

ABSTRACT

Title of Dissertation: **SIGNAL PROCESSING AND FORWARD
MODELING OF SPACE DEBRIS DETECTION
VIA PLASMA SOLITONS**

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The nonlinear interaction of objects in low Earth orbit with the space plasma environment has been hypothesized to cause precursor soliton plasma waves. These plasma-object interactions may lead to unique engineering applications, especially the detection of hazardous sub-centimeter orbital debris that is undetectable by conventional methods. This nonlinear perturbation is currently modeled by the forced Korteweg – de Vries (fKdV) equation. This thesis aims to understand and characterize these waves through simulation beyond the fKdV model while progressing space-based and ground detection schemes. Ultimately this technique may play an important role in the problem of detecting small space debris.

Three aspects of this detection scheme are developed. This includes two unconventional methods of detecting solitons. First the inverse scattering transform (IST), a mathematical spectral technique for decomposing a time series, is shown to automatically detect solitons from data. A numerical experiment using the fKdV model is performed to demonstrate this ability. The IST

is suitable as an in situ detection method. It could be the basis of a debris collision early warning system for spacecraft. Second, the existing technique of ionospheric sensing using Global Navigation Satellite System (GNSS) is extended to detecting spacecraft plasma wakes. Traditionally, it is used for global scale space weather monitoring. An experiment is carried out using a known target, the International Space Station, on existing GNSS receivers that measure the ionospheric irregularity associated with the spacecraft. This experiment shows that there is a modification to the total electron content (TEC) when the ISS flies through the radio line-of-sight. Using models that are compared to the experiment, a multi-point sensor is proposed that would resolve the diffraction pattern from these plasma structures.

This work uses multi-fluid plasma simulation to refine the fKdV model of soliton generation from debris. In particular, we find that the range of ion acoustic Mach numbers that are conducive to precursor soliton generation is larger than predicted by the fKdV equation. A new theory that matches the multi-fluid simulation results is derived using pressure balances to predict the supercritical Mach number. This new theoretical understanding of the critical Mach numbers predicts a wider range of orbits that will create precursor solitons than in previous studies. In addition, several new details of precursor solitons are discovered and characterized with multi-fluid simulation. This includes changes in the amplitude scaling of the periodicity of soliton generation (the “intersoliton interval”). Importantly, corrections to the first order results of the fKdV equation which couple fluid velocity, density, and electrostatic potential are identified. A theory that explains this in the small amplitude limit is derived. For debris detection, this effect impacts how the soliton is detected. The same soliton will manifest different amplitudes in each plasma species, contrary to the result of the fKdV equation. Thus, a model error in inferring debris properties from solitons has been discovered.

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by

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¹and our cats Eevee and Puddle

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List of Abbreviations

AFRL	Air Force Research Laboratory
ASAT	Anti-satellite weapon
CAM	Collision avoidance maneuver
C/NOFS	Communication / Navigation Outage Forecasting System
ESW	Electrostatic solitary wave
FCC	Federal Communications Commission
fKdV	Forced Korteweg - de Vries
GEO	Geostationary orbit
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
IST	Inverse scattering transform
KdV	Korteweg - de Vries
LEO	Low Earth orbit
MEO	Middle Earth orbit
PIST	Periodic inverse scattering transform
RRI	Radio Receiver Instrument
SPD	Space Policy Directive
SSN	Space Surveillance Network
TISE	Time independent Schrödinger equation
WMM	World Magnetic Model

Chapter 1: Introduction

1.1 Contributions

This thesis deals with the interaction between small fragments of spacecraft with the Earth's ambient plasma environment. The main contributions are as follows:

1. Developing an open-source code to automatically detect solitons from data using a novel signal processing technique, the inverse scattering transform.
2. Performing simulations and developing theory that highlight and explain a discrepancy in the velocity bounds of ion acoustic solitons.
3. Developing a novel detection strategy for small scale ionospheric irregularities associated with transiting spacecraft. An experiment was carried out to verify this technique.

1.2 Publications

These contributions have been (or will be) published in the following publications and conference talks. The following disclaimer applies to this dissertation and these publications.

Disclaimer: This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof,

nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

1.2.1 Journal Publications:

1. **I.M. DesJardin** & C.M. Hartzell, *Radiometric Eclipsing by Ionospheric Wakes for Detection of Transiting Spacecraft*, Radio Science (in preparation).
2. **I.M. DesJardin**, C.M. Hartzell, & J.S. Wrieden, *Multifluid Simulation of Ion Acoustic Solitons Arising from a Charged Source and Comparison to the Forced Korteweg - de Vries Model* (in preparation).
3. **I.M. DesJardin** and C.M. Hartzell, *Nonlinear spectral analysis of ion acoustic solitons arising from a streaming charged object using the numerical inverse scattering transform*, Physics of Plasmas 29, 112116 (2022) <https://doi.org/10.1063/5.0122918>.

1.2.2 Talks:

1. *On the velocity bounds of the forced Korteweg – de Vries equation applied to ion-acoustic solitons and GNSS detection of plasma wakes*, Invited Seminar, Naval Research Laboratory, Washington, D.C., April 2024.
2. *The critical velocity bounds of ion acoustic soliton generation by a charged supersonic grain*, Plasma Physics Seminar, College Park, MD, April 2024.
3. *Ionospheric Interaction Based Detection of Sub-centimeter Space Debris*, Advanced Maui Optical and Space Surveillance Technologies Conference, Maui, HI, Sept. 2023.
4. *Nonlinear spectral analysis of Ion Acoustic Solitary Wave solutions to the forced Korteweg-de Vries equation*, American Physical Society Division of Plasma Physics 64th Annual Meeting, Spokane, WA, Oct. 2022.

The rest of this chapter will introduce three unrelated concepts: (1) what is the ambient plasma environment around the Earth, (2) why are there small fragments of spacecraft (debris) orbiting the Earth, and (3) what is nonlinear wave theory. In subsequent chapters, we will tie these three concepts together and ultimately build to the established work of the community. I've written this chapter for the audience of (a) future aerospace engineering PhD students who are learning the background material for understanding this work, (b) my friends and family who are

trying to understand what I do, and (c) my past self who wished he could have read this. None of this chapter is my work. It is the best summary I can give of the background material. For the committee members who are familiar with the background material, the literature review and presentation of the work in this thesis begins in Chapter 2.

1.3 Plasmas in space

Plasma is the fourth state of matter. It occurs when some of the electrons from an ordinary molecule are removed. The removal process happens due to some event that energizes the (often outermost) electron above the *ionization energy*. Much of the plasma in this thesis is either from the Sun or energized parts of Earth’s atmosphere. The composition of Earth’s atmosphere is summarized in Table 1.1 [14]. Overwhelmingly, the atmosphere is composed of nitrogen and oxygen. Argon and carbon dioxide¹ are in smaller amounts. Other species are only present in trace quantities². The near-Earth plasma composition will reflect this.

As seen in Table 1.1, the dominant species all have first ionization energies between 12 and 16 eV. The wavelength (λ) of a photon is related to its energy (E) by $\lambda = hc/E \approx 1,239.8(nm \cdot eV)/E$ where h is the Planck constant and c is the speed of light. Thus the lowest energy photon

¹Finding an agreed upon mole fraction for CO₂ in the atmosphere is a nontrivial task. The NOAA Global Monitoring Laboratory Monthly Average Global CO₂ lists the surface CO₂ composition of the atmosphere at 416.83 ppm = 0.42% in September 2023. This differs from the value in Davies [14] (published in 1965) and in Table 1.1. This is due to historic increases in the amount of CO₂ in the Earth’s atmosphere since at least 1958 [15] when it started being recorded at Mauna Loa or probably longer [16].

²Similarly, finding a precise mole fraction for the trace quantities is a complicated discussion. The Wikipedia article on *Atmosphere of the Earth* uses identical quantities to Davies as of January 2023. They cite *Allen’s Astrophysical Quantities* as their source and leave the following footnote: “Two recent reliable sources cited here have total atmospheric compositions, including trace molecules, that exceed 100%... Both exceed 100% because their CO₂ values were increased to 345 ppmv, without changing their other constituents to compensate. This is made worse by the April 2019 CO₂ value, which is 413.32 ppmv. Although minor, the January 2019 value for CH₄ is 1866.1 ppbv (parts per billion). Two older reliable sources have dry atmospheric compositions, including trace molecules, that total less than 100%: U.S. Standard Atmosphere, 1976 (99.9997147%); and *Astrophysical Quantities* (1976, 99.9999357%).” The four species in Table 1.1 sum to 99.997%.

Table 1.1: Surface composition of the Earth's atmosphere.

Species	Mole Fraction (%) [17]	First Ionization Energy (eV) [18]
N ₂	78.084	15.5808
O ₂	20.946	12.0697
Ar	0.934	15.75962
CO ₂	0.033	13.773

that can ionize one of the primary species of the Earth's atmosphere has a wavelength of $\sim 75 - 100nm$. This corresponds to extreme-ultraviolet (EUV) light. As EUV and higher energy X-rays and gamma-rays from the Sun hit the top of the Earth's atmosphere, a layer of plasma forms. This is called the *ionosphere* (from ionized atmosphere). This process primarily happens on the day-side of the Earth where EUV flux is highest.

As ions and electrons are introduced to the atmospheric gas, they start to interact via *Coulomb forces*. If there are two charged particles at a distance d from each other, they will exert a force given by Eqn. 1.1 on each other. A negative force implies attraction while a position force implies repulsion. ϵ_0 is the permittivity of free space. It has a value in SI units of $\epsilon_0 = 8.8542 \times 10^{-12} F/m$ [19]. q_1 and q_2 are the charges of the particles in units of Coulomb (C).

$$F_{\text{Coulomb}} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{d^2} \quad (1.1)$$

Ions are generally positively charged while electrons are negatively charged. The ions will repel each other while attracting the electrons. Similarly, the electrons repel each other while attracting the ions. The thermal energy in each species (ion or electron) creates a pressure that balances the tendency of the ions and electrons to mix. There is a length scale that naturally arises from this equilibrium known as the *Debye length* (λ_D). This is given in Eqn. 1.2 [19] for

a collisionless two-temperature plasma. q_e is the elementary/electron charge ($1.602 \times 10^{-19}C$), k_B is the Boltzmann constant and n_0 is the number density of the plasma (in particles/ m^3). In a region of space smaller than the Debye length, there may be uneven amounts of ions and electrons. Above the Debye length, the overall ratio of ions to electrons is one, i.e. there is no net charge in the plasma.

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{q_e^2 n_0}} \quad (1.2)$$

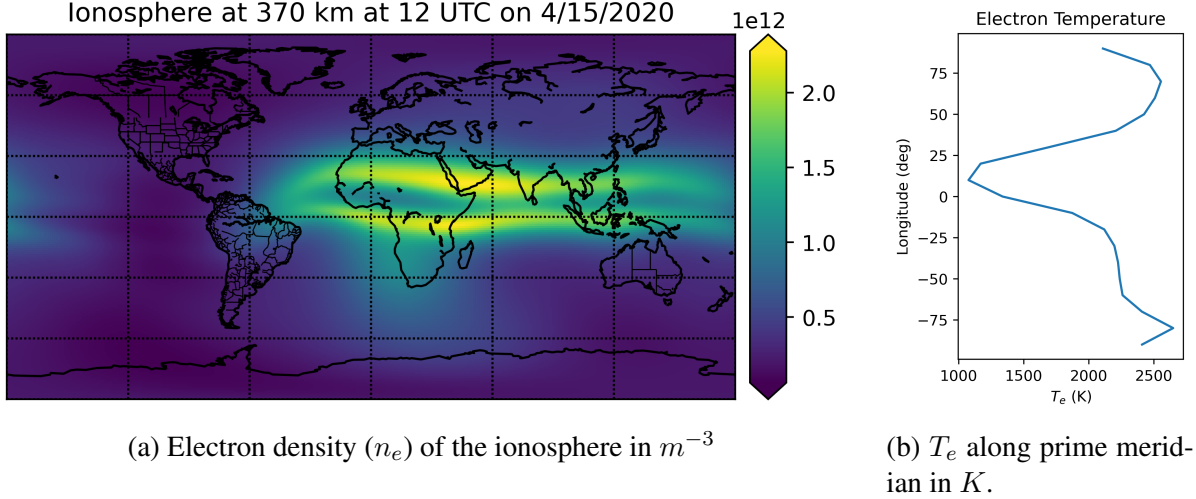


Figure 1.1: Both generated by the 2020 International Reference Ionosphere [3,4] with F10.7=100. The two main equatorial bands can be seen centered on the prime meridian. At 12 UTC, the sun is over the prime meridian.

A map of the ionosphere is shown in Fig. 1.1. The time of day is 12 UTC, so the Sun is approximately over the prime meridian. This is why the highest electron density is centered over Africa. Using the max and min values of $n_0 = n_e$ and T_e shown (which may not be at necessarily the same place), the Debye length could vary from 1.5 mm to 1.5 cm *on that given day at that altitude*. In later chapters of this thesis, plots will be given in units of the Debye length instead of a more conventional unit, say meters. This is because the Debye length will vary around the

Earth and between different plasmas. However, the physics will stay the same scaled by the local value of the Debye length. Generally speaking, at high latitudes and high altitudes the electron temperature goes up and the Debye length increases.

Each species in the plasma can have its own temperature (T). Mathematically, this is defined as proportional to the “average”³ (or thermal) velocity squared (i.e. units of kinetic energy, often scaled by the Boltzmann constant). In order to have a single temperature describe the entire fluid, the particles must frequently exchange energy. In a neutral gas (i.e. air) this happens via frequent collisions between the various types of molecules. However, in a collisionless plasma, energy exchange between the species is weaker especially if there is a significant mass difference. For this reason, the ions often have one temperature (T_i) and the electrons can have their own (T_e).

Given a temperature (“average” speed) and length scale (Debye length), there is an associated time scale. Imagine a particle is tugged from its Debye sphere⁴. The other particles in the Debye sphere will rearrange to attract it back in. When the particle is released, it will shoot into the Debye sphere and keep going where the process reverses. In this simple picture, the particle will oscillate back and forth like a pendulum⁵. Now let the “tugging” process be the thermal speed (v_t). The frequency of oscillation will be: $\omega \sim v_t/\lambda_D$. Substituting $v_t = \sqrt{\frac{k_B T}{m}}$ and λ_D in yields Eqn. 1.2, one arrives at the common value for *plasma frequency* (Eqn. 1.3). Since the ions and electrons have their own temperatures and masses, there are *two* plasma frequencies, the

³There are multiple mathematical ways to conceptualize the gist of “average” speed partially depending on the number of spatial dimensions the gas is allowed to move in. The term is being used loosely here.

⁴A sphere with a radius = 1 Debye length.

⁵There is a thought experiment that frequently appears in science communication books and videos about falling through a hole that goes through the entire Earth. In this case, the unfortunate person will bounce back and forth from one side of the Earth to the other. Sometimes the Earth is hollow. Falling through the Debye sphere is very mathematically analogous to this situation.

electron plasma frequency ($\omega_{p,e}$) and the ion plasma frequency ($\omega_{p,i}$) that respectively use the electron mass (m_e) and charge (q_e) or the ion mass (m_i) and charge (q_i) in the equation below:

$$\omega_p^2 = \frac{n_0 q^2}{\epsilon_0 m} \quad (1.3)$$

Since there are at least two species, plasmas have the property of supporting a hybrid sound speed, i.e. one with the Debye length set by the electrons but the plasma frequency set by the ions. Physically this corresponds to the situation where an ion is moved out of the Debye sphere, but the counter-force to pull it back in is set primarily by the Coulomb force of the electron reconfiguration. The ion will oscillate at the frequency set by their mass, but the length scale of this oscillation will be set by how far the electrons spread out when they respond. As can be seen in later derivations, this situation arrives when the electrons are much hotter than the ions ($T_e \gg T_i$). This hybrid speed is known as the ***ion acoustic speed*** and the associated mode is called the ion acoustic mode. The formula for the ion acoustic speed (v_{IA})⁶ is:

$$v_{IA} = \sqrt{\frac{k_B T_e}{m_i}} \quad (1.4)$$

Throughout this study, the results will be presented in non-dimensional quantities. The natural units of length, time, and velocity are respectively the Debye length (λ_D), inverse ion plasma frequency $\omega_{p,i}^{-1}$, and ion acoustic velocity v_{IA} . When velocity is divided by the ion acoustic velocity, this will be referred to as the ion acoustic Mach number (M_{IA}) or simply Mach number (M). The traditional Mach number in terrestrial aerodynamics has a very specific behavior near

⁶In some textbooks, the ion acoustic speed is called the “sound speed” and denoted by c_s . This notation requires the reader to know when c_s refers to the ion acoustic speed, ion thermal speed, or electron thermal speed since they are all “sound speeds.”

Mach 1 ($M = 1$) where the flow transitions from subsonic to supersonic. No terrestrial sound wave can travel faster than the classical speed of sound, so shock waves form. However there are three sound speeds in this regime of plasma physics. Hence, the $M = 1$ boundary is softer. If $m_e < m_i$ (as is the case in hydrogen and oxygen plasmas), then $v_{t,e} > v_{IA}$. Thus electron sound waves⁷ can travel faster than the ion acoustic speed. Later in this study, ion acoustic solitons will travel faster than $M = 1$. This apparent contradiction is due to the electron thermal speed being greater than $M = 1$.

Magnetic Fields

Plasmas are strongly influenced by magnetic fields. Individual particles will feel a force of $\mathbf{F} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B}$, where q is their charge, $\mathbf{v}$ is their velocity and $\mathbf{E}$ & $\mathbf{B}$ are the electric and magnetic fields respectively. The net effect is that, to first approximation, particles spiral around magnetic field lines at a distance of the **Larmor radius** ($r_L = \frac{mv_{\perp}}{qB}$) and at a **cyclotron frequency** ($\omega_c = qB/m$). Particles in this perpetual spiral motion are considered to be “trapped” along field lines. They can move freely along field lines, but will not move off of the field line unless some extenuating circumstance causes it to slip (some examples include macroscopic electric fields, bending magnetic field lines, rotating field lines). In near-Earth space⁸, the source of magnetic field is the one generated by the Earth. It has a dipolar structure with an elongated tail, as seen in Fig. 1.2.

The magnetic field lines intersect with the Earth at the magnetic north and south poles. Particles which are trapped by magnetic fields plunge into the atmosphere near the poles. This gives

⁷Sometimes called *Langmuir waves*.

⁸Often called *geospace*.

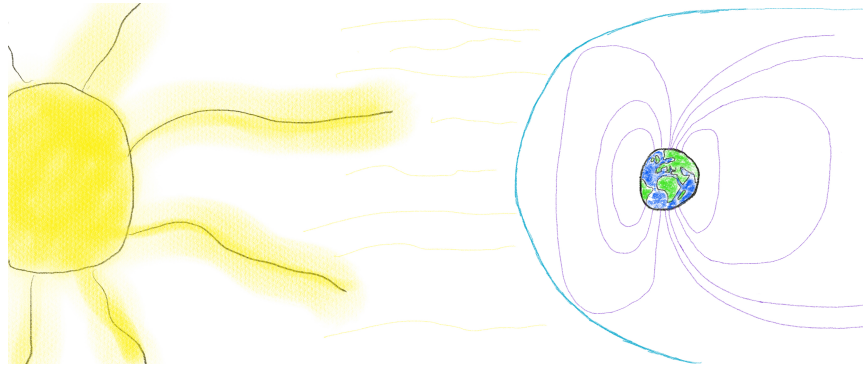


Figure 1.2: Earth's magnetic field interacting with the solar wind. The light blue line is the bow shock - where the solar wind is rapidly decelerated as it nears Earth.

rise to the auroras. The area inside the Earth's magnetic field is known as the *magnetosphere*.

Finally, there is another source of plasma in space — the Sun! Plasma constantly streams off of the surface of the sun. This is known as the *solar wind*. The wind streams past the Earth where it interacts with the Earth's magnetic field in a complicated fashion. One interaction mechanism is via magnetic reconnection on the day side of the Earth [20–22]. Eventually, some of the solar wind plasma is trapped along the Earth's field lines and migrates into the inner magnetosphere. *Distance* in this environment means how many magnetic field lines separate a location from where the solar wind was absorbed. Thus being at a high latitude or a high altitude at a low latitude achieves the same result. Often, literature will quantify this idea by expressing location as an L-shell. The L-shell, refers to the distance the local magnetic field line intersections the plane of the Earth's equator in units of the Earth's radius. Along the equator, L-shell simply refers to your altitude. However, at high altitudes the L-shell value is often much higher than the local altitude. In those places, the plasma composition around the Earth is primarily hydrogen with trace amounts of Helium, reflecting that the plasma originated in the solar wind. In middle-Earth-orbits (MEO) or geostationary orbits (GEO), both of which are well above the top of the ionosphere, the plasma is mostly solar wind in origin. Just as the ionospheric composition mirrors

the Earth’s atmospheric composition, the solar wind mirrors the Sun’s composition. Therefore, it’s mostly ions of hydrogen and helium.

The study of how the solar wind, magnetosphere, and ionosphere interact is commonly called space physics. Sometimes when the role of the Sun is being emphasized, it is known as heliophysics. When this problem is approached from a forecasting perspective, it is known as space weather. For the purposes of this thesis, these plasmas are the background conditions in which the space debris interacts with. Table 1.2 summarizes the various locations. From Table 1.2, it can be see that in general $T_e > T_i$, although by varying amounts, $r_{L,e} < \lambda_D < r_{L,i}$, and v_{IA} is on the scale of several km/s. As we shall see in the next chapter, this is comparable to orbital speed. The electron thermal speeds tend to be $\mathcal{O}(10^5 - 10^6)$ m/s. The length scale of interest is debris mm to cm in size. This makes it so that the length scale is of the order of the Debye length, but below the ion gyroradius.

Table 1.2: Space plasma properties in common orbits. From Refs. [1–3] and the Department of Defense World Magnetic Model (WMM). For the Larmor radii, the thermal velocity is taken as the perpendicular velocity, i.e. $v_{\perp} = v_t$ (smallest case).

Location	Species	T_e (eV)	T_i (eV)	v_{IA} (km/s)	λ_D (m)	$r_{L,i}$ (m)	$r_{L,e}$ (m)
Equatorial 400 km	O^+	0.25	0.01	2-10	0.01 - 0.1	2	0.05
Equatorial 700 km	O^+	0.25	0.16	0.8	9	0.04	10^{-6}
Polar 950 km	H^+	0.4	0.2	6	0.14	1.2	0.03

1.4 Debris in space

The other denizens of Earth orbit are the objects that humankind has put there since the launch of the first object into orbit, Sputnik 1, in 1957. These include spent rocket bodies, satellites, manned spacecraft, and space stations. There are also more exotic objects such as

tools that floated away from astronaut’s grip or lost thermal blankets from Space Shuttle missions. Finally, there are the remains of satellites that have been broken up into several small fragments. This can happen from intentional kinetic weapons⁹ (“anti-satellite weapons”), unintentional satellite-satellite collisions, or accidental explosions often from failing batteries. The spent rocket bodies, any defunct intact satellites, human byproducts, and satellite fragments are all considered *space debris*. All objects in orbit must travel at orbital speeds, often in excess of $10 \text{ km/s} \approx 22,400 \text{ mph}$ ¹⁰. Kinetic energy is given by $KE = \frac{1}{2}mv^2$ where m is the object’s mass and v is its velocity. The energy scales with velocity squared while only linear in mass. Since *speed* is fixed while mass differs, collisions between orbital objects involve a lot of energy between objects of all masses.

As Earth orbit becomes more and more crowded, the risk of collisions becomes greater. As of January 6th, 2024, according to the publicly available catalog of tracked space objects managed by the US Space Force (the US Space Surveillance Network / SSN) [24], of the total amount of objects in Earth’s orbit (44,600) about 42% (19,000) are considered debris. This has grown historically as more and more objects have been launched. A detailed chronology of launches from 1957-1998 can be found in Ref. [25]. In Subsection 1.4.2, recent debris-causing events are outlined. This debris poses a danger to existing satellites and astronauts. As will be explained further in Subsection 1.4.1, debris tends to create more debris through collisions with working satellites.

Satellites need to perform collision avoidance maneuvers (CAM) to prevent damage from debris–satellite collisions. As the amount of debris in a given orbit increases, the number of

⁹So far kinetic warheads have been used to destroy satellites. Other concepts have been developed using a laser or nuclear warhead.

¹⁰For a speed comparison, the SR-71 Blackbird jet once set a speed record flying from Los Angeles to Washington DC at an average speed of $2,124 \text{ mph} \approx 0.95 \text{ km/s}$ [23]. 10 km/s is a full 10x faster than this.

collision avoidance maneuvers needed to operate the spacecraft correspondingly increases. For example, Fig. 1.3 shows the historical number of collision avoidance maneuvers the International Space Station has had to do. In the early 2000s it was only necessary to do 1 CAM / year although in some years it was not necessary. In the 2020s, about 3 CAMs / year is typical. In August 2023, it was necessary for 2 CAMs in response to debris from events 1 and 3 in Subsection 1.4.2 (two different ASAT tests) [26]. In practice, the way this process works is the SSN catalog is analyzed for a probabilistic prediction of likely collision in the near future (or as it is often called an *orbital conjunction*). In principle, other catalogs can exist, but the SSN is the primary catalog used. Mathematically this process involves finding the intersection of ellipsoids centered around the locations of tracked objects where the size of the ellipsoid is set by its size and uncertainty in its orbit [27–29]. Warnings of high probability collisions are issued to satellite operators¹¹. Hopefully, a suitable CAM can be planned in time to avoid the collision.

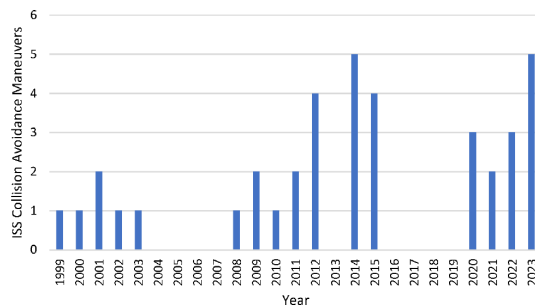


Figure 1.3: Collision avoidance maneuvers performed by the ISS during its lifetime. From [5]

CAMs prevent collisions. Without them, the lifetime of satellites would be shorter. However, *a CAM cannot stop collisions with objects that are not part of the orbital catalog*. Current research supports that there exists a large number of debris below 1 cm in size that are currently untracked. This population is indirectly inferred to exist by hypervelocity impacts observed on

¹¹<https://celestrak.org/SOCRATES/> is a public issuer of high risk collisions.

spacecraft (e.g. in solar panels or radiators) [30] and limited radar observation [31, 32]. These small debris – satellite collisions frequently occur [33, 34]. This population (along with the entire debris population) is expected to grow and be self-sustaining in the future from collisions [7, 35].

This dissertation is on the *detection* problem of sub-centimeter space debris in order to avoid it. The fundamental limit on detection size is driven by the technology available to use (primarily radars and telescopes). This work advances a concept for using plasma wave generation to detect debris. Instead of looking for the debris directly, look for signatures it creates in the plasma environment around Earth. Section 1.5 describes this debris – plasma interaction in more detail.

This dissertation is *not* on the remediation or management of existing debris. A full solution to the problem is outside the scope of a single person’s work. Space debris is a byproduct of humankind’s entire enterprise in space. It will require the cooperation of that enterprise (e.g. national space agencies, private companies, spacecraft manufacturers) to solve it through diplomacy, regulation, and coordinated cleanups. Subsection 1.4.4 goes through some strategies that are being attempted to either regulate the creation of or remove existing debris.

1.4.1 Kessler Syndrome

The logical conclusion of the growth of space debris is the formation of an artificial debris ring around the Earth. This process is not unique in the solar system. Potentially, Saturn’s rings formed by the collision of two moons [36, 37]. Some of Jupiter’s rings may be ejecta from small moons [38]. Around each of these planets a highly collisional process created a large population of small particles that now orbit around the planets as rings. In the late 1970’s, Donald Kessler

and Burton Cour-Palais carried out a thought experiment. *If the source of large objects was current Earth satellites and the collision mechanism was them naturally running into each other, when would a ring start to form?* More specifically, given some population of small objects in Earth orbit, when would that population naturally start to increase from collisions? This situation is now known as *Kessler syndrome*¹².

We shall now outline the main result of Ref. [6], Kessler and Cour-Palais' seminal paper on the concept. The notation used in this section may conflict with notation in later chapters. Let I denote the number of collisions on a particular satellite. σ is the collision cross section in distance squared, n is the number density of objects in objects / volume, and $\bar{v}$ is the average inter-object distance. Below 2,000 km, Kessler uses $\bar{v} = 7$ km/s. Note that this formula is identical to how it is defined for gas dynamics. For the physicists reading this, Kessler's treatment is a kinetic theory for space objects where the satellite catalog is used as a distribution function. The collision rate for an individual satellite can be approximated by Eqn. 1.5.

$$\frac{dI}{dt} = n\sigma\bar{v} \quad (1.5)$$

In a given element of volume (dV), one expects the total number of collisions per unit time (dC/dt) to equal the integral of individual collisions, i.e. Eqn. 1.6. The 1/2 prefactor is there to neglect self collisions. σ is an effective average cross-section of all orbital objects. Kessler uses a value of 4 m² in the 1978 paper.

¹²Donald J. Kessler is now retired. During his career as NASA (1962-1996), he created the Orbital Debris Program Office. On his website (<https://aquarid.physics.uwo.ca/kessler/iwp5.html>), he maintains a historical account of the term and work around it. This includes a two hour video on the early days of international collaboration on setting standards to mitigate orbital debris. Presumably, the video is Kessler's personal footage taken during the negotiation process while traveling to non-American space agencies to present his results and solicit cooperation. See Ref. [39].

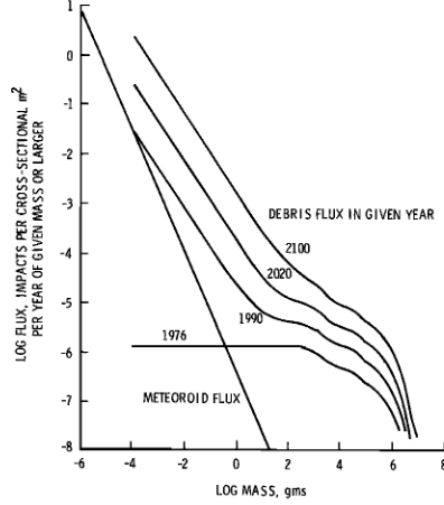


Figure 1.4: Fig. 6 from Ref. [6]. Average debris flux between 700 and 1200 km assuming that the input satellite injection rate is 510/yr and no atmospheric drag. This was published using data from an initial state in 1976.

$$\frac{dC}{dt} = \int \frac{dI}{dt} n dV = \frac{1}{2} \int n^2 \sigma \bar{v} dV \quad (1.6)$$

All objects in the same orbit move at the same speed regardless of their mass. For every collision, new objects are created. This means that the evolution of n depends on dC/dt . However, this relation *is* dependent on mass. It is found that for impacts at orbital speed, the number of *new* objects created at a given mass (M) depends on the total mass of the collision M_e in a power law form: $N = 0.8(M/M_e)^{-0.8}$. Finally, we define the *flux* of debris in a volume as $F = \int n dV \bar{v}$. In the absence of human-intervention, the source of debris flux is the natural meteoroids that come close to Earth. Kessler and Cour-Palais conducted simulations of the global debris distribution using the collision rates above and the debris generation law to obtain Fig. 1.4.

In Fig. 1.4, it can be seen that the artificial debris flux very quickly outweighs the background meteoroid flux. This was important because structural designs of spacecraft needed to take into account *future* orbital conditions with orders of magnitude higher expected collisions.

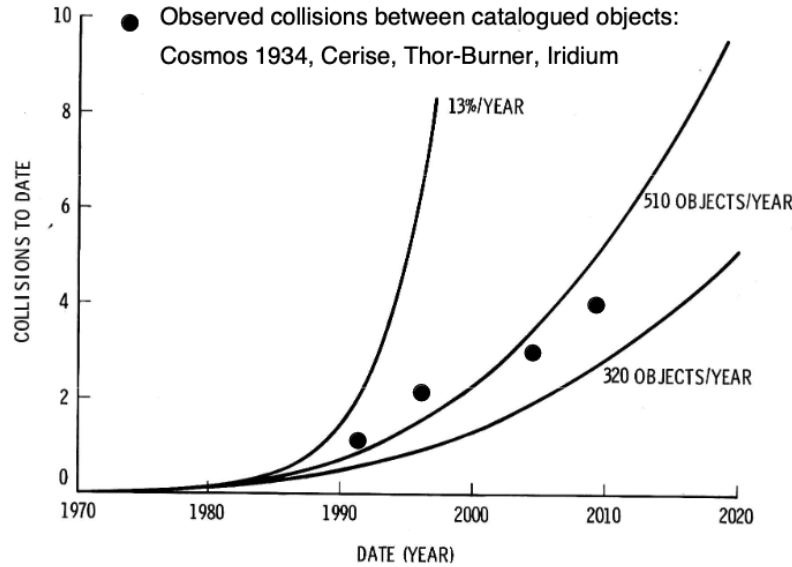


Figure 1.5: Fig. 2 from Ref. [7]. Dots indicate actual collision events. Lines are original predictions from the 1978 paper [6].

Secondly, it was found that the primary driver of future collisions came from debris generated by *current* satellites. The system will naturally grow even if satellites stopped being launched into orbit.

In 2010, Kessler and collaborators compared the predictions from the 1978 paper to what actually transpired. The actual number of collision events that happened matched almost perfectly with the computer simulations, as seen Fig. 1.5. The actual growth rate of space objects at the time of Fig. 1.5's publication was 300 / year. In the 2020's, it is an order of magnitude higher, as evident by Fig. 1.7. The last 5 year trend is from megaconstellation construction. However, it is expected that the debris driven growth of the new population will occur soon. In any case, it is well established that the growth rates predicted by Kessler and collaborators have been largely correct over the past 46 years¹³.

¹³Sputnik 1 was launched in 1957. Correctly predicting the growth of space objects for 46 years when humankind has only been in space for 67 years is about $2/3^{\text{rds}}$ of the entire space age.

Table 1.3: Orbital parameters of Fengyun-1C sun synchronous orbit.

Orbital Parameter	Value
Periapsis Altitude (km)	832.3
Apoapsis Altitude (km)	851.7
Inclination (deg)	99.0
Eccentricity	0.00134

1.4.2 Recent Major Debris Creation Events

Although this dissertation is not about policy nor litigation of debris creating parties, it is prudent to talk about the history and causes of the current debris environment in order to frame the importance of detection. Between 2005 and 2024, there have been three major debris creating events. We shall quickly outline the who, what, and where.

1: Fengyun-1C

In 2007, the People’s Republic of China conducted an ASAT test that destroyed the Fengyun-1C satellite, an existing Chinese weather satellite. Fengyun-1C had the orbital parameters¹⁴ given in Table 1.3. Fengyun-1C was in a sun synchronous orbit which is a special orbit with a very high inclination. This makes it such that the precision of the right ascension of the ascending node matches the period of a sidereal day. The practical effect of this is that the satellite is always viewing the dayside of the Earth. For optical measurements of the weather this is essential to carrying out the mission of that satellite.

¹⁴<https://en.wikipedia.org/wiki/Fengyun>

2: Iridium-Kosmos

In 2009, the Iridium 33 (private American company) and Kosmos 2251 (Russian military) satellites accidentally collided with each other. The geometry of the collision was similar to a T-bone geometry in automotive accidents. The orbital parameters of Iridium 33 were given by Table 1.4¹⁵, and the orbital parameters of Kosmos 2251 were given by Table 1.5¹⁶. Iridium 33 is estimated to have a mass of 700 kg. Kosmos 2251 is estimated to weigh 900 kg. Both satellites broke up almost completely during the collision. So in all, this generated about 1.5 metric tonnes of debris.

Table 1.4: Orbital parameters of Iridium 33

Orbital Parameter	Value
Periapsis Altitude (km)	779.6
Apoapsis Altitude (km)	799.9
Inclination (deg)	86.4
Eccentricity	0.3106

Table 1.5: Orbital parameters of Kosmos 2251

Orbital Parameter	Value
Periapsis Altitude (km)	783
Apoapsis Altitude (km)	821
Inclination (deg)	74.0
Eccentricity	0.4724

3: Kosmos-1408

In 2021, the Russian Federation conducted an ASAT test against the Kosmos-1408 satellite. Kosmos-1408 was a military satellite launched by the Soviet Union in 1982 and is widely

¹⁵From: https://en.wikipedia.org/wiki/Iridium_33

¹⁶https://en.wikipedia.org/wiki/Kosmos_2251

understood to have been inactive when it was destroyed. Its orbital parameters were given by Table 1.6. Unlike the first two major events, the Kosmos 1408 was in a lower orbit.

Table 1.6: Orbital parameters of Kosmos-1408.

Orbital Parameter	Value
Periapsis Altitude (km)	465
Apoapsis Altitude (km)	490
Inclination (deg)	82.60
Eccentricity	0.5878

Since ASAT tests compose of 2/3 of the major debris creating events, one may conclude that only two ASAT tests occurred in this time period. This is not so. The militaries of the United States of America and Republic of India both used ASAT weapons in 2008 [40] (Operation Burnt Frost) at 247 km and 2019 at 283 km respectively. This is at substantially lower altitudes than the Chinese and Russian tests where atmospheric drag shortens the lifetime of the debris from decades to months. In August 2019 (5 months after the Indian test), it was believed that only 50 tracked pieces of debris remained [41]. Since the size distribution of debris follows a power law and debris < 1 cm is not tracked, this does not capture the full amount of debris.

Fig. 1.6 shows the distribution of cataloged objects by orbital altitude. In Fig. 1.6, the peak of orbital object density is at ~ 800 km altitude. This peak increased between 2008 - 2010 by a factor of ~ 2 and from 1999 - 2008 also by a factor of ~ 2 . These two increases are due to the Iridium-Kosmos and Fengyun-1C debris creating events respectively. The majority of tracked objects in these orbits is debris. The bulk of space debris is currently located at ~ 800 km altitude.

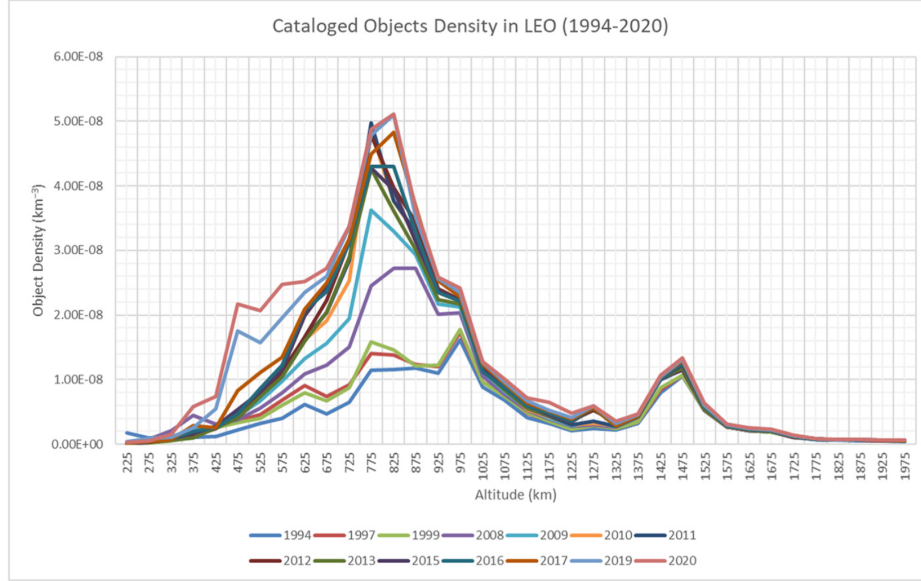


Figure 1.6: Distribution of cataloged objects in Earth orbit by altitude in 50 km bins. Different colors indicate different years. Source: Ref. [42].

1.4.3 Megaconstellations

A *constellation* is a group of several individual satellites that orbit around the same body. Often they are used to repeatedly image the same spot on the ground multiple times per day. A *megaconstellation* is a term in use to describe proposed constellations with thousands of individual satellites¹⁷. Several megaconstellations are currently funded projects by majors companies. Often their stated business case is to provide space-Earth internet service. Users could potentially only require a user terminal consisting of an antenna on a swivel mount that tracks the megaconstellation above. The size of the megaconstellation is driven by the requirement of having one (or multiple for redundancy) satellites overhead at all times.

As of the writing of this dissertation chapter, there is one main megaconstellation under construction. This is the *SpaceX Starlink* megaconstellation. Fig. 1.7, shows the number of ob-

¹⁷In the industry, sometimes the word *bird* is used to describe an individual satellite while the whole constellation is the *flock*. The company Planet, who runs an Earth observation constellation, calls its individual satellites *doves*.

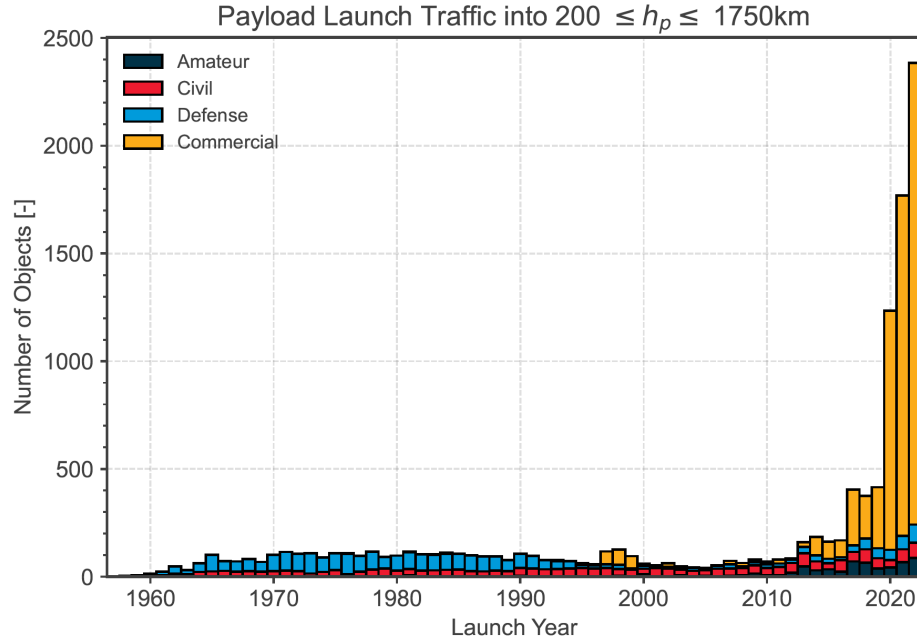


Figure 1.7: Launch traffic to orbits of periapsis (h_p) between 200 – 1750 km. This is colloquially known as *Low Earth Orbit* (LEO). Taken from Ref. [8]

jects launched to Low Earth Orbit (LEO) from all countries. It can clearly be seen that almost all launched objects in LEO happened since 2015. This is almost singularly due to the construction of the Starlink megaconstellation.

It is expected that the *completion of this megaconstellation will substantially increase the daily need for satellite operators to use collision avoidance maneuvers*. While orbital debris has been a risk in the past, these current projects will increase the total number of objects in orbit. This will in turn increase the likelihood that a major collision event occurs and so on¹⁸.

¹⁸The previous section discussed government monitoring of objects in orbit. In the United States there is a public version of the catalog at space-track.org. SpaceX and many other satellite operators voluntarily share orbital ephemeris (high precision orbital) data with the US Space Force that is posted to the public on that website.

1.4.4 Solutions to Space Debris

This dissertation is also *not* an account of all of the solutions that are being attempted. However, the following will be provided as background material for the interested reader.

