Technical Research Report

A Summary of Satellite Orbit Related Calculations

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1. Satellite Orbit Calculations (Satellite Orbit-Plane Coordinate System)

The path of a satellite orbiting around the earth takes the form of an ellipse. This section describes some of the quantities and properties related to the satellite orbit. For simplicity, a satellite orbit-plane coordinate system is chosen, so that the satellite orbit lies in the x-y plane, and we can deal with a 2-D instead of a 3-D system. The following are some of the parameters and equations that describe this path.

S – satellite

E – earth

O – geometric center of orbit

P_h – perifocus (point at which, satellite is closest to earth)

A_h – apofocus (point at which, satellite is furthest from earth)

a – semimajor axis

e – eccentricity (circle: $e = 0$, ellipse: $0 < e < 1$)

b – semiminor axis $b = a\sqrt{1-e^2}$

p – semiperimeter $p = a(1-e^2)$

v – true anomaly

$\boldsymbol{r}$ – position vector

r – distance between earth and satellite

$\mathbf{V}$ – velocity vector

V – speed of satellite

$\mu = \dfrac{\text{mass of earth} + \text{mass ofsatellite}}{\text{mass of earth}} \approx 1$

k – gravitational constant of earth $k \approx 1.9965 \times 10^7 \,\mathrm{m}^{3/2} / \mathrm{s}$

t – current time

T – time of perifocal passage (time at which, the satellite was at P_h)

P – period of revolution (time taken by satellite to revolve once around the earth)

$\mathbf{P}$, $\mathbf{Q}$, $\mathbf{W}$ – unit vectors along x_ω, y_ω and z_ω respectively

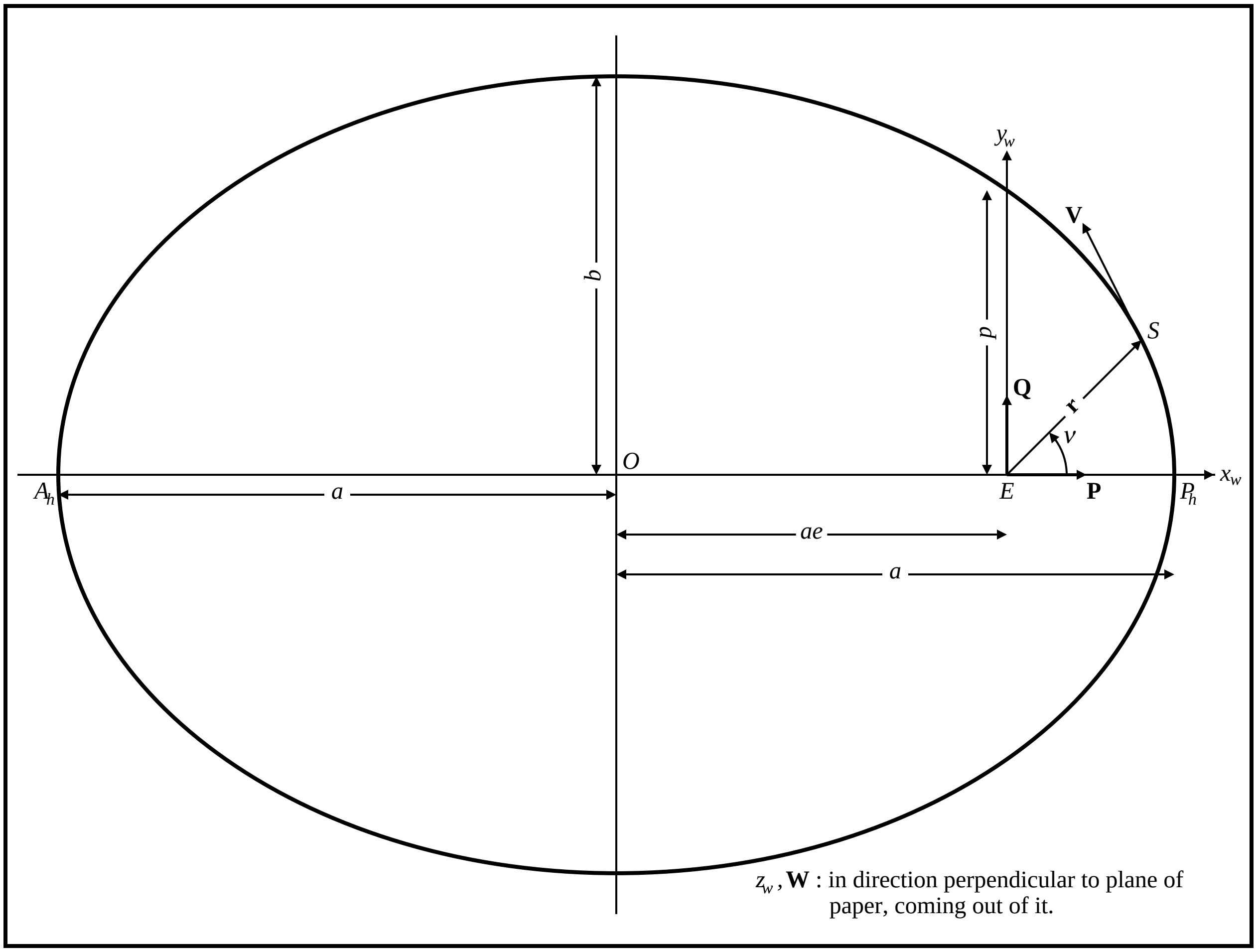

Fig. 1. Orbital plane view of a satellite orbit around the earth.

1.1. Satellite Orbit Specification

To specify the shape of the satellite orbit, and position of the satellite in the orbit, the following parameters need to be given: a, e, and T.

