

ABSTRACT

Title of Dissertation: **HEAT FLUX SENSORS AND
RECONSTRUCTION TECHNIQUES FOR
ATMOSPHERIC ENTRY SPACECRAFT**

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Characterizing the heat loads on spacecraft during atmospheric entry is essential to evaluate the performance of thermal protection systems (TPS), investigate aerothermal phenomena, and validate computational models. As the result of a growing push to obtain thermal diagnostic data during entry, descent, and landing (EDL), recent atmospheric entry spacecraft have hosted extensive instrumentation suites embedded in the TPS, including heat flux sensors and temperature probes. Current commercially available heat flux sensors, however, often have survivability limitations which constrain their use to less extreme regions of the spacecraft. Furthermore, the post-flight analysis techniques used by the space exploration community to reconstruct atmospheric entry heating conditions are often resource intensive, and impractical to implement in analyses that are either computationally expensive or time-sensitive. To address these engineering challenges, this thesis details the development of heat flux sensors designed for extreme thermal

environments and low cost analysis techniques to reconstruct the surface heating conditions on spacecraft during EDL.

Heat flux sensors tailored for moderate and high temperatures are fabricated using the transverse Seebeck effect (TSE) in antimony and rhenium single crystals. The thermoelectric properties and commercial availability of single crystal antimony make it an ideal platform to use for investigating the TSE operating mechanism in a robust sensor package, whereas the high-temperature compatibility of single crystal rhenium positions it as an optimal transducer material for extreme environment sensors. The single crystal antimony sensor demonstrates a linear heat flux-to-voltage transduction and is used to probe the effects of the package configuration on sensor performance characteristics. The rhenium-based heat flux sensor exhibits a responsivity that both increases in magnitude as the conditions become more challenging and remains stable after long term exposure to temperatures as high as 1000 °C. The temperature-dependence of the rhenium sensor's responsivity, characterized using a newly-developed calibration facility, agrees well with analytical predictions, and underscores how calibrations of heat flux sensors at application-relevant temperatures are necessary for accurate measurements.

An efficient Green's function-based inverse heat transfer (IHT) algorithm is developed to reconstruct the heating conditions on spacecraft during atmospheric entry. The Green's function formalism is adapted to accommodate nonlinear heat conduction phenomena in ablative TPS materials, including the effects of pyrolysis and internal gas transport. To improve the stability of the reconstructed heating conditions, a sensor fusion-based regularization methodology is proposed whereby measurements from collocated heat flux sensor and temperature probe instrumentation are embedded within the IHT formulation. The Green's function IHT approach is applied to the reconstruction of the heating conditions on the backshell of the Mars 2020 atmospheric entry

spacecraft. The reconstructed heating conditions are in good agreement with results from a high-fidelity IHT code used by the space exploration community, but are achieved with a greater-than 2 order of magnitude reduction in computation cost. These results demonstrate the Green's function framework as a promising IHT approach to reconstruct atmospheric entry heat loads from thermal measurements embedded in ablative TPS, and highlight how these techniques can enable certain post-flight analyses, such as uncertainty quantification, that have been hindered by the prohibitive cost of current methods.

HEAT FLUX SENSORS AND RECONSTRUCTION TECHNIQUES FOR ATMOSPHERIC ENTRY SPACECRAFT

by

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Dedication

To the black sheep.

Acknowledgments

I would first like to extend my utmost gratitude to my life partner Breanne and our furry companion Freddie, whose everlasting support and encouragement has kept me grounded over these past few years. Sharing the highlights of this journey with them will be what I cherish most.

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Table of Contents

Dedication	ii
Acknowledgements	iii
Table of Contents	vi
List of Tables	x
List of Figures	xi
Chapter 1: Introduction	1
Chapter 2: Extreme Environment Heat Flux Sensors - Review of the Literature	5
Chapter 3: A Heat Flux Sensor Leveraging the Transverse Seebeck Effect in Elemental Antimony	14
3.1 Heat Flux Sensor Design and Fabrication	15
3.2 Steady State Sensor Characterization	17
3.2.1 Steady-State Response	17
3.2.2 Analysis of the Sensor's Steady-State Response	21
3.3 Transient Sensor Characterization	25
3.3.1 Transient Response	25
3.3.2 Analysis of the Sensor's Transient Response	26
3.4 Sensor Response to Localized Heating	27
3.5 Summary	29
Chapter 4: A High-Temperature Heat Flux Sensor Using the Transverse Seebeck Effect in Elemental Rhenium	31
4.1 Heat Flux Sensor Design and Fabrication	32
4.2 Predicted Responsivity	33
4.3 Temperature-Dependent Sensor Calibration	37
4.3.1 Analysis of the Sensor's Temperature-Dependent Responsivity	38
4.4 High Temperature Resilience	42
4.5 Transient Response	44
4.5.1 Step Heat Flux	44
4.5.2 Amplitude Frequency Response	45
4.5.3 Analysis of the Sensor's Transient Response	46

4.6	Sensor Response to Localized Heating	48
4.7	Summary	50
Chapter 5:	Development of a High-Temperature Heat Flux Sensor Calibration Facility	51
5.1	Heat Flux Sensor Calibration Background	52
5.2	High-Temperature Calibration Facility Design Motivation	54
5.3	Calibration System Overview	56
5.4	Setup Considerations and Testing Protocols	60
5.4.1	Relative Sensor Alignment	61
5.4.2	Temperature Control	62
5.4.3	Mitigation of Convective Heat Transfer During Calibration	63
5.4.4	Baseline Heat Flux Measurement	64
5.5	Calibration Procedure	65
5.6	Heat Flux Calculation and Band Heater Correction	66
5.6.1	Analytical Heat Flux Calculation	67
5.6.2	Characterization of the Temperature Control Apparatus	70
5.7	Data Reduction	73
5.7.1	Response Curve	75
5.7.2	Calibration Uncertainty	76
5.8	Characterization of the IR Lamp Heat Source	78
5.8.1	Characterization of the IR Lamp Output Versus Lamp Input Power	78
5.8.2	Characterization of the IR Lamp Output Versus Sensor Position	80
5.8.3	Sensor Alignment Tolerances	84
5.8.4	Rhenium Sensor Calibration Setup Parameters	87
5.9	Summary	88
Chapter 6:	Reconstruction of Thermal Protection System Aeroheating from Embedded Thermal Sensors - Review of the Literature	90
Chapter 7:	Green's Function Formulation for Inverse Heat Transfer Problems	97
7.1	Green's Function Formulation	98
7.1.1	Green's Function Solution Equation	98
7.1.2	Green's Function Construction	100
7.2	Numerical Treatment and IHT Implementation	103
7.2.1	Discretization	103
7.2.2	Inversion of the Linear System	105
7.3	Modifications to the Green's Function IHT Framework for Entry Spacecraft	106
7.4	Summary	108
Chapter 8:	Sensor-Fusion Based Regularization for Ill-Posed Inverse Heat Transfer Problems	109
8.1	Motivation for a New Regularization Approach	110
8.2	Sensor Fusion Regularization Approach	114
8.3	Mitigation of Singularities in the Regularization Surrogate	116
8.4	Application of the Sensor Fusion Approach to a 1D Model System	118

8.4.1	Problem Definition	118
8.4.2	Results	121
8.4.3	Comparison Against Tikhonov Regularization	124
8.5	Summary	127
Chapter 9: A Green's Function Sensor Fusion Approach for 2D-Axisymmetric Model Systems		128
9.1	Reconstruction Approach for the 2D-Axisymmetric Model System	129
9.1.1	Green's Function Representation of the TPS-Sensor System	130
9.1.2	Recovery of the Net Hot-Wall Heat Flux	139
9.2	Results	141
9.2.1	Verification of the Reconstruction Algorithm on the 2D-Axisymmetric Model System	143
9.2.2	Effect of Varying the Radial and Axial Basis Function Orders	144
9.2.3	Effect of Varying the Temperature Probe Position	145
9.2.4	Robustness of the Sensor Fusion Approach Applied to the 2D-Axisymmetric Model System	148
9.2.5	Mitigation of Singularities	153
9.3	Summary	154
Chapter 10: Adaptation of the Green's Function Framework for Ablative Thermal Protection Systems		156
10.1	Heat Conduction in Pyrolyzing Solids	157
10.1.1	Conservation of Energy	159
10.1.2	Decomposition Modeling	164
10.2	Reconstruction Algorithm	166
10.2.1	Green's Function Formulation for Ablative TPS	167
10.2.2	Recovery of the Net Hot-Wall Heat Flux	177
10.2.3	Aeroheating Environment Reconstruction	179
10.2.4	Computation Cost	181
10.3	Numerical Verification of the Green's Function Adaptation for Ablative TPS Materials	182
10.3.1	Numerical Integration Resolution	185
10.3.2	Galerkin Basis Function Order	187
10.4	Summary	188
Chapter 11: Reconstruction of Thermal Protection System Aeroheating Using Ground Test and Flight Measurements		190
11.1	FIAT_Opt	190
11.2	Ground Test Validation	191
11.3	Mars 2020 Backshell Reconstruction	195
11.3.1	Nominal Flight Reconstruction	197
11.3.2	Uncertainty Quantification	199
11.4	Summary	210

Chapter 12: Conclusions, Contributions, and Future Work	211
12.1 Conclusions	211
12.2 Contributions	213
12.2.1 List of Publications	215
12.3 Future Work	217
12.3.1 Durability of the High-Temperature TSE Heat Flux Sensor in Extreme Environments.	217
12.3.2 Improving the Responsivity of TSE-Based Heat Flux Sensors	218
12.3.3 Modeling TPS Recession Using the Green’s Function-Based IHT Algorithm	219
Appendix A: Welding Process for W-26Re Wire on Rhenium Substrate	220
Appendix B: Detailed Engineering Drawings	222
B.1 Rhenium-based TSE Heat Flux Sensor	222
B.2 High-Temperature Calibration Facility	227
Bibliography	235

List of Tables

3.1	Seebeck coefficient tensor components of single crystal bismuth, antimony, and arsenic at room temperature.	15
3.2	Summary of material parameters for the TSE heat flux sensor.	18
4.1	Uncertainty bounds for the parameters in Eq. 4.1.	36
5.1	Summary of commercially-sourced calibration system components.	56
5.2	Temperature control apparatus configurations.	63
5.3	Tabulated band heater temperature and heat flux data from the calibration of the rhenium sensor detailed in Chapter 4 (Sweep 3, Fig. 4.6). The data includes the heat flux calculated with and without the band heater correction, corresponding with q_{raw} (Eq. (5.10)) and q_{corr} (Eq. (5.6)), respectively. The two heat flux values are compared in the right-most column. Note that, in the 100 °C calibration, the measured band heater temperatures may be higher than the set point value. In the latter two heat fluxes, air cooling was used to maintain the sensor at 100 °C.	71
5.4	Temperature control apparatus component temperature fit coefficients.	73
5.5	Map of the calibration heat flux versus lamp input power and heat source-sensor distance. Heat flux values represent the incident heat flux measured by the reference sensor with units of W/cm ² . To calculate the heat flux transmitted through the 0.5 mm thick sapphire radiometer window, multiply the tabulated values by a factor of 0.81.	82
5.6	Relative alignment tolerances for the sensors for near-field (between 2 and 4 cm) and far-field (≥ 4 cm) heat source-sensor distances. Tolerances correspond with a 3% allowable deviation in the heat flux.	86
8.1	Input temperature and heat flux sensor measurement noise parameters.	120
9.1	Input temperature and heat flux sensor measurement noise parameters for the 2D-axisymmetric model system.	148
10.1	Numerical verification heat shield stackup parameters.	182
10.2	Total computation time for various numerical integration orders N_x	186
10.3	Total computation time for various Galerkin basis function orders N	188
11.1	Panel arc-jet test campaign 162 run parameters [1].	192
11.2	Input uncertainty scaling parameters from [2]. Both uniform (U) and normal (N) distributions were used in this analysis.	200

List of Figures

2.1	Operating principle of a Gardon gauge [3] (reprinted with permission from SAGE Publications Ltd.). A heat flux produces a radial temperature gradient in a thin Constantan foil. As a result, a thermoelectric voltage difference is induced between two copper-Constantan junctions, one at the center of the foil and one at the edge.	7
2.2	a) Operating principle of a Schmidt-Boelter or thermopile-type heat flux sensor (reprinted from [4], with the permission of AIP Publishing.). A series of dissimilar material junctions is produced by alternating two thermoelectric materials on opposing sides of a thermal resistance layer. A temperature difference induced between the junctions on the top and bottom of the thermal resistance layer produces a thermoelectric voltage. b) Schematic of a wire-wound Schmidt-Boelter gauge [5] (reprinted by permission of the American Institute of Aeronautics and Astronautics, Inc.).	7
2.3	a) High temperature heat flux sensor using type-K thermoelectric materials [6] (reprinted with permission from Elsevier). The active thermopile legs act as both a transducer and thermal resistance layer. b) High temperature heat flux sensor using type-N thermocouple materials [7] (reprinted with permission, ©2025 IEEE). The hot and cold junction pairs are separated by a ceramic thermal resistance layer.	8
2.4	Schematic of the transverse Seebeck effect in an anisotropic single crystal. A voltage is generated between two probe locations separated by a distance L along the x -direction in response to a temperature gradient induced along the z -direction. In the schematic, CP and IP represent the cross-plane and in-plane crystal orientations, respectively, and θ represents the relative tilt between the coordinate systems defined by the crystal structure and the transducer geometry.	9
2.5	TSE-based heat flux sensor using a YBCO thin film, adapted from [8] (reprinted with permission from Elsevier). A incident heat flux produces a voltage difference orthogonal to the direction of the induced temperature gradient in the transducer.	11
2.6	Multilayer TSE device [9] (reprinted with permission from Springer Nature). Thermopower anisotropy is generated by layering two types of thermoelectric materials. The TSE is induced through rotation of the layered monolith.	11
2.7	High temperature transducer for a TSE heat flux sensor based on single crystal 4H-SiC (reprinted (adapted) with permission from [10]. Copyright 2025 American Chemical Society).	13

3.1	Exploded view of the sensor. The front end is a painted brass heat spreader, placed over the ceramic housing which accommodates two prismatic antimony transducers and tungsten wires. Two auxiliary thermocouples that probe the temperature at two antimony/tungsten junctions extend from the back end.	17
3.2	a) Optical microscope image of the sensor before attaching the brass plate. b) Completed sensor mounted on a brass holder.	18
3.3	Experimental response curve for the antimony-based TSE heat flux sensor. Data sets obtained with different radiative heat sources are overlaid. The linear fit using the totality of data is displayed as a solid line, with the corresponding equation shown in the plot.	20
3.4	Steady-state finite element simulations. a) Temperature contour map of the finite element model. A uniform heat flux q_0 is imposed on the top surface and the bottom surface is held at a constant temperature T_0 . b) Heat flux streamlines on the $y - z$ symmetry plane through the center of the model. The streamline colors represent the magnitude of heat flux. The boundaries between the components are superimposed as black lines.	23
3.5	Illustration of the polarity of the Seebeck voltage along the x -axis (sign symbols and blue arrows) in a pair of transducers with crystallographic tilt angles of $+\theta$ and $-\theta$ about the y -axis, respectively (parallel hatching). The heat flux is represented by a red arrow and the temperature gradient by a color gradient. a) A heat flux q_z induces a temperature gradient along the z -axis and generates transverse Seebeck voltages with opposite polarities. The voltages add up along the circuit loop. b) A heat flux q_x induces a temperature gradient along the x -axis and generates Seebeck voltages with equal polarities. The voltages offset each other along the circuit loop.	24
3.6	Transient response of the antimony-based TSE heat flux sensor to a step change in heat flux. The horizontal line indicates the signal corresponding to 63.2% of the steady-state voltage. The intersection between the voltage transient and the horizontal line was used to determine the time constant of the sensor ($\tau = 4.4$ s).	26
3.7	Laser scan data. a) Sensor response as a function of the position of the heating laser spot with respect to the center of the heat spreader. b) Outline of the antimony prisms and the tungsten wiring overlaid with the laser scan data. The sensor response is presented using a color scale. Note that all the voltage values are positive.	29
4.1	Exploded view of the high-temperature TSE heat flux sensor. Two rhenium transducers are connected in series using tungsten-rhenium alloy electrical wiring and mounted in a ceramic package. Four auxiliary thermocouples monitor the temperature near each transducer and electrical wire junction. The front face of the sensor is coated using a high absorptivity black paint. An Inconel bracket is placed over the front of the sensor for mounting to a heat sink.	33
4.2	Optical image of the back side of the TSE heat flux sensor during assembly, prior to sealing with cement. Visible components include the two transducers, electrical wiring, and thermocouples. Scale bar: 2 mm.	34

4.3	High-temperature TSE heat flux sensor mounted to a copper heat sink. The heat sink is spray-coated in boron nitride to inhibit oxidation. The heat sink has integrated coolant channels for water or air (not shown).	35
4.4	Thermoelectric transport properties of rhenium. The Seebeck coefficient anisotropy $\Delta S = S_{CP} - S_{IP}$ [11] and thermal conductivity k [12] are shown relative to their value at 25 °C, ΔS_0 and k_0 , respectively.	36
4.5	Predicted responsivity C_{TSE} of the TSE heat flux sensor from Eq. (4.1) (solid line). The dashed lines represent upper and lower bounds for the predicted responsivity due to the uncertainties of the parameters in Eq. (4.1). The shaded region bounded by the dashed lines captures responsivity values that are within 1 standard deviation of the nominal prediction.	36
4.6	TSE heat flux sensor calibration sweep from room temperature to 500 °C. Error bars represent the uncertainty of the voltage measurement multiplied by a coverage factor of 2 ($\pm 2\sigma$).	39
4.7	Temperature-dependent sensor responsivity from a sequence of calibration sweeps. Error bars represent a 95% confidence interval. The predicted responsivity from Fig. 4.5 is overlaid on the data for comparison. The shaded region represents responsivity values that fall within the $\pm 1\sigma_t$ uncertainty bounds of the predicted responsivity.	39
4.8	Schematic of the transducer circuit subject to both z - and x -oriented temperature gradients. The net temperature difference between the ends of the transducers ΔT_L is estimated using measurements from the four auxiliary thermocouples embedded in the package. The hash marks represent the orientations of the (0001) basal planes in each transducer prism.	41
4.9	Plot of $\Delta T_L/q$ versus absorbed heat flux for each calibration sweep. The 500 °C calibration datapoints are circled for the first sweep.	42
4.10	Front surface of the rhenium TSE heat flux sensor following heat treatment at 1000 °C for 10 hours under flow of ultra high purity argon.	43
4.11	Temperature-dependent sensor responsivity of the TSE heat flux sensor following heat treatment at 1000 °C. Error bars represent a 95% confidence interval. Also shown is the calibration data from the third sweep in Fig. 4.7, performed prior to the 1000 °C heat treatment.	43
4.12	Composite transient of the TSE heat flux sensor voltage output in response to a step heat flux. The data represent an average of 36 repeat transients. The horizontal line corresponds to 63.2% of the steady-state voltage.	45
4.13	Amplitude–frequency response of the TSE heat flux sensor. Voltage amplitudes A are normalized by the sensor’s steady-state output voltage A_0 . The line fit through the data represents a first-order response model with a time constant τ_{AFR} of 0.15 s. The dashed line represents an attenuation of -3 dB.	47
4.14	Laser scan data. a) Isometric view of the TSE heat flux sensor voltage output as a function of the position of the localized laser heat probe with respect to the centroid of the sensor. b) Contour plot of the TSE heat flux sensor voltage output with the outline of the rhenium transducers and wiring overlaid on top of the data. The dashed contour lines represent a measured voltage of 0 V.	49

5.1	Optical image of the high-temperature calibration facility. a) Isometric view of the calibration system with the prototype sensor in the heat source line-of-sight. b) Top-down view of the calibration system with the prototype sensor positioned in front of the water-cooled reference plate. The reference sensor and prototype sensor are mounted to the sensor rack, which simultaneously controls the positions of both sensors relative to the heat source.	57
5.2	Schematic of the high-temperature calibration facility. a) Network diagram of the calibration facility. The electrical leads of each sensor are connected to separate channels of a nanovoltmeter. The free ends of thermocouples embedded in each sensor are connected to separate channels of a thermocouple DAQ. The band heater surrounding the prototype sensor is connected to a PID controller. Within the prototype sensor heat sink, cooling channels are available for water or air. An additional channel is used to flow nitrogen directly into the optical tube and radiometer window cavity. The reference sensor, which must remain near room temperature during calibration, has channels for cooling water. The sensors are exposed to an IR lamp, which is controlled using a standalone controller. The IR lamp has channels for cooling water to prevent overheating. When out of the heat source line-of-sight, the prototype sensor is exposed to a water-cooled reference plate. b) Section-view schematic of the prototype sensor and temperature control apparatus configuration. c) Section-view schematic of the reference sensor configuration. The reference sensor assembly, including heat sink, thermopile, temperature-sensing thermocouple, and radiometer window, is purchased as a single unit from the manufacturer.	58
5.3	Sensor rack configuration. The prototype sensor and temperature control apparatus are mounted to the sensor rack using a stainless steel bracket. The reference sensor is mounted to the sensor rack using a precision alignment stage for alignment with the prototype sensor in the x - and z -directions. The sensors are mounted to a sliding rail (colored in blue) to allow for positioning in and out of the heat source line-of-sight. An additional precision alignment stage with adjustments in the y - and z -directions allows for simultaneous adjustment of both sensors relative to the heat source. The sensor rack is mounted to sliding rails aligned with the x -axis (not shown) to adjust the heat source-sensor distance. Inset: view of the backside of the $y - z$ alignment stage.	59
5.4	Prototype sensor and temperature control apparatus. a) Exploded view of the temperature control apparatus. A band heater is attached over a stainless steel optical tube, which threads directly onto the heat sink. The optical tube can accommodate radiometer windows 1 inch in diameter. b) Cut-away view of the temperature control apparatus.	60
5.5	Relative alignment of the prototype and reference heat flux sensors. Alignment targets are used with a stationary laser to determine the position of both sensors when positioned in the heat source line-of-sight. a) Prototype sensor positioned in the heat source line-of-sight. b) Reference sensor positioned in the heat source line-of-sight.	62

5.6	Surface energy balance on the prototype sensor during calibration. a) Sensor positioned in front of the water-cooled reference plate. b) Sensor positioned in front of the calibration heat source.	67
5.7	Experimental setup for characterizing the temperature control apparatus component temperatures. Thermocouples were used to measure the temperature of the heat sink, optical tube, and radiometer window, labeled correspondingly.	72
5.8	Temperature control apparatus component temperatures versus band heater set point.	73
5.9	Example of raw calibration data. The measurement corresponds with the 500 °C calibration in the third sweep for the rhenium TSE-based heat flux sensor detailed in Section 4 (Fig. 4.6). a) Prototype sensor output signal. b) Prototype sensor temperature. The data is represented as the average temperature measured by the four auxiliary thermocouples in the rhenium sensor. c) Reference sensor output signal. In each plot, the red circles mark the time at which a 10 s average measurement is sampled with the respective sensor positioned in the heat source line-of-sight. The blue squares in a) and b) mark the time at which a baseline measurement is taken for the prototype sensor.	74
5.10	Example of a heat flux vs. voltage response curve. The measurement corresponds with the 500 °C calibration in the third sweep for the rhenium TSE-based heat flux sensor detailed in Section 4 (Fig. 4.6) and with the raw data shown in Fig. 5.9. . .	76
5.11	IR lamp heat flux versus lamp input power at a heat source-sensor distance of 1.8 cm.	79
5.12	Diagram of the sensor aligned with the centerline axis of the IR lamp.	81
5.13	IR lamp output versus heat source-sensor distance.	81
5.14	Spatial map of the IR lamp heat flux in the a) $x - y$ symmetry plane and b) $x - z$ symmetry plane. The datapoints correspond with measurements from the reference heat flux sensor scan. Heat flux values in each plot are normalized by the maximum value.	85
5.15	Contour plot of the IR lamp heat flux in the a) $x - y$ symmetry plane and b) $x - z$ symmetry plane. The IR lamp filament is shown schematically in a) and b). An outline of the parabolic collimating reflector is shown as a dashed line in b). The schematic of the IR lamp in a) and b) is not drawn to scale.	85
6.1	Behavior of Tikhonov regularization on a reconstructed sample mass distribution from local gravity field measurements as a function of the user input regularization coefficient λ [13] (reprinted with permission from Springer Nature). At large values of λ , the reconstructed solution is over-smoothed; conversely, at low values of λ , the solution contains significant instabilities. Accurate solutions arise from a bounded region of λ values.	96
8.1	Comparison of the singular values of the Green's function matrix for three different materials with temperature probe locations of a) $x = 0.7L$, b) $x = 0.8L$, and c) $x = 0.9L$. Due to numerical round-off errors, a number of trailing zero singular values are omitted from the SLA561 series in a) and b).	113
8.2	Schematic of the TPS-sensor system	114

8.3	Schematic of the 1D verification test case. A 1D rod (labeled TPS) is exposed to a step heat flux and the temperature response is measured by a temperature probe at x_T . The face at $x = 0$ is held at a constant temperature $\tilde{T}_0$. A collocated heat flux sensor (labeled HFS) measures an augmented heat flux step.	119
8.4	Reconstruction algorithm inputs for the 1D verification test case. a) Simulated temperature probe measurement. b) Simulated heat flux sensor measurement. . .	120
8.5	Measurement inputs with added noise, plotted at 20 Hz. a) Examples of temperature probe measurements. b) Examples of heat flux sensor measurements. . . .	120
8.6	RMSE values for low, medium, and high noise measurement inputs as a function of the regularization coefficient using the sensor fusion approach. Datapoints represent the mean RMSE value from the 50 repeat sweeps. The shaded region represents a ± 1 standard deviation envelope.	122
8.7	Reconstructed heat flux profiles using high levels of input measurement noise with various regularization coefficient values. a) $\lambda_2 = 5 \times 10^6$. b) $\lambda_2 = 10^7$. c) $\lambda_2 = 4 \times 10^7$. The data is plotted at a rate of 20 Hz.	123
8.8	Reconstructed heat flux profiles using varying levels of heat flux sensor noise. a) Low heat flux sensor noise. b) Medium heat flux sensor noise. c) High heat flux sensor noise. In all reconstructions, a temperature measurement input with a high level of noise was used. The data is plotted at a rate of 20 Hz.	124
8.9	RMSE values for low, medium, and high noise measurement inputs as a function of the regularization coefficient using Tikhonov regularization. Datapoints represent the mean RMSE value from the 50 repeat sweeps. The shaded region represents a ± 1 standard deviation envelope.	125
8.10	Selected reconstructed heat flux profiles for different values of measurement input noise using Tikhonov regularization. Optimal regularization coefficients of $\lambda = 37, 60$, and 90 were used for the a) Low noise, b) Medium noise, and c) High noise scenarios, respectively. The data is plotted at a rate of 20 Hz.	126
9.1	2D axisymmetric model of the TPS-sensor system. The heat flux sensor is labeled as domain 1 and the TPS is labeled as domain 2. a) Isometric view of the model system. b) $r - z$ cross-section view of the model system. The top surface is exposed to the external environment. The bottom surface is held at a constant temperature $\tilde{T}_0$. The temperature is sampled from a probe embedded in the TPS. .	129
9.2	Convective heat flux boundary conditions. a) Sinusoidal pulse of the freestream temperature relative to $\tilde{T}_0$. b) Convective heat transfer coefficient with sharp ramp at $t - t_0 = 15$ s.	142
9.3	FEA-generated measurement inputs for the 2D-axisymmetric model system. a) Simulated temperature probe measurement. b) Simulated heat flux sensor measurement.	142
9.4	Reconstructed net hot-wall heat flux using noise-free input measurements (solid blue line). The dashed black line represents the FEA-generated TPS surface heat flux.	143
9.5	Reconstructed heating profiles for the 2D-axisymmetric model system using various combinations of radial $\mathcal{N}$ and axial $\mathcal{M}$ basis function orders. In each plot, the FEA reference solution is shown as a dashed line.	146

9.6	Reconstructed heating profiles with the temperature probe positioned at various radial positions r_T in the TPS. In all scenarios, the probe is positioned along the z -direction at $0.9L$. The dashed line represents the FEA-generated surface heat flux sampled from the same radial location as the temperature probe. The dashed-dot line represents the FEA-generated surface heat flux sampled from the edge of the model system at $r = R_2$. The heat flux sensor and TPS interface is at $r = 0.25R_2$.	147
9.7	2D-axisymmetric model system measurement inputs with added measurement noise. a) Examples of simulated temperature probe measurements. b) Examples of simulated heat flux sensor measurements.	149
9.8	Reconstruction results from sweeps of λ_2 values using the sensor fusion-based regularization approach. a) RMSE. Datapoints represent the average RMSE value, calculated for each sweep individually, with the shaded region enveloping ± 1 standard deviation. b) Error in the magnitude of peak heating. Datapoints represent the peak heat flux error in the mean solution from the 50 repeat sweeps. The shaded region represents a ± 1 standard deviation envelope from the distribution of the 50 repeat sweeps at the peak heat time. c) Error in the peak heat time. Datapoints represent the error in the peak heat time in the mean profile from the 50 repeat sweeps.	150
9.9	Reconstruction results from sweeps of λ values using Tikhonov regularization. a) RMSE. Datapoints represent the average RMSE value, calculated for each sweep individually, with the shaded region enveloping ± 1 standard deviation. b) Error in the magnitude of peak heating. Datapoints represent the peak heat flux error in the mean solution from the 50 repeat sweeps. The shaded region represents a ± 1 standard deviation envelope from the distribution of the 50 repeat sweeps at the peak heat time. c) Error in the peak heat time. Datapoints represent the error in the peak heat time in the mean profile from the 50 repeat sweeps.	151
9.10	Selected heat flux profiles stabilized using the sensor fusion-based regularization approach. a) Low noise. b) Medium noise. c) High noise	151
9.11	Selected heat flux profiles stabilized using Tikhonov regularization. a) Low noise. b) Medium noise. c) High noise. Reconstructions are shown using the optimal values of λ for each level of measurement noise.	152
9.12	Reconstructed net hot-wall heat flux profiles a) with the added parameter $C_0(t)$ and b) without the added parameter.	153
10.1	1D model system schematic including the TPS, adhesive, and substructure. The temperature measurement is sampled from a probe embedded in the TPS at x_T , where the origin of the coordinate system is defined at the TPS-adhesive interface. The internal surface of the stackup is modeled as insulating and the external surface is exposed to the aeroheating environment. A net hot-wall heat flux q_{HW} is conducted into the TPS solid.	167
10.2	Flow chart for the reconstruction algorithm.	180
10.3	Numerical verification inputs. a) Radiative (solid line) and convective (dashed line) boundary conditions imposed on the 1D model system. For the convective boundary condition, the bulk enthalpy is fixed at a constant value of 1 MJ/kg and the film coefficient is pulsed. b) Simulated temperature probe measurement.	183

10.4	Numerical verification results. a) Reconstructed net hot-wall and incident heat flux. b) Reconstructed surface temperature. Reference solutions are shown by the dashed line.	184
10.5	Numerical verification results. a) Reconstructed through-thickness temperature distribution. b) Reconstructed through-thickness composition. The temperature probe is located at $x/L = 0.915$. Reference solutions are shown by the dashed lines.	185
10.6	Reconstructed solutions for various numerical integration orders N_x . a-d) Reconstructed surface heating conditions. e-h) Reconstructed pyrolysis gas mass flux at the TPS surface. Reference solutions are shown by the dashed lines.	186
10.7	Reconstructed solutions for various Galerkin basis function orders N . Reference solutions are shown by the dashed lines.	188
11.1	Panel Test Facility campaign 162 test articles before (left) and after (right) arc-jet exposure. a) Test 1. b) Test 2. c) Test 3. d) Test 4. Adapted from [14].	193
11.2	Reconstructed net hot-wall (blue) and incident (orange) heating results for instrumented arc-jet test articles from PTF test campaign 162 a) Test 1, b) Test 2, c) Test 3, and d) Test 4. The solid series represent results from the Green's function approach and the dashed series represent results from FIAT_Opt. In each plot, the black dashed lines represent the target cold-wall or adiabatic-wall heat flux, labeled respectively.	194
11.3	Reconstructed surface temperatures for instrumented arc-jet test articles from PTF test campaign 162 a) Test 1, b) Test 2, c) Test 3, and d) Test 4. The solid series represent results from the Green's function approach and the dashed series represent results from FIAT_Opt.	196
11.4	Schematic of the Mars 2020 backshell showing the MEDLI2 instrumented TC plug, heat flux sensor, and radiometer locations (adapted from [15]). Highlighted sensor labels indicate those used in this work.	197
11.5	Reconstructed heat flux profiles from the Mars 2020 backshell. a) Results from the MTB01/MTB07 sensor pair. b) Results from the MTB02/MTB08 sensor pair.	198
11.6	Reconstructed surface temperatures from the Mars 2020 backshell. a) Results from the MTB01/MTB07 sensor pair. b) Results from the MTB02/MTB08 sensor pair.	199
11.7	Results of the Monte Carlo simulations for the reconstructed surface heating conditions for the a) MTB01/MTB07 and b) MTB02/MTB08 sensor pairs. The solid line represents the mean solution. The shaded envelope represents a 95% confidence interval.	201
11.8	Convergence plots of the peak heating total Sobol indices for the a) net hot-wall heat flux and b) incident heat flux for the MTB02/MTB08 sensor pair.	203
11.9	Total Sobol index calculations for the reconstructed net hot-wall heat flux for the MTB01/MTB07 sensor pair. Labeled datapoints at the respective peak heating times are the total Sobol indices calculated using the machine learning approach described in [2]. The solid black line is the nominal reconstructed net-hot wall heat flux solution from Fig. 11.5a. Inset: Magnified region for the total Sobol indices at the peak heating time.	204

11.10	Total Sobol index calculations for the reconstructed net hot-wall heat flux for the MTB02/MTB08 sensor pair. Labeled datapoints at the respective peak heating times are the total Sobol indices calculated using the machine learning approach described in [2]. The solid black line is the nominal reconstructed net-hot wall heat flux solution from Fig. 11.5b. Inset: Magnified region for the total Sobol indices at the peak heating time.	205
11.11	Total Sobol index calculations for the reconstructed incident heat flux for the MTB01/MTB07 sensor pair. Labeled datapoints at the respective peak heating times are the total Sobol indices calculated using the machine learning approach described in [2]. The solid black line is the nominal reconstructed incident heat flux solution from Fig. 11.5a. Inset: Magnified region for the total Sobol indices at the peak heating time.	206
11.12	Total Sobol index calculations for the reconstructed incident heat flux for the MTB02/MTB08 sensor pair. Labeled datapoints at the respective peak heating times are the total Sobol indices calculated using the machine learning approach described in [2]. The solid black line is the nominal reconstructed incident heat flux solution from Fig. 11.5b. Inset: Magnified region for the total Sobol indices at the peak heating time.	207
A.1	Schematic of the weld setup. The tungsten-rhenium electrical wire is pressed onto the transducer using an electrode. The transducer is placed on an aluminum substrate. A separate welding electrode is pressed against the aluminum substrate.	221
A.2	Removal of copper deposited from the welding electrode. a) Rhenium transducer circuit after welding, showing significant copper deposition on the weld region. b) Transducer circuit after baking in air at 300 °C for 30 minutes. All exposed copper is oxidized. c) Transducer circuit after etching in glacial acetic acid for ~20 minutes. All the exposed copper oxide is removed, revealing the underneath layer of bare copper. d) Transducer circuit after repeating the bake and etch process until all copper is removed.	221
B.1	High-temperature TSE-based heat flux sensor package: detailed drawing 1 of 3. .	223
B.2	High-temperature TSE-based heat flux sensor package: detailed drawing 2 of 3. .	224
B.3	High-temperature TSE-based heat flux sensor package: detailed drawing 3 of 3. .	225
B.4	Inconel bracket: detailed drawing 1 of 1.	226
B.5	High-temperature heat sink: detailed drawing 1 of 4.	228
B.6	High-temperature heat sink: detailed drawing 2 of 4.	229
B.7	High-temperature heat sink: detailed drawing 3 of 4.	230
B.8	High-temperature heat sink: detailed drawing 4 of 4.	231
B.9	Optical tube: detailed drawing 1 of 1.	232
B.10	Heat sink mounting bracket: detailed drawing 1 of 1.	233
B.11	Sensor rack $y - z$ alignment stage mounting bracket: detailed drawing 1 of 1. . .	234

Chapter 1: Introduction

During entry, descent, and landing (EDL), spacecraft are exposed to extreme heat loads. As a vehicle slows down in the upper atmosphere, its kinetic energy is dissipated in the form of heat, ideally transported around the vehicle body and into the wake region [16, 17]. Some of the heat generated, however, becomes absorbed by the spacecraft. Specialized materials, termed *thermal protection systems* (TPS), have been developed since the inception of supersonic flight to withstand these heat loads through various physical mechanisms, such as pyrolysis (i.e., decomposition of the TPS) or ablation (i.e., removal of TPS material) [16, 18]. Designing and sizing TPS for each mission architecture, however, remains a large challenge due to the overall uncertainties of predicted atmospheric entry heating conditions and the material response behavior of the TPS. To this end, even TPS designed for modern spacecraft are sized conservatively [19], accounting for unnecessary mass and volume in payload allocations that are already stringent.

Characterizing the heat loads experienced by spacecraft during atmospheric entry is critical to evaluate the performance of TPS, investigate aerothermal phenomena, and validate computational models, ultimately to reduce the margins associated with EDL. Recent space exploration missions have featured extensive arrays of thermal instrumentation to measure atmospheric entry heat rates and evaluate TPS performance [15, 20, 21]. These TPS-embedded instrumentation suites have contained combinations of direct heat flux sensors, thermocouple plugs with one or

more temperature probes embedded at various depths, and discrete temperature sensors. Among these, heat flux sensors, which exhibit a linear transduction of an absorbed heat flux to an electrical signal, offer the most direct platform for atmospheric entry heat flux measurements. Sparse implementations of Schmidt-Boelter-type heat flux sensors in the Mars 2020, Schiaparelli, and LOFTID entry spacecraft have demonstrated the value of direct thermoelectric heat flux measurement systems in practice, however survivability limitations have often constrained the commercially available sensors to less extreme areas of the spacecraft, such as the backshell [14, 21]. Because of their more complex constructions, direct heat flux sensors typically carry maximum operating temperatures much lower than the temperature of the surrounding TPS, which is often in excess of 800 °C [2], and require passive cooling infrastructure that is invasive to the TPS structure [21]. As an additional consequence, because these sensors must remain cool throughout the entry heat pulse, the interpretation of the measurement data is complicated by local time-varying heating augmentations at the cold-wall heat flux sensor surface, requiring significant post-processing efforts [14]. These drawbacks motivate the development of heat flux sensors that are more compatible with extreme atmospheric entry environments.

As a complement to measurements from direct heat flux sensors, inverse heat transfer (IHT) techniques—using in-depth temperature measurements to reconstruct the surface heat flux boundary condition—can be used to reconstruct atmospheric entry heating conditions which more closely represent the thermal loads experienced by a hot-wall TPS. IHT techniques are currently the most pervasive tool for post-flight assessment of TPS thermal loading, owed to a much broader adoption of temperature sensors versus direct heat flux sensors for EDL measurements [2, 22–24]. Current state-of-the-art IHT reconstruction methodologies, however, demand substantial computational resources. These techniques often rely on time-marching multiphysics

simulations embedded within an iterative objective function minimization framework, requiring many (>1000) individual simulations to converge on a single reconstructed heating profile [2]. While this cost is not prohibitive for one-off reconstructions, more intensive analyses (e.g., uncertainty quantification) or time-sensitive processes (e.g., real-time health monitoring) become inaccessible, even with access to modern computational tools. As such, there is a need for alternative computational methods that are more efficient and optimized for the aerothermal environment reconstruction end-use case.

To advance the field of thermal metrology and address the engineering challenges associated with characterizing atmospheric entry heating, this thesis aims to

1. Design, fabricate, and characterize robust heat flux sensors tailored for use in extreme high-temperature environments.
2. Develop novel inverse heat transfer techniques that enable more accurate, reliable, and efficient post-flight assessment of atmospheric entry heat loads than the current state of the art.

In particular, the contributions detailed throughout the following chapters aim to equip the space exploration community with a more capable set of tools to better understand the heat loads and aerothermodynamic phenomena that drive the design of entry spacecraft. Furthermore, while this thesis is motivated by the problem of atmospheric entry, the contributions have a broad applicability to other fields, such as energy production, combustion, and propulsion, and can generally be applied where heat transfer measurements or reconstructions are sought in hostile environmental conditions.

This thesis is structured as follows: Chapters 2-5 investigate the transverse Seebeck effect

(TSE) in anisotropic single crystals as a transduction platform for extreme environment heat flux sensors. Chapter 2 provides a review of the literature with a focus on the transverse Seebeck effect and prior heat flux sensors designed for high-temperature applications. Chapter 3 details the design and characterization of a heat flux sensor leveraging the TSE in elemental antimony for use at moderate temperatures, with emphasis on the integration of the transducers within a robust sensor package. Chapter 4 details the development of a high-temperature heat flux sensor using the TSE in elemental rhenium, fabricated using components with maximum operating temperatures exceeding 1000 °C. To enable characterization of the rhenium sensor’s temperature-dependent responsivity, a new state-of-the-art heat flux sensor calibration facility is developed, detailed in Chapter 5.

Chapters 6-11 detail the development of a Green’s function-based inverse heat transfer framework to efficiently reconstruct the heating conditions on spacecraft TPS during atmospheric entry. Chapter 6 consists of a review of the literature focused on the inverse heat transfer methodologies used by the space exploration and aerothermodynamics ground test communities. The Green’s function framework is introduced in Chapter 7, establishing a basis for the reconstruction algorithm developed in this thesis. In Chapters 8 and 9, a novel sensor fusion approach is introduced to increase the stability of IHT problems when temperature measurements are sampled from insulating materials. An adaptation of the Green’s function IHT framework is developed for ablative TPS materials in Chapter 10, allowing for the nonlinear effects of decomposition and internal gas transport to be included in the reconstruction of the surface heating conditions. The algorithm is used in Chapter 11 to reconstruct the surface heating conditions on arc jet ground test articles and on the backshell of the Mars 2020 spacecraft during atmospheric entry.

Chapter 2: Extreme Environment Heat Flux Sensors - Review of the Literature

This chapter serves to introduce the concept of heat flux sensing and survey the current state of the art of sensors developed for extreme high-temperature environments. A heat flux sensor is a device that outputs an electrical voltage in direct response to an absorbed heat flux. When exposed to the environment of interest, an energy exchange occurs at the sensor surface, usually through convection, radiation, or a combination of the two mechanisms. This exchange results in a net heat flux absorbed into the gauge, which becomes directly correlated with the measured electrical voltage. Strictly, the heat flux that produces an output signal is purely conductive, representing the absorbed heat flux; through calibration and sensor design (e.g., radiometer windows) the absorbed heat flux can be correlated with various heat flux quantities of interest, such as the radiative heat flux incident on the sensor surface.

Direct heat flux sensors typically operate using thermoelectric effects. In these sensors, often categorized as Gardon or Schmidt-Boelter gauges, the conventional Seebeck effect (CSE) is used as the heat flux-to-voltage transduction mechanism. As heat flows into the device, a temperature difference established between two dissimilar material junctions produces a thermoelectric voltage. This voltage V is a function of the difference in the Seebeck coefficients of the materials S_{ab} , the temperature difference between junctions ΔT , and the number of serial junctions present

N [4]

$$V = NS_{ab}\Delta T \quad (2.1)$$

In a Gardon gauge (Fig. 2.1), a copper wire is attached to the center of a thin, circular Constantan foil. The outer edge of the Constantan foil is attached to an annular copper heat sink. In this configuration, the heat flux absorbed by the foil establishes a radial temperature gradient. The arrangement of the copper wire, Constantan foil, and heat sink creates an electrical circuit with two serial junctions (i.e., at the center and edge of the foil) at two different temperatures [3]. In a Schmidt-Boelter or, generally, thermopile-type gauge, a series of alternating thermoelectric materials (termed *legs*) are connected on two opposing sides of a intermediate thermal resistance layer (Fig. 2.2a) [3, 25]. A temperature difference is induced between the junctions closest to the sensor surface (i.e., the hot side) and the junctions closest to the gauge body or heat sink (i.e., the cold side). Traditionally, a Schmidt-Boelter gauge consists of a Constantan wire wrapped around an insulating core (Fig. 2.2b). One of half of the wind is electroplated with copper; because the copper-Constantan leg has different effective transport properties versus pure Constantan, two sets of dissimilar material junctions are created.

While CSE-based sensors have been traditionally limited by low maximum operational temperatures, recently developed thermopile-type heat flux sensors have demonstrated good compatibility with extreme environments by utilizing high-temperature thermoelectric materials, alloys, and ceramics. Throughout the early 2010s, Diller *et al.* have reported extensively on a high-temperature heat flux sensor using type-K thermocouple materials (Chromel/Alumel), which function as both the thermopile legs and the thermal resistance layer (Fig. 2.3a) [26–29]. The heat flux sensor, supported by ceramic electrical insulators and Inconel packaging, can op-

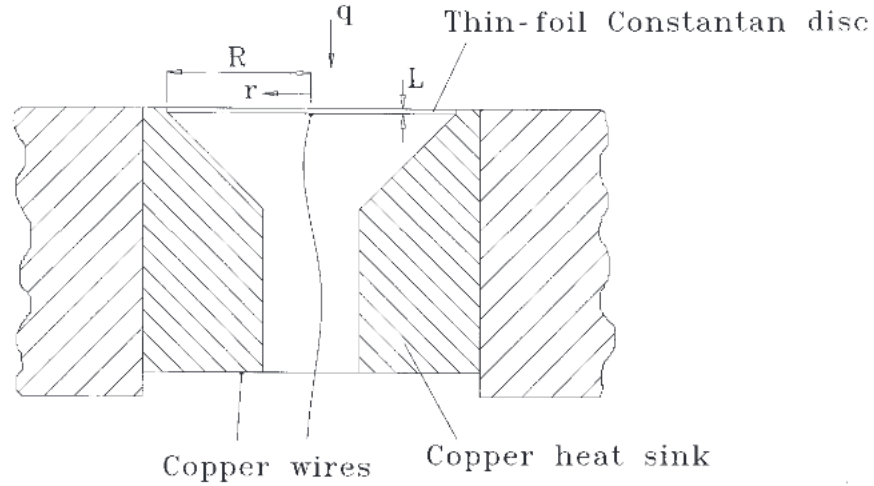


Figure 2.1: Operating principle of a Gardon gauge [3] (reprinted with permission from SAGE Publications Ltd.). A heat flux produces a radial temperature gradient in a thin Constantan foil. As a result, a thermoelectric voltage difference is induced between two copper-Constantan junctions, one at the center of the foil and one at the edge.

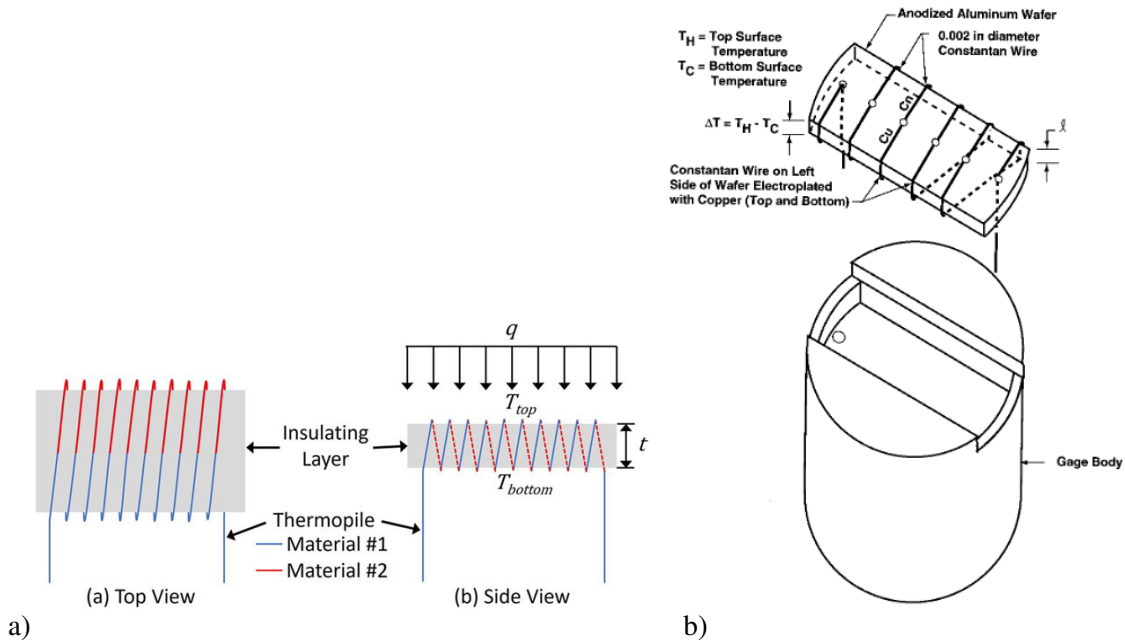


Figure 2.2: a) Operating principle of a Schmidt-Boelter or thermopile-type heat flux sensor (reprinted from [4], with the permission of AIP Publishing.). A series of dissimilar material junctions is produced by alternating two thermoelectric materials on opposing sides of a thermal resistance layer. A temperature difference induced between the junctions on the top and bottom of the thermal resistance layer produces a thermoelectric voltage. b) Schematic of a wire-wound Schmidt-Boelter gauge [5] (reprinted by permission of the American Institute of Aeronautics and Astronautics, Inc.).

erate at temperatures as high as 1000 °C [6,30]. Similar high-temperature thermopile-type heat flux sensors have since followed suit; Trelewicz *et al.* developed a heat flux sensor using type-N thermocouple materials (nickel-chromium-silicon/nickel-silicon) with a yttria-stabilized zirconia thermal resistance layer (Fig. 2.3b) [7]. The sensor can operate at temperatures as high as 860 °C and has been used within scramjet combustors to probe wall heating conditions during ground testing [31]. Other sensors have adopted similar concepts, utilizing platinum/platinum-rhodium [32, 33] and tungsten-(5%)rhenium/tungsten-(26%)rhenium [34] thermopile junctions separated over a ceramic thermal resistance layer to target operating temperatures up to 1000 °C. These sensors, however, all have complex constructions and must rely on dissimilar material junctions that are at two different temperatures, including on the hot side of the sensor. At high temperatures, the junctions become susceptible to elemental diffusion and thermomechanical stress, both of which can lead to degradation of the device. Furthermore, because the output signal scales with the number of dissimilar material junctions, thermopile-type heat flux sensors will always have many single points of failure. These drawbacks are intrinsic to heat flux sensors that utilize the CSE.

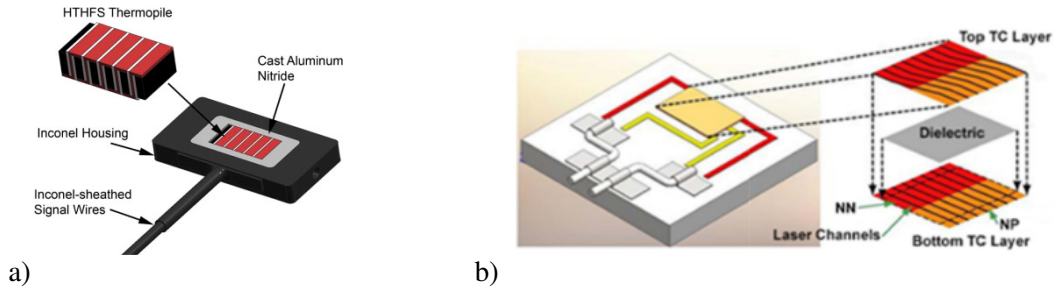


Figure 2.3: a) High temperature heat flux sensor using type-K thermoelectric materials [6] (reprinted with permission from Elsevier). The active thermopile legs act as both a transducer and thermal resistance layer. b) High temperature heat flux sensor using type-N thermocouple materials [7] (reprinted with permission, ©2025 IEEE). The hot and cold junction pairs are separated by a ceramic thermal resistance layer.

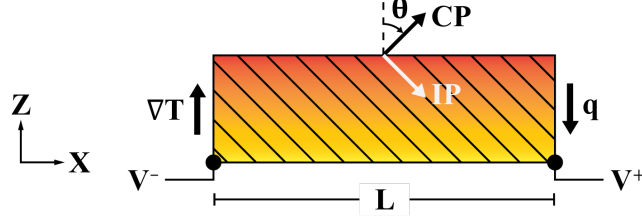


Figure 2.4: Schematic of the transverse Seebeck effect in an anisotropic single crystal. A voltage is generated between two probe locations separated by a distance L along the x -direction in response to a temperature gradient induced along the z -direction. In the schematic, CP and IP represent the cross-plane and in-plane crystal orientations, respectively, and θ represents the relative tilt between the coordinate systems defined by the crystal structure and the transducer geometry.

Alternatively, heat flux sensors can utilize the transverse Seebeck effect [35,36]. The TSE, illustrated schematically in Fig. 2.4, occurs in an anisotropic single crystal when the principal crystallographic axes are tilted with respect to the direction of an induced temperature gradient. Assuming a uniform temperature gradient in the z -direction in Fig. 2.4, the voltage generated along the x -direction due to the TSE is given by [37]

$$V_x = q_z \frac{S_{CP} - S_{IP}}{k} L \sin \theta \cos \theta \quad (2.2)$$

In Eq. 2.2, S_{IP} and S_{CP} are the in-plane (IP) and cross-plane (CP) components of the anisotropic Seebeck coefficient tensor, respectively, θ is the inclination of the CP axis (corresponding to $\langle 0001 \rangle$ in Miller-Bravais notation) from the z -axis towards the x -axis (Fig. 2.4), and k is the thermal conductivity (assumed isotropic). The parameter L is the length of the transducer along the x -direction, or cumulative length of all transducers if multiple are wired in series [38]. The parameter q_z is the heat flux parallel to the z -axis and is related to the temperature gradient $\partial T / \partial z$ via Fourier's law (Eq. 2.3)

$$q_z = -k \frac{\partial T}{\partial z} \quad (2.3)$$

The coupling between an orthogonal voltage and heat flux via Eq. (2.2) allows TSE-based heat flux sensors to elicit a number of advantages. Heat flux sensors using the TSE only require a single thermoelectric transducer material to generate a voltage output, in contrast with the two required for CSE-based sensors. The voltage can be scaled up by increasing the overall length of the transducer or connecting multiple in series. Furthermore, all electrical wire connections can be integrated on the cold side of the transducer, avoiding the placement of any weld or solder joints near the hottest portions of the sensor. Generally, heat flux sensors using the TSE exhibit more simple and robust constructions compared with their thermopile counterparts, yielding good compatibility with extreme environments.

TSE-based heat flux sensors have been developed using thin films, layered composites, and bulk single crystals. Renk *et al.* first reported on the TSE as a heat flux-to-voltage transduction mechanism in epitaxial $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (YBCO) thin films grown on miscut single crystal substrates (Fig. 2.5) [8, 39]. The development of similar thin film sensors, using YBCO and other layered oxides [37, 40–48], have since been a focus of the thermal measurement community due to the high spectral bandwidth (>100 kHz) that can be achieved by reducing the transducer thickness to the sub-micrometer scale. In fact, Knauss *et al.* have demonstrated a YBCO-based sensor with a time constant of $0.5\ \mu\text{s}$ [49], which is sufficient to resolve second-mode instabilities in hypersonic boundary layers. The original thin film TSE heat flux sensor reported on by Renk *et al.*, and further developed by Roediger [50], has matured into a commercial product marketed as the Atomic Layer Thermopile (ALTP, ForTech HTS GmbH). The ALTP has been deployed experimentally to probe boundary layer phenomena in combustion and high-speed aerodynamic test facilities [51–53]. Generally, however, YBCO-based thin film sensors are limited by low maximum operating temperatures and degrade rapidly in humid environments. Advances in the field

have, indeed, extended TSE-based sensing technologies toward high-temperature applications, culminating with the recent demonstration of a $\text{La}_{1-x}\text{Ca}_x\text{MnO}_3$ thin film heat flux sensor operating at 1000 °C [54]. Yet, sensors based on thin film transducers (0.1 to 1 μm thick) will always be more susceptible to damage from dust or micrometeoroid impact compared to those using bulk materials, thereby limiting their application in atmospheric entry spacecraft and particle-laden ground test facilities.

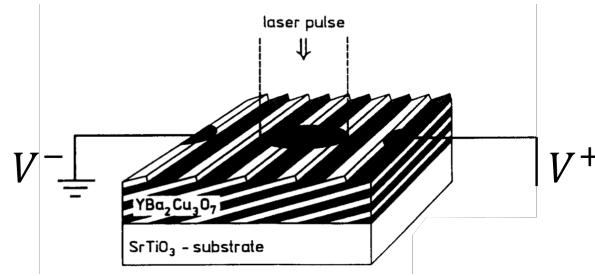


Figure 2.5: TSE-based heat flux sensor using a YBCO thin film, adapted from [8] (reprinted with permission from Elsevier). A incident heat flux produces a voltage difference orthogonal to the direction of the induced temperature gradient in the transducer.

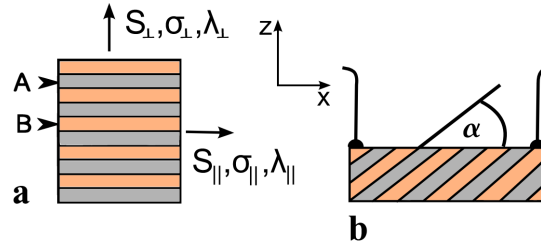


Figure 2.6: Multilayer TSE device [9] (reprinted with permission from Springer Nature). Thermopower anisotropy is generated by layering two types of thermoelectric materials. The TSE is induced through rotation of the layered monolith.

For extreme environment applications, multi-layered composite structures and bulk single crystals offer a potentially more robust transducer platform for TSE heat flux sensors. High temperature TSE-based sensors which use tilted multilayer composite structures have become increasingly prevalent within the past two decades. In these sensors, thermopower anisotropy (the

prerequisite for the TSE) is produced by layering two types of thermoelectric materials to create a monolithic composite (Fig. 2.6) [9]. Mityakov *et al.* [55] first reported on the use of type-K thermocouple materials (Chromel/Alumel) to construct a TSE heat flux sensor which can operate at temperatures exceeding 600 °C. A similar type-K sensor was later reported by Meier *et al.* [56] and others have followed, including alternative transducer types which leverage multilayer stainless steel/nickel composites [57]. Bulk single crystals which exhibit thermopower anisotropy due to their crystal structure have also been reported previously as transducers for TSE-based heat flux sensors. Sapozhnikov *et al.* have reported on a TSE heat flux sensor which leverages elemental bismuth single crystals [58]. The sensor exhibits a minimalistic configuration, consisting solely of a series of bismuth single crystal transducers and electrical wiring, and has been used to probe low-speed convection phenomena [59] and characterize shock tube performance [60]. More recently, Konopko *et al.* have developed TSE-based sensors using glass-coated crystalline bismuth microwires. In these sensors, the active sensing element is grown shaped as a continuous wire with a potential length exceeding 10 meters, eliminating the need for electrical wiring interfaces between individual transducers, as is required for most sensors [61, 62]. However, in spite of their very simple constructions, these sensors are limited in extreme environment applications due to the relatively low melting point of bismuth (270 °C). Alternative single crystal TSE-based heat flux sensors have been explored using Bi_2Te_3 , however with only marginal gains in the theoretical maximum operating temperature (up to 585 °C) and without a robust package [63].

Within the thermoelectrics community, there is a growing interest to identify single crystal materials with large anisotropies in their Seebeck coefficient tensor for use in transverse thermoelectric devices [64]. While a considerable number of these materials carry caveats that make them impractical for extreme environment heat flux sensors, such as low chemical stability in

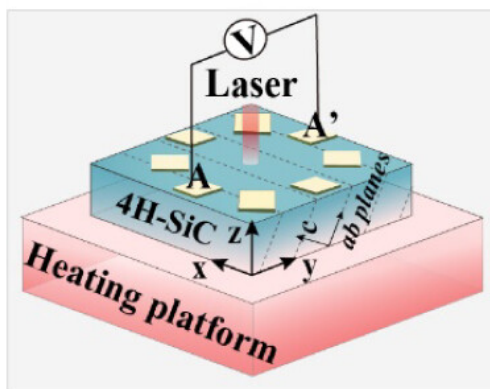


Figure 2.7: High temperature transducer for a TSE heat flux sensor based on single crystal 4H-SiC (reprinted (adapted) with permission from [10]. Copyright 2025 American Chemical Society).

air, a select few are promising, including rhenium [11], Re_4Si_7 [65], and 4H-SiC [66]. To-date, only 4H-SiC has been investigated as a single crystal transducer for high temperature TSE heat flux sensors (Fig. 2.7); recent contributions investigating the TSE in crystalline 4H-SiC have, in fact, demonstrated good compatibility of the transducer platform with temperatures as high as 1000 °C [10,67,68]. Due to the high thermal conductivity of 4H-SiC, however, these sensors will intrinsically have responsivities that are low (Eq. (2.2)). Furthermore, testing has exposed how the responsivities of 4H-SiC sensors degrade with increasing sensor temperature [10], limiting their performance under application-relevant operating conditions. For heat flux sensors tailored for used in atmospheric entry spacecraft, other anisotropic single crystal materials may be better suited.

Chapter 3: A Heat Flux Sensor Leveraging the Transverse Seebeck Effect in Elemental Antimony

Among the bulk single crystal TSE heat flux sensors introduced in Chapter 2, those using crystalline bismuth were most prevalent. While its melting point is low, bismuth has one of the largest Seebeck coefficient anisotropies $|S_{33} - S_{11}|$ of the pure elements, yielding large responsivities in sensors tailed for room- or low-temperature applications. Other semimetals with rhombohedral crystal structures, including antimony and arsenic, also exhibit large Seebeck coefficient anisotropies at room temperature (Table 3.1) [69, 70], but have not been used to the same extent for TSE-based devices. Notably, however, both antimony and arsenic have much higher melting points (631 °C and 816 °C, respectively) compared to bismuth (270 °C) and are, thus, well-positioned alternatives for sensors designed for moderate temperatures. Furthermore, these materials are generally inexpensive and can be grown routinely into large crystals. While arsenic has the higher melting point, it is generally avoided due to its toxicity, leaving antimony as the natural choice to investigate further as a TSE transducer for heat flux sensing applications.

In this chapter, the TSE in single crystal antimony is demonstrated as a practical transduction mechanism in a sealed and robust heat flux sensor. The sensor design and fabrication is detailed with an emphasis on the structural components used to ruggedize the device. While most of the TSE-based heat flux sensors introduced in Chapter 2 have been used without a ro-

Table 3.1: Seebeck coefficient tensor components of single crystal bismuth, antimony, and arsenic at room temperature.

