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Systematic Enumeration of 1:-1 Constant-Velocity Shaft Couplings

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SYSTEMATIC ENUMERATION OF 1:-1 CONSTANT-VELOCITY SHAFT COUPLINGS

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Abstract

A systematic methodology is employed for the enumeration of 1:-1 constant-velocity shaft couplings. As a result, four potential 1:-1 constant-velocity shaft couplings that are free from singular conditions are enumerated. A promising mechanism is selected for the dynamic analysis. The analysis shows that the mechanism does have the potential for use as a 1:-1 constant-velocity shaft coupling and as a gearless differential.

1 Introduction

The creation of mechanisms to achieve a given function, which is the conceptual phase, is perhaps the most difficult phase of mechanical design. The classification of mechanisms according to kinematic structure and function has been a useful aid in the creation of mechanisms. During the mid nineteen-sixties, it has been shown that kinematic structure can be represented by graphs and graphs can be enumerated systematically using the theory of graphs and combinatorial analysis (Dobrjanskyj and Freudenstein, 1967). This is a powerful tool for the creation of mechanisms in a relatively simple but systematic manner. The basic idea underlying this method is the separation of structure from function (Freudenstein and Maki, 1979, 1983, 1984). In this study, a modified approach is used as shown in Fig. 1 (Chatterjee and Tsai, 1994a and 1994b). In this approach, some of the functional requirements of a mechanism are translated into structural characteristics, which are then incorporated in a "Generator" for the enumeration of mechanism structures. The remaining functional requirements are implemented in a "Tester" for the evaluation of the structures. The mechanisms that pass the initial screening are called potential mechanisms. The advantage of this method over the Freudenstein and Maki's (1979) method is that the search space will be greatly reduced since some of the functional requirements are entered into the structural characteristics of the mechanism.

In the following study, this approach is used to derive 1:-1 constant-velocity (CV) shaft couplings. Based on the kinematic structure and function, potential mechanisms are enumerated. A detailed analysis is later done on one of these potential mechanisms by using the software package DADS (CADSI, 1988).

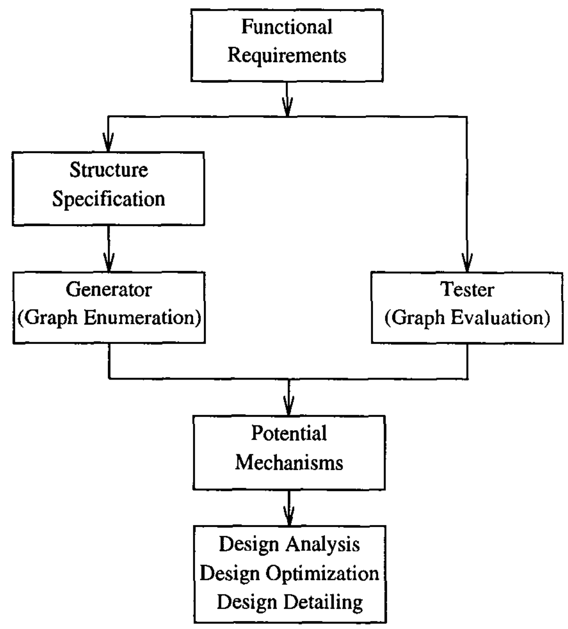

Figure 1: Systematic design of mechanisms

One important application of a 1:-1 CV shaft coupling is that, with a little modification, it can be made as a differential mechanism. Differential mechanisms are used in automobiles to produce the differential motion that is necessary for making a turn. A 1:-1 CV shaft coupling and its corresponding differential mechanism is shown in Fig. 2. In the coupling (Fig. 2(a)), a 1:-1 angular velocity ratio is obtained between the input and output shafts by fixing the cage (carrier) to the ground. In this case, the mechanism works as a one degree-of-freedom (dof) mechanism. An inversion and an addition of one coaxial link to Fig. 2(a) results in a two-dof mechanism as shown in Fig. 2(b). In Fig. 2(b), the cage serves as the input link and the two coaxial shafts serve as the output of the mechanism.

