ABSTRACT

Title of Document: EXISTENCE OF UNUSED MANAGED CAPACITY ON DEDICATED LANES AND AN ALTERNATIVE ON HOW TO SELL IT VIA AUCTIONS

Rafael Ernesto Olarte Valdivieso, Doctor of Philosophy in Civil Engineering, 2016

Directed By: Professor Ali Haghani, Professor, Department of Civil and Environmental Engineering

This dissertation verifies whether the following two hypotheses are true: (1) High-occupancy/toll lanes (and therefore other dedicated lanes) have capacity that could still be used; (2) such unused capacity (or more precisely, "unused managed capacity") can be sold successfully through a real-time auction. To show that the second statement is true, this dissertation proposes an auction-based metering (ABM) system, that is, a mechanism that regulates traffic that enters the dedicated lanes. Participation in the auction is voluntary and can be skipped by paying the toll or by not registering to the new system. This dissertation comprises the following four components: a measurement of unused managed capacity on an existing HOT facility, a game-theoretic model of an ABM system, an operational description of the ABM system, and a simulation-based evaluation of the system. Some other and more specific contributions of this dissertation include the following: (1) It provides a

definition and a methodology for measuring unused managed capacity and another important variable referred as “potential volume increase”.

(2) It proves that the game-theoretic model has a unique Bayesian Nash equilibrium.

(3) And it provides a specific road design that can be applied or extended to other facilities.

The results provide evidence that the hypotheses are true and suggest that the ABM system would benefit a public operator interested in reducing traffic congestion significantly, would benefit drivers when making low-reliability trips (such as work-to-home trips), and would potentially benefit a private operator interested in raising revenue.

EXISTENCE OF UNUSED MANAGED CAPACITY ON DEDICATED LANES AND AN ALTERNATIVE ON HOW TO SELL IT VIA AUCTIONS

By

Rafael Ernesto Olarte Valdivieso

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Advisory Committee:
Professor Ali Haghani
Professor Paul Schonfeld
Professor Lei Zhang
Professor Peter Cramton
Professor Barton Forman

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List of Acronyms

ABM	Auction-based metering
CE	Certainty equivalence
CEP	Certainty equivalent payment
DAR	Direct access ramp
GP	General purpose (used only when referring to the GP lanes)
HOT	High-occupancy/toll
HOV	High-occupancy vehicle
MC	Managed capacity (used only when referring to the MC interval)
OBU	On-board unit
RBM	Raffle-based metering
SOV	Single-occupant vehicle

List of Notation

$\lfloor \quad \rfloor$	Floor function (lowest nearest integer)
$\propto$	Level of constant absolute risk averseness
δ	Certainty equivalent payment
Δq	Potential volume increase
π	Volume pie
ρ_1	Maximum Fraction of SOVs that Would Choose to Enter the HOT lanes
ρ_2	Fraction of vehicles on the GP lanes that can switch to the HOT lanes without losing their path to their original destination
$\mathbb{A}$	Set of `advanced SOVs
a	player belonging to set $\mathbb{A}$
$\bar{a}$	number of players in set $\mathbb{A}$
$\mathbb{B}$	Set of basic SOVs
c^*	Optimal cutoff value
c_{-a}	Cutoff value of players other than a
c_a	Player a's cutoff value
d	Rank of a private value among the other private values (used only in raffles)
E	Expected value
e	Euler's constant
F	Cumulative probability distribution of private values
G	Probability of winning auction
G_{R}	Probability of winning raffle

$\mathbb{H}$ Set of HOVs

$\bar{k}$ Number of lottery items

m_{C} Managed capacity at a given entrance to dedicated lanes

$\mathbb{O}$ Set of drivers who cannot enter the HOT lane or whose destination cannot be reached via the HOT lane

P Probability of an event

p Toll price

$\bar{p}$ Maximum proper price (applies only to raffles)

$\underline{p}$ Maximum proper price (applies only to raffles)

q_{AFTER} Potential volume at a given entrance to dedicated lanes

q_{BEFORE} Volume at a given entrance to dedicated lanes

r Reserve price

U Expected utility when choosing to play auction (it does not have to have a monetary value)

u Utility function (it does not have to have a monetary value)

u_{MC} Unused managed capacity at a given entrance to dedicated lanes

U_{R} Expected utility when choosing to play raffle (it does not have to have a monetary value)

$\underline{v}$ Minimum private value

$\bar{v}$ Maximum private value

v_a Player a's private value

x Lottery outcome (expressed in monetary units)

$Y_{i:n}$ i^{th} order statistic of a sample of size n

1. Introduction

This dissertation adopts the term “dedicated lane” to refer to high-occupancy vehicle (HOV) lanes, high-occupancy/toll (HOT) lanes, and dedicated bus lanes. It is common to hear other terms to refer in a general manner to these lanes such as restricted use lanes, exclusive lanes, or special use lanes. The last term was used by Olarte and Haghani (2013; 2016).

Dedicated lanes are lanes that restrict their use to certain categories of vehicles usually with the objective of offering a faster or more reliable travel time. As a result of the restriction, not all the vehicle capacity is used but just a fraction. While in an ordinary road, vehicle capacity is the threshold that demand is not supposed to exceed, in a dedicated lane, the fraction aforementioned is the threshold that is not supposed to be exceeded, and it is referred in this dissertation as managed capacity. Exceeding this threshold would not allow users to perceive a faster or more reliable travel time when using the dedicated lanes than when using the ordinary road (more technically referred as, general purpose lanes or GP lanes). But, while using more managed capacity than what is available is undesirable, not using it in its entirety can also be undesirable. HOT lanes for example, are sometimes expected to use the entirety of their managed capacity so that the most number of vehicles can be taken away from the congested GP lanes. As the evidence in this dissertation reveals, HOT lanes are not capable of using the managed capacity in its entirety, leaving in this way gaps. These gaps are referred in this document as “unused managed capacity”[1]. If HOT lanes have indeed unused managed capacity, it is to be expected that HOV and dedicated bus lanes have also unused managed capacity since these two do not have the price metering mechanism that HOT lanes do have for making a better use of managed capacity.

The objective of this document is to present evidence that corroborate the validity of the following two hypotheses:

Hypothesis 1. HOT lanes (and therefore other dedicated lanes) have unused managed capacity.

Hypothesis 2. Such unused managed capacity can be sold successfully through a real-time auction.

For the first hypothesis, measurements were carried out after calibrating a microsimulation model of a section of an HOT facility in Minneapolis. As the measurements reveal, an even more useful concept than unused managed capacity is the concept of “potential volume increase”. While unused managed capacity measures the available slots that could be sold, the potential volume increase is the fraction of those slots that can actually be sold given the number of drivers who could, instead of remaining on the GP lanes, choose to buy those slots.

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[1] The term “excess managed capacity” was introduced previously by Olarte and Haghani (2013) to refer to the same concept. Nonetheless, the word “excess” may give the false impression that the concept refers to highways that were overdesigned for a much higher demand.

For the second hypothesis, this document proposes an auction-based metering system that could be added to HOT facilities. The proposed auction-based metering (ABM) system regulates the traffic inflow that enters the HOT lane (and by extension other types of dedicated lanes) thanks to the implementation of a "buyout auction". A buyout auction is a mechanism in which drivers are granted access to the HOT lane if they choose to pay the toll price. But they can also choose to participate in an auction (where submission of bids takes place in advance). If a participant wins the auction, she becomes authorized to detour to the HOT lane. Otherwise, that participant keeps driving on the same lane which will take her to the GP lanes. Participants in the auction compete for several available entrance slots (or auction items). Ideally, the number of slots (or to be more precise, the number of slots of several auctions) should be equal to the unused managed capacity because such is the main goal of the ABM system: to eliminate unused managed capacity.

The auction that is proposed can be explained to the general public as follows: at a certain location in the road and while driving, a set of drivers compete for a set of entrance slots where only the drivers with the highest bid amounts are allowed to enter the HOT lanes. Drivers can only submit one bid amount and that submission is to be made many minutes in advance before arriving to the location aforementioned, or perhaps even before getting into their vehicles. All winners of the auction will pay the highest price among those that did not win the auction. Thus, winners will pay the amount that they declared or less. Other aspects of the auction can later be explained to the public once they start using the system. For example, all drivers will be told that it is best for them to truthfully declare the maximum amount that they would be willing to pay for entering the HOT lanes. They will also be told that a lot of times the auction does not take place because there are not available slots in the HOT lanes. Therefore, even having submitted a very high bid amount will not guarantee entrance to the HOT lanes.

The auction is similar to a single-unit Vickrey auction and, as the simplicity of the previous paragraph suggests, it is not expected to cause major confusion among the general public. Nonetheless, this auction offers multiple identical items simultaneously while only one item can be accepted by each winner. This document refers to this auction as “extended Vickrey auction”. A general definition of this auction is provided in the terminology section (Section 1.1) and a mathematical definition is provided in Chapter 4 (Section 4.3). Now, since previous to the start of the extended Vickrey auction, participants have the option of paying the toll price and avoid the auction, the resulting game is referred in this document as a buyout auction (no theoretical analysis has been made previously to this kind of buyout auction). It is important to highlight that this game also has the important feature of having unlimited supply to drivers who choose to buy. In other words, just as HOT lanes operate today, all drivers who choose to pay the toll are granted access.

Before explaining in Section 1.4 how this document is organized, the following three sections do a review of the key concepts that are used throughout the document (Section 1.1), explain why selling unused managed capacity via auction-based metering (ABM) would be an idea worth implementing (Section 1.2), and specifies the contributions of this dissertation (Section 1.3).

1.1. Terminology

This document relies on the following key concepts. When indicated, some concepts are explained in more detail in the appendices.

(1) **SOV** and **HOV**. SOV stands for single-occupant vehicle. HOV stands for high-occupancy vehicle. While the SOV only has one passenger (the driver), the HOV has more than one passenger.

(2) **HOV lanes** and **HOT lanes**. HOV lanes are only allowed to be used by HOVs. They are lanes, usually within highways, although they can also be located in arterials. Likewise, HOT lanes, where HOT stands for high-occupancy/toll, are allowed to be used by HOVs too. But unlike HOV lanes, HOT lanes can also be used by SOVs if paying a toll price. Thus, in a HOT lane, SOVs can enter if paying a toll and HOVs can enter for free (or in some cases, after paying a fixed toll price lower than the one that is paid by SOVs). One of the goals of HOV lanes and HOT lanes is to incentivize drivers to carpool so that the number of vehicles on roads decreases. In some HOV and HOT facilities, motorcycles, buses, and hybrid vehicles are also given free or cheaper access.

Now, there are some technological advancements in HOT lanes that set them apart from HOV lanes. In most HOT lanes, the toll price varies continuously (every 3 or 15 minutes depending on the facility). Typically, the HOT facility has an algorithm (or a system) that calculates the toll price based on the level of congestion on the HOT lane: the higher the level of congestion on the HOT lane, the higher the toll price is; and the lower the level of congestion on the HOT lane, the lower the toll price becomes. Algorithms in some facilities may follow more sophisticated rules but in essence they all follow the approach aforementioned. Nonetheless, in some very few facilities, the toll price simply varies according to the time of the day, such as in the 91 Express Lanes in California (91 Express Lanes 2015). For a more detailed explanation of these concepts see Appendix A.

(3) **General purpose lanes**. General purpose (GP) lanes, also known as mixed lanes, are simply the ordinary roads that are not dedicated lanes. Usually, parallel to every dedicated lane, there is a set of parallel GP lanes that allows drivers to take to at least the same destinations as the dedicated lanes.

(4) **OBU**. On-board unit (OBU) is the generic term used to refer to any gadget or technological tool located inside the vehicle that allows it to communicate with the operator of the HOT facility. Examples are the so-called transponders which receive the name of “EZ-Pass” in the northeast of the United States, “Sunpass” in Florida, “Peachpass” in Georgia, etc. Unless specified, in this dissertation, the term OBU excludes smartphones, given that it has been argued that their use within vehicles is unsafe.

(5) **Direct access ramps**. Direct access ramps (DARs) are connections from outside the HOV or HOT facility to the HOV or HOT lanes. They do not require drivers to enter the GP lanes before entering the HOV or HOT facility. They are explained in more detail in Appendix A.

(6) **Capacity, vehicle capacity, managed capacity, and level of service**.

Operators of road facilities make sure that the traffic volume (also referred as traffic flow, volume or flow) that travels along a road segment does not exceed a certain threshold known as “vehicle capacity”, which is traditionally expressed in number of vehicles per hour. Otherwise, it can be said that traffic has reached a “breakdown” or it has collapsed in terms of congestion. To avoid loose concepts such as “breakdown” or “collapse”, the highway capacity manual (Transportation Research Board 2010) developed several methods to determine when breakdown starts exactly (or in other words, what threshold defines vehicle capacity). One way to define such breakdown is by relying on the concept of “level of service”. The highway capacity manual (Transportation Research Board 2010) introduced in 1950 the concept of level of service which later in 1963 evolved into six defined levels of service (for a historical background, see Roess & Prassas 2014, p.12). The first level of service or level A refers to the condition in which drivers perceive almost no congestion. Level B and mostly level C are the conditions that HOV and HOT lane facilities are supposed to offer to its users in order to make them more attractive than the GP lanes. Level F indicates that the traffic has reached breakdown. **Thus, it can be said that level of service A is “better” than level of service B or E.** When referring to a freeway, each level of service can be defined by its density as shown in Table 1.

Table 1. Levels of service as defined by (Transportation Research Board 2010, p.10.9) for 6-mile segments on freeways with no weaving.

Level of Service	Density [passenger-car/mile/lane]
A	less than or equal to 11
B	greater than 11 and less than or equal to 18
C	greater than 18 and less than or equal to 26
D	greater than 26 and less than or equal to 35
E	greater than 35 and less than or equal to 45
F	greater than 45 or when volume exceeds capacity

According to the table, reaching level of service F and having volume exceed capacity are not equivalent in theory, but in practice, they can be considered as the same.

Instead of using density, each level of service can be associated to a so-called “service flow rate”. Thus, for level F, the service flow rate would be the vehicle capacity. For level C, the service flow rate would be a fraction of the vehicle capacity.

If the operator’s objective is to guarantee that the HOT lanes (or any other dedicated lanes) have a certain level of service or better, then the service flow rate for that level of service is referred in this document as managed capacity. The term managed capacity has been used loosely in the literature (HNTB Corporation & Booz Allen Hamilton 2007, p.15, 45; Booz Allen Hamilton & HNTB Corporation 2008, p.15; Perez & Sciara 2003, p.64) but, in this dissertation, it has the specific definition provided above. A more succinct definition is the following: **Managed capacity is the service flow rate that corresponds to the level of service that the operator wants to guarantee on the dedicated lanes.** Thus, if the goal of the operator is to maintain a service flow rate no worse than the level of service C on the dedicated lanes, then that service flow rate constitutes the managed capacity.

Now, it is important to note that, unless indicated otherwise, this dissertation only focuses on one entrance when measuring managed capacity. For this reason, the reader should refer to Chapter 3 in order to know which assumptions are made in this document when the managed capacity is measured at one entrance and not at several entrances simultaneously. The managed capacity at a specific entrance to dedicated lanes is referred by the notation m_{C}.

Finally, the term "capacity" is almost the same as the term "vehicle capacity" but it also takes into account the number of people. This concept is important when applied to HOV, HOT, and dedicated bus lanes. Nonetheless, for simplicity, this document only focuses on vehicle capacity and managed capacity.

(7) **Dedicated bus lanes**. These are lanes that only allow buses. Although most dedicated bus lanes are part of hybrid systems that also allow HOVs or paying SOVs, there are also those that are purely dedicated to buses. It is worth mentioning that bus rapid transit (BRT) lanes are indeed dedicated bus lanes. Nonetheless, the BRT community would have to open their concept to allow paying private vehicles. Because this could be a strong change for the BRT community to consider, this document does not make a strong case for selling their unused managed capacity to private vehicles, although it could be done successfully.

(8) **Dedicated lanes**. This is not a technical term but is the term adopted in this dissertation to lanes that can only be used by certain types of vehicles. This restriction makes them faster or more reliable than GP lanes. HOV lanes, HOT lanes, and dedicated bus lanes fall in this category. As argued in Figure 3 (page 20), Chapter 3, and Appendix B, successful dedicated lanes have unused managed capacity. Truck lanes and dynamic shoulders are types of lanes not covered in this document but that could also fall in this category. A regular shoulder on a highway or major arterial would not fall in this category because, although it offers exclusivity to emergency vehicles, it can also have vehicles that are standing still for long periods of time. Dedicated lanes constitute a smaller concept than the concept of **managed lanes**. Managed lanes are explained in Appendix A (Section A.4).

(9) **Lottery**. The concept of lottery in expected utility theory covers many events or scenarios. For, example, lotteries can be simple or compound. They can also be degenerate. In this document, the term lottery will simply refer to any set of outcomes that can happen to an individual, where each outcome has a probability of occurrence less than one. Thus, the lottery is defined by two sets of the same cardinality: one of outcomes, and one of probabilities. The sets can be continuous or discrete, finite or infinite. In the context of a driver, examples of outcomes could be being granted free access to a HOT lane, being granted access to a HOT lane but asked to pay a certain amount of money, or (in a perhaps theoretical lottery) receiving an amount of money. The uncertainty of the outcome (which is measured by the probability) does not need to be caused

from interaction or competition with other individuals. Now, depending on the individual, there will be a corresponding utility function. Such function $u(x)$ (where x belongs to the set of lottery outcomes) can be ordinal or cardinal. Unlike ordinal utility functions, in a cardinal function not only matters which outcome is preferred over which but also by how much. In this document, utility functions are considered to be ordinal and lottery outcomes are assumed to be expressed in dollars or cents.

(10) **Risk-averseness**, **risk-neutrality**, **constant absolute risk-averseness**, and **CARA functions**. Risk-averseness is a behavior that individuals reveal when choosing between a certain outcome that generates a utility and an uncertain outcome that generates the same utility (although in this case, it would be an expected utility). Faced to this dilemma, a risk-averse individual would prefer the certain outcome. A risk-neutral individual would be indifferent between the two. Individuals who have a cardinal utility function $u(x)$ with concave shape reveal risk-averse behavior. And when such function $u(x)$ contains a parameter that allows quantifying the risk-averse behavior, and such behavior is constant and independent from the value of x, then it is said that the individual has constant absolute risk-averseness, and a constant absolute risk-averse (CARA) function.

(11) **Certainty equivalence** and **certainty equivalent payment**. In his seminal work on risk aversion, Pratt (1964) introduced the concept of “cash equivalence” which would be later referred by others as “certainty equivalent” (Mas-Colell et al. 1995, p.186) or “certainty equivalence” (Hershey & Schoemaker 1985,

p.1214). Certainty equivalence (CE) is the amount of money that, when given to an individual, would give her the same utility than (playing) the lottery. If the individual is risk-averse, then her CE is less than the lottery's expected utility. If the individual is risk-neutral, then her CE is equal to the lottery's expected utility. Now, when a lottery is composed by outcomes that differ among themselves not by the amount of money that an individual would receive but by an amount of money that an individual would have to pay, this dissertation uses the same concept used by Reynolds and Wooders (2009) which is certainty equivalent payment (CEP). Examples of this kind of outcome sets are {-\$0.25, -\$0.25, -\$0.50} or {v_a −\$0.25, v_a −\$0.25, v_a −\$0.50} (where v_a depends on the individual, not the outcome). CEP is defined as the amount of money that an individual is willing to pay in order to avoid (playing) the lottery.

(12) **Second-price sealed-bid auction** or **standard Vickrey auction**. The second-price sealed-bid auction or standard Vickrey auction is a game in which bidders submit one bid amount each, in secret or simultaneously. The bidder with the highest bid is granted one item and she has to pay the amount that was submitted by the second highest bid. In case of a tie among the highest bidders, the item is assigned at random among those bidders. Although Vickrey (1961) also applied a similar mechanism when having multiple items, this document adopts the terms "second-price sealed-bid auction" and "standard Vickrey auction" to refer to those that only offer one item. As explained in Appendix D, this auction has the important feature of incentivizing bidders to submit their truthful valuation of the item. This feature is also present in the so-called

English auction which is very common and known by the general public. It can be said that the standard Vickrey auction and the English auction are strategically equivalent.

(13) **Private value**, **private valuation**, **common value** and **willingness to pay**. Private value and private valuation both refer to the amount of money that a player is willing to pay for acquiring a lottery item. The private value of a player is independent from the private value of her rivals. Common value is also the amount of money that a player is willing to pay but it depends on (or is influenced by) the amount that others are willing to pay. "Willingness to pay" is a term used frequently in transportation to refer to the amount of money that a person is willing to pay for a transportation service. In this document, since it is always assumed that people have private values and not common values, the terms private value, private valuation, and willingness to pay will be used interchangeably. And unless specified otherwise, they refer to the willingness to pay for having the right to enter the HOT lanes.

(14) **Extended Vickrey auction** and **single-unit demand auction**. There are multiple extensions or generalization of the standard Vickrey auction for multiple items (Vickrey 1961; Clarke 1971; Groves 1973; Leonard 1983; Ausubel & Cramton 2004). Nonetheless, in this document, the term "extended Vickrey Auction" is used exclusively to refer to the auction that is composed of the following features: (1) Multiple identical items are offered simultaneously. (2) Each bidder submits one sealed-bid amount which indicates how much she is willing to obtain for any of the items. (3) The items are given to the highest

bidders but each bidder can only receive at most one item. (4) The winners have to pay the highest bid amount among those that did not win. Vickrey (1961) did work on this specific auction. Others have referred to it indirectly. But it is analyzed in this dissertation with the specific label of extended Vickrey auction. This auction is a specific case of the one proposed by Leonard (1983) in which items were non-identical and bidders could submit a different bid amount for each item. It can also be seen as a specific case of the multi-unit auctions (Krishna 2009, p.180) with limited demand where the demand is equal to one item. For this reason, this dissertation also refers to the extended Vickrey auction as a single-unit demand auction. Nonetheless, the term single-unit demand auction is less precise because it does not indicate how much winners are supposed to pay or whether it is a sealed-bid auction or not. Appendix D presents the proof that in the extended Vickrey auction, the weakly dominant strategy is also to submit their truthful valuation of the item. Although the proof can be derived from other generalizations of the standard Vickrey auction, it is not easily available in the literature for the extended Vickrey auction used in this dissertation. Figure 1 presents a simple comparison between the standard Vickrey auction and the extended Vickrey auction.

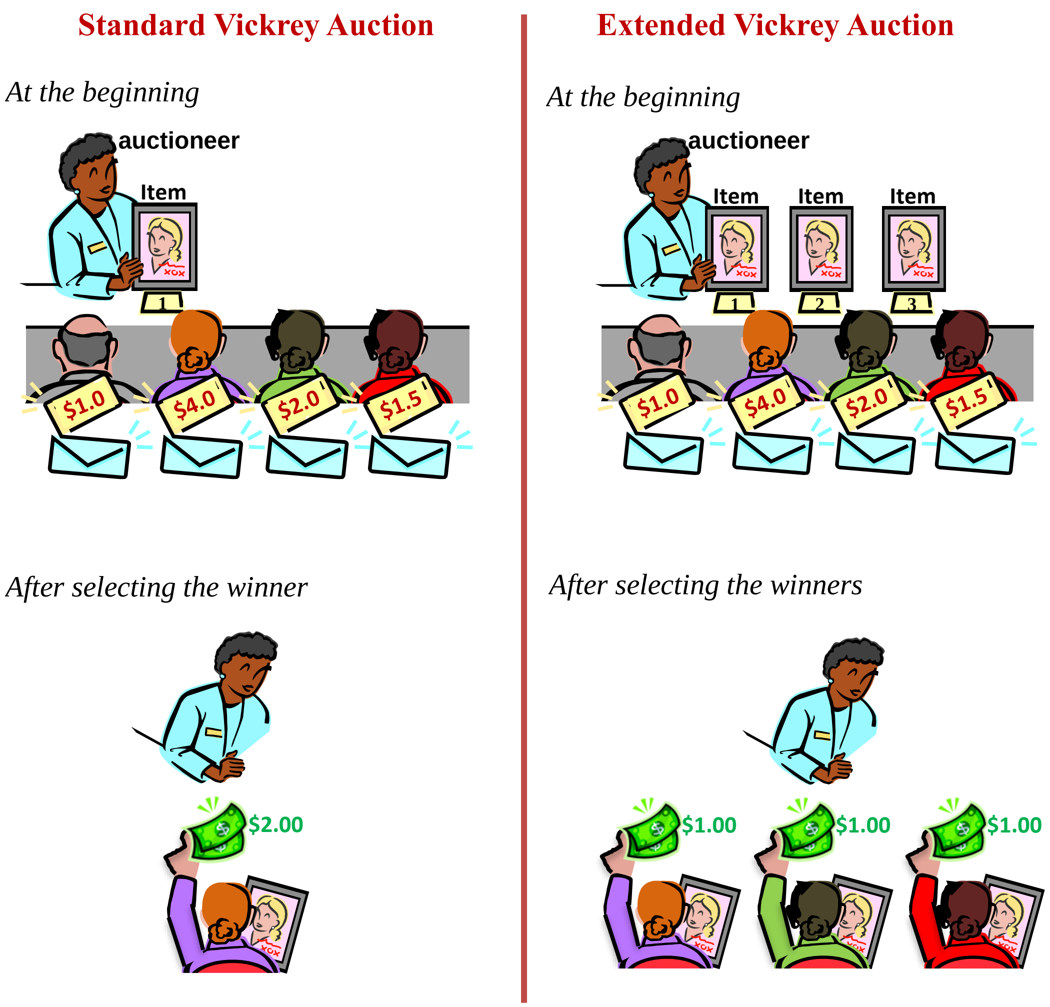


Figure 1. Comparison between the standard Vickrey auction and extended Vickrey auction (using the definitions adopted in this dissertation). In red: bid amounts. In green: amount that bidders end up paying after winning the auction.

Hereafter, whenever this document uses the term "auction", it is implied that it is referring to an extended Vickrey auction unless otherwise is specified.

(15) **Extended raffle**. In this dissertation, an extended raffle is defined as a random selection of $\bar{k}$ individuals from a group of $\bar{a}$ individuals, where $\bar{k} \leq \bar{a}$ and where each individual is granted one item. The individuals can also be referred as players, and it is assumed that they do not need to make any payment to

participate in the game. The word "extended" refers to the fact that the raffle can have more than one item. Using this definition, an extended raffle can be considered as a special case of an extended Vickrey auction where all players have submitted the same bid amount. Figure 2 presents the relation of an extended raffle with extended Vickrey auctions and lotteries, using the definitions presented above. All three definitions presented so far assume that players do not have to pay an amount in advance in order to be able to participate.

Hereafter, whenever this document uses the term "raffle", it is implied that it is referring to an extended raffle unless otherwise is specified.

(16) **Reserve price**. The games of extended raffle, extended Vickrey auction, and lottery may include reserve prices. A reserve price is a small amount of money that players must pay if they win the lottery. It is not a price for participating in the lottery. The amount is considered small if it is less than the minimum amount that any players would accept to pay if winning the game.

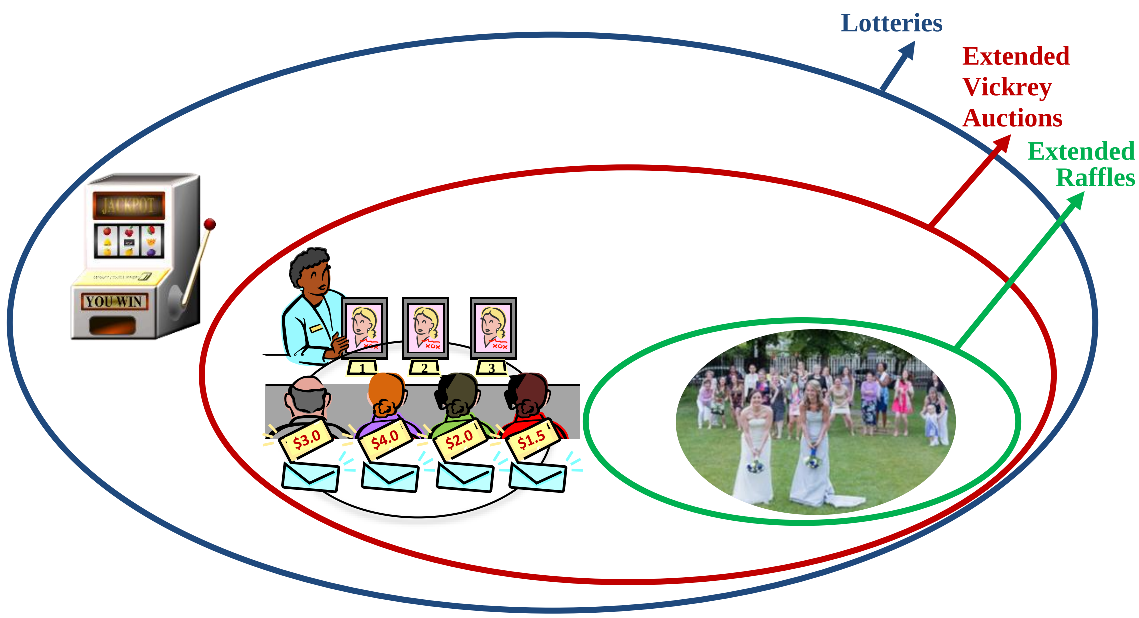


Figure 2. Relationship between the games of extended raffle, extended Vickrey auction, and lottery (using the definitions adopted in this dissertation). Depicted example of an extended raffle is the throwing of two bouquets at a multiple wedding. Thus, the example could be referred by the shorter label "raffle". In the three types of games, it is assumed that players are not required to pay for participating.

(17) **Nash equilibrium** and **Bayesian Nash equilibrium**. In simple terms, a Nash equilibrium is a state that is reached when individuals (or players in a game) have chosen a particular strategy (or move) and none of these individuals has an incentive (or an increase in their utility) by individually switching to a different strategy. The Bayesian Nash equilibrium differs from the Nash Equilibrium in the sense that the utilities that the individuals have are actually expected utilities (they take into account uncertainty). In an auction, the uncertainty is caused by the fact that each individual does not know how much utility the other individuals experience when they acquire an auction item.

(18) **Regret**. It is the additional utility that a player would acquire if choosing a different strategy. This document discards the possibility of having a negative regret. In other words, regret is always non-negative.

1.2. Why auction-based metering

If it is true that HOT lanes have unused managed capacity, then why should it be sold via auction-based metering? This section focuses on who would benefit, and how these benefits fall in a larger list of six arguments for implementing ABM in HOT lanes.

If ABM is capable of eliminating unused managed capacity, and if the HOT facility is operated by a public entity, then this entity who would benefit by having less traffic on the GP lanes (without degrading the level of service currently offered on the HOT lanes).

The second likely beneficiary is a certain category of potential new HOT users. Today, HOT lanes offer a reliable trip. Studies have shown that reliability is one of the main factors that attract HOT-lane users (Goodall and Smith, 2010; Liu et al., 2011). But perhaps there is another market that arises whenever there are trip purposes that do not need as much reliability. Such is the case of commuters who pass by the HOT facility on a regular basis when driving from work to home (not vice versa). In many of those trips, drivers do not have to arrive to their destination at a particular time, and could welcome an occasional faster trip (one that is free or with a lower price than the toll). If such unserved market exists, then these new users would benefit by occasionally using the HOT lane without degrading the minimum level of service that HOT lanes should offer.

The third beneficiary (but perhaps the less likely one) is a private entity who is in charge of operating the HOT facility. Without degrading the minimum level of service (or in other words, without exceeding the managed capacity), this operator may see an increase in revenue by acquiring new customers or increasing the loyalty of existing ones, where all these customers would belong to the market aforementioned. (Subsection 6.2.3, Figure 36 will suggest that the only situation in which there would be no increase is if having a raffle instead of an auction and if such raffle has no reserve prices).

The three benefits presented above fit within a list of six arguments that support the need for applying ABM to dedicated lanes. Figure 3 presents those arguments. The first three have already been mentioned. These arguments are discussed in more detail in Appendix B. In its discussion, the appendix presents an interesting theory about the existence of noise in the demand curves. It also presents studies that suggest that the unserved market aforementioned exists.

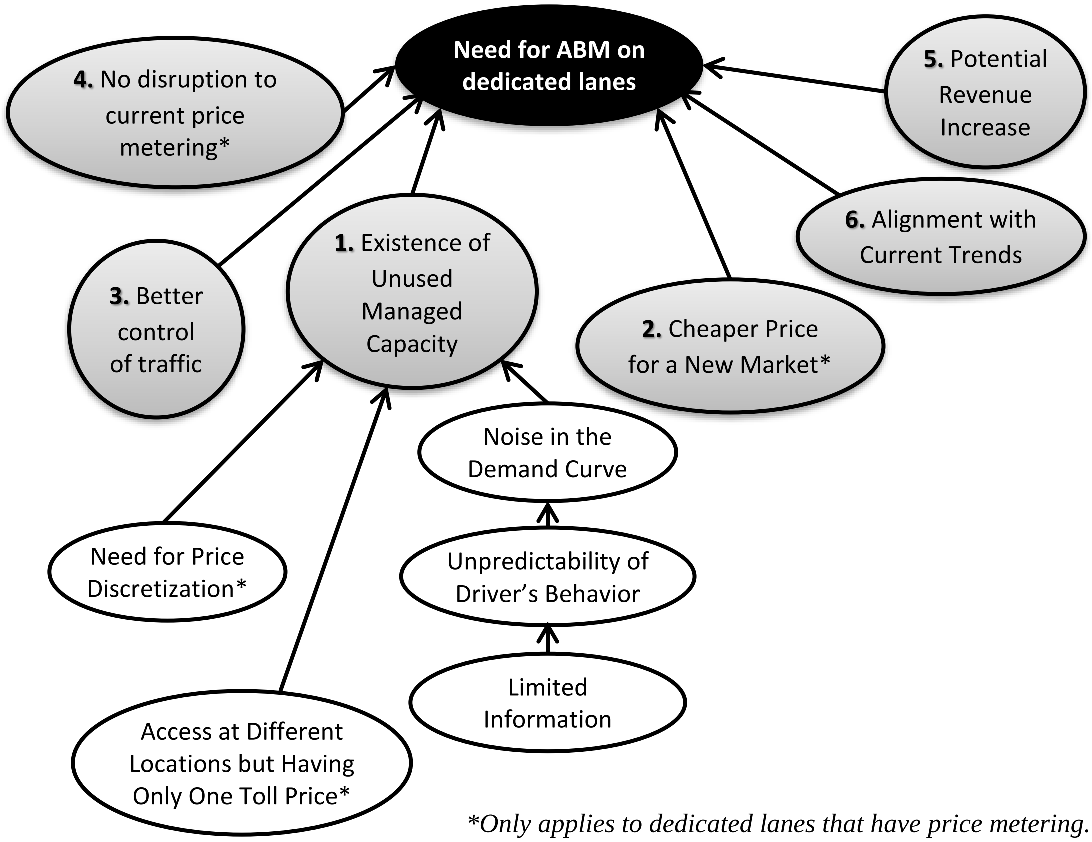


Figure 3. Six main arguments (highlighted in gray circles) and their supportive arguments for adding ABM to dedicated lanes.

Finally, it is worth making the following two observations regarding Figure 3. The third argument states that ABM does not imply overhauling current HOT facilities. Current HOT users who decide not to participate in auctions should not see their current driving experience affected. Also, the system can be tested at just one entrance and later be scaled up. Regarding the sixth argument, although new technologies such as satellite tolling, connected vehicles, and autonomous vehicles would optimize ABM, they are not key for its feasibility. The proposed system relies on the adaptation of existing technologies used at current HOT facilities and weight stations.

1.3. Objectives and contributions

The two general objectives of this dissertation are to provide evidence to the validity of the two hypotheses mentioned at the start of these chapter. In doing so, some of the specific contributions that this dissertation makes to the field of transportation are the following:

(1) It provides a definition and a methodology for measuring managed capacity (u_{MC}) and potential volume increase (Δq).

(2) It provides measurements of u_{MC} and potential volume increase Δq on a typical day of operations on a section of an HOT facility in Minneapolis.

(3) It provides a game-theoretic model for a buyout extended raffle and another one for a buyout extended Vickrey auction.

(4) It proves that each of the above two models has a unique Bayesian Nash equilibrium.

(5) It provides a design (with recommended design private values) of an ABM system (as well as an RBM system).

(6) It evaluates the aforementioned design in terms of congestion reduction and revenue changes.

1.4. Organization of this document

This document is organized as follows. Chapter 2 presents a literature review concerning HOT lanes and auction theory. Chapter 3 addresses the first hypothesis, by presenting quantitative evidence from an HOT facility in Minneapolis. Chapters 4 to 6 address the second hypothesis. Chapter 4 proposes a game-theoretic model. Chapter 5 proposes a generic auction-based metering (ABM) system by focusing on what the driver would experience and what the operator should do in order to make it function. In doing so, it describes the design parameters that the system requires. Chapter 6 applies the ABM system to the HOT facility in Minneapolis by presenting a microsimulation model and various numerical results. Such results allow determining the sensitivity of the congestion reduction and revenue increase to the design parameters and to a lesser extent, to driving behavioral parameters intrinsic to the traffic microsimulation component. The results also evaluate how much regret drivers experience when adopting the safer strategy instead of the optimal strategy recommended by the game-theoretic model. Chapter 7 and Chapter 8 conclude.

This dissertation has several appendices that allow the reader to focus on certain specific subjects mentioned throughout the main content of the dissertation. Of special importance is Appendix D. Appendix A defines in more detail aspects of dedicated lanes that were mentioned in the terminology section (Section 1.1). Appendix B and Appendix C look deeper at why certain assumptions were made and certain policies were followed for providing a framework before proposing the ABM system that is explained in this dissertation. Appendix D presents the proofs to the lemmas and propositions presented in Chapter 4 as well as classical properties of the standard and extended Vickrey auctions. Finally, Appendix E presents auction-based road systems that were considered initially but were discarded due to several difficulties that they presented.

2. Literature Review

This chapter looks at previous research conducted in the field of auction theory (with buyout auctions) in particular, and in the field of HOT lanes. It then looks at the few research that has focused on both fields simultaneously. This chapter does not provide references to dedicated lanes other than HOT lanes due to the fact that HOT lanes represent the fundamental challenge for applying the system proposed in this dissertation. Nonetheless, Appendix A does provide information about various types of dedicated lanes.

2.1. Auctions

Auctions constitute an ancient business practice. But it was not until 1961 with the work of William Vickrey that they were studied from a game theoretic approach. After his seminal work *Counterspeculation, Auctions, and Competitive Sealed Tenders* (1961), auction theory grew, its literature became extensive, several textbooks were written (such as, Krishna 2009), and it became a main component of microeconomic theory (such in Mas-Colell et al. 1995).

Regarding the specific subfield of buyout auctions, as pointed out by Gallien and Gupta (2007, p.815), there was not any theoretical work prior to the year 2000. Academic interest in this subfield was later sparked by the adoption of buyout auctions in online businesses such as eBay and Yahoo. Since 2001, different models have been proposed to explain the dynamics of these online buyout auctions. Figure 4 presents these models. Figure 4 also presents how the buyout auction that will be proposed in Section 4.3 fits within these other auctions.

	Number of Bidders			Number of Items			Particip-ants' Properties		Buyout Option	
	Two	Arbitrary	Unknown	Single-Unit	Multi-Unit (as a Sequential Sale)	Single-Unit Demand	Risk-Averse	Time-Sensitive	Temporary	Permanent
Budish & Takeyama (2001)	■			■			■			■
Reynolds & Wooders (2003)	■			■			■			
Kirkegaard & Overgaard (2003)	■			■	■				■	
Mathews (2003; 2004; 2006)		■		■			■	■		■
Reynolds & Wooders (2009)		■		■			■			
Ivanova-Stenzel & Kröger (2005; 2008)		■		■			■		■	
Hidvégi et al. (2006)		■		■			■			■
Kirkegaard & Overgaard (2008)		■		■	■				■	
Model in this dissertation		■				■	■		■	
Caldentey & Vulcano (2007)			■	■				■		
Gallien & Gupta (2007)			■	■				■		

Figure 4. Mathematical models on buyout auctions and common features with the proposed research. Note that only the model from this dissertation includes single-unit demand.

Previous research, shown in, deal with ascending auctions (like in an English auction). For this reason, some of that research take into account the time sensitivity (or impatience) of the bidders as the auction progresses in time or as the auction gets close to its end as determined by an auction clock. Only in cases where the research does not include time sensitivity, then they can be modeled as a standard Vickrey auction (as recommended by the adopted policies that are stated later in Section 4.1). Some models in Figure 4 are able to tackle unknown number of bidders but require knowledge of their arriving pattern. All of the previous work shown in Figure 4 look at answering how to set the buyout price in order to achieve an equilibrium. Since they are focused on the online business, some of them look at, if possible, how to set up the buyout price in order to maximize revenue.

Figure 4 uses the terms “single-unit”, “multi-unit” and “single-unit demand”. Figure 5 shows how these concepts relate to each other. The terms ¨single-unit¨ and “multi-unit” refer to how many items are being offered. When having multi-unit auctions sometimes the demand is limited, that is, it is not feasible or it is not allowed for any player to acquire all the items. When the restriction only allows one item, then one would have a “multi-unit auction with one-unit demand”. But for short, this document adopts the term “single-unit demand”. As shown in Figure 4 and Figure 5, the single-unit demand, when combined with a buyout option, has not been modeled before.

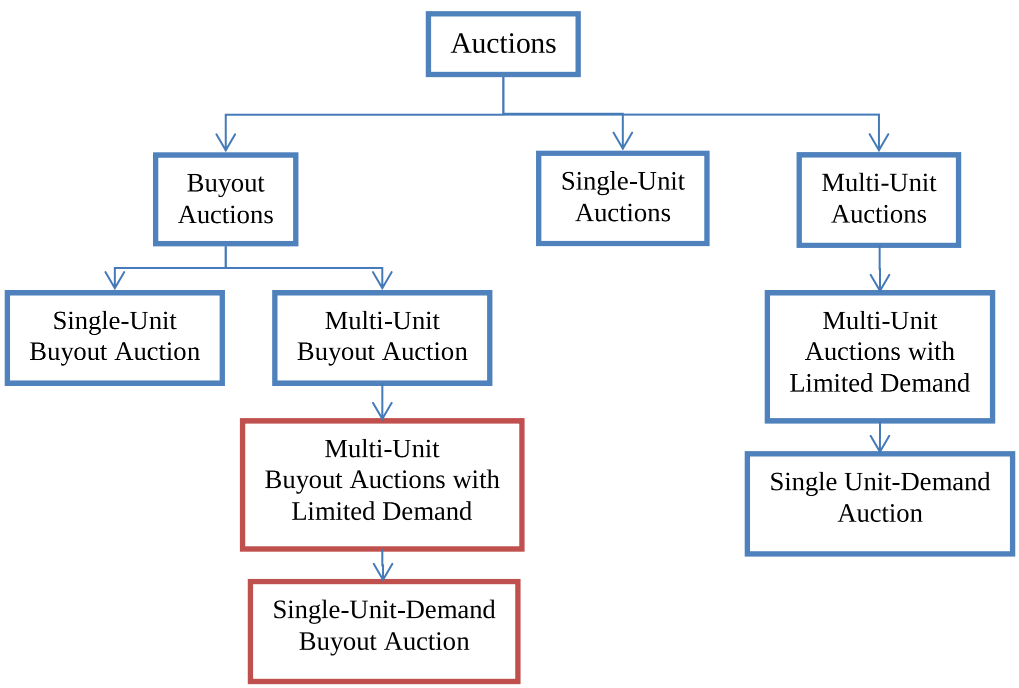


Figure 5. Types of auctions (with and without buy option) based on the number of items. Auctions that appear in red boxes have not been studied in any previous research.

Figure 4 makes the distinction between a buy option that is temporary and one that is permanent. These terms are not important for the purpose of this study, but for completion, they are explained in Appendix C.

Of the models shown in Figure 4, the models proposed by Reynolds and Wooders (2003; 2009) are very instrumental for this dissertation. They do not take into account the time factor, they take into account the risk-averseness of the bidders, and they indicate how the buyout price should be set in order to achieve an efficient equilibrium. But their model is not single-unit demand and does not possess other features that the proposed model requires (as recommended by the adopted policies that are stated later in Section 4.1).

An aspect that Figure 4 does not capture is the fact that in all of the existing models, the probability that two participants choose to buy at the same instant is zero. Or in the temporary buyout auction model by Reynolds and Wooders (2003; 2009), if two participants choose the option of buying at the same time instead of continuing bidding, then the auctioneer grants the item to just one buyer by carrying out a random selection. In an HOT facility, if a participant chooses to buy instead of to bid, which is very probable, then the operator should grant a space to each of them. That participant should not be subject to any subsequent random selection.

Outside the scope of this dissertation are studies that, motivated by the online auctions, try to find out the advantages for the seller to add a buyout price to their ascending auctions (Wang et al. 2008; Yu et al. 2006; Popkowski Leszczyc et al. 2009).

Finally, there is a different branch of studies from those considered here that looks at behavioral traits from bidders through experimental data. Following this methodology, they try to predict bidders' choices and provide answers to sellers regarding the benefits of adding the buyout auction. For example, Wan, Teo and Zhu (2003) found that online sellers who implement an auction with a temporary buy option would have a higher revenue if they (1) set a longer duration for their auctions, (2) are able to set a buyout price that is closer to the minimum bid, and (3) set a lower buyout price in reference to other online prices. Angst, Agarwal, and Kuruzovich (2008) provides a literature review in this area.

2.2. HOT lanes

Before reviewing the existing literature, it is important to note that literature that openly explains the algorithms used by current HOT operators is perhaps non-existent. It seems that most operators do not reveal their algorithms to the public. One can only find reports or hints in some papers that reveal criteria that they use but not the specific algorithms. For example, (Munnich & Buckeye 2007) reveal that the toll price is set based on the congestion on the HOT lane of the I-394 MnPass Express Lanes. One report states that the displayed price for the I-15 Express Lanes is set based on the overall congestion and travel time savings (HNTB Corporation 2006, p.5). One study that provides a hint on the possibility that hypothesis 1 is true in today's HOT facilities is the work of Munich and Buckeye (2007, p.51) which makes the following statement:

> *When the I-394 MnPASS project opened, the algorithm, a formula developed to determine the price for each segment based on the level of congestion in the MnPASS lane, was causing wildly fluctuating prices. The price would rise rapidly with the detection of added vehicles in the lane, which would serve to price drivers off the facility. When the number of users diminished, the price would decline. As drivers saw the price fall, they would again get back on the lane, and the cycle would start over. Recognizing that these wide fluctuations in prices caused erratic flows and likely kept many potential users off the facility, Mn/DOT modified the algorithm to provide less radical changes in toll prices, which in turn provided a more steady and predictable number of users and* ***kept the average toll price slightly higher****.*

Therefore, by setting the toll price higher, the instability is avoided but should create in some cases unused managed capacity.

The literature in the field of HOT lanes can be categorized in three fields: (1) economic models that propose analytical tools or optimal strategies for setting the toll price, (2) operational strategies and algorithms, and (3) studies that evaluate existing HOT facilities. Sometimes the boundary between the first two fields becomes blurry when a detailed system is proposed but it is highly connected to a macroscopic model.

Economic models that have been applied to HOT lanes were developed by Chu (1995), Arnott et al. (1999), Yang and Huang (1997), and Liu and McDonald (1999). Laval et al. (2015) and Rambha and Boyles (2016) later proposed models for improving the system optimum. As indicated by Yin and Lou (2009, p.45) when referring to the work preceding theirs, those models fall short in predicting the impact of HOT lanes on congestion relief because they assume hypothetical scenarios such as the knowledge of travel demand functions. A more data-driven model was developed by Li and Govind (2002) where users' sensitivity to the toll price and willingness to pay were derived from a survey. Economic models that provide analytical tools have been developed by Lu et al. (2008) who provided a user equilibrium comparison tool, by Ardekani et al. (2011) who proposed macroscopic speed-flow models and by (Gardner et al. 2013) who proposed a choice model for HOT lanes.

In terms of operational strategies and algorithms, where data is captured (mostly in real time) to allow the operator make quick decisions, the following literature can be found. Zhang et al. (2008) proposed using a logit model for estimating the dynamic toll prices. It included a feedback-based algorithm. Yin and Lou (2009) provided an algorithm that has a feedback-based controller as well as a self-learning controller. Lou et al. (2011) complemented the above algorithm by improving the self-learning controller and it incorporates a hybrid traffic model proposed by Laval and Daganzo (2006). Michalaka et al. (2011) incorporated into that model uncertainty in the travel demand and field data from the Interstate 95 HOT facility in Florida. Two different toll pricing strategy were also proposed by Morgul and Ozbay (2011), Zhang et al. (2014) and by Phan et al. (2015).

In terms of evaluation of existing facilities, the research literature includes many studies. It mainly focuses on determining the willingness to pay, the value of time, the value of reliability, and they employ discrete choice models. It focuses on the SR-91 Express Lanes, on the I-15 Express Lanes, and on HOT facilities outside California.

The evaluation studies of the SR-91 Express Lanes measure the value of travel time savings and the value of travel time reliability. Li, Hensher and Rose (2010) summarized their numerical results as shown in Table 2.

Table 2. Measurements of time savings and time reliability as summarized by Li, Hensher and Rose (2010). The value of time savings is the median and not the average obtained from the observations. For state preference (SP) surveys, reliability is simply the standard deviation of the observations. For revealed preference (RP), reliability is the difference between the 90th or 80th percentile and the median of travel time. "2009$" refers to dollars adjusted for inflation to 2009.

Study	Date Collection Period	Data	# of Respon-dents	Value of Time Savings [2009$/hour]	Value of Reliability
(Small et al. 1999)	1995	SP	N/A	5.1	17.8 2009$/hour
(Lam & Small 2001)	1997& 1998	RP	332	30.5	Male: 31.9 2009$/hour Female: 42.5 2009$/hour
(Small et al. 2005)	1999& 2000	RP& SP	548	RP: 27.5 SP: 15.2	RP: 25.0 2009$/hour SP: 6.9 2009$/incident
(Brownstone & Small 2005)	1999& 2000	RP& SP	81	SP: 16.1	6.4 2009$/incident

The first three studies in Table 2 also provide discrete choice models that estimate the percentage of users who choose the HOT lanes based on the toll rate, travel time savings and travel time reliability. Two important observations should be made from the studies shown in Table 2. One is the big discrepancy between the median values for the time savings between the state preference surveys and the revealed preference surveys. The other observation, as pointed out by Li, Hensher and Rose (2010), when defining reliability, not only they use different definitions but they ignore the three main frameworks used for travel time reliability: the mean-variance model, the scheduling model, and the mean lateness model.

The evaluation studies that focus on the I-15 Express Lanes quantify the value of travel time savings (Brownstone et al. 2003; Steimetz & Brownstone 2005). Brownstone et al. (2003) provided various numerical results. For example, they estimated that the median value of travel time savings was $30/hour, with a lower quartile of $23/hour to an upper quartile of $43/hour. But this study, as pointed out by Steimetz and Brownstone (2005, p.869), relied heavily on the travel speeds obtained from the loop detectors, a practice that can lead to a lot of errors. This forced Steimetz and Brownstone (2005) to complement these speeds with on-ramp queue times and floating car data. Thus, Steimetz and Brownstone (2005, p.869) obtained a median value of time of $29.68 (equal to the previous study) but with an uncertainty that makes it range between a lower quartile of $18.81/hour to an upper quartile of $45.69/hour. Steimetz and Brownstone (2005, p.879) also point out that the I-15 Express Lanes do not allow measuring travel time reliability because, since its tolling system is fully dynamic, then travel time savings and travel time reliability have high degree of collinearity.

The evaluation studies that focus on HOT-lane systems outside California are the following. One is the revealed preference survey carried out on the I-394 MnPass Express lanes (Goodall & Smith 2010) and another one is the revealed preference survey carried out on the SR-96 (Liu et al. 2011). They provide choice models for determining the percentage of users who choose HOT lanes and they provide hypotheses to explain their results. These results are the following. Goodall and Smith (2010, fig.6) provide percentages of vehicles that choose the HOT lane based on their travel time savings. Liu et al. (2011, fig.3a) provides two empirical distributions that describe how users value the time savings that HOT lanes offer. This distribution will be used later in this dissertation (Subsection 4.5.3). Zhang et al. (2009) also evaluated the same HOT facility but applied a simulation model for their findings.

Finally, outside of the three categories mentioned at the start of this subsection, is the recent work of Paleti et al. (2016). They look at the possibility that income inequality exists on today's HOT lanes. In response, they propose designing income-equitable toll prices.

2.3. HOT lanes with auctions

There is a small but growing line of research that has explored auctions on roads. Teodorović et al. (2008) proposed a system in which, for a given urban area, an operator assigns time slots via a combinatorial auction. The time slots could be three to five minutes long within one day or several days. The number of winners that could be assigned to each slot is determined by the parking spaces and the roads in the urban area. Their system does not offer the option of buying a time slot and it cannot be carried out in real time. Thus, drivers are expected to arrive within the time slot that they indicated previously at the auction. Zhou and Saigal (2014) later improved the combinatorial auction by not making winners pay the amount they bid but less. This system, following the approach of the Vickrey-Clarke-Grove mechanism (Nisan et al. 2007, sect.9), incentivizes bidders to submit their true valuation. Zhou and Saigal (2014) focused on auctioning road trips and suggested how to apply it to HOT lanes. They proposed real-time communication between vehicles and infrastructure, but they were not specific in the design. Collins et al. (2015) looked at the previous work and sought to replace the Vickrey-Clarke-Grove mechanism with a simpler standard Vickrey auction (Krishna 2009). Again, they did not focus on the implementation design and they did not include the buyout option in their mathematical model.

In 2013, the first work on lotteries (specifically extended raffles) with a buy option was presented by Olarte and Haghani. They also proposed a generic design for a highway. In 2016, they proposed and evaluated a design for an entrance on the I-394 MnPass Express Lanes that would allow all kinds of lotteries with a buy option (that is, buyout extended raffles but buyout extended Vickrey auctions). They also proposed a mathematical model for the buyout raffle. The evaluation focused on the raffle but not on auctions. This work was fundamental for the production of this dissertation.

Finally, there is a line of research that emerged from the interest of removing traffic lights and stop signs from street intersections. This line assumes that vehicles are connected and autonomous. It seems to have started with the work of Dresner and Stone (2004; 2005) which proposed a tile-based reservation control mechanism which allows vehicles from different directions traverse the intersection simultaneously. Schepperle et al. (2007), and Schepperle and Böhm (2008) proposed a different reservation control mechanism that allows second-price auctions. They also consider allowing vehicles increase their bid amount by taking subsidies from vehicles behind. Vasirani and Ossowski (2011) takes the tile-based reservation control mechanism, allows combinatorial auctions, and expands the concept to networks relying on user-equilibrium assignment. Levin and Boyles (2015) simplifies the original tile-based reservation control mechanism, applies first-price auctions, and expands the concept to networks using dynamic traffic assignment.

Interestingly, looking at the above line of research, Raphael et al. (2015a; 2015b) proposed having auctions at intersections without the need of autonomous vehicles. They propose using traffic lights.

3. Existence of Unused Managed Capacity and Potential Volume Increase

The objective of this chapter is to provide evidence on the validity of the first hypothesis. The first hypothesis states that current HOT lanes have unused managed capacity (u_{MC}). This chapter also provides evidence that a more important concept, potential volume increase (Δq), is also significant. The main difficulty for obtaining this evidence, as it will be revealed in detail in the mathematical definitions of both measures, is that it requires measuring first the managed capacity.

The terminology section (Section 1.1, page 7) already defined managed capacity as the service flow rate corresponding to the level of service that the operator wants to guarantee on the dedicated lanes. Now, because this dissertation only looks at managed capacity at a specific entrance (unless indicated otherwise), it is important to clarify the following aspects:

- Managed capacity at a specific entrance (and managed capacity in general) can be measured at any time interval. This dissertation suggests measuring it at time intervals that are shorter than the time interval that is used by the current dynamic toll pricing.
- This dissertation adopts the notation m_{C} when referring to managed capacity at a specific entrance (and not at several entrances during the same time interval).
- At the given entrance, m_{C} changes at every time interval depending on congestion occurring downstream, on the number vehicles that entered the HOT lanes at

upstream entrances, and also on future number of vehicles that will enter at those upstream entrances.

- In order to define m_C, it has to be agreed how far downstream to the entrance the density is to be measured.
- In order to define m_C, it could be agreed that the value of m_C would not only be restricted by the level of service that is to be guaranteed on the HOT lanes but also by a prioritization among the various entrances. For example, if the given entrance has a higher priority over an upstream entrance, then m_C may be higher than the managed capacity of the upstream entrance.

Two approaches for measuring m_C at a specific entrance are the following:

(1) Use a calibrated traffic micro-simulation model, and feed the HOT entrance with different combinations of traffic volumes: The combination that generates the maximum density without violating the targeted level of service will constitute the managed capacity.

(2) Again, use a calibrated traffic micro-simulation model, but feed the HOT entrance with the real traffic volume for the first time interval. Then, assuming that the HOT operator has an algorithm for calculating the managed capacity in the next interval, implement that algorithm in the model for estimating the managed capacity in the next interval and feed the entrance with a volume equal to that estimation. Apply the method again for the next time interval.

