ABSTRACT

Title of dissertation:	EQUILIBRIUM ANALYSIS AND CONTROL FOR DESIGN OF PACKET RESERVATION MULTIPLE ACCESS PROTOCOLS
	Amirali Sharifi, Doctor of Philosophy, 2010
Dissertation directed by:	Professor Eyad H. Abed Department of Electrical and Computer Engineering

The Packet Reservation Multiple Access (PRMA) protocol and its variants have been considered as possible access schemes for communication media for indoor communications, terrestrial communications and satellite communications. Most recently, PRMA (and its variants) has been considered for applications such as beyond third generation and/or fourth generation communication systems, cooperative communication, and multimedia communication in dynamic environments.

In this dissertation, equilibrium behavior of general voice and/or data systems employing PRMA are studied along with means for control of this behavior. The main objective is to determine conditions guaranteeing a unique equilibrium for these systems, as multistability can result in an unacceptable user experience. Systems considered include voice systems, voice and data systems, and voice systems with high propagation delay (these are studied both for an error-free channel and a random error channel). Also, various control schemes are introduced and their ef-

fect on these system is analyzed at equilibrium. Control schemes considered include a price based control, state estimation-based control, and control using multiple transmission power and capture. For each type of control, the effect of the control on the equilibrium structure of the system is studied, in the spirit of the methodology of bifurcation control. In bifurcation control, the number and nature of steady state solutions of a system are managed by appropriate design of system control laws. Several sufficient conditions for uniqueness of operating points of the PRMA systems under the studied control schemes is determined. Numerical analysis of the equilibrium equations of the systems is provided to support the analytical studies. The equilibrium behavior of voice systems and voice-data systems employing frame-based PRMA is also studied. Effects of price based control on these systems is analyzed. Further, the price based control studied in conjunction with the PRMA systems is extended to a finite buffer finite user slotted ALOHA system, and the equilibrium behavior of the system is studied using a tagged user approach.

Among the contributions of the dissertation are analytical sufficient conditions guaranteeing a unique equilibrium point for the various classes of systems studied, control law designs that result in improved system capacity, and extensive numerical studies including comparisons with two previously proposed approaches. Analysis is also given proving the Markovian nature of the system's stochastic dynamics (under some basic assumptions) and the existence of a unique stationary probability law.

Equilibrium Analysis and Control for Design of Packet Reservation Multiple Access Protocols

by

Amirali Sharifi

Dissertation submitted to the Faculty of the Graduate School of the University of Maryland, College Park in partial fulfillment of the requirements for the degree of
Doctor of Philosophy
2010

Advisory Committee:

Professor Eyad H. Abed, Chair/Advisor
Professor Armand Makowski
Professor Richard La
Professor Balakumar Balachandran
Professor Dana Nau

Dedication

To My Parents, Soussan and Touraj

To My Wife, Yoss

To My Sister, Maryam

Acknowledgments

I owe my gratitude to all the people who have made this thesis possible and because of whom my graduate experience has been one that I will cherish forever.

First and foremost I would like to express my sincere gratitude to my advisor, Professor Eyad H. Abed for his support, guidance, and advise during my graduate studies at the University of Maryland. I value his friendship and appreciate all I have learned from him.

I would like to thank my committee members, Professor Balachandran, Professor La, Professor Makowski, and Professor Nau for their time and valuable feedback. Thanks are also due to the faculty and staff of the Department of Electrical and Computer Engineering and the Institute for Systems Research for their constant support and help.

I owe my deepest gratitude to my mother and father who have always stood by me, gave me endless patience and love, and guided me through my life. Words cannot express the gratitude I owe them. Deep thanks are also due to my dear sister.

Special and deep thanks to my wife, Yoss. She has always surrounding me with care and love. I will never forget her support, encouragement, and patience throughout my graduate studies.

I would also like to thank all my great friends for their invaluable support. I am afraid I cannot name them here as they are too many. I would like to thank them all and wish them success wherever they are.