Conventional differential mechanisms are made by using gears. One main disadvantage of using gear-pairs is that gear-pairs has a line contact, which results in a severe wear and tear of the gear-pairs. The life of the mechanism is re-

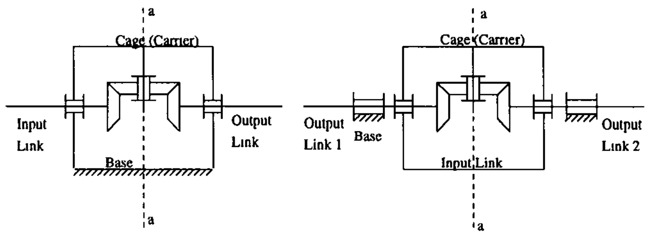

Figure 2: (a) 1:-1 CV Coupling, (b) Corresponding differential mechanism

duced quite a bit due to this fact. Another disadvantage is the higher cost of manufacturing and potential noisiness of the gear-pairs. Thus there is a need to replace these differential gears with differential linkages which have surface contact. In the following study, gearless couplings and differentials are enumerated, to overcome the above mentioned problems.

A lot of previous work has been done on CV shaft couplings and differentials. Hunt (1973) gives a general theory for the construction of a CV coupling. The journal literature on 1:-1 CV coupling and gearless differentials is sparse even though a lot of work has been done on differential gears (Yan and Hsieh, 1994). Habil and Altmann (1950) explains coupler transmissions for uniform transmission ratios and especially concentrates on 1:-1 coupler transmissions.

In the aforementioned literature, there is no indication as to whether the authors have conducted a systematic exploration of all possible 1:-1 CV couplings. In this investigation we develop a systematic search procedure to identify all the 1:-1 couplings within a list of search specifications. By applying the evaluation criteria on the mechanisms so obtained, four potential mechanisms are obtained. This process is described in greater detail below.

2 Principles of 1:-1 CV shaft couplings

The first step is to identify the functional requirements of a 1:-1 CV shaft coupling. Some of these are then converted to structure specifications while the rest is used as the evaluation criteria as shown in Fig. 1.

2.1 Functional Requirements

The basic functional requirement of a CV shaft coupling is to obtain a constant angular velocity ratio between the input and output shafts, in this case the ratio being 1:-1. Another important functional requirement is the ability to transform such a 1:-1 CV coupling to its corresponding differential mechanism.

2.2 Structural Characteristics

Before enumerating the mechanisms, a decision on the class of mechanisms to be enumerated is made. One of the most important problems that arises in a 1:-1 CV shaft coupling is the singularity condition. Habil and Altmann (1950) emphasises mainly on the occurence of singular points and different ways of avoiding it. A simple example explained below illustrates the occurence of singular points in 1:-1 CV shaft couplings.

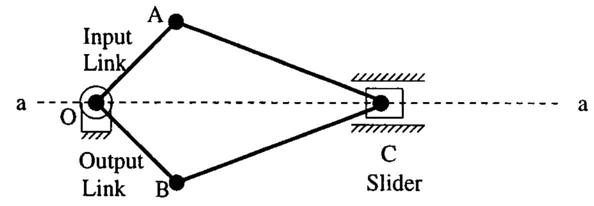

Figure 3: Existence of singularity conditions

For the planar six-link mechanism shown in Fig. 3, when the input link OA rotates in one direction about pivot O, the output link OB will rotate in the opposite direction with the same angular speed, thus acting as a 1:-1 CV coupling. But at the instant of time when the slider C is at one of its extreme positions, i.e. both OA and OB are aligned with the axis a-a, link OB has the capability to rotate in two directions, either along with link OA thus becoming a 1:1 CV shaft coupling or in the opposite direction of link OA thus remaining as a 1:-1 CV shaft coupling. This condition is called the singular condition and the two extreme points are called the singular points. Singularity free condition is a very important functional requirement as it may eliminate a whole class of mechanisms which possess singular points. Habil and Altmann (1950) came up with a few spatial mechanisms which overcome this singular condition and recently, Ricardo Consulting Engineers Ltd. developed a spatial gearless differential mechanism from one of them.

