{ar_c-r_e}{\Delta}$$

$$Y_{31} = \frac{ar_c-r_c}{\Delta} \qquad Y_{32} = -\frac{r_e}{\Delta} \qquad Y_{33} = \frac{-ar_c+r_c+r_e}{\Delta}$$

$$Z_{11} = r_e+r_b \qquad Z_{12} = +r_b \qquad Z_{13} = +r_e$$

$$Z_{21} = +ar_c+r_b \qquad Z_{22} = r_b+r_c \qquad Z_{23} = -ar_c+r_c$$

$$Z_{31} = -ar_c+r_e \qquad Z_{32} = +r_c \qquad Z_{33} = r_e+r_c-ar_c$$

In the figures, terminal 1 represents emitter, terminal 2 collector, and terminal 3 base.

As an illustration, the grounded base could be found from the admittance matrix by crossing out the upper gyrator section, and line and column 3, resulting in the matrices below and Fig. 6.

$$Y = \frac{1}{\Delta}\begin{bmatrix} r_b+r_c & -r_b \\ -(r_b+ar_c) & r_e+r_b \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} r_b+r_c & -(r_b+\frac{ar_c}{2}) \\ -(r_b+\frac{ar_c}{2}) & r_e+r_b \end{bmatrix} + \frac{ar_c}{2\Delta}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

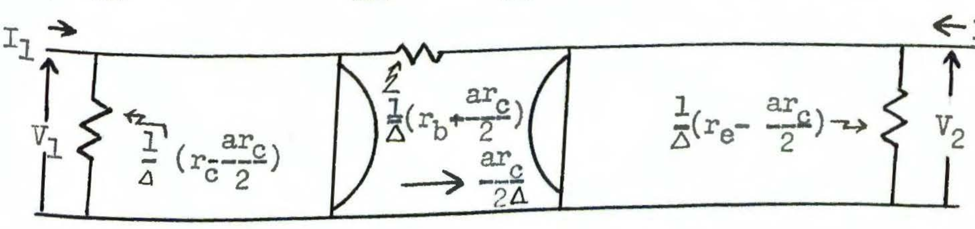

Fig. 6 Gyrator Equivalent for the Grounded Base Transistor

Similarly, the others could be found for the impedance and admittance basis.

In addition to the impedance and admittance indefinite matrices, a combined matrix can be used for the hybrid parameters,

$$\begin{array}{lllll}
e_1 & \Delta i_1 & r_b e_2 & 0 & r_c\,(1-a)e_3 \\
i_2 & -(r_b+ar_c)i_1 & e_2 & 0 & 0 \\
e_2 & 0 & 0 & \Delta i_2 & r_e e_3 \\
i_3 & -r_c i_1 & 0 & (-r_e-ar_c)i_2 & e_3 \;.
\end{array}$$

For grounded base: Cross out rows and columns 3, 4 and use matrix multiplier $\frac{1}{r_b+r_c}$.

For grounded emitter: Cross out rows and columns 1, 2 and use multiplier $\frac{1}{r_e+r_c(1-a)}$.

For grounded collector: Cross out rows and columns 2, 3 and use multiplier $\frac{1}{r_e+r_c(1-a)}$.

SECTION IV

LOADING TO SIMULATE A GYRATOR

Loading can be accomplished by placing a conductance in parallel with each one shown for the indefinite case.[10] If the two are opposite in sign, the pure gyrator will remain. For the indefinite impedance figure, series resistors of opposite sign to those of equal value in the circuit would do the same thing. Then, one terminal could be grounded, and the opposite gyrator could be removed resulting in a 4 terminal ideal gyrator. In this process, using actual transistor values some compensating elements may be of negative value. These would require some auxiliary equipment, and under certain conditions may not be constant enough. Therefore, a practical method of loading would be to examine the three terminal gyrator, and determine what simple loading with positive resistors, or conductances would produce a gyrator with a residual amount of loss.

Considering an actual point contact transistor

$$Z_A = r_c - \frac{ar_c}{2} = 0 \text{ when } \quad a = 2$$

which is not difficult to obtain. Z_B can be balanced out by a series resistor,

$$Z_B{}' = r_c - r_e \text{ for } \quad a = 2.$$

For an admittance basis, the same is true when

$$Y_B{}' = \frac{r_c - r_e}{\Delta} \quad \text{when } a = 2 \text{ and } \Delta \text{ is greater than } 0.$$

But, when Δ is less than 0, the only positive resistance to add is

$$Y_C{}' = \frac{r_b + r_c}{-\Delta} \quad \text{when } \quad a = 2.$$

If it happens that $a = 2$ will make $\Delta = 0$, then the admittance matrices are undefined.

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10. Jacob Shekel. The Gyrator as a 3 Terminal Element. Proc. I.R.E. 41:1014-1016

These changes, applied to the matrices, can be seen by changing the element affected in the matrix. For the admittance matrix with $a = 2$,

$$Y_B' = \frac{r_c - r_e}{r_b(r_e - r_c) + r_e r_c} \,.$$

The result for the symmetrical part of the indefinite matrix can be found as follows: Y_{23} and Y_{32} become zero, Y_{22} and Y_{33} have Y_B' added to them. The result is

$$\frac{1}{\Delta_1}\begin{bmatrix} r_b + r_c & -r_b - r_c & 0 \\ -(r_b + r_c) & r_b + r_c & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

where $\Delta_1 = \Delta$ with $a = 2$. The 3 terminal matrix for the unsymmetrical gyrator remains the same.

