ABSTRACT

Title of Dissertation: Stress-Controlled Versus Strain-Controlled Triaxial Testing of Sand

Samir Ahmed Alqutri, Doctor of Philosophy, 1994

Dissertation directed by: Deborah J. Goodings, Associate Professor
Department of Civil Engineering

The purpose of this research was to compare the strength characterizations of Mystic White Silica Sands using stress-controlled loading versus strain-controlled loading in a standard compression triaxial tests. To this end one hundred sixty-six tests were conducted involving two types of quartz sand, one fine MWSS45 and one medium coarse MWSS18, tested at three low to intermediate confining stresses of 14 kN/m^2, 28 kN/m^2 and 55 kN/m^2 with only one specimen diameter size of 71.1 mm. Of the one hundred sixty-six tests, eighty-six were stress-controlled tests and eighty were strain-controlled tests. All specimens were dry, but both loose and dense specimens were tested. The results were evaluated individually and as group. Comparison of the two types of loading tests were evaluated for repeatability, stress-strain characteristics and strength parameters.

The plots show that stress-controlled loading in general gives more reproducible results with smoother, steeper stress-strain plots and a larger average deviator stresses at failure than strain-controlled loading at all three levels of confining stresses for both sands. This results in somewhat larger values of ϕ'. Stress-controlled specimens were stiffer and failed with a clear cut failure surface while strain-controlled specimens mostly barreled.

STRESS-CONTROLLED VERSUS STRAIN-CONTROLLED TRIAXIAL TESTING OF SAND

by

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DEDICATION

Dedicated to my loving parents
and sisters

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I would like to express my deepest gratitude to Professor Deborah J. Goodings for her support, patience, and dedication throughout the course of this research. Mr. Roger Mackubin, Dr. Chaun-Ping Lin and Mr. Charlie Babendrier's help in experimental preparation is highly appreciated. Also the author is indebted to the graduate committee members, the staff and faculty of the Civil Engineering Department at the University of Maryland.

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NOMENCLATURE

c'	= cohesion
C_c	= coefficient of concavity = $D_{30}{}^{2} / (D_{60})(D_{10})$
C_u	= uniformity coefficient = D_{60} / D_{10}
Dr, D_r, D_R	= relative density = $(e_{max} - e) / (e_{max} - e_{min})$
D_{10}	= effective size = sieve diameter through which 10% of the total soil sample mass passes
D_{50}	= sieve diameter through which 50% of the total soil sample mass passes
e	= void ratio = V_V / V_S
e_o	= initial void ratio
G_s	= specific gravity of soil grains
n	= porosity = $e/(1+e) = V_V / V_T$
n_o	= initial porosity
N	= number of experiments
ϕ'	= internal friction angle
θ	= angle between plane on which failure occurs measured relative to plane on which major principal stress applied
ε, ε_a	= axial strain
ε_v	= volumetric strain
ε_2	= strain in the intermediate principal direction
$(\sigma_1 - \sigma_3)$	= deviator stress
σ_{1f}	= major principal stress at failure
V_V	= volume of voids between soil solids
V_S	= volume of soil solids
V_T	= $V_S + V_V$

CHAPTER ONE
INTRODUCTION

The American Society for Testing and Materials (1988) recommends in D2850-87, that triaxial testing of cohesive soils be conducted in a strain-controlled manner. There is, notably, no similar standard specified for cohesionless soils. Nonetheless, strain-controlled testing is the norm for them also. Bishop and Henkel (1962) noted that strain-controlled testing is recommended for four reasons: the rate of strain at failure is known; the influence of rheological factors on the strength can be considered; the stress-strain characteristics after peak strength can be measured; and the duration of the test can be predicted. Their recommendation acknowledged standard practice, which persists today. Even as far back as the 1950 ASTM meetings on the Triaxial Testing of Soils and Bituminous Mixtures, eight authors reported triaxial testing of soils including research conducted by Taylor (1950) at M.I.T., by Johnson (1950) at the U.S. Army Waterways Experiment Station, by Wagner (1950) at the U.S. Bureau of Reclamation, and Barber and Sawyer (1950) at the Bureau of Public Roads, and all used strain-controlled testing. Recent experimental work on development of shear bands makes use of various types of apparatus (triaxial, true triaxial, and biaxial) but in every instance strain-controlled testing is used to control rate of development of shearing to ensure that its development can be recorded.

However, while there may be constraints on the magnitude of strain or to some degree on rate of strain occurring in full scale geotechnical configurations, loading is principally stress-controlled. If the characterization of soil strength using strain-controlled testing can be applied correctly to stress-controlled loading conditions, then it is an acceptable, and, in fact, preferable technique to use strain-controlled characterization of soil strength. If it is not, however, then there is reason to examine differences in strength behavior according to loading conditions.

These differences may or may not be important in design, where a safety factor may obscure the importance of such differences. But in situations where accurate characterization of soil strength is critical, as it is in small physical models which may be extrapolated for full scale design, such as are tested in the geotechnical centrifuge, differences in behavior may become important. Here repeatability of results and differences in stress-strain response in the development of strain and in the stress causing shear failure can be extremely important. If these factors are influenced by the method of loading in a soil model and in triaxial testing of the soil then this should influence physical model design and soil strength characterization.

The first experimental evidence that differences in shearing resistance may occur when loading conditions are varied was published by Dennis (1988). Dennis worked with uniform Ottawa Banding sand with $D_{50} = 0.18$ mm (shown in Figure 1.1), in standard compression loading, with large confining stresses of 207 kN/m^2, 345 kN/m^2, 483 kN/m^2, 1035 kN/m^2 -- these would be typical of lateral stresses existing in situ at depths from about 20 m to 100 m. He tested saturated triaxial specimens by strain-controlled and stress-controlled loading, and found first that " specimen preparation technique affected the slope of the steady state line for stress-controlled loading, but the effect of specimen preparation technique was insignificant for strain-controlled loading, " and second that "specimens tested using strain-control had significantly higher strengths at steady state than specimens tested using stress-control when void ratios were similar." Other preliminary work at the University of Maryland by Stephanos (1989) presented results of 24 standard triaxial tests that were performed on a uniform Mystic White Silica Sand No.10 (MWSS10) with confining stresses of 68.9 kN/m^2 and 206.8 kN/m^2, these simulate lateral stresses in situ at depths of about 6 m and 20 m. He concluded that stress and strain controlled loadings produce broadly similar stress-strain characteristics up to the peak strength for these two levels of confining stresses. However, he found the variability of the strain-

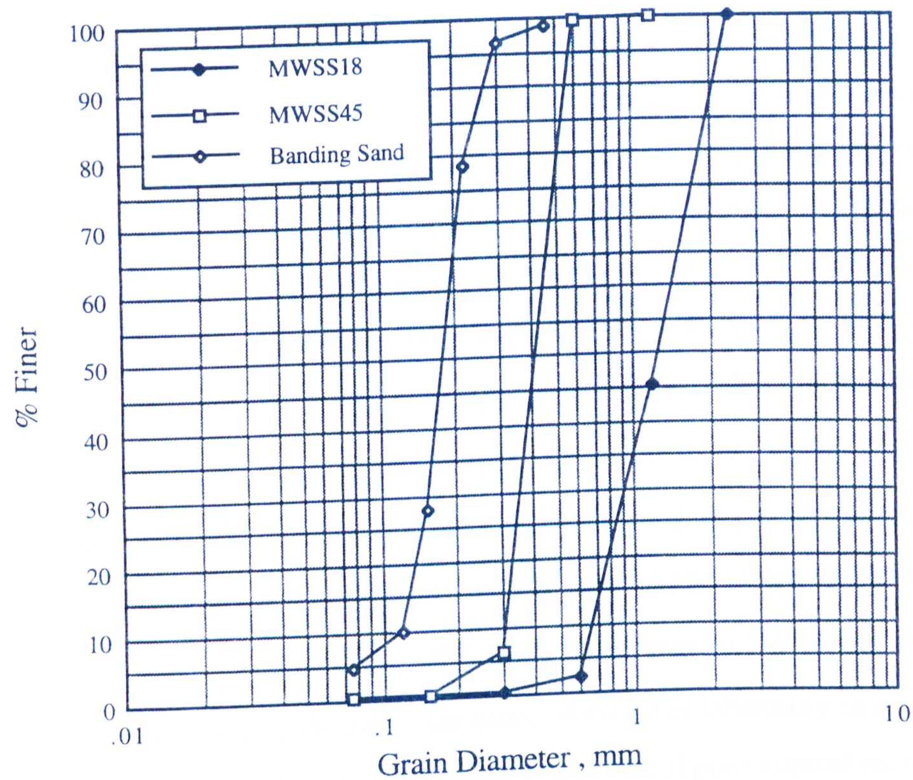


Fig. 1.1 - Gradation Curve Vs. % Finer for all Sands

controlled loading tests to be less than that of the stress-controlled loading tests, and the maximum deviator stresses are higher for strain-controlled loading than for stress-controlled loading. Further, plots of stress versus strain were smoother for stress-controlled tests than for strain-controlled, as shown in Figures 1.2 and 1.3.

In geotechnical centrifuge modeling, Corte et al. (1988) reported on physical models of failure of circular foundations on loose saturated sand conducted at three different centrifuge laboratories. The purpose of the work was to test for reproducibility of results. Method of loading in the models was not specified in the initial plan and as it turned out, three different methods were used, one stress-controlled, one strain-controlled, and one intermediate method. Reproducibility within a series for a single laboratory was very good for the laboratory using strain-controlled foundation loading. Those seven models intended to be identical, varied in their results by no more than 12% from the average, of 437 kN/m^2, ranging in values of foundation stress at failure from 382 kN/m^2 to 491 kN/m^2. Reproducibility was the poorest in the laboratory using stress-controlled foundation loading. The range of foundation stresses at failure varied from 490 kN/m^2 to 612 kN/m^2 with a variation up to 28% from the average of 558 kN/m^2 over four model tests. In spite of the long experience of the latter laboratory in centrifuge modeling, they speculated that the problem was one of poor control on void ratios in the specimens. The reason for conducting the research in this study was to check whether it could be, either, instead, or also a result of the method of loading.

The purpose of this work is to move from the small body of previous triaxial testing, to assess the influence, if any, of loading technique on the recorded stress-strain behavior using a large body of tests on dry sands in standard compression loading in the triaxial apparatus. Emphasis was placed on identifying the differences in peak strength measured, and on statistical scatter in data of these two different methods of loading the soil, at lateral stresses typical of those in geotechnical engineering design. These objectives were accomplished by

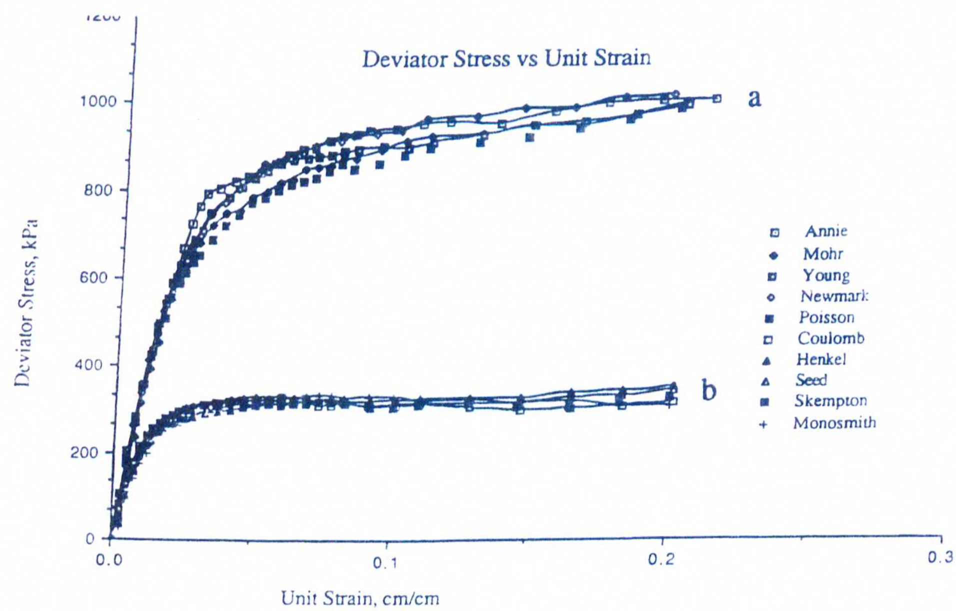


Fig. 1.2 - Strain Controlled Tests - (a) 206.83 kPa (b) 68.94 kPa

(Stephanos, 1989)

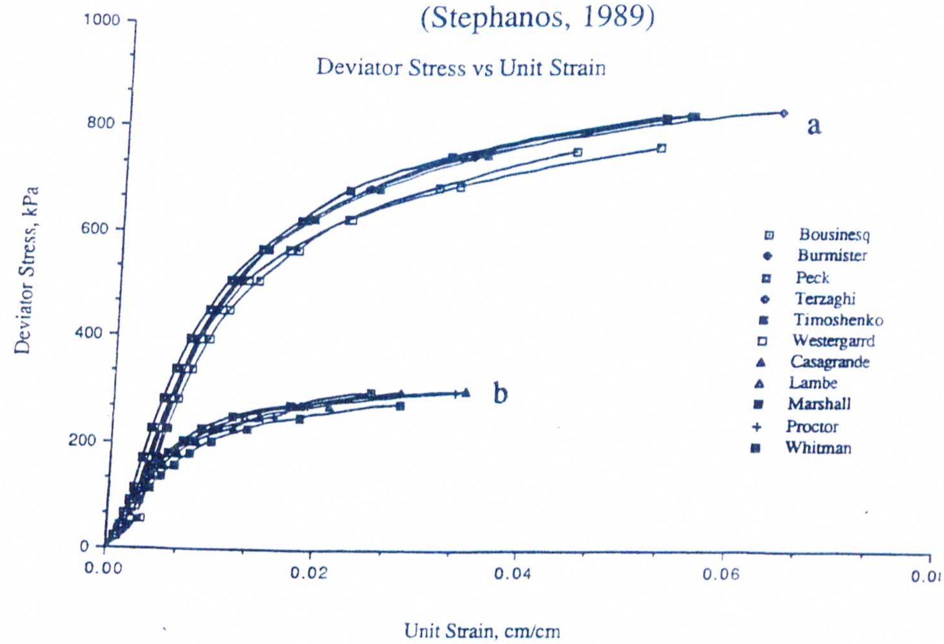


Fig. 1.3 - Stress Controlled Tests - (a) 206.83 kPa (b) 68.94 kPa

(Stephanos, 1989)

conducting a large body of triaxial tests using two uniformly graded cohesionless soils, one fine Mystic White Silica Sand No.45 and one medium coarse Mystic White Silica Sand No.18, both of which came from Mystic, Connecticut. The values of D_{50} for the two sands varied by a factor of 3, to give some measure of grain size effects, although the diameters of the triaxial specimens were equal to at least 30 grain diameters. Their grain size distributions are shown in Figure 1.1. Three levels of confining stresses were used: 14 kN/m^2, 28 kN/m^2, 55 kN/m^2. These were selected to simulate in situ lateral stresses more typically relevant to surface structures in geotechnical design, existing at depths between 1.5 m and 5 m, in contrast to the much larger lateral stresses selected by most other researchers. The experiments were all standard compression loading, the usual loading in the triaxial apparatus, and all were conducted using the T-1500 triaxial apparatus from Soil Test, Inc.

In this dissertation, chapter two presents a review of the literature related to this study with emphasis on the factors affecting the stress-strain characteristics of sand. Chapter three describes the materials, equipment and test methodology used in this study. Chapters four and five present the results of the testing program of this study, and the analysis of these results, respectively. Finally, chapter six presents the summary and conclusions of this study, and proposed recommendations for further research.

CHAPTER TWO
LITERATURE REVIEW

2.1 CHARACTERIZATION OF STRENGTH OF SANDS

The conventional characterization of the effective stress shear strength of dry or saturated sands tested under drained conditions is $c' = 0$ and ϕ' equal to a constant, typically between 30° and 45° (Lambe and Whitman, 1969); while this may be used in design, the true Mohr-Coulomb failure envelope is not described accurately in this way. The following sections review experimental work which highlights the various factors influencing measurement of soil strength.

2.1.1 CURVATURE OF THE STRENGTH ENVELOPE

At very high confining stresses (> 100,000 kN/m^2), the strength envelope for sand is curved, with ϕ' falling due to crushing of particles at their contacts (Vesic and Clough, 1968). At lower confining stresses, more typical in geotechnical design, sand will behave with some characteristics similar to a drained overconsolidated clay with marked differences between peak and ultimate strengths. The magnitude of those differences will depend on the initial void ratio of the specimen, and the confining stress and loading path, which together influence the tendency for the dilation to occur, and the possibility for that dilation to develop or be suppressed. This dilation accounts for the differences between peak and ultimate strengths in sands. The fact that the peak envelope is a function of the stresses and the void ratio means that a single soil can have a family of peak failure envelopes in the Mohr-Coulomb representation of failure, depending on void ratio, and those envelopes will be curved, particularly at low confining stresses.

Vesic and Clough (1968), for example, reported triaxial tests, both standard compression as well as constant average stress loading tests ($\Delta\sigma_1 + \Delta\sigma_2 + \Delta\sigma_3 = 0$), carried out to failure at confining stresses ranging from 40 kN/m^2 to 63,300 kN/m^2 on Chattahoochee River sand. Figure 2.1 compares the test results of their loose and dense specimens. At high stresses, where the mean normal stress was greater than 10,000 kN/m^2, the failure envelopes of both loose and dense specimens (e = 0.67 to 1.00) of the same sand could be characterized satisfactorily as $c' = 0$ and $\phi' = 33°$. At stresses less than 7,550 kN/m^2, however, the failure envelope for dense specimens (e = 0.64 to 0.75) was curved. A conventional straight line peak strength envelope would be said to be varying significantly, with values of ϕ' increasing as the median stress of the test fell, if c' were set to zero.

Earlier, Vesic and Barksdale (1963) also worked with Chattahoochee River sand, testing nineteen specimens at different initial void ratios over a wide range of confining stresses, using the same triaxial loading procedures as Vesic and Clough (1968). The resulting failure envelopes, plotted in Figure 2.2, and presumably based on peak strength show clearly that curved failure envelopes result for dense soils with Dr = 0.78 and Dr = 1.00, when σ is less than 5.5 tons/ft^2 or 527 kN/m^2. If the critical state concept (Schofield and Wroth, 1968) is assumed for sands then one would speculate that the ultimate strength failure envelopes of loose and dense sands would be one and the same at all stresses.

Ponce and Bell (1971) worked on uniform quartz sand, comparing the strength envelopes of loose and dense specimens of sands at confining stresses varying from 1.4 kN/m^2 to 241.2 kN/m^2. When confining stress fell below 29.5 kN/m^2 both loose and dense specimens showed marked increases in ϕ' which

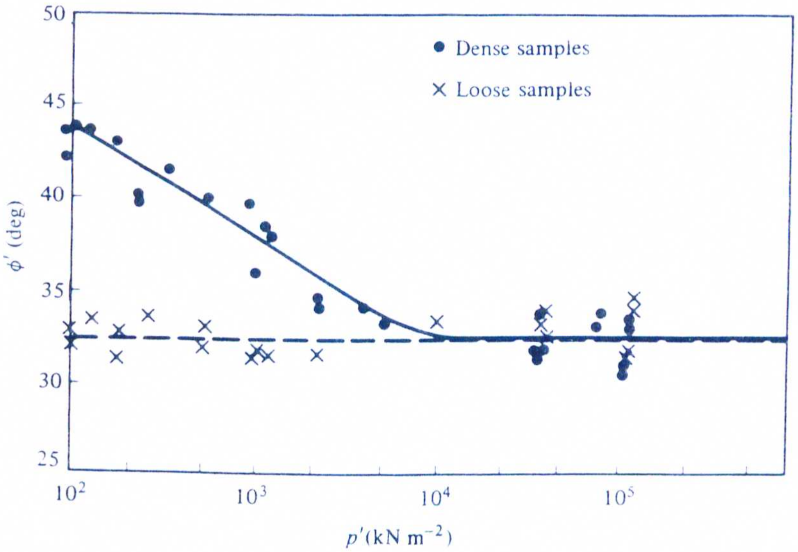


Fig. 2.1 - Angle of Internal Friction for Chattahoochee River Sand Tested at Different Stress Levels in the Triaxial Apparatus (Vesic and Clough, 1968)

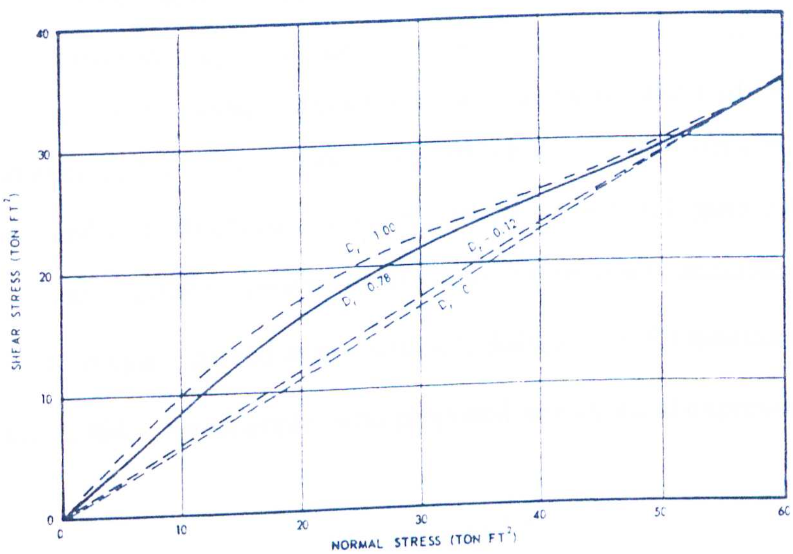


Fig. 2.2 - Shape of Mohr's Envelope for Sand (Vesic and Barksdale, 1963)

confirms that the Mohr-Coulomb failure envelop is curved, and that curvature depends on void ratio and confining stress. Figure 2.3 shows their data, replotted by Fukushima and Tatsuoka (1984), which highlights this trend. Fukushima and Tatsuoka (1984) themselves worked with Tayoura sand at confining stresses between 2 kN/m^2 and 400 kN/m^2, testing both loose and dense specimens in drained triaxial compression tests. They, too, found some curvature which was a function of void ratio and confining stress, although after correction for membrane effects, at low stresses they found the curvature to be small. Finally, Marachi et al. (1972), attempting to model behavior of rockfill using smaller particle sizes, detected marked curvature to failure envelopes when confining stresses were varied from 207 kN/m^2 to 4479 kN/m^2. Their results are shown in Figure 2.4.

In an attempt to develop expressions that represent accurately this envelope in τ - σ space curved at low stresses, Baligh (1976) mentioned work by Yareshenko (1964), for example, who proposed an empirical expression in the form of:

$$\tau = (k\sigma)^{1/n} \tag{2.1}$$

where:

τ = shear resistance on the failure plane at failure

σ = normal stress on the failure plane at failure

k & n = constants defining the envelope curvature

This expression was used by several researchers such as Berezantsev et al. (1968), as reported by Baligh (1976), in the investigation of the influence of the curvature of the failure envelope on calculations of the bearing capacity of shallow foundations. Baligh (1976) noted that determining the values of the variables k and n for an actual failure envelope from data for a sand may be

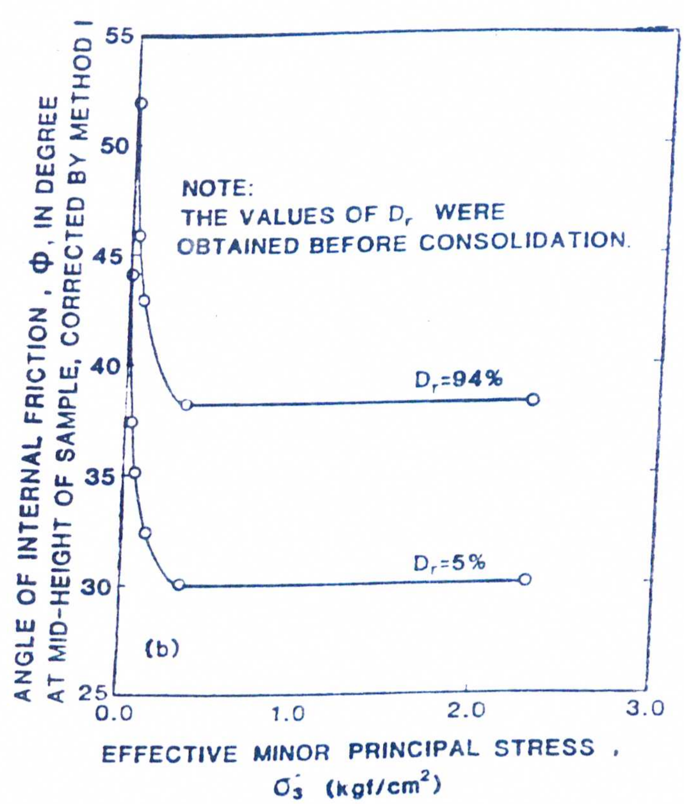


Fig. 2.3 - Changing of Angle of Internal Friction by the Change of Effective Minor Principal Stress in Triaxial Compression Test; reproduced from Ponce and Bell (1977): (Fukushima and Tatsuoka, 1984)

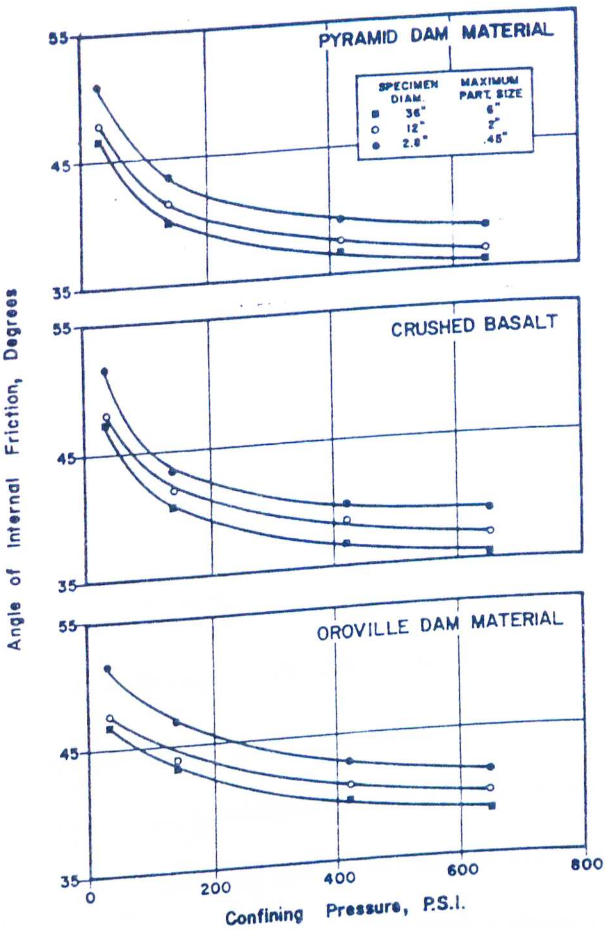


Fig. 2.4 - IC-D Shear Test Results with Modeled Rockfill Materials
(Marachi et al., 1972)

difficult, since there there may be many combinations of these two constants , all of which may fit, more or less, a given set of experimental results. He recommended, instead, equation 2.2 for the failure envelope, which he noted involves soil parameters that can be measured following his techniques.

$$\tau = \sigma \left| \tan \phi_0 + \tan \alpha \left(\frac{1}{2.3} - \log \frac{\sigma}{\sigma_0} \right) \right| \qquad (2.2)$$

where :

ϕ_0 = constant angle ($\phi_0 > 0$)

α = constant angle ($\alpha \geq 0$)

σ = normal stress

σ_0 = an arbitrary reference stress

He tested this equation on results of eight series of tests conducted by other researchers. He set α to vary between 0° and 12° (the higher value for denser soils) and then tested his equation against the results from those other researchers as shown in Figure 2.5. When normal stress exceeded about 10,000 kN/m^2 (= 100 kg/cm^2) most of the sands had constant ϕ = 33°. When normal stress was less than 10,000 kN/m^2, ϕ increased as normal stress fell for all sands except the loosest sand with Dr = 20%. He was satisfied with his predictions.

A curved peak envelope for sand at low stresses, then, is typical and accepted as an accurate characterization of its strength, even if it is not typically assumed in design.

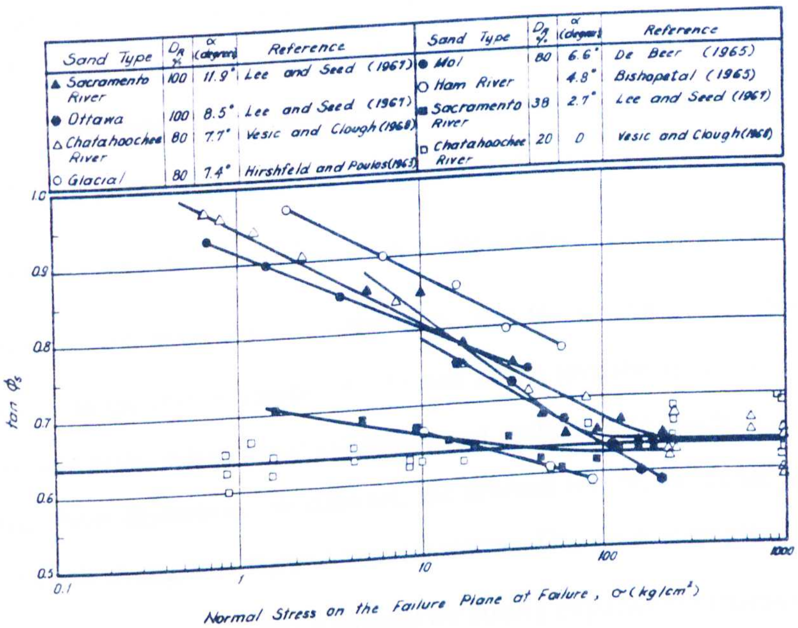


Fig. 2.5 - Effect of Normal Stress on Strength of Friction for Different Granular Materials (Baligh, 1976)

2.1.2 INFLUENCE OF VOID RATIO

It is apparent that void ratio and stress level both affect the peak strength envelope of sand and they are interconnected because they both affect particle interlocking and dilatancy of the sand, which leads to peak strength, followed by strain softening post peak behavior. Over forty years ago Eldin (1951) performed standard drained triaxial compression tests on loose (e = 0.84) and dense (e = 0.64) specimens of Brasted sand with results as plotted in Figure 2.6. Specimens were tested under strain-controlled standard compression triaxial loading with confining stress of 211 kN/m^2. The drained test on the loose specimen shows a stress-strain curve which reaches an ultimate deviator stress(σ_1 - σ_3), of about 500 kN/m^2 at 20% axial strain. At the same time the sample compresses substantially as the test proceeds. At the end of the test, the specimen has reached some ultimate state, when shear resistance and volume both reach constants during continuing shear distortion. In contrast, the drained test on the dense specimen exhibits a peak strength at about 5% axial strain. This peak strength is preceded by a slight contraction (ε_v = 0.2%) followed by strong expansion continuing until the end of the test when ε_v exceeds -5%. The deviator stress decreases after the peak, first sharply, then somewhat more gradually with further axial strain. At 20% strain, deviator stress is less than 600 kN/m^2. The critical state theory (see Schofield and Wroth, 1968) would argue that it is decreasing to reach an ultimate value equal to 500 kN/m^2 or the same ultimate deviator stress of the same soil, prepared loose, but tested at the same confining stress.

It is this dilation and interlocking which lead to development of the peak strength over and above what may be thought of as inherent shear resistance. The difference between peak strength and ultimate strength will be greatest when soil is

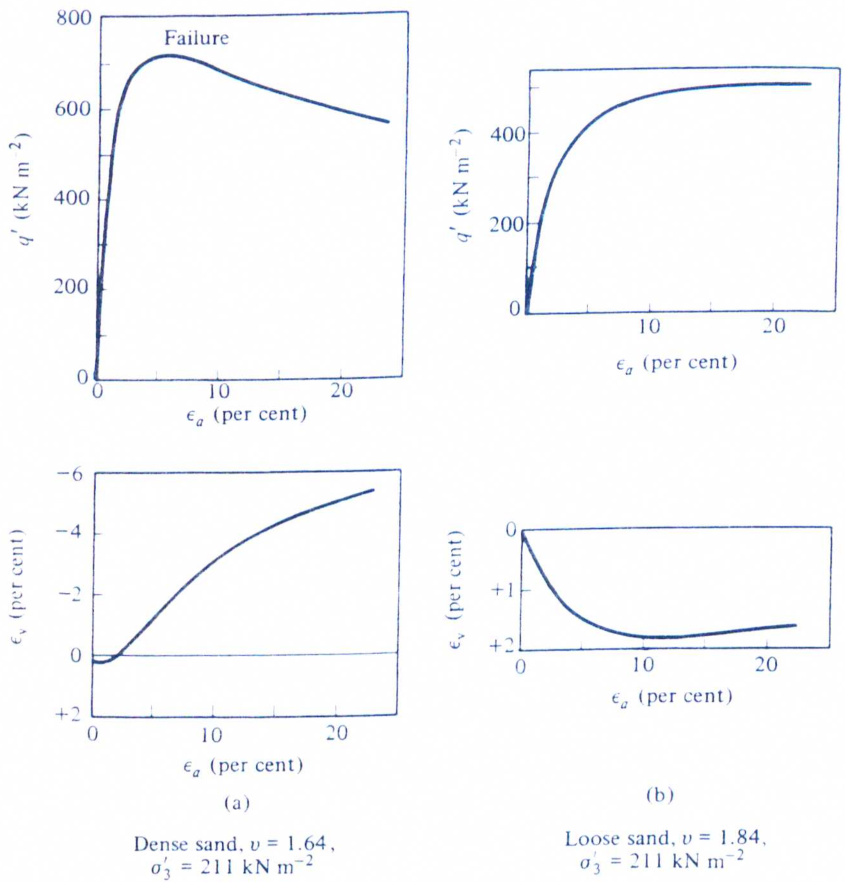


Fig. 2.6 - The Results of Drained Triaxial Tests on (a) a dense sample and (b) a loose sample of Brasted Sand (after Bishop and Henkel, 1962)

dense and stress is small since densely packed soil has little interlocking, and large stresses suppress dilation. These factors account for the family of curved peak strength envelopes at low to intermediate stresses. In the tests of this study, every attempt was made to limit ranges of void ratios in sets of tests which were to be compared.

2.2 OTHER FACTORS AFFECTING SOIL STRENGTH

2.2.1 DRY VERSUS SATURATED CONDITIONS

There are various other factors that influence shear resistance of sands and their measurement. The behavior of sands as cohesionless soils in both the dry and the saturated states has been examined by various researchers with varying conclusions.

The conventional assumption is that there is no difference in the effective stress behaviors due to the presence of water. Nash (1953) tested medium-fine quartz river sand in a series of triaxial tests and found drained shear strength behavior of air dry and saturated specimens to be indistinguishable (Figure 2.7). This same behavior was identified by Whitman and Healy (1962).

