ABSTRACT

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Emerging markets are more volatile and face different types of shocks, in size and nature, compared to their developed counterparts. Accurate identification of the stochastic properties of shocks is difficult. We show evidence suggesting that uncertainty about the underlying stochastic process is present in commodity prices. In addition, we build a dynamic stochastic general equilibrium model with informational frictions, which explicitly considers uncertainty about the nature of shocks. When formulating expectations, the economy assigns some probability to the shocks being temporary even if they are actually permanent. Parameter instability in the stochastic process implies that optimal saving levels (debt holdings) should be higher (lower) compared to a process with fixed parameters. Imperfect information about the nature of shocks matters when commodity GDP shares are high. Thus, economic policies based on misperception of the underlying regime can lead to substantial over/under saving with important associated costs.

Later, I introduce the first example of a particular class of preferences characterized by a negative third derivative and a constant and invariant coefficient of relative prudence in the sense of Kimball (1990). This particular feature enables us to isolate the effect of risk aversion on precautionary savings. Furthermore, I use this particular class of preferences to assess the effects of volatility, risk aversion, interest rates and intertemporal distortions on precautionary savings in finite and infinite horizon models of a small open economy. The effects of risk aversion, intertemporal distortions and interest rates on average assets holdings are qualitatively identical as the ones observed for CES preferences. Using an infinite horizon model I can evaluate the effects of persistence and volatility of shocks on precautionary savings and verify that these are qualitatively identical to the ones observed with CES preferences.

PRECAUTIONARY SAVINGS IN SMALL OPEN ECONOMIES.

By

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# Dedication

To Anahi, Eduardo, Valentina and Cuca

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# Chapter 1

# Introduction

Macroeconomic theory and empirical evidence show that uncertainty has important effects on macroeconomic performance (i.e. economic growth, saving and investment).[1] In particular, saving behaviour and current account balances in emerging markets are affected in very different ways depending on the type of shocks (i.e. temporary versus permanent). This phenomenon is particularly pervasive among commodity exporters, where high levels of volatility and uncertainty are common.[2]

However, most of the economic literature explaining saving behaviour in emerging markets has not focused on studying the relationship between uncertainty about the nature of shocks and optimal saving levels. Thus, a comprehensive understanding, a more realistic characterization of uncertainty, and its effects on economic performance and saving behaviour, are all still unexplored issues in the profession.

In terms of the specific determinants of precautionary savings in small open economies, in the face of uncertain shocks, it is important to identify the most relevant features that explain this phenomenon. As is now standard in micro founded models in macroeconomics, agents' preferences play a key role. There seems to be

[1] Ramey and Ramey (1995) find a significantly negative impact of volatility on economic growth. Mendoza (1997) provides an early contribution on the effects of terms of trade uncertainty on precautionary savings and economic growth. See also Aghion et al (2010) for a model put emphasis on the interaction between uncertainty and credit constraints and their effect on productivity enhancing investments.

[2] See Broda (2004), Kose (2002) and Mendoza (1995) on this issue.

some confusion in the profession regarding the main determinants of precautionary savings. Theoretical results in two-period models (i.e., Leland (1968)), point out that the sign of the third derivative is the key to determine whether precautionary savings are positive or negative. In particular, a positive third derivative is necessary and sufficient to generate positive precautionary savings when agents live only for two periods. On the other hand, as shown by Huggett and Ospina (2001) the sign of the third derivative is completely irrelevant in an infinite horizon model with heterogeneous agents and independent and identically distributed (i.e., iid) shocks.

This dissertation takes two approaches to the study of precautionary savings in small open economies. The first motivation is based on the observed disparity and disagreement on the behavior of commodity prices and their implications for optimal saving levels. The fact that there is still no agreement in the profession, leads us to develop a theoretical framework that can help explain the consequences of this type of uncertainty on optimal saving (debt) levels. Chapter 2 is devoted to presenting this discussion. We then take a step back and look at a more general issue: the determinants of precautionary savings in a standard open economy. In Chapter 3, I present different exercises and examples that help in elucidating this question.

# Chapter 2

# Imperfect Information and Saving in a Small Open Economy[3]

## 2.1 Introduction

The behaviour of commodity prices constitutes an econometric "puzzle" still unresolved. The empirical evidence presented below shows that "regime" changes (i.e., changes in parameters characterizing the underlying stochastic process such as persistence and/or volatility of innovations) in commodity prices are frequent and sizable. They are subject to large and unexpected fluctuations, and it is difficult to identify the statistical properties of the time-series (in particular, whether they are stationary or non-stationary processes). In this paper, we show some relevant features related to persistence of shocks and regime switches in these time series, using a sample of sixty two commodities over fifty years from IMF's *International Financial Statistics*.

Our model then builds on these facts and explores the optimal savings decisions in a small open economy where the representative agent is subject to uncertainty with respect to the regime in which commodity prices currently are (i.e., low persistence or high persistence regime). We assume that the agent knows the underlying law of motion between regimes and uses Bayesian learning to predict the state of the economy. Our main interest is to examine how optimal saving decisions are affected by this additional level of uncertainty.

[3] This chapter is joint work with Christian Daude.

Recent research shows that certain aspects of business cycles in developing countries are very different from business cycles in industrialized countries.[4] In particular for commodity exporters, the persistence of shocks is very relevant because the implications in terms of the required economic adjustment (i.e. fiscal and external balances) and optimal saving levels are potentially very different. There are many countries in which the business cycle is mainly driven by fluctuations in commodity prices. Furthermore, this is also relevant from a fiscal point of view, given that fiscal revenues, royalties or direct income from state-owned enterprises are large in many developing countries. It is not accidental that Chile (the largest copper exporter in the world) has developed a structural balance rule that especially tries to identify transitory copper windfalls and save them for “rainy days”. The present paper presents a framework to better understand these challenges from an analytical viewpoint.

One key element when studying optimal saving behaviour in emerging markets is to take into account the possibility of uncertainty about the type of shocks and regime changes (i.e., whether the economy is in a state of high or low persistence of shocks, and whether the economy is in a high or low volatility state). To be clear, we will refer to “the current regime” of commodity prices, as the one associated to a particular stochastic process, and to Markov switching between alternative regimes, as the process which allows switches in persistence and/or volatility of innovations. In terms of persistence of shocks, basic open macroeconomic principles (see e.g. Chapter 2 in Obstfeld and Rogoff, 1996) establish that a small open economy should

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[4] See Aguiar and Gopinath (2007).

finance temporary shocks and adjust to permanent shocks. However, these models in general are based on a perfect foresight environment where uncertainty does not play any relevant role in terms of optimal decision rules of economic agents. But uncertainty is definitely relevant due to its effect on optimal decisions and the importance of precautionary savings. Models with uncertainty have been used to study the consequences of considering alternative stochastic processes but always with complete information about their statistical properties and without considering any variation in persistence nor volatility over time. The most commonly used process to introduce uncertainty in DSGE models is a first-order autoregressive process with a given persistence and a given variance for the innovations. This means that shocks are not only, always transitory, but also, that the variance of the innovations is exactly the same across time.

It is well understood that differences in the stochastic process characterising uncertainty can have different effects on the level of optimal savings as well as other macroeconomic variables and their cycles. The goal of this paper is to explore what are the consequences of explicitly considering that the current regime of the economy is uncertain and that it can change over time. In other words, we explicitly introduce uncertainty about whether the persistence of shocks is high or low and potentially also changes the level of uncertainty, at each point in time. To capture changes in persistence across time, we use a particular stochastic process in an otherwise standard small open economy model. We will study two alternative processes with different characteristics about their stationarity properties in order to assess how uncertainty about shocks being temporary or permanent affects optimal saving levels.

In this way we introduce an additional layer of uncertainty which is precisely uncertainty not only about the particular realization of the shock (usual source of uncertainty), but also about the process (i.e., the nature of shocks).

Our framework constitutes a normative tool suitable to assess optimal saving behaviour in a realistic environment in which agents never know the "true" persistence of shocks. Following Reinhart and Wickham (1994): "…the key is how shocks are perceived by agents." In our model, agents use a learning technology which enables them to infer probabilities for the economy being in one regime or the other. To the best of our knowledge, this is the first study providing a simple theoretical framework to analyse how uncertainty about changes in persistence of shocks over time affect optimal saving levels. It has the advantage of being comparable with the standard DSGE model for a small open economy and constitutes a first step in the direction of understanding how this type of uncertainty affects saving levels and external balances of small open economies. In principle, taking into account this type of uncertainty is important for commodity exporters and seems to be the appropriate basic setup to think about optimal policy in the presence of realistic shock processes and informational frictions.

There is a vast empirical literature about commodity prices which has uncovered some stylized facts about their behaviour. Deaton and Laroque (1992) emphasise the existence of rare but large explosions in prices coupled with high degrees of persistence in more normal times. Grilli and Yang (1998) as well as Reinhart and Wickam (1994) argue that most commodity prices (in real terms) have a tendency to trend down in the long run. Leon and Soto (1995) claim that shocks to commodity prices exhibit relatively low persistence and there is room for stabilisation mechanisms (i.e. commodity stabilisation funds). On the other hand, Cashin et al (1999) present evidence supporting the existence of long lasting commodity price shocks and therefore argue that the costs of stabilisation funds might offset their benefits. Engel and Valdes (2001) conclude that there is no conclusive econometric evidence about processes with temporary or permanent shocks to better characterize copper prices. A good summary of these stylized facts can be found in Deaton (1999).

It has proven to be extremely difficult to characterize the long run as well as the short run behaviour of commodity prices. One of the reasons, is that it is almost impossible that the persistence and volatility of the shocks be the same in 1930 and in 1995, no matter what commodity are we talking about. Another important reason is the fact that these prices exhibit large and unexpected swings even in the short run (Cashin et. al, 1999). Therefore, given that shocks cannot always be transitory or permanent a single data generating process (i.e., an AR1) would in principle be unable to provide a good characterization of the actual behaviour of commodity prices.

In terms of related theoretical studies, Deaton (1991) and Carroll (2008) provide theoretical foundations to appropriately write and define a particular type of dynamic stochastic problem in which, at least, one of the variables is not stationary. These studies focus on uncertainty and precautionary savings and provide the necessary tools to formulate and then solve a model with permanent shocks. Ghosh and Ostry (1994) develop a precautionary savings model to study export instability and the external balance in developing countries. They explore the implications of changes in the variance of export earning shocks and analyze how this type of uncertainty affects optimal saving levels and the external balance. Our model builds on this literature but considers an additional layer of uncertainty with respect to the type of stochastic process that drives commodity prices, allowing for changes over time in the persistence of shocks and their volatility.

Aguiar and Gopinath (2007) consider a model in which shocks “hitting” the economy have a trend and a cycle component. They match two business cycle facts of emerging markets that are difficult to match with the standard small open economy models (e.g. Mendoza, 1991) – countercyclical trade balances and a higher volatility of consumption versus output. The authors argue that this is due to the prevalence of trend shocks. A related paper that incorporates learning about the trend and cycle components is Boz et al (2008). They show that once learning is included in the model, the prevalence of trend shocks is no longer needed. Van Nieuwerburgh and Veldkamp (2006) propose an explanation about how is the transition between booms and busts based on agents learning about productivity. They use a Bayesian filter to forecast the future realization of productivity. Boz (2009) also uses informational frictions as an explanation to emerging market crisis, and has the exact same device (learning about productivity) as van Nieuwerburgh and Veldkamp.

These papers have in common that the uncertainty is about decomposing total factor productivity shocks into permanent and transitory components. And the problem with their approach is that they focus on productivity shocks, about which there is some controversy in the profession. In contrast, uncertainty in our paper is about the underlying parameters, regimes or structure that is driving commodity prices. In this respect, our approach is more realistic, since commodity shocks are easier to observe. We allow autocorrelation coefficients of shocks and their variance to change over time rather than having a signal extraction problem regarding different realizations of shocks. This set-up is more relevant for commodity prices where – as we show in section II – regime switching between high and low volatility periods and changes in the persistence describe the statistical properties of the underlying stochastic process better than a "trend plus cycle" model with fixed parameters.

To study the effects of this type of uncertainty on saving behaviour in the simplest possible way, we will consider the standard DSGE small open economy model with a one-good endowment, adding two features. First, we will explicitly consider informational frictions. Second, we will consider two alternative stochastic processes, different from the standard AR1 commonly used in the literature. In particular, we study two alternative specifications to characterise and introduce uncertainty in the model. We first consider a stationary stochastic process (AR1 with regime switching) and then a non-stationary process.

The remainder of the paper is structured as follows. The next section presents empirical evidence on time varying persistence and volatility for sixty two commodity prices. In section III, we present the model economies. Section IV presents the quantitative analysis and the solution method. Section V discusses the main results. Section VI describes the policy implications. Finally, section VII concludes.

## 2.2 Commodity Prices

This section presents the most salient features about commodity prices acknowledging that both permanent and transitory components are potentially present and may be time varying. We consider a sample of 58 annual commodity price time series[5] over fifty years (1957-2007) from the IMF's International Financial Statistics. All original prices are in nominal US dollars, which we deflate by the US CPI. As standard unit root tests have very low power, whether commodity prices are better characterized by stationary or non-stationary processes is still an unresolved question which we do not directly address. Table A.1 in the appendix shows that the moments for commodity prices do vary significantly over time. This is a first indication of time variation in persistence and volatility in commodity prices.

Following as similar approach as Reinhart and Wickham (1994), as we are also interested in looking at the behaviour of trends and variances for each commodity across time, we show that the permanent shocks are present (and fluctuate over time) in every commodity considered in our sample (Table A.2 and Figure A.1 in the appendix). Furthermore, decomposing the total volatility for the price of each commodity, into a permanent and a transitory component, we are able to disentangle their relative importance. Figure A.1 shows also that there is a distinct difference between soft commodities (like food and beverages) and non-reproducible industrial inputs, oil, or metals and ores. While the first tend to exhibit a downward trend, as Reinhart and Wickham (1994) also argue, the second group either does not exhibit a

[5] The appendix presents the complete list.

trend with most commodities presenting an upward trend in recent years. Regarding the behaviour of the variances of the series, we show that there are substantial changes (Figure A.2 in the appendix) across time in all commodity prices in our sample.

To study the permanent and cyclical components of commodity prices we decompose the series in two parts (trend and cycle) using the Hodrick-Prescott filter with a smoothing parameter equal to 100. Figure A.1 shows that despite the heterogeneity among different commodities, there is one common characteristic among all. Trends change a lot over time, and this feature is present in every commodity considered in our sample.

How much of the total volatility is due to the permanent component? To address this issue, we use Cochrane's (1988) methodology to quantify the importance of permanent shocks. Specifically, suppose the variable $p_t$ has the following representation:

$$p_t = \alpha p_{t-1} + \varepsilon_t \text{ with } \varepsilon_t \sim N(0, \sigma^2).$$

If $\alpha$=1 and the disturbance term is white noise, then $p_t$ follows a random walk and the variance of its k-differences grows linearly with the lag difference:

$$\mathrm{var}(p_t - p_{t-k}) = k\sigma^2 .$$

If $\alpha < 1$, $p_t$ is a stationary process and the variance of its k-differences is given by:

$$\mathrm{var}(p_t - p_{t-k}) = \sigma^2 \frac{1-\alpha^{2k}}{1-\alpha^2}.$$

Therefore, the variance ratio $\frac{1}{k}\frac{\mathrm{var}(p_t - p_{t-k})}{\mathrm{var}(p_t - p_{t-1})}$ is equal to one if $p_t$ is a random walk. If $p_t$ is stationary, all shocks will eventually die out, hence the variance ratio will converge to zero. If $p_t$ is a general I(1) process, which has both permanent and transitory (stationary) components, then the ratio will converge to the ratio of the variance of the permanent shock to the total variance of the process. Therefore, the closer that ratio is to unity, the larger is the size of the unit root component and the lower is the relative weight of the temporary shocks.

Table A.2 in the appendix presents the main results. The values of k range between 1 and 20 years. There is substantial heterogeneity, but despite the different magnitudes, it is worth mentioning that the permanent component accounts for more than 30 percent of total volatility for thirteen commodities in the sample. Examples of this are coffee, iron ore, petroleum and tin, which seem to have substantial trend shocks over time. Despite the differences and relative importance of each component in each commodity, it is easy to see that permanent shocks are always present and can be a significant part of overall volatility in many cases.

To study the behaviour of the cyclical component,[6] we explore if there is any evidence of "parameter instability" over time. We want to evaluate if there is any evidence of time variation in persistence and volatility of innovations coming from the cyclical component of commodity prices. It is not our aim to determine what factors are causing these switches nor to identify or link particular episodes or states to exogenous variables causing this behaviour. The objective here is to see if this regime switching happens for a wide variety of commodities, as the cyclical component of their prices appears to have different persistence and volatility of innovations over time.

We analyse the behaviour of the cyclical component of commodity prices over time estimating a Markov switching regime model a-la Hamilton (1989). For

[6] A similar characterisation results if we model price changes instead of the cyclical component.

simplicity, we allow only two possible states for the parameters of the process. In particular, the two alternative models are the following. First, the Markov switching model is given by:

$$p_t = \mu_t + \rho_t p_{t-1} + \varepsilon_t, \tag{1}$$

where $\varepsilon_t \sim N(0, \sigma_t^2)$.

Notice that in this model we allow the mean ($\mu_t$), the persistence ($\rho_t$), and the volatility of the innovations ($\sigma_t$) to change over time. To keep it simple we will only allow two possible values for each parameter. So we estimated a 2-state Markov switching regime model. An alternative way to present this model is as follows:

$$p_t = s_t[\mu^A + \rho^A p_{t-1} + \varepsilon_t^A] + (1 - s_t)[\mu^B + \rho^B p_{t-1} + \varepsilon_t^B], \tag{2}$$

where for some periods $s_t$ is an indicator function with a transition probability matrix given by:

$$M = \begin{pmatrix} m_{11} & 1 - m_{11} \\ 1 - m_{22} & m_{22} \end{pmatrix} = \begin{pmatrix} \Pr(s_{t+1} = 1|s_t = 1) & \Pr(s_{t+1} = 0|s_t = 1) \\ \Pr(s_{t+1} = 1|s_t = 0) & \Pr(s_{t+1} = 0|s_t = 0) \end{pmatrix}.$$

Thus, we estimate 8 parameters: $\mu^A$, $\mu^B$, $\rho^A$, $\rho^B$, $\sigma^A$, $\sigma^B$, $m_{11}$, and $m_{22}$.

Second, we estimate a standard AR(1) given by:

$$p_t = \mu + \rho p_{t-1} + \varepsilon_t, \tag{3}$$

where $\varepsilon_t \sim N(0, \sigma^2)$, such that the mean, the persistence and volatility of innovations are fixed over time.

We select forty four commodities for which we have sufficiently long monthly observations (Table A.3 in the appendix) from the IMF's International

Financial Statistics database to conduct our estimations. We use monthly data in order to have more observations.[7] The time range considered is 1957M1-2008M12. The cyclical component is obtained using the Hodrick-Prescott filter to de-trend the series, with a smooth parameter equal to 129,600 (following Ravn and Uhlig, 2002).

As shown in Table A.3, the results indicate that for twenty-eight of these commodities, the estimated Markov switching model is a good characterization of its cyclical behaviour. For the other commodities it is often the case that one or more coefficients (out of ten), were not statistically significant in the estimation, so we excluded them from the comparison. What is even more interesting is that for all these commodities the estimated model is undoubtedly superior (provides a better fit) than a standard AR1 model. The criterion to determine which of these two econometric models was better to characterise cyclical movements in commodity prices was to compare the log likelihood for each model and conduct the likelihood ratio test to check that these differences are statistically significant. Thus, the results of the estimation indicate that the regime switching approach seems to be a better characterisation compared to the widely used first order autoregressive model. It is important to emphasise that we are not claiming that this econometric model is the best among all possible models to characterize commodity prices. We just show that a model that allows regime switching in persistence and volatility seems to be more favoured by the data than the usual AR1 model. The intuition for this is that it allows more flexibility compared to other processes where persistence and volatility can only assume one particular constant value over time. Given our estimation results we

[7] Recall that for a two state Markov switching model in which we are allowing all the parameters to switch we are estimating 10 parameters.

conclude that there is evidence of time variation in persistence and volatility in the cyclical component of many commodity prices.

From this section, we conclude that there seems to be fluctuations in deep parameters of commodity price series over time. In particular, the evidence suggests the existence of sizable changes in both, persistence and volatility.

## 2.3 Model Economies

### 2.3.1 Stationary Model

We consider a simple dynamic stochastic general equilibrium (DSGE) model of a small open economy. There is one tradable good (i.e., commodity GDP) which can be sold in international markets at a given price. We consider a constant endowment of non-commodity GDP, for calibration purposes as explained below. The price of the tradable good is the only source of uncertainty in our model economy. The representative agent can borrow and lend in international capital markets at a time-invariant real interest rate. Markets are incomplete, such that the only financial instrument available is a one-period non-contingent bond that pays the world's real interest rate.

We start considering the simplest possible model where the price of the commodity follows a stationary AR(1) process and will use it as our benchmark. We then extend this model to incorporate two additional features.

First, the price is stochastic and is a combination of two stochastic processes, each of these, with different persistence and volatility. Second, we introduce informational frictions. To be precise, we will consider *process uncertainty*. The representative agent observes the actual realization of the commodity price, but she doesn't know the true properties (i.e., mean, persistence and standard deviation) of the process which generated it. Within every regime, we have the standard uncertainty of which particular shock hits the economy each period.

Suppose, to simplify, that there are only two states of the world (A and B). Each of these is characterized by a given distribution with well defined moments.[8] We will first analyse how net foreign asset positions should be in this world, compared to the benchmark. Then we will add an extra layer of complication in order to analyze optimal debt levels under process uncertainty. Given this informational friction, the agent solves a learning problem. Using a Bayesian learning technology, she is updates beliefs and infers probabilities for the price coming from each of the possible distributions.

At the beginning of every period, the agent observes the realization of the price, updates her beliefs, and infers the corresponding probabilities for each possible distribution. Then, she chooses consumption and the level of net foreign assets she wants to hold.

The representative agent's preferences are given by:

$$u(C_t) = \frac{C_t^{1-\theta}}{1-\theta}, \quad (4)$$

[8] One could think about this as periods of high volatility versus periods of low volatility. Another alternative is to think about periods of "persistent shocks" vs. periods of "less persistent shocks" (i.e., periods of permanent shocks vs periods of temporary shocks).

where $\theta$ is the coefficient of relative risk aversion. The agent maximizes the expected present discounted value of utility subject to the following resource constraint:

$$C_t = P_t Y + A - B_{t+1} + RB_t, \tag{5}$$

where $C_t$ , $Y$ and $B_t$ denote consumption, the commodity endowment and the net foreign asset position in period $t$, respectively; while $R = (1 + r)$, where $r$ is the world real interest rate, which is assumed to be given and constant. The parameter $Y$ is the endowment of commodity goods available in the economy, while $A$ is the non-commodity GDP, which is also assumed to be constant. We introduce this parameter only for our quantitative analysis to calibrate the share of commodity GDP in total GDP.

For our benchmark model, the stochastic process for the commodity price is given by:

$$P_t = P_{t-1}^{\rho} e^{\mu + \varepsilon_t},$$

where $\mu$ and $\rho$ are both constant over time, and the error terms ($\varepsilon_t$) is assumed to be i.i.d. normal $N(0, \sigma^2)$.