The second approach would reveal managed capacity that is closer to what an operator can implement in real time. It takes into account the fact that the operator only has estimates of the future demand and not real values like in the first approach. And it relies on algorithms that are currently used by operators of ramp metering. Nonetheless, for this document, no HOT operator was found that had or would provide an algorithm as the one described above. Thus, the first approach was adopted for this document. Note that the first approach does not make any prioritizations among entrances.

In both approaches, a microsimulation model is needed in order to recreate any congestion (and subsequent reduction of capacity) that may be created downstream, especially where volume on the HOT lanes merges back to the GP lanes. But this need for a microsimulation model makes the process resource intensive. For this reason, instead of gathering evidence from various HOT facilities and various days, this chapter focuses on one typical work day of operations on a section of one HOT facility.

Results in this chapter will reveal that, although unused managed capacity (u_{MC}) is relevant for determining if there are available slots to be sold, potential volume increase (Δq) is more relevant because it comprises the available slots that would actually be sold given the existing volume. This chapter provides evidence of their existence by providing a lower bound for them. This chapter will also analyze the magnitudes obtained based on the assumptions that were made.

3.1. Description of the HOT facility

The HOT facility that was chosen is known as the I-394 MnPass Express lanes. It is operated by the Minnesota Department of Transportation. It was opened to the public in 2003. It has the following important features:

- HOVs who want to enter the HOT lane do not need to carry an on-board unit (OBU). SOVs who want to enter the HOT lane do have to carry an OBU.
- The aim of the operator is to have a LOS C at all times.
- The operator updates the toll price every 3 minutes.
- The toll price is discretized in intervals of 0.25 dollars (many other facilities discretize at every 0.01 dollars).
- The toll price has a minimum value of 0.25 dollars.
- The number of minutes that would be spent (or saved) by taking the HOT lane does not appear on any road sign.
- The price typically varies between 0.25 and 8 dollars. The average fee during peak period is between 1 and 4 dollars.

Additional information about the whole facility is presented in Appendix A.

3.2. Testbed, time period of analysis, calibration and validation

The testbed that was modeled is shown schematically in Figure 6 using the microsimulation application VISSIM 5.4 (PTV Planung Transport Verkehr, AG 2012). The testbed corresponds to the so-called “reversible section” of the HOT facility. The GP lanes were modeled but only used during the calibration process. The analysis considered only the time period in which the section is open in the westbound direction.

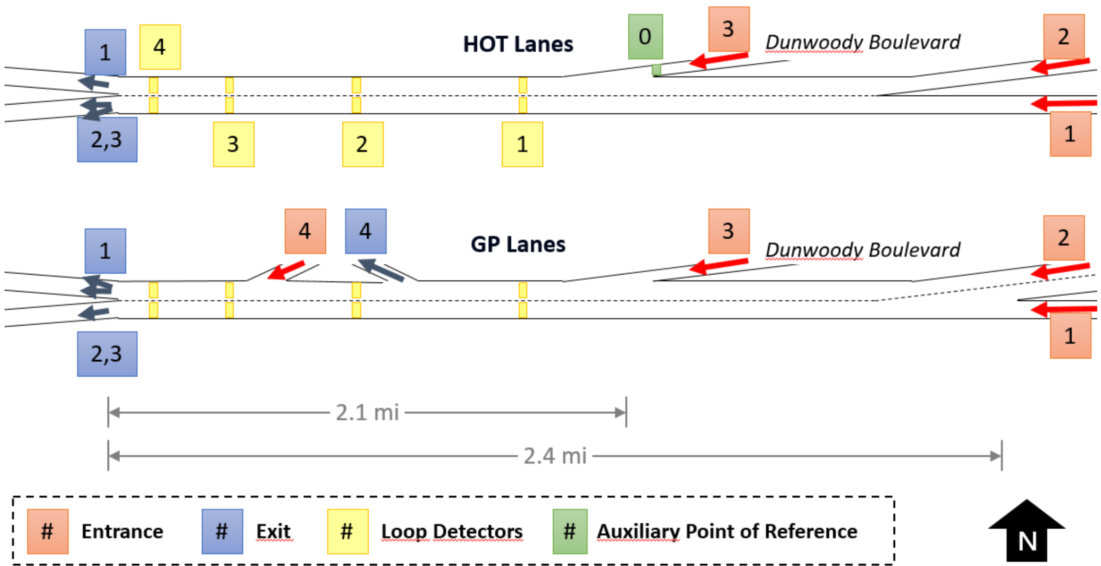


Figure 6. Schematic representation of the reversible section of the I-394 MnPass Express lanes and the GP lanes parallel to them. All entrances to the HOT lanes are direct-access ramps (DARs).

The hours of operation of the reversible section for the weekdays are as follows:

- Eastbound: from 6 AM to 1 PM.
- Westbound: from 2 PM to 5 AM.

The time period of analysis corresponds to Tuesday, March 1st 2016 from 12:15 AM to 05:00 AM, and from 2:00 PM to 11:45 PM. According to weather information provided by Minnesota Department of Transportation (Vaisala, Inc 2016), no precipitation occurred during that day.

Regarding the road characteristics of the section, it is important to highlight two aspects. First, all the entrances to the HOT lane have a DAR configuration. Second, the GP lanes allow vehicles to arrive to the same three destinations plus one additional one.

The car-following and lane-changing sub-components embedded in the VISSIM application contain a large number of driving behavior parameters. Nineteen of these parameters were calibrated using the methodology described in Appendix F. It is very important to clarify that the calibration mentioned here is not a calibration of the design parameters of the ABM system that will be proposed in Chapter 5 and modeled in Chapter 6. It is the calibration of nineteen driving behavioral parameters. Required data was obtained from Minnesota Department of Transportation (2011). These data included traffic volume, speed, and density at five-minute, and one-minute intervals. Since no origin-destination matrix was available, the following approach was adopted. Speeds obtained from the loop detectors at yellow box 4 in Figure 6 were used to apply a feature from the VISSIM application called “reduced speed zones”. As vehicles in the model approached those zones, they accelerated or decelerated in order to meet the real speed. In this way, the possible real congestion created by weaving at the exits was replaced with possible simulated congestion created by the reduced speed zones. As mentioned in the appendix, the calibration and validation were satisfactory.

3.3. Methodological approach

This section explains how unused managed capacity (u_{MC}) and potential volume increase (Δq) was measured at the HOT entrance. Whenever this section refers to "HOT entrance", it is referring to entrance 4 on the HOT lanes, as shown in Figure 6. And whenever it refers "GP entrance", it is referring to entrance 4 on the GP lanes, as shown in the same figure.

The first and most challenging step for calculating the two measures aforementioned is to calculate the managed capacity (m_{C}). This first step consists in finding the sequence of volumes (one volume for each one-minute interval) that makes the system get the closest to the targeted level of service. And so, it is tempting to use a metaheuristic algorithm. Nonetheless, metaheuristics such as genetic algorithms require evaluating several rounds (or generations) of at least ten solutions each. In this problem, each solution, which consists of one volume sequence, takes 30 hours to obtain (if using one Vissim license-seat and nine seeds for each solution). For this reason, the following approach was adopted instead. First, the volumes and densities were plotted as shown in Figure 7.

Before explaining how the figure below can be used, several aspects are worth clarifying. 26 vehicles/mile/lane is the maximum density at which the HOT lanes can be in order to comply with the requirement of having a level of service C or better. The marks on the light green line correspond to the maximum density obtained from measuring density at the locations indicated by the four yellow boxes and the one green box shown in Figure 6.

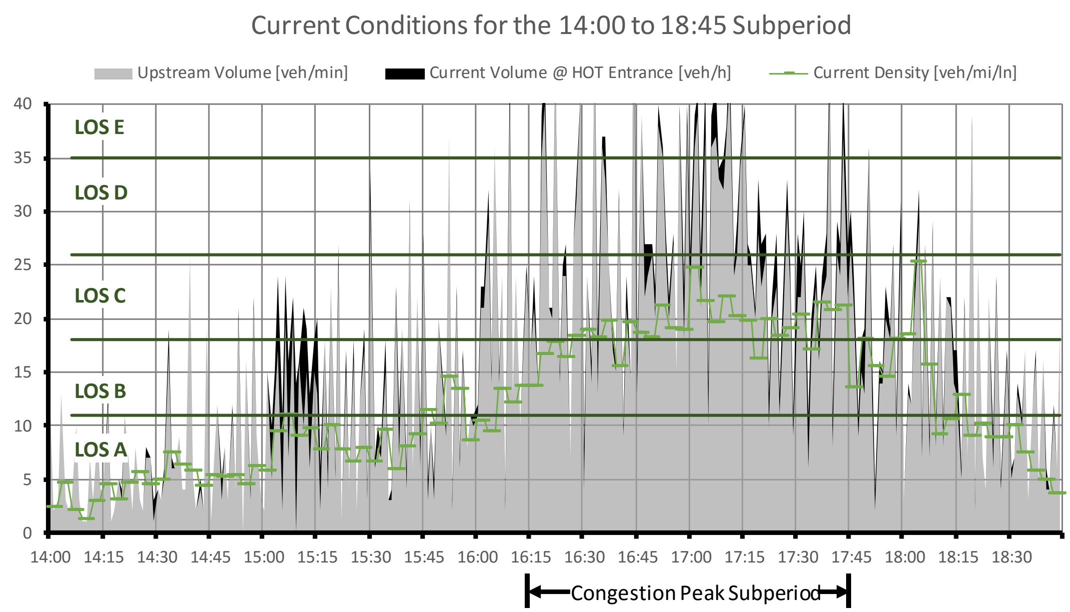


Figure 7. Shown as stacked areas: (in black) current volume at HOT entrance, and (in gray) incoming volume from the two upstream entrances. Shown as simple plots: (in green) current maximum density found on the HOT section of analysis, and (in dark green) density thresholds that define the levels of service. Figure shows that no density is above the LOS C threshold. Only one subperiod (14:00-18:45) of the whole period of analysis is shown in the figure. Also shown in the figure, congestion peak subperiod.

The next step in this methodological approach is to find a volume sequence to be pushed through the HOT entrance without violating the level C threshold. To aid this search, the figure above is used as follows: if at an interval the density is far away from the level C threshold, then a solution that pushes more vehicles at that interval should be tried. But there is no way to know exactly how many more vehicles can be pushed without violating the targeted level of service. For this document, eight different solutions were tried until reaching a volume sequence that had a density relatively close to the density threshold. It is important to note that this whole approach can be followed because in the figure above, there was no violation of the threshold. Otherwise, one would have to make assumptions on how the volumes that surpassed the density threshold should spill to adjacent intervals. The estimation of the managed capacity that was obtained is shown in Figure 8 and Figure 9 below. During the first three minutes, no estimation is carried out because it is used as a warm-up subperiod for the simulation model.

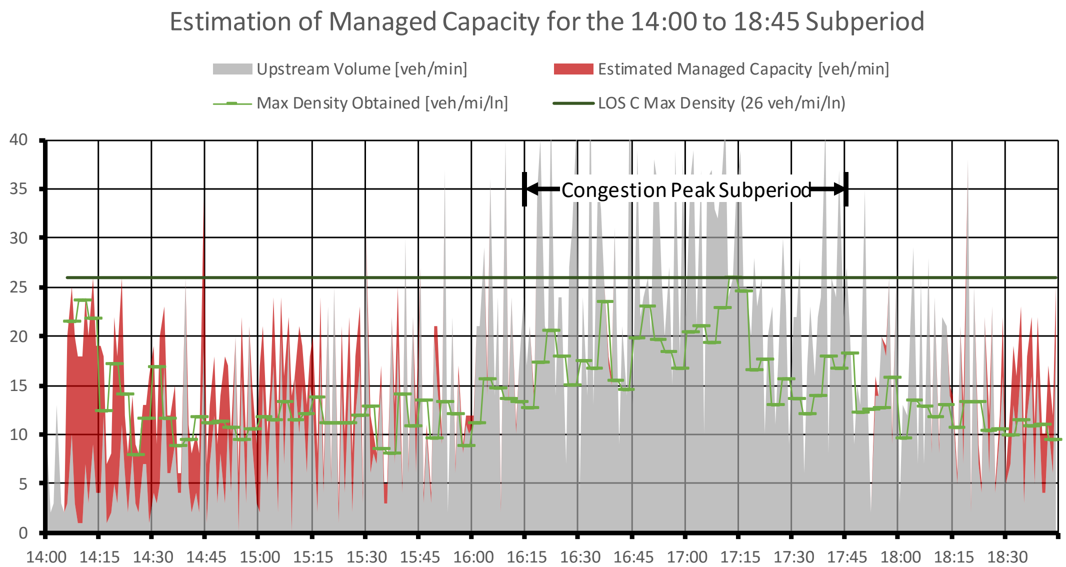


Figure 8. Shown as stacked areas: (in gray) incoming volume from the two upstream entrances, and (in red) estimated managed capacity. Shown as simple plots: (in green) maximum density obtained for the estimated managed capacity, and (in dark green) LOS C's maximum density. Only one subperiod of the whole period of analysis is shown in the figure. Whenever there is a great distance between the maximum density obtained and the LOS C threshold, then the managed capacity is much higher than the estimated one.

Figure 8 suggests that, for the subperiod that starts at 14:00 and ends at 18:45, the managed capacity is clearly significant except for the congestion-peak subperiod. For the congestion-peak subperiod, it could be significant because in several short intervals, the distance between the maximum density obtained and the LOS C threshold is greater than 8 veh/mi/ln (a value that is approximately the average difference between two contiguous levels of service). Figure 8 also reveals that the traffic microsimulation model tends to overestimate the maximum density, given in this way more conservative measurements of unused managed capacity (u_{MC}). This overestimation can be observed after noticing that in the congestion peak subperiod, in Figure 6, no volume was pushed at the HOT entrance, but it generated greater densities. Nonetheless, the differences seem to be less than 8 veh/mi/ln.

Figure 9 presents the results for the subperiod that starts at 18:45 and ends at 23:45. The figure suggests that the managed capacity (u_{MC}) that was obtained is significant. But the great distance between the maximum density obtained and the LOS C threshold suggests that the managed capacity is much higher. For the 00:15 to 05:00 subperiod, similar results to those shown if Figure 9 were obtained.

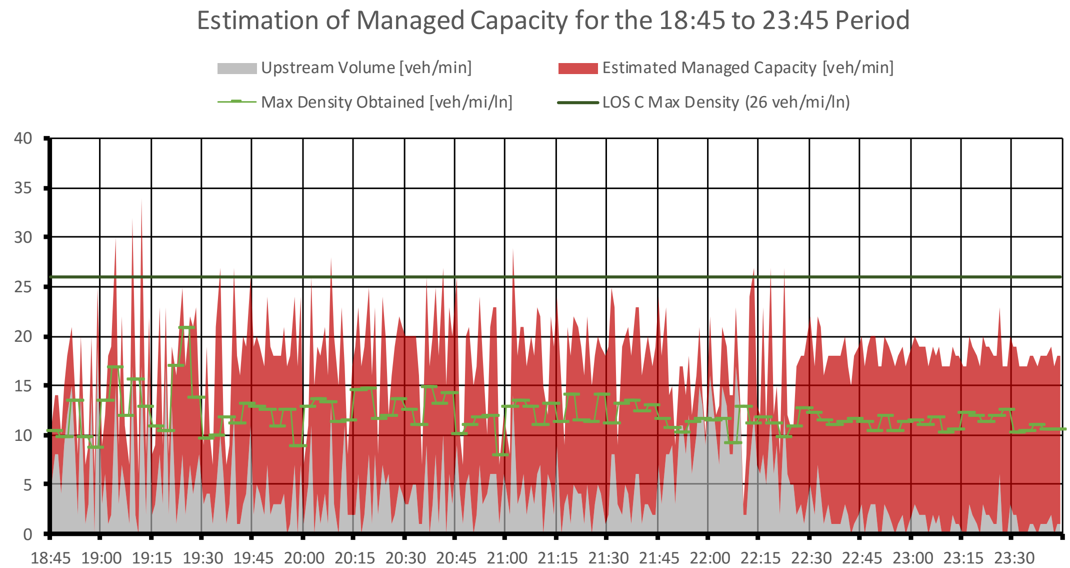


Figure 9. Same information that was provided in Figure 8 but for the subperiod starting at 18:00 and ending at 23:45. Again, whenever there is a great distance between the maximum density obtained and the LOS C threshold, then the managed capacity is actually much higher than the estimated one.

Now, having estimated the managed capacity, the methodological approach becomes straightforward. The unused managed capacity (u_{MC}) and potential volume increase (Δq) are then calculated using the table shown in Figure 10. As shown in the figure, measurements and calculations are made for every minute. Volumes at the HOT entrance (q_{BEFORE}) and at the GP entrance (q_{GP}) correspond to the locations indicated by the two red boxes labeled in Figure 6 as number three. The notation "$\lfloor \quad \rfloor$" refers to the nearest lower integer or floor function.

$\min(m_C, \max\lfloor q_{BEFORE}, \rho_1 \cdot \pi \rfloor)$

$\lfloor m_C - q_{BEFORE} \rfloor$ $q_{BEFORE} + \rho_2 \cdot q_{GP}$ $q_{AFTER} - q_{BEFORE}$

	m_C	q_{BEFORE}	u_{MC}	q_{GP}	π	q_{AFTER}	Δq
End of Interval [yyyy-mm-dd hh:mm]	Managed Capacity	Volume at HOT Entrance	Unused Managed Capacity	Volume at GP Entrance	Volume Pie	Potential Volume at HOT Entrance	Potential Volume Increase
2016-03-01 00:21	18	0	18	0	0.00	0	0
•••	•••	•••	•••	•••	•••	•••	•••
2016-03-01 14:43	6	1	5	15	15.31	6	5
2016-03-01 14:44	6	0	6	1	0.95	0	0
2016-03-01 14:45	15	0	15	4	3.81	2	2
•••	•••	•••	•••	•••	•••	•••	•••
2016-03-24 23:45	17	0	17	0	0	0	0
TOTAL [veh]	9,553	488	9,362	2,629	2,920	433	242
	666 veh/h	34 veh/h	652 veh/h	183 veh/h	203 veh/h	30 veh/h	17 veh/h

861 min

Unused Managed Capacity Percentage = 652/666 = 98%

Potential Volume Increase Percentage = 17/34 = 50%

Figure 10. Calculation of unused managed capacity and potential volume increase at the HOT entrance. Both values are also expressed as percentages at the bottom of the figure. Values correspond to one day of operations in which the HOT lanes are open to westbound traffic (March 1st 2016 from 12:15 AM to 05:00 AM, and from 2:00 PM to 11:45 PM). The two columns on the right rely on a factor ρ_1 of 0.4. Green boxes indicate formulas used for calculating columns.

Figure 10 reveals that u_{MC} for the time period of analysis is equal to 652 veh/h, or 98% when expressed as a percentage of m_C. These results are analyzed in the following section.

The potential volume increase (Δq) requires estimating first the volume pie which is defined as follows:

$$\pi = q_{BEFORE} + \rho_2 \cdot q_{GP} \tag{1}$$

The adoption of the name "pie" is due to the fact that that quantity represents all the volume that is to be distributed between the HOT entrance and the GP entrance. The calculation of π relies on the parameter ρ_2, which is positive and not greater than one. ρ_2 is the fraction of vehicles on the GP lanes that can switch to the HOT lanes without losing their path to their original destination. As mentioned in the previous section (3.2), there is only one destination (exit 4 in Figure 6) that can be reached on the GP lanes but that cannot be reached on the HOT lanes. To estimate ρ_2, it was assumed that for each five-minute interval, ρ_2 is equal to the fraction of vehicles on the GP lanes that did not take the first exit. Thus, all one-minute intervals within a same five-minute interval would have the same value of ρ_2.

The parameter that determines how π is to be distributed is ρ_1. This parameter is defined as the fraction of the volume pie that chooses the HOT entrance. Since π sometimes leaves out some vehicles that are at the arterial, ρ_1 can be described as the maximum fraction of vehicles that would choose to enter the HOT lanes.The value of 0.4 used for ρ_1 in Figure 10 suggests that there could be a general preference for using the GP lanes due to the fact that a large number of drivers do not acquire an OBU. That factor could also suggest that the GP lanes are expected to be more congested than the HOT lanes.

Now, although the definition of ρ_1 suggests that the resulting volume that would go to the HOT entrance (q_{AFTER}) should be simply equal to $\rho_1\pi$, there should actually be three adjustments as shown in this definition:

$$q_{\text{AFTER}} = \min(m_{\text{C}}, \max\lfloor q_{\text{BEFORE}}, \rho_1 \cdot \pi \rfloor) \tag{2}$$

First, it is possible that the HOT lanes already have a volume q_{BEFORE} greater than $\rho_1 \pi$. Then, the maximum function between the two should be used as long as (second adjustment) it is not greater than m_{C}. The third adjustment is the use of the integer value of $\rho_1 \pi$ so that the result be also integer. q_{BEFORE} is always an integer, but equation (2) puts it inside of the floor function just to reduce the number of parentheses in the expression.

Finally, having the value of q_{AFTER}, the calculation of the potential volume increase (Δq) is easily obtained through the following subtraction:

$$\Delta q = q_{\text{AFTER}} - q_{\text{BEFORE}} \tag{3}$$

Like with u_{MC}, and as shown at the bottom of Figure 10, it can also be expressed as a percentage of the managed capacity. The importance of Δq is that it actually represents the number of slots that can be sold because there are enough users who can buy them. The proposed ABM system seeks selling as many Δq slots as possible.

3.4. Final results and analysis

Table 3 presents the unused managed capacity that was obtained for the whole period using the methodology explained above. These values are the same values presented in Figure 10. In addition, the table presents the results for the peak-congestion subperiod. As argued latter in this subsection, these results should be much higher when having a better measurement of managed capacity.

Table 3. Lower bound of managed capacity, unused managed capacity, and unused managed capacity percentage for the whole period (00:15 - 05:00; 14:00 - 23:45) and the congestion peak subperiod (16:15 - 17:45). Also in the table, the average volume at the HOT lanes (which includes all three HOT entrances).

Time Period	Average Volume on HOT lanes	Managed Capacity (m_{C})	Unused Managed Capacity (u_{MC})	Unused Managed Capacity Percentage
Whole Period	224 veh/h/ln	666 veh/h	652 veh/h	98.0 %
Peak Subperiod	826 veh/h/ln	2 veh/h	1 veh/h	0.1 %

Table 4 presents the potential volume increase (Δq) assuming different values for parameter ρ_1. The values for the congestion peak-subperiod are not presented since Δq in that subperiod is always equal to zero due to the underestimation of the managed capacity.

Table 4. Lower bounds of potential volume increase percentage as a function of ρ_1 for the whole period (00:15 - 05:00; 14:00 - 23:45) and the congestion peak subperiod (16:15 - 17:45). Volume at HOT entrance is q_{BEFORE} = 34 veh/h.

Maximum Fraction of Vehicles that Would Choose to Enter the HOT lanes (ρ_1)	Potential Volume @ HOT Entrance (q_{AFTER})	Potential Volume Increase (Δq)	Potential Volume Increase Percentage
0.25	19 veh/h	6 veh/h	18%
0.40	30 veh/h	17 veh/h	50%
0.49	34 veh/h	21 veh/h	62%
0.60	45 veh/h	32 veh/h	93%

It is important to explain now why in Table 4 q_{AFTER} is sometimes less than q_{BEFORE}, given that the goal it to attract more vehicles to the HOT lane (by improving the facility with the implementation of auctions). When ρ_1 is equal to 0.25, q_{AFTER} is less than q_{BEFORE} due to the requirement that q_{AFTER} be always less than m_{C}. Unlike q_{AFTER}, q_{BEFORE} is in some intervals greater than m_{C} because of the inefficiencies of the current system or because, in our case, we have underestimated m_{C}.. Notice also that although q_{AFTER} is less than q_{BEFORE}, improving the system can potentially still have 6 new vehicles entering the HOT lanes (Δq = 6). These 6 new vehicles would enter at intervals where there is indeed sufficient m_{C} and not where m_{C} is currently been exceeded. Now, in the third scenario where ρ_1 is equal to 0.49, due to the same rationale, there are 21 potential new vehicles that can enter the HOT lanes despite the fact that q_{AFTER} is equal to q_{BEFORE}. Thus, the policy maker should interpret Table 4 as follows: If the system were to be improved, there are indeed 6 to 32 vehicles that can potentially be attracted (via the improvement of the facility). But the potential volume (q_{AFTER}) would increase, not simply when Δq is greater than zero, but when ρ_1 is greater than 0.49. And the policy maker should also take into account that the results in Table 4 are guaranteeing a better attractive level of service on the HOT lanes than the current one (due to the underestimation of the m_{C}).

Figure 11 sums up the two arguments that suggest that the final values presented in Table 3 and Table 4 could be significantly higher. First, as shown previously in Figure 8, there is at each one-minute interval a great distance between the maximum density and the LOS C threshold, which implies that m_{C} could be higher and therefore Δq could be higher. Only in the time intervals close to 14:00 and 17:15, the maximum density has been estimated exactly or almost exactly. Second, only in the time range of 14:00 to 14:16, Δq may not be much higher because the size of π (and therefore the size of $\rho_1\pi$) is not very large.

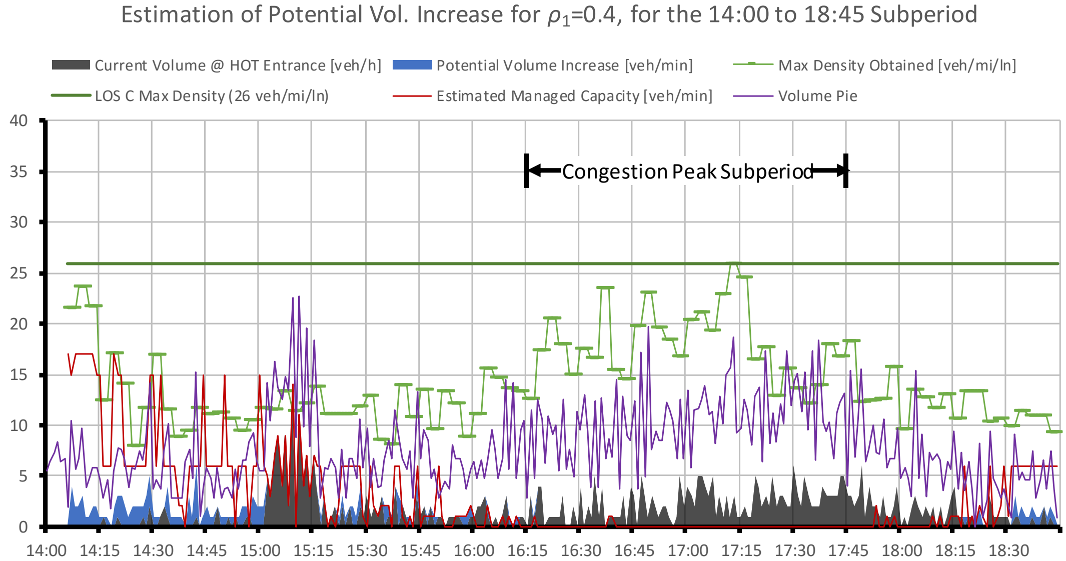


Figure 11. Shown as stacked areas: potential volume increase, and current volume at HOT entrance. Shown as simple plots: volume pie, (estimated) managed capacity, maximum density obtained for such managed capacity, and LOS C's threshold. Figure indicates that, outside the congestion peak subperiod, if the real managed capacity were estimated, the volume pie would be enough for obtaining an even greater estimation of the potential volume increase.

Figure 12 shows again that the quantification of Δq as presented in Table 4 could be higher. Nonetheless, it cannot be much higher as in Figure 11 because now the size of π is much smaller.

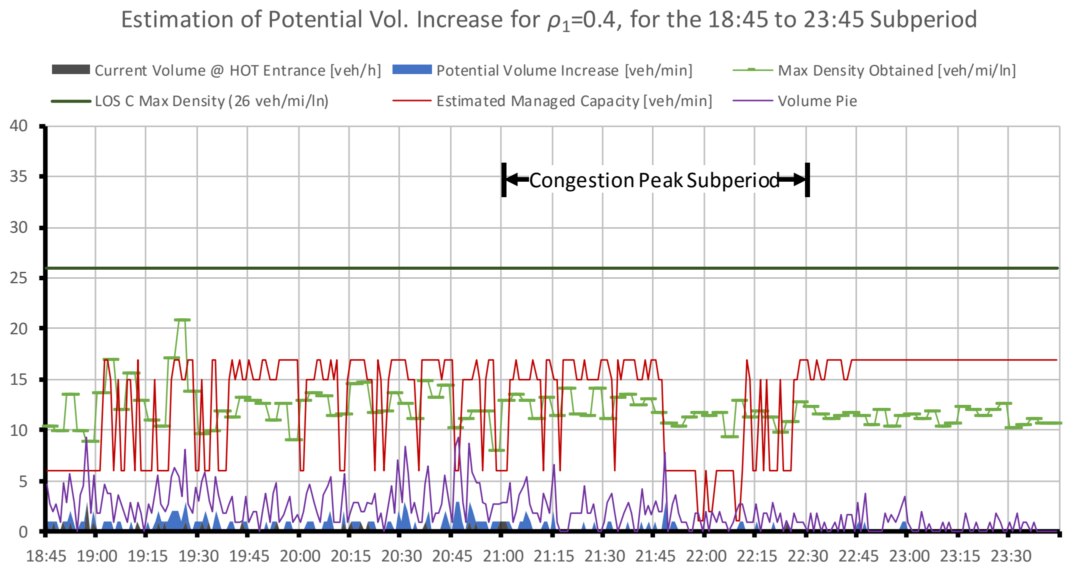


Figure 12. Same information that was provided in Figure 11 but for the subperiod starting at 18:00 and ending at 23:45. Unlike in Figure 9, if in this subperiod the actual managed capacity were estimated, the volume pie could deter obtaining a much greater estimation of the potential volume increase.

For the early morning subperiod of 00:15 to 05:00, the potential volume increase was zero because the size of π was insignificant.

In general, the values in Table 4 present a lower bound of the real Δq due to the fact that the managed capacity (m_C) could be much higher and due to the fact that the microsimulation model tends to overestimate the maximum density on the HOT section of analysis. Nonetheless this lower bound of Δq is already significant. Finally, the value of the managed capacity (m_C) shown in Table 3 is already very large, but as shown in Figure 11 and Figure 12, its influence in the magnitude of Δq is limited to what the size of π is.

4. Auction-Based Metering:

Adopted Policies and Mathematical Model

The objectives of this chapter are to present a game-theoretic model that precisely defines the buyout auction that takes place within the proposed ABM system, and to lay out the optimal strategy for its participants. It also analyzes a simpler game, that is, the buyout raffle that takes place in an RBM system. This chapter starts (in Section 4.1) by proposing a set of policies that narrows the different kinds of games that could have been proposed. Then, the chapter provides a formulation (Section 4.2) which defines the game in simple terms but which falls short in providing a method for estimating the optimal strategy for the players. Then, it provides the complete game formulation in Section 4.3, a section that does not require previous reading of the previous sections since it introduces all of its notation and assumptions. Before providing a section that estimates the optimal strategy, this chapter first provides (Section 4.4) the formulation of a simpler version of an ABM system: a raffle-based metering (RBM) system. This latter system will be useful later on in explaining the worst case scenario for a private operator for who seeks to maximize revenue (see Chapter 6, Subsection 6.2.3). It is also useful in understanding the calculation of the payoff functions of the ABM system. Finally, Section 4.5 provides the optimal strategy for players of an RBM system and an ABM system, and analyses implementation aspects. A major contribution of this dissertation is proof of the equilibrium on those two systems which allow computing the optimal strategy. The proofs are presented in detail in Appendix D. As the appendix reveals, the proof in the RBM system facilitates the proof in the ABM system.

4.1. Adopted policies

The mathematical model that will be described in this chapter assumes that the following policies have been adopted:

1. Players are assumed to be rational.
2. Players are assumed to have private values, not common values. In other words, the willingness of a player to pay a toll is independent of how much other players are willing to pay.
3. The auction should have some resemblance to an English auction since such type of auction is easy to explain to the general public.
4. The auction should include a "buy option". In other words, a classical auction that removes the possibility of gaining access through the payment of the toll should be ruled out.
5. Drivers who choose the "buy option" should always be granted access.
6. The auction should have single-unit demand (see terminology section, Section 1.1, for the definition of single-unit demand auctions).
7. Participation in this new system should be voluntary.
8. The goal of maximizing efficiency (whether operational or economic efficiency) should prevail over any goal of maximizing revenue.

The justification for having adopted the above policies is explained in detail in Appendix C. Appendix C assumes that the last six policies are key for making the implementation of the ABM system successful.

4.2. Initial formulation of game

This section formulates the buyout auction as a simple game by defining its player set, rules surrounding the game, its strategy set, and its payoff functions. Nonetheless, given its simplicity, it has limitations for determining the optimal strategy and it does not explain how it would interact within the dynamic setting that happens at an HOT facility.

4.2.1. Player set

In a HOT system, there are different kinds of users. They can be represented by the following sets:

- $\mathbb{A}$: Set of "advanced SOVs".
- $\mathbb{B}$: Set of "basic SOVs".
- $\mathbb{H}$: Set of HOVs.
- $\mathbb{O}$: Set of drivers who cannot enter the HOT lane (because are not HOVs or do not have the OBU necessary to enter as an SOV) or whose destination cannot be reached via the HOT lane.

"Advanced SOVs" and "basic SOVs" are single occupant vehicles that carry a technological device (also known as onboard unit or OBU) that allows them to pay the toll price. But unlike basic SOVs, the OBU that advanced SOVs carry also allows them to participate in the auction.

From the above sets, the set that is indeed considered as the player set of the game is $\mathbb{A}$. The other sets are assumed to be, not players, but features of the surrounding environment that creates uncertainty in some of the variables of the game.

4.2.2. Strategy set and mechanics of the game

The strategy set for the players in set $\mathbb{A}$ is composed by two strategies: to buy or to wait. Similar to the definitions provided by Reynolds and Wooders (2009), to buy in the ABM environment consists in paying the toll price, and to wait consists in playing the auction (when a player chooses to play the auction, she has to wait for the result). Here, the term "auction" refers to the extended Vickrey auction as defined in the terminology section (Section 1.1). And wining the auction (or wining the auction item) in the context of the HOT facility is to gain access to the HOT facility.

Figure 13 describes the mechanics of the ABM system from the perspective of a player. It also shows how each of the two elements in the strategy set trigger other actions in the system. Nonetheless, those other actions are not part of the two-element set. It is important to notice that the amount of money to be bid is not part of the two-element set. As mentioned in the terminology section (Section 1.1) and proved in Appendix D, the dominant strategy for bidders is to submit a bid amount equal to their true private value, that is, the true amount that they are willing to pay for entering the HOT lane. Therefore, the two-element strategy set assumes that players have submitted in advance their private value (the mechanism for this submission is explained in detail in Chapter 5).

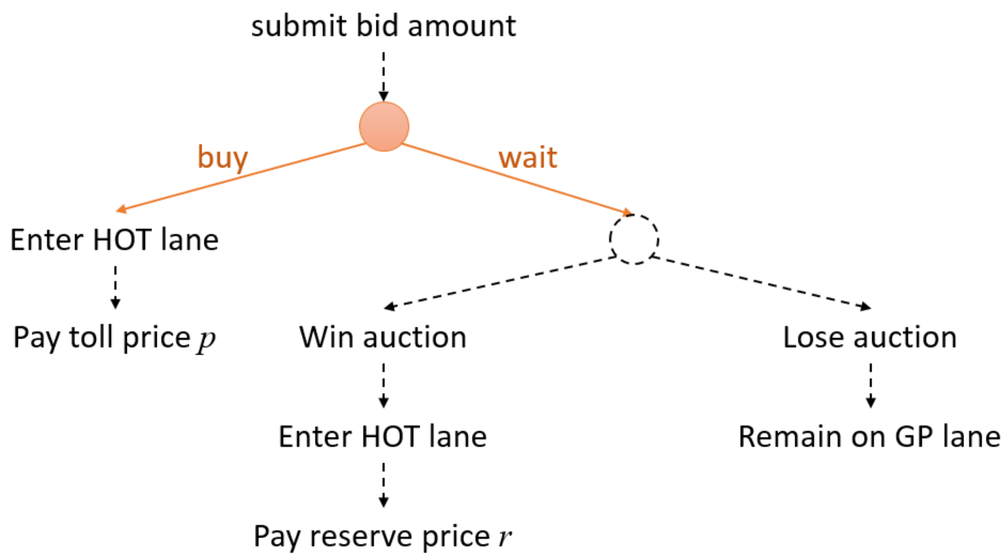


Figure 13. Mechanics of the buyout auction from the perspective of a player, and how the two-strategy set {buy, wait} fits within those mechanics. The term “auction” refers to the extended Vickrey auction as defined in the terminology section (Section 1.1).

As will be mentioned later in Section 5.1 and in Appendix B, Section 6, the step of having drivers to submit a bid amount is not supposed to happen while driving but before the start of the trip. In this way, the so-called “districted driving” is not encouraged.

4.2.3. <u>Payoff functions</u>

Any SOV who buys is granted access to the HOT lane. The payoff function for any player a that chooses to buy is defined later in detail in a further subsection. For the moment, it can be summarized as the utility of the subtraction of her private value minus the toll price.

The payoff function of choosing to wait depends on whether she wins or loses the auction. If she loses is zero. If she wins, it can be summarized as the utility of her private value minus the reserve price.

4.2.4. Limitations of this formulation

The two-element strategy set proposed so far does not facilitate the analysis for determining what the optimal strategy is. Specifically, it does not allow the player to anticipate whether she would win or lose auction if she chooses to wait. Thus, the following complete formulation is provided.

4.3. Final formulation of game

The player set, strategy set, and payoff set for the buyout auction are defined in this section. Also, mechanics surrounding the strategy set as well as assumptions are provided. All notation needed for this formulation is provided in this section.

4.3.1. Player set and assumptions

The player set is composed by advanced SOVs, that is, those single occupant vehicles whose OBU allows to pay the toll as well as to play the auction. Let $\mathbb{A}$ refer to the player set, let a refer to any player in that set, and let $\bar{a}$ refer to the number of elements in the set.

The following eight assumptions are made:

1. Each player a knows that each other player is rational.

2. Each player a knows the total number $\bar{a}$ of players in set $\mathbb{A}$.
3. The game offers at least $\bar{k}$ items where $\bar{k}$ is known by all players in set $\mathbb{A}$ (an item corresponds to the right to enter the HOT lanes).
4. Each player a has a private value v_a indicating the amount that she is willing to pay for entering the HOT lane. Each private value is assumed to be a realization of a random variable which has a known cumulative probability distribution F with support $[\underline{v}, \bar{v}]$. $\underline{v}$ can be as small as zero, and $\bar{v}$ can be as large as needed.
5. Each player a knows her private value v_a, does not know the private value of others, but knows that the other players' values are identically and independently distributed according to F.
6. Prices are continuous and positive. Let $r \in [\underline{v}, \bar{v}]$ refer to the reserve price and let $p \in [r, \bar{v}]$ refer to the toll price.
7. Each player a knows that the utility function assumed for all players is described by the following expression:

$$u(x; \propto) = \begin{cases} \dfrac{1 - e^{-\propto \cdot x}}{\propto}, & \text{if } \propto > 0 \\ x, & \text{if } \propto = 0 \end{cases} \tag{4}$$

where parameter $\propto$ is the level of risk averseness assumed to be equal among all players, and $x = 0$ if player a does not receive any of the $\bar{k}$ items, and x is equal to v_a minus any payment that she is required to make if she receives an item. The discussion at the end of this section argues the validity of assuming this utility function, and assuming that all players have the same value of $\propto$.
8. All players in $\mathbb{A}$ move (buy or wait) simultaneously. This assumption is based on the following two other assumptions:

- Each player a does not know which player chooses to buy or wait because she is not able to identify if a vehicle is indeed a player $a \in \mathbb{A}$ or if it is a vehicle from another category such as HOV, basic SOV, etc. (for an extended description of all categories of vehicles that are not part of the game, see previous section 4.2.1).
- The instance at which the game starts is unknown (Section 5.2 will reveal why this is true in the proposed ABM system).

Assumptions 2 and 3 are strong assumptions. One of the assumptions behind assumption 8 suggests that players in $\mathbb{A}$ are not able to recognize each other. Therefore, the only way assumption 2 would be valid is if any player a receives communication from an "external agent". Assumption 3 also supposes information from an external agent. As Section 5.2 will reveal, even if someone had all traffic information available (as it is the case when using a microsimulation software), it would be difficult to predict the value of $\bar{k}$ and $\bar{a}$. Subsection 4.5.3 will analyze what approach to follow when assumptions 2 and 3 are lifted.

The utility function defined in expression (4) is proposed in this game because it contains the following three desirable properties:

- It describes someone who has risk aversion. Nonetheless, it makes the simplification that such risk aversion is constant and absolute (for a brief definition of functions with constant absolute risk averseness or CARA functions, see the terminology section, Section 1.1, and for a detailed definition see Appendix D).

- Unlike other CARA functions (such as $-e^{-\propto x}$ and $-e^{-\propto x}/\propto$), it makes the two reasonable assumptions of having a zero function value for $x = 0$, and a function value equal to x for $\propto = 0$.
- Unlike the CARA function $-e^{-\propto x}/\propto$, it is a continuous function over $\propto \in [0, \infty)$ thanks to the following property:

$$\lim_{\propto \to 0} \frac{1 - e^{-\propto \cdot x}}{\propto} = x \tag{5}$$

The last two properties can be observed in Figure 14. To assume that the utility function's risk averseness is constant and absolute is a simplification. Arrow (1971, p.96), in his classical work on risk averseness, suggested that people (mostly in insurance and banking scenarios) have decreasing absolute risk aversion (DARA) and increasing relative risk aversion. Equation (4) does not have these two arguably desirable properties but it is used here as a simplification (Reynolds & Wooders 2009 also made this simplification). It is possible that the CARA assumption is valid because the scenario considered here does not involve high investment levels as is often the case in the banking and insurance sectors.

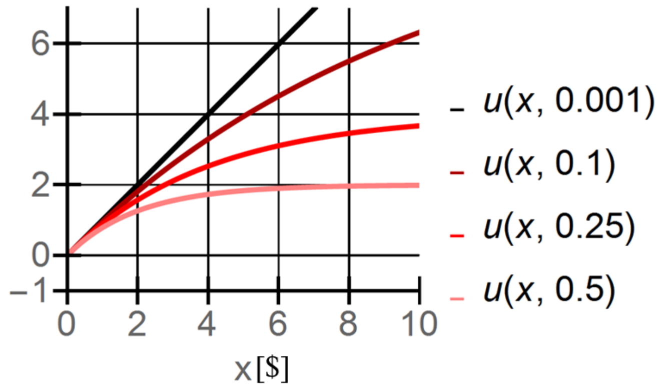


Figure 14. Utility function (chosen for this study) for different levels of risk-averseness.

Whether the assumption that all players have the same level of risk averseness is a strong one is an open question. Since the ABM system has not been implemented, there is no good knowledge about the risk attitudes for this system. Nonetheless, the proposed mathematical model may serve as the basis for more complicated models that consider DARA functions or variable risk aversion.

4.3.2. Strategy set and mechanics of the game

As in Subsection 4.2.2, here the term "to buy" is used to refer to the choice of paying the toll, while the term "to wait" is used to refer to the choice of playing the extended Vickrey auction. But unlike in the formulation of that subsection, this formulation follows the recommendation made by Reynolds and Wooders (2009) of assuming that players make their decision of whether to buy or to wait based on the "cutoff" strategy that they choose. Player a adopts a cut-off strategy c_a when she chooses to buy if her private value v_a is above c_a and she chooses to wait if her v_a is below c_a. Under this strategy, if $v_a = c_a$, then she would be indifferent between both options.

Given that player a would never have a positive utility if she buys with a v_a below p, then the set of all possible cutoff strategies is defined by the range $[p, \bar{v}]$.

Figure 15 presents how a cutoff strategy can be considered as an extension of what one typically does when buying items at a regular store. While in the store, one bases the decision on the price, at a buyout auction, one bases the decision on the cutoff where the cutoff is equal to the price plus a certain (usually small) amount. The small amount takes into account the fact that even if one's private value is greater than the price, it may still be more profitable to wait. As it will be seen in section 4.3.3, the optimal cutoff strategy will not only take into account the price but the other variables such $\bar{k}$, $\bar{a}$ and $\propto$.

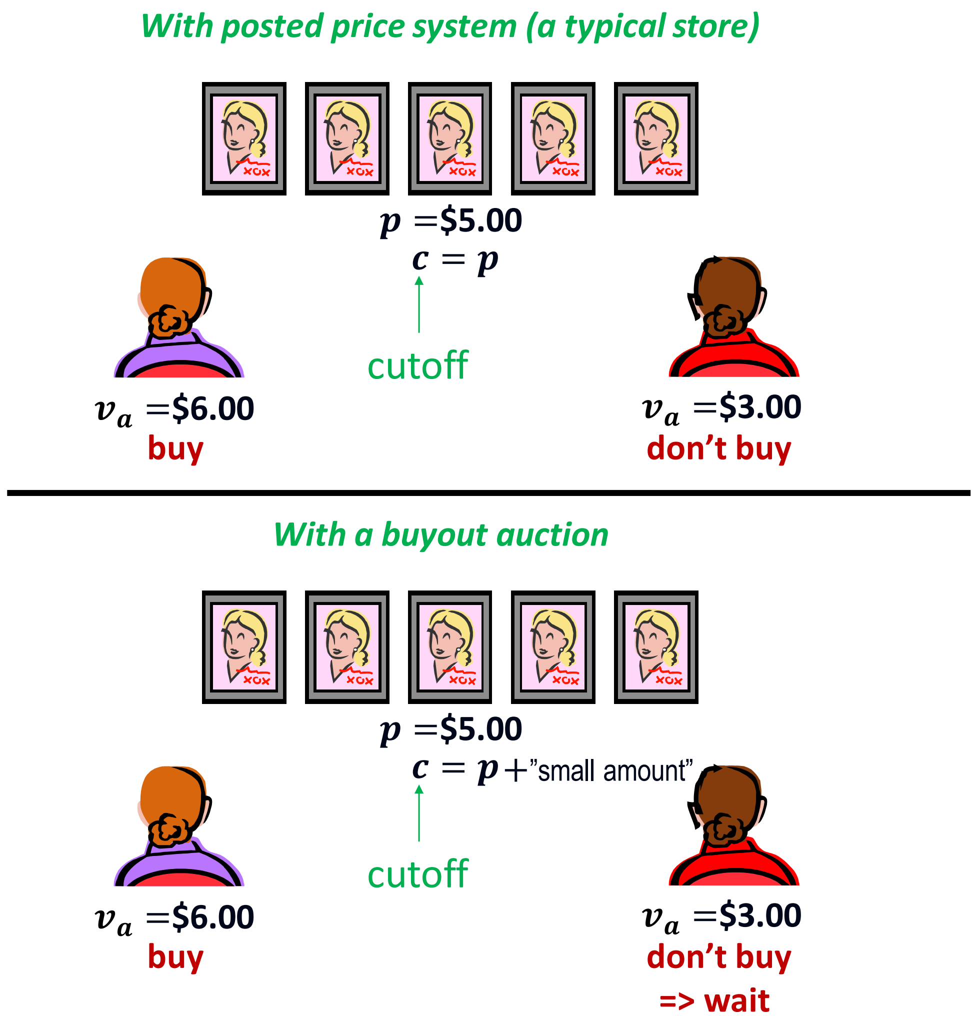


Figure 15. Example of how a cutoff strategy in a buyout auction is also applied at a typical store that sells items using the traditional posted price system with the difference that with a posted price, the cutoff is simply equal to the price. The figure assumes that the auction is an extended Vickrey auction and that the two people shown are just a sample of a greater set of players.

Figure 16 presents the infinite strategy set and how it fits within the mechanics of the ABM system.

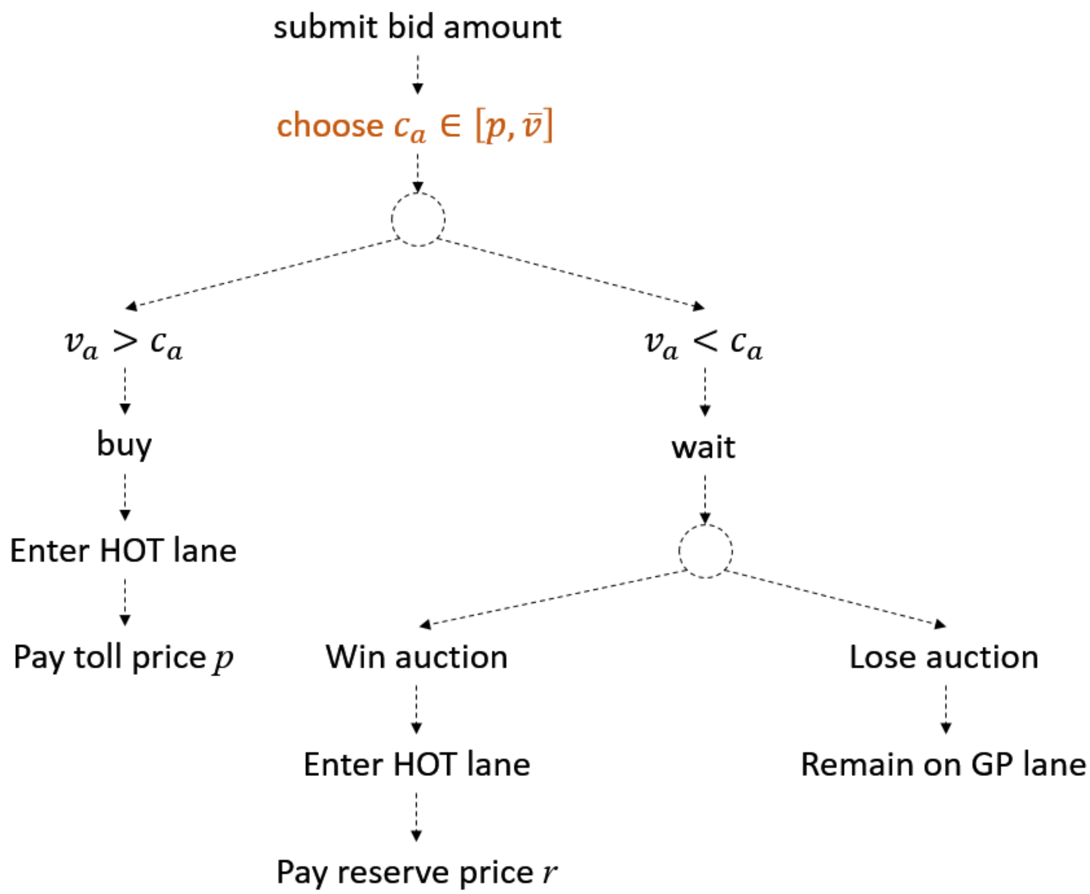


Figure 16. Mechanics of the buyout auction from the perspective of a player, and how the infinite strategy set $[p, \bar{v}]$ fits within those mechanics. The term "auction" refers to the extended Vickrey auction as defined in the terminology section (Section 1.1). Not shown in the figure is the fact that the player is indifferent between to buy or to wait if her private value is equal to the cutoff.

Regarding the mechanics of the game, one important aspect needs to be specified. To guarantee an item to any player who chooses to buy, the following rule needs to be applied. If it were the case that there were no more $\bar{k}$ items to be given to a buyer, the buyer would still obtain an item from another source, that is, an item drawn from a following time period.

4.3.3. *Payoff functions*

As stated previously, any SOV who buys is granted access to the HOT lane. Therefore, equation (4) constitutes the payoff function for any player a that chooses to buy, where x is equal to the private value minus the toll price, that is, $u(v_a - p; \propto)$.

Now, if player a chooses to wait, since she can sometimes win and sometimes lose, her payoff function is actually an expectation function. It can be obtained from the following rationale.

Assume player $a \in \mathbb{A}$ wants to know her best cutoff strategy $c_a \in [p, \bar{v}]$. Assume that all her rivals in set $\mathbb{A}$ play the same cutoff strategy $c_{-a} \in [p, \bar{v}]$ not necessarily equal to c_a. If player a chooses to wait, she would win the auction if the two events happen at the same time:

(1) The cutoff c_{-a} is greater than the $\bar{k}^{\text{th}}$ highest private value among the $\bar{a} - 1$ rivals (otherwise, player a would be left with no items to acquire in an auction).

(2) Her private value v_a is greater than the $\bar{k}^{\text{th}}$ highest private value among the $\bar{a} - 1$ rivals.

Notice that the key is to look at how player a's private value compares to the player with the $\bar{k}^{\text{th}}$ highest private value. And notice that the $\bar{k}^{\text{th}}$ highest private value is a random variable. Let $Y_{i:n}$ denote the i^{th} order statistic of a sample of size n. Then, the $\bar{k}^{\text{th}}$ highest private value is $Y_{\bar{k}:\bar{a}-1}$, that is, the $\bar{k}^{\text{th}}$ order statistic of a sample of size $\bar{a} - 1$ rivals.

Using the properties of order statistics, the probability of the first event is equal to the following expression:

$$P\{Y_{\bar{k}:\bar{a}-1} < c_{-a}\} = \sum_{k=\bar{a}-\bar{k}}^{\bar{a}-1} \binom{\bar{a}-1}{k} \cdot [F(c_{-a})]^{k} \cdot [1 - F(c_{-a})]^{\bar{a}-1-k} \tag{6}$$

The intersection of the two events is obtained by considering the minimum between c_{-a} and v_a. The probability G of the two events is therefore as follows:

$$G\big(\min\{v_a, c_{-a}\}\,; \bar{k}, \bar{a}\big) = P\{Y_{\bar{k}:\bar{a}-1} < \min\{v_a, c_{-a}\}\} =$$

$$\sum_{k=\bar{a}-\bar{k}}^{\bar{a}-1} \binom{\bar{a}-1}{k} \cdot [F(\min\{v_a, c_{-a}\})]^{k} \cdot [1 - F(\min\{v_a, c_{-a}\})]^{\bar{a}-1-k} \tag{7}$$

Let U be the expected utility for player a if choosing to wait. Using equation (7), such utility is the following:

$$U\big(v_a, c_{-a}; \bar{k}, \bar{a}, \propto, 0\big) = \int_{y=\underline{v}}^{y=\min(v_a, c_{-a})} u(v_a - y; \propto)\mathrm{d}G\big(y; \bar{k}, \bar{a}\big) \tag{8}$$

The null parameter in the above equation indicates that that definition assumes a reserve price equal to zero. If the reserve price is positive, then the expected utility is the following:

$$U\big(v_a, c_{-a}; \bar{k}, \bar{a}, \propto, r\big)$$

$$= \int_{y=r}^{y=\min(v_a, c_{-a})} u(v_a - y; \propto)\mathrm{d}G\big(y; \bar{k}, \bar{a}\big)$$

$$+ u(v_a - r; \propto) \int_{y=\underline{v}}^{y=r} \mathrm{d}G\big(y; \bar{k}, \bar{a}\big)$$

$$U\left(v_a, c_{-a}; \bar{k}, \bar{a}, \propto, r\right) = \int_{y=r}^{y=\min(v_a, c_{-a})} u(v_a - y; \propto)\mathrm{d}G\left(y; \bar{k}, \bar{a}\right) + u(v_a - r; \propto)G\left(r; \bar{k}, \bar{a}\right) \tag{9}$$

4.4. Final formulation of a simpler game

In this section, the formulation of the simpler buyout-raffle game is presented. Here, as is the case with the buyout auction, the player set remains being $\mathbb{A}$. Nonetheless, the OBUs of this advanced SOVs allow them to play extended raffles instead of extended Vickrey auctions. Thus, the system is no longer an ABM but a raffle-based metering system. The assumptions are the same as those presented in Subsection 4.3.1.

The strategy set remains being the infinite set $[p, \bar{v}]$. The mechanics are the same with the difference that there is no submission of bid amounts at the start, and that there is a raffle instead of a lottery. Figure 17 presents the resulting mechanics.

