ABSTRACT

Title of Dissertation:	ENHANCING RESILIENCE OF COMPLEX NETWORKS: WASHINGTON D.C. URBAN RAIL TRANSIT AS A CASE STUDY
	Yalda Saadat, Doctor of Philosophy, 2020
Dissertation directed by:	Professor Bilal M. Ayyub Department of Civil and Environmental Engineering

According to the United Nation's Department of Economic and Social Affairs Population Division, 66% of the world's population will reside in urban areas by 2050; a boost from 30 % in 1950. Urbanization has indeed triumphed and its speed has brought innovation and economic growth. Its synergies within infrastructure systems are undeniable and have increased the demand for such systems. However, urbanization is one reason infrastructure systems are knocked out of equilibrium and show complex dynamical behavior. Most infrastructure systems have been designed without planning for this magnitude of potential demographic changes; thus redesigns are long overdue. Also, climate change looms. Resource scarcity and host of other factors leave their impacts; all pose some incidence of perturbation in the state of the infrastructure system. These perturbations can affect the system's resilience, which is a defining property of each system for remaining functional in the midst of disruption

from adverse event. Therefore, it is essential to develop appropriate metrics and methods to enhance the resilience of infrastructures at the network level. Such enhancements are critical for sustainable infrastructure development that is capable of performing satisfactorily through intentional and/or stochastic disruptions. A resilience evaluation of a network typically entails assessing vulnerability and robustness as well as identifying strategies to increasing network efficiency and performance and offering recovery strategies ideally taken in a cost-effective manner.

This dissertation uses complex network theory (CNT) as the theoretic basis to enhance the resilience of large-scale infrastructure networks, such as urban rail transit systems. Urban rail transit infrastructures are heterogeneous, complex systems consisting of a large number of interacting nodes and links, which can imitate a network paradigm. Any adverse event leading to a disruption in the interaction and connectivity of network components would dramatically affect the safety and wellbeing of commuters, as well as the direct and indirect costs associated with performance loss. Therefore, enhancing their resilience is necessary.

Using the Washington D.C. Urban rail transit as a case study, this dissertation develops a methodology to analyze network topology, compute its efficiency, vulnerability and robustness in addition to provide a unified metric for assessing the network resilience. The steps of methodology are applied to two models of weighted and unweighted networks. For the weighted model two novel algorithms are proposed to capture the general pattern of ridership in the network, and to reflect the weights on assessing network efficiency, respectively.

This dissertation then proposes an effective strategy to increase the network resilience prior to a disruptive event, e.g., a natural disaster, by adding several loop lines in the network for topological enhancement. As such, adding a loop line can create redundancy to the vulnerable components and improve network resilience. Expanding on this, the dissertation offers comparative recovery strategies and cost model in the case of disruption. An effective recovery strategy must demonstrate rapid optimal restoration of a disrupted system performance while minimizing recovery costs.

In summary, the systematic methodology described above, assesses and enhances the network resilience. The initial results rank the most vulnerable and robust components of the network. The algorithms developed throughout the study advance the weighted network analysis state of art. The topological enhancement strategy offered basis to justify capital improvement. Post failure recovery analysis and the cost model serves to inform decision makers in identifying best recover strategies with special attention not only to restoring performance of a system but also on reducing associated failure and recovery costs. The use of the methodology proposed in this dissertation may lead to significant societal benefits by reducing the risk of catastrophic failures, providing references for mitigation of disruption due to adverse events, and offering resilience- based strategies, and related pursuits.

ENHANCING RESILIENCE OF COMPLEX NETWORKS: WASHINGTON D.C. URBAN RAIL TRANSIT AS A CASE STUDY

by

Yalda Saadat

Dissertation proposal submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
2020

Advisory Committee:
Professor Bilal M. Ayyub, Chair/Advisor
Professor Amr Baz, Dean's Representative
Assistant Professor Michelle Bensi
Dr Jennifer Helgeson, National Institute of Standards and Technology
Associate Professor Brian Phillips, University of Florida

Dedication

A portion of this dissertation has been written during the pandemic due to COVID-19 outbreak in 2020. At the beginning of the year, Jan. 8, 2020, some of my fellow Iranian Ph.D. students went to the airport, waved their hands to their families to take off by PS752 flight to pursue their dreams and they, along with other passengers, never saw the arrival time. These events along with many others spread sorrow. I, like many, grasped more than ever how, we, the people, are a connected network and we are impacted by a single moment of human suffering or bliss. This dissertation, which is all about the network, is dedicated to all those who have ever seen the people as an intertwined network and realized that the pain and pleasure of each one are related to the agony and ecstasy of others…

Acknowledgements

I would first like to express my deepest gratitude to my advisor Dr. Bilal Ayyub for his continuous advice and support during my Ph.D. I have been fortunate to work under the direction of this wonderful and remarkable person who has spent a great deal of his life on advancing the engineering field in many ways. He has inspired me to become an independent researcher, and provided opportunities for me to develop teaching, communication, mentorship, and supervision skills. He has helped me to realize the power of critical and structural thinking that help to be a bright and determined scientist. He has shaped my perspective on how to propose a research idea and develop that. It is his belief in my abilities and his kind care that made this dissertation happen. I hope that my current and future work provide him a peace of mind in that I am one of his legacies, following his path.

My sincere thanks must also go to the members of my thesis committee who are the most incredible committee members one can hope for. Dr. Amr Baz, one of the jewels of the University of Maryland, has openhandedly lent me his experience and showed me how to triumph. His continuous encouragement and support has been a resource of hope for me to be more certain of my professional goals. Dr. Michelle Bensi, who is one of the most talented and admirable faculty I know. She has generously given her time to offer me valuable comments toward improving my work and has supported me in my other pursuits fondly; doubtlessly, she is one of my role models. Dr. Jennifer Helgeson, an honorable researcher who provided me the opportunity to join her team at the National Institute of Standards and Technology

(NIST) to work on the exciting project that she was leading; her immense trust in me is appreciated and I am proud of the successful project that we delivered. She has also provided me with help and valuable insight that improved my work. Dr. Brian Philips, an esteemed faculty, whose teaching style I found deeply inspiring, his professional competencies and his positive attitude toward work are uplifting and I am grateful for his insightful suggestions, which have enhanced my dissertation.

I owe a very important debt to Dr. Joshua Epstein, one of the pioneers in the field of agent based modeling simulation for having a positive impact on my view toward research and for giving me heart to be a confident scientist. I cannot leave UMD without conveying my appreciation to Dr. David Lovell for showing me the great power of Monte-Carlo simulation through lengthy and enjoyable discussions.

I have received generous support and advice from my brilliant and best friends Dr. Kiana Aran and Dr. Parastoo Delgoshaei. My dissertation is enhanced through discussion with bright friends Dr. Shabnam Tafreshi, Dr. Anahita Mansouri and Dr. Mojtaba kheiri. My special gratitude goes to my friends Dr. Semira Nikuo, Dana Romero, and Norin Walker for reviewing this dissertation. Thank to friends who endured my weary times, cheered me on and celebrated each accomplishment: Aran Ghader Pour, Dr. Mana Norouzpour, Dr. Nasim Shahbazian, Dr. Iman Nasseri, Saloomeh Asghari, Farnaz Seifi, Valia Shoja, Dr. Shahriar Setoodeh, Maria Ben Abbas, Sara Shokri, Hilda Hashem pour, Poopak Rad, Maryam Almasi, Rojano Ezzati and my very special appreciation to my best and oldest friend Mandana

Mansouri for continuously supporting and encouraging me and lifting my spirit through this work. I cannot forget the kindness and support of my high-school literature teacher Shaban Alaie who has followed me and my progress through many years, applauding me for every professional victory. Many thanks to Dr. Tahereh Zohravi and Dr. Makon Fardis for providing me insightful advice, assisting me to be resilient.

I also thank my colleagues: Dr.Yanjie Zhang, Dr. David Webb, Dr. Yating Zhang, Dr. Huiling Hu, Dr. Ashkan Keivan, Hamed Ghaedi, and Sachraa Borjigin who I shared the moments of hard work with them and Sally Saleem, Siqi Cao, Yujie Mao and Majed Hamed who have been the greatest mentees and we learned from each other extensively. I also acknowledge the kind words that I received from UMD undergraduate students for whom I have been their TA. Those words reinforced my genuine interest for empowering others.

I am also grateful to all my in-law's family for the great and kind support. I would like to extend my appreciation to my uncles, my only aunt , and all my awesome cousins for being a source of constant encouragement. I lost an amazing grandma during my Ph.D., however, I cannot forget moments in my childhood when she was sitting on her green sofa with her one hand ready and blood pressure cuff in another hand, asking me: Dr. Yalda, would you please gauge my blood pressure? I wish, she could feel festive for this moment in which I receive my doctorate degree, although, she conferred it to me many years ago.

There are no proper words to convey my love and gratitude to my parents, Fahimeh Khoshmaram and Kioumars Saadat for their unconditional trust and love, boundless encouragement, and endless patience. They raised me up in the middle of the absurd Iran-Iraq war, yet provided me the safe environment to breathe, the intellectual atmosphere to thrive and a place full of love and inspirations. They are indeed my true role models. I am also grateful to my only brother, Babak who has been so generous in sending me encouraging vibes and for all the philosophical and helpful discussion we have had despite the long distance between us.

Finally, my thank with love goes to Mehdi Ghalambor, my husband, whom I promised a couple of months ago to complete this dissertation on his fortieth birthday and now I am loyal to my promise. I deeply admire, that he has understood me as an ambitious Ph.D., yet offered me his love, gentleness, kindness, patience, his unique sense of humor, his friendship and companionship. He has always been the first, and sometimes only person I could turn to, during this distressing yet inspiring period.

This dissertation is completed on November 10th, 2020.

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List of Abbreviation

ABM	Agent Based Modeling
ASCE	American Society of Civil Engineers
ASME	American Society of Mechanical Engineers
BN	Bayesian Network
CI	Critical Infrastructure
CNT	Complex Network Theory
DHS	Department of Homeland Security
DOT	Department of Transportation
FEMA	Federal Emergency Management Agency
FHWA	Federal Highway Administration
FV	Future Value
PPCIP	President′s Commission on Critical Infrastructure Protection
PPD	Presidential Policy Directive
PV	Present Value
LCCA	Life Cycle Cost Analysis
SC	Supply Chain
TRB	Transportation Research Board
NA	Network Analysis
LCCA	Life Cycle Cost Analysis
PPD	Presidential Policy Directive

Chapter 1: Introduction

Resilient infrastructure networks are important assets of any urban area; they impact the environment and support the life and well-being of its inhabitants. This chapter provides an overview of definitions of infrastructure networks, resilience and its attributes as well as an introduction to networks, their form and characteristic, and network analysis methods. In addition, this chapter describes the knowledge gaps and objectives of this study.

1.1 Infrastructure Systems

1.1.1 Definition of infrastructure systems

Infrastructures are the engine and backbone of urban life. They provide populations in urban areas with amenities and services. "Vast networks of energy, water, wastewater, transportation, landscapes, and communications are the fundamental means by which the society sustains life and well-being" (Pollalis et al. 2012). In its broadest sense, infrastructure consists of various systems and essential amenities and services that assist an organization or society to function. The Merriam-Webster dictionary (1828) defines the term "*infrastructure*" as the "underlying foundation or basic framework" enabling an organization or system to operate, as well as "the resources (such as personnel, buildings, and equipment)" required for relevant activities.

1.1.2 Types of Infrastructure systems based on their functions

In agreement with the definition of infrastructure, any asset containing physical, economic, social, political, cyber and other dimensions that support the fundamental need of society could be considered as infrastructure systems (Pollalis et al. 2012). For instance, roadways, railways, airports, power grids, water pipelines, communication systems, backup generators, landscapes, hospitals, schools, workforce, etc., are subcategories of physical infrastructure systems. Supply chain, however, could even be categorized as economic and nonphysical infrastructure types. Figure 1.1 shows some examples of major physical infrastructure systems that are essential for the built environment.

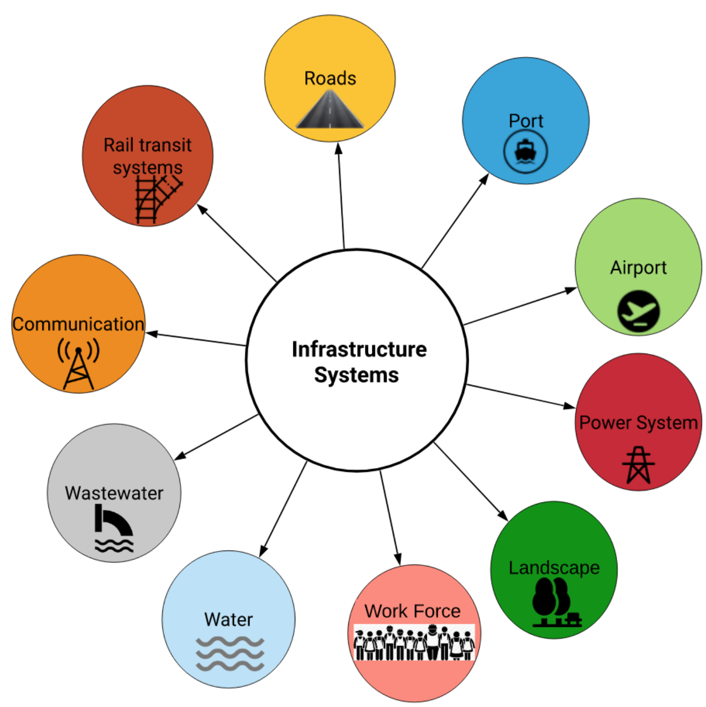


Figure 1.1. Urban major physical infrastructure systems that are essential for the built environment.

Infrastructure systems can also be categorized based on their criticality to the society, which this classification is based on governmental evaluation.

- **Critical Infrastructures**

According to Department of Homeland security (DHS, 2018), the term critical infrastructure refers to any infrastructure system that is considered vital to the society. DHS (2018) classifies 16 critical infrastructure sectors including chemical, commercial facilities, communications, critical manufacturing, dams, defense industrial base, emergency infrastructure, energy, financial services, food and agricultural, government facilities, healthcare and public health, information technology, nuclear reactors/materials/waste, transportation systems, and water and wastewater systems.

- **Others**

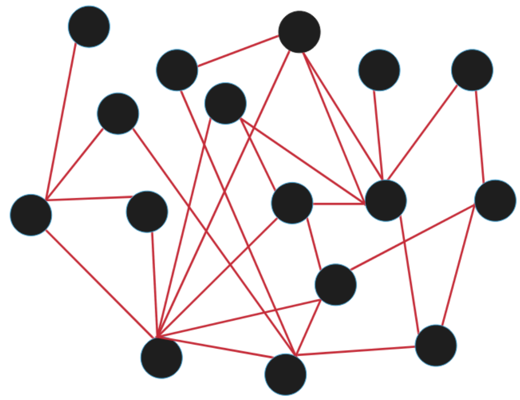
Any infrastructure system that is not included in the critical infrastructure systems identified by Department of Homeland security (DHS, 2018) is not considered as critical infrastructures by governments.

1.1.3 Infrastructure classification based on distribution mechanisms

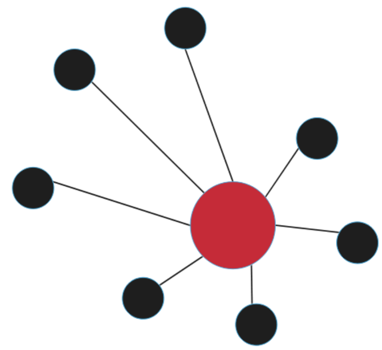
Infrastructure systems could be categorized as networks, nodal points, or the combination of those two in terms of their distribution mechanisms and their geometries (Pollalis et al. 2012). When an infrastructure system is categorized as a network, network components– which are basically several nodes connected by several links –and patterns of connections among nodes are responsible for distribution or collection purposes. For example, an urban water system could be considered as an infrastructure network in which water plants represent nodes and water pipelines indicate links.

In nodal point infrastructure types, a key component of the infrastructure system is/are one or few points that distribute or collect the resources to/from a variety of locations. For example, ports and airports are good examples of nodal points infrastructures.

Whether or not an infrastructure system is a network or a nodal point system, will be based on a project's size, the target of a study, and planning characteristics (e.g., centralized or decentralized). For example, in analyzing the reliability of a power system for one facility, a power system is modeled as a nodal point infrastructure. However, modeling power plants and power grids in measuring the vulnerability of the power system in a city level requires modeling power systems based on network characterization. Figure 1.2 (a-b) displays a network configuration and nodal point representation of infrastructure systems.



(a)



(b)

Figure 1.2. (a) Network configuration, and (b) Nodal point representation of infrastructure systems.

Many infrastructure systems are combinations of network and nodal point types. The one that is dominant determines the modeling method to fulfil the objective of the study. Understanding the distinction between the distribution mechanisms, i.e., network or nodal point types, could help in developing the resilience metric and measuring the vulnerability and robustness of a system.

This dissertation will focus on assessing the resilience of infrastructure networks through transportation system infrastructure as a case study. Thus, sections 1.2 and 1.3 and their subsections provide an in-depth overview introduction to networks in general as well as resilience definition and its attributes, i.e, performance, robustness and vulnerability, as well as analysis methods to assess the network resilience.

1.2 Introduction to networks

1.2.1 Definition of a network

According to Cambridge dictionary (1995) "a *network* is a large system consisting of many similar parts that are connected together to allow movement or communication between or along the parts, or between the parts and a control center."

A network or a graph, in its simplest form, is a collection of points joined together in pairs by lines. Points are referred to as *vertices* or *nodes* and lines are termed *links* or *edges* (Newman 2010). In fact, a network is a powerful abstract representation of a particular simplified system that breaks the system into nodes and edges and the pattern of connections. The behavior of any given network is highly dependent in those patterns of connections and interactions.

For instance, an urban rail transit system could be a tangible example of a network paradigm. Stations represent nodes and all other segments like tunnels, bridges, above grounds and etc., represent links. The performance of an urban rail transit system could depend on the connectivity pattern among the stations.

Networks or graphs that are used interchangeably in this dissertation are also perceived as a mathematical representation to model pairwise connectivity among nodes.

1.2.2 Types of networks

- **Directed and undirected networks**

A network could be *directed* or *undirected.* Directed graph or *digraph* includes a set of links that have directions associated with them displayed by arrows on the links to indicate the direction of those links. If the network does not have such characteristic, it is called an undirected graph.

- **Weighted and unweighted networks**

In an *unweighted* network, representation of a network is binary and links either exist or not without carrying any strength. In contrast, a *weighted* network is a type of graph in which a link carries some strengths and is given a numerical value.

- **Simple and non-simple networks**

A *simple* network or a *strict* graph contains neither multi-links nor self-links meaning there is not more than one link between any pair of nodes as well as nodes cannot be connected to themselves. Simple graphs are undirected and unweighted networks (Gibbons 1985;West et al. 2001; Newman 2011). Figure 1.3 shows a simple graph in which S_1, S_2...,S_6 represent nodes and the lines that connect the nodes together represent links.

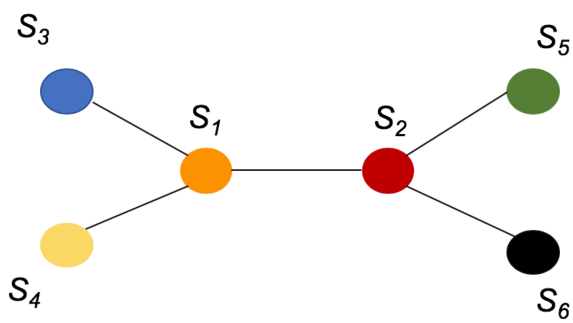


Figure 1.3. An example of a simple graph.

Non-simple networks are not simple and could include loops and/or multiple links between any pair of nodes.

- **Bipartite Networks**

A bipartite graph or *bigraph* is defined as a graph with the collection of its nodes partitioned into two or more disjointed subsets of nodes, such that the nodes in the same subset are not interconnected together, while a node in a subset is connected to one or more nodes in the other subset as shown in Fig. 1.4.

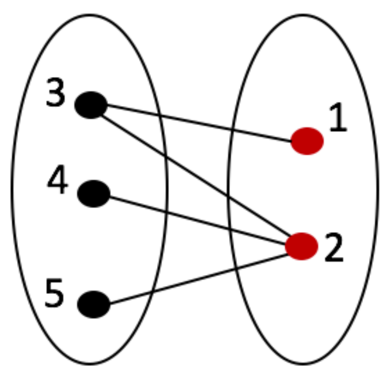


Figure 1.4. An example of a bipartite graph.

Thus, defining network properties and utilizing methods and tools to determine the network patterns of connection are necessary.

- **Single layer and multi-layer networks**

The distinction between a single layer network and a multi-layer network is based on the number of layers that are connected together. A single layer network consists of a one-layer graph, whereas a multilayer network consists of multiple layers of single-layer networks (Boccalettiet al. 2014; Kivelä et al. 2014; Danziger et al. 2014). Figure 1.5 (a-b) illustrates the single layer network and the multilayer network, respectively. Where the various layers of a multilayer network are intended to have a particular relationship, the nodes in the different graph layers could be connected by links to represent that relationship. Such a multilayer network is also called a "*network of networks*".

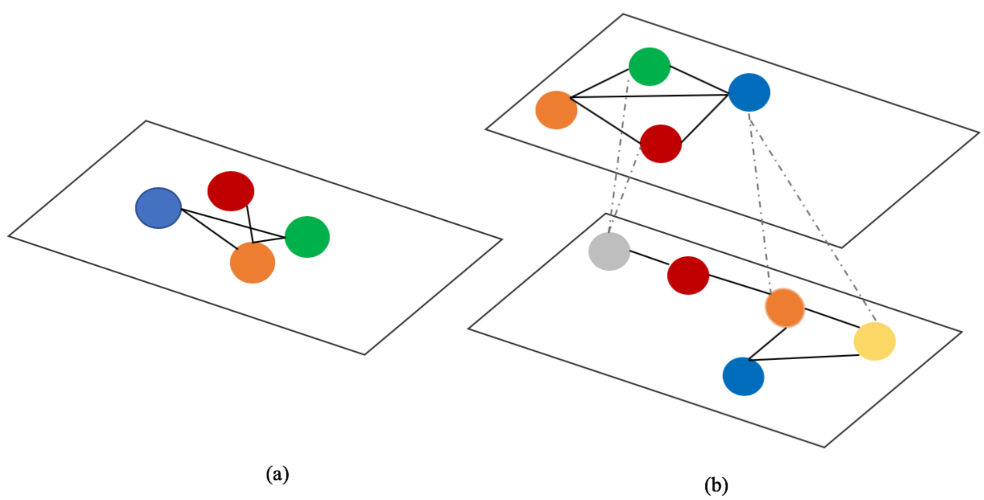


Figure 1.5. An example of a: (a) Single layer network; (b)Multilayer network.

1.3 Network Analysis and Application Domains Overview

Any system of interest that is composed of homogenous or/and heterogenous components linked together exemplifies a network. Thus, a broad range of network applications exists (e.g., ecosystem, human societies, spread of epidemics, collection of computers linked by data, internet and infrastructure networks to name a few). Many aspects of these networks could be better understood by studying the structure of such networks.

To study the structure of a network and how its components work together, there are extensive integrated techniques, mathematical tools, and computational programs available that might well be useful. Network analysis (NA) is using these tools and techniques to depict the relationship and pattern of connection among network components.

1.3.1 Methods of network analysis

- **Complex Network Theory**

Complex network theory (CNT) is a basic method that has been widely used in modeling urban critical infrastructure networks, such as power grids, water distribution systems, transportation networks, etc. Complex network theory is modeling a real system of interest in a form of graph with non-trivial topological elements that traditional networks do not possess (Lu et al. 2013; Chen et al.2012; Thai & Pardalos, 2012; Nielsen, 2011; Easley & Kleinberg 2010, Newman, 2010). The topology of a network is a key determining factor in CNT method. Topological characteristic indicators assist to calculate desired network attributes. CNT has been widely used in critical urban infrastructure networks, including grid networks (Albert et al. 2004; Wang and Rong 2009; Winkler et al. 2010, Ezzeldin and El-Dakhakhni, 2019), pipeline networks (Ouyang et al. 2008; Carvalho et al. 2009), water distribution networks (Simone et al. 2018), transport networks (e.g., roadway (Wu et al. 2007)), and airway networks (Zhang et al. 2010)).

- **Bayesian Network**

Bayesian networks (BN) are probabilistic graphical models for representing multivariate probability distributions and are used to capture the unknown and uncertainty disguised in the network. In BN models, nodes representing random variables and directed links describing probabilistic dependencies (Bensi et al. 2011). The method is explicitly underlined on vast conditional dependencies of network components. BN includes principles from different theories: graph theory, probability theory, computer science, and statistics (Gopnik &Tenenbaum, 2007). BNs can be used for assessing a wide range of attributes that are of paramount to infrastructure networks. For instance, Johansen and Tien (2017) employed BN to model the interdependencies among various infrastructure systems such as water, gas, and power systems. Hosseini & Barker (2016) used BN to model waterway ports. Tien & Kiureghian (2016) developed an algorithm for BN modeling and reliability of infrastructure systems and Bensi et al. (2009) assessed the performance of spatially distributed infrastructure systems by use of a Bayesian network methodology.

- **Simulation Based Analysis**

Simulation based analyses are techniques whereby specific software programs or a number of algorithms imitate the behavior of a network and models the interactions of its components. Simulation is an approach to forecast or measure the performance of large, complex and stochastic systems (Flood, 1998). A simulation-based analysis could be used as a single approach of analysis or it could be integrated with other approaches to capture a holistic view of it. There are typically two types of simulation approaches used to model networks, i.e., discrete event simulation (DES) and continuous simulation (CE). DES models the operation of a stochastic network as a sequence of discrete events and sets of variables. CE also models sets of events and variables, however, the changes in the variables take place continuously. Simulation methods take random process and probabilistic analysis into account and empower users to study the interactions and component relationships of a network in details before implementing it in the real world (Flood 1998).

Simulation-based methods have been widely used in network analysis. Tako and Robinson (2012) used discrete event simulation as decision support system in supply chain (SC) networks. Schmitt &Singh (2012) developed a simulation model to capture an actual network for consumer-packaged goods that a company is used for the analysis. Parker and Epstein (2014) applied agent-based modeling simulation to model the spread of an epidemic disease outbreak in a social network consisting several billion agents. Chen et al. (2015) simulated the behavior of pedestrian evacuation in a metro due to fire. Abdalla et al. (2007) simulated the performance of the water network using the data collected after flooding.

- **Others**

Several other methods could be used to analyze a network based on the target and the objective of the study. The methods outlined above also could be integrated and used in analyzing a network.

1.3.2 Complex network theory selection for analyzing urban infrastructure networks

Since this dissertation widely focuses on network topological analysis, it primarily uses the CNT method of modeling infrastructure networks. Therefore, elaborating the method, and its advantages and disadvantages, seems necessary.

Typically, two models are used under the CNT method; the model that is able to capture the network flow of the network; and the model that only focuses on the network topology. In other words, the CNT method could be employed to model the infrastructure network as a weighted network or an unweighted one. Both models are acceptable and have been widely used in relevant literature.

The focus of the weighted model is on both the network flow and the network topology, while the unweighted network merely focuses on the network topology. A weighted network could reflect more dimensions of the network of interest, and so it can be particularly helpful in transportation networks where not only the topology is important, but traffic flow is also paramount.

CNT method could simplify a highly complex system to the network consisting nodes and links. It works well in modeling the structural features of infrastructure networks and their pattern of connectivity and it provides an acceptable approximation for considering the flow of the network. It does not pose a heavily computational burden in the analytical process. However, some limitations exist in the use of complex network theory. For example, each network comes from or is based on a specific domain with particular characteristics. Analyses results from the CNT method need to be verified using domain specific models. In addition, while CNT could provide an acceptable approximation for considering the flow of the network, it fails to consider the actual distribution with associated dynamics of the flow. Furthermore, the weighted model typically works well for single-layer networks and yet cannot easily model multi-layers networks.

The CNT method also fails to detect or consider any structural defects as a source of hazards in the network prior to any failure. CNT could be integrated with other methods used in the field of network analysis to capture actual dynamics associated with the network flow. Table 1.1 summarizes the advantages and disadvantages of CNT.

Table 1.1 Advantages and disadvantages of complex network theory (CNT)

Complex Network Theory	
Advantages	**Disadvantages**
It is a powerful tool in modeling the structure features of infrastructure networks and their pattern of connectivity.	It is sensitive to the domain specific models.
Does not require a high computational work.	It fails to capture the actual dynamic of the network.
It could simplify a highly complex system to the network consisting nodes and links.	The weighted model cannot be easily used for multi-layer networks.
Provides an acceptable approximation for considering the flow of the network	CNT method also fails to detect or consider any structural defects as a source of hazards in the network prior to any failure.

This dissertation as previously stated, aims to model the critical infrastructure system as a network and use complex network theory as a primary method to assess the network resilience and its attributes. The next section, defines resilience and its attributes and describe what the infrastructure network resilience is?

1.4 Resilience of Infrastructure Networks

1.4.1 Definition of resilience

The term *resilience* appears in different fields and disciplines ranging from ecology, biology, physics, physiology, sociology, engineering to networks and infrastructure networks (Ayyub 2014b; Ayyub 2015; Hosseini et al 2016). It has been defined based on its application and use in some certain domains.

- In ecology, resilience is the ability of a system to absorb changes in variables and parameters and still persists (Holling 1973; Ayyub 2014b).
- Gao et al. (2016) defined resilience as a fundamental property of each complex system such as a biological system and its ability to remain functional when errors, failures and environmental changes occur.
- In physics, "resilience is the ability of an elastic material (such as rubber or animal tissue) to absorb energy (such as from a blow) and release that energy as it springs back to its original shape" (Merriam-Webster dictionary 1828).
- In physiology, resilience is an individual tendency to mentally or emotionally cope with disturbance and crisis and return to pre-crisis state quickly (Terte and Stephens 2014).

- In sociology, resilience is the tendency of groups or communities to cope with stress and adversity caused by social, political or environmental changes (Adger 2000).
- According to a Presidential Policy Directive (PPD 21) (Presidential Policy Directive (PPD) 2013) on Critical Infrastructure Security and Resilience, “the term resilience means the ability to prepare for and adapt to changing conditions and withstand and recover rapidly from disruptions. Resilience includes the ability to withstand and recover from deliberate attacks, accidents, or naturally occurring threats or incidents.” Ayyub (2015) further proposed the definition for the resilience of a system as “the persistence of its functions and performances under uncertainty in the face of disturbances.” This definition can be used in a variety of areas.

Additionally, resilience could be defined as the state that is expected as the recovery process emerges. Resilience addresses all activities through recovery such as prevention, protection from, mitigation, respond to and recovery from a threat or hazard that pose a great risk to a system (FEMA 2011).

1.4.2 Definition of resilience attributes

In many cases, evaluating the resilience of a system require measuring system *performance*, *efficiency*, *robustness*, *vulnerability* and some other attributes. Thus, the general definitions of most applicable attributes are provided as follows:

- Performance: According to Cambridge dictionary (1995) *performance* is the overall quality of a piece of work or activity that a person, machine, etc. is able to perform. In the context of network analysis, performance is the network's ability to provide services by proper work of all its components together. For instance, in a power grid network, performance is the quality of transmitting power to different parts of the network by proper cooperation of all power network components such as power stations, transmission towers, power grids, distribution systems, electrical circuits, etc.
- Efficiency: The background and etymology of the word *efficiency* springs back to the Latin word "*efficientia*" meaning to produce or increase some aspects of a system immediately (Merriam-Webster dictionary 1828). Hence, efficiency is the measure of the degree that a system is able to produce. In discussing a basis of network analysis, its definition as proposed by Latora & Marchiori (2001) is a measure of the flow efficiency between two nodes and it highly depends on the connectivity of those two nodes together. The concept of network efficiency can be used to both local and global scales. In fact, the global efficiency is an indicator of the node connectivity of a whole network.
- Robustness: The quality of being strong enough to not to fail in any adverse event is *robustness*. It is the ability of system components to sustain external shocks without significant degradation of performance (Tierney & Bruneau 2007; Ayyub 2015). When it is transposed into a

network, robustness refers to the ability of a network to withstand residual node connectivity after a disruptive event.

- Vulnerability: Contrary to robustness, *vulnerability* is the capability of being wounded, either physically or emotionally. First known use of vulnerability in English literature is back to early 1600. It is ultimately derived from the Latin word "*vulnus*" and "*vulnerar*" that originally mean "wound" and "to wound", respectively (Merriam-Webster dictionary, 1828). Paul (2014) defined the vulnerability in a system as an internal risk factor of system components that are exposed to external shocks. Vulnerability is the degree of susceptibility of a system to disruptive events; a higher vulnerability is associated with a greater susceptibility for adverse effects. In the context of network analysis, vulnerability relates to the negative changes in the global connectivity of a network after perturbations.

1.4.3 Definition of infrastructure network resilience

Infrastructure networks can be complex, and it is sometimes difficult to predict their resilience to changing conditions. Typically, a network with a significant number of connections can more easily adapt to changes. If a single component fails, the network can compensate to some degree. However, as more components fail, the network grows weaker and the number of remaining functioning components gradually decreases. With respect to each network, there is a particular point at which the network can no longer compensate for further changes and its functionality disappears. This point represents the network resilience (Nature video 2016).

In the case of civil infrastructure networks, resilience is associated with the ability of the infrastructure network to deliver a certain level of service even after extreme events occur (Bocchini et al. 2013). In other words, the infrastructure network continues to deliver services up to the point that is considered the infrastructure network resilience. After reaching that point, the infrastructure network loses its serviceability.

1.4.4 Resilience assessment and sustainable development

Brundtland (1987) defined the sustainable development as the development that meets the need of the present generation and the impacts on the future generation. This definition is in agreement with the sustainability definition itself which is preserving the environment, natural resources, the health of the residents and other basic requirements for the present and future generations to use.

Bocchini et al. (2013) described resilience and sustainability concepts consist of large similarities and as two paramount complementary attributes of any system which they should be taken into consideration in an integrated perspective. In a sense, resilience assessment is part of a sustainable development. Bocchini et al. (2013) further clarified the difference between sustainability and resilience is related to the consequence associated with events imposed to a system. Resilience considers large consequences of an extreme event with small probability of occurrence, while sustainability focuses on certain consequences distributed over the lifecycle of a system. Obtaining the sustainable state of a system, therefore, requires a system to be resilient.

According to Nelson et al. (2019) the concepts of vulnerability, resilience, sustainability that are frequently used to frame assessments related to system quality are interrelated; disregarding each of these concepts in the assessment could result in negative and unintended consequences.

Now the main question that arise is, how can we further develop resilient and sustainable infrastructure networks?

1.5 Knowledge gaps, research questions and objectives

The U.N.'s Human Settlements Program predicts a near doubling of the world's population living in urbanized areas by 2050 (Lederer 2013). Such foresight has increased the demand for infrastructure systems. Widespread use of the CIs subjects them to a diverse range of disturbances. A synergistic rapid growth of urban population concurrent with great increases in infrastructure systems' use may lead to perturbations in the state of such systems. Epidemic/pandemic diseases, natural disasters, climate change, extreme events, resource scarcity, human interventions, foreseeable future changes and other factors may pose some incidence of disturbance in the state of the infrastructure system and are the basis of the failure modes. Such cascading failure events in the state of infrastructure systems, in particular, the network type infrastructures, could extend throughout regional or/and national level. For example, the global COVID-19 outbreak in 2019-20 has impacted infrastructure networks and caused significant economic and social disruptions (World economic forum 2020). As another example, Hurricane Maria ─which until 2019 was the deadliest natural disaster in the United States for about hundred years─ led to one of the largest infrastructure failure events in Puerto Rico (Center for Puerto Rico 2018). Climate change impacts such as heat wave often leads to asphalt melting and concrete hogging, in roadways and also provides disturbance in rail infrastructure systems (McEvoy et al. 2012). Worldwide extreme events such as 2001 World Trade Center Attack which caused damage to numerous buildings and full collapse of the twin towers (Mendonca & William 2006), or the 2007 UK floods that struck most parts of the country for approximately two months (Bloomfield et al. 2009). Major black out that commonly affects millions of Americans every four month is the example of resource scarcity (Ouyang 2014). Water shut-off due to infrastructure aging such as water pipes crumbling in the Great Lakes cities led to a serious crisis for residents (APM Reports 2019). All these failure events caused a huge number of mortalities, spread the sorrow, and cost billions of direct and indirect losses.

Such disturbances can affect the system resilience to the extent that it would no longer be capable of performing acceptably and satisfactory (Sun et al. 2018).

Critical infrastructures are the backbone of the society and the economy and their resilience is an essential and vital property of CIs. Resilience enhancement and protection of CIs have been a growing concern in the recent years and considered as national priorities. For instance, the Executive Order (EO)13636 and Presidential Policy Directive (PPD) on protecting critical infrastructures are the remarks on the increasing need to enhance their resilience and to identify the successful practice and strategies to do so.

