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An Analytical Model of Epidemic Routing with Immunity for Disruption Tolerant Networks

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***Abstract*—We study the epidemic routing scheme with an immunity mechanism. The immunity mechanism enables more efficient utilization of limited buffer space at the nodes, by allowing nodes to remove outstanding copies of messages that have already been delivered. We develop a new analytical model for estimating the message delivery ratio (MDR) and the average delivery delay (ADD) under a variant of epidemic routing with the immunity mechanism, which we call an *immunity routing scheme* (IRS). The proposed model is based on a continuous-time Markov chain and takes into finite buffer sizes at the nodes.**

I. Introduction

Recently there have been growing interests in Disruption Tolerant Networks (DTNs), including military applications [11], [17], [18]. One of salient features of DTNs is that one-hop connectivity of the network between nodes is assumed to be sparse or intermittent. A consequence of this intermittent/sparse connectivity is that an end-to-end route between an information source and its intended destination is unlikely to be available when needed. For this reason traditional mobile ad-hoc network routing protocols (e.g., ad-hoc on-demand distance vector [22] or dynamic source routing [12]) that assume the availability of an end-to-end route are no longer suitable.

In addition to sparse connectivity, in general, a pair of nodes in a network may never encounter each other. Therefore, even when infinite delay is allowed, some nodes may never be able to deliver messages directly to their destinations. Hence, in some cases nodes may not be able to count on a single source or relay node to deliver messages to intended destinations, and multiple relay nodes may be required. For these reasons, some routing schemes (e.g., epidemic routing [30] and spray-and-wait routing [27]) allow multiple copies of messages in the network in order to increase the fraction of messages successfully delivered to their destinations, called a message delivery ratio (MDR), and/or to reduce average delivery delays (ADDs). This is generally done at the expense of increased storage requirements at the nodes and higher resource needs necessary to forward multiple copies.

A. A short survey of related work

There are several existing routing schemes for DTNs and studies on their analysis (e.g., [10], [18], [27], [28], [30], [31]). In this section, we limit our discussion to the studies most relevant to our study and provide a motivation for the study.

It is clear that the achievable performance of a routing scheme in DTNs depends on (i) the time-varying network topology (i.e., one-hop connectivity) and (ii) the information available to the scheme. On one hand, if the mobility of the nodes is deterministic and the contact times between the nodes are known in advance, a set of links can be scheduled ahead for transmission at *different* times to offer end-to-end delivery of messages. This is one possible operational mode of original *delay* tolerant networks. On the other hand, if the mobility is stochastic, which is the scenario of interest to us, only time-varying (local) one-hop connectivity information may be available to the nodes for forwarding decisions.

In some cases, the random mobility of the nodes may be (quasi-)stationary or predictable to a large extent. For instance, Song et al. [26] recently studied the mobility patterns of cellular phone users and showed that *human mobility* is, for the most part, very predictable (93 percent potential predictability). Another example is the *UMass DieselNet* [3], which consists of approximately 40 buses. Since the bus schedules are fixed, their mobility and resulting meeting times are largely predictable. In these cases the statistical information of the mobility processes may be estimated and exploited for message routing.

In the other scenarios, however, the mobility may be more *unpredictable* and/or *non-stationary* with time-varying statistical parameters (e.g., military operations in hostile or uncharted environments). In these scenarios, it may be difficult to gather accurate statistical information from the nodes' mobility and the network topology. As a result, the nodes will unlikely be able to learn and make use of the statistical properties of the nodes' mobility. In fact, when the statistical parameters of the mobility change faster than the nodes can estimate, relying on inaccurate and potentially misleading estimates of the mobility parameters, due to their time varying nature, may be harmful and can lead to worse performance (than not using them at all). We assume that this is the case for our study.

When the statistical information regarding nodes' mobility is not available for the reasons stated above, a natural approach to message forwarding is flooding or controlled flooding of messages, in hopes that one of the copies will reach the intended destination. It is clear that, if the message drop rates at the nodes caused by buffer overflows are negligible, the probability of delivery will increase with the number of relay

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nodes carrying a copy of messages. Therefore, in order to maximize the MDR, the message forwarding scheme should attempt to generate as many copies of messages as possible.

At one end of the spectrum, a simple approach to maximizing the number of nodes carrying a copy is to forward a copy to every node that comes in contact with another node with a copy. This is the basic idea behind *epidemic routing* [30], which mimics the way an infectious disease propagates throughout a population. Such a scheme increases storage requirements at the nodes. Hence, when the buffer size is finite, it leads to a higher message drop rate at the nodes, thereby reducing the stay times of the messages at the nodes. This in turn affects the MDR.

A variant of epidemic routing, called *spray-and-wait* [27], attempts to control the maximum number of copies in the network. The key idea behind the scheme is that once a sufficient number of nodes carry a copy, the benefits from generating additional copies are marginal. Limiting the maximum number of copies curbs the message drop rate at the nodes with a finite buffer. Consequently, the spray-and-wait routing scheme allows messages to remain in the buffer for a longer period, thereby increasing their chances of reaching their destinations before being dropped by buffer overflows. These observations suggest that there is a trade-off between the number of copies of messages produced and their stay times at the nodes when the buffer sizes are finite. This is explored by the (tunable) maximum allowed number of copies in the spray-and-wait routing scheme.

There is another dimension to the problem of designing an efficient routing scheme for DTNs: The copies of messages forwarded to other nodes *after* the messages have already been delivered, while they consume resources (e.g., buffer space), do not improve the MDR or the ADD. Thus, minimizing the proliferation of messages after their delivery will reduce unnecessary resource consumption and, in doing so, increase the stay times of messages without affecting the number of messages forwarded *before* delivery.

This is the basic observation exploited by the *antipackets* proposed by Haas and Small [7] and the *immunity* mechanism [20]. Although these two mechanisms are similar in nature and goals, to be precise, we study the immunity mechanism studied in [20]. The immunity mechanism provides a means for the nodes to propagate the information on the set of messages that have already been delivered, with the aim of curtailing wasteful, additional circulation of delivered messages.

B. Motivation for development of analytical models

First, we note that a similar immunity concept has been introduced earlier: Haas and Small [7] discuss the impact of deleting obsolete information in the context of an infostation model for sensor network applications. The identifier (ID) for a delivered or offloaded packet is called an *antipacket*, and they propose several different methods (called IMMUNE, IMMUNE_TX and VACCINE), based on how antipackets are used. Another closely related study is the work by Zhang et al. [31] on epidemic routing and its variants, including one similar to the epidemic routing with immunity. In addition, Matsuda and Takine [19] analyze the distribution of delivery delays under a class of (p, q)-epidemic routing schemes, including the epidemic routing with vaccine.

All of these studies examine delivery delays and/or buffer requirements, based on either a Markov chain model [7], [19] or ordinary differential equations (ODEs) for the fluid limits as the number of nodes increases [31]. When they analyze the buffer requirements or the delivery delays, however, they assume that the buffer size is infinite and is not a performance bottleneck. Thus, they do not explicitly model the message drops at individual nodes caused by buffer overflows. As a result, their findings cannot be used to predict the performance when the buffer sizes are finite and are not large enough to avoid buffer overflows.