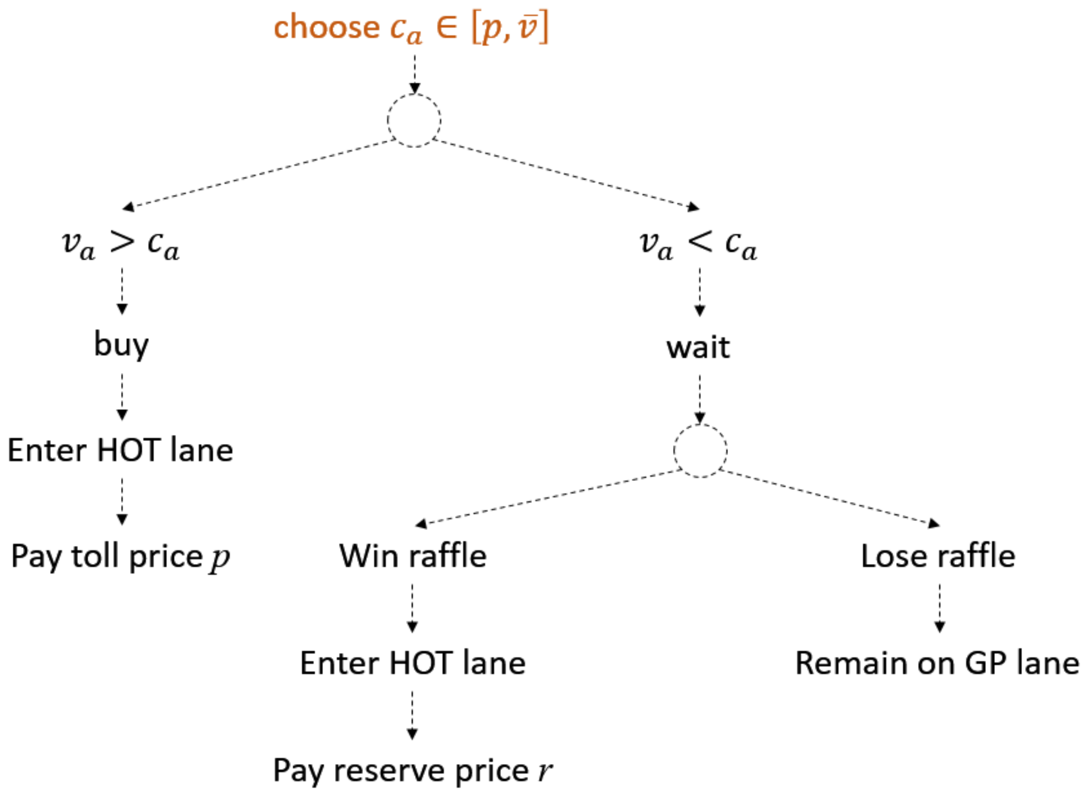


Figure 17. Mechanics of the buyout raffle from the perspective of a player, and how the infinite strategy set $[0, c]$ fits within those mechanics. The term "raffle" was defined in the terminology section (Section 1.1). Not shown in the figure is the fact that the player is indifferent between to buy or to wait if her private value is equal to the cutoff.

Regarding the payoff functions, if player a chooses to buy, the payoff function continues being equal to equation (4). But if she chooses to wait, the payoff changes significantly. The expectation function can be obtained from the following rationale, as proposed by Olarte and Haghani (2016).

Assume player $a \in \mathbb{A}$ wants to know her optimal cutoff strategy $c_a \in [p, \bar{v}]$. Assume that all her rivals in set $\mathbb{A}$ play the same cutoff strategy $c_{-a} \in [p, \bar{v}]$ not necessarily equal to c_a. If player a chooses to wait, she could be selected as one of the winners of the raffle at any of the five mutually exclusive events described below:

(1) All her rivals wait. Under this event, player a could be selected as one of the $\bar{k}$ winners from a pool of $\bar{a}$ players that wait.

(2) All her rivals wait except for one rival who buys. Under this event, player a could be selected as one of the $\bar{k} - 1$ winners from a pool of $\bar{a} - 1$ players that wait.

Before continuing with the list of five mutually exclusive events, notice that in the second event, the rival who chooses to buy must be the rival with the highest private value. If instead, it had been the case that the rival with the 2nd highest (or the 3rd highest) private value had chosen to buy, then it should had had also been the case that the rival with the highest value had preferred to buy due to the fact that all rivals are playing the same cutoff strategy c_{-a}. Therefore, if just one rival chooses to buy, then it must be the one with the highest value and no one else. Thus, the second exclusive event can be restated as:

(2) $\bar{a} - 2$ rivals wait, and the rival with the highest private value buys. Under this event, player a could be selected as one of the $\bar{k} - 1$ winners from a pool of $\bar{a} - 1$ players that wait.

Notice that if $\bar{a} - 2$ rivals wait, and given that all the rivals play the same strategy c_{-a}, then it should be equivalent to say that the rival with the 2nd highest private value buys. Thus, the second event can be restated once again as:

(2) The rival with the 2nd highest private value waits, and the rival with the highest private value buys. Under this event, player a could be selected as one of the $\bar{k} - 1$ winners from a pool of $\bar{a} - 1$ players that wait.

As it will be seen later, the last two restatements will be useful to express the events in mathematical form. The remaining mutually exclusive events are the following:

(3) The rival with the 3rd highest private value waits, and the rival with the 2nd highest private value buys. Under this event, player a could be selected as one of the $\bar{k} - 2$ winners from a pool of $\bar{a} - 2$ players that wait.

(4) The rival with the $(d + 1)^{\text{th}}$ highest private value waits, and the rival with the d^{th} highest private value buys. Under this event, player a could be selected as one of the $\bar{k} - d$ winners from a pool of $\bar{a} - d$ players that wait, where $d \in [3, \bar{k} - 2]$.

(5) The rival with the $\bar{k}^{\text{th}}$ highest private value waits, and the rival with the $(\bar{k} - 1)^{\text{th}}$ highest private value buys. Under this event, player a could be selected as the only winner from a pool of $\bar{a} - (\bar{k} - 1)$ players that wait.

Notice that if it were the case that the rival with the $\bar{k}^{\text{th}}$ highest private value buys, then the raffle would have no items and player a could not become a winner.

Each of the five mutually exclusive events described above have an independent probability of occurrence. Thus, the overall probability that player a becomes a winner is equal to the addition of the probabilities of the five events. Let G_{R} be such probability. Using once again the notation $Y_{i:n}$ to refer to the i^{th} order statistic of a sample of size n, and noting that the probability $P(\{Y_{i:n} < y\} \cap \{Y_{i+1:n} > y\})$ is equal to $\binom{n}{i} \cdot F(y)^i \cdot [1 - F(y)]^{n-i}$, then the probability G_{R} is equal to:

$$
\begin{aligned}
G_{\mathrm{R}}\left(c_{-a};\bar{k},\bar{a}\right) \quad &= \frac{\bar{k}}{\bar{a}}\cdot P\{Y_{\bar{a}-1:\bar{a}-1} < c_{-a}\} \\
&+ \frac{\bar{k}-1}{\bar{a}-1}\cdot P(\{Y_{\bar{a}-2:\bar{a}-1} < c_{-a}\}\cap\{Y_{\bar{a}-1:\bar{a}-1} > c_{-a}\}) \\
&+ \frac{\bar{k}-2}{\bar{a}-2}\cdot P(\{Y_{\bar{a}-3:\bar{a}-1} < c_{-a}\}\cap\{Y_{\bar{a}-2:\bar{a}-1} > c_{-a}\}) + \cdots \\
&+ \frac{\bar{k}-d}{\bar{a}-d}\cdot P\left(\{Y_{\bar{a}-(d+1):\bar{a}-1} < c_{-a}\}\cap\{Y_{\bar{a}-d:\bar{a}-1} > c_{-a}\}\right) + \\
&+ \frac{1}{\bar{a}-\bar{k}+1}\cdot P(\{Y_{\bar{a}-\bar{k}:\bar{a}-1} < c_{-a}\}\cap\{Y_{\bar{a}-\bar{k}+1:\bar{a}-1} > c_{-a}\})
\end{aligned}
$$

$$
\begin{aligned}
G_{\mathrm{R}}\left(c_{-a};\bar{k},\bar{a}\right) \quad &= \frac{\bar{k}}{\bar{a}}\cdot F(c_{-a})^{\bar{a}-1} + \cdots \\
&+ \frac{\bar{k}-d}{\bar{a}-d}\cdot \binom{\bar{a}-1}{d}\cdot F(c_{-a})^{\bar{a}-d-1}\cdot[1-F(c_{-a})]^{d} + \cdots \\
&+ \frac{1}{\bar{a}-\bar{k}+1}\cdot \binom{\bar{a}-1}{\bar{k}-1}\cdot F(c_{-a})^{\bar{a}-\bar{k}}\cdot[1-F(c_{-a})]^{\bar{k}-1}
\end{aligned}
$$

$$
\begin{aligned}
G_{\mathrm{R}}\left(c_{-a};\bar{k},\bar{a}\right) \quad &= \frac{\bar{k}}{\bar{a}}\cdot F(c_{-a})^{\bar{a}-1} \\
&+ \sum_{d=1}^{\bar{k}-1}\frac{\bar{k}-d}{\bar{a}-d}\cdot \binom{\bar{a}-1}{d}\cdot F(c_{-a})^{\bar{a}-d-1}\cdot[1-F(c_{-a})]^{d}
\end{aligned} \tag{10}
$$

Thus, probability G_{R} can be defined as in equation (10). It can also be defined more succinctly as in equation (11), but giving special attention to the case in which c_{-a} is equal to $\bar{v}$:

$$
G_{\mathrm{R}}\left(c_{-a};\bar{k},\bar{a}\right) = \begin{cases} \sum_{d=0}^{\bar{k}-1}\frac{\bar{k}-d}{\bar{a}-d}\cdot \binom{\bar{a}-1}{d}\cdot F(c_{-a})^{\bar{a}-d-1}\,[1-F(c_{-a})]^{d}, & \text{if } c_{-a} < \bar{v} \\ \frac{\bar{k}}{\bar{a}}, & \text{if } c_{-a} = \bar{v} \end{cases} \tag{11}
$$

$G_{\mathrm{R}}\left(c_{-a};\bar{k},\bar{a}\right)$ is equal to 0 at $c_{-a} = 0$, and is equal to $\bar{k}/\bar{a}$ at $c_{-a} = \bar{v}$. And it has a non-negative slope. Using equation (11), the expected profit of waiting is as follows:

$$
U_{R}\left(v_{a}, c_{-a};\bar{k},\bar{a},\propto,r\right) = u(v_{a}-r;\propto)\cdot G_{R}\left(c_{-a};\bar{k},\bar{a}\right) \tag{12}
$$

4.5. Optimal strategy

In this section, the optimal strategy is presented for the buyout raffle and for the buyout auction. Recall that while the buyout raffle involved an extended raffle, the buyout auction involves an extended Vickrey auction.

4.5.1. Buyout raffle

If all players in set $\mathbb{A}$ choose to play the same cutoff c^*, then there is a unique symmetric Bayesian Nash equilibrium if and only if for each player $a \in \mathbb{A}$, these two conditions hold:

1. if $v_a \in [\underline{v}, c^*)$, then the payoff of buying is less than the payoff of waiting, and
2. if $v_a \in (c^*, \bar{v}]$, then the payoff of buying is greater than the payoff of waiting, where the payoffs are defined by equations (4) and (12).

A unique cutoff c^* would then exist where the two payoff functions intersect as defined by the following expression:

$$u(c^* - p; \propto) = U_{\mathrm{R}}\left(c^*, c^*; r, \propto, \bar{k}, \bar{a}\right) \tag{13}$$

Figure 18 presents three examples where function u and U_{R} meet for different probability distributions F: from one where every private value v_a is equally distributed among the population to one where most of the population has a lower private value. In the left column, a uniform distribution is used. As shown in Figure 18(a1), it is clearly not a good fit of the data obtained from Minnesota. Nonetheless, it is used here because it allows observing the extreme scenario in which private values are equally distributed. The parameter “maximum” used for this distribution corresponds to the maximum value that the HOT facility is allowed to charge.

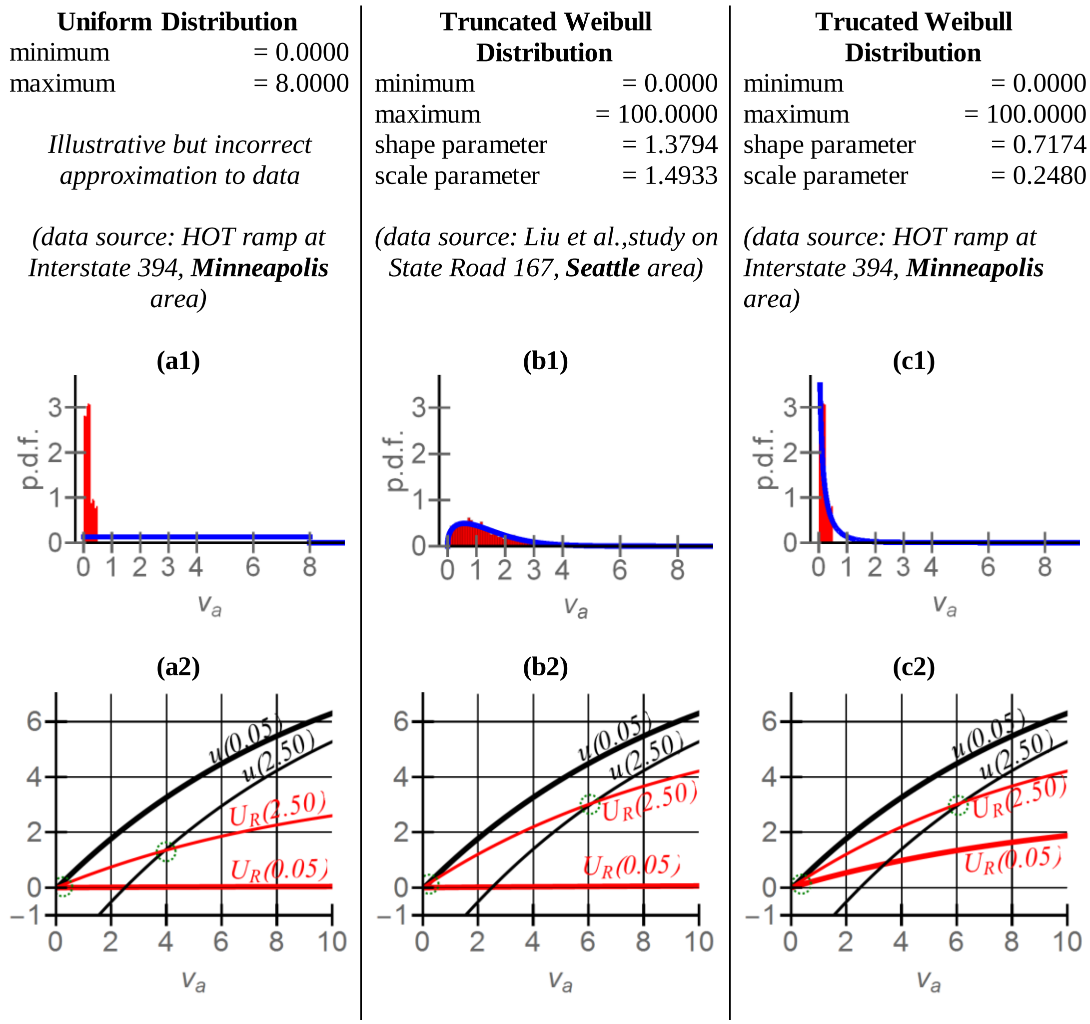


Figure 18. For different probability distributions, and represented by a green dashed circle: intersection of the payoff of buying and the payoff of waiting. Figure shows that when the price increases, so does the private value where the intersection occurs, also known as the optimal strategy c^*. Input values: $\bar{k} = 2$ items, $\bar{a} = 3$ players, $r = 0.00$ dollars, and $\propto = 0.10$.

The histogram from Seattle is an estimation that was made by Liu et al. (2011, fig.3) from the HOT facility on the Washington State Route 167. Each v_a in Figure 18(b1) corresponds to what users are willing to pay for 15 minutes of travel savings. The histogram from Minneapolis was obtained following the procedure described later in Subsection 6.2.4 which relies on historical toll prices and a microsimulation model. It is important to note that the resulting curve shown in Figure 18(c1) is more skewed to the left that it should really is due to the fact that the period of analysis only had toll prices no greater than 50 cents. But for the purpose of this analysis is acceptable because it illustrates the extreme case in which most of the population has a lower private value.

Figure 18 shows that when the toll price p increases from 5 cents to 2.50 dollars, the optimal strategy increases from $c^* \approx 0$ to $c^* \approx 4$ or $c^* \approx 6$ depending on the probability distribution.

Broadly speaking, this research found that there is always a unique symmetric equilibrium cutoff as long as the toll price falls within two thresholds. This research introduces the concepts of “maximum proper price” and “minimum proper price” to refer to those thresholds. The concepts themselves are independent of the CARA utility function that is used.

Definition 1. Consider the buyout extended raffle which offers at least $\bar{k}$ items and has a reserve price $r \geq 0$. Suppose $\bar{a} > \bar{k}$ players with any CARA utility function u_{CARA}, with level of risk aversion $\propto > 0$, and with private values v_a no greater than $\bar{v}$. Then, the *maximum proper price* $\bar{p}$ is implicitly defined by equation (14):

$$u_{CARA}(\bar{v}-\bar{p};\propto)=\frac{\bar{k}}{\bar{a}}u_{CARA}(\bar{v}-r;\propto) \tag{14}$$

■

The explicit calculation of $\bar{p}$ will be provided later after considering a specific CARA function. Notice that the above definition not only it is not restricted to the utility function defined in equation (4), but also, it does not depend on the cumulative distribution function F. As it will be shown later (in Proposition 1), if the price p goes above $\bar{p}$, there would be a unique symmetric equilibrium, but one in which no player would choose the buyout option.

Definition 2. Consider the buyout extended raffle which offers at least $\bar{k}$ items and has a reserve price $r \geq 0$. Suppose $\bar{a} > \bar{k}$ players with any CARA utility function u_{CARA}, with level of risk aversion $\propto > 0$, and with private values v_a no greater than $\bar{v}$. Then, the *minimum proper price* $\underline{p}$ for a certain private value $\tilde{v}$ is the minimum price less than $\tilde{v}$ such that:

$$\frac{\partial}{\partial v_a}u\left(v_a-\underline{p};\propto\right)>\frac{\partial}{\partial v_a}U_R\left(v_a,v_a;\bar{k},\bar{a},\propto,r\right) \qquad \forall v_a\in\left[\underline{p},\tilde{v}\right) \tag{15}$$

■

In other words, the minimum proper price $\underline{p}$ is the minimum price p that guarantees that the slope of the payoff of buying is always greater than the payoff of waiting (when v_a is equal to the cutoff c_{-a} of the rivals) for all private values greater than $\underline{p}$

and less than a given limit. As will be seen, the most common values for used for $\tilde{v}$ are $\bar{v}$ and $\bar{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r)$. Without using any specific CARA function, plotting the payoff of buying versus the payoff of waiting for different p values seems to be the only method for estimating $\underline{p}$. This research has not found a simpler method even for estimating an upper bound. Sometimes a $\underline{p}$ lower than $\tilde{v}$ cannot be found. In those cases, it is said that a $\underline{p}$ for that $\tilde{v}$ does not exist.

The following two lemmas allow calculating $\bar{p}$ explicitly when the CARA function is equal to the utility function u as defined in equation (4). Their proofs appear in Appendix D.

Lemma 1. For a given level of risk averseness $\propto$, the utility function u, defined by equation (4), is always less than $1/\propto$. ■

The above lemma allows proving the following two lemmas.

Lemma 2. Consider the buyout extended raffle which offers at least $\bar{k}$ items and has a reserve price $r \geq 0$. Suppose $\bar{a} > \bar{k}$ players with utility function u as defined by equation (4), with level of risk aversion $\propto > 0$, and with private values no greater than $\bar{v}$. Then, the maximum proper price $\bar{p}$ can be explicitly calculated by the following equation:

$$\bar{p}(\bar{v};\bar{k},\bar{a},\propto,r) = \bar{v} + \frac{1}{\propto} log\left[1 - \propto \frac{\bar{k}}{\bar{a}} u(\bar{v}-r;\propto)\right] \tag{16}$$

■

Notice that thanks to Lemma 1, equation (15), the value inside the natural logarithmic function is always less than one. Therefore, the logarithmic function value is negative and $\bar{p}$ is greater than $\bar{v}$. Equation (16) is obtained by replacing u_{CARA} with the utility function u defined in equation (4).

Using the formulation of the game provided in Section 4.4, the following proposition states in a formal manner that c^* exists, that it is unique, and that it has certain properties (such as the behavior just described against p). Its proof is presented in Appendix D.

Proposition 1. Consider the buyout extended raffle which has a buy price p, offers at least $\bar{k}$ items, has a reserve price $r \geq 0$. Suppose $\bar{a} > \bar{k}$ players with utility function defined by equation (4), with level of risk aversion $\propto > 0$, and with private values drawn from a cumulative distribution function F with range $[\underline{v}, \bar{v}]$. Suppose that the minimum proper price $\underline{p}(\bar{v};\bar{k},\bar{a},\propto,r)$ exists, and that $\underline{p}(\bar{v};\bar{k},\bar{a},\propto,r) < p$.

(i) If $p < \bar{p}(\bar{v};\bar{k},\bar{a},\propto,r)$,

(a) then there is a value c^* defined by equation (31),

$$u(c^* - p;\propto) = U_R(c^*, c^*; r, \propto, \bar{k}, \bar{a}) \tag{17}$$

(b) where such c^* has the following properties:

(1) It belongs to the range $(p, \bar{v})$.

(2) It defines a unique equilibrium cutoff (which is symmetric, and inefficient).

(3) It is increasing in p.

(4) It is increasing in $\bar{k}$.

(5) It is decreasing in $\bar{a}$.

(6) It is decreasing in r.

(7) It is decreasing in $\propto$.

(ii) If $p > \bar{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r)$, then there is a value c^* equal to $\bar{v}$, which defines an equilibrium cutoff that is unique, symmetric, and inefficient (and where the price is never accepted by the players). ■

In the two cases described in Proposition 1, the equilibrium is inefficient because whenever there are players choosing the raffle, there is the possibility of having players that acquire items while others with higher private value do not. Also notice that if the price is less than $\bar{p}$ and $\underline{p}$ does not exist, then the equilibrium is not guaranteed to be unique. Finally, notice that Proposition 1 does not state that there is an equilibrium for risk neutral players. Although it is very likely that an equilibrium exists when players are risk neutral, a concept different from minimum proper price would be needed. Appendix D presents a formal proof of Proposition 1.

Proposition 1 suggests that players should have knowledge of many of the variables involved in the game. Otherwise, they would not be able to calculate the optimal strategy. Subsection 4.5.3 looks at to what extent this suggestion is true by looking at how sensitive the optimal strategy is to those variables.

The following corollary simplifies one of the conditions of Proposition when the toll price is less than $\bar{p}(\bar{v};\bar{k},\bar{a},\propto,r)$. Its proof is almost identical to Propostiion 0 and therefore, it is not included in this dissertation.

Corollary 1. Consider the buyout extended raffle which has a buy price p, offers at least $\bar{k}$ items, has a reserve price $r \geq 0$. Suppose $\bar{a} > \bar{k}$ players with utility function defined by equation (4), with level of risk aversion $\propto > 0$, and with private values drawn from a cumulative distribution function F with range $[\underline{v},\bar{v}]$. Suppose that the minimum proper price $\underline{p}(\bar{v};\bar{k},\bar{a},\propto,r)$ exists, and that $\underline{p}(\bar{v};\bar{k},\bar{a},\propto,r) < p$.

(i) If $p < \bar{p}(\bar{v};\bar{k},\bar{a},\propto,r)$,

(a) then there is a value c^* defined by equation (31),

$$u(c^* - p;\propto) = U_R(c^*,c^*;r,\propto,\bar{k},\bar{a}) \tag{18}$$

(b) where such c^* has the following properties:

(1) It belongs to the range $(p,\bar{v})$.

(2) It defines a unique equilibrium cutoff (which is symmetric, and inefficient).

(3) It is increasing in p.

(4) It is increasing in $\bar{k}$.

(5) It is decreasing in $\bar{a}$.

(6) It is decreasing in r.

(7) It is decreasing in $\propto$.

If $p > \bar{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r)$, then there is a value c^* equal to $\bar{v}$, which defines an equilibrium cutoff that is unique, symmetric, and inefficient (and where the price is never accepted by the players). ■

The goal of being able to explain the optimal strategy to the general public could be reached by guaranteeing that in the RBM system, the optimal strategy only depends on the price (perhaps because the other variables do not change significantly over time or because they can be anticipated before the start of the trip). If such were the case, the player's decision process could follow the one proposed by Figure 19, which was first presented by Olarte and Haghani (2013). It assumes that the player knows how to calculate the optimal strategy using equation (13) or a simplification of such equation. In the example, the player has a private value $v_a = 8.50$ dollars. Using that value, she will be able to obtain a toll price that will be referred as the "trigger price". Thus the decision process would consist simply in following this rule: Always choose to buy unless the toll goes over the trigger price. This rule is the adaptation of the rule explained in Figure 15 to the case in which the cutoff c is in fact the optimal c^*. Notice that if the HOT facility relies on discrete prices such is the case in Minneapolis, the curve could actually be converted to a simple table.

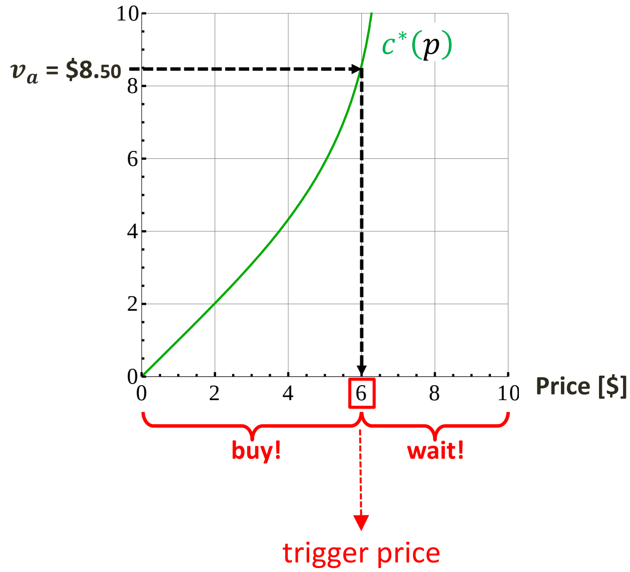


Figure 19. Example of how an advanced SOV, with private value $v_a = 8.50$ dollars, would decide between buying or waiting using an optimal cutoff c^*. The driver always chooses to buy unless the price goes above the trigger price of 6.00 dollars. After the trigger price, the driver maximizes her utility by waiting (participating in the raffle). Example taken from Olarte and Haghani (2013). Notice that if HOT facility had discrete prices, the figure could be replaced by a (perhaps short) table.

4.5.2. Buyout auction

Almost identical to the buyout raffle, in a buyout auction, if all players in set $\mathbb{A}$ choose to adopt the same cutoff c^*, then there is a unique symmetric Bayesian Nash equilibrium if and only if for each player $a \in \mathbb{A}$ the same two conditions hold:

1. if $v_a \in [\underline{v}, c^*)$, then the payoff of buying is less than the payoff of waiting, and
2. if $v_a \in (c^*, \bar{v}]$, then the payoff of buying is greater than the payoff of waiting, where the payoffs are defined by equations (4) and (9).

The only difference with the conditions of the buyout raffle is that the payoff function of waiting is defined by equation (9) instead of equation (12).

Like in the buyout raffle, the unique cutoff c^* would fall where the two payoff functions intersect and this cutoff would be defined by the following expression, which is analogous to equation (13):

$$u(c^* - p; \propto) = U\left(c^*, c^*; r, \propto, \bar{k}, \bar{a}\right) \tag{19}$$

Figure 20 presents three examples where function u and U meet for the same three probability distributions F. Like with Figure 18, Figure 20 shows that the optimal cutoff c^* increases with the toll p. Nonetheless, it shows the importance of the concept the certainty equivalent payment δ. In the buyout raffle, whether all players choose to wait or not depended on whether the price p was greater or less than $\bar{v}$. But in the buyout auction, the price p needs to be compared against the certainty equivalent payment $\delta(\bar{v})$ and not $\bar{v}$, where $\delta(\bar{v})$ can be much lower than δ. In Figure 20 (b2) all players choose to wait when $p = 2.50$ dollars because such price is greater than $\delta(\bar{v} = 100) \approx 0.85$ dollars, where the calculation of $\delta(\bar{v})$ is explained below. In Figure 20 (c2), the same happens because $\delta(\bar{v} = 100) \approx 0.12$ dollars. It is important to note that the values of δ are given in an approximate manner due to the difficulties of its computation.

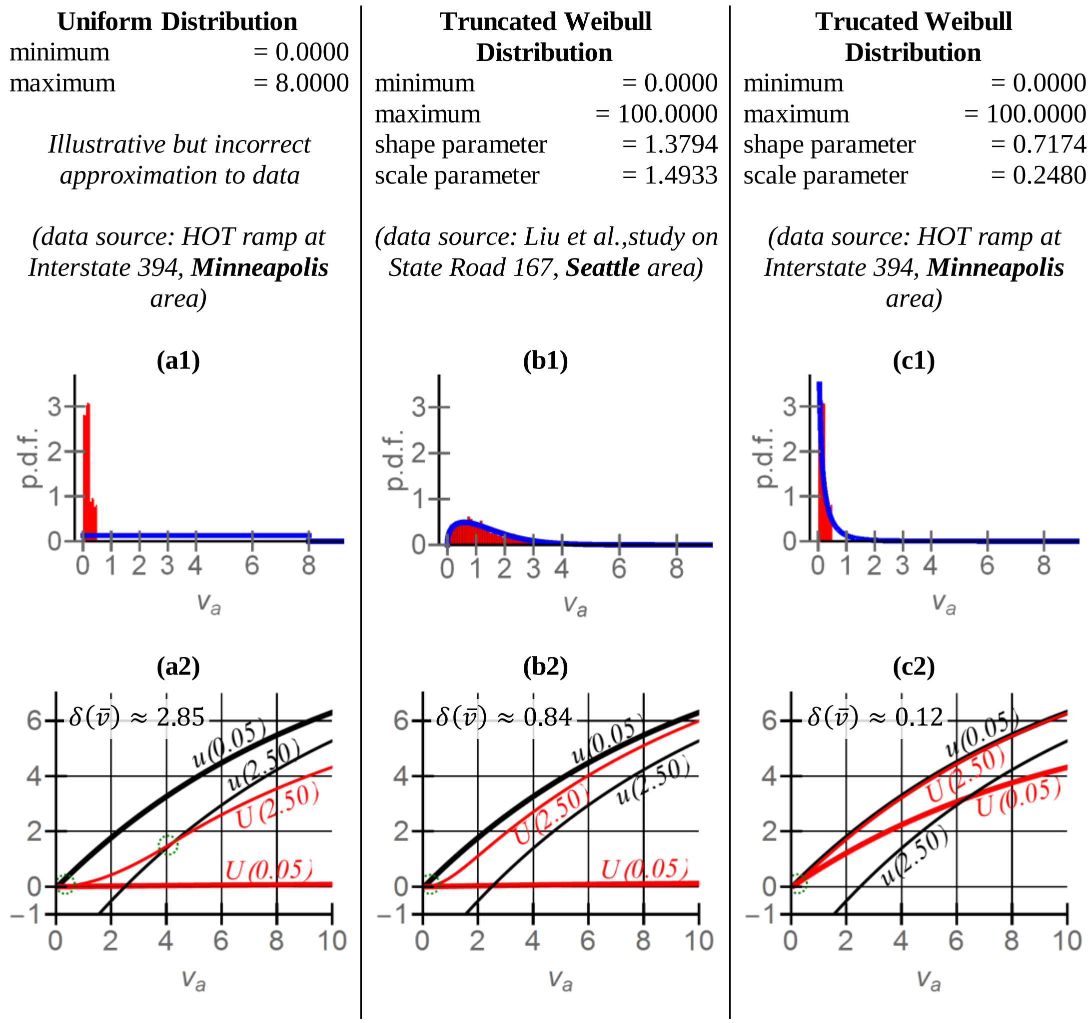


Figure 20. For different probability distributions, and represented by a green dashed circle: the optimal strategy c^*. Figure (a2) shows that when the price increases to 2.50 dollars, so does the cutoff c^*. Figure (b2) and (c2) show that a rise to $p = 2.50$ dollars does not allow the two payoffs to intersect due to the fact that $p > \delta(\bar{v})$. Input values: $\bar{k} = 2$ items, $\bar{a} = 3$ players, $r = 0.00$ dollars, and $\propto = 0.10$.

While in the buyout raffle, it is recommended that the toll price p be greater than a certain threshold (denoted by $\underline{p}$, and referred as minimum proper price), in the buyout auction such verification is not necessary. In the buyout auction, the equilibrium is always guaranteed to exist and to be unique.

But as in the buyout raffle, it is interesting to see if the price is below a certain threshold in order to find out whether at equilibrium nobody would accept the buy option. Here, the threshold that will be used is not the value $\bar{p}$ but the function δ when evaluated at $\bar{v}$. δ refers to certainty equivalent payment and is a concept used by Reynolds and Wooders (2009) for one of the types of auctions that they analyzed. As mentioned in the terminology section (Section 1.1), δ has also a practical meaning. It is the amount of money that a player with a certain $\propto$ and v_a would be willing to pay in order to acquire an item instead of taking the chance of acquiring it after playing and winning a lottery (such as an auction). The concept does not rely on an any specific CARA (or risk-averse) function. And unlike δ, it does not rely on any specific auction (or similar types of lotteries). Nonetheless, a formal definition of the concept is presented here for the CARA function u defined in equation (4) and for the extended Vickrey auction.

Definition 3. Consider an extended Vickrey auction which offers at least $\bar{k}$ items and has a reserve price $r \geq 0$. Suppose $\bar{a} > \bar{k}$ players with any CARA utility function u_{CARA}, with level of risk aversion $\propto > 0$, and with private values no greater than $\bar{v}$. Then, the *certainty equivalent payment* δ of a player with a private value equal to v_a is implicitly defined by equation (20).

$$u\big(v_a - \delta\big(v_a; \bar{k}, \bar{a}, \propto, r\big); \propto\big) = E\left[u\left(v_a - \max(r, \boldsymbol{Y})\,; \propto\right) | \boldsymbol{Y} \leq v_a\right] \tag{20}$$

■

The certainty equivalent payment δ presents the following six properties. Again, the notation δ assumes hereafter that the utility function is the one defined by equation (4) and that the lottery is the extended Vickrey auction.

Lemma 3. If bidders are risk neutral, that is, $\propto = 0$, then the certainty equivalent payment satisfies

$\delta(v_a; \bar{k}, \bar{a}, 0, r) = E[\max\{r, \boldsymbol{Y}\} | \boldsymbol{Y} \leq v_a]$

Lemma 4. If bidders are constant absolute risk averse, that is, $\propto > 0$, then the certainty equivalent payment δ satisfies

$$e^{\propto \cdot \delta(v_a; \bar{k}, \bar{a}, \propto, r)} = \frac{1}{G(y; \bar{k}, \bar{a})} \int_{\underline{v}}^{v} e^{\propto \cdot y} \, \mathrm{d}G(y; \bar{k}, \bar{a})$$

Lemma 5. $\delta(v_a; \bar{k}, \bar{a}, \propto, r)$ is less than v_a if $v_a > r$. $\delta(v_a; \bar{k}, \bar{a}, \propto, r)$ is equal to v_a if $v_a = r$.

Lemma 6. $\delta(v_a; \bar{k}, \bar{a}, \propto, r)$ is increasing in v since its derivative is positive and can be defined as follows**:**

$$\frac{\partial}{\partial v_a} \delta(v_a; \bar{k}, \bar{a}, \propto, r) = \frac{G'(v_a; \bar{k}, \bar{a})}{G(v_a; \bar{k}, \bar{a})} \frac{u(v_a - \delta(v_a; \bar{k}, \bar{a}, \propto, r); \propto)}{u'(v_a - \delta(v_a; \bar{k}, \bar{a}, \propto, r); \propto)}$$

where, for a function f with an independent variable x and a set $\mathbb{P}$ of parameters, the notation $f'(x; \mathbb{P})$ refers to $\frac{\partial}{\partial x} f(x; \mathbb{P})$ (Thus, $u'(v_a - \delta(v_a; \bar{k}, \bar{a}, \propto, r); \propto)$ is different from $\frac{\partial}{\partial v_a} \mathrm{u}(v_a - \delta(v_a; \bar{k}, \bar{a}, \propto, r); \propto)$).

Lemma 7. $\delta(v_a; \bar{k}, \bar{a}, \propto, r)$ is increasing in $\propto$.

Lemma 8. The expectation function of waiting can be defined as follows:

$U\left(v_a, c_{-a}; \bar{k}, \bar{a}, \propto, r\right)$

$$= u\left(v_a - \delta\left(\min(v_a, c_{-a})\,; \bar{k}, \bar{a}, \propto, r\right); \propto\right) \cdot G\left(\min(v_a, c_{-a})\,; \bar{k}, \bar{a}\right)$$

■

Reynolds and Wooders (2009) proved or stated as true the above lemmas, but they used $G(y; 1, \bar{a})$ instead of $G\left(y; \bar{k}, \bar{a}\right)$. Despite that difference, the above lemmas are still true because the proofs that Reynolds and Wooders (2009) presented simply require G to be any cumulative distribution function. These lemmas are not proved in Appendix D.

As shown in Lemma 8, the concept of certainty equivalent payment allows an alternative definition for the expectation function of waiting, equivalent to equation (9). This allows using the following equation instead of equation (19) for characterizing the intersection of the payoff of buying and the payoff of waiting:

$$u(c^* - p; \propto) = u\left(c^* - \delta\left(c^*; \bar{k}, \bar{a}, \propto, r\right); \propto\right) \cdot G\left(c^*; \bar{k}, \bar{a}\right) \tag{21}$$

Using the formulation of the game provided in section 4.3, the following proposition states in a formal manner that c^* exists, that it is unique, and that it has certain properties:

Proposition 2. Consider the buyout auction which has a buy price p, offers at least $\bar{k}$ items, has a reserve price $r \geq 0$. Suppose $\bar{a} > \bar{k}$ players with utility function defined by equation (4), with level of risk aversion $\propto > 0$, and with private values drawn from a cumulative distribution function F with range $[\underline{v}, \bar{v}]$.

(i) If $p < \delta(\bar{v}; \bar{k}, \bar{a}, \propto, r)$,

(a) then there is a value c^* defined by equation (44),

$$u(c^* - p; \propto) = U(c^*, c^*; r, \propto, \bar{k}, \bar{a}) \tag{22}$$

or equivalently by equation (45),

$$u(c^* - p; \propto) = u\left(c^* - \delta(c^*; \bar{k}, \bar{a}, \propto, r); \propto\right) \cdot G(c^*; \bar{k}, \bar{a}) \tag{23}$$

(b) where such c^* has the following properties:

(1) It belongs to the range $(p, \bar{v})$.

(2) It defines a unique equilibrium cutoff (which is symmetric, and efficient).

(3) It is increasing in p.

(4) It is increasing in $\bar{k}$.

(5) It is decreasing in $\bar{a}$.

(6) It is decreasing in r.

(7) It is decreasing in $\propto$.

(ii) If $p \geq \delta(\bar{v}; \bar{k}, \bar{a}, \propto, r)$, then there is a value c^* equal to $\bar{v}$, which defines an equilibrium cutoff that is unique, symmetric, and efficient (and where the price is never accepted by the players). ■

In the two cases described in Proposition 2, the equilibrium is efficient. In Proposition 2(i), when all players participate in the auction, if a bidder acquires an item, bidders with higher private values would have also acquired them. In Proposition 2(ii), all players who chose to buy and acquired the item, would have higher private values than those who bid. And among those who bid, the same efficiency would happen as described above for case (i). Appendix D presents a formal proof of Proposition 2. The proof follows the same strategies used for proving the buyout raffle. The resulting proof is very similar to the one provided by Reynolds and Wooders (2009) but, unlike theirs, it pays close attention to the following facts: their function when choosing the buy option is different from equation (4), they use a probability of winning the auction different from equation (12), and their model does not explain how c^* relates to the new variables of $\bar{k}$ and $\bar{a}$ (for example, should the unique equilibrium cease to exist with a different $\bar{k}$ and $\bar{a}$.

4.5.3. Sensitivity analysis and recommended alternatives

Proposition 1 and Proposition 2 indicate that calculating the optimal strategy requires knowing the values of the following input variables: number of items $\bar{k}$, number of players $\bar{a}$, risk averseness $\propto$, reserve price r, and cumulative distribution F. From these variables, only the first two the player will have trouble in anticipating, given the particular characteristics of the system that will be introduced in Chapter 5. The value for the third variable, the risk averseness, can be anticipated from public field surveys provided that the assumption of its homogeneity among users is valid.

The aim of this subsection is to suggest how the optimal strategy could be implemented or communicated to the general public when the values of the input variables are not well known, especially $\bar{k}$ and $\bar{a}$, which are the least predictable. To achieve this, this section looks at typical input values obtained at the HOT facility, using the testbed and the microsimulation model described previously in section 3.2.

As Chapter 6 (Subsection 6.2.6) will explain in detail, the lotteries (whether raffles or auctions) with the highest chances of occurring have the following number of items and players: $\{\bar{k} \geq \bar{a}, \bar{a}\}$, $\{\bar{k} = 2, \bar{a} = 3\}$, $\{\bar{k} = 1, \bar{a} = 2\}$, $\{\bar{k} = 1, \bar{a} = 3\}$, and $\{\bar{k} = 0, \bar{a}\}$ where the first pair and the last pair are the two most likely scenarios.

Figure 21 first looks at the buyout raffle that would take place in an RBM system. It analyzes how the optimal strategy varies for $\{\bar{k} \geq \bar{a}, \bar{a}\}$, $\{\bar{k} = 2, \bar{a} = 3\}$, $\{\bar{k} = 1, \bar{a} = 2\}$, $\{\bar{k} = 1, \bar{a} = 3\}$, and $\{\bar{k} = 0, \bar{a}\}$ using the same three distributions that have been used throughout this chapter. Although only the distribution shown in Figure 21(c1) is the one that would really apply for the HOT facility analyzed later in Chapter 6, the other two distributions are provided for illustration purposes. Figure 21 also looks at how the optimal strategy varies with the reserve price and the level of risk averseness.

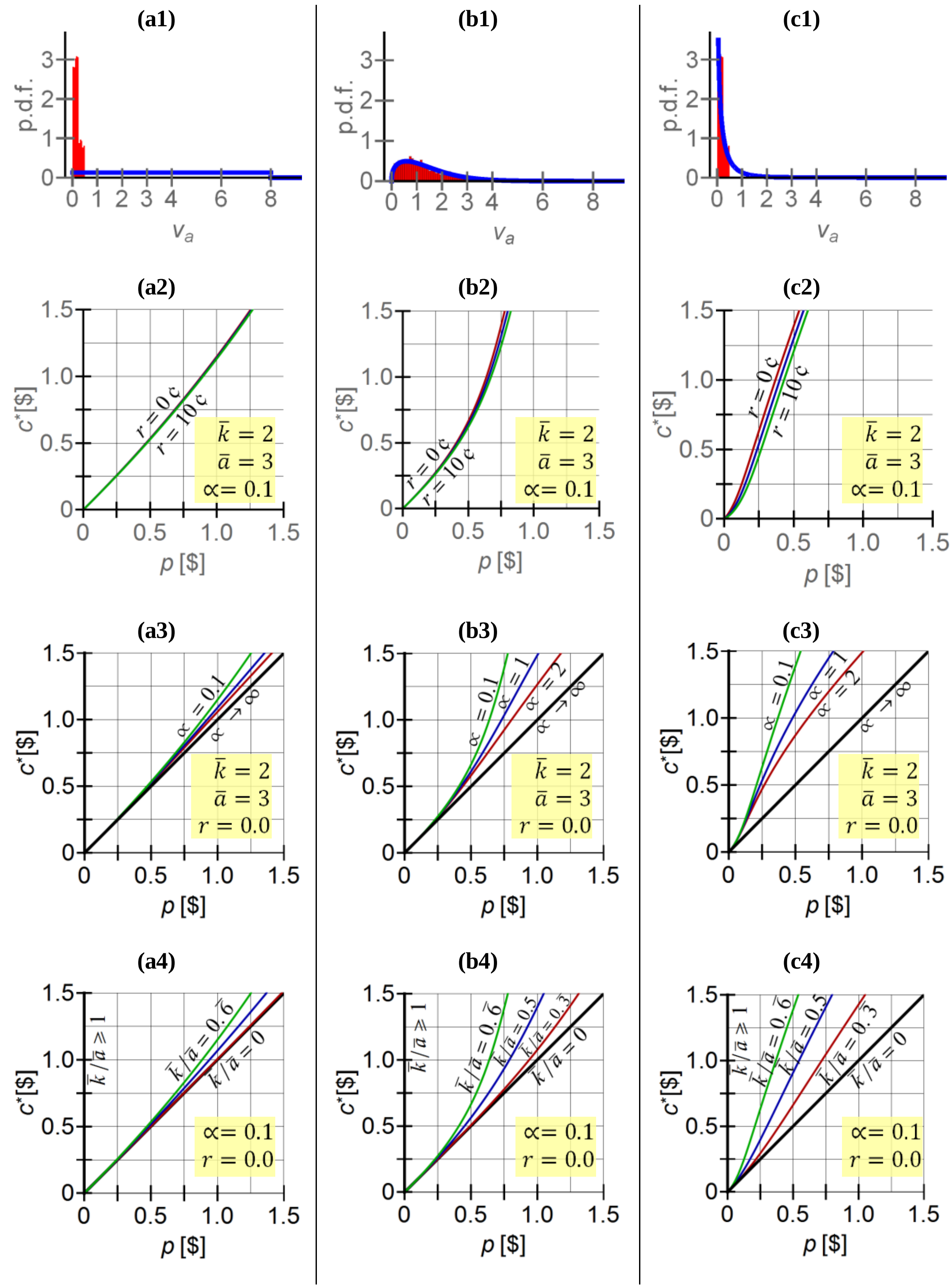


Figure 21. How the optimal strategy c^* in the buyout raffle changes to variations in the reserve price r, risk averseness $\propto$, and $\bar{k}/\bar{a}$ ratio. Each result is presented for the same three probability distributions of analysis. Yellow boxes indicate other input variables.

Figure 21 shows that the behavior of the optimal strategy against its input variables fits the description indicated by Proposition 1. Except for the typical values that would be used for the reserve price, the optimal strategy seems to be highly sensitive to the input variables. Figure 21(a4) to (c4) suggest that instead of analyzing the sensitivity to $\{\bar{k}, \bar{a}\}$ pairs, one could simplify the analysis by just looking at $\bar{k}/\bar{a}$ ratios. As Figure 22 indicates, this simplification is valid for all optimal strategies that are under 0.50 dollars. But when having higher values of c, this simplification was not valid in Figure 22(b4).

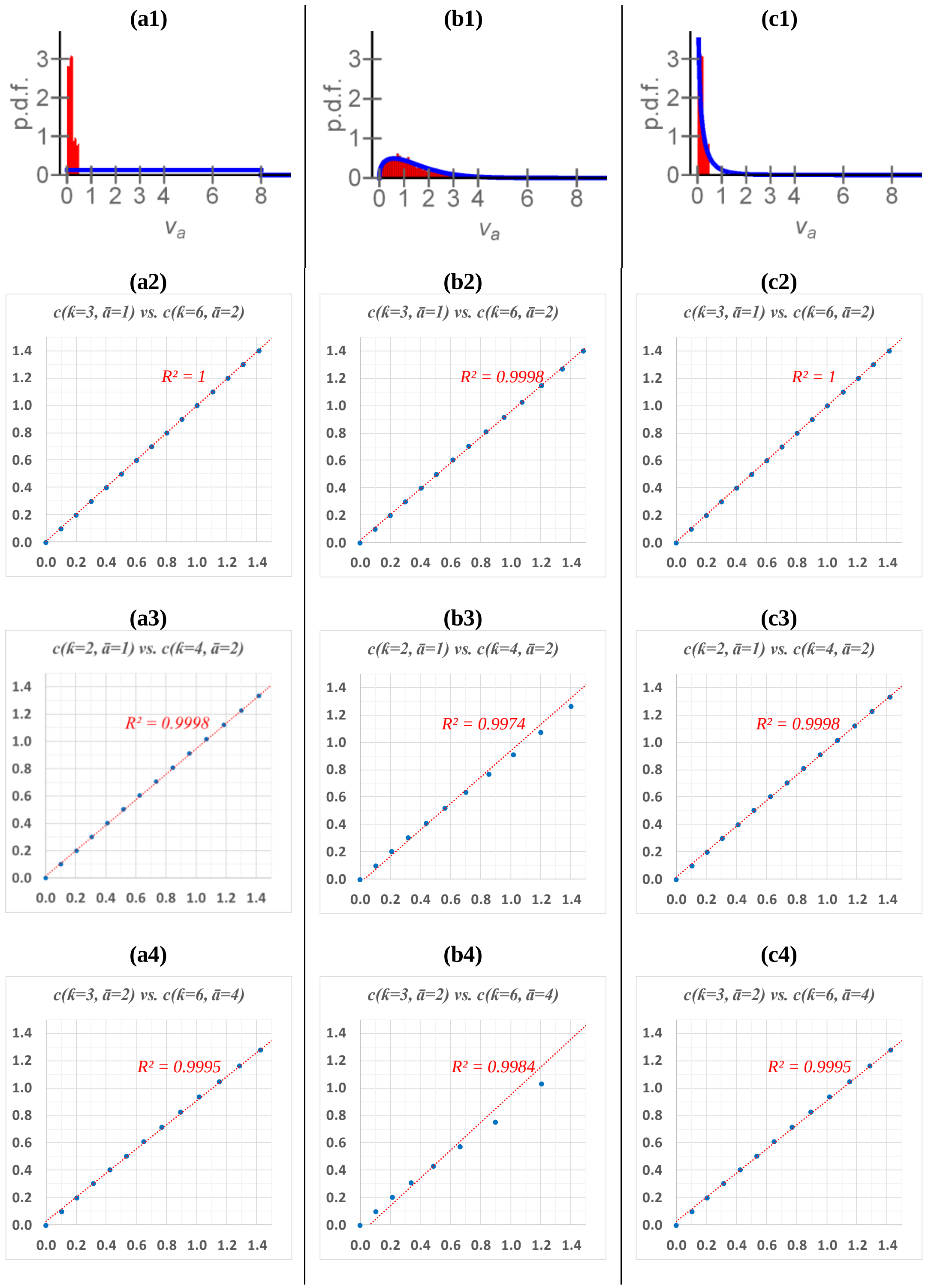


Figure 22. For the same three probability distributions, correlation between the optimal strategy for a buyout raffle with parameters $\bar{k}$, and $\bar{a}$, and the optimal strategy for the same game but with parameters $2\bar{k}$, and $2\bar{a}$. In the correlation figures, abscissas and ordinates represent toll prices in dollars.

Figure 23 carries out the same analysis made in Figure 21 but for the buyout auction that would take place in an ABM system. The results are similar except for the third column where the curves are much steeper. This is explained by the fact that the values for the maximum certainty equivalent payment, $\delta(\bar{v};\propto,r)$ are much lower than much of the toll prices considered. While the prices range from 0 to 1.5 dollars, the maximum δ values for that probability distribution are much lower. These values are not presented in the figure but are presented in the following list:

- $\delta\left(\bar{v}=100;\bar{k}=3,\bar{a}=2,\propto=0.10,r=0.00\right)\approx$ 0.12 dollars
- $\delta\left(\bar{v}=100;\bar{k}=3,\bar{a}=2,\propto=0.10,r=0.05\right)\approx$ 0.13 dollars
- $\delta\left(\bar{v}=100;\bar{k}=3,\bar{a}=2,\propto=0.10,r=0.10\right)\approx$ 0.16 dollars
- $\delta\left(\bar{v}=100;\bar{k}=3,\bar{a}=2,\propto=1.00,r=0.00\right)\approx$ 0.13 dollars
- $\delta\left(\bar{v}=100;\bar{k}=2,\bar{a}=1,\propto=0.10,r=0.00\right)\approx$ 0.32 dollars
- $\delta\left(\bar{v}=100;\bar{k}=3,\bar{a}=1,\propto=0.10,r=0.00\right)\approx$ 0.51 dollars

Figure 23 and Figure 21 allow introducing the concept of "safest strategy". The safest strategy is the optimal strategy for the worst of the possible cases. In Figure 23(a3) to (c3), the worst case happens when the level of risk averseness in the users is very high. In those cases, the safest strategy is depicted by the 45-degree black line. In Figure 23(a4) to (c4), the worst case happens when there are no auction items. Again, here the safest strategy is depicted by the 45-degree black line. Thus, the safest strategy suggested by Figure 23 and Figure 21 is $c^{*}=p$.

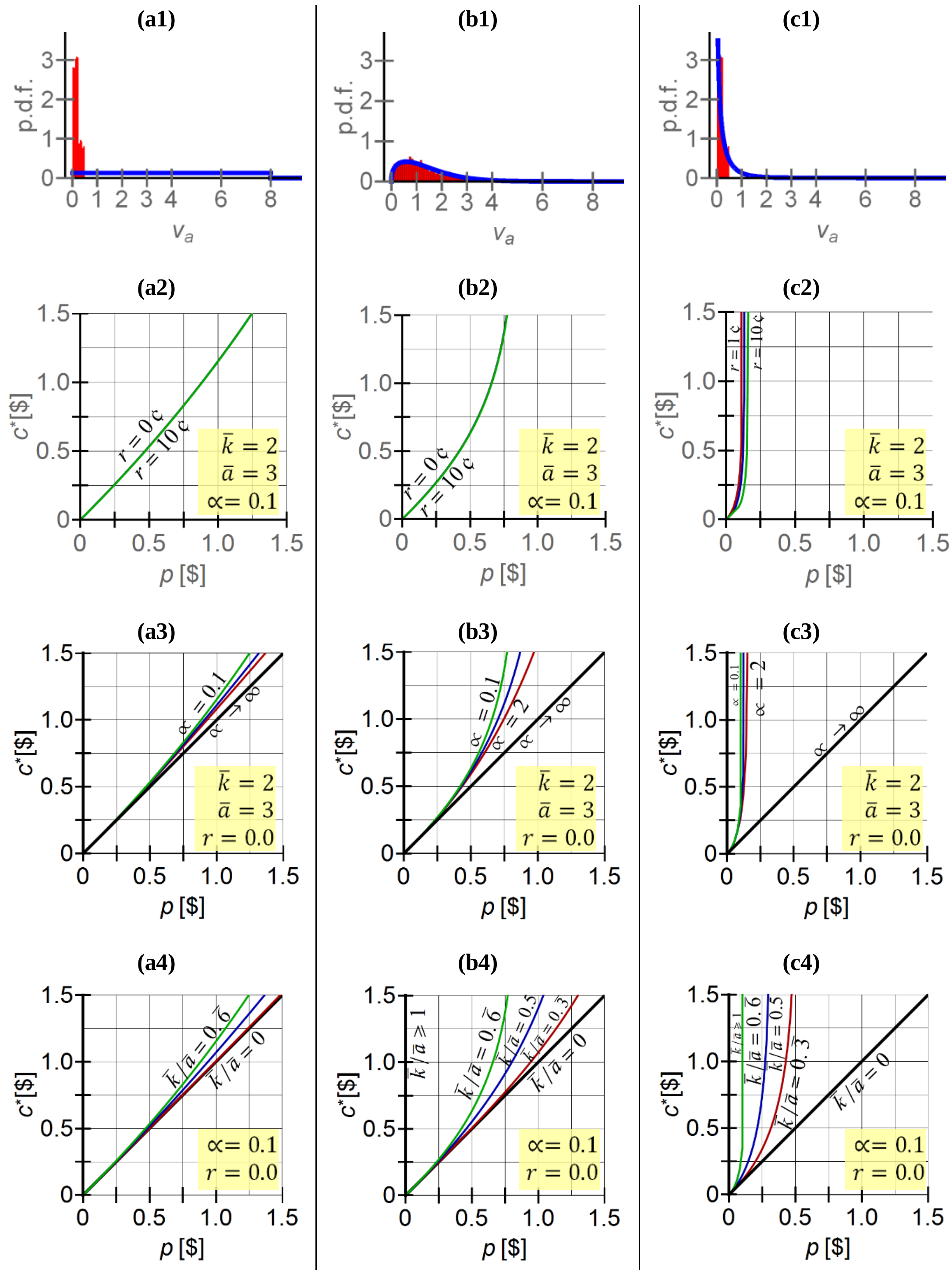


Figure 23. How the optimal strategy c^* in the buyout auction changes to variations in the reserve price r, risk averseness $\propto$, and $\bar{k}/\bar{a}$ ratio. Each result is presented for the same three probability distributions of analysis. Yellow boxes indicate other input variables.

The concept of safest strategy is perhaps equivalent to what are commonly known as "minimax regret" and "maximin" strategies in the field of game theory. Nonetheless, this dissertation does not extend into formalizing the concept of safest strategy as one of "minimax regret" strategy and "maximin" strategy.

In terms of whether the sensitivity could be simplified by looking at $\bar{k}/\bar{a}$ ratios instead of individual $\{\bar{k}, \bar{a}\}$ pairs, Figure 24 looks at whether this simplification is valid. When the private values have a uniform distribution (left column), the simplification is valid. Then, as the distribution becomes more skewed to the left (middle column), the simplification stops being valid in some cases when the optimal cutoff is greater than 0.40 dollars. And when the distribution becomes much more skewed to the left (third column), the simplification is valid in very few cases. It is important to clarify that a lot of points are not shown in the third column because they would drastically affect the red tendency line or because they were difficult to compute.

In conclusion, the different $\{\bar{k}, \bar{a}\}$ pairs that are most likely to happen are limited to a very small set and that in some cases, the $\bar{k}/\bar{a}$ ratio may be considered instead. Also, the results suggest that the safest strategy is $c^* = p$. In consequence, this dissertation proposes the following alternatives for implementing the optimal strategy or for communicating it to the general public.

(1) The operator should recommend the safest strategy.