Material	S_{11} ($\mu\text{V/K}$)	S_{33} ($\mu\text{V/K}$)	Reference
Bismuth	-50	-105	[69]
Antimony	44	21	[69]
Arsenic	20	-3	[71, 72]

bust sealed package, these components are essential for protection from chemical and physical damage, albeit at the cost of impairing the flow of heat through the transducer; such effects are analyzed in the proceeding sections. The sensor’s responsivity is characterized subject to a uniform steady heat flux and juxtaposed with calculations from an analytical model used to predict the theoretical responsivity. The transient response of the sensor is characterized subject to a step heat flux. The impact of the package design parameters on the sensor’s response time is investigated to establish design guidelines for future sensors. Using a localized heat source, the relative contributions of the TSE and CSE in the device are characterized, highlighting an output signal that is dominated by the TSE.

3.1 Heat Flux Sensor Design and Fabrication

The components of the antimony-based TSE heat flux sensor are shown in Fig. 3.1, with component material properties summarized in Table 3.2. The components include two prismatic antimony transducers, the ceramic housing, two auxiliary thermocouples, wiring, and the heat spreading cover plate. Two transducers measuring $5.1 \times 2.1 \times 1.45$ mm were sectioned using a dicing saw from a cylindrical antimony single crystal pellet (Goodfellow Corp., purity 99.999%, diameter 11 mm, height 2.1 mm, axis orientation $[11\bar{2}0]$). To obtain the crystal orientation which maximizes the TSE, dicing planes were at a 45° angle to both the $[0001]$ and $[1\bar{1}00]$ crystallo-

graphic directions. The ceramic housing (Boron Nitride PCBN1000, Precision Ceramics USA Inc.) was machined to outer dimensions of $16 \times 14 \times 3$ mm. PCBN1000 was chosen as the housing material because its thermal conductivity is similar to that of antimony, minimizing the housing's impact on the heat flow through the transducers. The transducers were mounted in a cavity $5.4 \times 4.5 \times 1.75$ mm using cement (Sauereisen No. 3). Tungsten wires (McMaster-Carr, 99.95%, 0.2 mm) were attached to the prisms using PELCO colloidal silver (Ted Pella 16034). Two auxiliary 40 AWG type-K thermocouples (Omega Engineering 5SC-TT-K-40-36-ROHS) were attached near the contacts between the tungsten wires and the transducers. The gaps between the transducers and the housing were filled with cement. Cement was also used to attach the brass heat spreading cover plate (McMaster-Carr, ultra-Machinable 360 Brass), measuring $16 \times 14 \times 2$ mm, to the corresponding face of the ceramic housing near the transducers. Fig 3.2 shows the sensor before and after attaching the brass plate. The outward facing surface of the brass plate was spray-coated with high temperature black paint (Rust-Oleum 248903). The black face is the heat flux-facing end of the sensor (i.e., the front end). On the back end of the sensor, a water-cooled heat sink was attached. The interface between the sensor and heat sink was filled with thermal compound (MX-5, ARCTIC) to improve heat dissipation. The free ends of the tungsten wires were connected to the electrical leads of a nanovoltmeter (Keithley 2182A). The auxiliary thermocouples were connected to a data acquisition module (Pico Technology TC-08). The as-built heat flux sensor measured $16 \times 14 \times 5.3$ mm with a frontal area of 224 mm^2 .

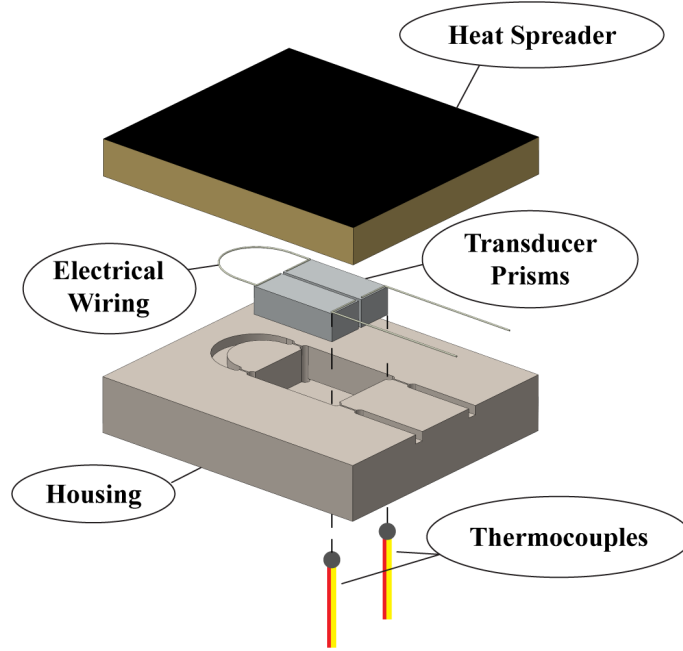


Figure 3.1: Exploded view of the sensor. The front end is a painted brass heat spreader, placed over the ceramic housing which accommodates two prismatic antimony transducers and tungsten wires. Two auxiliary thermocouples that probe the temperature at two antimony/tungsten junctions extend from the back end.

3.2 Steady State Sensor Characterization

3.2.1 Steady-State Response

The heat flux sensor was calibrated against a reference gauge using a transfer calibration technique [73, 74]. In this method, the response of the TSE heat flux sensor is compared to the response of the reference gauge with both sensors exposed sequentially to an identical heat flux. A NIST-traceable Schmidt-Boelter heat flux sensor (Medtherm 64-3-20) was used as the reference gauge. Two different radiative heat sources, a propane heater and a blackbody furnace (Oriel 67032), were used during the calibration campaign. The separation between the heat source and the sensors was varied from 1 to 100 cm to produce a range of heat fluxes. The furnace temperature was adjusted to achieve heat fluxes up to 7 W/cm^2 .

Table 3.2: Summary of material parameters for the TSE heat flux sensor.

Property	Value at room temperature	Reference
Brass, thermal cond.	1.16 W/(cm-K)	
Ceramic, thermal cond.	0.21 W/(cm-K)	
Cement, thermal cond.	0.033 W/(cm-K)	
Antimony, thermal cond.	0.22-0.244 W/(cm-K)	[12, 69]
Antimony, Seebeck coeff.	$S_{11} = 44 \mu\text{V/K}$, $S_{33} = 21 \mu\text{V/K}$	[69]
Antimony, resistivity	$44 \mu\Omega\text{-cm}$	This work
Tungsten wire, Seebeck coeff.	$-0.12 \mu\text{V/K}$	This work
Tungsten wire, resistivity	$6 \mu\Omega\text{-cm}$	This work
Resistance of sensor	8-16 Ω	This work

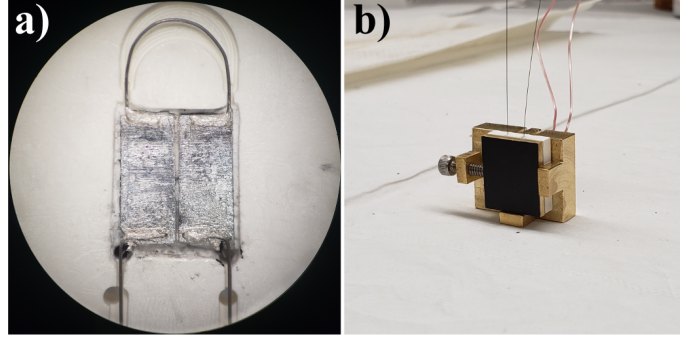


Figure 3.2: a) Optical microscope image of the sensor before attaching the brass plate. b) Completed sensor mounted on a brass holder.

The sensors were mounted on a translation stage that allows for repeatable positioning (within 0.05 cm) of each sensor at the same location in the line-of-sight of the heat source. Once the heat source setpoint was established, a sensor was positioned in the line-of-sight. The other sensor was positioned behind a reflective barrier. After 60 to 90 s, when a steady signal was achieved, the sensor positions were interchanged. Three repeat measurements were performed and averaged for each sensor.

The front surface temperature of each sensor and the ambient temperature were monitored during calibration. To account for differences in the convective heat flux from the ambient environment to each sensor, the total measured heat flux q_{in} due to both convective and radiative

heating was considered in the calibration.

$$q_{in} = \alpha q_{rad} + h(T_{\infty} - T_s) \quad (3.1)$$

In Eq. (3.1), α is the absorptivity of the sensor surface, q_{rad} is the incident radiative heat flux, h is the convective heat transfer coefficient, and T_{∞} and T_s are the ambient and sensor surface temperatures, respectively. The absorptivity of both sensor surfaces is estimated as $\alpha \simeq 0.95$ [75], thus the radiative heating term in Eq. (3.1) is equal for both sensors. The convective heat transfer coefficient, given in $W/(cm^2 \cdot K)$, was approximated using Eq. (3.2), which is based on empirical parameters determined from an analogous propane heater calibration setup and Schmidt-Boelter heat flux sensor [76].

$$h = 0.00078(T_{\infty} - T_s)^{0.36} \quad (3.2)$$

Combining Eqs. (3.1) and (3.2), the total heat flux measured by the TSE heat flux sensor is expressed as

$$q_{in,TSE} = q_{in,ref} + 0.00078 \left[(T_{\infty} - T_{s,TSE})^{1.36} - (T_{\infty} - T_{s,ref})^{1.36} \right] \quad (3.3)$$

where the subscripts *TSE* and *ref* are used to denote parameters of the TSE and reference sensors, respectively. The parameter $q_{in,ref}$ was determined from the average reference sensor voltage using the manufacturer-supplied response curve. The maximum difference in convective heat flux between the TSE and reference sensors was 7% of $q_{in,ref}$.

A total of 105 data points were used to construct the TSE heat flux sensor response curve

in the range of $0 - 7 \text{ W/cm}^2$ (Fig. 3.3). A straight line was fit to the data, yielding a sensor responsivity (i.e., slope) of $16.8 \text{ } \mu\text{V}/(\text{W/cm}^2)$ with a 95% confidence interval of $16.7 - 17.0 \text{ } \mu\text{V}/(\text{W/cm}^2)$, assuming normally distributed measurement errors. The sensor response is linear from 0.06 to 7 W/cm^2 , with the upper limit of the calibration restricted by the design limits of the Medtherm reference sensor. At equilibrium without incident heat flux, the TSE heat flux sensor's baseline voltage offset is $0.080 \text{ } \mu\text{V}$, corresponding to a detection threshold of $5 \times 10^{-3} \text{ W/cm}^2$.

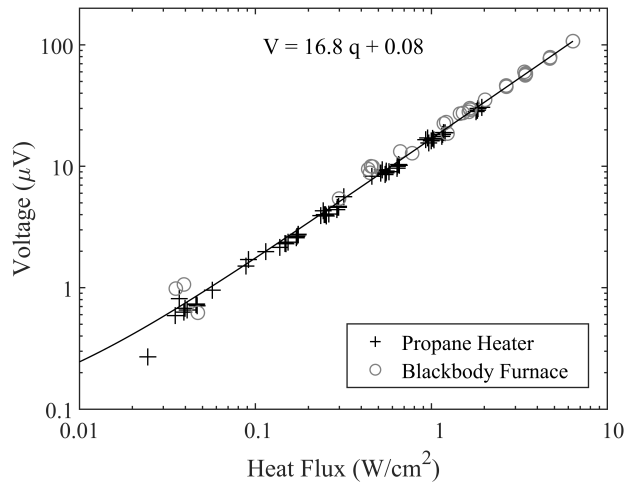


Figure 3.3: Experimental response curve for the antimony-based TSE heat flux sensor. Data sets obtained with different radiative heat sources are overlaid. The linear fit using the totality of data is displayed as a solid line, with the corresponding equation shown in the plot.

Throughout the calibration campaign, spanning four months, no change was observed in the responsivity, demonstrating good stability of the TSE heat flux sensor. This stability may be attributed to the fact that the temperature of the sensor never exceeded the maximum temperature used during assembly ($100 \text{ } ^\circ\text{C}$, while curing the black paint). At the high end of the heat fluxes tested, the sensor was exposed to ambient temperatures of up to $350 \text{ } ^\circ\text{C}$, however, the interior temperature, probed by the auxiliary thermocouples, did not exceed $74 \text{ } ^\circ\text{C}$ due to the water-cooled heat sink.

3.2.2 Analysis of the Sensor's Steady-State Response

In single crystal antimony, the Seebeck coefficient anisotropy $|S_{33} - S_{11}|$ and thermal conductivity (which is nearly isotropic) have been reported in the range of $23 - 26 \mu\text{V/K}$ [69, 77] and $0.22 - 0.24 \text{ W/(cm-K)}$, respectively. The transducers were fabricated in the optimal orientation of $\theta = \pi/4$ to maximize the TSE. Following Eq. (2.2), the theoretical responsivity of the single crystal antimony transducer is a least $52.5 \mu\text{V/(W/cm}^2\text{)}$ per cm length between voltage contacts.

Equation (2.2), however, may only be applied if the heat flux through the transducers is parallel to the z -axis. In practice, this cannot be perfectly satisfied in a multi-component/multi-material device. Lateral heat flows within the sensor induced as a result of its design have two important consequences: I) heat flux components perpendicular to q_z will arise within transducers, thereby reshaping the voltage distribution via the TSE and CSE, and II) the average value of q_z within the transducers will vary compared to the value of q_{in} absorbed by the front surface of the sensor. Generally, the measured voltage V_x will remain proportional to q_{in} , however the responsivity will deviate from Eq. (2.2) due to the augmentation or attenuation of the average value of q_z and the CSE contributions within the transducers. Therefore, it is crucial to consider the impact of the design and materials selection for each component of the sensor on the heat flow in and around the transducers. When considering the as-build sensor holistically,

$$V_x = 52.5\epsilon q_{in}L \quad (3.4)$$

where q_{in} is the uniform heat flux absorbed by the front surface of the sensor and ϵ is the gain, defined as the ratio of the average z -component heat flux within the volume of the transducers to

the absorbed heat flux

$$\epsilon = \frac{\text{mean}(q_z)}{q_{in}} \quad (3.5)$$

The value of ϵ was established using finite element analysis simulations (ANSYS APDL). A model of the sensor was generated utilizing the as-built sensor geometry and the corresponding material properties. The model was subjected to boundary conditions that mimic laboratory conditions, as illustrated in Fig. 3.4a: A uniform heat flux q_{in} was imposed on the front surface of the heat spreader and the back surface of the ceramic housing was held at a constant temperature. Nodal solutions of q_z within the volume of the transducers were extracted from the steady-state thermal analysis solution to determine the gain, with each nodal solution of q_z weighed proportional to the average volume of adjacent elements to account for the non-uniform mesh density. For the as-built sensor, a gain of $0.70 - 0.85$ was found, with the range of values accounting for the uncertainties in the cement properties. As the thermal conductivity of the package and transducer components are similar, the low gain is primarily attributed to the low thermal conductivity of the cement layers on the front and back of the transducers. The streamlines in Fig. 3.4b show the simulated magnitude and direction of the heat flux along the $y - z$ symmetry plane in the center of the sensor. The streamlines deflect from the vertical path to avoid the additional thermal resistance imposed by the excess thickness of the cement layers in the center region. The heat flux entering the transducers is lower (green in Fig. 3.4b) and that entering the housing is higher (orange) compared to the heat flux entering the brass (yellow). Because the streamlines in Fig. 3.4b follow the path of the least thermal resistance, it can be stated generally that a high gain is favored when the ratio between the transducer thermal conductivity and housing thermal conductivity is large, and when the cement thickness is low.

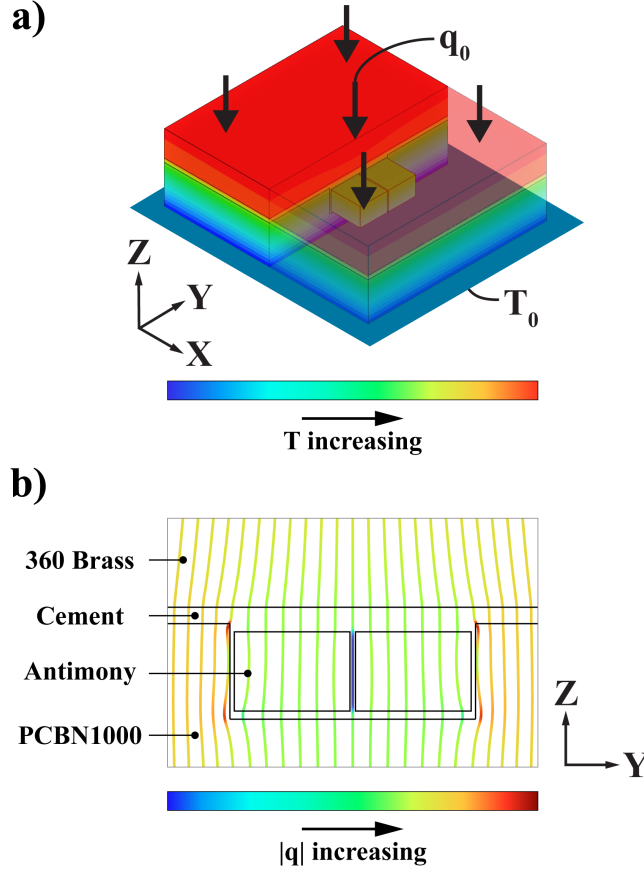


Figure 3.4: Steady-state finite element simulations. a) Temperature contour map of the finite element model. A uniform heat flux q_0 is imposed on the top surface and the bottom surface is held at a constant temperature T_0 . b) Heat flux streamlines on the $y - z$ symmetry plane through the center of the model. The streamline colors represent the magnitude of heat flux. The boundaries between the components are superimposed as black lines.

Taking into account the attenuation of the heat flux through the transducers relative to q_{in} , the responsivity is predicted to be $30 - 37 \mu\text{V}/(\text{W}/\text{cm}^2)$ for a total length of 0.82 cm between voltage contacts (Eq. (3.4)). The measured value of $16.8 \mu\text{V}/(\text{W}/\text{cm}^2)$ is lower, with the decrement attributed to an overestimation of the black paint absorptivity, imperfections in the construction of the sensors, and uncertainty in the Seebeck coefficient tensor components. Contributions from the CSE were ruled out by monitoring the temperature near the transducer and electrical wire junctions using the embedded auxiliary thermocouples. The impact of temperature gradients

along the x -axis were deliberately minimized using two identical transducers, as shown in Fig. 3.2a, where the crystallographic orientations are related by a 180° rotation around the z -axis. As such, q_z -related transverse Seebeck voltages generated in the two prisms add together, while equal and opposite CSE voltages offset, as illustrated in Fig. 3.5 [38, 58].

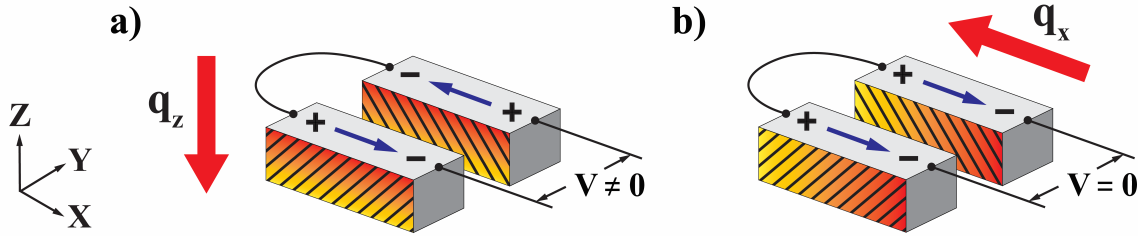


Figure 3.5: Illustration of the polarity of the Seebeck voltage along the x -axis (sign symbols and blue arrows) in a pair of transducers with crystallographic tilt angles of $+\theta$ and $-\theta$ about the y -axis, respectively (parallel hatching). The heat flux is represented by a red arrow and the temperature gradient by a color gradient. a) A heat flux q_z induces a temperature gradient along the z -axis and generates transverse Seebeck voltages with opposite polarities. The voltages add up along the circuit loop. b) A heat flux q_x induces a temperature gradient along the x -axis and generates Seebeck voltages with equal polarities. The voltages offset each other along the circuit loop.

During the calibration campaign, the temperature of the heat flux sensor varied between room temperature and 74°C in response to the large range of heating conditions. Both experimental and theoretical studies have shown that the responsivities of heat flux sensors can depend on the sensor temperature [4, 6, 7, 78]. In these thermopile-based sensors, responsivity values have been shown to vary by 10% [78] and 14% [7] from room temperature to 80°C . In TSE-based sensors, the temperature-dependence of the responsivity contains two main contributions: I) the transducer properties which appear in Eq. (2.2) and II) the gain of the sensor (Eqs. (3.4) and (3.5)). According to literature data, the Seebeck coefficient tensor anisotropy [79] and thermal conductivity [12, 69] of single crystal antimony increase and decrease, respectively, as the temperature increases from room temperature. This contribution is predicted to increase the re-

sponsivity by 12% when the sensor temperature is raised from 19 °C to 74 °C (upon exposure to 7 W/cm², with water cooling), and should result in a nonlinear convex calibration curve. The data in Fig. 3.3, however suggests a much lower variation in the responsivity. This is likely because the increase in the transducer responsivity is offset by a decrease in sensor gain over the same temperature range. The lower gain is a result of a decrease in the thermal conductivity of antimony relative to the thermal conductivity of the ceramic housing, diverting a larger portion of the heat flux away from the transducers.

3.3 Transient Sensor Characterization

3.3.1 Transient Response

The transient response of the heat flux sensor was analyzed by subjecting the sensor to a step change in heat flux. The blackbody furnace, held at 1000 °C, was used as the heat source. The heat flux sensor was positioned in the furnace line-of-sight at a distance of 15 cm. The step change in heat flux was achieved using a reflective shield 7.5 cm from the furnace which could be quickly removed. The sensor voltage and temperature data were recorded at 5 Hz. Following each step change, the signals were allowed to reach steady-state. The shield was inserted and removed four times.

A representative plot of voltage versus time delay for four transients is shown in Fig. 3.6. The time constant τ is defined as the rise time for the sensor to reach 63.2% of the steady-state voltage. The average value of τ for the TSE heat flux sensor was 4.4 ± 0.3 s. Under the same conditions, the time constant of the reference Medtherm 64-3-20 heat flux sensor was 0.45 ± 0.1 s. The time constant was independent of the heat flux.

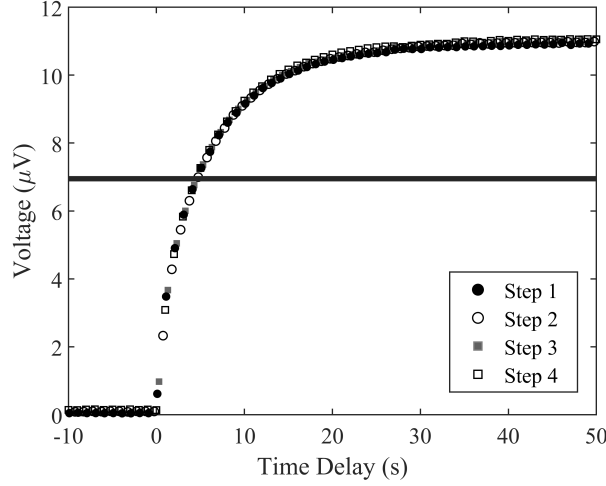


Figure 3.6: Transient response of the antimony-based TSE heat flux sensor to a step change in heat flux. The horizontal line indicates the signal corresponding to 63.2% of the steady-state voltage. The intersection between the voltage transient and the horizontal line was used to determine the time constant of the sensor ($\tau = 4.4$ s).

3.3.2 Analysis of the Sensor's Transient Response

The transient response of a heat flux sensor is dependent on the position of the transducers within the housing and the materials used in the sensor construction. In the TSE heat flux sensor, the transducers are placed behind a high thermal conductivity heat spreader and a low thermal conductivity cement layer, and are separated from the water-cooled heat sink by a ceramic housing. Transient finite element simulations were conducted to evaluate the influence of these components on τ . In these simulations, a step change in heat flux was imposed on the front surface of the heat spreader and a constant temperature was imposed on the back surface of the ceramic housing. The heat spreader characteristics had the largest impact on τ , reflecting observations from prior studies on thin foil gauges [80]. The time constant is linearly proportional to the product of its thickness and specific heat, all other parameters unchanged. The thermal conductivity of the heat spreader has an insignificant effect. The time constant τ is only weakly

impacted by the thermal properties of the cement and ceramic components. With the temperature constrained at the back surface of the lower thermal conductivity ceramic housing, the transient response is dominated by the time needed for the temperature to rise at the front of the sensor. This rise time is governed by the thermal mass of the heat spreader, i.e., the product of its volume, density, and specific heat. As an experimental confirmation, an applied heat flux step of 0.79 W/cm^2 caused the temperature near the front of the transducers to rise by 5.2 K . With a thermal mass per area of 0.714 (J/K)/cm^2 , the heat spreader temperature would increase by 5.2 K in 4.8 s when losses are ignored, in good qualitative agreement with the experimental time constant. If shorter response times are required, the design of the heat flux sensor package can be modified by using a thinner, lower thermal mass heat spreader.

3.4 Sensor Response to Localized Heating

A laser scanning technique was used to stimulate the heat flux sensor with a localized heat probe that produces temperature gradients both normal and parallel to the front surface of the sensor. A 808 nm continuous-wave diode laser with an optical power of 156 mW and a 2 mm beam diameter was utilized as the heat source. The location of the beam incident on the front face of the sensor was controlled using a precision translation stage. The laser beam was scanned at normal incidence across a $10.2 \times 7.6 \text{ mm}$ area near the center of the heat flux sensor. The sensor's steady-state voltage was recorded for 195 discrete locations. Figure 3.7 summarizes the results of the laser scan experiment. Notably, in Fig. 3.7a one can recognize, in the position-encoded response of the sensor, 4 quadrants of alternating high and low voltage values arranged around the center point of the sensor. The localized heat source generates temperature gradients in the

transducers both parallel and orthogonal to the sensor's surface, such that both the CSE and the TSE contribute to the measured voltage. CSE contributions are most prominent when the laser beam is positioned directly over an antimony transducer and tungsten wire junction, with the sign of the CSE voltage contribution dependent on the polarity of the junction (i.e., positive with antimony oriented towards the positive voltage terminal and tungsten towards the negative voltage terminal). This is highlighted in Fig. 3.7b by superimposing the laser scan results over the layout of the transducers and wires. The CSE contributions generate the 4 quadrants of alternating high and low voltage values on top of a gently and radially varying TSE voltage contribution. The positive sign of the measured voltage irrespective of heating probe location indicates that the voltage contributions from the TSE are larger in magnitude than those from the CSE, even in conditions tailored to enhance the CSE contributions. Therefore, these results validate the assertion that the TSE is the dominant transduction mechanism operating in the heat flux sensor under uniform heating conditions (i.e., when the temperature gradients are predominantly normal to the surface of the sensor).

The laser scan data also reveals that, along the symmetry axes of the TSE heat flux sensor, the TSE voltage decreases radially at a slow rate. The voltage decreases from the centroid value by an average of 20% at a distance of 5 mm from the center. By comparison, when performing the laser scan on the surface of the reference Medtherm 64-3-20 heat flux sensor, the voltage decreases by 50% at a distance of 1 mm from its center, and by more than 99.5% at a distance of 3 mm from the center. The design of the TSE heat flux sensor facilitates a high uniformity in the sensor response over an area that far exceeds that of the transducers.

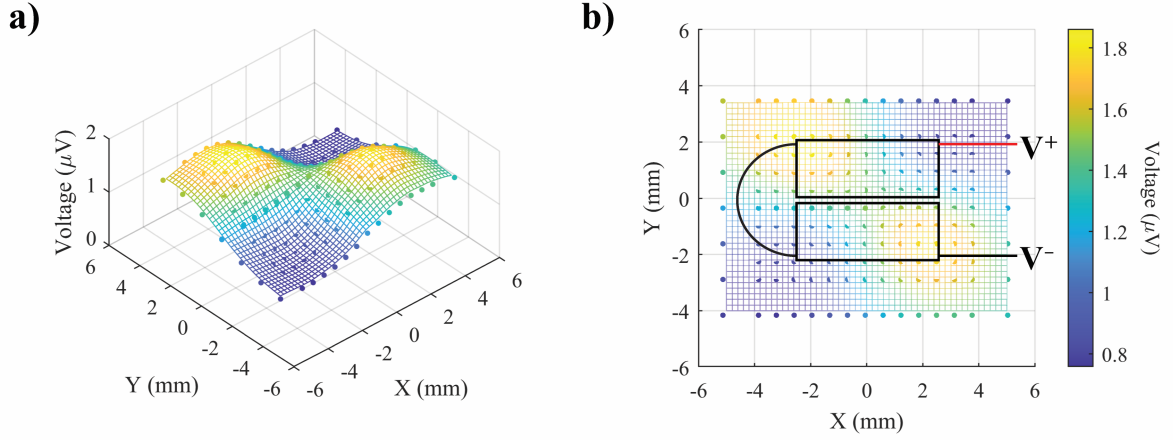


Figure 3.7: Laser scan data. a) Sensor response as a function of the position of the heating laser spot with respect to the center of the heat spreader. b) Outline of the antimony prisms and the tungsten wiring overlaid with the laser scan data. The sensor response is presented using a color scale. Note that all the voltage values are positive.

3.5 Summary

A heat flux sensor based on the transverse Seebeck effect in single crystal antimony demonstrated a linear transduction of the heat flux to an electrical voltage signal in the range of $0.06 - 7 \text{ W/cm}^2$, with even higher heat fluxes possible. The theoretically high responsivity of antimony and its stability over a wide range of environmental conditions make it a promising material for TSE-based sensors. Nevertheless, incorporating the antimony transducers into a rugged sensor package may be necessary, and can have a significant impact on the sensor's performance parameters. The ruggedization of the heat flux sensor achieved design objectives that included improved stability, lower output noise, a larger active sensing area, and a high degree of uniformity of the heat flux through the transducers. These benefits came at the cost of attenuating the heat flux through the transducers and increasing the sensor's time constant. Data from experiments and simulations illustrated how other performance-oriented design criteria could be met,

for example, how to modify the package to produce a higher responsivity, shorten the transients in the response due to a sudden change in input value, or achieve a wider temperature range of operation. The design principles highlighted here are compatible with a wide range of materials and applications that target extreme high temperatures.

Chapter 4: A High-Temperature Heat Flux Sensor Using the Transverse Seebeck Effect in Elemental Rhenium

To advance the capabilities of heat flux sensors towards higher temperatures, novel transducer platforms must be investigated. In Chapter 2, single crystal rhenium was suggested as a potential candidate for high temperature TSE-based heat flux sensors. Rhenium is a refractory metal with extensive heritage in harsh environments, exhibiting one of the highest melting points among the pure elements ($> 3100\text{ }^{\circ}\text{C}$). Critically, rhenium has a hexagonal closed-packed crystal structure with anisotropic Seebeck coefficient tensor components along the in-plane ($\langle 1\bar{1}00 \rangle$ and $\langle 11\bar{2}0 \rangle$) and cross-plane ($\langle 0001 \rangle$) directions, fulfilling the prerequisite for the TSE. Furthermore, rhenium has a unique seebeck coefficient anisotropy that increases with increasing temperature [11]; thus, a sensor based on crystalline rhenium has the potential to become *more* responsive in extreme environment conditions, contrasting the behavior of all high-temperature heat flux sensors surveyed in Chapter 2.

In this chapter, the TSE in single crystal rhenium is demonstrated as a viable transduction mechanism in a robust heat flux sensor fabricated using components with maximum operating temperatures exceeding $1000\text{ }^{\circ}\text{C}$. The responsivity of the sensor is characterized at application-relevant temperatures and compared with predictions using as-built device parameters and literature-based transport properties. The resilience of the sensor is investigated when exposed long term

to temperatures as high as 1000 °C. The transient response of the sensor is evaluated subject to dynamic heating conditions. Lastly, the relative contributions of the TSE and CSE in the sensor are investigated using a laser scan technique, as introduced in Chapter 3.

4.1 Heat Flux Sensor Design and Fabrication

The high-temperature TSE heat flux sensor, shown in Fig. 4.1, consists of two prismatic rhenium transducers, electrical wiring, a ceramic housing, and four auxiliary thermocouples. All components were fabricated using alloys and ceramics with maximum operating temperatures exceeding 1000 °C. The two prismatic single crystals of elemental rhenium (Princeton Scientific), measuring 5.3 x 2.1 x 1.3 mm (L x W x H) and with a crystal inclination of $\theta = -\pi/4$ (following the convention in Fig. 2.4), were secured in the ceramic housing (Aremco Aremcolox Bisque-Fired Alumina 96%) using cement (Sauereisen Tempseal No. 3) (Fig. 4.2). Tungsten-rhenium alloy wires (Omega Engineering W-26Re, 0.25 mm diameter) were spot-welded on the back side of the rhenium transducers, towards the heat sink. Four auxiliary type-K thermocouples (Omega Engineering model 5SRTC-GG-K-30-36-ROHS) were embedded in the sensor near each transducer and electrical wire junction (Fig. 4.2). The front face of the sensor, facing the heat source, was spray-coated with high-temperature black paint (Rust-Oleum model 248903). Following fabrication, the sensor was baked at 300 °C for approximately 1 hour to fully cure the black paint. To avoid adhesives, a front-side bracket (McMaster-Carr, Alloy X) and screws were used to secure the sensor to a heat sink (Fig. 4.3). The complete sensor has a frontal area of 196 mm² and a thickness of 2.1 mm. The two rhenium transducers are connected in series, augmenting the sensor response with a cumulative length of 8.3 mm between welded electrical contacts

(out of a combined transducer length of 10.6 mm). The sensor's voltage signal is collected by connecting the tungsten-rhenium electrical wires to the terminals of a nanovoltmeter (Keithley model 2182A). The auxiliary thermocouples were connected to a data acquisition module (Pico Technology TC-08).

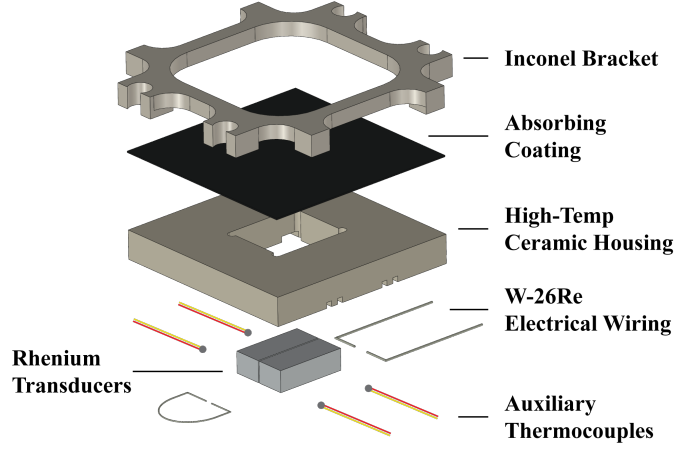


Figure 4.1: Exploded view of the high-temperature TSE heat flux sensor. Two rhenium transducers are connected in series using tungsten-rhenium alloy electrical wiring and mounted in a ceramic package. Four auxiliary thermocouples monitor the temperature near each transducer and electrical wire junction. The front face of the sensor is coated using a high absorptivity black paint. An Inconel bracket is placed over the front of the sensor for mounting to a heat sink.

4.2 Predicted Responsivity

Equation (2.2) provides a means for modeling the responsivity C_{TSE} of the rhenium TSE heat flux sensor when the thermoelectric properties of the transducer material are known. Defining the sensor responsivity as the ratio between the measured voltage V_x and the heat flux absorbed by the front face $-q_z$,

$$C_{TSE} = -\frac{V_x}{q_z} = -\frac{S_{CP} - S_{IP}}{k} L \sin \theta \cos \theta \quad (4.1)$$

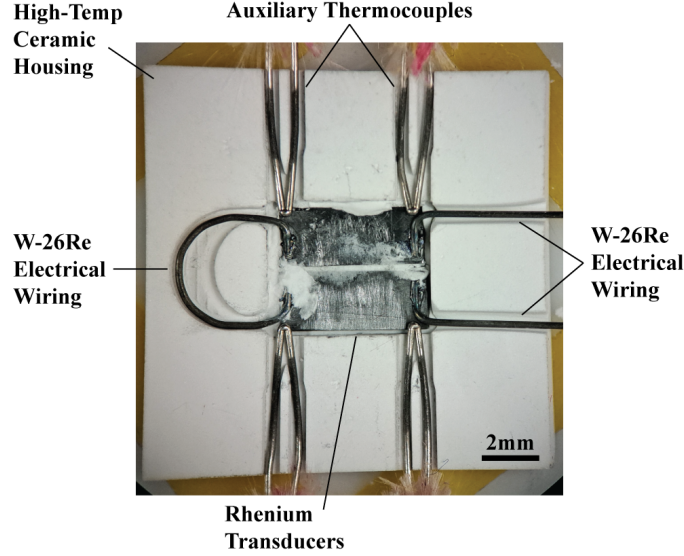


Figure 4.2: Optical image of the back side of the TSE heat flux sensor during assembly, prior to sealing with cement. Visible components include the two transducers, electrical wiring, and thermocouples. Scale bar: 2 mm.

In rhenium, the Seebeck coefficient anisotropy $\Delta S = S_{CP} - S_{IP}$ varies with temperature, decreasing from a value of $1.5 \mu\text{V/K}$ at 25°C to $0 \mu\text{V/K}$ at 275°C , and then to $-8.3 \mu\text{V/K}$ at 1000°C [11]. Between 300°C and 1000°C , the Seebeck coefficient anisotropy decreases (increases in magnitude) linearly with increasing temperature. Over the same temperature range, the thermal conductivity (assumed nearly isotropic) varies between 0.44 and 0.48 W/cm-K [12]. The Seebeck coefficient anisotropy and thermal conductivity of rhenium are shown in Fig. 4.4, normalized by their value at 25°C , ΔS_0 and k_0 , respectively. Figure 4.4 illustrates that the temperature-dependence of the predicted responsivity of the TSE heat flux sensor is dominated by the temperature-dependence of ΔS in rhenium. Applying Eq. (4.1) to the as-built heat flux sensor, with an effective transducer length of 8.3 mm and an orientation of $-\pi/4$, the responsivity of the sensor, shown in Fig. 4.5, is predicted to be $1.3 \mu\text{V}/(\text{W/cm}^2)$ at 25°C , decreasing in value to $-7.4 \mu\text{V}/(\text{W/cm}^2)$ at 1000°C , and changing sign from positive to negative at 275°C .

The uncertainty of the predicted responsivity was estimated to establish upper and lower

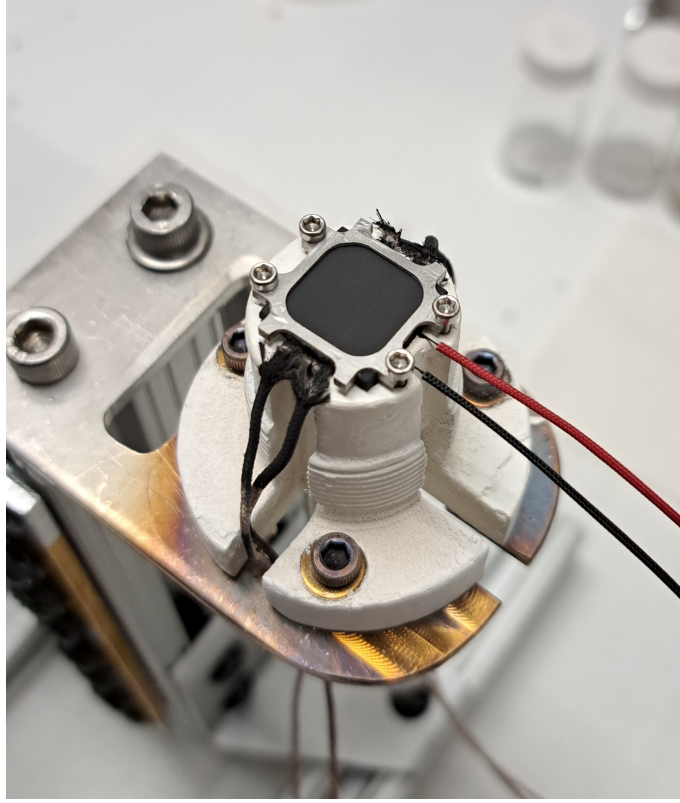


Figure 4.3: High-temperature TSE heat flux sensor mounted to a copper heat sink. The heat sink is spray-coated in boron nitride to inhibit oxidation. The heat sink has integrated coolant channels for water or air (not shown).

bounds for comparison against experimental calibration results (Section 4.3). The total uncertainty σ_t is calculated by propagating the uncertainties of each parameter χ_i in Eq. (4.1) via a root-sum-square

$$\sigma_t = \sqrt{\sum_{i=1}^5 \left[\frac{\partial C_{TSE}}{\partial \chi_i} \sigma_{\chi_i} \right]^2} \quad (4.2)$$

Uncertainty bounds for the five parameters in Eq. (4.1) are tabulated in Table 4.1, where each parameter is assumed to be normally distributed about its mean value. The upper and lower bounds of the predicted temperature-dependent responsivity, representing the nominal value of $C_{TSE} \pm 1\sigma_t$, are shown in Fig. 4.5.

Table 4.1: Uncertainty bounds for the parameters in Eq. 4.1.

Parameter (χ_i)	Uncertainty (σ_{χ_i})	Reference
S_{IP}, S_{CP}	$\pm 0.5 \mu\text{V/K}$	[81]
k	$\pm 5\%$ ($T < 200 \text{ }^\circ\text{C}$) or $\pm 15\%$ ($T \geq 200 \text{ }^\circ\text{C}$)	[12]
θ	$\pm \pi/180$	This work
L	$\pm 0.1 \text{ mm}$	This work

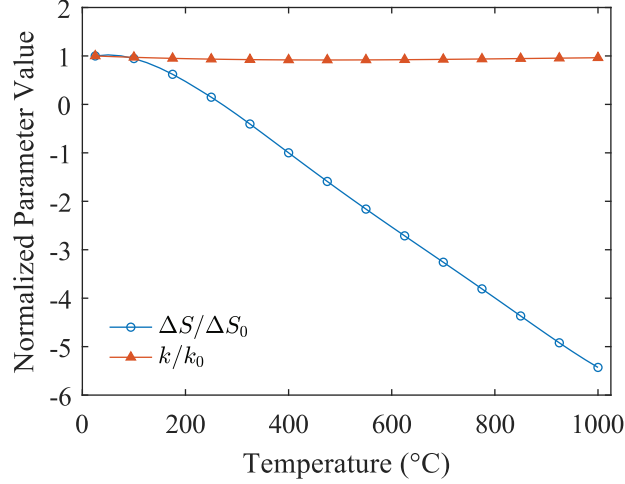


Figure 4.4: Thermoelectric transport properties of rhenium. The Seebeck coefficient anisotropy $\Delta S = S_{CP} - S_{IP}$ [11] and thermal conductivity k [12] are shown relative to their value at 25 °C, ΔS_0 and k_0 , respectively.

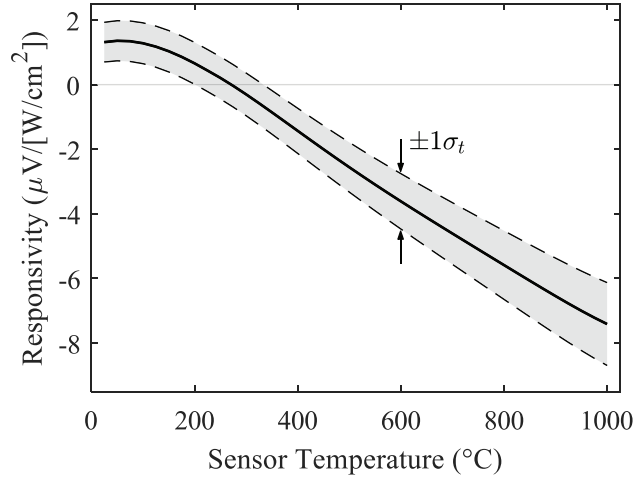


Figure 4.5: Predicted responsivity C_{TSE} of the TSE heat flux sensor from Eq. (4.1) (solid line). The dashed lines represent upper and lower bounds for the predicted responsivity due to the uncertainties of the parameters in Eq. (4.1). The shaded region bounded by the dashed lines captures responsivity values that are within 1 standard deviation of the nominal prediction.

4.3 Temperature-Dependent Sensor Calibration

The responsivity of the rhenium TSE heat flux sensor was characterized from room temperature to 500 °C repeatedly over a period of 9 months, during which the sensor was exposed to temperatures ranging from 20 °C to 1000 °C. Calibrations were performed using a high-temperature calibration facility. The facility, detailed in Chapter 5, uses a transfer calibration technique, where the output of the TSE heat flux sensor is compared to that of a reference Schmidt-Boelter heat flux sensor (Medtherm model 64P-25SB-24K, serial number 234941) with the two exposed to an identical heat flux [73, 74]. The temperature of the TSE heat flux sensor is modulated using a PID-controlled cylindrical band heater while the reference sensor is maintained at room temperature using cooling water. At temperatures above 350 °C, the sensor is maintained under industrial grade nitrogen gas, and above 520 °C under ultra high purity argon. An adjustable high-output infrared lamp (Precision Control Systems Inc. StripIR 5306B, 1000 W) is used to generate varying levels of heat flux. When exposed to the infrared lamp, the TSE heat flux sensor is maintained at a steady temperature, as measured by the auxiliary thermocouples, by controlling the power to the band heater. An additional water-cooled reference plate is used to provide the TSE heat flux sensor with a stable baseline heating condition when positioned out of the line-of-sight of the infrared lamp. The facility can perform accurate calibrations of heat flux sensors from room temperature to 500 °C and, notably, can maintain the sensor temperature to within 1% of the target set point value when exposed to different levels of heat flux.

For each set point temperature, a response curve (voltage vs. heat flux) was generated by exposing both sensors sequentially to four different heat fluxes ranging from 0.5 to 2.5 W/cm². Heat fluxes were calculated from the reference sensor voltage using the manufacturer-supplied

response curve. For each heat flux, the TSE heat flux sensor voltage was taken as the difference in the steady-state output signal between when the sensor is exposed to the infrared lamp and when it is positioned in front of the water-cooled reference plate; by comparing the two states, the output signal due to the exposure to the infrared lamp is isolated (Chapter 5). Response curves for the six sensor temperatures (referred to hereafter as a *calibration sweep*) are shown in Fig. 4.6. Over the tested sensor temperature range, the response is linear from 0 to 2.5 W/cm². The slope of a line fit to each set of calibration data yields the temperature-specific responsivity of the TSE heat flux sensor.

The responsivity of the sensor as a function of temperature is shown in Fig. 4.7 for three calibration sweeps. During the initial calibration sweep, the TSE heat flux sensor exhibited a room temperature responsivity of 1.3 $\mu\text{V}/(\text{W}/\text{cm}^2)$ and a responsivity of -3.2 $\mu\text{V}/(\text{W}/\text{cm}^2)$ at 500 °C, decreasing monotonically with increasing temperature. In the second sweep, a global decrease in the responsivity of the sensor was observed. During the third sweep, the responsivity of the sensor remained unchanged from the second sweep, indicating that the sensor was stable thereafter.

4.3.1 Analysis of the Sensor's Temperature-Dependent Responsivity

In general, the trend of the measured responsivity with increasing temperature is consistent with predictions from Section 4.2, with good quantitative agreement during the first calibration sweep for temperatures up to 400 °C. All calibrations, considering a 95% confidence interval, were within the $\pm 1\sigma_t$ uncertainty bounds of the predicted responsivity. In the latter two sweeps, the measured responsivity values become, overall, more negatively biased than the nominal pre-

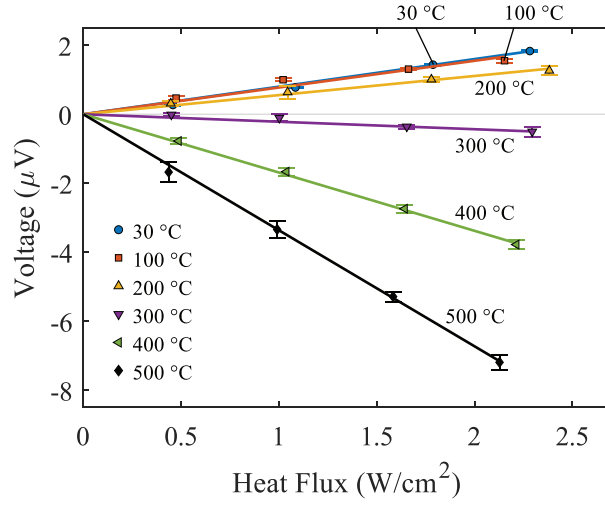


Figure 4.6: TSE heat flux sensor calibration sweep from room temperature to 500 °C. Error bars represent the uncertainty of the voltage measurement multiplied by a coverage factor of 2 ($\pm 2\sigma$).

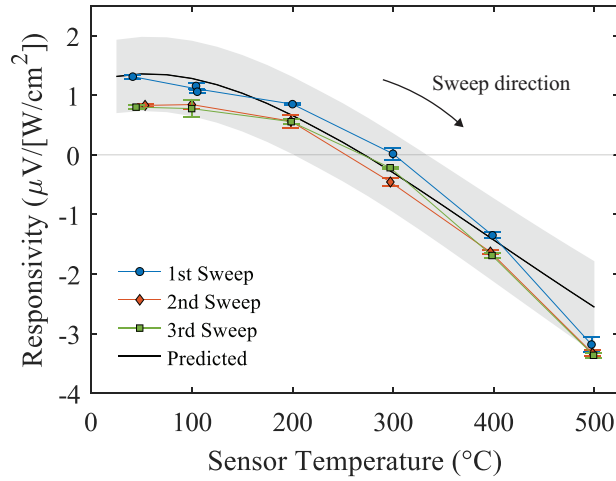


Figure 4.7: Temperature-dependent sensor responsivity from a sequence of calibration sweeps. Error bars represent a 95% confidence interval. The predicted responsivity from Fig. 4.5 is overlaid on the data for comparison. The shaded region represents responsivity values that fall within the $\pm 1\sigma_t$ uncertainty bounds of the predicted responsivity.

dictions. This behavior is explained next by considering the contributions of the CSE to the output voltage.

In the calculation of the predicted responsivity (Eq. (4.1)), it was assumed that the temperature gradient within the transducers was uniform and oriented in the z -direction. This is rarely achieved in practice, as any structural asymmetry, due to variations in the cement thickness, for example, will introduce non-uniform lateral temperature variations in the transducers. These lateral temperature variations may lead to CSE contributions in the output voltage. For two transducers connected in series and arranged such that their crystal orientations are symmetric with respect to a 2-fold rotation about the z -axis (Fig. 4.8), the CSE voltage V_{CSE} is proportional to the net temperature difference between the ends of the transducers ΔT_L

$$V_{CSE} = -(S_{IP} \cos^2 \theta + S_{CP} \sin^2 \theta - S_{wire}) \Delta T_L \quad (4.3)$$

where

$$\Delta T_L = (T_4 - T_3) - (T_1 - T_2) \quad (4.4)$$

In Eq. (4.3), S_{wire} represents the Seebeck coefficient of the tungsten-rhenium alloy wires, measured to be $-0.8 \mu\text{V/K}$ at room temperature. In Eq. (4.4), T_1 through T_4 are the temperatures of the transducer and electrical wire junctions, as labeled in Fig. 4.8. The value of ΔT_L was estimated using measurements from the four auxiliary thermocouples embedded near each transducer and electrical wire junction (Fig. 4.2). For each calibration, the trend of ΔT_L was linear with respect to the absorbed heat flux. In the first calibration sweep, the value of ΔT_L for each measurement was between -0.3 and $-0.47 \text{ }^\circ\text{C}$ per 1 W/cm^2 of absorbed heat flux, except at $500 \text{ }^\circ\text{C}$ (Fig. 4.9).

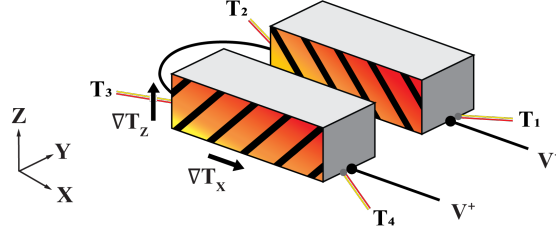


Figure 4.8: Schematic of the transducer circuit subject to both z - and x -oriented temperature gradients. The net temperature difference between the ends of the transducers ΔT_L is estimated using measurements from the four auxiliary thermocouples embedded in the package. The hash marks represent the orientations of the (0001) basal planes in each transducer prism.

During the first calibration at 500 °C, ΔT_L decreased to values between -0.48 and -0.6 °C per 1 W/cm² of absorbed heat flux. The large change in ΔT_L was attributed to the settling of sensor components in the package. In the second and third sweeps, the values of ΔT_L were generally more negative than those during first sweep, but remained narrowly clustered without significant outliers; this suggests no further changes in the sensor during subsequent sweeps. It should be noted that, prior to each sweep, the mounting pressure on the sensor from the Inconel bracket (Fig. 4.3) was adjusted to deliberately minimize the magnitude of ΔT_L . Thus, sweep-to-sweep variations in ΔT_L are attributed more-so to the adjustment of the mounting pressure, rather than changes in the heat conduction within the sensor due to additional settling of components.

From Eq. (4.3), it is anticipated that a decrease in ΔT_L —due to changes in the sensor during the first 500 °C calibration—will lead to a value of V_{CSE} that is more negative, decreasing the total voltage. This trend is reflected in the results shown in Fig. 4.7, where all temperature-specific responsivity values in the second and third sweeps are more negatively biased versus earlier data and, generally, the nominal predictions. Furthermore, the timing of the decrease of ΔT_L provides a reasonable explanation as to why the responsivity at 500 °C changed less from the first sweep to the second sweep versus all other temperatures; during the first 500 °C calibration,

the measured voltage already included the altered CSE voltage contributions.

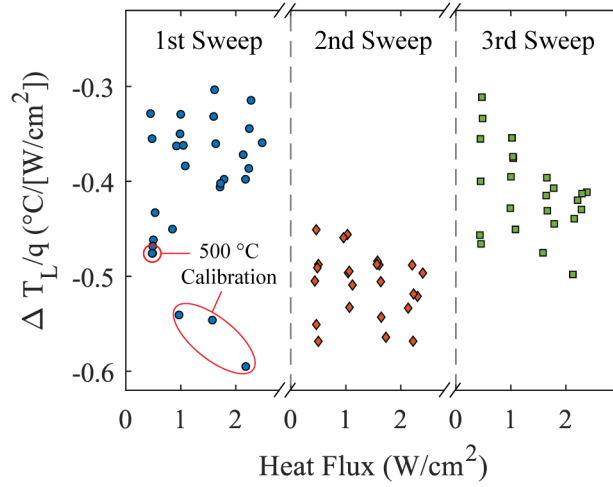


Figure 4.9: Plot of $\Delta T_L/q$ versus absorbed heat flux for each calibration sweep. The 500 °C calibration datapoints are circled for the first sweep.

4.4 High Temperature Resilience

The rhenium-based TSE heat flux sensor is fabricated using high temperature materials with maximum operating temperatures exceeding 1000 °C, however the calibration facility used in this work has an upper limit of 500 °C. To investigate the resilience of the sensor to high temperatures, the sensor was heat treated in a tube furnace at 1000 °C for 10 hours under flow of ultra high purity argon. The furnace was ramped from room temperature to 1000 °C at a rate of 10 °C/min. Following the 10 hour exposure, the furnace was allowed to cool to room temperature prior to removal of the sensor.

The resistance of the transducer circuit at room temperature, measured across 16 cm of electrical lead wire (8 cm per lead), remained the same (1.7 Ω) before and after the heat treatment, suggesting no degradation of the electrical wire and transducer weld joints. However, during the heat treatment, the black paint on the front surface of the sensor degraded significantly, as

shown in Fig. 4.10. Because the surface texture of the paint can alter the sensor's emissivity, the remaining paint was removed and a new coating was applied prior to further testing. Other than the degradation of the black paint, the sensor did not change visually.

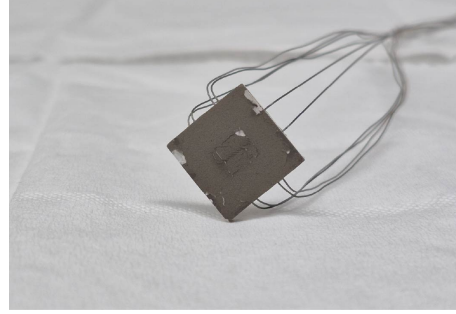


Figure 4.10: Front surface of the rhenium TSE heat flux sensor following heat treatment at 1000 °C for 10 hours under flow of ultra high purity argon.

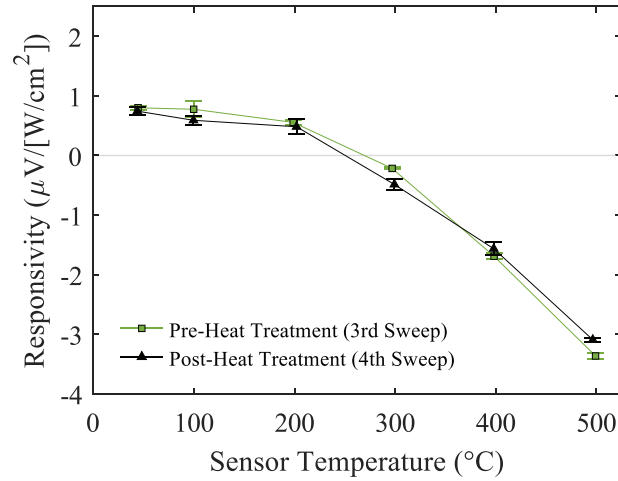


Figure 4.11: Temperature-dependent sensor responsivity of the TSE heat flux sensor following heat treatment at 1000 °C. Error bars represent a 95% confidence interval. Also shown is the calibration data from the third sweep in Fig. 4.7, performed prior to the 1000 °C heat treatment.

The rhenium-based TSE heat flux sensor was re-calibrated from room temperature to 500 °C following the 1000 °C heat treatment. The results of the post-heat treatment calibration are shown in Fig. 4.11, along with the third calibration sweep from Fig. 4.7, performed prior to the heat treatment. The sensor's responsivity is largely unchanged, demonstrating good stability. The

deviations in the post-heat treatment responsivity from the third sweep are less than the initial change in the responsivity during the first 500 °C calibration (Fig. 4.7), indicating minimal changes in the sensor due to further settling of components. These results demonstrate that the TSE heat flux sensor, as a whole, is compatible with temperatures of at least 1000 °C.

4.5 Transient Response

The transient response of the rhenium TSE heat flux sensor was investigated at room temperature subject to dynamic heating conditions. The sensor's time response was recorded for multiple heating scenarios, including a step change in heat flux and a sweep of sinusoidal heat flux profiles with varying frequencies. The former measurement was used to determine the characteristic time constant of the sensor [33, 43, 82]. The latter set of measurements were used to characterize the bandwidth of the sensor via its amplitude–frequency response [37, 49].

4.5.1 Step Heat Flux

The response of the rhenium TSE heat flux sensor was characterized subject to a step change in heat flux. A 808 nm continuous-wave diode laser (Thorlabs model L808P500MM) with a 5 x 5 mm square beam positioned over the centroid of the sensor was used as the heat source. To generate the heat flux step, the optical power of the laser was modulated between 0 and 144 mW as a square waveform with a frequency of 0.05 Hz. The sensor's voltage was amplified by a factor of 1,000 using a nanovoltmeter (Keithley model 2182) and output to a digital oscilloscope (Pico Technology Picoscope) where it was recorded at 300 Hz. A beam splitter was used to divert 10% of the laser power to a photodiode sensor to detect the onset of

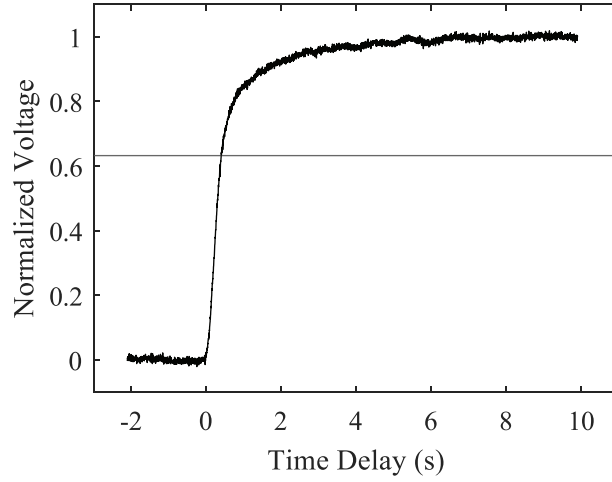


Figure 4.12: Composite transient of the TSE heat flux sensor voltage output in response to a step heat flux. The data represent an average of 36 repeat transients. The horizontal line corresponds to 63.2% of the steady-state voltage.

each heat flux step.

A total of 36 step transients were recorded for the TSE heat flux sensor. All transients were synchronized and averaged together, yielding the composite step response shown in Fig. 4.12. The step response time constant τ_s is defined as the rise time for the sensor to reach 63.2% of the steady-state voltage [33, 43, 82]. From Fig. 4.12, a time constant of $0.4 \text{ s} \pm 0.05 \text{ s}$ is inferred for the TSE heat flux sensor. An identical result (not shown) was obtained by stepping the power down from 144 to 0 mW.

4.5.2 Amplitude Frequency Response

The amplitude–frequency response (AFR) [49] of the rhenium TSE heat flux sensor was characterized. In this test, the response of the sensor is measured subject to sinusoidal heating conditions over a wide range of frequencies. The power output of the 808 nm diode laser was modulated as a sinusoidal waveform with a frequency adjusted from 0.01 to 20 Hz in discrete

steps; for all frequencies, the optical power of the laser was maintained at a constant amplitude of 120 mW peak-to-peak. The amplitude of the TSE heat flux sensor's voltage signal at the frequency of the heat pulse was measured using a lock-in amplifier (Stanford Research Systems model SR810).

The AFR test results for the TSE heat flux sensor are shown in Fig. 4.13. The response amplitude is constant for heat pulse frequencies up to 0.1 Hz and is attenuated by -3 dB at a frequency between 0.5 and 1 Hz, decaying quickly thereafter. The AFR time constant τ_{AFR} is determined by fitting the data using a first-order response model, for which the response amplitude A decays with increasing frequency according to Eq. (4.5) [49].

$$\frac{A}{A_0} = \frac{1}{\sqrt{1 + (2\pi f \tau_{AFR})^2}} \quad (4.5)$$

In Eq. (4.5), f is the frequency of the heat pulse waveform and A_0 is the amplitude of the TSE heat flux sensor response in the limit of $f \rightarrow 0$. An AFR time constant of 0.15 s produces an optimal fit of Eq. (4.5) to the data in Fig. 4.13.

4.5.3 Analysis of the Sensor's Transient Response

The antimony-based TSE heat flux sensor detailed in Chapter 3 had a time constant of 4.4 s, while the rhenium sensor detailed here is faster by an order of magnitude. The fast response time and high bandwidth of the rhenium sensor is attributed to the placement of the transducers close to the surface of the sensor and elimination of any heat spreading components that were included on the antimony-based TSE device.