Similar results will occur for the other methods of loading.

SECTION V

CIRCUIT PROPERTIES OF THE LOADED TRANSISTOR

Circuit properties generally used for transistors with resistance loads and sources are input resistance, output resistance, current gain, voltage gain and transducer power gain. For the grounded collector, using series loading of the emitter, results are shown in Fig. 7.

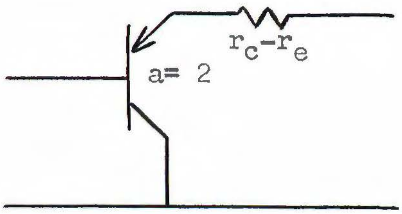


Fig. 7a Actual Loaded Transistor

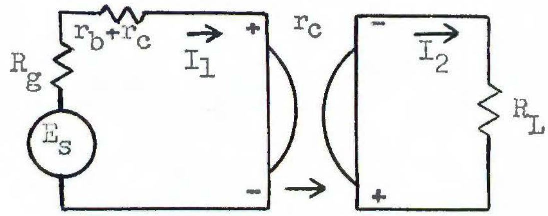


Fig. 7b Gyrator Equivalent

$$\text{Input Resistance} = r_b + r_c + \frac{r_c^2}{R_L} .$$

$$\text{Output Resistance} = \frac{r_c^2}{r_b + r_c + R_g} .$$

$$\text{Current Amplification} = -\frac{r_c}{R_L} .$$

$$\text{Voltage Amplification} = -\frac{r_c R_L}{r_c^2 + (r_b + r_c)\, R_L} .$$

$$\text{Transducer Gain} = \frac{r_c^2}{r_c^2 + (r_b + r_c)\, R_L} .$$

This illustrates the simplicity of calculations using the gyrator concept. The input resistance is the loss plus the reflected load. The output resistance is the reflected input. The current amplification is the value found from the gyrator matrix.

$$V_{out} = I_2 R_L = -sI_1 = -r_c I_1 .$$

$$\text{Current Amplification } (A_i) = \frac{I_2}{I_1} = -\frac{r_c}{R_L} .$$

$$\text{Voltage Amplification } (A_v) = A_i \frac{R_L}{R_{in}} = -\frac{r_c}{R_L} \cdot \frac{R_L}{r_b + r_c + \frac{r_c^2}{R_L}} = -\frac{r_c R_L}{r_c^2 + (r_b + r_c) R_L} .$$

$$\text{Transducer Power Gain } (G_t) = A_i\ A_v = \frac{r_c}{R_L} \cdot \frac{r_c\ R_L}{r_c^{\,2}+(r_b+r_c)R_L} = \frac{r_c^{\,2}}{r_c^{\,2}+(r_b+r_c)R_L}$$

The procedure to use for shunt loading is to select the point contact transistor that will operate at a = 2. At that operating point check the four terminal parameters and compute Δ .

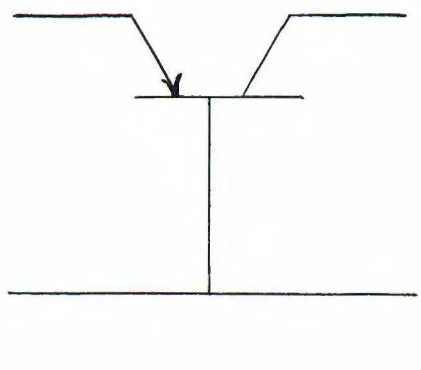
Examples of loading with the resulting characteristics are shown in Tables 2 and 3.