In contrast, Lee et al. (1967) presented the results of triaxial compression tests on sand dredged from the Sacramento River near Antioch, California, tested at various initial densities and confining stresses ranging from 100 kN/m^2 to 14,000 kN/m^2. Their results showed that the strengths of oven dry specimens were greater than the drained strengths of the same soil tested saturated, and the strengths of the air dry specimens fell somewhere between the two. The differences between behaviors became less at higher confining stresses. Lee et al. (1967) also presented a summary of direct shear tests on powdered mica performed by Horn and Deere (1962). They found oven dry specimens to have a friction angle larger by 9° when compared to similar tests using saturated drained

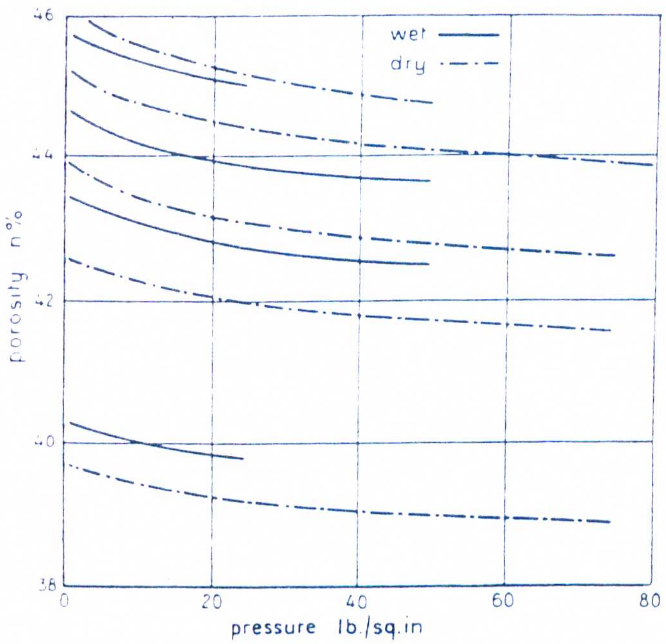


Fig. 2.7 - Compressibility Curves (Nash, 1953)

specimens. The same behavior was found by Bishop and Eldin (1953), who preformed drained triaxial compression tests on a fine to medium clean sand in both dry and saturated conditions using a confining stress of 42 kN/m^2. Their tests were performed on specimens having a wide range of densities. They found that the friction angle was higher for dry sand specimens than it was for the saturated sand specimens: the difference was estimated to be about 5° higher for the dense specimens and about 2° for the loose sand specimens.

Taken together, then, the results are largely inconclusive. Air dry sand was used in this study, which is typical in commercial laboratory testing of soil.

2.2.2 PARTICLE SIZE EFFECTS ON MEASUREMENT

Since soil is assumed to be a continuum in the field, the size of the test specimen, small by comparison with the field must still be large enough to behave as a continuum. Data by Holtz and Gibbs (1956) provided some indication of the influence of triaxial specimen size on the results obtained in triaxial compression tests on granular materials. They found that small diameter specimens (35.6 mm) may show interference of grain size when specimen diameter is equal to or less than 7.33 times D_{100} for uniformly graded soil but they also found evidence that in some cases specimens of diameter equal to only four grain diameters may not have adverse grain size effects.

Fukuoka (1957), Siddiqi (1984), and Baladi and Wu (1988) all recommended the use of a ratio of 6:1 between specimen diameter and maximum particle size based on their experiments. Marachi et al. (1972), who performed a total of 40 saturated drained triaxial compression tests on three models of rockfill materials, also kept specimen diameter equal to six times D_{100}, testing different parallel gradings, but they formed no conclusion about the suitability of this ratio. Unpublished work by Skinner (1987) at Imperial College, University of London suggested also that grading influences what is an acceptable ratio, but that something between 12 and 20 grain diameters is a minimum dimension for the diameter of a triaxial specimen if particle size effects are to be absent.

In this research, the minimum ratio of the specimen size to D_{100} for the uniformly graded soils is much greater than six, being a minimum of 30 and a maximum of 60.

2.2.3 MEMBRANE EFFECTS AT LOW CONFINING STRESSES

If pore pressure is to be controlled and confining stress to be applied by fluid pressure in the triaxial apparatus then a membrane must surround the soil specimen. If curvature of the strength envelope is greatest at low confining stress, when soil strength is to be measured, lateral confining stress of the membrane itself may interfere with measurement of soil strength by applying a lateral stress over and above that of the cell pressure. This membrane effect is a function of the modulus and initial diameter of the membrane and of the specimen. Although negligible at high confining stresses, a membrane effect can become significant when tests are conducted at low confining stresses, especially on soft soils.

The deformation of the specimen and the resulting interaction between the soil and the rubber membrane depends on specimen deformation as well as on the intrinsic properties of the membrane. If a specimen bulges or barrels, the influence of the membrane is distributed over a section of the specimen. Henkel and Gilbert (1952) preformed a series of undrained triaxial compression tests on remolded London clay, employing rubber membranes of three different thickness. They observed that the strength contributed by the membrane is proportional to the stiffness of the membrane and is independent of the cell pressure. However, when they used only the rubber membrane to confine the specimens with no lateral pressure, the effect was smaller than in the normal triaxial tests. From these observations, they suggested two theories for rubber membrane correction in triaxial compression tests.

In the first, they assumed the rubber membrane and the test specimen deform as a unit, which may be reasonable at high enough confining cell pressure. In this case the membrane is assumed to act as a reinforcing compression shell, and can be estimated by the following equation, recommended by ASTM Standard D2850-83:

$$\sigma_r = \frac{\pi D M \varepsilon (1 - \varepsilon)}{A_0} \tag{2.3}$$

where:

D = specimen diameter

M = rubber membrane modulus

ε = axial stain

A_o = initial area of specimen

In the second theory, they applied " Hoop Tension Theory " This is relevant to situations where the rubber membrane buckles as the specimen deforms, which is more typical of soil tested at low cell pressures. They estimated a correction for this case by the following equation:

$$\sigma_r = \frac{2M}{D} \frac{(1 - \sqrt{1 - \varepsilon})}{1 - \varepsilon} \tag{2.4}$$

where:

M = rubber membrane modulus

ε = axial strain

D = initial specimen diameter

In comparing the two membrane corrections recommended, the correction for specimens tested under high confining stresses exceeds that for low confining stresses for equal axial strains. This is because the membrane is pressed more tightly into the spaces between particles in the former instance, whereas, it deforms more independently of the soil in the latter (Henkel and Gilbert, 1952). As a comparison of the relative magnitudes of corrections, at 1.5% axial strain, the correction for membrane effects in tests under high confining cell pressures (equation 2.3) is 3.9 times that for low confining cell pressures (equation 2.4). This ratio decreases with increasing axial strain: at 10% strain the ratio is 3.2, and at 20% strain the ratio is 2.4. The actual magnitudes depend on specimen diameter (D), and rubber modulus (M). For example, for a specimen of diameter 36 mm with a membrane of modulus, M = 0.5 kN/m, the radial membrane correction at 10% strain from equation 2.3 is 4.9 kN/m^2 and from equation 2.4 is 1.5 kN/m^2. Axial strains leading to failure for specimens tested at low confining cell pressures are likely to be greater than for specimens tested at high confining cell pressures, and for the example given here 4.9 kN/m^2 may not be a negligible correction.

La Rochelle et al. (1988) mentioned an analytical solution by Duncan and Seed (1967) for the membrane correction in bulging failure. They took into account in their studies the volumetric and axial strains and proposed membrane corrections for both axial and lateral stresses. Later they compared their correction with experimental results by Henkel and Gilbert (1952) and found similar results, especially at low strains, as shown in Figure 2.8.

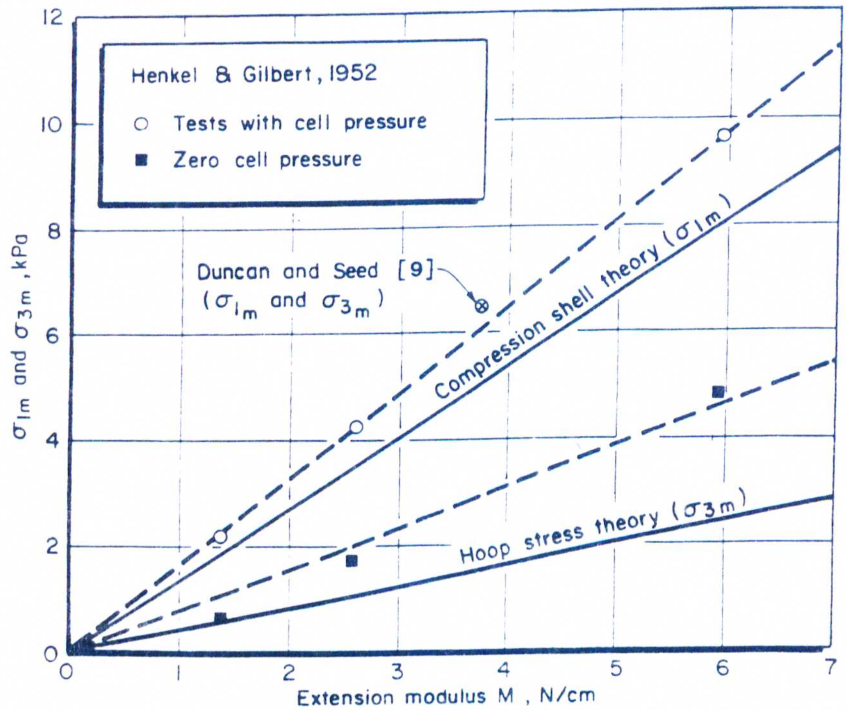


Fig. 2.8 - Experiment Results Obtained at 15% Strain on Specimens of Remolded Clay (La Rochelle et al., 1988)

Ponce and Bell (1971) considered membrane corrections in their triaxial compression tests of clean quartz sand. They speculated in their tests that at a confining stress of 241 kN/m^2, the membrane correction amounts to less than 1% of the confining stress, which they considered to be negligible. For tests conducted at the lowest confining stresses (< 1.4 kN/m^2), however, the membrane corrections amounted to 10% of the uncorrected values of the principal stresses and were not considered negligible.

Finally, Fukushima and Tatsuoka (1984) conducted triaxial tests on Tayoura sand at low confining stresses and used three methods for membrane corrections for specimens failing by bulging which included equations 2.3 and 2.4. They concluded, consistent with other researchers, that loose specimens tested at low confining stresses experience the greatest membrane effects because they undergo larger deformations prior to failure than specimens tested at high confining stresses. In their tests, their corrections for membrane effects were substantial, as shown in Figure 2.9.

In recognition of membrane effects, the modulus of elasticity for the triaxial membrane used in this study (specimen diameter = 71.1 mm and membrane thickness = 0.33 mm) was measured to be 0.48 kN/m using the method of Henkel and Gilbert (1952) as described by Bishop and Henkel (1957) and Head (1982). Further discussion of this influence is given in section 3.6.

2.3 DIRECT SHEAR VERSUS TRIAXIAL TESTS RESULTS

For many years, researchers have disputed the relative use of the direct shear apparatus to find the shear strength of soils compared to the use of the triaxial apparatus, the former being much easier and faster to operate. Taylor (1939) conducted a study on the comparison of results obtained using the direct shear apparatus versus the triaxial apparatus. He based his comparison on the shear

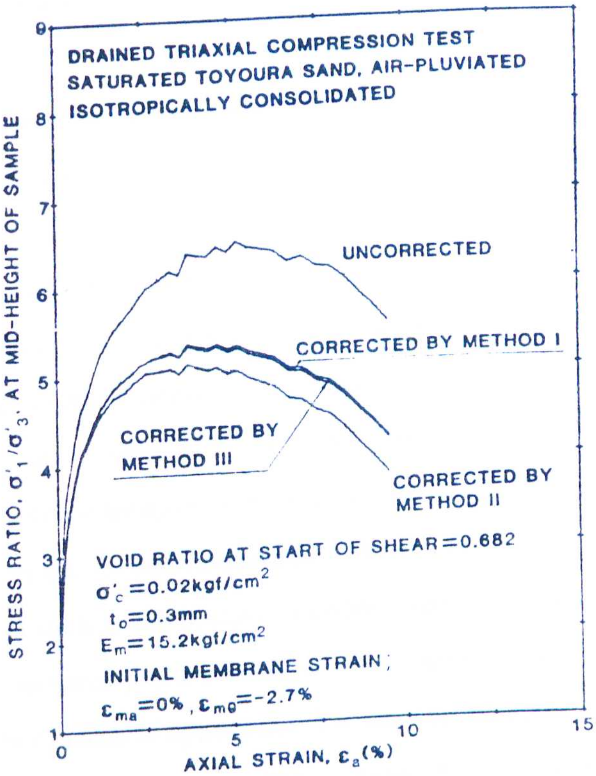


Fig. 2.9 - Stress-Strain Relationship for Stresses Corrected or Uncorrected for Membrane Forces (Fukushima and Tatsuoka, 1984)

strength properties of four sands prepared with initial void ratios varying by as much as 0.25 for a given soil with normal stresses ranging between 138 kN/m^2 and 413 kN/m^2. The peak friction angle was at most 1 or 2 degrees more for the direct shear method than for the triaxial method. Characteristics of stress distribution and strain and of changes in void ratios were different in the two methods, due at least in part to the considerable confinement on the direct shear specimen, which imposes plane strain condition with only one narrow zone in which the failure surface may develop.

Whereas Rowe (1969) recommended the use of the direct shear test for sands where stability problems are those most similar to plane strain, he recommended that a comprehensive understanding of soil behavior is gained by using triaxial apparatus. Section 5.5, which considers development of shear bands also includes some consideration of the importance of the stress conditions imposed by the apparatus on the strength measured. There are, of course, other types of apparatus available such as the hollow cylinder, plain strain, the true triaxial and the biaxial apparatus, to name just a few, all of which seek to improve upon deficiencies in other pieces of apparatus. But the triaxial apparatus has become a standard piece of equipment in geotechnical research laboratories, and in most laboratories for geotechnical design purposes. To ensure that the results are applicable to practitioners, the triaxial apparatus was used in this research.

CHAPTER THREE
EXPERIMENTAL METHOD

3.1 INTRODUCTION

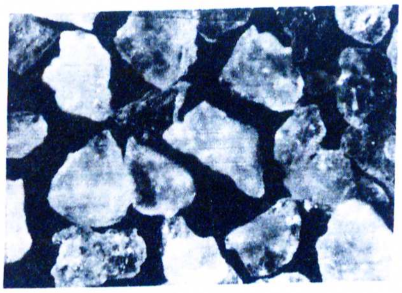
This chapter describes the testing procedure followed by the testing program of this study. This includes description and properties of sands tested, description of equipment and apparatus, sample preparation and setup, testing methodology and data calculation methods.

3.2 DESCRIPTION AND PROPERTIES OF SANDS

Two uniformly graded, clean, quartz sands were used in this study. Both came from Mystic, Connecticut and are sold by U.S.Silica Co. They have subrounded to subangular grains. Mystic White Silica Sand Number 18 (MWSS18) was the coarser of the two sands and Mystic White Silica Sand Number 45 (MWSS45) was roughly parallel in grading but finer in grain size by a factor of about 3. Microphotographs of the sand particles are shown in Figure 3.1. The grain size distributions of the sands are shown in Figure 3.2. The maximum and minimum void ratios (measured according to ASTM D4253-83 and ASTM D4254-83 respectively), D_{10}, D_{50}, C_C, C_U and G_S are provided in Table 3.1 for both.

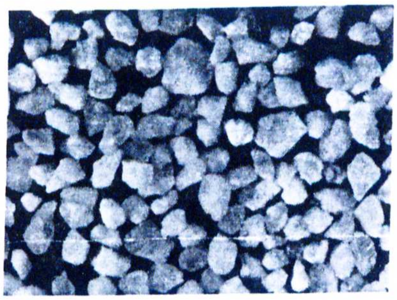
3.3 TEST EQUIPMENT

The shear strength behavior of the sands was investigated using the single unit T-1500 Back Pressure Triaxial Apparatus manufactured by Soil Test



20X Magnification

Microphotographs of MWSS18



20X Magnification

Microphotographs of MWSS45

Fig. 3.1 - Microphotographs of Sands Tested

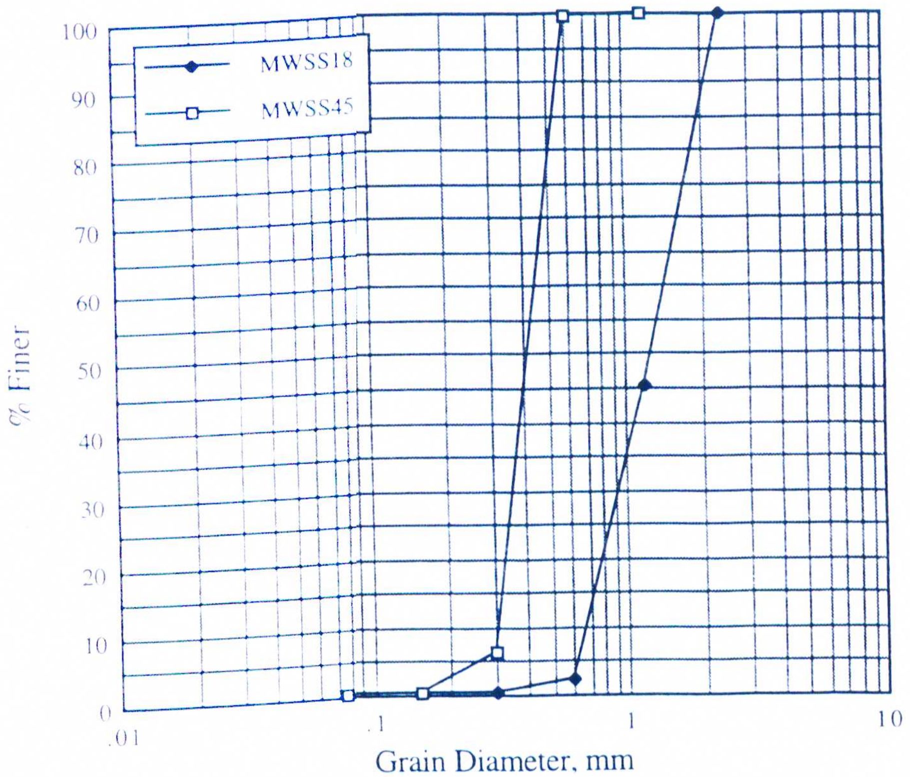


Fig. 3.2 - Gradation Curve Vs. % Finer for all Sands

Table 3.1 Index Properties of Sands

Sand Type	D_{10} (mm)	D_{50} (mm)	C_C	C_U	e_{max}	e_{min}	G_S
MWSS 18	0.69	1.35	0.95	2.07	0.86	0.69	2.57
MWSS 45	0.30	0.42	0.95	1.55	1.00	0.73	2.57
Banding Sand (Dennis, 1988)	0.12	0.18	1.07	1.59	0.82	0.51	2.65

(Figure 3.3), although four modifications were made to the original apparatus. The first modification was replacement of the original load ring by a Geotest Instrument Corporation load cell with a maximum loading capacity of 2.22 kN, which translates to a vertical deviator stress of 556 kN/m^2 on a specimen 71.1 mm in diameter. The second modification was the use of an LVDT (Linearly Variable Displacement Transducer) to measure the axial deformation replacing the original dial gauge. The third modification was the introduction of a pressure transducer to record the confining stress at all times during a test, even though confining stress was intended to be constant. The fourth and final modification was a change in the method of application of the dead load in the stress controlled tests. Whereas the first 24 tests were conducted by manual addition of the dead loads to the hanger on the apparatus, which is the expected means in stress-controlled loading, there was concern about any jarring to the specimen and the rate of application of loading by this method. As a result the dead load hanger was replaced for this study by a water loading system. That system consists of a lever arm extending 30 in. to the rear of the triaxial apparatus applying a load with a 10:1 ratio. The bucket hanging from that arm was then filled with water using a hose (Figure 3.4).

Readings from the load cell and the two transducers were recorded on the Optim Megadac 2200C Data Acquisition and Control System (Figure 3.5). This system has a maximum sampling rate of 20,000 samples per second and a capacity of up to 128 channels of input. The modular construction and external synchronization facility enable the usage of several sub-systems in parallel, effectively multiplying the sampling rates. A sampling rate of three samples per minute for each of the three channels was used throughout the investigation.

3.4 SAMPLE PREPARATION

The technique for specimen preparation followed was based on a

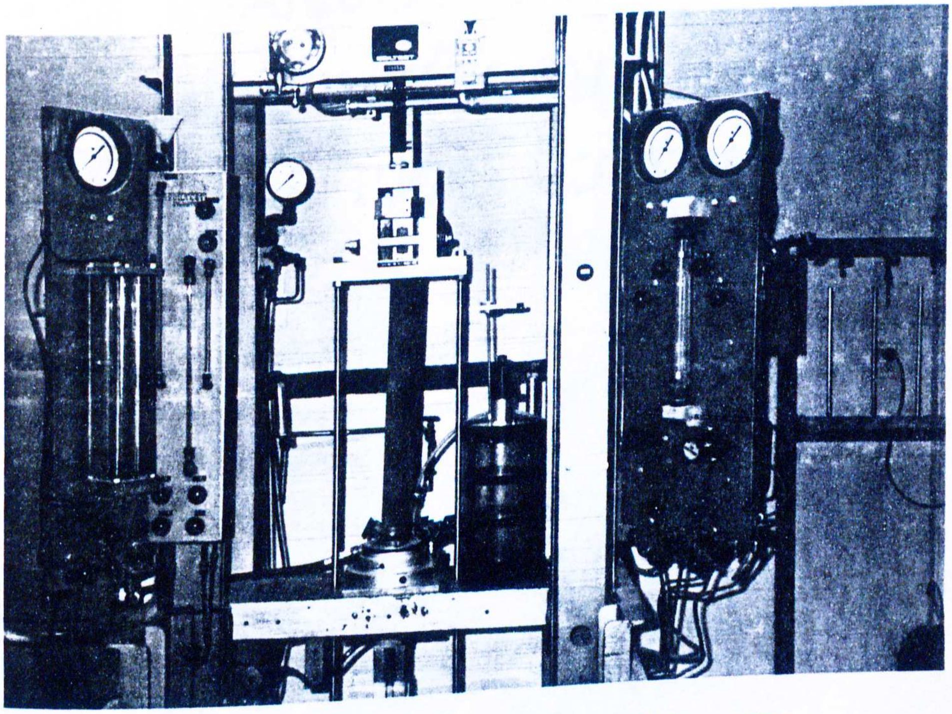


Fig. 3.3 - T-1500 Back Pressure Triaxial Apparatus

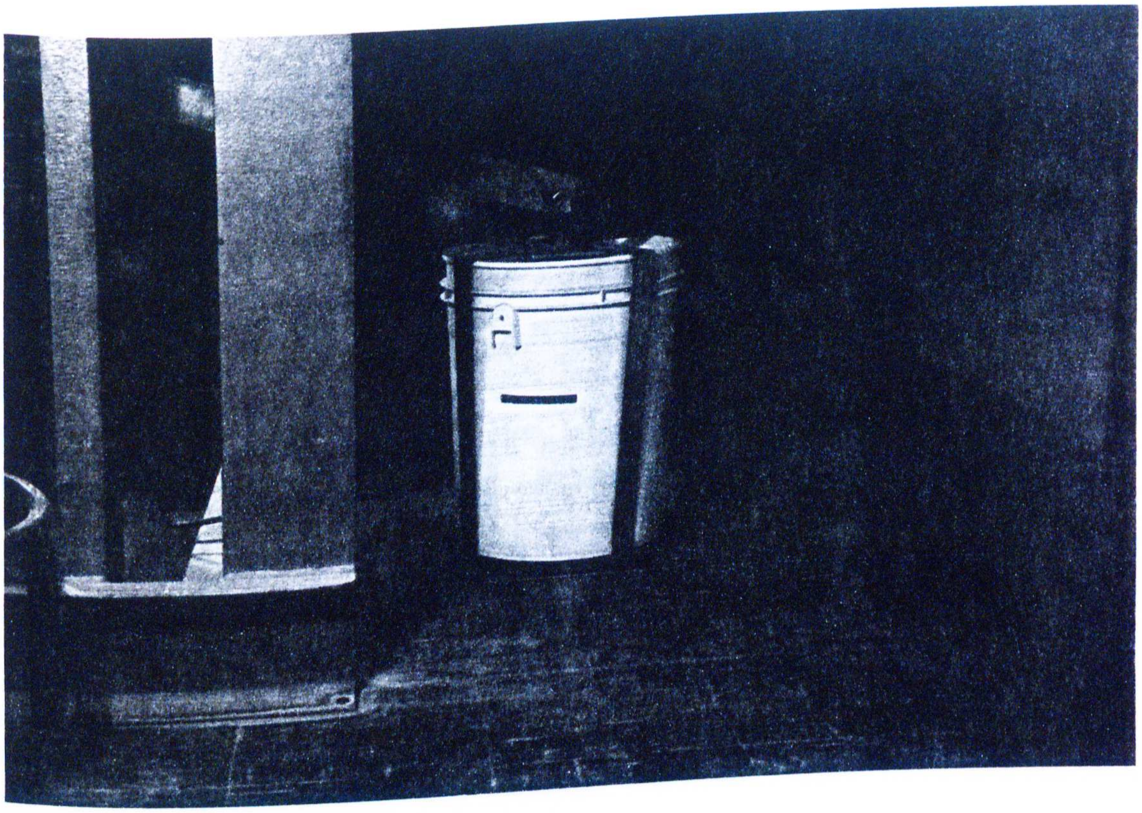


Fig. 3.4 - The Bucket of Water

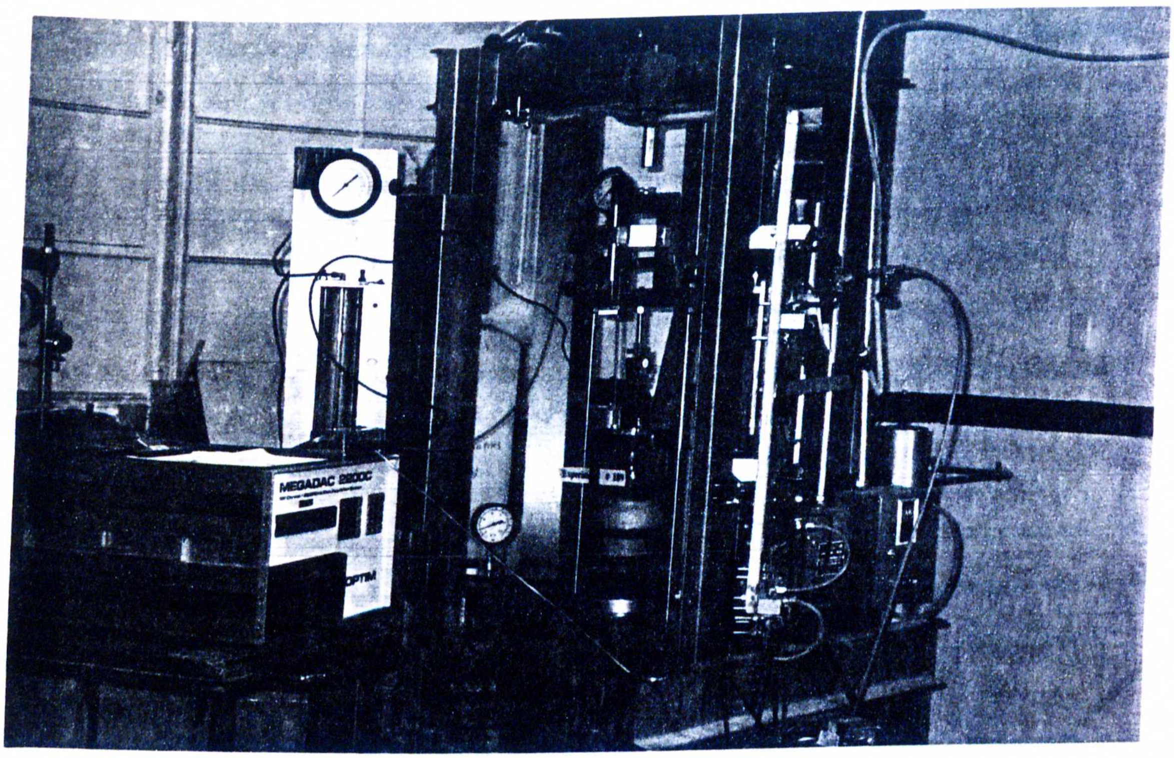


Fig. 3.5 - The Optim Megadac 2200C Data Acquisition and Control System

combination of ASTM D2850, and those recommended by Bowles (1986), and Bishop and Henkel (1962). Specimens were prepared in the standard split mold, which was clamped together and fitted around the pedestal of the triaxial apparatus. A 0.33 mm-thick latex membrane was used to enclose each specimen (Figure 3.6).

Specimens with loose particle packing were formed by pouring dry sand into the mold from a small height approximately one inch from the bottom of a small funnel above the advancing soil surface. After the mold was filled and the top surface of the specimen leveled, the top loading cap was placed and the membrane sealed around it using two O-rings.

Dense specimens were formed by pouring three, equal, known weights of sand into the forming mold and compacting each layer with an 11 lb. hammer (Figure 3.7) dropped from a height of 100 mm onto the soil surface. A small vacuum between 2 and 5 kN/m^2 (less than the minimum confining pressure of any test) was applied to the base of the specimen to give the specimen sufficient rigidity to stand while the split mold was removed (Figure 3.8). The height and diameter of the specimen were then measured to within 0.1 mm using Vernier Calipers (Figure 3.9). To calculate specimen volume and then average void ratio, two or three measurements of each dimension were taken, accounting for the membrane thickness. The average of the readings was then used to calculate the average void ratio.

To complete the specimen setup, the acrylic chamber of the triaxial cell was put in place and the cell sealed by screwing down the top collar to the top plate. The triaxial cell was then installed in its test position in the loading frame. After the cell was filled with water and a small confining pressure was applied, the vacuum in the sample was released, so that the pressure in the specimen was restored to atmospheric before proceeding with the triaxial test. The drainage valve was left open to atmospheric pressure throughout the test.

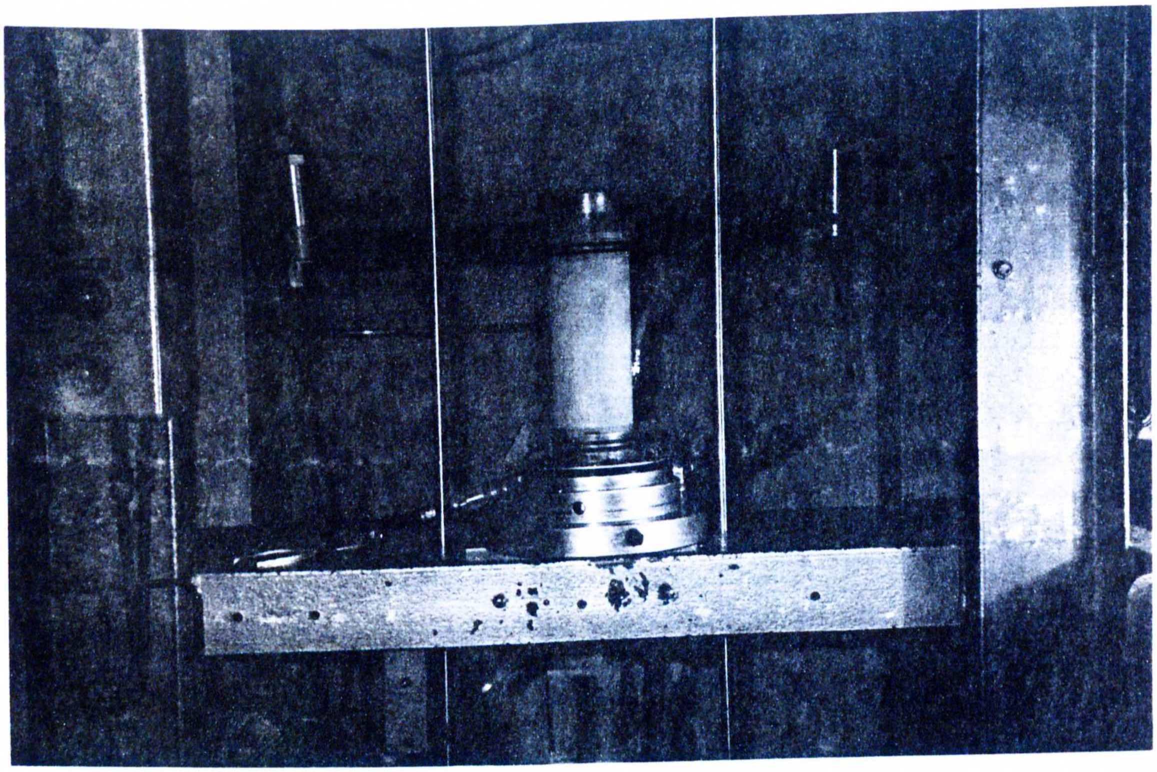


Fig. 3.6 - 71.1 mm Diameter Specimen Enclosed in Latex Membrane

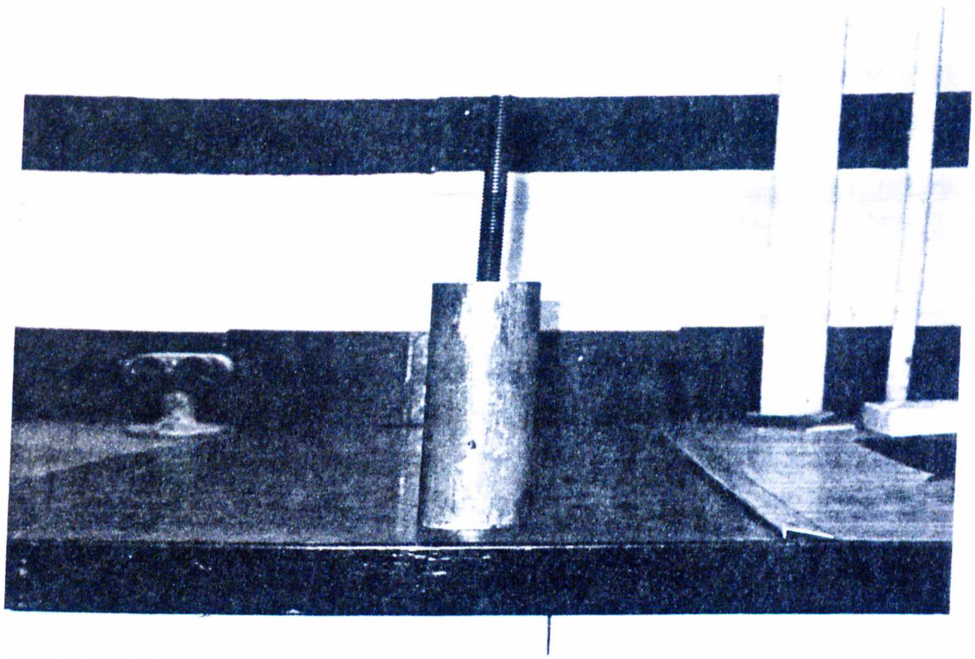


Fig. 3.7 - Hammer Used For Compaction of Dense Specimens

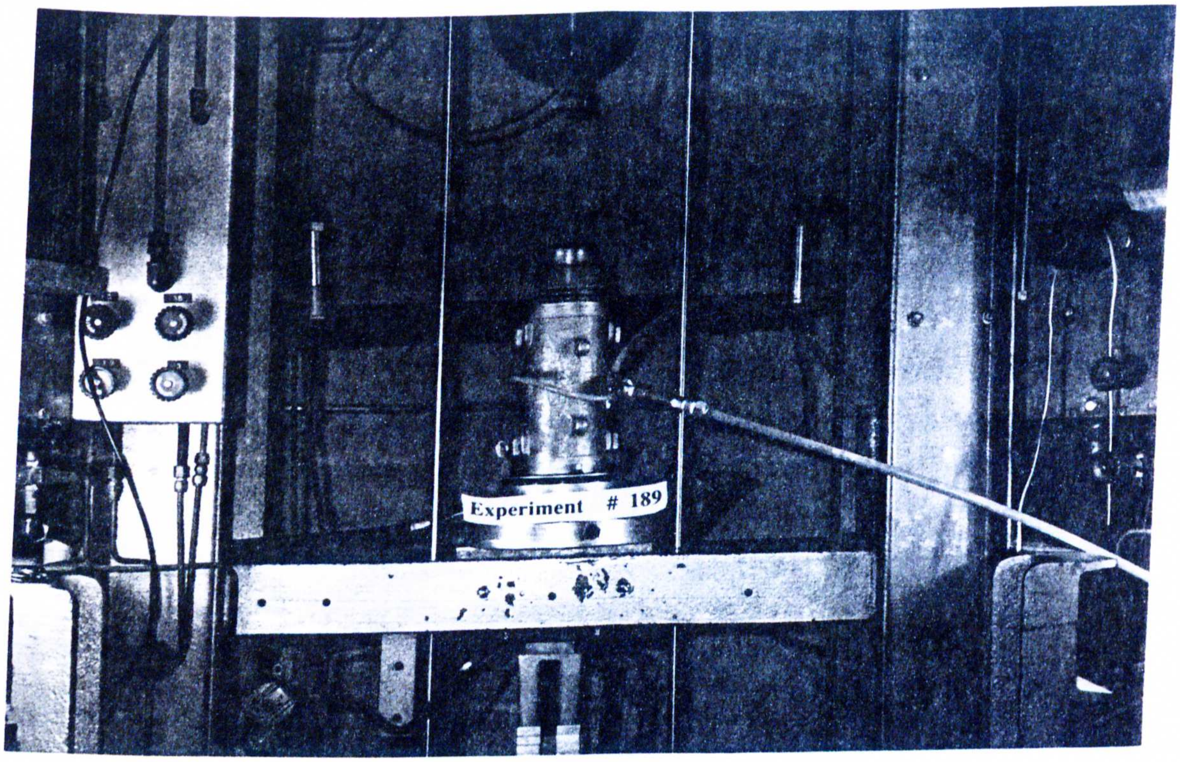


Fig. 3.8 - Specimen in Split Mold

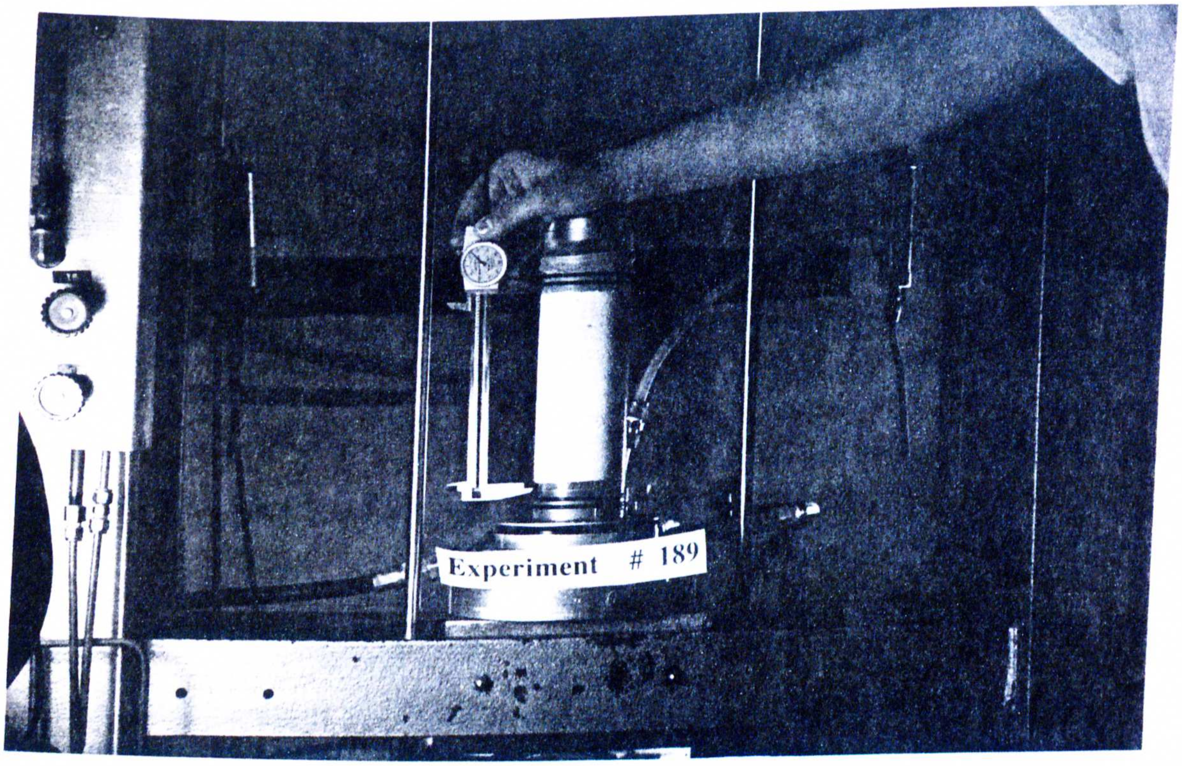


Fig. 3.9 - Specimen Measurements

3.5 TEST VOID RATIOS

Because shear strength of sand is so sensitive to void ratio, especially at low confining stresses, only specimens within a narrow range of void ratios were tested. For MWSS18, loose specimens were tested with average initial void ratios between 0.87 and 0.92 (Dr = -6% and -35% respectively). It is noteworthy that these negative relative densities indicate void ratios greater (looser) than the loosest packing, e = 0.86, measured using the ASTM method to determine maximum void ratio. This is attributed to membrane effects in specimen preparation. Loose specimens prepared in the triaxial forming jacket with no triaxial membrane present had a maximum void ratio of e = 0.86, which is equal to the maximum void ratio for MWSS18 using the ASTM method with the larger (standard) mold of diameter 152.4 mm. This agrees with speculation by Valid and Negussey (1988) on membrane effects on void ratio in loose sand. For MWSS45, loose specimens were tested with initial void ratios between 0.90 and 0.98 (Dr = +37% and +7% respectively), using the same exact method of sample preparation used for MWSS18 loose specimens. This finer sand did not show the same effects of a membrane on loose packing.