In this study, the search is restricted to one-dof spatial mechanisms since planar mechanisms either result in having singular points or too many prismatic pairs (see Habil and Altmann, 1950) both of which are undesirable. Also work has been done previously on planar mechanisms (though not systematically) but not in spatial 1:-1 CV shaft couplings.

The joints to be considered are turning pairs (R), prismatic pairs (P), cylindrical pairs (C), spherical pairs (S) and ball-in-cylinder (Bc) pairs. This is due to the high load-carrying capability of these joints. The Bc-pair is also included because of its ease of manufacture even though it is not a lower pair. Also since its freedom is equal to four, it helps in reducing the number of links and thus reducing the complexity and inertia of a mechanism.

Hunt (1973) describes the necessity of symmetry of a mechanism about the plane bisecting the input and the output shaft axes for it to act as a CV shaft coupling. This holds true for a single loop mechanism with the desired velocity ratio being 1:1. It turns out that this same necessary condition holds for a 1:-1 CV coupling with two independent loops. It can be seen from Fig. 2(a) that the mechanism is symmetric about the axis a-a and it has two loops. A detailed explanation as to why this works is given below.

Mechanisms with two independent loops are considered as they replicate a standard differential gear mechanism as shown in Fig. 2, the only difference being the gear pairs are replaced by the pairs chosen above. The main reason behind this decision is that these two loop couplings can be easily transformed to a differential mechanism which is of practical interest. Fig. 2 shows the presence of a middle-joint (an R-pair in this case) and a middle link which transmits motion to the two shafts when the cage is given an input. Fig. 2 also shows that the two output links of the differential mechanism are coaxial and that they both are supported by the same carrier. All these facts are incorporated into the structural characteristics of 1:-1 couplings.

A general mechanism satisfying the condition of symmetry with a joint in the middle is shown in Fig. 4. Because of the symmetry, the mechanism can be split into two halfs as shown in Fig. 5. We note that the two halves share two ternary links and one common joint. One of the ternary links serves as the carrier while the other serves as the lever to transmit motion from one half to the other half. In addition, each half of the mechanism is a one-dof kinematic chain.

Consider the left half of the mechanism as shown in Fig. 5(a). As the input link I1 rotates, the lever M1 will produce some motion. Now consider the motion of the lever M2 as the input to the right half of the mechanism as shown in Fig. 5(b). Since the right half of the mechanism is the mirror image of the left half about the plane of symmetry, the output link I2 of the right half will rotate in the opposite direction at the same rate as the link I1 of the left half. Hence the entire mechanism works as a 1:-1 CV shaft coupling.

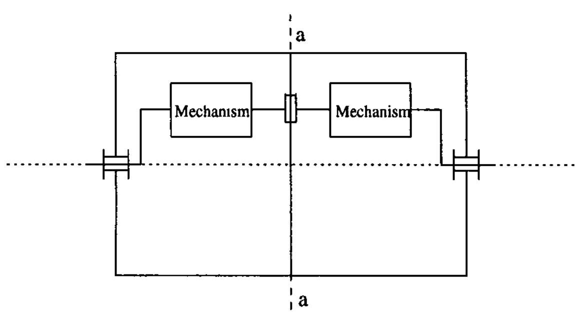

Figure 4: A general 1:-1 CV coupling

Next, a decision on the type of middle joint is made. Both R-pair and P-pair cannot be the middle joint since the resulting mechanisms will possess singular condition. The reason is that if R-pair is the middle joint, each half will function as a crank-and-rocker mechanism, which implies that it has two extreme singular positions. Similar reason holds for a mechanism with a P-pair in the middle. If the middle joint is an S-pair, then the middle link is capable of rotating about any axis with respect to the cage. The symmetry of such a mechanism will be lost when the middle link rotates about an axis that is not parallel to the plane of symmetry. Losing the symmetry implies that the mechanism will no longer function as a 1:-1 CV shaft coupling. So S-pair as the middle joint is rejected. Similar problem is faced with Bc-pair as