1.2. Period of Revolution

Given a, we can calculate P:

$$P = \frac{2\pi}{k\sqrt{\mu}}\, a^{3/2}.$$

Vice-versa, given P, we can calculate a:

$$a = \left(\frac{kP\sqrt{\mu}}{2\pi}\right)^{2/3}.$$

1.3. Distance of satellite from the center of the earth

Given the true anomaly, the distance of the satellite from the earth is

$$r = \frac{a(1-e^2)}{1+e\cos\nu}.$$

Satellite is closest to the earth at $\nu = 0$, $\cos\nu = 1$:

$$r_{P_h} = a(1-e)$$

and furthest from the earth at $\nu = \pi$, $\cos\nu = -1$:

$$r_{A_h} = a(1+e).$$

1.4. Displacement vector

$$\boldsymbol{r} = \left[\frac{a(1-e^2)\cos\nu}{1+e\cos\nu}\right]\mathbf{P} + \left[\frac{a(1-e^2)\sin\nu}{1+e\cos\nu}\right]\mathbf{Q}$$

1.5. Speed of Satellite

$$V = k\sqrt{\frac{\mu}{a}\left(\frac{1+2e\cos\nu+e^2}{1-e^2}\right)}$$

Speed is maximum at perifocus $\nu = 0$, $\cos\nu = 1$, and minimum at apofocus $\nu = \pi$, $\cos\nu = -1$:

$$\mathcal{V}_{\max} = k\sqrt{\frac{\mu}{a}\left(\frac{1+e}{1-e}\right)} \qquad \mathcal{V}_{\min} = k\sqrt{\frac{\mu}{a}\left(\frac{1-e}{1+e}\right)}$$

1.6. Velocity of Satellite

Tangential component:

$$V_t = k(1+e\cos\nu)\sqrt{\frac{\mu}{a}\frac{1}{1-e^2}}$$

Radial component:

$$V_r = ke\sin\nu\sqrt{\frac{\mu}{a}\frac{1}{1-e^2}}$$

Velocity vector:

$$\mathbf{V} = \left[-k\sin\nu\sqrt{\frac{\mu}{a}\frac{1}{1-e^2}}\right]\mathbf{P} \quad + \quad \left[k(\cos\nu+e)\sqrt{\frac{\mu}{a}\frac{1}{1-e^2}}\right]\mathbf{Q}$$

1.7. Satellite position at time t

Given the current time t, first solve for the apparent anomaly E using

$$E\left(\frac{\pi}{180}\right) - e \sin E = \frac{k\sqrt{\mu}}{a^{3/2}}\,(t - T).$$

To solve this, use the following algorithm:

(i) Set $E_0 \leftarrow \left(\frac{180}{\pi}\right)\frac{k\sqrt{\mu}}{a^{3/2}}\,(t-T);\, k \leftarrow 0.$

(ii) Set $E_{k+1} \leftarrow \dfrac{e\sin E_k - eE_k\cos E_k + E_0\,\frac{\pi}{180}}{\frac{\pi}{180} - e\cos E_k}$. If $\dfrac{|E_{k+1}-E_k|}{|E_k|} < 10^{-5}$, stop; otherwise, set $k \leftarrow k+1$, and repeat step (ii).

Note: The above algorithm assumes that E is in radians.

Then, solve for the true anomaly v from

$$\cos v = \frac{\cos E - e}{1 - e\cos E}; \qquad \sin v = \frac{\sqrt{1-e^2}\,\sin E}{1 - e\cos E}$$

Then, use v in the equations above.

2. Latitude-Longitude-Altitude Coordinate System

The earth is not a perfect sphere, but rather closer to an ellipsoid.

a_e – equatorial radius (semimajor axis) $a_e \approx 6.37815 \times 10^6$m

b_e – polar radius (semiminor axis)

f – flattening $f = \dfrac{a_e - b_e}{a_e} = \dfrac{1}{298.30}$

Usually, on earth, the location of a point is specified by two angular coordinates (*latitude–longitude*) and the *altitude* above/below the adopted reference ellipsoid. Because the earth is an ellipsoid, there are three kinds of latitude defined—*geocentric latitude, geodetic latitude* and *astronomical latitude*.

2.1. Latitude

ϕ' (Geocentric latitude) – The acute angle measures perpendicular to the equatorial plane between the equator and a line connecting the geometric center of the earth with the point formed by the intersection of the surface of the refence ellipsoid and the normal to a tangent plane touching the reference ellipsoid that contains the point.

ϕ (Geodetic latitude) – The acute angle measures perpendicular to the equatorial plane between that normal to a tangent plane touching the reference ellipsoid, and passing through the point, and the equatorial plane.

ϕ_a (Astronomical latitude) – The acute angle measured perpendicular to the equatorial plane formed by the intersection of a gravity ray with the equatorial plane.

Since the astronomical latitude is a function of the local gravitational field, it is affected by local surface anomalies like mountains, seas, etc.. It is usually used as an approximation to geodetic latitude, when the latter is not available.

Geodetic latitude is the one commonly used in maps, etc.. When not specified, geodetic latitude is assumed.

Conversion between geodetic and geocentric latitudes are as follows:

$$\phi = \tan^{-1}\left(\frac{1}{(1-f)^2}\,\tan\phi'\right); \qquad \phi' = \tan^{-1}\left((1-f)^2\,\tan\phi\right).$$

2.2. Longitude

There are two kinds of longitude used:

λ_E (East longitude) – Angle measured towards the east, in the equatorial plane between the prime meridian (0°) and the meridian crossing the surface point.

λ_W (West longitude) – Angle measured towards the west, in the equatorial plane between the prime meridian (0°) and the meridian crossing the surface point.

East longitude is the more common of the two. Conversion between the two is as follows:

$$\lambda_W = 360° - \lambda_E.$$

2.3. Altitude

H (Altitude) – Height of the object above/below the reference ellipsoid, measured normal to a tangent plane touching the surface; the normal passing through the point and the point of contact of the tangent plane with the reference point.