For MWSS18, dense specimens were tested with initial void ratios between 0.69 and 0.74 (Dr = 100% and 71% respectively), and for MWSS45 dense specimens the initial void ratios ranged from 0.73 to 0.79 (Dr = 100% and 78% respectively).

Because all specimens were tested dry, and the changes in air pressure resulting from changes in specimen volume were too small to measure there was no direct measurement of changes in specimen volumes and thus void ratios during the tests. Atkinson and Bransby (1978) represented compressibility of sand in the consolidometer at stresses up to 100 kN/m^2 for loose specimens to be roughly zero. And Vesic and Clough (1968) who conducted triaxial tests on dry sands, showed that at confining stresses of 100 kN/m^2, which is greater than applied in these tests, there is no change in void ratio arising from the application of that stress for dense specimens and less than 1% change for loose specimens. Their results are shown in Tables 3.2 and 3.3. Finally, Hettler and Vardoulakis (1984) also conducted several triaxial tests on dry sand samples, assuming that void ratios achieved in preparation were equal to the void ratios after application of the isotropic confining stress, and proving it was so for isotropic loading up to 50 kN/m^2. The initial void ratios then were assumed to exist also after application of isotropic confining pressure in the triaxial. The small changes in volume in dry sand specimens recorded by other researchers is typically obtained in true triaxial or plane strain apparatus. This was not feasible here in dry specimens.

3.6 TEST PROCEDURE

After specimen preparation and connection of all appropriate transducers to the data acquisition system, the air bleed valve to the triaxial cell was opened. The cell base was opened and water was allowed into the cell from the supply line. The connection from the constant pressure line to the cell was opened, and the pressure in the cell increased gradually to the required confining pressure for the test. All tests were conducted as standard compression loading, which requires the confining pressure to stay constant. The pressure regulator was manually adjusted only if the pressure transducer indicated change in pressure. Variation in the cell pressure was never more than 0.025 kPa ($\leq 0.2\%$), occurring only after peak strength was reached. This reinforces the assumption that change in specimen void ratio is very small during testing, and occurs after peak strength is reached.

Tests were conducted either as strain controlled, following the ASTM standard (ASTM D2850; Bowles, (1986); and Bishop and Henkel, (1962)), or

Table 3.2 Standard Triaxial Test Results on Dense Samples (Vesic and Clough, 1968)

Test Number	Initial Void Ratio	Void Ratio After Consolidation	Cell Pressure, σ_3, in kilograms per square centimeter	Axial Stress at Failure, σ_1, in kilograms per square centimeter	Axial Strain at Failure, in percent	Volumetric Strain at Failure, in percent	Mean Normal Stress at Failure, σ_0, in kilograms per square centimeter	Principal Stress Ratio at Failure (σ_1/σ_3)	Secant Angle, ϕ_s, in degrees	Initial Tangent Modulus, E, in kilograms per square centimeter
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
A-1	0.71	0.33	633	1,954	21.5	8.9	1,073	3.09	30.7	8,770
A-2	0.72	0.30	633	2,103	21.0	9.2	1,123	3.32	32.6	10,000
A-3	0.71	0.32	633	2,142	20.0	9.3	1,136	3.39	33.0	11,870
A-4	0.67	0.28	633	1,969	16.7	6.0	1,078	3.11	30.9	17,020
A-5	0.68	0.28	633	2,190	23.0	9.6	1,152	3.46	33.5	11,510
A-6	0.72	0.31	633	2,149	23.0	9.7	1,138	3.40	33.0	15,200
A-7	0.66	0.30	422	1,482	22.0	9.2	775	3.51	33.8	7,200
A-8	0.64	0.28	422	1,432	22.6	9.1	759	3.40	33.1	6,620
A-9	0.70	0.47	211	702	27.9	13.0	375	3.33	32.6	3,200
A-10	0.72	0.43	211	685	20.5	12.2	369	3.25	32.0	3,490
A-11	0.67	0.38	211	738	25.2	23.8	387	3.50	33.8	3,260
A-12	0.68	0.44	211	682	30.0	13.0	368	3.23	31.8	4,060
A-13	0.70	0.62	28.1	96.4	26.8	9.5	51	3.44	33.3	1,410
A-14	0.69	0.63	21.1	74.9	19.1	5.5	39	3.55	34.1	1,190
A-15	0.75	0.75	1.0	4.52	7.2	-2.0	2.2	4.52	39.7	275
A-16	0.74	0.74	1.0	4.63	7.3	-2.7	2.2	4.63	40.1	400

Table 3.3 Standard Triaxial Test Results on Loose Samples (Vesic and Clough, 1968)

Test Number (1)	Initial Void Ratio (2)	Void Ratio After Consolidation (3)	Cell Pressure, σ_3, in kilograms per square centimeter (4)	Axial Stress at Failure, σ_1, in kilograms per square centimeter (5)	Axial Strain at Failure, in percent (6)	Volumetric Strain at Failure, in percent (7)	Mean Normal Stress at Failure (σ_0) (8)	σ_1/σ_3 (9)	Secant Angle (ϕ) (10)	Initial Tangent Modulus, E, in kilograms per square centimeter (11)
B-1	0.96	0.31	633	2,053	19.6	9.0	1,106	3.24	31.9	9,750
B-2	1.00	0.35	633	2,025	19.8	8.7	1,096	3.20	31.6	12,180
B-3	0.99	0.35	633	2,129	22.4	8.8	1,132	3.36	32.8	11,310
B-4	0.96	0.26	633	2,299	26.4	9.4	1,188	3.63	34.6	10,010
B-5	0.99	0.27	633	2,085	21.3	9.6	1,117	3.29	32.3	11,290
B-6	0.99	0.48	211	714	26.5	13.0	379	3.38	32.9	3,670
B-7	1.01	0.47	211	726	29.5	13.2	383	3.44	33.4	3,670
B-8	1.00	0.45	211	751	28.3	13.6	391	3.56	34.2	4,590
B-9	1.05	1.04	1	3.16	22.6	4.0	1.7	3.16	31.4	200
B-10	1.02	1.01	1	3.32	21.0	3.3	1.8	3.32	32.6	125

as stress controlled. In strain controlled tests, the slowest axial strain rate possible on the apparatus was used, 2.1 mm/minute. Both loose and dense specimens of MWSS18 and MWSS45 reached completion of their testing in about 16 minutes, corresponding to a displacement of 33.60 mm, for a strain of 22.40% in large specimens originally 150 mm high; peak strength was reached after about 5 minutes. In the 80 strain controlled tests conducted and considered useful, this was more than adequate to reach and then go beyond the strain necessary for maximum shear resistance for the soil, about 15%.

In stress controlled tests, the first dead load hanger system was used only on loose specimens of MWSS18. Those specimens were loaded at 20 second intervals by initially increasing the applied axial stress in increments of 15% of the expected failure load. As the specimen approached failure, at about 80% of the failure deviator stress which occurred at about 1% strain, the loading increment was reduced to 2% of the expected failure load. Specimens loaded in this fashion typically reached peak strength in 2 to 4 minutes with subsequent failure occurring in less than two-tenths of a second, with final strain approaching 15%. The new water loading system was used in stress controlled tests on dense specimens of MWSS18 and all specimens of MWSS45. This loading method produced smoother stress-strain plots compared to the first dead load hanger system, the rate of loading was constant throughout the test and could be matched more closely to that of the strain controlled system. Loose specimens of MWSS45 loaded in this fashion reached their peak strength in 2 to 4 minutes and the final ultimate strengths at a strain approaching 15%. For dense specimens of both MWSS18 and MWSS45, the peak strengths were reached in 3 to 5 minutes at strains approaching 5% with subsequent failure occurring abruptly.

As discussed in section 2.2, the rubber membrane enclosing the triaxial specimen may have an effect on the specimen and this effect is a function of the elastic modulus and initial diameter of the membrane and of the specimen, and of the subsequent soil deformations. Although negligible at high confining stresses, membrane effects can become significant when testing at low confining stresses, especially on soft soils, which was not the case here. The elastic modulus for the rubber membrane used in this research was measured to be equal 0.48 kN/m (see Figure 3.10). At 10% axial strain, the membrane correct may be as much as 2.5 kN/m^2 on a 71.1 mm diameter specimen, if the ASTM standard D2850-83 correction is calculated. No membrane corrections were applied in the final analyses here, since they were more or less constant at a given confining stress for a given soil, at a given axial strain, and the objective of this research was to compare the effects of the two methods of loading on shear resistance measured all other factors being held constant.

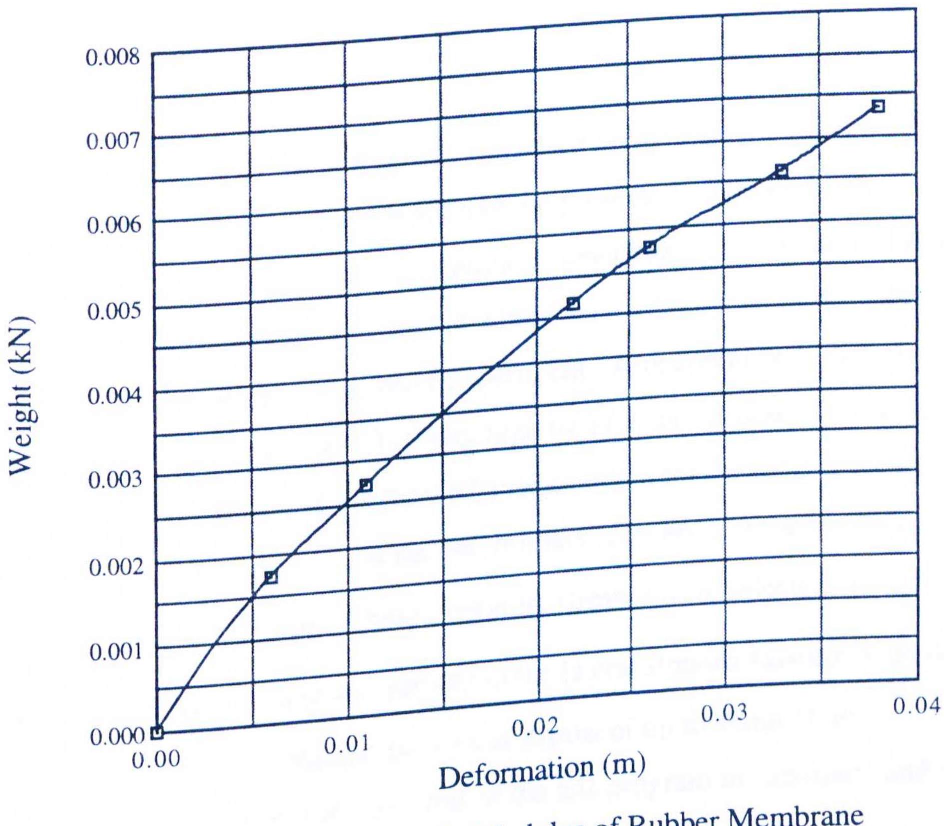


Fig. 3.10 - Elastic Modulus of Rubber Membrane

For axial Strain of 15% $x = (0.5)(0.15)(223.43) = 16.76$ mm
From The Above Graph, Load For 0.0168 m extension = 0.00365 kN
Modulus = $0.00365/(2)(0.15)(25) = 0.00048$ kN/mm = 0.48 kN/m

CHAPTER FOUR
RESULTS

4.1 INTRODUCTION

As mentioned in chapter one, the purpose of this research is to compare the characterization of the shear strength of dry sands using strain versus stress controlled loading in standard compression triaxial tests. In this chapter all the data of the major testing program are presented with some discussion. More thorough discussion and comparison of these data to published data from other researchers is reserved for chapter five.

4.2 TEST PROGRAM

The results of the 166 primary standard compression drained triaxial tests performed on MWSS18 and MWSS45 are presented. Results of 58 specimens were discarded because of equipment operations difficulties, or because the void ratio of the specimen did not fall within the narrow range of the values that was settled upon as acceptable as more experiments were conducted. All test specimens were approximately 150.0 mm high by 71.1 mm in diameter; these dimensions satisfy the ASTM (D2850-87) requirement for the ratio of length to diameter to be between 2 to 3. Of the 166 primary tests 86, were stress-controlled and 80 were strain-controlled. Three confining stresses were selected: 14 kN/m^2, 28 kN/m^2, 55 kN/m^2. These are relevant to the lateral stresses existing in geostatic conditions due to self-weight soil stresses at depths of up to about 10 m.

Tables 4.1 to 4.8 present data of the test program in summary, and plots of deviator stress (σ_1 - σ_3) versus strain are provided and discussed.

Table 4.1 Summary of Data for MWSS18 Loose Specimens

Test #	Specimen Diameter (mm)	Loading Condition	σ_3 (kPa)	σ_1 (kPa)	Deviator Stress (kPa) at 15% Strain	Initial Void Ratio
31	71.12	Stress Control	14.09	53.76	39.76	0.87
37			14.11	58.10	44.10	0.87
43			14.05	54.90	40.85	0.87
49			14.05	50.80	36.75	0.89
55			14.06	54.35	40.29	0.88
61			14.01	50.82	36.81	0.88
67			14.06	54.76	40.76	0.88
73			14.06	49.00	35.00	0.89
219			14.07	48.50	34.50	0.92
220			14.06	48.21	34.21	0.92
Average			14.06	52.32	38.30	0.89
C_v			0.00213	0.06348	0.08657	0.02129
12	71.12	Strain Control	14.02	45.55	31.53	0.90
16			14.07	50.55	36.48	0.89
18			13.98	51.88	37.88	0.87
20			13.89	42.00	28.00	0.92
50			13.98	44.73	30.73	0.90
56			14.00	44.90	30.90	0.90
62			13.99	47.61	33.61	0.89
74			13.97	48.13	34.13	0.89
Average			13.99	46.91	32.91	0.90
C_v			0.00357	0.06961	0.09896	0.01580
27	71.12	Stress Control	28.06	114.20	86.20	0.87
39			28.15	116.02	88.02	0.87
51			28.06	116.01	88.01	0.87
57			28.05	106.86	78.86	0.92
63			28.05	109.28	81.28	0.89
69			28.03	110.23	82.20	0.88
75			28.08	108.13	80.13	0.91
79			27.95	109.55	81.55	0.90
221			28.10	99.62	71.62	0.92
222			27.95	110.16	82.16	0.88
Average			28.05	110.01	82.00	0.89
C_v			0.00214	0.04411	0.05917	0.02273
22	71.12	Strain Control	28.03	90.59	62.59	0.92
28			28.00	105.90	77.90	0.88
46			27.95	95.97	67.97	0.91
52			28.01	112.42	84.42	0.87
58			28.01	103.85	75.85	0.88
64			27.93	104.95	76.95	0.87
70			28.03	96.63	68.63	0.89
82			27.91	97.88	69.97	0.89
Average			27.98	101.07	73.04	0.89
C_v			0.00179	0.06874	0.09544	0.02034
23	71.12	Stress Control	55.67	244.17	188.50	0.88
29			55.59	246.15	191.15	0.88
35			55.14	243.35	188.35	0.90
41			55.01	245.35	190.34	0.87
53			54.88	250.87	195.87	0.87
59			54.91	244.91	190.00	0.89
65			55.09	249.10	194.01	0.87
71			55.09	252.47	197.47	0.87
223			54.97	231.97	176.97	0.92
224			55.05	242.88	187.88	0.88
Average			55.17	245.12	187.05	0.88
C_v			0.00544	0.02299	0.05225	0.01853
24	71.12	Strain Control	55.50	225.92	170.92	0.92
30			55.00	214.18	159.18	0.92
36			55.19	236.30	181.30	0.91
42			54.96	236.62	181.62	0.90
48			54.96	237.40	182.40	0.91
60			55.14	239.03	183.89	0.91
66			55.08	232.46	177.46	0.91
83			55.05	241.53	186.53	0.88
Average			55.11	232.93	177.91	0.91
C_v			0.00327	0.03829	0.05506	0.01412

Table 4.2 Summary of Statistics for Data from MWSS18 Loose Specimens

	Deviator Stress at 15 % Strain						Initial Void Ratio		
	No. of Tests	Average	Standard Deviation	Coefficient of Variation	R^2	Range	Average	Standard Deviation	Range
Stress Controlled 14 kPa	10	38.30	3.32	8.7%	53.5%	34.21 - 44.10	0.89	0.01889	0.87 - 0.92
Stress Controlled 28 kPa	10	82.00	4.85	5.9%	83.5%	71.62 - 88.02	0.89	0.02025	0.87 - 0.92
Stress Controlled 55 kPa	10	187.05	9.77	5.2%	46.8%	176.97 - 197.47	0.88	0.01636	0.87 - 0.92
Strain Controlled 14 kPa	8	32.91	3.26	9.9%	89.3%	28.00 - 37.88	0.90	0.0142	0.87 - 0.92
Strain Controlled 28 kPa	8	73.04	6.94	9.5%	81.6%	62.59 - 84.42	0.89	0.0181	0.87 - 0.92
Strain Controlled 55 kPa	8	177.91	9.80	5.5%	49.3%	159.18 - 186.53	0.91	0.0129	0.88 - 0.92

Table 4.3 Summary of Data for MWSS45 Loose Specimens

Test #	Specimen Diameter (mm)	Loading Condition	σ_3 (kPa)	σ_1 (kPa)	Deviator Stress (kPa) at 15% Strain	Initial Void Ratio
104	71.12	Stress Control	14.09	35.41	21.41	0.97
114			14.11	52.31	38.31	0.92
120			14.05	47.10	33.10	0.94
125			14.05	48.69	34.69	0.94
131			14.06	49.70	35.70	0.93
137			14.01	35.09	21.09	0.97
143			14.06	42.73	28.73	0.96
149			14.06	34.62	20.62	0.98
Average			14.06	43.21	29.21	0.95
C_v			0.00213	0.16847	0.24921	0.02278
105	71.12	Strain Control	14.02	47.32	33.32	0.93
109			14.07	39.94	25.94	0.94
115			13.98	36.70	22.70	0.95
126			13.89	48.75	34.75	0.92
132			13.98	33.66	19.66	0.96
138			14.00	50.71	36.71	0.91
144			13.99	38.14	24.14	0.95
150			13.97	39.40	25.40	0.94
Average			13.99	41.82	27.83	0.94
C_v			0.00357	0.14931	0.22441	0.01780
110	71.12	Stress Control	28.06	105.23	77.23	0.96
116			28.15	103.45	75.45	0.96
121			28.06	107.86	79.86	0.93
127			28.05	106.91	78.91	0.91
133			28.05	93.88	65.88	0.97
139			28.03	104.95	76.95	0.96
145			28.08	102.10	74.10	0.95
151			27.95	106.47	78.47	0.93
Average			28.05	103.83	75.86	0.95
C_v			0.00214	0.04253	0.05854	0.02183
111	71.12	Strain Control	28.03	100.98	72.98	0.93
117			28.00	101.16	73.16	0.93
122			27.95	83.86	55.86	0.97
128			28.01	97.78	69.78	0.95
134			28.01	82.91	54.91	0.98
140			27.93	87.35	59.35	0.96
146			28.03	89.96	61.96	0.95
152			27.91	80.57	52.57	0.98
Average			27.98	90.60	62.57	0.96
C_v			0.00179	0.09233	0.13331	0.02087
112	71.12	Stress Control	55.67	225.14	170.14	0.95
118			55.59	212.77	157.77	0.97
123			55.14	242.11	187.11	0.91
129			55.01	237.23	182.33	0.92
135			54.88	230.42	175.42	0.94
141			54.91	217.13	162.13	0.96
147			55.09	216.23	161.23	0.96
153			55.09	208.11	153.11	0.98
Average			55.17	227.39	168.64	0.95
C_v			0.00544	0.07261	0.07210	0.02547
113	71.12	Strain Control	55.50	202.48	147.48	0.95
119			55.00	212.22	157.22	0.93
124			55.19	202.41	147.41	0.95
130			54.96	181.17	126.17	0.98
136			54.96	200.41	145.41	0.95
142			55.14	208.29	153.29	0.93
148			55.08	187.74	132.74	0.97
154			55.05	182.41	127.41	0.98
Average			55.11	197.14	142.14	0.96
C_v			0.00327	0.05999	0.08320	0.02094

Table 4.4 Summary of Statistics for Data from MWSS45 Loose Specimens

		Deviator Stress at 15 % Strain					Initial Void Ratio		
	No. of Tests	Average	Standard Deviation	Coefficient of Variation	R^2	Range	Average	Standard Deviation	Range
Stress Controlled 14 kPa	8	29.21	7.3	24.9%	96.0%	20.62 - 38.31	0.95	0.02164	0.92 - 0.98
Stress Controlled 28 kPa	8	75.86	4.48	5.9%	54.8%	65.88 - 79.86	0.95	0.02074	0.91 - 0.97
Stress Controlled 55 kPa	8	168.64	12.14	7.2%	95.7%	153.11 - 182.33	0.95	0.02420	0.91 - 0.98
Strain Controlled 14 kPa	8	27.83	6.23	22.4%	95.5%	19.66 - 36.71	0.94	0.01673	0.91 - 0.96
Strain Controlled 28 kPa	8	62.57	8.32	13.3%	97.4%	52.57 - 73.16	0.96	0.02040	0.93 - 0.98
Strain Controlled 55 kPa	8	142.14	11.80	8.3%	98.2%	126.17 - 157.22	0.96	0.02010	0.93 - 0.98

Table 4.5 Summary of Data for MWSS18 Dense Specimens

Test #	Specimen Diameter (mm)	Loading Condition	σ_3 (kPa)	σ_1 (kPa)	Deviator Stress (kPa) at Peak	Initial Void Ratio
187	71.12	Stress Control	14.09	57.51	43.51	0.69
191			14.11	53.45	39.45	0.73
195			14.05	55.62	41.62	0.70
199			14.05	57.70	43.70	0.69
203			14.06	55.59	41.59	0.70
207			14.01	55.05	41.05	0.72
211			14.06	54.59	40.59	0.71
215			14.06	50.95	36.95	0.74
Average			14.06	55.06	41.06	0.71
C_v			0.00213	0.03959	0.05310	0.02608
188	71.12	Strain Control	14.02	55.50	41.50 35.45*	0.69
192			14.07	56.62	42.62 37.69*	0.69
196			13.98	53.70	39.70 33.20*	0.72
200			13.89	49.26	35.26 33.53*	0.73
204			13.98	55.97	41.97 34.80*	0.70
208			14.00	54.82	40.82 34.03*	0.71
212			13.99	46.44	32.44 31.29*	0.74
216			13.97	50.97	36.97 25.97*	0.73
Average			13.99	52.91	38.91 33.25*	0.71
C_v			0.00357	0.06891	0.09370 0.1044*	0.02694
189	71.12	Stress Control	55.67	292.14	237.14	0.69
193			55.59	267.11	212.11	0.72
197			55.14	277.08	222.08	0.71
201			55.01	256.04	201.04	0.74
205			54.88	275.84	220.84	0.71
209			54.91	261.18	206.18	0.74
213			55.09	263.40	208.40	0.73
217			55.09	282.84	227.84	0.70
Average			55.17	271.95	216.95	0.72
C_v			0.00544	0.04468	0.05600	0.02554
190	71.12	Strain Control	55.50	281.30	226.30 185.93*	0.69
194			55.00	245.63	190.63 166.84*	0.74
198			55.19	275.54	220.54 187.20*	0.69
202			54.96	270.10	215.10 174.92*	0.71
206			54.96	274.44	219.77 182.20*	0.70
210			55.14	262.15	207.15 196.37*	0.73
214			55.08	265.06	210.06 149.51*	0.72
218			55.05	254.10	199.10 192.20*	0.74
Average			55.11	266.84	211.08 179.40*	0.72
C_v			0.00327	0.04472	0.05637 0.08518*	0.02895

(*) At 15% Strain

Table 4.6 Summary of Statistics for Data from MWSS18 Dense Specimens

		Deviator Stress at Peak					Initial Void Ratio		
	No. of Tests	Average	Standard Deviation	Coefficient of Variation	R^2	Range	Average	Standard Deviation	Range
Stress Controlled 14 kPa	8	41.06	2.18	5.3%	90.2%	36.95 - 43.70	0.71	0.01852	0.69 - 0.74
Stress Controlled 55 kPa	8	216.95	12.15	5.6%	96.7%	201.04 - 237.14	0.72	0.01832	0.69 - 0.74
Strain Controlled 14 kPa	8	38.91	3.65	9.4%	85.2%	32.44 - 42.62	0.71	0.01932	0.69 - 0.74
Strain Controlled 55 kPa	8	211.08	11.89	5.7%	90.7%	190.63 - 226.30	0.72	0.02070	0.69 - 0.74

Table 4.7 Summary of Data for MWSS45 Dense Specimens

Test #	Specimen Diameter (mm)	Loading Condition	σ_3 (kPa)	σ_1 (kPa)	Deviator Stress (kPa) at Peak	Initial Void Ratio
155	71.12	Stress Control	14.09	57.21	43.21	0.73
159			14.11	53.43	39.43	0.75
163			14.05	50.05	36.05	0.79
167			14.05	53.07	39.07	0.76
171			14.06	53.47	39.47	0.75
175			14.01	56.21	42.21	0.73
179			14.06	52.75	38.75	0.77
183			14.06	54.72	40.72	0.74
Average			14.06	53.86	39.86	0.75
C_v			0.00213	0.04103	0.05545	0.02728
156	71.12	Strain Control	14.02	55.59	41.59 34.91*	0.74
160			14.07	49.21	35.21 32.39*	0.77
164			13.98	52.54	38.54 30.67*	0.76
168			13.89	54.21	40.21 32.06*	0.76
172			13.98	48.10	34.10 31.34*	0.77
176			14.00	55.64	41.64 36.94*	0.73
180			13.99	51.83	37.83 33.26*	0.75
184			13.97	51.44	37.44 29.80*	0.78
Average			13.99	52.32	38.32 32.67*	0.76
C_v			0.00357	0.05300	0.07237 0.0713*	0.02203
157	71.12	Stress Control	55.67	272.62	217.62	0.73
161			55.59	274.24	219.24	0.73
165			55.14	257.10	202.10	0.77
169			55.01	251.71	196.77	0.79
173			54.88	264.34	209.34	0.75
177			54.91	254.50	199.50	0.78
181			55.09	274.73	219.73	0.73
185			55.09	269.67	214.67	0.74
Average			55.17	264.86	209.87	0.75
C_v			0.00544	0.03523	0.04440	0.03236
158	71.12	Strain Control	55.50	297.55	242.55 202.31*	0.73
162			55.00	255.39	200.39 185.47*	0.77
166			55.19	251.12	196.12 183.85*	0.79
170			54.96	266.99	211.99 176.65*	0.75
174			54.96	273.67	218.67 173.87*	0.73
178			55.14	255.95	200.95 178.78*	0.78
182			55.08	268.18	213.18 180.62*	0.74
186			55.05	265.89	210.89 194.53*	0.76
Average			55.11	266.84	211.84 184.51*	0.76
C_v			0.00327	0.05466	0.06885 0.05188*	0.02994

(*) At 15% Strain

Table 4.8 Summary of Statistics for Data from MWSS45 Dense Specimens

		Deviator Stress at Peak					Initial Void Ratio		
	No. of Tests	Average	Standard Deviation	Coefficient of Variation	R^2	Range	Average	Standard Deviation	Range
Stress Controlled 14 kPa	8	39.86	2.21	5.5%	92.3%	36.05 - 43.21	0.75	0.02053	0.73 - 0.79
Stress Controlled 55 kPa	8	209.87	9.32	4.4%	97.0%	196.77 - 219.73	0.75	0.02435	0.73 - 0.79
Strain Controlled 14 kPa	8	38.32	2.77	7.2%	60.0%	34.10 - 41.64	0.76	0.02203	0.73 - 0.78
Strain Controlled 55 kPa	8	211.84	14.58	6.9%	71.8%	196.12 - 242.55	0.76	0.02995	0.73 - 0.79

4.3 MWSS18: LOOSE SPECIMENS

4.3.1 STRESS AND STRAIN CONTROLLED TESTS AT 14 kN/m^2

Figures 4.1 and 4.2 show plots of data of ten tests conducted on sand MWSS18 under stress-controlled conditions, and eight tests under strain-controlled conditions with confining stress of 14 kN/m^2. The two additional stress-controlled tests were conducted to check on any effects of the smooth water applied stress-controlled loading used for models numbered 104 or greater compared to the initial dead load stress-controlled loading.

A narrow range of initial void ratio was achieved between 0.87 and 0.92, for the 18 tests. The stress-strain relationships are as one would expect for a loose sand, where stress increases rapidly at small strain, and then almost reaches a plateau with only small further increases in stress with large increases in strain; there is no peak/post-peak behavior. Specimens barreled as vertical stress was increased in the case of strain-controlled loading but developed a distinct failure surface in the case of stress-controlled loading, (Figures 4.3 and 4.4). All tests were continued to at least 15% strain and so deviator stresses for all specimens were taken for that level of strain. A strain of 15% is large for sands and most studies end testing at strains of 10% or less, because of stress discontinuities.

For stress-controlled tests the average deviator stress at 15% strain was 38.30 kN/m^2 varying from 34.21 kN/m^2 to 44.10 kN/m^2, which gives a standard deviation over ten tests of 3.32 kN/m^2 and a coefficient of variation of 8.7%; e_0 varied between 0.87 and 0.92. For the strain-controlled tests the average stress at 15% strain was 32.91 kN/m^2, 14% lower than the deviator stress for the stress-controlled tests, varying from 28.00 kN/m^2 to 37.88 kN/m^2 over eight tests with a standard deviation of 3.26 kN/m^2 and a coefficient of variation of 9.9%; e_0 also varied from 0.87 to 0.92.