Prior works have advanced the state-of- the-art solutions to this need, which have identified the different failure scenarios and have considerably enhanced the resilience of infrastructure systems and mitigated the risk. Many studies that are of conceptual and recommendation type, rather than modeling and simulation approaches typically belong to governmental reports. For example, Department of Homeland Security (DHS) categorized critical infrastructures sectors and provided protective plans for them. President′s Commission on Critical Infrastructure Protection (PCCIP) recommended some policies, research program establishments and technology developments to protect CIs. Specific other governmental reports also provided some infrastructure recovery plans for infrastructure systems following a failure event (FEMA 2018). However, governmental reports have not offered any technical approaches or discussed modeling to protect CIs, assess their resilience and mitigate their risk. The technical parts have been addressed by many scholars in this area and the concept of resilience have been used in design and operation of CIs

(O’Rourke et al. 2002; Ostfled 2005; Nelson at al. 2007; O’Rourke et al. 2007; Santora and Wilson 2008, Nazif et al. 2009; Omer et al. 2009).

Many studies used historical failure or empirical data to detect the CIs area of vulnerability and assess the system resilience (Utne et al. 2011; Kjølle et al. 2012; Ezell et al. 2000a; Ezell et al. 2000b; Alger et al. 2004; Abdalla et al. 2007; McDaniels et al. 2007; McDaniels et al. 2008; Singha and Kalita 2013; Wang and Taylor 2015; Duan et al. 2017; Hasnat et al. 2018). A large number of studies used probabilistic analysis in infrastructure systems to measure the risk and/or to develop the resilience index (Henley and Kumamoto 1996; Ayyub et al. 2009a; Ayyub et al. 2009b; Ayyub 2014a; Modarres et al. 2016; Eldosouky et al. 2017). Simulation-based analysis also have been widely used in modeling infrastructure systems and measuring the resilience of such systems (Abdalla et al. 2007; Barrett et al. 2010; Chen et al. 2015; Chen et al. 2017).

As previously stated network modeling such as Bayesian method (Bensi et al. 2009; Bensi et al. 2011; Gopnik and Tenenbaum, 2007; Hosseini and Barker 2016; Tien and Kiureghian 2016; Johansen and Tien 2017) and complex network theory (Albert et al., 2004; Wu et al. 2007; Ouyang et al. 2008; Carvalho et al., 2009; Wang and Rong 2009; Winkler et al. 2010; Zhang et al. 2010; Simone et al 2018; Ezzeldin and El-Dakhakhni 2019) have been largely used to assess the resilience and reliability of infrastructure systems.

The method of Infrastructure modeling and resilience assessments are not limited to aforementioned methods outlined above; however, the focus of this dissertation is on network-based methods and still there are gaps and challenges in existing work; therefor, future enhancement is necessary.

- **Gaps in Exiting Works**

The gaps exist in different steps of network analysis. For example, most of the network-based methods rely on network topology analysis; yet there are some topological characteristics (i.e., small world phenomenon that is related to connectivity of the network components) that are not fully explored and understood. This dissertation advanced the topological analysis method to better understand such characteristics.

Many literatures measured the area of vulnerability and robustness of the network when it is subjected to node failures (Zhang et al. 2017; Piacenza et al. 2017; Scherb et al. 2017; Coar et al. 2019) while network links are also important network components and their failure could pose different responses/challenges in the network. The link failure in addition to node failure is investigated in this dissertation and the vulnerability and robustness of the networks are assed under those two different failure scenarios.

The network representation could be binary, meaning that the network components either exist or not and the network is an unweighted model. However, the more precise model occurs when the network links carry some weight; there are some methods proposed to analyze a weighted network (Newman 2010; Guidotti et al. 2017) , however, some of the algorithms contain heavy computationally work and/or are still insufficient to capture the variant weights, i.e., negative weights, in the network. This dissertation proposes the novel algorithm that is more inclusive and also has less computational burden.

In the end, strategies that have been offered to enhance the resilience of the infrastructure network typically refer to the post-failure state of the network (Henry and Ramiz-Marquez 2012; Zhang et al. 2017; FEMA 2018), whereas strategies prior to failure mostly provide recommendations for protection (DHS, PCCIP). It is critical to technically apply some hypothetical strategy scenarios to the network to investigate its response to failure events. This dissertation investigates some strategies to enhance resilience of the network prior to failure as well as improves the current recovery strategies.

Measuring the resilience loss during or/and following an adverse event, provides a better understanding of sustainable development. Sustainable development goes toward forming the next generation of infrastructure systems, which trading friendly with the environment.

- **Research Questions**

One could perceive the earth as a giant and intricate complex infrastructure network with connected components. There has been much effort done to develop methods to increase the sustainability and the resilience of the components of this network in many ways.

In this dissertation, I aim to apply a similar view to infrastructure projects and describe each relevant infrastructure system as a complex network, using the urban rail transit network as a case study.

Sustainable infrastructure systems are resilient systems. Measurement methods and performance evaluation are key factors in designing and operating such complex systems. Methods of resilience measurement need to employ various fields of science and engineering and cannot be obtained through one discipline alone. A sense of interdisciplinary work should be applied to the challenge of infrastructure projects and their roles in the society which they function.

To this end, given the importance of resilient and sustainable infrastructure development as well as the gaps discussed in the existing works, this dissertation uses knowledge from engineering, probability and statistics, economic, computer science, and network analysis to address, the four following categories of questions that may arise in a network sense:

Category I

Which events/components are shaping the infrastructures networks and which events are shaped by them? How can we enhance the topological analysis of these networks as a fundamental step? And what would be a set of metrics we can develop to preserve the resilience of infrastructure networks?

Category II

How do these networks react in the event of failure? Does network response vary for various failure scenarios?

Category III

What is the dynamic/flow of the network and how the network dynamic/flow impact the network resilience ?

Category IV

Are there any precautious strategies we can employ to enhance the resilience of infrastructure networks? If the functionality of the system is disturbed, are there any strategies to help restore the system? Are these strategies cost-effective?

- **Research objectives**

It is through these enthusiastic questions that this dissertation begins to set the following five objectives:

Objective I

The size and complexity of networks and an analysis of its topology are responsible for the capacity of precisely determined metrics associated with the resilience of networks. Thus, the first step is defining an infrastructure system of choice in a network-based framework and analyzing its topology and assessing network efficiency which are the basis of developing the network resilience. Success determines which networks form and all the components that shape those networks.

Objective II

The different failure scenarios could make changes in the network vulnerability and robustness. For example, failing of different components of a network, i.e., node, link, may lead to a different response in the whole state of the network. Thus, the failure analysis is necessary to explore these changes. The failure analysis leads to assessing vulnerability of the network and developing resilience metric.

Objective III

Network resilience depends not only on the network's topology, but also on the weights that the network components carry. There are obvious inadequacies in considering only a network topology, where the whole state of the network is assumed to depend on a single parameter. The more useful and precise resilience function is a multi-dimensional manifold over the complex parameter space characterizing the network. Any perturbation applied to the network could change the connectivity of the network topology as well as disrupt the network flow. Therefore, the metrics developed in the earlier stage relating to objective II should be modified to reflect the network flow as well. Several factors, such as physical distances, in the networks or dynamical variations could be considered as network flow. The further a factor is seen; the better an assessment is made.

Objective IV

Increasing the resilience of the network by enhancing its topology prior to any failure, is one of the objectives of this dissertation. In addition, identifying proper post-failure recovery strategies, with special attention not only to restoring connectedness but also to minimize the total cost associated with a disruptive event resulting in resilience loss, is extensively elucidated.

Objective V

In the final step people will become part of the network directly and indirectly and their interaction with infrastructure networks will be considered. They increase scale and complexity of the network and the method in the objective II integrating with other methods, will be explored to find more precise resilience assessment.

1.6 Proposed Work

Since one of the most tangible examples of a complex infrastructure network is large scaled urban rail transit networks, the theoretical work of this dissertation to fulfill objectives outlined above, has been applied to urban rail transit networks. As such, for the purpose of illustration, Washington D.C. Metro network is used as a case study. In the rest of the dissertation, urban rail transit, metrorail and metro are used interchangeably.

The serviceability of an urban rail transit network categorized as a critical infrastructure has a significant impact on resolving cities' public transportation issues. It is an attractive, suitable and timely network, which has synergies with urbanization resulting in economic growth. It can redirect cities from poverty to prosperity.

To sustain the serviceability of such infrastructure systems, this dissertation examines the metrorail network resilience and associated metrics with well-defined relationships to vulnerability and tied to efficiency. This examination includes developing a metro rail model in a graph form and obtaining its basic features by network topology analysis. As such, Washington D.C. Metro is presented as a network paradigm by defining all its network components. Its topological characteristics are identified, calculated and used as a basis for further analyses. Two models of unweighted and weighted networks are used to measure the vulnerability and robustness of the network for each model in any failure event. These models are compared to evaluate accuracy of each model. The resilience index is developed to assess the network resilience and some strategies are offered to enhance the resilience of the network prior to and following failure due to any disruptive event, i.e., natural disasters. These strategies include topology enhancement and recovery strategies identification with respect to minimizing the performance loss and total recovery cost. In the end methods are proposed to evaluate how human interaction can be part of the network and influence the network resilience.

The proposed methodology of this dissertation is able to effectively predict the resilience loss in a metro network and measure its robustness, which is associated with passengers' safety.

Nonetheless, this dissertation is proposing the improved resilience assessment applied for rail transit networks, can be used for any other infrastructure network, and even not just infrastructure; it has great potential to be generalized for any network to increase its safety and sustainability.

1.7 Organization of the dissertation

Chapter 1 provides the definitions of infrastructure, network, resilience and its attribute, i.e., performance, efficiency, vulnerability, and robustness. Also describes the impact of resilience loss on the infrastructure, environment and society. A literature review is included to introduce the methods and models employed by network studies. Knowledge gaps, objectives, and research questions addressed in this dissertation are elaborated.

Chapter 2 is a brief chapter that demonstrates the overall methodology of the dissertation. Demonstrating the overall methodology allows to articulate the objectives and steps of the analysis in a condensed manner.

Chapter 3 describes the network topology analysis that was part of a work published in the ASME-ASCE journal part B, in 2019. In graph theory, particularly using complex network theory method, network topology analysis is the basis of any further analysis and it is worthy to allocate a full chapter for that. The topology of a network has a huge influence on the insights that can be derived from a network and all the network characteristics depend on the network topology. In this chapter the Washington D.C. Metrorail is presented as a topological graph and the network components are defined. This chapter then calculates the topological network characteristic indicators that are important for the rest of this dissertation.

Chapter 4 describes the study that was published in the ASME-ASCE journal part B, in 2019. It calculates metro network efficiency and evaluates the network vulnerability under some failure scenarios due to disruption in the network. Evaluation of the efficiency and vulnerability of a metrorail network following disruption requires consideration of two primary failure events, i.e., the failure of a metro station or the failure of a metro segment between stations. The nature of this disruption is not important in this chapter and the full failure of network components is considered one at a time. The vulnerability analysis is the basis for resilience assessment. Therefore, the resilience metric is developed and described. The examination and assessment in this chapter build on developing a metrorail model in a graph form and obtaining its basic features by network topology analysis that is demonstrated in Chapter 3. The evaluations in Chapter 4 are performed considering merely the topology of the network and when the network is modeled as an unweighted one.

Chapter 5 presents the study assessing the robustness and vulnerability of a weighted metro network, in which the ridership of the metro network is considered as an important factor through the calculations in addition to the network topological characteristics. This study has been submitted to Sustainable and Resilient Infrastructure Journal and currently is in the second round of review. Since ridership is a primary factor in defining the metrorail network performance, this chapter proposes a general ridership pattern and uses a novel methodology with developing a new algorithm to quantitatively measure the weighted-network robustness and vulnerability, which incorporates ridership throughout the Washington D.C. Metro as a case study. Therefore, such evaluations are no longer based on binary representation of the network. The weights that are carried by network components play an important role here.

Chapter 6 offers strategies to increase the resilience of the metro network that were published in ASCE-ASME journal part A, in 2020. Part of the work presented in this chapter was also presented in International Mechanical Engineering Congress and Expositions (IMECE 2018). In this chapter, the results obtained through previous chapters are taken one step further to demonstrate the methodology for resilient-based topology enhancement and a post-failure recovery strategies provision. In a sense, the resilience could be increased by enhancing its topology prior to any failure, such as adding an interloop (loop line) to the network. Thus, several loop lines are examined to project the different scenarios to enhance the network topology. When a network topology enhances, so does the network resilience. In addition, an approach to identify proper post-failure recovery strategies with special attention not only to restoring connectedness, but also on minimizing the total cost associated with a disruptive event resulting in resilience loss is extensively elucidated. Moreover, this chapter proposes future work that can be done to enhance the resilience of rail transit networks in a timely manner.

Chapter 7 outlines the conclusion, synthesis findings, major contributions and the future directions of this dissertation.

Chapter 2: Overall Methodology

2.1 Introduction

This brief chapter proposes a systematic overall research methodology to achieve objectives outlined in Chapter 1. Developing a research methodology is a critical task which allows to understand underlying steps and actions required to fulfil the objectives— as well as the study of boundaries and limitations—in a complete and comprehensive manner. The overall methodology is subsequently broken down in the following chapters to provide a detailed process of the set of interrelated methods/models used to accomplish each chapter objective using Washington D.C. Urban rail transit network as a case study.

2.2 Overall Methodology for Resilience Enhancement of Infrastructure Networks

2.2.1 Framework to assess the resilience attribute and develop resilience index

The framework to assess the network resilience and its attributes selects CNT method as a main method of use. This framework that is applied on two models of weighted and unweighted networks is summarized in Fig. 2.1 and includes following steps:

1. Defining a system as a network by specifying the network components and pattern of connectivity within that network. This step is fundamental for analyzing network topology;

2. Analyzing network topology that is the primary step for any further assessment; and results in computing network characteristic indicators and network efficiency;

3. Assessing network efficiency for an unweighted network;

4. Creating a weighted network by assigning weights to the network links. In this step, specifically for a metro network, a new algorithm is developed to model the general ridership pattern in the network and assign the ridership as weights on network links accordingly;

5. Assessing network efficiency for a weighted network. The efficiency assessment for an unweighted network indicated in step 3 merely depends on the network topological characteristics. Whereas efficiency assessment for a weighted model in step 5 requires reflecting of weights carried by network components in addition to network topological characteristics. A novel algorithm is developed in this step to quantitatively reflect link weights on network efficiency formulation ;

6. Assessing network vulnerability and robustness which are based on negative changes in efficiency as a result of a failure event, and residual network efficiency remaining following a failure in the network, respectively; and

7. Evaluating the network resilience index which is based on network performance and efficiency.

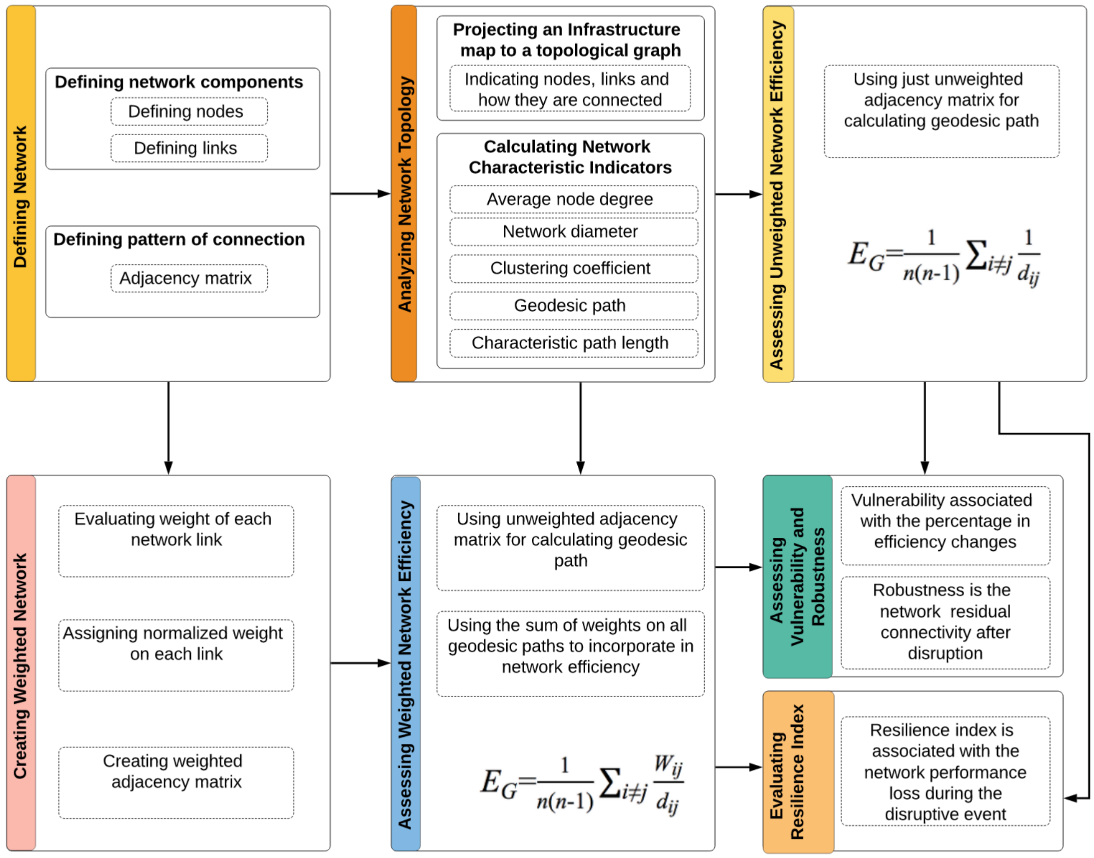


*Figure 2.2 is covered in detail in Chapters 3, 4 and 5.

Figure 2.1. A framework for resilience assessment of an infrastructure network through metro network as a case study.

2.2.2 Framework to identify strategies to enhance the resilience of the network

The framework in this section offers strategies to enhance the resilience of a network according to the following two scenarios:

- Identifying strategies to enhance resilience prior to failure; and
- Identifying post failure strategies to enhance resilience.

The resilience of a network prior to failure could be improved by enhancing network topology. As such, this dissertation examines adding several loop lines to create redundancy for critical network components and consequently decrease the network vulnerability and increase the network efficiency. The position of these loop lines has a considerable impact on the network efficiency. The goal is to protect most critical components of the network. If inserting a loop line in the network is cost effective, this topological enhancement is a promising strategy.

The resilience enhancement following a failure, however, requires identifying a series of effective strategies with respect to minimizing performance loss and recovery cost, along with other considerations, i.e. practicality. To minimize the performance loss an effective method used to ranks the recovery sequences based on resilience restoration. To minimize the recovery cost, a comprehensive cost model is proposed through an example of a failure event within a metro network.

Determining resilience enhancement strategies for both scenarios outlined above, requires vulnerability and robustness magnitudes as prerequisite terms and unified resilience index as a fundamental element. The framework to identify resilience enhancement strategies prior to and following failure is systematically followed by the framework to assess the network resilience demonstrated in the previous section and is shown in Fig. 2.2.

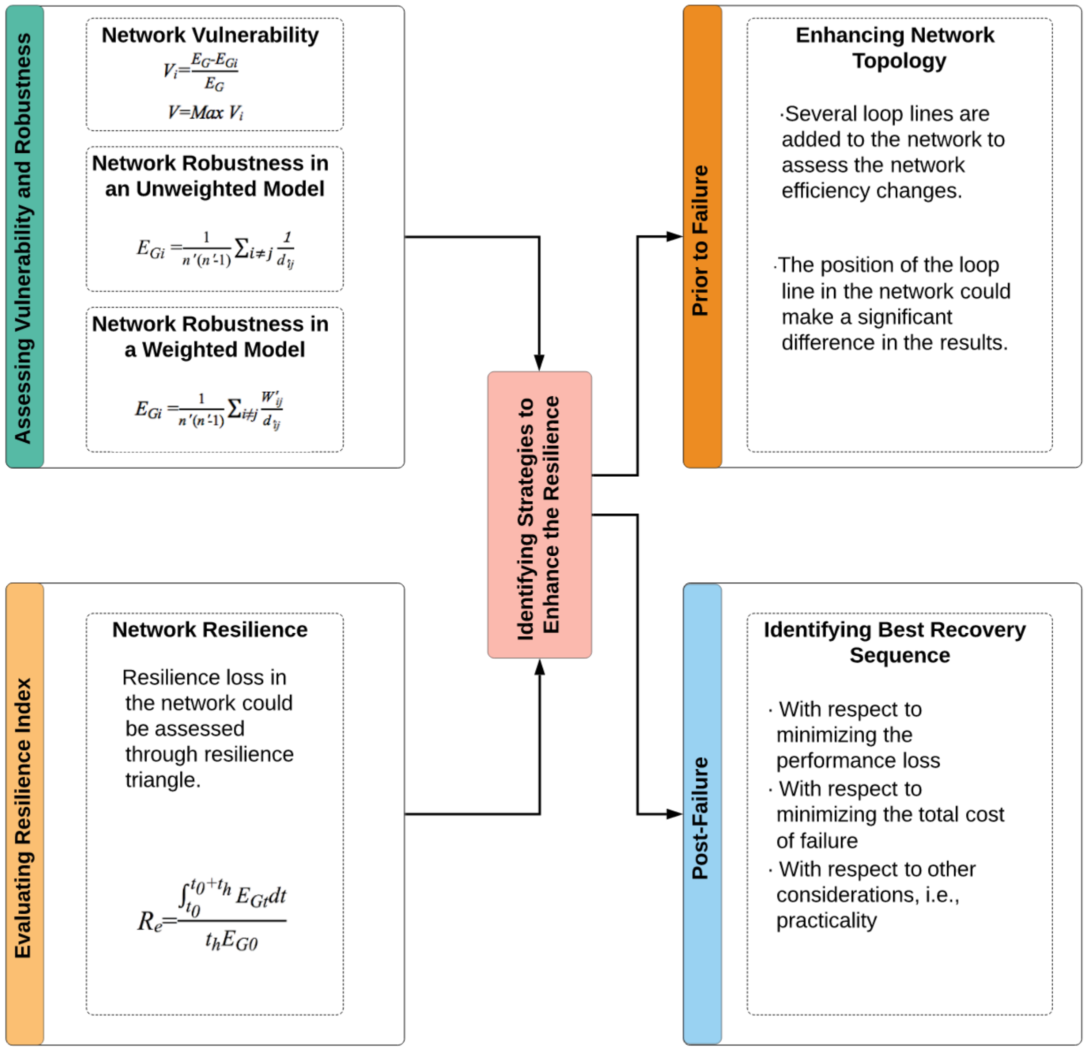


*Figure 2.2 is covered in detail in Chapter 6.

Figure 2.2. Identifying strategies to enhance the resilience of the network prior to and following failure.

2.2.3 Human interaction with infrastructure and its impacts on network resilience

Human interactions with infrastructure networks add another dimension in assessing network resilience. Thus, it is necessary to reflect this interaction in the resilience metric. The methodology to capture human behavior in the network and assess its impact on network resilience is under development and will be studied and added to this chapter in the coming months. Detailed research questions and research objectives of this section are listed in Chapter7.

2.3 Implication of the Methodology Results

The overall methodology of this dissertation assesses and potentially improves the resilience of urban infrastructures that have a network-based paradigm, such as urban rail transits. The results of the study can lead to the best use of infrastructure networks by providing guidance to protect network vulnerable areas or by offering strategies to enhance the network resilience. The methodology also accounts for human interaction with the network in the resilience metric. While the methodology is examined specifically in the context of large scaled urban rail transits, however, is applicable for any other complex networks.

Chapter 3: Topological Analysis of Metrorail Transit Networks, Washington D.C. Metro as a Case Study *

3.1 Introduction

Topology of a network refers to the properties of a network geometry and network structure. It demonstrates not only the network form and components, but rather the way that the components are connected together. In other words, a network topology describes the arrangement of network components (Grant 2014) and connectivity properties. In order to analyze the network topology, acquaintance with networks and their fundamental features is necessary. The structure of a network has an impact on how efficient the system is in meeting its functions. Studying the networks structure helps addressing many aspects of their functional purposes, such as estimating maximum flow, assessing routes of interest, exploring shortest paths, identifying the presence of hubs (i.e., nodes with high node degree), examining the impacts of attacks on or disruptions to the network. Also, such studies contribute to defining and quantifying associated costs, consequences of node or link failures, robustness, efficiency and resilience.

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* **Paper published or submitted**

Saadat, Y., Ayyub B.M., Zhang Y., Zhang D., & Huang H., 2019, "Resilience of Metrorail Networks: Quantification with Washington D.C. as a case study." ASCE. ASME J. Risk Uncertainty Part B. 2019, doi:10.1115/1.4044038.

Since this chapter exemplifies metro networks in general and Washington D.C. Metro network, in particular, the chapter initiates by stating the reasoning of why a metro system can be defined as a network and describes the advantage of defining a metrorail system in the framework of a network; then followed by introducing the Washington D.C. Metro network including data and information collected for the purposes of a case study. Introducing the case study at an early stage helps to illustrate the steps of topology analysis in this chapter and to demonstrate the methodology of the following chapters. This chapter terminates by network topological analysis applied to the Washington D.C. Metro.

3.2 Defining Metrorail Systems as Networks

The function of metrorail systems is meeting the transportation needs in cities by raising the capacity of public transportation, transferring people from one station to another in different parts of the cities. A metrorail system shares the general features of a network structure. It consists of stations (called nodes), links and patterns of interaction among nodes. Nodes and links are the essential elements to define a system in a network form. The connectivity properties are fundamental in understanding the network characteristics.

Since a metrorail system consists of a large number of interacting nodes and links, it could be perceived as a network. Studying a metro network's structure provides a basis for examining its functionality. This paper defines a metrorail system in a network form in order to understand its topology and to assess its efficiency, vulnerability and robustness, and quantify its resilience. Results from the proposed methodology, therefore, could boost public transportation leading to an increase in utilization and improvement in ridership and economic growth of a city.

3.3 The Washington D.C. Metrorail Network

This section introduces a case study at this stage in order to use it to illustrate the methodology throughout its development. Washington D.C. Metro is one of the busiest public transportation systems in the U.S. relative to the city's population. According to American Public Transportation Association (2016), the Washington D.C. Metro is the third-busiest rapid transit system in the U.S. with respect to its number of passenger trips. Washington Metropolitan Area Transit Authority (WMATA 2016) reported that roughly 640,000 passenger trips in the city were made on a daily basis in the year of 2016.

The Washington D.C. Metro is a critical and active public transportation system that serves different districts of the city along with suburban areas of Virginia and Maryland. The metro in Maryland and Virginia mostly operates on surface or elevated levels, while it is mainly subway in District of Columbia. This heavy rail rapid transit network contains 91 stations—9 of which are transfer stations that enable individuals to exchange across metro lines within the network if needed—and 140 links that are distributed over six color-coded lines, Red, Blue, Orange, Yellow, Green and Silver. Figure 3.1 displays the metro network and distinguishes among solid lines operating on a full-time schedule and dotted Yellow lines operating on a limited schedule. The shaded part of Silver line is under construction as of 2018. Table 3.1 shows the counts of stations for each line as well as the number of links in each color-coded line. These counts exclude components that do not operate on a full-time schedule.

By observing the Washington D.C. Metro map, one can detect some color-coded lines that share the same tracks and metro stations along these routes. These shared components are treated as respective single links in studying the Washington D.C. Metro in a network form.

Table 3.1 Number of stations and links in each color-coded line

Color-coded lines	Number of stations	Number of links
Red	27	26
Orange	26	23
Blue	27	26
Green	21	21
Yellow	17	16
Silver	28	28

Figure 3.3. Washington D.C. Metro network (Washington D.C. Metro Fiscal Year Budget 2018).

3.4 Network Topological Analysis

Topological analysis begins with defining the network components, such as that of the Washington D.C. Metro; then followed by a brief introduction to networks sufficient to define the theoretical foundation necessary to analyze the network topology. Therefore, the topological analysis herein is described as the following four steps:

1. Mapping the Washington D.C. Metro in the form of a graph, defining its components, and studying the fundamental elements at the framework level;
2. Introducing general features of networks and defining their characteristics;
3. Analyzing the topological characteristics and properties of the Washington D.C. Metro; and
4. Investigating the presence of small-world and scale-free phenomena in networks, explicitly for the Washington D.C. Metro network.

Figure 3.2 shows these four steps of topological analysis.

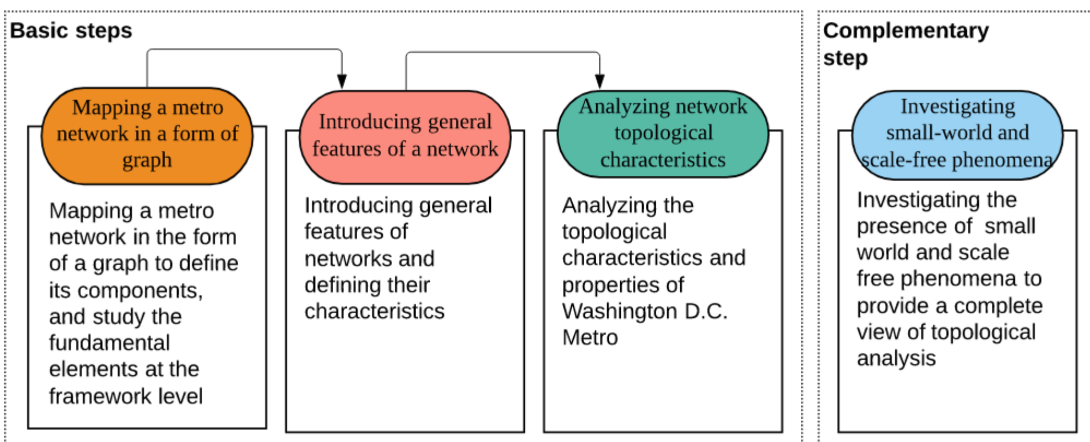

Figure 3.4. The metro network topological analysis in four steps.

3.4.1 Metrorail network in a form of graph and its basic features

As previously stated in Chapter 1, a network is a collection of some nodes together that can be connected in many ways through links. Links could be directed or undirected and carry weight or not. This section examines the nature and different categories of nodes and links in a metro network through topological mapping of a metrorail network and characterizes the network type.

In the public transportation network (PTN) literature, various categories of topological graphs are recognized to model a transportation network. Based on the pattern of connections of links among nodes, four types of topological graphs containing: L-space, B-space, P-space and C-space are distinguished for representing a bus transportation networks (BTN) as provided by Von Ferber (2009). In a bus transportation network,

- L-space graph denotes each station by a node and each path by a link in a manner that there is only one link to connect any two consecutive nodes of *i* and *j*. An L-space graph is shown in Fig. 3.3 a.
- B-space graph, shown in Fig. 3.3 b characterizes both routes and stations as nodes and somewhat is similar to a bipartite graph. In a B-space graph, a subset of square shape nodes that represents routes and another subset of circle shape nodes that represents stations are structured such that square shape nodes connect to only circle shape nodes of another subset. This is consistent with the structure of a bipartite graph.
- P-space graph shown in Fig. 3.3 c is a projection of B-space graph to the set of circle shape nodes representing stations; and

- C-space is a projection of B-space graph to the set of square shape nodes representing routes in a network as shown in Fig. 3.3 d.

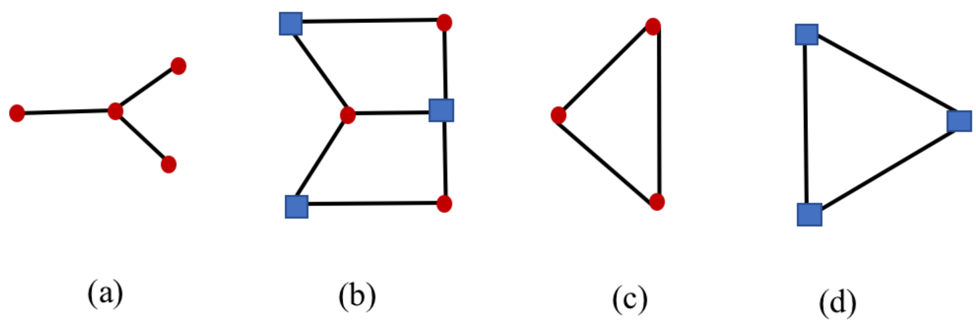

Figure 3.5. Four types of topological graphs in transportation studies: (a) L-space, (b) B-space, (c) P-space, (d) C-space.

Although the above categories are defined for a bus transportation network, an L-space graph is analogous to the nodes and links arrangement of a metro network representation and is a suitable topological representation to be adopted for a metro network. Thus, a metrorail network in the form of a graph illustrates stations by nodes, while links refer to metro tunnels, bridges, above grounds rail tracks and any other components connecting the stations together. Derrible and Kennedy (2009) summarized two categories for nodes in a metro network including *transfer stations* and *terminal stations*. Also, two types of links incorporating *single-use link* or *single-link* and *multiple-use links* or *multi-links*. Transfer or interchange stations often correspond to the locations in different places of a line where the train stops for a few moments to allow passengers to transit to a chosen district or change the route of travel, while terminal stations are at the end of a route where a train can launch or terminate a trip. Single-use link applies to a track that connects two sequential stations; whereas multiple-use links are two or more parallel segments that connect two consecutive stations.

To analyze the topology of Washington D.C. Metro network, mapping the metro network into topological graph is necessary as described by Zhang et al. (2018). Figure 3.4 illustrates the topological graph of the Washington D.C. Metro network by assigning a number to each station. This network shares the characteristics of a typical L-space network.

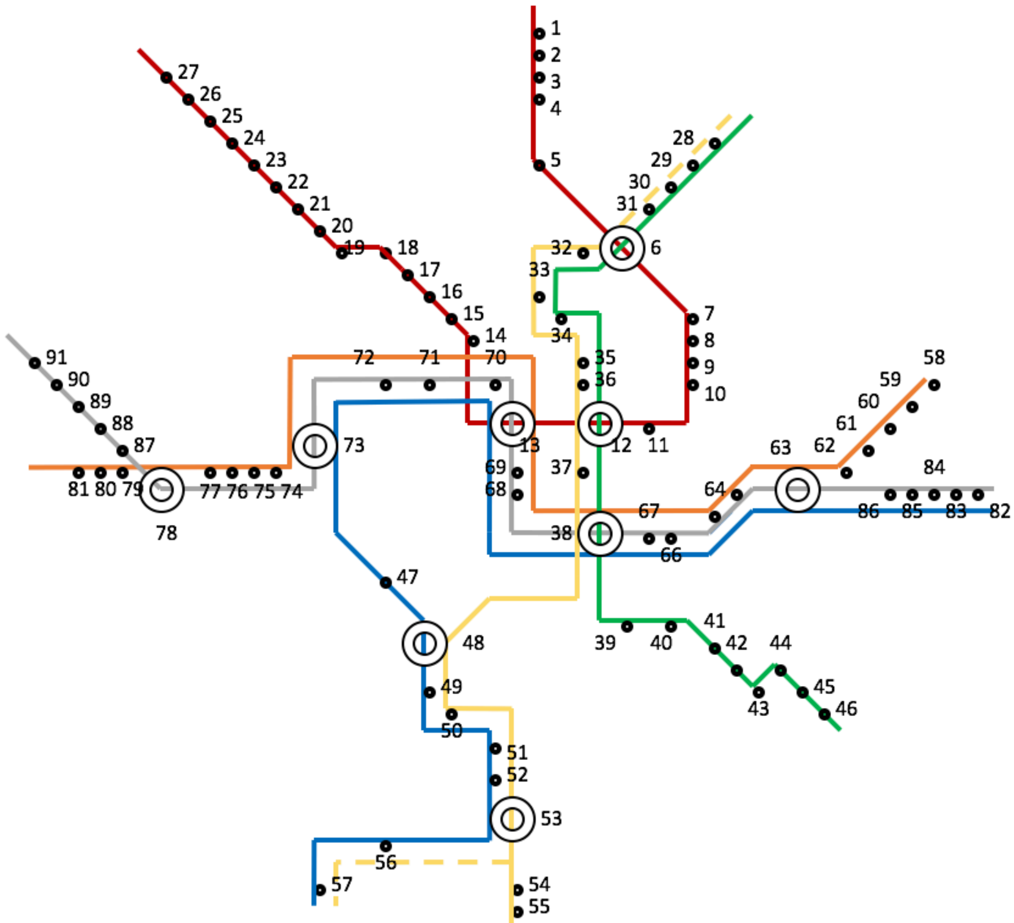

Figure 3.6. Topological graph of Washington D.C. Metro network.

3.4.2 Basic definitions and measurements necessary to analyze a network topology

As previously discussed, the behavior of a given network is highly dependent on those patterns of connection and interaction. Thus, defining network properties and utilizing methods and tools to determine the network patterns of connection are necessary. In order to illustrate the mathematical details of a network's characteristics in a computationally explicit and contained manner, this section introduces a notional network that is a portion of the Washington D.C. Metro topological graph with 17 nodes and 27 links as shown in Fig 3.5 for illustrating computations.