A popular approach to evaluating the impact of finite buffer sizes, especially when buffer sizes present a resource constraint, is by simulation (e.g., [11], [23], [24]). Simulation studies, however, are limited in that the results are valid only for the scenarios evaluated in the studies, and it is not easy to extrapolate the findings to other *unexamined* scenarios with different parameters. In addition, in order to obtain the results that reflect the scenarios of interest, they require appropriate simulation models with correct statistical properties, which is not always easy to ensure – simulation results produced with incorrect models can offer a misleading guidance for network engineers. Moreover, since we may not know in advance the exact settings under which routing schemes will be asked to perform, a wide range of scenarios with varying parameters must be examined. Unfortunately, running simulation for all scenarios of potential interest can be time consuming.

Recently, several studies evaluated the performance of existing and proposed message routing schemes, using empirical (mobility) traces from a limited set of experimental scenarios (e.g., [3], [8], [15]). These studies provide a glimpse of how the evaluated schemes may perform when operating in settings *similar* to the experimental settings. However, as mentioned earlier, some DTN networks are expected to operate in many, diverse environments, including hostile and/or *unanticipated* environments. Unfortunately, it is difficult, if not impossible, to collect a large number of mobility traces with the right statistical properties for all scenarios of interest for a meaningful performance evaluation. Furthermore, extrapolating the performance metrics to other settings for which mobility traces are not available is difficult.

Another approach to evaluating the performance of routing schemes is via analytical models; they allow us to *estimate* the performance of routing schemes over a range of parameter settings, without having to run time-consuming simulations or collecting mobility traces. This is especially important when the network size is large; large-scale simulation or trace-based studies are in general difficult, while a scalable model is often possible to develop and analyze. Moreover, mathematical models can oftentimes offer additional insights that are hard to acquire from simulation or trace-based studies. However, they are in general developed under a set of simplifying assumptions for tractability. For this reason, when (some of) the assumptions are violated, the numbers predicted by the models become less reliable.

These three approaches described above are complementary in nature; they have their own pros and cons and can provide us with valuable information about the performance of different schemes under varying settings. In this paper, we explore the last approach and propose a new analytical model for the epidemic routing scheme with an immunity mechanism. The model takes into account finite buffer sizes at the nodes and provides a method to estimate the MDR and the ADD. In general, finite buffer sizes introduce several challenges to developing a good mathematical model for performance evaluation and an estimation of the MDR and the ADD.

The rest of the paper is organized as follows: A detailed description of the immunity mechanism is provided in Section II. The mathematical setup and an analytical model of the IRS are outlined in Sections III and IV, respectively. Sections V and VI explain how the analytical model can be used to estimate the MDR and the ADD.

II. Description of the immunity mechanism

In the original epidemic routing, an exchange of messages between nodes takes place as follows [30]: When two nodes meet, each node prepares a summary vector with a list of messages it is currently carrying. They exchange the summary vectors, and by comparing the two vectors, each node determines the messages it does not have. They then request a copy of those messages from the other node.

In the immunity mechanism outlined in [20], following an encounter between two nodes, each node sends (i) a message list (m-list), which takes the role of the summary vector in the epidemic routing, and (ii) an immunity list (i-list). Both lists consist of message IDs. The m-list holds the IDs of the messages the node is currently carrying, and the i-list contains the IDs of the messages that have already been delivered to their destination.

Using the two lists, the nodes identify the set of messages to request from the other node. In addition, they identify the messages to be removed from their buffers, based on the i-list from the other node. After receiving the requested messages, they modify their m-list and i-list. We refer an interested reader to a preliminary simulation study reported in [20].

The purpose of the i-list [20] is to keep track of the list of already delivered messages, in order to reduce their proliferation after their delivery and to free up buffer space at the nodes for future message exchanges. For instance, when a node, say i, encounters another node j with a message that is on its i-list, node i does not request the message even if it does not have a copy of the message in its buffer. Furthermore, upon receiving the i-list from node i, node j removes its copy of the message. Hence, the (exchange of) immunity information prevents an unnecessary transmission of the message from node j to node i, and also removes the superfluous copy at node j, freeing up scarce buffer space. Note that the epidemic routing would allow the unnecessary transmission of the message and, barring buffer overflows, the stay of the unneeded copy of the message at node j's buffer.

III. Mathematical setup

We are interested in estimating the MDR and the ADD experienced by successfully delivered messages under the IRS with a finite buffer size at the nodes. To this end, we first develop a simple analytical model based on a Markov chain [4]. Rather than attempting to model and keep track of all the messages in the network, which will likely suffer from the *curse of dimensionality*, we focus on a *single* message and model the evolution of the *number of outstanding copies* of the message in the network over time.

Let $\mathcal{N} := \{1, 2, \ldots, N\}$, $N \geq 2$, be the set of mobile nodes in the network, which move on a domain $\mathbb{D}$. The location of node i at time $t \in \mathbb{R}_+ := [0, \infty)$ is denoted by $X_i(t)$. The mobility process of node $i \in \mathcal{N}$ is given by $\mathbb{X}_i := \{X_i(t);\ t \in \mathbb{R}_+\}$. We assume that the mobility processes of the nodes, $\mathbb{X}_i, i \in \mathcal{N}$, are mutually independent and stationary for the purpose of analysis.

For every pair of distinct nodes i and j in $\mathcal{N}$, we introduce a $\{0,1\}$-valued *reachability* process $\{\zeta_{ij}(t);\ t \in \mathbb{R}_+\}$ with the interpretation that $\zeta_{ij}(t) = 1$ if node i can communicate directly *to* node j at time $t \geq 0$, and $\zeta_{ij}(t) = 0$ otherwise. When $\zeta_{ij}(t) = 1$, we say that the communication link from node i to node j is 'up'. Otherwise, the communication link is 'down'. We assume that the communication links are bidirectional, i.e., $\zeta_{ij}(t) = \zeta_{ji}(t)$. The process $\{\zeta_{ij}(t);\ t \in \mathbb{R}_+\}$ is simply an alternating on-off process, with successive up and down time durations given by the random variables (rvs) $\{U_{ij}(k),\ k \in \mathbb{N}\}$ and $\{D_{ij}(k),\ k \in \mathbb{N}\}$, respectively, where $\mathbb{N} := \{1, 2, \ldots\}$. Note that the rvs $\{D_{ij}(k); k \in \mathbb{N}\}$ denote the intermeeting times between nodes i and j.

In order to make progress we introduce the following assumption on the intermeeting times between nodes:

Assumption 1: The intermeeting times $\{D_{ij}(k); k \in \mathbb{N}\}$ between two nodes $i, j \in \mathcal{N}$ are given by a sequence of independent and identically distributed (i.i.d.) exponential rvs with mean $(\nu_\star)^{-1}$.

It has been reported (e.g., [5], [16]) that the distribution of intermeeting times between a pair of nodes can be approximated by an exponential distribution (i) when nodes move according to a common mobility model, such as the random direction (RD) [2] and random waypoint (RWP) [13] mobility models, on a bounded domain or (ii) when the intermeeting times can be represented as a delayed geometric sum of i.i.d. rvs [14]. The same assumption was introduced in [31].

