SRC TR 86-3

Reliability Optimization Schemes for Convectively Cooled PCBs

by

D. Dancer, M. Pecht, and M. Palmer

TR-86-3

RELIABILITY OPTIMIZATION SCHEMES FOR CONVECTIVELY COOLED PCBs

David Dancer, Michael Pecht, Milton Palmer
Mechanical Engineering Department
University of Maryland
College Park, MD 20742

ABSTRACT

This article discusses the optimum placement of a single row of convectively cooled electronic components in order to reduce failures. It is shown that this problem is analogous to the classical operations research problem of the optimum time scheduling of n jobs on a single machine, where for each job there is a time dependent completion penalty. Several optimization schemes for solving the idealized problem are compared as to their accuracy and computational speed. For the dynamic programming scheme a new compact labelling procedure is proposed.

INTRODUCTION

The failure rates of electronic components are highly sensitive to temperature. Equations for predicting the failure rates typically have the form:

$$\lambda_i = C_i e^{-A_i/T_{j_i}} + D_i \tag{1}$$

where i is the failure rate of component i; A_i, C_i, and D_i are parameters dependent on the component properties and circuit characteristics; and T_{j_i} is the absolute junction temperature. For CMOS components with a supply voltage in excess of 5 volts, equation (1) is modified to

$$f_i = C_i e^{-A_i/T_{j_i}} + B_i T_{j_i} + D_i \tag{2}$$

where B_i is a parameter dependent on voltage. For optimization, the temperature independent terms, D_i, in equations (1) and (2) need not be considered.

In convectively cooled printed circuit boards (PCBs), components are for the most part located in rows across which forced air passes. As an approximation, the thermal analysis routines often assume the rows are thermally independent. This is especially valid if a fin structure is used to sandwich PCBs, as is the case with coplanar boards. In this article, an idealized situation is considered in which n components are to be placed in a single row and cooled by forced convection with a coolant inlet temperature T_0, mass flow rate W, and specific heat C_p. The heat generation rate of the ith component is q_i. The thermal resistance from the component junction to the coolant is assumed to be independent of position and is denoted by Θ_i. This resistance is the sum of the resistance of the junction to the board and the resistance of the board to the coolant. Heat balance shows that the fluid temperatures, T_{f_i}, are:

$$T_{f_1} = T_o + \frac{q_1}{WC_p} \tag{3}$$

$$T_{f_i} = T_{f_{i-1}} + \frac{q_i}{WC_p} \qquad s \leqslant i \leqslant n \tag{4}$$

Equations (3) and (4) can be combined to yield:

$$T_{f_i} = T_o + \sum_{k=1}^{i} \frac{q_k}{WC_p} \qquad 1 \leqslant i \leqslant n \tag{5}$$

The junction temperatures are given by:

$$T_{j_i} = T_{f_i} + q_i \, \Theta_i \tag{6}$$

From equation (6), the failure rates of equations (1) and (2) can be generalized as:

$$\lambda_i = \lambda_i \, (T_{j_i}) = \lambda_i \, (T_{f_i})$$

The objective is to minimize

$$\sum_{i=1}^{n} \lambda_i \, (T_{f_i}) \; .$$

Equations (3) to (6) are similar to those developed by Mayer [1] except Mayer assumes that

$$T_{f_i} = T_{f_{i-1}} + \frac{q_i + q_{i-1}}{2WC_p}$$

In the special case where the temperature sensitivity of the components are the same, it is obvious that the components should be positioned such that $q_1 \leq q_2 \leq \ldots \leq q_n$. Similarly, if the heat generation rates are the same, it is obvious that the most temperature sensitive components should be placed nearer the coolant inlet. In the general case of unequal heat generation rates and temperature sensitivities, it is often difficult to determine which of the n factorial permutations of n components in n locations is optimal.

The problem discussed above is analogous to the classical operations research problem of scheduling n jobs on a single machine. In the latter problem, each job has a processing time, P_i. Therefore, the time at which a job is completed is $T_i = \sum_{k=1}^{i} P_k$. Associated with each job is a completion time penalty, analogous to f_i, and completion time analogous to T_{f_i}.

A literature review indicated that 4 techniques have been used to solve the n job, single machine optimization problem: enumeration, dynamic programming, priority indexing, and branch and bound. The first 3 techniques will be discussed in this article. The branch and bound technique will be covered in an article.

Enumeration consists simply of considering all possible permutations and selecting the optimum. Although conceptually simple, enumeration is prohibitively compute intensive even for moderately sized problems. A dynamic programming (DP) algorithm has been developed by Held and Karp [2] to solve the n job, single machine problem. A brief description is included in Appendix A. It is noted that DP requires considerably less additions and comporisons than enumeration. Table 1 compares DP with enumeration.