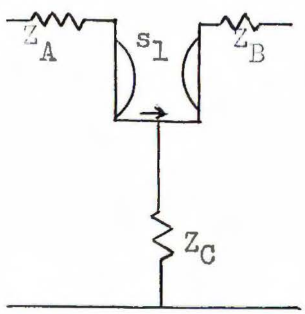
TABLE II

GYRATOR EQUIVALENTS

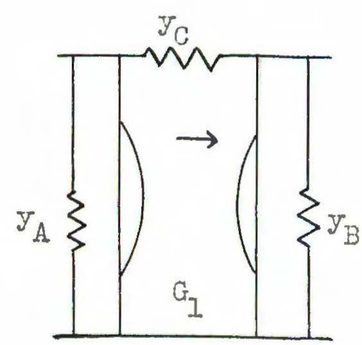
Type A

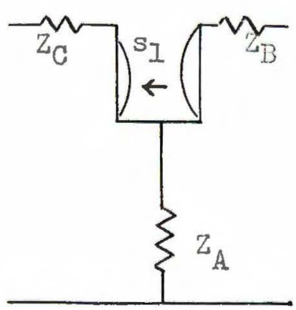
Actual



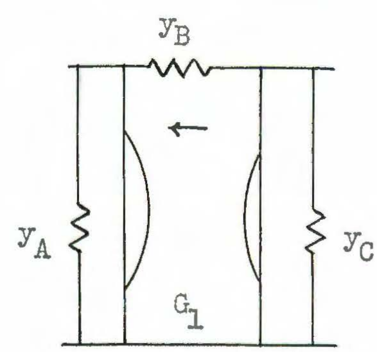


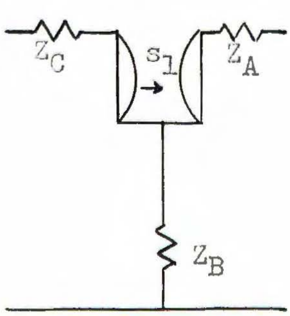
Grounded Base





Grounded Emitter





Grounded Collector

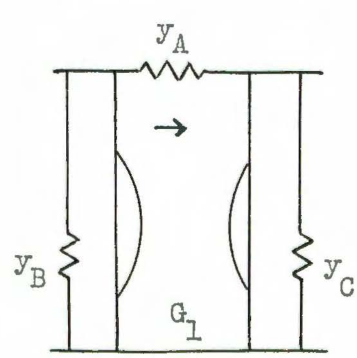
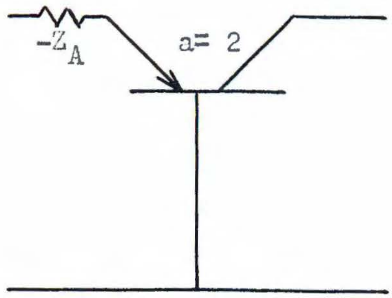


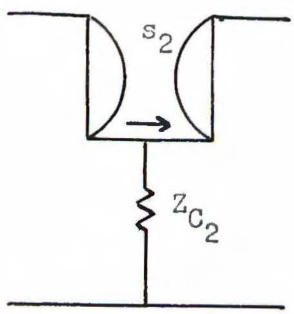
TABLE II (Cont.)

GYRATOR EQUIVALENTS

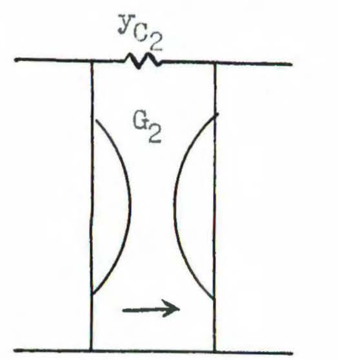
Type B

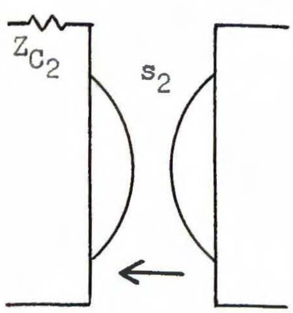
Actual



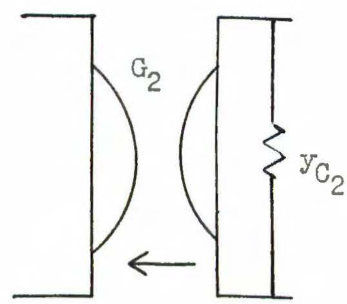


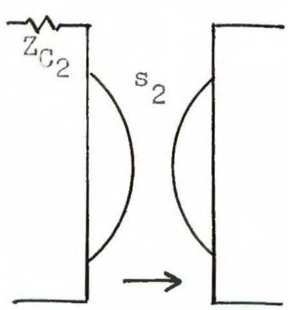
Grounded Base





Grounded Emitter





Grounded Collector

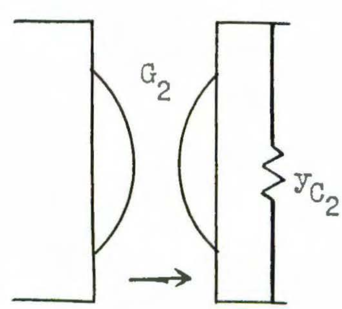
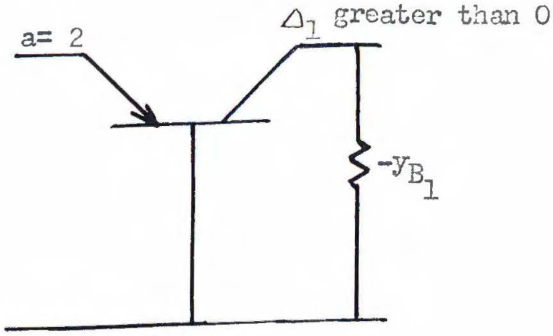


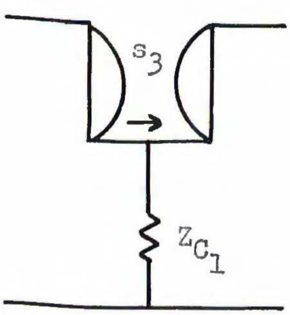
TABLE II (Cont.)