For every distinct pair $i, j \in \mathcal{N}$, define $\mathbb{M}_{ij} := \{M_{ij}^k; k \in \mathbb{Z}_+\}$, where $M_{ij}^0 = 0$, M_{ij}^k $(k \geq 1)$ denotes the time at which the k-th meeting between nodes i and j takes place after time 0^+, and $\mathbb{Z}_+ := \{0, 1, 2, \ldots\}$. From the meeting times $M_{ij}^k, k \in \mathbb{Z}_+$, we can define another sequence of rvs $\mathbb{I}_{ij} = \{I_{ij}^k; k \in \mathbb{N}\}$, where $I_{ij}^k = M_{ij}^k - M_{ij}^{k-1}$. When contact times $U_{i,j}(k)$, $k \in \mathbb{N}$, (i.e., the amount time during which the communication link between them is up) are much shorter than intermeeting times $D_{ij}(k)$, $k \in \mathbb{N}$, from Assumption 1, we can approximate $I_{ij}^k, k \geq 2$, as i.i.d. exponential rvs with a parameter $\nu \simeq \nu_\star$.

Assumption 2: The rvs $I_{ij}^k, k \geq 2$, are i.i.d. exponential rvs with a parameter ν. Furthermore, $\mathbb{I}_{ij}, i, j \in \mathcal{N}$, are mutually independent.

Assumption 2 implies that a node meets other nodes at the rate of $(N-1)\ \nu$, with the average amount of elapsed time between two consecutive meetings equal to $1/((N-1)\ \nu)$.

New messages arrive at node $i \in \mathcal{N}$ according to a Poisson process $\mathbb{B}_i$ with rate λ_i.[1] For our analysis, we assume that the new message arrival rates λ_i are the same, i.e., $\lambda_i = \lambda$ for all $i \in \mathcal{N}$ for some $\lambda > 0$, and that the new message arrival processes $\mathbb{B}_i, i \in \mathcal{N}$, are mutually independent.

IV. Markov chain-based model

As mentioned earlier, we focus on a *single* message generated by some node and examine how the number of outstanding copies of the message evolves over time until either (i) a copy of the message reaches its destination or (ii) the message is purged from the network.[2] Without loss of generality, we assume that the message is created at time $t_0 = 0$.

Let $Y_C(t)$, $t \in \mathbb{R}_+$, denote the number of copies of the message (i.e., the number of nodes carrying a copy of the message) at time t. In order to develop a tractable model we introduce following simplifying assumptions:

Assumption 3: (i) An exchange of messages between two nodes following an encounter takes place instantaneously, and all transmissions are successful. Further, either node is equally likely to request messages from the other node first. (ii) Suppose that two nodes i and j meet at time $t \in \mathbb{R}_+$, and a copy of message m requested by node j from node i causes a buffer overflow at node j. Then, every message present in node j's buffer just prior to the meeting is equally likely to be dropped. (iii) Messages lost to buffer overflows at different nodes are selected independently. (iv) The buffer is full at every node *at steady state*.

Assumption 3(i) means that the contact times following encounters are long enough to complete the exchange of messages between the nodes. Assumption 3(ii) is introduced for technical convenience so that we do not have to keep track of the position of each message in the buffer of every node. Removal of this assumption, however, leads to an intractable model because we need to keep track of not only the number of copies in the network, but also the position of *every* copy of the message in the buffer of its carrier. When some other buffer management scheme (e.g., First-In-First-Out (FIFO)) is employed, this will cause some discrepancy, especially when the buffer sizes are small. Assumption 3(iv) is a reasonable assumption when the buffer size is a performance bottleneck, which is the scenario of interest to us.

Recall from Assumptions 1 through 3 that the intermeeting times between nodes are given by i.i.d. exponential rvs and new messages are generated by the nodes according to mutually independent Poisson processes. Thus, we can model $\mathbf{Y}_C := \{Y_C(t); t \in \mathbb{R}_+\}$ using a continuous-time Markov chain (MC) [4]. The state space of the MC is given by $\mathcal{S} := \{0, 1, \ldots, N-1, D\}$, where (i) $Y_C(t) = k, k = 0, 1, \ldots, N-1$, means that the message has not been delivered and there are k nodes with a copy of the message at time t, and (ii) $Y_C(t) = D$ indicates that a copy of the message has been delivered to its destination. Once the message is eliminated from the network, i.e., the MC $Y_C(t)$ reaches state 0, the MC stays there forever. Similarly, once a copy of the message is delivered to the destination, the MC remains at state D for good.

[1] New messages here refer to the messages generated by node i, not including those received from other nodes.

[2] Keeping track of the number of copies after delivery results in an *intractable* model because each copy needs to be monitored separately, based on its position in the buffer of its carrier at the time the message is delivered.

A. Generator of the continuous-time Markov chain

Let us define q to be the probability that a message present in the buffer of a node remains in the buffer after the node encounters another node and exchanges messages. Note from Assumption 3 that q is also the average fraction of messages in the buffer at the time of an encounter, which are not lost to buffer overflows during the ensuing exchange of messages. Define $q_c := 1 - q$.

Denote the buffer size at the nodes by B. Assuming that each message is destined for a single destination, the off-diagonal elements of the generator of the continuous-time MC $\mathbf{Y}_C$ [4], denoted by $G = [g_{k,\ell};\ k, \ell \in \mathcal{S}]$, are given by

$$g_{k,\ell} = \begin{cases} k\ (N-k-1)\ \nu\ q \\ \qquad \text{if}\ \ k = 1, 2, \ldots, N-2\ \ \text{and}\ \ \ell = k+1, \\ k\ \nu\ \frac{1+q}{2} \\ \qquad \text{if}\ \ k = 1, 2, \ldots, N-1\ \ \text{and}\ \ \ell = D, \\ \frac{k\ (k-1)}{2}\ \nu\ (q_c)^2 \\ \qquad \text{if}\ \ k = 2, 3, \ldots, N-1\ \ \text{and}\ \ \ell = k-2, \\ \frac{\lambda}{(B\ q)/2} + \frac{(N-1)\ \nu\ q_c}{2} \\ \qquad \text{if}\ \ k = 1\ \ \text{and}\ \ \ell = 0, \\ \frac{k\ \lambda}{(B\ q)/2} + \frac{k\ (N-k)\ \nu\ q_c}{2} +\ k\ (k-1)\ \nu\ q\ q_c \\ \qquad \text{if}\ \ k = 2, 3, \ldots, N-1\ \ \text{and}\ \ \ell = k-1, \\ 0 \qquad \text{otherwise.} \end{cases} \quad (1)$$

The diagonal elements of G are given by $g_{k,k} = -\sum_{\ell \neq k} g_{k,\ell}$ for all $k \in \mathcal{S}$.

Let us explain the transition rates in (1): Recall from Assumption 2 that the meetings between a pair of nodes occur at the rate of ν.

