Abstract

Title of Thesis: The Performance of Multivariate Quality-Control Charts for Autocorrelated Bivariate Data

Chen-Hsiang Wu, Master of Science, 2006

Thesis Directed by: Assistant Professor Paul Zantek

To monitor the mass production process, several quality control charts are constructed. Two of the most recognized schemes are the multivariate exponentially weighted moving average (MEWMA) and multivariate cumulative sum (MCUSUM) schemes. Originally, we assume that the observations from the production process are independent. However, sometimes the observations are autocorrelated. In this article, a vector autoregressive model VAR (m) is applied. Here we want to study the impact of autocorrelations on both schemes. We also want to know about which scheme is more efficient when the observations are autocorrelated.

The Performance of Multivariate Quality-Control Charts for Autocorrelated Bivariate Data

By

Chen-Hsiang Wu

Thesis submitted to the Faculty of the Graduate School
of the University of Maryland in partial fulfillment
of the requirement for the degree of
Master of Science
2006

Advisory Committee:

Assistant Professor Paul Zantek, Chair/Advisor
Associate Professor Frank B. Alt
Associate Professor Paul J. Smith

ACKNOWLEDGEMENTS

I would like to express my sincerest appreciation and gratitude to Dr. Paul Zantek, the chairman of my thesis committee, for his inspiration, counseling, and guidance through the whole research. My appreciation also extends to the other members of my thesis committee: Dr. Frank B. Alt and Dr. Paul Smith. They all give me beneficial and valuable comments and suggestions.

Finally, I would like to dedicate this thesis to my parents for their love, sacrifices, and continuous supports through my life.

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CHAPTER 1

INTRODUCTION

The rapid evolution of online computers and data-gathering equipment has helped manufacturers enable to monitor the quality of multivariate processes during mass production. To assist engineers scrutinizing production processes, various types of multivariate quality control charts have been proposed. One of the most recognized multivariate quality-control schemes is the Multivariate Exponentially Weighted Moving Average (MEWMA) chart proposed by Lowry, Woodall, Champ, and Rigdon (1992). Another scheme introduced by Crosier (1988) is the Multivariate CUmulative SUM (MCUSUM) procedure.

Suppose we have a series of $p \times 1$ random vectors X_1, X_2, X_3, … each containing the observations on the p quality characteristics to be monitored. These vectors can contain either individual observations or sample mean vectors. A common assumption is that the X_i, i = 1, 2, … , are independent multivariate normal random vectors with mean vectors μ_i, i = 1, 2, … , respectively. For simplicity, we assume that each X_i has a known covariance matrix Σ_X.

Most quality-control charts are designed for the scenario where the observation vectors are independent of each other. However, in some manufacturing processes the observations are autocorrelated.

This thesis studies the effect of autocorrelation on the performance of the MEWMA and MCUSUM schemes. We seek to (a) characterize the performance of these schemes in the presence of autocorrelation and (b) determine which scheme performs better when the data is autocorrelated.

CHAPTER 2

LITERATURE REVIEW

For traditional multivariate quality-control schemes, one usually assumes that the observation vectors are given by

$$X_i = \mu_i + u_i, \; i = 1, 2, \ldots, \tag{2.1}$$

where u_i is an independent multivariate normal random vector with mean vector of zeros and covariance matrix Σ_u :

2.1 Multivariate Exponentially Weighted Moving Average Scheme

The MEWMA procedure proposed by Lowry et al. (1992) is designed specifically for detecting small and moderate changes in the mean vector. The MEWMA vector is given by

$$Z_i = RX_i + (I - R)Z_{i-1}, \; i = 1, 2, \ldots , \tag{2.2}$$

where $Z_0 = 0$ and $R = \mathrm{diag}(r_1, r_2, \ldots , r_p)$, $0 < r_j \leqq 1$, $j = 1, 2, \ldots , p$.

The MEWMA chart gives an out-of-control signal when

$$T_i = Z_i' \Sigma_{Zi}^{-1} Z_i > h_4,$$

where h_4 (>0) is a constant chosen to satisfy a specified in-control ARL and Σ_{Zi} is the covariance matrix of Z_i. If there is no special reason to weight the previous observations differently for the p quality-control variables, we can use the same smoothing constant for each variable, i.e., $r_1 = r_2 = \ldots = r_p = r$. In this case, the ARL performance of MEWMA depends only on the noncentrality parameter $\lambda = (\mu_i' \Sigma_X^{-1} \mu_i)^{1/2}$ and we can rewrite the MEWMA scheme as

$$Z_i = rX_i + (1 - r)Z_{i-1},\ i = 1, 2, \ldots \tag{2.3}$$

and the covariance matrix of Z_i is

$$\Sigma_{Zi} = \{r[1 - (1 - r)^{2i}]/(2 - r)\} \Sigma_X \tag{2.4}$$

2.2 Multivariate Cumulative Sum Scheme

Crosier (1988) proposed two multivariate CUSUM charts. The first procedure transforms a multivariate observation vector into a scalar, and a CUSUM statistic is calculated from this scalar. The other CUSUM procedure forms a CUSUM statistic directly from the observation vector. The latter procedure has the better ARL performance and is based on the statistic

$$C_i = \{(S_{i-1} + X_i)' \ \Sigma_X^{-1}(S_{i-1} + X_i)\}^{1/2},\ i = 1, 2, \ldots \tag{2.5}$$

where $S_i = 0$, if $C_i \leqq k_1$,

$= (S_{i-1} + X_i)(1-k_1/C_i)$, if $C_i > k_1$, $i = 1, 2,\ldots,$

where $S_0 = 0$ and $k_1 > 0$.

The procedure gives an out-of-control signal if

$$Y_i = \{S_i' \Sigma_X^{-1} S_i\}^{1/2} > h_2,$$

where $h_2 > 0$. Like the MEWMA procedure, the ARL of the multivariate CUSUM depends on the mean and covariance matrix of X_i only through the noncentrality parameter λ .

2.3 Monitoring Autocorrelated Processes

1. Univariate Process

In the univariate (p = 1) case, Alwan and Roberts (1988) suggest using the ARIMA model[1] to describe autocorrelated observations. Then, they construct two basic quality control charts: (1) Common-Cause Chart and (2) Special-Cause Chart. The first chart is applied to the fitted values from the ARIMA model to monitor the whole process. The

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[1] *Autoregressive Integrated Moving Average ARIMA (p, q, d) model is given by* $(1-\sum_{i=1}^{p}\Phi_i B^i)(1-B)^d X_t = (1+\sum_{j=1}^{q}\Theta_j B^j)u_t$, *which B is a backshift operator, i.e.,* $B^j u_t = u_{t-j}$ *(Box and Jenkins, 1976).*

second one uses the residuals from the fitted ARIMA model to detect changes in the process. From Wardell, Moskowitz, and Plante (1994), a general ARMA model is utilized in describing the autocorrelated observations. Also, a special-cause chart is used to monitor the process. In Lu, W. C. and Reynolds, M. R. (2001), they utilize the VAR (1) model to describe a univariate autocorrelated process. Then the CUSUM scheme can be implemented to monitor the autocorrelated process.

