ABSTRACT

Title of dissertation: ESSAYS ON FRICTIONS AND ECONOMIC FLUCTUATIONS

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I study how information frictions, in the forms of limited information capacity or asymmetric information, affects the firm's production and physical capital accumulation decisions, and how it can help a dynamic general equilibrium model to account for selected empirical characteristics over business cycle frequencies.

In the first chapter, I explore how limited information capacity affects fished-good inventory accumulation by firms. I use rational inattention to understand the responses of inflation and output inventories after nominal shocks. In the data, output inventories move less than sales in the U.S. manufacturing and trade sectors. To reconcile the model with the data, some studies have suggested that variation in price markups, rather than cost rigidities, must account for the bulk of the real effects of nominal shocks. I propose that this conclusion does not necessarily hold when the firms decisions are affected by an information capacity constraint. In my model, firms observe aggregate conditions with idiosyncratic noise. Paying more attention helps a decision maker to reduce the noise, but also incurs information

costs. Firms allocate their attention between pricing and production decisions, and their decision rules deviate from the first-best rules. This friction serves as a force to hinder drastic movements in production and inventory accumulation. I show that inventories can move less than sales even when the marginal cost of production is rigid. Numerical results suggest that inattention on the firm side can qualitatively match empirical impulse responses and business cycle moments. The fit with data is better than a staggered pricing model with elastic cost pressure, in terms of both matching impulse responses and simulated moments.

In the second chapter, I study how information asymmetry about the quality of used capital affects capital reallocation. Empirical studies of business cycle dynamics indicate that the reallocation of capital, or the movement of capital input from less productive firms to more efficient firms, is procyclical, whereas the dispersion of marginal product of capital is countercyclical. I build a model of endogenous partial capital irreversibility, which stems from both capital specificity and information asymmetry in the market for used capital. The resale price and average quality of used capital in the market vary with aggregate productivity shocks. In the model, imperfect substitution between new and used capital and information asymmetry interact to generate procyclical reallocation. Preliminary numerical results show procyclical reallocation and countercyclical dispersion of capital returns.

ESSAYS ON FRICTIONSAND ECONOMIC FLUCTUATIONS

by

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Dedication

To my parents and my loved one, whose encouragement made this work possible.

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List of Abbreviations

LHS	Left hand side
RHS	Right hand side
DSGE	Dynamic stochastic general equilibrium
IRF	Impulse response function
VAR	Vector autoregression
ARMA	Auto-regressive moving average
MA	Moving average
RI	Rational inattention
PPP	Purchasing Power Parity
FFR	Fed Funds Rate
PP&E	Property, Plants, and Equipment
TFP	Total factor productivity
MPK	Marginal product of capital

Chapter 1: The Behavior of Inventories and Marginal Cost under Rational Inattention

1.1 Introduction

This paper studies inventory and inflation behavior in response to monetary shocks. I focus on the relation between cost rigidities, inflation and inventory responses in monetary transmission. Estimates from vector autoregressive (VAR) models suggest that monetary tightening leads to a persistent decline in output, a muted and sluggish decline in inflation, and a rise in the inventory-sales ratio. Even though inventories decline after a monetary tightening, the movement is muted compared to the decline in sales, so that the inventory-sales ratio increases after a monetary tightening. In a standard model, the movement of inventory investment is tied to the elasticity of the marginal cost of production with respect to output. If marginal cost falls sharply with the fall in output, then the decline in inventories will indeed be limited, because firms' lower marginal cost spurs firms to accumulate more inventories. Therefore, inventories can indeed move less then sales. However, if the marginal cost of production does not respond much to sales, there is no incentive for intertemporal substitution in firms' production to counteract the effect of a spike in the real interest rate, and the latter will kick start a large disinvestment of inventories, since the real interest rate is a major determinant of the carrying cost of inventories (along with the depreciation rate).

For this reason, existing monetary business cycle models have a hard time explaining the joint behavior of finished good inventories relative to sales/shipments, inertial inflation, and the persistent response of output simultaneously. Models that assume that marginal cost is sluggish and not very responsive to changes in output can explain why the decline in the price level is muted and why output falls following a monetary tightening, but predict that inventories fall much more than sales, as the spike in the real interest rate translates into an increase in the cost of carrying inventories. Models in which the marginal cost of production falls sharply after a monetary tightening can explain why inventories move less than sales, but imply a more abrupt and temporary downward movement of the inflation rate even with a high degree of nominal rigidity, unless the model also assumes a high degree of rigidity (strategic complementarity) among firms' pricing decisions. Furthermore, elastic cost pressure and rigid prices translates to countercyclical price markups, which means firms expand their profit margin after a monetary tightening. However, in the data we do not find evidence of countercyclical markup or profits.[1] This calls into question viewing countercyclical markup variation as the key transmission mechanism.

[1] See the empirical results in section 2. Also, Loecker and Eeckhout (2018) studies the evolution of markups based on firm-level data. They find that markups are stable before 1980. After the 1980s the average markup is on a rising trend. Anderson et al. (2018) studies the behavior of markups in the U.S. retail sector. They find that markups are relatively stable over time and mildly procyclical.

I propose that this conundrum involving the responses of inventories, inflation, and marginal production cost can be reconciled when I embed information frictions in the inventory model in place of nominal rigidity in firms' pricing decisions. I show that when it takes time (and possibly other resources) for the firm to figure out the true state of the economy, then inventories can move less than sales even without a pronounced response of marginal cost, and inflation can be sluggish (due to both the information friction and the sluggish movement of production cost). The intuition is simple: information frictions affect both the firm's pricing and production decisions, unlike nominal rigidities, which only work through the pricing decisions. In other words, information frictions on the firm side generate sluggishness of both inflation and inventories. Compared to a model in which produced goods are not storable and thus a firm's relative price directly determines demand for its goods and its production level, in an inventory model unsold goods are storable and the firm's production is not directly pinned down by its relative price (though pricing and production decisions are still linked through optimality conditions). Under nominal rigidities and full information, a high responsiveness of production cost is required to discipline the movement of production (and inventories). Instead, if I assume limited information capacity, the information friction itself serves to hinder the impact response of prices and production to shocks. The model can thus accommodate a rigid cost of production, which adds to the persistence of the real effects of monetary shocks.

The information friction adopted in this paper follows Mackowiak and Wiederholt (2015) and Mackowiak and Wiederholt (2009).[2] I consider endogenous inattentive behavior, which affects the pricing and production decisions of a firm. In the model, firms face both aggregate shocks and idiosyncratic noise shocks. Firms cannot attend perfectly to all information by tracking each source of disturbance without error, because paying more attention to a particular shock incurs a flow information cost, which can be thought of as time devoted to information processing. Firms thus face a constraint on information flow and optimally decide the amount of attention allocated to different sources of shocks, as well as to their pricing and production decisions. One contribution of the paper is to examine the relative attention allocated to the two decisions and how it affects the inventory behavior. I argue that the firm pays more attention to the pricing decision because it is more relevant to profit maximization. Therefore the firm allows the inventory accumulation decision to deviate more from the first-best action.[3] To the best of my knowledge, this is the first study that explores the joint dynamics of prices and inventories from the perspective of information flows.

1.2 Literature Review

This paper is related to studies comparing information frictions and staggered pricing frictions à la Calvo, including Trabandt (2007), Korenok and Swanson

[2] As in Mackowiak and Wiederholt's papers, I only consider how inattentive firms' allocation of attention affects the firm-level price setting. Recent works such as Pasten and Schoenle (2014) and Stevens (2019) have explored how endogenous information acquisition affects product-level prices and how it determines aggregate inflation dynamics.

[3] As will become clear later, the amount of goods a firm produces determines its inventory accumulation *ex ante*. Throughout this paper I use the three terms "production decision", "inventory decision", and "Z_j decision" interchangeably.

(2007), Collard and Dellas (2010) and Mackowiak et al. (2009). In particular, Trabandt (2007) runs a horse race between sticky information and sticky prices with dynamic indexation. He concludes that both the sticky information model and the Calvo time-dependent pricing model with dynamic inflation indexation can account for inflation inertia, and suggests considering sticky information as a micro foundation for the hard-wired assumption of time-dependent pricing and indexation to past inflation. In this paper, inattentiveness on the firm side affects both the firm's price setting decision and its production decision (which affects inventory accumulation), and thus information frictions and time-dependent pricing frictions do have very different implications regarding the underlying force driving inventory movements, which affects the model's ability to fit the data.

This paper is also related to the debate about the relative importance of variations in price markups and cost rigidities. Staggered pricing and countercyclical price markups have been the key mechanism generating real effects of monetary shocks in New Keynesian models. Chari et al. (2000) raises the persistence problem facing staggered pricing models, in that empirically plausible degrees of price rigidity generate only a modest degree of output persistence in response to monetary shocks, and cannot account for the estimated output persistence in the U.S. economy. This persistence problem has led researchers to adopt real rigidities to explain the real effects of monetary shocks. Cost rigidity is an external source of real rigidity that reduces the responsiveness of firms' profit-maximizing prices to variation in aggregate output. Christiano et al. (2005) find that cost rigidity is more important than nominal price rigidities in accounting for the observed inertia of inflation and persistence of output. Dotsey and King (2006) reach a similar conclusion, showing that supply side frictions, including produced inputs and variable capacity utilization, that reduce the elasticity of marginal cost with respect to output serve to reduce the extent of price adjustment and generate persistent output responses to monetary shocks.

The other class of real rigidity are internal real rigidities, assumptions on preferences and technology that make it costly for a firm to charge prices that are too different from those of its competitors, and thus reinforce strategic complementarity and countercyclical markups. Examples include a quasi-kinked demand curve, firm-specific capital, and firm-specific labor. Exactly which class of real rigidity is more important has been controversial. Karabarbounis (2014) finds that variations in the gap between the real wage and household's marginal rate of substitution between labor and leisure, which may arise from sticky wages, explain 80% of the countercyclical fluctuations in the labor wedge in the U.S., while variations in price markups account for a small proportion. Closely related to my paper is Kryvtsov and Midrigan (2013). They utilize the tight link between inventory movements, markups and marginal cost in a theoretical model to infer that countercyclical price markups, rather than cost rigidities, must account for most (as high as 90%) of the real effects of aggregate demand shocks. I show that information frictions can weaken this tight link, and thus conclusions about the elasticity of marginal cost with respect to output drawn from inventory behavior need to be assessed with more care. In section 5 I apply a negative money growth shock and show that even when markup variation is relatively low (accounting for only 24% of the real effect), inventories can move less than sales because of the information capacity constraint. The implication of this paper is in line with Basu and Fernald (1997), which finds that the average 2-digit industry appears to produce with constant or even increasing returns, so that markups should be stable or even procyclical. This paper is also in line with the findings of Anderson et al. (2018), which finds that although markups have regional dispersion, they are relatively stable over time.

The relative importance of cost rigidity and markup variation has policy implications besides the persistence problem. Nominal price rigidity points to the importance of frictions in the product market that affect the firm's pricing decision, such as menu costs of price adjustment, a quasi-kinked demand curve, or firm-specific labor or capital. Different forms of strategic complementarity have different welfare implications for steady state inflation or deflation, as pointed out by Levin et al. (2006). For example, consider factor specificity which gives rise to an upward sloping marginal cost schedule. With factor specificity, non-adjusting firms suffer from steady state inflation as the distortion of their lower relative prices increases their cost of production significantly. Generally, when the distortions exists only in the product market, complete stabilization of the price level is sufficient to eliminate inefficient price dispersion and close the output gap, thus meeting the central bank's dual mandate.

In contrast, a rigid marginal cost points to the existence of frictions in factor markets. For example, wage rigidity can result from collective wage bargaining by trade unions or implicit wage contracts. The frictions in the labor market create a new source of distortions, and stabilizing the price level alone becomes suboptimal for welfare maximization. Just as variation in the price level creates price dispersion and inefficient distortions, volatility in wage inflation creates an inefficient distribution of employment across households. The central bank thus need to stabilize price inflation, wage inflation, and the output gap. Erceg et al. (2000) points out that it is impossible for monetary policy to attain a Pareto optimum unless either prices or nominal wages are completely flexible to keep the real wage at its Pareto optimum level. The optimal rule will induce greater flexibility in the more flexible nominal variable. Thomas (2008) studies optimal monetary policy in a setting with labor search and matching frictions and both staggered price and staggered wage setting. It reaches a similar conclusion as Erceg et al. (2000). Nominal wage rigidity gives rise to a rigid real wage, which generates inefficient job creation and inefficient unemployment fluctuations. A zero inflation policy is no longer optimal, and policy makers should use price inflation to bring the real wage closer to the flex-wage target. Generally, frictions in wage setting imply a case against strict price stability. Thus, the source and extent of cost rigidity matters for normative analysis.

In my model of limited information capacity, the deviation of the actual price response from the first-best response has significant persistence and dies out after 25 quarters. Relating to the international economics literature discussing the adjustment of a country's exchange rate toward Purchasing Power Parity, Taylor and Taylor (2004) points out that relative PPP only appears to hold in the long run (after taking a long-run average). Rogoff (1996) notes that the estimated half-life of adjustments is between 3 to 5 years, and the slow speed of adjustment is due to the persistence in nominal variables such as nominal wages and prices. Bergin and

Feenstra (2001) shows that trans-log preferences and pricing to market can significantly increase endogenous persistence. In my model, persistence is generated by both the limited information capacity and nominal wage rigidity.

1.3 The Model

I construct a DSGE model that incorporates inventory investment. This section first lays out the model without pricing frictions or information constraints on the firm side (in subsection 2.2), and then introduces an information friction in the form of a limited information capacity for firms (in subsection 2.3). To facilitate the comparison with Kryvtsov and Midrigan (2013), I assume that the firms' inventory holding motive comes from stock-out avoidance. Firms cannot observe the demand for their goods before production. Since production takes time, inventories are used as a buffer for higher-than-expected demand. The economy consists of final goods firms, intermediate goods firms, households, labor unions, and a government sector.

1.3.1 Households

Household $i \in [0,1]$ chooses consumption $C_t(i)$, hours worked $L_t(i)$, and bonds $B_t(i)$ to maximize its lifetime utility:

$$\max_{\{C_t(i),L_t(i),B_t(i)\}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[\frac{1}{1-\sigma_c}\big(C_t(i) - hC_{t-1}\big)^{1-\sigma_c} - \frac{\psi_l}{1+\sigma_l} L_t(i)^{1+\sigma_l}\Big]$$

subject to the budget constraint:

$$C_t(i) + \frac{B_t(i)}{P_t R_t} + \frac{T_t}{P_t} = \frac{B_{t-1}(i)}{P_t} + \frac{W_t^h L_t(i)}{P_t} + \frac{\Pi_t}{P_t} + \frac{Div_t}{P_t} \tag{1.1}$$

where T_t is a nominal lump-sum tax. The parameter h captures the degree of external habit formation. Household income comes from wages (with nominal wage rate W_t^h), risk-free interest income from bonds, the profit from the intermediate goods sector Π_t, and dividends rebated from labor unions, Div_t.

Since households are homogeneous, they will make the same choices for consumption, hours worked, and bond holdings. The first order conditions are listed as follows (dropping the i index):

$$(\partial C_t) \qquad \Xi_t = \left(C_t - hC_{t-1}\right)^{-\sigma_c} \tag{1.2}$$

$$(\partial L_t) \qquad \psi_l L_t^{\sigma_l} = \Xi_t \frac{W_t^h}{P_t} \tag{1.3}$$

$$(\partial B_t) \qquad \Xi_t = \beta R_t \mathrm{E}_t \left[\frac{\Xi_{t+1}}{\pi_{t+1}} \right] \tag{1.4}$$

where Ξ_t is the Lagrange multiplier associated with the budget constraint.

1.3.2 Final Goods Firms

There is a unit measure of final goods firms. Final goods firms are competitive. They combine differentiated intermediate goods into the final good S_t, which can be used for consumption by the household sector and government. The final goods firms bundle intermediate goods with the following Dixit-Stiglitz aggregator:

$$S_t = \Big[\int_0^1 v_{jt}^{\frac{1}{\theta}} S_{jt}^{\frac{\theta-1}{\theta}}\Big]^{\frac{\theta}{\theta-1}}, \quad \theta > 1 \tag{1.5}$$

where θ is the elasticity of demand for firm j's good and v_{jt} is an *i.i.d.* taste shock for type j good that follows a lognormal distribution $\log(v_{jt}) \sim \mathcal{N}(-\frac{\sigma_v^2}{2}, \sigma_v^2)$ with distribution function $G(\cdot)$. Therefore, the problem of a final goods firm is as follows:

$$\max_{\{S_{jt}\}_{j\in[0,1]}} P_t S_t - \int_0^1 P_{jt} S_{jt} dj$$

subject to equation (1.5) and the no stock out constraint:

$$S_{jt} \leq Z_{jt} \ \forall j \tag{1.6}$$

where Z_{jt} is intermediate goods firm j's quantity of goods available for sale at time t after production.

Solving the above problem gives the demand for intermediate good j as

$$S_{jt} = v_{jt}\Big[\frac{P_{jt} + \gamma_{jt}}{P_t}\Big]^{-\theta} S_t \tag{1.7}$$

where γ_{jt} is the Lagrange multiplier associated with stock out constraint (1.6). The aggregate price P_t is:

$$P_t = \Big[\int_0^1 v_{jt}\big[P_{jt} + \gamma_{jt}\big]^{1-\theta}\Big]^{\frac{1}{1-\theta}} \tag{1.8}$$

1.3.3 Intermediate Goods Firms

Intermediate goods firms are monopolistically competitive. They produce with labor only and are subject to aggregate productivity shocks (as well as the taste shock v_{jt} introduced in equation (1.5)). The production function is

$$Y_{jt} = \exp^{a_t} L_{jt}^{\alpha} \tag{1.9}$$

where a_t is the aggregate productivity. The usual economic logic for specifying decreasing returns in labor inputs is that physical capital, which this paper abstracts from, cannot be adjusted smoothly in the short run. The parameter $\alpha \in (0, 1)$ controls the returns to scale of the production technology, which also determines the sensitivity of real marginal cost to changes in output, and will thus play a crucial role in our estimation.[4]

Timing: I assume that an intermediate goods firm cannot see the realization of its taste shock v_{jt} before its pricing and production decisions are made. After the firm decides on its price and production quantity, v_{jt} is realized and the firm will try to meet all its demand, subject to the no stock out constraint (1.6). After production in period t, the firm will have a quantity of goods available for sale, denoted Z_{jt}, which is the sum of undepreciated inventory stocks carried from the last period,

[4] The purpose of specifying a decreasing returns to scale production technology is to control the responsiveness of the marginal cost to changes in output. Jung and Yun (2006) showed that the movement of inventories will be too sharp and severe after a monetary shock compared to the data without adding an adjustment friction. Alternatively, I assume that the production technology is decreasing returns to scale, as in Kryvtsov and Midrigan (2013), to increase the elasticity of the marginal cost.

$(1-\delta_z)inv_{j,t-1}$, and goods produced this period Y_{jt}. Unsold goods become this period's inventories. Depending on the realization of v_{jt}, there is a chance that the firm may run out of goods to supply, i.e., stock out. Given the demand from the final goods sector (equation (1.7)), the sales of firm j are:

$$S_{jt} = \min\Big\{Z_{jt}, v_{jt}\Big(\frac{P_{jt}}{P_t}\Big)^{-\theta} S_t\Big\} \tag{1.10}$$

Thus, firm j will stock out if the quantity demanded exceeds its quantity available for sale. The stock of goods available for sale, Z_{jt}, and its inventories at the end of the period, inv_{jt}, evolve over time according to the following:

$$Z_{jt} = (1-\delta_z)inv_{j,t-1} + Y_{jt} \tag{1.11}$$

$$inv_{jt} = Z_{jt} - S_{jt} \tag{1.12}$$

Marginal Cost of Production

Given the production function, the (nominal) marginal cost of production of intermediate goods firm j, denoted as Λ_{jt}, can be derived as:

$$\Lambda_{jt} = \frac{W_t}{\alpha \exp^{\frac{a_t}{\alpha}}} Y_{jt}^{\frac{1-\alpha}{\alpha}} \tag{1.13}$$

where W_t is the aggregate nominal wage. Note that the higher is α, the less responsive is marginal cost to fluctuations in production. In the extreme case when α is equal to one (constant returns to scale), marginal cost of production is not influenced by the output level.

Intermediate Firm's Problem without Information Friction

In the absence of pricing and informational frictions, the intermediate firm's problem can be characterized as:

$$\max_{\{Z_{jt},P_{jt}\}} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{P_0 \Xi_t}{P_t \Xi_0} \Big\{ P_{jt} \int \min(v_{jt} \big(\frac{P_{jt}}{P_t}\big)^{-\theta} S_t, Z_{jt}\big) dG(v) - W_t L_{jt} \Big\} \tag{1.14}$$

subject to

$$Y_{jt} = Z_{jt} - (1-\delta_z) \Big[Z_{j,t-1} - \min(v \big(\frac{P_{j,t-1}}{P_{t-1}}\big)^{-\theta} S_{t-1}, Z_{j,t-1}\big) \Big] \tag{1.15}$$

The firm's expected profit is expected sales revenue net of the cost of production. Let v^*_{jt} denote the threshold taste shock level beyond which firm j would stock out given goods available for sale Z_{jt}. Then

$$v^*_{jt} = \frac{Z_{jt}}{\big(\frac{P_{jt}}{P_t}\big)^{-\theta} S_t} \tag{1.16}$$

Intuitively, when the firm has a higher stock of finished goods Z_{jt} or less demand for its goods, its threshold taste shock is higher and it is less likely to hit the stock-out constraint.

Expected sales revenue can be derived as:

$$
\begin{aligned}
P_{jt}\int \min(v(\frac{P_{jt}}{P_t})^{-\theta}S_t, Z_{jt})dG(v) &= P_{jt}(\frac{P_{jt}}{P_t})^{-\theta}S_t\int \min(v, v^*_{jt})dG(v) \\
&= P_{jt}(\frac{P_{jt}}{P_t})^{-\theta}S_t\Big[\int_0^{v^*_{jt}} vdG(v) + v^*_{jt}(1 - F(\log v^*_{jt} + \frac{\sigma_v^2}{2}))\Big] \\
&= P_{jt}(\frac{P_{jt}}{P_t})^{-\theta}S_t F\big(\log v^*_{jt} - \frac{\sigma_v^2}{2}\big) + P_{jt}Z_{jt}\Big[1 - F(\log v^*_{jt} + \frac{\sigma_v^2}{2})\Big] \qquad (1.17)
\end{aligned}
$$

where $F(\cdot)$ is the distribution function of $\mathcal{N}(0, \sigma_v^2)$, and $1 - F(\log v^*_{jt} + \frac{\sigma_v^2}{2})$ is the probability of a stock out.

Decision Rules

Substituting the above expression for expected sales (1.17) into the firm's problem and with some algebraic manipulation, I can obtain the optimality conditions with respect to finished goods stock Z_{jt} and price P_{jt} as:[5]

$$
(\partial Z_{jt}) \qquad 1 - F\big(\log v^*_{jt} + \frac{\sigma_v^2}{2}\big) = \frac{\lambda_{jt} - \beta(1-\delta_z)\mathrm{E}_t\Big[\frac{\Xi_{t+1}}{\Xi_t}\lambda_{j,t+1}\Big]}{\frac{P_{jt}}{P_t} - \beta(1-\delta_z)\mathrm{E}_t\Big[\frac{\Xi_{t+1}}{\Xi_t}\lambda_{j,t+1}\Big]} \qquad (1.18)
$$

$$
(\partial P_{jt}) \qquad \frac{P_{jt}}{P_t} = \frac{\epsilon_{jt}}{\epsilon_{jt}-1}\frac{\beta(1-\delta_z)}{\alpha}\mathrm{E}_t\Big(\frac{\Xi_{t+1}}{\Xi_t}\lambda_{j,t+1}\Big) \qquad (1.19)
$$

where $\lambda_{jt} = \Lambda_{jt}/P_t$ is the real marginal cost of production, and

$$
\epsilon_{jt} = \frac{\theta\int_0^{v^*_{jt}} vdG(v)}{\int \min(v, v^*_{jt})dG(v)} \qquad (1.20)
$$

[5] The derivation is relegated to the appendix.

is θ times the fraction of sales in all states in which the firm does not stock out. Note that ϵ_{jt} can be viewed as the firm's effective elasticity of demand.

The economic intuition of the two optimal decision rules (equations (1.18) and (1.19)) is as follows. The LHS of equation (1.18) is the probability of a stock out, which decreases in Z_{jt}. Note that the numerator on the RHS of equaion (1.18) is the net cost of producing one more unit of goods this period (net of the cost saving from the need to produce next period). The product of the denominator and the LHS is the net benefit of producing one more unit of goods this period: a higher Z_{jt} means the firm is less likely to stock out and more likely to increase its profit made from selling an additional unit of goods, which is the price minus the expected discounted marginal cost next period. The intuition of (1.19)) is that the firm's optimal price is affected by the discounted marginal cost of production in the next period. Furthermore, the higher is the effective elasticity of substitution, the lower is the price-cost markup.

1.3.4 Labor Unions and Wage Rigidity

Staggered wage setting is modeled as in Smets and Wouters (2007).[6] I assume that households supply their homogeneous labor to intermediate labor unions, indexed by $l \in [0, 1]$. The labor union differentiates labor services supplied by households, and sets the nominal wage $W_t(l)$ for differentiated labor service $L_t(l)$, subject

[6] The purpose of incorporating sticky wages is to give rise to a delayed response of the real wage (and thus of real marginal cost) to monetary shocks. Without frictions in wage setting, the real wage is determined by equation (1.3), which tracks changes in output perfectly.

to a Calvo friction.[7] Each labor union supplies differentiated labor to a competitive **labor packer**, which packages differentiated labor through a composite function:

$$L_t = \Big[\int_0^1 L_t(l)^{\frac{\theta_w - 1}{\theta_w}} dl\Big]^{\frac{\theta_w}{\theta_w - 1}}, \qquad \theta_w > 1$$

and supplies L_t to the intermediate goods firms with composite wage rate W_t. The labor packer takes W_t and $W_t(l)$ as given and chooses $\{L_t(l)\}_{l\in[0,1]}$ to maximize its profit:

$$\max_{\{L_t(l)\}_{l\in[0,1]}} W_t\Big[\int_0^1 L_t(l)^{\frac{\theta_w - 1}{\theta_w}} dl\Big]^{\frac{\theta_w}{\theta_w - 1}} - \int_0^1 W_t(l)L_t(l)dl$$

The FOC of the labor packer gives

$$L_t(l) = (\frac{W_t(l)}{W_t})^{-\theta_w} L_t \tag{1.21}$$

From the zero profit condition for the labor packer one obtains the wage cost for the intermediate good producers:

$$W_t = \Big[\int_0^1 W_t(l)^{1-\theta_w}\Big]^{\frac{1}{1-\theta_w}} \tag{1.22}$$

Labor unions are subject to a Calvo wage setting friction, which is the source of nominal wage rigidity in the model. In any given period, each labor union can readjust $W_t(l)$ with probability $1 - \xi_w$. If the union is not allowed to adjust, the wage rate it charges is indexed to past inflation rates, with degree of indexation ℓ_w.

[7] Note that households supply homogenous labor services and receive homogenous wage rate W_t^h in return. It is labor unions that differentiate labor services and set wages $W_t(l)$ for differentiated labor.

If the union is allowed to readjust, the problem is to choose an optimal wage $\tilde{W}_t(l)$ that maximizes the discounted wage income in all future states in which the union is not allowed to adjust (taking the wage rate paid to households $\{W^h_{t+s}\}$ as given):

$$\max_{\tilde{W}_t(l)} \mathrm{E}_t \Big\{ \sum_{s=0}^{\infty} \xi_w^s \beta^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} \Big[W_{t+s}(l) - W^h_{t+s} \Big] L_{t+s}(l) \Big\}$$

subject to

$$L_{t+s}(l) = \big(\frac{W_{t+s}(l)}{W_{t+s}}\big)^{-\theta_w} L_{t+s} \tag{1.23}$$

where

$$W_{t+s}(l) = \tilde{W}_t(l) X_{t,t+s} \tag{1.24}$$

$$X_{t,t+s} = \begin{cases} 1 & \text{for } s = 0, \\ \Pi_{l=1}^{s} \pi_{t+l-1}^{\ell_w} & \text{for } s = 1, \ldots, \infty \end{cases}$$

The first order condition is:

$$(\partial \tilde{W}_t(l)) \qquad \mathrm{E}_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} \Big[X_{t,s} L_{t+s}(l) - \theta_w \big(\tilde{W}_t(l) X_{t,s} - W^h_{t+s} \big) \cdot \Big(\frac{X_{t,s} \tilde{W}_t(l)}{W_{t+s}} \Big)^{-\theta_w - 1} \big(\frac{X_{t,s}}{W_{t+s}} \big) L_{t+s} \Big] = 0$$

Substituting in equation (1.23), the above FOC can be simplified as:

$$\mathrm{E}_t \sum_{s=0}^{\infty} \xi_w^s \beta^s \frac{\Xi_{t+s} P_t}{\Xi_t P_{t+s}} (\theta_w - 1) L_{t+s}(l) \Big[\frac{\theta_w}{\theta_w - 1} W_{t+s}^h - X_{t,s} \tilde{W}_t(l) \Big] = 0 \tag{1.25}$$

Since all the unions allowed to adjust face the same problem and will set the same optimal wage rate, I can write $\tilde{W}_t(l)$ as $\tilde{W}_t$. The aggregate wage can be written as:

$$W_t = \big[(1 - \xi_w) \tilde{W}_t^{1-\theta_w} + \xi_w (\pi_{t-1}^{\ell_w} W_{t-1})^{1-\theta_w} \big]^{\frac{1}{1-\theta_w}} \tag{1.26}$$

The above equations (1.25) and (1.26) give rise to the wage Phillips Curve, and parameters ξ_w as well as ℓ_w control the degree of wage stickiness.

1.3.5 Government

At time t, the government pays back B_{t-1}, issues new bonds, B_t, collects lump-sum tax T_t and consumes G_t. The government budget constraint is

$$P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t} \tag{1.27}$$

where G_t is real government expenditure, assumed to be a constant fraction of aggregate output:

$$G_t = g_* Y_t$$

The monetary authority follows a feedback nominal interest rate rule that responds to the lagged nominal interest rate, current inflation, and the current deviation of output from its steady state level:

$$\frac{R_t}{R_*} = \Big(\frac{R_{t-1}}{R_*}\Big)^{\rho_R} \Big[\big(\frac{\pi_t}{\pi_*}\big)^{\phi_\pi} \big(\frac{Y_t}{Y_*}\big)^{\phi_Y} \Big]^{1-\rho_R} \exp(\varepsilon_{r,t}) \tag{1.28}$$

where $\varepsilon_{r,t}$ is the interest-rate shock that follows

$$\varepsilon_{r,t} \sim \mathcal{N}(0, \sigma_r^2)$$

.

1.3.6 Resource Constraint

Substituting the government budget constraint (1.27) into the household budget constraint (1.1), and integrating across all households, one obtains:

$$C_t + G_t = \frac{W_t^h L_t}{P_t} + \frac{\Pi_t}{P_t} + \frac{Div_t}{P_t} \tag{1.29}$$

Aggregate profit earned by the intermediate goods firms, Π_t, can be expressed as:

$$\Pi_t = \int_0^1 \Pi_{tj} dj = \int_0^1 P_{jt} S_{jt} - \int_0^1 W_t L_{jt}$$

Using the inventory evolution equations (1.11) and (1.12), we have:

$$S_{jt} = Y_{jt} - \big[inv_{jt} - (1-\delta_z) inv_{j,t-1} \big]$$

Thus, aggregate profit can be written as:

$$\Pi_t = \int_0^1 P_{jt} Y_{jt} dj - \int_0^1 P_{jt} \big[inv_{jt} - (1-\delta_z) inv_{j,t-1} \big] dj - W_t L_t \tag{1.30}$$

where L_t is total labor supplied by the labor packer, which should be equal to aggregate labor demand by firms:

$$L_t = \int_0^1 L_{jt} dj \tag{1.31}$$

Using the labor packer's zero profit condition

$$W_t L_t = \int_0^1 W_t(l) L_t(l) dl = W_t^h L_t + Div_t \tag{1.32}$$

the resource constraint (equation (1.29)) becomes

$$C_t + G_t + \int_0^1 \frac{P_{jt}}{P_t} \big[inv_{jt} - (1-\delta_z) inv_{j,t-1} \big] dj = \int_0^1 (\frac{P_{jt}}{P_t}) Y_{jt} dj \tag{1.33}$$

Thus, market clearing requires aggregate output be equal to the sum of aggregate consumption, government expenditure, and inventory investment.

1.3.7 Shocks

Other than the *i.i.d.* (across firms and time) taste shocks that follow lognormal distribution $\log(v_{jt}) \sim \mathcal{N}(-\frac{\sigma_v^2}{2}, \sigma_v^2)$, there are two exogenous aggregate shocks in the model:

1. The interest rate shock $\varepsilon_{r,t}$ in the nominal interest rate rule follows a Gaussian white noise process: $\varepsilon_{r,t} \sim \mathcal{N}(0, \sigma_r^2)$

2. The aggregate technology shock a_t follows a first order autoregressive process with mean zero:

$$a_t = \rho_a a_{t-1} + \varepsilon_{a,t}, \quad \varepsilon_{a,t} \sim \mathcal{N}(0, \sigma_a^2)$$

The innovations $\epsilon_{r,t}$ and $\varepsilon_{a,t}$ are mutually independent.

1.3.8 Intermediate Goods Firms : Rational Inattention

In the model presented in the previous subsection, decision makers in intermediate goods firms make optimal pricing and production decisions without frictions. In the numerical experiments in section 4, I impose either a Calvo pricing friction or an information friction on the intermediate goods firms, and compare the impulse responses generated by these two assumptions. The information friction is modeled as rational inattention, in that intermediate goods firms are inattentive to shocks, as opposed to a setting in which the firms have perfect information but may not be allowed to change prices in each period. The two settings are identical in all aspects except for the exact friction affecting the firm's decisions.

Inattentive firms are modeled as follows. I assume that intermediate firms devote limited attention to aggregate shocks hitting the economy, which include the aggregate TFP shocks and the nominal interest rate shocks.[8] They optimally

[8] The detailed description of the processes of aggregate TFP and the interest rate shocks is

allocate their attention to various sources of shocks, as well as to their pricing and production decisions, subject to a flow of information constraint.[9]

Attention Problem

The decision maker in firm j chooses the amount of attention allocated to the pricing and production decisions. He chooses the laws of motion for the price p_{jt} and stock of goods for sale Z_{jt} to maximize the expected discounted sum of profit minus the cost of paying attention:

$$\max_{\kappa,\{B_k(L),C_k(L)\}_{k=1,2}} \Big\{ \mathrm{E}_{j,-1}\Big[\sum_{t=0}^{\infty} \beta^t \big(\frac{1}{2}(\boldsymbol{x}_{jt} - \boldsymbol{x}_{jt}^{\dagger})' H_{x,0} (\boldsymbol{x}_{jt} - \boldsymbol{x}_{jt}^{\dagger}) + (\boldsymbol{x}_{jt} - \boldsymbol{x}_{jt}^{\dagger})' H_{x,1} (\boldsymbol{x}_{j,t+1} - \boldsymbol{x}_{j,t+1}^{\dagger}) \big) \Big] - \frac{\mu}{1-\beta}\kappa \Big\} \tag{1.34}$$

subject to the law of motion for the optimal action under perfect information

$$\boldsymbol{x}_{jt}^{\dagger} = \boldsymbol{x}_{jt}^{\dagger R} + \boldsymbol{x}_{jt}^{\dagger a} \tag{1.35}$$

$$= A_1(L)\varepsilon_{r,t} + A_2(L)\varepsilon_{a,t} \tag{1.36}$$

where $\boldsymbol{x}_{jt}^{\dagger} \equiv [p_{jt}^{\dagger}; Z_{jt}^{\dagger}]'$ denotes the firm's optimal actions with full information,[10] which depend on current and lagged values of the underlying shocks to the inter-

deferred to section 2.8.