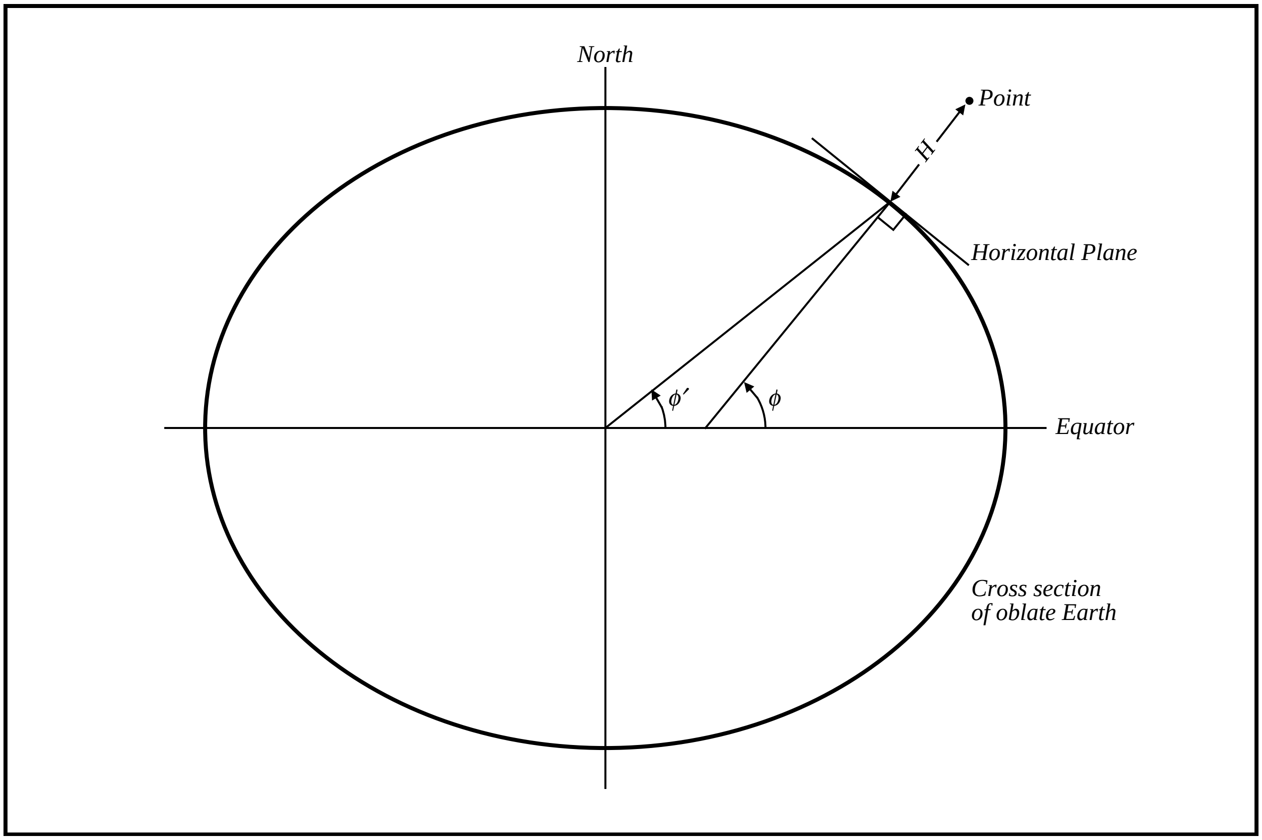

Fig. 2. Geodetic and Geometric latitude of a point.

3. Right Ascension-Declination Coordinate System

Calculations in space are best carried out in an inertial or fixed coordinate system. The right ascension-declination system is the basic astrodynamical system. This section explains this coordinate system and the next section shows how to specify a satellite orbit in this coordinate system.

The origin is fixed at the center of the earth. The x-axis points to the vernal equinox (a certain distant star) in the equatorial plane, the y-axis is perpendicular to the x-axis and also lies in the equatorial plane, and the z-axis points to the north-pole.

I, **J**, **K** – unit vectors along the x, y, and z axes respectively.

α (Right Ascension) – Angle measured in the plane of the equator from the vernal equinox to a plane normal to the equator (meridian) that contains the object. $0° \le \alpha \le 360°$.

δ (Declination) – Angle between object and equator measured in a plane normal to the equator, which contains the object and the origin. $-90° \le \delta \le 90°$.

r (Radial distance) – The distance between the origin and the object.