Table of Contents

List of Figures

Chapter 1

Background, Motivation, and Outline of Dissertation

This chapter serves to provide motivation for our research and necessary background information for the rest of this dissertation.

Packet Reservation Multiple Access (PRMA) was introduced in [1] as a combination of slotted-ALOHA [2], [3], [4] and TDMA (Time Division Multiple Access Protocol) [5]. However, PRMA protocol can also be viewed as a variation of reservation ALOHA with added features of speech activity detection and packet drop probability. It is assumed that a speech activity detector in the PRMA protocol can divide activities of a voice terminal into "on" or "off" states. Further, the PRMA protocol, in order to consider delay sensitivity of speech, can drop packets waiting for transmission more that a predetermined threshold.

Also, it is noted that reservation ALOHA can be considered as an explicit reservation protocol, meaning reservation packets are used for making reservation. However, the PRMA protocol can be considered as an implicit reservation protocol since no reservation channel is allocated. Also, as discussed later, in the PRMA protocol, we assume that only speech terminals can make reservation (data terminals cannot reserve time slots). In contrast, in the reservation ALOHA no difference exists between speech and data terminals.

Since the PRMA protocol is based on ALOHA protocol, same nonlinear behavior and instability of ALOHA type protocols is seen in the PRMA protocols. For example, based on a given set of parameters, a stable PRMA protocol is designed. The stable protocol has an acceptable throughput and delay. However, it has been shown that if parameters of the system and/or protocol changes, the behavior of the system can dramatically change. In one example, a stable system designed based on the PRMA protocol can have a unique operating point with small delay and hight throughput. However, minor changes in the parameters of the system can result in multiple operating points, some with very high delay and low throughput.

Combination of contention algorithm for burst-type data, reservation algorithm for periodic-type data, and use of speech activity detector, makes PRMA based protocols attractive. The PRMA protocol was introduced and has been widely considered and researched for providing speech and data communications in a terrestrial microcell system. Different variations of the PRMA protocol have been proposed and studied, which a set of these variations is summarized in this chapter. Also, modified versions of the PRMA protocol have been the subject of research, with the goal of application to low earth orbit mobile satellite systems (LEO-MSSs). These modified versions (such as the PRMA with hindering states (PRMA-HS)) have taken into account high round trip delays that exist in satellite communication. More recently the PRMA protocol and/or modifications of the protocol have been studied for, for example, cooperative packet speech communications [6], multimedia communication in environments with dynamic nature (such as motorways and airports) [7], [8], and [9], and beyond third generation and/or fourth generation of communication systems [10] and [11]. Utilizing speech activity detector of the PRMA protocol and deploying a relay node in [6] introduces a novel cooperative multiple access protocol for speech packets. Further, a centralized Mini-slot PRMA protocol based on OFDMA/TDD (orthogonal frequency division multiplexing / time division duplex) for access network architecture and media access control technique for beyond 3G systems is introduced and studied in [10] based on the system requirements.

Our goal in this dissertation is to address nonlinear behavior and instabilities of the PRMA protocol. We will consider several variations of the PRMA protocol (PRMA voice only, PRMA voice-data, PRMA-HS voice only, PRMA-HS voice-data, Framed PRMA voice only, and Framed PRMA voice-data) and study bifurcations that occur in their equilibrium state. Further, we consider several schemes to control the bifurcations. These schemes will include price based control, control using multiple power level and capture effect, and state estimation-based control. Moreover, we extend our analysis to PRMA protocols over random error channels.

Since ALOHA type systems are backbone of the PRMA protocols, in this chapter we, first, review nonlinear instabilities and bistable behavior of ALOHA type systems. Later, we review background information and modeling of PRMA and Packet Reservation Multiple Access with Hindering States (PRMA-HS) protocols and their stability issues.