[9] Since the taste shocks v_{jt}, which intermediate firms cannot observe before making decisions, are i.i.d. across firms and time, intermediate firms do not consider taste shocks when making their decisions.

[10] In this particular subsection, as well as in Appendix C, I abuse notation for ease of exposition to let variables (except for shocks) with time subscripts denote log deviations from the non-stochastic steady state values of the corresponding variables introduced in subsection 2.2. Therefore, $\boldsymbol{x}_{jt}^{\dagger}$ and $\boldsymbol{x}_{jt}$ are vectors of log deviations, as is the vector $\boldsymbol{\zeta}_t$ introduced later.

est rate and technology, and $H_{x,0} \equiv H_{\boldsymbol{x}_{jt},\boldsymbol{x}_{jt}}$ and $H_{x,1} \equiv H_{\boldsymbol{x}_{jt},\boldsymbol{x}_{j,t+1}}$ are matrices of second derivatives of the lifetime discounted profit function. In the above equation $\{A_k(L)\}_{k=1,2}$ (as well as $\{B_k(L), C_k(L)\}_{k=1,2}$ introduced below) are absolutely summable matrices of lag operators. Note that the firm's optimal decisions (and actual decisions) do not need to respond to its idiosyncratic taste shock, since taste shocks are *i.i.d.* across time and each period the firm has to make decisions before the taste shock is realized.

Following Mackowiak and Wiederholt (2015), I assume that the decision maker inherently cannot observe the realization of structural shocks $\varepsilon_{r,t}$ and $\varepsilon_{a,t}$ with full precision, but faces idiosyncratic noise shocks denoted by ν_{jt}^R and ν_{jt}^a respectively. Those noise shocks are assumed to be mutually independent (and also independent of the underlying structural shocks) Gaussian white noises with unit variances. The decision maker chooses the law of motion for the actual actions (signals):

$$\boldsymbol{x}_{jt} = \underbrace{B_1(L)\varepsilon_{r,t} + C_1(L)\nu_{jt}^R}_{\boldsymbol{x}_{jt}^R} + \underbrace{B_2(L)\varepsilon_{a,t} + C_2(L)\nu_{jt}^a}_{\boldsymbol{x}_{jt}^a} \tag{1.37}$$

where $\boldsymbol{x}_{jt} \equiv [p_{jt}; Z_{jt}]'$ denotes the actual decisions, which can be decomposed into two parts: the actual responses to the interest rate shock x_{jt}^R and to the aggregate productivity shock x_{jt}^a.[11] The term κ is the information capacity limiting the

[11] Note that this decomposition holds since I assume the structural shocks and noise shocks are mutually independent.

information flow between the firm's optimal decisions and actual decisions:

$$\mathcal{I}\Big(\big\{\boldsymbol{x}_{jt}^{\dagger R}, \boldsymbol{x}_{jt}^{\dagger a}\big\}_{t=0}^{\infty}; \big\{\boldsymbol{x}_{jt}^{R}, \boldsymbol{x}_{jt}^{a}\big\}_{t=0}^{\infty}\Big) \leq \kappa \tag{1.38}$$

where $\mathcal{I}\big(\{\boldsymbol{x}_{jt}^{\dagger}\}, \{\boldsymbol{x}_{jt}\}\big)$ is the mutual information between the two stochastic processes $\{\boldsymbol{x}_{jt}^{\dagger}\}_{t=0}^{\infty}$ and $\{\boldsymbol{x}_{jt}\}_{t=0}^{\infty}$, and can be defined as the reduction in entropy:

$$\begin{aligned}\mathcal{I}\big(\{\boldsymbol{x}_{jt}^{\dagger}\}, \{\boldsymbol{x}_{jt}\}\big) &= \lim_{T\to\infty}\frac{1}{T}\big[I(\boldsymbol{X}^{\dagger}) - I(\boldsymbol{X}^{\dagger}|\boldsymbol{X})\big] \\ &= \lim_{T\to\infty}\frac{1}{T}\Big[\frac{1}{2}\log\big(\frac{\det\Omega_{\boldsymbol{X}^{\dagger}}}{\det\Omega_{\boldsymbol{X}^{\dagger}|\boldsymbol{X}}}\big)\Big]\end{aligned}$$

where $I(\boldsymbol{X}^{\dagger})$ denotes the entropy of the random vector of optimal actions up to $T-1$, $\boldsymbol{X}^{\dagger} = (\boldsymbol{x}_0^{\dagger}, \ldots, \boldsymbol{x}_{T-1}^{\dagger})$; $I(\boldsymbol{X}^{\dagger}|\boldsymbol{X})$ denotes the conditional entropy of $\boldsymbol{X}^{\dagger}$ conditional on the information of $\boldsymbol{X} = (\boldsymbol{x}_0, \ldots, \boldsymbol{x}_{T-1})$; and $\Omega_{\boldsymbol{X}^{\dagger}}$ and $\Omega_{\boldsymbol{X}^{\dagger}|\boldsymbol{X}}$ denote the covariance matrix of $\boldsymbol{X}^{\dagger}$ and the conditional covariance matrix of $\boldsymbol{X}^{\dagger}$ conditional on $\boldsymbol{X}$ respectively. The second equality follows from the assumption that the shocks affecting the firm's decision are Gaussian.

The sum of the first and second terms in the firm's problem (1.34) is the second-order approximation to the difference of lifetime discounted profit from following the law of motion of the actual actions rather than that of the optimal actions. For a decision maker in a firm, the benefit of paying more attention to different sources of disturbances is that his actual actions (and thus the firm's profit) become closer to the optimal actions. However, paying more attention means the decision maker needs to process more information, which incurs a per-period marginal flow of information cost μ, assumed to be a constant. A decision maker compares the benefit and cost of paying attention and optimally allocates information flow. Since the marginal cost of information flow μ is assumed to be constant, and the disturbances coming from different sources are assumed to be independent, the optimal amount of attention allocated to each disturbance is independent of the amount of attention allocated to other shocks. This simplifies the computation process as I can focus on a particular shock when I simulate the firm's optimal and actual decisions.

Note that it is implicitly assumed that the signals observed by the decision maker of a firm will contain some noise, represented by ν_{jt}'s. Indeed, if firms can observe $\varepsilon_{r,t}$ and $\varepsilon_{a,t}$ without effort, then the decision maker can easily follow optimal decision rules. The essence of inattention is that the decision maker can endogenously choose the information structure of its signals through choosing the polynomials of lag operators $B_k(L)$ and $C_k(L), \quad k = 1, 2$. If there is no information capacity constraint, the decision maker can potentially make the signals extremely precise, so that $B_k(L) = A_k(L)$ and $C_k(L) = 0, \quad k = 1, 2$. However, in the presence of the capacity constraint, doing so would incur too much information cost. The firm will thus focus more on the shock that matters the most for its objective. The following proposition characterizes the optimal decision rules $\boldsymbol{x}_{jt}^{\dagger} \equiv [p_{jt}, Z_{jt}]'$ as driven by the vector of state variables exogenous to the firm $\boldsymbol{\zeta}_t = [w_t, S_t, \Xi_t, a_t]'$ (and its one-period lead and lag), which the firm also cannot observe with full precision.[12]

[12] Appendix C contains the derivation of the firm's objective function (1.34) and the proof of this proposition.

Proposition 1.

Let $\boldsymbol{\zeta}_t = [w_t, S_t, \Xi_t, a_t]'$ denote the vector of variables exogenous to the firm. Optimal decisions $\{\boldsymbol{x}^\dagger_{jt} \equiv [p^\dagger_{jt}, Z^\dagger_{jt}]'\}_{t=-1}^{\infty}$ of the firm are determined by the firm's optimal stock condition (equation (1.18)) and optimal pricing condition (equation (1.19)), and satisfy:

(i) The economy starts from the non-stochastic steady state: $\boldsymbol{x}^\dagger_{j,-1} = (0,0)'$.

(ii) In each period $t \geq 0$, $\boldsymbol{x}^\dagger_{jt}$ satisfies

$$\boldsymbol{x}^\dagger_t = H^{-1}_{\boldsymbol{x},0}\Big[\frac{1}{\beta}H_{\boldsymbol{x},1}\boldsymbol{x}^\dagger_{t-1} + H_{\boldsymbol{x}\boldsymbol{\zeta},-1}\boldsymbol{\zeta}_{t-1}\Big] + H^{-1}_{\boldsymbol{x},0}H_{\boldsymbol{x}\boldsymbol{\zeta},0}\boldsymbol{\zeta}_t + H^{-1}_{\boldsymbol{x},0}\mathrm{E}_t\Big[H_{\boldsymbol{x},1}\boldsymbol{x}^\dagger_{t+1} + H_{\boldsymbol{x}\boldsymbol{\zeta},1}\boldsymbol{\zeta}_{t+1}\Big] \tag{1.39}$$

where $H_{\boldsymbol{x},-1} \equiv H_{\boldsymbol{x}_{jt},\boldsymbol{x}_{j,t-1}}$, $H_{\boldsymbol{x}\boldsymbol{\zeta},0} \equiv H_{\boldsymbol{x}_{jt},\boldsymbol{\zeta}_t}$, $H_{\boldsymbol{x}\boldsymbol{\zeta},-1} \equiv H_{\boldsymbol{x}_{jt},\boldsymbol{\zeta}_{t-1}}$, and $H_{\boldsymbol{x}\boldsymbol{\zeta},1} \equiv H_{\boldsymbol{x}_{jt},\boldsymbol{\zeta}_{t+1}}$ are Hessian matrices from the firm's optimality conditions evaluated at the steady state. $\mathrm{E}_t[\cdot]$ is the expectation operator conditional on all available information in period t (including the evolution of $\{\boldsymbol{\zeta}_t\}$ up to the current period t).

Equation (1.39) can replace equation (1.36) and help pin down optimal decision rules in our algorithm to solve for the equilibrium.

1.3.9 Equilibrium under Rational Inattention

Definition. The equilibrium of the model with inattentive firms consists of the decision rules of households $\{C_t(i)\}$, $\{L_t(i)\}$, $\{B_t(i)\}$; firms $\{P^\dagger_{jt}\}$,$\{Z^\dagger_{jt}\}$, $\{P_{jt}\}$,$\{Z_{jt}\}$

$\{S_{jt}\}$, $\{inv_{jt}\}$, $\{Y_{jt}\}$, $\{L_{jt}\}$; labor unions $\{L_t(l)\}_{l\in[0,1]}$; and the government $\{G_t\}$, $\{B_t\}$, $\{T_t\}$; aggregate allocations $\{Y_t\}$,$\{L_t\}$,$\{\Pi_t\}$, $\{Div_t\}$ and prices $\{P_t\}$, $\{W_t^h\}$, $\{W_t\}$; the realization of aggregate and idiosyncratic states $\{R_t\}$, $\{a_t\}$ $\{v_{jt}\}$, as well as the realizations of firm specific signals, such that:

(i) Given the wage rate $\{W_t^h\}$, the nominal interest rate $\{R_t\}$, and the inflation rate $\{\pi_t\}$, each household's decision rule $\{C_t(i)\}$, $\{L_t(i)\}$, $\{B_t(i)\}$ satisfies her optimality conditions (1.2) - (1.4).

(ii) For each intermediate goods firm j, given aggregate sales, S_t, prices $\{P_t\}$, wage $\{W_t\}$, and the realization of shocks and signals, the optimal decision rules $\{P_{jt}^{\dagger}\}$ and $\{Z_{jt}^{\dagger}\}$ evolve according to equation (1.39), and the actual decision rules $\{P_{jt}\}$ and $\{Z_{jt}\}$ evolve according to equation (1.37); and the absolutely summable coefficients $\{B(L), C(L)\}$ maximize the firm's objective (1.34) subject to the information capacity constraint (1.38). Furthermore, given the initial inventory holding $inv_{j,-1}$, $\{P_{jt}\}$, $\{Z_{jt}\}$, $\{S_{jt}\}$, $\{inv_{jt}\}$, $\{Y_{jt}\}$, and $\{L_{jt}\}$ evolve according to equations (1.9)-(1.12).

(iii) Given aggregate labor demand $\{L_t\}$, aggregate price $\{P_t\}$, aggregate wage $\{W_t\}$, the wage paid to households $\{W_t^h\}$, and the backward indexation scheme, a labor union's decisions $\{\tilde{W}_t(l)\}$, $\{W_t(l)\}$ and $\{L_t(l)\}$ satisfy equations (1.23) - (1.25).

(iv) Given initial bond issuance B_{-1} and initial nominal interest rate R_{-1}, aggregate output $\{Y_t\}$, and the inflation rate $\{\pi_t\}$, variables $\{G_t\}$, $\{B_t\}$ and $\{T_t\}$

satisfy the government budget constraint (equation (1.27)), and $\{R_t\}$ satisfies the interest rate rule (1.28).

(v) The aggregate price $\{P_t\}$ satisfies equation (1.8), and the aggregate wage $\{W_t\}$ satisfies equation (1.26).

(vi) Labor demand equals labor supply:

$$\left[\int_0^1 L_t(l)^{\frac{\theta_w - 1}{\theta_w}} dl\right]^{\frac{\theta_w}{\theta_w - 1}} = L_t = \int_0^1 L_{jt} dj$$

In addition, aggregate profit $\{\Pi_t\}$ satisfies equation (1.30); $\{W_t^h\}$ and $\{Div_t\}$ satisfy equation (1.32). And the aggregate resource constraint (equation (1.33)) holds (goods market clears).

1.4 Comparative Statics & Discussion

1.4.1 Comparative Statics

The two optimality conditions of the frictionless intermediate firms' problem, which are reprinted here for convenience as (1.40) and (1.41), deserve more elaboration, since they will provide useful intuition for the comparison of model dynamics under the two distinct frictions. Here I present the comparative statics of an interest rate hike and a drop in the real marginal cost.

$$(\partial Z_{jt}) \qquad 1 - F\Big(\log v_{jt}^* + \frac{\sigma_v^2}{2}\Big) = \frac{\lambda_{jt} - \beta(1-\delta_z)\mathrm{E}_t\Big[\frac{\Xi_{t+1}}{\Xi_t}\lambda_{j,t+1}\Big]}{\frac{P_{jt}}{P_t} - \beta(1-\delta_z)\mathrm{E}_t\Big[\frac{\Xi_{t+1}}{\Xi_t}\lambda_{j,t+1}\Big]} \tag{1.40}$$

$$(\partial P_{jt}) \qquad \frac{P_{jt}}{P_t} = \frac{\epsilon_{jt}}{\epsilon_{jt}-1}\frac{\beta(1-\delta_z)}{\alpha}\mathrm{E}_t\Big(\frac{\Xi_{t+1}}{\Xi_t}\lambda_{j,t+1}\Big) \tag{1.41}$$

When the real interest rate increases, the discount factor $\mathrm{E}_t\Big[\frac{\Xi_{t+1}}{\Xi_t}\Big]$ decreases (through the household's Euler equation). Therefore, the right-hand side of equation (1.40) increases, which means that *ceteris paribus* the net cost of producing goods becomes higher relative to the net benefit. The firm responds by decreasing Z_{jt}, increasing the probability of a stock out. Thus, production and the inventory stock would fall. The intuition is that the real interest rate is the opportunity cost of accumulating and carrying inventories. As the opportunity cost increases, firms draw down their inventories. Conversely, if the current marginal cost λ_{jt} falls relative to the expected future marginal cost, then all else equal the right-hand side of equation (1.40) decreases, the net cost of producing goods decreases relative to the net benefit, and firms produce more and hold more Z_{jt}. It turns out that these inter-temporal substitution effects make Z_{jt} very sensitive to movements in the discount factor and to movements in the relative marginal cost of production. Therefore, when a monetary shock hits and the real interest rate goes up, for Z_{jt} to decrease by less than sales, current real marginal cost λ_{jt} must fall by more than the expected discounted marginal cost next period. This is why Kryvtsov and Midrigan (2013) reach the conclusion that for inventories to move slowly, marginal cost must be very elastic on impact with respect to nominal disturbances.

Figure 1.1 plots the movement of the Z_j surface, which I define as the LHS of equation (1.40) minus the RHS, for different combinations of Z_{jt} and relative price P_{jt}/P_t, when there is a rise in the real interest rate, holding all other variables exogenous to the firm at their steady state values (under a generic set of parameters). The darker shape is the original Z_j surface and the lighter shape is the surface after the interest rate change. We can see that a higher interest rate shifts the surface downward. Conversely, figure 1.2 shows the effect of a lower current real marginal cost on the Z_j surface. We can see that lower marginal cost shifts the surface from its original position upward and to the right. These comparative statics confirm the intuition described above that marginal cost must be elastic and fall sufficiently following an interest rate hike to curb the movement of Z_{jt} (and thus the change in inventories).

It is clear from equation (1.41) that, characteristic of a monopolistically competitive environment, the firm's optimal price is a markup over the expected value of discounted real marginal cost in the next period. The difference is that the effective elasticity of substitution ϵ_{jt} is θ times the share of sales in states in which the firm does not stock out, which reflects the non-linearity coming from the stock out constraint. The firm's markup depends on the probability of a stock out, which decreases with a higher threshold level v_{jt}^* as firm j has higher Z_{jt} for sale. The optimal price depends on expected next-period marginal cost instead of current marginal cost. Movements in real marginal cost change the shadow value of goods, and the firm optimally adjusts its price in the absence of frictions. Figure 1.3 shows

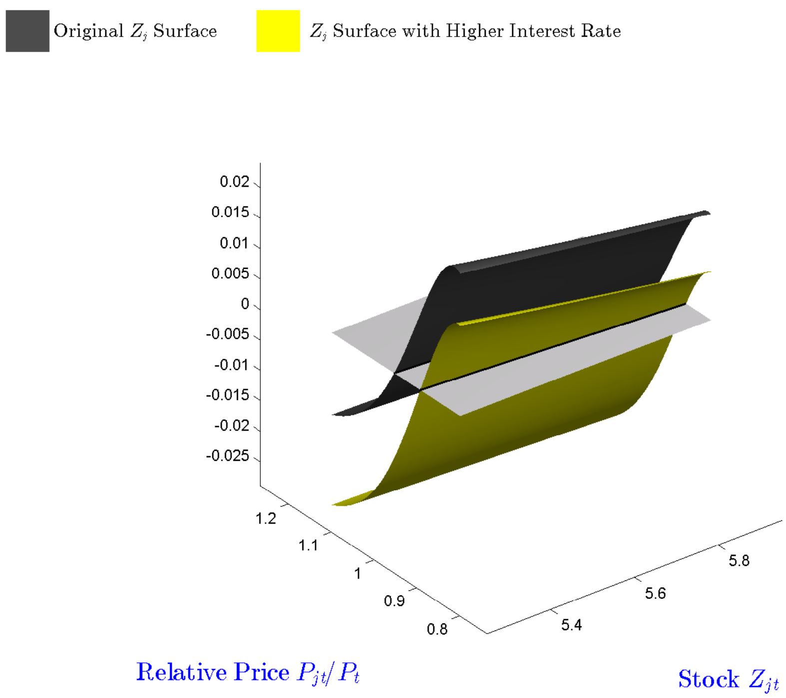

Fig. 1.1: Production Decision: Comparative Statics of Higher Real Interest Rate

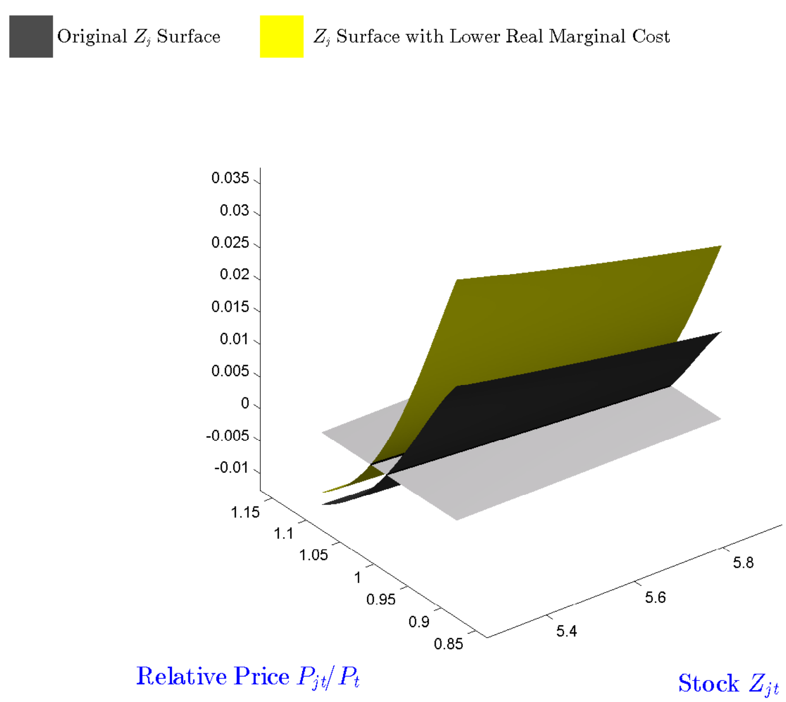

Fig. 1.2: Production Decision: Comparative Statics of Lower Real Marginal Cost

the effect of a lower marginal cost of production on the P_j surface, which I define as the LHS of equation (1.41) minus its RHS. The darker shape is the original P_j surface and the lighter shape is the surface after the drop in real marginal cost when all other variables exogenous to the firm are kept at the steady state levels. We can see that a drop in marginal cost shifts the surface upward, so *ceteris paribus* movements in real marginal cost push the firm's price in the same direction.

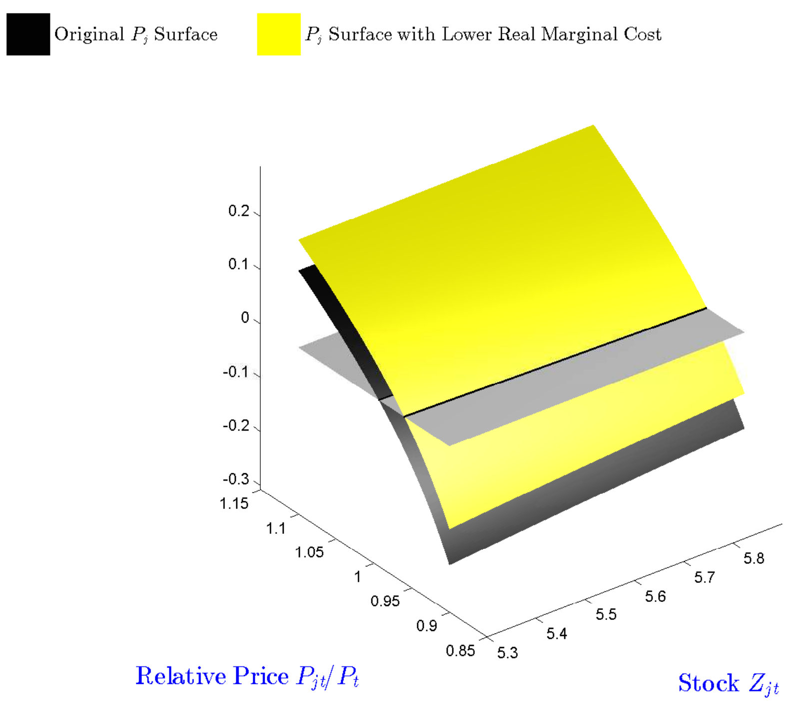

Fig. 1.3: Pricing Decision: Comparative Statics of Lower Real Marginal Cost

Another result is that the quantity of Z_{jt} affects the firm's optimal markup. When Z_{jt} falls, the threshold level v^*_{jt} decreases, increasing the probability of a stock out. This reduces the effective elasticity of demand ϵ_{jt}, and thus increases the markup. This fact is reflected by the negative slope of the line representing the intersection of the P_j surface with the zero plane. When marginal cost is rigid, the optimal price can actually increase after a tightening monetary shock due to the endogenous change in the desired markup induced by the stock-out constraint.[13]

Having established the effects of a change in the real interest rate and a change in the marginal cost on the optimality conditions w.r.t. Z_j and P_j, I combine the two optimality conditions in two-dimensional graphs to show the equilibrium comparative static effects. Figure 1.4 shows the equilibrium *ceteris paribus* effect of a rise in real interest rate, where the darker lines are the (Z_j, P_j) combinations that satisfy optimality condition (1.40) before and after the rise in the real interest rate, and the lighter lines correspond to the optimality condition (1.41). One can see that a rise in the real rate pushes both the Z_j line and P_j line to the left. This will cause a drop in Z_j (and thus production and ex post inventory stock), but the equilibrium price does not necessarily drop. Figure 1.5 shows the *ceteris paribus* effect of a drop in the real marginal cost. This will push the Z_j line to the right and push the P_j line further to the left, causing a drop in the equilibrium price. This is the reason that a drop in marginal cost can balance the effect of a rise in the real interest rate on Z_j. However, it also inevitably creates cost pressure for firms to mark down their price. The information capacity constraint introduced in subsection 2.3 will resolve this conundrum by making the actual responses of firms' decisions deviate from the comparative statics depicted here. In other words, actual movements in Z_j and P_j lines will be smaller under information capacity constraints.

[13] This mechanism implies that the rational inattention model, under rigid cost of production, can potentially account for the "price puzzle", the perverse temporary rise in the price level after monetary tightening that has been documented in some empirical studies.

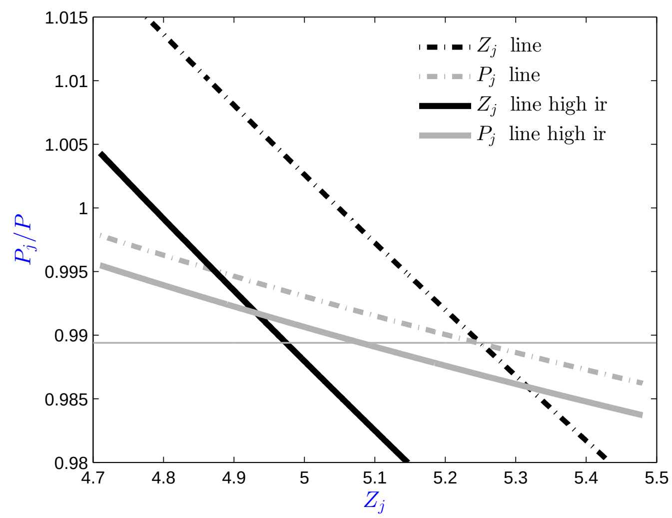

Fig. 1.4: Comparative Statics of Higher Interest Rate

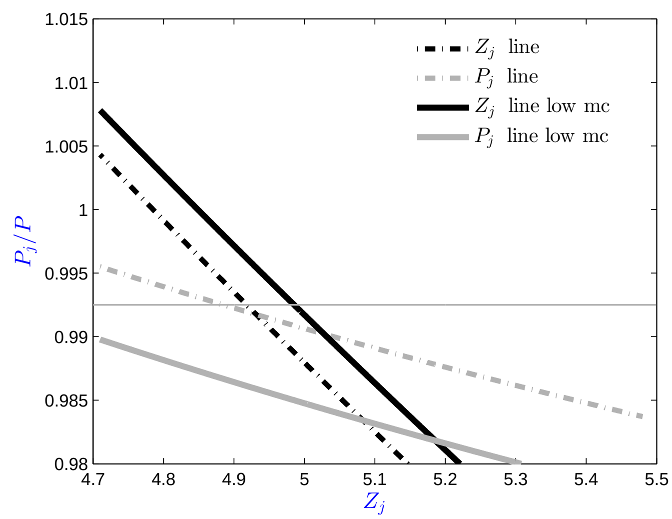

Fig. 1.5: Comparative Statics of Lower Real Marginal Cost

1.4.2 The Role of Information Frictions

The above exposition and comparative statics establish that when the real interest rate moves, the firm would adjust Z_j abruptly in the opposite direction unless there is a corresponding offsetting movement in real marginal cost. In other words, theory predicts a tight connection between inventory dynamics and the movement of marginal production cost following an interest rate shock. Common frictions in the literature to regulate the movement of inventory investment, such as decreasing returns to scale, labor adjustment costs, and the quadratic cost of deviating from a target stock-sales ratio, all serve to increase the elasticity of the marginal cost to the quantity of output, and thus all imply strong procyclicality of marginal production cost. This is why inventory behavior is seen as an evidence for elastic marginal cost and countercyclical markup variation (Kryvtsov and Midrigan (2013)). In this paper, I argue that information frictions serve to limit the response of inventories without requiring marginal cost of production to be elastic. Without perfect information about real input prices and the real discount factor, P_{jt} and Z_{jt} (and thus the inventory stock) do not fully adjust in the short run to the relative movements in the real interest rate and the marginal cost of production.

The intuition is quite simple. To make the inventory decision, firms not only need to observe their current demand and input prices, but they also need to know the aggregate price level and need to form expectation about future marginal costs. If firms cannot observe the aggregate variables in real time or cannot disentangle the exact sources of the shocks pushing aggregate variables, their reactions will be delayed. To fix ideas, consider for the moment a sticky information environment à la Mankiw and Reis (2002), in which firms update their information sporadically. Suppose that each period a firm can update its information with probability $(1-\phi)$, $0 < \phi \leq 1$. Therefore, each period there are a mass of $(1-\phi)\phi^{\tau}$ firms who make decisions based on stale information sets with vintage τ, $\tau = 1, 2, \ldots, \infty$. Assume $\alpha = 1$ so that all intermediate firms face the same marginal cost of production. After log linearization of the optimality condition w.r.t Z_j (equation (1.40)) and summing over all firms, I get the following expression for the log deviation of the stock-sales ratio from its steady state value:

$$
\begin{aligned}
-\eta(z_t - s_t) =& \frac{1}{1-\beta(1-\delta_z)}(1-\phi)\sum_{\tau=0}^{\infty}\phi^{\tau}\mathrm{E}_{t-\tau}(mc_t) \\
&+ \frac{(p_* - mc_*)\beta(1-\delta_z)}{[1-\beta(1-\delta_z)][p_* - \beta(1-\delta_z)mc_*]}(1-\phi)\sum_{\tau=0}^{\infty}\phi^{\tau}\mathrm{E}_{t-\tau}(rr_t - mc_{t+1}) \\
&+ \eta\Big[s_t - (1-\phi)\sum_{\tau=0}^{\infty}\phi^{\tau}\mathrm{E}_{t-\tau}(s_t)\Big]
\end{aligned}
$$

where lower case letters denote log deviations from the steady state values, and mc_t and rr_t denote the log deviations of the real marginal cost λ_t and the real interest rate respectively. $\eta > 0$ is a coefficient whose value depends on the parameters of the model. The final term on the right hand side is an expectation error term. When $\phi = 0$ the above equation collapses to its full information counterpart. The larger is ϕ, the stickier is the information and Z_j responds less to the relative variation in the real interest rate and the real marginal cost.

Instead of assuming sticky information, this paper follows Sims (2003) in assuming that firms face an information processing capacity constraint and are rationally inattentive. Different sources of shocks (such as productivity shocks and interest rate shocks) may imply different persistence of the variation in marginal cost, and firms need to disentangle the exact sources and magnitudes of the structural shocks to correctly forecast future marginal cost. Even if firms can observe the nominal interest rate R_t, they need to forecast π_{t+1} to know the real interest rate they face. In short, intermediate firms observe aggregate variables with noise. They make decisions P_j and Z_j based on imprecise information, and due to the information capacity constraint their actual decisions deviate from their first-best actions (defined as the actions under no information capacity constraint, $P_j^{\dagger}$ and $Z_j^{\dagger}$). Therefore, even if the aggregate conditions dictate a significant drop in $Z_j^{\dagger}$ (for example, after an increase in the interest rate), the firm's actual change in Z_j may be smaller. In the appendix, I derive a linear quadratic representation of the distance of the firm's actual profit from its first-best profit, which gives us the firm's objective under rational inattention (1.34). The distance depends on $P_j - P_j^{\dagger}$, $Z_j - Z_j^{\dagger}$, and the interaction between the two decisions. Other than being less ad hoc than the sticky information assumption, a rational inattention setting enables one to analyze how the firm allocates attention (information flow) between P_j and Z_j.

1.5 Empirical & Numerical Results

1.5.1 Empirical Results: Vector Autoregression

I estimate the dynamic response of key aggregates to monetary policy shocks in the data using a vector autoregression (VAR). The specification is based on a standard monetary VAR that appears in Christiano et al. (1996) and Evans and Marshall (1998), augmented by two variables pertaining to inventory accumulation, namely the inventory-sales ratio and finished good inventories in the manufacturing and trade sector. It is assumed that the monetary policy instrument is the Fed Funds rate, denoted as FF_t, which is determined by the following relationship

$$FF_t = f(\Omega_t) + \epsilon_{r,t}$$

where f is a linear function that describes the monetary authority's reaction to the state of the economy, Ω_t is the information set available to the monetary authority at time t, and $\epsilon_{r,t}$ is an exogenous shock to the monetary policy reaction function, which reflects non-systematic factors that affect monetary policy decisions. Define

$\mathcal{Y}_t$ as the 9-dimensional vector of the variables included in the VAR:

$$\mathcal{Y}_t = \begin{pmatrix} \ln(\text{Real GDP}_t) \\ \Delta\ln(\text{PCE Price Index}_t) \\ \ln(\text{Non-Durable Consumption}_t) \\ \ln(\text{Inventories}_t/\text{Sales}_t) \\ \ln(\text{Inventories}_t) \\ \ln(\text{Unit Labor Cost}_t) \\ \ln(\text{Non-Fin. Corporate Profit/GDP}_t) \\ (\text{PPI Commodity Annual Growth}_t) \\ \text{FF}_t \\ (\text{NBR/TR}_t) \\ (\text{M2 Money Annual Growth}_t) \end{pmatrix}$$

The last two variables in $\mathcal{Y}_t$ are the ratio of non-borrowed reserves to total reserves of depository institutions and the log annual growth rate of monetary aggregate M2. The information set available to the monetary authority at time t, Ω_t, includes time t values of real GDP, the PCE deflator, the inventory-sales ratio, the finished good inventory stock, unit labor cost, and the commodity price index. As in Christiano et al. (1996), $\epsilon_{r,t}$ is assumed to be orthogonal to all variables in Ω_t. Therefore the identification strategy is recursive because the variables in Ω_t are not contemporaneously affected by $\epsilon_{r,t}$.

I use quarterly data and construct a VAR with 3 lags. The lag length is chosen by a corrected Akaike information criterion developed by Hurvich and Tsai (1993) specifically designed for VARs. The reduced-form VAR is specified as the following:

$$\mathcal{Y}_t = \Gamma_0 + \sum_{k=1}^{3} \Gamma_k \mathcal{Y}_{t-k} + u_t = \sum_{k=1}^{3} \Gamma_k \mathcal{Y}_{t-k} + De_t \tag{1.42}$$

where Γ_0 is an 11×1 vector of constants; Γ_1, Γ_2, and Γ_3 are 11×11 matrices; e_t is an 11×1 vector of serially uncorrelated structural shocks, and D is an 11×11 matrix. Because of the recursive identification assumption, D is assumed to be a lower triangular matrix, which can be obtained from a Cholesky decomposition of the variance-covariance matrix :

$$DD' = \mathrm{E}[u_t u_t']$$

Most data series come from the FRED database, including Real GDP (FRED code GDPC96), the personal consumption expenditure price index (FRED code PCEPI), the producer price index for all commodities (FRED code PPIACO), non-borrowed reserves, total reserves, and monetary aggregate M2. Sales are real sales of the manufacturing and trade sector, obtained from NIPA underlying detail tables 2AUI and 2BU. Inventories are real finished-goods inventories of the manufacturing sector plus the inventories of the trade sector, also obtained from the NIPA database.[14] Unit labor cost is constructed following Gali and Gertler (1999). Non-Financial corporate profits before tax come from the FRED database

[14] Inventories of the manufacturing sector and the trade sector account for 85% of U.S. inventory stocks. The rest are in mining, utilities, and construction.