From the univariate process cases, we can have some ideas about the performance of quality control schemes on different univariate autocorrelated data processes. Also, we can find out which autocorrelated data process is easier to implement into the quality control schemes.

2. Multivariate Processes

In the multivariate situation, Chen (1994) discusses two univariate special cases of vector autoregressive moving average models: VAR (1) and MA (1) models. Also, a multivariate MA (1) model is presented in the paper. These autoregressive models are utilized to implement into the multivariate MEWMA scheme.

In this thesis, the experience of setting up the multivariate MA (1) model from Chen (1994) is applied to help us setting up the multivariate VAR (1) model. We can also learn how to make the adjustments in the multivariate MEWMA scheme when the observations are autocorrelated.

CHAPTER 3

DESIGN OF STUDY

3.1. Vector Autoregressive Model

We assume that the observations from the process follow an autoregressive model. Under the vector autoregressive (VAR (m)) model presented in Reinsel (1993), the observation vectors $\{X_i\}$ are generated as follows

$$X_i = \mu_i + \sum_{j=1}^{m} \Phi_j(X_{i-j} - \mu_i) + u_i, \text{ for } i = 1,2,\ldots, \quad (3.1)$$

where μ_i is the mean vector at time i, u_i is a vector of independent normal random variables with zero mean vector and covariance matrix Σ_u, and Φ_j is a $p \times p$ matrix that contains the autocorrelations. We assume that a first-order (VAR (1)) model (m = 1) in this thesis. Hence, the model simplifies to

$$X_i = \mu_i + \Phi(X_{i-1} - \mu_i) + u_i \quad (3.2)$$

The autocorrelation matrix Φ will influence the covariance matrix of the observation vectors $\{X_i\}$. Let Γ(i, i+s) be the cross-covariance matrix between observations X_i and X_{i+s}. Its (k, l) element is:

$$\gamma_{kl} = E\{(X_{i,k} - \mu_{i,k})(X_{i+s,l} - \mu_{i+s,l})\}. \quad (3.3)$$

Because the process is assumed to be stationary, μ_i is a constant vector μ and Γ(i, i+s) is a function only of the delay s; thus, we can express the matrix as Γ(s) for convenience.

To calculate the cross-covariance matrices through time, a series of Yule-Walker equations are needed. In the special case that $\mu = 0$, i.e., $X_i = \Phi X_{i-1} + u_i$, for i = 1, 2, …, the Yule-Walker equations for the VAR (1) model can be written in matrix form as

$$\Gamma(k) = \Phi\Gamma(k-1) \text{ for } k \geqq 1 \tag{3.4}$$

$$\text{and } \Gamma(0) = \Phi\Gamma(0)\Phi' + \Sigma_u \tag{3.5}$$

Then by solving the equations above, we can obtain the cross-covariance matrix $\Gamma(0)$ at delay 0 when Φ and Σ_u are known. The cross-covariance matrices $\Gamma(s)$ should be nonnegative definite. That is, for all integers $n > 0$ and p-dimensional vectors $\{g_i\}$ i = 1, 2, …n, we have

$$\sum_{i=1}^{n} \sum_{j=1}^{n} g_i'\Gamma(i-j)g_j \geqq 0.$$

In this study, we consider the bivariate (p = 2) case and several cases of the autocorrelation matrix elements. We have to be cautious in choosing the elements in the autocorrelation matrix Φ . For the VAR (1) process to be stationary, all the eigenvalues of Φ must be less than 1 in absolute value[2].

[2] *Stationarity condition: Similar to the invertibility for the MA process, from the equation*

I. The stationary VAR (1) model presented in Reinsel, (1993) in which the autocorrelation matrix $\Phi = \begin{bmatrix} 0.8 & 0.7 \\ -0.4 & 0.6 \end{bmatrix}$ and the mean vector and covariance matrix Σ_u of error terms u_i are equal to (0, 0)' and $\begin{bmatrix} 4 & 1 \\ 1 & 2 \end{bmatrix}$, respectively.

II. The autocorrelation matrix $\Phi = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.1 \end{bmatrix}$. Here we use the same mean vector and covariance matrix Σ_u of error terms u_i as Case I.

III. Same as Case II, except the autocorrelation matrix $\Phi = \begin{bmatrix} -0.9 & 0 \\ 0 & -0.1 \end{bmatrix}$.

IV. The autocorrelation matrix $\Phi = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$. Here we use the same mean vector and covariance matrix Σ_u of error terms u_i as Case I.

V. Same as Case IV, except the diagonal elements of autocorrelation matrix $\Phi_{11} = \Phi_{22} = -0.5$

$\Phi(B)^{-1} = (1/\det\{\Phi(B)\})Adj\{\Phi(B)\}$, $\Phi(B)^{-1}$ will be convergent for $|B| \leq 1$ if all roots of $\det\{\Phi(B)\} = 0$ are greater than 1 in absolute value ($\Phi(B) = \sum_{j=1}^{\infty} \Phi_j B^j$), which B is a backshift operator, i.e., $B^j u_t = u_{t-j}$. The VAR process will be stationary if this condition is satisfied.

3.2 Multivariate Exponentially Weighted Moving Average Scheme Setup

In this study, we begin with the mean vector on-target μ = (0, 0)' and shift both variables by 0.5, 1, 1.5, 2, 2.5 and 3 to the off-target value.

In the MEWMA scheme, we need to calculate the test statistic $T_i = Z_i' \Sigma_{Zi}^{-1} Z_i$. The formula for calculating Σ_{Zi} is, from Chen (1994),

$$\Sigma_{Zi} = [r/(2-r)](1-(1-r)^{2i})\Gamma(0) + 2\Sigma_{j=1}^{i-1}[r/(2-r)](1-r)^j(1-(1-r)^{2(i-j)})\Gamma(j)$$

$$= [r/(2-r)][(1-(1-r)^{2i})\Gamma(0) + 2\Sigma_{j=1}^{i-1}(1-r)^{2(i-1)}\Phi^j\Gamma(0)]. \qquad (3.7)$$

For our study of on-target ARL, we consider two sets of the smoothing parameter and critical value, which are r = 0.1, h_4 = 8.79, and r = 0.2, h_4 = 9.65[3].

For off-target ARL comparisons, we fix the smoothing parameter r, and then adjust the control limit h_4 to obtain an in-control ARL of 200 by running SAS simulations repeatedly.

3.3 Multivariate Cumulative Sum Scheme Setup

For the on-target ARL study, we consider two parameter settings from Crosier (1988): k_1 = 0.5 with control limit h_2 = 5.485 and k_1 = 1.0 with control limit h_2 = 2.99. Each of

[3] *There are two sets of r and h_4 both for on-target ARL = 200 in the independent observation cases from Lowry et al (1992). The sets of design constants are from simulations in the paper.*

these k_1, h_2 pairs yields an in-control ARL of 200 in the case of independent observations ($\Phi = 0$).

For the comparisons of off-target ARL, we fix k_1 and then adjust the control limit h_2 to make the in-control ARL equal to 200 by running SAS simulations repeatedly.

CHAPTER 4

RESULTS

On-Target ARL

We first assess the impact of autocorrelation on the in-control ARLs of the MEWMA and MCUSUM schemes. For each of the six cases of Φ, Tables 1 and 2 present the in-control ARL of both schemes (different parameter settings).