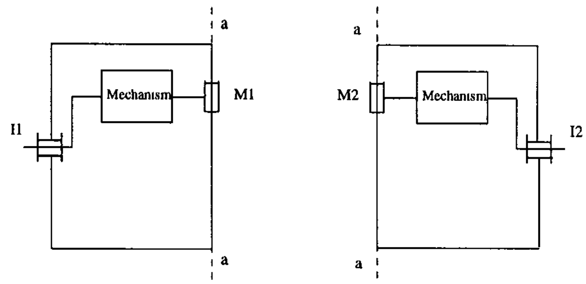

Figure 5: (a) The left half, (b) The right half

the middle joint. With C-pair as the middle joint, the above problems are overcome since the C-pair is able to slide and rotate at the same time. This makes it possible for the two ends of the lever to describe a full circle on a cylinder per rotation of the input link. Since the lever can perform a continuous motion, there will be no singular points. Hence the only feasible middle joint is a C-pair. All the above characteristics are summarized as the search specifications in Table 1 while a list of evaluation criteria (Tester) are summarized in Table 2.

Table 1: Search Specifications

- The search is limited to spatial mechanisms.
- The joints to be considered are turning pairs (R), prismatic pairs (P), spherical pairs (S), cylindrical pairs (C) and ball in cylinder pairs (Bc).
- The search is restricted to only mechanisms with two independent loops.
- The cage (base-link) is a ternary link (i.e., it has three joints). This link is actually the input link of a differential mechanism. The middle joint connects the cage to the middle link which is the other ternary link. All the other links are binary.
- The input link and the output link must be coaxial and they must both be supported by the same cage.
- The joints connecting the input and output links with the cage are R-pairs. This means that the cage has two inline R-pairs (the third joint is the middle joint).
- The middle joint is a C-pair with its joint axis perpendicular to the joint axis of the input and output links.
- The maximum number of joints in a mechanism is limited to seven. This controls the size of a mechanism from becoming too big thus making the mechanism more portable.
- To achieve a 1:-1 ratio from the input to the output link, the symmetry of a mechanism about the plane bisecting the shaft axes must be maintained. This plane of symmetry splits the mechanism into two sub-mechanisms, each of them has one degree-of-freedom (satisfying the general degree-of-freedom equation).
- There is one common joint (the middle joint) and two common links (the middle link and the cage) between these two sub-mechanisms.

Table 2: Evaluation Criteria

- The maximum number of prismatic and cylindrical pairs cannot exceed three. (This limits the amount of sliding in a mechanism, thus reducing frictional energy loss, wear and tear).
- No link can have more than one sliding pair. (This again limits the amount of sliding in a mechanism).

3 Enumeration of 1:-1 CV shaft couplings

Based on the structural characteristics listed in Table 2, two partially labelled graphs representing all possible 1:-1 CV shaft couplings are shown in Figs. 6 and 7, where X and Y denote the unknown joints that are to be enumerated. The correspondence between the graphs and their structural diagrams is that the links are represented by vertices, the joints by edges, and the joint connection of links correspond to the edges connection of vertices. The edges are labelled according to the joint type and the fixed link is identified with a circle around the vertex.

Each graph contains two loops and each loop represent one-dof mechanism which satisfies the general degree-of-freedom equation:

$$F = \lambda(l - j - 1) + \sum_{i=1}^{j} f_i \tag{1}$$

where
F degree of freedom of a mechanism, $F = 1$
λ mobility number $\lambda = 6$ for spatial mechanisms
l number of links
j number of joints
f_i degree of freedom permitted by i^{th} joint
For each loop,

$$j = l \tag{2}$$

Substituting equation (2), $F = 1$, and $\lambda = 6$ into equation (1) yields

$$\sum_{i=1}^{j} f_i = 7 \tag{3}$$

That is the sum of degress of freedom associated with all the joints in each loop must be equal to seven.

