ABSTRACT

Title of Thesis:	ANALYSIS OF TRANSFER COORDINATION IN FLEXIBLE-ROUTE BUS SERVICES
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Urban residents are often confronted with transportation predicaments. The inconvenience caused by the conventional fixed-route bus system has led to excessive reliance on private cars, worsening traffic congestion and air pollution. However, a flexible bus route can provide passengers with a convenient, expedient, cost-effective commuting option. This thesis studies a flexible bus system with Many-to-One (M-1) and Many-to-Many (M-M) demand patterns, comprising multiple rectangular residential zones and a central terminal. The total cost of flexible route bus service is modeled and modified for coordinated and uncoordinated headway conditions. Among them, the demand between each service zone and the central terminal, and the demand among service zones are analyzed to optimize headways in order to minimize total system cost. Finally, the sensitivity analyses are conducted to explore the impact of parameter changes on the results. The comparison of baseline and sensitivity analysis results shows that more benefits can be achieved when coordinating headways under low-demand conditions.

ANALYSIS OF TRANSFER COORDINATION IN FLEXIBLE-ROUTE BUS SERVICES

by

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Chapter 1: Introduction

1.1 Background

Urban residents often face numerous challenges in transportation and commuting. The current public transportation system is often deemed inconvenient, which has resulted in most people relying heavily on private cars. However, this overreliance on private cars has led to severe traffic congestion, causing people to worry about being late and endure long waits in traffic jams. Private car owners also face a range of issues, such as fluctuating fuel prices, difficulty finding parking spots, and other related problems. Even during leisure travel, people have to worry about parking availability at airports and train stations and the possibility of traffic congestion on highways, which may cause delays in their flights and trains. Governments must also pay close attention to the spin-off environmental and health issues that arise as a result of private car usage, such as air pollution and energy consumption. Hence, it is imperative for people to address these issues and look for better alternatives to make transportation, and particularly commuting, more convenient, efficient, and sustainable for everyone. Many-to-One (M-1) and Many-to-Many(M-M) Flexible-route Bus Service are innovative public transportation modes that aim to reduce the use of private cars by providing commuters with a convenient and fast way to reach their destinations. This service offers a flexible route tailored to the passengers' needs, ensuring that they can reach their destinations efficiently and reliably. Unlike conventional buses, the M-1 and M-M Flexible-route Bus Service are designed to provide a more accessible, reliable, and personalized travel experience for commuters.

1.2 Objectives

The purpose of this thesis is to propose and analyze a flexible route bus service which serves multiple residential zones and avoids unnecessary transfer delays at its central terminal. The aim is to provide commuters with a more dependable, efficient, and seamless mode of transportation. Additionally, this proposal aims to develop a flexible bus cost model and evaluate the cost implications associated with flexible-route buses, with or without headway coordination. The mathematical model is used to analyze some characteristics of the three cases and, if possible, transfer on the shortest path to avoid detours to the central terminal. The objective function of the Flexible-Route Bus service is the sum of user costs and operating expenses. Sensitivity analyses will be conducted to investigate the impact of varying factors such as population density, bus operating costs, value of time, headway, and bus speed on the total cost of Flexible Route bus systems.

1.3 Scope

This thesis analyzes the concept of interconnecting service zones of flexible route buses through a central terminal. The flexible route buses operate between zones and the central terminal, including transfers through other zones before arriving at the central terminal. Among them, the central terminal can serve as a transfer terminal for passengers traveling between different zones. This thesis extends the analysis to multiple zones and optimizes the headway of the public transportation system while minimizing costs by analyzing conditions for coordinating route headways. It also covers Many-to-One, One-to-Many, and Many-to-Many service demand patterns.

1.4 Organization

In this thesis, Chapter 2 reviews the literature most relevant to this study. It includes studies on flexible route services, coordinated transfers, and residential zones in transport services. Chapter 3 explains the basic assumptions, formulates the total cost function of flexible route buses to cover multiple residential zones, and separately analyzes the cases with and without headway coordination. The numerical methods are used to optimize the headway. Chapter 4 presents simplified multiple results, including distance, population, bus trip time, passenger travel time, number of stops, and costs, and performs multiple sensitivity analyses to show how parameter changes affect the cost results. Finally, Chapter 5 provides a summary of the thesis and presents recommendations for future research directions.

Chapter 2: Literature Review

2.1 Timed transfers

There is a considerable literature on schedule coordination for timed transfers, which includes notable contributions from the University of Maryland. The studies begin with some descriptive analyses. Sullivan (1975) proposed the philosophy of route layout for light rail transit and coordinated services throughout the day. He considered that the concept of timed transfers in a suitable network structure setting could improve passenger mobility and increase revenue for transport operators. Sullivan (1980) introduced the timed transfer experience in a Canadian city. He suggested it would be better to use "clock" headway, which are buses leaving at the same time of the hour, and slack times to ensure the reliability of the public system. Nelson et al. (1982) evaluated 11 bus transfer policy components and included timed transfers. They classified timed transfers into four different types: simple timed transfers, pulse scheduling, line-ups, and neighborhood pulse. Headways on different routes need to be synchronized by changing route lengths and/or modifying layovers, resulting in multiple actions required by the operator. They estimated that pulse-timed transfers could increase ridership by about 5-12%. Vuchic et al. (1983) provided a detailed description of timed transfer systems (TTS) and can replace individual bus lines in low-density areas where transfers are inconvenient. The timed transfer system can bring convenience to passengers traveling between any two points in the service area. Many factors such as route length, operating speed, headway and number of vehicles are taken into consideration.

Most studies neglected the impact of randomness of vehicle travel times on timed transfers, causing inconvenience and missed connections. Some early studies aimed to optimize public transit systems with coordinated transfers using mathematical models. Thus, Rapp and Gehner (1976) developed a coordinated four-stage interactive graphic process for operational transportation planning and mainly studied computerized transfer systems. A computerized transfer system has successfully reduced passenger transfer times in Switzerland's Basel bus system without increasing costs. Salzborn (1980) studied a bus system with integrated connections between feeder lines and one trunk route. He devised a method to generate timetables based on preset values for minimum and maximum transfer times, transfer stop times, and terminal times. This approach can be used for transferring between intercity and feeder bus routes. Later, Hall (1985) developed and evaluated an analytical optimization model that depends on average vehicle delays and headways on transfer routes and obtained a closed-form solution for a timed transfer system with exponentially distributed bus arrivals. Bakker et al. (1988) proposed a simple timed transfer concept to modify transport networks to suit modern cities. The proposal emphasized the importance of aligning route headways with pulsed headways or multiples thereof. They also found that linear scheduling optimization must yield time transfer control in long headway systems. Voß (1992) formulated the schedule synchronization problem in public transport networks as a quadratic semi-assignment problem (QSAP) and determined the departure times of buses and/or trains to minimize waiting times between passenger transfers. They utilized heuristic algorithms for finding initial feasible solutions and improved them using tabu search methods. He demonstrated the effectiveness of his solutions by solving real-world problems and testing them on randomly generated data. Chakroborty et al. (1995) used a genetic algorithm to minimize passengers' total transfer time (TT) or initial waiting time (IWT), or both, in solving a transfer station scheduling problem. Constraints such as fleet size, minimum and maximum parking times, and maximum transit time limited optimal scheduling. Ting and Schonfeld (2005) developed a model for scheduling coordination to improve service quality and reduce wait times in public transportation networks. In this model, vehicles on different routes arrive at the transfer station at the same time or almost at the same time, which can save users waiting time at the transfer station. In optimizing headway and slack time, they found that slack time changes with variables such as headway, vehicle arrival time difference, transfer volume and passenger time value.

In order to improve public transport systems, this thesis aims to minimize the total transfer time and cost. To achieve this for Flexible-route bus services with headway coordination, the headway of each route must be equal. The round-trip time should also be a multiple of the headway to compare and optimize the total cost of taking the Flexible-Route bus.

2.2 Flexible-route versus Conventional bus services

Considerable research has also been conducted regarding flexible-route services, conventional bus services (i.e., with fixed routes and schedules), and the possible competitive or synergistic relations among these two service types. Thus, Adebisi (1980) developed a model to estimate the expectation and variance of operating times for partially and fully flexible bus route services. In partially flexible bus route service, some passengers are served on fixed route service while others are served at the doorsteps. The fully flexible route service serves all passengers at their doorsteps. His model considered the number of passengers in a vehicle and their location on the rectangular grid road network in a stochastic-based scenario. Qui et al. (2015) explored a new demi-flexible operating policy in transportation policy to bridge the gap between flexible route services and conventional fixed-route systems. The bus service does not offer complete curbside assistance, but it still provides some flexibility for passengers. This policy helps optimize low-demand operating environments in sparse communities such as suburbs and rural areas. Zheng et al. (2018) compared route deviation policy and point deviation policy by using two analytical models to assist planners in making decisions. It was found that the point deviation policy is more effective in low-demand scenarios. In situations of low-to-moderate demand, the route deviation policy is preferable compared to the point deviation policy. Stiglic et al. (2015) introduced meeting points into the ride-sharing system. In this system, passengers can be picked up and dropped off at the departure or destination point, or at a meeting point within a certain distance from the departure point or destination. This model offers increased flexibility for both the driver and passengers while reducing the driving distance. The only downside was that passengers must reach a meeting point within a certain distance of the origin or destination before the vehicle arrives. Yu et al. (2015) proposed a bi-level nonlinear mixed integer programming model to optimize the circulator service network design problem (joint routing and stopping optimization problem). The model aimed to minimize passengers' walking time and total travel cost by comparing the efficiency of demand-responsive transportation services to conventional bus operations. Demand response services have been noted to improve mobility by reducing passengers' perceived travel time.

This thesis aims to enhance the transportation experience of passengers residing in multiple residential zones. It will primarily focus on conducting a detailed analysis of the costs incurred by the Flexible Route bus under different circumstances, including those with and without headway coordination. The study intends to identify the most efficient methods to improve passenger mobility and reduce total costs.

There has also been considerable research on the synergy between flexible route services and conventional bus services in different situations. The relative advantages of fixed-route, flexible-route and variable-type bus services are analyzed by Chang & Schonfeld (1991a) and by Kim and Schonfeld (2012). Chang and Schonfeld (1991a) proposed a many-to-one service model for comparing fixed-route conventional buses to flexible-route bus systems. The model optimizes vehicle and service area sizes to minimize total costs. The relative advantage of subscribing to a bus service increases when the service area is smaller, the bus travels at higher speeds, higher values of access and wait time, and the value of in-vehicle time is lower. According to Kim and Schonfeld's (2012) study, when considering different demand scenarios, the average cost per passenger of flexible-route services was lower than conventional fixed-route bus operations in conditions of lower demand density. They also analyzed variable-type services that switch bus services between conventional bus services during peak hours and flexible route bus services during low-demand periods. Kim and Schonfeld (2013) used a genetic algorithm and analytic optimization to integrate fixed and flexible bus services, while also considering demand density to determine the optimal type of service. Chen and Nie (2017) analyzed a hybrid transportation system that combines fixed-route and demand-adaptive services using simulation. They used the flexibility of demand-adaptive services to connect passengers from origin/destination, thereby increasing the accessibility of fixed-route bus systems. This new hybrid transportation system can closely combine fixed-route services with demand-adaptive services to reduce total system costs.

2.3 Impact of residential areas on the public transportation system

In some studies, the shape, size, and level of demand for residential areas have an impact on the choice between flexible and conventional buses. Chang and Schonfeld (1993) developed a model for optimizing the dimensions of service area and headway independently or jointly, but only for fixed-route bus services. The model formulations for fixed and flexible route services differed considerably. It was assumed that the zone shapes are rectangles. They found that zone length, width, and headway should increase with distance from the main terminal. Chang and Schonfeld (1991a), and Kim and Schonfeld (2013, 2014) assumed that the service zones are rectangular and divide large service regions into multiple zones based on demand levels. It should be noted that efficient flexible route services planning is highly dependent on zone size, regardless of whether the zones are rectangular. Chang and Schonfeld (1991a, 1991b), Kim and Schonfeld (2013, 2014) compared conventional bus service and flexible route bus service in terms of total cost and integrated conventional and flexible route bus service. They provided methods for determining the type of bus service required based on the density of demand, service zone size, headway, and other factors. Flexible route bus service has the advantage of a lower average cost in the case of lower demand density.

Kim et al. (2019) set the service area or single area model as an irregular grid shape for flexible route buses. However, they did not specify how the origin-to-destination matrix for inter-regional trips should be determined and how transfer times should be modeled. Chen and Schonfeld (2022) expanded on the previous study by examining a flexible route bus system in two specific zones. However, the study did not address scenarios where there are more than two zones and transfers are not limited to a single central station.

In this thesis, the model was further extended and served by a flexible route bus system serving multiple zones. Compared to previous studies, this study identifies some routes where transfers at central stations can be avoided within multiple zones. The shape of each zone is rectangular, and the headway separations may differ between service zones.

In urban public transportation, conventional bus services have challenges in satisfying the people's growing travel needs. Converting conventional bus services to flexible routes can offer passengers more convenient transportation in multiple zones. In this model, headway is optimized to minimize the average cost per trip. In addition to further extending the flexible route bus system service to multiple zones, the model also considers routes that avoid transfers at the central terminal in the shortest path. The thesis models both coordinated and uncoordinated operations and provides numerical examples to compare them.