	Average Run Length	
	MEWMA ($r = 0.1$)	**MCUSUM ($k_1 = 0.5$)**
$\Phi = 0$	200.51	199.59
Case I ($\Phi = \begin{bmatrix} 0.8 & 0.7 \\ -0.4 & 0.6 \end{bmatrix}$)	35.00	22.38
Case II ($\Phi = diag(\Phi_{11}, \Phi_{22})$, $\Phi_{11} \neq \Phi_{22}$, positive Φ)	20.50	13.68
Case III ($\Phi = diag(\Phi_{11}, \Phi_{22})$, $\Phi_{11} \neq \Phi_{22}$, negative Φ)	242.95	304.29
Case IV ($\Phi = diag(\Phi_{11}, \Phi_{22})$, $\Phi_{11} = \Phi_{22}$, positive Φ)	25.67	21.96
Case V ($\Phi = diag(\Phi_{11}, \Phi_{22})$, $\Phi_{11} = \Phi_{22}$, negative Φ)	1816.2	4600.9

Table 1. In-Control ARLs (MEWMA: $r = 0.1$; MCUSUM: $k_1 = 0.5$, each ARL is estimated from 10000 simulated run lengths)

	Average Run Length	
	MEWMA (r= 0.2)	**MCUSUM** (k_1 = 1.0)
$\Phi = 0$	200.33	200.01
Case I ($\Phi = \begin{bmatrix} 0.8 & 0.7 \\ -0.4 & 0.6 \end{bmatrix}$)	22.05	11.09
Case II (Φ =diag(Φ_{11}, Φ_{22}), $\Phi_{11} \neq \Phi_{22}$, positive Φ)	18.37	13.39
Case III (Φ =diag(Φ_{11}, Φ_{22}), $\Phi_{11} \neq \Phi_{22}$, negative Φ)	222.75	167.00
Case IV (Φ =diag(Φ_{11}, Φ_{22}), $\Phi_{11} = \Phi_{22}$, positive Φ)	20.55	19.58
Case V (Φ =diag(Φ_{11}, Φ_{22}), $\Phi_{11} = \Phi_{22}$, negative Φ)	1196.2	499.98

Table 2. In-Control ARLs (MEWMA: r = 0.2; MCUSUM: k_1 = 1, each ARL is estimated from 10000 simulated run lengths)

From Table 1 and 2, we see that in the cases of positive autocorrelation, the in-control ARL is much smaller than in the case of independent observations (Φ = 0). Thus, the control limits for independent observations must be modified for a positively autocorrelated process in order to have the desired in-control ARL. For the cases of negative autocorrelation, the in-control ARL is actually higher than when Φ = 0.

Off-Target ARL (Failtime[4] = 0)

Tables 3 and 4 show the ARLs of MEWMA and MCUSUM procedures when mean

[4] *Failtime is the number of observations before the mean shift occurs.*

shifts occur.

	MeanShift[5] = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
ARL	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**
Case I	212.76	199.60	106.02 (79.81)	110.26 (90.71)	50.64 (29.2)	46.95 (32.87)	25.70 (14.03)	25.56 (16.67)
Case II	209.10	200.32	187.72 (85.2)	166.31 (96.25)	146.98 (30.89)	121.06 (36.07)	96.77 (15.42)	82.58 (18.35)
Case III	199.29	199.97	125.15 (85.2)	162.44 (96.25)	36.41 (30.89)	55.89 (36.07)	7.87 (15.42)	16.66 (18.35)
Case IV	211.91	200.07	115.45 (28.66)	81.22 (33.5)	39.27 (8.86)	26.25 (10.95)	16.71 (4.57)	15.04 (6.35)
Case V	202.18	200.31	14.49 (28.66)	33.01 (33.5)	3.06 (8.86)	7.37 (10.95)	2.58 (4.57)	4.23 (6.35)

Table 3. Zero State Off-Target ARLs (MEWMA: r = 0.1; MCUSUM: k_1 = 0.5)

The ARLs of independent case (Φ = 0) with the same λ are in parentheses

	MeanShift = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
ARL	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**
Case I	198.73	200.01	128.31 (85.26)	151.86 (100.11)	70.13 (38.92)	83.89 (42.56)	36.11 (25.77)	43.99 (30.00)
Case II	201.27	200.56	158.82 (95.34)	175.73 (102.35)	144.95 (37.84)	124.18 (52.11)	95.24 (21.02)	86.55 (28.39)
CaseIII	201.24	200.20	152.50 (95.34)	182.74 (102.35)	87.94 (37.84)	138.86 (52.11)	31.48 (21.02)	83.66 (28.39)
Case IV	199.21	200.07	129.75 (41.53)	127.28 (47.76)	55.12 (17.00)	40.87 (21.45)	22.43 (10.21)	15.52 (12.72)
Case V	200.30	199.89	36.65 (41.53)	116.41 (47.76)	6.56 (17.00)	20.11 (21.45)	3.17 (10.21)	5.59 (12.72)

Table 4. Zero State Off-Target ARLs (MEWMA: r = 0.2; MCUSUM: k_1 = 1)

The ARLs of independent case (Φ = 0) with the same λ are in parentheses

[5] *When mean shift = 1 occurs, the on-target mean vector (0,0)' changes to off-target mean vector (1,1)'. The noncentrality parameters λ are not equal in different cases. However, in each case, the noncentrality parameters are equal in both MEWMA and MCUSUM schemes. A list of noncentrality parameters λ in each case is shown in Appendix I.*

We can see that autocorrelations can delay the quality-control procedures to detect the off-target signal. In Case I, both procedures have similar results in spotting the out-of-control signal. And in Case II and IV with positive autocorrelations, the MEWMA scheme has worse results but not far away from MCUSUMs'. However, when negative autocorrelations affect the process, the MEWMA scheme has better performance than MCUSUM scheme.

Off-Target ARL (Failtime $\neq$ 0)

Here we observe how efficient the MEWMA and MCUSUM schemes with some pre-runs can detect the out-of-control signals in the autocorrelated observation case. The cases are divided into two categories: Positive autocorrelations (Case II and IV) and Negative autocorrelations (Case III and V). The tables and figures below present the ARL performances of the MEWMA (smoothing parameter r = 0.1 and 0.2) and MCUSUM (k_1 = 0.5 and 1.0) schemes when the mean vectors of processes are out of control:

	MeanShift = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
ARL	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**
Case I	99.14	216.79	54.27	113.00	21.99	45.11	11.57	23.71
Case II	87.66	187.90	83.93	167.22	66.66	121.62	54.07	82.11
CaseIII	155.71	197.92	109.90	156.41	31.40	53.50	5.80	15.19
Case IV	136.13	191.22	89.53	80.29	25.67	26.16	12.70	15.27
Case V	179.82	208.00	11.93	31.72	2.81	6.67	2.09	3.70

Table 5(a) Steady State (Failtime = 5) Off-Target ARLs (MEWMA: $r = 0.1$; MCUSUM: $k_1 = 0.5$)