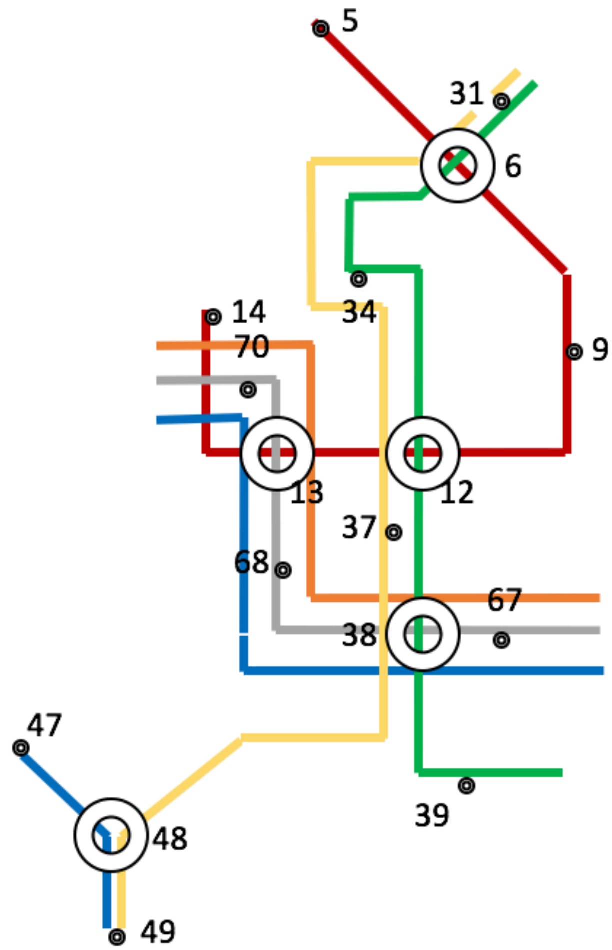

Figure 3.7. A notional topological graph of a portion of the Washington D.C. Metro network.

For a network, a topology vector as a graph G is specified as:

$$G= [S, E] \tag{3.1}$$

where G represents a graph, S is the number of all nodes, i.e., stations, and E is the number of all links, i.e., edges. Identifying the nodes by unique integers in S, such as:

$$S= [s_i|i=1,2,3, 4,.. n] \tag{3.2}$$

is beneficial for representing the network mathematically. Then, e_{ij} denotes the link that connects node i to node j expressed as:

$$E= [e_{ij}|i, j=1,2,3, 4, .. n] \tag{3.3}$$

Equally each link can be expressed also as $e_{ij} =(i,j)$. Such an arrangement provides a *link list* for the network. Referring to Fig. 3.5, the vector G has 17 nodes and 27 links as:

$$G= [17, 27] \tag{3.4}$$

For complex networks, however, this illustration might be ambiguous. An improved representation of a network that effectively shows the state of the connection between any two nodes is the *adjacency matrix*. The adjacency matrix denoted as A_{ij} gives the association between each pair of nodes. In another word, the adjacency matrix defines the pattern of connectivity between each pair of nodes in mathematical terms. Different mathematical terms are available that are suitable for *simple networks* as well as for multi-links or *self-links*, connecting two nodes by more than one link or connecting a link to itself, respectively. Newman (2010) summarized two adjacency matrices for simple graphs and non-simple graphs. In a simple graph, the elements in A_{ij} take on values of one if there is a link between nodes i and j and the elements are equal to zero if there is no link between i and j.

Zhang et al. (2018) defined a simple adjacency matrix that is applicable to simple and non-simple networks specifically in transportation, denoted as $A_{ij} = [a_{ij}]_{n \times n}$ where:

$$a_{ij} = \begin{cases} \infty & \text{for nodes } i \text{ and } j \text{ not connected directly} \\ 1 & \text{for a direct link between nodes } i \text{ and } j \\ 0 & \text{for } i{=}j\text{, connecting a node with itself} \end{cases} \qquad (3.5)$$

The adjacency matrix for the network in Fig. 3.5 is:

$$A_{ij}= \begin{array}{cc} & \begin{array}{ccccccccccccccccc} S_5 & S_6 & S_9 & S_{12} & S_{13} & S_{14} & S_{31} & S_{34} & S_{37} & S_{38} & S_{39} & S_{47} & S_{48} & S_{49} & S_{67} & S_{68} & S_{70} \end{array} \\ \begin{array}{c} S_5 \\ S_6 \\ S_9 \\ S_{12} \\ S_{13} \\ S_{14} \\ S_{31} \\ S_{34} \\ S_{37} \\ S_{38} \\ S_{39} \\ S_{47} \\ S_{48} \\ S_{49} \\ S_{67} \\ S_{68} \\ S_{70} \end{array} & \begin{bmatrix} 0 & 1 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ 1 & 0 & 1 & \infty & \infty & \infty & 1 & 1 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & 1 & 0 & 1 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 1 & 0 & 1 & \infty & \infty & 1 & 1 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 1 & 0 & 1 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 & 1 \\ \infty & \infty & \infty & \infty & 1 & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & 1 & \infty & \infty & \infty & \infty & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & 1 & \infty & 1 & \infty & \infty & \infty & 0 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 1 & \infty & \infty & \infty & \infty & 0 & 1 & \infty & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 & 0 & 1 & \infty & 1 & \infty & 1 & 1 & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 & 0 & \infty & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 & 1 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 & \infty & 1 & 0 & 1 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 & 0 & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 1 & \infty & \infty & \infty & \infty & 0 & \infty & \infty \\ \infty & \infty & \infty & \infty & 1 & \infty & \infty & \infty & \infty & 1 & \infty & \infty & \infty & \infty & \infty & 0 & \infty \\ \infty & \infty & \infty & \infty & 1 & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & \infty & 0 \end{bmatrix} \end{array} \quad (3.6)$$

Many of the networks studied in different technical areas have used binary entries of 0 or 1 in such adjacency matrices. The concept of network links, however, varies across fields, such as transportation versus ecological systems. In these instances, variations in interaction strengths are essential to the networks ability to carry on respective basic (Yook 2001).

In non-simple graphs, the elements contain a number equal to the multiplicity of links between two nodes to represent multi-links, and a self-link is denoted by two since a self-linking a node connects to itself by one link including its two ends. A non-simple graph and its correspondent adjacency matrix are shown in Fig. 3.6.

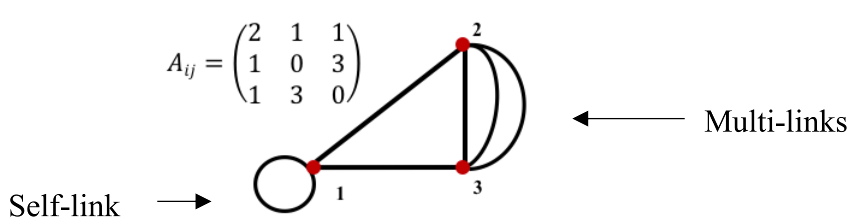

Figure 3.8. A non-simple graph and its correspondent adjacency matrix.

Representing link strength variation requires the use of a weighted adjacency matrix in which the elements are no longer binary. Weighted networks are often perceived to be more difficult to analyze than unweighted equivalents. In many cases, however, one may apply standard techniques for unweighted graphs to weighted ones by using a mapping from a weighted network to an unweighted multigraph (Newman 2004).

The adjacency matrix of a digraph has a slightly different form compared to the adjacency matrix of simple graph previously introduced. The elements of a simple graph are one if there is a connection between each pair of *i* and *j*. However, the elements of a digraph adjacency matrix are one if there is a link between node *j* with an arrow indicating a direction to node *i,* and is zero if there is no link between them. Figure 3.7 comparatively demonstrates the adjacency matrices of a simple graph and a digraph containing three nodes. It should be noted that values of one correspond to the inflows to a node. In the case of multi-links directed networks, elements in an adjacency matrix are greater than one. However in self-link directed networks, diagonal elements are one instead of being two as the commonly depicted in the representation for self-link in adjacency matrices of undirected networks. Analyzing a directed network often entails converting a directed network to undirected one using conversion mechanisms as provided by Newman (2010). In the current work, the notional network of the portion of Washington D.C. Metro is an unweighted, undirected network.

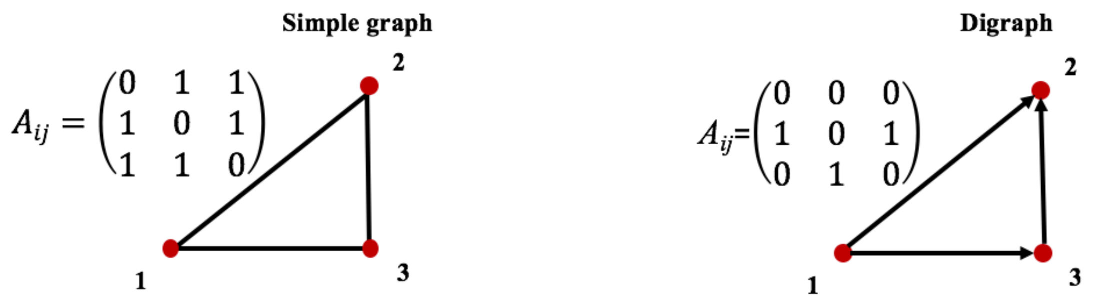

Figure 3.9 Comparison of simple graph and digraph and their correspondent adjacency matrices.

Network topological analysis additionally requires consideration of several fundamental quantitative properties applicable to transportation networks, such as the *node degree*, *average node degree*, *walk*, *path*, *path length*, *geodesic path, network's diameter*, *characteristic path length* or *average path length, density, global clustering coefficient, local clustering coefficient* and network average clustering coefficient as subsequently described.

The degree of node i (or the valency of node) specified by K_i is the number of nodes that have the direct connection with it or the number of adjacency nodes connected to the i^{th} focal node. The node degree can be computed in terms of the adjacency matrix in an undirected network as follows:

$$K_i = \sum_{j=1}^{n} A_{ij} \tag{3.7}$$

where i is the focal node, j is the number of adjacency nodes, and n is the total nodes in the network. The average degree or mean degree of an undirected graph denoted by $\overline{K}$ is the average of degree for all nodes as follows:

$$\overline{K}=\frac{1}{n}\sum_{i=1}^{n} K_i \tag{3.8}$$

Using Eqs. (3.7) and (3.8), the average node degree for the network shown in Fig. 3.5 equates to 2.12 which means every station is linked to 2.12 other stations.

In graph theory, a walk denoted as W is a finite non-null sequence of $W=s_1e_{12}s_2e_{23}....s_re_{rr+1}$ whose terms are alternating between nodes and links such that for $1 \leq i \leq r$, the ends of e_{ii+1} are s_i and s_{i+1}; which means each link endpoints are the two nodes adjacent to it (Bondy 1976). A walk between node s_i and s_r is expressed as a (s_i-s_r)-walk, and the nodes s_i and s_r are the origin and terminus of W, respectively. The length of W, in this case, is the integer r. For example in the graph shown in Fig. 3.5, the (s9-s14)-walk is described as:

$$(s_9\text{-}s_{14})\text{-walk} = s_9\ e_{9,12}\ s_{12}\ e_{12,13}\ s_{13}\ e_{13,14}\ s_{14} \tag{3.9}$$

In any case, where the nodes are distinct, the walk defines a path. Thus, a path is a sequence of any nodes in the network such that every consecutive pair of nodes in the sequence is connected by a link (Newman 2004). In some literature, a path in a directed graph is called *dipath*. A dipath is also a sequence of any nodes in the graph where every sequence of nodes connected by directed links and with the restriction that all links have the same direction.

Path length is the number of links in a specific path, and the geodesic path (or the shortest path) is a path with the minimum number of links. Thus, the minimum number of links that need to be traversed along the path between two nodes of s_i and s_j defines the geodesic path, denoted as d_{ij}. It offers a basis to introduce the concept of a network's diameter labeled as D, defined as the maximum number of links to navigate along all possible paths.

The characteristic path length (or average path length) L is the mean over all geodesic path lengths d_{ij} for all possible pairs of nodes in a network as follows:

$$L=\frac{1}{n(n-1)}\sum_{i\neq j} d_{ij} \tag{3.10}$$

It is necessary to avoid including the unconnected nodes in the characteristic path length formulation to prevent divergence. Costa et al. (2007) noted that even without including the unconnected nodes, this definition of characteristic path length still possesses a problem as it introduces distortion to the network. Nonetheless, this definition for simple graphs works well.

The maximum number of links in a simple graph is the number of pairs of distinct nodes; thus, in a network with n nodes, the maximum number of links (or edges) denoted by E_{Max} is:

$$E_{Max} = \binom{n}{2} = \frac{n(n-1)}{2} \tag{3.11}$$

Network density denoted as ρ is a fraction of the number of links with respect to the possible maximum number of links expresses as:

$$\rho = \frac{E}{E_{Max}} \tag{3.12}$$

Clustering as an important property of a network identifies a subset of nodes that are linked significantly stronger to each other than to nodes outside the subset. It is computed by graph clustering algorithms (Hartmann et al. 2014). A clustering coefficient C_i for a node i is computed based on the likelihood of cliquishness of two connected nodes among a greater group of connected nodes. Watts and Strogatz (1998) asserted that the clustering coefficient C_i measures the cliquishness of a typical neighborhood, whereas characteristic path length L measures the typical separation in the graph. The clustering coefficient occurs at two levels: local and global. Global clustering coefficient C_G is defined based on open and closed triplets of nodes. An open triplet is a three-node case connected by two links, and a closed triplet is a three-node connected by three links producing a triangle. Global clustering coefficient, therefore, is the fraction of all closed triplets to the number of total open or closed triplets in a network (Ostroumova Prokhorenkova and Samosvat 2014). This definition is applicable for both directed and undirected networks. On the other hand, a local clustering coefficient in undirected graphs C_i is defined as the fraction of possible interconnections between the neighbors of s_i over the total potential links for node s_i. Node s_i with a node degree of K_i has at most K_i (K_i -1)/2 links among the nodes. Denoting e_{ni} as the number of links between neighbors of s_i, the local clustering coefficient is expressed as:

$$C_i = \frac{2e_{ni}}{K_i\,(K_i - 1)} \tag{3.13}$$

Network average clustering coefficient $\overline{C}$ is defined as the mean of local clustering coefficient C_i for all the nodes as:

$$\overline{C} = \frac{1}{n} \sum_{i=1}^{n} C_i \tag{3.14}$$

In the existence of some node/nodes with node degree 1 in the network the local clustering coefficient is not well-defined. In some network literatures, it is common to remove those nodes with node degree of 1 in the local clustering coefficient calculation. However, this approach is not used in this paper. Instead, in the presence of node/nodes with node degree 1, which means the absence of well-defined clustering coefficient, the definition of the global network efficiency is used to represent the average clustering coefficient. Another important basic property related to distance is the *global efficiency* (E_G) (also called efficiency). Its definition as proposed by Latora and Marchiori (2001) is a measure of the flow efficiency between nodes i and j. In fact, the global efficiency is an indicator of node connectivity of a network and it is proportional to the reciprocal of shortest distance. It is quantified as follows:

$$E_G = \frac{1}{n(n-1)} \sum_{i \neq j} \frac{1}{d_{ij}} \tag{3.15}$$

Equation (3.15) employs the adjacency matrix provided in Eq. (2.6) that is appropriate only for L-space networks. Floyd (1962) provided an algorithm for computing network efficiency. Also, Eqs. (2.10) and (2.14) are for computing the characteristic path length and average clustering coefficient of the network shown in Fig. 3.5. The characteristics of the portion of Washington D.C. Metro network are shown in Table 3.2.

Table 3.2 Characteristics of the portion of Washington D.C. Metro network

Characteristics of metro Network	Calculated value for the Washington D.C. Metro
Node n	17.00
Link	27.00
Average node degree $\overline{K}$	2.120
Characteristic path length L	3.270
Diameter of network D	7.000
Network cluster coefficient $\overline{C}$	Is not defined
Network efficiency E_G	0. 423

3.4.3 Washington D.C. Metro topological analysis

This section systematically analyzes the Washington D.C. Metro network topology. The vector $G = [S, E]$ of Eq. (3.1) is used to represent the stations and links in this metro network. Demonstrating the state of connections between any pairs of stations in a metro network requires an adjacency matrix introduced by Eq. (3.5). The network characteristics, such as characteristic path length, clustering coefficient, efficiency, etc., can be obtained using the topological graph of Washington D.C. Metro shown in Fig. 3.5 and considering the pattern of connection among nodes. For this purpose, the adjacency matrix $A_{ij} = [a_{ij}]_{91\times91}$ for the Washington D.C. Metro network was developed. The results of subsequent topological analyses of the metro network are based on this adjacency matrix. Using Eqs. (3.7) and (3.8) and assuming that the network unweighted, the average node degree $\overline{K}$ of the Washington D.C. Metro network are computed. The Washington D.C. Metro network average node degree equates to 2.0439. Thus, every station is linked to 2.0439 other stations. Figure 3.8 shows the distribution of node degree for 91 stations (blue columns) in the Washington D.C. Metro network.

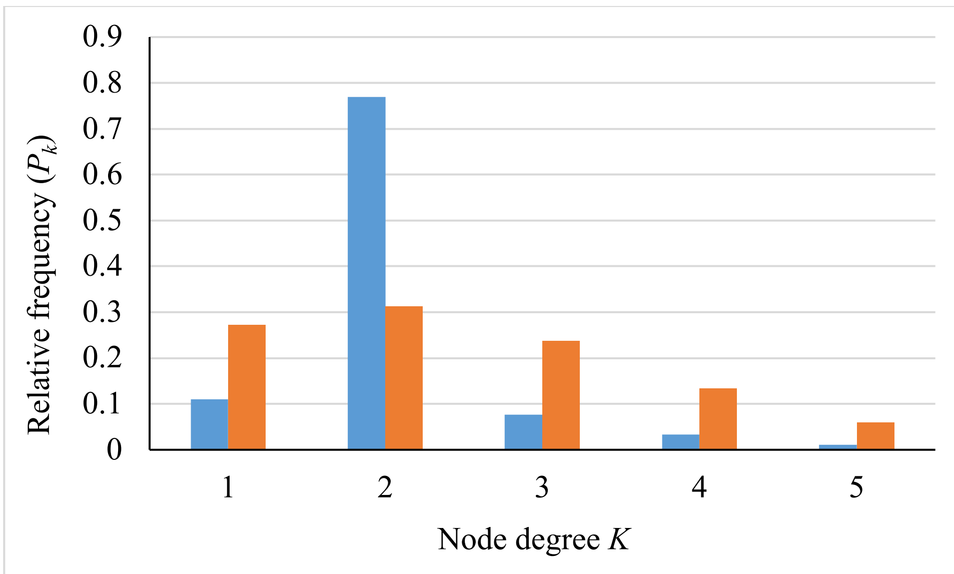

Figure 3.10. Distribution of node degree for the Washington D.C. Metro network (blue columns) and the truncated binomial distribution (orange columns).

Based on the adjacency matrix, utilizing the algorithm developed by Floyd (1962), and using Eq. (3.10), the characteristic path length of the Washington D.C. Metro network is calculated. Equations (3.14) and (3.15) provide the clustering coefficient and efficiency of the network. Table 3.2 presents the results of these calculations. The characteristic path length for the Washington D.C. Metro is 11.30 as the shortest path between any two stations, i.e., a path needs to pass through 11.30 stations on the average. Also, the longest path includes 28 stations defining the diameter of network.

Since there are some nodes with node degree 1 in Washington D.C. Metro, its local clustering coefficient is not well-defined. Thus, in this paper global clustering coefficient is used to represent Washington D.C. Metro network average clustering coefficient. The global clustering coefficient is 0.038.The global clustering coefficient is 0.038. Network efficiency of Washington D.C. Metro network is equal to 0.14320. This measurement corresponds to the state of the network without any components failure. Initial metro network efficiency is the basis of calculating vulnerability, robustness and resilience of the metro in the case of failure and disruption. It should be noted that the dotted lines in Fig. 3.1 is not included in this study to compute the characteristic path length. However, the dotted Yellow line is included in calculating the clustering coefficients. The stations in the shaded region of the Sliver line are under construction and therefore are excluded from the topological analysis.

Table 3.3 Initial Characteristics of Washington D.C. Metro network

Characteristics of metro Network	Calculated value for the Washington D.C. Metro	Interpretations
Number of nodes n	91	91stations
Number of links	140	93 links with lines sharing the same tracks treated as a singly link
Average node degree $\overline{K}=\frac{1}{n}\sum_{i=1}^{n} K_i$	2.0439	Each Station averagely connects to 2.0439 other station
Characteristic path length $L=\frac{1}{n(n-1)}\sum_{i\neq j} d_{ij}$	11.30	The average shortest path between any two stations is 11.30 links
Diameter of network D	28	The longest geodesic path in link count, among all network possible geodesic paths
Network cluster coefficient $\overline{C}=\frac{1}{n}\sum_{i=1}^{n} C_i$	0.0380*	The average of local clustering of all nodes as the fractions of neighboring connections to node s_i to all possible links connected to node s_i
Network efficiency $E_G=\frac{1}{n(n-1)}\sum_{i\neq j}\frac{1}{d_{ij}}$	0.1432	An indicator of node connectivity that is proportional to the reciprocal of shortest distance

* This measurement is based on using global clustering coefficient due to absence of well-defined local clustering coefficient in Washington D.C. Metro network.

3.4.4 Small-World and Scale-Free phenomena in networks

Beyond the basic definitions of networks discussed earlier, it is necessary to investigate the presence of a *small-world* network and a *scale-free* network as two important phenomena in analyzing networks' topology. In order to understand a small-world network, it is beneficial to describe the characteristic of *Regular lattice* networks and *Random* networks shown in Fig. 3.9. Regular lattice is one kind of an extreme graph that has the minimum heterogeneity. In fact, by looking at the degree probability distribution P_K, which gives the probability as a frequency of having a node with node degree equal to K_i, the level of *heterogeneity* can be easily recognized (Sole andValverde 2004). In regular lattice network's structure, nodes are only connected to their neighbors and the lowest randomness is observed in their pattern of connections. In other words, the probability of randomly rewiring links of the network is close to zero (P_r =0). In its counterpart of a random network, the nodes are rewired randomly or the probability of randomly rewiring links is close to 1 (P_r =1).

Watts and Strogatz (1998) used similar networks shown in Fig. 3.9 to describe the small-world network. As it is shown, ring regular lattice here is highly clustered and presents large characteristic path length. Characteristic path length is proportional to ($n/2\overline{K}$). However, the random network is not clustered and clustering coefficient is almost zero. Also, characteristic path length is short and proportional to $\ln(n)/\ln(\overline{K})$. They showed that small-world networks are an intermediate case between regular and random networks with $0 < P_r < 1$.

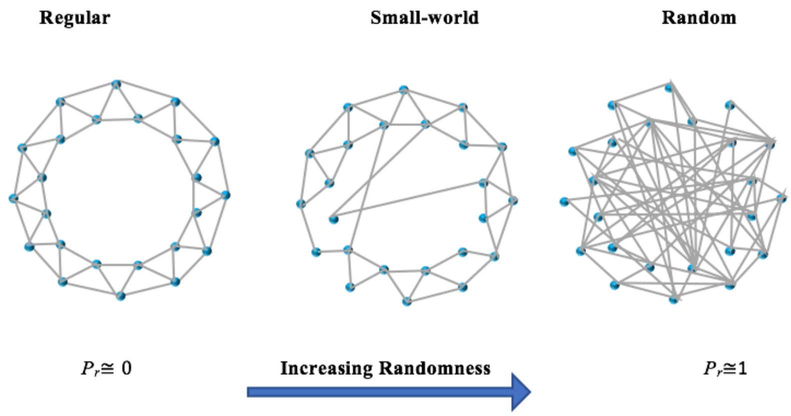

Figure 3.11. Regular network, small-world network and random network.

Despite the large network size, small-world networks tend to have a high clustering coefficient $\overline{C}$ along with a small characteristic path length L. Thus, in most networks and in particular social networks, a small-world network should meet two conditions:

$$L \cong> \left(\frac{\ln(n)}{\ln(\overline{K})}\right) \quad (3.16a)$$

$$\overline{C} > \left(\frac{\overline{K}}{n}\right) \quad (3.16b)$$

Latora and Marchiori (2002) justified characterizing a small-world network by only its clustering coefficient and characteristic path length on the basis of demonstrated suitability for social networks; while it suffers from some limitations in order to be used in the realm of weighted transportation networks and/or disconnected networks, i.e., networks with at least a pair of nodes such that no path has those nodes as its endpoints. The limitations refer to the cases when the characteristic path length and clustering coefficient are not well defined, such as in the case of disconnected and weighted networks. In addition, in the circumstance where the physical length of the links is important or the size of the network is small, the above method fails to draw any conclusion. For investigating the small-world phenomenon in those networks, other formulations based on network efficiency are necessary. In general, small-world and random networks may have a distribution of node degree following a binomial distribution (Porekar 2002) as:

$$P_K = \binom{n-1}{k} P^k (1-P)^{n-1-k} \tag{3.17}$$

where P_K = probability (P) of node degree K among all its possible values, n = number of nodes, and p = probability of two nodes being randomly connected. According to Barabasi (2016) the appropriate type of the degree distribution of a random network is the binomial distribution. However, when the number of nodes in the network is relatively large and the average node degree is small, the binomial distribution may be approximated well by a Poisson distribution.

For the Washington D.C. Metro network, considering an average node degree of 2.0439, each node has (n-1) trials to be randomly connected to other nodes. Each trial correspond to the characteristic of the Bernoulli trial that satisfies the following assumptions:

1. Each trial has two possible outcomes of being randomly connected to other nodes or being not connected;
2. All (n-1) trials are independent of each other;
3. The probability of two nodes being randomly connected remain constant from trial to trial and could be calculated as:

$$p = \frac{\text{Average node degree}}{(n\text{-}1)} = 0.023 \tag{3.18}$$

The third property of the binomial distribution might not hold well in the case of metro networks since a node is more likely to be connected directly to adjacent nodes than others far removed. Implementing p=0.023 in Eq. (3.17) and having k to take on values of (0,1,2,…90), the binomial distribution model could be easily produced. However, in the Washington D.C. Metro network the values of k are 1, 2, 3, 4 and 5. Therefore, a truncated binomial distribution in the domain of real data is a more appropriate model. The truncated binomial distribution P_{KT} is

$$P_{KT} = \frac{\binom{n-1}{k} P^k (1-P)^{n-1-k}}{1-(P_K(k=0)+\sum_{i=6}^{i=90}(P_K(i))} = \frac{\binom{n-1}{k} P^k (1-P)^{n-1-k}}{\sum_{i=1}^{i=5}(P_K(i))} = \frac{\binom{n-1}{k} P^k (1-P)^{n-1-k}}{0.86} \tag{3.19}$$

For a truncated distribution, Eq. (3.18) does not apply, and p should be calculated based on the mean value definition for discrete random variables as follows:

$$\text{Average node degree} = \sum_{i=1}^{i=5} (iP_{KT}(i)) \tag{3.20}$$

1. Using Eq. (3.20) p is recalculated as 0.0252 and subsequently the truncated binomial distribution is reproduced based on the new p. The truncated binomial distribution model (orange columns) are shown in Fig. 3.8.

The chi-square goodness of fit test is performed to assess the reliability of the model. The comparison of the observed (O_f) frequencies of the network with frequencies expected (E_f) from the binomial distribution model (without the truncation) provides the values for computing chi-square distribution statistic shown as χ^2:

$$\chi^2 = \sum_{i=1}^{90} \frac{(O_{fI} - E_{fi})^2}{E_{fI}} = 1.36 \tag{3.21}$$

The critical value of the chi-square test statistic with degrees of freedom of 4, i.e., sample size- number of binomial distribution parameters -1, and a level of significance of 5% is 9.49. This critical value is larger than computed χ^2 and, therefore, supports the hypothesis that the Washington D.C. Metro network follows a binomial distribution. The truncated distribution enhances this fit.

In addition, Eqs. (3.16a) and (3.16b) can be used roughly to better determine whether or not the Washington D.C. Metro network is a small-world network. Table 3.4 shows the two necessary conditions and their values that relate L and $\overline{C}$. For the Washington D.C. Metro network, the global clustering coefficient is used as the network clustering coefficient and it is greater than random network clustering coefficient. The characteristic path length fulfills the correlation of Eq. (3.16a). Thus, according to the formalism mentioned above, Washington D.C. Metro network is a small-world network.

Table 3.4 Conditions on L and $\overline{C}$ for the Washington D.C. Metro network

Conditions on L and $\overline{C}$	Calculated value for Washington D.C. Metro
$L > (\frac{\ln(n)}{\ln(\overline{K})})$	$(L{=}11.30) > (\frac{\ln(n)}{\ln(\overline{K})}{=}6.31)$
$\overline{C} > (\frac{\overline{K}}{n})$	$(\overline{C}{=}0.038) > (\frac{\overline{K}}{n}{=}0.022)$

Regardless of categorizing a network to be of a small-world or not, in the presence of a power model for node degree (K) and probability (P_K), the network is most likely to be a scale-free network. Scale-free networks are characterized by a highly heterogeneous degree distribution, which follows a power-law (Barabasi and Albert 1999).

$$P_K = \alpha(K)^{-\gamma} \tag{3.22}$$

where α and γ are the parameters of the power curve and K is a possible node degree value. Using nonlinear curve fitting, α and γ can be obtained where the latter signifies the slope of the line on logarithmic scales, and frequently varies between 2 and 3, although sometimes it may lie beyond these limits.

For scale-free networks, the power-law indicates that the majority of nodes have small node degree, and the probability of nodes with small node degree is higher than otherwise. Thus, a few nodes with significant node degree exist which are easily identifiable in the network. The nodes with high node degree are called *hubs*; therefore, scale-free networks demonstrate the existence of hubs in a network. The removal of nodes with small node degree does not change the pattern of connectivity and the structure of other nodes, hence, has no effect on the whole topology of the network (Albert et al. 2000). In the scale-free networks, nodes tend to remain connected and the characteristic length tends to remain small. This property is consistent with the definition of network's robustness. Thus, scale-free networks display robustness under a random node removal even by a high failure rate, yet are vulnerable to intentional attack (Crucitti et al. 2004). The network exhibits a low level of robustness under the removal of few vital nodes with a higher node degree.

It is noteworthy to mention that a network can show the characteristics of small-world and scale-free phenomena at the same time, and these two properties do not have any conflict with each other. Investigating the existence of small-world network in a metro system depends on the network being weighted or unweighted and connected or disconnected. For weighted transportation networks or disconnected transportation networks, alternative technique and mathematical formalism are necessary (Latora and Marchiori 2002). Exploring the presence of scale-free and small-world phenomena in the metro network enhances the understanding of the nature of the network as illustrated in the rest of this section.

Furthermore, nonlinear regression graphical analysis illustrated in Fig. 3.10 displays the nonlinear fitting of the relative frequency P_K versus the node degree value K. The fitted model is:

$$P_K = 0.12(K)^{-1.34} \tag{3.23}$$

The nonlinear relationship produced $\alpha = 0.12$ and $\gamma = 1.34$ with the latter smaller than 2. However, this fit is not a reliable one; typically power law graphs are not reliable when the range of the data and sample size are small. Nonetheless, the Washington D.C. Metro network indeed has a few nodes with large node degree and the majority of nodes have small node degree. Thus, with a reasonable approximation, the Washington D.C. Metro is considered as a scale-free network too. As discussed earlier, scale-free networks show more robustness when they are subjected to random failures compared to intentional disruptions.

The basic prerequisite information presented in this section is necessary for developing metrics for vulnerability, robustness and resilience of a network.

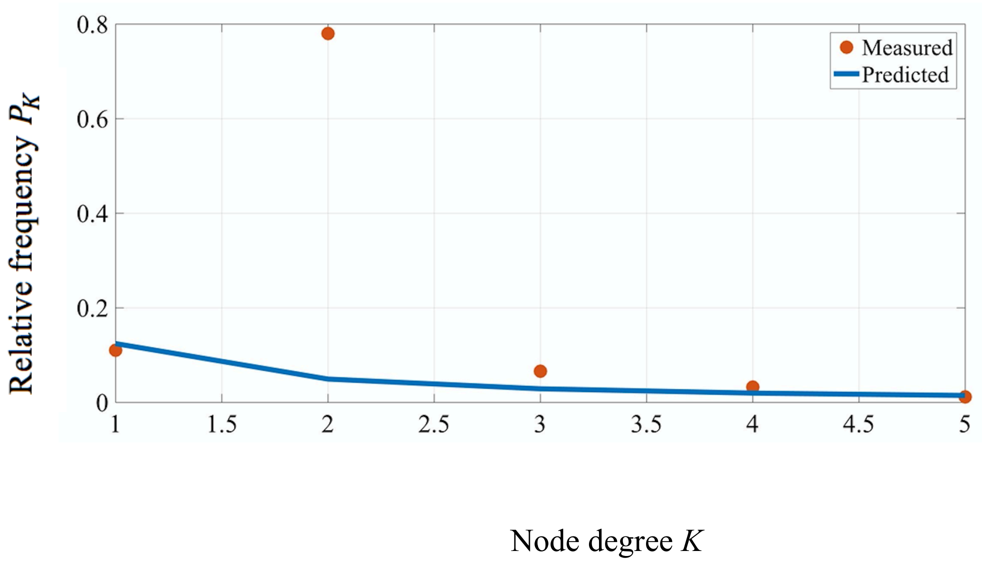

Figure 3.12. Nonlinear fitting of the relative frequency with node degree for Washington D.C Metro network.

3.5 Concluding Remarks and Contribution

According to Derrible and Kennedy (2010), an efficient approach and a coherent basis to characterize the safety and robustness of transportation network require topological analysis. A topology analysis explores network form and defines the components that shape the network. It also describes the network connectivity and its characteristics. It is a primary step of network analysis and is the basis for assessing the network resilience and its attributes. This chapter serves as a network topological analysis of the Washington D.C. Metro and includes developing a metro rail model in a form of a graph and obtaining its basic features by network topology analysis. The analytical work demonstrates that the Washington D.C. Metro with its 91 stations and 140 links is an L-shaped network with the presence of small-world and scale-free phenomena. This chapter proposes a new probabilistic method to investigate the presence of small-world in the network. The outcome of Chapter 3 helps to better understand the characteristics that this network hold.

Chapter 4: Vulnerability and Resilience of Metrorail Networks, Quantification with Washington D.C. as a Case Study*

4.1. Introduction

Infrastructures systems provide populations in urban areas with amenities and services. While the need for infrastructure systems in societies is well recognized, great attention should be paid toward enhancing the ability and resilience of infrastructure systems to deliver a desired quality level of service. Many numbers of infrastructure systems have been designed and implemented without much consideration for inevitable future changes such as rapid urban areas development. The U.N.'s Human Settlements Program predicts a near doubling of the world's population living in urbanized areas by 2050 (Lederer 2013). This statistic alone illustrates the dire need for infrastructures development and improvement in order to address imminent challenges facing communities.

Urban rail transit networks, also termed metrorail or metro networks, are essential transportation infrastructures, which have attracted users in increasing numbers worldwide by facilitating transportation in urban life. The growth of a metro network

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* **Paper published or submitted**

Saadat, Y., Ayyub B.M., Zhang Y., Zhang D., & Huang H., 2019, "Resilience of Metrorail Networks: Quantification with Washington D.C. as a case study." ASCE. ASME J. Risk Uncertainty Part B. 2019, doi:10.1115/1.4044038.

is inextricably intertwined with the growth of the city and its economy. Metro networks are complex systems consisting of a large number of interacting nodes and links representing stations and connecting rail segments, respectively. They are characterized by steady speed, schedule predictability and large capacity, and typically form the artery of urban public transportation (Latora and Marchiori 2002; Angelouis and Fisk 2006; Derrible and Kennedy 2010).

The London metro network, which opened in 1863, was the first metro network with the ridership up to approximately 30,000 passengers daily. In 2014, there were metro networks in more than 157 cities in the world, with roughly 150 million passenger trips being made on a daily basis (UITP 2015). Metrorail networks have made a great contribution to city expansions, intensified urbanization, and economic growth. In return, urbanization has propelled the development and growth of metro networks, thereby introducing complexity to such networks.

In a complex system, disruptions or even lesser perturbations may curtail functionality of the system. In particular, any disruption of such an interaction and equilibrium potentially impacts commuters' wellbeing, has a dramatic effect on the safety of commuters, and influences the resilience and reliability of Metro operation (Cadarsoa et al. 2013; Nguyen, Bdugin and Marais, 2015), in addition to direct and indirect losses associated with performance loss. For example, the December 2017 derailing of an Amtrak train traveling from Seattle to Portland resulted in three fatalities and about 100 non-fatal injuries (CNN 2017). In October 2012, Superstorm

Sandy knocked out of commission several New York City subway lines due to tunnel flooding severely diminishing the number of normal weekday riders (FEMA 2013). The September 2011 accident in the Shanghai Metro resulted in 271 injuries and significant recovery costs (Mu 2011). The National Transportation Safety Board (NTSB 2010) reported an accident in June 2009 on an aboveground track on the Red Line of the Washington D.C. Metro that resulted in 52 injuries, nine fatalities, and 12 million US dollars in direct losses. These events, like many, demonstrate the types of threats to citizens, facilities and infrastructures. Additionally, they illustrate the importance of maintaining the safe operation of metro networks and the need for enhancing metro resilience and robustness by identifying the vulnerability of the network.

The concept of vulnerability is increasingly being used to quantify the impact of any adverse event on the performance of transportation networks. In a metrorail network, robustness refers to the ability of a network to withstand residual node connectivity following a failure event, and vulnerability relates to the negative changes in the global efficiency of a network in a post-failure state. Compared to reliability analysis, which is a probabilistic measure of the network connectivity; robustness and vulnerability analyses are more focused on the consequences due to abnormal events (Jenelius et al. 2006; Jenelius and Mattsson, 2015). According to Nelson, Gillespie-Marthaler et al. (2019) the concepts of robustness, vulnerability, and reliability are frequently used to frame assessments related to system quality. For example, Rokhneddin et al. (2013) evaluated the reliability of large aging bridge networks;

Yang et al. (1996) assessed the reliability of water distribution systems. Li and He (2002) proposed a method to evaluate seismic system reliability of large lifeline systems. Improving the reliability and robustness of a system by identifying vulnerable components and preserving them is of paramount importance to enhance the system resilience and is a path to sustain a reliable system.