Chapter 3: Methodology

This thesis presents a model for the flexible route bus transportation mode. The flexible route bus shown in Figure 1 serves Many-to-One (M-to-1) and Many-to-Many (M-to-M) demand patterns, where users travel between zones and central terminals, as well as between zones and zones. Passengers can transfer to their destination using either the central terminal or zones as transfer stations. This chapter formulates the necessary relations for computing the total cost with coordinated or uncoordinated headway. Additionally, it presents a calculus-based method for optimizing headways on connecting routes and minimizing total cost.

3.1 Notation and Baseline value

The variables and parameters used in the analysis, along with their symbols, units and baseline values are shown in Table 1.

Table 1: Variables and Parameters

Variable	Definition	Baseline value
α	Constant coefficient	~0.03
A	Zone area $= \frac{LW}{N_{zone}}$	-
B	Unit Bus operator cost ($/unit hr)	75.0
c_1	Coefficient for travel time	12.0
c_2	Coefficient for trip price	1.0
C_o	Operating cost ($/hr)	-
C_{pij}	Out of pocket cost ($/hr)	0
C_t	Total cost ($/hr)	-
C_v	In-vehicle cost ($/hr)	-
C_w	Waiting cost ($/hr)	-
d	Total distance (miles)	-
D	Population density (persons/sq. mile)	20
g	Number of persons per group	1
G	General cost/impedance cost ($/one way trip)	-
h	Headway (hrs/veh)	-

i	Origin zone i	0, …, I (7)
j	Destination zone j	0, …, I (7)
J_i	Line-haul distance (miles)	J_1=4.0 J_2=5.0 J_3=6.0 J_4=7.0 $J_5 = 5.0$ $J_6 = 4.0$ $J_7 = 5.0$
k	Stein's constant for rectilinear space	1.15
L	Length of service zone (miles)	L_1=4.0 L_2=6.0 L_3=5.0 L_4=6.0 $L_5 = 5.0$ $L_6 = 6.0$ $L_7 = 5.0$
L_{max}	Maximum load factor	1.2
m	Number of the transfer stations from zone i to zone j	-
n	Number of stops per tour per zone	-
N_{zone}	Number of zones	1.0
P	Population (persons)	$P_{service\ zone\ 1} = 240$ $P_{service\ zone\ 2} = 600$ $P_{service\ zone\ 3} = 400$ $P_{service\ zone\ 4} = 600$ $P_{service\ zone\ 5} = 400$ $P_{service\ zone\ 6} = 600$ $P_{service\ zone\ 7} = 400$ $P_{terminal} = 200$
P'	Impedance	-
Q	Number of trips per person hour	-
R_b	Bus round-trip time (hr)	-
S_c	Vehicle capacity (seats/vehicle)	50
t_s	Stopping time (hr/stop)	25/3600
t_t	Transfer time (hr/stop)	120/3600
τ	Parameter for coordination (ratio average wait time to headway)	0.5
U_{ij}	Binary variable. ($U_{ij} = 0$ if the transfer occurs at zone 0. $U_{ij} = 1$ if transfer occurs elsewhere)	0 or 1
v	Value of in-vehicle time (\$/passenger hr)	12.0
v_w	Value of waiting time (\$/passenger hr)	24.0
V	Bus speed in local service zone (mi/hr)	20.0

V_f	Bus speed in line-haul (mi/hr)	40.0
W	Width of service zone (miles)	W_1=3.0 W_2=5.0 W_3=4.0 W_4=5.0 W_5=4.0 W_6=5.0 W_7=4.0
X	Integer (the smallest integer that the headway can multiply. The new round-trip time is just larger than the round-trip time obtained under the coordination of the headway.)	-
z	Exponent of impedance function	1
Z_i	Tour length within the zone i	-

3.2 Assumptions

The following assumptions are made for the multiple service zones with one terminal:

1. Destinations and origins are fairly uniformly distributed over time and space within each service zone.
2. Passenger pickups and drop-offs are randomly intermingled within each tour.
3. The demand in the central terminal zone is assumed to be concentrated at a single point, i.e., with zero area.
4. The average waiting time is a constant fraction τ of the headway. The default value of τ is 0.5, reflecting uniform arrivals over time of passengers and buses.
5. The transfer time depends on the zone where the transfer is made. If the transfer is made in zone 0 (the central terminal), the route headways may be coordinated, bus arrivals and departures from different routes are approximately synchronized, and the transfer time is assumed to be 0. If there is coordination, but the shortest path avoids zone 0 (terminal), the transfer time per passenger is

assumed to be $\tau \times h$. If there is no coordination, the transfer time is assumed to be $\tau \times h_i$, where h_i is the headway of the next route.

6. In the case of headway coordination, it is assumed that headways are equal in all zones. When the headway is the same in each zone, headway coordination is only implemented at zone 0, while the other zones are not coordinated.

Figure 1 depicts a central terminal that is represented as a single point. This terminal is connected to J rectangular residential zones through J line haul road links. In this case J = 7. Each of these J residential zones is considered a distinct area, with a rectilinear street network, within which movements are limited to vertical and horizontal directions. The outer zones in Figure 1 are served by routes that pass through the inner zones, and passengers are allowed to transfer in the inner zones. The analysis method and cost function in this thesis are not restricted to 7 residential zones but are applicable to multiple residential zones.

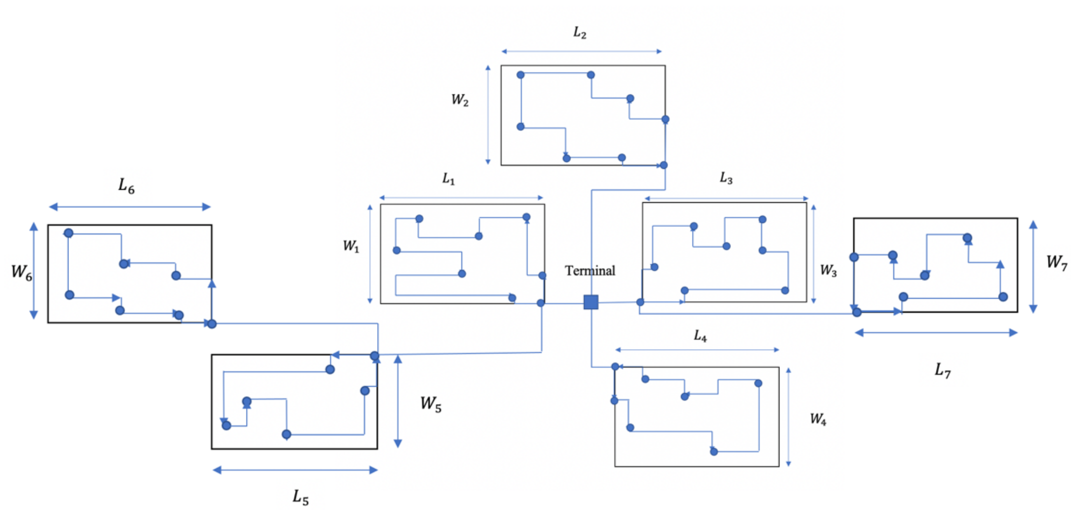

Figure 1 Flexible-route Bus Services with seven service zones

3.3 Problem Formulation

For the total cost of flexible route buses, bus operating costs, user in-vehicle costs, and waiting costs are considered. Among them, the user waiting cost includes the transfer cost. Flexible buses also offer door-to-door services with negligible user access costs compared to conventional buses.

The total cost C_t of flexible bus service is sum of the vehicle operation costs C_o, the in-vehicle costs C_v, the waiting time costs C_w.

$$C_t = C_o + C_v + C_w \tag{1}$$

The units of population density are persons/sq. mile. The population density is assumed to be fixed. Therefore, the population in zone i, denoted by P_i, is formulated as:

$$P_i = D_i L_i W_i = D_i A_i \qquad \text{(i=0, …, I)} \tag{2}$$

where D_i is population density, A_i is the area of the service zone.

The number of trips Q_{ij} (in person trips/hour) between zone i to zone j is assumed as a function of the square root of the population product $P_i P_j$ of the two zones and divided by the impedance function. There are no internal trips from zone i to zone i.

$$Q_{ij} = \frac{\alpha\sqrt{P_i P_j}}{{P'_{ij}}^{z}} \qquad (i \neq j) \qquad \text{(i=0,…, I, j =0,…, I)} \tag{3}$$

where α is constant coefficient and z is the exponent of impedance function.

Here, the impedance function is denoted as P'_{ij}. The impedance function is formulated as:

$$P'_{ij} = c_1 \frac{d_{ij}}{speed} + c_2 C_{pij} \tag{4}$$

where d_{ij} is trip distance from zone i to zone j, $speed$ is flexible route bus speed, C_{pij} is out-of-pocket cost (including trip price), $c_1$and c_2 are the coefficients of travel time and out-of-pocket cost respectively.

The number of stops n_i during the tour requires the number of passengers per hour and the number of people per group:

$$n_i = \frac{\sum_j Q_{ij} + \sum_j Q_{ji}}{g} h \tag{5}$$

where g is the number of people per group/stop. The product of hourly demand and headway provides the number of passengers per hour.

According to Stein (1978) and Daganzo (1984), the tour length within the zone i is formulated as:

$$Z_i \cong k\sqrt{n_i A_i} \tag{6}$$

where A_i is the area of zone I, k ($k = 1.15\ for\ the\ rectilinear\ space$) is Stein's constant for this tour length relation. For the operator cost C_o , the average round-trip time R_{ij} of buses has to be determined. A bus travels from the terminal to the zone i at a express speed V_f for a distance J. Finally, the tour length within the zones divided by the local speed, the number of stops n_i multiplied by the stopping time t_s and the number of transfer stations from terminal to zone i multiplied by the transfer time t_t must be added to obtain the vehicle round-trip time R_i.

$$R_i = 2\frac{J_{0i}}{V_f} + \frac{k\sqrt{n_i A_i}}{V} + n_i t_s + (2m+1)t_t \tag{7}$$

where J_{0i} is the line haul distance from central terminal to zone i. It should be noted that zone 0 can serve as a transfer station.

To determine the in-vehicle cost of passengers, the passenger one-way travel time should be computed. It can be found by adding line haul distance J_{ij} from zone i to zone j divided by line haul speed V_f, the half tour length within the zone i and zone j divided by the local speed V, the half number of stops in zone i and zone j multiplied by the stopping time t_s, and the number of transfer stations from zone i to j multiplied by the transfer time t_t.

$$T_{ij} = \frac{J_{ij}}{V_f} + \frac{k\sqrt{n_i A_i}}{2V} + \frac{k\sqrt{n_j A_j}}{2V} + (\frac{n_i+n_j}{2})t_s + mt_t \quad (8)$$

For the bus operation service cost C_o, the required bus fleet size is vehicle round-trip time R_i divided by headway h. The bus operations service cost is formulated as:

$$C_o = \sum_i \frac{R_i}{h_i} B \quad (9)$$

where B is the bus operator cost.

The total hourly in-vehicle cost C_v for the flexible bus service is:

$$C_v = \sum_i \sum_j T_{ij} Q_{ij}\, v \quad (10)$$

where T_{ij} is passenger one way travel time, v is value of time.

The average waiting time is assumed as τ multiplied by the headway. The hourly user waiting time is obtained by multiplying the average waiting time by the value of waiting time and the number of trips between zone i to zone j. As stated earlier, the average waiting time here is assumed to be half the headway. Here, τ is assumed to be 0.5.

In this thesis, the waiting cost also encompasses the transfer cost. This study analyzes service zones and determines the impact of coordinated headway compared to uncoordinated headway based on planning needs. When headways are coordinated, each route will have the same headway. When the headways are not coordinated, there will be different headways according to the characteristics of each route. In the case of headway coordination or no coordination, there are two scenarios to consider:

The hourly user waiting cost without headway coordination is formulated as:

$$C_w = \tau v_w \sum_i h_i \sum_j Q_{ij} \qquad (i=0 \dots I, j = 0 \dots I) \tag{11}$$

In the case of headway coordination, it is necessary to separately consider whether the shortest path of the trips can avoid zone 0 as a transfer station. When a trip needs to pass through zone 0, the transfer time is negligible. The hourly user waiting cost with headway coordination is formulated as:

$$C_w = \tau v_w h \sum_i \sum_j (Q_{ij} + Q_{ij} \times U_{ij}) \quad (i = 0 \dots I, j = 0 \dots I) \tag{12}$$

$$U_{ij} = \begin{matrix} 0,\ if\ the\ transfer\ occurs\ at\ zone\ 0 \\ 1,\ if\ transfer\ occurs\ elsewhere \end{matrix} \tag{13}$$

where U_{ij} is a binary variable. Headway coordination is only implemented at zone 0, while the other zones are not coordinated.

Therefore, the total cost C_t is the sum of operation cost C_o, in-vehicle costs C_v, waiting time costs C_w . waiting cost which include the transfer cost.