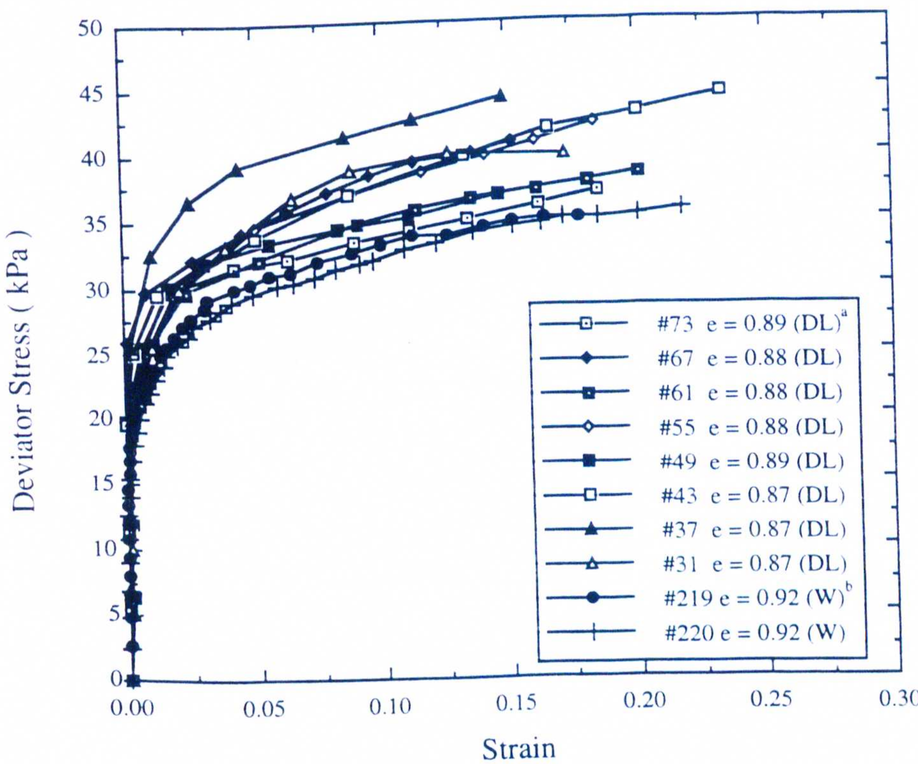


Fig. 4.1 - Stress Controlled Tests : MWSS18
Confining Pressure = 14 kPa; Loose Specimens

(a) DL = Dead Load
(b) W = Water Loading System

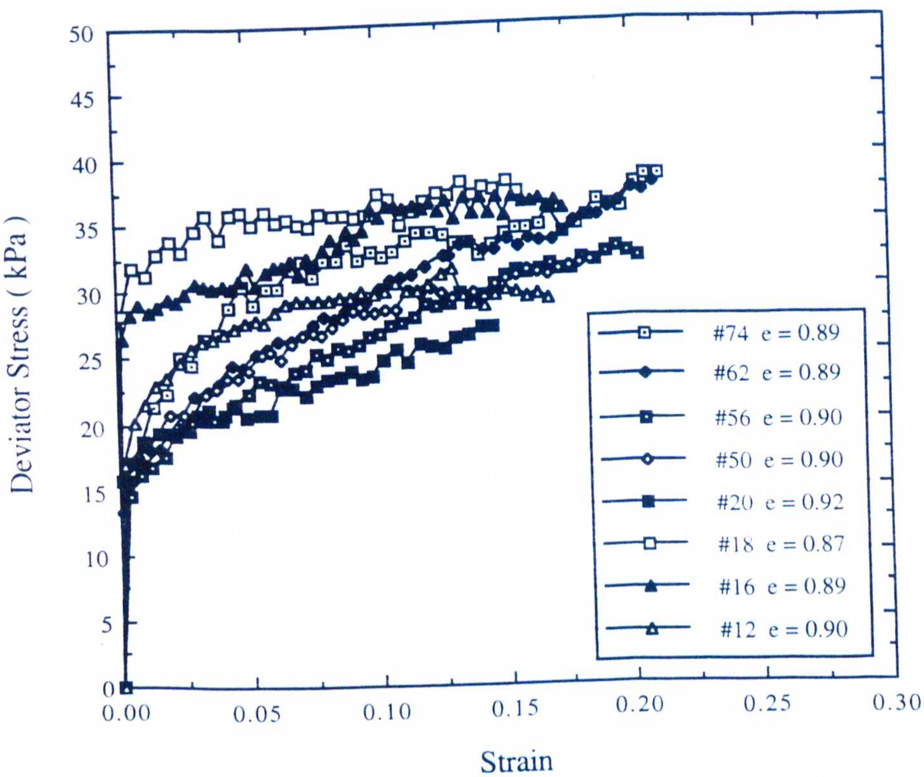


Fig. 4.2 - Strain Controlled Tests : MWSS18
Confining Pressure = 14 kPa; Loose Specimens

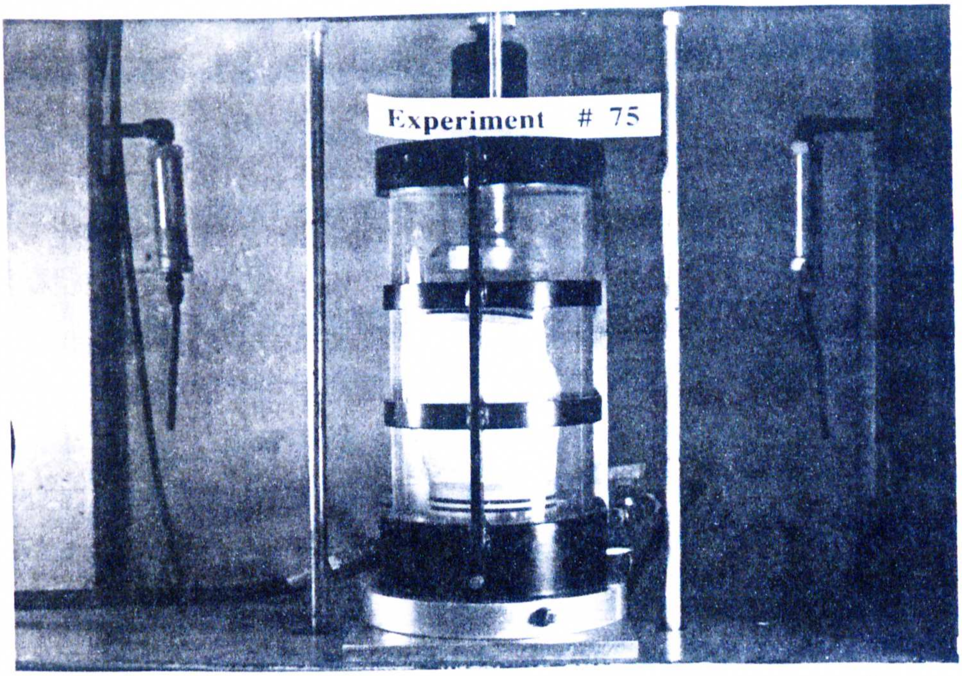


Fig. 4.3 - Specimen After Failure Under Stress Controlled Loading
(MWSS 18; Loose Specimen)

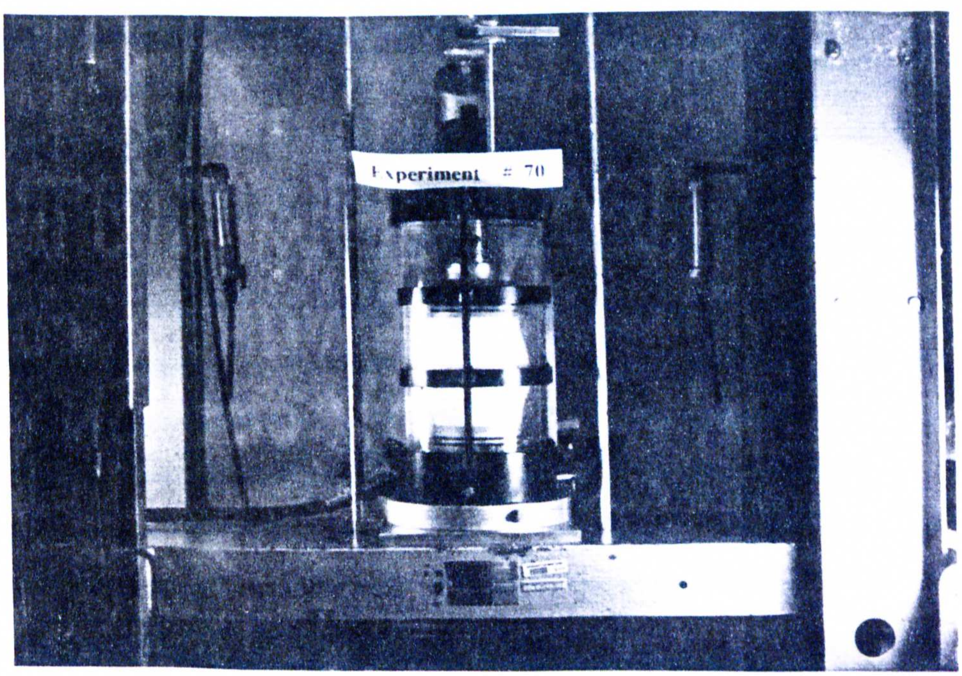


Fig. 4.4 - Specimen After Failure Under Strain Controlled Loading
(MWSS 18; Loose Specimen)

An examination of Figures 4.1 and 4.2 highlights the differences in stress-strain patterns of stress and strain controlled tests. Stress-controlled tests show very small strains, less than 1%, for stresses up to 20 kN/m^2. Even after 20 kN/m^2 the pattern of development of strain with increasing stress is similar for all specimens as they approach some asymptotic value. The data of strain-controlled tests also show very little strain during initial stressing, but this time up to 15 kN/m^2. There is greater dissimilarity between specimens in their stress-strain characteristics, and while strain increases dramatically, several specimens do not seem to be approaching an asymptotic value for stress, even at 15% strain. These relative behaviors are what might be expected for stress-controlled tests in which a build-up of stress with negligible inter-particle movement is followed by marked movements (or strain) at small subsequent stress increments and development of a more distinct failure plane. Results of the strain-controlled tests indicate, instead, less build-up of stress at particle contacts, but rather steady and controlled rearrangement of particles as further increments in stress are applied and more continuous development of barreling in the specimen.

Variation in void ratio between specimens may be expected to account for some variation in behavior. For specimens behaving lightly overconsolidated, as these are, this effect will be much less than for heavily overconsolidated specimens where strength, especially in granular soils, is extremely sensitive to small changes in void ratio. Nonetheless, to control for this effect one may plot deviator stress against void ratio over a small range of void ratios and this may be assumed to be a straight line over a small range of e, such as in this case. For stress-controlled tests, as shown in Figure 4.5, the relationship between the deviator stress (Y) and the void ratio (X) is Y = 142.27 - 117.09X, where the void ratio varied from 0.87 to 0.92. The regression analysis on the data for that straight line gives a coefficient of determination, R^2, of 53.50%, where 100% indicates a perfect fit. For the strain-controlled tests, as shown in Figure 4.6, a similar

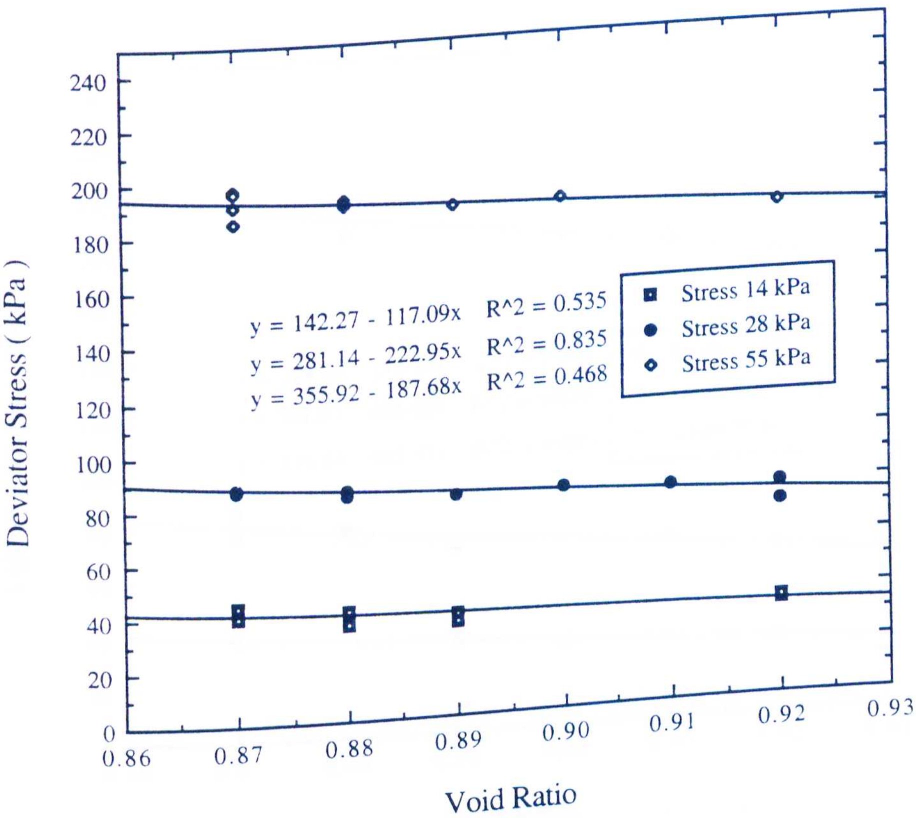


Fig. 4.5 - Deviator Stress Vs. Void Ratio
Stress Controlled Tests : MWSS18; Loose Specimens

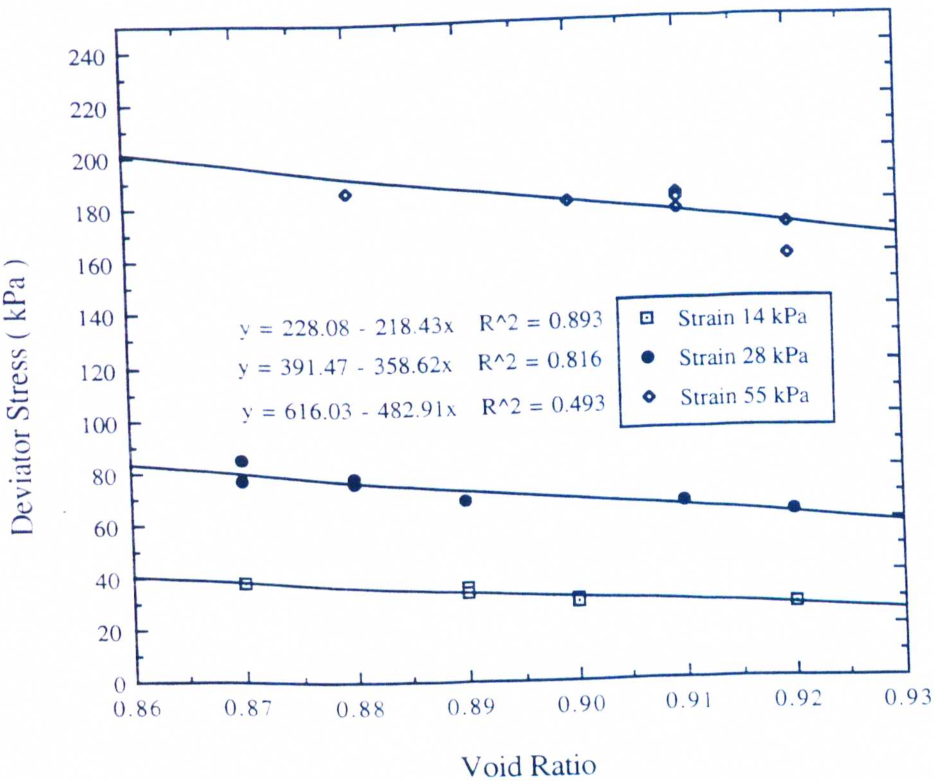


Fig. 4.6 - Deviator Stress Vs. Void Ratio
Strain Controlled Tests : MWSS18; Loose Specimens

relationship is observed between the deviator stress and the void ratio which varied from 0.87 to 0.92. The equation of the straight line is Y = 228.08 - 218.43X with R^2 of 89.30%. This shows a better fit of data for the strain-controlled tests but with slightly greater influence of void ratio on deviator stress reflected in the greater slope of the line. The large differences in Y intercept values are a result of differences in slope over the very narrow range of X (or e) in these tests and are not significant to the deviator stresses at 15% strain over these narrow ranges.

Statistical analysis of the results for loose specimens of MWSS18 tested at a confining stress of 14 kN/m², conducted using a standard error of estimate (see for example, McCuen, 1985), showed that error in deviator stress at failure from the results of three tests may be quite large varying from about 12 kN/m² to 23 kN/m². Six tests, however, showed a marked decline in the error in deviator stress, and so the eight tests per condition in this work, then, is very adequate to give an error in deviator stress of no more than 5 kN/m². These values are given in Table 4.9 and plotted in Figure 4.7. The technique is shown in the appendix.

4.3.2 STRESS AND STRAIN CONTROLLED TESTS AT 28 kN/m²

Figures 4.8 and 4.9 show plots of data of ten tests conducted on loose specimens of MWSS18 under stress-controlled conditions (eight tests with dead load and two tests with the water loading system), and eight tests under strain-controlled conditions with confining stress of 28 kN/m². The same narrow range of initial void ratios, 0.87 to 0.92 acceptable for the 14 kN/m² confining stress tests was achieved for the 28 kN/m² tests, and in this case there was also a full range of initial void ratios in tests of both stress and strain-controlled specimens. The stress-strain relationships followed the same pattern the 14 kN/m² confining stress

Table 4.9 Estimated Error as a Function of Sample Size for Deviator Stress as a Function of Void Ratio for MWSS18 Loose Specimens

No. of Experiment (N)	14 kPa Tests		28 kPa Tests		55 kPa Tests	
	Stress (kPa)	Strain (kPa)	Stress (kPa)	Strain (kPa)	Stress (kPa)	Strain (kPa)
3	23.135	11.931	13.279	34.443	40.875	70.338
8	2.728	1.407	1.566	4.062	4.821	8.297
12	2.208	1.046	1.164	3.020	3.584	6.167
40	1.100	0.567	0.631	1.637	1.943	3.343

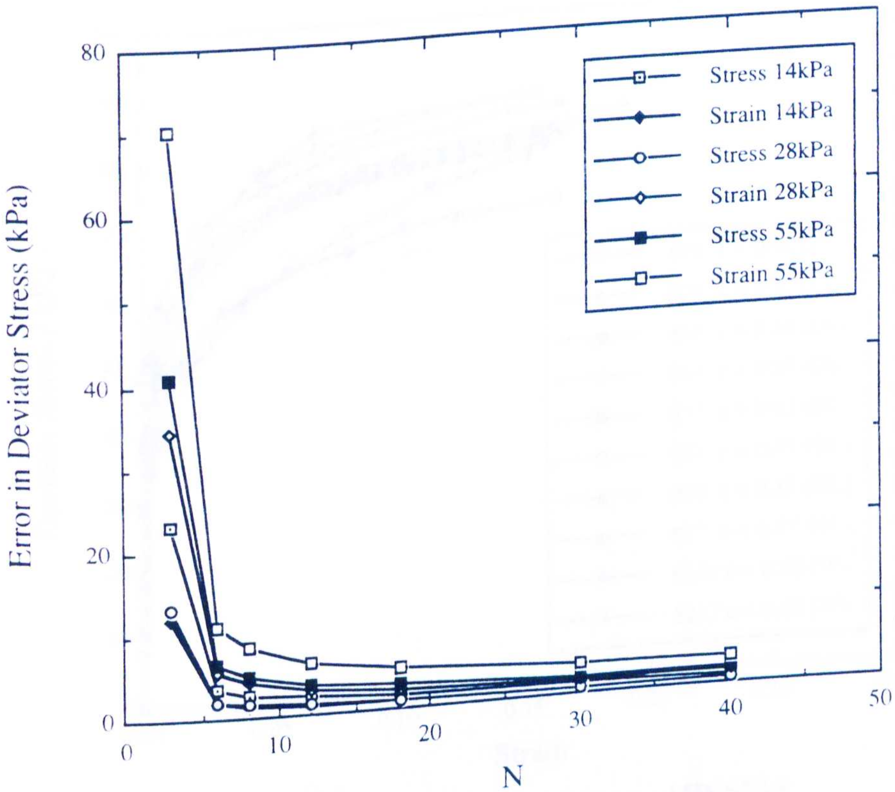


Fig. 4.7 - No. of Samples Vs. Error: MWSS18 Loose Specimens

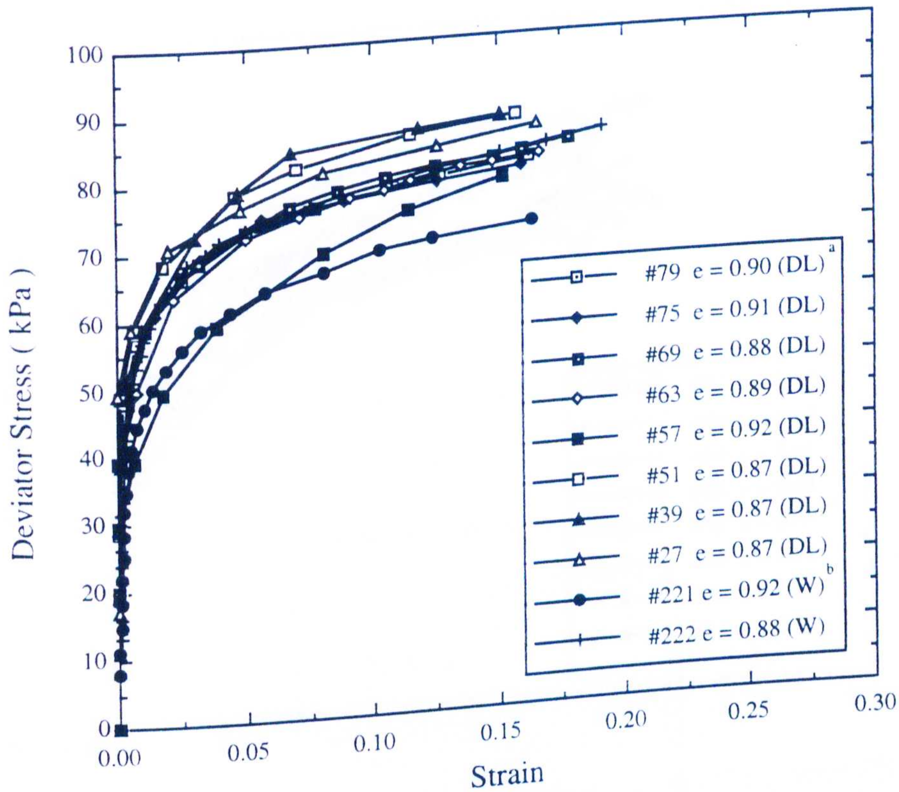


Fig. 4.8 - Stress Controlled Tests : MWSS18
Confining Pressure = 28 kPa; Loose Specimens

(a) DL = Dead Load
(b) W = Water Loading System

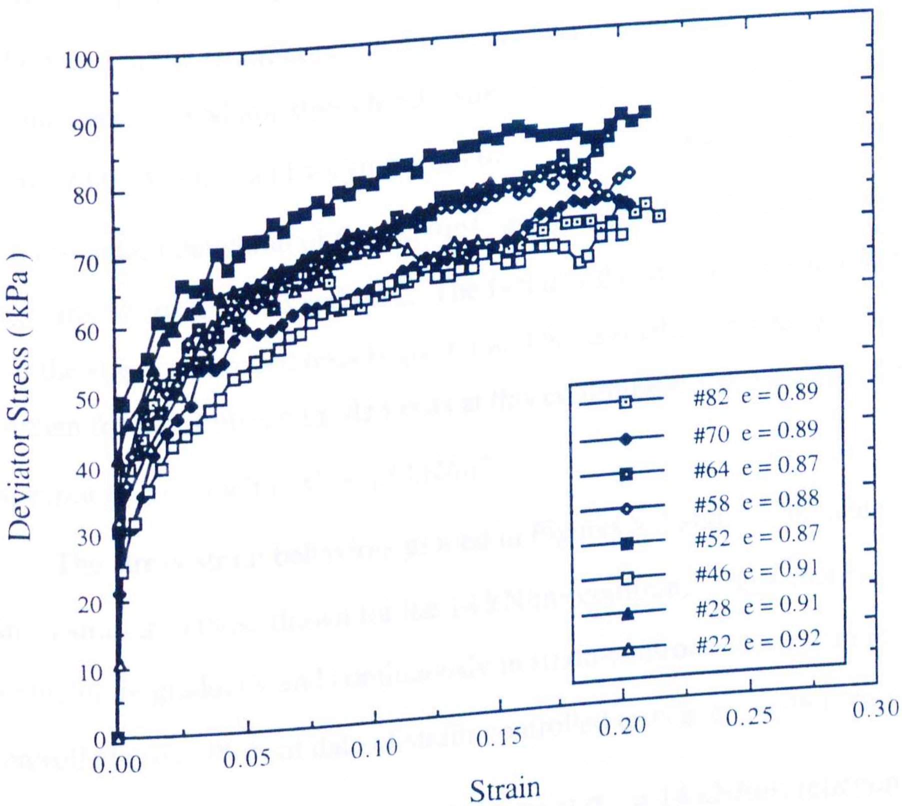


Fig. 4.9 - Strain Controlled Tests : MWSS18
Confining Pressure = 28 kPa; Loose Specimens

specimens did, but the curves are smoother. Final deviator stress was taken at 15% axial strain in all cases.

For stress-controlled tests the average deviator stress was 82.00 kN/m^2, varying from 71.62 kN/m^2 to 88.02 kN/m^2, which gives a standard deviation over ten tests of 4.85 kN/m^2 and a coefficient of variation of 5.9%; e_o varied between 0.87 and 0.92. For the strain-controlled tests the average deviator stress was again less than the average deviator stress for the stress-controlled tests, by approximately 11%, being 73.04 kN/m^2, and varying from 62.59 kN/m^2 to 84.42 kN/m^2 over 8 tests with a standard deviation of 6.94 kN/m^2 , and a coefficient of variation of 9.5%; e_o varied between 0.87 and 0.92. The fact that the average deviator stress at failure for the stress-controlled tests is greater and has a smaller coefficient of variation than for the strain-controlled tests at this confining stress, was also true for the specimens tested with σ'_c = 14 kN/m^2.

The stress-strain behaviors plotted in Figures 4.8 and 4.9, indicate conclusions similar to those drawn for the 14 kN/m^2 confining stress, that is that strain occurs more gradually and continuously in strain-controlled tests than in stress controlled tests. Plots of data of strain-controlled tests at σ'_c = 28 kN/m^2 are somewhat smoother than for strain-controlled tests at σ'_c = 14 kN/m^2, reflecting the influence of greater confinement.

Figures 4.5 and 4.6 give the equations of the straight line relationship between deviator stress (Y) and void ratio (X) for the two test conditions. For stress controlled tests, Y = 281.14 - 222.95X with R^2 = 83.5%, which is better than R^2 for stress-controlled tests conducted at σ'_c = 14 kN/m^2, and Y = 391.47 - 358.62X with R^2 = 81.6% for the strain-controlled tests, which is about the same as R^2 for $\sigma'_c = 14$ kN/m^2. Stress-controlled tests, then, give higher deviator stress at failure, and again void ratio shows less influence on that deviator stress, than in strain-controlled tests. Again, statistical analysis on these results shows a similar conclusion that eight tests produce results that are acceptable in terms of error (see Figure 4.7).

4.3.3 STRESS AND STRAIN CONTROLLED TESTS AT 55 kN/m^2

Figures 4.10 and 4.11 show plots of data of ten tests conducted on MWSS18 under stress-controlled conditions (eight with dead load stress increments and two with water applied stress increments), and eight tests conducted under strain-controlled conditions with confining stress of 55 kN/m^2. The acceptable range of the initial void ratios remained the same for the specimens tested with 55 kN/m^2 confining stress as it was for the specimens tested with 14 kN/m^2 and the 28 kN/m^2 confining stresses, being between 0.87 and 0.92, but initial void ratios for strain-controlled tests varied from 0.88 to 0.92. Again, tests were carried out to an axial strain value of 15% or more until failure occurred.

Examination of the stress-strain relationships for the 55 kN/m^2 confining stress illustrated in Figures 4.10 and 4.11 again indicate agreement with the trends observed in the two previous conclusions for the 14 kN/m^2 and 28 kN/m^2 confining stresses, regarding development of strain with stress. For the stress-controlled tests, the average deviator stress at 15% strain was 187.05 kN/m^2, varying from 176.97 kN/m^2 to 197.47 kN/m^2, which gives a standard deviation value of 9.77 kN/m^2 over ten tests with a coefficient of variation of 5.2% for an initial range of void ratios between 0.87 and 0.92. For the strain-controlled tests, the average deviator stress at 15% strain was 177.91 kN/m^2, approximately 5%

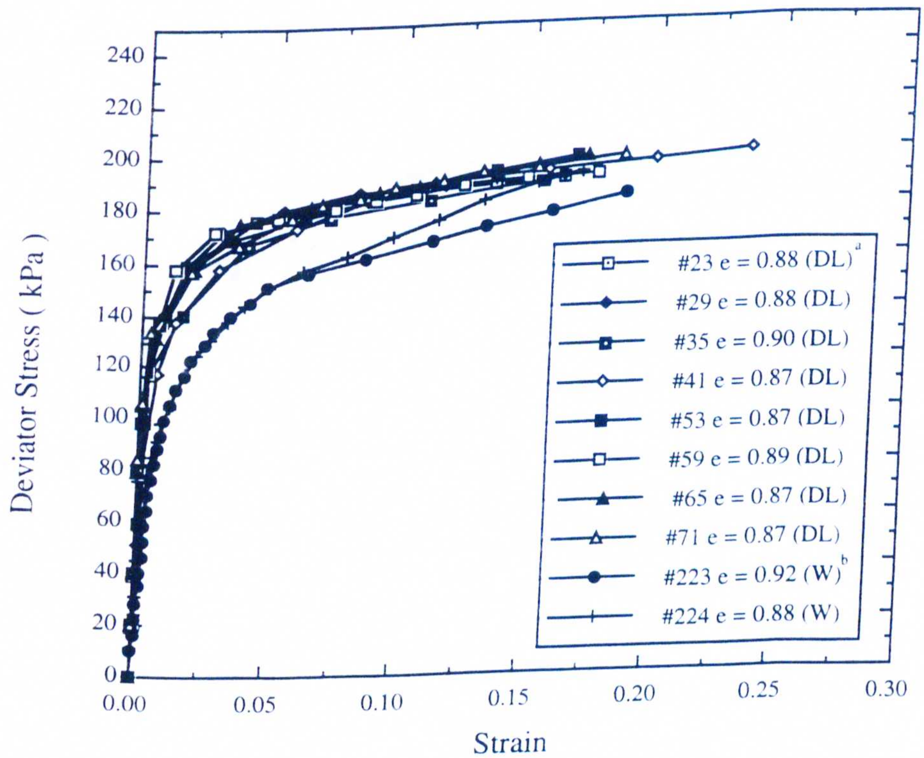


Fig. 4.10 - Stress Controlled Tests : MWSS18
Confining Pressure = 55 kPa; Loose Specimens

(a) DL = Dead Load

(b) W = Water Loading System

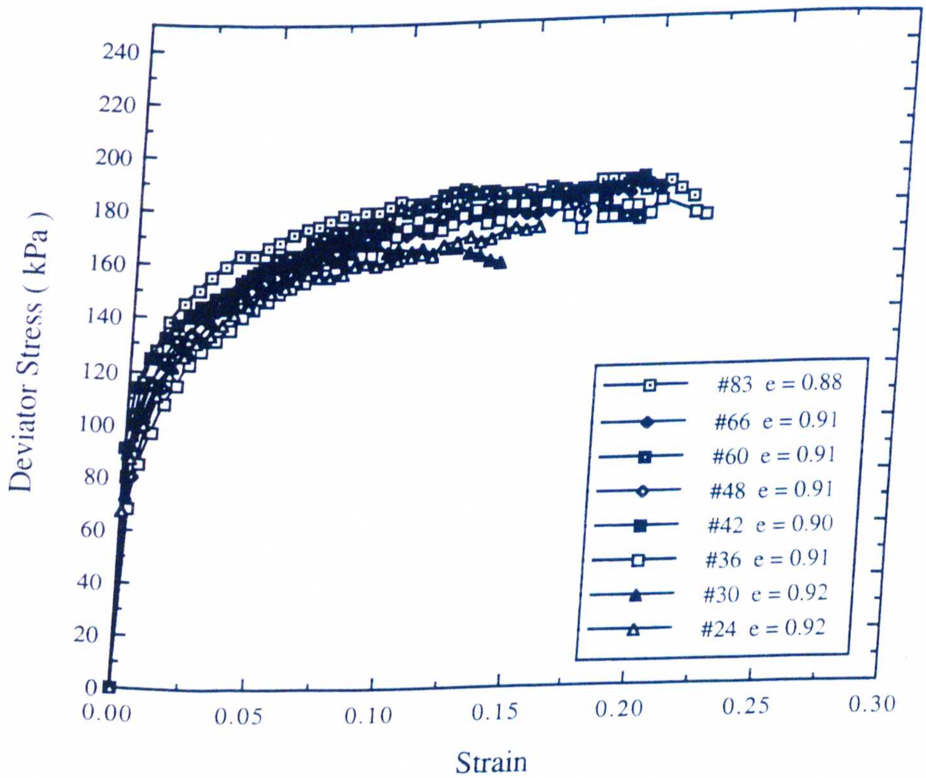


Fig. 4.11 - Strain Controlled Tests : MWSS18
Confining Pressure = 55 kPa; Loose Specimens

less than the average deviator stress for the stress-controlled tests, varying from 159.18 kN/m^2 to 186.53 kN/m^2 over eight tests with a standard deviation value of 9.80 kN/m^2 and a coefficient of variation of 5.5% over a similar range of initial void ratios, between 0.88 and 0.92. The fact that the average deviator stress at failure for stress-controlled tests is greater and has a smaller coefficient of variation than for strain-controlled tests at the same confining stress was also the case for tests with σ'_c equal to 14 kN/m^2 and 28 kN/m^2.

Figures 4.5 and 4.6 illustrate the relationship between the initial void ratio (X) and the deviator stress (Y) for these 55 kN/m^2 tests as a straight line. For the stress-controlled tests, Y = 355.92 - 187.68X with a coefficient of determination of R^2 = 46.8% , and Y = 616.03 - 482.91X with a coefficient of determination about the same, R^2 = 49.3%, for the strain-controlled tests. After eight tests, an error of 4.8 kN/m^2 is predicted for stress-controlled tests, and an error of 8.3 kN/m^2 for strain-controlled tests as shown in Figure 4.7. These values are summarized in Table 4.9. Again eight tests are acceptable.

Figures 4.12 and 4.13 show a sample of stress-strain plots at the three levels of confining stresses, 14 kN/m^2, 28 kN/m^2, 55 kN/m^2, all with e = 0.89. The final stress point shown in Figure 4.12 is the deviator stress immediately before sudden and complete failure occurred by the application of the next stress increment.