Many attempts have been made to assess not only the resilience of networks in general, but also of metrorail networks particularly. Derrible and Kennedy (2010) suggested network topological analysis as a fundamental initial step to assess the resilience of a network. Newman (2010) introduced basic and advanced features of a network. Watts and Strogatz (1998) investigated the existence of small-world phenomenon in a network, and Sen and Dasgupta (2003) examined this phenomenon in the Indian railway network. Latora and Marchiori (2002) also studied the existence of a small-world property for the Boston subway. Statistical analysis of 22 public transportation networks in Polish cities revealed the complex nature of their topological structures (Sienkiewicz and Holyst 2005). Derrible and Kennedy (2009)analyzed subway systems using updated graph theory. Haznagy et al. (2015) identified similarities and differences within urban public transportation systems of five Hungarian cities using complex network theory by comparing their network descriptors. In the paper by Haznagy et al. (2015) critical nodes are ranked based on their greatest centrality measures that typically refer to the nodes in central positions or transfer nodes in the network where exchanging route possibilities exist. Wu et al. (2016) contrasted the network properties of five metro systems of Beijing, Hong

Kong, London, Paris and Tokyo and compared their relative network efficiencies. Network efficiency is the indicator of network connectivity. Wang (2015) quantified the robustness of metro networks subjected to random failures and targeted attacks. Recently, a general framework for assessing a large-scale metro was suggested based on analyzing its vulnerability and recovery rapidity within a unified metric(Zhang et al. 2017). The assessment by Zhang et al. (2017) primarily focused on analyzing the changed connectivity after the removal of only network node(s) without considering link removal.

This chapter presents a method based on graph theory for analyzing network resilience using the Washington D.C. Metro of 91 stations and 140 links as a case study. To achieve this main objective, this chapter uses topology analysis results of a metro network that previously is mapped in a form of a graph in Chapter 3. Mapping a metro network into a graph and analyzing its characteristics provides a basis to evaluate its efficiency means by which the measure of efficient flow between any two nodes, and vulnerability and robustness as primary components for assessing network resilience. Then, this chapter examines the changes in connectivity and measures the resilience loss in the network after one-at-a-time node and link removal. Previous resilience assessments of metrorail networks have not accounted for link removal in the analysis. This lack of investigation might be a result of relying on the assumption that link removal insignificantly impacts L-space networks. However, this paper demonstrates that links removal could lead to considerable resilience loss in metrorail networks.

The chapter is organized as follows: The next section describes the vulnerability assessment and resilience metric development methodology used in this chapter, which includes three subsections of proposing a general framework to assess the network resilience, network efficiency prior to and following any failure event, network vulnerability under two circumstances of node and link failures, and the network resilience index; the chapter 4 methodology outline is shown in Fig. 4.1.The last section concludes the study and discusses the implications of the findings as well as future expansion applied to this topic.

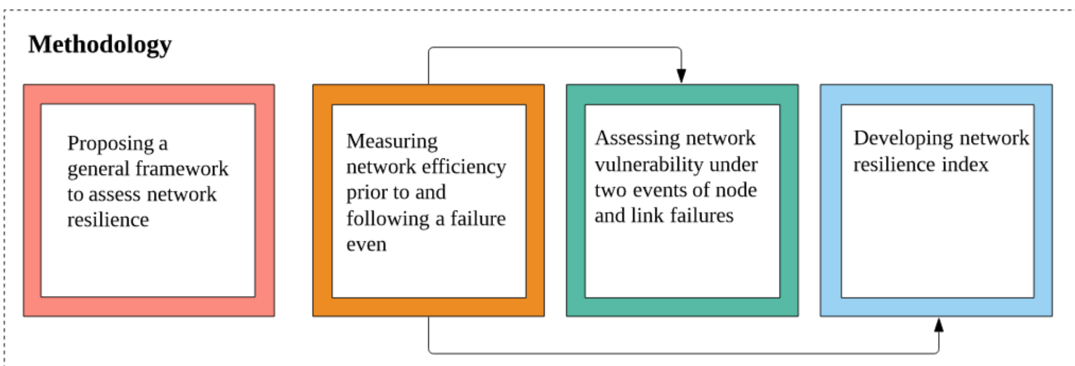

Figure 4.1 A proposed vulnerability assessment and resilience metric development methodology

4.2. Methodology for Unweighted Network Resilience Assessment

4.2.1. General frame work to assess the network resilience

The objective of this chapter is to develop a general framework to assess the resilience of a metrorail network. As previously stated, in order to assess the resilience of a network and understand how the corresponding network works, studying its network structure is necessary. A systematic approach for assessing the resilience of a network requires characterizing its form, size, dependencies and interdependencies under normative sources of disruption. Size and complexity of a system, along with a topological analysis, determine appropriate metrics for the resilience of a network. Therefore, the general resilience frame work consists of the following three steps:

1. Defining a system in a network-form that includes signifying nodes and links and the pattern of connectivity among nodes by links;
2. Analyzing topology by computing node degree, average node degree and adjacency matrix offering a basis for calculating characteristic path length and network efficiency; and
3. Assessing resilience based on vulnerability and robustness analysis by computing a resilience index. The vulnerability, robustness and resilience index are the basis on network efficiency.

Figure 4.2 shows these three steps and the analytical framework.

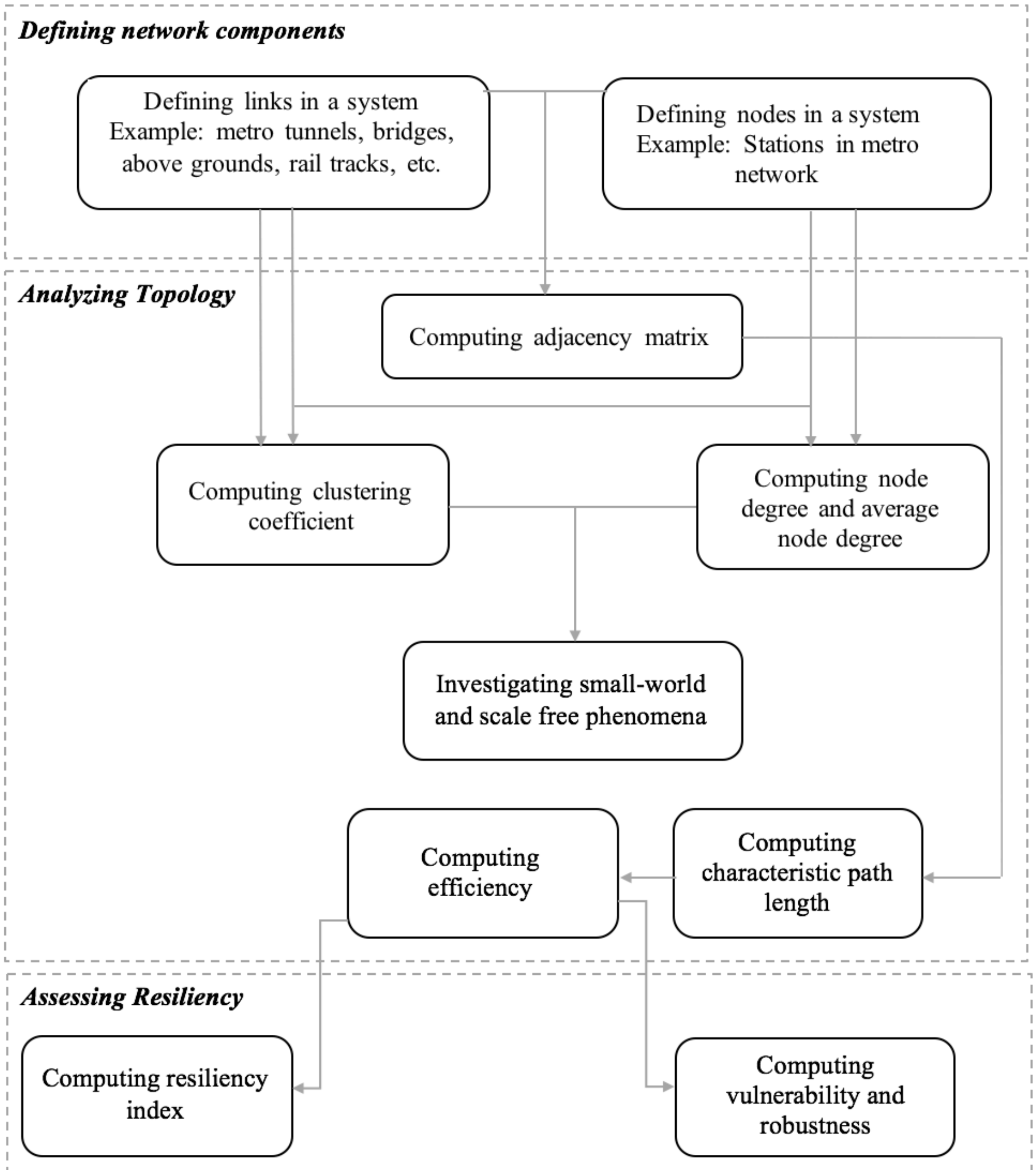

Figure 4.2. A general framework to assess the network resilience including three steps of defining a system in a network-form, analyzing topology, and assessing resilience.

4.2.2. Network efficiency prior to and following failure events

Measuring the changes of the network efficiency prior to and following any disruptive event is the basis for further calculations, i.e., vulnerability, robustness, resilience index, herein. Changes in network connectivity alter the network topology, and hence its efficiency. The efficiency of large metro networks is affected by either node or link failures. Node failures can be linked to any disruption that results in the removal of one or more stations from a metro network. Link failures are other sources of disruption, such as tunnel or rail failures. Thus, two failure cases are considered in this study:

1. One- at-a-time node failure in the network
2. One- at-a-time link failure in the network

The network efficiency for an unweighted network is calculated using Eq. (3.15). Referring to the network efficiency result presented in Table 3.3, Washington D.C. Metro's initial network efficiency is 0.1432.

4.2.3. Model metro network vulnerability, robustness and resilience

Evaluating the vulnerability of a network requires measuring changes between the initial network connectivity, i.e., prior to any failures, and post-failure network connectivity. Hence, the vulnerability can be associated with network efficiency and quantified as follows, respectively, for node *i* and the network's entirety:

$$V_i = \frac{E_G - E_{Gi}}{E_G} \tag{4.1a}$$

$$V = Max\ V_i \tag{4.1b}$$

where E_G is the efficiency of a network before node or link removal, and E_{Gi} is the efficiency of a metro network after removal of nodes or links in a metro network. The residual connectivity after node or link removal E_{Gi} can be considered as a measure of its robustness. The vulnerability of network after removal of a node or link is V_i, and V is the vulnerability of a whole network.

4.2.3.1. Washington D.C. Metro vulnerability with node failures

The vulnerability assessment of the Washington D.C. Metro network covers two cases: (1) node removal one at a time, and (2) link removal one at a time. For both cases, Eqs. (4.1a) and (4.1b) are used to calculate the vulnerability. In Eq. (4.1a), E_{Gi} refers to the efficiency of a network after a node removal or a link removal providing measures of respective robustness. Characterizing the node connectivity is the key element for measuring robustness, which is a necessary pursuit for vulnerability assessment. In order to calculate E_{Gi}, Eq. (3.15) is used with a new set of d_{ij} after node or link removal and regenerating the network.

Transitioning the focus in this section to the vulnerability assessment in the event of a node failure, the efficiency of the network after removing each station is evaluated using an improved Floyd algorithm; this algorithm has been developed in such a way as to assess all new geodesic paths d_{ij} for each of the 91 stations by running the code only once. The developed algorithm automatically evaluates the new adjacency matrix and subsequently creates new geodesic paths and network efficiency. The size of the adjacency matrix after one node removal is 90 times 90. Consequently, the end result is expressed as a vector $B=[b_{ij}]_{1X91}$, whose b_{ij} corresponds to E_{Gi} after removing each station. Figure 4.3 displays the vulnerability profile of the metro network when subjected to node removal for all 91 nodes. Also, Table 4.1 lists the ten most critical stations along with their associated network vulnerability values.

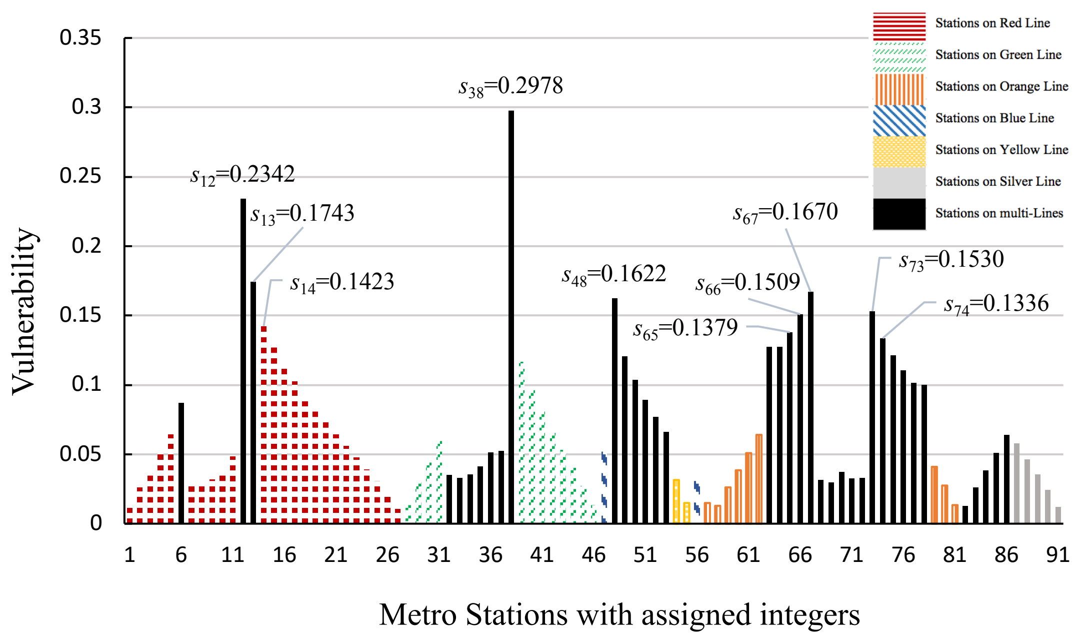

Figure 4.3.Vulnerability of Washington D.C. Metro network subjected to node removal for all 91 stations.

Table 4.1 The ten most critical stations in the Washington D.C Metro network

Rank order	Name of the station	Station number	Node degree	Network Vulnerability Magnitude (%)
1	L' Enfant Plaza	38	5	29.78
2	Gallery Place	12	4	23.42
3	Metro Center	13	4	17.43
4	Federal Center SW	67	2	16.70
5	Pentagon	48	3	16.22
6	Rosslyn	73	3	15.30
7	Capital South	66	2	15.09
8	Farragut North	14	2	14.23
9	Eastern Market	65	2	13.79
10	Court House	74	2	13.36

Equation (4.1b) equates the network vulnerability to the maximum vulnerability among all nodes removed once at a time. Thus, the vulnerability of the network is almost 0.30 and *L' Enfant Plaza Station* is of the greatest vulnerability in the Washington D.C. Metro network. By removing the *L' Enfant Plaza Station*, the efficiency of the network would be drastically reduced by 29.78%. While a considerable likelihood of correlation between the efficiency reduction and the removal of the station with higher node degree exists, it is not always the case. By removing the *Federal Center SW* station with a node degree of 2, the connectivity would be reduced by 16.70%; this reduction is 15.30% for *Rosslyn* station which has a higher node degree of 3. In addition, *Fort Totten* station (node 6) has a node degree of 4 and is not ranked among the ten most critical stations. The majority of the critical nodes are the transfer stations located in the downtown area and are heavily used by local commuters. The other critical nodes are connected to transfer stations and are responsible for tying all four quadrants of Washington D.C. to the downtown area. In the event of a critical station removal, the impacted quadrants of the city would fail to be connected to the downtown area.

4.2.3.2. Washington D.C. metro vulnerability analysis with link failures

In this section, the vulnerability assessment is enhanced to account for link removals in the network as presented herein. In this assessment, by using the link removal method, i.e., removing one link at a time from the network, the regenerated network is developed, after which the associated network efficiency is derived using Eq. (3.15). Measuring network efficiency in this case is based on the Floyd (1962) algorithm by changing the adjacency matrix after each link removal and not changing the size of the adjacency matrix. Table 4.2 shows the vulnerability measurements for the fifteen most critical links in the Washington D.C. Metro network. In Table 4.2, (i,j) is the indicator for a link between node i and node j.

Table 4.2 The fifteen most critical links in the Washington D.C Metro network

Rank order	Link	Station names at two ends of each link	Color-coded lines	Vulnerability Magnitude (%)
1	(13,69)	(Metro center, Federal Triangle)	Blue-Orange-Silver	16.07
2	(68,69)	(Smithsonian, Federal Triangle)	Blue-Orange-Silver	14.42
3	(38,68)	(L' Enfant Plaza, Smithsonian)	Blue-Orange-Silver	13.14
4	(73,74)	(Rosslyn, Court House)	Orange-Silver	12.78
5	(38,67)	(L' Enfant plaza, Federal Center SW)	Blue-Orange-Silver	12.07
6	(13,14)	(Metro Center, Farragut North)	Red	11.71
7	(48, 49)	(Pentagon, Pentagon City)	Blue-Yellow	11.57
8	(74,75)	(Court House, Clarendon)	Orange-Silver	11.50
9	(63,64)	(Stadium Armory, Potomac Ave)	Blue-Orange-Silver	11.14
10	(75,76)	(Clarendon, Virginia-Sq-GMU)	Orange-Silver	10.42
11	(14,15)	(Farragut north, Dupont Circle)	Red	10.14
12	(49,50)	(Pentagon City, Crystal City)	Blue-Yellow	9.85
13	(76,77)	(Virginia-Sq-GMU, Ballston-MU)	Orange-Silver	9.50
14	(15,16)	(Dupont Circle, Woodley Park)	Red	8.86
15	(38,39)	(L' Enfant plaza, Waterfront)	Green	8.78

Based on Table 4.2, the most critical lines in the Washington D.C. Metro network are the Blue-Orange-Silver lines, which share the same track and therefore are considered as one single-use link. The most vulnerable segment of those lines is where they connect east Washington D.C. to the downtown area. The portion of Orange-Silver lines connecting western Washington D.C. to downtown and the Blue-Yellow lines connecting the southern part of the city to the downtown area are vulnerable as well. In addition, the Red line where several links connect the northwest quadrant of Washington D.C. to the downtown area and a segment of the Green line responsible for connecting the southeast quadrant of the city to the downtown area are considered the most vulnerable links. Notably, in analyzing link removal, the segments connected to one of the transfer stations are exposed to protentional threats and are ranked among the most critical links in the Washington D.C. Metro network.

4.2.4. Resilience index

In the network analysis literature, usually the network connectivity, which is also represented by the network efficiency, is the key criterion to express a network performance. Tracing performance changes in the network could also provide the basis to quantify the network resilience. Improving network resilience helps enhance safety, and requires appropriate resilience metrics. Resilience, defined as the persistence of performance under uncertainty due to a disruptive event, fundamentally can be measured by defining performance, a performance loss profile due to disruption, and a recovery profile. The following two key measures necessary to quantify network resilience as follows: (1) network performance before and after a disruptive event, and (2) network performance recovery to the initial or some other level of functionality. The changes in the network performance constitute the resilience loss in the network (Bruneau et al. 2003; Bruneau et al. 2007). In addition, Bruneau et al. (2003) proposed the concept of a resilience triangle to quantify the resilience of a system. Resilience triangle in a network represents the changes in a network efficiency from the time that the disruptive event happens to the time that the network efficiency recovers to its initial state.

Figure 4.4 illustrates the concept of a *resilience triangle*, where Q_t on the vertical axis is the network performance. The shaded area presents the resilience loss expressed as follows:

$$R_e = \frac{\int_{t_0}^{t_0+t_h} Q_t dt}{t_h Q_0} \tag{4.2}$$

where R_e is the resilience index, Q_t is the performance of the disrupted system, Q_0 is the performance before disruption, t_0 is the time when the system is subjected to disruption, and t_h is the period of time that the system needs to return to initial performance.

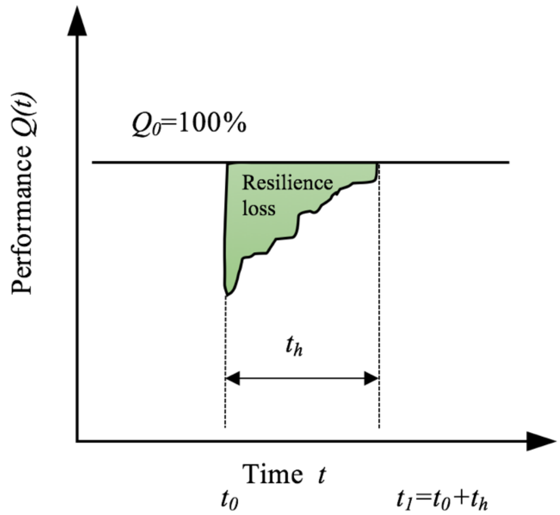

Figure 4.4. Resilience properties and resilience triangle in a system.

Further, Ayyub (2014b) provided a schematic representation of a system performance loss due to an adverse event in the system followed by a recovery procedure to revert the system to the original level of system performance. Figure 3.5 shows the simplified version of the aforementioned schematic, which is tailored for network analysis purposes. The vertical axis represents the network efficiency index E_{Gt} at time t and the horizontal axis indicates the time t. In Fig. 3.5 the adverse event is assumed to lead to a failure in the network and the recovery process is assumed to restore the full initial global efficiency E_{G0} of the network.

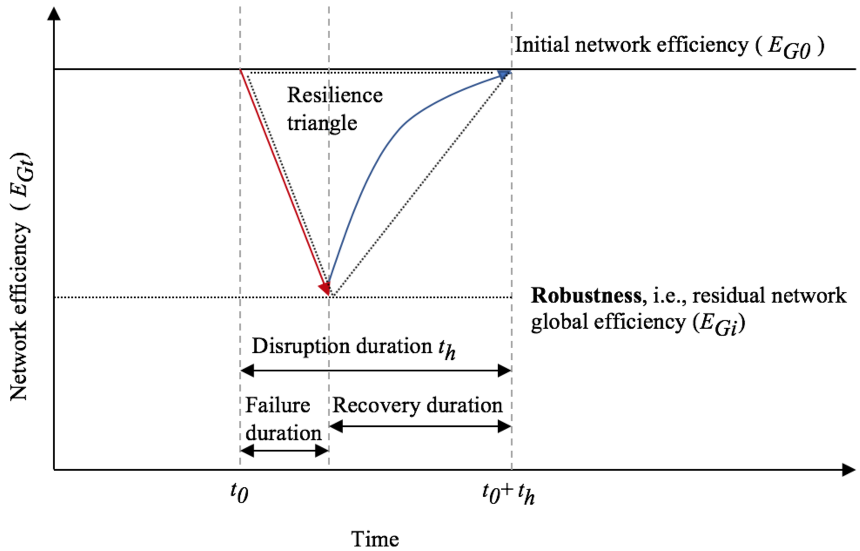

Figure 3.5. Schematic representation of resilience triangle and robustness in a network.

The global network efficiency could be used to represent the network performance. Hence, the resilience triangle for a metro network is:

$$R_e = \frac{\int_{t_0}^{t_0+t_h} E_{Gt} dt}{t_h E_{G0}} \tag{4.3}$$

where E_{Gt} is the network efficiency at time t, and E_{G0} is the initial efficiency of the metro network. During the time interval of t_h, the diminished performance system could be recovered to its initial performance level. Major considerations in enhancing network resilience are recovery strategies and associated costs.

Henry and Ramiz-Marquez (2012)investigated the importance of recovery strategy in resilience analysis. An effective recovery strategy not only depends on the best recovery sequence, but also on the recovery cost. The associated recovery cost is correlated with the cost of income loss due to network disruption and the cost of repairs. Considering the factors of recovery sequence and recovery cost, decision makers can embrace an effective sequential recovery scheme. Identifying an effective recovery strategy is necessary to enhance the resilience of a network and provides a better safety assessment, a more accurate cost-benefit evaluation, and risk reduction in the network. A detailed recovery analysis of the Washington D.C. Metro is described in chapter 6.

4.3. Synthesis of Findings

A metro system provides for effective transportation means with synergies in urban areas resulting in economic growth and equity. Enhancing the resilience of metro systems contributes to enhancing commuters' safety. Methods to measure metro network performance, robustness and vulnerability provide a basis for measuring and enhancing metro resilience, operation and maintenance. This chapter adopts the Washington D.C. Metro as a case study and uses the topology analysis results presented in Chapter 3 to propose a method for analyzing vulnerabilities and resilience based on complex network theory. Complex network theory is a basic method that has been widely used in modeling urban critical infrastructure networks, such as transportation networks. Metro networks have non-trivial topological characteristics and typically the pattern of connections in their networks are not merely regular nor random. Thus, a metro network can be viewed as a complex system of a large number of interactive units, and is a realization of a spatial network.

Therefore, complex network theory works well in modeling the structural features of a metro network and its pattern of connectivity.

Assessing the vulnerability and resilience of a metro network requires modeling its topology using complex network theory. Through the analytical and computational work presented, the vulnerability of the Washington D.C. Metro is evaluated by method of connectivity changes due to either node removal or link removal, one at a time for each type. In addition, the connectivity changes between the initial network connectivity (i.e., prior to any failures) and network connectivity measures after failure along with the resilience triangle concept are used to develop a unified metric of network resilience. The Washington D.C. Metro network was assessed comprehensively by considering two scenarios, i.e., the metro network subjected to node removal and link removal. The node removal vulnerability assessment identified the *L' Enfant Plaza Station* to have the greatest value. Removing the *L' Enfant Plaza Station* drastically reduces the network efficiency by 29.74%. Additionally, examining the link removal identified the cases, in the multi color-coded lines responsible for connecting east, west and south of Washington D.C. to the downtown area, as well as the Red and Green metro lines connecting the north-west quadrants of the city and the south-east quadrant of the city to the downtown area, with the most vulnerable links in this metro system. The reductions in efficiency for these links range from 16.07 % to 8.78% for the most critical links of the network. Therefore, exclusive attention should be given to these most vulnerable stations and metro links. Hence, in order to enhance the resilience of the network, it is recommended to add one circle route, which protects the stations located in the downtown area, and also has sufficient connections to the four quadrants of the city. This idea will be examined in Chapter 6 of this dissertation.

Enhancing the network's resilience requires a substantial investment, yet great benefits and the robust network are the outcome and .

Chapter 5: Failure Analysis of Weighted Urban Rail Transit Networks Incorporating General Ridership Patterns*

5.1 Introduction

In any non-linear system as complex as an urban rail transit network or metrorail network, some incidence of perturbations of its state is inevitable. These perturbations can highly affect the networks' resilience. Increasing the ability of metrorail networks to withstand such perturbations and improving their operational efficiency and performance require robustness and vulnerability assessments as key attributes of resilience. Most models developed for this purpose associate a network's failures to binary representations of the failure of its components without incorporating weight factors; while all networks are best described by considering respective weights assigned to links.

For transportation systems, most studies from the perspective of network topology have focused on the vulnerability under stochastic failures or deliberate attacks by randomly or selectively removing some nodes or links from the network (Holme and Kim 2002; Laporte et al. 2010; Lin and Ban 2013; Deng et al. 2013; Zhang et al. 2018; Saadat et al. 2019); whereas few studies have considered the characteristics of

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* **Paper published or submitted**

Saadat, Y., Ayyub B.M., Zhang Y., Zhang D., & Huang H. (2020). " Urban Rail Transit Networks Toward Reliable Development: Failure Analysis Incorporating General Ridership Patterns ." Submitted to Reliability Engineering and System Safety Journal, under second round of review.

traffic flow through a network. This limitation may result in an incomplete view of transportation networks leading to suboptimal or erroneous decisions. The network's structure and function are the two key concerns in the study of complex transportation networks, and traffic conditions are the direct embodiment of transportation network function. Therefore, such models could be enhanced by considering a weighted transportation network using traffic data for analyzing its characteristics. For instance, Bagler (2008) used the number of flights as the weight to analyze the distribution characteristics of weighted complex network eigenvalues in the airport network of India. Sun et al. (2015) considered the passenger flow influence in measuring the functional vulnerability and serviceability of the Shanghai metrorail, although from the perspective of line operation and not as a weighted network. Feng et al. (2017) analyzed the traffic flow pattern in the Beijing subway system based on trip data and travel time to create the weighted network. The previous studies, however, have not considered the actual ridership traveling on each link of the network, while ridership has significant influence on transportation network attributes.

One challenge in assessing ridership-weighted transportation network attributes is characterizing passenger flow patterns. Reported spatiotemporal distribution patterns of passenger flow or traffic conditions are typically based on databases of real networks (Kerner and Klenov 2004; Ahas et al. 2010; Gao et al. 2013; Li et al. 2013). Anbaroğlu et al. (2015) modeled the heterogeneity of London roadways, and Wu et al. (2016) obtained the traffic volume pattern of Bellevue roadways in Washington state based on collecting real data. Kurant and Thiran (2006) extracted the physical traffic flow from timetables of public mass transportation networks as a data source.

In metrorail networks, smart metrorail cards could help in collecting ridership data, however, may not fully capture the number of passengers traveling on each link. The ideal state is to obtain ridership data on each link. Typically, collecting such data is not straightforward and even if it is obtainable, it would isolate the paper to a specific case study or some set of data. Hence, there is a need for a general model that can provide a good approximation for assessing ridership traveling on metrorail links. Ideally, such a model is universal in nature and applicable by calibration to any network.

In the actual operation of a metrorail network, ridership directly affects the status of a station or links in the network in terms of connectivity importance. Correspondingly, it is urgent to efficiently reflect on the number of passengers that each metrorail link carries when analyzing metrorail network attributes during or following network connectivity failures.

This chapter aims to address two gaps as outlined previously by proposing models to (1) characterize the general ridership pattern in an urban rail transit system, and (2) analyze a ridership-weighted network defined by associated link weights. Ridership serves as the basis for computing weight factors for links. These models were used for two disruption cases resulting from one failure at a time of a node or a link. The main objective of the chapter is to utilize such failures to analyze the network efficiency, robustness, and vulnerability among other attributes by employing a novel methodology and using the Washington D.C. Metro network as a case study. By comparing the network efficiency before and after the failure of each node or link, the vulnerability as well as robustness were quantitatively calculated and the highly vulnerable nodes and links were identified. Assessing network vulnerability and robustness offers a basis to enhance the network resilience.

5.2 Proposed Methodology for Assessing Ridership-Weighted Metrorail Network Attributes

Analyzing a weighted metrorail network's attributes starts by characterizing the topological indicators without the weights. Then, the number of passengers on each link is normalized[1]and assigned to the corresponding link as weight factors. The link-weighted network model, i.e., weighted network, offers a basis for calculating network efficiency by which to measure the flow between any pair of nodes in a network. Finally, a vulnerability assessment is conducted by calculating the changes in the metrorail network efficiency due to one-at-a-time node and link removals from the weighted network. Different methods are available to analyze the efficiency of weighted networks. For instance, Newman (2004) suggested an effective technique of mapping weighted networks on to unweighted multigraphs and analyzing those

[1] It should be noted that the weights need not be normalized in general. For the specific application of this chapter and to make it concise to the expression of weighted global network efficiency, weights have been used in a normalized sense.

unweighted multigraphs instead. Guidotti et al. (2017) proposed using unweighted axillary nodes as nodal and link weights for assessing topology-based reliability of a network. In the case of a metrorail network with ridership considered as weights on links, the size of corresponding multigraphs or network with axillary nodes may become relatively large and could lead to analytical and computational challenges. This chapter proposes a novel model for evaluating the efficiency of a weighted metrorail network reflecting its ridership by concurrently incorporating two matrices: (1) an unweighted adjacency matrix, i.e., a matrix for demonstrating a pattern of connection in a network, and (2) weighted adjacency matrix in which the matrix elements represent normalized weights on links between any two nodes. The unweighted adjacency matrix is used to determine the shortest path, i.e., geodesic path between any two stations, hereafter indicated by d_{ij}. Weighted adjacency matrix is used to calculate the network efficiency by undertaking the sum of link weights on each geodesic path. The proposed methodology as shown in Fig. 5.1 consists of the following four steps:

1. Mapping a metrorail system into a topological graph and defining nodes, links and pattern of connection, i.e., adjacency matrix;
2. Analyzing topology of an unweighted metrorail network and calculating the geodesic path between any two stations along with other network characteristic indicators;

3. Evaluating ridership for each link of the metrorail network by using the data available on each station in order to generate the weighted adjacency matrix; and
4. Assessing efficiency, robustness and vulnerability of the metrorail network after failure events.

Steps 1 and 2 are already achieved in Chapter 2 and the procedure is not replicated in this or following chapters. Rather, the results are used to calculate the necessary terms hereafter.

These steps are detailed in subsequent sections.

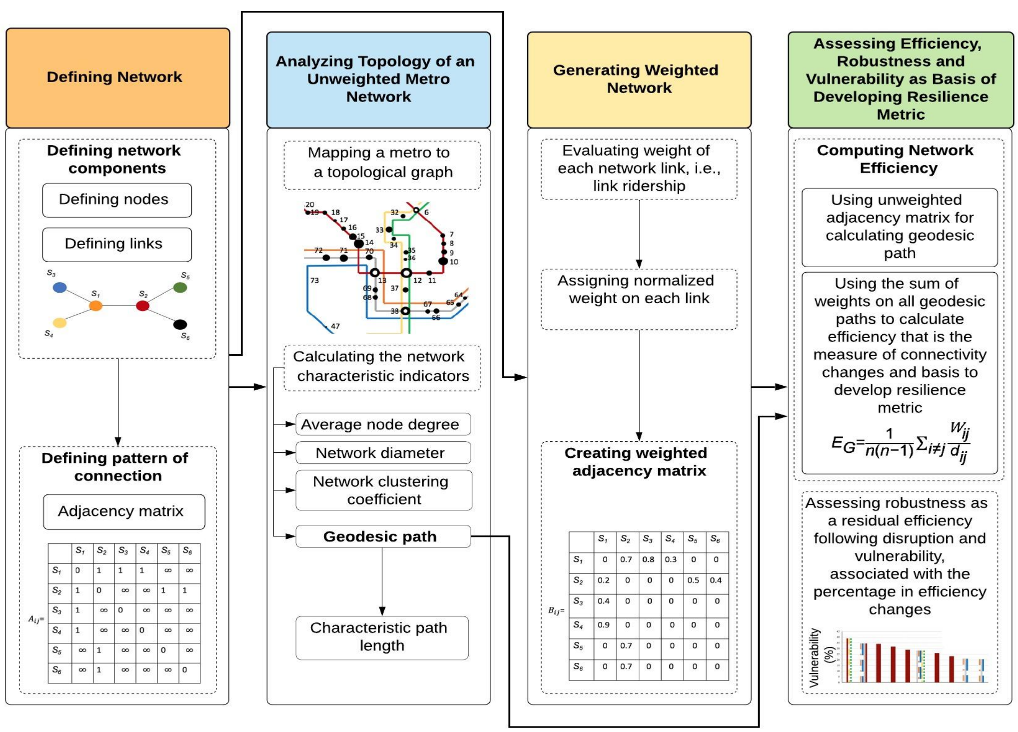

Figure 5.1. A proposed methodology for assessing ridership-weighted metrorail network attributes.

5.2.1 Defining a network and generating unweighted adjacency matrix

According to Chapter 2, an urban metrorail transit network could be seen as a realization of a spatial network. In mapping a metrorail system into a topological graph thereof, each node represents a metrorail station, while the link between nodes signifies the metrorail segments, i.e., tunnel, bridge, underground, connecting track, etc. Network components are defined by Eq. (3.1) and network unweighted adjacency matrix is described by Eq. (3.5). In complex network theory, an adjacency matrix is a square $n \times n$ matrix used to illustrate the state of connection in an entire network. It is the key element for network topological analysis and generating it is the initial step in any further investigating the network characteristics.

5.2.2 Generating a weighted Network and weighted adjacency matrix

5.2.2.1 Ridership data as weights for links

The ridership data of the Washington D.C. Metro stations can be found on Washington Metropolitan Area Transit Authority (WMATA, 2016) archives. Figure 5.2 shows the ridership distribution of a total of 91 stations comprising the Washington D.C. Metrorail. It is found that 44% of stations have less than 5,000 passengers on average each day and 36% of stations have between approximately 5,000 to 10,000 passengers on a daily basis. There are few stations that have a relatively large number of passengers, which is shown in Table 5.1. The data indicates that ridership in the Washington D.C. Metro is not distributed uniformly.