Considering a transient response that is limited by the thermal inertia of the material in

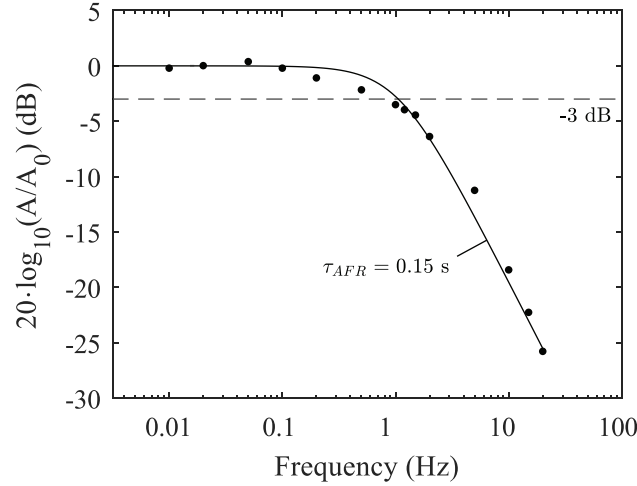


Figure 4.13: Amplitude–frequency response of the TSE heat flux sensor. Voltage amplitudes A are normalized by the sensor’s steady-state output voltage A_0 . The line fit through the data represents a first-order response model with a time constant τ_{AFR} of 0.15 s. The dashed line represents an attenuation of -3 dB.

front of the transducers (here, an approximately 0.2 mm thick cement layer), the theoretical time constant of the rhenium TSE heat flux sensor for a step change in heat flux is predicted to be approximately 0.1 s [80]. This, however, does not consider heat conduction from the cement layer into the transducers, which will prolong the rise time of the sensor’s response to steady-state conditions. Thus, a time constant of 0.1 s is considered a lower bound for the TSE heat flux sensor and in good qualitative agreement with the experimental results.

If a faster response time is desired, the thickness of the cement layer on the front face of the sensor can be reduced, or eliminated altogether. Without the coating layer, the transient response of the TSE heat flux sensor would become dominated by heat conduction within the rhenium transducers [80, 83], and can result in a sensor with a time constant as low as 0.03 s. The coating, however, plays an important role protecting the transducer material from environmental effects (i.e. oxidation, sublimation, and erosion) that can reduce the sensor’s lifespan. Thus, while fast response times can be achieved by tailoring the configuration of sensor components, it

is important to consider the end-use case in the design.

4.6 Sensor Response to Localized Heating

The response of the rhenium TSE heat flux sensor at room temperature to a localized heat source was probed using a laser scanning technique. The technique is similar to that described in Chapter 3, Section 3.4. The localized heat source produces temperature gradients both normal and parallel to the front surface of the sensor to investigate the relative voltage contributions from the TSE and CSE. An 808 nm continuous-wave diode laser (Thorlabs model L808P500MM) with an optical power of 119 mW and a focused 2 mm beam diameter was used as the heat source. The sensor was mounted to an automated translation stage. The laser was scanned at normal incidence across a 12 x 12 mm area around the centroid of the heat flux sensor. The voltage output of the sensor was recorded for 576 discrete locations using a nanovoltmeter (Keithley model 2182).

The results of the laser scan experiment are shown in Fig. 4.14. Consistent with results from the antimony-based TSE heat flux sensor detailed in Chapter 3, the scan produces a voltage profile characterized by alternating quadrants of high and low values superimposed over a positive voltage that decreases radially from the centroid (Fig. 4.14a). The extrema are positioned near the transducer and electrical wire junctions (Fig. 4.14b) and are driven by contributions from the CSE. The sign of the CSE contributions depends on the polarity of the junction (i.e., whether the electrical wire is oriented towards the positive or negative terminal of the nanovoltmeter), yielding the alternating high/low profile in Fig. 4.14a.

With the laser positioned over the centroid of the sensor, maximizing the TSE and minimizing the CSE, a voltage of 0.46 μV was measured. This is a factor of 6 lower than the magnitude

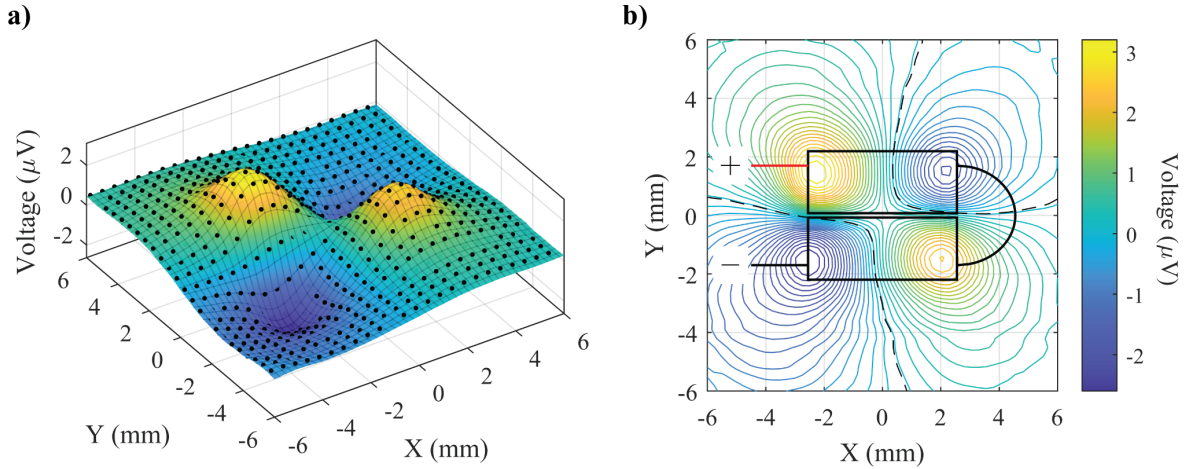


Figure 4.14: Laser scan data. a) Isometric view of the TSE heat flux sensor voltage output as a function of the position of the localized laser heat probe with respect to the centroid of the sensor. b) Contour plot of the TSE heat flux sensor voltage output with the outline of the rhenium transducers and wiring overlaid on top of the data. The dashed contour lines represent a measured voltage of 0 V.

of the maximum and minimum voltage values at the extrema, 3.1 and -2.6 μV , respectively. By comparison, in the antimony TSE heat flux sensor, the voltage was purely positive over the entire scan area, even at the local extrema. There, the data suggested a sensor response that was dominated by the TSE, even when subject to heating conditions that were tailored to elicit a strong CSE contribution. In rhenium, the relatively low Seebeck coefficient anisotropy at room temperature renders the TSE comparatively smaller than the CSE. As such, these results highlight the importance of using at least two transducers arranged in series with 2-fold rotational symmetry (Fig. 4.8); when subject to a lateral temperature gradient that is uniform, the net contributions of the CSE to the output voltage are zero, even though CSE voltages at each junction may be significant. With this configuration, only when non-uniform temperature gradients are present in the sensor, as was investigated in Section 4.3.1, do CSE voltages begin to contribute to the output voltage.

4.7 Summary

A high-temperature heat flux sensor was developed using the transverse Seebeck effect in single crystal rhenium. The responsivity of the sensor is predicted to more than quintuple in magnitude from 25 °C to 1000 °C, from 1.3 $\mu\text{V}/(\text{W}/\text{cm}^2)$ to -7.4 $\mu\text{V}/(\text{W}/\text{cm}^2)$ with a sign change at 275 °C, respectively, owed to the unique temperature-dependent behavior of rhenium's Seebeck coefficient tensor components. The temperature-dependent responsivity of the sensor was characterized from room temperature to 500 °C and was found to be in good agreement with analytical predictions. Over repeat calibration sweeps, the sensor's responsivity demonstrated good reproducibility following an initial exposure to 500 °C, highlighting the stability of the sensor once properly cured at application-relevant temperatures. The sensor was heat treated at 1000 °C for 10 hours in argon to investigate its resilience to high temperatures. Following the heat treatment, the sensor's responsivity remained unchanged from the pre-heat treatment calibration, demonstrating good compatability of the sensor with extreme thermal environments. The sensor exhibits a time constant of 0.4 s when exposed to a step increase in heat flux and can resolve dynamic heating conditions with frequencies up to 0.1 Hz without attenuation of its output signal, adequate for resolving long time-scale transient heating phenomena during high-speed flight and dynamic energy production.

Chapter 5: Development of a High-Temperature Heat Flux Sensor Calibration Facility

Thermoelectric-based heat flux sensors exhibit responsivities that vary strongly with sensor temperature [4, 28]. In the rhenium-based TSE heat flux sensor detailed in Chapter 4, the magnitude of the responsivity is predicted to grow by a factor of five from room temperature to 1000 °C. This strong temperature dependence is in no way unique to the sensor platform studied in this thesis; in fact, all high-temperature heat flux sensors reported in the literature have responsivities that vary with temperature [6, 7, 54]. This general behavior is due to the strong temperature-dependence of the transport properties of thermoelectric materials, i.e. the Seebeck coefficient and thermal conductivity, which directly influence the sensor's responsivity via Eqs. (2.1) and (2.2). To minimize measurement errors in extreme environments, it is, thus, essential that sensors are calibrated over a temperature range relevant to the sensing application.

This chapter details the development of a calibration facility to enable accurate, temperature-dependent heat flux sensor calibrations of the rhenium-based sensor developed in this thesis. A background on heat flux sensor calibration methodologies is given first, with an emphasis on temperature-dependent calibration setups to motivate the design of a new type of facility. The calibration system is detailed in subsequent sections, highlighting the various subsystem and infrastructure components used to increase the quality of sensor calibrations. Setup considerations

and testing protocols are introduced to reduce the interactions between the sensor and ambient environment. Guidelines are established for the sensor calibration procedure and data reduction process, including quantification of the measurement uncertainty using continuously measured output data from the sensor. Through characterization of the calibration system, setup tolerances are developed to minimize measurement errors.

5.1 Heat Flux Sensor Calibration Background

In the calibration of a heat flux sensor with an unknown responsivity (referred to hereafter as a *prototype sensor*), the sensor's voltage is measured while exposed to a known heat flux. Heat flux sensor calibration methodologies fall under one of two categories [73, 74]: I) absolute calibration or II) transfer calibration. In the absolute calibration approach, the calibration heat flux is either characterized a priori or calculated using the temperature and emissivity of the heat source via the Stefan-Boltzmann equation. In the transfer calibration approach, the calibration heat flux is measured in situ using a reference standard with the prototype and reference heat flux sensors exposed sequentially to identical heating conditions. Generally, transfer calibrations lead to more accurate calibrations with higher reproducibility, as they are less susceptible to off-nominal variations in the heating conditions and errors in the assumptions used to calculate the heat flux emitted from the source [74].

To perform temperature-dependent heat flux sensor calibrations, both the sensor temperature and calibration heat flux must be modulated in some fashion. Pullins and Diller have reported on the development of multiple high-temperature calibration methodologies, both absolute and transfer, in which the calibration heat source is used directly to modulate the temperature of a

prototype sensor [6, 30]. In the wide angle radiation calibration system [6], a prototype sensor is embedded in a temperature-controlled hot plate, adjacent from a water cooled reference sensor. During calibration, the voltage of the prototype sensor is measured while the reference sensor measures the radiative emission from the prototype sensor at different hot plate temperatures. At each temperature, the responsivity of the sensor is determined as the ratio of the prototype sensor voltage to the heat flux emitted from the sensor surface. A similar concept is executed in the in-cavity calibration system [30], where a prototype sensor is inserted directly into the cavity of a temperature-controlled graphite furnace. The prototype sensor is air-cooled to maintain it at a lower temperature than the furnace, thus creating a non-zero heat flux at the sensor surface. The heat flux is estimated by calculating the net surface energy balance at the sensor surface, accounting for losses due to convection and radiative emission. By modulating the temperature of the furnace, the prototype sensor temperature and heat flux are directly increased or decreased. Both methods are simple, and have been shown to generate reproducible calibrations of high-temperature heat flux sensors up to 900 °C [7]. However, due to the strong coupling between the sensor temperature and calibration heat source, only one heat flux set point can be accessed per sensor temperature. Thus, these methods are susceptible to instrumentation bias errors and large uncertainties at low sensor temperatures, where the both the calibration heat flux and prototype sensor voltage are weak.

Preferably, temperature-dependent calibrations should be performed through independent modulation of the sensor temperature and calibration heat flux. The primary advantage of this approach is that multiple calibration heat flux set points can be accessed for a given sensor temperature, which allows the sensor's responsivity to be derived via linear regression, reducing the influence of measurement bias errors. Miller and Alpert [4] have reported on the use of an en-

vironmental chamber to independently control the temperature of a prototype heat flux sensor during calibration at sub-ambient temperatures, where an adjustable infrared lamp housed within the chamber was used as the calibration heat source. A similar concept has been demonstrated by Chen *et al.* [54], using a muffle furnace to modulate the sensor temperature up to 1000 °C. In this approach, a laser directed through a small aperture in the furnace wall was used as the calibration heat source. In both setups, a reference heat flux sensor maintained at room temperature was used to measure the heat flux from the calibration heat source; thus, they are both considered transfer calibration techniques. These facilities, however, only maintain control of the environment surrounding the heat flux sensor, rather than the temperature of the sensor itself. Consequently, when exposed to large heat fluxes over a significant time period, the temperature of the sensor may deviate from the target temperature. For the rhenium-based TSE heat flux sensor developed in this thesis, which has a responsivity that exhibits a strong temperature-dependence, a calibration facility with direct control of the prototype sensor temperature is desired.

5.2 High-Temperature Calibration Facility Design Motivation

As defined in Chapters 3 and 4, the responsivity C of a heat flux sensor at a constant temperature is the ratio of the sensor voltage V to the absorbed heat flux q

$$C = \frac{V}{q} \quad (5.1)$$

In room-temperature transfer calibration facilities, q is measured directly using a reference heat flux sensor. However, when calibrating a heat flux sensor at elevated temperatures, the additional heating contributions from radiative emission, hot calibration system components, and convective

heat transfer with the ambient environment must also be considered; critically, the heat flux measured by the reference sensor, which must often remain at room temperature, will not be an accurate representation of the heat flux absorbed by the prototype sensor.

For high-temperature calibrations, it becomes more tenable to calculate the responsivity as the change in voltage with respect to the change in heat flux between two states

$$C = \frac{V_{LOS} - V_0}{q_{LOS} - q_0} \quad (5.2)$$

In Eq. (5.2), quantities subscripted with *LOS* are defined with the prototype sensor positioned in the line-of-sight of a calibration heat source and quantities subscripted with 0 are defined with the prototype sensor exposed to a baseline condition. The main advantage of Eq. (5.2) is that, if the prototype sensor is maintained at the same temperature between the line-of-sight and baseline states, contributions from radiative emission and convection to q_{LOS} and q_0 are identical; as such, when subtracting q_0 from q_{LOS} , these contributions are negated entirely. Thus, the denominator in Eq. (5.2) is representative of the heat flux measured by the reference sensor, even when the two sensors are maintained at different temperatures. Considering this, a high degree of temperature-dependent calibration accuracy can be obtained through I) active control of the prototype sensor temperature when exposed to different heating conditions and II) exposure of the prototype sensor to a constant baseline heating condition for each calibration heat flux. These criteria drive the design and configuration of the high-temperature calibration facility.

Table 5.1: Summary of commercially-sourced calibration system components.

Component	Manufacturer	Model
IR Lamp	Precision Control Systems inc.	StripIR 5306B-02
IR Lamp Controller	Precision Control Systems inc.	5420-120
Nanovoltmeter	Keithley	2182A
Thermocouple DAQ	Pico Technology	TC-08
Band Heater	Tempco	MPP51101
Band Heater Controller	Tempco	TPC10063
Reference Heat Flux Sensor	Medtherm	64P-25SB-24K
Radiometer Window	Pelco	16005-1020

5.3 Calibration System Overview

The heat flux sensor calibration facility developed for the rhenium-based sensor reported in this thesis is shown in Fig. 5.1 and represented schematically in Fig. 5.2. The system consists of an adjustable 1 kW infrared (IR) lamp and controller, water-cooled reference plate, sensor rack, nanovoltmeter, thermocouple DAQ, band heater temperature controller, and infrastructure for supplying water, air, or nitrogen to various components. The sensor rack (Fig. 5.3) contains mounting interfaces for both the prototype and reference heat flux sensors, as well as the band heater and other components used to control the temperature of the prototype sensor. The electrical leads of both sensors are connected to separate channels in the nanovoltmeter (Fig. 5.2a). The free ends of the thermocouples from both sensors are connected to separate channels in the thermocouple DAQ (Fig. 5.2a). Data from the nanovoltmeter and thermocouple DAQ are recorded digitally using a custom LabView program. Details for each commercially-sourced component are tabulated in Table 5.1.

The prototype sensor is mounted to the sensor rack via a copper heat sink with internal channels for cooling water or air (Fig. 5.4). The copper heat sink is spray coated in boron

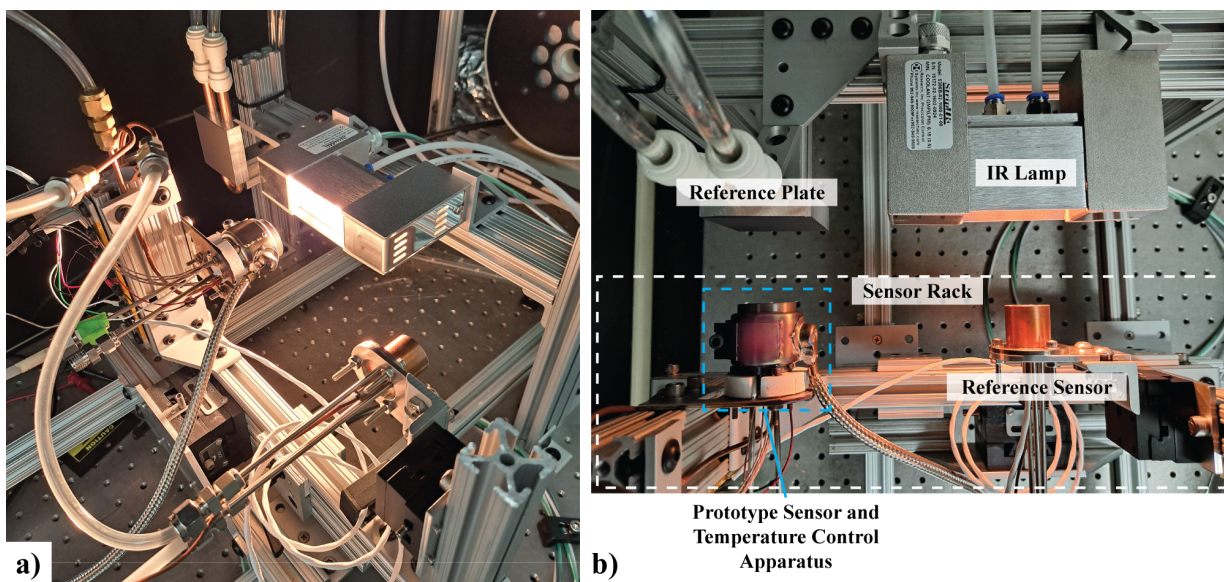
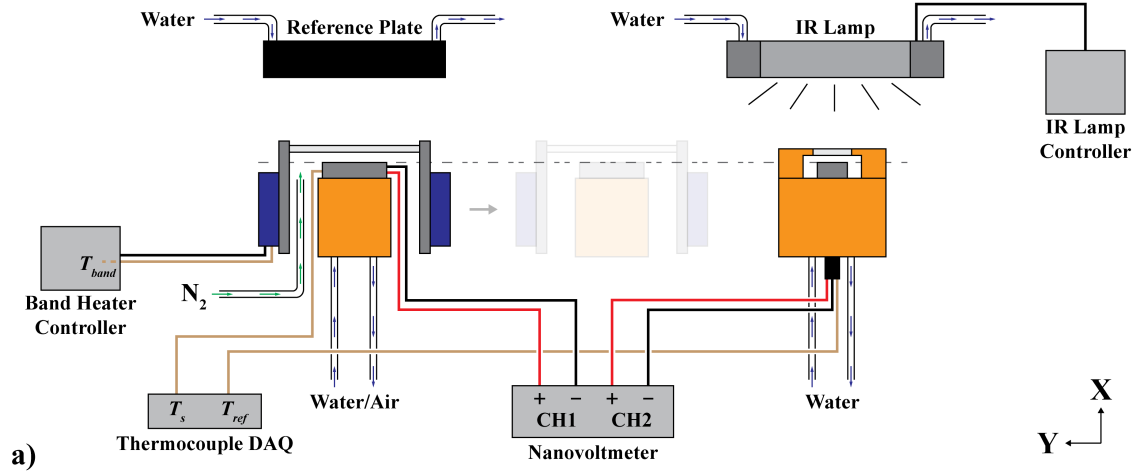


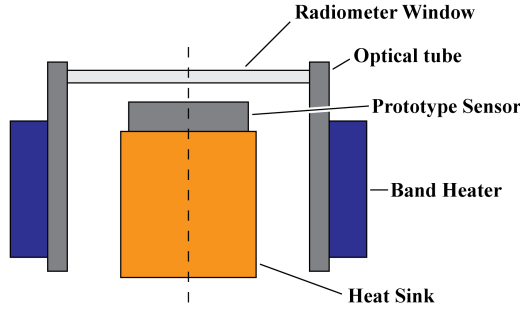
Figure 5.1: Optical image of the high-temperature calibration facility. a) Isometric view of the calibration system with the prototype sensor in the heat source line-of-sight. b) Top-down view of the calibration system with the prototype sensor positioned in front of the water-cooled reference plate. The reference sensor and prototype sensor are mounted to the sensor rack, which simultaneously controls the positions of both sensors relative to the heat source.

nitride to inhibit oxidation at high temperatures (See Fig. 4.3 from Chapter 4). The heat sink is attached to the sensor rack using a machined 316 stainless steel bracket. The band heater is mounted to a 316 stainless steel optical tube that screws directly onto the copper heat sink (Fig. 5.4). Together, the copper heat sink, optical tube, and band heater form the *temperature control apparatus*. The optical tube can accommodate radiometer windows 1 inch in diameter. When used during calibration, the radiometer window and optical tube assembly create a cavity in front of the sensor, in which an inert gas (e.g., nitrogen or argon) can be flowed to prevent oxidation of sensor and calibration system components (Fig. 5.2a). Detailed drawings for the heat sink, heat sink mounting bracket, and optical tube are included in Appendix B.

The reference standard used for calibrations is a water-cooled Medtherm 64-series Schmidt-Boelter heat flux sensor (Medtherm 64P-25SB-24K, SN 234941) with an added sapphire ra-



Prototype Sensor Configuration



Reference Sensor Configuration

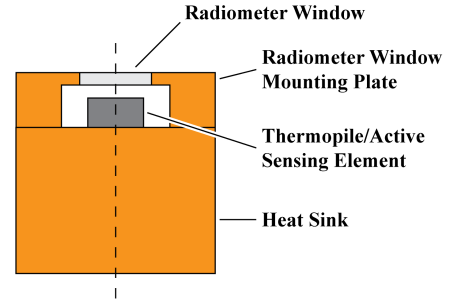


Figure 5.2: Schematic of the high-temperature calibration facility. a) Network diagram of the calibration facility. The electrical leads of each sensor are connected to separate channels of a nanovoltmeter. The free ends of thermocouples embedded in each sensor are connected to separate channels of a thermocouple DAQ. The band heater surrounding the prototype sensor is connected to a PID controller. Within the prototype sensor heat sink, cooling channels are available for water or air. An additional channel is used to flow nitrogen directly into the optical tube and radiometer window cavity. The reference sensor, which must remain near room temperature during calibration, has channels for cooling water. The sensors are exposed to an IR lamp, which is controlled using a standalone controller. The IR lamp has channels for cooling water to prevent overheating. When out of the heat source line-of-sight, the prototype sensor is exposed to a water-cooled reference plate. b) Section-view schematic of the prototype sensor and temperature control apparatus configuration. c) Section-view schematic of the reference sensor configuration. The reference sensor assembly, including heat sink, thermopile, temperature-sensing thermocouple, and radiometer window, is purchased as a single unit from the manufacturer.

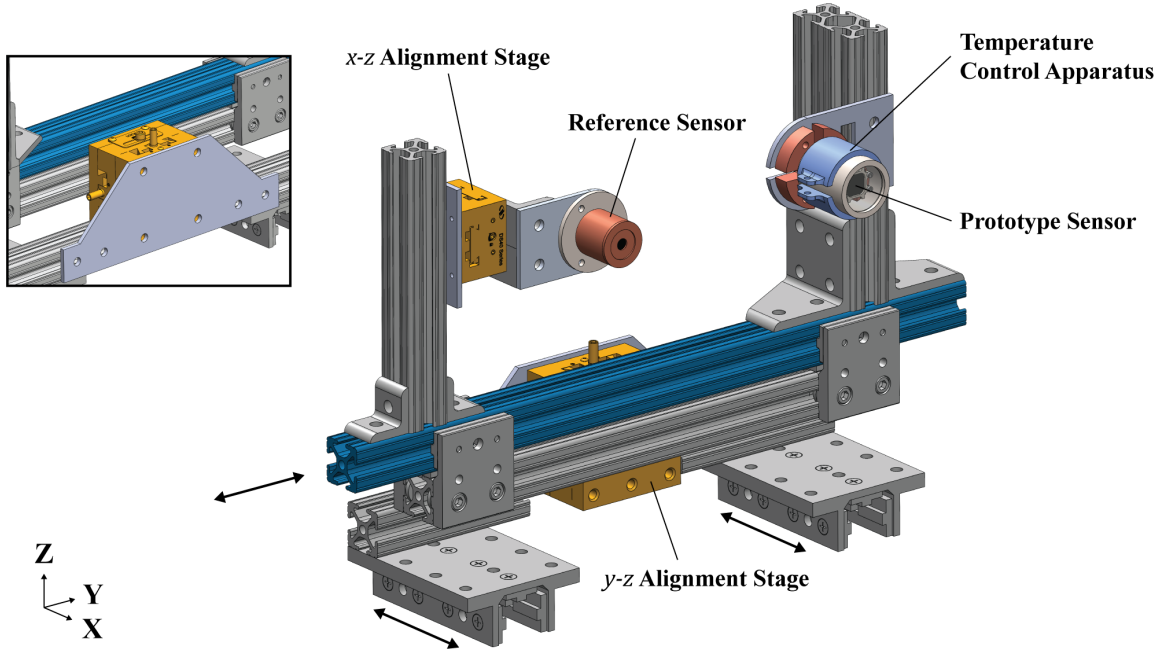


Figure 5.3: Sensor rack configuration. The prototype sensor and temperature control apparatus are mounted to the sensor rack using a stainless steel bracket. The reference sensor is mounted to the sensor rack using a precision alignment stage for alignment with the prototype sensor in the x - and z -directions. The sensors are mounted to a sliding rail (colored in blue) to allow for positioning in and out of the heat source line-of-sight. An additional precision alignment stage with adjustments in the y - and z -directions allows for simultaneous adjustment of both sensors relative to the heat source. The sensor rack is mounted to sliding rails aligned with the x -axis (not shown) to adjust the heat source-sensor distance. Inset: view of the backside of the $y - z$ alignment stage.

diameter window attachment (Fig. 5.2c). The sensor has a heat flux measurement range from 0 to 28 W/cm² and was procured with room-temperature calibration data from the manufacturer both with and without the sapphire radiometer window installed. Both calibrations are traceable to NIST standards. The reference sensor is mounted to the sensor rack on a 2-axis precision translation stage with degrees of freedom in the x - and z -directions for alignment with respect to the prototype sensor (Fig. 5.3). The sensors are mounted to an additional 2-axis precision translation stage on the sensor rack that adjusts the position of both sensors simultaneously in the y - and z -directions for alignment with respect to the heat source. The sensor rack is mounted on

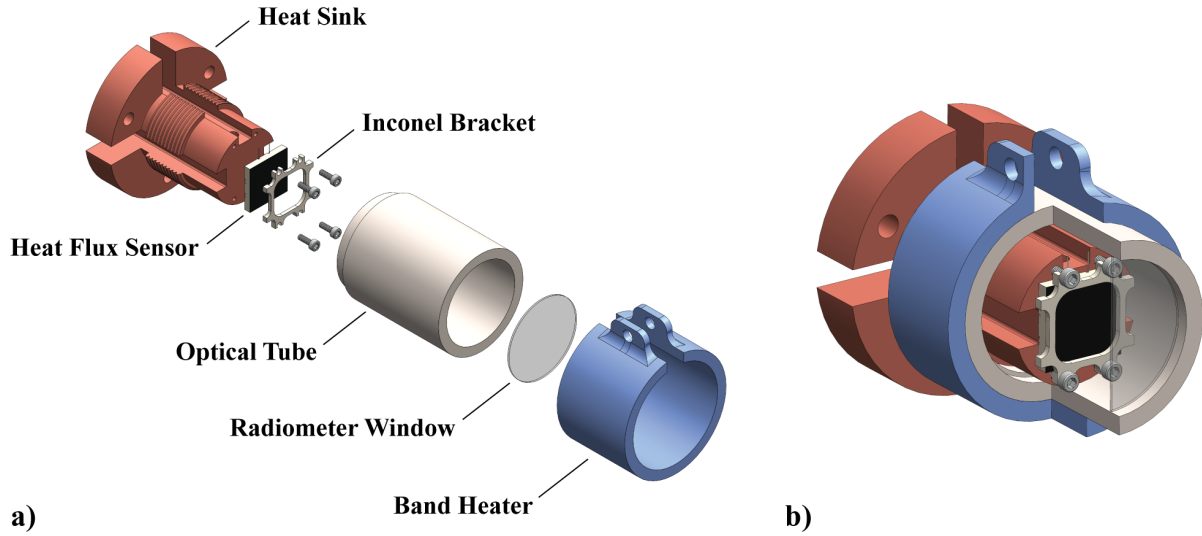


Figure 5.4: Prototype sensor and temperature control apparatus. a) Exploded view of the temperature control apparatus. A band heater is attached over a stainless steel optical tube, which threads directly onto the heat sink. The optical tube can accommodate radiometer windows 1 inch in diameter. b) Cut-away view of the temperature control apparatus.

sliding rails oriented in the x -direction to control the distance between the heat source and the sensors (referred to as the *heat source-sensor distance*). An orthogonal sliding rail in the sensor rack, oriented in the y -direction, is used to sequentially position each sensor into the heat source line-of-sight during calibration procedures.

5.4 Setup Considerations and Testing Protocols

The high-temperature calibration facility introduces multiple system components, controls, and process steps that are not normally considered in heat flux sensor calibrations. This includes the temperature control apparatus and its configuration for each test, the radiometer window and optical tube assembly, and the water-cooled reference plate. Each component and subsystem serves to improve the accuracy of heat flux sensor calibrations at elevated temperatures, either by increasing controllability of the sensor temperature, or by mitigating interactions between the

sensor and ambient environment. In this section, the unique testing protocols and setup considerations for the high-temperature calibration system, including design rationales, are detailed.

5.4.1 Relative Sensor Alignment

The heat flux measured by each sensor will be a strong function of the position of the sensor with respect to the heat source. Alignment of the prototype heat flux sensor with respect to the reference sensor is, thus, crucial to ensure that each sensor is exposed to the same heat flux during calibration. Specifically, the absorbing surfaces of the active sensing regions of both sensors (Figs. 5.2b and 5.2c) should be at the exact same position when positioned in the heat source line-of-sight.

Alignments are performed for all 3 axes. For the y - and z -axes, alignment targets are used to determine the positions of each sensor relative to a laser spot that remains at a fixed location (Fig. 5.5). The laser spot is initially aligned with the centroid of the prototype sensor in the line-of-sight position (Fig. 5.5a). The y -position of the reference sensor when in the line-of-sight position (Fig. 5.5b) is adjusted using a limit screw on the sensor rack translation stage (colored blue in Fig. 5.3). The z -position of the reference sensor is adjusted using the corresponding lead screw on the reference sensor precision translation stage.

The x -axis alignment of the sensors is considerably more challenging. This is because the front surface of the active sensing element of the reference sensor is recessed from the front surface of the radiometer window mounting plate, which is not easily removed (Fig. 5.2c). The alignment is first performed by adjusting the x -axis lead screw on the precision translation stage of the reference sensor until the front surface of the radiometer window mounting plate is flush

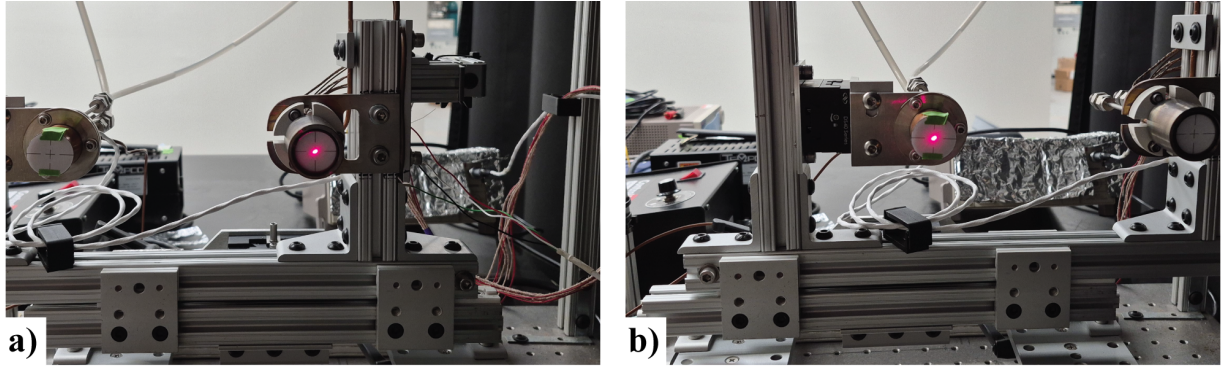


Figure 5.5: Relative alignment of the prototype and reference heat flux sensors. Alignment targets are used with a stationary laser to determine the position of both sensors when positioned in the heat source line-of-sight. a) Prototype sensor positioned in the heat source line-of-sight. b) Reference sensor positioned in the heat source line-of-sight.

with the front absorbing surface of the prototype sensor. This is verified by removing the optical tube from the temperature control apparatus and placing a soft datum, as to not damage the absorbing coating on the prototype sensor, in contact with the front of the sensors in the heat source line-of-sight. To correct for the recessed position of the active sensing element, the precision translation stage is adjusted by 3.3 mm (distance between the front of the active sensing element and radiometer window mounting plate, measured with the radiometer window mounting plate removed) in the x -direction, corresponding with 10.5 counterclockwise turns of the lead screw.

5.4.2 Temperature Control

Active control of the prototype sensor temperature during calibration is essential to ensure that the sensor remains close to the target temperature when exposed to a large range of heating conditions. The temperature control apparatus is designed to modulate, and maintain, the sensor temperature at any value between room temperature and 500 °C. For high temperature calibrations (≥ 150 °C), the power delivered to the band heater is modulated via control of the band heater temperature set point to maintain the sensor temperature at the target value.

For calibrations at sensor temperatures below 150 °C, but above room temperature, a combination of band heater temperature control and air cooling of the heat sink is required. This is because, at high heat flux values ($> 1 \text{ W/cm}^2$), the heat soak from the calibration heat source will cause the temperature of the sensor to increase beyond the target temperature, unless an additional dissipation mechanism is used. Control of the sensor temperature via the band heater should be used first. When the sensor temperature can no longer be maintained at the target set point, even when the band heater controller is not delivering power to the heater, air cooling must be used. When air cooling the heat sink, the inlet air pressure is adjusted in the range of 5-30 psi using an in-line regulator until a stable temperature at the target set point is achieved.

For calibrations at room temperature, the heat sink is water-cooled and the band heater is not used. Generally, the maximum flow rate of water that can be achieved, typically in the range of 2.9-3.5 mL/s, should be used to prevent the sensor temperature from increasing much higher than room temperature. A summary of the temperature control apparatus configurations for desired calibration conditions is given in Table 5.2.

Table 5.2: Temperature control apparatus configurations.

Prototype Sensor Temperature	Temperature Control Method
Room temperature	Water cooling
50 - 150 °C	Band heater/air cooling
150 - 500 °C	Band heater

5.4.3 Mitigation of Convective Heat Transfer During Calibration

During calibration, radiometer windows of the same type (e.g., sapphire) and thickness, if available, are used in between both sensors and the calibration heat source. The primary function

of the radiometer windows is to remove the effects of convective heat transfer between the sensors and the ambient environment. With the prototype sensor at an elevated temperature and the reference sensor at room temperature, convective heating contributions could lead to large disparities between the heat flux absorbed by each sensor. While these effects, ideally, are minimized through the use of a baseline measurement (Eq. (5.2)), this assumes that the surrounding ambient environment remains constant when positioned in and out of the heat source line-of-sight. By using a radiometer window installed in the stainless steel optical tube, the gaseous environment surrounding the prototype sensor in the cavity is at a similar temperature as the sensor and temperature control apparatus; thus convective contributions are largely diminished altogether. The radiometer window in front of the reference sensor ensures that the heat flux incident on the active sensing region of each sensor from the calibration heat source is identical.

5.4.4 Baseline Heat Flux Measurement

Accurate calibration of a heat flux sensor using Eq. (5.2) requires two well-defined heating conditions, one for the line-of-sight measurement and one for the baseline measurement. In room-temperature calibration facilities, a baseline heat flux measurement can be achieved with the sensor positioned far out of the line-of-sight of the heat source, where the sensor is at thermal equilibrium with the surrounding environment. In the high-temperature calibration facility, however, this is not enough to guarantee an accurate baseline measurement, as the sensor and temperature control apparatus will be radiating strongly, heating the surrounding environment. To achieve a stable, recurring baseline heating condition, the sensor surroundings must be designed intentionally. A large water-cooled reference plate with a high emissivity coating is, thus,

used to provide a reliable baseline heating condition when measuring q_0 .

5.5 Calibration Procedure

Sensor calibrations are performed by measuring the response of the prototype sensor to varying heat fluxes, with the sensor held at a constant temperature. Prior to the measurement, the temperature of the prototype sensor is increased to the target value by adjusting the set point temperature of the band heater. Once steady conditions are achieved (~ 30 to 40 minutes), the prototype sensor is positioned in front of the water-cooled reference plate and the reference sensor is placed in the line-of-sight of the heat source (Fig. 5.1b). The sensor rack is positioned at the desired heat source-sensor distance and locked in place.

During the calibration measurement, the prototype and reference sensors are exposed sequentially to the same heat flux. For each heat flux set point, the baseline measurement is performed first. In this position, the baseline measurement for the prototype sensor is performed simultaneously with the measurement of the calibration heat flux via the reference sensor. The input power to the IR lamp is adjusted to the desired set point value and the output of the lamp is allowed to reach steady-state (~ 60 s). The voltage and temperature of each sensor are continuously recorded at a rate of 1 Hz. After a steady voltage signal is achieved for the reference sensor, the system is held at steady conditions for an additional 60 to 300 s. Although not a strict requirement, the longer waiting period is recommended to allow for identification of drifts in the prototype sensor's baseline voltage (see Section 5.7.2). After the steady-state waiting period, a 10 -second average voltage measurement is recorded for each sensor. The prototype sensor is then positioned in the heat source line-of-sight. To maintain the sensor at the target temperature, the

band heater set point temperature is adjusted downwards. As mentioned in Section 5.4, the use of air cooling may also be necessary if calibrating the prototype sensor at a temperature $< 150\text{ }^{\circ}\text{C}$. Once both the temperature and voltage of the prototype sensor have reached a steady state, the system is held at steady conditions for at least 60 s. After 60 s, a 10-second average voltage measurement is recorded. The prototype sensor is then repositioned in front of the water-cooled reference plate. To prevent deviations in the prototype sensor temperature from the target value, the band heater set point temperature is adjusted back upwards. The measurement process is repeated for different heat fluxes, each time allowing the temperature of the prototype sensor to reach a steady state at the target value, including when positioned in front of the water-cooled reference plate.

5.6 Heat Flux Calculation and Band Heater Correction

The reference sensor measures the heat flux incident on the sensors from the calibration heat source. During calibration, the prototype sensor is exposed to additional radiative heating from a small section of the hot optical tube and radiometer window positioned in front of the sensor (Fig. 5.4). As the prototype sensor is positioned in and out of the heat source line-of-sight, the temperature of the band heater is adjusted to maintain the sensor at the desired temperature. As a result, the temperature of the optical tube and radiometer window varies between each position, which alters the additional heat flux incident on the prototype sensor. To account for this variation, the heating contributions from the optical tube and radiometer window components are estimated for each sensor position, and used to more thoroughly calculate the actual heating conditions for Eq. (5.2).

5.6.1 Analytical Heat Flux Calculation

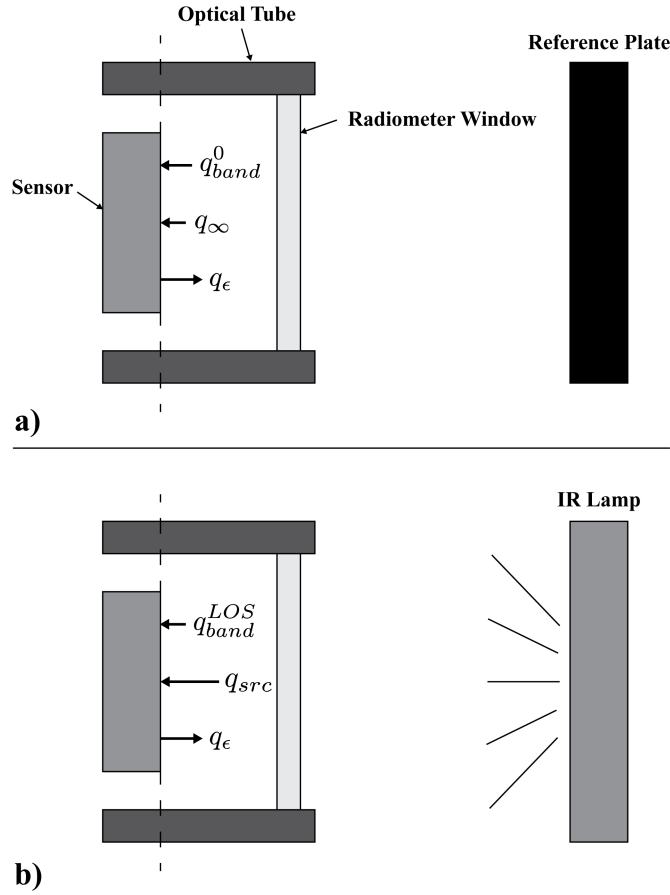


Figure 5.6: Surface energy balance on the prototype sensor during calibration. a) Sensor positioned in front of the water-cooled reference plate. b) Sensor positioned in front of the calibration heat source.

Figure 5.6 schematically represents the relevant heating terms on the prototype sensor surface during calibration. The net heat flux absorbed by the sensor when positioned in front of the water-cooled reference plate q_0 is given by the surface energy balance in Fig. 5.6a

$$q_0 = \alpha_{sens}(q_\infty + q_{band}^0) - q_\epsilon \quad (5.3)$$

In Eq. (5.3), α_{sens} is the absorptivity of the prototype sensor surface, q_∞ is the heat flux transmit-

ted through the radiometer window from the reference plate, and q_{band}^0 is the heat flux incident on the sensor from the optical tube and radiometer window when positioned in front of the reference plate. The heat flux emitted from the front surface of the sensor q_ϵ is defined as

$$q_\epsilon = \epsilon_{sens} \sigma_{SB} T_{surf}^4 \quad (5.4)$$

where ϵ_{sens} is the emissivity of the prototype sensor surface, equal to α_{sens} , σ_{SB} is the Stefan-Boltzmann constant, and T_{surf} is the sensor surface temperature, approximated as the sensor package temperature T_s . The net heat flux absorbed by the prototype sensor when positioned in the line-of-sight of the heat source q_{LOS} is given by the surface energy balance in Fig. 5.6b

$$q_{LOS} = \alpha_{sens}(q_{src} + q_{band}^{LOS}) - q_\epsilon \quad (5.5)$$

In Eq. 5.5, q_{src} is the heat flux transmitted through the sapphire window from the calibration heat source and q_{band}^{LOS} is the heat flux incident on the sensor from the surrounding optical tube and radiometer window when positioned in the heat source line-of-sight.

For use in Eq. (5.2), the difference in heat flux between the two states must be calculated. Subtracting Eq. (5.3) from Eq. (5.5) yields

$$q_{LOS} - q_0 = \alpha_{sens}(q_{src} - q_\infty) + \alpha_{sens}(q_{band}^{LOS} - q_{band}^0) \quad (5.6)$$

In Eq. 5.6, the term $q_{src} - q_\infty$ is equivalent to the heat flux incident on the active sensing region of the reference heat flux sensor (i.e., after transmission through the radiometer window) when positioned in the heat source line-of-sight. The quantity $q_{src} - q_\infty$ is, therefore, calculated from

the reference sensor voltage using the manufacturer-supplied response curve, calibrated as a total heat flux sensor, without a radiometer window, to directly capture the post-transmission heat flux. The second term in Eq. 5.6 is calculated using the geometries and temperatures of the calibration system components. For each position, q_{band} is calculated via

$$q_{band}^{(l)} = (1 - F_{s,w}) \epsilon_{OT} \sigma_{SB} T_{OT}^4 + F_{s,w} \int_0^\infty \pi \epsilon_w(\lambda) I_\lambda(\lambda, T_w) d\lambda \quad (5.7)$$

where $l = LOS$ or 0. The first term on the left-hand side of Eq. 5.7 represents the heat flux incident on the sensor from the portion of the optical tube which extends beyond the front surface of the sensor. The stainless steel optical tube is approximated as a gray body with an emissivity ϵ_{OT} of 0.5 [84, 85] and temperature T_{OT} . The second term represents the heat flux incident on the sensor from the radiometer window, where ϵ_w represents the spectral emissivity of sapphire [86], approximated as independent of the window temperature T_w , and I_λ represents the Planck blackbody distribution. In Eq. (5.7), $F_{s,w}$ represents the view factor of the window from the sensor surface approximating the two components as parallel disks

$$F_{s,w} = \frac{1}{2} \left[\Gamma - \sqrt{\Gamma^2 - 4 \left(\frac{r_w}{r_s} \right)^2} \right] \quad (5.8)$$

where

$$\Gamma = 1 + \frac{d_{sw}^2 + r_w^2}{r_s^2} \quad (5.9)$$

In Eqs. (5.8) and (5.9), r_w and r_s are the radii of the radiometer window and sensor, respectively, and d_{sw} is the distance between the sensor surface and the radiometer window. In the calibration of the rhenium heat flux sensor in Chapter 4, $F_{s,w} \approx 0.86$, corresponding with an effective sensor

radius of 2.5 mm, a radiometer window radius of 12.7 mm, and a distance of 5 mm between the two components.

The magnitude of the band heater correction (second term on the right-hand side of Eq. (5.6)) varies depending on the calibration temperature. Table 5.3 compares calculated heat flux values for the third calibration sweep of the rhenium sensor in Chapter 4. In Table 5.3, q_{raw} is the heat flux calculated without the band heater correction

$$q_{raw} = \alpha_{sens}(q_{src} - q_{\infty}) \quad (5.10)$$

and q_{corr} is the heat flux calculated with the correction using Eq. (5.6), representative of $q_{LOS} - q_0$. Generally, the magnitude of the correction increases with increasing sensor temperature. While, above 100 °C, the difference in the band heater temperature between the baseline T_{band}^0 and line-of-sight T_{band}^{LOS} positions is similar for each corresponding heat flux, the sensitivity of q_{band} (Eq. (5.7)) to varying temperature increases as the calibration system becomes hotter. From 400 °C to 500 °C, however, the magnitude of the correction does not change. This is due to the spectral emissivity of sapphire, which decreases sharply with decreasing wavelength at a value of ~ 6.5 μm [86]. From 400 °C to 500 °C, the peak blackbody emission decreases from a wavelength of 4.3 μm to 3.7 μm ; thus, at high temperatures, the radiometer window becomes less emissive.

5.6.2 Characterization of the Temperature Control Apparatus

The temperature of the window T_w and the optical tube T_{OT} were determined experimentally to evaluate the expressions q_{band}^0 and q_{band}^{LOS} . The temperature control apparatus, including a sapphire radiometer window, was assembled as if performing a calibration, but without a proto-

Table 5.3: Tabulated band heater temperature and heat flux data from the calibration of the rhodium sensor detailed in Chapter 4 (Sweep 3, Fig. 4.6). The data includes the heat flux calculated with and without the band heater correction, corresponding with q_{raw} (Eq. (5.10)) and q_{corr} (Eq. (5.6)), respectively. The two heat flux values are compared in the right-most column. Note that, in the 100 °C calibration, the measured band heater temperatures may be higher than the set point value. In the latter two heat fluxes, air cooling was used to maintain the sensor at 100 °C.

T_s (°C)	T_{band}^{LOS} (°C)	T_{band}^0 (°C)	q_{raw} (W/cm ²)	q_{corr} (W/cm ²)	% Difference
100	94	104	0.48	0.47	-1.07
	87	104	1.03	1.02	-0.83
	90	104	1.67	1.66	-0.42
	97	104	2.16	2.15	-0.17
200	196	207	0.46	0.45	-2.23
	188	207	1.06	1.04	-1.63
	176	207	1.81	1.78	-1.52
	165	207	2.42	2.38	-1.50
300	298	312	0.47	0.45	-4.11
	290	312	1.03	1.00	-2.90
	279	312	1.70	1.65	-2.60
	272	312	2.35	2.29	-2.25
400	399	412	0.51	0.49	-4.65
	391	412	1.08	1.04	-3.53
	381	412	1.70	1.64	-3.27
	375	412	2.28	2.21	-2.88
500	496	506	0.46	0.44	-4.94
	492	506	1.02	0.99	-3.10
	485	506	1.63	1.58	-2.89
	479	506	2.19	2.13	-2.75

type heat flux sensor installed. Thermocouples were inserted into the optical tube and radiometer window cavity through the backside of the heat sink to measure the temperature at three locations (Fig. 5.7): I) the front face of the copper heat sink, II) the center of the radiometer window on the inner surface, and III) the inner surface of the optical tube near edge of the radiometer window. In the former location, the thermocouple was attached to a prototype sensor mounting point using a screw. In the latter two locations, the thermocouples were held in place against the components

by applying pressure to the thermocouple lead wires externally. An additional thermocouple was placed against the center of the radiometer window on the outer surface; however, due to the large amount of convective cooling of the thermocouple junction from the ambient environment, this measurement was deemed not representative of the radiometer window temperature.

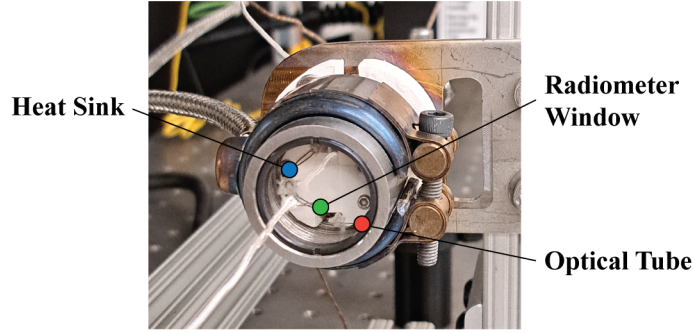


Figure 5.7: Experimental setup for characterizing the temperature control apparatus component temperatures. Thermocouples were used to measure the temperature of the heat sink, optical tube, and radiometer window, labeled correspondingly.

The temperature of the band heater was varied from room temperature to 550 °C in 100 °C increments, with two temperatures tested in smaller increments above 500 °C. At each band heater temperature, the temperature control apparatus was allowed to reach a steady state and the temperatures of each component were recorded. Figure 5.8 shows the measured temperatures of the heat sink, radiometer window, and optical tube as a function of the band heater temperature. The trend of all component temperatures is linear with respect to the band heater temperature T_{band} . A line is fit to each series using the equation

$$T_i = 20 \text{ }^{\circ}\text{C} + m_i(T_{band} - 20 \text{ }^{\circ}\text{C}) \quad (5.11)$$

where m_i is the linear fit coefficient for component i . Fit coefficients for all three temperature control apparatus components are tabulated in Table 5.4.

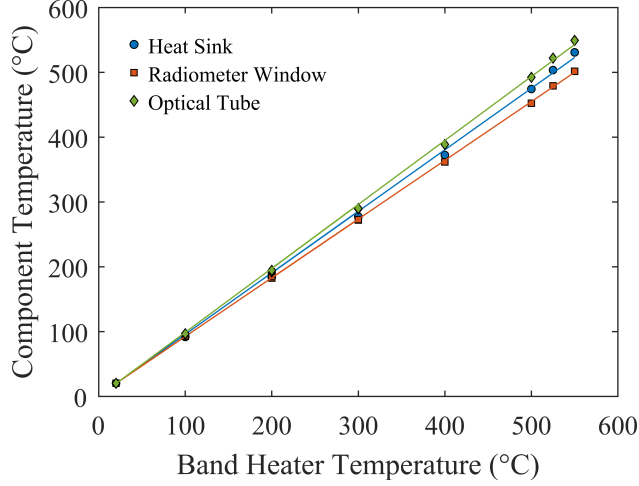


Figure 5.8: Temperature control apparatus component temperatures versus band heater set point.

Table 5.4: Temperature control apparatus component temperature fit coefficients.

Component	m_i
Heat Sink	0.949
Radiometer Window	0.906
Optical Tube	0.987

As anticipated, the temperature difference between the band heater and optical tube is minimal, due to the two components being metallic and in good thermal contact. The heat sink temperature is slightly lower than the band heater temperature due to the effect of heat dissipation to the environment. The temperature of the radiometer window at the center is the lowest due to the low thermal conductivity of sapphire. The linearity of the data allows for the evaluation of T_w and T_{OT} via the band heater set point temperature.

5.7 Data Reduction

The calibration procedure in Section 5.5 generates a large time series of continuously sampled data that must be post-processed. Figure 5.9 shows an example of raw calibration data,

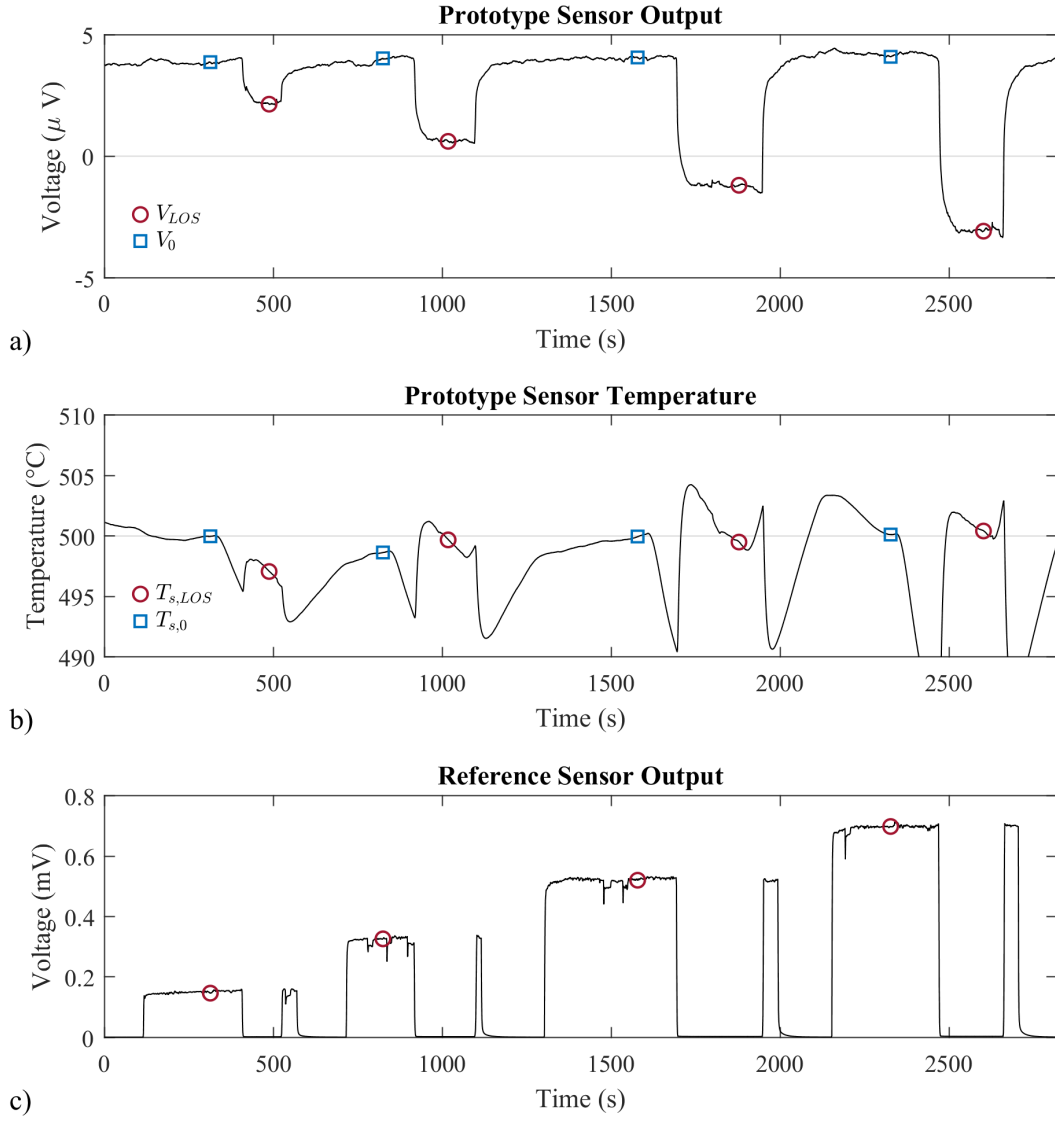


Figure 5.9: Example of raw calibration data. The measurement corresponds with the 500 $^{\circ}\text{C}$ calibration in the third sweep for the rhenium TSE-based heat flux sensor detailed in Section 4 (Fig. 4.6). a) Prototype sensor output signal. b) Prototype sensor temperature. The data is represented as the average temperature measured by the four auxiliary thermocouples in the rhenium sensor. c) Reference sensor output signal. In each plot, the red circles mark the time at which a 10 s average measurement is sampled with the respective sensor positioned in the heat source line-of-sight. The blue squares in a) and b) mark the time at which a baseline measurement is taken for the prototype sensor.

including the prototype sensor output voltage, prototype sensor temperature, and reference sensor output voltage. The points marked with symbols in Fig. 5.9 represent the times at which the 10 s average measurement is sampled from each sensor, with line-of-sight measurements marked as red circles and baseline measurements marked as blue squares. Multiple observations can be drawn from the data in Fig. 5.9: I) The voltage measurement time series near each data point is at a steady state (Fig. 5.9a), II) the temperature of the prototype sensor does not deviate more than 1% from the target temperature (500 °C), except for brief moments when the sensor is moved in and out of the heat source line-of-sight (Fig. 5.9b), and III) brief signal drops are present in the voltage output time series of both sensors (Figs. 5.9a and 5.9c). The last observation has been attributed to the IR lamp controller responding to fluctuations in the main input power line, and is not indicative of the behavior of either sensor. Nonetheless, care should be taken to avoid sampling within 10 s of the signal drops.

5.7.1 Response Curve

For a constant sensor temperature, a response curve is generated using multiple voltage vs. heat flux data points. An example response curve is shown in Fig. 5.10. For each data point, the voltage is taken as the difference in the steady-state 10 s average output signal between when the sensor is exposed to the heat source and when it is positioned in front of the water-cooled reference plate (Section 5.5). The heat flux is calculated using Eq. (5.6). The slope of a line fit to the calibration data yields the temperature-specific responsivity of the prototype heat flux sensor.

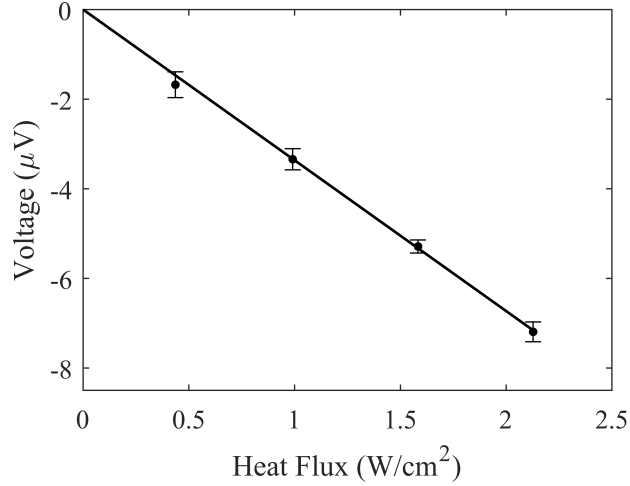


Figure 5.10: Example of a heat flux vs. voltage response curve. The measurement corresponds with the 500 °C calibration in the third sweep for the rhenium TSE-based heat flux sensor detailed in Section 4 (Fig. 4.6) and with the raw data shown in Fig. 5.9.

5.7.2 Calibration Uncertainty

For each data point, the uncertainty is influenced by the uncertainties in both the line-of-sight and baseline voltage measurements. When the sensor is positioned in the heat source line-of-sight, the uncertainty in the voltage output σ_{LOS} is driven by short time-scale fluctuations in the IR lamp output (excluding the signal drops noted previously) and band heater power. When the sensor is positioned in front of the water-cooled reference plate, the uncertainty in the voltage output σ_0 is driven by long time-scale drifts. Both quantities are estimated using the continuously measured voltage of the prototype sensor, sampled during steady-state conditions (Fig. 5.9). The parameter σ_{LOS} is calculated using the standard deviation of the voltage sampled 60 s prior to the line-of-sight measurement time. The parameter σ_0 is calculated using the standard deviation of the voltage sampled 300 s prior to the baseline measurement time. If the steady-state period prior to either measurement is shorter in duration than the standard deviation sample window, the sample region is truncated to only capture the variation in the voltage while at steady-state

conditions.

The total uncertainty of a data point σ_T is estimated by propagating the uncertainties of the line-of-sight and baseline voltage measurements

$$\sigma_T = \sqrt{\sigma_{LOS}^2 + \sigma_{\infty}^2} \quad (5.12)$$

The error bars for each data point in Fig. 5.10 represent the total uncertainty calculated using Eq. (5.12), multiplied by a coverage factor of 2 ($\pm 2\sigma_T$).

If the uncertainties of the data points in the response curve are uniform (i.e., homoscedastic), a line is fit to the data using an ordinary least squares regression. This is often the case for room-temperature calibrations, where the measurement is performed over a much shorter duration and the band heater is not used. Generally, for calibrations at elevated temperatures, the measurement uncertainties will be non-uniform (i.e., heteroscedastic), especially if the responsivity is low. In this case, a weighted least squares regression is used to fit a straight line to the calibration data, where the slope (i.e., responsivity) C is calculated via

$$C = (\mathbf{q}^T \mathbf{w} \mathbf{q})^{-1} \mathbf{q}^T \mathbf{w} \mathbf{V} \quad (5.13)$$

In Eq. (5.13), $\mathbf{q}$ and $\mathbf{V}$ are vectors of length N corresponding with the calculated heat flux and voltage differences (Eq. (5.2)), and $\mathbf{w}$ is a diagonal matrix of weights calculated using the uncertainty of each voltage measurement (Eq. (5.12))

$$w_i = \frac{1}{(\sigma_{T,i}/V_i)^2} \quad (5.14)$$

Equation (5.13) naturally forces the linear fit through the origin. A 95% confidence interval is calculated for the linear fit coefficient $C_{0.95}$ to establish the uncertainty of the calibration

$$C_{0.95} = C \pm \hat{t}_{0.05/2} \text{SE} \quad (5.15)$$

In Eq. (5.15), $\hat{t}_{0.05/2}$ represents the critical t-value for a 95% confidence interval and SE is the standard error

$$\text{SE} = \sqrt{\text{MSE} \times (\mathbf{q}^T \mathbf{w} \mathbf{q})^{-1}} \quad (5.16)$$

where MSE represents the mean squared error of the fit

$$\text{MSE} = \frac{\sum_{i=1}^N w_i (V_i - \hat{V}_i)^2}{N - 1} \quad (5.17)$$

In Eq. (5.17), $\hat{V}_i$ represents the predicted voltage using the fit coefficient determined in Eq. (5.13).