(Code:A464RC1Q027SBEA). I use the augmented Dickey-Fuller test on each series to check for possible unit roots. For those series for which I cannot reject the null hypothesis of the existence of a unit root, I distill their transient components as the 2-year ahead projection errors, as suggested by Hamilton (2018). Series that are free from a unit root include changes in the ln(PCE index), ln (Inventory/Sales), and the PPI commodity annual growth rate.[15][16]

Figure 1.6 reports the impulse responses to a 60 basis-point (annualized) tightening monetary policy shock estimated using data from the 1st quarter of 1970 to the 4th quarter of 2007.[17] The shaded areas around the point estimates are 95% confidence intervals obtained from 5,000 bootstraps. I also plot the impulse responses generated by the local projection method of Jordá (2005) (the bubbled line) as a robustness check. A monetary tightening shock decreases real GDP and pushes the inventory-sales ratio up. The point estimates for the response of inventory stock appear to fall after 2 quarters, although this response is not statistically significant as the confidence interval contains zero. The inventory-to-sales ratio rises and peaks in the second quarter at around 0.35% above its steady state level. The rise in the inventory-sales ratio is statistically significant. The inflation rate has a very stagnant response. The point estimates indicate a falling inflation rate, with a trough

[15] A common practice in the literature to treat possibly non-stationary series is to use the Hodrick-Prescott Filter. Hamilton (2018) points out possible problems associated with HP-filtering and suggests constructing projection errors by running linear regression on lagged values to avoid the problems.

[16] An alternative is to take a first difference of the possibly I(1) series. The VAR results thus obtained are qualitatively the same as the results from HP- filtering the series, but the responses of output and consumption become less persistent.

[17] The 60 basis point magnitude of the Fed Funds Rate shock is in line with the ones adopted in Christiano et al. (2005) and Christiano et al. (1996).

10 quarters after the shock at around -0.03%, but the confidence bands indicate that the responses are not statistically different from zero. The response of unit labor cost does not exhibit strong procyclicality (unit labor cost actually appears to rise following the tightening monetary shock). The point estimates suggest that the non-financial corporate profit margin decreases after the tightening, and it is statistically significant at the third quarter. In summary, the inventory-sales ratio indeed goes up, but judging from the responses of unit labor cost and corporate profits, one cannot find evidence for elastic marginal cost or a rising price-cost markup after the monetary tightening. As a robustness check for the behavior of the inventory-sales ratio, I run the same VAR for the post-Volcker period, from Q1 1982 to Q4 2007. The results are depicted in figure 1.7. The point estimates of the impulse responses are similar, although the confidence bands are wider compared to figure 1.6 due to a smaller sample size.

1.5.2 Calibration & Estimation

The parameter values used in the numerical experiments are listed in table 1.1. The parameters used in either the Calvo Pricing model or the rational inattention model (abbreviated as RI henceforth) are divided into 2 subsets. The first subset are either chosen on the basis of existing literature, or are calibrated to match the steady state values of the model to long-run averages of the U.S. data. They include $\{\beta, \mu, \theta, \theta_w, \psi_l, g_*, \delta_z, \sigma_v, \rho_R, \phi_\pi, \phi_y, h\}$. It is worth mentioning that in the numerical experiments presented below, the Calvo sticky price model and the RI model share

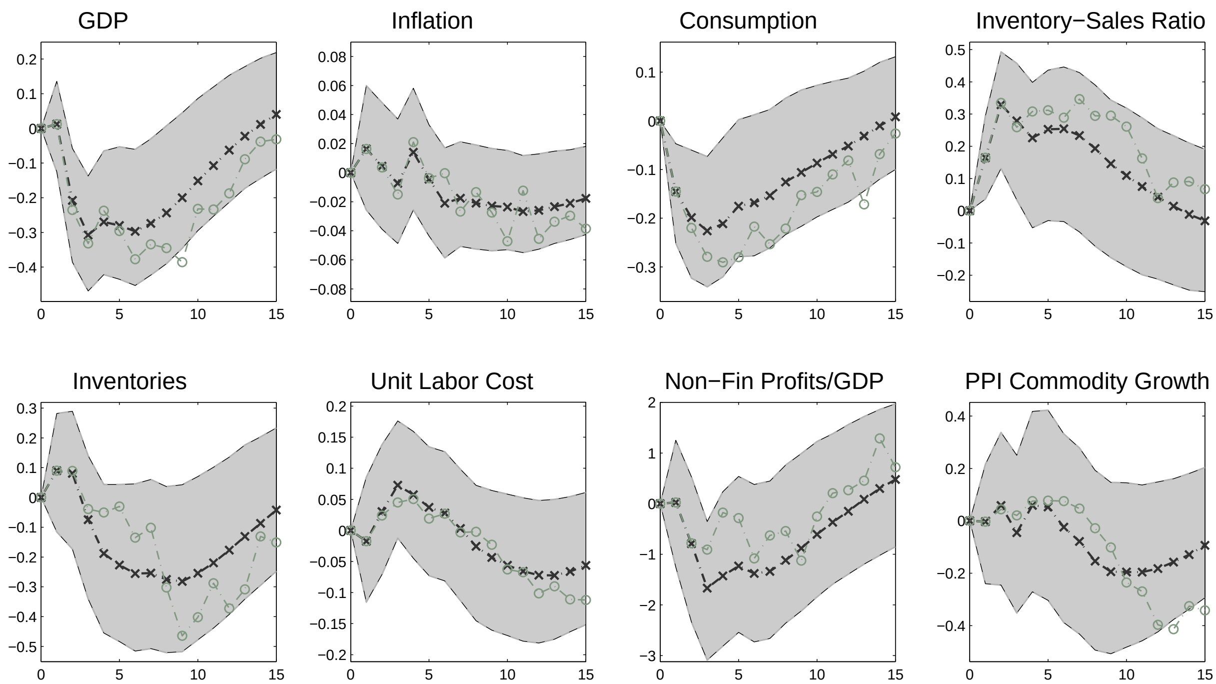

Fig. 1.6: This figure displays the impulse responses to a 0.60% (annualized) shock to the Fed Funds Rate from estimated recursive vector autoregression (VAR). The shaded areas are 95% confidence bands, obtained by 5,000 bootstraps. The bubbled lines show the points estimates of impulse responses using the local projection method of Jordá (2005).

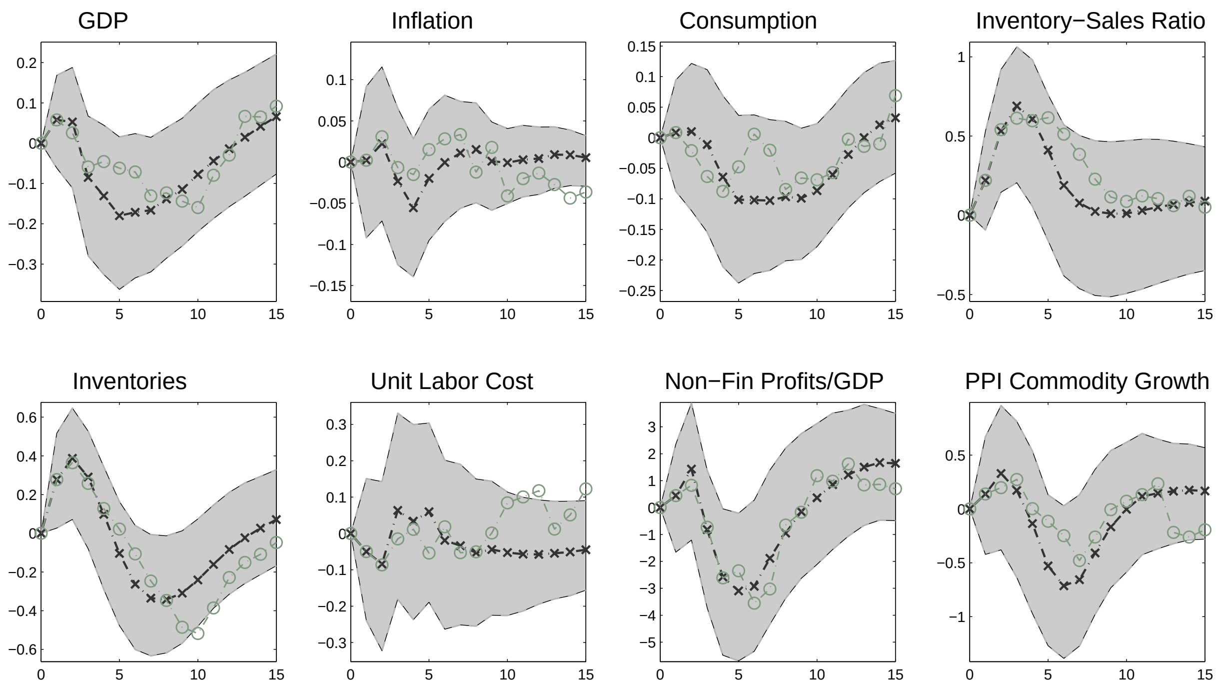

Fig. 1.7: This figure displays the impulse responses to a 0.60% (annualized) shock to the Fed Funds Rate from estimated recursive vector autoregression (VAR). The shaded areas are 95% confidence bands, obtained by 5,000 bootstraps. The bubbled lines show the points estimates of impulse responses using the local projection method of Jordá (2005).

the same parameter values for parameters in the first subset (if applicable). The remaining set of parameters are estimated, and will differ across those two models in our numerical exercise. Parameters in the second subset are $\{\xi_p, \ell_p, \xi_w, \ell_w,\ \sigma_c, \sigma_l, \alpha\}$. Those parameters are chosen to be estimated because they either govern the behavior of firms and households or have implications for model selection and comparison.[18] They control the degree of price and wage stickiness, the returns to scale of firms' production function (which also governs the degree of strategic complementarity), and the household's inter-temporal elasticities of substitution and labor supply.

The predetermined parameter values are determined as follows. The discount factor β is set equal to 0.9898, consistent with long-run averages of the Fed Funds Rate and the inflation rate. $g_* = 0.16$ is consistent with the long run average ratio of government final consumption expenditure to output in the United States. I set θ and θ_w, the elasticities of demand for differentiated goods and labor, to be equal to 5, following Kryvtsov and Midrigan (2013). This implies that the firm's markup is 20%. The standard deviation σ_v of the idiosyncratic taste shock is set equal to 0.33 and the quarterly depreciation rate of inventories δ_z is 1.8%. These two values jointly determine the quarterly frequency of stock outs as 16.3%, and the steady state end-of-period inventory-sales ratio as 0.498. Both values, when converted to monthly terms, roughly match those considered in Kryvtsov and Midrigan (2013). Also, the inventory depreciation rate δ_z implies an average change in the ratio of finished good inventories to output of 0.9%, which roughly matches the corresponding ratio in the

[18] The habit persistence parameter h governs the response of consumption and is a source of real rigidity in both models. To make the comparison between the information friction model and the staggered pricing model clear, I let both models have the same habit persistence.

data for the U.S. manufacturing and trade sectors. For the RI model, the marginal cost of flow of information μ is set equal to 0.0525% of the intermediate goods firm's sales each period. This value is lower than the 0.058% adopted in Mackowiak and Wiederholt (2015). The monetary policy rule coefficients ρ_R, ϕ_π, and ϕ_y, as well as exogenous habit formation in consumption h, come from Smets and Wouters (2007).

Estimation

I use U.S. quarterly data to estimate the second set of parameters. The estimation strategy is to match model implied impulse response functions of chosen variables to the impulse responses of the estimated VAR model described in the previous subsection. The estimation method can be summarized as follows. Let $\vartheta = [\xi_p, \ell_p, \xi_w, \ell_w,\ \sigma_c, \sigma_l, \alpha]$ denote the vector of estimated parameters, and let $\Psi(\vartheta)$ be the model impulse response functions depending on the parameter vector. Let $\hat{\Psi}$ denote the impulse responses generated by the VAR. The estimation criterion is

$$J = \min_{\vartheta} (\hat{\Psi} - \Psi(\vartheta))' \, V^{-1} \, (\hat{\Psi} - \Psi(\vartheta)) \tag{1.43}$$

where V is the sampling uncertainty of $\hat{\Psi}$. I use a weighted variance-covariance matrix to construct V. Let n be the number of variables whose impulse responses are included in $\hat{\Psi}$, and m denote the number of periods tracked for each variable's impulse response. Then $\hat{\Psi}$ can be written as a $1 \times nm$ vector:

$$\hat{\Psi} = [\tau_{1,1}, \dots, \tau_{1,m}, \tau_{2,1}, \dots, \tau_{2,m}, \dots, \tau_{n,1}, \dots, \tau_{n,m}]$$

The matrix V is constructed to be the following:

$$V = \begin{bmatrix} \Sigma_1 & 0 & \dots & 0 \\ 0 & \Sigma_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & \dots & \Sigma_n \end{bmatrix}$$

where submatrices Σ_i along the diagonal are the variance-covariance matrices of the individual impulse responses for each variable. Therefore, I consider only the covariance of each individual variable's impulse responses across periods, but omit the covariance in the impulse responses between any two different variables. For each $\Sigma_i, i = 1, \dots, n$, I use a weighted variance-covariance matrix of the following form:

$$\Sigma_i = \begin{bmatrix} \sigma_{i,1}^2 & \frac{\sigma_{i,12}}{2} & \dots & \frac{\sigma_{i,1(m-1)}}{m-1} & \frac{\sigma_{i,1m}}{m} \\ \frac{\sigma_{i,21}}{2} & \sigma_{i,2}^2 & \dots & & \frac{\sigma_{i,2m}}{m} \\ \vdots & \vdots & \ddots & & \vdots \\ & & & \sigma_{i,m-1}^2 & \frac{\sigma_{i,(m-1)m}}{m} \\ \frac{\sigma_{i,1m}}{m} & & \dots & & \sigma_{i,m}^2 \end{bmatrix}$$

The weighted variance-covariance matrices account for the idea that it is more important to match the first periods of the theoretical and empirical impulse responses rather than the later ones. Each Σ_i can be estimated by applying the Monte Carlo method. I use the originally estimated VAR model to simulate a series of artificial data. Then, I use the artificial data to reestimate the VAR model and generate the impulse responses. This serves as one Monte Carlo draw. I run 5,000 Monte

Carlo draws and $\Sigma_i, i = 1, \ldots, n$ can be obtained from the variation in the impulse responses of each variable across all draws.

I estimate the second set of parameters ϑ for the Calvo Sticky Price model. Due to the computational burden of the RI model, it is not viable to run the impulse response matching exercise. I thus apply the values of the posterior mode of corresponding parameters reported in Smets and Wouters (2007). In particular, I estimate parameters to match the Calvo Pricing model and VAR impulse responses for 6 variables, including output, inflation, non-durable consumption, manufacturing and trade inventory-sales ratio, finished-good inventories, and the real wage (unit labor costs). I then apply the estimated parameters to the RI model, provide an initial guess for the first-best decisions $\{P_{jt}^{\dagger}\}^{(1)}$ and $\{Z_{jt}^{\dagger}\}^{(1)}$, and let the firm choose the precision of their decisions by choosing $B(L)$ (the response to current and lagged realizations of structural shocks) and $C(L)$ (the response to current and lagged idiosyncratic noises) that maximize the firm's objective (1.34), and thus obtains the actual decisions $\{P_{jt}\}^{(1)}$ and $\{Z_{jt}^{\dagger}\}^{(1)}$. Then using equation (1.39) in Proposition 1, I obtain the second guess for first-best decisions $\{P_{jt}^{\dagger}\}^{(2)}$ and $\{Z_{jt}^{\dagger}\}^{(2)}$, and run the optimization of the firm's objective again to obtain $\{P_{jt}\}^{(2)}$ and $\{Z_{jt}\}^{(2)}$. I keep iterating until successive guesses for the first-best decision rules converge and the equilibrium is obtained. I then generate impulse responses and simulate various moments from the RI equilibrium. The horizon of each variable's response being matched is chosen as $m = 16$.[19]

[19] Due to the identification assumption for the Fed Funds Rate shock in the VAR, the impulse responses in the first period cannot be matched, so for each variable I generate 16 periods of impulse responses and try to match 15 periods, starting from the second period.

Note that through this estimation exercise, I give the Calvo Pricing model the best possible chance to fit the data impulse responses, by searching over the feasible range of parameters to obtain parameter values that best fit 6 impulse response functions estimated from the VAR. For computational reasons, I cannot directly estimate the parameters of the rational inattention model using the same impulse response matching exercise. Instead, I handicap the rational inattention model by assigning it parameters that come from Smets and Wouters (2007), which would give the model higher cost rigidity (through higher values of ξ_w, ℓ_w, and α). This procedure builds in an advantage for the Calvo Pricing model over the RI model, since the RI model's goodness of fit can potentially be improved further by searching over the parameter space.

The lower half of table 1.1 lists estimated parameter values. From the estimated parameters listed in the lower half of the 2nd column, notice that the impulse response matching exercise of the Calvo Pricing model obtains a low degree of wage stickiness, making the real wage and the real marginal cost responsive to the interest rate shock. This is because the model tries to match the responses of finished-good inventories and the inventory-sales ratio by incorporating an elastic marginal cost of production (and thus a countercyclical price markup) to hinder excessive inventory accumulation, corresponding to the intuition depicted in the comparative statics (figure 1.4 and figure 1.5). On the other hand, the exercise also obtains an α lower than 1, meaning that there are decreasing returns in production. Lower α implies more strategic complementarity in firms' pricing and production decisions. With an (individual) elastic marginal cost schedule firms will be more reluctant to adjust prices in response to cost pressure, since a smaller α means that a given magnitude of price adjustment brings about a larger change in the firm-specific marginal cost and profit. One may wonder why the model estimates imply a relatively low degree of price rigidity ($\xi_p = 0.5059$ and $\ell_p = 0.01$). This is because the model also needs to match the response of unit labor cost. If prices are too sticky, the response of the real wage will be too dramatic and deviate too much from the empirical impulse responses, as the nominal wage is very flexible in the estimated model.

	RI	Calvo Pricing	Description
Predetermined:			
β	$0.96^{\frac{1}{4}}$	$0.96^{\frac{1}{4}}$	Discount factor
μ	0.0525%	–	Flow cost of info. (% of sales)
θ	5.0	5.0	Kryvtsov and Midrigan (2013)
θ_w	5.0	5.0	Kryvtsov and Midrigan (2013)
δ_z	1.8%	1.8%	Inventory depreciation (quarterly)
σ_v	33%	33%	s.d. of demand shock.
ψ_l	1.2	1.2	Labor disutility coefficient
g_*	0.16	0.16	Avg. gov't consumption/GDP
ρ_R	0.81	0.81	AR(1) coefficient of interest rate rule
ϕ_π	2.03	2.03	Policy response to π_t
ϕ_Y	0.22	0.22	Policy response to output gap
π_*	1.0	1.0	Steady state gross inflation
h	0.71	0.71	Consumption habit persistence
	SW (2007):	**Estimated**:	
ξ_p	–	0.5059	Degree of price stickiness
ℓ_p	–	0.01	Degree of price indexation
ξ_w	**0.82**	**0.1398**	Degree of wage stickiness
ℓ_w	**0.69**	**0.1155**	Degree of wage indexation
σ_c	1.3981	1.2469	Inverse of IES
σ_l	0.40	0.3823	Inverse Frisch elasticity
α	**1.0**	**0.8633**	Returns to scale

Tab. 1.1: Parameter Values

1.5.3 Discussion

I apply the parameters listed in table 1.1 for the Calvo Sticky Price model and RI model, run numerical experiments, and compare the results obtained from the two different assumptions about the frictions affecting the intermediate goods firm's price setting and production decisions. Note that compared to the Calvo Pricing model, the parameter values applied to the RI model entail a much higher cost rigidity. Real wages are highly sticky, and there is constant short-run returns to scale in production. In the first numerical experiment, the economy starts from the non-stochastic steady state and is perturbed by a positive 0.60% (annualized) shock to the nominal interest rate. I compare the model generated impulse responses and the 95% confidence bands of the estimated VAR model (the sample period is from Q1 1970 to Q4 2015) described in section 4.1.

Comparing Theoretical Models to the Estimated VAR: Impulse Response Matching

Figure 1.8 compares the impulse responses generated from the RI model and the Calvo Pricing model to those of the VAR, applying the parameter values listed in table 1.1. The first thing to note is that the response of output generated by the RI model is more in line with the empirical impulse response compared to the Calvo pricing model. The Calvo pricing model generates an immediate decline of output of around 0.12% on impact of the shock, followed by a rapid rebound back to the steady state value. On the contrary, the response of the RI model is larger and more persistent, with output falling initially by 0.18% and remaining below steady state for 15 quarters. The two models also generate significant disparities in the responses of inflation and the real wage. The response of the real wage is more modest and stagnant in the RI model, while the real wage falls by more than 0.25% on impact in the Calvo Pricing model, because of the estimated low degree of wage stickiness ($\xi_w = 0.1398$). This translates to a much more dramatic response of real marginal cost in the Calvo Pricing model than in the RI model.[20] As we can see, the drop in real marginal cost on impact in the Calvo Pricing model is almost 6 times as large as in the RI model. Thus, the cost pressure in the Calvo Pricing model makes its inflation response more pronounced and short-lived. Inflation goes down by about 0.14% in the Calvo Pricing model, while for the RI model the inflation response is extremely modest and lies within the 95% confidence bands of the VAR impulse response for most periods, partly due to the stagnant real marginal cost, and partly due to the rigidity generated by the information processing constraint. In the last subplot I plot the response of real profits (defined as the price-cost markup times the amount of sales) generated from the two models. It is clear that the Calvo Pricing model generates a sharp rise in the firm's real profit, whereas in the RI model the response of real profit is mild but much more persistent.

Compared to the Calvo model, the RI model has a larger and more persistent consumption response, in line with data, even though the parameter of habit per-

[20] Since RI model is constant returns to scale ($\alpha = 1$), the response of the real wage equals the response of its marginal cost of production.

sistence is the same across the two models. Once again, this is due to the disparity in the elasticities of real marginal cost. As expected, the more elastic marginal cost in the Calvo model makes the real effects of monetary shocks more short-lived. Regarding inventory dynamics, consistent with the data, both models generate a rising inventory-sales ratio and a declining inventory stock after the monetary tightening shock, but the RI model's response is larger and more persistent. Comparing the overall goodness of fit, the value of the estimation criterion is 221.6 for the Calvo Pricing model and 146.0 for the RI model. In other words, the deviation from the VAR impulse responses implied by the Calvo Pricing model is 52% larger than that of the RI model. Thus, in terms of matching impulse responses, the RI model with an inelastic response of marginal cost fits the data better than the Calvo Pricing model. Table 1.2 provides a decomposition of the contribution to the estimation criterion for the 6 matched series across the two models. The RI model performs better for 5 out of the 6 series being matched. Not only does it match the inventory-sales ratio and inventory dynamics better, it also excels significantly in matching the persistent responses of output, inflation, and consumption. Note that the two models have the same parameter value for habit persistence, but the slow-moving marginal production cost increases the overall response persistence in the RI model. The reason that the Calvo model performs better in matching the unit labor cost is mainly due to the abrupt and short-lived response of the real wage in the Calvo model, and due to the limitation that the immediate large deviation when the shock hits is not accounted for in the estimation criteria because of the identification specification in the VAR (that the Fed Funds Rate shock does not immediately affect the variables ranked before the FFR).

(figure 1.8)	RI	Calvo Pricing
Output	30.90*	56.12
Inflation	18.06*	38.39
Consumption	8.83*	39.45
Inventory-Sales	10.93*	22.00
Inventories	36.96*	38.78
Unit Labor Cost	40.29	26.87*
Sum	146.0*	221.6

Tab. 1.2: Decomposition of Estimation Criterion

To make the comparison between the two models clearer, I re-estimate the Calvo Pricing model by matching output, inflation, consumption, and unit labor cost, dropping inventories and inventory-sales ratio. The results are presented in figure 1.9 (which also lists the estimated parameter values). We can observe that if the Calvo Pricing model is not required to match the inventory dynamics, the model would want to parameterize a high degree of nominal price and wage stickiness. The degree of wage stickiness even surpasses price stickiness, resulting in inertial marginal cost. However, the Calvo Pricing model will perform poorly in matching the inventory dynamics (the 4th and 5th subplots).

The RI model under an acyclical price markup achieves a better fit than the Calvo model because rational inattention on the firm side makes the firm's actual decisions deviate from its first-best decisions. The information friction adopted in this paper serves as a mechanism to limit the response of inventory investment/disinvestment and production to nominal shocks, and thus the model does not need variation in marginal cost and price markups to counteract the effect on inventories of real interest rate movements as in a model of full information. Figure 1.10 plots the actual response of stock for sale Z_{jt} (the solid line) in the RI model to a positive 0.60% (annualized) interest rate shock and the corresponding first-best structural response $Z_{jt}^{\dagger}$ (the dash-dotted line). In the absence of information constraints, the firm should decrease Z_{jt} by about 1.0% on impact. The firm's actual adjustment is much smaller compared to its optimal rule. Figure 1.11 depicts the same comparison between the actual and the first-best responses for prices. The deviation of the actual price response from the first-best response has significant persistence and dies out after 25 quarters. Note that the magnitude of the distance $Z_{jt} - Z_{jt}^{\dagger}$ is much larger than the magnitude of $P_{jt} - P_{jt}^{\dagger}$. This is because the firm's profit is much more sensitive to its pricing decision than to its Z_j decision around the steady state. With our benchmark parametrization, the value of the Hessian matrix $H_{x,0}$ in the firm's objective (1.34) is:

$$H_{x,0} = \begin{bmatrix} -3.418 & -0.365 \\ -0.365 & -0.088 \end{bmatrix}$$

The profit deviation is more than 34 times more sensitive to $(P_{jt} - P_{jt}^{\dagger})^2$ than to $(Z_{jt} - Z_{jt}^{\dagger})^2$. The reason is that the pricing decision determines the demand for the firm's goods and thus both directly determines the firm's revenue and indirectly affects the probability of a stock out, while Z_{jt} only affects the firm's current period profit indirectly, by affecting the marginal probability of a stock out, multiplied by the forgone price markup (profit) in the case of a stock out. As the probability of a stock out is not high in the steady state, this term has a small value. Therefore, firms tend to let Z_{jt} deviate from $Z_{jt}^{\dagger}$ by more.

Value of Estimation Criterion-
RI: 146.0, Calvo Pricing: 221.6

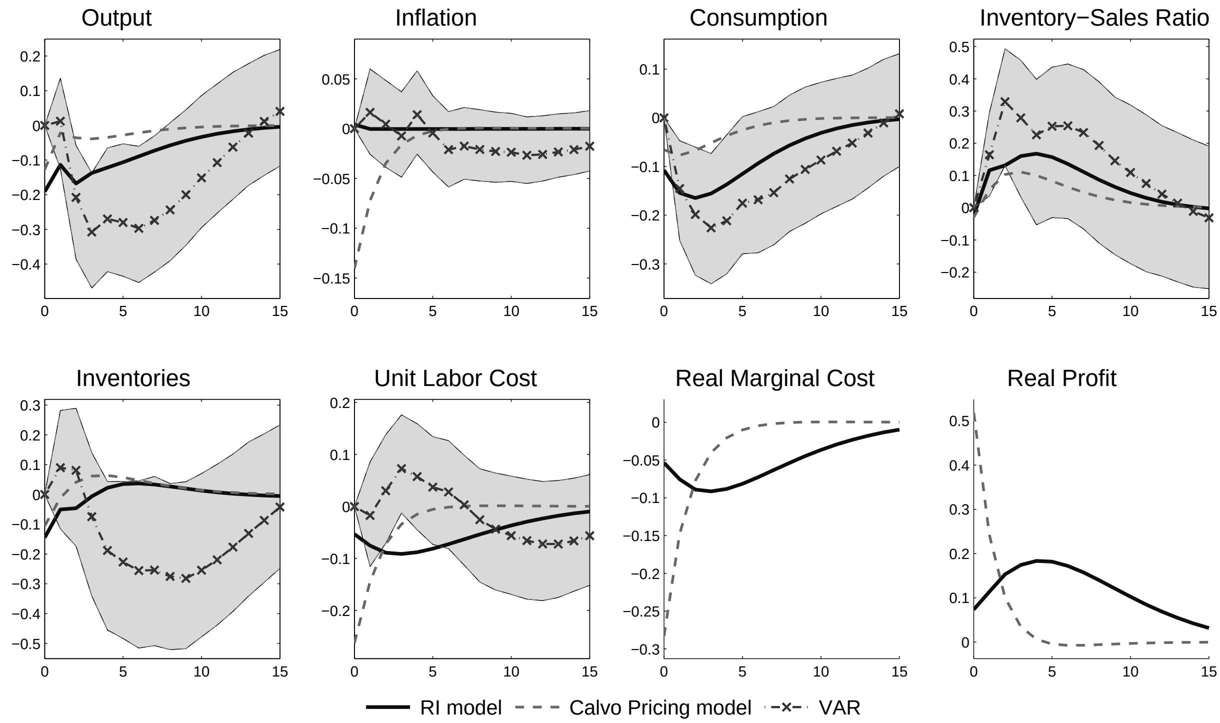

Fig. 1.8: This figure compares the impulse responses generated by the Rational Inattention model (the solid lines), the Calvo Pricing model (dashed line), and the vector autoregression (VAR) model (the dashed-cross lines). The areas within the dashed lines are the 95% confidence bands obtained from 5,000 bootstraps.

Estimated Parameters (matching 4 variables):

$\xi_p = 0.854$, $\ell_p = 0.590$, $\xi_w = 0.953$, $\ell_w = 0.882$, $\alpha = 0.994$, $\sigma_c = 1.102$, $\sigma_l = 0.341$

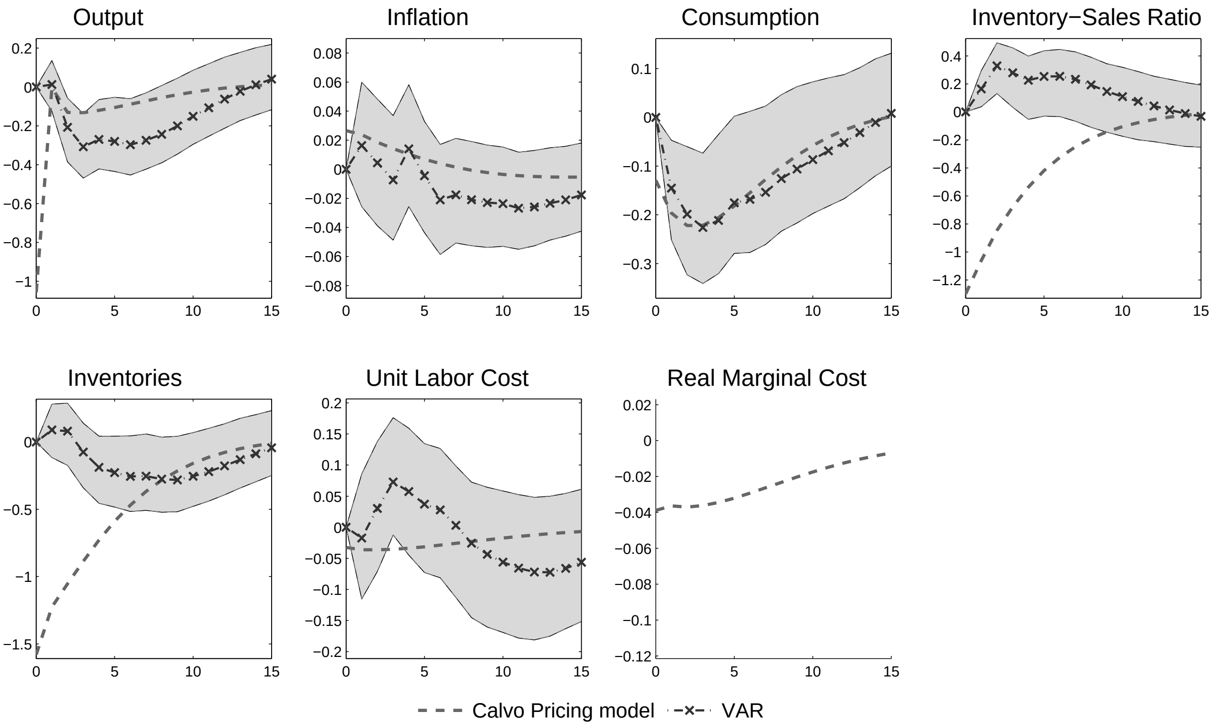

Fig. 1.9: This figure compares the impulse responses generated by the Calvo Pricing model (dashed line) and the vector autoregression (VAR) model (the dashed-cross lines). The areas within the dashed lines are the 95% confidence bands obtained from 5,000 bootstraps. The parameter values for the Calvo model are obtained by matching only 4 variables, including output, inflation, consumption, and unit labor cost.

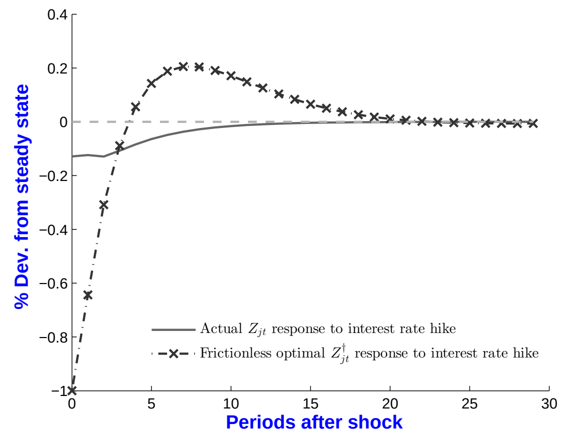

Fig. 1.10: First-Best and Actual Response of Stock for Sale Z_{jt} under rational inattention.

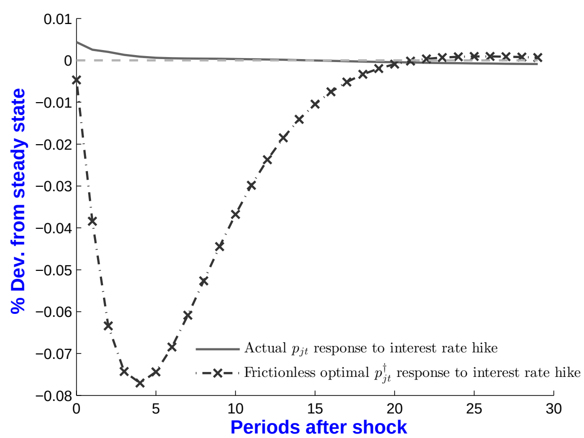

Fig. 1.11: First-Best and Actual Response of Price p_{jt} under rational inattention
The upper and lower figures compare the optimal and actual response of stock available for sale Z_{jt} and relative price p_{jt} to a positive 0.12% nominal interest rate shock respectively. The dash-dotted line is the optimal response and the solid line is the actual response.