	MeanShift = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
ARL	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**
Case I	145.61	221.67	76.81	112.36	33.62	45.58	16.75	23.91
Case II	129.13	191.28	108.61	166.23	101.76	125.46	67.41	91.90
CaseIII	162.84	197.90	95.23	156.64	27.94	53.91	5.21	15.10
Case IV	131.99	189.88	80.50	77.32	29.07	26.03	12.55	15.39
Case V	159.64	207.01	10.41	31.34	2.53	6.60	1.86	3.62

Table 5(b) Steady State (Failtime = 25) Off-Target ARLs (MEWMA: $r = 0.1$; MCUSUM: $k_1 = 0.5$)

	MeanShift = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
ARL	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**
Case I	165.33	222.76	83.98	112.45	31.31	45.05	15.92	23.76
Case II	144.86	191.56	108.92	172.64	103.25	131.86	71.61	98.29
CaseIII	173.47	200.75	108.38	160.66	27.11	54.48	5.17	15.21
Case IV	151.99	188.60	83.11	78.55	27.54	26.45	12.93	15.39
Case V	163.88	209.54	9.59	31.50	2.41	6.61	1.82	3.62

Table 5(c) Steady State (Failtime = 50) Off-Target ARLs (MEWMA: $r = 0.1$; MCUSUM: $k_1 = 0.5$)

	MeanShift = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
ARL	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**
Case I	143.97	244.91	94.68	169.39	42.00	83.58	23.23	40.87
Case II	99.36	188.43	103.57	170.93	80.61	130.65	59.67	90.81
CaseIII	180.06	206.47	162.03	186.25	79.98	135.07	23.62	79.91
Case IV	153.67	198.37	107.76	125.26	48.43	42.03	20.60	16.67
Case V	216.83	224.10	36.17	123.07	5.61	19.00	2.51	4.68

Table 5(d) Steady State (Failtime = 5) Off-Target ARLs (MEWMA: $r = 0.2$; MCUSUM: $k_1 = 1.0$)

	MeanShift = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
ARL	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**
Case I	193.28	264.65	121.52	181.39	53.19	91.45	28.22	44.93
Case II	175.99	216.01	162.38	192.96	130.83	154.82	100.99	112.05
CaseIII	192.99	207.31	162.55	186.62	69.95	140.02	25.85	83.82
Case IV	159.69	199.18	114.26	129.48	48.79	42.43	19.65	17.14
Case V	253.67	221.37	37.61	121.51	5.78	19.29	2.42	4.74

Table 5(e) Steady State (Failtime = 25) Off-Target ARLs (MEWMA: $r = 0.2$; MCUSUM: $k_1 = 1.0$)

	MeanShift = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
ARL	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**	**MEWMA**	**MCUSUM**
Case I	179.48	266.38	126.09	181.22	60.35	90.35	29.70	44.72
Case II	185.30	218.67	155.40	198.18	135.32	152.66	98.99	112.27
CaseIII	172.62	212.20	156.52	185.84	77.43	138.97	25.63	85.08
Case IV	171.58	200.26	108.27	126.82	49.40	41.88	19.97	16.98
Case V	248.92	220.38	37.46	123.03	5.51	19.36	2.44	4.74

Table 5(f) Steady State (Failtime = 50) Off-Target ARLs (MEWMA: $r = 0.2$; MCUSUM: $k_1 = 1.0$)

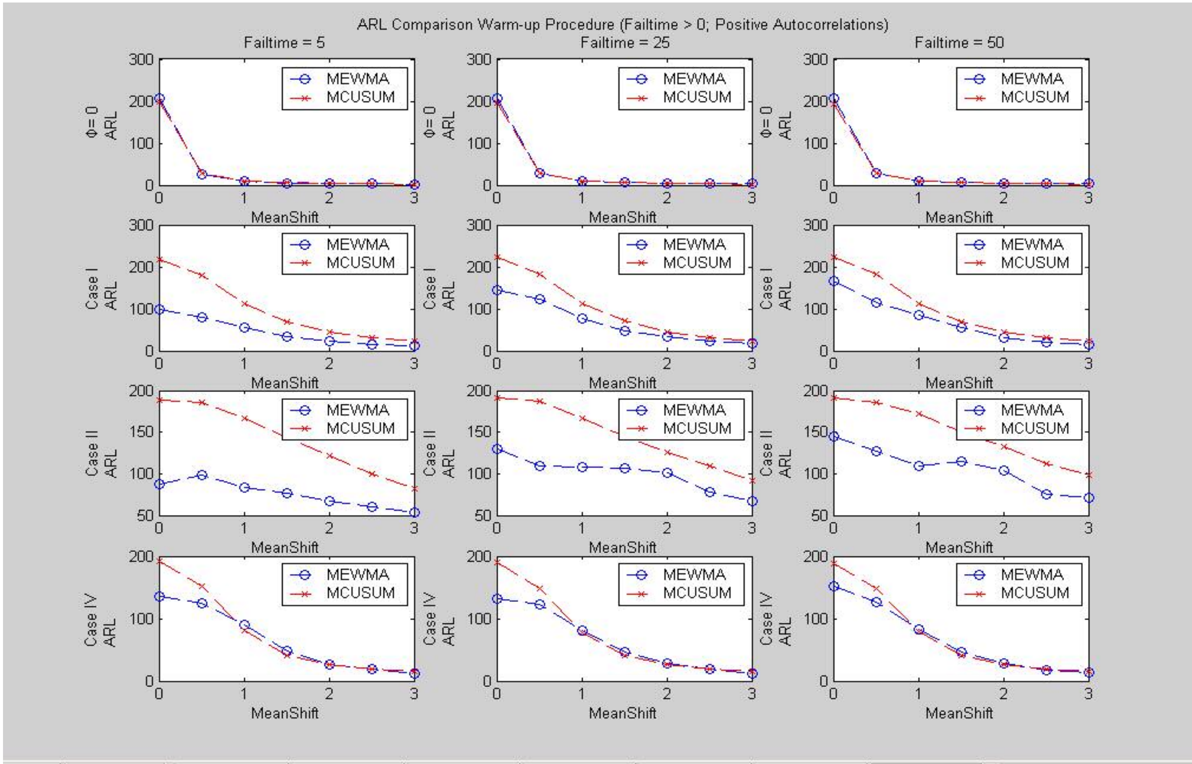

Figure 1. Steady State Off-Target ARLs (Positive Autocorrelation; MEWMA: r = 0.1 and MCUSUM: k_1 = 0.5)