5.8 Characterization of the IR Lamp Heat Source

The IR lamp used in the calibration system was characterized to determine the range of heat fluxes that can be generated and the spatial uniformity of the heat flux. The results of the characterization experiments are used to develop setup guidelines to minimize calibration errors.

5.8.1 Characterization of the IR Lamp Output Versus Lamp Input Power

The output of the IR lamp is modulated by controlling the input power via the IR lamp controller, where the user sets a percentage value relative to the full-scale input power. The heat

flux output from the IR lamp was characterized as a function of the lamp input power. The reference Schmidt-Boelter radiometer was used to measure the incident heat flux from the IR lamp. The sensor was positioned in the line-of-sight of the heat source at a heat source-sensor distance of 1.8 cm, representing the distance between the front surface of the active sensing element of the reference sensor (Fig. 5.2c) and the front face of the IR lamp reflector. The IR lamp input power was adjusted from 15% to 80% in 10% increments. At each lamp input power, the voltage of the reference sensor was measured after allowing the output to reach a steady-state value. The sensor voltage is converted to incident heat flux using the manufacturer-supplied response curve.

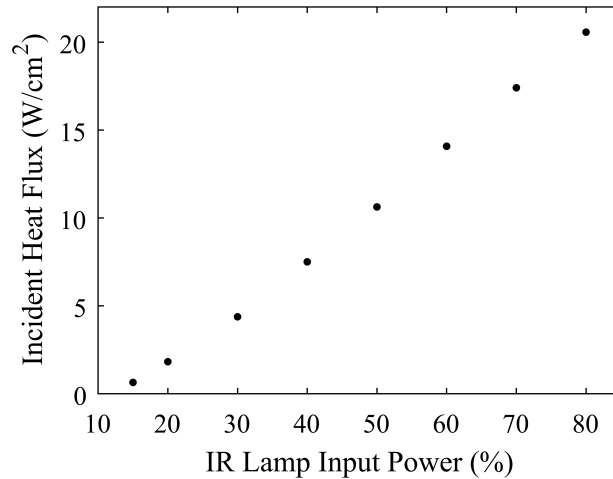


Figure 5.11: IR lamp heat flux versus lamp input power at a heat source-sensor distance of 1.8 cm.

Figure 5.11 shows the measured incident heat flux from the IR lamp versus input power. Below a lamp input power of 15%, the controller is not able to drive current through the lamp filament. The relationship between the measured heat flux and the lamp input power is linear for input powers higher than 20%. At a heat source-sensor distance of 1.8 cm, the heat flux sensor measures a maximum incident heat flux of 20.6 W/cm².

5.8.2 Characterization of the IR Lamp Output Versus Sensor Position

The incident heat flux from the IR lamp was characterized as a function of the sensor position to determine the sensitivity of the measured heating conditions to potential misalignment of the prototype and reference sensor. During calibration, each sensor is sequentially positioned into the same nominal location with respect to the line-of-sight of the IR lamp. Realistically, it is anticipated that some misalignment will exist between the two sensors, such that they are positioned into different locations with respect to the IR lamp when exposed. Using the measured spatial distribution of the IR lamp output, relative alignment tolerances in all three axes are developed to bound the error in the calibration heat flux (Section 5.8.3). Additionally, the data is used to generate a map of achievable heat flux values for a set of calibration setup conditions.

5.8.2.1 Heat Flux Versus Heat Source-Sensor Distance

The heat flux from the IR lamp was determined as a function of the heat source-sensor distance. In this experiment, the reference Schmidt-Boelter radiometer was used to measure the incident heat flux at each distance. The y - and z -positions of the reference heat flux sensor were adjusted to align its centroid with the centerline axis of the IR lamp (i.e., the x -axis intersecting the IR lamp centroid, Fig. 5.12), geometrically approximated as a 2×1 inch (5.1×2.5 cm) rectangle. The IR lamp input power was set to a constant value of 30%. The heat source-sensor distance was varied from 1.9 to 7.8 cm in approximately 0.5 cm increments. At each position, the steady-state voltage of the reference sensor was recorded. The sensor voltage was converted to incident heat flux using the manufacturer-supplied response curve.

The heat flux measured by the reference heat flux sensor versus the heat source-sensor

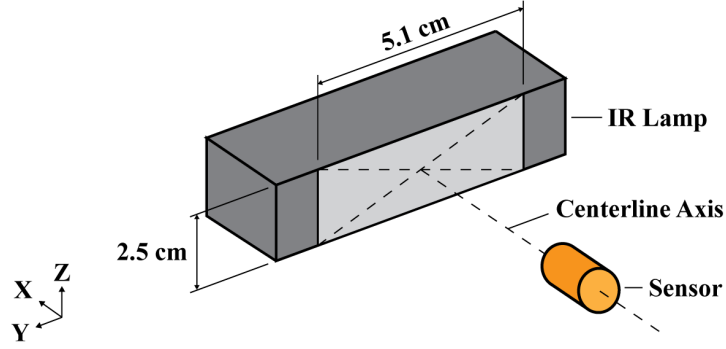


Figure 5.12: Diagram of the sensor aligned with the centerline axis of the IR lamp.

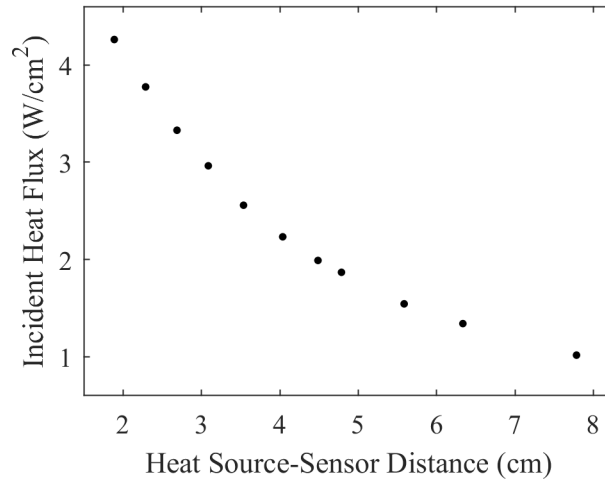


Figure 5.13: IR lamp output versus heat source-sensor distance.

distance is shown in Fig. 5.13. With increasing distance, the heat flux decreases monotonically; however, the rate of the heat flux attenuation also decreases monotonically. Thus, to minimize errors due to sensor misalignment, the sensors should be placed at the maximum distance from the heat source that still allows access to the desired range of heat flux values for calibration.

To aid in the setup of each calibration to achieve a desired range of heating conditions, Table 5.5 presents a map of heat flux values corresponding with various heat source-sensor distances and IR lamp input powers. The values in Table 5.5, in units of W/cm^2 , are derived via interpolation of the results in Fig. 5.13, and assuming that the trend shown in Fig. 5.11 of heat

flux-vs-lamp input power applies to all heat source-sensor distances. Values in Table 5.5 represent incident heat flux; for conversion to the heat flux transmitted through the 0.5 mm thick sapphire radiometer window, values in Table 5.5 are multiplied by a factor of 0.81.

It should be noted that the data in Table 5.5 is based on measurements performed when the IR lamp was brand new. Subsequent measurements performed approximately 1 year after the initial dataset was generated (with approximately 75 hrs of use) showed a 5% decrement in the heat flux for a given combination of lamp power and heat source-sensor distance. This is possibly due to dust accumulation on the lamp reflector, which remained uncovered, or wear of the tungsten filament, which has a stated lifetime of 400 hrs.

Table 5.5: Map of the calibration heat flux versus lamp input power and heat source-sensor distance. Heat flux values represent the incident heat flux measured by the reference sensor with units of W/cm^2 . To calculate the heat flux transmitted through the 0.5 mm thick sapphire radiometer window, multiply the tabulated values by a factor of 0.81.

Distance (cm)	IR Lamp Input Power (%)						
	20	30	40	50	60	70	80
2.0	1.72	4.12	7.06	10.00	13.24	16.37	19.35
2.5	1.47	3.53	6.05	8.57	11.35	14.04	16.59
3.0	1.27	3.04	5.21	7.37	9.77	12.08	14.27
3.5	1.08	2.59	4.43	6.28	8.32	10.28	12.15
4.0	0.94	2.25	3.86	5.47	7.24	8.96	10.58
4.5	0.83	1.98	3.40	4.81	6.37	7.87	9.31
5.0	0.74	1.78	3.05	4.32	5.72	7.07	8.35
5.5	0.66	1.58	2.70	3.82	5.07	6.26	7.40
6.0	0.60	1.43	2.45	3.47	4.59	5.68	6.71
6.5	0.54	1.30	2.23	3.16	4.18	5.17	6.11
7.0	0.50	1.19	2.04	2.89	3.82	4.73	5.59
7.5	0.45	1.08	1.85	2.62	3.46	4.28	5.06
8.0	0.40	0.97	1.66	2.34	3.10	3.84	4.54

5.8.2.2 Heat Flux Versus Planar Misalignment

The heat flux from the IR lamp was determined as a function of the y - and z -axis position of the sensors with respect to the lamp's centerline axis. During the experiment, the IR lamp input power was set to a constant value of 30%. The reference Schmidt-Boelter radiometer was scanned in the y (left-right) and z (up-down) directions over a region extending ± 1 cm from the centerline axis of the IR lamp. The output signal of the sensor was sampled every 0.25 cm along the scan path, converted to incident heat flux using the manufacturer-supplied response curve. Due to the finite area of the active sensing region of the reference sensor, the measured heat flux is interpreted as the incident heat flux averaged over a region extending ± 1 mm from the centroid of the sensor, corresponding with a 2 mm diameter active sensing region. The scans were performed independently for each direction; while sweeping along one axis, the sensor was fixed at the IR lamp center position along the other axis (e.g., when sweeping in the y -direction, the sensor was fixed at $z = 0$ cm). Due to the geometry of the IR lamp reflector, the heat flux distributions along the y and z axes are assumed to vary little with respect to the orthogonal direction near the centerline axis. For each scan, the sensor was held at a constant heat source-sensor distance. Scans were performed for multiple heat source-sensor distances.

The results of the scans are shown in Fig. 5.14 as a surface plot and in Fig. 5.15 as contours¹. Also included in Fig. 5.14 are the data from Fig. 5.13. Heat flux values in Fig. 5.14 and 5.15 are normalized by the maximum value. The spatial uniformity of the IR lamp is deduced

¹In the scan along the $x - z$ symmetry plane (Fig. 5.14b and Fig. 5.15b), data for $x = 1.9$ and $x = 5.8$ cm, representing the closest and farthest distances, respectively, were collected approximately one year after the initial experiment. In that time, a decrement of approximately 5% was observed in the output power of the IR lamp for any given input power along the IR lamp centerline axis. Data for $x = 1.9$ and $x = 5.8$ cm are, thus, scaled by a factor of 1.05 in Fig. 5.14b and Fig. 5.15b to align with the prior data at the centerline position.

from the variation in the heat flux at a given heat source-sensor distance (i.e., the variation along a vertical slice in Fig. 5.15.). In the $x - y$ symmetry plane, the heat flux from the IR lamp is very uniform at a constant heat source sensor distance, indicated by nearly vertical contour lines in Fig. 5.15a. In the $x - z$ symmetry plane, the heat flux varies more significantly at a given heat source-sensor distance. At distances less than 3 cm, the heat flux is non-monotonic as the sensor is swept in either direction along the z -axis. This behavior is attributed to the combined heating from radiation emitted directly from the lamp filament and from radiation reflected off the collimating parabolic mirror inside the IR lamp. The heat flux is most uniform along the z -direction for distances near 3 cm. Beyond a heat source-sensor distance of approximately 3.5 cm, the spatial variation of the heat flux along the z -axis does not change appreciably with increasing heat source-sensor distance. At these distances, the majority of the heat flux is from the collimated beam reflected off the lamp's mirror.

5.8.3 Sensor Alignment Tolerances

Using the results from Section 5.8.2, alignment tolerances are defined to ensure that the heat flux incident on the prototype and reference sensors varies no more than a specified amount. Alignment tolerances corresponding with a 3% allowable deviation in the incident heat flux between the two sensors are tabulated in Table 5.6. Tolerances for the y - and z -axes in Table 5.6 are defined as the y - or z -axis offset (Figs. 5.14 and 5.15), respectively, in which the incident heat flux deviates from the centerline axis value by approximately 3%. Tolerances for the y - and z -axes assume that one of the sensors is aligned with the centerline axis of the IR lamp and that both sensors are placed at the same heat source-sensor distance. Tolerances for the x -axis

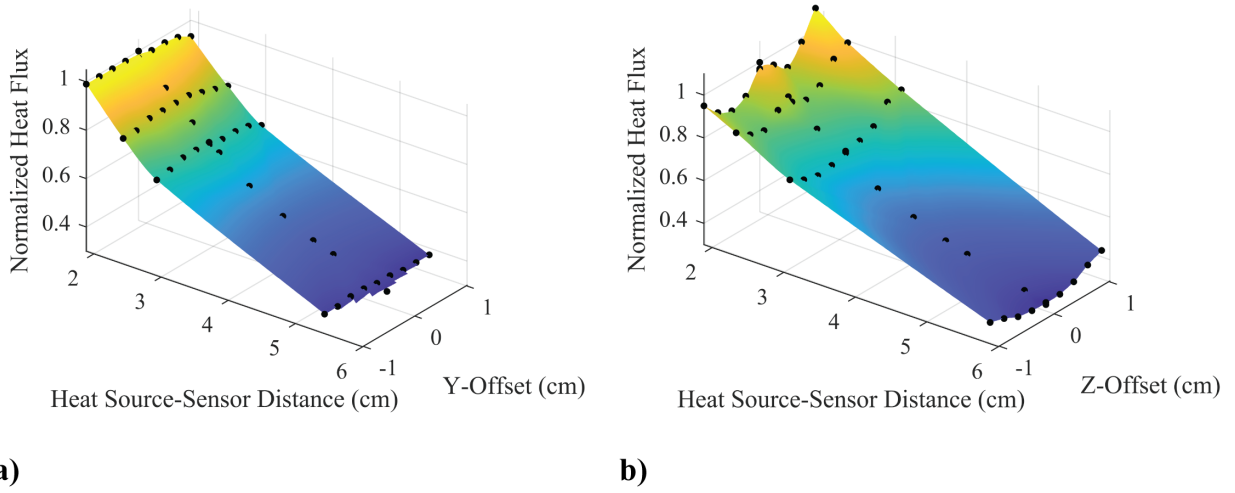


Figure 5.14: Spatial map of the IR lamp heat flux in the a) $x - y$ symmetry plane and b) $x - z$ symmetry plane. The datapoints correspond with measurements from the reference heat flux sensor scan. Heat flux values in each plot are normalized by the maximum value.

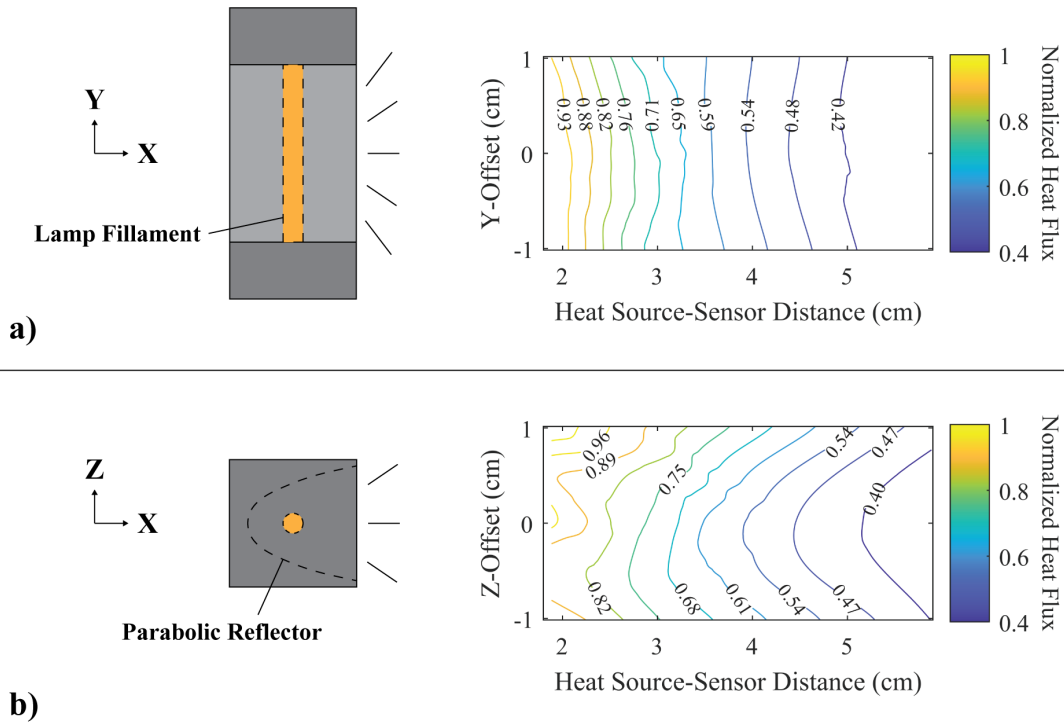


Figure 5.15: Contour plot of the IR lamp heat flux in the a) $x - y$ symmetry plane and b) $x - z$ symmetry plane. The IR lamp filament is shown schematically in a) and b). An outline of the parabolic collimating reflector is shown as a dashed line in b). The schematic of the IR lamp in a) and b) is not drawn to scale.

are defined as the distance from a specific heat source-sensor distance in which the incident heat flux deviates by approximately 3%, determined using the data in Fig. 5.13. Due to the increase in the variation of the heat flux with respect to distance as the sensor is positioned closer to the heat source, alignment tolerances are defined separately for near-field (between 2 and 4 cm) and far-field (≥ 4 cm) heat source-sensor distances.

Table 5.6: Relative alignment tolerances for the sensors for near-field (between 2 and 4 cm) and far-field (≥ 4 cm) heat source-sensor distances. Tolerances correspond with a 3% allowable deviation in the heat flux.

Axis	alignment tolerance (mm)	
	Near-Field	Far-Field
x	± 1	± 1.2
y	± 7.5	± 7.5
z	± 2.5	± 2.5

The alignment tolerances in Table 5.6 are defined independently for each axis. Considering possible misalignments in all 3 axes simultaneously, the sensors should be positioned within an ellipsoidal region defined by

$$\left(\frac{\Delta x}{\delta_x}\right)^2 + \left(\frac{\Delta y}{\delta_y}\right)^2 + \left(\frac{\Delta z}{\delta_z}\right)^2 = 1 \quad (5.18)$$

where Δx , Δy , and Δz are the relative misalignments between the sensors for each direction, and δ_x , δ_y , and δ_z are the tolerances in Table 5.6 corresponding with each axis.

It should be noted that the y - and z -axis alignment tolerances assume that the active sensing regions of each sensor are of a similar size. For a prototype sensor with a larger active sensing region than the reference sensor, the distribution of the heat flux from the IR lamp will be smoothed compared to the results in Fig. 5.14 and 5.15. Conservatively, the tolerance limits in Table 5.6

can be adjusted using the relative difference in the dimensions of the active regions of each sensor

$$\bar{\delta}_i = \delta_i - 0.5(d_i - d_{0,i}) \quad (5.19)$$

where $i = y$ or z . In Eq. (5.19), $\bar{\delta}_i$ is the reduced tolerance for axis i , and d_i and $d_{0,i}$ are the dimensions of the active sensing regions of the prototype and reference sensors, respectively, along either the y - or z -axis. For the Schmidt-Boelter reference sensor used in the calibration system, a value of $d_0 = 2$ mm is used for both y - and z -dimensions. If the active sensing region of the prototype sensor is large, such that $\bar{\delta}_i \leq 0$, the heat flux incident on the sensor will deviate by more than $\pm 3\%$ over its area, even when the sensors are perfectly aligned. In this instance, tolerances corresponding with a larger deviation in the heat flux (e.g. 5%) can be derived or a different heat source with a more uniform heat flux output can be used.

5.8.4 Rhenium Sensor Calibration Setup Parameters

In the the calibration of the rhenium-based TSE heat flux sensor detailed in Chapter 4, the sensor rack was positioned with a heat source-sensor distance of approximately 5.9 cm. At this distance, the IR lamp output power was varied between 20% and 44% to achieve a range of absorbed heat flux values from 0.5 to 2.5 W/cm². The heat source-sensor distance was chosen to minimize potential calibration errors due to misalignment of the sensors along the x -axis (Section 5.8.2.1), while still granting access to a range of calibration heat fluxes up to 2.5 W/cm².

It is worth juxtaposing the calibration setup introduced in Chapter 3 for the antimony-based sensor with the calibration system detailed in this chapter, used for the rhenium-based sensor. In Chapter 3, a blackbody reference furnace was used to achieve calibration heat fluxes up to 7

W/cm². This, however, required a partial insertion of the prototype and reference sensors into the aperture of the blackbody furnace housing (≈ 2.54 cm diameter), which is not achievable when using the band heater on the temperature control apparatus. Furthermore, calibrations using the blackbody furnace can also be more susceptible to fore-aft misalignment of the sensors along the x -axis, as the sensors must be positioned close to the furnace to achieve high heat fluxes². However, it is worth noting that the heat flux output from the blackbody reference furnace is significantly more stable and less noisy than the IR lamp used in the calibration setup detailed in this chapter. Thus, if the maximum desired heat flux is low (<1 W/cm²), the use of a blackbody reference furnace as the heat source might be preferred.

5.9 Summary

A high-temperature transfer calibration facility is developed to characterize the responsivities of heat flux sensors from room temperature to 500 °C. The system is modular and can be adapted to test a wide variety of heat flux sensors of different types and form factors for extreme environment applications. A calibration methodology is developed to increase the accuracy of calibrations through independent modulation of the sensor temperature and incident heat flux. Testing protocols are introduced to reduce error in the measurements, either by increasing the control authority over the sensor temperature or by reducing interactions between the sensor and surrounding environment. By mapping the spatial distribution of the heat flux emitted by the calibration heat source, setup tolerances are established to ensure that the variation in heat flux between the prototype and reference sensor is less than 3%.

²For completeness, it is worth noting that, as the heat source-sensor distance becomes very small, the heat flux output from a diffuse emitter becomes insensitive to distance (see Eq. (5.8) as $d_{sw} \rightarrow 0$). This, however, occurs at a much smaller distance than that used in the antimony-based TSE heat flux sensor calibration.

Because the heat source and sensor temperature control are entirely independent, different radiation heat sources, such as lasers, blackbody reference furnaces, and solar reflectors, can be implemented seamlessly without major changes to the testing protocols and procedures. The facility introduces a first-of-its-kind capability for high-temperature sensor calibrations. For the rhenium-based TSE heat flux sensor developed in Chapter 4, the calibration system allows for detection of minor changes ($< 1 \mu\text{V}/(\text{W}/\text{cm}^2)$) in the sensor's temperature-dependent responsivity following subsequent exposures to high temperatures, highlighting a resolution that has not yet been demonstrated using prior methodologies.

Chapter 6: Reconstruction of Thermal Protection System Aeroheating from Embedded Thermal Sensors - Review of the Literature

This chapter serves as an introduction to inverse heat transfer (IHT) problems and techniques, and surveys the current state-of-the-art IHT approaches used by the space exploration, and adjacent, communities. In IHT problems, the surface heat flux boundary condition on a solid is reconstructed using the time history of a temperature probe measurement and the known material properties of the solid medium. The temperature measurement can be sampled either from the surface or in-depth. Generally, the goal of an IHT method is to define a time-varying surface heat flux that minimizes the difference between the temperature measurement T_{meas} and the predicted temperature at the probe location $T_{predict}$. More formally, the heat flux should minimize the sum of the squared residuals between the two temperature time series [87]

$$\min \sum (T_{meas} - T_{predict})^2 \quad (6.1)$$

In Eq. (6.1), the summation term to-be-minimized is often referred to as the *objective function*. While the particular nuances of each IHT algorithm vary, all approaches, with few exceptions for direct analytical methods, aim to satisfy some form of Eq. (6.1).

Current IHT methods for the analysis of heat loads on spacecraft during atmospheric

entry typically rely on high-fidelity multiphysics simulations (to calculate $T_{predict}$) embedded within a nonlinear objective function minimization framework (to satisfy Eq. (6.1)). The two most prevalent algorithms used by the atmospheric entry community are FIAT_Opt [2] and INHEAT [88, 89]. FIAT_Opt leverages the Fully Implicit Ablation and Thermal analysis program (FIAT) [90, 91], developed at NASA Ames, within a Gauss-Newton or Levenberg-Marquardt optimization routine. INHEAT leverages the CHarring Ablator Response code (CHAR) [92], developed at NASA Johnson, embedded in an adaptation of the function specification method formalized by Beck [87]. The two codes have been developed within the past 12 years in response to a growing adoption of TPS-embedded thermal instrumentation on entry spacecraft, both within the scope of the Artemis program and interplanetary science missions. While applications of INHEAT to space exploration missions are sparse in the public literature (owed to its prevailing use for crewed spaceflight, which is heavily regulated), Alpert *et al.* have reported on the use of FIAT_Opt (and its underlying methodology developed by Mahzari [93]) to reconstruct the heat loads on the Mars 2020 [2] and Mars Science Laboratory [22] atmospheric entry spacecraft.

The current IHT methods used by the atmospheric entry community, however, demand substantial computational resources. This cost is rooted in the complex material response behavior of ablative TPS, which drives the use of well-established high-fidelity multiphysics codes, such as FIAT and CHAR, in the IHT formulation. Both codes employ a spatially discretized mesh and implicit time-marching finite volume or finite element schemes, which increase the computational overhead of the IHT problem in two ways: I) although the objective function only considers the thermal response at the temperature measurement location, the whole-domain solution is required at each time step, and II) the solution must be calculated using iterative nonlinear optimization

routines, which require the evaluation of local objective function gradients—i.e., the change in the objective function with respect to change in the surface heat flux boundary condition—for each iteration [88]. The main implication of the latter being that, for nonlinear material systems, many (>1000) individual simulations may be needed to converge on a single heat flux solution [2]. Generally, these drawbacks persist in all iterative IHT algorithms that rely on mesh discretization and time-marching schemes [94–96], but are exacerbated in approaches tailored for ablative TPS materials due to the comparatively large cost of single solution evaluations.

Meshless IHT techniques offer more computationally efficient alternatives well-suited to address the high cost of current reconstruction methods for atmospheric entry applications. The most simple, yet, pervasive, meshless IHT methods are those which use semi-infinite approximations to model heat conduction within the solid. In short-duration ground test facilities, semi-infinite methods have been used extensively since the 1960s to calculate the heating conditions on test articles directly from surface temperature measurements. Early data reduction techniques from Cook and Felderman [97], Schultz and Jones [98], and, more recently, Merski [99] treated temperature sensors placed on the test article surface as thermally massless, yielding a straightforward calculation of the heat flux via analytical solutions of the 1D heat conduction equation. Liu *et al.* extended this approach to include the effects of a finite-thickness polymer layer coated on top of a semi-infinite test article, allowing for more accurate correlations between measured temperature-sensitive paint intensities and the true temperature of the surface [100]. Semi-infinite methods, however, assume that the through-thickness propagation of the surface heat flux is shallow relative to the thickness of the test article. These assumptions become invalid in atmospheric entry scenarios, where the relevant time scales are to the order of seconds to minutes and the temperature measurements are sampled below the surface. Furthermore, semi-infinite analytical

methods are typically reserved for linear systems with constant thermal properties, limiting their use in more complex material systems.

Within the ground test community, alternative meshless IHT methods, such as the Non-integer System Identification (NISI) method [101–103] and the Green’s function approach [87, 104], have been used to more thoroughly account for subsurface heat conduction effects that are not captured by semi-infinite approximations. These methods are both impulse response-based, where the thermal response at the temperature probe location is coupled directly with the surface heat flux boundary condition, enabling efficient IHT reconstructions. Battaglia *et al.* developed the NISI method [101] to solve IHT problems in systems with unknown material properties, where the impulse response of the test article at the temperature probe location is calibrated experimentally [102]. In particular, the NISI method is advantageous for geometries that are intractable to model analytically, and for when the material properties and temperature probe location are not known with a large degree of certainty. Most recently, the approach was used within high-enthalpy plasma test facilities to reconstruct the heat flux on the sharp leading edge of a composite fin [103]. The NISI method, however, has limited amenability to nonlinear material systems, as the impulse response used in the IHT formulation is valid only for the state in which it is calibrated in.

The Green’s function approach may offer a more versatile and adaptable framework for efficient IHT reconstructions in spacecraft TPS. In the Green’s function approach, the impulse response at the temperature measurement location is calculated from the known system parameters, such as the probe location, test article geometry, and test article material properties. Beck *et al.* first introduced Green’s functions for IHT problems in the 1990s to validate the heating conditions on carbon-composite specimens during thermal properties characterization [87, 105].

This formulation carried over into the aerothermodynamics community, where Green’s function-based IHT algorithms have been developed more recently to efficiently solve for the surface heating conditions on multi-layered test articles in hypersonic test facilities. Whalen *et al.* leveraged the Green’s function formulation via Laplace transform to recover the surface heat flux on two- and three-layer test articles from surface temperature measurements [106]. Compared with semi-infinite approaches, this formulation allows for more accurate IHT solutions in long-duration experiments, where the finite thickness and construction of a test article significantly impact the surface temperature measurement. Massa *et al.* developed an alternative Green’s function IHT approach via the Galerkin method to solve for the global surface heating conditions on complex, multimaterial three-dimensional geometries using measurements from distributed temperature sensors [107–109]. Notably, in the body of work of Massa *et al.*, the Green’s function IHT formalism was advanced to accommodate temperature-dependent thermal properties, demonstrating the approach as a viable candidate for IHT problems in nonlinear material systems. Beyond temperature-dependent thermal properties, however, Green’s function-based IHT techniques have not been matured to handle the complex heat conduction phenomena present in ablative TPS materials, motivating the development of new techniques to further adapt the approach.

A general caveat of IHT problems not yet discussed is their susceptibility to developing large instabilities in the solution if the input temperature measurement contains small errors or noise [13]. These instabilities arise due to the diffusive nature of heat conduction, which acts to physically suppress high frequency information in the thermal response, thereby amplifying high frequencies in the inverse reconstruction. In atmospheric entry applications, instabilities are especially prevalent in IHT solutions due to the low thermal diffusivity of spacecraft TPS,

and by the fact that temperature measurements are sampled in-depth, away from the surface. Regularization techniques, such as Tikhonov regularization [110] or truncated singular value decomposition [13], are often used to damp the amplification of measurement errors. Regularization methods have, in fact, been implemented in most IHT methodologies introduced in this review, including FIAT_Opt [2], INHEAT [88], the Green's function IHT techniques developed by Whalen [106] and Massa [111], and the NISI method [103], with all but the latter specifically utilizing Tikhonov regularization. However, regularization incurs a trade off between solution stability and the retention of sharp features [13, 112] (Fig. 6.1), which may lead to the suppression of quickly varying temporal features in the reconstructed heating solution, such as the advancement of a boundary layer transition front over the location of a temperature probe. While other novel regularization approaches have been proposed that may be better suited for resolving sharp features, such as the combined Tikhonov/total variation regularization approach proposed by Gholami and Hosseini [113], or the inhomogeneous regularization approach proposed by Han and Lee [114], these approaches often involve assumptions of the solution behavior a priori or the definition of extra user-defined hyperparameters which need to be optimized for each measurement application; generally, for atmospheric entry reconstructions, more robust methods are desired.

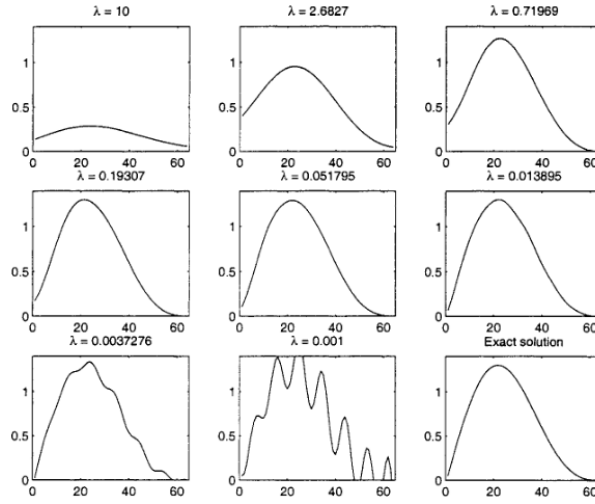


Figure 6.1: Behavior of Tikhonov regularization on a reconstructed sample mass distribution from local gravity field measurements as a function of the user input regularization coefficient λ [13] (reprinted with permission from Springer Nature). At large values of λ , the reconstructed solution is over-smoothed; conversely, at low values of λ , the solution contains significant instabilities. Accurate solutions arise from a bounded region of λ values.

Chapter 7: Green's Function Formulation for Inverse Heat Transfer Problems

A Green's function-based inverse heat transfer algorithm is developed in this thesis to efficiently reconstruct the heating conditions experienced by spacecraft during atmospheric entry. In this chapter, the foundational basis of the IHT algorithm is introduced, from which adaptations are made in subsequent chapters to extend its capabilities for atmospheric entry reconstructions. The Green's function formalism detailed in this chapter follows the work of Beck and Haji-Sheikh, who have made extensive contributions to the field of Green's functions for heat conduction problems over the past four decades [87, 104, 115–118], and borrows some more recent elements from the work of Massa *et al.* [107, 108]. The numerical implementation of the IHT approach borrows heavily from the work of Hansen, who has contributed significantly to the analysis of ill-posed inverse problems [13, 112, 119, 120]. Equations are defined in 3D coordinates to simplify the notation and maintain generality.

The IHT algorithm seeks to recover the heat flux on the surface of a spacecraft using a TPS-embedded temperature probe measurement and TPS material properties as inputs. The approach consists of two main steps: first, a Green's function formulation is developed to efficiently model the thermal response of the TPS at the temperature measurement location. In the second step, the Green's function representation of the TPS thermal response is inverted to recover the surface heating condition.

7.1 Green's Function Formulation

7.1.1 Green's Function Solution Equation

The Green's function framework for heat conduction considers the solutions to two linear, inhomogeneous partial differential equations [104]

$$\frac{1}{\alpha} \frac{\partial \tilde{T}}{\partial t} - \nabla^2 \tilde{T} = \frac{1}{k} g(\mathbf{x}, t) \quad (7.1)$$

$$\frac{1}{\alpha} \frac{\partial G}{\partial t} - \nabla^2 G = \frac{1}{\alpha} \delta(\mathbf{x} - \mathbf{x}_0, t - \tau) \quad (7.2)$$

Equation (7.1) is the linear heat conduction equation with an inhomogeneous volumetric energy generation term $g(\mathbf{x}, t)$. The parameter $\tilde{T}$ is temperature, k represents the thermal conductivity, and α represents the thermal diffusivity. Equation (7.2) is termed the *auxiliary equation* and models Green's function $G(\mathbf{x}, t, \mathbf{x}_0, \tau)$ as the response of a linear system at $\mathbf{x}$ and t to an inhomogeneous Dirac delta forcing term at $\mathbf{x}_0$ and τ . Equations (7.1) and (7.2) are expressed as functions of general coordinates $\mathbf{x}$, which can take the form of a Cartesian $\{x, y, z\}$, cylindrical $\{r, \theta, z\}$, or spherical $\{r, \theta, \phi\}$ basis.

The derivation of the Green's function solution equation from Eqs. (7.1) and (7.2) is extensive, involving numerous identities, algebraic manipulation, and heuristics, and thus will not be detailed here (see Cole *et al.* [104], pp. 64-67). Principally, Eq. (7.1) and Eq. (7.2) are recast and

arranged as a series of integral convolutions which yield the solution of $\tilde{T}$

$$\begin{aligned}
\tilde{T}(\mathbf{x}, t) = & \int_V \tilde{T}(\mathbf{x}_0, 0) G(\mathbf{x}, t, \mathbf{x}_0, 0) dV_0 \\
& + \int_0^t \int_V \frac{\alpha}{k} g(\mathbf{x}_0, \tau) G(\mathbf{x}, t, \mathbf{x}_0, \tau) dV_0 d\tau \\
& + \int_0^t \int_S \alpha \nabla_0 \tilde{T}(\mathbf{x}_0, \tau) G(\mathbf{x}, t, \mathbf{x}_0, \tau) \cdot \hat{\mathbf{n}} dS_0 d\tau \\
& - \int_0^t \int_S \alpha \tilde{T}(\mathbf{x}_0, \tau) \nabla_0 G(\mathbf{x}, t, \mathbf{x}_0, \tau) \cdot \hat{\mathbf{n}} dS_0 d\tau
\end{aligned} \tag{7.3}$$

Equation (7.3) is the standard form of the *Green's function solution equation*. The components on the right-hand side of Eq. (7.3) capture the effects of various heat conduction problem elements on the temperature solution at $\mathbf{x}$ and t . The first term on the right-hand side of Eq. (7.3) accounts for inhomogeneous initial conditions, where the integrand is evaluated over the system volume with respect to a differential volume element in dummy coordinates (e.g., $dV_0 = dx_0 dy_0 dz_0$ in Cartesian coordinates). The second term captures the effects of volumetric energy generation phenomena on the temperature solution. The third and fourth surface integral terms capture the effects of boundary conditions, either Neumann (third integral term) or Dirichlet (fourth integral term), where $\hat{\mathbf{n}}$ is the surface normal vector. Note that the gradient operator in the third and fourth integrals ∇_0 is expressed in terms of dummy variables.

For a broad range of linear heat conduction problems with uniform initial conditions (i.e., $\tilde{T}(\mathbf{x}, 0) = \tilde{T}_0$) subject to a heat flux boundary condition $q(\mathbf{x}, t)$ on an exposed surface, Eq. (7.3) can be reduced to the form of

$$T(\mathbf{x}, t) = - \int_0^t \int_S \frac{\alpha}{k} q(\mathbf{x}_0, \tau) G(\mathbf{x}, t, \mathbf{x}_0, \tau) dS_0 d\tau \tag{7.4}$$

where T is defined as the temperature rise from initial conditions $T = \tilde{T} - \tilde{T}_0$. The heat flux on the surface q is related to the temperature gradient $\nabla\tilde{T}$ via Fourier's law

$$q = -k\nabla\tilde{T} \cdot \hat{\mathbf{n}} \quad (7.5)$$

It is important to note that the form of Eq. (7.3) captures the specific application of the Green's function auxiliary equation (Eq. (7.2)) to the linear heat conduction equation (Eq. (7.1)). Realistically, any linear, parabolic, inhomogeneous differential equation (e.g., Fick's law) can be coupled with Eq. (7.2) using an appropriate diffusion constant to generate a form of the Green's function solution equation that corresponds with a different problem of interest (e.g., species transport). This includes mathematical abstractions of the heat conduction equation, which will become essential to capture nonlinear transport phenomena in ablative TPS.

7.1.2 Green's Function Construction

To define Green's function, one can simply recognize that the solutions to Eq. (7.1) and Eq. (7.2) are identical if the boundary conditions are homogeneous and the inhomogeneous forcing term in Eq. (7.1) is defined as [104]

$$\frac{1}{k}g(\mathbf{x}, t) = \frac{1}{\alpha}\delta(\mathbf{x} - \mathbf{x}_0, t - \tau) \quad (7.6)$$

Thus, Green's functions can be defined explicitly using the solution of Eq. (7.1) under these conditions. Solutions are often sought using one of three methods [104]: I) separation of variables, II) Laplace transform, and III) the Galerkin method. The former two methods are usually imple-

mented in systems with simple geometries and uniform material properties, where solutions to the heat conduction equation with homogeneous boundary conditions are analytically accessible. While both methods are extensible to more complex, multi-layer systems via numerical approximations [105, 106, 121, 122], they can become intractable when considering higher-dimensional geometries (e.g., 2D, 2D-axisymmetric, and 3D).

For arbitrarily complex systems, the latter approach (i.e., the Galerkin method) is most flexible, and will be the one utilized throughout this thesis. In the Galerkin method, Green's function is constructed using a finite sum of eigenfunctions that approximate the solution of Eq. (7.1) given the condition in Eq. (7.6). Adapted from [104] and [107],

$$G(\mathbf{x}, t, \mathbf{x}_0, \tau) = \sum_{n=1}^N \frac{k}{\alpha} \phi_n(\mathbf{x}) \phi_n(\mathbf{x}_0) e^{-\Lambda_{n,n}(t-\tau)} \quad (7.7)$$

In Eq. (7.7), $\Lambda_{n,n}$ represents the eigenvalue accompanying each eigenfunction ϕ_n . The eigenfunctions are approximated using an N -ordered set of linearly independent basis functions $\hat{\mathbf{f}}$

$$\phi_n = \sum_{j=1}^N \psi_{j,n} \hat{f}_j \quad (7.8)$$

In Eq. (10.45), basis functions $\hat{f}_j$ are required to satisfy all homogeneous boundary and, for multimaterial systems, interface continuity conditions. Basis functions are scaled by the eigenvector ψ_n accompanying each eigenfunction ϕ_n . Eigenvalues $\Lambda_{n,n}$ and eigenvectors ψ_n in Eq. (7.7) and (7.8) are solutions to the generalized eigenvalue problem [104, 107]

$$(\mathbf{A} + \Lambda \mathbf{B}) \Psi = 0 \quad (7.9)$$

where $\mathbf{A}$ and $\mathbf{B}$ are termed *interaction matrices* [108]

$$\mathbf{A} = - \int_V k \nabla \hat{\mathbf{f}} \nabla \hat{\mathbf{f}}^T dV \quad (7.10)$$

$$\mathbf{B} = \int_V \frac{k}{\alpha} \hat{\mathbf{f}} \hat{\mathbf{f}}^T dV \quad (7.11)$$

and Ψ is a matrix of eigenvectors

$$\Psi = [\psi_1 \ \dots \ \psi_N] \quad (7.12)$$

Massa *et al.* have shown that a high reconstruction accuracy can be achieved using primitive polynomial basis function sets (e.g., $\hat{f}_j = x^j$) to construct Green's function [107]. This is notable, as these basis functions do not strictly satisfy the criteria that *all* boundary conditions must be homogeneous; instead, they are often defined only to enforce the homogeneous boundary conditions in the physical problem. Nonetheless, primitive polynomial functions are particularly attractive for multimaterial systems, where global basis functions can be defined using combinations of polynomial basis function sets across domain boundaries [104]. Furthermore, Massa *et al.* have shown that polynomial basis functions yield faster convergence of IHT problems using substantially fewer terms compared to trigonometric basis functions, which form an identical construction to Green's functions derived using separation of variables [107]. For this reason, Green's functions for all model systems studied in the subsequent chapters will be constructed using various adaptations of primitive polynomial basis function sets.

7.2 Numerical Treatment and IHT Implementation

7.2.1 Discretization

In the second step of the reconstruction approach, the Green's function representation of the measured temperature rise T (Eq. (7.4)) is inverted to recover the unknown surface heat flux boundary condition q . For measurements sampled at a finite rate, Eq. (7.4) must be discretized into the form of a linear system [13, 106]

$$\mathbf{T} = -\frac{\alpha}{k} \mathbf{G} \mathbf{q} \quad (7.13)$$

For a measurement time series of length N_t , $\mathbf{T}$ is a $N_t - 1$ sized vector representing the measured temperature rise, disregarding the temperature measurement at $t = 0$. In Eq. (7.13) $\mathbf{G}$ is an $N_t - 1 \times N_t - 1$ lower triangle matrix where each element represents an incremental integration of Green's function [13]

$$G_{i,j} = \int_{t_{j-1}}^{t_j} \int_S G(\mathbf{x}, t_i, \mathbf{x}_0, \tau) dS_0 d\tau \quad (7.14)$$

for $j \leq i$. It is important to note that the construction of $\mathbf{G}$ via Eq. (7.14) assumes no spatial variation of q over the surface and constant temporal behavior between t_{i-1} and t_i (i.e., piecewise-constant behavior over the entire time domain). The former assumption is relaxed in Chapter 9, where q varies spatially in a piecewise fashion over a 2D-axisymmetric, multi-material model system.

If the measurement sample rate is coarse, a linear interpolation of q between t_{i-1} and t_i

$$q(t)|_{t_{i-1} \rightarrow t_i} = q_{i-1} \left(\frac{t_i - t}{\Delta t} \right) + q_i \left(1 + \frac{t - t_i}{\Delta t} \right) \quad (7.15)$$

may be embedded in the definition of $\mathbf{G}$, where Δt is the time between measurement samples.

Expanding the incremental integration of the right-hand side of Eq. (7.4) to include the heat flux from Eq. (7.15), yields

$$\begin{aligned} \int_{t_{j-1}}^{t_j} \int_S G(\mathbf{x}, t_i, \mathbf{x}_0, \tau) q(\tau) dS_0 d\tau \\ = q_{j-1} \int_{t_{j-1}}^{t_j} \int_S G(\mathbf{x}, t_i, \mathbf{x}_0, \tau) \left(\frac{t_j - \tau}{\Delta t} \right) dS_0 d\tau \\ + q_j \int_{t_{j-1}}^{t_j} \int_S G(\mathbf{x}, t_i, \mathbf{x}_0, \tau) \left(1 + \frac{\tau - t_j}{\Delta t} \right) dS_0 d\tau \end{aligned} \quad (7.16)$$

Evaluating the terms on the right-hand side of Eq. (7.16) for $j \leq i \in \{1, 2, 3, \dots, N_t - 1\}$ produces the linear system

$$\mathbf{T} = -\frac{\alpha}{k} (\mathbf{G}_1 \mathbf{q}^- + \mathbf{G}_2 \mathbf{q}) \quad (7.17)$$

where $q_j^- = q_{j-1}$, and $\mathbf{G}_1$ and $\mathbf{G}_2$ represent the matrix integrals corresponding with the first and second terms on the right-hand side of Eq. (7.16), respectively. Applying a column shift to $\mathbf{G}_1$ to align the indices of $\mathbf{q}^-$ and $\mathbf{q}$ then simplifying gives

$$\mathbf{T} = -\frac{\alpha}{k} (\mathbf{G}_1^+ + \mathbf{G}_2) \mathbf{q} \quad (7.18)$$

Therefore, when seeking a solution where $q(t)$ is linearly interpolated between measurement

points, $\mathbf{G} = \mathbf{G}_1^+ + \mathbf{G}_2$. The elements of $\mathbf{G}_1^+$ can be calculated directly as

$$G_{1_{i,j}}^+ = \int_{t_j}^{t_{j+1}} \int_S G(\mathbf{x}, t_i, \mathbf{x}_0, \tau) \left(\frac{t_{j+1} - \tau}{\Delta t} \right) dS_0 d\tau \quad (7.19)$$

for $j \leq i$.

7.2.2 Inversion of the Linear System

Once discretized, Eq. (7.13) is inverted to recover the solution of $\mathbf{q}$. Explicitly, the solution of $\mathbf{q}$ seeks to minimize the squared l^2 -norm of the measurement residual

$$\min \left\{ \left\| \mathbf{T} + \frac{\alpha}{k} \mathbf{G} \mathbf{q} \right\|_2^2 \right\} \quad (7.20)$$

In practice, solutions derived using Eq. (7.20) are susceptible to large instabilities if noise is present in the input temperature measurement time series $\mathbf{T}$. Therefore, an additional stability mechanism is needed to adequately recover a physical solution. Regularization, introduced in Chapter 6, is an effective and widely used technique to mitigate the amplification of measurement noise present in $\mathbf{T}$ [13, 87, 106, 113], typically implemented by introducing an additional penalty function to Eq. (7.20). The most common approach is Tikhonov regularization, where the solution of Eq. (7.13) seeks to simultaneously minimize the squared l^2 -norms of the measurement residual and the solution itself, such that [13, 110]

$$\min \left\{ \left\| \mathbf{T} + \frac{\alpha}{k} \mathbf{G} \mathbf{q} \right\|_2^2 + \|\lambda \mathbf{q}\|_2^2 \right\} \quad (7.21)$$

In Eq. (7.21), the regularization coefficient λ is introduced to tune the strength of the penalty function embedded in the second l^2 -norm.

For linear systems with constant material properties, the unknown heat flux q is calculated directly using Eq. (7.21) via least squares. In this scenario, the Green's function formalism allows for the computational cost of an IHT problem to be reduced to that of a linear regression with $N_t - 1$ variables. The approach is, thus, inherently optimized for IHT problems, and exceptionally well-positioned to reduce the cost of post-flight reconstruction efforts. When nonlinearities, such as temperature-dependent thermal properties or source terms, are introduced into the Green's function IHT approach, the solution of q via Eq. (7.21) may become iterative. However, by utilizing various forms of the Green's function solution equation to account for these nonlinearities, the added computational cost is minimal.

7.3 Modifications to the Green's Function IHT Framework for Entry Spacecraft

A primary objective of this thesis is to apply the Green's function IHT framework to the reconstruction of atmospheric entry heat loads from measurements embedded in ablative TPS. The formalism established in this chapter, however, is tailored for linear systems with constant thermal properties, with only recent contributions adapting it for nonlinear systems [109]. Furthermore, the numerical techniques introduced in this chapter for stabilizing ill-posed IHT problems have notable drawbacks when applied to TPS materials, which are typically more insulating than the materials used in aerothermodynamic ground test facilities that have been a focal point of the IHT community. To extend the Green's function IHT framework for atmospheric entry applications, two novel adaptations are investigated in subsequent chapters of this thesis:

1. *A sensor fusion-based regularization approach to stabilize ill-posed IHT problems for spacecraft TPS.*

To improve the stability of IHT solutions from measurements embedded in insulating TPS materials, an alternative sensor fusion-based regularization approach is investigated in Chapters 8 and 9 for regions of spacecraft which contain collocated heat flux and temperature probe instrumentation. The approach leverages the intrinsic stability of the heat flux sensor measurement and properties of the parabolic heat conduction equation to solve the IHT problem via a measurement surrogate.

2. *A modification of the Green's function framework to accommodate nonlinear heat conduction phenomena in ablative TPS.*

Ablative TPS materials exhibit many complex, nonlinear material response phenomena that strongly influence the IHT solution. Current methods, however, fall short of being able to model ablative TPS materials using the Green's function approach. In Chapter 10, the Green's function formalism is amended to accommodate ablative TPS phenomena, such as pyrolysis and internal gas transport, in the IHT formulation.

The following chapters are organized to gradually introduce the two concepts into the IHT algorithm. The sensor fusion approach is developed first in Chapter 8 and investigated on a 1D linear system. In Chapter 9, the Green's function formalism is applied to a 2D-axisymmetric model system with temperature-dependent TPS properties to study the sensor fusion approach on a more complex configuration. In Chapter 10, heat conduction phenomena in ablative TPS materials are thoroughly introduced and infused within the Green's function approach to recover the surface heating conditions on spacecraft TPS. The algorithm components developed in Chapters

8-10 are used in Chapter 11 to reconstruct the heating conditions on experimental ground test and flight systems, including the Mars 2020 atmospheric entry spacecraft.

7.4 Summary

The Green's function framework for IHT problems is introduced. The approach leverages the Green's function solution equation to efficiently model the thermal response of the TPS at the temperature probe location and recovers the surface heat flux boundary condition from the discretized Green's function solution equation via least squares linear regression. In the reconstruction algorithm, Green's functions are constructed using the Galerkin method. For measurements sampled at coarse temporal intervals, a technique is introduced to discretize the Green's function solution equation in a way that embeds a linear interpolation of the surface heat flux boundary condition between sample points into the IHT solution. The IHT solution is stabilized using solution regularization. The formalism established in this chapter forms the basis of the IHT approach studied in this thesis, from which two novel adaptations are made in subsequent chapters to extend the methodology into atmospheric entry applications.

Chapter 8: Sensor-Fusion Based Regularization for Ill-Posed Inverse Heat Transfer Problems

In the Green's function framework, the transient heat conduction problem is cast as a linear system of equations of the form $\mathbf{Ax} = \mathbf{b}$, where the surface heat flux boundary condition is recovered directly through least squares inversion of the linear system. As stated in Chapters 6 and 7, this operation, however, is ill-posed, and requires an additional regularization mechanism to damp noise-induced instabilities in the solution. This behavior is not characteristic of the Green's function framework; in fact, all IHT problems which aim to recover surface heating conditions from subsurface temperature measurements will encounter similar instabilities. Rather, the amplification of noise in IHT problems is due to characteristics of the governing parabolic heat conduction equation: as information diffuses from the surface into the solid depth, high-frequency components are damped most severely. In essence, solids act as strong low-pass filters. As a result, in IHT problems, high frequency components in the temperature measurement, such as noise, become selectively amplified in the reconstructed heating solution, requiring additional mechanisms to obtain stable solutions.

This chapter details the development of a sensor fusion-based regularization approach to stabilize ill-posed IHT problems using collocated heat flux sensor and temperature probe instrumentation, leveraging existing measurement configurations flown aboard atmospheric entry

spacecraft. The motivation for the new approach is given first, highlighting the potential shortfalls of applying conventional regularization methods to highly insulating TPS materials. The basis for the sensor fusion methodology is introduced next and applied to the Green’s function IHT framework detailed in Chapter 7. The performance of the regularization methodology is evaluated on a 1D linear numerical verification test case with input measurements perturbed with noise. The approach is benchmarked against Tikhonov regularization, the most commonly used regularization approach in the IHT community, to demonstrate the benefits of leveraging collocated heat flux and temperature probe instrumentation in IHT problems when both are accessible.

8.1 Motivation for a New Regularization Approach

To motivate the development of a new regularization approach, a more thorough introduction of ill-posed inverse problems is given with added context for spacecraft TPS materials. Hansen [13] develops a rigorous mathematical intuition to the ill-posed nature of deconvolution problems (of the same nature as the IHT problem explored here) using a singular value decomposition (SVD). In the linear system $\mathbf{Ax} = \mathbf{b}$, the SVD of $\mathbf{A}$ encodes information about the spectral content of the linear system

$$\mathbf{A} = \sum_{i=1}^N \mathbf{u}_i \sigma_i \mathbf{v}_i^T \quad (8.1)$$

where σ_i are the singular values of $\mathbf{A}$, size $N \times N$, and $\mathbf{u}_i$ and $\mathbf{v}_i$ are the left and right singular vectors, respectively. With increasing size of $\mathbf{A}$, the singular values σ_i approach zero while the singular vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ concurrently become more oscillatory. As a consequence, in the direct inversion of the linear system [13]

$$\mathbf{x} = \sum_{i=1}^N \frac{\mathbf{u}_i^T \mathbf{b}}{\sigma_i} \mathbf{v}_i \quad (8.2)$$

the high-order oscillatory singular vectors couple with high-frequency components in $\mathbf{b}$ and are amplified proportional to the inverse of their corresponding near-zero singular values. In the Green's function approach, $\mathbf{A}$ represents the Green's function matrix $\mathbf{G}$ (ignoring a scaling factor of $-\alpha/k$), $\mathbf{b}$ represents the temperature measurement time series $\mathbf{T}$, and $\mathbf{x}$ is the unknown surface heat flux boundary condition $\mathbf{q}$.

A number of regularization approaches have been explored by the numerical analysis community to stabilize ill-posed problems. In Chapters 6 and 7, the concept of Tikhonov regularization, perhaps the most pervasive regularization approach, was introduced [110], where the minimization of the solution residual is balanced by an additional term penalizing the squared l^2 -norm of the solution itself

$$\min \{ \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2 + \|\lambda\mathbf{x}\|_2^2 \} \quad (8.3)$$

where the regularization coefficient λ tunes the magnitude of the penalty term. The solution of Eq. (8.3) can be expressed in terms of its SVD [13]

$$\mathbf{x} = \sum_{i=1}^N \frac{\sigma_i^2}{\sigma_i^2 + \lambda^2} \frac{\mathbf{u}_i^T \mathbf{b}}{\sigma_i} \mathbf{v}_i \quad (8.4)$$

where the behavior of the regularization approach is more explicit. From Eq. (8.4), it can be seen how Tikhonov regularization eliminates noise-induced instabilities by artificially suppressing high-frequency solution components through definition of the parameter $\lambda^2 > 0$. Generally, this is the underlying operating mechanism behind all *spectral regularization* methods, which

have solutions occupying the form of

$$\mathbf{x} = \sum_{i=1}^N f_i \frac{\mathbf{u}_i^T \mathbf{b}}{\sigma_i} \mathbf{v}_i \quad (8.5)$$

where f is often termed a *filter function*. Other examples include truncated SVD [13], Landweber iteration [123], and conjugate gradient least squares [124]. While spectral regularization techniques are very effective at damping noise-induced instabilities, they contain a number of drawbacks:

1. Regularization parameters (e.g., λ in Tikhonov regularization) must be tuned for measurement characteristics such as noise level, sample rate, and solution magnitude.
2. The optimal regularization parameters are often constrained within a relatively tight window. Too low, and the solution error is dominated by non-physical oscillations. Too high, and the solution error is dominated by errors in the steady regions, as low frequency solution components become attenuated.
3. Sharp features in the solution (e.g., step changes in heat flux) are smoothed, even when optimal regularization parameters are defined.

The first and second drawbacks are general caveats of spectral regularization methods for all reconstruction applications. The third drawback, however, is exacerbated when applied to spacecraft TPS materials, as TPS materials are, by design, highly insulating. Because of this, they behave as very strong low-pass filters and are more likely to amplify noise from the input temperature measurement.

To further demonstrate the challenges associated with applying spectral regularization tech-

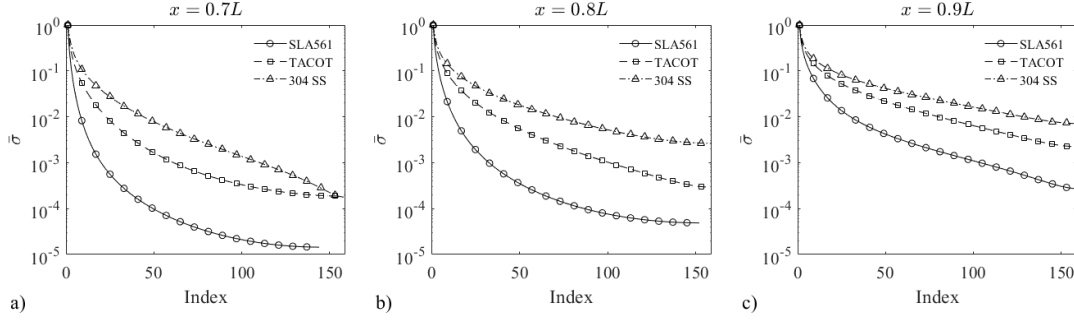


Figure 8.1: Comparison of the singular values of the Green's function matrix for three different materials with temperature probe locations of a) $x = 0.7L$, b) $x = 0.8L$, and c) $x = 0.9L$. Due to numerical round-off errors, a number of trailing zero singular values are omitted from the SLA561 series in a) and b).

niques to IHT problems for spacecraft TPS, Fig. 8.1 compares the singular values of a Green's function matrix $\mathbf{G}$ for three different materials, Viking-era SLA561 [125], TACOT [126], and 304 Stainless Steel (304 SS), at three different temperature probe depths. The former two materials are representative of common low-density ablative TPS, while the latter material is commonly used for wind tunnel test articles. Singular values in Fig. 8.1 are normalized by the largest singular value for each material.

$$\bar{\sigma} = \frac{\sigma}{\max(\sigma)} \quad (8.6)$$

In Fig 8.1, singular values corresponding with the two TPS materials are shown to decay much more rapidly than those corresponding with 304 SS. Considering Eq. (8.5), a much stronger filter function will be required to obtain a stable solution in TPS material systems for the same temperature measurement time series. Figure 8.1 also underscores the effect of temperature probe location: the further a temperature probe is placed from the surface, the more unstable the IHT problem becomes. For highly insulating materials, the decrease in stability with increasing probe depth is rapid. Generally, the behavior in Fig. 8.1 implicates that spectral regularization techniques will generate oversmoothed solutions when applied to spacecraft TPS, out of neces-

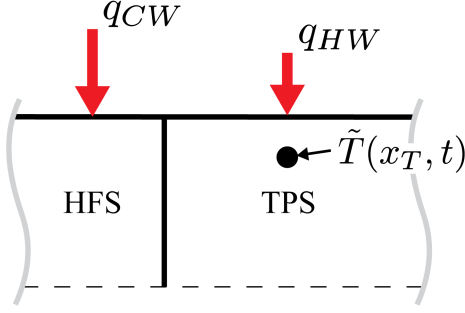


Figure 8.2: Schematic of the TPS-sensor system

sity for stability. Thus, alternative regularization methods are sought to improve the robustness of IHT methods for atmospheric entry applications.

8.2 Sensor Fusion Regularization Approach

Many recent and future atmospheric entry spacecraft are configured with both heat flux sensors and temperature probe instrumentation at select locations. Examples include the backshell shoulder region of the Mars 2020 spacecraft [15], the forebody heat shield of the LOFTID flight demonstrator [20], and the backshell of the Dragonfly spacecraft [127]. A schematic of a collocated heat flux sensor (HFS) and TPS-embedded temperature probe is shown in Fig. 8.2, herein referred to as the *TPS-sensor system*. During transient heating scenarios, the disparate thermal properties between the heat flux sensor and the TPS drives a large divergence between the surface temperatures of each component [14, 28, 128]; while the surface of the insulating TPS becomes hot, the heat flux sensor surface remains cold, close to initial conditions. As a result, a difference arises in the net heat flux absorbed into the cold-wall heat flux sensor, q_{CW} , and the hot-wall TPS, q_{HW} , through various heat transfer mechanisms, such as convection and radiative dissipation. This is one of the primary reasons why the interpretation of measurements from di-

rect heat flux sensors in both atmospheric entry spacecraft and high-enthalpy ground test facilities requires extensive post processing to extract representative TPS heating values [14, 129].