GYRATOR EQUIVALENTS

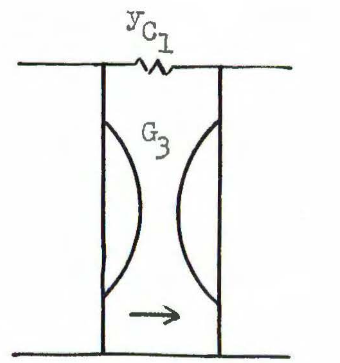
Type C

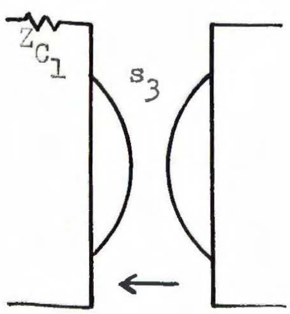
Actual



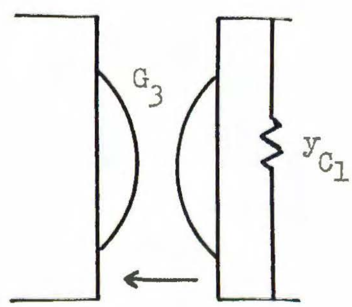


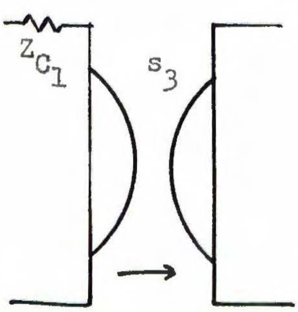
Grounded Base





Grounded Emitter





Grounded Collector

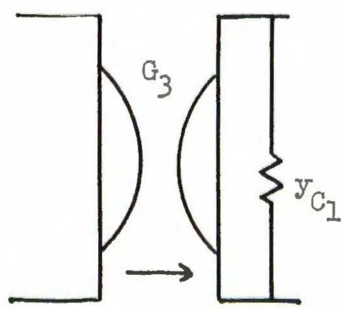
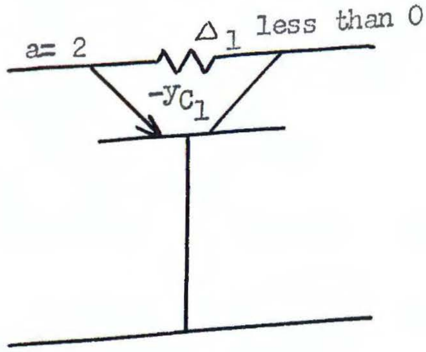


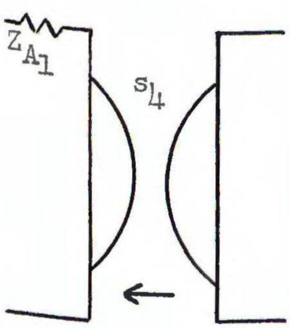
TABLE II (Cont.)

