ABSTRACT

Title of Thesis: ANALYSES OF INFLUENTIAL FACTORS FOR ACCURATE DETERMINATION OF PEAK RATE FACTORS AND TIMES TO PEAK OF UNIT HYDROGRAPHS

Tianming Zhao, Master of Science, 2017

Thesis Directed By: Professor Richard H. McCuen, Department of Civil and Environmental Engineering

Despite the availability of a number of sophisticated hydrologic models, the unit hydrograph (UH) is still one of the most widely used models for computing runoff hydrographs. The two-parameter gamma UH can be fully characterized by two parameters: the peak rate factor (PRF) and the time to peak (tp_{UH}). Currently, obtaining accurate estimates of UH parameters is still a problem, especially for ungauged watersheds. The goal of this research was to analyze factors that influence estimates of UH parameters and to develop general guidelines that can assist in estimating UH parameters more accurately. A calibration model was developed for evaluating PRFs and tp_{UH}s simultaneously from rainfall-runoff data. The effects of various influential factors were identified and investigated based on analyses of both synthetic and measured rainfall-runoff data. Results showed that the accuracy of calibrated UH parameters is affected by the rainfall characteristics, the time offset, the nonuniformity of rainfall, the extent of nonlinear watershed processes, and the flexibility of the gamma probability distribution function. Guidelines were developed to assist UH users in interpreting the calibration results and calibrating UH parameters more accurately.

ANALYSES OF INFLUENTIAL FACTORS FOR ACCURATE DETERMINATION OF PEAK RATE FACTORS AND TIMES TO PEAK OF UNIT HYDROGRAPHS

By

Tianming Zhao

Thesis submitted to the Faculty of the Graduate School of the
University of Maryland, College Park, in partial fulfillment
of the requirements for the degree of
Master of Science
2017

Advisory Committee:

Professor	Richard H. McCuen, Chair
Associate Professor	Kaye L. Brubaker
Assistant Professor	Barton A. Forman

ACKNOWLEDGEMENTS

First of all, I would like to thank my advisor, Dr. McCuen, for his patience, assistance, and tireless guidance during my two years of study. I appreciate and will always be grateful for the numerous hours that he spent providing me instructions in doing research and commenting on academic writing. His diligence and professionalism will always inspire me as I become either an engineer or a researcher after graduation.

I would like to thank Dr. Brubaker for letting me be the TA for her undergraduate classes. These were invaluable experiences for me. I learned how to use ArcGIS and solidified my knowledge of fluid mechanics. I would also like to thank her for the wonderful modeling techniques that I learned from her classes. These techniques will benefit me a lot.

I would like to thank Dr. Forman for his efforts in helping me achieve my education goals. His classes were always challenging, but these challenges helped me realize what I can achieve and they greatly built my confidence. I also appreciate his encouragement in my presentation and the patience in answering my questions.

I would like to thank my friends, Jing Wang, Yuan Xue, Yilu Feng, Lu Liu, and Meg Ryan. Thank you all for giving me the assistance and encouragement during the past two years.

Finally, I would like to thank my parents, Suqing Miao and Jicheng Zhao, and my girlfriend, Yanyi Zhang. Without their support, this thesis could not have been completed.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: INTRODUCTION

1.1. INTRODUCTION

With increased urbanization and land use change, stormwater quantity and quality controls have become increasingly important. Studies on stormwater drainage systems in hydrology are not just limited to the estimation of peak discharge rates, but attempt to simulate and estimate the entire watershed response to rainfall. Runoff hydrographs, which directly reflect watershed responses to rainfall events, are widely used in hydrologic design, especially when a watershed has considerable storage or significant nonhomogeneities and varies in soil type and land use. Due to the importance of runoff hydrographs in design, the simulation of rainfall-runoff processes and the accurate estimation of storm runoff hydrographs are essential in hydrology. While a number of sophisticated and comprehensive hydrologic models have been proposed, the unit hydrograph, due to its simplicity and practicality, is still one of the most widely used models to compute storm runoff hydrographs, especially for ungauged areas. With given rainfall excess and the unit hydrograph for a watershed, the runoff hydrograph can be predicted by convolving the rainfall excess hyetograph with the unit hydrograph.

The concept of the unit hydrograph was first introduced by Sherman (1932). It is the discharge hydrograph that results from 1-inch of effective rainfall (i.e., rainfall excess) distributed uniformly, at a uniform rate, over a drainage area in a specific time period. The unit hydrograph can be viewed as a smoothing function that transforms a rainfall excess hyetograph into a direct runoff hydrograph. The unit hydrograph has been widely used and presented in many different forms. The Snyder's unit hydrograph (Snyder, 1938), the Clark unit hydrograph (Clark, 1945), the two-parameter gamma unit hydrograph (Edson, 1950; Nash, 1959; Aron and White, 1982), the NRCS (formally SCS) unit hydrograph (1972 and 2007), and the three-parameter beta unit hydrograph are notable examples.

The NRCS dimensionless unit hydrograph was developed based on the analysis of a large number of unit hydrographs from watersheds varying in size and locations. It has been incorporated into the hydrologic models, TR-20 and TR-55, and has been widely used in engineering design. A NRCS dimensionless unit hydrograph can be fully characterized and dimentionlized by two UH parameters, the peak rate factor (PRF), and the time to peak (tp_{UH}). It can be represented by a two-parameter gamma unit hydrograph, with the UH parameters defined by the scale and shape parameters. Although the NRCS dimensionless unit hydrograph or the two-parameter gamma unit hydrograph has been used and studied for decades, many problems still need to be addressed.

1.1.1. Problem #1: Accurate Estimation of PRFs

A PRF is required for developing a unit hydrograph, as it directly affects the proportion under the rising limb. A standard value of 484 (see section 2.2.4 for derivations) was adopted by the NRCS and has been widely used for decades. A value of 284 is recommended for coastal (Delaware, Maryland, and Virginia) areas.

The unit hydrograph serves as a transformation function that transforms an input rainfall excess hyetograph into a direct runoff hydrograph. The PRF should vary with watershed and storm characteristics. The most recent NRCS engineering manual

(2007) suggested that the PRF should vary from about 700 in steep terrain to 100 or less in flat, swampy areas. However, specific instructions or guidelines for selecting a peak rate factor for specific watershed have not been proposed. Some researchers and hydrologic engineers have been estimating the PRFs though calibrations of measured rainfall and runoff data. However, few studies have been conducted to assess the rationality and accuracy of these calibrated PRFs.

1.1.2. Problem #2: Accurate Estimation of tp_{UH}s

The tp_{UH} is another required parameter for developing a unit hydrograph. The tp_{UH} is the time from the start of the UH to its peak and is usually computed using the time of concentration derived from watershed characteristics. However, studies have shown that the time of concentration is not only affected by the watershed characteristics but also affected by the characteristics of storms. The hydraulic radius of a flow is dependent on the storm magnitude while the time of concentration is dependent on the hydraulic radius. Therefore, it is necessary to develop a method that shows the influence of both storm and watershed characteristics on tp_{UH}. In addition, the rationale and accuracy of these calibrated tp_{UH}s need to be assessed.

1.1.3. Problem #3: Analyses of Complex Storms

Existing methods for calibrating UH parameters are based on analyses of simple storms that usually have one burst of rainfall that is often quite uniformly distributed. This limits the number of storm events that are available for analyses, but more importantly, limits our understandings of the way that watersheds respond to complex storms. Measured rainfall events usually have multiple peaks and considerable variation, and therefore, it is necessary to develop a method for extracting UH parameters from complex storms.

1.1.4. Problem #4: Influence of Various Factors on the Accuracy of Calibrated UH Parameters

A number of factors can affect the accuracy of calibrated UH parameters. Many studies have revealed that UH parameters could be a function of watershed characteristics (e.g., the area, the slope, and the LCLU). But few studies have investigated the effect of watershed characteristics on the accuracy of UH parameters. Rainfall characteristics, such as the rainfall depth, the number of peaks, and the duration, can also influence the values and accuracy of the calibrated UH parameters since they influence the temporal watershed condition. The distance between the rainfall and runoff gauges and the nonuniformity of rainfall over a watershed are other influential factors. Past studies on gamma unit hydrographs have focused on the methods for calibrating UH parameters, as well as the development of relations between UH parameters and watershed characteristics. Influential factors, such as watershed and rainfall characteristics, may influence parameter estimation and the accuracy of calibrated values. This has not been systematically studied.

1.2. GOALS AND OBJECTIVES

To solve the above problems, the goal of this research is to investigate and understand factors that influence empirical estimates of gamma UH parameters and to develop general guidelines that can assist in calibrating UH parameter more accurately. To accomplish this goal, the following objectives direct this research:

(1) To develop a calibration model for extracting PRFs and $\mathrm{tp}_{\mathrm{UH}}$s simultaneously from measured complex rainfall-runoff data;

(2) To investigate the effects of various factors that may influence the calibration accuracy of UH parameters by generating synthetic rainfall-runoff data and using the results to develop general guidelines for UH modeling; and

(3) To assess the effectiveness and usefulness of the general guidelines by analyzing measured rainfall-runoff data.

Meeting these general objectives can benefit those trying to understand factors that influence unit hydrograph calibration. This should improve design accuracy and better protect the public from poor estimates of floods.

1.3. IMPLICATIONS

The utilization of unit hydrographs for engineering design requires highly accurate unit hydrographs. The accuracy of the form of a unit hydrograph is dependent to the accuracy and certainty of UH parameters, e.g., the PRF and the $\mathrm{tp}_{\mathrm{UH}}$. Current users of the NRCS unit hydrograph often determine the UH parameters by referring to suggested values by the NRCS (e.g., the standard PRF of 484 or 284 for coastal areas) or calibrating UH parameters from simple storms; however, users lack knowledge of the accuracy of calibrated UH parameters as well as the interpretation of calibration results. Someone who is calibrating a unit hydrograph needs general guidelines to make the best use of the available measured rainfall-runoff data (both simple and complex storms) to calibrate the UH parameters, and to interpret the calibration results more accurately. Fulfilling the above objectives will lead to a calibration model that can be used with both simple and complex storms; general guidelines on the factors that influence unit hydrograph calibration will result. These guidelines will assist users in assessing the accuracy of UH parameters, interpreting the calibration results, and consequently determining UH parameters more accurately.

CHAPTER 2: LITERATURE REVIEW

2.1. INTRODUCTION

Previous work on unit hydrographs focused on the derivation and parameter estimation for a variety of unit hydrograph models. In this chapter, the structure and methods of parameter estimation of a number of commonly used unit hydrographs are discussed. Advantages and constraints for certain unit hydrographs are also discussed.

2.2. DEVELOPMENT OF UNIT HYDROGRAPHS

Although a number of rainfall-runoff models have been proposed, unit hydrograph theory is still widely used in hydrologic planning and design due to its simplicity and reasonable physical basis. The concept of a unit hydrograph was first presented and used by Sherman (1932) to transfer a rainfall excess hyetograph to a storm runoff hydrograph. He defined a unit hydrograph as the discharge hydrograph that results from 1-inch of effective rainfall (i.e., rainfall excess) distributed uniformly, at a uniform rate, over a drainage area in a specific time period. The theory of unit hydrographs greatly improved the understanding and simulation of rainfall-runoff processes. In hydrologic designs, if the rainfall excess and the unit hydrograph of a watershed are given, the direct runoff hydrograph can be predicted by convolving the rainfall excess hyetograph with the unit hydrograph. This process is called convolution. In analysis, if the actual rainfall excess and direct runoff data are given, the unit hydrograph can be predicted through the process of deconvolution (i.e., calibration).

2.2.1. Snyder's Unit Hydrograph

Snyder (1938) provided a method for developing synthetic unit hydrographs for ungaged watersheds in sizes from 10 to 10,000 squares miles. This method, which requires five input parameters, includes the drainage area, A; the length of the main channel, L; the length from the outlet to the centroid of the watershed area, L_c; the watershed storage coefficient, C_t; and the empirical coefficient, C_p. The parameters were used to determine seven points that characterized his unit hydrograph. Synder provided a series of formulas to estimate these parameters. Then a smooth unit hydrograph could be graphed over the seven points, and the total runoff adjusted to a 1-inch depth of effective rainfall. This approach suffered because of its subjectivity. The other seem more objective.

2.2.2. Clark Unit Hydrograph

Clark (1945) proposed his unit hydrograph method, which was based on utilization of a translation hydrograph followed by a linear reservoir routing process. This method requires three parameters: the watershed time of concentration, t_c; a storage coefficient, R; and a time-area curve. The time of concentration is the time that the runoff will take to move from the hydraulically most distant point to the watershed outlet. The storage coefficient reflects the temporal storage capacity of the watershed, and it has a unit of time. The time-area curve is a cumulative curve that defines the cumulative portion of the drainage area that contributes runoff to the watershed outlet as a function of time from the start of the rainfall excess. The three parameters can be estimated based on the analyses of measured rainfall and runoff data for gauged areas. For ungauged areas, the t_c and R can be approximated by analyzing watershed characteristics. In spite of the availability of a number of methods for predicting the t_c and R (e.g., Sabol, 1988; Straub et al., 2000), most of these methods have specific assumptions and constraints. Commonly accepted estimation methods have not been developed.

2.2.3. Two-Parameter Gamma Unit Hydrograph

Due to similarity in shape between the probability density functions (PDF) and a typical unit hydrograph, many studies have been conducted in an attempt to derive unit hydrographs using a variety of probability density functions (e.g., the gamma, beta, and log-normal distributions). The most notable advantage of using a pdf as a unit hydrograph is that it provides a reasonably smooth shape and guarantees the area under unit hydrograph is always equal to 1 area-inch. For distribution that are unbounded in the right tail, a criterion for limiting its extent is necessary

Edson (1950) presented a formula for a unit hydrograph expressed by the two-parameter gamma distribution. The two-parameter gamma distribution can be expressed by the following equation:

$$f(t) = \frac{t^{c-1}e^{-t/b}}{b^c\Gamma(c)} \tag{2-1}$$

in which $f(t)$ is the runoff depth per unit time per effective rainfall $[1/T]$; b and c are the scale and shape parameters, respectively; and $\Gamma(c)$ is the value of gamma function for c. The discharge rate, q(t), can be computed as:

$$q(t) = Vf(t) = \mathrm{CA}\frac{t^{c-1}e^{-t/b}}{b^c\Gamma(c)} \tag{2-2}$$

in which $q(t)$ is the discharge rate (cfs) at time t (hr), V is the unit runoff volume in cfs-hr, C is a constant unit converter, and A is the drainage area. C is equal to 1.008

when the drainage area is in acres and the unit effective rainfall is 1-inch. When $q(t)$ reaches the peak (i.e., when t is equal to tp_{UH}) , the relation between $q(tp_{UH})$, tp_{UH}, b, and c can be expressed as:

$$tp_{UH} = b(c-1) \tag{2-3}$$

$$q(tp_{UH}) = \frac{CA(c-1)^{c-1}e^{1-c}}{b\Gamma(c)} \tag{2-4}$$

Nash (1959) and Dooge (1959) derived unit hydrographs that had the same formula as Edson (1950) by assuming the watershed is acting as a series of linear reservoirs with equal storage and delay times when responding to effective rainfall. Aron and White (1982) also used the gamma pdf to represent a UH and noted that the time to peak of a UH, tp_{UH}, could be computed using the time of concentration, t_c. If the $q(tp_{UH})$ and the tp_{UH} are known, the gamma parameters b and c can be derived from equations (2-3) and (2-4); therefore, the form a gamma UH can be fully characterized.

The two-parameter gamma unit hydrograph has drawn the attention of researchers and has been widely studied since 1950. Studies have focused on (1) ways to accurately derive estimates of the two parameters, b and c, from the observed rainfall and runoff data; (2) correlations of the two parameters with watershed characteristics; and (3) ways to develop regionalized or synthetic unit hydrographs for ungauged areas that have similar physical characteristics with the studied area.

In terms of the estimation of the two parameters of the gamma unit hydrograph, Nash (1957) used the method of moments. He noticed that this method resulted in more error near the peak of the unit hydrograph and suggested that more accurate methods are needed. Gary (1961) used the method of maximum likelihood to estimate the two parameters. Singh (1976) estimated the two parameters using the linear programing and least squares methods. The estimating methods provided by Gary and Singh may give very rough approximations since they are only based on the minimization of error in the peak flow. Boufadel (1998) derived the two parameters by minimizing the errors of all ordinates based on nonlinear constrained optimization.

In terms of regionalization, Gary (1960) developed synthetic unit hydrographs for 42 selected watersheds by using the two-parameter Nash model and investigated the relationships between the two parameters and watershed hydrologic and topographic characteristics. Wu (1963) used the two-parameter Nash model to derive synthetic unit hydrographs in the state of Indiana. He correlated the two parameters with the watershed characteristics (i.e., area, length of main stream, slope of main stream, shape factor, and valley shape coefficient) based on geomorphological studies and regression analyses, and developed several regression equations that were applicable in the state of Indiana. Cruise (1980) developed regression equations for the two parameters for urbanized watersheds. He found that, the extent of imperviousness was more influential to the calibrated shape parameter c than to the scale parameter b.

2.2.4. NRCS Dimensionless Unit Hydrograph

The Natural Resources Conservation Service (NRCS, 1972), formerly known as the Soil Conservation Service (SCS), published a dimensionless unit hydrograph (DUH) based on analyses of a large number of unit hydrographs from watersheds varying in size and location. The NRCS DUH procedure is incorporated in NRCS hydrologic model programs, TR-20 and TR-55, which are now widely used to derive synthetic unit hydrograph for ungauged watersheds. The basic concept of the NRCS DUH was initially developed by Mockus (1957). The unit hydrographs were averaged and then made dimensionless by dividing all discharge rates by the peak discharge and time ordinates by the time to peak. The curvilinear DUH (see Figure 2-1) had a time base that is 5 times the time to peak, t_p, and approximately 3/8 (37.5%) of the total runoff volume occurred before the time of peak discharge q_p. The curvilinear DUH can be approximately simplified by an equivalent triangular unit hydrograph, in which the time base is 8/3 of the time to peak and the runoff volume under the rising limb is assumed exactly 3/8 (37.5%) of the total runoff volume.

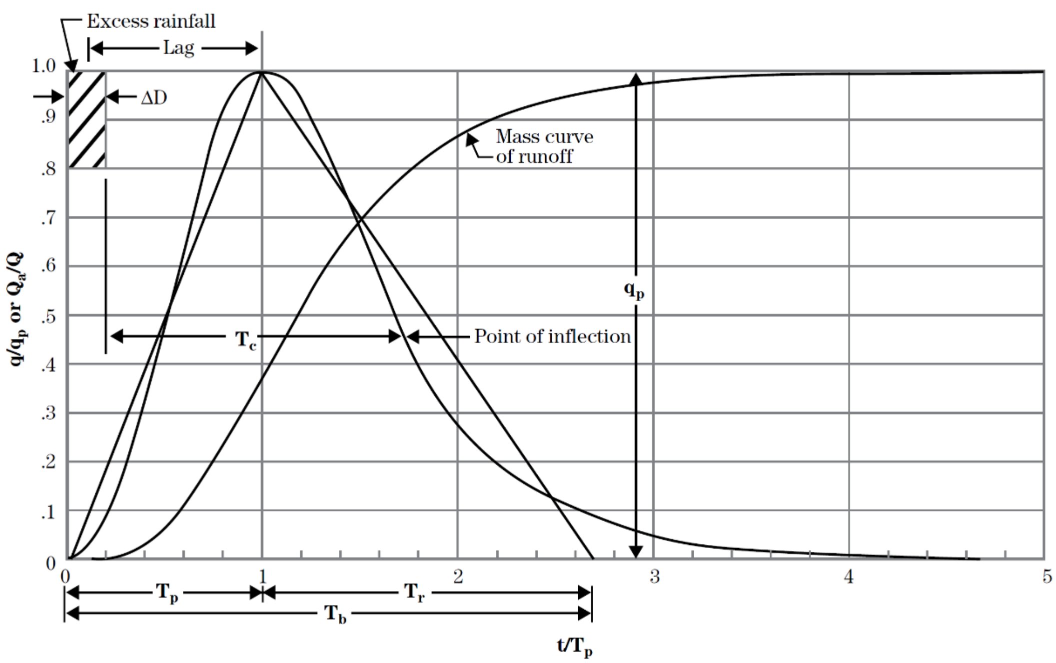

Figure 2-1. Typical diagrams of NRCS dimensionless curvilinear and triangular unit hydrographs [Reproduced from NRCS National Engineering Book Chapter 16].

For the equivalent triangular unit hydrograph, the total volume of direct runoff Q is 1 inch and the relation between Q, tb_{UH}, tp_{UH}, and q_p can be expressed by the following equations:

$$Q = \frac{1}{2} q_p \, tb_{UH} = \frac{1}{2} q_p (\, tp_{UH} + t_r) \tag{2-5}$$

$$q_p = \frac{2Q}{tp_{UH} + t_r} \tag{2-6}$$

in which tb_{UH} is the time base in hr; t_r is the recession time in hr, with t_r being approximately 1.67 times tp_{UH}. In order to have the q_p in cfs, the drainage area A (mi^2) is included in this equation, with the equation (2-6) expressed as:

$$q_p = \frac{645.33 \times 2 \times A \times Q}{(1+1.67) \times tp_{UH}} = \frac{645.33 \times k \times AQ}{tp_{UH}} = \frac{484AQ}{tp_{UH}} = \frac{PRF \times AQ}{tp_{UH}} \tag{2-7}$$

$$PRF = 645.33 \, k = \frac{q_p tp_{UH}}{AQ} \tag{2-8}$$

in which the constant 645.33 is a function of the unit conversion factor; k is the shape parameter of a unit hydrograph; and PRF is the peak rate factor. The constant 484 is known as the standard PRF and depends on 37.5% of the volume being under the rising limb. Other relations between the unit duration D, the time of concentration, t_c, and tp_{UH}, can be expressed through the graphic analysis:

$$1.7 \, tp_{UH} = t_c + D \tag{2-9}$$

$$tp_{UH} = \frac{D}{2} + 0.6 t_c \tag{2-10}$$

Solving D and t_p yields

$$tp_{UH} = \frac{2}{3} t_c \tag{2-11}$$

$$D = 0.133 t_c = 0.2 \, tp_{UH} \tag{2-12}$$

When using the NRCS DUH, the t_c is often computed using the watershed characteristics. The tp_{UH} can then be computed with equation (2-11). The q_p can then be determined based on equation (2-7). Once the tp_{UH} and the q_p are determined, the NRCS DUH can be dimensionalized to a synthetic unit hydrograph based on the Figure 2-1.

The peak rate factor PRF is the most prominent feature of the NRCS DUH. From equation (2-7), the peak discharge rate is linearly related to the PRF. The constant PRF of 484 results from the assumption that $t_r = 1.67tp_{UH}$ and the runoff volume under the rising limb is 3/8 of the total runoff volume. McCuen and Bondelid (1983) reported that the constant PRF of 484 is not sufficient to demonstrate variation in watershed characteristics and, therefore, the PRF should be a function of both rainfall and watershed characteristics. The most recent NRCS engineering manual (2007) suggested that the PRF should vary from about 700 in steep terrain to 100 or less in flat, swampy areas. However, specific guidelines for adjusting the DUH with was not provided.

Many studies, after the publication NRCS DUH procedure, have been conducted with a focus on determining the accurate PRFs and the forms of NRCS DUH for different watersheds with specific characteristics based on regional analyses of measured rainfall and runoff data. Welle et al. (1980) developed an alternative DUH for the coastal plain watersheds based on data from four gauged watersheds. This alternative DUH known as the Delmarva DUH was later adopted by NRCS to be the regionalized DUH for the coastal regions of Delaware, Maryland, and Virginia, in which the peak factor is 284 and the runoff volume prior to the peak discharge is 22%

of the total runoff volume. McCuen and Bondelid (1983), based on analysis of measured data of six watersheds, reported that the standard value of 484 is too large for coastal watersheds on the Delmarva peninsula, and pointed out the Delmarva DUH with a PRF of 284 gave a more rational shape than the standard NRCS DUH, but was not applicable for all watersheds in such areas. Meadows and Chestnut (1983) reported that using a standard NRCS DUH would significantly overestimate the peak discharge for coastal areas. Capece et al. (1986) found that the appropriate PRFs ranged from 75 to 100 for Florida flatwoods watersheds. His results indicated that PRFs for rainfall events less than 0.50 inches were 20-30% higher than PRFs for larger events. Studies conducted by Sheridan et al. (1993, 2002) also indicated significant variation of the PRF for coastal and flatwoods watersheds in Georgia, Texas, and Florida, and peak discharges would be overestimated by using standard value of 484 in such areas. Fang et al. (2005) reported an average PRF of 370 for central Texas watersheds with a standard deviation of 76.

The PRF should vary on a case-by-case basis. The shape of the NRCS unit hydrograph would need to be change to vary with a changing PRF. Studies have found that the NRCS unit hydrograph, with a varying PRF, can be transferred to a two-parameter gamma unit hydrograph. The gamma UH parameters, c and b, can be defined by the PRF and tp_{UH}, respectively, by using the conversion equations (McCuen 1998). Once the PRF and the tp_{UH} are given, the parameters, c and b, can be computed using equations (2-13) and (2-14). The NRCS unit hydrograph can then be developed based on equation (2-2).

2.2.5. Other Unit Hydrographs

Unit hydrographs based on other probability distribution functions (pdfs) have also drawn attention. The three-parameter beta unit hydrograph is a notable example. It has the advantage of being able to produce all possible shapes of a UH due to its considerable flexibility, but such considerable flexibility may cause irrationality of the calibrated UH parameters and the loss of an additional degree of freedom. Bhunya et al. (2007) investigated the potential of gamma, chi-square, Weibull, and beta distributions as synthetic unit hydrographs. They found that beta and Weibull pdfs are more flexible than the gamma and chi-square pdfs in reproducing the shape of a UH because the skew can vary on both tails of the pdf curve.

In summary, despite the availability of a number of proposed unit hydrographs, the NRCS unit hydrograph is still the most widely used unit hydrograph in engineering design. The NRCS unit hydrograph can be essentially represented by a two-parameter gamma UH. Therefore, in this research, the two-parameter gamma UH was studied in depth, in terms of accurate calibration of the UH parameters (i.e., either the parameters b and c, or the PRF and tp_{UH}) and interpretation of factors that influence calibration accuracy and uncertainty.

CHAPTER 3: DEVELOPMENT OF GAMMA UNIT HYDROGRAPH CALIBRATION MODEL

3.1. INTRODUCTION

To extract the best-fit gamma unit hydrographs from actual storm data, the first task is to develop a unit hydrograph analysis model that analyzes rainfall-runoff data and calibrates the watershed unit hydrograph. With given rainfall-runoff data, the model output will be the calibrated unit hydrograph that produces the best goodness of fit. The method described in this chapter will be used to analyze measured rainfall hyetograph and runoff hydrograph data for the purpose of identifying the unit hydrograph that results in the best representation of the measured runoff hydrograph. The primary steps of the analysis are: (1) separate baseflow from direct runoff; (2) remove the initial abstraction; (3) separate the remaining rainfall into losses and rainfall excess such that the volume of rainfall excess is equal to the volume of the direct runoff; (4) assume a set of unit hydrograph parameters and convolve the rainfall excess and the unit hydrograph to obtain a computed runoff hydrograph; (5) compute the goodness-of-fit statistics between the computed and measured hydrographs; (6) repeat step 4 and 5 for various combinations of the UH parameters until the best set is found. All of these steps are discussed in this chapter.

3.2. RAINFALL AND RUNOFF DATA

Measured rainfall and runoff data in this study were obtained from the Hydrologic Data for Experimental Agricultural Watersheds in the United States (provided by U.S. Department of Agriculture) which presents a number of storm events with associated rainfall and runoff data for different watersheds. The location, area, slope, land use, and antecedent moisture for each of these watersheds is provided. The watershed morphologies and locations of rainfall and runoff gages for some of these watersheds are also presented. For the measured storm data, the rainfall intensity is recorded in inches per hour; the rainfall depth is then computed in inches; the runoff rate is recorded in cubic feet per second or inches per hour; and the computed runoff depth is in inches. For the purpose of this study, the selected rainfall and runoff data will be converted to inches per minute.

3.3. DEVELOPMENT OF DIRECT RUNOFF HYDROGRAPH

A measured runoff hydrograph can be modeled as two parts, the direct runoff that results from rainfall excess and the baseflow that is discharged from ground water. In studies of the rainfall-runoff process and hydrograph analysis, the direct runoff hydrograph is computed by separating the baseflow from total runoff hydrograph based on a model of the baseflow separation. The shape of the measured direct runoff hydrograph depends on the model that is used to separate the total runoff hydrograph.

A number of techniques have been proposed for separating direct runoff and baseflow. These techniques reveal different conceptualizations of the runoff process. Selection of the separation technique depends on the type and amount of measured data available, the desired accuracy of hydrologic designs, and the effort that a researcher wants to expand (McCuen 1998). The commonly used separating methods are constant-discharge baseflow separation, constant-slope baseflow separation, and concave baseflow separation (see Figure 3-1). For the purpose of this study, the constant-slope baseflow separating method was used.

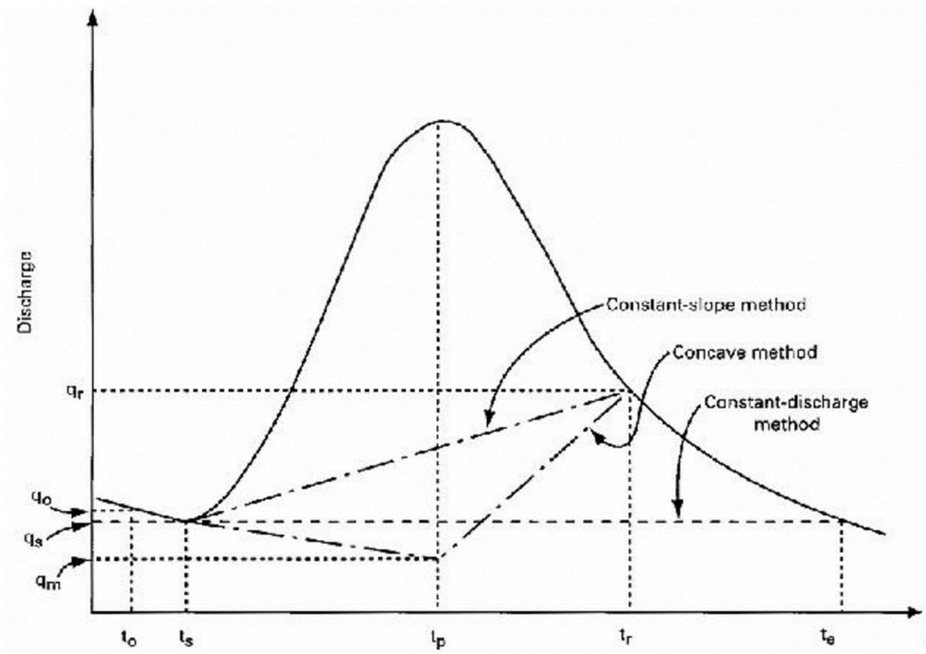

Figure 3-1. Typical methods for separating baseflow from total runoff (McCuen 1998).

For the constant-slope baseflow separation method, direct runoff is assumed to begin at the time the minimum discharge occurs and end at the time at which an inflection point on recession limb of the total runoff hydrograph is observed. The starting time of rainfall excess is assumed to be the same as the ending time of rainfall initial abstraction (which will be discussed in section 3.3). The inflection point is usually the point where a total runoff hydrograph changes from a concave curve to a convex curve (i.e., the slope changes from being greater than 1 to less than 1). For many actual storm data, however, an inflection point is not evident and thus is difficult to identify; therefore, it is often selected subjectively on the recession limb by researchers. After the starting and ending times of rainfall excess are determined, baseflow and direct runoff can be separated by connecting a straight line from the starting point of direct runoff to the infection point on the recession limb. The part above the straight line will be the direct runoff while the part below the straight line will be the baseflow (see Figure 3-1). The computation of baseflow can be expressed as:

$$q_b = \begin{cases} q_{total} & \text{for } t < t_s \\ q_s + (t - t_s) * \frac{C - q_s}{t_r - t_s} & \text{for } t_s \leq \ t \leq t_r \\ q_{total} & \text{for } t > t_r \end{cases} \tag{3-1}$$

where t_s and t_i are the starting time and the time of the inflection point, respectively; q_b, q_s, q_r, and q_{total} represent the baseflow, discharge at the starting time (t_s) of direct runoff, discharge at the time (t_i) of inflection point, and the discharge of total runoff hydrograph, respectively.

3.4. DEVELOPMENT OF RAINFALL EXCESS

Once the direct runoff is separated from the total runoff, the rainfall excess can then be separated from the total rainfall. In the analysis procedure, a measured rainfall hyetograph can be viewed as consisting of three parts: initial abstraction, losses, and rainfall excess. The initial abstraction refers to that part of rainfall that is used to initially fill the surface depressions and saturate the surface layer prior the start of direct runoff. The rainfall losses refer to that part rainfall that fills the surface depressions and infiltrates into the ground after the start of direct runoff and it reflects the watershed natural storage and capability of retaining water. The rainfall excess is that part of rainfall that causes direct runoff and, therefore, it has the same volume with direct runoff. In order to obtain the rainfall excess, the initial abstraction and losses need to be separated from the total rainfall hyetograph.

Although many empirical methods have been developed for estimating the initial abstraction, there is no accurate and widely accepted method for identifying the initial abstraction. The initial abstraction is usually subjectively estimated by researchers based on personal experience and by observing rainfall hyetograph and runoff hydrograph. It is usually that small part of rainfall that does not cause a significant increase in the runoff hydrograph. An example of separation of initial abstraction is shown in Figure 3-2.

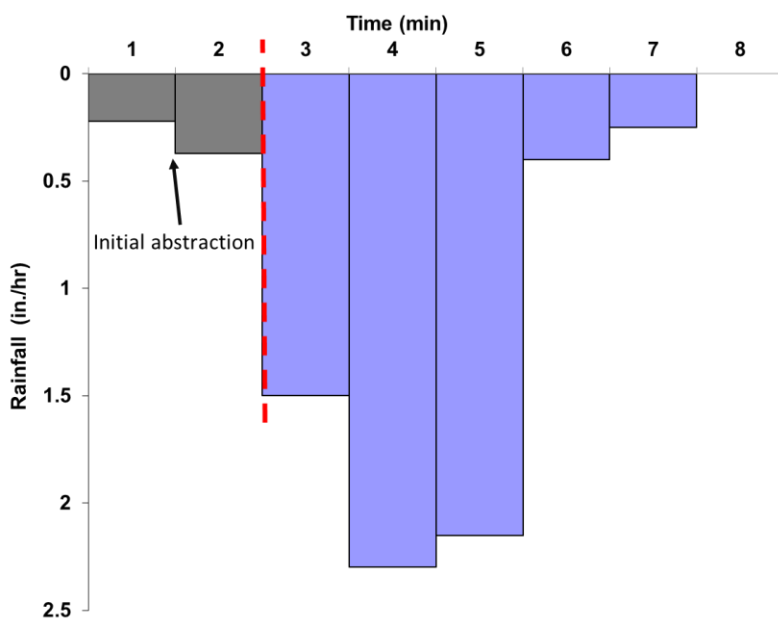

Figure 3-2.   An example of initial abstraction.

Once initial abstraction is separated from total rainfall, the remaining part of rainfall can then be separated into rainfall losses and rainfall excess. The rainfall excess has the same volume as the direct runoff. The volume of the rainfall losses is simply the difference between the total rainfall and the sum of initial abstraction and rainfall excess.

In this study, the phi-index method was used to separate rainfall losses and rainfall excess (see Figure 3-3). The phi-index ($\emptyset$) method is based on the assumption that rainfall loss occurs at a constant rate such that the rainfall excess has the same volume as the direct runoff. The steps for conducting phi-index separation are shown as follows:

(1) Compute the volumes of total rainfall (V_p) and direct runoff (V_d).

(2) Make an initial estimation of the phi-index:

$$\emptyset = \frac{V_p - V_d}{Tb} \tag{3-2}$$

where V_p is the volume of total rainfall; V_d is the volume of direct runoff; Tb is the time duration from the time at which rainfall excess occurs (or initial abstraction ends) to the ending time of the rainfall event; $\emptyset$ is the average rate of rainfall losses.

(3) Compute the rate of rainfall losses for each ordinate on the rainfall hyetograph by using the flowing conditional function,

$$L(t) = \begin{cases} \emptyset \ \text{ if } \emptyset \leq P(t) \\ P(t) \ \text{ if } \emptyset > P(t) \end{cases} \tag{3-3}$$

where L(t) is rate of rainfall losses at time t ; $P(t)$ is the rainfall intensity at time t.

(4) Compute the total volume of rainfall losses (V_{loss}):

$$V_{loss} = \sum [L(t) * \Delta t] \tag{3-4}$$

(5) Compute the intensity ($P_e(t)$) of rainfall excess for each ordinate on the rainfall hyetograph excluding the part of initial abstraction:

$$\mathrm{Pe(t)} = \mathrm{P(t)} - \mathrm{L(t)} \tag{3-5}$$

(6) Compare V_{loss} and $V_p - V_d$ to check whether the previous estimate of $\emptyset$ needs to be adjusted:

a. If $V_{loss} = V_p - V_d$, the previous estimate of $\emptyset$ is correct and the rate of rainfall losses is exactly same with initial $\emptyset$; and go to Step 8.

b. If $V_{loss} < V_p - V_d$, the previous estimate of $\emptyset$ needs to be adjusted by computing the phi-index correction, $\Delta\emptyset$:

$$\Delta\emptyset = \frac{V_p - V_d - V_{loss}}{Tb_1} \tag{3-6}$$

where Tb_1 represent the time duration when $P(t)$ is greater than $\emptyset$.

(7) Adjust the phi-index: $\emptyset_{adjusted} = \emptyset_{previous} + \Delta\emptyset$; and return to Step 3.

(8) Separation is finished and use the latest value of $\emptyset$ to represent rainfall losses.

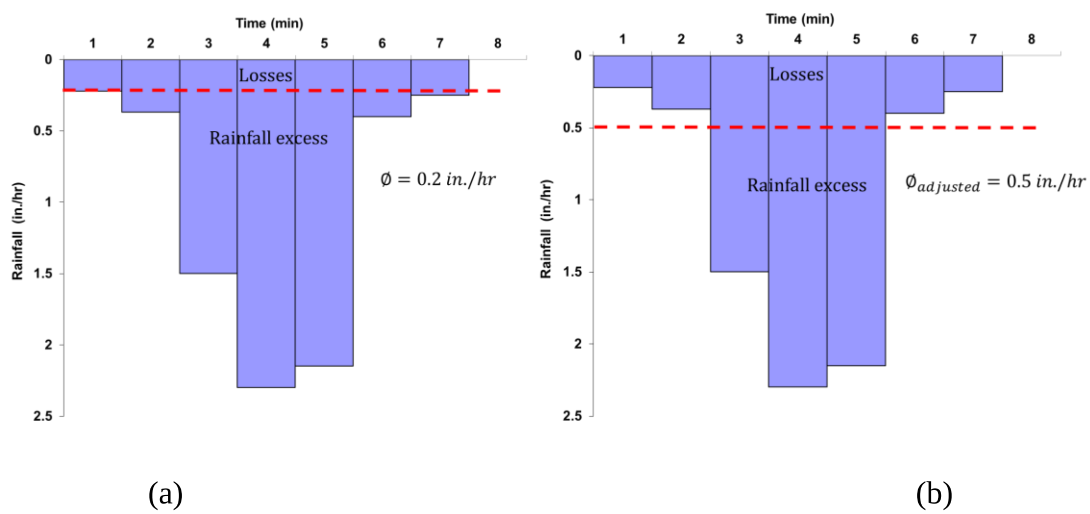

Figure 3-3. Phi-index separation method: (a) initial estimate of $\emptyset$ is correct; (b) initial estimate of $\emptyset$ needs to be adjusted.

3.5. GENERATION OF GAMMA DISTRIBUTION UNIT HYDROGRAPH

In hydrograph analysis, a unit hydrograph can be viewed as a transfer function or a smoothing function that transfers input rainfall excess to output direct runoff. t reflects the effects of watershed characteristics (e.g., area, slope, land use, natural storage) and watershed conditions (e.g., antecedent moisture) on rainfall excess. During a rainfall event, the unit hydrograph is usually considered as being temporally constant and always has a volume of one inch. A number of unit hydrographs have been proposed based on different assumptions and statistic distributions. For the purpose of this study, the two-parameter gamma distribution is used to represent the unit hydrograph (gamma UH) and is used to transfer rainfall excess into computed direct runoff because it is the mostly used unit hydrograph in current hydrologic designs.

The form of a gamma UH is controlled by a shape parameter, c, and a scale parameter, b. As discussed in Chapter 2, the parameters c and b can be defined by the peak rate factor (PRF) and the time to peak ($\mathrm{tp_{UH}}$) of the UH, respectively, by using the conversion equations (McCuen 1998):

$$\mathrm{c} = 1.006 + 1.104 * 10^{-3} * \mathrm{PRF} + 1.267 * 10^{-5} * \mathrm{PRF}^2 + 1.646 * 10^{-9} * \mathrm{PRF}^3 \tag{3-7}$$

$$\mathrm{b} = \mathrm{tp_{UH}}/(\mathrm{c} - 1) \tag{3-8}$$

Once a PRF and a $\mathrm{tp_{UH}}$ are given, the parameters c and b can be computed by using equations (3-7) and (3-8); and each ordinate of the gamma UH can then be computed by:

$$f(t) = \frac{t^{b-1} e^{-t/b}}{b^c \Gamma(c)} \tag{3-9}$$

where $f(t)$ is the ordinate of the unit hydrograph at time t; $\Gamma(c)$ is the value of gamma function for shape parameter c. Since the gamma distribution is a bounded distribution ($t \in (0, +\infty)$), the right side of the gamma UH has to be truncated to produce a rational number of ordinates of the gamma UH. McCuen (2016), based on the assumption that 99.9 % of the area under the gamma UH is retained, proposed an equation to compute the number of ordinates of a UH, n_{UH}:

$$n_{UH} = 6434.7/PRF^{1.191} * tp_{UH} \tag{3-10}$$

in which n_{UH} rounds a number to the next smaller integer. In terms of the gamma parameter, equation (3-10) can be expressed as:

$$n_{UH} = 6.146 e^{0.01536c} c^{0.5} b \tag{3-11}$$

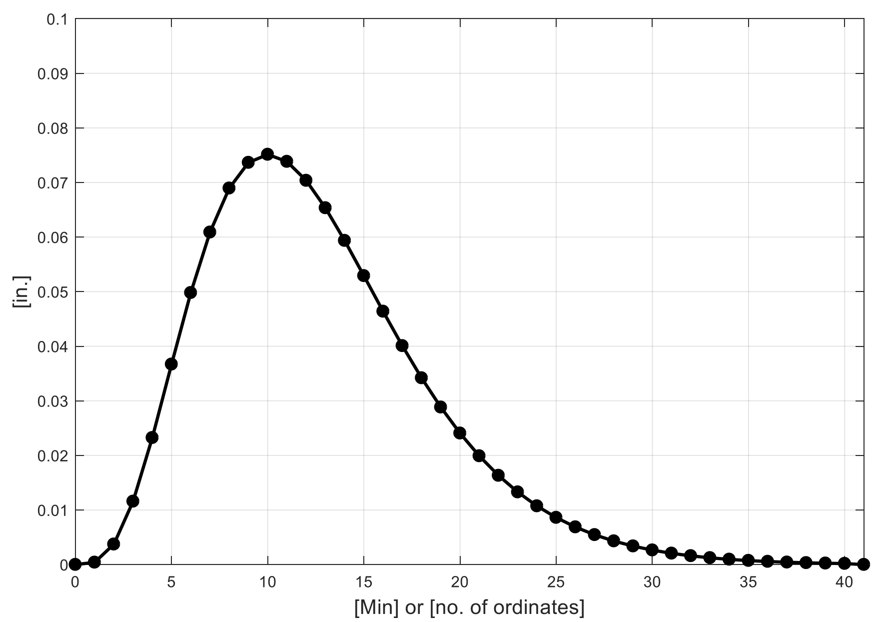

Figure 3-4. A gamma UH with a PRF of 484 and a tp_{UH} of 10 minutes.