The first non standard feature that we will consider is "parameter instability" or time variation in persistence and volatility of the innovations. To this end we will consider the following stochastic process for the price:

$$P_t = P_{t-1}^{\rho_t} e^{\mu_t + \varepsilon_t},$$

where now $\mu_t$ and $\rho_t$ are both allowed to change over time, while $\varepsilon_t$ is assumed to be i.i.d. normal $N(0, \sigma_t^2)$, such that the volatility of innovations is also allowed to change over time. To keep it simple we will only assume two possible stationary processes.

$$P_t = \begin{cases} P_{t-1}^{\rho^A} e^{\mu^A + \varepsilon_t^A} & \text{if } s_t = 1 \\ P_{t-1}^{\rho^B} e^{\mu^B + \varepsilon_t^B} & \text{if } s_t = 0 \end{cases}, \tag{6}$$

where both $|\rho^A| < 1$ and $|\rho^B| < 1$, $\varepsilon_t^A \sim N(0, (\sigma^A)^2)$ and $\varepsilon_t^B \sim N(0, (\sigma^B)^2)$ are both i.i.d., and $s_t$ is an unobserved "latent" variable which evolves according to an exogenous stationary Markov process.

#### 2.3.1.1 Learning Problem

There are two "types" of uncertainty. First, there is process uncertainty. This means that shocks can be generated by distribution A or distribution B at each moment in time, and second, there is the usual uncertainty about the actual realization of the price.

The unobserved "latent" variable $s_t$ follows a two state Markov process with transition matrix given by:

$$M = \begin{pmatrix} m_{11} & 1 - m_{11} \\ 1 - m_{22} & m_{22} \end{pmatrix},$$

which we assume is known by the agent. That is, $s_t \in \{0,1\}$ where $s_t = 1$, corresponds to shocks coming from distribution *A* and $s_t = 0$ corresponds to shocks coming from distribution *B*. We also assume that the agent knows these distributions with certainty; she knows the mean, persistence and standard deviation of innovations of each possible distribution; what she cannot observe is $s_t$ and therefore whether the shock did actually came from *A* or *B*. Following Boz(2007) we assume an irreducible Markov chain for the "latent" variable $s_t$, such that all elements are strictly positive and strictly smaller than 1.

At the beginning of each period the agent observes the actual price but do not observe past or present values of the latent variable. Therefore she uses the information revealed by the price to infer the probability of the shock in the current period coming from *A* or *B*.

Beliefs are defined as:

$$\tilde{s}_t = E[s_t | I_t^U],$$

where $I_t^U$ is the information set which includes the entire history of realizations of the endowment observed by the agent, given by:

$$I_t^U = \{P_t, P_{t-1}, P_{t-2}, \dots\}.$$

We will refer to this information structure as "imperfect information". The belief $\tilde{s}_t$ is formed by updating the previous period's belief $\tilde{s}_{t-1}$ using Bayes' rule:

$$\Pr(s_{t-1} = s^A | I_t^U) = \frac{\Pr(P_t^i P_{t-1}^j | s^A) \Pr(s_{t-1} = s^A | I_{t-1}^U)}{\Pr\left(P_t^i P_{t-1}^j \middle| s^A\right) \Pr(s_{t-1} = s^A | I_{t-1}^U) + \Pr(P_t^i P_{t-1}^j | s^B) \Pr(s_{t-1} = s^B | I_{t-1}^U)} \qquad (7)$$

The first probability in the numerator is the probability of observing the price at time *t* given that the economy is in state $s^A$, while the second is the probability corresponding to $\tilde{s}_t$, given the one-to-one mapping between beliefs and probabilities in this set-up. Thus, we have:

$$[\Pr(s_t = s^A | I_t^U) \quad \Pr(s_t = s^A | I_t^U)] = [\Pr(s_{t-1} = s^A | I_t^U) \quad \Pr(s_{t-1} = s^A | I_t^U)]M \qquad (8)$$

Consequently:

$$\tilde{s}_t = [\Pr(s_t = s^A | I_t^U) \quad \Pr(s_t = s^A | I_t^U)] \begin{bmatrix} s^A \\ s^B \end{bmatrix} \qquad (9)$$

We denote the evolution of the agent's beliefs as $\tilde{s}_{t+1} = \phi(\tilde{s}_t, P_{t+1}, P_t)$. When the agent makes her decisions at date *t*, $P_{t+1}$ is not known, but its distribution (conditional on $\tilde{s}_{t+1}$) is. It is in this way that she can form her expectations about shocks coming from one distribution or the other, using all the information available in period *t*.

## 2.3.2 Non Stationary Model

In this section we will study a more general process to characterise the evolution of the stochastic price over time. The main motivation behind this process is partly based on section II, and also based on the inconclusive evidence in the empirical literature about the stationarity or non-stationarity of commodity prices. Given these observations, we consider a stochastic process where stationarity can change over time. To be precise, the stochastic process for the price will be stationary during some periods of time and non-stationary during others.

$$P_t = \begin{cases} P_{t-1}^{\rho} e^{\mu+\varepsilon_t^A} & \text{if } s_t = 1 \\ P_{t-1} e^{\mu+\varepsilon_t^B} & \text{if } s_t = 0 \end{cases}, \tag{10}$$

where $0 < \rho < 1$ and $\varepsilon_t \sim N(0, \sigma^2)$ is independently and identically normally distributed and $s_t$, is an unobserved "latent" variable which evolves according to an exogenous stationary Markov process, as before.

For simplicity, we will assume that both, the mean and the standard deviation of innovations is always the same regardless of whether the process for the price is stationary or non-stationary. This will enable us to focus only on the effects of time varying persistence on optimal saving levels.

Since the overall process for the price is a combination of two processes, and given that one of these is non-stationary, the overall process is non-stationary. In order to be able to solve the model, we need to normalize all variables to induce stationarity.

Let $\hat{C}_t = \alpha_t C_t$ , $\hat{Y}_t = \alpha_t Y_t$, $\hat{B}_t = \alpha_t B_t$, with $\alpha_t = \frac{1}{P_{t-1}}$. Therefore, after de-trending, the resource constraint becomes:

$$\hat{C}_t = \hat{P}_t\left(Y - \hat{B}_{t+1}\right) + R\hat{B}_t + \hat{A}_t, \tag{11}$$

where $\hat{P}_t$ is given by:

$$\hat{P}_t = \begin{cases} P_{t-1}^{\rho-1} e^{\mu+\varepsilon_t} \text{ if } s_t = 1 \\ e^{\mu+\varepsilon_t} \qquad\quad \text{if } s_t = 0 \end{cases} \tag{12}$$

Now, the sequential problem for the agent is:

$$\max_{\{\hat{C}_t, \hat{B}_{t+1}\}_{t=0}^{\infty}} E_0^P \left[ E_0 \left[ \sum_{t=0}^{\infty} \beta^t \left( P_{t-1}^{1-\theta} \right) \frac{\hat{C}_t^{1-s}}{1-s} \middle| I_0^U \right] \right], \tag{13}$$

subject to equation (11). $E_0^P$ is the expectations operator with respect to the process/regime, while $E_0$ is the expectations operator with respect to the particular realisation of the endowment. The first order conditions for the competitive equilibrium are:

$$\beta^t \left( P_{t-1}^{1-\theta} \right) u'\left( \hat{C}_t \right) - \lambda_t = 0,$$

$$-\lambda_t \hat{P}_t + E_t^P \{ E_t [\lambda_{t+1} R | I_0^U ] \} = 0,$$

which combined yield the Euler equation:

$$u'\left( \hat{C}_t \right) = R\beta \left( P_t^{1-\theta} \right) E_t^p \{ E_t [ u'\left( \hat{C}_{t+1} \right) | I_0^U ] \},$$

which has the usual interpretation. The marginal benefit of saving an additional unit of the endowment is equal to the marginal cost of not consuming that unit. The expectation on the right-hand side can be written as:

$$\Pr(s_{t+1} = 0 | I_t^U)\, E_t^{AR}\left[u'\left(\hat{C}_{t+1}\right) | I_0^U\right] + \Pr(s_{t+1} = 1 | I_t^U)\, E_t^{UR}\left[u'\left(\hat{C}_{t+1}\right) | I_0^U\right],$$

such that it is a weighted average of the expectations under transitory shocks(AR1) and permanent shocks (UR process).

**Definition:** A *competitive equilibrium* is given by allocations $\hat{B}_{t+1} = b(\hat{B}_t, \hat{P}_t, \tilde{s}_t)$, $\hat{C}_{t+1} = c(\hat{B}_t, \hat{P}_t, \tilde{s}_t)$, such that:

(*i*) Agents maximise expected utility (13) subject to their budget constraint (11).

(*ii*) Goods and assets markets clear.

## 2.4 Quantitative Analysis

### 2.4.1 Computation

#### 2.4.1.1 Stationary Case

The recursive representation of the agent's problem is:

$$V(B, P, \tilde{s}) = \max\{u(C) + \beta E[V(B', P', \tilde{s}') | P, \tilde{s}]\}, \qquad (14)$$

subject to:

$$C = PY + A - B' + RB.$$

The solution algorithm includes the following steps:

Discretise the state space. We use 200 equally spaced nodes for $B$, 5 grid points for the price and 20 equally spaced nodes for $\tilde{s}_t$.

Evaluate the evolution of beliefs $\tilde{s}_{t+1} = \phi(\tilde{s}_t, P_{t+1}, P_t)$ using equations (7) – (9).

Solve the dynamic programming problem described in (14) using value function iterations in order to get $B_{t+1} = b(B_t, P_t, \tilde{s}_t)$, $C_{t+1} = c(B_t, P, \tilde{s}_t)$.

### 2.4.1.2 Non Stationary Case

The recursive representation of the agent's problem is:

$$V(\hat{B}, \hat{P}, \tilde{s}) = \max\left\{u(\hat{C}) + \beta(\hat{P})^{1-\theta}\left[p^S(\hat{P}, \tilde{s}) \int V(\widehat{B'}, \widehat{P'}, \tilde{s}')dF(\widehat{P'}|\hat{P}) + p^{NS}(\hat{P}, \tilde{s}) \int V(\widehat{B'}, \widehat{P'}, \tilde{s}')dG(\widehat{P'}|\hat{P})\right]\right\}, \quad (15)$$

subject to

$$\hat{C} = \hat{P}\left(Y - \widehat{B'}\right) + R\hat{B} + \hat{A},$$

where $F(.)$ and $G(.)$ are the stationary and non-stationary distributions for the endowment respectively, $p^S$ and $p^{NS}$ are the conditional probabilities for the distribution being stationary and non-stationary respectively.

The solution algorithm includes the following steps:

Discretise the state space. We use 200 equally spaced nodes for assets, 5 grid points for the commodity price, and 20 equally spaced nodes for the latent state variable. To discretise the stationary and non-stationary stochastic processes we use Tauchen's (1986) method.

Evaluate the evolution of beliefs $\tilde{s}_{t+1} = \phi(\tilde{s}_t, P_{t+1}, P_t)$ using equations (7) – (9).

Solve the dynamic programming problem described in (15) using value function iterations in order to get $\hat{B}_{t+1} = b(\hat{B}_t, \hat{P}_t, \tilde{s}_t)$, $\hat{C}_{t+1} = c(\hat{B}_t, \hat{P}_t, \tilde{s}_t)$.

### 2.4.2 Calibration

For the stationary model, we will use data for copper prices and the Chilean economy. For preferences and the risk free interest rate we use standard parameters in the literature. The stochastic process (AR(1) in this case) is estimated using data from the IMF International Financial Statistics (IFS). Since we divide total GDP in commodity GDP and non-commodity GDP, we will use a parameter A to calibrate the share of copper GDP in total GDP. On average, between 1993 and 2009, copper has accounted for around 7 percent of total GDP. The resulting parameters are presented in Table 1.

Table 1. Parameters for AR1 copper economy

| Parameter | Parameter value |
|---|---|
| $\beta$ | 0.98 |
| $\theta$ | 2 |
| $r$ | 0.017 |
| $\sigma$ | 0.063 |
| $\rho$ | 0.91 |
| $\mu$ | 0.001 |
| $Y$ | 1 |
| $A$ | 12 |

Table 2 shows the parameters for the stationary model with time varying parameters. These correspond to our estimation results of a regime switching AR(1) model. For simplicity, we allow the mean, the persistence and the volatility of innovations to take two possible values over time. The share of copper GDP in total GDP continues to be 7 percent, in order to make it comparable to the benchmark.

Table 2. Parameters for AR1 Markov switching economy

| Parameter | Parameter value |
|---|---|
| $\beta$ | 0.98 |
| $\theta$ | 2 |
| $r$ | 0.017 |
| $\rho^A$ | 0.9316 |
| $\rho^B$ | 0.8877 |
| $\sigma^A$ | 0.0359 |
| $\sigma^B$ | 0.0925 |
| $\mu^A$ | -0.0034 |
| $\mu^B$ | 0.0091 |
| $Y$ | 1 |
| $A$ | 12 |
| $m_{11}$ | 0.94 |
| $m_{22}$ | 0.90 |

For the discount rate, the risk free interest rate and the coefficient of relative risk aversion, we use standard values used in the literature.

For the switching model between a stationary and non-stationary model we calibrate both the benchmark and the more general model to Mexico and oil prices. For the Benchmark model we use the parameter A to match the share of oil GDP in total GDP, which is also around 7 percent for the Mexican economy. We choose the persistence and volatility of innovations for the price process in order to match the actual volatility and persistence of petroleum prices. Table 3 shows the resulting parameters for the AR1 benchmark model.

Table 3. Parameters for AR1 Oil economy

| Parameter | Parameter value |
|---|---|
| $\beta$ | 0.98 |
| $\theta$ | 2 |
| $r$ | 0.017 |
| $\sigma$ | 0.14 |
| $\rho$ | 0.78 |
| $\mu$ | 0 |
| $Y$ | 1 |
| $A$ | 12 |

For the non-stationary model with permanent and transitory shocks, the non-stationary part has a unit root by construction. We choose the persistence and the volatility of innovations of the stationary part, as well as the transition matrix (between the stationary and non-stationary parts) to match the actual persistence and volatility in petroleum prices. For simplicity, and also to isolate the effect of persistence, we assume that the volatility of innovations of the stationary part and the non-stationary parts are exactly the same. The share of petroleum GDP in total GDP continues to be 7 percent. The parameters are shown in Table 4.

Table 4. Parameters for stationary/non-stationary Markov switching economy

| Parameter | Parameter value |
| --- | --- |
| $\beta$ | 0.98 |
| $\theta$ | 2 |
| $r$ | 0.017 |
| $\rho^{AR1}$ | 0.687 |
| $\rho^{UR}$ | 1 |
| $\sigma$ | 0.18 |
| $\mu$ | 0 |
| $Y$ | 1 |
| $A$ | 12 |
| $m_{11}$ | 0.97 |
| $m_{22}$ | 0.80 |

## 2.5 Results

In this section we present and explain the main results obtained for each of the two models described above. Our main interest is to study how "parameter instability" affects optimal saving levels. The non-stationary model can be viewed as a case in which the economy faces both permanent and transitory shocks and our interest is to assess how the presence of this alternating stochastic process between temporary and permanent shocks affects optimal average assets holdings.

### 2.5.1 Stationary Model

In Table 5, we present the moments for the AR1 model with perfect information. In this case, all the results of the standard small open economy textbook model hold. First, consumption volatility is smaller than total output volatility. Second, there is a positive correlation between total output and consumption. Third, the correlation between assets holdings and output is positive. This means that in "good times" the economy is saving and in "bad times" it is dissaving, due to the consumption smoothing motive. Fourth, the correlation between output and the current account (CA) is positive. And fifth, on average, the CA is zero, which means that debts are always repaid. Notice also that total output is uncertain, and consumers are prudent (Kimball, 1990), therefore they have a precautionary motive to save. This basically implies saving for a rainy day. It is important to highlight the fact that there are two main motives to save. First, the consumption smoothing motive because consumers are risk averse and want to smooth consumption over time. Second, there is a precautionary savings motive because consumers are prudent.[9]

Table 5. Moments of the benchmark AR1 model with perfect information

| Moments | $PY$ | $B$ | $C$ | $TB$ | $CA$ |
|---|---|---|---|---|---|
| Mean | 0.070 | -0.4719 | 0.9918 | 0.0082 | 0 |
| Std Deviation | 0.1288 | 0.2372 | 0.0088 | 0.0445 | 0.0438 |
| Autocorrelation | 0.8404 | 0.9622 | 0.872 | 0.692 | 0.6892 |
| | $corr(B,Y)$ | $corr(C,Y)$ | $corr(B,C)$ | $corr(TB,Y)$ | $corr(CA,Y)$ |
| Correlation | 0.4784 | 0.9448 | 0.4842 | 0.5245 | 0.4802 |

Let us consider now the model with parameter instability, meaning that the mean, the persistence and the volatility of innovations are time varying, with the parameters of Table 2. Table 6 shows the corresponding moments for the case where the agents have perfect information with respect to the regime in which the economy currently is.

[9] It is worth mentioning that consumers may very well be risk averse but imprudent (see Roitman, 2010).

Table 6. Moments of the AR1 model with perfect information and regime switching

| Moments | $PY$ | $B$ | $C$ | $TB$ | $CA$ |
|---|---|---|---|---|---|
| Mean | 0.070 | -0.2931 | 0.9953 | 0.0047 | -0.001 |
| Std Deviation | 0.1877 | 1.4427 | 0.0143 | 0.2852 | 0.2813 |
| Autocorrelation | 0.7605 | 0.9736 | 0.7499 | 0.6941 | 0.6915 |
| | $corr(B,Y)$ | $corr(C,Y)$ | $corr(B,C)$ | $corr(TB,Y)$ | $corr(CA,Y)$ |
| Correlation | 0.3713 | -0.0974 | 0.514 | 0.7569 | 0.6089 |

Table 7. Moments of the AR1 model with imperfect information and regime switching

| Moments | $PY$ | $B$ | $C$ | $TB$ | $CA$ |
|---|---|---|---|---|---|
| Mean | 0.070 | -0.4483 | 0.9924 | 0.0076 | -0.0002 |
| Std Deviation | 0.1877 | 0.3941 | 0.0114 | 0.0808 | 0.0798 |
| Autocorrelation | 0.7605 | 0.9593 | 0.8071 | 0.6717 | 0.6689 |
| | $corr(B,Y)$ | $corr(C,Y)$ | $corr(B,C)$ | $corr(TB,Y)$ | $corr(CA,Y)$ |
| Correlation | 0.4103 | 0.93 | 0.4203 | 0.6926 | 0.5589 |

The first and most important difference with the benchmark is that in this case, average assets holdings are higher (i.e., debt is lower). The economy is holding one third of the debt in comparison to the benchmark case. Intuitively, in this world, the representative agent knows that the price follows a stochastic process with parameter instability. This implies that there can be big jumps when there is a change in persistence or volatility. Furthermore, there is uncertainty about when a particular jump is going to take place. The agent knows this and since he is interested in having a smooth consumption path, the optimal thing to do is to accumulate a buffer stock of assets that enables her to save in order to prevent big fluctuations in the optimal consumption path. The rest of the results are qualitatively the same. Consumption volatility is lower than total output volatility. Assets holdings go up in good times and down in bad times, consumption is positively correlated with total output and the current account is pro-cyclical.

Going one step further, we are interested in the effects of process uncertainty on optimal saving levels. Table 7 presents the moments for the case where parameters are the same as those for Table 6 (i.e. parameters correspond to Table 2), but there is *process/regime uncertainty*, such that the agent does not know the true state of the economy. Notice that in this case the level of average assets holdings is more than one and a half times lower than in the case of perfect information. While the debt/GDP ratio under perfect information is 29.3 percent, it is 44.8 percent under imperfect information. At the same time, net foreign assets are higher compared to the benchmark (i.e., the debt/GDP levels is about 6 percentage points lower).

Let us first analyze why under imperfect information the optimal debt level is higher compared to the case of perfect information. When the agent can only observe the shock but does not know from which distribution it is coming from, she needs to form beliefs (with the corresponding associated probabilities) in order to infer the distribution which generated the observed realization of the price. These beliefs (and probabilities) are used to form expectations which are in turn used to decide the amount of net foreign assets to hold. Intuitively, one could identify two effects. On the one hand, the fact of not knowing for sure where the observed realization is coming from (and because of the way beliefs are formed) makes the agent behave as if it were coming from the average between the two possible distributions. It is as if the economy were facing a process characterized by the average mean, persistence and volatility of innovations. On the other hand, there is an additional effect which would in principle induce agents to save more, and this is the *process uncertainty* effect. It turns out that the effect of the former is bigger than the latter.

The rest of the moments are qualitatively similar as under perfect information and the benchmark.

## 2.5.2 Stationary/Non-stationary Regime Switching Model

For the case where the commodity price can alter between a stationary and a non-stationary regime, the exercise will be to establish a benchmark – in this case for the case of oil in the Mexican economy – and then look at the effects of considering a more general process, with particular focus on average assets holdings levels. The resulting moments are presented in Table 8. For this benchmark model the results are qualitatively the same as for the case of Chile. Notice however that a key assumption here is that oil prices are stationary (i.e., they are characterized by a first order autoregressive process).

Table 8. Moments of the benchmark model with perfect information for the oil economy

| Moments | $PY$ | $B$ | $C$ | $TB$ | $CA$ |
|---|---|---|---|---|---|
| Mean | 0.070 | -0.445 | 0.9925 | 0.0075 | -0.0002 |
| Std Deviation | 0.200 | 0.1945 | 0.0067 | 0.1212 | 0.12 |
| Autocorrelation | 0.380 | 0.8014 | 0.4508 | 0.3835 | 0.3829 |
| | $corr(B,Y)$ | $corr(C,Y)$ | $corr(B,C)$ | $corr(TB,Y)$ | $corr(CA,Y)$ |
| Correlation | 0.408 | 0.8262 | 0.1311 | 0.9071 | 0.4116 |

Consider now the model in which the process is allowed to be, some periods stationary and some periods non stationary. We can see three striking differences with the benchmark. First, average assets holdings are substantially higher. While under the AR1 assumption the stationary debt-to-GDP ratio is 44.5 percent, under the regime switching model, where the oil price can alter between a stationary and non-stationary regime, average debt-to-GDP is just 5.2 percent, as shown in Table 9. Second, the difference between consumption and total output volatility are substantially smaller compared to the benchmark. This is not that surprising, given that the correlation between consumption and output is almost 1. Third, the current account is countercyclical. The agent knows that the changes in persistence could have dramatic consequences because in one of the regimes the process is non-stationary. The best forecast as of today that the agent can have, conditional on shocks coming from that process, is today's realization. This induces the agent to save considerably more than in the case in which he always faces temporary shocks.

Table 9. Moments of the Markov switching model with perfect information for the oil economy

| Moments | $PY$ | $B$ | $C$ | $TB$ | $CA$ |
|---|---|---|---|---|---|
| Mean | 0.070 | -0.0526 | 1.0029 | -0.0029 | -0.0038 |
| Std Deviation | 0.200 | 6.5759 | 0.169 | 9.1458 | 8.8968 |
| Autocorrelation | 0.380 | -0.0091 | 0.5581 | -0.4454 | -0.4429 |
| | $corr(B,Y)$ | $corr(C,Y)$ | $corr(B,C)$ | $corr(TB,Y)$ | $corr(CA,Y)$ |
| Correlation | -0.6604 | 0.9901 | -0.6153 | 0.4735 | -0.2599 |

As before, we are also interested in assessing the effects of process uncertainty on optimal saving levels, now under permanent and transitory shocks. The results are presented in Table 10. Interestingly, there is no effect whatsoever. In other words, process uncertainty is not an issue here and this is due to the low share of Oil GDP in total GDP. But the important result is that regardless of whether there is process uncertainty or not, average assets holdings are ten times higher compared to a model in which oil prices are assumed to be stationary. Thus, under permanent and transitory shocks, process uncertainty is not as relevant as explicitly considering that shocks can be temporary or permanent.