Locally, however, both regions in Fig. 8.2 are exposed to the same external heating conditions throughout the entry heat pulse. Therefore, it can be expected that the time series of q_{CW} , measured by the heat flux sensor, and q_{HW} , to-be-determined, will exhibit correlated features. Critically, it can also be assumed that the discrepancy between q_{HW} and q_{CW} , which is driven by differences in the surface temperatures of the two regions, will evolve at a finite rate. This is necessary to maintain consistency with the parabolic behavior of the governing heat conduction equation, which mandates that the temperature of a diffusive solid must evolve smoothly in time. To this end, the measured time series of q_{CW} can be used as a surrogate for q_{HW} by enforcing smoothness in the relationship between the two quantities

$$q_{HW} = C(t)q_{CW} \quad (8.7)$$

In Eq. (8.7), $C(t)$ is introduced as an unknown, smooth, time-varying function which establishes a relationship between q_{HW} and q_{CW} . Recalling Eq. (7.13) from the preceding chapter, within the Green's function framework, the solution of q_{HW} is calculated from the input temperature probe measurement sampled at x_T via the least squares solution of

$$\mathbf{T} = -\frac{\alpha}{k}\mathbf{G}\mathbf{q}_{HW} \quad (8.8)$$

For the purposes of introducing the sensor fusion-based regularization approach, heat conduction is assumed to be one-dimensional in the TPS domain near the temperature probe. Substituting

Eq. (8.7) into Eq. (8.8), the IHT problem is recast into the form of

$$\mathbf{T} = -\frac{\alpha}{k} \mathbf{G} \text{diag}(\mathbf{q}_{CW}) \mathbf{C}(\mathbf{t}) \quad (8.9)$$

where $\text{diag}(\mathbf{q}_{CW})$ is a square matrix with $\mathbf{q}_{CW}$ arranged along the diagonal elements. The regularized solution of $\mathbf{C}(\mathbf{t})$ aims to simultaneously minimize the squared l^2 -norms of the solution residual and the 2nd derivative of $\mathbf{C}(\mathbf{t})$, in effect enforcing a smooth time evolution between q_{HW} and q_{CW}

$$\min \left\{ \left\| \mathbf{T} + \frac{\alpha}{k} \mathbf{G} \text{diag}(\mathbf{q}_{CW}) \mathbf{C}(\mathbf{t}) \right\|_2^2 + \|\lambda_2 \mathbf{L}_2 \mathbf{C}(\mathbf{t})\|_2^2 \right\} \quad (8.10)$$

In Eq. (8.10), the regularization coefficient λ_2 is introduced to tune the magnitude of the 2nd derivative penalty term, where $\mathbf{L}_2$ is a discrete 2nd derivative finite difference operator acting on $\mathbf{C}(\mathbf{t})$ [13]

$$\mathbf{L}_2 = \frac{1}{\Delta t^2} \begin{bmatrix} 1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2 & 1 \end{bmatrix} \quad (8.11)$$

Following the solution of $\mathbf{C}(\mathbf{t})$ from Eq. (8.10), q_{HW} is recovered via Eq. (8.7).

8.3 Mitigation of Singularities in the Regularization Surrogate

Equation (8.7) and the commensurate restatement of the IHT problem (Eq. (8.10)) are valid if the ratio $q_{CW}/q_{HW} \gg 0$ for the full measurement time series. However, if the ratio q_{CW}/q_{HW} becomes small, singularities can develop in $C(t)$, amplifying noise from the cold-wall heat flux

sensor measurement in the reconstructed net hot-wall heat flux. These conditions may develop in pulsed heating scenarios or when the TPS-sensor system is subject to sharply varying boundary conditions.

To accommodate all heating conditions generally, an additional surrogate parameter $C_0(t)$ is added to Eq. (8.7)

$$q_{HW} = C(t)q_{CW} + C_0(t)\bar{q}_{CW} \quad (8.12)$$

where $C_0(t)$ is an unknown, smooth, time-varying function. The coefficient $C_0(t)$ scales the quantity $\bar{q}_{CW}$, which represents the average value of q_{CW} up to time $t + \tau_{HFS}$, where τ_{HFS} is the response time of the heat flux sensor. Substituting Eq. (8.12) into Eq. (8.8) and recasting the minimization statement in the same fashion as before yields

$$\min \left\{ \left\| \mathbf{T} + \frac{\alpha}{k} \mathbf{G} [\text{diag}(\mathbf{q}_{CW}) \text{diag}(\bar{\mathbf{q}}_{CW})] \begin{Bmatrix} \mathbf{C}(\mathbf{t}) \\ \mathbf{C}_0(\mathbf{t}) \end{Bmatrix} \right\|_2^2 + \left\| \begin{Bmatrix} \lambda_2 \\ \lambda_2^{(0)} \end{Bmatrix} \cdot \begin{bmatrix} \mathbf{L}_2 & 0 \\ 0 & \mathbf{L}_2 \end{bmatrix} \begin{Bmatrix} \mathbf{C}(\mathbf{t}) \\ \mathbf{C}_0(\mathbf{t}) \end{Bmatrix} \right\|_2^2 \right\} \quad (8.13)$$

The regularized solution of $\mathbf{C}(\mathbf{t})$ and $\mathbf{C}_0(\mathbf{t})$ now aims to simultaneously minimize the squared l^2 -norms of the solution residual and the 2nd derivatives of $\mathbf{C}(\mathbf{t})$ and $\mathbf{C}_0(\mathbf{t})$, in effect enforcing smoothness in both parameters. In Eq. (8.13), $\lambda_2^{(0)}$ is an added parameter to tune the magnitude of the 2nd derivative penalty term for $\mathbf{C}_0(\mathbf{t})$. Once the solutions of $\mathbf{C}(\mathbf{t})$ and $\mathbf{C}_0(\mathbf{t})$ are obtained, q_{HW} is recovered via Eq. (8.12).

8.4 Application of the Sensor Fusion Approach to a 1D Model System

8.4.1 Problem Definition

A computational verification test case was constructed to analyze the robustness of the sensor fusion-based IHT algorithm to input measurements perturbed with noise and evaluate its sensitivity to varying regularization parameters. The verification test case models a one-dimensional linear system with constant material properties subject to step heat flux boundary conditions. A schematic of the 1D test case is shown in Fig. 8.3. A rod (labeled TPS) of thickness $L = 10$ mm, thermal conductivity $k = 0.05$ W/m-K, and thermal diffusivity $\alpha = 2 \times 10^{-7}$ m²/s, representative of Viking-era SLA-561 at -73°C [125], is embedded in a perfectly insulating housing adjacent to, but thermally isolated from, a heat flux sensor. The rod has an initial temperature of $\tilde{T}_0$ and is exposed to a step heat flux q_{HW} with an amplitude of $\Delta q_{HW} = 1$ kW/m² at $x = L$ and $t = t_0$. The rod is held at a constant temperature of $\tilde{T}_0$ at $x = 0$. The heat flux sensor is exposed to the same step heat flux profile, but with a larger amplitude of $\Delta q_{CW} = 1.5 \times \Delta q_{HW}$ to simulate a cold-wall boundary condition. A temperature probe is embedded in the rod at $x_T = 0.8L$.

The sensor data used in the reconstruction algorithm consist of discrete measurements imprinted with realistic transient response characteristics. The temperature probe measurement is simulated using the analytical solution of the transient heat conduction problem. The heat flux sensor measurement q_{HFS} , used as a proxy for q_{CW} , is simulated with a first-order exponential

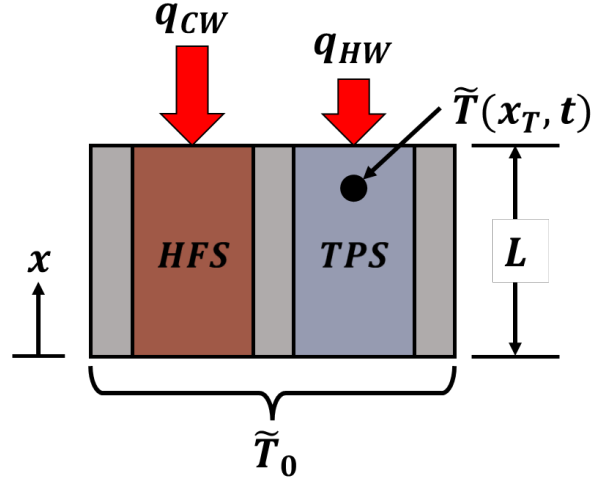


Figure 8.3: Schematic of the 1D verification test case. A 1D rod (labeled TPS) is exposed to a step heat flux and the temperature response is measured by a temperature probe at x_T . The face at $x = 0$ is held at a constant temperature $\tilde{T}_0$. A collocated heat flux sensor (labeled HFS) measures an augmented heat flux step.

rise to Δq_{CW} with a time constant of τ_{HFS}

$$q_{HFS}(t) = \begin{cases} 0 & t < t_0 \\ \Delta q_{CW} [1 - \exp(-t/\tau_{HFS})] & t \geq t_0 \end{cases} \quad (8.14)$$

Ideal temperature measurement inputs are shown in Fig. 8.4a, calculated up to 20 seconds after initiation of the step. The heat flux sensor measurement input is shown in Fig. 8.4b, calculated using a time constant τ_{HFS} of 0.2 s.

To simulate errors in the measurements, three levels of Gaussian noise are added directly to the ideal inputs in Fig. 8.4. Examples of noisy input data are shown in Fig. 8.5. Noise parameters are tabulated in Table 8.1, where σ_T and σ_{HFS} represent the standard deviations of the noise applied to the temperature probe and heat flux sensor measurements, respectively. Each measurement was sampled at a rate of 100 Hz (2500 total measurement points) to achieve

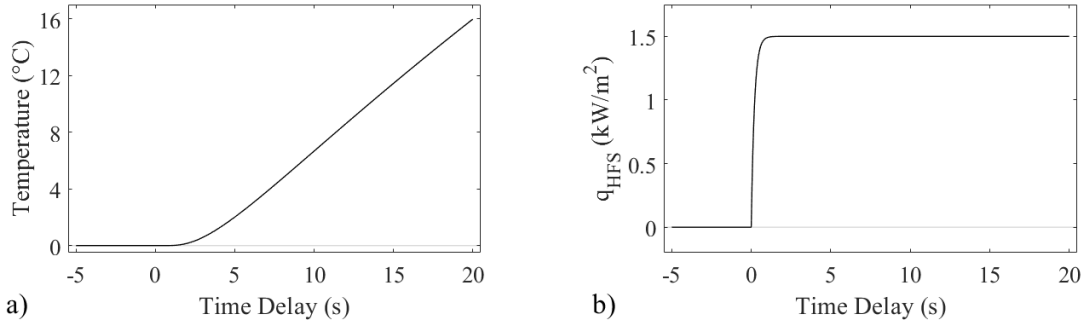


Figure 8.4: Reconstruction algorithm inputs for the 1D verification test case. a) Simulated temperature probe measurement. b) Simulated heat flux sensor measurement.

reproducible noise distributions for all reconstructions.

Table 8.1: Input temperature and heat flux sensor measurement noise parameters.

Noise Level	σ_T (°C)	σ_{HFS} (W/m²)
Low	$0.1 + 0.025 \times (\tilde{T} - \tilde{T}_0)$	37.5
Medium	$0.2 + 0.05 \times (\tilde{T} - \tilde{T}_0)$	75
High	$0.4 + 0.1 \times (\tilde{T} - \tilde{T}_0)$	150

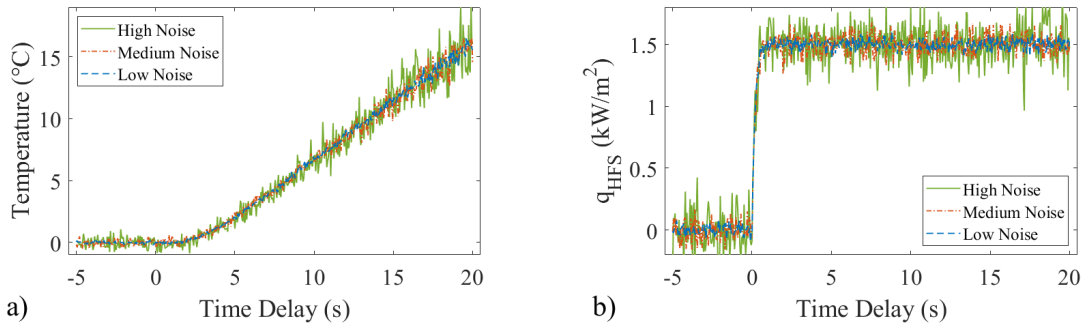


Figure 8.5: Measurement inputs with added noise, plotted at 20 Hz. a) Examples of temperature probe measurements. b) Examples of heat flux sensor measurements.

The heat flux absorbed by the rod at $x = L$ was recovered using simulated measurements from Fig. 8.5 as inputs to Eq. (8.13) and compared with the imposed step heat flux boundary condition. In the reconstruction algorithm, Green's function was constructed using an 10th-order

set of monomial basis functions (Eq. (7.8)) [107]

$$\hat{f}_j = x^j \quad (8.15)$$

and discretized assuming piecewise-constant temporal behavior of the heat flux (Eq. (7.14)).

Reconstructed heating profiles were calculated using a range of λ_2 values spanning from $\lambda_2 = 4 \times 10^5$ to $\lambda_2 = 2 \times 10^8$. The parameter $\lambda_2^{(0)}$ was defined as a large multiple of λ_2

$$\lambda_2^{(0)} = 10^3 \times \lambda_2 \quad (8.16)$$

For each level of input measurement noise and value of λ_2 , the quality of the reconstruction was quantified by calculating the root-mean-squared-error (RMSE) of the reconstructed heat flux profile q_{HW}

$$\text{RMSE} = \sqrt{\frac{1}{\text{length}(q_{HW})} \sum_{i=1}^{\text{length}(q_{HW})} \left[q_{HW}^{(i)} - q_{HW,ref}^{(i)} \right]^2} \quad (8.17)$$

where $q_{HW,ref}$ is the exact heat flux profile imposed on the surface of the rod. In all scenarios, RMSE values were calculated for the duration of the measurement beginning 2 s after the heat flux step to exclude transients near the step region. Reconstructions were repeated 50 times for each combination of noise level and λ_2 value using independently generated noise inputs to produce distributions of RMSE values.

8.4.2 Results

Results of the parametric sweeps of λ_2 are shown in Fig. 8.6 for all three noise levels. Individual datapoints represent the mean RMSE values from the 50 repeat sweeps per measurement

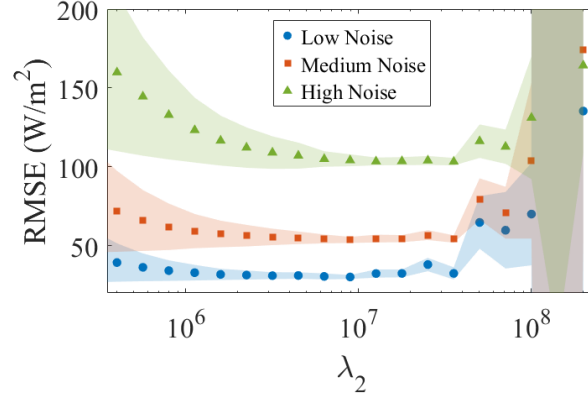


Figure 8.6: RMSE values for low, medium, and high noise measurement inputs as a function of the regularization coefficient using the sensor fusion approach. Datapoints represent the mean RMSE value from the 50 repeat sweeps. The shaded region represents a ± 1 standard deviation envelope.

noise level, with the shaded regions enveloping two standard deviations of calculated RMSE values for all sweeps. In the reconstruction algorithm, the effect of increasing λ_2 from small values is a gradual increase in the accuracy and stability of the reconstruction. At regularization coefficient values of approximately $\lambda_2 = 3 \times 10^6$ and $\lambda_2 = 5 \times 10^6$, RMSE values become insensitive to variations in the regularization coefficient for the low/medium and high input measurement noise scenarios, respectively. The reconstruction accuracy remains high as the regularization coefficient is increased to a value $\lambda_2 = 4 \times 10^7$, with all mean RMSE values falling below 105 W/m² (10.5% full-scale heat flux). Increasing the regularization coefficient beyond $\lambda_2 = 4 \times 10^7$ leads to a sharp increase in the reconstruction error for all levels of measurement noise, indicating that the solution has become unstable.

The range of stability achieved by the sensor fusion approach suggested in Fig. 8.6 is demonstrated in Fig. 8.7, where reconstructed heat flux profiles are shown using high noise measurement inputs with values of λ_2 spanning from $\lambda_2 = 5 \times 10^6$ to $\lambda_2 = 4 \times 10^7$. Qualitatively, all reconstructions appear similar. Immediately following the step, a slight overshoot in the recon-

struction is present due to the finite-rate heat flux sensor response. In the reconstructions using higher values of λ_2 (Figs. 8.7b and 8.7c), a slight down-slope in the reconstructed heating profile is observed, where the heat flux is underpredicted versus the target value at later time steps. The error in the undershoot is minor in magnitude, however, and is not observed for all repeat reconstructions (see, for example, Fig. 8.8c).

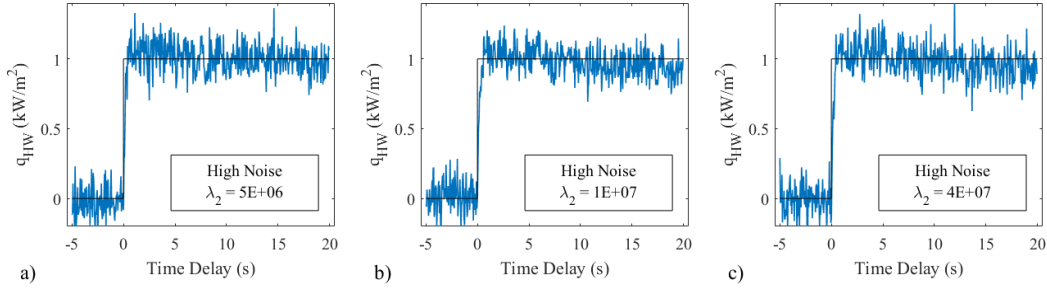


Figure 8.7: Reconstructed heat flux profiles using high levels of input measurement noise with various regularization coefficient values. a) $\lambda_2 = 5 \times 10^6$. b) $\lambda_2 = 10^7$. c) $\lambda_2 = 4 \times 10^7$. The data is plotted at a rate of 20 Hz.

The stratification of RMSE values by input measurement noise level in Fig. 8.6 is a result of the smooth coupling enforced between q_{HW} and q_{CW} (Eq. (8.12)), which causes noise from the heat flux sensor measurement proportional to q_{HW}/q_{CW} to propagate into the reconstruction. To illustrate this behavior, Fig. 8.8 presents a series of reconstructed heating profiles using high levels of temperature measurement noise, but with varying levels of heat flux sensor noise. A regularization coefficient of $\lambda_2 = 10^7$ is used for all reconstructions. As evident in Fig. 8.8, the noise in the input heat flux sensor measurement strongly dictates the noise in the reconstructed heat flux profile. Furthermore, the results in Fig. 8.8 highlight that, using the sensor fusion approach, a high quality IHT reconstruction can be achieved even when the temperature measurement is perturbed with significant amounts of noise, as long as the noise in the heat flux sensor measurement is minimal (Fig. 8.8a).

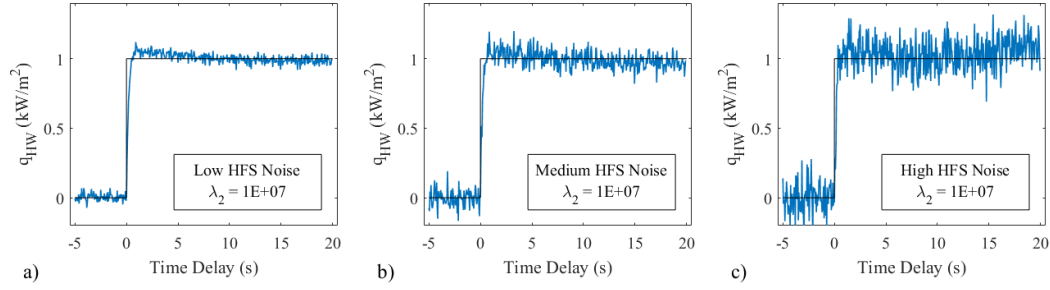


Figure 8.8: Reconstructed heat flux profiles using varying levels of heat flux sensor noise. a) Low heat flux sensor noise. b) Medium heat flux sensor noise. c) High heat flux sensor noise. In all reconstructions, a temperature measurement input with a high level of noise was used. The data is plotted at a rate of 20 Hz.

8.4.3 Comparison Against Tikhonov Regularization

The performance of the sensor fusion approach is compared against the performance of Tikhonov regularization applied to the same numerical verification test case. Results from a parallel series of sweeps using the same temperature measurement inputs (Fig. 8.5a), but stabilized using Tikhonov regularization, are presented in Fig. 8.9. Increasing the regularization coefficient λ from small values leads to a sharp decrease in the RMSE of the reconstructed heat flux profile. RMSE values continue to decrease to a local minimum at separate locations for each level of input measurement noise before increasing thereafter. Minimal RMSE values occur for the low, medium, and high measurement noise levels with regularization coefficient values of $\lambda = 37, 60$, and 90 , respectively. With large values of λ , the reconstructed heat flux profiles coalesce towards the same RMSE values. In this region, all noise-induced instabilities have become damped, and increasing the value of λ results in the attenuation of real, slowly-varying solution components.

A juxtaposition of the results shown in Fig. 8.6 and 8.9 illustrates the increase in stability that may be achieved through the use of a regularization surrogate. When using Tikhonov regularization (Fig. 8.9), and, more broadly, conventional spectral regularization techniques, the

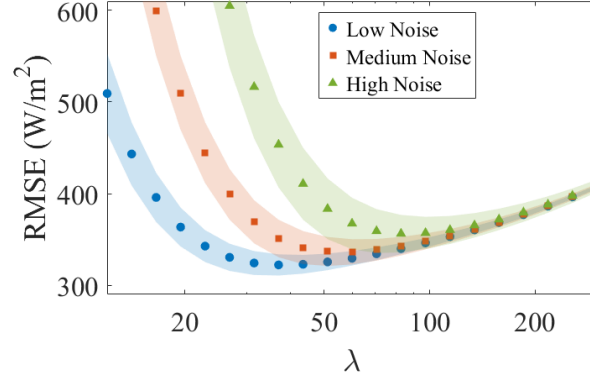


Figure 8.9: RMSE values for low, medium, and high noise measurement inputs as a function of the regularization coefficient using Tikhonov regularization. Datapoints represent the mean RMSE value from the 50 repeat sweeps. The shaded region represents a ± 1 standard deviation envelope.

regularization coefficient must be tailored for each measurement condition to achieve optimal results. In contrast, when using the sensor fusion approach, optimal results for the current numerical test case may be accessed using a range of λ_2 spanning from $\lambda_2 = 5 \times 10^6$ to $\lambda_2 = 4 \times 10^7$ for all levels of measurement noise. This level of robustness alleviates the need to define an optimal value of λ_2 for each reconstruction scenario and generalizes the IHT approach for many different measurement conditions.

To qualitatively compare the reconstruction features from both approaches, a set of reconstructed heat flux profiles using Tikhonov regularization are shown in Fig. 8.10. For each level of input measurement noise, the optimal regularization coefficient, as per the minimum RMSE, was used. The results in Fig. 8.10 illustrate the effects of increasing the input measurement noise on the optimized solutions. While the instabilities are sufficiently damped in all scenarios, the step at $t - t_0 = 0$ s becomes progressively smoothed as the optimal value of λ increases with increasing measurement input noise. Furthermore, in all scenarios, the reconstructed heat flux profile near the end of the measurement ($t - t_0 > 15$ s) diverges from the steady state value. By

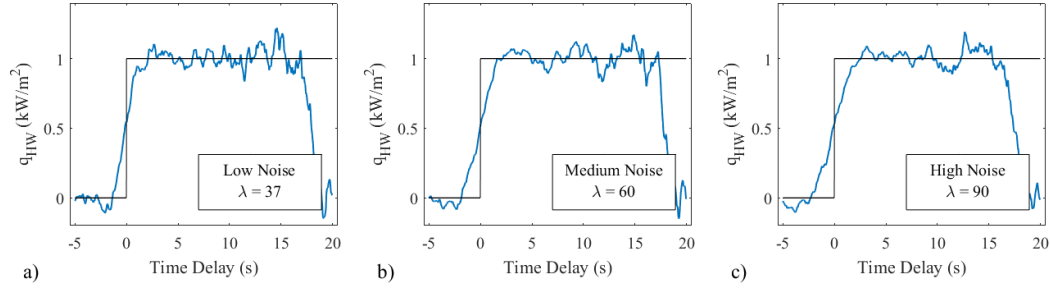


Figure 8.10: Selected reconstructed heat flux profiles for different values of measurement input noise using Tikhonov regularization. Optimal regularization coefficients of $\lambda = 37$, 60, and 90 were used for the a) Low noise, b) Medium noise, and c) High noise scenarios, respectively. The data is plotted at a rate of 20 Hz.

contrast, using the sensor fusion approach, the step feature is resolved adequately for all reconstructed heating profiles in Figs. 8.7 and 8.8 under a broader range of regularization coefficient values and measurement conditions.

While the heating conditions used in this numerical test case are simple, the results offer valuable insight into the unique behavior of the sensor fusion approach. By coupling the measured value of q_{CW} with the unknown value of q_{HW} , the solution is stabilized without explicitly damping high frequency solution components, leading to a better retention of sharp features. Furthermore, because the spectral content of the solution is not modified directly by the regularization coefficient, a large range of regularization coefficient values emerge in which the solution becomes virtually insensitive to variations in λ_2 . This robustness positions the sensor fusion approach as a promising candidate to stabilize the recovery of heating conditions on entry spacecraft, which already feature thermal instrumentation suites with the requisite collocated sensor configurations.

8.5 Summary

A sensor fusion-based regularization methodology is developed to improve the stability of reconstructed heating conditions on spacecraft TPS using measurements from collocated heat flux sensor and temperature probe instrumentation. The approach enforces a smooth time evolution in the relationship between the unknown net hot-wall heat flux absorbed by the TPS and the cold-wall heat flux measured by the heat flux sensor, which is physically justified by the parabolic, diffusive nature of heat conduction. The performance of the sensor fusion approach is evaluated on a 1D numerical test case using an analytically calculated input temperature probe measurement and a heat flux sensor measurement imprinted with real measurement characteristics. Subject to a large degree of noise in the input measurements, the approach is able to sufficiently recover the input heat flux on the model system without a significant sensitivity to suboptimal choice of the user input regularization coefficient. While noise from the collocated heat flux sensor measurement is propagated into the reconstruction using the sensor fusion approach, the solutions are generally more stable than those calculated using Tikhonov regularization.

Chapter 9: A Green's Function Sensor Fusion Approach for 2D-Axisymmetric Model Systems

In Chapter 8, a sensor fusion-based regularization methodology was introduced to improve the stability of IHT problems by leveraging measurements from collocated heat flux sensor and TPS-embedded temperature probe instrumentation. The formulation was evaluated on a model system that assumed one-dimensional heat conduction within the TPS domain. This is a valid assumption for sensor configurations in which the sensors are spaced conservatively to limit thermal interactions. However, if the temperature probe is placed in the TPS close to the heat flux sensor, multidimensional heat conduction effects may need to be accounted for.

In this chapter, the Green's function IHT approach is extended to reconstruct the heating conditions on a 2D-axisymmetric representation of the TPS-sensor system, thereby capturing the effects of multidimensional heat conduction in the TPS domain. To model the TPS-sensor system more consistent with the TPS materials used on atmospheric entry spacecraft, the effects of temperature-dependent thermal properties are introduced into the Green's function formulation. A new polynomial basis function set is developed to construct Green's functions for the multi-material model system. The robustness and performance of the Green's function sensor fusion approach, combining the nonlinear 2D-axisymmetric Green's function IHT formulation with the sensor fusion-based regularization methodology, is evaluated using a simulated heat pulse with a

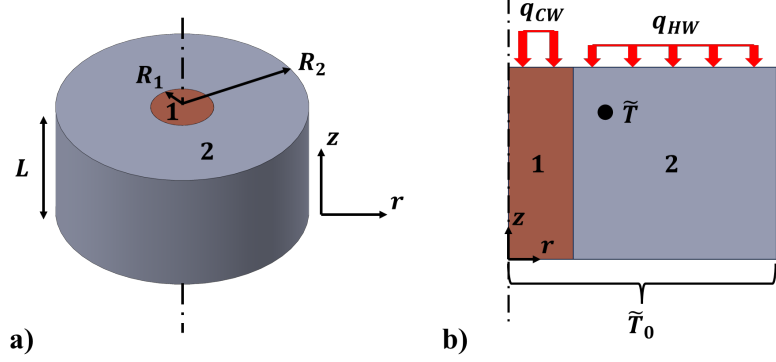


Figure 9.1: 2D axisymmetric model of the TPS-sensor system. The heat flux sensor is labeled as domain 1 and the TPS is labeled as domain 2. a) Isometric view of the model system. b) $r - z$ cross-section view of the model system. The top surface is exposed to the external environment. The bottom surface is held at a constant temperature $\tilde{T}_0$. The temperature is sampled from a probe embedded in the TPS.

sharp, transient heating feature.

9.1 Reconstruction Approach for the 2D-Axisymmetric Model System

A schematic of the 2D-axisymmetric model system is shown in Fig. 9.1. The model system consists of a cylindrical heat flux sensor (domain 1) with a radius R_1 and thickness L , surrounded by TPS (domain 2) with an outer radius R_2 . The heat flux sensor domain is modeled using constant thermal properties, whereas the TPS domain is modeled using temperature-dependent thermal properties. The temperature probe is embedded in the TPS at $\mathbf{x}_T$. The bottom surface of the model system is held at a constant temperature $\tilde{T}_0$ while the top surface is exposed to the external environment. The heat flux sensor absorbs a net cold-wall heat flux q_{CW} that is augmented versus the net hot-wall heat flux q_{HW} absorbed by the TPS due to the difference in the surface temperatures of the two domains throughout transient heating scenarios. The heat flux over the surface of each domain is assumed to be constant with respect to the radial position.

The reconstruction algorithm for the 2D-axisymmetric TPS-sensor system consists of two

main steps, largely based on the approach outlined in Chapter 7. First the TPS-sensor system is modeled using the Green's function solution equation to establish a relationship between the net hot- and cold-wall heat flux boundary conditions and the thermal response of the TPS at the temperature probe location. Due to the temperature-dependent thermal properties of the TPS domain, an additional step precedes the formulation of the Green's function solution equation to linearize the heat conduction equation. Second, the Green's function representation of the TPS-sensor system is inverted to recover the net hot-wall heat flux on the TPS surface, leveraging the sensor fusion regularization approach detailed in Chapter 8 to stabilize the solution.

9.1.1 Green's Function Representation of the TPS-Sensor System

9.1.1.1 Nonlinear Heat Conduction

The temperature distribution in a solid with temperature-dependent material properties is governed by the nonlinear heat conduction equation

$$\rho C_p(\tilde{T}) \frac{\partial \tilde{T}}{\partial t} - \nabla \cdot \left(k(\tilde{T}) \nabla \tilde{T} \right) = g(\mathbf{x}, t) \quad (9.1)$$

For use in the Green's function framework, Eq. (9.1) must be linearized. To eliminate the nonlinear terms produced by expansion of the divergence operator in Eq. (9.1), a Cole-Hopf transform $T = f(\theta)$ is employed, where T represents the temperature change from an initial reference temperature $T = \tilde{T} - \tilde{T}_0$ and θ is introduced as a transformation variable. Following [130], the

Cole-Hopf transformation $f(\theta)$ is the solution to the following homogeneous differential equation

$$\frac{d}{d\theta} \left[k(\tilde{T}) \frac{df}{d\theta} \right] = 0 \quad (9.2)$$

which allows Eq. (9.1) to be recast in terms of the transformation variable θ

$$\frac{1}{\alpha(\tilde{T})} \frac{\partial \theta}{\partial t} - \nabla^2 \theta = \frac{g(\mathbf{x}, t)}{A} \quad (9.3)$$

In Eq. (9.3), A is an arbitrary constant that arises after integrating Eq. (9.2)

$$k(\tilde{T}) \frac{df}{d\theta} = A \quad (9.4)$$

For an explicit definition of the transformation from T to θ , Eq. (9.4) is integrated indefinitely via separation of variables

$$\theta = \frac{1}{A} \int k(\tilde{T}) dT \quad (9.5)$$

Equation (9.5) is a general form of the more common Kirchhoff transformation, where A would be defined as the thermal conductivity at a reference value $A = k_0$ [130]. In fact, the Kirchhoff transformation has been used in prior Green's function-based IHT methods to accommodate temperature-dependent thermal properties [107]. However, when the thermal conductivity $k(\tilde{T})$ can be approximated by a quadratic or linear polynomial in terms of T , the Cole-Hopf transformation has the advantage that the inverse of Eq. (9.5) is significantly more tractable, due to the flexibility afforded to the arbitrary integration constant A .

To further linearize Eq. (9.3), the thermal diffusivity $\alpha(\tilde{T})$ is expanded about a reference

value $\alpha_0 = \alpha(\tilde{T}_0)$ [130, 131]

$$\alpha(\tilde{T}) = \alpha_0 + \alpha_T(T) \quad (9.6)$$

Substituting Eq. (9.6) into Eq. (9.3) and simplifying yields

$$\frac{1}{\alpha_0} \frac{\partial \theta}{\partial t} - \nabla^2 \theta = \frac{g'(\mathbf{x}, t)}{A} \quad (9.7)$$

where

$$\frac{g'(\mathbf{x}, t)}{A} = \frac{\alpha_T(T)}{\alpha_0} \nabla^2 \theta + \frac{\alpha_0 + \alpha_T(T)}{\alpha_0} \frac{g(\mathbf{x}, t)}{A} \quad (9.8)$$

Equation (9.7) is in the form of a linear, inhomogeneous partial differential equation, where the effects of temperature-dependent material properties are captured within the transformation $T = f(\theta)$ and by the inhomogeneous source term (Eq. (9.8)).

9.1.1.2 Green's Function Representation of the TPS-Sensor System

The Green's function solution equation for Eq. (9.7) is analogous to that defined in Chapter 7, Eq. (7.3)

$$\begin{aligned} \theta(\mathbf{x}, t) = & \int_V \theta(\mathbf{x}_0, 0) G(\mathbf{x}, t, \mathbf{x}_0, 0) dV_0 \\ & + \int_0^t \int_V \frac{\alpha_0}{A} g'(\mathbf{x}_0, \tau) G(\mathbf{x}, t, \mathbf{x}_0, \tau) dV_0 d\tau \\ & + \int_0^t \int_S \alpha_0 \nabla_0 \theta(\mathbf{x}_0, \tau) G(\mathbf{x}, t, \mathbf{x}_0, \tau) \cdot \hat{\mathbf{n}} dS_0 d\tau \\ & - \int_0^t \int_S \alpha_0 \theta(\mathbf{x}_0, \tau) \nabla_0 G(\mathbf{x}, t, \mathbf{x}_0, \tau) \cdot \hat{\mathbf{n}} dS_0 d\tau \end{aligned} \quad (9.9)$$

In Eq. (9.9), the first volume integral represents the effects of initial conditions. The second volume integral captures the effects of temperature-dependent material properties and internal volumetric source terms. The third integral accounts for the effects of Neumann boundary conditions. The last integral term accounts for Dirichlet boundary conditions.

Within the Cole-Hopf transformation, Fourier's law relates the gradient of θ normal to the system boundary with the absorbed heat flux into the solid q via [130]

$$q = -A\nabla\theta \cdot \hat{\mathbf{n}} \quad (9.10)$$

and allows the third integral term in Eq. (9.9) to be expressed as a function of q

$$- \int_0^t \int_S \frac{\alpha_0}{A} q(\mathbf{x}_0, \tau) G(\mathbf{x}, t, \mathbf{x}_0, \tau) dS_0 d\tau \quad (9.11)$$

Because the only inhomogeneous boundary condition in the TPS-sensor system is captured by the third term in Eq. (9.9) and the initial conditions are zero by definition of $T = \tilde{T} - \tilde{T}_0$, the first and last integral terms in Eq. (9.9) vanish. Expanding the remaining volume and surface integrals to separate the heat flux sensor and TPS domains, the transformed temperature distribution within the TPS is modeled via

$$\begin{aligned} \theta(\mathbf{x}, t) = & - \int_0^t \int_{S_{TPS}} \frac{\alpha_{0,TPS}}{A_{TPS}} q_{HW}(\tau) G_{TPS,TPS}(\mathbf{x}, t, \mathbf{x}_0, \tau) dS_0 d\tau \\ & - \int_0^t \int_{S_{HFS}} \frac{\alpha_{0,HFS}}{A_{HFS}} q_{CW}(\tau) G_{TPS,HFS}(\mathbf{x}, t, \mathbf{x}_0, \tau) dS_0 d\tau \\ & + \int_0^t \int_{V_{TPS}} \frac{\alpha_{0,TPS}}{A_{TPS}} g'(\mathbf{x}_0, \tau) G_{TPS,TPS}(\mathbf{x}, t, \mathbf{x}_0, \tau) dV_0 d\tau \end{aligned} \quad (9.12)$$

The surface integral terms on the right-hand side of Eq. (9.12) represent individual contributions

of q_{HW} and q_{CW} to the transformed temperature rise measured within the TPS. The additional volume integral term captures the effects of the temperature-dependence of the TPS material properties on the transformed temperature rise within the TPS. The parameter G represents the Green's function of the TPS due to a heat flux boundary condition or source term imposed on the TPS ($G_{TPS,TPS}$) or on the heat flux sensor ($G_{TPS,HFS}$). In the Green's function sensor fusion approach, Green's functions are constructed for the TPS-sensor system using the Galerkin method to approximate the linear heat conduction eigenfunctions ϕ_n . Adapted from Eq. (7.7) in Chapter 7

$$G_{l,m}(\mathbf{x}, t, \mathbf{x}_0, \tau) = \sum_{n=1}^N \frac{k_{0,m}}{\alpha_{0,m}} \phi_n^{(l)}(\mathbf{x}) \phi_n^{(m)}(\mathbf{x}_0) e^{-\Lambda_{n,n}(t-\tau)} \quad (9.13)$$

where $l, m = \text{TPS or HFS}$, and $k_{0,m}$ represents the thermal conductivity of domain m evaluated at $\tilde{T}_0$. Eigenfunctions ϕ_n are expressed as the sum of an N -ordered set of scaled basis functions $\hat{\mathbf{f}}^{(l)}$

$$\phi_n^{(l)} = \sum_{j=1}^N \psi_{j,n} \hat{f}_j^{(l)} \quad (9.14)$$

where $l = \text{TPS or HFS}$. In Eq. (9.13), the transformed temperature-of-interest is located in domain l and the heat flux boundary condition or source is imposed on domain m . In Eq. (9.14), basis functions $\hat{f}_j$ are required to satisfy the homogeneous boundary and interface continuity conditions imposed on the system, and are defined separately for each domain in Section 9.1.1.3. Eigenvalues $\Lambda_{n,n}$ and eigenvectors ψ_n are solved using the approach described in Chapter 7, where the interaction matrices $\mathbf{A}$ (Eq. (7.10)) and $\mathbf{B}$ (Eq. (7.11)) are defined piecewise, as per [108].

$$\mathbf{A} = \sum_{l \in \{HFS, TPS\}} - \int_{V_l} k_{0,l} \nabla \hat{\mathbf{f}}^{(l)} \nabla \hat{\mathbf{f}}^{(l)T} dV \quad (9.15)$$

$$\mathbf{B} = \sum_{l \in \{HFS, TPS\}} \int_{V_l} \frac{k_{0,l}}{\alpha_{0,l}} \hat{\mathbf{f}}^{(l)} \hat{\mathbf{f}}^{(l)T} dV \quad (9.16)$$

9.1.1.3 Basis Functions for the 2D-Axisymmetric Model System

Polynomial basis functions are defined for the 2D-axisymmetric model system to approximate the eigenfunctions of the transformed heat conduction equation (Eq. (9.7)). While the basis functions are defined in θ -space, they must be tailored to satisfy continuity of temperature and heat flux across domain boundaries in the physical problem. Thus, the basis functions and Cole-Hopf transformation for each domain are defined concurrently. The Cole-Hopf transformation for the TPS domain is defined using a high-order polynomial representation of the TPS thermal conductivity $k_{TPS}(\tilde{T})$

$$k_{TPS}(\tilde{T}) = k_{0,TPS} + k_T T + k_{TT} T^2 + \dots \quad (9.17)$$

where $k_T, k_{TT}, \dots$ are polynomial coefficients. Substituting Eq. (9.17) into Eq. (9.5) and integrating yields

$$\theta = \frac{1}{A_{TPS}} \left[k_{0,TPS} T + \frac{k_T T^2}{2} + \frac{k_{TT} T^3}{3} + \dots \right] \quad (9.18)$$

which is valid for $R_1 \leq r \leq R_2$. To maintain a similar transformation space at the TPS-sensor interface, the Cole-Hopf transformation for the heat flux sensor domain is defined as

$$\theta = \frac{1}{A_{HFS}} (k_{0,HFS} T) \quad (9.19)$$

for $0 \leq r < R_1$. In Eq. (9.19), A_{HFS} is defined relative to the TPS domain transformation parameters A_{TPS} and $k_{TPS}(\tilde{T})$

$$A_{HFS} = A_{TPS} \frac{\int_0^{\bar{T}_{12}} k_{0,HFS} dT}{\int_0^{\bar{T}_{12}} k_{TPS}(\tilde{T}) dT} \quad (9.20)$$

where $\bar{T}_{12}$ is the estimated average temperature rise from $\tilde{T}_0$ at the TPS-sensor domain interface.

If $\bar{T}_{12}$ is estimated to be small, then A_{HFS} reduces to

$$A_{HFS} = A_{TPS} \frac{k_{0,HFS}}{k_{0,TPS}} \quad (9.21)$$

It is worth reemphasizing that A_{TPS} is arbitrary and can be defined using any constant value, excluding 0.

By defining Eq. (9.18) and (9.19) as such, continuity of temperature at the domain interface is sufficiently approximated by the transformed continuity condition

$$\theta_{HFS}|_{r=R_1} = \theta_{TPS}|_{r=R_1} \quad (9.22)$$

Maintaining continuity of heat flux across the domain interface in transformed space is simply satisfied using Eq. (9.10)

$$A_{HFS} \nabla \theta_{HFS} \cdot \hat{\mathbf{n}} \Big|_{r=R_1} = -A_{TPS} \nabla \theta_{TPS} \cdot \hat{\mathbf{n}} \Big|_{r=R_1} \quad (9.23)$$

In addition to the interface continuity conditions, basis functions must satisfy the following

homogeneous boundary conditions

$$\theta|_{z=0} = 0 \quad (9.24a)$$

$$A_{HFS} \nabla \theta \cdot \hat{\mathbf{n}} \Big|_{r=0} = 0 \quad (9.24b)$$

$$A_{TPS} \nabla \theta \cdot \hat{\mathbf{n}} \Big|_{r=R_2} = 0 \quad (9.24c)$$

which correspond with the following physical boundary conditions

$$T|_{z=0} = 0 \quad (9.25a)$$

$$k_{0,HFS} \nabla T \cdot \hat{\mathbf{n}} \Big|_{r=0} = 0 \quad (9.25b)$$

$$k_{TPS}(\tilde{T}) \nabla T \cdot \hat{\mathbf{n}} \Big|_{r=R_2} = 0 \quad (9.25c)$$

Equation (9.24a) enforces a constant temperature boundary condition at $z = 0$. Equation (9.24b) enforces a smooth radial temperature distribution along the center axis within the heat flux sensor domain. Eq. (9.24c) enforces an adiabatic wall condition on the outer circumference of the TPS.

Once the transformed interface continuity and boundary conditions are well defined, basis functions are constructed. The basis functions $\hat{f}_j^{(l)}$ are decomposed into components aligned along the orthogonal directions r and z

$$\hat{f}_j^{(l)}(r, z) = \mu_j^{(l)}(r) \nu_j^{(l)}(z) \quad (9.26)$$

The axial boundary condition (Eq. (9.24a)) is satisfied by defining ν_j for both domains as a

monomial basis function

$$\nu_j^{(HFS)} = \nu_j^{(TPS)} = z^{\xi_j} \quad (9.27)$$

where ξ is a set of integers up to order $\mathcal{M}$. The radial boundary and interface continuity conditions are satisfied by recasting $\mu_j^{(HFS)}$ and $\mu_j^{(TPS)}$ in terms of a global basis function μ_j and perturbation μ'_j [104, 115]

$$\mu_j^{(HFS)} = \mu_j \quad (9.28)$$

$$\mu_j^{(TPS)} = \mu_j + \Gamma \mu'_j \quad (9.29)$$

where μ'_j is designed to vanish at the $r = R_1$ interface and Γ is specified to satisfy Eq. (9.23). The global basis function must satisfy both adiabatic constraints given by Eqs. (9.24b) and (9.24c), while the perturbation basis function must only satisfy the adiabatic constraint given by (9.24c). The global basis function is defined as a polynomial expression which contains derivatives with respect to r that vanish at $r = 0$ and $r = R_2$, adapted from [104]

$$\mu_j = \frac{(2\eta_j - 2)r^{2\eta_j}}{R_2^{2\eta_j-1}} - \frac{2\eta_j r^{2\eta_j-2}}{R_2^{2\eta_j-3}} \quad (9.30)$$

In Eq. (9.30), η is a set of integers up to order $\mathcal{N}$ specified to complement the set ξ , such that all permutations of the elements of η and ξ are represented, e.g., $\eta = \{1, 2, 3, 1, 2, 3\}$ and $\xi = \{1, 1, 1, 2, 2, 2\}$. The permutations of η and ξ form a total Galerkin basis function order of $N = \mathcal{N} \times \mathcal{M}$.

The perturbation basis function μ'_j must be designed to vanish at $r = R_1$, however its first derivatives must be non-zero for all terms at $r = R_1$ to maintain continuity of heat flux across the domain boundary. These constraints, along with the adiabatic conditions at $r = R_2$, are satisfied

with the following polynomial modified from [115]

$$\mu'_j = R_1 R_2 \eta_j \left[1 - \left(\frac{r}{R_1} \right)^{\eta_j + 1} \right] - R_2^2 (\eta_j + 1) \left[1 - \left(\frac{r}{R_1} \right)^{\eta_j} \right] \quad (9.31)$$

Substituting Eqs. (9.31) and (9.30) into Eqs. (9.28) and (9.29), differentiating with respect to r , and evaluating at the interface (Eq. (9.23)) yields the definition of Γ

$$\Gamma = \frac{\left(\frac{A_{HFS}}{A_{TPS}} - 1 \right) 2\eta_j (2\eta_j - 2) \left(\frac{R_1}{R_2} \right)^{2\eta_j - 3} \left[\frac{R_1^2}{R_2^2} - 1 \right]}{R_2 \eta_j (\eta_j + 1) \left[\frac{R_2}{R_1} - 1 \right]} \quad (9.32)$$

9.1.2 Recovery of the Net Hot-Wall Heat Flux

In the second step of the reconstruction algorithm, the net hot-wall heat flux is recovered from the Green's function representation of the TPS-sensor system. The net hot-wall heat flux is first isolated from all other quantities in Eq. (9.12). For tractability, measured quantities (i.e., the transformed temperature and cold-wall heat flux) are combined into a single parameter $\gamma(\mathbf{x}, t)$

$$\gamma(\mathbf{x}, t) = \theta(\mathbf{x}, t) + \int_0^t \int_{S_{HFS}} \frac{\alpha_{0,HFS}}{A_{HFS}} q_{cw}(\tau) G_{TPS,HFS}(\mathbf{x}, t, \mathbf{x}_0, \tau) dS_0 d\tau \quad (9.33)$$

Likewise, the source terms which capture the effects of temperature-dependent material properties are represented by a single parameter $g_{tot}(\mathbf{x}, t)$

$$g_{tot}(\mathbf{x}, t) = \int_0^t \int_{V_{TPS}} \frac{\alpha_{0,TPS}}{A_{TPS}} g'(\mathbf{x}_0, \tau) G_{TPS,TPS}(\mathbf{x}, t, \mathbf{x}_0, \tau) dV_0 d\tau \quad (9.34)$$

Equation (9.12) is then expressed as

$$\gamma(\mathbf{x}, t) - g_{tot}(\mathbf{x}, t) = - \int_0^t \int_{S_{TPS}} \frac{\alpha_{0,TPS}}{A_{TPS}} q_{HW}(\tau) G_{TPS,TPS}(\mathbf{x}, t, \mathbf{x}_0, \tau) dS_0 d\tau \quad (9.35)$$

where the unknown net hot-wall heat flux q_{HW} is isolated on the right-hand side. Following the algorithm formulation outlined in Chapter 7, Eq. (9.35) is recast as a discrete linear system

$$\gamma - \mathbf{g}_{tot} = - \frac{\alpha_{0,TPS}}{A_{TPS}} \mathbf{G} \mathbf{q}_{HW} \quad (9.36)$$

Substituting Eq. (8.12) into Eq. (9.36) yields the sensor fusion-based form of the linear system

$$\gamma - \mathbf{g}_{tot} = - \frac{\alpha_{0,TPS}}{A_{TPS}} \mathbf{G} [\text{diag}(\mathbf{q}_{CW}) \text{diag}(\bar{\mathbf{q}}_{CW})] \begin{Bmatrix} \mathbf{C}(\mathbf{t}) \\ \mathbf{C}_0(\mathbf{t}) \end{Bmatrix} \quad (9.37)$$

where $\mathbf{C}(\mathbf{t})$ and $\mathbf{C}_0(\mathbf{t})$ are solved by simultaneously minimizing the l^2 -norms of the measurement residual and the 2nd derivatives of $\mathbf{C}(\mathbf{t})$ and $\mathbf{C}_0(\mathbf{t})$

$$\min \left\{ \left\| \gamma - \mathbf{g}_{tot} + \frac{\alpha_{0,TPS}}{A_{TPS}} \mathbf{G} [\text{diag}(\mathbf{q}_{CW}) \text{diag}(\bar{\mathbf{q}}_{CW})] \begin{Bmatrix} \mathbf{C}(\mathbf{t}) \\ \mathbf{C}_0(\mathbf{t}) \end{Bmatrix} \right\|_2^2 + \left\| \begin{Bmatrix} \lambda_2 \\ \lambda_2^{(0)} \end{Bmatrix} \cdot \begin{bmatrix} \mathbf{L}_2 & 0 \\ 0 & \mathbf{L}_2 \end{bmatrix} \begin{Bmatrix} \mathbf{C}(\mathbf{t}) \\ \mathbf{C}_0(\mathbf{t}) \end{Bmatrix} \right\|_2^2 \right\} \quad (9.38)$$

Because g' in g_{tot} is an implicit function of $\nabla^2 \theta$, g_{tot} must be calculated using iterative solutions of q_{HW} . This process is detailed more thoroughly in Chapter 10, however a summary

is given here: The solution of Eq. (9.38) is first calculated assuming $g_{tot} = 0$. The initial reconstructed time series of q_{HW} (via $C(t)$ and $C_0(t)$) is then used to calculate $\nabla^2\theta$ at select locations in the TPS by applying the Laplacian operator ∇^2 to the Green's function solution equation (Eq. (9.12)). Calculated values for $\nabla^2\theta$ in the TPS domain are then summed in a Gaussian quadrature scheme to approximate the integral g_{tot} , which is then used in Eq. (9.38) to calculate the updated solution. The calculation of q_{HW} and g_{tot} is repeated until the solution of q_{HW} converges, typically in under 10 iterations.

9.2 Results

A computational test case was constructed to analyze the accuracy of the reconstruction algorithm applied to the 2D-axisymmetric model system. The test case models a heat flux sensor with radius $R_1 = 4$ mm embedded in the TPS domain with an outer radius $R_2 = 16$ mm and thickness $L = 10$ mm. The heat flux sensor is modeled as copper with constant material properties. The TPS is modeled using temperature-dependent thermal properties representative of Viking-era SLA-561 [125]. The model system has an initial temperature of $\tilde{T}_0 = -73$ °C and is exposed to a 40 s duration sinusoidal convective heat pulse at t_0 through variation of the freestream temperature $\tilde{T}_\infty$ at the top surface (Fig. 9.2a). To simulate a boundary layer transition event, the convective heat transfer coefficient h undergoes a sharp increase (three-fold rise over 0.25 s) prior to the peak of the freestream temperature (Fig. 9.2b).

Input temperature and heat flux measurements are generated for the reconstruction algorithm from finite element analysis (FEA) simulations (Ansys MechanicalTM). The temperature measurement (Fig. 9.3a) is simulated using the FEA-generated nodal temperature solu-

tion located at $\mathbf{x}_T = \{0.4R_2, 0.9L\}$. The heat flux sensor measurement q_{HFS} (Fig. 9.3b) is retrieved from the FEA-generated heat flux solution along the center axis of the model at $\mathbf{x}_{HFS} = \{0, 0.95L\}$. Both measurements are sampled at a rate of 8 Hz.

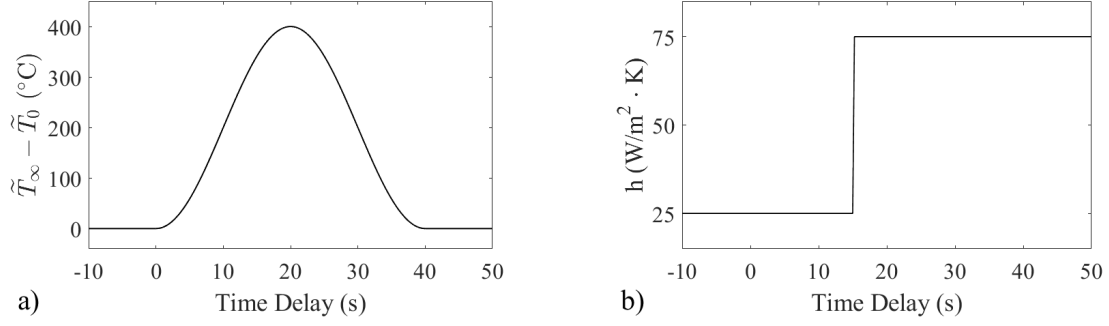


Figure 9.2: Convective heat flux boundary conditions. a) Sinusoidal pulse of the freestream temperature relative to $\tilde{T}_0$. b) Convective heat transfer coefficient with sharp ramp at $t - t_0 = 15$ s.

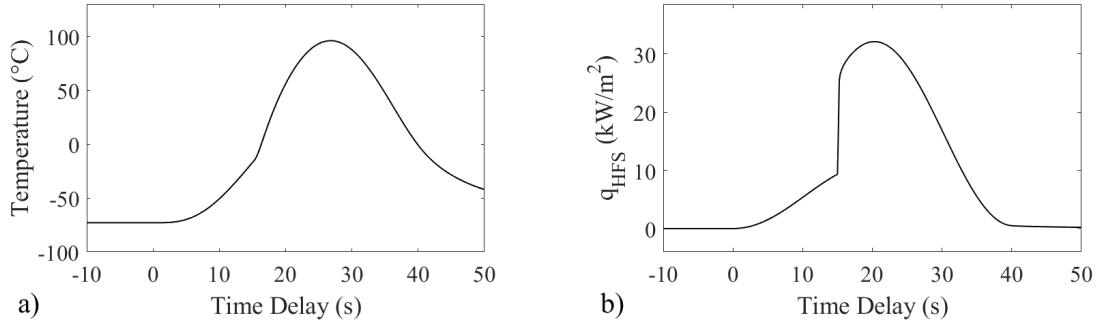


Figure 9.3: FEA-generated measurement inputs for the 2D-axisymmetric model system. a) Simulated temperature probe measurement. b) Simulated heat flux sensor measurement.

The heat flux sensor measurement is used directly to represent the cold-wall heat flux q_{CW} absorbed at the heat flux sensor surface. The location $\mathbf{x}_{HFS}$ is sufficiently close to the surface such that the lag between q_{HFS} and q_{CW} is governed by a time constant of ~ 40 ms, far less than the interval between datapoints (see, for example, [80]).

9.2.1 Verification of the Reconstruction Algorithm on the 2D-Axisymmetric Model System

The accuracy of the reconstruction algorithm is first evaluated on the 2D-axisymmetric model system with noise-free measurement inputs. Basis function orders of $\mathcal{N} = 9$ and $\mathcal{M} = 9$, corresponding to the radial and axial components, respectively, were used to construct Green's function. Green's function was discretized using the linear interpolation scheme outlined in Section 7.2.1 (Eq. (7.16)). Regularization coefficient values of $\lambda_2 = 10^3$ and $\lambda_2^{(0)} = 10^3 \times \lambda_2$ were used to stabilize the solution.

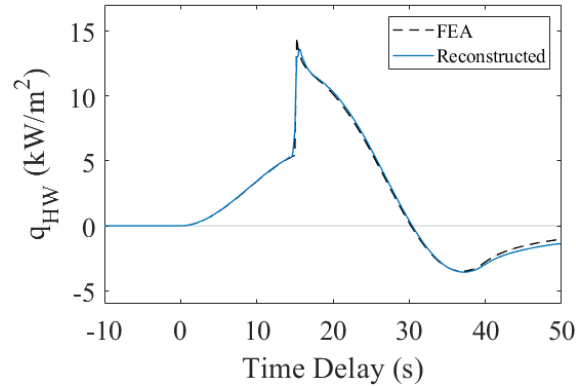


Figure 9.4: Reconstructed net hot-wall heat flux using noise-free input measurements (solid blue line). The dashed black line represents the FEA-generated TPS surface heat flux.

The reconstructed net hot-wall heat flux profile using noise-free measurement inputs is shown in Fig. 9.4, juxtaposed with the FEA-generated solution shown as a dashed line. The FEA-generated heat flux profile is extracted from the TPS surface at the same radial location as the temperature probe, i.e., at $\mathbf{x} = \{0.4R_2, L\}$. In the FEA-generated heat flux profile, the increase in the heat transfer coefficient during the simulated boundary layer transition event at $t - t_0 = 15$ s introduces a proportional increase in the surface heat flux. Directly following the

transition event, the TPS surface temperature increases rapidly towards the freestream temperature, reducing the magnitude of the heat flux. The heat flux continues to decrease following the peak in the freestream temperature (Fig. 9.2a) and changes sign as the freestream temperature falls below the TPS surface temperature.

In the reconstructed net hot-wall heat flux profile, the sharp jump in heating during the simulated boundary layer transition event is resolved to within 6% of the FEA-generated value (13.6 kW/m² reconstructed (solid blue line) vs. 14.5 kW/m² FEA (dashed black line)) with a delay of 0.375 s in the temporal location of peak heating (hereafter termed the *peak heat time*). Following peak heating, and through the remainder of the heat pulse, the reconstruction algorithm closely predicts the FEA-generated net hot-wall heat flux profile.

9.2.2 Effect of Varying the Radial and Axial Basis Function Orders

The effect of varying the radial and axial Galerkin basis function orders, $\mathcal{N}$ and $\mathcal{M}$, respectively, on the the reconstructed surface heat flux is analyzed. Figure 9.5 shows a series of reconstructed heating profiles for all permutations of $\mathcal{N}, \mathcal{M} \in \{5, \dots, 9\}$. For low values of both $\mathcal{N}$ and $\mathcal{M}$, the reconstructed heat flux is accurate in the steady regions, however the peak heating feature is damped. Exceptions to this behavior include the reconstructed heating profiles for $\mathcal{M} = 5$, which are able to better resolve the peak heating feature, but are generally less accurate in the steady region. In the peak heating region, the reconstructed heat flux profiles are primarily sensitive to the choice of axial basis function order $\mathcal{M}$, where higher values lead to a sharper reconstructed peak feature, with the exception noted for $\mathcal{M} = 5$. For a constant value of $\mathcal{M}$, incremental improvements in the reconstruction are obtained by increasing $\mathcal{N}$ (along the rows in

Fig. 9.5).

9.2.3 Effect of Varying the Temperature Probe Position

The objective of the 2D-axisymmetric Green's function formulation is to resolve multidimensional heat conduction phenomena within the TPS near the heat flux sensor and TPS interface. The performance of the reconstruction algorithm is analyzed subject to variations in the temperature probe location near the interface. Figure 9.6 shows a series of reconstructed heating conditions (Fig. 9.6a-d) and temperature measurement inputs (Fig. 9.6e-h) for various radial temperature probe locations r_T , ranging from $r_T = 0.3R_2$ to $r_T = 0.5R_2$. In all scenarios, the temperature measurement is sampled from the same axial location ($z = 0.9L$). Also presented in Fig. 9.6a-d are the FEA-generated heat flux solutions sampled from the surface at the radial position of the temperature probe ($r = r_T$) and at the outer edge of the TPS ($r = R_2$), where the heat flow is entirely one-dimensional.

As shown in Fig. 9.6e-h, the temperature measurement changes substantially as the probe is moved towards the heat flux sensor interface ($r = 0.25R_2$), suggesting the development of large multidimensional heat flows within the TPS at the probe location. The surface heat flux sampled from $r = r_T$ also begins to vary closer to the interface, indicating that the FEA-generated surface heating profile is not constant over the entire TPS domain. The reconstructed heat flux profiles (Fig. 9.6a-d), however, only vary slightly. Far away from the heat flux sensor and TPS interface, the reconstruction algorithm recovers the $r = r_T$ and $r = R_2$ surface heating profiles almost exactly (Fig. 9.6d). Close to the heat flux sensor and TPS interface, the reconstruction algorithm recovers a heating profile in between the heat flux profiles sampled from $r = r_T$ and $r = R_2$

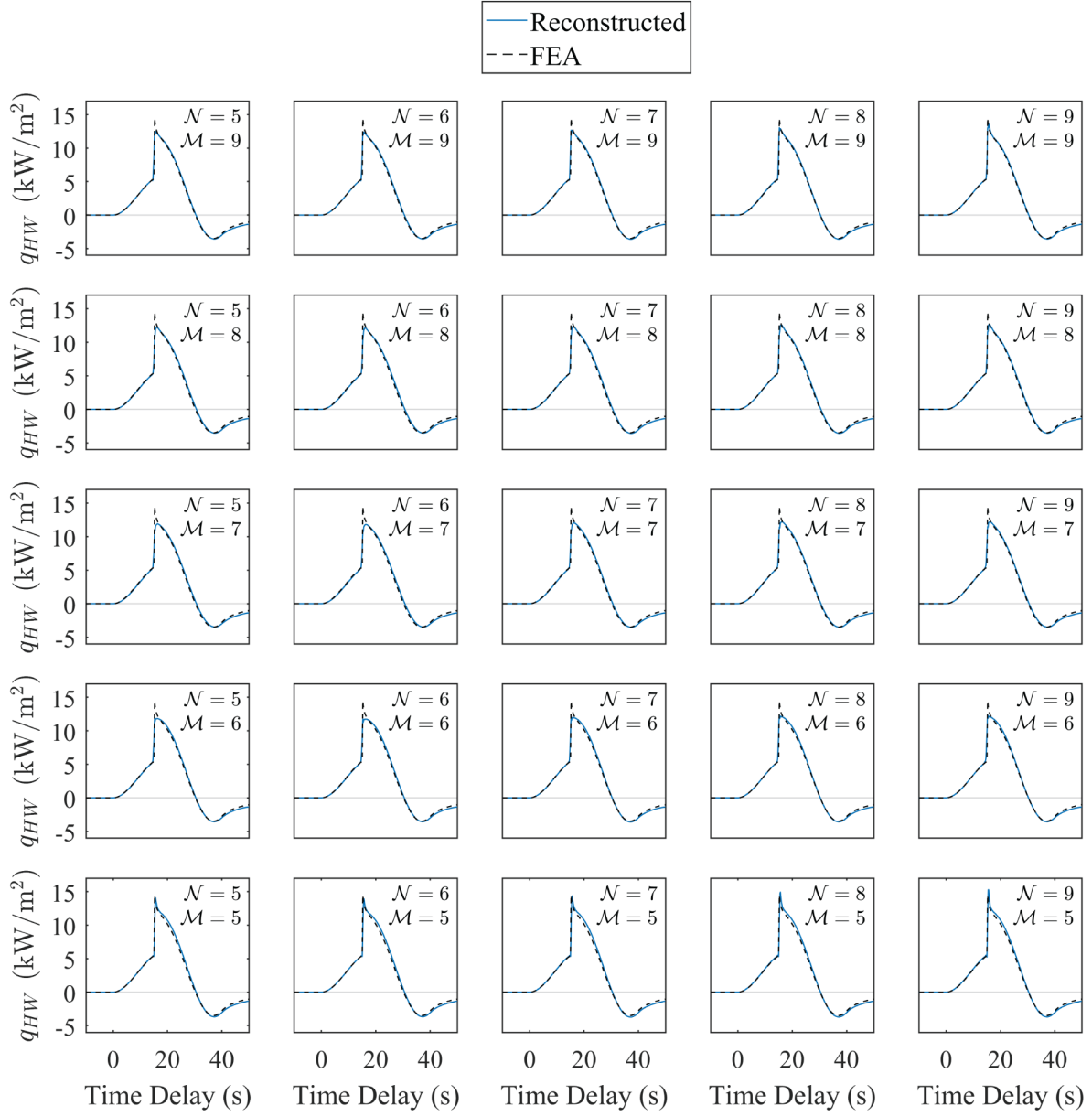


Figure 9.5: Reconstructed heating profiles for the 2D-axisymmetric model system using various combinations of radial $\mathcal{N}$ and axial $\mathcal{M}$ basis function orders. In each plot, the FEA reference solution is shown as a dashed line.