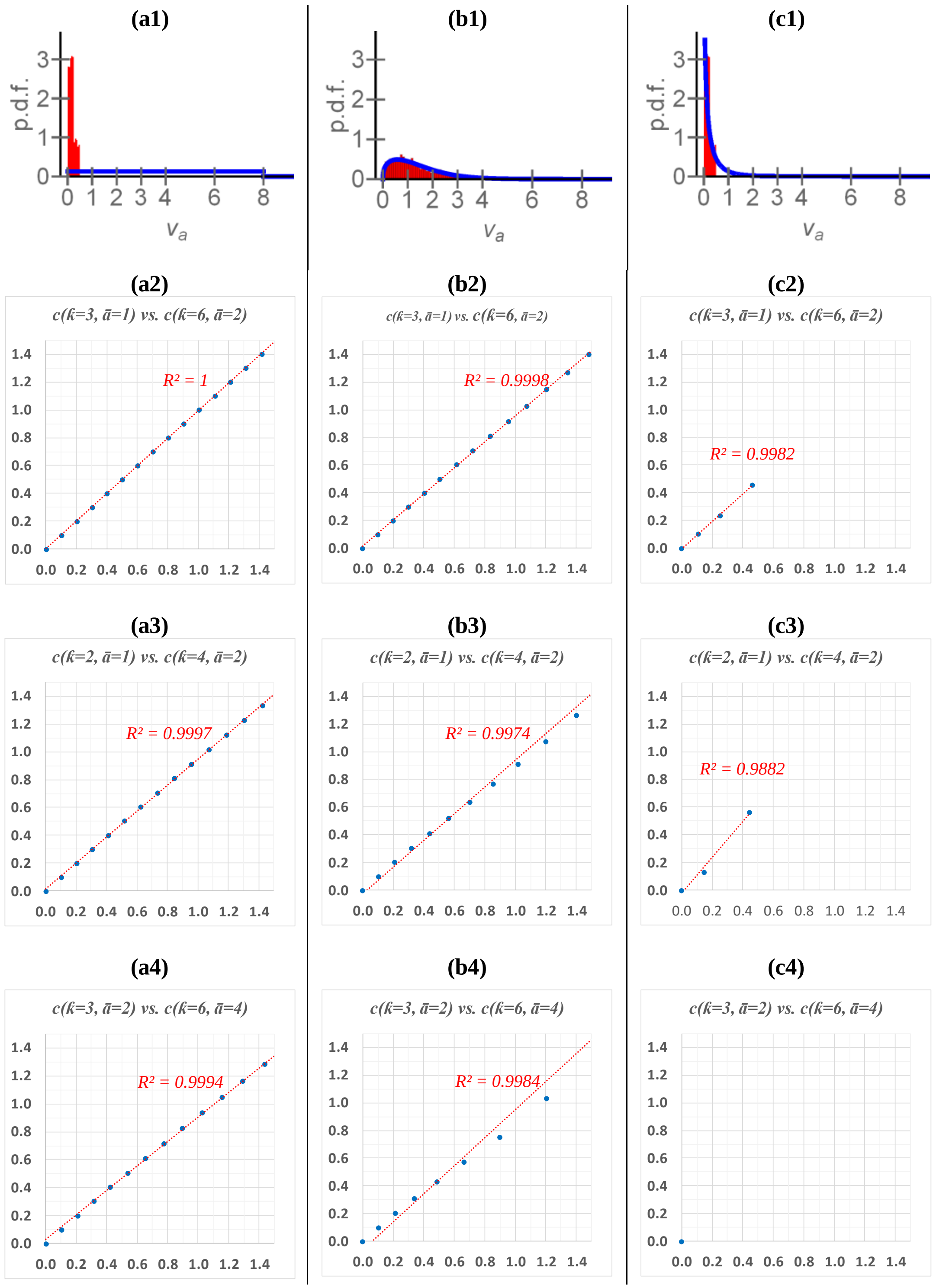


Figure 24. For the same three probability distributions, range of optimal strategies in which the R^2 value is close to 1.0 when comparing the optimal strategy for a buyout auction with parameters $\bar{k}$, and $\bar{a}$, and the optimal strategy for the same game but with parameters $2\bar{k}$, and $2\bar{a}$. Figure c4 only shows one value due to the closeness of $\delta[\bar{v};\bar{k},\bar{a},\propto,r] \approx 0.12$ and $\delta[\bar{v};2\bar{k},2\bar{a},\propto,r] \approx 0.09$ to the origin. Input values: $p = 2.50$ dollars, $r = 0.00$ dollars, and $\propto = 0.10$.

(2) The operator should maintain a consistent $\bar{k}/\bar{a}$ ratio depending on the time of the day and recommend the optimal strategy for that ratio. Maintaining a consistent $\bar{k}/\bar{a}$ ratio would imply artificially increasing $\bar{a}$ or altering $\bar{k}$ as needed for every game.

(3) The system should be modified in order to allow drivers to know the different variables $\bar{k}$ and $\bar{a}$ in advance.

The first alternative is perhaps the preferred one if one considers the results that Chapter 6 will reveal. Using the concept of regret, and the estimation procedure indicated in Proposition 2, Chapter 6 will reveal that the probability of a player experiencing regret when playing the safest strategy is very low.

Regarding the second alternative, it may be attractive in the sense that it leaves more room for choosing the wait option. Artificially increasing $\bar{a}$ is in a sense equivalent to making the same assumption of the first alternative, that is, to assume a worse case. But decreasing $\bar{k}$ may reduce the operational efficiency because it would not take advantage of the whole unused managed capacity. Increasing $\bar{k}$ would also reduce the operational efficiency because it may lead to exceeding the unused managed capacity.

The third alternative simply proposes trying to devise a better system than the one proposed in Chapter 5 where $\bar{k}$ and $\bar{a}$ can be known in advance by the players. The third alternative may involve also improving the mathematical model by allowing different values in the risk averseness.

5. Auction-Based Metering:

Operational Description and Technological Features

This chapter describes the proposed ABM system for a HOT entrance. Before implementing the system, the main requirement is that the entrance must be fed by a DAR that originates at a two-lane arterial (nonetheless, extensions of the system are later mentioned in the further research chapter, Chapter 8). The HOT facility described briefly in Section 3.2 (and described in detail in Appendix A) is used as an example in order to provide a sense of the possible design values that would characterize the system at other entrances. The following description of the operations of the system is self-contained, although illustrative animations, recently posted by Olarte (2016), are also helpful to understand the system. Olarte and Haghani (2016) provided the same description but restricted to the RBM system.

Figure 25 presents the entrance as it existed until the summer of 2013 but with seven new elements that allow the proposed ABM system to work. Shown in the figure is Linden Avenue West. Not shown is the Van White Memorial Boulevard which was opened in the summer of 2013. Both roads have very low congestion. For this reason, and to simplify the concept, the proposed ABM system ignores any traffic coming from them.

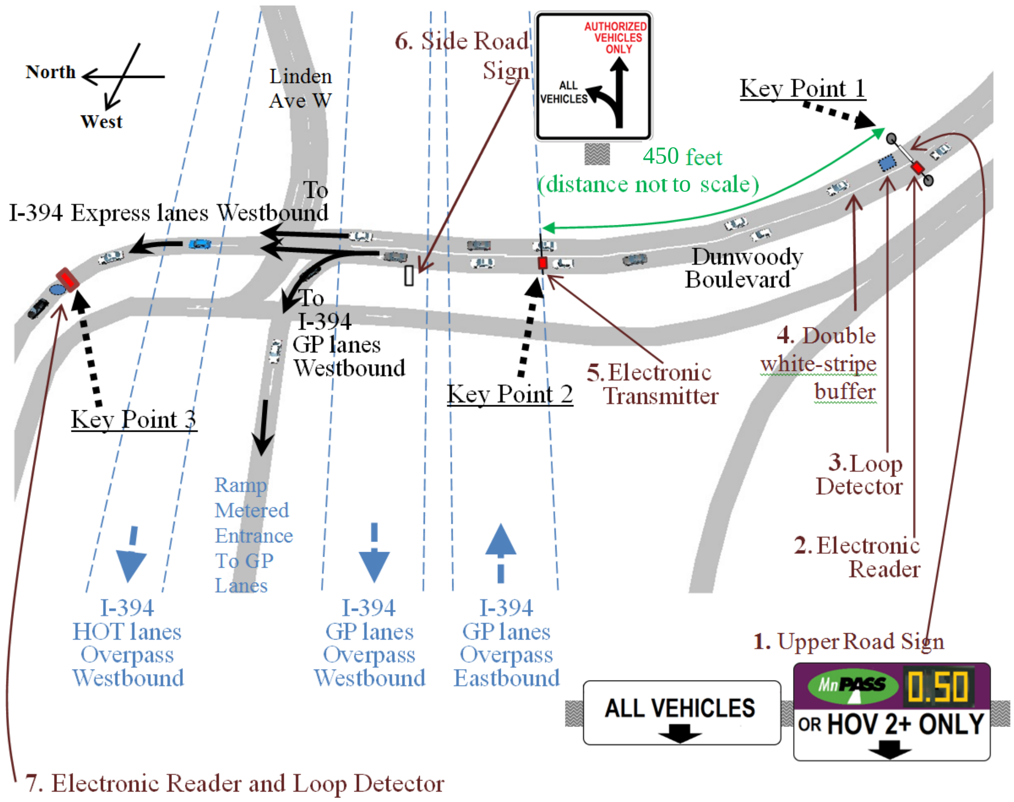


Figure 25. Proposed ABM system for entrance at Dunwoody Boulevard to HOT facility. Elements labelled in brown font indicate the 7 new elements that convert the existing road into the ABM system. Key points 1 to 3 are shown simply to facilitate the explanation of how the system works.

Key points 1 to 3 are not elements but points that facilitate the explanation hereafter. The length of 450 feet is a design parameter for which specific values are discussed in depth in Subsection 6.2.2. It could be as short as 80 feet. This parameter defines the length of one important component of the system: the “auction zone”. It also defines the length of the “non-auction zone” which is parallel to the action zone. Despite the fact that the above design can only be applied to entrances that are fed with a two-lane arterial, it is a design that has fewer traffic unknowns than the generic design proposed earlier by Olarte and Haghani (2013).

5.1. Description of the system from the perspective of the user

Vehicles driving on Dunwoody Boulevard wanting to enter Interstate 394 need to pass by key point 1. At this point, HOVs wanting to enter the HOT lanes take the right lane. SOVs wanting to enter the HOT lanes and willing to buy (that is, pay the toll) take the right lane too. SOVs wanting to enter the HOT lanes but not willing to buy take the left lane. After choosing the left lane, SOVs pass by an electronic reader that senses their OBU (if they have one) and enter into what is referred here as the auction zone. Every five seconds, the system looks at the SOVs carrying OBUs within that zone, and selects as winners the SOVs with the highest private values. As mentioned in the terminology section (Section 1.1), the private value is the amount that an SOV is willing to pay for getting access to a HOT lane. The private value would have to be input at the latest 15 minutes in advance via an online account (The interstate 85 express lanes in Georgia follow this approach in order to encourage doing the submission outside the vehicle and avoid the so-called distracted driving). As SOVs arrive to key point 2, an electronic transmitter sends a message to their OBUs in case they won the auction. If they win, they continue straight. If they lose, they must turn left and enter the GP lanes. The communication at key point 2 is enabled by replacing the OBU used today with one similar to the PrePass® Plus which is deployed by the not-for-profit organization HELP Inc. (2016). The system may include letting winners verify past results online, and charging winners a reservation price. Finally, drivers not willing to enter the HOT lanes take the left lane at key point 1 and turn left at key point 2.

Some SOVs may opt to remain with today's OBUs. And so, they would not be able to play auctions but they would still be able to enter the HOT lane by paying the toll. It is assumed that this set of SOVs, referred as basic SOVs, would not see their driving experience being perturbed since only three of the seven new elements would be highly visible: the displaced upper road sign, the new double white-stripe buffer, and the new side road sign. SOVs whose OBUs allow both options (to buy or to wait) are referred as advanced SOVs. (The terms basic SOVs and advanced SOVs were introduced previously in Subsection 4.2.2).

Briefly, in terms of how drivers could game the system, it is reasonable to assume that they would not have a rational incentive to switch lanes along the auction zone. Nonetheless, it cannot be discarded the idea of separating it from the non-auction zone with plastic pylons or wider separations (as is currently done in some HOT facilities). For drivers who enter the HOT lane without carrying the right transponder or after losing the auction, the control of those violations should be done with a very similar law enforcement system to the one used in most of today's facilities.

5.2. Description of the system from the perspective of the operator

Figure 26 presents the algorithm that makes the ABM system function. As stated all along, the auction is the extended Vickrey auction formulated in Section 4.3. At every time step t, a new cycle begins. In the microsimulation model that is presented in Chapter 6, each time step t had a length of one tenth of a second. The system requires that there already exists a mechanism for reading the managed capacity (m_{C}), that is, the maximum number of vehicles that can pass during a certain time interval (called "MC interval") without making the HOT lane go above the desired level of service. At every time step t, the system looks at the vehicles that passed by key points 1, 2, and 3 during the last MC interval. At each of the key points, a different set of steps are executed for each vehicle. The steps at key point 1 are numerous, and so, they were grouped into one sub-algorithm that is illustrated in Figure 27.

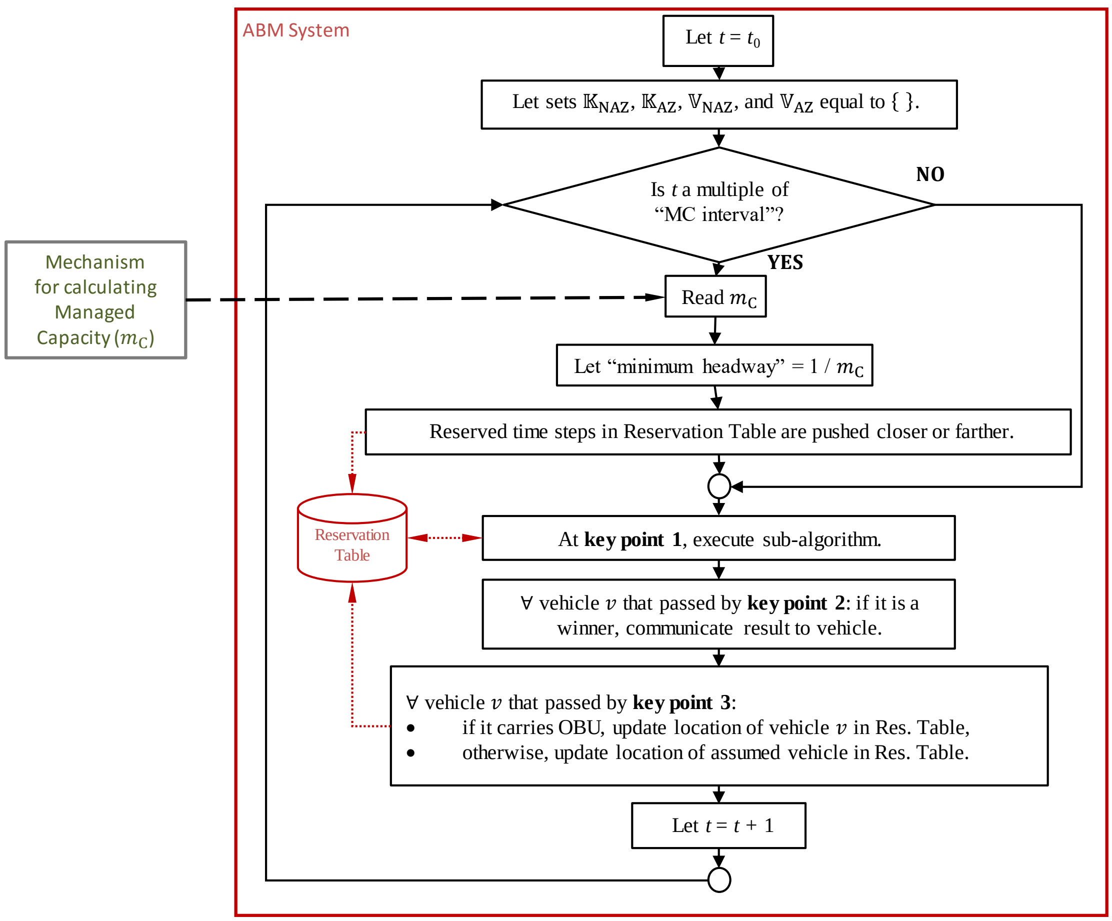


Figure 26. Algorithm that describes how the ABM system works.

Other key elements shown in Figure 26, are the use of a minimum headway, the existence of a reservation table, and the need for defining a time resolution for such table, as well as the use of sets $\mathbb{K}_{\mathrm{NLS}}$, $\mathbb{K}_{\mathrm{LS}}$, $\mathbb{V}_{\mathrm{NLS}}$, and $\mathbb{V}_{\mathrm{LS}}$, where "LS" refers to “lottery segment”, a name used previously by Olarte and Haghani (2016) for referring to the auction zone. These key elements are better understood with the following description of the sub-algorithm at key point 1.

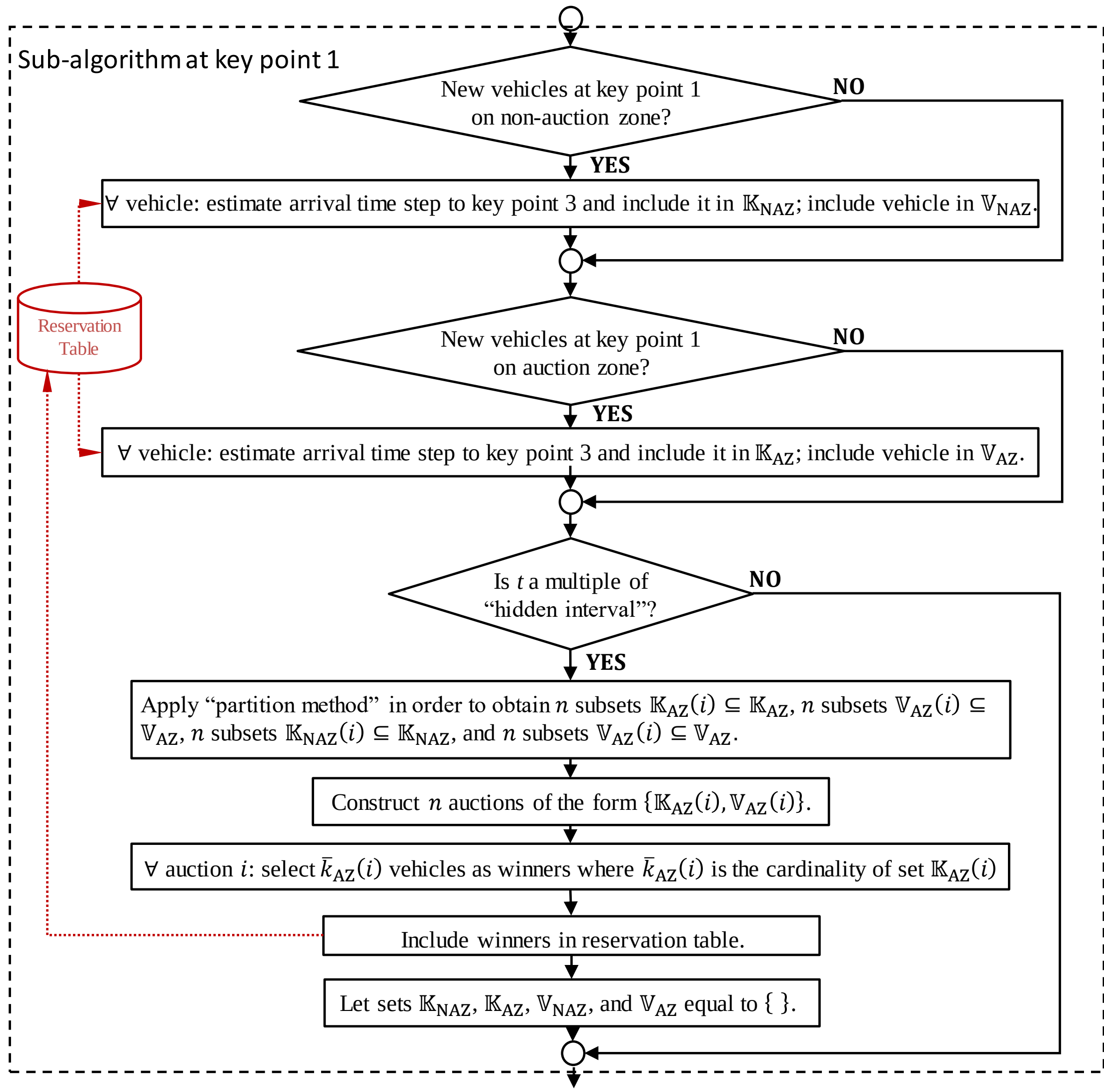


Figure 27. Sub-algorithm that is executed at every time step at key point 1.

Figure 27 presents the sub-algorithm that takes place at key point 1. Like with the rest of the algorithm, several steps involve reading information from or storing information on the so-called reservation table. This table consists of records, one for each future time step. Every time t that a vehicle passes by key point 1, the time step at which the vehicle is forecasted to pass by key point 3 (if choosing to buy or if winning the auction) is estimated. Then at a later step in the sub-algorithm, the forecasted arrival time is used to make a reservation in the reservation table. One important aspect of the table is that the distance between the reserved time steps should not be less than the minimum headway that was calculated in Figure 26. That is why in Figure 26, at the end of each MC interval, the reserved time steps in the reservation table need to be pushed closer or farther apart depending on whether the minimum headway decreases or increases at the end of each MC interval. This minimum headway, which according to the m_{C} values obtained later in Chapter 6 for Minneapolis is usually equal to 5 seconds and is never lower than 2 seconds, reduces the need for making a very accurate forecast at key point 1. The use of the minimum headway and the mechanism in which reservations are pushed closer or farther apart constitute the so-called “reservation rule”. The reservation rule adds robustness by reducing the need of accurate forecasting of the arrivals to key point 3.

Sets $\mathbb{V}_{\mathrm{NLS}}$, and $\mathbb{V}_{\mathrm{LS}}$ are used to store the vehicles that pass by key point 1 (but they are independent of the reservation table). Sets $\mathbb{K}_{\mathrm{NLS}}$ and $\mathbb{K}_{\mathrm{LS}}$ store the time steps that are reserved for the vehicles that pass by key point 1. Not shown in Figure 27 (or in Figure 26) is that whenever the reservation table gets updated within the sub-algorithm or in the algorithm, $\mathbb{K}_{\mathrm{NLS}}$ gets updated too.

One design parameter that the sub-algorithm has is the “hidden interval”. This time interval derives its name from the fact that it is very short (1 to 8 seconds) and therefore, it is difficult for drivers to know, as they drive, when it starts or ends. As explained below, this interval triggers the start of a group of one or more auctions.

Whenever time t coincides with the end of the hidden interval, the system takes the following steps in order to execute one or several simultaneous auctions. First, the set $\mathbb{K}_{\mathrm{LS}}$ is constructed. Its elements constitute all the items that will be given away via auctions.

It is very important to note that all the elements in $\mathbb{K}_{\mathrm{LS}}$ cannot be given away in just one auction. For example, assume that the first advanced SOV in the hidden interval is able to reach key point 3 at $t = 100$ and later at $t = 150$ while the last advanced SOV is only able to reach it at $t = 150$. It is clear that the last vehicle should not compete against the first vehicle for $t = 100$. But it could be possible to have the first vehicle compete for $t = 100$ and $t = 150$ if the last vehicle is assigned to compete for a later $t = 200$. In theory, set $\mathbb{K}_{\mathrm{LS}}$ can to be partitioned in different ways as long as cases such as the one described above are taken into account. In practice, partitioning $\mathbb{K}_{\mathrm{LS}}$ is not hard due to the small cardinality of $\mathbb{K}_{\mathrm{LS}}$ and $\mathbb{V}_{\mathrm{LS}}$ (this finding was obtained in Subsection 6.2.6). But still, defining a rule for partitioning $\mathbb{K}_{\mathrm{LS}}$ (referred hereafter as “partition method”) is not straightforward. The following subsection (Subsection 5.2.2) looks at partition methods in detail.

For the ABM system to work, sets $\mathbb{K}_{\mathrm{NLS}}$ and $\mathbb{V}_{\mathrm{NLS}}$ do not need to be partitioned because just the n subsets from the partitions of $\mathbb{K}_{\mathrm{NLS}}$ and $\mathbb{V}_{\mathrm{NLS}}$ allow defining the auctions that need to be executed. Nonetheless, Figure 27 proposes partitioning $\mathbb{K}_{\mathrm{LS}}$ and $\mathbb{V}_{\mathrm{LS}}$ in case the partition method requires or in case there is a theoretical need to analyze them.

Lastly, two general observations need to be made. First, the ABM system never uses the toll price as input. It does not interfere with the existing mechanism for calculating the toll. Its only link with the toll is that it may be better if the interval for calculating the toll is a multiple of the MC interval. Second, despite this weak link, the facility still has the dynamic toll that changes in reaction to the congestion level. But while the price-based metering happens at every one or couple of MC intervals, the ABM system executes lotteries at the much smaller hidden intervals.

5.2.1. Recap of design parameters

The above description of the ABM system shows that there are certainly some design parameters that need to be taken into account. Table 8 presents those parameters, the values recommended later in Section 6.2 for the specific entrance shown in Figure 25, and the possible range of values that they could adopt in similar HOT entrances.

Table 8. (PREVIEW) Design parameters of the ABM system, recommended values for HOT entrance as presented in Figure 25, and possible range of feasible values.

Design Parameter	**Recommended Value**	**Possible Range of Feasible Values**
MC interval length	1 minute	30 seconds to 1.5 minutes, less than the interval used for calculating the toll price
hidden interval length	5 seconds	1 to 8 seconds
auction zone length	450 feet	80 to 650 feet
time resolution	$^1/_{10}$ seconds	$^1/_{20}$ seconds to 2 seconds
partition method	greedy	Any partition method that doesn't group non-adjacent vehicles
capacity release ($\boldsymbol{d}_{\mathbf{C}}$) smoothing	None for low volume, linearized for other volume levels	None, linearized, quadratic, etc.
reserve price ($\boldsymbol{r}$)	Greater than zero if having an RBM system.	Lower than the toll price (p)*

**The difference between the r and p should be such that the average user perceives a real difference between paying one amount or the other.*

The reserve price and the first three parameters have already been explained in this chapter. The time resolution defines the length of the time step t shown in Figure 26 and Figure 27 as well as the time steps corresponding to the records in the reservation table. The partition method is explained in the next subsection. The smoothing of the so-called "capacity release" is a tool that Section 6.2 recommends for mid to high traffic volume. Subsection 5.2.3 explains it in detail.

5.2.2. Partition method

Defining a rule for partitioning $\mathbb{K}_{\mathrm{LS}}$ (referred hereafter as "partition method") is not straightforward due to the very different cases that can arise, partly because different objectives could be adopted (revenue maximization, minimization of average distance among players in subsets, etc.). In theory, an NP-hard problem could be defined. But as just mentioned, solving it as an NP-hard partition problem is not necessary due to the few number of reserved slots and players.

Olarte and Haghani (2016) implemented a partition method that they referred as the "$\bar{k} = 1$" method. It consists of the simple rule defined as follows: $\mathbb{K}_{\mathrm{LS}}$ is partitioned into n subsets such that vehicles in $\mathbb{V}_{\mathrm{LS}}$ and $\mathbb{V}_{\mathrm{NLS}}$ are assigned to just one reserved time step in $\mathbb{K}_{\mathrm{LS}}$, that is, the first one that allows them to reach key point 1. Because in this rule, all subsets have just one element, this method is referred as "$\bar{k} = 1$". This rule was simple to code and therefore, Olarte and Haghani (2016) chose it for their work. But it is operationally inefficient because it does not take advantage of including more items at each auction.

In this dissertation, a more efficient method is introduced and is referred hereafter as the "greedy" method. It is partially described in the Figure 28.

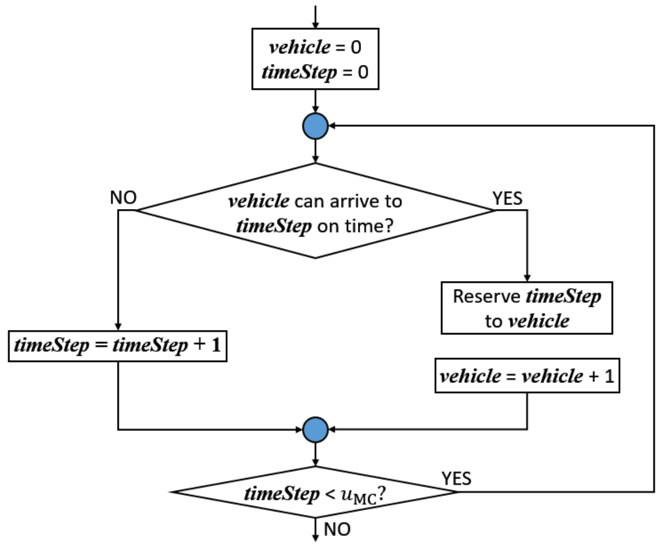


Figure 28. The "greedy" partition method (a partial description). The variable "vehicle" refers to all the vehicles in $\mathbb{V}_{\mathrm{LS}}$ and $\mathbb{V}_{\mathrm{NLS}}$ but sorted according to their arrival to the HOT ramp. The variable "timeStep" refers to time steps that are still available in the reservation table.

5.2.3. Smoothing of capacity release

One specific but important aspect of the ABM system, not shown in the previous figures, is the following. The minimum headway is the key tool for impeding the operator from giving away more auction items (or time slots) than allowed by the managed capacity. Nonetheless, the managed capacity may still be exceeded if once it is all used before the end of the MC interval, there are vehicles that enter the HOT lane as HOVs or after paying the toll. Another cause for exceeding the managed capacity is a possible decrease in m_{C} at the end of the MC interval. If before the interval, significant amount of m_{C} has been reserved, then a sudden decrease in m_{C} may force some reservations to be cancelled but not in time to communicate that cancellation to the auction winners. For this reason, the size of the auctions $\mathbb{K}_{\mathrm{LS}(i)}$

needs to be shrunk right before the selection of the winners using one of the two following rules:

$$0 \leq \bar{k} \leq \max(m_{\mathrm{C}} - r_{C}, 0) \tag{24}$$

$$0 \leq \bar{k} \leq \max(d_{\mathrm{C}}(t) - r_{\mathrm{C}}, 0) \tag{25}$$

where $\bar{k}$ is the cardinality of a given subset $\mathbb{K}_{\mathrm{LS}(i)}$, r_{C} is the reserved capacity or number of reserved time steps since the start of the MC interval, and d_{C} is the discharged capacity (or capacity release), that is, a function of t that, at the beginning of the MC interval, starts from a value lower than m_{C}, and increases at every time step t until reaching m_{C} by the end of the MC interval. Expression (25) is more stringent than (24) because it does not allow all the managed capacity to be used at the beginning of the MC interval. Note that both rules recognize that even with these controls, the reserved capacity may still surpass m_{C} or d_{C} due to the same reasons already explained above.

In expression (24), it will be said hereafter that this expression refers to having no smoothing of the capacity release. And expression (25) is the general rule to be applied for smoothing the capacity release. But there are several ways of smoothing it. The simplest way is the one applied later in Chapter 6, which is referred hereafter as linearized capacity release. Here, the capacity release (or d_{C}) is a linear function of t, being $d_{\mathrm{C}}(t) = 0$ at the start of the MC interval, and $d_{\mathrm{C}}(t) = m_{\mathrm{C}}$ at the end of the MC interval. It could be possible that the smoothing of the capacity release could be quadratic or even more elaborated depending on certain features of the entrance.

6. Auction-Based Metering: Microsimulation Model and Results

This chapter explains the microsimulation model that applied the ABM system to an HOT entrance. The model comprises two components mainly: the ABM system itself which has seven design parameters, and a traffic microsimulation component which has a different set of parameters, mostly related to driving behavior. After explaining the model in Section 6.1, Section 6.2 presents simulation results that look at the sensitivity of the system to two important design parameters: whether and how the system smooths the capacity release (Subsection 6.2.1), and the length of the auction zone (Subsection 6.2.2). Once appropriate values are recommended for those two parameters, the section presents simulation results in terms of congestion reduction and variation in the operator's revenue (Subsection 6.2.3). All these results are obtained under the assumption that advanced SOVs always adopt the safest strategy. Then, Subsections 6.2.4 and 6.2.5 present results that indicate that the percentage of drivers that experience regret for adopting the safest strategy instead of the optimal strategy is miniscule. Subsection 6.2.6 then presents the most likely auction scenarios that advanced SOVs face, in terms of number of auction items and number of rivals, again after assuming that they adopt the "safest strategy". It is important to note that the results in Subsections 6.2.4 and 6.2.6 were key in providing implementation recommendations back in Subsection 4.5.3. Finally, Section 6.3 recaps the design parameter values recommended for the HOT entrance used in the analysis.

6.1. Microsimulation model

The microsimulation model comprises the two components shown in green in Figure 29, that is, the traffic microsimulation component and the ABM system. The whole system was developed in Visual Basic.NET but the traffic component was developed using the microsimulation application VISSIM 5.4 (PTV Planung Transport Verkehr, AG 2012).

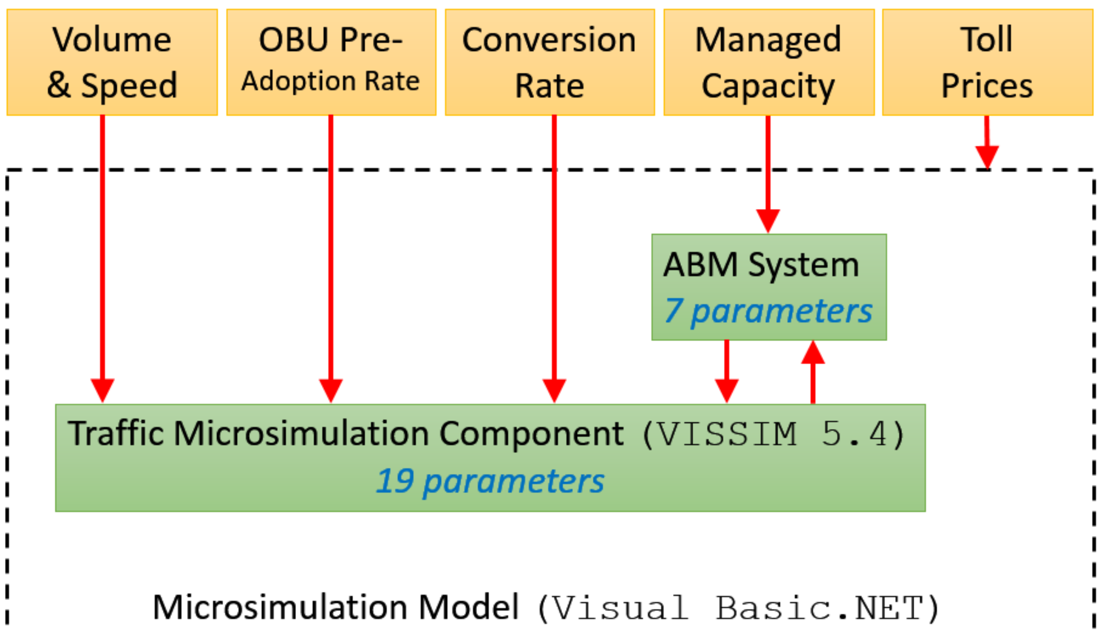


Figure 29. Model inputs and model components (including number of parameters and software platforms)

As indicated by Figure 29, toll prices are not generated by the model but are obtained from historical values provided by personnel from the Minnesota Department of Transportation. As the figure suggests, although the prices change every three minutes, a simplification is made in which those prices do not change in response to effect that the microsimulation model has in shifting vehicles from the GP lanes to the HOT lanes. Ideally, this simplification should be avoided by developing a model (external to the microsimulation model) that mimics how the operator currently makes those changes. But the simplification may produce in general more conservative values. If the simplification were lifted, toll prices may go higher as the current price mechanism detects more vehicles on the HOT lanes. This in turn would deter more vehicles from paying the toll price and choosing the auction. Thus, in the end, the results may produce an increase in the volume that enters the HOT lane and perhaps an increase in revenue. The subsections within this section explain in detail the other inputs shown in the figure.

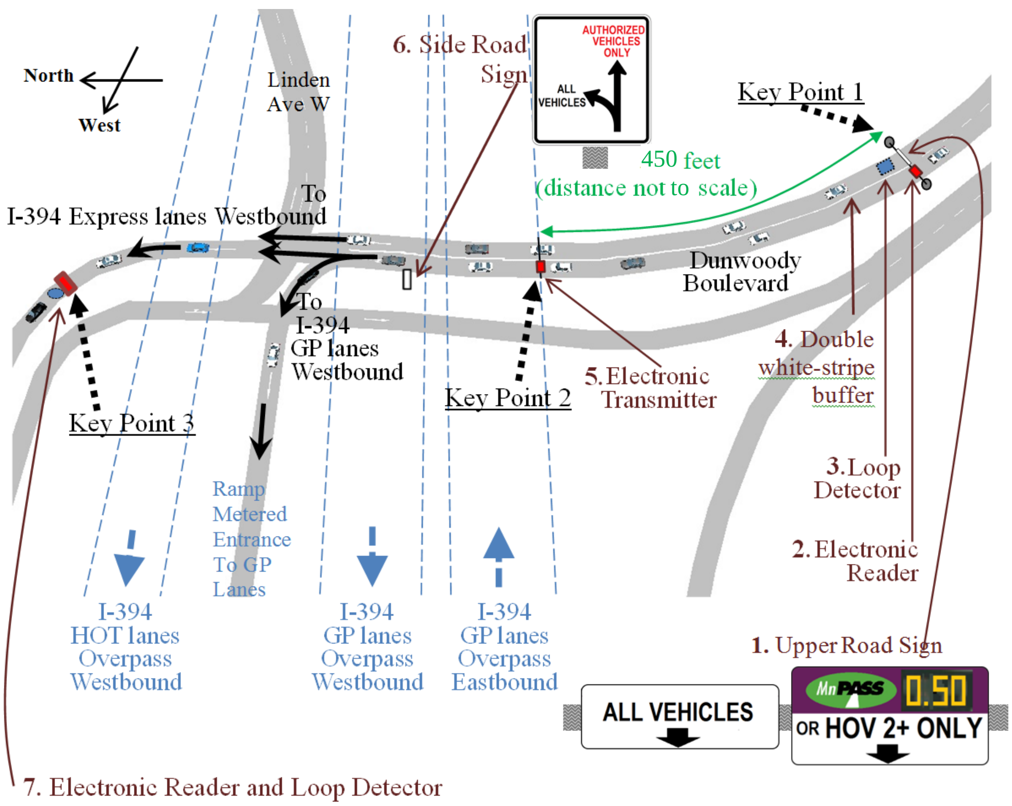


Figure 25 (DUPLICATE). Proposed ABM system for entrance at Dunwoody Boulevard to HOT facility. Elements labelled in brown font indicate the 7 new elements that convert the existing road into the ABM system. Key points 1 to 3 are shown simply to facilitate the explanation of how the system works.

The model applied the ABM system to the entrance described in Figure 25. The figure was already presented in Chapter 5 to describe how an ABM system functions.

As indicated in Figure 25, the ABM system has seven design parameters. The adopted parameter values used for this chapter are indicated in Table 5 (Note that Section 6.3 will recap which values are finally recommended for the entrance shown in Figure 25).

Table 5. Design parameter values used for the results analyzed at each of the subsections in this section. Below each subsection number, type of result that subsection seeks to obtain

Parameter used, and other features of model	**6.2.1** sensitivity to d_C smoothing	**6.2.2** sensitivity to length of auction zone	**6.2.3** impact on congestion and revenue	**6.2.5** percentage of players with regret	**6.2.6** most likely $\{\bar{k}, \bar{a}\}$ scenarios
MC interval length [minutes]	1	1	1	1	1
hidden interval length [seconds]	7	1 to 7	5	5	7
auction zone length [feet]	580	50 to 650	450	450	580
time resolution [seconds]	$1/_{10}$	$1/_{10}$	$1/_{10}$	$1/_{10}$	$1/_{10}$
partition method	"$\bar{k} = 1$"	greedy	greedy	greedy	"$\bar{k} = 1$"
capacity release (d_C) smoothing	none, linearized	none, linearized	linearized	linearized	none, linearized
reserve price [dollars]	0.00	0.05	0.00, 0.01, 0.5	0.05	0.00
system implemented	RBM	ABM	ABM, RBM	ABM	RBM
Number of seeds per result	1	3, 9	9	9	4

Table 5 shows that different values were used for each subsection in this section. In addition, it shows that sometimes an RBM system was implemented instead of an ABM system. The reason for this variation is two-fold. On one hand, in some subsections, a sensitivity against various parameter values was made. On the other hand, the development of the microsimulation model started from a simple model (an RBM system, with a "$\bar{k} = 1$" partition method, and with a reserve price equal to zero) at a time where there was uncertainty on which parameter values to use, to a more sophisticated one (an ABM system, with a greedy partition method, and which allows different values in the reserve price) at a time when there was better knowledge of the parameter values to be used. Ideally, the variation of parameter values should be restricted to only results where a sensitivity analysis was conducted. Nonetheless, the production of simulation results was costly. As indicated in the table, only results in Subsections 6.2.3 and 6.2.5 were obtained from 9 random seeds since these subsections did not look so much at tendencies but at specific numerical results. It is important to note that implementing an RBM system instead of an ABM system should not generate different results as long as those results are not related to revenue collection or regret, and as long as advanced SOVs always adopt the safest strategy.

The testbed used comprises the entrance depicted in Figure 25 plus the HOT facility depicted in the Figure 6 (see duplicate figure below). Figure 6, as well as the testbed and the calibration process of the traffic microsimulation component were already described briefly in Section 3.2. Appendix A presents a detailed description of the HOT facility and Appendix F presents a detailed description of the calibration and validation process. Although the microsimulation component has a large number of parameters, this dissertation applied the approach followed by Williams et al. (2010)

in which they calibrated only 19 of those parameters for the calibration of an HOT facility.

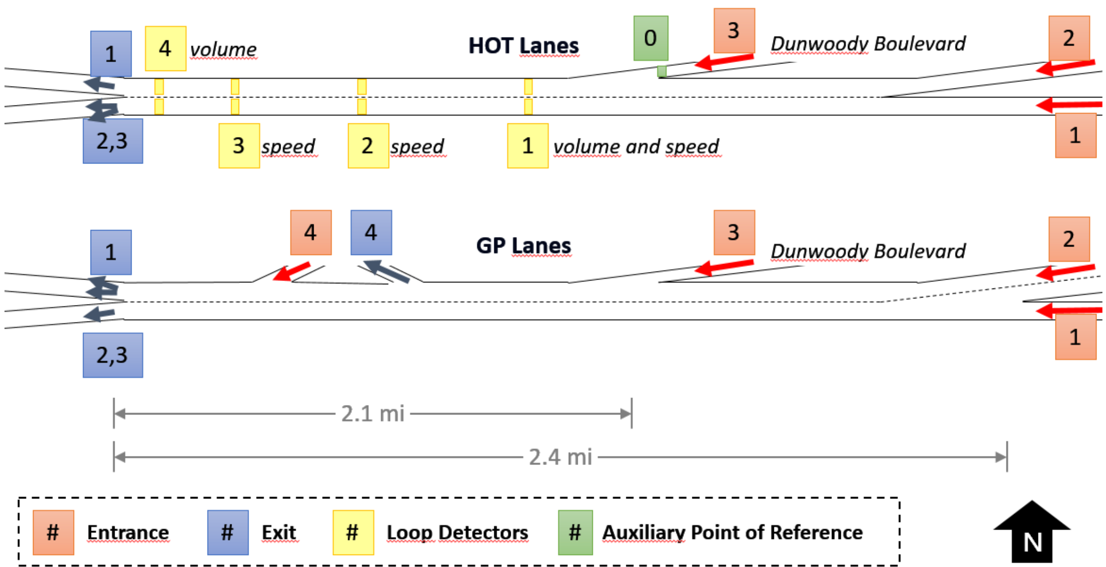


Figure 6 (DUPLICATE). Schematic representation of the reversible section of the I-394 MnPass Express lanes and the GP lanes parallel to them. All entrances to the HOT lanes are DARs. Yellow boxes with numbers represent location of detectors used to calibrate VISSIM parameters, and indicate type of measurement that was included in the objective function.

The study period starts on Thursday, May 11th 2012 at 20:15 and ends on the same day at 21:00. The date chosen corresponds to one of the few dates in which traffic manual counts are publicly available online for Dunwoody Boulevard (City of Minneapolis 2015). This date was also chosen because years later, a loop detector that was needed for recording speed stopped providing data (that loop detector is mentioned in the subsection that refers to input speed data, that is, Subsection 6.1.4)

6.1.1. *Input volumes*

Input volumes for the HOT lanes and the GP lanes were obtained from the Minnesota Department of Transportation (2011). Depending on the magnitude of the volumes, the following two cases were defined:

1. Case $1V$ or low-congestion case (176 veh/h): For the first two entrances to the HOT lanes and the GP lanes (shown in Figure 6), the volumes were obtained from the loop detectors located at those entrances. The volumes for the third entrances were not read from the loop detectors but from the manual counts obtained at Dunwoody Boulevard. Because the volume at that arterial was so low, it was decided to use the volume corresponding to the interval 16:30 to 17:15 which was higher (but outside the study period defined above). Still, volumes in this case are low overall. In this case, the volume that passes by the two lanes on the arterial (and that then enters to the HOT lanes or GP lanes) is equal to 132 vehicles, which equates to 176 vehicles per hour.
2. Case $6V$ or mid-congestion case (956 veh/h): Same as in previous case but with the difference that the volume in the arterial was multiplied by six. Nonetheless, some of the volume that entered the arterial from some approaches was not increased. In this case, the volume that passes by the two lanes on arterial is equal to 717 vehicles, which equates to 956 vehicles per hour.

Figure 30 presents the volumes for both cases, as vehicles pass by key point 3 (key point 3 is indicated in Figure 25). To have a sense of whether the volumes used are representative of today's traffic levels, Figure 30 also presents the volume for a

20:15-to-21:00 interval in which its total volume constitutes the 95th percentile of all the 20:15-to-21:00 intervals in 2015. The figure suggests that case $6V$ is acceptable for representing the highest volume scenarios that the arterial currently experiences. Figure 30 also presents the managed capacity that was estimated for the testbed and which is explained in the subsection below.

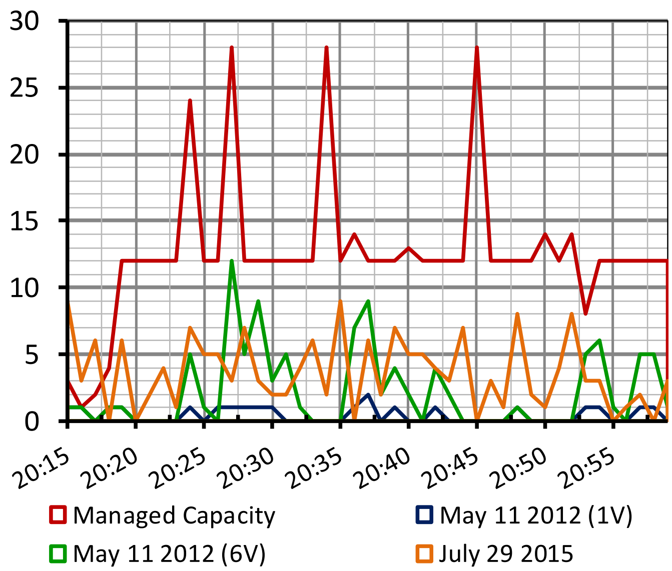


Figure 30. Volume (in vehicles per minute) at key point 3 for the cases considered: $1V$ and $6V$. Also in the figure: volume recorded on a day that registered the 95th percentile total volume of 2015, and a possible managed capacity for the study period of May 11th 2012.

6.1.2. Managed capacity

As explained in the description of the ABM system (Section 5.2), the scope of the system does not involve calculating the managed capacity (m_C). This is a value that needs to be read from an external system. Therefore, it was required, before obtaining the simulation results, to calculate m_C. Given the limitations in resources, it was decided to calculate it by carrying out simulations in which as many vehicles as possible were assigned to enter the HOT entrance without reaching a level of service on the HOT lanes that is worse than "level of service B" (a stricter limit than the "level of service C" that the facility uses today). A genetic algorithm composed of 10 generations and 10 solutions per generation was used. The resulting m_{C} is shown by the red line in Figure 30.

For the mid-congestion case ($6V$), one could have assumed that not only the volume at the HOT entrance denoted as Keypoint 3 in Figure 6. Instead, one could have assumed that the volume at all the entrances should have resulted from multiplying the original volume by a factor close to six. Doing so would have likely produced a lower m_{C} than the one shown in Figure 30. Nonetheless, doing so would have also compromised the validity of the driving behavior parameters that were found for the HOT facility with the original volume and therefore, a new calibration and validation process would have been required.

6.1.3. <u>OBU pre-adoption rate, and conversion rate</u>

Variations in congestion reduction and revenue collection depend on how many SOVs (with or without OBU) would decide to acquire an OBU that enables auctions once the ABM is implemented. This number constitutes an important input for the microsimulation model. To measure it, the concept of "conversion rate" is introduced to refer to the percentage of SOVs (whether they already have or did not have a regular OBU) that acquire an OBU that enables them to participate in auctions once the ABM system is put in place. One could also think about percentage of HOVs that switch to the new system but for simplicity, only SOVs are considered in this dissertation.

Another assumption became important for feeding the model due to the fact that not all the necessary data was available as the following explanation indicates. Personnel from the Minnesota Department of Transportation were key in providing the actual number of vehicles that carry an OBU and use the HOT lanes. Using these values, it was easy to estimate how many HOT lane users were HOVs and how many were SOVs. Nonetheless, it was unknown how many of the GP lane users were SOVs carrying an OBU. But the data did reveal a lower bound of 4%, that is, at least 4% of SOVs during the study period carried an OBU. So, two other higher percentages were also considered. These percentages constitute the concept that this dissertation introduces as “OBU pre-adoption rate”. For example, if the OBU pre-adoption rate us 36%, then it means that before the implementation of the ABM system, 36% of SOVs already carried had an OBU (that is, an OBU that does not allow auctions).

The microsimulation model considered three OBU pre-adoption rates: 4%, 36%, and 68%. And it considered three conversion rates: 0%, 25%, and 75%.

6.1.4. Input speed data

In theory, speed data is not necessary if one already has a calibrated microsimulation component. Nonetheless, speed data was important in order to compensate the fact that no OD matrix was available and therefore, the possible congestion at yellow box

4 in Figure 6 could not be simulated directly. Instead, provided by Minnesota Department of Transportation (2011), one-minute interval speeds were obtained from the loop detectors located at yellow box 4. These speeds were used to apply a feature from the VISSIM application called “reduced speed zones”. As vehicles in the model approached those zones, they accelerated or decelerated in order to meet the real speed. In this way, the possible real congestion created by weaving at the exits was replaced with possible simulated congestion created by the reduced speed zones.

It is important to note the following in regards to the speed on the arterial. The traffic manual counts did not provide this speed. But loop detector data provided an average speed of 34 miles per hour at the location indicated in Figure 31. It was assumed that vehicles at that point are not speeding as they would in the highway. In addition, the loop detector not only allows to obtain an average speed but also a distribution that can be applied among the vehicles that pass by that point. Since the average speed of 34 miles per hour seemed reasonable, this loop detector data was used.

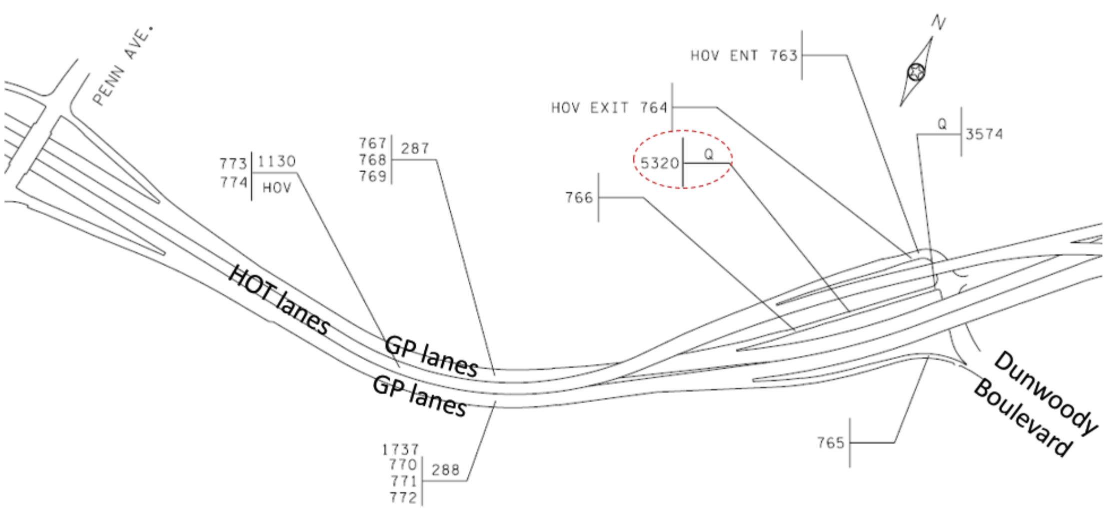


Figure 31. In red, location of the loop detector used for estimating the speed distribution on the arterial. Loop detector estimated an average speed of 34 miles per hour.

6.2. Simulation results

After developing the calibrated microsimulation model, the following results were obtained.

6.2.1. <u>General behavior and sensitivity to smoothing of capacity release</u>

Before analyzing the sensitivity of the ABM system to different design parameters and its impact on traffic and revenue, it is important to first observe a general result of how the system works. Figure 32 presents such result by looking at the number of vehicles per minute that chose the HOT lanes instead of the GP lanes. To simplify this analysis, the plots shown in the figure were obtained from one random seed. Any of the nine plots presented in the figure shows that when the conversion rate is zero (that is, when nobody has switched to using the new system), the volume that enters the HOT ramp is much below the managed capacity threshold. Thus, there are plenty of gaps (unused managed capacity) on the HOT lanes. But when SOVs switch to OBUs that allow them to participate in the auctions, the volume increases substantially because the system is now allowed to fill the gaps to some extent, especially in the mid-congestion case. It is important to note that, although the results presented in Figure 32 were obtained from an RBM system instead of an ABM system, this difference should not affect the volume that enters the HOT lane, given that it is assumed that advanced SOVs always adopt the safest strategy.

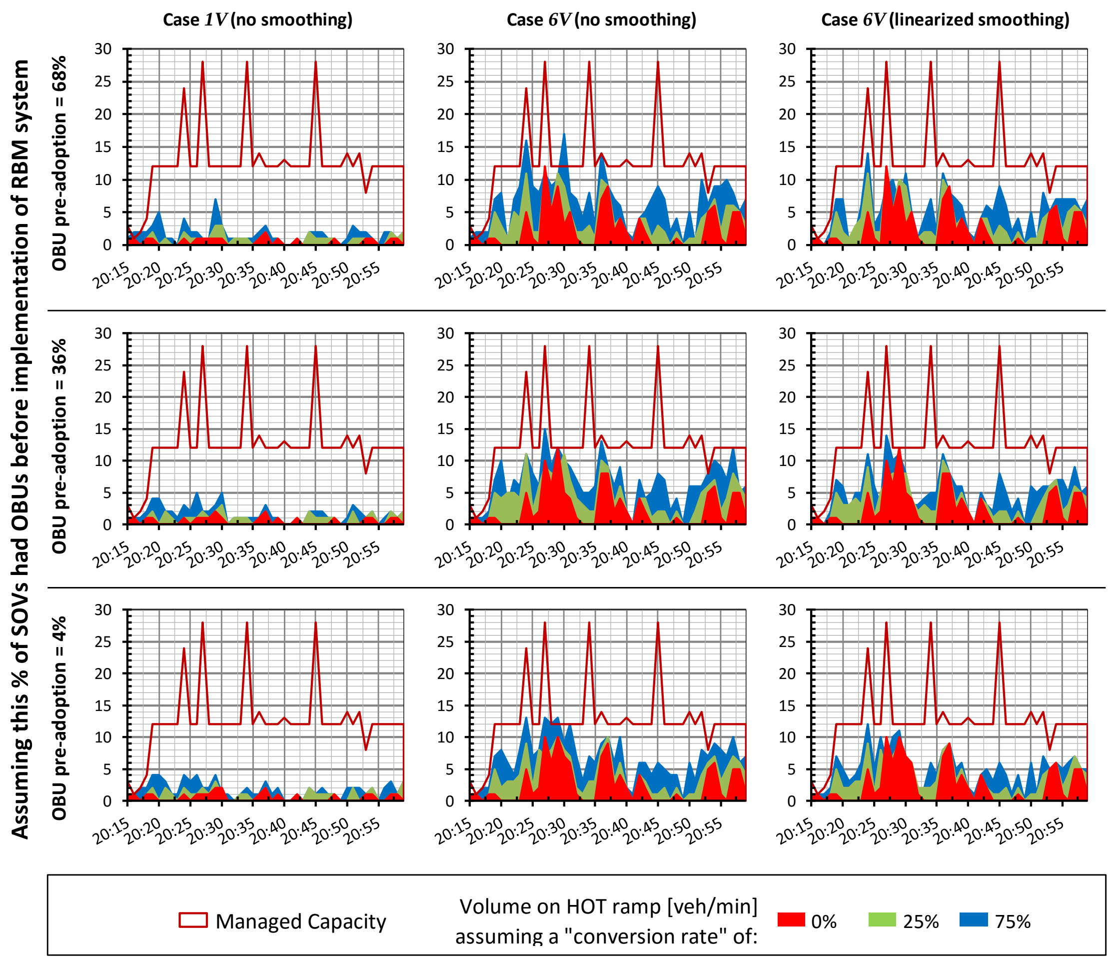


Figure 32. Volume at key point 3 after applying the RBM system to cases $1V$ and $6V$ where only in the latter, linearized smoothing of capacity release was applied. For each of the three resulting cases, three OBU pre-adoption rates (4%, 36%, and 68%), and three conversion rates (0%, 25%, and 75%) are considered. Design parameter values used: hidden interval = 7 seconds, auction zone length = 580 feet, partition method = "$\bar{k} = 1$", reserve price = 0 dollars. Number of random seeds per result: 1.