1.4.4.1 Monitoring and Policy

In the late 2010s and early 2020s, several governments have begun to update policy on managing the Earth orbital environment. Unfortunately, the author is biased towards knowledge of American efforts in this area. In the United States, the Trump administration issued a memorandum, Space Policy Directive 3/SPD-3, in 2018 which directed American agencies to find ways to actively work on “space traffic management” [43]. A 2021 Senate subcommittee testimony on the topic occurred [44] during the Biden administration. One effect has been the creation of the *Office of Space Commerce* within the Department of Commerce as a civilian organization to manage collision avoidance. This office uses existing government catalogs and commercial data it purchases to create collision models. These models are communicated to satellite operators who then perform small collision avoidance maneuvers. In many ways, this resembles the function of a control tower at an airport. This effort has been echoed in other countries. The European Space Agency (ESA) has a *ESA Space Debris Office* that has published an annual Space Environment Report since 2016 [8]. In 2015, the Chinese National Space Administration (CNSA) established the *CNSA Space Debris Observation and Data Application Center* [45] as part of the National Astronomical Observatory of the Chinese Academy of Sciences. The Indian Space Research Organization (ISRO) built a dedicated facility called IS4OM for SSA purposes [46]. In 2022, ISRO performed 22 collision avoidance maneuvers on its fleet of 124 satellites [47]. With the

omission of the Russian space program, this establishes that all major space faring governments have established an orbital debris office of some sort within the last decade.

1.4.4.2 Regulation

In the United States, the primary mechanism for regulating spacecraft is via the communications licensing. All spacecraft to some degree are remotely controlled via radio. In order to gain access to a given frequency, the spacecraft operator needs to apply to the Federal Communications Commission (FCC) for a license. In the past, a deorbit plan has been a part of the FCC licensing process. For example, the FCC filing for the SpaceX Starlink constellation [48] specifies an “orbital debris mitigation plan” that the company must comply with to obtain its license. This includes a 125 km buffer in orbital altitude between Starlink and the OneWeb constellation and lowering the altitude to between 540 - 570 km [49]. The FCC has recently modified their guideline so that all new satellites in low Earth orbit must nominally deorbit within 5 years [50]. Previously, that guideline was 25 years.

Additionally, there are industry standards for designing spacecraft missions such that orbital debris generation is mitigated. One set of standards is published by NASA [51–53]. This includes guidelines for picking orbits such that for the frontal area of the satellite (which determines the atmospheric drag) the lifetime is under the specified amount and ensuring probabilistic limits of “debris-generating explosions for all credible failure modes of each spacecraft and upper stage (excluding small particle impacts) is less than 0.001 (1 in 1,000).” An example of a debris-generating explosion would be a battery explosion or leftover propellant. For leftover propellant the guideline is to vent unspent fuel that is no longer necessary. For spacecraft in geosynchronous

orbit (GEO) where atmospheric disposal is prohibitively fuel expensive the guideline is to, “maneuver to an orbit with perigee altitude sufficiently above 35,986 km (upper boundary of the GEO zone) to ensure the structure remains outside the GEO zone for at least 100 years.” These guidelines and FCC regulations amount to spacecraft manufacturers being required by law to design their spacecraft bus and mission profile to these standards. At the time of FCC license application the manufacturer has to establish that they have followed the guidelines.

1.4.4.3 Active Removal

Several researchers have proposed strategies for removing existing debris. The dominant technical issue is that it would take an entire (or multiple) rocket(s) to launch a spacecraft with enough fuel to latch onto one piece of debris and then perform a maneuver to burn itself and the debris up in the atmosphere. Latching onto a piece of debris requires proximity operations with an uncontrolled target, requiring potentially more fuel. These issues make debris remediation a costly endeavor. Nonetheless, several experimental missions have been or are soon to be launched to validate the concept including (but not an exhaustive list): Astroscale ADRAS-J [54–56], Astroscale ELSA-d [57], the Northrop Grumman Mission Extension Vehicle (MEV) [58, 59], and ESA ClearSpace-1 [60–62]. This concept is largely targeted for removing single large pieces of debris due to the one-satellite to one-debris ratio. There have been two MEVs launched. Both attached to functioning GEO satellites and either took over propulsion and pointing or acted as a extra fuel tank. Neither disposed of debris and thus far it has been a proof-of-concept. ADRAS-J rendezvoused with a rocket upper stage. It did not attach nor remove the rocket body. ELSA-d launched with its own test piece of debris that it successfully controlled to deorbit. ClearSpace-1

has yet to launch. Thus far, none of these concepts have rendezvoused and docked with a piece of *existing* debris, let alone deorbited with it.

The dock-and-use-thrusters-on-the-docking-spacecraft-to-deorbit-both concept is highly limited by propellant use. Since debris removal doesn't require a sophisticated trajectory or tight tolerances (simply impart force in the anti-velocity vector direction) experimental propellantless space propulsion techniques have been studied as candidates. Two have been proposed and tested to varying degrees of fidelity: solar sails and space tethers. A ***solar sail*** uses photon pressure on a large surface area to move a spacecraft. One prominent example was the LightSail-2 mission that successfully used photon pressure to extend its orbital lifetime 10 months longer than if there was no sail [63,64]. While this was a successful on-orbit demonstration of the solar sail technology, it did not deorbit any debris. TugSat was a cubesat concept with the intention to prototype a concept of using small solar sail spacecraft to deorbit larger geostationary satellites [65]. Numerous theoretical studies have been taken to understand the performance and control aspects of this problem [66–68]. ***Space tethers*** use the Lorentz force between the current down a long wire and the Earth's magnetic field to create a force. Similar to solar sails, space tethers have been experimentally flown and shown to some degree to be a feasible concept. The Tethered Satellite System (TSS-1R) mission successfully demonstrated current collection along the tether consistent with the physics expected [69,70]. However, after deploying to ≈ 20 km in length, the tether snapped 5 hours into the mission. A recent advance in the design of these devices is the use of a low work function coating. This allows passive thermionic emission instead of active potential control devices. This iteration of the space tether is currently the subject of active research [71–80].

1.5 Nonlinear waves and solitons

Now we begin again with a new topic. Plasmas contain waves. It is a complicated topic where there have been books written about mostly plasma waves: [17, 81]. In this section, we will introduce the concept of waves in plasma with a focus on ion acoustic waves. Then we will develop the idea of a *nonlinear* ion acoustic wave. All of these concepts exist in their own right independently of being generated by space debris.

1.5.1 Ion acoustic wave (linear analysis)

We will start by ignoring the magnetic field ($\mathbf{B} = 0$). A five moment fluid description of a plasma system is written down in Eqns. 1.7 - 1.9. S is a source of free charge present in the system. S will be used to represent the charge that accumulates on a piece of debris. Electrons are assumed to behave adiabatically, i.e. $n_e = n_e \exp \left[\frac{e\phi}{k_B T_e} \right]$. ρ_i is the ion mass density, $n_i = \rho_i / m_i$ is the ion number density, m_i is the ion mass, Z is the ion charge number, ϕ is the electrostatic potential, u_i is the ion bulk velocity, k_B is the Boltzmann constant, T_e is the electron temperature, p_i is the ion pressure, and $\mathbf{E}$ is the electric field. e is the elementary charge and ε_0 is the permittivity of free space.

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i u_i) = 0 \quad (1.7)$$

$$\frac{\partial (\rho_i u_i)}{\partial t} + \nabla \cdot (\rho_i u_i^2) + \nabla p_i = \rho_i \left(\frac{q_i}{m_i} \right) \mathbf{E} \quad (1.8)$$

$$\frac{\partial^2 \phi}{\partial \mathbf{x}^2} + \frac{e}{\varepsilon_0} \left(Z n_i - n_e \exp \left[\frac{e\phi}{k_B T_e} \right] \right) = S \quad (1.9)$$

Each variable is expanded as $\rho = \rho_0 + \hat{\rho}$. Temperatures are constant. $u_{i,0} = u_{e,0} = u_0$.

$$ik\rho_{i,0}\hat{u}_i = i \underbrace{(\omega - ku_0)}_{w_d \text{ (Doppler)}} \hat{\rho}_i \quad (1.10)$$

$$ik \frac{\hat{\rho}_i}{m_i} T_i - i\omega \hat{\rho}_i u_0 - i\omega \rho_0 \hat{u}_i + ik \hat{\rho}_i u_0^2 + ik 2u_0 \rho_0 \hat{u}_i = -ik \left(\frac{\rho_i q_i}{m_i} \right) \hat{\phi} \quad (1.11)$$

$$-k^2 \hat{\phi} - \frac{e}{\varepsilon_0} \left(\frac{Z}{m_i} \rho_{0,i} + \frac{Z}{m_i} \hat{\rho}_i - \frac{\rho_{0,e}}{m_e} - \frac{\rho_e}{m_e} \frac{e\hat{\phi}}{k_B T_e} \right) = S \quad (1.12)$$

Simplifying and enforcing quasi-neutrality on $\rho_{i,0}$ and $\rho_{e,0}$ yields the following:

$$k\rho_{i,0}\hat{u}_i = w_d \hat{\rho}_i \quad (1.13)$$

$$ik \frac{\hat{\rho}_i}{m_i} T_i - i\hat{\rho}_i u_0 \omega_d - i\rho_{0,i} \hat{u}_i (\omega_d - ku_0) = -ik \left(\frac{\rho_i q_i}{m_i} \right) \hat{\phi} \quad (1.14)$$

$$- \left(\frac{\rho_e q_e^2}{m_e k k_B T_e} + k^2 \right) \hat{\phi} + \frac{q_i}{m_i \varepsilon_0} \hat{\rho}_i = S \quad (1.15)$$

We can expand the momentum equation and simplify to get the following:

$$\hat{\rho}_i \left(1 - \frac{k^2 k_B T_e}{m_i \omega_d^2} \right) = \frac{k^2}{\omega_d^2} \left(\frac{\rho_{0,i} q_i}{m_i} \right) \hat{\phi} \quad (1.16)$$

Substituting into Poisson's equation with $S = 0$ and using $v_{IA} = \sqrt{k_B T_e / m_i}$ and λ_D (defined in

Eqn. 1.2) yields:

$$\left[1 - \frac{k^2 v_{IA}^2}{\omega_d^2} \left(\frac{T_i}{T_e} \right) \right] \left[\frac{1}{\lambda_D^2} + k^2 \right] = \left(\frac{\omega_{p,i}}{\omega_d} \right)^2 k^2$$

Rearranging and simplifying yields the dispersion relation. This includes an electron/ion temperature difference term and the Doppler effect, which are often absent from textbook derivations.

$$\omega_d^2 = (\omega - ku_0)^2 = k^2 v_{IA}^2 \left(\frac{T_i}{T_e} \right)^2 + \frac{(k\lambda_D)^2}{1 + (k\lambda_D)^2} \omega_{p,i}^2 \quad (1.17)$$

Often $T_e \gg T_i$, so the ion acoustic speed term on the right-hand side is small. Damping can *not* be derived from fluid equations even though it is physical. A kinetic approach has to be used that results in Landau damping.

1.5.2 Ion acoustic soliton (KdV approach)

The previous section considered the linear dispersion relation of ion acoustic waves. Now, a *nonlinear* wave equation will be derived in limit where dispersion balances nonlinear steepening. This was originally done in Ref. [82] and the derivation here is largely from [83]. Consider again Eqns. 1.7 - 1.9. In one dimension and nondimensionalized, they can be written as Eqns. 1.18 - 1.20. n is the nondimensionalized ion number density, v is the nondimensional velocity (the ion acoustic Mach number), and ϕ is now nondimensionalized.

$$\frac{\partial^2 \phi}{\partial x^2} = \exp \phi - n \quad (1.18)$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} (nv) = 0 \quad (1.19)$$

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial x} = - \frac{\partial \phi}{\partial x} \quad (1.20)$$

This can be written in a frame moving at the sound speed (i.e. $x \rightarrow x - t$). In the new

coordinate system $\partial/\partial x \rightarrow \partial/\partial x$ and $\partial/\partial t \rightarrow \partial/\partial t - \partial/\partial x$.

$$\frac{\partial^2 \phi}{\partial x^2} = \exp \phi - n \quad (1.21)$$

$$\frac{\partial n}{\partial t} - (1 - v) \frac{\partial n}{\partial x} + n \frac{\partial v}{\partial x} = 0 \quad (1.22)$$

$$\frac{\partial v}{\partial t} - (1 - v) \frac{\partial v}{\partial x} = -\frac{\partial \phi}{\partial x} \quad (1.23)$$

As seen in Eqn. 1.17, ion acoustic waves all essentially move at the ion acoustic speed (v_{IA}) but are modified by temperature ratio (T_e/T_i) and the wavelength to Debye length ratio ($k\lambda_D$). If one assumes that $T_e \gg T_i$, $\omega_d = \omega$, and that $k\lambda_D$ is small, the dispersion relation can be Taylor expanded as:

$$\omega = v_{IA}k \left(1 - \frac{(k\lambda_D)^2}{2} \right) \quad (1.24)$$

In the frame moving at the sound speed, we assume that the time scale of the *solution* is the same time scale as *dispersion* that is also the same time scale as *nonlinearity*. Therefore, $\partial/\partial t \sim k^3 \sim v\partial v/\partial x$. If one assumes that the time evolution from these secondary effects is small, then this resolves an ordering for the system of Eqns. 1.21 - 1.23. Namely, it is assumed that $\partial/\partial t \sim \epsilon$. $\partial/\partial x \sim k \sim \epsilon^{1/3}$. Each variable is expanded out in powers of ϵ as in Eqns. 1.25 - 1.27. $\exp \phi$ can be expanded out to $\exp \phi \approx 1 + \phi + \frac{1}{2}\phi^2 + \dots = 1 + (\epsilon\phi_1 + \epsilon^2\phi_2 + \dots) + \frac{1}{2}(\epsilon\phi_1 + \epsilon^2\phi_2 + \dots)^2$.

$$n = 1 + \epsilon n_1 + \epsilon^2 n_2 + \dots \quad (1.25)$$

$$v = \epsilon v_1 + \epsilon^2 v_2 + \dots \quad (1.26)$$

$$\phi = \epsilon \phi_1 + \epsilon^2 \phi_2 + \dots \quad (1.27)$$

$\mathcal{O}(\epsilon^{3/2})$:

$$-\frac{\partial n_1}{\partial x} + \frac{\partial v_1}{\partial x} = 0 \quad (1.28)$$

$$-\frac{\partial v_1}{\partial x} = -\frac{\partial \phi_1}{\partial x} \quad (1.29)$$

$\mathcal{O}(\epsilon)$:

$$0 = \phi_1 - n_1 \quad (1.30)$$

From these two expansions, we find that $\boxed{n_1 = \phi_1 = v_1}$.

$\mathcal{O}(\epsilon^{5/2})$:

$$\frac{\partial n_1}{\partial t} - \frac{\partial n_2}{\partial x} + \frac{\partial}{\partial x} (n_1 v_1) + \frac{\partial v_2}{\partial x} = 0 \quad (1.31)$$

$$\frac{\partial v_1}{\partial t} - \frac{\partial v_2}{\partial x} + v_1 \frac{\partial v_1}{\partial x} = -\frac{\partial \phi_2}{\partial x} \quad (1.32)$$

$\mathcal{O}(\epsilon^2)$:

$$\frac{\partial^2 \phi_1}{\partial x^2} = \phi_2 - n_2 + \frac{1}{2} \phi_1^2 \quad (1.33)$$

Differentiate Eqn. 1.33 and substitute into the sum of Eqns. 1.31 and 1.32. The lower order

terms are:

$$2\frac{\partial u_1}{\partial t} + \frac{\partial^3 u_1}{\partial x^3} + 2u_1\frac{\partial u_1}{\partial x} \quad (1.34)$$

Finally, divide by 2. We are left with the Korteweg – de Vries (KdV) equation. The KdV equation is a ubiquitous 1D wave equation that describes dispersive fluid systems with the effect of weak nonlinearity.

$$\frac{\partial u_1}{\partial t} + u_1\frac{\partial u_1}{\partial x} + \frac{\partial^3 u_1}{\partial x^3} = 0 \quad (1.35)$$

1.5.3 Ion acoustic soliton (Sagdeev pseudopotential approach)

Another approach to deriving the soliton solution that is more general can be constructed from the Poisson equation. The electron charge density will be given by the standard Boltzmann distribution, $n_e = n_0 \exp\left(\frac{e\phi}{k_B T_e}\right)$. The ions will assumed to be cold, i.e. their energy conservation equation will be given by: $\frac{1}{2}m_i v^2 - e\phi = E_i$. Assuming that the flux of ions ($\Gamma_i \equiv n_i v_i$) is constant, an equation for n_i can be solved for. Substituting these two assumptions of charge density into the Poisson equation and dividing through by constants yields Eqn. 1.36, where the nondimensional quantities are given by: $\psi = \frac{e\phi}{T_e}$, $\Gamma = \frac{Z\Gamma_i}{\sqrt{2n_{e,0}v_{ia}}}$, $\Gamma_i = n_i v_i$, $\lambda_D^2 = \frac{\epsilon_0 T_e}{e^2 n_{e,0}}$, $E = \frac{E_i}{ZT_e}$, $E_i = \frac{m_i}{2}v_i^2 - e\phi$, and $v_{ia}^2 = \frac{ZT_e}{m_i}$.

$$-\frac{\lambda_D^2}{2} \left(\frac{d}{dx} \psi \right)^2 = -2\Gamma \sqrt{E - \psi} - \exp \psi + c \quad (1.36)$$

Eqn. 1.36 can be rewritten as:

$$\frac{1}{2} \left(\frac{d}{dx} \psi \right)^2 + U(\psi) = 0 \quad (1.37)$$

Where

$$U(\psi) = -2\lambda_D^{-2}\Gamma\sqrt{E-\psi} - \lambda_D^{-2}\exp\psi + c \quad (1.38)$$

Eqn 1.37 has the form of an energy conservation equation for a particle in a potential well given by U . This potential well is given by Eqn. 1.38 (the “pseudopotential”). Periodic solutions in the potential well give spatially periodic solutions of the ion acoustic equation. In the limit of small perturbations near the minima, we shall show that this reduces to the linearized small magnitude solutions. Outside of the small limit, the full *nonlinear* solution can be obtained. This approach was first published by R. Sagdeev [84], so in literature this approach is often coined the *Sagdeev pseudopotential*.

For convenience let's adjust how the constant is introduced. If $U(0) = 0 = -\lambda_D^{-2}2\Gamma\sqrt{E} - \lambda_D^{-2} + c$, then $c = \lambda_D^{-2}2\Gamma\sqrt{E} + \lambda_D^{-2}$. The degree of freedom of the constant can be introduced by $c = \lambda_D^{-2}2\Gamma\sqrt{E} + \lambda_D^{-2} + H$, where H is another constant. The pseudopotential is now given as:

$$U(\psi) = 2\lambda_D^{-2}\Gamma \left(\sqrt{E} - \sqrt{E-\psi} \right) + \lambda_D^{-2} (1 - \exp\psi) - H \quad (1.39)$$

If $\tilde{U} = 2\lambda_D^{-2}\Gamma \left(\sqrt{E} - \sqrt{E-\psi} \right) + \lambda_D^{-2} (1 - \exp\psi)$ (i.e. the pseudopotential with a constant chosen so that $\tilde{U}(0) = 0$), then this can be written in a more suggestive form:

$$\frac{1}{2} \left(\frac{d\psi}{dx} \right)^2 + \tilde{U}(\psi, E, \Gamma) = H \quad (1.40)$$

We can identify H with a the “pseudoenergy” of this fictitious mechanical system. H will be conserved along a given solution $\psi(x)$. We can proceed by analyzing the shape of $\tilde{U}$ for different E and Γ without having to linearize the system. We shall assume that we are in the wave frame moving at speed V . In that case, the flux (Γ) is known: $\Gamma_i = n_{i,0}V = n_{e,0}V$. In that frame, $E_i = \frac{m_i}{2}V^2$. $E/\Gamma = \frac{1}{\sqrt{2}Z} \frac{V}{v_{ia}} = \mathcal{O}(V/v_{ia})$. V/v_{ia} shall be defined as the ***ion acoustic Mach number*** ($M_{ia} \triangleq V/v_{ia}$). The ratio of E to Γ corresponds to admissible wave solutions at a particular Mach number. Correspondingly this will change the qualitative behavior of the pseudo-potential.

Fig. 1.8 graphs the pseudopotential. The periodic solutions to Eqn. 1.36 correspond to possible oscillations (i.e. valleys) in the pseudo-potential. E and Γ are fixed for each pseudopotential curve. Γ is fixed while E increases. When $E/\Gamma > 1$, the local valley moves rightward. For more information, see a recent publication on interpreting these pseudopotentials [85].

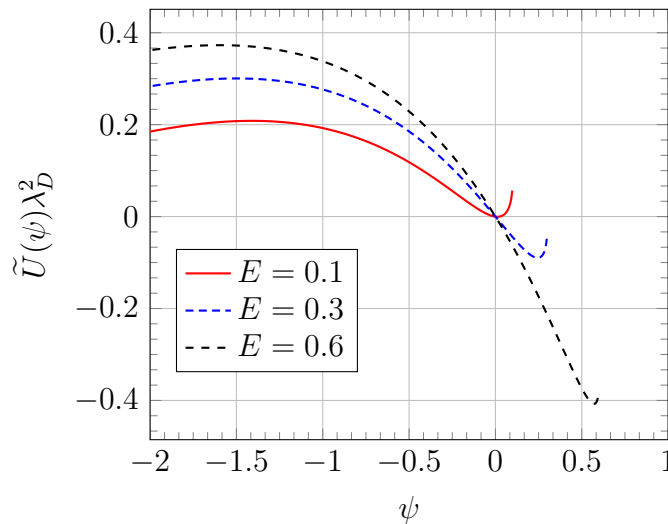


Figure 1.8: Sagdeev pseudopotential for cold ion system all with $\Gamma = 0.3$.

Using the same classical potential theory, contours in a pseudo-phase space ($d\psi/dx$ vs ψ) can be plotted because $d\psi/dx = \sqrt{-2U(\psi)}$. At each $(\psi, \frac{d\psi}{dx})$ location in the phase portrait, a vector can be associated with where a “pseudo-particle” (i.e. a particle in the pseudo-potential) would go. This vector will be $(u, v) = \left(\frac{d\psi}{dx}, -\frac{\partial U}{\partial \psi}\right)$ where $\frac{\partial U}{\partial \psi}$ is given by Eqn. 1.41.

$$\frac{\partial U}{\partial \psi} = \frac{\lambda_D^{-2} \Gamma}{\sqrt{E - \psi}} - \lambda_D^{-2} \exp \psi \quad (1.41)$$

In a phase portrait (e.g. Fig. 1.9), it is clear that a particle that starts on $(\psi, \frac{d\psi}{dx}) = (0, 0)$ will have a periodic solution that is completely positive. For a lower value of E (corresponding to the red curve in Fig. 1.8), this orbit would be a positive and negative ψ .

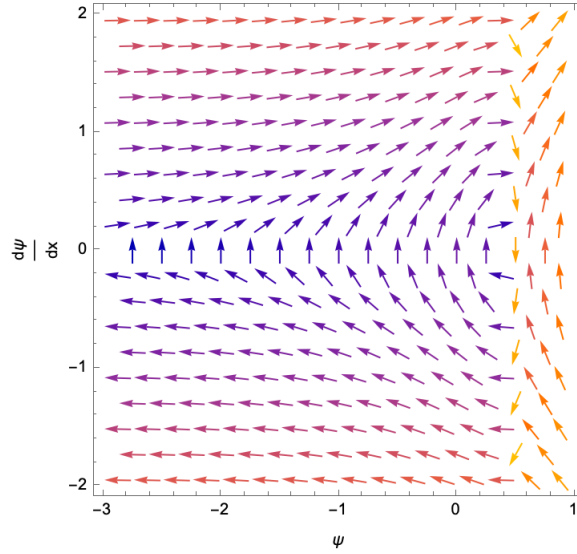


Figure 1.9: Phase diagram of Sagdeev pseudopotential. The quivers are the tangent vectors along trajectories of the pseudoparticle. This is for that case of $\Gamma = 0.3$ and $E = 0.6$.

Soliton Solutions

Now we shall outline how to go from the pseudopotential to soliton solutions of the cold ion equations¹⁹. In the paradigm of the pseudopotential, solitary solutions are solutions where $d\psi/dx \rightarrow 0$ and $\psi = 0$. We expand $\tilde{U}$ out to third order and find that for $Z = 1$ in Eqn. 1.42. $M_{ia} \triangleq V/v_{ia}$ is the ion acoustic Mach number of the background flow.

$$\lambda_D^2 \tilde{U}(\psi) = \frac{1}{2} (1 - M_{ia}^{-2}) \psi^2 + \frac{1}{2} \left(\frac{1}{3} - M_{ia}^{-4} \right) \psi^3 + \mathcal{O}(\psi^4) \quad (1.42)$$

The zeroth and first order terms in the Taylor expansion are zero. Eqn. 1.36 can now be solved analytically. $\frac{1}{3} - M_{ia}^{-4} < 0$ for $1 \leq M_{ia} \leq 1.316$. Since we are interested in solutions near Mach 1, we shall rewrite that term so it is positive.

$$\int_{x_0}^x x' dx' = \int_0^\psi \frac{\lambda_D d\psi'}{\sqrt{(1 - M_{ia}^{-2}) (\psi')^2 - (M_{ia}^{-4} - \frac{1}{3}) (\psi')^3}} \quad (1.43)$$

The simplest way to solve Eqn. 1.43 is through a substitution with hyperbolic trig functions.

Rearranging:

$$x - x_0 = \int_0^\psi \frac{\lambda_D d\psi'}{(1 - M_{ia}^{-2})^{1/2} \psi' \sqrt{1 - \left(\frac{M_{ia}^{-4} - \frac{1}{3}}{1 - M_{ia}^{-2}} \right) \psi'}} \quad (1.44)$$

This trick hinges on the following properties of hyperbolic trig functions: $1 - \text{sech}^2 x =$

¹⁹While it is not shown in the work, this is how one goes from Eqn. 38 – Eqn. 40 in [84]. Sagdeev truncates the coefficient of the third order term from $1 - 3M_{ia}^{-4} \rightarrow 1$.

$\tanh^2 x$ and $\frac{d}{dx}(\text{sech}^2 x) = -2 \text{sech}^2 x \tanh x$. Therefore, we will set:

$$\text{sech}^2 w = \left(\frac{M_{ia}^{-4} - \frac{1}{3}}{1 - M_{ia}^{-2}} \right) \psi'$$

$$-2dw \text{sech}^2 w \tanh w = \left(\frac{M_{ia}^{-4} - \frac{1}{3}}{1 - M_{ia}^{-2}} \right) d\psi'$$

Substituting these in:

$$x - x_0 = - \int_{w(\psi'=0)}^{w(\psi'=\psi)} \frac{\lambda_D 2 \text{sech}^2 w \tanh w}{(1 - M_{ia}^{-2})^{1/2} \text{sech}^2 w \sqrt{1 - \text{sech}^2 w}} dw$$

where $w(\psi' = 0)$ and $w(\psi' = \psi)$ are the adjusted boundaries of the integral in w space.

The numerator and denominator largely cancel, leaving us with:

$$x - x_0 = \frac{2\lambda_D}{(1 - M_{ia}^{-2})^{1/2}} \int_{w(\psi'=0)}^{w(\psi'=\psi)} dw \quad (1.45)$$

Eqn. 1.45 is trivial. The integral is just w evaluated at the boundary in w space. The exact meaning of $w(0)$ is ambiguous since $\text{sech}^2 x \rightarrow 0$ at $+\infty$ and $-\infty$. We shall say that $w(\psi' = 0) = 0$ because the lower bound of the integral was arbitrary in the first place. The upper bound comes from the definition of the variable w .

$$w(\psi' = \psi) = \text{sech}^{-2} \left[\left(\frac{M_{ia}^{-4} - \frac{1}{3}}{1 - M_{ia}^{-2}} \right) \psi \right]$$

Eqn. 1.45 becomes:

$$-\frac{1}{2}(1 - M_{ia}^{-2})^{1/2} \frac{x - x_0}{\lambda_D} = \text{sech}^{-2} \left[\left(\frac{M_{ia}^{-4} - \frac{1}{3}}{1 - M_{ia}^{-2}} \right) \psi \right] \quad (1.46)$$

Finally, we arrive at a soliton solution. sech^2 is an even function, so the minus sign in the argument can go away. Writing in terms of the ϕ , the physical electrostatic potential:

$$\phi(x) = \frac{T_e}{e} \left(\frac{1 - M_{ia}^{-2}}{M_{ia}^{-4} - \frac{1}{3}} \right) \text{sech}^2 \left[\frac{1}{2} (1 - M_{ia}^{-2})^{1/2} \left(\frac{x - x_0}{\lambda_D} \right) \right] \quad (1.47)$$

This is the same solution as in [84]. If one further Taylor expands around $M_{ia} = 1$ for a small parameter $\delta M = M_{ia} - 1$, let $x_0 = 0$, and transforms the spatial coordinate back to the lab frame, then one arrives at Eqn. 1.48.

$$\boxed{\phi(x) = \frac{3T_e}{e} \delta M \text{sech}^2 \left[\sqrt{\frac{\delta M}{2}} \left(\frac{x - Vt}{\lambda_D} \right) \right]} \quad (1.48)$$

Eqn. 1.48 is the *solitary wave solution to the Sagdeev pseudo-potential for cold ions*. The speed shows up as the controlling factor for the amplitude [$\mathcal{O}(\delta M)$] and the width [$\mathcal{O}(\delta M^{-1/2})$]. *Faster solitons are taller and skinnier*. The opposite is also true. In literature, experimentalists will multiply the observed amplitude of a soliton with the square of its width. This quantity is known as the *soliton parameter*. Theoretically, it should be $\mathcal{O}(\delta M \delta (M^{-1/2})^2) = \mathcal{O}(1)$ for all speed solitons. In [9, 86, 87], this property has been experimentally verified to be a constant. Figs. 4.19b and 4.18 show the observed soliton parameters in the 1D multifluid simulation that we conducted with an uncertainty bound derived from estimating the uncertainty of the width. The uncertainty for the width is so high because often the solitons overlap in their tails making it hard to judge where one stops and the other begins. The background flow is set by the boundary

conditions. A Gaussian electrostatic potential is imposed in the middle of the domain to represent the plasma sheath from the debris pushing on the flow. As discussed during the predefense, solitons either propagate ahead (precursor) or move at the same speed as the debris (pinned). There are three subplots because the amplitude and width of the soliton can be measured as either a density wave in ions and electrons or as an electrostatic potential wave as is presented in Eqn. 1.48.

Korteweg – de Vries equation soliton

Eqn. 1.48 is also the one-soliton solution that can be derived from the KdV equation that describes ion acoustic waves. Without deriving the KdV equation from the fluid equations (see [82, 83] for example), we shall state that the KdV equation for nonlinear ion acoustic waves is given by Eqn. 1.49. ψ is the same nondimensionalized variable as previously defined. This equation is in a frame moving at $V = v_{ia}$.

$$\frac{\partial \psi}{\partial t} + \psi \frac{\partial \psi}{\partial x} + \frac{1}{2} \frac{\partial^3 \psi}{\partial x^3} = 0 \quad (1.49)$$

Looking for solitary solutions to Eqn. in 1.49 in the frame $x - \delta M t$ where $d/dt = 0$ in that frame, we can transform this PDE into an ODE given by Eqn. 1.50

$$-\delta M \frac{\partial \psi}{\partial x} + \psi \frac{\partial \psi}{\partial x} + \frac{1}{2} \frac{\partial^3 \psi}{\partial x^3} = 0 \quad (1.50)$$

Using successive methods of quadrature and integrating, we arrive at:

$$\frac{1}{2} \left(\frac{\partial \psi}{\partial x} \right)^2 = \delta M \psi^2 - \frac{1}{3} \psi^3 \quad (1.51)$$

Notice how the right hand side of Eqn. 1.51 looks exactly like the pseudo-particle in a pseudo-potential, but with a potential that is already very simple. This is because in deriving the KdV equation from the plasma fluid equation, the concept of being in a small limit has already been incorporated into the equations. The only nonlinearity that is allowed to remain is the same nonlinearity that is in the second and third order of the Taylor series of the Sagdeev pseudopotential. We will use the same hyperbolic trig substitution as before. Let

$$\text{sech}^2 x = \frac{1}{3\delta M} \psi$$

Thus

$$-2dx \text{sech}^2 x \tanh x = \frac{1}{3\delta M} d\psi$$

$$x - x_0 = \int_0^\psi \frac{d\psi'}{\sqrt{2\delta M} \psi' \sqrt{1 - \frac{1}{3\delta M} \psi'}} = \int_0^\psi \frac{-2 \text{sech}^2 x \tanh x}{\sqrt{2\delta M} \text{sech}^2 x \sqrt{1 - \text{sech}^2 x}} dx = \int_{x(0)}^{x(\psi)} \frac{-\sqrt{2}}{\sqrt{\delta M}} dx$$

Finally:

$$\psi = 3\delta M \text{sech}^2 \left(\sqrt{\frac{\delta M}{2}} x \right) \quad (1.52)$$

The KdV equation is already in nondimensionalized units, so x in this context is already

in units of Debye length. This solution is in a frame moving at speed $v_{ia} + \delta M v_{ia}$. Under these circumstances, Eqn. 1.52 and 1.48 are identical.

Interpretation

As it has been shown, the “classical” soliton result can either be derived from taking the small amplitude solitary solutions of the cold ion Sagdeev pseudo-potential or as the solitary solution of the Korteweg – de Vries equation. This is a one-parameter solution, i.e. for a fixed amplitude soliton, there is only one solitary solution that matches that.

Chapter 2: Literature Review

Now that the concepts of the space plasma, space debris, and nonlinear waves have been established, the current understanding of nonlinear waves generated by space debris moving through space plasma shall be discussed. This largely summarizes the work in Refs. [1, 9, 10, 88, 89]. This forms the basis for the new work discussed in Chapters 3 - 5 of this dissertation.

2.1 Predicted Soliton Generation

2.1.1 Derivation of fKdV Equation

Sen et al (2015) [89] introduced a source term into the KdV equation to represent the debris. The result will be the forced KdV (fKdV) equation. Let us return to the multifluid description of a plasma with Boltzmann electrons, however this time there is allowed to be a source of free charge denoted by S .

$$\frac{\partial \rho_i}{\partial t} = \nabla \cdot (\rho_i u_i) \quad (2.1)$$

$$\frac{\partial (\rho_i u_i)}{\partial t} + \nabla \cdot (\rho_i u_i^2) = -\rho_i \left(\frac{q_i}{m_i} \right) \frac{\partial \phi}{\partial x} \quad (2.2)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{q_e}{\epsilon_0} \left(\frac{Z}{m_i} \rho_i - \frac{\rho_e}{m_e} \exp \left[\frac{q_e \phi}{k_B T_e} \right] \right) = S \quad (2.3)$$

This restricted system can be further reduced into the forced Korteweg – de Vries (fKdV - Eqn. 2.27) in the weakly nonlinear limit around by expanding around a small parameter $\varepsilon = (v_{\text{orbit}} - v_{co})/v_{ia} - 1 \equiv M_{IA} - 1$ where τ and ξ are stretched time and space coordinates, respectively. Throughout the rest of this study, M will be used as shorthand for M_{IA} . $\xi = \varepsilon^{1/2} \left(x - \sqrt{\frac{k_B T_e}{m_i}} t \right)$ and $\tau = \varepsilon^{3/2} t$. These are chosen to balance dispersion $\mathcal{O}(k^3)$ and nonlinearity $\mathcal{O}(uk)$ in the frame moving at the ion acoustic speed [83]. k is the wavenumber. In this solution, nonlinearity, dispersion, and speed difference from ion acoustic Mach 1 are all assumed to be of $\mathcal{O}(\varepsilon)$. This is termed “weak nonlinearity.” The fluid quantities are expanded as in Eqns. 2.4 - 2.9.

$$\rho_i = \rho_{i,0} + \varepsilon \rho_{i,1} + \varepsilon^2 \rho_{i,2} + \mathcal{O}(\varepsilon^3) \quad (2.4)$$

$$\rho_e = \rho_{e,0} + \varepsilon \rho_{e,1} + \varepsilon^2 \rho_{e,2} + \mathcal{O}(\varepsilon^3) \quad (2.5)$$

$$u_i = 0 + \varepsilon u_{i,1} + \varepsilon^2 u_{i,2} + \mathcal{O}(\varepsilon^3) \quad (2.6)$$

$$u_e = 0 + \varepsilon u_{e,1} + \varepsilon^2 u_{e,2} + \mathcal{O}(\varepsilon^3) \quad (2.7)$$

$$\phi = 0 + \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \mathcal{O}(\varepsilon^3) \quad (2.8)$$

$$S = \varepsilon^2 S_2 \quad (2.9)$$

First order: Expanding Eqns. 2.4 - 2.9 our to first order in ε , one finds Eqns. 2.10 - 2.12.

$$\frac{\partial}{\partial t} \left(\frac{\rho_{i,1}}{\rho_{i,0}} \right) = \frac{\partial u_{i,1}}{\partial x} \quad (2.10)$$

$$\frac{\partial}{\partial t} u_{i,1} = - \left(\frac{q_i}{m_i} \right) \frac{\partial \phi_1}{\partial x} \quad (2.11)$$

$$0 = \frac{\epsilon_0 m_e}{q_e Z \rho_{e,0}} \frac{\partial^2 \phi_1}{\partial x^2} + \frac{m_e \rho_{i,1}}{m_i \rho_{e,0}} - \frac{q_e}{k_B T_e Z} \phi_1 \quad (2.12)$$

Let us move to moving reference frame with speed v_f that moves at the wave speed. In this frame, let us assume that time evolution happens slowly in the same ordering as the small quantity, i.e. $\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t_2} - v_f \frac{\partial}{\partial x} = \varepsilon \frac{\partial}{\partial t} - v_f \frac{\partial}{\partial x}$. Substituting this assumption about the time derivative into Eqns. 2.4 - 2.9 yields Eqns. 2.13 - 2.15. Due to the lack of time derivatives in the Poisson equation, 2.12 is identical to 2.15.

$$\frac{\partial}{\partial x} \left(\frac{\rho_{i,1}}{\rho_{i,0}} \right) = \frac{\partial}{\partial x} \left(\frac{u_{i,0}}{v_f} \right) \quad (2.13)$$

$$v_f \frac{\partial}{\partial x} u_{i,1} = \left(\frac{q_i}{m_i} \right) \frac{\partial \phi_1}{\partial x} \quad (2.14)$$

$$0 = \frac{\epsilon_0 m_e}{q_e Z \rho_{e,0}} \frac{\partial^2 \phi_1}{\partial x^2} + \frac{m_e \rho_{i,1}}{m_i \rho_{e,0}} - \frac{q_e}{k_B T_e Z} \phi_1 \quad (2.15)$$

Now nondimensional quantities shall be introduced to simplify Eqns. 2.13 - 2.15. Let $\tilde{n} = \frac{\rho_{i,1}}{\rho_{i,0}} = \frac{m_e \rho_{i,1}}{m_i \rho_{e,0}}$, $\tilde{\phi} = \frac{q_e \phi_1}{k_B T_e Z}$, $\xi^2 = x^2 \frac{\rho_{i,0} q_e^2}{\epsilon_0 k_B T_e} \equiv (x/\lambda_D)^2$, $\tilde{u} = \frac{u_{i,1}}{v_f}$. Since the frame speed is the ion acoustic speed, $v_f = v_{IA}$, some terms go to unity.

$$\frac{\partial \tilde{n}}{\partial \xi} = \frac{\partial \tilde{u}}{\partial \xi} \quad (2.16)$$

$$\frac{\partial \tilde{u}}{\partial \xi} = \frac{\partial \tilde{\phi}}{\partial \xi} \quad (2.17)$$

$$0 = \frac{\partial^2 \tilde{\phi}}{\partial \xi^2} + \tilde{n} - \tilde{\phi} \quad (2.18)$$

From Eqns. 2.16 and 2.17 one can argue that the density, velocity, and electrostatic potential perturbations are all equal in nondimensional units, i.e. $\tilde{n} = \tilde{u} = \tilde{\phi}$.