For example, if the PRF is 484 and the tp_{UH} is 10 minutes, respectively, n_U will equal to 40.82 (use 41). The parameters c and b will equal 3.6950 and 2.7064, respectively. The values of the ordinates and the shape of the gamma UH are shown in Figure 3-4. Other characteristics of the gamma UH are the inflection point on the rising and falling limbs:

$$t_i = b(c-1) \pm b\sqrt{c-1} \qquad (3\text{-}12)$$

and the half-width length, which is the time between the points on the rising and falling limb at a discharge of 50% of the peak discharge:

$$W_h = 1.59e^{-0.023c}c^{0.75}b \qquad (3\text{-}13)$$

3.6. CONVOLUTION TO DEVELOP COMPUTED DIRECT RUNOFF HYDROGRAPH

The process by which a rainfall excess hyetograph is converted into a computed direct runoff via unit hydrograph is called convolution. It is also referred as linear superposition and is a process of multiplication, translation with time, and addition that can be expressed as:

$$q(t) = \sum_{i=1}^{t} Pe(i) \times UH(t-i+1) \qquad (3\text{-}14)$$

where $q(t)$ is the time distributed computed direct runoff; $Pe(i)$ is the time distributed rainfall excess hyetograph; $UH(t)$ is the time distributed unit hydrograph. The number of ordinates for the time base of the computed direct runoff (n_{CDRO}) is given by:

$$n_{CDRO} = n_{pe} + n_{UH} - 1 \tag{3-15}$$

where n_{pe} and n_{UH} are the time bases of rainfall excess hyetograph and unit hydrograph, respectively. An example of the convolution process is illustrated in Table 3-1.

Table 3-1. An example of convolution process.

Time [min]	*Pe(t)* [in./min]	*UH(t)* [in./min]	Convolution				*q(t)* [in./min]
			*Pe(1)*UH* [in./min]	*Pe(2)*UH* [in./min]	*Pe(3)*UH* [in./min]	*Pe(4)*UH* [in./min]	
0	0	0.000	0.000				0.000
1	1	0.125	0.125	0.00			0.125
2	2	0.250	0.250	0.25	0.00		0.500
3	4	0.500	0.500	0.50	0.50	0.000	1.500
4	3	0.125	0.125	1.00	1.00	0.375	2.500
5	0	0.000	0.000	0.25	2.00	0.750	3.000
6				0.00	0.50	1.500	2.000
7					0.00	0.375	0.375
8						0.000	0.000

3.7. CALIBRATION AND GOODNESS OF FIT

The calibration process aims to extract the gamma UH that produces the best goodness of fit. Once the rainfall excess hyetograph and the observed direct runoff hydrograph are separated from the measured rainfall-runoff data, a number of gamma UHs based on different combinations of peak rate factor (PRF) and time to peak (tp_{UH}) would be convolved with the rainfall excess hyetograph to generate a number of computed direct runoff hydrographs, which are then compared to the observed direct runoff hydrograph. The UH that yields the best goodness of fit is assumed to be the correct UH. Since the gamma UH can be fully characterized by the peak rate factor and time to peak, calibration of best-fit gamma UH is essentially the calibration of UH parameters.

The calibration criteria was based on a combination of different goodness-of-fit statistics which includes the standard error (Se), relative standard error (Se/Sy), the bias, and the relative bias. The standard error or root mean square error is a direct measure of fitting between the computed direct runoff hydrograph and the observed direct runoff hydrograph, with a lower standard error indicating higher prediction accuracy. The relative standard error indicates the relative improvement in prediction accuracy provided by a model compared to using the mean value of the observed data as the model. A relative standard error equal to or greater than 1 suggests that no improvement in prediction accuracy is provided by the model. The bias and relative bias indicate that whether the computed values are overestimating or underestimating the observed values. The bias and relative bias should optimally be close to zero. Calibration was conducted with the goal of minimizing the standard error and relative standard error as well as the bias and relative bias. For the purpose of this study, the relative standard error was used as the major calibration criteria for prediction accuracy.

In summary, the steps for conducting the calibration can be expressed as follows:

(1) Separate rainfall excess and direct runoff hydrograph from measured rainfall-runoff data (see sections 3.3 and 3.4);

(2) Develop gamma UHs based on different tp_{UH}s and PRFs.

(3) Convolve the rainfall excess hyetograph of step (1) with gamma UHs of step (2) to develop computed direct runoff hydrographs;

(4) Compare the ordinates of the computed direct runoff hydrographs in step (3) with the ordinates of the observed direct runoff hydrograph in step (1) to identify the best-fit (minimum Se/Sy) computed direct runoff hydrograph; the gamma UH that results in the best-fit computed direct runoff hydrograph is considered to have the correct UH parameters;

(5) Report the calibrated UH parameters and goodness-of-fit statistics.

CHAPTER 4: EFFECT OF CAUSAL FACTORS ON UNIT HYDROGRAPH CALIBRATION

4.1. INTRODUCTION

In the process of calibration, the UH with the PRF and tp_{UH} that results in the best goodness of fit (i.e., minimum Se/Sy) is considered to be the calibrated UH for the study watershed. However, with the same data base, a number of UHs with different combinations of tp_{UH} and PRF may also produce relatively similar goodness of fit with the calibrated UH, even though the values of tp_{UH} and PRF may be considerably different. Different combinations of UH parameters (i.e., PRF and tp_{UH}) that produce similar goodness of fit can be presented as a response surface that shows an ellipse for the same value of Se/Sy equaling to 0.1. The character of the ellipse is a measure of the calibration uncertainty of the peak rate factor and the time to peak. In this chapter, a value of 0.1 will be considered as an empirically justified objective criteria for measuring uncertainty; however, for other analyses, other values may be more appropriate. The calibration uncertainty of UH parameters affect the ability to find the true optimum, with larger ranges of UH parameters indicating higher uncertainty and that the inherently (or underlying) true UH parameters would be more difficult to determine.

In order to calibrate the UH parameters more accurately and understand problems in interpreting the calibration results, it is necessary to fully understand the effects of various factors on the calibration of UH parameters in terms of calibration accuracy (i.e., the Se/Sy) and calibration uncertainty (i.e., the size of the ellipse).

The objective of this chapter is to investigate the factors that influence the accuracy and uncertainty of calibrated UH parameters. The effects of the temporal watershed characteristics, the time offset of runoff to rainfall, and the rainfall characteristics (i.e., rainfall complexity and rainfall peakedness) are studied based on analyses of synthetic data. The effects of rainfall nonuniformity and dynamic convolution are investigated based on analyses of measured rainfall-runoff data. The flexibility and limitations of the gamma distribution unit hydrograph both to fit and to reflect the hydrologic response of a watershed are discussed. General guidelines are then be presented to assist users to understand the effects of various factors on UH calibration and to assist in the determination of a reasonable UH for a watershed.

4.2. EFFECT OF INHERENT UH PARAMETES ON UH CALIBRATION

Understanding the characteristics of the gamma UH is critical when conducting a calibration study. The shape and scale of a gamma UH depends on its parameters, i.e., shape and scale parameters, while the characteristics of a computed direct runoff hydrograph depends on both the characteristics of the UH and the characteristics of the rainfall excess hyetograph. Thus, before using a calibrated UH, it is necessary to fully understand the effects of both UH parameters. This led to the following three research questions:

(1) How do the UH parameters control the form of a gamma UH?

(2) How do the UH parameters further influence the characteristics of a computed direct runoff hydrograph?

(3) What conditions influence the uncertainty in the accuracy of a computed unit hydrograph？

These three questions will be addressed in this section .

4.2.1. Effect of UH Parameters on the Shape of a Gamma UH

In probability theory and statistics, the gamma distribution is a two-parameter family of distributions that are controlled by a shape parameter and a scale parameter. The gamma distribution parameters can be transformed to UH parameters, i.e., the PRF and the $\mathrm{tp}_{\mathrm{UH}}$. The $\mathrm{tp}_{\mathrm{UH}}$ is often used to reflect the temporal extent of the quantities of the unit hydrograph. The PRF reflects the temporal distribution of volumes under the quantiles of the UH. The objective of this section is to investigate the way that the UH parameters control the form of a gamma UH.

The form of a gamma UH can be basically characterized by a peak value and a half-width of the UH (i.e., width at one-half the peak height) (see Figure 4-1). The peak value and the half-width of a UH can be computed using the equations 3-12 and 3-13, respectively. Both the PRF and the $\mathrm{tp}_{\mathrm{UH}}$ affect the peak value and the half-width of a UH. Theoretically, the peak of a gamma UH increases with either increasing PRF or decreasing $\mathrm{tp}_{\mathrm{UH}}$, while the half-width of a UH increases with either decreasing PRF or increasing $\mathrm{tp}_{\mathrm{UH}}$. Since the peak value and half-width of a UH are controlled by both the PRF and the $\mathrm{tp}_{\mathrm{UH}}$, different combinations of the PRF and $\mathrm{tp}_{\mathrm{UH}}$ could produce similarly shaped UHs. The following equation can be used as a metric that indicates the relative peakedness of a gamma unit hydrograph:

$$R_p = \frac{(c-1)^{c-1}e^{1-c}}{1.59b^2\Gamma(c)^{0.75}e^{-0.023c}} \tag{4-1}$$

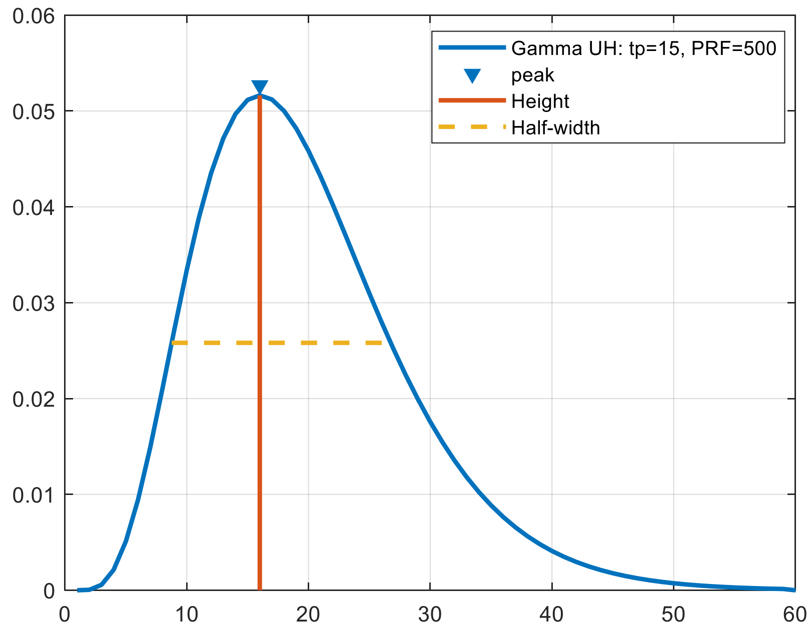

Figure 4-1. A diagram of the peak value (height) and the half-width of a UH.

To illustrate the effects of the PRF and tp_{UH} on the form of the UH, the half-widths and peak values of a number of UHs were computed for different combinations of PRF and tp_{UH} (see Figure 4-2). For example, a gamma UH with a PRF of 200 and a tp_{UH} of 5 has a half-width of 8.5 and a peak discharge of 0.06. In Figure 4-2, the PRF ranges from 100 to 1000, which basically covers the most likely values of the PRF (it can be less than 100 or greater than 1000); the tp_{UH} ranges from 3 to 30 in any unit of time (e.g., 1 min, 2 min, 1 hr) that is consistent with the cell size (or time interval) of the rainfall hyetograph used to illustrate the effect of the half-width. Analyses of Figure 4-2 show the effects of the UH parameters on the form of the gamma UH; these can be summarized as follows.

(1) The PRF and the tp_{UH} are highly intercorrelated in that different combinations of the PRF and tp_{UH} can produce the same peak value or the same half-width

of a gamma UH.

(2) A higher peak and a smaller half-width can be obtained by increasing the PRF and/or decreasing the tp_{UH}.

(3) The peak of a gamma UH changes more rapidly at large PRFs and small tp_{UH}s than at low PRFs and large tp_{UH}s. This occurs because a UH with a large PRF and a small tp_{UH} is more peaked and distributed over a shorter time base.

(4) The half-width of a gamma UH varies more at low PRFs and large tp_{UH}s than at large PRFs and small tp_{UH}s. This occurs because a UH with a low PRF and a large tp_{UH} is very flat and distributed over a long time base.

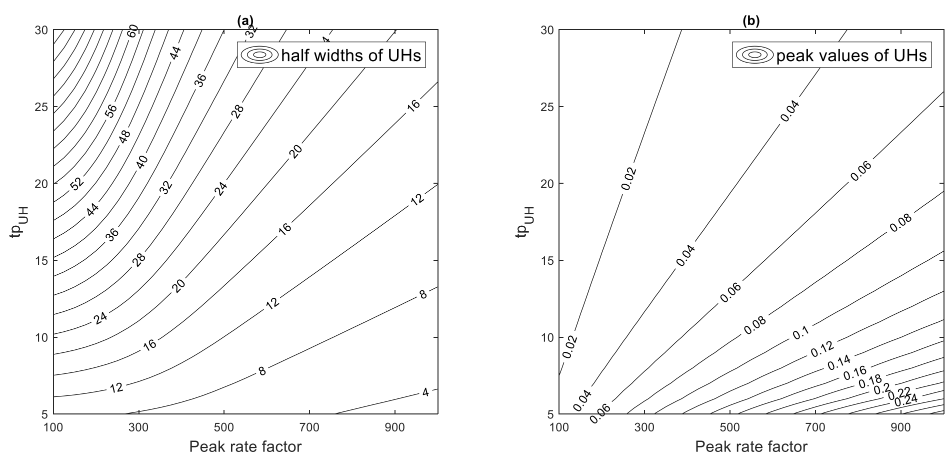

Figure 4-2. The (a) half widths and (b) peak values of gamma UHs generated by using different combinations of tp_{UH} and PRF.

4.2.2. Effect of UH Parameters on the Form of a Computed Direct Runoff

A computed direct runoff hydrograph is generated by convolving a rainfall excess hyetograph with a UH. Thus, the form of the computed direct runoff is influenced by both the rainfall excess hyetograph and the UH parameters. In this section, the effect of PRF and tp_{UH} on the form of computed direct runoff will be discussed. The effect of rainfall excess hyetograph characteristics on the computed direct runoff and on the UH calibration will be discussed in section 4.4.

Nine computed direct runoff hydrographs (see Figure 4-3) were generated by convolving a synthetic one-peaked triangular rainfall excess hyetograph with nine gamma UHs with different combinations of PRF and tp_{UH}. As the PRF increases (see cases 1, 4, and 7) and/or the tp_{UH} decreases (see cases 3, 2, and 1), the gamma UH becomes more peaked and the computed direct runoff hydrograph also becomes more peaked. When the rainfall excess hyetograph is convolved with a peaked UH (i.e., large PRF and/or short tp_{UH}) that has a relatively short time duration (compared with the rainfall excess hyetograph), the computed direct runoff hydrograph can basically reproduce the form of the rainfall excess hyetograph (see Figure 4-3, case 7). When the rainfall excess hyetograph is convolved with a flat UH (i.e., low PRF and/or long tp_{UH}) that has a relatively long time duration, the form of the computed direct runoff is flat and the characteristics of rainfall excess hyetograph (e.g., peak, width) are basically smoothed out (see Figure 4-3, case 3).

In summary, the effect of UH parameters on the form of a computed direct runoff is similar to the effects on the form of a gamma UH as discussed in section 4.2.1. The form of the computed direct runoff is dominated by the flatness of the UH compared with that of rainfall.

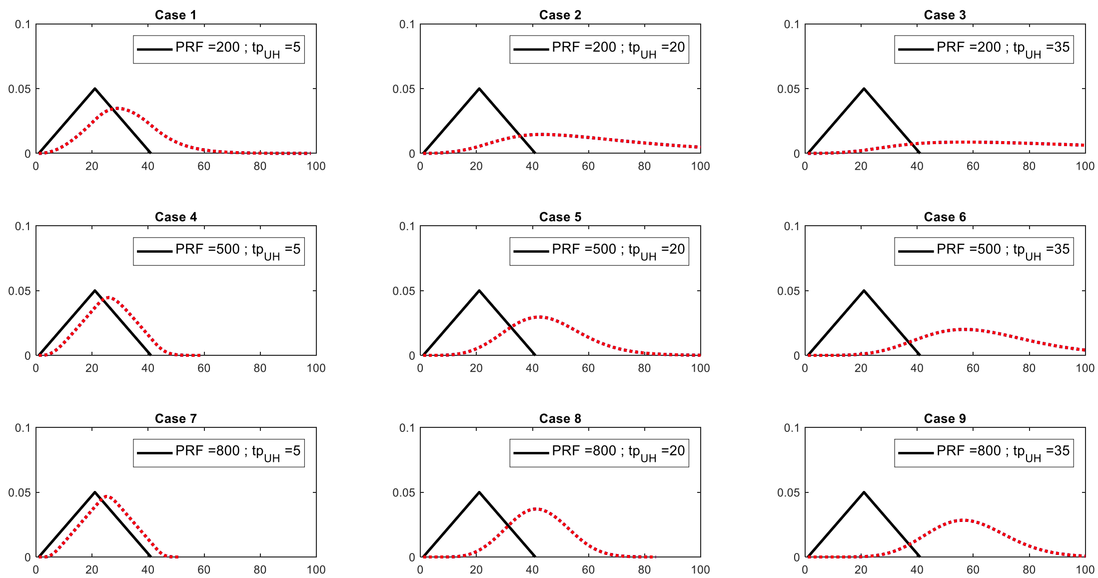

Figure 4-3. Synthetic one-peaked triangular rainfall excess (solid line) and computed direct runoff hydrographs (dashed line) based on different combinations of PRF and tp_{UH}.

4.2.3. Effect of Temporal Watershed Characteristics on Uncertainty of Calibrated UH Parameters

A unit hydrograph is intended to reflect the temporal effects of watershed characteristics and watershed conditions on the draining of rainfall excess, and it can be characterized by a UH model and UH parameters. The inherently true UH parameters are imbedded within the actual rainfall-runoff data. The purpose of calibration is to extract the best estimates of the inherent UH parameters from the measured data. The inherent UH parameters should reflect the temporal watershed characteristics. A large PRF and/or a small tp_{UH} are often indicative of a steep watershed with little storage capacity, while a low PRF and/or a large tp_{UH} are indicative of a watershed with a shallow slope or large storage capacity. Before analyzing actual rainfall-runoff data, the accuracy and uncertainty of calibrated UH parameters under different temporal watershed characteristics (different inherent UH parameters) should be assessed.

To assess the accuracy and uncertainty of the calibrated UH parameters, the true UH parameters need to be known. Since the true UH parameters are imbedded in the measured data and not obtainable, it is necessary to conduct such assessments using synthetic data. In section 4.2.1, Figure 4-3 showed computed direct runoff hydrographs that were generated by convolving a one-peaked triangular rainfall hyetograph with nine UHs based on different combinations of PRFs and tp_{UH}s. Each case in Figure 4-3 should be viewed as measured rainfall-runoff data under specific temporal watershed conditions. For example, in case 7, the computed direct runoff hydrograph can be viewed as a measured runoff hydrograph of a steep watershed (i.e., an inherent PRF of 800 and an inherent tp_{UH} of 5), while in case 3, the computed direct runoff hydrograph can be viewed as a measured runoff hydrograph of a low sloped watershed (i.e., an inherent PRF of 200 and an inherent tp_{UH} of 35) with significant natural storage. Now with the given measured rainfall and runoff data, a Se/Sy response surface for each case in Figure 4-3 can be developed (see Figure 4-4). According to these Se/Sy response surfaces, the accuracy (i.e., minimum Se/Sy) and uncertainty (i.e., the range of the PRFs and the range of tp_{UH}s within the ellipse of Se/Sy of 0.1) were summarized and presented in Table 4-1 (see column 6 to column 9).

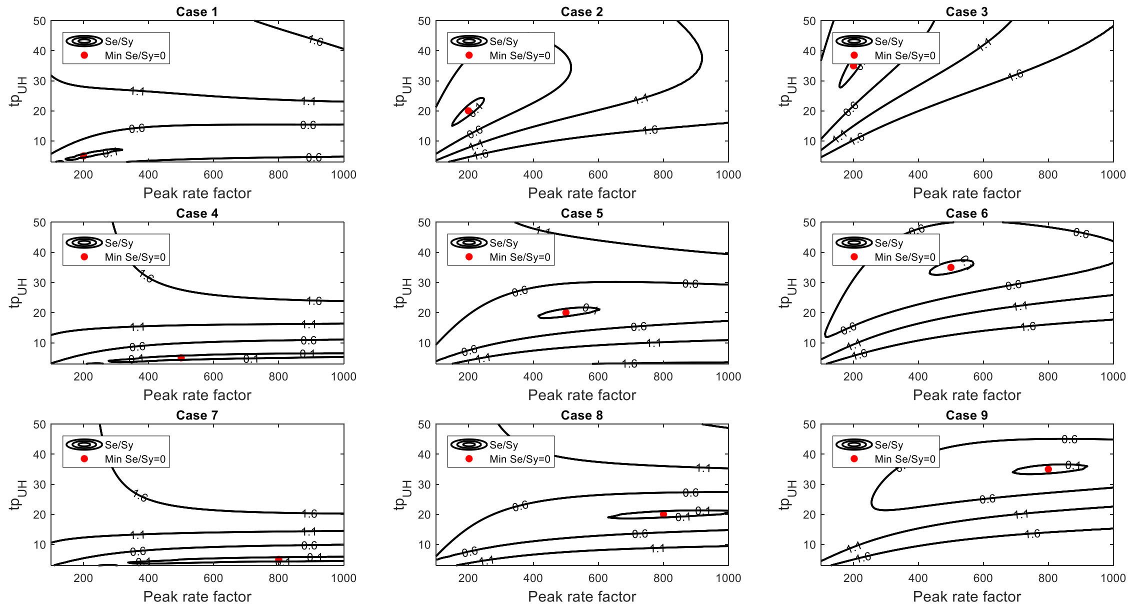

Figure 4-4. The Se/Sy response surfaces for the rainfall and runoff data in Figure 4-3.

Table 4-1. Approximate calibration uncertainty for the rainfall and runoff data in Figure 4-3.

Case No.	Wathershed Condtions		Calibration Uncertainty			
	Inherent PRF	Inherent tp_{UH}	Range of PRFs	Coefficient of Variation of PRF	Range of tp_{UH}s	Coefficient of Variation of tp_{UH}s
	(1)	(2)	(3)	(4)	(5)	(6)
1	200	5	205	1.03	4	0.80
2	200	20	95	0.48	9	0.45
3	200	35	80	0.40	13	0.37
4	500	5	790	1.58	3	0.60
5	500	20	190	0.38	2	0.10
6	500	35	135	0.27	4	0.11
7	800	5	750	0.94	2	0.40
8	800	20	370	0.46	2	0.10
9	800	35	230	0.29	2	0.06
Column 3 = [Max PRF - Min PRF] within in Se/Sy=0.1						
Column 5 = [Max tp_{UH} - Min tp_{UH}] within in Se/Sy=0.1						
Column 4 = range/mean = (column 3 /Column 1)						
Column 6 = range/mean = (column 5 /Column 2)						

In terms of calibration accuracy, the minimum Se/Sy for each case is essentially zero and the calibrated UH parameters are exactly equal to the assumed UH parameters. This occurs because in this experiment, the synthetic measured direct runoff hydrographs were generated using the assumed parameters. In this scenario, the calibration accuracy (i.e., minimum Se/Sy) are identical for the different watershed characteristics (i.e., different inherent UH parameters) and will not be further discussed; instead, the focus is on the parameter uncertainty.

The Se/Sy response surfaces are of value in reflecting the uncertainty of the calibrated UH parameters. It is evident that the calibration uncertainty varies with the inherent UH parameters. For cases 4 and 7, the ellipses of the Se/Sy response surfaces are greatly elongated in the PRF direction (see Figure 4-4), which shows high uncertainty of the calibrated PRF. Specifically, the ranges of PRFs are 790 and 750 (see column 3 in Table 4-1). Conversely, in cases 1, 2, and 3, the ellipses are greatly elongated in the $\mathrm{tp_{UH}}$ direction which shows a high uncertainty of the calibrated $\mathrm{tp_{UH}}$. Cases 5, 6, and 9 seem to produce relative low uncertainty for both the calibrated PRF and $\mathrm{tp_{UH}}$. Assessing the contents of Figure 4-4 and Table 4-1 leads to the following three general guidelines:

(1) As the inherent $\mathrm{tp_{UH}}$ increases, the uncertainty of the calibrated PRF decreases (see column 3 in Table 4-1). This occurs because the ellipse of the Se/Sy response surface become less elongated in the PRF direction (see Figure 4-3). This can be seen by comparing column 3 of Table 4-1 for cases 1, 2, and 3, as the inherent $\mathrm{tp_{UH}}$ increases from 5 to 35, the range of calibrated PRFs decreased from 205 to 80; for cases 4, 5, and 6, the range of calibrated

PRFs decreased significantly from 790 to 135; and for cases 7, 8, and 9, the range of calibrated PRFs also significantly decreased from 750 to 230.

(2) Similarly, as the inherent PRF increases, the uncertainty of the calibrated tp_{UH} decreases (see column 5 in Table 4-1). This occurs because the ellipse of the Se/Sy response surface become less elongated in the tp_{UH} direction (see Figure 4-3). This can be seen by comparing column 5 of Table 4-1 for cases 1, 4, and 7, as the inherent PRF was increased from 200 to 800, the range of the calibrated PRFs decreased from 4 to 2; for cases 2, 5, and 8, the range of calibrated PRFs decreased from 9 to 2.; and for cases 3, 6, and 9, the range of calibrated PRFs decreased from 13 to 2.

(3) Comparing columns 4 and 6 of Table 4-1, it is evident that both changes in the tp_{UH} or in the PRF can lead to significant changes in the Se/Sy. This phenomenon revels that the Se/Sy is very sensitive to both the PRF and the tp_{UH}. This occurs because the tp_{UH} can directly affect the time to peak of the computed direct runoff hydrograph, while the PRF can directly influence the temporal distribution of volumes of the computed direct runoff hydrograph.

In summary, calibration accuracy as measured by Se/Sy is very sensitive to both the PRF and the tp_{UH}. Therefore, when analyzing measured rainfall-runoff data, the PRF and the tp_{UH} should be calibrated simultaneously from the measured hyetographs and hydrographs.

4.3. EFFECT OF RAINFALL CHARACTERISTICS ON UH CHARACTERISTICS

Many studies on measured rainfall-runoff data have shown that the storm-to-storm calibrated UHs for the same watershed can be quite different. This is because each set of rainfall-runoff data indicates a temporal state of watershed characteristics that is not only affected by the inherent watershed characteristics (e.g., slope, form) but also affected by the temporal watershed condition (e.g., antecedent soil moisture) and the rainfall characteristics (i.e., depth or intensity, the number of peaks, and the peakedness of peaks). Even for the same watershed, the antecedent soil moisture at the beginning of each rainfall event will be different. The greater the antecedent soil moisture, the more likely the calibrated tp_{UH} will be short and have a rapid rise. Also, a more intense rainfall is more likely to produce a UH with a shorter tp_{UH} and a larger PRF since the heavy rainfall quickly fills the surface depressions and saturates the ground.

Generally, any factor that affects the flow properties (e.g., flow rate, flow type) and the watershed surface conditions (e.g., moisture, storages, roughness, infiltration rate) can lead to differences in calibrated PRFs and tp_{UH}s. In actual storm events, both the flow properties and surface conditions of the watershed vary significantly spatially and temporally, but unfortunately, such variabilities cannot be measured and used in the calibration of a UH. The watershed response is generally assumed to be constant through the duration of a rainfall event. As mentioned previously, the theory of the constant UH for a watershed itself is only an approximation. However, it is still very important to study the effects of rainfall characteristics on calibrated UHs.

In this section, the effects of rainfall characteristics are investigated with the focus on their effects solely on the UH calibration process without considering the variabilities of antecedent soil moisture and the state of the watershed, i.e., flow properties and surface conditions. These analyses generate the following research questions: (1) How do rainfall characteristics affect the ability to calibrate a UH? (2) At the same level of accuracy (i.e., the same Se/Sy), which rainfall pattern is more likely to produce the UH that has the least uncertainty (i.e., the smallest size of ellipse)? Since the inherently true UH for a watershed is imbedded within measured data and not obtainable, in order to address the above questions, it is necessary to use synthetic rainfall-runoff data where the true values of UH parameters would be known.

Before generating synthetic data, the first step is to determine the rainfall characteristics that needed to be studied. Since linear convolution would be applied, variation in the rainfall depth will not influence the variability of the computed runoff hydrograph. Thus, only the rainfall characteristics that potentially influence the form of the rainfall will be studied. In actual storms, the form of rainfall hyetograph can be quite variable and hard to characterize, but in general, the rainfall characteristics can be basically represented in terms of the peakedness (or flatness) and the number of peaks. The number of peaks of the rainfall hyetograph will reflect the degree of storm complexity, with a greater number of peaks indicating a rainfall hyetograph that is more variable and complex. The peakedness of a rainfall hyetograph reflects the temporal uniformity of the rainfall. If the volumes of rainfall are constant, a rainfall hyetograph that has a higher peak and a shorter time base would be considered as the more peaked rainfall. Thus, in this section, these two major rainfall characteristics (i.e., rainfall complexity and rainfall peakedness) are investigated sequentially with respect to their effects on the calibration of UH parameters.

4.3.1. Effect of Rainfall Complexity on UH Calibration

The objective of this section is to investigate the effect of rainfall complexity on the calibration of UH parameters. In this section, synthetic one-peaked, two-peaked, and three-peaked hyetographs were used to represent the rainfall events with different levels of complexity. The complexity of a rainfall hyetograph can influence the complexity of a computed hydrograph and, therefore, the accuracy of the fit. The steps for generating the synthetic observed direct runoff hydrographs, computed direct runoff hydrographs, and Se/Sy response surfaces are as follows:

(1) Develop one-peaked, two-peaked, and three-peaked rainfall excess hyetographs, Pe(t), each with the same volume of 1 inch and 40 ordinates, which are shown in Figure 4-5.

(2) Develop assumed true gamma UHs based on selected PRFs of 200 (flat, swampy terrain), 500 (moderate terrain), 800 (steep rocky terrain), and selected tp_{UH}s of 5, 20, and 35 ordinates.

(3) Convolve each Pe(t) of step (1) with each gamma UH of step (2) to obtain nine observed runoff hydrographs that are assumed to be the measured direct runoff hydrograph, RO(t) (see Figure 4-6). These would reflect the true, but unknown, watershed responses.

(4) Develop gamma UHs based on different tp_{UH}s (from 3 to 50, with a time interval of 1) and PRFs (from 100 to 1000, with an interval of 5).

(5) Convolve the Pe(t) of step (1) with gamma UHs of step (4) to obtain the computed direct runoff hydrographs, $\widehat{RO}$. These would represent the computed direct runoff hydrographs in the calibration process, while the direct runoff hydrograph of step (3) would reflect the true unknown watershed response.

(6) For each set of rainfall and observed runoff data, compare the RO of step (3) and the $\widehat{RO}$ of step (5) to obtain a Se/Sy response surface and a minimum Se/Sy (see Figure 4-7).

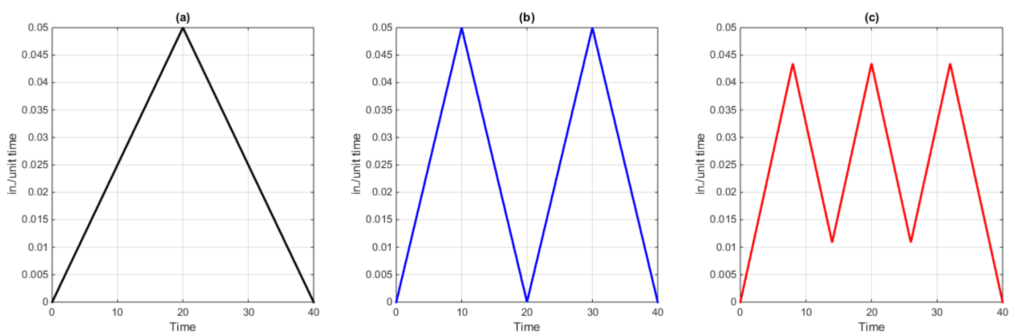

Figure 4-5. Generated one-peaked (a), two-peaked (b), and three peaked (c) rainfall hyetographs.

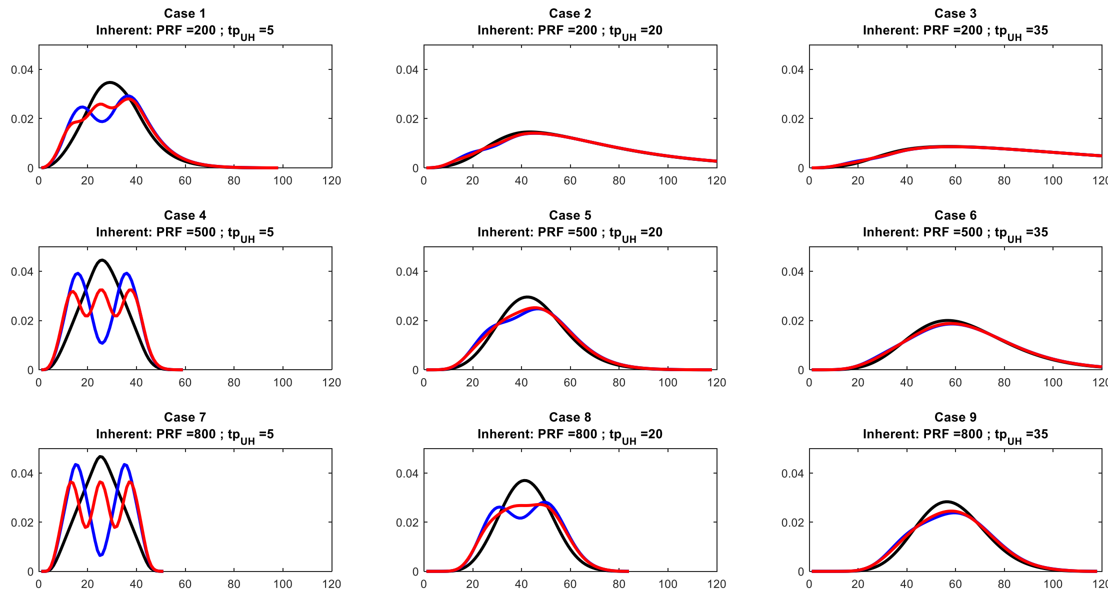

Figure 4-6. Generated observed direct runoff hydrographs based on different inherent PRFs and tp_{UH}s for different rainfall patterns: the black solid lines represent the observed direct runoff for one-peaked rainfall hyetograph; the blue solid lines represent the observed direct runoff for two-peaked rainfall hyetograph; the black solid lines represent the observed direct runoff for three-peaked rainfall hyetograph.

As presented in section 4.2.2, the form of a computed direct runoff hydrograph is controlled by both the rainfall characteristics and the gamma UH. This finding is supported by Figure 4-6, where either the inherent tp_{UH} increases or the inherent PRF decreases, the observed direct runoff hydrograph becomes flatter and the characteristics of the hyetograph become less influential on the direct runoff hydrograph. In terms of the effect of rainfall complexity on UH calibration, the Se/Sy response surfaces (see Figure 4-7) from different rainfall patterns show great dissimilarity in distributions of the isolines, especially when the tp_{UH} is low while the PRF is high (see case 7 in Figure 4-7). This result indicates that rainfall complexity is a factor that can influence UH calibration.

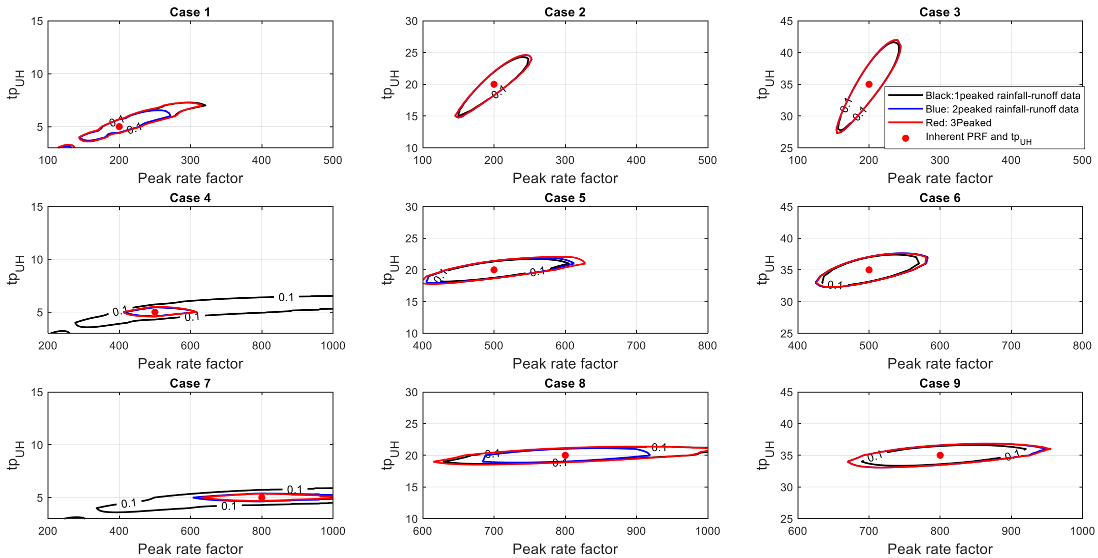

Figure 4-7. Se/Sy response surfaces for each set of rainfall-runoff data shown in Figure 4-6: the black contour lines represents the Se/Sy responses developed from the one-peaked rainfall-runoff data; the blue contour lines represents the Se/Sy responses developed from the two-peaked rainfall-runoff data; the red contour lines represents the Se/Sy responses developed from the three-peaked rainfall-runoff data.

To further investigate such effects, the calibration uncertainty for different types of rainfall data were computed and summarized in Table 4-2. The bold and underlined numbers indicate the minimum ranges of tp_{UH}s and PRFs among the three rainfall patterns, which can be viewed as uncertainty of calibrated UH parameters for a study watershed.

Table 4-2. Uncertainty (the range of PRFs and the range of tp_{UH}s within the ellipse of the minimum Se/Sy +0.1 of the Se/Sy response surface) of calibrated UH parameters for different rainfall patterns.

Case No.	Inherent PRF	Inherent tp_{UH}	Calibration Uncertainty					
			Range of PRFs within Se/Sy=0.1 [Max PRF - Min PRF]			Range of Tps within Se/Sy=0.1 [Max tp_{UH} - Min tp_{UH}]		
			1pk	2pk	3pk	1pk	2pk	3pk
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	200	5	205	**150**	195	4	**3**	4
2	200	20	**95**	100	100	**9**	**9**	**9**
3	200	35	**80**	85	85	**13**	**13**	**13**
4	500	5	790	**195**	200	3	**0**	**0**
5	500	20	**190**	205	230	**2**	3	4
6	500	35	**135**	155	150	**4**	**4**	**4**
7	800	5	750	390	**365**	2	**0**	**0**
8	800	20	370	**230**	380	**2**	**2**	**2**
9	800	35	**230**	275	285	**2**	**2**	**2**
1pk = one-peaked rainfall; 2pk = two-peaked rainfall; 3pk = three-peaked rianfall								

Assessment of the effect of rainfall complexity on the UH calibration is based on the comparison of the calibration uncertainty for different rainfall patterns (see Table 4-2) and the similarity between the rainfall excess hyetograph and the direct runoff hydrograph (i.e., whether the rainfall characteristics are reflected in the runoff hydrographs). Comparing the Figures 4.5, 4.6, and 4.7, it is evident that, when the observed direct runoff hydrograph basically reflects the variation in the rainfall hyetograph (see case 1, 4, 7, and 8 in Figure 4-6), the calibrated UH for this watershed will be more likely to have a small tp_{UH} and/or a large PRF, and the calibrated UH parameters tend to have less uncertainty for the more complex rainfall. This is evident by comparing the ranges of tp_{UH}s (columns 3, 4, and 5 in Table 4-2) and PRFs (columns 6, 7, and 8) for different rainfall patterns. For example, in case 7, where the runoff hydrographs greatly reflect rainfall characteristics, the two-peaked rainfall and three peaked rainfall produce smaller ranges of PRFs (390 and 365, respectively) and tp_{UH}s (0 and 0, respectively) than the one-peaked rainfall, which has a range of PRFs of 750 and a range of tp_{UH}s of 2. However, a greater complexity of a rainfall hyetograph does not always guarantee less uncertainty of calibrated UH parameters. For example, in case 1, the ranges of PRFs (150) and tp_{UH}s (3) of the two-peaked rainfall are even smaller than the ranges of PRFs (195) and tp_{UH}s (4) of the three-peaked rainfall. This occurs because, in this case, the form of the three-peaked direct runoff hydrograph shows less variation (or more uniformity) in the shape than the form of the two-peaked runoff.