4.3.4 DEAD LOAD VERSUS WATER LOADING SYSTEM

Only loose specimens of MWSS18 were subjected to increments by dead loading in stress-controlled tests. When the applied stress reached 80% of the expected deviator stress at failure, the size of the stress increments was reduced to

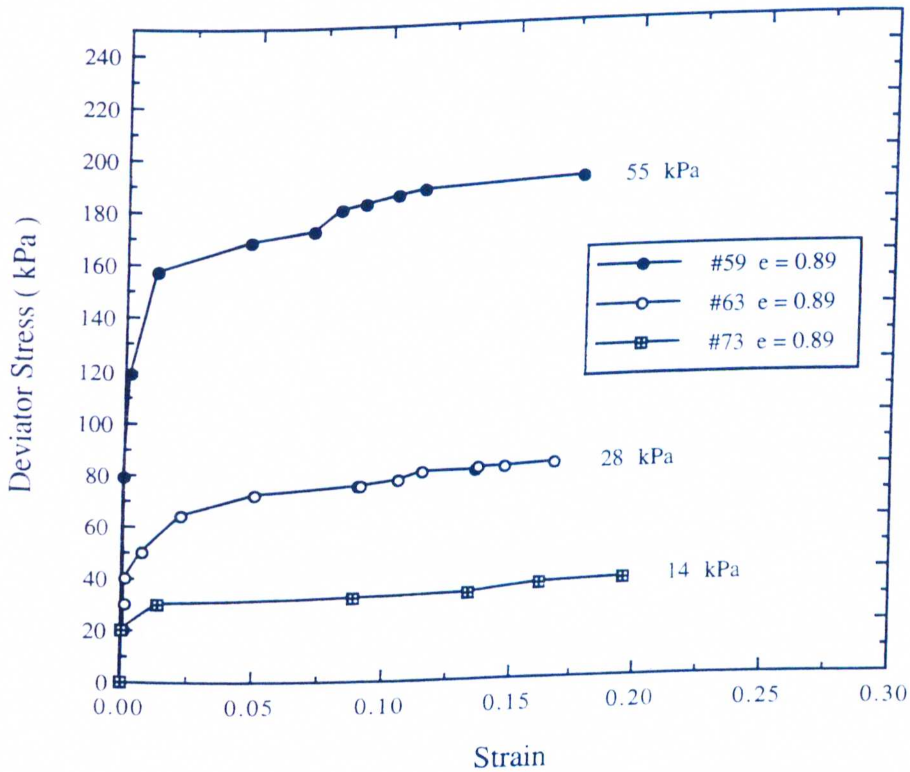


Fig. 4.12 - Deviator Stress Vs. Strain: Loose Specimens Stress Controlled Tests (Dead Load Application: MWSS18)

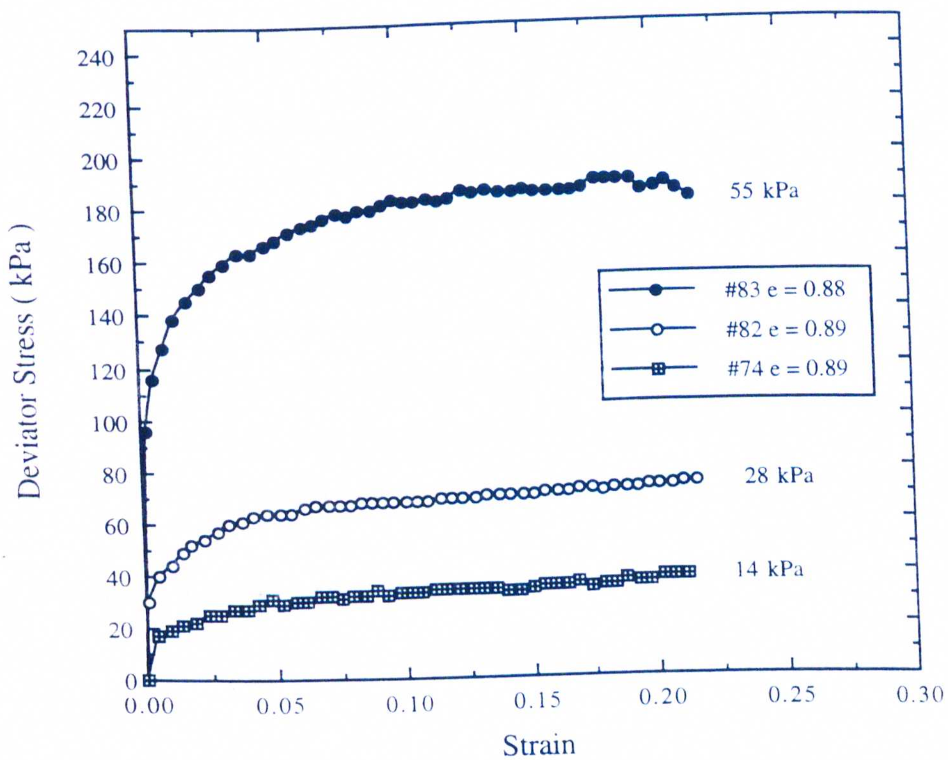


Fig. 4.13 - Deviator Stress Vs. Strain; Loose Specimens
Strain Controlled Tests : MWSS18

2% of the expected stress at failure. This meant, of course, that the second last increment added may have been just slightly less than the true failure stress, and the next and final increment may have exceeded by almost 2% the actual deviator stress necessary for failure. This immediately introduces a possibility for greater coefficient of variation, which the water loading system removes, and may have accounted for the poorer values of R^2 for those groups of tests when void ratio is controlled for in Figure 4.5.

Stress-strain plots of stress-controlled tests show the water loading system to give very reproducible stress-strain plots with final deviator stresses in the midst of these measured using the dead load system, although strains developed somewhat earlier and more continuously when a water loading system was introduced. The water loading system was adapted to provide a loading method easier to control, with more reproducible results for stress-controlled loading. It is therefore a recommended modification when stress-controlled loading is required.

4.4 MWSS45: LOOSE SPECIMENS

4.4.1 STRESS AND STRAIN CONTROLLED TESTS AT 14 kN/m²

Figures 4.14 and 4.15 show plots of data of eight tests conducted on the finer sand MWSS45, under stress-controlled conditions, and eight tests under strain-controlled conditions with confining stress of 14 kN/m^2. The stress-controlled tests were all conducted with the water loading method of deviator stress-controlled increments. A similar range of initial void ratio was achieved for MWSS45, between 0.91 and 0.98, for the 16 tests. This range of void ratios indicates loose particle packing, although the relative density is greater for this soil at this void ratio than for MWSS18 at the same void ratio.

The stress-strain relationships are broadly similar to the plots shown in Figures 4.1 and 4.2 obtained for MWSS18 with strain-controlled testing showing

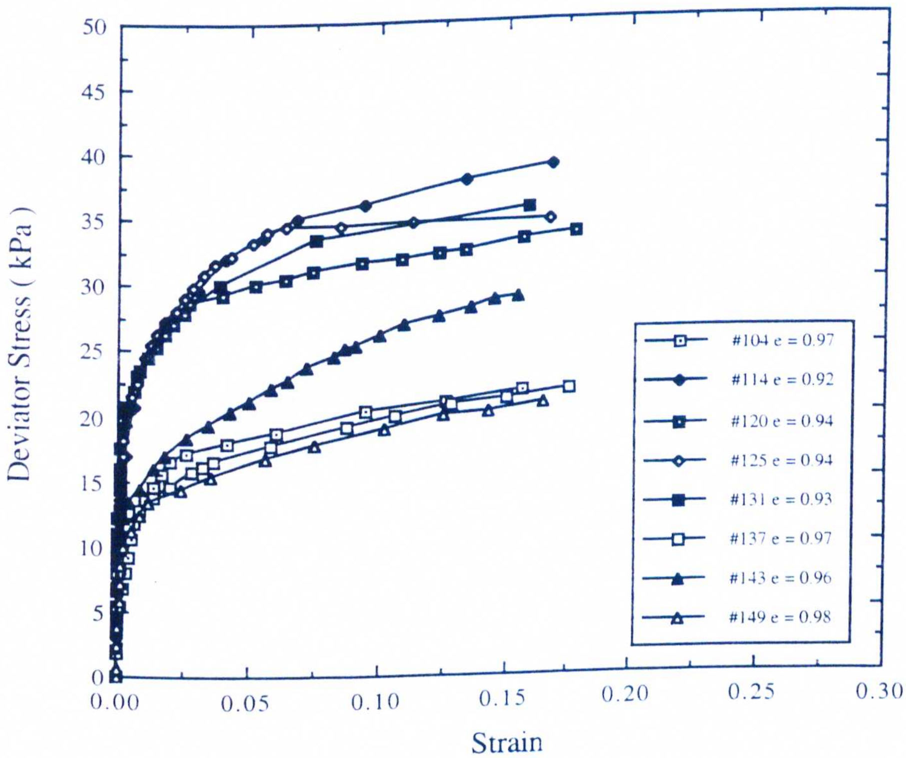


Fig. 4.14 - Stress Controlled Tests : MWSS45
Confining Pressure = 14 kPa; Loose Specimens

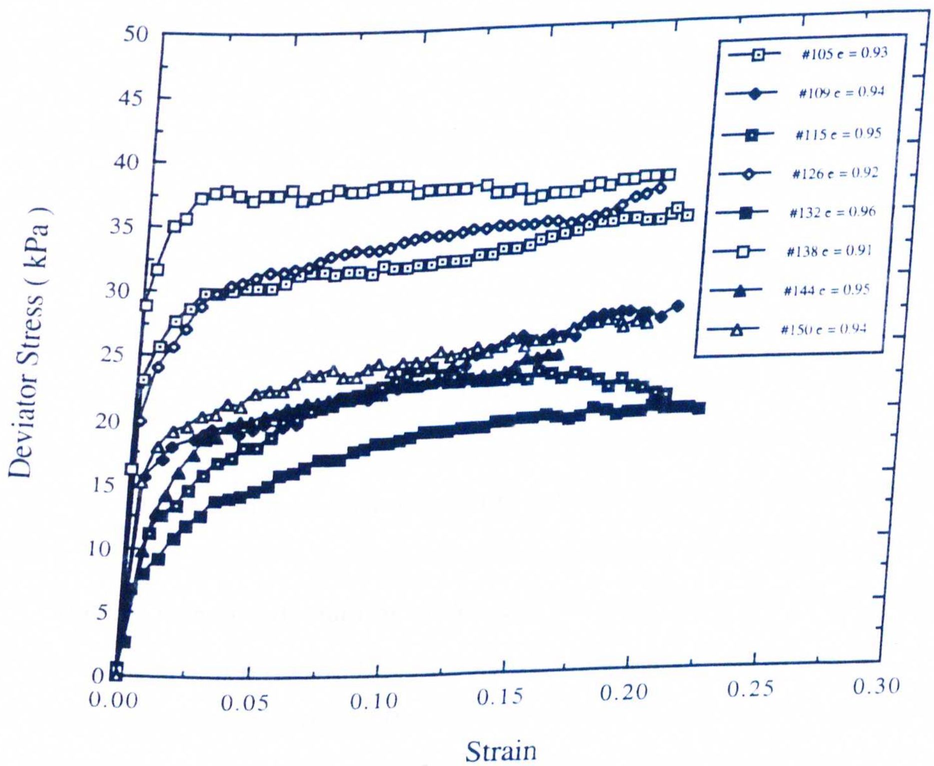


Fig. 4.15 - Strain Controlled Tests : MWSS45
Confining Pressure = 14 kPa; Loose Specimens

less uniformity from test to test in development of stress with strain than observed in stress-controlled testing. Development of failure was also similar to specimens of MWSS18, where the stress-controlled specimens developed a more distinct failure surface and the strain-controlled specimens bulged as vertical stress was increased.

Tables 4.3 and 4.4 show the average deviator stress at 15% strain for eight stress-controlled tests to be 29.21 kN/m^2 varying from 20.62 kN/m^2 to 38.31 kN/m^2, and standard deviation of 7.3 kN/m^2, which give a coefficient of variation of 24.9%; e_o varied from 0.92 to 0.98. For strain-controlled tests the average deviator stress was 27.83 kN/m^2, which is approximately 5% lower than the average of the stress-controlled tests over a range from 19.66 kN/m^2 to 36.71 kN/m^2, and a standard deviation of 6.23 kN/m^2 over eight tests, with a coefficient of variation of 22.4%; e_o varied over a range from 0.91 to 0.96.

The relative magnitudes of average deviator stresses at failure for stress and strain controlled testing is the same for loose specimens of MWSS18 and MWSS45, but the standard deviations of deviator stress at failure are much larger for MWSS45 than for MWSS18. For stress-controlled tests as shown in Figure 4.16 the relationship between the deviator stress (Y) and the void ratio (X) is $Y = 341.21 - 327.69X$, where the void ratio varied over a wide range from 0.92 to 0.98. The coefficient of determination, $R^2 = 96.0\%$, which is better than R^2 for stress-controlled tests for MWSS18. For strain controlled tests as shown in Figure 4.17 the straight line relationship between the deviator stress (Y) and the void ratio (X) is $Y = 392.15 - 387.12X$, for a narrower range of void ratio from 0.91 to 0.96, with $R^2 = 95.5\%$, which is almost the same as the R^2 for strain-controlled tests for MWSS18. These measures of repeatability and coefficient of determination are not contradictory, but rather show that for this soil, which is loose but still denser than MWSS18, deviator stress at failure is very sensitive to void

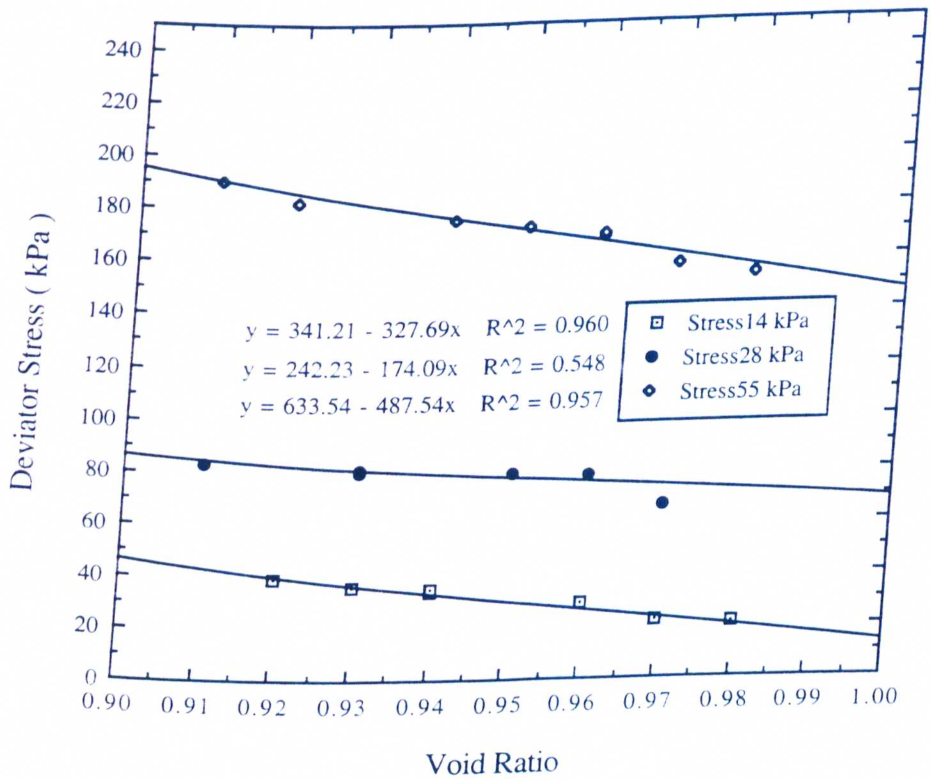


Fig. 4.16 - Deviator Stress Vs. Void Ratio
Stress Controlled Tests : MWSS45; Loose Specimens

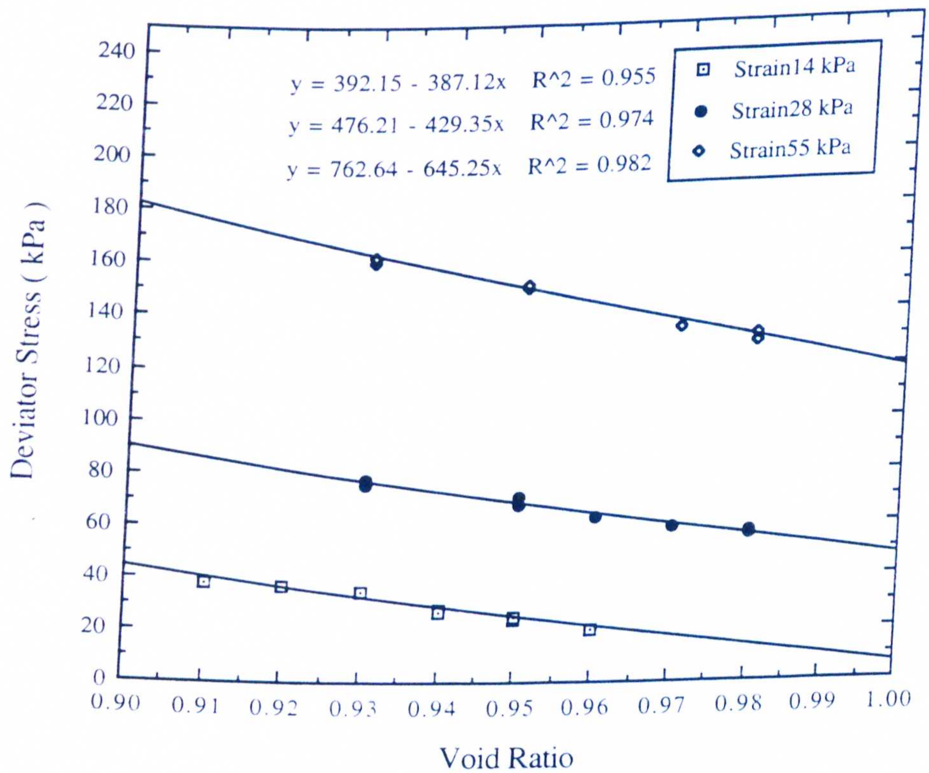


Fig. 4.17 - Deviator Stress Vs. Void Ratio
Strain Controlled Tests : MWSS45; Loose Specimens

ratio and repeatability is good when one controls for e. MWSS18, which was very loose, behaves like a lightly overconsolidated soil, for which one expects less sensitivity of deviator stress to e. This is typical for sands (see Atkinson and Bransby, 1978).

4.4.2 STRESS AND STRAIN CONTROLLED TESTS AT 28 kN/m^2

Figures 4.18 and 4.19 show plots of eight stress-controlled tests, and eight strain-controlled tests respectively, with confining pressure of 28 kN/m^2. The same narrow range of initial void ratios between 0.91 and 0.98 obtained for the 14 kN/m^2 tests was achieved for the tests with 28 kN/m^2 confining pressure. The 28 kN/m^2 tests produced smoother stress-strain plots than the same soil tested at a confining stress of 14 kN/m^2. Again, deviator stress was taken at 15% axial strain in all cases as shown in Tables 4.3 and 4.4.

For the eight stress-controlled tests the average deviator stress was 75.86 kN/m^2, varying from 65.88 kN/m^2 to 79.86 kN/m^2. The initial void ratios ranged from 0.91 to 0.97. The standard deviation over the eight tests was 4.48 kN/m^2, with a coefficient of variation of 5.9%. For the strain-controlled tests the average deviator stress over eight tests was approximately 17% lower than the average deviator stress for the stress-controlled tests being 62.57 kN/m^2 and varying over a wider range of stress, from 52.57 kN/m^2 to 73.16 kN/m^2. The standard deviation was 8.32 kN/m^2 with a coefficient of variation of 13.3%; e_o varied from 0.93 to 0.98.

Comparing the plots on Figures 4.18 and 4.19 gives the same conclusion that was made for tests conducted with 14 kN/m^2 confining pressure, that is the stress-controlled tests produced smoother stress-strain plots than

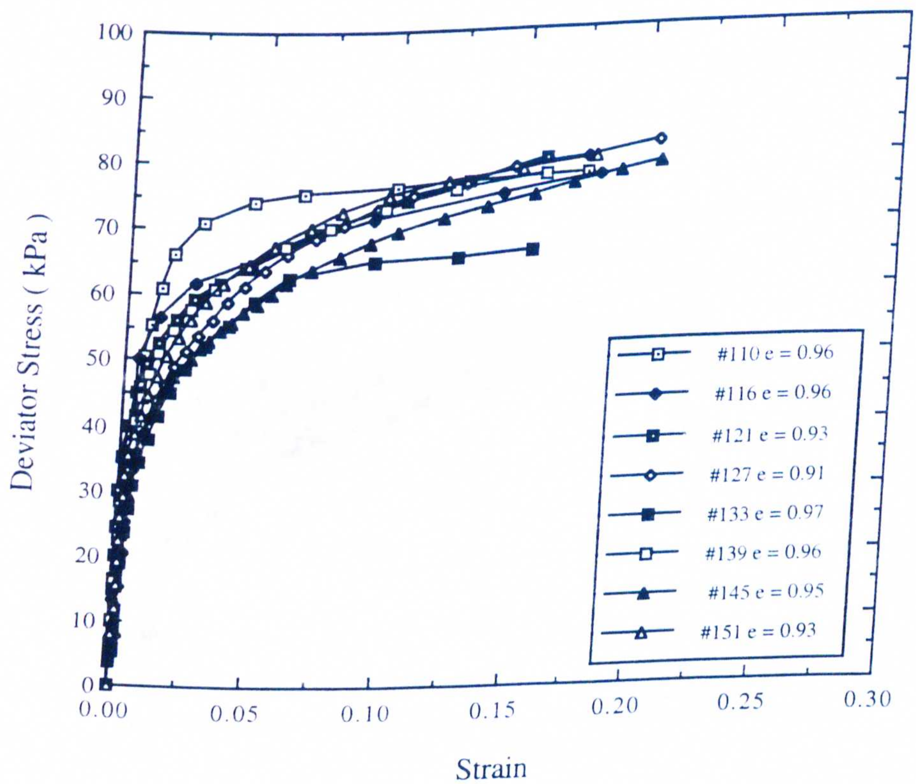


Fig. 4.18 - Stress Controlled Tests : MWSS45
Confining Pressure = 28 kPa; Loose Specimens

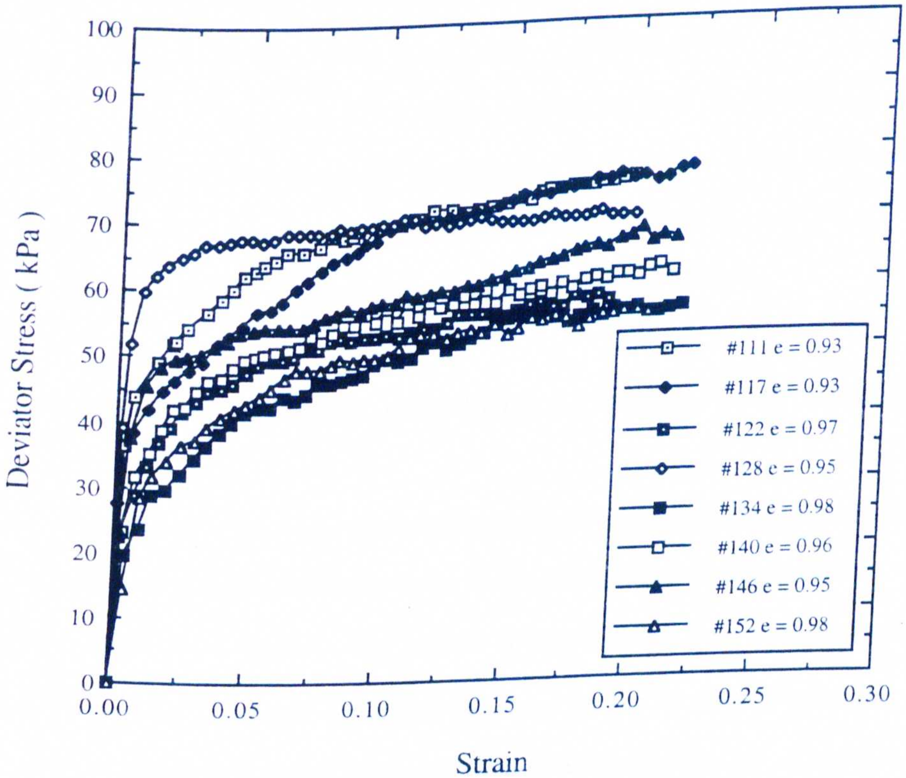


Fig. 4.19 - Strain Controlled Tests : MWSS45
Confining Pressure = 28 kPa; Loose Specimens

recorded in strain-controlled tests. This was not observed in the case for the 28 kN/m^2 confining pressure of MWSS18 (Figures 4.5 and 4.6), due to the method of loading as previously explained in section 4.3.4.

The relationship between the deviator stress (Y) and the void ratio (X) is shown as straight lines for both types of loading as shown in Figures 4.16 and 4.17. For the eight stress-controlled tests the linear equation is $Y = 242.23 - 174.09X$ with a poor coefficient of determination of 54.8%, which is approximately 34% lower than R^2 for stress-controlled tests for MWSS18 at the same confining stress. For the eight strain-controlled tests the linear equation is $Y = 476.21 - 429.35X$, with a much better coefficient of determination of 97.4%, which is 16% higher than R^2 for strain-controlled tests MWSS18.

4.4.3 STRESS AND STRAIN CONTROLLED TESTS AT 55 kN/m^2

Figures 4.20 and 4.21 show plots of data of eight tests conducted under stress-controlled conditions and eight tests under strain-controlled conditions with confining pressure of 55 kN/m^2. The narrow range of initial void ratios acceptable for the 14 kN/m^2 and 28 kN/m^2 remained the same for the 55 kN/m^2 confining pressure, and all test were carried out to an axial strain value of 15% or more.

The stress-strain relationships for the 55 kN/m^2 confining stress illustrated in Figures 4.20 and 4.21 again indicate a strong agreement with the trends observed in the conclusions for this soil tested at confining stresses of the 14 kN/m^2 and 28 kN/m^2 confining stresses, about development of strain with stress.

For the stress-controlled tests, the average deviator stress at 15% strain for eight tests was 168.64 kN/m^2, varying from 153.11 kN/m^2 to 182.33 kN/m^2,

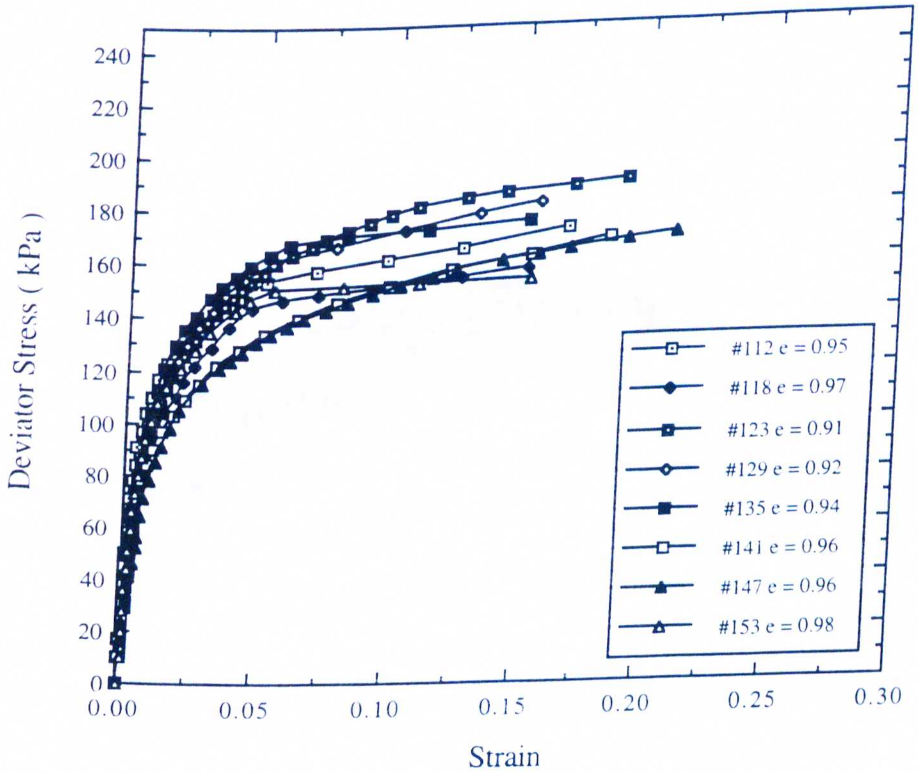


Fig. 4.20 - Stress Controlled Tests : MWSS45
Confining Pressure = 55 kPa; Loose Specimens

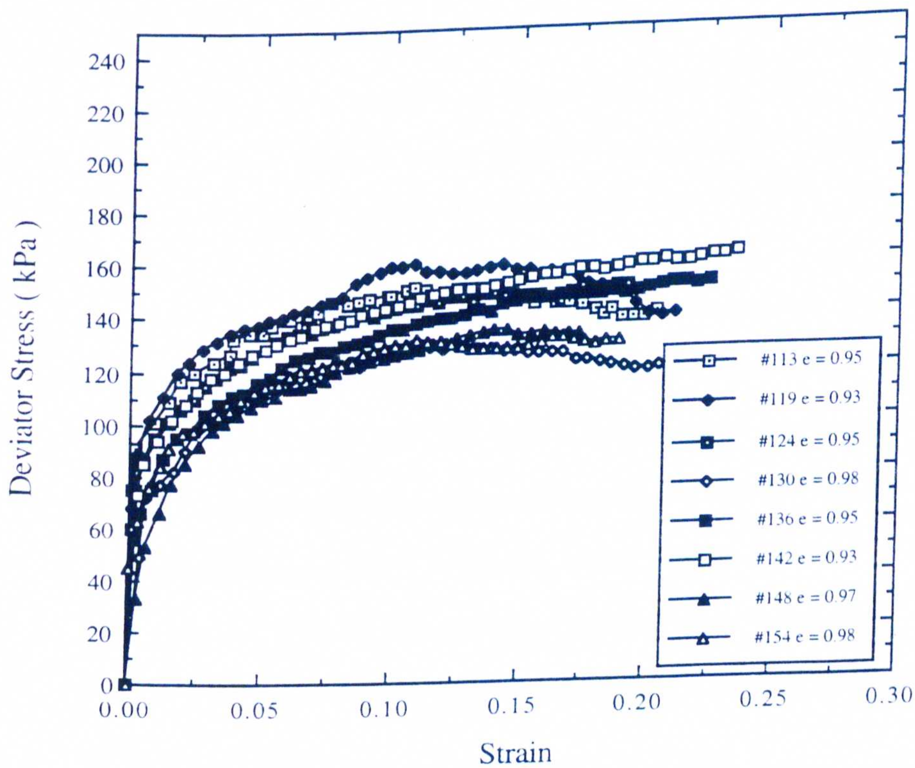


Fig. 4.21 - Strain Controlled Tests : MWSS45
Confining Pressure = 55 kPa; Loose Specimens

which gives a standard deviation value of 12.14 kN/m^2 and a coefficient of variation of 7.2% for an initial range of void ratios between 0.91 to 0.98. For the strain-controlled tests, the average deviator stress at 15% strain was 142.14 kN/m^2, approximately 16% lower than the average deviator stress for the stress-controlled tests, varying from 126.17 kN/m^2 to 157.22 kN/m^2 over eight tests with a standard deviation value of 11.80 kN/m^2 and a coefficient of variation of 8.3% over a narrower range of initial void ratios, between 0.93 and 0.98. The fact that the average deviator stress at failure for stress-controlled tests is greater and has a smaller coefficient of variation than for strain-controlled tests at the same confining stress here was the case also for MWSS18.

Figures 4.16 and 4.17 illustrate the straight line relationships between the initial void ratio (X) and the deviator stress (Y) for the 55 kN/m^2 tests. For the stress-controlled tests, $Y = 633.54 - 487.54X$ with a coefficient of determination of $R^2 = 95.7\%$, being 51% higher than stress-controlled tests for MWSS18 at the same level of confining pressure, and $Y = 762.64 - 645.25X$ with a better coefficient of determination, $R^2 = 98.2\%$, for the strain-controlled tests, also approximately being 51% higher than for strain-controlled tests for MWSS18 at the same level of confining pressure.

Figures 4.22 and 4.23 show a sample of stress-strain plots at the three levels of confining pressures, 14 kN/m^2, 28 kN/m^2, 55 kN/m^2.

4.5 MWSS18: DENSE SPECIMENS

4.5.1 STRESS AND STRAIN CONTROLLED TESTS AT 14 kN/m^2

The second phase of the testing program was to examine the effects of the type of triaxial loading on dense specimens for the same sands. Here, two

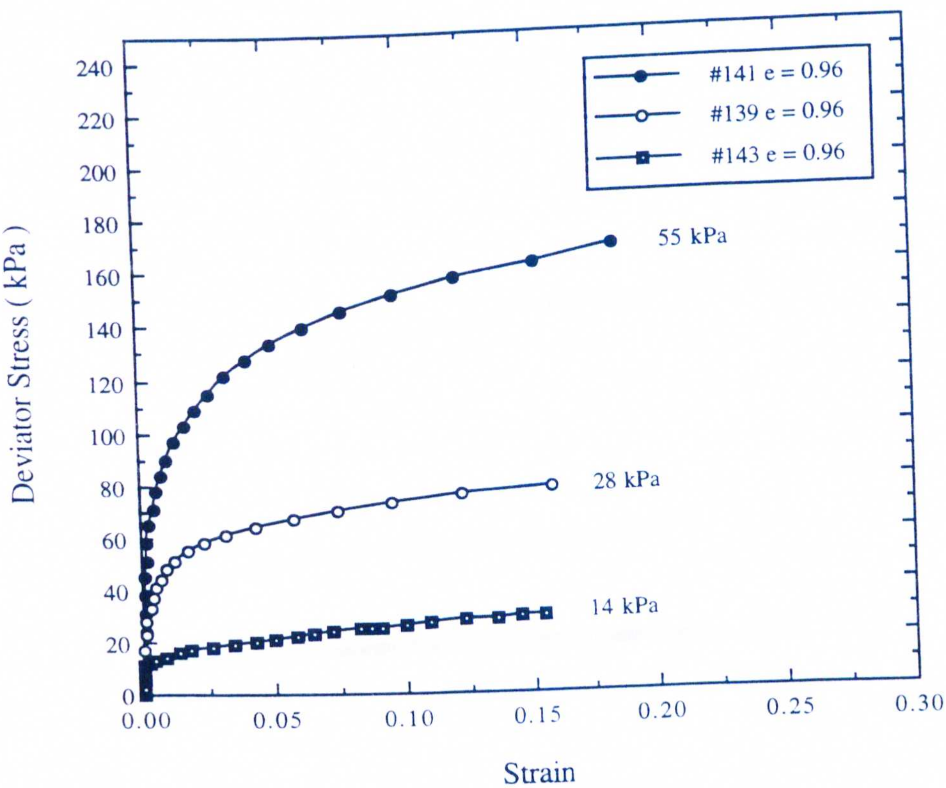


Fig. 4.22 - Deviator Stress Vs. Strain; Loose Specimens Stress Controlled Tests (Water System Loading: MWSS45)

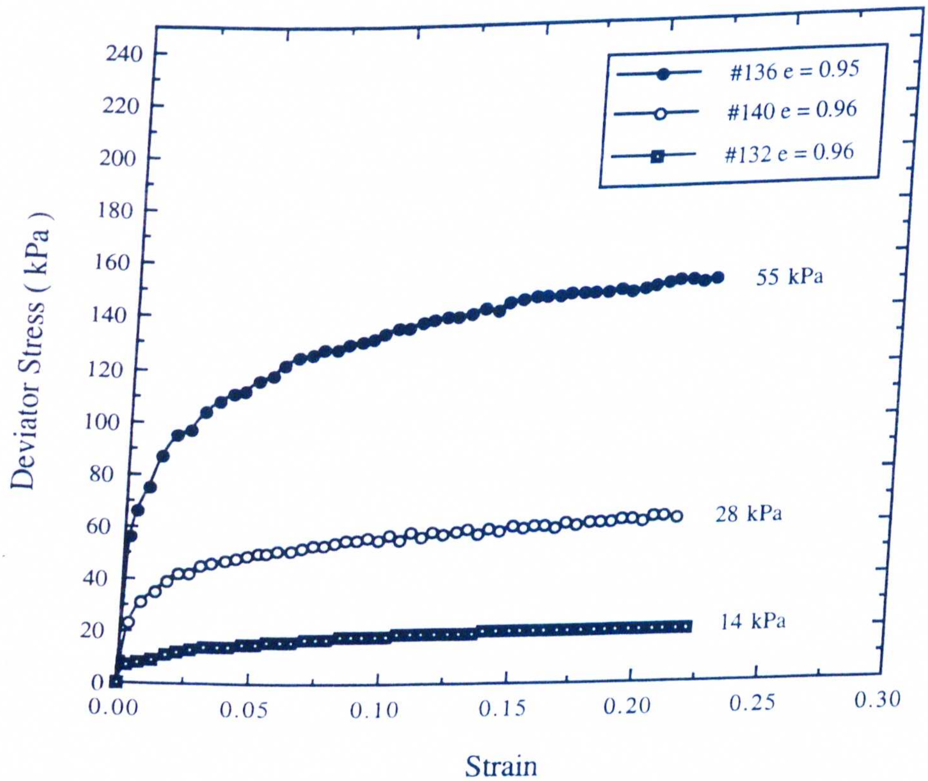


Fig. 4.23 - Deviator Stress Vs. Strain; Loose Specimens
Strain Controlled Tests : MWSS45

levels of confining pressures were used, 14 kN/m^2 and 55 kN/m^2.

Figures 4.24 and 4.25 show plots of eight stress-controlled tests, and eight strain-controlled tests respectively, with confining pressure of 14 kN/m^2 for MWSS18.

The range of initial void ratios was between 0.69 and 0.74 for the 16 tests. This range shows the sand to be in a dense state, having relative densities between 71% and 100%. All stress-controlled specimens showed failure before 5% strain had occurred, and most before 3%. All strain-controlled tests showed peak strength occurring at 5% strain or less. Post/peak deviator stress can only be recorded for the strain-controlled tests and was done at 15% strain.