Second order: Now expanding Eqns. 2.4 - 2.9 our to second order in ε and changing the time derivative, one finds Eqns. 2.19 - 2.21.

$$\frac{\partial}{\partial t_2} \rho_{i,1} - v_f \frac{\partial}{\partial x} \rho_{i,2} = \frac{\partial}{\partial x} (\rho_{i,1} u_{i,1}) + \rho_{i,0} \frac{\partial}{\partial x} u_{i,2} \quad (2.19)$$

$$\rho_{i,0} \frac{\partial}{\partial t_2} u_{i,1} - v_f \frac{\partial}{\partial x} (\rho_{i,0} u_{i,2}) - v_f \frac{\partial}{\partial x} (\rho_{i,1} u_{i,1}) = -\rho_{i,1} \left(\frac{q_i}{m_i} \right) \frac{\partial \phi_1}{\partial x} - \rho_{i,0} \left(\frac{q_i}{m_i} \right) \frac{\partial \phi_2}{\partial x} \quad (2.20)$$

$$\frac{\partial^2 \phi_1}{\partial x^2} + \frac{q_e Z}{\epsilon_0 m_i} \rho_{i,2} - \frac{q_e^2 \rho_{e,0}}{\epsilon_0 m_e k_B T_e} \phi_2 = S_2. \quad (2.21)$$

Now the equations can be nondimensionalized. The nondimensional t_2 is τ . The nondimensional second order quantities will be denoted with a 2 subscript.

$$\frac{\partial \tilde{n}}{\partial \tau} - \frac{\partial \tilde{n}_2}{\partial \xi} = \frac{\partial}{\partial \xi} (\tilde{n} \tilde{u}) + \frac{\partial \tilde{u}_2}{\partial \xi} \quad (2.22)$$

$$\frac{\partial \tilde{u}}{\partial \tau} - \frac{\partial \tilde{u}_2}{\partial \xi} = -\frac{\partial}{\partial \xi} (\tilde{n} \tilde{u}) - \tilde{n} \frac{\partial \tilde{\phi}}{\partial \xi} - \frac{\partial \tilde{\phi}_2}{\partial \xi} \quad (2.23)$$

$$\frac{\partial^2 \tilde{\phi}}{\partial \xi^2} + \tilde{n}_2 - \tilde{\phi}_2 = \tilde{S}_2 \quad (2.24)$$

Taking the derivative of Eqn. 2.24 and substituting it into the addition of Eqns. 2.22 and 2.23 finally yields the forced KdV equation.

$$\frac{\partial \tilde{n}}{\partial \tau} + \tilde{n} \frac{\partial \tilde{n}}{\partial \xi} + \frac{1}{2} \frac{\partial^3 \tilde{n}}{\partial \xi^3} = \frac{1}{2} \frac{\partial \tilde{S}_2}{\partial \xi} \quad (2.25)$$

As just shown in the derivation, the density perturbations, bulk velocity, and electrostatic potential are coupled, i.e. they are all equal to a nondimensional quantity U defined by Eqn. 2.26. Some studies incorporate the third order solutions that give rise to extra terms in the KdV equation. Sometimes these solutions are called “dressed solitons.” Even in this situation, U is still equal to these four quantities [90].

$$U = \frac{(u - v_{IA})}{v_{IA}} = \frac{n_e - n_0}{n_0} = \frac{n_i - n_0}{n_0} = \frac{e\phi}{k_B T_e} \quad (2.26)$$

Substituting U into Eqn. 2.25, the full fKdV equation is found (Eqn. 2.27). In Eqn. 2.27, the coefficients are now general. For the ion acoustic fluid equations, $\alpha = 1$ and $\beta = \frac{1}{2}$. When a κ distribution is used that is derived from spacecraft measurements, these coefficients change in

the third decimal place [1]. Since this is a small modification, it is neglected for this study.

$$\frac{\partial U}{\partial \tau} + \alpha U \frac{\partial U}{\partial \xi} + \beta \frac{\partial^3 U}{\partial \xi^3} = \frac{1}{2} \frac{\partial S}{\partial \xi} \quad (2.27)$$

2.1.2 Derivation of Velocity Bounds

Shen (1993) [91] and *Grimshaw et al (1986) [92]* provided analytical formulae for the generation regimes of precursor and pinned solitons. Precursor solitons are created for $M_{sup} > M > M_{sub}$. If $M > M_{sup}$, pinned solitons are created [91]. A is the spatial integral of the forcing function. For the Gaussian source of width G and magnitude Φ used in this study, $A = \Phi G \sqrt{\pi}$.

$$M_{sub} = 1 - \left(\frac{3 (\alpha/2)^2 A^2}{4\beta} \right)^{1/3} \quad (2.28)$$

$$M_{sup} = 1 + \left(\frac{3 (\alpha/2)^2 A^2}{16\beta} \right)^{1/3} \quad (2.29)$$

A derivation of Eqns. 4.1 and 4.2 will now be shown. Consider the stationary solutions to Eqn. 2.27. In that case, the time derivative may be neglected. Integrating once and transforming to the frame moving δM yields Eqn. 2.30. This is reminiscent of an elliptic integral with the extra inclusion of $S(\xi)$. C is a constant of integration.

$$-\delta M U + \frac{\alpha}{2} U^2 + \beta \frac{\partial^2 U}{\partial \xi^2} = \frac{1}{2} S + C \quad (2.30)$$

Let $U^\pm$ be the values of U as $U \rightarrow \pm\infty$. $S(\xi) \rightarrow 0$ as $\xi \rightarrow \infty$. In the case that $U^\pm \neq 0$, it can be shown that $\delta M = \frac{\alpha}{2}(U^- + U^+)$. The constant $C = -\frac{\alpha}{2}U^-U^+$. Suppose that S is given

by $S = S_0\delta(\xi)$ where δ is a Dirac delta function. The solution on each side of the δ function is given by a sech^2 function $\left[U^\pm = \frac{3\delta M}{2\alpha} \text{sech}^2\left(\sqrt{\frac{\delta M}{4\beta}}(\xi - L^\pm)\right)\right]^1$. The δ function source leads to a condition across the source:

$$\frac{\partial U}{\partial \xi}\Big|_{0^+} - \frac{\partial U}{\partial \xi}\Big|_{0^-} = \frac{S_0}{2\beta} \quad (2.31)$$

Now consider localized forcing, i.e. forms of $S(\xi)$ that go $\rightarrow 0$ at $\pm\infty$ but are not δ functions with $\delta M > 0$. Let l be the length scale of the source. Ref. [92] showed that in the limit as $l \rightarrow 0$, then the δ source solution matches the solution with non-delta S if one sets $S_0 = \int_{-\infty}^{\infty} S(\xi)d\xi$. Let's refer to this integrated quantity as A . There are now two equations (the constant and the delta source jump condition) to solve for the boundary conditions as a function of the source properties. Combining these equation yields the following condition:

$$-\sqrt{\frac{\delta M}{\beta}}U(0) \left[-\tanh\left(\sqrt{\frac{\delta M}{4\beta}}L^+\right) + \tanh\left(\sqrt{\frac{\delta M}{4\beta}}L^-\right) \right] = \frac{A}{2\beta} \quad (2.32)$$

If $A > 0$, then $L^+ \neq L^-$. From continuity across the delta source, $L^+ = -L^-$. Eqn. 2.32 can be written as a polynomial in terms of $f = \tanh\left(\sqrt{\frac{\delta M}{4\beta}}L^+\right)$:

$$f^3 - f - \frac{A\alpha}{3\delta M} \frac{1}{\sqrt{\beta\delta M}} = 0 \quad (2.33)$$

The roots of this polynomial determine the type of solution. The critical value of δM that gives Eqn. 2.33 a double real root is $\delta M_{\text{crit}} = \left(\frac{3(\alpha/2)^2 A^2}{16\beta}\right)^{1/3}$.

If $\delta M < 0$, this same problem has a different solution through a related polynomial. This

¹See the solutions to the homogeneous KdV equation in Chapter 1.

critical value of $\delta M_{\text{crit}} = \left(\frac{3(\alpha/2)^2 A^2}{4\beta} \right)^{1/3}$. Recall that $\delta M = M - 1$. Substituting these two critical δM values yields Eqns. 4.1 - 4.2.

2.1.3 Correspondence of Velocity Bounds to Debris Orbits

Truitt and Hartzell (2020) performed analysis to link these nondimensionalized quantities to orbits. Given the bounds below, what debris orbits create precursors?

$$1 - \left(\frac{3A^2}{8} \right)^{1/3} < \frac{v_d - v_{\text{plasma}}}{v_{IA}} < 1 + \left(\frac{3A^2}{32} \right)^{1/3} \quad (2.34)$$

2.1.3.1 What is the “average” speed of an orbit?

From the vis-viva equation, one can predict the debris velocity will be given by $v_d = \sqrt{2\mu(r^{-1} - (2a)^{-1})} = \sqrt{\frac{\mu}{a} \left((1 + e \cos \nu) \left(\frac{2}{1-e^2} \right) - 1 \right)}$ where μ is the gravitational parameter of the Earth, r is the current radial coordinate of the debris from the Earth’s center, e is the eccentricity, ν is the true anomaly, and a is the semimajor axis of the orbit. The true anomaly goes from $\nu \in [0, 2\pi)$. For circular orbits (i.e. as $e \rightarrow 0$), the circular orbital speed $v_d = \sqrt{\mu/a}$ is recovered. In Ref. [1] and most studies, an assumption is made that the circular orbital speed = the average orbital speed = a characteristic speed of the orbit. We will quickly go on a diversion to more rigorously establish this fact. In doing so, a less common mathematical object, the *elliptic integral* will be introduced. This (in a different context) plays an important role in Chapter 3 of this dissertation. Hopefully this will help the reader gain an intuition for what the elliptic integral is.

In principle, one could spatially averaging over an orbit in order to establish the “average”

orbital speed². This can be done by integrating the vis-viva velocity from $\nu = 0$ to $\nu = 2\pi$:

$\hat{v}_d = \frac{1}{2\pi} \int_0^{2\pi} v_d(\nu) d\nu$ where $\hat{v}_d$ is the spatially averaged orbital velocity. Substituting this in and rearranging it can be shown that:

$$\hat{v}_d = \frac{1}{2\pi} \int_0^{2\pi} v_d(\nu) d\nu \quad (2.35)$$

$$= \frac{1}{2\pi} \sqrt{\frac{\mu}{a}} \int_0^{2\pi} \sqrt{(1 + e \cos \nu) \left(\frac{2}{1 - e^2} \right) - 1} d\nu \quad (2.36)$$

The integral $\int \sqrt{\cos x + cx} \, dx$ ($c \neq 1$) is not expressible in elementary functions. However, it can be put in a standard form where it is well characterized. This standard form is called the (incomplete) *elliptic integral* (of the second kind). It is defined in Eqn. 2.37.

$$E(\phi, k) = \int_0^\phi \sqrt{1 - k^2 \sin^2 \theta} d\theta \quad (2.37)$$

Eqn. 2.36 can be put in the standard form of Eqn. 2.37 by using half-angle identities ($2 \cos^2 x/2 = 1 + \cos x$) and the Pythagorean identity ($\sin^2 x + \cos^2 x = 1$). Performing these on Eqn. 2.36 yields:

²Typically averages in orbital dynamics are done by averaging over one period (i.e. integrating in time) not over true anomaly. This is a constructed example to explain the elliptic integral.

$$\hat{v}_d = \frac{1}{2\pi} \sqrt{\frac{\mu}{a}} \int_0^{2\pi} \sqrt{\frac{2e}{1-e^2} \cos \nu + \left[\frac{2}{1-e^2} - 1 \right]} d\nu \quad (2.38)$$

$$= \frac{1}{2\pi} \sqrt{\frac{\mu}{a}} \int_0^{2\pi} \sqrt{\frac{4e}{1-e^2} \cos^2 \left(\frac{\nu}{2} \right) + \left[\frac{2}{1-e^2} - \frac{2e}{1-e^2} - 1 \right]} d\nu \quad (2.39)$$

$$= \frac{1}{2\pi} \sqrt{\frac{\mu}{a}} \int_0^{2\pi} \sqrt{\frac{(1+e)^2}{1-e^2} - \frac{4e}{1-e^2} \sin^2 \left(\frac{\nu}{2} \right)} d\nu \quad (2.40)$$

$$= \frac{1}{\pi} \sqrt{\frac{\mu}{a}} \frac{1+e}{\sqrt{1-e^2}} \int_0^\pi \sqrt{1 - \frac{4e}{(1+e)^2} \sin^2 \phi} d\phi \quad (2.41)$$

$$= \frac{1}{\pi} \sqrt{\frac{\mu}{a}} \frac{1+e}{\sqrt{1-e^2}} E \left(\pi, \frac{2\sqrt{e}}{1+e} \right) \quad (2.42)$$

It is common to define the **elliptic modulus** (m) as $m = k^2$ (where k is the k in Eqn. 2.37). Furthermore, the elliptic integral from $[0, \pi/2]$ is the same as from $[\pi/2, \pi]$. Thus there domain of integration can be reduced. When the integral bounds go from 0 to $\pi/2$ the elliptic integral is said to be complete. The complete elliptic integral (of the second kind) is defined by Eqn. 2.43. In this notation, a single argument elliptic function takes m as input. A double argument takes k . Sometimes different notations are used.

$$E(m) = \int_0^{\pi/2} \sqrt{1 - m \sin^2 \theta} d\theta = 2 \int_0^\pi \sqrt{1 - m \sin^2 \theta} d\theta \quad (2.43)$$

Using this notation, Eqn. 2.42 can be further simplified to Eqn. 2.44. Note that $E(0) = \frac{\pi}{2}$. Therefore, as expected, when $e \rightarrow 0$, the circular velocity is recovered. Fig. 2.1 plots Eqn. 2.44 normalized by the circular orbital velocity ($v_{circ} = \sqrt{\frac{\mu}{a}}$). As the eccentricity of the orbit increases, the average velocity increases. For further intuition, Eqn. 2.44 has a simple Taylor series expansion: $\frac{\hat{v}_d}{v_{circ}} = 1 + \frac{3}{4}e^2 + \frac{33}{64}e^4 + \mathcal{O}(e^6)$.

$$\hat{v}_d = \frac{2}{\pi} \sqrt{\frac{\mu}{a}} \frac{1+e}{\sqrt{1-e^2}} E\left(\frac{4e}{(1+e)^2}\right) \quad (2.44)$$

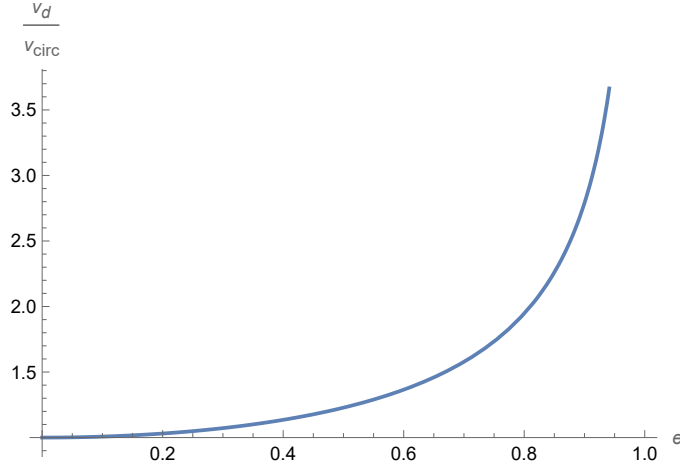


Figure 2.1: True anomaly averaged orbital velocity as a function of eccentricity.

However, at $e = 1$ the change of the true anomaly averaged orbit from the mean circular orbit (using the semimajor axis) is only 3.7. For order of magnitude analysis, this error is permissible. Therefore the assumption of just using the circular orbit speed is justified.

Returning to the main thread of this derivation, let's assume that the debris speed is the circular speed $v_d = \sqrt{\mu/a}$. The plasma speed in Eqn. 2.34 will be given by the corotation velocity of the atmosphere. This corotation velocity is given by the following where $t_{sd} = 86164$ (the number of seconds in a sidereal day) and θ_{lat} is the geographic latitude.

$$v_{co} = \frac{2\pi r \cos \theta_{lat}}{t_{sd}} \quad (2.45)$$

Substituting the circular debris velocity and Eqn. 2.45 into Eqn. 2.34 yields a physical criterion for precursor soliton generation.

$$1 - \left(\frac{3A^2}{8}\right)^{1/3} < \frac{\sqrt{\frac{\mu}{a}} - \frac{2\pi r \cos \theta_{lat}}{t_{sd}}}{\sqrt{\frac{k_B T_e}{m_i}}} < 1 + \left(\frac{3A^2}{32}\right)^{1/3} \quad (2.46)$$

The task remaining is to quantify A . For $S = \Phi_{p,n} \exp(-(\xi/G)^2)$, $A = \Phi_{p,n} G \sqrt{\pi}$. Let the debris have characteristic size $G\lambda_D$. Therefore, G will be the nondimensionalized characteristic size of the debris. $\Phi_{p,n} = \phi_C \exp(-1)$ is the magnitude of the plasma sheath evaluated at one Debye length nondimensionalized by the electron temperature. The charging model of an idealized conducting sphere [93] is utilized to set the magnitude of the applied potential. The charging model is given by an implicit equation (Eqn. 4.28) in dimensionalized units. Φ_s is the surface potential. ϕ_C is set to the nondimensionalized sheath potential at one Debye length, i.e. $\phi_C = \exp(-1) \frac{e\Phi_s}{k_B T_e}$.

$$\exp\left[\frac{e\Phi_s}{k_B T_e}\right] = \sqrt{m_e/m_i} \left(\frac{G+1}{G}\right)^2 \left[1 - \frac{(G+1)^2 - G^2}{(G+1)^2} \exp\left(\frac{G^2}{(G+1)^2 - G^2} \frac{e\Phi_s}{k_B T_e}\right)\right] \quad (2.47)$$

Using these models of velocity bounds, surface potential, and orbital speed Ref. [1] used models of the ionospheric composition to estimate where precursor solitons would be created. Figs. 2.2 and 2.4 illustrate where precursor solitons are expected to be created. The precursor soliton generation region forms a V-shape where the lowest altitude is centered on the current solar longitude. During the day-night cycle this V moves with the sun. During the time shown in the figures, the lowest altitude is ~ 1000 km. As the size of the debris decreases, A decreases. Thus the physical size of the precursor generation region also decreases. The opposite is true as well. Larger debris will have a larger V-shape.

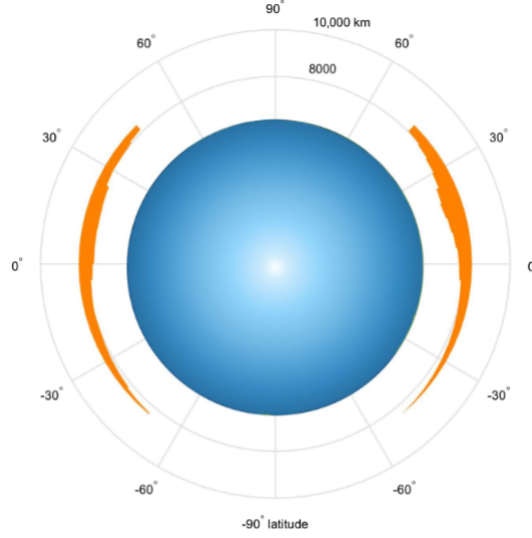


Figure 2.2: Precursor soliton generation region for 5 mm radius debris (in orange) around the Earth during winter. The sun is shining into the page. From Ref. [1].

2000 km altitude forms the upper boundary of where ionospheric models stop. However, in Fig. 2.4 it appears that the precursor generation regime extends upwards from this boundary. The next higher region is considered middle Earth orbit (MEO). The plasma populations there are well less characterized than in LEO. Assuming a constant temperature of $T_e = T_i = 1eV$, a H^+ plasma, and a linearly decreasing plasma density from $10^{10} - 10^8 m^{-3}$ (which is characteristic of the plasmasphere), Ref. [1] found precursor generation regime in Fig. 2.4. The eccentricity is allowed to be non-circular. If at some point along the orbit the situation in Eqn. 2.46 arises, then the orbit is said to produce solitons. Above 4000 km, only eccentric orbits can create solitons.

2.2 Laboratory Experiments

Ion acoustic solitons from flow greater than $M = 1$ have been created in laboratory environments. Jaiswal *et al* (2016) [9] first created precursor solitons in a dusty plasma. Fig. 2.5 shows one set of results from this experiments. A plasma seeded with dust (“Dust Flow” in the

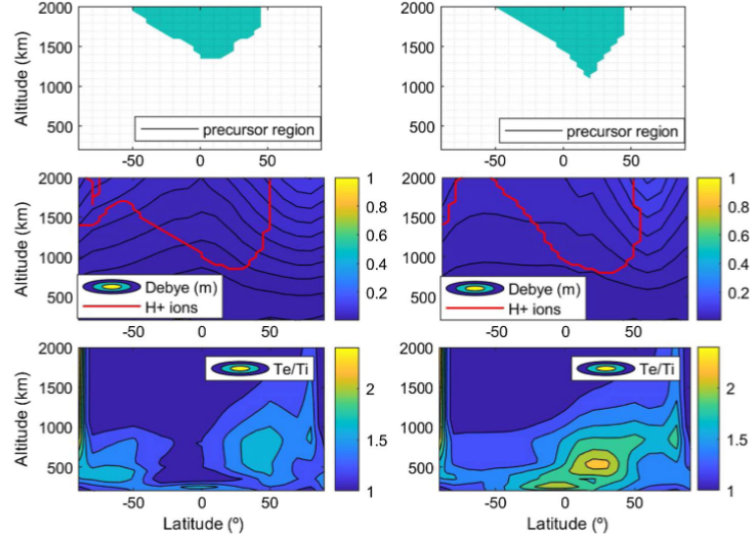


Figure 2.3: Precursor soliton generation region and plasma parameters for 5 mm radius debris (in orange) around the Earth during winter at 06:00 UTC. From Ref. [1].

figure) is flowed from right to left where it encounters a wire than can have an arbitrary potential. This experiment demonstrated for the first time that (a) precursor solitons could be created in a plasma in the lab and (b) that they were well modeled by the fKdV equation. The inclusion of dust particles is a different physical situation than the pure ion/electron plasma that has been discussed thus far. The ion density was 10^{14} m^{-3} and the dust temperature was 10^9 m^{-3} . Instead of the ion acoustic speed, the dust acoustic speed was used to quantify the Mach number. In this experiment, a sharp transition was noted from $M < 1$ where no precursor solitons are created to $M > 1$ where precursor solitons were created.

Follow on experiments have been conducted that support this basic physical picture. This includes further experiments in the same dusty plasma setup [94, 95] and more recently in the Naval Research Laboratory Space Simulation Chamber [96]. In many of these setups, one drawback is imprecise control of the experimental conditions. Specifically, controlling the electron temperature and flow speed at the same time seems to be challenging. However, this is impor-

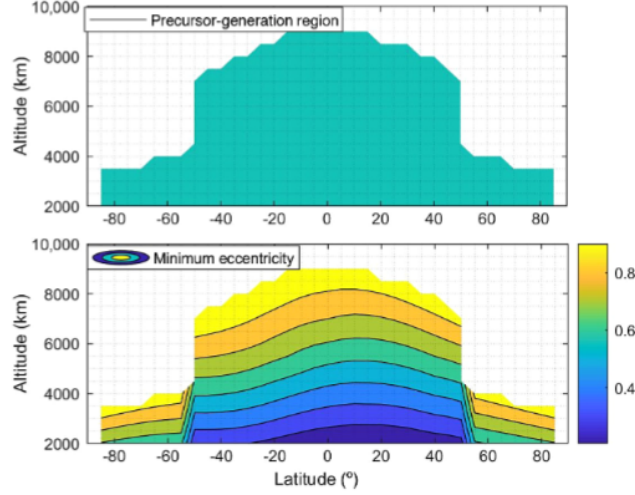


Figure 2.4: Precursor soliton generation region and plasma parameters for 5 mm radius debris (in orange) around the Earth during winter at 06:00 UTC in MEO. Lower subplot shows minimum eccentric orbit (maximum is always) one in the precursor region. From [1].

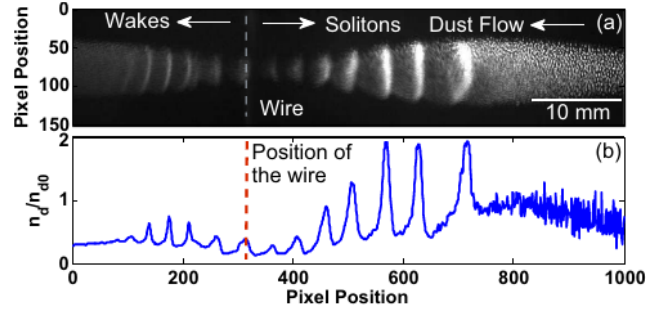


Figure 2.5: Image from dusty plasma chamber. (a) shows the raw image. (b) shows the intensity profile along the centerline. From [9].

tant because these two quantities set the Mach number. To our knowledge, there has not been a parameter study of one probe under a controlled sweep of Mach numbers.

2.3 Satellite Observations

Bernhardt et al (2023) [10] examined data from the Vector Electric Field Instrument (VEFI) and Radio Receiver Instrument (RRI) aboard the Air Force Research Laboratory (AFRL) Communication / Navigation Outage Forecasting System (C/NOFS) satellite during colocation down-

stream of a Space Shuttle engine burn and near various LEO satellites (Starlink 2672, Starlink 2521, COSMOS 2251 debris, and Iridium 911). As seen in Fig. 2.6, C/NOFS saw an increase in spectral power primarily between the ion cyclotron and lower hybrid frequency while near Starlink 2521. Although less in magnitude, there was also power in the ion sound frequency. This is characteristic of the observations seen in other conjunctions. That study coined the term “FLASH” to qualitatively describe this observational phenomenon. Due to the colocation, it is presumed that the RRI FLASH is due to wave activity emanating from the target spacecraft.

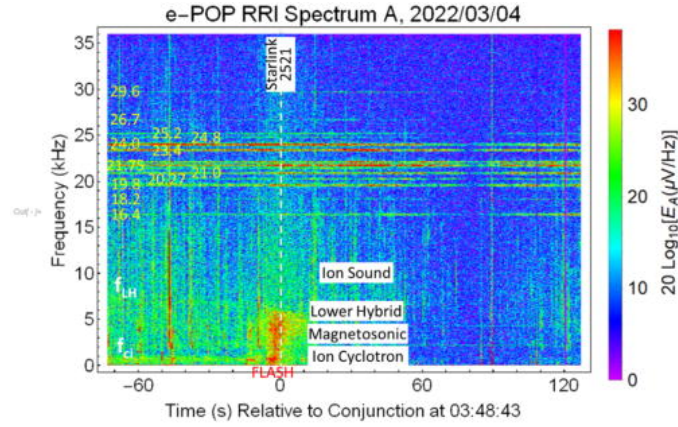


Figure 2.6: RRI data during conjunction with Starlink 2521. From [10].

Due to the preponderance of the spectral power occurring near the magnetosonic frequency, this study suggests that magnetic effects are important to understand when interpreting spacecraft data. If a soliton is picked up by a radio receiver, how it shows up in frequency space could be a potentially complicated question. One theoretical approach is examined in Chapter 3 of this dissertation.

2.4 Theory and Simulation

In order to better include magnetic effects, *Sen et al (2023)* [88] derived a different fKdV equation in the limit of dispersive Alfvén waves. This predicts a new type of soliton, the “pinned magnetosonic soliton.” Compared to the ion acoustic, the magnetosonic would have scales of the ion skin depth, which is typically much larger in the ionosphere. It is expected that both structures exist, but in what geometries and relative proportion is unknown.

Tiwari and Sen (2016) [97] and *Dharodi et al (2023)* [98] respectively used a Boltzmann electrons / ion fluid code and particle-in-cell code to examine ion acoustic solitons. These showed good agreement with the fKdV model in 1D. Ref. [98] in particular showed that precursor solitons could be created in 2D from a blunt body or flat plate in a plasma flow. Fig. 2.7 shows the precursor solitons generated from a blunt body.

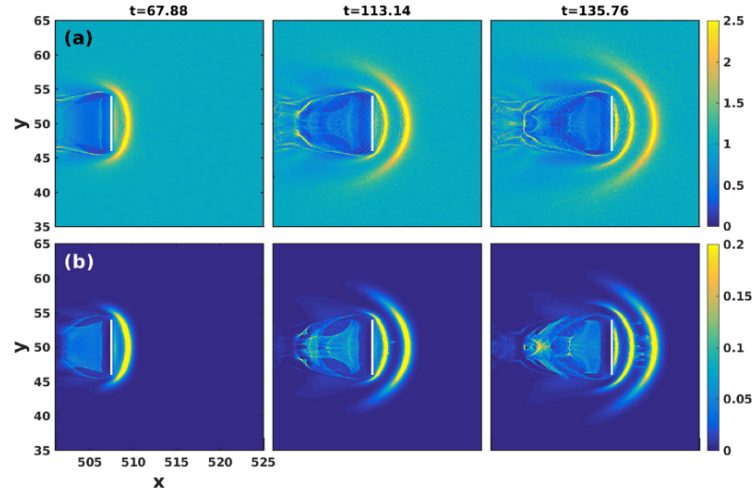


Figure 2.7: Ion charge density (a) and kinetic energy (b) at various times in particle-in-cell simulation of precursor solitons.

Chapter 3: Automatic Detection of Solitons using the Inverse Scattering Transform

This chapter is based on the journal publication *DesJardin & Hartzell (2022)* [99] with an additional subsection “Interpreting Hyperelliptic Functions”.

3.1 Inverse Scattering Transform

The problem of detecting soliton sources naturally splits into two subproblems: automatically detecting the solitons and inferring properties of the source. This chapter is concerned with the former. The latter problem needs to consider the soliton changes due to inhomogeneities of the plasma environments, damping rates, direction of propagation in all three dimensions, sensitivity of source shape to generated soliton amplitude, etc. Furthermore, the automatic detection method outlined in this study cannot distinguish between precursor and pinned solitons. Some parts of the inference problem have been considered in [1, 100]. However, an inverse source model from measurements is still an open problem.

In spacecraft observations of electrostatic solitary waves, solitons manifest as broadband data on a spectrogram even though they are a single coherent mode [10, 101] due to their large amplitude and correspondingly high nonlinearity. The solution of a partial differential equation is only a superposition of Fourier modes if it is linear in the solution. Using a traditional

spectrogram implicitly supposes that all coherent modes are Fourier modes. Previous attempts at dealing with coherent wave nonlinearity have relied upon statistical correlation between Fourier modes, termed bispectral analysis [102, 103]. This method has been applied to sinusoidally and cnoidally¹ driven KdV systems [104]. However, bispectral analysis still decomposes the soliton into many harmonics. Wavelet analysis has been used to detect optical solitons [105] which are governed by a different PDE and in different context to understand space plasma [106, 107]. Although not the focus of this chapter, they also may provide a solution to the automatic detection problem.

The homogeneous KdV equation, even though it is nonlinear, supports an exact spectra found through the inverse scattering transform (IST) [108, 109]. The spectra consists of a series of cnoidal waves. The IST reduces to the Fourier transform in the limit of small nonlinearity [110]. An IST filter could be used to screen for solitons from data in much the same way a spectrogram can isolate linear modes. In this study, we use a numerical implementation of the IST as a spectral analysis tool to identify solitons from simulated data. This technique is demonstrated to be robust to noise.

The IST was created for the homogeneous KdV equation and has only recently been analytically extended to certain sources of the forced KdV (fKdV) equation [111–114]. Streaming localized sources produce solitary structures that are similar in form to the homogeneous solutions [115]. For many of these sources, the IST decomposition is still a series of cnoidal waves. However, the mode’s time evolution may be different from the homogeneous case. This study demonstrates for the first time that the numerical IST can identify solitons in the forced case. Furthermore, the homogeneous IST has helped interpret the low frequency power spectrum and

¹A cnoidal wave is a periodic solution to the KdV equation expressible by the Jacobi elliptic function cn .

filter high frequency noise in a physically meaningful way in the nonlinear ocean waves community [116, 117] because the KdV equation is also a model of surface water waves in the shallow depth limit. We believe that the method outlined in this study can be used in the plasma physics community to filter and understand ion acoustic solitons, especially in the context of automatic detection by spacecraft and ion acoustic turbulence experiments, in a similar manner. This need has been remarked upon in hollow cathode experiments [118]. In this study we demonstrate that the IST is a better interpretable spectral technique than the Fourier transform for a simulated ion acoustic soliton system. We provide the source code and dataset used in this publication for the IST with the aim of it being used in future analysis (see Data Availability section).

3.1.1 Inverse Scattering Transform

The homogeneous KdV equation (Equation 1.35) is exactly solvable through a transformation from the KdV to the time independent Schrodinger equation (TISE) [108]. The TISE is interpreted as a scattering problem, where asymptotically planar waves $\psi \sim \exp[\pm ikx]$ are modified by a localized scattering potential $u(x)$ given by Equation 3.1. Throughout this study, we shall denote the scattering potential in the TISE as the TISE potential and the physical electrostatic potential as the electric potential. After nondimensionalization, these two quantities are actually the same through the KdV-TISE transformation, but they correspond to different uses in the two problems. The TISE is a functional eigenvalue problem in x at every t where $E = E(t)$ and $\psi = \psi(x, t)$. If $u(x, t)$ is a solution of the KdV equation, then $E_t = 0$. The eigenvalues of the associated $\psi(x, t)$ are the same at every time and all of the time dependence is in ψ .

$$\psi_{xx} = (u - E)\psi \quad (3.1)$$

In the TISE scattering problem, solving for $\psi(x, t)$ is not done directly for nontrivial TISE potentials. Rather, the problem is reformulated as a set of coefficients that relate the asymptotic solutions from the left and right of the scattering potential. These coefficients are functions of $E = k^2$ and t . E is often called the energy level because in quantum mechanics E is the physical energy level of a wavefunction. There is no physical correspondence to energy here. One common set of coefficients consists of a the transmission coefficient and b the reflection coefficient. Under the condition that $u(x, t)$ obeys the KdV equation and is localized ($\lim_{x \rightarrow \pm\infty} u = 0$), the time dependence of the scattering coefficients is explicitly given by Equations 3.2 and 3.3 [119]. The process of finding the rooted eigenvalues k , and coefficients $a(k, 0)$ and $b(k, 0)$ for an initial value to the KdV equation $u(x, 0)$ is known as forward scattering.

$$a(k, t) = a(k, 0) \quad (3.2)$$

$$b(k, t) = b(k, 0) \exp(8ik^3t) \quad (3.3)$$

The scattering data and eigenvalues are sufficient to reconstruct the scattering potential through the Gelfand-Levitan-Marchenko equation. This process is called inverse scattering and is not used directly in this publication. Together the inverse scattering, forward scattering, and scattering data time integration is called the IST.

In the forced case, the time evolution of the scattering coefficients is not known. However,

it has been shown that sources of certain analytical form are also solvable by the IST albeit with a different time evolution equation [111–114, 120].

3.2 Methods

3.2.1 Numerical Inverse Scattering Transform

3.2.1.1 Overview

The Osborne algorithm [110] is used to compute the IST of a discrete signal with periodic boundary conditions. Periodic boundary conditions are natural for signal processing. The numerical implementation of the IST will be denoted the PIST to be consistent with other literature [121]. The term for ISTs for all nonlinear integrable equation is the nonlinear Fourier transform (NFT) [122]. Note that some authors denote PIST as the KdV-Nonlinear Fourier Transform (KdV-NLFT) [117]. PIST relies on a transfer matrix formulation of the TISE, defined in Equation 3.4. The scattering matrix $\mathbf{M}$ is a complex matrix with unity determinant which takes the role of a and b . The reflection and transmission coefficients can be defined from $\mathbf{M}$ by $a = |M_{12}|^2$ and $b = |M_{11}|^2$.

$$\psi_{\text{right}} = \mathbf{M}\psi_{\text{left}} \quad (3.4)$$

ψ describes the relative amplitudes of the scattered waves. ψ contains the amplitude of both the left and right going waves evaluated at an arbitrary location. Both $\mathbf{M}$ and ψ will take different forms depending on a choice of basis functions which is not unique. In the simplest formulation, the choice of basis is planar waves. The full solution will be a superposition of $A \exp(ikx)$

and $B \exp(-ikx)$. We therefore define ψ_i as Equation 3.5. For a constant scattering potential, Equation 3.6 gives the transfer matrix, where $k_n = \sqrt{k^2 - u}$. We use the subscript notation because this treatment of the scattering matrix is conducive to a piecewise constant representation of the signal, such as in the context of a discrete digital representation of a continuous signal. The T superscript indicates the transpose.

$$\psi_i = [A(x_i), B(x_i)]^T \quad (3.5)$$

$$\mathbf{M}(k) = \begin{bmatrix} \exp(ik_n(k)L) & 0 \\ 0 & \exp(-ik_n(k)L) \end{bmatrix} \quad (3.6)$$

Similarly, for across a step in TISE potential $\mathbf{M}$ is given by Equation 3.7, where k^- refers to the k_n before the step and k^+ refers to the k_n after. Using these two basic solutions, the total $\mathbf{M}$ can be constructed sequentially, i.e. $\mathbf{M}_{x_0 \rightarrow x_2} = \mathbf{M}_{x_1 \rightarrow x_2} \mathbf{M}_{x_0 \rightarrow x_1}$.

$$\mathbf{M} = \frac{1}{2} \begin{bmatrix} 1 + \frac{k^-}{k^+} & 1 - \frac{k^-}{k^+} \\ 1 - \frac{k^-}{k^+} & 1 + \frac{k^-}{k^+} \end{bmatrix} \quad (3.7)$$

As detailed in [110, 123], the planar wave choice of basis for ψ , while being the most physically intuitive basis, is not the simplest one to deal with algebraically nor numerically. A computationally simpler basis is $[1, 0]^T$ and $[0, 1]^T$ [124]. Now the step transfer matrix is given by Equation 3.8, where k_n has the same definition as above.

$$\mathbf{M}_{\text{step}} = \begin{bmatrix} \cos(k_n \Delta x) & \frac{\sin(k_n \Delta x)}{k_n} \\ -k_n \sin(k_n \Delta x) & \cos(k_n \Delta x) \end{bmatrix} \quad (3.8)$$

k_n is complex, but lies on either the real or imaginary axes. $\sin(i\operatorname{Re}(x)) = i \sinh(\operatorname{Re}(x))$ and $\cos(i\operatorname{Re}(x)) = \cosh(\operatorname{Re}(x))$, so the following substitutions can be made if $\operatorname{Im}(k_n) \neq 0$:

$$\cos(k_n \Delta x) \rightarrow \cosh(|k_n| \Delta x) \quad (3.9)$$

$$-k_n \sin(k_n \Delta x) \rightarrow |k_n| \sinh(|k_n| \Delta x) \quad (3.10)$$

$$\frac{\sin(k_n \Delta x)}{k_n} \rightarrow \frac{\sinh(|k_n| \Delta x)}{|k_n|} \quad (3.11)$$

This allows for the evaluation of the forward scattering problem without the need for complex algebra by checking the sign of k_n^2 at every loop iteration. $\mathbf{M}$ is now real.

Recall that $\mathbf{M} = \mathbf{M}(k)$. It has been shown that for the periodic boundary conditions in the KdV equation, the associated TISE scattering problem can be simplified by using Bloch states [125]. The result, given by Equation 3.12, is that the solution to the KdV equation can be written as a *finite* linear superposition of hyperelliptic modes given by the zeros of the transfer matrix. $\mu_j(x; x_0, t)$ are hyperelliptic functions that contain all of the space and time dependence. E_j satisfy $\frac{1}{2} \operatorname{tr} \mathbf{M}(k = E_j) \pm 1 = 0$ and $M_{12}(k = \mu_j) = 0$. $N \in \mathbb{N}$ is the number of “degrees of freedom” in the problem.

$$\lambda u(x, t) = -E_1 + \sum_{j=1}^N 2\mu_j(x; x_0, t) - E_{2j} - E_{2j+1} \quad (3.12)$$

3.2.1.2 Interpreting Hyperelliptic Functions

The spatial structure of the hyperelliptic functions is given by Eqn. 3.13. There are two possible solutions depending on the sign of the square root. σ_j can be ± 1 to differentiate between the two solutions. Unlike in linear analysis, the shape of the hyperelliptic functions depends on the values of the other hyperelliptic functions in the denominator.

$$\frac{d\mu_j}{dx} = \frac{2i\sigma_j \sqrt{\prod_{k=1}^{2N+1} (\mu_j(x) - E_k)}}{\prod_{k=1, j \neq k}^N (\mu_j(x) - \mu_k(x))} \quad (3.13)$$

The time evolution of the hyperelliptic functions is given by a coupled ODE system involving all of the μ_j and E_j at time t . It is omitted from this study since it is not used. Each $\mu_j(x, t)$ can be assigned a modulus $m_j = (E_{2j+1} - E_{2j}) / (E_{2j+1} - E_{2j-1})$ and an amplitude $a_j = E_{2j+1} - E_{2j}$. Each soliton will manifest itself as a separate μ_j in this decomposition with $m_j = 1$. When m_j is calculated numerically, if $m_j \geq 0.999$ that mode is considered a soliton. This is an arbitrary choice based on numerical precision and benchmarking cases. All other modes are “radiation” modes. Thus the forward scattering transform reduces to finding the zeros of $\text{tr} \mathbf{M}(k) \pm 1$ and $M_{12}(k)$ with the initial data.

The meaning of the modulus can be seen by taking the limit of Eqn. 3.13 where there is a single hyperelliptic function. The denominator will be 1. There will be one degree of freedom, i.e. $N = 1$. In this case, the spatial structure of the single mode will be governed by:

$$\frac{d\mu_1}{dx} = 2i\sigma_1 \sqrt{(\mu_1(x) - E_1)(\mu_1(x) - E_2)(\mu_1(x) - E_3)} \quad (3.14)$$

This equation can be solved by separation of variables. The subscript on μ and σ is dropped

for simplicity. x_0 is the constant of integration from the x integral. This integral resembles the integral used in solving for sech^2 solutions to the KdV equation using a Sagdeev potential. As we shall show, this is also an elliptic integral similar to the one introduced in the discussion of calculating the mean orbital velocity of an elliptical orbit.

$$\frac{1}{2i\sigma} \int \frac{d\mu}{\sqrt{(\mu - E_1)(\mu - E_2)(\mu - E_3)}} = x - x_0 \quad (3.15)$$

If one substitutes $\mu \rightarrow \sin^2 \theta$, it is possible to rewrite Eqn. 3.15 in the form of an elliptic integral. Solutions can be read from a mathematical reference book (e.g. Abramowitz & Stegun 17.4.61² [11]). The general solution to an integral of the form $\int \frac{dt}{\sqrt{P}}$ where P is a trinomial in the form of $P(t) = (t - \beta_1)(t - \beta_2)(t - \beta_3)$ where the roots (β) are real. If $\beta_1 > \beta_2 > \beta_3$, then in general the result is given by Eqn. 3.16 where $\lambda = \frac{1}{2}(\beta_1 - \beta_3)^{1/2}$ and $F(x, m)$ is an incomplete elliptic integral of the first kind. m is the modulus of said elliptic integral. Since $P(t)$ has three zeros, $\frac{1}{\sqrt{P(t)}}$ is singular in three places. $\phi(t)$ depends on the domain the integral is in.

$$\lambda \int_{\beta_1}^x \frac{dt}{\sqrt{P(t)}} = F(\phi(t), m) \quad (3.16)$$

Using this result to evaluate Eqn. 3.15 yields:

$$\frac{1}{i\sigma(E_3 - E_1)^{1/2}} F(\phi, m) = x - x_0 \quad (3.17)$$

The table of possibilities from Abramowitz & Stegun is shown in Fig. 3.1. As can be seen, the shape broadly depends on the amplitude compared to the energy levels. The modulus is set

²<https://personal.math.ubc.ca/cbm/aands/frameindex.htm>

17.4.61		
$\lambda = \frac{1}{2} (\beta_1 - \beta_2)^{1/2}, m = \sin^2 \alpha = \frac{\beta_2 - \beta_3}{\beta_1 - \beta_3}$		
$m_1 = \cos^2 \alpha = \frac{\beta_1 - \beta_2}{\beta_1 - \beta_3}$		
17.4.62		
$\lambda \int_{\beta_3}^x \frac{dt}{\sqrt{P}}$	$F(\varphi \setminus \alpha)$	$\sin^2 \varphi = \frac{x - \beta_3}{\beta_2 - \beta_3}$
17.4.63		
$\lambda \int_x^{\beta_2} \frac{dt}{\sqrt{P}}$	$F(\varphi \setminus \alpha)$	$\cos^2 \varphi = \frac{(\beta_1 - \beta_2)(x - \beta_3)}{(\beta_2 - \beta_3)(\beta_1 - x)}$
17.4.64		
$\lambda \int_{\beta_1}^x \frac{dt}{\sqrt{P}}$	$F(\varphi \setminus \alpha)$	$\sin^2 \varphi = \frac{x - \beta_1}{x - \beta_2}$
17.4.65		
$\lambda \int_x^{\infty} \frac{dt}{\sqrt{P}}$	$F(\varphi \setminus \alpha)$	$\cos^2 \varphi = \frac{x - \beta_1}{x - \beta_2}$
17.4.66		
$\lambda \int_{-\infty}^x \frac{dt}{\sqrt{-P}}$	$F(\varphi \setminus (90^\circ - \alpha^\circ))$	$\sin^2 \varphi = \frac{\beta_1 - \beta_2}{\beta_1 - x}$
17.4.67		
$\lambda \int_x^{\beta_3} \frac{dt}{\sqrt{-P}}$	$F(\varphi \setminus (90^\circ - \alpha^\circ))$	$\cos^2 \varphi = \frac{\beta_1 - \beta_2}{\beta_1 - x}$
17.4.68		
$\lambda \int_{\beta_2}^x \frac{dt}{\sqrt{-P}}$	$F(\varphi \setminus (90^\circ - \alpha^\circ))$	$\sin^2 \varphi = \frac{(\beta_1 - \beta_2)(x - \beta_2)}{(\beta_1 - \beta_2)(x - \beta_3)}$

Figure 3.1: Table of domain dependent solutions to elliptic integral of first kind. From Ref. [11]

by the width of between the energy levels.

The inverse of $F(\phi, m)$ is the Jacobi elliptic function $\text{sn}(x, m)$, i.e. $F(\text{sn}(x, m), m) = x$.

Thus:

$$\phi(\mu) = \text{sn} \left(i\sigma (E_3 - E_1)^{1/2} (x - x_0), m \right) \quad (3.18)$$

Inverting this equation in full generality is complex. Following the approach in Ref. [126], one can show that for small amplitude (and hence small elliptic modulus) solutions to Eqn. 3.13 can be approximated by Eqn. 3.19 where $k = \Delta k = \frac{2\pi}{L}$, L is the domain length, $r = (E_3 - E_2)/2$ and $\bar{E} = (E_2 + E_3)/2$.

$$\left(\frac{d(\mu - E_1)}{dx} \right)^2 = k^2 [r^2 - (\mu - \bar{E})] \quad (3.19)$$

The solution to Eqn. 3.19 is $\mu(x) = E_1 + r \cos(k(x - x_0))$. This corresponds to the Fourier solution.