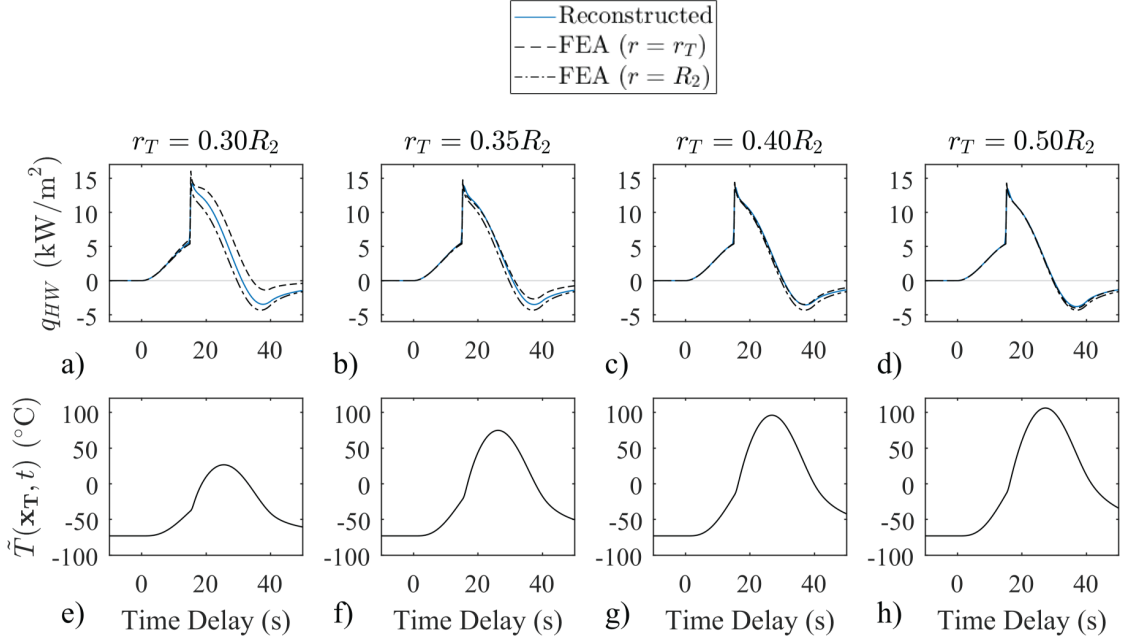


Figure 9.6: Reconstructed heating profiles with the temperature probe positioned at various radial positions r_T in the TPS. In all scenarios, the probe is positioned along the z -direction at $0.9L$. The dashed line represents the FEA-generated surface heat flux sampled from the same radial location as the temperature probe. The dashed-dot line represents the FEA-generated surface heat flux sampled from the edge of the model system at $r = R_2$. The heat flux sensor and TPS interface is at $r = 0.25R_2$.

(Fig. 9.6a). This behavior is attributed to the assumption of a uniform heat flux over the entire TPS surface in the reconstruction algorithm, which has the effect of recovering a heat flux profile that represents a spatially weighted average of q_{HW} , influenced most by the heating conditions closest to the temperature probe. Thus, from the results in Fig. 9.6, it can be inferred that the main source of reconstruction error in the numerical test case originates from a lack of information on the spatial variation of q_{HW} near the interface.

9.2.4 Robustness of the Sensor Fusion Approach Applied to the 2D-Axisymmetric Model System

The sensitivity of the reconstructed net hot-wall heat flux to variations in the user input regularization parameter is analyzed for the 2D-axisymmetric model system with measurements perturbed with noise. The input measurements from Fig. 9.3, corresponding with a temperature probe positioned at $\mathbf{x}_T = \{0.4R_2, 0.9L\}$, were perturbed using three levels of noise (Fig. 9.7), tabulated in Table 9.1. Reconstructed net hot-wall heat flux profiles were calculated using a range of regularization coefficient values spanning from $\lambda_2 = 10^6$ to $\lambda_2 = 10^9$ for each level of measurement noise. In each reconstruction, the parameter $\lambda_2^{(0)}$ was defined as $\lambda_2^{(0)} = 10^3 \times \lambda_2$. Green's function was constructed using basis function orders of $\mathcal{N} = 9$ and $\mathcal{M} = 9$.

Table 9.1: Input temperature and heat flux sensor measurement noise parameters for the 2D-axisymmetric model system.

Noise Level	σ_T ($^{\circ}\text{C}$)	σ_{HFS} (kW/m^2)
Low	$1 + 0.025 \times (\tilde{T} - \tilde{T}_0)$	$0.2 + 0.025 \times q_{HFS}$
Medium	$2 + 0.05 \times (\tilde{T} - \tilde{T}_0)$	$0.4 + 0.05 \times q_{HFS}$
High	$4 + 0.1 \times (\tilde{T} - \tilde{T}_0)$	$0.8 + 0.1 \times q_{HFS}$

For each combination of input measurement noise and regularization coefficient λ_2 , the root-mean-squared-error (RMSE, Eq. (8.17)) of the entire reconstructed heating profile, error in the reconstructed peak heat flux, and error in the peak heat time were calculated. Sweeps of λ_2 were repeated 50 times for each level of measurement noise to generate distributions of results. The errors in the peak heat flux and peak heat time were calculated from the distributions of the 50 repeat sweeps for each combination of λ_2 and measurement noise level to remove the influence

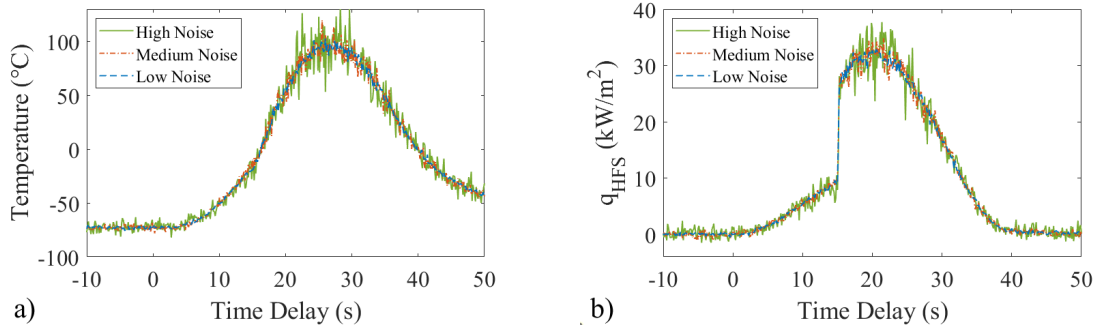


Figure 9.7: 2D-axisymmetric model system measurement inputs with added measurement noise. a) Examples of simulated temperature probe measurements. b) Examples of simulated heat flux sensor measurements.

of local noise fluctuations on the interpretation of the two quantities.

Results of the parametric sweeps of λ_2 are shown in Fig. 9.8. Consistent with the 1D results presented in Chapter 8 (Fig. 8.6), the results from the 2D-axisymmetric model system outline a region of regularization coefficient values where the RMSE, peak heat flux error, and delay in the peak heat time are insensitive to variations in λ_2 . This region of stability is bounded by approximately $\lambda_2 = 10^7$ and $\lambda_2 = 10^8$ for all levels of measurement noise (Fig. 9.8a). Within these bounds, the peak heat flux error remains small, within 0.5 kW/m² (Fig. 9.8b), and the peak heat time is predicted to within 0.75 s for all levels of measurement noise (Fig. 9.8c). Beyond a value of $\lambda_2 = 1.5 \times 10^8$, the reconstructed heat flux diverges and becomes unstable.

Results from the sensor fusion approach are compared next with results from a parallel series of sweeps using the same temperature measurement inputs (Fig. 9.7a), but stabilized using Tikhonov regularization (Fig. 9.9). Sweeps were performed using a range of regularization coefficient values spanning from $\lambda = 100$ to $\lambda = 2000$. In the reconstructed heat flux profiles, RMSE values are lowest around a different regularization coefficient value for each level of input measurement noise, specifically $\lambda = 380$, 530, and 740 for the low, medium, and high noise

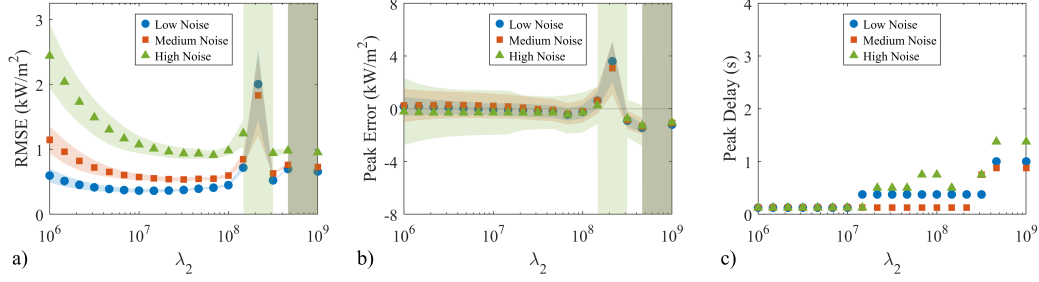


Figure 9.8: Reconstruction results from sweeps of λ_2 values using the sensor fusion-based regularization approach. a) RMSE. Datapoints represent the average RMSE value, calculated for each sweep individually, with the shaded region enveloping ± 1 standard deviation. b) Error in the magnitude of peak heating. Datapoints represent the peak heat flux error in the mean solution from the 50 repeat sweeps. The shaded region represents a ± 1 standard deviation envelope from the distribution of the 50 repeat sweeps at the peak heat time. c) Error in the peak heat time. Datapoints represent the error in the peak heat time in the mean profile from the 50 repeat sweeps.

scenarios, respectively (Fig. 9.9a). However, when only considering minimization of the RMSE values for optimal results, significant errors emerge in the reconstructed peak heating values. For example, at the optimum value of λ for the low measurement noise scenario, as per the minimum RMSE, the peak heat flux is underpredicted by 3 kW/m² (20% of peak heat flux), with a delay of 1.625 s in the peak heat time (Figs. 9.9b and 9.9c).

To analyze the solution features from each regularization method, Fig. 9.10 and Fig. 9.11 show families of reconstructed heat flux profiles using the sensor fusion approach (Fig. 9.10) and Tikhonov regularization (Fig. 9.11). In each scenario, the average value of q_{HW} at each timestep (solid blue line) and the standard deviation of q_{HW} (blue shaded region) were calculated from the 50 repeat reconstructions. A regularization coefficient value of $\lambda_2 = 2.2 \times 10^7$ was used in the sensor fusion approach for all input measurement noise levels (Fig. 9.10). Optimal regularization coefficient values, as per the minimum RMSE, were used in the Tikhonov regularization-based reconstructions (Fig. 9.11).

For all levels of input measurement noise, the sensor fusion approach can resolve the peak

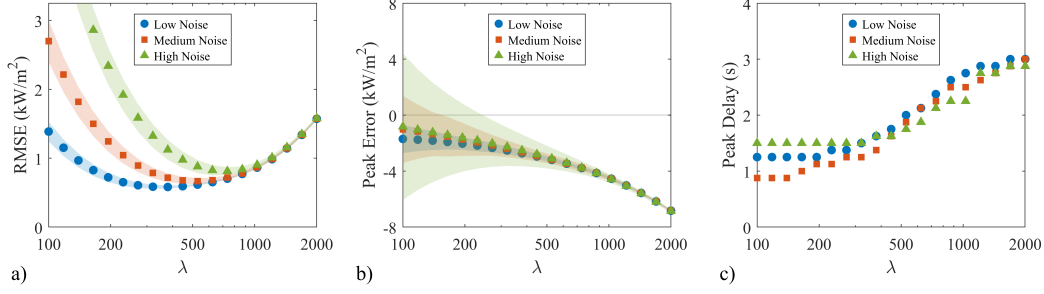


Figure 9.9: Reconstruction results from sweeps of λ values using Tikhonov regularization. a) RMSE. Datapoints represent the average RMSE value, calculated for each sweep individually, with the shaded region enveloping ± 1 standard deviation. b) Error in the magnitude of peak heating. Datapoints represent the peak heat flux error in the mean solution from the 50 repeat sweeps. The shaded region represents a ± 1 standard deviation envelope from the distribution of the 50 repeat sweeps at the peak heat time. c) Error in the peak heat time. Datapoints represent the error in the peak heat time in the mean profile from the 50 repeat sweeps.

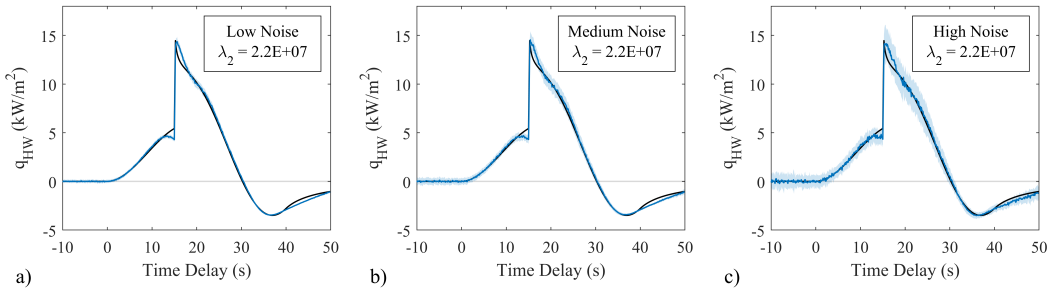


Figure 9.10: Selected heat flux profiles stabilized using the sensor fusion-based regularization approach. a) Low noise. b) Medium noise. c) High noise

heat flux to within 0.5 kW/m^2 (4% of peak heat flux) with a delay equal to, or less than, 0.5 s in the peak heat time (Fig. 9.10). Qualitatively, all mean reconstructed heat flux profiles in Fig. 9.10 are similar. The effect of increasing the input measurement noise is an increase in the noise and variance of the reconstructed heat flux values, notably at, and directly following, the region of peak heating. This is primarily attributed to the increase in the heat flux sensor noise, as discussed in Chapter 8. Prior to the sharp increase in heat flux at $t - t_0 = 15 \text{ s}$, a dip occurs in all reconstructed heat flux profiles. For large values of λ_2 , which act to add stiffness to the behavior of $C(t)$ and $C_0(t)$, features in q_{HW} that are not represented in q_{CW} , such as the sharp peak in

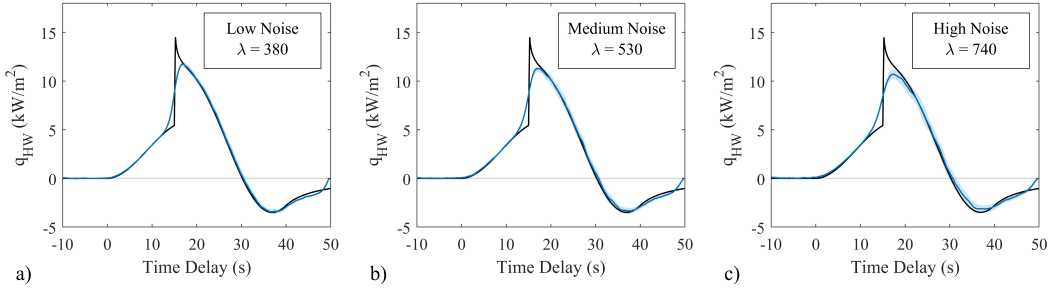


Figure 9.11: Selected heat flux profiles stabilized using Tikhonov regularization. a) Low noise. b) Medium noise. c) High noise. Reconstructions are shown using the optimal values of λ for each level of measurement noise.

the heat flux following the boundary layer transition event, may lead to reconstruction errors nearby in the time series. Yet, these errors do not have a demonstrable impact on other solution features, such as the sharp heating augmentation at $t - t_0 = 15$ s and smooth cool-down phase following $t - t_0 = 20$ s, which are well-resolved in the reconstructed heat flux profiles for all levels of measurement noise. The reconstructed heat flux profiles using Tikhonov regularization, by contrast, further underscore the benefits of the sensor fusion approach when reconstructing the heating conditions on highly insulating TPS materials. When the regularization coefficient is optimized around a minimum RMSE value, all sharp solution features are damped, with the degree of attenuation increasing with the regularization coefficient value.

A broader implication of these results is that, using spectral regularization techniques, the interpretation of the reconstructed peak heat flux, which largely influences the design and sizing of TPS, varies depending on the noise in the input temperature measurement. In the sensor fusion approach, a uniqueness of the solution is obtained, where the reconstruction characteristics become insensitive to the choice of regularization coefficient and the uncertainties of the peak heat flux are purely due to measurement uncertainties. This robustness is extremely valuable for space exploration, where experimental datasets are sparse and the results derived from them are

often difficult to validate.

9.2.5 Mitigation of Singularities

In the sensor fusion-based regularization approach, the solution is stabilized by enforcing smooth behavior in $C(t)$. In Eq. (8.12), an additional parameter $C_0(t)$ was introduced to correct for singularities that arise when the ratio q_{CW}/q_{HW} becomes small. In the current numerical test case, this condition occurs at the end of the heat pulse, where the heat flux sensor measurement (Fig. 9.3b) quickly approaches zero while the TPS surface heat flux continues to vary (Fig. 9.4) as the surface temperature more slowly converges towards the free stream temperature (Fig. 9.2a).

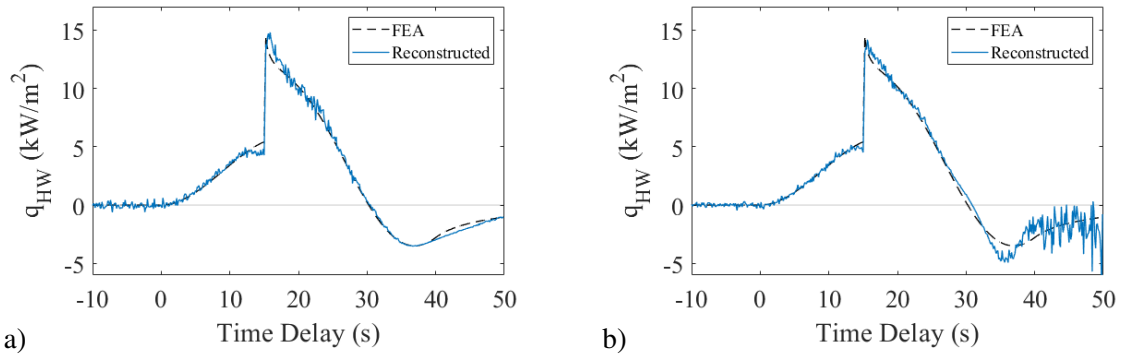


Figure 9.12: Reconstructed net hot-wall heat flux profiles a) with the added parameter $C_0(t)$ and b) without the added parameter.

Figure 9.12 compares two reconstructed heating conditions using the sensor fusion approach with both parameters $C(t)$ and $C_0(t)$ (Eq. (8.12)) and with only $C(t)$ (Eq. (8.7)). Reconstructions are performed using low noise measurement inputs and with a regularization coefficient of $\lambda_2 = 2.2 \times 10^7$. In the sensor fusion approach, large values of the 2nd derivative of $C(t)$ are penalized. Without the added parameter $C_0(t)$, $C(t) = q_{HW}/q_{CW}$ (Eq. (8.7)), which diverges to infinity along with its 2nd derivative. Thus, the penalty function becomes large, such

that minimization of the solution residual (i.e., the first l^2 -norm in Eq. (9.38)) is favored and instabilities develop in the reconstructed heating profile, as if the solution was not regularized. Furthermore, as $C(t)$ becomes large, noise in the heat flux sensor measurement is amplified more severely in the reconstructed net hot-wall heat flux profile (Fig. 9.12b). The inclusion of the additional parameter $C_0(t)$ embeds an intrinsic offset into the definition of q_{HW} via $\bar{q}_{CW}$ and avoids singularities when q_{CW} becomes small, leading to more stable solutions (Fig. 9.12a).

9.3 Summary

A Green's function sensor fusion approach is developed to reconstruct the heating conditions on spacecraft TPS using measurements from collocated heat flux sensor and temperature probe instrumentation. The Green's function framework is applied to a 2D-axisymmetric, multi-material model system with temperature-dependent thermal properties to capture the effects of multidimensional heat conduction in the reconstruction. Basis functions are developed for the multi-material system to construct Green's functions via the Galerkin method. The performance of the approach is evaluated on a numerical test case with measurements generated using finite element analysis simulations. To probe the capabilities of the reconstruction algorithm, the test case consisted of a heating profile with a sharp transient feature, emulating a boundary layer transition event. Analysis of the reconstructed heating conditions highlighted how the Green's function sensor fusion approach can resolve sharp features in the imposed heating profile, even when subject to measurements that are perturbed with substantial noise. Generally, the reconstructed heating conditions using the sensor fusion approach were more accurate than those stabilized using Tikhonov regularization and less sensitive to suboptimal choice of the user input regularization

coefficient, reinforcing the findings in Chapter 8. This robustness positions the algorithm for use in a vast range of measurement applications without the need to optimize algorithm parameters for a specific end-use case, and more broadly demonstrates the improvements that may be gained over current methods by stabilizing ill-posed systems with surrogate measurements.

Chapter 10: Adaptation of the Green’s Function Framework for Ablative Thermal Protection Systems

The Green’s function solution equation in Chapter 7 (Eq. 7.3) is derived considering the linear heat conduction equation for a system with constant material properties. In the aerothermodynamics community, recent advances by Massa *et al.* have extended the Green’s function formalism to accommodate material systems with temperature-dependent thermal properties via a Kirchhoff transform [107]. In Chapter 9, the Cole-Hopf transformation, a general form of the Kirchhoff transform, was used in a similar fashion to linearize the heat conduction equation in a model system with temperature-dependent thermal properties for use in the Green’s function IHT formulation.

Ablative TPS used on entry spacecraft, however, elicit much more complex material response behavior. As the temperature of the TPS increases during atmospheric entry, the virgin material—usually composed of a rigid porous matrix impregnated with phenolic resin—decomposes into char, creating pyrolysis gases. These pyrolysis gases diffuse through the porous structure, to the surface, and into the boundary layer. In addition to the strong temperature- and composition-dependence of the properties of ablative TPS materials, multiple additional material response behaviors and thermal transport mechanisms need to be considered when attempting to reconstruct the surface heat flux from embedded temperature measurements:

1. The phase change from virgin to char creates three material regions: I) virgin material closest to the internal spacecraft structure, II) fully charred material near the TPS surface, and III) a gradient of partially charred material between the fully virgin and fully charred regions. The boundaries between the regions are often indistinct and constantly changing.
2. The pyrolysis reaction introduces an endothermic energy sink (or, in some instances, an exothermic energy source) mechanism into the heat conduction equation.
3. Heat transport within the TPS includes convection due to the transport of pyrolysis gases.

As such, prior methods used to linearize the governing heat conduction equation, such as the Kirchhoff or Cole-Hopf transformations, are not currently compatible with these material systems, motivating the development of new techniques to further amend the Green's function IHT approach for atmospheric entry applications.

In this chapter, the Green's function-based IHT algorithm is adapted to reconstruct the heating conditions on atmospheric entry spacecraft from temperature measurements embedded in ablative TPS. A novel multi-parameter Cole-Hopf transformation is introduced to render the governing heat conduction equation for porous, pyrolyzing solids compatible with the Green's function formalism. The performance of the approach is investigated on a representative atmospheric entry heating scenario using simulated measurements generated from a high-fidelity material response code and an ablative material database.

10.1 Heat Conduction in Pyrolyzing Solids

In the material response modeling community, significant efforts have been devoted to accurately predicting the behavior of ablative materials for the development low margin, high-

performance TPS. In the late 1960s, the Charring Material Ablation program (CMA) was developed to model the mass and energy conservation equations of ablative solids. The general formalism of CMA has remained a good predictor of ablative TPS behavior to-date, with successive codes, such as the Fully Implicit Ablation and Thermal response analysis program (FIAT) [90] and the CHarring Ablator Response code (CHAR) [92], adopting similar, but more thorough and numerically stable, formulations. Recent high fidelity material response codes, including the Porous material Analysis Toolbox based on OPENFoam (PATO), have further advanced the material response modeling state of the art, now capturing the effects of complex gas-solid interactions and equilibrium chemistry in the decomposition of porous materials [132–134].

The objective of this chapter is to adapt the current ablative material response models used by the space exploration community for the Green’s function IHT approach, so that the effects of material decomposition and pyrolysis gas transport are sufficiently accounted for in the reconstruction of the surface heating conditions. The formulation detailed hereafter borrows heavily from the heritage framework developed by Moyer and Rindal, CMA [135], and adopts some more recent elements from Lachaud *et al.* [132], Amar *et al.* [92], and Chen and Milos [90]. The material response modeling concepts are introduced throughout the remainder of this section through derivation of the governing heat conduction equation. In contrast to the heat conduction equation introduced in Chapter 7, the heat conduction equation considered in this chapter is derived for a one-dimensional system, consistent with FIAT and CMA.

10.1.1 Conservation of Energy

To model the temperature distribution within porous, pyrolyzing solids, the heat conduction equation must be derived from the governing energy conservation equation

$$\frac{\partial}{\partial t} (\rho_s h_s + \phi \rho_g e_g) = \frac{\partial}{\partial x} \left(k \frac{\partial \tilde{T}}{\partial x} \right) - \frac{\partial}{\partial x} (h_g \dot{m}_g) \quad (10.1)$$

In Eq. (10.1), the term on the left-hand side represents the time rate of change of the solid and gaseous energy stored in a differential control volume, where h_s and ρ_s are the enthalpy and density of the solid, respectively, e_g and ρ_g are the internal energy and density of the pyrolysis gas, respectively, and ϕ is the porosity of the solid [132]. Pyrolysis gases within the TPS are assumed to be in local thermal equilibrium with the surrounding solid. The first term on the right-hand side of Eq. (10.1) represents energy transport due to heat conduction, where k is the thermal conductivity¹ and $\tilde{T}$ represents the absolute temperature. The second term on the right-hand-side of Eq. (10.1) represents energy transport due to mass diffusion of the pyrolysis gas $\dot{m}_g$ through the solid, i.e., internal convection, where h_g is the enthalpy of the pyrolysis gas.

10.1.1.1 Solid Energy Storage

Equation (10.1) is first rearranged to separate the solid and gaseous energy storage terms

$$\frac{\partial}{\partial t} (\rho_s h_s) = \frac{\partial}{\partial x} \left(k \frac{\partial \tilde{T}}{\partial x} \right) - \frac{\partial}{\partial t} (\phi \rho_g e_g) - \frac{\partial}{\partial x} (h_g \dot{m}_g) \quad (10.2)$$

¹Note that the thermal conductivities of porous TPS materials are often defined as a weak function of pressure, and thus implicitly contain information about the thermal conductivity of both the solid and gas phase.

In pyrolyzing TPS materials, properties of the solid phase vary as the material decomposes from a virgin to a charred state. Typically, solid properties are defined using an effective char volume fraction χ [92]

$$\chi = \frac{\rho_s - \rho_v}{\rho_c - \rho_v} \quad (10.3)$$

where ρ_v and ρ_c represent the density of the fully virgin and fully charred TPS material, respectively. The solid energy storage term on the left-hand side of Eq. (10.2) is expanded considering the solid as a volume-averaged composite of virgin and char phases

$$\rho_s h_s = (1 - \chi) \rho_v h_v + \chi \rho_c h_c \quad (10.4)$$

Equation (10.4) is substituted into the left-hand side of Eq. (10.2) and expanded

$$\frac{\partial}{\partial t} (\rho_s h_s) = \rho_v \frac{\partial h_v}{\partial t} - \rho_v \chi \frac{\partial h_v}{\partial t} - \rho_v h_v \frac{\partial \chi}{\partial t} + \rho_c \chi \frac{\partial h_c}{\partial t} + \rho_c h_c \frac{\partial \chi}{\partial t} \quad (10.5)$$

Noting that

$$\frac{\partial h}{\partial t} = \frac{\partial h}{\partial \tilde{T}} \frac{\partial \tilde{T}}{\partial t} \quad (10.6)$$

where $\partial h / \partial \tilde{T}$ represents the specific heat C_p , Eq. (10.5) simplifies to

$$\frac{\partial}{\partial t} (\rho_s h_s) = [(1 - \chi) \rho_v C_{p,v} + \chi \rho_c C_{p,c}] \frac{\partial \tilde{T}}{\partial t} + (\rho_c h_c - \rho_v h_v) \frac{\partial \chi}{\partial t} \quad (10.7)$$

The bracketed term on the right-hand side of Eq. (10.7) is equivalent to $\rho_s C_p(\tilde{T}, \chi)$, where the specific heat of the solid $C_p(\tilde{T}, \chi)$ is modeled by interpolating between the specific heats of the

fully virgin and fully charred phases, $C_{p,v}$ and $C_{p,c}$, respectively [92]

$$C_p(\tilde{T}, \chi) = (1 - \chi) \frac{\rho_v}{\rho_s} C_{p,v}(\tilde{T}) + \chi \frac{\rho_c}{\rho_s} C_{p,c}(\tilde{T}) \quad (10.8)$$

The time rate of change of the char volume fraction $\partial\chi/\partial t$ in Eq. (10.7) is solved by differentiating Eq. (10.3)

$$\frac{\partial\chi}{\partial t} = \frac{1}{\rho_c - \rho_v} \frac{\partial\rho_s}{\partial t} \quad (10.9)$$

In Eq. (10.9), $\partial\rho_s/\partial t$ is the decomposition rate of the solid TPS, which will be addressed in Section 10.1.2. Substituting Eq. (10.9) into Eq. (10.7), the solid energy storage term is reduced to [135]

$$\frac{\partial}{\partial t} (\rho_s h_s) = \rho_s C_p(\tilde{T}, \chi) \frac{\partial\tilde{T}}{\partial t} + \frac{\rho_c h_c - \rho_v h_v}{\rho_c - \rho_v} \frac{\partial\rho_s}{\partial t} \quad (10.10)$$

10.1.1.2 Internal Convection

The convective term in Eq. (10.1) is handled next. First, expanding the derivative yields

$$\frac{\partial}{\partial x} (h_g \dot{m}_g) = h_g \frac{\partial\dot{m}_g}{\partial x} + \dot{m}_g \frac{\partial h_g}{\partial x} \quad (10.11)$$

The value and gradient of the pyrolysis gas mass flux, $\dot{m}_g$ and $\partial\dot{m}_g/\partial x$, respectively, are approximated by considering mass conservation of the gas phase

$$\frac{\partial\rho_g}{\partial t} + \frac{\partial\dot{m}_g}{\partial x} = \dot{w}_g \quad (10.12)$$

where $\dot{w}_g$ represents the chemical production rate of pyrolysis gas, balanced by the solid decomposition rate

$$\dot{w}_g = -\frac{\partial \rho_s}{\partial t} \quad (10.13)$$

Assuming quasi-steady flow,

$$\frac{\partial \dot{m}_g}{\partial x} = -\frac{\partial \rho_s}{\partial t} \quad (10.14)$$

To approximate $\dot{m}_g$, the flow of the pyrolysis gases is assumed to be one-dimensional and towards the TPS surface. Thus,

$$\dot{m}_g = \int_0^x \frac{\partial \dot{m}_g}{\partial x'} dx' = - \int_0^x \frac{\partial \rho_s}{\partial t} dx' \quad (10.15)$$

10.1.1.3 Governing Heat Conduction Equation

After a full accounting of all volumetric source terms, the governing energy conservation equation (Eq. (10.1)) reduces to the form of the nonlinear, inhomogeneous heat conduction equation

$$\rho_s C_p(\tilde{T}, \chi) \frac{\partial \tilde{T}}{\partial t} - \frac{\partial}{\partial x} \left(k(\tilde{T}, \chi) \frac{\partial \tilde{T}}{\partial x} \right) = g(x, t) \quad (10.16)$$

The thermal conductivity in Eq. (10.16) is defined in a similar fashion as the specific heat, interpolated between the thermal conductivities of the fully virgin and fully charred phases, k_v and k_c , respectively [92]

$$k(\tilde{T}, \chi) = (1 - \chi) \frac{\rho_v}{\rho_s} k_v(\tilde{T}) + \chi \frac{\rho_c}{\rho_s} k_c(\tilde{T}) \quad (10.17)$$

In Eq. (10.16), $g(x, t)$ captures the effects of pyrolysis

$$g(x, t) = \left(h_g - \frac{\rho_c h_c - \rho_v h_v}{\rho_c - \rho_v} \right) \frac{\partial \rho_s}{\partial t} - \frac{\partial}{\partial t} (\phi \rho_g e_g) + \frac{\partial h_g}{\partial x} \int_0^x \frac{\partial \rho_s}{\partial t} dx' \quad (10.18)$$

The terms in Eq. (10.18) more explicitly represent the various mechanisms in which decomposition of the solid TPS impacts the TPS temperature distribution. The first term represents the net rate of enthalpy change as the virgin TPS decomposes into a mixture of solid char and pyrolysis gas. The second term captures the added thermal inertia of the pyrolysis gas during transient heating scenarios. The third term accounts for the effects of pyrolysis gas convection as gases diffuse through the porous solid and towards the surface.

Thermodynamic properties of the pyrolysis gas phase (i.e., density, enthalpy, and internal energy) are calculated as a function of temperature, but at constant pressure. Naturally, the production of pyrolysis gases in the TPS will produce pressure gradients which drive gaseous species towards the surface and, in higher dimensions, other regions of the TPS. Resolving these pressure gradients, however, requires an accounting of momentum conservation in the gas phase. For 1D systems, this level of fidelity is only expected to marginally improve the accuracy of the reconstruction algorithm, but with substantial added effort. Thus, momentum conservation of the gas phase will not be considered in this thesis.

Finally, it should be noted that the above derivation is defined for a fixed coordinate system and geometry, and, thus, does not consider recession of the TPS. During atmospheric entry, accurate modeling of the TPS recession behavior requires detailed a priori knowledge of the external entry environment and boundary layer composition; thus, the IHT problem would become under-defined. For this reason, recession is typically neglected even when using state-of-the-art

IHT codes for post-flight analysis [2]. While not necessarily accurate for Earth entry applications, neglecting recession is a valid approximation for many high-profile space exploration missions that will either descend into non-oxidizing environments (e.g., Dragonfly or Uranus Orbiter and Probe), or have coatings which greatly inhibit the TPS recession at less extreme heating rates ($<100 \text{ W/cm}^2$, e.g., polysiloxane coatings on the forebody heat shields of Mars 2020 and Mars Science Laboratory [136–138]). For reconstructions of backshell heating conditions, the assumption of negligible surface recession is generally valid based on the heat rates these areas are expected to experience [2]. Thus, the level of fidelity captured by Eq. (10.16) is applicable to a wide range of mission architectures of interest to the space exploration community.

10.1.2 Decomposition Modeling

For closure of Eq. (10.16) and Eq. (10.18), the decomposition rate of the TPS $\partial\rho_s/\partial t$ must be captured accurately. In the literature and broader TPS modeling community, the level of detail used to capture pyrolysis and material decomposition varies significantly. Heritage frameworks, such as FIAT [90, 91] and CMA [135] adopt an empirical Arrhenius rate formulation to model the composite constituent density change as a function of temperature. These methods are still used currently to model pyrolysis in spacecraft TPS. More recent efforts have aimed to resolve gas-solid interactions and individual chemical reactions in the decomposition of porous materials [132, 133]. These approaches typically rely on first principles over empirical fit parameters, and thus may be more versatile when applied to different material systems. They are also significantly more complex and computationally expensive than their heritage counterparts, which makes them impractical for the current Green’s function IHT approach. Therefore, the pyrolysis and material

decomposition model detailed herein will closely follow that which is implemented in FIAT, as described by Chen and Milos [90].

The formation of char from the pyrolysis of virgin TPS material first requires a coarse definition of the solid phase composition. The solid TPS is considered as a three-constituent mixture with a total density ρ_s

$$\rho_s = (1 - \phi) [\Gamma_v (\rho_A + \rho_B) + (1 - \Gamma_v) \rho_C] \quad (10.19)$$

In Eq. (10.19), ρ_A and ρ_B are densities of the resin constituents, and ρ_C is the density of the material comprising the solid matrix. The quantity Γ_v defines the resin volume fraction in the virgin composite material. The decomposition reaction of component i is modeled using a modified Arrhenius rate equation [90].

$$\frac{d\rho_i}{dt} = -\bar{A}_i \rho_{0,i} \exp\left(-\frac{E_{A,i}}{R\tilde{T}}\right) \left[\frac{\rho_i - \rho_{r,i}}{\rho_{0,i}}\right]^{\psi_i} \quad (10.20)$$

In Eq. (10.20), $\bar{A}$ is a pre-exponential factor and ρ_0 and ρ_r are the initial density and residual density of component i , respectively. In the exponential term, E_A is the activation energy for the decomposition of component i , and R is the universal gas constant. The final bracketed term is raised to the power ψ , specific for each constituent. The reaction parameters in Eq. (10.20) are typically determined experimentally via thermogravimetric analysis [139].

Eq. (10.20) is a first-order ODE that is solved by marching forward in time from the initial uncharred reference state. For each time step, the density of each constituent is calculated and substituted into Eq. (10.19) to determine the composite density as a function of time.

10.2 Reconstruction Algorithm

Using the governing heat conduction equation derived in Section 10.1 (Eq. (10.16)), the Green's function-based IHT algorithm is leveraged to recover the surface heating conditions on ablative TPS materials. The new approach consists of three steps, with the first two analogous to those described in Chapter 7. First, a Green's function formulation is developed to efficiently model the transient heat conduction within the TPS at the temperature measurement location. Due to the strong temperature- and composition-dependence of the thermophysical properties of pyrolyzing TPS materials, the governing heat conduction equation is pretreated with a multi-parameter Cole-Hopf transform to render it compatible with the Green's function formalism. In the second step, the Green's function representation of the TPS thermal history is inverted to recover the net hot-wall heat flux absorbed at the TPS surface. In the third step, the reconstructed net hot-wall heat flux is used to calculate the TPS surface temperature and char state to predict the heat flux solely from the external aeroheating environment.

The reconstruction algorithm is developed for the multi-material 1D model system shown in Fig. 10.1. The model system is representative of common TPS stackup configurations flown on entry spacecraft, and consists of an ablative TPS, an adhesive layer, and a substructure with thicknesses of L , L_{ad} , and L_{sub} , respectively. The adhesive and substructure domains do not decompose and are modeled using constant thermal properties. The origin of the coordinate system ($x = 0$) is set at the interface between the TPS and the adhesive. The temperature probe is embedded in the TPS at a depth of $L - x_T$. The external surface of the TPS is exposed to the aeroheating environment, which results in a net hot-wall heat flux q_{HW} conducted into the solid. The internal surface of the stackup is modeled as insulating.

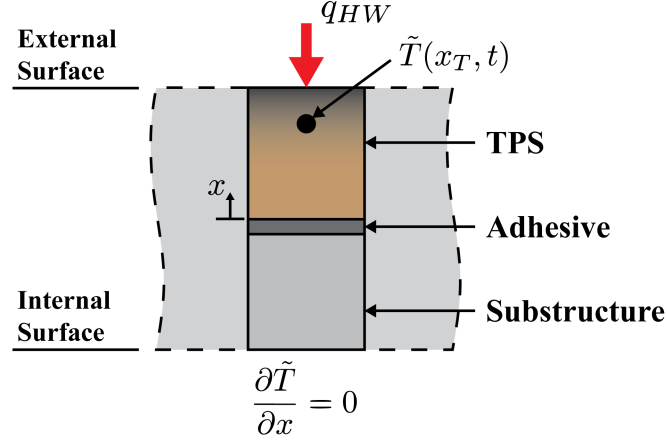


Figure 10.1: 1D model system schematic including the TPS, adhesive, and substructure. The temperature measurement is sampled from a probe embedded in the TPS at x_T , where the origin of the coordinate system is defined at the TPS-adhesive interface. The internal surface of the stackup is modeled as insulating and the external surface is exposed to the aeroheating environment. A net hot-wall heat flux q_{HW} is conducted into the TPS solid.

10.2.1 Green's Function Formulation for Ablative TPS

10.2.1.1 Pretreatment of the Governing Heat Conduction Equation

To model the material response of the TPS via the Green's function formalism, Eq. (10.16) must be linearized. To eliminate the nonlinear terms produced by expansion of the divergence term on the left-hand side of Eq. (10.16), a multi-parameter Cole-Hopf transform is introduced

$$\{T, \chi\} = \{f_1(\theta), f_2(f_1(\theta))\} \quad (10.21)$$

In Eq. (10.21), T represents the temperature change from an initial reference temperature $T = \tilde{T} - \tilde{T}_0$ and θ is introduced as a transformation variable (hereafter referred to as the *transformed*

temperature). The transformation in Eq. (10.21) must satisfy the following condition

$$\left[k \frac{\partial^2 f_1}{\partial \theta^2} + \frac{\partial k}{\partial f_1} \left(\frac{\partial f_1}{\partial \theta} \right)^2 + \frac{\partial k}{\partial f_2} \frac{\partial f_2}{\partial f_1} \left(\frac{\partial f_1}{\partial \theta} \right)^2 \right] = 0 \quad (10.22)$$

which arises after substituting Eq. (10.21) into Eq. (10.16) and expanding. Equation (10.22) is itself a chain-rule expansion of the following homogeneous differential equation

$$\frac{\partial}{\partial \theta} \left[k(\tilde{T}, \chi) \frac{\partial f_1}{\partial \theta} \right] = 0 \quad (10.23)$$

which is integrated to yield

$$k(\tilde{T}, \chi) \frac{\partial f_1}{\partial \theta} = A \quad (10.24)$$

In Eq. (10.24) A is an arbitrary constant. Equation (10.24) defines the required behavior for the transformation between T and χ , and θ . To accommodate the linear transformation from $\tilde{T}$ to T , the thermal conductivity $k(\tilde{T}, \chi)$ is defined relative to a reference thermal conductivity value $k_0 = k(\tilde{T}_0, 0)$ as a function of the temperature change T

$$k(\tilde{T}, \chi) = k_0 + k_T(T, \chi) \quad (10.25)$$

It is important to recognize from Eq. (10.24) that f_1 , and the associated inverse transformation $\theta = f_1^{-1}(T)$, has no closed-form solution, which is in contrast to the conventional Cole-Hopf transformation used in Chapter 9, where only the temperature-dependence of the thermal conductivity was considered [107, 130]. Instead of defining f_1 and f_2 explicitly, it is preferred to define

an approximate inverse transformation

$$\theta = \frac{1}{A} \left(1 + \beta \frac{\rho_c}{\rho_s} \chi \right) \int_0^T [k_0 + k_T(T', 0)] dT' \quad (10.26)$$

where β is a constant which best satisfies the condition in (10.24) for a local temperature time history. The procedure used to determine β is detailed in Section 10.2.1.2. In the limit as $\beta \rightarrow 0$, Eq. (10.26) reduces to a single-parameter Cole-Hopf transformation defined for the virgin TPS. Thus, Eq. (10.26) can be considered as a modification of the virgin material single-parameter Cole-Hopf transformation, scaled using the term enclosed in parenthesis to capture the effects of decomposition. Uniqueness of the transformation from θ to T and χ is guaranteed due to the strong dependence of χ on the local temperature time history and the irreversible behavior of the TPS decomposition reaction.

As a result of the multi-parameter Cole-Hopf transform, the transformed heat conduction equation becomes

$$\frac{1}{\alpha(\tilde{T}, \chi)} \frac{\partial \theta}{\partial t} - \frac{\partial^2 \theta}{\partial x^2} = \frac{g(x, t)}{A} \quad (10.27)$$

To fully linearize Eq. (10.27), the thermal diffusivity $\alpha(\tilde{T}, \chi)$ is expanded relative to a reference value $\alpha_0 = \alpha(\tilde{T}_0, 0)$ as a function of the temperature change T

$$\alpha(\tilde{T}, \chi) = \alpha_0 + \alpha_T(T, \chi) \quad (10.28)$$

which, after substituting into Eq. (10.27) and rearranging, yields the following linear, inhomoge-

neous partial differential equation

$$\frac{1}{\alpha_0} \frac{\partial \theta}{\partial t} - \frac{\partial^2 \theta}{\partial x^2} = \frac{g'(x, t)}{A} \quad (10.29)$$

In Eq. (10.29), all nonlinear effects due to temperature- and composition-dependent thermophysical properties and pyrolysis are captured either within the multi-parameter Cole-Hopf transformation or as a virtual source term g' , where

$$\frac{g'(x, t)}{A} = \frac{\alpha_T(T, \chi)}{\alpha_0} \frac{\partial^2 \theta}{\partial x^2} + \frac{\alpha_0 + \alpha_T(T, \chi)}{\alpha_0} \frac{g(x, t)}{A} \quad (10.30)$$

Equations (10.27)-(10.30) are analogous to Eqs. (9.3)-(9.8) derived in Chapter 9, but now specified for 1D heat conduction and for thermal properties that vary with both temperature and composition.

10.2.1.2 Determination of Multi-Parameter Cole-Hopf Transformation Parameters

While the form of the multi-parameter Cole-Hopf transform in Eq. (10.26) is general, the parameter β must be defined locally and for each temperature time history separately. For tractability, the effective char mass fraction χ_m is used in place of the char volume fraction χ in Eq. (10.26), defined as

$$\chi_m = \frac{\rho_c}{\rho_v - \rho_c} \left(\frac{\rho_v}{\rho_s} - 1 \right) \quad (10.31)$$

The effective char mass and volume fractions are related via

$$\chi_m = \frac{\rho_c}{\rho_s} \chi \quad (10.32)$$

Substituting Eq. (10.32) into Eq. (10.26) and differentiating both sides with respect to θ yields

$$A = \left\{ (1 + \beta \chi_m) k(\tilde{T}, 0) + \beta \frac{\partial \chi_m}{\partial T} \int_0^T [k_0 + k_T(T', 0)] dT' \right\} \frac{\partial T}{\partial \theta} \quad (10.33)$$

In Eq. (10.33), the term in curly braces is defined as $\bar{k}$, yielding

$$\bar{k} \frac{\partial T}{\partial \theta} = A \quad (10.34)$$

Therefore, the condition established by Eq. (10.24) is best satisfied when β is defined such that

$$\frac{\bar{k}}{k} \approx 1 \quad (10.35)$$

Substituting Eq. (10.25) into Eq. (10.35) and rearranging gives

$$\left\{ \frac{\chi_m k(\tilde{T}, 0) + \frac{\partial \chi_m}{\partial T} \int_0^T [k_0 + k_T(T', 0)] dT'}{k(\tilde{T}, \chi) - k(\tilde{T}, 0)} \right\} \beta \approx 1 \quad (10.36)$$

The term in curly braces consists of quantities that can be estimated from a single input temperature time series. As such, β is evaluated via least squares to best approximate the linear system

$$\mathbf{X}_\beta \beta = \mathbf{1} \quad (10.37)$$

where $\mathbf{X}_\beta$ is an $N_t - 1 \times 1$ vector which represents the bracketed terms in Eq. (10.36). The time series $\mathbf{X}_\beta$ requires the local solution of T , ρ_s , and $\partial\chi_m/\partial T$. For a given temperature time history, $\partial\chi_m/\partial T$ can be calculated directly. Differentiating Eq. (10.31) with respect to T yields

$$\frac{\partial\chi_m}{\partial T} = -\frac{\rho_c}{\rho_v - \rho_c} \frac{\rho_v}{\rho_s^2} \frac{\partial\rho_s}{\partial T} \quad (10.38)$$

where, from Eq. (10.19) and Eq. (10.20)

$$\frac{\partial\rho_s}{\partial T} = (1 - \phi) \left[\Gamma_v \left(\frac{\partial\rho_A}{\partial T} + \frac{\partial\rho_B}{\partial T} \right) + (1 - \Gamma_v) \frac{\partial\rho_C}{\partial T} \right] \quad (10.39)$$

and

$$\frac{\partial}{\partial T} \left(\frac{\partial\rho_i}{\partial t} \right) = -\bar{A}_i \rho_{0,i} \exp \left(-\frac{E_{A,i}}{R\tilde{T}} \right) \left[\frac{\rho_i - \rho_{r,i}}{\rho_{0,i}} \right]^{\psi_i} \left\{ \frac{E_{A,i}}{R\tilde{T}^2} + \frac{\psi_i}{\rho_{0,i}} \left[\frac{\rho_i - \rho_{r,i}}{\rho_{0,i}} \right]^{-1} \frac{\partial\rho_i}{\partial T} \right\} \quad (10.40)$$

Collecting the terms outside of the curly braces on the right-hand side of Eq. (10.40) as $\partial\rho_i/\partial t$ and rearranging the differential terms on the left-hand side gives

$$\frac{\partial}{\partial t} \left(\frac{\partial\rho_i}{\partial T} \right) = \frac{\partial\rho_i}{\partial t} \left\{ \frac{E_{A,i}}{R\tilde{T}^2} + \frac{\psi_i}{\rho_{0,i}} \left[\frac{\rho_i - \rho_{r,i}}{\rho_{0,i}} \right]^{-1} \frac{\partial\rho_i}{\partial T} \right\} \quad (10.41)$$

Equation (10.41) is solved forward in time to yield the time series of $\partial\rho_s/\partial T$ (Eq. (10.39)) and $\partial\chi_m/\partial T$ (Eq. (10.38)) for the calculation of β via Eq. (10.37).

10.2.1.3 Green's Function Solution Equation

Once pretreated into the form given by Eq. (10.29), the solution to the governing heat conduction equation at the temperature probe location can be accessed via the Green's function solution equation. Within the Green's function formalism, the transformed temperature within the TPS is represented as a sum of two integrals

$$\theta(x, t) = - \int_0^t \frac{\alpha_0}{A} q_{HW}(\tau) G(x, t, L, \tau) d\tau + \int_0^t \int_0^L \frac{\alpha_0}{A} g'(x_0, \tau) G(x, t, x_0, \tau) dx_0 d\tau \quad (10.42)$$

The first integral term on the right-hand side of Eq. (10.42) captures the contributions of the net hot-wall heat flux conducted into the TPS q_{HW} , where G represents the Green's function of the TPS domain and A is the arbitrary constant defined in the multi-parameter Cole-Hopf transform (Eq. (10.24)). Within the multi-parameter Cole-Hopf transformation, q_{HW} is related to $\partial\theta/\partial x$ via

$$q_{HW} = -A \frac{\partial\theta}{\partial x} \quad (10.43)$$

which is analogous to Fourier's law. The second integral on the right-hand side of Eq. (10.42) captures the effects of pyrolysis, internal gas transport, and nonlinear material properties. It should be noted that, while the 1D model system (Fig. 10.1) contains additional domains for the adhesive layer and substructure, these components do not pyrolyze and are modeled using constant material properties; thus, they are not included in the volumetric integral in Eq. (10.42).

The TPS domain Green's function $G(x, t, x_0, \tau)$ is constructed in a similar fashion as in Chapter 9, using a finite sum of eigenfunctions approximated with the Galerkin method. Adapted

from [104] and [107],

$$G(x, t, x_0, \tau) = \sum_{n=1}^N \frac{k_0}{\alpha_0} \phi_n(x) \phi_n(x_0) e^{-\Lambda_{n,n}(t-\tau)} \quad (10.44)$$

In Eq. (10.44), $\Lambda_{n,n}$ represents the eigenvalue accompanying each TPS-domain eigenfunction ϕ_n . The eigenfunctions are approximated using an N -ordered set of scaled basis functions $\hat{f}$

$$\phi_n = \sum_{j=1}^N \psi_{j,n} \hat{f}_j \quad (10.45)$$

In Eq. (10.45), basis functions $\hat{f}_j$ are required to satisfy the homogeneous boundary and interface continuity conditions of the model system in Fig. 10.1. Eigenvalues $\Lambda_{n,n}$ and eigenvectors ψ_n in Eqs. (10.44) and (10.45) are calculated using the approach described in Chapter 9 (Eqs. (9.15) and (9.16), adapted from [108]). Finally, it is important to emphasize that, while the eigenvalues and eigenvectors are defined *globally* for the entire model system, basis functions and, consequently, the eigenfunctions are defined *locally* for each domain; in the TPS domain, the effects of the adhesive layer and substructure on the TPS material response are captured within the behavior of the basis functions $\hat{f}_j$, considering continuity of temperature and heat flux across interfaces.

10.2.1.4 Basis Functions for the Multi-Layered Model System Stackup

Basis functions are defined to satisfy the homogeneous boundary and interface continuity conditions of the multi-material 1D model system (Fig. 10.1). Considering that the origin is defined at the TPS-adhesive interface, the adiabatic boundary condition imposed on the inner

surface of the substructure is satisfied via

$$k_{sub} \frac{\partial T}{\partial x} \Big|_{x=-(L_{ad}+L_{sub})} = 0 \quad (10.46)$$

where k_{sub} is the thermal conductivity of the substructure. At the TPS-adhesive interface, continuity of temperature and heat flux is maintained via

$$T_{TPS} \Big|_{x=0} = T_{ad} \Big|_{x=0} \quad (10.47a)$$

$$k(\tilde{T}, \chi) \frac{\partial T_{TPS}}{\partial x} \Big|_{x=0} = k_{ad} \frac{\partial T_{ad}}{\partial x} \Big|_{x=0} \quad (10.47b)$$

noting again that $k(\tilde{T}, \chi)$ is defined for the TPS domain. In Eq. (10.47b), k_{ad} is the thermal conductivity of the adhesive layer. Similarly, for the adhesive-substructure interface

$$T_{ad} \Big|_{x=-L_{ad}} = T_{sub} \Big|_{x=-L_{ad}} \quad (10.48a)$$

$$k_{ad} \frac{\partial T_{ad}}{\partial x} \Big|_{x=-L_{ad}} = k_{sub} \frac{\partial T_{sub}}{\partial x} \Big|_{x=-L_{ad}} \quad (10.48b)$$

Prior to constructing the set of basis functions for each domain, the boundary and interface continuity conditions must be transformed. Because the adhesive and substructure domains are approximated using constant material properties, a transformation is applied to each domain such that

$$\theta_{TPS} \Big|_{x=0} = \theta_{ad} \Big|_{x=0} \quad (10.49a)$$

$$\theta_{ad} \Big|_{x=-L_{ad}} = \theta_{sub} \Big|_{x=-L_{ad}} \quad (10.49b)$$

This condition is satisfied by

$$\theta_{ad} = \frac{1}{A_{ad}} (k_{ad}T) \quad (10.50a)$$

$$\theta_{sub} = \frac{1}{A_{sub}} (k_{sub}T) \quad (10.50b)$$

where

$$A_{ad} = A \frac{k_{ad}}{k_0} \quad (10.51a)$$

$$A_{sub} = A \frac{k_{sub}}{k_0} \quad (10.51b)$$

reiterating that k_0 is the TPS thermal conductivity defined at a reference state $k(\tilde{T}_0, 0)$ and A is the arbitrary constant defined in the multi-parameter Cole-Hopf transformation (Eq. (10.24)). The transformations in Eq. (10.50) are valid if the temperature at the TPS-adhesive interface does not vary substantially from initial conditions (< 200 K).

Maintaining continuity of heat flux across domain boundaries is satisfied using the analogue of Fourier's law (Eq. (10.43))

$$A \frac{\partial \theta_{TPS}}{\partial x} \Big|_{x=0} = A_{ad} \frac{\partial \theta_{ad}}{\partial x} \Big|_{x=0} \quad (10.52a)$$

$$A_{ad} \frac{\partial \theta_{ad}}{\partial x} \Big|_{x=-L_{ad}} = A_{sub} \frac{\partial \theta_{sub}}{\partial x} \Big|_{x=-L_{ad}} \quad (10.52b)$$

which, using Eq. (10.51), simply reduces to

$$k_0 \frac{\partial \theta_{TPS}}{\partial x} \Big|_{x=0} = k_{ad} \frac{\partial \theta_{ad}}{\partial x} \Big|_{x=0} \quad (10.53a)$$

$$k_{ad} \frac{\partial \theta_{ad}}{\partial x} \Big|_{x=-L_{ad}} = k_{sub} \frac{\partial \theta_{sub}}{\partial x} \Big|_{x=-L_{ad}} \quad (10.53b)$$

The basis functions which satisfy the adiabatic boundary condition (Eq. (10.46)) and interface continuity conditions (Eq. (10.49) and (10.53)) are, thus, defined as

$$\hat{f}_j = \hat{f}_j^{(ad)} + \Gamma_{TPS} \left[1 - \left(1 + \frac{x}{L_{sub} + L_{ad}} \right)^j \right] \quad (10.54a)$$

$$\hat{f}_j^{(ad)} = \hat{f}_j^{(sub)} + \Gamma_{ad} \left[1 - \left(1 + \frac{L_{ad}}{L_{sub}} + \frac{x}{L_{sub}} \right)^j \right] \quad (10.54b)$$

$$\hat{f}_j^{(sub)} = (x + L_{sub} + L_{ad})^{2j-2} \quad (10.54c)$$

where

$$\Gamma_{TPS} = (L_{sub} + L_{ad}) \left[\frac{2j-2}{j} (L_{sub} + L_{ad})^{2j-3} - \frac{\Gamma_{ad}}{L_{sub}} \left(1 + \frac{L_{ad}}{L_{sub}} \right)^{j-1} \right] \left[1 - \frac{k_{ad}}{k_0} \right] \quad (10.55a)$$

$$\Gamma_{ad} = \frac{2j-2}{j} L_{sub}^{2j-2} \left[1 - \frac{k_{sub}}{k_{ad}} \right] \quad (10.55b)$$

for $j \in \{1, 2, 3, \dots, N\}$.

10.2.2 Recovery of the Net Hot-Wall Heat Flux

In the second step of the reconstruction algorithm, the net hot-wall heat flux is recovered from the Green's function formulation of the TPS thermal response. The net hot-wall heat flux is first isolated from all other quantities in Eq. (10.42)

$$\theta(x, t) - g_{tot}(x, t) = - \int_0^t \frac{\alpha_0}{A} q_{HW}(\tau) G(x, t, L, \tau) d\tau \quad (10.56)$$

where $g_{tot}(x, t)$ represents the integrated source term

$$g_{tot}(x, t) = \int_0^t \int_0^L \frac{\alpha_0}{A} g'(x_0, \tau) G(x, t, x_0, \tau) dx_0 d\tau \quad (10.57)$$

For measurements sampled at a finite rate, Eq. (10.56) is recast as a discrete linear system

$$\boldsymbol{\theta} - \mathbf{g}_{tot} = -\frac{\alpha_0}{A} \mathbf{G} \mathbf{q}_{HW} \quad (10.58)$$

Equation (10.58) is of the same form as the ill-posed linear systems investigated in Chapters 7-9. Thus, the solution requires regularization for stability, either using a conventional approach, such as Tikhonov regularization, or the sensor fusion approach introduced in Chapter 8.

When θ and g_{tot} are well defined, q_{HW} can be accessed directly via the least-squares solution of Eq. (10.58). The parameter g_{tot} , however, is initially unknown, and must be calculated using iterative solutions of q_{HW} . The iterative solution process is shown schematically in Fig. 10.2. For each iteration of q_{HW} , a Gaussian quadrature scheme is used to approximate g_{tot} by calculating the integrand term in Eq. (10.57) at a small number N_x of discrete locations ξ_i within the TPS [140, 141]

$$\int_0^L g'(x_0, \tau) G(x, t, x_0, \tau) dx_0 = \int_{-1}^1 g'(\xi_0, \tau) G(\xi, t, \xi_0, \tau) d\xi_0 \approx \sum_{i=1}^{N_x} w_i g'(\xi_i, \tau) G(\xi, t, \xi_i, \tau) \quad (10.59)$$

where w_i are standard Gaussian quadrature weights and ξ_i are the roots of the N_x -order Legendre polynomial. In Eq. (10.59), ξ is mapped to x through transformation of the TPS domain from $x \in [0, L]$ to $\xi \in [-1, 1]$. The components of g' at each integration point ξ_i are derived from the

local solutions of θ , $\partial\theta/\partial x$, or $\partial^2\theta/\partial x^2$, which are efficiently calculated using Eq. (10.42), or its derivatives with respect to x . The local solutions of θ and $\partial\theta/\partial x$ are transformed to yield T , χ , and $\partial T/\partial x$, which are then used to compute all other parameters in g' .

10.2.3 Aeroheating Environment Reconstruction

In the final step of the reconstruction algorithm, the solution of q_{HW} is used to predict the heat flux absorbed by the TPS solely from the aeroheating environment. At the surface, multiple heat transfer mechanisms contribute to q_{HW} via the surface energy balance

$$q_{HW} = q_{conv} + \alpha_s q_{rad} - \epsilon_s \sigma_{SB} (\tilde{T}_s^4 - \tilde{T}_\infty^4) \quad (10.60)$$

In the right-hand side of Eq. (10.60), the first term represents the convective heat flux through the viscous boundary layer. The second term represents the absorbed heat flux from the shock layer radiation, where α_s is the absorptivity of the TPS surface and q_{rad} is the total radiative flux incident on the TPS. The third term represents the radiative emission from the TPS, where ϵ_s is the emissivity of the TPS surface, σ_{SB} is the Stefan-Boltzmann constant, $\tilde{T}_s$ is the TPS surface temperature, and $\tilde{T}_\infty$ represents the background temperature.