Comparing Simulated Moments

Table 1.3 compares various simulated business cycle moments (conditional on monetary policy shocks) generated from the VAR, the RI model and the Calvo Pricing model. Overall, the RI model generates business cycle moments that qualitatively match the VAR generated moments, except for the correlation between output and inflation, which is negative in the RI model but positive in the data. For the Calvo Pricing model, all the generated correlations qualitatively match the data. In terms of the correlations of the inventory-sales ratio with output and sales respectively, both the RI model and the Calvo Pricing model match the negative signs of the correlations seen in the data, although in terms of magnitude of the correlations, the RI model fits better. Both the RI model and the Calvo Pricing model can match the high correlation between output and sales, but the RI model fits slightly better in terms of magnitude. Overall, we observe that the RI model matches the correlation between inventory dynamics and other variables better then the Calvo model. In terms of matching the ratios of relative standard deviations of inflation to output, the real wage to output, and the inventory-sales ratio to sales, the RI model clearly fits better, as the ratios for the Calvo Pricing model are far above those in the data, because the Calvo Pricing model generates too much variation in inflation and the real wage and too little variation in output and sales.

	VAR	RI	Calvo Pricing
$\rho(IS_t, S_t)$	-0.898	-0.881	-0.653
$\rho(IS_t, Y_t)$	-0.942	-0.787	-0.381
$\rho(Y_t, \Delta I_t)$	0.812	0.120	0.415
$\rho(Y_t, S_t)$	0.963	0.970	0.860
$\rho(Y_t, \pi_t)$	0.374	-0.291	0.888
$\rho(\Delta\pi_t, \Delta\pi_{t-1})$	-0.220	-0.491	-0.232
$\sigma(\pi_t)/\sigma(y_t)$	0.095	0.012	1.098
$\sigma(IS_t)/\sigma(S_t)$	0.527	0.981	1.656
$\sigma(w_t)/\sigma(y_t)$	0.285	0.622	2.095

Tab. 1.3: Reported simulated moments are conditional on interest rate shocks. All series are quarterly. IS_t, S_t, ΔI_t, w_t, and Y_t stand for inventory-sales ratio, real sales, inventory investment, real wage, and output.

1.6 Robustness & Extension

To further characterize the RI model solution and check its robustness and sensitivity, I perform three experiments. In the first experiment, I check the sensitivity of Z_j and the inventory-sales ratio to different marginal information costs. Next, I examine the model's performance in the presence of larger interest rate shocks. Finally, I consider a shock to the aggregate money supply (an aggregate demand shock) instead of a shock to the interest rate rule, and then calculate the relative contribution of cost rigidity and variation in price markups in accounting for the changes in aggregate consumption, as in Kryvtsov and Midrigan (2013).

1.6.1 Sensitivity to Information Cost

The benchmark parametrization applies a modest marginal information cost μ, corresponding to 0.0525% of sales revenue each period. Since there is not an empirical target corresponding to the information processing cost in the model, this parameter is chosen rather arbitrarily. One might be curious how sensitive are inventory dynamics to different values of μ. Figure 1.12 depicts how the response of the actual stock for sale Z_{jt} is affected by different levels of marginal information cost μ. Intuitively, a lower information cost loosens the information capacity limit and brings the response of Z_{jt} closer to the first-best response. As a result, a lower marginal information cost amplifies the response of Z_{jt} and alters the response of the inventory-sales ratio as shown in figure 1.13. As we can see, when information processing becomes less costly, the stock for sale Z_j has a more pronounced downward movement with inertial marginal production cost following a tightening interest rate shock, pushing the inventory-sales ratio downward as the theory predicts. Table 1.4 lists the firm's profit deviation and information flows as a function of the marginal information cost μ. Consistent with intuition, the lower is the marginal information cost, the higher are the information flows to pricing and production decisions, and the smaller is the absolute value of profit deviation (the difference of the firm's profit with the information capacity constraint from its first-best level without such a constraint). Notice that the firm always allocates more attention to its pricing decision than to its production decision.

Marginal info. cost μ	0.0525%	0.0263%	0.0132%
Profit deviation	0.0025	0.0014	0.0013
Info. flow to P_j	0.0768	0.2264	0.1779
Info. flow to Z_j	0.0596	0.1038	0.1310

Tab. 1.4: Sensitivity analysis to marginal information costs.

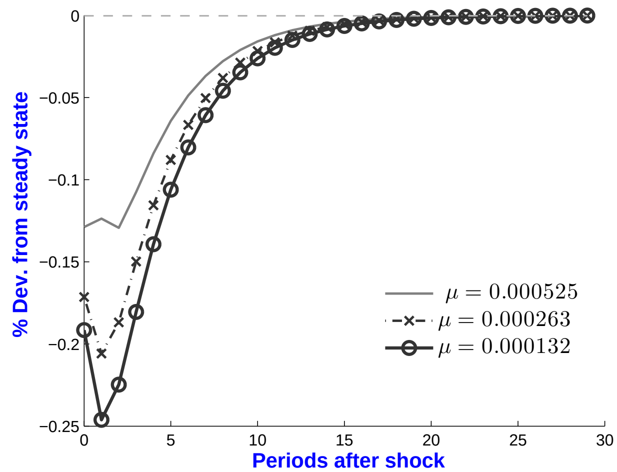

Fig. 1.12: The Sensitivity of Z_{jt} to Marginal Information Costs μ

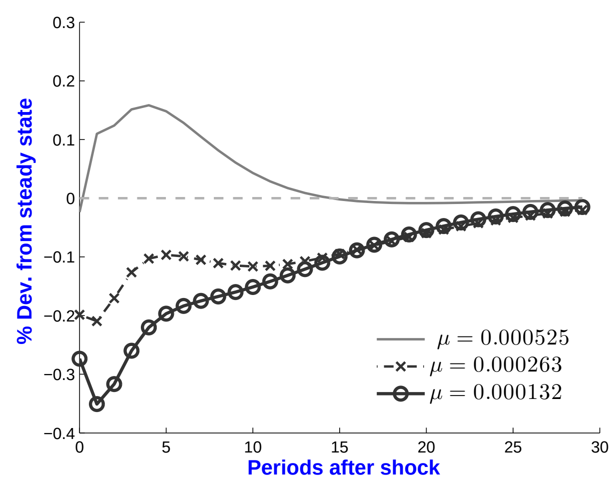

Fig. 1.13: The Sensitivity of Inventory-Sales Ratio to Marginal Information Costs μ

1.6.2 Larger Interest Rate Shock

One may wonder whether the model can still give rise to a limited movement of inventories when the standard deviation of the shock becomes larger. From information theory, one would predict that as the aggregate shocks become more volatile, firms should pay more attention to aggregate conditions. Paying more attention may improve the precision of their decisions, and therefore larger shocks might amplify the response of Z_j and dampen the effect of the information capacity constraint. The standard deviation of the interest rate shock in the benchmark calibration is set as 60 basis points (in the quarterly interest rate) in the previous section. Here I vary the standard deviation and plot the inventory-sales ratio in figure 1.14.

The figure shows that the inventory-sales ratio increases by more when the magnitude of the shock becomes larger. This is mainly due to a larger drop of sales. The response of Z_j also become larger, but is not as sensitive to the magnitude of the shock compared to the response of sales. I list the absolute values of the deviation in profit (from the first-best level) and the information flows allocated to P_j and Z_j decisions respectively in the following table 1.5. The table shows that even though the firms increase their information flows to both decisions in response to larger shocks, the deviation in profit widens. Therefore, even though firms pay more attention to track the aggregate conditions, their actual decisions deviate by even more from the first-best decisions (which are also amplified by larger shocks). Notice that in the firm's problem (1.34), the interaction between P_j and Z_j also affects the profit deviation. If overall the firm's P_j and Z_j decisions deviate from the first-best responses in opposite directions, their interaction makes the firm's profit closer to the first best. The intuition is that, from the firm's optimality condition equation (1.18), the firm wants to equate the probability of stock out to the ratio between the net cost and the net benefit of producing one more unit in period t. A lower P_j (relative to the first best $P_j^{\dagger}$) induces larger demand for the firm's product and thus increases the chance of a stock out, and in this case the firm would want a higher Z_j (relative to the first best $Z_j^{\dagger}$) to decrease the chance of a stock out. The (negative) contribution of the interaction terms to the profit deviation becomes larger in magnitude with an increased magnitude of the shock, which means that larger deviations of P_j and Z_j from the first-best decisions also contain a counteracting force in their interaction that benefits the firm's profit. The interaction term thus provides a limiting force to the firm's information allocation.

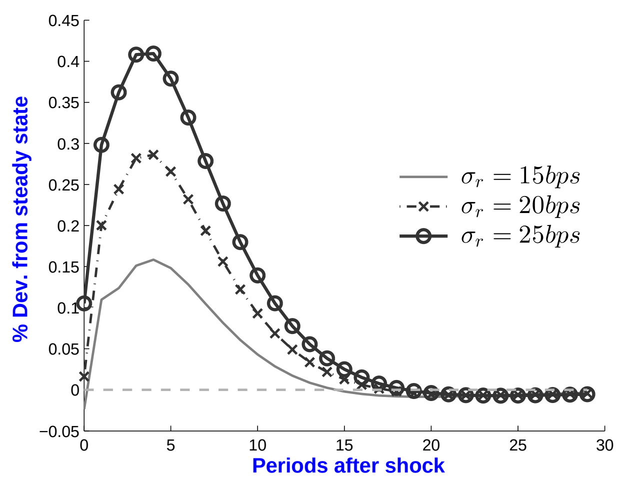

Fig. 1.14: The Sensitivity of Inventory-Sales Ratio to σ_r

Standard deviation σ_r	15 bps	20 bps	22 bps	25 bps
Profit deviation	0.0025	0.0047	0.0052	0.0072
Info. flow to P_j	0.0768	0.1009	0.1055	0.1164
Info. flow to Z_j	0.0596	0.0878	0.1009	0.1039
Contribution of interaction to profit deviation	-15.5%	-17.2%	-18.8%	-19.1%

Tab. 1.5: Sensitivity analysis to larger standard deviation of interest rate shock.

1.6.3 Money Growth Shock

The model employed in this paper assumes that monetary policy is represented by the feedback interest rate rule (equation (1.28)). Kryvtsov and Midrigan (2013) instead consider a shock to the monetary aggregate. As aggregate nominal spending equals the nominal money supply, they can decompose the real effects of the nominal money growth shock into a cost rigidity term and a price markup term:

$$P_t C_t = M_t \quad \Longrightarrow$$

$$\Delta \ln(C_t) = \underbrace{\Delta\big[\ln(M_t) - \ln(\Lambda_t)\big]}_{\text{cost rigidity term}} + \underbrace{\Delta\big[\ln(\Lambda_t) - \Delta \ln(P_t)\big]}_{\text{markup term}}$$

They show that the model with staggered pricing and staggered wage setting can match the countercyclical inventory-sales ratio only when nominal marginal cost Λ_t tracks the changes in money supply M_t closely and is much more responsive than P_t, or in other words, when the markup is strongly countercyclical. They conclude that in view of the model, the markup term contributes as much as 90% to changes in aggregate expenditure $\Delta \ln(C_t)$. To provide a direct comparison of the RI model results to those of Kryvtsov and Midrigan (2013), I set government consumption to zero so that aggregate expenditure is equal to private consumption C_t. Also, I assume $h = 0$ (no habit persistence,) as in their paper. I replace the interest rate rule with a cash-in-advance constraint in the consumer's problem and let the monetary authority control the nominal money supply. I assume $\xi_w = 0.90$ to give the model higher cost rigidity. Other parameters are the same as those listed in the second column of table 1.1.

I shock the economy by a 0.6% decrease in the nominal money supply and plot the impulse responses in figure 1.15. In the upper-left subplot, one can see that changes in the price markup (the distance between nominal marginal cost and the price level) are small compared to the fall in money supply. In the upper right subplot we see that inventories move less than sales, resulting in a countercyclical inventory-sales ratio. The RI model stands in sharp contrast to the staggered pricing model of Kryvtsov and Midrigan (2013), which can generate a countercyclical inventory-sales ratio only when nominal marginal cost closely tracks the shifts in nominal money supply. In their paper, the markup variation contributes as much as 90% to the change in aggregate expenditure. Figure 1.16, I apply the parameter values listed in table 1.1 (except $h = 0$) to the Calvo Pricing model with a money growth shock. One can see stark differences from figure 1.15 in that, nominal marginal cost tracks the change in money supply closely in the first subplot. In the RI model (figure 1.15), the cost rigidity term accounts for the bulk of the changes in consumption, whereas the variation in the price markup only accounts for about 24%. Table 1.6 reports various moments of the RI model and their data counterpart (which are taken from TABLE 3 in Kryvtsov and Midrigan (2013)).

Conditional on a nominal money supply shock, the RI model with rigid marginal cost does a satisfactory job matching the data moments.

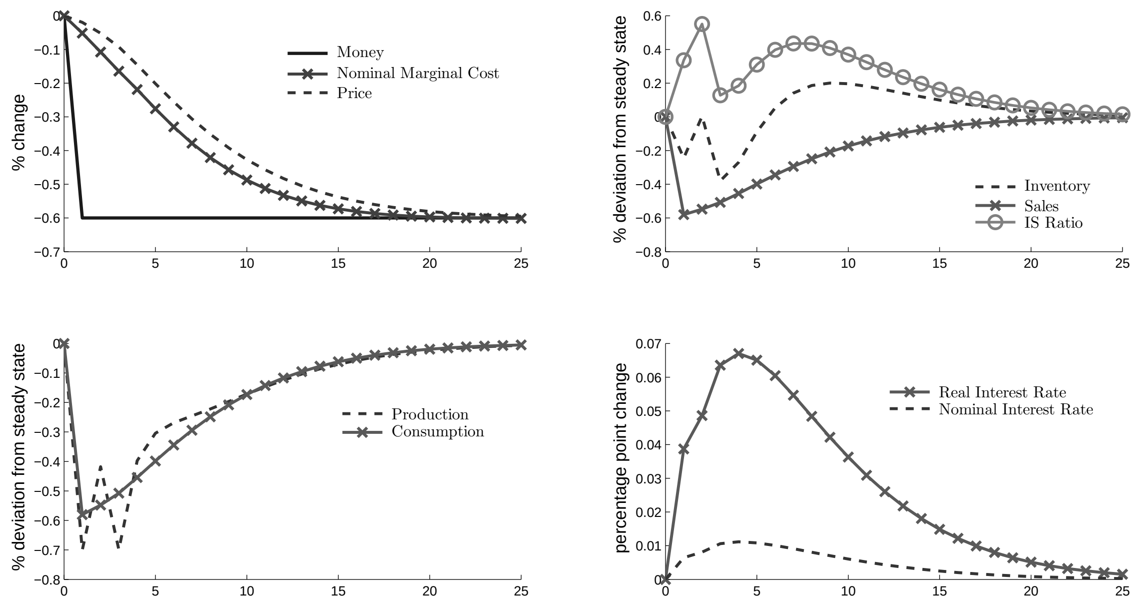

Fig. 1.15: Impulse Responses to a −0.6% Money Growth Shock, RI Model

	Data	RI
$\rho(IS_t, S_t)$	-0.710	-0.8255
$\rho(IS_t, IS_{t-1})$	0.880	0.876
$\rho(Y_t, \Delta I_t)$	0.630	0.246
$\sigma(IS_t)/\sigma(S_t)$	0.93	1.018
$\sigma(y_t)/\sigma(S_t)$	1.110	1.022
Elast. IS_t to S_t	-0.660	-0.842
Elast. I_t to S_t	0.340	0.158

Tab. 1.6: Simulated moments of the RI model and moments in the data (taken from Kryvtsov and Midrigan (2013)). IS_t, S_t, ΔI_t, w_t, and Y_t stand for the inventory-sales ratio, real sales, inventory investment, the real wage, and output.

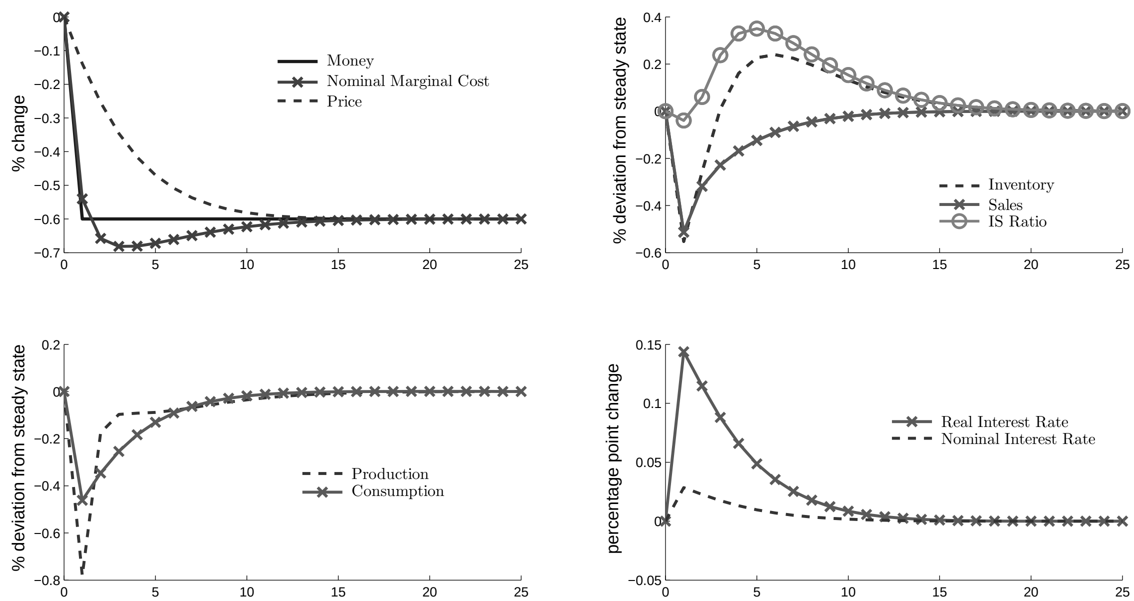

Fig. 1.16: Impulse Responses to a -0.6% Money Growth Shock, Calvo Model

1.7 Conclusion

I construct a medium-scale DSGE model with cost rigidities that incorporates inventory investment, and reexamine what inventory adjustment implies for the elasticity of real marginal cost with respect to output in response to monetary shocks. I compare a Calvo staggered pricing friction with an information capacity friction that assumes firms are inattentive to shocks hitting the economy and endogenously choose the information content in their actions. In a staggered pricing setting with full information, inventories are highly sensitive to the costs of inventory acquisition (the real marginal cost of production) and inventory holding (the real interest rate). Therefore, such a model predicts that real marginal cost needs to be highly elastic and sensitive to economic fluctuations, to counteract the direct effect of changes in interest rates and explain why the inventory-sales ratio rises following a monetary tightening. In contrast, this paper finds that with an information capacity constraint, the impact of the relative movements in the real interest rate and real marginal cost on the response of inventories is weakened, as the information capacity constraint implies that firms may deviate from their first-best decision rules. The capacity constraint serves as a force to limit excessive movements in inventories. Thus, the model can deliver a countercyclical inventory-sales ratio after a monetary shock even if marginal cost is rigid. Numerical results demonstrate that a model with inattentive firms and high cost rigidity generates more persistent real effects, and fits the empirical VAR impulse responses and simulated moments better than the Calvo Pricing model with elastic marginal cost.

Chapter 2: Endogenous Irreversibility and Capital Reallocation under Information Asymmetry

2.1 Introduction

Some empirical studies of business cycle dynamics indicate that the reallocation of capital, or the movement of capital input from less productive firms to more efficient firms, is procyclical. Eisfeldt and Rampini (2006) compiled empirical evidence showing strong procyclicality of the reallocation of physical capital among US firms, as represented by acquisitions and sales of plants, property and equipment.[1] Another related important fact is that the cross-sectional standard deviation of capital productivity is countercyclical according to several measures, which means that the benefit of reallocating physical capital across establishments to more productive uses is potentially counter-cyclical; in other words, the benefit is greater in periods of recessions. These two facts point to a puzzle: why does the amount of capital reallocation evolve in almost exactly the opposite way to what the "creative destruction" or the "cleansing effect" point of view regarding business cycle downturns would predict? Eisfeldt and Rampini (2006) postulate that there

[1] The authors provide an updated online version of their empirical evidence of the comovement of capital reallocation and the cyclical component of GDP at the following URL `https://sites.google.com/site/andrealeisfeldt/home/capital-reallocation-and-liquidity`

are costs or frictions involved in reallocating capital, and that those frictions and costs are countercyclical. If we define the time-varying reallocation cost as the inverse of the liquidity of capital, which might stem from adverse selection or financial frictions, then liquidity is expected to be procyclical. A calibrated measurement in Eisfeldt and Rampini (2006) finds that the cost of reallocation is 2.6 times higher in recessions than on average. On the relative importance of reallocation, defined as the sum of acquisitions and sales of property, plants and equipment (PP&E), with respect to aggregate investment activity, Eisfeldt and Rampini (2006) report that reallocation is about 28.6% on average for publicly traded firms in Compustat data, with acquisitions accounting for 19.5% and sales of PP&E about 9.1%.[2] Using Census data, Eisfeldt and Rampini (2007) find that smaller firms and more financially constrained firms rely more on used capital in total capital expenditure.

This paper attempts to fill the gap in the literature by identifying possible sources of frictions that impede capital reallocation, and studying the implications of such frictions for the dynamics of reallocation following a productivity shock. I model the time-varying cost of reallocating capital as an endogenous partial capital irreversibility that stems from both capital specificity and information asymmetry in the used capital market. I extend the idea of Kurlat (2013) and build a general equilibrium model with heterogeneous entrepreneurs producing an identical final good. Entrepreneurs hold capital, a fixed proportion of which is subject to an exogenous separation into capital goods of bad quality, or lemons, which are of

[2] Both acquisitions and sales of PP&E appear to be proyclical for publicly traded firms. Acquisitions do exhibits more procyclicality than sales of PP&E. The correlation between acquisitions and output 0.675, while the correlation between sales of PP&E and output is 0.329. Both statistics are significant.

no use either for production or consumption. Entrepreneurs can directly trade existing capital in the used-capital market. The market is plagued by information asymmetry, since only the sellers know the true quality of used capital they bring to the market, and the buyer cannot verify the quality traded in the market *ex ante*. The pooling price of used capital thus depends on the rationally expected proportion of lemons traded in the market.

As in Kurlat (2013)'s model, heterogeneity is essential here, as it determines the demand and supply of used capital and thus the market equilibrium. I assume that entrepreneurs can buy and sell their existing capital in the used capital market, but due to partial specificity of used capital, they must combine used investment goods with some new investment goods in a bundle to install productive capital. This assumption, together with information asymmetry, create a gap between the buying and selling prices of investment goods and thus partial investment irreversibility, which fluctuates with aggregate shocks.

The Mechanism. The mechanism works as follows. The gap between the price of new investment goods and the price of used capital creates partial irreversibility in plant-level capital accumulation. Because idiosyncratic technological shocks are persistent, depending on the current period idiosyncratic shock and capital holdings, an entrepreneur falls into one of three situations. If he is hit by a low idiosyncratic shock, he will downsize the production scale by selling some good (non-lemon) capital. Conversely, an entrepreneur with a higher idiosyncratic productivity shock expands production scale by acquiring new and used investment goods and supplies no non-lemon used capital to the market. An entrepreneur with an intermediate idiosyncratic shock is characterized by inaction. It neither acquires new capital nor sells any non-lemon capital. The threshold that splits the inaction region from the other two regions depends on the gap between the buying and selling prices of investment goods, which is endogenously affected by aggregate shocks because they affect the decisions of the agents. This mechanism could potentially make capital reallocation procyclical if the inaction region widens during downturns. In turn, procyclical reallocation potentially amplifies the effect of a negative aggregate shock on output, since if less capital flows to the more productive plants, measured aggregate TFP declines. I also extend the model to include a fixed borrowing limit, which does not exceed the natural borrowing constraint. This credit market friction compounds capital irreversibility and further exacerbates capital misallocation, since a borrowing limit hinders the investment undertaken by small scale entrepreneurs with relatively high productivity, and drags down the demand for investment goods. This model also creates a countercyclical dispersion of the marginal product of capital, as small and productive firms expand by less and large unproductive firms downsize by less in recessions, meaning the inherent benefit of capital reallocation is countercyclical.

2.2 Literature Review

Kiyotaki and Moore (2012) were the first papers to introduce the concept of liquidity into a DSGE model. Kiyotaki and Moore (2012) defines the illiquidity of private assets as the degree of impediment to their transactions. In Kiyotaki and

Moore (2012), firms receive an idiosyncratic investment opportunity each period. To finance favorable investment projects, a firm can either issue financial claims to the new project (capital stock) or liquidate existing capital stock. A firm can only issue claims against a fixed fraction of the future returns from the new capital, so that internal funds are needed. Further, existing private claims issued by other firms held on the firm's balance sheet are not fully resalable. Kiyotaki and Moore (2012) models liquidity shocks as changes in the fraction of existing private claims that a firm can sell to finance new investments. In Kiyotaki and Moore (2012), this fraction is *exogenously* determined and fluctuates randomly. When this fraction decreases, the amount of resources that can be efficiently transferred is reduced, causing aggregate investment and output to drop.

One major criticism of Kiyotaki and Moore (2012) is that the exogenous liquidity shock is ad hoc. Some recent studies try to provide a microfoundation to endogenize fluctuations of asset resaleability. Kurlat (2013) builds a model in which entrepreneurs face idiosyncratic investment technology shocks and part of their existing capital becomes lemons each period. Information asymmetry between buyers and sellers of capital regarding its quality generates an adverse selection problem. The model can generate an endogenous liquidity crunch following a shift in the distribution of investment efficiencies. Bigio (2015) builds a related model with two frictions: a limited enforcement friction either in labor contracts or investment inputs (or both), and information asymmetry between sellers and buyers of existing capital (where the buyers are financial intermediaries) regarding the quality of the capital. The limited enforcement friction amounts to a working capital constraint.

In the spirit of Kiyotaki and Moore (2012), entrepreneurs need to sell existing capital stocks to finance working capital (to relax the enforcement constraint). Information asymmetry is also key in this model to generate endogenous liquidity fluctuations. The major difference from Kurlat (2013) is that the endogenous liquidity crunch in Bigio (2015) comes from a shock to the dispersion of asset quality.

Another strand of literature deals with capital irreversibility and its implications. Eisfeldt and Rampini (2006) provide empirical evidence that the amount of capital reallocation is highly procyclical while the inherent benefit of reallocation is countercyclical, meaning that some frictions must exist that prevent the resale of capital in recessions. Cooper and Haltiwanger (2006) point out that in the presence of capital irreversibility, represented as a gap between the buying and selling price of capital, the firm will respond to an adverse shock by holding on to capital to avoid the loss associated with reselling. Further, because of caution the firm will not build its capital stock as quickly. Two papers closely related to mine are Cui (2014) and Lanteri (2015). Cui (2014) builds a DSGE model with constant partial capital irreversibility. Firms hit with negative shocks will hold on to capital at first, and then start disinvesting when the internal return of capital falls below the resale price. Since disinvesting firms face a constant resale price, Cui (2014)'s model cannot generate procyclical capital reallocation without introducing an exogenous tightening of credit constraints. Cui (2014) thus models procyclical capital reallocation as the result of exogenous credit shocks. Lanteri (2015) constructs a model with heterogeneous firms and endogenously time-varying capital irreversibility that stems from partial specificity of used capital.

My model combines both the specificity of used capital, as in Lanteri (2015), and information asymmetry in the market for used capital, as in Kurlat (2013). Similar to Kurlat (2013), this paper has the characteristic that a negative shock affects both the investment/disinvestment decisions of agents and the average quality of assets traded in the market, which feeds back to affect agents' decisions. In my model, however, imperfect substitutability compounds the information asymmetry problem in the asset market, as specificity makes quantity demanded not sensitive enough to price changes, exacerbating the adverse selection effect and pushing average quality of assets to worsen and the equilibrium price to fall more following a negative shock. Capital specificity interacts with adverse selection to determine the resale price of capital and the price gap, so that the model can potentially generate more significant comovement and fluctuation of reallocation. In addition, Lanteri (2015)'s model features perfect capital markets without frictions, whereas I consider a fixed borrowing limit. Khan and Thomas (2013) also study the interplay between credit frictions and capital irreversibility. In their model, capital irreversibility is exogenously given and they focus on studying the role of exogenous credit shocks.

Caveats. This paper, as other studies cited here, does not distinguish between acquisitions and sales of PP&E. However, acquisitions tend to be qualitatively different from organic transfers of used capital, since acquisitions can be merely ownership changes without significant physical reshuffling at the establishment level. The model developed in this paper is more suited to account for physical transfers of PP&E. Furthermore, establishment entry and exits are not considered in my study. Endogenous firm exits can be an important factor shaping reallocation fluctuations.

In the following section, I use a simple one-period model to study analytically the qualitative relationship between the aggregate TFP shock, the price of used capital and capital reallocation. Section 3 develops a dynamic general equilibrium model with labor input and credit constraints.

2.3 A Simple Model of Capital Reallocation

I first construct a simple one-period model of heterogeneous entrepreneurs facing idiosyncratic productivity shocks. The model features information asymmetry in capital quality in the market for used capital, and imperfect substitutability between new and used investment goods. The model is based jointly on the work of Kurlat (2013) and Lanteri (2015). For simplicity I abstract from the labor market and assume that capital is the sole production input. Entrepreneurs own capital and are heterogeneous in their idiosyncratic productivity. As in Kurlat (2013), a fixed proportion (λ) of existing capital units turn at the beginning of the period into bad quality assets or "lemons", which are of no use either as a production input or for consumption. I assume that trading of used capital is conducted in a centralized market. Only the seller of used capital knows the quality it supplies to the market. Buyers are not able to verify the quality before the transaction takes place, and thus the equilibrium price of used capital is a pooling price, which depends both on the demand and supply of used capital, as well as on the average quality of capital traded in the market. Firms that draw a low idiosyncratic productivity will downsize by supplying some good-quality capital on the market. On the other hand, entrepreneurs that wish to expand can do so either by purchasing new investment goods or by acquiring used capital. Used capital and new investment goods are not perfect substitutes, but differ along two dimensions. First, they differ in their quality, as the average quality of used capital depends on the fraction of lemons supplied to the market. Second, used capital has some degree of specificity (exogenously given) so that to install used capital bought in the market the entrepreneur has to combine it with some new investment goods. A negative aggregate shock, such as a TFP shock, affects both the demand and supply of used capital and changes the average quality traded in the market and thus its price, which feeds back to affect agents' decisions. This section serves as a qualitative assessment of how exogenous shocks affect capital reallocation. I use the simple model to derive some analytical results on the response of price and aggregate reallocation of capital to exogenous changes of aggregate productivity.

2.3.1 The Setting

The economy consists of a continuum of entrepreneurs $j \in [0,1]$ normalized to a unit measure. Each entrepreneur owns the same initial capital level k_0. An entrepreneur produces a homogeneous final output good with the production function

$$y_j = z_j Z k_j^{\alpha} \tag{2.1}$$

where Z is aggregate total factor productivity, z_j is the entrepreneur's idiosyncratic productivity shock with distribution function $F(z)$, and $\alpha \in (0,1)$ is the elasticity of output with respect to capital. The fraction of capital goods of bad quality is fixed and given by $\lambda \in (0, 1)$. I assume that capital is infinitely divisible and that the exogenous separation of capital into good and bad quality applies to new investment goods as well.

At the beginning of the period, the entrepreneur observes his idiosyncratic productivity shock and aggregate TFP. The entrepreneur can adjust his capital before production. Apparently, since capital goods of bad quality have no use for production, all entrepreneurs will sell them on the used-capital market, as long as the pooling price of used capital is greater than zero. Firms that draw a low idiosyncratic productivity will find it optimal to disinvest some non-lemon capital, as long as the marginal product of keeping an additional unit of capital is lower than the market price of used capital. On the other hand, an entrepreneur with higher idiosyncratic productivity would like to expand his production scale. It can do so by purchasing new investment goods (assumed to be supplied inelastically by the households) or acquiring used capital from other entrepreneurs. I follow Lanteri (2015) in assuming that used capital is partially specific to its previous owners. Therefore, expanding entrepreneurs cannot invest by buying used capital only. Used capital goods must be bundled with some new investment goods to make them specific to the entrepreneur. Partial specificity makes new investment goods and used capital imperfectly substitutable. This assumption serves two purposes. Firstly, it makes the buying price of capital different from the selling price of capital even if the entrepreneur expands by purchasing used capital, and thus creates partial capital irreversibility. Secondly, it also helps the model generate procyclical reallocation and a countercyclical average market fraction of lemon capital. In a recession, more firms would like to disinvest and downsize, so that the price of used capital tends to fall. However, when used capital is partially specific, expanding firms cannot fully take advantage of a falling price, so that the price effect on quantity demanded is limited by this imperfect substitutability. This makes the price of used capital fall deeper in recession and the inaction region to widen. This point will become clearer in later discussion.

Imperfect Substitution. In my model, units of new and used capital do not enter the capital accumulation function as perfect substitutes even after accounting for quality, as in equation (3) below. This is a key assumption underpinning the mechanism in this paper. Imperfect substitution through a CES aggregator is a crude formulation which reflects some factors not explicitly considered in the model. First, new and used equipment may have different maintenance schedule. After acquiring plant and equipment from liquidators, a firm generally still needs to make maintenance amendments and/or replacements to make the plant ready for use. Another related consideration is the vintage of used capital. Vintage is not explicitly modeled here, but used machines may become less substitutable for new machines as they age, since technological changes may be embedded in new machines. There is also some empirical evidence in the literature supporting imperfect substitutability between new and used assets. Edgerton (2011) uses a tax credit affecting only new capital to test the assumption that used and new capital are perfect substitutes. If they are indeed perfectly substitutable, they must sell at the same after-tax price, and tax provisions favoring new capital should imply a lower price for existing used capital. However, his test showed that the estimated effect of the bonus depreciation on the price of used construction machinery is close to zero, and his estimates for the elasticity of substitution are between 1 and 10. Furthermore, if perfect substitutability holds, even a small increase in the price of used capital would cause investing firms to demand only new capital, which is not consistent with the data. Perfect substitutability is also hard to reconcile with the fact that the price of used investment goods is more volatile than the price of new capital. Garvazza (2011) finds that the resale price of more specific models of aircraft is significantly more volatile than that of more flexible models.

Investment Technology. I assume that an expanding entrepreneur does not have the option to buy only new investment goods. He must combine both new and used investment goods to increase his capital stock. In the model, the price of new investment goods is normalized to one.

The investment technology is given by a CES aggregator of new investment and used capital investment:

$$k_j - (1-\lambda)k_0 = g(i_{j,new}, i_{j,used}) \tag{2.2}$$

$$g(i_{j,new}, i_{j,used}) = \left\{\eta^{\frac{1}{\epsilon}}[(1-\lambda)i_{j,new}]^{\frac{\epsilon-1}{\epsilon}} + (1-\eta)^{\frac{1}{\epsilon}}[(1-\lambda^M)i_{j,used}]^{\frac{\epsilon-1}{\epsilon}}\right\}^{\frac{\epsilon}{\epsilon-1}} \tag{2.3}$$

where $i_{j,new}$ denotes new investment goods and $i_{j,used}$ is used capital purchased by expanding entrepreneurs; λ is the aggregate fraction of bad quality capital (which applies to new investment goods as well) and λ^M is the fraction of bad quality used capital traded in the market. $\eta \in (0,1)$ is a parameter that determines the average ratio between new and used investment, and $\epsilon \in (0,\infty)$ is the elasticity of substitution between new and used investment goods, which also serves as an inverse measure of capital specificity. When $\epsilon \to \infty$, new and used investment goods are perfectly substitutable; when $\epsilon = 0$, the investment technology is Leontief and does not allow any substitutability between new and used capital. Note that new investment does not suffer specificity issues. Investing entrepreneurs bundle new investment goods ($i_{j,new}$) with used investment goods ($i_{j,old}$) via the CES aggregator.

I normalize the price of new investment goods to be 1 and let p denote the equilibrium price of used capital. The following corollary gives the CES price index of a bundle of new and used investment goods.