GYRATOR EQUIVALENTS

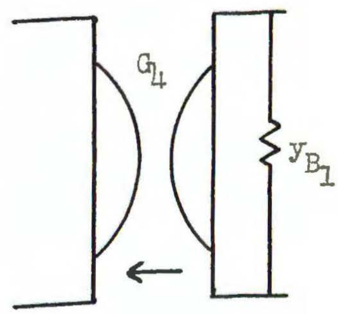
Type D

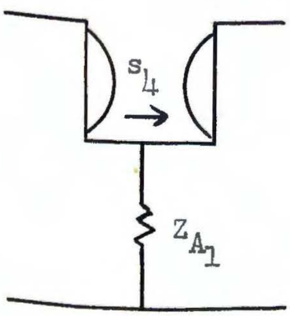
Actual



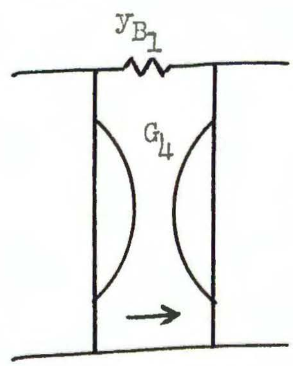


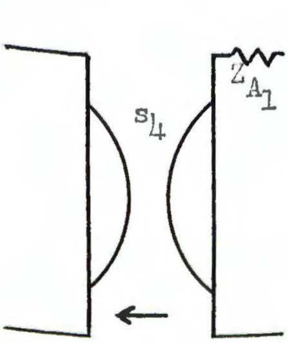
Grounded Base





Grounded Emitter





Grounded Collector

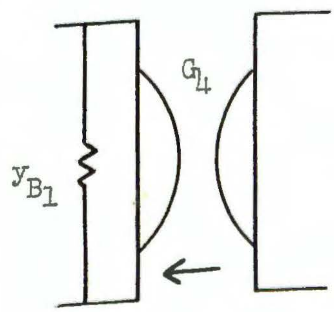


TABLE III

SUMMARY OF CHARACTERISTICS

Type	Case 1 Grounded Base	Case 2 Grounded Emitter	Case 3 Grounded Collector
		CURRENT AMPLIFICATION	
A	$\frac{r_b + ar_c}{r_b + r_c + R_L}$	$\frac{r_e - ar_c}{r_c(1-a) + r_e + R_L}$	$\frac{r_c}{r_c(1-a) + r_e + R_L}$
B	$\frac{r_b + 2r_c}{r_b + r_c + R_L}$	$-\frac{r_c}{R_L}$	$\frac{r_c}{R_L}$
C	$\frac{\Delta_1(r_b + 2r_c)}{\Delta_1(r_b + r_c) + r_c^2 R_L}$	$-\frac{\Delta_1}{r_c R_L}$	$\frac{\Delta_1}{r_c R_L}$
D	$-\frac{(-\Delta_1)}{r_c R_L}$	$\frac{(-\Delta_1)(2r_c - r_e)}{(-\Delta_1)(r_c - r_e) + r_c^2 R_L}$	$\frac{-(-\Delta_1)r_c}{(-\Delta_1)(r_c - r_e) + r_c^2 R_L}$
E		$\frac{s}{R_L}$	
		VOLTAGE AMPLIFICATION	
A	$\frac{(ar_c + r_b)R_L}{\Delta + (r_e + r_b)R_L}$	$-\frac{(ar_c - r_e)R_L}{\Delta + (r_e + r_b)R_L}$	$\frac{+r_c R_L}{\Delta + (r_b + r_c)R_L}$
B	$\frac{(r_b + 2r_c)R_L}{r_c^2 + (r_b + r_c)R_L}$	$\frac{-r_c R_L}{r_c^2 + (r_b + r_c)R_L}$	$\frac{r_c R_L}{r_c^2 + (r_b + r_c)R_L}$
C	$\frac{(r_b + 2r_c)R_L}{\Delta_1 + (r_b + r_c)R_L}$	$\frac{-r_c R_L}{\Delta_1 + (r_b + r_c)R_L}$	$\frac{r_c R_L}{\Delta_1 + (r_b + r_c)R_L}$
D	$\frac{-r_c R_L}{(-\Delta_1) + (r_c - r_e)R_L}$	$\frac{(2r_c - r_e)R_L}{(-\Delta_1) + (r_c - r_e)R_L}$	$-\frac{r_c R_L}{(-\Delta_1)}$
E		$\frac{R_L}{s}$	

TABLE III (Cont.)

SUMMARY OF CHARACTERISTICS

Type	Case 1 Grounded Base	Case 2 Grounded Emitter	Case 3 Grounded Collector
		INPUT RESISTANCE	
A	$\frac{\Delta+(r_e+r_b)R_L}{r_b+r_c+R_L}$	$\frac{\Delta+(r_e+r_b)R_L}{r_e+r_c(1-a)+R_L}$	$\frac{\Delta+(r_b+r_c)R_L}{r_e+r_c(1-a)+R_L}$
B	$\frac{r_c{}^2+(r_b+r_c)R_L}{r_b+r_c+R_L}$	$\frac{r_c{}^2+(r_b+r_c)R_L}{R_L}$	$\frac{r_c{}^2+(r_b+r_c)R_L}{R_L}$
C	$\frac{\Delta_1\left[\Delta_1+(r_b+r_c)R_L\right]}{\Delta_1(r_b+r_c)+r_c{}^2R_L}$	$\frac{\Delta_1\left[\Delta_1+(r_b+r_c)R_L\right]}{r_c{}^2R_L}$	$\frac{\Delta_1\left[\Delta_1+(r_b+r_c)R_L\right]}{r_c{}^2R_L}$
D	$\frac{-\Delta_1\left[-\Delta_1+R_L(r_c-r_e)\right]}{r_c{}^2R_L}$	$\frac{-\Delta_1\left[-\Delta_1+R_L(r_c-r_e)\right]}{-\Delta_1(r_c-r_e)+r_c{}^2R_L}$	$\frac{\Delta_1{}^2}{-\Delta_1(r_c-r_e)+r_c{}^2R_L}$
E	$\frac{s^2}{R_L}$		
		OUTPUT RESISTANCE	
A	$\frac{\Delta+(r_b+r_c)R_g}{r_e+r_b+R_g}$	$\frac{\Delta+\left[r_e+r_c(1-a)\right]R_g}{r_e+r_b+R_g}$	$\frac{\Delta+\ r_e+r_c(1-a)R_g}{r_b+r_c+R_g}$
B	$\frac{r_c{}^2+(r_b+r_c)R_g}{r_b+r_c+R_g}$	$\frac{r_c{}^2}{r_b+r_c+R_g}$	$\frac{r_c{}^2}{r_b+r_c+R_g}$
C	$\frac{\Delta_1\left[\Delta_1+(r_b+r_c)R_g\right]}{\Delta_1(r_b+r_c)+r_c{}^2R_g}$	$\frac{\Delta_1{}^2}{\Delta_1(r_b+r_c)+r_c{}^2R_g}$	$\frac{\Delta_1{}^2}{\Delta_1(r_b+r_c)+r_c{}^2R_g}$
D	$\frac{\Delta_1{}^2}{-\Delta_1(r_c-r_e)+r_c{}^2R_g}$	$\frac{-\Delta_1\left[-\Delta_1+(r_c-r_e)R_g\right]}{-\Delta_1(r_c-r_e)+r_c{}^2R_g}$	$\frac{-\Delta_1\left[-\Delta_1+(r_c-r_e)R_g\right]}{r_c{}^2R_g}$
E	$\frac{s^2}{R_g}$		