- $g_{k,k+1}$ – When there are k $(1 \leq k \leq N-2)$ nodes with a copy of the message, from the assumed mutual independence of $\mathbb{I}_{ij}$ (Assumption 2) the meetings between the nodes with a copy of the message and other nodes without a copy, excluding the destination, take place at the rate of $k(N-k-1)\nu$. Since a node with a copy, when it encounters a node without a copy, will successfully deliver a copy to the other node and not lose its own copy with probability q, the transition rate $g_{k,k+1}$ equals $k(N-k-1)\nu\ q$.
- $g_{k,D}$ – Analogous to the previous case, when k nodes have a copy of the message, say m, they will meet the destination of message m at the rate of $k \cdot \nu$. When a node i with a copy of message m meets the destination, it will successfully deliver message m if (i) it delivers the messages requested by the destination, including message m, first or (ii) it first receives new messages it requested from the destination without dropping its copy of message m. Since we assume that either node will request messages from the other node first with equal probability of 1/2, the probability that node i will successfully deliver message m to the destination upon encounter is given by $(1+q)/2$.

• $g_{k,k-2}$ – When there are k nodes with a copy, these nodes meet with each other at rate $(k\ (k-1)\ \nu)/2$, where $(k\ (k-1))/2$ is the number of different pairs of the nodes that can meet amongst the k nodes. When two of these nodes meet, the copy at each node will be lost with probability q_c, independently of each other (Assumption 3(iii)). Thus, the rate at which two copies are lost to buffer overflow equals $(k\ (k-1)\ \nu\ \ (q_c)^2)/2$.

• $g_{k,k-1}$ – There are two separate cases to consider:
(i) $k = 1$ – If only a single node has a copy of the message, the message could be lost in two different ways. First, the copy may be lost due to a buffer overflow caused by generation of new messages at the carrier of the copy between meetings with other nodes. Since we do not know the exact position of the message in the buffer, we assume that it is in the middle of the $B \cdot q$ messages that survived the last meeting with another node (hence, $B \cdot q/2$) and approximate the rate of the event as $\lambda/((B \cdot q)/2)$.

Second, when the node with the only copy meets another node, it may take on some of the messages being carried by the other node which are absent in its buffer and, in the process, drop the only copy in the network in order to free up enough buffer space for requested messages before it had an opportunity to deliver the message to the other node. This will happen with probability $q_c/2$ because the probability that the carrier will request messages from the other node first is 1/2 and, given that it does, the only copy of the message will be dropped from the buffer with probability q_c. This yields the rate $((N-1)\ \nu\ q_c)/2$ for the second case.
(ii) $k > 1$ – When there are $k > 1$ copies in the network, the number of copies can decrease by one in three different ways. The first two are the same as in the case of $k = 1$. The third case arises when two nodes with a copy of the message meet (at rate $(k\ (k-1)\ \nu)/2$ as explained earlier) and one of the two copies is dropped, which happens with probability $2\ q \cdot q_c$ (i.e., one copy is lost to a buffer overflow while the other copy survives the exchange of message(s)).

B. Embedded discrete-time Markov chain

Let $\{t_n; n = 1, 2, \ldots\}$ denote the sequence of times at which the continuous-time MC $\mathbf{Y}_C$, starting at state 1 at time $t = 0$, makes a transition to another state. Then, we can define a discrete-time MC $\mathbf{Y}_D := \{Y_D(n); n \in \mathbb{Z}_+\}$ with initial state $Y_D(0) = 1$ and $Y_D(n) = Y_C(t_n^+)$, embedded in the continuous-time MC with the same state space $\mathcal{S}$.

The one-step transition probabilities of the discrete-time MC $\mathbf{Y}_D$ can be found from the transition rates of the continuous-time MC in (1): The entries of the one-step transition matrix $\mathbf{P} = [P_{k,\ell}; k, \ell \in \mathcal{S}]$ of $\mathbf{Y}_D$ are equal to

$$P_{k,\ell} = \begin{cases} \frac{g_{k,\ell}}{-g_{k,k}} & \text{if } \ g_{k,\ell} > 0, \\ 1 & \text{if } \ k = \ell = 0 \ \text{ or } \ k = \ell = D, \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

The discrete-time MC $\mathbf{Y}_D$ is shown in Fig. 1.

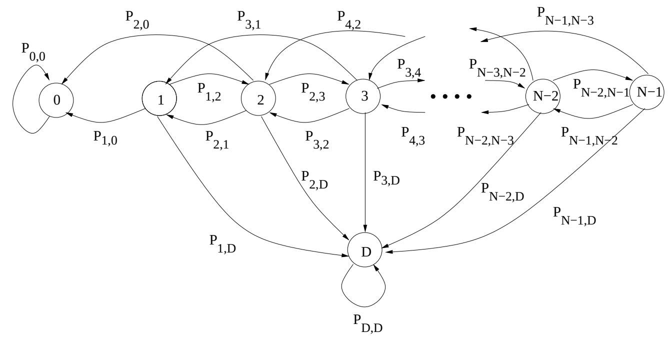

Fig. 1. Discrete-time Markov chain $\mathbf{Y}_D$.

V. Estimation of message delivery ratios and average delivery delays

Suppose that the parameters (B, N, Λ, ν, q), where $\Lambda = N \cdot \lambda$ is the aggregate new message generation rate at all nodes, are known. We describe how we can estimate both the MDR and the ADD under the IRS, using the continuous-time and discrete-time MCs described in the previous section. We denote the MDR by p_{MDR} and the ADD by D_{avg}.

A. Estimation of message delivery ratios (MDRs)

First, note that states 0 and D of the discrete-time MC $\mathbf{Y}_D$ are the only two absorbing states, and the other states are transient. This tells us that, starting at state 1, the MC will reach one of these two absorbing states at some finite $n \in \mathbb{Z}_+$ with probability one. Hence, the probability that a copy of the message is successfully delivered to its destination is the probability that the MC $\mathbf{Y}_D$, starting with $Y_D(0) = 1$, eventually reaches state D (instead of reaching state 0).

Let $f_i, i \in \mathcal{S}$, denote the probability that the MC $\mathbf{Y}_D$ will reach state D, starting at state $i \in \mathcal{S}$. It is obvious from the definition that $f_0 = 0$ and $f_D = 1$, and the MDR is given by $p_{MDR} = f_1$. For each state $k \in \mathcal{S}$, by conditioning on the first transition out of state k, we obtain

$$f_k = \sum_{\ell \in \mathcal{S}} P_{k,\ell}\ f_\ell. \tag{3}$$

Eq. (3) yields the following set of linear equations.

$$\begin{aligned} f_1 &= P_{1,2}\ f_2 + P_{1,0}\ f_0 + P_{1,D}\ f_D = P_{1,2}\ f_2 + P_{1,D}, \\ f_k &= P_{k,k+1}\ f_{k+1} + P_{k,k-1}\ f_{k-1} + P_{k,k-2}\ f_{k-2} \\ &\quad + P_{k,D}\ f_D \\ &= P_{k,k+1}\ f_{k+1} + P_{k,k-1}\ f_{k-1} + P_{k,k-2}\ f_{k-2} + P_{k,D}, \\ &\qquad \text{for } \ k = 2, 3, \ldots, N-2, \ \text{and} \end{aligned} \tag{4}$$

$$\begin{aligned} f_{N-1} &= P_{N-1,N-2}\ f_{N-2} + P_{N-1,N-3}\ f_{N-3} + P_{N-1,D}\ f_D \\ &= P_{N-1,N-2}\ f_{N-2} + P_{N-1,N-3}\ f_{N-3} + P_{N-1,D} \end{aligned}$$

Note that there are $N-1$ unknowns $\{f_1, \ldots, f_{N-1}\}$ and $N-1$ linearly independent equations. Hence, we can solve for the unknowns as follows.