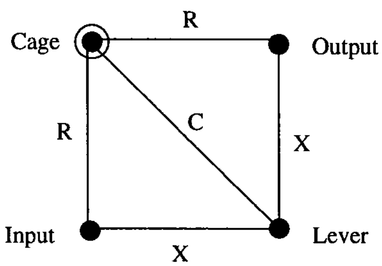

Figure 6: Four-link mechanism

Applying equation (3) to the mechanism shown in Fig. 6, we obtain $f_x + 1 + 2 = 7$, where f_x denotes the degrees of freedom permitted by the X joint. Hence $f_x = 4$ and X can only be a Bc pair. Only one joint combination is possible and it is RBcCBcR.

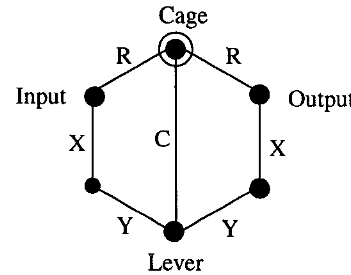

Figure 7: Six-link mechanism

For the mechanism shown in Fig. 7, we have $f_x + f_y + 1 + 2 = 7$, where f_x and f_y denotes the degrees of freedom permitted by the X and Y joints respectively. Hence, $f_x = f_y = 2$, or $f_x = 3$ and $f_y = 1$, or $f_x = 1$ and $f_y = 3$. With these values of f_x and f_y, the five joint combinations are possible. They are RCCCCCR, RRSCSRR, RSRCRSR, RPSCSPR and RSPCPSR.

These are the only possible mechanisms that can be obtained using the search specifications. The graphs and their corresponding couplings are shown in Fig. 8.

The following notation applies for all diagrams:

1	fixed link (cage or carrier)
2	input link
3 and 5	intermediate coupler links
4	lever (middle link)
6	output link

Each of the structures is a one degree-of-freedom mechanism consisting of two loops connected together by the middle pair (C-pair) and the symmetry about the plane bisecting the shaft axes is responsible for the 1:-1 velocity ratio between the input and the output shafts.

4 Evaluation of the enumerated mechanisms

Fig. 8 shows a total of six 1:-1 CV shaft couplings that have been enumerated. Each of these mechanisms are evaluated based on the evaluation criteria in Table 2. The results of the evaluation are listed in Table 3. The application of the evaluation guidelines on these mechanisms results in four acceptable mechanisms.

An interesting feature is noted about the mechanisms enumerated. The degrees of freedom for the whole mechanism, calculated by using the general degree-of-freedom equation (1), is zero. These set of mechanisms which do not satisfy the general degree-of-freedom equation are called overconstrained mechanisms (see Mavroidis and Roth, 1995a and 1995b). Thus all the mechanisms enumerated are overconstrained mechanisms.