Now, in terms of whether the capacity release needs to be smoothed or not, Figure 32 indicates that it is clearly not necessary for case $1V$. But in case $6V$, only when the OBU pre-adoption rate was 36%, it was unnecessary to have smoothing in the capacity release. Thus, the figure suggests that it is safer to adopt linearized capacity release only in the mid-congestion. The figure also suggests that although the OBU pre-adoption rate affects the volume on the HOT ramp, it does not affect it in a major way.

The smoothing of the capacity release does cause a reduction in the volume that passes by the HOT ramp and on revenue collection. Instead of conducting a formal quantification of this impact, to simplify the analysis, this dissertation assumes that smoothing should always be avoided for the low-congestion case and linearized smoothing should always be applied for the mid-congestion case. Making that assumption, the next question to answer is the following: how does volume and revenue change when varying the length of the auction zone?

6.2.2. Sensitivity to length of auction zone

Before generating results for different lengths, one must determine what the appropriate length of the hidden interval is for each of those lengths. For example, if the seven seconds are adopted for the hidden interval, but the system has a length of 50 feet, auction winners will not receive their message on time when they get to key point 2. After testing different values for different lengths, the recommended hidden intervals were obtained. These are summarized in Table 6.

Table 6. Recommended values of hidden intervals to different lengths of the auction zone. Parameter values used: partition method = greedy, d_C smoothing = linearized, reserve price = 0.05 dollars. Number of random seeds per result: 9.

Length of Auction Zone [feet]	**Length of Hidden Interval [seconds]**
650	7
550	6
450	5
350	4
250	3
200	2
100	1

Now, Figure 33 looks at how the auction zone length impacts revenue and volume entering the HOT ramp (that is, the volume that passes by key point 3). For each of the lengths, the hidden interval recommended in Table 6 was used. The figure considers three OBU pre-adoption rates and three conversion rates. It focuses on the mid-congestion case. Although the study of analysis comprised only 45 minutes, results were linearly extrapolated to one hour.

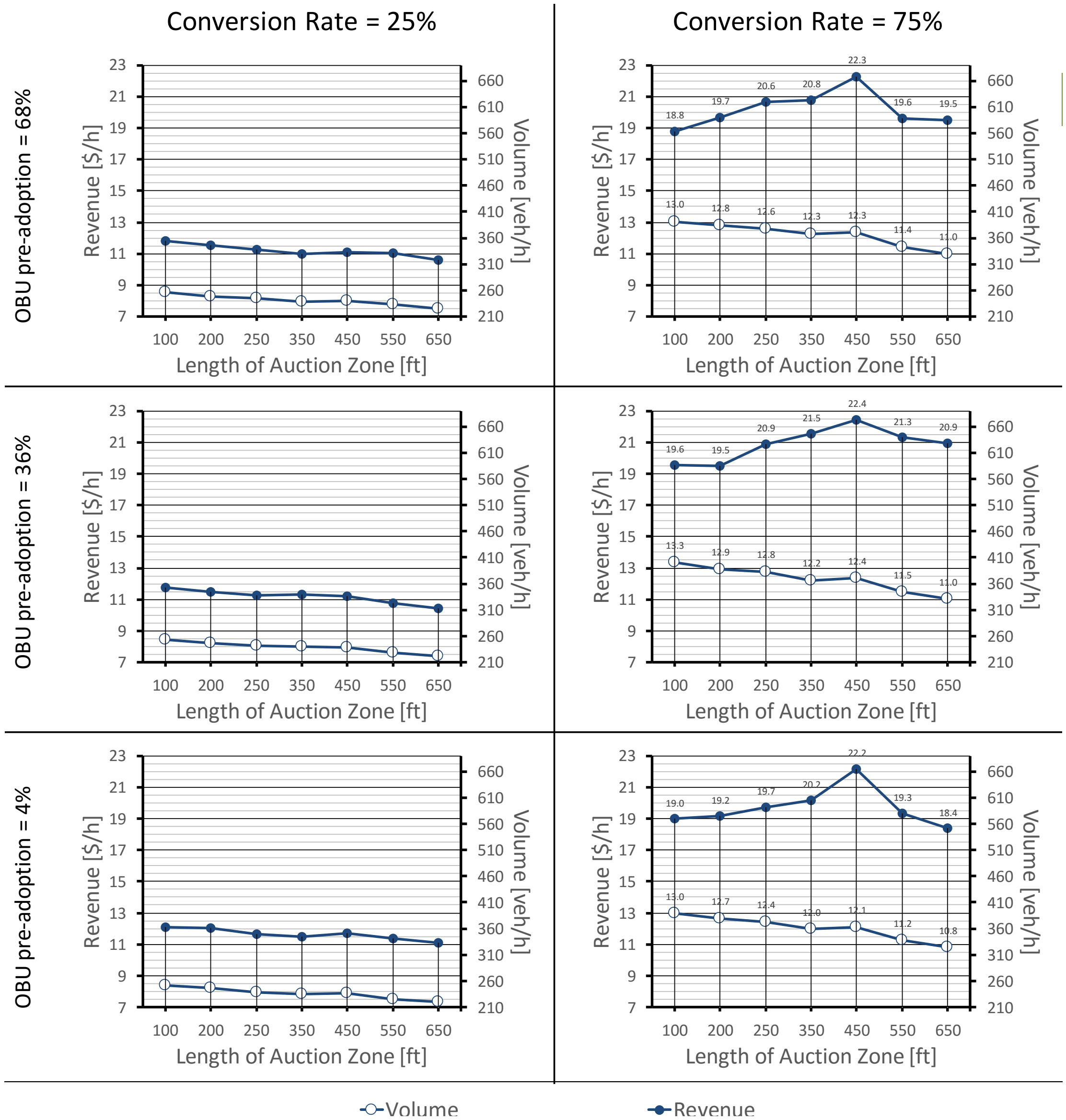


Figure 33. How traffic volume at HOT ramp (key point 3) and revenue changes to different lengths of the auction zone when having mid-congestion on the arterial (case 6*V*). Three OBU pre-adoption rates and three conversion rates are considered. Each point corresponds to average of 3 random seeds and 2 traffic parameter value sets (one calibrated set and one uncalibrated set). Design parameter values used: hidden interval length = 5 seconds, auction zone length = 450 feet, partition method = greedy, d_C smoothing = linearized, reserve price = 0.05 dollars.

The results in Figure 33 suggest that when the conversion rate is high, volume that enters the HOT ramp decreases slightly with the auction zone length while revenue increases significantly until reaching a peak. The decrease in the volume is explained by the fact that, as the auction zone becomes longer, the system loses the ability to find unused managed capacity. More specifically, the forecast that the system has to make about when vehicles will pass by key point 3 becomes less accurate and so, those inaccuracies reduces the ability of the system to find available slots within the reservation table. Nonetheless, although available slots become wasted with a longer auction zone, each of the auctions start having more players competing among themselves pushing the payments higher. But after a certain length, the decrease in the volume pushes the revenue down again.

When the conversion rate is low, there is also a decrease in the volume as the auction zone becomes longer. But because the volume in general is so low, a longer auction zone is not capable of obtaining higher revenues. The results suggest, especially when the conversion rate is of 75%, that a length of 450 feet maximizes revenue collection without having a major loss in the volume that enters the HOT lane.

Figure 34 looks at whether the previous results change dramatically when the calibration of the microsimulation component is not optimal. On the right column, the VISSIM default values were used for the 19 parameters. As the figure indicates, there are no changes in the result except for a slight variation when the OBU pre-adoption rate is 36%.

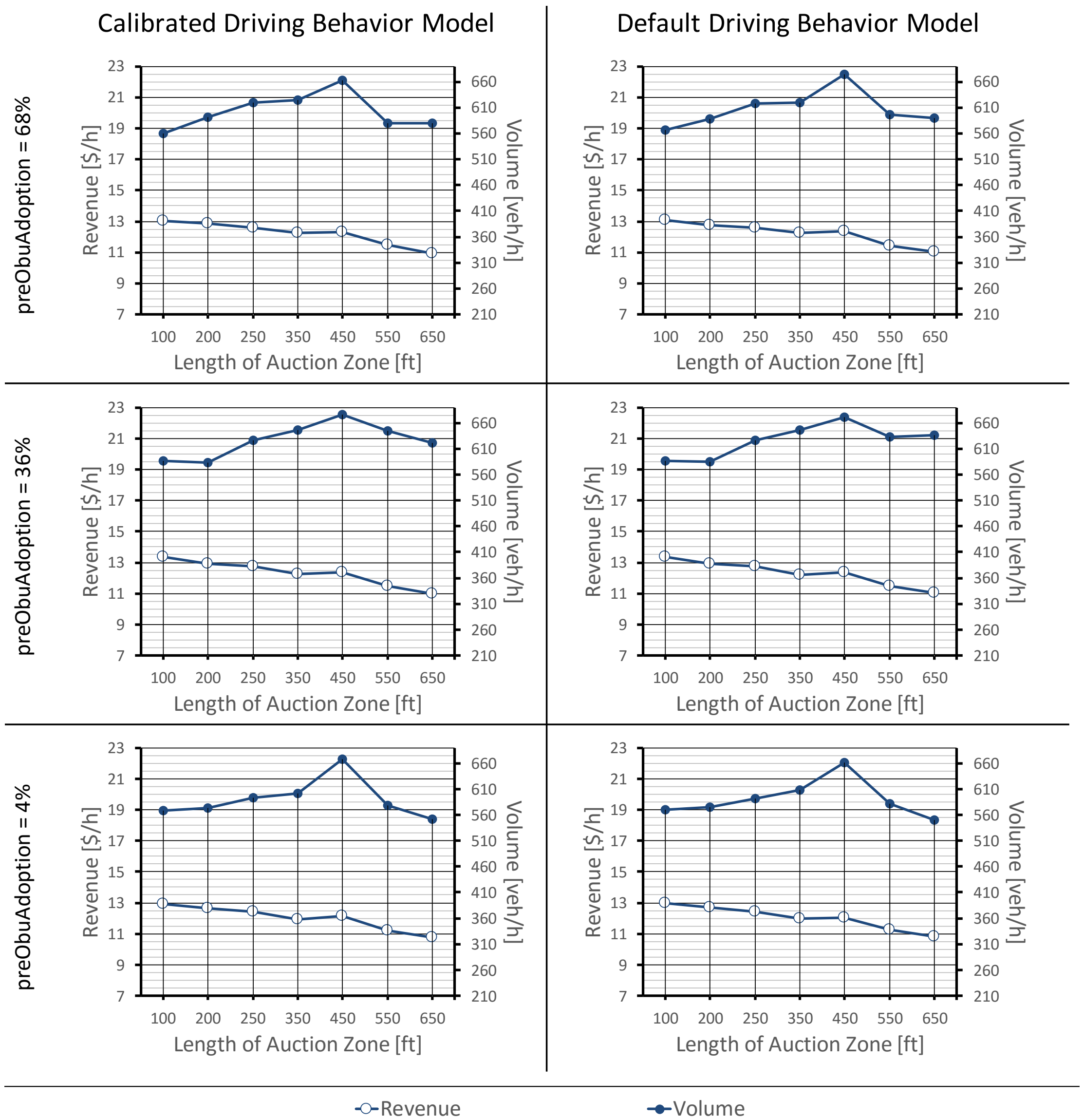


Figure 34. Comparison between results obtained for the calibrated model and an uncalibrated model, in terms of volume at HOT ramp (key point 3) and revenue collection. Mid-congestion on the arterial (case 6*V*), and three OBU pre-adoption rates are considered. Design parameter values used: hidden interval length = 5 seconds, auction zone length = 450 feet, partition method = greedy, d_C smoothing = linearized, reserve price = 0.05 dollars. Number of random seeds per result: 3.

6.2.3. *Impact on traffic congestion and revenue collection*

Figure 35 looks at how volume at the HOT ramp and revenue collection change with the adoption of the ABM system. Each cell is obtained from averaging nine seeds.

Con-version Rate	Pre-Adoption Rate	Volume ¶				Revenue †			
		No Auction	Auction (r = 0 ¢)	Auction (r = 1 ¢)	Auction (r = 5 ¢)	No Auction	Auction (r = 0 ¢)	Auction (r = 1 ¢)	Auction (r = 5 ¢)
25	68	105.6	258.2	256.0	239.3	5.85	6.09	7.1	11.09
	36	105.6	255.3	251.6	237.8	5.85	7.11	7.9	11.21
	4	105.6	251.8	250.9	236.0	5.85	6.92	8.0	11.73
Average		**105.6**	**255.1**	**252.8**	**237.7**	**5.9**	**6.7**	**7.7**	**11.3**
Percent Change			**142**	**139**	**125**		**15**	**31**	**94**
75	68	105.6	396.4	394.0	370.4	5.85	10.07	13.0	22.29
	36	105.6	399.3	394.7	370.7	5.85	10.47	12.6	22.45
	4	105.6	388.2	385.1	362.9	5.85	9.80	12.2	22.17
Average		**105.6**	**394.7**	**391.3**	**368.0**	**5.9**	**10.1**	**12.6**	**22.3**
Percent Change			**274**	**271**	**248**		**73**	**115**	**281**

¶ *Average volume at HOT ramp (HOVs and SOVs) [veh/h]*
† *Average revenue (in dollars) at HOT Ramp [$/h]*

Figure 35. Total volume and total revenue on key point 3 after applying ABM system to mid-congestion case $6V$. For each of the three cases, three OBU pre-adoption rates (4%, 36%, and 68%), and two conversion rates (25%, and 75%) are compared against the conversion rate of 0%. For the total revenue, three values for the reserve price (r) are considered. Design parameter values used: hidden interval length = 5 seconds, auction zone length = 450 feet, partition method = greedy, d_C smoothing = linearized. Number of random seeds per result: 9.

Figure 35 shows that when implementing the ABM system, the HOT ramp experiences an increase volume of 125% to 248% depending on the conversion rate that is assumed and on the reserve price that is adopted. These two factors also play an important role in the revenue collection which can increase from 15% to 281%.

Once again, these results show that the OBU pre-adoption rate does not play a significant role in affecting volume at the HOT ramp and revenue.

The results in Figure 35 suggest that there is always a revenue increase. But they always make the assumption that at least 25% of all SOVs would acquire the auction-enabled OBU. What if the following hypothesis were true? “If an ABM system is implemented, then the operator would collect less revenue because the only users who would acquire the auction-enabled OBUs would be those who already have an OBU, and therefore, the usage of the HOT lanes would be the same but its users would pay less.”

Perhaps the best way to verify that hypothesis is by looking at one of the worst case scenarios for the operator (in terms of revenue collection). One of the worst case scenarios would happen if instead of having an ABM system, the operator implements an RBM system with a reserve price of zero dollars. In such case, winners of the auction would enter for free instead of paying the toll price. In addition, assume that the SOVs who did not have an OBU remain without an OBU once the RBM system is implemented. Thus, it can be said that the “conversion rate for SOVs without an OBU” is 0%. And assume that 25% percent of the SOVs who already had an OBU switch to auction-enabled auctions (that is, they become advanced SOVs) because they are interested in paying a cheaper price. Figure 36 looks at this case and at the case in which the “conversion rate for SOVs without an OBU” is 25%.

Conversion Rate for SOVs		Pre-Adoption Rate	Volume ¶		Revenue †	
with OBU	without OBU		No Raffle	Raffle (r = 0 ¢)	No Raffle	Raffle (r = 0 ¢)
25	0	68	105.6	229.0	5.85	5.40
		36	105.6	188.4	5.85	5.87
		4	105.6	147.9	5.85	6.63
Average			**105.6**	**188.4**	**5.85**	**5.97**
Percent Change				**78**		**2**
25	25	68	105.6	259.1	5.85	6.02
		36	105.6	257.4	5.85	6.82
		4	105.6	257.7	5.85	7.02
Average			**105.6**	**258.1**	**5.85**	**6.62**
Percent Change				**144**		**13**

¶ *Average volume at HOT ramp (HOVs and SOVs) [veh/h]*
† *Average revenue (in dollars) at HOT Ramp [$/h]*

Figure 36. Total volume and total revenue on key point 3 after applying RBM system to mid-congestion case $6V$. For each of the three cases, three OBU pre-adoption rates (4%, 36%, and 68%) and two conversion rates are considered (0% and 25%), where the conversion rates look discriminately at SOVs who already had an OBU and those who did not. Design parameter values used: hidden interval length = 5 seconds, auction zone length = 450 feet, partition method = greedy, d_C smoothing = linearized. Number of random seeds per result: 9.

Focusing on the upper part of Figure 36, that is, when the conversion rate for SOVs who did not have an OBU is 0%, the results suggest that the revenue would stay relatively the same. The slight increase of 2% seems to be due to the aleatory nature of the simulation results. But when the conversion rate for SOVs who did not have an OBU is 25%, the operator collects more revenue. In all cases in the figure and as expected, the results show an increase in volume at the HOT ramp.

6.2.4. <u>Empirical cumulative distribution function</u>

The game-theoretic model requires knowledge of the cumulative distribution function F. The microsimulation model was used to generate an empirical distribution for that function. The empirical distribution will allow measuring regret in the next subsection. It was also needed for previous Subsection 4.5.3, where the practical consequences of the game-theoretic model were analyzed.

To obtain the empirical function F, the following method was used. For every OBU pre-adoption rate and for every minute, the fraction of SOVs that entered the HOT lanes versus those that entered the GP lanes was obtained from the historical traffic data. Then, during the simulation process, at every interval, the corresponding fraction value fed a random binary variable which decided which SOV would choose the non-auction zone and which one, the auction zone. This method eliminated the need to assume a certain distribution for the value of time or apply a mode choice model. Then, a histogram was obtained from the choices that the SOVs made under the toll prices that the ABM system had, and assuming that each driver's willingness to pay is always below 10 dollars. For example, if an SOV chose to take the GP lanes when the toll price was equal to 0.25 dollars, then the microsimulation model would assign randomly for that vehicle a private value between 0 and 0.24 dollars. But if it had chosen the HOT lanes instead, then it would have been assigned a private value between 0.25 and 9.99 dollars. The resulting histogram was approximated by a truncated Weibul distribution using the function "EstimatedDistribution" in the application Mathematica 10.2 (and using its default parameters). The resulting histogram, probability distribution function, cumulative distribution function, and parameters are presented in Figure 37.

Trucated Weibull Distribution	
minimum	= 0.0000
maximum	= 100.0000
shape parameter	= 0.7174
scale parameter	= 0.2480

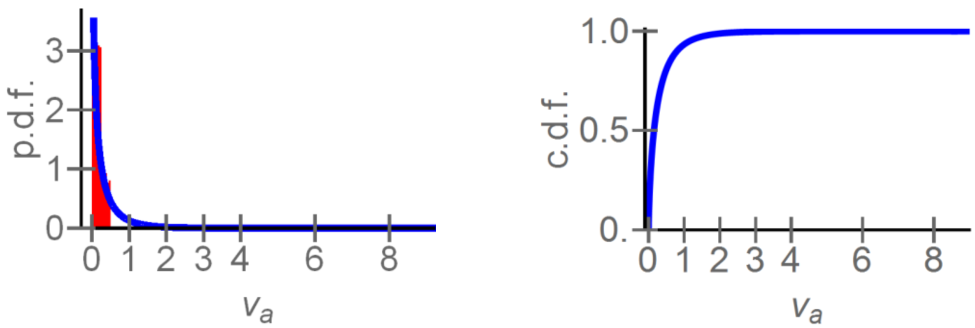


Figure 37. Left figure: probability distribution function and histogram of the data obtained at HOT facility. Right figure: resulting cumulative probability distribution (or function F). Box on top: the parameters of the distribution. Number of random seeds per sample point: 4.

The methodology adopted released a probability distribution that is skewed to the left. A lower value for the parameter "maximum" may reduce the skewness.

6.2.5. <u>Regret for not playing the optimal strategy</u>

The results obtained so far assume that advanced SOVs always adopt the safest strategy. But it is natural to ask whether some of them may experience regret and if so, how great that regret may be. The latter question is harder to respond given that it is difficult to give a practical description to a difference in utility. But the former question is indeed addressed in this subsection.

For the comparison that is analyzed in this subsection, the microsimulation model measures regret using equations (26) and (27), where a player a is adopting the safe strategy. Equation (26) focuses on the regret that player would experience when its private value v_a is equal to or greater than its cutoff (which is equal to the price). In other words, the equation focuses on the event that the safest strategy indicates to her that she should buy.

$$\tilde{r}\{v_a \geq p\} = \begin{cases} u(v_a - \max\{y, r\}; \propto) - u(v_a - p; \propto), \text{if } c^* < p \wedge v_a > y \\ u(v_a - \max\{y, r\}; \propto) - u(v_a - p; \propto), \text{if } c^* < p \wedge v_a = y \wedge \text{"}a \text{ wins"} \\ u(v_a - p; \propto), \text{if } c^* < p \wedge v_a = y \wedge \text{"}a \text{ loses"} \\ u(v_a - p; \propto), \text{if } c^* < p \wedge v_a < y \\ 0, \text{if } c^* = p \end{cases} \quad (26)$$

where $\tilde{r}$ is the regret for not choosing the optimal strategy, y is the highest value among the players that do not win the auction, "a wins" refers to the event of winning a random selection when player a's private value is equal to y, and "a loses" refers to the event of losing the random selection when her private value is equal to y.

Because the optimal cutoff c^* is never higher than p, then, when the safest strategy instructs to wait, the optimal strategy would never instruct to buy. Therefore, the regret after waiting is defined by the following simple expression:

$$\tilde{r}(v_a < p) = 0 \quad (27)$$

The microsimulation model calculated for each advanced SOV using equations (26) and (27). It also had to anticipate an estimation of the variable $\bar{k}$ and $\bar{a}$ in order to compute the optimal c^*. The results obtained are presented in Table 7.

Table 7. For an OBU pre-adoption rate of 100%, a conversion rate of 75% and different risk-averseness levels, percentage of advanced SOVs that experience regret when adopting the safest strategy instead of the optimal strategy. Also shown in table, certainty equivalent payment for an advanced SOV whose private value v_a is equal to 4 dollars. Parameter values used: hidden interval length = 5 seconds, auction zone length = 450 feet, partition method =greedy, d_C smoothing = linearized, reserve price = 0.05 dollars. Number of random seeds per result: 4.

Risk Averseness $(1/\propto)$	**Certainty Equivalent Payment** $\tilde{\delta}(v_a = 4)$	**Regret (RBM)**	**Regret (ABM)**
0.13	2.25	0.00%	0.07%
0.26	2.50	0.00%	0.07%
0.61	3.00	0.00%	0.07%
2.77	3.75	0.00%	0.07%

Table 7 shows that a very small fraction of advanced SOV experience regret. It also suggests that measuring the regret (and not the instances of having regret) is less relevant given that the number of instances is so small. To try to give a more practical sense of what each level of risk averseness signifies, the table presents the certainty equivalent payment for each level, for a player whose private value (v_a) is equal to 4 dollars.

Table 7 has one column that is not very important but which was included in order to give a more practical sense of what each level of risk averseness signifies. This column is the one that presents an example of what the certainty equivalent payment (CEP) would be for each of those levels. Instead of relying on Definition 3 (page 87) for defining CEP, Table 7 relies on the broader definition used in the terminology section (Section 1.1) where the lottery does not have to be specifically an extended Vickrey auction. This explains why the table uses the symbol $\tilde{\delta}$ instead of δ. Recall that within the simulation, the number of items and rivals varies. Therefore, players in the simulation face different specific extended Vickrey auctions. Due to this variation, Table 7 cannot use them for showing a standard relation between risk averseness and CEP. Table 7 calculates the CEP from a more straightforward example. Suppose that an individual is faced between giving away four dollars and keeping the money in her pocket, where each of the two outcomes has the same probability. The CEP for this lottery would be obtained from equation (29), which derives from equation (28), assuming that v_a is equal to 4 dollars.

$$u\left(v_a - \tilde{\delta}; \propto\right) = \tag{28}$$

$$\frac{1}{2}(\text{"utility of giving 4 dollars"}) + \frac{1}{2}(\text{"utility of keeping the money"})$$

$$u\left(v_a - \tilde{\delta}; \propto\right) = \frac{1}{2} u(v_a - v_a; \propto) + \frac{1}{2} u(v_a - 0; \propto)$$

$$u\left(v_a - \tilde{\delta}; \propto\right) = \frac{1}{2} u(v_a; \propto)$$

$$\frac{1 - e^{-\propto\left(v_a - \tilde{\delta}\right)}}{\propto} = \frac{1 - e^{-\propto(v_a)}}{2 \propto}$$

$$1 - e^{-\propto\left(v_a - \tilde{\delta}\right)} = \frac{1}{2} - \frac{e^{-\propto(v_a)}}{2}$$

$$e^{-\propto\left(v_a - \tilde{\delta}\right)} = \frac{1}{2} + \frac{e^{-\propto(v_a)}}{2}$$

$$\propto \left(\tilde{\delta} - v_a\right) = \log \frac{1 + e^{-\propto(v_a)}}{2}$$

$$\tilde{\delta} = v_a + \frac{1}{\propto} \log \frac{1 + e^{-\propto(v_a)}}{2} \tag{29}$$

Thus, using this example, when Table 7 refers to individuals with an $\propto$ equal to 0.13, it is referring to individuals who would be willing to give away 2.25 dollars for sure instead of running the risk of having to give away four dollars.

6.2.6. *Most likely auction scenarios*

Initial results suggested that the game-theoretic model had an optimal cutoff c^* that was almost identical to the price or that it could be easily approximated by one or two straight lines in a c^* versus p plot (see for example Figure 19, a curve that was obtained by Olarte and Haghani in 2013). But it was later noticed that such approximations were only valid for auctions or raffles that had more than 7 items (that is, $\bar{a} \geq 7$), and certain specific $\bar{k}/\bar{a}$ ratios. For this reason, it became important to have better knowledge of what the most probable auctions or raffles are in terms of number of players and number of items.

Figure 38 focuses on the low-congestion case ($1V$) and looks at what the most probable number of players in an RBM system that has a $\bar{k} = 1$ partition method. The results should give a very good picture of what would happen in an ABM system that also had a $\bar{k} = 1$ partition method. The figure suggests that it is very unlikely to have more than one player. At the same time, the figure indicates that roughly 30 percent of the times, the raffle does not occur. To say that a raffle does not occur means that there is at least one player but there are no items because there is not unused managed capacity at that particular instance. If the $\bar{k} = 1$ partition method were to be changed to the greedy method, it would be reasonable to expect that entrance would be almost guaranteed (if there is unused managed capacity at that instance) because raffles, and therefore auctions, would have more items.

Conver-sion Rate	OBU pre-adoption rate	Frequency of Raffle Occurrence	Frequency of Having this Number of Players in Set $\mathbb{A}$ in Raffle				
			1	2	3	4	>4
25	68	72.2%	98.9%	1.1%			
	36	73.8%	100.0%				
	4	71.0%	98.7%	1.3%			
75	68	72.0%	95.3%	4.7%			
	36	71.2%	98.4%	1.6%			
	4	69.4%	97.4%	2.2%	0.5%		
	Average	**71.6%**	**98.1%**	**1.8%**	**0.1%**	**0.0%**	**0.0%**

	10th Percentile‡	65.6%	94.7%	0.0%	0.0%	0.0%	0.0%
	50th Percentile‡	70.3%	100.0%	0.0%	0.0%	0.0%	0.0%
	90th Percentile‡	77.7%	100.0%	5.1%	0.0%	0.0%	0.0%
	100th Percentile‡	91.7%	100.0%	7.1%	1.9%	0.0%	0.0%

‡ percentile obtained from all simulated frequencies (not from the averages appearing in rows above)

Figure 38. Frequency of occurrence of raffles, and frequency of number of players within each raffle after applying RBM system to low-congestion case ($1V$). For each frequency, three OBU pre-adoption rates (4%, 36%, and 68%) and two conversion rates (25%, 75%) are considered. Design parameter values used: hidden interval length = 7 seconds, auction zone length = 580 feet, partition method = "$\bar{k} = 1$", d_C smoothing = none, reserve price = 0.00 dollars. Number of random seeds per result: 4.

Figure 39 focuses on the mid-congestion case ($6V$). Here, the chances of having more than one player are still low but significant. But most of the times, the raffle does not occur. Again, if the $\bar{k} = 1$ partition method were to be changed to the greedy method, the entrance would be very high because raffles, and therefore auctions, would have more items. In the mid-congestion case, the highest risk is not about losing a raffle or auction. It is about entering a raffle or auction but not being granted access because there is no unused managed capacity.

Conver-sion Rate	OBU pre-adoption rate	Frequency of Raffle Occurrence	Frequency of Having this Number of Players in Set $\mathbb{A}$ in Raffle				
			1	2	3	4	>4
25	68	40.4%	90.8%	7.8%	1.4%		
	36	38.8%	91.7%	7.3%	1.0%		
	4	40.0%	91.2%	7.8%	1.0%		
75	68	37.5%	76.1%	16.7%	5.5%	1.2%	0.6%
	36	37.7%	74.2%	17.1%	6.0%	2.5%	0.2%
	4	37.5%	75.6%	15.5%	6.4%	2.1%	0.4%
	Average	**38.6%**	**83.3%**	**12.0%**	**3.5%**	**1.0%**	**0.2%**

10th Percentile‡	34.9%	73.7%	4.9%	0.0%	0.0%	0.0%
50th Percentile‡	38.5%	80.1%	13.3%	2.9%	0.0%	0.0%
90th Percentile‡	43.0%	95.1%	17.8%	7.5%	2.9%	0.8%
100th Percentile‡	43.4%	98.1%	19.5%	9.3%	4.2%	0.9%

‡ percentile obtained from all simulated frequencies (not from the averages appearing in rows above)

Figure 39. Frequency of occurrence of raffles, and frequency of number of players within each raffle after applying RBM system to mid-congestion case ($6V$). For each frequency, three OBU pre-adoption rates (4%, 36%, and 68%) and two conversion rates (25%, 75%) are considered. Design parameter values used: hidden interval length = 7 seconds, auction zone length = 580 feet, partition method = "$\bar{k} = 1$", d_C smoothing =none, reserve price = 0.00 dollars. Number of random seeds per result: 4.

It is reasonable to assume, based on the results of Figure 39, that when having a system with a greedy partition method, the most likely scenarios, in terms of number of items and players, are the following $\{\bar{k} \geq \bar{a}, \bar{a}\}$, $\{\bar{k} = 2, \bar{a} = 3\}$, $\{\bar{k} = 1, \bar{a} = 2\}$, $\{\bar{k} = 1, \bar{a} = 3\}$, and $\{\bar{k} = 0, \bar{a}\}$ where the first pair and the last pair are the two most likely scenarios.

6.3. Recommended values for design parameters

Based on the results obtained in Table 6 for the hidden interval, based on the sensitivity to the capacity release smoothing and to the auction zone length, and based on how the reserve price impacts revenue significantly in some cases (as suggested by Figure 36), Table 8 presents recommended values for the design parameters.

Table 8. Design parameters of the ABM (or the RBM) system, recommended values for HOT entrance as presented in Figure 25, and possible range of feasible values.

Design Parameter	Recommended Value	Possible Range of Feasible Values
MC interval length	1 minute	30 seconds to 1.5 minutes, less than the interval used for calculating the toll price
hidden interval length	5 seconds	1 to 8 seconds
auction zone length	450 feet	80 to 650 feet
time resolution	$^{1}/_{10}$ seconds	$^{1}/_{10}$ seconds to 2 seconds
partition method	greedy	Any partition method that doesn't group non-adjacent vehicles
capacity release ($\boldsymbol{d_C}$) smoothing	None for low volume, linearized for other volume levels	None, linearized, quadratic, etc.
Reserve price ($\boldsymbol{r}$)	Greater than zero if having an RBM system.	Lower than the toll price (p)*

**The difference between the r and p should be such that the average user perceives a real difference between paying one amount or the other.*

The recommendations for the two parameters not mentioned above are based on the following analysis. The length of one minute for the MC interval is recommended because it is shorter than the three minutes used today for changing the toll price. If using an even shorter interval such as 20 seconds, the potential gains may be lost with the inaccuracy of estimating the managed capacity. The time resolution of one tenth of a second corresponds to the maximum resolution that the application VISSIM 5.4 (PTV Planung Transport Verkehr, AG 2012) can handle. A higher resolution does not seem to be necessary.

7. Conclusion

This dissertation verified whether the following two hypotheses were true: (1) present-day HOT lanes have unused managed capacity; (2) this unused managed capacity can be sold successfully to users by adding a bid option to the current pay-to-enter mechanism. As mentioned in the introduction, if HOT lanes have unused managed capacity, then other categories of dedicated lanes have unused managed capacity too such as HOV lanes, and dedicated bus lanes. This chapter presents the general conclusions regarding the validity of the two hypotheses. It then presents specific conclusions regarding the auction-based metering (ABM) system that is proposed as evidence of the validity of the second hypothesis. And it finalizes by addressing questions that were supposed to be outside the scope of the project but became answered as a byproduct of the development of this dissertation.

7.1. General conclusions

Regarding the first hypothesis, Chapter 3 quantified the unused managed capacity (u_{MC}) on the Interstate 394 MnPass Express Lanes in Minnesota, on the westbound direction of the so-called "reversible section". This process required the use of traffic microsimulation software. One typical weekday of operations was considered for carrying out this quantification. The dissertation found that the section of study has a managed capacity (m_{C}) of 666 vehicles per hour during the whole period of analysis. From that amount, 98% is not being unused. In other words, it has an unused managed capacity of 652 vehicles per hour on average. When focusing on the congestion peak subperiod, the managed capacity and the unused managed capacity are insignificant due to the vehicles that are already on the HOT lanes from upstream entrances. One big caveat about this result is that the calculation of the managed capacity can be improved if more time is invested in conducting traffic simulations. Therefore, it is to expect that in a better analysis, the unused managed capacity during the congestion-peak subperiod is significant. The chapter revealed that a much more important quantity than the unused managed capacity is the potential volume increase (Δq) which indicates the number of time slots that can actually be sold (via a new system such as the ABM) given the existing volume. If expressed as a percentage of the current HOT users, the potential volume increase ranges from 18% to 93% depending on the factor ρ_1 that is assumed. The factor ρ_1 is the maximum fraction of total vehicles, at the entrance that feeds the HOT and GP lanes, that would choose to enter the HOT lanes at a given time interval.

Regarding the second hypothesis, Chapters 4 to 6 proposed an auction based metering (ABM) system that in effect allows to sell the unused managed capacity. Specifically, Chapter 4 proposed a strategy for users to follow, Chapter 5 described the whole system from an operational perspective (including a user's perspective), and Chapter 6 looked at the effectiveness of the system in terms of congestion reduction and revenue increase. Chapter 6 also looked at how sensitive the system is to certain design parameters and to traffic behavioral parameters, and provided likely scenarios that allowed Chapter 4 to provide practical recommendations in regards to the mathematical model. The seven main characteristics of the proposed ABM system are the following:

1. Although the auction constitutes the main novelty of the system, it does not remove the possibility of paying the toll price in order to have immediate access to the HOT lanes. For this reason, the system is in fact a buyout auction, where the term “buyout” refers to the fact that registered users can skip the auction by paying the toll price.
2. The ABM regulates (meters) the vehicles that enter the HOT lanes using two mechanisms. One is the existing pay-to-enter mechanism that increases or decreases the displayed toll price in order to limit or increase the usage of the HOT lanes. The other one is the auction that only allows highest bidders to enter the HOT lanes. These two components of the metering system assure that the HOT lanes always provide a reliable minimum level of service.
3. The auction is a multiple unit auction with single-unit demand, or for short, a “single-unit demand auction”. This means that it offers several identical items (or entrance slots) at once but bidders are only allowed to get one item.
4. The auction can be considered as a generalization of the standard Vickrey auction for single-unit auctions. As with a standard Vickrey auction, the proposed auction (referred in this dissertation as “extended Vickrey auction”) has silent bids and the amount of money that each of the winners are to pay is not equal to what they silently declare. It is in fact lower and it is equal to the highest bid amount among the losers.

5. The resulting extended Vickrey auction with buy option (referred throughout the dissertation simply as “buyout auction”) has not been analyzed by the academic literature and perhaps has never been applied. But it can be easily explained to the public as an auction where the highest bidders are granted access to the HOT lanes after paying at most what they declared to pay. And drivers who choose to pay the toll price, can skip the auction because they get immediate access to the HOT lanes.
6. Participation in the auctions is voluntary.
7. The goal of reducing congestion prevails over the goal of increasing revenue.

These seven characteristics are in line with the last six of the eight adopted policies presented in Section 4.1.

It is important to clarify that because the system always guarantees the right to enter the HOT lane if paying the toll, the system follows the following rule: if a vehicle pays the toll, she takes away one auction item; and if there are no auction items left, then the next time a vehicle pays the toll, she will take an item away from the game that follows.

Chapters 4 to 6 also looked at the possibility of implementing a raffle-based metering (RBM) system instead of an ABM system. Raffles operate like auctions in which all bidders bid the same amount (or in other words, where winners are simply obtained by random selection). An RBM system is useful to analyze because it is operationally very similar and quicker to deploy (making it suitable as an interim solution), and because it allowed looking in detail (in Subsection 6.2.3) at possible revenue changes.

As explained in the next subsection (Subsection 7.2), Chapters 4 to 6 allow drawing the following three general conclusions concerning the question of whom would benefit from the system:

1. ABM would benefit public operators by reducing unused managed capacity.
2. ABM would likely benefit work-to-home trips (or other trips of low reliability needs), by offering a new alternative (without degrading its targeted level of service to current users).
3. ABM could benefit private operators by increasing revenue.

The above three general conclusions are in line with the arguments proposed in the introduction chapter (Section 1.2).

7.2. Specific conclusions regarding auction-based metering

This section explains the findings in chapters 4 to 6 that give support to the three general conclusions aforementioned regarding the beneficiaries of auction-based metering. It also presents other more specific conclusions that can be drawn from those chapters.

Chapter 4 presents a game-theoretic model that defines the rules of the game, allows determining the optimal strategy (by finding the Bayesian Nash equilibrium), and allows estimating how much registered users would lose from not adopting that strategy. The chapter introduces the terms “advanced SOV” and “basic SOVs” to distinguish registered from unregistered users. “Advanced SOVs” and “basic SOVs” are single occupant vehicles that carry a technological device (also known as onboard unit or OBU) that allows them to pay the toll price. But unlike basic SOVs, the OBU that advanced SOVs carry also allows them to participate in the auction. The chapter considered not only ABM systems but also RBM systems. The game-theoretic model in both systems suggests that the optimal strategy can be defined in terms of a variable referred as cutoff or c. Chapter 4 proved (by referring to Appendix D) that each model has a unique Bayesian Nash equilibrium. This equilibrium is used to compute the optimal cutoff c^*. The computation and main characteristics of this unique optimal strategy are presented in Proposition 1 (page 80) for the buyout raffle of an RBM system and in Proposition 2 (page 89) for the buyout auction of an ABM system. The optimal strategy is similar to another strategy, the so-called “safest strategy”, which this research found to be very important. The safest strategy is defined as follows: if the willingness of the driver to pay the toll price is equal to or above the toll price p, then she always chooses to pay the toll price (to buy); otherwise, she always chooses to play the auction (to wait). Likewise, the cutoff strategy has the same definition with the exception that instead of basing the decision on p, the decision is based on c^*, where c^* is equal to or greater than p. The three drawbacks that the model present is that the computation of c^* requires (1) that the driver knows in real time the number of rivals $\bar{a}$ and the number of auction (or raffle) items $\bar{k}$, (2) that the level of risk averseness is equal among all players, and (3) that the computation of c^* is not trivial. The last drawback can be easily overcome by having high discretization of toll prices (such is the case in Minneapolis where tolls are in multiples of 25 cents). In such case, the computation of c^* can be replaced with perhaps a small look-up table. The second drawback could be avoided only if a field survey shows that all users in the population or within few broad segments of the population have in fact the same and constant level of risk averseness. If this risk averseness is not the same, a more robust mathematical model should be conceived. The first drawback cannot be easily overcome because, as described in Chapter 5, anticipating $\bar{a}$ and $\bar{k}$ is an intrinsic challenge in the proposed ABM system. In fact, the system benefits from this challenge. Given this difficulty, and relying on sensitivity analysis, Subsection 4.5.3 recommends adopting one of these three alternatives:

(1) The operator should recommend advanced SOVs to adopt the safest strategy.

(2) The operator should maintain a consistent $\bar{k}/\bar{a}$ ratio depending on the time of the day and recommend the optimal strategy for that ratio. Maintaining a consistent $\bar{k}/\bar{a}$ ratio would imply artificially increasing $\bar{a}$ or altering $\bar{k}$ as needed for every game.

(3) The system should be modified in order to allow drivers to know the different variables $\bar{k}$ and $\bar{a}$ in advance.

This dissertation assumes that the first recommendation is the preferred one given that the results in Chapter 6 suggest that if all advanced SOVs adopt the safest strategy, less than 0.1% of them would experience some level of regret.

Chapter 5 described the proposed ABM system from an operational perspective. Although the system can only be applied to two-lane arterials that allow direct entrance to HOT lanes, it does not require overhauling the existing facility. And it leaves room for possible expansion to wider arterials (due to the robustness of the so-called "reservation rule") and possible interaction with other arterials (by addition of traffic signals). Other key features of the system are the noninterference with the existing mechanism used for calculating the toll price; the metering of inflow at time intervals that are smaller than those required by the calculation of the toll price and by the calculation of the managed capacity; and the conservation of the existing pay-to-enter mechanism for SOVs who are not interested in having auction-enabled OBUs. In regards to the toll price, because the system does not interfere with the existing mechanism, the toll price still changes with the number of vehicles, just as today's HOT facilities do. But while this price-based metering happens at every "MC interval" (or at longer intervals), the ABM system executes auctions at much smaller intervals known as "hidden intervals" (usually less than 9 seconds). Finally, it is important to highlight that the existence of hidden intervals, and the variable number of auctions that are carried out at every hidden interval has one good advantage from the operational point of view but one drawback from the mathematical point of view. From the operational point of view, these two features prevent advanced SOVs from knowing who exactly they are competing against. In consequence, they will find no incentive in accelerating or decelerating (or in changing lanes if the system is scaled up to more lanes). And the variable number of auctions guarantees that drivers who cannot arrive at the same time to the HOT ramp are not included in the same auction. But from the mathematical point of view, the system cannot anticipate the number of players and items due to the apparent complexity of the partition problem that it needs to solve. Thus, anticipating $\bar{k}$ and $\bar{a}$ is indeed an intrinsic challenge of the ABM system from the mathematical point of view.

Chapter 6 described the microsimulation model that was applied to the entrance in Minneapolis in order to observe how the ABM system behaves in terms of congestion reduction and revenue changes, assuming that vehicles always adopt the safest strategy. These two measurements provided answers to the important questions of how the current public operator and a hypothetical private operator would benefit. Specifically, Subsection 6.2.3, Figure 35 suggests that ABM, when having mid-congestion, reduces unused managed capacity considerably by allowing 125% to 248% more vehicles to enter the HOT lanes (better results would be obtained when lowering the reserve price of 0.05 dollars). The same figure suggests an increase in revenue by 15% if only 25% of current SOVs become advanced SOVs and if having a reserve price of 0.00 dollars. The figure also suggests much higher increases if more SOVs become advanced SOVs or if having higher reserve prices. Thus, the figure suggests that the general conclusion of having a public operator that would benefit from ABM by congestion reduction is true. The figure also suggests that the general conclusion of having a private operator who would benefit by having revenue increase is true. But the following figure (Figure 36) looked at an extreme case that may observe no revenue changes or perhaps revenue decrease. Such extreme case happens when implementing an RBM system instead of an ABM system, when none of the SOVs who did not have an OBU become advanced SOVs, and when only 25% of SOVs who had an OBU become advanced SOVs.

Regarding the low congestion case, this dissertation adopted a less systematic approach in order to simplify the analysis. Figure 32 suggests that indeed there is an increase in the traffic volume that pass by the HOT ramp for the low-congestion case. And in terms of revenue, the results for the mid-congestion case suggest that the revenue should also increase in the low congestion case unless an RBM system is adopted instead of an ABM system and unless no new SOVs obtain an OBU.

The sensitivity analyses of the impacts on congestion and revenue that Chapter 6 presented indicate that the “OBU pre-adoption rate” (percentage of SOVs that were already using the HOT lanes) does not play a major role in affecting the results. The “conversion rate” (percentage of SOVs that switched to an auction-enabled OBU) has a much greater influence. Only when analyzing revenue for RBM systems with no reserve prices, the OBU pre-adoption rate had certain influence. It is also important to note about these results that if, besides the microsimulation model, one also models the mechanism that generates prices in response to the congestion on the HOT lane, results may improve in terms of having more vehicles on the HOT lanes. Results would also show higher toll prices (if adjustments are not made to such price mechanism). But it is not clear if results would also show higher revenue.

Chapter 6 found that the impacts to congestion and revenue were sensitive to the design parameters. Therefore, it recommended design values for a specific HOT entrance (see Table 8). When adopting the recommended design values, Chapter 6 did not find sensitivity to the behavioral parameters that were fed into the traffic microsimulation component.

Chapter 6 also found that the regret that advanced SOVs would experience from choosing the safest strategy instead of the optimal strategy is miniscule.

Finally, Chapter 6 was instrumental in characterizing the size of the buyout auctions in terms of number of auction items and number of players. These measurements were not obtained directly from an ABM system but indirectly from an RBM system when applied to the same HOT facility. Subsection 6.2.6 suggests that the most likely combinations of number of players $\bar{a}$ and number of slots $\bar{k}$ can be limited to just five: $\{\bar{k} \geq \bar{a}, \bar{a}\}$, $\{\bar{k} = 2, \bar{a} = 3\}$, $\{\bar{k} = 1, \bar{a} = 2\}$, $\{\bar{k} = 1, \bar{a} = 3\}$, and $\{\bar{k} = 0, \bar{a}\}$ where the first one and the last one are the two most likely scenarios. This result was useful in conducting sensitivity analysis of the game-theoretic model and led to the three recommended alternatives indicated above in this chapter (page 151).

7.3. Answers that were originally outside of the scope

At the start of this research, it was decided that the following questions were outside of the scope of the dissertation. Nonetheless, answers were obtained as a byproduct of the development of this dissertation:

(1) What probability distributions best describe the private values of the users? Answer: The microsimulation model developed for Chapter 6 used a methodology for estimating the private values. It generated an empirical cumulative distribution based on the choices that the SOVs made under the historical toll price values. The method is explained in Subsection 6.2.4. Nonetheless, this method created a curve that was perhaps too skewed to the left because there were not any high toll prices during the period of analysis.

(2) Which specific design should be employed in order to have a successful system? Answer: The results and tests obtained during the development of this dissertation evolved into a recommendation of specific design parameter values in Section 6.3. Of the seven parameters, one is directly related to geometric design.

(3) How does the mathematical model change if considering discrete prices instead of continuous prices? Answer: As suggested by Figure 19, the recommended curve that describes the optimal strategy could still be applied if having discrete prices. In fact, having discrete prices would allow replacing the curve with a simple look-up table. Nonetheless, the mathematical model behind the curve relies on the assumption that two players cannot have the same private value (because that probability is equal to zero when using continuous probability functions). Therefore, the idea of developing a model that assumes discrete private values may be an attractive one.

8. Further Research

Regarding the game-theoretic model, the following research ideas are worth pursuing:

(1) Provide a formal verification that the safest strategy is equivalent to a "minimax regret" strategy or a "maximin" strategy.

(2) Expand the model to one that includes the uncertainty in the number of vehicles and the number of slots. Perhaps use the current model as one that describes a subgame within a larger subgame.

(3) Verify whether less restrictive assumptions can be made in regards to the level of risk averseness, especially, allowing it to be different among the population.

Regarding the microsimulation model, results would be more reliable with better field data, including surveys on how users would use the system. Results regarding revenue change would also be more reliable if in addition to the microsimulation model, the mechanism that is used today for generating tolls is also modeled. Ideally, the toll generation should not simply mimic today's mechanism but should also provide an expected managed capacity value for each toll value.

Regarding the revenue effects, it would be interesting to simulate cases where basic SOVs pay the toll on a certain day at the expense of not being able to pay it on the following days. This may show that ABM can increase revenue even when the reserve price is zero.

All the recommendations just mentioned assume preserving the ABM system as described in Chapter 5. Nonetheless, that system could be adapted to other entrances or HOT facilities. It could be improved by adapting it to more lanes (taking advantage of the robustness of the so-called “reservation rule”), and by allowing it to interact with other incoming arterials (through addition of traffic signals or by forcing vehicles to stop when they lose the auction). It could also be improved by considering HOT corridors with multiple exits which would require the operator to estimate the origin-destination matrix at different time intervals. This extension could also include several ABM entrances. Similar challenges would arise when considering HOT networks.

As stated in the introduction, although not essential, technologies such as satellite tolling, vehicle-to-vehicle communication, and autonomous vehicles could improve the proposed ABM system. Therefore, testing such technologies for ABM can be encouraged. Improvements can also be made while looking at how other trends would benefit from ABM such as HOT and managed networks, greater expectation on travel time reliability, and mechanisms for reducing distracted driving.

Instead of looking at how autonomous vehicles and connected vehicles can improve ABM, one can also think of how ABM could improve those domains. For example, there is current interest in applying autonomous and connected vehicles to street intersections in order to remove traffic lights and stop signs. One of the goals of ABM is to keep a vehicle stream maintain a certain level of service over others. This goal could be applied to current research in street intersections when priority is given to one direction (or set of lanes) over the other ones. Also, current research does not look at the possibility of guaranteeing access to drivers who pay a fix toll. Poole and Swenson (2012) do propose how to guarantee this kind of direct access but without optimizing the calculation of the toll price or without relying on the latest technologies. Therefore, the proposed dissertation could be used to improve their intersections by including auctions (which may require vehicle-to-infrastructure communication, and perhaps connected autonomous vehicles).

Appendix A.Dedicated Lanes: A Background

This appendix explains the concepts of high-occupancy vehicle (HOV) lanes, high-occupancy/toll (HOT) lanes, and dedicated bus lanes. These three types of lanes have been referred in this dissertation as dedicated lanes. In addition, this appendix presents in detail the HOT facility that was used for developing the microsimulation model, that is, the I-394 MnPass Express Lanes. Also the concept of managed lanes is not covered in this dissertation, this appendix ends by comparing it to dedicated lanes.

A.1. High-occupancy vehicle lanes

For decades, traffic officials have adopted the approach of dedicating some lanes of highways exclusively to vehicles with two or more occupants (also known as high-occupancy vehicles or HOVs). Solo drivers (also known as single-occupant vehicles or SOVs) are not allowed to use these lanes. In this manner, transportation authorities have had the expectation that by encouraging people to carpool, HOV lanes would reduce traffic congestion. Sometimes, HOV lanes are subcategorized into HOV-2+ or HOV-3+. HOV-3+ lanes refer to those lanes that only accept vehicles with at least three passengers. Likewise, HOV-2+ lanes only accept vehicles with at least two passengers. Making sure that solo drivers do not enter the HOV lanes is usually enforced by the police. The main goal is to reduce congestion by discouraging people from driving alone and offering a more reliable time to those who carpool.

A.2. High-occupancy/toll lanes

Although HOV lanes constitute a valid approach for reducing congestion, it has been observed that HOV lanes have in many cases “unused managed capacity”, that is, empty space (or available slots) that could be occupied by additional vehicles and still offer a more reliable travel time. Because the term “unused managed capacity” is introduced in this dissertation, the literature uses instead other less specific terms to refer to this concept. For example, the Federal Highway Administration uses the expression “excess capacity available”. And they explain that its existence in the HOV lanes is due to the following reason: “because [occupancy requirements] must be set at whole numbers, the [HOV] lanes often wind up with significant excess capacity available” (Federal Highway Administration 2008, p.16).

Due to the existence of unused managed capacity, during the last decade, traffic officials started to implement high-occupancy/toll (HOT) lanes. HOT lanes, like HOV lanes, allow access to HOVs. In addition, HOT lanes allow access to solo drivers. But unlike HOV lanes, HOT lanes require solo drivers to pay a toll by using an in-vehicle device, also known as onboard unit (OBU). This toll does not require traditional toll booths.

The first HOT lanes were built in 1998 but their objective was not to reduce unused managed capacity. These first HOT lanes, the “SR-91 Express Lanes” in Orange County, California, were adopted as a means to repay private investors who built these roads. But after the implementation of these HOT lanes, a trend did start of converting existing HOV lanes to HOT lanes in order to make better use of their capacity. Other HOT lanes were the result of building additional lanes to existing highways that did not have HOV lanes. Figure 40 provides a list of most of the HOT-lane systems operating by 2013. All HOT facilities that are mentioned in this chapter appear in that figure.

Figure 40. HOT facilities in the United States operating by the end of 2013. Source: (Perez 2013)

A.2.1. <u>Dynamic tolling systems</u>

Besides providing free access to HOVs and tolling solo drivers, the second most important feature in a HOT lane is that the toll rate changes throughout the day in response to the level of congestion. When the demand for the HOT lane increases, the toll rate rises. When the demand for the HOT lane decreases, the toll rate decreases. In some HOT lanes, such as the SR-91 Express Lanes, the price changes at few points in the day following a pre-established schedule. Figure 41 provides an example of this schedule for the SR-91 for the year 2011.

91 Express Lanes — **Toll Schedule** *Effective January 1, 2011* — **Eastbound** *SR-55 to Riverside Co. Line*

	Sun	M	Tu	W	Th	F	Sat
Midnight	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
1:00 am	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
2:00 am	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
3:00 am	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
4:00 am	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
5:00 am	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
6:00 am	$1.30	$2.05	$2.05	$2.05	$2.05	$2.05	$1.30
7:00 am	$1.30	$2.05	$2.05	$2.05	$2.05	$2.05	$1.30
8:00 am	$1.65	$2.05	$2.05	$2.05	$2.05	$2.05	$2.05
9:00 am	$1.65	$2.05	$2.05	$2.05	$2.05	$2.05	$2.05
10:00 am	$2.50	$2.05	$2.05	$2.05	$2.05	$2.05	$2.50
11:00 am	$2.50	$2.05	$2.05	$2.05	$2.05	$2.05	$2.50
Noon	$3.00	$2.05	$2.05	$2.05	$2.05	$3.10	$3.00
1:00 pm	$3.00	$2.85	$2.85	$2.85	$3.10	$4.90	$3.00
2:00 pm	$3.00	$4.10	$4.10	$4.10	$4.20	$3.10	$3.00
3:00 pm	$2.50	$4.40	$3.70	$4.45	$5.95	$10.25	$3.00
4:00 pm	$2.50	$5.55	$7.80	$8.30	$9.95	$9.35	$3.00
5:00 pm	$2.50	$4.85	$7.25	$8.50	$9.80	$8.00	$3.00
6:00 pm	$2.50	$4.40	$3.60	$3.60	$4.90	$5.30	$2.50
7:00 pm	$2.50	$3.10	$3.10	$3.10	$4.50	$4.95	$2.05
8:00 pm	$2.50	$2.05	$2.05	$2.05	$2.85	$4.50	$2.05
9:00 pm	$2.05	$2.05	$2.05	$2.05	$2.05	$2.85	$2.05
10:00 pm	$1.30	$1.30	$1.30	$1.30	$1.30	$2.05	$1.30
11:00 pm	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30

91 Express Lanes — **Toll Schedule** *Effective January 1, 2011* — **Westbound** *Riverside Co. Line to SR-55*

	Sun	M	Tu	W	Th	F	Sat
Midnight	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
1:00 am	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
2:00 am	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
3:00 am	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
4:00 am	$1.30	$2.40	$2.40	$2.40	$2.40	$2.40	$1.30
5:00 am	$1.30	$3.95	$3.95	$3.95	$3.95	$3.80	$1.30
6:00 am	$1.30	$4.10	$4.10	$4.10	$4.10	$3.95	$1.30
7:00 am	$1.30	$4.55	$4.55	$4.55	$4.55	$4.40	$1.75
8:00 am	$1.75	$4.10	$4.10	$4.10	$4.10	$3.95	$2.05
9:00 am	$1.75	$3.25	$3.25	$3.25	$3.25	$3.25	$2.50
10:00 am	$2.50	$2.05	$2.05	$2.05	$2.05	$2.05	$2.50
11:00 am	$2.50	$2.05	$2.05	$2.05	$2.05	$2.05	$2.90
Noon	$2.50	$2.05	$2.05	$2.05	$2.05	$2.05	$2.90
1:00 pm	$2.90	$2.05	$2.05	$2.05	$2.05	$2.05	$2.90
2:00 pm	$2.90	$2.05	$2.05	$2.05	$2.05	$2.05	$2.90
3:00 pm	$2.90	$2.05	$2.05	$2.05	$2.05	$2.50	$2.90
4:00 pm	$3.05	$2.05	$2.05	$2.05	$2.05	$2.50	$3.05
5:00 pm	$3.05	$2.05	$2.05	$2.05	$2.05	$2.50	$3.05
6:00 pm	$3.05	$2.05	$2.05	$2.05	$2.05	$3.00	$2.50
7:00 pm	$2.50	$1.30	$1.30	$1.30	$1.30	$2.05	$2.05
8:00 pm	$2.50	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
9:00 pm	$2.50	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
10:00 pm	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30
11:00 pm	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30	$1.30

Figure 41. Example of a Toll Schedule for the HOT lanes in the SR-91 in California (Source: OCTA 2011)

One interesting example is worth mentioning here. The I-95 Express Toll Lanes close to Baltimore have a pre-established schedule too. But they do not offer any special advantages to HOVs. Thus, they are not technically HOT lanes.