The heat flux imposed by the aerothermal environment during atmospheric entry is represented by the sum of the first two terms on the right-hand side of Eq. (10.60), which will be referred to hereafter as the *incident heat flux* q_{inc} . To recover q_{inc} , q_{HW} is corrected by the radiative emission from the TPS surface, such that

$$q_{inc} = q_{HW} + \epsilon_s \sigma_{SB} (\tilde{T}_s^4 - \tilde{T}_\infty^4) \quad (10.61)$$

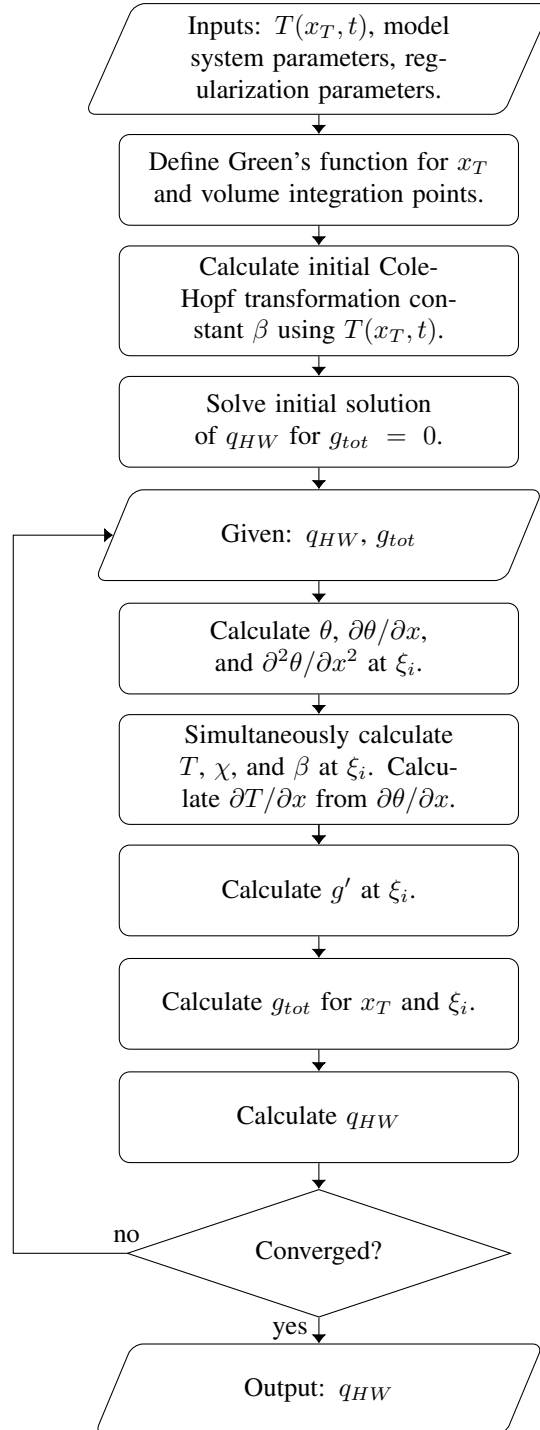


Figure 10.2: Flow chart for the reconstruction algorithm.

The emissivity of the TPS ϵ_s depends on both temperature and composition [92]

$$\epsilon_s(\tilde{T}, \chi) = (1 - \chi) \frac{\rho_v}{\rho_s} \epsilon_v(\tilde{T}) + \chi \frac{\rho_c}{\rho_s} \epsilon_c(\tilde{T}) \quad (10.62)$$

where ϵ_v and ϵ_c are the emissivities of the virgin and charred TPS material, respectively. The surface temperature $\tilde{T}_s$ and char fraction are calculated via Eq. (10.42), evaluated at the surface using the converged solution of q_{HW} .

10.2.4 Computation Cost

The total cost of the reconstruction algorithm is sensitive to the size of the measurement time series N_t and the order of the numerical integration scheme N_x used to approximate g_{tot} . In the limiting case, when the effects of pyrolysis and nonlinear material properties are negligible, the total algorithm cost is dominated by the least squares solution of q_{HW} in Eq. (10.58). This operation is purely sensitive to N_t . As pyrolysis effects become more significant, the iterative solution of q_{HW} begins to drive the total reconstruction cost, notably due to both the calculation of g' for N_x integration points, as well as the least squares solution of q_{HW} for each iteration. Nonetheless, most steps in the reconstruction algorithm consist of basic matrix operations that can be efficiently solved with minimal computing resources. For the reconstruction of the Mars 2020 atmospheric entry heating conditions (Chapter 11), which considers an input measurement time series with $N_t = 600$ sample points and a $N_x = 14$ order numerical integration scheme, the reconstruction algorithm routinely converges in less than 8 seconds with fewer than 10 iterations using a consumer-grade personal computer. By comparison, conventional time-marching-based IHT methods generate solutions for the same number of sample points in approximately 30 min-

utes using state-of-the-art computing clusters and significant code optimization [2, 88].

10.3 Numerical Verification of the Green’s Function Adaptation for Ablative TPS Materials

A computational verification test case was constructed to analyze the accuracy of the reconstruction algorithm in an atmospheric entry heating scenario. Using a simulated temperature probe measurement in response to prescribed boundary conditions, the objective of the reconstruction algorithm is to recover the TPS surface heating conditions, surface temperature, and through-thickness material response (i.e., components of g' in Eq. (10.30)) in close agreement with results from a high-fidelity material response code, which will be referred to as the *reference solution*. The heat shield stackup and associated parameters used in the verification test case are summarized in Table 10.1. The TPS was modeled using the Theoretical Ablative Composite for Open Testing (TACOT) material database [126] with no surface recession. The adhesive was modeled as HT424 and the substructure was modeled as aluminum with a reduced density and thermal conductivity to emulate a honeycomb configuration (34 kg/m^3). The temperature probe was placed in the TPS domain at a depth of 0.254 cm from the external surface.

Table 10.1: Numerical verification heat shield stackup parameters.

Component	Material	Thickness (cm)	Note
TPS	TACOT	3.0	No recession
Adhesive	HT424	0.14	Constant properties
Substructure	Aluminum Honeycomb	1.27	Constant properties

Initially, the system is held at a uniform temperature and pressure of 300 K and 10 kPa,

respectively. In the verification scenario, the external surface of the TPS is exposed to a combined convective and radiative heat pulse (Fig. 10.3a). Throughout the heat pulse, the film coefficient C_H and incident radiative heat flux q_{rad} are modulated as a sine wave. The pressure at the external surface of the TPS remains fixed at 10 kPa and the bulk enthalpy is fixed at 1 MJ/kg. The TPS-embedded temperature probe measurement (Fig. 10.3b) and reference surface heat flux, surface temperature, and through-thickness material response solutions were calculated using the Porous material Analysis Toolbox based on OpenFOAM (PATO) [142]. In the reconstruction algorithm, a basis function order of $N = 12$ was used to construct Green's function (Eq. (10.44)) and a numerical integration order of $N_x = 36$ was used to evaluate g_{tot} in the TPS domain (Eq. (10.57)). Because the heating conditions are smoothly varying and the input measurements do not contain noise, Tikhonov regularization was used to stabilize the solution [13].

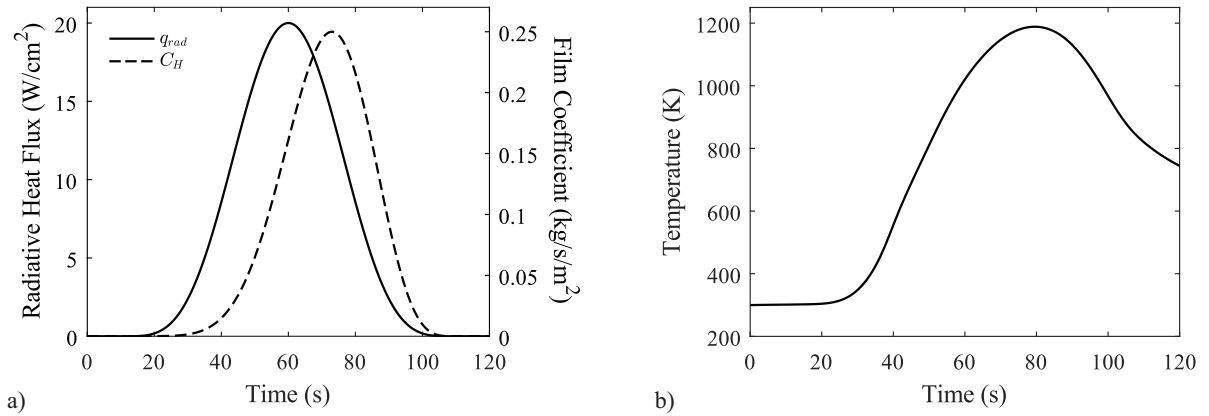


Figure 10.3: Numerical verification inputs. a) Radiative (solid line) and convective (dashed line) boundary conditions imposed on the 1D model system. For the convective boundary condition, the bulk enthalpy is fixed at a constant value of 1 MJ/kg and the film coefficient is pulsed. b) Simulated temperature probe measurement.

The results of the verification test case are shown in Fig. 10.4. Reconstructed net hot-wall and incident heating profiles are shown in Fig. 10.4a and the reconstructed surface temperature is shown in Fig. 10.4b. In Fig. 10.4a, the blue line represents the net hot-wall heat flux q_{HW} and the

orange line represents the incident heat flux q_{inc} (Eq. (10.61)). Reconstructed heat flux profiles using the Green's function approach are shown as solid lines and PATO-generated outputs (i.e., the reference solution) are shown using dashed lines.

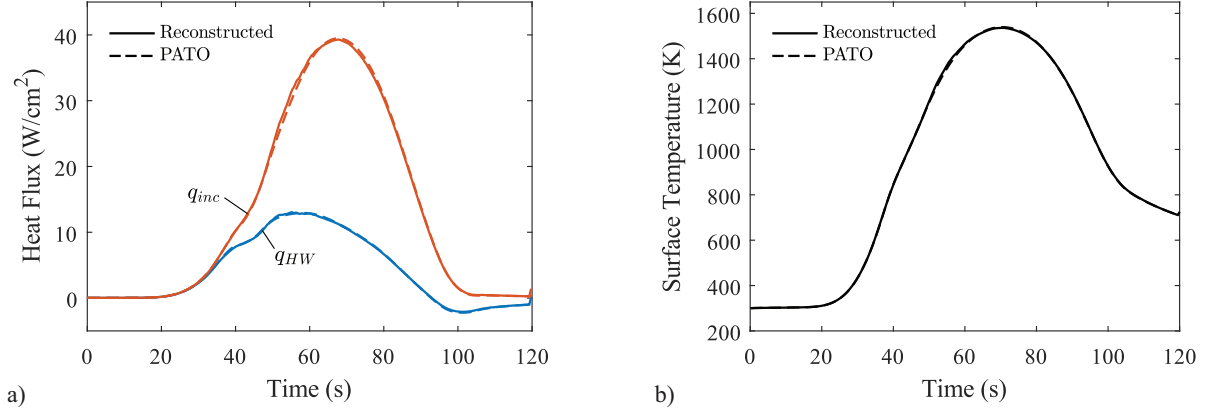


Figure 10.4: Numerical verification results. a) Reconstructed net hot-wall and incident heat flux. b) Reconstructed surface temperature. Reference solutions are shown by the dashed line.

The reconstruction algorithm recovers the TPS surface heating conditions in good agreement with the reference solution. The approach is accurate to within 5% throughout the significant heating region (from $t = 45$ s to $t = 90$ s), with errors of 2% for both the peak net hot-wall and incident heat flux values. The surface temperature rise from initial conditions is reconstructed to within 1% of the PATO reference solution.

The reconstructed through-thickness material response is compared next against the reference solution to assess the algorithm's ability to capture global heat conduction and thermo-physical phenomena in pyrolyzing TPS materials. Figure 10.5 compares the through-thickness temperature distribution (Fig. 10.5a) and decomposition, represented as $1 - \chi$ (Fig. 10.5b), for every third integration point in the numerical solution of g_{tot} . Similar to the reconstructed surface heat flux and temperature, the through-thickness material response is captured in excellent agreement with the reference solution.

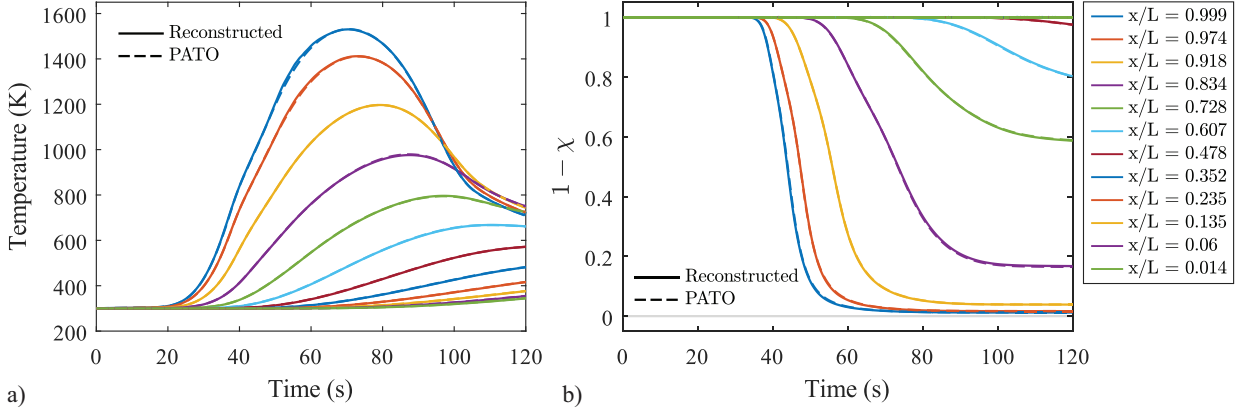


Figure 10.5: Numerical verification results. a) Reconstructed through-thickness temperature distribution. b) Reconstructed through-thickness composition. The temperature probe is located at $x/L = 0.915$. Reference solutions are shown by the dashed lines.

10.3.1 Numerical Integration Resolution

Due to the highly nonlinear behavior and numerical stiffness of the TPS decomposition reaction, accurate reconstructions which consider the effects of pyrolysis and internal gas transport require an adequate resolution of numerical integration points near the surface. If the resolution of the numerical integration scheme is coarse in relation to the length scale of the decomposition front (typically designated as the region where $0.02 < \chi < 0.98$), sharp spatial variations in $\partial \rho_s / \partial t$ can lead to large nonphysical oscillations in the pyrolysis gas mass flux (Eq. (10.15)), negatively impacting the reconstruction accuracy.

Figure 10.6 shows the reconstructed heating conditions (Fig. 10.6a-d) and pyrolysis gas mass flux at the TPS surface (Fig. 10.6e-h) for four different values of N_x . Gaussian quadrature schemes generate exact integral solutions for polynomials of order $2N_x - 1$ or less [140, 141]. Thus, the lowest value of $N_x = 12$ is the minimum numerical integration order that would yield an exact solution to the integral of Green's function (approximated using a 22nd-order polynomial

for $N = 12$) in g_{tot} , assuming no spatial variation in g' .

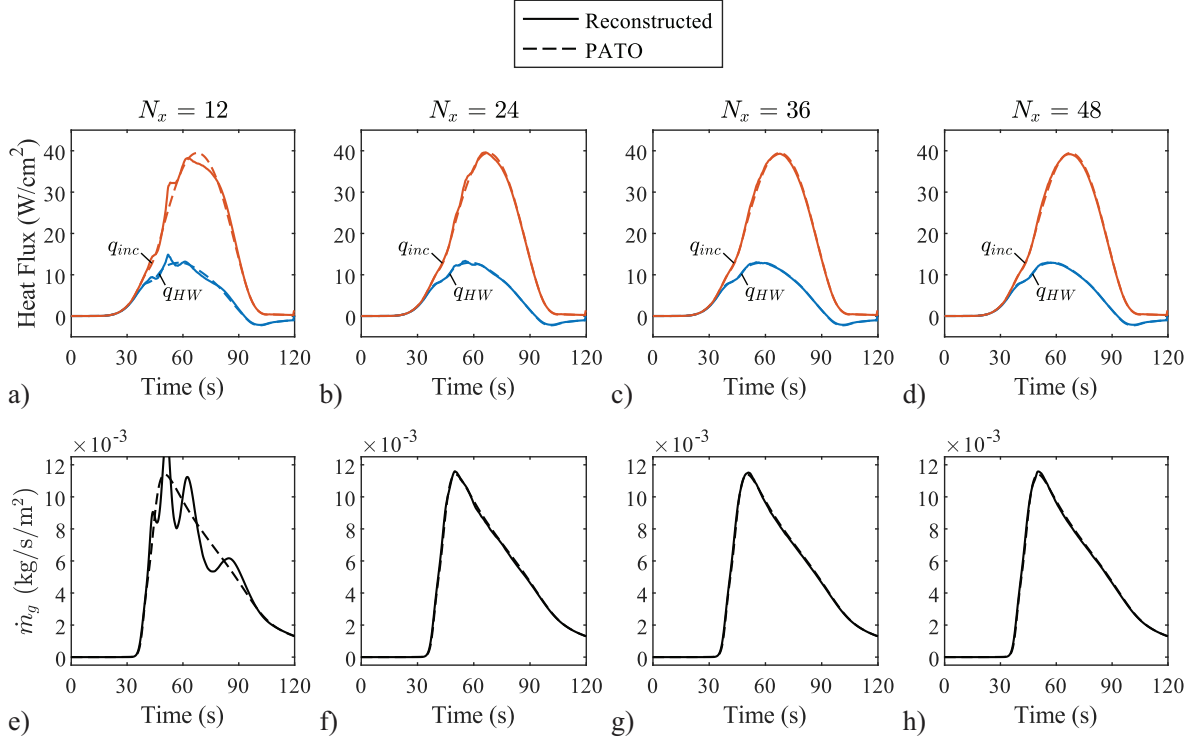


Figure 10.6: Reconstructed solutions for various numerical integration orders N_x . a-d) Reconstructed surface heating conditions. e-h) Reconstructed pyrolysis gas mass flux at the TPS surface. Reference solutions are shown by the dashed lines.

Table 10.2: Total computation time for various numerical integration orders N_x .

N_x	Computation Time (s)
12	9.2
24	14.2
36	23.1
48	34.9

Large oscillations are visible in the solution of the lowest quadrature order (Fig. 10.6a and Fig. 10.6e), which become progressively smoothed as the resolution of the numerical integration scheme increases. Beyond a numerical integration order of $N_x = 36$, representative of the verification results shown previously, the reconstructed heating conditions and material response

become insensitive to refinement. Solution times for the four reconstructions shown in Fig. 10.6, calculated using a consumer-grade PC, are tabulated in Table 10.2. The increase in computation time demonstrates that, for the same sized measurement time series, the total cost of the reconstruction algorithm becomes highly dependent on the number of integration points required to accurately calculate g_{tot} .

10.3.2 Galerkin Basis Function Order

In prior work, criteria for the minimum order N of the Galerkin basis function set used to approximate Green's function was established using the dimensionless thermal penetration depth [143]

$$\alpha_0 t_m / (L - x_T)^2 \quad (10.63)$$

where t_m is the characteristic time of the measurement time series. Considering a characteristic measurement time equal to half of the test scenario time (i.e., $t_m = 60$ s), the parameter $\alpha_0 t_m / (L - x_T)^2$ is equal to 13.4. From [143], it is expected that the Green's function approach will yield accurate solutions when $N \geq 5$ for these conditions. Reconstructed heating conditions for four different values of the Galerkin basis function order N are shown in Fig. 10.7. For all reconstructions in Fig. 10.7, a numerical integration order of $N_x = 36$ was used.

The results in Fig. 10.7 confirm the criteria outlined in [143]. For Galerkin basis function orders of $N = 6$ and above, the reconstructed heating solutions are indistinguishable. However, in contrast with the behavior of increasing the numerical integration order, there is no significant increase in computation time when increasing the Galerkin basis function order, as shown in Table 10.3. In fact, for solutions using higher-order basis functions ($N \geq 9$), the computation

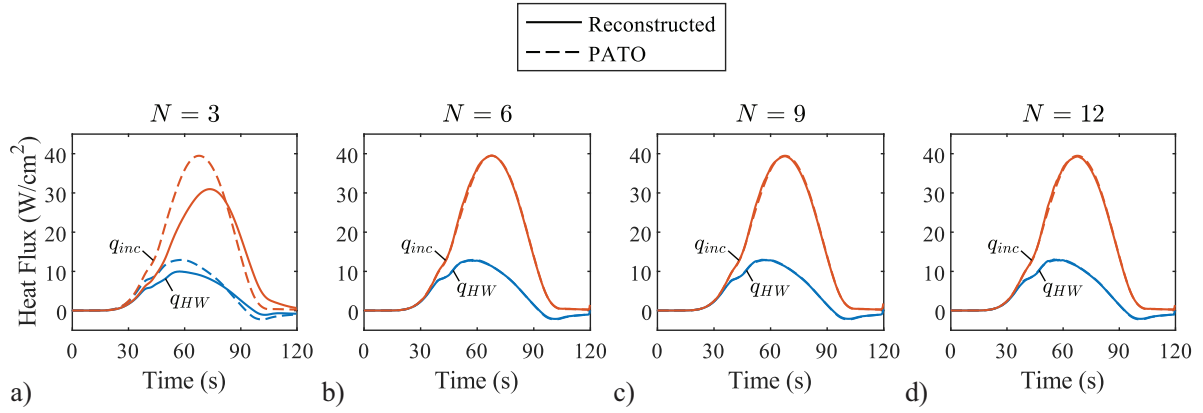


Figure 10.7: Reconstructed solutions for various Galerkin basis function orders N . Reference solutions are shown by the dashed lines.

time decreases due to faster convergence. Thus, as long as the basis function order N is above the minimum threshold, the precise choice of N is unlikely to have a considerable impact on the solution quality and speed.

Table 10.3: Total computation time for various Galerkin basis function orders N .

N	Computation Time (s)
3	17.7
6	21.6
9	20.8
12	23.1

10.4 Summary

A Green's function-based inverse heat transfer algorithm demonstrated accurate and efficient computation of the surface heating conditions on ablative TPS materials. The approach leverages a novel multi-parameter Cole-Hopf transformation to render the governing heat conduction equation for pyrolyzing solids compatible with the linear Green's function formalism. Nonlinear TPS material response phenomena, such as pyrolysis and internal gas transport, are

captured within the Green's function framework to account for their effects on the reconstructed heating conditions. In addition to the net-hot wall heat flux conducted into the TPS solid, the algorithm also calculates the surface temperature and composition, the incident heat flux from the external aeroheating environment, and the through-thickness decomposition and temperature distributions within the TPS. The performance of the approach was evaluated using a numerical test case which simulated the response of an ablative TPS, TACOT, subject to a combined convective and radiative heat pulse. A simulated temperature probe measurement and reference solution were generated using the high-fidelity material response code, PATO. Subject to representative atmospheric entry heating conditions, the approach was accurate to within 5% of the high-fidelity reference solution. Analysis of the reconstructed heating conditions subject to varying algorithm input parameters demonstrated how the accuracy of the approach is most sensitive to the resolution of the numerical integration scheme used to capture the effects of pyrolysis gas transport globally, primarily driven by the short length-scale of the TPS decomposition front.

Chapter 11: Reconstruction of Thermal Protection System Aeroheating Using Ground Test and Flight Measurements

Chapters 7-10 detailed the development of a robust Green's function-based inverse heat transfer approach to reconstruct the heating conditions on spacecraft during atmospheric entry. The approach introduced two novel innovations: I) a sensor fusion-based regularization methodology to improve the stability of reconstructed heating conditions, and II) an adaptation of the Green's function formalism to accommodate nonlinear heat conduction phenomena in ablative TPS materials. In the preceding chapters, the performance of each innovation was evaluated using numerical verification test cases generated via computational simulations. In this chapter, experimental measurements are used to validate the Green's function-based reconstruction approach and benchmark its performance against the current state-of-the-art inverse heat transfer code, FIAT_Opt. The data sets used were generated from both ground testing and flight missions. These include measurements from arc-jet test articles and from the backshell of the Mars 2020 atmospheric entry spacecraft.

11.1 FIAT_Opt

Throughout this chapter, reconstructions of the TPS surface heating conditions using the Green's function approach are compared to those generated using the state-of-the-art IHT code

FIAT_Opt. FIAT_Opt, developed at NASA Ames, utilizes NASA's one-dimensional thermal energy transport modeling tool, FIAT [90], introduced in Chapter 10. FIAT takes the aerothermal environment and aeroshell stackup as inputs, and outputs the material response of the TPS throughout its depth. In FIAT_Opt, embedded temperature measurements and the material stackup are used instead as inputs, and the program adjusts the surface heating profile to minimize the difference between the FIAT-generated temperature predictions and the input measurements. FIAT_Opt offers the user several options for objective function minimization and regularization; in this analysis, a Gauss-Newton method was used to minimize the difference between the predicted and measured temperatures, and Tikhonov regularization was used to damp instabilities.

11.2 Ground Test Validation

During the development and qualification of the Mars Entry Descent and Landing Instrumentation 2 (MEDLI2) sensor suite and TPS architecture used on the Mars 2020 spaceflight mission, numerous panel arc-jet tests were performed in the Panel Test Facility (PTF) at NASA Ames. PTF test campaign 162 consisted of four SLA-561V panels instrumented with a thermocouple (TC) plug, direct heat flux sensor, and radiometer [1, 14] (Fig. 11.1). The TC plug and heat flux sensor were separated by a distance of 10 cm along the spanwise flow direction. The TPS test panels were coated with a white thermal control spray, except for within a 5 cm radius of each heat flux sensor and TC plug, to mimic the Mars 2020 backshell configuration. The sensors were sampled at a rate of 20 Hz (downsampled to 5 Hz in the Green's function approach and 4 Hz in FIAT_Opt). Nominal test conditions are detailed in [1] and shown in Table 11.1. The first two tests represent low heating conditions, whereas the latter two tests represent high heating

conditions.

Due to the availability of measurements from both sensors, the sensor fusion-based regularization methodology (Chapter 8) was used to stabilize the reconstructed heating conditions, with regularization coefficients of $\lambda_2 = 10^6$ and $\lambda_2^{(0)} = 10^9$ used for all scenarios. The Galerkin basis function set defined in Chapter 10 of order $N = 12$ was used to construct Green's function, discretized assuming piecewise-constant temporal behavior of the heat flux (Eq. (7.14)). A numerical integration order of $N_x = 14$ was used to calculate g_{tot} (Eq. (10.57)).

Reconstructed heat flux profiles are shown in Fig. 11.2 for the low heating (Figs. 11.2a and 11.2b) and high heating (Figs. 11.2c and 11.2d) test conditions. Each figure displays the net hot-wall heat flux q_{HW} conducted into the solid and the incident heat flux q_{inc} from the arc-jet test environment (Eq. (10.61)), comparing the reconstructed heat flux profiles using the Green's function approach with those using FIAT_Opt. The plots also indicate the target cold-wall and adiabatic-wall heat flux values for each arc-jet test for comparison with the reconstructed incident heat flux.

Table 11.1: Panel arc-jet test campaign 162 run parameters [1].

Test	Target Adiabatic-Wall Heat Flux (W/cm ²)	Target Cold-Wall Heat Flux (W/cm ²)	Steady Test Duration (s)
1 & 2	4.3	5.4	63
3 & 4	8.6	9.7	25

Multiple stages of the arc-jet test sequence are evident in the heat flux profiles, including the tunnel startup, test article insertion (including initial transients), steady test period, and test article extraction phases (labeled in Fig. 11.2a). Most critical is the performance of the reconstruction algorithm during the steady test period, which begins at approximately $t = 45$ s and

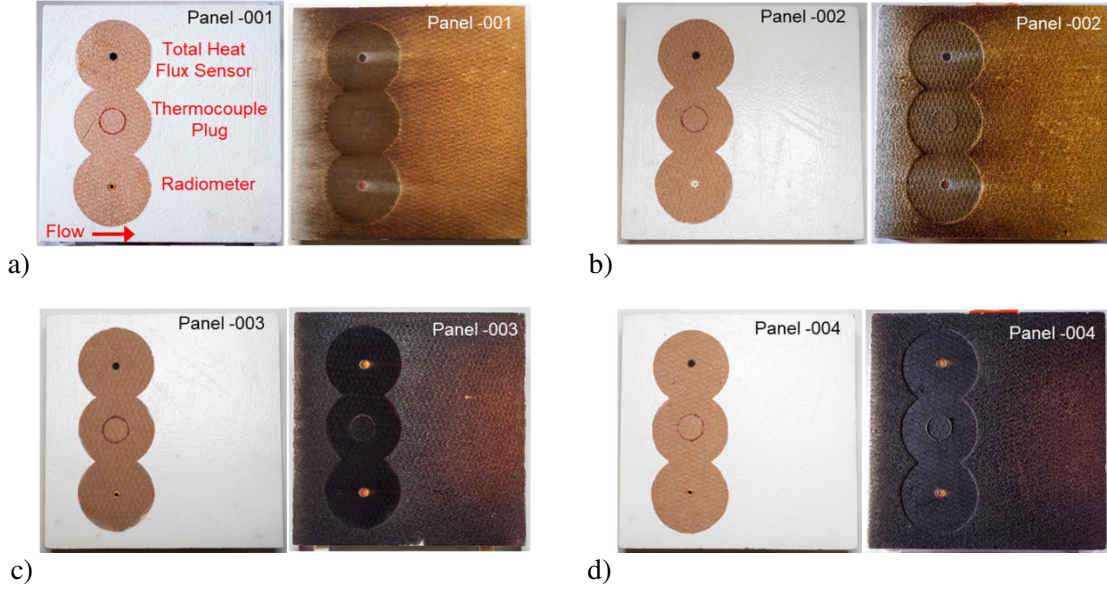


Figure 11.1: Panel Test Facility campaign 162 test articles before (left) and after (right) arc-jet exposure. a) Test 1. b) Test 2. c) Test 3. d) Test 4. Adapted from [14].

ends with a drop in the incident heating upon test article extraction. Throughout the steady test period, the net hot-wall heat flux decreases with time for all test scenarios as the TPS surface approaches an adiabatic-wall condition ($q_{HW} = 0$). Across all test scenarios, the reconstruction algorithm predicts the net hot-wall and incident heat flux to within 12% and 6% of FIAT_Opt, respectively. In the low heating scenarios, both methods predict q_{inc} within the target cold-wall and adiabatic-wall bounds during the steady test period. In the high heating scenarios, both methods overpredict the target cold-wall heating conditions, by up to 15% using FIAT_Opt and up to 10% using the Green's function approach. It should be noted, however, that the stated uncertainty for the calibrated PTF cold-wall heat flux is $\pm 15\%$ [144]. Considering expanded cold-wall and adiabatic-wall bounds due to facility calibration uncertainties, both reconstruction methods adequately predict the incident heating conditions in the high heating scenarios.

The discrepancies between the incident heat flux calculated using the Green's function approach and FIAT_Opt are correlated with differences in the reconstructed surface temperatures $\tilde{T}_s$,

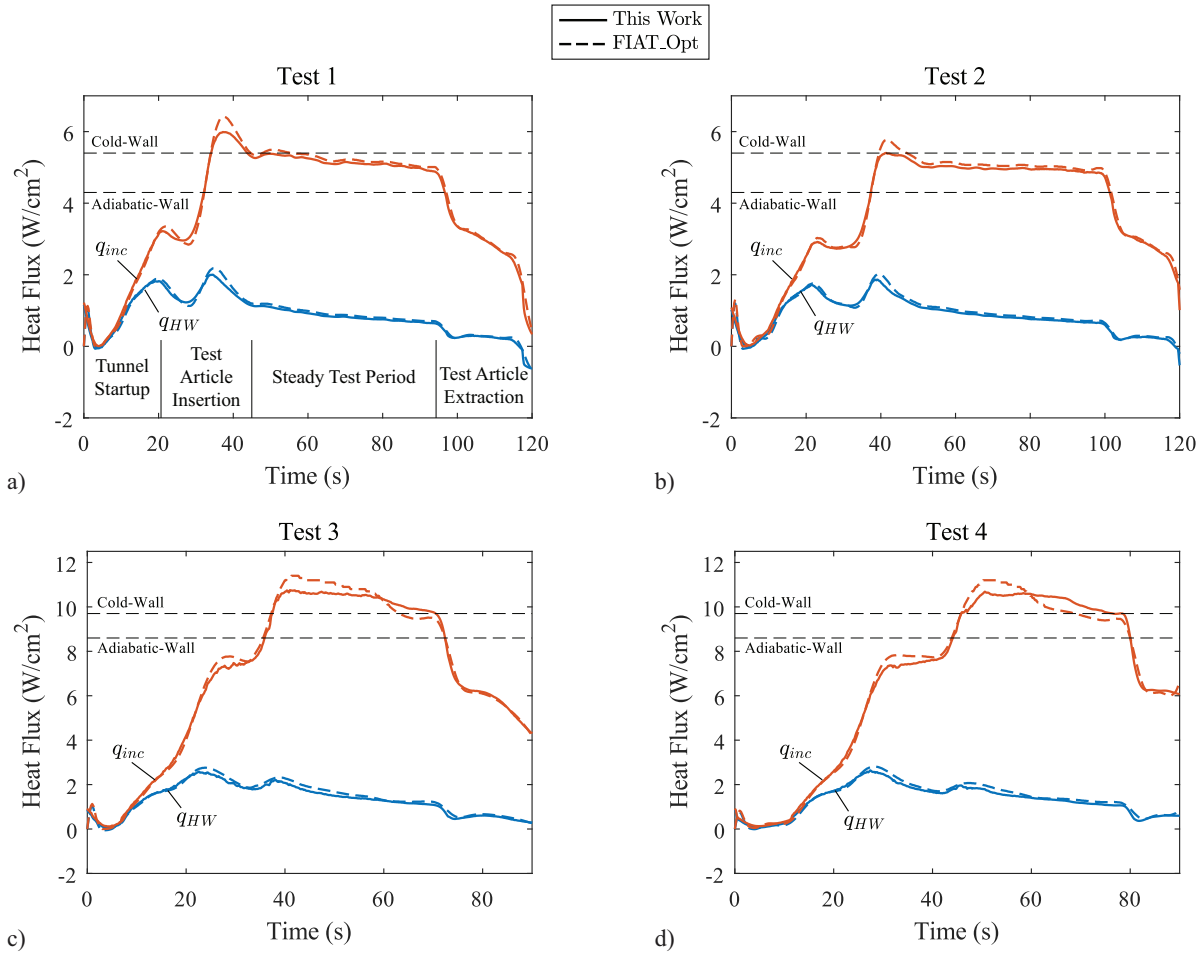


Figure 11.2: Reconstructed net hot-wall (blue) and incident (orange) heating results for instru-mented arc-jet test articles from PTF test campaign 162 a) Test 1, b) Test 2, c) Test 3, and d) Test 4. The solid series represent results from the Green's function approach and the dashed series represent results from FIAT.Opt. In each plot, the black dashed lines represent the target cold-wall or adiabatic-wall heat flux, labeled respectively.

notably for the high heating conditions. Figure 11.3 shows the reconstructed surface temperature profiles for all four tests. In the low heating conditions (Figs. 11.3a and 11.3b), the Green’s function approach recovers the surface temperature to within 1% of FIAT_Opt during the steady test region, with only minor discrepancies during the transient test article insertion phase. In the high heating scenarios (Figs. 11.3c and 11.3d), the Green’s function approach slightly underpredicts the TPS surface temperature, to within 2%, up to halfway through the steady test period (approximately at $t = 60$ s). In the latter half of the steady test period, the FIAT_Opt-reconstructed surface temperature is more dynamic and drops below that predicted by the Green’s function approach, commensurate with the behavior of q_{inc} in Figs. 11.2c and 11.2d. While the difference in the reconstructed surface temperature between the two approaches is small compared to the overall temperature rise, the radiative emission from the hot TPS surface scales as a function of $\tilde{T}_s^4$. Thus, in scenarios with high heating rates, small discrepancies in the reconstructed surface temperatures become amplified in the calculation of q_{inc} (Eq. (10.61)). Nonetheless, the results demonstrate that the Green’s function approach can reconstruct the heating conditions on ablative TPS materials in high-enthalpy flows in good agreement with the high-fidelity IHT methods used by the space exploration community.

11.3 Mars 2020 Backshell Reconstruction

The Mars 2020 atmospheric entry spacecraft flew an extensive set of TPS-embedded thermal instrumentation as part of the MEDLI2 sensor suite [15]. Among these were two pairs of collocated heat flux sensors and TC plugs located on the backshell shoulder region: the MTB01/MTB07 TC plug/heat flux sensor pair, embedded in the wind-side region of the back-

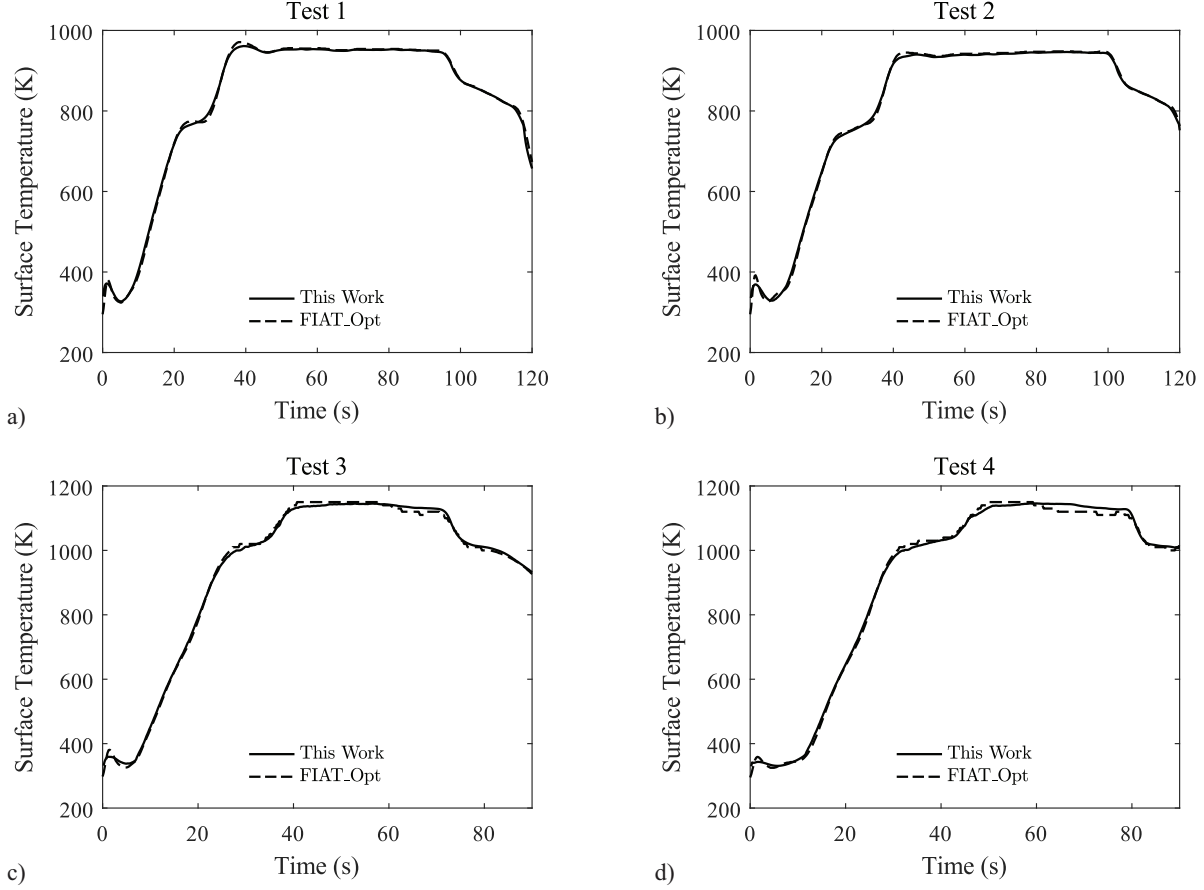


Figure 11.3: Reconstructed surface temperatures for instrumented arc-jet test articles from PTF test campaign 162 a) Test 1, b) Test 2, c) Test 3, and d) Test 4. The solid series represent results from the Green's function approach and the dashed series represent results from FIAT_Opt.

shell, and the MTB02/MTB08 sensor pair, embedded in the lee-side region of the backshell. Both sensor pairs are highlighted in Fig. 11.4. In the region of the sensors, the nominal thickness of the TPS was 1.27 cm [19]. Within each TC plug, the shallowest TC was placed at a nominal depth of 0.254 cm from the external TPS surface [2]. During atmospheric entry, measurements from the TC plugs and heat flux sensors were sampled at a rate of 8 Hz and 16 Hz, respectively. For both the Green's function approach and FIAT_Opt, all measurements were downsampled to 4 Hz. In the Green's function-based reconstruction algorithm, the same set of user input parameters (i.e., λ_2 , $\lambda_2^{(0)}$, N , and N_x) defined in the preceding section (Section 11.2) for PTF test 162 are used

throughout this section for the Mars 2020 flight reconstruction.

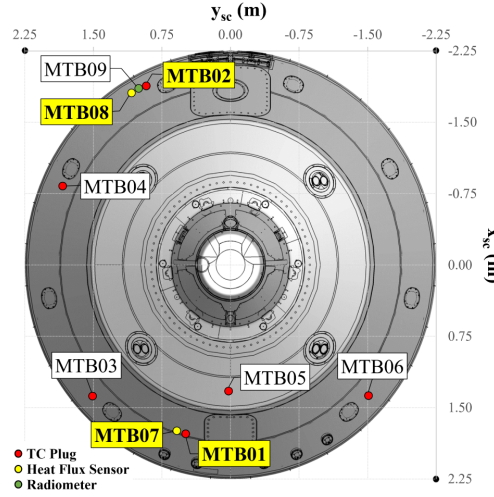


Figure 11.4: Schematic of the Mars 2020 backshell showing the MEDLI2 instrumented TC plug, heat flux sensor, and radiometer locations (adapted from [15]). Highlighted sensor labels indicate those used in this work.

11.3.1 Nominal Flight Reconstruction

Reconstructed net hot-wall and incident heat flux profiles are shown in Fig. 11.5 for both sensor pairs. Using the Green’s function approach, the results converged in less than 8 s, versus over 30 minutes using FIAT_Opt [2]. Consistent with prior analysis, the MTB02/MTB08 sensor pair on the leeward side experienced the highest incident heating conditions on the backshell, with a reconstructed peak heat flux of 4.8 W/cm^2 . For both sensor pairs, a bimodal peak region is present in the reconstructed heat flux profiles, suggesting the presence of separate radiative and convective peak heating regimes.

The Green’s function approach predicts a similar net hot-wall heat flux q_{HW} profile compared with FIAT_Opt, with peak values recovered to within 0.1 W/cm^2 for both sensor pairs. At the peak incident heating time, the Green’s function approach overpredicts the FIAT_Opt-

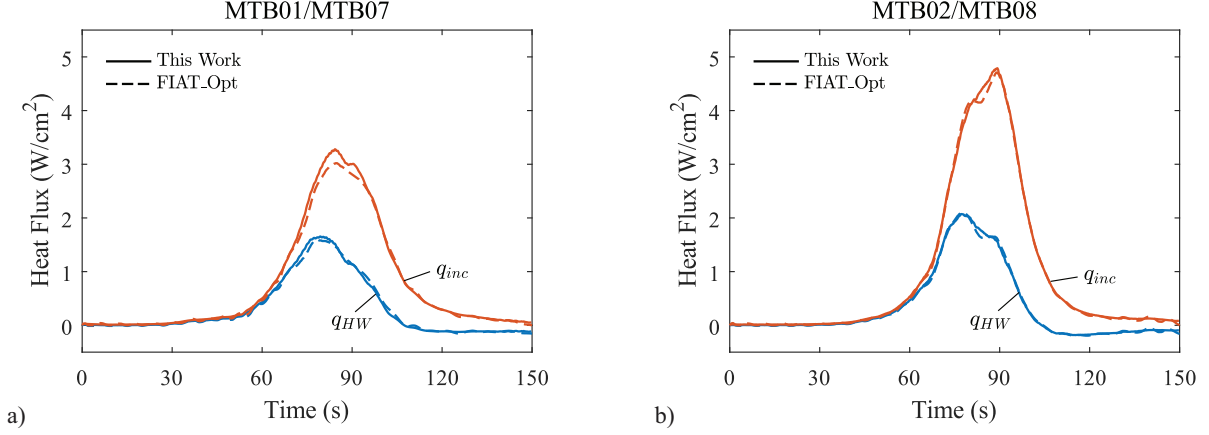


Figure 11.5: Reconstructed heat flux profiles from the Mars 2020 backshell. a) Results from the MTB01/MTB07 sensor pair. b) Results from the MTB02/MTB08 sensor pair.

generated incident heat flux q_{inc} by 0.3 W/cm^2 and 0.1 W/cm^2 for the MTB01/MTB07 and MTB02/MTB08 sensor pairs, respectively. The higher estimated peak incident heating using the Green's function approach is correlated with a higher reconstructed TPS surface temperature versus FIAT_Opt. Reconstructed surface temperature profiles using both approaches are shown in Fig. 11.6. For both sensor pairs, the Green's function approach overpredicts the surface temperature near the peak heating time, by 23 K and 8 K for the MTB01/MTB07 and MTB02/MTB08 sensor pairs, respectively. The amplification of the reconstructed surface temperature discrepancies in the calculation of q_{inc} can be explained in a similar fashion as in Section 11.2, i.e. the radiative emission from the TPS surface scales with $\tilde{T}_s^4$, amplifying small errors in T_s in the calculation of q_{inc} . Nonetheless, these results reinforce the conclusions from the preceding section, and demonstrate that the Green's function approach can capture the relevant heat conduction phenomena in spacecraft TPS during atmospheric entry, generating reconstructed heating solutions in close agreement with state-of-the-art IHT methods. Furthermore, the Green's function approach can compute such in over two orders of magnitude less time using inferior computing resources.

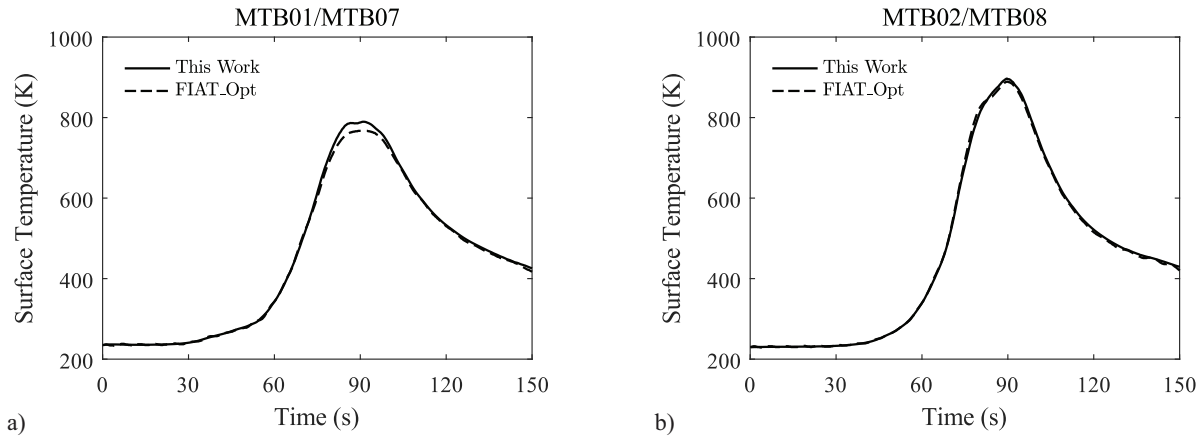


Figure 11.6: Reconstructed surface temperatures from the Mars 2020 backshell. a) Results from the MTB01/MTB07 sensor pair. b) Results from the MTB02/MTB08 sensor pair.

11.3.2 Uncertainty Quantification

In the post-flight analysis of Mars 2020, Monte Carlo analyses were used to determine the uncertainty of the reconstructed heating conditions. While Monte Carlo simulations are sufficient to estimate the total reconstruction uncertainty, more detailed analyses are required to decompose the individual contributions of various input parameters to the total reconstruction uncertainty. These analyses are critical to guide materials characterization efforts and reduce the margins of post-flight reconstructed atmospheric entry environments in the most cost-effective manner. Variance-based sensitivity analyses—such as the calculation of first order or total Sobol indices—are often used to quantify input parameter uncertainty contributions [145]. As a metric, the total Sobol index of an input parameter is defined as the remaining model output variance when all other parameters are fixed, normalized by the total model variance [146]. More generally, the total Sobol index of a parameter can be considered as the fractional contribution of the uncertainty of that parameter to the total model output uncertainty. Due to the computational cost of such analyses, however, calculating total Sobol indices organically using FIAT_Opt is not a practical

endeavor. In prior work, a machine learning-based approach was implemented to overcome the computational burden by developing a surrogate model to predict the peak heat flux and heat load at each TC plug location [2]. While this technique enabled the calculation of total Sobol indices with respect to the peak heat flux and heat load, the analyses did not give any insight with respect to other periods or features throughout the entry heat pulse.

In this section, the Green’s function approach is used to calculate the uncertainty of the reconstructed heating conditions, as well as total Sobol indices for each input parameter, throughout the entire atmospheric entry flight duration. Input parameter uncertainty distributions for the Mars 2020 flight lot backshell TPS and the embedded temperature probe are tabulated in Table 11.2. Uncertainty distributions in Table 11.2 are normalized by the mean value, unless otherwise noted, and are representative of those used in prior MEDLI2 post-flight analyses [2]. The variations in virgin and char densities were coupled using a single scaling distribution. For material properties that are bounded, such as emissivity, a truncated normal distribution was used to avoid non-physical inputs.

Table 11.2: Input uncertainty scaling parameters from [2]. Both uniform (U) and normal (N) distributions were used in this analysis.

	ρ	C_p	ϵ_v	ϵ_c	k	x_T
Distribution	U	N	N*	N*	N	N
$2\sigma/\mu$	$\pm 0.018^\dagger$	0.15	0.15	0.15	0.13	$0.0076 \text{ cm}^\ddagger$

* Truncated normal distribution

† Upper/lower bounds

‡ Not normalized by mean

Monte Carlo simulations (5000 samples) were used to compute the overall uncertainty of the reconstructed heating values. Resulting distributions of the reconstructed net hot-wall and incident heat flux for each sensor pair are shown in Fig. 11.7. In Fig. 11.7, the overall solution

distribution is represented as a mean solution with a 95% confidence interval envelope ($\pm 1.96\sigma$) for both the net hot-wall (blue) and incident (orange) heat flux.

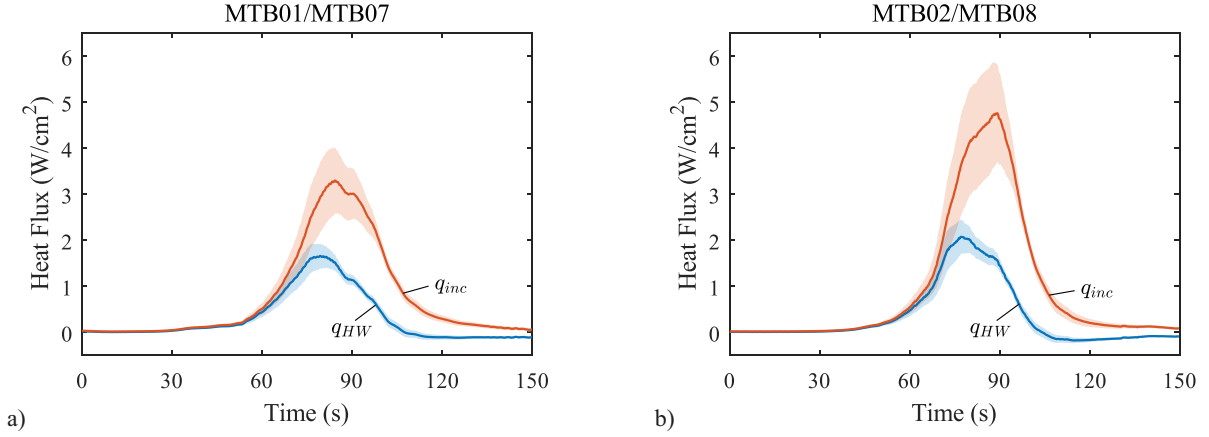


Figure 11.7: Results of the Monte Carlo simulations for the reconstructed surface heating conditions for the a) MTB01/MTB07 and b) MTB02/MTB08 sensor pairs. The solid line represents the mean solution. The shaded envelope represents a 95% confidence interval.

At the peak heat time, the 95% confidence interval for the reconstructed incident heat flux spans 1.4 W/cm² for the MTB01/MTB07 sensor pair (Fig. 11.7a) and 2.1 W/cm² for the MTB02/MTB08 sensor pair (Fig. 11.7b), in good agreement with prior analysis [2]. In both sensor pairs, the largest variation in both q_{HW} and q_{inc} coincided with the respective peak heating times. The margins of the reconstructed heating conditions due to input parameter uncertainties are, at least, a factor of 5 greater than the discrepancies in the nominal solutions using the Green's function approach and FIAT_Opt (Fig. 11.5).

Next, total Sobol indices for the reconstructed heating conditions were calculated using the Monte Carlo estimator proposed in [147]

$$S_{T,i} = \frac{1}{2} \frac{\sum_{j=1}^{N_s} [q(\mathbf{A}_s)_j - q(\mathbf{A}_B^{(i)})_j]^2}{\sum_{j=1}^{N_s} [q(\mathbf{A}_s)_j - q_0]^2} \quad (11.1)$$

In Eq. (11.1), $S_{T,i}$ is the total Sobol index for input parameter i , N_s represents the number of

samples per input parameter, q represents the model output, either the net hot-wall or incident heat flux, and q_0 is the mean solution. The variables $\mathbf{A}_s$ and $\mathbf{A}_B^{(i)}$ are sampling matrices with dimensions of $N_s \times k_s$, where k_s is the number of input parameters. The matrix $\mathbf{A}_B^{(i)}$ is constructed by replacing column i in matrix $\mathbf{A}_s$ with column i in matrix $\mathbf{B}_s$, where $\mathbf{B}_s$ is an independent sampling matrix with the same dimensions as $\mathbf{A}_s$. Thus, for N_s samples, $N_s \times (k_s + 1)$ independent reconstructions are required.

Total Sobol indices were calculated at each time step during the entry heat pulse for the reconstructed net hot-wall and incident heat flux for both sensor pairs using $N_s = 5000$ samples per input parameter in Table 11.2 ($k_s = 6$). Convergence plots of the peak heating total Sobol indices for the MTB02/MTB08 sensor pair are shown in Fig. 11.8. For both q_{HW} and q_{inc} , total Sobol indices at the peak heating time were fully converged after approximately $N_s = 4000$ samples. Total Sobol indices for q_{HW} are plotted in Figs. 11.9 and 11.10 for the MTB01/MTB07 and MTB02/MTB08 sensor pairs, respectively. Likewise, total Sobol indices for q_{inc} are plotted in Figs. 11.11 and 11.12 for the MTB01/MTB07 and MTB02/MTB08 sensor pairs, respectively. In each plot, labeled datapoints at the respective peak heating times represent the total Sobol indices calculated using the machine learning approach described in [2].

For each sensor pair, total Sobol indices for q_{HW} (Figs. 11.9 and 11.10) remained relatively constant until the peak heating region, with the reconstruction uncertainty dominated by the uncertainty in specific heat ($S_{T,C_p} > 0.9$). At the peak of q_{HW} , total Sobol indices calculated using the current approach show good agreement with those calculated using the prior machine learning approach for both sensor pairs (solid circles in Figs. 11.9 and 11.10). Following the peak heating region, total Sobol indices vary drastically. As the heat pulse subsides, $S_{T,k}$ increases substantially, up to 0.95 at $t = 94$ s for both sensor pairs, and S_{T,C_p} falls sharply, remaining low

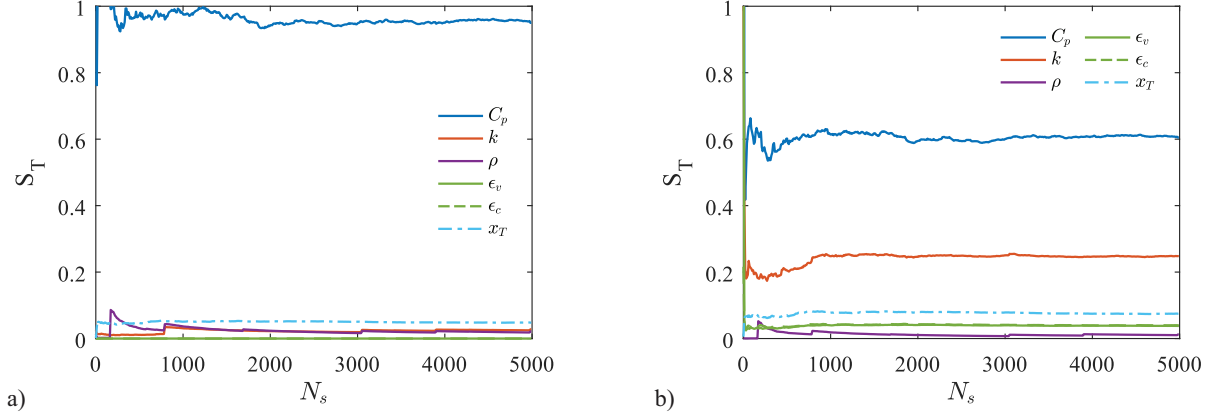


Figure 11.8: Convergence plots of the peak heating total Sobol indices for the a) net hot-wall heat flux and b) incident heat flux for the MTB02/MTB08 sensor pair.

well into the cooling phase of the heat pulse.

The strong dependence of the net hot-wall heat flux uncertainty on the uncertainty of the specific heat can be explained by considering how specific heat, or thermal mass ($\rho_s C_p$), dominates the transient response characteristics of the TPS thermal history near the embedded temperature probe (Eq. (10.16)). The effect of thermal mass is to impart a delay into the temperature response when the TPS is subject to transient surface heating conditions. Up to peak heating, the heat flux boundary condition is increasing rapidly, such that any variation in the delay introduced in the reconstruction relative to the mean solution will induce the most significant variation in the model output. The large subsequent increase in $S_{T,k}$ following the peak heating region may be explained by considering the heat soak within the TPS. Due to the highly insulating properties of the Mars 2020 backshell TPS, the short entry heat pulse likely achieved a very shallow thermal penetration depth, leading to significant thermal energy storage near the surface in the form of a hot layer. After the regime of maximum heating, the rate of heat dissipation from the near-surface hot region into the inner TPS will occur proportional to the thermal conductivity, leading to a larger propagation of thermal conductivity uncertainty into the total output uncertainty. For

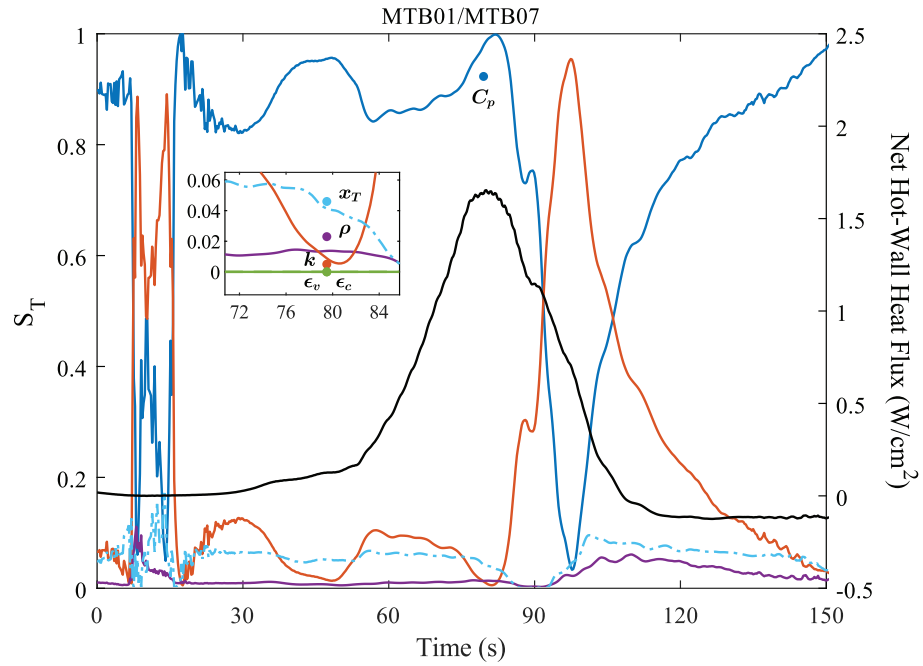


Figure 11.9: Total Sobol index calculations for the reconstructed net hot-wall heat flux for the MTB01/MTB07 sensor pair. Labeled datapoints at the respective peak heating times are the total Sobol indices calculated using the machine learning approach described in [2]. The solid black line is the nominal reconstructed net-hot wall heat flux solution from Fig. 11.5a. Inset: Magnified region for the total Sobol indices at the peak heating time.

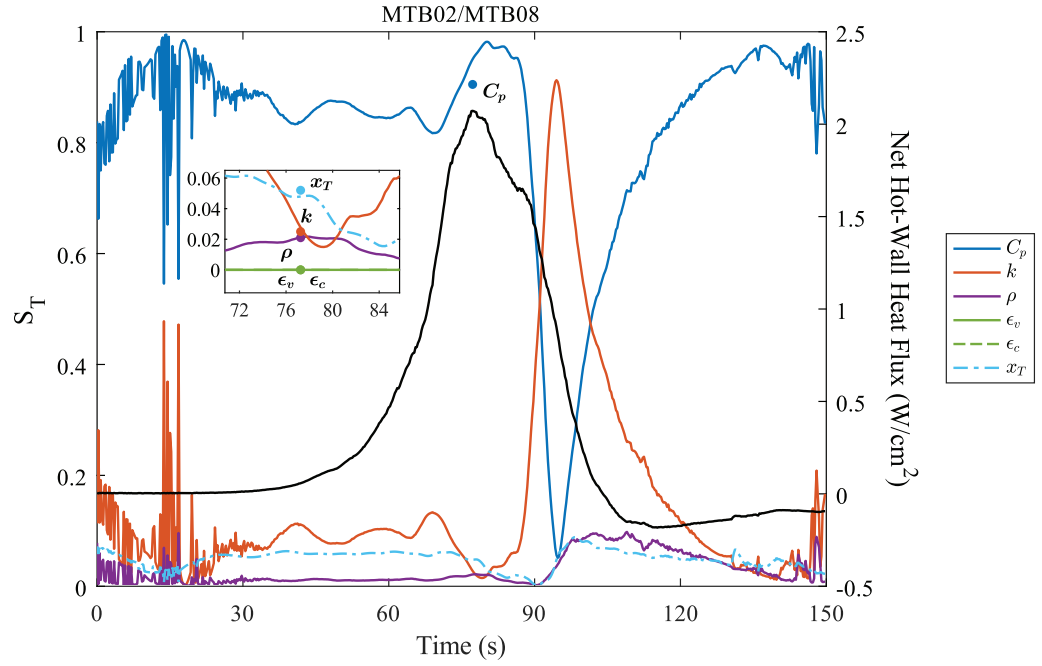


Figure 11.10: Total Sobol index calculations for the reconstructed net hot-wall heat flux for the MTB02/MTB08 sensor pair. Labeled datapoints at the respective peak heating times are the total Sobol indices calculated using the machine learning approach described in [2]. The solid black line is the nominal reconstructed net-hot wall heat flux solution from Fig. 11.5b. Inset: Magnified region for the total Sobol indices at the peak heating time.