Corollary 1. *Given a bundle g of new and used investment goods, the CES price index associated with the bundle is*

$$\rho = \Big[\eta\big(\frac{1}{1-\lambda}\big)^{1-\epsilon} + (1-\eta)\big(\frac{p}{1-\lambda^M}\big)^{1-\epsilon}\Big]^{\frac{1}{1-\epsilon}} \tag{2.4}$$

and the optimal composition of new and used investment goods in the bundle is

$$i_{new} = \eta(1-\lambda)^{\epsilon-1}\rho^{\epsilon}g \tag{2.5}$$

$$i_{used} = (1-\eta)(1-\lambda^M)^{\epsilon-1}p^{-\epsilon}\rho^{\epsilon}g \tag{2.6}$$

Proof: See the appendix.

The CES aggregator gives a well defined optimal ratio of new to used investment, given the values of p and λ^M. From corollary 1, it is clear that the higher is the price of used capital p, the lower is the weight of used investment goods in the bundle. On the other hand, as long as the elasticity of substitution of new and used investment goods ϵ is greater than one, a rise in λ^M, the fraction of bad quality capital traded in the market, decreases the relative weight of used investment goods in the bundle. This means that when the perceived average quality of used investment goods is higher, an investing entrepreneur is more willing to substitute used investment goods for new investment goods. Furthermore, note that since $\lambda^M > \lambda$, $\frac{p}{1-\lambda^M} > \frac{p}{1-\lambda}$ always holds. As long as $p \leq 1$ (which is always true throughout the paper, since 1 is the price of new investment goods), I can deduce that $\rho > p/(1-\lambda)$ and thus $\rho > p$ always holds. This means that there is a gap between the buying and selling price of investment goods, and this gap endogenously changes with the price of used capital p and the market fraction of bad quality investment goods λ^M.

Note that rather than modeling the market for new capital explicitly, I make the assumption that expanding entrepreneurs are subject to the investment technology to bundle new investment goods with old ones, and the price of new investment goods is fixed at 1. This is a reduced form way to link investment activities to equilibrium in the market for used capital. It also helps reconcile the model with the empirical fact that a significant proportion of the investment of production units consists of investment in used capital (acquisitions and sales of property, plant and equipment).[3] This assumption implicitly posits that the supply of new investment

[3] Eisfeldt and Rampini (2007) point out that smaller and more credit constrained entrepreneurs

goods is fully elastic, so that the price of new capital goods doesn't change. In the real world, the price of new investment goods does fluctuate, although its volatility is smaller than that of used capital prices.[4] Suppose that I allow the entrepreneurs the option to expand by purchasing new investment goods only, and denote the price of new capital by P_t^N. Then, equilibrium prices ρ_t, p_t, λ^M, and P_t^N should make expanding entrepreneurs indifferent between purchasing new capital goods only or choosing the bundle. This leads to a modified version of equation (2.4):

$$\rho_t = \Big[\eta\big(\frac{P_t^N}{1-\lambda}\big)^{1-\epsilon} + (1-\eta)\big(\frac{p_t}{1-\lambda^M}\big)^{1-\epsilon}\Big]^{\frac{1}{1-\epsilon}} = \frac{P_t^N}{1-\lambda}$$
$$\implies \quad \frac{p_t}{1-\lambda^M} = \frac{P_t^N}{1-\lambda}$$

Hence, the price of new investment goods P_t^N will likely move in the same direction as p_t and in the same direction as λ^M.

2.3.2 Entrepreneur's Optimal Investment Decision

In the one-period model, an entrepreneur simply maximizes one period output, plus the net proceeds from the sales of used capital and the purchase of investment goods. After observing the aggregate TFP and his own idiosyncratic productivity

tend to invest in a higher proportion of used capital when they make investment. They rationalize this phenomenon by the timing of cash flows that constitute used capital investment. I abstract from this consideration here and assume that the same degree of partial specificity applies to all entrepreneurs.

[4] Lanteri (2015) used a dataset on the value of commercial aircraft of different vintages, and reported that the standard deviation of used aircraft prices is more than two times as large as new aircraft prices.

shock, an entrepreneur chooses his optimal capital level for production. Under the same assumptions that lead to corollary 1, I can characterize the optimization problem of a generic entrepreneur j as

$$\max_{s_{b,j},s_{g,j},k_j,g_j} z_j Z k_j^{\alpha} - \rho g_j + p(s_{b,j} + s_{g,j})$$

$$\text{subject to } k_j = \big[(1-\lambda)k_0 - s_{g,j}\big] + g_j$$

$$g_j \geq 0, \quad s_{b,j} \in [0, \lambda k_0], \quad s_{g,j} \in [0, (1-\lambda)k_0]$$

where g_j is the entrepreneur's investment, $s_{b,j}$ are his sales of bad quality capital and $s_{g,j}$ are his sales of good quality capital. Since capital goods of bad quality (lemons) are not useful for production, as long as $p > 0$ the entrepreneur will sell all of his bad capital to the market. Therefore, $s_{b,j} = \lambda k_0$. Furthermore, since $\rho > p$, it is never optimal to sell good capital and buy investment goods at the same time. An entrepreneur can either set $g_j > 0$ or ($s_{g,j} > 0$), but not both. If the entrepreneur chooses neither to invest ($g_j = 0$) nor to disinvest $s_{g,j} = 0$, the amount of good capital that enters the production function is $(1-\lambda)k_0$. As the optimization problem is static, an entrepreneur's optimal investment or disinvestment decision can be easily characterized:

- If the entrepreneur has high enough productivity, he will invest by buying a bundle of new and used investment goods. The entrepreneur will invest if his idiosyncratic productivity z_j is greater than a threshold $z^I \equiv \frac{\rho}{\alpha Z \left[(1-\lambda)k_0\right]^{\alpha-1}}$.

The optimal capital level is given by

$$k_j = \left(\frac{\alpha z_j Z}{\rho}\right)^{\frac{1}{1-\alpha}}$$

and his investment is given by $g_j = k_j - (1-\lambda)k_0$.

- If the entrepreneur is hit by a sufficiently low idiosyncratic productivity, he will disinvest by selling some good quality capital. The entrepreneur will disinvest if his idiosyncratic productivity z_j is lower than a threshold $z^D \equiv \frac{p}{\alpha Z\left[(1-\lambda)k_0\right]^{\alpha-1}}$. The optimal capital level is given by

$$k_j = \left(\frac{\alpha z_j Z}{p}\right)^{\frac{1}{1-\alpha}}$$

and his sales of good capital are given by $s_{g,j} = (1-\lambda)k_0 - k_j$.

- Firms with intermediate idiosyncratic productivity $z^D \leq z_j < z^I$ will neither buy investment goods nor sell any good quality capital, as their marginal product of capital lies between the purchasing price ρ and selling price p.

2.3.3 Equilibrium in the Market for Used Capital

Under Assumption 1, given a chosen amount of total investment, investing entrepreneurs buy a bundle of new and used capital. The result in corollary 1 gives a well-defined fraction of used investment goods relative to the whole bundle. The market demand for used capital can be obtained by integrating over the measure of investing entrepreneurs:

$$D_{used} = (1-\eta)(1-\lambda^M)^{\epsilon-1}\big(\frac{p}{\rho}\big)^{-\epsilon}\int_{z^I}\Big[\big(\frac{\alpha z_j Z}{\rho}\big)^{\frac{1}{1-\alpha}} - (1-\lambda)k_0\Big]dF(z_j) \qquad (2.7)$$

The total supply of used capital is the sum of the supply of lemon used capital λk_0 from all entrepreneurs, and the supply of good-quality used capital:

$$S_{used} = \lambda k_0 + \int^{z^D}\Big[(1-\lambda)k_0 - \big(\frac{\alpha z_j Z}{p}\big)^{\frac{1}{1-\alpha}}\Big]dF(z_j) \qquad (2.8)$$

The market fraction of bad-quality capital λ^M, which must coincide with the entrepreneur's expected value, is given by

$$\lambda^M = \frac{\lambda k_0}{\lambda k_0 + \int^{z^D}\Big[(1-\lambda)k_0 - \big(\frac{\alpha z_j Z}{p}\big)^{\frac{1}{1-\alpha}}\Big]dF(z_j)} \qquad (2.9)$$

The denominator of the above equation is lower than k_0, so $\lambda^M > \lambda$ must hold. In other words, the market fraction of bad capital is always larger than the given fraction λ. Market clearing in the used capital market requires that the total demand for used investment goods is equal to the total supply. This condition defines implicitly the equilibrium price of used capital $p(Z;\epsilon)$ as a function of aggregate productivity Z and the elasticity of substitution between new and used investment goods ϵ. The following proposition summarizes the relationship between this elasticity and the effect of aggregate shocks on capital irreversibility and reallocation.

Proposition 1. *Provided that a regularity condition holds, there exists an*

$\bar{\epsilon} > 0$ *such that for* $\epsilon < \bar{\epsilon}$ *the elasticity of* p *with respect to* Z *is greater than 1, and reallocation is increasing in* Z.

The proof of this result is in the appendix, and the regularity condition basically ensures that the market excess demand for used capital is decreasing in the price of used capital p, which is always satisfied in our quantitative exercise. The intuition behind proposition 1 is as follows. Consider the case when a negative aggregate TFP shock hits the economy. At the original price p and λ^M, quantity demanded will be lower as fewer firms want to expand, and used capital supplied will be higher as more firms want to downsize. Therefore, there is excess supply in the market and the price for used capital tends to fall. However, a higher supply of good quality used capital also drives down the market fraction of lemons, which tends to induce entrepreneurs to substitute more used investment goods for new investment goods because of the higher average quality. This is the indirect effect of Z on excess demand through λ^M, and can be represented by a outward shift in the demand curve for used capital in the price-quantity space. Provided that this indirect effect is not too large, there is still excess supply in the market, and the price of used capital has to fall. When the elasticity of substitution between new and used capital is low (that is, when specificity of used capital is significant), investing entrepreneurs cannot fully take advantage of the falling price since they are limited by this imperfect substitutability. Therefore quantity demanded is not very sensitive to price changes, and p has to fall further to eliminate excess demand. However, a falling p decreases the supply of non-lemon used capital and increases λ^M, which induces market demand to shift inward, reducing the equilibrium quantity of capital reallocated and further exacerbating the decline in price p. The additional thrust of the average quality λ^M on market equilibrium is the major difference between the mechanism in this paper and that of Lanteri (2015). This mechanism of adverse selection can potentially generate more volatile capital reallocation dynamics in business cycles.

2.4 A Full DSGE Model

In this section, I present an infinite-horizon general equilibrium model with a representative household and a unit measure of heterogeneous entrepreneurs producing an identical final good. This model incorporates the static effect of an aggregate productivity shock on capital reallocation described in the previous section.

2.4.1 Households

The economy is populated by a unit measure of identical households and a unit measure of heterogeneous entrepreneurs. Households consume and supply labor to entrepreneurs to earn wage income. Since my focus is on capital reallocation and investment in the entrepreneurial sector, I keep the Household's problem simple by assuming that they do not accumulate wealth and are therefore hand to mouth. Their period utility function is given by the GHH preference:

$$U(c_h, l_h) = \frac{1}{1-\tau}\Big(c_h - \psi \frac{l_h^{1+\theta_h}}{1+\theta_h}\Big)^{1-\tau} \tag{2.10}$$

$$\text{subject to} \quad c_h = w(Z,\mu) l_h$$

where $w(Z, \mu)$ is the equilibrium wage rate as a function of aggegate productivity Z and the cross-sectional distribution of entrepreneurs over idiosyncratic states, which households take as given. θ_h is the reciprocal of the Frisch elasticity of labor supply, which governs wage stickiness in this model. Let $C_h(Z, \mu)$ denote the household decision rule for current consumption, and $L_h(Z, \mu)$ the decision rule for labor supply.

2.4.2 Entrepreneurs

There is a continuum of heterogeneous entrepreneurs with unit measure. Entrepreneurs differ from one another in three dimensions: their idiosyncratic productivity shock z, bond holdings b, and capital at hand k. Let $\mu(k, b, z)$ denote the distribution of entrepreneurs over capital, bond holding, and idiosyncratic productivity. As in the one-period model, entrepreneurs hold capital and trade used capital with one another. Entrepreneurs produce a homogeneous output good with the following decreasing-returns-to-scale technology:[5]

$$y_{jt} = z_{jt} Z_t k_{jt}^{\alpha} n_{jt}^{\gamma} \tag{2.11}$$

with $\alpha + \gamma < 1$. The entrepreneur chooses current labor demand, the future level of capital, and his current consumption to maximize his lifetime utility. I assume that

[5] I assume the production technology to be decreasing instead of constant returns to scale for computational purposes. As will become clear, if the technology is CRS the entrepreneurial profit will be linear in capital, and so an entrepreneur's investment/disinvestment decision will depend only on his idiosyncratic productivity, independent of his capital holdings. In a numerical experiment, since the stochastic process for the entrepreneur's idiosyncratic shock needs to be discretized, this property makes the quantity demanded and supplied less sensitive to price adjustments.

the idiosyncratic shock follows a first-order autoregressive process:

$$\log z_{jt} = \rho_z \log z_{j,t-1} + \varepsilon_{zt} \tag{2.12}$$

where ε_{zt} is an *i.i.d.* perturbation with mean zero and variance σ_ε.

Timing. At the start of period t, an entrepreneur observes his state variables k_{jt}, b_{jt}, and z_{jt}, and aggregate productivity Z_t. He hires labor and uses existing capital for production. After production, the markets for used investment goods and bonds open. Existing capital depreciates after production at a depreciation rate δ. There is also an exogenous scrappage of bad capital at a given rate λ. For installed capital, I assume that capital depreciation and scrappage happens at the same time, and scrapped capital is part of depreciated capital, so that $\delta \geq \lambda$ holds. The entrepreneur decides his optimal consumption, investment/disinvestment and bond holdings, and trades used capital with other entrepreneurs. At the end of the period all lemon capital and the depreciated capital are retired from the economy and the entrepreneur carries the remaining capital into the next period $t+1$.

Entrepreneurs take the equilibrium wage rate $w(Z_t, \mu)$, gross interest rate $R(Z_t, \mu)$, the market fraction of bad-quality used investment goods λ_t^M, and the market price of used capital p_t as given. They can borrow with a one-period bond, but the amount that they can borrow is subject to a fixed borrowing limit $\underline{b}$. An entrepreneur chooses current labor, future capital and bond holding to maximize his lifetime utility. Since there is no labor market friction, the labor hiring decision is intratemporal and can be separated from the investment or disinvestment decision in a convenient way. I first solve for the labor decision and derive the implied return on capital.

Given the amount of capital at hand, an entrepreneur's labor demand is determined by equating the marginal product of labor to the wage rate:

$$n_{jt} = \left[\frac{\gamma z_{jt} Z_t k_{jt}^{\alpha}}{w_t}\right]^{\frac{1}{1-\gamma}} \tag{2.13}$$

and entrepreneurial output net of the wage bill is

$$\begin{aligned} y_{jt} - w_t n_{jt} &= \left[\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} - w_t\left(\frac{\gamma}{w_t}\right)^{\frac{1}{1-\gamma}}\right]\left(z_{jt} Z_t k_{jt}^{\alpha}\right)^{\frac{1}{1-\gamma}} \\ &= \pi(w_t)\left(z_{jt} Z_t k_{jt}^{\alpha}\right)^{\frac{1}{1-\gamma}} \end{aligned} \tag{2.14}$$

where $\pi(w_t) = \left[\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} - w_t\left(\frac{\gamma}{w_t}\right)^{\frac{1}{1-\gamma}}\right]$. Since I assume that the production technology is decreasing returns to scale, i.e. $\alpha + \gamma < 1$, the return to capital is concave in the entrepreneur's existing capital k_{jt}. Since optimal labor demand is an intratemporal decision, this return on capital function can be incorporated into the entrepreneur's intertemporal consumption, saving and investment decisions.

The entrepreneur's intertemporal problem is recursive in the state vector (k_{jt}, b_{jt}, z_{jt}), where b_{jt} are his holdings of one-period bonds. After observing the state, each entrepreneur makes a discrete decision of investing, disinvesting (by selling some good-quality capital), or inaction. He also chooses current consumption and next-period bond holding. Let subscript i denote investing, d denote disinvesting, and n denote inaction. I describe a generic entrepreneur j's intertemporal problem in recursive form, suppressing the j subscript for ease of notation:

$$V(k,b,z;Z,\mu) = \max \left\{ V^i(k,b,z;Z,\mu), V^d(k,b,z;Z,\mu), V^n(k,b,z;Z,\mu) \right\} \tag{2.15}$$

The value of an investing entrepreneur is

$$\begin{aligned} V^i(k,b,z;Z,\mu) = &\max_{c,k'>(1-\delta)k,b',s_b} \quad \log(c) + \beta \mathrm{E}\big[V(k',b',z';Z',\mu')|z,Z\big] \\ \text{subject to} \quad & c + \frac{b'}{R} = \pi(w_t)\Big(z_{jt}Z_t k_{jt}^{\alpha}\Big)^{\frac{1}{1-\gamma}} - \rho[k' - (1-\delta)k] + p\, s_b + b \\ & b' \geq -\underline{b} \\ & s_b \in [0, \lambda k] \end{aligned} \tag{2.16}$$

The value of a disinvesting entrepreneur is

$$\begin{aligned} V^d(k,b,z;Z,\mu) = &\max_{c,k'<(1-\delta)k,b',s_b} \quad \log(c) + \beta \mathrm{E}\big[V(k',b',z';Z',\mu')|z,Z\big] \\ \text{subject to} \quad & c + \frac{b'}{R} = \pi(w_t)\Big(z_{jt}Z_t k_{jt}^{\alpha}\Big)^{\frac{1}{1-\gamma}} - p[k' - (1-\delta)k] + p\, s_b + b \\ & b' \geq -\underline{b} \\ & s_b \in [0, \lambda k] \end{aligned} \tag{2.17}$$

The value of inaction is

$$
\begin{aligned}
V^n(k, b, z; Z, \mu) = \max_{c, b'} \quad & \log(c) + \beta \mathrm{E}\big[V((1-\delta)k, b', z'; Z', \mu')|z, Z\big] \qquad (2.18)\\
\text{subject to} \quad & c + \frac{b'}{R} = \pi(w_t)\Big(z_{jt} Z_t k_{jt}^{\alpha}\Big)^{\frac{1}{1-\gamma}} + p\, s_b + b \\
& b' \geq -\underline{b} \\
& s_b \in [0, \lambda k]
\end{aligned}
$$

where $\underline{b}$ is an exogenously given borrowing limit that does not exceed the natural borrowing constraint, and s_b is the entrepreneur's sales of bad-quality capital.

2.4.3 Market Clearing & Recursive Equilibrium

There are four markets in the model: the market for used investment goods, the market for labor, the market for final goods, and the market for one-period bonds. Market clearing in the market for used investment goods is imposed in a similar way to the one-period model in section 2. Investing entrepreneurs build new investment goods and bundle them with used investment goods via a CES aggregator. The demand for used investment goods is obtained by solving a CES expenditure minimization problem. The pooling price of used capital p_t equates the total demand for used investment goods and total supply. The wage rate w_t equates aggregate labor demand from entrepreneurs and labor supply from households. The gross interest rate R_t clears the market for one-period bonds (such that aggregate net bond holding of the entrepreneurial sector is zero).

The following definition describes a recursive competitive equilibrium.

Definition 1. (Recursive Competitive Equilibrium). *A recursive competitive equilibrium is defined as a set of prices* $\{p, \rho, R, w\}$*; the market fraction of bad capital* λ^M*; a law of motion* Γ *for the cross-sectional distribution* $\mu(k, b, z)$*; value functions of entrepreneurs* $\{V^i, V^d, V^n, V\}$*; decision rules for entrepreneurs* $\{c, k', b', n, s_b, g, i_{new}, i_{used}, d\}$ *and decision rules for households* $\{C_h, L_h\}$*, such that*

(i) Given wage rate w*,* $\{C_h, L_h\}$ *maximizes period utility of the households.*

(ii) The entrepreneur's labor demand $n(k, b, z)$ *satisfies (13).*

(iii) The value functions V^i, V^d, V^n *and* V *solve (15)-(18) and* c, k', b' *are the associated policy functions.*

(iv) For investing entrepreneurs, the investment is $g(k, b, z) = k'(k, b, z) - (1 - \delta)k$ *where* k' *solves (16).* $g(k, b, z)$ *is allocated to* i_{new} *and* i_{used} *according to (5) and (6).*

(v) For disinvesting entrepreneurs, the disinvestment (supply of good used capital) is $d(k, b, z) = (1 - \delta)k - k'(k, b, z)$*, where* k' *solves (17).*

(vi) The market for final output clears:

$$C_h + \int c(k, b, z)d\mu + \int i_{new}(k, b, z)d\mu = Z \int zk^{\alpha}n(k, b, z)^{\gamma}d\mu$$

(vii) The labor market clears: $L_h = \int n(k, b, z)d\mu$

(viii) Aggregate net bond holding is zero: $\int b'(k, b, z)d\mu = 0$.

(ix) The market for used capital clears:

$$\lambda \int k d\mu + \int d(k, b, z)d\mu = \int i_{used}(k, b, z)d\mu$$

(x) The market fraction of bad quality capital satisfies

$$\lambda^M = \frac{\lambda \int k d\mu}{\lambda \int k d\mu + \int d(k,b,z) d\mu}.$$

Furthermore, λ^M*, the price of used capital* p *and the price of investment bundle* ρ *satisfy (4).*

(xi) The idiosyncratic shock z_j *follows the AR(1) process* $\log z_{jt} = \rho_z \log z_{j,t-1} + \varepsilon_{zt}$*. The law of motion* Γ *defines the evolution of the distribution* μ *of entrepreneurs over* k, b *and idiosyncratic productivity* z*, given the policy functions and the transition probability of* z*.*

2.4.4 Analysis & Computation

Households

Since households do not accumulate wealth, their problem is static and the first order condition of (10) with respect to l_h gives the following simple equilibrium conditions:

$$l_h = \Big(\frac{w(Z,\mu)}{\psi}\Big)^{\frac{1}{\theta_h}} \tag{2.19}$$

$$c_h = w(Z,\mu) l_h \tag{2.20}$$

Equations (2.19) and (2.20) fully characterize the solution to the household's problem. Note that the higher is the Frisch elasticity $1/\theta_h$, the more sensitive is labor supply to changes in the wage rate.

Entrepreneurs

The entrepreneur's problem involves discrete choices arising from investment irreversibility, three-dimensional heterogeneity in capital holding, saving, and idiosyncratic productivity, and an occasionally binding borrowing constraint, and is therefore too computationally demanding to solve with standard VFI. I apply the endogenous grid method (EGM) first proposed by Carroll (2006) to solve this dynamic programming problem. The essence of EGM is to predetermine the optimal value of the endogenous state variable and use the first order optimality condition to directly solve for the decision rule for the control variable, thus circumventing the most time-consuming procedure of root finding of the first order optimality conditions in standard VFI. However, one major difficulty in applying EGM to the entrepreneur's problem stems from the mixed discrete-continuous choice arising from partial capital irreversibility. While EGM is more efficient and accurate than standard VFI, standard EGM cannot be used when the value function is non-concave, as the first-order condition is necessary but not sufficient to guarantee an optimal solution, as in the entrepreneur's problem considered here, which involves a mixed discrete-continuous choice in capital holdings. I use the EGM algorithm developed by Fella (2014), which can deal with non-smooth and non-concave problems. In what follows I briefly describe how the algorithm can be used to solve the entrepreneur's problem.

All entrepreneurs trivially sell their lemon capital. The investing entrepreneur's flow budget constraint can be written as

$$c_t + \frac{b_{t+1}}{R_t} = \pi(w)\Big(z_{jt}Z_t k_{jt}^{\alpha}\Big)^{\frac{1}{1-\gamma}} + b_t - \rho_t[k_{t+1} - (1-\delta)k_t] + p_t\lambda k_t$$

and the disinvesting entpreneur's flow budget constraint is

$$c_t + \frac{b_{t+1}}{R_t} = \pi(w)\Big(z_{jt}Z_t k_{jt}^{\alpha}\Big)^{\frac{1}{1-\gamma}} + b_t - p_t[k_{t+1} - (1-\delta)k_t] + p_t\lambda k_t$$

The borrowing constraint is $b_{t+1} \geq -\underline{b}$. Apply a change of variable to get:

$$a_{t+1} = b_{t+1} + \underline{b} \geq 0 \tag{2.21}$$

where a_{t+1} denotes the entrepreneur's *end-of-period* asset holding adjusted for untapped borrowing capacity. Let

$$m(k_{t+1}; a_t, k_t, z_{jt}) = \pi(w)\Big(z_{jt}Z_t k_{jt}^{\alpha}\Big)^{\frac{1}{1-\gamma}} + a_t - h(k_{t+1}, k_t) + p_t\lambda k_t - \frac{R_t - 1}{R_t}\underline{b}$$

be the total resources available for consumption, conditional on choosing k_{t+1}, where

$$h(k_{t+1}, k_t) = \begin{cases} \rho_t[k_{t+1} - (1-\delta)k_t] & \text{if } k_{t+1} > (1-\delta)k_t \\ p_t[k_{t+1} - (1-\delta)k_t] & \text{otherwise} \end{cases}$$

I can rewrite the entrepreneur's budget constraint as:

$$c_t + \frac{a_{t+1}}{R_t} = m(k_{t+1}; a_t, k_t, z_{jt}) \tag{2.22}$$

An entrepreneur's recursive problem in a stationary equilibrium without aggregate uncertainty can be rewritten as:

$$\mathbb{V}(a, k, z) = \max_{k' \geq 0, a' \in [0, m(k'; a, k, z)R]} \log(m(k'; a, k, z) - \frac{a'}{R}) + \tilde{V}(a', k', z) \tag{2.23}$$

where $\mathbb{V}(a, k, z)$ is the value function of the entrepreneur in state (a, k, z) and

$$\tilde{V}(a', k', z) = \beta \mathrm{E}_z \mathbb{V}(a', k', z')$$

denotes the expectation of the continuation value.

Let $\Omega(\cdot, \cdot; a, k, z)$ denote the feasibility set given state variables $a, k,$ and z_t, with $\Omega(a', k'; a, k, z)$ being one specific point in it and

$$\Omega(\cdot, \cdot; a, k, z) = \{a', k' : a' \in [0, m(k'; a, k, z)], k' > 0\}$$

Denote the per-period utility function as

$$U(a', k'; a, k, z) = \log(m(k'; a, k, z) - \frac{a'}{R})$$

The specification of the per-period utility functions satisfy the following two conditions:

Condition 1. *The per-period utility function U(a',k'; a,k,z) is differentiable with respect to* a' *on the interior of* $\Omega(\cdot, k'; a, k, z)$ *and with respect to* a *on the interior of* $\Omega(a', k'; \cdot, k, z)$.

Condition 2. *The second derivative* $U_{a'a'}(a', k'; a, k, z)$ *is strictly negative.*

The first order condition of (23) with respect to a' is:

$$-\frac{1}{R[m(k'; a, k, z) - \frac{a'}{R}]} + \tilde{V}_a(a', k', z) = 0 \tag{2.24}$$

I state and prove the following corollary that underpins solving the entrepreneur's problem in the quantitative exercise:

Corollary 2. *The feasibility set* $\Omega(\cdot, \cdot; a, k, z)$ *is convex, and the first order condition (24) is a necessary and sufficient condition for a maximum of* a'.

Proof: Consider any two points (a'_1, k'_1) and (a'_2, k'_2) that belong to the feasibility set $\Omega(\cdot, \cdot; a, k, z)$. To show that the feasibility set is convex, it suffices to show that any linear combination of (a'_1, k'_1) and (a'_2, k'_2) still lies in the feasibility set. In other words, we want to show that for any $\theta \in (0, 1)$,

$$\theta a'_1 + (1 - \theta)a'_2 \leq m(\theta k'_1 + (1 - \theta)k'_2; a, k, z)R \tag{2.25}$$

It is straightforward to see that if both k'_1 and k'_2 are both larger than $(1-\delta)k$, then linear combinations of (a'_1, k'_1) and (a'_2, k'_2) satisfy the above inequality. The same holds true if both k'_1 and k'_2 are both less than or equal to $(1 - \delta)k$. Without loss of generality, let $k_1' > (1-\delta)k$ and $k_2' \leq (1-\delta)k$. The proof can be divided into two cases.

Case 1: $\theta k_1' + (1-\theta)k_2' > (1-\delta)k$; In this case, since in equilibrium $0 < p_t < \rho_t$ and since $k_2' \leq (1-\delta)k$, I have

$$
\begin{aligned}
&-\theta\rho_t[k_1' - (1-\delta)k] - (1-\theta)p_t[k_2' - (1-\delta)k] \\
&\quad \leq -\theta\rho_t[k_1' - (1-\delta)k] - (1-\theta)\rho_t[k_2' - (1-\delta)k] = -\rho_t[\theta k_1' + (1-\theta)k_2' - (1-\delta)k]
\end{aligned}
$$

Thus, inequality (25) is satisfied.

Case 2: $\theta k_1' + (1-\theta)k_2' \leq (1-\delta)k$; In this case, since in equilibrium $0 < p_t < \rho_t$ and since $k_1' > (1-\delta)k$, I have

$$
\begin{aligned}
&-\theta\rho_t[k_1' - (1-\delta)k] - (1-\theta)p_t[k_2' - (1-\delta)k] \\
&\quad \leq -\theta p_t[k_1' - (1-\delta)k] - (1-\theta)p_t[k_2' - (1-\delta)k] = -p_t[\theta k_1' + (1-\theta)k_2' - (1-\delta)k]
\end{aligned}
$$

Thus, inequality (25) is still satisfied. This establishes that the feasibility set $\Omega(\cdot, \cdot; a, k, z)$ is convex. By Theorem 4.11 in Stokey et al. (1989), I know that the first order condition (24) is a necessary and sufficient condition for a maximum of a'.

Given corollary 2, I can apply the endogenous grid method (EGM) developed by Barillas and Fernandez-Villaverde (2007) to run value function iteration more efficiently.

2.4.5 Amplification Mechanism of Reallocation Friction

In this section, I argue that procyclical capital reallocation can amplify an exogenous TFP shock, in the sense that measured aggregate TFP falls by more than the magnitude of the exogenous TFP shock itself. Let Z_A denote measured aggregate TFP. An econometrician usually measure aggregate TFP by the residual method. He would first assume an aggregate production function

$$Y = Z_A K^\alpha N^\gamma \implies Z_A = \frac{Y}{K^\alpha N^\gamma}$$

where Y, K, and N denote aggregate capital, labor, and output respectively. The econometrician would get estimates of α and γ, and use observed Y, K, and N to back out measured aggregate TFP, Z_A. Measured TFP is affected both by the exogenous TFP shock, Z, and by the allocative efficiency of capital and labor across heterogeneous entrepreneurs. Suppose, for simplicity, that the entrepreneurs' idiosyncratic productivity can only be either z_h or z_l, which denote the high and low idiosyncratic shock respectively, with $z_h > z_l$. Further, suppose that the measure of entrepreneurs facing high and low shocks is equal, and that the production function is constant returns to scale, $\alpha = 1 - \gamma$. Let actual aggregate TFP, Z, be 1. Let k_h and k_l denote capital holdings of entrepreneurs facing high and low idiosyncratic shocks respectively. The following corollary argues heuristically that more severe capital misallocation lowers measured aggregate TFP.

Corollary 3. *Under the above assumptions, the lower is $\frac{K_h}{K_l}$, the lower is Z_A*

Proof: By definition

$$Z_A = \frac{Y}{K^\alpha N^{1-\alpha}} = \frac{y_h + y_l}{(k_h + k_l)^\alpha (n_h + n_l)^{1-\alpha}}$$

Labor demand and output are expressed in equations (2.13) and (2.14) as functions of idiosyncratic productivity, the wage rate, and capital holding. Plugging them back into the above equation, I get

$$\begin{aligned} Z_A &= \frac{(\frac{\gamma}{w})^{\frac{1-\alpha}{\alpha}} (z_h^{\frac{1}{\alpha}} k_h + z_l^{\frac{1}{\alpha}} k_l)}{(k_h + k_l)^\alpha (\frac{\gamma}{w})^{\frac{1-\alpha}{\alpha}} (z_h^{\frac{1}{\alpha}} k_h + z_l^{\frac{1}{\alpha}} k_l)^{1-\alpha}} = \left(\frac{z_h^{\frac{1}{\alpha}} k_h + z_l^{\frac{1}{\alpha}} k_l}{k_h + k_l}\right)^\alpha \\ &= \left(z_h^{\frac{1}{\alpha}} - \frac{z_h^{\frac{1}{\alpha}} - z_l^{\frac{1}{\alpha}}}{\frac{k_h}{k_l} + 1}\right) \end{aligned}$$

Since $z_h > z_l$, from the above equation it is clear that when $\frac{k_h}{k_l}$ is lower, measured TFP is lower. This captures the amplification mechanism of procyclical reallocation frictions. In a recession, the price gap widens and entrepreneurs do not adjust to their efficient capital level. Entrepreneurs with low idiosyncratic productivity hold too much capital, which results in an efficiency loss of the economy.

The endogenous allocative component of measured TFP can be measured as

$$TFP_{end} = \log(Z_{A,t}) - \log(Z_t)$$

My numerical results in the next section show that TFP_{end} falls following a negative TFP shock.

2.5 Numerical Results

This section presents some numerical results in general equilibrium of the economy without aggregate uncertainty. The economy is without aggregate uncertainty in the sense that it stays at the stationary equilibrium until the shock hits. The shock is unanticipated by the agents and after the perturbation periods agents expect no more shocks in the future, so that the economy transitions back to the stationary equilibrium eventually. To arrive at the stationary equilibrium, I keep aggregate TFP (Z) fixed at 1, and start from an initial cross-sectional distribution μ_0. Market clearing prices p, w, r are solved iteratively with a bisection method until excess demand/supply is within an error bound, and λ^M is obtained with adaptive updates in the inner loop. I numerically solve the entrepreneur's problem, obtain his policy functions and compute the equilibrium variables which clear the market for used capital, the labor market, and the bond market. Then the cross-sectional distribution is updated and I iterate on this process until the cross-sectional distribution converges to the stationary distribution. After the stationary distribution is reached, I hit the model with negative TFP shocks and compute the impulse responses of equilibrium variables.

2.5.1 Parameters

Table 1 lists the key parameters used in the numerical experiment. The parameters are set generally in correspondence with past numerical exercises. The investment technology is defined by two parameters: η and ϵ. I follow Lanteri

(2015) and set $\eta = 0.70$. This gives a 31.7% steady-state ratio of used capital to total capital purchased by investing firms, which is approximately the upper bound of the estimates found by Eisfeldt and Rampini (2007). The elasticity of substitution between new and used investment goods ϵ is a key parameter in this model. Edgerton (2011) exploits a tax credit (bonus depreciation incentive) which affected only new investment to provide estimates of this elasticity using data from construction equipment, aircraft and farming equipment. His estimated values range between 1 and 10. I intentionally set this elasticity to be low in my benchmark case to examine the effects of a low elasticity of substitution in this model. Since there is limited empirical evidence on this parameter, I try a higher value of ϵ to see how the results are affected. I also try different values of the borrowing limit $\underline{b}$ to numerically assess the effect of a tighter or looser borrowing constraint.

Parameter	Value
β	0.95
γ	0.55
α	0.24
δ	0.065
λ	0.002
η	0.7
ϵ	2.5
$\underline{b}$	0.35
θ_h	0.4
ψ	2.0
τ	2.0
ρ_z	0.9
σ_ε	0.11

Tab. 2.1: Benchmark parameters.

The benchmark calibration gives rise to the following steady-state values:

Y	C	I	K	N	p	λ^M	Capital Reallocation
0.557	0.480	0.077	1.190	0.262	0.944	6.84%	0.035

Tab. 2.2: Steady state values under benchmark calibration

The borrowing constraint is about 63% of production in the steady state. The natural borrowing limit is the production lowest realization of the idiosyncratic productivity shock divided by the net interest rate $(R_t - 1)$.