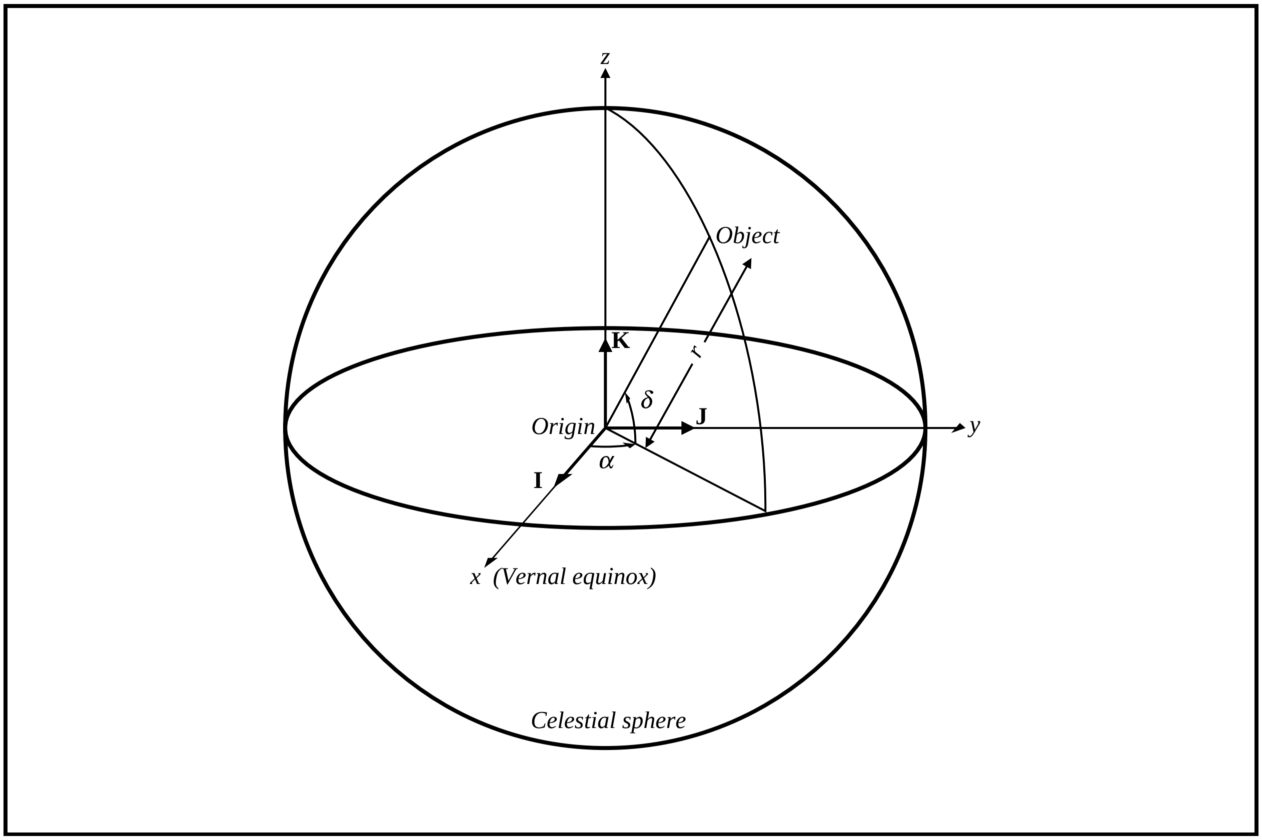

Fig. 3. Right Ascension-Declination Coordinate System.

3.1. Conversions

Given $[x, y, z]$ coordinates in the right ascension-declination system, computation of $[\alpha, \delta, \boldsymbol{r}]$ is as follows:

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\text{Solve for } \delta \text{ from } \cos\delta = \frac{\sqrt{x^2+y^2}}{r}; \qquad \sin\delta = \frac{z}{r}$$

$$\text{Solve for } \alpha \text{ from } \cos\alpha = \frac{x}{\sqrt{x^2+y^2}}; \qquad \sin\alpha = \frac{y}{\sqrt{x^2+y^2}}$$

Given $[\alpha, \delta, \boldsymbol{r}]$ coordinates in the right ascension-declination system, computation of $[x, y, z]$ is as follows:

$$x = r\cos\delta\cos\alpha$$

$$y = r\cos\delta\sin\alpha$$

$$z = r\sin\delta$$

4. Satellite Orbit Orientation in Space

In Section 1, we discussed satellite orbit calculations in the orbital-plane coordinate system $[\mathbf{P}, \mathbf{Q}, \mathbf{W}]$. In this section, we discuss how the satellite orbit is actually oriented in space (i.e., with respect to an inertial frame of reference). We shall specify orbit orientation in the right ascension-declination system.

ι (Orbital Inclination) – Angle between orbital and equatorial planes measured in a plane perpendicular to a line defining their respective intersection. $0° \le i \le 180°$

Ω (Longitude of the ascending node) – The angle measured in the equatorial plane between the principal axis (vernal equinox) and the line defining the intersection of the orbital and equatorial planes, as a point passes through the equator from $-z$ to $+z$. $0 \le \Omega \le 360°$

ω (Argument of the perigee) – Angle measured in the orbital plane from the line joining defined by the longitude of the ascending node to another line in the orbital plane, which contains the focus and passes through the perifocus. $0 \le \omega \le 360°$

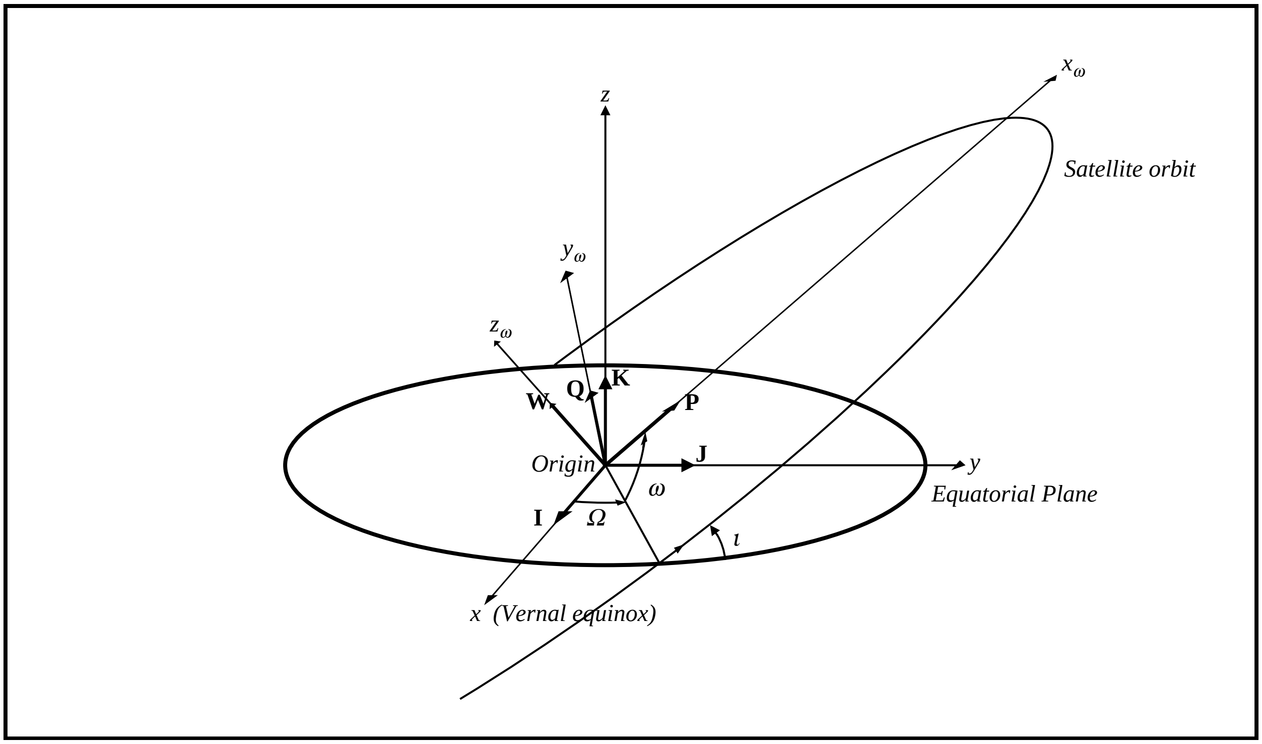

Fig. 4. Satellite Orbit Orientation in the Right Ascension-Declination Coordinate System.