When the rainfall characteristics are smoothed out by the UH or not evidently reflected in the direct runoff hydrograph (see cases 2, 3, 5, 6, and 9), the calibrated UH will be more likely to have a large tp_{UH} and/or a low PRF; under such conditions, the UH calibration will be minimally influenced by the rainfall pattern, and the rainfall hyetograph that leads to a direct runoff hydrograph with a greater peakedness will produce a slightly smaller uncertainty of the calibrated parameters. This occurs because a direct runoff hydrograph with a greater peakedness shows more variation (or less uniformity) in the shape, which is more sensitive to changes in UH parameters. For example, in case 5, where the runoff hydrographs from different rainfall patterns show the basically same form but the runoff hydrograph for a one-peaked rainfall shows relatively greater peakedness; the one-peaked rainfall even has slightly smaller ranges of PRFs (190) and tp_{UH}s (2) than the two-peaked rainfall, which produced a range of PRFs of 205 and a range of tp_{UH}s of 3, and the three-peaked rainfall, which produced a range of PRFs of 230 and a range of tp_{UH}s of 4. In case 3, where all of the runoff hydrographs for different rainfall patterns show significant flatness and similarity, the ranges of tp_{UH}s and PRFs for different rainfall patterns are essentially the same.

In summary, the effects of rainfall complexity on UH calibration can be summarized in two general guidelines:

(1) When an observed direct runoff hydrograph greatly reflects the variation of the rainfall hyetograph, the watershed is more likely to have a steep slope and/or little natural storage. This produces a UH with either an inherently large PRF or a short tp_{UH} (compared with rainfall duration), or both. For such watersheds, the calibration uncertainty is less for the more complex rainfall excess hyetograph.

(2) Conversely, when an observed direct runoff hydrograph shows a more uniform form and barely reflects the variation of the rainfall hyetograph, the watershed is more likely to have a shallow slope and/or considerably more natural storage. When analyzing the measured data, these conditions indicate that the UH would have an inherently low PRF and/or a long tp_{UH}. For such watersheds, UH calibration is minimally influenced by the rainfall pattern and the more peaked rainfall hyetograph will produce slightly less calibration uncertainty.

4.3.2. Effect of Rainfall Peakedness on UH Calibration

The objective of this section is to investigate the effect of rainfall peakedness on the variation of the calibrated UH parameters. Synthetic one-peaked triangular rainfall hyetographs with time bases of 10, 20, and 40 ordinates were developed (see

Figure 4-8). Since each hyetograph has a unit volume, the peakedness of the rainfall hyetograph is inversely proportional to the time base.

The steps for generating the synthetic observed direct runoff hydrographs, computed direct runoff hydrographs, and Se/Sy response surfaces were the same as that used in section 4.3.1. The synthetic observed direct runoff hydrographs and the Se/Sy response surfaces are shown in Figures 4.9 and 4.10, respectively.

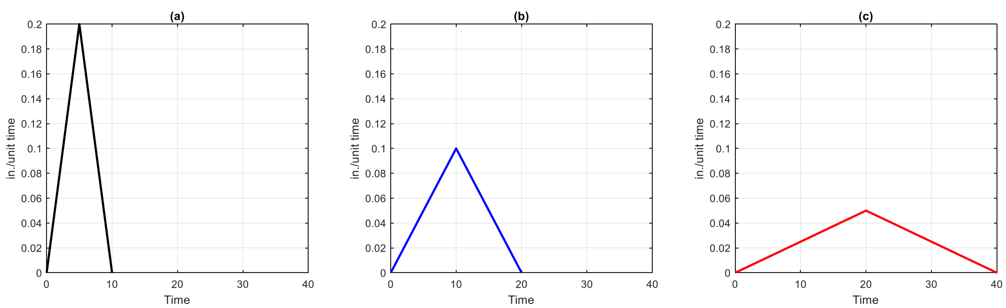

Figure 4-8. Generated one-peaked rainfall hyetographs with time bases of 10, 20, and 40 ordinates.

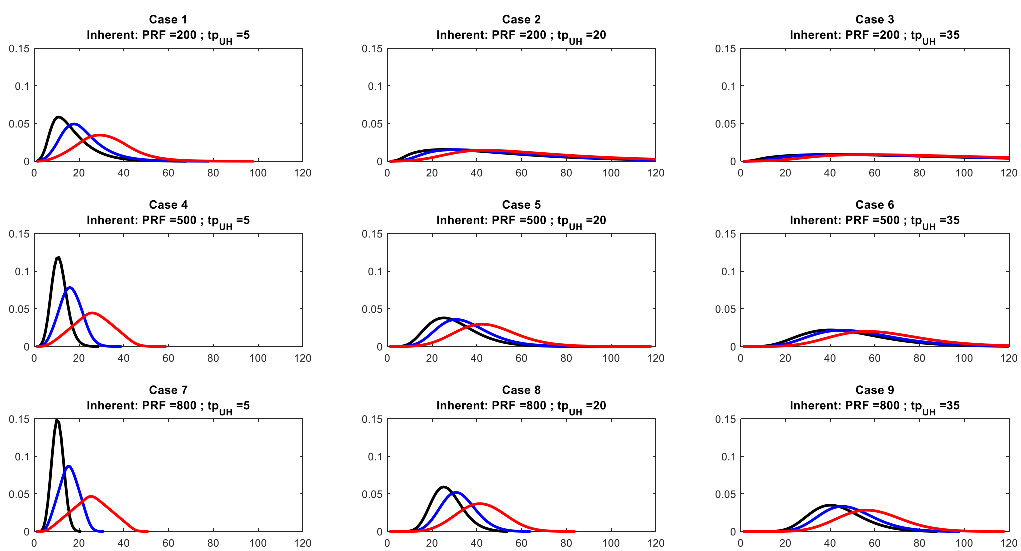

Figure 4-9. Generated observed direct runoff hydrographs based on different inherent PRFs and tp_{UH}s for different rainfall patterns: the black, blue and red solid lines represent the observed direct runoff hydrographs for rainfall hyetographs with time bases of 10, 20, and 40 ordinates, respectively.

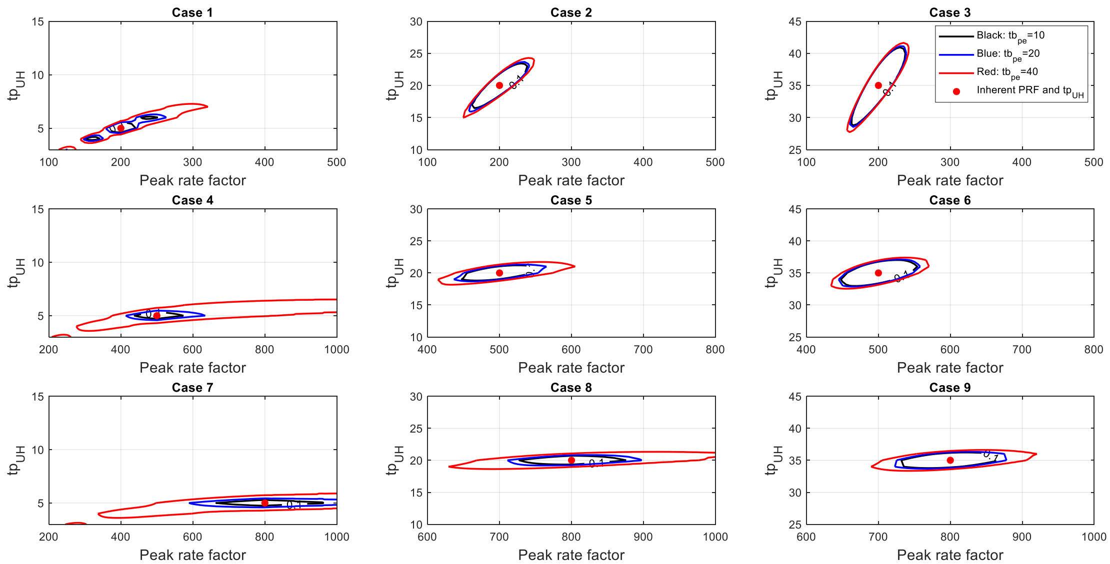

Figure 4-10. Se/Sy response surfaces for each set of rainfall-runoff data shown in Figures 4.8 and 4.9: the black, blue and red solid lines represent the Se/Sy response surfaces developed from the rainfall hyetographs with time bases of 10, 20, and 40 ordinates, respectively.

Assessment of the effect of rainfall peakedness on UH calibration is based on the comparison of the calibration uncertainty for these rainfalls with different peakedness. Results shows that a more peaked rainfall is more likely to produce calibrated UH parameters with greater certainty. In Table 4-3, it is evident that that the ranges of PRFs (see column 3) and tp_{UH}s (see column 6) for the rainfall hyetograph with a time base of 10 ordinates are always equal to or less than those (see columns 4, 5, 7, and 8) for the rainfall hyetographs with time bases of 20 and 40 ordinates. This occurs because a more peaked rainfall is more likely to produce a more-peaked (or less uniform) direct runoff hydrograph, which is more sensitive to the changes in UH parameters. This result is most evident when a watershed has an inherently large PRF and/or an inherently small tp_{UH} (see cases 1, 4, 7, and 8) and least evident when a watershed has an inherently low PRF and/or an inherently large

tp_{UH} (see cases 2, 3, 6, and 9). For example, in case 7, where the watershed has an inherently large PRF and an inherently long tp_{UH}, the rainfall with a time base of 10 ordinates has much smaller ranges of PRFs (295) and tp_{UH}s (0) than the rainfall both with time bases of 20 ordinates (with a range of PRFs of 410 and a range of tp_{UH}s of 0) and 40 ordinates (with a range of PRFs of 750 and a range of tp_{UH}s of 2). While in case 3, where the watershed has an inherently low PRF and a long tp_{UH}, all of the ranges of PRFs and tp_{UH}s for different rainfalls are basically the same. This is because when the peakedness of rainfall is gradually smoothed in the direct runoff hydrographs, calibration accuracy is minimally influenced by the rainfall pattern.

Table 4-3. Uncertainty (the range of PRFs and the range of tp_{UH}s within the ellipse of the minimum Se/Sy +0.1 of the Se/Sy response surface) of calibrated UH parameters for different rainfall patterns.

Case No.	Inherent PRF	Inherent tp_{UH}	Calibration Uncertainty					
			Range of PRF within Se/Sy=0.1 [Max PRF - Min PRF]			Range of Tps within Se/Sy=0.1 [Max tp_{UH} - Min tp_{UH}]		
			Tb_{Pe}=10	Tb_{Pe}=20	Tb_{Pe}=40	Tb_{Pe}=10	Tb_{Pe}=20	Tb_{Pe}=40
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	200	5	**95**	135	205	**2**	3	4
2	200	20	**70**	80	95	**6**	7	9
3	200	35	**70**	75	80	**11**	12	13
4	500	5	**130**	215	790	**0**	**0**	3
5	500	20	**100**	125	190	**2**	**2**	**2**
6	500	35	**100**	110	135	**4**	**4**	**4**
7	800	5	**295**	410	750	**0**	**0**	2
8	800	20	**145**	180	370	**0**	**0**	2
9	800	35	**130**	150	230	**2**	**2**	**2**

Tb_{pe} =10, 20, 40 :Rainfalls with time bases of 10,20, and 40 ordinates

In summary, rainfall peakedness can significantly influence the uncertainty of calibrated UH parameters. More peaked storm events (i.e., greater peakedness and/or shorter time base) will produce calibrated UH parameters with greater certainty.

4.4. EFFECT OF TEMPORAL OFFSET ON UH CALIBRATION

Unit hydrographs analyzed using measured rainfall and runoff data will depend on the extent to which the rainfall hyetograph is in synchronization with the runoff hydrograph. Ideally, the rain gauge will be located at the center of the watershed. Unfortunately, it is common for rain gauges to be located outside of the watershed. The spatial locations of rain gauges can affect the temporal agreement between the hyetograph and the hydrograph as the velocity of the storm influences the time difference between the start of the rainfall and the start of the measured runoff. According to UH theory, the direct runoff hydrograph begins at the same time that the rainfall excess begins. In actual measured rainfall-runoff data, however, the direct runoff hydrograph may start before or after the beginning time of the rainfall excess depending on both the spatial locations of rainfall and runoff gauges and the velocity of the storm. For example, if the storm cell moves over the rain gauge one hour before it passes over the watershed, this time difference may be evident in the hydrograph. If it is not accounted for in the analysis, the timing and form of a calibrated unit hydrograph may be greatly distorted from the true watershed response.

The objective of this section is to investigate the effect of temporal mismatch between the timings of rainfall and runoff on the calibration of UH parameters. The analysis was conducted based on synthetic rainfall excess and direct runoff data without considering the effects of separating the initial abstraction and baseflow. The synthetic rainfall excess hyetograph has a one-peaked triangular shape. The synthetic observed direct runoff hydrograph was developed by convolving the one-peaked rainfall excess with a gamma UH developed using a PRF of 400 and a tp_{UH} of 7. The time offset was investigated in terms of its effect on the calibration accuracy (i.e., the Se/Sy) and uncertainty (i.e., the size of ellipse of the response surface).

When data experience a time offset, the timing of the direct runoff may not be coordinated with the timing of the rainfall excess (see Figure 4-11). In this case, the assumption was made that the velocity of the storm cell introduced a time offset of 8 time units. Since the computed direct runoff begins at the same time as the start of the rainfall excess, the PRF and tp_{UH} of the calibrated UH had to be distorted in order to make the computed direct runoff hydrograph resemble the form of the observed direct runoff hydrograph. As a result of calibration, a PRF of 700 and tp_{UH} of 15 were fitted with a Se/Sy of 0.2727. A mismatch between the observed direct runoff hydrograph and the best-fit computed direct runoff hydrograph is evident. When the time offset was corrected (see Figure 4-12), a PRF of 400 and a tp_{UH} of 7 were calibrated with a Se/Sy of essentially zero (i.e., perfect fit). This experiment indicates that the time offset can significantly influence the values of calibrated UH parameters and the calibration accuracy.

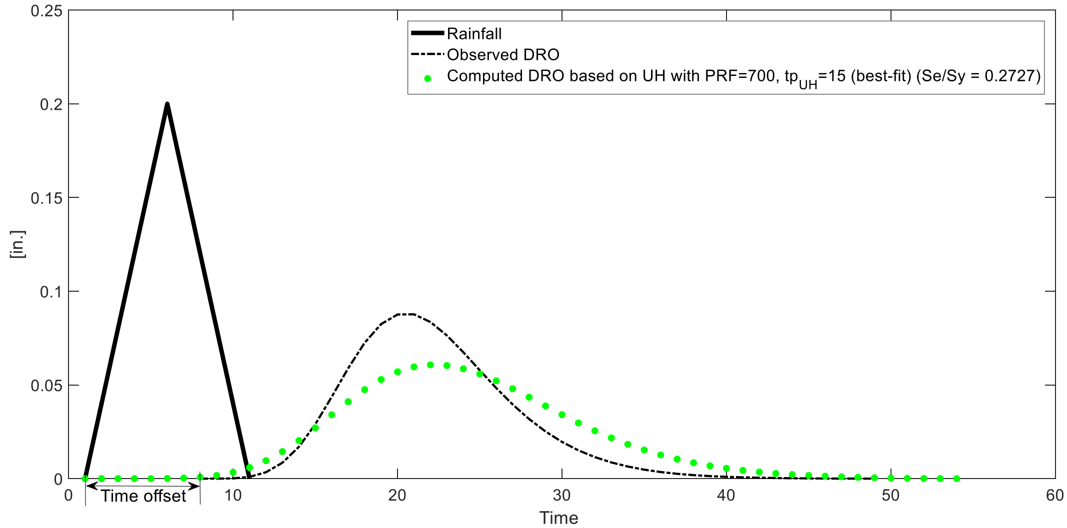

Figure 4-11. Rainfall and computed and observed direct runoff hydrograph with a time offset of 8.

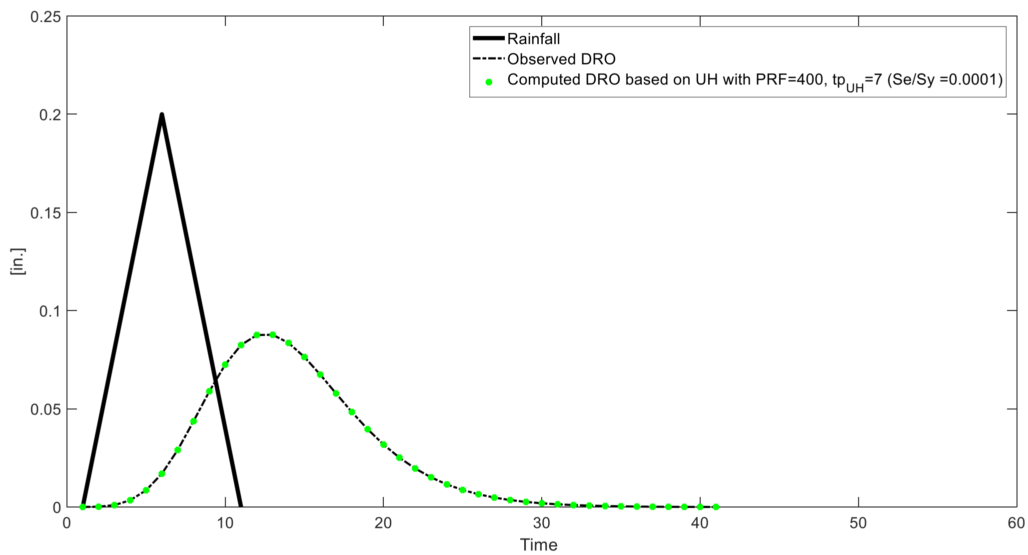

Figure 4-12. Rainfall and observed direct runoff hydrograph with time offset corrected.

In order to further investigate the effect of a time offset on UH calibration, different values of the time offset were assumed and tested. Table 4-4 shows that, as the incorrect time offset increases, the actual PRF and tp_{UH} of the calibrated UH are greatly distorted and the calibration accuracy significantly worsens. By plotting the time offset versus the calibrated UH parameters (see Figures 4-13 and 4-14), it is evident that both the calibrated PRF and tp_{UH} are linearly correlated with the time offset. The slight distortion from linearity is the result of forcing the times to peak to be integer values. To minimize the error and mimic the form of observed direct runoff hydrograph, the tp_{UH} had to be increased to keep the peak of the computed direct runoff hydrograph as close as possible to the peak of observed direct runoff hydrographs. Simultaneously, the PRF had to be increased to maintain a high peak discharge rate. The time offset has a greater effect on the computed tp_{UH} than on the PRF. This is illustrated by the higher correlation in Figure 4-14 than that of Figure 4-13.

Table 4-4. Calibrated UH parameters and values of minimum Se/Sy developed from rainfall and runoff data with various time offsets where the true PRF and tp_{UH} are equal to 450 and 7, respectively.

Time offset	Calibrated PRF	Calibrated tp_{UH}	Se/Sy
0	450	7	0.000
2	520	9	0.095
4	600	11	0.193
8	700	15	0.273
12	780	19	0.363
16	880	23	0.416
20	1200	29	0.409

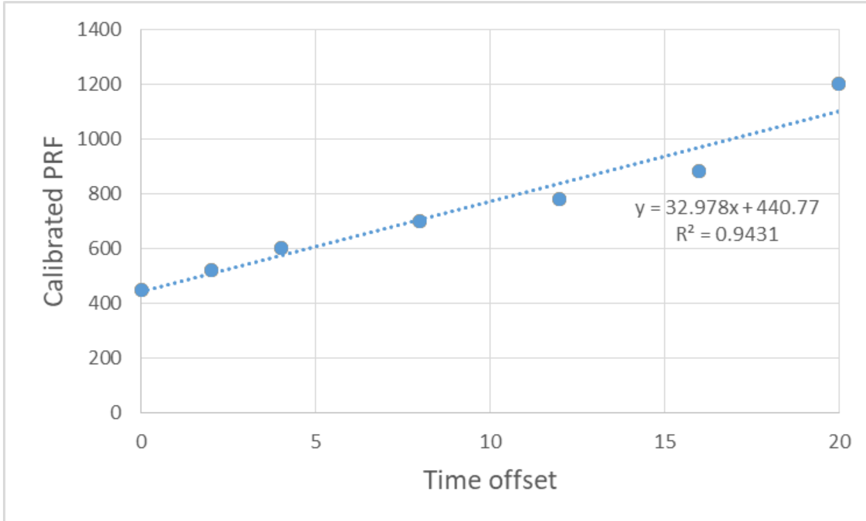

Figure 4-13. Linear regression of calibrated PRF and time offset.

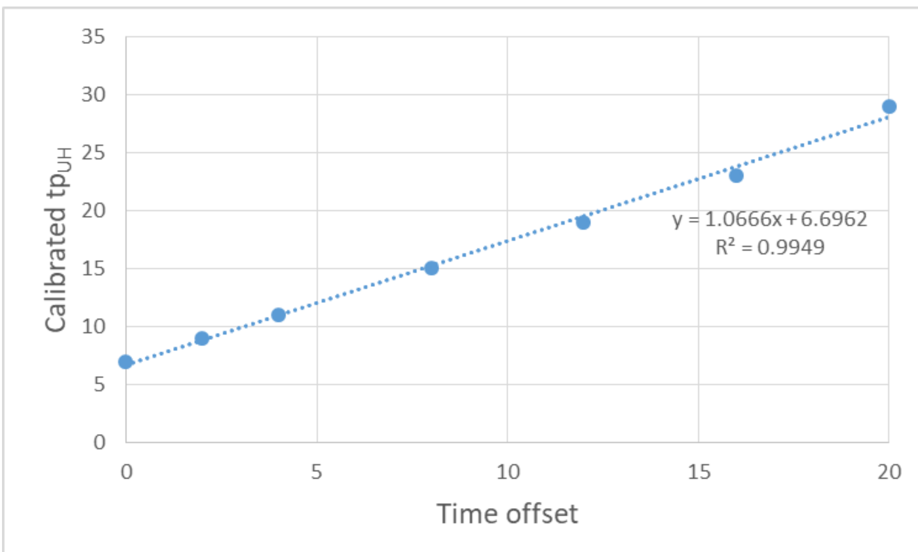

Figure 4-14. Linear regression of calibrated tp_{UH} and time offset.

Calibration uncertainty also depends on the time offset, which is reflected in the size of ellipse of a Se/Sy response surface. Figure 4-15 shows the Se/Sy response surfaces developed from the rainfall-runoff data with time offsets of 4, 8, 12, and 16. The size of the ellipse of a response surface depends on the time offset. An increasing ellipse indicates greater uncertainty. The four plots in Figure 4-15 show the effect of an incorrect time offset. For example, if timing was off by 16 ordinates, the ellipse is very large, which indicates large parameter uncertainty. As the time offset was gradually corrected (i.e., smaller offsets), smaller ellipses resulted because the parameter uncertainty was less.

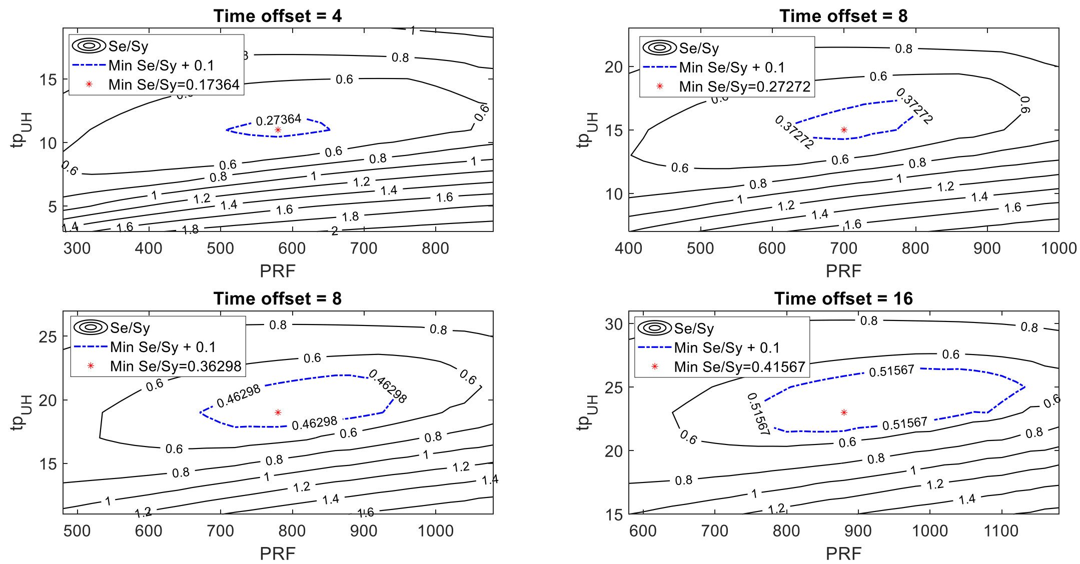

Figure 4-15. Se/Sy response surfaces developed from the rainfall-runoff data with temporal offsets of 4, 8, 12, and 16.

In summary, a time offset can have a significant influence on UH calibration. In almost all cases, the data needed to accurately identify the existence of a time offset is not available. Therefore, adding a time offset by moving the rainfall excess either nearer or farther from the direct runoff hydrograph can either improve or worsen the calibration accuracy. If a time offset exists but is not corrected in the calibration, the actual UH parameters can be significantly distorted, with reductions in both accuracy and certainty. Thus, in order to accurately identify the UH parameters, the time offset should be assessed and allowed to vary when calibrating a UH. Where the necessary data (i.e., watershed characteristics, locations of rain gauges) are not available, the sensitivity of the calibrated UH should be assessed by trying several different values of time offset. If the accuracy is sensitive to time offset, a more complete sensitivity analysis will be necessary.

4.5. FLEXIBILITY AND LIMITATIONS OF GAMMA UNIT HYDROGRAPHS FOR FITTING DIFFERENT DIRECT RUNOFF HYDROGRAPHS

In the previous experiments, the true UH parameters were based on assumed values and the values of minimum Se/Sy were essentially zero. In the analysis of measured data, however, the minimum Se/Sy will usually be much greater than zero. Therefore, before analyzing the actual rainfall-runoff data, the conditions that result in poor fits (i.e., high Se/Sy) need to be investigated. Theoretically, the poor fit can be caused by either the dissimilarity between the rainfall excess hyetograph and the direct runoff hydrograph or the inability of the gamma unit hydrograph to fit the direct runoff hydrograph. The dissimilarity can be easily assessed by comparing the forms of the rainfall excess and direct runoff. A poor fit can be expected if the forms of the rainfall excess and direct runoff show great dissimilarity. Therefore, in this section, the focus is on the flexibility and limitation of the gamma unit hydrograph for fitting different direct runoff hydrographs.

The gamma pdf is a function that can adapt to different forms and scales. The extent of the variation is regulated by its parameters. The function itself controls its flexibility to adapt to different forms. The inability to adapt to different forms and scales will be reflected in the values of goodness-of-fit statistics. The values of the two-parameter gamma distribution parameters establish the extent to which the assumed population model matches measured system responses.

When the gamma distribution is used as a model of a unit hydrograph, its ability to provide accurate estimates of measured runoff hydrographs depends on its ability to match the form and scale characteristics of the observed direct runoff hydrograph. The failures to match is evident from the goodness-of-fit statistics, but the same value of a statistic can result from quite dissimilar hydrographs. In this section, effects of the skewness, width of a measured direct runoff hydrograph on fitting were discussed. The effect of the closeness between the times to peak of the rainfall excess hyetograph and the measured direct runoff hydrograph on fitting was also discussed.

4.5.1. Effect of Skewness of Measured Direct Runoff Hydrographs on Fitting

In probability theory and statistics, skewness is a measure of the asymmetry of the probability distribution. A direct runoff hydrograph can have an either left-tail skewness (i.e., the left tail is longer) or a right-tail skewness (i.e., the right tail is longer). Figure 4-16 shows four direct runoff hydrographs that reflect the runoff for different watershed conditions. Case A is a right-tail skewed hydrograph, and case B is a symmetric hydrograph; case C is left-tail skewed. Case D is also symmetric, but relatively flat. A single-peaked, triangular rainfall excess hyetograph with a time base of 20 ordinates was assumed to have generated each of the four direct runoff hydrographs. The rainfall excess and each of the direct runoff hydrographs were deconvolved to identify the best-fit gamma unit hydrograph, with the Se/Sy used to assess the calibration accuracy. Various combinations of the gamma UH parameters were tired until the best fit was found.

A gamma distribution was used to represent each unit hydrograph. The gamma parameters were calibrated and the set that caused the computed hydrograph to most closely match the assumed hydrograph was considered the true values. While cases A and B have similar values of Se/Sy (i.e., 0.48 for A and 0.38 for B), the degree of fit is influenced by different factors. The hydrograph of case A has a relatively narrow half-width for the time of peak and, therefore, was not able to provide a perfect fit; thus, it is more of an issue of form. Conversely, the accuracy of case B is limited because of a scale issue, namely that a gamma distribution has problems being fit to a center loaded hydrograph. In spite of these issues, the degrees of fit, as indicated by the low Se/Sy values, are both reasonably good.

Cases C and D of Figure 4-16 have similar Se/Sy values (i.e., 0.97 for C and 0.92 for D) but the causes of the poor fits differ. Since the gamma distribution is not very flexible, it cannot take on the scale of case C or the form of case D. The peak of case C occurs at a time that is beyond the center point, and the gamma distribution is not sufficiently flexible to take on a left-tail skewed shape, as the scale of the quantiles differs from the functional form. The fit of the trapezoidal form of case D is poor because values of the shape and scale parameters of the gamma distribution cannot reproduce the flat-topped form of case D, especially when the tail of the distribution is not long. The shape and scale problems that are evident in these two cases prevent good agreement.

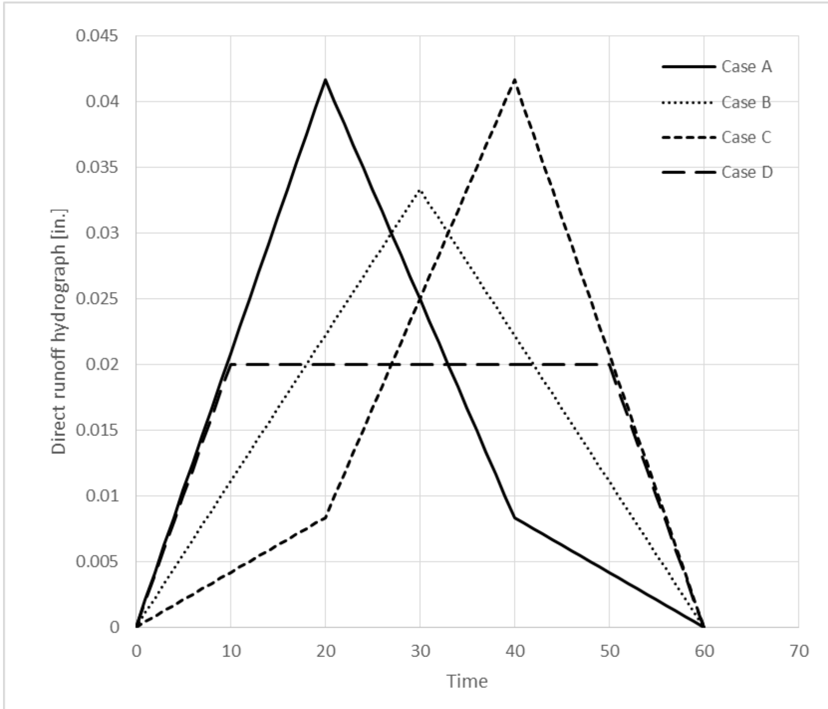

Figure 4-16. Synthetic measured direct runoff hydrographs with different characteristics: case A is right-skewed; case B is symmetric; case C is left-skewed; and case D is symmetric rectangular.

In summary, the ability of gamma UH to fit measured direct runoff hydrographs is limited by its inherent flexibility, and it is more likely to produce reasonably better goodness of fit when fitting direct runoff hydrographs with a relative wide half-width and a right-tail skewed shape.

4.5.2. Effect of Widths of Measured Direct Runoff Hydrographs on Fitting

The analyses in section 4.5.1 indicated that the half-width of the measured direct runoff hydrograph may be an important factor that influence the fitting. In order to further investigate the way that the width of the measured direct runoff hydrograph affects the goodness of fit, a general measured direct runoff hydrograph form was developed (see Figure 4-17). The form is based on six points with linearity assumed between points. Ordinates were then computed from the model at a 1-min increment. The six points were varied to enable the measured direct runoff hydrograph to have four different forms with different widths (see Table 4-5). All of these forms have a time to peak of 14 ordinates and magnitude of 18, with a time base of 60 ordinates. Each hydrograph was transformed to a unit volume of 1 (i.e., the sum of the discrete ordinates equaled to 1). A single-peaked, triangular rainfall excess hyetograph with a time to peak of 5 and a time base of 10 ordinates was assumed. Each of the four alternative direct runoff hydrographs were assumed to have resulted from the same rainfall excess hyetograph.

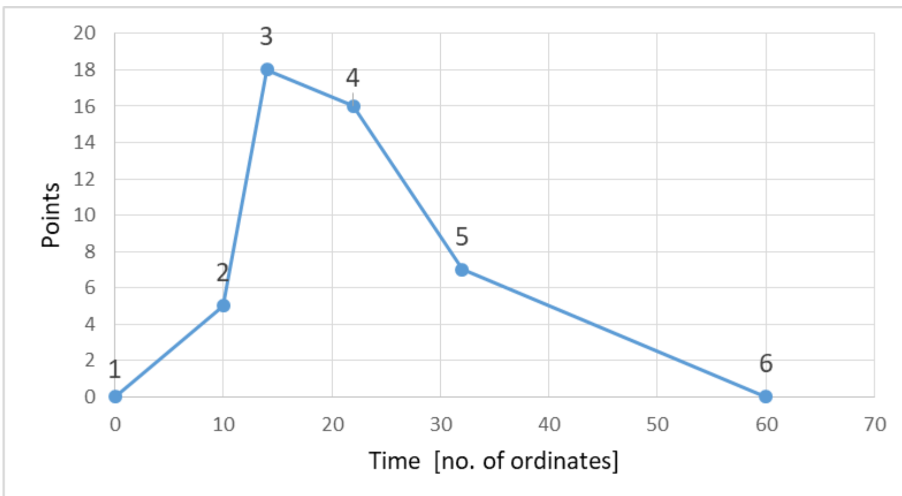

Figure 4-17. Alternative measured direct runoff hydrograph.

Table 4-5. Locations and values of the six points of each of the measured direct runoff hydrographs: t and q are the time and discharge rate at specific point, respectively.

Point	Thin		Moderate		Wide		Very Wide	
	t	q	t	q	t	q	t	q
1	0	0	0	0	0	0	0	0
2	12	5	8	5	4	5	4	12
3	14	18	14	18	14	18	14	18
4	18	16	22	16	32	16	32	16
5	40	7	40	7	50	7	50	10
6	60	0	60	0	60	0	60	0
Half-width	22.50		26.15		38.92		48.00	

Table 4-6. Calibration results for the thin, moderate, wide, and very wide direct runoff hydrographs with varying time offsets.

Time Offset	Thin			Moderate			Wide			Very Wide		
	tp_{UH}	PRF	Se/Sy	tp_{UH}	PRF	Se/Sy	tp_{UH}	PRF	Se/Sy	tp_{UH}	PRF	Se/Sy
5	22	456	0.309	21	445	0.171	22	335	0.114	20	230	0.167
4	21	455	0.305	19	405	0.161	20	305	0.114	18	200	0.145
3	20	430	0.303	18	380	0.152	19	285	0.110	16	175	0.142
2	19	410	0.301	17	360	0.145	18	265	0.122	14	145	0.174
1	18	390	0.300	16	340	0.142	16	235	0.137	11	110	0.239
0	16	345	0.294	14	300	0.136	15	215	0.166	9	90	0.330
Note: Times to peak were fitted to the nearest integer; peak rate factors were fitted to ±5												

The rainfall excess and each of the hydrographs were deconvolved to identify the best-fit gamma unit hydrographs, with the Se/Sy used to assess the goodness of fit. It is evident that the width of the runoff hydrograph has a significant influence on the fitting (see Table 4-6). The following results are evident:

(1) Both the thin and very wide runoffs did not achieve good accuracy, with the Se/Sy>0.29. The gamma pdf has a set half-width that depends on the shape and scale parameters. Generally, the gamma pdf will not provide an accurate fit to data that have a relatively narrow or relative wide runoff hydrograph. This constraint is the result of the gamma UH being dependent on only two parameters and its lack of flexibility.

(2) The calibrated times to peak for the moderate runoff hydrographs are reasonable; the tp_{UH} for the wide and thin are also reasonable but slightly larger.

(3) The PRF increases as the direct runoff hydrograph gets thinner. This is necessary because the gamma shape parameter must increase to produce a narrower peak area. For a very wide shape, both the tp_{UH} and the PRF must be decreased because the peak discharge is relative low.

(4) The degree of fit partially reflects the ability of the gamma pdf to match the form of the measured direct runoff hydrograph. In these cases, the assumption of linearity also contributes to the computed value of Se/Sy.

The effect of time offset was also investigated in this section through the assessment on the goodness of fit and calibrated UH parameters by adding time offsets up to 5 time units to move the rainfall excess hyetograph away from direct runoff hydrograph. In terms of time offset, the degree of fit can be poorer (thin and moderate runoff hydrographs) or variable (improve and then get poorer for wide and very wide runoff hydrographs). It is difficult to fit the wide and very wide runoff hydrographs without an offset. As the offset increases, the Se/Sy improves for the wide and very wide runoff hydrographs because a relatively low PRF allows the UH to spread out.

In summary, the gamma UH will not fit data with either a relatively narrow or relatively wide measured runoff hydrographs. Adding or minimizing the temporal offset between rainfall and runoff may not lead to a higher calibration accuracy, but it is always worth consideration.

4.5.3. Effect of the Closeness between the Times to Peak of Rainfall Excess Hyetograph and the Measured Direct Runoff Hydrograph

The closeness between the time to peak of the rainfall excess (tp_{pe}) and the time to peak of the direct runoff hydrograph (tp_{DRO}) can directly influence the accuracy of a fitted unit hydrograph. Since the watershed smooths the rainfall excess, the computed direct runoff hydrograph usually has a flatter shape and a later peak than that of the rainfall excess hyetograph. If the difference between the $\mathrm{tp_{pe}}$ and $\mathrm{tp_{DRO}}$ is too small, it is difficult for the unit hydrograph to provide a good fit to the runoff. Therefore, in this section, the effect of the difference between the $\mathrm{tp_{pe}}$ and $\mathrm{tp_{DRO}}$ on the calibrated UH parameters was further investigated.

To study the effect of the time difference in the peaks, three direct runoff hydrographs, DRO(t), with the same time base ($\mathrm{tb_{DRO}}$) of 100 but different times to peak ($\mathrm{tp_{DRO}}$) of 24, 50, and 76 were generated using the beta distribution (see Figure 4-18). Three single-peaked triangular rainfalls (see Figure 4-19) with time bases ($\mathrm{tb_{pe}}$) of 10, 20, and 40 and times to peak of ($\mathrm{tp_{pe}}$) 5, 10, and 20 were assumed to result in each of the three alternative direct runoff hydrographs. The rainfall excess and each of the hydrographs were deconvolved to identify the best-fit gamma unit hydrograph, with the Se/Sy used to assess the goodness of fit. The results of deconvolution and calibration are shown in Figure 4-20.

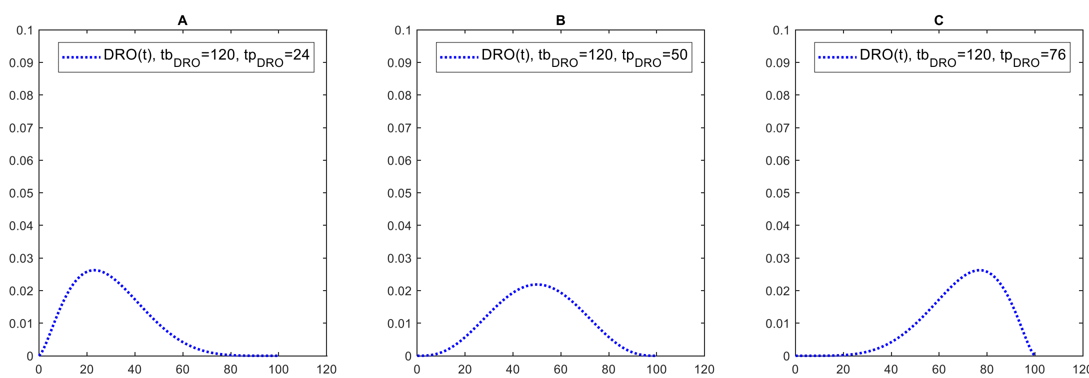

Figure 4-18. Synthetic measured direct runoff hydrographs with different times to peak.

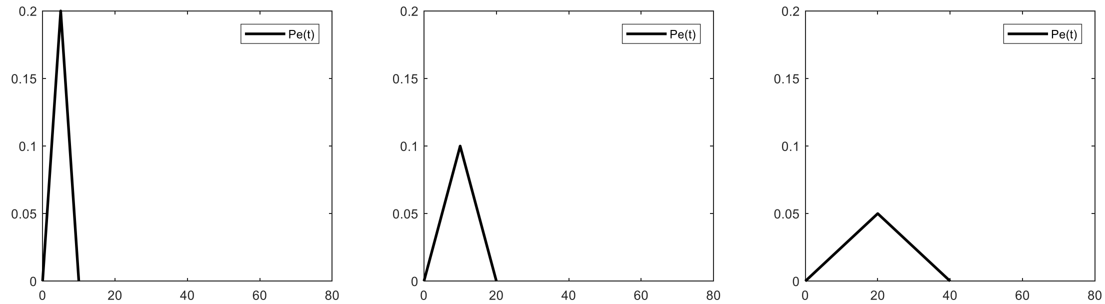

Figure 4-19. Rainfall excess hyetographs with different times to peak and time bases.