Table 1 - Evaluation of Enumeration and DP Additions and Comparisons

Number of Components	Enumeration Additions	DP Maximum	Enumeration Comparisons	DP Maximum Comparisons
4	72	41	23	41
10	3.27×10^7	5,174	3.63×10^6	5174
15	1.83×10^{13}	16,503	1.31×10^{12}	16,503
20	4.61×10^{19}	1,048,785	2.43×10^{18}	1,048,785
n	$(n-1)n!$	n^{2n-1} + small	$n!-1$	n^{2n-1} + small

DP has two drawbacks only: First, it requires 2^{n-1} storages where n is number of components. For IBM PC Basic, cases with more than 13 components require auxiliary storage. Second, the subscript notation of DP is not well suited for computers and compact labelling must be used. Schrage and Baker [3] describe one compact labelling scheme.

The work of Smith [4], McNaughton [5], and Rothkopf [6] show that if the failure rate is either a linear or exponential function of temperature, i.e.

$$f_i = q_i + h_i \, (T_f - T_o) \tag{7}$$

$$f_i = k_i - \frac{L_i}{r} e^{-r(T_f - T_o)} \tag{8}$$

then simple priority indexing rules exist for the optimum sequencing of the components. In equation (8) it should be emphasized that r must be the same for all components. Rothkopf [7] has shown that no functions, other than linear or exponential, will result in simple rules. Neither equation (1) or (2) is of the form of equation (7) or (8). However, Appendix B shows how equation (1) can be linearized or exponentialized. For the linear case, the components should be in decreasing order of

$$\frac{A_i C_i WC_p}{T_{r_i}^{\;2}\; q_i}\, e^{\frac{-A_i}{T_n}} ,$$

where

$$T_{r_i} = T_o + \frac{\sum_{k=1}^{n} \frac{q_k}{WC_p}}{2} + q_i\, \theta_i ,$$

For the exponential case, the components should be in decreasing order of

$$\frac{\dfrac{A_i^{\,2} C_i}{T_r^{\,4}}\, e^{\frac{-A_i q}{WC_p T_{r_i}^{\,2}}}}{1 - e^{\frac{-A_i q}{WC_p T_{r_i}^{\,2}}}}$$

SIMULATION ANALYSIS

This section describes computer simulation experiments in optimizing the placement of n components in a single row. The computer used was an IBM PC and the language was IBM PC Basic. The optimizatin schemes tested were enumeration, dynamic programming, and linear priority index. It was found that the compact labelling scheme of Schrage and Baker [3] was inefficient. Therefore, a new compact labelling scheme was developed and is described in Appendix C; Appendix D has listings of the computer programs used and Appendix E has sample results.

Table 2 compares the computer time required for the various optimization schemes. As can be seen in Table 2, DP is considerably faster than enumeration, while the Smith (linear) priority index is considerably faster than DP

Table 2 - Comparison of Computer Times (minutes)

Number of Components	Time Enumeration	Time DP	Time Smith
4	0.4	0.4	0.3
7	36.8	1.4	0.5
9	-	5.4	0.6
13	-	122.7	0.7

In all instances it was found that the results from enumeratiOn and DP were identical. For those problems in which the dimensionless quantity $\frac{A_i q}{WC_p T_{r_i}^2}$ was less than about 0.2 the Smith index gave results close to DP and enumeration.

PLANS FOR NEXT REPORTING PERIOD

In the next reporting period it is planned to refine the DP and Smith (linear) index programs and to investigate the Rothkopf exponential scheme and various branch and bound techniques.

ACKNOWLEDGEMENT

This work is partially supported by the Institute for Defense Analysis, and the Systems Research Center under NSF Grant No. CDR-85-00108.

REFERENCES

1. Mayer

2. Held, M., and R. Karp, 1962, A Dynamic Programming Approach to Sequencing Problems, J. Soc. Indust. Appl. Math., 10, 196-210.

3. Schrage and Baker

4. Smith, W.E., 1956, Various Optimizers for Single Stage Production, Naval Res. Logist. Quart, 3, 59-66.

5. McNaughton, R., 1959, Scheduling with Deadlines and Loss Functions, Mgmt. Sci. 6, 1-12.

6. Rothkopf, M.H., 1966, Scheduling Independent Tasks on Parallel Processors, Mgmt. Sci. 12, 437-447.

7. Rothkopf, M.H., 1964, Scheduling Tasks on One or More Processors, Ph.D. dissertation, M.I.T., School of Industrial Management, also Interim Technical Report No. 2, Operations Research Center, M.I.T.