4.1. Conversions

Conversion from the $[\mathbf{P}, \mathbf{Q}, \mathbf{W}]$ to the $[\mathbf{I}, \mathbf{J}, \mathbf{K}]$ coordinate system is as follows:

$$\begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \\ \mathbf{W} \end{bmatrix} = \begin{bmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\iota & \sin\iota \\ 0 & -\sin\iota & \cos\iota \end{bmatrix} \begin{bmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{J} \\ \mathbf{K} \end{bmatrix}$$

Conversion from the $[\mathbf{I}, \mathbf{J}, \mathbf{K}]$ to the $[\mathbf{P}, \mathbf{Q}, \mathbf{W}]$ coordinate system is as follows:

$$\begin{bmatrix} \mathbf{I} \\ \mathbf{J} \\ \mathbf{K} \end{bmatrix} = \begin{bmatrix} \cos\Omega & -\sin\Omega & 0 \\ \sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\iota & -\sin\iota \\ 0 & \sin\iota & \cos\iota \end{bmatrix} \begin{bmatrix} \cos\omega & -\sin\omega & 0 \\ \sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{P} \\ \mathbf{Q} \\ \mathbf{W} \end{bmatrix}$$

5. Calculation of Local Siderial Time

Relating an earth-fixed coordinate system to the right ascension-declination system requires the calculation of *local siderial time* of any point on the earth's surface.

θ_g – siderial time (in degrees)

λ_E – East longitude of observer (known)

θ – local siderial time (in degrees)

L – number of days in a tropical year

T_u – Number of Julian Centuries since 1900

J.D. – Julian date

U.T. – Universal Time (Greenwich Mean Time and Date)

Δt – Time of day, in seconds

Local siderial time is given by

$$\theta = \theta_g + \lambda_E$$

where siderial time is given by

$$\begin{aligned}
\theta_g &= \theta_{g_0} + \Delta t \frac{d\theta}{dt} \\
\theta_{g_0} &= (99.69098329 + 36000.76893 T_u + 3.87080 \times 10^{-4} T_u^2)^\circ \\
\frac{d\theta}{dt} &= \frac{1}{240}\left(1 + \frac{1}{L}\right) \quad {}^\circ/\,\text{sec} \\
L &= 365.24219879 - 6.14 \times 10^{-6} T_u \quad \text{days} \\
T_u &= \frac{\text{J.D.} - 2415020.0}{36525} \\
\text{J.D.} &= 2415020.0 + \text{Number of days since Jan 0, 1900}
\end{aligned}$$

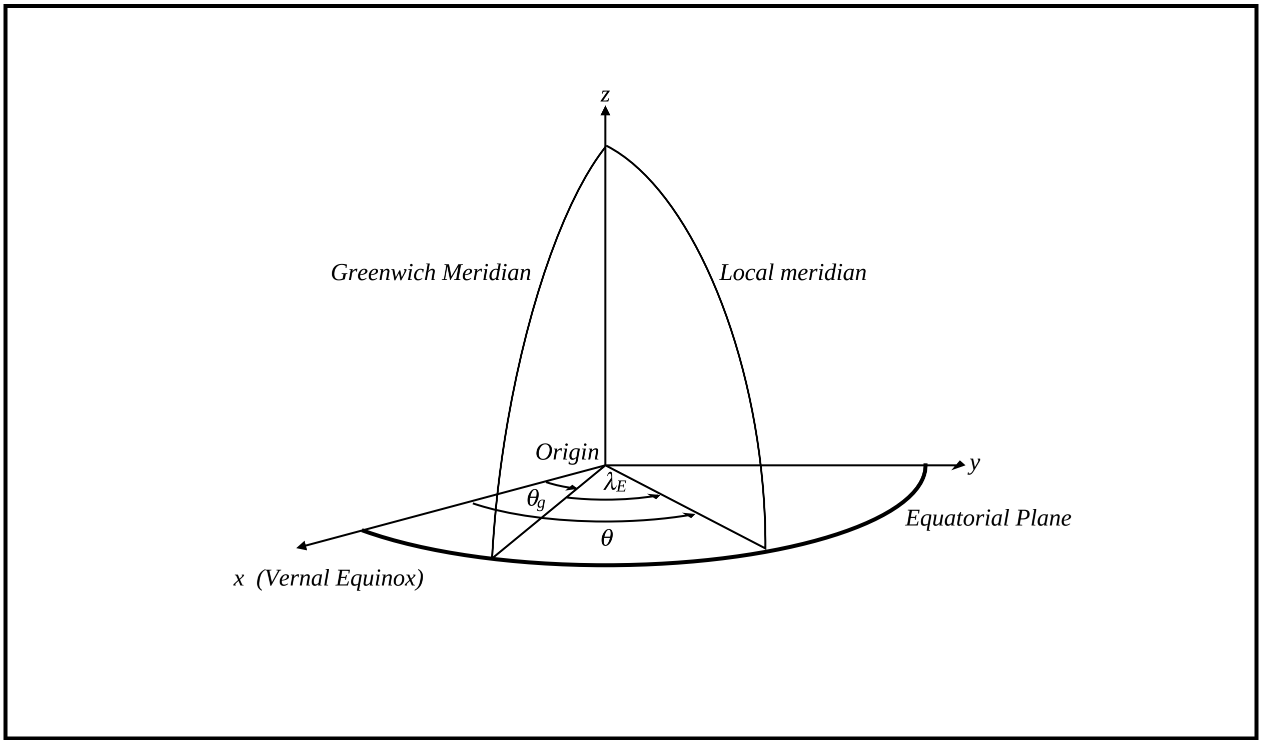

Fig. 5. Siderial time and Local Siderial time.