It is evident that a poor fit can result when the $\mathrm{tp_{DRO}}$ is close to the $\mathrm{tp_{pe}}$ (see case 3 in Figure 4-20). In this case, the UH is forced to have a small $\mathrm{tp_{UH}}$ and low PRF. Since the unit hydrograph is a smoothing function, the computed direct runoff will always have a longer time to peak and longer time base than that of the rainfall excess; therefore, if the time to peak of the rainfall excess is too close or even longer than the time to peak of the measured direct runoff, there will be a mismatch in times to peak between the computed direct runoff and the measured direct runoff. The poor fit can also occur when the measured DRO(t) is skewed left and the rainfall duration is short (see case 7 in Figure 4-20). This result further supports the previous finding (refer to section 4.5.1) that the gamma UH is not sufficiently flexible to convolve with a short rainfall into a left-skewed runoff hydrograph. The Se/Sy improves for longer duration storms (see case 9 in Figure 4-20) because the rainfall is smoothed.

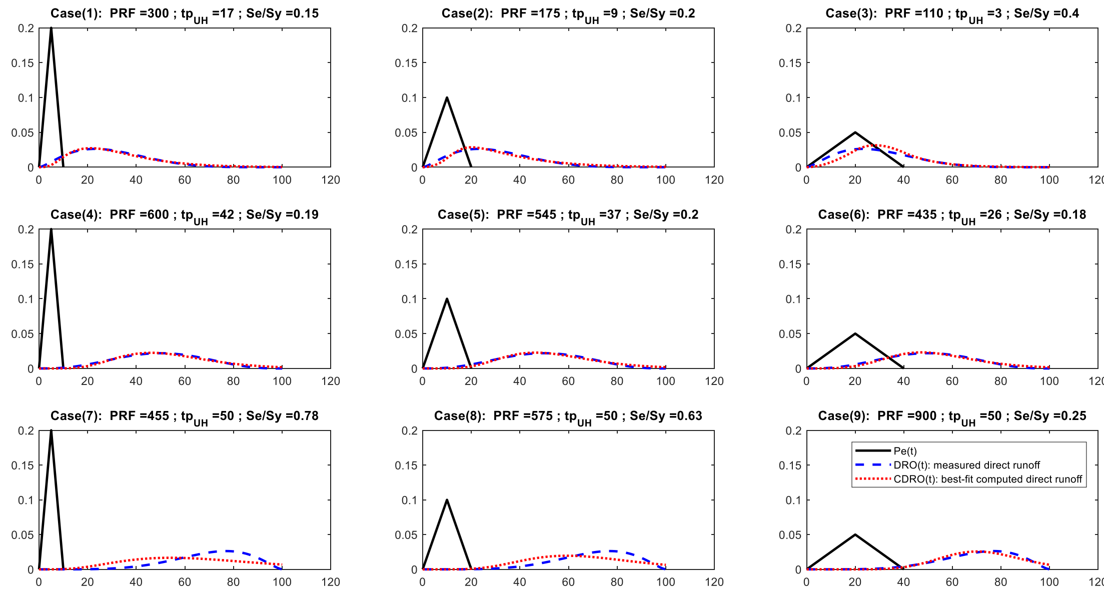

Figure 4-20. Calibrated UH parameters and goodness of fits: cases 1-3 have different Pe(t) and the same DRO(t) =A (see Figure 4-19); cases 4-6 have different Pe(t) and the same DRO(t) =B; and case 7- 9 have different Pe(t) and the same DRO(t) =C.

It seems that the tp_{pe} is not a problem for a more center-loaded DRO(t) (see cases 4, 5, and 6). In these cases, the calibrated UH parameters (PRF and tp_{UH}) vary significantly (from 435 to 600 and from 26 to 42, respectively) but the Se/Sy values are very similar (0.18 to 0.19). This result indicates that the gamma UH is sufficiently flexible to fit the more center-loaded DROs(t).

In summary, the analyses herein showed that the difference between the tp_{pe} and tp_{DRO} has a significant effect on the accuracy of UH calibration. The analyses also support the previous finding (refer to section 4.5.1) that the gamma UH is more likely to produce reasonably better goodness of fit when fitting direct runoff hydrographs that have center-loaded or right-tail skewed shapes. Poor goodness of fit can result from either fitting left-tail skewed direct runoff hydrographs or short differences in time between the times to peak of a rainfall excess hyetograph and a measured direct runoff hydrograph. One important implication of this finding is that time offsetting may not be a good idea if it overly shortens the time between the times to peak of the rainfall excess hyetograph and runoff hydrograph. But if it is known that the rainfall excess and direct runoff are not synchronized, than time offset adjustment is necessary.

4.6. EFFECT OF RAINFALL NONUNIFORMITY ON VARIATIONS OF CALIBRATED UH PARAMETERS

In section 4.3, the analyses on synthetic data showed that uncertainty of calibrated UH parameters can be influenced by rainfall characteristics (i.e., rainfall complexity and peakedness). In addition to the effects of the rainfall complexity and peakedness, one common issue of measured data that should be investigated is the nonuniformity of the rainfall over the watershed. Since a watershed may have multiple rain gauges that are separately distributed over the drainage area, such nonuniformity of rainfall may cause these rain gauges to produce different rainfall hyetographs but share one measured runoff hydrograph. Consequently, when calibrating the UH parameters from such sets of rainfall-runoff data, variations in the computed UH parameters may resulted from the different rain gauge measurements. Thus, the objective of this section is to assess the potential variations in the computed UH parameters when the rainfall is not spatially uniform over the watershed.

This study was based on analyses of data available for two selected watersheds. Each watershed has multiple rain gauges within the watershed boundary. Ralston Creek (area=1926 ac= 3.01 mi^2) includes five rain gauges, with rainfall data available for three of the five gauges; only total depths were available for the other two gauges. The Hastings, NB (411 acres), watershed includes five rain gauges with rainfall hyetographs available for just two of the five gauges. The same runoff hydrograph was used with each hyetograph. Gamma UHs were calibrated, and the tp_{UH} and PRF were computed for each combination of P(t) and RO(t).

The Ralston Creek watershed has five rain gauges (see Figure 4-21). The weight assigned to each gauge based on a Thiessen analysis is shown in Table 4-7. At the individual gauge locations, the total storm depths ranged from 2.13 in. to 4.30 in., with a weighted mean of 2.81 in. For the three rain gauge locations where temporally distributed rainfall data were available, the weighted mean rainfall was 2.88 in. The calibrated values of the time to peak and the peak rate factor are shown in Table 4-8. The weighted averages are also given. For this watershed, the three hyetographs for the same storm yielded very little difference in the times to peak (i.e., 1 to 4). The three PRFs were small, i.e., 60 or less, but showed as much as a 57% error from the weighted mean.

Table 4-7. Data characteristics of each rain gauge in Ralston Creek watershed.

Gage	d_{cm}	d_0	A_i	w_i	p_i	$w_i p_i$
1	3300	14400	431	0.2239	2.62	0.5866
2	6800	36900	286	0.1483	4.30	0.6377
3	3200	29800	538	0.2793	3.43	0.8208
4	4500	15100	492	0.2556	2.13	0.5444
5	7600	17100	179	0.0929	2.36	0.2192
			1926	1.0000		2.8087

d_{cm}= distance (ft) from gage to watershed center of mass;
d_0 = distacne (ft) from gage to watershed outlet; A_i=Thiessen area (ac) assocaited with gage *i*; w_i=Thiessen weight for gage *i*; p_i=rainfall depth (in.) measured at gage *i*.

Table 4-8. Calibration results for rain gauges in Ralston Creek watershed.

Gage	A_i	w_i	p_i	$w_i p_i$	tp_{UH}	PRF	$w_i tp_{UH}$	$w_i PRF$
2	274	0.1423	4.30	0.612	3	46	0.427	6.54
3	649	0.3370	3.43	1.156	4	60	1.348	20.22
4	1003	0.5207	2.13	1.109	1	22	0.521	11.46
	1926	1.0000		2.877			2.296	38.22

Data for a second watershed (see Figure 4-22) where rainfall hyetographs were available at multiple gauges were analyzed. During the 07/03/1959 storm event on watershed W-5 at Hastings, NB (411 acres), rainfall hyetographs were available at gauges C-45-R and D-45-R. The latter gage is at the watershed outlet near the stream gauge. Gauge C-45-R is located in the upper reaches of the watershed and may be more representative of rainfall falling on the watershed because the upper portion of the watershed is larger than the area near the outlet. While the same maximum intensity, 5.4 in./hr, was measured at the two gauges, the total rainfall depth at gage D-45-R was about 15% greater than the total depth at gage C-45-R (i.e., 2.46 in. vs 2.11 in.).

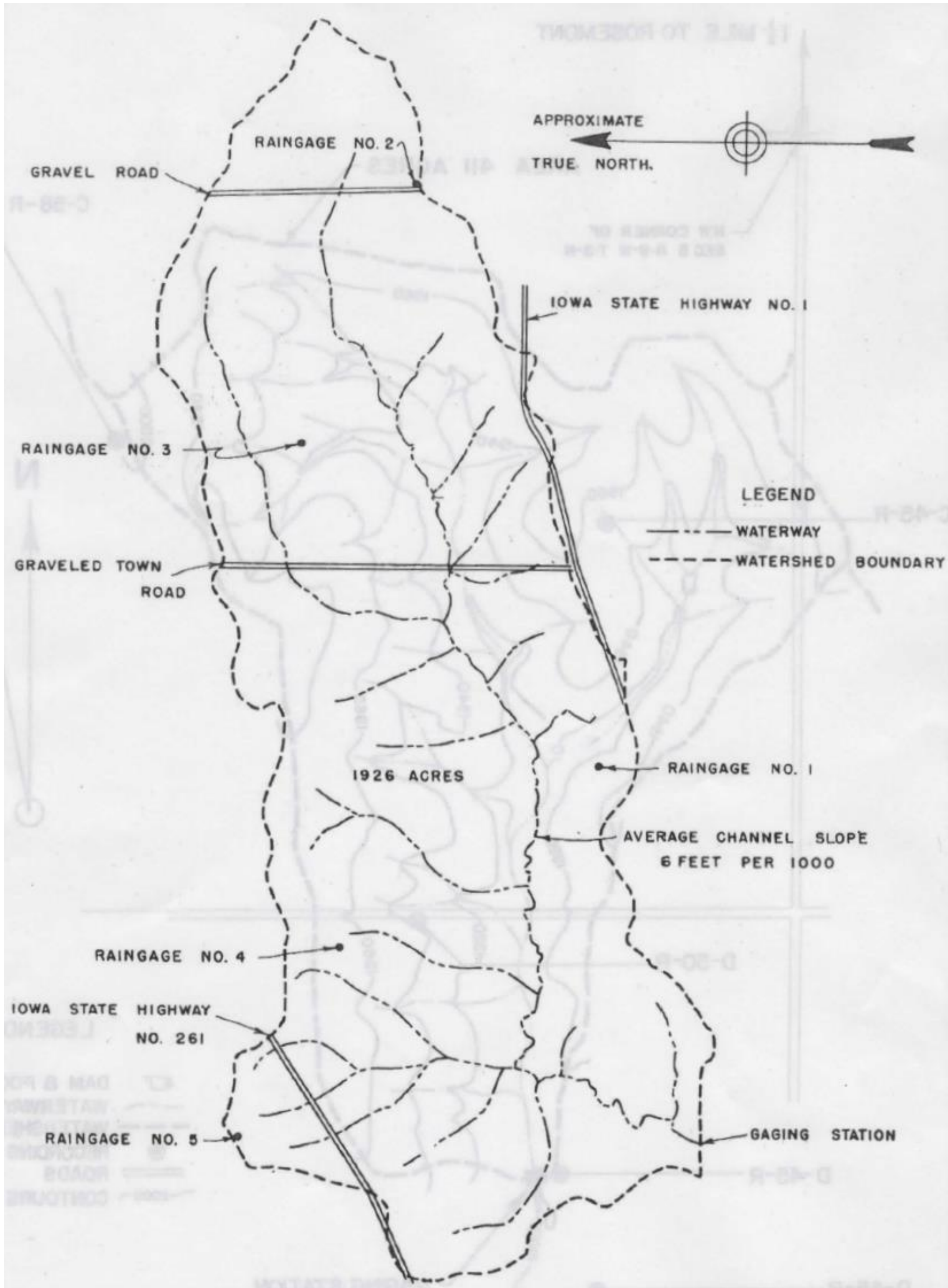

Figure 4-21. Topographic map of Ralston creek watershed [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in The United States, 1959].

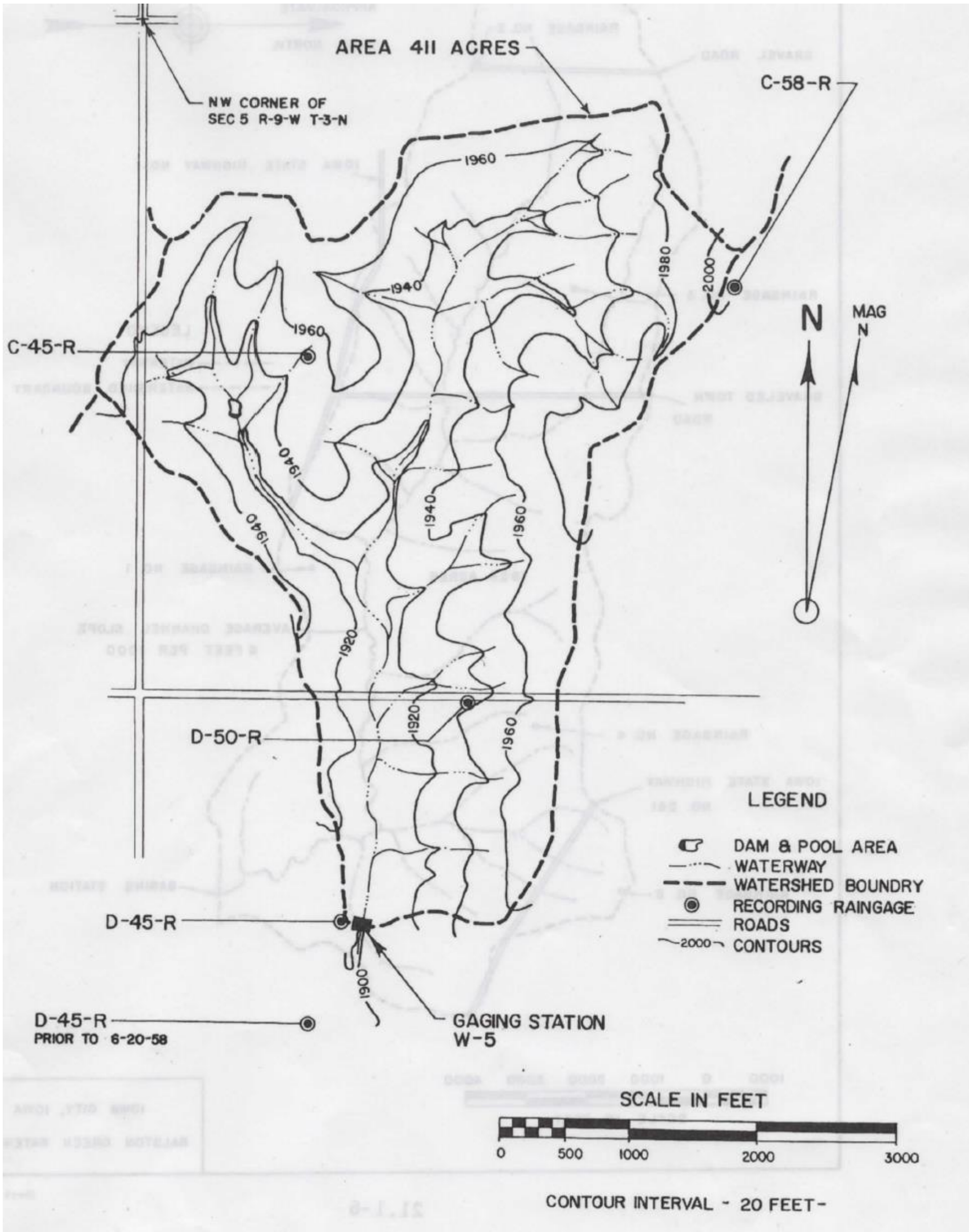

Figure 4-22. Topographic map of Hastings, Nebraska watershed [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in The United States, 1959].

Using the P(t) data for rain gauge C-45-R resulted in an optimum tp_{UH} of 29 minutes and a PRF of 330, with Se/Sy = 0.374. Using the P(t) data for rain gauge D-

45-R resulted in corresponding values of 24 minutes and 274. These represent differences of 19% each.

In summary, based on the analyses of storm data from these two watersheds, the rainfall hyetographs can show sufficient variation as to produce significant changes in the UH parameters. If such data are available, each hyetograph should be analyzed to assess the potential variations in the UH parameters.

4.7. EFFECT OF DYNAMIC CONVOLUTION ON UH CALIBRATION

The previous analyses on synthetic data have shown various factors, i.e., the time offset, the limited flexibility of gamma pdf, can cause the data set to yield poor goodness-of-fit statistics. When analyzing measured data, the reason for a poor fit is commonly difficult to identify. One possible reason might be that the watershed processes are nonlinear, but the unit hydrograph model is assumed to be linear, i.e., static rather than dynamic convolution. Therefore, in an attempt to explore the likelihood that this assumption of linearity can be a factor in limiting accuracy, three data sets that had produced poor goodness of fit were evaluated using a dynamic convolution model.

Dynamic convolution has the advantage of increasing the flexibility of the UH model because it introduces a third parameter. It has the disadvantage of reducing the degrees of freedom. Linear convolution is based on the model:

$$DRO(t) = \int_0^t Pe(t)U(\tau)d\tau \tag{4-2}$$

in which τ is the variable of integration. The unit hydrograph is the same for each burst of rainfall excess. In the case of dynamic convolution, the static unit hydrograph model $U(\tau)$ is replaced with a UH model that varies with time:

$$DRO(t) = \int_0^t Pe(t)U(t,\tau)d\tau \tag{4-3}$$

While a number of options are available for incorporating time into the UH model, the most basic approach is to make one of the UH parameter a function of time. In terms of the gamma UH, either the shape or the scale parameter can be represented by a function. A basic linear model could be used.

While the scale parameter is generally the more sensitive parameter, it is usually quantified using the time of concentration. Therefore, in this exploratory study, nonlinearity was introduced by way of the shape parameter. As an exploratory analysis, the constant value of the shape parameter c was replaced by a linear function of time:

$$c = c_2 + c_3 t \tag{4-4}$$

in which c_2 and c_3 are constant but c varies with time. As time progresses, the shape parameter changes by an amount reflected in the parameter c_3. For each minute within the storm, the rainfall excess is convolved with a unit hydrograph that is specific to the time increment.

Three watersheds were analyzed to assess the potential for dynamic convolution to help explain either a relatively poor fit of the gamma UH model or irrational parameters. The rainfall excess hyetograph and the direct runoff hydrograph were analyzed using the measured data. Numerical least squares was used to estimate the parameter values for the gamma UH model with static and dynamic convolution.

The first test used the data from the 08/26/1971 event on watershed W-10 at Oxford, MS (see Figure 4-23). The static UH analysis produced the following values: b = 28.75, c = 1.034, $\bar{e}/\bar{y}$ = -0.014, Se/Sy = 0.711, and R^2 = 0.5005. These values of b and c correspond to a tp_{UH} of 1 minute and a PRF of 21. Both of these values seem low. The dynamic model produced the following results: b = 41.7, c_2 = 1.009, c_3= -0.00225, $\bar{e}/\bar{y}$ = -0.014, Se/Sy = 0.666, and R^2 = 0.561. While the PRF varies with time, the initial PRF, i.e., c_2, correspondence to a value of 40. Thus, along with an improvement in goodness of fit, the parameters based on the dynamic analysis appear to be more reasonable estimates. The time base of the UH was 64 UH minutes. In spite of these improvements, the overall model did not provide a good fit to the measured data, which could be due to storm complexity or a time offset problem.

The second analysis used the first part of the 07/08/1967 storm event (see Figure 4-24) on watershed 7-H at Hastings, Nebraska. With a static UH, the computed rainfall excess and direct runoff produced the following results: tp_{UH}= 44 min, PRF = 96, $\bar{e}/\bar{y}$ = 0.010, Se/Sy = 0.4767, and R^2 = 0.786. The b and c values correspond to tp_{UH} of 44 and a PRF of 96. Given a time base of the DRO(t) of 66 minutes, the tp_{UH} of 44 minutes seemed long, in spite of the reasonably good fit. Using the dynamic convolution, the analysis yielded the following results: b = 92, c_2 = 1.161, c_3 = 0.0140, $\bar{e}/\bar{y}$ = 0.010, Se/Sy = 0.486, and R^2 = 0.777. Thus, the overall fit of the dynamic analysis was slightly poorer than that of the static analysis. The value of the scale parameter was not rational and the positive value of c_3 is not reasonable. For this analysis, the use of dynamic convolution did not improve the model.

A third analysis was made on the watershed W-1 at Stillwater, OK, using data for the 06/27/1957 event (see Figure 4-25). Calibration using static convolution produced the following results: tp_{UH} =1 min, PRF = 95, $\bar{e}/\bar{y}$ = -0.003, Se/Sy = 0.803, and $R^2 = 0.413$. These correspond to gamma parameters of b = 4.412 and c = 1.227. Using dynamic convolution, the gamma UH model resulted in the following: b = 54.5, $c_2 = 1.02$, $c_3 = -0.0019$, $\bar{e}/\bar{y} = -0.003$, Se/Sy = 0.582, and $R^2 = 0.679$. The initial value of c_2 corresponds to a PRF of about 8. While the goodness of fit improved very significantly, the model parameters are still not reasonable.

These analyses suggest that using time-dependent convolution can lead to improved goodness of fit and in some cases more reasonable coefficients. Improvements were not found in all of these analyses, possibly because of rainfall characteristics, the length of the storm, the time offset, or the antecedent watershed conditions. But as an exploratory assessment, the approach appears to have merit and needs additional study.

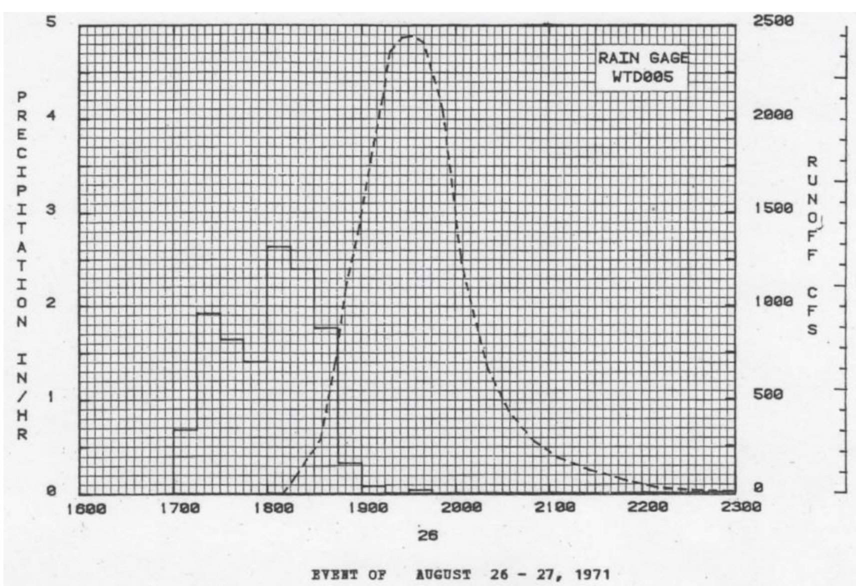

Figure 4-23. Storm event occurred on 08/26/1971 on watershed W-10 at Oxford, Mississippi [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in The United States, 1971].

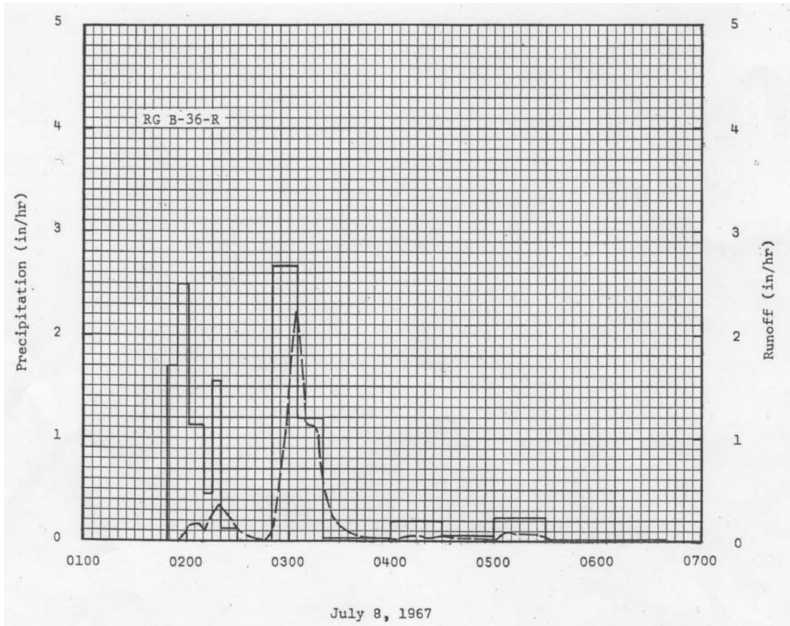

Figure 4-24. Storm event occurred on 07/08/1967 on watershed 7-H at Hastings, Nebraska [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in The United States, 1967].

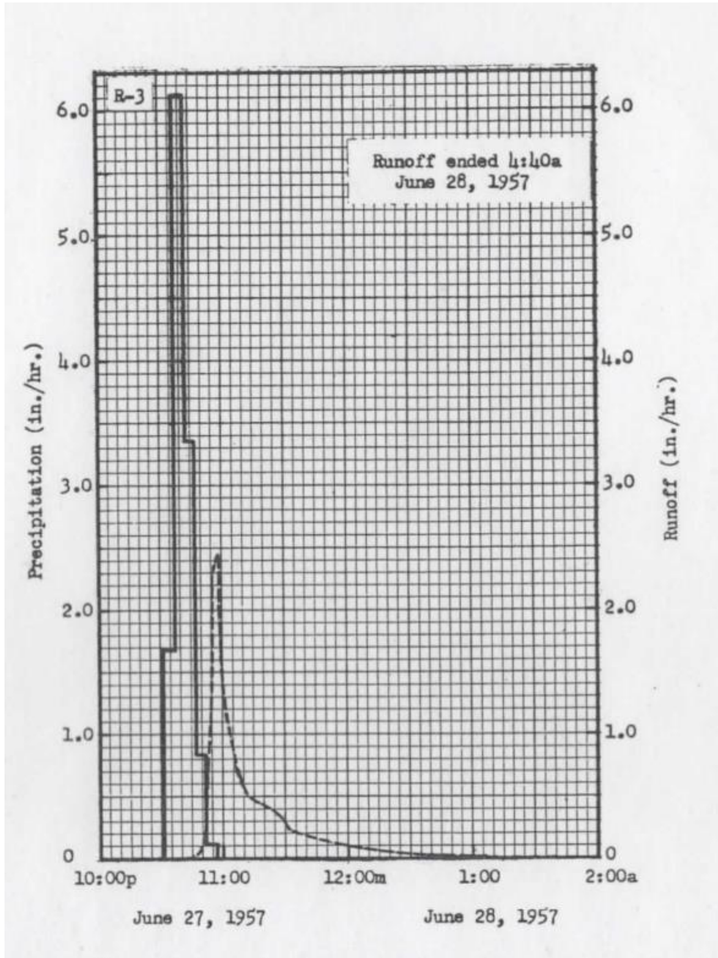

Figure 4-25. Storm event occurred on 06/27/1957 on watershed W-1 at Stillwater, Oklahoma [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in The United States, 1957].

4.8. GENERAL GUIDELINES FOR UH CALIBRATION

The analyses conducted yielded general guidelines regarding unit hydrograph (UH) calibration. These guidelines were developed based on the results from the analyses of the previously conducted experiments. Awareness and acknowledgement of these guidelines will not only improve the understanding of the calibration process, but also improve the calibration accuracy and diminish the calibration uncertainty of calibrated times to peak (tp_{UH}) and peak rate factors (PRF) of unit hydrographs.

4.8.1 Guideline: The form of the direct runoff hydrograph can be greatly influenced by the form of the unit hydrograph.

A runoff hydrograph represents the direct response of watershed to a storm event. In actual measured rainfall and runoff data, the rainfall characteristics (e.g., the number of peaks, peakedness) can be reflected in the runoff hydrograph, but sometimes, a rainfall hyetograph and a runoff hydrograph may show little similarity because the watershed processes smooth the variation of the rainfall excess hyetograph. For example, a multi-peaked rainfall hyetograph often produces a single-peaked runoff hydrograph, or a very peaked rainfall hyetograph produces a flat runoff hydrograph. These phenomena need to be fully understood when analyzing the results of unit hydrograph calibrations from measured data.

In the analyses reported herein, the results indicate that the form of the direct runoff hydrograph is influenced by the form of the unit hydrograph. As the peak rate factor decreases and/or the time to peak of the unit hydrograph increases, the gamma unit hydrograph becomes flatter, which then cause the computed direct runoff hydrograph to become flatter. When the rainfall excess is convolved with a narrow unit hydrograph (compared with the width of the rainfall), the computed direct runoff hydrograph will more than likely reflect the characteristics of the rainfall excess hyetograph. The physical implication of such a condition is that the watershed will more likely have a steep slope and/or little natural storage (e.g., a steep rocky terrain). When rainfall excess is convolved with a flat unit hydrograph, the computed direct runoff hydrograph will more likely be flat and the characteristics of rainfall excess will be smoothed. The physical implication of such a condition is that the watershed will more likely have a shallow slope and/or a considerable natural storage (e.g., a flat swampy watershed).

Awareness of this guideline is very important as it allows the user to estimate the possible ranges of the UH parameters before conducting the calibration and check the rationality of the calibrated UH parameters. For example, a direct runoff hydrograph that greatly reflects the variation in the rainfall excess indicates that the underlying unit hydrograph has either a high peak rate factor and/or a short time to peak. In this case, before the calibration, the possible ranges of UH parameters can be approximately estimated. If calibrated UH parameters are within the estimated possible ranges, then the calibration result is likely reasonable. However, if calibrated UH parameters are not consistent with the estimated values, then the calibrated result may not be rational and may need recalibration. This may be caused by other factors, such as time offset and the limited flexibility of gamma UH. Thus, awareness and implementation of this guideline is very important.

4.8.2 <u>Guideline</u>: While the time to peak of a unit hydrograph is the more critical parameter, it should be calibrated simultaneously with the peak rate factor.

In most cases, the peak rate factor and the time to peak are correlated. The analyses of previous studies (see section 4.2) have shown that changing the time to peak of a unit hydrograph can lead to significant and usually a greater change in Se/Sy than from change in the peak rate factor, which indicates that the Se/Sy may be more sensitive to the time to peak of the unit hydrograph than to the peak rate factor.

This results because the time to peak of the unit hydrograph directly affects the time to peak of the computed direct runoff hydrograph, and any mismatch in timing between the computed direct runoff hydrograph and the observed direct runoff hydrograph will cause large errors, which significantly increases the Se/Sy. All of these results indicate that the time to peak of the unit hydrograph is a critical parameter and must be calibrated simultaneously with the peak rate factor.

Awareness of this guideline is very important for accurately determining unit hydrographs from measured data. The NRCS dimensionless unit hydrograph is widely used in hydrologic designs. The time to peak of the unit hydrograph is treated as an external parameter and is usually estimated using the time of concentration, which itself is an inaccurate parameter even though many methods have been proposed for estimating watershed times of concentration. Thus, in order to improve estimation accuracy, the time to peak of a unit hydrograph must be calibrated simultaneously with the peak rate factor. Additionally, any factors that may affect the calibrated value of the time to peak, such as a temporal offset or the separation of the initial abstraction, should be considered and fully analyzed before beginning the unit hydrograph calibration process.

4.8.3 <u>Guideline</u>: Rainfall characteristics can significantly affect the uncertainty of calibrated UH parameters.

Many studies on measured rainfall-runoff data have shown that the storm-to-storm unit hydrographs for the same watershed can be quite different. The accuracy and uncertainty of the calibrated UH parameters for these unit hydrographs may also be different. Therefore, it is necessary to understand reasons that rainfall characteristics affect the ability to calibrate a UH.

Two major rainfall characteristics, i.e., rainfall complexity and rainfall peakedness, were selected and investigated in section 4.3 in terms of their effects on the uncertainty of the calibrated UH parameters. Analyses of the results showed that rainfall characteristics can significantly affect the uncertainty of the calibrated UH parameters. Generally, more complex and more peaked rainfall tend to produce calibrated UH parameters with less uncertainty.

Awareness of these guidelines is important as it allows the user to approximately assess the available measured data prior to calibration. When the rainfall excess characteristics are minimally reflected in the direct runoff hydrograph, the calibration uncertainty is minimally affected by the rainfall pattern. Under such conditions, more peaked rainfall hyetographs will provide less calibration uncertainty. While when the rainfall characteristics are evidently reflected in the direct runoff hydrograph, the calibration uncertainty is less for more complex rainfall excess hyetographs.

In summary, based on the analyses of synthetic rainfall-runoff data, when selecting storm events to use as the identification of the underlying watershed UH, it can be advantageous to select more complex and more peaked storm events.

4.8.4 Guideline: A time offset, which is likely to have a physical basis, can improve or distort estimations of actual UH parameters and significantly change the calibration accuracy and uncertainty; therefore, the time offset should be tested by allowing it to vary as one phase of the optimization process.

Unit hydrographs analyzed from measured rainfall and runoff data will depend on the extent to which the measured rainfall hyetograph is in synchronization with the runoff hydrograph. According to linear UH theory, the direct runoff hydrograph begins at the same time as the rainfall excess occurs. In actual measured rainfall-runoff data, however, the observed direct runoff hydrograph often occurs before or after the beginning time of the rainfall excess due to the spatial locations of the rainfall and runoff gauges and the velocity of the storm. Previous analyses (see section 4.4) have shown that, if a time offset is existing but not considered when conducting the calibration, it can significantly distort the actual UH parameters and worsen the calibration accuracy. When analyzing measured data, in most cases, the information about the spatial locations of the rainfall and runoff gauges as well as the velocity of the storm are insufficient and hard to quantify. Therefore, adding a time offset by moving the rainfall excess more closely to or apart from the direct runoff hydrograph can either improve or worsen the calibration accuracy. If a time offset seems to exist in the measured storm data, several values of time offsets should be tested when conducting calibration to see whether or not the accuracy is sensitive to time offset. If the accuracy is sensitive to time offset, then the sensitivity to the offset must be made.

Awareness of this guideline is critical for UH calibration. The time offset is a common issue with measured data and can have significant influence on the calibrated UH parameters. Optimizing the offset will yield the more accurate and less uncertain UH parameters.

4.8.5 <u>Guideline</u>: The ability of the gamma distribution to accurately represent the hydrologic response of a watershed is limited by its flexibility; therefore, the selection of data sets to use in calibrating the gamma UH model needs to respect this limitation of the model.

The gamma pdf is generally a right-tail skewed model with a set half-width relation that is dependent on the values of the two parameters. The two parameters limit the flexibility of the gamma UH. A gamma UH cannot have more than 50% under its rising limb. The flexibility can be increased by adding a location (time offset) parameter:

$$f(t) = \frac{(t-t_0)^{c-1} e^{-(t-t_0)/b}}{b^c \Gamma(c)} \qquad (4\text{-}5)$$

The gamma UH is not sufficiently flexible to adjust to some conditions. Generally, poor goodness-of-fit statistics can result if

(1) The data characteristics are too complex to be fitted with a two-parameter model (i.e., a great dissimilarity between a rainfall excess hyetograph and observed direct runoff hydrograph);

(2) The existing time offset between rainfall excess and direct runoff is ignored or not corrected during calibration;

(3) The observed direct runoff hydrograph shows a left-tail skewed shape rather than a right-tail skewed shape (see section 4.5.1);

(4) The width of the observed direct runoff hydrograph is either too thin or two wide (see section 4.5.2); and

(5) The difference in times to peak of the rainfall excess hyetograph and the observed direct runoff hydrograph is too short (see section 4.5.3).

Awareness of this guideline is critical when analyzing the measured data. It provides model users with tools to understand the calibration results and assist in the selection of rainfall-runoff data sets.

4.8.6 <u>Guideline</u>: The spatial nonuniformity of rainfall over a watershed can cause variations of the calibrated UH parameters; therefore, if several rainfall hyetographs are available from multiple rain gauges, each hyetograph should be analyzed to assess the potential variations in the UH parameters for that watershed.

During actual storm events, the spatial nonuniformity of rainfall is a common factor. This phenomenon is most evident when a watershed has a large area. In terms of UH calibration, the analyses in section 4.6 showed that rainfall nonuniformity could be a significantly influential factor and cause variations of the calibrated UH parameters. Rainfall nonuniformity is evident by the variability of the hyetographs provided by rain gauges at different locations. In order to understand the calibration results and determine the most reasonable and representative UH parameters, each hyetograph should be used to calibrate UH parameters and then to use a weighted averaged value of these calibrated UH parameters.

4.8.7 <u>Guideline</u>: Given the assumption of linearity of the UH model, the extent of the nonlinearity of the watershed processes can reduce the calibration accuracy. The dynamic convolution model, which has more flexibility than the static model, can lead to but not guarantee improved goodness of fit.

Considering the variability and complexity of the watershed processes during a rainfall event, the watershed processes can easily be nonlinear. Thus, the UH model would need to vary as time progresses over the duration of the event. When calibrating a UH from measured data, a dynamic UH model can sometimes improve the accuracy over that provided by the static model. The analyses conducted in section 4.7 showed that using time-dependent convolution can lead to improved goodness of fit and in some cases more reasonable coefficients. However, it should be noted that the flexibility of the dynamic model may also be insufficient to capture the watershed processes due to the variability and the complexity of the watershed processes. The dynamic model requires an additional variable to allow the UH parameters to vary with time.

In summary, the dynamic model has the potential to provide more accurate calibrated parameters. Awareness of this general guideline is very important since it points out another limitation of the static convolution model and provides another potential reason for the poor fitting when analyzing measured data.

CHAPTER 5: ANALYSIS OF MEASURED DATA

5.1. INTRODUCTION

The objective of this chapter is to investigate the effectiveness and usefulness of the general guidelines that were developed in Chapter 4 using measured rainfall and runoff data. A number of measured storm data from various watersheds were selected to calibrate unit hydrographs. A detailed analysis for each storm event was presented in an attempt to assess the practicability of the guidelines in understanding unit hydrograph calibration results and in accurately estimation of UH parameters. A summary of analyses of these measured data was presented.

It would be difficult to develop general guidelines from measured storm event due to the inherent variability of both the time to peak and the PRF of the UH. The variability was evident from the size of ellipse in the Se/Sy response surface. This same variability would be inherent to measured data, but other factors may actually increase the uncertainty beyond that inherent to the synthetic data.

5.1.1. Objectives of Analyzing Measured Data

The ultimate objective of this research is to develop general guidelines that can help researchers to calibrate the UH parameters (i.e., the tp_{UH} and PRF). Having guidelines for analyzing measured storm data could lead to greater accuracy and to better understand the problems in interpreting the calibration results. In chapter 4, the effects of various factors on UH calibration were discussed. General guidelines that are relevant to these factors were developed. Most of these general guidelines were developed based on analyses of synthetic data with the assumption that the true UH parameters were known. Additionally, they depend on the assumption that the initial abstraction and baseflow were already separated from the rainfall excess. In analyzing measured data, however, the following confounding factors are inherent to UH analysis: (1) the true UH parameters are embedded in the measured data and not obtainable; (2) the initial abstraction and baseflow are subjectively determined; and (3) rainfall hyetographs and runoff hydrographs are much more variable in form than that showed in synthetic data. Considering the variability and complexity of measured storm event data, analyzing measured rainfall hyetographs and runoff hydrographs data is much more complex than analyzing synthetic data. Therefore, efforts are needed to assess the effectiveness and practicability of these general guidelines when they are used in analyzing the measured rainfall and runoff data. Specifically, the objectives of analyzing measured data can be summarized as:

(1) to assess the ability of the complete UH analysis model to derive UH parameters from measured data;

(2) to evaluate the ability of the general guidelines of chapter 4 to assist in interpreting the calibration results (e.g., large/small PRF, long/short tp_{UH}, good/poor accuracy); and

(3) to incorporate additional guidelines derived from analyses of measured data with previous general guidelines; this should provide a more comprehensive set of guidelines.

5.1.2. Measured Rainfall and Runoff Data

Measured rainfall and runoff data in this study were obtained from the small watershed data base of the USDA for Hydrologic Data for Experimental Agricultural Watersheds in the United States. This data base includes many storm events with associate rainfall and runoff data for different watersheds at many locations. In this database, the location, area, slope, land use, and antecedent moisture for each of these watersheds are provided. The watershed morphologies and locations of rainfall and runoff gauges for some of these watersheds are also presented. For the measured storm data, the rainfall intensity is recorded in inches per hour; the rainfall depth is then computed in inches; the runoff rate is recorded in cubic feet per second or inches per hour; and transformed to runoff depths in inches. For the purpose of the reported analyses, the units of the selected rainfall and runoff data will be converted to inches per minute.

In this chapter, 29 storm events were selected from 25 small agricultural watersheds, located in Virginia, Mississippi, Ohio, Iowa, Wisconsin, Missouri, Nebraska, Oklahoma, Texas, Colorado, New Mexico, and Arizona. Each of the storm events had a rainfall depth greater than 0.7 inch, with most of them (25 of 29) having a rainfall depth greater than 1 inch. The maximum rainfall depth was 4.17 inches.

5.2. METHODS OF ANALYSIS

For each storm event, the UH parameters were calibrated with goodness-of-fit statistics reported (i.e., relative bias and Se/Sy). A detailed analysis of each event was then made in an attempt to assess the practicability of the general guidelines in understanding the calibration result and in calibrating the UH parameters more accurately. It should be noted that the relative bias was essentially zero for every case as the UH depths were unity. Small biases resulted in some analyses because the calibration required the tp_{UH} to be an integer.

The steps for conducting the calibration and analyses are well specified in Chapter 3. In summary, the primary steps can be can be expressed as follows:

(1) Separate the rainfall excess and the direct runoff from the measured rainfall-runoff data (see sections 3.3 and 3.4);

(2) Develop a gamma UH based a tp_{UH} and a PRF;

(3) Convolve the rainfall excess hyetograph of step (1) with the gamma UH of step (2) to develop a computed direct runoff hydrographs;

(4) Compare the computed direct runoff hydrograph in step 3 with the observed direct runoff hydrograph of step 1 and calculate the Se/Sy and relative bias;

(5) Repeat steps (2) to (4) for all reasonable UHs to find the best-fit (minimum Se/Sy) computed direct runoff hydrograph; the gamma UH that resulted in the best-fit computed direct runoff hydrograph was assumed to be the watershed gamma UH with the corresponding calibrated UH parameters;

(6) Report the calibrated UH parameters and goodness-of-fit statistics;

(7) Analyze the accuracy and the rationality of the storm data and the calibration results.