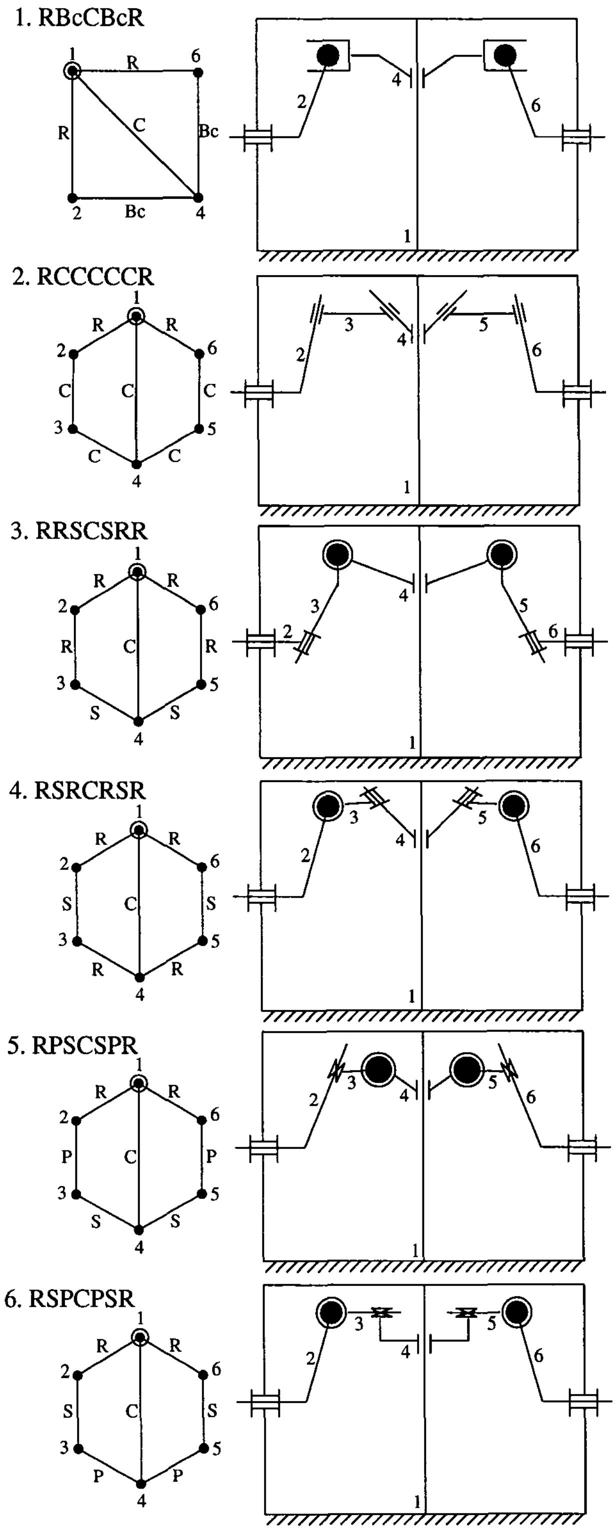

Figure 8: Enumerated graphs and corresponding mechanisms

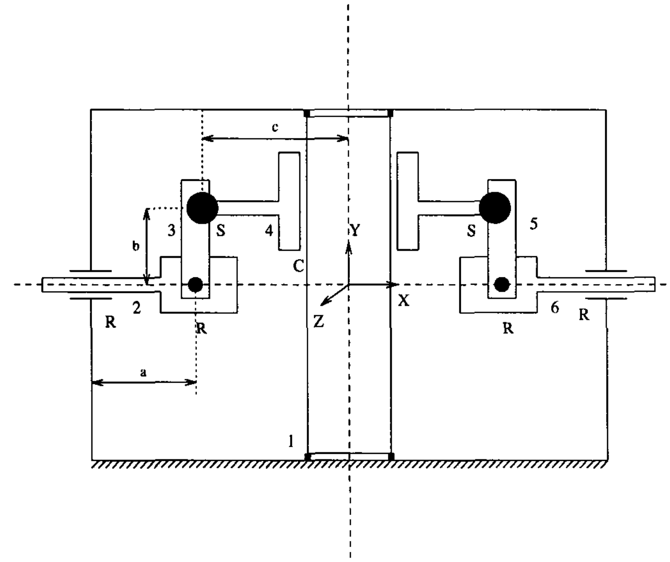

Figure 9: Simulation model

An examination of the potential mechanisms shows that the RBcCBcR mechanism is the mechanism used in the Ricardo differential. Furthermore, the RRSCSRR mechanism appears to be a better alternative since it has fewer sliders in it. The number of sliders is an important criterion especially if the coupling is used as a differential mechanism.

Table 3: Evaluation

1. Acceptable as a potential mechanism. Even though this mechanism has two Bc-pairs (which are not lower pairs), the less number of links in the mechanism is an advantage.
2. Reject : The presence of too many cylindrical pairs in the mechanism can cause excessive sliding friction.
3. Acceptable as a potential mechanism.
4. Acceptable as a potential mechanism.
5. Acceptable as a potential mechanism.
6. Reject : The lever has three sliders.