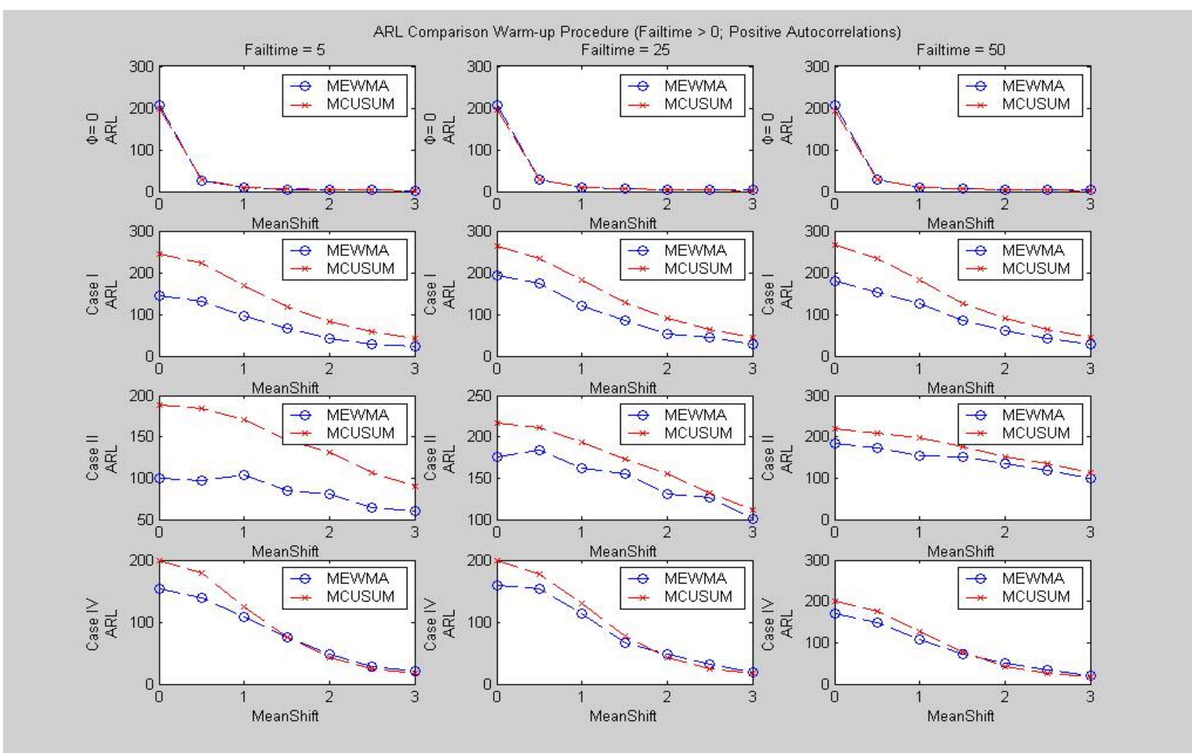

Figure 2. Steady State Off-Target ARLs (Positive Autocorrelation; MEWMA: r = 0.2 and MCUSUM: k_1 = 1.0)

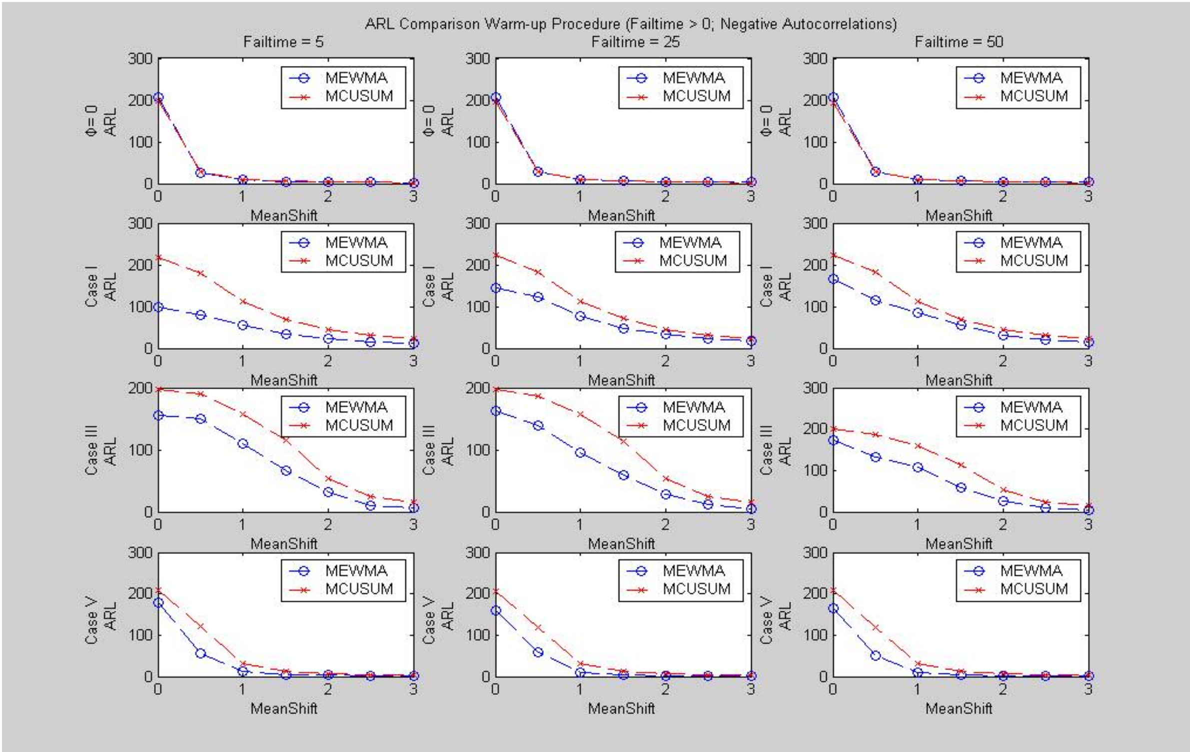

Figure 3. Steady State Off-Target ARLs (Negative Autocorrelation; MEWMA: r = 0.1 and MCUSUM: $k_1 = 0.5$)

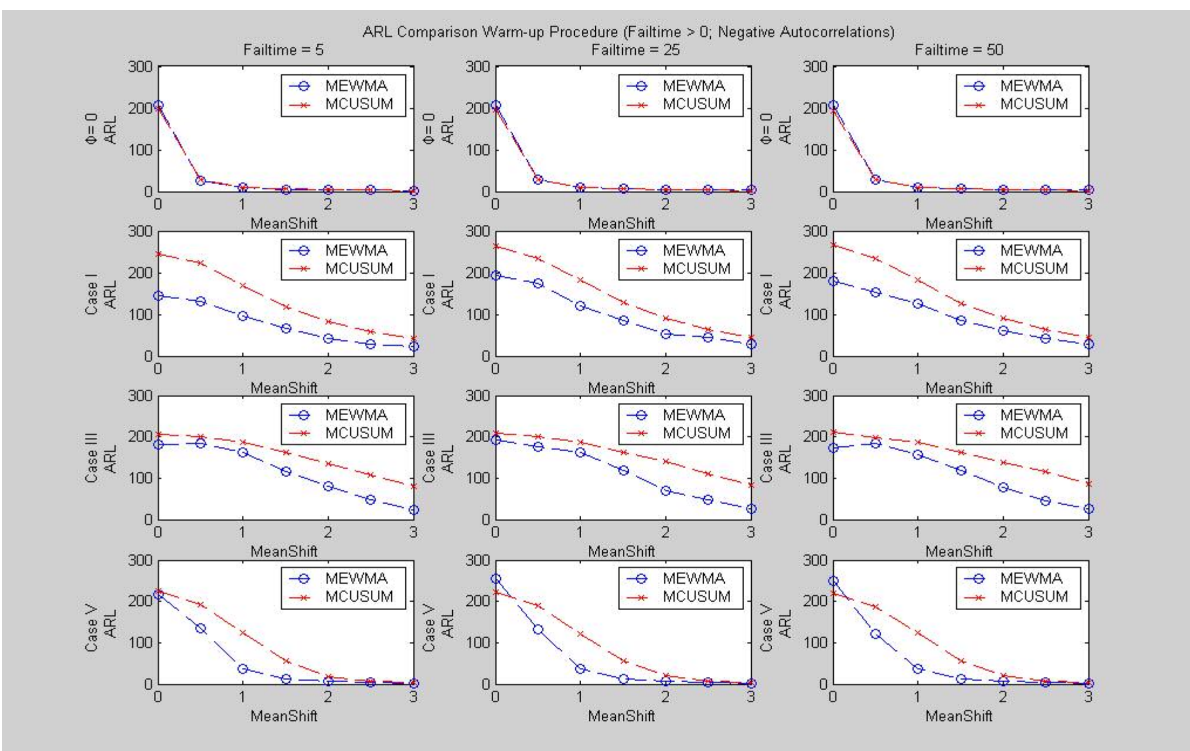

Figure 4. Steady State Off-Target ARLs (Negative Autocorrelation; MEWMA: r = 0.2 and MCUSUM: $k_1 = 1.0$)