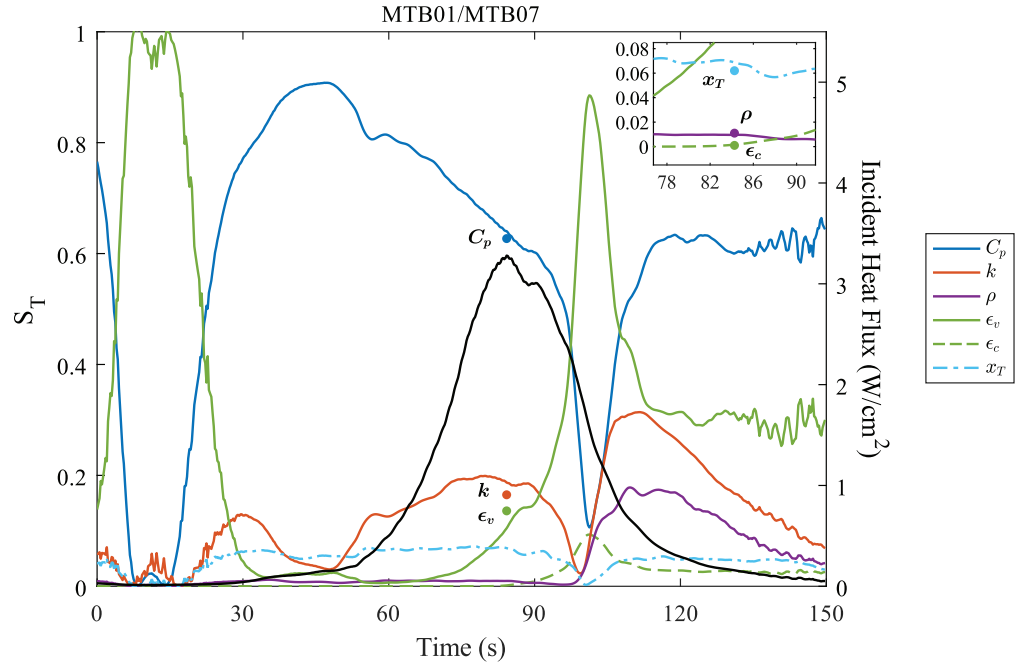


Figure 11.11: Total Sobol index calculations for the reconstructed incident heat flux for the MTB01/MTB07 sensor pair. Labeled datapoints at the respective peak heating times are the total Sobol indices calculated using the machine learning approach described in [2]. The solid black line is the nominal reconstructed incident heat flux solution from Fig. 11.5a. Inset: Magnified region for the total Sobol indices at the peak heating time.

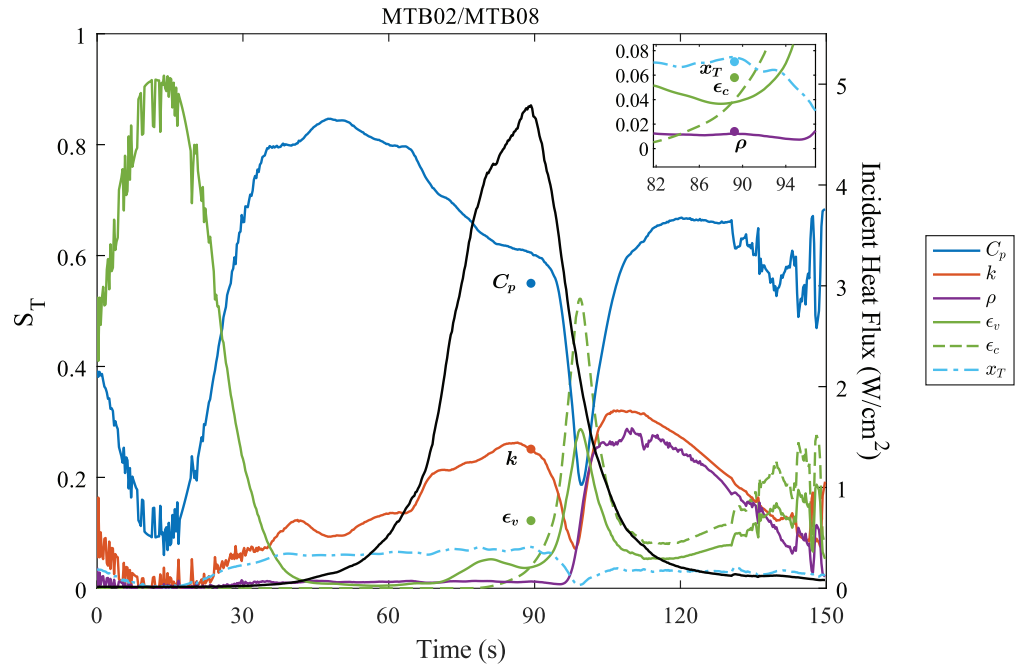


Figure 11.12: Total Sobol index calculations for the reconstructed incident heat flux for the MTB02/MTB08 sensor pair. Labeled datapoints at the respective peak heating times are the total Sobol indices calculated using the machine learning approach described in [2]. The solid black line is the nominal reconstructed incident heat flux solution from Fig. 11.5b. Inset: Magnified region for the total Sobol indices at the peak heating time.

both sensor pairs, as the hot layer cools, the reconstruction uncertainty becomes less sensitive to variations in the thermal conductivity, leading to a steady decay of $S_{T,k}$ with time.

For both sensor pairs, the uncertainty of q_{inc} (Figs. 11.11 and 11.12) up to peak heating is dominated by the uncertainty in the specific heat, albeit to a lesser extent than the reconstructed net hot-wall heat flux. Generally, at the peak incident heat flux, $S_{T,C_p} \approx 0.6$ and $S_{T,k} \approx 0.2$, both of which are in good agreement with results from the prior machine learning approach. Other total Sobol indices at the peak incident heating time are also in good agreement with the prior approach, with the exception of S_{T,ϵ_v} for the MTB02/MTB08 sensor pair (Fig. 11.12).

Immediately following the peak incident heating region, S_{T,C_p} and $S_{T,k}$ drop rapidly, and S_{T,ϵ_v} and S_{T,ϵ_c} rise. For the MTB01/MTB07 sensor pair, the virgin emissivity uncertainty begins to dominate the total output uncertainty, with $S_{T,\epsilon_v} = 0.9$ at $t = 100$ s. For the MTB02/MTB08 sensor pair, the char emissivity uncertainty begins to dominate the total output uncertainty, with $S_{T,\epsilon_c} = 0.5$ at $t = 100$ s. Following their respective peaks, S_{T,ϵ_v} and S_{T,ϵ_c} decay quickly and S_{T,C_p} , $S_{T,k}$, and $S_{T,\rho}$ increase.

The incident heating results (Figs. 11.11 and 11.12) show a similar trend as the net hot-wall heating results (Figs. 11.9 and 11.10), where the uncertainty in heat capacity is the main driver of the reconstruction uncertainty up to peak heating. For q_{inc} , the larger value of $S_{T,k}$ up to peak heating is attributed to the effects of radiative emission at the TPS surface: while the transient thermal response characteristics of the TPS may be more strongly coupled with its thermal mass, the magnitude of the temperature rise at the surface, which contributes to the radiative emission by $\tilde{T}_s^4$, is dominated more-so by its thermal conductivity. Thus, the variation in q_{inc} becomes sensitive to the variation in the thermal conductivity. Following peak heating, the corresponding spike in either S_{T,ϵ_v} or S_{T,ϵ_c} at $t = 100$ s (Figs. 11.11 and 11.12) is attributed to the estimated

char state at the TPS surface: due to the increased heating at the MTB02/MTB08 sensor pair location, the surface of the MTB02 TC plug at $t = 100$ s is estimated to be closer to a fully charred state, whereas the MTB01 TC plug surface is estimated to be only slightly pyrolyzed, resulting in a different sensitivity to variation in virgin and char properties. After $t = 100$ s the increase of $S_{T,\rho}$ can be directly attributed to how decomposition of the TPS is coupled with other model system input parameters: during pyrolysis, all thermophysical properties change as a function of the char volume fraction χ , which is, itself, purely a function of the solid density ρ_s relative to the densities of the fully virgin and charred states, ρ_v and ρ_c , respectively (Eq. (10.3)). Thus, if ρ_v and ρ_c are varied, all effective thermophysical properties during decomposition of the TPS become varied as a direct consequence, changing the output solution.

For the Mars 2020 backshell TPS, reductions in the uncertainty of the specific heat and thermal conductivity will yield the most effective reduction in the reconstructed aeroheating uncertainty in the region of peak heating. Further reductions in the uncertainty of the reconstructed heating conditions following the region of peak heating can be gained by reducing the uncertainty of the emissivity and density. These results highlight how the uncertainty margins attributed to post-flight aerothermal environment reconstructions are dynamically sensitive to variation in the different input parameters throughout the entry heat pulse. While the peak heating total Sobol indices expose the input parameters which drive the uncertainty of the aeroheating reconstruction within the region of the highest heat flux and the largest reconstruction uncertainty, they are not necessarily applicable to other regions which are also considered in the design and sizing of TPS. This is a significant consideration when adapting uncertainty quantification efforts to other missions, such as Dragonfly, which utilizes a similar TPS architecture to Mars 2020 but features a prolonged convective cooling phase during atmospheric entry [148].

11.4 Summary

A novel Green's function-based IHT algorithm was used to reconstruct the surface heating conditions on experimental ground test and flight systems using measurements from TPS-embedded thermal instrumentation. The approach was validated using arc-jet measurements obtained during the development and qualification of the Mars Entry Descent and Landing Instrumentation 2 sensor suite, and applied to the reconstruction of the heating conditions on the Mars 2020 backshell during Martian atmospheric entry. In all scenarios, the Green's function approach was benchmarked against the current state-of-the-art IHT code, FIAT_Opt. Owing to the inherent efficiency of the Green's function approach, the algorithm demonstrated a > 2 order of magnitude reduction in computation time versus FIAT_Opt, all without significant discrepancies in the reconstructed heating conditions between the two methods. The capabilities of the approach enabled a full flight variance-based sensitivity analysis of the reconstructed heating conditions on the Mars 2020 backshell during atmospheric entry, highlighting, for the first time, how the uncertainties of the specific heat, thermal conductivity, and emissivity of the Mars 2020 flight lot TPS were dominant drivers of the total reconstruction uncertainty at different times during the entry heat pulse.

Chapter 12: Conclusions, Contributions, and Future Work

12.1 Conclusions

The objectives of this thesis were to advance the capabilities of heat flux sensing technologies for extreme environments and develop efficient inverse heat transfer techniques for the post-flight reconstruction of atmospheric entry heating. Moderate- and high-temperature heat flux sensors were developed using the transverse Seebeck effect (TSE) in single crystal antimony and single crystal rhenium. The antimony-based heat flux sensor demonstrated the TSE as a practical transduction mechanism for sensors with a ruggedized package and highlighted the various design elements that influence the output signal. Principally, the results suggest how the sensor's steady state responsivity is impacted primarily by the thermal conductivity and arrangement of sensor package components, and how the sensor's transient response time is most sensitive to the thermal mass of the heat spreading components in front of the transducers. These behaviors apply generally to heat flux sensors that are integrated within a robust package and can be used to guide the design of future devices for better performance.

The rhenium-based TSE heat flux sensor showcased promising high-temperature performance characteristics and good compatibility with extreme environments. Due to the high melting points of rhenium and the tungsten-rhenium alloy electrical wiring used in the transducer circuit (~ 3000 °C), the sensor has a maximum operating temperature only restricted by the

limits of the materials used in the sensor packaging. The sensor exhibits good resilience when exposed long term to temperatures as high as 1000 °C and, notably, has a responsivity that increases in magnitude when used at elevated temperatures. The strong temperature-dependence of the sensor's responsivity emphasizes the importance of calibrating heat flux sensors under conditions similar to their intended use. These results demonstrate the TSE in refractory metal single crystals as an enabling technology to fill the gap between current sensing capabilities and the requirements of atmospheric entry, and highlight the potential performance that can be achieved by these sensors in extreme environments.

A Green's function-based inverse heat transfer (IHT) algorithm was developed to efficiently reconstruct the heat loads experienced by spacecraft during atmospheric entry. The algorithm introduces a sensor fusion-based regularization approach to improve the stability of reconstructed heating conditions and an adaptation of the Green's function formalism to accommodate nonlinear heat conduction phenomena in ablative TPS materials. The integration of measurements from collocated heat flux and temperature sensors into the same IHT framework yielded significant improvements over current regularization methods, including a better recovery of sharp solution features and increased robustness of the reconstruction to sub-optimal choice of user-defined regularization parameters. The adaptation of the Green's function formulation for ablative TPS is enabled by a novel multi-parameter Cole-Hopf transformation, where the nonlinear effects of temperature- and composition-dependent TPS thermal properties are largely removed from the governing equations. Applied to a numerical verification scenario simulating an atmospheric entry heat pulse, the approach accurately reconstructs the heating conditions, surface temperature, pyrolysis gas blowing rate, and through-thickness distributions of the TPS temperature and composition from a single input temperature measurement. These results demonstrate

the Green's function framework as a viable technique for modeling heat conduction in ablative material systems, with flexibility to accommodate other nonlinear material response behavior, such as recession and gas-surface interactions.

The Green's function-based IHT algorithm was used to reconstruct the heat loads on experimental arc jet ground test articles and atmospheric entry spacecraft, further highlighting its capabilities when applied to material systems relevant to space exploration and high-speed flight. Applied to the reconstruction of the heat loads on the backshell of the Mars 2020 flight vehicle, the Green's function approach recovers the surface heating conditions and surface temperature in good agreement with the current state-of-the-art IHT algorithm, FIAT_Opt, but can do so in > 2 orders of magnitude less time with inferior computing resources. The inherent efficiency of the Green's function approach enabled a full flight variance-based sensitivity analysis of the reconstructed Mars 2020 backshell heating conditions, lending insight into the major contributing factors to the reconstruction uncertainty outside of the peak heating region that were not captured by prior analyses. Overall, access to more efficient reconstruction techniques will help to reduce TPS sizing margins, improve material models, and guide materials characterization efforts at a substantially reduced cost than with existing methods.

12.2 Contributions

This thesis establishes multiple contributions to the field of thermal metrology. Chapter 3 detailed the first TSE-based heat flux sensor to leverage elemental antimony and, more broadly, is the first investigation of a TSE-based heat flux sensor with a ruggedized package. The high-temperature heat flux sensor detailed in Chapter 4 was the first TSE heat flux sensor to leverage

refractory metals for transduction. The transducer circuit in the sensor has the highest theoretical maximum operating temperature of all heat flux sensors studied in the literature (~ 3000 °C). Furthermore, the temperature-dependent behavior of the rhenium-based TSE heat flux sensor is unique, and is the only high-temperature heat flux sensor in the literature to exhibit a responsivity that monotonically increases in magnitude towards extreme high temperatures. The stability of the sensor's responsivity following long term heat treatment at 1000 °C, albeit, in an inert atmosphere, is also a novel result in the field, and bodes well for the use of the sensing technology outside of atmospheric entry applications, such as in energy generation facilities, e.g., for the detection of long time-scale dynamics in combustion processes due to energy demand fluctuations. The calibration facility developed in Chapter 5 is the current state-of-the-art facility in the field for temperature-dependent heat flux sensor calibrations up to 500 °C, and is the first to introduce closed-loop, independent control of the sensor temperature and the calibration heat flux.

The Green's function-based reconstruction algorithm represents a first-of-its-kind adaptation of the Green's function approach for modeling the material response of ablative thermal protection systems. The formalism detailed in Chapter 10 extends the capabilities of impulse response-based IHT methods from material systems with only temperature-dependent thermal properties to porous, decomposing, multi-phase solids. The sensor fusion regularization approach developed in Chapter 8 is the first use of a direct heat flux sensor measurement as a surrogate to stabilize ill-posed IHT problems, with the potential capabilities of the approach highlighted in Chapter 9. Chapter 11 represents the first use of a Green's function-based IHT algorithm to reconstruct the heat loads on atmospheric entry spacecraft from flight measurements. Juxtaposed with the current IHT methods used by the space exploration community, the Green's function approach developed in this thesis represents a >2 order of magnitude reduction in computation cost

over the current state of the art, enabling post-flight analyses that have previously been inaccessible due to the large associated cost. One such analysis is uncertainty quantification; in Chapter 11, the Green’s function approach was used to perform a variance-based sensitivity analysis of the reconstructed Mars 2020 backshell heat flux for the full flight duration. This is the first known instance of a sensitivity analysis being performed for the full flight duration for reconstructed atmospheric entry heating conditions.

12.2.1 List of Publications

Journal Publications:

1. McAfee, K., Sunderland, P. B., & Rabin, O. “A Heat Flux Sensor Leveraging the Transverse Seebeck Effect in Elemental Antimony.” *Sensors and Actuators A: Physical* **363**, 114729 (2023).
2. McAfee, K., Sunderland, P. B., & Rabin, O. “A High-Temperature Heat Flux Sensor Using the Transverse Seebeck Effect in Elemental Rhenium.” *Sensors and Actuators Reports* **10**, 100391 (2025).
3. McAfee, K., Song, Z., Sunderland, P. B., & Rabin, O. “Transfer Calibration of Heat Flux Sensors Up to 500 °C.” *International Journal of Heat and Mass Transfer* (2025, in preparation).
4. McAfee, K., Alpert, H. S., & Rabin, O. “Reconstruction of Ablative Thermal Protection System Aeroheating Using a Green’s Function Approach.” *Journal of Spacecraft and Rockets* (2025, in preparation).
5. McAfee, K., Alpert, H. S., & Rabin, O. “A Sensor Fusion-Based Regularization Approach for Stabilizing Inverse Heat Transfer Problems for Atmospheric Entry Spacecraft.” *Journal of Thermophysics and Heat Transfer* (2025, in preparation).

Conference Publications

1. McAfee, K., Alpert, H. S., & Rabin, O., “Reconstruction of Thermal Protection System Aeroheating Using a Green’s Function Approach,” AIAA 2025-2704. In *AIAA SCITECH FORUM 2025*. American Institute of Aeronautics and Astronautics, Orlando, FL (2025). doi:10.2514/6.2025-2704.

2. McAfee, K., Alpert, H. S., Sunderland, P. B., & Rabin, O., “A Green’s Function Sensor Fusion Approach for Evaluating Spacecraft Entry Heating From On-Board Thermal Instrumentation,” AIAA 2024-4033. In *AIAA AVIATION FORUM AND ASCEND 2024*. American Institute of Aeronautics and Astronautics, Las Vegas, NV (2024). doi:10.2514/6.2024-4033.

Oral Seminars

1. McAfee, K., “Green’s Functions for Efficient Inverse Heat Transfer Solvers.” NASA Ames Research Center, Moffett Field, CA (2025).
2. McAfee, K., “Design and Characterization of a Transverse Seebeck Effect Heat Flux Sensor.” Leibniz Institute for Solid State and Materials Research, Dresden, Germany (2023).

Conference Presentations

1. McAfee, K., Alpert, H. S., & Rabin, O., “Reconstruction and Uncertainty Quantification of Atmospheric Entry Heat Loads Using a Green’s Function Approach.” The 36th NASA Thermal Fluids and Analysis Workshop, San Jose, CA (2025). Poster Presentation
2. McAfee, K., Alpert, H. S., & Rabin, O., “A Green’s Function Sensor Fusion Approach for Reconstructing the Heat Loads on Atmospheric Entry Spacecraft.” The 22nd International Planetary Probe Workshop, Stuttgart, Germany (2025). Poster Presentation.
3. McAfee, K., Sunderland, P. B., & Rabin, O., “Extreme Environment Heat Flux Sensors for Planetary Entry Spacecraft.” The 22nd International Planetary Probe Workshop, Stuttgart, Germany (2025). Poster Presentation.
4. McAfee, K., Alpert, H. S., Sunderland, P. B., & Rabin, O., “Direct Reconstruction of Ablative Thermal Protection System Aeroheating Using a Green’s Function Approach.” The 14th Ablation Workshop, Laurel, MD (2024). Poster Presentation.
5. McAfee, K., Sunderland, P. B., & Rabin, O., “Robust Through Simplicity: Leveraging the Transverse Seebeck Effect for Extreme Environment Heat Flux Sensing.” The WE-Heraeus Seminar on Transverse Effects in Thermoelectric Systems, Bonn, Germany (2024). Poster Presentation. *Awarded Best Poster.*
6. McAfee, K., Sunderland, P. B., & Rabin, O., “Robust Heat Flux Sensors for Power Plant Extreme Environments.” The 2024 APS March Meeting, Minneapolis, MN (2024). Oral Presentation.
7. McAfee, K., Sunderland, P. B., & Rabin, O., “Design and High-Temperature Characterization of an Extreme Environment Heat Flux Sensor.” The 2023 JANNAF Joint Subcommittee Meeting, Salt Lake City, UT (2023). Oral Presentation.

8. McAfee, K., Sunderland, P. B., & Rabin, O., “Leveraging the Transverse Seebeck Effect for Robust Heat Flux Sensors.” The 34th NASA Thermal and Fluids Analysis Workshop, College Park, MD (2023). Poster Presentation. *Awarded Best Poster*.
9. McAfee, K., Sunderland, P. B., & Rabin, O., “Design and Characterization of a Single Crystal Antimony Heat Flux Sensor Utilizing the Transverse Seebeck Effect.” The 39th Annual International Conference on Thermoelectrics, Seattle, WA (2023). Oral Presentation.
10. McAfee, K., Sunderland, P. B., & Rabin, O., “Robust Heat Flux Measurement Systems via the Transverse Seebeck Effect.” Burger’s Program and Combustion Institute Summer School on Fire Safety Science, College Park, MD (2023). Poster Presentation.
11. McAfee, K., Sunderland, P. B., & Rabin, O., “Utilizing the Transverse Seebeck in Anisotropic Single Crystals for Novel Heat Flux Sensing Devices.” The 30th International Materials Research Congress, Virtual (2022). Oral Presentation.
12. McAfee, K., Sunderland, P. B., & Rabin, O., “Leveraging Thermoelectric Anisotropy in Single Crystal Antimony for Novel Heat Flux Sensing Devices.” The 64th Electronic Materials Conference, Columbus, OH (2022). Poster Presentation.

12.3 Future Work

12.3.1 Durability of the High-Temperature TSE Heat Flux Sensor in Extreme Environments.

The high-temperature TSE heat flux sensor was calibrated up to 500 °C and exposed to 1000 °C for a significant duration in an inert atmosphere. Otherwise, the limitations of the sensor’s durability were not tested in this thesis. Significant work remains to evaluate the capabilities of the rhenium-based sensor in extreme environment conditions, such as its resilience to long-duration thermal cycling (>1 cycle), thermal shock, exposure to oxidative environments, and vibrational loading. To support this type of testing, the high-temperature calibration facility could be upgraded to increase the maximum calibration temperature. Currently, the maximum calibration temperature is limited by the heat sink, which is made of copper and contains coolant

channels for air or water, in addition to external tubing, that are sealed using a brazing compound with a solidus of ~ 660 °C. The fabrication of a dedicated high-temperature stainless steel heat sink without coolant channels would increase the maximum calibration temperature of the facility up to 760 °C, which corresponds with the maximum operating temperature of the band heater.

12.3.2 Improving the Responsivity of TSE-Based Heat Flux Sensors

Generally, the TSE-based heat flux sensors developed in this thesis had responsivities that are much lower than conventional heat flux sensors, and even a few select high-temperature thermopile-based sensors reported in the literature. While a large responsivity was not the main objective of this work, it should be considered for future generation devices, as a low responsivity may limit the adoption and use of these sensors in engineering applications. To this end, the responsivity of a TSE-based sensor may be improved through the use of alternative transducer materials with large anisotropies in their Seebeck coefficient tensor. Within the field of thermoelectrics, a recent surge of interest in materials with anisotropic charge carrier transport properties has highlighted a few novel materials that are exceedingly well positioned for TSE-based devices, including Re_4Si_7 [149], WSi_2 [150], and PdSe_2 [151]. Most notable among these is Re_4Si_7 , which has a Seebeck coefficient anisotropy 2 orders of magnitude higher and a thermal conductivity that is a factor of 10 lower than elemental rhenium [65], and, thus, would generate an output signal $1000\times$ larger for the same geometry used in the high-temperature TSE heat flux sensor. While the maximum operating temperature of Re_4Si_7 (~ 1700 °C [152]) is lower than that of rhenium (> 3000 °C), it remains higher than that of any thermopile-based sensor using type-K or type-N thermocouple materials. As such, a heat flux sensor using Re_4Si_7 could potentially

demonstrate pioneering capabilities in both extreme environment compatibility and responsivity, once growth processes are sufficiently advanced to produce large form factor crystals that can be sectioned into transducers.

12.3.3 Modeling TPS Recession Using the Green's Function-Based IHT Algorithm

While the Green's function-based IHT algorithm was developed to accurately model the effects of decomposition and pyrolysis gas transport in ablative materials, the formulation did not consider recession of the TPS domain. This is a significant limitation when considering Earth entry applications, as recession of the char layer via oxidation has a considerable impact on the surface energy balance and in-depth TPS thermal response. Even if the recession profile can be measured explicitly, for example, during ground testing, a large degree of work remains to advance the Green's function IHT formalism for materials that ablate. This behavior is fundamentally contradictory to the definition of Green's functions, which are constructed using a stationary eigensystem. However, methods have been detailed in the literature in which the geometry of an ablating solid is preserved at the initial state in a transformed space [153], suggesting that further adaptations of the Green's function formalism for ablative TPS are possible.

Appendix A: Welding Process for W-26Re Wire on Rhenium Substrate

A fabrication process was developed to weld fine-gauge tungsten-rhenium alloy wires to the single crystal rhenium transducers. The wires were welded to the substrate using a spot welder (US Solid USS-BSW07). A copper electrode was used to apply pressure to the tungsten-rhenium wire down towards the substrate (Fig. [A.1](#)). The substrate was placed on a conductive aluminum plate. To complete the weld circuit, a second electrode was pressed against the aluminum plate. Argon was flowed into the weld region to prevent oxidation. A pulse time of 0.6 ms on the welder was used.

Due to the high melting points of rhenium and tungsten-rhenium alloys, some of the copper from the electrode inevitably becomes deposited on the weld region (Fig. [A.2a](#)). To remove the copper, a fabrication process step was implemented. The welded transducer circuit was baked in air at 300 °C for approximately 30 minutes to oxidize the deposited copper (Fig. [A.2b](#)). The transducer circuit was then submerged in glacial acetic acid at 50 °C for approximately 20 minutes, which selectively etches the copper oxides (Fig. [A.2c](#)). The bake and etch process step was repeated until no copper remained in the region of the weld joint (Fig. [A.2d](#)).

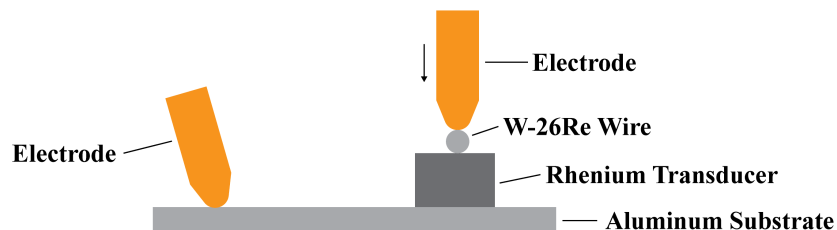


Figure A.1: Schematic of the weld setup. The tungsten-rhenium electrical wire is pressed onto the transducer using an electrode. The transducer is placed on an aluminum substrate. A separate welding electrode is pressed against the aluminum substrate.

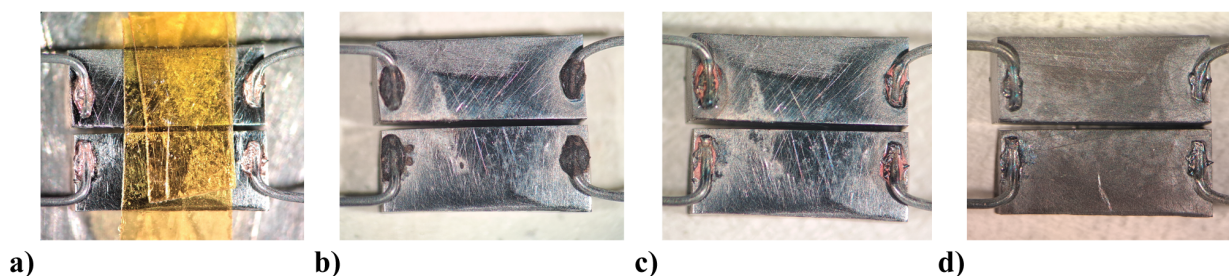


Figure A.2: Removal of copper deposited from the welding electrode. a) Rhenium transducer circuit after welding, showing significant copper deposition on the weld region. b) Transducer circuit after baking in air at 300 °C for 30 minutes. All exposed copper is oxidized. c) Transducer circuit after etching in glacial acetic acid for ~20 minutes. All the exposed copper oxide is removed, revealing the underneath layer of bare copper. d) Transducer circuit after repeating the bake and etch process until all copper is removed.

Appendix B: Detailed Engineering Drawings

B.1 Rhenium-based TSE Heat Flux Sensor

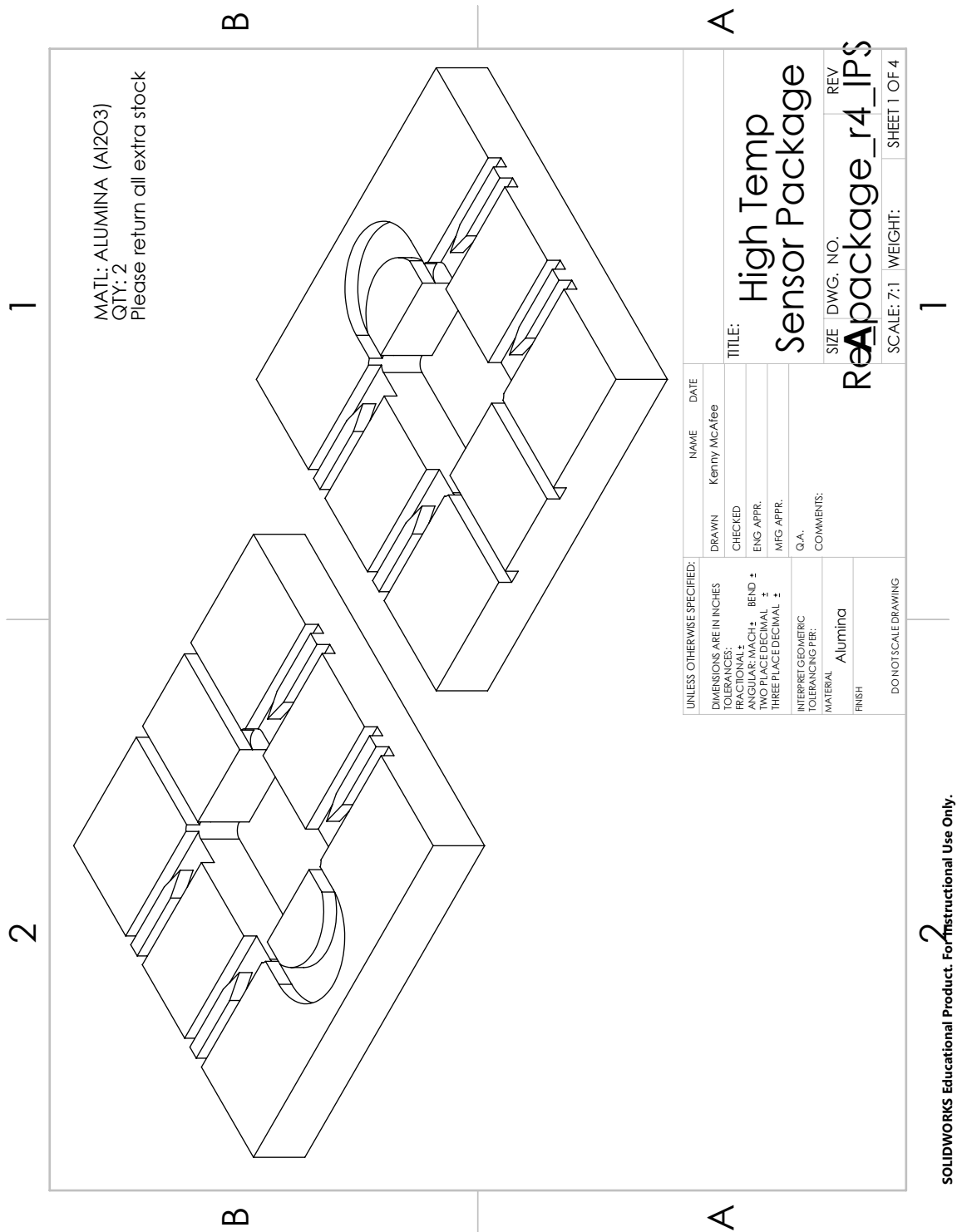


Figure B.1: High-temperature TSE-based heat flux sensor package: detailed drawing 1 of 3.

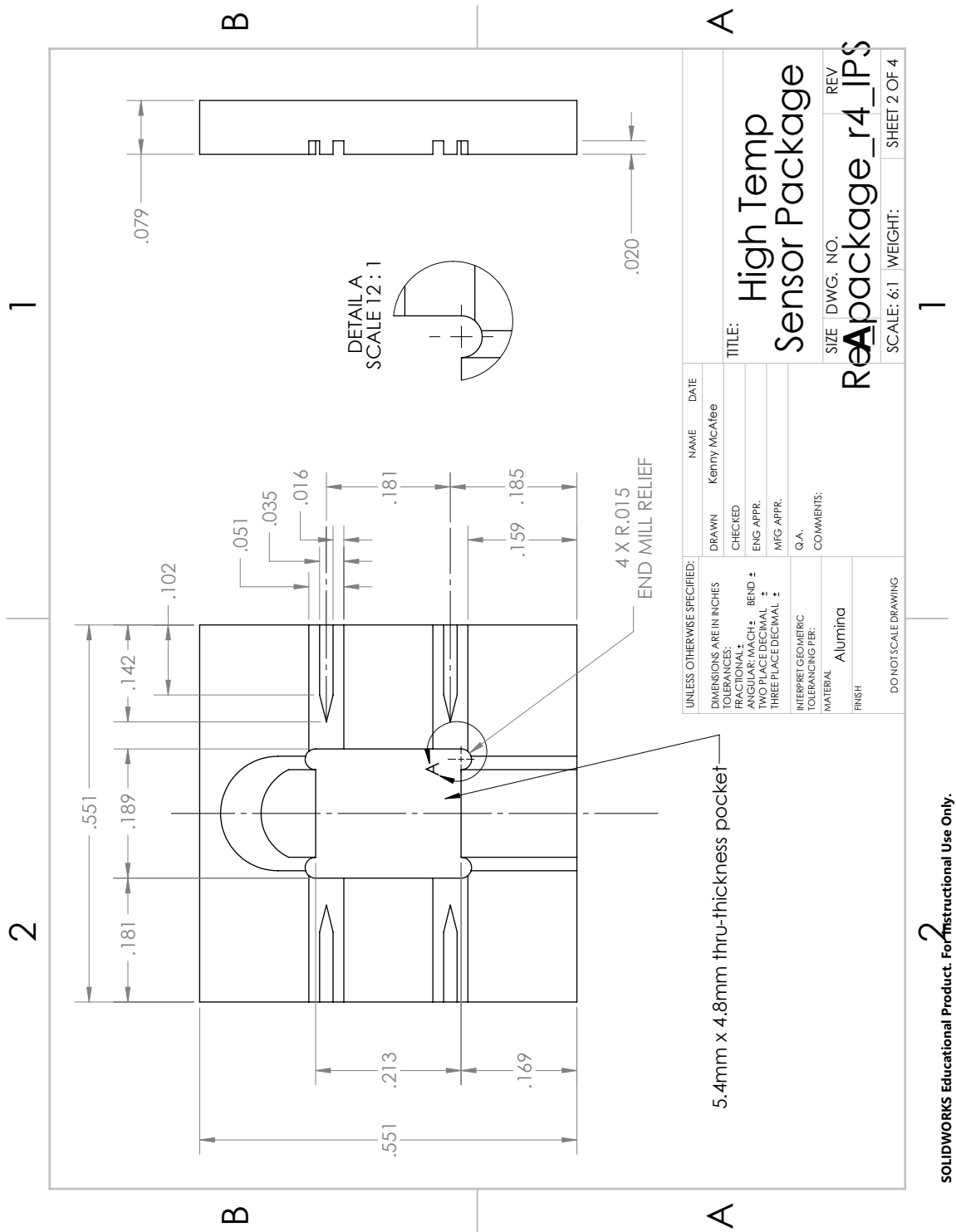
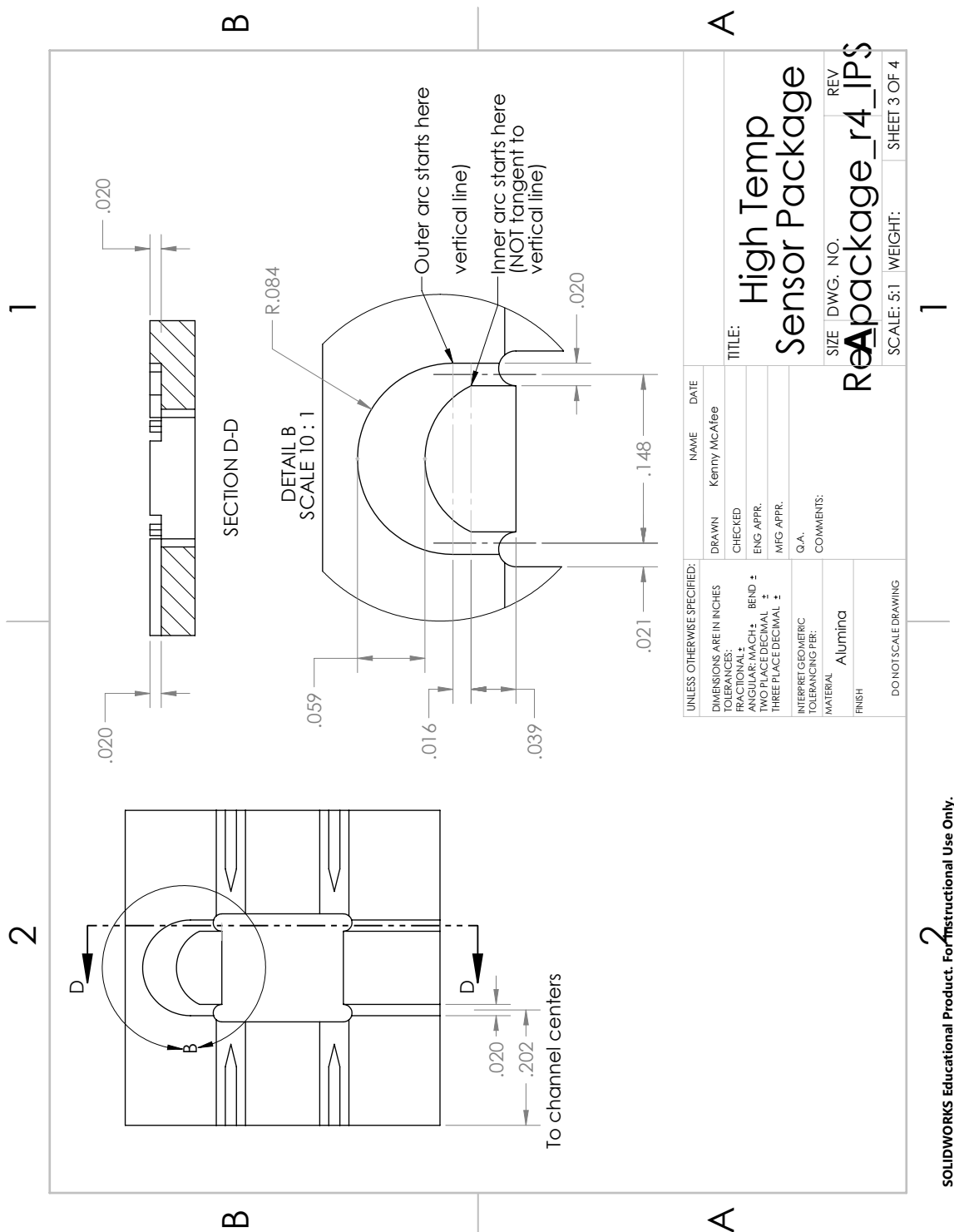


Figure B.2: High-temperature TSE-based heat flux sensor package: detailed drawing 2 of 3.



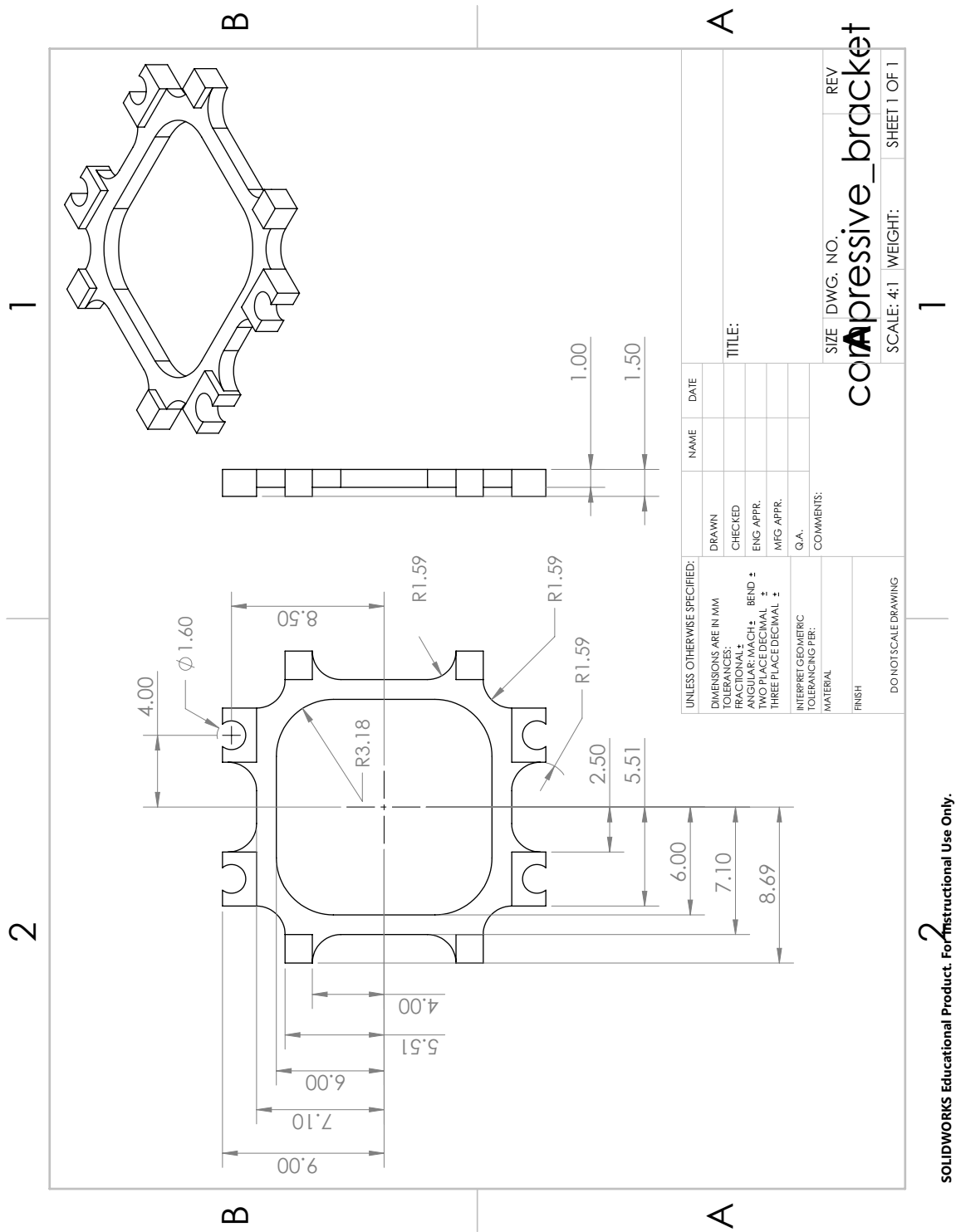


Figure B.4: Inconel bracket: detailed drawing 1 of 1.

B.2 High-Temperature Calibration Facility

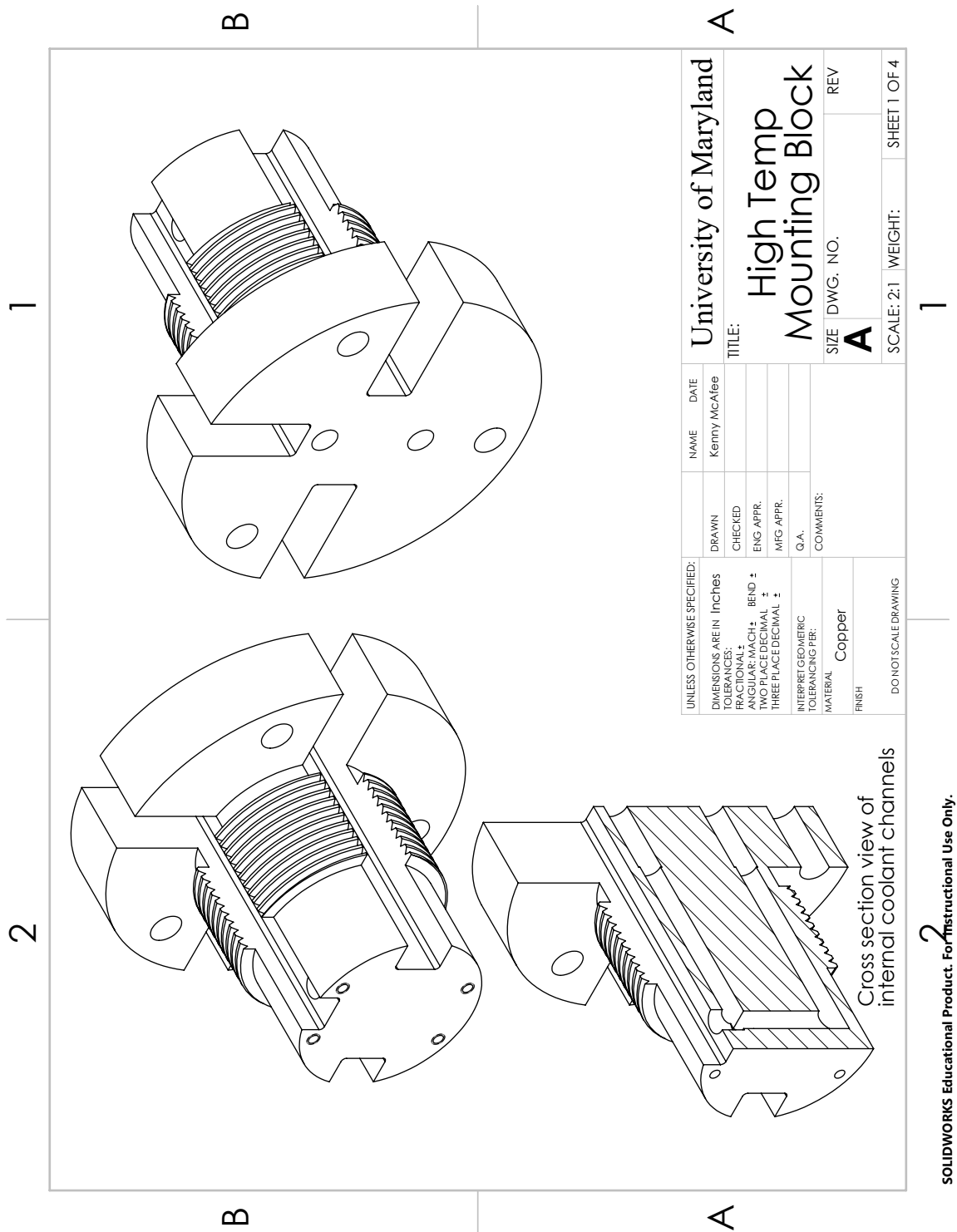


Figure B.5: High-temperature heat sink: detailed drawing 1 of 4.

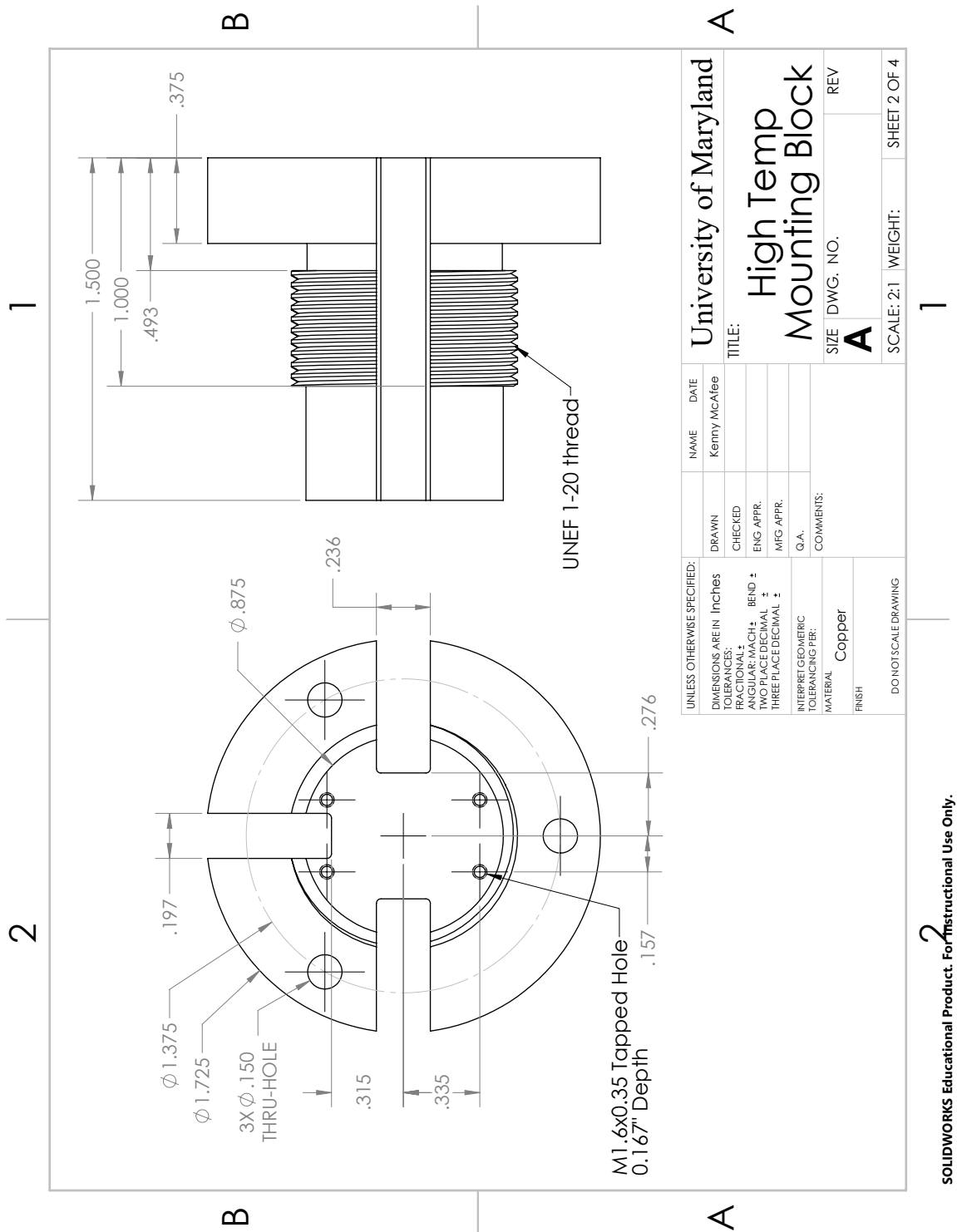


Figure B.6: High-temperature heat sink: detailed drawing 2 of 4.

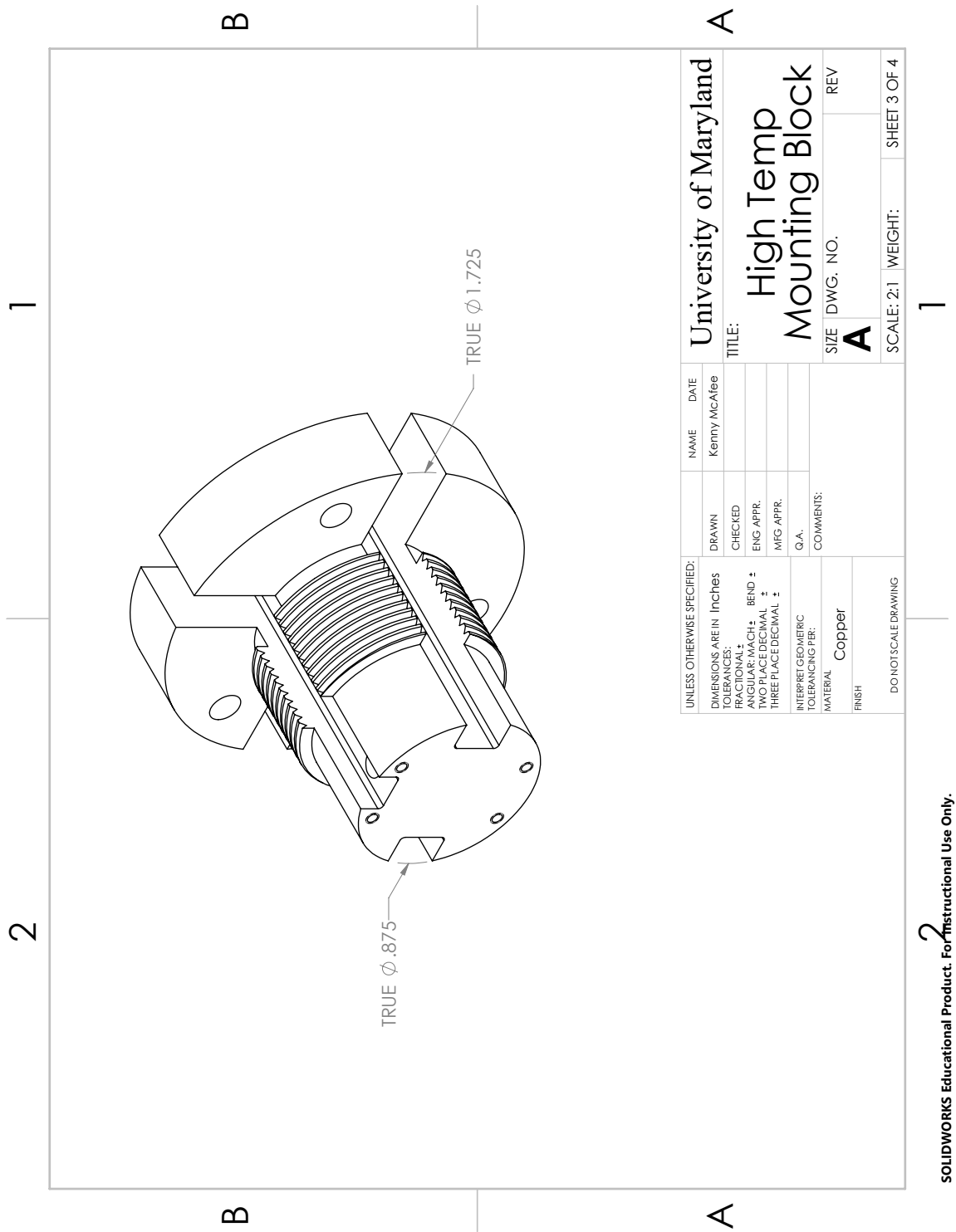


Figure B.7: High-temperature heat sink: detailed drawing 3 of 4.

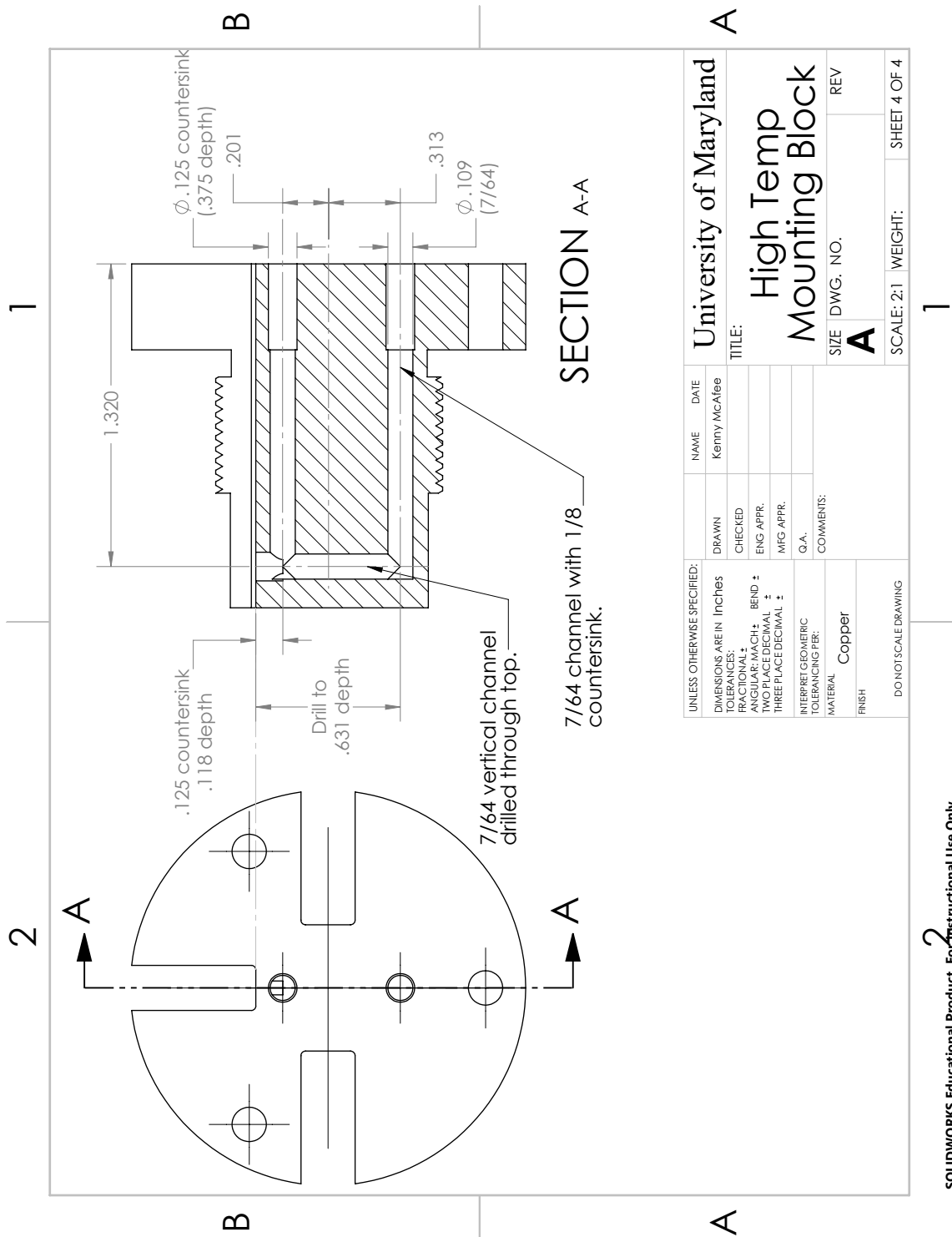


Figure B.8: High-temperature heat sink: detailed drawing 4 of 4.

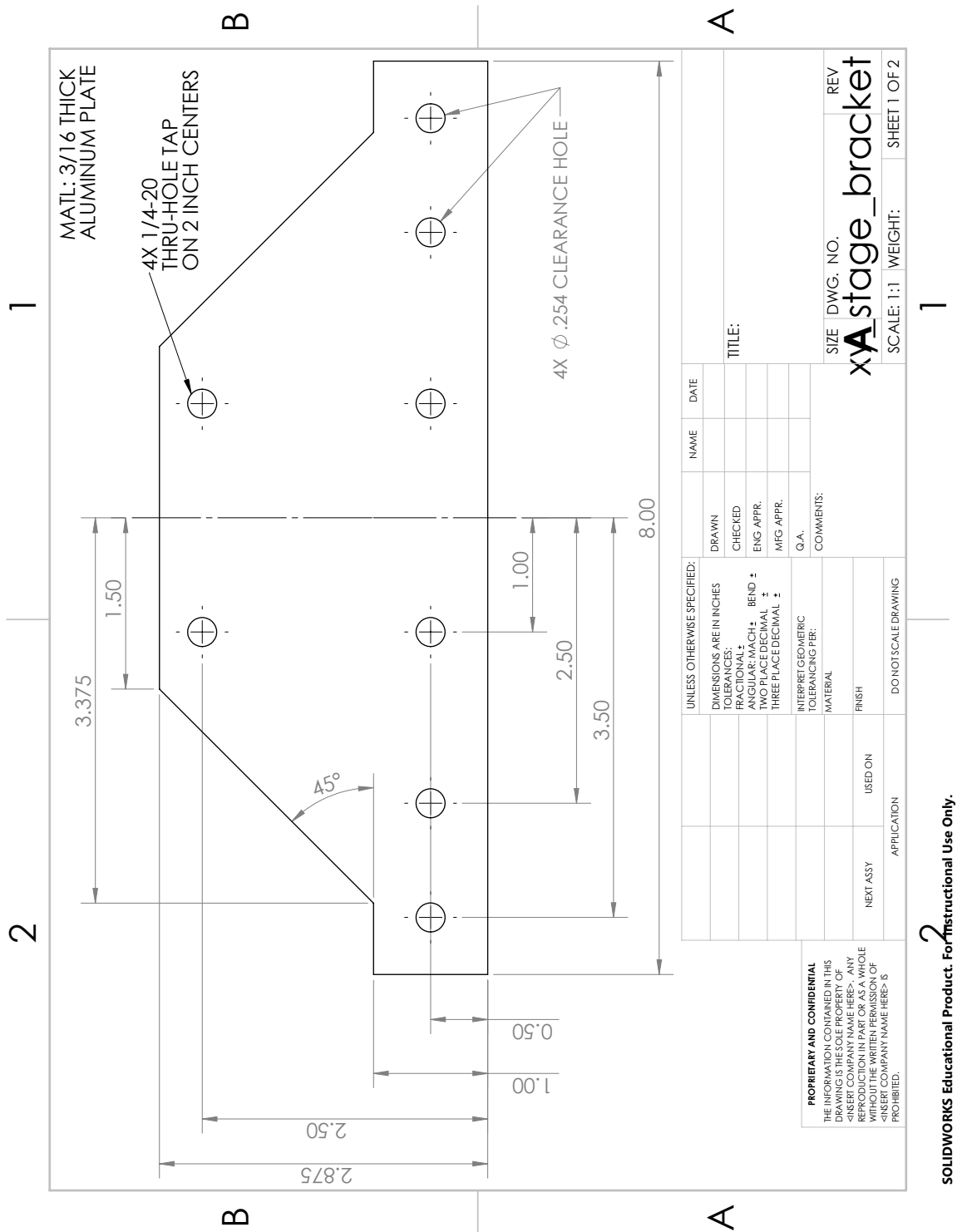


Figure B.11: Sensor rack $y - z$ alignment stage mounting bracket: detailed drawing 1 of 1.

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