5.3. RESULTS OF MEASURED DATA ANALYSES

The analysis of each event is summarized in Table A-1 in Appendix A. The primary goodness-of-fit criterion is the Se/Sy. If the Se/Sy < 0.3, the accuracy is good; if the 0.3 < Se/Sy < 0.6, the accuracy is relatively good; if the 0.6 < Se/Sy < 0.75, the accuracy is relatively poor; if the Se/Sy > 0.75, the accuracy is poor; and if Se/Sy > 1, the accuracy is extremely poor. The detailed analysis for each storm event were presented as follows:

Watershed: Chestnut Branch   Location: Blacksburg, Virginia   Date: 07/11/1965

The P(t) and the RO(t) show great similarity in form (see Figure 5-1). As evident from the data, both of the P(t) and the RO(t) have abrupt and steep rising limbs, as well as relatively shallow and long recession limbs. The similarity in form indicates that the watershed either had little natural storage or had been nearly saturated prior to the rainfall event. Thus, the intense rainfall at the beginning of the storm event caused an abrupt increase of runoff. When calibrating the UH, either a short tp_{UH} or a high PRF or both should be expected because the runoff from such a UH should basically reflect the characteristics of the rainfall.

Calibration of the UH provided a short tp_{UH} of 3 minutes and a PRF of 110, but with a very poor fit (Se/Sy=1.042). In this case, this result indicates that other factors had a greater influence on the goodness of fit than did the similarity in form. Firstly, as evident from the data, the RO(t) occurred approximately one hour later than the P(t). Such a delay in time (i.e., temporal offset between rainfall and runoff) caused most of the P(t) to be separated as initial abstraction or losses (0.772 in.) and only a very small part of the P(t) could be actually used as Pe(t) (0.008 in.). Due to the limited volume, the Pe(t) would not be able to retain the characteristics of the P(t) and would further result in a severely distorted computed DRO(t) (i.e., the volume of rainfall = the volume of direct runoff). Therefore, a time offset could be tested to more closely align the P(t) and the RO(t). Secondly, since the RO(t) has a narrow half-width, the gamma UH is likely not sufficiently flexible to fit such form.

In summary, the poor fit was caused by both the temporal offset time between the rainfall and runoff and the limited flexibility of the gamma pdf. Due to the poor accuracy, the calibrated UH parameters for this storm event are not representative of the temporal watershed response.

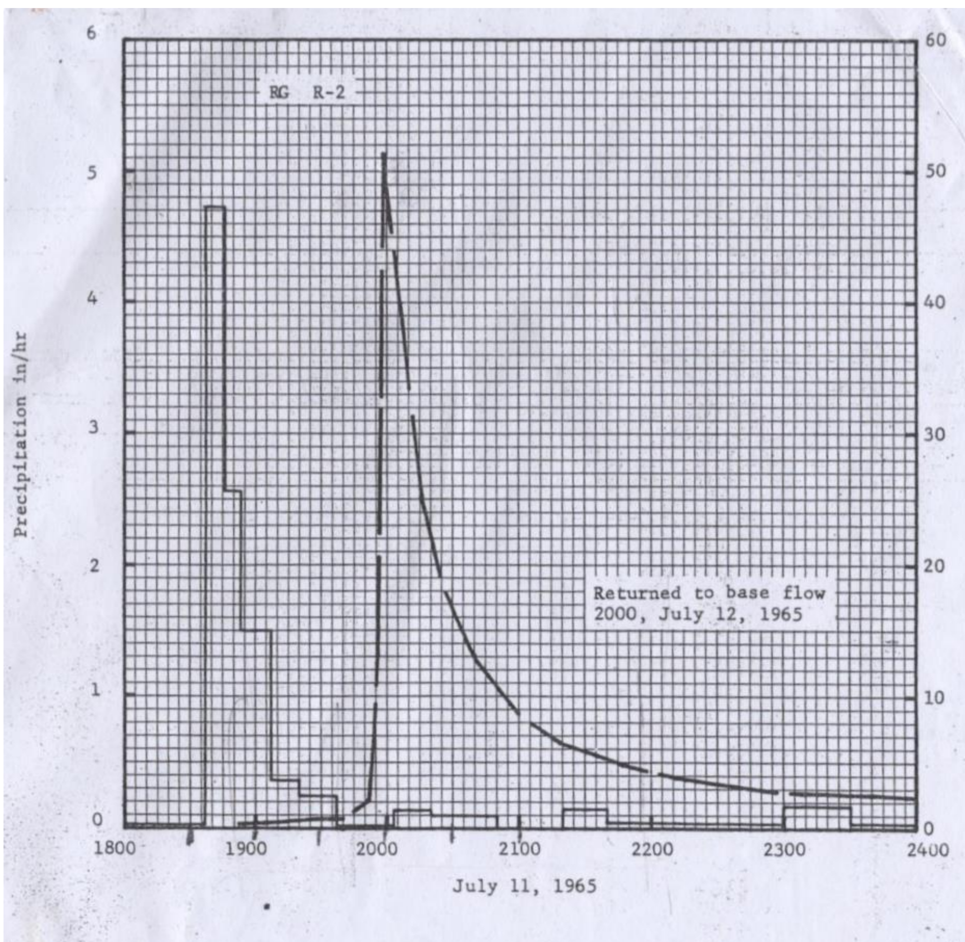

Figure 5-1. Storm event occurred on 07/11/1965 on watershed Chestnut Branch, at Blacksburg, Virginia [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1965].

<u>Watershed</u>: 130 **<u>Location</u>: Coshocton, Ohio** **<u>Date</u>: 06/12/1957**

The P(t) and the RO(t) have similar half-widths (see Figure 5-2) and two evident peaks. Most of the P(t) extended into the RO(t). The first and the second major peaks of the P(t) seemed to cause the first and second peaks of the the RO(t), respectively.

In calibration, the first major peak of the P(t) was used as initial abstraction, with most of the second peak of the P(t) appearing as Pe(t) (n_{pe} = 26 min; v_{pe} = 1.208 in.). The Pe(t) and DRO(t) have similar times to peak of 26 minutes and 31 minutes, respectively. Calibration yielded a very long tp_{UH} of 94 minutes and a low PRF of 110, with relatively good accuracy (Se/Sy = 0.4781). The tp_{UH} seems too long, given the small difference between tp_{Pe} and tp_{DRO}. The low PRF seems irrational considering the similarity in form between the P(t) and the RO(t). The PRF is not consistent with the proportion (65.4%) under the rising limb of the DRO(t). In summary, although a good fit resulted, the calibrated UH parameters are not reasonable.

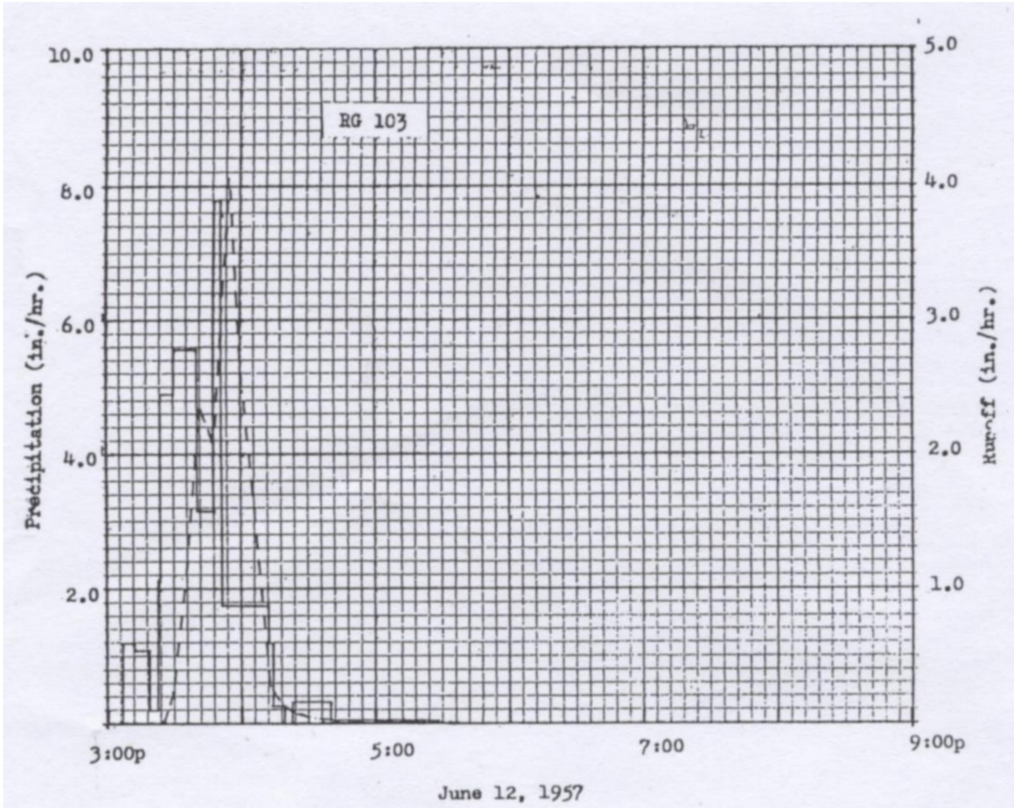

Figure 5-2. Storm event occurred on 06/12/1957 on watershed 130, at Coshocton, Ohio [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1957].

<u>Watershed</u>: 130 <u>Location</u>: Coshocton, Ohio <u>Date</u>: 06/28/1957

The P(t) shows more variation than the RO(t) (see Figure 5-3). The last burst of the P(t) seemed to cause the RO(t). The P(t) greatly extends into the RO(t).

The P(t) that occurred before 6:00 pm was used as the initial abstraction, with approximately one-fifth of the P(t) appearing as Pe(t) (n_{pe} = 56 min; v_{pe} = 0.446 in.). The difference in the times to peak (tp_{Pe} = 36 min; tp_{DRO} = 49 min) of the Pe(t) and DRO(t) is relatively small. The duration of the DRO(t) is 30 minutes longer than that of the Pe(t). Calibration yielded a very long tp_{UH} of 132 minutes and a low PRF of 131, with poor accuracy (Se/Sy = 0.7726). The tp_{UH} seems too long, given the relatively small difference between tp_{Pe} and tp_{DRO}. The PRF is too low and not consistent with the proportion (79.0%) under the rising limb of the DRO(t). The poor fit was probably caused by the considerable overlap between the P(t) and the DRO(t). In summary, the calibrated UH parameters are not sufficiently representative of the watershed response.

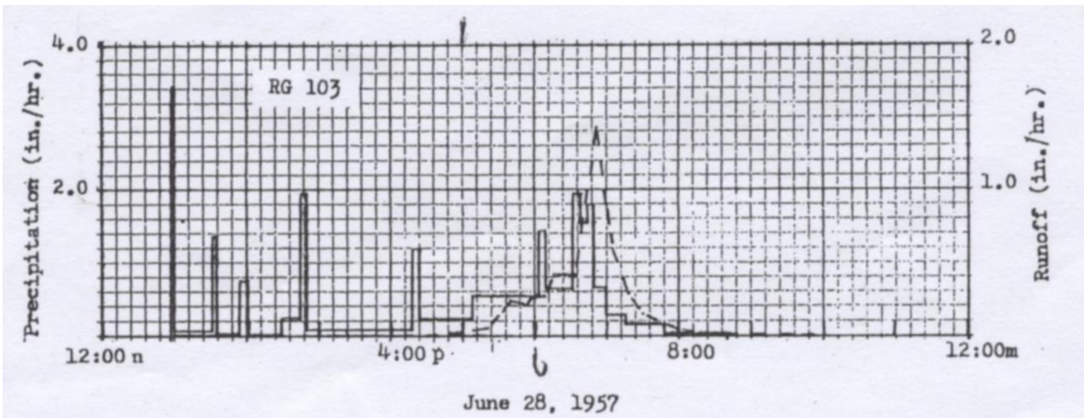

Figure 5-3. Storm event occurred on 06/28/1957 on watershed 130, at Coshocton, Ohio [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1957].

<u>Watershed</u>: 132 <u>Location</u>: Coshocton, Ohio <u>Date</u>: 05/13/1964

The P(t) and the RO(t) show great dissimilarity in form (see Figure 5-4). The P(t) has a center-loaded form with a relatively short time base of 40 minutes. The RO(t) has a flat form with an extremely long time base of more than 6 hours. This dissimilarity in form indicates that the watershed had large natural storage at the time of this event. Therefore, either a low PRF or a long tp_{UH} or both should be expected because the UH basically smoothed out the rainfall characteristics.

When calibrating the UH, the delay in time between the P(t) and the RO(t) caused most of the P(t) to be initial abstraction or losses (0.77 in.), with only a small part of rainfall appearing as rainfall excess (0.42 in.). Consequently, the Pe(t) has a peaked form, while the DRO(t) had an extremely flat form. Calibration yielded a short tp_{UH} of 2 minutes and a very low PRF of 10, with a poor fit (Se/Sy=0.7726). The tp_{UH} is short and likely due to the small difference between the tp_{Pe} (12 minutes) and the tp_{DRO} (28 minutes). The low PRF is reasonable because it forces the computed DRO(t) to have a flat form that would be needed to mimic the flat form of the measured direct runoff. The poor fit was likely due to the long time base (382 minutes) of the measured direct runoff.

In summary, the poor fit resulted from the extremely long time base of the RO(t). Due to the poor accuracy, the calibrated UH parameters for this storm event are not representative of the temporal watershed response.

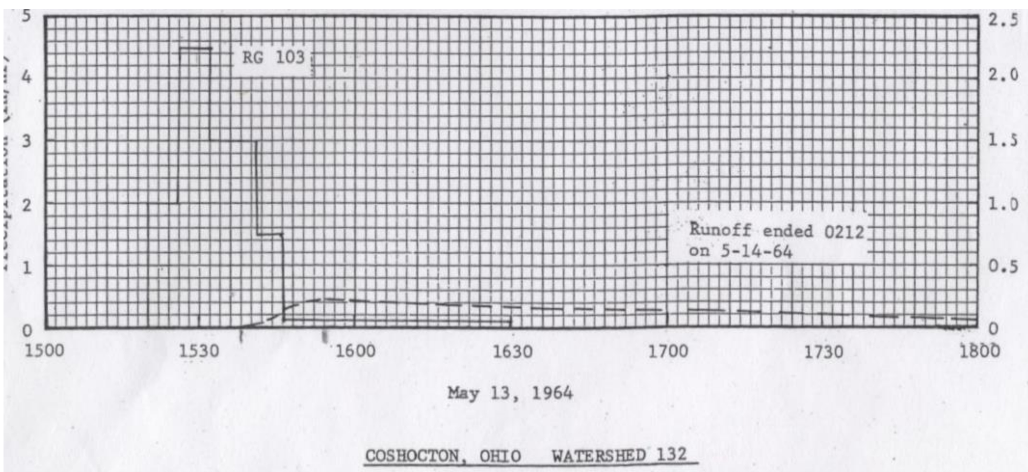

Figure 5-4. Storm event occurred on 05/13/1964 on watershed 132, at Coshocton, Ohio [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1964].

Watershed: 196 Location: Coshocton, Ohio Date: 05/13/1964

Both the P(t) and the RO(t) have similar half-widths and abrupt increases at the beginning the storm event (see Figure 5-5),. The recession limb of the P(t) extended into the RO(t). The RO(t) had a much longer recession limb than that of the

P(t). The RO(t) occurred about 30 minutes following the P(t) and the total depth of P(t) is low. A time offset may exist.

Most of the P(t) was used as initial abstraction with only a small part of the P(t) appearing as Pe(t) (n_{pe} = 20 min; v_{pe} = 0.1780 in.). The Pe(t) and DRO(t) have times to peak of 14 minutes and 33 minutes, respectively. Calibration yielded a short tp_{UH} of 7 minutes and a low PRF of 164, with a reasonable goodness of fit (Se/Sy = 0.317). The short tp_{UH} was likely due to the small difference between tp_{Pe} and tp_{DRO} since the calibration model would force the UH to minimize the mismatch between peaks of the computed and measured DRO(t)s. The low PRF probably resulted from the short time base (i.e., 20 min) of Pe(t) and the long time base (i.e., 128 min) of the DRO(t) because n_{DRO}-n_{pe}+1 was likely large. The low PRF was consistent the small proportion (13%) under the rising limb of the DRO(t).

In summary, the calibrated UH parameters are reasonable, with a relatively good fit. A time offset could be tested to more closely align the P(t) and the RO(t).

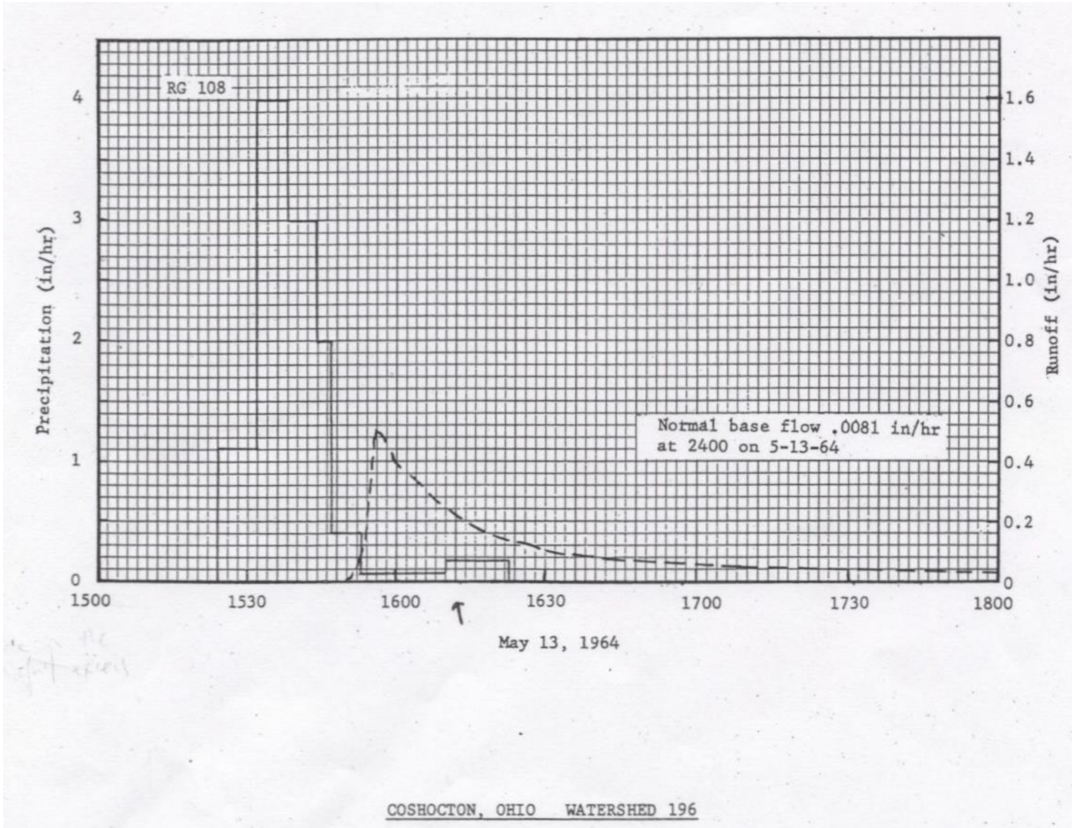

Figure 5-5. Storm event occurred on 05/13/1964 on watershed 196, at Coshocton, Ohio [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1964].

Watershed: 196 Location: Coshocton, Ohio Date: 06/16/1946

As shown in Figure 5-6, the P(t) has three evident peaks while the RO(t) has two less evident peaks. The second and third peaks of the PE(t) seemed to cause the two peaks of the RO(t). Both the P(t) and the RO(t) have very long time bases. The P(t) greatly extends through the RO(t). The half-width of RO(t) is slightly narrower than that of the P(t).

The first peak and the long tail of the P(t) were cut off as initial abstraction. The part of P(t) that overlapped with the RO(t) was separated as Pe(t) (n_{pe} = 57 min;

v_{pe} = 0.668 in.). The Pe(t) and DRO(t) have close times to peak (tp_{Pe} = 44 min; tp_{DRO} = 57 min). The calibration results show that a very long tp_{UH} of 57 minutes and a relatively small PRF of 132, with an irrationally poor fit (Se/Sy = 2.51). For this storm data, multiple factors have caused in the poor fit. Firstly, the great complexity and dissimilarity in form (e.g., from three peaks to two peaks) between the P(t) and the RO(t) might suggest a nonlinear process in the watershed that cannot be captured by the assumed linear convolution model. Secondly, since the P(t) greatly extends through the RO(t), the UH model might have difficulty in reproducing the RO(t) due to the fitting flexibility of gamma pdf. Additionally, the closeness between tp_{Pe} and tp_{DRO} might be another factor that limited the ability to fit.

In summary, the extremely poor fit was caused by multiple factors including the complexity of storm data, the dissimilarity between P(t) and R(t), and the limited flexibility of the gamma pdf. Due to the extremely poor accuracy, the calibrated UH parameters for this storm event are not representative of the temporal watershed response.

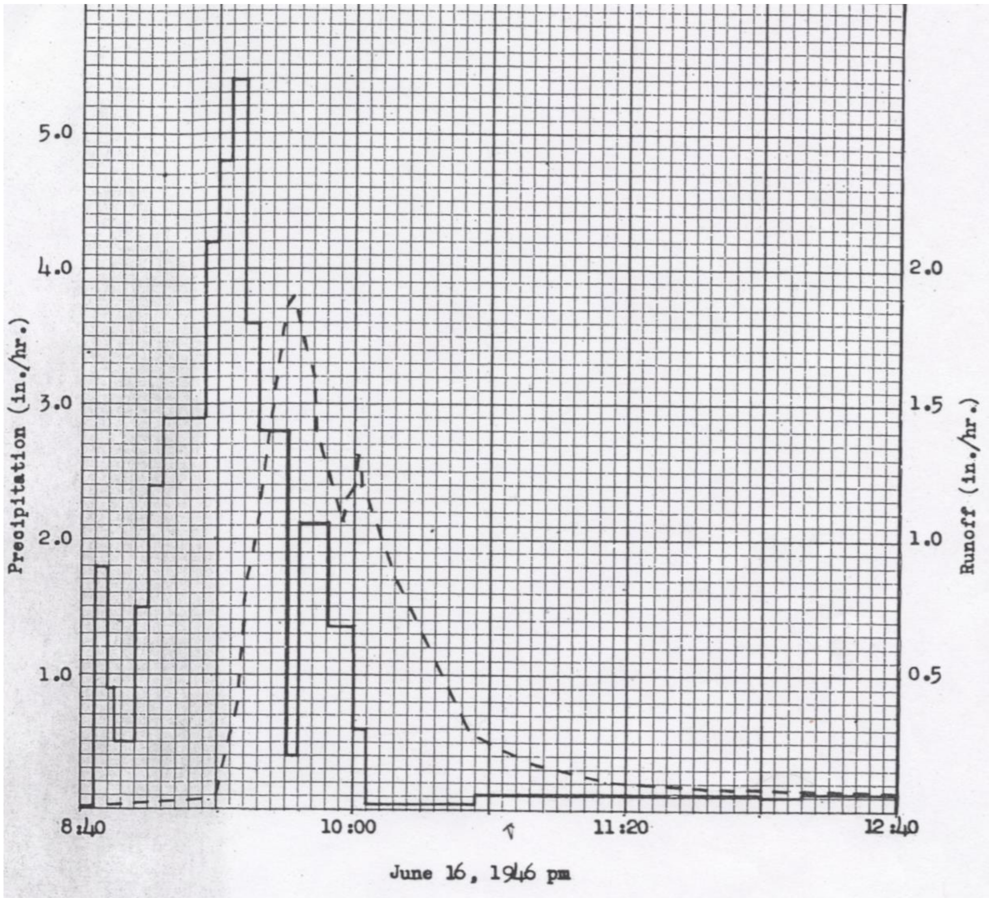

Figure 5-6. Storm event occurred on 06/16/1946 on watershed 196, at Coshocton, Ohio [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1946].

Watershed: WC-1 Location: Oxford, Mississippi Date: 07/08/1965

As shown in the data (see Figure 5-7), both the P(t) and the RO(t) have two peaks and an abrupt increase at the beginning of the rainfall event. The P(t) greatly extends through the RO(t). The half-width of RO(t) is slightly narrower than that of the P(t). The RO(t) occurs about 8 minutes later than the P(t), which indicates a time offset might exist.

In calibration, a small part of P(t) that occurred before the start of the RO(t) was used as initial abstraction, with most of the rainfall used as Pe(t) (n_{pe} = 44 min;

v_{pe} = 1.214 in.). The Pe(t) and DRO(t) have essentially the same times to peak (tp_{Pe} = 13 min; tp_{DRO} = 11 min) and time bases (n_{pe} = 44 min; n_{DRO}= 43 min). A very long tp_{UH} of 62 minutes and a PRF of 484 were calibrated, with a very poor fit (Se/Sy = 1.03), For this storm data, the poor fit resulted from two principal factors, the closeness of times to peak and the closeness of time bases. Generally, since a UH model is a smoothing function, the tp_{DRO} and n_{DRO} should be greater than tp_{Pe} and n_{pe}, respectively. In this case, however, both of tp_{Pe} and n_{pe} were even greater than those of the RO(t), which consequently yielded irrational UH parameters with poor accuracy. In order to test the effect of time offset, a 20-min time offset was added to more closely align the P(t) and the RO(t). The result of adding a temporal offset did not improve calibration accuracy.

In summary, the poor fit was caused by the closeness of the times to peak and the closeness of the time bases. The calibrated UH parameters for this storm event are not representative of the temporal watershed response.

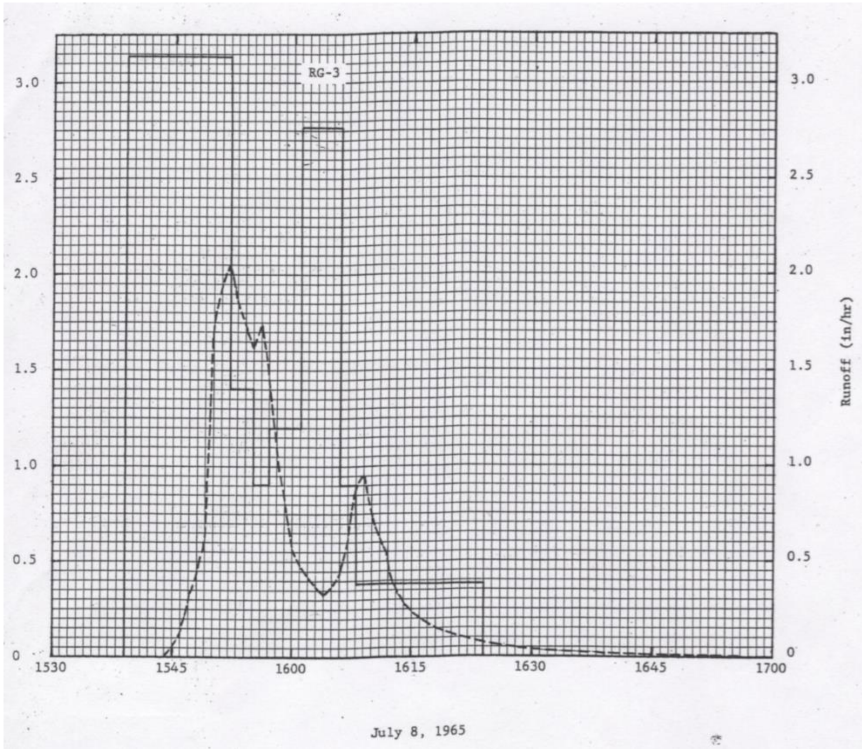

Figure 5-7. Storm event occurred on 07/08/1965 on watershed WC-1, at Oxford, Mississippi [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1965].

Watershed: WC-1 Location: Oxford, Mississippi Date: 07/09/1967

By comparing the rainfall and runoff data (see Figure 5-8), it is evident that the P(t) shows much more variation than that of the RO(t). The RO(t) retains the basic characteristics (i.e., half-width, peak rate) of the P(t) but most of the small peaks of the P(t) were smoothed out.

The majority of the P(t) was used as Pe(t) (n_{pe} = 75 min; v_{pe} = 1.113 in.). The Pe(t) and DRO(t) have close times to peak (tp_{Pe} = 62 min; tp_{DRO} = 66 min). Calibration yielded an irrational long tp_{UH} of 109 mins (approximately two times longer than the Pe(t) and DRO(t)) and an extremely large PRF of 944, with a very poor fit (Se/Sy = 1.51). For this storm data, the poor fit probably resulted from the complexity of rainfall and runoff data and the closeness of times to peak. Firstly, the gamma UH based on a linear convolution model might not be sufficiently flexible to fit the complex (i.e., multiple peaks, less uniformity) storm data that might be caused by a nonlinear watershed processes. Secondly, in most cases, the closeness of times to peak of the Pe(t) and DRO(t) will generally force a small tp_{UH}; This would allow the model to match the peaks of the Pe(t) and DRO(t) with minimum errors. In this case, since the rainfall data have a complex, multi-peaked form, an irrational tp_{UH} resulted. A 20-min time offset was added to more closely align the P(t) and the RO(t) to seek a better accuracy. The result, however, indicated that adding temporal offset did not improve but even worsened the accuracy (Se/Sy changed from 1.51 to 1.60). This results support the previous findings that adding temporal offset does not always guarantee improvement in calibration accuracy.

In summary, the poor fit was caused by the limitation of the gamma UH model to fit complex data. Due to the poor accuracy, the calibrated UH parameters for this storm event are not representative of the temporal watershed response.

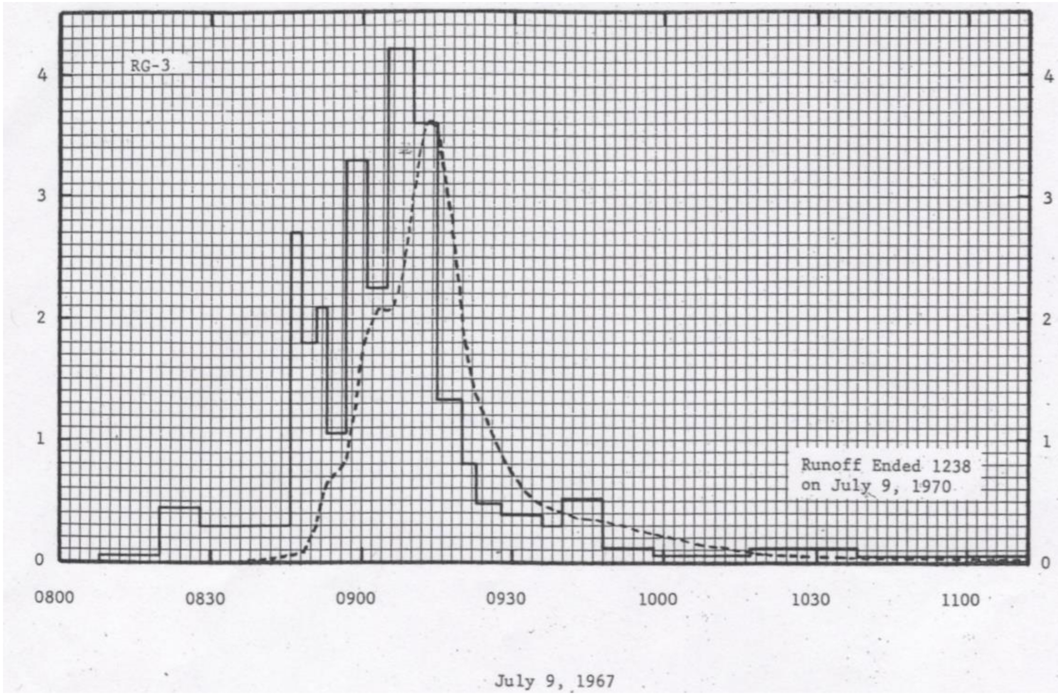

Figure 5-8. Storm event occurred on 07/09/1967 on watershed WC-1, at Oxford, Mississippi [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1967].

Watershed: WC-2 Location: Oxford, Mississippi Date: 07/08/1965

As shown in Figure 5-9, both the P(t) and the RO(t) show two peaks. Most of the P(t) extends into the RO(t). Compared with the P(t), the RO(t) has a flatter, uniform form. Since the RO(t) occurs about 20 minutes later than the P(t), a time offset might exist.

The entire first peak and a small part of the second peak of the P(t) was considered to be initial abstraction with the remaining part of the P(t) being Pe(t) (n_{pe} = 27 min; v_{pe} = 0.321 in.). The Pe(t) and DRO(t) have similar times to peak (tp_{Pe} = 13 min; tp_{DRO} = 18 min). Calibration yielded a very short tp_{UH} of 1 minute and a very small PRF of 43, with somewhat poor accuracy (Se/Sy = 0.687). The short tp_{UH} is reasonable and probably resulted from the closeness of the tp_{Pe} and tp_{DRO}. As mentioned previously, the closeness of the times to peak between Pe(t) and DRO(t) will generally force a small tp_{UH} because the calibration will emphasize the peaks of the Pe(t) and DRO(t). The small PRF is most likely caused by the large difference in the time bases (n_{pe} = 27; n_{DRO} = 75).

Convolution forces a small value of the PRF to generate a long time base of UH (n_{UH} = n_{DRO}-n_{pe} +1=49). Therefore, the calibrated PRF is reasonable and consistent with the small proportion under the rising limb (11.4%). The poor accuracy was likely due to the dissimilarity in form between the Pe(t) and DRO(t). The first peak of the Pe(t) was extracted while the RO(t) still maintained the two-peaked form. Fitting a one-peaked Pe(t) to a two-peaked RO(t) reduced the accuracy. In order to improve the accuracy, a time offset could be tested to utilize more P(t) as the Pe (t). The calibrated UH parameter are reasonable given the dissimilarities of the rainfall and the runoff.

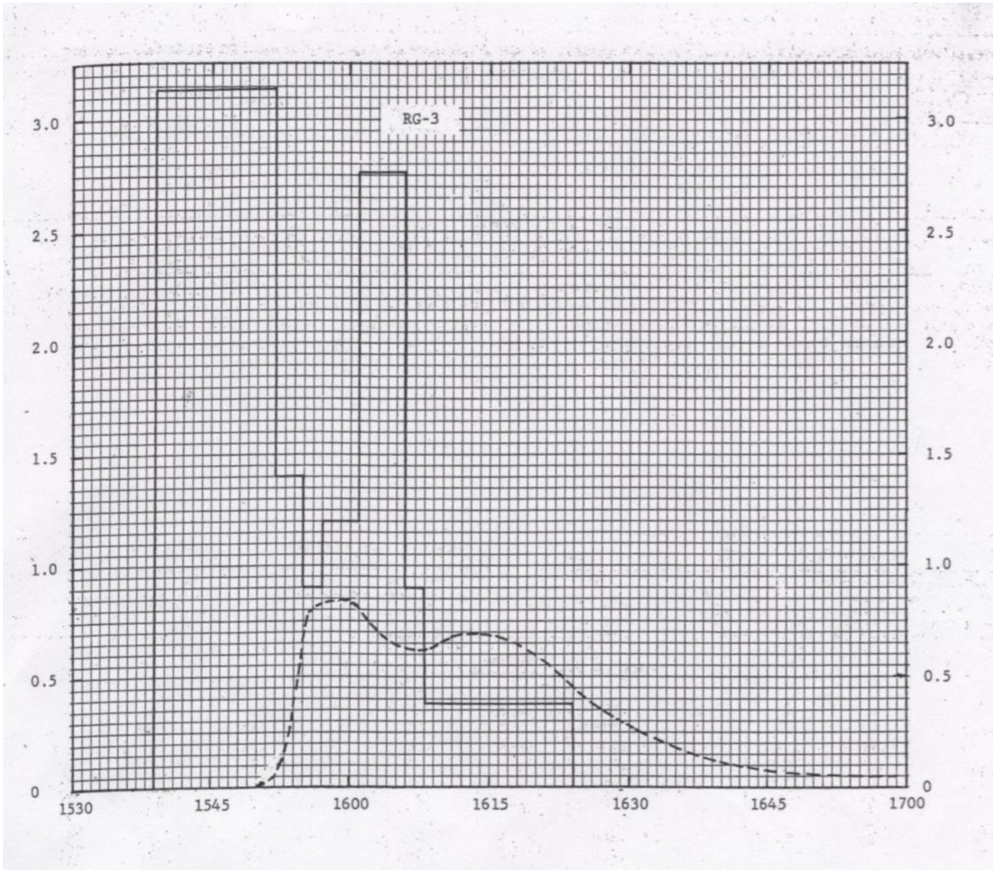

Figure 5-9. Storm event occurred on 07/08/1965 on watershed WC-2, at Oxford, Mississippi [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1965].

Watershed: W-5 Location: Oxford, Mississippi Date: 05/31/1967

As evident from Figure 5-10, both the P(t) and the RO(t) show similar one-peaked forms. The RO(t) has a much longer time base and a slightly wider half-width than that of the P(t).

During calibration, most of the P(t) was used as initial abstraction and losses, with only a small part of the P(t) assumed as Pe(t) (n_{pe} = 60 min; v_{pe} = 0.352 in.). The Pe(t) and DRO(t) have large differences in the times to peak (tp_{Pe} = 60 min; tp_{DRO} = 100 min) and in the time bases (n_{pe} = 75 min; n_{DRO}=315 min). Calibration yielded a very small tp_{UH} of 1 minute and an extremely low PRF of 8, with a very poor accuracy (Se/Sy = 1.501). The calibrated tp_{UH} seemed too short, while the calibrated PRF was too small and is not consistent with the proportion (34%) under the rising limb of the DRO(t). The poor fit probably resulted from the difficulty in fitting the extremely long tail of DRO(t). Since only a small part of the P(t) was used as the Pe(t), a time offset could be tested to move the P(t) more closely to the RO(t).

In summary, the calibrated UH parameters are irrational and not convincing. The poor fit was mostly likely due to the limited flexibility of the gamma pdf and the extremely long tail of the DRO(t).

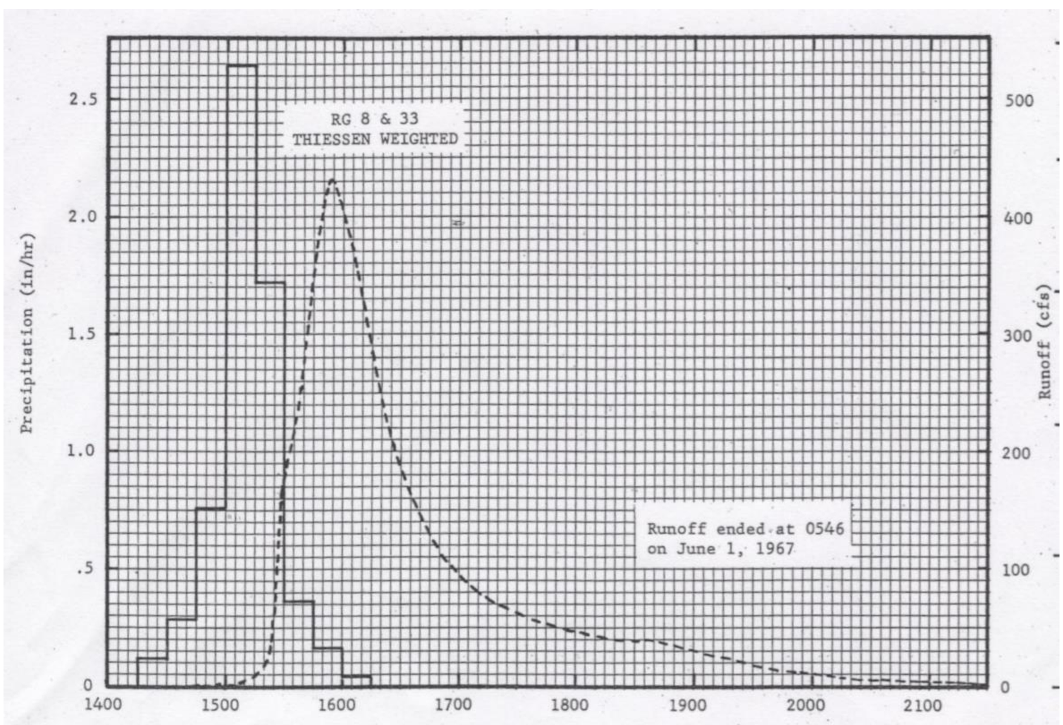

Figure 5-10. Storm event occurred on 05/31/1967 on watershed W-5, at Oxford, Mississippi [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1967].

Watershed: W-10 **Location: Oxford, Mississippi** **Date: 05/31/1967**

As shown in Figure 5-11, the P(t) and the RO(t) show considerable dissimilarity in form. The P(t) has a two-peaked complex form while the RO(t) has a smooth one-peaked center-loaded form. The RO(t) has a much longer time base than that of the P(t). Since the P(t) started at approximately the same time as the P(t), a time offset was not tested.

In calibration, most of the P(t) was initial abstraction and losses and only a small part of P(t) appeared as Pe(t) (n_{pe} = 75 min; v_{pe} = 0.188 in.). The Pe(t) and DRO(t) have large differences in both the times to peak (tp_{Pe} = 75 min; tp_{DRO} = 124 min) and the time bases (n_{pe} = 75 min; n_{DRO}=195 min). Calibration yielded a tp_{UH} of 36 minutes and a PRF of 204, with relatively good accuracy (Se/Sy = 0.3916). The UH parameters seem reasonable and consistent with the observations on the storm data. The good fit was probably due to the relatively simple forms of the Pe(t) and DRO(t).