3.2.1.3 Algorithm Summary

Given a discrete signal u_n which is of length N , the associated transfer matrix $\mathbf{M}$ for energy level $E = k^2$ can be calculated with Algorithm 1. An intermediate scattering matrix at every piecewise step $\mathbf{T}$ is used to accumulate the signal. This is defined in Alg. 1. The implementation of $\mathbf{M} = \mathbf{T}\mathbf{M}$ uses Ballard's algorithm [127] for improved numerical stability because $\sinh x$ and $\cosh x$ both increase very quickly for small x . Matrix multiplication is done with CBLAS level 3 matrix multiplication subroutines. The $\mathbf{M}$ matrix calculation is implemented in C, while the overall zero finding and spectra construction passes are done in MATLAB. The MATLAB coder functionality is used to call the C subroutines.

Algorithm 1 Forward Scattering Osborne Algorithm

```

procedure FORWARDSCATTER( $k, U_n, \Delta x, N$ )
   $n \leftarrow 1$ 
   $\mathbf{M} \leftarrow \mathbf{I}_{2 \times 2}$ 
  while  $n \neq N$  do
     $k_n^2 \leftarrow k^2 + u_n$ 
     $k_n \leftarrow \sqrt{|k_n^2|}$ 
    if  $k_n^2 \geq 0$  then
       $\mathbf{T} \leftarrow \begin{bmatrix} \cos(k_n \Delta x) & \frac{\sin(k_n \Delta x)}{k_n} \\ -k_n \sin(k_n \Delta x) & \cos(k_n \Delta x) \end{bmatrix}$ 
    else
       $\mathbf{T} \leftarrow \begin{bmatrix} \cosh(k_n \Delta x) & \frac{\sinh(k_n \Delta x)}{k_n} \\ k_n \sinh(k_n \Delta x) & \cosh(k_n \Delta x) \end{bmatrix}$ 
    end if
     $\mathbf{M} = \mathbf{T}\mathbf{M}$ 
     $n \leftarrow n + 1$ 
  end while
end procedure

```

Finding the zeros of M_{12} and $\text{tr}\mathbf{M} \pm 1$ is done with a four pass method outlined in Osborne,

Chapter 17 [110]. The method relies on the fact that between each zero of $M_{11} = E_j$, there are one or two μ_i and between each E_i and E_{i+1} , there is one zero of $\text{tr}(\mathbf{M}) + 1$ and one zero of $\text{tr}(\mathbf{M}) - 1$. In the first pass, an accounting function $S(x_n, E) = \sum_{i=0}^n |\text{sign}(M_{11}(x_{i+1}, E)) - \text{sign}(M_{11}(x_i, E))|/2$ is evaluated on a fixed resolution E grid from E_{min} to E_{max} . $S(x_n, k)$ gives the number of sign changes in M_{11} from x_1 to x_i which is assumed to be the same as the number of zeros of M_{11} on a sufficiently fine E grid. E_{min} and E_{max} are chosen such that the graph of $M_{11}(E)$ has no zeros left of E_{min} and E_{max} is in a region where $|M_{11}| \leq 1$. Osborne [110] discusses some theoretical properties analogous to Nyquist sampling theorems which constrain E_{min} and E_{max} . This application focuses on low frequency modes, so the sampling frequency is not rigorously considered. In the second pass, the exact zeros of M_{11} are found using an optimization solver. The location of the sign change in S is used as an initial guess. The third pass finds the locations of μ_i through a bisection method root solver with $M_{11,i}$ and $M_{11,i+1}$ as the bounds. The sign of $M_{11,i}$ and $M_{11,i+1}$ are checked. If they are dissimilar, there is assumed to be one root, and the bisection solver is directly used. If the signs are the same, it is presumed that there are two roots. A third point of a dissimilar sign is searched for in-between the two bounds by first guessing the midpoint and then randomly guessing in that interval. Once a third point of dissimilar sign is found at E_3 , two bisection searches between $[M_{11,i}, E_3]$ and $[E_3, M_{11,i+1}]$ are conducted. The fourth pass finds the zeros of $\text{tr}(\mathbf{M}(E)) + 1$ and $\text{tr}(\mathbf{M}(E)) - 1$ between each μ_i and μ_{i+1} with two bisection searches. If $\max S = N - 1$, then there are N μ_i and $2N + 1$ E_j . N is the number of modes in the hyperelliptic decomposition.

Figure 3.2 shows the intermediate calculations of the numerical IST of $u(x) = \text{sech}^2(x/\sqrt{2})$ using the four pass process. This is an analytical solution to the homogeneous KdV equation corresponding to a single soliton moving at nondimensionalized speed $c = 2$. The variable speed in

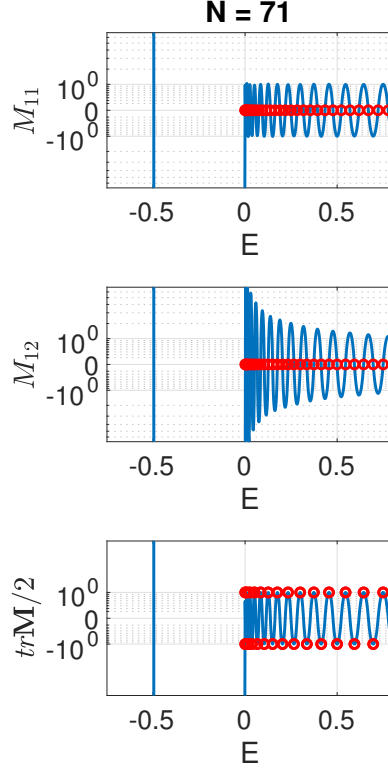


Figure 3.2: Intermediate calculations in the IST process. The circles are the numerically found zeros of M_{11} , $\mu_i = M_{12}$, and $E_j = tr\mathbf{M} \pm 1$. The solid blue lines are the respective functions evaluated on the E grid. The y axis are on a symmetric log scale [12]. Not shown are the values of the zeros at $E \approx 0.5$. For $E = -0.4951$, $M_{11} = -9.632 \times 10^{13}$, $M_{12} = -4.5940 \times 10^{13}$, $(tr\mathbf{M}/2 + 1) = 6.7791 \times 10^{13}$, and $(tr\mathbf{M}/2 - 1) = 6.7791 \times 10^{13}$.

single soliton solution is discussed and given later by Equation 3.21. The signal has 150 points from $[-12.5, 87.5]$. The fixed resolution E grid is 3000 points from $[-2, 5]$. The nonlinearity parameter $\lambda = 1$. The zero with $E \approx 0.5$ represents the soliton mode. All components of $\mathbf{M}$ vary from 10^{60} to -10^{27} over a small width near $E \approx -0.5$. -10^{27} is a local minimum of this function, not a floating point representation of negative infinity. The zeros in that interval are not found to great precision because that function is steeper than double floating precision can resolve. However, this is acceptable for our purposes. In general, the leftmost zero is subject to this problem because all components of $\mathbf{M} \rightarrow \infty$ as $E \rightarrow -\infty$.

3.2.2 fKdV Simulation

The solutions to the fKdV equation are solved using a pseudo-spectral fluid simulation which uses the Chan–Kerkhoven pseudospectral scheme [128] for time stepping the solution. The time evolution equation is given by Equation 3.20 where k are the wavenumbers of the Fourier modes, $s = \pi/L$, Δt is the timestep, L is the length of the spatial domain, and $\mathcal{F}$ represents a discrete Fourier transform. The timestep Δt is chosen to ensure a numerical stability condition.

$$u^{n+1} = \mathcal{F}^{-1} \left[\frac{1}{1 - \Delta t \beta i s^3 k^3} ((1 - \Delta t \beta i s^3 k^3) \mathcal{F}(u^{n-1}) - \Delta t \alpha i s k \mathcal{F}(u^n)^2 + \Delta t i s k \mathcal{F}(S)) \right] \quad (3.20)$$

This has been demonstrated to be accurate to $u_{\text{numerical}} - u_{\text{analytical}} \approx 10^{-4}$ on known solutions to the fKdV equation [1, 100].

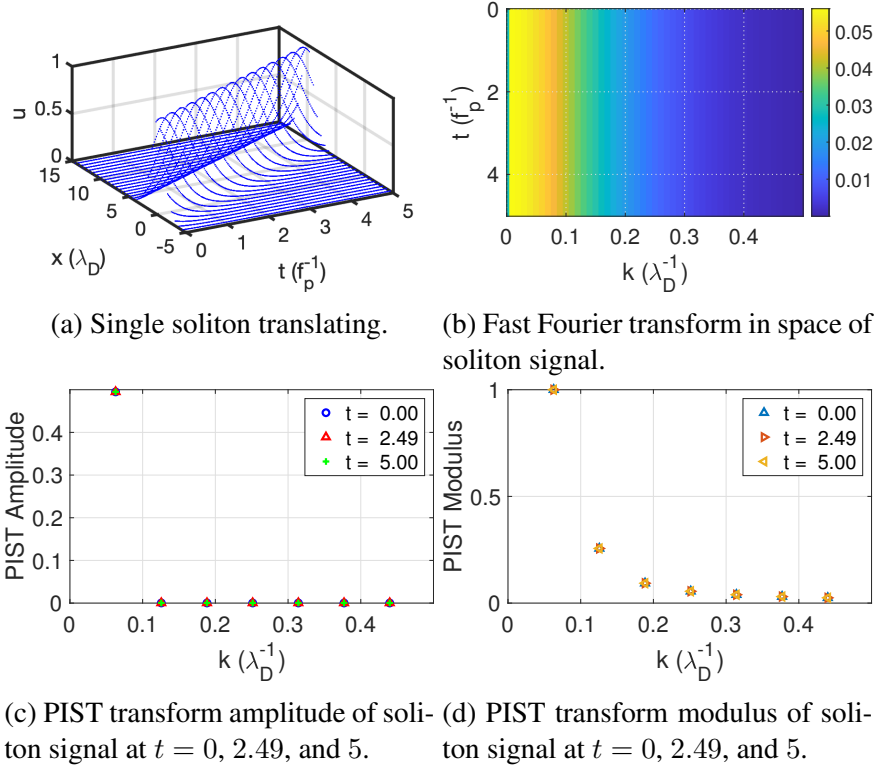


Figure 3.3: Spectrum of $u(x) = \text{sech}^2\left(\frac{\sqrt{2}}{2}(x - 2t)\right)$ using the FFT and PIST. [Associated dataset available at <https://doi.org/10.5281/zenodo.7017245>] (Ref. [13])

3.3 Results

3.3.1 Homogeneous Cases

3.3.1.1 Single Soliton

The single soliton solution to the KdV equation is given by Equation 3.21. c is the soliton speed and x_0 is the soliton initial location. For this numerical experiment, $x_0 = 0$, $c = 2$, $\lambda = 1$, $x \in [-L/8, 7L/8]$ with 150 points where $L = 100$. There are 200 time points with $t \in [0, 5]$. The numerical solution is visualized in Figure 3.3a. In Figures 3.3b, 3.3c, & 3.3d k is the spectral wavenumber associated with the transforms. The PIST modes have a wavenumber associated

with them. For the low modulus modes, this corresponds exactly to the wavenumber of equivalent Fourier mode. An interesting discussion of this can be found in Osborne & Bergamasco [126]. For the high modulus modes, this is related to the eigenvalue of the Schrödinger equation.

$$u(x, t) = \frac{c}{2} \operatorname{sech}^2 \left[\frac{\sqrt{c}}{2} (x - ct - x_0) \right] \quad (3.21)$$

The PIST of this solution yields a single dominant mode with a modulus $m > 0.999$ indicating a soliton mode in Figure 3.3c. The PIST is taken at three times to indicate the spectrum is constant (i.e. $d/dt(E) = 0$) even though the soliton is spatially translating. $a_1 \approx 0.5$, which corresponds to a u amplitude of $2a_1 \approx 1$. From Equation 3.12 with $\lambda = 1$ this a agrees with the analytical solution. The Fourier transform of the same signal is a constant broadband spectrum as seen in Figure 3.3b. The PIST is interpretable as a single solitary wave directly from the decomposition.

3.3.1.2 Three Solitons

The initial condition $u(x, 0) = 12 \operatorname{sech}^2(x)$ results in three solitons of differing heights as seen in Figure 3.4b. This spectral simulation has KdV parameters $\alpha = 6$, $\beta = 1$, $\lambda = 1$. From this numerical example forward, all data comes from an fKdV simulation, in this case with zero forcing. The full spatial domain is $x \in [-300, 300]$ discretized into 12,288 points. The time domain is $t \in [0, 3]$. The height of a soliton determines its speed, so these three solitons will spread out. PIST can identify these three solitons *even when they are overlapping* such as during the initial condition. Table 3.1 shows the first six modes of the PIST at $t = 0$. Three modes have a soliton modulus. The physical amplitude of these modes also corresponds to the amplitude of

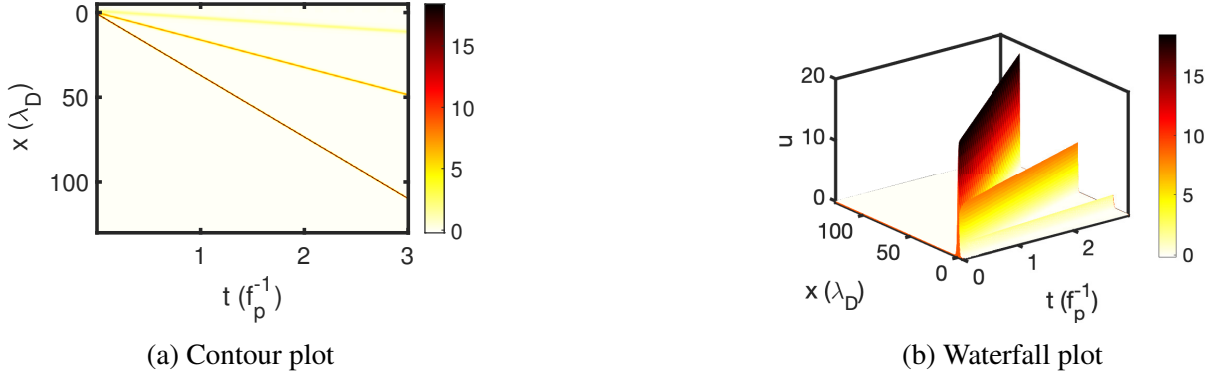


Figure 3.4: $u(x, 0) = 12 \text{sech}^2(x)$. This initial condition will propagate into three solitons. [Associated dataset available at <https://doi.org/10.5281/zenodo.7017245>] (Ref. [13])

Table 3.1: Magnitude and moduli of the six highest amplitude IST modes of $u(x) = 12 \text{sech}^2(x)$. The first three largest magnitude modes are all solitons while the rest are radiation.

Mode	a	m
1	5.1600	1.0000
2	2.9672	1.0000
3	0.9869	1.0000
4	0.0009	0.0108
5	0.0017	0.0073
6	0.0025	0.0068

the resultant solitons in Figure 3.4a and 3.4b. Recall that $u_{\max} = 2a$.

3.3.2 Stability to Noise

Here we demonstrate that PIST is robust to small noise motivated by ionospheric conditions. Solitary structures governed by the KdV equation will be a sech^2 pulse in ϕ , not the electric field $\mathbf{E}$. In one dimension $E = -\frac{\partial \phi}{\partial x}$, which would make a bidirectional electric field. However, we will assume that the strength of the nondimensional KdV signal $u \sim \phi \sim |\mathbf{E}|$ to discuss the relative strength of ion acoustic solitons to other plasma waves in the ionosphere. Table 3.2 shows the typical electric fields for several phenomena in the ionosphere.

Table 3.2: $|\mathbf{E}|$ for several space plasma phenomena.

Signal	Magnitude (mV/m)	Source
Large Magnitude Whistler	140	Ref. [129]
Magnetopause Crossing	30	Ref. [130]
Solar Wind Turbulence	0.25	Ref. [131]
Ionospheric Soliton	500	Ref. [1]

The largest noise content will come from large magnitude whistler waves which are 30% as large as the ionospheric soliton. If the signal defined in Equation 3.22 is superimposed on the single transiting soliton, then PIST still recovers that there is a single soliton. The first term represents a 5 kHz whistler wave with dimensionless frequency $\omega' = (5\text{kHz})\lambda_D/v_{IA}$, where $\lambda_D = 3.6$ cm and $v_{IA} = 5.5$ km/s. The second term is white noise with an amplitude one order of magnitude smaller than the whistler wave. This represents lower order noise signals.

$$U_{\text{noise}} = 0.3 \cos(2\pi\omega' t) + 0.01 * \mathcal{N}(0.0, 1) \quad (3.22)$$

Under this noise perturbation $u \rightarrow u + u_{\text{noise}}$, PIST decomposes the signal into a soliton and many low amplitude modes as seen in Figures 3.5 and 3.6. The question may arise, is there an effective signal to noise ratio dependent upon the signal discretization or fundamental to the IST? This is harder to establish due to the nonlinear nature of the governing equation. If one adds a significantly large noise perturbation, the noise perturbation itself will quickly break into a train of solitons. An example of this is the original Zabusky-Kruskal problem [132] where a large amplitude cosine wave was observed to break into many solitons. This analysis simply demonstrates that the method is robust to *small* perturbations which have not steepened enough for the nonlinear term to be dominant.

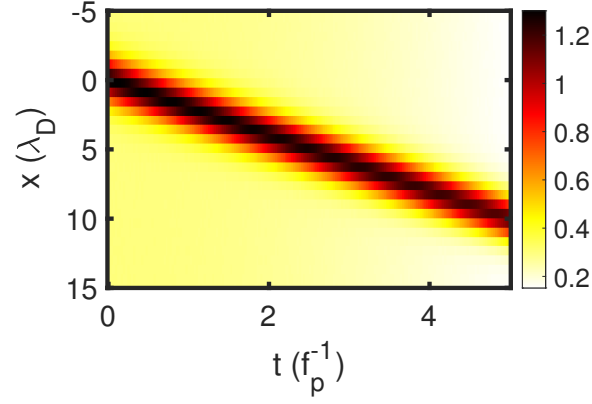


Figure 3.5: Single soliton transiting through a noisy background environment. [Associated dataset available at <https://doi.org/10.5281/zenodo.7017245>] (Ref. [13])

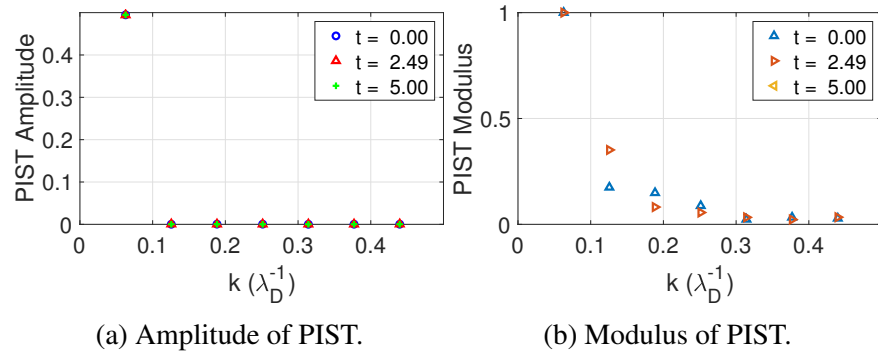


Figure 3.6: PIST Transform of noisy single soliton at three times. The dominant soliton mode ($k \approx 0.06$) is constant with time.

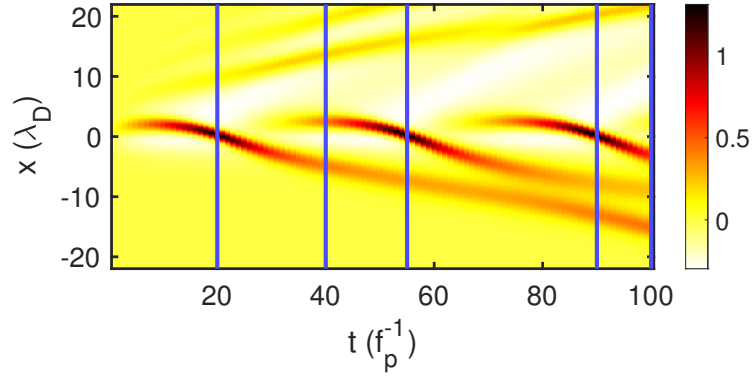


Figure 3.7: Solitons growing and detaching from the forcing term. The blue lines indicate time cuts where a signal is filtered. [Associated dataset available at <https://doi.org/10.5281/zenodo.7017245>] (Ref. [13])

3.3.3 Inhomogeneous Cases

When localized forcing is added to the KdV equation, this models a moving debris object with a nonzero surface potential. Due to plasma sheath conditions, it is very likely that a debris object immersed in plasma will have a nonzero surface potential. For a spherical object in a Maxwellian plasma, the surface potential is determined by the relative length scales of the sphere and the Debye length of the plasma it is immersed in [93]. We apply PIST to the plasma near the debris to search for automatic detection of this signature. Through our numerical experiments, we have found for the first time that the PIST decomposition finds the correct number of solitons on data from the fKdV equation with localized forcing.

3.3.3.1 sech^2 forcing

Wu showed [115] that a sech^2 choice of forcing, given by Equation 3.23, with an initial condition of $u_0(x) = a \text{sech}^2(Kx)$ results in a stable non-moving soliton solution where $u(x, t) = u_0(x)$. In the case of a zero initial condition, this forcing results in the growth of a

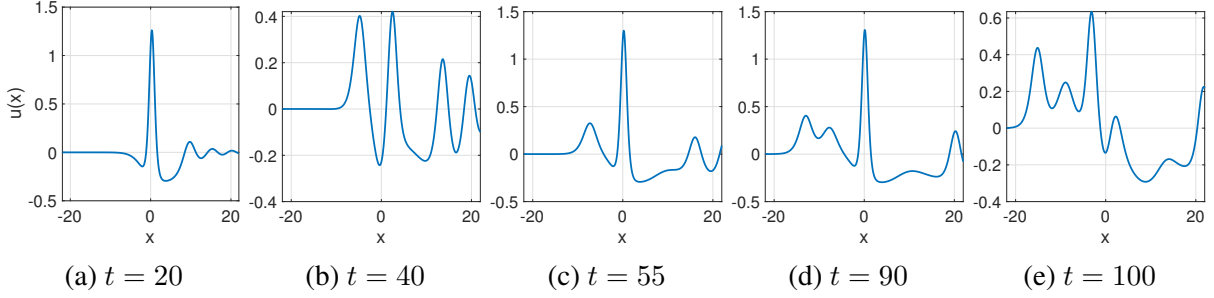


Figure 3.8: Spectral simulation of sech^2 fKdV equation. Fig. 3.7 shows the full simulation. (a-e) are the clipped signals from each of the blue lines in Fig. 3.7.

soliton at $x = 0$ and then subsequent detachment once it reaches a critical amplitude. The growth and detachment cycle results in a periodic generation of solitons from the source.

The PIST is capable of detecting a soliton, even in a forced setting. In Figure 3.7, the fKdV equation with a forcing term given by Equation 3.23 with $b = -0.1250$ and $K = 0.6124$.

$$S(x) = 2b \text{sech}^2(Kx) \quad (3.23)$$

The discretization parameters were $x \in [-250, 250]$ with 2048 points, $t \in [0, 2]$ with $\Delta t = 0.001$. $\alpha = -3/2$ and $\beta = -1/6$ so $\lambda = 3/2$. The fixed E grid is set from $E \in [-1.4, 2]$ with $\Delta E = 6.8 \times 10^{-5}$. Figure 3.7 shows the simulation which is spatially clipped near the nonzero signal. The generation and detachment pattern happens every $\Delta t \approx 45$. The solitons then propagate in the $-x$ direction. For analysis, truncated spatial cuts are taken at noteworthy times. $t = 20, 55$, and 90 are times when the precursor soliton initially detaches from the forcing term (Figures 3.8a, 3.8c, and 3.8d). At $t = 40$, the first soliton has already detached and the second one is forming (Figure 3.8b). At $t = 100$, a fourth soliton just starts to form and the first three are cleanly detached (Figure 3.8e). Figures 3.9a - 3.9e show the PIST mode amplitudes and moduli. The moduli m are near $m = 1$, so $\sigma(m)$ is plotted where $\sigma(m) =$

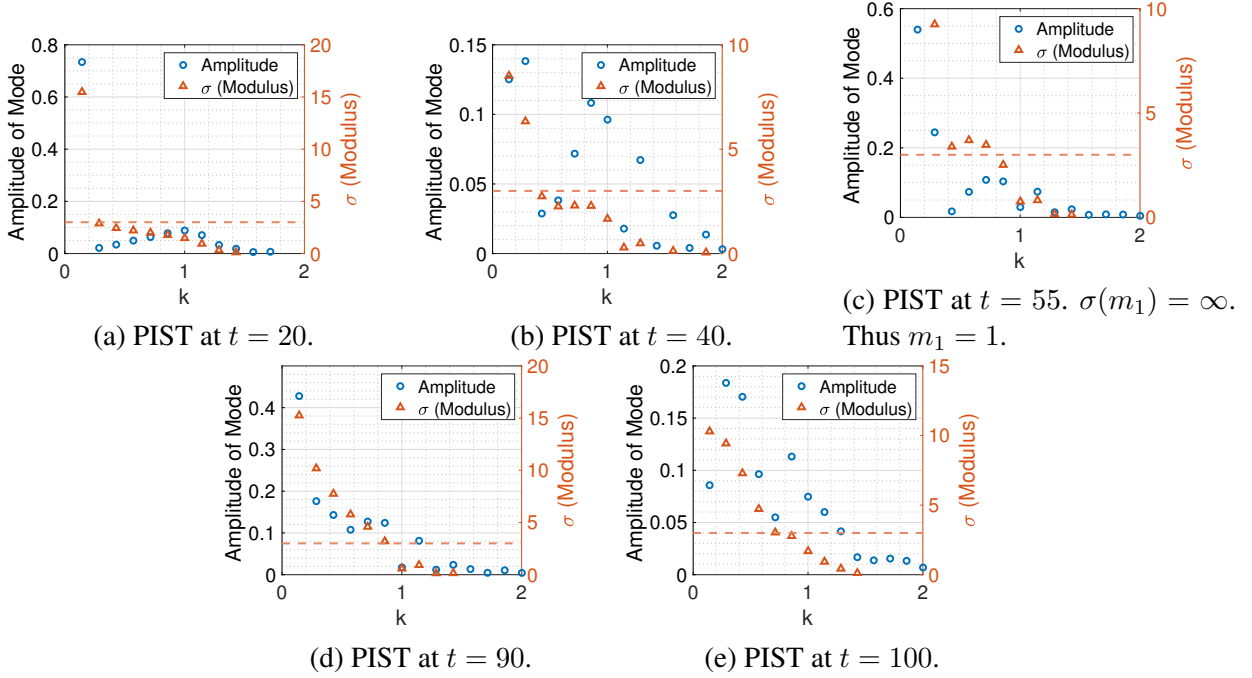


Figure 3.9: PIST of fKdV equation with sech^2 forcing taken at different times. The dashed line indicates $\sigma(m) = 3$ which is taken as the boundary between soliton and not-soliton modes.

$\log(1 - 9(m/9)) / \log(1/10)$. This defines a logarithmic scale which counts the number of post decimal point nines. For example: $\sigma(1) = \infty$, $\sigma(0.9) = 1$, $\sigma(0.999) = 3$, and $\sigma(0) = 0$. Under the definition that a soliton is defined as any mode with $m > 0.999$, then $\sigma(m) > 3$ is a soliton. This is plotted as horizontal line on Figures 3.9a - 3.9e.

Figure 3.9a and 3.9b show that at $t = 20$ and $t = 40$ respectively, there are one and two solitons seen in the system by PIST. This matches the visual observation in Figure 3.8a and 3.8b. The amplitudes for the single soliton in PIST ($2 * 0.75 = 1.5$) is close to the amplitude of the observed signal. What is observed may be partially the superposition of the other modes of the system and the single soliton, therefore this is good agreement. In the two soliton case (Figure 3.8c) it is not clear that the amplitudes of the observed solitons match the PIST readings. However, the non-solitonic modes are large amplitude during this cut, as seen by the large negative of value of the soliton. A KdV soliton, by definition, must be single signed. At $t = 55$, there are

five solitons in the decomposition, even though there are only two in the simulation. Most of the energy of the system is in the first two solitons. Similarly, at $t = 90$, there are six solitons even though the system only has three. The energy is mostly contained in the first three modes. After the third soliton detaches, the decomposition at $t = 100$ shows four solitons as expected by visually inspecting Figure 3.8e. At $t = 90$ the solitons are nonlinearly interacting. During the time where the PIST number of solitons does not match the simulation, the solitons lose coherence because they are interacting with each other. This suggests that in the forced setting, PIST can identify solitons as long as they are not interacting with each other.

3.3.3.2 Gaussian forcing

In dusty plasmas Gaussian forcing is used to approximate the surface charge on debris objects [1] such as in Equation 3.24 where Φ_{surf} is a surface potential determined by sheath theory. a is a characteristic size of the debris and V is the relative speed in the plasma medium. This is discussed in Truitt & Hartzell [100].

$$S(x, t) = \Phi_{\text{surf}} * \exp \left[-\frac{x - Vt}{a} \right] \quad (3.24)$$

Figure 3.10 shows a simulation of such a forcing term centered at $x = 0$. Qualitatively, it has the same behavior of growing and detaching solitons from the forcing source as was shown in the prior examples.

The simulation has the following physical parameters: $\alpha = 1.0051$, $\beta = 0.4925$, $a = 0.2783$, $\lambda_D = 3.6\text{cm}$, $v_{IA} = 6.1\text{ km/s}$, and $V = 1.1240$. a and V are nondimensional quantities normalized by λ_D and v_{IA} respectively. Relevant numerical parameters are: $t_{\text{max}} = 200$, $\Delta t =$

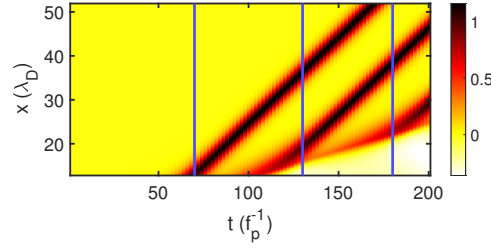


Figure 3.10: Simulation of Gaussian fKdV equation. Three solitons grow and detach from a moving source. The vertical blue lines indicate time cuts where the PIST is taken. [Associated dataset available at <https://doi.org/10.5281/zenodo.7017245>] (Ref. [13])

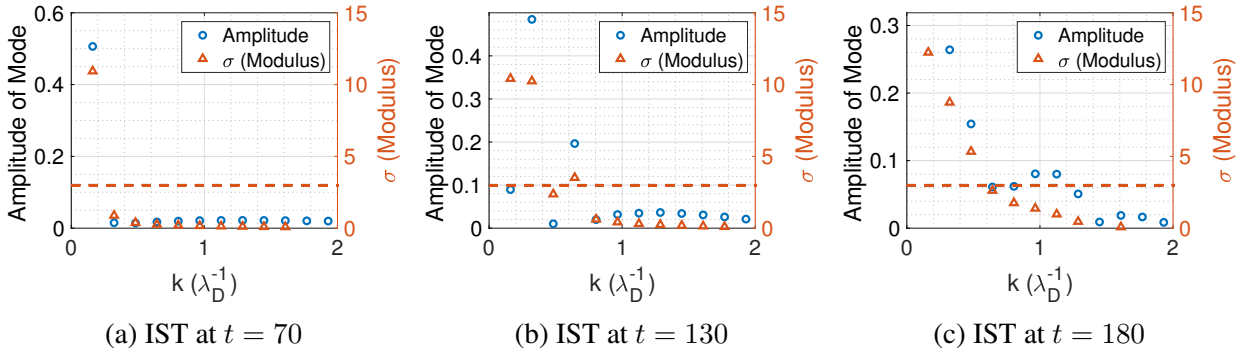


Figure 3.11: PIST of Gaussian forced fKdV equation. This models a 1cm charged debris object generating solitons in the ionosphere at 2000km . The dashed line indicates $\sigma(m) = 3$ which is taken as the boundary between soliton and not-soliton modes.

0.001 , $L = 300$, $N = 512 \times 15 = 7680$, $E \in [-1.5, 3]$, and $\Delta E = 1.5 \times 10^{-3}$. These quantities correspond to a 1 cm sized debris object in a 2000km altitude circular Earth orbit. The variation in α and β is to account for the observed κ value of electrons in Earth magnetospheric models [1] at that location [1]. The data used by the PIST is clipped from $x = 20.625$ to $x = 59.6875$ to reduce the PIST run time and isolate the precursor solitons.

It can be seen that in each of these three cuts, the correct number of solitons is identified. At $t = 70$, a single soliton mode is detected in Figure 3.11a. At $t = 130$, two solitons are detected in Figure 3.11b. Finally at $t = 180$, three solitons are detected in Figure 3.11c. It was observed that if the signal is not clipped close to the source and the wake behind the moving debris is part of the signal, the PIST decomposition becomes less accurate.

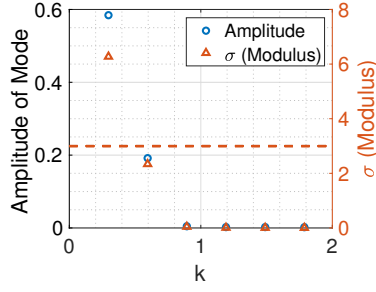


Figure 3.12: IST at $t = 130$ but the domain has been clipped from 30.78125 instead of 12.8125. The dashed line indicates $\sigma(m) = 3$ which is taken as the boundary between soliton and not-soliton modes.

3.3.3.3 Gaussian forcing amplitude mismatch

When two solitons are present in the frame, the PIST does not recreate the correct amplitude as seen in Figure 3.11b. One soliton mode has the correct amplitude while the other one is significantly lower than the truth. However, when the second soliton is clipped out of the frame, the IST reports the remaining soliton amplitude correctly. An example of this is shown in Figure 3.12. This phenomena also occurs in the unforced setting. It shall be investigated in future work.

In the use case where the IST would be used to post process spacecraft data, multiple domain windows could be used to analyze the same data. Since the full IST identifies the correct number of solitons, the full domain filtering would still be admissible as an automated soliton detector.

3.4 Conclusions

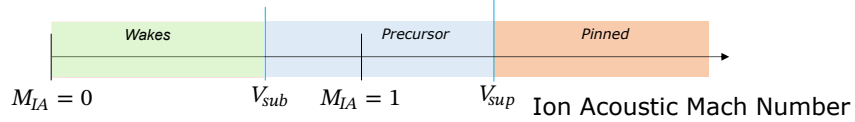
This study applied a numerical nonlinear spectral analysis tool, the inverse scattering transform, to the forced and unforced Korteweg De-Vries equation in the context of an ion acoustic wave generated from a debris object. For the unforced KdV equation, the PIST provides a more

useful decomposition than a Fourier transform because it correctly identifies nonlinear coherent modes as a single mode instead of broadband noise. It can also identify multiple colocated modes which visually look similar. This decomposition is shown to be stable to small perturbations in the signal. For some localized forcing functions the PIST was found to identify the correct number of solitons, even though it is derived for homogeneous solutions to the KdV equation. Specifically, we have shown that the IST correctly identifies solitons predicted to be produced by charged orbital debris.

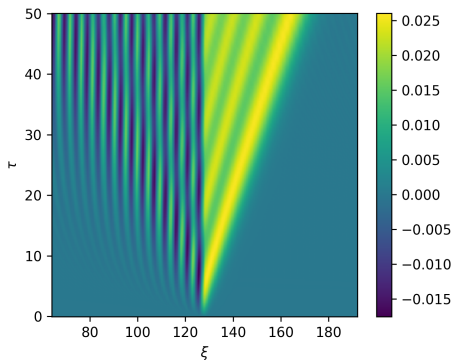
Chapter 4: Multifluid Simulation of Ion Acoustic Solitons Arising from a Charged Source and Comparison to the Forced Korteweg – de Vries Model

The text of this chapter is in preparation for journal submission.

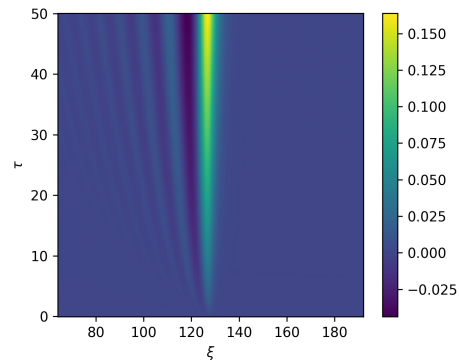
The forced Korteweg – de Vries equation (fKdV) [91] has three flow regimes, which we shall denote “wake” (sometimes called linear in this chapter), “precursor”, and “pinned.” These are diagrammed in Fig. 4.1a. Some authors refer to the “precursor” regime as the “transcritical” regime. They are synonymous.



(a) Flow transition points



(b) Precursor regime.



(c) Pinned regime.

Figure 4.1: Flow regimes of the forced Korteweg - de Vries equation. The x -axes in b and c are a stretched spatial coordinate that moves with the forcing function: $\xi = \epsilon^{1/2} (x - M_{IA}t)$. The y -axes in b and c are a stretched time coordinate: $\tau = \epsilon^{3/2}t$. The colors indicate U , the dependent variable in the fKdV equation. These are explained in further detail in Chapter 2.

In the wake regime, small amplitude waves are generated by the background plasma flow interacting with the source. When the flow velocity increases beyond the *subcritical speed* (M_{sub}), solitons are created that move faster than the source. These solitons are periodically generated at the source and then propagate up the background flow, i.e. upstream. Fig. 4.1b shows a direct simulation of the fKdV equation whose solution is in the precursor regime. The source is located at $\xi = 128$ in the middle of the shown domain. Solitons are created every ~ 20 time steps. When the background speed becomes greater than the *supercritical speed* (M_{sup}), the system enters the pinned regime. In the pinned regime, solitons move at the same speed as the source. Thus, there is a single soliton that is fixed on the source. Fig. 4.1c shows a simulation of the fKdV at a different speed than 4.1b where the solution is in the pinned regime. The source is again at $\xi = 128$ in the middle of the domain. The pinned soliton grows until around $\tau \approx 40$. The specifics of the direct fKdV simulation are detailed in the next section.

In this chapter, *ion acoustic* solitons in a collisionless hydrogen plasma are studied. The solitons are ion acoustic because the dispersion portion of the balance comes from the ion acoustic wave mode. The nonlinearity comes from the nonlinear term in the ion momentum equation. Ion acoustic solitons are stable under collisions with other solitons [132] and have been shown to have long propagation distances when damping effects (such as Landau and collisional damping for ion acoustic plasma solitons) are included [100]. Damping effects are not considered in this study since they only impact solitons once they have been generated. If they were to be considered, a modified simulation scheme would be required [133–135]. As demonstrated throughout this study, solitons are periodically generated by a source of free charge. They have been studied experimentally in a dusty plasma chamber [9, 136, 137]. For these reasons, they are seen as an attractive mechanism of detecting the source. One proposed application of this is indirectly

observing sub-centimeter space debris [1, 10, 88, 89, 97, 99, 100, 138, 139].

4.1 Theory

The fKdV equation has been shown to model the generation of precursor and pinned solitons. Precursor solitons are created for $M_{sup} > M > M_{sub}$. If $M > M_{sup}$, pinned solitons are created [91]. A is the spatial integral of the forcing function [91, 92]. For the Gaussian source of width G and magnitude Φ used in this study, $A = \Phi G \sqrt{\pi}$.

$$M_{sub} = 1 - \left(\frac{3 (\alpha/2)^2 A^2}{4\beta} \right)^{1/3} \quad (4.1)$$

$$M_{sup} = 1 + \left(\frac{3 (\alpha/2)^2 A^2}{16\beta} \right)^{1/3} \quad (4.2)$$

In this study we find the subcritical and supercritical velocity boundaries from the multifluid plasma scenario and compare them to the theoretical bounds given by Eqns. 4.1 and 4.2. This influences the orbits where space debris will create precursor or pinned ion acoustic solitons, which in turn impacts the feasibility of using the aforementioned plasma-based methods to detect space debris.

4.1.1 Ion Acoustic Dispersion Relation Derivation and Streaming Instability

Consider the linearized continuity and momentum equations for ions and electrons and the Poisson equation, given by Eqns. 4.3 - 4.7. Here, n_i , n_e , u_i , u_e , and ϕ are the perturbed quantities. The ions and electrons move with bulk velocity u_0 . The electrons can move by an

additional velocity, u_δ .

$$ku_0m_in_i + kn_0m_iu_i - \omega m_in_i = 0 \quad (4.3)$$

$$kn_iT_i - n_0m_iu_i(\omega - ku_0) = -kn_0Z\phi q_e \quad (4.4)$$

$$km_en_eu_\delta + ku_0m_en_e + kn_0m_eu_e - \omega m_en_e = 0 \quad (4.5)$$

$$kn_eT_e - n_0m_eu_e(\omega - k(u_\delta + u_0)) = kn_0\phi q_e \quad (4.6)$$

$$k^2\phi = \frac{q_e(Zn_i - n_e)}{\epsilon_0} \quad (4.7)$$

Simplifying this system, Eqns. 4.3 - 4.7 can be put into a matrix form given by Eqn. 4.8.

$v_{t,i}$ is the thermal speed of the ions, ($v_{t,i}^2 = \frac{k_B T_i}{m_i}$). For simplicity of comparison, Z is taken to be unity ($Z = 1$).

$$\begin{pmatrix} 1 - \frac{(\omega - ku_0)^2}{\omega_{p,i}^2} + \frac{k^2 v_{t,i}^2}{\omega_{p,i}^2} & -1 \\ -1 & 1 - \frac{k^2 v_{IA}^2}{\omega_{p,i}^2} - \frac{m_e}{m_i} \frac{(\omega - ku_\delta - ku_0)^2}{\omega_{p,i}^2} \end{pmatrix} \begin{pmatrix} n_i \\ n_e \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (4.8)$$

If n_i and n_e are nonzero, then the determinant of the matrix must be zero. This yields the dispersion relation. In the limit of $m_e \rightarrow 0$ and $T_i \rightarrow 0$, the standard Doppler shifted ion acoustic dispersion relation is recovered (Eqn. 4.9). Even though this is *not* taken in the limit as $u_\delta \rightarrow 0$, the dependence on u_δ in the dispersion relation disappears when $m_e \rightarrow 0$. In the absence of inverse Landau damping, this limit is mathematically stable.

$$k^2 \omega_{p,i}^2 v_{IA}^2 = (\omega - ku_0)^2 (k^2 v_{IA}^2 + \omega_{p,i}^2) \quad (4.9)$$

When $m_e \neq 0$, then a u_δ dependence is introduced to the dispersion relation that can form a streaming instability. This is not present when Maxwellian electrons are assumed because the Doppler shifted frequency term in the electron term of the dispersion relation is neglected (as modeled in Ref. [97]). In order to understand the basic streaming instability, for now it shall be assumed that $T_i \rightarrow 0$, $T_e \rightarrow 0$, and $u_0 \rightarrow 0$.

$$\omega^2 - \frac{m_e}{m_i} (\omega - k u_\delta)^2 \left(\frac{\omega^2}{\omega_{p,i}^2} - 1 \right) = 0 \quad (4.10)$$

Let $x = \omega/\omega_{p,i}$, $m^* = m_e/m_i$ and $\alpha = k u_\delta/\omega_{p,i}$. Eqn. 4.10 can be rewritten as Eqn. 4.11.

$$x^2 - m^* (x - \alpha)^2 (x^2 - 1) = 0 \quad (4.11)$$

Writing in polynomial in order of powers of x this becomes Eqn. 4.12. m^* and α are positive constants.

$$x^4 - 2\alpha x^3 + (\alpha^2 - 1 - m^{*-1})x^2 + 2\alpha x - \alpha^2 = 0 \quad (4.12)$$

Fig. 4.2 shows the roots of Eqn. 4.12 while varying α . The max growth rate (γ) occurs at $\approx \sqrt{m_i/m_e}$ which is 42 for hydrogen and has a value of ≈ 2 . This is a well-known result, sometimes known as the Buneman instability.