Table 10. Moments of the Markov switching model with imperfect information for the oil economy

| Moments | $PY$ | $B$ | $C$ | $TB$ | $CA$ |
|---|---|---|---|---|---|
| Mean | 0.070 | -0.0524 | 1.003 | -0.003 | -0.0039 |
| Std Deviation | 0.200 | 6.6649 | 0.1687 | 9.3076 | 9.0799 |
| Autocorrelation | 0.380 | -0.0196 | 0.5604 | -0.4512 | -0.4486 |
| | $corr(B,Y)$ | $corr(C,Y)$ | $corr(B,C)$ | $corr(TB,Y)$ | $corr(CA,Y)$ |
| Correlation | -0.6563 | 0.9895 | -0.6087 | 0.4728 | -0.2581 |

## 2.6 Sensitivity Analysis

It is interesting to assess whether the direction of the results presented above change if some key parameters change. We will focus our attention in comparing the models with and without process uncertainty (i.e., perfect versus imperfect information) in the non-stationary model presented above. First we check whether saving levels are higher under imperfect information (compared to the case of perfect information) as the share of commodity GDP is higher than 7percent. Second, we check how the transition matrix between the two processes affects average assets holdings.

In order to assess how important is the magnitude of the share of GDP in our results we solved the model for a commodity GDP share of 20 percent and 66 percent. We find that when the share is 20 percent, it makes no difference whether you face process uncertainty or not, average assets holdings are the same.[10] As Table 11 shows, for a share of 66 percent[11], it happens that savings are higher under process uncertainty. This seems to suggest that when the proportion of output which is volatile is relatively high, then process uncertainty can (and should) matter a lot. The level of debt under perfect information is 12 percent higher compared to the case of process uncertainty.

Table 11. Moments of the Markov switching model with higher share of commodity sector in GDP

| *Perfect Information* | | | | | |
|---|---|---|---|---|---|
| Moments | *PY* | *B* | *C* | *TB* | *CA* |
| Mean | 0.660 | -0.4605 | 0.9856 | 0.0144 | 0.0063 |
| Std Deviation | 0.200 | 3.679 | 0.2012 | 3.1696 | 3.1165 |
| Autocorrelation | 0.380 | 0.575 | 0.4919 | 0.0015 | 0.0002 |
| | *corr(B,Y)* | *corr(C,Y)* | *corr(B,C)* | *corr(TB,Y)* | *corr(CA,Y)* |
| Correlation | -0.1994 | 0.9375 | -0.0332 | -0.2602 | -0.2459 |

| *Imperfect Information* | | | | | |
|---|---|---|---|---|---|
| Moments | *PY* | *B* | *C* | *TB* | *CA* |
| Mean | 0.660 | -0.4188 | 0.9859 | 0.0141 | 0.0067 |
| Std Deviation | 0.200 | 2.7395 | 0.1954 | 2.3092 | 2.2755 |
| Autocorrelation | 0.380 | 0.6172 | 0.5509 | 0.0439 | 0.0433 |
| | *corr(B,Y)* | *corr(C,Y)* | *corr(B,C)* | *corr(TB,Y)* | *corr(CA,Y)* |
| Correlation | -0.1656 | 0.9647 | -0.0477 | -0.3356 | -0.2821 |

With respect to the transition matrix between regimes, it is easy to argue that for small shares of commodity GDP, it does affect average assets holdings, such that there is no difference between facing process uncertainty or perfect information (just compare the tables presented in the last section with the ones presented below). But

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10 Results are not reported (but are available upon request) due to space considerations.

11 Notice that many oil exporting countries (i.e., Saudi Arabia, Libya) have extremely high ratios of oil GDP.

for relatively high shares of commodity GDP, there are two interesting results (see Table 12). First, a transition matrix with all its elements equal to 0.5 provides no information at all about whether shocks are temporary or permanent, therefore we observe that for both perfect and imperfect information cases assets go up (i.e., debt go down). Second, it is always the case that under process uncertainty, average assets holdings are higher compared to the perfect info case. For a share of 66 percent, the debt level under perfect information is 42 percent higher compared to the case of process uncertainty. The transition matrix in the tables below has all its elements equal to 0.5

Table 12. Moments of the Markov switching model without learning ($m_{ij}$=0.5)

| *Perfect Information* | | | | | |
|---|---|---|---|---|---|
| Moments | *PY* | *B* | *C* | *TB* | *CA* |
| Mean | 0.660 | -0.3745 | 0.9795 | 0.0205 | 0.0139 |
| Std Deviation | 0.2828 | 101.9228 | 0.289 | 108.5891 | 106.7542 |
| Autocorrelation | 0.4515 | 0.501 | 0.5814 | -0.0357 | -0.0394 |
| | *corr(B,Y)* | *corr(C,Y)* | *corr(B,C)* | *corr(TB,Y)* | *corr(CA,Y)* |
| Correlation | -0.0862 | 0.9573 | 0.0152 | -0.1924 | -0.1489 |

| *Imperfect Information* | | | | | |
|---|---|---|---|---|---|
| Moments | *PY* | *B* | *C* | *TB* | *CA* |
| Mean | 0.660 | -0.2672 | 0.9829 | 0.0171 | 0.0124 |
| Std Deviation | 0.2828 | 47.0486 | 0.2722 | 44.3206 | 43.9041 |
| Autocorrelation | 0.4515 | 0.5573 | 0.6499 | -0.0032 | -0.0085 |
| | *corr(B,Y)* | *corr(C,Y)* | *corr(B,C)* | *corr(TB,Y)* | *corr(CA,Y)* |
| Correlation | 0.0776 | 0.9876 | 0.1143 | -0.2733 | -0.1285 |

## 2.7 Policy Implications

In a small open economy which chooses consumption levels and assets/debt positions across time in an optimizing framework, taking explicitly into account the existence of “parameter instability” seems to be crucial to determine optimal debt levels. Identification of temporary and permanent shocks poses serious challenges for policy makers because optimal reactions in terms of consumption/saving levels are completely different. For a small open economy with access to international capital markets, it is optimal to finance temporary shocks and adjust to permanent shocks.

The results of our simulations indicate that policy makers should be cautious when choosing policy rules. There has been a big debate regarding fiscal policy rules, both in policy and academic circles, with mixed experiences. The case of Chile, with the copper stabilization fund and the explicit fiscal rule is a successful example of countercyclical fiscal policy in Latin America. In terms of the model presented above, one could think about optimal fiscal policy financed with external debt, as has been the case in many developing countries. Setting a particular target level for external debt, a debt ceiling, or a balanced budget rule is not optimal. On the contrary, an optimal rule should be based first, on the current level of external debt, second, the state of the economy (i.e. good times or bad times) and third, the policy maker's “beliefs” about the state of the economy or the policy maker’s “beliefs” about the persistence of shocks at a particular point in time. In other words, optimal fiscal rules should be state contingent and should put some kind of weights, or probabilities of regime shifts that make current price levels more or less permanent, as well as more volatile. It would be a big mistake to “take a stand”, and assume, for simplicity, that prices follows a simple process and that there are no regime switches.

Moreover, wrong perceptions (or assumptions) about the nature of the process could lead to substantial over or under spending with the associated high or (unnecessarily) low levels of debt.

In practice, though, state contingent policy rules are difficult to implement because oftentimes they are hard to explain to politicians, congressmen, or the public in general. They are also costly, because they imply continuous monitoring and assessments of the state of the economy and commodity prices, as well as continuous forecasting and prediction about output gaps or persistence and volatility of the relevant stochastic process (i.e. commodity prices) driving economic fluctuations. In spite of this, and given actual uncertainty about the true stochastic process, we want to emphasise that forecasts or predictions are important and necessary in order to set and implement sensible saving or debt rules over time. This, of course, has immediate consequences on consumption volatility, which is an important concern in many developing countries.

In terms of commodities stabilization funds, the model suggests some room for them, since accumulating a buffer stock of foreign assets can help stabilize economic fluctuations over time and therefore increase welfare.

## 2.8 Conclusions

In this paper, we have shown that trends in commodity prices change over time for almost all commodities considered in our sample. At the same time, the volatility of commodity prices also displays substantial changes over time.

Furthermore, to assess the relative importance of parameter instability and process uncertainty, we showed that the permanent component in commodity prices can, in some cases, account for more than half of the total volatility. Regarding the cyclical (transitory) component, we estimated a Markov switching model and found that it can better fit the data compared to the standard AR(1) model, usually used in the literature. Based on these findings, we build a dynamic stochastic general equilibrium model with parameter instability and informational frictions to explicitly capture uncertainty about the underlying process in terms of persistence and volatility. This model has two particular features compared to the standard intertemporal model for a small open economy.

First, we explicitly model changes in persistence of shocks across time. This adds an extra layer of uncertainty (i.e., process uncertainty) on top of the standard one, regarding the particular realization of the shock. Second, agents have a learning technology and use it to infer probabilities about the nature of the process. In this way they form the appropriate expectations and are able to choose optimally, how much to borrow/lend and therefore how much to consume over time. We focus our attention in assessing first, how this model compares to the standard textbook model of a small open economy and second, the effects of *process uncertainty* on optimal saving (debt) levels. We show that parameter instability in the stochastic process implies that optimal saving levels (debt holdings) should be higher (lower) compared to a process with fixed parameters. Imperfect information about the stochastic process matters when commodity GDP shares are high, therefore informational frictions (i.e., imperfect information) imply that optimal saving (debt) levels should be higher (lower) compared to the perfect information case.

If policymakers suffer from "misperception", they will use inappropriate policy rules. They will under/over save compared to the case in which they acknowledge the existence of differences in the regime of the stochastic process of commodity prices. The consequences of misperception can be devastating for commodity exporters. They could end up overspending and accumulating high (and often times unsustainable) levels of debt, and this could eventually create other problems like pro-cyclical fiscal spending and default. On the other extreme they could end up over-saving with the associated and forgone opportunity cost of funds. Either extreme is dangerous and that is why it is important to take into account *process uncertainty* at the time of making saving and spending decisions at government levels. This type of uncertainty can also have major effects on the fiscal and external balances and that is precisely why it is important to incorporate it when thinking about optimal policy.

# Chapter 3

# Precautionary Saving in a Small Open Economy Revisited

## 3.1 Introduction

The aim of this chapter is twofold. First, this is the first study providing an example of a class of preferences corresponding to risk averse but imprudent agents.[12] On top of that, it is the first study providing an example of preferences characterized by a constant and invariant coefficient of relative prudence. Second, having an explicit utility function for this kind of preferences is useful to assess the effects of volatility, risk aversion, interest rates, intertemporal distortions and persistence on precautionary savings levels in a small open economy.

Inspired by Huggett and Ospina (2001) I can show that, regardless of the structure of the shocks, in an infinite horizon small open economy model with a representative agent, the third derivative is completely irrelevant to generate positive precautionary savings. Because, as pointed out by Aiyagari (1994) the key determinants of precautionary savings are: the infinite horizon and a borrowing constraint, and not the sign of the third derivative.

This paper provides an example of a particular class of preferences characterized by (i) a negative third derivative and (ii) a constant and invariant coefficient of relative prudence, which are going to be useful to study precautionary

[12] From now on, every time I use the word prudence or imprudence, I will be refering to the definition used in Kimball (1990).

saving behavior in a small open economy.[13] These two features will enable me to focus on two particular questions. First, is it possible to have positive precautionary savings in the presence of a negative third derivative? Second, what are the key elements driving this result? As is now standard in economic theory, the class of preferences considered in this study display risk aversion but at the same time they also display a non-standard feature, imprudence (in the sense of Kimball (1990)). On the one hand, risk aversion provides incentives to increase precautionary savings but on the other hand imprudence provides an incentive to save less in the face of uncertainty. The novelty of this class of preferences is that a constant and invariant coefficient of relative prudence will enable to isolate how changes in risk aversion (without changing the degree of prudence) affect precautionary savings.[14]

Interestingly enough, CES preferences are also characterized by a constant coefficient of relative prudence (CCRP) but it is the same parameter affecting risk aversion that also determines relative prudence, so it is difficult to disentangle whether precautionary savings are say, higher, because of higher risk aversion, higher relative prudence or both. The class of preferences considered in this study display a CCRP which is completely independent of the particular parameters defining the degree of risk aversion. This provides a clear example that risk aversion does not necessarily imply prudence, in fact, in this case agents are risk averse and imprudent. Furthermore, depending on the structure of the environment (i.e., shocks, interest

[13] The coefficient of relative prudence (as defined by Kimbal (1990)) is not only constant, but also, it does not depend on particular parameters characterizing preferences.

[14] See appendix for the exact definition and computation of relative risk aversion and relative prudence.

rates, intertemporal distortions), precautionary savings can be higher or lower compared to an economy without uncertainty.

It is extremely important to distinguish between the concept of "risk aversion" and "prudence". Risk aversion refers to the fact that agents dislike risk (i.e., uncertainty) and like to smooth consumption across time, and prudence refers to the fact that agents like to be prepared for a very bad outcome (i.e., having a buffer stock of assets would enable them to dissave instead of reducing consumption). The degree of risk aversion is determined by the concavity of the utility function used to represent preferences, whereas the degree of prudence is determined by the convexity of the marginal utility.

As highlighted by Aiyagari (1994), in a two period model, the borrowing constraint can be ignored by making suitable assumptions about the time profile of the endowment, but in an infinite horizon model, the borrowing constraint cannot be ignored. The combination of uncertainty, infinite horizon and a borrowing constraint implies that precautionary savings will always be higher compared to the case of perfect foresight regardless of the sign of the third derivative (i.e., regardless of whether agents are prudent or imprudent). Intuitively, when an economy faces borrowing constraints, in an infinite horizon model under uncertainty, it fears getting a sufficiently large sequence of bad shocks (i.e., low endowment realizations) which would push it towards the constraint and force it to consume its income without the possibility of smoothing consumption.[15] Section 2 presents the utility function corresponding to the particular class of preferences used in this paper. Section 3

[15] Notice that prudence (i.e., a positive third derivative) is not necessary to generate precautionary saving behavior.

shows that in a two period model with risk averse but imprudent agents, average assets holdings are lower in an environment with uncertainty than without. Section 4 considers a three period model identical to the one in Section 3 except for the fact of having one extra period. In Section 5 I conduct a set of experiments in order to determine how average assets holdings are affected by changes in volatility, risk aversion, interest rates and intertemporal distortions. In section 6, I construct a particular example where even in spite of a negative third derivative, savings levels are higher under uncertainty. Section 7 presents an infinite horizon version of the basic small open economy model under uncertainty calibrated for Mexico and uses this model to assess the effects of persistence and volatility of shocks as well as interest rates and risk aversion on precautionary savings levels. Section 8 concludes.

## 3.2 Invariant Relative Prudence

We consider the particular class of preferences characterized by the following utility function:

$$u(c_t) = ac_t - bc_t^3 \tag{1}$$

Notice that this function is increasing, strictly concave and has a negative third derivative (i.e., $u'''<0$). One salient feature of these preferences is that they have a constant and invariant coefficient of relative prudence which is completely unrelated to the degree of risk aversion. As a matter of fact, it is constant and independent of the particular parameters defining the concavity of the utility function.[16] (See the appendix for details).

Intuitively, this particular class of preferences is characterized by risk aversion and imprudence and constitutes, to the best of my knowledge, the first explicit example that risk aversion does not necessarily implies prudence.

## 3.3 Two Period Model

I consider a 2 period small open economy model where the endowment of period two is stochastic. Using this model I will show that the sign of the third derivative determines whether precautionary savings are positive or negative.

There is only one tradable good and the economy is perfectly integrated into world capital markets (i.e., agents can borrow and lend to/from the rest of the world at a given real interest rate). The real interest rate is taken as given ($r \geq 0$). The budget constraint for period two is

$$0 = (1+r)b_1 + y_2^L - c_2^L \tag{2}$$

$$0 = (1+r)b_1 + y_2^H - c_2^H \tag{3}$$

where $y_2^H$ and $y_2^L$ are the endowments received in period two in the good and bad states of nature respectively. There is only one non-contingent bond ($b_1$) and consumption in the good and bad states of nature are given $c_2^H$ and $c_2^L$ respectively.

The budget constraint for period one is:

[16] Note that for the CES class of preferences, the coefficient of relative prudence is constant but it depends on the same parameter defining the degree of risk aversion.

$$b_1 = (1+r)b_0 + y_1 - c_1 \tag{4}$$

where initial assets ($b_0$) are given.

Preferences are given by:

$$U = u(c_1) + \beta u(c_2) \tag{5}$$

where ($\beta \in [0,1]$) is the subjective discount factor and the period utility is given by the utility function presented in section 3.2.

The economy maximizes (5), choosing $c_1$ ,$c_2^H$ and $c_2^L$ subject to (2), (3), and (4);

### 3.3.1 Results

In order to solve the model, and without loss of generality, we assume the following parameterization.

Table 13. Parameters. 2 period model.

| $a$ | $b$ | $r$ | $\beta$ | $y_1$ | $y_2^L$ | $y_2^H$ | $b_0$ |
|---|---|---|---|---|---|---|---|
| 6 | 1/3 | 0.01 | 0.98 | 1.5 | 1 | 2 | 0 |

Following Durdu et. al. (2007), precautionary savings are defined as the difference between average assets holdings under uncertainty and its counterpart under no uncertainty. As we can see, under no uncertainty, the economy has a perfectly smooth consumption path and average assets holdings are zero.

Table 14. No Uncertainty. 2 period model.

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.5064 | 0 |
| 2 | 1.5 | -0.0064 | 1.4936 | -0.0064 |

As pointed out in Leland (1968), introducing uncertainty will generate positive precautionary savings only if the third derivative is positive. As can be seen in table 2 below, given that the class of preferences considered in this study display a negative third derivative, average assets holdings are negative and therefore precautionary savings are negative. This economy chooses to hold more debt under uncertainty compared to the case of no uncertainty. Intuitively, since agents are imprudent, they choose to save less (i.e., hold more debt) under uncertainty than under no uncertainty.

Table 15. Uncertainty. 2 period model.

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.7530 | 0 |
| 2 | 1.0 | -0.2530 | 0.7444 | -0.2530 |
| | 2.0 | | 1.7444 | |

This result can be better understood considering two elements; the Euler equation and the sign of the third derivative.

$$u'(c_1) = pu'(c_2^H) + (1-p)u'(c_2^L) = E\ [u'(c_2)] \tag{6}$$

Notice that depending on whether u′(c) is linear, strictly convex or strictly concave,

$$E\ [u'(c_2)] = u'[E(c_2)] \tag{7}$$

$$E\ [u'(c_2)] > u'[E(c_2)] \tag{8}$$

$$E\ [u'(c_2)] < u'[E(c_2)] \tag{9}$$

and this in turn depends on whether $u'''(c) = 0$, $u'''(c) > 0$ or $u'''(c) < 0$ respectively. Using these two pieces of information, it is easy to see that:

$$u'(c_1) = E\ [u'(c_2)] \tag{10}$$

$$u'(c_1) > E\ [u'(c_2)] \tag{11}$$

$$u'(c_1) < E\ [u'(c_2)] \tag{12}$$

Therefore, the Euler equation and the sign of the third derivative determine whether precautionary savings are positive or negative.

## 3.4 Three Period Model

In this section, and in order to show that the result of the previous section holds when the time horizon is finite, I consider the exact same model of a small open economy, but with one more time period.

### 3.4.1 Results

The parameterization used to solve the model is the following:

Table 16. Parameters. 3 period model.

| $a$ | $b$ | $r$ | $\beta$ | $y_1$ | $y_2^L$ | $y_2^H$ | $y_3^L$ | $y_3^H$ | $b_0$ |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 1/3 | 0.01 | 0.98 | 1.5 | 1 | 2 | 1 | 2 | 0 |

As we can see from table 5 below, the economy still achieves full consumption smoothing under no uncertainty.

Table 17. No Uncertainty. 3 period model.

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.5 | 0 |
| 2 | 1.5 | -0.0085 | 1.4957 | -0.0085 |
| 3 | 1.5 | -0.0043 | 1.4957 | -0.0043 |

Table 18. Uncertainty. 3 period model.

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.5767 | 0 |
| 2 | 1.0 | -0.0767 | 1.2613 | -0.0767 |
| | 2.0 | -0.0767 | 1.7487 | |
| 3 | 1.0 | -0.3387 | 0.6579 | -0.0824 |
| | 2.0 | 0.1739 | 2.1757 | |

It then follows that introducing uncertainty makes the economy save less (i.e., hold more debt) than in the case of no uncertainty. Average assets holdings (fifth column) are negative both in periods two and three. And again, this result is caused by the sign of the third derivative.

## 3.5 Volatility, Intertemporal distortions, Risk Aversion and

In order to better understand how strong the "imprudence effect" is, this section presents four experiments. I will study the effects of the volatility of the endowment, the mean of the endowment (i.e., intertemporal distortion), risk aversion and interest rates on precautionary savings levels.

### 3.5.1 Volatility

Does higher volatility increases or decreases precautionary savings? In order to answer this question, we will use the benchmark parameterization (Table 4). To assess the effect of volatility I will change the variance of the distribution of the endowment in periods two and three preserving the mean (i.e., mean preserving spread). So in this case

Table 19. Parameters. Volatility.

| $y_2^L$ | $y_2^H$ | $y_3^L$ | $y_3^H$ |
|---|---|---|---|
| 0.7 | 2.3 | 0.7 | 2.3 |

Table 20. Uncertainty (mean preserving spread)

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.6841 | 0 |
| 2 | 0.7 | -0.1841 | 1.1632 | -0.1841 |
| | 2.3 | -0.1841 | 1.8966 | |
| 3 | 0.7 | -0.6491 | 0.0444 | -0.2158 |
| | 2.3 | 0.2175 | 2.5196 | |

Notice that in this case, higher volatility implies lower savings (i.e., the debt is higher). In Table 4 total average assets holdings is -0.1591 whereas now, with a higher variance in the endowment, total average assets holdings is -0.3999.

This result could capture the empirical observation that developing countries having more volatile output than industrialized countries, hold, on average, more debt. This is a result of the concavity of the first derivative of the utility function.

### 3.5.2 Intertemporal Distortions

In this case the idea is to evaluate whether a lower endowment in period three provides incentives to save more in good times in order to smooth consumption across time. Notice that in this case $y_3^H = y_3^L = 0$.

Table 21. No uncertainty (intertemporal distortion)

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.0218 | 0 |
| 2 | 1.5 | 0.4782 | 0.9964 | 0.4782 |
| 3 | 0 | 0.9866 | 0.9964 | 0.9866 |

Table 22. Uncertainty (intertemporal distortion)

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.0422 | 0 |
| 2 | 1 | 0.4578 | 0.7348 | 0.4578 |
|  | 2 | 0.4578 | 1.2373 |  |
| 3 | 0 | 0.7276 | 0.7348 | 0.9763 |
|  | 0 | 1.2251 | 1.2373 |  |

As we can see in Table 6 and 7, the "imprudence effect" is stronger than the intertemporal distortion effect and this is why total average assets holdings are still lower under uncertainty (1.4341) compared to the no uncertainty case (1.4648). But at the same time, it is easy to see that an intertempral distortion involving a lower endowment in period 3 implies higher savings. To see this, one should compare Tables 6 and Table 3 for the no uncertainty case and Tables 7 and 4 for the cases with uncertainty. In both cases, a lower endowment in period 3 generates higher incentives to save in period 1 and 2 and this is caused by the concavity (i.e., risk aversion) of preferences. Consumers might be imprudent by they are still risk averse and as a consequence they want to smooth consumption.[17]

### 3.5.3 Risk Aversion

In this case a=5, which implies a higher relative risk aversion. Our interest here is to compare these results with the benchmark.

Table 23. No uncertainty (risk aversion)

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.5062 | 0 |
| 2 | 1.5 | -0.0062 | 1.4968 | -0.0062 |
| 3 | 1.5 | -0.0031 | 1.4968 | -0.0031 |

[17] Notice also that average consumption under uncertainty in periods two and three is the same.