Given a matrix $\mathbf{A}$, we denote the submatrix of $\mathbf{A}$ containing rows r1 through r2 and columns c1 through c2 by $\mathbf{A}_{r1:r2,c1:c2}$. When the submatrix contains a single row or a column, we simply write $\mathbf{A}_{r1,c1:c2}$ or $\mathbf{A}_{r1:r2,c1}$. We can rewrite the relationship in (4) in the following simpler matrix form:

$$\mathbf{f} = \mathbf{P}_{1:N-1,1:N-1}\ \mathbf{f} + \mathbf{P}_{1:N-1,D}\ ,$$

where $\mathbf{f} = (f_1, f_2, \ldots, f_{N-1})^T$, and $\mathbf{P}$ is the one-step transition matrix of the discrete-time MC in (2). Hence, we obtain

$$\mathbf{f} = (\mathbf{I}_{N-1,N-1} - \mathbf{P}_{1:N-1,1:N-1})^{-1}\mathbf{P}_{1:N-1,D}\ , \quad (5)$$

where $\mathbf{I}_{N-1,N-1}$ is an $(N-1)\times(N-1)$ identity matrix.

B. Estimation of average delivery delays (ADDs)

We define the end-to-end delivery delay of a successfully delivered message (i.e., a copy of the message reaches its destination) to be the amount of time it takes after the generation of the message for the destination to receive a copy of the message. Then, the ADD D_{avg} experienced by successfully delivered messages is equal to the expected amount of time it takes for the continuous-time MC $\mathbf{Y}_C$, starting at state 1, to reach state D, *conditional* on the event that it reaches D. This expected delay can be computed using the MCs in a similar way we computed the MDR.

First, since we are dealing only with the messages that are successfully delivered, the MC must reach the absorbing state D (instead of state 0). Hence, we need to modify the transition probabilities of the discrete-time MC as follows:
When the MC $Y_D(n)$ is at state 1, it jumps either to state 2 with probability $(N-2)\ q/((N-2)\ q+1)$ or to state D with probability $1/((N-2)\ q+1)$. Note that these are the conditional probabilities $P_{1,2}/(1-P_{1,0})$ and $P_{1,D}/(1-P_{1,0})$, respectively. Similarly, when the MC is at state 2, it is not allowed to jump to state 0 and we need to modify the transition probabilities out of state 2 accordingly.

Let us define an $N\times N$ matrix $\mathbf{P}^\star = [P^\star_{k,\ell}; k,\ell \in \mathcal{S}^\star]$, where $\mathcal{S}^\star := \mathcal{S}\setminus\{0\}$, and

$$P^\star_{k,\ell} = \begin{cases} P_{k,\ell}/(1-P_{k,0}) & \text{if} \quad k=1,2 \quad \text{and} \quad \ell\in\mathcal{S}^\star, \\ P_{k,\ell} & \text{otherwise.}\end{cases} \quad (6)$$

Define $d(k) = -(g_{k,k})^{-1}$, $k = 1,2,\ldots,N-1$, to be the expected amount of time the continuous-time MC spends at state k after it enters the state till the next jump out of the state. Suppose that $ED(k), k = 1,2,\ldots,N-1$, denotes the expected delivery delay till a copy of the message is delivered, starting with k copies of the message in the network, minus $d(k)$. It is clear that $ED(D) = 0$ by definition. Then, by conditioning on the first transition out of the state under consideration, we obtain, for every $k\in\mathcal{S}^\star$,

$$ED(k) = \sum_{\ell\in\mathcal{S}^\star} P^\star_{k,\ell}\ ED(\ell) + d(k). \quad (7)$$

Define $\mathbf{ED} = (ED(1),\ldots,ED(N-1))^T$ and $\mathbf{d} = (d(1),\ldots,d(N-1))^T$. Then, we can rewrite the relation in (7) in the following matrix form:

$$\mathbf{ED} = \mathbf{P}^\star_{1:N-1,1:N-1}\ \mathbf{ED} + \mathbf{d}$$

or, equivalently,

$$\mathbf{ED} = (\mathbf{I}_{N-1,N-1} - \mathbf{P}^\star_{1:N-1,1:N-1})^{-1}\mathbf{d}\ . \quad (8)$$

The ADD experienced by successfully delivered messages is then given by $D_{avg} = ED(1) + d(1)$.

VI. Performance estimation under the immunity routing scheme

Given the parameters (B, N, Λ), if the meeting rate between a pair of distinct nodes, ν, and the probability q are known, the MDR and the ADD can be computed using (5) and (8), respectively. However, much of difficulty in estimating these performance measures under the IRS lies in the calculation of q. In this section we explain how we can approximate ν and q in order to estimate p_{MDR} and D_{avg} for the IRS.

A. Estimation of probability q

In this subsection we first assume that the meeting rate ν is known and describe how we estimate q. Approximation of ν is detailed in the following subsection. First, we show that, given a fixed value of q, there are two constraints (dependent on other fixed system parameters) which must be satisfied by the average message arrival rate at a node. Let χ_n be the message arrival rate at a node, including messages generated by the node and those received from other nodes. We denote χ_n that satisfies the first (resp. second) constraint by $\chi^1_n(q)$ (resp. $\chi^2_n(q)$). We then find $q^\star$ that satisfies both constraints, i.e., $\chi^1_n(q^\star) = \chi^2_n(q^\star)$, and use $q^\star$ to estimate p_{MDR} and D_{avg}.

(i) Constraint 1: Suppose that α is the probability that a node with a copy of a message, conditional on the event that the message is successfully delivered to the destination, will be immunized before it loses its copy to a buffer overflow and that μ_n is the buffer overflow rate at a node (i.e., a time average of the number of messages lost to buffer overflows per unit time). Then, we have the following relation:

$$\mu_n = \chi_n\ (1 - p_{MDR}\cdot\alpha)\ , \quad (9)$$

where the right-hand side is the message arrival rate times the probability that a message will be dropped due to buffer overflows (i.e., one minus the probability that the message will be removed successfully via immunization before being dropped by a buffer overflow).

The average number of messages dropped by a node due to buffer overflows *per* meeting with other nodes, denoted by Σ, can be computed as follows: As mentioned in Section IV-A, there are two types of events that cause buffer overflows. First, when the buffer of a node is full, any message generated by the node causes a buffer overflow. Secondly, when a node encounters another node and receives new messages currently absent in its buffer, it causes buffer overflow(s) if there is not enough buffer space for the new messages.