With the independent assumption being satisfied, the ARL performances of MEWMA and MCUSUM schemes are similar with some pre-runs before a mean shift occurs. Then in these simulations with autocorrelations, we can observe that in both positive and negative autocorrelated circumstance the MEWMA scheme with the warm-up steps can send out the out-of-control signals faster than the MCUSUM scheme when the mean shift occurs. Also, compare to the $\Phi = 0$ case with same noncentrality parameter λ in each case (see Table 3, 4 and Appendix II), the MEWMA and MCUSUM schemes perform worse in almost all autocorrelated observation cases (only in negative autocorrelation case III and V, the performances of MEWMA and MCUSUM are slightly better than $\Phi = 0$ case when the mean shift is large).

CHAPTER 5

CONCLUSION

Practically, since the MEWMA and MCUSUM procedures both have their own setting of the reference parameters, the average run length comparisons are usually difficult to perform. From Crosier (1988) and Lowry et al (1992), we know that in the independent observation case the ARL performances of the MEWMA and MCUSUM schemes will only depend on the noncentrality parameter $\lambda = (\mu' \Sigma_X^{-1} \mu)^{1/2}$. In this study, when the autoregressive VAR (1) model is applied in the simulations, we make the ARL comparisons by only controlling the mean vector and covariance matrix.

In this article, first we apply the smoothing parameters r = 0.1 (h_4 = 8.79), r = 0.2 (h_4 = 9.65) for MEWMA schemes and k_1 = 0.5 (h_2 = 5.485), k_1 = 1.0 (h_2 = 2.99) for MCUSUM scheme from the independent observation case to observe the impact of autocorrelation. Then from Table 1 and 2, we find that the on-target ARLs of the MEWMA and MCUSUM schemes in the positive autocorrelation cases are shorter than in the independent observation case ($\Phi = 0$). In contrast, the on-target ARLs in the negative autocorrelation cases are much longer than in the case $\Phi = 0$.

Second, in the off-target ARL comparison, we have two states to discuss: zero state

(Failtime = 0) and steady state (Failtime $\neq$ 0). In zero state (results shown in Table 3 and 4), we can observe that the MCUSUM scheme has slightly better (or equal) off-target ARL performance than MEWMA in the positive autocorrelation cases. However, the MEWMA scheme has the lead in all negative autocorrelation cases. In steady state (results shown in Table 5 and Fig. 1, 2, 3, and 4), the MEWMA scheme out performs the MCUSUM scheme in both positive and negative autocorrelation cases. For this reason, practitioners may prefer the MEWMA scheme to the MCUSUM scheme.

From the previous MEWMA results, we also found an interesting result. Compare the steady state off-target ARLs of Case II with Case III's, we can find out that the larger mean shifts (MeanShift > 1, same noncentrality parameters, Appendix II) are easier to detect in the negative autocorrelation cases. Also, same results are found between the Case IV and V comparison. In the MCUSUM results, the difference between positive and negative autocorrelation cases is not obvious as MEWMAs'. However, the out-of-control signals are simpler to be identified in the negative autocorrelation cases. From the results of run length standard deviations (Appendix III), we also can observe that the run lengths have smaller standard deviations in the negative autocorrelation cases when the mean shifts are larger. Therefore, we can conclude that the MEWMA and MCUSUM schemes are more efficient in the negative autocorrelation cases.

APPENDIX

APPENDIX I – Noncentrality Parameter λ

Noncentrality Parameter λ	Mean Shift = 0.5	Mean Shift = 1	Mean Shift = 1.5	Mean Shift = 2	Mean Shift = 2.5	Mean Shift = 3
Case I	0.1169	0.2339	0.3508	0.4677	0.5847	0.7016
Case II	0.1106	0.2211	0.3317	0.4422	0.5528	0.6633
Case III	0.1106	0.2211	0.3317	0.4422	0.5528	0.6633
Case IV	0.2315	0.4629	0.6944	0.9258	1.1573	1.3887
Case V	0.2315	0.4629	0.6944	0.9258	1.1573	1.3887

Table 6. Noncentrality Parameter λ in Each Case

APPENDIX II – ARLs of Φ = 0 Case with Same λ of Autocorrelation Cases

Failtime / λ	0	5	10	15	20	25	30	35	40	45	50
0.1169	147.66	151.94	149.42	150.59	151.95	152.21	151.16	151.26	154.11	153.84	152.20
0.2339	79.81	82.23	84.16	83.34	84.16	83.42	83.48	82.62	83.98	82.83	83.03
0.3508	45.27	47.46	47.67	47.57	49.03	47.47	47.80	47.36	47.39	48.39	47.32
0.4677	29.20	30.14	30.54	30.29	30.54	31.17	30.50	30.46	30.33	30.58	30.40
0.5847	19.11	20.89	21.29	21.63	21.21	21.55	21.62	21.61	21.36	21.31	21.20
0.7016	14.03	15.67	16.08	16.15	16.32	16.21	16.43	16.23	16.22	16.09	16.17

Table 7(a). MEWMA ($r = 0.1$, $h_4 = 8.79$) ARLs of $\Phi = 0$ with the λ from Case I

Failtime / λ	0	5	10	15	20	25	30	35	40	45	50
0.1169	152.80	151.25	150.11	147.80	149.09	150.60	150.92	150.40	151.82	151.80	150.41
0.2339	90.71	87.52	87.33	87.62	85.65	86.36	88.09	87.45	87.85	88.34	87.46
0.3508	52.09	51.01	49.69	50.55	50.52	50.53	49.24	50.75	50.14	50.21	51.04
0.4677	32.87	31.82	31.42	31.18	31.66	31.31	31.77	31.86	31.23	31.86	31.52
0.5847	22.63	21.85	21.46	21.62	21.43	21.43	21.68	21.77	21.62	21.56	21.83
0.7016	16.67	16.12	15.92	16.04	15.84	15.97	15.82	16.03	16.06	16.08	15.89

Table 7(b). MCUSUM ($k_1 = 0.5$, $h_2 = 5.485$) ARLs of $\Phi = 0$ with the λ from Case I