2.5.2 Numerical Results

The entrepreneur's idiosyncratic shock process is discretized using the Tauchen's method into 5 different values. I assume that the economy is at the stationary equilibrium at $t = 0$, and I perturb the economy with negative aggregate TFP shocks of -1.00% at $t = 1$ and -0.90% at $t = 2$, after which TFP reverts back to $Z = 1$.

The figures 2.1, 2.2, and 2.3 present the impulse responses of various model variables in general equilibrium, whereas figures 2.4, 2.5, and 2.6 present impulse responses of allocative efficiency.[6] Besides IRFs under the presence of information asymmetry, I also plot the impulse responses under no information asymmetry ($\lambda = 0$), in which case the model collapses to that of Lanteri. The figures 2.1 and 2.4 are under the benchmark specification with the parameter values listed in table 2.1. In figures 2.2 and 2.5, I consider a higher elasticity of substitution ($\epsilon = 3.5$), and I consider the effect of a tighter borrowing limit ($\underline{b} = 0.25$) in figures 2.3 and 2.6.

In the benchmark case, as the shock hits, there's only a small drop in the price of used investment goods. However, following the fall in p, the market fraction of

[6] The IRFs of the market fraction λ^M and of aggregate sales of good capital S_g are mirror images, as $\lambda_M = \frac{S_b}{S_b+S_g}$, and S_b is determined by the aggregate capital stock, which is quite stable.

bad capital rises substantially, which is consistent with what the model predicts. The decrease in price and the increase in λ^M exacerbates the drop in both the supply of good-quality investment goods and the demand for used capital, causing capital reallocation to fall. Total reallocation stands for $S_b + S_g$. It is clear that with the information asymmetry, the drop in total reallocation is about 2 times as large as under no information asymmetry, confirming the model's implication that adding information asymmetry can substantially increase the procyclicality of capital reallocation. One can also see that the magnitude of the fall in investment is significantly larger under information asymmetry, output is marginally lower, and it takes longer for aggregate capital to climb back to the pre-shock steady state level. As we can see in figure 2.2, considering a higher elasticity of substitution does not alter the above observation, but the effects of the negative TFP shocks on capital reallocation and on aggregate investment are slightly more muted. Figure 2.2 suggests that the procyclicality of capital reallocation may be robust to a range of the elasticity of substitution. Figure 2.3 considers a tighter borrowing limit. One can see that the fall in capital reallocation and aggregate investment is now larger, compared to figure 2.1, both in the case of my model and in that of Lanteri (2015). The intuition is that some investing entrepreneurs will be credit constrained and cut back ther capital acquisition, causing their demand for both the new and used capital to fall. However, the overall qualitative distinction of the responses under the two cases does not change significantly.

Allocative Efficiency In figures 2.4 - 2.6, I look at the change in two measures of allocative efficiency following the negative TFP shocks. The first measure is mean MPK ratio, defined as the mean marginal product of capital of entrepreneurs with the highest idiosyncratic shock divided by the mean MPK of entrepreneurs receiving the second lowest idiosyncratic shock. The second measure is the endogenous allocative efficiency component of measured TFP discussed in section 3.5. Figure 2.4 shows that the mean MPK ratio rises on impact of the negative shock, and it increases more in the presence of information asymmetry. One can make a similar conclusion by looking at the responses of endogenous TFP. These results show that the dispersion of the marginal return of capital widens and allocative efficiency worsens following the shock. Figure 2.5 corresponds to a higher elasticity of substitution. The results are similar to figure 4, but the responses are slightly dampened. In 2.6, we can observe that, with a tighter borrowing limit, the effect of the shocks on the mean MPK ratio is much larger, while the responses of endogenous TFP are almost identical whether or not there is information asymmetry or not. These results suggests that a tighter borrowing limit might exert a significant effect on allocative efficiency.

2.6 Conclusion

This paper explores the cyclicality of capital reallocation. The model incorporates two key features: imperfect substitutability between new and used capital and information asymmetry in the resale market of used investment goods. Impulse response functions of equilibrium variables show that the interaction of these two ingredients does generate procyclicality of reallocation, countercyclical dispersion of capital returns, and procyclical allocative efficiency. A higher elasticity of substitution ϵ seems to dampen the procyclical property of reallocation, while a tighter borrowing limit tends to increase the procyclicality of capital reallocation. However, the effect of information asymmetry per se seems to be robust under different parameter values considered in my exercise.

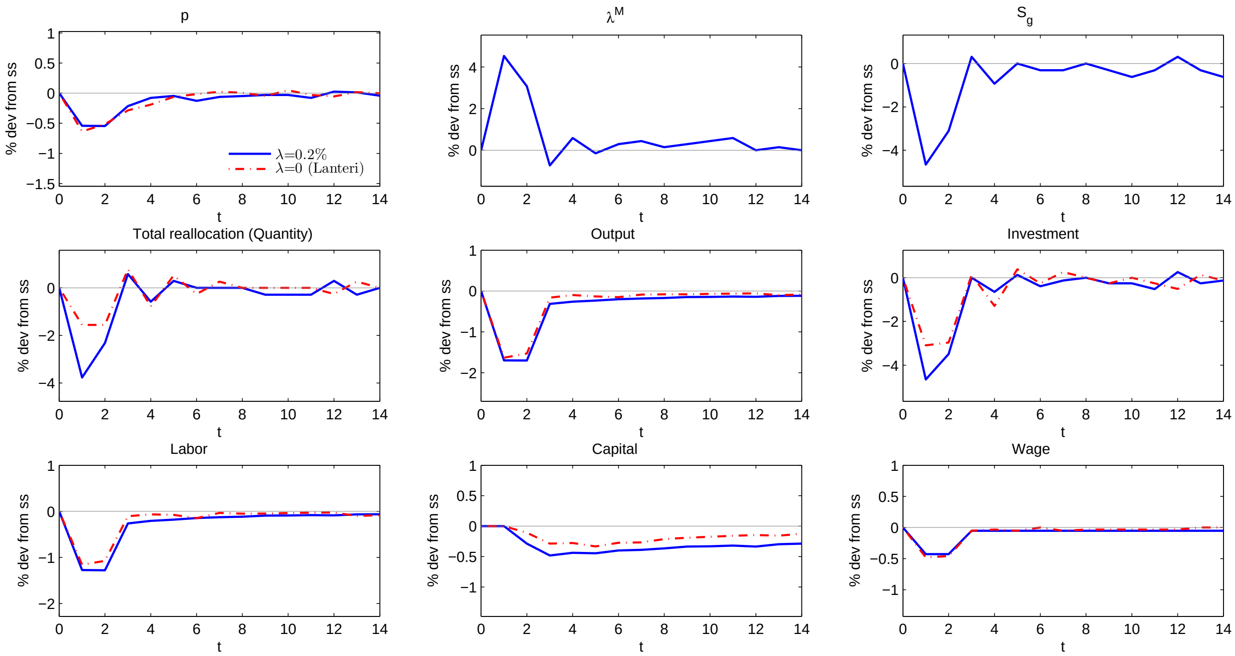

Fig. 2.1: Impulse responses to negative TFP Shocks that last for 2 periods with $\epsilon = 2.5$ and $\underline{b} = 0.35$. The magnitudes of the shocks are 1.0% in the first period and 0.9% in the second period.

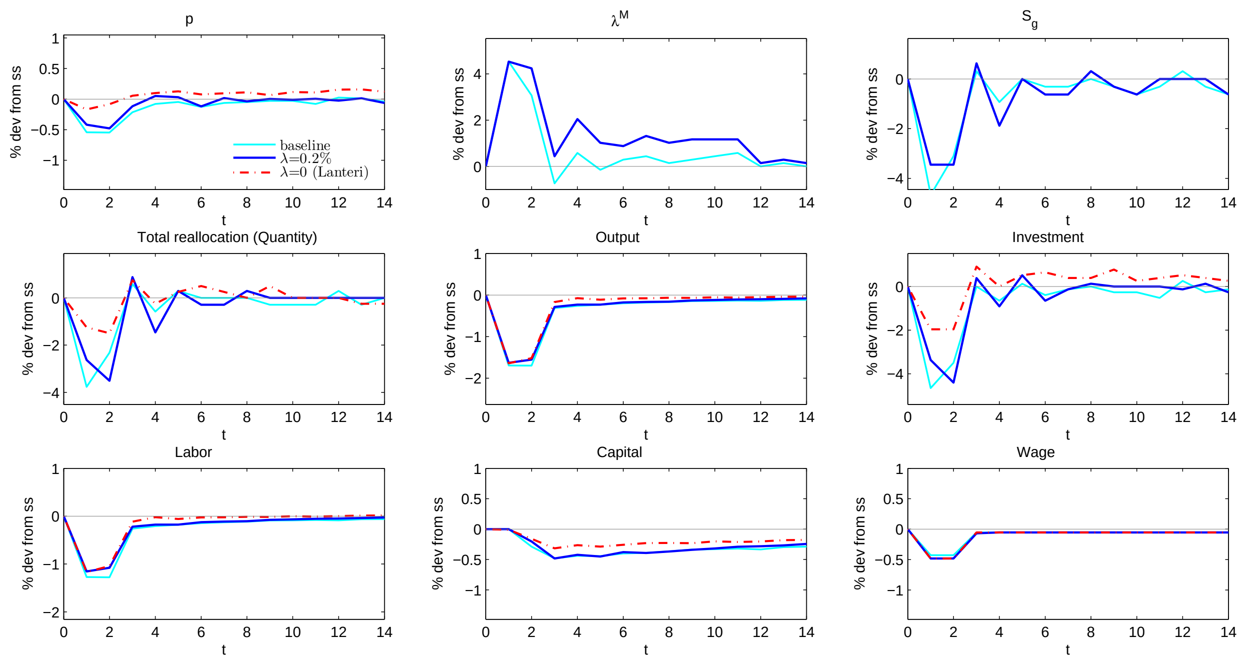

Fig. 2.2: Impulse responses to negative TFP Shocks that last for 2 periods with $\epsilon = 3.5$ and $\underline{b} = 0.35$. The magnitudes of the shocks are 1.0% in the first period and 0.9% in the second period.

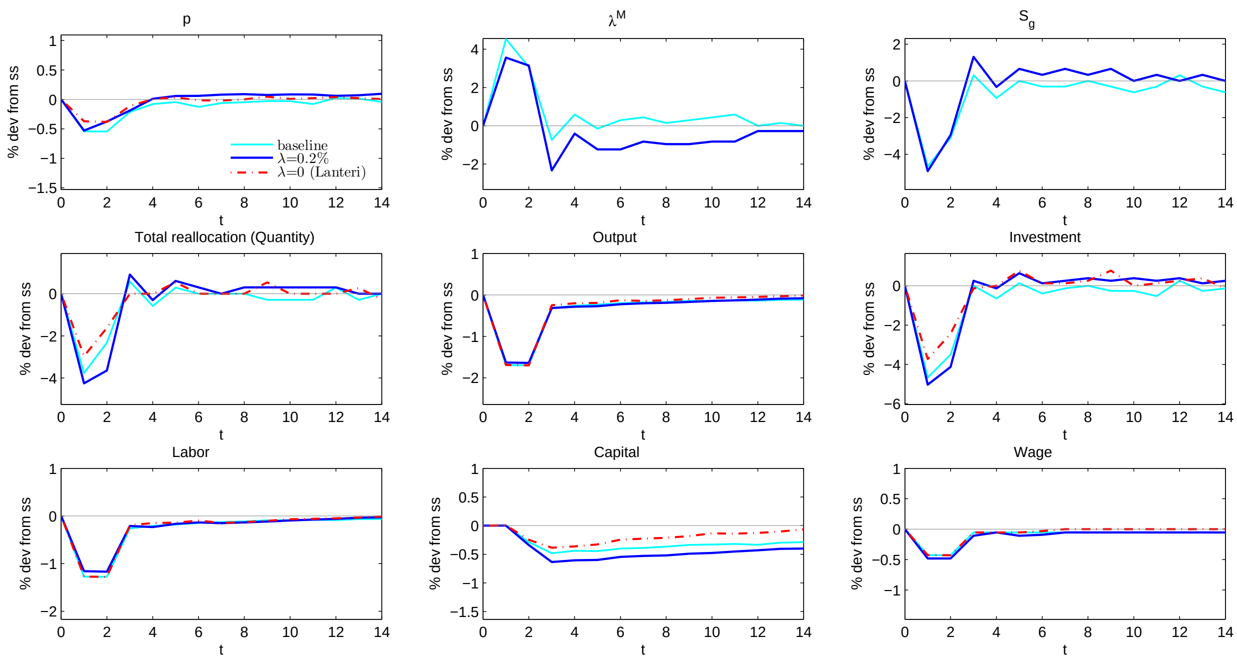

Fig. 2.3: Impulse responses to negative TFP Shocks that last for 2 periods with $\epsilon = 2.5$ and $\underline{b} = 0.25$. The magnitudes of the shocks are 1.0% in the first period and 0.9% in the second period.

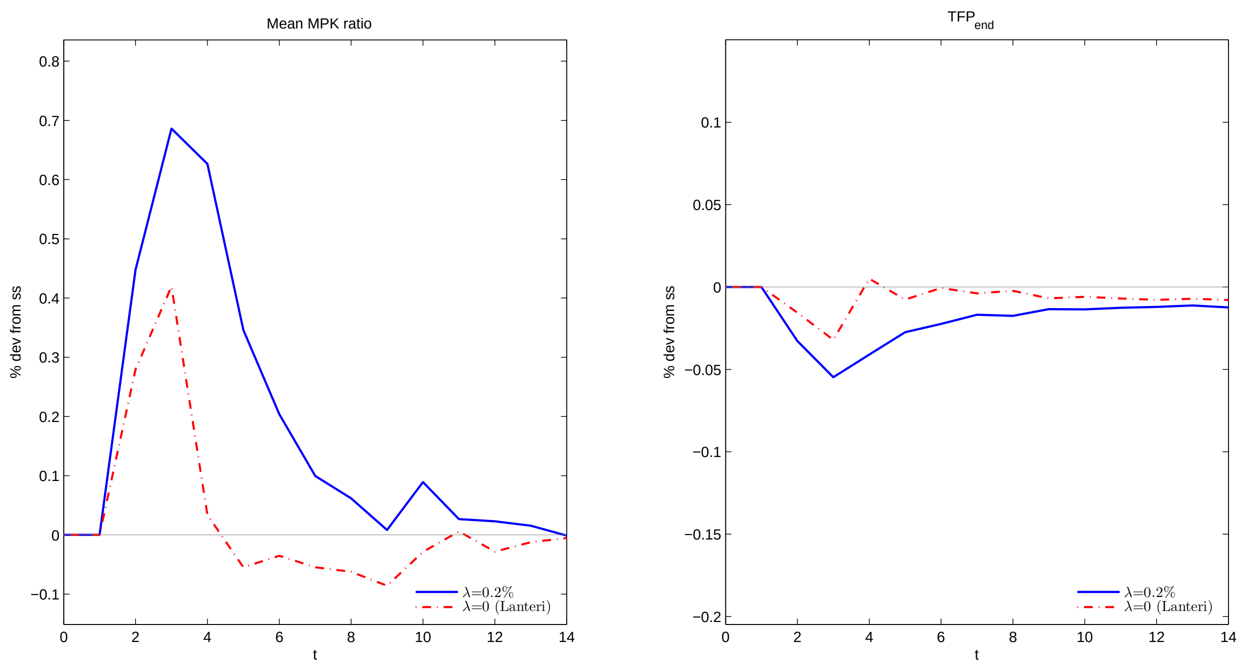

Fig. 2.4: Impulse responses of allocative efficiencies to negative TFP Shocks that last for 2 periods with $\epsilon = 2.5$ and $\underline{b} = 0.35$. The magnitudes of the shocks are 1.0% in the first period and 0.9% in the second period.

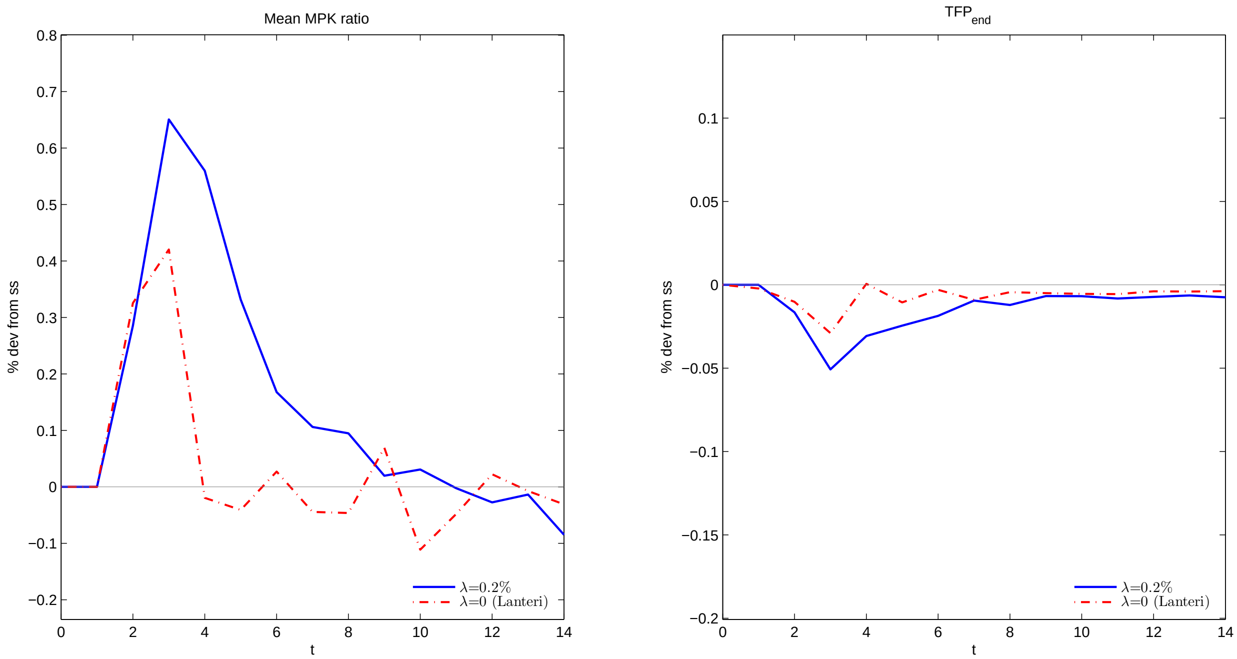

Fig. 2.5: Impulse responses of allocative efficiencies to negative TFP Shocks that last for 2 periods with $\epsilon = 3.5$ and $\underline{b} = 0.35$. The magnitudes of the shocks are 1.0% in the first period and 0.9% in the second period.

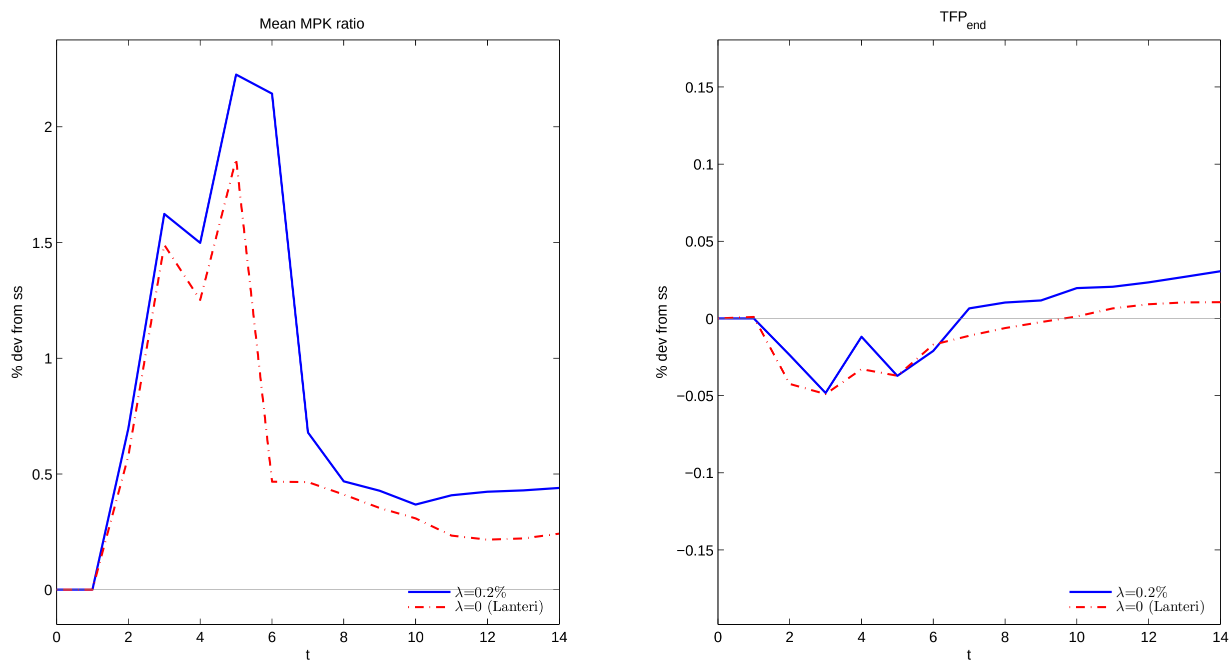

Fig. 2.6: Impulse responses of allocative efficiencies to negative TFP Shocks that last for 2 periods with $\epsilon = 2.5$ and $\underline{b} = 0.25$. The magnitudes of the shocks are 1.0% in the first period and 0.9% in the second period.

Chapter A: Appendix for Chapter 1

A.1 Firm's Optimality Conditions without Frictions

Plug the expression for expected sales:

$$\int \min(v(\frac{P_{jt}}{P_t})^{-\theta}S_t, Z_{jt})dG(v) = (\frac{P_{jt}}{P_t})^{-\theta}S_t \int \min(v, v^*_{jt})dG(v)$$
$$= (\frac{P_{jt}}{P_t})^{-\theta}S_t F\big(\log v^*_{jt} - \frac{\sigma_v^2}{2}\big) + Z_{jt}\Big[1 - F(\log v^*_{jt} + \frac{\sigma_v^2}{2})\Big]$$

and the firm's constraint (1.15) back into the firm's objective (1.14) and take partial derivatives with respect to Z_{jt} yield (note that the terms involving the partial derivative on the distribution function $F(\cdot)$ would cancel out):

$$(\partial Z_{jt}) \qquad 0 = \Xi_t \frac{P_{jt}}{P_t}\Big[1 - F\big(\log v^*_{jt} + \frac{\sigma_v^2}{2}\big)\Big] - \frac{\Xi_t}{P_t}\frac{\partial(\Lambda_{jt}Y_{jt})}{\partial Y_{jt}}\frac{\partial Y_{jt}}{\partial Z_{jt}}$$
$$-\beta \mathrm{E}_t\Big[\frac{\Xi_{t+1}}{P_{t+1}}\frac{\partial(\Lambda_{j,t+1}Y_{j,t+1})}{\partial Y_{j,t+1}}\frac{\partial Y_{j,t+1}}{\partial Z_{jt}}\Big]$$

Since $\partial(\Lambda_{jt}Y_{jt})/\partial Y_{jt} = \Lambda_{jt}/\alpha$, I can rewrite the above equation as

$$\Xi_t \frac{\lambda_{jt}}{\alpha} = \Xi_t \frac{P_{jt}}{P_t}\Big[1 - F\big(\log v_{jt}^* + \frac{\sigma_v^2}{2}\big)\Big] + \beta(1-\delta_z)F\big(\log v_{jt}^* + \frac{\sigma_v^2}{2}\big)\mathrm{E}_t\Big[\Xi_{t+1}\frac{\lambda_{j,t+1}}{\alpha}\Big] \tag{A.1}$$

Dividing equation (A.1) by Ξ_t and then subtracting both sides by $\beta(1-\delta_z)\mathrm{E}_t\Big[\frac{\Xi_{t+1}}{\Xi_t}\frac{\lambda_{j,t+1}}{\alpha}\Big]$ yields:

$$\frac{\lambda_{jt}}{\alpha} - \beta(1-\delta_z)\mathrm{E}_t\Big[\frac{\Xi_{t+1}}{\Xi_t}\frac{\lambda_{j,t+1}}{\alpha}\Big] = \Big[1 - F\big(\log v_{jt}^* + \frac{\sigma_v^2}{2}\big)\Big]\Big\{\frac{P_{jt}}{P_t} - \beta(1-\delta_z)\mathrm{E}_t\Big[\frac{\Xi_{t+1}}{\Xi_t}\frac{\lambda_{j,t+1}}{\alpha}\Big]\Big\}$$

which after rearranging gives the expression equation (1.18).

Taking derivative with respect to P_{jt} yields:

$$(\partial P_{jt}) \qquad 0 = \frac{\Xi_t}{P_t}\big(\frac{P_{jt}}{P_t}\big)^{-\theta}S_t\int \min(v, v_{jt}^*)dG(v) - \theta\frac{\Xi_t}{P_t}\big(\frac{P_{jt}}{P_t}\big)^{-\theta}S_tF\big(\log v_{jt}^* - \frac{\sigma_v^2}{2}\big) - \beta\Xi_{t+1}\frac{\partial(\lambda_{j,t+1}Y_{j,t+1})}{\partial Y_{j,t+1}}\frac{\partial Y_{j,t+1}}{\partial P_{jt}}$$

$$\Longrightarrow \qquad 0 = \frac{\Xi_t}{P_t}\big(\frac{P_{jt}}{P_t}\big)^{-\theta}S_t\int \min(v, v_{jt}^*)dG(v) - \theta\frac{\Xi_t}{P_t}\big(\frac{P_{jt}}{P_t}\big)^{-\theta}S_tF\big(\log v_{jt}^* - \frac{\sigma_v^2}{2}\big) + \frac{\theta\beta(1-\delta_z)}{\alpha}\Xi_{t+1}\frac{\lambda_{j,t+1}}{P_t}\big(\frac{P_{jt}}{P_t}\big)^{-\theta-1}S_tF\big(\log v_{jt}^* - \frac{\sigma_v^2}{2}\big)$$

$$\implies \quad 0 = \frac{\Xi_t}{P_t}\left(\frac{P_{jt}}{P_t}\right)^{-\theta} S_t \int \min(v, v^*_{jt}) dG(v) - \theta \frac{\Xi_t}{P_t}$$
$$- \frac{\theta}{P_{jt}}\left(\frac{P_{jt}}{P_t}\right)^{-\theta} S_t \Big[\Xi_t \frac{P_{jt}}{P_t} - \frac{\beta(1-\delta_z)}{\alpha}\Xi_{t+1}\lambda_{j,t+1}\Big] F\Big(\log v^*_{jt} - \frac{\sigma_v^2}{2}\Big) \tag{A.2}$$

Multiplying equation (A.2) by P_{jt} and dividing by $(\frac{P_{jt}}{P_t})^{-\theta} S_t$, and using the fact that

$$F\Big(\log v^*_{jt} - \frac{\sigma_v^2}{2}\Big) = \int_0^{v^*_{jt}} v dG(v)$$

I get:

$$\Xi_t \frac{P_{jt}}{P_t} \int \min(v, v^*_{jt}) dG(v) - \theta \Big[\Xi_t \frac{P_{jt}}{P_t} - \frac{\beta(1-\delta_z)}{\alpha} \mathrm{E}_t\Big(\Xi_{t+1} \frac{\Lambda_{t+1}}{P_{t+1}}\Big)\Big] \int_0^{v^*_{jt}} v dG(v) = 0$$

Dividing the above equation by $\Xi_t \int \min(v, v^*_{jt}) dG(v)$ and rearranging yields:

$$\frac{P_{jt}}{P_t}\Big[\frac{\theta \int_0^{v^*_{jt}} v dG(v)}{\int \min(v, v^*_{jt}) dG(v)} - 1\Big] = \frac{\theta \int_0^{v^*_{jt}} v dG(v)}{\int \min(v, v^*_{jt}) dG(v)} \frac{\beta(1-\delta_z)}{\alpha} \mathrm{E}_t\Big(\frac{\Xi_{t+1}}{\Xi_t} \frac{\Lambda_{j,t+1}}{P_{t+1}}\Big)$$

which gives optimal pricing condition equation (1.19).

A.2 Aggregate Price Index

In equilibrium the aggregate price level (equation (1.8)) can be expressed in terms of v^*_{jt}. One can use the fact that:

$$S_{jt} = v_{jt}\Big[\frac{P_{jt} + \gamma_{jt}}{P_t}\Big]^{-\theta} S_t = v^*_{jt}\Big[\frac{P_{jt}}{P_t}\Big]^{-\theta} S_t \quad \text{if } v_{jt} > v^*_{jt}$$

to write

$$\left[P_{jt} + \gamma_{jt}\right]^{1-\theta} = \left(\frac{v_{jt}^*}{v_{jt}}\right)^{\frac{\theta-1}{\theta}} P_{jt}^{1-\theta} \quad \text{if } v_{jt} > v_{jt}^*$$

Substituting the above expression into the aggregate price level (equation (1.8)), we have:

$$\begin{aligned}
P_t^{1-\theta} &= \Big[\int P_{jt}^{1-\theta} \int_0^{v_{jt}^*} v_{jt} dG(v) dj + \int P_{jt}^{1-\theta} \int_{v_{jt}^*}^{\infty} v_{jt} \Big(\frac{v_{jt}^*}{v_{jt}}\Big)^{\frac{\theta-1}{\theta}} dG(v) dj\Big] \\
&= \int P_{jt}^{1-\theta} \Big[\int_0^{v_{jt}^*} v_{jt} dG(v) + {v_{jt}^*}^{\frac{\theta-1}{\theta}} \int_{v_{jt}^*}^{\infty} v_{jt}^{\frac{1}{\theta}} dG(v)\Big] dj \\
&= \int P_{jt}^{1-\theta} \Big\{F\big(\log v_{jt}^* - \frac{\sigma_v^2}{2}\big) + {v_{jt}^*}^{\frac{\theta-1}{\theta}} \exp\big(\frac{\sigma_v^2(1-\theta)}{2\theta^2}\big)\Big[1 - F\big(\log v_{jt}^* - \frac{\sigma_v^2}{\theta} + \frac{\sigma_v^2}{2}\big)\Big]\Big\} dj \\
&\equiv \int P_{jt}^{1-\theta} \Phi(v_{jt}^*) dj \qquad \text{(A.3)}
\end{aligned}$$

A.3 Non-Stochastic Steady State

I list and show how to solve for the steady state of the economy. For nominal variables, let the lower case letters denote real variables. For example, $w_t = W_t/P_t$, $\tilde{w}_t = \tilde{W}_t/P_t$ and $p_{jt} = P_{jt}/P_t$. Let asterisk subscripts denote steady state values. The equation for aggregate wage (1.26), together with the first-order condition for the optimal wage equation (1.25), imply that in the steady state

$$\tilde{w}_* = w_* = \frac{\theta_w}{\theta_w - 1} w_*^h \tag{A.4}$$

The inventory evolution equations (1.11) and (1.12) imply

$$S_* + \delta_z inv_* = Y_* \tag{A.5}$$

$$inv_* = Z_* - S_* \tag{A.6}$$

The production function of intermediate goods firms implies

$$Y_* = L_*^{\alpha} \tag{A.7}$$

From the FOCs of the household's problem one obtains:

$$\Xi_* = \big[(1-h)C_*\big]^{-\sigma_c} \tag{A.8}$$

$$L_* = \big[(1-h)C_*\big]^{-\frac{\sigma_c}{\sigma_l}}\big[\frac{w_*^h}{\psi_l}\big]^{\frac{1}{\sigma_l}} \tag{A.9}$$

$$r_* = \frac{R_*}{\pi_*} = \frac{1}{\beta} \tag{A.10}$$

Intermediate goods firms' optimal decisions (A.1) and (A.2) imply the following equations in the steady state:

$$1 - F\big(\log v_*^* + \frac{\sigma_v^2}{2}\big) = \frac{\lambda_*[1-\beta(1-\delta_z)]}{\alpha p_* - \beta(1-\delta_z)\lambda_*} \tag{A.11}$$

$$(\theta-1)p_*\frac{\int \min(v, v_*^*)dG(v)}{v_*^*} = \frac{\theta\lambda_*}{\alpha}\Big[1+\beta(1-\delta_z)\Big(\frac{\int \min(v, v_*^*)dG(v)}{v_*^*} - 1\Big)\Big] \tag{A.12}$$

The aggregate price (A.3) implies

$$p_*^{1-\theta}\Big\{F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big) + v_*^{*\frac{\theta-1}{\theta}}\exp\big(\frac{\sigma_v^2(1-\theta)}{2\theta^2}\big)\big[1 - F\big(\log v_*^* - \frac{\sigma_v^2}{\theta} + \frac{\sigma_v^2}{2}\big)\big]\Big\} = 1 \tag{A.13}$$

Finally, the aggregate resource constraint (1.29) in the steady state is:

$$C_* + g_*Y_* + p_*\delta_z inv_* = p_*Y_* \tag{A.14}$$

Using (A.11), (A.12), and (A.13) I can solve for v_*^*, p_*, and λ_*. Since $v_*^* = \frac{Z_*}{S_*} p_*^\theta$ I also obtain $\frac{Z_*}{S_*}$. Using (A.4), (A.7) and (A.9), Y_* can be expressed as a function of C_*:

$$Y_* = \Big\{ \big[(1-h)C_*\big]^{-\frac{\sigma_c}{\sigma_l}} \big[\frac{(\theta_w - 1)w_*}{\theta_w \psi_l}\big]^{\frac{1}{\sigma_l}} \Big\}^{\alpha} \tag{A.15}$$

Using (A.5) and (A.6) Y_* can be expressed as a function of S_*:

$$Y_* = \Big[1 + \delta_z (\frac{Z_*}{S_*} - 1)\Big] S_* \tag{A.16}$$

Using the equation for marginal cost (1.13) I get:

$$w_* = \alpha \lambda_* Y_*^{\frac{\alpha-1}{\alpha}} \tag{A.17}$$

The aggregate resource constraint can be written as:

$$C_* + g_* Y_* + p_* \delta_z S_* \big(\frac{Z_*}{S_*} - 1\big) = p_* Y_* \tag{A.18}$$

I can jointly solve for C_*, S_*, Y_*, and w_* From (A.15), (A.16), (A.17), and (A.18). Thus, I also obtain w_*^h and Z_*, as well as inv_* and L_*.