Recall that, by Assumption 3(iv), we assume a buffer is always full at steady state. Since the overall buffer overflow rate of a node is μ_n and the rate at which messages are lost to the first type of buffer overflow is $\lambda = \Lambda/N$, the buffer overflow rate due to the second type equals $\mu_n - \lambda$. Obviously, the rate $\mu_n - \lambda$ is equal to the average number of messages dropped by a node per meeting, namely Σ, times the rate at which the node meets other nodes, $(N-1)\ \nu$. Therefore,

$$\Sigma\ (N-1)\ \nu = \mu_n - \lambda \quad \text{or} \quad \Sigma = \frac{\mu_n - \lambda}{(N-1)\ \nu}\ . \quad (10)$$

From (10), the fraction of messages in a buffer lost during an exchange of messages following a meeting, q_c, is given by

$$q_c = \frac{\Sigma}{B} = \frac{\mu_n - \lambda}{(N-1)\ \nu\ B}. \quad (11)$$

Substituting (9) in (11) for μ_n and solving for p_{MDR} yields

$$p_{MDR} = \frac{1}{\alpha} - \frac{\lambda}{\chi_n\ \alpha} - \frac{q_c\ (N-1)\ \nu\ B}{\chi_n\ \alpha}. \quad (12)$$

As explained in the previous section, for a given value of q, we can compute p_{MDR} from (5). Thus, we are interested in finding χ_n and α that satisfy (12). To this end, we first rewrite α as a function of p_{MDR} and χ_n and then solve for χ_n.

Suppose that a message m is delivered to its destination at time t_D. The immunization delay of a node i for message m is defined to be the delay incurred until node i receives the immunity for message m after t_D. The expected immunization delay, denoted by ξ^{-1}, can be computed using a simple continuous-time MC, based on the meeting rates between nodes. This is explained in Appendix A.

Assume that node i has a copy of the message at time t_D. The *residual life* of message m at node i refers to the additional stay time after t_D the message would spend at node i till it is removed by a buffer overflow *if there were no immunity*. Assume that we can model both the immunization delay of node i and the residual life of message m at node i as independent exponential rvs with parameter ξ and B/μ_n, respectively.[3] Then, α is equal to the probability that node i will be immunized before message m is lost to a buffer overflow. This is given by

$$\alpha = \frac{\xi}{\xi + \mu_n/B} = \frac{\xi}{\xi + \chi_n\ (1 - p_{MDR} \cdot \alpha)/B}, \quad (13)$$

where the second equality follows from (9). We can solve (13) for α and obtain

$$\alpha = \frac{\xi\ B + \chi_n - \sqrt{(\xi\ B + \chi_n)^2 - 4\ \chi_n\ p_{MDR}\ \xi\ B}}{2\ \chi_n\ p_{MDR}}. \quad (14)$$

Note that α depends only on χ_n and p_{MDR} (for fixed B and ξ). Therefore, given q (hence, $p_{MDR}(q)$), we can find a unique value of χ_n that satisfies (12) and (14). We denote this value by $\chi_n^1(q)$.

(ii) Constraint 2: Let $C^\star$ be the average number of copies generated of messages, including the original copy of the messages. Similarly, $C_D^\star$ and $C_{UD}^\star$ denote the average number of copies generated of successfully delivered messages and that of undelivered messages, respectively. Then, we have

$$C^\star = p_{MDR}\ C_D^\star + (1 - p_{MDR})\ C_{UD}^\star\ . \quad (15)$$

We can write $C_D^\star$ as the sum $C_D^b + C_D^a$, where C_D^b (resp. C_D^a) is the average number of copies of successfully delivered messages generated before (resp. after) they are delivered to the destinations.

The aggregate message arrival rate at *all* nodes must satisfy $N \cdot \chi_n = \Lambda \cdot C^\star$. This, with (15), gives us

$$\chi_n = \frac{\Lambda}{N} \left(p_{MDR}\ (C_D^b + C_D^a) + (1 - p_{MDR})\ C_{UD}^\star \right). \quad (16)$$

[3]In general, these rvs are not exponential. However, we make this assumption to simplify the computation of α.

Hence, given q, in order to calculate χ_n we need to compute C_D^b, C_D^a, and $C_{UD}^\star$.

1. Computation of C_D^b – We can compute C_D^b by following the same steps used to compute D_{avg} in Section V-B: Let $C_k^*, k = 1, 2, \ldots, N-1$, denote the expected number of copies produced of a successfully delivered message *until* it is delivered to the destination, starting with k copies in the network, and $C_D^* = 0$. Then, for every $k = 1, 2, \ldots, N-1$, by conditioning on the first transition out of the state, we obtain the relation

$$C_k^* = \sum_{j=1}^{k-1} P_{k,j}^\star\ C_j^* + \sum_{j=k+1}^{N-1} P_{k,j}^\star\ (C_j^* + 1). \quad (17)$$

This relation states that the number of copies generated increases by one with each transition from state k ($k = 1, 2, \ldots, N-2$) to $k+1$.[4] In a matrix form, (17) becomes

$$\mathbf{C}^* = P_{1:N-1,1:N-1}^\star\ \mathbf{C}^* + \mathbf{p}^\dagger\ ,$$

where $\mathbf{C}^* = (C_1^*, \ldots, C_{N-1}^*)^T$, and $\mathbf{p}^\dagger = (P_{1,2}^\star, P_{2,3}^\star, \ldots, P_{N-2,N-1}^\star, 0)^T$. Therefore, $C_D^b = C_1^*$ can be computed from

$$\mathbf{C}^* = (I_{N-1,N-1} - P_{1:N-1,1:N-1}^\star)^{-1}\ \mathbf{p}^\dagger.$$

2. Computation of $C_{UD}^\star$ – In order to compute $C_{UD}^\star$, we first calculate the expected number of copies produced of a message (not necessarily successfully delivered) until either a copy of the message is delivered or the message is eradicated from the network without being delivered, which we denote by $C^{\star\star}$. It is clear that

$$C^{\star\star} = p_{MDR}\ C_D^b + (1 - p_{MDR})\ C_{UD}^\star\ . \quad (18)$$

Let $C_k^\ddagger, k = 1, 2, \ldots, N-1$, denote the expected number of copies generated of a message till either a copy reaches the destination or all copies disappear from the network, starting with k copies in the network. Clearly, $C^{\star\star} = C_1^\ddagger$. Again, by conditioning on the first transition out of state k, we have a relation similar to (17): For every $k = 1, 2, \ldots, N-1$,

$$C_k^\ddagger = \sum_{j=1}^{k-1} P_{k,j}\ C_j^\ddagger + \sum_{j=k+1}^{N-1} P_{k,j}\ (C_j^\ddagger + 1) \quad (19)$$

Eq. (19) is equivalent to $\mathbf{C}^\ddagger = P_{1:N-1,1:N-1}\ \mathbf{C}^\ddagger + \mathbf{p}^\ddagger$, where $\mathbf{C}^\ddagger = (C_1^\ddagger, \ldots, C_{N-1}^\ddagger)^T$, and $\mathbf{p}^\ddagger = (P_{1,2}, P_{2,3}, \ldots, P_{N-2,N-1}, 0)^T$. Hence,

$$\mathbf{C}^\ddagger = (I_{N-1,N-1} - P_{1:N-1,1:N-1})^{-1}\ \mathbf{p}^\ddagger.$$