Now let us make no assumptions on taking the determinant of Eqn. 4.8. The result is the following expression:

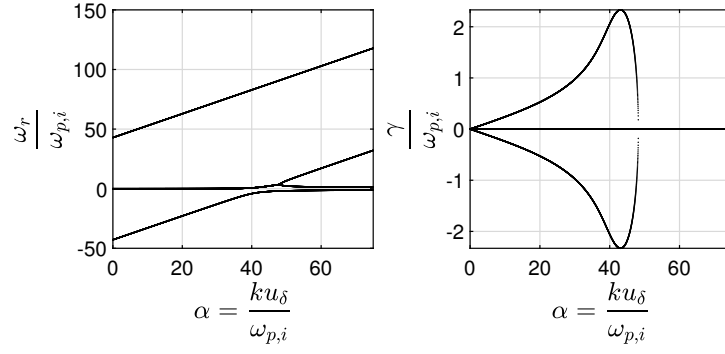


Figure 4.2: Growth rate of cold streaming instability for $m_i/m_e = 1836.1$.

$$\begin{aligned}
& mx^4 - \\
& x^3(2\alpha m + 4\beta m) + \\
& x^2(-\nu^2 + \alpha^2 m + 6\alpha\beta m + 6\beta^2 m - \mu^2 m - m - 1) + \\
& x(2\beta\nu^2 + 2\beta - 2\alpha^2\beta m - 6\alpha\beta^2 m + \\
& 2\alpha\mu^2 m + 2\alpha m - 4\beta^3 m + 2\beta\mu^2 m + 2\beta m) \\
& - \beta^2\nu^2 - \beta^2 + \mu^2\nu^2 + \mu^2 + \nu^2 + \alpha^2\beta^2 m - \alpha^2\mu^2 m - \\
& \alpha^2 m + 2\alpha\beta^3 m - 2\alpha\beta\mu^2 m - 2\alpha\beta m + \beta^4 m - \beta^2\mu^2 m - \beta^2 m = 0
\end{aligned}$$

The dimensionless quantities are defined as follows $\mu^2 = \frac{k^2 v_{t,i}^2}{\omega_{p,i}^2}$, $\nu^2 = \frac{k^2 v_{IA}^2}{\omega_{p,i}^2}$, $\alpha = \frac{ku_\delta}{\omega_{p,i}}$, $\beta = \frac{ku_0}{\omega_{p,i}^2}$, $x = \frac{\omega}{\omega_{p,i}}$, and $m = \frac{m_e}{m_i}$. This follows the same conventions as Eqn. 4.12. In the limit of $\mu = \nu = \beta = 0$, the instability is identical. For $\mu = 0.1$, $\nu = 1$, $\beta = 1$, and $m^{-1} = 1836.1$, Fig. 4.3 shows the roots of the full dispersion relation. One can see that the most unstable mode is still near $\alpha = 42$, although it has slightly increased. The growth rate is still $\gamma \approx 2\omega_{p,i}$.

In the classical plasma sheath, the ions are accelerated to the ion acoustic speed (sometimes

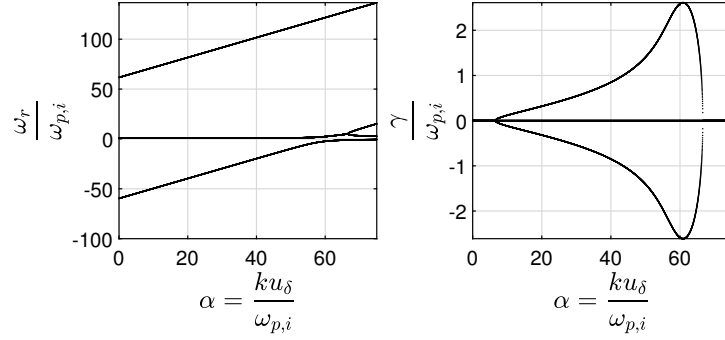


Figure 4.3: Growth rate of warm streaming instability for $m_i/m_e = 1836.1$.

called the Bohm velocity in this context) while the electrons are repulsed. So the difference in ion and electron streaming velocity can be approximated as $u_\delta \sim u_{IA}$. This means for $k < \lambda_D^{-1}$, an instability will be excited. It will grow at a rate comparable to the plasma frequency. As seen by comparing Figs. 4.3 and Fig. 4.2, the basic two stream instability is sufficient to explain the basic order of magnitude physics.

4.2 Methods

4.2.1 Numerical Methods

The multifluid description of a plasma is given by Eqns. 4.13 - 4.15. This is equivalent to the first three moments of the Vlasov equation with a Maxwellian velocity distribution function in each species. Perturbations along the magnetic field line are considered. Thus the multifluid equations can be placed in the electrostatic limit, $\mathbf{B} = 0$. A scalar pressure closure is used, i.e. $p_s = n_s k_B T_s$. $n_s = \rho_s / m_s$, where p_s is the pressure, ρ_s is the mass density, m_s is the species mass, k_B is the Boltzmann constant, n_s is the number density, and T_s is the temperature. The s subscript indicates the species (*ion* or *electron*). In the equations below, q_s is the species charge, $\mathbf{u}_s$ is the species bulk velocity, and $\mathcal{E}_s$ is the species total energy. Instead of directly enforcing a

Poisson equation, the field is allowed to evolve from Maxwell's equations. The species energy is also evolved in time.

$$\frac{\partial \rho_s}{\partial t} = \nabla \cdot (\rho_s \mathbf{u}_s) \quad (4.13)$$

$$\frac{\partial (\rho_s \mathbf{u}_s)}{\partial t} + \nabla p_s + \nabla \cdot (\rho_s \mathbf{u}_s \otimes \mathbf{u}_s) = \rho_s \left(\frac{q_s}{m_s} \right) (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) \quad (4.14)$$

$$\frac{\partial \mathcal{E}_s}{\partial t} + \nabla \cdot (\mathbf{u}_s (p_s + \mathcal{E}_s)) = \rho_s \left(\frac{q_s}{m_s} \right) \mathbf{u}_s \cdot \mathbf{E} \quad (4.15)$$

The `Gkeyll` v2.0 five-moment multifluid solver is used to solve the full multifluid equations. A summary of the numerical methods is given in [140, 141]. `Gkeyll` uses a finite volume scheme on the homogeneous parts of the fluid equations. Two nonphysical potentials (ψ and ζ) are introduced into Maxwell's equations to ensure the time evolution equations are hyperbolic, yielding Eqns. 4.16 - 4.19. As η and $\chi \rightarrow \infty$ (i.e. as the propagation speeds of the fictitious potentials become infinitely fast), this set of equations goes exactly to Maxwell's equations.

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} + \eta \nabla \psi = 0 \quad (4.16)$$

$$\epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} + \chi \nabla \zeta = -\mu_0 \mathbf{J} \quad (4.17)$$

$$\frac{1}{\chi} \frac{\partial \zeta}{\partial t} + \nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (4.18)$$

$$\frac{\epsilon_0 \mu_0}{\eta} \frac{\partial \psi}{\partial t} + \nabla \cdot \mathbf{B} = 0 \quad (4.19)$$

This is advantageous for computation. `Gkeyll` sets the propagation speeds of the fictitious

potentials to multiples of the speed of light so that they are numerically close to ∞ . Poisson's equation (Eqn. 2.3) is implicitly obeyed when $\mathbf{B} = 0$.

Fig. 4.4 demonstrates how Poisson's equation is implicitly obeyed by the `Gkeyll` simulation. From the free-stream flow component of the electric field (E_x), we calculate the left-hand side of Poisson's equation $\left(\frac{\partial^2 \phi}{\partial x^2}\right)$ with a finite differencing method. This is expected to be accurate to $\mathcal{O}(\Delta x^2)$ where Δx is the grid size. This is subtracted from the net free charge, which is just the difference in ion and electron densities. As seen in the rightmost subplot of Fig. 4.4, the remainder is at most on the same order as the error in the finite differencing of the field solution except near the source at $x = 64\lambda_D$. At the source, there is an implicit free charge that gives rise to the applied electric field.

4.2.2 Unforced Ion Acoustic Behavior

Previous work studying ion acoustic solitons initialize simulations with a density hump in the domain that is shown to decay into a train of ion acoustic solitons [140, 142, 143]. In this section, it is shown that this multifluid setup creates a similar situation that is very well modeled by the KdV equation. In this case, the initial conditions of the multifluid simulation are set to the following in the lab frame:

$$n_{i,e} = 1 + \exp\left(-\frac{|x - x_c|}{2}\right) \quad (4.20)$$

$$\mathcal{E}_{i,e} = \frac{n_{i,e} k_B T_{i,e}}{\gamma - 1} \quad (4.21)$$

$$u_{i,e} = M = 0 \quad (4.22)$$

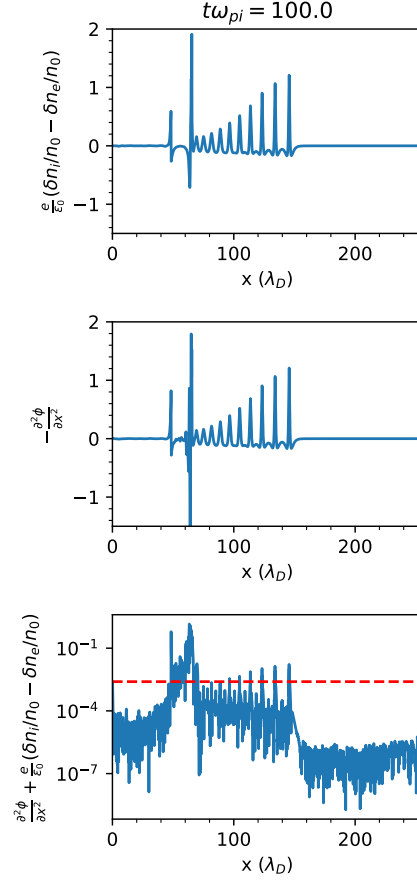


Figure 4.4: The top and middle plots indicate the left- and right-hand sides respectively of Poisson's equation in a multifluid simulation with parameters $G = 0.72$, $M = 1$, and $\phi_C = -0.975$ (quantities defined in *Simulation Setup*). The bottom subplot indicates the absolute error. The dashed red line in the rightmost subplot indicates $(\Delta x)^2$. This is the order of the expected error of $\partial^2 \phi / \partial x^2$ from taking the derivative using a second order finite difference method.

x_c is the x -coordinate of the source and γ is the gas constant. In the homogeneous KdV simulation, the initial condition is set to $u_0 = u_{0,\text{KdV}} \exp\left(-\frac{|x-x_c|}{2}\right)$. It was found that setting $u_{0,\text{KdV}} = 0.3$ shows good agreement with the multifluid simulation. Due to the transformations that convert the lab frame coordinates and dependent variables to the KdV frame coordinates and dependent variables, $u_{i,\text{lab frame}} = u_{\text{KdV}} + 1$. In the results, the initial condition is labeled in the lab frame to avoid confusion in comparing the plots.

The KdV space/time coordinates are also different from the multifluid simulation. The

KdV coordinates are $\xi = \epsilon^{1/2}(x - t)$, $\tau = \epsilon^{3/2}t$ where x is normalized by the Debye length and t is normalized by the plasma frequency. Thus, the ξ coordinate is moving at the ion acoustic velocity. The frame moves with speed $+u$ and has coordinates (t', x') . The computed solution is given on some uniform rectangular mesh. In this case, the mesh is (50×256) . The lab frame is defined by:

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} t' \\ x' - ut' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -u & 1 \end{pmatrix} \begin{pmatrix} t' \\ x' \end{pmatrix} \quad (4.23)$$

This can be done with an image shear mapping operation. The definition of a shear is given by Eqn. 4.24.

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x \\ mx + y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ m & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \quad (4.24)$$

If the following maps are made $(x', y') \rightarrow (t, x)$ and $(x, y) \rightarrow (\tau, \xi)$, then this equation reads:

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ m & 1 \end{pmatrix} \begin{pmatrix} \tau \\ \xi \end{pmatrix} \quad (4.25)$$

Comparing Eqns. 4.24 and 4.25 it is obvious that a shear with $m = -u$ is equivalent to this coordinate transform. By adding in the scale factor ϵ and the effect of working on a discrete grid, we arrive at the image transformation given by Eqn. 4.26. The `scikit-image` `AffineTransform` class is used to perform these image shifts [144].

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} \epsilon^{3/2} & 0 \\ -(\Delta t/\Delta x) \epsilon^{1/2} & \epsilon^{1/2} \end{pmatrix} \begin{pmatrix} \tau \\ \xi \end{pmatrix} \quad (4.26)$$

Fig. 4.5 shows a numerical example of this transformation. A solution in a frame moving with $c = 1$ is sheared into the lab frame. The upper left hand of the lab frame domain is not present in the moving frame domain, so it is filled in with zeros. Section 4.2.3.

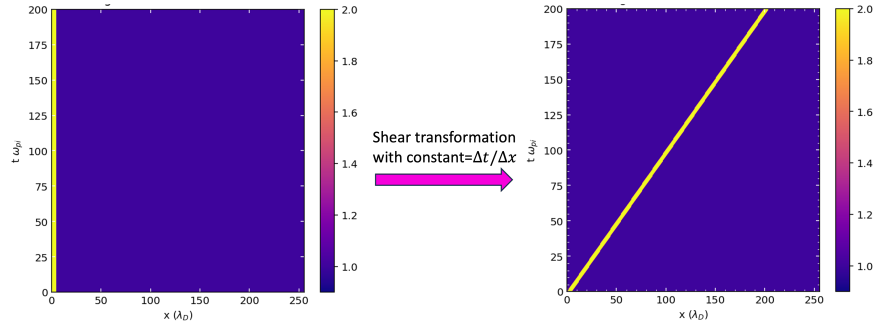


Figure 4.5: Example of shear transformation to go from a moving to a lab reference frame on a rectangular grid. The left subplot is moving with a speed of $c = 1\Delta x/\Delta t$ relative to the lab frame (right).

4.2.3 Forced Ion Acoustic Behavior

In the second part of this study, the density hump initialization is replaced with an applied electric field that represents the charged source. The background plasma is initialized with a leftward moving flow. Precursor and pinned solitons will be created from this setup. The size of the source and speed of the background flow are varied to numerically compute the properties of the created solitons in different flow regimes.

4.2.3.1 Simulation Setup

A static electric field is imposed in the domain that approximates the potential on the debris surface. In a frame moving with the debris, this electric potential is given by a Gaussian, $\phi_{\text{applied}} = \phi_C e^{-(x/G)^2}$ where $G = a/\lambda_D$, a is the debris radius, and ϕ_C is the charged potential of the debris (often $\phi_C < 0$). The magnetic vector potential can be taken as $\mathbf{A} = 0$, so the electric field can be written as simply the gradient of the electric potential, $E = -d\phi/dx$. The applied electric field is set to $-d\phi_{\text{applied}}/dx$, which is given by Eqn. 4.27.

$$E_{\text{applied}} = -\frac{d\phi_{\text{applied}}}{dx} = -\frac{2\phi_C}{G^2} \exp[-(x/G)^2] \quad (4.27)$$

The charging model of an idealized conducting sphere [93] is utilized to set the magnitude of the applied potential. The charging model is given by an implicit equation (Eqn. 4.28) in dimensionalized units. Φ_s is the surface potential. ϕ_C is set to the nondimensionalized sheath potential at one Debye length, i.e. $\phi_C = \exp(-1) \frac{e\Phi_s}{k_B T_e}$.

$$\exp\left[\frac{e\Phi_s}{k_B T_e}\right] = \sqrt{m_e/m_i} \left(\frac{G+1}{G}\right)^2 \left[1 - \frac{(G+1)^2 - G^2}{(G+1)^2} \exp\left(\frac{G^2}{(G+1)^2 - G^2} \frac{e\Phi_s}{k_B T_e}\right)\right] \quad (4.28)$$

In the setup used in this study, the Python script `solve_surf_potential.py` solves Eqn. 4.28 using a zero-finding routine. ϕ_C is provided to the `Gkeyll` input file. This charging model does *not* take into account $T_e \neq T_i$ effects. This may lead to inaccuracy of the following results in mapping spherical debris to soliton size. However, since the focus of this study is on

the velocity bounds, this effect is unimportant. The fKdV equation mapping of source function size to soliton characteristics *already* assumes that $T_e \gg T_i$. The effect of adding in temperature effects is just to add a damping coefficient to represent Landau damping. Furthermore, there is the possibility that the local non-Maxwellian behavior of the particle velocity distribution functions (i.e. from solitons) may itself change the particle fluxes and charging equilibrium. The effect of this possible feedback loop is not accounted for in this setup as this is an activate debate in the community.

The initial energy of each species is set to $\mathcal{E}_{s,0} = \frac{1}{2}\rho_{s,0}u_{s,0}^2 + p_{s,0}/(\gamma - 1)$. The following quantities are set as the initial conditions in the simulation:

1. $\epsilon_0 = 1$

2. $m_{\text{proton}} = 1$

3. $n_{e,0} = n_{i,0} = 1$

4. $T_e = 1$

5. $\gamma = 5/3$

6. $T_i = T_e/100 \ll T_e$

7. $m_{\text{electron}} = m_{\text{proton}}/1836.15267 = 1/1836.15267$

8. $n_{i,e} = n_{(e,i),0} + \exp\left(-\frac{|x-x_c|}{2}\right)$

9. $\mathcal{E}_{i,e} = \frac{1}{2}\rho_{i,e}u_{i,e}^2 + \frac{n_{i,e}k_B T_{i,e}}{\gamma-1}$

10. $u_{i,e} = -M$

11. $E_x = 0$

The applied field is set to the value in Eqn. 4.27. Using these values for fundamental physical constant, the Debye length (λ_D) and ion plasma frequency ($\omega_{p,i}^{-1}$) are both equal to one.

4.2.3.2 Analyzing Precursor Simulation Data

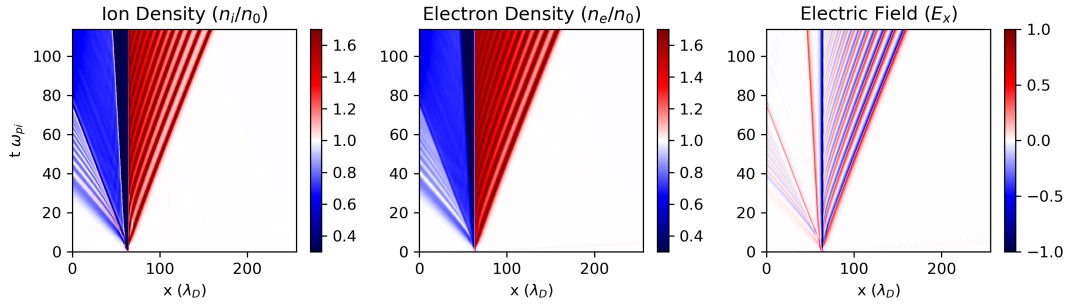
Fig. 4.6 shows a representative simulation in the precursor velocity regime. The source center is located at $x_c = 64\lambda_D$. A copy boundary condition is used on both boundaries. It sets ghost cells outside the boundary to copy the cells in the boundary. Copy boundary conditions on the left boundary absorb the left-running wake structure. On the right boundary this has the effect of continuously injecting plasma at a constant flow rate into the simulation. Precursor solitons can create nonphysical reflections at the right boundary. In cases where this is observed, only the time steps before the boundary reflection are used for data analysis. The simulation in Fig. 4.6 ends before the precursors can reach the right boundary.

To lend a physical perspective, we shall briefly comment on representative dimensional quantities. Since the relation from debris properties and location in the ionosphere to nondimensional quantities is not unique, we shall at first consider a particular location in the same orbital plane as the Iridium-Kosmos collision. Using the International Reference Ionosphere [3], at 145° longitude on June 11, 2024 at 10:00 UTC, 10° latitude, and 800 km altitude the electron density was $1.2 \times 10^5 \text{ cm}^{-3}$, the electron temperature was 1867 K, and the ion temperature was 1342 K. The primary ion species was O^+ . This would make the Debye length 9 mm and the ion acoustic speed 1750 m/s. The electric potential (ϕ) is given in units of $k_B T_e / e \sim 0.16 \text{ V}$ in this plasma. Under these conditions, the simulation in Fig. 4.6 corresponds to a 6 mm radius piece of debris moving 1750 m/s relative to the background plasma that itself corotates with the Earth at a speed

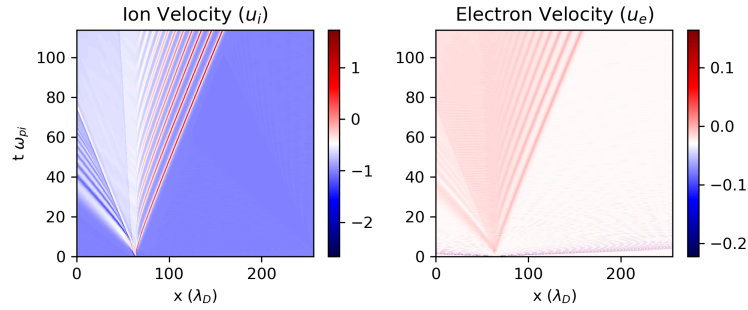
≤ 520 m/s at 800 km altitude. If the orbit was aligned with the corotation, then the debris is going 2270 m/s. However, this depends on the inclination and eccentricity of the orbit. The electric field in Fig. 4.6a is given in units of $0.16 \text{ V}/\lambda_D \sim 27 \text{ V/m}$. One caveat is that this simulation was done with Hydrogen as the dominant ion species instead of Oxygen. This modifies the charging and sound speed slightly. At 800 km altitude, a circular orbit would have a speed of 7500 m/s. The semimajor axis of an orbit with a speed of 2270 m/s at 800 km is below the surface of the Earth. So this represents a suborbital object at the site of the Iridium-Kosmos collision. As the periapsis of this hypothetical debris increases, the Mach number correspondingly increases. Similarly, at higher altitudes the electron temperature increases while the orbital speed decreases. This has the effect of lowering the Mach number. As found in Ref. [100], above 1200 km the Mach number lowers enough to create precursor solitons using the velocity bounds that are being evaluated in this chapter.

For each simulation in this part of the study, the following characteristics are recorded: soliton amplitude, soliton width, observed flow regime, precursor soliton speed, and precursor limit cycle. The soliton amplitude and width is recorded in ion density, electron density, and electrostatic potential. Fig. 4.7 shows how the amplitudes and widths are obtained from the simulation data. A peak finding library is used to identify the local maxima of the solitons. The average of the last four maxima is recorded as the amplitude. The average of the distances between the last four maxima is recorded as the width. The standard deviation of both measurements is recorded as the uncertainty. The solitons take time to develop to their maximum amplitude. This is why only the last four peaks are used.

The limit cycle is the period between the generation of precursor solitons. This is found by marking the times of the start of the precursor soliton and averaging over the interval between



(a) Density colors are limited to the colorbars shown. These were chosen to center around one (for n_i and n_e) and zero (for E_x) while illustrating the global maximum and minimum values.



(b) Bulk velocities of the ion and electron species (u_i and u_e).

Figure 4.6: Generation of precursor solitons in multifluid simulation. In this instance, $G = 0.72$, $M = 1$, and $\phi_C = -0.975$. The background flow condition is in the $-x$ direction (right to left).

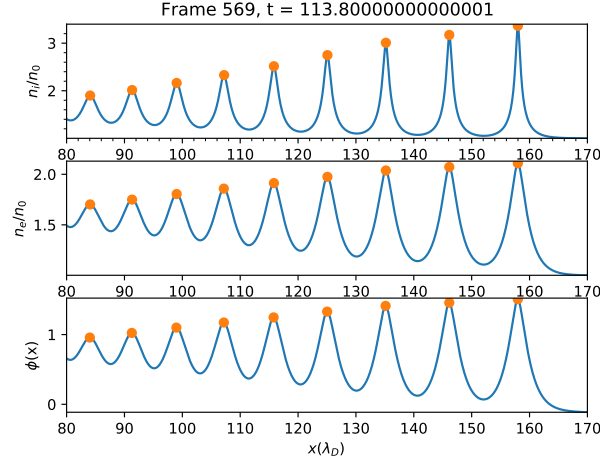


Figure 4.7: Peak-finding the amplitudes of the solitons for the same simulation as in Fig. 4.6. The rightmost four peaks are used to find the amplitude of the ion, electron, and electrostatic potential solitons. The x-value is used to find the width. The standard deviations of both are used to find the uncertainty of the measurement.

these times. The “start” of the precursor soliton is subjectively where the precursor intersects with the line $x = 64\lambda_D$ where the source is centered. The uncertainty is found by taking the standard deviation of these intervals. Near the source, the “start” is not always clearly identifiable. This is a major source of noise in this quantity. The observed flow regime refers to distinguishing between small amplitude linear waves (wakes), precursor solitons, and pinned solitons.

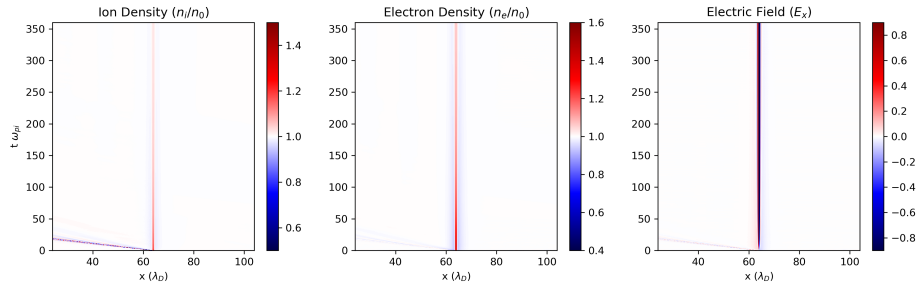
4.2.3.3 Error Analysis of Soliton Speed

The speed of the soliton is calculated by measuring the slope from two points on the same soliton on a spacetime heatmap plot of the ion density, such as in Fig. 4.6a. The uncertainty in this measurement is driven by the width of the soliton, i.e. by visually identifying the center. If the slope $m = \frac{y_2 - y_1}{x_2 - x_1}$ has only one uncertain measurement, y_2 , then $\sigma_m \approx |m|\sigma_{y_2}/(y_2 - y_1) \approx \sigma_{y_2}/(x_2 - x_1)$. σ_m is used to calculate the error bars in Fig. 4.21.

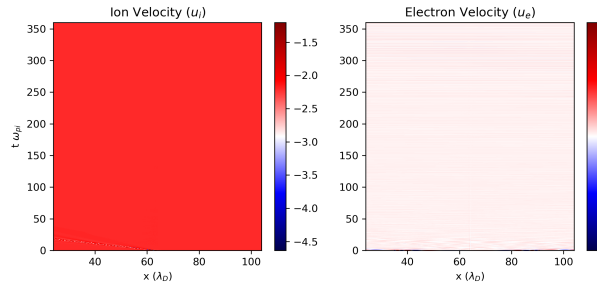
4.2.3.4 Analyzing Pinned Simulation Data

Fig. 4.8 shows a characteristic example of a simulation in the pinned soliton regime. As can be seen in Fig. 4.8a, there is a constant density structure centered on the source at $x = 64$. This structure is largely a positive hump in the ion density and electron density. The electric field has a bipolar structure from the gradient of a Gaussian potential. Fig. 4.8b demonstrates that the density structure is not apparent in the bulk velocity. The soliton is in fact pinned to the source. Except for a high frequency electron plasma oscillation, the background bulk velocity is the initial condition. Interestingly, this appears to violate the KdV theory (Eqn. 2.26) because the fluid velocities do not follow the density perturbations. If the same plasma conditions and assumptions are used as in the discussion around Fig. 4.6 (800 km altitude, $n_e = 1.2 \times 10^5 \text{ cm}^{-3}$, and $T_e = 1867\text{K}$) then Fig. 4.8 describes a 3 mm radius debris moving 4.37 km/s. The electric potential in Fig. 4.9 would have an amplitude of -0.14 V .

Fig. 4.9 demonstrates how the amplitude and width of the pinned soliton is calculated. A local extremum library is again used to find the local maximum and minimum. These are sorted and the domain is clipped so the maximum at the top of the pinned structure is found in the ion and electron densities. The minimum is found in the electrostatic potential. These are recorded as the soliton amplitude. The height difference between the maximum and the value next to it is stored as the uncertainty. The local minimum on the right of the hump is found for the ion and electron densities as is the local maximum for the electrostatic potential. The difference between these and the maximum of the hump is stored as the half-width of the pinned soliton. The width reported is the half-width measured multiplied times two. The grid spacing is used as the uncertainty.



(a) Density colors are limited to the colorbars shown. These were chosen to center around one (for n_i and n_e) and zero (for E_x) while illustrating the global maximum and minimum values. The far left and right of the domain are clipped in order to zoom in on the source at $x = 64$.



(b) Bulk velocities of the ion and electron species (u_i and u_e). The far left and right of the domain are clipped in order to zoom in on the source at $x = 64$.

Figure 4.8: Generation of pinned solitons in multifluid simulation. In this instance, $G = 0.36$, $M = 2.2$, and $\phi_C = -0.94$. The background flow condition is in the $-x$ direction (right to left).

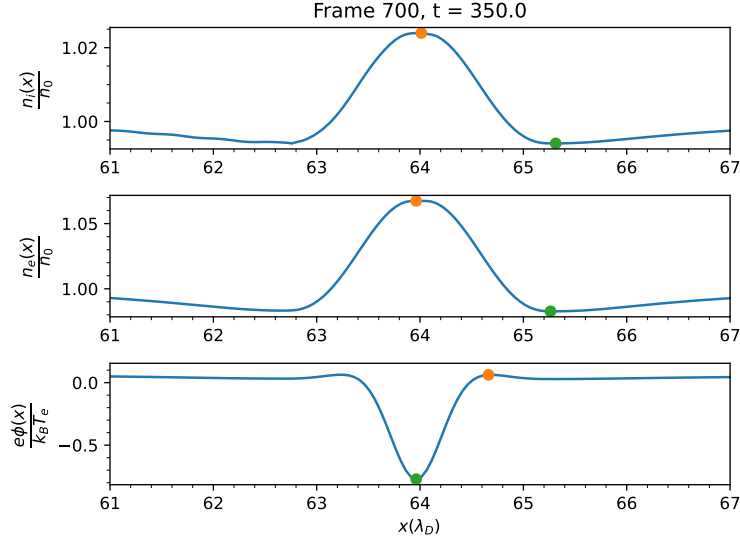


Figure 4.9: Local maximum and minimum are used to calculate the amplitude and half-width of the pinned soliton.

4.3 Results

Unforced Ion Acoustic Behavior

In this section, results from initializing the simulation with the density hump given by Eqn. 4.22 are shown and compared to direct KdV simulations with the coordinates adjusted to be in the same reference frame. The change of reference frame procedure is given in the methods section.

As can be seen in Fig. 4.10, the KdV simulation and the multifluid simulation with a density hump initial condition are in good agreement with the behavior of the linear ion acoustic waves and the ion acoustic soliton. The KdV simulation is transformed into the lab frame with a shear operation, and thus, does not wrap around the periodic domain as in the multifluid simulation. The remaining portion of the center hump after the solitons have detached is not accounted for by the KdV equation nor does it interact with the solitons in the multifluid simulation. The left-running and right-running fan of smaller waves trailing the two solitons are also recreated in

the multifluid simulation. The KdV simulation has periodic boundary conditions in its original domain. However, when it is shifted using the shear transformation, the periodicity is lost. This is why it does not wrap around the domain at $t\omega_{pi} \approx 50$ and $t\omega_{pi} \approx 145$. The domain $t\omega_{pi} \geq 150$ is blank in the KdV simulation because the inverse map of the shear transformation does not map to a point in the original KdV domain.

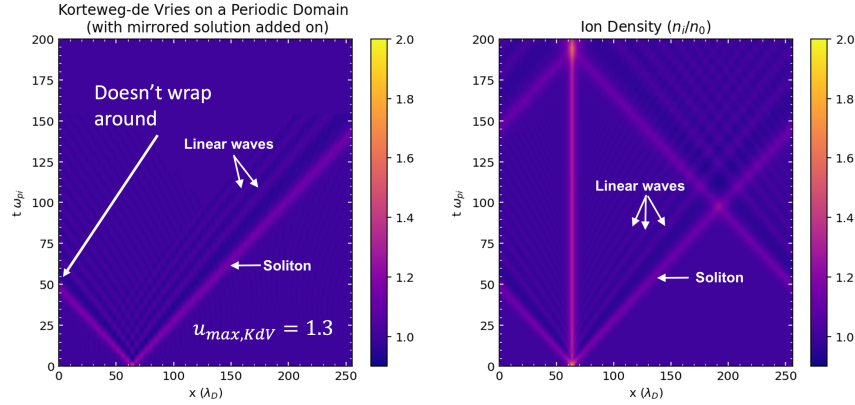


Figure 4.10: Comparison of unforced KdV equation transformed to lab frame (left) and multifluid simulation (right). Both are simulated with periodic boundary conditions, but the left subplot's boundary is no longer periodic after the frame transformation.

In the left subplot of Fig. 4.10, the KdV solution is shown with its mirror solution about the axis $x = 64$ added on. This is because the KdV equation can only capture the solitons moving in *one direction*. In this case, the KdV equation only captures the right-running solitons even when left-running and right-running solitons are actually generated by the multifluid simulation.

In comparison to previous literature [140, 142], it has been shown that the ion acoustic solitons decaying from the initial hump are *exactly* describable by the KdV equation *given the appropriate coordinate transform*, rather than characteristically modeled. However, the KdV equation only describes the moving solitons in a particular direction. It does *not* describe the stationary solutions *in the lab frame*. Curiously, the KdV equation is Galilean invariant [83]. This suggests that it holds true in all reference frames disregarding relativistic effects. However,

the fixed relation between the amplitude of the soliton and the speed of the soliton suggests that there is a preferred reference frame $[\xi = \epsilon^{1/2}(x - Mt)]$ where this relation is true. We suggest that *while the KdV equation is Galilean invariant, it only describes density perturbations in one reference frame.*

Forced Ion Acoustic Behavior

Now the forced simulation results shall be presented. These detail the observed precursor and pinned soliton generation from a charged source under varying flow conditions. Where these results differ from the fKdV model will be highlighted and discussed.

Fig. 4.11 shows the fluid energy and temperature of the ions and electrons in the same simulation as Fig. 4.6. The precursor solitons can be seen to have a higher temperature than the initial condition. No temperature difference is discernible with the ions. However, the ions do have a lower energy than the free stream (mostly kinetic) energy. The wake can be seen to have a near region of warm ions and a far region of cooler ions, although they are still hotter than the free stream.

Fig. 4.12 shows the electrostatic potential and electron density from the same simulation. The $\exp \phi$ line is the effective Boltzmann electrons calculated with the electrostatic potential and electron temperature. It can be seen that this curve is in good agreement with the actual electron density, indicating that their behavior is largely Boltzmann.

Taken together, Figs. 4.11 and 4.12 indicate that the electrons follow a Boltzmann distribution, however they are heated. In the previously used simulation approach of Ref. [97], there is no mechanism for the electrons to heat. Therefore the produced soliton would have a higher

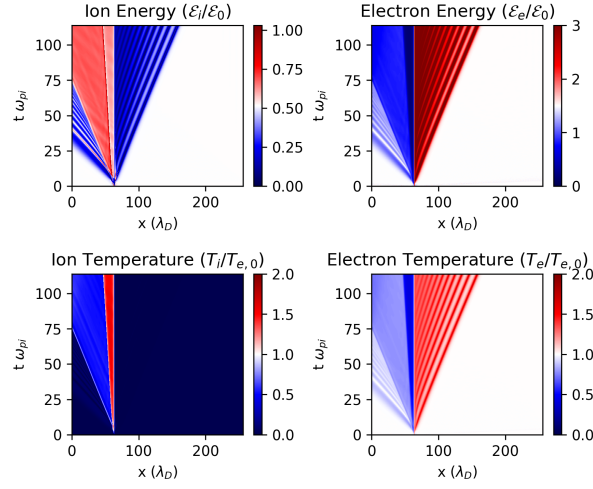


Figure 4.11: Fluid energy and temperatures of ions and electrons with $G = 0.72$, $M = 1$, and $\phi_C = -0.975$. $\mathcal{E}_0$ refers to the nondimensionalization constant. In this case $\mathcal{E}_0 = \frac{n_0 k_B T_e}{\gamma - 1}$.

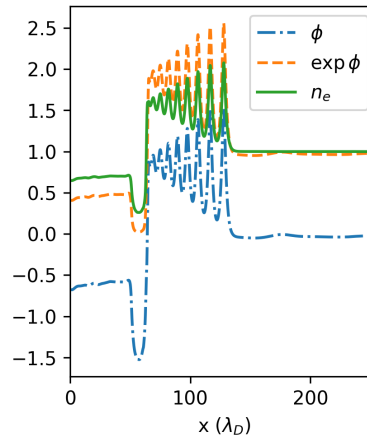


Figure 4.12: Comparison of nondimensionalized electrostatic potential, electron density, and effective Boltzmann electron density at $t = 80/\omega_{p,i}$ in multifluid simulation with $G = 0.72$, $M = 1$, and $\phi_C = -0.975$.

electron density than what is observed here.

The properties of the solitons generated by the applied field representing a charged source are now presented as a function of the parameter sweep. The subcritical and supercritical velocities (M_{sub} and M_{sup}) are also estimated to be between the linear wave and precursor soliton speeds and between the precursor and pinned soliton speeds, respectively. For each of the sizes considered in Table 4.1, simulations were run in increments of $0.1M$ from $0.1 - 1.7M_{\text{sup}}$ with other points as needed to resolve the velocity bounds. For some source sizes, these bounds are extended in either direction to observe a given transition point or trend more precisely. Using the same plasma conditions at 800 km that were considered earlier, this corresponds to debris with a radius of 0.6 - 8 mm radius moving from 0.02 - 2.8 km/s relative to the corotating plasma. The full list of simulation parameters is given in the dataset. The full dataset can be obtained using the methods in the “Data Availability” section.

Table 4.1: Theoretical subcritical (M_{sub}) and supercritical (M_{sup}) velocities using Eqns. 4.1 & 4.2 for various debris sizes (G) used in this study. Velocities are in units of ion acoustic velocity and rounded to the fourth decimal place.

G	M_{sub}	M_{sup}
0.07	0.8300	1.0771
0.18	0.6798	1.2017
0.20	0.6562	1.2166
0.36	0.4870	1.3232
0.72	0.1658	1.5255
0.91	0.0115	1.6227

Fig. 4.13 reports the observed amplitudes of the precursor solitons in this study as a function of their predicted supercritical velocity found using the method in section 4.2.3.2. In general, as the speed of the background flow increases, the amplitude of the precursor solitons generated also increases. Similarly, as the size of the source increases, the amplitude of the precursor soli-

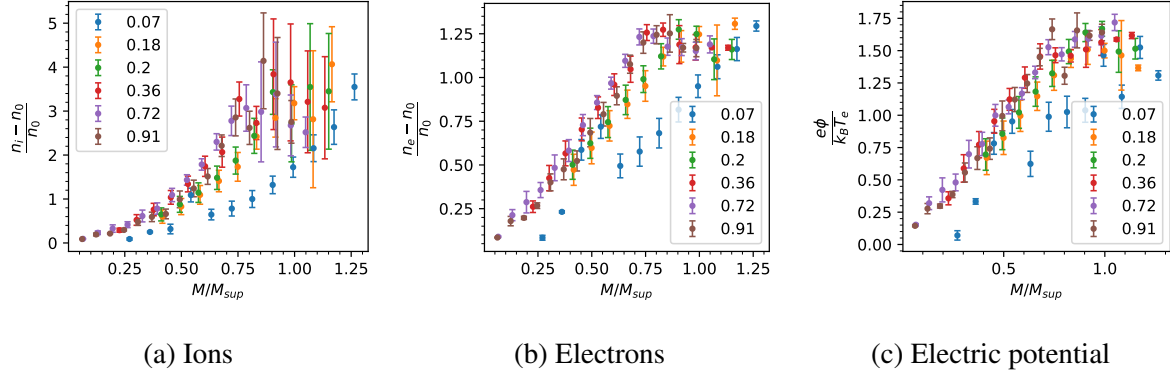


Figure 4.13: Amplitudes of precursor solitons as seen by the ions, electrons, and potential. The error bars indicate the standard deviation of averaging over the last four solitons in a frame. The legend indicates the amplitude for various values of source size, G .

tons also increases. It can be seen that for all source sizes, there exists points where $M/M_{\text{sup}} > 1$. The M_{sup} used to normalize the flow velocity for each source size is the same as the theoretical M_{sup} in Table 4.1. If a precursor amplitude is recorded for $M > M_{\text{sup}}$, then precursor solitons are occurring above the expected flow regime. Quantifying and explaining this is done later in this study.

In the electron amplitude, there is a change in the trend of larger precursor solitons for increased flow speed at a given source size between $0.8 < M/M_{\text{sup}} < 1$ for the source sizes > 0.1 . Instead, the soliton amplitudes level off. This is also observable, although less pronounced, in the ion and potential amplitude plots. This occurs near the precursor to pinned transition speed. As explained later and seen in Fig. 4.16, there is a secondary effect that occurs which resembles wave breaking. This effect happens when the ion amplitude increases while the electron amplitude levels out.

Fig. 4.14 shows the amplitude-flow speed relations of the pinned solitons. Larger sources produce larger pinned solitons. Conversely to the precursors, as the flow speed increases, the pinned soliton amplitude decreases. Both of these behaviors are consistent with directly simu-

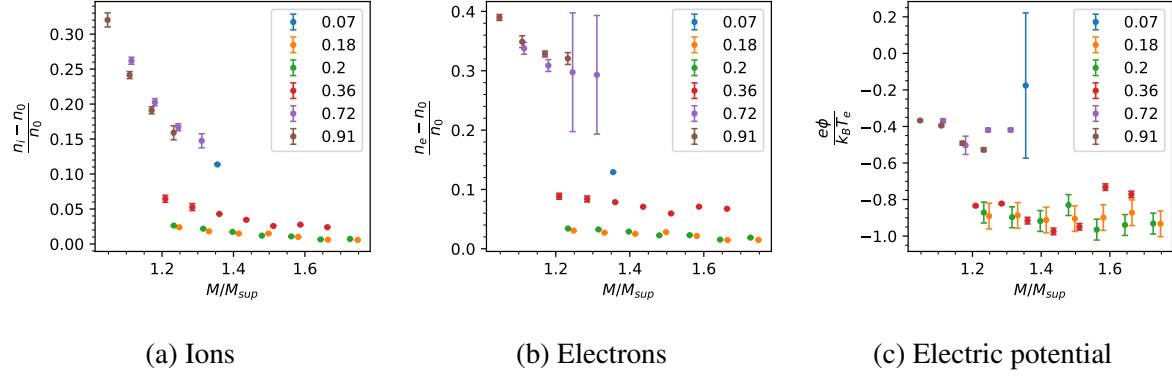


Figure 4.14: Amplitudes of pinned solitons as seen by the ions, electrons, and potential. The legend indicates the source size, G .

lating the fKdV equation as in Ref. [1]. The electrostatic potential is normalized at each time so that $\frac{1}{L} \int \phi dx = 0$, even in the “far-field” where there is no wake nor solitons resolved in the simulation domain. This probably leads to an inconsistency in the electrostatic potential since it is being set to 0 in the wake as if it is far from the source. Thus, the trend in the right subplot of Fig. 4.14 is not as smooth as the ion or electron amplitude.