Table 24. Uncertainty (risk aversion)

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.5745 | 0 |
| 2 | 1 | -0.0745 | 1.2623 | -0.0745 |
| | 2 | -0.0745 | 1.7498 | |
| 3 | 1 | -0.3375 | 0.6591 | -0.0812 |
| | 2 | 0.175 | 2.1768 | |

Total average assets holding under NO uncertainty (Table 8) are -0.0093, and under uncertainty (Table 9) -0.1557. Comparing these results to the benchmark, we can see that under NO uncertainty, total average assets holdings are -0.0127 (Table 3) and under uncertainty (Table 4) are -0.1591. Thus, we can see that higher relative risk aversion implies higher savings.

### 3.5.4 Interest Rates

One would expect that higher interest rates generate an increase on average assets holdings. In this case, r=0.02.

Table 25. No uncertainty (higher interest rate)

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.5003 | 0 |
| 2 | 1.5 | -0.0003 | 1.4998 | -0.0003 |
| 3 | 1.5 | -0.0002 | 1.4998 | -0.0002 |

Table 26. Uncertainty (higher interest rate)

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 1.5 | 0 | 1.5688 | 0 |
| 2 | 1 | -0.0688 | 1.2632 | -0.0688 |
| | 2 | -0.0688 | 1.7531 | |
| 3 | 1 | -0.3333 | 0.66 | -0.0783 |
| | 2 | 0.1767 | 2.1803 | |

We can see that savings are lower under uncertainty, compared to the no uncertainty case. But comparing Table 11 and Table 4, it is easy to see that higher interest rates do generate higher average assets holdings. In this case (Table 11) total average assets holdings under uncertainty are -0.1471, which are higher than -0.1591 (Table 4).

So the main message of this section is to highlight that except for an increase in volatility, increases in risk aversion, interest rates and intertemporal distortions have all the exact same qualitative effects as the CES preferences. Obviously, a mean preserving spread increase in volatility generates lower savings instead of higher (which is what one would expect under CES preferences) because of the concavity of the first derivative of the utility function.

## 3.6 Higher Savings under Uncertainty

A legitimate question is whether it is possible to generate higher savings under uncertainty for this particular class of preferences (i.e., positive precautionary savings despite $u'''<0$).

As we saw in the previous section, intertemporal distortions or increases in risk aversion are not enough to undo the effect of imprudence (i.e., lower savings under uncertainty) . For this reason, we construct an example in which there is an intertemporal distortion in period 3 ($y_3^H = y_3^L = 0$) but also a change in relative risk aversion across time, so in this case the paremeter a in the utility function assumes a different value for period 3 ($a_1$ =49, $a_2$=49, $a_3$=3).

Table 27. No uncertainty

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 2.5 | 0 | 4.8123 | 0 |
| 2 | 2.5 | -2.3123 | 0.0827 | -2.3123 |
| 3 | 0 | 0.0819 | 0.0827 | 0.0819 |

Table 28. Uncertainty

| period | endowment | assets | consumption | E(assets) |
|---|---|---|---|---|
| 1 | 2.5 | 0 | 4.4756 | 0 |
| 2 | 2 | -1.9756 | 0.0017 | -1.9756 |
| | 3 | -1.9756 | 0.5048 | |
| 3 | 0 | 0.003 | 0.003 | 0.2514 |
| | 0 | 0.4998 | 0.5048 | |

It is possible to see from Tables 12 and 13 that savings are higher under uncertainty. So, in order to undo the "imprudence effect" it is necessary to combine a drastic increase in risk aversion and at the same time a "negative shock" in endowment in the last period.[18]These two forces together are stronger than imprudence and therefore the economy ends up saving more under uncertainty (Table 13) compared to the case of no uncertainty (Table 13).

[18] Notice that in period three there is no uncertainty. Recall that for these class of preferences lower volatility increases savings.

## 3.7 Infinite Horizon Model

This section main focus is showing that in an infinite horizon small open economy model, the sign of the third derivative is irrelevant to generate positive precautionary savings. There are two basic features of infinite horizon models under uncertainty that are key to understand how these kind of models differ from their finite horizons counterparts. First, the relationship between the real interest rate and the rate of time preference and second, the borrowing constraints implied by either Inada conditions or non-negativity constraints (in consumption).

It is important to understand that under uncertainty, a stationary equilibrium exists only if the real interest rate is lower than the subjective rate of time preference. When horizons are finite, whether the rate of time preference equals or exceeds the interest rate will only affect the shape of the consumption path. But this will not affect the existence of a well defined equilibrium. As highlighted by Aiyagari (1994) under infinite horizon, if the interest rate is equal or higher than the rate of time preference, agents will choose to accumulate an infinite amount of assets and average assets holdings will be infinite. Intuitively, when the real interest rate is higher than the rate of the preference, consumers want to postpone consumption to the future and be lenders. This will also be true in the case that the rate of time preference equals the real interest rate. Intuitively, under infinite horizon, there is always a positive probability of getting a sufficiently long string of bad endowment realizations and in order to maintain a smooth marginal utility across time, agents would accumulate an arbitrarily large amount of assets to buffer bad realizations of the shocks.[19]

Given this, it is required that the real interest rate is lower than the rate of time preference for assets to be finite. This is a well understood feature of infinite horizon small open economy models under uncertainty. In other words, if the subjective discount factor is equal to the real interest rate, the model induces a random walk component in the equilibrium marginal utility of consumption and net foreign assets. This result is completely independent of the sign of the third derivative and it is only caused by the infinite horizon and a borrowing constraint.

It is also important to note that Inada conditions on preferences (i.e., CES preferences) implicitly introduce a borrowing constraint because consumption can never be zero. This is usually called the natural borrowing limit. Having an ad-hoc borrowing limit (usually for calibration purposes) affects average assets holdings, but does not affect the qualitative behavior of the economy.

Consider a simple small open economy inhabited by a representative agent. There is only one tradable good. The representative agent can borrow and lend in international capital markets at a given real interest rate. Markets are incomplete, since the only financial instrument available is a one-period non-contingent bond that pays the world's real interest rate.

Agent's preferences are given by:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right] \qquad (21)$$

[19] See Chamberlain and Wilson (1984) or Ljungqvist and Sargent (2004) for details.

where the period utility function is the one presented in section 2.

The economy chooses consumption and foreign assets to maximize (21) subject to the following constraints. The resource constraint,

$$c_1 = (1+r)b_t + y_t - b_{t+1} - A \tag{22}$$

where $C_t$, Y, and $B_t$ denote consumption, endowment and net foreign assets position in period t respectively and R=(1+r), where r is the world's real interest rate which is taken as given and constant. A is a positive parameter needed in order to insure that consumption is never higher than $\bar{c} = \sqrt{a/3b}$. Following Durdu et. al. (2007), this parameter can be thought as lump sum absorption.

Since this particular class of preferences doesn’t display an Inada condition, we impose a non-negativity constraint given by $C_t \geq 0$. Note that $C_t \geq 0$ automaticaly implies a lower bound in the assets space. In other words, $C_t \geq 0$ is implicitly assuming that there is a borrowing constraint. Thus,

$$b_t \geq \phi$$

where  is the borrowing limit for net foreign assets.

The economy's income, Y, is subject to random shocks, which follow a first-order Markov chain.

### 3.7.1 Equilibrium

If the borrowing limit is not binding, the optimality condition for the competitive equilibrium is

$$u'(c_t) = (1+r)\beta E\ [u'(c_{t+1})] \tag{23}$$

which has the usual interpretation. The marginal benefit of saving an additional unit of the endowment is equal to the marginal cost of not consuming that unit.

A competitive equilibrium is defined by stochastic sequences [$C_t$, $B_{t+1}$] such that the Euler equation (23) and the resource constraint (22) are both satisfied for all t.

### 3.7.2 Parameterization

The parameterization is exactly the same as in Durdu et al. (2007) except for the borrowing limit and the lump sum absorption (A) which were chosen to match the level of net foreign assets and the level of consumption for the Mexican economy. The preference parameters (i.e., risk aversion) were chosen to match the standard deviation of consumption.

Table 29. Parameters. Infinite Horizon Model.

| Notation | Parameter/ variable | value |
|---|---|---|
| $\beta$ | discount factor | 0.94 |
| r | interest rate | 0.059 |
| a | preference parameter | 8.35 |
| b | preference parameter | 0.333 |
| $\sigma_e$ | Std dev of GDP innovations | 0.026 |
| $\rho$ | Autocorrelation of GDP | 0.597 |
| $\phi$ | Ad-hoc debt limit | -1.34 |
| A | Lump sum absorption | -0.77 |

### 3.7.3 Results

In this section I present and explain qualitative and quantitative results obtained with the model economy described above.

I first show that precautionary savings are positive despite the fact that preferences display a negative third derivative. Second, following Durdu, Mendoza and Terrones (2007), it is easy to see that using this particular class of preferences does not make a substantial difference regarding saving levels and cyclical behavior of macroeconomic variables in the economy.

Our main interest is to assess the effects of (i) volatility, (ii) peristence, (iii) risk aversion and (iv) interest rates, on the level of average assets holdings and therefore precautionary savings.

Table 30. Moments.

| | Baseline | rho=0.7 | sd=5% | sd=2.5% | a=6 | r=6.3% |
|---|---|---|---|---|---|---|
| Precautionary Savings | 0.92 | 0.94 | 1.05 | 0.88 | 0.98 | 1.12 |
| NFA ratios | 0.31 | 0.30 | 0.22 | 0.34 | 0.27 | 0.16 |
| **Means** | | | | | | |
| Output | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| Consumption | 0.69 | 0.69 | 0.70 | 0.68 | 0.69 | 0.70 |
| Foreign Assets | **-0.42** | **-0.40** | **-0.29** | **-0.46** | **-0.36** | **-0.22** |
| Trade Balance | 0.31 | 0.31 | 0.29 | 0.32 | 0.31 | 0.30 |
| Current Account | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| **Standard Deviation** | | | | | | |
| Output | 3.29 | 3.62 | 6.26 | 2.47 | 3.22 | 3.19 |
| Consumption | 3.26 | 3.79 | 5.58 | 2.64 | 2.80 | 2.41 |
| Foreign Assets | 17.56 | 22.36 | 42.09 | 9.49 | 25.82 | 34.81 |
| Trade Balance | 5.48 | 5.75 | 11.87 | 3.59 | 6.40 | 7.24 |
| Current Account Account | 5.24 | 5.41 | 11.27 | 3.44 | 6.06 | 6.70 |
| **Output Correlations** | | | | | | |
| Consumption | 0.79 | 0.81 | 0.72 | 0.85 | 0.68 | 0.55 |
| Foreign Assets | 0.51 | 0.49 | 0.46 | 0.57 | 0.42 | 0.31 |
| Trade Balance | 0.75 | 0.69 | 0.78 | 0.69 | 0.79 | 0.84 |
| Current Account | 0.47 | 0.53 | 0.49 | 0.42 | 0.50 | 0.53 |
| **Autocorrelations** | | | | | | |
| Output | 0.55 | 0.64 | 0.54 | 0.55 | 0.55 | 0.54 |
| Consumption | 0.76 | 0.80 | 0.79 | 0.73 | 0.82 | 0.87 |
| Foreign Assets | 0.93 | 0.95 | 0.95 | 0.91 | 0.96 | 0.97 |
| Trade Balance | 0.47 | 0.57 | 0.49 | 0.40 | 0.51 | 0.55 |
| Current Account | 0.45 | 0.55 | 0.47 | 0.39 | 0.49 | 0.52 |

Column one shows the baseline, which uses the parameterization described above and the class of preferences considered along this study. The first thing to notice is that precautionary savings are positive regardless of the sign of the third derivative. This is shown in rows one and two of table 15.[20]And the intuition for this result is that in an infinite horizon model with a borrowing constraint, agents take into account that many different sequences of endowments are possible to realize, in particular, there is always a positive probability to receive a very long sequence of the

[20] The second row is just the level of average assets holdings divided by the ad-hoc borrowing limit.

worst possible shock. If this is the case, the optimal response is to accumulate a large enough buffer stock of assets in order to smooth consumption (i.e., dissave) when needed. Therefore, average assets holdings are always higher under uncertainty compared to a situation in which the endowment is completely certain. And this result is independent of whether the marginal utility is convex (i.e., $u'''>0$), linear (i.e., $u'''=0$) or concave (i.e., $u'''<0$) and it is only a consequence of the infinite horizon and the borrowing constraint.

The qualitative effects of higher peristence, higher volatility, higher degree of risk aversion and higher interest rates go in the standard direction. Higher peristence affects the volatility of GDP (higher persistence implies higher GDP volatility since $\sigma_y^2 = \sigma_\varepsilon^2 /(1-\rho_y^2)$) and therefore precautionary savings increase. As is standard for almost every class of preferences, and in particular, for the ones with positive third derivative, it is almost always the case (with the exeption of very particular examples shown in Huggett (2004)) that higher volatility increases precautionary savings (i.e., lower debt for this particular calibration). Regarding risk aversion, for the class of preferences considered in this study, for a given b, it is the a parameter that affects the degree of risk aversion (see appendix for details) and from table 15 it is easy to observe that higher risk aversion (i.e., lower a) increases the level of precautionary savings. The most interesting thing to notice is that the coefficient of relative prudence (CRP) is constant and does not depend on any preference parameters. This particular feature enables us to isolate the effect of risk aversion in the determination of precautionary savings.[21]

Regarding the effect of interest rates, it is also possible to observe that higher interest rates increase average assets holdings (i.e., lower debt in this case). Interestingly enough, for an increase of less than one percentage point in the interest rate, the economy goes from having a 42% debt (as a fraction of GDP) to a 22% debt. The reason for this inmense change is the particular relationship between average assets holdings and interest rates highlighted in Aiyagari (1994).[22]

## 3.8 Is the infinite horizon necessary?

A legitimate question is whether it is possible to generate precautionary savings in a finite horizon model without changing the utility function across time. And the answer is yes, provided that the time horizon is long enough. It is easy to show that in a multiperiod model, if time horizon is long enough, then, agents will behave in the same way as in an infinite horizon model. And the intuition for this is the following. It is very well understood that in these types of models, it is optimal to hit the borrowing constraint at certain moments in time for certain realizations of the endowment. Once the economy hits the borrowing limit, then it is not possible to do

[21] Notice that for CES preferences or exponential preferences this is not the case and the same parameters affecting the degree of risk aversion, also affect the degree of prudence.

[22] As the real interest rate approaches the rate of time preference from below, average assets holdings go to infinity.

consumption smoothing until either a buffer stock of assets is build or the debt is reduced. Obviously, whether the economy actually hits the borrowing limit, depends on the particular realization of the endowment shocks, but the fact of having a finite number of periods implies a lower probability of hitting the constraint compared to the case of having infinite periods. Therefore, if the time horizon is relatively short; the higher the variance of the endowment, the fewer the incentives to save, given the preferences introduced in section 3.2. Moreover, if the number of periods is not very large, then the chances to get a sufficiently large stream of bad shocks are smaller compared to the case in which the time horizon is infinite.

In order to illustrate this result we solve the small open economy model with finite horizon using the following parameterization:

Table 31. Parameters. Finite Horizon Model.

| Notation | Parameter/ variable | value |
|---|---|---|
| $\beta$ | discount factor | 0.98 |
| r | interest rate | 0.02 |
| a | preference parameter | 6 |
| b | preference parameter | 0.333 |
| $\sigma_e$ | Std dev of GDP innovations | 0.05 / 0.35 |
| $\rho$ | Autocorrelation of GDP | 0.6 |
| $\phi$ | Ad-hoc debt limit | -2 |

We solve the model under two alternative scenarios: low (0.05) and high (0.35) variance for the innovations of the endowment. We first solve a forty period model (T=40) and then a two hundred period model (T=200) and compare the net foreign asset position corresponding to low and high level of uncertainty within each model.

Table 32. $\sigma_e$ = 0.05 T=40

| | Y | NFA | C | CA |
|---|---|---|---|---|
| mean | 1.4999 | -0.0756 | 1.4988 | -0.0004 |
| std dev | 0.0584 | 0.1673 | 0.0275 | 0.0497 |

Table 33. $\sigma_e$ =0.35 T=40

| | Y | NFA | C | CA |
|---|---|---|---|---|
| mean | 1.5002 | -0.3501 | 1.4941 | -0.001 |
| std dev | 0.4198 | 0.6057 | 0.2818 | 0.2221 |

Table 34. $\sigma_e$ =0.05 T=200

| | Y | NFA | C | CA |
|---|---|---|---|---|
| mean | 1.4997 | -0.6772 | 1.4863 | -0.0002 |
| std dev | 0.0699 | 0.5432 | 0.0309 | 0.0621 |

Table 35. $\sigma_e$ = 0.35 T=200

| | Y | NFA | C | CA |
|---|---|---|---|---|
| mean | 1.5033 | -0.5447 | 1.4925 | -0.0001 |
| std dev | 0.4881 | 0.9056 | 0.3467 | 0.242 |

As we can see from Tables 32 and 33, the agent has no incentives to save more under higher uncertainty simply because the chances of hitting the borrowing constraint in such a short period of time (T=40) are relatively low. There is not enough time to hit the constraint. On the other hand, from tables 34 and 35, it is easy to see that for a longer time horizon (T=200), even with a negative third derivative, the agent has incentives to save. The intuition for this result is that the effect of uncertainty is larger than the effect of imprudence on precautionary savings. Despite the fact of being imprudent, and knowing that the time horizon is relatively long, the agent decides to increase the amount of net foreign assets (i.e., reduce its debt) whenever uncertainty is higher (i.e., higher volatility of shocks).

## 3.9 Conclusion

This paper presents the first example of a particular class of preferences never considered before neither in the macroeconomic nor in the precautionary savings literature. These preferences are characterized by a concave utility function which displays two salient features; first, a negative third derivative and second, a constant but invariant relative prudence coefficient (in the sense of Kimball (1990)).

Intutively, agents are risk averse but imprudent. The advantage of this particular utility function is twofold. First, it enables us to assess, both qualitatively and quantitatively, the effects of changes in volatility and persistence of shocks (so it is possible to analyze, not only iid shocks, but more general structures for shocks) , risk aversion, interest rates and intertemporal distortions on the levels of precautionary savings in a small open economy. And second, it is possible to isolate the effect of an increase in risk aversion on precautionary savings (for a given and constant degree of prudence). This is a crucial difference with all other classes of preferences (i.e., CES or exponential) in which parameters affecting the degree of risk aversion also affect the degree of relative prudence.

As shown in the numerical exercises conducted above, this particular class of preferences enables to focus and highlight the importance of different determinants of precautionary savings both in finite and infinite horizons models. Unfortunately the wide use of preferences with a positive third derivative has blurred the importance of isolating risk aversion as a determinant of precautionary savings. This lead to the mistaken belief that it is the sign of the third derivative what determines precautionary savings. This is only true in two period models. In models with more than two periods one can build examples where the combination of increasing risk aversion (across time), intertemporal distortions and sufficiently high volatility offset the effect of imprudence on precautionary savings. Moreover, provided the time horizon is long enough, the effect of uncertainty is larger than the effect of imprudence and higher uncertainty implies higher savings. Regarding infinite horizon models, the key ingredient is the interaction of three elements; uncertainty, infinite horizon and borrowing constraints, regardless of the sign of the third derivative.

# Appendix A

# Overview

**Table** A.1 **Descriptive statistics of commodity prices**

| **Commodity** | | **57-69** | **70-79** | **80-89** | **90-99** | **00-07** |
|---|---|---|---|---|---|---|
| ALUMINUM | Mean | 16.266 | 16.192 | 14.682 | 11.840 | 12.231 |
| | SD | 0.467 | 2.248 | 4.486 | 1.538 | 2.093 |
| | SD/Mean | 0.029 | 0.139 | 0.306 | 0.130 | 0.171 |
| | Autocorrelation | 0.108 | 0.775 | 0.345 | 0.132 | 0.742 |
| BEEF | Mean | 1.164 | 1.457 | 1.076 | 0.814 | 0.709 |
| | SD | 0.224 | 0.317 | 0.128 | 0.173 | 0.044 |
| | SD/Mean | 0.192 | 0.218 | 0.119 | 0.212 | 0.062 |
| | Autocorrelation | 0.911 | 0.478 | 0.732 | 0.887 | -0.228 |
| BUTTER | Mean | 1.039 | 0.927 | 0.765 | 0.705 | 0.529 |
| | SD | 0.148 | 0.144 | 0.157 | 0.080 | 0.070 |
| | SD/Mean | 0.142 | 0.155 | 0.205 | 0.114 | 0.132 |
| | Autocorrelation | 0.374 | 0.245 | 0.746 | 0.274 | 0.126 |
| COCOA BEANS | Mean | 19.315 | 30.657 | 19.943 | 10.953 | 10.320 |
| | SD | 5.409 | 14.856 | 4.834 | 1.469 | 2.242 |
| | SD/Mean | 0.280 | 0.485 | 0.242 | 0.134 | 0.217 |
| | Autocorrelation | 0.611 | 0.719 | 0.602 | 0.206 | 0.333 |
| CACAO | Mean | 17.564 | 29.900 | 18.621 | 10.251 | 11.304 |
| | SD | 5.146 | 15.831 | 4.239 | 1.687 | 2.342 |
| | SD/Mean | 0.293 | 0.530 | 0.228 | 0.165 | 0.207 |
| | Autocorrelation | 0.609 | 0.603 | 0.488 | 0.573 | 0.096 |
| COCONUT OIL:PHILIPPINES | Mean | 9.786 | 10.226 | 6.034 | 4.857 | 3.723 |
| | SD | 1.225 | 3.713 | 2.386 | 1.036 | 0.900 |
| | SD/Mean | 0.125 | 0.363 | 0.395 | 0.213 | 0.242 |
| | Autocorrelation | 0.245 | -0.046 | 0.259 | 0.577 | 0.297 |
| COFFEE:OTHER MILDS | Mean | **1.336** | 1.807 | 1.376 | 0.932 | 0.588 |
| | SD | 0.227 | 0.804 | 0.279 | 0.298 | 0.118 |
| | SD/Mean | 0.170 | 0.445 | 0.203 | 0.320 | 0.201 |
| | Autocorrelation | 0.729 | 0.571 | -0.127 | 0.361 | 0.588 |
| COFFEE:BRAZIL (NEW YORK) | Mean | 1.261 | 1.952 | 1.532 | 0.868 | 0.509 |
| | SD | 0.226 | 0.894 | 0.498 | 0.290 | 0.133 |
| | SD/Mean | 0.179 | 0.458 | 0.325 | 0.334 | 0.262 |
| | Autocorrelation | 0.596 | 0.496 | 0.208 | 0.453 | 0.574 |
| COFFEE:BRAZIL: US CENTS/LB | Mean | 1.066 | 1.538 | 1.088 | 0.714 | 0.440 |
| | SD | 0.166 | 0.729 | 0.374 | 0.266 | 0.131 |
| | SD/Mean | 0.156 | 0.474 | 0.344 | 0.372 | 0.298 |
| | Autocorrelation | 0.580 | 0.570 | -0.195 | 0.518 | 0.561 |
| COFFEE:UGANDA | Mean | 1.039 | 1.637 | 1.157 | 0.623 | 0.321 |
| | SD | 0.139 | 0.792 | 0.282 | 0.225 | 0.104 |
| | SD/Mean | 0.134 | 0.484 | 0.244 | 0.362 | 0.323 |
| | Autocorrelation | 0.494 | 0.569 | 0.256 | 0.474 | 0.753 |
| COPPER | Mean | 28.218 | 28.031 | 18.035 | 18.281 | 21.909 |
| | SD | 9.996 | 7.891 | 4.820 | 3.603 | 12.509 |
| | SD/Mean | 0.354 | 0.282 | 0.267 | 0.197 | 0.571 |
| | Autocorrelation | 0.820 | 0.420 | 0.613 | 0.494 | 0.864 |
| COPRA:PHILIPPINES | Mean | 6.127 | 6.509 | 3.934 | 3.145 | 2.456 |
| | SD | 0.780 | 2.608 | 1.365 | 0.602 | 0.635 |
| | SD/Mean | 0.127 | 0.401 | 0.347 | 0.192 | 0.258 |
| | Autocorrelation | 0.132 | 0.022 | 0.264 | 0.483 | 0.295 |