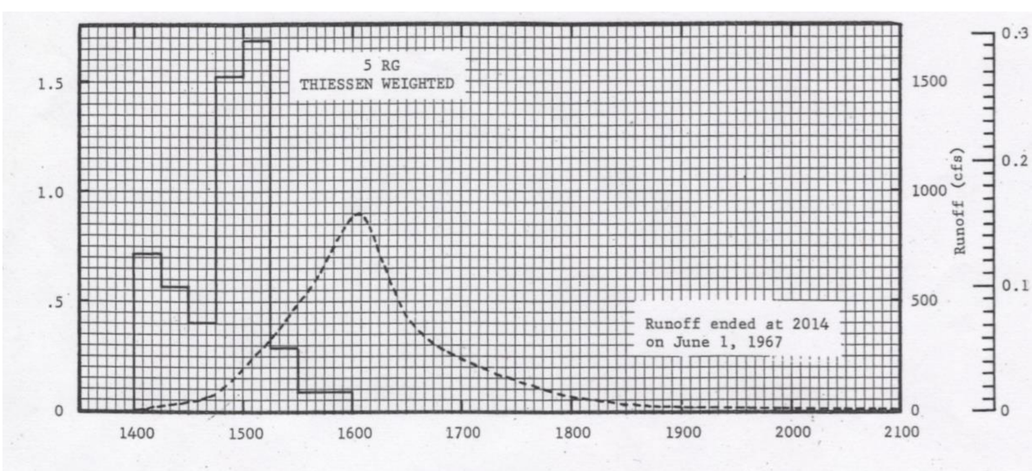

Figure 5-11. Storm event occurred on 05/31/1967 on watershed W-10, at Oxford, Mississippi [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1967].

<u>Watershed</u>: W-10 <u>Location</u>: Oxford, Mississippi <u>Date</u>: 08/26/1971

By comparing the rainfall and runoff data (see Figure 5-12), it is evident that the P(t) shows a more complex form than that of the RO(t). The P(t) has a complex multiple-peaked form while the RO(t) has a simply one-peaked form. Since the RO(t) occurs about 1 hr later than the P(t), a time offset might exist.

In calibration, most of the P(t) was used as initial abstraction and losses and only a small part of the P(t) was measuredly used as Pe(t) (n_{pe} = 105 min; v_{pe} = 0.506 in.). The Pe(t) and DRO(t) have large differences in both the times to peak (tp_{Pe} = 75 min; tp_{DRO} = 147 min) and the time bases (n_{pe} = 105 min; n_{DRO}=310 min). As a result of calibration, a tp_{UH} of 1 minute and a PRF of 21, with a relatively poor accuracy (Se/Sy = 0.711). The calibrated tp_{UH} seems too short. The calibrated PRF is too small and is not consistent with the proportion (0.449) under the rising limb of the DRO(t). The poor fit was probably caused by the low depth of the Pe(t). Due to the limited volume, the Pe(t) would not be able to retain the characteristics of the P(t) and would further result in a severely distorted measured direct runoff. In order to involve more rainfall into calibration, a 1-hour time offset was used to align the P(t) and the RO(t) more closely. However, the result shows that the time offset did not improve the accuracy as it forced the P(t) to extend into the DRO(t). In summary, the calibrated UH parameters are not accurate.

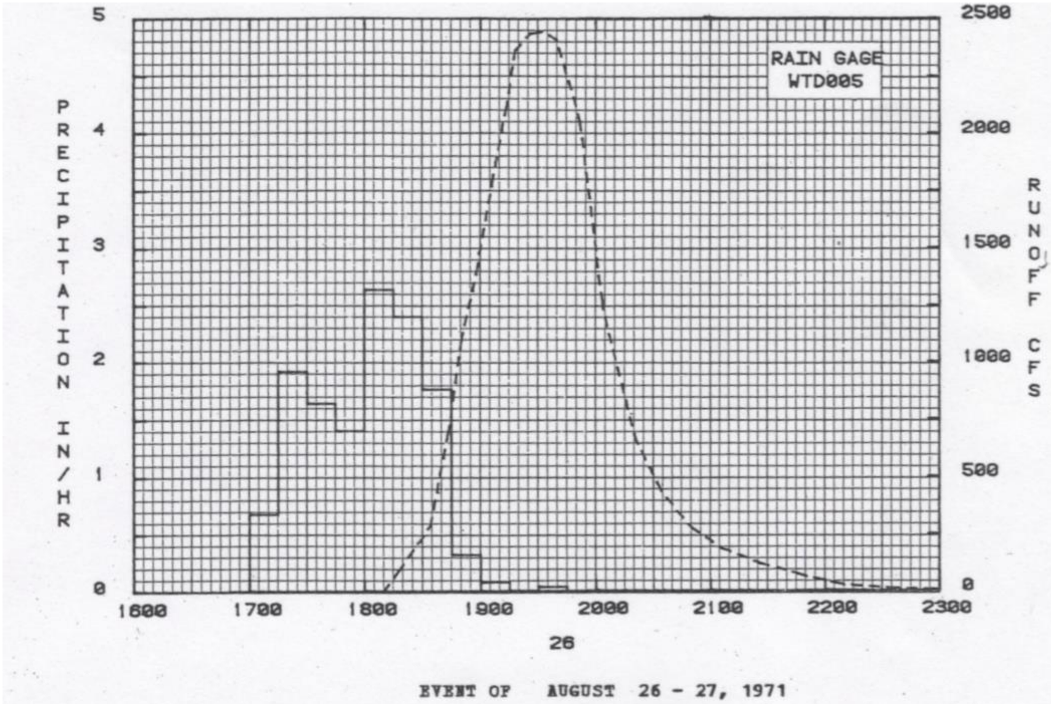

Figure 5-12. Storm event occurred on 08/26/1971 on watershed W-10, at Oxford, Mississippi [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1971].

<u>Watershed</u>: W-17 <u>Location</u>: Oxford, Mississippi <u>Date</u>: 03/04/1964

As evident in Figure 5-13, the P(t) shows more complexity in the form than that of the RO(t), which has a simple center-loaded form. The P(t) does not extend into the RO(t). The half-width of RO(t) is wider than that of P(t).

In calibration, most of the P(t) was used as initial abstraction and losses, with approximately one-third of P(t) appearing as Pe(t) (n_{pe} = 269 min; v_{pe} = 0.575 in.). The Pe(t) and DRO(t) have large differences in both the times to peak (tp_{Pe} = 255 min; tp_{DRO} = 444 min) and the time bases (n_{pe} = 269 min; n_{DRO} = 793 min). Calibration yielded a very long tp_{UH} of 200 minutes and a relatively large PRF of 398, with very good accuracy (Se/Sy = 0.1835). Given the large differences in the times to peak and in the time bases, both the long tp_{UH} and high PRF are reasonable.

The good accuracy probably resulted from the fact that the half-width of RO(t) is rationally wider than that of P(t), and the P(t) does not greatly extend into the RO(t). Under these conditions, the gamma pdf is sufficiently flexible to fit the data. In summary, the calibrated UH parameters are reasonable.

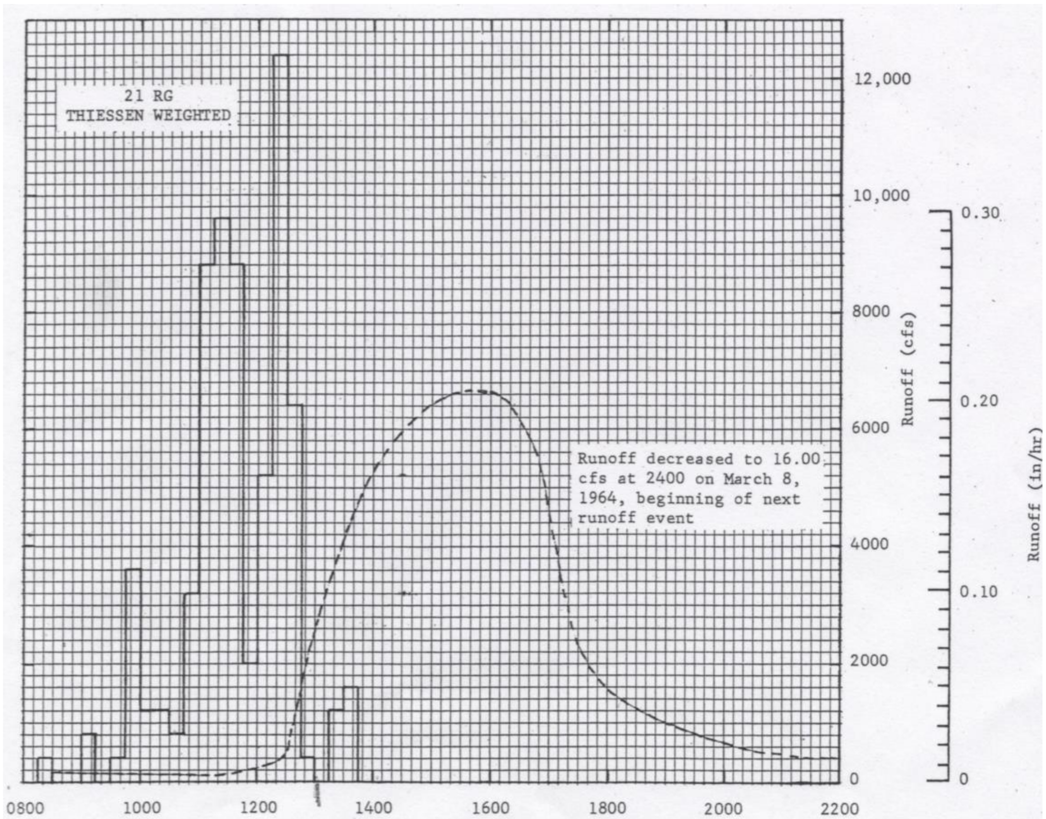

Figure 5-13. Storm event occurred on 03/04/1964 on watershed 17, at Oxford, Mississippi [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1964].

<u>Watershed</u>: W-32 <u>Location</u>: Oxford, Mississippi <u>Date</u>: 06/18/1971

Both the P(t) and the RO(t) show simple center-loaded forms with little variation (see Figure 5-14). Compared with the P(t), the RO(t) shows a much wider half-width and a longer time base. There is basically no overlap between the P(t) and the RO(t) and thus a time offset may exist.

In calibration, most of the P(t) was used as initial abstraction and losses with only a small part of the P(t) appearing as Pe(t) (n_{pe} = 45 min; v_{pe} = 0.323 in.). The Pe(t) and DRO(t) have large differences in both the times to peak (tp_{Pe} = 45 min; tp_{DRO} = 136 min) and the time bases (n_{pe} = 45 min; n_{DRO}=315 min). A tp_{UH} of 11 minutes and a PRF of 65 were calibrated, but with a relatively poor Se/Sy of 0.7670. The poor accuracy was probably due to the low depth of the Pe(t). The limited volume of Pe(t) would not be able to retain the characteristics of the P(t) and would further result in a severely distorted measured direct runoff. The extremely long time base of the RO(t) may be another contributing factor to the poor accuracy since the gamma pdf is not sufficiently flexible to match the extremely long tail of the DRO(t).

In order to involve more rainfall and increase the volume of Pe(t) used in analysis, a time offset of 30 minutes was added to align the P(t) and the RO(t) more closely. The result, however, indicates that adding time offset did not significantly improve the calibration accuracy (i.e., Se/Sy changed from 0.7670 to 0.7588). The poor fit was likely due to the low depth of the Pe(t) and the limited flexibility of the gamma pdf to fit an extremely long-tailed DRO(t). Due to the poor accuracy, the calibrated UH parameters are not representative of the watershed response.

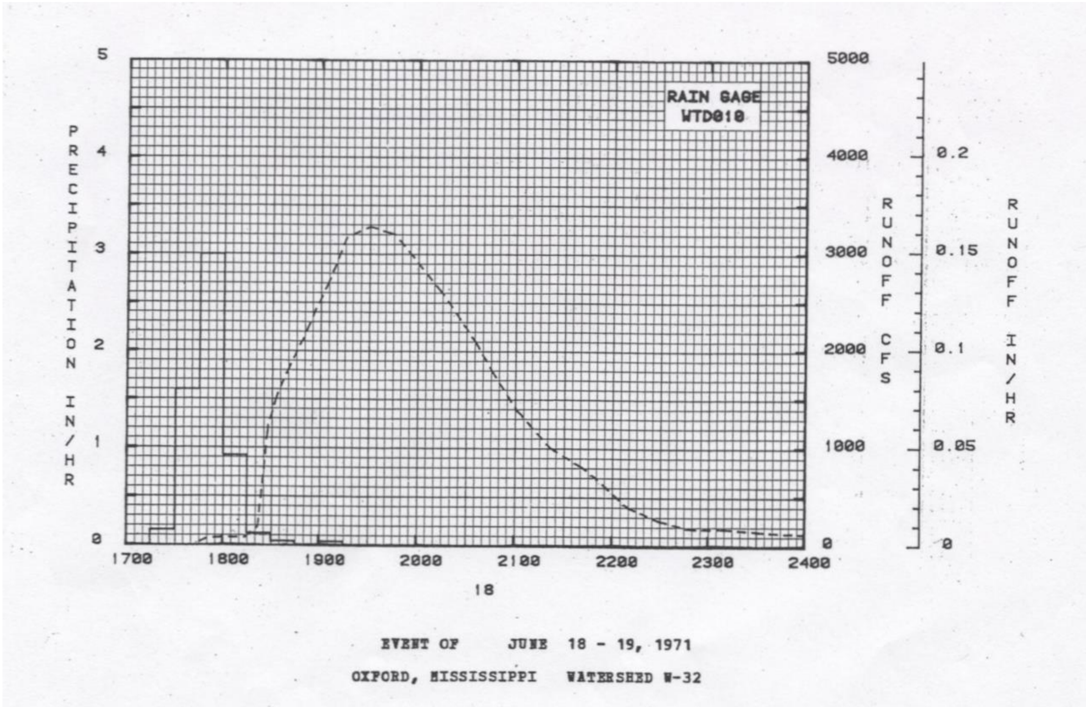

Figure 5-14. Storm event occurred on 06/18/1971 on watershed 32, at Oxford, Mississippi [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1964].

<u>Watershed</u>: W-35A **<u>Location</u>: Oxford, Mississippi** **<u>Date</u>: 06/18/1971**

As evident from Figure 5-15, both the P(t) and the RO(t) show simple center-loaded forms. The P(t) does not greatly extend into the RO(t). The half-width of RO(t) is only slightly wider than that of P(t).

In calibration, the middle part of P(t) was used as Pe(t) (n_{pe} = 30 min; v_{pe} = 0.696 in.). The Pe(t) and DRO(t) have a large difference in the times to peak (tp_{Pe} = 30 min; tp_{DRO} = 109 min) and the time bases (n_{pe} = 30 min; n_{DRO}=270 min). Calibration yielded a tp_{UH} of 28 minutes and a PRF of 231, with good accuracy (Se/Sy = 0.2401). Given the large differences in the times to peak and time bases, both the long tp_{UH} and PRF are reasonable. The good accuracy probably resulted from the facts that (1) both the P(t) and the RO(t) has simple forms; (2) the half-width of RO(t) is wider than that of P(t); and (3) the P(t) does not greatly extend into the RO(t). Under these conditions, the gamma pdf is sufficiently flexible to fit the data. In summary, the calibrated UH parameters are rational.

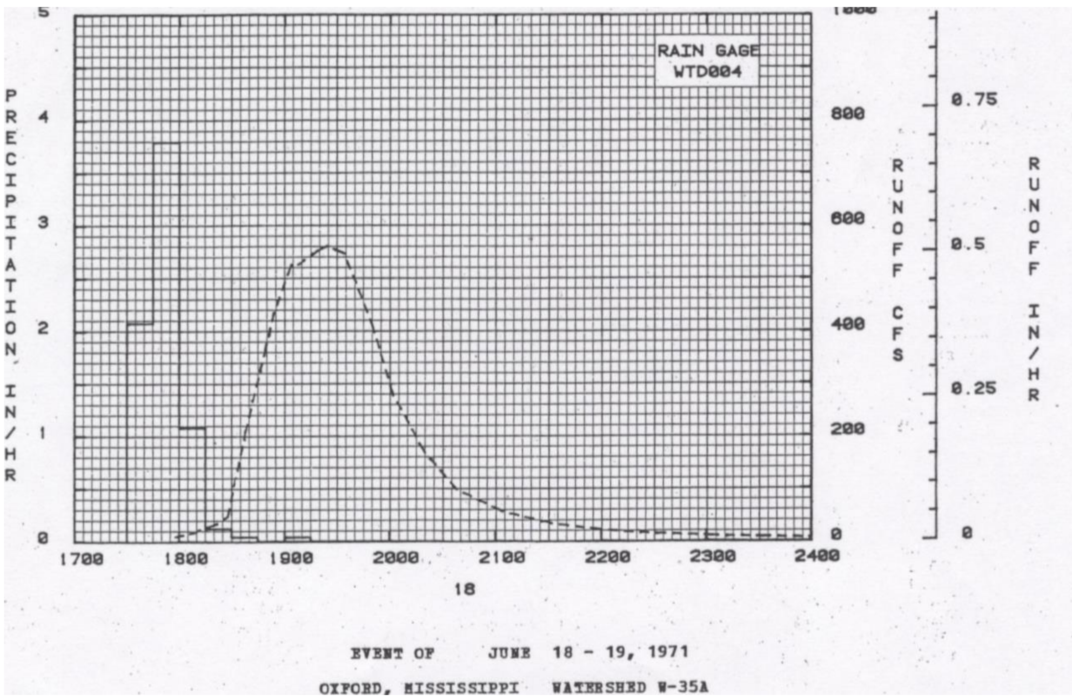

Figure 5-15. Storm event occurred on 06/18/1971 on watershed 35A, at Oxford, Mississippi [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1971].

Watershed: W-4 Location: Fennimore, WISCONSIN Date: 08/12/1943

As evident from Figure 5-16, both the P(t) and the RO(t) show similar half-widths, right-skewed forms, and sharp rises at the beginning of the storm event. The P(t) and the RO(t) show some temporal overlap.

In calibration, most of the P(t) was used as initial abstraction and losses and only a small part of the P(t) was assumed as Pe(t) (n_{pe} = 10 min; v_{pe} = 0.369 in.). The Pe(t) and DRO(t) have a small difference in the times to peak (tp_{Pe} = 5 min;

tp_{DRO} = 16 min) and a large difference in the time bases (n_{pe} = 10 min; n_{DRO}=72 min). Calibration yielded a very small tp_{UH} of 3 minutes and a low PRF of 107, with good accuracy (Se/Sy = 0.2127). The short tp_{UH} is likely due to the small difference between tp_{Pe} and tp_{DRO} since the calibration model would force the UH to minimize the mismatch between peaks of the computed and the measured DRO(t). The low PRF of 107 probably resulted from the large difference in the time bases because n_{DRO}-n_{pe}+1 was likely large. The low PRF of 107 is also consistent with the small proportion (14.6%) under the rising limb of the RO(t). In summary, the calibrated UH parameters are reasonable representations of the storm event with a good fit.

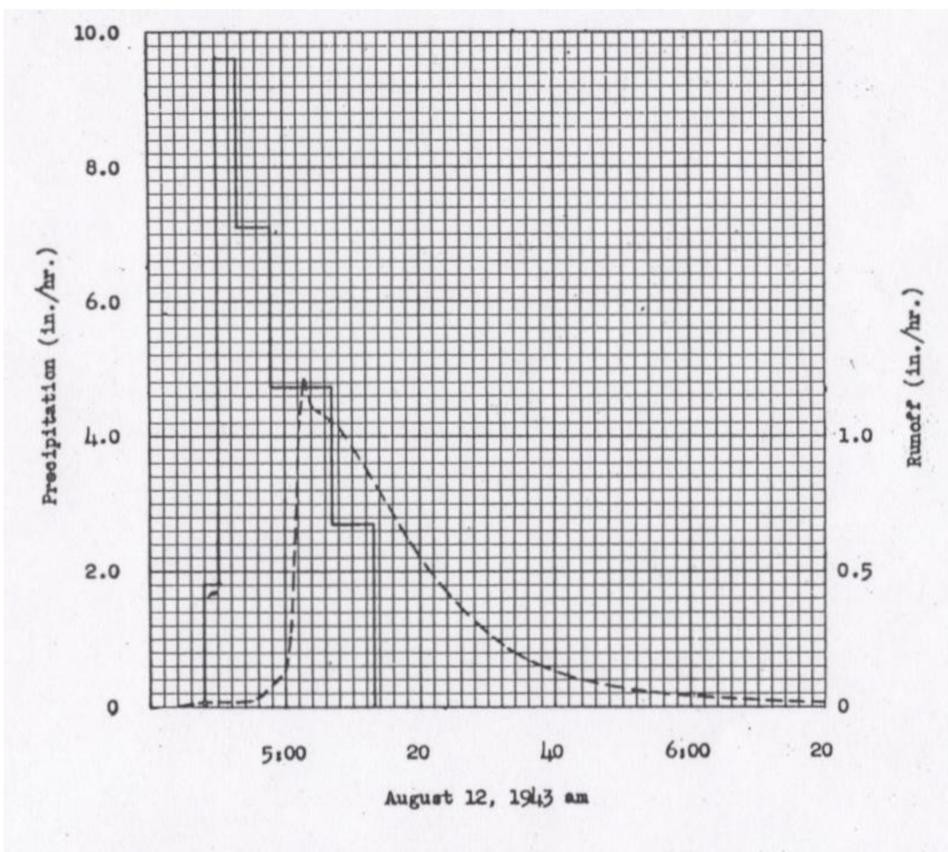

Figure 5-16. Storm event occurred on 08/12/1943 on watershed W-4, at Fennimore, Wisconsin [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1943].

Watershed: **Location**: **Iowa City, Ralston Creek** **Date**: **07/18/1956**

As is evident in Figure 5-17, both the P(t) and the RO(t) show right-skewed forms and sharp increases at the beginning of the storm event. Compared with the RO(t), the P(t) shows more variation in form. Since little P(t) overlapped into the RO(t), a temporal offset may be needed.

In calibration, most of the P(t) was used as initial abstraction and losses, with only a small part of the P(t) appearing as Pe(t) (n_{pe} = 44 min; v_{pe} = 0.686 in.). The Pe(t) and DRO(t) have large differences in both the times to peak (tp_{Pe} = 24 min; tp_{DRO} = 125 min) and time bases (n_{pe} = 44 min; n_{DRO}=224 min). Calibration yielded a very small tp_{UH} of 2 minutes and a very low PRF of 32, with good accuracy (Se/Sy = 0.1942). Since only a small part of the P(t) was used as the Pe(t), the Pe(t) would not be able to retain the characteristics of the P(t). Therefore, even though good accuracy resulted, the calibrated UH parameters are not representative of the watershed response.

In order to increase the volume of Pe(t) into the analysis, a time offset of 30 minutes was used to align the P(t) and the RO(t) more closely. The results, however, indicated that using a time offset did not improve but even worsened the calibration accuracy (Se/Sy changed from 0.1942 to 0.7689). This is probably because more of the runoff extended into the rainfall; therefore, the data may be more difficult to fit because of the limited flexibility of gamma pdf. This is another example that shows that adding a time offset cannot guarantee better accuracy but can even worsen the accuracy depending on the extent of overlap of the rainfall and runoff as well as the effects of other factors. In summary, the calibrated UH parameters are not representative of this watershed.

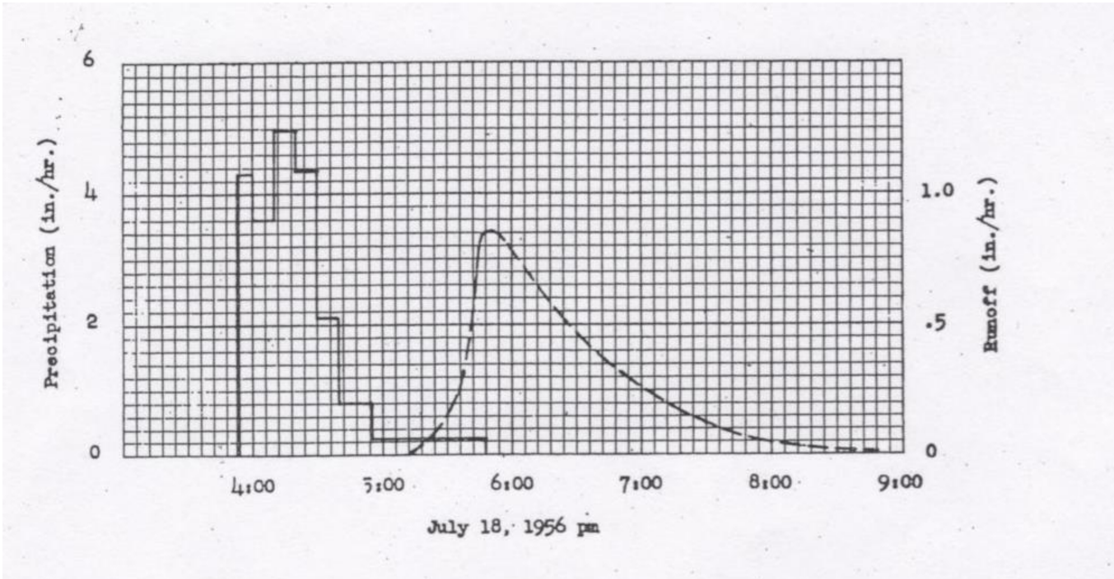

Figure 5-17. Storm event occurred on 07/18/1956 on watershed Iowa City, Ralston Creek [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1956].

<u>Watershed</u>: W-1 <u>Location</u>: McCredie, MO <u>Date</u>: 08/19/1949

The P(t) and the RO(t) show great dissimilarity in form (see Figure 5-18). The P(t) has multiple small peaks and a long flat tail that extends into the RO(t). The RO(t) shows two major peaks and a long tail. Since the P(t) and the RO(t) basically occurred at the same time such that a time offset seems not required.

In calibration, most of the P(t) was used as initial abstraction, with only a small part of the P(t) appearing as Pe(t) (n_{pe} = 29 min; v_{pe} = 0.174 in.). The Pe(t) and DRO(t) have a small difference in the times to peak (tp_{Pe} = 18 min; tp_{DRO} = 44 min) but a large difference in the time bases (n_{pe} = 29 min; n_{DRO}=103 min). Calibration yielded a very small tp_{UH} of 3 minutes and a very low PRF of 31, with a relatively poor accuracy (Se/Sy = 0.6943). The poor fit was probably due to the limited flexibility of the gamma UH to transform a nearly one-peaked rainfall into a two-peaked runoff. Since the UH is a smoothing function, converting a one-peaked form to a two-peaked form by using gamma pdf is theoretically impossible with linear convolution. Another contributory factor was the considerable overlap between the P(t) and the RO(t). In summary, the calibrated UH parameters are not representative and convincing due to the poor accuracy.

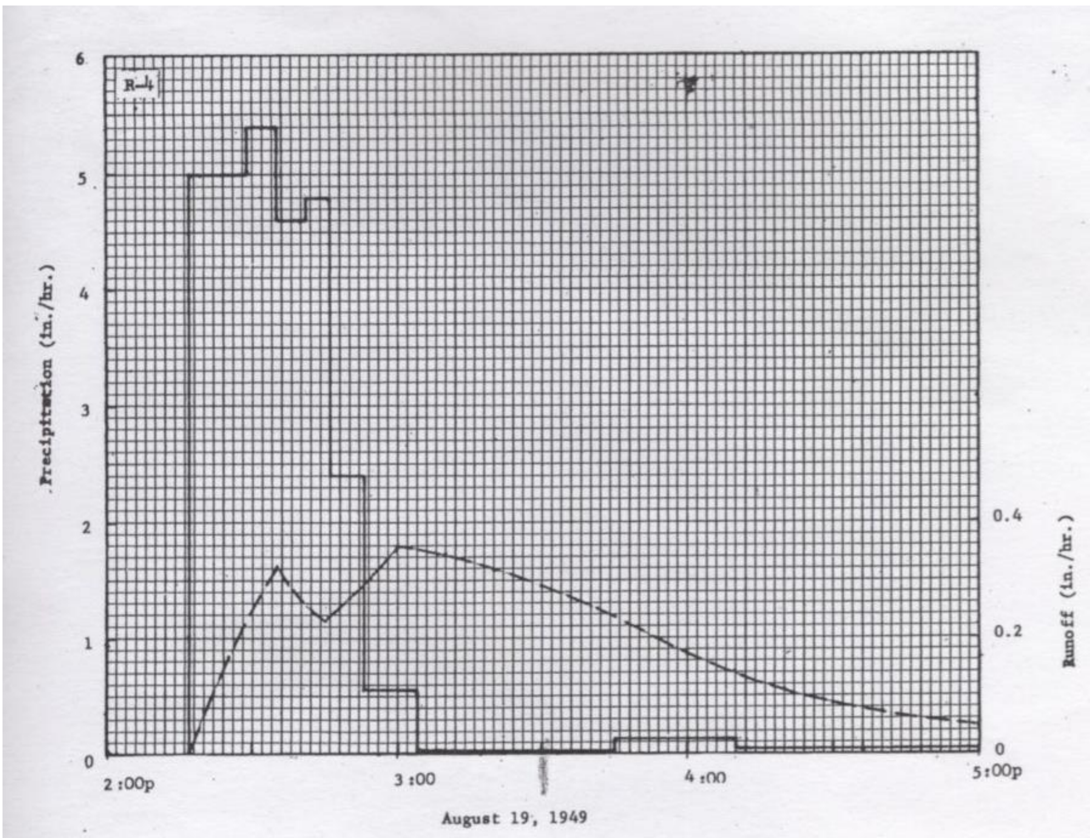

Figure 5-18. Storm event occurred on 08/19/1949 on watershed W-1, at McCredie, Missouri [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1949].

Watershed: W-4H **Location: Hastings, Nebraska** **Date: 07/08/1967**

As is shown in the Figure 5-19, both the P(t) and the RO(t) show two separate peaks. Both of the two peaks of P(t) greatly extended into the RO(t). The UH analyses were conducted for the individual section of the event.

In the case of the first section, a small part of the P(t) was used as the Pe(t) (n_{pe} = 13 min; v_{pe} = 0.103 in.). The Pe(t) and DRO(t) have a small difference in the times to peak (tp_{Pe} = 13 min; tp_{DRO} = 29 min) and a relatively large difference in the time bases (n_{pe} = 13 min; n_{DRO}=49 min). Calibration yielded a very short tp_{UH} of 2 minutes and very low PRF of 45, with very poor accuracy (Se/Sy = 0.9549). The short tp_{UH} resulted from the small difference between tp_{Pe} and tp_{DRO}. The poor fit was probably due to the low depth of the Pe(t) and the considerable overlap between the Pe(t) and DRO(t).

In the case of the second peak, the major part of the P(t) was retained as the Pe(t) (n_{pe} = 29 min; v_{pe} = 0.550 in.). The Pe(t) and DRO(t) have similar times to peak (tp_{Pe} = 15 min; tp_{DRO} = 14 min) and a relatively large difference in the time bases (n_{pe} = 29 min; n_{DRO}=54 min). The calibration produced an extremely short tp_{UH} of 1 minute and a relatively high PRF of 528, with relatively good accuracy (Se/Sy = 0.4523). The short tp_{UH} is reasonable given the small difference between tp_{Pe} and tp_{DRO}.

Comparing these two analyses, the calibration of the second peak shows better accuracy than that of the first part of the event. This occurred because most of the first peak of the rainfall was lost at the beginning of the event to saturate the soil surface, with little Pe(t); for the second part of the rainfall, however, since the surface is relatively saturated, a greater part of the rainfall occurred as the Pe(t), which produced a direct runoff that could retain more characteristics of the rainfall.

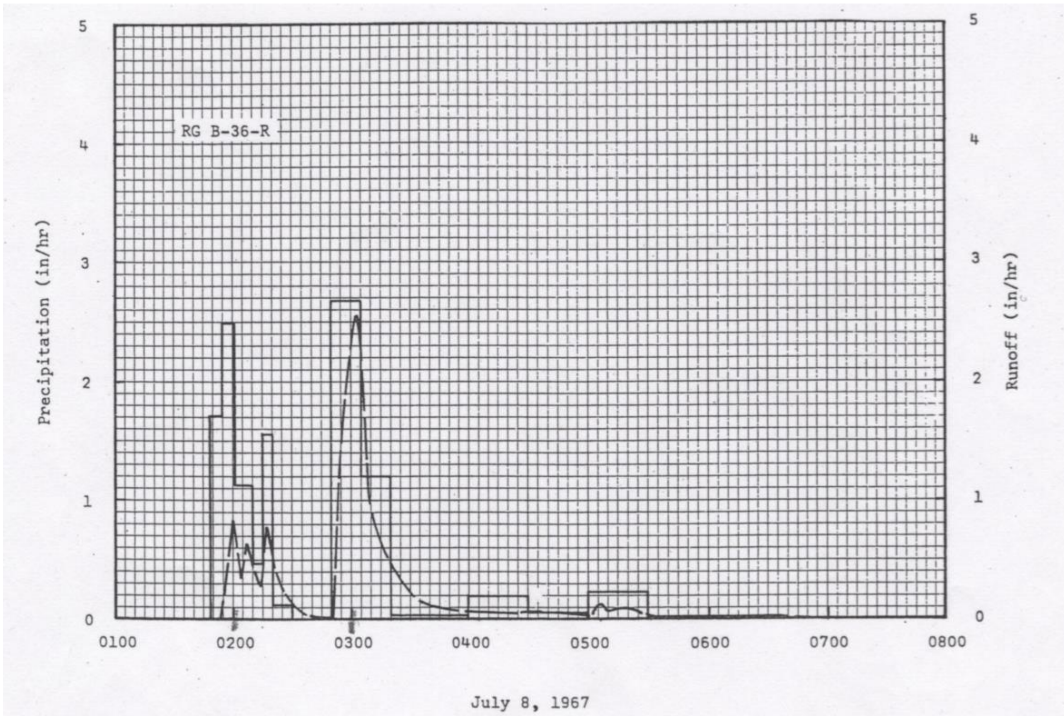

Figure 5-19. Storm event occurred on 07/08/1967 on watershed W-4H, at Hastings, Nebraska [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1967].

<u>Watershed</u>: 511 <u>Location</u>: Chickasha, Oklahoma <u>Date</u>: 05/29/1970

The P(t) and the RO(t) show great dissimilarity in form (see Figure 5-20). Compared with the P(t), the RO(t) shows a much wider half-width and longer time base. Additionally, little rainfall extended into the RO(t). Since the P(t) and the RO(t) are greatly separated, a time offset may exist.

In calibration, most of the P(t) was used as initial abstraction and losses, with only a small part of the P(t) appearing as Pe(t) (n_{pe} = 19 min; v_{pe} = 0.076 in.). The Pe(t) and DRO(t) have extremely large differences in both the times to peak (tp_{Pe} =

19 min; tp_{DRO} = 379 min) and the time bases (n_{pe} = 19 min; n_{DRO}=540 min). This forced a long tp_{UH} of 86 minutes and a moderate PRF of 266, with relatively good accuracy (Se/Sy = 0.3983). The calibrated UH parameters seem reasonable. However, due to the limited volume of the Pe(t), the Pe(t) may not reflect the typical watershed response.

In order to involve more rainfall into the analysis, a time offset of 2 hours was added to more closely align the P(t) and the RO(t). The time offset did not significantly improve the calibration accuracy (i.e., Se/Sy changed from 0.3983 to 0.3812). In summary, due to the limited volume of Pe(t), the calibrated UH parameters may not sufficiently representative of the temporal watershed response.

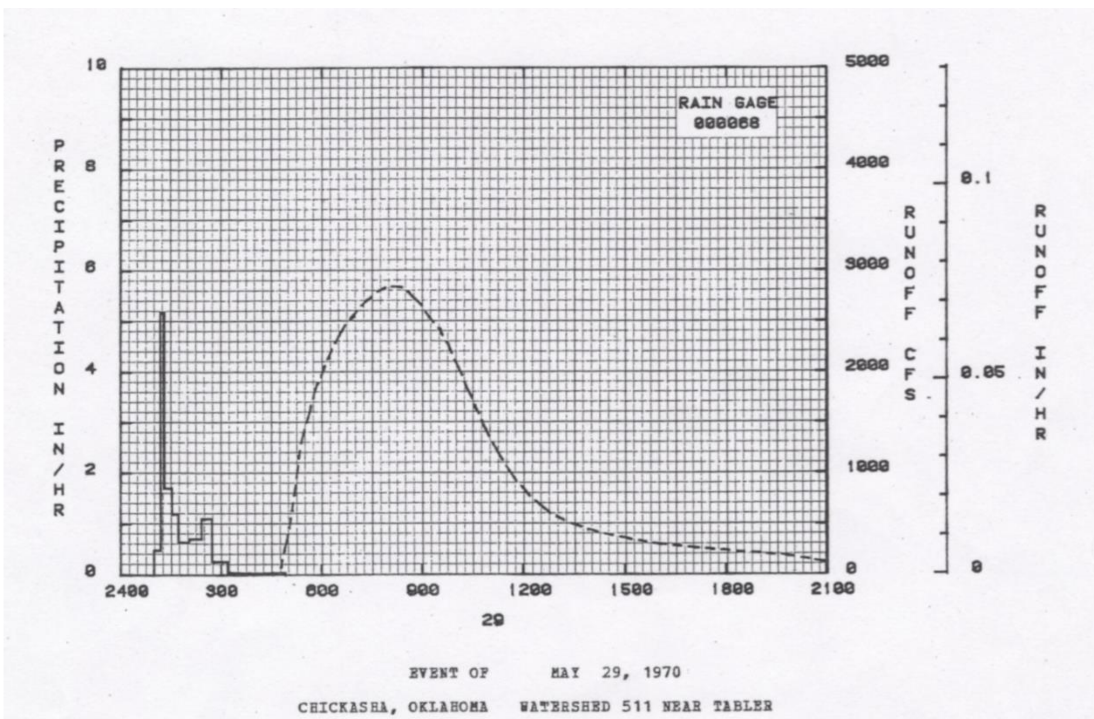

Figure 5-20. Storm event occurred on 05/29/1970 watershed 511, at Chickasha, Oklahoma [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1970].

<u>Watershed</u>: 621 <u>Location</u>: Chickasha, Oklahoma <u>Date</u>: 05/10/1964

The P(t) and the RO(t) show great dissimilarity in form (see Figure 5-21). The P(t) has a multiple-peaked form with only a small part extending into the RO(t). The RO(t) shows a simple center-loaded form with an extremely long tail. Compared with the P(t), the RO(t) shows a much wider half-width. Since the P(t) and the RO(t) are greatly separated, a time offset may be needed to accurately model the event.

During calibration, most of the P(t) was lost due to the initial abstraction and losses, with only a small part of the P(t) appearing as Pe(t) (n_{pe} = 31 min; v_{pe} = 0.199 in.). The Pe(t) and DRO(t) have considerable differences in both the times to peak (tp_{Pe} = 9 min; tp_{DRO} = 109 min) and the time bases (n_{pe} = 31 min; n_{DRO}=204 min). Calibration yielded a long tp_{UH} of 38 minutes and a PRF 352, with relatively good accuracy (Se/Sy = 0.3223). Both of the tp_{UH} and PRF seem reasonable.

In order to involve more rainfall into the analysis, a time offset of 1 hour was added to more closely align the P(t) and the RO(t). The result indicated that adding time offset significantly improved the calibration accuracy (i.e., Se/Sy changed from 0.3223 to 0.2112). The calibrated the tp_{UH} and PRF are 40 minutes and 452, respectively. The calibrated PRF is consistent with the proportion (42.65%) under the rising limb of the DRO(t). In summary, if the time offset has a hydrometeorological cause, the calibrated UH parameters are reasonable and representative of the temporal watershed response.

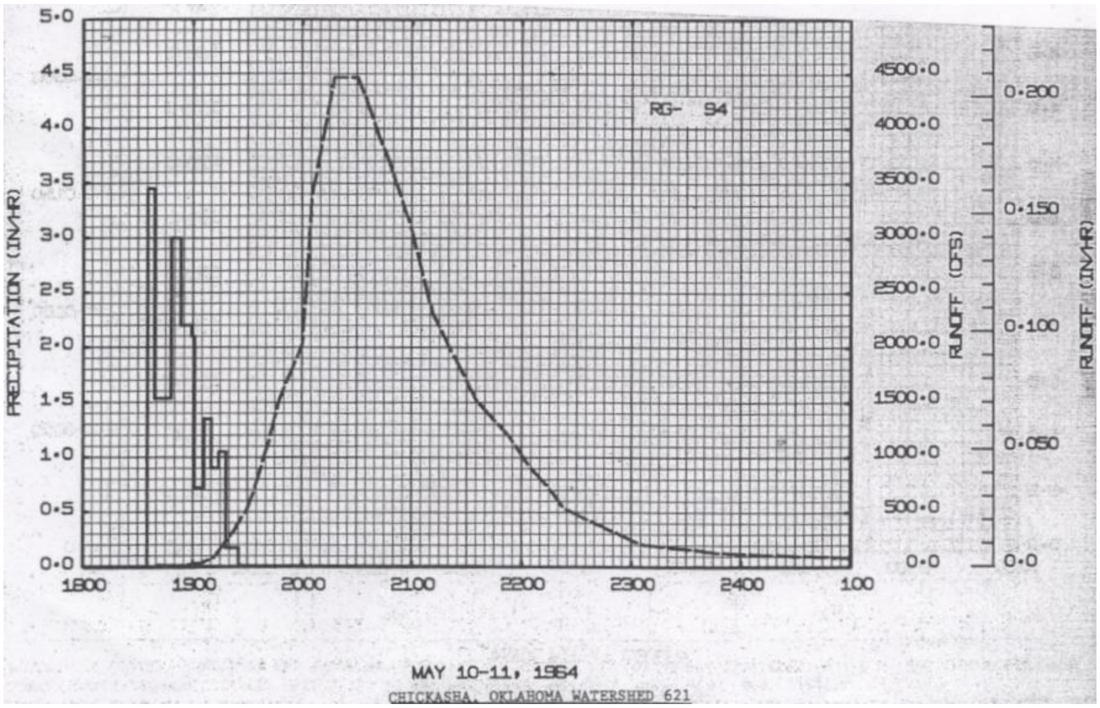

Figure 5-21. Storm event occurred on 05/10/1964 watershed 621, at Chickasha, Oklahoma [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1964].

<u>Watershed</u>: C-8 **<u>Location</u>: Chickasha, Oklahoma** **<u>Date</u>: 09/19/1965**

The P(t) and the RO(t) show great dissimilarity in form (see Figure 5-22). The P(t) has a multiple-peaked form with a long tail that extends into the RO(t). The RO(t) shows a simple center-loaded form. Compared with the P(t), the RO(t) shows a slightly wider half-width.