Failtime / λ	0	5	10	15	20	25	30	35	40	45	50
0.1106	152.65	155.65	153.07	154.84	156.71	156.49	155.60	157.61	156.50	158.08	155.85
0.2211	85.20	87.94	90.12	88.66	89.19	88.23	90.28	89.48	87.97	89.27	87.51
0.3317	49.52	51.73	51.68	51.83	52.72	51.93	51.87	51.33	51.77	52.21	51.66
0.4422	30.89	33.12	33.58	33.11	33.28	34.15	33.26	33.18	33.33	33.13	33.44
0.5528	21.01	22.81	23.20	23.72	23.28	23.53	23.62	23.36	23.21	23.25	23.26
0.6633	15.42	17.11	17.52	17.68	17.76	17.64	17.74	17.78	17.42	17.74	17.71

Table 7(c). MEWMA ($r = 0.1$, $h_4 = 8.79$) ARLs of $\Phi = 0$ with the λ from Case II and III

Failtime / λ	0	5	10	15	20	25	30	35	40	45	50
0.1106	156.66	155.64	153.77	150.37	153.79	152.27	156.38	155.87	156.15	154.44	155.27
0.2211	96.25	92.45	93.44	93.02	92.33	91.74	93.50	93.41	93.04	93.96	94.46
0.3317	56.68	55.03	54.86	55.09	54.76	54.85	53.94	55.25	55.01	55.23	54.90
0.4422	36.07	34.70	34.53	34.31	34.74	34.67	34.97	34.69	34.46	34.91	34.35
0.5528	24.91	23.95	23.70	23.65	23.70	23.83	23.65	23.82	23.81	23.60	24.09
0.6633	18.35	17.53	17.57	17.54	17.36	17.58	17.35	17.54	17.52	17.56	17.57

Table 7(d). MCUSUM ($k_1 = 0.5$, $h_2 = 5.485$) ARLs of $\Phi = 0$ with the λ from Case II and III

Failtime / λ	0	5	10	15	20	25	30	35	40	45	50
0.2315	80.89	83.29	85.19	84.44	85.38	83.95	84.59	84.19	84.74	83.95	83.80
0.4629	28.66	30.79	30.98	30.85	30.95	31.69	31.16	30.98	30.99	31.02	30.93
0.6944	14.27	15.94	16.29	16.47	16.51	16.47	16.72	16.42	16.51	16.35	16.40
0.9258	8.86	10.29	10.66	10.82	10.81	10.88	10.93	10.96	10.95	10.79	10.93
1.1573	6.12	7.53	7.90	8.09	8.06	8.12	8.06	8.06	8.10	8.06	8.00
1.3887	4.57	5.94	6.27	6.42	6.42	6.45	6.42	6.45	6.47	6.46	6.44

Table 7(e). MEWMA ($r = 0.1$, $h_4 = 8.79$) ARLs of $\Phi = 0$ with the λ from Case IV and V

Failtime / λ	0	5	10	15	20	25	30	35	40	45	50
0.2315	91.61	88.63	88.23	88.75	86.69	87.77	88.79	88.65	89.11	89.72	88.96
0.4629	33.50	32.26	31.92	31.71	32.29	31.82	32.33	32.51	31.80	32.55	32.08
0.6944	17.02	16.36	16.21	16.32	16.03	16.30	16.04	16.28	16.29	16.20	16.06
0.9258	10.95	10.60	10.42	10.48	10.42	10.42	10.44	10.43	10.52	10.43	10.52
1.1573	8.02	7.74	7.70	7.63	7.66	7.64	7.64	7.65	7.69	7.70	7.68
1.3887	6.35	6.10	6.10	6.04	6.04	6.08	6.06	6.05	6.02	6.08	6.06

Table 7(f). MCUSUM ($k_1 = 0.5$, $h_2 = 5.485$) ARLs of $\Phi = 0$ with the λ from Case IV and V

APPENDIX III – Design Constants for On-Target ARL 200

	MEWMA		MCUSUM	
Case I	$r = 0.1$ $h_4 = 15.40$	$r = 0.2$ $h_4 = 19.31$	$k_1 = 0.5$ $h_2 = 9.13$	$k_1 = 1$ $h_2 = 7.087$
Case II	$r = 0.1$ $h_4 = 46.35$	$r = 0.2$ $h_4 = 37.375$	$k_1 = 0.5$ $h_2 = 29.80$	$k_1 = 1$ $h_2 = 16.04$
Case III	$r = 0.1$ $h_4 = 8.265$	$r = 0.2$ $h_4 = 9.435$	$k_1 = 0.5$ $h_2 = 4.987$	$k_1 = 1$ $h_2 = 3.094$
Case IV	$r = 0.1$ $h_4 = 27.26$	$r = 0.2$ $h_4 = 27.57$	$k_1 = 0.5$ $h_2 = 14.60$	$k_1 = 1$ $h_2 = 8.476$
Case V	$r = 0.1$ $h_4 = 5.835$	$r = 0.2$ $h_4 = 6.735$	$k_1 = 0.5$ $h_2 = 3.545$	$k_1 = 1$ $h_2 = 2.632$

Table 8. Design Constants of MEWMA and MCUSUM schemes

APPENDIX IV – Standard Deviation of RLs

	Standard Deviation	
	MEWMA ($r = 0.1$)	MCUSUM ($k_1 = 0.5$)
$\Phi = 0$	202.68	193.49
Case I	49.06	30.81
Case II	20.94	16.01
Case III	250.43	312.57
Case IV	23.54	19.56
Case V	1832.55	4640.33

Table 9(a). Standard Deviations of On-Target RLs (MEWMA: $r = 0.1$; MCUSUM: $k = 0.5$)

	Standard Deviation	
	MEWMA (r= 0.2)	MCUSUM (k_1 = 1.0)
Φ= 0	203.11	199.25
Case I	32.12	19.92
Case II	20.03	15.89
Case III	244.73	172.46
Case IV	20.34	17.34
Case V	1164.81	501.22

Table 9(b). Standard Deviations of On-Target RLs (MEWMA: r = 0.2; MCUSUM: k = 1.0)

	MeanShift = 1		MeanShift = 2		MeanShift = 3	
StD	MEWMA	MCUSUM	MEWMA	MCUSUM	MEWMA	MCUSUM
Φ= 0	5.02	4.71	1.36	1.26	0.66	0.66
Case I	112.84	109.98	43.68	38.22	18.12	16.59
Case II	211.88	168.05	148.81	119.93	94.69	83.81
Case III	149.48	161.26	38.10	55.89	5.61	4.94
Case IV	106.06	65.01	28.85	12.00	7.48	4.98
Case V	17.37	25.34	1.29	2.43	0.83	1.07

Table 10(a). Standard Deviations of Zero-State Off-Target RLs (MEWMA: r = 0.1; MCUSUM: k = 0.5)

	MeanShift = 1		MeanShift = 2		MeanShift = 3	
StD	MEWMA	MCUSUM	MEWMA	MCUSUM	MEWMA	MCUSUM
Φ= 0	6.23	8.44	4.75	7.23	3.01	5.22
Case I	136.58	117.02	66.87	88.10	31.80	40.08
Case II	174.67	196.72	159.64	143.19	117.15	102.18
Case III	187.27	185.38	93.33	141.65	30.64	82.72
Case IV	126.50	125.01	49.45	33.83	16.60	8.55
Case V	39.41	121.95	4.13	16.32	1.20	2.18