| | | | | | | |
|---|---|---|---|---|---|---|
| COTTON:LIVERPOOL | Mean | 0.904 | 1.046 | 0.729 | 0.600 | 0.388 |
| | SD | 0.058 | 0.190 | 0.161 | 0.114 | 0.046 |
| | SD/Mean | 0.064 | 0.182 | 0.221 | 0.190 | 0.119 |
| | Autocorrelation | 0.239 | 0.110 | 0.494 | 0.401 | 0.024 |
| FISHMEAL | Mean | 8.929 | 12.651 | 7.391 | 4.460 | 4.978 |
| | SD | 1.981 | 4.478 | 1.945 | 0.874 | 1.176 |
| | SD/Mean | 0.222 | 0.354 | 0.263 | 0.196 | 0.236 |
| | Autocorrelation | 0.419 | 0.078 | 0.675 | 0.244 | 0.770 |
| GROUNDNUTS:NIGERIA | Mean | 5.750 | 8.059 | 9.394 | 7.154 | 5.708 |
| | SD | 0.495 | 2.253 | 3.308 | 1.258 | 0.699 |
| | SD/Mean | 0.086 | 0.280 | 0.352 | 0.176 | 0.122 |
| | Autocorrelation | 0.038 | 0.086 | 0.778 | 0.161 | -0.268 |
| GROUNDNUT OIL | Mean | 9.412 | 12.888 | 7.462 | 7.232 | 6.630 |
| | SD | 1.019 | 2.852 | 2.068 | 1.055 | 1.495 |
| | SD/Mean | 0.108 | 0.221 | 0.277 | 0.146 | 0.226 |
| | Autocorrelation | 0.105 | -0.002 | 0.359 | 0.119 | 0.246 |
| HIDES | Mean | 0.418 | 0.580 | 0.587 | 0.679 | 0.507 |
| | SD | 0.083 | 0.194 | 0.163 | 0.059 | 0.095 |
| | SD/Mean | 0.198 | 0.334 | 0.278 | 0.087 | 0.187 |
| | Autocorrelation | 0.018 | 0.386 | 0.852 | 0.470 | 0.871 |
| IRON ORE:BRAZIL (US CENTS/DMTU) | Mean | 0.508 | 0.345 | 0.270 | 0.249 | 0.316 |
| | SD | 0.110 | 0.039 | 0.031 | 0.028 | 0.120 |
| | SD/Mean | 0.217 | 0.112 | 0.116 | 0.112 | 0.380 |
| | Autocorrelation | 0.915 | 0.002 | 0.642 | 0.719 | 0.897 |
| JUTE:BANGLADESH | Mean | 7.595 | 6.155 | 3.549 | 2.690 | 1.997 |
| | SD | 1.529 | 1.034 | 1.012 | 0.550 | 0.245 |
| | SD/Mean | 0.201 | 0.168 | 0.285 | 0.204 | 0.123 |
| | Autocorrelation | 0.222 | 0.744 | 0.218 | 0.394 | 0.439 |
| LAMB:NEW ZEALAND | Mean | 0.933 | 1.208 | 1.019 | 1.009 | 1.014 |
| | SD | 0.084 | 0.175 | 0.206 | 0.097 | 0.110 |
| | SD/Mean | 0.090 | 0.145 | 0.202 | 0.096 | 0.109 |
| | Autocorrelation | 0.199 | 0.577 | 0.854 | -0.017 | 0.587 |
| LEAD | Mean | 7.114 | 9.170 | 5.717 | 4.859 | 6.138 |
| | SD | 1.451 | 2.548 | 1.919 | 1.013 | 3.917 |
| | SD/Mean | 0.204 | 0.278 | 0.336 | 0.208 | 0.638 |
| | Autocorrelation | 0.478 | 0.259 | 0.776 | 0.254 | 0.903 |
| LINSEED OIL | Mean | 7.445 | 9.070 | 5.495 | 4.526 | 4.695 |
| | SD | 1.128 | 4.772 | 1.377 | 0.888 | 1.479 |
| | SD/Mean | 0.152 | 0.526 | 0.251 | 0.196 | 0.315 |
| | Autocorrelation | 0.773 | 0.471 | 0.412 | -0.004 | 0.434 |
| MAIZE: US | Mean | 1.611 | 1.741 | 1.116 | 0.924 | 0.740 |
| | SD | 0.121 | 0.372 | 0.224 | 0.148 | 0.098 |
| | SD/Mean | 0.075 | 0.214 | 0.200 | 0.160 | 0.133 |
| | Autocorrelation | 0.488 | 0.554 | 0.620 | 0.273 | 0.133 |
| MAIZE:Thailand | Mean | 1.677 | 1.846 | 1.242 | 1.473 | 1.411 |
| | SD | 0.118 | 0.395 | 0.286 | 0.458 | 0.451 |
| | SD/Mean | 0.070 | 0.214 | 0.230 | 0.311 | 0.319 |
| | Autocorrelation | 0.515 | 0.482 | 0.769 | 0.539 | -0.777 |
| NICKEL | Mean | 54.933 | 75.868 | 65.482 | 56.730 | 97.575 |
| | SD | 3.752 | 4.395 | 32.187 | 11.960 | 57.291 |
| | SD/Mean | 0.068 | 0.058 | 0.492 | 0.211 | 0.587 |
| | Autocorrelation | 0.825 | -0.073 | 0.561 | 0.521 | 0.931 |
| PALM OIL:MALAYSIA | Mean | 6.238 | 6.818 | 4.325 | 3.699 | 2.737 |
| | SD | 1.088 | 1.521 | 1.193 | 0.821 | 0.646 |
| | SD/Mean | 0.174 | 0.223 | 0.276 | 0.222 | 0.236 |
| | Autocorrelation | 0.665 | 0.273 | 0.413 | 0.450 | 0.224 |

| | | | | | | |
|---|---|---|---|---|---|---|
| PETROLEUM:AVERAGE CRUDE PRICE | Mean | 0.057 | 0.159 | 0.251 | 0.150 | 0.274 |
| | SD | 0.004 | 0.103 | 0.092 | 0.025 | 0.093 |
| | SD/Mean | 0.078 | 0.644 | 0.366 | 0.163 | 0.340 |
| | Autocorrelation | 0.922 | 0.649 | 0.865 | 0.362 | 0.934 |
| PETROLEUM:DUBAI | Mean | 0.057 | 0.156 | 0.243 | 0.137 | 0.256 |
| | SD | 0.004 | 0.102 | 0.098 | 0.021 | 0.091 |
| | SD/Mean | 0.067 | 0.655 | 0.404 | 0.152 | 0.355 |
| | Autocorrelation | 0.894 | 0.655 | 0.881 | 0.087 | 0.932 |
| PETROLEUM:UK BRENT | Mean | 0.067 | 0.182 | 0.260 | 0.151 | 0.278 |
| | SD | 0.003 | 0.105 | 0.100 | 0.027 | 0.097 |
| | SD/Mean | 0.048 | 0.575 | 0.386 | 0.180 | 0.348 |
| | Autocorrelation | 0.333 | 0.602 | 0.887 | 0.447 | 0.935 |
| PHOSPHATE ROCK:MOROCCO | Mean | 0.391 | 0.523 | 0.386 | 0.322 | 0.309 |
| | SD | 0.041 | 0.309 | 0.074 | 0.036 | 0.047 |
| | SD/Mean | 0.105 | 0.591 | 0.193 | 0.110 | 0.152 |
| | Autocorrelation | 0.735 | 0.489 | 0.862 | 0.615 | -0.136 |
| POTASH | Mean | 0.802 | 0.949 | 0.867 | 0.920 | 0.945 |
| | SD | 0.092 | 0.185 | 0.198 | 0.038 | 0.119 |
| | SD/Mean | 0.115 | 0.195 | 0.229 | 0.041 | 0.126 |
| | Autocorrelation | 0.454 | 0.358 | 0.730 | -0.036 | 0.818 |
| RICE:THAILAND (BANGKOK) | Mean | 4.779 | 5.186 | 2.927 | 2.339 | 1.625 |
| | SD | 0.766 | 2.033 | 1.075 | 0.225 | 0.228 |
| | SD/Mean | 0.160 | 0.392 | 0.367 | 0.096 | 0.140 |
| | Autocorrelation | 0.635 | 0.406 | 0.783 | 0.164 | 0.756 |
| RICE:THAILAND | Mean | 3.794 | 4.138 | 2.593 | 2.663 | 1.887 |
| | SD | 0.483 | 1.848 | 0.768 | 0.536 | 0.210 |
| | SD/Mean | 0.127 | 0.447 | 0.296 | 0.201 | 0.111 |
| | Autocorrelation | 0.503 | 0.335 | 0.710 | -0.270 | 0.438 |
| RUBBER:MALAYSIA | Mean | 0.774 | 0.554 | 0.455 | 0.366 | 0.383 |
| | SD | 0.183 | 0.122 | 0.110 | 0.105 | 0.153 |
| | SD/Mean | 0.236 | 0.221 | 0.243 | 0.288 | 0.400 |
| | Autocorrelation | 0.784 | 0.257 | 0.441 | 0.617 | 0.926 |
| RUBBER:THAILAND | Mean | 0.693 | 0.500 | 0.424 | 0.334 | 0.323 |
| | SD | 0.156 | 0.111 | 0.103 | 0.093 | 0.125 |
| | SD/Mean | 0.225 | 0.223 | 0.244 | 0.278 | 0.385 |
| | Autocorrelation | 0.702 | 0.300 | 0.396 | 0.615 | 0.912 |
| SHRIMP: U.S. GULF | Mean | 0.058 | 0.110 | 0.121 | 0.114 | 0.083 |
| | SD | 0.017 | 0.024 | 0.016 | 0.012 | 0.023 |
| | SD/Mean | 0.289 | 0.219 | 0.131 | 0.103 | 0.276 |
| | Autocorrelation | 0.816 | 0.482 | -0.158 | 0.610 | 0.922 |
| SILVER | Mean | 3.832 | 7.119 | 8.998 | 3.966 | 4.766 |
| | SD | 1.034 | 2.927 | 5.333 | 0.379 | 1.707 |
| | SD/Mean | 0.270 | 0.411 | 0.593 | 0.096 | 0.358 |
| | Autocorrelation | 0.783 | 0.602 | 0.646 | 0.324 | 0.867 |
| SISAL:EAST AFRICA | Mean | 7.188 | 8.735 | 5.847 | 5.713 | 5.307 |
| | SD | 2.338 | 4.521 | 1.058 | 0.750 | 0.371 |
| | SD/Mean | 0.325 | 0.518 | 0.181 | 0.131 | 0.070 |
| | Autocorrelation | 0.621 | 0.407 | 0.858 | 0.344 | 0.337 |
| SORGHUM:US | Mean | 1.407 | 1.634 | 1.061 | 0.882 | 0.746 |
| | SD | 0.118 | 0.338 | 0.227 | 0.128 | 0.097 |
| | SD/Mean | 0.084 | 0.207 | 0.213 | 0.145 | 0.131 |
| | Autocorrelation | 0.660 | 0.605 | 0.748 | 0.302 | 0.131 |
| SOYBEANS: US | Mean | 3.019 | 3.768 | 2.345 | 1.866 | 1.528 |
| | SD | 0.279 | 0.922 | 0.365 | 0.225 | 0.242 |
| | SD/Mean | 0.092 | 0.245 | 0.156 | 0.121 | 0.158 |
| | Autocorrelation | 0.404 | 0.238 | 0.342 | 0.295 | 0.098 |

| | | | | | | |
|---|---|---|---|---|---|---|
| SOYBEAN MEAL | Mean | 2.353 | 3.122 | 2.019 | 1.694 | 1.435 |
| | SD | 0.193 | 1.079 | 0.351 | 0.289 | 0.176 |
| | SD/Mean | 0.082 | 0.346 | 0.174 | 0.170 | 0.122 |
| | Autocorrelation | 0.234 | 0.010 | 0.424 | 0.242 | 0.037 |
| SOYBEAN OIL | Mean | 6.454 | 7.578 | 4.807 | 4.172 | 3.392 |
| | SD | 1.479 | 2.153 | 0.953 | 0.531 | 0.689 |
| | SD/Mean | 0.229 | 0.284 | 0.198 | 0.127 | 0.203 |
| | Autocorrelation | 0.571 | 0.173 | 0.240 | 0.142 | 0.404 |
| SUGAR: US CENTS/LB | Mean | 0.130 | 0.217 | 0.109 | 0.099 | 0.061 |
| | SD | 0.032 | 0.146 | 0.057 | 0.022 | 0.012 |
| | SD/Mean | 0.245 | 0.669 | 0.523 | 0.224 | 0.190 |
| | Autocorrelation | 0.357 | 0.505 | 0.819 | 0.570 | 0.231 |
| SUGAR:EU | Mean | 0.173 | 0.196 | 0.192 | 0.233 | 0.191 |
| | SD | 0.012 | 0.046 | 0.029 | 0.012 | 0.008 |
| | SD/Mean | 0.071 | 0.234 | 0.149 | 0.051 | 0.044 |
| | Autocorrelation | 0.760 | 0.686 | 0.575 | 0.558 | 0.225 |
| SUGAR:CARIBBEAN | Mean | 0.111 | 0.208 | 0.108 | 0.086 | 0.060 |
| | SD | 0.062 | 0.145 | 0.083 | 0.018 | 0.013 |
| | SD/Mean | 0.553 | 0.694 | 0.767 | 0.207 | 0.220 |
| | Autocorrelation | 0.353 | 0.438 | 0.838 | 0.508 | 0.168 |
| SUGAR:US | Mean | 0.193 | 0.257 | 0.219 | 0.181 | 0.144 |
| | SD | 0.016 | 0.119 | 0.041 | 0.009 | 0.014 |
| | SD/Mean | 0.085 | 0.461 | 0.187 | 0.048 | 0.098 |
| | Autocorrelation | 0.258 | 0.337 | -0.301 | 0.593 | 0.771 |
| SUGAR:PHILIPPINES | Mean | 0.186 | 0.219 | 0.163 | 0.155 | 0.112 |
| | SD | 0.021 | 0.118 | 0.026 | 0.019 | 0.023 |
| | SD/Mean | 0.110 | 0.539 | 0.159 | 0.120 | 0.206 |
| | Autocorrelation | 0.036 | 0.538 | 0.412 | 0.259 | 0.520 |
| TEA | Mean | 4.053 | 2.823 | 2.108 | 1.645 | 1.442 |
| | SD | 0.612 | 0.525 | 0.502 | 0.205 | 0.189 |
| | SD/Mean | 0.151 | 0.186 | 0.238 | 0.125 | 0.131 |
| | Autocorrelation | 0.941 | 0.162 | 0.197 | 0.510 | 0.332 |
| TEA:SRI LANKA | Mean | 3.492 | 2.483 | 1.973 | 1.825 | 1.736 |
| | SD | 0.501 | 0.400 | 0.419 | 0.259 | 0.087 |
| | SD/Mean | 0.143 | 0.161 | 0.213 | 0.142 | 0.050 |
| | Autocorrelation | 0.992 | 0.257 | 0.363 | 0.578 | -0.238 |
| TIMBER:HARDWOOD LOGS:SARAWAK | Mean | 1.031 | 1.400 | 1.208 | 1.920 | 1.356 |
| | SD | 0.132 | 0.324 | 0.319 | 0.615 | 0.119 |
| | SD/Mean | 0.128 | 0.231 | 0.264 | 0.320 | 0.088 |
| | Autocorrelation | 0.751 | 0.180 | 0.435 | 0.415 | 0.239 |
| TIN | Mean | 86.615 | 133.402 | 108.629 | 47.080 | 47.827 |
| | SD | 18.363 | 38.654 | 41.196 | 3.490 | 17.429 |
| | SD/Mean | 0.212 | 0.290 | 0.379 | 0.074 | 0.364 |
| | Autocorrelation | 0.826 | 0.779 | 0.873 | 0.159 | 0.558 |
| TIN:MALAYSIA | Mean | 84.557 | 126.963 | 108.963 | 47.417 | 47.361 |
| | SD | 17.754 | 36.660 | 40.422 | 4.445 | 16.959 |
| | SD/Mean | 0.210 | 0.289 | 0.371 | 0.094 | 0.358 |
| | Autocorrelation | 0.830 | 0.823 | 0.872 | 0.404 | 0.484 |
| TIN:BOLIVIA | Mean | 85.980 | 132.432 | 108.483 | 43.001 | 33.913 |
| | SD | 18.138 | 35.741 | 41.219 | 9.672 | 14.589 |
| | SD/Mean | 0.211 | 0.270 | 0.380 | 0.225 | 0.430 |
| | Autocorrelation | 0.835 | 0.801 | 0.839 | 0.673 | 0.607 |
| TIN:THAILAND | Mean | 66.555 | 126.324 | 108.001 | 46.477 | 47.472 |
| | SD | 22.617 | 35.748 | 40.673 | 3.468 | 17.796 |
| | SD/Mean | 0.340 | 0.283 | 0.377 | 0.075 | 0.375 |
| | Autocorrelation | 0.928 | 0.826 | 0.877 | 0.307 | 0.544 |

| | | | | | | |
|---|---|---|---|---|---|---|
| WHEAT U.S. GULF | Mean | 1.915 | 2.151 | 1.476 | 1.218 | 1.084 |
| | SD | 0.137 | 0.654 | 0.260 | 0.204 | 0.188 |
| | SD/Mean | 0.071 | 0.304 | 0.176 | 0.168 | 0.174 |
| | Autocorrelation | 0.315 | 0.566 | 0.833 | 0.489 | 0.518 |
| WHEAT:ARGENTINA | Mean | 1.845 | 2.066 | 1.378 | 1.085 | 0.952 |
| | SD | 0.133 | 0.706 | 0.405 | 0.181 | 0.131 |
| | SD/Mean | 0.072 | 0.342 | 0.294 | 0.166 | 0.138 |
| | Autocorrelation | 0.361 | 0.539 | 0.903 | 0.183 | 0.043 |
| WOOL:AUSTRALIA:48:COARSE | Mean | 6.926 | 6.253 | 4.596 | 3.116 | 3.567 |
| | SD | 1.350 | 1.732 | 0.767 | 0.541 | 0.942 |
| | SD/Mean | 0.195 | 0.277 | 0.167 | 0.174 | 0.264 |
| | Autocorrelation | 0.586 | 0.262 | 0.631 | 0.333 | 0.412 |
| WOOL:AUSTRALIA:64:FINE | Mean | 9.603 | 9.026 | 7.812 | 5.596 | 4.915 |
| | SD | 1.277 | 3.689 | 2.276 | 1.233 | 0.491 |
| | SD/Mean | 0.133 | 0.409 | 0.291 | 0.220 | 0.100 |
| | Autocorrelation | 0.213 | 0.266 | 0.532 | 0.070 | -0.227 |
| ZINC | Mean | 7.660 | 11.887 | 9.023 | 9.307 | 10.200 |
| | SD | 1.318 | 5.099 | 2.369 | 1.622 | 5.732 |
| | SD/Mean | 0.172 | 0.429 | 0.263 | 0.174 | 0.562 |
| | Autocorrelation | 0.468 | 0.499 | 0.734 | 0.176 | 0.705 |