5.1. Example

What is the local siderial time at Wantig, West Antigua ($\lambda_E = 298.2213°$) at U.T. of 1962 October 12^{day} 10^{hr} 15^{min} 30^{sec}.

Step 1: Calculate J.D. and Δt

$$\begin{aligned} J.D. &= 2415020.0 && \text{to Jan 0, 1900} \\ &+22645.0 && \text{to Jan 0, 1962} \\ &+273.0 && \text{to Oct 0, 1962} \\ &+0.5 && \text{to Oct 1, 1962} \\ &+11 && \text{to Oct 12, 1962} \\ &= 2437949.5 && \text{days} \end{aligned}$$

$$\Delta t = 10^{\text{hr}} \quad 15^{\text{min}} \quad 30^{\text{sec}} = 36930\text{sec}$$

Step 2: Calculate T_u

$$T_u = 0.62777550$$

Step 3: Calculate L

$$L = 365.2421949$$

Step 4: Calculate $d\theta / dt$

$$\frac{d\theta}{dt} = 4.178075 \times 10^{-3\circ} / \text{sec}$$

Step 5: Calculate θ_{g_0}

$$\theta_{g_0} = 22700.09171°$$

Step 6: Calculate θ_g

$$\theta_g = 22854.38801°$$

Step 7: Calculate θ

$$\theta = 22854.38801 + 298.2213 = 23152.60931 \equiv 112.6093 \mod 360°$$

Therefore, local siderial time is $\theta = 112.6093°$.

6. Azimuth-Elevation Coordinate System

The Azimuth-elevation coordinate system places a point on the surface of the earth (observer) at the origin of the coordinate system. The coordinate system is fixed with reference to the earth, and therefore, rotates as the earth rotates. In this coordinate system, the position of any object is described by the following three quantities:

h (Elevation) – Angular elevation of an object above the observer's horizon. $-90° \leq h \leq 90°$

A (Azimuth) – The angle from the North to the object's meridian, measured in the observer's horizontal plane. $0° \leq A \leq 360°$

ρ_h (Slant Range) – Distance between the object and the observer.

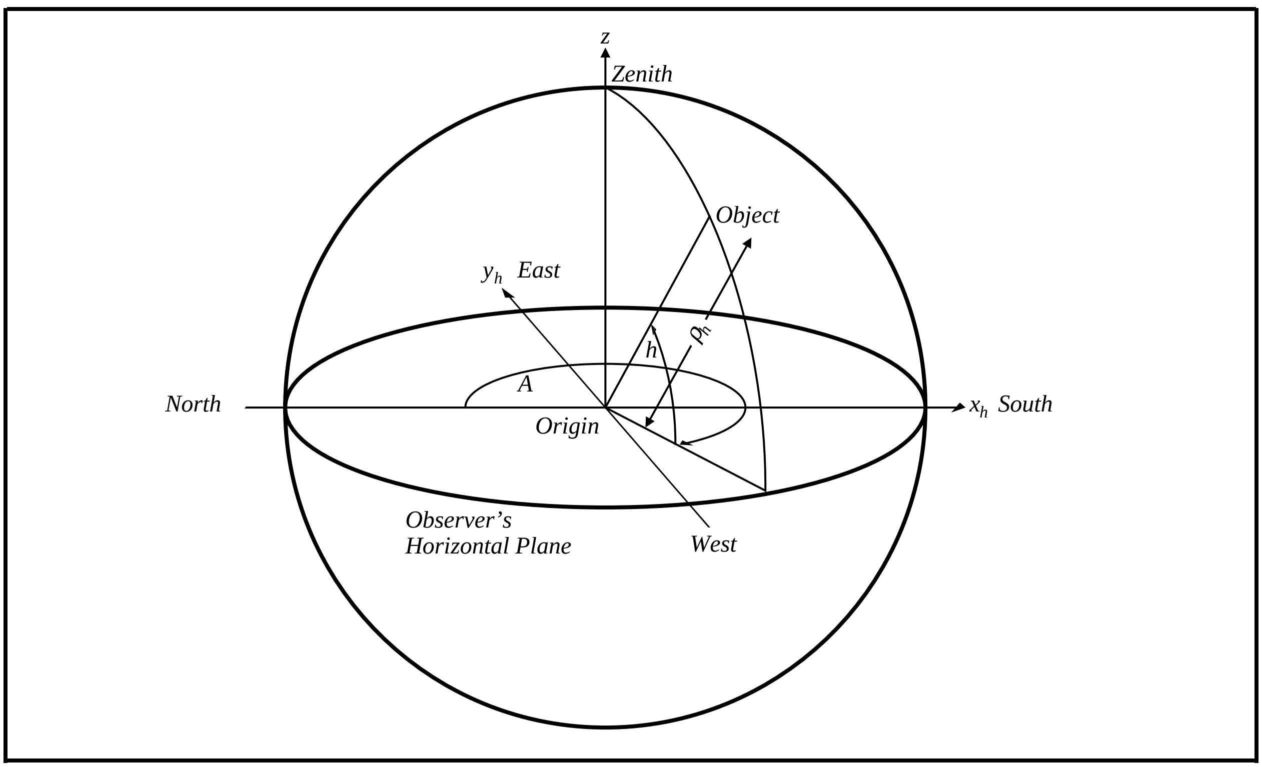

Fig. 6. Azimuth-Elevation Coordinate System

7. Conversions

In this section, we give the transformations between the three coordinate systems discussed in previous sections. Calculation of certain parameters require that the satellite and/or observer (ground station) be in one of the coordinate systems. For example, antenna direction and performance computation can be best done if the satellite position is represented in the azimuth-elevation coordinate system. Footprint calculation is best done when the satellite is represented in the latitude-longitude-altitude coordinate system. We have already discussed how to represent the satellite position in the right ascension-declination system. Therefore, in this section, we give transformations between this system and the others.