TABLE III (Cont.)
SUMMARY OF CHARACTERISTICS

Type	Grounded	GAIN
A	Base	$\frac{(ar_c+r_b)^2 R_L}{(r_b+r_c+R_L)\,[\Delta+(r_b+r_e)R_L]}$
A	Emitter	$\frac{(ar_c-r_e)^2 R_L}{(r_e+r_c-ar_c+R_L)\,[\Delta+(r_b+r_e)R_L]}$
A	Collector	$\frac{r_c^2 R_L}{(r_e+r_c-ar_c+R_L)\,[\Delta+(r_b+r_c)R_L]}$
B	Base	$\frac{(r_b+2r_c)^2 R_L}{(r_b+r_c+R_L)\,[r_c^2+(r_b+r_c)R_L]}$
B	Emitter	$\frac{r_c^2}{r_c^2+(r_b+r_c)R_L}$
B	Collector	$\frac{r_c^2}{r_c^2+(r_b+r_c)R_L}$
C	Base	$\frac{\Delta_1(r_b+2r_c)^2 R_L}{[\Delta_1+(r_b+r_c)R_L]\,[\Delta_1(r_b+r_c)+R_L r_c^2]}$
C	Emitter	$\frac{\Delta_1}{\Delta_1+(r_b+r_c)R_L}$
C	Collector	$\frac{\Delta_1}{\Delta_1+(r_b+r_c)R_L}$
D	Base	$\frac{(-\Delta_1)}{(-\Delta_1)+(r_c-r_e)R_L}$
D	Emitter	$\frac{(-\Delta_1)(2r_c-r_e)^2 R_L}{[(-\Delta_1)(r_c-r_e)+r_c^2 R_L]\,[(-\Delta_1)+(r_c-r_e)R_L]}$
D	Collector	$\frac{r_c^2 R_L}{(-\Delta_1)(r_c-r_e)+r_c^2 R_L}$
E		1

In the tables, the following abbreviations were used, with the current and voltage conventions of the admittance basis. Case E is an ideal gyrator.

$$Z_A = r_e - \frac{ar_c}{2} \qquad Z_B = r_c - \frac{ar_c}{2} \qquad Z_C = r_b + \frac{ar_c}{2}$$

$$\Delta = r_e r_b + r_b r_c + r_c r_e - ar_c r_b$$

$\Delta_1 = \Delta$ when $a = 2$ or $\Delta = r_e r_b + r_e r_c - r_b r_c$

$$y_A = \frac{1}{\Delta}\left(r_c - \frac{ar_c}{2}\right) \qquad y_B = \frac{1}{\Delta}\left(r_e - \frac{ar_c}{2}\right) \qquad y_C = \frac{1}{\Delta}\left(r_b + \frac{ar_c}{2}\right)$$

$$y_{B_1} = \frac{1}{-\Delta_1}(r_c - r_e) \qquad y_{C_1} = \frac{1}{\Delta_1}(r_b + r_c)$$

$$Z_{A_1} = \frac{(-\Delta_1)(r_c - r_e)}{r_c^2} \qquad Z_{C_2} = r_b + r_c \qquad Z_{C_1} = \frac{\Delta_1 (r_b + r_c)}{r_c^2}$$

$$y_{C_2} = \frac{r_b + r_c}{r_c^2}$$

$$G_1 = \frac{ar_c}{2\Delta} \qquad s_1 = \frac{ar_c}{2}$$

$$G_2 = \frac{1}{r_c} \qquad s_2 = r_c$$

$$G_3 = \frac{r_c}{\Delta_1} \qquad s_3 = \frac{\Delta_1}{r_c}$$

$$G_4 = \frac{r_c}{-\Delta_1} \qquad s_4 = -\frac{\Delta_1}{r_c}$$

The tables show various results of loading and can be analyzed as follows. The input and output resistance of all types of loading are always positive which is logical as all the negative resistances were absorbed by the loading. With all types of loading the absolute value of current gain varied inversely with the load resistance, which is the property of the gyrator. The current gain and voltage gain for all types of loading have the same sign, which can be seen by inspecting the equivalent gyrator for each case. The sign is + if the arrow is left to right from input to output which is the normal direction for the gyrator. The power gain is always + and less than or equal to one for all types of loading, due to the circuit not being active, i.e. no negative resistances.

SECTION VI

EXPERIMENTAL RESULTS

To check the gyrator action, the gyrator property of impedance inversion was used. The input resistance was measured for grounded base, grounded emitter and grounded collector, using series loading for two point contact transistors, 2N32 and 2N32A. The method used was to adjust a series decade box until the vacuum tube voltmeter reading across it was equal to the vacuum tube voltmeter reading across the input of the loaded transistor.[11] The circuit used for the grounded base is shown in Fig. 8.