For stress-controlled tests the average deviator stress at failure was 41.06 kN/m^2, which is approximately 7% higher than the deviator stress (38.30 kN/m^2) for stress-controlled tests on loose specimens of MWSS18 at the same level of confining pressure. Deviator stresses for stress-controlled tests varied over a very narrow range from 36.95 kN/m^2 to 43.70 kN/m^2, which gives a standard deviation over eight tests of 2.18 kN/m^2 and a coefficient of variation of 5.3%; e_o varied between 0.69 and 0.74. For strain-controlled tests the deviator stress at peak ($\leq$ 5% strain) varied from 32.44 kN/m^2 to 42.62 kN/m^2 with average value of 38.91 kN/m^2, 5% lower than the average deviator stress for stress-controlled tests, and a standard deviation value of 3.65 kN/m^2 and coefficient of variation of 9.4%. Void ratios were calculated to vary between 0.69 and 0.74. At 15% axial strain the average deviator stress was 33.25 kN/m^2, which is approximately 1% higher than the deviator stress for strain-controlled tests at the same level of confining pressure on loose specimens of MWSS18 and 19% lower than the average deviator stress for stress-controlled MWSS18 dense specimens. Post/peak deviator stress varied from 25.97 kN/m^2 to 37.69 kN/m^2, with a

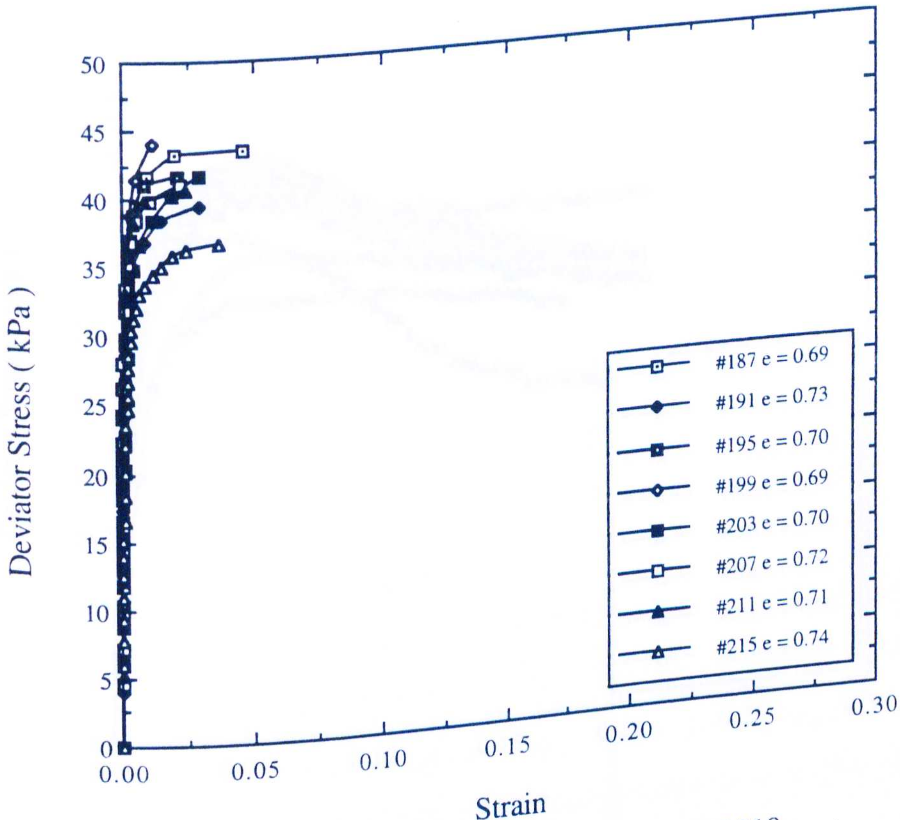


Fig. 4.24 - Stress Controlled Tests : MWSS18
Confining Pressure = 14 kPa; Dense Specimens

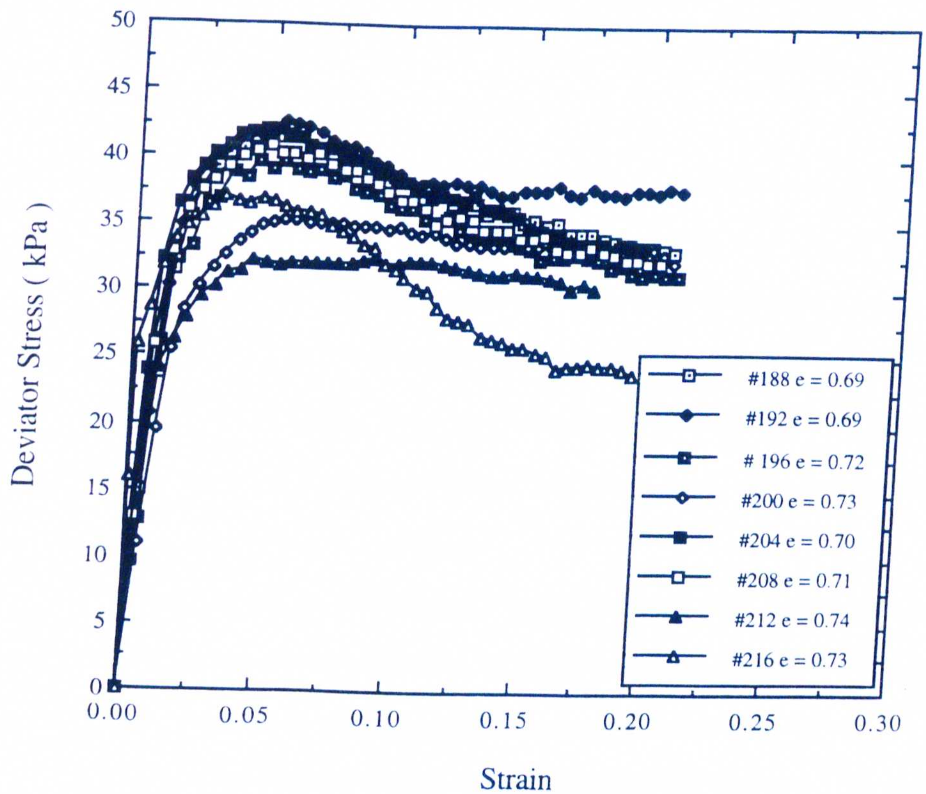


Fig. 4.25 - Strain Controlled Tests : MWSS18
Confining Pressure = 14 kPa; Dense Specimens

standard deviation value of 3.47 kN/m^2 and a coefficient of variation of 10.4%.

Figures 4.26 and 4.27a give the linear equations for stress-controlled tests at failure as Y = 120.43 - 111.79X, with R^2 = 90.1%, and Y = 164.31 - 175.69X, with a lower coefficient of determination, R^2 = 85.8% for strain-controlled tests at peak. For the post/peak behavior (at 15% strain) the linear equation is Y = 129.09 - 134.28X, with R^2 = 55.2% (Figure 4.27b).

4.5.2 STRESS AND STRAIN CONTROLLED TESTS AT 55 kN/m^2

Figures 4.28 and 4.29 show plots of eight tests conducted under stress-controlled conditions and eight tests under strain-controlled conditions with confining pressure of 55 kN/m^2. The narrow range acceptable for the 14 kN/m^2 confining pressure remained the same for the 55 kN/m^2. Also, all test were carried out to an axial strain value of 15% or more.

The stress-strain relationship in Figures 4.28 and 4.29 indicate a strong agreement with the trends observed for the 14 kN/m^2 confining pressure, regarding the development of strain with stress and the failure patterns development.

For the stress-controlled tests, the average deviator stress at 15% axial strain for eight tests was 216.95 kN/m^2, varying from 201.04 kN/m^2 to 237.14 kN/m^2, which is approximately 14% higher than the average deviator stress at 15% strain for MWSS18 loose specimens at the same level of confining stress. The standard deviation was 12.15 kN/m^2, with a coefficient of variation value of 5.6% for an initial void ratios values between 0.69 and 0.74. For the strain-controlled tests, the average deviator stress at peak ($\leq$ 5% axial strain) was 211.08 kN/m^2 which is 3% lower than the peak for the stress-controlled tests of the same sand. At peak, the deviator stress varied from 190.63 kN/m^2 to 226.30 kN/m^2.

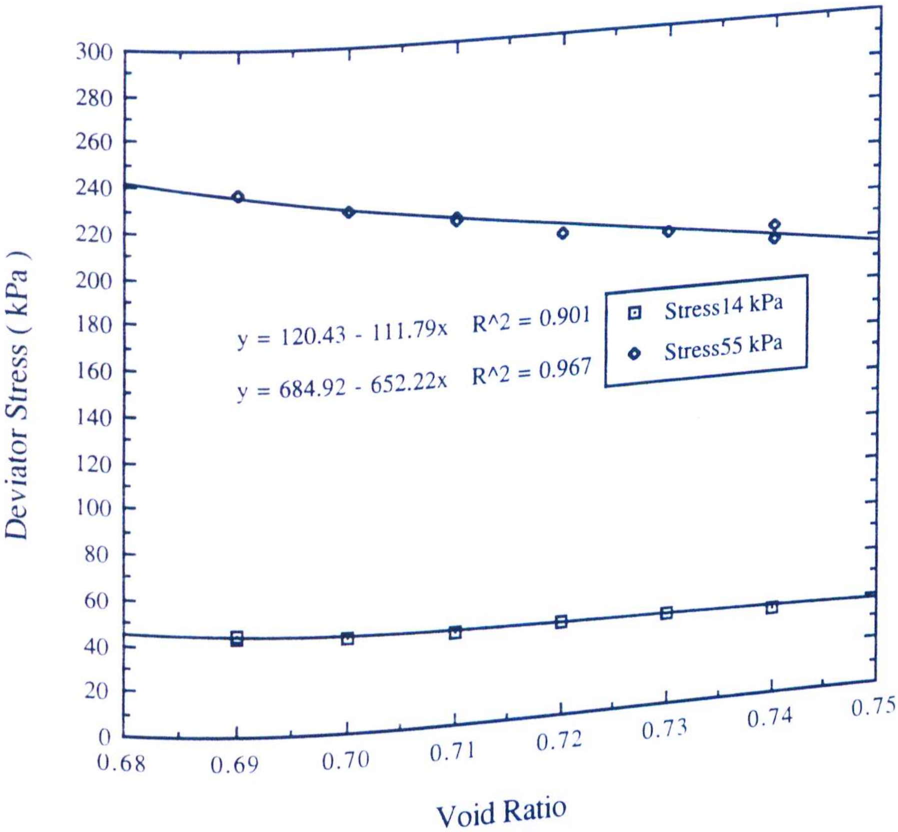


Fig. 4.26 - Deviator Stress Vs. Void Ratio
Stress Controlled Tests : MWSS18; Dense Specimens

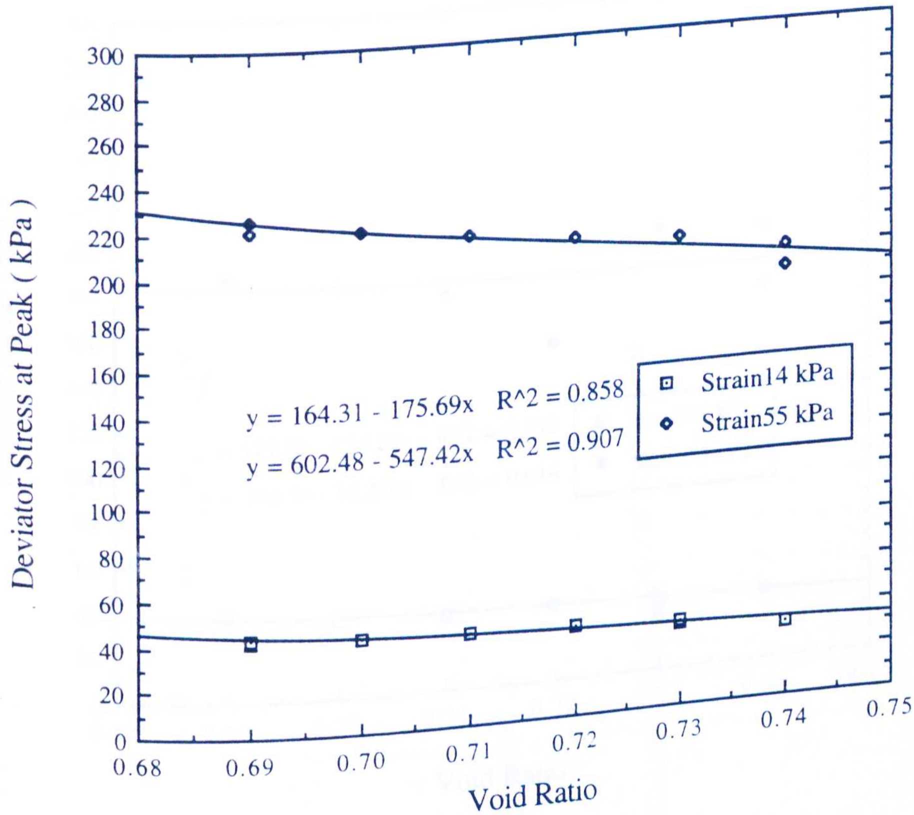


Fig. 4.27a - Deviator Stress Vs. Void Ratio
Strain Controlled Tests: MWSS18: Dense Specimens at Peak

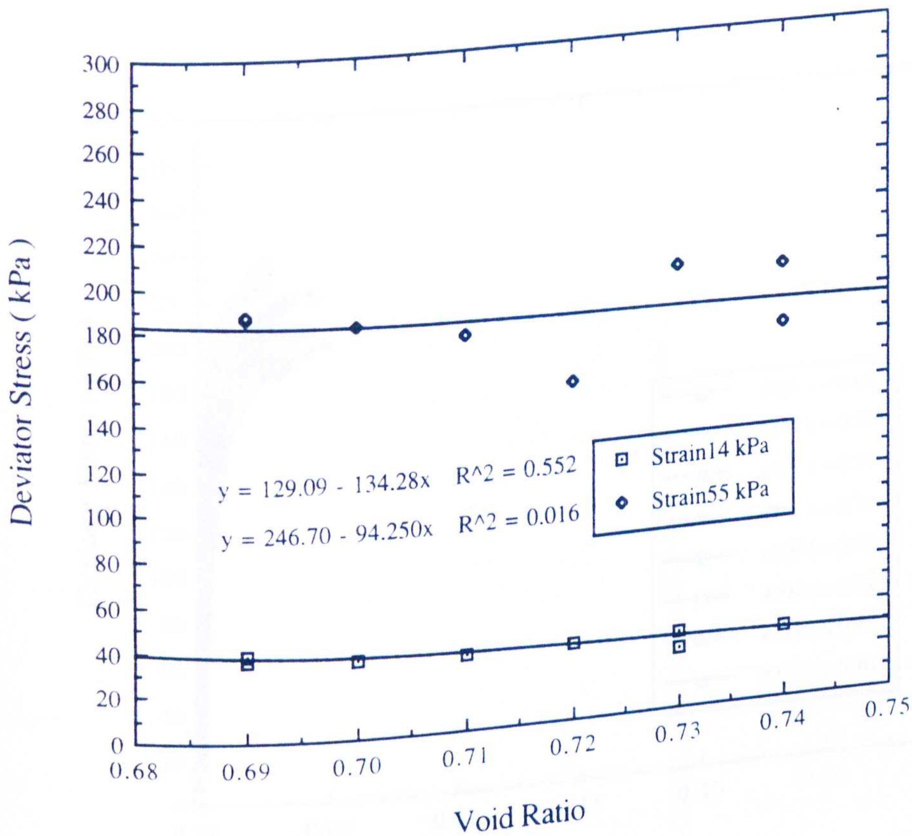


Fig. 4.27b - Deviator Stress Vs. Void Ratio
Strain Controlled Tests : MWSS18; Dense Specimens at 15% Strain

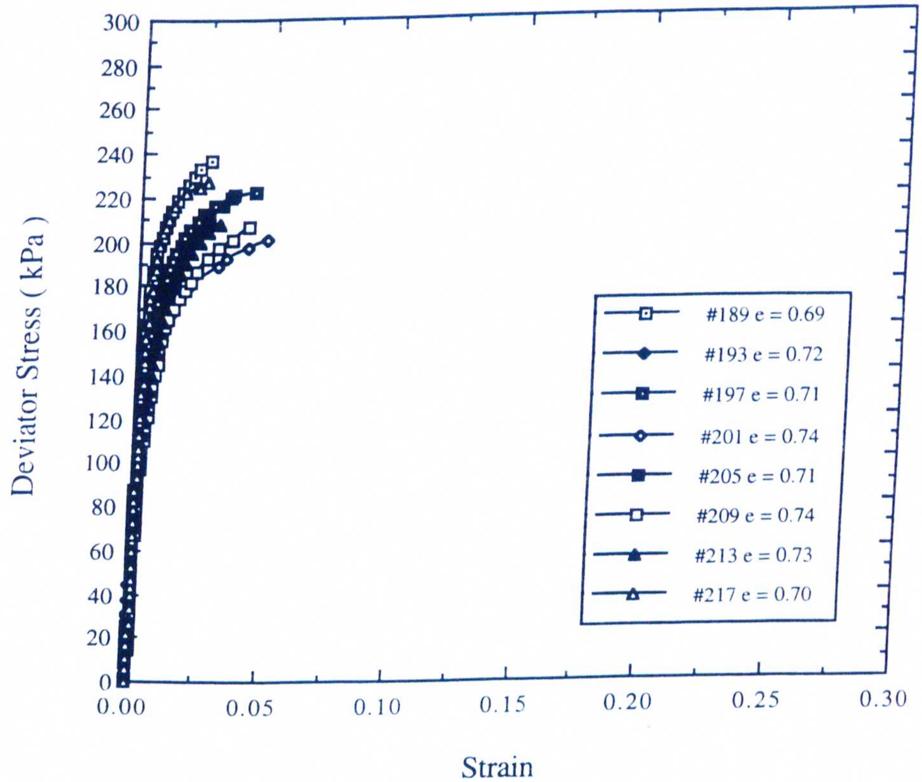


Fig. 4.28 - Stress Controlled Tests : MWSS18
Confining Pressure = 55 kPa; Dense Specimens

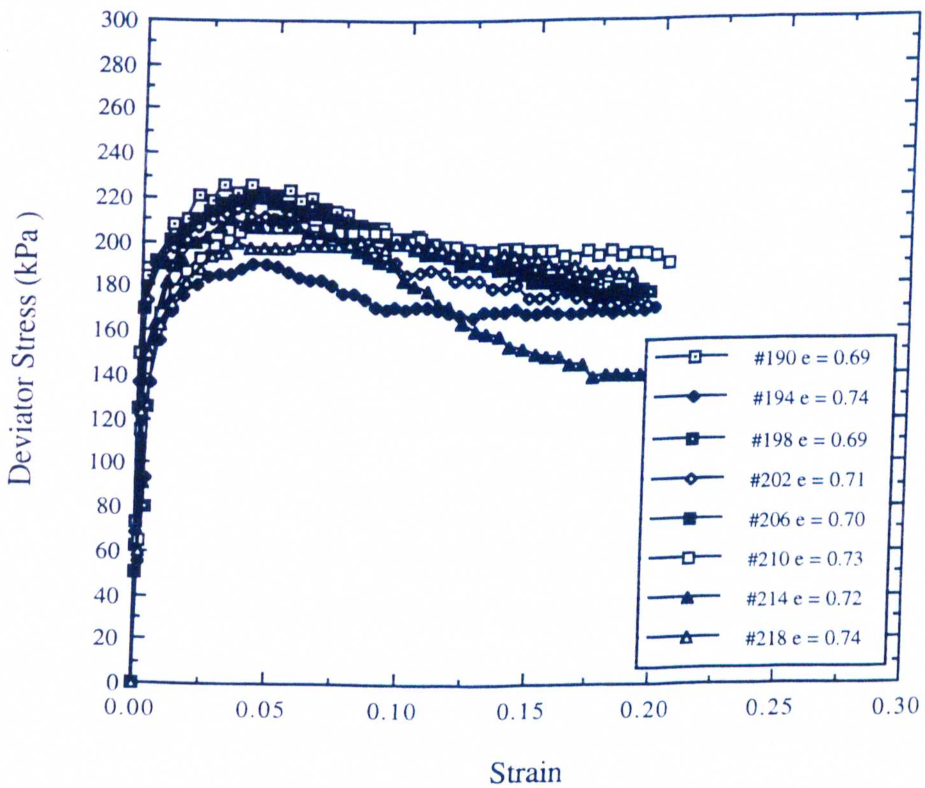


Fig. 4.29 - Strain Controlled Tests : MWSS18
Confining Pressure = 55 kPa; Dense Specimens

The standard deviation was 11.89 kN/m^2 and the coefficient of variation was 5.7%. At 15% axial strain the average deviator stress was less than 1% higher than the average of MWSS18 loose specimens at the same level of confining pressure, being 179.40 kN/m^2, and 17% lower than the average deviator stress at peak for dense specimens of MWSS18. The range in deviator stress was from 149.51 kN/m^2 to 192.20 kN/m^2. The standard deviation for the eight tests was 15.28 kN/m^2, with a coefficient of variation value of 8.5%. The fact that the average deviator stress at failure for stress-controlled tests is greater and has a smaller coefficient of variation than for strain-controlled tests at the same confining stress was also noted the case for tests with σ'_c equal to 14 kN/m^2 for both peak and post/peak behavior.

Figures 4.26 and 4.27a illustrate the linear relationship over this small range of void ratios. For stress-controlled tests at failure Y = 684.92 - 652.22X with a coefficient of determination, R^2 = 96.7%, and Y = 602.48 - 547.42X with a close value of the coefficient of determination, R^2 = 90.7%, over a full range of initial void ratios between 0.69 to 0.74 for strain-controlled tests at peak. At 15% strain, Y = 246.70 - 94.25X, with much lower value for R^2 = 1.6%.

Figures 4.30 and 4.31 show a sample of stress-strain plots at the two levels of confining pressures, 14 kN/m^2, 55 kN/m^2. Also, as was the case with MWSS18 and MWSS45 loose specimens, the final stress point shown in Figure 4.30 is the last deviator stress before sudden and complete failure caused by the next increment.

4.6 MWSS45: DENSE SPECIMENS

4.6.1 STRESS AND STRAIN CONTROLLED TESTS AT 14 kN/m^2

Figures 4.32 and 4.33 show plots of data of eight tests that were

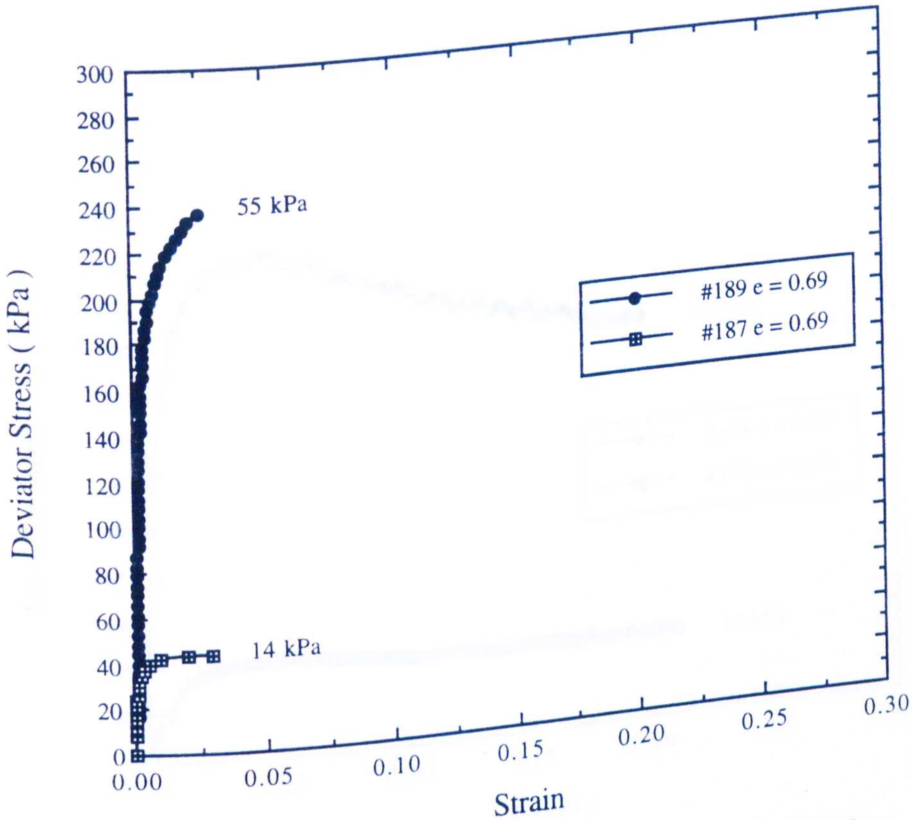


Fig. 4.30 - Deviator Stress Vs. Strain; Dense Specimens
Stress Controlled Tests (Water System Loading: MWSS18)

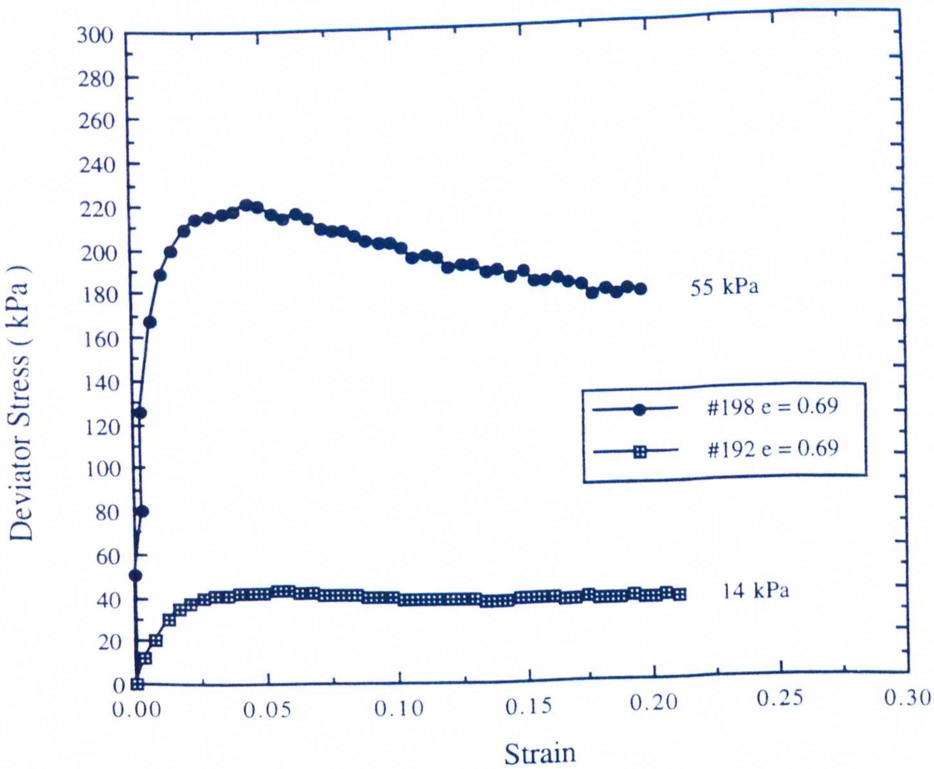


Fig. 4.31 - Deviator Stress Vs. Strain; Dense Specimens
Strain Controlled Tests : MWSS18

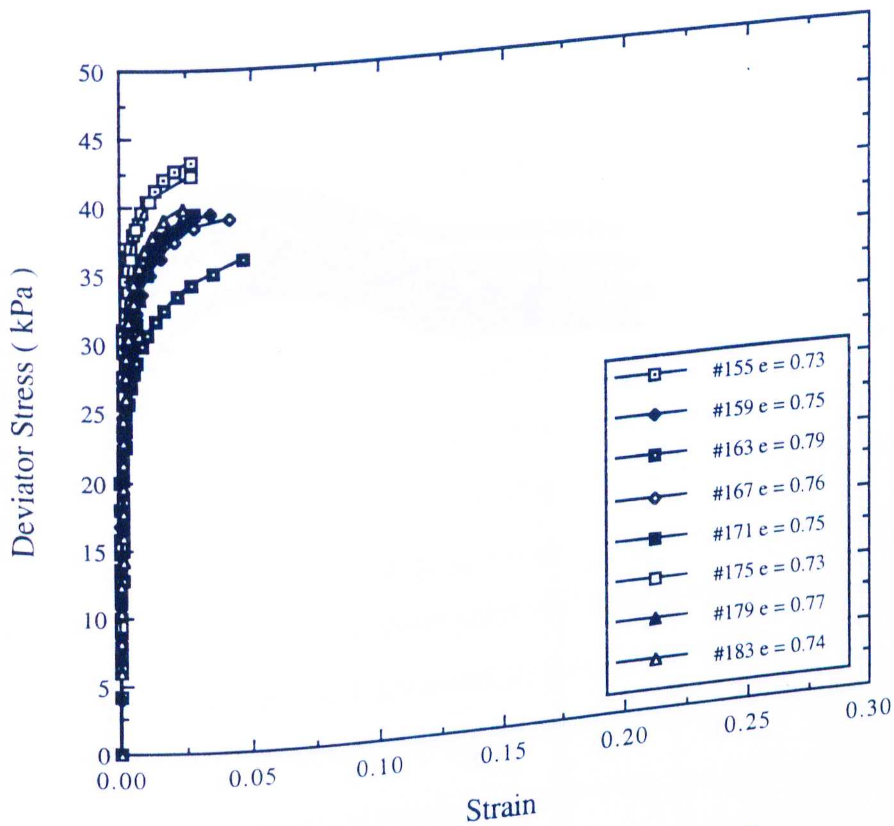


Fig. 4.32 - Stress Controlled Tests : MWSS45
Confining Pressure = 14 kPa; Dense Specimens

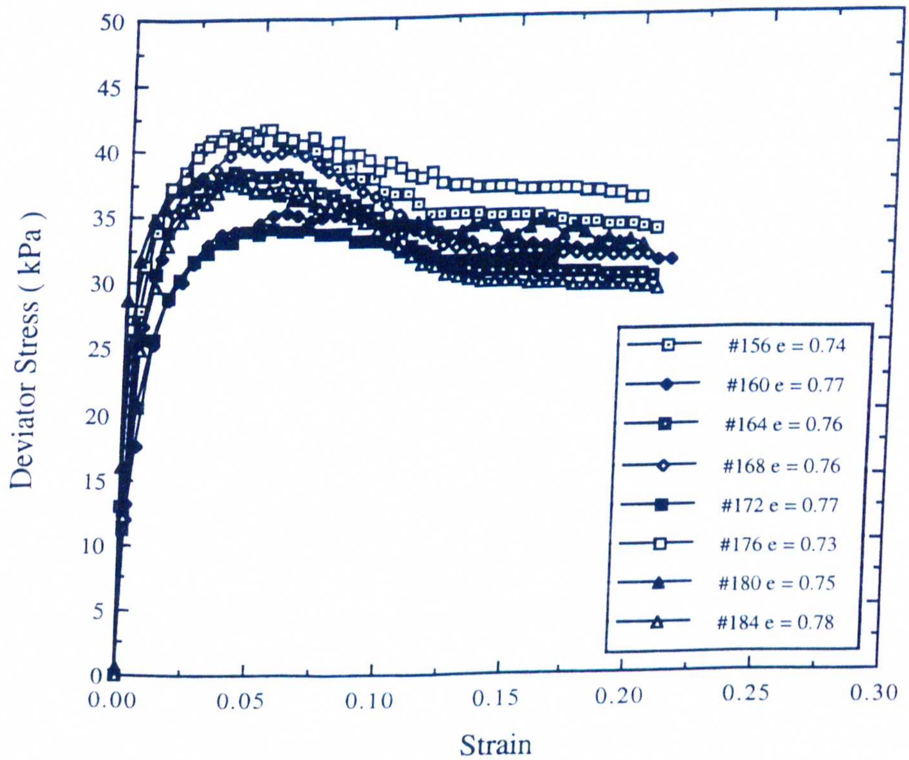


Fig. 4.33 - Strain Controlled Tests : MWSS45
Confining Pressure = 14 kPa; Dense Specimens

conducted on the finer sand, MWSS45, under stress-controlled conditions and eight tests under strain-controlled conditions at a low confining stress. A narrow range of initial void ratios was achieved, between 0.73 and 0.79, for the 16 tests. This range shows the soil to be dense, having relative densities between 78% and 100%.

For stress-controlled tests the peak (and failure) deviator stresses occurred at between 3% and 5% axial strain. These deviator stresses varied over a narrow range from 36.05 kN/m^2 to 43.21 kN/m^2, with an average of 39.86 kN/m^2. This is 27% greater than the deviator stress at 15% strain for loose specimens of MWSS45 tested at the same confining stress. These data give a standard deviation over eight tests of 2.21 kN/m^2 and a coefficient of variation of 5.5%. For strain-controlled tests the deviator stress at peak resistance also occurred at between 3% and 5% axial strain, and the average value was 38.32 kN/m^2, Which is 4% lower than the average deviator stress at peak for stress-controlled tests of the same sand. Peak deviator stress for strain-controlled tests varied from 34.10 kN/m^2 to 41.64 kN/m^2, with a standard deviation of 2.77 kN/m^2 and a coefficient of variation of 7.2%. At 15% axial strain the average deviator stress was 32.67 kN/m^2, which is approximately 15% greater than the deviator stress for strain-controlled tests at the same level of confining pressure for loose specimens of MWSS45. At 15% strain, deviator stress varied from 29.80 kN/m^2 to 36.94 kN/m^2, with a standard deviation value of 2.33 kN/m^2 and a coefficient of variation of 7.1%; e_o varied over a slightly smaller range between 0.73 and 0.78.

An examination of Figures 4.32 and 4.33 shows the differences, which are very small, in stress-strain patterns of stress versus strain controlled tests. Data of stress-controlled tests show a steeper initial climb in stress-strain plots than in the plot for strain-controlled tests. Specimens showed a well defined failure plane. Strain- controlled specimens barreled as vertical stress was increased but the failure area stayed in the middle third of the specimens (Figures 4.34 and 4.35).

The relationship between the deviator stress (Y) and the void ratio (X) remained a straight line (Figures 4.36 and 4.37a) over the small range of initial void ratios as was the case for MWSS18 and MWSS45 loose specimens. For stress- controlled tests the deviator stress Y = 117.70 - 103.44X with a coefficient of determination value of 92.3%. For strain-controlled tests Y = 135.82 - 128.72X with a coefficient of determination, R^2 = 60.0% at peak, and at 15% axial strain Y = 130.67 - 129.37X, with R^2 = 85.8% (Figure 4.37b). The fact that the average deviator stress at failure for stress-controlled tests is greater and has a smaller coefficient of variation than for strain-controlled tests at the same confining stress was also noted to be the case for tests with σ'_c equal to 14 kN/m^2.

4.6.2 STRESS AND STRAIN CONTROLLED TESTS AT 55 kN/m^2

Figures 4.38 and 4.39 show plots of eight tests conducted under stress-controlled conditions and eight tests under strain-controlled conditions with confining pressure of 55 kN/m^2. The narrow range acceptable for the initial values of e used in the tests with 14 kN/m^2 confining pressure was also achieved for these tests. Also, all tests were carried out to an axial strain of 15% or more.

The stress-strain relationship in Figures 4.38 and 4.39 indicate a strong agreement with the trends observed for the 14 kN/m^2 confining pressure, regarding the development of strain with stress and the peak/post-peak type of behavior and the failure patterns.