**Table** A.2 **Variance Ratios**

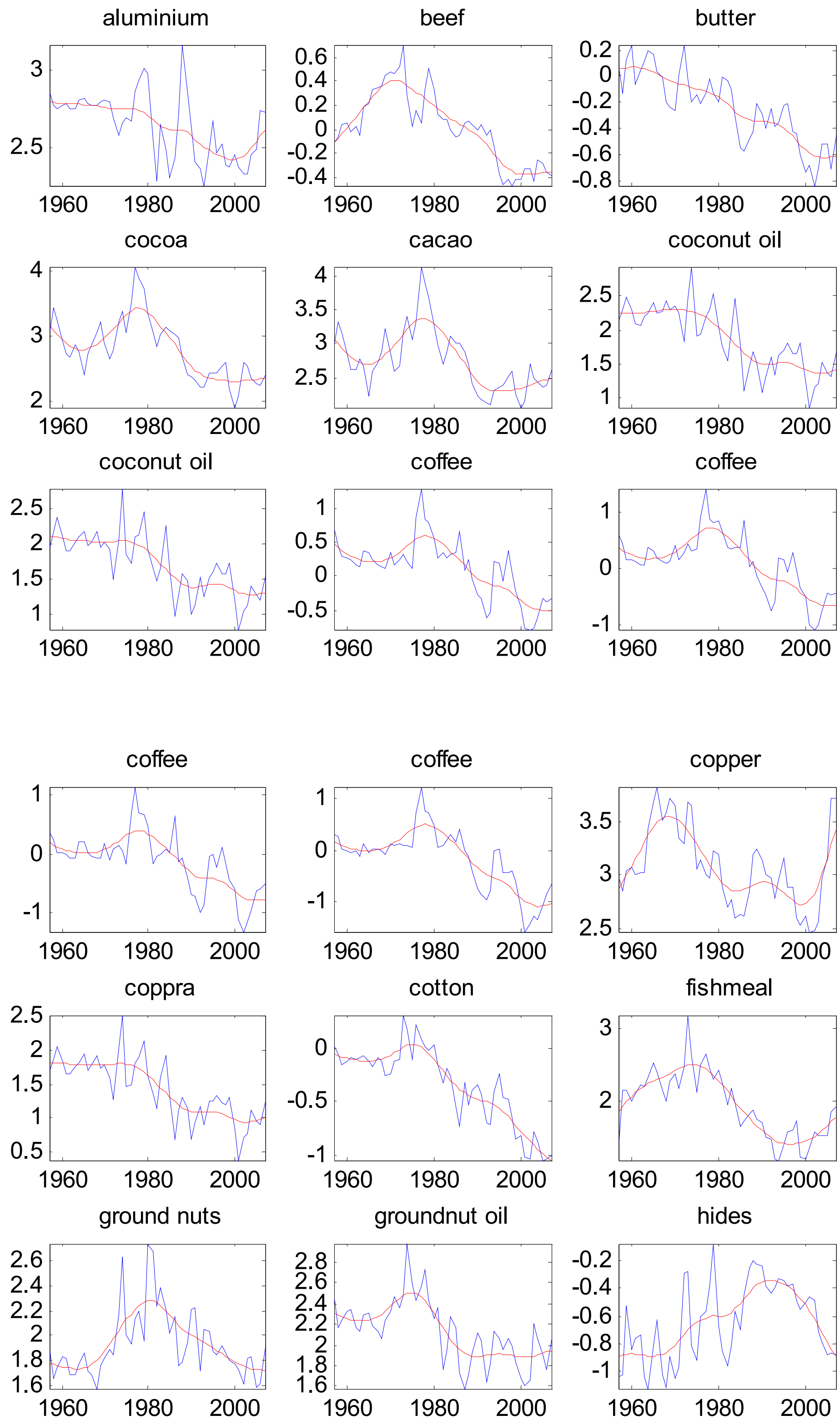
| | **Commodity Price** | **Lags (*k*)** | | | | | |
|---|---|---|---|---|---|---|---|
| | | **2** | **4** | **8** | **10** | **12** | **20** |
| 1 | ALUMINUM | 0.972 | 0.649 | 0.189 | 0.195 | 0.275 | 0.157 |
| 2 | BEEF:AUSTRALIA | 0.985 | 0.845 | 0.733 | 0.665 | 0.590 | 0.391 |
| 3 | BUTTER:NEW ZEALAND | 0.761 | 0.545 | 0.241 | 0.191 | 0.161 | 0.147 |
| 4 | COCOA BEANS | 1.040 | 0.796 | 0.572 | 0.613 | 0.712 | 0.362 |
| 5 | CACAO: US$/MT | 0.967 | 0.769 | 0.554 | 0.581 | 0.676 | 0.297 |
| 6 | COCONUT OIL:PHILIPPINES | 0.516 | 0.259 | 0.196 | 0.150 | 0.174 | 0.083 |
| 7 | COFFEE:OTHER MILDS | 0.908 | 0.829 | 0.352 | 0.336 | 0.383 | 0.203 |
| 8 | COFFEE:BRAZIL (NEW YORK) | 0.920 | 0.871 | 0.442 | 0.434 | 0.472 | 0.317 |
| 9 | COFFEE:BRAZIL: US CENTS/LB | 0.915 | 0.848 | 0.305 | 0.290 | 0.344 | 0.169 |
| 10 | COFFEE:UGANDA | 1.169 | 1.053 | 0.520 | 0.517 | 0.586 | 0.418 |
| 11 | COPPER | 1.110 | 0.962 | 0.648 | 0.565 | 0.480 | **0.274** |
| 12 | COPRA:PHILIPPINES | 0.542 | 0.256 | 0.186 | 0.147 | 0.177 | 0.091 |
| 13 | COTTON:LIVERPOOL | 0.584 | 0.505 | 0.422 | 0.364 | 0.300 | 0.172 |
| 14 | FISHMEAL | 0.562 | 0.499 | 0.371 | 0.362 | 0.422 | 0.263 |
| 15 | GROUNDNUTS:NIGERIA | 0.440 | 0.411 | 0.265 | 0.297 | 0.271 | 0.224 |
| 16 | GROUNDNUT OIL | 0.623 | 0.406 | 0.359 | 0.345 | 0.269 | 0.137 |
| 17 | HIDES | 0.685 | 0.483 | 0.372 | 0.247 | 0.206 | 0.142 |
| 18 | IRON ORE:BRAZIL | 1.028 | 0.855 | 0.547 | 0.581 | 0.450 | 0.423 |
| 19 | JUTE:BANGLADESH | 0.688 | 0.267 | 0.297 | 0.172 | 0.191 | 0.124 |
| 20 | LAMB:NEW ZEALAND | 0.937 | 0.688 | 0.452 | 0.492 | 0.400 | 0.235 |
| 21 | LEAD | 1.022 | 0.810 | 0.359 | 0.372 | 0.379 | 0.314 |
| 22 | LINSEED OIL | 0.919 | 0.466 | 0.316 | 0.314 | 0.279 | 0.103 |
| 23 | MAIZE: US | 0.831 | 0.568 | 0.409 | 0.341 | 0.348 | 0.144 |
| 24 | MAIZE: Thailand | 0.407 | 0.339 | 0.242 | 0.217 | 0.180 | 0.072 |
| 25 | NICKEL | 0.916 | 0.841 | 0.516 | 0.369 | 0.349 | 0.260 |
| 26 | PALM OIL:MALAYSIA | 0.615 | 0.485 | 0.314 | 0.269 | 0.243 | 0.114 |
| 27 | PETROLEUM:AVERAGE CRUDE PRICE | 0.977 | 0.961 | 1.181 | 1.140 | 1.090 | **0.594** |
| 28 | PETROLEUM:DUBAI | 0.895 | 0.906 | 1.137 | 1.090 | 1.035 | 0.557 |
| 29 | PETROLEUM:UK BRENT | 0.865 | 0.891 | 1.086 | 1.044 | 1.012 | 0.608 |
| 30 | PHOSPHATE ROCK:MOROCCO | 0.957 | 0.612 | 0.377 | 0.356 | 0.330 | 0.133 |
| 31 | POTASH | 0.853 | 0.486 | 0.313 | 0.290 | 0.227 | 0.127 |
| 32 | RICE:THAILAND (BANGKOK) | 1.107 | 0.699 | 0.553 | 0.459 | 0.396 | 0.221 |
| 33 | RICE:THAILAND | 0.832 | 0.509 | 0.402 | 0.312 | 0.239 | 0.111 |
| 34 | RUBBER:MALAYSIA | 0.994 | 0.876 | 0.355 | 0.285 | 0.283 | 0.199 |
| 35 | RUBBER:THAILAND | 0.983 | 0.848 | 0.324 | 0.263 | 0.269 | 0.189 |
| 36 | SHRIMP: U.S. GULF | 0.623 | 0.723 | 0.599 | 0.546 | 0.531 | 0.555 |
| 37 | SILVER | 0.930 | 0.768 | 0.811 | 0.804 | 0.805 | 0.720 |
| 38 | SISAL:EAST AFRICA | 1.091 | 0.819 | 0.419 | 0.285 | 0.272 | 0.137 |
| 39 | SORGHUM:US | 0.919 | 0.640 | 0.531 | 0.466 | 0.483 | 0.222 |
| 40 | SOYBEANS: US | 0.721 | 0.531 | 0.386 | 0.355 | 0.366 | 0.187 |
| 41 | SOYBEAN MEAL | 0.546 | 0.365 | 0.291 | 0.271 | 0.259 | 0.117 |
| 42 | SOYBEAN OIL | 0.632 | 0.434 | 0.250 | 0.259 | 0.194 | 0.103 |
| 43 | SUGAR: US CENTS/LB | 0.968 | 0.626 | 0.478 | 0.403 | 0.386 | 0.142 |
| 44 | SUGAR:EU | 0.989 | 1.040 | 0.791 | 0.541 | 0.309 | 0.235 |
| 45 | SUGAR:CARIBBEAN | 0.977 | 0.718 | 0.476 | 0.391 | 0.347 | 0.171 |
| 46 | SUGAR:US | 0.697 | 0.357 | 0.264 | 0.225 | 0.232 | 0.130 |
| 47 | SUGAR:PHILIPPINES | 0.937 | 0.504 | 0.355 | 0.263 | 0.213 | 0.085 |
| 48 | TEA | 0.669 | 0.429 | 0.300 | 0.269 | 0.160 | 0.063 |
| 49 | TEA:SRI LANKA | 0.823 | 0.474 | 0.416 | 0.339 | 0.220 | 0.140 |
| 50 | TIMBER:HARDWOOD | 0.705 | 0.634 | 0.505 | 0.415 | 0.272 | 0.138 |
| 51 | TIN | 0.869 | 0.936 | 0.949 | 0.957 | 1.007 | 0.923 |
| 52 | TIN:MALAYSIA | 0.847 | 0.927 | 0.952 | 0.942 | 0.988 | 0.937 |
| 53 | TIN:BOLIVIA | 0.822 | 0.871 | 0.891 | 0.840 | 0.921 | 0.979 |
| 54 | WHEAT U.S. GULF | 1.094 | 0.675 | 0.436 | 0.372 | 0.349 | 0.153 |
| 55 | WHEAT:ARGENTINA | 1.018 | 0.635 | 0.467 | 0.448 | 0.376 | 0.144 |
| 56 | WOOL:AUSTRALIA:48:COARSE | 1.019 | 0.694 | 0.292 | 0.321 | 0.235 | 0.158 |
| 57 | WOOL:AUSTRALIA:64:FINE | 0.823 | 0.484 | 0.233 | 0.216 | 0.191 | 0.078 |
| 58 | ZINC | 1.033 | 0.719 | 0.343 | 0.375 | 0.336 | 0.161 |

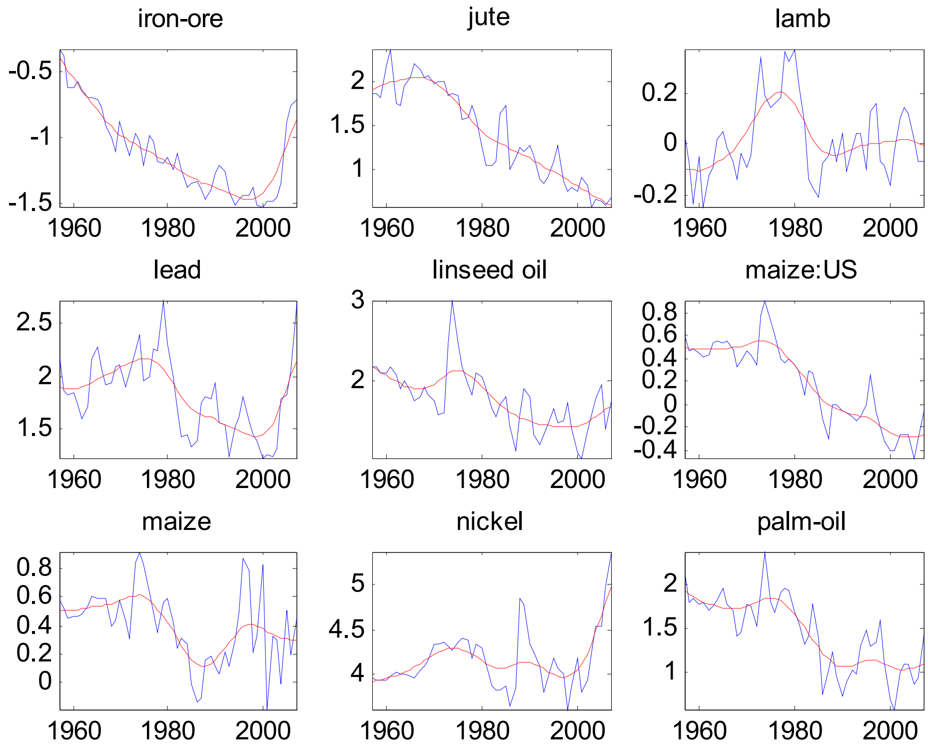
**Table** A.3 **Goodness of fit and log-likelihoods**

| | Commodity | Good Fit | Log like MS | Log like AR1 |
|---|---|---|---|---|
| 1 | ALUMINUM | * | 1324.05 | 1052.43 |
| 2 | BEEF:AUSTRALIA | * | 1085.09 | 971.21 |
| 3 | COCOA BEANS | * | 846.12 | 810.19 |
| 4 | COCONUT OIL:PHILIPPINES | * | 808.64 | 743.51 |
| 5 | COFFEE:OTHER MILDS | | | |
| 6 | COFFEE:BRAZIL (NEW YORK) | * | 917.59 | 704.98 |
| 7 | COFFEE:UGANDA | * | 933.26 | 801.41 |
| 8 | COPPER | * | 914.05 | 808.96 |
| 9 | COPRA:PHILIPPINES | * | 788.28 | 697.84 |
| 10 | COTTON:LIVERPOOL | | | |
| 11 | FISHMEAL | * | 975.12 | 802.26 |
| 12 | GROUNDNUTS:NIGERIA | | | |
| 13 | GROUNDNUT OIL | * | 994.11 | 888.2 |
| 14 | HIDES | | | |
| 15 | IRON ORE:BRAZIL (US CENTS/DMTU) | * | 1658.8 | 979.55 |
| 16 | JUTE:BANGLADESH | * | 1114.39 | 834.96 |
| 17 | LAMB:NEW ZEALAND | * | 1029.71 | 976.75 |
| 18 | LEAD | * | 924.03 | 825.94 |
| 19 | LINSEED OIL | * | 913.65 | 787.38 |
| 20 | MAIZE: US | * | 1046.49 | 984.09 |
| 21 | NICKEL | * | 1236.08 | 825.63 |
| 22 | PALM OIL:MALAYSIA | * | 856.08 | 753.84 |
| 23 | PETROLEUM:AVERAGE CRUDE PRICE | * | 1149.59 | 742.43 |
| 24 | PETROLEUM:DUBAI | | | |
| 25 | PETROLEUM:UK BRENT | | | |
| 26 | PHOSPHATE ROCK:MOROCCO | * | 1685.7 | 798.76 |
| 27 | POTASH | * | 1476.11 | 521.27 |
| 28 | RICE:THAILAND (BANGKOK) | | | |
| 29 | RUBBER:MALAYSIA | | | |
| 30 | SHRIMP: U.S. GULF | * | 808.74 | 700.73 |
| 31 | SISAL:EAST AFRICA | | | |
| 32 | SOYBEANS: US | * | 1026.59 | 894.21 |
| 33 | SOYBEAN MEAL | | | |
| 34 | SOYBEAN OIL | * | 884.6 | 845.38 |
| 35 | SUGAR:EU | | | |
| 36 | SUGAR:CARIBBEAN | | | |
| 37 | SUGAR:US | * | 1284.04 | 904.44 |
| 38 | TEA | * | 758.34 | 686.76 |
| 39 | TIMBER:HARDWOOD LOGS:SARAWAK | | | |
| 40 | TIN | | | |
| 41 | WHEAT U.S. GULF | | | |
| 42 | WOOL:AUSTRALIA:48:COARSE | * | 1065.21 | 1001.78 |
| 43 | WOOL:AUSTRALIA:64:FINE | | | |
| 44 | ZINC | * | 973.82 | 873.43 |

* This test is if all Markov switching coefficient are statistically significant at a 10%. The fourth and fifth column show the log likelihoods for the Markov switching and the AR1 processes, respectively.

**Figure** A.1 **Commodity Prices (constant US dollars)**

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-1.5
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1980
2000
jute
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1.5
1
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1980
2000
lamb
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0
-0.2
1960
1980
2000
lead
2.5
2
1.5
1960
1980
2000
linseed oil
3
2
1960
1980
2000
maize:US
0.8
0.6
0.4
0.2
0
-0.2
-0.4
1960
1980
2000
maize
0.8
0.6
0.4
0.2
0
1960
1980
2000
nickel
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1980
2000
palm-oil
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1
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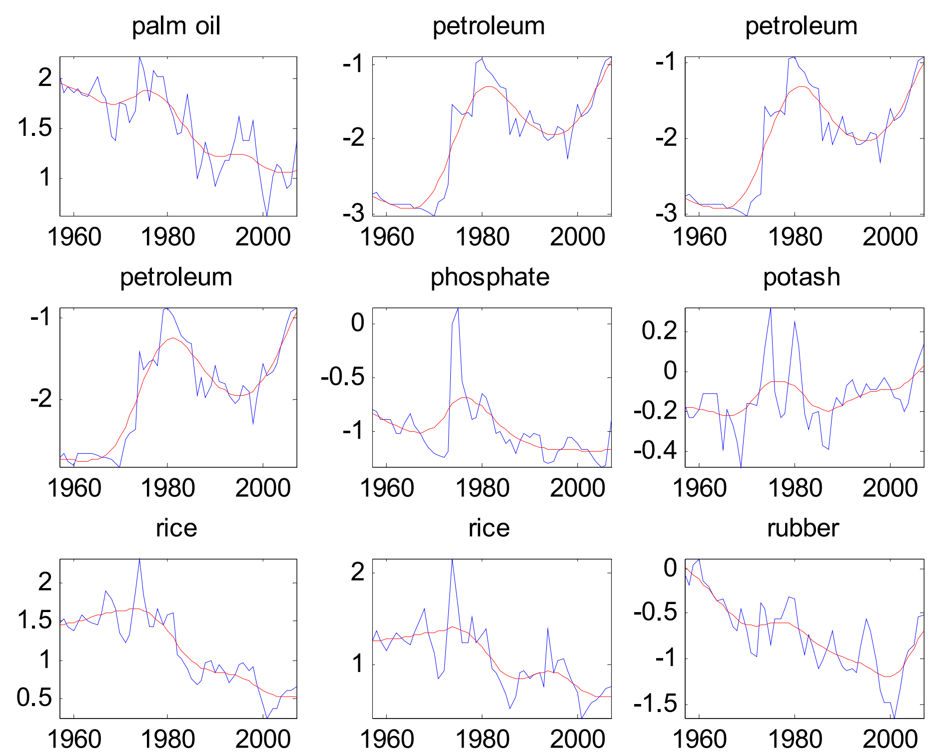
1980
2000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1
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1980
2000
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-2
-3
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1980
2000
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-2
-3
1960
1980
2000
petroleum
-1
-2
1960
1980
2000
phosphate
0
-0.5
-1
1960
1980
2000
potash
0.2
0
-0.2
-0.4
1960
1980
2000
rice
2
1.5
1
0.5
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1980
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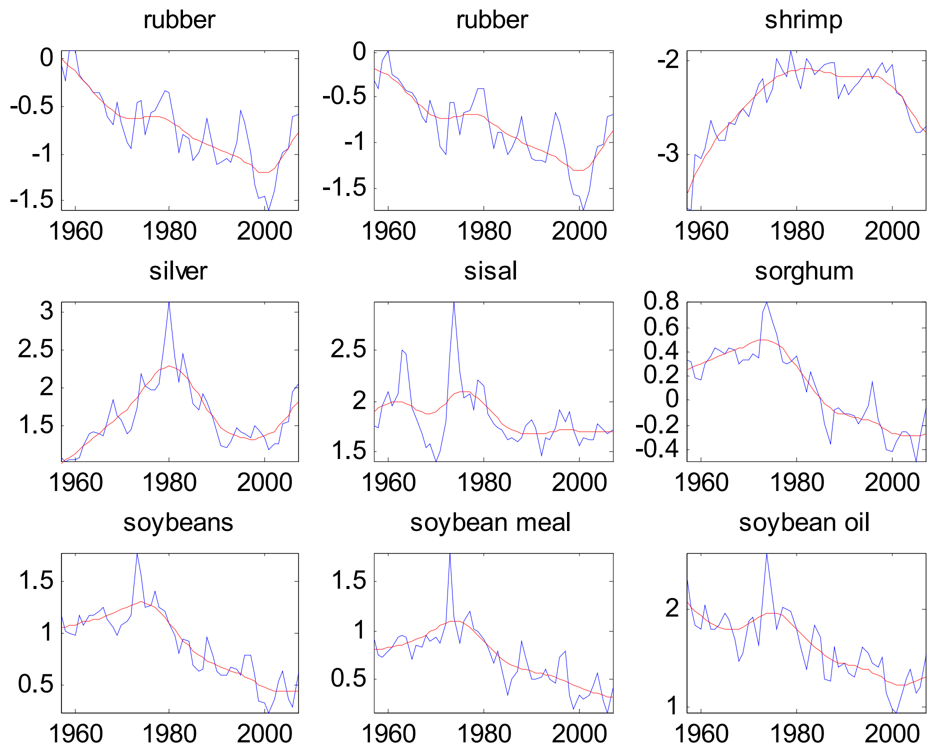
1980
2000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1980
2000
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1980
2000
shrimp
-2
-3
1960
1980
2000
silver
3
2.5
2
1.5
1960
1980
2000
sisal
2.5
2
1.5
1960
1980
2000
sorghum
0.8
0.6
0.4
0.2
0
-0.2
-0.4
1960
1980
2000
soybeans
1.5
1
0.5
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1980
2000
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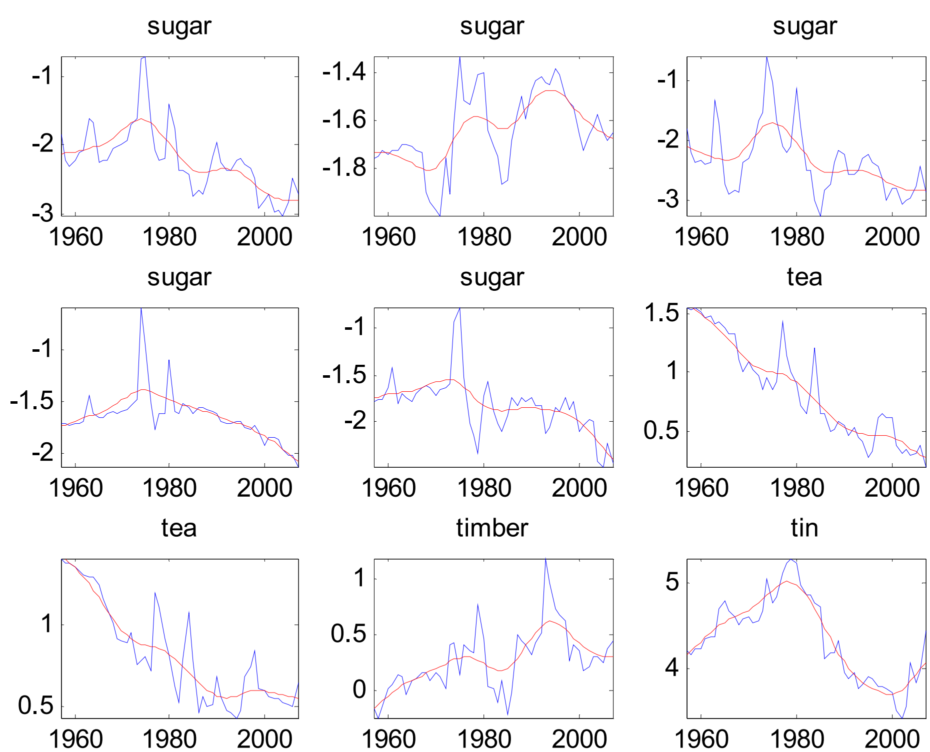
2
1
1960
1980
2000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-3
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1980
2000
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1960
1980
2000
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-2
-3
1960
1980
2000
sugar
-1
-1.5
-2
1960
1980
2000
sugar
-1
-1.5
-2
1960
1980
2000
tea
1.5
1
0.5
1960
1980
2000
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1980
2000
timber
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1980
2000
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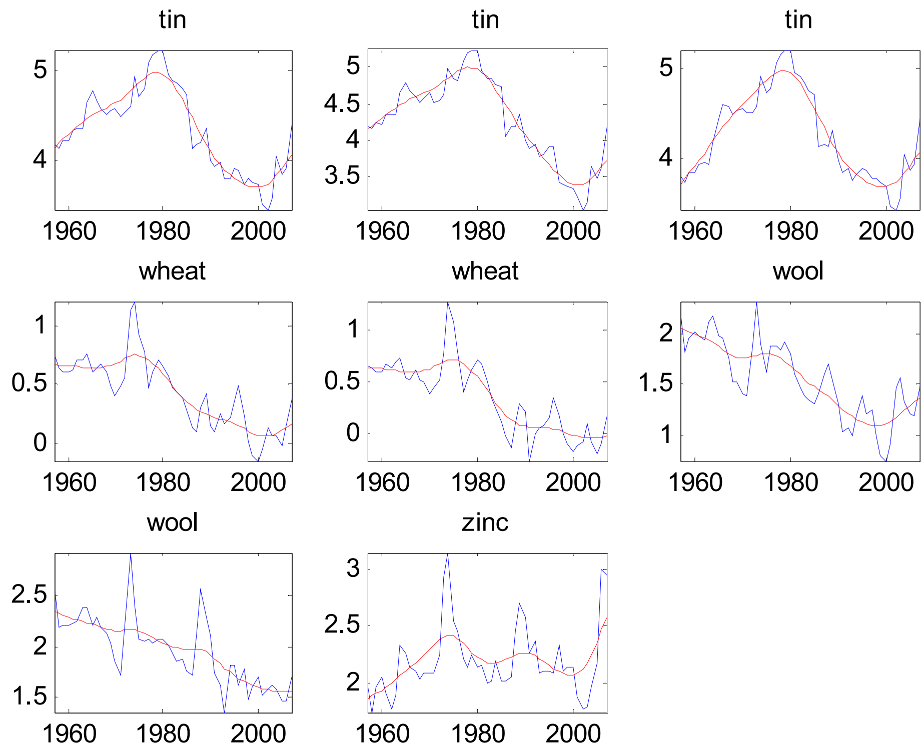
5
4
1960
1980
2000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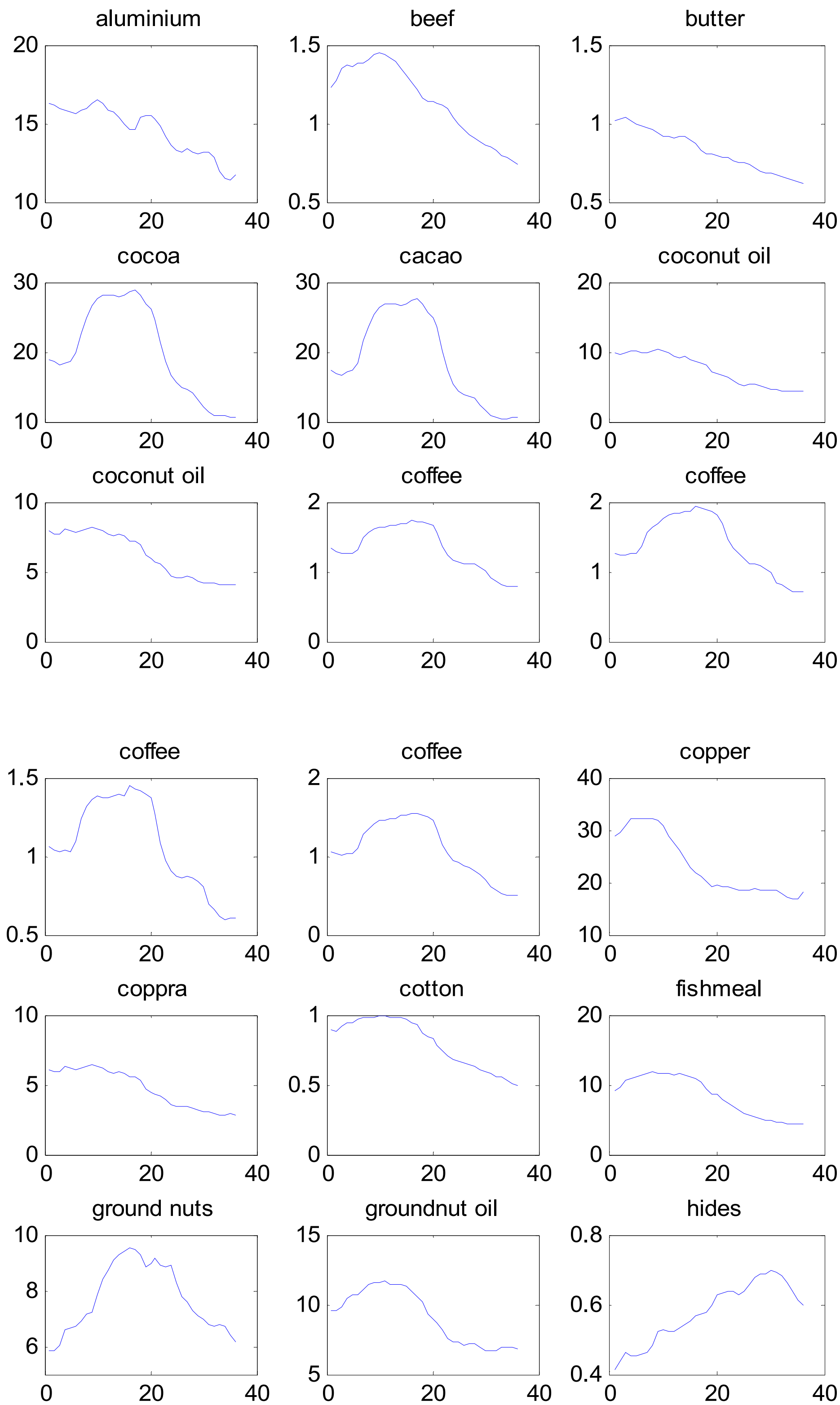
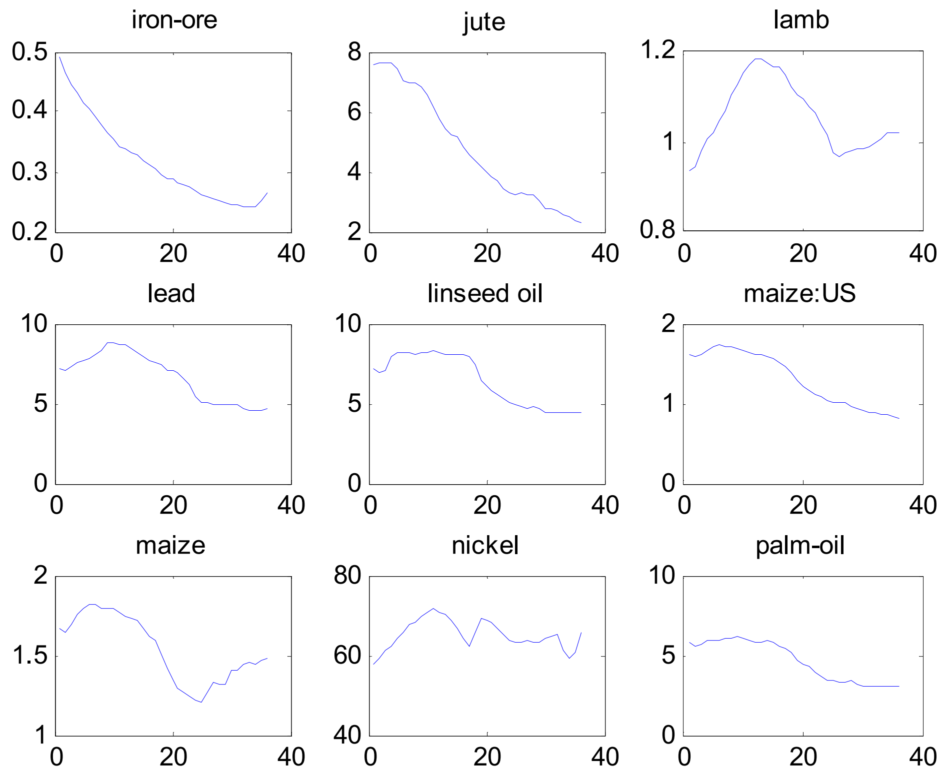
tin
tin
wheat
wheat
wool
wool
zinc