The total cost C_t without headway coordination is formulated as:

$$C_t = \sum_i \frac{R_i}{h_i} B + \sum_i \sum_j T_{ij} Q_{ij}\, v + \tau v_w \sum_i h_i \sum_j Q_{ij} \tag{14}$$

Equation 14a expresses the complete revenue of each route in case the headways are not coordinated.

$$C_{ti} = \frac{R_i}{h_i} B + \left(\frac{J_i}{V_f} + \frac{k\sqrt{n_i A_i}}{2V} + \left(\frac{n_i}{2}\right) t_s\right) v \sum_j (Q_{ij} + Q_{ji}) + \tau v_w h_i \sum_j (Q_{ij} + Q_{ji}) \tag{14a}$$

where n_i is the number of stops in route i and it is formulated as:

$$n_i = \frac{\sum_j Q_{ij} + \sum_j Q_{ji}}{g} h_i \tag{14b}$$

The total cost C_t with headway coordination is formulated as:

$$C_t = \sum_i \frac{R_i}{h} B + \sum_i \sum_j T_{ij} Q_{ij}\, v + \tau v_w h \sum_i \sum_j (Q_{ij} + Q_{ij} \times U_{ij}) \tag{15}$$

<u>3.4 Optimization Method</u>

In this thesis, Figure 1 shows a flexible bus system that serves seven zones with a central terminal. Four service zones (1-4) connect directly to the central terminal (zone 0). The other three service zones, namely zones 5, 6, and 7, must be connected to the central terminal through other zones acting as transfer stations. This study aims to estimate the optimal headway value for the flexible route bus in Figure 1, based on cost analysis, and whether the headway is coordinated or uncoordinated.

Case 1: Service zones without headway coordination

In this case, where the headway of zone 1, zone 2, zone 3, zone 4, zone 5, zone 6 and zone 7 are seven decision variables (h1, h2, h3, h4, h5, h6, h7 respectively). Assume that the parameter of coordination τ is 0.5 and the average transfer time is half the headway of the next bus route the passenger takes. The total cost without headway coordination is formulated as:

$$C_t = \sum_i \frac{R_i}{h_i} B + \sum_i \sum_j T_{ij} Q_{ij}\, v + \tau v_w \sum_i h_i \sum_j Q_{ij} \tag{16}$$

In order to optimize the total cost, the first derivative of the total cost C_t with respect of h_i is set equal to 0. The optimal headway and total cost are obtained from the following equations.

$$\frac{\partial C_t}{\partial h_i} = -\frac{R_i}{{h_i}^2} B + \tau v_w \sum_j (Q_{ij} + Q_{ji}) = 0 \qquad (i = 1, \ldots, 7\ (I)) \tag{17}$$

It is necessary to analyze the second derivative of the total cost C_t with respect of h_i. When the second derivative of total cost C_t with respect of h_i is always positive in the feasible region, the function C_t is strictly convex and has a unique optimal value.

$$\frac{\partial^2 C_t}{\partial {h_i}^2} = \frac{2R_i}{{h_i}^3} B > 0 \qquad (i = 1, \dots, 7\ (I)) \tag{18}$$

The optimal positive headway can be determined by solving Equation 18 and 17. If the optimal headway is used in Equation 16, the minimum total cost can be achieved in this case.

Case 2: Service zones with headway coordination

When the headways are coordinated, there is only one decision variable $h_i = h\ for\ all\ i$ left. Assume that the parameters of the coordination τ is 0.5 and the transfer time is (or close to) 0 if the trips go through the central terminal (zone 0). In this thesis, the coordination of headway is implemented in zone 0.

The cost of transfer will depend on whether the shortest path of the trip passes through the central terminal. The total cost with headway coordination is formulated as:

$$C_t = \sum_i \frac{R_i}{h} B + \sum_i \sum_j T_{ij} Q_{ij}\, v + \tau v_w h \sum_i \sum_j (Q_{ij} + Q_{ij} \times U_{ij}) \tag{19}$$

The first derivative of the total cost C_t with respect of h equal to 0 is formulated as:

$$\frac{\partial C_t}{\partial h} = -\sum_i \frac{R_i}{h^2} B + \tau v_w \sum_i \sum_j (Q_{ij} + Q_{ij} \times U_{ij}) = 0 \tag{20}$$

It is important to check the condition for a function C_t to be strictly convex in order to confirm that it meets the criteria. The second derivative of the total cost C_t with respect of h is:

$$\frac{\partial^2 C_t}{\partial h^2} = \sum_i \frac{2R_i}{h^3} B > 0 \tag{21}$$

Headway coordination can eliminate the cost of transfers, but it also loses the flexibility that can be brought by optimizing each headway according to the characteristics of each route. In this thesis, the optimized costs obtained are compared with and without headway coordination, and the preferred option is the one with the lower cost.

Chapter 4: Numerical Results

Numerical studies were conducted to compare the results and costs of two cases: with and without headway coordination. Baseline parameter values are provided in Table 1. This chapter optimizes the headways of bus routes and discusses two sub-cases based on whether the headways between routes are coordinated. When headways are not coordinated, they are divided into different headways based on the number of zones. In the case of headway coordination, all headways must be equal to reduce transfer delays.

Table 2 presents the assumed line-haul distances and areas obtained with baseline values for seven service zones.

Table 2: Line haul distance and area for each zone

	Zone 1	Zone 2	Zone 3	Zone 4	Zone 5	Zone 6	Zone 7
J_i (miles)	4	5	6	7	5	4	5
A_i (mi^2)	12	30	20	30	20	30	20

The service zones are predetermined, which also means that the areas and line haul distance are given. There is no area represented for the central terminal.

4.1 Case one: baseline results without headway coordination

Tables 3 and 4 present the population, distance, round-trip time for vehicles, one-way passenger travel time, average number of stops, and headways without headway coordination obtained with baseline values.

Table 3: Baseline results without headway coordination

	Route 1	$Route$ 2	$Route$ 3	$Route$ 4	$Route$ 5	$Route$ 6	$Route$ 7

Population in each zone	Zone 1 240	Zone 2 600	Zone 3 400	Zone 4 600	Zone 5 400	Zone 6 600	Zone 7 400
Distance (miles)	4	5	6	7	9	13	11
Round-trip time (hours)	0.9158	1.3845	1.2567	1.5069	1.5767	2.1314	1.6480
Passenger one-way travel time (hours)	0.4412	0.6756	0.6117	0.7368	0.7717	1.0490	0.8074
Avg. number of stops	9.5649	10.6386	10.8713	11.0483	13.2192	14.8067	12.5479
Headway (hours)	0.1665	0.1744	0.1856	0.2129	0.2014	0.2461	0.2793

Table 4: Passenger one-way travel time for the shortest path avoids zone 0 without headway coordination

	T_{16} or T_{61}	T_{15} or T_{51}	T_{37} or T_{73}	T_{56} or T_{65}
Passenger one-way travel time (hours)	1.2569	0.9796	1.0857	1.2707

From Table 3, it should be noted that the population refers to the population of zones that can be reached by each route. Distance refers to the distance from the central terminal to the separate zones. However, passengers traveling on routes 5, 6, and 7 must transfer at specific zones to reach their final destination. Passenger one-way travel time refers to the time it takes for a passenger to travel one-way on different routes. It can be found that when the demand and distance of a certain route increase, the travel time and stopping points required for a flexible route bus trip also increase. Headways are optimized by minimizing the total cost. Table 4 shows the travel times for passenger one-way trips between zones without transferring at the central terminal. When traveling through special zone-to-zone itineraries without transferring at the central terminal, it's important to consider both the stopping time and tour time in both zones simultaneously. This often results in longer travel times compared to trips starting from central terminals.

4.2 Case two: baseline results with headway coordination

Tables 5 and 6 list the population, distance, vehicle round-trip time, one-way passenger travel time, average number of stops and headway based on the baseline values in the case of headway coordination.

Table 5: Baseline results with headway coordination

	Route 1	$Route$ 2	$Route$ 3	$Route$ 4	$Route$ 5	$Route$ 6	$Route$ 7
Population in each zone	Zone 1 240	Zone 2 600	Zone 3 400	Zone 4 600	Zone 5 400	Zone 6 600	Zone 7 400
Distance (miles)	4	5	6	7	9	13	11
Round-trip time (hours)	1.0447	1.5537	1.3652	1.5484	1.6481	2.0778	1.5437
Passenger one-way travel time (hours)	0.5057	0.7602	0.6659	0.7575	0.8074	1.0222	0.7552
Avg. number of stops	13.0851	13.8925	13.3398	11.8231	14.9504	13.7052	10.2336
Headway (hours)	0.2278						

Table 6: Passenger one-way travel time for the shortest path avoid the zone 0 with headway coordination

	T_{16} or T_{61}	T_{15} or T_{51}	T_{37} or T_{73}	T_{56} or T_{65}
Passenger one-way travel time (hours)	1.2946	1.0797	1.0878	1.2796

The first row and column of Tables 5 and 6 show similar content, but different results as Tables 3 and 4 for each route such as population, distance, and round-trip time. It should be noted that headway coordination can make travel faster and more efficient for both passengers and vehicles, especially in situations with low population and long distances. Compared with the case of uncoordinated headway distance, headway coordination can reduce vehicle trip time, passenger travel time, and the number of stops in low-demand and long-distance situations. If demand increases and distance decreases, the benefits brought by headway coordination will be reduced, resulting in increased vehicle trip time, passenger travel time, and number of stops compared to uncoordinated headway.

Figure 2 shows the sensitivity of each cost to the headway in the case of headway coordination. From the figure, it can be found that the total cost decreases sharply at first and then gradually increases as the headway increases. The minimum total cost in Figure 2 also shows the optimality of the headway in Table 5. Within a certain period of time, as the headway grows, the number of needed buses decreases, resulting in a decrease in bus fleet size and operation cost. In this thesis, with fixed demand. increases in headways increase the passenger travel times and waiting times since there are more passengers and stops per tour. This also leads to increased operation and waiting costs.

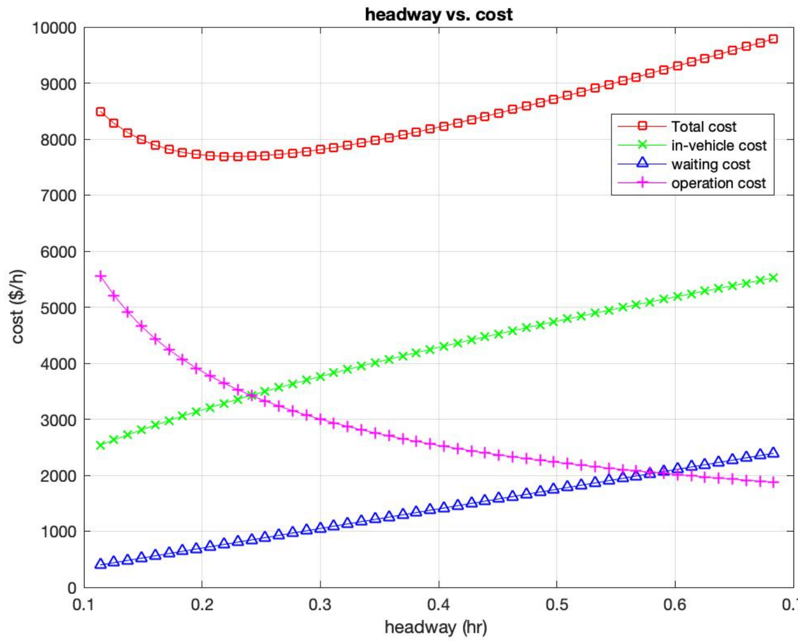

Figure 2 Headway vs. costs with headway coordination

4.3 Case three: baseline results with headway coordination and integer ratios of round-trip time/headway

Table 7 presents the vehicle round-trip time, one-way passenger travel time, and average number of stops obtained based on the baseline values when the vehicle round-trip time is an integer multiple of the headway.

In this case, the equation 7 can be rewritten as:

$$R_i = h * X \tag{22}$$

where X is the smallest integer for which $h * X$ exceeds the vehicle round-trip time on route i. The new round-trip time is just larger than the round-trip time obtained under the coordination of the headway.

Table 7: Results based on integer ratios of round-trip time/headway

	Route 1	$Route\ 2$	$Route\ 3$	$Route\ 4$	$Route\ 5$	$Route\ 6$	$Route\ 7$
Round-trip time (hours)	1.1390	1.5946	1.3668	1.5946	1.8224	2.2780	1.5946

Passenger one-way travel time (hours)	0.5057	0.7602	0.6659	0.7575	0.8074	1.0222	0.7552
Avg. number of stops	13.0851	13.8925	13.3398	11.8231	14.9504	13.7052	10.2336

In order to minimize the total transfer time and cost, it is desirable to relate the complete vehicle round-trip time to the headway of each route. The headway ($h = 0.2278$) used in case three is the same as the case of headway coordination, and then the vehicle round-trip time is adjusted to the smallest integer multiple of the headway that allows a complete vehicle round-trip.

4.4 Costs for three cases

Table 8 shows the operation cost, in-vehicle cost, waiting cost, and total cost with headway coordination, without headway coordination, and with headway coordination based on round-trip time with integer headway.