5 Analysis of the 1:-1 CV coupling

The RRSCSRR mechanism which has the least number of sliders is chosen for the analysis using the software package DADS (CADSI, 1988). The schematic of an RRSCSRR mechanism is shown in Fig. 9.

The mechanism is modelled as a real world system resembling an existing differential gear box. The link lengths are chosen as $a = 3cm$, $b = 2cm$, and $c = 4cm$. For simplicity, all the links are assumed to be cylinders, their axes with respect to the global coordinate system XYZ at the instant shown in the Fig. 9 is explained below. The local Y-axis of the input and output links are along the global X-axis, the local X-axis of the couplers are along the global Y-axis and the local X-axis of the lever is along the global X-axis. The mass moment-of-inertias of all the links are calculated with respect to their local coordinate systems. Since all the links are cylinders, the center of mass of all the links is the center of the cylinders which makes the I_{xy}, I_{yz} and I_{xz} components of the moment-of-inertia for all the links equal to zero. The masses and moment-of-inertia properties of the links are shown in Table 4.

Table 4: Masses and moment-of-inertias of the links

Link	*Mass* (g)	I_{xx} $(g.cm^2)$	I_{yy} $(g.cm^2)$	I_{zz} $(g.cm^2)$
Input	6.36	4.77	0.8	4.77
Coupler1	1.06	0.033	0.353	0.353
Lever	18.9	2.36	100.8	100.8
Coupler2	1.06	0.033	0.353	0.353
Output	6.36	4.77	0.8	4.77

These values are inputted into the software package and the dynamic analysis is performed with respect to the global frame of reference shown in Fig. 9. A constant angular velocity of 28π *rad/s* was given to the input link 2. The analysis resulted in the output link 6 rotating at the same speed but in the opposite direction of the input link 2, thus behaving as a 1:-1 CV shaft coupling.

An important factor that affect the kinematics of the mechanism is the motion of the lever (middle link). The lever has motion only about the global Y-axis (both translation and rotation). For one rotation of the input shaft, the angular and linear displacements as well as velocities of the lever about the global Y-axis are plotted in Figs. 10 and 11. The displacements indicate that the lever oscillates ± 0.5 *rad* and slides ± 2 *cm*. The maximum angular velocity of the lever occurs at the top and bottom dead-center positions while the maximum linear velocity occurs at the mid-position of the lever, where the angular velocity is zero.