Both Fig. 4.13 & 4.14 show that to first order $e\phi/(k_B T_e) \sim \delta n_i/n_0 \sim \delta n_e/n_0$. This is also predicted by the fKdV equation. This study supports that this approximation is mainly true with the fulfillment of Poisson’s equation noted below. It should also be noted that pinned solitons have a much smaller amplitude than precursor solitons. This was observed in Ref. [1] using direct simulations of the fKdV equation.

The ion to electron max soliton amplitude also increases with overall soliton size. This is to be expected if $e\phi/(k_B T_e) \sim \delta n_i/n_0 \sim \delta n_e/n_0$ in order to satisfy Poisson’s equation. Fig. 4.15 plots this phenomenon. It should be noted that the code does not directly solve for the electrostatic potential. ϕ is derived from the simulation results for E . As mentioned above, the electrostatic potential (ϕ) is set so the mean value in every frame is zero. In the latter time steps

of some simulations, this is not expected to match $\phi \rightarrow 0$ at $x = \infty$ due to the wake occupying 1/4 of the domain. This is because in the wake, $\phi \neq 0$ due to the species imbalance.

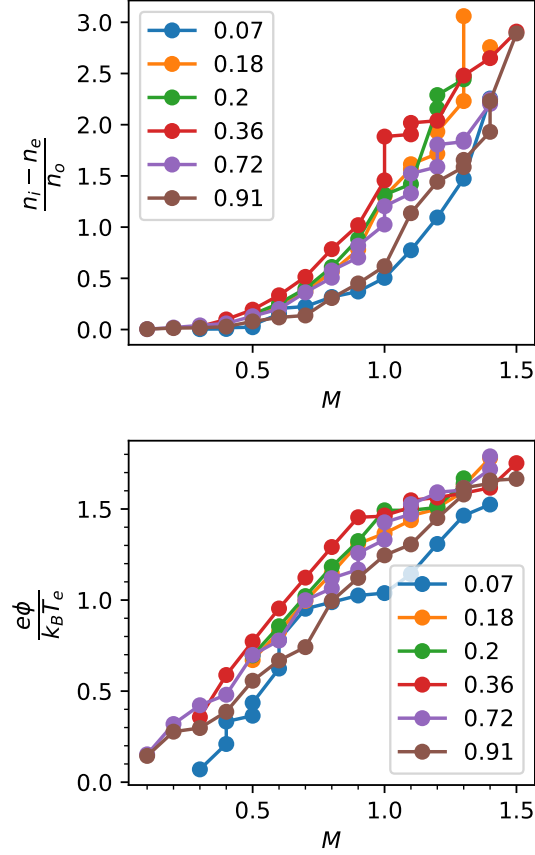


Figure 4.15: Difference between $n_i/n_0 - 1$ and $n_e/n_0 - 1$ of precursor soliton maxima in the top subplot. The bottom subplot indicates the maximum potential. The error bars in the amplitude are not shown. The legend indicates values of source size, G .

Just before the transition from precursor to pinned soliton generation, the amplitude of the solitons, especially the ion amplitude, is highly uncertain. This is due to an observed effect where the solitons will grow and collapse periodically. Fig. 4.16 shows time steps from a simulation where this effect is seen. The behavior resembles nonlinear steepening followed by wave breaking which has been reported for large magnitude ion acoustic waves [145, 146]. This behavior would violate the weakly nonlinear assumption in deriving the KdV equation and thus, this phys-

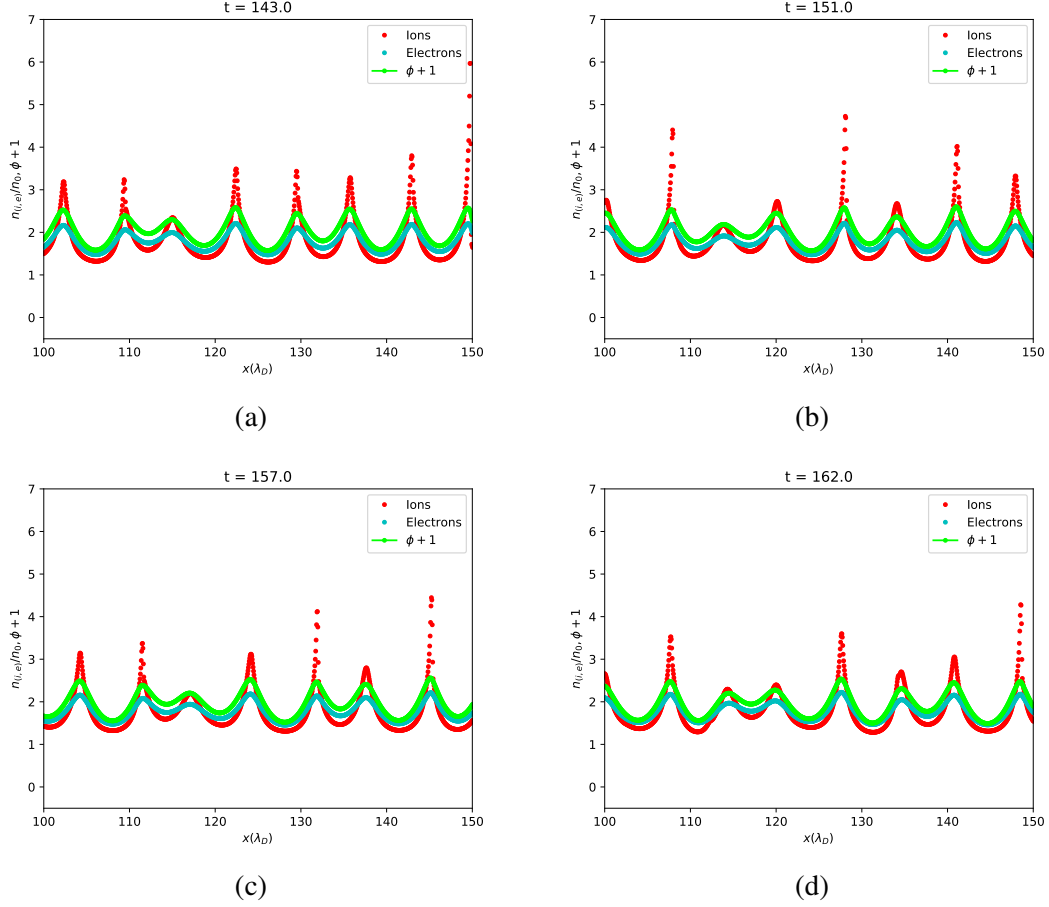


Figure 4.16: Ion density, electron density, and electrostatic potential for a source of size $G = 0.36$, magnitude $\phi_C = -0.9404$, and a background flow of $M = 1.5$. Each subplot is a different time step. The x -domain is zoomed in far from the source in order to observe the “jumping” behavior of the solitons. The numerical solution is represented with dots instead of lines in order to better observe the steepening on the right side of the ion density.

ical effect cannot be observed using the KdV equation. However, the effect can be observed using the multifluid simulation as demonstrated by the very sharp gradient along the right side of the steepened solitons in Fig. 4.16. Further work should consider kinetic simulations to directly observe this potential wave breaking. For the purposes of the remainder of this study, this phenomenon is considered to be noise. Since this effect is observed after the solitons detach from the source, it is not expected to impact the generation regimes of the solitons.

The observed peak-to-peak widths of the precursor solitons are plotted in Fig. 4.17. As the

flow decreases and/or the source gets smaller, the width of the precursor soliton increases. This is the opposite of the amplitude relations. The large error bars may be due to the solitons being close enough together that they overlap slightly. This would cause the peak-to-peak width to be an underestimate of the actual soliton width.

A feature of the one soliton solution is that the soliton parameter, defined as the amplitude times the width squared ($\Pi = a_s w^2$) is a constant. This can be seen to be a constant by comparing the coefficient in front of and argument of the sech^2 term in Eqn. 1.52. This was predicted theoretically [82, 147] and observed experimentally [9] 50 years later. Therefore, the width and amplitude should trend opposite of each other. Fig. 4.18 shows the soliton parameter observed in simulation with error bars derived from the amplitude and width error bars. Using the standard formula for propagating Gaussian errors, the soliton parameter error is estimated to be $\sigma_\Pi = \sqrt{\left(\frac{\partial \Pi}{\partial w}\right)^2 \sigma_w^2 + \left(\frac{\partial \Pi}{\partial a_s}\right)^2 \sigma_{a_s}^2} = \sqrt{4a_s^2 w^2 \sigma_w^2 + w^4 \sigma_{a_s}^2}$. σ_{a_s} is the uncertainty in the amplitude. σ_w is the uncertainty in the width. The error dominates the soliton parameter in the precursor solitons. In using the soliton parameter as a comparison to fKdV theory, there appears to be nothing contradictory. However, these simulations do not resolve the width of the solitons well enough to substantiate any claims about the soliton parameter. This result is important to report because future simulations will need to treat this analysis carefully. Current literature does not address this difficulty.

The width of the pinned solitons is shown in Fig. 4.19a along with the soliton parameter in Fig. 4.19b. The same inverse of the amplitude trends is observed here. It can also be seen that within the uncertainty bounds of the soliton parameter, the soliton parameter for a given source size is constant over all M/M_{sup} . This is further confirmation that the structures in the simulation are indeed solitons and is consistent with literature. The width uncertainty for the pinned solitons

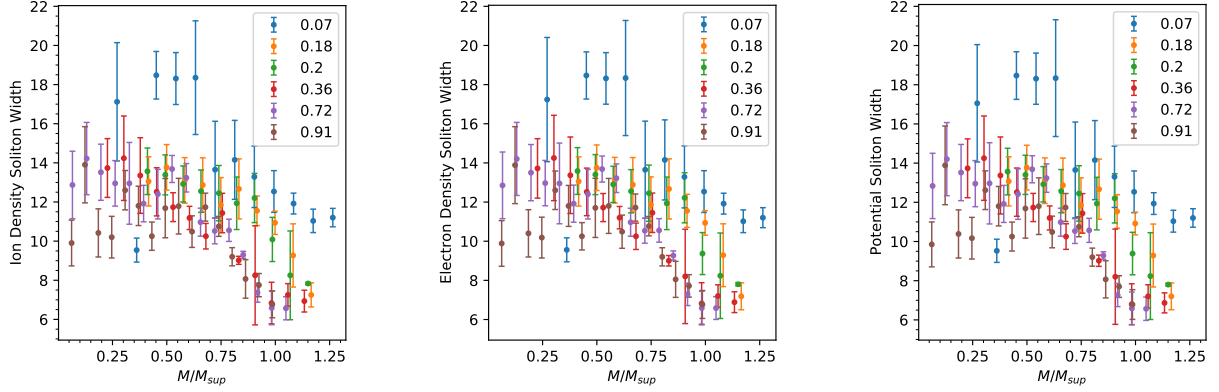


Figure 4.17: Width of precursor solitons as measured peak to peak of local maxima. Error bar is standard deviation of last 4 soliton peak-to-peak measurements. Legend indicates source size, G .

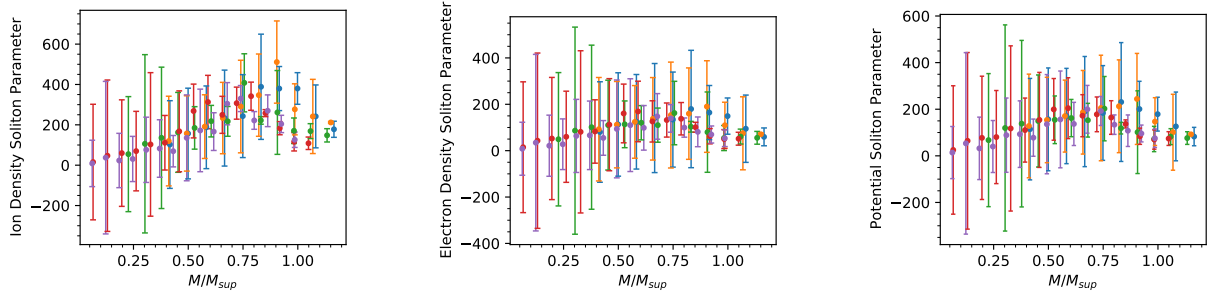
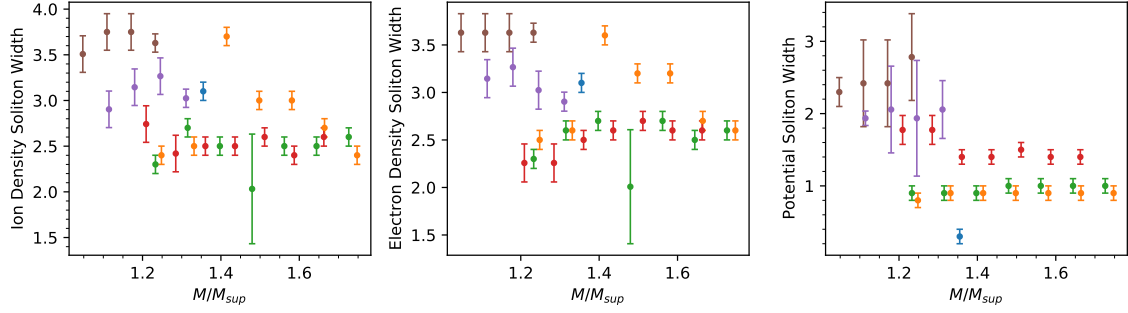


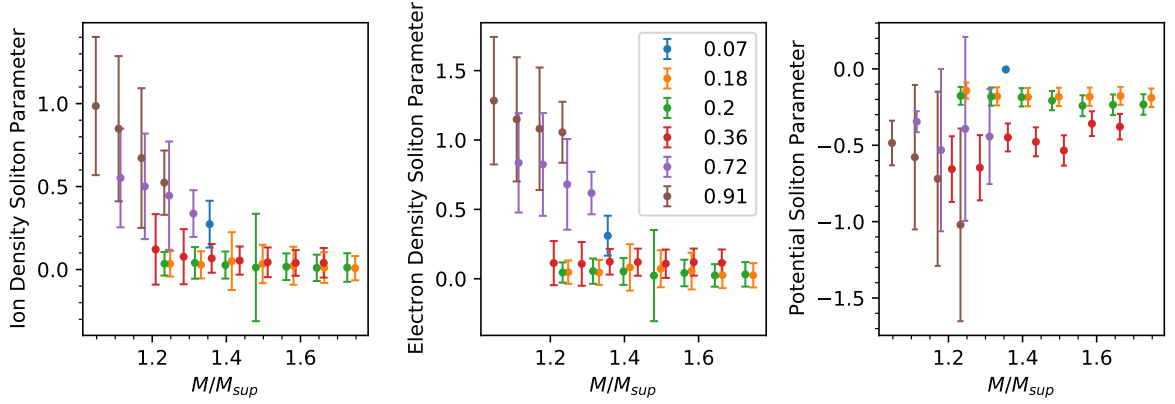
Figure 4.18: Soliton parameter ($\Pi = a_s w^2$) observed in simulation of the precursor solitons. The coloring indicates the size and uses the same legend as Fig. 4.17. The error in the soliton parameter is derived from the error in the amplitude and widths.

is lower than the precursor solitons because there is only one. The solitons can't have some finite overlap that would add uncertainty.

Fig. 4.20 shows the limit cycle period and time to first generation trends observed in the multifluid simulation. The limit cycle period is defined as the time between emissions of precursor solitons from the source. The results here contradict results from direct simulation of the fKdV equation in Ref. [1]. In Ref. [1], a U-shaped trend was observed in the limit cycle and the time to first generation had a super-logarithmic monotonically increasing behavior. However, in this study the limit cycle is approximately linearly increasing while the time to first generation is more or less constant except a few outliers near M_{sup} . The U-shaped trend in Ref. [1] varied

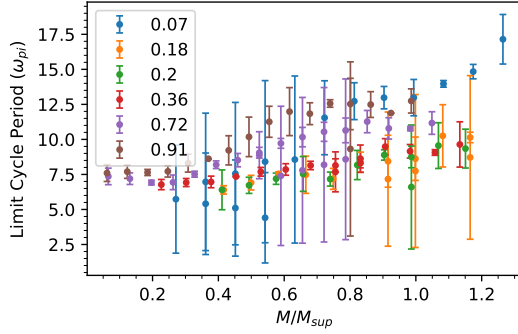


(a) Observed width of the pinned solitons. Instead of peak-to-peak, these are measured peak-to-trough and then multiplied by two.

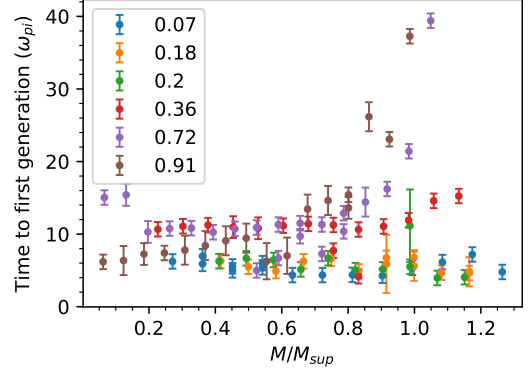


(b) Soliton parameter ($\Pi = a_s w^2$) observed in simulation of the pinned solitons. The error in the soliton parameter is derived from the error in the amplitude and width.

Figure 4.19: Width properties of the pinned solitons. The legend in subplot (b) - middle pane is the same in all subplots and panes in this figure.



(a) Generation periodicity (limit cycle period) of precursor solitons.



(b) Time to first generation of precursor soliton

Figure 4.20: Temporal properties of precursor solitons.

over two orders of magnitude unlike what is observed here. This discrepancy is unexplained. It was found in Ref. [91] that for a Dirac delta source, the fKdV limit cycle scales $\mathcal{O}(\phi_C^{-1})$ and $\mathcal{O}(M^{-3/2})$. This is also inconsistent with the multifluid simulation.

From first principles the limit cycle period should be $\sim \gamma^{-1}$ where γ is the growth rate of the unstable mode. As shown in Appendix A, the two stream instability has a growth rate of $\gamma \approx 2\omega_{p,i}$, even in the presence of multiple species temperature and a constant species common streaming velocity. In the classical plasma sheath, the velocity difference between the ions and electrons is constant regardless of the size or amplitude of the sheath. Therefore, if the mechanism of instability is a two streaming instability created by the plasma sheath, then the constancy of the limit cycle is reasonable.

Fig. 4.21 shows the observed precursor speeds in the simulation. The speed is measured by taking the slope of the ion density on a $t\omega_{pi}$ vs x/λ_D plot such as in Fig. 4.6a. The width of the soliton line is used to calculate the error bar. The amplitude and speed of the soliton are predicted to be related by the classical (one soliton solution) sech^2 soliton solution [82, 83]. Fig. 4.21 and Fig. 4.13 demonstrate that as the amplitude increases, the speed of the soliton

increases. The classical Sagdeev pseudo-potential approach predicts that the maximum speed of an ion acoustic soliton is $M = 1.6$ [83]. A different approach using a second order PDE reduction of the multifluid equations predicts a max speed of $M = \sqrt{3} \approx 1.73$ [145]. In this study, the maximum speeds of precursor solitons are consistent with the $\sqrt{3}$ bound except for one data point as seen in Fig. 4.21. This point correlates with the previously observed wave-breaking, which contributes to the error.

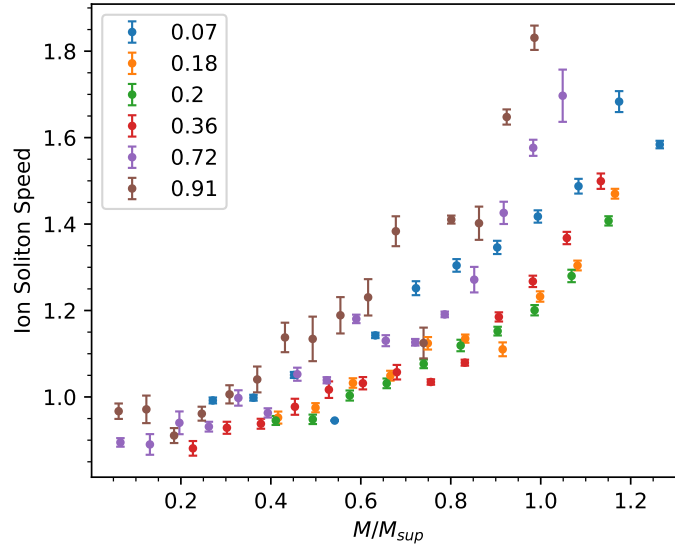


Figure 4.21: Propagation speed of precursor solitons observed in simulations. The error in the speed is estimated to be $\sigma = \sigma_{y2}/(x_2 - x_1)$ where (x_1, y_1) and (x_2, y_2) are the points used to calculate the speed of the precursor soliton.

Now we shall compare the observed and fKdV theory velocity bounds between flow regimes. As seen in Fig. 4.22, only one debris size ($G = 0.07$) stopped producing solitons below a tested flow speed. We tested down to $M = 0.05$ for $G = 0.18$. This would indicate that the subcritical velocity boundary as predicted by fKdV theory (Eqn. 4.2) is much too high. The solitons produced in the subsonic flows have a supersonic speed. Laboratory studies using a dusty plasma (which is *not* the proton – electron plasma used here) remark that no precursor solitons are ob-

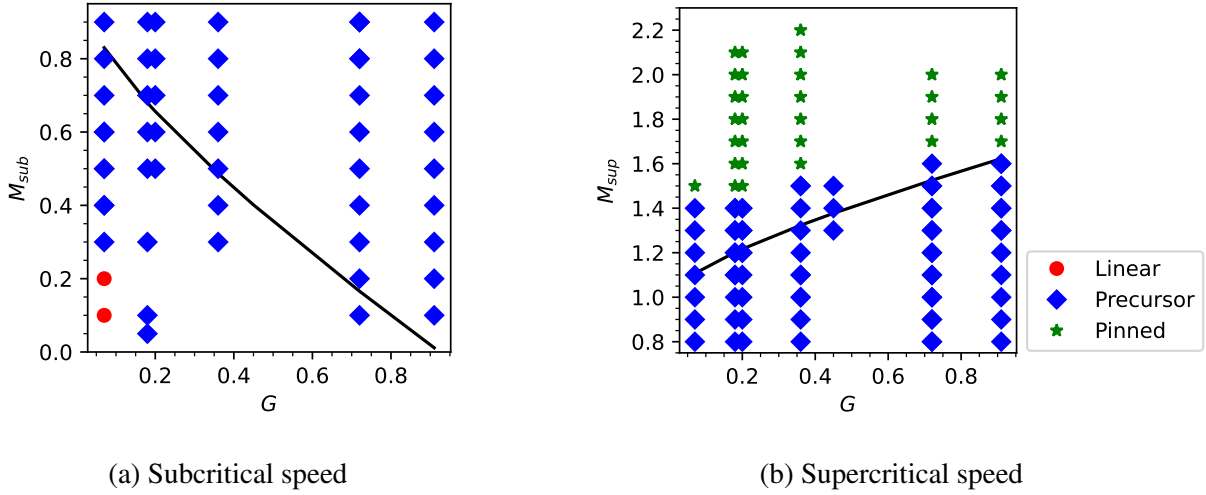


Figure 4.22: Subcritical and supercritical speeds as observed in the simulation (dots) and predicted by KdV (black line) as a function of debris size G . The color and style of dots indicates the type of flow behavior. Large debris sizes agree with the predicted supercritical speed. At low debris sizes, the supercritical speed differs from theory. Dots with the linear shape indicate the “wake” flow regime.

served until the flow is supersonic [9]. If the dusty plasma in these experiments has any shared behavior to the simulated plasma system, this is surprising.

Similarly, the supercritical bound predicted by theory seems to be too low for the smaller debris sizes. The $G = 0.91$ size debris is in line with the predicted theory. Given that the fKdV derivation assumes that the relevant length scales are all above the Debye length, it is sensible that the theory is most incorrect when this assumption is violated. Surprisingly, the debris size seems to have a very small effect on the supercritical velocity. The supercritical velocity varies from $1.45 - 1.65$ over an order of magnitude of debris sizes.

Theory of Supercritical Velocity Bound

It is clear that the fKdV theory does not match the observed velocity bounds in the multi-fluid simulation of ion acoustic precursor solitons. We now describe a new theory that predicts

the supercritical velocity by balancing pressures. Suppose that the formation of the soliton occurs as follows. First, the plasma sheath creates an electrostatic pressure on the flow. In the ram region (where the incoming flow impinges upon the sheath), the bulk flow stagnates. A streaming instability in the sheath causes the local plasma to increase in density, and hence static pressure. As the density at the ram increases, a soliton forms that may or may not propagate ahead of the source. The pressure comes to equilibrium when the static pressure balances the flow and sheath pressure. At this point, the density hump ejects and propagates upstream as a precursor soliton.

To model this, consider the one dimensional ion momentum equation in a conservative form given by Eqn. 4.29.

$$\frac{\partial}{\partial t} (\rho_i u_i) + \nabla \left(p_i + \rho_i u_i^2 + \rho_i \frac{Ze}{m_i} \phi \right) = 0 \quad (4.29)$$

In the moment when the soliton reaches steady-state, the flux term is conserved. The dynamic pressure ($\rho_i u_i^2$) is exerted downstream against the plasma sheath. The electrostatic term ($\rho_i \frac{Ze}{m_i} \phi$) represents an upstream pressure exerted by the plasma sheath counter to the flow. Since static pressure (p_i) is equal to $n_i k_B T$, then a pressure balance can be given in Eqn. 4.30.

$$n_i k_B T + \rho_i \frac{Ze}{m_i} \phi = \rho_i u_i^2 \quad (4.30)$$

This can be rearranged for the ion density (Eqn. 4.31) which shall be interpreted as the maximum ion density of the soliton. Let $T = T_e$ because it is the dominant temperature and use the nondimensionalized form of the surface potential evaluated at one Debye length away ϕ_C . Let $\rho_i = n_0 m_i$. n_0 is used for the electrostatic and dynamic pressures because the potential and bulk velocity respectively dominate those terms. Recall that the surface potential is an implicit

function of the charging equation (Eqn. 4.28). Suppose we can write $\phi_C = \phi_C(G)$ so the surface potential is a function of the debris size. Dividing Eqn. 4.30 by $n_0 k_B T_e$ gives Eqn. 4.31.

$$\frac{n_i}{n_0} = M^2 - \phi_C(G) \quad (4.31)$$

For debris with a size near the Debye length ($G \sim 1$), Eqn. 4.28 can be Taylor expanded out around $G = 1$ to Eqn. 4.32. The more common Taylor expansions of charging equations [93] for $G \ll 1$ and $G \gg 1$ are poor approximations here because they become invariant to G .

$$\begin{aligned} \exp[\exp(1)\phi_C] = & -\frac{4}{3}(G-1) \left(\exp\left[\frac{\exp(1)\phi_C}{3} + 1\right] \phi_C - 3 \exp\left[\frac{\exp(1)\phi_C}{3}\right] + 3 \right) \sqrt{\frac{m_e}{m_i}} \\ & - \left(\left(3 \exp\left[\frac{\exp(1)\phi_C}{3}\right] - 4 \right) \sqrt{\frac{m_e}{m_i}} \right) \end{aligned} \quad (4.32)$$

Now a procedure for Taylor expanding this further shall be outlined. If Eqn. 4.32 or Eqn. 4.28 is solved numerically, we can find $\phi_C(G = 1)$ for a given mass ratio. However, it is dependent on the mass ratio, i.e. species. We will illustrate this procedure using hydrogen as an example. If Eqn. 4.32 is Taylor expanded around $\phi_C \approx \phi_C(G = 1)$ for hydrogen, then Eqn. 4.33 is derived.

$$\phi_C(G) = \frac{(-0.307504G - 0.965406)\sqrt{m_e/m_i} + 0.18489}{(G - 1.82773)\sqrt{m_e/m_i} - 0.135165} \quad (4.33)$$

When Eqn. 4.33 is expanded around $G = 1$, we find a linear expression for the potential as a function of debris size. For hydrogen, this is given by Eqn. 4.34 rounded to the first decimal place. It should be noted that this formula is only valid for objects near the Debye length in hydrogen plasma.

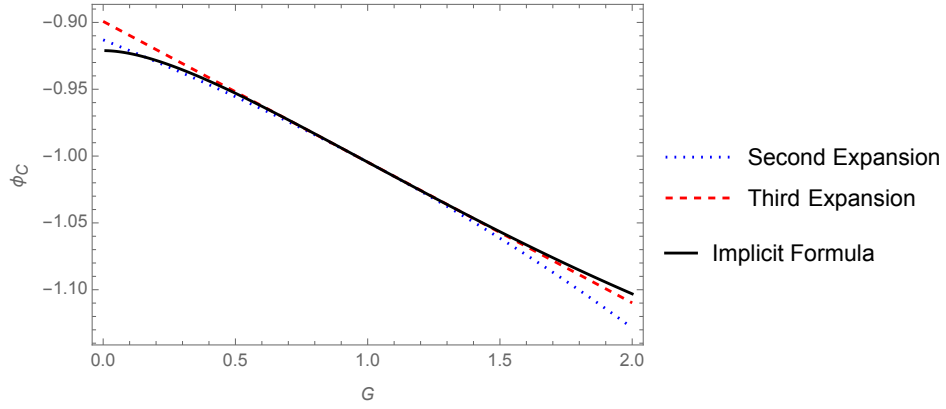


Figure 4.23: Solution to implicit charging formula for a mass ratio of H^+ . The second expansion corresponds to Eqn. 4.33 and the third expansion corresponds to Eqn. 4.34.

$$\phi_C(G) = -0.1G - 0.9 \quad (4.34)$$

These two approximate solutions are compared to the numerically computed implicit solution in Fig. 4.23. For the range of debris sizes ($G \in [0.18, 0.91]$) considered in this study, Eqn. 4.34 is sufficiently accurate for use in this theory. This same procedure can be used for different charging models and different species.

We now find an expression for the soliton amplitude (A) as a function of debris size and background flow speed by substituting Eqn. 4.34 into Eqn. 4.31. For this theory, the ion density will be used as the amplitude. The result is given by Eqn. 4.35.

$$a_s = \frac{n_i - n_0}{n_0} = M^2 + 0.1G - 0.1 \quad (4.35)$$

If the soliton is given by the one soliton solution then $M_{soliton} = a_s/3 + 1$. In this study, it has also been shown how the one soliton solution only roughly approximates the solution. Eqn. 4.35 is only as accurate as the one soliton solution. Additionally, in classical soliton theory

using the Sagdeev pseudopotential ion acoustic solitons can not go faster than $M_{soliton} = 1.6$. Hence, there is a maximum possible amplitude of $a_s = 1.8$. We can reflect this in Eqn. 4.35 by converting it into a piece-wise function:

$$a_s = \begin{cases} M^2 + 0.1G - 0.1 & \text{if } a_s \leq 1.8, \\ 1.8 & \text{otherwise} \end{cases} \quad (4.36)$$

The speed of this soliton is $a_s/3 + 1$:

$$M_{soliton} = \begin{cases} 0.33M^2 + 0.033G + 0.97 & \text{if } M^2 + 0.1G - 0.1 \leq 1.8, \\ 1.6 & \text{otherwise} \end{cases} \quad (4.37)$$

This theory can be seen to be in agreement with simulation for low speed solitons by plotting Eqn. 4.37 in Fig. 4.24. The theoretical supercritical bound is where the theory lines intersect with the line $M_{soliton} = M$. The soliton becomes pinned when its speed matches that of the background flow. For all debris sizes considered, this theory predicts that $M_{sup} = 1.6$. This invariance to debris size is closer to what is seen in simulation than what is observed in Fig. 4.22b. It can also be seen in Fig. 4.24 that at high soliton speed, the theoretical speed is most wrong. Since the one soliton solution breaks down at large amplitude solitons, this is to be expected. If the line of intersection with $M_{soliton} = M$ where the *observed* soliton speeds is used, it can be seen that they are in agreement with the observed supercritical speed (i.e. there are no observed precursor speeds to the right of $M_{soliton} = M$).

If a least squares regression is performed on the data in Fig. 4.24 using a polynomial of the form of Eqn. 4.37, then the result is Eqn. 4.38 which has an average prediction error of 0.9%

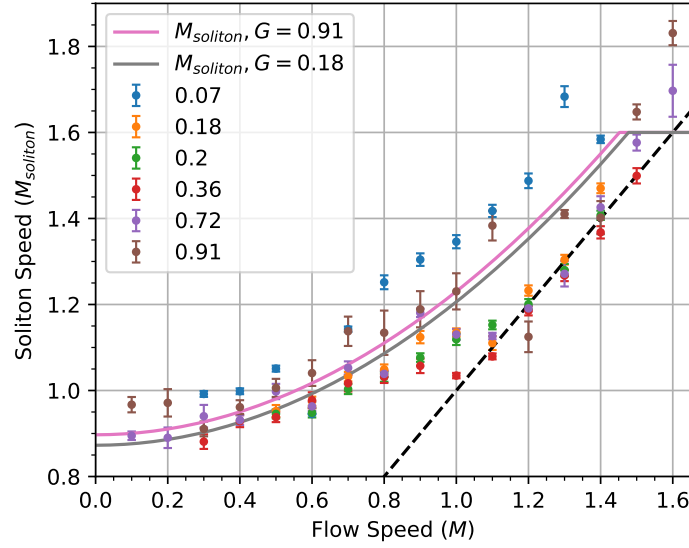


Figure 4.24: Speed of precursor solitons as a function of background flow. The lines indicate the theoretical value given by Eqn. 4.37. The dots indicate the value observed in simulation for various debris sizes. The dashed line indicates where the soliton speed matches the flow speed ($M_{soliton} = M$). The color of the dot indicates debris size G .

and $R^2 = 0.82$. The theoretical model (Eqn. 4.37) has an average error of 2%. The first and last coefficients in Eqn. 4.38 match the theoretical values to the first significant figure. The G coefficient is two orders of magnitude less than compared to the theoretical value. The Taylor expansion error of the charging model is at most $\sim 5\%$ off. This discrepancy is surprising and warrants further investigation.

$$M_{soliton} = 0.28M^2 + 2 \cdot 10^{-4}G + 0.9 \quad (4.38)$$

Theory of Next Order Nonlinearity

Consider the deviations of the simulation results from the one soliton solution of KdV theory. One phenomenon was that the amplitude of the ion density did not match the electron

density for a given soliton. We now seek to predict this. If the ions are cold and the electrons are adiabatic, then the densities for each can be expressed by Eqn. 4.39 & 4.40. Note that these expressions for ion and electron density were not prescribed in the simulation and are only used here for the following derivation.

$$n_i = \frac{n_0}{\sqrt{1 - 2 \frac{Ze\phi}{k_B T_e} M^{-2}}} \quad (4.39)$$

$$n_e = n_0 \exp\left(\frac{Ze\phi}{k_B T_e}\right) \quad (4.40)$$

Substituting the one soliton solution (Eqn. 1.52) into Eqn. 4.39 and 4.40 results in Eqn. 4.41.

$$\frac{n_i - n_e}{n_0} = \frac{1}{\sqrt{1 - 6\delta M M^{-2}}} - \exp(3\delta M) \quad (4.41)$$

The M in Eqn. 4.41 refers to the speed of the soliton, not the speed of the background flow. Fig. 4.25 compares this formula to the simulation results. For low speed solitons (and hence small magnitude) the theoretical relation is in agreement with the simulation results. As the speed of the soliton increases, the one soliton solution breaks down further and Eqn. 4.41 is no longer valid.

4.4 Conclusions from Multifluid Simulation

In this chapter, we have demonstrated that the forced Korteweg - de Vries model approximates the behavior of the multifluid 1D electrostatic plasma system. The multi-phase behavior

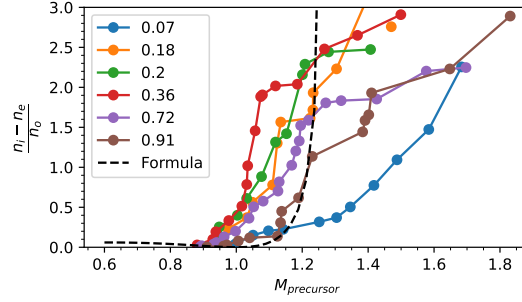


Figure 4.25: Difference in magnitude of precursor solitons between the ion and electron densities as a function of the observed precursor speed. The dashed line indicates the theoretical value given by Eqn. 4.41. The color indicates the debris size G .

of the fKdV equation is seen in simulation. However, the velocity boundary of this phase transition from precursor to pinned is higher in multifluid simulation than predicted by fKdV theory for sub-Debye length debris. In Ref. [148], electron density perturbations were observed by the EISCAT incoherent scatter radar near the known trajectory of the CSTB 1 Cubesat. The target is smaller than the electron density data product, so a probabilistic approach was taken to correlate electron density fluctuations with cubesat location. It was found that the expected pinned soliton did not explain the presence of electron density variation. However, this could be explained by the generation of precursor solitons rather than the pinned soliton. This may occur if the supercritical velocity was higher than expected as observed in this study.

The phase transition from linear to precursor in this study is significantly lower than predicted by fKdV theory indicating that precursors are more likely to be generated than previously thought. The supercritical transition agrees for large debris, but is higher for small sized debris. A new theory based on a pressure balance argument was derived here and shown to accurately predict the speed of the precursor solitons to within 2% error. This same theory predicts a constant supercritical velocity for all debris sizes. This is approximately what is seen in simulations. No theory is presented for the subcritical velocity bound.

Finally, we observed that the first order asymptotic results in deriving the fKdV equation, namely that $\delta n_i/n_0 \sim \delta n_e/n_0 \sim e\phi/(k_B T_e)$, are only approximate in the full fluid simulations. To our knowledge, this is the first reporting of the difference in fluid quantities. In the detection of ion acoustic solitons, simultaneous measurements of n_i and ϕ ought to produce different values *even in the non-dimensional variables*. Large amplitude solitons will have the largest deviation. A theory for small amplitude deviations was derived here and shown to be accurate at low soliton speeds.

Ref. [99] suggested the characteristics of solitons could be used to not only detect debris, but also infer properties of the source. Ref. [1] found for a source of size $G = 0.36$ at $M/M_{\text{sup}} \approx 0.75$ that the soliton would have an amplitude of 1. This study found for the same conditions, $(n_i - n_0)/n_0 = 2.05$, $(n_e - n_0)/n_0 = 1.03$, and $e\phi/(k_B T_e) = 1.45$. In this study, all debris sizes considered at that speed produce precursors with amplitudes of 1 ± 1 . An infinitely precise measurement of a precursor of 1 does not narrow down what size debris could have created the precursor soliton alone in this instance. Based on this study, constructing a map of (soliton amplitude, soliton width) $\rightarrow (G, M/M_{\text{sup}})$ is ill-posed. Space missions who seek to directly observe ion acoustic solitons for the purpose of detecting properties about their source will need to observe several of these quantities at once in order to differentiate a large slow source from a fast small source. Observing a single soliton provides insufficient data.

4.5 Data Availability

The simulation input files, simulation output data files, and analysis scripts can be found at <https://zenodo.org/doi/10.5281/zenodo.10688762>. An Excel worksheet that tabulates all of the simulation parameters and summarizes the results in each is also included in the compressed file. This is also included as supplemental material. We will also provide a basic input script to the Gkeyll team who will put it in their Github repository: <https://github.com/ammahakim/gkyl-paper-inp> once this has undergone peer review.

Chapter 5: Radiometric Eclipsing by Ionospheric Wakes for Detection of Transiting Spacecraft

The content of this chapter is based on a manuscript being prepared for journal publication.

5.1 GNSS Scintillation Based Detection

Ionosphere induced scintillation of GNSS (Global Navigation Satellite Systems) is studied to increase the accuracy of GNSS PVT (position, velocity, time) solutions and to understand the Sun-magnetosphere-ionosphere system coupling which is fundamental to phenomena such as the aurora and geomagnetically induced ground currents. Some studies have extended these techniques to search for signatures of transient phenomena such as earthquakes [149–151], underground nuclear explosions [152–154], tsunamis [155], and rocket exhaust plumes [156, 157]. The seismic activities generate atmospheric gravity waves that affect the ionosphere. Ionospheric disturbances have been previously studied in the context of space weather including small scale (< 30 km) ionospheric disturbances associated with particle precipitation [158, 159] and large scale ionospheric disturbances from geomagnetic storms [160]. It is also known that spacecraft create local changes to the ionosphere. This has been measured on the Space Shuttle [161] and near the Cluster spacecraft [162]. *This study examines using GNSS receivers to detect spacecraft by detecting their ionospheric interactions.*

Compared to previous applications of GNSS ionosphere sensing, orbiting spacecraft are a very small scale (< 1 km) and fast disturbance (< 1 sec). Many of these aforementioned phenomena appear on many receivers with a time delay. With the relative sparsity of current networks, it is not expected that a single event will cross multiple sensors frequently. This makes searching for these events difficult. In this study, we will use orbital dynamics to find geometries where a very small scale ionospheric disturbance cross the radio line-of-sight. For this *known* event, the data from the ground receiver will be presented to understand how this phenomenon affects the radio signal.

It has also been shown that GNSS, specifically GPS, can be used as a bistatic radar. This is a related concept in the sense that L-band radio from a GNSS transmitter is interacting with an object. However, this technique relies on searching for the Doppler shifted scattered wave instead of the diffracted incident wave. GNSS bistatic radar is used for maritime surveillance [163–165] and analyzing helicopter blade dynamics [166]. Some work has been done to extend this concept to spacecraft detection, although it appears to be in the development stage [167–169]. The scattered wave off of spacecraft is substantially weaker in a noisier background than that off of ships reflecting up. The effect of the plasma wake around a spacecraft on this technique has not been articulated in the literature. This could explain the anomalously large radar cross section observed in over-the-horizon HF radar studies of the International Space Station [170].

This eclipsing physics could be applied as a sensing strategy for detecting spacecraft below GPS orbit (20,200 km). The advantage of this strategy is the scalability of the concept. Each ground station often has a simultaneous link with ~ 10 GPS satellites ($<$ half of the 32 operational ones). This could be extended to include other GNSS systems, i.e. GLONASS (24 satellites), BeiDou (30 satellites), or Galileo (30 satellites). After including these other systems a

given ground station would have a link with ~ 40 GNSS satellites. Many national governments already have networks of reference GNSS ground stations. For example, the American NOAA CORS network¹ has ~ 1600 operational GNSS stations at the time of writing. In practice, the geometrical constraint described would have to be satisfied by one of the $\sim 64,000$ links in this network.

Radio propagation through a plasma is strongly influenced by the local properties of the plasma. The index of refraction in a plasma neglecting magnetic and Landau damping effects is given by Eqn. 5.1. This is a limit of the Appleton-Hartree equation. ω_p is the plasma frequency where $\omega_p^2 = \frac{n_e(\mathbf{x})e^2}{\epsilon_0 m_e}$ in SI units. m_e is the electron mass. For frequencies below the plasma frequency ($\omega \ll \omega_p$), the relative permittivity is negative ($\epsilon_r < 0$), so the index of refraction is imaginary ($\text{Im}(n) > 0, \text{Re}(n) = 0$). Transionospheric propagation is not possible at those frequencies. This study is interested in frequencies much larger than the plasma frequency ($\omega \gg \omega_p$). In this limit, $0 < \epsilon_r < 1$, thus $0 < n < 1$.

$$n^2 = \epsilon_r = 1 - \frac{\omega_p^2}{\omega^2} \quad (5.1)$$

n is a function of the local electron density ($n_e(\mathbf{x})$). Local structure in $n_e(\mathbf{x})$ will lead to classical optics phenomena such as scattering and refraction resulting from the changing electron density. Thus, radio wave propagation will be affected when transmitted through the electron density variations that are generated by the passage of a spacecraft or piece of debris. This predicted modification of radio waves may provide an alternate method to detect debris, rather than traditional methods that rely on directly observing the debris itself. In this study, we focus

¹<https://geodesy.noaa.gov/CORS/about/contributors.shtml>

on GNSS signals as the probing radio transmission because they have global coverage and are well characterized. However, this technique is not unique to GNSS signals. Ionospheric disturbances have been indirectly observed by radio telescopes using astronomical radio sources. Astronomical radio sources include Sagittarius A [171, 172], supernova remnants (such as Cassiopeia A [173] and the Crab Nebula [174]), and radio galaxies [175]. In particular, Cassiopeia A has been studied at 25.19 MHz [176]. The largest modifications to the Appleton-Hartree equation occur when $\omega = \omega_p$.