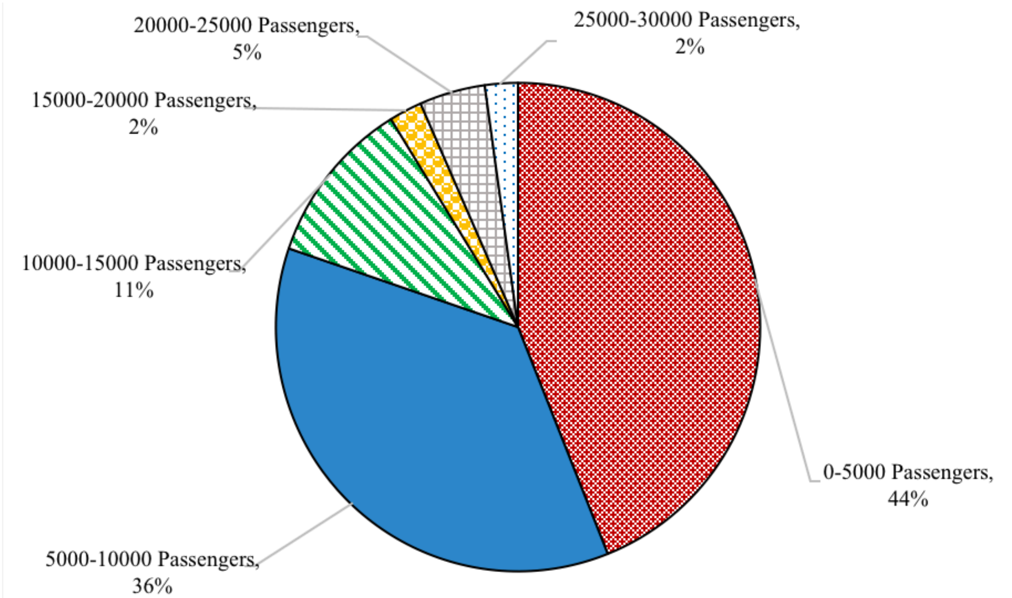

Figure 5.2. Distribution of station ridership in the Washington D.C. Metro.

Table 5.1 Stations that have large passenger volume in the metrorail network

Rank order	Name of the station	Station number	Number of passengers	Node degree
1	Union Station	10	29371	2
2	Gallery Place	12	25537	4
3	Farragut North	14	24597	2
4	Metro Center	13	24330	4
5	L' Enfant Plaza	38	19343	5
6	Dupont Circle	15	18653	2

Referring to the number of passengers for each station, Fig. 3.4, which is a topological graph of Washington D.C. Metro, is modified to Fig. 5.3, in which node sizes reflect the number of passengers flowing in each station.

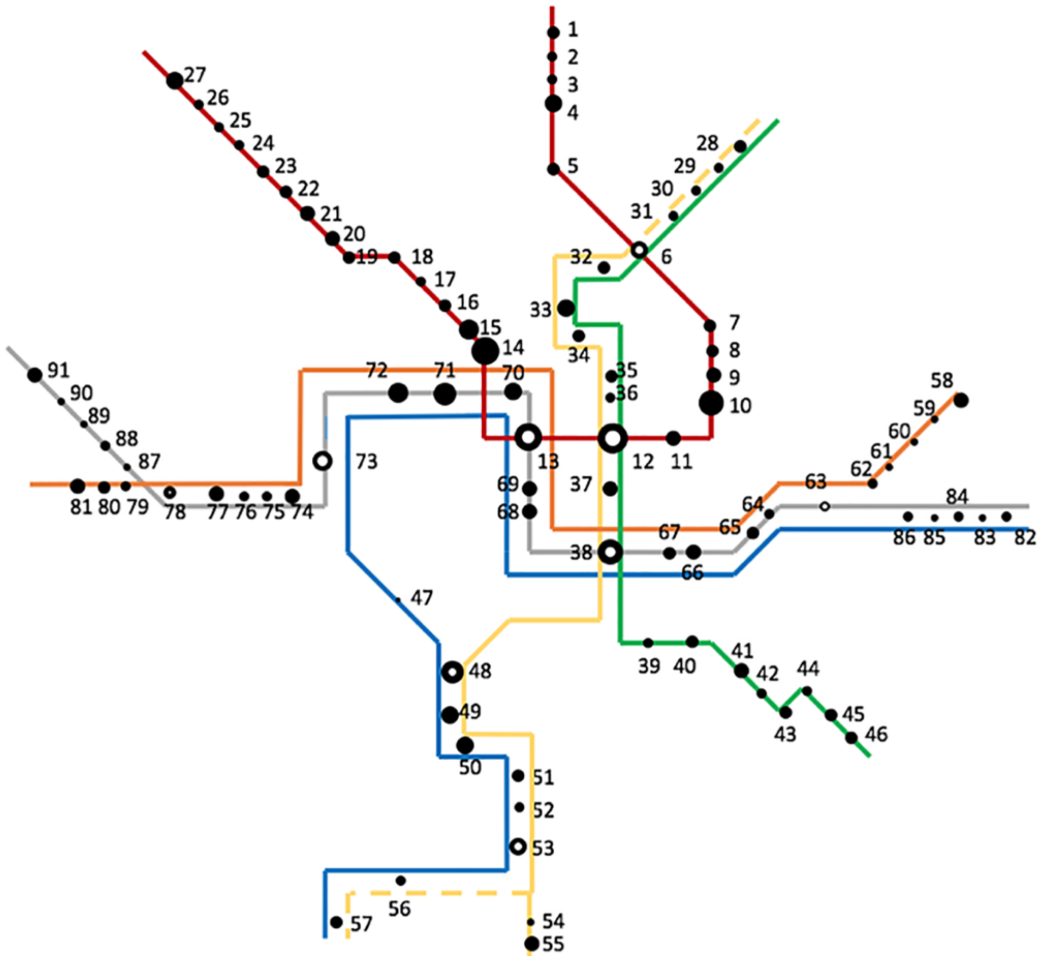

Figure 5.3. Topological graph of Washington D.C. Metro network (The node sizes reflect the number of passengers flowing in each station).

5.2.2.1 Weighted adjacency matrix

To generate a weighted metro network, ridership data provide the dominant basis for computing the weight factors that define the dynamic nature of the network. Collecting ridership data for each link is not a practical task in the real world and the behavior of passenger flow could be very complicated. However, passengers at any particular station must travel to one of its neighboring stations or exit the metro system. The current paper relies on following assumptions for computing the weight factors:

1. The ridership for each link is related to the number of passengers at two end stations of the corresponding link assuming the system is a closed system;
2. The number of passengers traveling from node *i* to node *j* frequently differs from the passengers traveling in the reverse direction of the link;
3. The network is undirected. It should be noted that directed networks stand for networks including a set of links with arrows indicating the direction of those links. Undirected networks are free of this characteristic and, therefore, their analyses are simpler;
4. Attractiveness of link is based on the density of passengers on the stations connected by associated links, and thus, it covers the case of a heavily used line;
5. The data used are based on the average ridership on a daily basis;
6. Different times of a day could attract different passenger density. However, the model proposed herein, concentrates on a general case considering the average number of daily passengers. Since the number of passengers might be different on different times of a day, for more realistic computational work, it would be beneficial to redo them for smaller time step based on the data availability.

Thus, to ease the evaluation, it is reasonable to assume the ridership on each link is proportional to the average of the sum of passengers in both directions on the link, i.e., $i \rightarrow j$, and $j \rightarrow i$. The more the passengers at the two neighboring stations, the larger the ridership on the link connecting the two stations. The equations for calculating ridership for each link can be expressed as:

$$R_{(i,j)} = \frac{1}{2}(R_{i \rightarrow j} + R_{j \rightarrow i}) \tag{5.1}$$

$$R_{(i,j)} = \frac{1}{2}\left(P_i \frac{P_j}{\sum P_{neighboring\ nodes\ to\ node\ i}} + P_j \frac{P_i}{\sum P_{neighboring\ nodes\ to\ node\ j}}\right) \tag{5.2}$$

where, $R\ (i, j)$ signifies the ridership per day on a link between two neighboring nodes i and j in the undirected network; P_i and P_j represent number of passengers at station i and station j, respectively. The passengers that flow from node i to node j, i.e., $R_{i \rightarrow j}$, is calculated as the ratio of multiplication of the number of passengers at station i by the number of passengers at station j to the sum of passengers at all neighboring stations to station i. Neighboring nodes mean the nodes that are directly connected to one specific node. The equivalent calculation is relevant for the ridership in the reverse direction, i.e., $R_{j \rightarrow i}$. It just needs modification of the denominator to passengers at all stations neighboring to station j.

Figure 5.4 demonstrates the logic behind the calculation of $R_{i \rightarrow j}$ and $R_{j \rightarrow j}$. The available data are the average number of daily passengers for each station. The ridership between any two neighboring stations, which is needed for this model, can be evaluated with respect to the total passengers of the initial station who would travel to all of its neighboring stations. For example, in Fig. 5.4, $R_{i \to j}$ and $R_{j \to j}$ are evaluated by Eqs. (5.3) and (5.4).

$$R_{i \to j} = P_i \frac{P_j}{(P_j + P_m + P_n)} \tag{5.3}$$

$$R_{j \to i} = P_j \frac{P_i}{(P_i + P_k + P_l)} \tag{5.4}$$

where P_m, P_n, P_l and P_k are the number of passengers at stations m,n,l and k respectively. The neighboring stations of station i are stations j, m and n; the neighboring stations of stations j are station i, k and l.

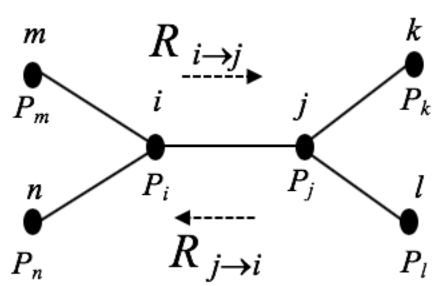

Figure 5.4. A theoretical example of a ridership calculation on a link for a simple network.

The weight w_{ij} assigned to each link could be achieved by normalizing the ridership at each link calculated as the ratio of $R\ (i, j)$ to the ridership of the link that carries the maximum flow as follow:

$$w_{ij} = \frac{R_{(i,j)}}{Max\ R_{(t,s)}} \tag{5.5}$$

where stations t and s are two neighboring stations belonging to the metro network located at two ends of a link that carry the maximum ridership.

The ridership for each link is calculated based on Eqs. (5.3) and (5.4), which demonstrate the number of passengers at each link $R\ (i, j)$ could be calculated as the average number of passengers who travel from node i to node j, and from node j to node i, i.e., $R_{i \rightarrow j}$, and $R_{j \rightarrow i}$. Also, the ridership in each direction could be estimated as the ratio of multiplication of the number of passengers at station i by the number of passengers at station j to the sum of passengers at all neighboring stations to station i. The weight w_{ij} assigned to each link, therefore is achieved and normalized using Eq. (5.5).

Based on the above calculations the weighted network is generated. Figure 5.5 demonstrates the normalized link ridership weight distribution for the Washington D.C. Metro network.

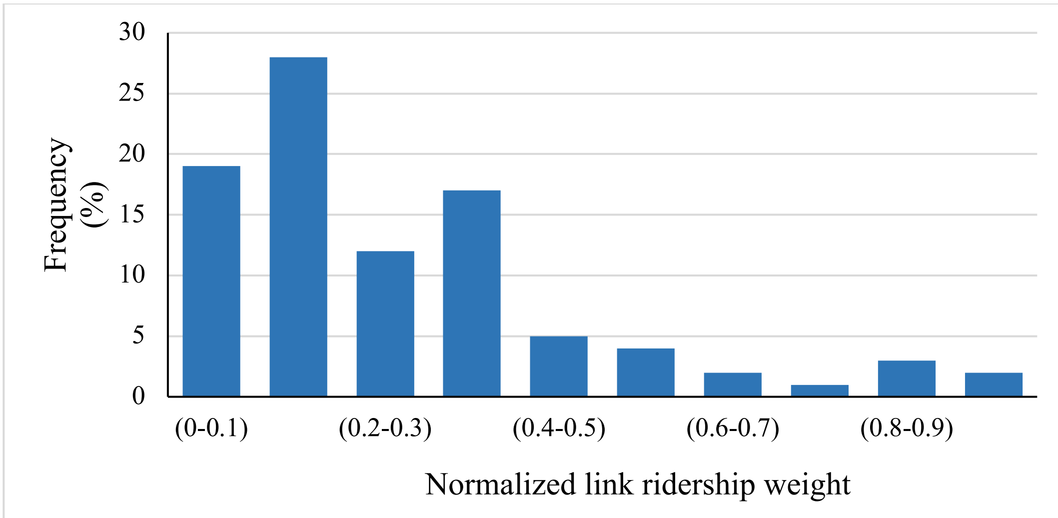

Figure 5.5. Distribution of link ridership weight for the Washington D.C. Metro.

Determining the weight on each link could lead to creating a weighted adjacency matrix. The elements of a weighted network adjacency matrix in the proposed methodology include either the weights in the case of direct link between two neighboring nodes or zero otherwise as follows:

$$a_{ij}=\begin{cases} w_{ij} & \text{Weight of connection link from nodes } i \text{ and } j \\ 0 & \text{When there is no direct link between node } i \text{ and j} \end{cases} \tag{5.6}$$

5.2.3 Assessing efficiency, robustness, and vulnerability

The network efficiency or global network efficiency of a metrorail network is a measure of how efficiently stations i and j connect within the network. The global efficiency of an unweighted network is quantified as Eq. (3.15). While, the global efficiency of a weighted network is expressed as:

$$E_{GW}=\frac{1}{n(n-1)}\sum_{i\neq j}\frac{W_{ij}}{d_{ij}} \tag{5.7}$$

where n is the number of stations and d_{ij} is the geodesic path or shortest path. In addition, W_{ij} alone is the sum of all w_{ij} on each geodesic path. The method used in this paper for evaluating network efficiency satisfies the following assumptions: (1) Passengers typically tend to use the geodesic path d_{ij} between their origin station and their destinations. (2) The geodesic path in this paper is the minimum number of links between any pair of nodes. Other dimensions that may affect the geodesic path assessment, such as link lengths and travel time are not considered herein. Thus, the path length which is the number of links in a specific path is the same for a weighted network and an unweighted network in this study. (3) The elements of a weighted network adjacency matrix in the proposed methodology are based on Eq. (5.6). (4) Although, each link in the network is assigned the weight that reflects its ridership, for analyzing the global network efficiency, weighted-links on the geodesic path between each pair of nodes have been taken into account.

In references to the assumptions outlined above, the $n \times n$ unweighted adjacency matrix is used in Floyd algorithm (Floyd 1962) to measure the geodesic path between any stations i and j in the network. Also, the $n \times n$ weighted adjacency matrix is used to incorporate in the calculation of global efficiency. Therefore, global network efficiency, using Eq. (5.7), by averaging the entire ratio of the sum of link weights on each geodesic path to the corresponding geodesic path for all possible pair of nodes in the network could be achieved.

As previously stated, network efficiency is usually defined as an indicator to quantify the network connectivity. The robustness of a metro network lies in its ability to resist and maintain residual network connectivity after a disruptive event. Accordingly, E_{Gi} is the residual network connectivity and could be considered as the network robustness. Thus, the network robustness E_{Gi} is the network efficiency after removal of network node and/or links and could be expressed as follows:

$$E_{GWi} = \frac{1}{n'(n'-1)} \sum_{i \neq j} \frac{W'_{ij}}{d'_{ij}} \tag{5.8}$$

where n' is the number of nodes after node removals in the network. In the case of merely link removals, the number of nodes remain unchanged. The new geodesic path is d'_{ij} and is calculated through a revised adjacency matrix which captures changes in the network's pattern of connection. Also, W'_{ij} is the sum of all w_{ij} on each new geodesic path d'_{ij}.

Evaluating the weighted network vulnerability relates to the changes on the weighted network efficiency due to disruptive events and can be quantified as:

$$V_i = \frac{E_{GW} - E_{GWi}}{E_{GW}} \tag{5.9a}$$

$$V = Max\ V_i \tag{5.9b}$$

where V_i is the network vulnerability after disruption, E_{GW} is the initial network efficiency of a network prior to any disruption in the network and E_{GWi} is the post network efficiency after removal of nodes or links in a metro network. Vulnerability assessment of a weighted metrorail herein also includes two cases:

Case I: One-at-a-time node removal due to a disruptive event; and

Case II: One-at-a-time link removal due to a disruptive event.

The factual example of such events could happen during natural hazards such as flooding. For instance, Superstorm Sandy (2012) knocked out several New York City subway links and stations due to flooding (FEMA 2013).

These two cases are described in the subsequent subsections in a structured manner.

5.2.3.1 Vulnerability assessment of the weighted network with node failures

When a station fails in a weighted metro network, passengers using that station may decide to pick other transportation modes. In this dissertation, an assumption is made that passengers choose other alternate transportation options excluding the metro systems, such as busses, for the purpose of illustration. Other variants to assumptions are possible; but will not affect the overall methodology and underlying models. This assumption offers a basis to reduce analytical and computational difficulties by not including the option within the metrorail network; however, this assumption could be removed in future works by linking networks of several transportation modes including the metrorail network. On this basis, all links on the entire network may also be affected by having their weights increased or decreased. Thus, the method of node removal for vulnerability assessment requires regenerating both the adjacency matrix and the weighted adjacency matrix. The size of the new adjacency matrix is $(n\text{-}1) \times (n\text{-}1)$. By using the new adjacency matrix in a modified Floyd algorithm (Floyd, 1962), all new geodesic paths between any pair of nodes can be derived. Also, the new $(n\text{-}1) \times (n\text{-}1)$ weighted adjacency matrix is used in Eq. (5.8) for the purpose of calculating the post global network efficiency after a disruptive event. Then, using Eqs. (5.9a) and (5.9b) the vulnerability of a weighted metro network is assessed. Table 5.2 lists the ten most vulnerable stations in the Washington D.C. Metro network. Also, Fig. 5.6 displays the vulnerability profile of those stations.

Table 5.2 The ten most vulnerable stations in the weighted Washington D.C. Metro network.

Rank order	Name of the station	Color-coded lines	Station number	Number of passengers	Node degree	Vulnerability Magnitude (%)
1	Gallery Place	Red-Yellow-Green	12	25537	4	39.19
2	Metro Center	Red-Orange-Silver-Blue	13	24330	4	34.32
3	Farragut North	Red	14	24597	2	34.26
4	Dupont Circle	Red	15	18653	2	31.84
5	Woodley Park	Red	16	5861	2	28.92
6	L' Enfant Plaza	Green-Orange-Silver-Blue-Yellow	38	19343	5	28.70
7	Cleveland Park	Red	17	3961	2	25.96
8	Van Ness	Red	18	6158	2	23.16
9	Rosslyn	Orange-Silver-Blue	73	13666	3	21.13
10	Federal Center SW	Orange-Silver-Blue	67	5697	2	20.94

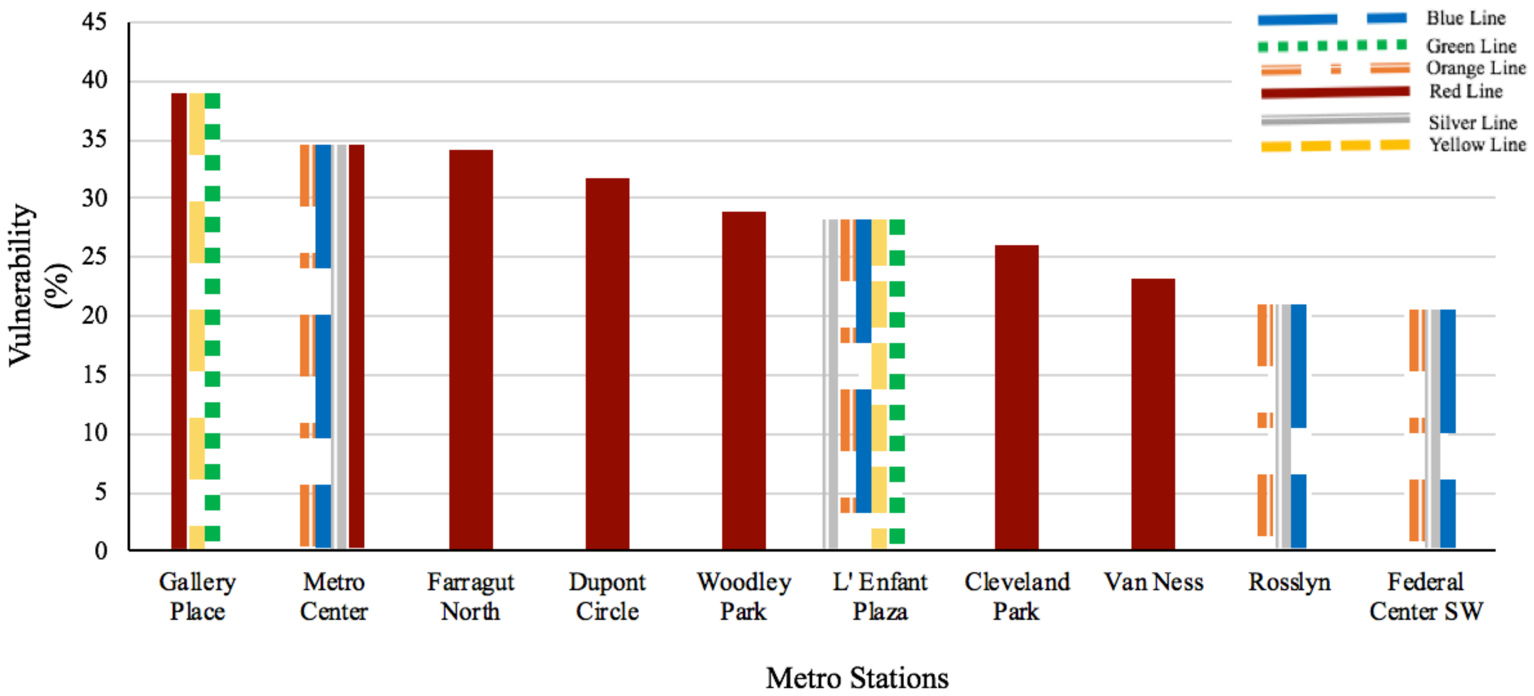

Figure 5.6. The ten most vulnerable stations in the weighted Washington D.C. Metro.

As demonstrated in Table 5.2, the most vulnerable station in the weighted Washington D.C. Metrorail is the Gallery Place station, which carries a relatively large number of passengers. According to Eq. (5.8) the initial global network efficiency for the weighted network is 0.1354. Removing Gallery Place reduces network global efficiency by 39.19%, which is considerable. The global network efficiency after the Gallery Place station removal would be 0.0823 and it is reduced by 0.0531, which is 39.19% of the value of the initial global network efficiency. Although fewer than half of the most vulnerable stations carry a large number of passengers and are heavily used by commuters, that is not always the case. There are some stations that carry a small number of passengers and are still considered among the most vulnerable stations like the Cleveland Park station. In counterpart, the Union Station, which carries the largest number of passengers, is not ranked among the most vulnerable stations. However, transfer stations that have large node degrees located in the central part of the city play a critical role and they are ranked among the most vulnerable stations in the metrorail network. Results show stations located on the northwest part of the Red Line are also among the most vulnerable stations. There is a strong likelihood of correlation between the most vulnerable stations and their locations in the network. Thus, it could be concluded that location plays a more significant role in vulnerability assessment rather than the number of passengers. Identifying vulnerable stations could raise the attention of planners and engineers for safety enhancement. Nevertheless, some stations, such as the Union Station, are not ranked among the most vulnerable stations yet, are strategically important in the city.

Therefore, their safety enhancement and resilience investment should also receive appropriate consideration.

5.2.3.2 Vulnerability assessment of the weighted network with link failures

The vulnerability assessment with link removal follows almost a similar procedure to the vulnerability analysis with node removal to examine the contribution of each link on the network global efficiency. The differences between node and link removal vulnerability assessment are associated with two important considerations:

- The size of the adjacency matrix does not change in the link removal case. When a link fails in the network, its two end stations are disconnected and there is no direct connection between those stations. Thus, the corresponding element in the adjacency matrix is substituted with infinity instead of one.
- The size of the weighted adjacency matrix in the link removal case remains the same. However, the weight of the failed link is set to zero.

The link vulnerability assessment of the Washington D.C. Metro network is similar to the node vulnerability analysis. Yet, the size of unweighted and weighted adjacency matrices remain as 91×91 and the entity for the failed link in the network.

The global efficiency after link removal is evaluated for all links on the network. Table 5.3 demonstrates the ten most critical links of the Washington D.C. Metro network, and Fig. 5.7 shows those ten most vulnerable links in the metro.

Table 5.3 The ten most critical links in the Washington D.C Metro network

Rank order	Link	Station names at two ends of each link	Color-coded lines	Robustness	Vulnerability Magnitude (%)
1	(13,14)	(Metro center, Farragut North)	Red	0.0866	36.04
2	(14,15)	(Farragut North, Dupont Circle)	Red	0.0900	33.57
3	(15,16)	(Dupont Circle, Woodley Park)	Red	0.0942	30.44
4	(16,17)	(Woodley Park, Cleveland Park)	Red	0.0983	27.44
5	(17,18)	(Cleveland Park, Van Ness)	Red	0.1019	24.74
6	(18,19)	(Van Ness, Tenley Town)	Red	0.1054	22.20
7	(73,74)	(Rosslyn, Court House)	Orange-Silver	0.1060	21.77
8	(66,67)	(Capital South, Federal Center SW)	Blue-Orange-Silver	0.1065	21.39
9	(74,75)	(Court House, Clarendon)	Orange-Silver	0.1084	19.98
10	(19,20)	(Friendship Heights, Bethesda)	Red	0.1087	19.71

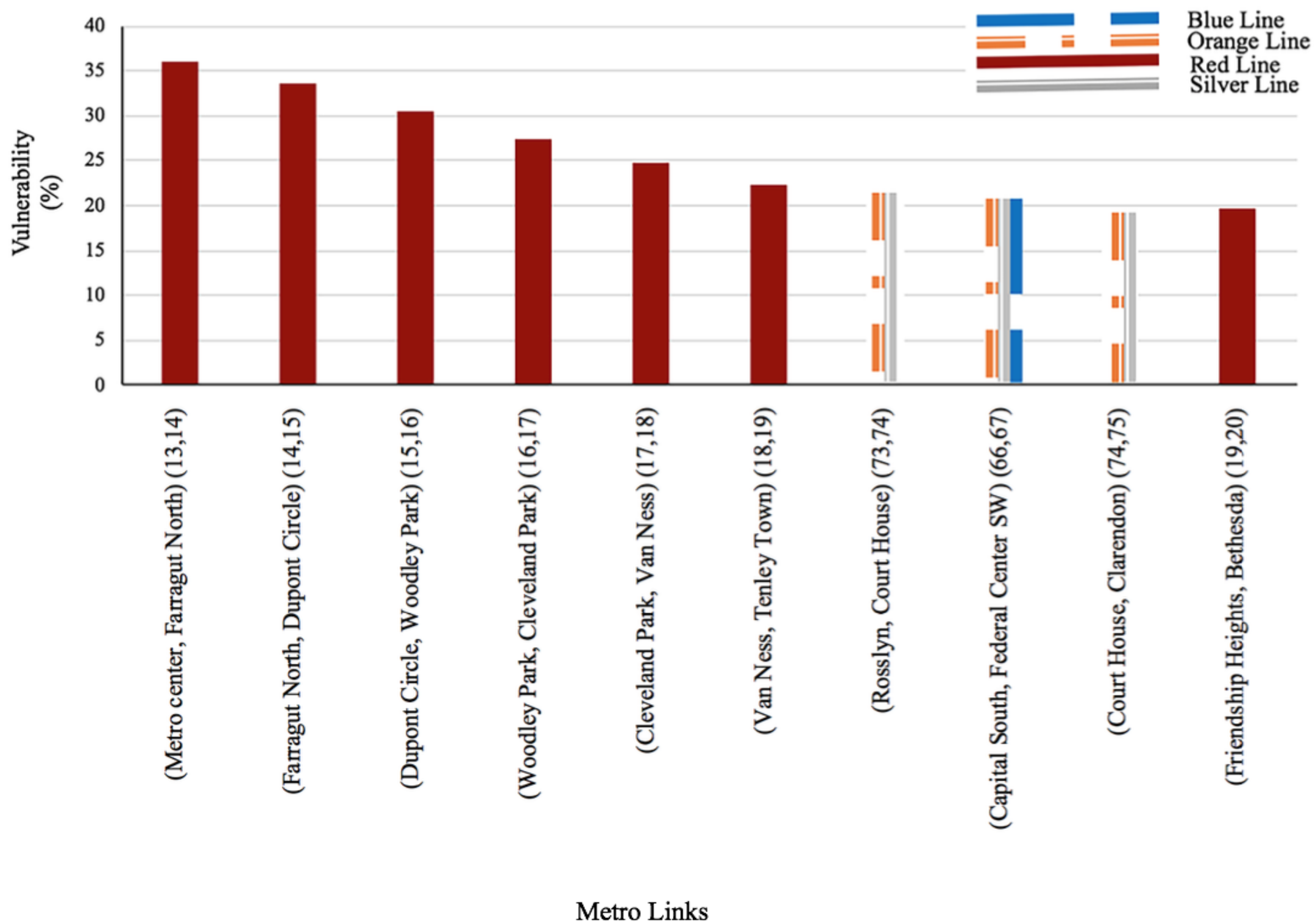

Figure 5.7. The ten most vulnerable links in the weighted Washington D.C. Metro.

5.2.4 Comparing network vulnerability using weighted and unweighted metrics

To assess the benefit of using a weighted network rather than an unweighted one, this section compares the robustness and vulnerability measurements of weighted and unweighted networks. In Chapter 4 the global efficiency, robustness, and vulnerability of unweighted Washington D.C. Metro network's components are calculated. Table 5.4 and 5.5 provide the comparison among critical nodes and links in the cases of unweighted and weighted networks, respectively.

Table 5.4 Comparison between the most critical nodes in the unweighted and weighted Washington D.C Metro networks

Unweighted network			Weighted network		
Name of the station (station number)	Robustness	Vulnerability Magnitude (%)	Name of the station (station number)	Robustness	Vulnerability Magnitude (%)
L' Enfant Plaza (38)	0.1005	29.78	Gallery Place (12)	0.0824	39.19
Gallery Place (12)	0.1096	23.42	Metro Center (13)	0.0890	34.32
Metro Center (13)	0.1182	17.43	Farragut North (14)	0.0891	34.26
Federal Center SW (67)	0.1192	16.70	Dupont Circle (15)	0.0923	31.84
Pentagon (48)	0.1199	16.22	Woodley Park (16)	0.0963	28.92
Rosslyn (73)	0.1212	15.3	L' Enfant Plaza (38)	0.0966	28.70
Capital South (66)	0.1215	15.09	Cleveland Park (17)	0.1003	25.96
Farragut North (14)	0.1228	14.23	Van Ness (18)	0.1041	23.16
Eastern Market (65)	0.1234	13.79	Rosslyn (73)	0.1068	21.13
Court House (74)	0.1240	13.36	Federal Center SW (67)	0.1071	20.94

Table 5.4 shows how link strengths, presented by their weights, may affect changing the order of components criticality. Almost half of critical stations are mutual in the two cases of unweighted and weighted networks. However, they have different orders and their robustness and vulnerability are considerably different. The Robustness measurements are lower in the weighted network and the vulnerability magnitudes are higher. Thus, a weighted network shows heightened sensitivity to a disruptive event and could have impact on planning and decision making for those critical components.

The results of critical links in unweighted and weighted networks are significantly different. A weighted network shows heightened criticality and illustrates the importance of considering link strengths in the calculation.

Table 5.5 Comparison between the most critical links in the unweighted and weighted Washington D.C Metro networks

Unweighted network			**Weighted network**		
Station names and numbers at two ends of each link	Robustness	Vulnerability Magnitude (%)	Station names and numbers at two ends of each link	Robustness	Vulnerability Magnitude (%)
(Metro center, Federal Triangle) (13,69)	0.1201	16.07	(Metro center, Farragut North) (13,14)	0.0866	36.04
(Smithsonian, Federal Triangle) (68,69)	0.1235	14.42	(Farragut North, Dupont Circle) (14,15)	0.0900	33.57
(L' Enfant Plaza, Smithsonian) (38,68)	0.1243	13.14	(Dupont Circle, Woodley Park) (15,16)	0.0942	30.44
(Rosslyn, Court House) (73,74)	0.1248	12.78	(Woodley Park, Cleveland Park) (16,17)	0.0983	27.44
(L' Enfant plaza, Federal Center SW) (38,67)	0.1259	12.07	(Cleveland Park, Van Ness) (17,18)	0.1019	24.74

Table 5.5 , Continued

Unweighted network			**Weighted network**		
Station names and numbers at two ends of each link	Robustness	Vulnerability Magnitude (%)	Station names and numbers at two ends of each link	Robustness	Vulnerability Magnitude (%)
(Metro Center, Farragut North) (13,14)	0.1264	11.71	(Van Ness, Tenley Town) (18,19)	0.1054	22.20
(Pentagon, Pentagon City) (48,49)	0.1266	11.57	(Rosslyn, Court House)(73,74)	0.1060	21.77
(Court House, Clarendon) (74,75)	0.1267	11.50	(Capital South, Federal Center SW) (66,67)	0.1065	21.39
(Stadium Armory, Potomac Ave) (63,64)	0.1272	11.14	(Court House, Clarendon) (74,75)	0.1084	19.98
(Clarendon, Virginia-Sq-GMU) (75,76)	0.1282	10.42	(Friendship Heights, Bethesda) (19,20)	0.1087	19.71

5.3 Conclusions and Contributions

Chapter 5 analyzes failure events resulting in node or link removal in weighted urban rail transit networks and assesses the network attributes such as efficiency, robustness, and vulnerability. Robustness and vulnerability assessment is a necessary means to analyze risk and resilience, and to identify potential threats to such a network. The origins of these threats may come from within or outside of the network and could pose risks to the network performance. To assess the network performance along other attributes, a weighted network model based on the CNT method is presented. A weighted matrix along with an adjacency matrix are used to evaluate the network global efficiency, i.e., prior to and post failure. The variation in global efficiency before and after disruption is the basis of vulnerability assessment. The identified vulnerable components of a weighted network are more reasonable and realistic than they would be without considering weight.

In order to conduct such an assessment, the methodology proposed in this chapter has the following novel approaches, which could advance this area of research:

- A practical model is proposed to calculate link ridership based on the passenger volume of stations. The model presented here is different from typical origin-destination models. It directly estimates the number of passengers who travel on each link. Network links intrinsically may carry different weights. Weights assigned to the network links in this dissertation are based on information collected at the network nodes. Thus, this method could be applied to estimate the weight strengths on the network links without changing the adjacency matrix and without posing higher computational burden.

- The topological connectivity for an unweighted network is analyzed in order to achieve a geodesic path between any two stations and also combined with the use of ridership data to construct a weighted network–in terms of the summation of link ridership weights on each geodesic path– to incorporate them into the global network efficiency equation.
- A global network efficiency expression is altered by adding a weight factor considering ridership for the metrorail network. Thus, the network global efficiency herein is computed based on the geodesic path between any pair of nodes and the sum of weights assigned to links of corresponding geodesic paths for all possible pairs of nodes.

The presented methodology addresses shortcomings in previous works as provided in the introduction that incorporated only the weighted adjacency matrix in their algorithm to calculate the geodesic paths and other metrorail network characteristics. While those models could work for the network that includes positive weights, it would fail to consider negative weights. Typically, the algorithms available for calculating topological network characteristics fail to integrate matrices with negative entities in calculating geodesic paths. Furthermore, including the weighted adjacency matrix as a direct input for calculating the geodesic path may not lead to achieving realistic results and may disorder the definition of geodesic path. Deliberating the modified definition of network efficiency herein could consider the negative weights in generic cases. Negative weights may not occur in the case of metro networks as provided in the case study; however, in general, negative values are possible. For example, in social studies of acquaintance networks when considering animosity among individuals, negative weight values are possible and their use is appropriate (Newman 2004).

Methods proposed in this chapter are not limited to the specific domain, e.g., metrorail networks, and could be generalized for any domain that has network paradigms. However, for the purpose of illustrating methodology in a structured manner, it is rendered through a metrorail network domain. Its application for a metrorail network can reflect not only connectivity in an applied way but also ridership, which are the two critical aspects of the network.

Network vulnerability for the case study of choice, Washington D.C. Metro, was assessed by considering two circumstances, i.e., the metro network subjected to node removal and link removal. Both cases demonstrate that the most critical components of the Washington D.C. Metro belong to the central part of the city as well as the northwest section of the Red Line. The results also demonstrate that some stations and links in the metrorail that carry a larger number of passengers compared to other stations and links ranked among the most vulnerable components of the metrorail. However, it is not a typical paradigm. More specifically, on the basis of Washington D.C. Metro network analysis, the removal of a link on the combined Blue-Orange-

Silver Line in the central part of the city with moderate passenger flow would negatively impact the total metrorail performance rather than removing a link on the single but heavily traveled Red Line; or removal of a station on the combined Green-Red-Yellow Line located in the central part of the city would drastically decrease vulnerability of the metro rather than the Union Station located on the Red Line with larger volume of passengers. Hence, considering only the weight factor in obtaining critical components is insufficient. Node degree and the positions of stations or links determine the contributions of components to the whole network's connectivity, have also their own impacts. The rankings according to criticality of stations and links is a combination result of the link position, station position, and the ridership of the network. Therefore, in the vulnerability assessment, location is as important as ridership and sometimes more influential. It is proposed that during the planning, construction, and operation of the metrorail transit network, great attention should be paid to the management and protection of these vulnerable positions. Accordingly, in future development, enhancement through risk reduction should be focused on the Red Line located in the northwest of the city as well as segments located in the central part of the city.