1.1 Nonlinear Instability in Slotted-ALOHA and Reservation-ALOHA

Packet switching has found many applications in communications because of its ability to handle traffic with a high ratio of peak-to-average. One application of packet switching to radio channels is the packet radio. However, one important behavior of packet radio is its bistability, which means that the system can possess two statistically stable equilibrium points, one in a desirable low-delay region, and the other in an undesirable high-delay region. Since stability is statistical in nature, the system oscillates between these two points. Unstable behavior of the synchronous systems (S-ALOHA) was originally predicted by Rettberg [12] using a deterministic model. Metacalfe [13] used a steady-state analysis to demonstrate existence of two stable equilibriums. Kleinrock and Lam [14] also developed models to include effects of system dynamics and control strategies. Carleial and Hellman [15] also developed similar models independently to study bistable behavior of S-ALOHA systems.

S-ALOHA: Consider a finite or infinite number of terminals transmitting packets on a shared communication medium to an access point (such as a base station). The channel is slotted and each slot is equal to transmission time of a packet. Packets arrive at terminals randomly and terminals can send their packets at the beginning of next time slot (in case of deferred first transmission). A transmission in a time slot is successful if a single terminal attempts transmission during the time slot. If a collision happens, terminals will go to a retransmission state and will retransmit their packets in future time slots according to some permission probability. If a finite number of terminals (M) are sharing the communication medium, this channel can be viewed as a discrete-time Markov chain with $M + 1$ states, corresponding to the number of terminals in retransmission mode. One-step transition probability matrix can be written easily. The Markov chain can be shown to be irreducible, aperiodic and positive-recurrent; therefore, stationary probability density function exists. The system's expected throughput at each state is calculated. Expected drift is an interesting indicator of the system behavior, which shows how the system tends to move in its random walk over the stat space. Depending on system parameters, a graph of expected drift may intersect a zero line at either one or three points. In the case of one point, the system has only one equilibrium point and it is stable because the expected drift's graph changes from positive to negative at that point. However, in the case of three points, the system has two stable fixed points at the two ends and one unstable equilibrium point in between. Starting from state zero, the system drifts toward the first stable equilibrium point. Note, however, that the concept of stability is deployed in the statistical sense. Therefore, if the system passes the second equilibrium point, which is unstable, it drifts toward the second stable equilibrium point, which is in a region of states with a high number of terminals in retransmission mode and high delay. Usually the system will not return to the desirable stable equilibrium point. The same behavior is seen in stationary probability density of the Markov chain. In the case of only one stable equilibrium point, the probability density is unimodal, with a peak around the stable point. However, when the system exhibits two stable equilibrium points, the probability density function becomes bimodal with two peaks around the stable points.

Kleinrock and Lam [14] used a fluid approximation to study the stability behavior of ALOHA protocol with an infinite population model. They calculated expected channel throughput at each state using an equilibrium contour. They also calculated a channel load line as channel input. The intersection of these two graphs is the equilibrium point(s) of the system. As Kleinrock and Lam defined, a slotted ALOHA channel is said to be stable if the equilibrium contour and its load line intersect exactly at one point. Otherwise, the channel is unstable. A point on the load line is said to be a stable equilibrium point if it acts as a "sink". If it is the only stable point, it is a globally stable equilibrium point. Otherwise, it is a locally stable equilibrium. Further, an equilibrium point is said to be an unstable equilibrium point if fluid flow emanates from it and this is the same bistable behavior previously observed using the fluid approximation by Kleinrock and Lam. Jenq [3] showed that both the input-output packet flow balance principle (used by Kleinrock and Lam) and the concept of expected drift (used by Carleial and Hellman [15]) used for stability analysis of the slotted ALOHA system are mathematically equivalent. Jenq also showed that the slotted ALOHA system can only have either one or three equilibrium points.