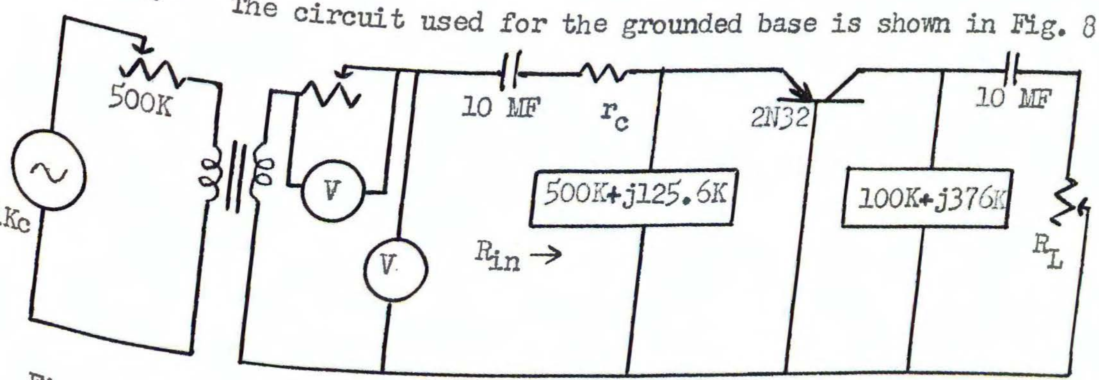


Fig. 8 Method of Testing

This is the procedure that was used. First, the operating point of a transistor was found with a=2 when the output was short-circuited. Then the value of collector resistance was found by measuring the open circuit output resistance, grounded base at the same operating point. This value is actually $r_b + r_c$, but as r_c is much greater than r_b, it is a good approximation to r_c.[12] Setting the series resistor in the emitter arm the input resistance of the loaded circuit is measured for grounded base, grounded

11. Leonard Krugman. Fundamentals of Transistors. John F. Rider & Co., New York, 1954, p. 68

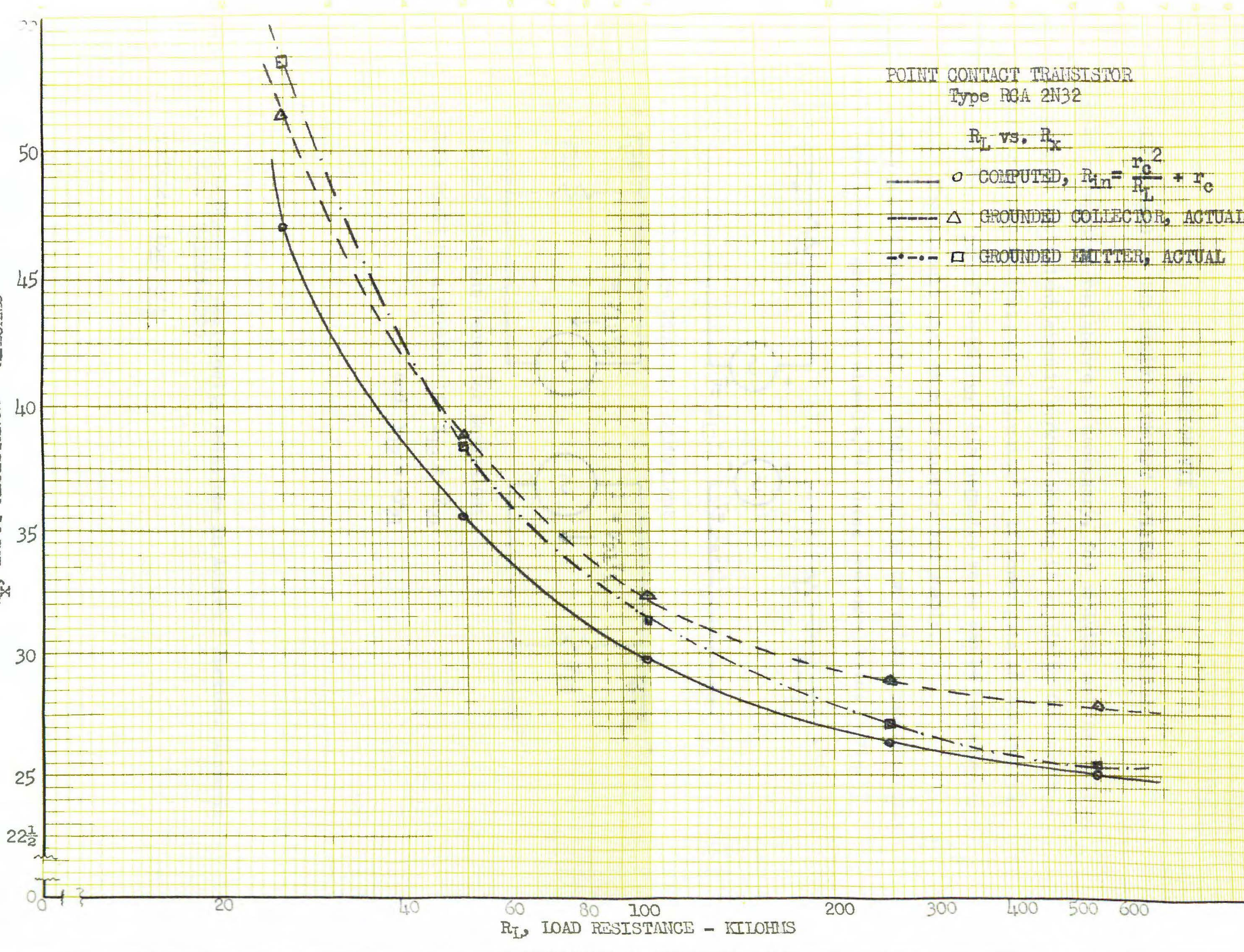
12. R. F. Shea. Principles of Transistor Circuits. John Wiley & Sons, New York, 1953, p. 337