Identifying the most vulnerable stations, tunnels, bridges, above-grounds and any other segments of a metro network provides insights to enhance metrorail safety, reduce the susceptibility of the vulnerable components, improve the resilience of the whole metro network and, ultimately, results in economic savings. Also, the results offer a basis to manage network operations, reconstitute ridership after an incident, and monitor vulnerable elements, which are significant for global network connectivity and/or carry a relatively larger ridership.

Chapter 6: Resilience-Based Strategies for Topology Enhancement and Recovery of Metrorail Transit Networks*

6.1 Introduction

This dissertation provides a structured methodology to assess a metro network's vulnerability and develops a resilience metric by utilizing the Washington D.C. Metro as a case study. Chapter 5, in particular, offers strategies for enhancing its resilience prior to any network failure as a planning activity or post failure as a recovery activity. On this basis, the contribution/novelty of this chapter could be expressed as follows:

- Improving the resilience of the network prior to failure through topology enhancement such as adding a loop line to the network and using the results as a basis for justifying capital improvement projects.
- Assessing the post failure recovery strategy based on minimizing performance loss in the network and identifying the best recovery sequences

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* **Paper published or submitted**

Saadat, Y., Ayyub, B.M., Zhang, Y., Zhang, D., Huang, H., (2020). "resilience-based Strategies for Topology Enhancement and Recovery of Metrorail Transit Networks." ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, DOI: 10.1061/AJRUA6.000105.

Saadat, Y., Zhang, Y., Zhang, D., Ayyub, B.M., Huang, H. (2018). "Post-Failure Recovery Strategies for Metrorail Transit Networks with Washington D.C. as a Case Study." International Mechanical engineering congress & Exposition, PA, U.S.

not only in the event of multiple station failures, but also in the event of multiple link failures.

- Providing a comprehensive cost-benefit model, proposing a cost breakdown hierarchy in the instance of a metro disruptive event and considering cost analysis as a complimentary work for post-recovery strategy.

6.2 A Proposed Methodology for Increasing Resilience of Metro Transit Network through topology enhancements and recovery strategies.

The strategies offered in this chapter to enhance resilience before failure is through network topology enhancement such as adding a loop line in the network. Adding a loop line creates redundancy for critical components of the metro and leads to significantly reducing the vulnerability of the network. However, a post failure state shifts the attention to employing effective recovery strategies based on minimizing performance loss. An effective recovery strategy is defined by identifying the recovery sequence of nodes or links that maximizes the resilience while taking into consideration the minimization of the recovery cost. Effective post-failure recovery strategies are defined by recovery sequences and recovery costs. The methodology herein is shown in Fig. 6.1 and consists of the following steps:

1. Defining a system as a network by its nodes, links and a pattern of connectivity;
2. Analyzing the network topology and computing the network characteristic indicators, i.e., node degree, average node degree, adjacency matrix and characteristic path length;

3. Assessing the network efficiency, which is the basis of measuring its vulnerability;

4. Evaluating the resilience metric as an index for the metro system;

5. Enhancing the topology of the network by individually examining several alternate loop lines as capital improvements to the network through creating redundancy to vulnerable components and comparing the states before and after adding a loop; and

6. Identifying effective recovery strategies on the basis of minimizing the performance loss during and after a disruptive event and minimizing the total cost associated.

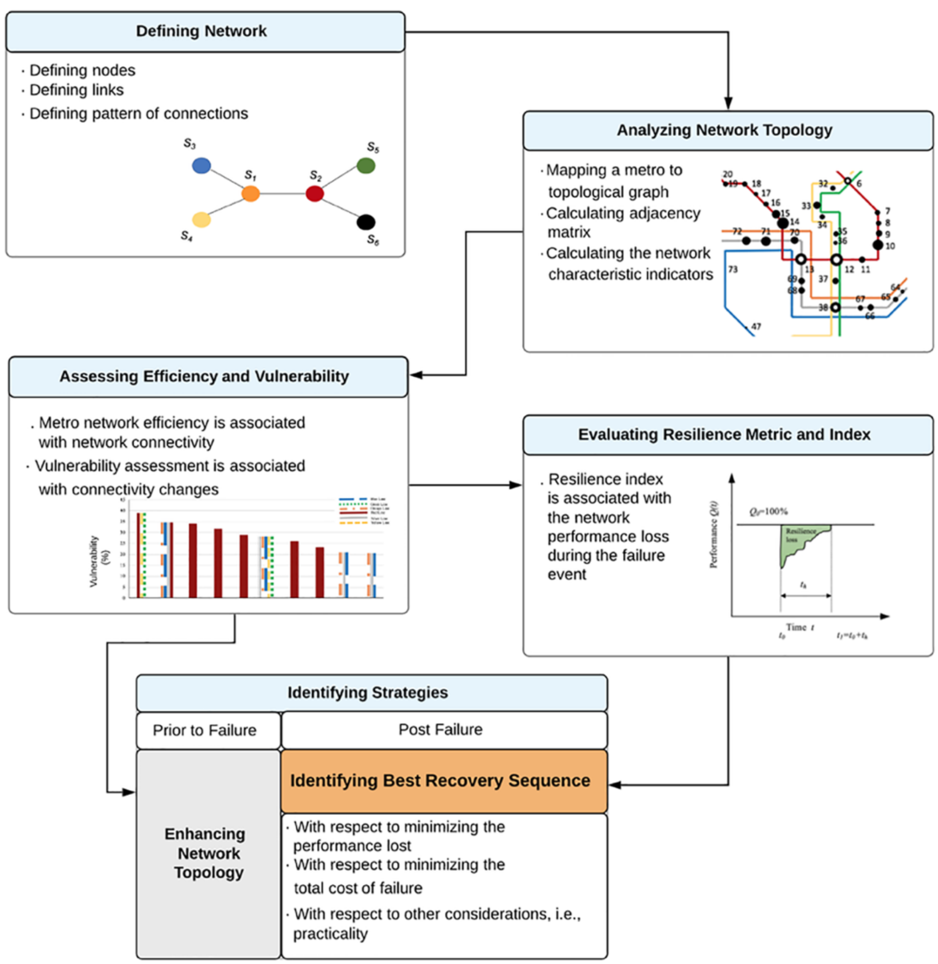


Figure 5.1. A proposed methodology for increasing resilience of metro transit network through topology enhancements and recovery strategies.

6.2.1 Enhancing metro topology by capital improvement

Topology enhancements offer a basis to create redundancy for vulnerable components of a network by capital improvement that may include loop lines through the network core. This section examines vulnerability changes of critical components after added loop lines and compares network efficiencies for several proposed loop lines. These loop lines are expected to not only decrease the network vulnerability but also allow passengers alternate tracks in cases of maintenance or emergency. They could increase the network connectivity by decreasing the metro characteristic path length. Additionally, when a transfer station fails in the network, a loop line offers alternate connection for the primary metro lines. Accordingly, this section demonstrates concepts introduced using the Washington D.C. Metro's critical components by adding three hypothetical loop line options. For each loop line, a new metro map was generated. Existing stations were examined with respect to their passenger flow and proximity to major job centers of the city to determine which stations are the proper location for a loop line to overlay through them in a way that enhance redundancy for critical components of the network. The global network efficiency and network vulnerability were recalculated. Comparisons were made between the original network and each network with an added loop line. The analytical model is laid out in detail in the rest of this section.

Figures 6.2a-6.2c show three metro maps of Washington D.C. each with a hypothetical loop line assumed as dotted lines.

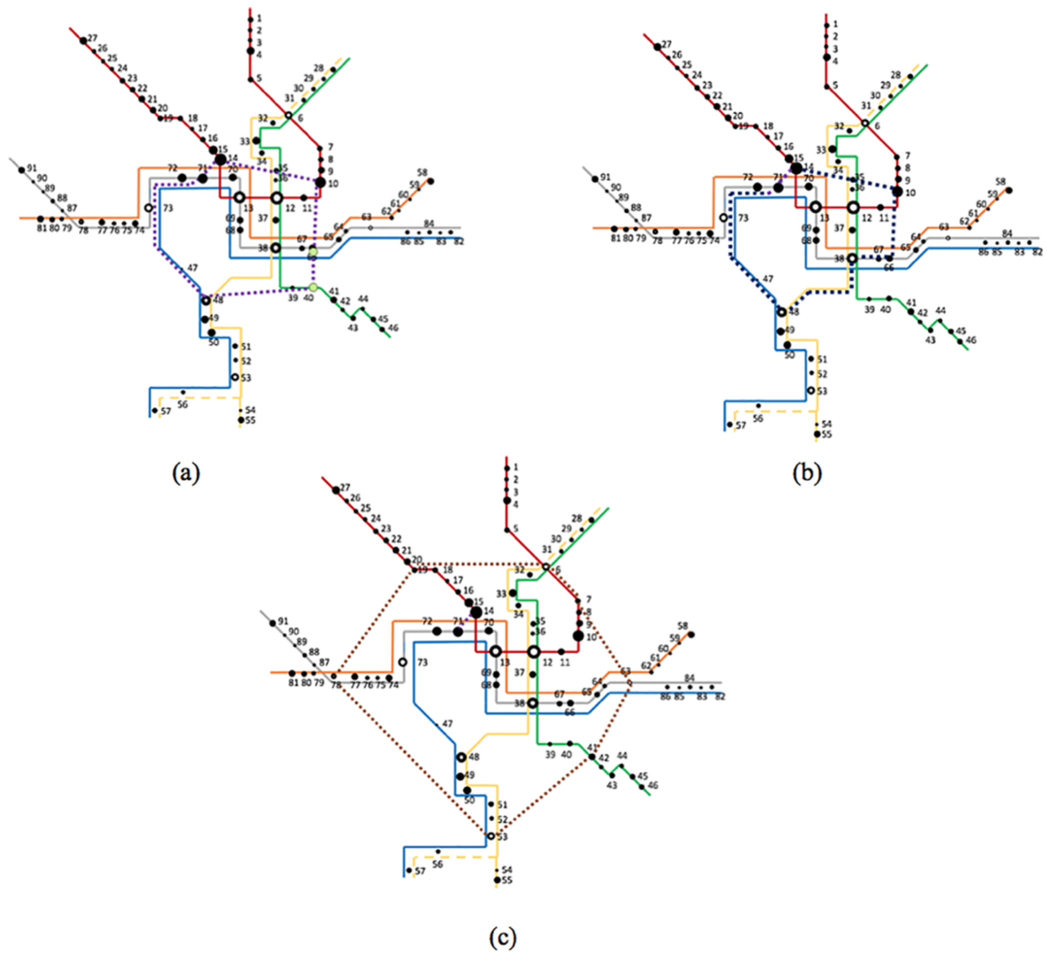


Figure 6.2. Three hypothetical loop lines added to Washington D.C. Metro as: (a) Loop line 1; (b) Loop line 2; (c) Loop line 3.

The first loop line in Fig. 6.2a connects existing metro lines together and passes through existing stations 10, 35, 14, 71, 72, 73, 47, 48, 39, 40 and 66. It increased the number of physical metro links by 12 regardless of how lines sharing the same tracks are taken into account. However, if they are considered, they are treated as a single link and the number of total links added to the network is equal to 6. The second loop line in Fig. 6.2b is a modified version of the first loop line by connecting stations 10, 35, 14, 71, 72, 73, 47, 48, 38, 67, and 66. It added 4 new separate links into the network that are not sharing the same tracks with other lines and mostly passes through exciting tunnel lines for the purpose of cost effectiveness. The stations above are chosen because they are in the core of the city around the most vulnerable components of the network and, referring to Fig. 6.3, they have significant passenger flow. For example, station 10 is *Union* station and according to Washington Metropolitan Area Transit Authority (WMATA 2016), it carries 29371 passengers on a daily basis, which is a relatively large number. Its location is strategically important in the downtown area, close to historical places, a block from the U.S. Capitol, and neighboring many leisure destinations in the city. Also, as a major train station and transportation hub, it is one of the most important stations that loop lines 1 and 2 pass through. The major difference between the first and second loop lines is the link between station 48 and 49, which would be an actual bridge built over the Potomac River and it may induce substantial construction costs to the network. The third loop line , i.e., loop line 3, passes through stations 6, 7, 32, 18,19,78, 53,41, 63 and inserts six additional links to the original network as shown in Fig. 6c. Loop line 3 is a longer interloop and enfolds a larger portion of the city. It also connects the four different quadrants of the city together. The stations for loop line 3 are chosen based on the passenger flow, location and access to important city's assets. For example, station 41 is Amtrak's Washington D.C. Train station in addition to the metro station. Loop line 3 is relatively lengthy and has the capacity to add some new stations into it. This paper does not designate additional stations on the loop lines for the purpose of reducing analytical difficulties. Adding such stations could be an option and may be investigated in future studies.

For the three loop lines, adjacency matrices were regenerated. The size of the adjacency matrix remains the same as the original adjacency matrix while its entries change. By using the new adjacency matrices in the modified Floyd algorithm (1962), all new geodesic paths between any pair of nodes were derived. Using Table 3.3 information and Eq. (3.15), characteristic path length and global network efficiency for each network were calculated, respectively, which are shown in Table 6.1. Accordingly, global network efficiency for all three loop lines is increased and the greatest measure is associated with metro network embracing loop line 3, which has the shortest characteristic path length. The global efficiency in the original network is 0.1432 while in the network with loop line 3 is 0.1918, with an increase of 33.9 %. Characteristic path length in the network with loop line 3 significantly drops from 11.50 to 6.94. Nevertheless, the efficiency changes in the metro with loop lines 1 and 2 are not as great as the one with loop line 3, and though they are less than ten percent, they still play important roles in the metro connectivity. Figures 6.3a-6.3b show the importance of loop line 1 as an example and how it assists to maintain the connectivity after failure of node 38, i.e., *L' Enfant plaza*. In Fig. 6.3a, node 38 is failed and east/south east parts of the metro are disconnected from the rest. Figure 6.3b shows that while node 38 is in a failed state, loop line 1 connects the four quadrants of the city.

Table 6.1 The network efficiency for the original network and for the network with each loop added

Rank order with respect to E_G	Network with different topology	Network global efficiency	Characteristic path length
1	Original network	0.1432	11.50
2	Network with loop line 1	0.1554	10.11
3	Network with loop line 2	0.1514	10.37
4	Network with loop line 3	0.1918	6.94

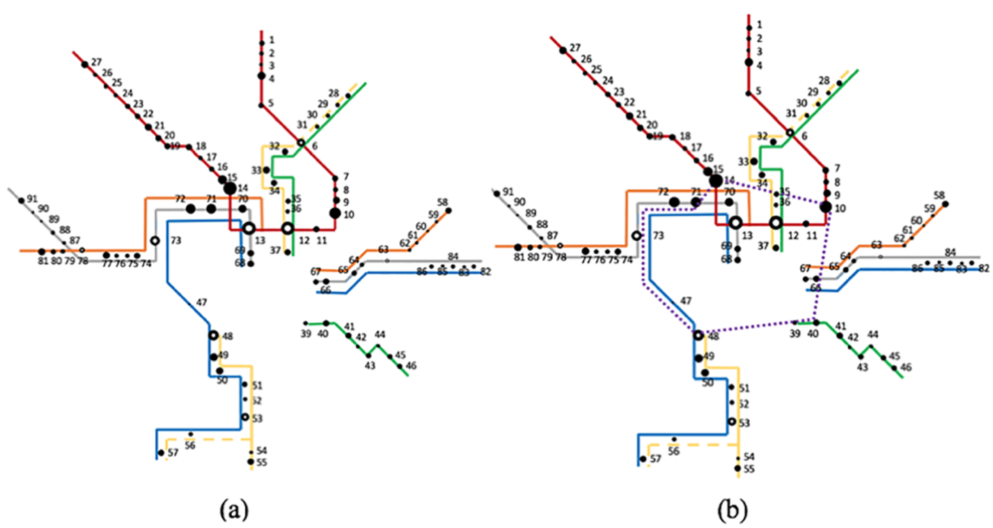


Figure 6.3. Comparison between metro connectivity after failure of node 38: (a) Original map after failure; (b) Map with added loop line 1 after failure.

Network efficiency is essential for vulnerability evaluation for both cases of one-at-a-time node and link failures using Eqs. (4.1a) and (4.1b). Table. 6.2 shows the vulnerability assessment results for the different networks in the case of node failures. Figure 6.4 display the vulnerability profile comparison of those stations.

Table 6.3 and Fig. 6.5 show the result in the case of link failures. The links are shown with numbers at their two ends. Some of them are from a single line, however, most of them are from multiple lines sharing the same tracks.

Table 6.2 Vulnerability of the ten most critical stations in the Washington D.C. Metro network for different network topologies

			Vulnerability Magnitude (%)			
Rank order with respect to V	Name of the station	Station's number	Original network	Network with loop line 1	Network with loop line 2	Network with loop line 3
1	L' Enfant plaza	38	29.78	5.17	16.3	6.43
2	Gallery Place	12	23.42	3.75	3.87	4.71
3	Metro Center	13	17.43	3.98	4.09	4.34
4	Federal Center SW	67	16.70	3.24	4.75	2.88
5	Pentagon	48	16.22	14.78	14.57	4.01
6	Rosslyn	73	15.30	13.13	15.28	3.26
7	Capital South	66	15.09	19.23	17.82	2.54
8	Farragut North	14	14.23	17.07	17.39	2.85
9	Eastern Market	65	13.79	15.39	15.31	2.74
10	Court House	74	13.36	13.13	13.28	2.70

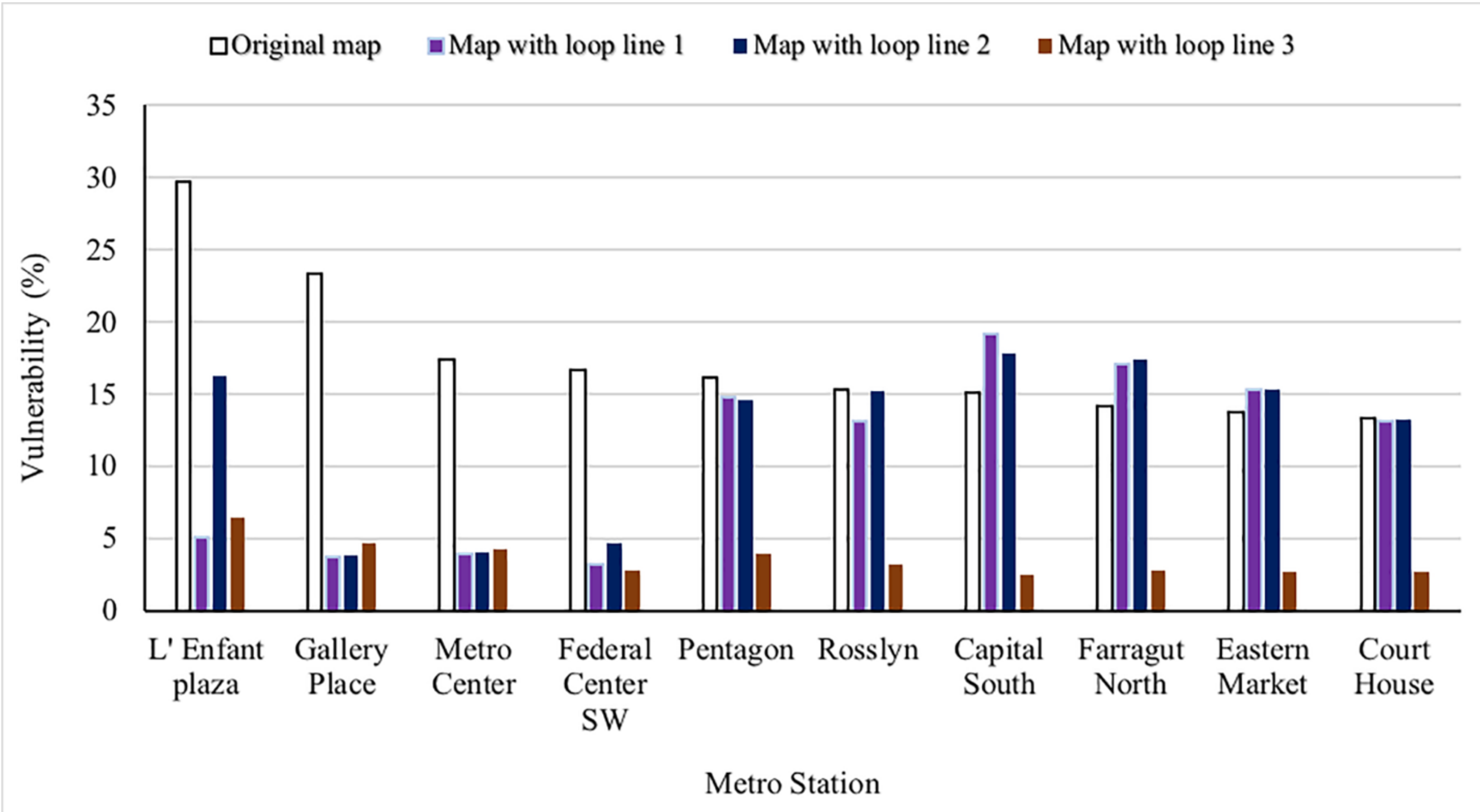


Figure 6.4. Comparison of the ten most vulnerable stations in the weighted Washington D.C. Metro for different metro topologies.

Table 6.3 Vulnerability of the ten most critical links in the Washington D.C Metro network for different network topologies

			Vulnerability Magnitude (%)			
Rank order with respect to V	Link	Color-coded lines	Original network	Network with loop line 1	Network with loop line 2	Network with loop line 3
1	(13,69)	Blue-Orange-Silver	16.07	0.74	0.70	0.52
2	(68,69)	Blue-Orange-Silver	14.42	0.52	0.46	0.45
3	(38,68)	Blue-Orange-Silver	13.14	0.96	0.90	0.93
4	(73,74)	Orange-Silver	12.78	12.48	12.61	1.14
5	(38,67)	Blue-Orange-Silver	12.07	0.00	0.00	0.00
6	(13,14)	Red	11.71	0.67	0.68	0.00
7	(48, 49)	Blue-Yellow	11.57	10.70	10.73	1.05
8	(74,75)	Orange-Silver	11.50	11.19	11.32	1.11
9	(63,64)	Blue-Orange-Silver	11.14	11.95	11.95	0.80
10	(75,76)	Orange-Silver	10.42	10.10	10.22	1.40

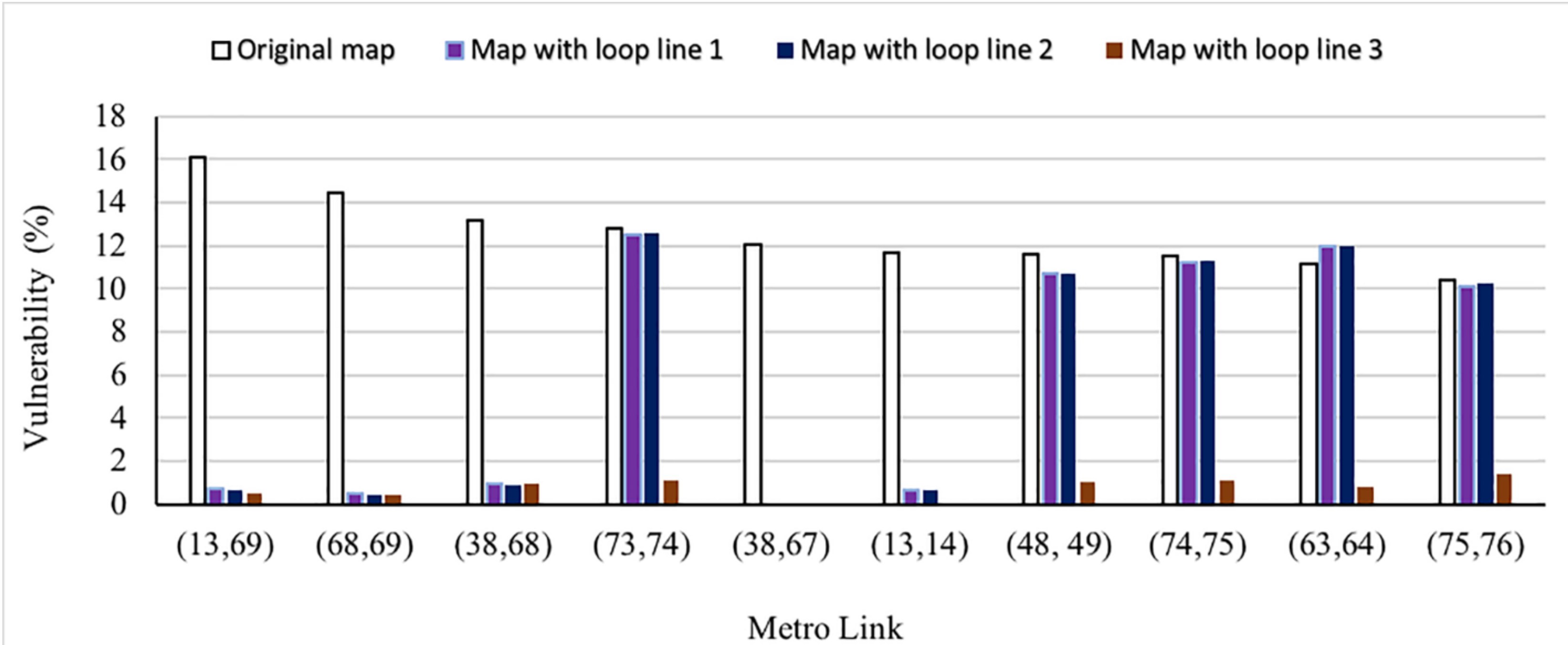


Figure 6.5. Comparison of the ten most vulnerable links in the weighted Washington D.C. Metro for different metro topologies.

With merely focusing on the vulnerability assessment, results demonstrate that inserting an appropriate loop line could significantly decrease the vulnerability of critical network components and enhance the network connectivity. For instance, loop line 1 notably decreases the vulnerability of the *L' Enfant Plaza* station from 29.78% to 5.17%. Loop line 3 reduces the vulnerability of aforementioned station to 6.43%. Although this change is not significant in the case of adding loop line 2, it is still meaningful. While loop lines 1 and 2 decrease the vulnerability of most critical stations, they expectedly have a slight impact on the vulnerability rise of stations, *Capitol South*, i.e., 66, *Farragut North*, i.e., 14, and *Eastern Market*, i.e., 65. After adding loop lines 1 and 2, stations 66 and 14 convert to transfer stations and station 65 would be connected to new transfer station, i.e., *Capitol South.* Transfer stations and stations that are connected to them usually ranked among most critical network components. Loop lines 1 and 2 reduce the vulnerability of all links, however, changes range from 15.96% to almost zero, noting that some cases have insignificant negative impacts. Loop line 3 considerably drops the vulnerability of all the 10 most critical stations in the network. Also, it induces an enormous positive impact on the vulnerability of most critical links.

All loop lines improve the most critical components of the network, affect the other network components and change their vulnerability ranking. If all components are considered with vulnerabilities assessed and still the whole network vulnerability is reduced, adding a loop line is promising. Table 6.4 summarizes the whole network vulnerabilities and their associated most vulnerable stations for the original metro network and networks with added loop lines.

Table 6.4 Vulnerability summary for all alternative Washington D.C Metro network with different network topologies

Rank order with respect to V	Network with different topology	Name of the Station	Station number	Network Vulnerability (%)
1	Original network	L' Enfant plaza	38	29.78
2	Network with loop line 1	Capital South	66	19.23
3	Network with loop line 2	Capital South	66	17.82
4	Network with loop line 3	Stadium-Armory	63	19.99

This chapter investigates three hypothetical loop lines for the purpose of illustration. Obviously, many other loop lines could be explored. The number of loop lines, adding or removing stations within them, changing the location of the loop lines, weighing construction cost and many other factors could change the preferences. In fact, deciding on the proper alternative is a typical trade-off problem. Identifying the optimum alternate loop line(s) requires methods of optimization and cost-benefit analysis which is beyond the scope of this paper.

6.2.2 Identifying effective recovery sequence

Following a failure, adopting appropriate recovery strategy could play an important role in the network resilience restoration. The best recovery strategy depends not only on the recovery sequence, but also the recovery cost. Parameters like practicality, legal requirements, risk management, risk informed decision, etc., could impact the sequence of recovery selected. This paper suggests steps needed to identify an appropriate recovery sequence based on the key points and metrics related to resilience restoration and recovery cost.

6.2.1.1. Recovery sequence preferences with respect to resilience restoration

Henry and Ramiz-Marquez (2012) discussed the importance of a sequential recovery strategy in the events that connectivity of the network need to be restored to the initial state. In the event of a transfer station failure or failure in multi-links, the sequential recover strategy is most effective. Assuming that only one component of a network can be repaired at a time, it is important to capture which sequential recovery provides the highest resilience restoration of the metro network. Thus, the network resilience during restoration for the following three cases using the Washington D.C. Metro is evaluated:

- Identifying the appropriate recovery sequence for a single disrupted metro station;

- Identifying the appropriate recovery sequence for multiple disrupted metro stations (i.e., the disruption of multiple stations with same node degree and the failure of multiple stations with a different node degree); and
- Identifying the appropriate recovery sequence for multiple disrupted metro links.

Each of these cases are expanded upon in the rest of this section assuming a disruptive event leads to full failure in the affected metro network station and links. Also, other transportation modes, such as busses, are not considered as elimination of connectivity interruption in such cases of disruptions.

In the first case, a single metro station is disrupted. With reference to Table 4.1, the most vulnerable station of the Washington D.C. Metro, which plays a significant role in the metro connectivity, is the *L' Enfant Plaza* station. It is shown by node 38 in the map and the node degree of this station is five, the highest node degree in the network. As for the node 38 shown in Fig. 6.6, in total, three lines, i.e., line 1, 2 and 3 connect to node 38; therefore, the possible recovery sequences to fully recover the three lines through the station are equal to the permutation of 3, i.e., P (3,3) =3! = 6, and each recovery sequence has a corresponding resilience index. In general, the number of possible recovery sequences for *m* lines at a station is equal to permutation of *m* in P(*m*,*m*). The permutations of possible recovery sequences are denoted m_1-m_2...m_n, where the order of recovery proceeds from left to right and the m_i (*i* from 1 to

n) denotes the number assigned to each line. Resilience index is calculated using Eq. (4.3) assuming that the recovery time is constant for all six choices. Effectively, Eq. (4.3) shows the relation between R_e, E_G and the recovery time. Hence, recovery profile in terms of efficiency restoration over time are produced to calculate resilience index. In Table 6.5, the resilience index is calculated for all six possible sequences recovery of *L' Enfant Plaza* station.

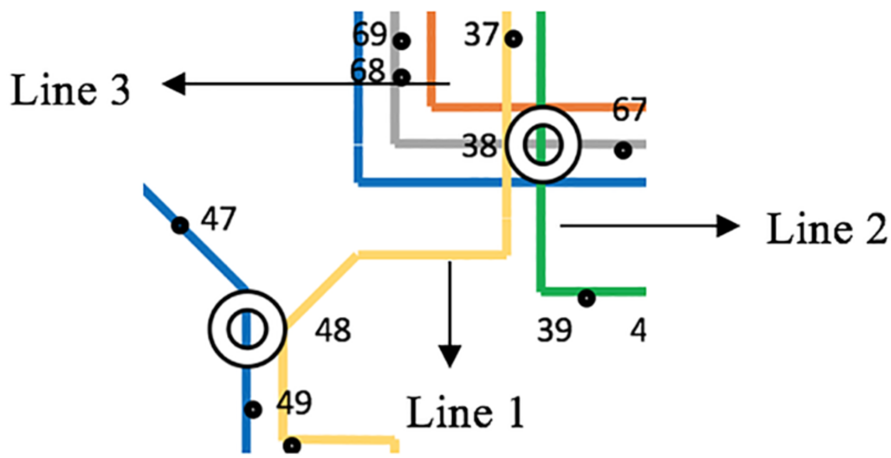


Figure 6.6. L' Enfant plaza station (node 38) and the lines1, 2 and 3 connected to this station.

Table 6.5 Resilience (R_e) for six possible recovery sequences of L' Enfant plaza station

Rank order with respect to R_e	Recovery sequence	R_e	Rank order with respect to R_e	Recovery sequence	R_e
1	2-3-1	0.865	4	2-1-3	0.831
2	3-2-1	0.858	5	1-3-2	0.815
3	3-1-2	0.835	6	1-2-3	0.796

Initial network efficiency is 0.1432 and the network connectivity after the *L' Enfant Plaza* station failure is 0.1005. The sequence desired is with the maximum value of resilience index. Table 6.5 reveals the largest R_e value is 0.865 corresponding to recovery sequence 2-3-1. Verifying that if *L' Enfant Plaza* station is disrupted, the first metro line to be recovered should be 2, followed by 3, and finally 1 to provide the highest resilience restoration. Conversely, the smallest R_e value from the table is 0.796 for the recovery sequence 1-2-3. The efficiency-time curves for sequence 2-3-1 with maximum R_e and sequence 1-2-3 with minimum R_e are shown in Fig. 6.7-a.

The proposed steps above can be used for the other two cases of multi-station and multi-link removal. The multi-station case includes two sub-cases where the stations either:

- Have different node degrees, or

- Have identical node degrees.

For sub-case one, stations *L' Enfant Plaza* (node 38) with node degree of 5 and *Fort Totten* (node 6) with node degree of 4 and *Pentagon* station (node 48) with node degree of 3 are selected to exemplify the multi-station disruption with different node degrees. Here also there are six permutations for the recover sequences; Table 5.6 shows R_e values corresponding to the six possible recovery sequences in addition to Fig. 6.7-b, which shows the efficiency-time recovery curves for best and worst recovery sequences. For sub-case two, three stations, *Fort Totten* (node 6), *Gallery Place* (node 12), and *Metro Center* (Node 13), all with identical node degrees of 4, are selected. Table 6.7 demonstrates all possible recovery sequences and their associated R_e for sub-case two of multi stations removal with same node degree. Figure 6.7-c shows efficiency-time curves for the two extreme recovery sequences in terms of R_e.

Table 6.6 Resilience (R_e) for six possible recovery sequences of multi stations disruption with different K_i

Rank order with respect to R_e	Recovery sequence	R_e	Rank order with respect to R_e	Recovery sequence	R_e
1	38-48-6	0.824	4	6-38-48	0.747
2	38-6-48	0.799	5	48-6-38	0.710
3	48-38-6	0.781	6	6-48-38	0.708

Table 6.7 Resilience (R_e) for six possible recovery sequences of multi stations disruption with same K_i

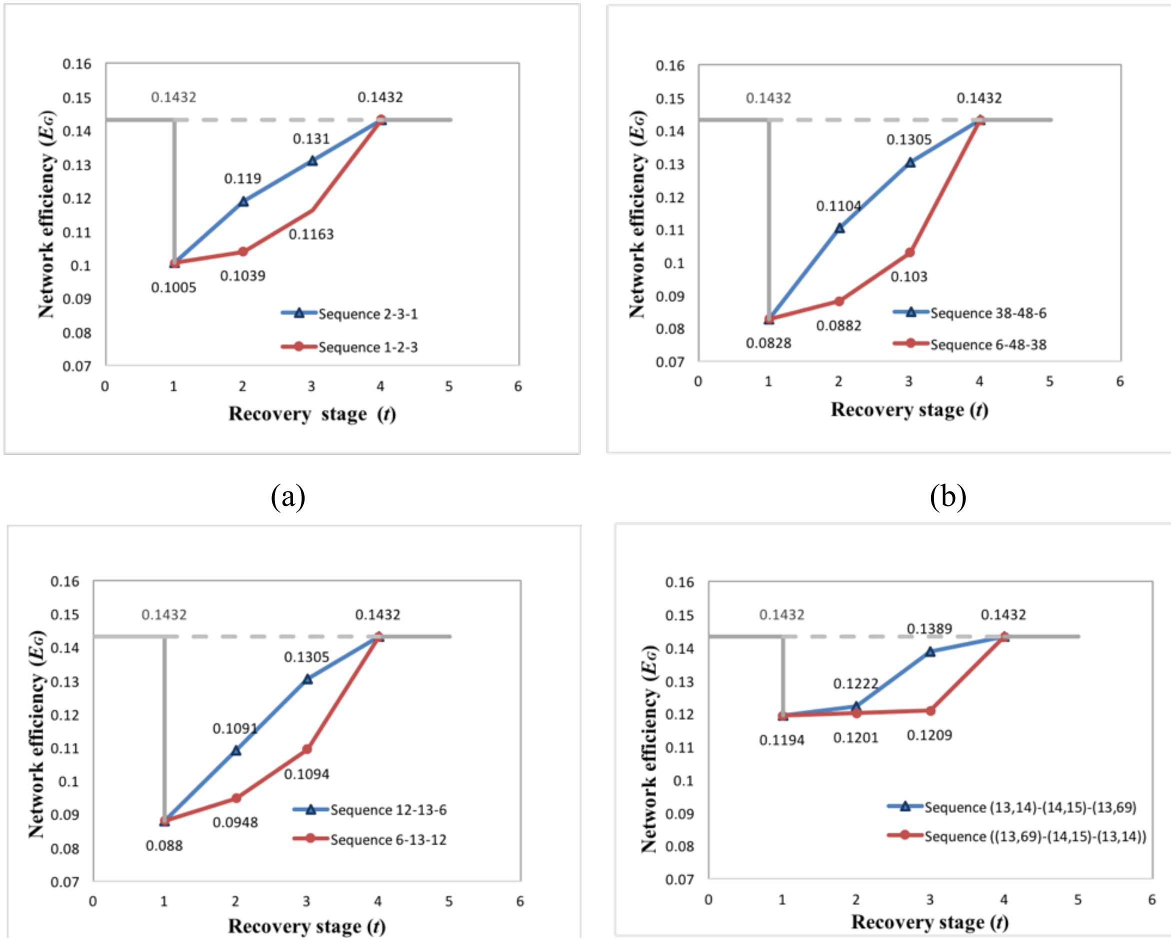
Rank order with respect to R_e	Recovery sequence	R_e	Rank order with respect to R_e	Recovery sequence	R_e
1	12-13-6	0.827	4	12-6-13	0.798
2	13-6-12	0.819	5	6-12-13	0.767
3	13-12-6	0.815	6	6-13-12	0.744

The final case investigates the best recovery sequence of multiple disrupted links. In demonstrating this case, three links of (13,69), (13,14) and (14,15) are selected. Table 6.8 demonstrates the resilience index measurements for six possible recovery sequences. Figure 6.7d also displays the *efficiency-time curves for the recovery sequences with* R_e *=0.913 and* R_e =0.866.