Table 8: Costs for three cases

	Without headway coordination	With headway coordination at zone 0	With headway coordination and integer ratios of round-trip time/headway
Vehicle operating cost ($/h)	3.7254×10^3	3.5496×10^3	3.7500×10^3
Passenger in-vehicle cost ($/h)	3.2036×10^3	3.3416×10^3	3.3416×10^3
Waiting cost ($/h)	992.3630	797.0354	797.0354
Total cost ($/h)	7.9214×10^3	7.6883×10^3	7.8886×10^3

The difference in total cost between headways coordinated without adjusting the vehicle round-trip times and the uncoordinated headways is formulated as:

$$Difference = \frac{C_{co} - C_{unco}}{C_{unco}} \times 100\% \quad (23)$$

where C_{co} is the total cost of headways coordinated without adjusting the vehicle round-trip times and C_{unco} is the total cost of headway uncoordinated. The difference in total cost between headways coordinated without adjusting the vehicle round-trip times and the uncoordinated headways is -2.9427%. By using the same equation and replacing the total cost of uncoordinated headway with the total cost of coordinated headway by adjusting the vehicle round-trip time, it can be found that the total cost difference between coordinated headway with and without adjusting the vehicle round-trip time is-2.5391%.

In the case of flexible route buses with uncoordinated headways, the average transfer time is τ multiply the headway of the next bus routes taken by the passenger. The waiting cost includes the expenses associated with waiting and transferring. In the case of headway coordination, all headways are equal, and bus arrivals and departures are synchronized at zone 0, which allows transfer times there to approach zero. However, if the passenger can avoid zone 0 as a transfer station while traveling between zones, transfer costs should be considered. Compared with case of uncoordinated headways, it can be found that the headway coordination condition reduces the transfer time generated by transfer through zone 0, resulting in a reduction in waiting costs. The reduction in total cost also shows that coordinated. In the case of headway coordination based on round-trip time with integer headway, the adjusted vehicle round-trip time affects the optimized operator cost, resulting in increased total cost. This shows that headway coordination without adjusting the vehicle round-trip time is preferred in this situation with low demand.

4.5 Sensitivity Analyses

This section studies how changes in input parameters in the optimization model affect the output. This is useful for understanding variables and reducing the cost of flexible route buses. This section provides sensitivity analysis based on demand (population density), value of waiting time/in-vehicle time, bus operator cost, stopping time, bus speed, line-haul distance, and number of persons per group. The objective is to analyze how these parameters affect the output costs.

4.5.1 Case one: Sensitivity analyses without headway coordination

Figure 3 shows that as population density increases from 10 persons/sq. mile to 60 persons/sq. mile. The cost of flexible route buses increases with population density due to higher passenger demand. The increase in in-vehicle cost is higher than waiting and bus operator costs because in-vehicle time has the greatest impact compared with other factors. Passengers need to spend the most time on the bus. As the population density increases, bus frequency also rises to meet the demand, leading to longer bus trip time and in-vehicle times. When the population density reaches around 37.72 persons/sq. mile, the increase in passengers affects in-vehicle time more than bus fleet size and causes in-vehicle cost to exceed operation cost.

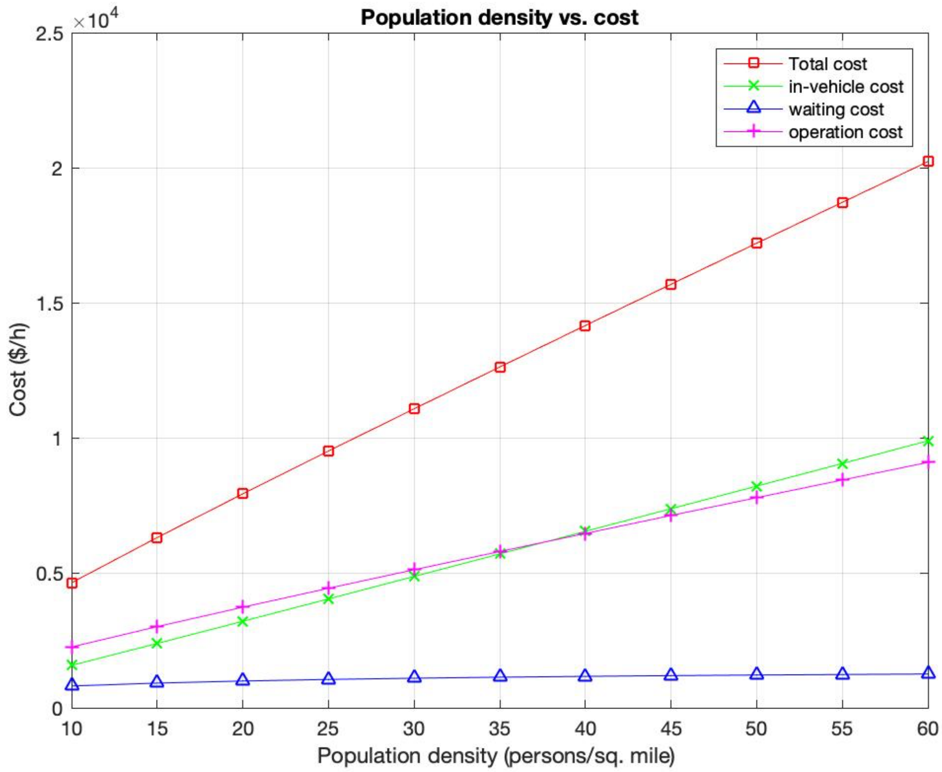

Figure 3 Costs vs. population density without headway coordination

Figure 4 illustrates that an increase in population density results in more trips being made, making it more cost-effective to allocate the cost to those trips. Therefore, the average waiting cost, average operation cost, and average total cost will decrease as the population density increases. As the number of trips increases, the average in-vehicle cost also increases due to longer passenger travel time.

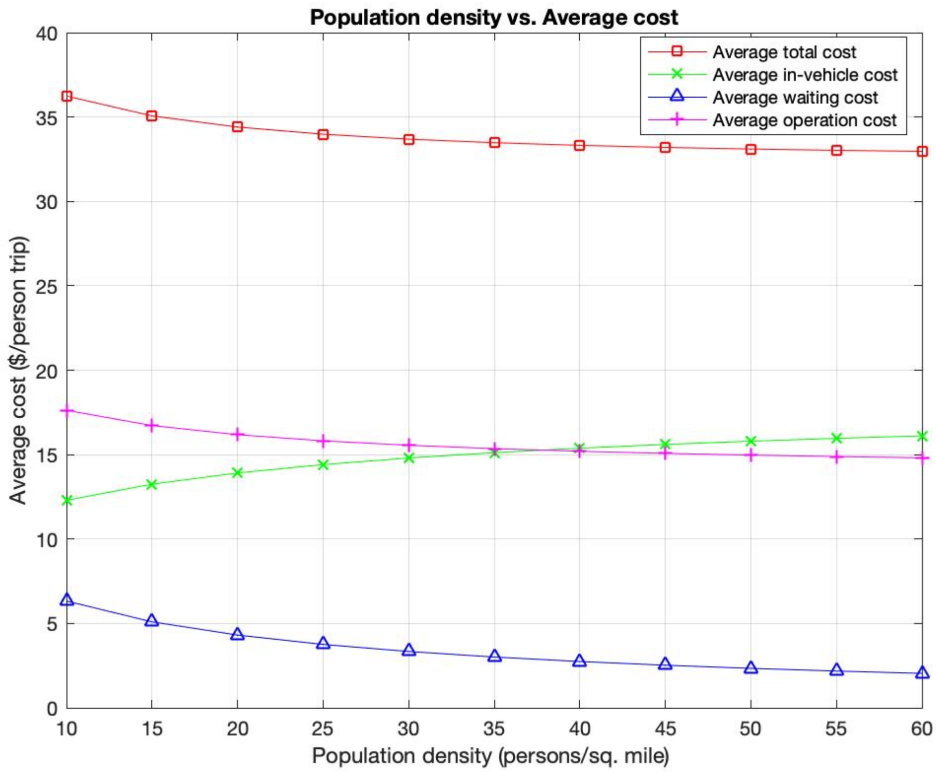

Figure 4 Average costs vs. population density without headway coordination

Figures 5 and 6 show the sensitivity of each cost to the value of waiting time and in-vehicle time without headway coordination. As can be seen from Figure 5, when the value of waiting time increases, the optimized headway decreases to ensure the optimal waiting cost. The reduction in headway indirectly reduces in-vehicle costs by decreasing the number of stops and tour length in each zone. As the value of waiting time increases, the operation cost, waiting cost, as well as total cost, will also increase. Figure 6 illustrates that the in-car cost, operation cost, and total cost increase as the value of in-vehicle time increases. As the value of in-vehicle time increases, the optimized headway decreases to minimize in-vehicle cost. Here, the demand and waiting time values are assumed to be fixed. As the frequency of buses decreases, the cost of waiting also decreases.

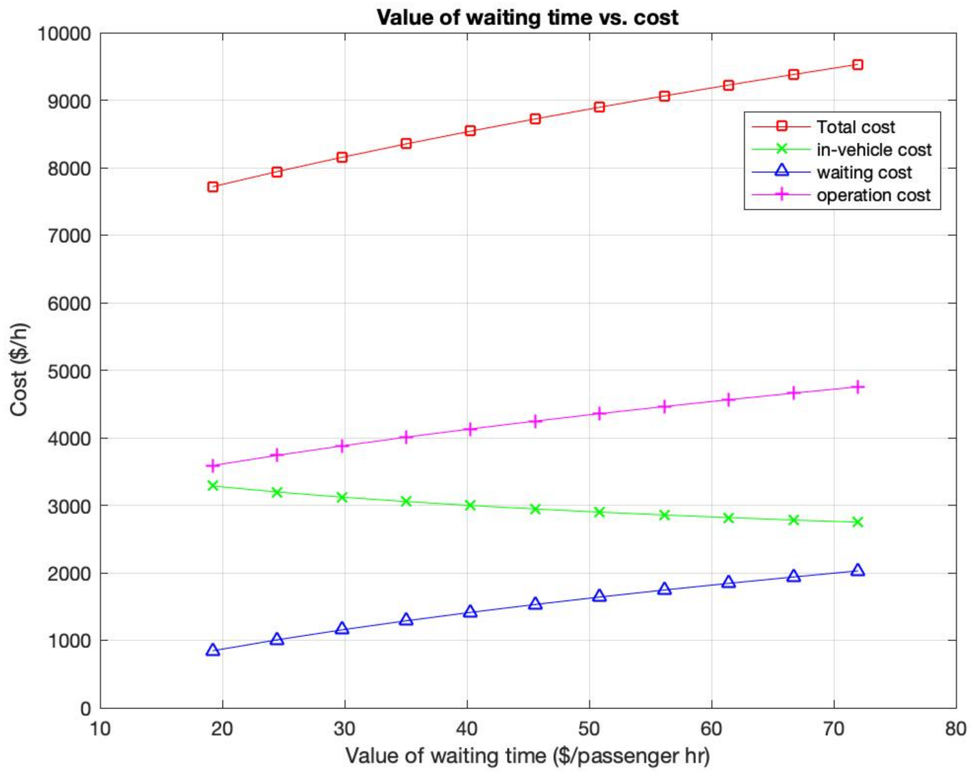

Figure 5 Costs vs. value of waiting time without headway coordination

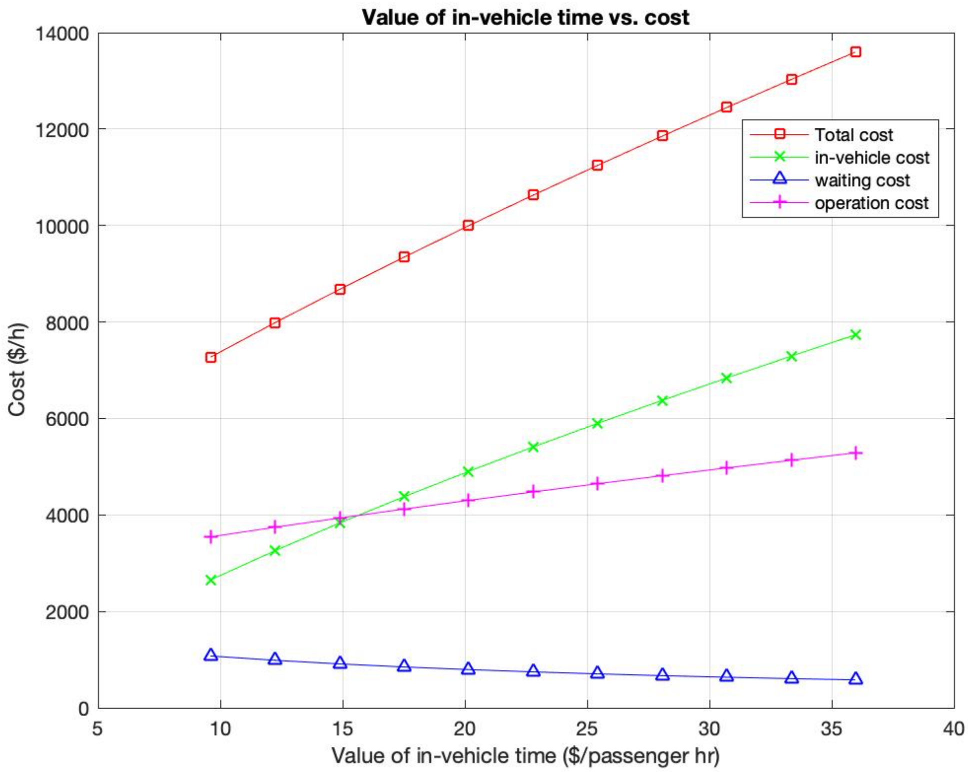

Figure 6 Costs vs. value of in-vehicle time without headway coordination

As illustrated in Figure 7, when the cost for bus operators increases, the operation cost, the in-vehicle cost and the total cost also increase. It should be noted that as the cost for the bus operator increases, the fleet size decreases accordingly in order to optimize operational cost, leading to an increase in headway. As the duration of passenger travel increases based on the increase in the number of stops, the in-vehicle costs will also increase correspondingly.