**Figure** A.2.1 **Rolling-window means (15-years)**

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lamb
lead
linseed oil
maize:US
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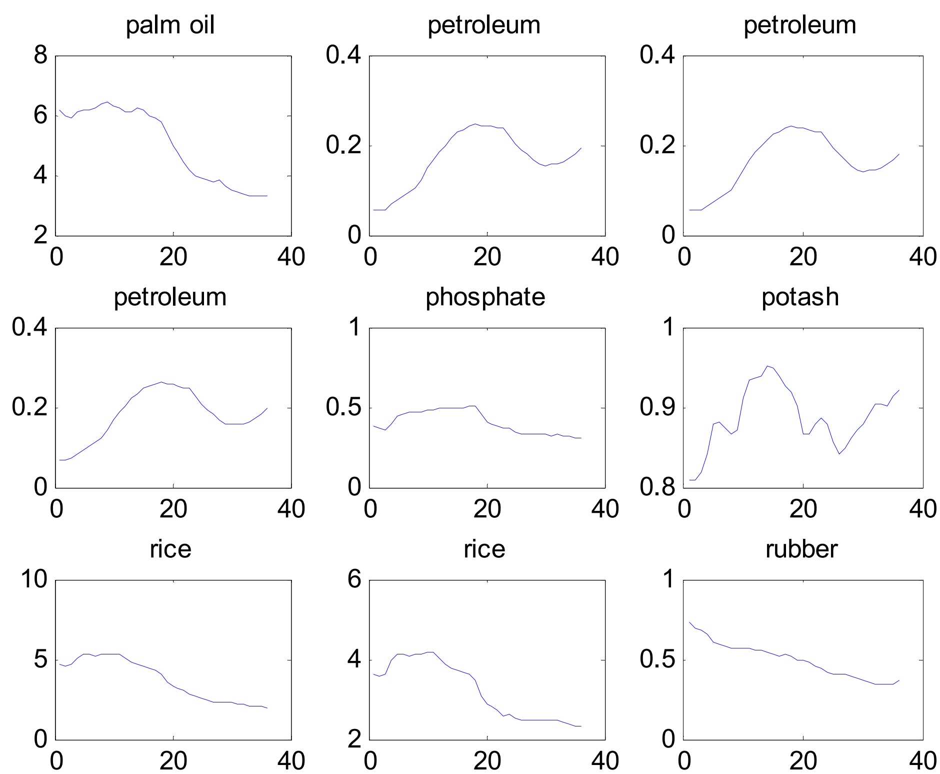
nickel
palm-oil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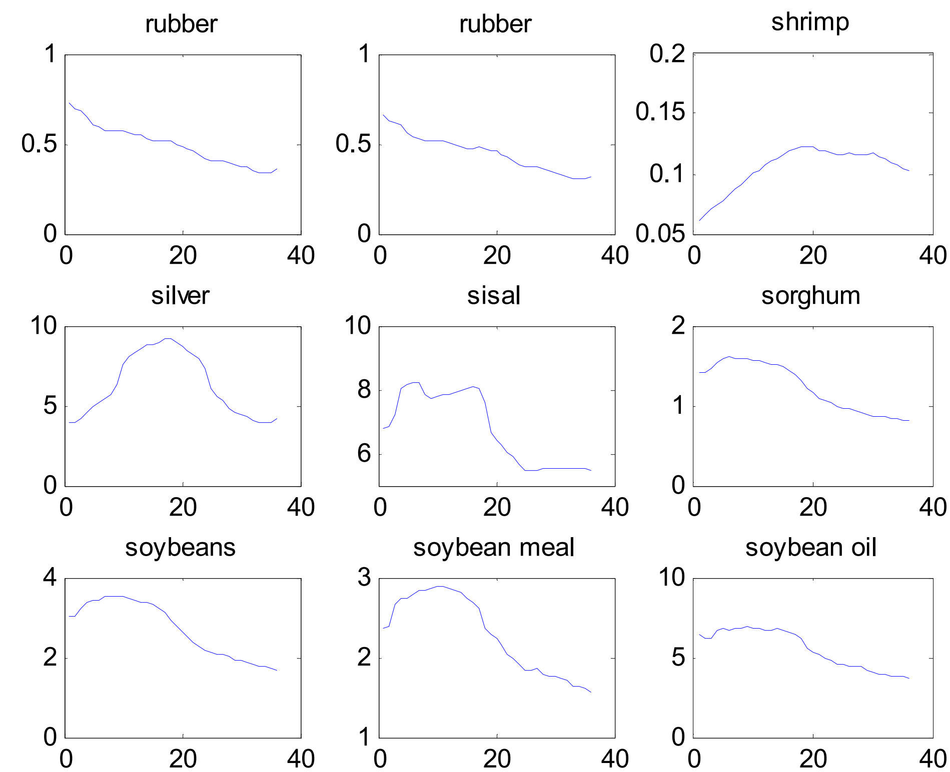
petroleum
petroleum
phosphate
potash
rice
rice
rubber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rubber
shrimp
silver
sisal
sorghum
soybeans
soybean meal
soybean oil
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0.15
0.2
2
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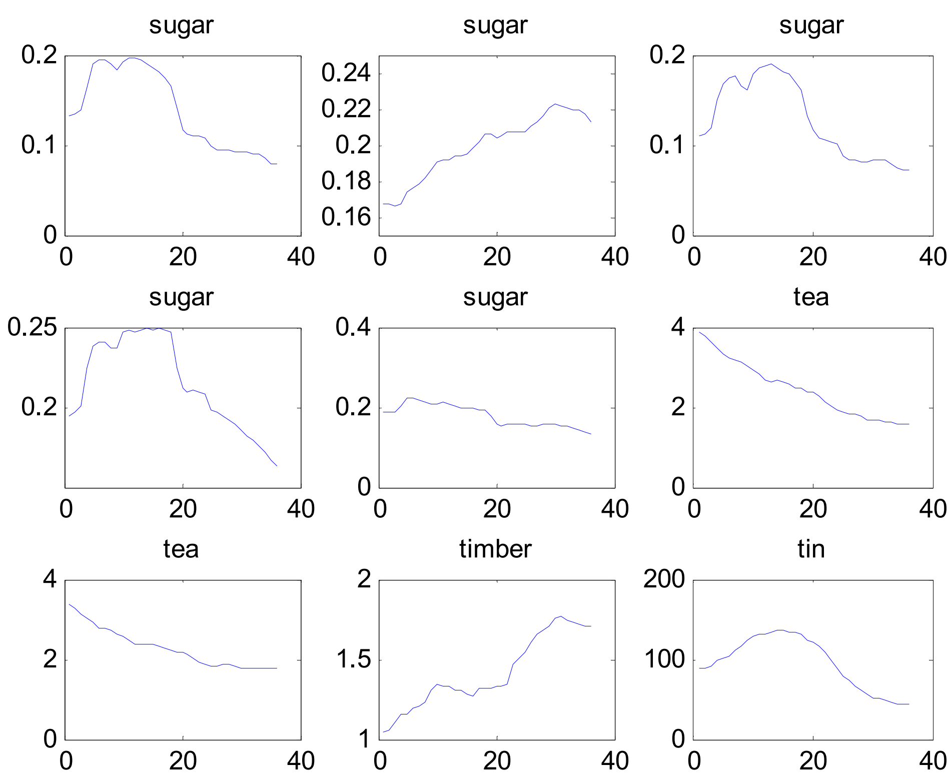
6
8
10
5
3
0
20
40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sugar
sugar
sugar
tea
tea
timber
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0.22
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0.25
0.4
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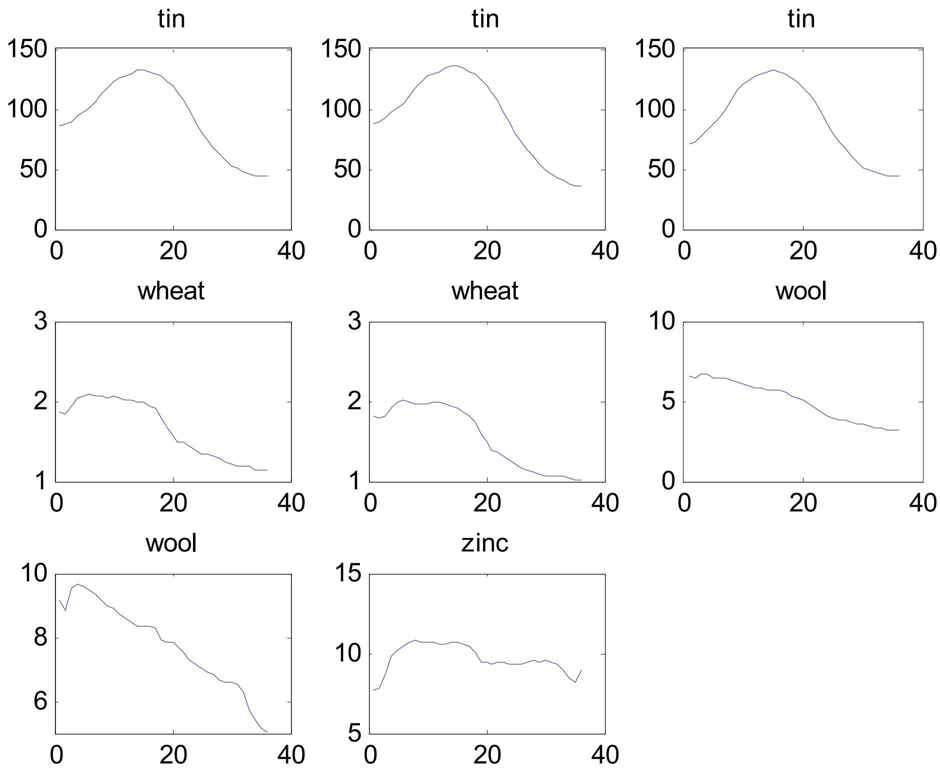
1.5
2
4
100
200
20
40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20
40
tin
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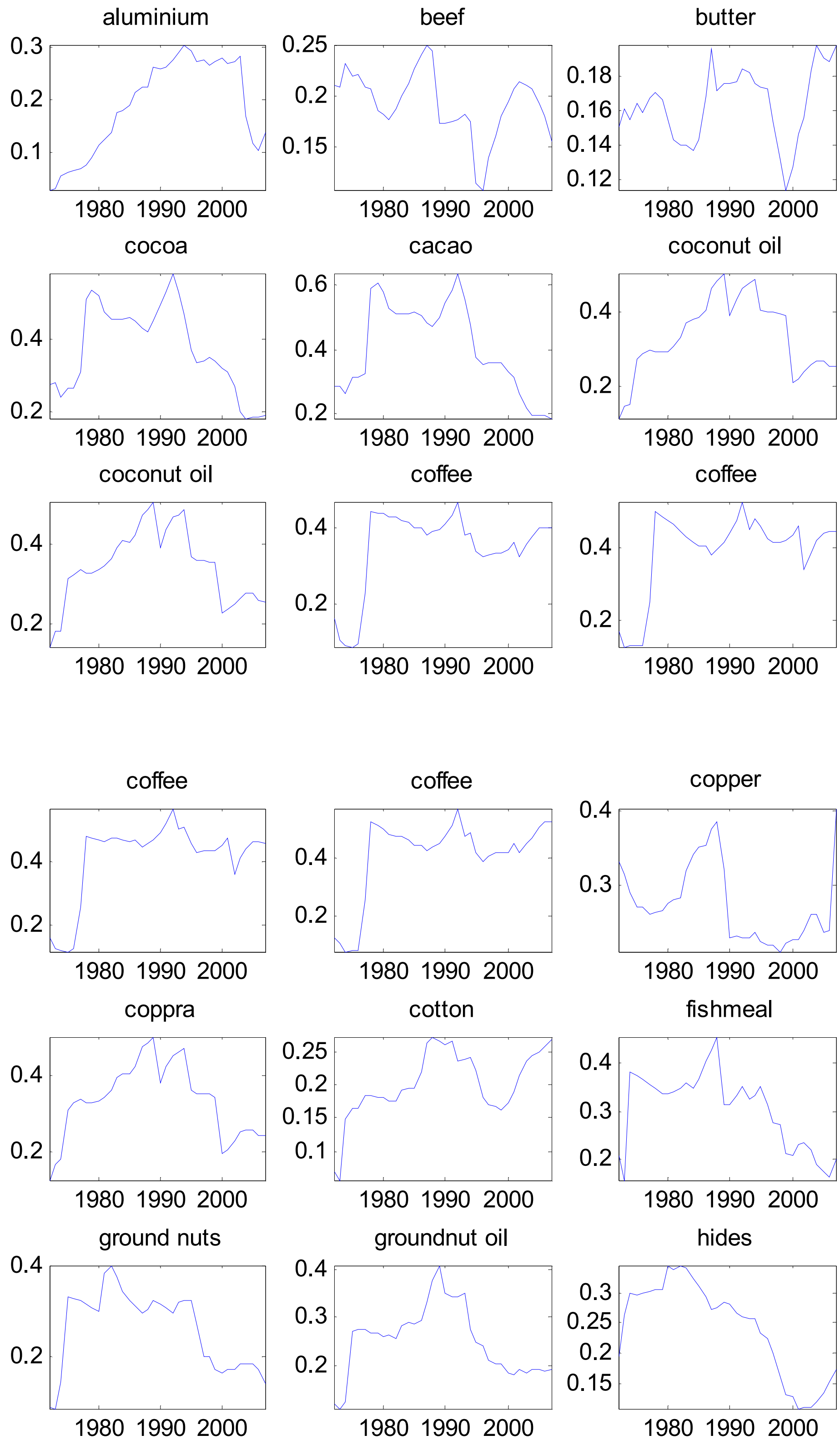
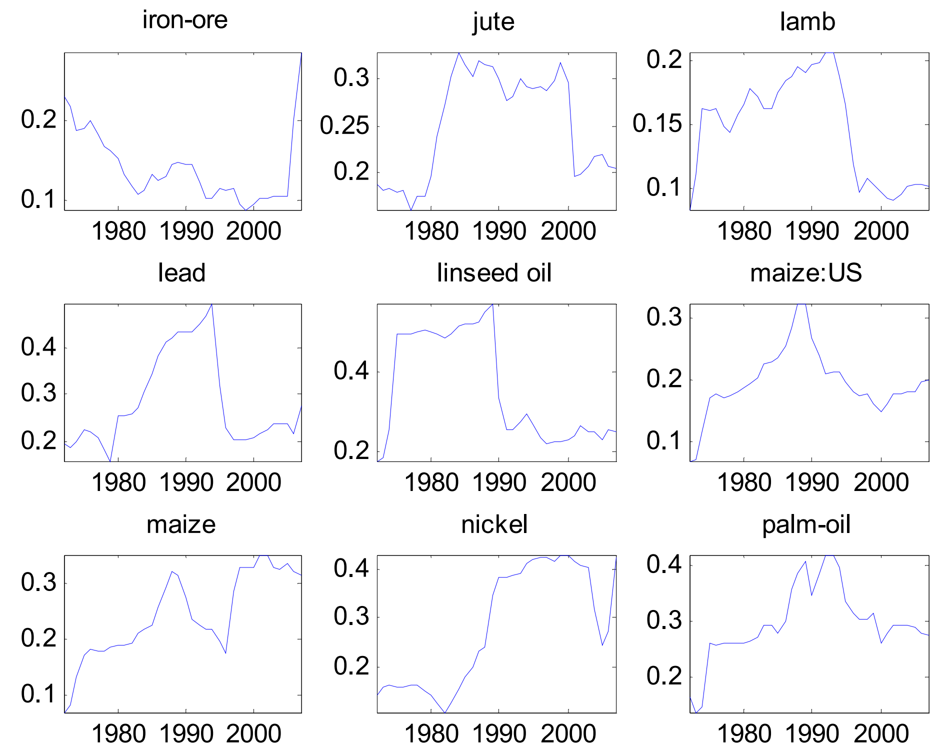
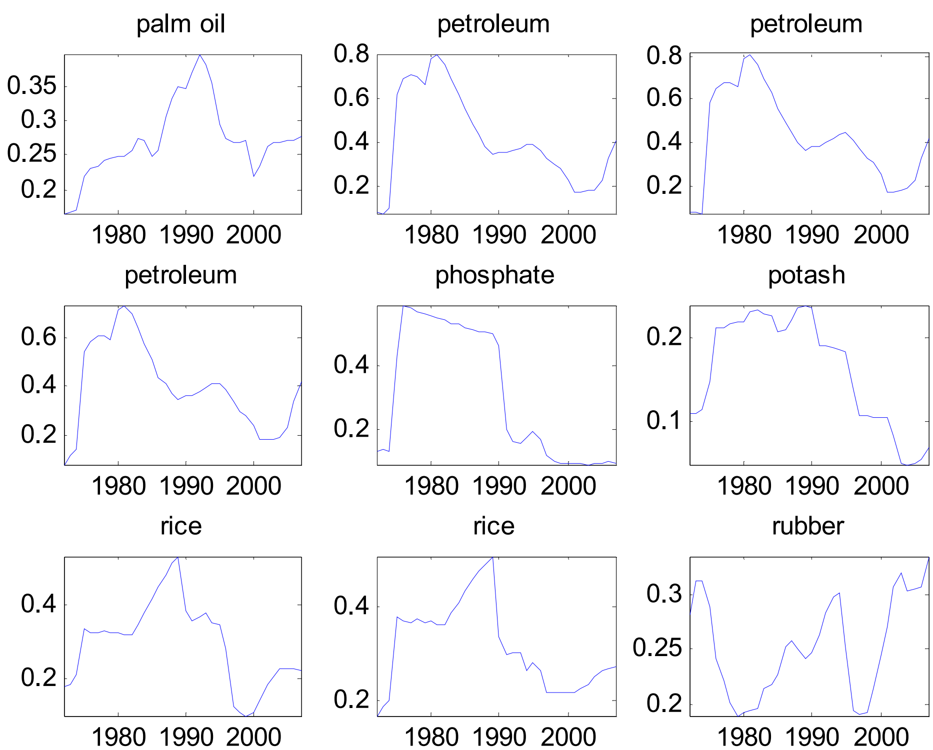
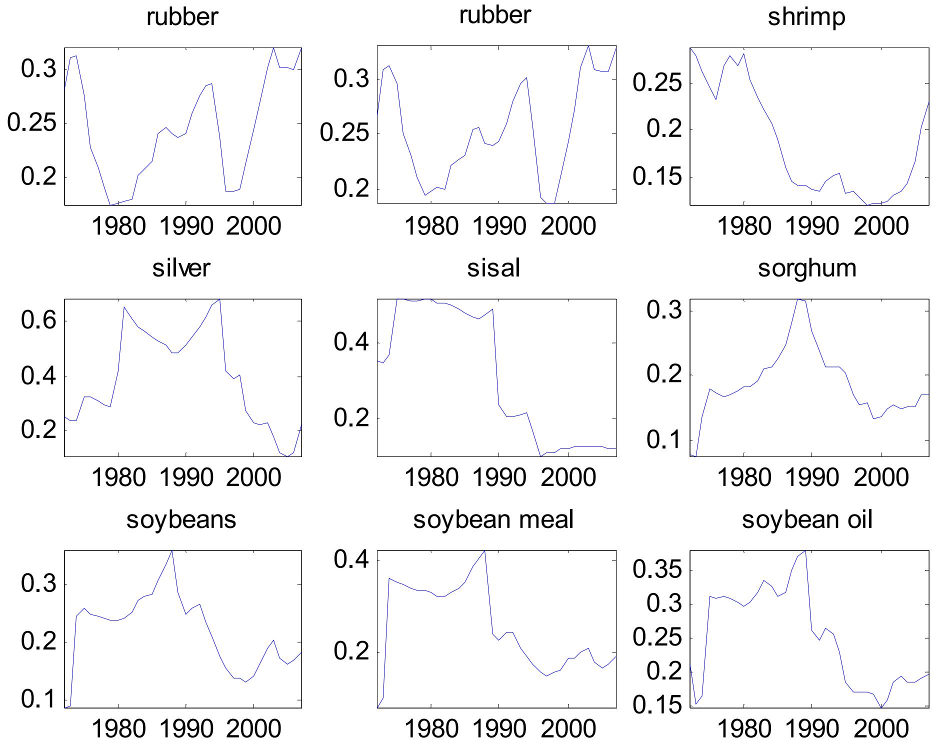
wheat
3
2
1
wheat
wool
10
5
wool
8
6
zinc
15