Table 5.1: Carrier frequencies of common GNSS systems

System	Frequency (MHz)
GPS L1	1575.42
GPS L2	1227.60
GPS L3	1381.05
GLOSNASS L1	1602.0
GLOSNASS L2	1246.0
BeiDou B1	1575.42
BeiDou B2	1191.795
BeiDou B3	1268.52
Galileo E1	1575.42
Galileo E5	1191.795
Galileo E6	1278.75

Scattering theory has been derived from the Appleton-Hartree equation [177]. The theory was created in response to observations of Sputnik 1, Explorer 1 and Explorer III scattering over-the-horizon radio broadcasts at night which were highly dependent on background ionosphere conditions. These bursts were used to track early satellites to varying degrees of success. How often these bursts would happen and whether they could always be attributed to spacecraft flying overhead was debated at the time.

5.1.1 Length Scale of the Plasma Wake around a Supersonic Object in the Ionosphere

This study shall use the length scale of the plasma wake as the appropriate “structure” that would cause the eclipse. However, other plasma density structures are created by supersonic flow. We ignore them because for now, it seems that the wake is the largest of them spatially. However, if something like a magnetosonic pinned soliton [88] were to exist, this could be a candidate to supercede the wake as the largest “structure” near the object. The wake size depends on the geometry of the object [178, 179] and the wake spreading angle depends on the relative sound speeds of the ions and electrons [162]. “Wing” structures of layered electron and ion enhancement from magnetization of the plasma have also been found to exist [180, 181]. For order of magnitude analysis, we shall consider the wake behind a conducting sphere as derived by [177, 182] given in Eqn. 5.2. M is the ion acoustic Mach number defined by $M^2 = \frac{m_i v_0^2}{k_B T_e}$ where m_i is the ion mass, T_e is the electron temperature, and v_0 is the speed of the sphere. I_0 is a modified Bessel function of the first kind. The motion of the sphere is in the $-\hat{z}$ direction. ρ is the radial distance from the z -axis. The radius of the sphere is a .

$$\delta n = -2n_0 \exp \left[-\frac{M^2}{2} \left(\frac{\rho}{z} \right)^2 \right] \int_0^{-\frac{aM}{\sqrt{2}z}} \tau \exp(-\tau^2) I_0 \left(-\frac{\rho}{z} \sqrt{2} M \tau \right) d\tau \quad (5.2)$$

From Eqn. 5.2, some appropriate length scales shall be established. The on-axis ($\rho = 0$) length of wake depends solely on the length parameter $aM/\sqrt{2}$ in the integral and decays $\mathcal{O}(\exp(-z^2))$. The off-axis width at $z = aM/\sqrt{2}$, the end of the wake, depends on $(\rho/a)^2$. If $M > 1$, then L (the characteristic length of the wake) is longer than the width. $L \sim aM$. For the

ISS, $v_0 \sim 7$ km/s and the ion acoustic velocity of the ionosphere $c_S \sim 1$ km/s. So $L \equiv Ma \sim 7a$.

5.2 Methods

5.2.1 GNSS Receiver Ionospheric Data

Phase Measurements

GNSS receivers produce two observables, the *pseudorange* (P) and *carrier phase* (Φ). The pseudorange is derived from codes that are modulated onto the carrier wave using binary phase shift keying [183]. A GNSS transmitter may transmit on multiple frequencies. Each frequency is a “signal.” Different signals contain one or more codes plus the carrier wave of the unmodulated signal. For GPS, the possible codes are the “coarse / acquisition” (C/A), “civilian” (C), “military” (M), and “precision” (P) code. There are three signals: L1, L2, and L5. However, L5 is a recent addition and is not used in this study. The L1 signal on the newest (GPS III) satellites contains the C, C/A, P, and M codes while the L2 signal contains the C, P, and M codes. The first GPS satellites had just C/A and P codes. Of the codes used for ranging, there are eight signal / code pairs (L1-C/A, L1-C, L1-P, L1-M, L2-L2C, L2-P, L2-M). The entire GPS space segment is a mix of old and newer satellites. The P and M codes are the most precise, however the L1-P pseudorange data was not available. The M code is also not widely available yet. Other GNSS systems (e.g. BeiDou, GLONASS, and Galileo) have their own sets of codes and signals. Generally speaking, each system has 2-3 signals and 2+ codes / signal.

To simplify this discussion of pseudoranges in this study, the L1-CA and L2-P codes are used only. Pseudoranges will be denoted by P . P_1 will refer to the L1-CA derived pseudorange.

Similarly, P_2 will refer to the L2-P pseudorange. A description of code based ranging can be found in Ref. [183].

The carrier phase (Φ) is the number of observed cycles in the radio signal. In principle, there is only a single carrier phase associated with each signal (i.e. L1 carrier phase and L2 carrier phase). Again, the L5 signal is ignored. However, the receiver records a carrier phase measurement associated with each signal-code pair due to how the demodulation process works internally in the receiver. For consistency, we use the L1-CA signal/code carrier phase for Φ_1 and the L2-P signal/code carrier phase for Φ_2 .

Each GPS observable (P_1 , Φ_1 , P_2 , and Φ_2) is the sum of the physical range plus errors. For convenience, units of phase are converted to distance by multiplying by the wavelength. The carrier phase differs from the actual range by an integer ambiguity (N) that must be resolved. Eqns. 5.3 and 5.4 are standard models of each observable where R is the physical range, $K_{P,R}$ is the receiver transmit delay, $K_{P,T}$ is the transmitter transmit delay, c is the speed of light, dt_R is the receiver clock offset, dt_T is the transmitter clock offset, T is the tropospheric error, $\mathcal{M}$ and m are the multipath errors for pseudorange and carrier phase respectively, $k_{P,R}$ and $k_{P,T}$ are the carrier phase instrument delay for the receiver and transmitter respectively, and ϵ_ρ and ϵ_Φ are the leftover errors. $(\kappa/f_i^2)TEC$ is the ionospheric range error and has different signs for the pseudorange and carrier phase. κ is a physical constant $= 40.3 \text{ m}^3/\text{s}^2$ and TEC is the total electron count. In discussions of global mapping of the ionosphere, a distinction is made between vertical and slant TEC . In this study all TEC refers to slant TEC .

$$P_i = R + c(dt_R - dt_T) + T + \frac{\kappa}{f_i^2} TEC + K_{P,R} - K_{P,T} + \mathcal{M} + \epsilon_\rho \quad (5.3)$$

$$\Phi_i = R + c(dt_R - dt_T) + T - \frac{\kappa}{f_i^2} TEC + k_{P,R} - k_{P,T} + m + \lambda_i N + \epsilon_\Phi \quad (5.4)$$

The ionospheric delay has to do with the total amount of plasma the radio wave propagates through. For small ionospheric irregularities from transiting spacecraft, this is expected to create a small change in the TEC . It is strongly frequency dependent, unlike the other terms in the error equation. From differencing GPS observables via Eqns. 5.5 or 5.6, the TEC can be indirectly measured. Of course if the other terms, namely the multipath have some frequency (and hence wavelength dependence) they will be an error source in these equations.

$$TEC = \frac{f_1^2 f_2^2}{\kappa(f_2^2 - f_1^2)} (P_1 - P_2) \quad (5.5)$$

$$TEC = \frac{f_1^2 f_2^2}{\kappa(f_2^2 - f_1^2)} (\Phi_2 - \Phi_1) \quad (5.6)$$

Each frequency may also have a different integer ambiguity (N). Removing this error by taking an average of the pseudorange TEC and matching the mean of the carrier phase TEC is known as “phase leveling.” Let DCP be the TEC calculated from differencing the carrier phase (Eqn. 5.6) and DPR be the TEC from differencing the pseudorange (Eqn. 5.5). $\langle x \rangle$ denotes the temporal average of x over one GPS arc. The TEC that is used is given by Eqn. 5.7.

$$TEC = DCP + \frac{\langle w(DCP - DPR) \rangle}{\langle w \rangle} \quad (5.7)$$

w is a weight parameter intended to calculate the offset using the least noisy data points. Local elevation (ψ) is used as a proxy for noise since multipath is the strongest near the horizon. The formula for the weight is given by $w = \sin \psi$. Of course other weights could have been used to quantify a “clear” signal. Several authors have proposed alternatives [184–186]. In this study, the absolute TEC is not important. Rather, we are interested in fast variations of TEC . If one were to use data from multiple ground stations, a more careful treatment may be required. This would entail using other techniques to solve for $K_{P,R}$, $K_{P,T}$, $k_{P,R}$, $k_{P,T}$, & T in Eqns. 5.5 and 5.6 at each ground station before applying Eqn. 5.7.

Amplitude Measurements

Diffraction below the Fresnel scale causes rapid changes in amplitude. Each GPS signal has two amplitude measurements associated with it. First, the devices report the *carrier-to-noise density* ratio (C/N_0). It is related to the signal-to-noise ratio by Eqn. 5.8 where BW is the bandwidth of the carrier wave.

$$SNR = \frac{C/N_0}{BW} \quad (5.8)$$

The raw I/Q values of the receiver are also used. The I/Q is a method of storing a sinusoid as two real values where I is the in-phase value and Q is the quadrature (90° out of phase) value. The signal amplitude is given by $\sqrt{I^2 + Q^2}$. The C/N_0 is provided at 1Hz and the I/Q values are provided at 50Hz. In the data, it can be seen that generally speaking the C/N_0 is a smoothed version of I/Q signal amplitude.

5.2.2 Prediction of Eclipsing Event from Orbital Data

To test the feasibility of using GPS to detect spacecraft-induced plasma features, we first consider a known and well-characterized target: the International Space Station (ISS). Fig 5.1 diagrams the geometry in 2D. The radio line of sight must intersect the GNSS transmitter, plasma wake, and ground station. Since the plasma wake's exact position is unknown, this study will search for when the GNSS transmitter, target spacecraft, and ground station are approximately colinear. The radio line of sight will *always* be defined between the GNSS transmitter and the ground station. The distance from the center of the target spacecraft to the closest point in the radio line of sight will be the “miss distance.”

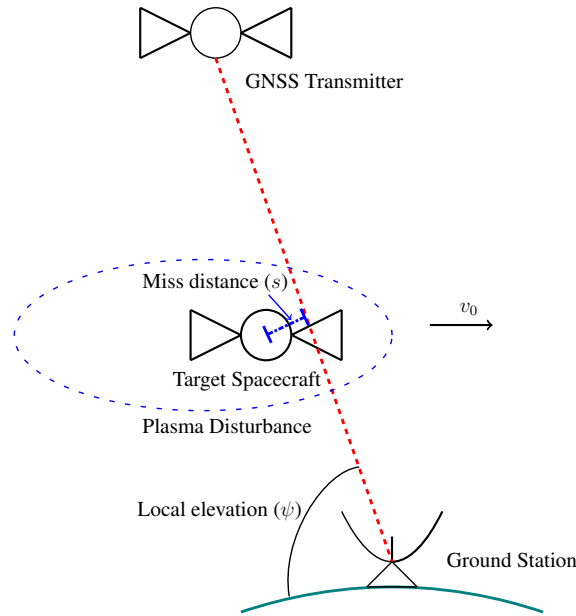


Figure 5.1: Geometry of eclipsing events. The dashed blue ellipse around the target spacecraft (in this study the ISS) indicates the plasma wake. The red dashed line indicates the radio line of sight from the GNSS transmitter to the ground station. The blue dashed line indicates the miss distance of the target spacecraft from the radio line of sight.

We search through historical ephemeris data of the GPS satellite and ISS to find a time when an eclipse event happened. We seek a time when the ground station, ISS, and GPS transmitter

are colinear. The precision of this geometric constraint is given by the time the spacecraft wake spends in the radio line-of-sight. For an object moving at v_0 , this timescale is approximately $\tau \sim L/v_0$ where L is the length scale of the wake defined above. For $v_0 \approx 7$ km/s and $L \approx 1$ km, this geometric happenstance happens for < 1 s.

From the point of view of a ground observer, this geometry occurs when the azimuth and elevation of these two objects in the sky are the same. To avoid confusion in these often overloaded terms, in this context azimuth is defined as the angle from local geodetic north. Elevation (ψ) is defined as the angle above the horizon. Throughout this study, the term “local elevation” will be used to differentiate from height above sea level. The difference in azimuth and elevation is quantified by calculating the angular difference on the local celestial hemisphere using law of cosines on a spherical triangle by Eqn. 5.9 where θ is the angular difference, ϕ_i is the co-altitude of object i and λ_i is the azimuth of object i both in radians. Co-altitude is defined as the complement of the elevation (i.e. $\phi = \frac{\pi}{2} - \psi$). An eclipsing is said to have happened when θ is less than a threshold. For the initial search, a threshold of 1° was used. From those initial findings, the eclipsing events with the smallest θ were explored.

$$\theta = \cos^{-1} [\cos(\phi_{ISS}) \cos(\phi_{GPS}) + \sin(\phi_{ISS}) \sin(\phi_{GPS}) \cos(|\lambda_{GPS} - \lambda_{ISS}|)] \quad (5.9)$$

The ephemeris of both the ISS and GPS satellites are used to generate the local azimuth and elevation. The ISS ephemeris comes from the JPL Horizons system². The GPS ephemeris comes from the NASA’s Crustal Dynamics Data Information System (CDDIS)³ via the `sp3`

²<https://ssd.jpl.nasa.gov/horizons/>

³https://cddis.nasa.gov/Data_and_Derived_Products/index.html

Python library⁴.

5.2.3 Physical Models of Diffraction

Having presented the data, this study shall now present first principles derived models of diffraction in order to understand the change in amplitude. For these data sets, the wavelength is much smaller than the length scale of the wake ($\lambda \ll L$), so a geometric optics approach can be taken. However, the ISS is one of the larger spacecraft in orbit. For spacecraft smaller than the ISS, $\lambda \sim L$. A weak scattering approach is derived for this case.

5.2.3.1 Optically Opaque Sphere

Sommargren and Weaver [187] found that for an opaque sphere of radius a , the intensity of the shadowed region of the diffraction pattern at the edge radial distance ($s' = a(r + d)/d$) is given by Eqn. 5.10. Due to the perfect symmetry of the problem, the point exactly on axis is bright with the same intensity as if the sphere wasn't there. r is the distance from the sphere to the observer. d is the distance from the sphere to a point source illuminating it.

$$I = \frac{1}{4(r + d)^2} \left[1 + J_0^2 \left(\frac{kas'}{r} \right) + 2J_0 \left(\frac{kas'}{r} \right) \cos \frac{kas'}{r} \right] \quad (5.10)$$

The without the sphere present, the illumination would be $I = (r + d)^{-2}$. Therefore the transmission coefficient is given by Eqn. 5.11. As $a \rightarrow 0$, $J_0(a^2) \rightarrow 1$ and $T \rightarrow 1$.

⁴<https://pypi.org/project/sp3/>

$$T^{\text{opaque sphere}} = \frac{1}{4} \left[1 + J_0^2 \left(\frac{ka^2}{r} \cdot \frac{r+d}{d} \right) + 2J_0 \left(\frac{ka^2}{r} \cdot \frac{r+d}{d} \right) \cos \left(\frac{ka^2}{r} \cdot \frac{r+d}{d} \right) \right] \quad (5.11)$$

It will be assumed that $(r + d)/d \approx 1$, i.e. that the point source is much farther behind the illuminated sphere than the observer is in front of it. Fig. 5.12 shows the solution to Eqn. 5.11 for $k = \frac{2\pi}{20 \text{ cm}}$ so it is representative of L-band GNSS signals.

5.2.3.2 Weak Scattering of a Plasma Sphere

The treatment above is only valid for objects larger than a wavelength ($\sim 20 \text{ cm}$). When the scale length of the object a begins to approach 10s of cm for GNSS signals, a different treatment must be given. In general, it is well known that when the wavelength becomes closer to the size of the object, the far-field electric field is given by Eqn. A.16. A derivation from Maxwell's equations can be found in Appendix A.

$$\mathbf{E}_1(\mathbf{r}) = \frac{E_0 \exp(ikr)}{4\pi r} \mathbf{k}' \times \left(\mathbf{k}' \times \int \exp(-i\mathbf{q} \cdot \mathbf{r}') \chi(\mathbf{r}') \hat{e} d^3\mathbf{r}' \right) \quad (5.12)$$

The susceptibility of a cold plasma is $\chi = -\frac{\omega_p^2}{\omega^2}$, where ω_p is the spatially varying plasma frequency. In SI units, $\omega_p^2 = \frac{n_e e^2}{\varepsilon_0 m_e}$, where $n_e = n_e(\mathbf{r})$ is the spatially varying electron density, e is the elementary charge, and m_e is the electron mass. Substituting the plasma susceptibility into Eqn. 5.12, yields Eqn. 5.13.

$$\mathbf{E}_1(\mathbf{r}) = \frac{E_0 e^2 \exp(ikr)}{4\pi \varepsilon_0 m_e \omega^2 r} \mathbf{k}' \times \left(\mathbf{k}' \times \int \exp(-i\mathbf{q} \cdot \mathbf{r}') n_e(\mathbf{r}') \hat{e} d^3\mathbf{r}' \right) \quad (5.13)$$

As a simple case, consider a spherically symmetric electron density.

$$n_e(\mathbf{r}) = n_e(r) = \delta n \exp(-(r/a)^2) + n_0 \quad (5.14)$$

The integral in Eqn. 5.13 can be evaluated under a spherically symmetric coordinate system along $\mathbf{q}$.

$$\int_0^{2\pi} \int_0^\pi \int_0^\infty \exp(-iqr' \cos(\theta)) [\delta n \exp(-(r/a)^2) + n_0] r'^2 \sin\theta dr' d\theta d\phi \quad (5.15)$$

The ϕ integral is trivial. $\int_0^\pi \exp(-ia \cos \theta) \sin \theta d\theta = \int_{-1}^1 \exp(iau) du = \frac{-1}{ia} (e^{ia} - e^{-ia}) = \frac{2}{a} \sin a$. Plugging these in leaves the r' integral.

$$2\pi \int_0^\infty \frac{2}{qr'} \sin qr' [\delta n \exp(-(r/a)^2) + n_0] r'^2 dr' \quad (5.16)$$

Finally we have:

$$2\pi\delta n \frac{\sqrt{\pi}a^3}{4} \exp\left(-\frac{1}{4}a^2q^2\right) + n_0\delta^3(\mathbf{q}) \quad (5.17)$$

The delta term will be ignored. It can be shown that $q = 2k \sin \theta_S/2$ where θ_S is the scattering angle (angle b/w $\mathbf{k}'$ and $\mathbf{k}$). Plugging this into Eqn. 5.13 yields:

$$\mathbf{E}_1(\mathbf{r}) = \frac{\delta n a^3 E_0 e^2 \sqrt{\pi} \exp(ikr)}{8\varepsilon_0 m_e \omega^2 r} \exp\left(-\frac{1}{4}a^2 \sin^2(\theta_S/2)\right) \mathbf{k}' \times (\mathbf{k}' \times \hat{e}) \quad (5.18)$$

Along the direct line of sight assuming a linearly polarized wave, i.e. $\theta_S = 0$, $\mathbf{k}' \times (\mathbf{k}' \times \hat{e}) = -k^2 \hat{e}$, this simplifies to:

$$\mathbf{E}_1(r) = -E_0 \frac{\delta n k^2 a^3 e^2 \sqrt{\pi}}{8 \varepsilon_0 m_e \omega^2} \frac{\exp(ikr)}{r} \hat{e} \quad (5.19)$$

At the point of the observer, $\omega = ck$, where c is the vacuum speed of light. So we are left with

$$\mathbf{E}_1(r) = -E_0 \frac{\delta n a^3 e^2 \sqrt{\pi}}{8 \varepsilon_0 m_e c^2} \frac{\exp(ikr)}{r} \hat{e} \quad (5.20)$$

Taking the incident field as $\mathbf{E}_0 = E_0 \frac{\exp(ikr)}{kr} \hat{e}$, we find that the transmission coefficient is:

$$T = \left(1 - \frac{\delta n k a^3 e^2 \sqrt{\pi}}{8 \varepsilon_0 m_e c^2} \right)^2 \quad (5.21)$$

Put more simply, if the classical radius of the electron is $r_e = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{m_e c^2}$, and $k = 2\pi/\lambda$ then T becomes:

$$T = \left(1 - \pi^{5/2} \delta n a^3 r_e / \lambda \right)^2 \quad (5.22)$$

Eqn. 5.22 describes the same Eqn. 5.11 at the first shadow.

5.2.4 Design of a GNSS Interferometry Sensor

5.2.4.1 Along-track spacing

A single observation does not provide enough information for initial orbit determination. For angles only orbit determination (i.e. Gauss's method), three or more observations are needed [188]. Similarly, for analyzing traveling ionospheric disturbances (TIDs), other authors have used radio interferometry techniques to image TIDs [189, 190]. Radio interferometry works by combining time series measurements from multiple receivers to produce an image. These previous works have used receivers separated by 10s to 100s of km, which is unsuitable for this purpose. We shall briefly outline what spacing of receivers would be suitable for performing orbit determination with this setup. Fig. 5.2 shows a 2D setup for analyzing this problem. The primed coordinates indicate the spacecraft at three different times. The unprimed coordinates indicate multiple ground stations. x_0 is the location of the GPS transmitter.

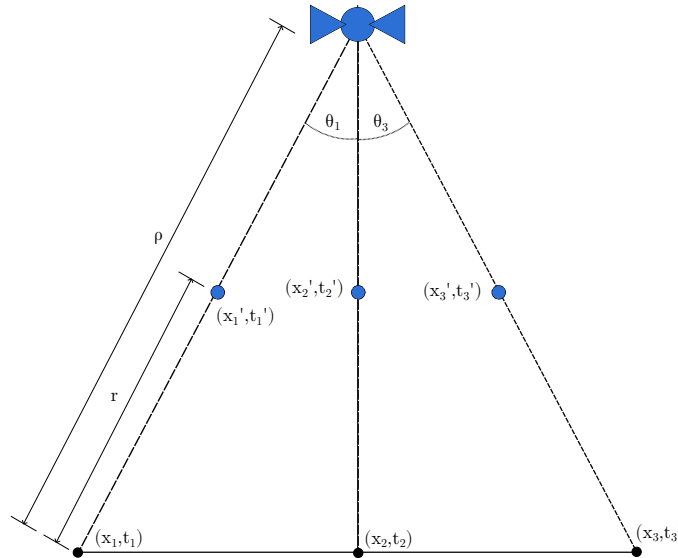


Figure 5.2: Planar geometric setup of multiple receivers.

Let $\rho(t_i)$ be the range of the GNSS transmitter to the ground station. Let $r(t_i)$ be the range

of the target spacecraft at time t_i to the ground station. The x - coordinate of the direct line of sight of the ground station and spacecraft at time t_i is given by Eqn. 5.23 where θ_i is the angle from directly to the surface from the GNSS transmitter.

$$x_i = \rho(t_i) \sin \theta_i + x_0 \quad \& \quad x'_i = \left(\rho(t'_i) - r(t'_i) \right) \sin \theta'_i + x_0 \quad (5.23)$$

Let us assume that the primed and unprimed times are the same due to $r \ll cf_s^{-1}$ where f_s is the sampling frequency (i.e. at 50 Hz, $cf_s^{-1} \approx 6,000$ km). In that case, one finds that the right hand side can be written neatly in terms of the ground station separation as in Eqn. 5.24.

$$x'_2 - x'_1 = (x_2 - x_1) - (r(t_2) \sin \theta_2 - r(t_1) \sin \theta_1) \quad (5.24)$$

Let us define the average ground station separation distance to be $d = x_2 - x_1$. The difference in primed coordinates is $v_0(t_2 - t_1)$ where v_0 is the speed of the spacecraft. Let us assume that $r(t_2) \approx r(t_1)$, i.e. $\left(\frac{r(t_2) - r(t_1)}{r(t_1)} = \frac{x_1 - x'_1}{r(t_1)} = \sin \theta_1 \ll 1 \right)$. Necessarily, θ_1 must be small and $\theta_2 = 0$ by construction. Thus we find Eqn. 5.25 where $\Delta t = t_2 - t_1$ and $\Delta \theta = \theta_2 - \theta_1$.

$$d = v_0 \Delta t + r \Delta \theta \quad (5.25)$$

Since all of the angles are small, $\Delta \theta = -\frac{d}{\rho}$. Let us assume that the eclipsing event happens for N samples, and the sampling rate of the sensor is f_s . Δt is chosen so that $\Delta t = Nf_s^{-1}$. Finally we arrive at Eqn. 5.26.

$$d = \left(1 + \frac{r}{\rho}\right)^{-1} v_0 N f_s^{-1} \quad (5.26)$$

5.2.4.2 Off-track spacing

In addition to the along track spacing needed to resolve the motion of the target spacecraft, the off-track spacing must be sufficiently dense to capture all possible angles of the spacecraft arc. If the width of the ground area affected by the spacecraft wake is given by w , then the off-track spacing s must be $s \leq \frac{w}{2}$. Fig. 5.3 diagrams this geometrical constraint.

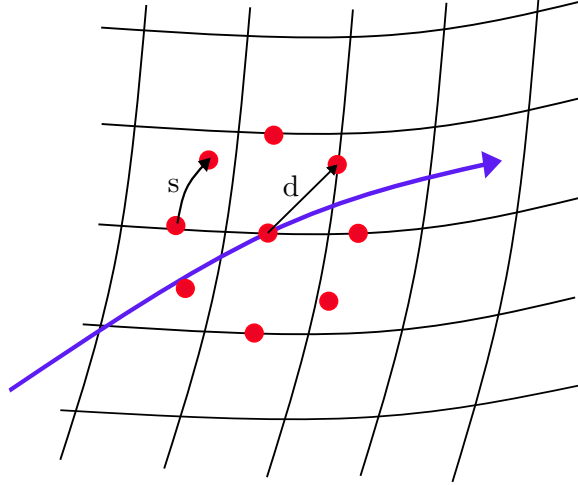


Figure 5.3: Cross track of target spacecraft intersecting with a circular pattern on ground stations. The ground track is the solid blue line. The red dots indicate the position of ground stations on a lat / lon grid. s is the spacing between the circular ground stations. d is the radius of the circle.

If the width of the wake is W , then the width of the projected ground area will be related by Eqn. 5.27.

$$\frac{W}{\rho - r} = \frac{w}{\rho} \quad (5.27)$$

$\frac{\rho}{\rho - r} \approx 1$ for a GNSS transmitter in MEO and a target spacecraft in LEO. Thus, w can be

approximated as $w = W$.

5.3 Results

5.3.1 Observation of International Space Station Radiometric Eclipsing

This study makes use of data from the Canadian High Arctic Ionospheric Network (CHAIN) [191]. CHAIN is a network of stationary ionospheric reference GNSS receivers spread over the high latitude regions of Canada for the purpose of scientific monitoring of the ionosphere. The geomagnetic north pole is currently located in Nunavut, Canada. Thus many solar – magnetic field interactions happen in the vicinity of CHAIN. Although this study is using very specialized receivers in a particular geographic location, this spacecraft detection technique is not unique to the polar regions. CHAIN is preferred because it has public records of raw data files that record at 50 Hz going back more than a decade. The 50 Hz data is processed from Septentrio binary files (the reference station manufacturer) using the `sb2f2ismr` computer program.

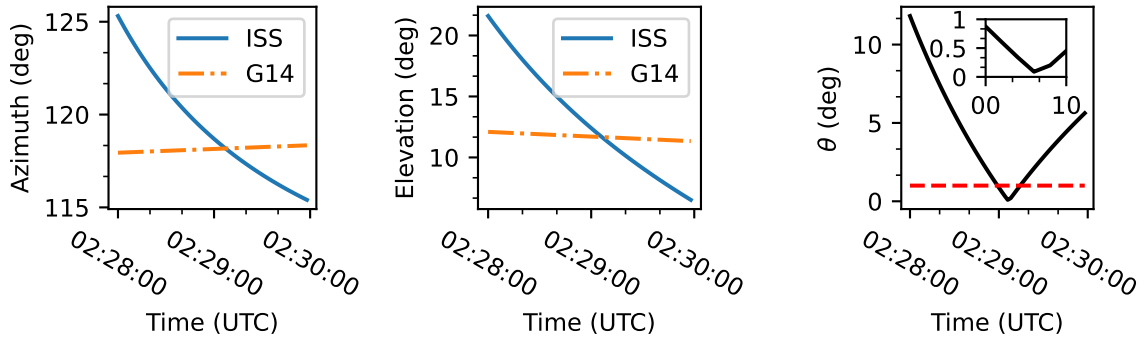
Table 5.2 lists the eclipsing events discussed in this study along with the angular difference (θ) and local elevation (ψ). A multitude of natural physical activities may cause changes to the ionosphere. To rule out space weather effects, the observations are taken from days with quiet geomagnetic activity. Table 5.2 has a column for Kp index, a scale from 0 - 9 where > 5 indicates significant geomagnetic activity. All four of these events happen on days with small Kp values. A coarser search was first used to make a small list of candidate times. From there, this list is reduced to just ones with small θ and low Kp index. The “Case Number” in Table 5.2 reflects the chronological order from a larger list that is not shown.

Table 5.2: Eclipsing events. θ indicates the angular difference between the ISS and GPS satellite calculated with Eqn. 5.9. ψ indicates the local elevation during the eclipse. PRN indicates the assigned pseudorandom noise code of the eclipsed GPS satellite.

Case Number	PRN	Ground Station	Date	θ ($^\circ$)	ψ ($^\circ$)	Kp
18	14	Ministik Lake	January 30, 2021	0.09	12	1
4	26	Ministik Lake	January 12, 2021	0.12	19	2
26	05	Ministik Lake	June 8, 2021	0.6	61	1-
27	23	Ministik Lake	June 13, 2021	0.2	20	0

5.3.1.1 Case 18

On January 30, 2021 at 2:29:06 UTC the ISS was 0.08997° away on the sky from the G14 GPS satellite. Fig. 5.4 shows the local azimuth, local elevation, and angular difference calculated with Eqn. 5.9. The ISS has an orbital period of 90 minutes while GPS satellites have an orbital period of 12 hours. Hence in Fig. 5.4, the G14 appears stationary while the ISS crosses its fixed location.



(a) Local azimuth of ISS and (b) Local elevations of ISS and G14 calculated from orbital ephemeris at ground station. (c) Angular difference between ISS and G14 when projected on local hemisphere. Inset is number of seconds from 02:29. Dashed line is at 1° .

Figure 5.4: Local azimuth, elevation, and angular difference during the ISS eclipsing event on January 30, 2021 from GPS coordinates $53.350818^\circ N$, $121.97384^\circ W$ (Ministik Lake ground station). PRN 14 was the GPS transmitter that was eclipsed. This corresponds to spacecraft USA-309 a Block III model GPS spacecraft.

The range of the ISS from the ground station is approximately $R = \frac{h}{\sin \psi}$ where ψ is the local elevation of the ISS. Using an elevation of $\psi = 11.7^\circ$ (from Fig. 5.4b) and an orbital altitude of $h = 413$ km, the range is $R = 2,037$ km. The arclength from the ISS to the center of the line of sight (s) is $s = R\theta = 3.2$ km.

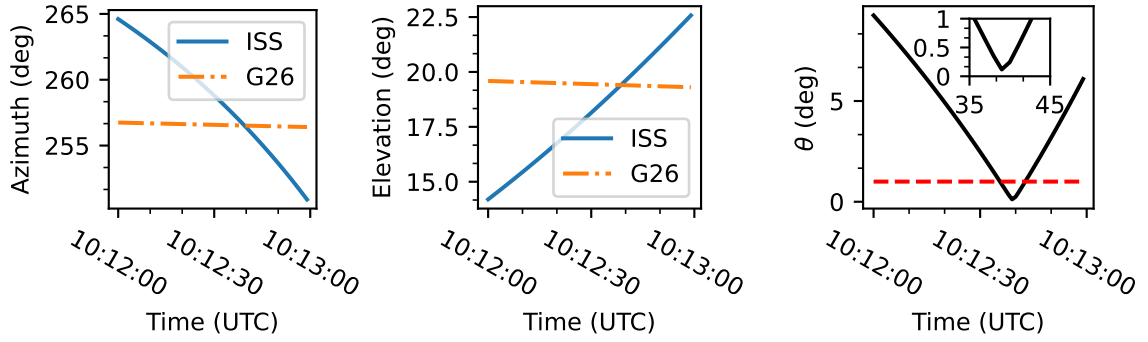
Fig. 5.6a shows data from the CHAIN_{edm} receiver. In this figure, the *TEC* is the phase leveled *TEC* defined by Eqn. 5.7. At 2:29:00 there is a 10 second long duration spike and drop in the *TEC* of amplitude 0.1 TECU in Fig. 5.6a. The one-second standard deviation of the *TEC* (labelled STD in the second row) is approximately 0.02 TECU throughout this time period. Similarly, there is a drop in the carrier to noise ratio (C/N0) of ~ -2 dB-Hz in Fig. 5.6a. In signal amplitude ($\sqrt{I^2 + Q^2}$) in Fig. 5.6a the same trend is resolved at a higher temporal frequency. From 2:29:30 onwards, there is a drop in *TEC*. The multipath (Fig. 5.6a) is not correlated with the spacecraft eclipsing. The multipath and signal strength have a long period (two minute) long sinusoid shape that match in duration. This would indicate that they come from a source of noise in the environment such as the terrain interacting with the slowly moving GPS line of sight.

The interpretation of this data is based on temporal correlation. In Fig. 5.4, it is established that a close encounter between the radio line of sight and the ISS occurred centered around 02:29:06 UTC. The ISS stayed within 1° of the radio line-of-sight for approximately 14 seconds. The rapid jump and fall in both *TEC* and signal strength in Fig. 5.6a is correlated in time to when the eclipsing event is predicted to occur based on the ephemeris data presented in Fig. 5.4. The one-second standard deviation in the *TEC* data is 1/5th the amplitude of the correlated *TEC* drop. Thus the measurement noise of the carrier phase is smaller than the modification from the eclipsing event. The longer period sinusoid in the signal strength and multipath that is not observed in *TEC* is not correlated in time or estimated duration with the predicted eclipsing. This

separation in time scale suggests there are two phenomena occurring during this three minute window.

5.3.1.2 Case 4

Fig. 5.5 shows observer azimuth and elevation from the Ministik Lake ground station in the CHAIN network. The International Space Station (ISS) and GPS transmitter G26 have a minimum angular difference of 0.12° at 2021-01-12, 10:12:39.000 UTC. This can be seen in Fig. 5.5c.



(a) Local azimuth of ISS and (b) Local elevations of ISS and G26 calculated from orbital ephemeris at ground station. (c) Angular difference between ISS and G26 when projected on local hemisphere. Dashed red line indicates $\theta = 1^\circ$. Inset x-axis is seconds after 10:12.

Figure 5.5: The ISS eclipses the G26 GPS satellite on January 12, 2021 at 10:12 AM UTC time. Subplots a & b show the local observer elevation and azimuth from $53.350818^\circ N, 112.97384^\circ W$ which is the location of the Ministik Lake ground station in the CHAIN network. PRN 26 corresponds to USA-260, a Block-IIF model GPS satellite.

Using the same ISS orbital altitude as before, the arclength from the radio beamline is 2.7 km. Fig. 5.6b shows the CHAIN data from the G26 – edm link for two minutes centered around the predicted eclipsing event. The TEC in Fig. 5.6b linearly decreases throughout the entire time period. The signal strength in C/N_0 and $\sqrt{I^2 + Q^2}$ in Fig. 5.6b is appreciably constant. There is

a several minute period background sinusoid in the multipath (Fig. 5.6b).

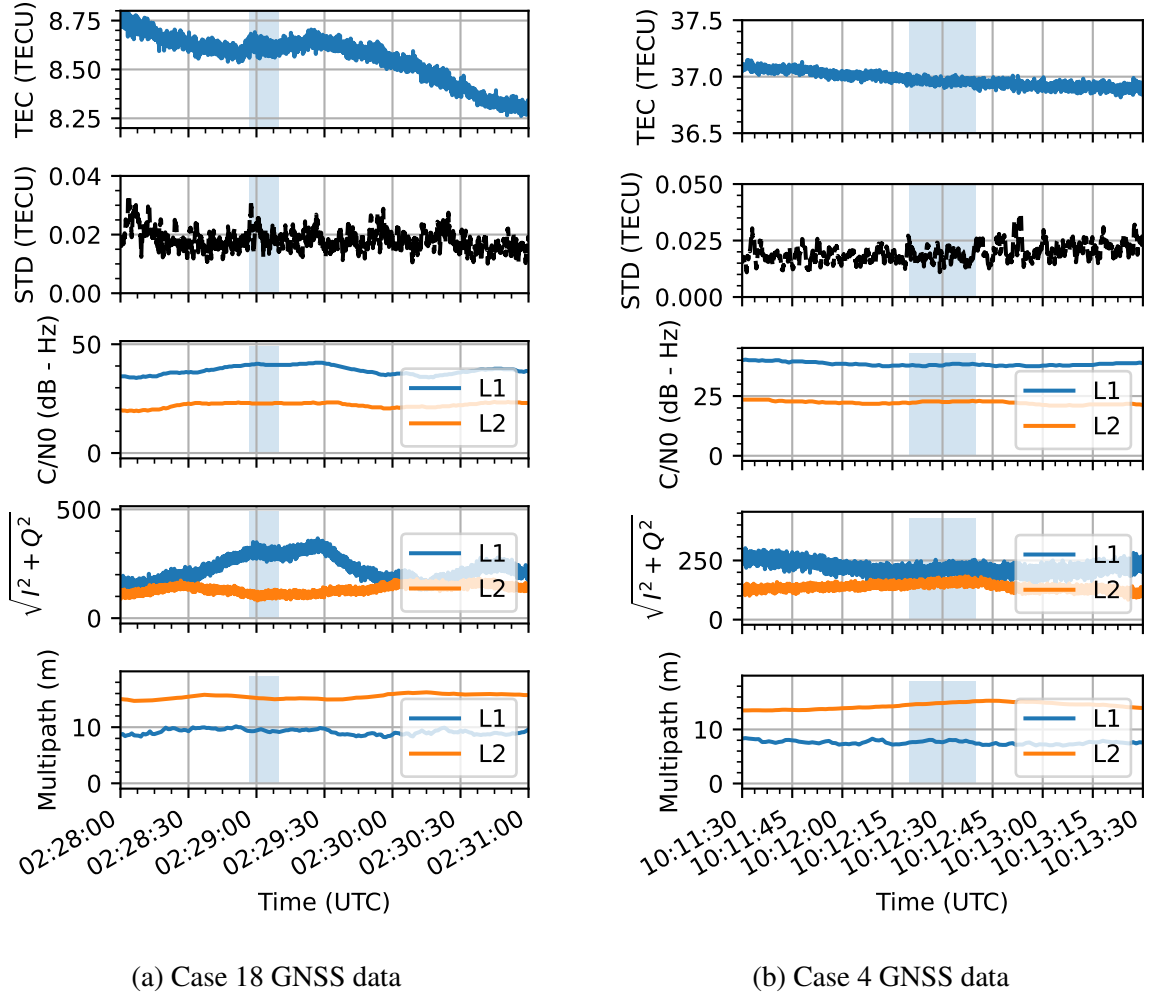


Figure 5.6: GNSS receiver data from edm ground station. STD is the 1 second standard deviation of the 50 Hz TEC . $\sqrt{I^2 + Q^2}$ is the signal amplitude calculated directly from I/Q data at 50 Hz. C/N0 is the signal to noise ratio reported by the receiver at 1Hz. The shaded region indicates the time of interest.

Fig. 5.7 shows the TEC calculated with the L1 and L2 *pseudorange* data at 1Hz. In this data, there is -10 TECU drop in only the $G26$ –edm link that occurs within 5 seconds of the eclipsing event calculated via the orbital ephemeris. This -10 TECU drop is only present in the pseudorange data, not the phase leveled carrier phase.

The -10 TECU drop in pseudorange is not correlated with a drop in signal strength or a

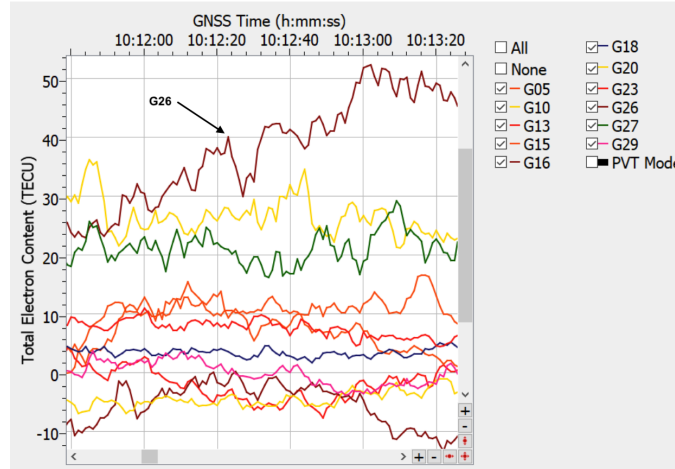


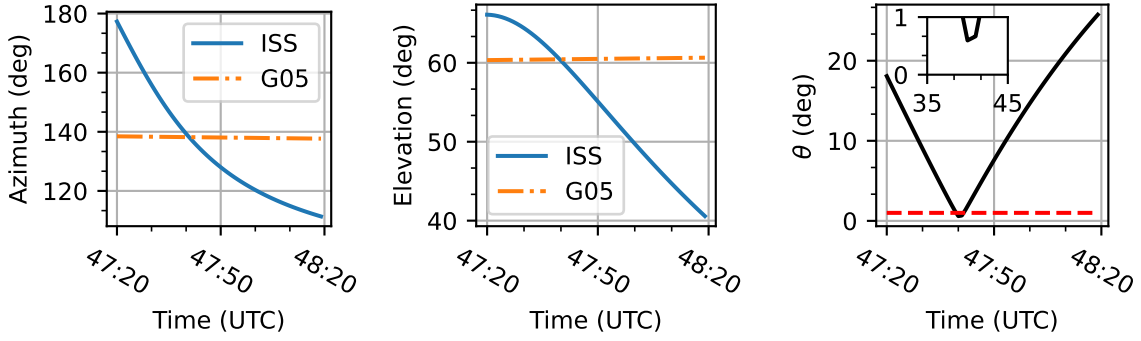
Figure 5.7: Total electron content calculated from pseudoranges by RxTools (receiver manufacturer’s proprietary software). The $G26$ – e_{dm} link is the topmost maroon line.

rapid change in multipath behavior. In other instances, the pseudorange seemed to trend with the multipath. However, in this case it seems to be anomalous. Perhaps this is just a coincidence.