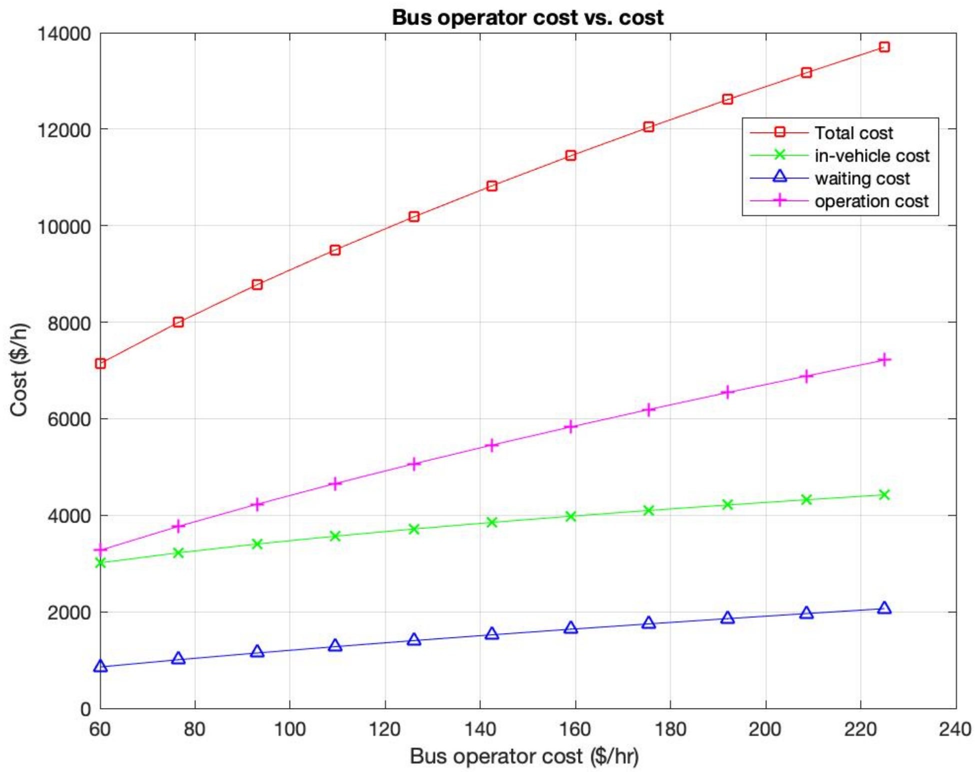

Figure 7 Costs vs. bus operator cost without headway coordination

Figure 8 shows the sensitivity of each cost to stopping time without headway coordination. The increase in stopping time affects bus trip time and passenger travel time, which in turn increases the operation cost and in-vehicle cost. In order to reduce operation and in-vehicle costs, the headway is optimized by reducing it. This also results in a reduction in waiting costs as the value of waiting time is fixed.

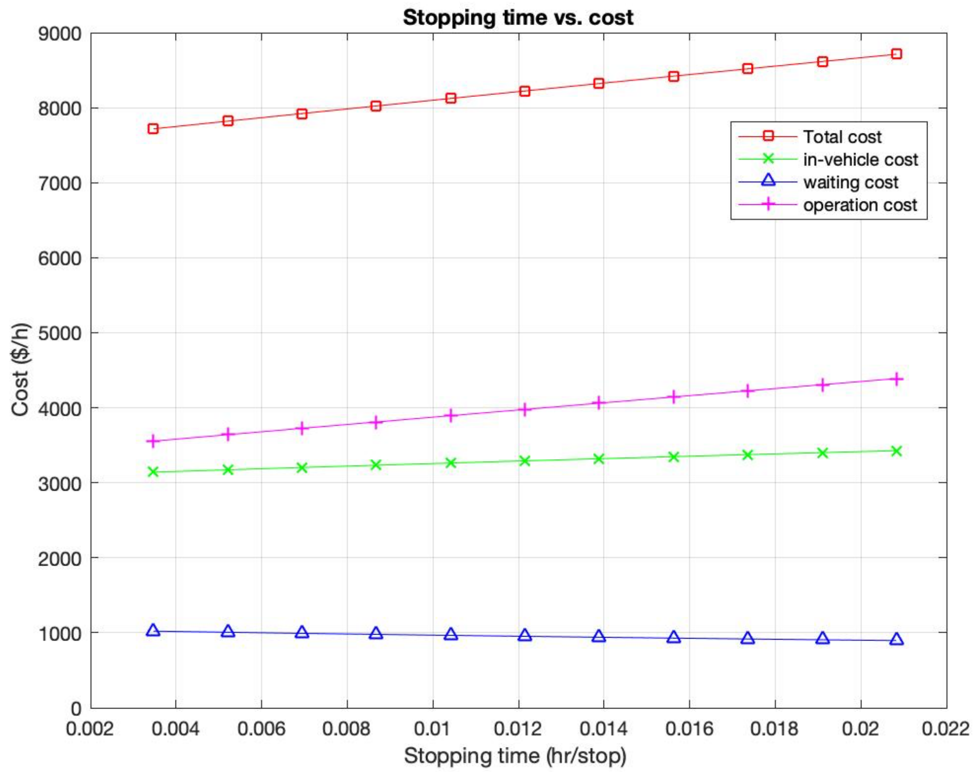

Figure 8 Costs vs. stopping time without headway coordination

Figures 9 and 10 show the sensitivity of each cost to bus speed in the local service zone and line haul without headway coordination. When the speed of the bus increases, the operation cost, in-vehicle cost, and the total cost decrease rapidly at first and then gradually stabilize. This is due to the fact that both the travel time of the bus and the time passengers spend on the bus are reduced. When the bus speed increases to a certain level, the impact on cost will gradually decrease because the change in time will also gradually decrease with the increase in speed. By comparing Figures 9 and 10, it is evident that an increase in bus speed within a local service zone will result in a more substantial cost reduction than an increase in bus speed in line haul. This is because the driving distance of buses in a local service zone is longer than that in line haul.

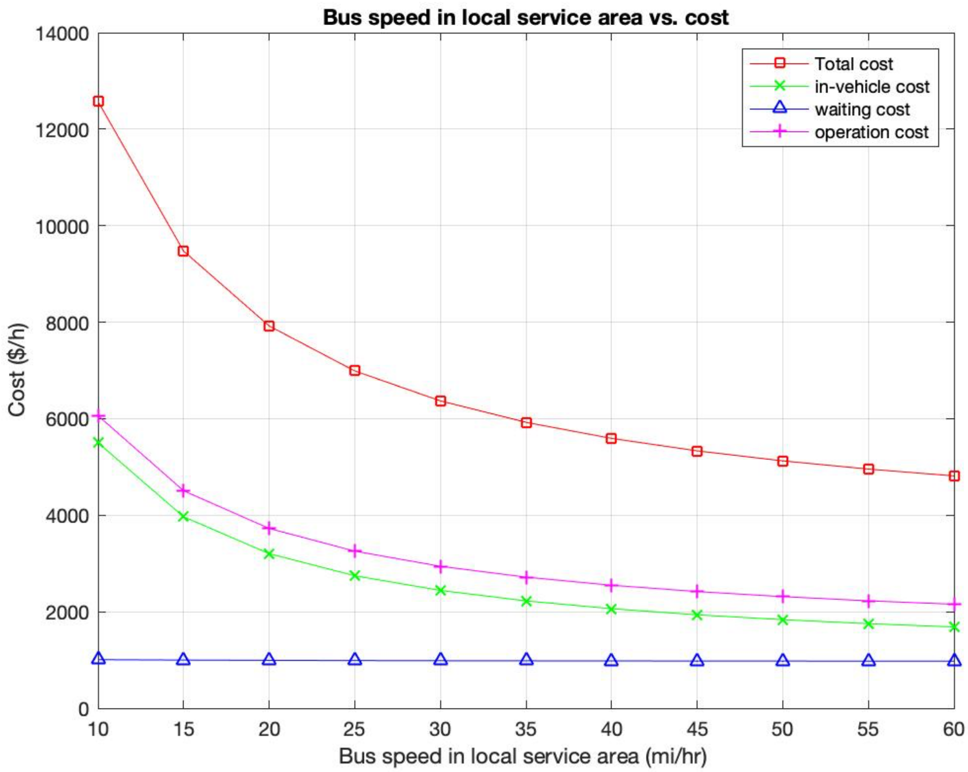

Figure 9 Costs vs. bus speed in local service zone without headway coordination

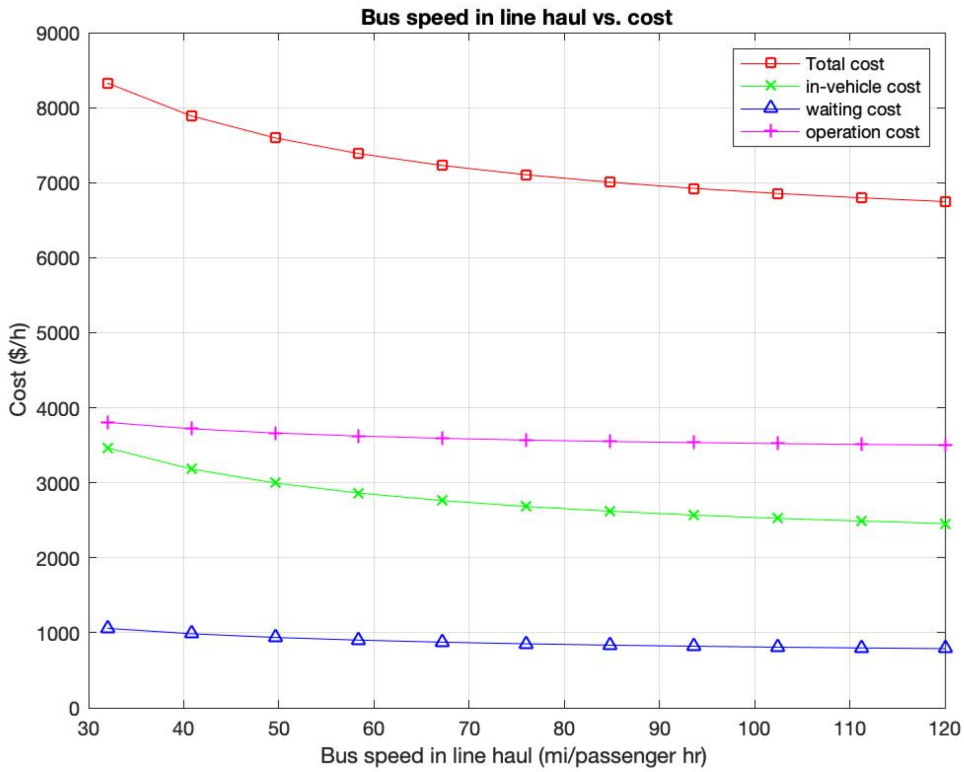

Figure 10 Costs vs. bus speed in line-haul without headway coordination

As shown in Figure 11, with an increase in line haul distance, there is also an increase in each cost. This is because the distance covered by the bus increases as the line haul distance increases. This also indirectly leads to an increase in bus trip time, bus fleet size, and passenger time on the bus. This also demonstrates that the growth rate of in-vehicle and operation costs is greater than that of waiting costs. When the line-haul distance change rate exceeds approximately 181.5%, the in-vehicle cost increase surpasses the operational cost. As the line haul distance increases, the time spent by buses on the line haul will gradually exceed the time spent on the local service zone.