In other HOT lanes, the price changes every few minutes such as is the case of the I-394 MnPass Express Lanes, where it is updated every three minutes. Under this system, a price is displayed on electronic signs located besides the highway. The driver makes the decision of entering the HOT lane after looking at that price. The exact mechanism on how the toll is charged to vehicles changes from project to project. Subsection A.2.3 describes this mechanism for the I-394 in Minnesota. In some cases, the price varies depending on the length of the trajectory. Every project uses a different algorithm for setting the price. But in general, the main input variable of that algorithm is the level of congestion on the HOT lanes.

A.2.2. *Types of entrances*

Typically, there are two ways of entering the HOT lanes: through Direct Access Ramps (or DARs) or through "gates". A gate is a generic term but this section employs it to refer to a connection between a GP lane and a HOT lane through a simple juxtaposition. Figure 42 depicts a typical gate on the I-15 FasTrak Lanes. It also shows the exit that could follow after the gate.

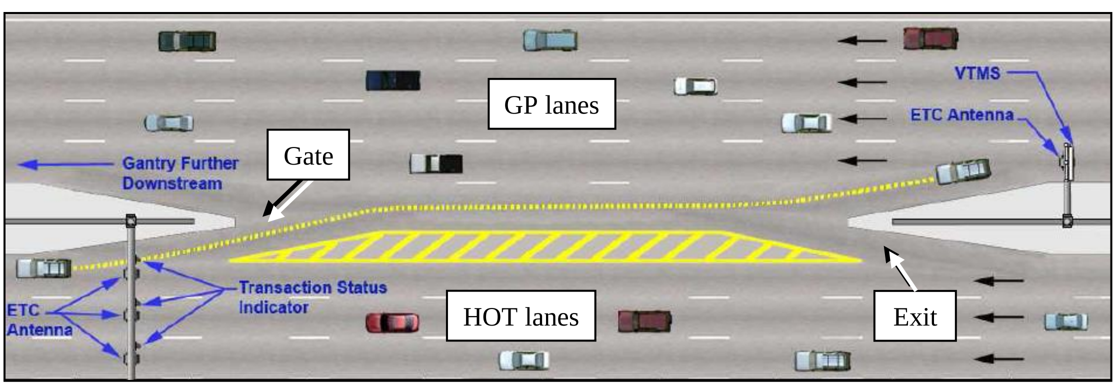


Figure 42. Typical gate (and exit) on the I-15 FasTrak Lanes in California (Source: HNTB Corporation 2006, p.9).

The term DAR used here is taken from the terminology that the authorities of the I-15 FasTrak Lanes use. DARs allow the direct entrance to the HOT lanes through exclusive ramps. In this manner, drivers can avoid entering the GP lanes before entering the HOT lanes. This apparent advantage to drivers comes with a caveat: before entering the ramp, they cannot make a visual comparison of the congestion on the GP lanes with the congestion on the HOT lanes. Gates on the other hand allow this visual comparison (if the HOT lanes do not have a high or thick barrier separating them from the GP lanes). Figure 43 depicts two DARs on the I-15 Express Lanes.

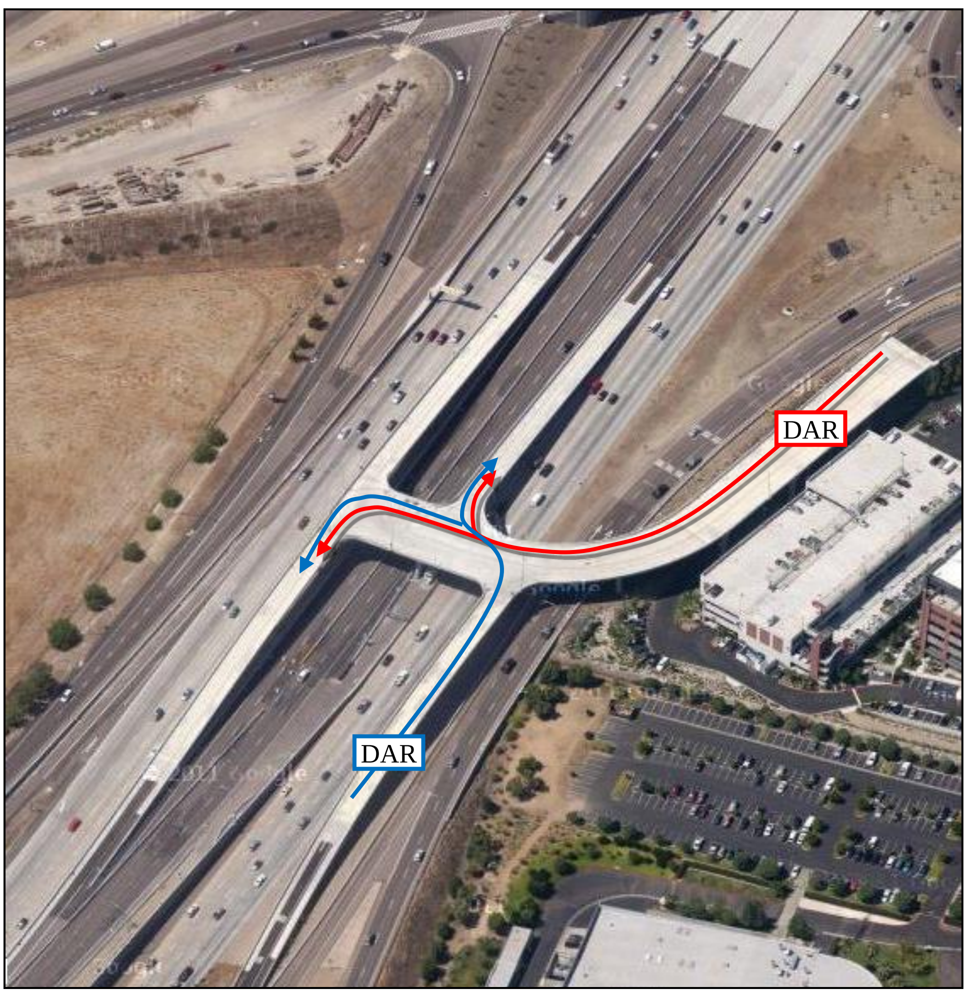


Figure 43. Direct access ramp (DAR) for entering (and exiting) the I-15 Express Lanes, California (Source: Google Maps 2011).

A.2.3. *I-394 MnPass Express Lanes*

This facility was originally an HOV system. It opened as a HOT lane system in May of 2005 and it is known to the public as “the I-394 MnPass Express Lanes”. The system is composed of two contiguous segments: 7 miles of two one-directional HOT lanes and 3 miles of two reversible HOT lanes. On the 7-mile segment, each one-directional HOT lane is separated from the GP lanes by a double-white-stripe-buffer. This feature, shown in Figure 44, was a novelty at its time (Halvorson & Buckeye 2006), avoiding the construction of a more intrusive barrier. The 7-mile segment is also known as the “diamond section”.

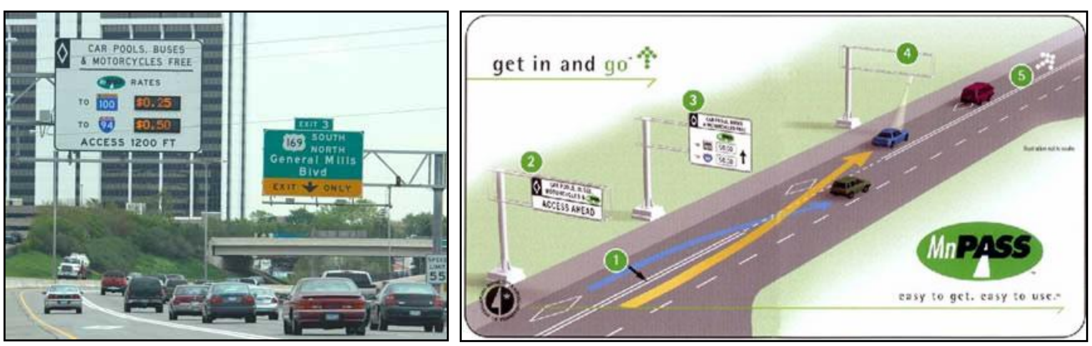


Figure 44. “Diamond section” of the I-394 MnPass Express Lanes. Right photograph depicts how vehicles can access or exit this section of the HOT lanes.

At some points, the double-white-line becomes an intermittent line, acting as a gate, allowing the exit from or entrance to the HOT lane.

The reversible section is approximately 3 miles long. Figure 45 shows the area where the diamond section ends and the reversible section starts.

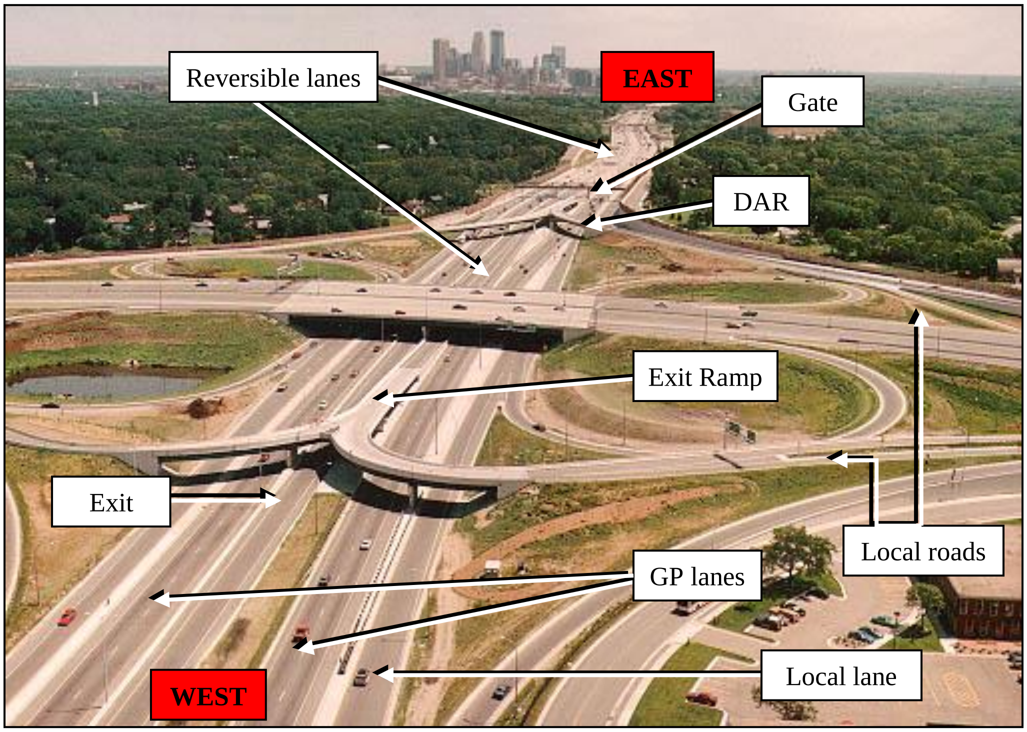
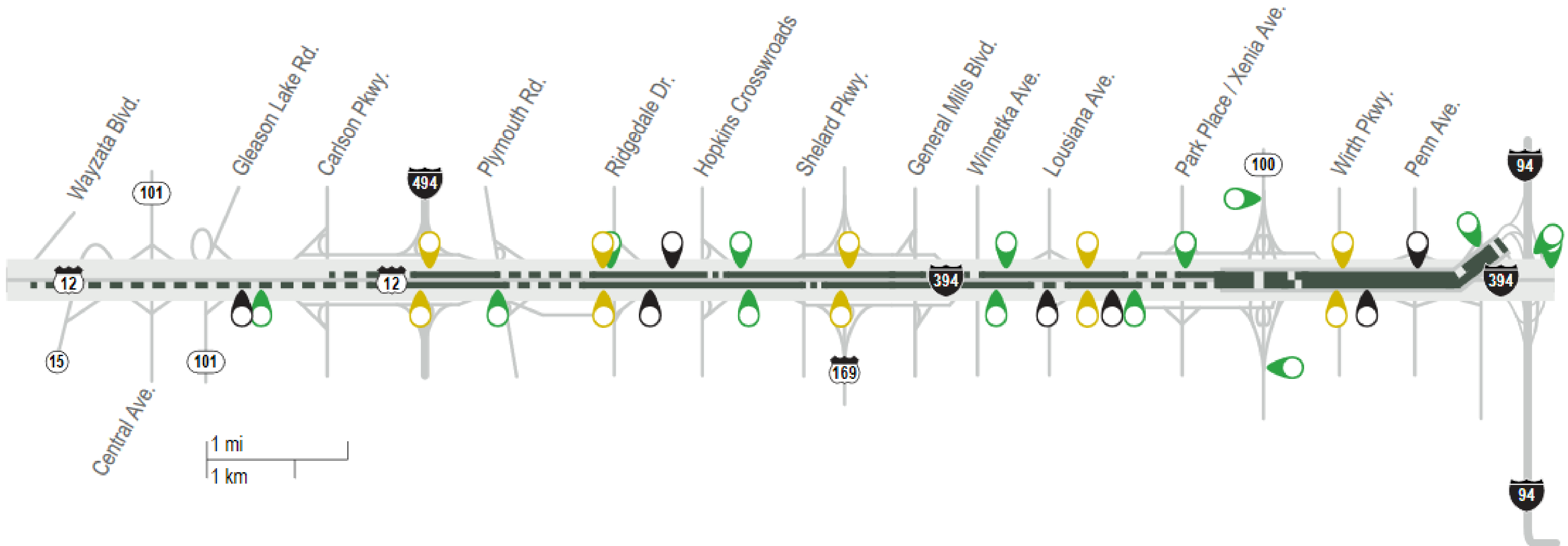


Figure 45. West extreme of the “reversible section” of the I-394 MnPass Express Lanes, and East extreme of “diamond section”. These HOT lanes can be accessed from local roads through direct access ramps (DARs) or from the GP lanes through a gate.

Figure 46 depicts the general configuration of the I-394 MnPass Express Lanes. The system stretches from West to East. A user of the HOT lanes who starts her trip on the West side and needs to go East, would need to enter the diamond section, then exit briefly and then enter the reversible lanes. She would also have to pay attention to the schedule that the reversible lanes have. Currently, this schedule is as follows: On a weekday, in order to go eastbound, the reversible section is open from from 6 a.m. to 1 p.m. Then it would close from 1 p.m. to 2 p.m. to allow the directional change. From 2 p.m. to 5.a.m. it opens for the westbound direction. On weekends, the reversible lanes open only for the eastbound starting at 8:30 am.

Anyone who enters the diamond section pays a minimum toll of $0.25 and anyone who enters the reversible section pays a minimum toll of $0.25. To the knowledge of the author, the algorithm that sets the displayed price is not published anywhere but it is based on the traffic density on the HOT lanes (Munnich & Buckeye 2007, p.51).

The I-394 MnPass Express Lanes also allow access to transit and special vehicles such as motorcycles. But their volume is low compared to HOVs and solo drivers that used them.



I-394 MnPASS Express Lanes Pricing Zones

Direction	Limits	Hours
Eastbound	Wayzata Blve. to Hwy. 100	6AM - 10AM, Mon - Fri
Reversible lane Eastbound	Hwy. 100 to I-94	6AM - 1PM, Mon - Fri
Westbound	Hwy. 100 to Carlson Pkwy.	2PM - 7PM, Mon - Fri
Reversible lane Westbound	I-94 to Hwy. 100	2PM - 5AM Mon - Fri

Legend

- Message sign
- Toll rate sign
- Tolling location
- MnPASS open access (broken line)
- MnPASS restricted access (double solid line)

N

Figure 46. General depiction of the I-394 MnPass Express Lanes to the general public. Source: (Minnesota Department of Transportation 2013)

A.3. Dedicated bus lanes

These are lanes that are allowed to be used only by buses. They are commonly referred in the United States as “bus only lanes” or simply “bus lanes”. The goal is to let buses to have a reliable travel time, especially when the adjacent general purpose lanes are congested. According to the National Transit Database (Federal Transit Administration 2013), the first street with a designated bus lane in the United States (and perhaps in the world) was in Chicago in 1939. One difficulty of using the concept of “dedicated bus lanes” is that whenever these lanes are implemented, most of the times one of the following two cases happen:

(1) The number of buses that use them is few, especially when they have few bus stops: In this case, other categories of vehicles are also allowed to enter so that the general public accept having set aside a lane from the rest of the traffic. Other categories of vehicles range from HOVs and paying SOVs, to motorcycles, taxis, and “right turners”.

(2) The number of buses is high: In this case, public officials tend to prefer having a system similar to a subway or metro service, that is, a system with high frequency transit service, and quick boarding among other features. Such system is what is known as “bus rapid transit” or BRT. Although these lanes are indeed dedicated to buses only, a rationale develops which states that no other kinds of vehicles should be allowed so that the reliability goal is met.

In any of the two cases above, this dissertation proposes allowing SOVs to enter the dedicated bus lanes. In the second case, the reliability would not be deteriorated because the system proposed in this dissertation has very high control on the number of vehicles that can enter the system.

Figure 47 presents an example of the first case. Here, the dedicated bus lane is also shared with HOVs and with right turners (vehicles who need to make a right turn in order to exit the street).

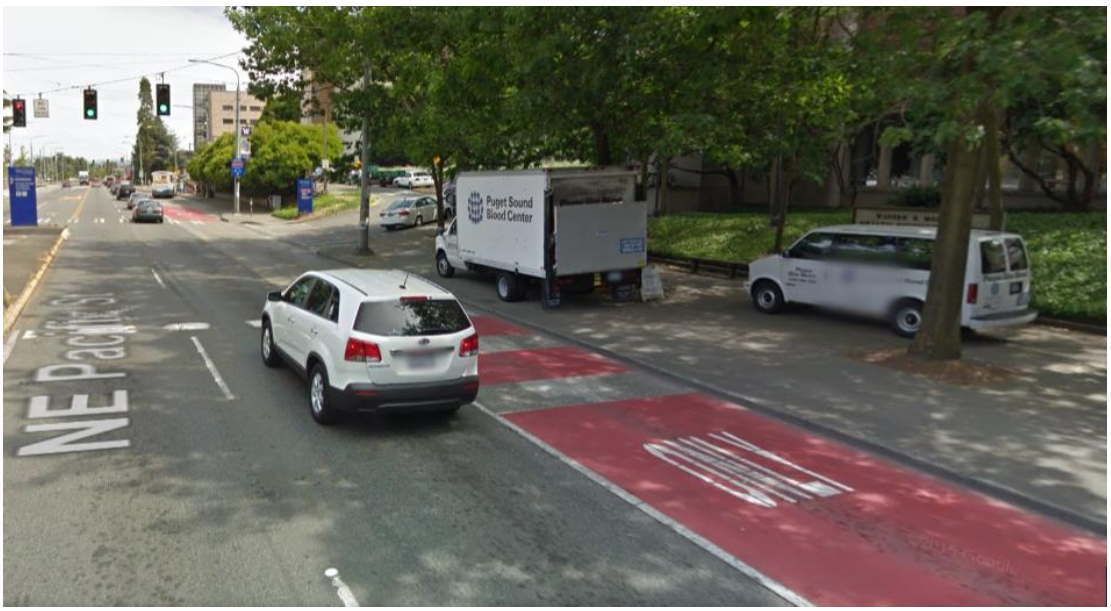


Figure 47. Example of a dedicated bus lane. Here, private vehicles are also allowed to enter if they are HOVs or if they intend to make a right turn. Location: NE Pacific Street & NE Pacific Place, Seattle, Washington. Source: Google Maps (2015).

Another example of the first case takes place in San Francisco. As shown in Figure 48, the lanes are shared with taxis.

Figure 48. Example of dedicated bus lane in San Francisco, California. Here, private vehicles are also allowed to enter if they are taxis. Source: Johnson (2014).

In theory, another example of the first case are the I-394 MnPass Express Lanes already explained in this appendix (Subsection A.2.3). They are restricted to buses, and also to motorcycles, HOVs and paying SOVs. But since their main feature is that they have dynamic pricing for SOVs, they are not referred as bus lanes or hybrid lanes but HOT lanes.

One interesting combination of both cases is the system in New York city. Although the intention is to have a BRT system, the lanes are also open to vehicles who need to make a right turn, to vehicles who need to make a short stop to drop off or pick somebody, and to emergency vehicles (Office of the Mayor, New York City 2016a). Figure 49 presents such system.

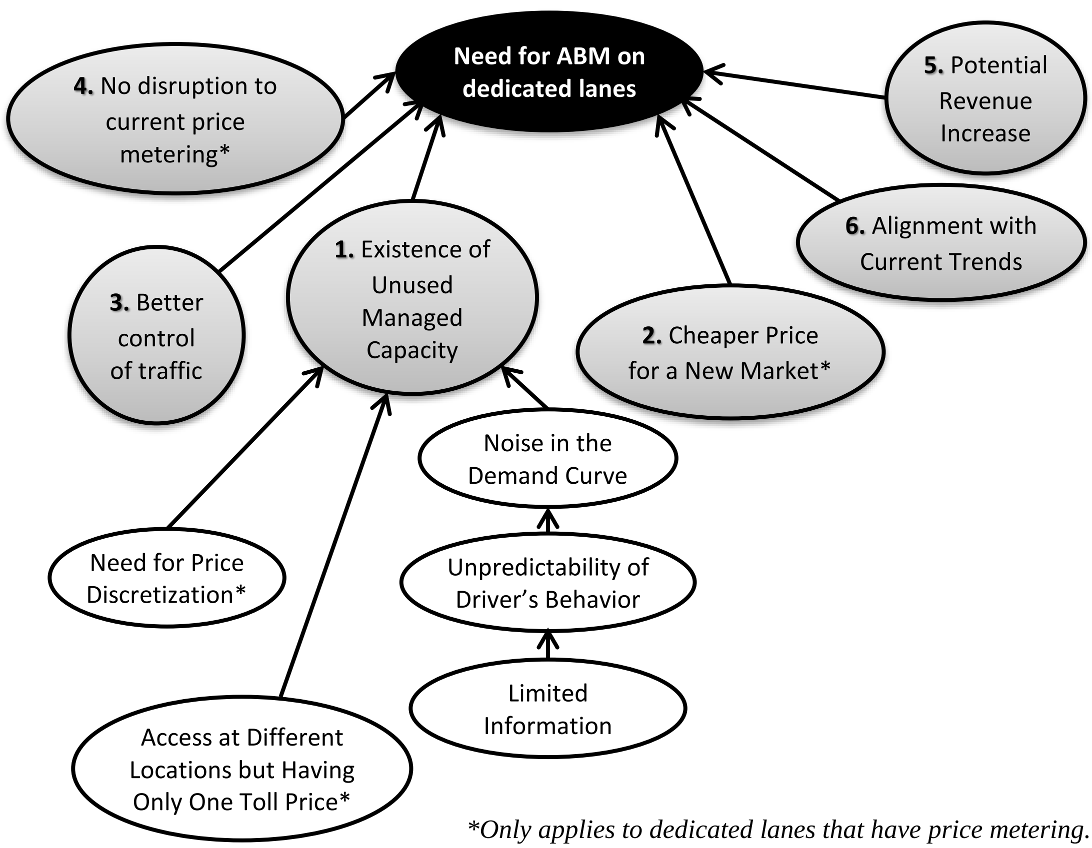
Figure 49. Example of dedicated bus lanes in New York City. Figure on the left, curbside bus lanes. Figure on the right, offset bus lanes. Source: Office of the Mayor, New York city (2016b).

A.4. Managed lanes

Managed lanes refer to a much broader concept than HOT lanes and dedicated lanes. They are not considered in this dissertation but included in this section due to its great importance. Managed lanes encompass any set of lanes, usually within a highway facility, that have mechanisms put in place to influence (or manage) how traffic behaves. Examples of managed lanes, other than dedicated lanes, are reversible lanes, dynamic variable speed lanes, lanes with ramp-metered access, etc. Managed lanes may involve a combination of those examples. A similar concept to managed lanes is active traffic management (ATM). Perhaps the difference between the two concepts, if any, is that in an ATM facility, there are protocols for more reactive operations (for example, response to accidents).

Appendix B. Arguments on Why Auction-Based Metering for Dedicated Lanes

Section 1.2 explained that the reason for having auction-based metering on dedicated lanes was to benefit three parties involved or potentially involved on a dedicated-lane facility: public operators, drivers when conducting trips that do require high reliability (for example, typical work-to-home trips), and private operators. This appendix frames those benefits within a list of six arguments, which are presented in the figure below. Arguments 1, 2, 3 and 5 are directly related to the three beneficiaries aforementioned. The other arguments are also related to those beneficiaries but indirectly.



**Only applies to dedicated lanes that have price metering.*

Figure 3 (REPEATED). Six main arguments (highlighted in gray circles) and their supportive arguments for adding ABM to dedicated lanes.

B.1. Existence of unused managed capacity

Perhaps the main reason for introducing an auction is the argument that current HOT lanes have unused managed capacity. Chapter 3's main role was to define unused managed capacity and to determine whether its quantity was significant. But this section looks at its causes. Figure 3 presents those causes.

This section does not focus on other dedicated lanes for the following simple reasons. Because HOV lanes do not have any price mechanism, they are even more prone to have unused managed capacity than HOT lanes. And dedicated bus lanes, they would use all their managed capacity, only in the ideal scenario that their lanes are completely used by buses and that transit demand is the same throughout the day. If such were the case, it would be expected that the policy makers opt for expanding the overall capacity of those lanes for future increments in the demand.

Figure 50 shows the concept of unused managed capacity and how it relates to a hypothetical demand curve on HOT facility. Here, the demand curve is the relation between the toll price and the number of solo drivers that would enter the HOT lane. Since solo drivers base their decision of whether to enter the HOT lane, not only on the price but on many other factors, the figure proposes including one of those other factors as a third dimension, that is, the level of service on the general purpose lanes (but as said, there should be additional dimensions). Now, say the operator sets the toll price at 1.50 dollars. According to the figure, a flow of 600 veh/h would enter the HOT lane. But if the operator lowers the toll to 1.25 dollars, the HOT lane would surpass the managed capacity. In HOT facilities such as the I-394 MnPass Express Lanes, price can be varied only in increments of 0.25 dollars. Say, Figure 50 also assumes that prices are discretized in 0.25 dollars. Then, according to the figure, there is an unused managed capacity of approximately 200 veh/h when the operator is forced to set the price at $1.50. Otherwise, lowering to 1.25 dollars would exceed the managed capacity. Hence, price discretization can generate unused managed capacity.

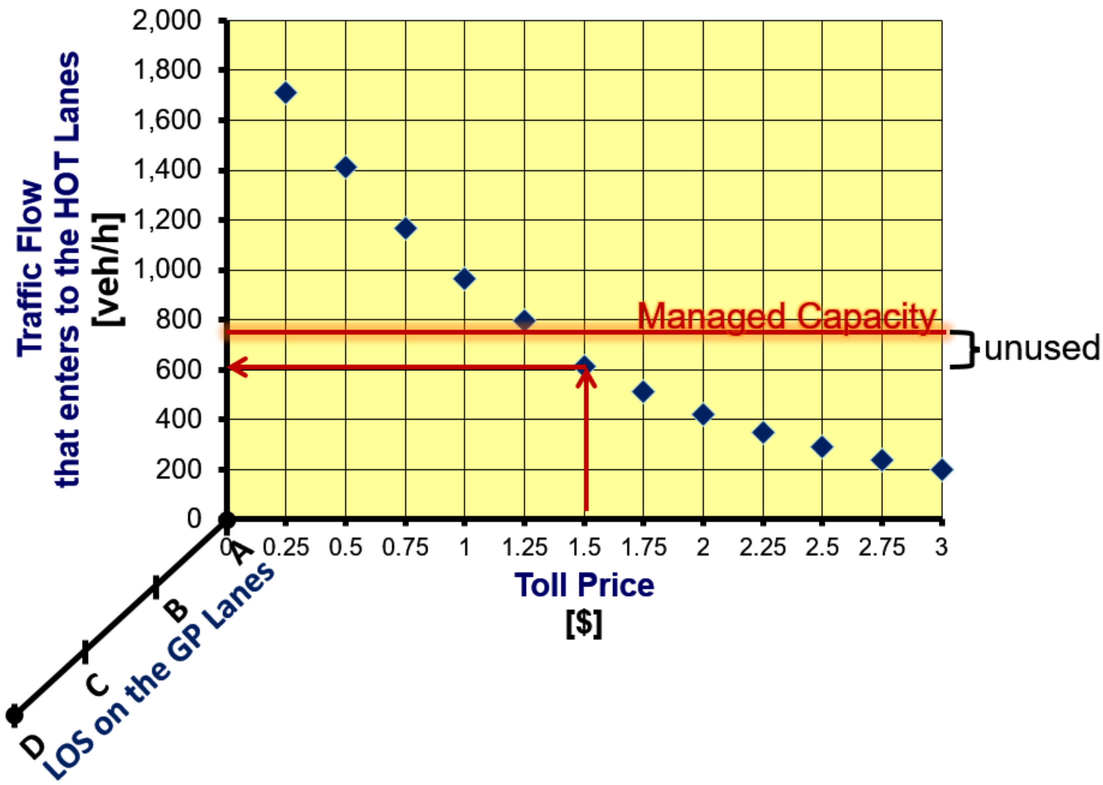


Figure 50. Example of unused managed capacity that is caused by discretization of toll prices. In this example, if the level of service on the GP lanes is A, and if the operator sets the price for entering the HOT lanes at 1.5 dollars, then 600 veh/h would to enter the HOT lane leaving close to 200 veh/h of unused managed capacity.

Another and perhaps more important source of unused managed capacity is the existence of noise on the demand curve. As illustrated in Figure 51, noise can be defined as variations in the demand curve due to factors not considered in the figure (for example, Figure 51 presents a demand that reacts to price and level of service, but how about weather factors). Due to the noise, the operator is forced to set prices that in many occasions would lead to unused managed capacity.

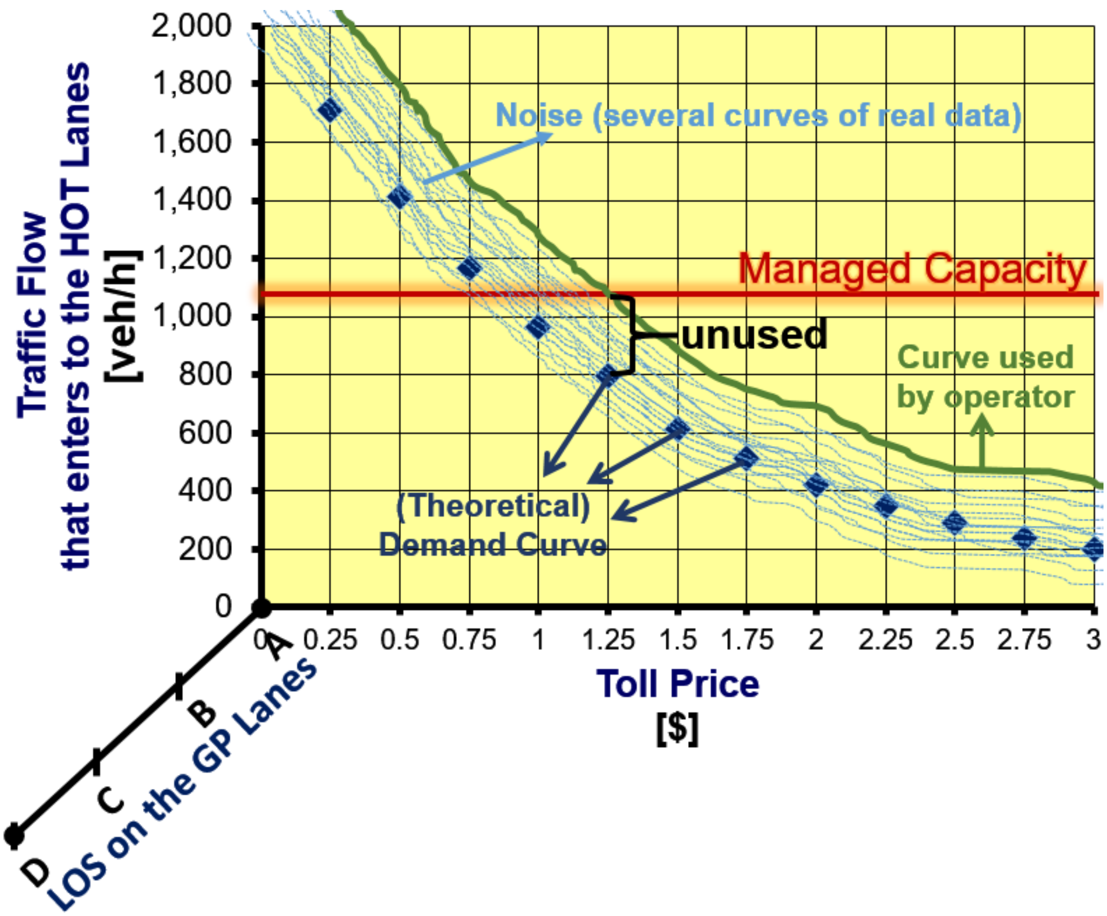


Figure 51. Example of unused managed capacity that is caused by noise in the demand curve. In the example, approximately 300 veh/h of unused managed capacity may exist if setting the toll price at 1.25 dollars. Also shown, the curve that the operator needs to apply in order to avoid exceeding the managed capacity on the HOT lanes.

As mentioned above, not considering enough variables in the demand curve leads to having noise in the demand curve. But some other factors could be the heterogeneity of the users or, as suggested by Figure 3, the unpredictability of users' behavior. Sometimes, that unpredictability is caused by the differences in information that users have in terms of congestion downstream and road conditions.

Finally, another reason for having unused managed capacity is the fact that several access points, located at different locations along the HOT lanes have one common toll price. For example, the reversible section on the I-394 MnPass Express lanes have three access points located one mile apart. But all three entrances display the same dynamic toll price. This feature of some HOT facilities creates the heterogeneity in the users and therefore, the operator needs to adopt measures as the one indicated in Figure 51.

B.2. Cheaper price for a new market

The bidding alternative, when not enforced but offered as a voluntary cheaper alternative, can become very attractive to some users. As suggested by the research presented in this section, these users may have not been served satisfactorily by today's HOT lanes. Suppose that the following type of user exists: a solo driver, who most of the time does not have urgency in arriving to her destination but is a frequent user of the given highway. This user, wanting to avoid the congestion that she faces almost daily, would consider the bidding alternative as attractive. When she does not win the auction, things would remain the same. But when she wins the auction, she would feel satisfied of having a less stressful commute. One could consider this type of users as a new market. Or the new market could comprise all types of users who, with different levels of frequency, are faced with trips that are not urgent. One common example of this kind of trips are the work-to-home trips. In a work-to-home trip, a faster trip (and cheap trip) is almost always welcome but not always needed.

The following research work presents evidence that trips with low level of urgency, or in other words, trips whose users are not needing high reliability, have not been served satisfactorily.

One revealed preference study (Steimetz & Brownstone 2005) talks about the existence of users with "flexible schedules". These type of users choose not to enter the HOT lane during congestion-peak hours when the toll price is very high. It is not clear whether these flexible users expect high reliability when driving during congestion non-peak hours. If they do not expect high reliability, then they would not have a problem in bidding with the risk of losing the auction. This possibility may be true since one important assumption that the study makes is that drivers' main reason for choosing the HOT lane was the potential travel time savings (and not reliability). There are two other revealed-preference studies (Goodall & Smith 2010; Liu et al. 2011) which tried to identify additional factors influencing solo drivers for choosing the HOT lane. Their results are somewhat contradictory but they both support the notion of the existence of a significant market of low-reliability trips. Interestingly, they also suggest that there are very personal notions on how each driver interprets a particular value in the displayed price. One big assumption that they both make is that users have perfect knowledge of the travel time (or speeds) of the HOT lanes and the GP lanes.

One of the two studies was conducted by Goodall and Smith (2010) for the I-394 Express Lanes. Their study argues that there are two kinds of SOV users: frequent users and infrequent users. Frequent users are those who drive mostly during the congested periods. And since during these congested times, the travel time is more unpredictable, they tend to choose the HOT lanes regardless of the toll rate. Infrequent users on the other hand would drive at shoulder hours and would choose the HOT lanes only if the price seems reasonable. Although the concept of reliability served to understand the behavior observed in the SOVs, reliability was not quantified.

The second study was conducted by Liu et al. (2011) on the SR 196 HOT Lanes in Washington State. They also arrived to the conclusion that during the congestion period, SOVs use the HOT lanes due to its high reliability. Thus, they both concluded that during that phase, demand does not diminish with higher prices. Where the two studies defer is that during the congestion phase, Goodall and Smith (2010) found that SOVs are insensitive to the toll price while Liu et al. (2011) found that demand increases with the toll price. And during the non-congestion period, Goodall and Smith (2010) found that demand decreases with the toll price while Liu et al. (2011) found that demand varies with travel savings and reliability but not the toll price. Liu et al. (2011) also found that, unlike Goodall and Smith (2010), frequent users and infrequent users have the same distribution in their value of time. The differences in the results are not only due to the quantification that Liu makes of reliability but to the differences between the HOT facilities that they studied. Goodall and Smith

(2010) focused on the reversible lanes of the I-394 MnPass Express Lanes which are two-lane HOT lanes and which is fed by various DARs. On the other hand, the HOT lanes in the SR-196 are connected to the GP lanes through simple gates. When the system has DARs, the driver has to make the choice (of entering or not entering the HOT lane) before being able to observe and compare the congestion on the GP lanes versus the congestion on the HOT lane.

Either of the two scenarios may benefit from a bidding alternative. If the population served presents the characteristics described by Goodall and Smith (2010), then the auction would provide an alternative for the users who usually take the GP lanes during shoulder hours and why not, to the frequent drivers too. On the other hand, if the population served presents the characteristics described by Liu et al. (2011), then the auction would constitute an additional alternative to the population that always abstained from entering the HOT lanes.

B.3. Better Control of Traffic

Auction-based metering does not eliminate but complements the way HOT operators set the toll price today, and in this way, there is a better way of controlling the number of vehicles that enter the HOT lanes. Current HOT lanes rely on algorithms with different levels of sophistication to set the toll price in order to have as much flow entering the HOT lane as possible without reaching an undesirable LOS on the HOT lane. Therefore, one could think that by improving these algorithms, the unused managed capacity could be almost reduced to zero. But, following the arguments presented previously in Section B.1, users decide to enter a HOT lane based on many external and internal variables that these algorithms cannot capture. So, in an auction-based metering (ABM) system, these algorithms can continue being implemented for setting the price, but it is only through an auction that the unused managed capacity can be sold. The reason why only an auction is capable of selling unused managed capacity is because when a user accepts to participate in an auction, she is also accepting that the operator tells her to or not to access the HOT lane. In very few cases in traffic management, a driver accepts the operator to tell her not to enter a particular lane or spot in a rather “arbitrary” manner (perhaps the only other case is through traffic signals). Thus, the auction grants a powerful tool to the operator to manage traffic.

In the case of an HOV lanes, they would have the same improvement as described above for HOT lanes. In the case of dedicated bus lanes, a good control of any SOV inflow is mandatory so that the transit headways do not become disrupted.

B.4. No Disruption to Current Price Metering

As mentioned in the previous section, auction-based metering does not eliminate but complements the way operators set the toll price today. Current users of HOT lanes (those who pay and will continue paying the full toll price) would experience a more reliable travel time due to the higher control that ABM adds to the price mechanism.

In addition, ABM should not imply overhauling the existing mechanism (notice for example that the ABM system proposed in Section 5.1 only requires the addition of three visible elements). And it also should not alter the way current SOVs use the HOT facility. If an SOV does not want to switch to an OBU that enables auctions, she does not have to do so. And she should not experience any addition of features in the road that confuse her.

B.5. Potential for Revenue Increase

In the case of HOV lanes and dedicated bus lanes, it is very probable that ABM would increase the operator's revenue by selling the unused a managed capacity to SOVs who are not willing to become HOVs or take transit.

In the case of HOT lanes, one may argue that the fact that some users bid instead of paying the toll price may decrease the operator's revenue. Therefore, increasing revenue is an attractive benefit but requires more analysis. But there are reasons to believe that the decrease should not happen. These reasons include the following:

(1) As section B.2 mentioned, the new bidders would not be existing payers but an unserved market.

(2) If such market exists but has already being paying the toll price, most likely, they currently use the HOT lane infrequently. With the ABM system, their frequency should rise.

(3) As results in Figure 38 and Figure 39 suggest, an SOV may submit a very high bid and still not be granted access to the HOT lane because at that particular

instance, there was not unused managed capacity. Therefore, they may not be willing to become bidders.

Still, it is reasonable to think that revenue should decrease or experience insignificant changes if all the new bidders are simply the same users who used to pay the toll price. Subsection 6.2.3 looked at this possibility in detail assuming that all SOVs are only driven by the toll price. It also looked at the extreme case in which, instead of having an auction, a raffle is implemented, and if the reserve price is equal to zero.

B.6. Alignment with Current Trends

In relation to the concept of HOT lanes, there are other ideas being developed or being implemented for managing traffic. As explained below, ABM would not interfere with these trends. In some cases, it would benefit from them. And in some others, it would contribute to them.

To begin with, the continuous adoption of HOT lanes has made some analysts predict the emergence of HOT networks (Roth 2006, p.451). Some of the new HOT lanes will intersect with the old ones leading to the formation of HOT networks. There are still no real HOT networks. A HOT network can present challenges to the operator: How to set prices? How to avoid complexity in the signage? ABM can be helpful for the operator in these two ways. First, an auction is a transparent way of discovering the market price. Second, a good design ABM system should not increase complexity to the system. In fact, an ABM system may reduce complexity, especially in a HOT network, by would allowing to input different bidding prices (in advance, prior to start of the trip) depending on the different variables in the system.

In the academic field (thanks to works like Brilon et al. 2005) as well as in practice (due to the increase of congestion, the wider availability of traffic information, and the existence of HOT lanes), the concept of travel time reliability has gained importance. It is no longer enough for the driver to receive information on how long her commute would be. Now, the user expects information on how reliable this information is. On one hand, an ABM system has a better control of the volume that enters the HOT lane and in this way, reduces scenarios in which HOT lanes are slower than GP lanes (as mentioned by Goodall & Smith 2010).

One important trend is the advent of “connected vehicles” which started with the label of vehicle-to-vehicle communication and IntelliDrive (Research and Innovative Technology Administration 2010). In its most broad sense, it allows vehicle-to-vehicle communication and vehicle-to-infrastructure communication through advanced sensor systems. This has the potential for facilitating how the information arrives to the OBU and perhaps remove for static booms and static auction zones (as suggested by Section 5.1). In a similar sense, the proposed ABM system would benefit from geotolling (or satellite tolling) and automated vehicles. Nonetheless, although these technologies would improve the design proposed in this dissertation, as Section 5.2 reveals, they are not critical for the success of ABM.

Finally, there are two trends that may be used for arguing against ABM. One is the emergence of “managed lanes”. As more clearly defined in Appendix A.4, they include dedicated lanes as well as lanes with features such as variable speeds, reversible sections, dynamic shoulders, etc. Managed lanes have a lot of benefits. But transportation planners are aware that implementing several of their features runs the risk of raising complexity in the driving experience. Therefore, the introduction of ABM could be perceived as another feature of a managed-lane system and in this manner, it could be perceived as something that would raise complexity in the driving experience. This wrong perception can be tackled in two ways. First, ABM should not make great modifications on the infrastructure and should make sure that the road signage is clear. Second, the operator should market the bidding alternative only as an optional feature to the customer who is interested in finding a way of saving money in the long run.

The second trend that could be argued against the implementation of ABM is the campaign that in 2010 Secretary Ray LaHood started against distracted driving (U.S. Department of Transportation 2010). Texting or using cell phones while driving is banned in many states of the United States. But as mentioned in Section 5.1, the user of an auction-enabled OBU would only need to interact with any electronic device while at standstill and not while driving. In addition, the design proposed in that same section reflect the need for using a street signage and geometric design that is clear to the driver.

Thus, current trends in traffic management, technology and social attitudes should not deter the implementation of ABM. And some trends, such as the higher demand for travel time reliability and connected vehicles, could benefit by the introduction of ABM.

Appendix C. Validity of Policies Adopted for Game-Theoretic Model

Before proposing a game-theoretic model, it was clear that there were many auction systems or mechanisms that could be proposed. For that reason, section 4.1 proposed a set of policies which allowed an initial framework. From those policies, which may include assumptions, additional assumptions were later adopted in Subsection 4.3.1. Finally, the game-theoretic model was proposed in Subsection 4.3.1 to 4.3.3.

This appendix explains the rationale behind the policies that were adopted. They are explained in the same order as they were presented in Section 4.1.

C.1. Players are assumed to be rational.

This assumption simplifies the modeling of the game-theoretic model. But how can this assumption be justified? Loosely speaking, rational players (in the context of game theory) or rational bidders (in the context of auction theory) refer to individuals who, given their personal preferences, try to maximize their satisfaction. When satisfaction is quantifiable, then it is expressed by a so-called “utility function”. The assumption of rationality is stronger when the stakes are high, that is, when making a bad decision in the game or bidding incorrectly in the auction can represent important loses or important gains. Now, it is safe to say that present-day HOT lanes do not charge more than 5.00 dollars per trip on average. This can imply that many participants in an auction system may be willing to submit bids in a careless way.

Nonetheless, it can be assumed that solo drivers will be trying to enter the HOT lane with a frequency that would compound their losses or their gains. This is one reason to think that users are rational. Another reason is that there may be a fixed cost or initial registration fee. This fixed cost would filter out users who do not have much interest in benefiting from the auction system.

C.2. Players are assumed to have private values, not common values.

In other words, the willingness of a player to pay a toll is independent of how much other players are willing to pay. Say, a driver when getting in her car, at standstill, is asked: What is the maximum amount that you are willing to pay if granted access to the HOT lane? To assume that the driver has a "private value" is to assume that she will answer such question based on the importance of the trip that she is going to make and not on how much her friends, colleagues or other drivers are willing to pay (otherwise, she would have a "common value"). This can be especially true if we assume that drivers would participate on an auction knowing in advance that regardless of how high they bid, there is always a chance that there will not be space for them at the HOT lane. Nonetheless, it should not be discarded the possibility that the driver can be provided with information about the "item" that she is being offered. For example, it is possible that she is willing to pay $5.00 if using the HOT lane will save her 5 minutes. Or it is possible that she is willing to pay $5.00 because there is 90% probability that the HOT lane will take her 20 minutes to arrive to her destination. Or it is possible that she is willing to pay $5.00 because there is 90% probability that it will save her 10 minutes. She could even submit several bid amount at standstill, one for each possible scenario or range of values. Nonetheless, the auction system should transform all those bid amounts into an amount that can be compared against each other such as “dollars per right to enter”, or “dollars per minute saved”.

C.3. The auction should not be complex. For this reason, an auction that has similarities to an English auction should be adopted.

This recommendation sets two important objectives: simplicity and safety. Although these objectives seem obvious, they can easily be compromised during the design of the auction when trying to achieve other objectives. Simplicity and safety should be the basis for any alternative considered. Since the players would include anyone capable of driving, then neither complex strategizing before the auction nor complex steps during the auction should be required to them. This simplicity can be achieved in part by not demanding too much information from the bidder. Ideally, the level of complexity should not make anybody feel in disadvantage because her knowledge or her experience in auctions is limited.

The need for simplicity is based on the following assumption: The general public would not support an auction in which bidders would feel forced to purchase from a third party an electronic device or a software package in order to set the best strategy or to respond to complex situations by submitting the best bids. Nonetheless, the general public would accept the idea of installing an electronic device, preferably small, which would automate some steps of the auction so that distracting driving does not happen or is minimized.

It is possible that for certain levels of complexity, the general public would perceive the effort required for learning or for bidding as part of a fair cost to those solo drivers who have a real interest in choosing the auction.

Following the idea of having a simple system, it may be true that an auction that resembles an English auction should be implemented, since the general public are familiarized with this format. The extended Vickrey auction proposed in this section resembles an English auction (see its general explanation in the terminology section, Section 1.1). As such, it can be explained to some people in a similar manner in which the company eBay explains it for its customers: this is a "proxy auction" in which the individual gives power to a "magical elf" who would bid for the customer (Lucking-Reiley 2000, p.12). Also, as explained in the terminology section (Section 1.1), the standard Vickrey auction and the extended Vickrey auction do not force the winner to pay all the amount that she is willing to pay. This feature is also found in the English auction.

C.4. The auction should include a "buy option".

Present-day drivers are used to the simple idea that, if they pay the displayed price, they will have access to the HOT lane. Also, if a solo driver has urgency to reach her destination, she can rely on the idea that she will, with all certainty, enter the HOT lane as long as she pays the displayed price. In a traditional auction, like an English or a Vickrey auction, the bidder does not know if she will obtain the item. Therefore, the auction in a HOT lane should offer the choice to the solo driver of paying the toll price if she wants to avoid the uncertainty of not knowing whether she will enter the HOT lane or not.

It is useful now to look at the concepts of temporary and permanent buy options. Currently, eBay and Yahoo offer English auctions with a buy option. eBay's auctions have a temporary buy option. Yahoo has a permanent buy option. A temporary buy option gives opportunity to the bidder to buy the item only at the beginning. If she submits her first bid, she loses her opportunity to execute the buy option. A permanent buyout auction allows the bidder to execute her buy option at any time during the bidding process.

With creativity, the two formats could be implemented in an auction-based metering system. Nonetheless, this dissertation proposes having a temporary buyout auction which seems to be the one that best adapts to the context of a HOT lane.

C.5. Drivers who choose the "buy option" should always be granted access.

All participants who execute the buy option should be granted entrance to the HOT lane. A participant who executes the buy option should not be denied access due to the lack of managed capacity. There should not be a limited offer of items as typical auctions have.

C.6. The auction should have multi-unit supply and single-unit demand.

Based on the number of items offered and on the number of items bidders are allowed to acquire, auctions can follow the classification presented in the figure below. It also includes types of auctions that offer a buy option.

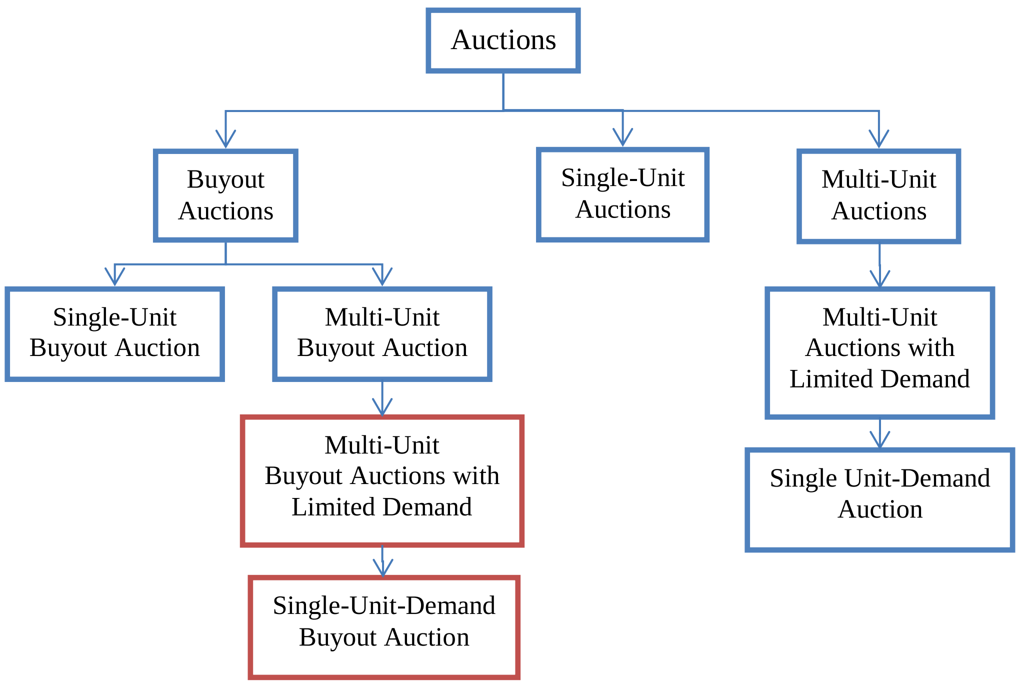


Figure 5 (REPEATED). Types of auctions (with and without buy option) based on the number of items. Auctions that appear in red boxes have not been studied in any previous research.

As explained in Section 2.1, this dissertation adopted the term "single-unit-demand auction" to refer to auctions of multiple supply and one-unit demand. It is different to classical "single-unit auction" such as the English and Dutch auctions.

This policy states that a single-unit demand auction should be adopted instead of selling the unused managed capacity through a sequence of single-unit auctions. A sequence of single-unit auctions would imply selling one item at a time (similar to the approach followed by Kirkegaard & Overgaard 2008). But in this case, every single-unit auction would have a lesser number of participating drivers. If the first auction has five participants and the last one only two, then the best strategies at the first auction might be very different from the best strategies at the last one. Still, one could apply a model that encompasses this sequence of auctions with decreasing number of participants and in this way, the model would allow determining a strategy that would perform the best on average.

In conclusion, the policy adopted is to have a single-unit demand auction. But whether it is better than having a sequence of single-unit auctions is still an open question.

C.7. Participation in the new system should be voluntary.

Drivers who want to continue with the traditional price mechanism should be able to do so, that is, choose to buy and have immediate access to the HOT lane, or choose not to buy and remain on the general purposed (GP) lanes. Also, drivers who do not drive alone (that is, HOVs) would also continue having free (or discounted) access to the HOT lane. Thus, drivers who choose to participate in the new auction system should do so only because they feel an incentive to do it (that is, the incentive of paying less than the toll price).

Making the buyout option voluntary also means not degrading the service to drivers who opt not to register to the new auction system. These drivers should not feel confused by infrastructure that is added to serve drivers who do register. And having a voluntary option should also mean that the current facility should not be overhauled. Although implementing the new system should have installation costs, it should not involve making major changes to the facility.

C.8. The goal of maximizing efficiency (whether operational or economic efficiency) should prevail over any goal of maximizing revenue.

Policy decision makers who are in favor of implementing HOT-lane systems are aware that if not being careful with the public, these systems can be seen as unfair to the public (The Washington Times 2011; Poole 2011). For this reason, designing an auction whose purpose is to maximize the revenue of the operator should be discarded in favor of goals that think on the benefit of the general public.

To start, consider the goal of reducing congestion. This could be a way of defining efficiency from an operational perspective. And it is reasonable to think that it is a goal that should prevail over the goal of maximizing revenue.

There is also a more technical definition of efficiency in the economics field. It has being argued that despite its importance, it has been defined in several ways that are not equivalent (see Paul Makdissi 2006). But still, most of the times, including in auction theory, economic efficiency refers specifically to the concept of Pareto efficiency: “an economic allocation or decision is efficient if and only if there is no other feasible allocation that makes some individuals better off without making other individuals worse off”. When dealing with auctions of incomplete information, the above concept is not specific enough and therefore, one refers to it as *ex post efficiency* as first used by Holmström and Myerson (1983). In simple terms, a decision on the allocation of an item (after executing an auction) is ex post efficient if the object ends up in the hands of the person who values it the most.

After looking at the definition of economic efficiency, it is also reasonable to think that the goal of reaching it should prevail over the goal of maximizing revenue. And as explained by Krishna (2009, p.5)., these two objectives are not always aligned. Therefore, one needs to be prioritized over the other one.

Finally, it is interesting to observe that there is economic efficiency if the new auction system does not deteriorate the good service that current HOT users experience. This efficiency was advocated by the previous policy too.

Appendix D.Mathematical Proofs

This appendix presents the mathematical proof of a proposition and a corollary that state that in the extended Vickrey auction and in the standard Vickrey auction (as defined in the terminology section, Section 1.1), the weakly dominant strategy for a bidder is to submit her true valuation. This appendix then ends by providing proofs to the lemmas and propositions laid out in Subsection 4.5.1 and Subsection 4.5.2. Before each proof, a comment is included in regards to whether the proof can be found in previous work, it is original to this dissertation or it is similar to an existing proof. The comment may explain some other aspects.