In order to compute $C_{UD}^\star$, we use the relationship (18). Note that C_D^b can be computed as explained above, and p_{MDR} can be obtained from (5) given q. Thus, we get

$$C_{UD}^\star = \frac{C^{\star\star} - p_{MDR}\ C_D^b}{1 - p_{MDR}}\ .$$

3. Computation of C_D^a – Suppose that a copy of message m is delivered to its destination at time t_D and that node j

[4]When a node with a copy meets another node without a copy, it is possible for a new copy to be generated in our model even when the MC stays at the same state. We discount these copies in calculation of $C^\star, C_D^\star$, and $C_{UD}^\star$ as they are small compared to the total number of copies generated.

does not have a copy of message m at time t_D. Without loss of generality, denote the set of nodes with a copy of the message at time t_D by $\{1, 2, \ldots, K\} =: \mathcal{K}$. To simplify our analysis, we assume that the immunization delays experienced by the nodes for message m can be modeled as i.i.d. exponential rvs with parameter ξ, which are independent of $\mathbb{I}_{ij}, i, j \in \mathcal{N}$.[5] Let $\mathcal{A}_K :=$ {node j does not receive a copy of message m from the K nodes in $\mathcal{K}$}, assuming that the copies at the nodes in $\mathcal{K}$ can be dropped only by an immunity message (but no buffer overflow).

Let $\Theta_i, i \in \mathcal{N}$, be the time at which node i receives the immunity for message m and $M^\star_{ij}, i \in \mathcal{K}$, the first time nodes i and j meet after t_D. Define $\mathcal{A}^i := \{\min(\Theta_i, \Theta_j) < M^\star_{ij}\}$. Note that $\mathcal{A}^i$ is the event that node j does not request message m from node i when they meet because either node i has dropped its copy after receiving immunity ($\Theta_i < M^\star_{ij}$) or node j has been immunized ($\Theta_j < M^\star_{ij}$) before $M^\star_{ij}$. Thus,

$$\mathbf{Pr}\left[\mathcal{A}_K\right] = \mathbf{Pr}\left[\bigcap_{i=1}^{K} \mathcal{A}^i\right] = \mathbf{Pr}\left[\bigcap_{i=1}^{K} \left\{\min(\Theta_i, \Theta_j) < M^\star_{ij}\right\}\right].$$

By conditioning on the immunization delay Θ_j of node j,

$$\begin{aligned}
&\mathbf{Pr}\left[\mathcal{A}_K\right] \\
&= \int_{\mathbf{R}_+} \mathbf{Pr}\left[\bigcap_{i=1}^{K} \left\{\min(\Theta_i, \Theta_j) < M^\star_{ij}\right\} \;\middle|\; \Theta_j = t\right] \\
&\qquad \times \xi \ \exp(-\xi \ t) \ dt \\
&= \int_{\mathbf{R}_+} \left(\prod_{i=1}^{K} \mathbf{Pr}\left[\min(\Theta_i, \Theta_j) < M^\star_{ij} \;\middle|\; \Theta_j = t\right]\right) \\
&\qquad \times \xi \ \exp(-\xi \ t) \ dt \\
&= \int_{\mathbf{R}_+} \left(\prod_{i=1}^{K} \mathbf{Pr}\left[\min(\Theta_i, t) < M^\star_{ij}\right]\right) \xi \ \exp(-\xi \ t) \ dt \ ,
\end{aligned} \tag{20}$$

where the second equality follows from the assumed conditional independence of $\mathcal{A}^i, i \in \mathcal{K}$, given Θ_j. Since $\Theta_i, i \in \mathcal{K}$, are i.i.d. by assumption, $\mathbf{Pr}\left[\min(\Theta_i, t) < M^\star_{ij}\right], i \in \mathcal{K}$, are the same and we only need to compute $\mathbf{Pr}\left[\min(\Theta_1, t) < M^\star_{1j}\right]$ in order to determine $\mathbf{Pr}\left[\mathcal{A}_K\right]$ using (20).

For every $t \in (0, \infty)$, let $\mathcal{E}(t) = \{M^\star_{1j} > t\}$ and $\mathcal{E}^c(t) = \{M^\star_{1j} \leq t\}$. Then, by the law of total probability,

$$\begin{aligned}
&\mathbf{Pr}\left[\min(\Theta_1, t) < M^\star_{1j}\right] \\
&= \mathbf{Pr}\left[\min(\Theta_1, t) < M^\star_{1j} \;\middle|\; \mathcal{E}(t)\right] \cdot \mathbf{Pr}\left[\mathcal{E}(t)\right] \\
&\quad + \mathbf{Pr}\left[\min(\Theta_1, t) < M^\star_{1j} \;\middle|\; \mathcal{E}^c(t)\right] \cdot \mathbf{Pr}\left[\mathcal{E}^c(t)\right].
\end{aligned}$$

Clearly, $\mathbf{Pr}\left[\min(\Theta_1, t) < M^\star_{1j} \;\middle|\; \mathcal{E}(t)\right] = 1$ and $\mathbf{Pr}\left[\mathcal{E}(t)\right] = \exp(-\nu t)$. Note that

$$\begin{aligned}
&\mathbf{Pr}\left[\min(\Theta_1, t) < M^\star_{1j} \;\middle|\; \mathcal{E}^c(t)\right] \cdot \mathbf{Pr}\left[\mathcal{E}^c(t)\right] \\
&= \mathbf{Pr}\left[\left\{\min(\Theta_1, t) < M^\star_{1j}\right\} \bigcap \mathcal{E}^c(t)\right].
\end{aligned}$$

[5] This assumed independence between the immunization delays and $\mathbb{I}_{ij}, i, j \in \mathcal{N}$, does not hold in practice as the immunization delays are tied to the meeting times between nodes.

By conditioning on $M^\star_{1j}$,

$$\begin{aligned}
&\mathbf{Pr}\left[\left\{\min(\Theta_1, t) < M^\star_{1j}\right\} \bigcap \mathcal{E}^c(t)\right] \\
&= \int_0^t \mathbf{Pr}\left[\min(\Theta_1, t) < M^\star_{1j} \;\middle|\; M^\star_{1j} = \tau\right] \ \nu \ \exp(-\nu\tau) \ d\tau \\
&= \int_0^t \mathbf{Pr}\left[\Theta_1 < \tau\right] \ \nu \ \exp(-\nu\tau) \ d\tau \\
&= \int_0^t \left(1 - \exp(-\xi\tau)\right) \ \nu \ \exp(-\nu\tau) \ d\tau \\
&= 1 - \exp(-\nu t) - \frac{\nu}{\xi + \nu}\left(1 - \exp(-(\xi + \nu)t)\right).
\end{aligned}$$

Therefore,

$$\begin{aligned}
&\mathbf{Pr}\left[\min(\Theta_1, t) < M^\star_{1j}\right] \\
&= \exp(-\nu t) + 1 - \exp(-\nu t) - \frac{\nu}{\xi + \nu}\left(1 - \exp(-(\xi + \nu)t)\right) \\
&= 1 - \frac{\nu}{\xi + \nu}\left(1 - \exp(-(\xi + \nu)t)\right).
\end{aligned} \tag{21}$$

The probability $\mathbf{Pr}\left[\mathcal{A}_K\right]$ can be obtained by substituting (21) in (20) and carrying out the integration.