7.1. From Right Ascension-Declination to Azimuth-Elevation

Given satellite coordinates $[x, y, z]$ (or $[\alpha, \delta, r]$) in the right ascension-declination system, and ground station coordinates $[\phi, \lambda_E, H, t]$ to compute the satellites azimuth-elevation coordinates with respect to the ground-station: $[A, h, \rho_h]$.

Satellite Coordinates

x, y, z – satellite's coordinates in the right ascension-declination system

Note: If the satellite's coordinates are given in the form $[\alpha, \delta, r]$, conversion to the $[x, y, z]$ form can be done as in Section 3.

Ground-station coordinates

ϕ – Station's geodetic latitude

λ_E – Station's east longitude

H – Station's elevation above sea-level

t – Universal time (U.T.)

Azimuth-Elevation coordinates

A – Azimuth angle of satellite from ground station

h – Elevation angle of satellite from ground station

ρ_h – Distance between ground station and satellite

First, compute station's siderial time θ, from λ_E and t, as shown in Section 5. Next, compute

$$
\begin{aligned}
G_1 &= \frac{a_e}{\sqrt{1-(2f-f^2)\sin^2\phi}} + H \\
G_2 &= \frac{a_e(1-f)^2}{\sqrt{1-(2f-f^2)\sin^2\phi}} + H \\
x_o &= G_1 \cos\phi \cos\theta \\
y_o &= G_1 \cos\phi \sin\theta \\
z_o &= G_2 \sin\phi
\end{aligned}
$$

$$
\begin{aligned}
\rho_x &= x - x_o \\
\rho_y &= y - y_o \\
\rho_z &= z - z_o
\end{aligned}
$$

Distance between satellite and ground station is therefore, given by

$$\rho_h = \sqrt{\rho_x^2 + \rho_y^2 + \rho_z^2}$$

$$\begin{bmatrix} L_{xh} \\ L_{yh} \\ L_{zh} \end{bmatrix} = \begin{bmatrix} \sin\phi\cos\theta & \sin\phi\sin\theta & -\cos\phi \\ -\sin\theta & \cos\theta & 0 \\ \cos\theta\cos\phi & \sin\theta\cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} \rho_x / \rho \\ \rho_y / \rho \\ \rho_z / \rho \end{bmatrix}$$

Elevation can be computed from

$$\sin h = L_{zh}$$

$$\cos h = \sqrt{1 - L_{zh}^2}$$

and Azimuth from

$$\sin A = \frac{L_{yh}}{\sqrt{1 - L_{zh}^2}}$$

$$\cos A = \frac{L_{xh}}{\sqrt{1 - L_{zh}^2}}$$

7.2. From Azimuth-Elevation to Right Ascension-Declination

Given satellite coordinates $[A, h, \rho_h]$ in the azimuth-elevation coordinate system with respect to the ground station with coordinates $[\phi, \lambda_E, H, t]$, to compute the satellite's coordinates: $[x, y, z]$ (or $[\alpha, \delta, r]$) in the right ascension-declination system.

Satellite Coordinates

$[A, h, \rho_h]$ – satellite's coordinates in the azimuth-elevation coordinate system

A – Azimuth angle of satellite from ground station

h – Elevation angle of satellite from ground station

ρ_h – Distance between ground station and satellite

Ground-station coordinates

ϕ – Station's geodetic latitude

λ_E – Station's east longitude

H – Station's elevation above sea-level

t – Universal time (U.T.)

Right Ascension-Declination coordinates $[x, y, z]$ – satellite's coordinates in the right ascension-declination coordinate system. These can be converted to $[\alpha, \delta, r]$ as in Section 3.

First, compute station's siderial time θ, from λ_E and t, as shown in Section 5. Next, compute

$$L_{xh} = -\cos A \cos h$$

$$L_{yh} = \sin A \cos h$$

$$L_{zh} = \sin h$$

$$G_1 = \frac{a_e}{\sqrt{1 - (2f - f^2)\sin^2\phi}} + H$$

$$G_2 = \frac{a_e(1-f)^2}{\sqrt{1-(2f-f^2)\sin^2\phi}} + H$$

$$x_o = G_1 \cos\phi \cos\theta$$

$$y_o = G_1 \cos\phi \sin\theta$$

$$z_o = G_2 \sin\phi$$

$$\begin{bmatrix} L_x \\ L_y \\ L_z \end{bmatrix} = \begin{bmatrix} \sin\phi\cos\theta & \sin\phi\sin\theta & -\cos\phi \\ -\sin\theta & \cos\theta & 0 \\ \cos\theta\cos\phi & \sin\theta\cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} L_{xh} \\ L_{yh} \\ L_{zh} \end{bmatrix}$$

$$\rho_x = \rho_h L_x$$

$$\rho_y = \rho_h L_y$$

$$\rho_z = \rho_h L_z$$

The satellite coordinates are then, given by

$$x = \rho_x + x_o$$

$$y = \rho_y + y_o$$

$$z = \rho_z + z_o$$

7.3. From Right Ascension-Declination to Latitude-Longitude-Altitude

Given the satellite's coordinates $[x, y, z]$ (or equivalently, $[\alpha, \delta, r]$) in the right ascension-declination coordinate, and the universal time (U.T.), compute its position in the latitude-longitude-altitude coordinate system.

Right Ascension-Declination Coordinates

x, y, z – satellite's rectangular coordinates in the right ascension-declination coordinate system.

Note: If the satellite's coordinates are given in the $[\alpha, \delta, r]$ form, they may be converted to this form as in Section 3.

Latitude-Longitude-Altitude Coordinates

U.T. – Universal time (U.T.) (GMT and date.)

ϕ – Satellite's geodetic latitude

λ_E – Satellite's east longitude

H – Satellite's altitude above sea-level

First, compute siderial time θ_g from the Universal Time (U.T.) as in Section 5. Next, compute α from the following equations

$$\sin\alpha = \frac{y}{\sqrt{x^2+y^2}}$$

$$\cos\alpha = \frac{x}{\sqrt{x^2+y^2}}$$

Then, compute the satellite's east longitude as

$$\lambda_E = \alpha - \theta_g \mod 360°$$

Next, compute ϕ and H from the following iterative algorithm:

(i) Step 1: Compute r and δ ($-90° \le \delta \le 90°$) from

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\sin\delta = \frac{z}{r}$$
$$\cos\delta = \frac{\sqrt{x^2 + y^2}}{r}$$

Set $\phi'_0 = \delta$, $k = 0$.