For the stress-controlled tests, the deviator stress at peak (and failure) occurred at between 2% and 5% axial strain. The average deviator stress for the eight tests was 209.87 kN/m^2, varying from 196.77 kN/m^2 to 219.73 kN/m^2,

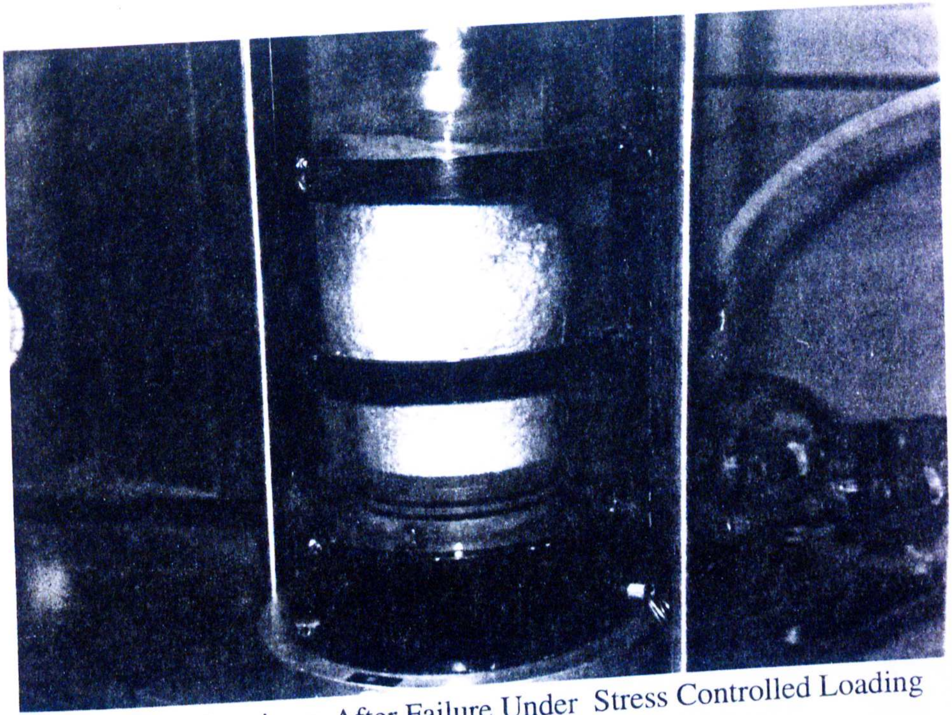


Fig. 4.34 - Specimen After Failure Under Stress Controlled Loading

(MWSS 18; Dense Specimen)

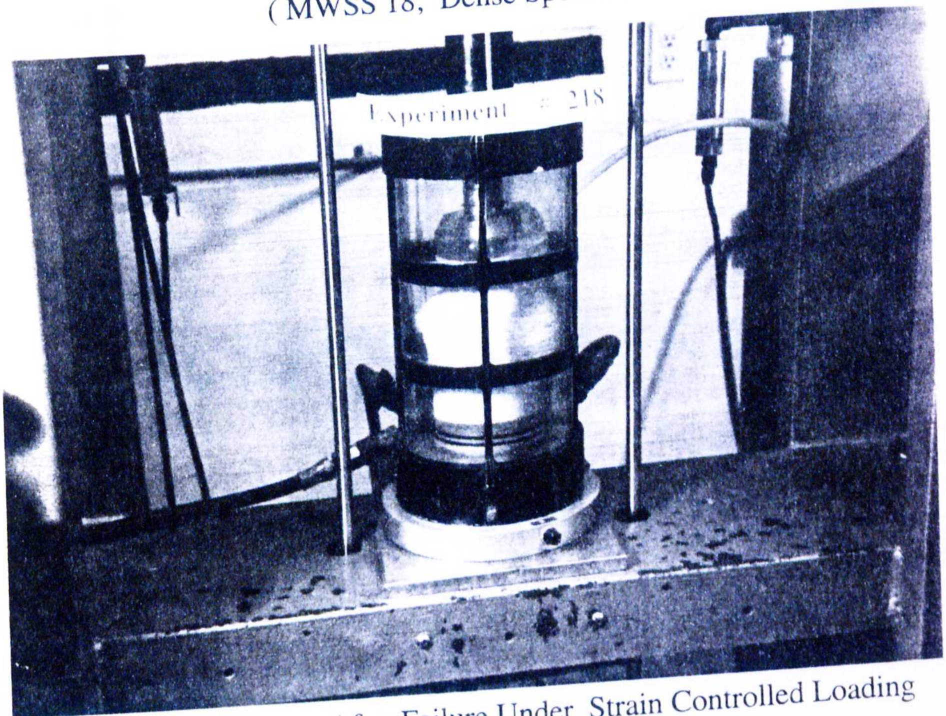


Fig. 4.35 - Specimen After Failure Under Strain Controlled Loading

(MWSS 18; Dense Specimen)

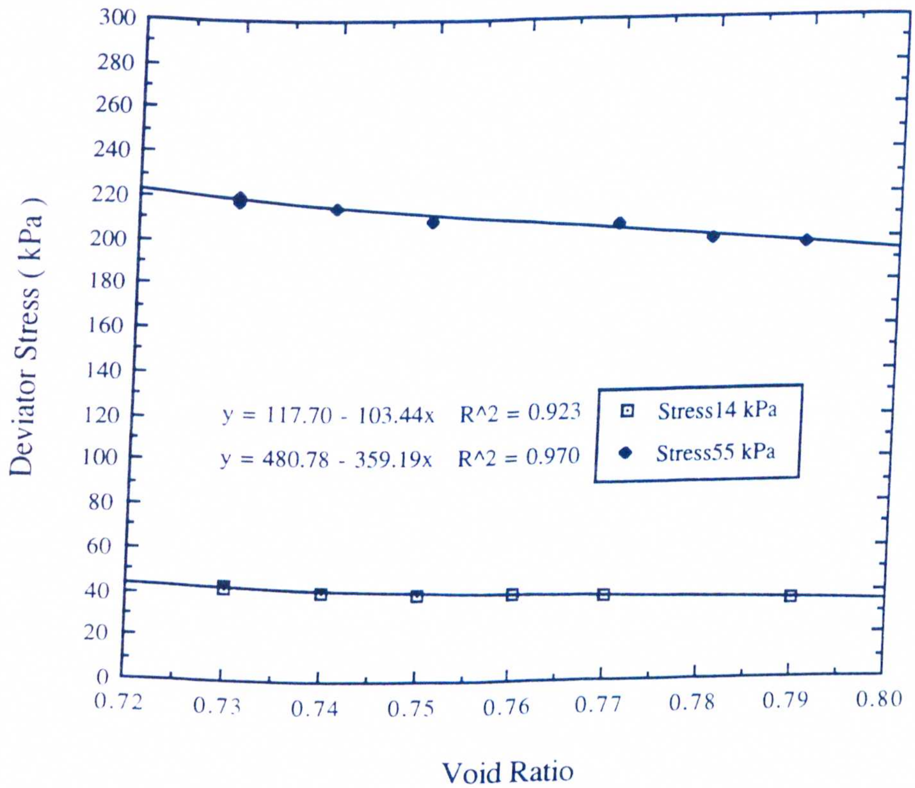


Fig. 4.36 - Deviator Stress Vs. Void Ratio
Stress Controlled Tests : MWSS45; Dense Specimens

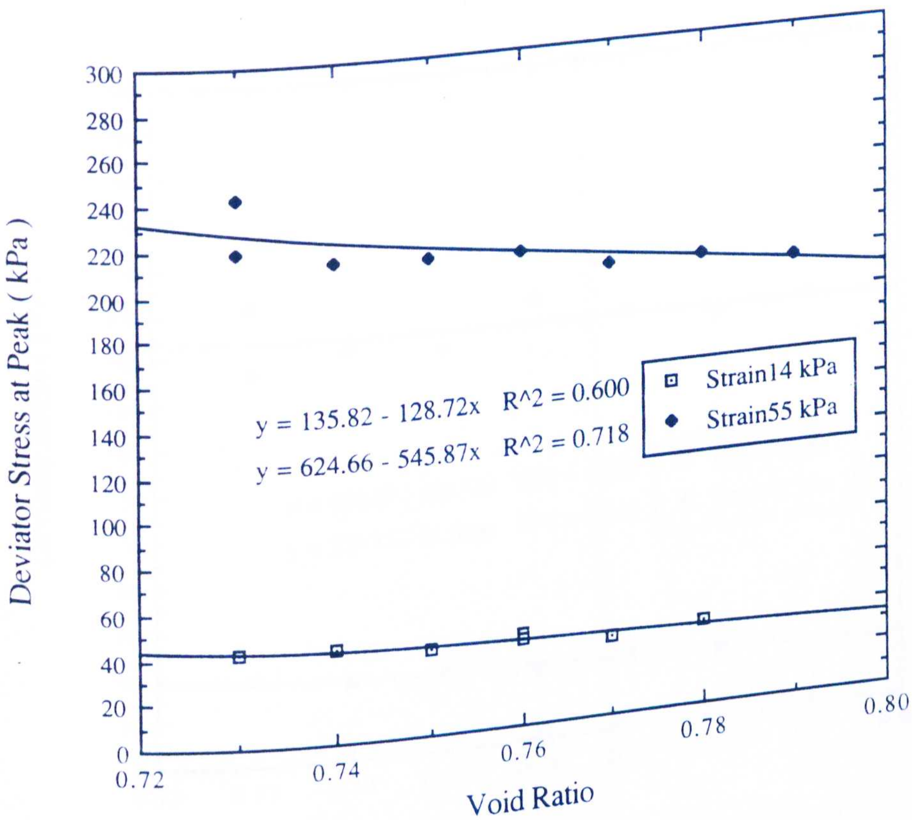


Fig. 4.37a - Deviator Stress Vs. Void Ratio
Strain Controlled Tests: MWSS45; Dense Specimens at Peak

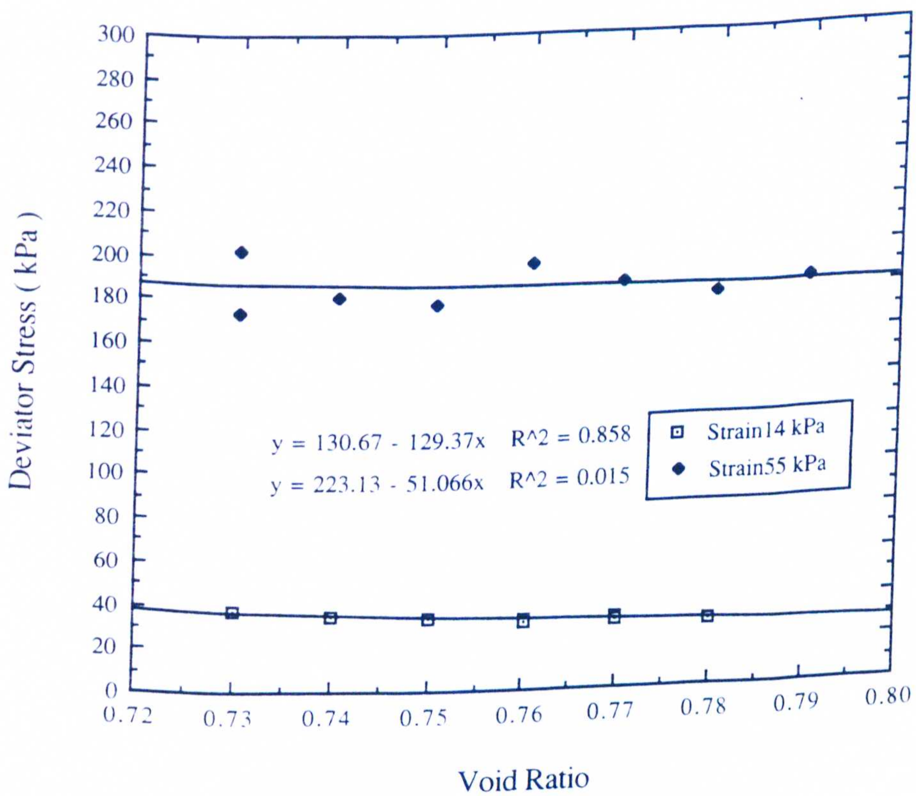


Fig. 4.37b - Deviator Stress Vs. Void Ratio
Strain Controlled Tests : MWSS45; Dense Specimens at 15% Strain

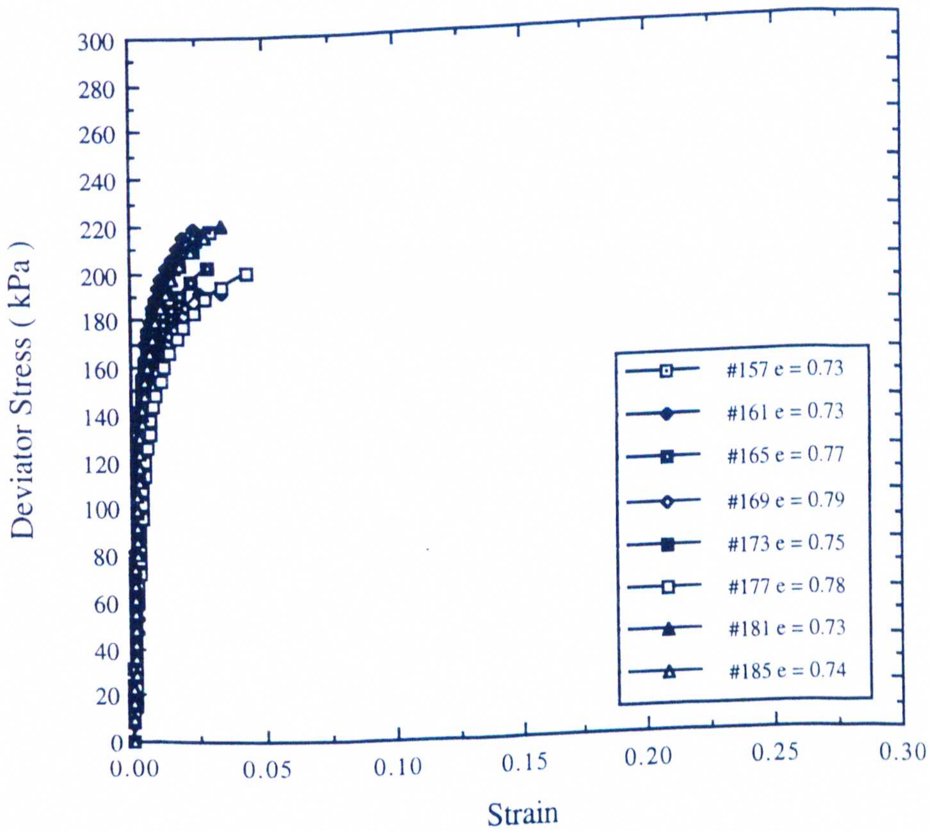


Fig. 4.38 - Stress Controlled Tests : MWSS45
Confining Pressure = 55 kPa; Dense Specimens

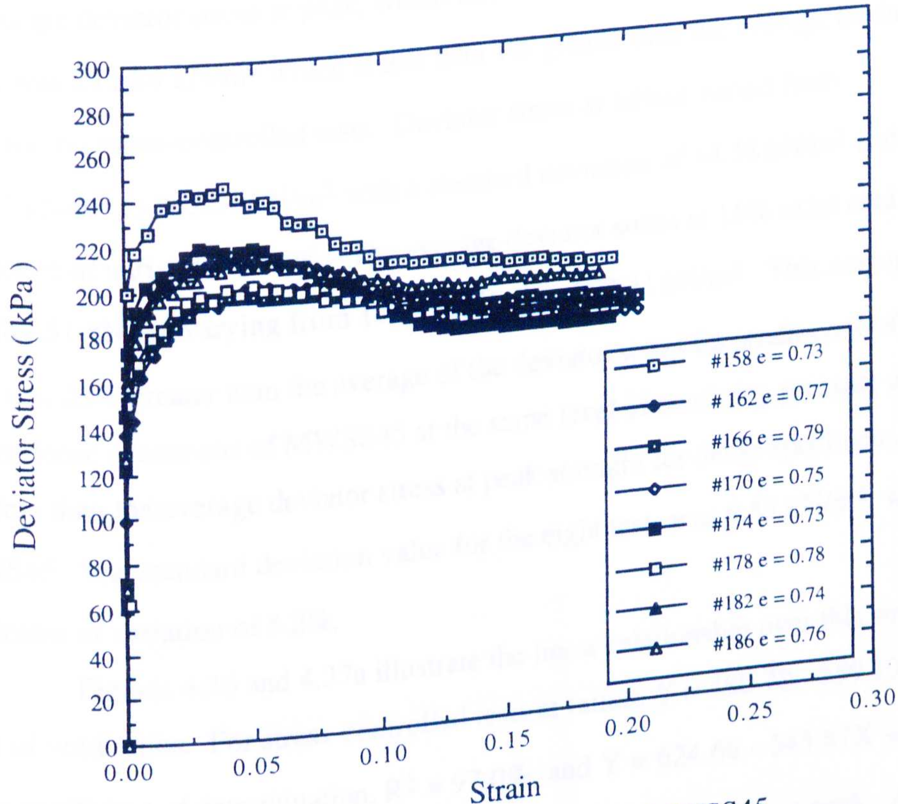


Fig. 4.39 - Strain Controlled Tests : MWSS45
Confining Pressure = 55 kPa; Dense Specimens

which was approximately 20% higher than the average deviator stress for loose specimens of MWSS45 tested at the same level of confining stress. The standard deviation was 9.32 kN/m^2, with a coefficient of variation value of 4.4%, over the range of initial void ratios values from 0.73 to 0.79. For the strain-controlled tests, the average deviator stress at peak, which also occurred between 2% and 5% axial strain, was 211.84 kN/m^2 which is less than 1% greater than the average deviator stress for the stress-controlled tests. Deviator stress at failure varied from 196.12 kN/m^2 to 242.55 kN/m^2 with a standard deviation of 14.58 kN/m^2 and a coefficient of variation of 6.9%. The average deviator stress at 15% axial strain was 184.51 kN/m^2 varying from 173.87 kN/m^2 to 202.31 kN/m^2. This average value was 23% greater than the average of the deviator stress for strain-controlled tests on loose specimens of MWSS45 at the same level of confining pressure and 12% less than the average deviator stress at peak strength for dense specimens of MWSS45. The standard deviation value for the eight tests was 9.57 kN/m^2, with a coefficient of variation of 5.2%.

Figures 4.36 and 4.37a illustrate the linear relationship over this small range of void ratios. For stress-controlled tests at failure $Y = 480.78 - 359.19X$ with a coefficient of determination, $R^2 = 97.0\%$, and $Y = 624.66 - 545.87X$ with a coefficient of determination, $R^2 = 71.80\%$ for strain-controlled tests at peak. At 15% strain $Y = 223.13 - 51.07X$, with $R^2 = 1.5\%$ indicating less good fit.

Figures 4.40 and 4.41 show a sample of stress-strain plots at the two levels of confining pressures, 14 kN/m^2, 55 kN/m^2. Also, as was the case with MWSS18 dense specimens, the final stress point shown in Figure 4.40 is the last deviator stress before sudden and complete failure caused by the next increment.

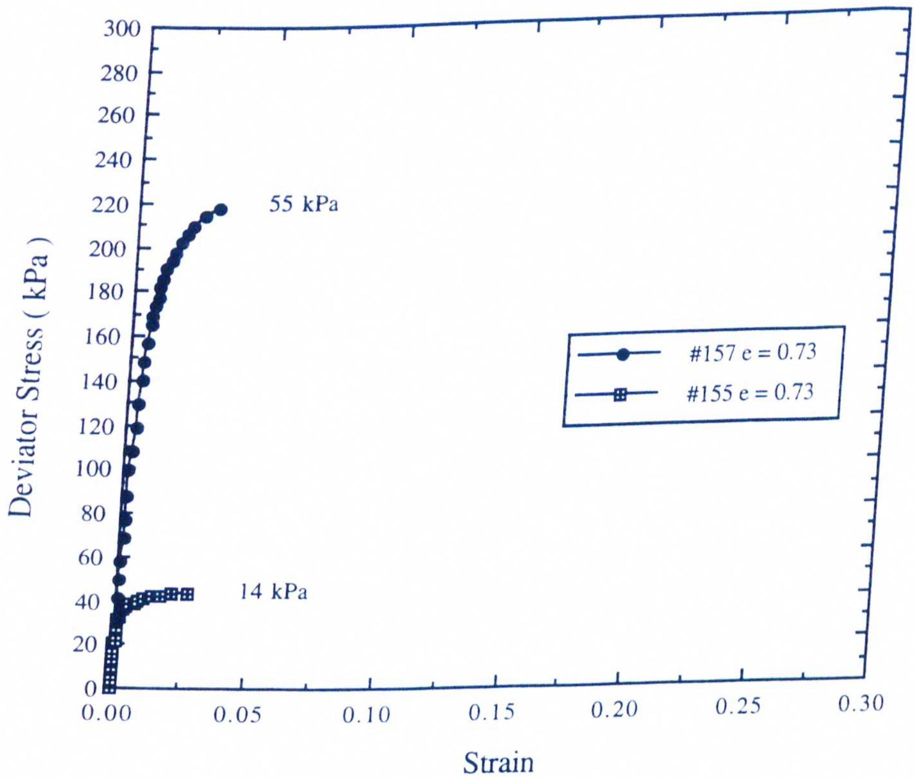


Fig. 4.40 - Deviator Stress Vs. Strain; Dense Specimen Stress Controlled Tests (Water System Loading: MWSS45)

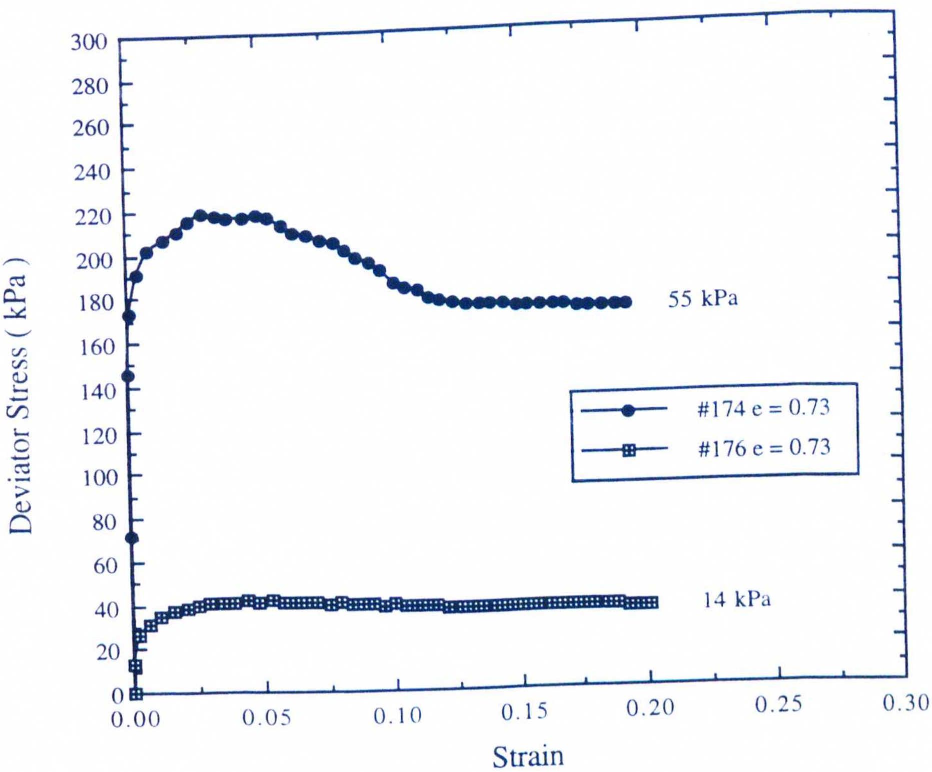


Fig. 4.41 - Deviator Stress Vs. Strain; Dense Specimens
Strain Controlled Tests : MWSS45

CHAPTER FIVE
DISCUSSION OF RESULTS

5.1 INTRODUCTION

This chapter is a discussion of the two major issues under investigation in this work which are first, how reproducible are the results, and second, are there differences in soil strength characterization according to the loading techniques.

5.2 REPRODUCIBILITY WITHIN GROUPS WITH IDENTICAL TEST CONDITIONS

Evidence from this work and from previous researchers working with theoretical models of soil strength and with actual tests of soil strength, as noted in section 2.1.2, make it clear that void ratio has a strong effect on soil strength especially when the soil is dense. Attention to achieving a desired void ratio, which is uniform throughout the specimen, is emphasized, so also is consistency in the method of preparation. Void ratio does not relate information about anisotropy that can be induced by the method of placement. For example, sample vibration is likely to lead to a different particle orientation than will tamping of the soil surface during preparation. That anisotropy affects soil strength measured in a triaxial specimen, and affects measured response in a geotechnical model.

Within this series of similar tests examining reproducibility and stress-strain response, all specimens were prepared by tamping the soil as it was placed. This is a common technique for sample preparation. Void ratios were limited to narrow ranges and specimens not falling within those ranges after preparation were discarded. Average void ratios for loose specimens of MWSS18 varied before testing from 0.87 to 0.92 (which gave negative relative densities) with standard deviations of e within a group of similar tests being no more than 0.020. Void ratios for dense specimens of MWSS18 varied before testing from 0.69 to 0.74 with the same maximum standard deviation of 0.020 for void ratio within a similar group of tests. Void ratios for loose specimens of MWSS45 varied from 0.91 to 0.98 with a slightly larger maximum standard deviation of 0.024. For dense specimens of MWSS45, void ratios before testing varied from 0.73 to 0.79 with a maximum standard deviation of 0.030 within a group of similar tests. Even with this largest standard deviation of 0.030, reproducibility of void ratios within any series of similar tests was very good, at least equal to what one might hope to be achieved in a typical but careful commercial laboratory. Results from researchers by Vesic and Clough (1968), shown in Tables 3.2 and 3.3 show a standard deviation in the void ratios at preparation within a given group of no more than 0.029. This uniformity will be more difficult to achieve and, arguably, more critical, in larger centrifuge models where stresses vary with location in the model, unlike the relative uniformity in a triaxial specimen.

Ignoring these small variations in void ratio and examining repeatability of the deviator stresses within a group of identical tests (same soil, same range of e, same confining stress, same stress path and same method of controlling loading), the coefficients of variation for deviator stresses, shown on Table 4.2, show that results from tests on the coarser soil, MWSS18, were more reproducible, having smaller coefficients of variation, than for tests on the finer soil, MWSS45. The largest coefficient of variation for deviator stresses at failure for MWSS18 was 9.9%, whereas for MWSS45 the largest coefficient of variation for deviator stress at failure was 24.9%. While the standard deviation will be the same for σ_1, at failure, as for deviator stress ($\sigma_1 - \sigma_3$) at failure, since σ_3 is held constant during a test, the coefficient of variation in σ_1 at failure will be less. The largest coefficient of variation for MWSS18 for major principal stress at failure is 7%, and similarly for MWSS45 is 17%.

Vesic and Clough (1968) who conducted eleven of their triaxial tests on sand with a confining stress of 62,000 kN/m^2 had a standard deviation in their major principal stresses at failure of 9,702 kN/m^2 and 10,584 kN/m^2 for six dense and five loose specimens, respectively. These correspond to coefficients of variation of 5.0% and 4.7%, which are in keeping with the lower values of coefficients of variation obtained in this work.

Of course certainty of a value in an experiment improves with the number of times the experiment is repeated Table 4.9 and Figure 4.7, discussed in section 4.3.1, demonstrated that whereas three trials may lead to quite large error in triaxial tests results (up to 40% of deviator stress at failure in the case of loose specimens of MWSS18 tested with σ'_c = 14 kN/m^2), six tests led to a dramatic reduction in error. These depend too on the skill of the researcher or the technician. Their results are shown in Tables 3.2 and 3.3.

Overall trends in reproducibility for tests of a single soil show, first, that: the coefficient of variation decreases (improves) when the confining stress increases; second, the coefficient of variation is the same or better (smaller) when dense specimens are tested rather than loose; and third, the stress-controlled tests tend to show a very slightly lower (better) coefficient of variation than the strain-controlled tests.

If repeatability of the deviator stresses at failure is examined controlling for void ratio, assuming that a straight line can be drawn through data of this narrow range of void ratios versus deviator stresses, then reliability is improved. The basis for this linear relationship between e and same measure of stress at failure for sands over a small range of e is discussed in Atkinson and Bransby (1978). The measure of repeatability is conformance to that straight line, using the coefficient of determination, R^2, as that measure, where $R^2 = 1.0$ indicates perfect adherence to a straight line.

In 14 out of 20 cases, R^2 was better than 80% when deviator stress at failure was plotted against void ratio, and the lowest value of R^2 was 46.8%. In loose specimens, soil subjected to strain-controlled loading showed better values of R^2 (closer to 1.0) than for stress-controlled loading. In dense specimens the reverse was true. However, the slope of the relationship between e and $(\sigma_1 - \sigma_3)$ is so flat that given the narrow ranges of e at which the two soils may exist, while characterization can be improved by accounting for e in that characterization, the improvement was minor, with the one exception, loose specimens of MWSS45 tested at low confining stress, 14 kN/m^2, had coefficients of variation above 20%, but values of R^2 above 95%. In research or in practice, then, if small coefficients of variation are required for test results to be satisfactory, then first attention should be given to keeping void ratios in specimens tested within a very narrow range of values, and second attention may be paid to developing the relationship between void ratio and stress at failure. This latter step is likely to be required only in laboratory research, when one considers the large variation naturally occurring in the void ratio in the field.

The report by Corte et al. (1988) on geotechnical centrifuge models of shallow foundations on loose fine sand (D_{50} = 0.17 mm), described in the introduction , showed a maximum variation from the average of four tests of up to 28% for models tested in stress-controlled loading. This is not out of line with variations in deviator stress at failure measured in these triaxial tests for stress-controlled loading of fine, loose sand tested at low confining stress.

5.3 SIMILARITY IN σ_{1f} OF STRESS VERSUS STRAIN CONTROLLED TESTING

For these soils tested at these void ratios and stress levels, stress-controlled testing showed, in general, better reproducibility than strain-controlled loading. But a second effect of loading conditions may be the relative values of stresses at failure.

It is possible to accept that the ranges in void ratios in each set of tests were very narrow, and to compare mean values of σ_1 at failure from each set as calculated in Tables 4.2, 4.4, 4.6, and 4.8. Alternatively, one may normalize a set of results for a constant e by using the various regression equations relating e and deviator stress, to calculate deviator stress at a constant e. This was done for each set of data, and is tabulated in Table 5.1. That Table shows in all instances but one, stress-controlled testing led to larger average values of deviator stress (and σ_1) at failure when corrected to a constant e for a group, than otherwise identical tests conducted under strain- controlled loading. This was also true for the means of each group without correcting for variations in e. For loose specimens of the coarser soil, MWSS18, the value of σ_1 at 15% strain for e = 0.89 was barely greater, only 1%, in stress-controlled testing than the value of σ_1 for strain-controlled testing when $\sigma'_c = 55$ kN/m^2. But when confining stress is less for tests on the same loose soil, the differences in the values of σ_1 at 15% strain for stress-controlled loading were 9% and 10% larger than for strain-controlled testing. For the finer soil, MWSS45, loose specimens showed the greatest effect of stress-controlled testing compared to strain-controlled testing. At 15% strain, loose specimens tested under stress-controlled loading had values of σ_1 corrected to e = 0.95 at failure between 9% and 14% greater than the values of σ_1 at failure for strain-controlled loading. This is consistent with the trends reported by Corte et al. (1988) in their centrifuge model studies of foundations on loose fine sand. Their models tested by stress-controlled loading showed 28% greater bearing capacity,

Table 5.1 Deviator Stress at the Same Void Ratio for Stress and Strain Controlled Loose and Dense Specimens of MWSS18 & MWSS45

SAND	Stress-controlled Tests	Strain-controlled Tests	$\frac{(\sigma_1 - \sigma_3)_{Stress}}{(\sigma_1 - \sigma_3)_{Strain}}$	$\frac{(\sigma_1)_{Stress}}{(\sigma_1)_{Strain}}$
MWSS18 loose specimens	Deviator Stress, kN/m^2 @ e = 0.89	Deviator Stress, kN/m^2 @ e = 0.89		
Confining Pressure KN/m^2				
14	38.06	33.68	1.13	1.09
28	82.71	72.30	1.14	1.10
55	188.88	186.24	1.01	1.01
MWSS45 loose specimens	Deviator Stress, kN/m^2 @ e = 0.95	Deviator Stress, kN/m^2 @ e = 0.95		
Confining Pressure KN/m^2				
14	29.90	24.39	1.23	1.14
28	76.84	68.33	1.12	1.09
55	170.38	149.65	1.14	1.10
MWSS18 dense specimens	Deviator Stress, kN/m^2 @ e = 0.71	Deviator Stress, kN/m^2 @ e = 0.71		
Confining Pressure KN/m^2				
14	41.06	39.57	1.04	1.03
55	221.84	213.81	1.04	1.03
MWSS45 dense specimens	Deviator Stress, kN/m^2 @ e = 0.76	Deviator Stress, kN/m^2 @ e = 0.76		
Confining Pressure KN/m^2				
14	39.09	37.99	1.03	1.02
55	207.80	209.80	0.99	0.99

$(\sigma_1 - \sigma_3)_{Stress}$ = Deviator Stress for Stress-controlled Test

$(\sigma_1 - \sigma_3)_{Strain}$ = Deviator Stress for Strain-controlled Test arguably closer to deviator stress than to σ_1 at failure, than models tested by strain-controlled loading, although the two types of loading were conducted at different laboratories.

In contrast to loose specimens, dense specimens in this study showed less effect, and unlike all other cases, at $\sigma'_c = 55$ kN/m^2 the corrected value of σ_1 for stress-controlled loading tests at failure was 1% less than that for strain-controlled loading.

A two-sample t test (see appendix) was preformed to determine if there is a statistically significant difference between the deviator stresses corrected for e of each type of loading at all levels of confining stresses. The standard errors of estimate (S_e) for the mean deviator stresses at failure corrected for e for stress and strain controlled tests was calculated for each of the 20 cases. The t test showed that for a 99% level of confidence ($\alpha = 0.01$) that at all levels of confining stresses for both sands in the loose states, there is a significant difference in average deviator stresses at failure between stress-controlled tests and strain-controlled tests. For dense specimens for both sands, the t test accepted the null hypothesis ($\mu_1 = \mu_2$) indicating, the reverse for what is concluded for loose specimens, that there is no statistically indicated difference in average deviator stresses at failure between stress-controlled and strain-controlled tests. This is not so surprising, given the small margins of differences in deviator stresses at failure corrected for e for dense specimens.

At failure σ_1 is expected to be greater for dense specimens than for loose specimens when both specimens are tested at the same confining stress, σ'_c. The difference in σ_1 associated with void ratio, and the strains at which these occur, have an impact on the design failure envelope (and safety factor) in practice, because they give insight into the possible dangers of progressive failure. Large differences between peak and ultimate strength or shear stresses for dense and loose specimens signal an increased risk of progressive failure in practice and in physical stress correct models. In all cases, strain-controlled loading shows a larger difference between σ_1 at failure for dense and loose specimens than do tests involving stress-controlled loading. For the coarser soil, MWSS18, strain-controlled tests show the average σ_1 at failure in a dense specimen (e = 0.71) to be 13% greater than in a loose specimen (e = 0.95) when both were tested at $\sigma'_c = 14$ kN/m^2. When $\sigma'_c = 55$ kN/m^2 the average value of σ_1 at failure was 14% greater when the specimen was dense (e = 0.71) compared to its behavior when specimens were loose (e = 0.95). For stress-controlled tests the differences are 6% when $\sigma'_c = 14$ kN/m^2 and 12% when $\sigma'_c = 55$ kN/m^2 for MWSS18 at the same average void ratios. For MWSS45, the finer soil, the differences are larger. Strain-controlled testing shows σ_1 at failure for dense specimens at e = 0.75 to be 25% greater than σ_1 at failure for loose specimens at e = 0.95, for stress-controlled loading when $\sigma'_c = 14$ kN/m^2 and 27% for strain-controlled loading at the same confining stress. For larger confining stress, $\sigma'_c = 55$ kN/m^2, the difference is larger -- stress-controlled tests show a difference in the average σ_1 at failure for dense specimens to be 19% greater than the average for loose specimens, whereas strain-controlled testing shows a difference of 36%. Strain-controlled loading, then, which in general gave a weaker picture of soil strength than stress-controlled testing, also gives a more conservative picture of loss of strength in progressive failure.

5.4 MOHR-COULOMB ENVELOPES AND DESIGN IMPLICATIONS

Design of geotechnical structures is still done most often by limit equilibrium analysis which normally deals with characterizing soil strength using c' and ϕ'. For this reason Mohr-Coulomb failure envelopes were determined for stress and strain controlled tests, examining the effect of variability of results and the effect of stress-controlled versus strain-controlled loading. In all cases a straight line fit to the data was imposed and the intercept, c', was made to be zero, reflecting the practice imposed by most designers for failure envelopes describing the behavior of sand. Typical values for medium coarse sand and fine sand are between 30° and 45° (Das, 1983).