5.3.1.3 Case 26

Fig. 5.8 shows the local azimuth, elevation, and angular difference between the ISS and G05 on June 8, 2021 at 21:47 UTC time. The minimum of Fig. 5.5c is at 2021-06-08, 21:47:40.000 UTC with $\theta = 0.594^\circ$. At this time the ISS had a local elevation of 60.3° . Additionally, the local elevation of the eclipsing event was $\sim 60^\circ$. Thus the line of sight was overhead instead of close to the horizon. This sky position corresponds to a range of $\approx 418 \text{ km} / \sin(60.3^\circ) = 481^\circ$. The miss distance from the line of sight is $\sim 5 \text{ km}$.

Fig. 5.9a shows the ground station data during the eclipsing event in Fig. 5.8 including the 50 Hz TEC . About 8 seconds before the eclipsing event (21:47:32.000), the TEC drops and rises 0.08 TECU over ~ 2 seconds. Following this, the TEC drops 0.1 TECU and rises slowly over ~ 10 seconds. There is a drop in the L1 C/N_0 of 2 dB-Hz at 21:47:33. There is no



(a) Local azimuth of ISS and (b) Local elevations of ISS and G05 calculated from orbital ephemeris at ground station. (c) Angular difference between ISS and G05 when projected on local hemisphere. Dashed red line indicates $\theta = 1^\circ$. Inset x-axis is seconds after 21:47.

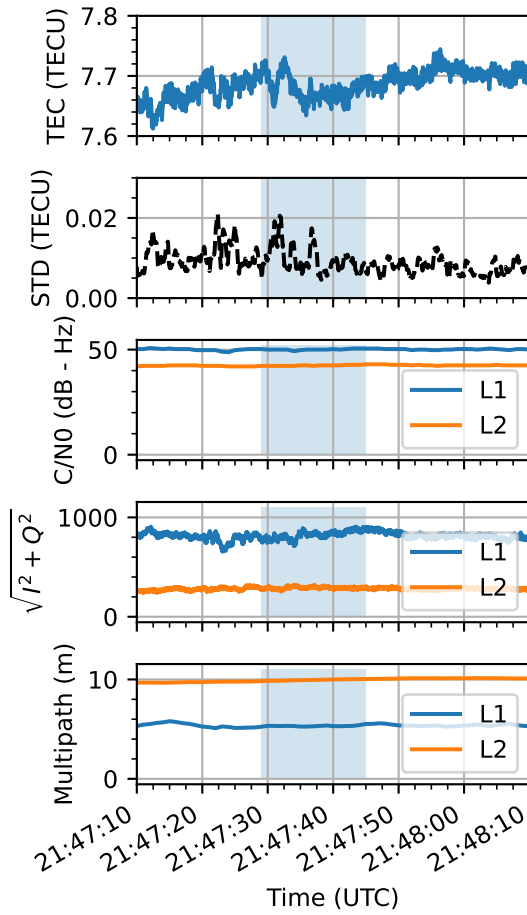
Figure 5.8: The ISS eclipses the G05 GPS satellite on June 8, 2021 at 21:47 UTC time. Subplots a & b show the local observer elevation and azimuth from $53.350818^\circ N, 112.97384^\circ W$ which is the location of the Ministik Lake ground station in the CHAIN network.

significant change in multipath in Fig. 5.9a. The one second standard deviation in the TEC is at most 0.02 TECU during this period. This indicates that the observed rapid change in TEC is physical and not measurement error.

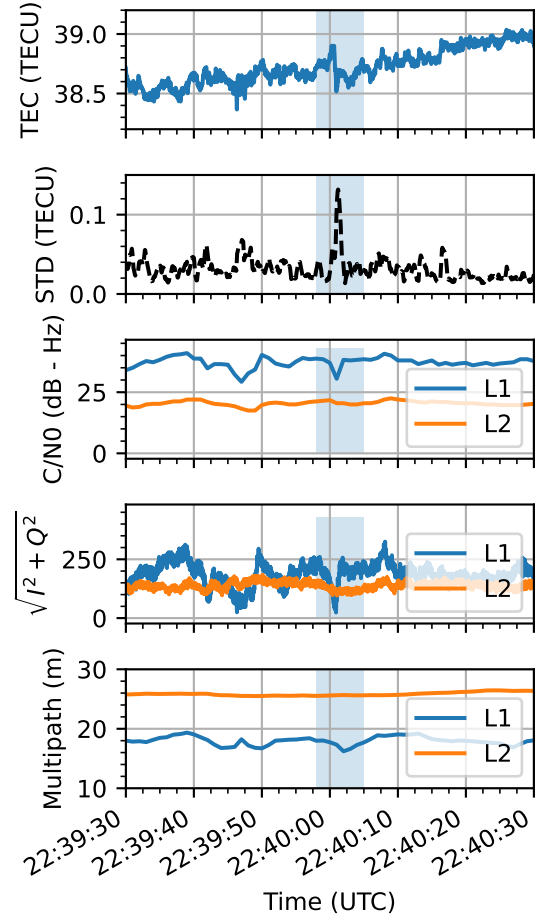
5.3.1.4 Case 27

Fig. 5.10 shows the local azimuth and elevation from the edm ground station. At 2021-06-13T22:40:02.733 UTC, the angular difference was 0.18° (the minimum of Fig. 5.5c) and the ISS local elevation was 20.3° . This corresponds to a ground station to ISS range of $\approx 418 \text{ km} / \sin(20.3) = 1,204 \text{ km}$ and a miss distance of $\approx 8 \text{ km}$.

Fig. 5.9b shows data from the ground station near the eclipsing event. All subplots have the same time axis. The larger background trend in Fig. 5.9b is a slow rise in TEC ($\sim 0.5 \text{ TECU/min}$). At the eclipsing time (22:40:03) the TEC rapidly rises 0.2 TECU and

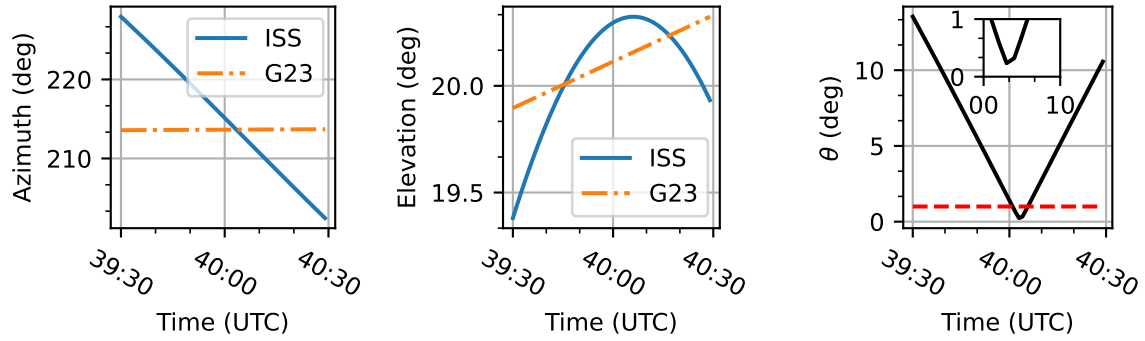


(a) CHAIN data from case 26.



(b) CHAIN data from case 27.

Figure 5.9: GNSS receiver data from edm ground station. STD is the 1 second standard deviation of the 50 Hz TEC . $\sqrt{I^2 + Q^2}$ is the signal amplitude calculated directly from I/Q data at 50 Hz. C/N0 is the signal to noise ratio reported by the receiver at 1Hz. The shaded region indicates the time of interest.



(a) Local azimuth of ISS and (b) Local elevations of ISS and G23 calculated from orbital and G23 calculated from orbital ephemeris at ground station site. (c) Angular difference between ISS and G23 when projected on local hemisphere. Dashed red line indicates $\theta = 1^\circ$. Inset x-axis is seconds after 22:40.

Figure 5.10: The ISS eclipses the G23 GPS satellite on June 13, 2021 at 22:40 UTC time. Subplots a & b show the local observer elevation and azimuth from $53.350818^\circ N, 112.97384^\circ W$ which is the location of the Ministik Lake ground station in the CHAIN network.

then drops 0.4 TECU. At the same time, there is a drop in L1 signal strength (Figs. 5.9b and 5.9b). There is a drop in L1 multipath, however it is small. Additionally, this eclipsing takes place at a local elevation of $\sim 20^\circ$ which is above the common threshold of 15° for avoiding multipath from the tree line and nearby buildings. There is a coincident increase in the one second standard deviation of the TEC , although this is still less than the magnitude of the TEC change.

5.3.2 Prediction of total electron content (TEC)

The total electron content is defined as the line integral of the electron density through the radio beam. For a change in electron density, one would expect a change in the TEC given by Eqn. 5.28. Following the analysis described in Ref. [192] the expected change in TEC for the observations from Case 18 shall be calculated.

$$\delta TEC = \int \delta n_e dl \quad (5.28)$$

For a Gaussian of characteristic width W , i.e. $\delta n_e = \alpha n_0 \exp\left(-\frac{l^2}{W^2}\right)$, the expected change in TEC is given by Eqn. 5.29. α is the percentage of the background density difference from the free stream in the wake. In Ref. [161], this was experimentally measured to be ≈ 0.99 . For the following analysis, α will be set to unity. The characteristic width of the Gaussian (W) depends on the angle that the radio-line-of-sight makes with the spacecraft wake. In general, it is safe to assume that $a < W < L$ where a and L are defined in Section 5.1.1.

$$\delta TEC = \alpha n_0 W \sqrt{\pi} \quad (5.29)$$

For each eclipsing event, let's define an observed length scale as $\hat{L} \equiv \frac{\delta TEC}{\sqrt{\pi} n_0}$. From comparison with Eqn. 5.29, it can be seen that this is the equivalent size of a Gaussian perturbation that would result in the observed δTEC . For each observation, we calculate this quantity and plot it in Fig. 5.11. Under the wake hypothesis, this quantity corresponds to the length of wake that the radio line of sight propagates through. In Fig. 5.11, a guess of $n_0 = 5 \times 10^{11} m^{-3}$ is used. It can be seen that the values for $\hat{L}$ are of the order of the expected wake length (single km). This also corresponds to the miss distance.

In Fig. 5.11, no error bars are provided because precisely quantifying the error bars requires more information than is available. There are two main possible sources of error, the miss distances and the background electron density. We shall briefly comment on each.

Precision of miss distance. From the IGS website⁵, the final ephemeris for GPS is accurate

⁵<https://igs.org/products/>

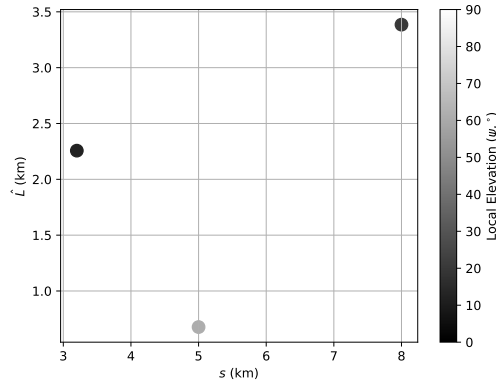


Figure 5.11: Length scale of TEC change inferred from observed δTEC ($\hat{L}$) as a function of miss distance (s). Shade indicates local elevation of eclipsing event.

to 2.5 cm. Thus it is reasonable to expect the miss distance error induced from GPS ephemeris error to be < 1 cm. The two other sources of error are the (i) GPS coordinates of the ground station and (ii) the ephemeris of the ISS. The GPS coordinates of the ground station are assumed to be correct to within the precision of the GPS ephemeris (< 1 cm). To our knowledge, JPL Horizons doesn't provide an error estimate for the ISS ephemeris. The JSC flight controllers provide the ephemeris to the public as well⁶. Presumably Horizons gets the ephemeris from them. JSC says that they update the ephemeris 3 times / week. Thus, error is allowed to accumulate for approximately 2 days. It is known that in drag dominated orbital environments, error can accumulate on the order of km / day [188, 193]. Therefore, the miss distance (s) is probably only accurate to within $\pm$ some amount of km, although further study of the precision of the ephemeris and exact error properties of atmospheric drag are needed to confirm this.

Precision of background electron density. The ISS goes through $+51^\circ$ to -51° latitude. Therefore, it experiences the equatorial and mid-latitude ionosphere. This can have a background electron density from $10^{10} - 10^{12} m^{-3}$. Without a local measurement at the ISS, the background

⁶https://spotthestation.nasa.gov/trajectory_data.cfm

density is unknown. Due to its possible range, the correct value could vary by a factor of 10 from the guess used in Fig. 5.11. Thus the error in $\hat{L}$, could be off by an order of magnitude.

Given the uncertainties that are inherent in this calculation of $\hat{L}$, the magnitude of these TEC changes match the expected value. In principle, if the orientation of the line of sight is known compared to the wake direction, δTEC can be computed analytically from integrating Eqn. 5.2 along the line of sight. However, our current measurement uncertainties are greater than the precision needed to test this result. A scientifically meaningful test would observe $\delta TEC(s)$ to a spatial resolution less than $< \frac{aM}{\sqrt{2N}}$ where N is the number of points along the wake that are measured. For the ISS with $N = 10$, $\frac{aM}{\sqrt{2N}} \approx 50$ m. Thus, 10 meter-level precision ephemeris would be appropriate for resolving this wake. It would also have a local measurement of n_0 at the time of eclipsing. Such probes exist on the ISS (e.g. NRL SPADE, USU FPMU), however the authors did not find public data from these instruments taken during the identified eclipsing events.

5.3.3 Comparison of amplitude to data

Fig. 5.12 plots Eqns. 5.11 and 5.22 with $\lambda = 20$ cm and $n_e = 5 \times 10^{11} m^{-3}$. The vertical dashed line indicates when the size of the sphere transitions from being described by “weak” (Eqn. 5.22) vs “strong” scattering (Eqn. 5.11). The point where the weak scattering intersects with the strong scattering is near where this dashed line is, confirming the theory.

In the examples shown above, the amplitude sometimes decreased by 1 – 10 dB-Hz. This corresponds to losing 30 – 90% of the power, i.e. $T = 0.1 - 0.7$. In Fig. 5.12, it can be seen that a km sized opaque sphere at the distance of the ISS would result in $T = 0.3$. However, due to

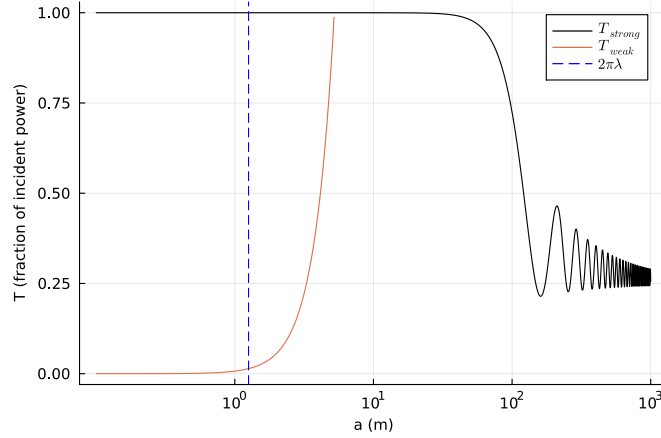


Figure 5.12: Transmission coefficient of sphere of size a . The black line indicates the transmission coefficient. The blue dashed line is 2π times the wavelength of GPS L1. The red line indicates the transmission coefficient of a dielectric sphere.

the alternating light and dark regions, this association cannot be trivially inverted. So in order of magnitude, there is agreement. However, this is not sufficient to uniquely determine the result. Multiple simultaneous measurements may add the precision needed to unambiguously infer scale size from amplitude change.

5.3.4 Extension to sub-cm Debris

Thus far, we have focused on the case study of the International Space Station to establish a proof-of-concept. We chose this target because its orbit is very well known, published publicly, and presumably it creates a larger plasma wake that is easier to detect. Extending this concept to smaller targets entails establishing the following parameters:

1. TEC sensitivity of the receiver
2. Sampling rate

In the following subsections, we shall attempt to scale the ISS observations to requirements

for sub-cm debris. Future experimental work should test these predictions.

5.3.4.1 TEC sensitivity of the receiver

First the **plasma wake** of sub-cm debris will be considered. From Eqn. 5.29, $\delta TEC \propto W$. If $W \propto a$ where a is again the scale size of object, then $\delta TEC_{1mm} = 10^{-5} \delta TEC_{100m}$. Thus, since the observed TEC changes were on the order of 0.1 TECU, the receiver would have to be sensitive to 10^{-6} TECU.

Electromagnetic pinned solitons have been proposed as a plasma-debris interaction to allow observation (instead of the wake). These are a fundamentally different plasma structure than the wake. A full discussion is outside the scope of this study. Electromagnetic pinned solitons can have TEC contents of $0.9n_0 \cdot 6\frac{c}{\omega_{pi}}$ where $\frac{c}{\omega_{pi}}$ is the ion skin depth [88]. For $n_0 \sim 10^{15}m^{-3}$ and ion skin depths of 1 km (as in [88]), the TEC would be $TEC \sim 2$ TECU. This is well within the feasibility of this concept, although these plasma conditions are also optimistic in using a plasma density on the upper end of what is observed.

Ion acoustic solitons are another proposed plasma-debris interaction to allow observation. As in [194], the TEC content would be $\sim 1n_0 \cdot 10\lambda_D$ where λ_D is the Debye length. The Debye length in many debris laden orbits is ~ 1 cm. With $n_0 \sim 10^{15}m^{-3}$, this would make the TEC of this disturbance $TEC \sim 0.01$ TECU. This is comparable to the observed TEC disturbances in this paper.

Now the **signal to noise ratio** (SNR) will be considered. As we've shown experimentally, a GNSS receiver seems to receive enough radio power to detect the disturbance from a large orbital object like the ISS. In that experiment, the noise was on the order of 0.001 TECU. This can be

understood theoretically by examining the carrier phase tracking. When a phased locked loop (a typical electronic circuit used for this purpose) is used to track the carrier phase, the phase error (σ_{PLL}) is given by the following equation:

$$\sigma_{PLL} = \sqrt{\frac{BW}{C/N_0} \frac{\lambda}{2\pi}} \quad (5.30)$$

In Eqn. 5.30, BW is the signal bandwidth and C/N_0 is the carrier to noise ratio. The smallest C/N_0 seen in the experimental results shown in Figs. 5.6 and 5.9 was ~ 20 dB-Hz. It varies signal to signal, but typically GNSS signal tracking loops have a bandwidth of roughly 15 Hz. At 20 cm and 20 dB-Hz signal strength, $\sigma_{PLL} = 0.02$ m. At $f = 1.2$ GHz, this corresponds to a TEC standard deviation of 0.004 TECU. This noise estimate is the same order of magnitude as the observed 1 second standard deviations in Figs. 5.6 and 5.9. The carrier to noise ratio is often larger than 20 dB-Hz so in the observations used in this study the observed noise stays below 0.004 TECU. For an electromagnetic pinned soliton, a SNR of 500 is expected. For ion acoustic solitons, a SNR of 2-5 is more realistic.

In summary, the TEC sensitivity strongly depends on the type of plasma disturbance. The wake of a sub-cm piece of space debris is probably not observable with current GNSS receivers using this method. However, both ion acoustic and electromagnetic pinned solitons would appear to be within the appropriate sensitivity. The noise observed (SNR ~ 3) in this experiment matches the noise predicted by Eqn. 5.30 and is below the magnitude of TEC changes predicted for solitons.

5.3.4.2 Sampling rate

Based on the current observations, it appears that a 1Hz sampling rate is *not* sufficient to resolve any changes in TEC that occur during an ISS radiometric eclipse due to the fast transient nature of the event. 50 Hz appears to resolve it. Many GNSS receivers in ionospheric monitoring networks decimate their data to a lower sampling rate in historical storage. One conclusion of this study is that this decimation obscures fast transient events in the data one would observe at the full sampling rate.

Sub-cm debris moves at the same orbital speed as the ISS. However, if the wake is shorter, then the TEC variations will have a shorter duration. Let the duration of the TEC change be τ . $\tau \sim v_0 W$ where v_0 is the speed of the target spacecraft. Since v_0 is constant, $\tau \sim W$. Thus, instead of seconds in duration, the event will be 10s of microseconds (five magnitudes smaller).

If instead of the wake, **ion acoustic** or **electromagnetic pinned** solitons are considered, $\tau \sim v_0 \lambda_D$ and $\tau \sim v_0 \frac{c}{\omega_{pi}}$ respectively. The ion skin depth has the same scale length as the wake of the ISS. Thus, for electromagnetic pinned solitons, the timescale would be comparable. Ion acoustic solitons would be several orders of magnitude shorter in duration since $\lambda_D \ll W_{ISS}$.

In conclusion, as with the sensitivity the GNSS eclipsing technique with current generation receivers would be unsuitable for detecting the wake of sub-cm debris. However, the same receivers used in this study would be suitable for observing electromagnetic pinned solitons. To detect ion acoustic solitons, the sampling rate would probably need to be increased to the order of kHz. Whether or not this is possible is outside the scope of this study.

5.3.5 Practicality of Multipoint GNSS Sensor for Detecting Spacecraft

Using the design criteria developed in subsection 5.2.4, for $f_s = 50$ Hz, $r = 410$ km, $\rho = 22,700$ km, $v_0 = 8$ km/s, and $N = 10$ the appropriate GNSS receiver spacing would be 1.5 km. For $N = 1$ the appropriate spacing would be 150 meters.

If a square grid of 20 x 20 receivers is used that are spaced 500 meters apart, the whole system would have a size of 10 km². This would be impractical for a dedicated site. However, if GNSS receivers are simply added on to existing buildings, this concept becomes much more feasible. The ideal location would be (a) in a low radio noise zone and (b) already have dispersed sites with this separation. In the United States, several state's department of transportation (e.g. Texas Department of Transportation, Missouri Department of Transportation, Ohio Department of Transportation) or geologic surveys (e.g. North Carolina Geodetic Survey, NCGS) operate static GNSS receivers dispersed throughout the state. These are sometimes used as data sources for the NOAA CORS network. For example, Fig. 5.13 shows the NCGS sites that are part of the CORS network. Many of the NCGS sites are colocated with other facilities such as highway maintenance facilities, road salt depots, and rest stops.

Charlotte to Greensboro along I-85 is approximately a 150 km journey. There are four stations along that route, making the inter ground station distance approximately 40 km. Since the grid seems uniform, we will estimate that the NCGS network has an average separation distance of $d = 40$ km. Among the states with complete CORS coverage, this spacing seems typical. However, as seen in Fig. 5.13, some states don't have this uniform spacing (i.e. Virginia). However, the North Carolina separation distance is still an order of magnitude too large.

In principle, one could construct a denser segment of GNSS receivers along an interstate

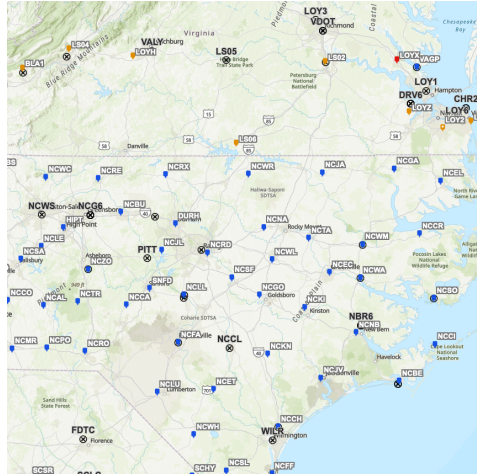


Figure 5.13: NOAA CORS network map. From <https://geodesy.noaa.gov/CORS/>

highway. Since state DOTs already operate similar stations, putting them along interstates may make acquiring the site land straightforward. It would also make access for maintenance easy. If one did this where multiple interstates meet, then the multi-point sensor would have multiple branches. If they meet at an approximately right angle and the condition in subsection 5.2.4.2, then $s \approx d\sqrt{2} \leq \frac{w}{2}$. For $w \approx 1$ km as in the ISS, this would mean $d \leq 350$ m. So even with a cross configuration, GNSS receivers were arranged every 100 m or so from the center of the intersection, this would be sufficient to meet the criteria established above. Some intersections on the continental United States that appear to be good candidates for this design include: I80 and I79, I57 and I70, I69 and I169, I35 and I90, I80-I74-I280, I135 and I70, I20 and I10, and I95 and I74. For example, Fig. 5.14 shows marked up satellite imagery of the intersection of I95 and I74 in southeastern North Carolina. The stars in Fig. 5.14 indicate possible locations of ground stations where the inter ground station spacing is between 100 - 200 meters. This concept doesn't account for the effect on highway safety. A highway interchange is an appropriate sizing for the proposed sensor.

The characteristic network of receivers in Fig. 5.14 contains 18 receivers. If one used



Figure 5.14: Satellite imagery from the intersection of I95 and I74. Stars indicate possible locations of ground stations. In these locations the lawn by the highway is mowed, indicating there is access to the site. Image credit in bottom of image.

the SparkFun RTK mosaic-X5 survey grade GNSS detector kit⁷ as the sensor, an approximate construction budget is given by Table 5.3. The antenna in this kit has been certified by the National Geodetic Survey with an available calibration. \$10,000 is assumed to be sufficient to design a power and networking system to power each unit and provide internet access to each receiver. This may entail a hookup to the local utility, extension cables to all of the sensors, and a cell tower based WiFi network. The specifics will depend on the exact site. The total comes to \$33,000. Since this isn't a complete budget, a conservative estimate of the construction cost for this multipoint sensor is on the order of \$50,000 USD. This does not include labor and the cost to develop software to detect eclipses from raw GNSS data.

Table 5.3: Potential budget for construction of GNSS ground sensor

Item	Quantity	Cost (USD)
GNSS Receiver + Antenna	18	1200
GNSS Antenna Mounting Kit	18	15
Survey Pole	18	110
Power & Networking	-	10,000
Total	-	33,850

⁷<https://www.sparkfun.com/products/23748>

More conventional ionospheric measurements are taken with radio telescopes and incoherent scatter radars. These are not commercial off-the-shelf products, so historical construction costs are used to estimate an order of magnitude cost. According to its public website⁸, the Greenbank telescope cost \$95,000,000 to construct. The Arecibo telescope was constructed for \$9,300,000 USD in 1963 with subsequent upgrades that also cost in the range of \$10,000,000⁹. Adjusted for inflation, \$9,300,000 is \$95,600,000. One of the first ISRs, the Jicamarca Radio Observatory, was constructed in 1959 for approximately \$1,000,000 USD [195]. Adjusted for inflation that cost is approximately \$10,000,000 in 2024 USD. The EISCAT 3D upgrade, a substantial construction to the existing EISCAT radars, is budgeted to cost \$79,300,000 EUR¹⁰. In summary, historical radar / radio telescope projects cost between \$10,000,000 - \$100,000,000 USD.

GNSS receivers, radio telescopes, and ISRs consist of the same basic components: antennas and receiving electronics. So why is there a substantial difference in construction cost? ISRs require a transmitter and large power supply (MW+) to receive any reflected signal. This cost isn't necessary for the GNSS system. Radio telescopes and ISRs have steerable dishes (either through a mechanical tracking mounts or phased array antenna) to achieve high integration times (by tracking a fixed object in the sky) or directionally high gains (to only receive the reflected signal and not other noise). GNSS can distinguish directionality of various carrier waves and isolate them from noise through multiplexing. The GNSS signal was carefully designed to allow multiple satellites to share a carrier frequency by using pseudorandom codes to distinguish the signals from different satellites (i.e. multiplexing). This allows the receiver to sort out direc-

⁸<https://greenbankobservatory.org/about-old/green-bank-facts/>

⁹https://ethw.org/Milestones:NAIC/Arecibo_Radiotelescope,_1963

¹⁰https://roadmap2021.esfri.eu/projects-and-landmarks/browse-the-catalogue/eiscat_3d/

tionality during the signal processing stage instead of by hardware design. The downside of this method is that a GNSS detector can only observe where a GNSS satellite is overhead. There are also limits to how many satellite signals can be multiplexed together. ISRs have a greater range of directionality. The antennas used in radio telescopes and ISRs are typically much higher gain than GNSS survey-grade antennas. For a parabolic antenna (like the main dish of Arecibo or Greenbank), the gain is given by:

$$G = \left(\frac{\pi d}{\lambda} \right)^2 \epsilon_A$$

d is the diameter of the antenna, λ is the wavelength, and ϵ_A is the antenna efficiency. Arecibo had a diameter of 305 m and Greenbank has a diameter of 100 m. For example, at a 20 cm wavelength with an efficiency of 0.6 (which may or may not be realistic), the gains for Arecibo and Greenbank are respectively 71 dB and 62 dB. A commercial surveying GNSS antenna typically has a 5 dB gain. Thus, an Arecibo or Greenbank telescope has a 55+ dB higher antenna gain than the GNSS system due to just the larger (and hence more expensive) antenna. By swapping the GNSS antenna for a large parabolic dish of Greenbank telescope size, the carrier to noise ratio could be boosted to approximately 75 dB-Hz. The error model given in Eqn. 5.30 predicts TEC measurement precision of 0.00008 TECU given the other parameters of the carrier phase tracking loop are kept constant. Given that the TEC modifications from solitons are expected to be substantially larger than this, the added precision is an inefficient tradeoff for the much greater increase in price.

5.4 Conclusions

In conclusion, we have demonstrated a proof-of-concept that the TEC does change when a large spacecraft (the ISS) flies through the radio beam. The amplitude of this TEC change is the same order of magnitude as treating the wake as a Gaussian, although due to the limitations of this experiment, this hypothesis can only be examined to the precision of an order of magnitude. This is the first time this effect has been reported.

Due to the nature of how much GNSS data is stored, it is also hypothesized that previous detections have been decimated by compression techniques. This study did not consider a multiple detections of the same eclipsing event due to the sparsity of the data. It would be interesting in future work to examine such a scenario to establish if triangulation of the eclipsing event is possible.

Using this proof-of-concept, we have expanded on this concept to both sizing a multi-point receiver such that it is sensitive to radiometric eclipsing. Additionally, using physical scaling laws the performance of this technique has been hypothesized for sub-cm debris which would be the most meaningful. In the case of a soliton based detection, this scheme is hypothesized to have the appropriate performance to detect solitons.

Chapter 6: Conclusions and Future Work

Space debris is an existing and growing problem for satellite operators and manned missions near Earth. As outlined in Chapter 1, the number of launches injecting human created material into Earth orbit is exponentially increasing. These new spacecraft are expected to contribute to and be more susceptible to debris populations. Therefore, new methods of detecting the debris are crucially important for the sustainable use of outer space. In particular, the debris population less than a centimeter in size is not visible to existing sensors. Indirectly observing sub centimeter space debris via plasma solitons is seen as a new technique to augment the Space Surveillance Network so that it can detect this untracked population [1, 196].

Detecting plasma solitons can either be done in-situ on a survey spacecraft or from the ground with a incoherent scatter radar or the GNSS technique discussed in Chapter 5. Plasma solitons may either stay with the debris (pinned) or propagate ahead of it (precursor). In this dissertation, we have explored several aspects of the detection problem. Firstly, the inverse scattering transform (Chapter 3 and published in [99]) is suited for detecting solitons both pinned and precursor from any source (ground or space based). This work shows that a novel fast method for computing the nonlinear spectrum of time series data (as developed in Ref. [110]) can be used to automatically detect the presence and properties of a soliton. This is also the first time this signal processing technique has been used on simulations of plasma systems. Solitons are created in

other plasma physics systems where this technique may also be useful.

Chapter 5 details a prototype of a ground based detection scheme that would be best suited for pinned solitons. This is necessary to associate the soliton with the orbit of the debris. The preliminary analysis of the radiometric eclipsing technique was published at the 2023 AMOS conference (Ref. [192]). This dissertation further develops this ideas with a proof-of-concept experiment where the International Space Station was shown to create a detectable signature in the total electron content measured by a GNSS ground station. This signal occurs on a timescale of seconds. This means that, at least for the detection of the ISS, existing networks of GNSS receivers are physically capable of detecting this radio disturbance, however the frequency the data is recorded (often at < 0.03 Hz) will need to change in order to resolve the fast timescale. Theoretical analysis was conducted that showed some agreement with the ISS observations. This was used to project how the technique would work for sub centimeter debris. It is predicted that detecting the solitons that are predicted to be created by debris may be in the regime of detectability using this technique. Future work will entail constructing a multi point receiver in order to more precisely understand the geometrical properties of the interaction between the spacecraft and the GNSS signal. Some discussion and sizing of this was considered in Chapter 5.

As shown in Ref. [1], the orbital locations where solitons are generated and their type (precursor or pinned) depends on the local ion acoustic Mach number. This changes what detection schemes are suited for different altitudes of orbital debris. In Chapter 4, it has been shown that the original estimates in Ref. [1] of the critical Mach numbers that govern this transition are incorrect for sub Debye length debris due to assuming the transition points were near Mach 1. This includes much sub centimeter debris. Chapter 4 presents two fluid simulations that were used to more accurately identify the transitions between linear, precursor, and pinned flow regimes.

A new theory was also derived that shows good agreement with the simulation results. For the purposes of space debris detection, the range of Mach numbers that produce precursor solitons was found to be wider than previously thought. This correlates to a wider range of orbits that would produce precursor solitons. In this theory, the sheath potential was shown to strongly control the supercritical Mach number. However, an analytical model was used in our studies that neglects any feedback from the solitons to the sheath. The natural extension to this investigation is to directly resolve the sheath dynamics. Is the sheath constant, as assumed in this simulation, due to the fast charging timescale compared to the ion acoustic dynamics? Due to the change in plasma properties in front of and behind the debris's direction of motion, is the charging or particle fluxes uniform over the surface? Future simulations that resolve the sheath boundary condition will shed light on this issue. It was also found that near the supercritical Mach number, a wave breaking behavior is observed in the precursor solitons. If physical, this would be a new semi-stable oscillation of the soliton. The next step would be to repeat the flow conditions in higher dimensions. The added dimensionality ($1D \rightarrow 2D$) may change the spreading of the soliton to suppress this behavior.

Appendix A: Derivation of Scattering Theory

Consider the macroscopic Maxwell's equations with no free charge nor currents with the standard constitutive relations $\mathbf{D} = \varepsilon(\mathbf{r}) \mathbf{E}$ and $\mathbf{H} = \frac{1}{\mu_0} \mathbf{B}$. We consider a spatially varying permittivity, but a constant permeability.

$$\nabla \cdot \mathbf{D} = 0 \tag{A.1}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} \tag{A.2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{A.3}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{A.4}$$

Taking the curl of Eqns. A.2 and A.4 results in the standard propagation equations with a source term from the spatially varying permittivity given by Eqns. A.5 and A.6. It helps to note that $\nabla \cdot \mathbf{D} = \nabla \varepsilon(\mathbf{r}) \cdot \mathbf{E} + \varepsilon(\mathbf{r}) \nabla \cdot \mathbf{E} = 0$. In vacuum, this reduces to wave equation for freely propagating light. This can be seen by taking $\nabla \varepsilon \rightarrow 0$ and $\varepsilon(\mathbf{r}) \rightarrow \varepsilon_0$.

$$\nabla^2 \mathbf{B} - \mu_0 \varepsilon(\mathbf{r}) \frac{\partial^2}{\partial t^2} \mathbf{B} = \mu_0 \frac{\partial}{\partial t} (\nabla \varepsilon(\mathbf{r}) \times \mathbf{E}) \quad (\text{A.5})$$

$$\nabla^2 \mathbf{E} - \mu_0 \varepsilon(\mathbf{r}) \frac{\partial^2}{\partial t^2} \mathbf{E} = -\nabla \left(\frac{1}{\varepsilon(\mathbf{r})} (\nabla \varepsilon(\mathbf{r}) \cdot \mathbf{E}) \right) \quad (\text{A.6})$$

Focusing on Eqn. A.6, we consider wave solutions on an infinite domain that take the form of $\mathbf{E} = \hat{\mathbf{E}}_\omega \exp[-i\omega t]$. Substituting this ansatz into Eqn. A.6 and dropping the hat on $\mathbf{E}$ gives A.7. In vacuum, $\mu_0 \varepsilon \omega^2 = (\omega/c)^2 = k^2$. This is related to the dispersion relation of the medium.

$$\nabla^2 \mathbf{E} + \mu_0 \varepsilon(\mathbf{r}) \omega^2 \mathbf{E} = -\nabla \left(\frac{\nabla \varepsilon(\mathbf{r})}{\varepsilon(\mathbf{r})} \cdot \mathbf{E} \right) \quad (\text{A.7})$$

We shall assume “weak” scattering. Instead of solving Eqn. A.7 in its entirety, we shall treat the left hand side as dominant with the right hand side as a weak source. More specifically, consider $\frac{\nabla \varepsilon(\mathbf{r})}{\varepsilon(\mathbf{r})} \ll 1$. Then, we can write the solution to Eqn. A.7 as $\mathbf{E} = \mathbf{E}_0 + \mathbf{E}_1$ with $\mathbf{E}_1 \ll \mathbf{E}_0$ and $\mathbf{E}_1 \sim \frac{\nabla \varepsilon(\mathbf{r})}{\varepsilon(\mathbf{r})}$. We shall also substitute $\mu_0 \varepsilon \omega^2 \rightarrow k^2$. This results in two equations. Eqn. A.8 describes the propagation of the incident light. Eqn. A.9 describes the scattered light. In the region away from the scattering (which is what shall be considered) $k(\mathbf{r}) \rightarrow \frac{2\pi}{\lambda}$ where λ is the wavelength of the light. We assume that the scattered and incident light have the same wavelength. For this reason, we drop the explicit $\mathbf{r}$ dependence on the left hand side of Eqns. A.8 and A.9.

$$\nabla^2 \mathbf{E}_0 + k^2 \mathbf{E}_0 = 0 \quad (\text{A.8})$$

$$\nabla^2 \mathbf{E}_1 + k^2 \mathbf{E}_1 = -\nabla \left(\frac{\nabla \varepsilon(\mathbf{r})}{\varepsilon(\mathbf{r})} \cdot \mathbf{E}_0 \right) \quad (\text{A.9})$$

Eqn. A.8 will be specified as a plane wave, i.e. for propagation in the z direction and polarized in the x direction $\mathbf{E}_0 = E_0 \exp(ikr) \hat{e}$. $\hat{e}$ is a unit vector in the direction of the incident electric field. Eqn. A.9 can be solved by the method of Green's functions.

$$\mathbf{E}_1(\mathbf{r}) = - \int G(\mathbf{r}, \mathbf{r}') \nabla \left(\frac{\nabla \varepsilon(\mathbf{r}')}{\varepsilon(\mathbf{r}')} \cdot \mathbf{E}_0(\mathbf{r}') \right) d^3 \mathbf{r}' \quad (\text{A.10})$$

The Green's function for the 3D Helmholtz operator $(\nabla^2 \mathbf{u} + k^2 \mathbf{u})$ is $G(\mathbf{r}, \mathbf{r}') = -\frac{\exp(-ik|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|}$.

Thus

$$\mathbf{E}_1(\mathbf{r}) = \int \frac{\exp(-ik|\mathbf{r}-\mathbf{r}'|)}{4\pi|\mathbf{r}-\mathbf{r}'|} \nabla \left(\frac{\nabla \varepsilon(\mathbf{r}')}{\varepsilon(\mathbf{r}')} \cdot \mathbf{E}_0(\mathbf{r}') \right) d^3 \mathbf{r}' \quad (\text{A.11})$$

At large distances from the scatterer compared to the light wavelength ($kr \gg 1$), $\frac{1}{|\mathbf{r}-\mathbf{r}'|} \approx \frac{1}{r}$ and $\exp(-ik|\mathbf{r}-\mathbf{r}'|) \approx \exp(ikr - i\mathbf{k}' \cdot \mathbf{r}')$ where $\mathbf{k}'$ is the wavevector of the scattered wave. If one uses the expression $\nabla(\nabla \ln \varepsilon \cdot \mathbf{E}_0)$ directly this matches exact solutions found in [197] (although they keep a spatially variable permeability).

$$\mathbf{E}_1(\mathbf{r}) = \frac{\exp(ikr)}{4\pi r} \int \exp(-i\mathbf{k}' \cdot \mathbf{r}') \nabla \left(\frac{\nabla \varepsilon(\mathbf{r}')}{\varepsilon(\mathbf{r}')} \cdot \mathbf{E}_0(\mathbf{r}') \right) d^3 \mathbf{r}' \quad (\text{A.12})$$

Let $\mathbf{P}_0(\mathbf{r}) = \varepsilon_0 \chi(\mathbf{r}) \mathbf{E}_0 = (\varepsilon(\mathbf{r}) - \varepsilon_0) \mathbf{E}_0$ be the volumetric induced dipole moment from

the incident electric field. The operator $\nabla \left(\frac{\nabla \varepsilon(\mathbf{r}')}{\varepsilon(\mathbf{r}')} \cdot \mathbf{E}_0(\mathbf{r}') \right)$ can be simplified by recognizing that the interior part is basically a series of differential operators acting on the induced dipole moment. Consider that $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$. It can be shown by taking the divergence of $\mathbf{D}$ that $\frac{\nabla \varepsilon(\mathbf{r})}{\varepsilon(\mathbf{r})} \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \nabla \cdot \mathbf{P}$. In the Feynman lectures on this topic, the right hand side is solved for in terms of the polarization directly: https://www.feynmanlectures.caltech.edu/II_32.html#Ch32-S6. Plugging this into Eqn. A.12 yields the physically clear Eqn. A.13. In the context of Fourier optics, $\nabla (\nabla \cdot \mathbf{P}_0)$ is known as the “aperture function.”

$$\mathbf{E}_1(\mathbf{r}) = \frac{\exp(ikr)}{4\pi\varepsilon_0 r} \int \exp(-i\mathbf{k}' \cdot \mathbf{r}') \nabla (\nabla \cdot \mathbf{P}_0) d^3\mathbf{r}' \quad (\text{A.13})$$

$\nabla (\nabla \cdot \mathbf{P}_0) = \nabla^2 \mathbf{P}_0 + \nabla \times (\nabla \times \mathbf{P}_0)$. However, we shall claim that $\nabla^2 \mathbf{P}_0$ is small.

Therefore, we are left with:

$$\mathbf{E}_1(\mathbf{r}) = \frac{\exp(ikr)}{4\pi\varepsilon_0 r} \int \exp(-i\mathbf{k}' \cdot \mathbf{r}') \nabla \times (\nabla \times \mathbf{P}_0) d^3\mathbf{r}' \quad (\text{A.14})$$

We assume that the $\nabla \chi(\mathbf{r}) \cdot \mathbf{E}_0 \ll \chi(\mathbf{r}) \nabla \times \mathbf{E}_0$. If the properties of the scattering media change on longer spatial scales than the wavelength, this is justified. Now, we can write the integral in terms of susceptibility:

$$\mathbf{E}_1(\mathbf{r}) = \frac{\exp(ikr)}{4\pi r} \mathbf{k}' \times \left(\mathbf{k}' \times \int \exp(-i\mathbf{k}' \cdot \mathbf{r}') \chi(\mathbf{r}') \mathbf{E}_0(\mathbf{r}') d^3\mathbf{r}' \right) \quad (\text{A.15})$$

Recall that $\mathbf{E}_0 = E_0 \hat{e} \exp(i\mathbf{k} \cdot \mathbf{r})$. Let's define $\mathbf{q} = \mathbf{k}' - \mathbf{k}$. Then:

$$\mathbf{E}_1(\mathbf{r}) = \frac{E_0 \exp(ikr)}{4\pi r} \mathbf{k}' \times \left(\mathbf{k}' \times \int \exp(-i\mathbf{q} \cdot \mathbf{r}') \chi(\mathbf{r}') \hat{e} d^3\mathbf{r}' \right) \quad (\text{A.16})$$

We can recognize $\int \exp(-i\mathbf{q} \cdot \mathbf{r}') \chi(\mathbf{r}') d^3\mathbf{r}'$ as the Fourier transform of the susceptibility of the scatterer. For a non-varying magnetic permeability, the optical index of refraction is $n = \sqrt{1 + \chi}$. So this integral is strongly related to the Fourier transform of the index of refraction.

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