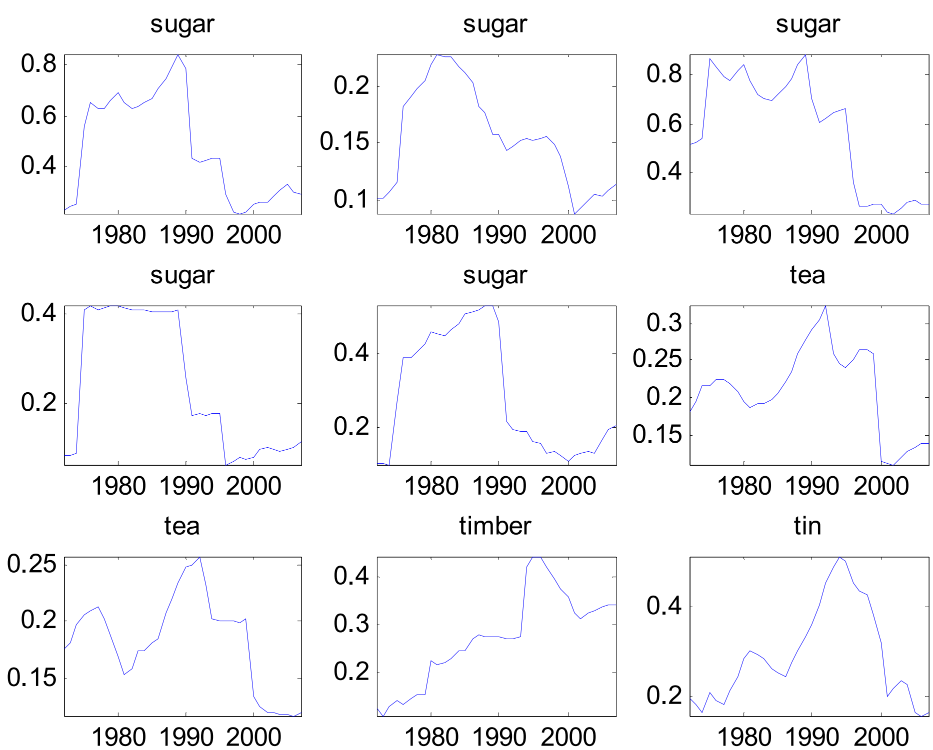
**Figure** A.2.2 **Rolling-window coefficients of variation (15-years)**




iron-ore
0.2
0.1
1980 1990 2000
jute
0.3
0.25
0.2
1980 1990 2000
lamb
0.2
0.15
0.1
1980 1990 2000
lead
0.4
0.3
0.2
1980 1990 2000
linseed oil
0.4
0.2
1980 1990 2000
maize:US
0.3
0.2
0.1
1980 1990 2000
maize
0.3
0.2
0.1
1980 1990 2000
nickel
0.4
0.3
0.2
1980 1990 2000
palm-oil
0.4
0.3
0.2
1980 1990 2000


palm oil
0.35
0.3
0.25
0.2
1980 1990 2000
petroleum
0.8
0.6
0.4
0.2
1980 1990 2000
petroleum
0.8
0.6
0.4
0.2
1980 1990 2000
petroleum
0.6
0.4
0.2
1980 1990 2000
phosphate
0.4
0.2
1980 1990 2000
potash
0.2
0.1
1980 1990 2000
rice
0.4
0.2
1980 1990 2000
rice
0.4
0.2
1980 1990 2000
rubber
0.3
0.25
0.2
1980 1990 2000


rubber
0.3
0.25
0.2
1980 1990 2000
rubber
0.3
0.25
0.2
1980 1990 2000
shrimp
0.25
0.2
0.15
1980 1990 2000
silver
0.6
0.4
0.2
1980 1990 2000
sisal
0.4
0.2
1980 1990 2000
sorghum
0.3
0.2
0.1
1980 1990 2000
soybeans
0.3
0.2
0.1
1980 1990 2000
soybean meal
0.4
0.2
1980 1990 2000
soybean oil
0.35
0.3
0.25
0.2
0.15
1980 1990 2000


sugar
0.8
0.6
0.4
1980 1990 2000
sugar
0.2
0.15
0.1
1980 1990 2000
sugar
0.8
0.6
0.4
1980 1990 2000
sugar
0.4
0.2
1980 1990 2000
sugar
0.4
0.2
1980 1990 2000
tea
0.3
0.25
0.2
0.15
1980 1990 2000
tea
0.25
0.2
0.15
1980 1990 2000
timber
0.4
0.3
0.2
1980 1990 2000
tin
0.4
0.2
1980 1990 2000

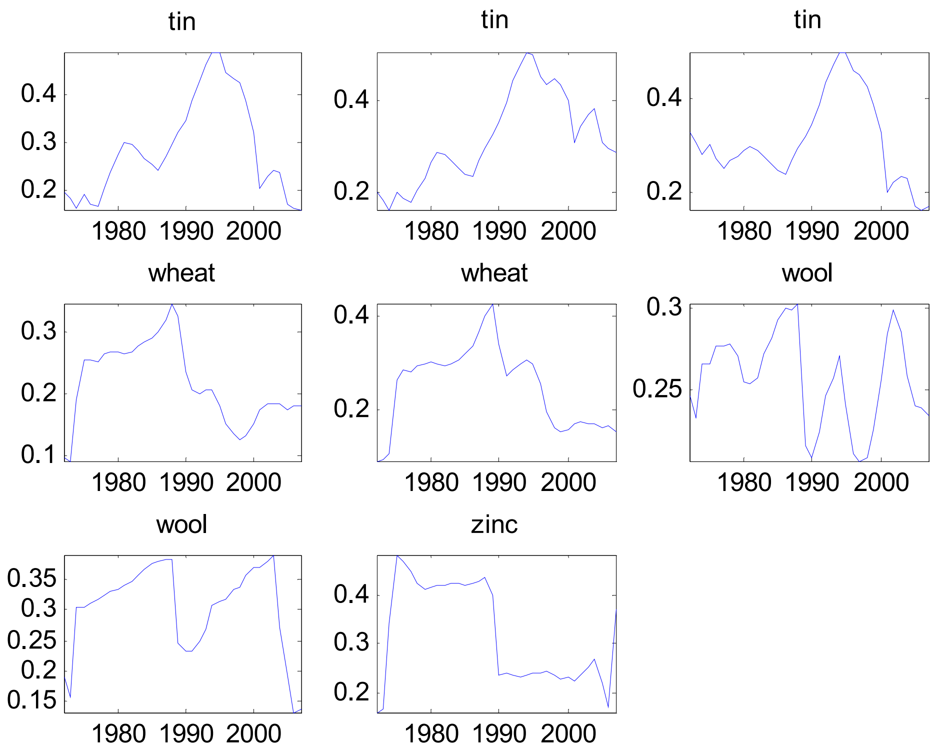
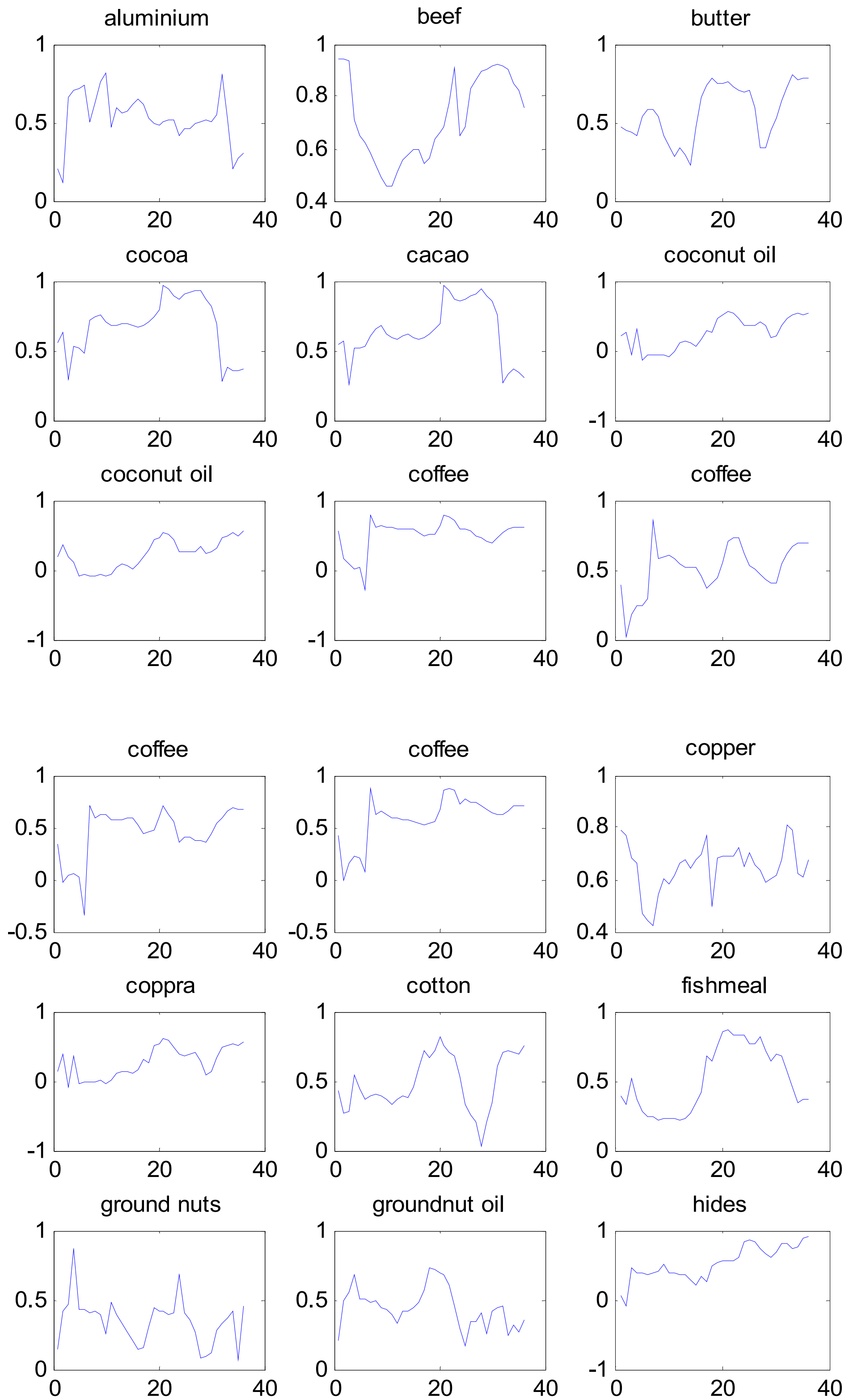
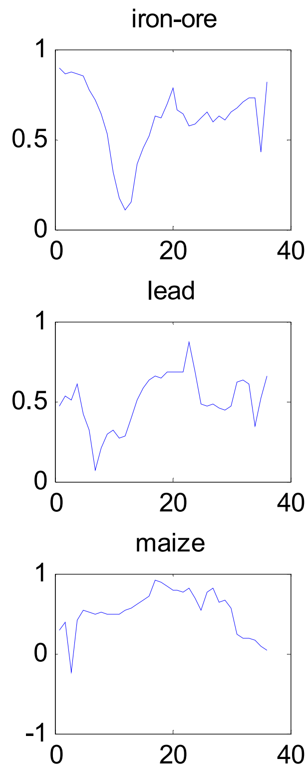
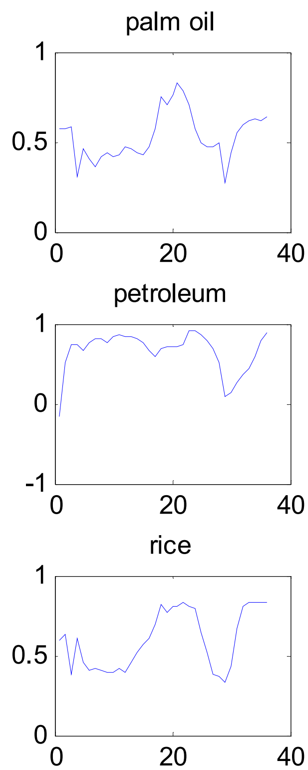
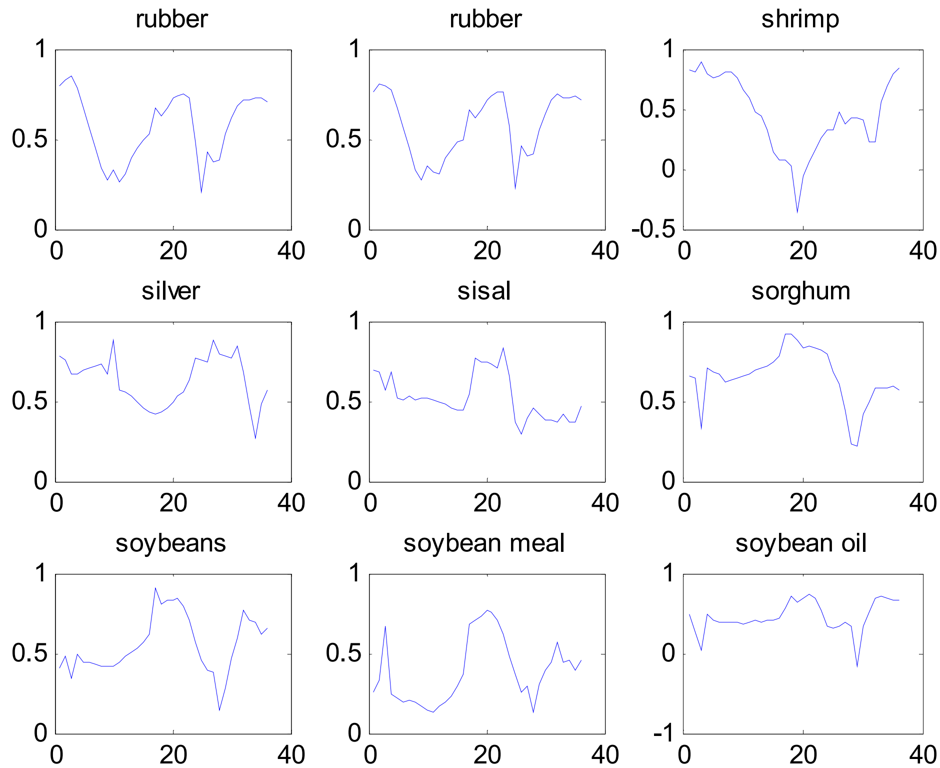

tin
0.4
0.3
0.2
1980 1990 2000
tin
0.4
0.2
1980 1990 2000
tin
0.4
0.2
1980 1990 2000
wheat
0.3
0.2
0.1
1980 1990 2000
wheat
0.4
0.2
1980 1990 2000
wool
0.3
0.25
1980 1990 2000
wool
0.35
0.3
0.25
0.2
0.15
1980 1990 2000
zinc
0.4
0.3
0.2
1980 1990 2000

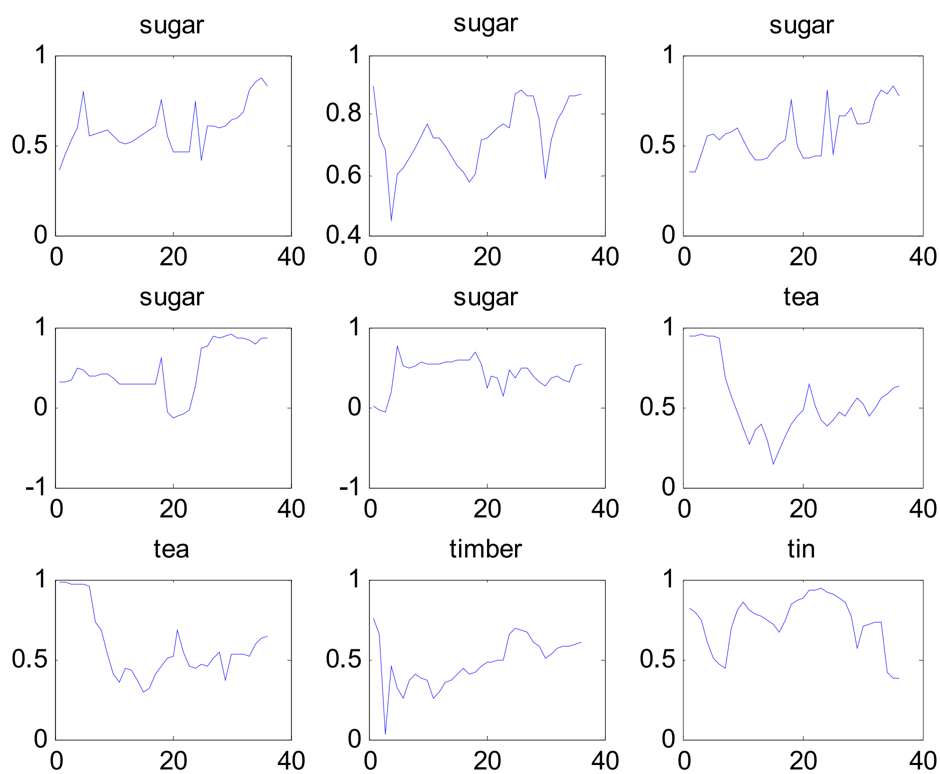
**Figure** A.2.3 **Rolling-window autocorrelations (15-years)**

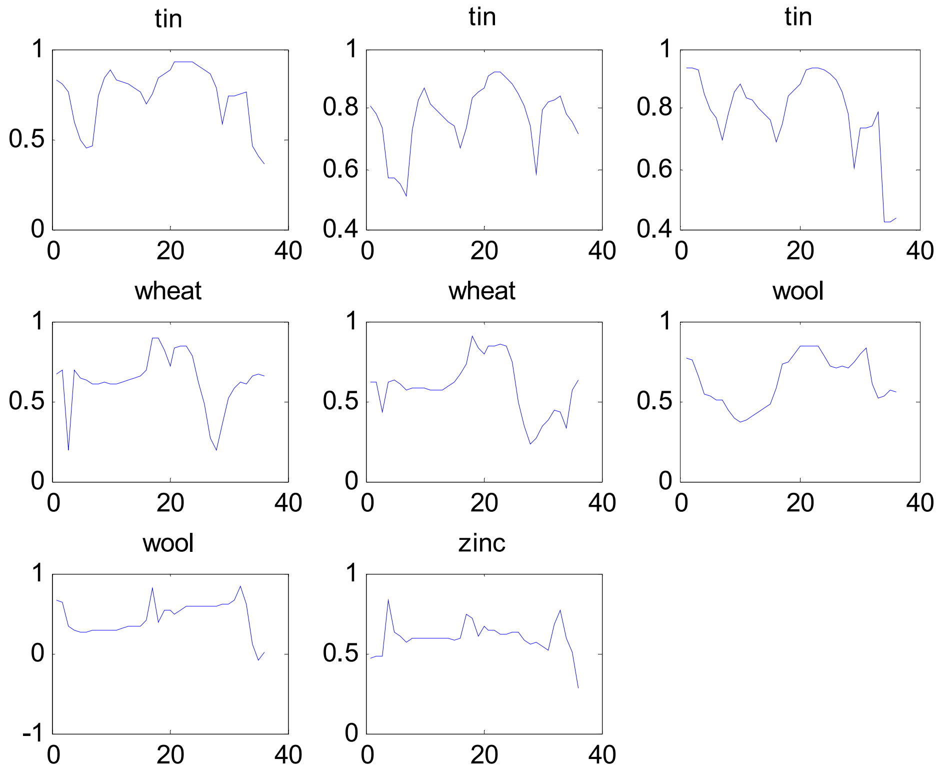



iron-ore
jute
lamb
lead
linseed oil
maize:US
maize
nickel
palm-oil


palm oil
petroleum
petroleum
petroleum
phosphate
potash
rice
rice
rubber


rubber
rubber
shrimp
silver
sisal
sorghum
soybeans
soybean meal
soybean oil


sugar
sugar
sugar
sugar
sugar
tea
tea
timber
tin


tin
tin
tin
wheat
wheat
wool
wool
zinc

# Appendix B

## Relative prudence and relative risk aversion

The first, second and third derivatives of the utility function presented in section 3.2 are:

$$u'(c_t) = a - 3bc_t^2$$
$$u''(c_t) = -6bc_t$$
$$u'''(c_t) = -6b$$

So, following Kimball (1990) the coefficient of relative prudence (CRP) is:

$$\eta_R(c) = -\frac{u'''(c_t)}{u''(c_t)}c$$

thus for this class of preferences, the coefficient of relative prudence is:

$$\eta_R(c) = -\frac{-6b}{-6bc}c = -1$$

So this utility function displays constant relative prudence. Furthermore, the coefficient of relative prudence is completely independent of the parameters defining the curvature and the degree of risk aversion.

Relative risk aversion is:

$$\theta_R(c) = -\frac{u''(c_t)}{u'(c_t)}c\,,$$

then for this class of preferences

$$\theta_R(c) = -\frac{-6bc}{[a-3bc^2]}c = \frac{6bc^2}{[a-3bc^2]}$$

so, as long as $[a-3bc^2] > 0$, a lower a implies a higher coefficient of relative risk aversion.

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