Loose specimens of MWSS18 had an average best fit Mohr-Coulomb failure envelope for stress-controlled loading of 39.4°, with a variation from a minimum angle of 37.7° to a maximum angle of 40°. Strain-controlled tests showed a lower average friction angle of 36.1°, a minimum of 35.6° and a maximum of 38.5°. Loose specimens of MWSS45 had a average friction angle of 35.3° with a minimum of 34° and a maximum of 39° for stress-controlled tests. For strain-controlled tests the average failure angle is 31.2° with the minimum and maximum angles being 29.6° and 35.5° respectively (see Table 5.2).

Dense specimens of MWSS18 had an average value of ϕ' at peak of 41.8° with a minimum of 39.5° and a maximum of 42.5° for stress-controlled tests. For strain-controlled tests the average ϕ' at peak is 39.8° and at post/peak is 36.4°, the minimum and maximum values at peak are 39.3° and 42.1°, respectively. For dense specimens of MWSS45 the average friction angle for stress-controlled tests at peak is 41° with a minimum of 39.5° and a maximum of 41.7°. For strain-controlled tests of the same sand the average ϕ' at peak is 40.8° and at post/peak is

Table 5.2 Best Fit Mohr-Coulomb Failure Envelopes with $c' = 0$

SAND	$\phi'_{min.}$		$\phi'_{max.}$		$\phi'_{average}$	
MWSS18	ϕ'_{stress}	ϕ'_{strain}	ϕ'_{stress}	ϕ'_{strain}	ϕ'_{stress}	ϕ'_{strain}
LOOSE	37.7°	35.6°	40.0°	38.5°	39.4°	36.1°
DENSE	39.5°	39.3°	42.5°	42.1°	41.8°	39.8°
MWSS45	ϕ'_{stress}	ϕ'_{strain}	ϕ'_{stress}	ϕ'_{strain}	ϕ'_{stress}	ϕ'_{strain}
LOOSE	34.0°	29.6°	39.0°	35.5°	35.3°	31.2°
DENSE	39.5°	40.0°	41.7°	43.6°	41.0°	40.8°

37.6° with a minimum of 40° and a maximum of 43.6° at peak.

The influence on ϕ' of the variability in deviator stresses at failure, then, was about 3° in most cases, and in loose MWSS45 was 5° and 6° when one compared maximum and minimum values of ϕ', for stress-controlled and strain-controlled, respectively, within groups of identical tests. But it is apparent that differences in variation within a group of identical tests which are identified as a function of a test being stress-controlled rather than strain-controlled, are not significant when the possible failure envelopes are drawn. Actual values of ϕ', however, are influenced by stress and strain control of tests. In some cases there is substantial overlap in ranges for ϕ', but the variation was the greatest for loose specimens, especially for the finer soil. Values of ϕ' for loose specimens of MWSS45 ranged from 29.6° to 35.5° for strain-controlled testing, to 34° to 39° for stress-controlled testing.

The failure envelopes are plotted in Figures 5.1 and 5.4 for both the average of the stress-controlled tests and the average of the strain-controlled tests. This variation is not insignificant even when the average values of ϕ' are considered. An angle of 39.4° (from stress-controlled testing) of MWSS18 for loosely deposited soil compared to 36.1° (from strain-controlled testing) would provide an immediate factor of safety of 1.13 if 36.1° were used in design, but if 39.4° were the relevant strength. If 36.1° were assumed to be the angle of friction and an additional factor of safety of 1.5 were applied, so that a designer assumed a design angle of 25.9°, then the resulting safety factor would be 1.69 if the true angle of friction were 39.4°. Similarly, if an angle of ϕ' of 35.3°, from stress-controlled testing of loosely deposited MWSS45 is compared to 31.2° from strain-controlled testing of the same, then there is an immediate factor of safety of 1.17, if

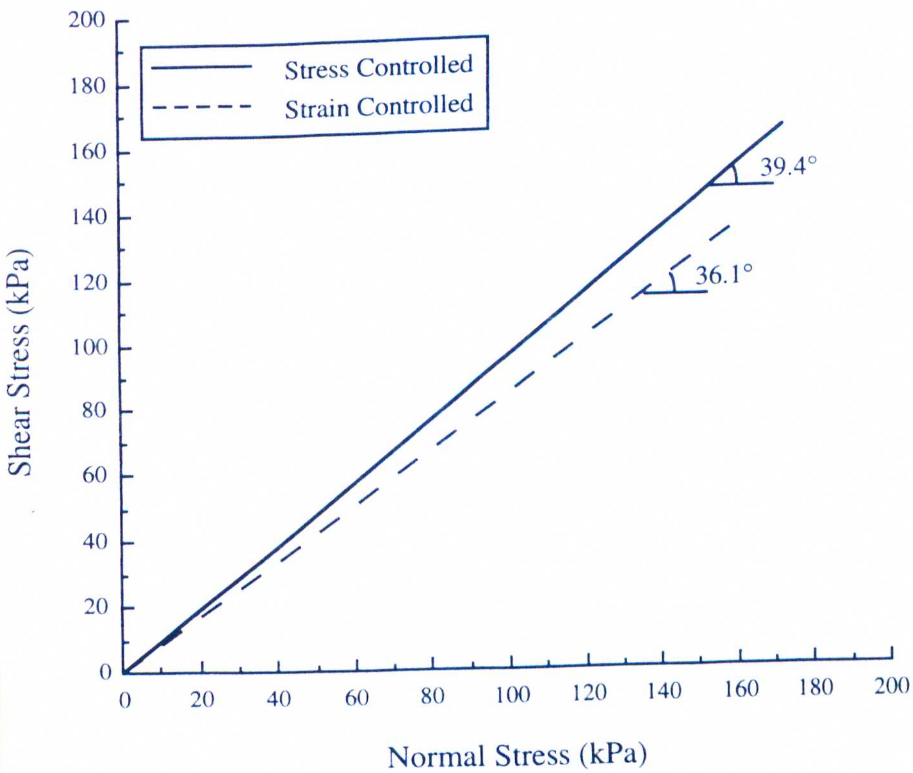


Fig. 5.1 - Average Failure Envelopes For MWSS18 Loose Specimens

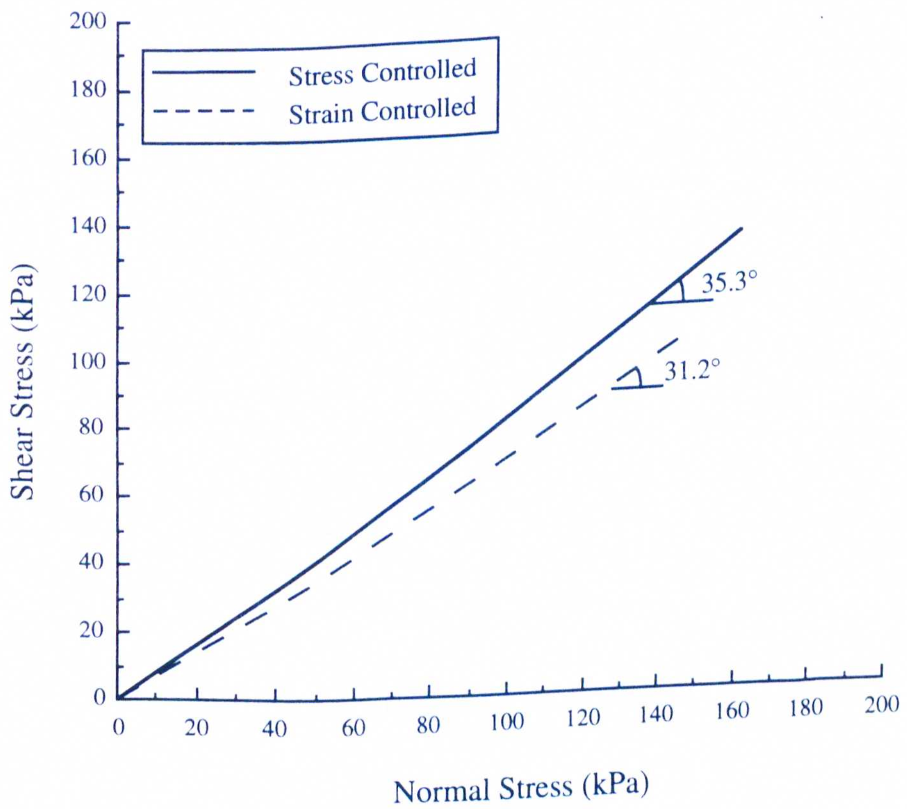


Fig. 5.2 - Average Failure Envelopes For MWSS45 Loose Specimens

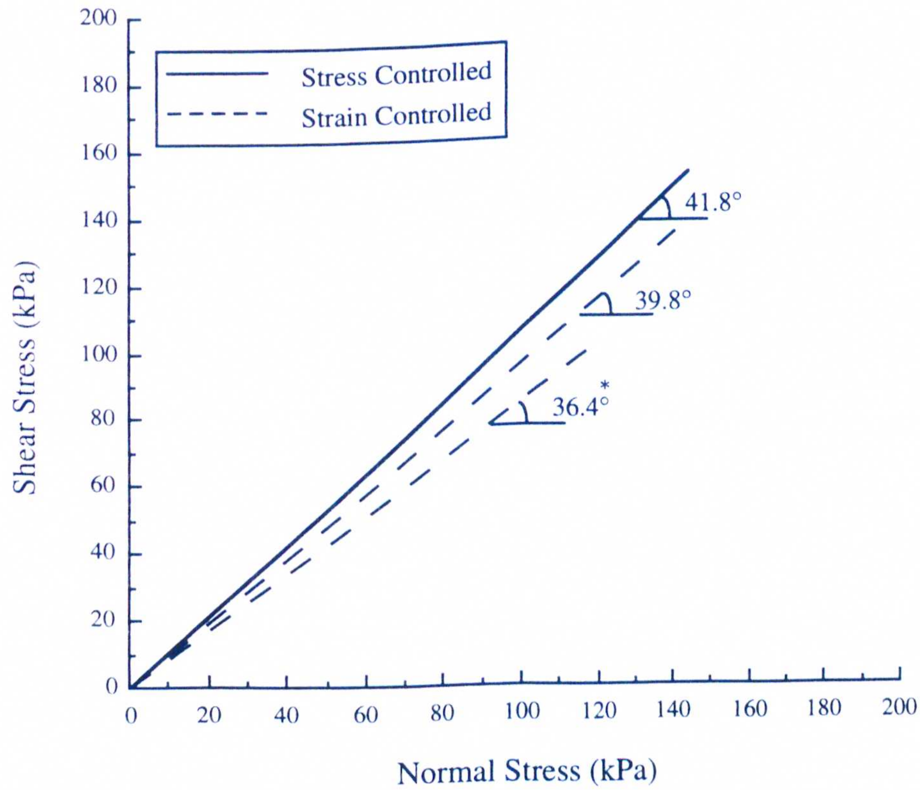


Fig. 5.3 - Average Failure Envelopes For MWSS18 Dense Specimens at Peak Strength

(*) Friction Angle At Post/Peak

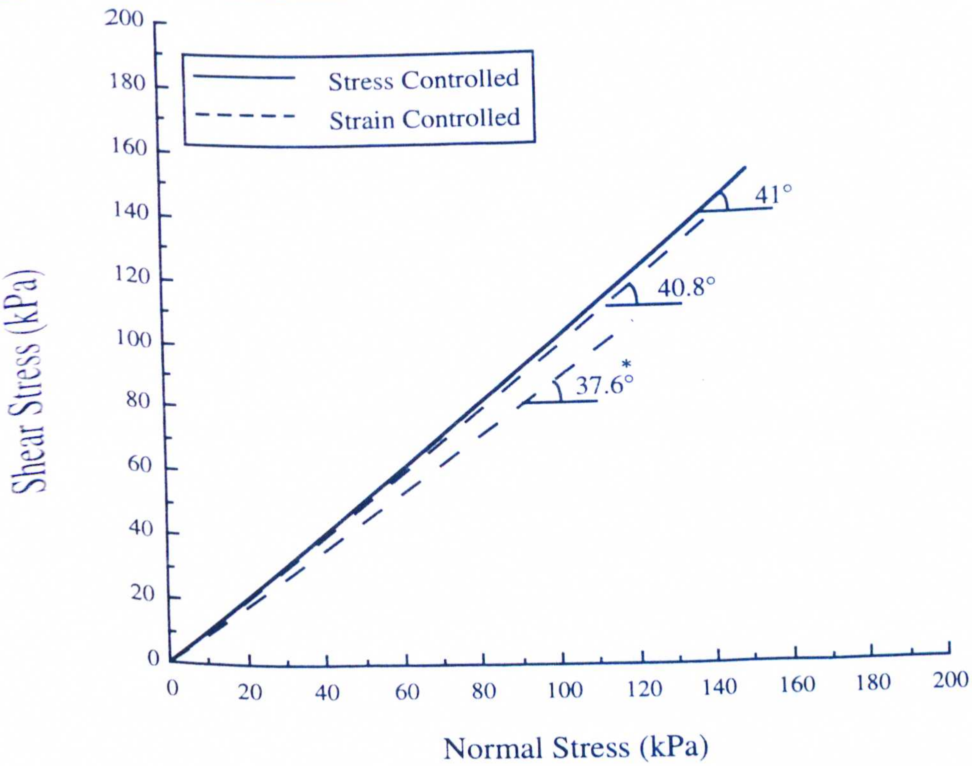


Fig. 5.4 - Average Failure Envelopes For MWSS45 Dense Specimens at Peak Strength

(*) Friction Angle At Post/Peak

35.3° is the relevant strength. If 31.2° is assumed to be the angle of friction and an additional factor of safety of 1.5 is applied, meaning that design assumes an angle of 22°, then if the true angle of friction is 35.3°, the resulting safety factor is 1.8. Differences are even greater for those specimens of the finer soil MWSS45.

But what implications does this have for design? It is always most prudent from the point of view of safety to use data which give a lower estimate of strength, but it may not be "good engineering" from the point of view of economical and appropriate design. A slope 10 m in height with a factor of safety of 1.3 will have an angle of 29.3° extending a horizontal distance from crest to the toe of 17.8 m, if ϕ' is assumed to be 36.1° at failure. The same slope height with the same factor of safety but ϕ' at failure assumed to be 39.4°, will have an angle of 32.3° and extend a horizontal distance of 15.8 m, 11% less than the previous calculations, and a difference in fill volume of 10 m^3/m length of slope. This affects, then, costs in fill and in land acquisitions. If the design includes supporting a bridge footing at the top of the slope, the implications for design angle and cost are even greater. Certainly the use of strain-controlled parameters is "safer" than stress-controlled parameters, and since most loadings in the field are stress-controlled, this provides a built-in additional failure of safety, if one can afford this.

In stress correct centrifuge physical models where real behavior of settlement and failure are simulated, if the method of loading the model can affect its stiffness and stresses at failure, then this must be considered in model design and then in analysis. This work indicates that method of model loading will affect model response in both regards.

Finally, the soil specimens behaved more stiffly in the initial stages of stressing under stress-controlled loading compared to strain-controlled. Table 5.3 shows that in every case, values of E are greater in stress-controlled testing than in strain-controlled testing, and in some cases one can be as great as twice the other

Table 5.3 Tangent and Secant Modulus at their Corresponding Strains for Loose and Dense Specimens of MWSS18 & MWSS45

SAND	Stress-controlled Tests		Strain-controlled Tests	
MWSS18 loose specimens	E_T kN/m^2	E_S* kN/m^2	E_T kN/m^2	E_S* kN/m^2
Confining Pressure KN/m^2				
14	4715.32 (0.00109)	373.07 (0.08912)	4013.17 (0.00410)	211.32 (0.16396)
28	24312.77 (0.00104)	1029.37 (0.07154)	18547.45 (0.00547)	783.86 (0.08382)
55	50503.41 (0.00232)	6469.27 (0.02649)	29235.91 (0.00346)	3155.24 (0.05363)
MWSS45 loose specimens	E_T kN/m^2	E_S* kN/m^2	E_T kN/m^2	E_S* kN/m^2
Confining Pressure KN/m^2				
14	5977.80 (0.00203)	261.14 (0.10021)	2289.41 (0.00299)	178.93 (0.10246)
28	25897.69 (0.00109)	928.13 (0.07557)	15551.24 (0.00351)	492.89 (0.11371)
55	29088.73 (0.00196)	1538.79 (0.09816)	26868.56 (0.00203)	1303.31 (0.10467)
MWSS18 dense specimens	E_T kN/m^2	E_S* kN/m^2	E_T kN/m^2	E_S* kN/m^2
Confining Pressure KN/m^2				
14	25886.74 (0.00116)	7603.16 (0.00515)	3624.17 (0.00805)	1496.57 (0.02586)
55	55512.63 (0.00257)	17996.80 (0.01189)	50767.64 (0.00301)	12500.44 (0.01560)
MWSS45 dense specimens	E_T kN/m^2	E_S* kN/m^2	E_T kN/m^2	E_S* kN/m^2
Confining Pressure KN/m^2				
14	25739.28 (0.00114)	5672.26 (0.00687)	11367.36 (0.00232)	2055.99 (0.01825)
55	60270.62 (0.00155)	13671.38 (0.01433)	49611.78 (0.00224)	13591.29 (0.01442)

(*) at 90% of Failure

for otherwise identical test condition. An indirect control on soil deformation is often achieved in design by application of a factor of safety to the failure envelope, rather than by specific calculation. This means that calculations of stress-controlled field deformations will be over estimated if based on strain-controlled test data. Conversely, when site behavior is monitored, strains to failure for stress-controlled field conditions will be less than anticipated from strain-controlled laboratory data, and this will be unconservative to some degree in most conditions. This stiffer behavior can also be observed in Corte et al.'s (1988) report on foundation loading in centrifuge models of sand.

5.5 DISCUSSION OF RESULTS IN CONTEXT OF DEVELOPMENT OF SHEAR BANDS

Clearly there is some difference in the development of failure when the soil is loaded by stress-control compared to strain-control. Identifying the mechanical response of soils has been of particular interest over the past 10 to 15 years. The mechanical response of a soil is influenced by its fabric as well as by flaws or discontinuities that may exist in the material. These flaws can lead to development of a shear band that spreads and forms a damaged zone in which most of the deformation is localized. Development of shear bands occurs in this way in metals, in rocks, and in most solids, but granular materials have specific constitutive properties such as pressure sensitivity which affect their behavior somewhat differently. Desrues and Chambon (1989) noted that shear band analysis is normally approached in one of two ways, either perturbation analysis which assumes a preexisting band of weakness, or a direct analysis which assumes a kinematically specified loss of uniqueness of the material's incremental response. They considered the direct analysis to be most useful for granular materials because it allows analysis of the transition from homogeneous deformation to localized deformation which develops with shear.

Muhlhaus and Vardoulakis (1987) identified from experiments on soil confined with zero lateral strain, the mean thickness of the final localized shear band to be about 16 times D_{50} (varying from 18.5 times D_{50} for fine sand to 13 times D_{50} for medium sand). Vardoulakis (1989) suggested also that the theoretical kinematic model of shear band development is more appropriate for granular media, since, he said, the statical theory, which predicts a thickness of the shear band of half that, or 8 times D_{50}, fails to account for the fact that even when lateral strain is made to be zero in a soil specimen, the grains are still three dimensional rather than an assembly of rods which are truly two-dimensional as Cundall (1989) assumed in his work.

Desures et al. (1985) suggested from stereophotogrammetric analysis of soil failed under biaxial ($\varepsilon_2 = 0$) stress conditions, that when a shear band develops, the failure is a result of localization of deformation beginning from a point followed by rapid local change in e and in strength. Deformations then become concentrated along a band. They did not discuss data of other concentrations of deformation in the specimens prior to development of the final shear band failure. Bardet (1991) reanalyzed Desrues's (1984) results for deformations leading up to failure. Prior to failure, development of concentrated deformation began on one surface, then moved to a second, but then moved back to the first, along which failure finally occurred. The angles of those changing potential planes varied from 20° to the horizontal to 33° in samples of the same soil. The soil, then, seems to be testing out different potential failure surfaces.

Shear band inclination influences the effects of principal stresses by its resulting effect on normal and shear forces. The Coulomb theory predicts from the classical solution that the inclination, θ, of the shear band (or the failure surface) to the plane on which σ_1 acts is at $45+\phi'/2$ for an isotropic material. Vardoulakis

(1980) considered Coulomb's analysis to have shortcomings in explaining stresses in the shear band, as the deformation developed at failure. His theoretical work and his analysis of experimental work by Arthur et. al (1977) who used biaxial stress conditions, showed for sand that $\theta = 45 + (\phi'_p + \upsilon'_p)/4$ gave good predictions for the inclination of the failure surface, where υ_p is the dilation angle at peak. Since dilatancy is a function of void ratio (=Vv/Vs) or porosity (= Vv/(Vv + Vs)), then, he concluded that peak values of ϕ' are a function of porosity, n, and from his collected data that $\sin\phi'_p = 1.45 - 1.97n_o$ gave a good prediction of ϕ'_p. Bardet (1991) also analyzed θ and developed an Extended Mohr-Coulomb theory which considered plasticity, but his predictions of θ did not show an improvement on predictions using Vardoulakis (1980) theory.

Most tests on development of shear bands are conducted using biaxial apparatus which allows for good identification of development of failure. But biaxial ($\varepsilon_2 = 0$) conditions are also different from failure in axisymmetric ($\varepsilon_2 \neq 0$) cases. Reads and Green (1976), and Desrues et al. (1985) found strain to failure in axisymmetric conditions, such as in the triaxial apparatus, to be roughly 3 times larger than the strain to failure in the biaxial case. Another difference between biaxial and axisymmetric conditions is that in the axisymmetric case there exists a third direction along which failure may occur, thereby, providing greater possibility of failure. In fact biaxial tests may simulate many field conditions better than axisymmetric conditions, for example for conditions of slope instability, strip footing failure, and failure along the axis of a tunnel. This means that if triaxial tests are conducted to determine failure parameters and expected strain to failure, and thus interpretation of strain data from the field, then strain to failure in an in situ biaxial condition will be under-estimated by those triaxial results, although predictions of settlement may be conservative, or predicted to be greater than observed.

Even though Vardoulakis (1980) looked to confirm his work with biaxial strain-controlled test data, his prediction equation ($\sin\phi'_p = 1.45 - 1.97n_o$) gives surprisingly good predictions for the strain-controlled triaxial tests of this study, especially for dense specimens, as shown in Table 5.4. The predictions become less close for loose specimens tested with strain-controlled loading, and are quite poor for stress-controlled testing. This suggests, then, that the observation by other researchers made regarding development of failure for strain-controlled loading are likely to be appropriate to strain-controlled loading in this work, but that stress-controlled loading which leads to different values of ϕ' and different shapes of failure may have quite different mechanisms applying to the details of failure. The only way to detect those differences is to conduct stereophotogrammetric biaxial tests under stress-controlled loading to failure, but the axisymmetric failure in the triaxial specimens is still likely to be different from the biaxial failure.

Another unresolved issue about development of failure focuses on the point at which localization of deformation and thus strain softening occurs. Desrues and Chambon (1989) predicted that this occurs before failure at peak, while Vardoulakis et al. (1978) predicted it occurs at peak strength, and Drescher and Vardoulakis (1982), and Hettler and Vardoulakis (1984) predicted this occurs after peak strength. All these researchers worked with strain-controlled loading, which allows for redistribution of stresses, rather than the sudden and complete failure which develops when peak shear stress is exceeded in truly stress-controlled tests with loading by dead weights, and not by hydraulic pistons. Stress concentrations and concentration of deformations may then develop along the first most critical shear band, whereas, strain-controlled loading gives opportunity for the soil to experience a shifting of stress and deformation concentrations over a variety of potential shear bands to find the most critical one. This may explain the

Table 5.4 Comparison withTheoretical Study from Biaxial Tests

Sand	Void Ratio	$\phi'_{predicted}$ (Vardoulakis, 1980)*	$\phi'_{measured}$ (This Study)
MWSS18	0.72	38.7°	38° (Strain-Controlled)
	0.72	38.7°	42° (Stress-Controlled)
	0.90	31.1°	34° (Strain-Controlled)
	0.89	31.5°	40° (Stress-Controlled)
MWSS45	0.70	39.7°	39° (Strain-Controlled)
	0.75	37.3°	41° (Stress-Controlled)
	0.95	29.4°	34° (Strain-Controlled)
	0.95	29.4°	38° (Stress-Controlled)

(*) Prediction based on results from strain-controlled biaxial tests greater variation and lower deviator stresses of soils tested under strain-controlled testing than stress-controlled. It may also explain why soils subjected to stress-controlled loading failed with well developed failure surfaces whereas, for soils tested under strain-controlled loading, failure was characterized by barreling, suggesting existence of more shear bands or at least a possibly wider zone of weakness.

CHAPTER SIX
SUMMARY AND CONCLUSIONS

6.1 SUMMARY

The purpose of this research was to examine the influence on measurement of soil strength made in the triaxial apparatus when soil is loaded by strain-control methods or stress-control methods. Repeatability, actual values of deviator stress at failure, and the influence on determinations of ϕ' when $c' = 0$, were the important issues.

To this end one hundred sixty-six useful triaxial tests were conducted involving two types of dry sands, one fine MWSS45 and one medium coarse MWSS18 at three low to intermediate confining stresses, 14 kN/m^2, 28 kN/m^2 and 55 kN/m^2, with specimen height 150 mm and specimen diameter 71.1 mm. This size meant that specimen diameter was a minimum of 30 grain diameters. Of the one hundred sixty-six tests, eighty-six were stress-controlled tests and eighty were strain-controlled tests. Loose specimens and dense specimens were tested.

A summary of the tests is as follows:

1. Reproducibility within groups with identical test conditions:

Reproducibility is better for loose specimens of coarser sand (MWSS18) than the loose specimens of the finer sand (MWSS45). For dense specimens of both sands reproducibility was the same. Coefficients of variation of deviator stress ($\sigma_1 - \sigma_3$) were lower (better) for stress-controlled tests than strain-controlled tests in almost every case, except for the 14 kN/m^2 confining stress for loose specimens of MWSS45 where the reverse occurred.

The equations developed relating void ratio and deviator stress using statistical analysis gave coefficients of determination, R^2, in 14 out of 20 cases higher than 80%. The lowest value, R^2, was 46.8%, obtained for loose specimens of MWSS18 at σ'_c = 55 kN/m^2, although the coefficient of variation for that set of data was very good -- 5.2%.

Overall, one can conclude that trends in reproducibility for tests of each sands are as follows:

(1) As the confining stress increases the coefficient of variation decreases (improves).

(2) For dense specimens the coefficient of variation is the same or better when compared to loose specimens.

(3) Stress-controlled tests show slightly better coefficients of variations than the strain-controlled tests.

(4) Loose specimens of stress-controlled tests, in general, have higher coefficients of determination, R^2, values than strain-controlled tests, but the reverse was true for dense specimens.

2. Similarity in σ_{1f} of stress versus strain controlled testing:

In general, at failure as one expects, σ_1 was greater for dense specimens than for loose specimens when both specimens are tested with the same confining stress, σ'_c. The final results in terms of relative values of σ_{1f} showed that the difference in σ_{1f} between stress and strain controlled testing corrected for e decreased for loose specimens of MWSS18 as the confining stress increased, while for loose specimens of MWSS45 the trend was less clear, but not contradictory. For dense specimens the values of σ_{1f} corrected for e were closer when comparing results of stress and strain controlled tests and in the one case of dense specimens of MWSS45 at $\sigma'_c = 55$ kN/m^2, strain-controlled testing showed very slightly greater strength than stress-controlled testing. A two-sample t test confirmed that there is a statistically significant difference between stress and strain controlled test results for loose soils, but not in the case of the two dense soils.

3. Mohr-Coulomb envelopes and design implications:

Mohr-Coulomb failure envelopes were determined for stress and strain controlled tests. Since the average deviator stress was greater for stress-controlled tests than strain-controlled tests, the friction angles were greater for data from stress-controlled than from strain-controlled tests. The difference was less for dense specimens than loose specimens, especially for MWSS45.

The influence on ϕ' with each test group showed that a difference of at least 3° existed between ϕ'_{max} and ϕ'_{min} and this was as high as 5° and 6° for stress and strain controlled testing respectively for loose specimens of MWSS45.

4. Discussion of results in context of development of shear bands:

There is some difference in the development of failure when the soil is loaded by stress-controlled methods compared to strain-controlled. The mechanical response of the soil is influenced by its fabrics as well as by other flaws or discontinuities that may be present in the material. Many researchers have worked in this area trying to understand the development of shear bands, their thickness, their inclination and their widths. Most of the tests explaining the development of shear bands have been conducted in strain-controlled manner using the biaxial apparatus ($\varepsilon_2 = 0$). In this research a comparison between Vardoulakis (1980) study and the results of the strain-controlled tests on the inclination of shear bands showed a closer agreement, although tests in this study were done using the axisymmetric (triaxial) condition ($\varepsilon_2 \neq 0$).

The thickness of the shear bands ranges from 18 times D_{50} for fine sand to 13 times D_{50} for medium sand as found by Muhlhaus and Vardoulakis (1987). No examination of shear bands development was done in this study due to the need for stereophotogrammetric equipment which was not available. There is evidence, however that stress-controlled axisymmetric loading leads to a different development of failure.

5. Finally, the development of a water loading system for stress-controlled loading instead of the original dead load hanger system proved to be accurate and produce superior results.

6.2 CONCLUSIONS

The conclusions of this work are as follows:

1. With respect to reproducibility, control of void ratio over a narrow range, and consistency of specimen preparation which influences anisotropy of the specimen, are more important for producing repeatable results than is the choice between stress-controlled versus strain-controlled loading. Reproducibility is best when specimens are tested at higher confining stress, and when soil particle packing is dense, but neither of these are parameters that can be selected by the engineer conducting the soil tests since both must be chosen based on true field conditions.

For eight identical tests of a soil with the largest standard deviation in void ratio of 0.030, the largest coefficient of variation for values of σ_1 at failure or at 15% strain in this series was 24.9%, and the smallest coefficient of variation for any group of identical tests was 4.4%, when the standard deviation in void ratio was 0.020. An examination of error showed that six identical tests reduced error to a very low level, if void ratio is tightly controlled.

2. With respect to the magnitude of the principal stress at failure, stress-controlled tests showed the soil to be stiffer and stronger than did strain-controlled

tests. This contradicts one aspect of findings by Dennis (1988) and Stephanos (1989), both of whom conducted tests at higher confining stresses. It is consistent however with the results of comparative centrifuge model studies reported by Corte et al. (1988). Relative values of σ_1 at failure corrected for differences in initial void ratios in these standard compression loading triaxial tests showed a maximum difference of 14% between stress-controlled and strain-controlled loading, but most differences were much less. Strain-controlled tests, then, tend to give a more conservative (or weaker) picture of soil strength at failure, and indicate stronger potential for progressive failure than do stress-controlled tests.

The greater strain at failure (and smaller modulus of elasticity) in strain-controlled tests means that settlement in a stress-controlled geotechnical condition, will be over predicted if the prediction is based on strain-controlled stress-strain tests data. This is usually conservative or safe. Conversely, deformations prior to failure recorded in the field for a true stress-controlled situation will be less than predicted by strain-controlled stress-strain test data, and this is usually unconservative and potentially unsafe, depending on the application.

3. With respect to values of ϕ' for a conventional straight line Mohr-Coulomb representation of soil strength, making $c' = 0$, differences in variability between stress-controlled versus strain-controlled tests are largely eliminated. In most cases ϕ' varies by no more than 1.5° for stress or for strain controlled test data, although this small variability is strongly affected by control over specimen preparation. Stress-controlled test data, however, gives larger value of ϕ', often by only a small margin, but sometimes by as large as 4°. Depending on the nature of the project, this may have a significant impact on the cost of an engineering project, and so consideration should be given at the laboratory stage as to the method of specimen loading. It may have a much greater effect in assessing behavior in

geotechnical centrifuge model research, where accurate definition of soil strength without the luxury of factors of safety, is a necessity both to understand mechanisms of failure and to explore results to full scale. This may have contributed to at least some of the variation between results in centrifuge models of bearing capacity failure of shallow foundations on sand from different geotechnical laboratories, reported by Corte et al. (1988).

4. Finally, other researcher have worked to define the development of failure for use in constitutive models of soil behavior, using observations from strain-controlled biaxial tests. The research in this study suggests that development of shear bands and failure in stress-controlled loading may be sufficiently different from that developing in strain-controlled loading that extrapolation from one to the other may not be strictly valid. The difference between triaxial axisymmetric stress conditions and biaxial stress conditions is a second possibly important, difference. There are, however, difficulties to be overcome to conduct stress-controlled shear band research, especially in a triaxial apparatus.

6.3 RECOMMENDATIONS FOR FURTHER RESEARCH

The important implications for this work are more for the researcher who requires accurate models of the stress-strain behavior of soils, than for the practitioner, who, by the very nature of geotechnical engineering practice has only a very approximate control over, and limited knowledge of the soil at a site. For the researcher involved in geotechnical centrifuge model studies, the constitutive modeler seeking to model exact behavior of soil, or the researcher involved in the particular mechanics of failure, the following further topics of research are recommended for consideration:

1. Exploration of variability, and stress compared to strain controlled testing of cohesive soils.

2. Comparison of model response using stress versus strain controlled loading in centrifuge models.
3. Examination of the development of shear band in stress-controlled loading compared to those developed in strain-controlled loading. Biaxial stress conditions have been applied by other researchers for a variety of important practical reasons, but axisymmetric triaxial conditions may show different development of failure.

APPENDIX
STATISTICAL ANALYSIS

A.1 Two-SAMPLE t TEST:

The two-sample t test is used to test for a significant difference between the means of two groups for a prespecified level of significance.

$$H_o: \mu_1 = \mu_2$$

$$H_A: \mu_1 \neq \mu_2 \quad \text{(two-tailed)}$$

$$\alpha = 0.01$$

The test statistic is:

$$t = \frac{\hat{Y}_1 - \hat{Y}_2}{S_p}$$

where:

$\hat{Y}_1$ = Deviator stress @ failure for stress-controlled test @ a constant (e)
$\hat{Y}_2$ = Deviator stress @ failure for strain-controlled test @ a constant (e)
S_p = The pooled standard error

$$S_p = \left[\left(\frac{(n_1 - 1)S_{e1}^2 + (n_2 - 1)S_{e2}^2}{n_1 + n_2 - 2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)\right]^{0.5}$$

where:

S_{e1} = Standard error of estimate for stress-controlled tests @ a constant (e)
S_{e2} = Standard error of estimate for strain-controlled tests @ a constant (e)
n_1 = Number of samples for stress-controlled tests
n_2 = Number of samples for strain-controlled tests

The results of the t test is shown in Table A.1.

Table A.1 Results of The Two-Sample t Test

MWSS18				MWSS45			
LOOSE (e = 0.89)		DENSE (e = 0.71)		LOOSE (e = 0.95)		DENSE (e = 0.76)	
Confining Pressure kN/m^2	Decision	Confining Pressure kN/m^2	Decision	Confining Pressure kN/m^2	Decision	Confining Pressure kN/m^2	Decision
14	Reject Ho	14	Accept Ho	14	Reject Ho	14	Accept Ho
28	Reject Ho	—	—	28	Reject Ho	—	—
55	Reject Ho	55	Accept Ho	55	Reject Ho	55	Accept Ho

A.2 DETERMINATION OF NUMBER OF SAMPLES FOR EACH TEST GROUP

The number of experiments was determined using the following statistical analysis: Confidence interval of the regression equation for a single point on the line.

$$Y \pm t_{\alpha/2} S_e \left[\frac{1}{n} + \frac{(X_o - \overline{X})^2}{\sum (X - \overline{X})^2} \right]^{0.5}$$

since,

$$(X - \overline{X}) = (X_o - \overline{X}), \text{ then } \sum (X - \overline{X})^2 = n(X - \overline{X})^2 = n(X_o - \overline{X})^2$$

$$\therefore \quad t_{\alpha/2} S_e \left[\frac{1}{n} + \frac{(X_o - \overline{X})^2}{n(X_o - \overline{X})^2} \right]^{0.5} = t_{\alpha/2} S_e \left[\frac{2}{n} \right]^{0.5}$$

$\therefore$ error $= \dfrac{t_{\alpha/2} S_e \sqrt{2}}{\sqrt{n}}$ is the CI on a mean when the regression is used

(see Fig. 4.7 & Table 4.9)

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