Table 10(b). Standard Deviations of Zero-State Off-Target RLs (MEWMA: r = 0.2; MCUSUM: k = 1.0)

	MeanShift = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
StD	MEWMA	MCUSUM	MEWMA	MCUSUM	MEWMA	MCUSUM	MEWMA	MCUSUM
Case I	177.61 (215.10)	219.16 (260.72)	97.43 (143.16)	109.65 (177.42)	35.63 (59.23)	37.91 (87.30)	16.81 (30.82)	16.73 (41.52)
Case II	167.93 (173.36)	192.65 (212.21)	157.40 (179.35)	170.12 (194.82)	118.69 (141.97)	122.75 (150.76)	96.40 (96.25)	80.74 (105.22)
Case III	211.47 (200.02)	195.78 (209.93)	143.40 (183.82)	154.86 (186.41)	43.17 (97.24)	41.38 (138.60)	6.66 (26.17)	5.20 (83.55)
Case IV	176.57 (184.87)	185.90 (196.56)	106.49 (115.21)	65.41 (123.64)	24.84 (52.39)	12.31 (34.91)	10.00 (17.81)	5.43 (8.97)
Case V	274.89 (252.94)	209.41 (222.40)	16.11 (37.64)	24.93 (121.01)	1.37 (3.87)	2.48 (15.47)	1.00 (1.19)	1.12 (2.17)

Table 10(c). Standard Deviations of Steady-State Off-Target RLs (Failtime = 5)

MEWMA: r = 0.1 and MCUSUM: k_1 = 0.5 (MEWMA: r = 0.2 and MCUSUM: k1 = 1.0 in the parentheses)

	MeanShift = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
StD	MEWMA	MCUSUM	MEWMA	MCUSUM	MEWMA	MCUSUM	MEWMA	MCUSUM
Case I	225.85 (248.44)	222.51 (263.86)	111.02 (140.70)	107.00 (179.80)	43.91 (58.89)	37.82 (88.31)	18.32 (30.50)	16.43 (42.25)
Case II	189.37 (191.15)	195.81 (221.91)	161.87 (200.70)	167.30 (194.14)	145.33 (142.77)	122.17 (150.87)	98.02 (117.17)	86.35 (106.63)
Case III	239.14 (216.36)	198.28 (206.68)	138.58 (194.20)	155.69 (184.71)	43.36 (83.90)	41.94 (140.49)	5.08 (29.11)	5.19 (83.80)
Case IV	178.46 (172.54)	190.63 (199.07)	102.01 (132.95)	64.45 (123.79)	31.32 (51.21)	12.75 (34.22)	10.59 (17.47)	5.68 (9.12)
Case V	264.79 (263.89)	204.60 (223.37)	15.45 (40.42)	24.79 (119.52)	1.60 (4.22)	2.52 (16.11)	0.99 (1.23)	1.10 (2.14)

Table 10(d). Standard Deviations of Steady-State Off-Target RLs (Failtime = 25)

MEWMA: r = 0.1 and MCUSUM: k_1 = 0.5 (MEWMA: r = 0.2 and MCUSUM: k1 = 1.0 in the parentheses)

	MeanShift = 0		MeanShift = 1		MeanShift = 2		MeanShift = 3	
StD	MEWMA	MCUSUM	MEWMA	MCUSUM	MEWMA	MCUSUM	MEWMA	MCUSUM
Case I	234.10	224.91	114.25	106.14	39.57	37.63	16.87	16.33
	(202.52)	(261.32)	(156.06)	(182.78)	(68.91)	(88.78)	(33.13)	(40.82)
Case II	208.07	192.46	157.43	168.01	149.13	125.01	95.68	87.36
	(217.28)	(217.54)	(177.80)	(196.54)	(150.62)	(148.41)	(112.35)	(105.37)
Case III	236.03	199.22	146.52	159.66	37.33	42.43	4.95	5.19
	(188.78)	(211.54)	(169.00)	(187.16)	(85.26)	(141.31)	(27.51)	(82.94)
Case IV	218.24	188.52	97.94	64.27	28.09	12.77	10.13	5.63
	(189.08)	(201.43)	(125.58)	(124.64)	(49.01)	(34.02)	(15.52)	(8.85)
Case V	276.19	208.21	13.98	25.07	1.32	2.49	0.99	1.11
	(283.11)	(217.90)	(42.23)	(122.97)	(4.16)	(16.05)	(1.23)	(2.16)

Table 10(e). Standard Deviations of Steady-State Off-Target RLs (Failtime = 50)

MEWMA: r = 0.1 and MCUSUM: k_1 = 0.5 (MEWMA: r = 0.2 and MCUSUM: k1 = 1.0 in the parentheses)

REFERENCES

Alt, F. B. (1984), “Multivariate Quality Control,” *The Encyclopedia of Statistical Sciences*, eds. S. Kotz, N. L. Johnson, and C. R. Read, New York: John Wiley, pp. 110-122.

Alt, F. B. and Smith N. D. (1988), “Multivariate Process Control,” *Handbook of Statistics*, VOL. 7, eds. P. R. Krishnaiah and C. R. Rao, Elsevier Science Publisher B. V., pp. 333-351.

Alwan, L. C. and Roberts, H. V. (1988), “Time-series Modeling for Statistical Process Control”. *Journal of Business and Economic Statistics*, VOL. 6(1), pp. 87-95.

Chen, Chou-Lin (1994), “Exponentially Weighted Moving Average Control Charts for Univariate and Multivariate Autocorrelated Data,” Unpublished Ph. D. Dissertation, University of Maryland College Park.

Crosier, R. B. (1988), “Multivariate Generalizations of Cumulative Sum Quality Control Schemes”. *Technometrics*, VOL. 30, pp. 291-303.

Jobson, J. D. (1991), *Applied Multivariate Data Analysis, VOL. I: Regression and Experimental Design*. New York: Springer-Verlag.

Lowry, C. A., Woodall, W. H., Champ C. W., and Rigdon, S. E. (1992), “A Multivariate Exponentially Weighted Moving Average Control Chart”. *Technometrics*, VOL. 34, pp. 46-53.

Lowry, C. A. and Montgomery, D. C. (1995), “A Review of Multivariate Control Charts,” *IIE Transactions* VOL. 27, pp. 800-810.

Lu, C. W. and Reynolds, M. R. (2001), “CUSUM Charts for Monitoring an Autocorrelated Process,” *Journal of Quality Technology*, VOL. 33 (3), pp. 316-334.

Prabhu, S. S. and Runger, G. C. (1997), “Designing a Multivariate EWMA Control Chart,” *Journal of Quality Technology*, VOL. 29(1), pp. 8-15.

Reinsel, G. C. (1997), *Elements of Multivariate Time Series Analysi*s, 2nd. New York: Springer.

Rencher, A. C. (2002), *Methods of Multivariate Analysis*, 2nd. New York: John Wiley.

Wardell, D. G., Moskowitz, H. and Plante, R. D. (1994), “Run-Length Distributions of Special-Cause Control Charts for Correlated Processes,” *Technometrics*, VOL. 36, pp. 3-17.