During calibration, most of the P(t) was lost due to the initial abstraction and losses, with only a small part of the Pe(t) appearing as the Pe(t) (n_{pe} = 19 min; v_{pe} = 0.139 in.). The Pe(t) and DRO(t) have relatively large differences in both the times to peak (tp_{Pe} = 19 min; tp_{DRO} = 59 min) and the time bases (n_{pe} = 32 min; n_{DRO}=115 min). Calibration yielded a tp_{UH} of 13 minutes and a PRF of 184, with good accuracy (Se/Sy = 0.2669). The calibrated tp_{UH} seems reasonable. The calibrated PRF seems smaller than expected based on the relatively large proportion (41.3%) under the rising limb of the DRO(t). The good accuracy was probably due to the reasonable half-width of the RO(t), which is wider than that of the P(t), and the relatively less complex forms of the P(t) and the RO(t). In summary, the calibrated UH seems to be a reasonably good model of the watershed response.

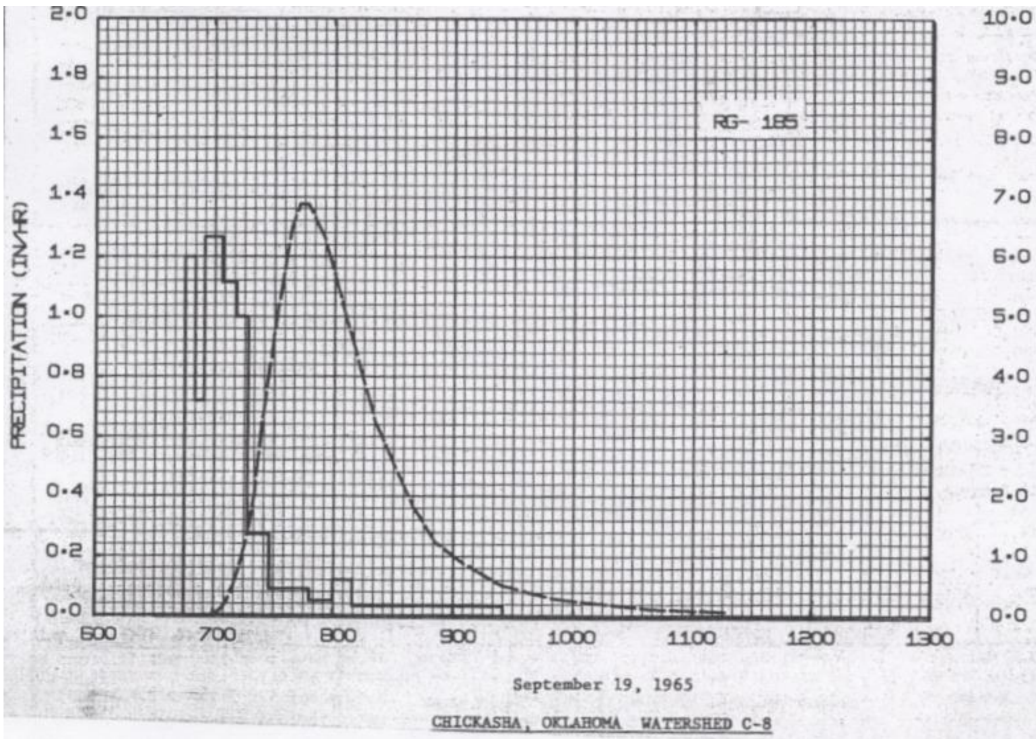

Figure 5-22. Storm event occurred on 09/19/1965 watershed C-8, at Chickasha, Oklahoma [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1965].

<u>Watershed</u>: W-4 <u>Location</u>: Stillwater, Oklahoma <u>Date</u>: 06/19/1972

As evident from Figure 5-23, the P(t) has a multiple-peaked form with a long flat tail that extends into the RO(t). The RO(t) shows a simple right-skewed form with a long tail and has a slightly wider half-width than that of the P(t).

During calibration, most of the P(t) was lost due to the initial abstraction and losses, with only a small part of the P(t) appearing as the Pe(t) (n_{pe} = 21 min; v_{pe} = 0.308 in.). The Pe(t) and DRO(t) have relatively large differences in both the times to peak (tp_{Pe} = 21 min; tp_{DRO} = 60 min) and the time bases (n_{pe} = 21 min; n_{DRO}= 121 min). Calibration yielded a small tp_{UH} of 6 minutes and a PRF of 131, with relatively good accuracy (Se/Sy = 0.3289). The calibrated tp_{UH} seems low based on the large difference between the tp_{Pe} and the tp_{DRO}. The relatively low PRF seems reasonable given the large difference in the time bases, and it is consistent with the proportion (23.1%) under the rising limb of the DRO(t). The good accuracy was probably due to the reasonable half-width of the RO(t) and the relatively less complex forms of the P(t) and the RO(t). In summary, the calibrated PRF is reasonable but the calibrated tp_{Pe} seems low.

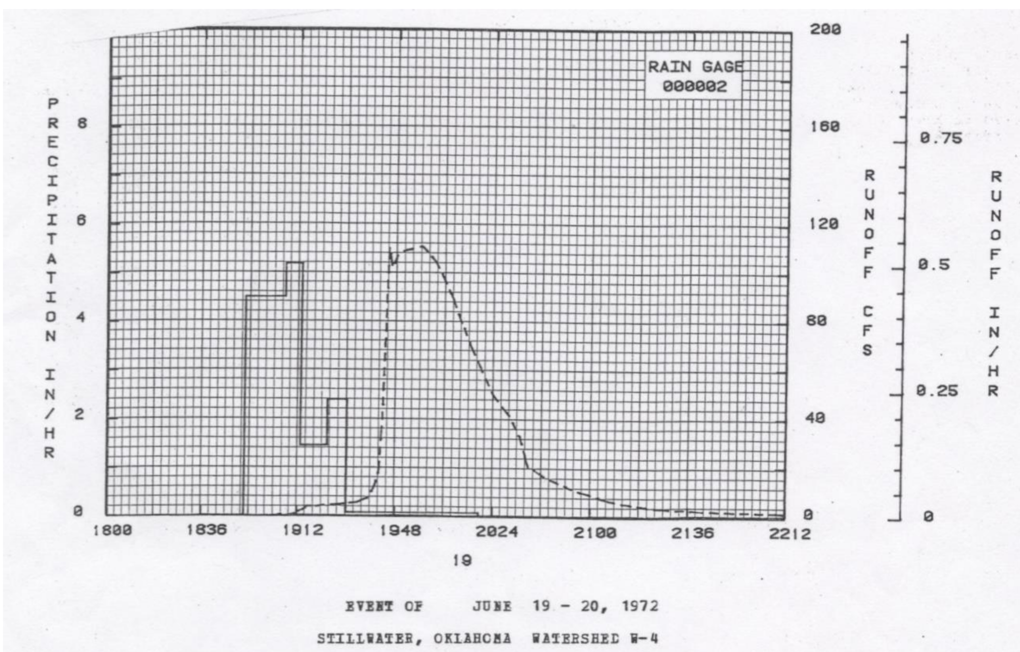

Figure 5-23. Storm event occurred on 06/19/1972 watershed W-4, at Stillwater, Oklahoma [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1972].

Watershed: W-1 **Location: Stillwater, Oklahoma** **Date: 12/29/1972**

Both the P(t) and the RO(t) show right-skewed forms (see Figure 5-24). The P(t) has one primary peak and another smaller peak. The RO(t) has a much wider half-width and a longer tail than that of the P(t).

The initial part of the first peak and the entire second peak of the P(t) were extracted as initial abstraction. Three-eighths of the total rainfall depth was assumed to be the Pe(t) (n_{pe} = 19 min; v_{pe} = 0.303 in.). The Pe(t) and DRO(t) have relatively large differences in both the times to peak (tp_{Pe} = 15 min; tp_{DRO} = 53 min) and the time bases (n_{pe} = 19 min; n_{DRO}= 201 min). Calibration yielded a tp_{UH} of 19 minutes and a PRF of 196, with a very good fit (Se/Sy = 0.1660). Both the calibrated tp_{UH} and PRF are reasonable. The PRF is consistent with the proportion (28.87%) under the rising limb of the DRO(t). The good accuracy was probably due to the reasonable half-width of the RO(t) and the relatively less complex forms of the P(t) and the RO(t). In summary, the calibrated UH parameters are representative of the of the watershed response.

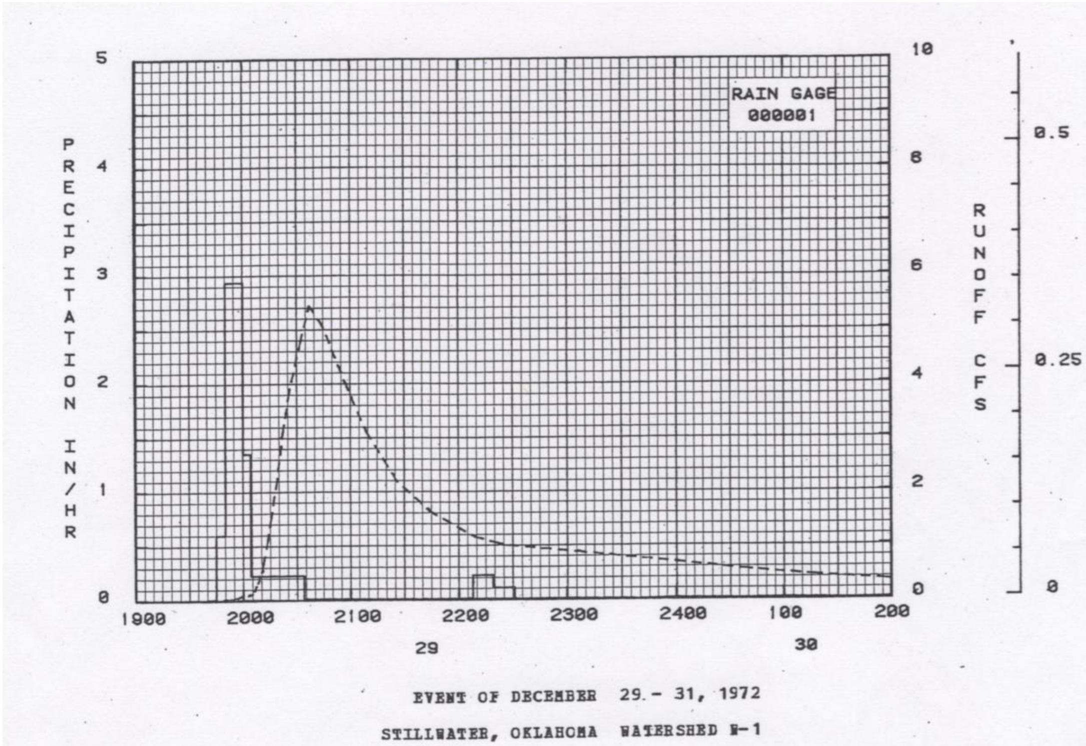

Figure 5-24. Storm event occurred on 12/29/1972 watershed W-1, at Stillwater, Oklahoma [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1972].

<u>Watershed</u>: D **<u>Location</u>: Riesel, Texas** **<u>Date</u>: 05/10/1965**

In Figure 5-25, the P(t) has a primary peak with a long flat tail that greatly extends into the RO(t). The RO(t) shows a simple right-skewed form with a long tail and a much wider half-width than that of the P(t).

Both the major sections of the first peak and the long tail of the P(t) were eliminated as initial abstraction or losses, which resulted in nearly half of total rainfall being the Pe(t) (n_{pe} = 19 min; v_{pe} = 1.497 in.). The Pe(t) and DRO(t) have relatively large differences in both the times to peak (tp_{Pe} = 16 min; tp_{DRO} = 51 min) and the time bases (n_{pe} = 26 min; n_{DRO}= 202 min). The calibration resulted in a tp_{UH} of 16 minutes and a low PRF of 85, with good accuracy (Se/Sy = 0.3062). Both the calibrated tp_{UH} and PRF are reasonable. The PRF is consistent with the proportion (19.77%) under the rising limb of the DRO(t). The good accuracy was probably due to the reasonable half-width of the RO(t) and the relatively less complex forms of the P(t) and the RO(t). In summary, the calibrated UH parameters are representative of the watershed response.

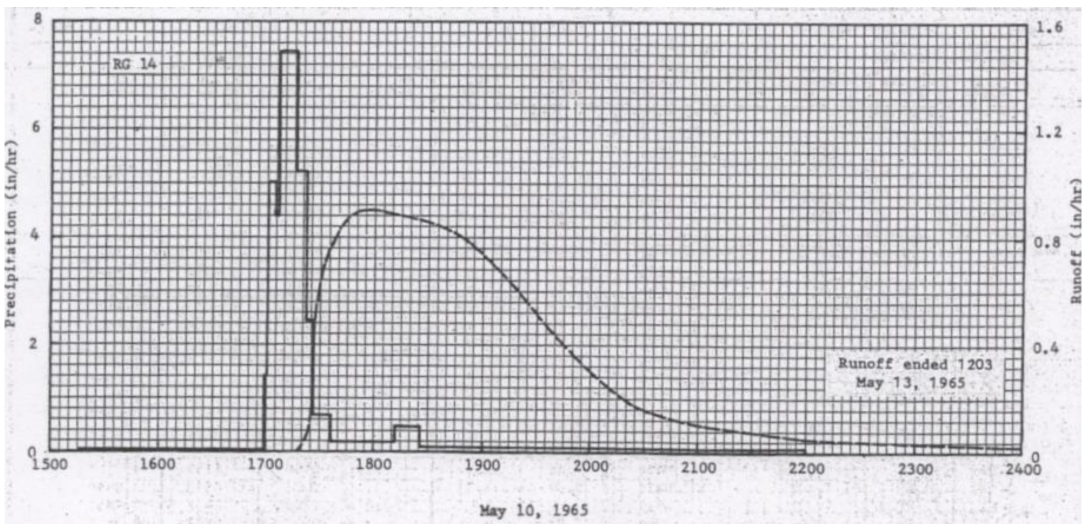

Figure 5-25. Storm event occurred on 05/10/1965 watershed D, at Riesel, Texas [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1965].

<u>Watershed</u>: W-1V **<u>Location</u>: Springs, Colorado** **<u>Date</u>: 08/02/1939**

The P(t) and the RO(t) show considerable dissimilarity in form (see Figure 5-26). The P(t) has multiple peaks and greatly extends into the RO(t). The RO(t) shows a single center-loaded form and a much narrower half-width than that of the P(t).

Most of the rainfall was lost due to the initial abstraction and losses, with only a small part of the rainfall appearing as the Pe(t) (n_{pe} = 21 min; v_{pe} = 0.302 in.). The Pe(t) and DRO(t) have a small difference in the times to peak (tp_{Pe} = 17 min; tp_{DRO} = 26 min) but a considerable difference in the time bases (n_{pe} = 21 min; n_{DRO}=111 min). Calibration yielded a small tp_{UH} of 6 minutes and a PRF of 217, with relatively good accuracy (Se/Sy = 0.4043). The calibrated tp_{UH} is reasonable and resulted from the small difference in the times to peak. The PRF seems reasonable because of the considerable difference in the time bases.

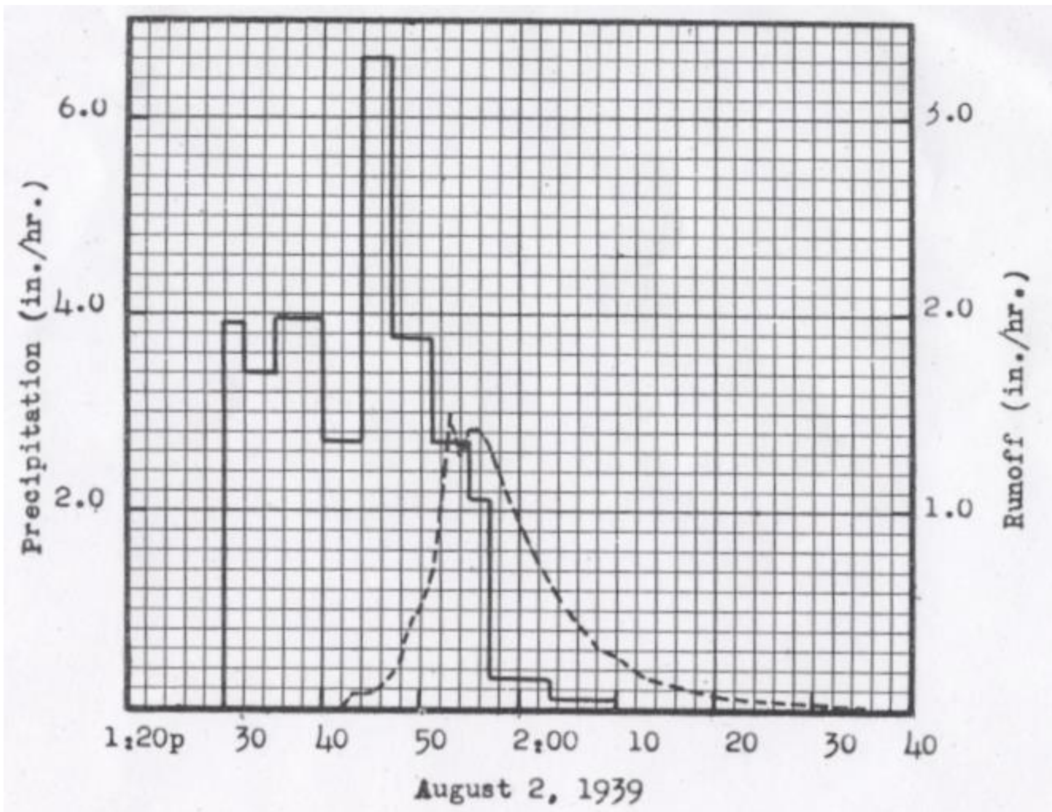

Figure 5-26. Storm event occurred on 08/02/1939 watershed W-1V, at Springs, Colorado [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1939].

<u>Watershed</u>: W-1 <u>Location</u>: Santa Fe, New Mexico <u>Date</u>: 08/25/1947

Figure 5-27 shows that the P(t) has a right-skewed form with a tail that extends into the RO(t). The RO(t) shows a simple right-skewed form with a long tail and a slightly wider half-width than that of the P(t).

Most of the P(t) was lost due to the initial abstraction and losses, with only a small part of the P(t) appearing as Pe(t) (n_{pe} = 12 min; v_{pe} = 0.341 in.). The Pe(t) and DRO(t) have a small difference in the times to peak (tp_{Pe} = 8 min; tp_{DRO} = 20 min) and a considerable difference in the time bases (n_{pe} = 12 min; n_{DRO}=92 min). Calibration yielded a small tp_{UH} of 14 minutes and a PRF of 352, with relatively good accuracy (Se/Sy = 0.4179). Both the calibrated tp_{UH} (14 min) and PRF (352) are reasonable. The PRF is also consistent with the proportion (25.8%) under the rising limb of the DRO(t). The good accuracy was probably due to the reasonable half-width of the RO(t) and the relatively less complex forms of the P(t) and the RO(t). In summary, the calibrated UH parameters are representative of the watershed response.

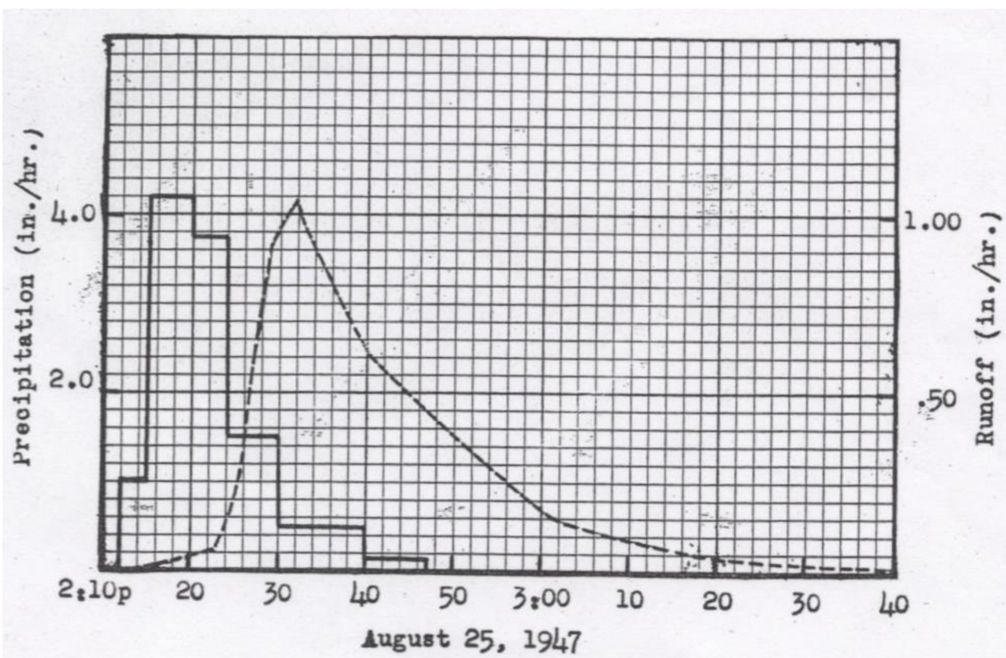

Figure 5-27. Storm event occurred on 08/25/1947 watershed W-1, at Santa Fe, New Mexico [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1947].

<u>Watershed</u>: 63.103 **<u>Location</u>: Tombstone, Arizona** **<u>Date</u>: 09/08/1970**

In Figure 5-28, both the P(t) and the RO(t) show simple center-loaded forms. The P(t) greatly overlaps the RO(t) and has a slightly wider half-width than that of the RO(t).

Most of the P(t) was assumed as Pe(t) (n_{pe} = 37 min; v_{pe} = 1.300 in.). The Pe(t) and DRO(t) have small differences in both the times to peak (tp_{Pe} = 21 min; tp_{DRO} = 13 min) and the time bases (n_{pe} = 37 min; n_{DRO}=45 min). Calibration resulted in a very short tp_{UH} of 1 minute and a large PRF of 575, with a very poor fit (Se/Sy = 0.9912). The short tp_{UH} resulted from the small difference in the times to peak since the calibration model would force the UH to minimize the mismatch between peaks of the computed and measured DRO(t)s. The high PRF was due to the small difference in the time bases because n_{DRO}-n_{pe}+1 was small. The poor fit occurred because the tp_{Pe} is even greater than the tp_{DRO}, which is impossible to reproduce by using gamma UH model. The narrow half-width of the RO(t) and the considerable overlap between the P(t) and the RO(t) might be other contributing factors to the poor fit. In summary, due to the poor accuracy, the calibrated UH parameters are irrational and not representative of the watershed response.

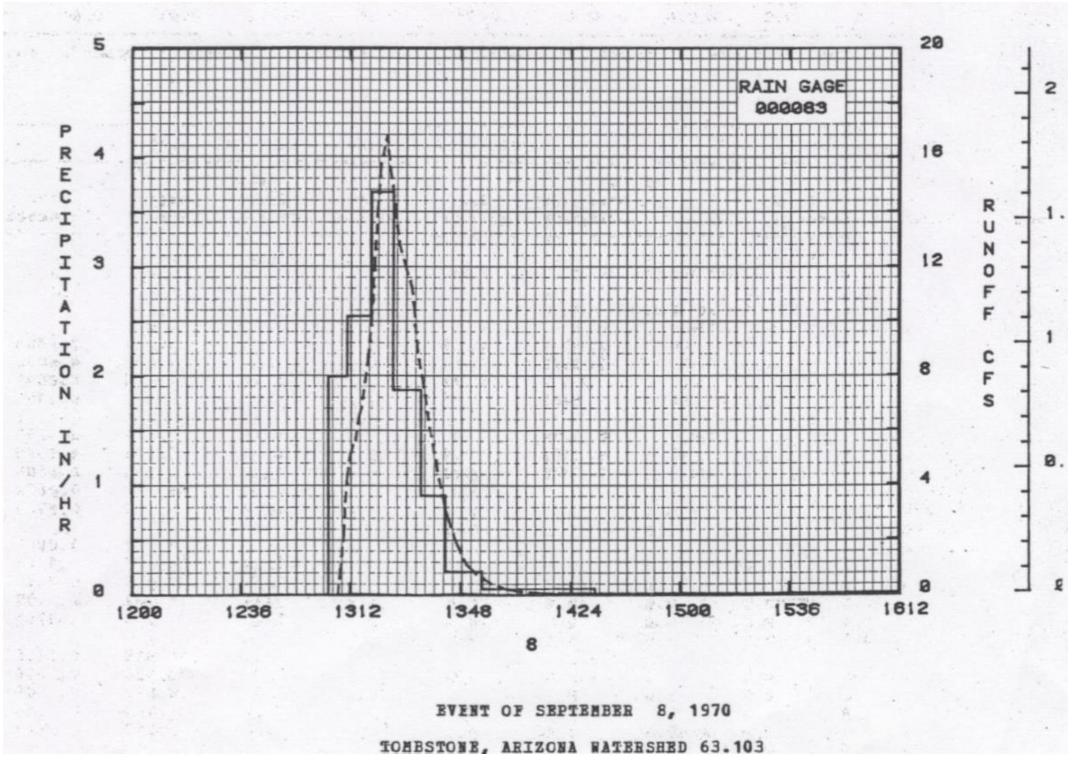

Figure 5-28. Storm event occurred on 09/08/1970 watershed 63.103, at Tombstone, Arizona [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1970].

<u>Watershed</u>: 45.005 <u>Location</u>: Safford, Arizona <u>Date</u>: 07/22/1955

Figure 5-29 shows that both the rainfall and runoff data have a simple center-loaded form. The P(t) extended into RO(t) and has a slightly narrower half-width than that of the RO(t). A time offset may exit.

Only a small part of the P(t) appeared as Pe(t) (n_{pe} = 37 min; v_{pe} = 0.355 in.). The Pe(t) and DRO(t) have relatively large differences in the times to peak (tp_{Pe} = 32 min; tp_{DRO} = 59 min) and in the time bases (n_{pe} = 37 min; n_{DRO}=87 min). Calibration yielded a short tp_{UH} of 6 minutes and a small PRF of 132, with very poor accuracy (Se/Sy = 0.9912). The calibrated tp_{UH} seems small given the relatively large difference (27 minutes) in tp_{Pe} and tp_{DRO}. The calibrated PRF seems low and is not consistent with the proportion (55.78%) under the rising limb of the DRO(t). The poor fit is probably due to the low depth of the Pe(t) and the potential time offset between the rainfall and runoff.

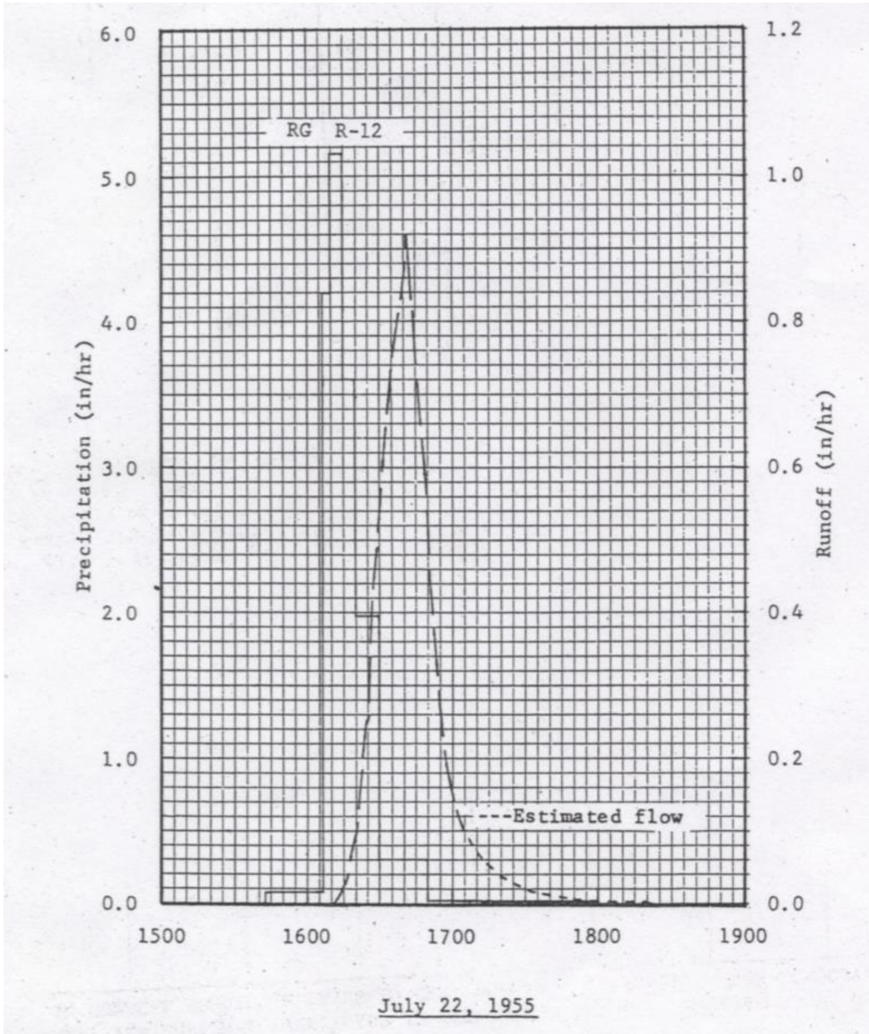

Figure 5-29. Storm event occurred on 07/22/1955 watershed 45.055, at Safford, Arizona [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1955].

5.4. SUMMARY OF ANALYSES

From the results of analyses, it is evident that analyzing the measured storm data was much more difficult than analyzing the synthetic data due to the complexity and variability of the storm data. The average Se/Sy was equal to a relatively bad accuracy of 0.6572. The poor fits might be caused by multiple factors (i.e., rainfall complexity, time offset), and it was often difficult to identify a principal factor.

In terms of the effectiveness and the practicability, the general guidelines played an important role in analyzing and understanding the calibration results. Although the calibration results were hard to explain explicitly, the possible associations suggested by the general guidelines were still quite useful when analyzing and understanding the calibration results. In summary, the gamma UH is not sufficiently flexible to adjust to many conditions. A poor fit can result under the following conditions.

(1) The rainfall excess either greatly extends or minimally extends into the direct runoff.

- Too much overlap (e.g., 09/08/1970, 63.103, Tombstone) between rainfall and runoff will worsen the accuracy due to a fitting issue, specifically the closeness of times to peak between rainfall excess and direct runoff; while minimal overlap (e.g., 06/18/1971, W-32, Oxford) will yield little rainfall excess that will not reflect the watershed response.

(2) Data characteristics are too complex to be fitted with a two-parameter model (i.e., complexity of storm data, a great dissimilarity between rainfall and runoff).

- For example, in the storm event (08/19/1949, W-1) at McCredie, MO, the poor fit resulted from fitting a two-peaked runoff with a one-peaked rainfall; in the storm event (07/09/1967, WC-1) at Oxford, Mississippi, the poor fit resulted from the complex characteristics (i.e., multiple peaks, great variation in form) of the rainfall; in the storm event (05/13/1964, WC-1) at Oxford, Mississippi, the poor fit is most likely caused by the extremely long time base of the direct runoff. Therefore, when analyzing the measured data, it is better to use simple storm data rather than more complex data.

(3) The existing time offset between rainfall excess and direct runoff is ignored.

- Time offset seems a common factor in measured data. It can directly affect the volume of rainfall excess and the difference in the times to peaks between the rainfall excess and direct runoff. It is important to know that using a time offset can either increase (e.g., 05/29/1970, 511, Chickasha) or decrease the accuracy (e.g., 07/18/1956, Iowa City) depending on the effects of other factors. Aligning the rainfall and runoff too closely may forces the tp_{UH} to be too short and the PRF to be too large which can cause too much overlap and worsen the accuracy. If the rainfall and runoff are offset, then most of the rainfall will be extracted as initial abstraction or losses, with little rainfall appearing as rainfall excess; thus, the rainfall excess will not sufficiently reflect the form the rainfall characteristics. If a time offset exists, an analysis on the geometry of the watershed is required, and multiple offset values should be tested. Ultimately, a reason for the offset needs to be provided.

(4) The direct runoff hydrograph shows a left-tail skewed form rather than a right-tail skewed form, or the runoff hydrograph has a right skewed form but with an abrupt increase at the beginning (e.g., 07/11/1965, Chestnut, Blacksburg).

- The ability of gamma UH to fit measured direct runoff hydrographs is limited by its inherent lack of flexibility. The gamma pdf can fit a right-tail skewed shape but cannot accurately reproduce a left-tail skewed shape. And it also cannot fit a shape that has a steep rising limb. Therefore, if the direct runoff shows either a left-tail skewed form or an abrupt increase at the beginning of the rising limb, poor accuracy is most likely due to the inability of the gamma UH to fit such shapes.

(5) The half-width of the observed direct runoff hydrograph is either too thin (e.g., 07/08/1965, WC-1, Oxford) or too wide (e.g., 06/18/1971, W-32, Oxford) compared with half-width of the rainfall excess.

- The half-width of the computed direct runoff hydrograph is determined by the half-width of the rainfall excess and the half-width of the gamma UH. Since the UH is a smoothing function, the half-width of the computed direct runoff hydrograph must be greater than that of the rainfall excess hyetograph. Therefore, if the half-width of the observed direct runoff hydrograph is too thin or even less than the half-width of the rainfall excess hyetograph, a poor fit will occur. Conversely, if the half-width of the observed direct runoff hydrograph is much wider than that of the rainfall excess hyetograph, then even a very flat UH cannot transform the rainfall excess to a computed direct runoff hydrograph that has a similar half-width as the observed direct runoff hydrograph; a poor fit will also occur.

(6) The difference in the times to peak of the rainfall excess hyetograph and the observed direct runoff hydrograph is too short (e.g., 07/08/1965, WC-2, Oxford; 07/08/1967, W-4H, Hastings NE).

- Since the UH is a smoothing function, the computed direct runoff hydrograph usually has a flatter shape and a later peak than that of the rainfall excess hyetograph. If the difference of the $\mathrm{tp_{pe}}$ and $\mathrm{tp_{DRO}}$ is too small, the $\mathrm{tp_{UH}}$ will be very short since the calibration model will force the UH to minimize the mismatch between peaks of the computed and measured direct runoff hydrographs; then a poor fit may occur. Therefore, when analyzing actual data, it should be noted that the small difference in the $\mathrm{tp_{pe}}$ and the $\mathrm{tp_{DRO}}$ can be a major contributing factor that can significantly distort the calibrated UH parameters and reduce the calibration accuracy.

(7) The nonlinear processes occur during the storm event.

- In the analyses of measured data, the watershed processes were assumed to be linear and the tp_{UH} and the PRF were assumed to be constant with time (i.e., static model). However, the watershed processes can be nonlinear easily due to the variability of the temporal watershed conditions. A poor fit can occur when the watershed processes are nonlinear, but the unit hydrograph model is assumed to be linear, i.e., static rather than a dynamic model. When analyzing the measured data, whether or not the watershed processes are linear cannot be identified by observing the rainfall hyetographs and runoff hydrographs. However, it can be expected that more complex rainfall and observed runoff are more likely caused by the nonlinear watershed processes. Therefore, as mentioned previously, if sufficient data are available, it is better to use storm events with simple rainfall to calibrate the UH parameters. Additionally, nonlinear processes can also be a contributing factor that reduces the calibration accuracy.

In summary, the general guidelines developed from the analyses on both synthetic and measured data are very useful and they can assist researchers in calibrating the UH parameters more accurately and in interpreting the calibration results more easily.

5.5. TIME-AREA CURVE ANALYSIS

A primary objective of this research was to identify factors that influence the characteristics of unit hydrographs. Past research (e.g., Dooge 1973) has shown that the shape of the watershed is a primary determinant of the watershed time-area curve. However, the time-area curve is usually associated with the instantaneous unit hydrograph (IUH), with storage routing necessary to transform the IUH into an UH for a non-instantaneous unit duration. In spite of this need for storage transformation, a time-area curve is still a reflection of the shape of a UH.

The immediate concern here is whether or not the gamma UH model can be sufficiently flexible or if it can limit the ability for regenerating the measured runoff. To address this concern, three time-area curves were developed from watershed topographic maps. One watershed, Hasting 4-H (see Figure 5-30), is narrow at the outlet and widens to the upper part of the watershed. A second watershed, Stillwater W-1 (see Figure 5-31), is wide near the outlet and tapers to a point at the top of the watershed. The third watershed, Fennimore W-4 (see Figure 5-32), is more circular in shape. The computed time-area curves for these three watersheds are shown in Figures 5-33, 5-34, and 5-35, respectively. The ordinates for each time-area curve are listed in Table 5-1. The connection between watershed shape and the shape of the time-area curve is evident. The expanding shaped watershed, Hasting, has a relatively late peak and wide half-width. The tapered watershed, Stillwater, has a time-area curve that has a relatively early peak and a moderate half-width. The more circular-shaped watershed, Fennimore, has a more triangular time-area curve with a moderate half-width and a relatively moderate time to peak. These characteristics are summarized in Table 5-2. The relative time to peak (R_{tp}) is closely related to the gamma distribution time to peak, $tp_{UH} = b(c - 1)$ (column 3 vs column 9).

The goodness of fit indicates the ability of the gamma distribution to adapt to different shapes. While the gamma distribution is generally right-skewed, it can adapt to near-symmetric shapes. However, the two-parameter gamma cannot take on left-skewed shapes. The gamma distribution was fitted to each of the time-area curves using numerical least squares and the goodness of fit (i.e., Se, Se/Sy, and R^2) assessed (see Table 5-2). While each fitted gamma distribution produced accurate results (e.g., $R^2 > 0.9$ and $Se/Sy < 0.3$), the statistics indicate that the form of the time-area curve influences the ability to fit. The time-area curve for the Fennimore watershed resulted in the best accuracy, while the wider time-area curve of the Stillwater watershed gave the poorest result. It appears that the gamma distribution is sufficiently flexible to provide a reasonably good fit to a variety of time-area shapes.

The goodness-of-fit statistics of Table 5-2 only indicate the ability of a time-area curve to be represented by a gamma distribution. They do not indicate the accuracy with which a gamma UH will produce the direct runoff hydrograph. Table 5-2 shows the goodness-of-fit statistics for the regeneration of direct runoff hydrographs for one storm on each of these three watersheds. Standard error ratios of 0.215, 0.954, and 0.803 resulted from the modeling of direct runoff for actual storms for Fennimore, Hasting, and Stillwater, respectively. While the Fennimore data showed the best fit in both cases, the standard error ratios for storm-event-regeneration of the other two the other two watersheds were poor (i.e., $Se/Sy > 0.75$). While some of this may be due to the time-area characteristics of the watershed, it seems that other contributing factors were more important, such as the temporally dynamic conditions of the watershed during the storm and any time-offset conditions.

Table 5-1. Ordinates of dimensionless time-area curves for Fennimore W-4, Hastings W-4-H, and Stillwater W-1 watersheds.

Fennimore W-4		**Hastings W-4-H**		**Stillwater W-1**	
t/t_b	Proportion	t/t_b	Proportion	t/t_b	Proportion
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.1110	0.0353	0.0910	0.0255	0.0910	0.0387
0.2220	0.0772	0.1820	0.0558	0.1820	0.1055
0.3330	0.1982	0.2730	0.0784	0.2730	0.1883
0.4440	0.1907	0.3640	0.1195	0.3640	0.1669
0.5550	0.1702	0.4550	0.1420	0.4550	0.1375
0.6660	0.1498	0.5450	0.1195	0.5450	0.1041
0.7770	0.0902	0.6360	0.1156	0.6360	0.0774
0.8880	0.0577	0.7270	0.1068	0.7270	0.0748
1.0000	0.0307	0.8180	0.0890	0.8180	0.0748
	1.0000	0.9090	0.0852	0.9090	0.0267
		1.0000	0.0627	1.0000	0.0053
			1.0000		1.0000

Table 5-2. Goodness-of-fit statistics of fitting the time-area curves with the two-parameter gamma UH.

Watershed	w_{hR}	R_{tp}	**Se**	**Se/Sy**	R^2	***b***	***c***	***b(c-1)***
Fennimore	0.456	0.33	0.0152	0.228	0.954	1.03	4.82	3.94
Hastings	0.464	0.45	0.0082	0.242	0.947	2.46	3.24	5.50
Stillwater	0.736	0.27	0.0171	0.298	0.920	1.50	3.41	3.63

w_{hR}= relative half weight = half width/ time base of time–area curve ;
R_{tp}= relative time to peak= time of peak/ time–base of time–area curve.

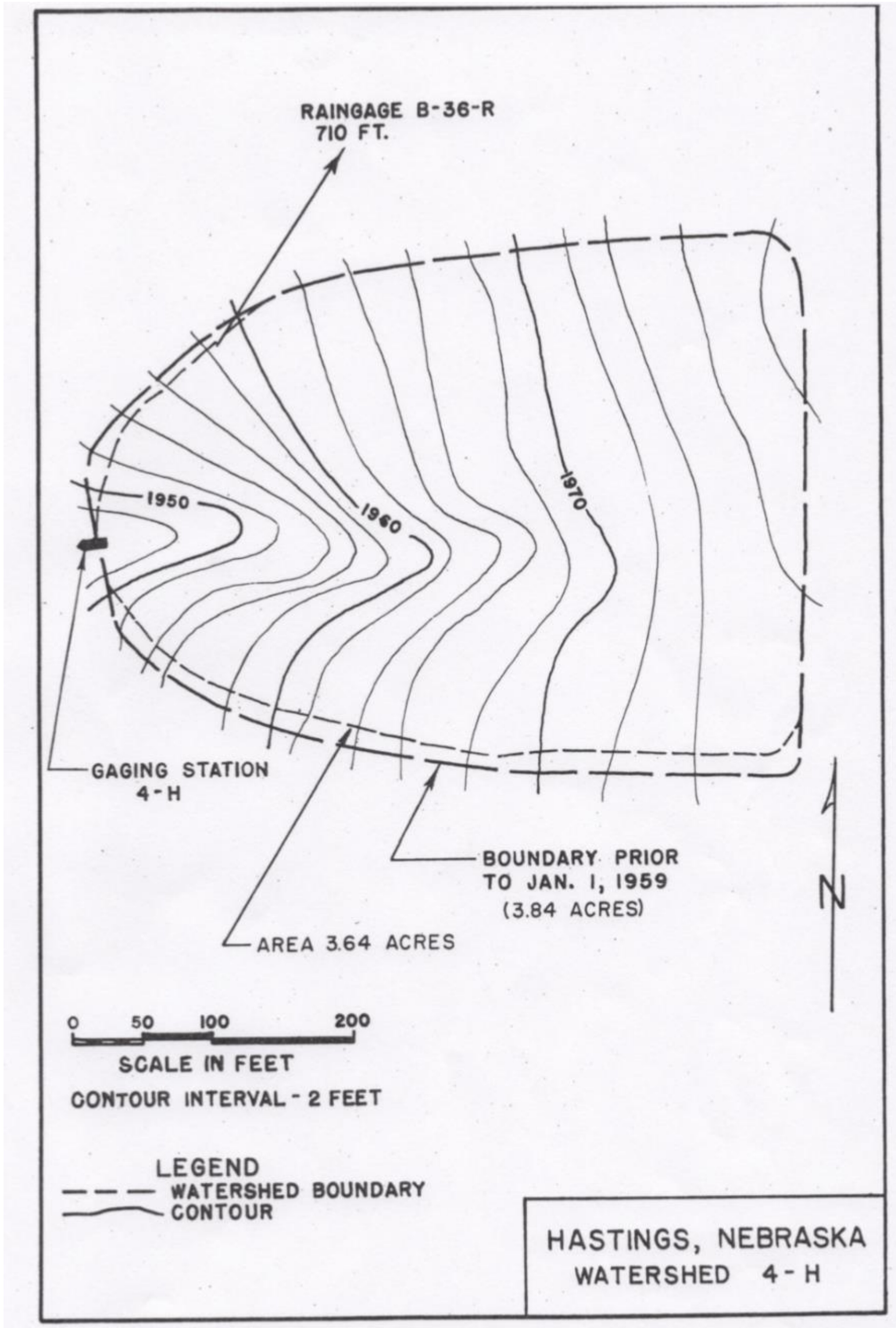

Figure 5-30. Topographic map of Hastings, Nebraska watershed [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1959].