Table 6.8 Resilience (R_e) for six possible recovery sequences of multiple links

Rank order with respect to R_e	Recovery sequence	R_e	Rank order with respect to R_e	Recovery sequence	R_e
1	(13,14)-(14,15)-(13,69)	0.913	4	(13,69)-(13,14)-(14,15)	0.871
2	(14,15)-(13,14)-(13,69)	0.908	5	(14,15)-(13,69)-(13,14)	0.866
3	(13,14)-(13,69)-(14,15)	0.876	6	(13,69)-(14,15)-(13,14)	0.866

Several recovery stages are shown in all Figs. 6.7a-6.7 d, starting with stage 0, which represents the network in its pre-failure state with network efficiency calculated as 0.1432. Total station failure is represented in stage 1 with a sharp reduction in network efficiency. Stages 2 and 3 represent the two intermediate steps of the recovery sequence and the final stage 4 represents a full recovery. In the event of a single node failure in the network where the node degree is greater than 2, the results indicate that the choice of a sequential recovery strategy is consequential and the best recovery strategy should be identified. In the case of multi-transfer stations disruption, the sequential recovery strategy has a significant effect on the resilience restoration; however, the resilience index did not change significantly when the authors examined the sequential recovery strategy for the multi-link disruption. Recovery cost, besides recovery sequence, is another key creation for decision maker to select a recovery order. The subsequent section explores the recovery cost.



(a) (b)

(c) (d)

Figure 6.7. Comparison of network efficiency under two different recovery sequences with maximum and minimum R_e for disruption cases of: (a) One transfer station; (b) Multi stations with different Ki; (c)Multi stations with same Ki; (d) Multiple links.

6.2.1.2. Recovery sequence preferences with respect to cost effectiveness

The total cost associated with a disruptive event of a metro system includes the cost associated with the decrease in revenue due to income loss, and the cost associated with the risk to passengers and workers, i.e., life losses/injuries, in addition to property loss, i.e., repairable/none-repairable damages and subsequent repairs. The cost represented by the decrease in revenue depends on the changes in volume of passengers using the metro line or station before and after their failure. The cost represented by life losses and injuries depend on the severity of the adverse event resulting in disruptive event; the assessment of property loss cost as a result of failure could be quantified in monetary terms using analytical model, historical data and expert judgment. The total cost of a disruptive event is defined as

$$C_{total} = C_{Income\ loss} + C_{Life\ loss/injuries} + C_{Property\ loss} \tag{6.1}$$

The cost of property loss $C_{Property\ loss}$ is the cost of the implementation of repairs, and consists of direct costs, i.e., cost of repairing and replacing equipment, machinery damage, consumption of engineering materials, content and inventory damage and also salary for the workforce in addition to the cost of non-repairable damages (also termed indirect cost which is caused by social related factors and could be seen as a portion of cost of repair.) It can be represented as

$$C_{Property\ loss} = C_{Repair} \tag{6.2}$$

Incorporating C_{Repair} in Eq. (6.1) lead to C_{total} as follows:

$$C_{total} = C_{Income\ loss} + C_{Life\ loss/injuries} + C_{Repair} \tag{6.3}$$

Assumptions related to the repaired state as provided by Ayyub (2015) in terms of the restored state, such as as good as new, as good as old, etc. Following the model proposed by Henry and Ramirez-Marquez (2012), the recovery cost during the disruption $C_{Recovery}$ is sum of the income loss due to network failure, and the cost of repair.

$$C_{Recovery} = C_{Income\ loss} + C_{Repair} \tag{6.4}$$

where $C_{Income\ loss}$ typically is decreasing in revenue due to income loss of metrorail and metrobus tickets, other metro access payment, parking fees, retail incomes, diminished personnel productivity due the metro closing, damage to metro's reputation, etc.

Ridership on Washington D.C. Metro comprises approximately 92 percent of the total metro revenue (Washington D.C. Metro Fiscal Year 2018 Proposed Budget). Therefore, the income loss due to a decline in metro ridership is defined as:

$$C_{Metro\ ticket} = \beta\ VOL_{loss} \tag{6.5}$$

where VOL_{loss} is the total loss in passenger volume after a disruptive event, and β is the average metro ticket price. The passenger volume, however, is expected to return to its initial state, $VOL_{n,}$ from disablement state, VOL_d , during the recovery period, (for $t \; in \; [t_0, t_1]$). Therefore, it is the linear function of time (Zhang et al. 2017) and described as:

$$VOL_{loss} = \frac{1}{2}\,(VOL_n - VOL_d)(\,t_1 - t_0) \qquad (6.6)$$

where t_0 is the time of station disablement and t_1 is the time of completion of recovery. Although the cost associated with passenger volume has a primary role in the income loss of a metro network, the other metro attributes, i.e., decrease in revenue due to metro bus passengers and parking fees, a decline in passengers on other metro access, and a decrease in metro ridership due to damage to the metro's reputation, a decrease in personnel productivity and metro retail could also be considered as an intensifying factor. According to Washington D.C. Metro Budget report Fiscal year (2018), all other discussed elements increase $C_{Metro\ ticket}$ by multiplying $\lambda \approx 1.56$ which is not insignificant. A more comprehensive analysis requires gathering data or simulation of the dynamics of the system coupled with alternate transportation modes available to the passengers.

Repair cost C_{Repair} also is a function of recovery time, t_1, and includes all the direct and indirect cost of reconstructing or restoring the property losses. Therefore, the recovery cost becomes:

$$C_{Recovery} = \frac{1}{2}\,\lambda\beta\,(VOL_n - VOL_d).(t_1 - t_0) + C_{Repair}\,t_1 \qquad (6.7)$$

Metro repairs typically take considerable time and often recovery time does not occur equally over the time that metro components are disrupted. Thus, for cost analysis might require the use of the time value of money termed in terms of a discount rate. A discount rate determines how money at the present time has different value compared to the equivalent amount in future time. Deliberating this, any sum of money in present is worth more than the same amount in future represented by a coefficient. Hence, the $C_{Recovery}$ in this case is as follows:

$$C_{Recovery} = [\frac{1}{2}\lambda\beta\ (VOL_n - VOL_d))(t_1 - t_0) + C_{Repair}\ t_1].(\frac{1}{1+i})^{t_1} \quad (6.8)$$

where i is the discount rate; the equation to express the relationship between present value (PV) and future value (FV) of the money is as follows:

$$P = F(1 + i)^{-N} \quad (6.9)$$

where P is the present amount for a future payment, F, in N number of compounding periods, i.e., typically years.

Analyzing the total cost in this dissertation also adopted the comparable mindset although not exact method of life cycle cost analysis (LCCA) as a supporting method. According to Federal Highway Administration (FHWA 2002), life cycle cost analysis is a method to quantify the total cost of alternatives options for investment or preservation in the given project and to make economically sound decisions perhaps by selecting the lower total cost option. LCCA is a decision support method which is composed of the different following steps (TRB 2014):

- Establishing different alternatives: In this step, several options are considered to fulfil the objectives of the given projects and the economic difference between those options are associated with their total costs;
- Determining each activity timing: Each activity duration for different alternatives of the project is estimated;
- Estimating costs: The cost that demonstrates the difference between each alternative is projected;
- Computing life cycle costs: In this step, the total cost associated with each alternative in its life cycle is assessed; and
- Analyzing the results: Referring to the previous steps, the results are interpreted and the present value (PV) and inflation rate of each alternative are discussed and the cost efficient option is selected.

The different steps of LCCA are illustrated in Fig. 6.8 . LCCA provides a mean to evaluate economic effectiveness of different alternatives.

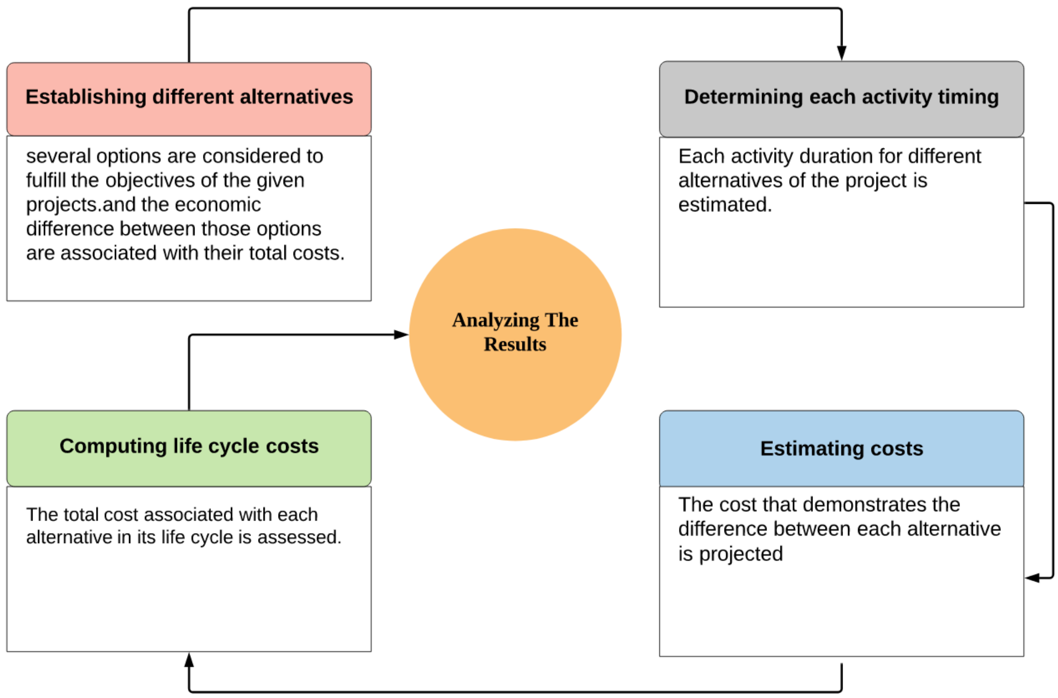


Figure 6.8. The different steps of Life cycle cost analysis LCCA.

For the purpose of illustration and interpretation of the cost model outlined above, stations 38, 12 and 6 are assumed to be disrupted due to flooding resulting in several fatalities and many injuries in each station. Figure 6.9 shows a possible cost breakdown hierarchy for the disruptive event, measuring each item’s contribution to the total cost.

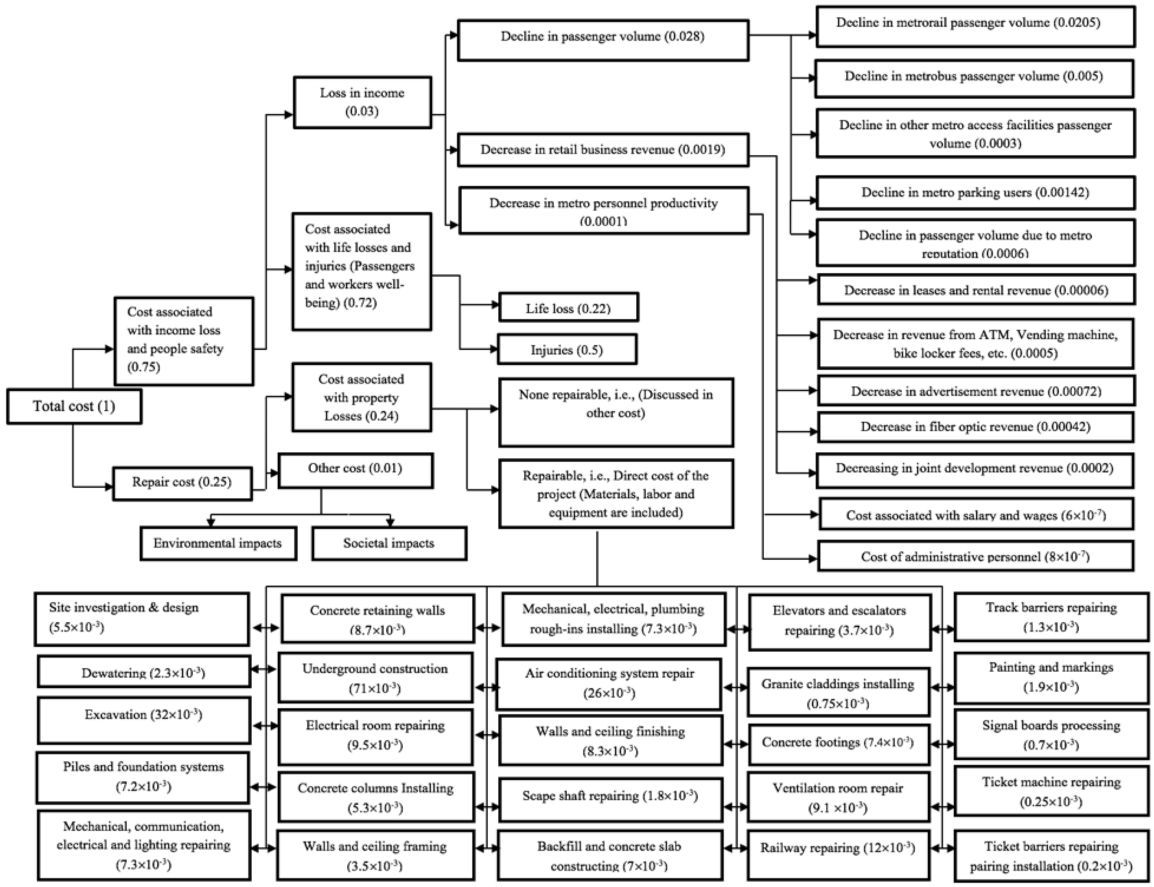


Figure 6.9. Illustrative cost breakdown hierarchy for a metro disruptive event in terms of their fractions with respect to the total cost based on the assumption of 9 fatalities and 100 Injuries in each station.

In this example, several steps of LCCA are occurred almost the same and one of the step with justified difference as follows:

- The different alternatives are available as there are three disrupted stations with similar level of services and need to be recovered;

- The duration for recovering each alternative is estimated and the duration of each activity for different alternatives are easy to determine. The period of time to assess the cost differences is not the same for alternatives, however, it is assumed to be a factor of each other's. Hence the results can be fairly compared;
- The cost that demonstrates the difference between each alternative is determined;
- Although sufficient data to assess the total cost of each alternative's life-cycle in the cost model of this study is not available, the total cost associated with disruptive events could be computed. The total life cycle cost consists of initial cost, service cost, maintenance cost, operation cost and disposal cost. The total cost associated with a disruptive event, however, includes the repair cost, cost associated with life loss injuries and decrease in revenue due to income loss. Based on assumption made for each project, the total cost associated with a disruptive event could be comparable with the cost associated with service, maintenance and operation which are the significant portion of total life cycle cost of each alternative. In this dissertation, the project is metro network and the network components should be preserved for many long-term service life. Therefore, it is reasonable to substitute computing the total cost associated with a disruptive event with computing life cycle costs in the steps outlined for LCCA concluding LCCA supports the cost model presented in this dissertation; and

- The results could be feasibly analyzed and compared to select the cost efficient alternative.

To analyze the total cost associated with disruptive event for each alternative (here each disrupted station), having realistic values for fatality and injury counts require making many other assumptions on the source and characteristic of the flooding event, time of the day, passenger population attributes, emergency response, other conditions beyond the stations, etc. Therefore, fatality and injury counts are indeterminate in realistic terms, and in order to illustrate the cost model, the values of 9 fatalities and 100 injuries for each station are used as an demonstrative case of an extreme scenario.

The U. S. Department of Transportation (DOT) regulation policy which includes rail roads, defines a Value of Statistical Life (VSL) of $9.6 million in 2016 US$ per fatality and compensation for injuries ranged from 3% to 60 % with a mean of 20% of VSL based on injuries severity (DOT 2016). Thus, this policy is used as a basis in quantifying the life losses and injuries in this paper and reflected in Fig. 6.8. The measurements of income losses are according to Washington D.C. Metro Budget report Fiscal year (2018), and the rough estimates of repair cost break down are based on communication with the operations manager of the FH Paschen company (2018), executing part of the Washington D.C. Metro construction. Considering variables such as the methods and equipment used in the project, as well as the construction crew, and also indirect costs may affect the repair cost and duration.

Repair cost and duration are different for the aforementioned stations. In this particular example, we assume the repair duration for the station 38 is t_1, for the station 12 is t_2 and for the station 6 is t_3. Based on further communication with the operations manager of the FH Paschen company (2019), the repair cost and duration of station 12 and 6 are roughly 0.7 and 0.4 of station 12, respectively. Income loss depends on the number of passengers in each station. For example, the ticket cost at *L' Enfant Plaza* station and using Eqs. (6.5) and (6.6) and factoring in the average ticket prices, $\beta = \$3.25$ (in 2018), the recovery cost is estimated to be 3.25(20394)t_1 = 0.66x10^5 t_1. Taking λ=1.56, then cost is equal to 0.103x10^6 t_1. Additionally, the income loss contribution to the total cost according to Fig. 6.8 is 0.03, thus the total cost is 3.45x10^6 t_1 and the recovery cost is calculated by deducting life losses and injury costs from the total cost. Thus, the recovery cost for the station 38 (*L' Enfant Plaza* station) is $C_{Recovery}$= 0.28xC_{total}= 0.96x$10^6 t_1$. Accordingly, the total cost for station 12 and 6 would be 3.79x10^6 t_2 and 1.48x10^6 t_3, respectively. Considering the relationship between t_1, t_2 and t_3, the recovery and total costs for the three stations, referencing Fig. 6.8 cost details, are displayed in Table 6.9 and Fig. 6.10.

Table 6.9 Recovery cost and total cost for stations L' Enfant plaza, Pentagon, Fort Totten

Station	Daily Passenger volume	C_{Recovery}	C_{total}
L' Enfant plaza (38)	20235	$0.966*10^6\,t_1$	$3.45*10^6\,t_1$
Gallery Place (12)	22427	$0.745*10^6\,t_1$	$2.65*10^6\,t_1$
Fort Totten (6)	8030	$0.165*10^6\,t_1$	$0.592*10^6\,t_1$

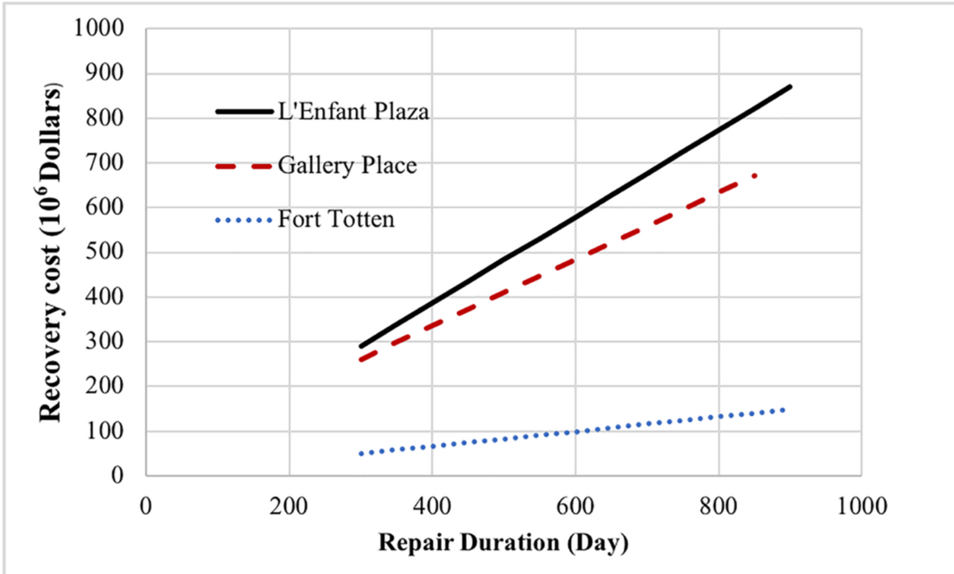


Figure 6.10. Comparison of time-recovery cost for restoring three stations in Washington D. C. Metro.

The example outlined above indicates identifying an appropriate recovery strategy depends not only on the recovery sequence and associated resilience, but also on the recovery cost. The recovery cost itself depends on recovery time t_I that is the dominant variables. Therefore, minimizing the recovery cost requires reducing the duration of recovery time, which basically is the repair duration. The repair cost in this paper is assumed to be different for the three stations considering their repair durations have linear relationship with their repair costs. However, looking further into the repair cost for each station with different sequences, construction methods, different work crews and materials, different direct and indirect cost would modify the relationship between repair cost and repair time. Previously stated consideration such as methods, equipment, work crew, accessibility and etc., could even change the linear relationship between repair cost and repair time and convert it to a nonmonotonic and nonlinear function, which recognizing the optimum recovery time requires solving a time-cost trade off problem. Thus, a complete view regarding repair cost and duration of each station requires data and the use of optimization. Such pursuits may be undertaken in future works.

6.3 Conclusions and Contributions

A metro network is a critical components of interconnected infrastructure systems. Many attempts have been made to increase the ability of metro networks to adapt to the future changing environmental or occasional conditions leading to disruptive events; yet, development and enhancement through risk reduction are required to improve metro network resilience. To rise to this challenge, this chapter provides a methodology for examining comparative strategies prior to or post failure by a disruptive event through topology enhancement, or recovery sequence and cost identification, respectively.

Chapter 6 uses the CNT method to illustrate the methodology and associated analytics for assessing the efficiency and vulnerabilities of the metro network. Accordingly, a topological network of nodes and links constitutes the basic framework of a metro network as a foundational step for risk and resilience analysis.

An efficiency and vulnerability assessment identifies the most critical components of the network that need to be enhanced.

In this basis, this chapter provides strategies to improve the network resilience prior to failure given the topology enhancement through capital improvement. The chapter examines implanting three loop line alternatives in the network with each inserted based on subjective considerations, e.g., location criticality, passenger flow, locational connectedness, etc. Results show the network vulnerability could be reduced significantly by adding a loop line. In some components, inserting one loop line could reduce the vulnerability by 24.6%, i.e., an influence of loop line 1 on *L′ Enfant Plaza* station.

Efficiency assessment also provides the platform to develop the resilience metric that is the basis of the recovery analysis for the stages that follow a failure. For this purpose, this chapter provides steps to include the analysis of recovery and associated costs. The chapter comparatively ranks the recovery sequences based on resilience restoration and illustrates the cost model through an example to show that identifying the most appropriate recovery strategies based on only resilience restoration is insufficient. A recovery sequence that is cost effective is desirable.

It should be pointed out that the metro network in chapter 5 assumed to be unweighted and undirected and all analyses are based on these assumptions. Although, this simplification method is practiced as an accepted and widely used method of assessment in network literatures, the better evaluation would be to consider weight on network components. Thus, several factors such as — physical length of each link, travel time, passenger flow, train volume and etc. — would be reflected as weight on the network components and form a weighted network. Such an assessment could be explored in future studies.

In addition, the comparative recovery analysis method presented in this chapter does not provide all the details necessary for practitioners, rather, it is intended to provide guidance to planners and decision-makers for selecting appropriate recovery sequence. Cost-time trade off analysis and optimizations are necessary for this purpose. Also, this chapter does not claim that adding a loop line is the most efficient way of enhancing the network resilience, yet preliminary results herein offer some insights to inform planners and decision-makers in managing resilience and risks.

Chapter 7: Conclusions and Future Directions

7.1. Conclusions and Findings

This dissertation offers a methodology and presents a series of efforts to enhance the resilience of infrastructure networks.

7.1.1 Enhancing the Network Topological Analysis

Chapter 3 studies the network topological analysis. Derrible and Kennedy (2010), indicated that an efficient approach to characterize the safety and robustness of the network is through topological analysis. Studying the size and complexity of the network indeed are fundamental steps to analyzing its topology and basis to determine metrics associated with the resilience of networks and the resilience attributes. Thus, Chapter 3 serves to define the infrastructure system of choice, Washington D.C. Metro rail transit, as a network paradigm; identify the network form and the components that shape the network, and to provide a new probabilistic methods to investigate the topological characteristic of the network and enhance the topological analysis. The results show that Washington D.C. Metro is an L-shaped network, contains 91 stations (nodes) and 140 links, and demonstrates the presence of small-world and scale-free.

7.1.2 Investigating Different Failure cases in the Network and Their Impacts on the Network Resilience

Failure analysis is the basis to assess the vulnerability of the network and to develop the resilience metric. Different failure cases may alter the network vulnerability and robustness not with the similar impact. For example, failing of different components of a network, i.e., node, link, may lead to a distinct response in the whole state of the network. Thus, as presented in Chapter 4 the failure analysis is implemented to explore the changes in the response that the network may have for each failure case. A set of analytical and computational work has been performed to evaluate the vulnerability of the Washington D.C. Metro using method of connectivity changes due to either node removal or link removal, one at a time for each case. The node removal identified the most critical stations are located in the downtown area. Furthermore, the link removal identified the most critical links, are positioned in the multi color-coded lines responsible for connecting east, west and south of Washington D.C. to the downtown area, as well as the Red and Green metro lines connecting the north-west quadrant of the city and the south-east quadrant of the city to the downtown area. Also, for both cases, the connectivity changes between the initial network connectivity (i.e., prior to any failures) and network connectivity measures after failure along with the resilience triangle concept are used to develop a metric of network resilience.

7.1.3 Considering the Network Dynamic

Network resilience depends on both network topology and network dynamic. The network dynamic or the flow of the network is referencing the weights that the network components carry. The dominant weight in the metro network is the passenger flow. Since counting the number of passengers who travel on each link is a challenging task, a practical, mathematical model is proposed in Chapter 5 to anticipate link ridership based on the passenger volume of the stations. This method could be applied to estimate the weight strengths on the network links without changing the adjacency matrix and without posing higher computational burden. Any perturbation applied to the network could change the connectivity of the network topology as well as disrupt the network flow. Chapter 5 also proposes a novel algorithm to consider the network flow in the network global efficiency calculation and to reflect it on the resilience assessment. In order to calculate the global network efficiency along other resilience metrics, the topological connectivity for an unweighted network is analyzed in order to achieve a geodesic path between any two stations and also combined with the use of ridership data to construct a weighted network ─in terms of the summation of link ridership weights on each geodesic path─ to incorporate them into the global network efficiency equation. A modified network global efficiency is also derived to reflect the network flow. The algorithm proposed in Chapter 5 is precise, straightforward and sensible. It leads to affordable and quick computational work, and also addresses some shortcomings in other methods of weighted network analyses, such as inability to calculate the negative weights and also disordering the definition of geodesic path.

The algorithm is used to evaluate most components of the weighted Washington D.C. Metro by assessing the vulnerability of the network in the cases of node and link removals. Both cases demonstrate that the most critical components of the Washington D.C. Metro belong to the central part of the city as well as the northwest section of the Red Line. The results also demonstrate that some stations and links in the metrorail that carry a larger number of passengers compared to other stations and links ranked among the most vulnerable components of the metrorail. However, it is not a typical paradigm as extensively discussed in Chapter 5. The rankings according to criticality of stations and links is a combination result of the link position, station position, and the ridership of the network. Therefore, in the vulnerability assessment, location is as important as ridership and sometimes more influential.

7.1.4 Offering strategy to enhance the network resilience prior to and following a failure

Chapter 6 offered some strategies to increase the resilience of the network either by enhancing its topology prior to any failure, or by identifying proper post-failure recovery strategies, with special attention not only to restore the network resilience but also to minimize the total cost associated with a disruptive event resulting in resilience loss. Strategies for improving the network resilience prior to failure are provided given the topology enhancement through capital improvement. Different loop lines are inserted in the network based on subjective considerations, e.g., location criticality, passenger flow, locational connectedness, etc. Results show the network vulnerability could be reduced significantly by adding a loop line.

Chapter 6 also includes the recovery analysis following a failure as an assertive strategy to increase the network resilience. It ranks the recovery sequences based on the resilience restoration and suggests the comprehensive cost model through an example to show that identifying the most appropriate recovery strategies based on only resilience restoration is insufficient. A recovery sequence that is cost effective is desirable.

7.2 Novelty and Contributions

The novelty and major contribution of this dissertation lies on:

- Providing the probabilistic method to investigate the presence of small-world characteristic in the network and advance the topological analysis;
- Analyzing several failure cases, i.e., node and link failures in the network, to calculate the network resilience attributes;
- Proposing a novel algorithm to model the general pattern of ridership in the metro network;
- Proposing a new algorithm to reflect the network components weights on calculating the network efficiency;
- Offering a strategy to enhance the network resilience prior to a failure by enhancing the network topology;
- Offering a post-failure recovery strategy and a comprehensive cost model to identify the best recovery sequence and maximize the network resilience;

7.3 Future Directions

There are many future directions that researchers can take following the results of the work presented in this dissertation:

1. This dissertation mainly focuses on passenger flow to form the weighted network. While, several factors from the tangible to abstract, such as — physical length of each link, slope of each link, travel time, train volume, reliability, energy, and, etc. — would be reflected as weight on the network components and form a weighted network. Such an assessment could be explored in future studies.
2. The cost model in this dissertation provided comprehensive guidance to planners and decision makers for comparative recovery analysis study and selecting appropriate recovery sequence. However, the model does not provide all the details necessary for practitioners. To enhance the cost analysis, more economic data could be added and cost-time trade off analysis and optimizations will provide the enhanced economic assessments.
3. One of the areas that impacts the infrastructure networks resilience, and has not yet been fully explored, is human interaction with infrastructure networks. Humans interact with infrastructure networks in such complex ways ranging from linear to nonlinear to obscure, which may positively or negatively affect a network's resilience. Quantifying the human behavioral response to disruption on the network resilience; and exploring the impact of a disruptive

event in the network on human well-being would be the interesting area of the future work.

4. This dissertation focuses on the engineering point of view to provide recommendations on the network modeling; while other dimensions - such as public policy, community demands, and other aspects - may impact the objective of the study and so modeling the network. For example, the objective of the investors of the metro network could lie more on saving money rather than delivering people to their destination more readily. Or, the metro might be needed by people in the poorest areas who lack other transport modes; however, those areas may be of less importance to those who fund metro. All of these objectives are also important and should be considered. The more items are seen, the better model is achieved. Thus, in large infrastructure systems, typically different experts collaborate with each other to make the optimum decisions.
5. Infrastructure networks are interdependent; the state of one infrastructure may depend on the state of others. Any disruption in the components of one infrastructure could propagate risk in other infrastructures and trigger cascading failures across different infrastructure networks. Therefore, the recovery of a certain network from failure events, may be influenced by the recovery of other networks. Exploring the infrastructure interdependencies and providing the generic and quantitative framework for modeling the interdependency of other main infrastructure networks, i.e., communication and power grid networks, within an urban rail transit network by leveraging

other network analysis methods such as Bayesian network method, would be another exciting area of the future pursuits.

Appendix A: Numerical Example to Demonstrate the Model Proposed for Assessing Link Weights*

To give a clear understanding of the logic and the method of the ridership calculation, a numerical example is provided to show how to calculate the ridership of one link based on the number of passengers at the two neighboring transfer stations connected by the link. Figure A-1. (a-c) demonstrates two, three and four nodes portion of metro network in abscissa. Each node has a specific number of passengers. The number of passengers in each node is indicated by Pi and ridership from node i to node j and node j to node i are specified by $R_{i \to j}$ and $R_{j \to i}$, respectively. In this example i and j are: i=[1,2,3,4] and j=[1,2,3,4].

* **Paper published or submitted**

Saadat, Y., Ayyub B.M., Zhang Y., Zhang D., & Huang H. (2020). " Urban Rail Transit Networks Toward Reliable Development: Failure Analysis Incorporating General Ridership Patterns ." Submitted to Reliability Engineering and System Safety Journal, under second round of review

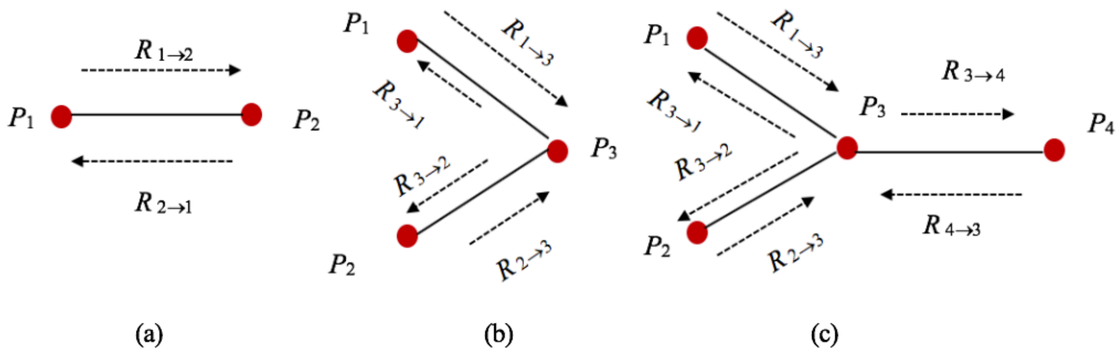


Figure A-2. The different network examples of two, three and four nodes and their associated link ridership and node passengers (The arrows are referring to the passengers who travel from node *i* to *j* and *j* to *i*)

First, assume two nodes are connected by a link such as Fig. A-1-(a), all passengers from node 1 (P_1) leaves station 1 $\Longrightarrow R_{1\to2} = P_1$. It can also be presented as, $R_{2\to1} = P_1 \times \frac{P_2}{P_2}$. Also, all passengers from node 2 (P_2) leaves station 2 $\Longrightarrow R_{2\to1} = P_2$, or, $R_{2\to1} = P_2 \times \frac{P_1}{P_1}$.

In Fig. A-1-(b), the number of nodes is increased and passengers in node 1, P_1, do not have any choice other than transferring to node 3. Thus, $R_{13}=P_1$, and with the same logic, $R_{2\to3} = P_2$. However, for node 3 the situation is different. Passengers in node 3 could move to either node 1 or node 2. Hence, $R_{3\to1}+R_{3\to2} = P_3$. This is a single equation with two $R_{j\to i}$ unknowns. However, a reasonable assumption can be made on the number of passengers that move to node 1 and the number of passengers that move to node 2 as being proportional to the count of passengers in node 1 and node 2, respectively. The portion or fraction of P_3 to move to node 1 is $\frac{P_1}{P_1+P_2}$ of passengers in node 1, and $\frac{P_2}{P_1+P_2}$ to node 2. Thus, multiplying these fractions by the number of passengers in node 3 gives the portion of passengers moving from node 3 to node 1 and from node 3 to node 2, respectively, as follows: $R_{3\to1} = (\frac{P_1}{P_1+P_2}) \times P_3$, $R_{3\to2} = (\frac{P_2}{P_1+P_2}) \times P_3$.

The process is repeated for 4 nodes according to Fig. A-1-(c) and the corresponding results are as follows: $R_{1\to3} = P_1$, $R_{2\to3} = P_2$, $R_{4\to3} = P_4$, $R_{3\to1} = (\frac{P_1}{P_1+P_2+P_4}) \times P_3$, $R_{2\to3} = (\frac{P_2}{P_1+P_2+P_4}) \times P_3$, and $R_{3\to4} = (\frac{4}{P_1+P_2+P_4}) \times P_3$. The results meet the condition of $R_{3\to1} + R_{3\to2} + R_{3\to4} = P_3$.

The 4-node case can be illustrated using the following values:

P_1	P_2	P_3	P_4
2	4	27	12

The results are:

$$R_{13}=P_1 \Longrightarrow R_{13}=2$$

$$R_{23}=P_2 \Longrightarrow R_{23}=4$$

$$R_{43}=P_4 \Longrightarrow R_{43}=12$$

$$R_{31}=(\frac{P_1}{P_1+P_2+P_4}) \times P_3 \Longrightarrow R_{31}==(\frac{2}{2+4+12}) \times 27=3$$

$$R_{23}=(\frac{P_2}{P_1+P_2+P_4}) \times P_3 \Longrightarrow R_{23}==(\frac{4}{2+4+12}) \times 27=6$$

$$R_{34}==(\frac{4}{P_1+P_2+P_4}) \times P_3 \Longrightarrow R_{34}==(\frac{12}{2+4+12}) \times 27=18$$

And it is expected to have $R_{31} + R_{32} + R_{34} = P_{3,}$ 3+6+18=27, which is fulfilled.