A.4 Proof of Proposition 1 and Quadratic Approximation of Profit Loss

Proof of Proposition 1:

I derive the linear quadratic approximation to the firm's lifetime discounted profit function, as in Mackowiak and Wiederholt (2015). The period profit function of an intermediate goods firm can be expressed in terms of log-deviations from the steady state. For convenience and clarity I abuse the notation here to let a variable with a time subscript denote the log deviation of the original variable (in the main text) from its non-stochastic steady state value. The period utility function can be written as:

$$
\begin{aligned}
&\Xi_* e^{\Xi_t} p_*^{1-\theta} S_* e^{(1-\theta)p_t+S_t} F\big(\log v_*^* - \frac{\sigma_v^2}{2} + Z_{jt} + \theta p_{jt} - S_t\big) \\
&+ \Xi_* e^{\Xi_t} p_* Z_* e^{p_{jt}+Z_{jt}} \big[1 - F\big(\log v_*^* + \frac{\sigma_v^2}{2} + Z_{jt} + \theta p_{jt} - S_t\big)\big] \\
&- \Xi_* e^{\Xi_t} \frac{w_*}{\alpha} Y_* e^{w_t - \frac{1}{\alpha}(a_t} \Big\{ Z_* e^{Z_{jt}} - (1-\delta_z)\Big(Z_* e^{Z_{j,t-1}} - p_*^{-\theta} S_* e^{-\theta p_{j,t-1}+S_{t-1}} \\
&\qquad\qquad \cdot F\big(\log v_*^* - \frac{\sigma_v^2}{2} + Z_{j,t-1} + \theta p_{j,t-1} - S_{t-1}\big) \\
&\qquad\qquad - Z_* e^{Z_{j,t-1}} \big[1 - F\big(\log v_*^* + \frac{\sigma_v^2}{2} + Z_{j,t-1} + \theta p_{j,t-1} - S_{t-1}\big)\big]\Big)\Big\}^{\frac{1}{\alpha}}
\end{aligned}
$$

Let $\boldsymbol{x}_t = [p_{jt}, Z_{jt}]'$ and let $\boldsymbol{\zeta}_t = (w_t, S_t, \Xi_t, a_t)'$ denote the vector of variables exogenous to the firm. Let $h(\boldsymbol{x}_{-1}, \boldsymbol{x}_0, \boldsymbol{\zeta}_0, \boldsymbol{x}_1, \boldsymbol{\zeta}_1, \dots)$ denote the function that is obtained by multiplying the period profit function by β^t and summing all over t from time zero to infinity. Then

$$
\begin{aligned}
&\mathrm{E}_{j,-1}\Big[h(\boldsymbol{x}_{-1},\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\ldots)\Big] \cong h(0,0,0,0,0,\ldots)\\
&+\sum_{t=0}^{\infty}\beta^t\mathrm{E}_{j,-1}\Big[h_x'\boldsymbol{x}_t+h_\zeta'\boldsymbol{\zeta}_t+\frac{1}{2}\boldsymbol{x}_t'H_{x,-1}\boldsymbol{x}_{t-1}+\frac{1}{2}\boldsymbol{x}_t'H_{x,0}\boldsymbol{x}_t+\frac{1}{2}\boldsymbol{x}_t'H_{x,1}\boldsymbol{x}_{t+1}\\
&+\frac{1}{2}\boldsymbol{x}_t'H_{x\zeta,-1}\boldsymbol{\zeta}_{t-1}+\boldsymbol{x}_t'H_{x\zeta,0}\boldsymbol{\zeta}_t+\frac{1}{2}\boldsymbol{x}_t'H_{x\zeta,1}\boldsymbol{\zeta}_{t+1}+\frac{1}{2}\boldsymbol{\zeta}_t'H_{\zeta x,-1}\boldsymbol{x}_{t-1}+\frac{1}{2}\boldsymbol{\zeta}_t'H_{\zeta x,1}\boldsymbol{x}_{t+1}+\frac{1}{2}\boldsymbol{\zeta}_t'H_\zeta\boldsymbol{\zeta}_t\Big]\\
&+\frac{1}{\beta}\big(h_x'\boldsymbol{x}_{-1}+\frac{1}{2}\boldsymbol{x}_{-1}'H_{x,0}\boldsymbol{x}_{-1}+\frac{1}{2}\boldsymbol{x}_{-1}'H_{x,1}\boldsymbol{x}_0+\frac{1}{2}\boldsymbol{x}_{-1}'H_{x\zeta,1}\boldsymbol{\zeta}_0+\frac{1}{2}\boldsymbol{\zeta}_{-1}'H_{\zeta x,1}\boldsymbol{x}_0\big)
\end{aligned}
\tag{A.19}
$$

where $\big(\beta^t h_x\big)$ is the vector of first derivatives of $h(\boldsymbol{x}_{-1},\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\ldots)$ with respect to $\boldsymbol{x}_t$ evaluated at the non-stochastic steady state, $\big(\beta^t h_\zeta\big)$ is the vector of first derivatives of $h(\ldots)$ with respect to $\boldsymbol{\zeta}_t$ evaluated at the non-stochastic steady state. $\big(\beta^t H_{x,\tau}\big)$ is the matrix of second derivative of $h(\ldots)$ with respect to $\boldsymbol{x}_t$ and $\boldsymbol{x}_{t+\tau}$ evaluated at the non-stochastic steady state, $\big(\beta^t H_{x\zeta,\tau}\big)$ is the matrix of second derivative of $h(\ldots)$ with respect to $\boldsymbol{x}_t$ and $\boldsymbol{\zeta}_{t+\tau}$ evaluated at the non-stochastic steady state, and $\big(\beta^t H_{\zeta x,\tau}\big)$ is the matrix of second derivative of $h(\ldots)$ with respect to $\boldsymbol{\zeta}_t$ and $\boldsymbol{x}_{t+\tau}$ evaluated at the non-stochastic steady state.

Note that $h_{\boldsymbol{x}_t}(\boldsymbol{x}_{-1},\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\ldots)=0$ corresponds to the firm's optimality conditions in period t. The process of optimal actions $\{\boldsymbol{x}_t^\dagger\}_{t=0}^\infty$ should satisfy a first order approximation to $h_{\boldsymbol{x}_t}(\boldsymbol{x}_{-1},\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\ldots)=0$ around the steady state, which can be obtained from taking the first derivative of the RHS of (A.19) with respect to $\boldsymbol{x}_t$ and set the derivative equal to zero. I get:

$$\mathrm{E}_t\big[h_x + H_{x,-1}\boldsymbol{x}^\dagger_{t-1} + H_{x,0}\boldsymbol{x}^\dagger_t + H_{x,1}\boldsymbol{x}^\dagger_{t+1} + H_{x\zeta,-1}\boldsymbol{\zeta}_{t-1} + H_{x\zeta,0}\boldsymbol{\zeta}_t + H_{x\zeta,1}\boldsymbol{\zeta}_{t+1}\big] = 0$$

which can be rewritten as:

$$\boldsymbol{x}^\dagger_t = H^{-1}_{x,0}h_x + H^{-1}_{x,0}\left[\frac{1}{\beta}H_{x,1}\boldsymbol{x}^\dagger_{t-1} + H_{x\zeta,-1}\boldsymbol{\zeta}_{t-1}\right] + H^{-1}_{x,0}H_{x\zeta,0}\boldsymbol{\zeta}_t + H^{-1}_{x,0}\mathrm{E}_t\left[H_{x,1}\boldsymbol{x}^\dagger_{t+1} + H_{x\zeta,1}\boldsymbol{\zeta}_{t+1}\right]$$

where $\mathrm{E}_t[\cdot]$ is the expectation operator conditional on all available information in period t (including the evolution of $\{\boldsymbol{\zeta}_t\}$ up to current period t). Thus, the process of optimal actions $\{\boldsymbol{x}^\dagger_t\}_{t=0}^\infty$ can be characterized by the following conditions:

(i) The economy starts from the non-stochastic steady state: $\boldsymbol{x}^\dagger_{-1} = [0, 0]'$.

(ii) In each period $t \geq 0$, $\boldsymbol{x}^\dagger_t$ satisfies:

$$\boldsymbol{x}^\dagger_t = H^{-1}_{\boldsymbol{x},0}h_{\boldsymbol{x}} + H^{-1}_{\boldsymbol{x},0}\left[\frac{1}{\beta}H_{\boldsymbol{x},1}\boldsymbol{x}^\dagger_{t-1} + H_{\boldsymbol{x}\boldsymbol{\zeta},-1}\boldsymbol{\zeta}_{t-1}\right] + H^{-1}_{\boldsymbol{x},0}H_{\boldsymbol{x}\boldsymbol{\zeta},0}\boldsymbol{\zeta}_t + H^{-1}_{\boldsymbol{x},0}\mathrm{E}_t\Big[H_{\boldsymbol{x},1}\boldsymbol{x}^\dagger_{t+1} + H_{\boldsymbol{x}\boldsymbol{\zeta},1}\boldsymbol{\zeta}_{t+1}$$

This establishes Proposition 1.

Quadratic Approximation of Profit Loss:

Using the following equivalence relations:

$$H_{x\zeta,0} = H'_{\zeta x,0}, \quad H_{x\zeta,1} = \beta H'_{\zeta x,-1}, \quad H_{\zeta x,1} = \beta H'_{x\zeta,-1}, \quad H_{x,1} = \beta H'_{x,-1}$$

I can rewrite the above equation as:

$$
\begin{aligned}
&\mathrm{E}_{j,-1}\Big[\tilde{h}(\boldsymbol{x}_{-1}, \boldsymbol{x}_0, \boldsymbol{\zeta}_0, \boldsymbol{x}_1, \boldsymbol{\zeta}_1, \ldots)\Big] = h(0,0,0,0,0,\ldots)\\
&+ \sum_{t=0}^{\infty} \beta^t \mathrm{E}_{j,-1}\Big[h_x' \boldsymbol{x}_t + h_\zeta' \boldsymbol{\zeta}_t + \frac{1}{2}\boldsymbol{x}_t' H_{x,0}\boldsymbol{x}_t + \boldsymbol{x}_t' H_{x,1}\boldsymbol{x}_{t+1}\\
&+ \boldsymbol{x}_t' H_{x\zeta,1}\boldsymbol{\zeta}_{t+1} + \boldsymbol{\zeta}_t' H_{\zeta x,1}\boldsymbol{x}_{t+1} + \boldsymbol{x}_t' H_{x\zeta,0}\boldsymbol{\zeta}_t + \frac{1}{2}\boldsymbol{\zeta}_t' H_\zeta \boldsymbol{\zeta}_t\Big]\\
&+ \frac{1}{\beta}\mathrm{E}_{j,-1}\Big[h_{-1}'\boldsymbol{x}_{-1} + \frac{1}{2}\boldsymbol{x}_{-1}' H_{x,-1}\boldsymbol{x}_{-1} + \boldsymbol{x}_{-1}' H_{x,1}\boldsymbol{x}_0 + \boldsymbol{x}_{-1}' H_{x\zeta,1}\boldsymbol{\zeta}_0 + \boldsymbol{\zeta}_{-1}' H_{\zeta x,1}\boldsymbol{x}_0\Big]
\end{aligned}
\tag{A.20}
$$

Multiplying the above equation by $(\boldsymbol{x}_t - \boldsymbol{x}_t^\dagger)'$, and using the fact that $(\boldsymbol{x}_t - \boldsymbol{x}_t^\dagger)$ is realized at time t yields

$$
\begin{aligned}
\mathrm{E}_t\big[(\boldsymbol{x}_t - \boldsymbol{x}_t^\dagger)'(h_x + H_{x,-1}\boldsymbol{x}_{t-1}^\dagger + H_{x,0}\boldsymbol{x}_t^\dagger + H_{x,1}\boldsymbol{x}_{t+1}^\dagger\\
+ H_{x\zeta,-1}\boldsymbol{\zeta}_{t-1} + H_{x\zeta,0}\boldsymbol{\zeta}_t + H_{x\zeta,1}\boldsymbol{\zeta}_{t+1})\big] = 0
\end{aligned}
$$

Using the law of iterated expectations and rearranging yields

$$
\begin{aligned}
&\mathrm{E}_{j,-1}\big[(\boldsymbol{x}_t - \boldsymbol{x}_t^\dagger)'(h_x + H_{x\zeta,-1}\boldsymbol{\zeta}_{t-1} + H_{x\zeta,0}\boldsymbol{\zeta}_t + H_{x\zeta,1}\boldsymbol{\zeta}_{t+1})\big]\\
&\quad = -\mathrm{E}_{j,-1}\big[(\boldsymbol{x}_t - \boldsymbol{x}_t^\dagger)'(H_{x,-1}\boldsymbol{x}_{t-1}^\dagger + H_{x,0}\boldsymbol{x}_t^\dagger + H_{x,1}\boldsymbol{x}_{t+1}^\dagger)\big]
\end{aligned}
\tag{A.21}
$$

Using (A.20) yields the difference in the firm's lifetime discounted profit when its actual actions deviates from the optimal actions as:

$$\begin{aligned}
&\mathrm{E}_{j,-1}\Big[h(\boldsymbol{x}_{-1},\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\ldots)\Big]-\mathrm{E}_{j,-1}\Big[h(\boldsymbol{x}^\dagger_{-1},\boldsymbol{x}^\dagger_0,\boldsymbol{\zeta}_0,\boldsymbol{x}^\dagger_1,\boldsymbol{\zeta}_1,\ldots)\Big]\\
&\cong\sum_{t=0}^{\infty}\beta^t\mathrm{E}_{j,-1}\Big[\frac{1}{2}\boldsymbol{x}'_tH_{x,0}\boldsymbol{x}_t+\boldsymbol{x}'_tH_{x,1}\boldsymbol{x}_{t+1}\frac{1}{2}\boldsymbol{x}^{\dagger'}_tH_{x,0}\boldsymbol{x}^\dagger_t-\boldsymbol{x}^{\dagger'}_tH_{x,1}\boldsymbol{x}^{\dagger'}_{t+1}\Big]\\
&\quad+\sum_{t=0}^{\infty}\beta^t\mathrm{E}_{j,-1}\Big[(\boldsymbol{x}_t-\boldsymbol{x}^\dagger_t)(h_x+H_{x\zeta,-1}\zeta_{t-1}+H_{x\zeta,0}\zeta_t+H_{x\zeta,1}\zeta_{t+1})\Big]\\
&+\frac{1}{\beta}\mathrm{E}_{j,-1}\Big[h'_{-1}\boldsymbol{x}_{-1}+\frac{1}{2}\boldsymbol{x}'_{-1}H_{x,-1}\boldsymbol{x}_{-1}+\boldsymbol{x}'_{-1}H_{x,1}\boldsymbol{x}_0+\boldsymbol{x}'_{-1}H_{x\zeta,1}\boldsymbol{\zeta}_0+\Big]\\
&-\frac{1}{\beta}\mathrm{E}_{j,-1}\Big[h'_{-1}\boldsymbol{x}^\dagger_{-1}+\frac{1}{2}\boldsymbol{x}^{\dagger'}_{-1}H_{x,-1}\boldsymbol{x}^\dagger_{-1}+\boldsymbol{x}^{\dagger'}_{-1}H_{x,1}\boldsymbol{x}^\dagger_0+\boldsymbol{x}^{\dagger'}_{-1}H_{x\zeta,1}\boldsymbol{\zeta}_0+\Big]
\end{aligned}\tag{A.22}$$

I assume that the economy starts from the non-stochastic steady state. Therefore $\boldsymbol{x}_{-1}=\boldsymbol{x}^\dagger_{-1}=(0,0)'$. Substituting (A.21) into (A.22) yields:

$$\begin{aligned}
&\mathrm{E}_{j,-1}\Big[h(\boldsymbol{x}_{-1},\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\ldots)\Big]-\mathrm{E}_{j,-1}\Big[h(\boldsymbol{x}^\dagger_{-1},\boldsymbol{x}^\dagger_0,\boldsymbol{\zeta}_0,\boldsymbol{x}^\dagger_1,\boldsymbol{\zeta}_1,\ldots)\Big]\\
&=\sum_{t=0}^{\infty}\beta^t\mathrm{E}_{j,-1}\Big[\frac{1}{2}\boldsymbol{x}'_tH_{x,0}\boldsymbol{x}_t+\boldsymbol{x}'_tH_{x,1}\boldsymbol{x}_{t+1}-\frac{1}{2}\boldsymbol{x}^{\dagger'}_tH_{x,0}\boldsymbol{x}^\dagger_t-\boldsymbol{x}^{\dagger'}_tH_{x,1}\boldsymbol{x}^{\dagger'}_{t+1}\Big]\\
-&\mathrm{E}_{j,-1}\big[(\boldsymbol{x}_t-\boldsymbol{x}^\dagger_t)'(H_{x,-1}\boldsymbol{x}^\dagger_{t-1}+H_{x,0}\boldsymbol{x}^\dagger_t+H_{x,1}\boldsymbol{x}^\dagger_{t+1})\big]+\frac{1}{\beta}\mathrm{E}_{j,-1}\Big[\boldsymbol{x}^{\dagger'}_{-1}F_{x,1}(\boldsymbol{x}_0-\boldsymbol{x}^\dagger_0)\Big]
\end{aligned}$$

Using that $H_{x,1}=\beta H'_{x,-1}$ and rearranging the above equation yields

$$\begin{aligned}
&E_{j,-1}\Big[\tilde{h}(\boldsymbol{x}_{-1},\boldsymbol{x}_0,\boldsymbol{\zeta}_0,\boldsymbol{x}_1,\boldsymbol{\zeta}_1,\ldots)\Big]-E_{j,-1}\Big[\tilde{h}(\boldsymbol{x}^\dagger_{-1},\boldsymbol{x}^\dagger_0,\boldsymbol{\zeta}_0,\boldsymbol{x}^\dagger_1,\boldsymbol{\zeta}_1,\ldots)\Big]\\
&=\sum_{t=0}^{\infty}\beta^tE_{j,-1}\Big[\frac{1}{2}(\boldsymbol{x}_t-\boldsymbol{x}^\dagger_t)'H_{x,0}(\boldsymbol{x}_t-\boldsymbol{x}^\dagger_t)+(\boldsymbol{x}_t-\boldsymbol{x}^\dagger_t)'H_{x,1}(\boldsymbol{x}_{t+1}-\boldsymbol{x}^\dagger_{t+1})\Big]
\end{aligned}$$

which forms the firm's objective function (1.34).

The Form of Hessian Matrices: Using the steady-state equations, I obtain:

$$
\begin{aligned}
h_x =&(0,0)',\\
H_{x,0} =&\Xi_* p_*^{-\theta} S_* \begin{bmatrix} A_{pp} & A_{pz} \\ A_{zp} & A_{zz} \end{bmatrix}
\end{aligned}
$$

where

$$
\begin{aligned}
A_{pp} =&(1-\theta)^2 p_* F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big) + p_* v_*^* \big[1 - F\big(\log v_*^* + \frac{\sigma_v^2}{2}\big)\big]\\
&- \theta^2 \big[p_* - \frac{\beta(1-\delta_z)}{\alpha}\lambda_*\big] f\big(\log v_*^* - \frac{\sigma_v^2}{2}\big) - \theta^2 \frac{\beta(1-\delta_z)}{\alpha}\lambda_* F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big)\\
&- \theta^2 \frac{\beta(1-\delta_z)^2}{\alpha}\frac{(1-\alpha)}{\alpha}\lambda_* \frac{p_*^{-\theta} S_*}{Y_*} F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big)^2\\
A_{pz} =&p_* v_*^* \Big[1 - F\big(\log v_*^* + \frac{\sigma_v^2}{2}\big)\Big] - \theta \Big[p_* - \frac{\beta(1-\delta_z)}{\alpha}\lambda_*\Big] f\big(\log v_*^* - \frac{\sigma_v^2}{2}\big)\\
&- \theta \frac{\beta(1-\delta_z)^2}{\alpha}\frac{(1-\alpha)}{\alpha}\lambda_* \frac{Z_*}{Y_*} F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big) F\big(\log v_*^* + \frac{\sigma_v^2}{2}\big)\\
A_{zp} =&A_{pz}\\
A_{zz} =&- v_*^* \big[p_* - \frac{\beta(1-\delta_z)}{\alpha}\lambda_*\big] f\big(\log v_*^* + \frac{\sigma_v^2}{2}\big)\\
&- \frac{(1-\alpha)}{\alpha^2} v_*^* \lambda_* \frac{Z_*}{Y_*}\big[1 + \beta(1-\delta_z)^2 F\big(\log v_*^* + \frac{\sigma_v^2}{2}\big)^2\big].
\end{aligned}
$$

$$
H_{x,1} =\Xi_* p_*^{-\theta} S_* \begin{bmatrix} 0 & B_{pp} \\ 0 & B_{zz} \end{bmatrix}
$$

where

$$
\begin{aligned}
B_{pp} =&\frac{\theta\beta(1-\delta_z)}{\alpha}\frac{(1-\alpha)}{\alpha}\frac{\lambda_*}{Y_*} Z_* F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big)\\
B_{zz} =&\frac{\beta(1-\delta_z)}{\alpha}\frac{(1-\alpha)}{\alpha}\frac{\lambda_*}{Y_*} Z_* v_*^* F\big(\log v_*^* + \frac{\sigma_v^2}{2}\big)
\end{aligned}
$$

$$H_{x\zeta,-1} = \Xi_* p_*^{-\theta} S_* \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{(1-\alpha)}{\alpha}\frac{(1-\delta_z)}{\alpha}\frac{\lambda_*}{Y_*} Z_* F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big) & 0 & 0 & 0 \end{bmatrix}$$

$$H_{x\zeta,0} = \Xi_* p_*^{-\theta} S_* \begin{bmatrix} 0 & D_{ps} & -\frac{\theta\beta(1-\delta_z)}{\alpha}\lambda_* F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big) & 0 & 0 \\ -\frac{\lambda_*}{\alpha} v_*^* & D_{zs} & -\frac{\beta(1-\delta_z)}{\alpha}\lambda_* v_*^* F\big(\log v_*^* + \frac{\sigma_v^2}{2}\big) & \frac{\lambda_*}{\alpha^2} v_*^* & \frac{\lambda_*}{\alpha^2} v_*^* \end{bmatrix}$$

where

$$\begin{aligned} D_{ps} &= -p_* v_*^* \big[1 - F\big(\log v_*^* + \frac{\sigma_v^2}{2}\big)\big] + \theta\Big[p_* - \frac{\beta(1-\delta_z)}{\alpha}\lambda_*\Big] f\big(\log v_*^* - \frac{\sigma_v^2}{2}\big) \\ &\quad + \frac{\theta\beta(1-\delta_z)^2}{\alpha}\frac{(1-\alpha)}{\alpha} p_*^{-\theta} S_* \frac{\lambda_*}{Y_*} F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big)^2 \\ D_{zs} &= v_*^* \Big[p_* - \frac{\beta(1-\delta_z)}{\alpha}\lambda_*\Big] f\big(\log v_*^* + \frac{\sigma_v^2}{2}\big) \\ &\quad + \frac{\beta(1-\delta_z)^2}{\alpha}\frac{(1-\alpha)}{\alpha}\frac{\lambda_*}{Y_*} Z_* F\big(\log v_*^* + \frac{\sigma_v^2}{2}\big) F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big). \end{aligned}$$

$$H_{x\zeta,1} = \Xi_* p_*^{-\theta} S_* \begin{bmatrix} E_p & 0 & E_p & -\frac{1}{\alpha}E_p & -\frac{1}{\alpha}E_p \\ E_z & 0 & E_z & -\frac{1}{\alpha}E_z & -\frac{1}{\alpha}E_z \end{bmatrix}$$

where

$$\begin{aligned} E_p &= \frac{\theta\beta(1-\delta_z)}{\alpha}\lambda_* F\big(\log v_*^* - \frac{\sigma_v^2}{2}\big) \\ E_z &= \frac{\beta(1-\delta_z)}{\alpha}\lambda_* v_*^* F\big(\log v_*^* + \frac{\sigma_v^2}{2}\big). \end{aligned}$$

A.5 Algorithm for the Rational Inattention Model

The rational inattention model is solved using an iteration procedure as in Mackowiak and Wiederholt (2015), with some slight deviations because in my model the firm decision rules involve both the pricing decision and the production decision. I solve the RI model with an outer iteration to pin down the first-best and actual decision rules and an inner iteration for equilibrium aggregation, divided into the following 5 steps:

Step 1: I make a guess for the law of motion for the optimal price $P_{jt}^{\dagger}$ and optimal stock for sale $z_{jt}^{\dagger}$. This involves guessing the equilibrium vector of first-best decision rules $\boldsymbol{x}_{jt}^{\dagger}$ in equation (1.36) that enter the firm's optimization problem (1.34). A good initial guess will be the equilibrium responses obtained from a full-information model.

Step 2: I solve for the firm's optimization problem (1.34) in chapter 1. As in Mackowiak and Wiederholt (2015), I turn the problem into a finite-dimensional problem, by parameterizing the infinite-order polynomials $B_k(L)$ and $C_k(L)$ in equation (1.37) as a lag polynomial of a finite-order ARMA process. Various nonlinear optimization packages can be used to solve the problem by choosing the coefficients for the ARMA process. I use the *csminwel* package developed by Chris Sims, available on his web page.

Step 3: This step involves the inner iteration for equilibrium aggregate sales. I first guess a path for the response of aggregate sales, use the firm's actual pricing decisions to obtain the aggregate price level using the log-linearized version of the equation for aggregate price (A.3), and use the firm's actual decision rule Z_{jt} to obtain the aggregate stock for sale Z_t. Given the guess for the sales series, I can obtain the responses of aggregate inventories and aggregate output. Then, I can use the interest rate rule (1.28) to obtain the responses of the nominal and real interest rates. Then, I use the households optimality conditions (1.2) and (1.4) to obtain the responses of aggregate consumption. Using equation (1.33) I can obtain a new guess for the aggregate sales series. The inner iteration continues until the aggregate sales series converges.

Step 4: I can use the wage Phillips curve (derived from equation (1.25) and equation (1.26)) to obtain the response of the aggregate nominal wage rate.

Step 5: I compute the law of motion for the optimal responses $\boldsymbol{x}_{jt}^{\dagger}$ using equation (1.39) in Proposition 1 of chapter 1. If the law of motion for the vector of first-best responses differs from the previous guess, I make the guess in the next outer iteration a weighted average of the previous guess and the solution obtained by equation (1.39). The outer iteration continues until a fixed point for $\boldsymbol{x}_{jt}^{\dagger}$ is obtained.[1]

[1] By assuming a constant marginal cost of attention μ, the RI model can be solved for each shock separately.

A.6 Solving the Sticky Price Model

Since the production function is (likely) decreasing returns to scale and firms can carry different levels of inventory stock, firms's pricing and production decisions will depend on their state variables. Since firms with different inventory stock may choose different produciton levels and the marginal cost they face would depend on the quantity of produciton, firms' marginal cost become firm specific. Therefore, firms that are allowed to re-optimize do not necessarily choose the same optimal price and optimal production. The standard technique used to linearize and solve the representative agent model around the steady state thus could not be completely applied in the model. In what follows, I describe the model under Calvo pricing and work out a solution method via the method of undetermined coefficients.

A.6.1 Conjectured Policy Rules

I conjecture (and later verify) two sets of linear policy functions for a firm's labor hiring, pricing, and inventory decisions, depending on whether the firm is allowed to re-optimize price or not. The form of the conjectured policy rules are listed in the following table:

The conjectured policy rules are log-linear. I use tilde to symbolize the deviation of any variable to symbolize its deviation from the mean value of all firms. For example, $\tilde{mc}_{jt}$ is the log deviation of firm j's marginal cost to the economy-wide average marginal cost. Note the optimizing and non-optimizing firms share the same policy rules except for the price decision, as non-optimizing firms could not reset its

Firms:	optimize p_{jt} w/ prob. $1-\xi_p$	non-optimize p_{jt} w/ prob. ξ_p
Policy Rules:		
$\tilde{l}_{jt} =$	$\psi_1^l \tilde{inv}_{j,t-1} + \psi_2^l \hat{p}_{jt}$	$\psi_1^l \tilde{inv}_{j,t-1} + \psi_2^l \hat{p}_{jt}$
$\hat{p}_{jt} =$	$\psi_1^p \tilde{inv}_{j,t-1} + \hat{p}_t^*$	$\hat{p}_{j,t-1} - \pi_t + \ell_p \pi_{t-1}$
$\tilde{Z}_{jt} =$	$\psi_1^z \tilde{inv}_{j,t-1} + \psi_2^z \hat{p}_{jt}$	$\psi_1^z \tilde{inv}_{j,t-1} + \psi_2^z \hat{p}_{jt}$
$\tilde{mc}_{jt} =$	$\psi_1^{mc} \tilde{inv}_{j,t-1} + \psi_2^{mc} \hat{p}_{jt}$	$\psi_1^{mc} \tilde{inv}_{j,t-1} + \psi_2^{mc} \hat{p}_{jt}$

Tab. A.1: Policy Rules

price other than indexation to inflation rate in the previous period. $\hat{p}_t^*$ denotes the average of optimal prices set by optimizing firms.

A.6.2 Expected Inventory Stock

From the log-linearized form of inventory evolution process (1.11) and (1.12) and after taking expectation, one get:

$$\mathrm{E}_t inv_{jt} = (1-\delta_z) inv_{j-1,t} + \frac{Y_*}{inv_*} y_{jt} - \frac{S_*}{inv_*} \mathrm{E}_t s_{jt} \tag{A.23}$$

Note that the expected sales of firm j is

$$\mathrm{E}_t S_{jt} = \mathrm{E}_t \min\Big(v_{jt}^* \Big(\frac{P_{jt}}{P_t}\Big)^{-\theta} S_t, Z_{jt}\Big) = \Big(\frac{P_{jt}}{P_t}\Big)^{-\theta} S_t \Psi(v_{jt}^*)$$

where

$$\Psi(v_{jt}^*) = F\Big(\log v_{jt}^* - \frac{\sigma_v^2}{2}\Big) + v_{jt}^* \Big[1 - F\big(\log v_{jt}^* + \frac{\sigma_v^2}{2}\big)\Big] \tag{A.24}$$

Therefore, the log-linearized equation for expected sales could be expressed as

$$
\begin{aligned}
\mathrm{E}_t s_{jt} &= \epsilon_\Psi \big[\theta p_{jt} + z_{jt} - s_t\big] - \theta p_{jt} + s_t \\
&= \epsilon_\Psi z_{jt} - \theta(1-\epsilon_\Psi) p_{jt} + (1-\epsilon_\Psi) s_t
\end{aligned}
$$

where $\epsilon_\Psi = \Psi'(v_*^*)v_*^*/\Psi(v_*^*)$ is the elasticity of $\Psi(\cdot)$ w.r.t v_{jt}^* in the steady state. Putting the above expression for expected sales as well as the log-linearized equations for y_{jt} and $z_{jt]}$ into (A.23), I get:

$$
\begin{aligned}
\mathrm{E}_t inv_{jt} =& (1-\delta_z) inv_{j-1,t} + \frac{Y_*}{inv_*}\big(a_t + \alpha l_{jt}\big) \\
& - \frac{S_*}{inv_*}\Big\{\epsilon_\Psi \Big[(1-\delta_z)\frac{inv_*}{Z_*} inv_{j,t-1} + \frac{Y_*}{Z_*}\big(a_t + \alpha l_{jt}\big)\Big] - \theta(1-\epsilon_\Psi)p_{jt} + (1-\epsilon_\Psi)s_t\Big\}
\end{aligned}
$$

After rearranging, the above equation becomes:

$$
\begin{aligned}
\mathrm{E}_t inv_{jt} =& (1-\delta_z)\Big(1 - \frac{S_*}{Z_*}\epsilon_\Psi\Big) inv_{j,t-1} + \frac{Y_*}{inv_*}\Big(1 - \frac{S_*}{Z_*}\epsilon_\Psi\Big)\big(a_t + \alpha l_{jt}\big) \\
& + \frac{S_*}{inv_*}\theta(1-\epsilon_\Psi)p_{jt} - \frac{S_*}{inv_*}\theta(1-\epsilon_\Psi)s_t
\end{aligned}
$$

Substracting the analogous equation for aggregate economic variables, one obtains:

$$
\begin{aligned}
\mathrm{E}_t \tilde{inv}_{jt} =& (1-\delta_z)\Big(1 - \frac{S_*}{Z_*}\epsilon_\Psi\Big) \tilde{inv}_{j,t-1} + \frac{Y_*}{inv_*}\Big(1 - \frac{S_*}{Z_*}\epsilon_\Psi\Big)\big(\alpha \tilde{l}_{jt}\big) \\
& + \frac{S_*}{inv_*}\theta(1-\epsilon_\Psi)\hat{p}_{jt}
\end{aligned}
$$

Plugging in the guessed policy rule for $\tilde{l}_{jt}$ from table 3, one can write expected inventory stock as:

$$\mathrm{E}_t \tilde{inv}_{jt} = \psi_1^{inv} \tilde{inv}_{j,t-1} + \psi_2^{inv} \hat{p}_{jt}$$

where

$$\begin{cases} \psi_1^{inv} & = (1-\delta_z)\Big(1 - \frac{S_*}{Z_*}\epsilon_\Psi\Big) + \frac{Y_*}{inv_*}\Big(1 - \frac{S_*}{Z_*}\epsilon_\Psi\Big)\alpha\psi_1^l \\ \psi_2^{inv} & = \frac{S_*}{inv_*}\theta(1-\epsilon_\Psi) + \frac{Y_*}{inv_*}\Big(1 - \frac{S_*}{Z_*}\epsilon_\Psi\Big)\alpha\psi_2^l \end{cases}$$

The log-linearized equation for stock for sale is

$$z_{jt} = (1-\delta_z)\frac{inv_*}{Z_*} inv_{j,t-1} + \frac{Y_*}{Z_*}(a_t + \alpha l_{jt})$$

After subtracting the analogous aggregate equation:

$$\tilde{z}_{jt} = (1-\delta_z)\frac{inv_*}{Z_*} inv_{j,t-1} + \frac{Y_*}{Z_*}(\alpha \tilde{l}_{jt})$$

So that

$$\begin{cases} \psi_1^z & = (1-\delta_z)\frac{inv_*}{Z_*} + \frac{Y_*}{Z_*}\alpha\psi_1^l \\ \psi_2^z & = \frac{Y_*}{Z_*}\alpha\psi_2^l \end{cases}$$

A.6.3 Expected Aggregate Price and Real Marginal Cost

The expression for nominal marginal cost is

$$\Lambda_{jt} = \frac{W_t}{\alpha \exp^{\frac{a_t}{\alpha}}} Y_{jt}^{\frac{1-\alpha}{\alpha}} = \frac{W_t}{\alpha \exp^{a_t}} L_{jt}^{1-\alpha}$$

Therefore, the log-linearized form for real marginal cost is:

$$mc_{jt} = w_t + a_t + (1-\alpha)l_{jt}$$

After subtracting aggregate variables:

$$\tilde{mc}_{jt} = (1-\alpha)\tilde{l}_{jt} \qquad \Longrightarrow$$

$$\begin{cases} \psi_1^{mc} & = (1-\alpha)\psi_1^l \\ \psi_2^{mc} & = (1-\alpha)\psi_2^l \end{cases}$$

The expected next-period price is

$$\mathrm{E}_t\hat{p}_{j,t+1} = \xi_p \mathrm{E}_t(\hat{p}_{jt} - \pi_{t+1} + \ell_p\pi_t + \bar{p}_t - \bar{p}_{t+1}) + (1-\xi_p)\Big[\psi_1^p \mathrm{E}_t(\tilde{inv}_{jt}) + \mathrm{E}_t(\hat{p}^*_{t+1})\Big]$$

Note that

$$\hat{p}^*_{t+1} = \tilde{p}^*_{t+1} - \bar{p}_{t+1}$$

and

$$\tilde{p}^*_{t+1} = \frac{\xi_p}{1-\xi_p}(\pi_{t+1} - \ell_p\pi_t) + \frac{1}{1-\xi_p}\big(\bar{p}_{t+1} - \xi_p\bar{p}_t\big)$$

Therefore the expected next-period price becomes

$$\mathrm{E}_t(\hat{p}_{j,t+1}) = \xi_p\hat{p}_{jt} + (1-\xi_p)\Big[\psi_1^p \mathrm{E}_t(\tilde{inv}_{jt})\Big]$$

The expected next-period real marginal cost is:

$$
\begin{aligned}
\mathrm{E}_t \tilde{mc}_{t+1} =& \psi_1^{mc} \mathrm{E}_t(\tilde{inv}_{jt}) + \psi_2^{mc} \mathrm{E}_t(\hat{p}_{j,t+1}) \\
=& \Big[\psi_1^{mc} + \psi_2^{mc}(1-\xi_p)\psi_1^p \Big] \mathrm{E}_t(\tilde{inv}_{jt}) + \psi_2^{mc}\xi_p \hat{p}_{jt} \\
=& \Big[\psi_1^{mc} + \psi_2^{mc}(1-\xi_p)\psi_1^p \Big] \psi_1^{inv} \tilde{inv}_{j,t-1} \\
& + \Big\{ \Big[\psi_1^{mc} + \psi_2^{mc}(1-\xi_p)\psi_1^p \Big] \psi_2^{inv} + \psi_2^{mc}\xi_p \Big\} \hat{p}_{jt} \qquad \text{(A.25)}
\end{aligned}
$$