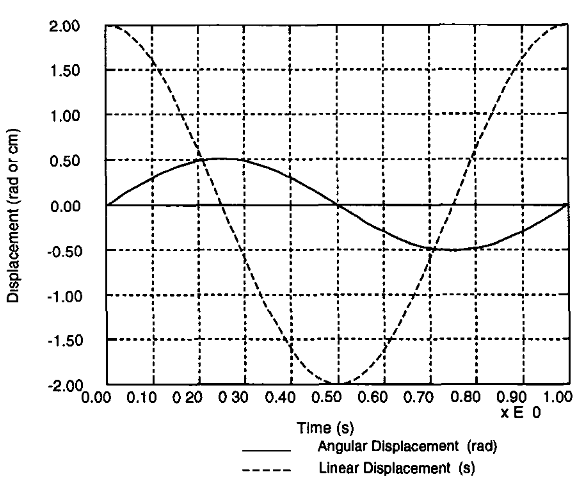

Figure 10: Angular and linear displacement of the lever

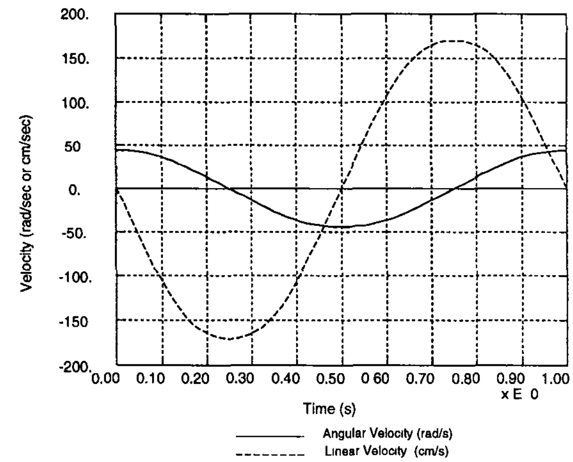

Figure 11: Angular and linear velocity of the lever

In the dynamic analysis, inertia effects of the links are considered while the effect due to gravity is neglected. An important factor to be considered is the reaction forces and torques on the carrier. The analysis produces force and torque acting on the carrier only about the global Y-axis as shown in Fig. 12. The presence of symmetry in the mechanism yields no forces and torques along the X and Z axes. Another important factor to be considered is the force exerted by the cage on the lever at the C-pair. Since the lever can slide freely along the Y-axis, there can be no force exerted on the lever in the Y-direction. The forces in the X and Z directions are plotted in Fig. 13.

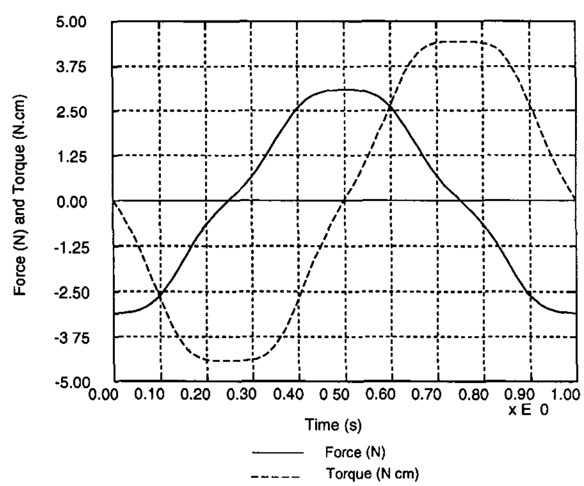

Figure 12: Reaction force and torque acting on the ground about the global Y-axis

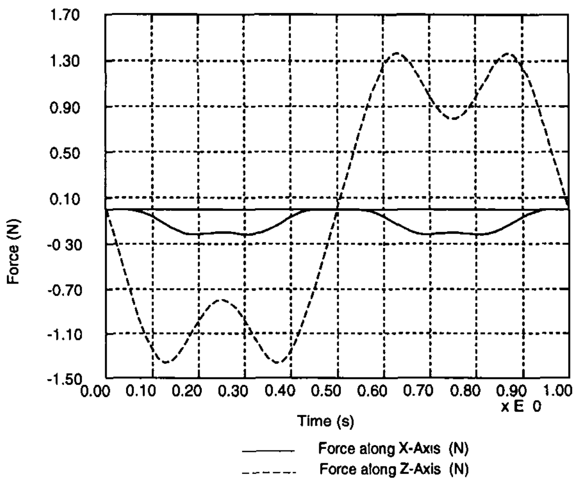

Figure 13: Force exerted by the C-pair on the lever along the global X and Z-axes

6 Conclusion

We have adopted an improved methodology for the enumeration of all possible 1:-1 CV shaft couplings which satisfy a set of search specifications. First, the functional requirements of a 1:-1 CV shaft coupling which is suitable for application as a gearless differential are defined. Then, most of these functional requirements are translated into structural characteristics and incorporated in a generator for the enumeration of mechanism structures. The remaining functional requirements are implemented in a tester for the evaluation of mechanism structures. As a result, we obtain four potential 1:-1 CV shaft couplings.

To perform a feasibility study, a promising candidate mechanism is chosen for the dynamic analysis. The analysis reveals that the mechanism can provide a 1:-1 velocity ratio between the input and the output shafts and the lever possesses a continuous sinusoidal motion. The rotation of the lever is out of phase with respect to its translation by 90 degrees and the shaking force and moment acting on the cage are reasonably small. Hence, the mechanism does have a potential for application as a 1:-1 CV shaft coupling and as a gearless differential. A prototype mechanism will be developed to establish a proof of the concept as a future study.

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