Appendix B: Average Daily Passenger Data

Table B.1 Average daily passenger data in each station for Washington D.C. Metro

Station number	Station name	Average daily passengers	Station number	Station name	Average daily passengers
1	Glenmont	5671	19	Tenley town-AU	6587
2	Wheaton	3864	20	Friendship Height	8503
3	Forest Glen	2230	21	Bethesda	9883
4	Silver spring	12269	22	Medical center	5591
5	Takoma	5329	23	Grosvenor-Strathmore	5206
6	Fort Totten	7543	24	White Flint	3641
7	Brookland-cua	6324	25	Twin brook	4163
8	Rhode Island Ave	7727	26	Rockville	4245
9	NoMa-Gallaudet U	9038	27	Shady Grove	11732
10	Union Station	29371	28	Greenbelt	5738
11	Judiciary Sq	8722	29	College park UMD	4068
12	Gallery Place	25537	30	Prince George's Plaza	4385
13	Metro Center	24330	31	West Hyattesville	3402
14	Farragut North	24597	32	Georgia Ave-petworth	6151
15	Dupont Circle	18653	33	Columbia Heights	11840
16	Woodley Park	5861	34	UST	6671
17	Cleveland Park	3961	35	Shaw-Howard	4989
18	Van Ness-UDC	6158	36	Shaw-Howard	4283

Table B-1, Continued

Station number	Station name	Average daily passengers	Station number	Station name	Average daily passengers
37	Archives	7829	55	Huntington	7002
38	L'Enfant Plaza	19343	56	Van Dorn ST	2970
39	Waterfront	4008	57	Franconia-Springfield	6821
40	Navy Yard-Ballpark	6834	58	New-Carrollton	7209
41	Anacostia	7799	59	Landover	1667
42	Congress Heights	2431	60	Cheverly	1153
43	Southern Ave	4751	61	Deanwood	1347
44	Naylor RD	2471	62	Minnesota Ave	2387
45	Suitland	4918	63	Stadium-Armory	2430
46	Branch Ave	5449	64	Potomac Ave	3635
47	Arlington Cemetery	363	65	Eastern Market	5500
48	Pentagon	14584	66	Capitol South	6957
49	Pentagon City	12558	67	Federal center SW	5697
50	Crystal City	11480	68	Smithsonian	7149
51	Ronald Reagan national airport	5631	69	Federal Triangle	7381
52	Braddock RD	4543	70	Mc-Pherson Sq	14340
53	King-St old town	7238	71	Farragut West	20917
54	Eisenhower Ave	1486	72	Foggy-Bottom-GWU	20121

Table B-1, Continued

Station number	Station name	Average daily passengers	Station number	Station name	Average daily passengers
73	Rosslyn	13666	83	Morgan BLVD	1849
74	courthouse	7074	84	Addison RD	2971
75	clarendon	4423	85	Capitol Heights	1893
76	Virginia Sq-GMU	3898	86	Benning RD	2823
77	Ballston-MU	10759	87	Mclean	1562
78	East falls church	3913	88	Tyson corner	2857
79	West Falls Church	2715	89	Greensboro	1079
80	West Falls Church	4981	90	Spring Hill	1042
81	Vienna	10005	91	Wiehle-Reston East	7306
82	Largo Town center	4435			

Bibliography

Abdalla, R., Tao, C. V., Cheng, Q., and Li, J. (2007). “A network-centric modeling approach for infrastructure interdependency.” Photogrammetric Engineering & Remote Sensing, 73(6),681–690.

Adger, N. (2000).” Social and ecological resilience: are they related?” Human Geography 24(3):347-364, DOI: 10.1191/030913200701540465.

Ahas, R., Aasa, A., Silm, S., & Tiru, M. (2010). Daily rhythms of suburban commuters’ movements in the Tallinn metropolitan area: Case study with mobile positioning data. Trans. Res. C-Emer. 18 (2010) 45–54.

Alger, M., Wilson, E., Gould, T., Whittaker, R., and Radulovic, N. (2004). “Real-time traffic monitoring using mobile phone data.” Vodafone Pilotentwicklung GmbH.

Albert, R., Jeong, H., Barabasi, A.L. (2000). “Error and Attack tolerance of complex networks.”Letters to Nature, 406, 378-382.

Albert, R., Albert, I., & Nakarado, G. L., (2004). Structural Vulnerability of the North American Power Grid. Physical Review E, 69, 025103.

American Public Transportation. (2016). Transit ridership report fourth quarter. http://www.apta.com/resources/statistics/Pages/ridershipreport.aspx (Accessed on October 31, 2018.)

Anbaroğlu, B., Cheng, T., & Heydecker, B. (2015). Non-recurrent traffic congestion detection on heterogeneous urban road networks. Transportmetrica A 11(2015) 754–771.

Angelouis, P., & Fisk, D. (2006). Large subway systems as complex networks. Physica A, 367, 553-558.

APMreports .(2019). “ So close, yet so costly.” https://www.apmreports.org/story/2019/02/07/great-lakes-water-shutoffs (Accessed on May 2020)

Ayyub, B. M., Foster, J., and McGill, W. L. (2009a). “Risk analysis of a protected hurricane-prone region. i: Model development.” Natural Hazards Review, 10(2), 38–53.

Ayyub, B. M., Foster, J., McGill, W. L., and Jones, H. W. (2009b). “Risk analysis of a protected hurricane-prone region. ii: Computations and illustrations.” Natural Hazards Review, 10(2),54–67.

Ayyub, B. M. (2014a). Risk analysis in engineering and economics. Crc Press.

Ayyub, B. M. (2014b). “Systems resilience for multi-hazard environments: Definition, metrics, and valuation for decision making.” Risk Analysis, 34(2), 340–355.

Ayyub, B. M. (2015). “Practical Resilience Metrics for Planning, Design, and Decision Making.” ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, 1(3), 04015008.

Bagler, G. (2008). Analysis of the airport network of India as a complex weighted network. Physica A, 387 (12), 2972-2980.

Barabasi, A. L., and Albert, R. (1999). “Emergence of scaling in random networks.” Science, 286(5439), 509-512.

Barabasi, A. L. (2016). “Network Science.” *Random Networks*, Cambridge University Press,

Barrett, C., Beckman, R., Channakeshava, K., Huang, F., Kumar, V. A., Marathe, A., Marathe M. V., and Pei, G. (2010). “Cascading failures in multiple infrastructures: From transportationto communication network.” CRIS, IEEE, 1–8.

Bensi, M., Kiureghian, K.D., Straub, D. (2011). “Bayesian network modeling of correlated random variables drawn from a Gaussian random field.” Structural Safety. 33: 317-332.

Bensi, M. T., D. Straub, P. Friis-Hansen, and A. Der Kiureghian (2009), Modeling infrastructure system performance using BN, in Proc. ICOSSAR'09, Osaka, Japan.

Bloomfield, R., Chozos, N., Nobles, P. (2009). “Infrastructure interdependency analysis: requirements, capabilities and strategy.” Adelard document reference: D/418/ 12101/3.

Boccaletti, S., Bianconi, G., Criado, R., Del Genio, C. I., Gómez-Gardenes, J., Romance, M.,Sendina-Nadal, I., Wang, Z., and Zanin, M. (2014). “The structure and dynamics of multilayer networks.” Physics Reports, 544(1), 1–122.

Bocchini, P., Frangopol, D. M., Ummenhofer, T., & Zinke, T. (2013). Resilience and sustainability of civil infrastructure: Toward a unified approach. Journal of Infrastructure Systems, 20(2), 04014004.

Bondy, J.A., Murty, U.S.R. (1976). " Graphs and Subgraphs." Graph theory with applications, the Macmillan press Ltd, N.Y., 12.

Bruneau, M., Chang, S. E., Eguchi, R. T., Lee, G. C., O'Rourke, T. D., Reinhorn, A. M., Shinozuka,M., Tierney, K., Wallace, W. A., and von Winterfeldt, D. (2003). "A framework to quantitatively assess and enhance the seismic resilience of communities." Earthquake Spectra, 19(4), 733-752.

Bruneau, M., and Reinhorn, A. (2007). "Exploring the concept of seismic resilience for acute carefacilities." Earthquake Spectra, 23(1), 41–62.

Brundtland, G. H. (1987). Our common future: Brundtland-report, Oxford University Press, Oxford.

Cadarsoa, L., Marín, Á., & Maróti, G. (2013). Recovery of disruptions in rapid transit networks. Transportation Research Part E: Logistics and Transportation Review, 53, 15-33.

Cambridge Dictionary (1995) https://dictionary.cambridge.org/us/dictionary/english/network (accessed on February 27, 2018.)

Cambridge Dictionary (1995) https://dictionary.cambridge.org/us/dictionary/english/performance (accessed on March 31, 2020.)

Carvalho, R., Buzna, L., Bono, F., Gutierrez, E., Just, W, & Arrowsmith, D. (2009). Robustness of Trans-European Gas Networks. Physical Review E, 80, 016106.

Chen, G., Wang, X. F., & Li, X. (2012). " Introduction to Complex Networks:

Models, Structures and Dynamics.” Beijing, China: High Education Press.

Chen, Y., Cai, Y., Li, P., and Zhang, G. (2015). “Study on evacuation evaluation in subway fire based on pedestrian simulation technology.” Mathematical Problems in Engineering, 2015.

Chen, S., Di, Y., Liu, S., and Wang, B. (2017). “Modelling and analysis on emergency evacuation from metro stations.” Mathematical Problems in Engineering, 2017.

CNN (Cable News Network). (2017). http://www.cnn.com/2017/12/19/us/amtrak-derailment washington/index.html. (December.19 , 2017)

Coar, M., Garlock, M., & Elhami Khorasani, N. (2019) Effects of water network dependency on the electric network for post-earthquake fire suppression, Sustainable and Resilient Infrastructure, DOI: 10.1080/23789689.2018.1563408.

Costa, L. da F., Rodrigues, F. A., Travieso, G., Villas Boas, P. R., (2007). “Characterization of complex networks: A survey of measurements.” Advances in Physics, 56(1), 167-242.

Crucitti, P., Latora, V., Marchiori, M., and Rapisarda, A. (2004). "Error and attacktolerance of complex networks." *Nature*, 340, 388-394.

Danziger, M. M., Bashan, A., Berezin, Y., Shekhtman, L. M., and Havlin, S. (2014). “An introduction to interdependent networks.” International Conference on Nonlinear Dynamics of Electronic Systems, Springer, 189–202.

Deng, Y., Li, Q. M., Lu, Y., & Yuan, J. F. (2013). Topological and measure of urban metro network: the case of Nanjing. Journal of Network, 8 (6),1350-1356.

Department of Homeland Security (2018). “Critical Infrastructure.” https://www.cisa.gov/homeland-security-presidential-directive-7 (Accessed on 2 April 2020)

Derrible, S., Kennedy, C., (2009). “Network Analysis of World Subway Systems Using Updated Graph Theory.” Transp. Res. Record: J. Transp. Res. Board, 2112, 17–25.

Derrible, S., and Kennedy, C. (2010). “The complexity and robustness of metro networks.” *Physica A*, 389(17), 3678-3691.

Dimitriou, D., Karlaftis, M., Kepaptsoglou, K. and Stathopoulos, M. (2006). “Public Transportation during the Athens 2004 Olympics: From Planning to Performance.", Transportation Research Board.

DOT (Depertment of Transportation). (2016). “Treatment of the Value of Preventing Fatalities and Injuries in Preparing Economic Analyses”. https://cms.dot.gov/sites/dot.gov/files/docs/2016%20Revised%20Value%20of%20a%20Statistical%20Life%20Guidance.pdf (2016). (Accessed on April 18, 2019).

Duan, Z., Lei, Z., Zhang, M., Li, W., Fang, J., and Li, J. (2017b). “Understanding evacuation and impact of a metro collision on ridership using large-scale mobile phone data.” IET Intelligent Transport Systems, 11(8), 511–520.

Easley, D., & Kleinberg, J. (2010). “Networks, Crowds, and Markets: Reasoning about a Highly Connected World.” London, UK: Cambridge University Press.

Eldosouky, A., Saad, W., and Mandayam, N. (2017). “Resilient critical infrastructure: Bayesiannetwork analysis and contract-based optimization.” arXiv:1709.00303.

EO (Executive Order). (2013). “ Improving Critical infrastructure Cyber security.” resilience.” EO 13636. https://www.cisa.gov/sites/default/files/publications/eo-13636-ppd-21-fact-sheet-508.pdf (Accessed on April 2020)

Ezell, B.C., Farr, J.V., Wiese. (2000). “Infrastructure risk analysis model. Journal of Infrastructure Systems.” 2000;6(3):114–7.

Ezell BC, Farr JV, Wiese. (2000). “Infrastructure risk analysis of municipal water distribution system. Journal of Infrastructure Systems.” 2000;6(3):118–22.

Ezzeldin, M., & El-Dakhakhni, W.E. (2019). Robustness of Ontario power network under systemic risks, Sustainable and Resilient Infrastructure, DOI: 10.1080/23789689.2019.1666340.

Federal highway administration (FHWA). (2002). “Life cost analysis premier”. https://www.fhwa.dot.gov/asset/lcca/010621.pdf(Accessed on November 9, 2020).

FEMA (Federal Emergency Management Agency). (2011). “ National Preparedness Goal.” FEMA P-1.

FEMA (Federal Emergency Management Agency). (2013). “ Mitigation Assessment Team Report: Hurricane Sandy in New Jersey and New York Building Performance Observation, Recommendation and Technical Guidenance.” FEMA P-942.

FEMA (2018). “Infrastructure interdependency assessment, Puerto Rico.” http://www.camarapr.org/Camara-en-Accion-18-19/17-nov-8/gob/PR-Infrastructure-Interdependency-Assessment-Report-Sept-2018.pdf (Accessed on February 12)

Feng, J., Li, X., Mao, B., Xu, Q., & Bai, Y. (2017). Weighted complex network analysis of the Beijing subway system: train and passenger flows. Physica A, 474:213–223.

FH Paschen Company (Communication with Tony Trost, Operations manager). (2018), Address: 7700 Leesburg Pike Suite 244, Falls Church, VA 22043. Phone: (703) 245-0280, https://www.fhpaschen.com/contact/ (personal communication with authors.)

FH Paschen Company (Communication with Tony Trost, Operations manager). (2019), Address: 7700 Leesburg Pike Suite 244, Falls Church, VA 22043. Phone: (703) 245-0280, https://www.fhpaschen.com/contact/ (personal communication with authors.)

Flood, J.E. (1998). “Telecommunications Switching, Traffic and Networks, Chapter 4: *Telecommunications Traffic*,” New York: Prentice-Hall.

Floyd, R. W. (1962). “Algorithm 97 (Shortest Path).” *Communications of the Acm*, 5(6), 345-345.

Grant, T. J., ed. (2014). “Network Topology in Command and Control,” Advances in Information Security, Privacy, and Ethics. IGI Global. pp. xvii, 228, 250.

Gao, J., Barzel, B., Barabási, A.L. (2016), “Universal resilience patterns in complex networks,” Nature, Letter, Vol. 530, doi:10.1038/nature16948.

Gao, S., Wang, Y., Gao, Y., & Liu.Y. (2013). Understanding urban traffic-flow characteristics: a rethinking of betweenness centrality. Environ. Plann. B 40 (2013)135–153.

Gibbons, A. (1985). Algorithmic graph theory. Cambridge university press.

Gopnik, A., Tenenbaum, J.B. (2007). “Bayesian special section Introduction: Bayesian networks, Bayesian learning and cognitive development,” Developmental Science 10:3 (2007), pp 281–287.

Guardian (2014). “A history of world metro systems in pictures.” theguardian.com/cities/gallery/2014/sep/10/-sp-history-metro-pictures-london-underground-new-york-beijing-seoul (Accessed on December 2017)

Guidotti, R., Gardoni, P., and Chen, Y. (2017). “Network reliability analysis with link and nodal weights and auxiliary nodes,” Structural Safety, 65, 12-26.

Hartmann, T., Kappes, A., Wagner, D. (2014). “Clustering Evolving Networks.” *Computing Research Repository (CoRR)*, abs/1401, 3516.

Hasnat, M.M., Islam, M. R., and Hadiuzzaman, M. (2018). “Application of gis for disaster response in dense urban areas: A case study for dhaka city.” Transportation Research Board 97th Annual Meeting.

Háznagy, A. , Fi, I., London, A., Nemeth, T. (2015). “Complex network analysis of public transportation networks: A comprehensive study.” International Conference on Models and Technologies for Intelligent Transportation Systems (MT-ITS), Budapest, pp. 371-378. doi: 10.1109/MTITS.2015.7223282.

Henry, D., and Ramirez-Marquez, J. E. (2012). “Generic metrics and quantitative approaches for system resilience as a function of time.” Reliab. Eng. Syst. Saf., 99, 114-122.

Hensher, D. and Brewer, A. (2002). “Going for gold at the Sydney Olympics: how did transport perform?", Transport Reviews , Vol. 22, pp. 381-399.

Holling CS. (1973). “Resilience and stability of ecological systems.” Annual Review of Ecology and Systematics, 1973; 4:1–23.

Holme, P., & Kim, B. J. (2002). Attack vulnerability of complex networks. Physical Review E, 65 (5), 105-109.

Hosseini, S.M., & Barker, K. (2016). “Modeling infrastructure resilience using Bayesian networks: A case study of inland waterway ports.” Computer & Industrial Engineering, Volume 93, March 2016, Pages 252-266. https://doi.org/10.1016/j.cie.2016.01.007

Hosseini, S.M., Barker, K., and Ramirez-Marquez, J. E. (2016). “A review of definitions and measures of system resilience.” Reliability Engineering & System Safety, 145, 47–61.

Jenelius, E., Petersen, T., & Mattsson, L. G. (2006). Importance and exposure in road network vulnerability analysis. Transportation Research Part A: Policy and Practice, 40 (7), 537-560.

Jenelius, E., & Mattsson, L. G. (2015). Road network vulnerability analysis: conceptualization, implementation and application. Computers, Environment and Urban Systems, 49 (1), 136-147.

Johansen, C., & Tien, I. (2017). "Probabilistic multi-scale modeling of interdependencies between critical infrastructure systems for resilience." Sustainable and Resilient Infrastructure, 1–15.

Jorje, J.,Washington Post. (2020). "Metro ridership was on major upswing before pandemic brought it crashing down." https://www.washingtonpost.com/transportation/2020/04/09/metro-ridership-was-major-upswing-before-pandemic-brought-it-crashing-down/ (Accessed on May 2020)

Kerner, B.S., & Klenov, S.L. (2004). Spatial–temporal patterns in heterogeneous traffic flow with a variety of driver behavioral characteristics and vehicle parameters. Physica A, 37 (2004) 8753–8788.

Kivelä, M., Arenas, A., Barthelemy, M., Gleeson, J. P., Moreno, Y., and Porter, M. A. (2014)."Multilayer networks." Journal of complex networks, 2(3), 203–271.

Kjølle, G.H., Utne, I.B., Gjerde, O. (2012). "Risk analysis of critical infrastructures emphasizing electricity supply and interdependencies." Reliability Engineering and System Safety 2012;105:80–9.

Kurant, M., & Thiran, P. (2006). Extraction and analysis of traffic and topologies of transportation networks. Phy. Rev. E 74 (2006) 036114.

Laporte, G., Mesa, J. A., & Perea, F. (2010). A game theoretic framework for the robust railway transit network design problem. Transportation Research Part B: Methodological, 44 (4), 447-459.

Latora, V., and Marchiori, M. (2001). “Efficient behavior of small-world networks.” Physical Review Letters, 87(19), 4.

Li, J.,& He, J. (2002). A recursive decomposition algorithm for network seismic reliability evaluation. Earthquake engineering & structural dynamics, vol. 31, no. 8, pp. 1525-1539, 2002.

Lin, J., & Ban, Y. (2013). Complex network topology of transportation systems. Transport Reviews, 33 (6), 658-685.

Li, L., Li. Y., & Li. Z. (2013). Efficient missing data imputing for traffic flow by considering temporal and spatial dependence. Transportation Research, part C-Emerging Technologies. 34 (2013) 108–120.

Latora, V., and Marchiori, M. (2001). “Efficient behavior of small-world networks.” *Physical Review Letters*, 87(19), 4.

Latora, V., Marchiori, M., (2002). “Is the Boston Subway a small-world network?” Physica A: Statistical Mechanics and its Applications, 314(1), 109-113.

Lederer, E. (2013). “UN predicts near doubling of city dwellers by 2050”. The San Diego Union Tribune, http://www.sandiegouniontribune.com/sdut-un-predicts-near-doubling-of-city-dwellers-by-2050-2013dec09-story.html (Accessed on December10, 2013)

Lu, J., Chen, C., Ogorzalek, M., Ljicljana,T. (2013). "Theory and applications of complex networks: Advances and challenges.”. IEEE International Symposium on Circuits and Systems (ISCAS2013). pp. 2291–2294. doi:10.1109/ISCAS.2013.6572335. ISBN 978-1-4673-5762-3.

Mendonca, D., William, A.W. (2006).“ Impacts of the 2001 world trade center attack on New York City critical infrastructures.” Journal of Infrastructure Systems 2006;12(4):260–70.

Merriam Webster Dictionary (1828): hhttps://www.merriam-webster.com/dictionary/infrastructure(Accessed on January 31, 2020)

Merriam Webster Dictionary (1828): [https://www.merriam-webster.com/dictionary/resilience.](https://www.merriam-webster.com/dictionary/resilience) (Accessed 25 Feb. 2020)

Merriam-Webster.com Dictionary, Merriam-Webster: [https://www.merriam-webster.com/dictionary/efficiency#h1 (](https://www.merriam-webster.com/dictionary/efficiency#h1)Accessed 2 Apr. 2020)

Merriam-Webster.com Dictionary, Merriam-Webster: https://www.merriam-webster.com/dictionary/vulnerable (Accessed 2 Apr. 2020)

McEvoy, D., Ahmed, I., and Mullett, J. (2012). “The impact of the 2009 heat wave on Melbourne’s critical infrastructure.” Local Environment, 17(8), 783 – 796.

McDaniels, T., Chang, S., Peterson, K., Mikawoz, J., Reed, D. (2007). “Empirical framework for characterizing infrastructure failure interdependencies.” Journal of Infrastructure Systems2007;13(3):175–84.

McDaniels, T., Chang, S., Reed, D. (2008). “Characterizing infrastructure failure interdependencies to inform systemic risk.” Wiley Handbook of Science and TechnologyforHomelandSecurity2008:1–16.

Mu, X. Q. (2011). “Two subway trains collide in Shanghai, over 270 injured.” Xinhuanet, Xinhuanews, Beijing.

Nature video (2016). “Network earth”: https://www.youtube.com/watch?v=xZ3OmlbtaMU (Accessed on 31 March. 2020)

Nazif, S. and Karamouz, M. (2009). “Algorithm for assessment of water distribution system’s readiness: planning for disasters.” J. of Water Resources Planning and Management, Vol. 135, No. 4, Special Issue: Remembering the contributions and influence of G.V. Loganathan, pp.244–252.

Nelson, D.R., Adger, W.N. and Brown, K. (2007) . “adaptation to environmental change: contributions from a resilience framework.” Annual Review of Environment and Resources, Vol. 32, pp.395–419.

Nelson, K. S., Gillespie-Marthaler, L., Baroud, H., Abkowitz, M., & Kosson, D. S. (2019). An integrated and dynamic framework for assessing sustainable resilience in complex adaptive systems. Sustainable and Resilient Infrastructure.

Newman, M.E.J. (2004). “Analysis of Weighted Networks.” Physical Review Letters,70,056131.

Newman, M.E.J. (2010). “Mathematics of Networks.” Networks an Introduction, Oxford university press ink, N.Y.,99-166.

Nguyen, T. P. K., Bdugin, J., & Marais, J. (2015). Method for evaluating an extended fault tree to analyze the dependability of complex systems: application to a satellite-based railway system. Reliability Engineering & System Safety, 133, 300-313.

Nielsen, M. (2011). “ Reinventing Discovery: The new era of networked science.”

Princeton, NJ: Princeton University Press, 2011.

NTSB (National Transportation Safety System Board). (2010). "NTSB Railroad Accident Report: Collision of Two Washington Metropolitan Area Transit Authority Metrorail Trains Near Fort TottenStation."(PDF).

NYT(New York Times). (2015). "Amtrak Train Derailed Going 106 M.P.H. on Sharp Curve; at Least 7 Killed ". https://www.nytimes.com/2015/05/14/us/amtrak-train-derails-crash-philadelphia.html

O'Rourke, T.D. (2007). "Critical infrastructure, interdependencies, and resilience', The Bridge, Vol. 37, No. 1, pp.22–29. Resilience of civil infrastructure systems 349.

O'Rourke, T.D., Lembo, A.J. and Nozick, L.K. (2002). "Lessons learned from the World Trade Center disaster about critical utility systems." Beyond September 11th: An Account of Post-Disaster Research, pp.269–290, Special Publication 39, Natural Hazards Research and Applications Information Center, Boulder, CO.

Omer, M., Nilchiani, R. and Mostashari, A. (2009). "Measuring the resilience of the global internet infrastructure system." International Systems Conference, 2009 IEEE International Systems Conference Proceedings, Inst. of Elec. and Elec. Eng. Computer Society, Vancouver, BC, Canada, 23–26 March, pp.156–162 [online] http://dx.doi.org/10.1109/SYSTEMS.2009.4815790 (Accessed 15 August 2018).

Ostfled, A. (2005). "Water distribution systems connectivity analysis." J. of Water Resources Planning and Management, Vol. 131, No. 1, pp.58–66.

Ostroumova Prokhorenkova, L., Samosvat, E. (2014). “Global Clustering Coefficient in Scale-free Networks”. Lecture Notes in Computer Science, 882, 47-58.

Ouyang, M., Yu, M. H., Huang, X. Z., & Luan, E. J. (2008). Emergency Response to Disaster Struck Scale-free Network with Redundant Systems. Physica A, 387, 4683-4691.

Ouyang, M., (2014). “ Review on modeling and simulation of interdependent critical infrastructure systems. Reliability engineering & System safety, 121:43-60, 2014.

Parker, J., Epstein, J. (2014). “ A Distributed Platform for Global-Scale Agent-Based Models of Disease Transmission,” ACM Trans Model Comput Simul. 2011 Dec; 22(1): 2. doi: 10.1145/2043635.2043637.

Paul, Sh. (2014). Vulnerability Concepts and its Application in various Fields: A Review on Geographical Perspective. Journal of Life Earth Science, 8, 63–81.

Piacenza, J.R., Proper, S., Bozorgirad, M., Hoyle, C., & Tumer, I.Y. (2017). Robust topology design of complex infrastructure systems. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering, 3(2):021006-021006-10. doi:10.1115/1.4036152.

PlanItMetro. (Metro’s planning blog). (2015). “ Cherry Blossoms 2015: The Secret to Beating the Crowds.” https://planitmetro.com/2015/03/30/cherry-blossoms-2015-the-secret-to-beating-the-crowds/ (Accessed on May 2020)

Porekar. J. (2002). “Random Networks.”

http://wwwf1.ijs.si/~rudi/sola/Random_Networks.pdf

Pollalis, S., Georgoulias, A., Ramos, S., Schodek, D. (2012). “Introduction: Sustainability-A Broad Perspective.” Infrastructure Sustainability and Design, Routledge, N.Y.,1.

PPD (Presidential Policy Directive). (2013). “Critical infrastructure security and resilience.” PPD 21, https://obamawhitehouse.archives.gov/the-press-office/2013/02/12/presidential-policy-directive-critical-infrastructure-security-and-resil(Accessed on February 2020)

President′s Commission on Critical Infrastructure Protection (PCCIP). Critical foundations: protecting America′s infrastructures: the report of the President ′s commission on critical infrastructure protection. Rep. No. 040-000-00699- 1, U.S. Government Printing Office, Washington, D.C.; 1997.

Report of Center for Puerto Rico Studies (October2018): https://centropr.hunter.cuny.edu/sites/default/files/data_briefs/Hurricane_maria_1YR.pdf (Accessed on August 2019)

Rokneddin, K., Ghosh, J., Duensa-Osorio, L., & Padgett. J.E. (2013). "Bridge retrofit prioritization for ageing transportation networks subject to seismic hazards". In: Structure and Infrastructure Engineering. 9.10 (2013), pp. 1050-1066.

Saadat, Y., Zhang, Y., Zhang, D., Ayyub, B.M., Huang, H. (2018). “Post-Failure Recovery Strategies for Metrorail Transit Networks with Washington D.C. as a Case Study.” International Mechanical engineering congress & Exposition, PA, U.S.

Saadat, Y., Ayyub B.M., Zhang Y., Zhang D., & Huang H., 2019, "Resilience of Metrorail Networks: Quantification with Washington D.C. as a case study." ASCE. ASME J. Risk Uncertainty Part B. 2019, doi:10.1115/1.4044038.

Saadat, Y., Ayyub, B.M., Zhang, Y., Zhang, D., Huang, H., (2020). "Resilience-based Strategies for Topology Enhancement and Recovery of Metrorail Transit Networks." ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part A: Civil Engineering, DOI: 10.1061/AJRUA6.000105.

Saadat, Y., Ayyub, B.M., Zhang, Y., Zhang, D., Huang, H., (Forthcoming 2020). "Urban Rail Transit Networks Toward Reliable Development:Faiulre Analysis Incorportating General Ridership Pattern." Reliability Engineering and System Safety journal (Under Review).

Sen, P., Dasgupta, A., Sreeam, P., Mukherjee, G., Manna, S.S (2003). "Small-World Properties of the Indian Railway Network." *Phisical Reveiew*, E67, 036106.

Schmitt, A. J., and Singh, M. (2012). "A quantitative analysis of disruption risk in a multi-echelon supply chain." International Journal of Production Economics, 139(1), 22–32.

Scherb, A., Garre, L., & Straub, D. (2017). Reliability and component importance in networks subject to spatially distributed hazards followed by cascading failures. ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering, 3(2):021007-021007-9. doi:10.1115/1.4036091.

Schlake, B. W., Barkan, C.P.L, and Edwards, J.,R. (2011). "Train delay and economic impact of in-service failures of railroad rolling stock."

Transportation Research Record: Journal of the Transportation Research Board, (2261):124-133.

Simone, A., Ridolfi, L., Beradi, L., & Laucelli, DB. (2018). Complex Network Theory for Water Distribution Networks Analysis. 13th International Conference on Hydro informatics, 3, 1971-1978.

Sienkiewicz, J., Holyst, J.A., (2005). "Statistical Analysis of 22 Public Transport Networks in Poland." *Physical Reveiew,* E72,046127.

Singha, M. R. and Kalita, B. (2013). "Mapping mobile phone network onto urban traffic network."to appear Proceeding of International Multi conference of Engineers and Computer Scientists.

Sole, R.V., Valverde, S. (2004). "Information Theory of Complex Networks: On Evolution and Architectural Constraints." *Lecture Notes in Physics,* Springer, Berlin, 650, 189-207.

Sun, Daniel (Jian) & Shituo, G. 2016. Measuring vulnerability of urban metro network from line operation perspective. Transportation Research Part A: Policy and Practice, Elsevier, vol. 94(C), pages 348-359.

Sun, W., Bocchini, P., & Davison, B.D. (2018) Resilience metrics and measurement methods for transportation infrastructure: the state of the art, Sustainable and Resilient Infrastructure, DOI: 10.1080/23789689.2018.1448663.

Tako, A.A., Robinson, S. (2012). " the application of discrete event simulation and system dynamics in the logistics and supply chain context," Decision Support Systems, Volume 52, issue 4, pages 802-815, https://doi.org/10.1016/j.dss.2011.11.015.

Terte, D., Stephens, C., (2014). "Psychological Resilience of Workers in High-Risk Occupations". Stress and Health. 30 (5): 353–355. doi:10.1002/smi.2627.

Thai, M. T., &Pardalos, P. M. (2012). " Handbook of optimization in complex networks: theory and applications." New York, NY: Springer.

Tien, I., Kiureghian, A.D. (2016). "Algorithms for Bayesian Network Modeling and Reliability Assessment of Infrastructure Systems." Reliability Engineering, System Safety 156, DOI: 10.1016/j.ress.2016.07.022.

Tierney, K., & Bruneau, M. (2007). "Conceptualized and measuring resilience." TR News 250, 14-17, (http://onlinepubs.trb.org/onlinepubs/trnews/trnews250_p14-17.pdf) (Accessed on Dec. 11, 2019)

Transportation Research Board (TRB): Azizinamini, A., Power, E.H., Myers, G.F., Celik Ozyildirim, H., Kline, E., Whitmore, D.W., Mertz, D.R. (2014). " Design guide for bridge for service life."

UITP (International Association of Public Transport). (2015). "World Metro Figures." http://www.uitp.org/sites/default/files/cck-focus-papers-files/UITP-Statistic%20Brief-Metro-A4-WEB_0.pdf (Accessed on December 2017)

United Nation's Department of Economic and Social Affairs (UN) .(2014). "World's population increasingly urban with more than half living in urban areas." https://www.un.org/en/development/desa/news/population/world-urbanization-prospects-2014.html (Accessed on July 2018)

Utne, I.B., Hokstad, P., Vatn J. (2011). “ A method for risk modeling of interdependencies in critical infrastructures.” Reliability Engineering and System Safety 2011;96:671–8.

Von Ferber, C., Holovatch, T., Holovatch, Y., Palchykov, V. (2009). “Public transport networks: empirical analysis and modeling.” European Physical Journal B, 68(2), 261-275.

Wang, J. W., & Rong, L. L. (2009). Cascade-based Attack Vulnerability on the US Power Grid. Safety Science, 47, 1332-1336.

Wang, Q. and Taylor, J. E. (2015). “Resilience of human mobility under the influence of typhoons.” Procedia engineering, 118, 942–949.

Wang, X., Koç, Y., Derrible, S., Ahmad, S.N., Kooij, R.E. (2015). “Quantifying the robustness of metro networks.” 6th International Symposium on Transportation Network Reliability, Nara, Japan.

Washington D.C. Metro Fisical Year (FY) Budget. (2018): https://www.wmata.com/about/records/public_docs/upload/Metro_FY2018_Proposed_Budget_15Dec16_v4.pdf (Accessed on October 31, 20182018)

WMATA (Washington Metropolitan Area Transit Authority). (2016). https://www.wmata.com/initiatives/plans/upload/2016_historical_rail_ridership.df. (accessed on May 31, 2019.)

Watts, D. J., Strogatz, S. H. (1998). “Collective dynamics of ‘small-world’networks.” *nature*, 393(6684), 440-442.

Winkler, J., Duenas-Osorio, L., Stein, R., & Subramanian, D. (2010). Performance Assessment of Topologically Diverse Power Systems Subject to Hurricane Events. Reliability Engineering and System Safety, 95, 323-336.

WMATA (Washington Metropolitan Area Transit Authority). (2016). https://www.wmata.com/initiatives/plans/upload/2016_historical_rail_ridership.pdf.(accessed on October 31, 2018.)

West, D. B. et al. (2001). Introduction to graph theory, Vol. 2. Prentice hall Upper Saddle River.

Winkler, J., Duenas-Osorio, L., Stein, R., & Subramanian, D. (2010). Performance Assessment of Topologically Diverse Power Systems Subject to Hurricane Events. Reliability Engineering and System Safety, 95, 323-336.

World economic forum (weforum). (2020). "How digital infrastructure can help us through the COVID-19 crisis." https://www.weforum.org/agenda/2020/04/digital-infrastructure-public-health-crisis-covid-19/ (Accessed on April 2020)

WTOP News. (Washington's Top News). (2016). "The metro lines most vulnerable to overcrowding, delays." ttps://wtop.com/dc-transit/2016/03/the-metro-lines-most-vulnerable-to-overcrowding-delays/ (Accessed on May 2020)

Wu, J. J., Gao, Z. Y., & Sun, H. J. (2007). Effects of the Cascading Failures on Scale-free Traffic Networks. Physica A, 387, 505-511.

Wu, Y., Chen, F., Lu, C., & Yang, S. (2015).Urban traffic flow prediction using a spatiotemporal random effects model. Intelligent Transportation System, 20 (2016) 282–293.

Wu, X. T., Tse, C. K., Dong, H. R., Ho, I. W. H. & Lau, F. C. M. (2016), A Network Analysis of World's Metro Systems, International Symposium on Nonlinear Theory and Its Applications, 606-609, NOLTA2016, Yugawara, Japan.

Yang, S.L., Hsu, N.S, Louie, P. W., & Yeh, W. W. (1996) .Water distribution network reliability: Connectivity analysis. Journal of Infrastructure Systems, vol. 2, no. 2, pp. 54-64, 1996.

Yook, S.H., Jeong, H., Barbasi, A.L.(2001). "Weighted Evolving Networks." *Physical review Letters*, 86, 5835-5838.

Zhang, D., Du, F., Huang, H., Zhang, F., Ayyub, B.M., Beer, M. (2017). "Resiliency Assessment Model of Urban Rail Transit Networks: Shanghai Metro as an Example". *Safety Science Elsevier.*

Zhang, J., Cao, X. B., Du, W. B., & Cai, K. Q. (2010). Evolution of Chinese Airport Network. Physica A, 389, 3922-3931.

Zhang, J., Zhao, M., Liu, H., Xu, X. (2013). "Networked characteristics of the urban rail transit networks." *Physica A: Statistical Mechanics and its Applications*, 392(6), 1538-1546.

Zhang, J., Wang. S., & Wang. X. (2018). Comparison analysis on vulnerability of metro networks based on complex network. Physica A, 496, 72-78.