(ii) Step 2:

$$\boldsymbol{r}_{c,k} = a_e \sqrt{\frac{1 - (2f - f^2)}{1 - (2f - f^2)\cos^2\phi'_k}}$$
$$\phi_k = \tan^{-1}\left[\frac{1}{(1-f)^2}\tan\phi'_k\right], \qquad -90° \le \phi_k \le 90°$$
$$H_k = \sqrt{r^2 - r_{c,k}^2 \sin^2(\phi_k - \phi'_k)} \quad - r_{c,k}\cos(\phi_k - \phi'_k)$$
$$\phi'_{k+1} = \delta - \sin^{-1}\left[\frac{H_k}{r}\sin(\phi_k - \phi'_k)\right]$$

(iii) Step 3: If $\dfrac{|\phi'_{k+1} - \phi'_k|}{|\phi'_{k+1}|} < 10^{-5}$, then go to Step (iv). Otherwise, set $k \leftarrow k+1$ and go to Step (ii).

(iv) Step 4: Set

$$H = H_k$$
$$\phi = \tan^{-1}\left[\frac{1}{(1-f)^2}\tan\phi'_{k+1}\right], \qquad -90° \le \phi \le 90°$$

7.4. From Latitude-Longitude-Altitude to Right Ascension-Declination

Given the satellite's coordinates $[\phi, \lambda_E, H, t]$ in the latitude-longitude-altitude coordinate system, compute its $[x, y, z]$ coordinates in the right ascension-declination coordinate.

Latitude-Longitude-Altitude Coordinates

t – Universal time (U.T.) (GMT and date.)

ϕ – Satellite's geodetic latitude

λ_E – Satellite's east longitude

H – Satellite's altitude above sea-level

Right Ascension-Declination Coordinates

$[\alpha, \delta, r]$ – satellite's coordinates in the right ascension-declination coordinate system.

Note: Once the satellite's $[\alpha, \delta, r]$ coordinates are found, they may be converted into the $[x, y, z][\alpha, \delta, r]$ form as in Section 3.

First, compute siderial time θ_g from the Universal Time (t) as in Section 5. Also, compute the satellite's geocentric latitude ϕ' from its geodetic latitude ϕ as in Section 2. Next, compute

$$r_c^2 = \frac{a_e^2\left[1-(2f-f^2)\right]}{1-(2f-f^2)\cos^2\phi'}$$

$$r = \sqrt{r_c^2 + H^2 + 2r_c H\,\cos(\phi-\phi')}$$

$$\alpha = \theta_g - (360^\circ - \lambda_E), \qquad 0^\circ \le \alpha \le 360^\circ$$

$$\delta = \phi' + \sin^{-1}\left[\frac{H}{r}\sin(\phi-\phi')\right], \qquad -90^\circ \le \delta \le 90^\circ$$

8. Ground-Station Computations

In this section, we discuss various parameters related to antenna adjustments from the ground station.

8.1. Satellite Visibility

To check whether the satellite is visible (above the local horizon) from the position of the ground-station, compute satellite coordinates in the azimuth-elevation coordinate system with the ground-station at the reference. If the angle of elevation is positive, the satellite is visible, and this angle gives the angle of elevation of the satellite above the horizontal at that point. If, on the other hand, the angle is negative, then the satellite is not visible from this ground-station at the current time.

8.2. Antenna Direction and Range of Satellite

To compute the directionin which to point the antenna, simply compute the satellite's coordinates in the azimuth-elevation system with the ground-station as the origin. The azimuth and elevation angles give the direction in which, to point the antenna. The distance of the satellite is given by the third coordinate.

8.3. Antenna Misdirection

Let the position of a satellite in the azimuth-elevation coordinate system with the ground station at the reference be $[A, h, \rho_h]$. Let the antenna at the ground station be pointed in the direction: Azimuth A' and elevation h'. To calculate the antenna mismatch angle, ε, which is the angular displacement between the antenna direction and the satellite direction, proceed as follows:

$$S_x = \cos A \cos h$$

$$S_y = \sin A \cos h$$

$$S_z = \sin h$$

$$S'_x = \cos A' \cos h'$$

$$S'_y = \sin A' \cos h'$$

$$S'_z = \sin h'$$

Compute ε from

$$\cos\varepsilon = S_x S'_x + S_y S'_y + S_z S'_z \qquad 0^\circ \le \varepsilon \le 180^\circ$$

9. Satellite Computations

In this section, we discuss various parameters related to the satellite's positioning with reference to the earth.

9.1. Satellite Position and Altitude

To compute the location on the earth's surface (latitude and longitude) over which, the satellite is directly overhead, simply compute the satellite's position in the latitude-longitude-altitude coordinate system. The satellite's altitude is given by the third coordinate.

9.2. Footprint radius

Let the satellite's position be specified in the latitude-longitude-altitude coordinate system as $[\phi, \lambda_E, H]$. Given an angle γ, the satellite footprint radius r_f is the maximum distance (along the earth's surface) from the coordinate $[\phi, \lambda_E]$ at which, the angle of elevation of the satellite is no less than γ. (Therefore, the footprint of the satellite consists of all those points on the earth surface where the antenna elevation required will be γ or more.) We compute r_f as follows:

$$
\begin{aligned}
c_e &= a_e\left(1-\frac{f}{2}\right)\\
\beta &= \cos^{-1}\left(\frac{c_e\cos\gamma}{H+c_e}\right)-\gamma, \qquad 0^\circ \le \beta \le 90^\circ\\
r_f &= c_e\left(\frac{2\pi}{360}\beta\right)
\end{aligned}
$$

References

1. Methods of Orbit Determination, Pedro Ramon Escobal, John Wiley and Sons, New York, 1965.