Proposition 0. Consider a set of bidders $\{1,2,\ldots,a,\ldots,\bar{a}\}$, where $a > 2$ and where each bidder a has a private value v_a. Consider one set of items $\{1,2,\ldots,k,\ldots,\bar{k}\}$ for auctioning where $1 \leq \bar{k} < \bar{a}$. Under the rules of an extended Vickrey auction (as defined in the terminology section, Section 1.1), the weakly dominant strategy for any bidder a in that game is to submit her true valuation, that is, to submit a bid amount b_a=v_a.

Comment. This auction can be found in Vickrey's classical work (Vickrey 1961). And the following proof may be derived from such work or subsequent work. Although the extended Vickrey auction that takes place in the ABM system makes the restriction that private values are continuous, the following proof removes that restriction. In other words, thanks to the following proof, Proposition 0 can be applied to continuous private values as well as discrete private values.

Proof. Consider the game as a confrontation of two players who move simultaneously: bidder a and the group composed of all her rivals and which is represented hereafter as "$-a$". Let b_{-a} denote the k^{th} highest bid amount within group $(-a)$. Thus, we can now represent the game as a confrontation of two players where one of them, group $(-a)$, can make three different moves: to submit a bid amount b_{-a} that is less than player's valuation v_a, to submit a b_{-a} equal to v_a, or to submit a b_{-a} greater than v_a. Notice that if it were the case that private values were continuous, the second move would have a probability of zero.

The above two-player description in which group $(-a)$ can make three different moves (or strategies) is represented in Figure 52 with a tree-like structure, or as is referred in the field of game theory, with an "extensive form representation". Nonetheless, unlike a typical extensive form representation, here Figure 52 does not indicate player a's payoff, not only because it is unclear how it should be defined but also because it is unnecessary for the purpose of this demonstration. The dashed lines indicate that player a cannot observe player (-a)'s move. Thus, the apparent sequential game suggested by the tree-like representation where player $(-a)$'s turn comes before player a's turn is actually a simultaneous game.

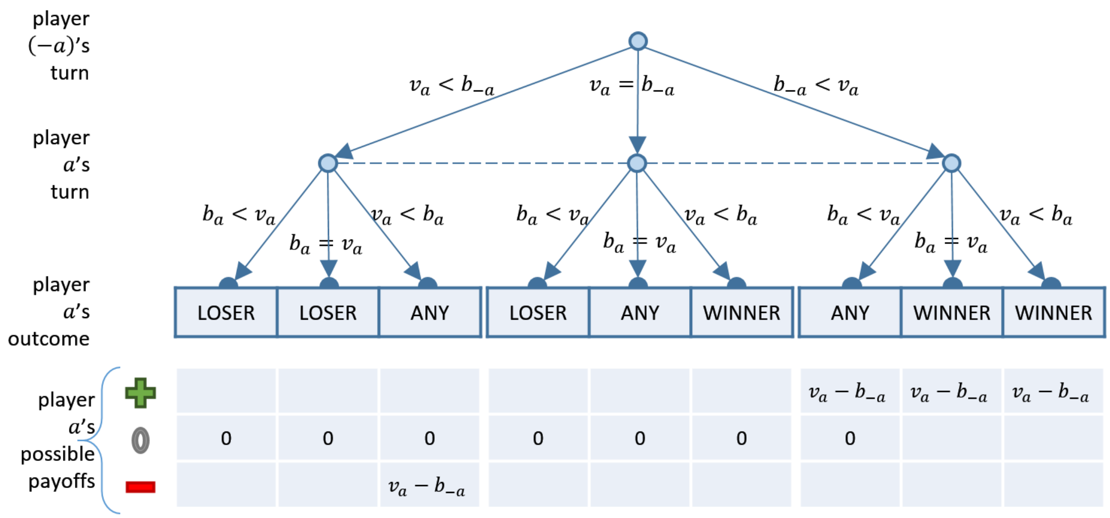


Figure 52.“Extensive-form representation” of the extended Vickrey auction as a simultaneous game of two players: player a, and group $(-a)$, composed of all her rivals. Payoffs for player $(-a)$ are not shown. Dashed lines indicate that player cannot observe player $(-a)$'s move.

Figure 52 also depicts the moves (or strategies) that player a can make. Notice that they differ, not on how player a's bid amount responds against b_{-a} but on how she responds against her private value v_a. Thus, for each move from group (-a), player has three responses: to bid below her private value ($b_a < v_a$), to bid her private value ($b_a = v_a$), and to bid above her private value ($b_a > v_a$). Although her move can lead her to winning an item and sometimes not, one actually needs to look at the payoffs to determine which of her three strategies is weakly dominant. For example, notice that although player a's payoff is always equal to $v_a - b_{-a}$ whenever she wins an item, that expression is sometimes positive and sometimes negative.

A strategy is weakly dominant if, at each strategy from the rival, such strategy provides a payoff that is always equal or greater than the payoffs from choosing the other strategies. Assume that $b_a = v_a$ is weakly dominant. Let us now verify if it is equal or greater than the other strategies for each of group $(-a)$'s three moves:

1. $v_a - b_a$: Strategy $b_a = v_a$ produces equal payoff to payoff from $b_a < v_a$. And it produces equal or less payoff than payoff from $v_a < b_a$.
2. $v_a = b_a$: Strategy $b_a = v_a$ produces equal payoff to the other strategies' payoff.
3. $v_a - b_a$: Strategy $b_a = v_a$ produces equal or greater payoff than payoff from $b_a < v_a$. And it produces equal payoff to payoff from $v_a < b_a$.

Therefore, it is indeed a weakly dominant strategy for a player to submit a bid amount that is equal to her bid amount.

Corollary 0. Consider a set of bidders $\{1,2,\dots,a,\dots,\bar{a}\}$, where $a > 2$ and where each bidder a has a private value v_a. Consider one item for auctioning. Under the rules of a standard Vickrey auction (as defined in the terminology section, Section 1.1), the weakly dominant strategy for any bidder a in that game is to submit her true valuation, that is, to submit a bid b_a=v_a.

Comment. This auction can be found in Vickrey's classical work (Vickrey 1961). And its proof has been presented by several authors. Nonetheless, the following proof is presented as a direct derivation of Proposition 0.

Proof. The standard Vickrey auction can be defined as an extended Vickrey auction where the number of auction items, $\bar{k}$, is equal to one. Since Proposition 0 holds for any $1 \leq \bar{k} < \bar{a}$, then it holds for the corollary.

Lemma 1. For a given level of risk averseness $\propto$, the utility function u, defined by equation (4), is always less than $1/\propto$.

Comment. It is very probable that other authors have provided a proof to the claim in this lemma.

Proof. Since the exponential function always render a positive value, then

$$e^{-\propto x} > 0$$

$$1 - e^{-\propto x} < 1$$

Since $\propto$ is always positive, then

$$\frac{1 - e^{-\propto x}}{\propto} < \frac{1}{\propto}$$

And by definition of u, then

$$u(x; \propto) < \frac{1}{\propto}$$

Thus, the lemma is proved.

Lemma 2. Consider the buyout extended raffle which offers at least $\bar{k}$ items and has a reserve price $r \geq 0$. Suppose $\bar{a} > \bar{k}$ players with utility function u as defined by equation (4), with level of risk aversion $\propto > 0$, and with private values no greater than $\bar{v}$. Then, the maximum proper price $\bar{p}$ can be explicitly calculated by the following equation:

$$\bar{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r) = \bar{v} + \frac{1}{\propto} log\left[1 - \propto \frac{\bar{k}}{\bar{a}} u(\bar{v} - r; \propto)\right] \tag{30}$$

Comment. The following proof is original to this dissertation.

Proof. Apply the definition of maximum proper price p to the utility function u defined in equation (4).

$$\frac{1 - e^{-\propto(\bar{v}-\bar{p})}}{\propto} = \frac{\bar{k}}{\bar{a}} u(\bar{v} - r; \propto)$$

$$e^{-\propto(\bar{v}-\bar{p})} = 1 - \propto \frac{\bar{k}}{\bar{a}} u(\bar{v} - r; \propto)$$

Since $u(\bar{v} - r; \propto)$ is less than $1/\propto$ and $\bar{k}/\bar{a}$ is less than one, then the right side is always positive. Therefore, the natural logarithm applied to the right side exists.

$$-\propto (\bar{v} - \bar{p}) = log\left[1 - \propto \frac{\bar{k}}{\bar{a}} u(\bar{v} - r; \propto)\right]$$

$$\propto (\bar{p} - \bar{v}) = log\left[1 - \propto \frac{\bar{k}}{\bar{a}} u(\bar{v} - r; \propto)\right]$$

$$\bar{p} = \bar{v} + \frac{1}{\propto} log\left[1 - \propto \frac{\bar{k}}{\bar{a}} u(\bar{v} - r; \propto)\right]$$

Thus, the lemma is proved. ■

Proposition 1. Consider the buyout extended raffle which has a buy price p, offers at least $\bar{k}$ items, has a reserve price $r \geq 0$. Suppose $\bar{a} > \bar{k}$ players with utility function defined by equation (4), with level of risk aversion $\propto > 0$, and with private values drawn from a cumulative distribution function F with range $[\underline{v}, \bar{v}]$. Suppose that the minimum proper price $\underline{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r)$ exists, and that $\underline{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r) < p$.

(i) If $p < \bar{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r)$,

(a) then there is a value c^* defined by equation (31),

$$u(c^* - p; \propto) = U_R(c^*, c^*; r, \propto, \bar{k}, \bar{a}) \tag{31}$$

(b) where such c^* has the following properties:

(1) It belongs to the range $(p, \bar{v})$.

(2) It defines a unique equilibrium cutoff (which is symmetric, and inefficient).

(3) It is increasing in p.

(4) It is increasing in $\bar{k}$.

(5) It is decreasing in $\bar{a}$.

(6) It is decreasing in r.

(7) It is decreasing in $\propto$.

(ii) If $p > \bar{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r)$, then there is a value c^* equal to $\bar{v}$, which defines an equilibrium cutoff that is unique, symmetric, and inefficient (and where the price is never accepted by the players).

Comment. The following proof is original to this dissertation. Nonetheless it follows several strategies used by Reynolds and Wooders (2009, p.27) for one of their buyout auctions.

Proof. Start by proving Proposition 1(i), that is, when $p < \bar{p}$. Before proving each of the statements, it is important to clarify here that, given the definition of reserve price, p belongs to the range $\left(r, \bar{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r)\right)$. This relation will be cited throughout this proof and therefore, it is stated here in expression (32):

$$p \in \left(r, \bar{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r)\right) \tag{32}$$

The fact that p is greater than $\underline{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r)$ will only become relevant when proving the uniqueness of the equilibrium.

Within Proposition 1(i), let us now prove statement (a). Using the definition of U_{R}, for any candidate cutoff c, equation (31) can be rewritten as follows:

$$u(c - p; \propto) = u(c - r; \propto) \cdot G_R(c; \bar{k}, \bar{a}) \tag{33}$$

In the above expression, although $G_{\mathrm{R}}(c; \bar{k}, \bar{a})$ is not a constant but a function of c, one can observe that if $G_{\mathrm{R}}(c; \bar{k}, \bar{a})$ were equal to one, then c would not exist because $u(c - p; \propto)$ and $u(c - r; \propto)$ describe the same curve but displaced by the distance $p - r$. This non-convergence of the two curves can be observed the in Figure 53. The figure also presents a case when the risk level is $\propto = 0$. Although Proposition 1 only applies to risk averse players, it is illustrative to see the curves when $\propto = 0$.

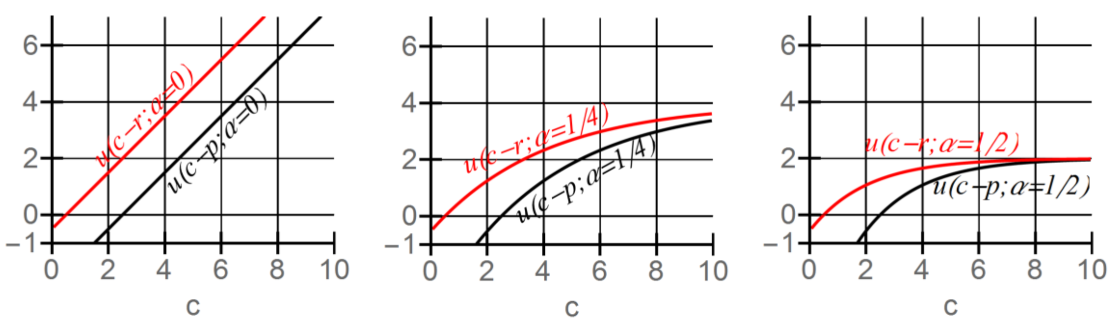


Figure 53. Functions $u(c - p; \propto)$ and $u(c - r; \propto)$ for $p = 2.50$ dollars, $r = 0.50$ dollars, and three values of $\propto$. The curves always keep horizontal distance of 2 dollars.

As c increases, as predicted by Lemma 1, the curves increase asymptotical to $1/\propto$ but never reach that value. Also, the curve is strictly increasing. For these two reasons, the two curves always keep the same distance $p - r$ and never intersect. Now, according to the definition of the probability G_{R}, that is equation (11), the only time in which G_{R} is equal to one, it is when $\bar{k} = \bar{a}$. But, since the proposition states that $\bar{k}$ is less than $\bar{a}$, then the curve $u(c - r; \propto)$ is decreased. Figure 54 now shows how the curves in Figure 53 would decrease.

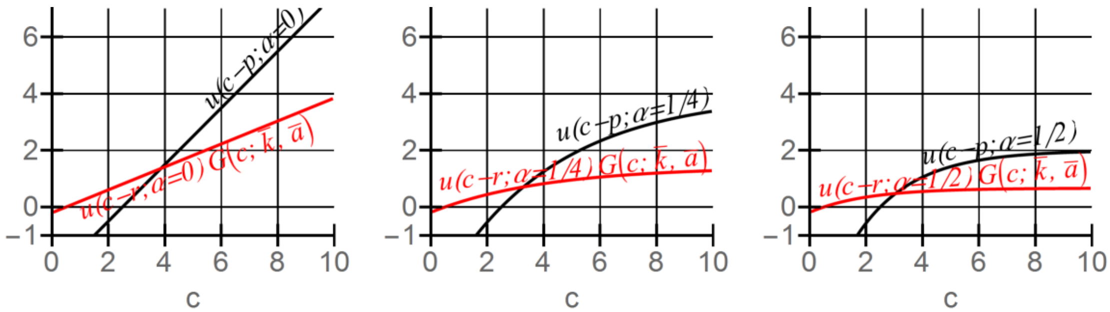


Figure 54. Payoff functions for $v = c$, $p = 2.50$ dollars, $r = 0.50$ dollars, $\bar{k} = 3$, $\bar{a} = 2$, and three values of $\propto$.

G_{R} has a non-negative slope, and then after $c = \bar{v}$, it becomes equal to the constant $\bar{k}/\bar{a}$. Therefore, the two payoff functions described in (33) should always intersect. Thus, there is at least one c satisfying equation (33). Nonetheless, it is not clear yet whether that intersection occurs at a c less than $\bar{v}$. Such verification is made in the following proof of property (b)(1).

Within Proposition 1(i), let us now prove the properties within statement (b). Start by property (b)(1). Define $\widehat{U}_{\mathrm{B}}(c) = u(c - p, \propto)$ and $\widehat{U}_{\mathrm{W}}(c) = u(c - r; \propto) \cdot G_{\mathrm{R}}\left(c; \bar{k}, \bar{a}\right)$. As indicated in Figure 55, since $\widehat{U}_{\mathrm{B}}(c)$ and $\widehat{U}_{\mathrm{W}}(c)$ are continuous, this proof consists in proving that $\widehat{U}_{\mathrm{W}}(p) > \widehat{U}_{\mathrm{B}}(p)$, and that $\widehat{U}_{\mathrm{B}}(p) > \widehat{U}_{\mathrm{W}}(\bar{v})$.

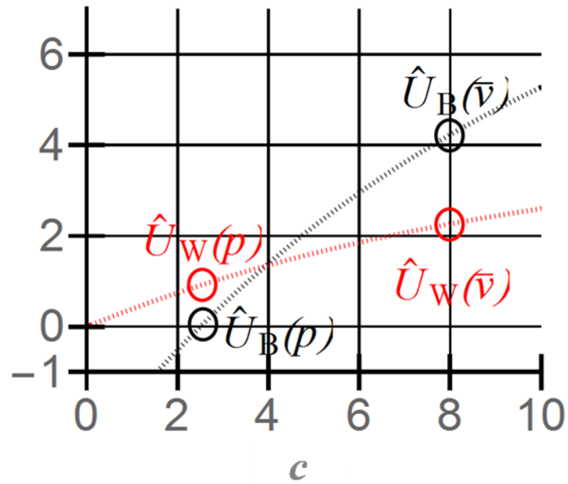


Figure 55. Figure presents two relations that need to be proved in order to prove that c falls within the range $(p, \bar{v})$: $\widehat{U}_{\mathrm{W}}(p) > \widehat{U}_{\mathrm{B}}(p)$, and $\widehat{U}_{\mathrm{B}}(\bar{v}) > \widehat{U}_{\mathrm{W}}(\bar{v})$.

Start by proving that $\widehat{U}_{\mathrm{W}}(p) > \widehat{U}_{\mathrm{B}}(p)$. According to the definition of $\widehat{U}_{\mathrm{B}}$, $\widehat{U}_{\mathrm{B}}(p)$ is equal to zero. Since expression (32) states that p is greater than r, then $\widehat{U}_{\mathrm{W}}(p) = \mathrm{u}(p - \mathrm{r}; \propto) \cdot G_{\mathrm{R}}(p; \bar{\mathrm{k}}, \bar{\mathrm{a}}) > 0$. Thus, $\widehat{U}_{\mathrm{W}}(p)$ is indeed greater than $\widehat{U}_{\mathrm{B}}(p)$.

We finalize by proving that $\widehat{U}_{\mathrm{B}}(\bar{v}) > \widehat{U}_{\mathrm{W}}(\bar{v})$. According to expression (32), the following inequality is true:

$$p < \bar{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r) \tag{34}$$

Apply the definition of $\bar{p}$.

$$p < \bar{v} + \frac{1}{\propto} log\left[1 - \propto \frac{\bar{k}}{\bar{a}} u(\bar{v} - r; \propto)\right]$$

Apply the definition of G_{R} for $c = \bar{v}$ as presented in equation (11).

$$p < \bar{v} + \frac{1}{\propto} log[1 - \propto G_R(\bar{v}; \bar{k}, \bar{a}) u(\bar{v} - r; \propto)]$$

$$p < \bar{v} + \frac{1}{\propto} log\left[1-\propto \widehat{U}_W(\bar{v})\right]$$

$$\bar{v} - p > -\frac{1}{\propto} log\left[1-\propto \widehat{U}_W(\bar{v})\right]$$

$$-\propto (\bar{v} - p) < log\left[1-\propto \widehat{U}_W(\bar{v})\right]$$

$$e^{-\propto(\bar{v}-p)} < 1-\propto \widehat{U}_W(\bar{v})$$

$$1 - e^{-\propto(\bar{v}-p)} > \propto \widehat{U}_W(\bar{v})$$

$$\frac{1 - e^{-\propto(\bar{v}-p)}}{\propto} > \widehat{U}_W(\bar{v})$$

$$u(\bar{v} - p, \propto) > \widehat{U}_W(\bar{v})$$

$$\widehat{U}_B(\bar{v}) > \widehat{U}_W(\bar{v}) \tag{35}$$

Therefore, c belongs to the interval $(p, \bar{v})$.

Let us now prove property (b)(2). To prove the uniqueness of c, it will be proved that the slope of $\widehat{U}_{\mathrm{B}}$ and the slope of $\widehat{U}_{\mathrm{W}}$ are both positive and that the slope of $\widehat{U}_{\mathrm{B}}$ is greater than the slope of $\widehat{U}_{\mathrm{W}}$. Regarding $\widehat{U}_{\mathrm{B}}$, it is clear that its slope is positive because it is equal to u, and u is strictly increasing. $\widehat{U}_{\mathrm{W}}$ has also a positive slope because it is equal to the multiplication of two functions (u and $\mathrm{G_R}$) which have positive slope. Now, the function describing the slope of $\widehat{U}_{\mathrm{B}}$ is defined as follows:

$$\frac{d}{dc}\widehat{U}_B(c) = \frac{\partial}{\partial c}u(c - p; \propto)$$

The function describing the slope of $\widehat{U}_{\mathrm{W}}$ is defined as follows:

$$\frac{d}{dc}\widehat{U}_W(c) = \frac{\partial}{\partial c}\left[u(c-r;\propto)\cdot G_R\left(c;\bar{k},\bar{a}\right)\right]$$

According to the assumptions made for the whole proposition, p is greater than the minimum proper price $\underline{p}$. Therefore, according to the definition of $\underline{p}$, the following is true:

$$\frac{d}{dc}\widehat{U}_B(c) > \frac{d}{dc}\widehat{U}_W(c) \qquad \forall c \in (p,\bar{v}) \tag{36}$$

Thus, the cutoff c is unique. Denote this cutoff c as c^*

Now, let us prove whether c^* describes a state of equilibrium. To prove this, one needs to look at how the slopes of the payoffs compare, not along different cutoffs c (as expression (36) does), but along different private values v_a for the cutoff c^*. Therefore, expression (37) must hold for all $v_a \in (p,\bar{v})$:

$$\frac{\partial}{\partial v_a}u(v_a-p;\propto) > \frac{\partial}{\partial v_a}\left[u(v_a-r;\propto)\cdot G_R\left(c^*;\bar{k},\bar{a}\right)\right] \tag{37}$$

$$\frac{\partial}{\partial v_a}u(v_a-p;\propto) > G_R\left(c^*;\bar{k},\bar{a}\right)\frac{\partial}{\partial v_a}(v_a-r;\propto)$$

Apply equation (4) (definition of u).

$$\frac{\partial}{\partial v_a}\frac{1-e^{-\propto(v_a-p)}}{\propto} > G_R\left(c^*;\bar{k},\bar{a}\right)\frac{\partial}{\partial v_a}\frac{1-e^{-\propto(v_a-r)}}{\propto}$$

$$\frac{-\propto}{\propto}(-e^{\propto p-\propto v_a}) > G_R\left(c^*;\bar{k},\bar{a}\right)\frac{-\propto}{\propto}(-e^{\propto p-\propto r})$$

$$\frac{e^{\propto p}}{e^{\propto v_a}} > G_R\left(c^*;\bar{k},\bar{a}\right)\frac{e^{\propto r}}{e^{\propto v_a}}$$

$$e^{\propto p} > G_R\left(c^*;\bar{k},\bar{a}\right)e^{\propto r} \tag{38}$$

Given that $e^{\propto p} > e^{\propto r}$ and given that $G_{\mathrm{R}}\left(c^{*}; \bar{k}, \bar{a}\right) < 0$, then it has now been proved that c^{*} describes a state of equilibrium.

Let us now prove property (b)(3). $\widehat{U}_{\mathrm{B}}$ is a function of p but $\widehat{U}_{\mathrm{W}}$ is not. Therefore, an increase in p, that respects expression (32), would displace $\widehat{U}_{\mathrm{B}}$ to the right but would not have an effect $\widehat{U}_{\mathrm{W}}$. Therefore, the new cutoff point would happen at a c^{*} that is higher. Thus, c^{*} is increasing in p.

Let us now prove property (b)(4) and (b)(5). An increase in $\bar{k}$ would increase the probability G_{R} which in turn would increase the function $\widehat{U}_{\mathrm{W}}$ but would not affect $\widehat{U}_{\mathrm{B}}$. Therefore, an increase in $\bar{k}$ would locate the new cutoff point at a c^{*} that is higher. Thus, c^{*} is increasing in $\bar{k}$. Following a similar rationale, it can be concluded that c^{*} is decreasing in $\bar{a}$.

Let us now prove property (b)(6). An increase in r would move $\widehat{U}_{\mathrm{W}}$ to the right while would not have effect on $\widehat{U}_{\mathrm{B}}$. Since $\widehat{U}_{\mathrm{W}}$ is an increasing function, the new cutoff point would happen at a c^{*} that is lower. Thus, the solution is decreasing in $\propto$.

Let us now prove property (b)(7). This requires using a notation for $\widehat{U}_{\mathrm{B}}$ and $\widehat{U}_{\mathrm{W}}$ that also shows explicitly their dependence on $\propto$. Let $\widetilde{U}_{\mathrm{B}}(c; \propto) = u(c - p, \propto)$ and let $\widetilde{U}_{\mathrm{W}}(c; \propto) = u(c - r; \propto) \cdot G_{\mathrm{R}}\left(c; \bar{k}, \bar{a}\right)$. Assume $\propto$ increases from $\propto'$ to $\propto''$. Let c' be the solution to $\widehat{U}_{\mathrm{B}}(c, \propto') = \widehat{U}_{\mathrm{W}}(c, \propto')$ and let c'' be the solution to $\widetilde{U}_{\mathrm{B}}(c, \propto'') = \widetilde{U}_{\mathrm{W}}(c, \propto'')$. Since $\widehat{U}_{\mathrm{B}}(p)$ is equal to zero, and $\widehat{U}_{\mathrm{W}}(p)$ is greater than zero, then, as illustrated in Figure 56, if it can be shown that $\widetilde{U}_{\mathrm{W}}(c'', \propto') > \widetilde{U}_{\mathrm{B}}(c'', \propto')$, then c'' has to be less than c', and therefore, property (b)(7) is true.

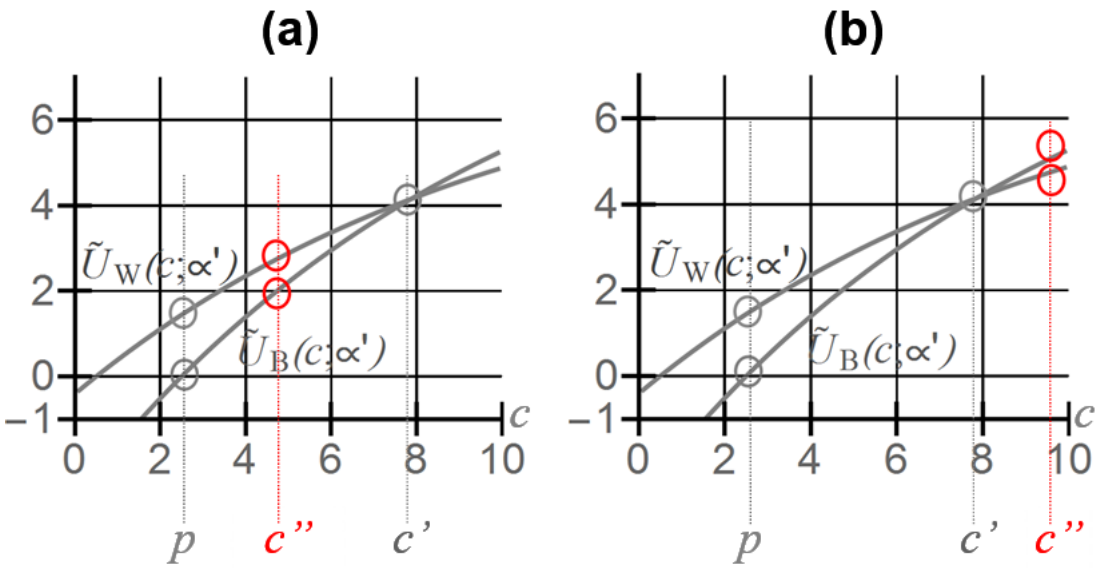


Figure 56. Functions $\widetilde{U}_{\mathrm{W}}(c, \propto')$ and $\widetilde{U}_{\mathrm{B}}(c, \propto')$ for different cutoffs c. c' represent the equilibrium cutoff. $\widetilde{U}_{\mathrm{B}}(p, \propto')$ is always equal to zero, $\widetilde{U}_{\mathrm{W}}(p, \propto')$ is always greater than $\widetilde{U}_{\mathrm{B}}(p, \propto')$. Figure (a) shows that, for $c'' \neq c'$, if $\widetilde{U}_{\mathrm{W}}(c'', \propto') > \widetilde{U}_{\mathrm{B}}(c'', \propto')$, then c'' has to be less than c'. Figure (b) shows that, for $c'' \neq c'$, if $\widetilde{U}_{\mathrm{W}}(c'', \propto') < \widetilde{U}_{\mathrm{B}}(c'', \propto')$, then c'' has to be greater than c'.

As suggested by Reynolds and Wooders (2009, p.28), it can be shown that, for x and y fixed and $x < y$, the relation $1 - e^{-\propto \cdot x}/1 - e^{-\propto \cdot y}$ is increasing in $\propto$. Therefore, the following statement is true, where $x = c'' - p$ and $y = c'' - r$:

$$\frac{1 - e^{-\propto'' \cdot (c'' - p)}}{1 - e^{-\propto'' \cdot (c'' - r)}} > \frac{1 - e^{-\propto' \cdot (c'' - p)}}{1 - e^{-\propto' \cdot (c'' - r)}}$$

$$\frac{\dfrac{1 - e^{-\propto'' \cdot (c'' - p)}}{\propto''}}{\dfrac{1 - e^{-\propto'' \cdot (c'' - r)}}{\propto''}} > \frac{1 - e^{-\propto' \cdot (c'' - p)}}{1 - e^{-\propto' \cdot (c'' - r)}}$$

Apply equation (4) (definition of u) on the left hand side.

$$\frac{u(c''-p;\propto'')}{u(c''-r;\propto'')} > \frac{1-e^{-\propto'\cdot(c''-p)}}{1-e^{-\propto'\cdot(c''-r)}}$$

Apply equation (33) on left hand side.

$$G_{\mathrm{R}}\left(c'';\bar{\mathrm{k}},\bar{\mathrm{a}}\right) > \frac{1-e^{-\propto''\cdot(c''-p)}}{1-e^{-\propto''\cdot(c''-r)}}$$

$$\left[1-e^{-\propto''\cdot(c''-r)}\right]\cdot G_{\mathrm{R}}\left(c'';\bar{k},\bar{a}\right) > 1-e^{-\propto''\cdot(c''-p)}$$

$$\frac{\left[1-e^{-\propto''\cdot(c''-r)}\right]}{\propto''}\cdot G_{\mathrm{R}}\left(c'';\bar{k},\bar{a}\right) > \frac{1-e^{-\propto''\cdot(c''-p)}}{\propto''}$$

Apply equation (4) (definition of u).

$$u(c''-r;\propto'')\cdot G_{\mathrm{R}}\left(c'';\bar{k},\bar{a}\right) > u(c''-p;\propto'')$$

$$\widetilde{U}_{\mathrm{W}}(c'',\propto') > \widetilde{U}_{\mathrm{B}}(c'',\propto')$$

Let us now end by proving Proposition 1(ii). Here, the uniqueness of c^* is proved not only by showing that $c^* = \bar{v}$ describes a state of equilibrium but also by showing that a c different from $\bar{v}$ cannot exist.

First it will be proved that a c different from $\bar{v}$ cannot describe a state of equilibrium. Specifically, it is assumed that $c \in [p,\bar{v})$. Once again, the notation f', $\widehat{U}_{\mathrm{B}}$ and $\widehat{U}_{\mathrm{W}}$ are used.

According to the definition of $\widehat{U}_{\mathrm{B}}$, $\widehat{U}_{\mathrm{B}}(p)$ is equal to zero. And according to the definition of $\widehat{U}_{\mathrm{W}}(p)$, $\widehat{U}_{\mathrm{W}}(p)$ is greater than zero. Now, according to the condition of the statement, we have:

$$p \geq \bar{p}\left(\bar{v}; \bar{k}, \bar{a}, \propto, r\right) \tag{39}$$

Applying the same rationale used for reaching expression (35) from expression (34), one obtains expression (40).

$$\widehat{U}_B(\bar{v}) \leq \widehat{U}_W(\bar{v}) \tag{40}$$

Thus, if c belongs to $[p, \bar{v})$, then one is confronted with one of the two scenarios shown in Figure 57.

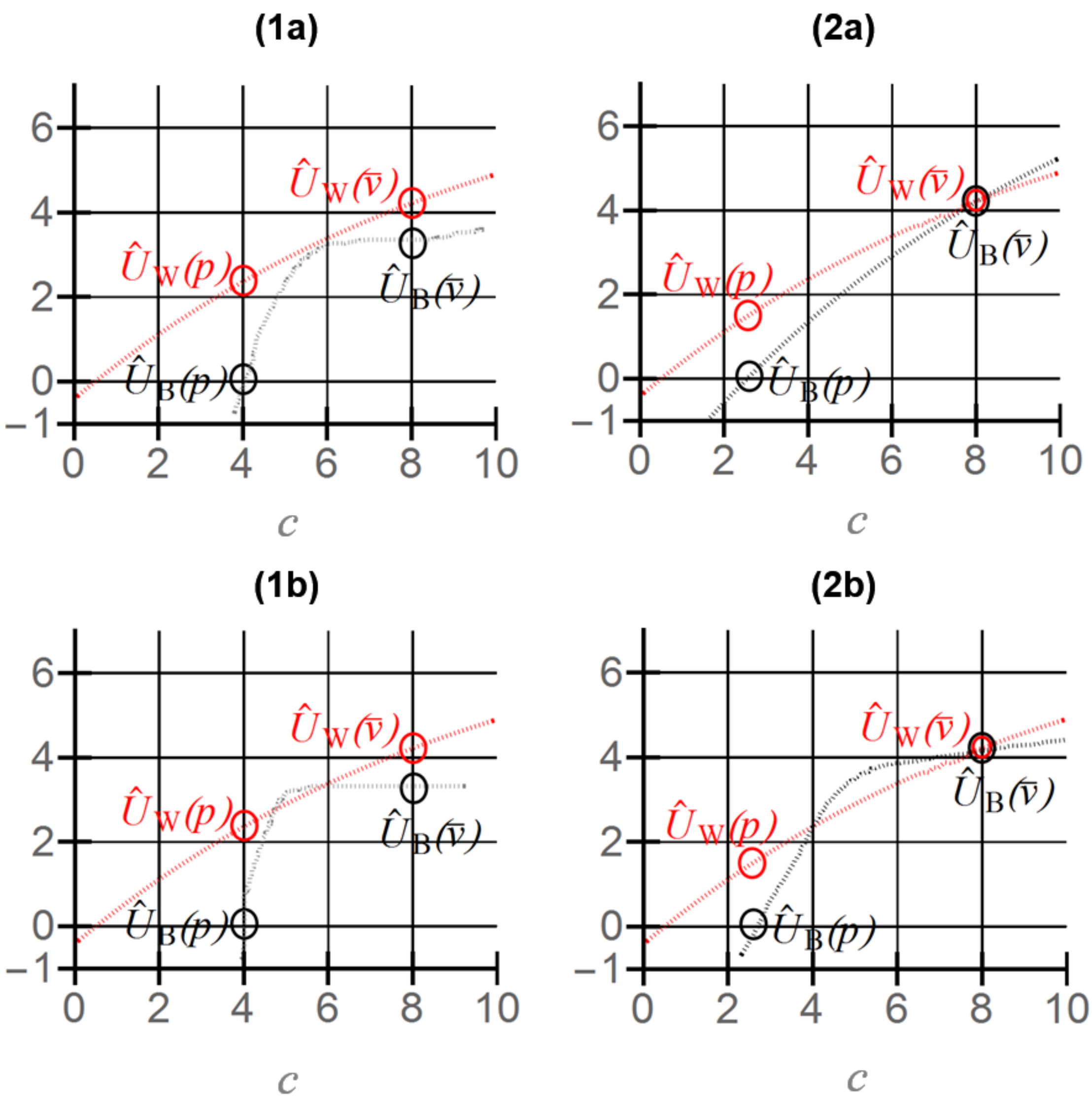


Figure 57. If it were true that there is a unique c less than $\bar{v}$ for $p \geq \bar{p}(\bar{v}; \bar{k}, \bar{a}, \propto, r)$, then $\hat{U}_{\mathrm{W}}(p) > \hat{U}_{\mathrm{B}}(p) = 0$, and $\hat{U}_{\mathrm{B}}(\bar{v}) < \hat{U}_{\mathrm{W}}(\bar{v})$ (as shown in graphs 1a and 1b) or $\hat{U}_{\mathrm{B}}(\bar{v}) = \hat{U}_{\mathrm{W}}(\bar{v})$ (as shown in graphs 2a and 2b). Notice that in any case, the two curves meet but do not cross each other (as shown in graphs 1a and 1b), or have to cross each other several times (as shown in graphs 2a and 2b).

Since p is greater than the minimum proper price $\underline{p}$, then, according to the definition of $\underline{p}$, expression (41) should be true.

$$\frac{d}{dc}\hat{U}_B(c) > \frac{d}{dc}\hat{U}_W(c) \qquad \forall c \in (p, \bar{v}) \tag{41}$$

But expression (41) is stating that where $\widehat{U}_B$ and $\widehat{U}_{\mathrm{W}}$ meet, they should cross each other, which contradicts any of the alternatives shown in Figure 57.

Finally, it will be proved that $c^* = \bar{v}$ describes a state of equilibrium. Here, one needs to look at how the slopes of the payoffs compare along different private values v_a for the cutoff $c^* = \bar{v}$ by looking at whether expression (42) holds:

$$\frac{\partial}{\partial v_a} u(v_a - p; \propto) > \frac{\partial}{\partial v_a}\left[u(v_a - r; \propto) \cdot G_R\left(c^*; \bar{k}, \bar{a}\right)\right] \tag{42}$$

Applying the same rationale used for reaching expression (38) from expression (37), one obtains expression (43).

$$e^{\propto p} > G_R\left(c^*; \bar{k}, \bar{a}\right) e^{\propto r} \tag{43}$$

Given that $e^{\propto p} > e^{\propto r}$ and given that $G_R\left(\bar{v}; \bar{k}, \bar{a}\right) < G_{\mathrm{R}}\left(c^*; \bar{k}, \bar{a}\right) < 0$, then $c^* = \bar{v}$ is indeed a state of equilibrium.

Proposition 2. Consider the buyout auction which has a buy price p, offers at least $\bar{k}$ items, has a reserve price $r \geq 0$. Suppose $\bar{a} > \bar{k}$ players with utility function defined by equation (4), with level of risk aversion $\propto > 0$, and with private values drawn from a cumulative distribution function F with range $\left[\underline{v}, \bar{v}\right]$.

(i) If $p < \delta\left(\bar{v}; \bar{k}, \bar{a}, \propto, r\right)$,

(a) then there is a value c^* defined by equation (44),

$$u(c^* - p; \propto) = U\left(c^*, c^*; r, \propto, \bar{k}, \bar{a}\right) \tag{44}$$

or equivalently by equation (45),

$$u(c^* - p; \propto) = u\big(c^* - \delta\big(c^*; \bar{k}, \bar{a}, \propto, r\big); \propto\big) \cdot G\big(c^*; \bar{k}, \bar{a}\big) \tag{45}$$

(b) where such c^* has the following properties:

(1) It belongs to the range $(p, \bar{v})$.

(2) It defines a unique equilibrium cutoff (which is symmetric, and efficient).

(3) It is increasing in p.

(4) It is increasing in $\bar{k}$.

(5) It is decreasing in $\bar{a}$.

(6) It is decreasing in r.

(7) It is decreasing in $\propto$.

(ii) If $p \geq \delta\big(\bar{v}; \bar{k}, \bar{a}, \propto, r\big)$, then there is a value c^* equal to $\bar{v}$, which defines an equilibrium cutoff that is unique, symmetric, and efficient (and where the price is never accepted by the players).

Comment. It should be acknowledged that the following proof is for the most part the same as the one already presented by Reynolds and Wooders (2009, p.27) but applied to a different cumulative probability distribution G. Nonetheless, the utility function used here for participants who choose the buy option is different from (and not really a specific case of) theirs. Also, in their proposition, they did not identify a relation between the equilibrium cutoff c* and the new variables introduced in this dissertation: $\bar{k}$ an $\bar{a}$. Thus, one cannot conclude from their proof whether the unique equilibrium would cease to exist with different values of $\bar{k}$ or $\bar{a}$.

Proof. Start by proving Proposition 2(i), that is, when $p < \bar{p}$. Before proving each of the statements, it is important to clarify here that, given the definition of reserve price,

p belongs to the range $\left(r, \delta\left(\bar{v}; \bar{k}, \bar{a}, \propto, r\right)\right)$. This relation will be cited throughout this proof and therefore, it is stated here in expression (46):

$$p \in \left(r, \delta\left(\bar{v}; \bar{k}, \bar{a}, \propto, r\right)\right) \tag{46}$$

Within Proposition 2(i), the existence of a cutoff c that satisfies expression (45) cannot be deduced intuitively from a plot due to the fact that instead of having the constant term r (such was the case in Proposition 1), one has the term δ which is a function of c and not a constant. Therefore, the existence of the cutoff will be proved as a byproduct of proving its location (which is the first property presented in statement (b)).

Within Proposition 2(i), let us now prove the properties within statement (b). Start by property (b)(1). Define $\widehat{U}_{\mathrm{B}}(c) = u(c - p, \propto)$ and $\widehat{U}_{\mathrm{W}}(c) = u\left(c^{*} - \delta\left(c^{*}; \bar{k}, \bar{a}, \propto, r\right); \propto\right) \cdot G\left(c^{*}; \bar{k}, \bar{a}\right)$. As indicated in Figure 58, since $\widehat{U}^{\mathrm{B}}(c)$ and $\widehat{U}_{\mathrm{W}}(c)$ are continuous, this proof consists in proving that $\widehat{U}_{\mathrm{W}}(p) > \widehat{U}_{\mathrm{B}}(p)$, and that $\widehat{U}_{\mathrm{B}}(p) > \widehat{U}_{\mathrm{W}}(\bar{v})$.

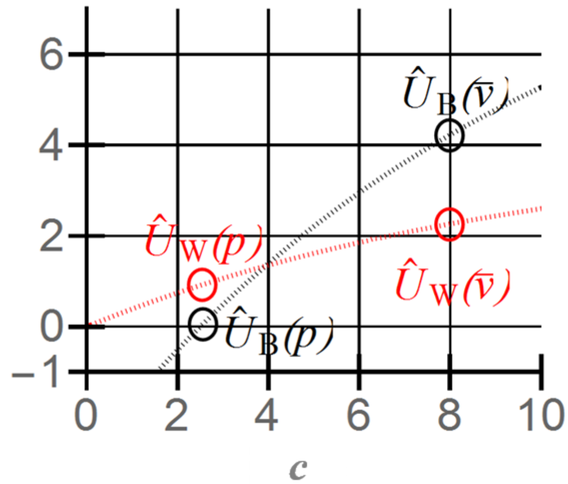


Figure 58. Figure presents two relations that need to be proved in order to prove that c falls within the range $(p, \bar{v})$: $\widehat{U}_{\mathrm{W}}(p) > \widehat{U}_{\mathrm{B}}(p)$, and $\widehat{U}_{\mathrm{B}}(\bar{v}) > \widehat{U}_{\mathrm{W}}(\bar{v})$.

Start by proving that $\widehat{U}_{\mathrm{W}}(p) > \widehat{U}_{\mathrm{B}}(p)$. According to the definition of $\widehat{U}_{\mathrm{B}}$, $\widehat{U}_{\mathrm{B}}(p)$ is equal to zero. Since expression (46) states that p is greater than r, then, according to Lemma 5 (page 88), $p > \delta\left(p; \bar{k}, \bar{a}, \propto, r\right)$. Therefore, $\widehat{U}_{\mathrm{W}}(p) = \mathrm{u}\left(p - \delta\left(p; \bar{k}, \bar{a}, \propto, r\right); \propto\right) \cdot G\left(p; \bar{\mathrm{k}}, \bar{\mathrm{a}}\right) > 0$. Thus, $\widehat{U}_{\mathrm{W}}(p)$ is indeed greater than $\widehat{U}_{\mathrm{B}}(p)$.

We finalize by proving that $\widehat{U}_{\mathrm{B}}(\bar{\mathrm{v}}) > \widehat{U}_{\mathrm{W}}(\bar{\mathrm{v}})$. According to Lemma 5 (page 88), $\bar{\mathrm{v}} > \delta\left(\bar{\mathrm{v}}; \bar{k}, \bar{a}, \propto, r\right)$. Thus, $\widehat{\mathrm{U}}_{\mathrm{B}}(\bar{\mathrm{v}}) = u(\bar{\mathrm{v}} - p; \propto) > u\left(p - \delta\left(\bar{\mathrm{v}}; \bar{k}, \bar{a}, \propto, r\right); \propto\right) \cdot 1 = \widehat{\mathrm{U}}_{\mathrm{W}}(\bar{\mathrm{v}})$. Therefore, c exists and belongs to the interval $(p, \bar{v})$.

Let us now prove property (b)(2). Here, the following notation needs to be introduced. For a function f with an independent variable x and a set $\mathbb{P}$ of parameters, let $f'(x; \mathbb{P})$ refer to $\frac{\partial}{\partial x} f(x; \mathbb{P})$. Thus, $u'\left(v_a - \delta\left(v_a; \bar{k}, \bar{a}, \propto, r\right); \propto\right)$ is different from $\frac{\partial}{\partial v_a} \mathrm{u}\left(v_a - \delta\left(v_a; \bar{k}, \bar{a}, \propto, r\right); \propto\right)$ or from $\frac{\partial}{\partial \propto} \mathrm{u}\left(v_a - \delta\left(v_a; \bar{k}, \bar{a}, \propto, r\right); \propto\right)$.

$u'\left(v_a - \delta\left(v_a; \bar{k}, \bar{a}, \propto, r\right); \propto\right)$ is equal to $\frac{\partial}{\partial\left(v_a - \delta\left(v_a; \bar{k}, \bar{a}, \propto, r\right)\right)} \mathrm{u}\left(v_a - \delta\left(v_a; \bar{k}, \bar{a}, \propto, r\right); \propto\right)$ and $\delta'\left(v_a; \bar{k}, \bar{a}, \propto, r\right)$ is equal to $\frac{\partial}{\partial v_a} \delta\left(v_a; \bar{k}, \bar{a}, \propto, r\right)$.

Now, we prove the uniqueness of c. To accomplish this, it will be proved that the slope of $\widehat{U}_{\mathrm{B}}$ is greater than the slope of $\widehat{U}_{\mathrm{W}}$ at the point where they intersect. At the intersection, c must comply with (44). Hence,

$$u(c - p; \propto) = U\left(c, c; r, \propto, \bar{k}, \bar{a}\right)$$

$$u(c - p; \propto) = u\left(c - \delta\left(c; \bar{k}, \bar{a}, \propto, r\right); \propto\right) \cdot G\left(c; \bar{k}, \bar{a}\right) \tag{47}$$

Since c is within the interval $(p, \bar{v})$ (as stated by property b(1)), then $\mathrm{G}\left(\mathrm{c}; \bar{\mathrm{k}}, \bar{\mathrm{a}}\right)$ is less than one. Then the two utility function values in expression (47) should comply with the following inequality.

$$u(c - p; \propto) < u\left(c - \delta\left(c; \bar{k}, \bar{a}, \propto, r\right); \propto\right) \tag{48}$$

Since u is concave, then the slope of $u(x; \propto)$ at $x = c - p$ should be steeper than the slope at $x = c - \delta\left(c; \bar{k}, \bar{a}, \propto, r\right)$. Then the derivatives of the two utility function values in expression (48) should comply with the following inequality.

$$u'(c - p; \propto) > u'\left(c - \delta\left(c; \bar{k}, \bar{a}, \propto, r\right); \propto\right) \tag{49}$$

Thus, at the point of intersection, expression (49) should hold.

Now, the function describing the slope of $\widehat{U}_{\mathrm{B}}$ is defined as follows:

$$\widehat{U}_B'(c) = \frac{\partial}{\partial c} u(c - p; \propto) \tag{50}$$

And the function describing the slope of $\widehat{U}^{\mathrm{W}}$ is defined as follows:

$$\widehat{U}_W'(c) = \frac{\partial}{\partial c}\left[u\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right) \cdot G\left(c;\bar{k},\bar{a}\right)\right]$$

$$\widehat{U}_W'(c) = G\left(c;\bar{k},\bar{a}\right) \cdot u'\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right) \cdot \left(1 - \delta'\left(c;\bar{k},\bar{a},\propto,r\right)\right)$$

$$+G'\left(c;\bar{k},\bar{a}\right) \cdot u\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right)$$

$$\widehat{U}_W'(c) = G\left(c;\bar{k},\bar{a}\right) \cdot u'\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right)$$

$$-G\left(c;\bar{k},\bar{a}\right) \cdot u'\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right) \cdot \delta'\left(c;\bar{k},\bar{a},\propto,r\right)$$

$$+G'\left(c;\bar{k},\bar{a}\right) \cdot u\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right)$$

Simplify second term in the right hand side after substituting $\delta'\left(c;\bar{k},\bar{a},\propto,r\right)$ with the equivalent expression suggested by Lemma 6 (page 88).

$$\widehat{U}_W'(c) = G\left(c;\bar{k},\bar{a}\right) \cdot u'\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right)$$

$$-G'\left(c;\bar{k},\bar{a}\right) \cdot u\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right)$$

$$+G'\left(c;\bar{k},\bar{a}\right) \cdot u\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right)$$

$$\widehat{U}_W'(c) = G\left(c;\bar{k},\bar{a}\right) \cdot u'\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right) \tag{51}$$

According to equations (50) and (51), in order to find out whether $\widehat{U}_{\mathrm{B}}'(\mathrm{c})$ is greater than $\widehat{U}_{\mathrm{W}}'(\mathrm{c})$, one must then verify whether the following inequality holds:

$$u'(v_a - p;\propto) > G\left(c;\bar{k},\bar{a}\right) \cdot u'\left(c - \delta\left(c;\bar{k},\bar{a},\propto,r\right);\propto\right) \tag{52}$$

Due to expression (49) and to the fact that $G\left(c;\bar{k},\bar{a}\right)$ is less than one, then expression (53) is true. Thus, the cutoff c is unique. Denote this cutoff c as c^*

Now, let us prove whether c^* describes a state of equilibrium. To prove this, one needs to look at how the slopes of the payoffs compare along different private values

v_a for the cutoff c^*. Therefore, and using the alternative definition of U from Lemma 8 (page 88), it must be true that expression (53) holds for all $v_a \in (p, \bar{v})$:

$$\frac{\partial}{\partial v_a} u(v_a - p; \propto) > \frac{\partial}{\partial v_a} \left[u\left(v_a - \delta\left(min(v_a, c^*); \bar{k}, \bar{a}, \propto, r\right); \propto\right) \cdot G\left(min(v_a, c^*); \bar{k}, \bar{a}\right) \right] \tag{53}$$

Due to the term $\min(v_a, \mathrm{c}^*)$ in expression (53), one has to analyze two cases: when $v_a > \mathrm{c}^*$ and when $v_a < \mathrm{c}^*$.

According to expression (47), and given that $G\left(c^*; \bar{k}, \bar{a}\right)$ is less than one, then $u(v_a - p; \propto)$ is less than $u\left(v_a - \delta\left(c^*; \bar{k}, \bar{a}, \propto, r\right); \propto\right)$. Therefore, expression (54) is true regardless of which of the two cases is considered.

$$p > \delta\left(c^*; \bar{k}, \bar{a}, \propto, r\right) \tag{54}$$

Now, consider the case when $v_a > \mathrm{c}^*$. Expression (53) becomes expression (55).

$$\frac{\partial}{\partial v_a} u(v_a - p; \propto) > G\left(c^*; \bar{k}, \bar{a}\right) \cdot \frac{\partial}{\partial v_a} u\left(v_a - \delta\left(c^*; \bar{k}, \bar{a}, \propto, r\right); \propto\right) \tag{55}$$

Expression (55) indeed true due to the fact that $G\left(c^*; \bar{k}, \bar{a}\right)$ is less than one, due to expression (54), and due to the fact that u is concave (along v_a). Therefore, expression (55) is true, then c^* does describe a state of equilibrium for $v_a > c^*$.

Consider the case when $v_a < \mathrm{c}^*$. Expression (53) becomes expression (56).

$$\frac{\partial}{\partial v_a} u(v_a - p; \propto) > \frac{\partial}{\partial v_a} \left[u\left(v_a - \delta\left(v_a; \bar{k}, \bar{a}, \propto, r\right); \propto\right) \cdot G\left(v_a; \bar{k}, \bar{a}\right) \right] \tag{56}$$

$$\frac{\partial}{\partial v_a} u(v_a - p; \propto) > G\big(v_a; \bar{k}, \bar{a}\big) \cdot u'\big(v_a - \delta\big(v_a; \bar{k}, \bar{a}, \propto, r\big); \propto\big) \cdot \Big(1 - \delta'\big(v_a; \bar{k}, \bar{a}, \propto, r\big)\Big)$$

$$+G'\big(v_a; \bar{k}, \bar{a}\big) \cdot u\big(v_a - \delta\big(v_a; \bar{k}, \bar{a}, \propto, r\big); \propto\big)$$

$$\frac{\partial}{\partial (v_a - p)} u(v_a - p; \propto) > G\big(v_a; \bar{k}, \bar{a}\big) \cdot u'\big(v_a - \delta\big(v_a; \bar{k}, \bar{a}, \propto, r\big); \propto\big)$$

$$u'(v_a - p; \propto) > G\big(v_a; \bar{k}, \bar{a}\big) \cdot u'\big(v_a - \delta\big(v_a; \bar{k}, \bar{a}, \propto, r\big); \propto\big) \tag{57}$$

Expression (57) comes from applying Lemma 6 (page 88). If expression 57 is true, then c^* does describe a state of equilibrium for $v_a < c^*$.

Due to expression (54) and since $\delta\big(v_a; \bar{k}, \bar{a}, \propto, r\big)$ is increasing in v (Lemma 6, page 88) then p is also greater than $\delta\big(c^*; \bar{k}, \bar{a}, \propto, r\big)$ for all $v_a < c^*$. Therefore $u'(v_a - p; \propto) > u'\big(v_a - \delta\big(v_a; \bar{k}, \bar{a}, \propto, r\big); \propto\big)$ for all $v_a < [p, c^*]$. Thus,

$$u'(v_a - p; \propto) > G\big(v_a; \bar{k}, \bar{a}\big) \cdot u'\big(v_a - \delta\big(v_a; \bar{k}, \bar{a}, \propto, r\big); \propto\big) \tag{58}$$

since $1 > G\big(c; \bar{k}, \bar{a}\big) \geq G\big(v_a; \bar{k}, \bar{a}\big)$. From expressions (57) and (44), it can then be concluded that c^* does describe a state of equilibrium for $v_a < c^*$

Let us now prove property (b)(3). $\widehat{U}_B$ is a function of p but $\widehat{U}_W$ is not. Therefore, an increase in p, that respects expression (46), would displace $\widehat{U}_B$ to the right but would not have an effect $\widehat{U}_W$. Therefore, the new cutoff point would happen at a c^* that is higher. Thus, c^* is increasing in p.

Let us now prove property (b)(4) and (b)(5). An increase in $\bar{k}$ would increase the probability G which in turn would increase the function $\widehat{U}_W$ but would not affect $\widehat{U}_B$.

Therefore, an increase in $\bar{k}$ would locate the new cutoff at a c^* that is higher. Thus, c^* is increasing in $\bar{k}$. Following a similar rationale, it can be concluded that c^* is decreasing in $\bar{a}$.

Let us now prove property (b)(6). An increase in r would move $\widehat{U}_{\mathrm{W}}$ to the right while would not have effect on $\widehat{U}_{\mathrm{B}}$. Since $\widehat{U}_{\mathrm{W}}$ is an increasing function, the new cutoff point would happen at a c^* that is lower. Thus, the solution is decreasing in $\propto$.

Let us now prove property (b)(7). This requires using a notation for $\widehat{U}_{\mathrm{B}}$ and $\widehat{U}_{\mathrm{W}}$ that also shows explicitly their dependence on $\propto$. Let $\widetilde{U}_{\mathrm{B}}(c;\propto) = u(c-p,\propto)$ and let $\widetilde{U}_{\mathrm{W}}(c;\propto) = u(c-r;\propto) \cdot G\left(c;\bar{k},\bar{a}\right)$. Assume $\propto$ increases from $\propto'$ to $\propto''$. Let c' be the solution to $\widehat{U}_{\mathrm{B}}(c,\propto') = \widehat{U}_{\mathrm{W}}(c,\propto')$ and let c'' be the solution to $\widetilde{U}_{\mathrm{B}}(c,\propto'') = \widetilde{U}_{\mathrm{W}}(c,\propto'')$. Since $\widehat{U}_{\mathrm{B}}(p)$ is equal to zero, and $\widehat{U}_{\mathrm{W}}(p)$ is greater than zero, then, as illustrated in Figure 59, if it can be shown that $\widetilde{U}_{\mathrm{W}}(c'',\propto') > \widetilde{U}_{\mathrm{B}}(c'',\propto')$, then c'' has to be less than c', and therefore, property (b)(7) is true.