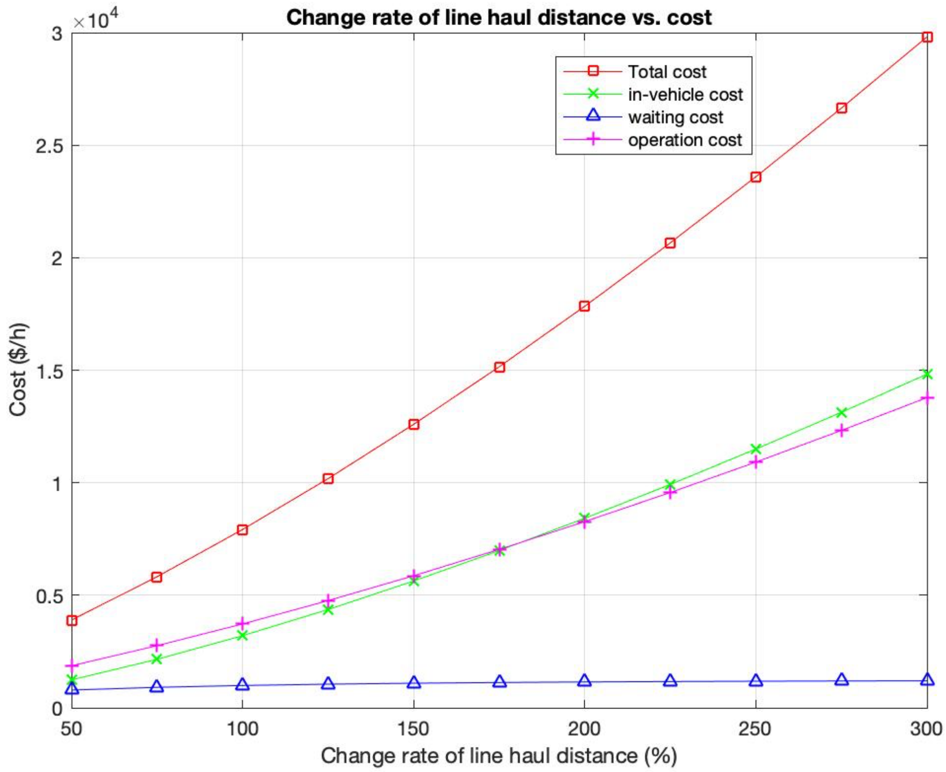

Figure 11 Costs vs. line-haul distance multiplier without headway coordination

Figure 12 shows the sensitivity of each cost to the number of people per group without headway coordination. In this instance, the value of headway remains unaltered. There is no impact on waiting costs with an increase in the number of people per group, as shown in the figure. This is because increasing the number of people per group only reduces bus trip time and passenger travel time, as well as operation and in-vehicle costs.

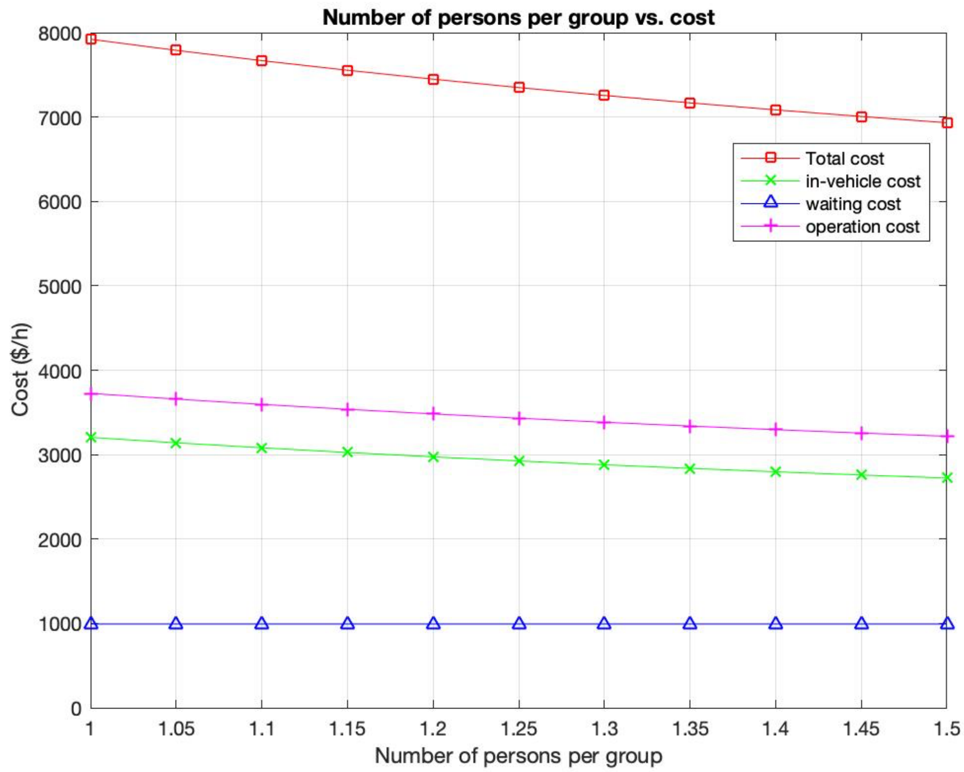

Figure 12 Costs vs. persons per group (without change the headway and headway coordination)

4.5.2 Case two: Sensitivity analyses with headway coordination

Figure 13 shows the sensitivity of each cost to population density with headway coordination. The variables' range and situation are similar to Figure 3 without head coordination. It should be noted that the in-vehicle cost exceeds the operation cost when the population density is about 26.14 persons/sq. mile. This indicates that the increase in passengers is likely to have a greater impact on travel time than the bus fleet size compared to the case without headway coordination.

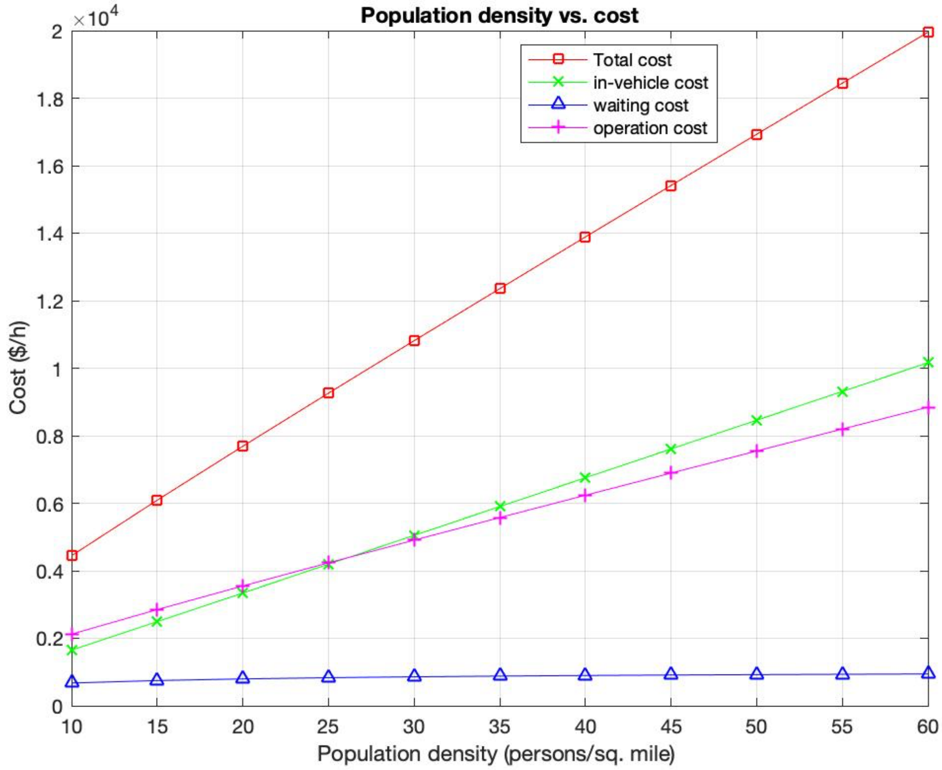

Figure 13 Costs vs. population density with headway coordination

Figures 14 to 16, respectively, show the sensitivity of each cost to the value of waiting time, value of in-vehicle time, and bus operator cost under headway coordination. In order to achieve the optimal waiting cost and in-vehicle cost, the decrease in the optimal headway is caused by an increase in the value of waiting time and in-vehicle time. In Figure 14, reducing the optimal headway indirectly reduces in-vehicle costs by decreasing the number of stops and overall tour length in each zone. In Figure 16, the bus fleet size will decrease due to the growing bus operating costs, resulting in an increase in optimal headway. When the population density is lower, Figures 14 to 16 show lower total costs compared to Figures 5 to 7 in case one. This indicates that the total cost is preferable with headway coordination and low demand.