A.6.4 Inventory-Sales Ratio

The first order condition w.r.t Z_{jt} gives rise to the following equation:

$$
\frac{\lambda_{jt} - \beta(1-\delta_z)\mathrm{E}_t\left(\frac{\Xi_{t+1}}{\Xi_t}\lambda_{j,t+1}\right)}{\frac{P_{jt}}{P_t} - \beta(1-\delta_z)\mathrm{E}_t\left(\frac{\Xi_{t+1}}{\Xi_t}\lambda_{j,t+1}\right)} = 1 - F(\log(v_{jt}^* + \frac{\sigma_v^2}{2})) = \Psi'(v_{jt}^*)
$$

After log-linearization, it becomes:

$$
\begin{aligned}
\varepsilon_{\Psi'}(\theta\tilde{p}_{jt} + z_{jt} - s_t) =& \frac{mc_{jt} - \beta(1-\delta_z)\mathrm{E}_t(d_{t,t+1} + mc_{j,t+1})}{1 - \beta(1-\delta_z)} \\
& - \frac{p_*\tilde{p}_{jt} - \beta(1-\delta_z)mc_*\mathrm{E}_t(d_{t,t+1} + mc_{j,t+1})}{p_* - \beta(1-\delta_z)mc_*}
\end{aligned}
$$

where

$$
\varepsilon_{\Psi'} = \frac{\Psi''(v_*^*)v_*^*}{\Psi'(v_*^*)} = \frac{-f(\log(v_{jt}^* + \frac{\sigma_v^2}{2})}{1 - F(\log(v_{jt}^* + \frac{\sigma_v^2}{2})}
$$

Rearranging the above equation gives:

$$\varepsilon_{\Psi'}(z_{jt} - s_t) = - \Big[\theta\varepsilon_{\Psi'} + \frac{p_*}{p_* - \beta(1-\delta_z)mc_*}\Big]\tilde{p}_{jt} + \frac{1}{1-\beta(1-\delta_z)}mc_{jt}$$
$$+ \Big[\frac{\beta(1-\delta_z)mc_*}{p_* - \beta(1-\delta_z)mc_*} - \frac{\beta(1-\delta_z)}{1-\beta(1-\delta_z)}\Big]\mathrm{E}_t(d_{t,t+1} + mc_{j,t+1})$$

Subtracting the analogous aggregate condition I obtain:

$$\varepsilon_{\Psi'}\tilde{z}_{jt} = - \Big[\theta\varepsilon_{\Psi'} + \frac{p_*}{p_* - \beta(1-\delta_z)mc_*}\Big]\hat{p}_{jt} + \frac{1}{1-\beta(1-\delta_z)}\tilde{mc}_{jt}$$
$$+ \Big[\frac{\beta(1-\delta_z)mc_*}{p_* - \beta(1-\delta_z)mc_*} - \frac{\beta(1-\delta_z)}{1-\beta(1-\delta_z)}\Big]\mathrm{E}_t(\tilde{mc}_{j,t+1})$$

Denote

$$B_1 = -\Big[\frac{\beta(1-\delta_z)mc_*}{p_* - \beta(1-\delta_z)mc_*} - \frac{\beta(1-\delta_z)}{1-\beta(1-\delta_z)}\Big]$$

Rearranging and using the expression for expected marginal cost (A.25), I obtain the following equations by matching coefficients:

$$\frac{\psi_1^{mc}}{1-\beta(1-\delta_z)} = \varepsilon_{\Psi'}\psi_1^z + B_1\Big[\psi_1^{mc} + \psi_2^{mc}(1-\xi_p)\psi_1^p\Big]\psi_1^{inv} \tag{A.26}$$

$$\frac{\psi_2^{mc}}{1-\beta(1-\delta_z)} = \varepsilon_{\Psi'}\psi_2^z + \theta\varepsilon_{\Psi'} + \frac{p_*}{p_* - \beta(1-\delta_z)mc_*}$$
$$+ B_1\Big\{\Big[\psi_1^{mc} + \psi_2^{mc}(1-\xi_p)\psi_1^p\Big]\psi_2^{inv} + \psi_2^{mc}\xi_p\Big\} \tag{A.27}$$

A.6.5 Pricing

The equation determining re-optimizing firm's optimal price is:

$$\mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\xi_p \beta)^i \frac{\Xi_{t+i}}{\Xi_t} \frac{P_{t+i}}{P_{jt}^*} S_{j,t+i} \Big[\frac{P_{jt}^* \Pi_{l=0}^{i-1} \pi_{t+l}^{\ell_p}}{P_{t+i}} - \beta(1-\delta_z) \mathrm{E}_{t+i} \big(\frac{\epsilon_{j,t+i}}{\epsilon_{j,t+i}-1} \frac{\Xi_{t+i+1}}{\Xi_{t+i}} \lambda_{j,t+i+1} \big) \Big] = 0 \tag{A.28}$$

where $\epsilon_{j,t+i}$ is the effective elasticity of demand as defined in (1.20), which I rewrite here:

$$\epsilon_{j,t+i} = \frac{\theta \int_0^{v_{j,t+i}^*} v dG(v)}{\int \min(v, v_{j,t+i}^*) dG(v)} = \theta \Big[1 - \frac{v_{j,t+i}^* \big(1 - F(\log(v_{j,t+i}^* - \frac{\sigma_v^2}{2})) \big)}{\Psi(v_{j,t+i}^*)} \Big]$$

Log-linearizing ϵ

$$\widehat{\frac{\epsilon_{j,t+i}}{\epsilon_{j,t+i}-1}} = \frac{-1}{\epsilon_* - 1} \widehat{\epsilon}_{j,t+i} = \frac{\frac{\epsilon_*}{\theta} - 1}{\frac{\epsilon_*}{\theta}} \Big[\hat{v}_{j,t+i}^* + \big(1 - F(\widehat{\log(v_{j,t+i}^*} - \frac{\sigma_v^2}{2})) \big) - \Psi(\hat{v_{j,t+i}^*}) \Big] \tag{A.29}$$

$$= \frac{-1}{\epsilon_* - 1} \frac{\epsilon_* - \theta}{\epsilon_*} \big(1 - \varepsilon_\Psi + \varepsilon_{\Psi'} \big) \big[\theta \tilde{p}_{j,t+i} + z_{j,t+i} - s_{t+i} \big] \tag{A.30}$$

$$= -D_\epsilon \big[\theta \tilde{p}_{j,t+i} + z_{j,t+i} - s_{t+i} \big] \tag{A.31}$$

where

$$D_\varepsilon \equiv \frac{1}{\epsilon_* - 1} \frac{\epsilon_* - \theta}{\epsilon_*} \big(1 - \varepsilon_\Psi + \varepsilon_{\Psi'} \big)$$

Log-linearizing Pricing Equation

Log-linearizing (A.28) gives

$$\mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \tilde{p}^*_{j,t+i} = \mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \mathrm{E}_{t+i}\Big[\frac{\widehat{\epsilon_{j,t+i}}}{\epsilon_{j,t+i}-1} + \frac{\widehat{\Xi_{t+i+1}}}{\Xi_{t+i}} - mc_{j,t+i+1}\Big]$$

Plugging (A.31) into the above equation and rearranging yields:

$$\mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \tilde{p}^*_{j,t+i}(1 + D_\epsilon\theta) = \mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \Big[- D_\epsilon(z_{j,t+i} - s_{t+i}) + \mathrm{E}_{t+i}\big(d_{t+i,t+i+1} + mc_{j,t+i+1}\big)\Big] \tag{A.32}$$

Deriving Phillips Curve

Now we work toward solving for the Phillips curve from the above linearized pricing equation (A.32). Note that from (A.25) I know

$$\mathrm{E}_{t+i}\tilde{mc}_{j,t+i+1} = \Phi_1 \begin{bmatrix} \tilde{inv}_{t+i-1} \\ \hat{p}_{j,t+i} \end{bmatrix}$$

where

$$\Phi_1 \equiv \begin{bmatrix} \Big[\psi_1^{mc} + \psi_2^{mc}(1-\xi_p)\psi_1^p\Big]\psi_1^{inv} \\ \Big[\psi_1^{mc} + \psi_2^{mc}(1-\xi_p)\psi_1^p\Big]\psi_2^{inv} + \psi_2^{mc}\xi_p \end{bmatrix}^T \tag{A.33}$$

Define the matrix Ψ_1 as:

$$\Psi_1 \equiv \begin{bmatrix} \psi_1^{inv} & \psi_2^{inv} \\ 0 & 1 \end{bmatrix}$$

For $i \geq 1$,

$$\begin{aligned}
\mathrm{E}_t^{\bar{p}} \begin{bmatrix} \tilde{inv}_{j,t+i-1} \\ \hat{p}_{j,t+i} \end{bmatrix} &= \mathrm{E}_t^{\bar{p}} \Big\{ \Psi_1 \begin{bmatrix} \tilde{inv}_{j,t+i-2} \\ \hat{p}_{j,t+i-1} \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} (\pi_{t+i} - \ell_p \pi_{t+i-1}) \Big\} \\
&= \mathrm{E}_t^{\bar{p}} \Big\{ \Psi_1^2 \begin{bmatrix} \tilde{inv}_{t+i-3} \\ \hat{p}_{j,t+i-2} \end{bmatrix} + \Psi_1 \begin{bmatrix} 0 \\ -1 \end{bmatrix} (\pi_{t+i-1} - \ell_p \pi_{t+i-2}) + \begin{bmatrix} 0 \\ -1 \end{bmatrix} (\pi_{t+i} - \ell_p \pi_{t+i-1}) \Big\} \\
&= \dots \\
&= \mathrm{E}_t^{\bar{p}} \Big\{ \Psi_1^i \begin{bmatrix} \tilde{inv}_{j,t-i} \\ \hat{p}_{j,t} \end{bmatrix} + \sum_{l=1}^{i} \Psi_1^{i-l} \begin{bmatrix} 0 \\ -1 \end{bmatrix} (\pi_{t+l} - \ell_p \pi_{t+l-1}) \Big\}
\end{aligned}$$

Therefore I can write:

$$\begin{aligned}
\mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \tilde{mc}_{j,t+i+1} &= \mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \Phi_1 \begin{bmatrix} \tilde{inv}_{j,t+i-1} \\ \hat{p}_{j,t+i} \end{bmatrix} \\
&= \mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \Phi_1 \Big\{ \Psi_1^i \begin{bmatrix} \tilde{inv}_{t-i} \\ \hat{p}_{j,t} \end{bmatrix} + \sum_{l=1}^{i} \Psi_1^{i-l} \begin{bmatrix} 0 \\ -1 \end{bmatrix} (\pi_{t+l} - \ell_p \pi_{t+l-1}) \Big\} \\
&= \mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} \Phi_1 (\beta\xi_p\Psi_1)^i \begin{bmatrix} \tilde{inv}_{j,t-i} \\ \hat{p}_{j,t} \end{bmatrix} + \mathrm{E}_t^{\bar{p}} \sum_{i=1}^{\infty} \Phi_1 (\beta\xi_p)^i \sum_{l=0}^{i-1} \Psi_1^{i-l-1} \begin{bmatrix} 0 \\ -1 \end{bmatrix} (\pi_{t+l+1} - \ell
\end{aligned}$$

$$\Longrightarrow$$

$$
\begin{aligned}
\mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \tilde{mc}_{j,t+i+1} &= \Phi_1 (\mathrm{I} - \xi_p \beta \Psi_1)^{-1} \begin{bmatrix} \tilde{inv}_{j,t-i} \\ \\ \hat{p}_{j,t} \end{bmatrix} \\
&\quad + \mathrm{E}_t^{\bar{p}} \sum_{l=0}^{\infty} \Phi_1 \Psi_1^{-l-1} (\beta \xi_p \Psi_1)^{l+1} (\mathrm{I} - \xi_p \beta \Psi_1)^{-1} \begin{bmatrix} 0 \\ \\ -1 \end{bmatrix} (\pi_{t+l+1} - \ell_p \pi_{t+l}) \\
&= \Phi_1 (\mathrm{I} - \xi_p \beta \Psi_1)^{-1} \Bigg\{ \begin{bmatrix} \tilde{inv}_{j,t-i} \\ \\ \hat{p}_{j,t} \end{bmatrix} + \mathrm{E}_t^{\bar{p}} \sum_{l=1}^{\infty} (\beta\xi_p)^l (\pi_{t+l} - \ell_p \pi_{t+l-1}) \begin{bmatrix} 0 \\ \\ -1 \end{bmatrix} \Bigg\}
\end{aligned} \tag{A.34}
$$

Define

$$
\Phi_2 \equiv \begin{bmatrix} \psi_1^z \\ \\ \psi_2^z \end{bmatrix}^T
$$

By the same token I can derive:

$$
\begin{aligned}
\mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \tilde{z}_{j,t+i} &= \mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \Phi_2 \begin{bmatrix} \tilde{inv}_{j,t+i-i} \\ \\ \hat{p}_{j,t+i} \end{bmatrix} \\
&= \Phi_2 (\mathrm{I} - \xi_p \beta \Psi_1)^{-1} \Bigg\{ \begin{bmatrix} \tilde{inv}_{j,t-i} \\ \\ \hat{p}_{j,t} \end{bmatrix} + \mathrm{E}_t^{\bar{p}} \sum_{l=1}^{\infty} (\beta\xi_p)^l (\pi_{t+l} - \ell_p \pi_{t+l-1}) \begin{bmatrix} 0 \\ \\ -1 \end{bmatrix} \Bigg\}
\end{aligned} \tag{A.35}
$$

For the price aggregation, I know that:

$$
\begin{aligned}
\mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \tilde{p}_{j,t+i} &= \tilde{p}_{jt} + \mathrm{E}_t^{\bar{p}} \sum_{i=1}^{\infty} (\beta\xi_p)^i \Big[\tilde{p}_{jt} + \ell_p \sum_{l=1}^{i} \pi_{t+l-1} - \sum_{l=1}^{i} \pi_{t+i}\Big] \\
&= \frac{1}{1-\beta\xi_p}\tilde{p}_{jt} - \mathrm{E}_t^{\bar{p}} \sum_{i=1}^{\infty} (\beta\xi_p)^i \sum_{l=1}^{i} (\pi_{t+l} - \ell_p \pi_{t+l-1}) \\
&= \frac{1}{1-\beta\xi_p}\tilde{p}_{jt} - \mathrm{E}_t^{\bar{p}} \sum_{l=1}^{\infty} (\pi_{t+l} - \ell_p \pi_{t+l-1}) \sum_{i=l}^{\infty} (\beta\xi_p)^i \\
&= \frac{1}{1-\beta\xi_p}\tilde{p}_{jt} - \mathrm{E}_t^{\bar{p}} \sum_{l=1}^{\infty} (\pi_{t+l} - \ell_p \pi_{t+l-1}) \frac{(\beta\xi_p)^l}{1-\beta\xi_p} \qquad \text{(A.36)}
\end{aligned}
$$

Slightly manipulating the RHS of (A.32) gives:

$$
\begin{aligned}
\mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \tilde{p}^*_{j,t+i}(1+D_\epsilon\theta) =& \mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \Big[\mathrm{E}_{t+i} \tilde{mc}_{j,t+i+1} - D_\epsilon \tilde{z}_{j,t+i}\Big] \\
&+ \mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \Big[- D_\epsilon (z_{t+i} - s_{t+i}) + \mathrm{E}_{t+i}\big(d_{t+i,t+i+1} + mc_{t+i+1}\big)\Big]
\end{aligned}
\qquad \text{(A.37)}
$$

Plugging (A.34) and (A.35) into the RHS, and (A.36) into the LHS of the above equation gives:

$$
\begin{aligned}
&\frac{(1+D_\epsilon\theta)}{1-\beta\xi_p}\tilde{p}^*_{jt} - \frac{(1+D_\epsilon\theta)}{1-\beta\xi_p}\mathrm{E}_t^{\bar{p}} \sum_{l=1}^{\infty} (\beta\xi_p)^l (\pi_{t+l} - \ell_p \pi_{t+l-1}) \\
&= (\Phi_1 - D\Phi_2)(\mathrm{I} - \xi_p\beta\Psi_1)^{-1} \Big\{ \begin{bmatrix} \tilde{inv}_{j,t-i} \\ \hat{p}^*_{jt} \end{bmatrix} + \mathrm{E}_t^{\bar{p}} \sum_{l=1}^{\infty} (\beta\xi_p)^l (\pi_{t+l} - \ell_p \pi_{t+l-1}) \begin{bmatrix} 0 \\ -1 \end{bmatrix} \Big\} \\
&\quad + \mathrm{E}_t^{\bar{p}} \sum_{i=0}^{\infty} (\beta\xi_p)^i \Big[- D_\epsilon (z_{t+i} - s_{t+i}) + \mathrm{E}_{t+i}\big(d_{t+i,t+i+1} + mc_{t+i+1}\big)\Big] \qquad \text{(A.38)}
\end{aligned}
$$

Integrating the above equation over all re-optimizing firms gives:

$$
\begin{aligned}
&\frac{(1+D_\epsilon\theta)}{1-\beta\xi_p}\tilde{p}_t^* - \frac{(1+D_\epsilon\theta)}{1-\beta\xi_p}\mathrm{E}_t^{\bar{p}}\sum_{l=1}^{\infty}(\beta\xi_p)^l(\pi_{t+l}-\ell_p\pi_{t+l-1}) \\
&= (\Phi_1 - D\Phi_2)(\mathrm{I}-\xi_p\beta\Psi_1)^{-1}\Bigg\{\begin{bmatrix}0\\ \\ \hat{p}_t^*\end{bmatrix} + \mathrm{E}_t^{\bar{p}}\sum_{l=1}^{\infty}(\beta\xi_p)^l(\pi_{t+l}-\ell_p\pi_{t+l-1})\begin{bmatrix}0\\ \\ -1\end{bmatrix}\Bigg\} \\
&\quad + \mathrm{E}_t^{\bar{p}}\sum_{i=0}^{\infty}(\beta\xi_p)^i\Big[-D_\epsilon(z_{t+i}-s_{t+i}) + \mathrm{E}_{t+i}\big(d_{t+i,t+i+1}+mc_{t+i+1}\big)\Big] \qquad \text{(A.39)}
\end{aligned}
$$

Now, subtracting (A.39) from (A.38) yields:

$$
\frac{(1+D_\epsilon\theta)}{1-\beta\xi_p}(\hat{p}_{jt}^* - \hat{p}_t^*) = (\Phi_1 - D\Phi_2)(\mathrm{I}-\xi_p\beta\Psi_1)^{-1}\begin{bmatrix}\tilde{inv}_{j,t-i}\\ \\ \hat{p}_{jt}^* - \hat{p}_t^*\end{bmatrix}
$$

Therefore, by matching coefficients, I have the following two equations:

$$
\frac{(1+D_\epsilon\theta)}{1-\beta\xi_p}\psi_1^p = (\Phi_1 - D\Phi_2)(\mathrm{I}-\xi_p\beta\Psi_1)^{-1}\begin{bmatrix}1\\ \\ \psi_1^p\end{bmatrix} \qquad \text{(A.40)}
$$

Equations (A.26), (A.27), and (A.40) are three equations to solve for three unknown coefficients ψ_1^l, ψ_2^l, and ψ_1^p.

Chapter B: Appendix for Chapter 2

B.1 Proof of Corollary 1

Consider the CES expenditure minimization problem of choosing new and used investment goods in a bundle:

$$\begin{aligned} &\min_{i_{new}, i_{used}} \; i_{new} + p\, i_{used} \\ &\text{subject to } \; \left\{\eta^{\frac{1}{\epsilon}}[(1-\lambda) i_{j,new}]^{\frac{\epsilon-1}{\epsilon}} + (1-\eta)^{\frac{1}{\epsilon}}[(1-\lambda^M) i_{j,used}]^{\frac{\epsilon-1}{\epsilon}}\right\}^{\frac{\epsilon}{\epsilon-1}} = g \end{aligned}$$

Combining the first order conditions, we have:

$$\begin{aligned} &\left(\frac{i_{new}}{i_{used}}\right)^{\frac{1}{\epsilon}} \left(\frac{1-\lambda}{1-\lambda^M}\right)^{\frac{\epsilon-1}{\epsilon}} \left(\frac{\eta}{1-\eta}\right)^{\frac{1}{\epsilon}} = \frac{1}{p} \\ \Longrightarrow \quad & i_{used} = i_{new} \frac{\eta}{1-\eta} \left(\frac{1-\lambda^M}{1-\lambda}\right)^{\epsilon-1} p^{-\epsilon} \end{aligned} \tag{B.1}$$

Plugging the above equation back into the constraint, we have:

$$\begin{aligned} & g = \left\{\eta^{\frac{1}{\epsilon}}[(1-\lambda) i_{j,new}]^{\frac{\epsilon-1}{\epsilon}} + (1-\eta)^{\frac{1}{\epsilon}}[(1-\lambda^M) i_{j,new}]^{\frac{\epsilon-1}{\epsilon}} \left(\frac{1-\lambda^M}{1-\lambda}\right)^{\frac{(\epsilon-1)^2}{\epsilon}} \left(\frac{1-\eta}{\eta}\right)^{\frac{\epsilon-1}{\epsilon}} p^{1-\epsilon}\right\}^{\frac{\epsilon}{\epsilon-1}} \\ \Longrightarrow \quad & g = (1-\lambda) i_{new} \left\{\eta^{\frac{1}{\epsilon}} + (1-\eta)\eta^{\frac{1-\epsilon}{\epsilon}} \left(\frac{1-\lambda^M}{1-\lambda}\right)^{\epsilon-1} p^{1-\epsilon}\right\}^{\frac{\epsilon}{\epsilon-1}} \end{aligned} \tag{B.2}$$

Multiplying both sides of the above equation by η and then $(1-\lambda)^{\epsilon-1}$, we have:

$$
\begin{aligned}
& (1-\lambda)i_{new}\Big\{\eta+(1-\eta)\big(\frac{1-\lambda^M}{1-\lambda}\big)^{\epsilon-1}p^{1-\epsilon}\Big\}^{\frac{\epsilon}{\epsilon-1}} = \eta g \\
\implies \quad & (1-\lambda)^{\epsilon}i_{new}\Big\{\eta+(1-\eta)\big(\frac{1-\lambda^M}{1-\lambda}\big)^{\epsilon-1}p^{1-\epsilon}\Big\}^{\frac{\epsilon}{\epsilon-1}} = \eta(1-\lambda)^{\epsilon-1}g \\
\implies \quad & i_{new}\Big\{\eta\big(\frac{1}{1-\lambda}\big)^{1-\epsilon}+(1-\eta)\big(\frac{p}{1-\lambda^M}\big)^{1-\epsilon}\Big\}^{\frac{\epsilon}{\epsilon-1}} = \eta(1-\lambda)^{\epsilon-1}g \qquad \text{(B.3)}
\end{aligned}
$$

Let $\rho = \Big\{\eta\big(\frac{1}{1-\lambda}\big)^{1-\epsilon}+(1-\eta)\big(\frac{p}{1-\lambda^M}\big)^{1-\epsilon}\Big\}^{\frac{1}{1-\epsilon}}$. Combining equations (B.1) and (B.3), we have:

$$
\begin{aligned}
i_{new} &= \eta(1-\lambda)^{\epsilon-1}\rho^{\epsilon}g \\
i_{used} &= (1-\eta)(1-\lambda^M)^{\epsilon-1}p^{-\epsilon}\rho^{\epsilon}g
\end{aligned}
$$

Therefore, the total expenditure can be expressed as

$$
\begin{aligned}
i_{new}+pi_{used} &= \Big[\eta\big(\frac{1}{1-\lambda}\big)^{1-\epsilon}+(1-\eta)\big(\frac{p}{1-\lambda^M}\big)^{1-\epsilon}\Big]rho^{\epsilon}g \\
&= \rho^{1-\epsilon}\rho^{\epsilon}g = \rho g
\end{aligned}
$$

which verifies that ρ is the CES price index.

B.2 Proof of Proposition 1

The market clearing condition can be written as follows:

$$
\begin{aligned}
E(p, Z; \epsilon) &= D_{used} - S_{used} \\
&= \theta(p, \lambda^M) \int_{z^I} \Big[\big(\frac{\alpha z_j Z}{\rho(p, \lambda^M)}\big)^{\frac{1}{1-\alpha}} - (1-\lambda)k_0 \Big] dF(z_j) \\
&\quad - \Big\{ \lambda k_0 + \int^{z^D} \Big[(1-\lambda)k_0 - \big(\frac{\alpha z_j Z}{p}\big)^{\frac{1}{1-\alpha}} \Big] dF(z_j) \Big\} = 0 \qquad \text{(B.4)}
\end{aligned}
$$

where

$$
\begin{aligned}
\rho(p, \lambda^M) &= \Big[\eta \big(\frac{1}{1-\lambda}\big)^{1-\epsilon} + (1-\eta) \big(\frac{p}{1-\lambda^M}\big)^{1-\epsilon} \Big]^{\frac{1}{1-\epsilon}}, \\
z^I &= \frac{\rho}{\alpha Z \big[(1-\lambda)k_0 \big]^{\alpha-1}} \\
z^D &= \frac{p}{\alpha Z \big[(1-\lambda)k_0 \big]^{\alpha-1}}, \\
\lambda^M &= \frac{\lambda k_0}{\lambda k_0 + \int^{z^D} \Big[(1-\lambda)k_0 - \big(\frac{\alpha z_j Z}{p}\big)^{\frac{1}{1-\alpha}} \Big] dF(z_j)}
\end{aligned}
$$

and

$$
\theta(p, \lambda^M) = (1-\eta)(1-\lambda^M)^{\epsilon-1} \big(\frac{p}{\rho}\big)^{-\epsilon}
$$

is the ratio of used investment to total investment for investing entrepreneurs. Note that the market fraction λ^M depends both on p and Z. Equation (B.4) defines the market clearing price of used investment goods p as an implicit function of the aggregate productivity Z and the elasticity of substitution between new and used investment goods ϵ. Using total differentiation of equation (B.4) to obtain the derivative of p with respect to Z in equilibrium, we get the relationship of the derivatives of excess demand with respect to p and Z:

$$E_p\, dp + E_Z\, dZ = 0 \Longrightarrow \frac{dp}{dZ} = -\frac{E_Z}{E_p}$$

with

$$\begin{aligned}
E_Z =& \frac{\theta}{(1-\alpha)Z}\int_{z^I}\left(\frac{\alpha z_j Z}{\rho}\right)^{\frac{1}{1-\alpha}} dF(z_j) + \frac{1}{(1-\alpha)Z}\int^{z^D}\left(\frac{\alpha z_j Z}{p}\right)^{\frac{1}{1-\alpha}} dF(z_j) \\
&+ \theta_{\lambda^M}\frac{\partial \lambda^M}{\partial Z}\int_{z^I}\Big[\left(\frac{\alpha z_j Z}{\rho}\right)^{\frac{1}{1-\alpha}} - (1-\lambda)k_0\Big] dF(z_j) \\
&- \frac{\theta \rho_{\lambda^M}}{(1-\alpha)\rho}\frac{\partial \lambda^M}{\partial Z}\int_{z^I}\left(\frac{\alpha z_j Z}{\rho}\right)^{\frac{1}{1-\alpha}} dF(z_j) \\
&+ \theta_\rho \rho_{\lambda^M}\frac{\partial \lambda^M}{\partial Z}\int_{z^I}\Big[\left(\frac{\alpha z_j Z}{\rho}\right)^{\frac{1}{1-\alpha}} - (1-\lambda)k_0\Big] dF(z_j)
\end{aligned}$$

The last three terms on the RHS of the above equation describe the indirect effect of Z on market demand of used capital through the market fraction λ^M.

$$\begin{aligned}
E_p =& \theta_p\int_{z^I}\Big[\left(\frac{\alpha z_j Z}{\rho}\right)^{\frac{1}{1-\alpha}} - (1-\lambda)k_0\Big] dF(z_j) - \frac{\theta \rho_p}{(1-\alpha)\rho}\int_{z^I}\left(\frac{\alpha z_j Z}{\rho}\right)^{\frac{1}{1-\alpha}} dF(z_j) \\
&- \frac{1}{(1-\alpha)p}\int^{z^D}\left(\frac{\alpha z_j Z}{p}\right)^{\frac{1}{1-\alpha}} dF(z_j) + \theta_{\lambda^M}\frac{\partial \lambda^M}{\partial p}\int_{z^I}\Big[\left(\frac{\alpha z_j Z}{\rho}\right)^{\frac{1}{1-\alpha}} - (1-\lambda)k_0\Big] dF(z_j) \\
&- \frac{\theta \rho_{\lambda^M}}{(1-\alpha)\rho}\frac{\partial \lambda^M}{\partial p}\int_{z^I}\left(\frac{\alpha z_j Z}{\rho}\right)^{\frac{1}{1-\alpha}} dF(z_j) \\
&+ \theta_\rho\big(\rho_p + \rho_{\lambda^M}\frac{\partial \lambda^M}{\partial p}\big)\int_{z^I}\Big[\left(\frac{\alpha z_j Z}{\rho}\right)^{\frac{1}{1-\alpha}} - (1-\lambda)k_0\Big] dF(z_j)
\end{aligned}$$

The last three terms on the RHS of the above equation describe the indirect effect of p on market demand of used capital through the market fraction λ^M.

Consider the case when $\epsilon = 0$ (new and used investment goods are perfect complements). In this case, $\theta = \frac{1-\eta}{1-\lambda^M}$, $\theta_\rho = 0$, $\theta_p = 0$, and $\theta_{\lambda^M} = \frac{1-\eta}{(1-\lambda^M)^2} > 0$. The price of the investment bundle is

$$\rho = \frac{\eta}{1-\lambda} + \frac{(1-\eta)p}{1-\lambda^M}, \quad \rho_p = \frac{(1-\eta)}{1-\lambda^M} > 0, \quad \rho_{\lambda^M} = \frac{(1-\eta)p}{(1-\lambda^M)^2} > 0.$$

Also, note that

$$\frac{\partial \lambda^M}{\partial Z} = \frac{(\lambda^M)^2}{\lambda k_0} \frac{1}{(1-\alpha)Z} \int_{z^D} \left(\frac{\alpha z_j Z}{p}\right)^{\frac{1}{1-\alpha}} dF(z_j) > 0,$$

and

$$\frac{\partial \lambda^M}{\partial p} = -\frac{(\lambda^M)^2}{\lambda k_0} \frac{1}{(1-\alpha)p} \int_{z^D} \left(\frac{\alpha z_j Z}{p}\right)^{\frac{1}{1-\alpha}} dF(z_j) < 0.$$

So that

$$Z\frac{\partial \lambda^M}{\partial Z} = -p\frac{\partial \lambda^M}{\partial p}.$$

Hence, when $\epsilon = 0$ the elasticity of p with respect to Z can be written as:

$$\phi_{p,Z}(0) = \frac{dp}{dZ}\frac{Z}{p} = -\frac{E_Z}{Ep}\frac{Z}{p} = \frac{(1-\alpha)ZE_Z}{-(1-\alpha)pEp} \tag{B.5}$$

where the numerator is

$$
\begin{aligned}
(1-\alpha)ZE_Z =& \frac{1-\eta}{1-\lambda^M}\int_{z^I}(\frac{\alpha z_j Z}{\rho})^{\frac{1}{1-\alpha}}dF(z_j) + \int^{z^D}(\frac{\alpha z_j Z}{p})^{\frac{1}{1-\alpha}}dF(z_j) \\
&+ \Big\{(1-\alpha)\frac{\partial\theta}{\partial\lambda^M}\frac{\partial\lambda^M}{\partial Z}Z\int_{z^I}\Big[(\frac{\alpha z_j Z}{\rho})^{\frac{1}{1-\alpha}} - (1-\lambda)k_0\Big]dF(z_j) \\
&- \frac{\theta\rho_{\lambda^M}}{\rho}\frac{\partial\lambda^M}{\partial Z}Z\int_{z^I}(\frac{\alpha z_j Z}{\rho})^{\frac{1}{1-\alpha}}dF(z_j)\Big\} \qquad \text{(B.6)}
\end{aligned}
$$

and the denominator is

$$
\begin{aligned}
-(1-\alpha)pE_p =& \frac{1-\eta}{1-\lambda^M}\frac{\frac{(1-\eta)p}{1-\lambda^M}}{\rho}\int_{z^I}(\frac{\alpha z_j Z}{\rho})^{\frac{1}{1-\alpha}}dF(z_j) + \int^{z^D}(\frac{\alpha z_j Z}{p})^{\frac{1}{1-\alpha}}dF(z_j) \\
&- \Big\{(1-\alpha)\frac{\partial\theta}{\partial\lambda^M}\frac{\partial\lambda^M}{\partial p}p\int_{z^I}\Big[(\frac{\alpha z_j Z}{\rho})^{\frac{1}{1-\alpha}} - (1-\lambda)k_0\Big]dF(z_j) \\
&- \frac{\theta\rho_{\lambda^M}}{\rho}\frac{\partial\lambda^M}{\partial p}p\int_{z^I}(\frac{\alpha z_j Z}{\rho})^{\frac{1}{1-\alpha}}dF(z_j)\Big\} \qquad \text{(B.7)}
\end{aligned}
$$

Note that the terms inside the big curly brackets in equations (B.6) and (B.7) are the indirect effect of Z (and p) on the demand of used investment goods through λ^M. The indirect terms in the numerator and the denominator are actually the same since $Z\frac{\partial\lambda^M}{\partial Z} = -p\frac{\partial\lambda^M}{\partial p}$. Furthermore, since $\big(\frac{(1-\eta)p}{1-\lambda^M}\big)/\rho < 1$, the sum of the first two terms on the RHS of the denominator equation (B.7) is less than the sum of the first two terms on the RHS of the numerator equation (B.6). Therefore, as long as **the indirect effect of p on the demand of used investment goods is smaller (in absolute value) than the direct effect**, both the numerator and the denominator are positive and $\phi_{p,Z}(0) > 1$. Now, since $\phi_{p,Z}(\epsilon)$ is continuous in ϵ, I can establish that there exists an $\bar{\epsilon} > 0$ such that $\phi_{p,Z}(\epsilon) > 1$ for $\epsilon < \bar{\epsilon}$. This completes the proof of the first part.

The meaningful reallocation of capital is the total supply of used investment goods of good quality, $S^g_{used} = \int^{z^D} \Big[(1-\lambda)k_0 - \big(\frac{\alpha z_j Z}{p}\big)^{\frac{1}{1-\alpha}}\Big] dF(z_j)$. Now, note that by the chain rule

$$\begin{aligned}\frac{dS^g_{used}}{dZ} &= \frac{dS^g_{used}}{d\big(\frac{Z}{p}\big)} \frac{d\big(\frac{Z}{p}\big)}{dZ} \\ &= \Big\{ \frac{-p}{(1-\alpha)Z} \int^{z^D} \Big[\big(\frac{\alpha z_j Z}{p}\big)^{\frac{1}{1-\alpha}} \Big] dF(z_j) \Big\} \frac{d\big(\frac{Z}{p}\big)}{dZ}\end{aligned}$$

and

$$\frac{d\big(\frac{Z}{p}\big)}{dZ} = \frac{1}{p} - \frac{Z}{p^2}\frac{dp}{dZ} = \frac{1}{p}\big[1 - \phi_{p,Z}(\epsilon)\big] < 0 \ \text{ for } \ \epsilon < \bar{\epsilon}$$

Therefore, reallocation of used capital increasing in Z.

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