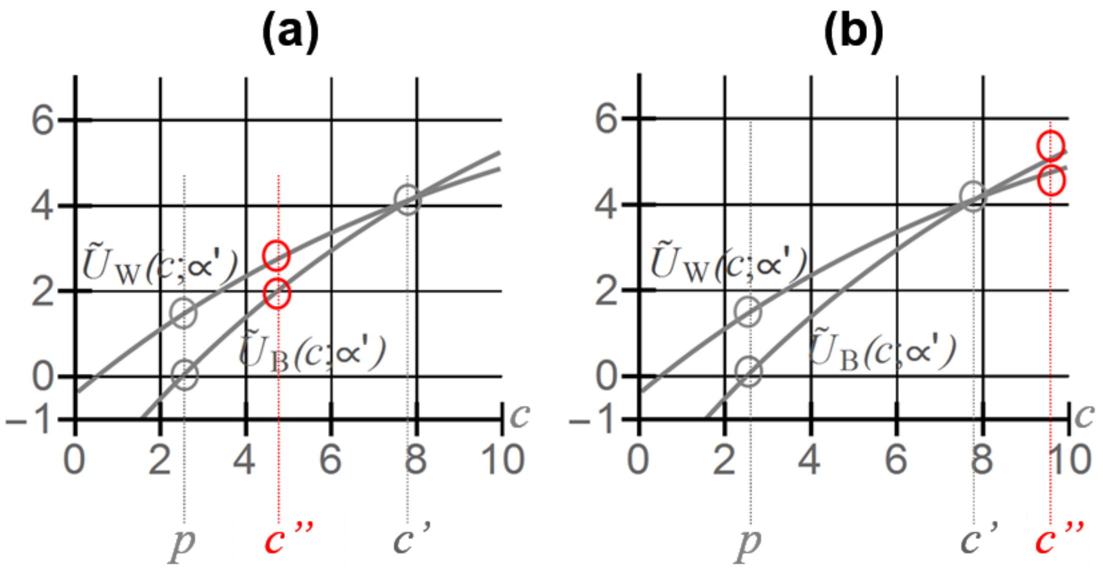


Figure 59. Functions $\widetilde{U}_{\mathrm{W}}(c,\propto')$ and $\widetilde{U}_{\mathrm{B}}(c,\propto')$ for different cutoffs c. c' represent the equilibrium cutoff. $\widetilde{U}_{\mathrm{B}}(p,\propto')$ is always equal to zero, $\widetilde{U}_{\mathrm{W}}(p,\propto')$ is always greater than $\widetilde{U}_{\mathrm{B}}(p,\propto')$. Figure (a) shows that, for $c'' \neq c'$, if $\widetilde{U}_{\mathrm{W}}(c'',\propto') > \widetilde{U}_{\mathrm{B}}(c'',\propto')$, then c'' has to be less than c'. Figure (b) shows that, for $c'' \neq c'$, if $\widetilde{U}_{\mathrm{W}}(c'',\propto') < \widetilde{U}_{\mathrm{B}}(c'',\propto')$, then c'' has to be greater than c'.

As suggested by Reynolds and Wooders (2009, p.28), it can be shown that, for x and y fixed and $x < y$, the relation $1 - e^{-\propto \cdot x}/1 - e^{-\propto \cdot y}$ is increasing in $\propto$. Therefore, the following statement is true, where $x = c'' - p$ and $y = c'' - r$:

$$\frac{1 - e^{-\propto'' \cdot (c''-p)}}{1 - e^{-\propto'' \cdot \left(c'' - \delta(c'';\bar{\mathrm{k}},\bar{\mathrm{a}},\propto,\mathrm{r})\right)}} > \frac{1 - e^{-\propto' \cdot (c''-p)}}{1 - e^{-\propto' \cdot \left(c'' - \delta(c'';\bar{\mathrm{k}},\bar{\mathrm{a}},\propto,\mathrm{r})\right)}}$$

$$\frac{\dfrac{1 - e^{-\propto'' \cdot (c''-p)}}{\propto''}}{\dfrac{1 - e^{-\propto'' \cdot \left(c'' - \delta(c'';\bar{\mathrm{k}},\bar{\mathrm{a}},\propto,\mathrm{r})\right)}}{\propto''}} > \frac{1 - e^{-\propto' \cdot (c''-p)}}{1 - e^{-\propto' \cdot \left(c'' - \delta(c'';\bar{\mathrm{k}},\bar{\mathrm{a}},\propto,\mathrm{r})\right)}}$$

Apply equation (4) (definition of u) on the left hand side.

$$\frac{u(c'' - p;\propto'')}{u\left(c'' - \delta\left(c'';\bar{k},\bar{a},\propto,r\right);\propto''\right)} > \frac{1 - e^{-\propto' \cdot (c''-p)}}{1 - e^{-\propto' \cdot \left(c'' - \delta(c'';\bar{k},\bar{a},\propto,r)\right)}}$$

Apply equation (45) on left hand side.

$$G\left(c'';\bar{k},\bar{a}\right) > \frac{1-e^{-\propto''\cdot(c''-p)}}{1-e^{-\propto''\cdot(c''-\delta(c'';\bar{k},\bar{a},\propto,r))}}$$

$$\left[1-e^{-\propto''\cdot\left(c''-\delta\left(c'';\bar{k},\bar{a},\propto,r\right)\right)}\right]\cdot G\left(c'';\bar{k},\bar{a}\right) > 1-e^{-\propto''\cdot(c''-p)}$$

$$\frac{\left[1-e^{-\propto''\cdot\left(c''-\delta\left(c'';\bar{k},\bar{a},\propto,r\right)\right)}\right]}{\propto''}\cdot G\left(c'';\bar{k},\bar{a}\right) > \frac{1-e^{-\propto''\cdot(c''-p)}}{\propto''}$$

Apply equation (4) (definition of u).

$$u\left(c''-\delta\left(c'';\bar{k},\bar{a},\propto,r\right);\propto''\right)\cdot G\left(c'';\bar{k},\bar{a}\right) > u(c''-p;\propto'')$$

$$\widetilde{U}_{\mathrm{W}}(c'',\propto') > \widetilde{U}_{\mathrm{B}}(c'',\propto')$$

Let us now end by proving Proposition 2(ii). Here, the uniqueness of c^* is proved not only by showing that $c^* = \bar{v}$ describes a state of equilibrium but also by showing that a c different from $\bar{v}$ cannot exist. Once again the notation f', $\widehat{U}_{\mathrm{B}}$ and $\widehat{U}_{\mathrm{W}}$ are used.

First it will be proved that a c different from $\bar{v}$ cannot describe a state of equilibrium. Specifically, it is assumed that If $c = p$ and that $c \in (p,\bar{v})$.

Let $c = p$. Then $u(\mathrm{v_a}-p;\propto) > U\left(\mathrm{v_a},\mathrm{c}^*;\mathrm{r},\propto,\bar{\mathrm{k}},\bar{\mathrm{a}}\right)$ for all $\mathrm{v_a} \in (p,\bar{v}]$. But as $\mathrm{v_a}$ approaches p, $u(\mathrm{v_a}-p;\propto)$ approaches zero, while $U\left(\mathrm{v_a},\mathrm{c}^*;\mathrm{r},\propto,\bar{\mathrm{k}},\bar{\mathrm{a}}\right)$ is strictly positive, which contradicts that $c = p$ is an equilibrium cutoff.

Let $c \in (p,\bar{v})$. Without loss of generality, let c be the largest such cutoff. In the proof of statement (b)(2), it was shown that $\widehat{U}_{\mathrm{B}}(\mathrm{v_a})$ is steeper than $\widehat{U}_{\mathrm{W}}(\mathrm{v_a})$ at the intersection, that is, when $\mathrm{v_a}$ is equal to c. Therefore, $\widehat{U}_{\mathrm{B}}(\mathrm{v_a}) > \widehat{U}_{\mathrm{W}}(\mathrm{v_a})$ for all $\mathrm{v_a} \in$

$(c, \bar{v})$. If $c = \bar{v}$, then either (i) $\widehat{U}_{\mathrm{B}}(\bar{v}) > \widehat{U}_{\mathrm{W}}(\bar{v})$, or (ii) $u(\bar{v} - p; \propto) = \widehat{U}_{\mathrm{W}}(\bar{v})$. Since $p \geq \delta(\bar{v}, \propto)$ then

$$\widehat{U}_{\mathrm{B}}(\bar{v}) = u(\bar{v} - p; \propto) \leq u\big(\bar{v} - \delta\big(\bar{v}; \bar{k}, \bar{a}, \propto, r\big); \propto\big) = \widehat{U}_{\mathrm{W}}(\bar{v}) \tag{59}$$

which contradicts (i). If $\widehat{U}_{\mathrm{B}}(\bar{v}) = \widehat{U}_{\mathrm{W}}(\bar{v})$ then $\widehat{U}_{\mathrm{B}}(\bar{v})$ is steeper thatn $\widehat{U}_{\mathrm{W}}(\bar{v})$ at $c = \bar{v}$, which contradicts $\widehat{U}_{\mathrm{B}}(\mathrm{v_a}) > \widehat{U}_{\mathrm{W}}(\mathrm{v_a})$ for all $\mathrm{v_a} \in (c, \bar{v})$.

Finally, it will be proved that $c^* = \bar{v}$ describes a state of equilibrium. Here, one needs to look at how the slopes of the payoffs compare along different private values v_a for the cutoff $c^* = \bar{v}$ by looking at whether expression (60) holds for $v_a < \bar{v}$:

$$\frac{\partial}{\partial v_a} u(v_a - p; \propto) > \frac{\partial}{\partial v_a}\big[u\big(v_a - \delta\big(\bar{v}; \bar{k}, \bar{a}, \propto, r\big); \propto\big) \cdot G\big(\bar{v}; \bar{k}, \bar{a}\big)\big] \tag{60}$$

Since $p > \ \delta\big(\bar{\mathrm{v}}; \bar{k}, \bar{a}, \propto, r\big)$ then

$$u(\bar{v} - p, \propto) \leq u\big(\bar{v} - \delta\big(\bar{\mathrm{v}}; \bar{k}, \bar{a}, \propto, r\big); \propto\big) = U(\bar{v}, \bar{v}, \propto) \cdot \tag{61}$$

Furthermore, $U'(v_a, \bar{v}, \propto) = G\big(v_a; \bar{k}, \bar{a}\big) \cdot u'\big(v_a - \delta(v_a, \propto)\big)$ and $u'(v_a - p, \propto) = u(v_a - p, \propto)$. Since $v_a < \bar{v}$, then

$$u'(v_a - p, \propto) \geq u'\big(v_a - \delta\big(v_a; \bar{k}, \bar{a}, \propto, r\big); \propto\big) > G\big(v_a; \bar{k}, \bar{a}\big) \cdot u'(v_a - \delta(v_a, \propto); \propto) \tag{62}$$

where the weak inequality follows from $p \geq \delta\big(\bar{\mathrm{v}}; \bar{k}, \bar{a}, \propto, r\big) > \delta\big(v_a; \bar{k}, \bar{a}, \propto, r\big)$ and the strict inequality follows from $G\big(v_a; \bar{k}, \bar{a}\big) < 1$. This establishes that expression (60) is valid for $v_a < \bar{v}$. Thus, $c^* = \bar{v}$ is indeed a state of equilibrium.

Appendix E. Other Road Designs

Before reaching the final design proposed in Section 5.1, other designs were tested for this dissertation. This appendix presents such designs. Overall, because they are not suggesting implementing systems on a "DAR-type" entrance but on a "gate-type" entrance (see Subsection A.2.2 for their definitions), they present the difficulty of creating weaving on the general purpose (GP) lanes. This characteristic not only it may create safety issues but it can be difficult to model making it difficult to measure the unused managed capacity.

To simplify the description of the operation of the designs, this appendix uses the sets presented in section 4.2.1 for referring to the different categories of vehicles. For convenience, those categories are presented here again:

- $\mathbb{A}$: Set of "advanced SOVs".
- $\mathbb{B}$: Set of "basic SOVs".
- $\mathbb{H}$: Set of HOVs.
- $\mathbb{O}$: Set of drivers who cannot enter the HOT lane (because are not HOVs or do not have the OBU necessary to enter as an SOV) or whose destination cannot be reached via the HOT lane.

E.1. Double entrance

Figure 60 presents a double entrance design. This is the only design in which simulation tests were carried out for this dissertation. It functions as follows. Drivers belonging to 𝔸, 𝔹 or ℍ, while driving on the GP lanes see the toll price p. Drivers from 𝔸 and 𝔹 who regard the toll price as affordable choose to enter the HOT lanes using the first entrance. Drivers from ℍ also use the first entrance. Drivers who regard p as unaffordable, continue driving on the GP lanes and pass by the row of electronic readers. These readers detect the OBUs that drivers in 𝔸 in this manner quantify the number of bidders. By crossing the row of readers, participants enter into what is called the "auction zone". Before any bidder reaches the row of electronic transmitters, the operator should be able to quantify also the unused managed capacity. When the first bidder reaches the row of transmitters, the operator will be able to tell her if she is one of the winners of the auction by sending a signal to her OBU. All participants who are told on the row of transmitters that they are winners use the second entrance to the HOT lane.

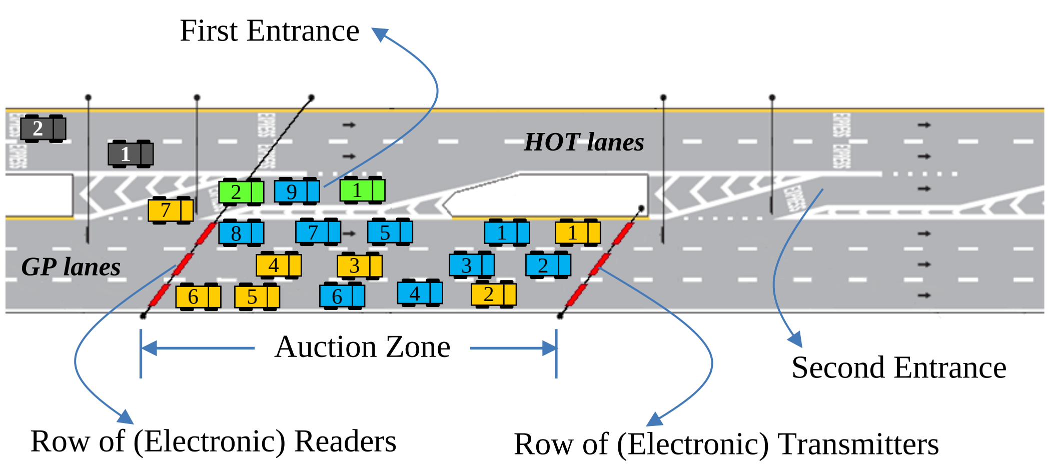
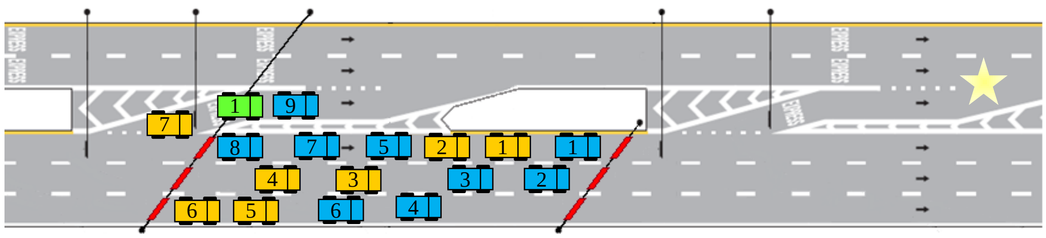


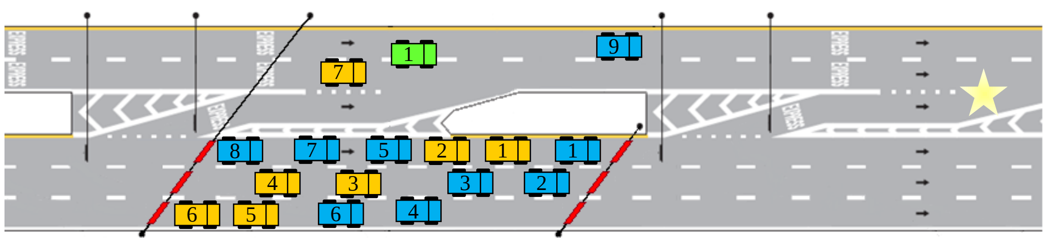
Figure 60. Double entrance design. Vehicles in set 𝔸 appear in amber, vehicles of 𝔹 appear in blue, vehicles in ℍ appear in harlequin (or bright green) and vehicles in ℚ appear in quartz (or dark gray). Signage that presents the toll price is not shown in the figure since it is upstream of the shown segment.

This design presented three challenges. One was the difficulty to predict which vehicles from the two entrances were going to meet. This difficulty is portrayed in Figure 61.

Scenario 1



Scenario 2



Scenario 3

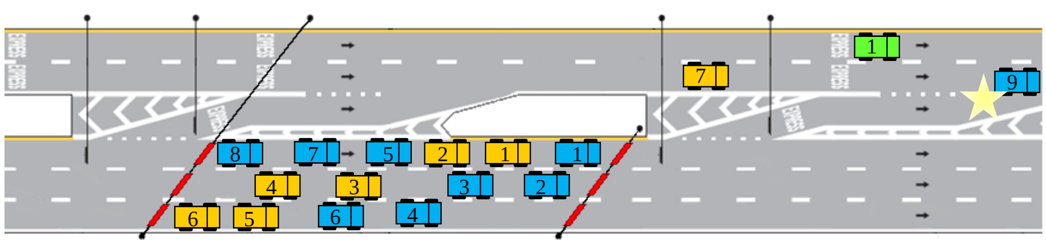


Figure 61. For a given instant in time, three scenarios depicting elements of $\mathbb{A}$ (in amber), $\mathbb{B}$ (in blue), and $\mathbb{H}$ (in harlequin or bright green). Only in the first scenario, vehicles from $\mathbb{A}_{\mathrm{BUY}}$, $\mathbb{B}_{\mathrm{BUY}}$ and $\mathbb{H}$ have a chance of physically meet at the point of convergence (shown as a star) with the winners from $\mathbb{B}_{\mathrm{BID}}$.

The second and unexpected difficulty can be described using Figure 61 too. In any of the scenarios shown in the figure, it can be observed that the queue of vehicles on the third lane of the GP lanes does not block the first entrance. The simulations showed that, after testing several lengths for the auction zone, that first entrance would always get blocked due to the merging that vehicles had to do at the second entrance.

The third challenge, but perhaps not as important as the second one, was the weaving movements occurring right after the row of transmitters when winners were told to use the second entrance.

The above three challenges incentivized the idea of designing an auction-based system for a DAR-type entrance and not for gate-type entrance.

E.2. GP lanes with discriminatory lanes

Figure 62 presents a design with discriminatory lanes on the GP lanes. It functions as follows. Again, drivers belonging to $\mathbb{A}$, $\mathbb{B}$ or $\mathbb{H}$, while driving on the GP lanes see the toll price p. But now, several yards after the toll price, they see another sign indicating that drivers wanting to enter the HOT lanes need to take the leftmost lane. Then, the GP lanes pass from having three lanes to four. Drivers from $\mathbb{O}$ and from $\mathbb{B}$ not willing to enter HOT lanes the toll must drive on the two rightmost lanes. All other drivers should drive on the two leftmost lanes. Several yards later, a sign indicates that drivers from $\mathbb{H}$, and drivers willing to pay the toll price must be on the leftmost lane while the others must be on the second leftmost lane.

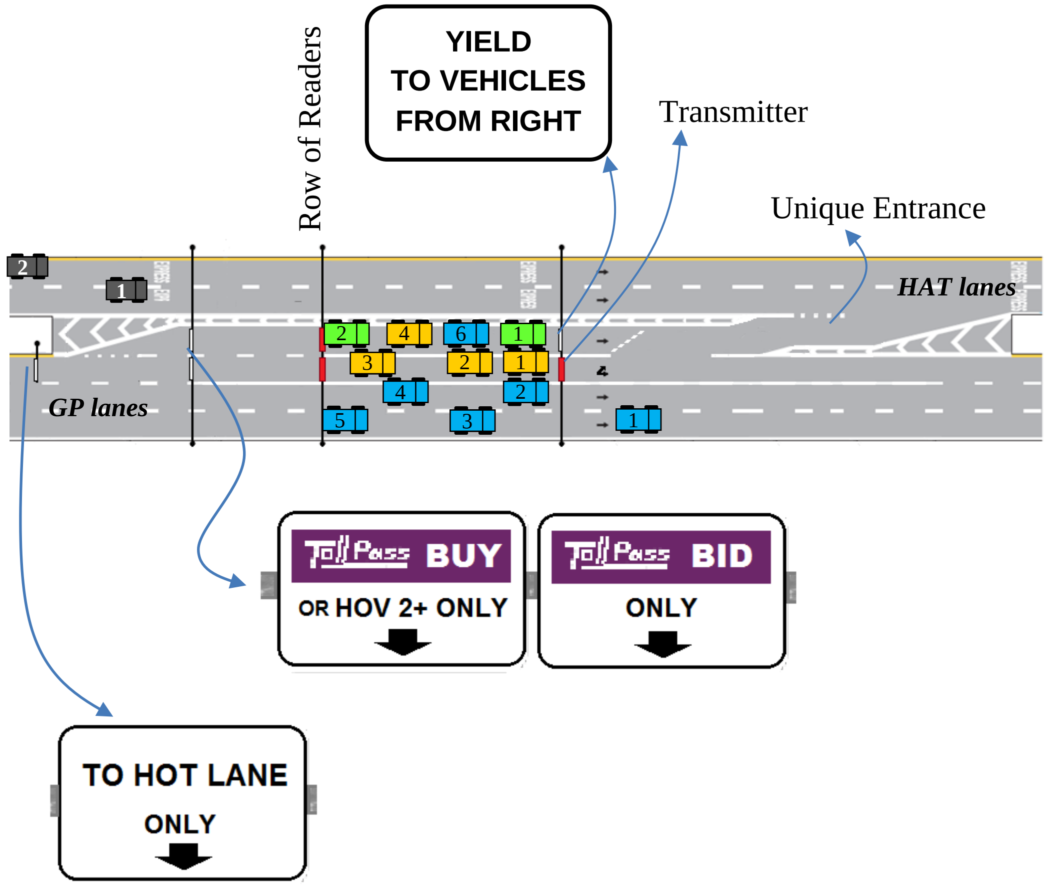


Figure 62. Design with GP lanes with discriminatory lanes. Vehicles in set 𝔸 appear in amber, in set 𝔹 appear in blue, in set ℍ appear in harlequin (or bright green). Signage that presents the toll price is not shown in the figure because it is upstream of the shown segment.

Several yards later, all drivers on the two leftmost lanes pass by a row of readers. These readers count the vehicles and records the willingness to pay of the vehicles on the second leftmost lane. Then, drivers from set 𝔸 on the second leftmost lane pass by the transmitter where it will be told of the auction was won. If they win, in order to reduce conflicts and in order to force the sets to drive together, they will have priority over the left lane when entering the HOT lane.

The goal of having discriminatory lanes is that it reduces weaving and it allows vehicles to declare if they are going to pay the toll, bid or simply avoid the HOT lanes.

E.3. Double entrance and GP lanes with discriminatory lanes

Figure 33 presents a design that functions as the previous one but does not require vehicles on the leftmost lane to do any yielding. Nonetheless, this simplification may make the measurement of unused managed capacity more difficult.

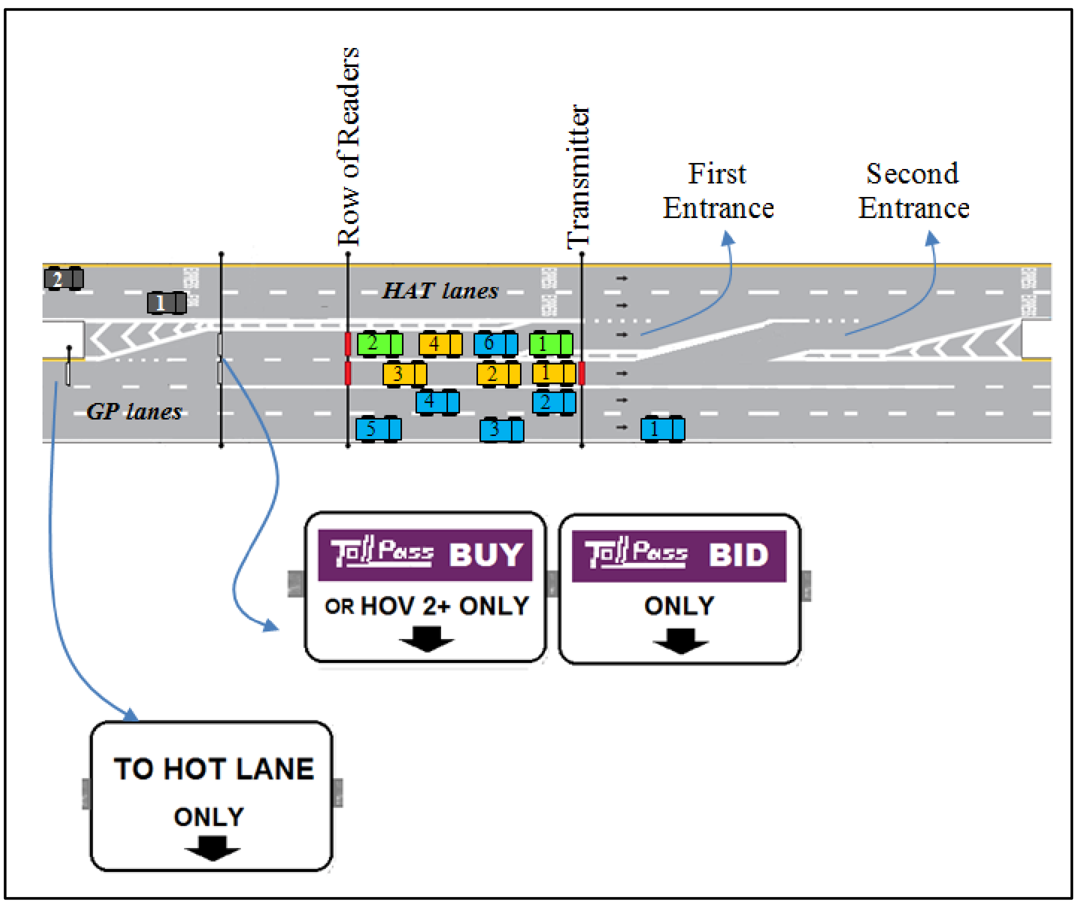


Figure 63. Similar design to Figure 62 but having two entrances in order to remove the yield sign.

E.4. Single entrance and GP lanes with no discriminatory lanes

Another possible design is implementing the previous one but with no discriminatory lanes. Although this approach has the benefit of removing some road signage, it requires all drivers to explicitly indicate if they will remain on the GP lanes through some sort of electronic device such as an improved OBU or smart phone (notice that this is one of the very few cases in which this dissertation considers using a smart phone as an OBU).

Figure 64 depicts this design with all the required signage. This signage follows the recommendations postulated by the Manual on Uniform Traffic Control Devices (Federal Highway Administration 2009) for HOT lanes. All the previous designs did not show all the recommended signage. This design also presents challenges due to the increased weaving activity.

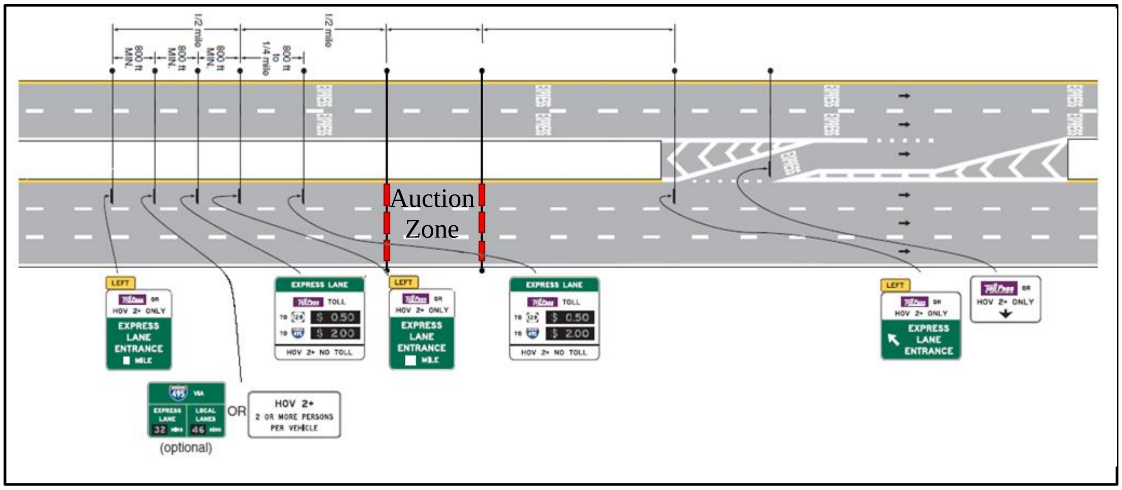


Figure 64. Design composed of single entrance and no discriminatory lanes on the GP lanes. Here, all the warning and information signs that drivers need are shown, as recommended by the Manual on Uniform Traffic Control Devices (Federal Highway Administration 2009) for HOT lanes.

Appendix F. Calibration and Validation Process

This appendix explains the calibration and validation that was carried out for the traffic microsimulation model. This model was used in Chapter 3 for measuring unused managed capacity. It was also used in Chapter 6 as a component of a larger model for testing the auction based metering system. The following process was summarized by Olarte and Haghani (2016).

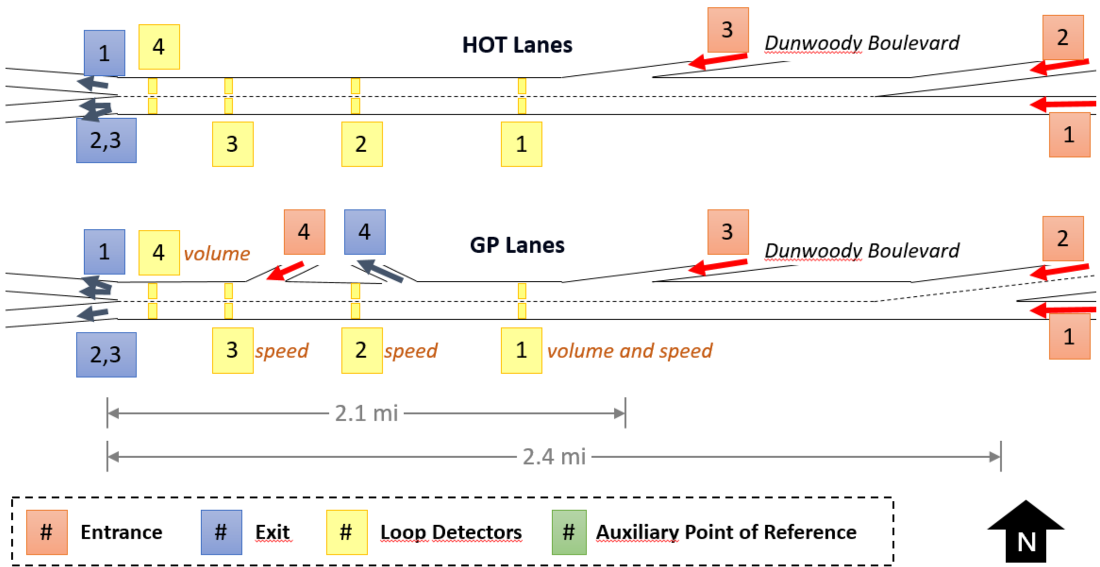


Figure 65. Schematic representation of the reversible section of the I-394 MnPass Express lanes and the GP lanes parallel to them. Labels in orange indicate the dimensions used for the calibration process at each of the loop detectors involved in the calibration process.

The objective function used for the calibration process was the mean absolute percentage error as defined by equation (63). In this equation, K refers to the total number of measurements, o_k refers to the observed measurement obtained from the loop detector data, and r_k refers to the simulated measurement. Each r_k represents an average of values from nine random seeds. Measurements correspond to intervals of five minutes.

$$\text{MAPE} = \frac{1}{K}\sum_{k=1}^{K}\left|\frac{o_k - r_k}{o_k}\cdot 100\right| \tag{63}$$

The observed measurements comprise speeds and volumes obtained from the sources shown in Figure 65. Because congestion on the HOT lanes was so low, only the GP lanes were used for a more meaningful calibration. Table 9 presents the specific time intervals used. The term "sub-model" is adopted here to indicate that only a partial section of the whole traffic model was used. Note that there was no calibration or validation on the arterials because it was not possible to conciliate the volumes provided from Dunwoody Boulevard with those from the GP and HOT lanes. Additional counts in 2013 on other arterials adjacent to Dunwoody Boulevard would have allowed such conciliation.

Table 9. Sub-model and time intervals (for Thursday, May 11th 2012) used for calibrating and validating the driving behavior parameters.

Sub-Model No.	Scope	Warm-Up Interval Used Before Calibration	Interval Used for Calibration	Warm-Up Interval Used Before Validation	Interval Used for Validation
1	GP Lanes	20:27-20:35	20:35-21:00	21:12-21:20	21:20-21:45
2	HOT Lanes	None	None	17:12-17:20	17:20-17:45

Two strategies were adopted to cope with the lack of knowledge of the origin-destination (OD) matrix. First, the time periods used for the calibration and validation were such that there were no vehicles taking the first exit out of the GP lanes. This reduced the number of dimensions in the OD matrix. Second, the speeds obtained from the loop detectors at yellow box 4 in Figure 65 were used to apply a feature from the VISSIM application called “reduced speed zones”. As vehicles in the model approached those zones, they accelerated or decelerated in order to meet the real speed. In this way, the possible real congestion created by weaving at the exits was replaced with possible simulated congestion created by the reduced speed zones. This explains why only volumes (and not speeds) from the loop detectors at yellow box 4 were used for calculating equation (63). As shown in Table 10, after conducting the calibration process, a MAPE of 4.6% was obtained. This is a very good value. Nonetheless, it was found that the default parameters and other behavioral parameters obtained for other facilities also produced a good MAPE value for the sub-models. This seems to suggest that the lack of weaving maneuvers in the sub-models or the fact that the sub-models are not more than three-mile long may have resulted in a somewhat insensitive traffic model.

Table 10. MAPE obtained when applying different parameters to sub-models.

Driving Behavior Parameter Values Obtained from	Sub-Model No. 1 (20:27-21:00)	Sub-Model No. 1 (21:12-21:20)	Sub-Model No. 2 (17:12-17:45)
Calibration of VISSIM Sub-Model No. 1	4.6	5.6	4.2
VISSIM 5.4 default values	5.0	4.9	4.0
Williams et al. (2010) Uncongested Scenario	5.1	5.0	4.0
Williams et al. (2010) Congested Scenario	4.8	4.8	4.1

References

91 Express Lanes, 2015. General Information: Toll Schedules. Available at: https://www.91expresslanes.com/schedules.asp [Accessed: May 6, 2016].

Angst, C.M., Agarwal, R. & Kuruzovich, J., 2008. Bid or buy? Individual shopping traits as predictors of strategic exit in on-line auctions. *International Journal of Electronic Commerce*, 13(1), p.59–84.

Ardekani, S.A., Ghandehari, M. & Nepal, S.M., 2011. Macroscopic Speed-Flow Models for Characterization of Freeway and Managed Lanes. *Buletinul Institutului Politehnic din lasi. Sectia Constructii, Arhitectura*, 57(1), p.149-159.

Arnott, R., de Palma, A. & Lindsey, R., 1999. Recent developments in the bottleneck model. In *Road Pricing, Traffic Congestion and the Environment: Issues of Efficiency and Social Feasibility*. Northhampton, MA, U.S.A.: Edward Elgar Pub, p. 79-110.

Arrow, K.J., 1971. *Essays in the theory of risk-bearing*, Markham Pub. Co.

Ausubel, L.M. & Cramton, P., 2004. Vickrey Auctions with Reserve Pricing. *Economic Theory*, 23(3), p.493-505.

Booz Allen Hamilton & HNTB Corporation, 2008. *FHWA Office of Operations - A Review of HOV Lane Performance in the United States: Final Report*, Washington, DC, U.S.A.: Federal Highway Administration. Available at: http://ops.fhwa.dot.gov/publications/fhwahop09029/index.htm [Accessed: June 7, 2013].

Brilon, W., Geistefeldt, J. & Regler, M., 2005. Reliability of Freeway Traffic Flow: A Stochastic Concept of Capacity. In *Transportation and Traffic Theory: Flow, Dynamics and Human Interaction, Proceedings of the 16th International Symposium on Transportation and Traffic Theory, University of Maryland, College Park, Maryland, 19-21 July 2005*. 16th International Symposium on Transportation and Traffic Theory, University of Maryland, College Park, Maryland, 19-21 July 2005. College Park, MD: Elsevier.

Brownstone, D. et al., 2003. Drivers' willingness-to-pay to reduce travel time: evidence from the San Diego I-15 congestion pricing project. *Transportation Research Part A: Policy and Practice*, 37(4), p.373-387.

Brownstone, D. & Small, K.A., 2005. Valuing time and reliability: assessing the evidence from road pricing demonstrations. *Transportation Research Part A: Policy and Practice*, 39(4), p.279-293.

Budish, E.B. & Takeyama, L.N., 2001. Buy prices in online auctions: irrationality on the internet? *Economics Letters*, 72(3), p.325-333.

Chu, X., 1995. Endogenous Trip Scheduling: The Henderson Approach Reformulated and Compared with the Vickrey Approach. *Journal of Urban Economics*, 37(3), p.324-343.

City of Minneapolis, 2015. Transportation Data Management System. *Transportation Data Management System*. Available at: http://minneapolis.ms2soft.com/tcds/tsearch.asp?loc=Minneapolis&mod= [Accessed: October 16, 2015].

Clarke, E.H., 1971. Multipart pricing of public goods. *Public choice*, 11(1), p.17–33.

Collins, A. et al., 2015. Exploring a Toll Auction Mechanic Enabled by V2I Technology. In *TRB 2015 Annual Meeting Compendium of Papers*. TRB Annual Meeting 2015. Washington, D.C., United States.

Dresner, K. & Stone, P., 2004. Multiagent Traffic Management: A Reservation-Based Intersection Control Mechanism. In *Proceedings of the Third International Joint Conference on Autonomous Agents and Multiagent Systems - Volume 2*. AAMAS ’04. Washington, DC, USA: IEEE Computer Society, p. 530–537. Available at: http://dx.doi.org/10.1109/AAMAS.2004.190 [Accessed: October 6, 2013].

Dresner, K. & Stone, P., 2005. Multiagent traffic management: an improved intersection control mechanism. In *Proceedings of the fourth international joint conference on Autonomous agents and multiagent systems*. AAMAS ’05. New York, NY, USA: ACM, p. 471–477. Available at: http://doi.acm.org/10.1145/1082473.1082545 [Accessed: October 6, 2013].

Federal Highway Administration, 2008. Economics: Pricing, Demand, and Economic Efficiency—A Primer - Table of Contents - FHWA Operations. Available at: http://ops.fhwa.dot.gov/publications/fhwahop08041/cp_prim4_00.htm [Accessed: June 3, 2016].

Federal Highway Administration, 2009. *Manual on Uniform Traffic Control Devices*, Federal Highway Administration. Available at: http://mutcd.fhwa.dot.gov/pdfs/2009/pdf_index.htm [Accessed: February 23, 2011].

Federal Transit Administration, 2013. What is the National Transit Database. Available at: http://www.ntdprogram.gov/ntdprogram/ntd.htm [Accessed: June 2, 2016].

Gallien, J. & Gupta, S., 2007. Temporary and Permanent Buyout Prices in Online Auctions. *MANAGEMENT SCIENCE*, 53(5), p.814-833.

Gardner, L.M., Bar-Gera, H. & Boyles, S.D., 2013. Development and comparison of choice models and tolling schemes for high-occupancy/toll (HOT) facilities. *Transportation Research Part B: Methodological*, 55, p.142-153.

Goodall, N. & Smith, B.L., 2010. What Drives Decisions of Single-Occupant Travelers in High-Occupancy Vehicle Lanes? *Transportation Research Record: Journal of the Transportation Research Board*, 2178(1), p.156-161.

Google Maps, 2011. I-15 & Ted Williams Parkway, California. Available at: http://maps.google.com/maps?f=q&source=s_q&hl=en&geocode=&q=san+diego,+ca&aq=&sll=37.0625,-95.677068&sspn=40.59616,67.763672&ie=UTF8&hq=&hnear=San+Diego,+California&ll=32.96326,-117.095675&spn=0.003724,0.00618&t=k&z=18 [Accessed: March 17, 2011].

Google Maps, 2015. NE Pacific Street & NE Pacific Place, Seattle, Washington. *Google Maps*. Available at: https://www.google.com/maps/@47.6506957,-122.3078851,3a,75y,145.88h,67.56t/data=!3m6!1e1!3m4!1sKM2SZl09kpAwJMqa_MRj-A!2e0!7i13312!8i6656 [Accessed: June 2, 2016].

Groves, T., 1973. Incentives in teams. *Econometrica: Journal of the Econometric Society*, p.617–631.

Halvorson, R. & Buckeye, K., 2006. High-Occupancy Toll Lane Innovations: I-394 MnPASS. *Public Works Management & Policy*, 10(3), p.242-255.

HELP Inc., 2016. About Us – PrePass. Available at: http://prepass.com/about-us/ [Accessed: May 24, 2016].

Hershey, J.C. & Schoemaker, P.J.H., 1985. Probability versus Certainty Equivalence Methods in Utility Measurement: Are They Equivalent? *Management Science*, 31(10), p.1213-1231.

HNTB Corporation, 2006. *Task 3.1 - Electronic Toll Collection System (ETCS) Concept of Operations (I-15 Managed Lane)*, SANDAG.

HNTB Corporation & Booz Allen Hamilton, 2007. *Considerations for High Occupancy Vehicle (HOV) Lane to High Occupancy Toll (HOT) Lane Conversions Guidebook - FHWA Operations*, Washington, D.C., United States: Federal Highway Administration. Available at: http://ops.fhwa.dot.gov/publications/fhwahop08034/ [Accessed: August 17, 2011].

Holmström, B. & Myerson, R.B., 1983. Efficient and Durable Decision Rules with Incomplete Information. *Econometrica*, 51(6), p.1799-1819.

Ivanova-Stenzel, R. & Kröger, S., 2005. *Price formation in a sequential selling mechanism*, SFB/TR 15 Governance and the Efficiency of Economic Systems, Free University of Berlin, Humboldt University of Berlin, University of Bonn, University of Mannheim, University of Munich. Available at: http://ideas.repec.org/p/trf/wpaper/92.html [Accessed: March 21, 2011].

Ivanova-Stenzel, R. & Kröger, S., 2008. Price formation in a sequential selling mechanism. *Journal of Economic Behavior & Organization*, 67(3-4), p.832-843.

Johnson, M., 2014. Red paint keeps drivers out of San Francisco's bus lanes. *Greater Greater Washington*. Available at: http://greatergreaterwashington.org/post/24268/red-paint-keeps-drivers-out-of-san-franciscos-bus-lanes/ [Accessed: June 2, 2016].

Kirkegaard, R. & Overgaard, P.B., 2008. Buy-out prices in auctions: seller competition and multi-unit demands. *The RAND Journal of Economics*, 39(3), p.770–789.

Kirkegaard, R. & Overgaard, P.B., 2003. Buy-Out Prices in Online Auctions: Multi-Unit Demand. *SSRN eLibrary*. Available at: http://papers.ssrn.com/sol3/papers.cfm?abstract_id=382180 [Accessed: March 21, 2011].

Krishna, V., 2009. *Auction Theory, Second Edition*, Academic Press.

Lam, T.C. & Small, K.A., 2001. The value of time and reliability: measurement from a value pricing experiment. *Transportation Research Part E: Logistics and Transportation Review*, 37(2-3), p.231-251.

Laval, J.A. et al., 2015. Real-time congestion pricing strategies for toll facilities. *Transportation Research Part B: Methodological*, 71, p.19-31.

Laval, J.A. & Daganzo, C.F., 2006. Lane-changing in traffic streams. *Transportation Research Part B: Methodological*, 40(3), p.251-264.

Leonard, H.B., 1983. Elicitation of Honest Preferences for the Assignment of Individuals to Positions. *Journal of Political Economy*, 91(3), p.461-479.

Levin, M.W. & Boyles, S.D., 2015. Intersection Auctions and Reservation-Based Control in Dynamic Traffic Assignment. *Transportation Research Record: Journal of the Transportation Research Board*, 2497, p.35-44.

Li, J. & Govind, S., 2002. An Optimization Model For Assessing Pricing Strategies of Managed Lanes. In *Paper Submitted to the 82nd Annual Meeting of the Transportation Research Board. June*.

Li, Z., Hensher, D.A. & Rose, J.M., 2010. Willingness to pay for travel time reliability in passenger transport: A review and some new empirical evidence. *Transportation Research Part E: Logistics and Transportation Review*, 46(3), p.384-403.

Liu, L.N. & McDonald, J.F., 1999. Economic efficiency of second-best congestion pricing schemes in urban highway systems. *Transportation Research Part B: Methodological*, 33(3), p.157-188.

Liu, X. et al., 2011. Quantifying the attractiveness of high-occupancy-toll lane under various traffic conditions using traffic sensor data. *Transportation Research Record*, 2229.

Lou, Y., Yin, Y. & Laval, J.A., 2011. Optimal dynamic pricing strategies for high-occupancy/toll lanes. *Transportation Research Part C: Emerging Technologies*, 19(1), p.64-74.

Lu, C.-C., Mahmassani, H.S. & Zhou, X., 2008. A bi-criterion dynamic user equilibrium traffic assignment model and solution algorithm for evaluating dynamic road pricing strategies. *Transportation Research Part C: Emerging Technologies*, 16(4), p.371-389.

Lucking-Reiley, D., 2000. Vickrey Auctions in Practice: From Nineteenth Century Philately To Twenty-First Century E-Commerce. *Journal of Economic Perspectives*, 14, p.183--192.

Mas-Colell, A., Whinston, M.D. & Green, J.R., 1995. *Microeconomic Theory*, Oxford University Press, USA.

Mathews, T., 2003. A Risk Averse Seller in a Continuous Time Auction with a Buyout Option. *Brazilian Electronic Journal of Economics*, 5(2). Available at: http://ideas.repec.org/a/bej/issued/v5y2002i2mathews.html [Accessed: March 11, 2011].

Mathews, T., 2006. Bidder Welfare In An Auction With A Buyout Option. *International Game Theory Review (IGTR)*, 8(4), p.595-612.

Mathews, T., 2004. The impact of discounting on an auction with a buyout option: a theoretical analysis motivated by eBay's buy-it-now feature. *Journal of Economics*, 81(1), p.25–52.

Michalaka, D., Lou, Y. & Yin, Y., 2011. Proactive and Robust Dynamic Pricing Strategies for High-1 Occupancy/Toll (HOT) Lanes. In *TRB 90th Annual Meeting Compendium of Papers*. TRB 90th Annual Meeting. Washington, D.C., United States.

Minnesota Department of Transportation, 2013. Express lane locations. Available at: http://www.mnpass.org/maps.html [Accessed: June 3, 2016].

Minnesota Department of Transportation, 2011. Mn/DOT Data Tools. *Mn/DOT Traffic Data*. Available at: http://data.dot.state.mn.us/datatools/ [Accessed: December13, 2011].

Morgul, E.F. & Ozbay, K., 2011. Simulation-Based Evaluation of Feedback Based Dynamic Congestion Pricing Strategy on Alternate Facilities. In TRB 90th Annual Meeting. Washington, D.C., United States.

Munnich, L. & Buckeye, K., 2007. I-394 MnPASS High-Occupancy Toll Lanes: Planning and Operational Issues and Outcomes (Lessons Learned in Year 1). *Transportation Research Record: Journal of the Transportation Research Board*, 1996(1), p.49-57.

Nisan, N., Roughgarden, T. & Tardos, É., 2007. *Algorithmic Game Theory* 1 edition., Cambridge ; New York: Cambridge University Press.

OCTA, 2011. Toll Schedules. Available at: http://www.91expresslanes.com/schedules.asp [Accessed: March 17, 2011].

Office of the Mayor, New York City, 2016a. Bus Rapid Transit - Bus Lane Rules. Available at: http://www.nyc.gov/html/brt/html/about/bus-lanes.shtml [Accessed: June 3, 2016].

Office of the Mayor, New York City, 2016b. Bus Rapid Transit - Select Bus Service Features. Available at: http://www.nyc.gov/html/brt/html/about/sbs-features.shtml#off-set-bus-lanes [Accessed: June 2, 2016].

Olarte, R., 2016. Lottery-based metering system for the I-394 MnPass Express Lanes. *Lottery-based metering system for the I-394 MnPass Express Lanes*. Available at: http://terpconnect.umd.edu/~olarte/i394/design1.html [Accessed: March 15, 2016].

Olarte, R. & Haghani, A., 2016. Introducing and testing a game-theoretic model for a lottery-based metering system in Minneapolis, United States. In *Proceedings of the 14th World Conference on Transport Research*. 14th World Conference on Transport Research. Shanghai, China.

Olarte, R. & Haghani, A., 2013. Metering for Special Use Lanes Via Lotteries. In *Proceedings of the 17th IRF World Meeting & Exhibition*. 17th IRF World Meeting & Exhibition. Riyadh, Saudi Arabia.

Paleti, C., He, X. & Peeta, S., 2016. Design of income-equitable toll prices for high occupancy toll lanes in a single toll facility. *Transportation Planning and*

Technology, 39(4), p.389-406.

Paul Makdissi, 2006. *On the Definition of Economic Efficiency*, CIRPEE. Available at: http://ideas.repec.org/p/lvl/lacicr/0644.html [Accessed: December 13, 2011].

Perez, B., 2013. FHWA Guide for Priced Managed Lane Development: Overview. Available at: http://managedlanes.org/fhwa-priced-managed-lanes-guide/ [Accessed: June 3, 2016].

Perez, B.G. & Sciara, G.-C., 2003. A Guide for Hot Lane Development. Available at: http://ntl.bts.gov/lib/jpodocs/repts_te/13668.html [Accessed: February 16, 2011].

Phan, T. et al., 2015. A Model-based Dynamic Toll Pricing Strategy for Controlling Highway Traffic. Available at: http://dspace.mit.edu/handle/1721.1/99197 [Accessed: May 30, 2016].

Poole, R., 2011. Reason Foundation - Surface Transportation Innovations #097. Available at: http://reason.org/news/show/surface-transportation-news-97 [Accessed: December 13, 2011].

Poole, R. & Swenson, C., 2012. Managed Arterials. *Transportation Research Record: Journal of the Transportation Research Board*, 2297, p.66-72.

Popkowski Leszczyc, P.T.., Qiu, C. & He, Y., 2009. Empirical testing of the reference-price effect of buy-now prices in Internet auctions. *Journal of Retailing*, 85(2), p.211–221.

Pratt, J.W., 1964. Risk Aversion in the Small and in the Large. *Econometrica*, 32(1/2), p.122-136.

PTV Planung Transport Verkehr, AG, 2012. *VISSIM 5.40 User Manual*.

Rambha, T. & Boyles, S.D., 2016. Dynamic pricing in discrete time stochastic day-to-day route choice models. *Transportation Research Part B: Methodological*. Available at: http://www.sciencedirect.com/science/article/pii/S0191261516000175 [Consulté le mai 29, 2016].

Raphael, J., Maskell, S. & Sklar, E., 2015a. First Steps Toward an Auction-Based Traffic Signal Controller. In Y. Demazeau et al., *Advances in Practical Applications of Agents, Multi-Agent Systems, and Sustainability: The PAAMS Collection*. Lecture Notes in Computer Science. Springer International Publishing, p. 300-303. Available at: http://link.springer.com/chapter/10.1007/978-3-319-18944-4_32 [Accessed: July 5, 2016].

Raphael, J., Maskell, S. & Sklar, E., 2015b. From Goods to Traffic: First Steps Toward an Auction-Based Traffic Signal Controller. In Y. Demazeau et al., éd. *Advances in Practical Applications of Agents, Multi-Agent Systems, and Sustainability: The PAAMS Collection*. Lecture Notes in Computer Science. Springer International Publishing, p. 187-198. Available at: http://link.springer.com/chapter/10.1007/978-3-319-18944-4_16 [Accessed: July 5, 2016].

Research and Innovative Technology Administration, 2010. Achieving the Vision: From VII to IntelliDrive. Available at: http://www.its.dot.gov/press/2010/vii2intellidrive.htm [Accessed: August 20, 2011].

Reynolds, S.S. & Wooders, J., 2009. Auctions with a buy price. *Economic Theory*, 38(1), p.9-39.

Reynolds, S.S. & Wooders, J., 2003. Auctions with a buy price. Available at: http://www.springerlink.com/content/h7pk11t29004r854/ [Accessed: March 11, 2011].

Roess, R.P. & Prassas, E.S., 2014. *The Highway Capacity Manual: A Conceptual and Research History: Volume 1: Uninterrupted Flow*, Springer Science & Business Media.

Roth, G., 2006. *Street Smart: Competition, Entrepreneurship, and the Future of Roads* illustrated edition., Transaction Publishers.

Schepperle, H. & Böhm, K., 2008. Auction-Based Traffic Management: Towards Effective Concurrent Utilization of Road Intersections. In *2008 10th IEEE Conference on E-Commerce Technology and the Fifth IEEE Conference on Enterprise Computing, E-Commerce and E-Services*. 2008 10th IEEE Conference on E-Commerce Technology and the Fifth IEEE Conference on Enterprise Computing, E-Commerce and E-Services. p. 105-112.

Schepperle, H., Böhm, K. & Forster, S., 2007. Towards Valuation-aware Agent-based Traffic Control. In *Proceedings of the 6th International Joint Conference on Autonomous Agents and Multiagent Systems*. AAMAS '07. New York, NY, USA: ACM, p. 185:1–185:3. Available at: http://doi.acm.org/10.1145/1329125.1329349 [Accessed: July 5, 2016].

Small, K.A. et al., 1999. *Valuation of travel-time savings and predictability in congested conditions for highway user-cost estimation*, Transportation Research Board, NCHRP.

Small, K.A., Winston, C. & Yan, J., 2005. Uncovering the Distribution of Motorists' Preferences for Travel Time and Reliability. *Econometrica*, 73(4), p.1367-1382.

Steimetz, S.S.C. & Brownstone, D., 2005. Estimating commuters' « value of time » with noisy data: a multiple imputation approach. *Transportation Research Part B: Methodological*, 39(10), p.865-889.

Teodorović, D. et al., 2008. Auction-Based Congestion Pricing. *Transportation Planning & Technology*, 31(4), p.399-416.

The Washington Times, 2011. Editorial: Georgia's tolling nightmare. Available at: http://www.washingtontimes.com/news/2011/nov/1/georgias-tolling-nightmare/print/ [Accessed: December 13, 2011].

Transportation Research Board, 2010. *Highway Capacity Manual* 5th edition., Washington, DC, U.S.A.: Transportation Research Board.

U.S. Department of Transportation, R., 2010. U.S. Transportation Secretary Ray LaHood Launches « Faces of Distracted Driving » Online Awareness Campaign | Department of Transportation. Available at: https://www.transportation.gov/briefing-room/us-transportation-secretary-ray-lahood-launches-%E2%80%9Cfaces-distracted-driving%E2%80%9D-online [Accessed: June 5, 2016].

Vaisala, Inc, 2016. SCAN Web 6.0. Available at: http://rwis.dot.state.mn.us/ [Accessed: May 8, 2016].

Vasirani, M. & Ossowski, S., 2011. A Computational Market for Distributed Control of Urban Road Traffic Systems. *IEEE Transactions on Intelligent Transportation Systems*, 12(2), p.313-321.

Vickrey, W., 1961. Counterspeculation, Auctions, and Competitive Sealed Tenders. *The Journal of Finance*, 16(1), p.8-37.

Wan, W., Teo, H.H. & Zhu, L., 2003. Determinants of Buy-It-Now Auction Choice on eBay. In *7th Pacific-Asia Conference on Information Systems*.

Wang, X., Montgomery, A. & Srinivasan, K., 2008. When auction meets fixed price: a theoretical and empirical examination of buy-it-now auctions. *Quantitative Marketing and Economics*, 6(4), p.339-370.

Yang, H. & Huang, H.-J., 1997. Analysis of the time-varying pricing of a bottleneck with elastic demand using optimal control theory. *Transportation Research Part B: Methodological*, 31(6), p.425-440.

Yin, Y. & Lou, Y., 2009. Dynamic tolling strategies for managed lanes. *Journal of Transportation Engineering*, 134(2), p.45–52.

Yu, H., Dang, C. & Wang, S.Y., 2006. Game theoretical analysis of buy-it-now price auctions. *International Journal of Information Technology and Decision Making*, 5(3), p.557–581.

Zhang, G. et al., 2008. A Feedback-Based Dynamic Tolling Algorithm for High-Occupancy Toll Lane Operations. *Transportation Research Record: Journal of the Transportation Research Board*, 2065, p.54-63.

Zhang, G., Ma, X. & Wang, Y., 2014. Self-Adaptive Tolling Strategy for Enhanced High-Occupancy Toll Lane Operations. *IEEE Transactions on Intelligent Transportation Systems*, 15(1), p.306-317.

Zhang, G., Yan, S. & Wang, Y., 2009. Simulation-Based Investigation on High-Occupancy Toll Lane Operations for Washington State Route 167. *Journal of Transportation Engineering*, 135(10), p.677-686.

Zhou, H. & Saigal, R., 2014. Combinatorial Auction to Allocate Traffic. *Transportation Research Record: Journal of the Transportation Research Board*, 2423, p.35-43.