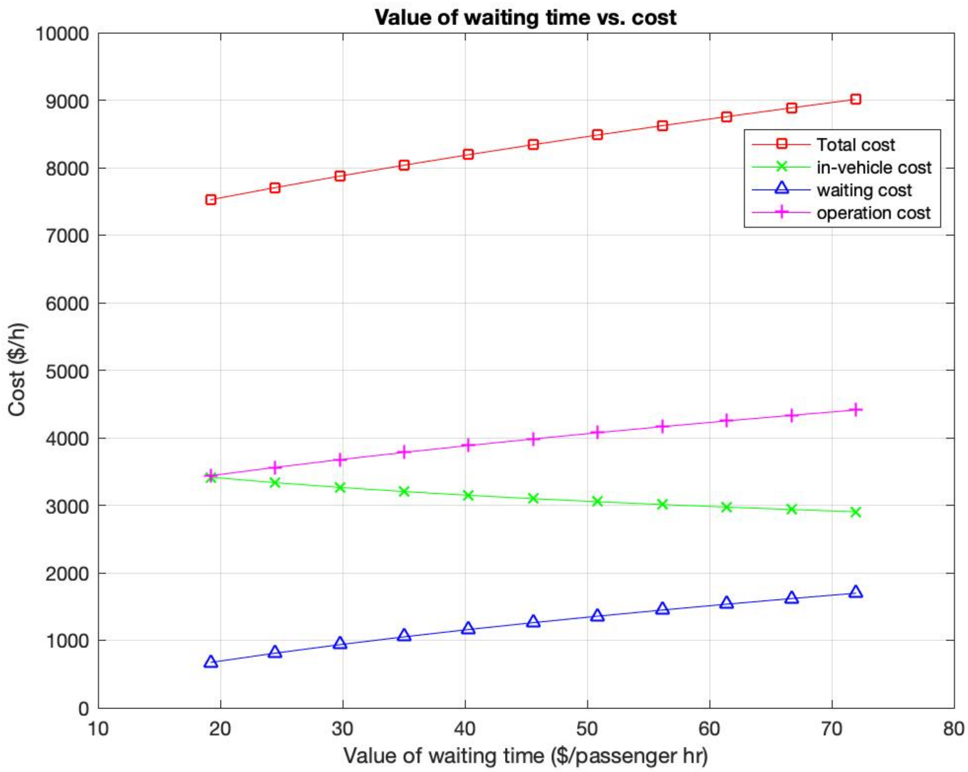

Figure 14 Costs vs. value of waiting time with headway coordination

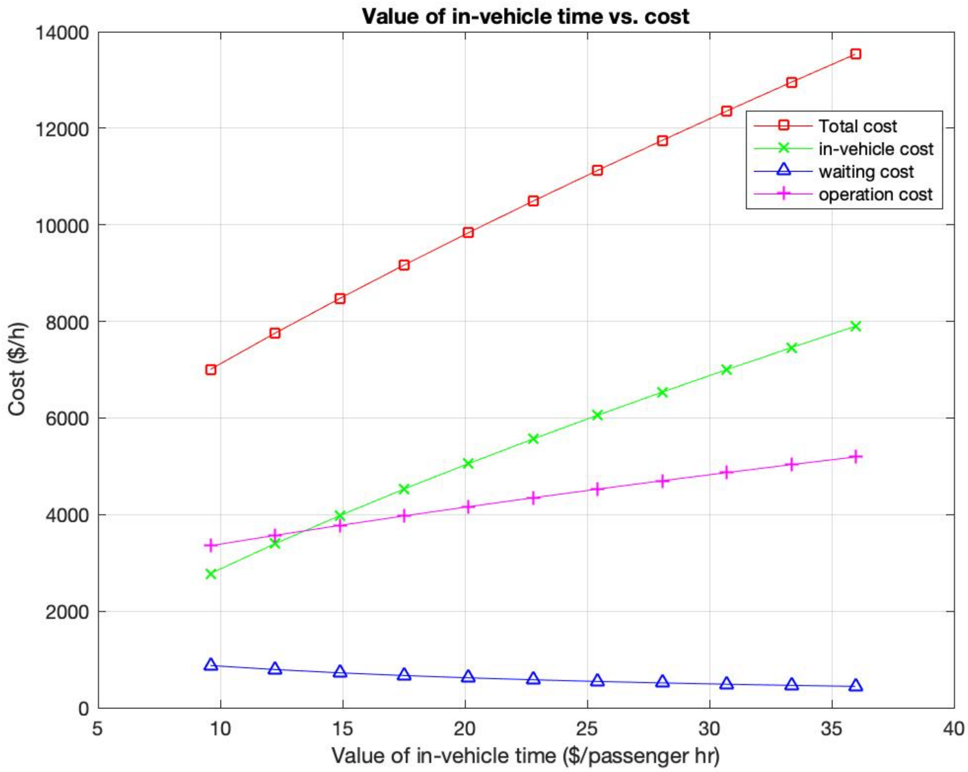

Figure 15 Costs vs. value of in-vehicle time with headway coordination

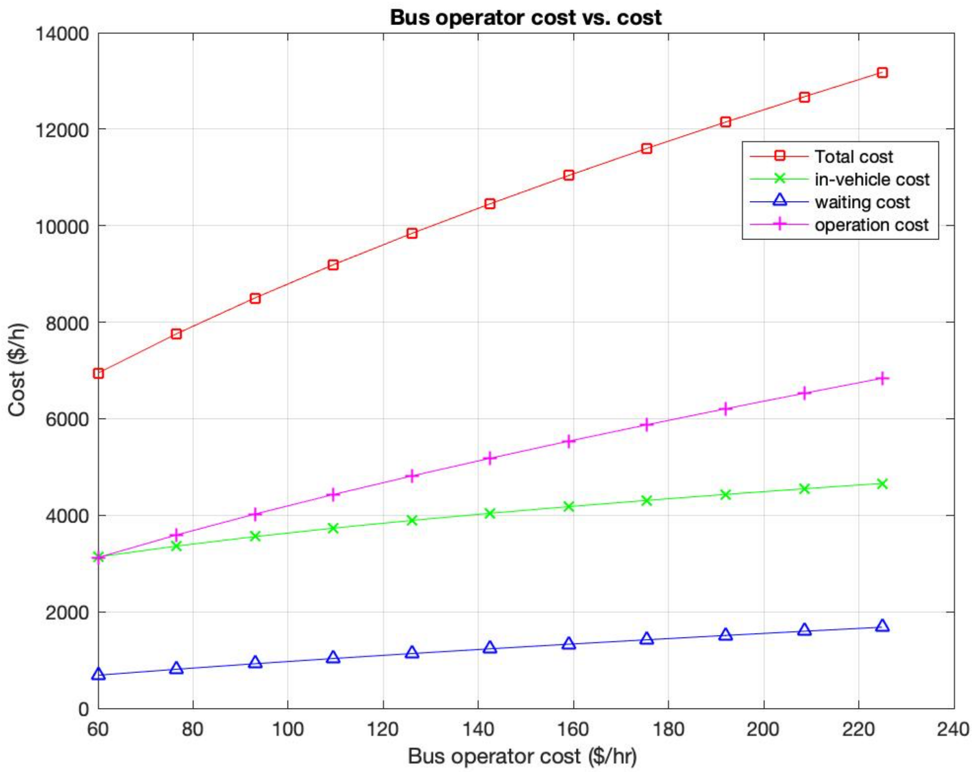

Figure 16 Costs vs. bus operator cost with headway coordination

Figures 17 to 21 show the sensitivity of each cost to stopping time, bus speed in local service zone/line haul, line haul distance, and number of persons per group with headway coordination. Once the headway is coordinated, the headway remains consistent across all routes. When the coordinated condition has a smaller headway than the uncoordinated condition, round-trip and passenger travel times decrease. In this thesis, coordinated headway is preferred over uncoordinated headway due to smaller distance, area, and demand, resulting in lower costs.

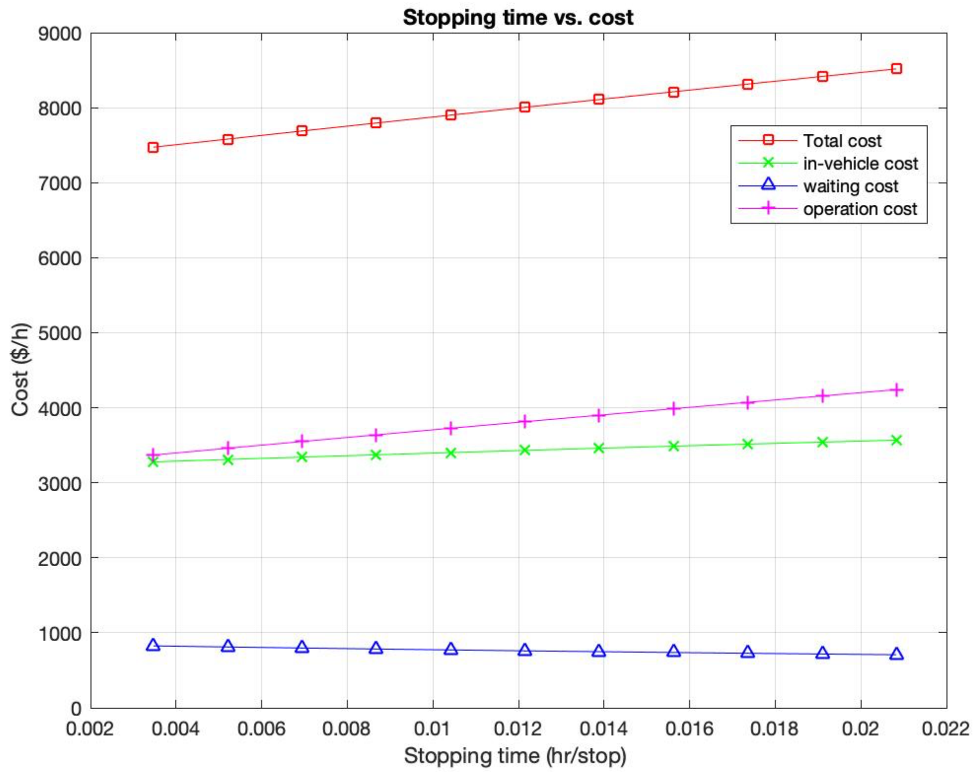

Figure 17 Costs vs. stopping time with headway coordination

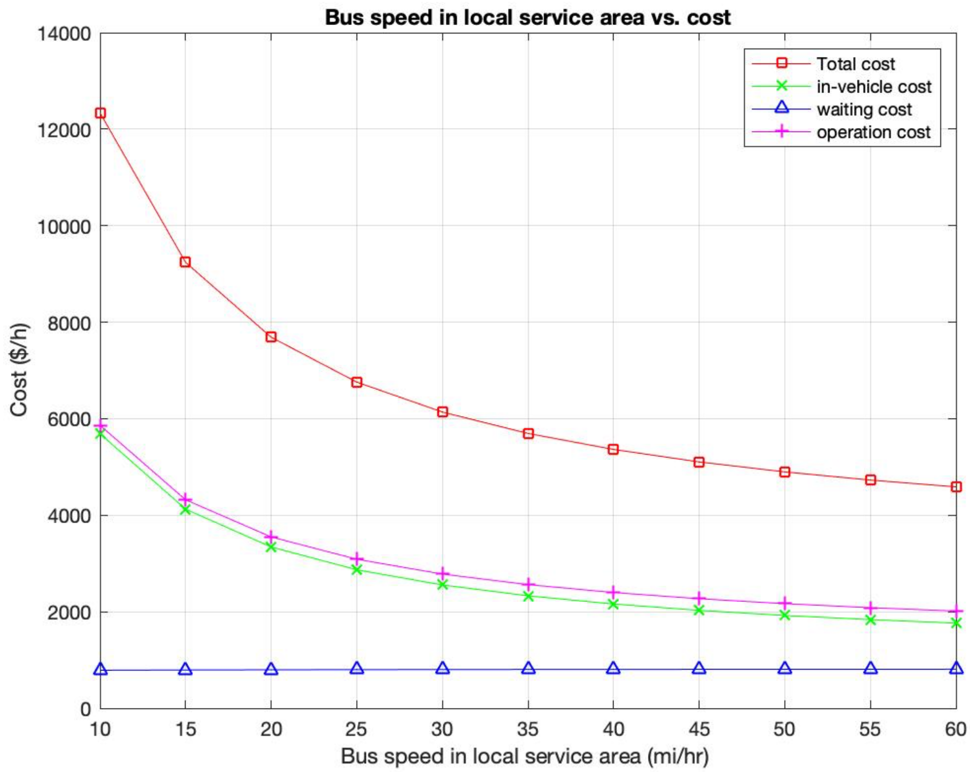

Figure 18 Costs vs. bus speed in local service zone with headway coordination

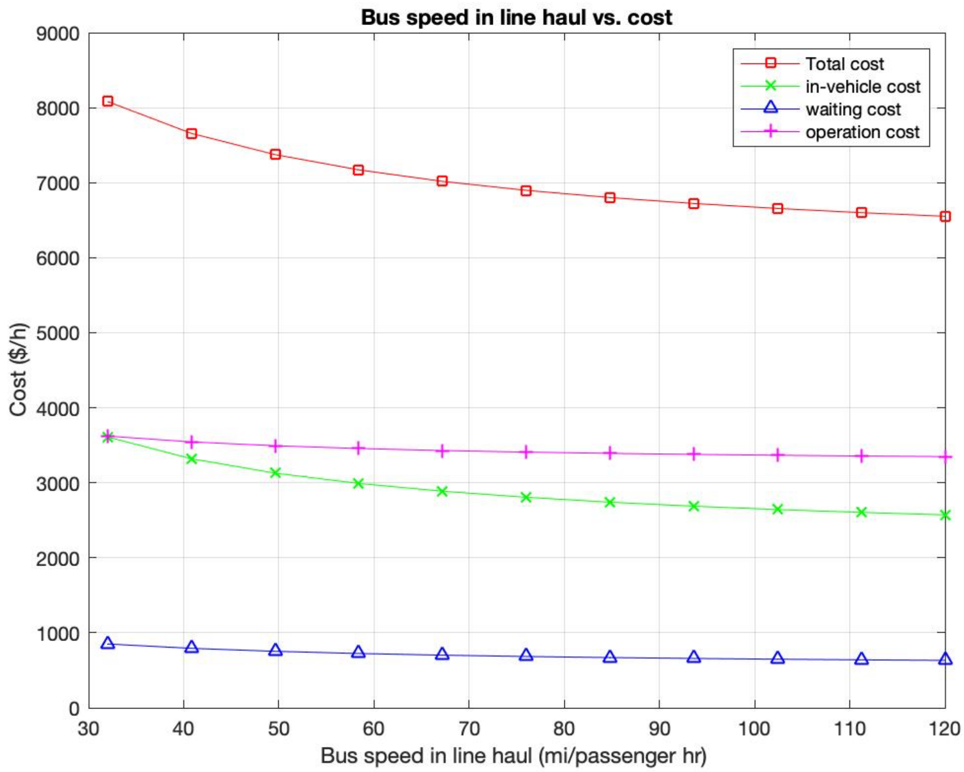

Figure 19 Costs vs. bus speed in line-haul with headway coordination

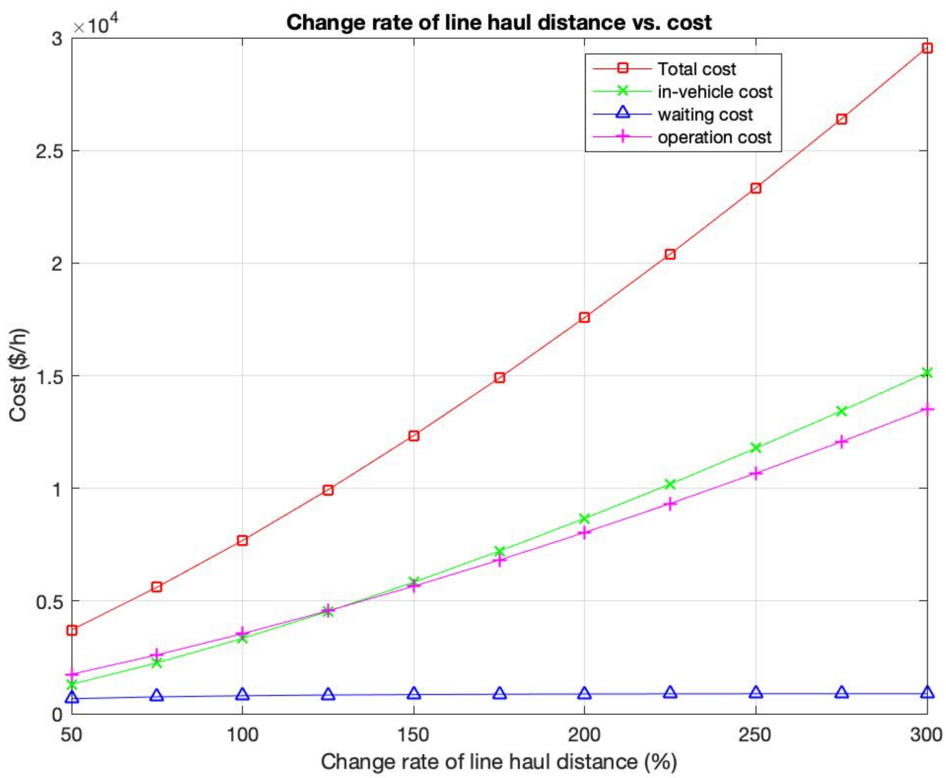

Figure 20 Costs vs. line-haul distance multiplier with headway coordination

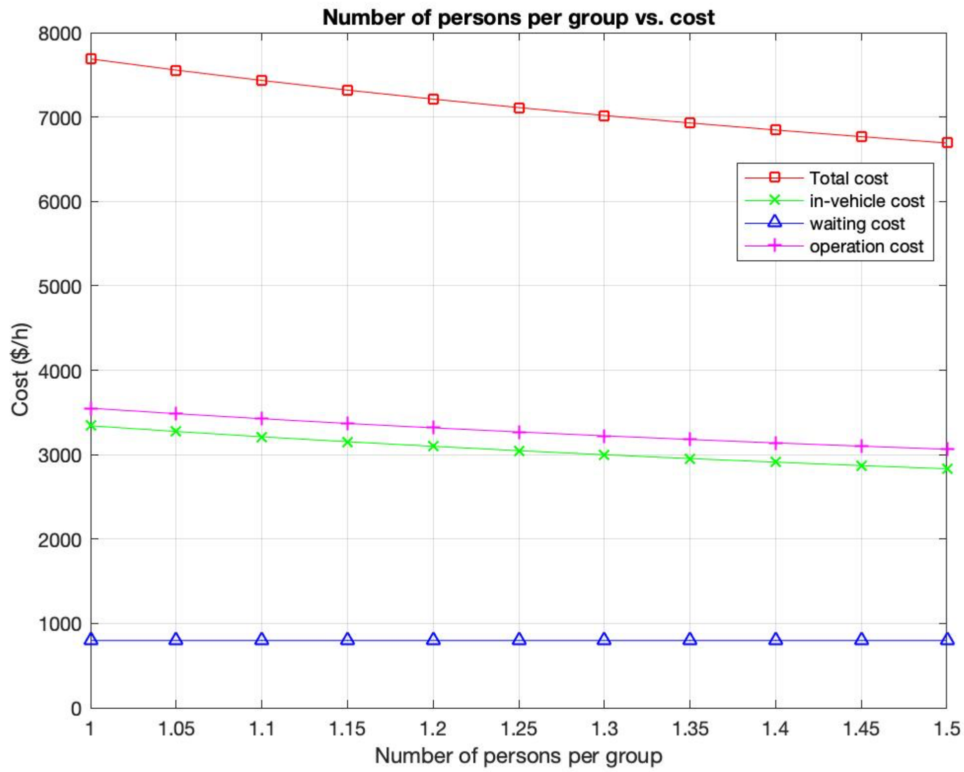

Figure 21 Costs vs. number of persons per group (without change the headway and with headway coordination)