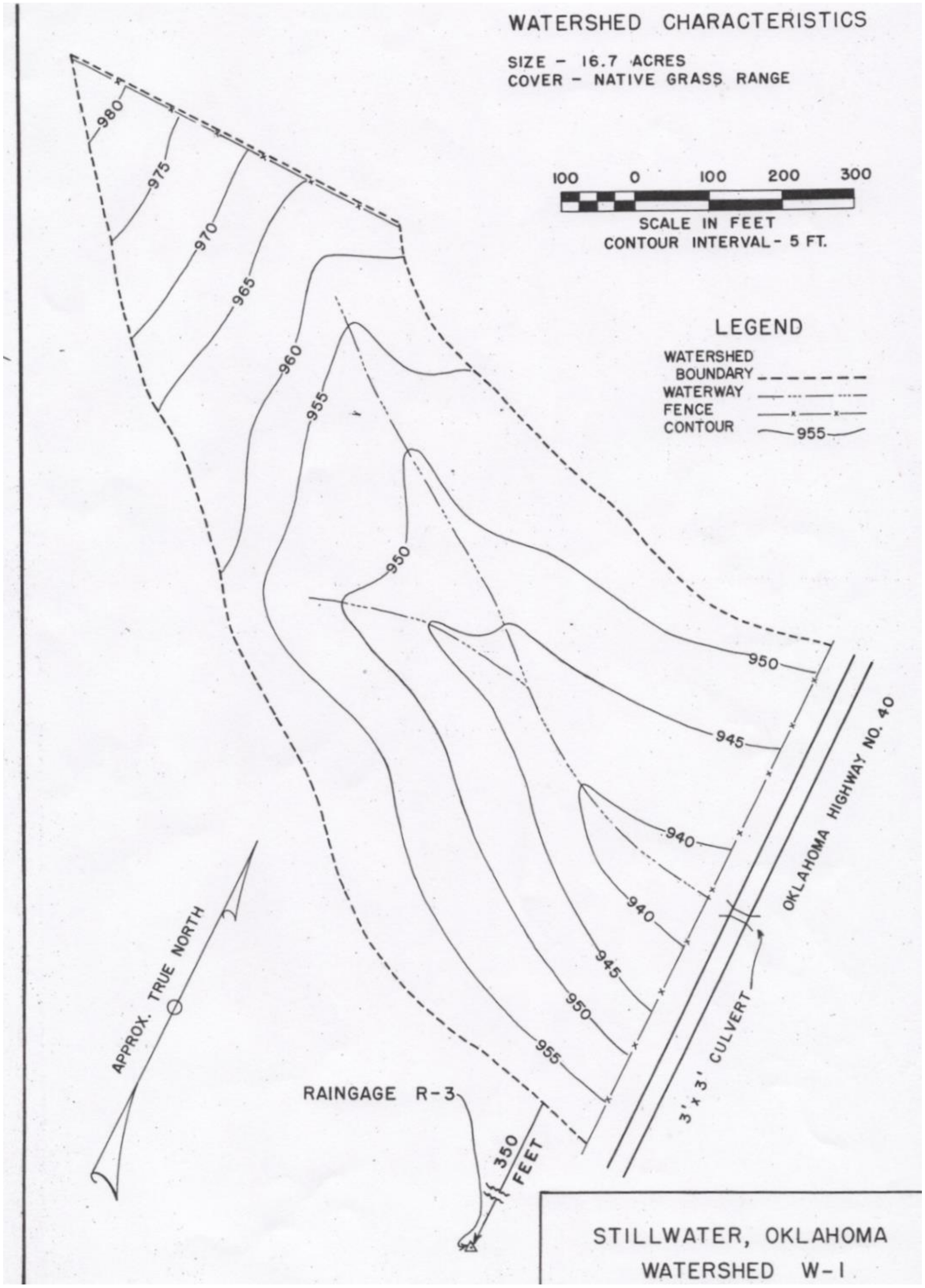

Figure 5-31. Topographic map of Stillwater, Oklahoma watershed [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1959].

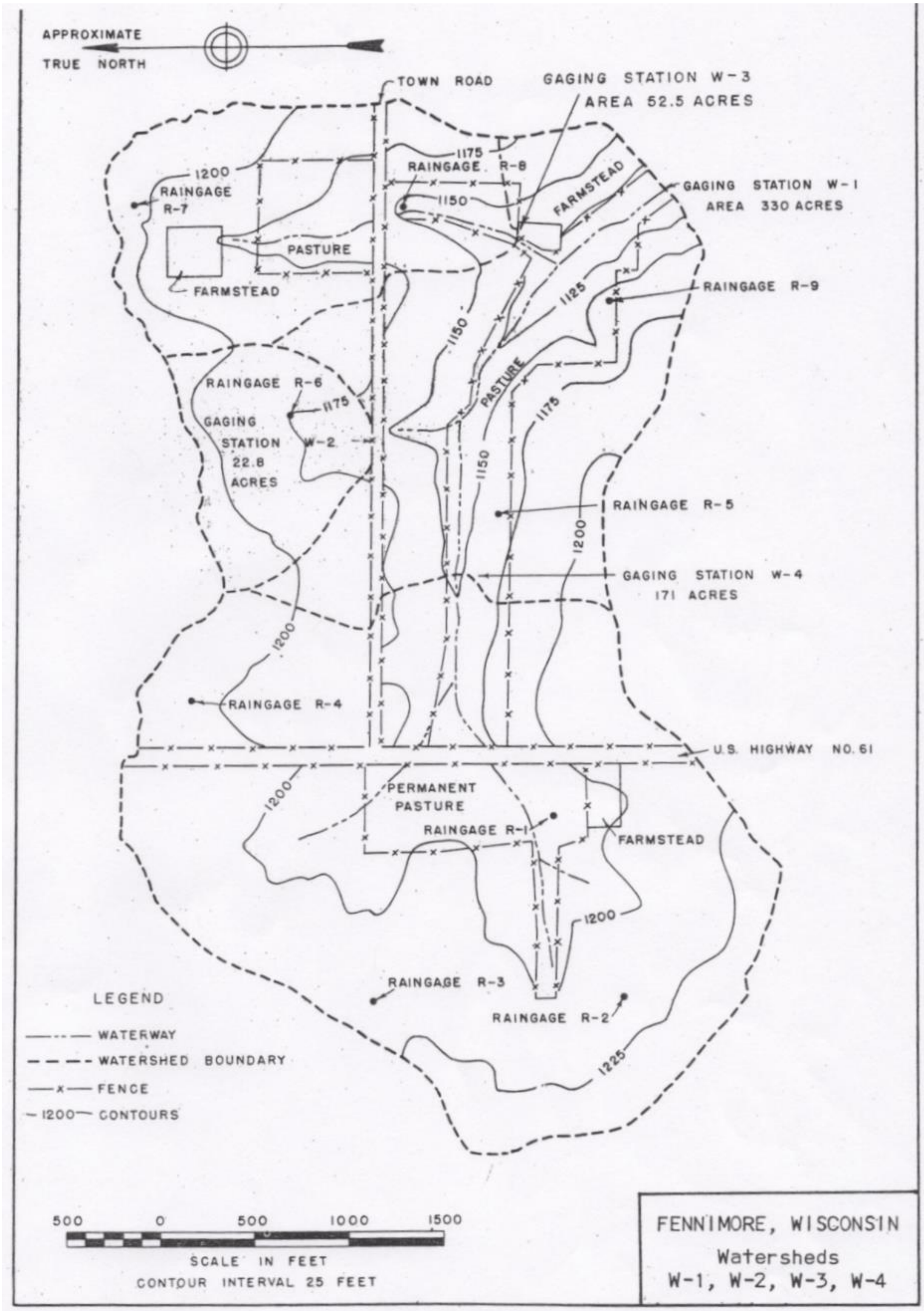

Figure 5-32. Topographic map of Fennimore, Wisconsin watershed [Reproduced from Hydrologic Data for Experimental Agricultural Watersheds in the United States, 1959].

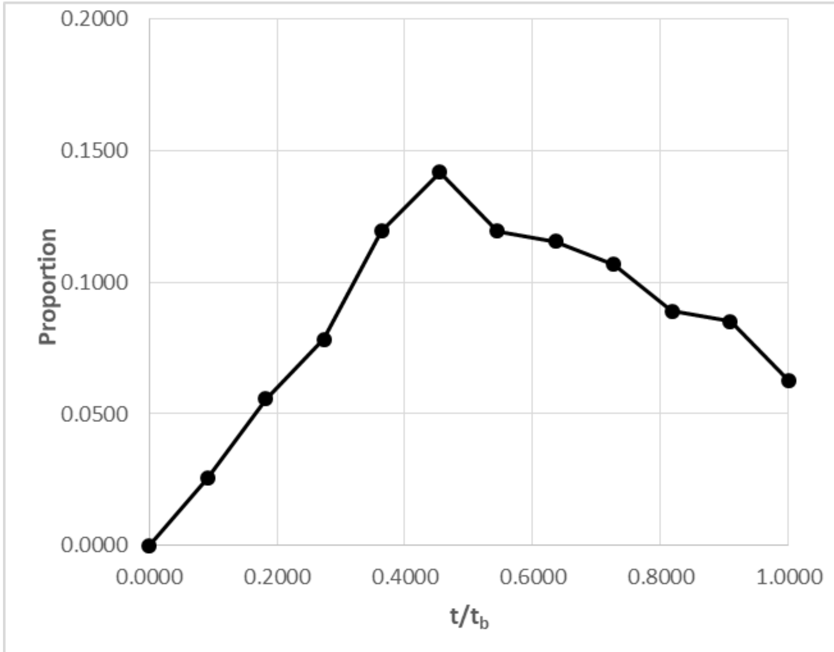

Figure 5-33. Dimensionless time-area curve for Hasting 4-H watershed.

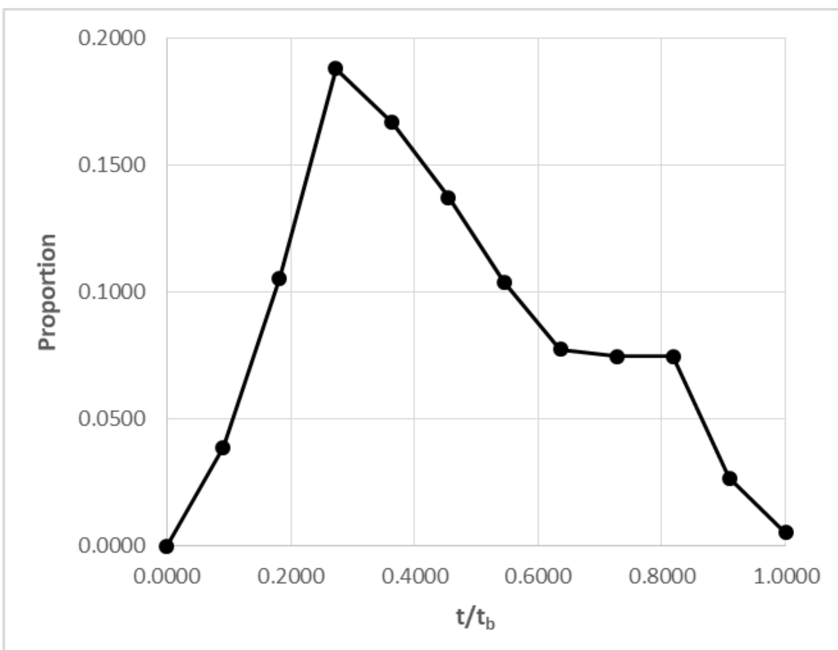

Figure 5-34. Dimensionless time-area curve for Stillwater W-1 watershed.

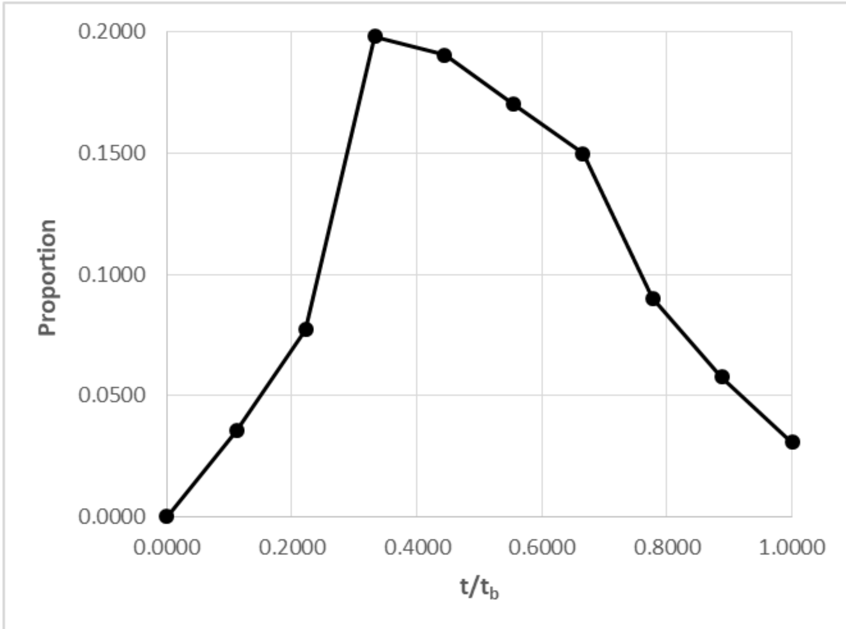

Figure 5-35. Dimensionless time-area curve for Fennimore W-4 watershed.

CHAPTER 6: CONCLUSIONS AND RECOMMENDATIONS

6.1. INTRODUCTION

The goal of this research is to investigate and understand factors that influence empirical estimates of the parameters of gamma unit hydrographs and to develop general guidelines that can help researchers to calibrate UH parameters more accurately. Although a number of unit hydrographs have been developed, the unit hydrograph based on the gamma pdf is still a widely used unit hydrograph due to its simplicity (i.e., only two parameters are required to determine the form of a gamma unit hydrograph) and reasonable physical basis. While past studies on gamma unit hydrographs have focused on the methods for predicting UH parameters, as well as the relation between UH parameters and watershed characteristics, factors that may influence the parameter estimation and the quality of fit have not been systematically studied. Therefore, in order to improve estimates of the gamma UH parameters, efforts are needed to identify, interpret, and assess the effects of these influential factors on the accuracy and uncertainty of the estimated UH parameters.

The goal of this research was achieved by sequentially accomplishing the following the objectives: (1) to develop a calibration model for extracting PRFs and tp_{UH}s from measured storm events; (2) to investigate the effects of various factors that may influence the calibration accuracy of UH parameters by analyzing synthetic rainfall-runoff data and from which general guidelines could be developed; and (3) to assess the effectiveness and usefulness of developed general guidelines by analyzing the measured rainfall-runoff data.

The gamma UH calibration model was based on three major steps: the separation of the initial abstraction, losses, and baseflow; generating rainfall excess hyetographs and direct runoff hydrographs; and calibrating the best-fit UH parameters, the PRF and tp_{UH}, with the reported goodness-of-fit statistics.

The effects of various factors on the accuracy and uncertainty of calibrated UH parameters were studied. The effects of the temporal watershed characteristics, the time offset, and the rainfall characteristics (i.e., rainfall complexity and rainfall peakedness) were studied based on analyses of synthetic data. The effects of both the rainfall nonuniformity and dynamic convolution were also investigated based on analyses of measured rainfall-runoff data. The flexibility and limitations of the gamma distribution unit hydrograph to fit and to reflect the hydrologic response of a watershed were also discussed. General guidelines regarding these influential factors were developed.

The measured rainfall-runoff data from 29 agricultural watersheds were analyzed in an attempt to assess the effectiveness and practicability of the developed general guidelines in understanding the UH hydrographs calibration results and in the estimation of correct UH parameters. Summary of analyses on measured data were presented. Additional general guidelines were developed.

6.2. CONCLUSIONS

With measured rainfall and runoff data, the gamma UH calibration model can successfully output the best-fit PRF and tp_{UH} with reported goodness-of-fit statistics. The procedures for separating the initial abstraction, losses, and baseflow, generating rainfall excess hyetographs and direct runoff hydrographs, and conducting calibration were proved feasible. Calibration results indicated that the calibration accuracy (i.e., Se/Sy) is sensitive to both the tp_{UH} and the PRF. Therefore, the tp_{UH} should be calibrated simultaneously with the PRF.

The general guidelines developed from the analyses of both synthetic and measured data were very useful. They improved the current state in interpretation of UH hydrograph calibration results and assisted in calibrating the UH parameters from measured storm event data more accurately. Most of the general guidelines were developed based on the analyses of synthetic data and then validated based on the analyses of measured data. Since the true UH parameters imbedded within the storm data are unknown, it would be difficult to develop general guidelines from measured storm event data due to the inherent uncertainty of both the PRF and the tp_{UH}. Additionally, when analyzing measured data, multiple contributing factors may influence the calibration results simultaneously. The principal contributory factor would be difficult to identify and assess. Therefore, utilization of synthetic data to investigate these factors, one at a time, was considered necessary.

According to the analyses on both synthetic and measured data, all of these influential factors have significant effects on the calibration results. In terms of the effect of rainfall and runoff characteristics, the more complex rainfall and runoff data seem to be more difficult to fit with the two-parameter gamma UH model. When analyzing many sets of measured rainfall-runoff data, it is common to have short storms where the rainfall hyetograph is characterized by sequences of short bursts of high intensity followed by short periods of low intensity; such a pattern is referred to as a complex time series. Complex rainfall events can be difficult to analyze and to produce a unit hydrograph that allows convolution to accurately reproduce the direct runoff hydrograph. However, unless the issue of complex storm events is understood, a significant number of measured data sets would be ignored even though they are likely needed to provide a sufficient data base.

In most cases, a rainfall hyetograph shows considerable complexity when the direct runoff hydrograph is much less complex. This can result from a combination of the short time interval used with the rainfall data and the smoothing of the runoff by the watershed. Such events need to be analyzed as they reflect an important set of watershed conditions.

One problem with unit hydrograph modeling with complex storms is the modeling requirement related to the number of ordinates:

$$n_{UH} = n_{DRO} - n_{pe} + 1 \tag{6-1}$$

where n is the number of ordinates and the subscripts UH, DRO, and pe refer to the unit hydrograph, the direct runoff, and the rainfall excess, respectively. If n_{DRO} is just slightly larger than n_{pe}, which is common, then it forces the n_{UH} to be small, which produces a relatively short tp_{UH} and large PRF. Thus, less smoothing takes place through convolution and the computed direct runoff hydrograph is relatively complex while the observed direct runoff derived from the total runoff is less complex because of watershed smoothing. In such cases, the goodness of fit would be relatively poor and the computed parameters are likely to be poor estimators of the true watershed values.

Several options for reducing this negative effect of complexity can be stated. First, the constraint of equation (6-1) needs to be resolved, possibly by allow n_{UH} to vary from the constraint. Second, a loss function that smooths out the rainfall excess could be developed, which would provide a rainfall excess that has a complexity that is more compatible with the complexity of the observed direct runoff hydrograph.

The simulated data of section 4.3 indicated that the fitting method used herein could yield accurate estimates of the two UH parameters; however, the rainfall had minimal complexity compared with many actual hyetographs. In some of the actual data sets of Chapter 5, complexity resulted in both good (i.e., Oxford W-17, 03/04/1964) and poor results (i.e., Coshocton 130, 06/28/1957). The factors discussed above interact with complexity in determining the ability to provide the goodness of fit.

The issue of complexity needs further investigation. The first step would be to develop a metric that could characterize the complexity of a hyetograph or hydrograph. Criteria based on this metric would need to be developed in order to avoid it being assessed subjectively. The metric should be useful in deciding whether or not a data set should be analyzed. Therefore, simple rainfall-runoff data are recommended for UH calibration, but the analyses on complex storm events are still required, especially when a sufficient supply of data is not available.

In terms of the effect of time offset, it seems to be a common issue in measured data and can potentially have a considerable effect on the calibration accuracy of UH parameters. A time offset can directly affect the volume of the computed rainfall excess and the difference in times to peaks between the rainfall excess and direct runoff. Including a time offset can either increase or decrease the accuracy depending on the spatial locations of the rainfall and runoff gauges.

Therefore, the time offset should be tested by allowing it to vary in the calibration process.

Regardless of the time offset, the spatial nonuniformity of rainfall over a watershed as well as the extent of the nonlinearity of the watershed processes are two other common factors that are inherent in measured data. Rainfall nonuniformity can cause variations in the calibrated UH parameters. Therefore, rainfall hyetographs from different rain gauges should be analyzed to assess the potential variations in the UH parameters. Watershed processes can be nonlinear. An assumption of linearity is the basis of the UH model as it is commonly applied. Nonlinear (dynamic) watershed processes can make it difficult to achieve good calibration accuracy. Utilization of a dynamic model can yield, but not guarantee, an improvement in calibration accuracy. However, the dynamic model is still not sufficiently sophisticated to simulate the complex and varying rainfall-runoff processes.

The flexibility of the gamma UH was investigated. It was found that the gamma UH cannot provide good fits under the following conditions:

(1) The data characteristics are too complex to be fitted with a two-parameter model (i.e., a great dissimilarity between a rainfall excess hyetograph and observed direct runoff hydrograph).

(2) The existing time offset between rainfall excess and direct runoff is ignored or not corrected.

(3) The observed direct runoff hydrograph shows a left-tail skewed form rather than a right-tail skewed form.

(4) The width of the observed direct runoff hydrograph is either too thin or too wide.

(5) The difference in times to peak of the rainfall excess hyetograph and the observed direct runoff hydrograph is too short or too long.

(6) The nonlinear watershed processes exist.

Therefore, the selection of data sets to use in calibrating UH parameters needs to respect the limited flexibility of the gamma UH model.

In summary, the developed gamma UH calibration model is effective and workable. The general guidelines listed the problems that researchers may have when calibrating unit hydrographs and identified the possible reasons why good or poor fits result. Model users will benefit from these generated guidelines when selecting the measured data for analysis, interpreting the calibration results (i.e., either good or poor fits), and identifying the correct UH parameters.

6.3. RECOMMENDATIONS FOR FUTURE STUDIES

The general guidelines developed herein have significantly improved the chance for success in calibrating UHs using available measured data and the interpretation of the calibration results. The general guidelines still need to be continually enriched and advanced, especially in terms of the selection of storm data and the fitting of the UH parameters. In order to improve the effectiveness and the practicability of the calibration model, the following three topics are recommended for future study.

6.3.1. Improve Model Structure

One disadvantage of the current calibration model is that the flexibility of the two-parameter gamma pdf limits the model from yielding a good fit under certain conditions. For example, the gamma UH cannot provide a good fit when an observed direct runoff hydrograph shows a left-tail skewed form. Therefore, the next step in improving the model structure will be to improve the fitting flexibility of the model. This can be done either by adding a third location parameter to the two-parameter gamma pdf or using a more complex pdf with more variability (e.g., the beta pdf). However, it should be noted that although adding a third parameter or using a more complex pdf can improve the fitting flexibility of the calibration model, irrationality of the calibrated parameters may occur due to the increased flexibility or loss of an additional degree of freedom. Therefore, utilization of these two methods need to assess and respect the rationality of the parameters.

6.3.2. Explore Effects of Other Factors on UH Calibration

In this research, the effects of the rainfall characteristics (i.e., complexity and peakedness), the time offset, the rainfall nonuniformity, and the nonlinearity of watershed processes on UH calibration, have been discussed and summarized in the general guidelines. The next step will be to explore and assess the effects of other influential factors, such as rainfall depth, rainfall duration, watershed characteristics (i.e., area, slope, land use), and antecedent soil moisture. Investigating these factors will require analyses of a large quantity of measured data from different watersheds or additional simulation using synthetic data. The effect of each of these factors should be investigated and incorporated into the general guidelines. The general guidelines will be gradually enriched and advanced in improving the knowledge in selecting measured data for analysis, interpreting the calibration results (i.e., either good or poor fits), and identifying the correct UH parameters.

6.3.3. Regionalize the UH Parameters

As more and more measured data will be analyzed, the calibrated UH parameters as well as the watershed and rainfall information can be correlated to develop regression equations, in which each of the UH parameters can be defined as a function of watershed characteristics and other variables. Regionalization of the UH parameters will reduce the calibration uncertainty by increasing the sample size. These generated regression equations will be very useful for engineering design, especially for ungauged areas where calibration of measured storm data is not available.

APPENDIX A

Table A-1. Detailed information of each storm event

Location	Area (ac)	Watershed	Storm date	Tot P	Tot Q	Q/P	V_{pe}	prop	tp_p	tp_{pe}	tp_{RO}	tp_{DRO}	n_{pe}	n_{DRO}
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
Blacksburg, Virginia	1058	Chest nut	7/11/1965	0.78	0.522	0.07	0.008	0.062	9	9	81	81	9	191
Coshocton, Ohio	1.63	130	6/12/1957	3.01	1.410	0.47	1.208	0.654	26	26	31	31	26	54
Coshocton, Ohio	1.63	130	6/28/1957	2.78	1.010	0.36	0.446	0.790	36	36	49	49	56	86
Coshocton, Ohio	0.59	132	5/13/1964	1.34	0.570	0.43	0.420	0.024	12	12	28	28	21	382
Coshocton, Ohio	303	196	5/13/1964	1.07	0.336	0.31	0.178	0.134	14	14	33	33	20	128
Coshocton, Ohio	303	196	6/16/1946	4.17	1.445	0.35	0.668	0.353	44	44	57	57	57	130
Oxford, Mississippi	3.88	WC-1	7/8/1965	1.22	0.460	0.38	1.214	0.996	13	13	11	11	44	43
Oxford, Mississippi	3.88	WC-1	7/9/1967	1.94	1.510	0.78	1.113	0.409	62	62	66	66	75	116
(offset 20 mins)							1.046	0.065	62	62	86	86	75	136
Oxford, Mississippi	1.45	WC-2	7/8/1965	1.22	0.475	0.39	0.321	0.114	13	13	18	18	27	75
Oxford, Mississippi	1130	W-5	5/31/1967	1.52	0.468	0.31	0.352	0.341	60	60	100	100	75	315
Oxford, Mississippi	5530	W-10	5/31/1967	1.33	0.231	0.17	0.188	0.533	75	75	111	124	75	195
Oxford, Mississippi	5530	W-10	8/26/1971	3.22	0.598	0.19	0.506	0.449	75	75	147	147	105	310
Oxford, Mississippi	32100	W-17	3/4/1964	1.70	1.058	0.62	0.575	0.510	255	255	444	420	269	793
Oxford, Mississippi	20000	W-32	6/18/1971	1.47	0.399	0.27	0.323	0.372	45	45	136	136	45	315
(offset 30 mins)							0.318	0.370	45	45	106	106	45	285
Oxford, Mississippi	1090	W-35A	6/18/1971	1.79	0.806	0.45	0.696	0.409	30	30	109	109	30	270
Fennimore, Wisconsin	171	W-4	8/12/1943	2.11	0.433	0.20	0.369	0.146	5	5	16	16	10	72
Iowa City	1926		7/18/1956	4.30	1.000	0.23	0.686	0.235	24	24	125	125	44	224
offset 30 mins							0.456	0.261	24	24	125	125	39	224
McCredie, Missouri	153	W-1	8/19/1949	3.02	0.602	0.20	0.174	0.511	18	18	44	44	29	103
Hastings, Nebraska (first burst)	3.64	W-4H	7/8/1967	2.07	1.090	0.53	0.103	0.650	13	13	26	29	13	49
(second burst)							0.550	0.502	15	15	14	14	29	54
Chickasha, Oklahoma	38910	511	5/29/1970	2.39	0.435	0.18	0.076	0.465	19	19	427	379	19	540
(offset 2 hr)							0.095	0.388	19	19	307	235	19	420
Chickasha, Oklahoma	21310	621	5/10/1964	1.42	0.338	0.24	0.199	0.346	9	9	109	109	31	204
(offset 1 hr)							0.129	0.427	9	9	49	60	23	144
Chickasha, Oklahoma	27.3	C-8	9/19/1965	0.73	0.274	0.38	0.139	0.413	19	19	59	59	32	115
Stillwater, Okalahoma	206	W-4	6/19/1972	2.22	0.410	0.18	0.308	0.231	21	21	60	60	21	121
Stillwater, Okalahoma	16.7	W-1	12/29/1972	0.81	0.599	0.74	0.303	0.289	15	15	53	53	19	201
Riesel,Texas	1110	D	5/10/1965	3.11	2.240	0.72	1.497	0.198	16	16	51	51	26	202
Springs, Colorado	35.6	W-1V	8/2/1939	1.68	0.346	0.21	0.302	0.427	17	17	25	26	21	111
Santa Fe, New Mexico	141	W-1	8/25/1947	0.90	0.381	0.42	0.341	0.258	8	8	18	20	12	92
Tombstone, Arizona	9.1	63.103	9/8/1970	1.44	0.604	0.42	1.300	0.674	21	21	13	13	37	45
Safford, Arizona	723	45.005	7/22/1955	1.42	0.380	0.27	0.355	0.558	32	32	59	59	37	87

NOTATION

	P(t) =	Rainfall hyetograph
	Pe(t) =	Rainfall excess hyetograph
	RO(t) =	Runoff hydrograph
	DRO(t) =	Direct runoff hydrograph
Column 5:	Tot P =	Rainfall depth (in.)
Column 6:	Tot Q =	Runoff depth (in.)
Column 7:	Q/P =	Ratio of Q to P
Column 8:	V_{pe} =	Depth of Pe(t) and DRO(t)
Column 9:	prop =	Proportion of DRO(t) under the rising limb
Column 10:	tp_P =	Minutes from start of P(t) to peak P(t)
Column 11:	tp_{Pe} =	Minutes from start of Pe(t) to peak Pe(t)
Column 12:	tp_{RO} =	Minutes from start of RO(t) to peak RO(t)
Column 13:	tp_{DRO} =	Minutes from start of DRO(t) to peak DRO(t)
Column 14:	n_{Pe} =	No. of ordinates in Pe(t)
Column 15:	n_{DRO} =	No. of ordinates in DRO(t)

Table A-1. Detailed information of each storm event [continued]

Location	Area (ac)	Watershed	Storm date	…	P_X	Pe_X	RO_X	DRO_X	Rb	Se/Sy	tp_{UH}	PRF	b	c
(1)	(2)	(3)	(4)		(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)
Blacksburg, Virginia	1058	Chest nut	7/11/1965		0.0789	0.0009	0.0007	0.0050	-0.0320	1.0420	3	110	10.60	1.28
Coshocton, Ohio	1.63	130	6/12/1957		0.1300	0.0907	0.0583	0.0544	0.0020	0.4781	94	110	332.00	1.28
Coshocton, Ohio	1.63	130	6/28/1957		0.0325	0.0226	0.0200	0.0161	0.0040	0.7726	132	131	355.00	1.37
Coshocton, Ohio	0.59	132	5/13/1964		0.0750	0.0426	0.0100	0.0100	-0.0040	0.7249	2	10	109.20	1.02
Coshocton, Ohio	303	196	5/13/1964		0.0667	0.0232	0.0075	0.0074	-0.0010	0.3168	7	164	13.08	1.54
Coshocton, Ohio	303	196	6/16/1946		0.0900	0.0486	0.0314	0.0254	-0.0020	2.5100	57	222	63.79	1.89
Oxford, Mississippi	3.88	WC-1	7/8/1965		0.0523	0.0523			-1.0000	1.0300	62	484	16.78	4.70
Oxford, Mississippi	3.88	WC-1	7/9/1967		0.0700	0.0598	0.0595	0.0557	-1.0000	1.5100	109	944	7.43	14.72
(offset 20 mins)					0.0700	0.0580	0.0595	0.0544	-1.0000	1.6000	154	144	356.00	1.43
Oxford, Mississippi	1.45	WC-2	7/8/1965		0.0523	0.0196	0.0137	0.0130	-0.0050	0.6886	1	43	12.98	1.08
Oxford, Mississippi	1130	W-5	5/31/1967		0.0440	0.0194	0.0058	0.0056	0.0080	1.5010	1	8	63.92	1.02
Oxford, Mississippi	5530	W-10	5/31/1967		0.0280	0.0076	0.0025	0.0024	-0.0180	0.3916	36	204	46.60	1.72
Oxford, Mississippi	5530	W-10	8/26/1971		0.0440	0.0161	0.0074	0.0070	0.0140	0.7110	1	21	28.75	1.04
Oxford, Mississippi	32100	W-17	3/4/1964		0.0207	0.0129	0.0034	0.0027	0.0200	0.1835	200	398	78.24	3.56
Oxford, Mississippi	20000	W-32	6/18/1971		0.0500	0.0215	0.0027	0.0025	0.0000	0.7670	11	65	83.50	1.13
(offset 30 mins)					0.0500	0.0212	0.0027	0.0025	0.0000	0.7588	11	66	81.77	1.14
Oxford, Mississippi	1090	W-35A	6/18/1971		0.0633	0.0375	0.0085	0.0082	0.0040	0.2401	28	231	29.25	1.96
Fennimore, Wisconsin	171	W-4	8/12/1943		0.1600	0.0724	0.0193	0.0188	0.0030	0.2127	3	107	11.06	1.27
Iowa City	1926		7/18/1956		0.1000	0.0412	0.0141	0.0128	0.0030	0.1942	2	32	36.79	1.05
offset 30 mins					0.1000	0.0328	0.0141	0.0110	-0.0020	0.7689	2	49	22.05	1.09
McCredie, Missouri	153	W-1	8/19/1949		0.0900	0.0132	0.0059	0.0033	-0.0010	0.6934	3	31	57.20	1.05
Hastings, Nebraska (first burst)	3.64	W-4H	7/8/1967		0.0417	0.0150	0.0100	0.0062	-0.0020	0.9549	2	45	24.54	1.08
(second burst)					0.0447	0.0309	0.0320	0.0309	0.0000	0.4523	1	528	0.23	5.36
Chickasha, Oklahoma	38910	511	5/29/1970		0.0862	0.0094	0.0012	0.0005	-0.0120	0.3983	86	266	70.08	2.23
(offset 2 hr)					0.0862	0.0118	0.0012	0.0005	-0.0100	0.3812	84	246	78.59	2.07
Chickasha, Oklahoma	21310	621	5/10/1964		0.0575	0.0239	0.0035	0.0029	-0.0040	0.3227	38	352	18.66	3.04
(offset 1 hr)					0.0575	0.0185	0.0035	0.0023	0.0030	0.2112	40	452	12.33	4.25
Chickasha, Oklahoma	27.3	C-8	9/19/1965		0.0211	0.0070	0.0042	0.0034	-0.0180	0.2669	13	184	20.05	1.65
Stillwater, Okalahoma	206	W-4	6/19/1972		0.0867	0.0228	0.0090	0.0084	0.0050	0.3289	6	131	16.14	1.37
Stillwater, Okalahoma	16.7	W-1	12/29/1972		0.0490	0.0292	0.0053	0.0057	-0.0050	0.1660	19	196	26.33	1.72
Riesel,Texas	1110	D	5/10/1965		0.1250	0.0899	0.0149	0.0139	0.0020	0.3062	16	85	83.16	1.19
Springs, Colorado	35.6	W-1V	8/2/1939		0.1100	0.0566	0.0230	0.0226	0.0020	0.4043	6	217	6.99	1.86
Santa Fe, New Mexico	141	W-1	8/25/1947		0.0700	0.0412	0.0163	0.0159	-0.0010	0.4179	14	352	6.88	3.04
Tombstone, Arizona	9.1	63.103	9/8/1970		0.0614	0.0599	0.8123	0.8111	0.0010	0.9912	1	575	0.19	6.14
Safford, Arizona	723	45.005	7/22/1955		0.0860	0.0410	0.0169	0.0168	0.0020	0.3063	6	132	15.95	1.38

NOTATION

P(t) = Rainfall hyetograph
Pe(t) = Rainfall excess hyetograph
RO(t) = Runoff hydrograph
DRO(t) = Direct runoff hydrograph

Column 16: P_x = Max ordinate (normalized) to peak P(t)
Column 17: Pe_x = Max ordinate (normalized) to peak Pe(t)
Column 18: RO_x = Max ordinate (normalized) to peak RO(t)
Column 19: DRO_x = Max ordinate (normalized) to peak DRO(t)
Column 20: Rb = Relative bias for optimum PRF,
Column 21: Se/Sy = Standard error ratio
Column 22: tp_{UH} = Time to peak of UH
Column 23: PRF = Optimum (calibrated) PRF
Column 24: b = Gamma scale parameter
Column 25: c = Gamma shape parameter

REFERENCES

Aron, G. and White, E. L. (1982). "Fitting a gamma distribution over a synthetic unit hydrograph". *Water Resources Bulletin*, 18(1):95-98.

Bhunya, P. K., Berndtsson, R., Ojha, C. S. P., and Mishra, S. K. (2007). "Suitability of Gamma, Chi-square, Weibull, and Beta distributions as synthetic unit hydrographs". *Journal of Hydrology*, 334(1):28-38.

Boufadel, M. C. (1998). "UNIT HYDROGRAPHS DERWED FROM THE NASH MODEL". *JAWRA Journal of the American Water Resources Association*, *34*(1):167-177.

Capece, J. C., Campbell, K. L., and Baldwin L. B. (1986). "Estimation of runoff peak rates and volumes from Flatwoods watersheds". Agricultural Engineering Department, Institute of Flood and Agricultural Sciences, University of Florida, Gainesville, Florida.

Clark, C. O. (1945). "Storage and the unit hydrograph". Transactions of the American Society of Civil Engineers, 110:1419–1446.

Cruise, J.F. (1980). "Unit hydrographs for urbanizing watersheds". *Journal of the Hydraulics Division,* ASCE, 106(HY3):440-445.

Dooge, J.C.I. (1959). "A general theory of the unit hydrograph". *Journal of Geophysical Research,* 64(1):241–256.

Edson, C. G. (1950). "Parameters for relating unit hydrograph to watershed characteristics" Transactions, American Geophysical Union, 32(4):591.

Fang, X., Khanal, P., Theodore, C., David, T., and Pratistha, P. (2005). "Revisit of NRCS unit hydrograph procedures". Proceedings of the ASCE Texas Section Spring Meeting, Austin, Texas.

Gray, D M. (1960). "Derivation of hydrographs for small watersheds from measurable physical characteristics". Retrospective Theses and Dissertations. Paper 2644.

Gray, D. M. (1961). "Synthetic Unit Hydrographs for Small Watersheds". *Journal of the Hydraulics Division*, ASCE, 87(HY4):33-54.

McCuen, R.H. (2005). Hydrologic Analysis and Design (Third Edition). Pearson Prentice Hall, Upper Saddle River.

McCuen, R.H. and Bondelid, T.R. (1983). "Estimating unit hydrograph peak rate factors". *Journal of Irrigation and Drainage Engineering*, ASCE, 109(2):238‒250.

Meadows, M. E., and Chestnut, A. L. (1983). "The curve number runoff model as an infiltration model for hydrograph simulation". In *Advances in Irrigation and Drainage: Surviving External Pressures* (pp. 298-307). ASCE.

Mockus, V. (1957). "Use of storm and watershed characteristics in synthetic hydrograph analysis and application". American Geophysical Union, Pacific Southwest Region, Sacramento, CA.

Nash, J.E. (1957). "The form of the instantaneous unit hydrograph". *International Association of Scientific Hydrology*, 45(3):114–121.

Nash, J. E. (1959). "Systematic determination of unit hydrograph parameters". *Journal of Geophysical Research*, 64(1):111-115.

Sabol, G. V. (1988). “Clark Unit Hydrograph and R-parameter estimation”. *Journal of Hydraulic Engineering*, 114:103–111.

Sheridan, J. M. and Merkel, W. H. (1993). “Determining design unit hydrographs for small watersheds”. U.S. Geological Survey, Water-Resources Investigations Report 93–4018. 8:42–49. Washington, D.C.

Sheridan, J.M., Merkel W.H., and Bosch D.D. (2002). “Peak rate factors for flatland watersheds”. *Applied Engineering in Agriculture*, 18:65-69.

Sherman, L. K. (1932). “Streamflow from rainfall by the unit-graph method”. *Engineering News Record*, 108:501-505.

Singh, K. P. (1976). “Unit hydrographs—a comparative study”. Water Resource Bulletin, 12(2):381–392.

Snyder, F.F. (1938). “Synthetic unit-graphs”. Transactions, American Geophysics Union, 19:447–454.

Straub, T.D., Melching, C.S., and Kocher, K.E. (2000). “Equations for estimating Clark unit hydrograph parameters for small rural watersheds in Illinois”. U.S. Geological Survey, Water-Resources Investigations Report 00-4184.

U.S. Department of Agriculture, Natural Resources Conservation Service. 1972. Hydrographs. National Engineering Handbook, Part 630, Chapter 16. Washington, DC.

U.S. Department of Agriculture, Natural Resources Conservation Service. 2007. Hydrographs. National Engineering Handbook, Part 630, Chapter 16. Washington, DC.

Welle, P., Woodward, D.E., and Fox Moody, H. (1980). “A dimensionless unit hydrograph for the Delmarva Peninsula”. American Society of Agricultural Engineers, Paper Number 80–2013, St. Joseph, MI.

Wu, I. (1963). “Design hydrographs for small watersheds in Indiana”. *Journal of the Hydraulics Division*, ASCE, 89(HY6):35−65.