emitter and grounded collector. This value was a good approximation to $r_c - r_e$.

Results for the 2N32A and 2N32 were comparable. Those for the 2N32 are shown.

For the grounded base, the variation of input with changes in load were too slight to be noticeable. However, results for grounded collector and grounded emitter came within 10% for the range from $R_L = ar_c$ to R_L approaching infinity. Below ar_c the error increased, due to accumulation of errors in maintaining $a = 2$, and the loading resistance approximation to r_c.

Generally, the supply impedances were sufficiently high to disregard, except for the errors introduced when the input resistance, or load resistance were too high.


POINT CONTACT TRANSISTOR
Type RCA 2N32
R_L vs. R_x
o COMPUTED, $R_{in} = \frac{r_c^2}{R_L} + r_c$
△ GROUNDED COLLECTOR, ACTUAL
□ GROUNDED EMITTER, ACTUAL
50
45
40
35
30
25
$22\frac{1}{2}$
0
0
20
40
60
80
100
200
300
400
500
600
R_L, LOAD RESISTANCE - KILOHMS

SECTION VII

USEFULNESS OF THE GYRATOR CONCEPT APPLIED TO TRANSISTORS

As a negative resistance is necessary, for activity and instability, the gyrator equivalent circuit method showing it explicity for one frequency will aid analysis of a circuit.[13] If power gain is not desired, just a non-reciprocal coupling, this negative value can be cancelled by loading. The circuit will then be passive and stable.

This equivalent circuit is similar to the two voltage generator circuit, or current generator circuit.[14] These are shown below in Fig. 9.

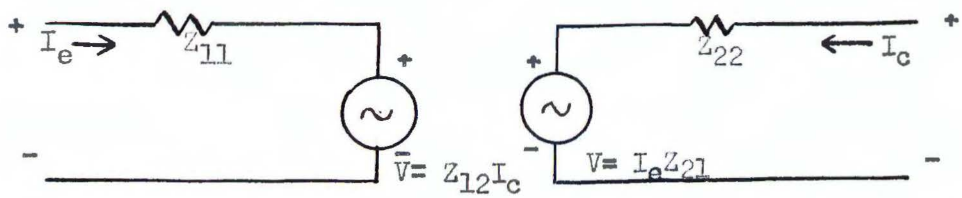


Fig. 9a Voltage Generator Equivalent

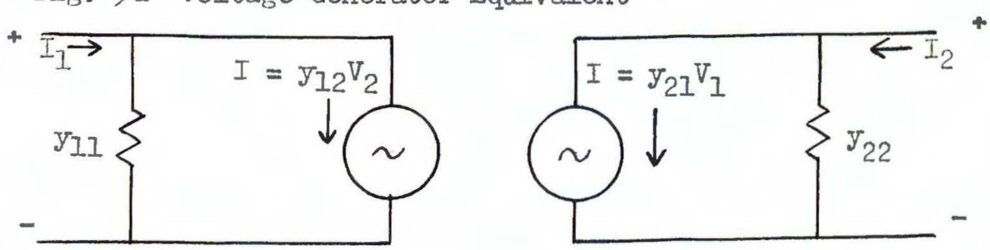


Fig. 9b Current Generator Equivalent

This would be a gyrator if $Z_{12} = -Z_{21}$

and $y_{12} = -y_{21}$

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13. Jacob Shekel. Reciprocity Relations in Active 3 Terminal Elements. Proc. I.R.E. 42:1268-1270

14. Vallese, loc. cit.

SECTION VIII

APPLICATIONS OF THE GYRATOR

The main use of the gyrator is as a non-reciprocal device. The transistor has a low reciprocal impedance inherently, compared to the vacuum tube. If the gyrator aspect of the transistor is emphasized more, the similarity between the transistor and vacuum tube can be made closer. As a buffer circuit, the more the circuit resembles a gyrator the better.

When the gyrator idea was first conceived a physical unit was constructed with disappointing results, however, the media necessary and the type of construction necessary were postulated. Since that time the field of ferrites has opened, and at microwave frequencies gyrators have been made for non-reciprocal couplings.

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____________, "Reciprocity Relations in Active Three-Terminal Elements", Proc. I.R.E. 42:1268-1270, August, 1954.

Tellegen, B. D. H., "The Gyrator, a New Electric Network Element", Phillips Research Report, 3:81-101, 1948.

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