4.5.3 Case three: Sensitivity analyses with headway coordination alternatives

Figure 22 shows the sensitivity of the average cost to population density under coordinated headway, uncoordinated headway, and coordinated headway based on integer ratios of round-trip time/headway. The average cost per trip is obtained by dividing the total cost by the number of trips. From the figure, it can be observed that the average cost rapidly decreases and then stabilizes with increasing population density. Due to the relatively small baseline values, such as the zone area and travel distance, the difference in average cost resulting from coordination or incoordination of headway is insignificant. As demand for transportation increases, the frequency of service will be adjusted to meet passenger needs. When demand is low, the total cost only needs to satisfy a small portion of passenger demand. Among them, bus operation service costs account for the largest proportion. The rate of increase in total cost is lower than the rate of increase in the number of trips. As demand increases, the total cost will be divided into more trips, resulting in a lower average cost. When the population density reaches a certain level, the proportion of in-vehicle cost gradually increases, as the impact of passenger travel time exceeds that of bus trip time. As demand increases, the number of trips increases less and results in a smaller reduction in average costs. In the case of headway coordination based on integer ratios of round-trip time/headway, the round-trip time changes only by the smallest integer multiple of the headway. It will appear that the round-trip time is not at the same time when rounded up, resulting in uneven results. When demand increases, the benefits caused by headway coordination will gradually decrease. When the demand is low, it is preferable to have flexible routes bus that are coordinated with headways.

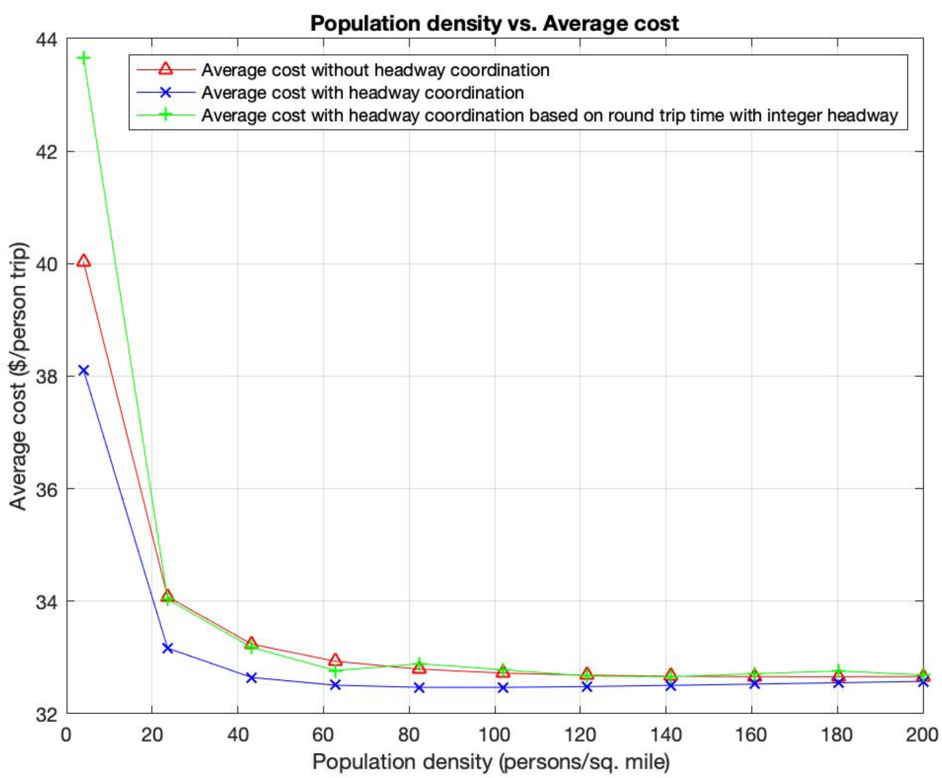

Figure 22 Average cost vs. population density without headway coordination, with headway coordination and with headway coordination and integer ratios of round-trip time/headway

Figure 23 shows the sensitivity of the average cost to the value of in-vehicle time under coordinated headway, uncoordinated headway, and coordinated headway based on integer ratios of round-trip time/headway. As the value of in-vehicle time increases, the average cost also increases. As the in-vehicle time value increases, the average cost of headway coordination and the average cost of headway incoordination gradually approach. This shows that when the total number of trips remains unchanged, as the value of in-vehicle time increases, the impact of the value of in-vehicle time on in-vehicle cost becomes more significant with headway coordination than without it. The uncoordinated headway is more competitive when the value of in-vehicle time is larger. Compared to headway coordination, coordinating headway based on integer ratios of round-trip time/headway incurs a higher average cost.

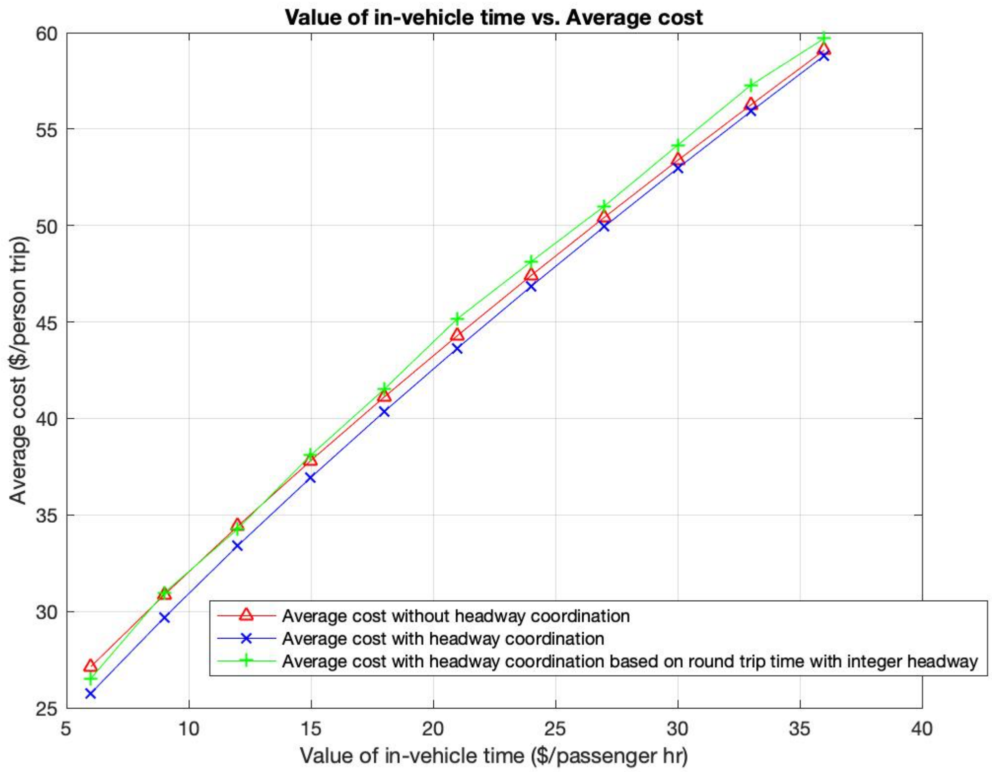

Figure 23 Average cost vs. value of in-vehicle time without headway coordination, with headway coordination and with headway coordination and integer ratios of round-trip time/headway

Figure 24 shows the sensitivity of the average cost to the exponent of the impedance function for coordinated headway, uncoordinated headway, and coordinated headway based on integer ratios of round-trip time/headway. z (the exponent of impedance function) is the elasticity of demand Q_{ij} with respect to impedance. From the figure, it can be observed that increasing the exponent of the impedance function leads to an increase in the average cost. The growth of the exponent of the impedance function affects demand, resulting in a reduction in the number of trips and an increase in headway. The average cost increases as the total cost is spread over fewer trips. As the exponent of the impedance function increases, the change in the number of trips decreases. Eventually, the number of trips approaches zero, resulting in average costs being closer across all three cases. Coordinating the headways of flexible route buses in low demand situations is still preferable.

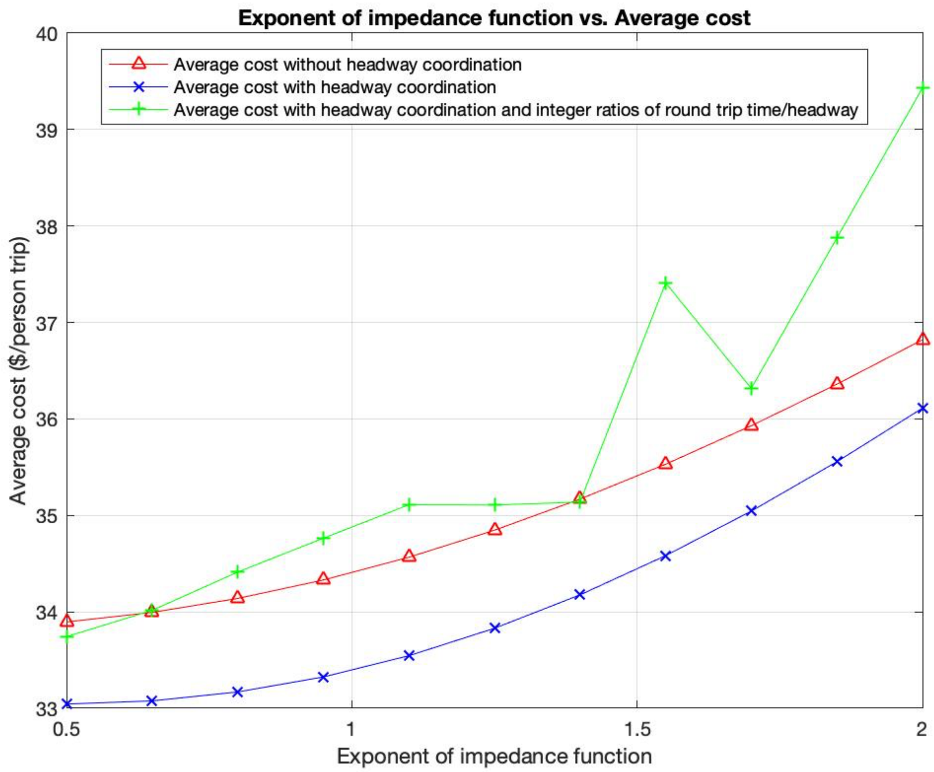

Figure 24 Average cost vs. exponent of impedance function without headway coordination, with headway coordination and with headway coordination and integer ratios of round-trip time/headway

Chapter 5: Conclusion

This thesis analyzes a flexible route bus system serving multiple rectangular-shaped zones and a central terminal to evaluate the impact of flexible route buses on the transportation system. A benchmark model is proposed, and the model is remodeled based on the extension of coordinated and uncoordinated operations. These explore the impact of coordinated and uncoordinated headways on flexible route buses. When demand decreases, coordination can lead to benefits such as decreasing costs due to reduced travel time. By using reasonable baseline parameter values and comparing the resulting travel time, number of stops, costs, and other parameters, the coordinated headway will result in less total cost compared to a non-coordinated headway. Finally, sensitivity analyses are comparatively conducted for the proposed modes. Sensitivity analyses indicate that increases in demand while keeping the zone area constant increase service frequency, thereby reducing the average cost. This study also has some limitations. The total demand will change with factors such as waiting time, in-vehicle time, and fares in the actual transportation bus system. It is necessary to revise the demand in this study for future research. In real-world scenarios, areas with lower demand in the zones affect bus system planning, leading to longer access and waiting times for some passengers. The assumption of uniform distribution in the zone should be modified in future research.

This study can also be improved in several aspects:

1. The model can be extended to include additional regions and network structures with varying shapes and multiple transfer terminals. This

thesis only analyzes the situation of multiple rectangular zones and one central terminal, which is very limited.

2. The time and additional cost caused by congestion can be considered.
3. Including conventional buses and other transportation modes for analysis and comparison is possible.
4. The modeling process can be updated based on changes in demand since demand changes over time in the real world.
5. Consider the variability of travel times and the use of optimized slack times to minimize transfer delays.
6. Consider real-time dispatching and other control decisions when vehicles deviate from their schedules.

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