We approximate the probability that a node without a copy of message m at the time of its delivery will not receive a copy by $\mathbf{Pr}\left[\mathcal{A}_K\right]$ with $K = C^b_D$ and

$$C^a_D \simeq (N - 1 - C^b_D) \cdot \left(1 - \mathbf{Pr}\left[\mathcal{A}_{C^b_D}\right]\right) .$$

In this approximation, we ignore two factors whose contributions to C^a_D tend to cancel each other out. First, other nodes that receive a copy after t_D can also forward a copy to node j. At the same time, although on the average C^b_D copies are generated before delivery, some of these copies may have been lost to buffer overflow before the delivery takes place. The first tends to increase to the probability that node j will receive a copy, while the latter decreases the probability. Simulation results suggest that the proposed model tends to slightly overestimate $C^\star$ due to these approximations.

Once we obtain $C^\star_D = C^b_D + C^a_D$ and $C^\star_{UD}$, we can compute χ_n using (16) as a function of q (through $p_{MDR}(q)$), which we denote by $\chi^2_n(q)$. Since the correct value of q must satisfy both (12) and (16), we can numerically find $q^\star \in [0, 1]$ that satisfies $\chi^1_n(q^\star) = \chi^2_n(q^\star)$.

B. Estimation of average intermeeting times and a meeting rate ν between two nodes

In the previous subsection we assumed that the meeting rate between two nodes, namely ν, was known. In practice, this quantity may be estimated by individual nodes, for example, by maintaining a record of meetings with other nodes. In this subsection, we explain how it can be approximated for our analysis when the one-hop connectivity between the nodes is determined by the distances between them: Recall from Section III that, for every pair of distinct nodes i and j, $\{\zeta_{ij}(t);\ t \in \mathbb{R}_+\}$ is the reachability process between the nodes and $\{U_{ij}(k); k \in \mathbb{N}\}$ and $\{V_{ij}(k); k \in \mathbb{N}\}$ are the sequence of contact times and intermeeting times, respectively, between them. When nodes' mobility is stationary, from elementary renewal theory [29],

$$\mathbf{Pr}\left[\zeta_{ij}(0)=1\right]=\frac{\mathbf{E}\left[U_{ij}(2)\right]}{\mathbf{E}\left[U_{ij}(2)\right]+\mathbf{E}\left[V_{ij}(2)\right]}. \tag{22}$$

Suppose that the spatial distribution $\mathcal{G}$ of the nodes is known. For example, the spatial distribution of the nodes under the RWP and RD mobility models has been investigated in the literature (e.g., [1], [9], [21]). If two nodes can communicate directly if and only if their distance is not larger than their transmission range γ, the probability $\mathbf{Pr}\left[\zeta_{ij}(0)=1\right]$ is equal to

$$\mathbf{Pr}\left[\zeta_{ij}(0)=1\right]=\int_{\mathbb{D}}\Big(\int_{\mathbb{D}\cap D_\gamma(\mathbf{x})} d\mathcal{G}(\mathbf{y})\Big)\ d\mathcal{G}(\mathbf{x})\ ,$$

where $\mathbb{D}$ is the mobility domain, and $D_\gamma(\mathbf{x})$ is the disk centered at $\mathbf{x}$ with a radius γ. Similar calculation can be performed under different one-hop connectivity models (e.g., the cost-based model [25]).

It is clear from (22) that the meeting rate ν, which is given by $\nu=(\mathbf{E}\left[U_{ij}(2)\right]+\mathbf{E}\left[V_{ij}(2)\right])^{-1}$, can be computed if we can find either $\mathbf{E}\left[U_{ij}(2)\right]$ or $\mathbf{E}\left[V_{ij}(2)\right]$. For instance, Han et al. [6] investigated the distribution and expected value $\mathbf{E}\left[U_{ij}(2)\right]$ of contact times under the RWP mobility model and illustrated how they can be estimated. The expected value $\mathbf{E}\left[V_{ij}(2)\right]$ is then obtained from (22) as

$$\mathbf{E}\left[V_{ij}(2)\right]=\mathbf{E}\left[U_{ij}(2)\right]\times\frac{1+\mathbf{Pr}\left[\zeta_{ij}(0)=1\right]}{\mathbf{Pr}\left[\zeta_{ij}(0)=1\right]}.$$

The meetings rates under the RD model can be calculated in a similar manner.

Appendix

Suppose that a message is delivered at time t_0, at which time the destination receives immunity. Without loss of generality assume that the destination is node 1 and $t_0=0$. Let $N_I(t), t\geq 0$, be the number of nodes with immunity at time t, with $N_I(0)=1$. Then, by Assumption 2, $\{N_I(t);\ t\geq 0\}$ is a continuous-time MC with state space $\tilde{\mathcal{S}}=\{1,2,\ldots,N\}$ and the generator $\tilde{G}=[\tilde{g}_{ij}; i,j\in\tilde{\mathcal{S}}]$, where

$$\tilde{g}_{i,j}=\begin{cases} i\ (N-i)\ \nu \\ \qquad \text{if } i=1,2,\ldots,N-1 \text{ and } j=i+1, \\ -i\ (N-i)\ \nu \\ \qquad \text{if } i=1,2,\ldots,N-1 \text{ and } j=i, \\ 0 \qquad \text{otherwise.}\end{cases}$$

This tells us that when $N_I(t)=k$, the number of nodes with immunity $N_I(t)$ jumps to state $k+1$ at rate $k(N-k)\nu$, which is the rate at which one of the k nodes with immunity meets one of the remaining $N-k$ nodes without immunity. Let $T_k, k=2,3,\ldots,N$, denote the time at which $N_I(t)$ jumps from $k-1$ to k, and $T_1=0$. Then, we have

$$\begin{aligned}\mathbf{E}\left[T_k\right]&=\mathbf{E}\left[T_{k-1}\right]+\frac{1}{(k-1)(N-k+1)\nu}\\ &=\sum_{l=2}^{k}\frac{1}{(\ell-1)(N-\ell+1)\nu}\ ,\qquad k=2,3,\ldots,N.\end{aligned}$$

We denote by Γ_i, $i\in\mathcal{N}$, the time at which node i receives immunity with $\Gamma_1=0$. It is plain that $\{T_k, k=2,\ldots,N\}$ are the order statistics of $\{\Gamma_i, i=2,\ldots,N\}$ [29]. Therefore,

$$\mathbf{E}\left[\sum_{i=2}^{N}\ \Gamma_i\right]=\mathbf{E}\left[\sum_{k=2}^{N}\ T_k\right]=\sum_{k=2}^{N}\left(\sum_{l=1}^{k-1}\frac{1}{l\ (N-\ell)\ \nu}\right),$$

and the average immunization delay is given by $\xi^{-1}=\mathbf{E}\left[\sum_{i=2}^{N}\ \Gamma_i\right]/(N-1)$.

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