R-ALOHA: A performance analysis of reservation ALOHA (R-ALOHA) was first done by Lam [16]. In that work he did not address stability issues of R-ALOHA. Later, Tasaka [17] studied stability and performance of the reservation ALOHA packet broadcast system. He used an approximate method called equilibrium point analysis (EPA) to study a multidimensional Markov chain. R-ALOHA is simple in principle and easy to implement like S-ALOHA, but it is more suitable for multi-packet messages. It is assumed that each terminal handles one message at a time and the number of packets in one message is geometrically distributed. In R-ALOHA, a channel is slotted and N time slots are grouped to a frame. In any time slot a terminal with no message generates a message with probability σ, and the number of packets in this message is distributed geometrically with average of $1/\gamma$. A terminal with a message to transmit cannot generate new messages. Time slots are either reserved or unreserved. The terminal will send its first packet immediately on an available time slot. If the packet arrives successfully at an access point, that time slot will be reserved for that terminal and the terminal will send the rest of its packets without contention in that slot in future frames. A terminal can only reserve one time slot in a frame. If a collision occurs, the terminal will attempt re-transmission in other available time slots with some probability. The R-ALOHA system can be modeled with a Markov chain. The Markov chain is irreducible, aperiodic, and positive-recurrent and therefore, has a unique stationary probability density function. However, because of the large number of states, it is difficult to use the technique of Markov analysis. Therefore, an approximate method, EPA, is used. Strictly speaking, an equilibrium point is defined as a point, which satisfies the condition that the expected increase in the number of users in each state is zero at that point. Tasaka showed that, like S-ALOHA, the R-ALOHA has either one stable equilibrium point or two stable and one unstable equilibrium points depending on system parameters. Tasaka also showed that the one equilibrium point correspond to a unimodal stationary distribution and the three equilibrium points corresponds to a bimodal stationary probability distribution. He also showed that error in the approximate method EPA is due to the shape of the stationary distribution around its peaks.

Onozato and Noguchi [18] developed a new tool to study behavior of a multi-access communication system. Based on a Markovian model introduced by Lam and Carleial, Onozato and Noguchi introduced an approximate birth and death for slotted ALOHA and gave a detailed analytical description of cusp catastrophe in S-ALOHA. They also introduced a measure for the bistable behavior of S-ALOHA as the ratio of the two peaks in the steady-state probability density function, and they studied the changes in this measure as system parameters change.

In another work, Onozato, Liu, and Noguchi [19] studied effect of capture on stability of slotted ALOHA systems. They assumed that terminals sharing the slotted ALOHA channel are divided into two groups and there is no capture effect among the terminals of the same group. Only when a collision between packets of different groups occurs, capture effect may occur. They studied stability of the system by modeling the capture effect in the concept of probability.

Recently Sakakibara *et al.* [20] and [21] studied how limiting the number of retransmissions affects the stability of slotted ALOHA systems with no capture. They showed that a slotted ALOHA system has a unimodal steady-state probability distribution, for any values of system parameters, if the number of retransmissions is limited to, at most, eight. They have also shown that increasing the number of retransmission trials enlarges the bistable region.

1.2 Packet Reservation Multiple Access (PRMA)

In this section, we present a brief review of the packet reservation multiple access protocol. A two-way wireless communication with star topology is considered in which terminals send their packets to an access point (such as a base station) (uplink) using PRMA protocol as medium access control scheme. The access point broadcasts a continuous stream of packets to the terminals (downlink). These packets contain feedback information, voice, and/or data packets. The uplink channel is subject to collisions but the downlink is not.

As noted above, the evolution of the PRMA protocol from slotted ALOHA is due to use of speech activity detector that can detect period of silence or talkspurt for voice terminals. Therefore, the PRMA protocol provides reservation for voice terminals that successfully transmit a packet. Further, in comparison to reservation ALOHA protocol, the PRMA protocol considers delay sensitivity of speech packets by dropping packets in the buffer of a voice terminal that have been waiting for transmission for more than a predetermined threshold. The PRMA protocol was mainly developed for voice terminals. However, it has been shown that the PRMA protocol can effectively support different traffic such as voice, data, multimedia, etc.

The PRMA channel is divided into time slots of duration τ seconds. N consecutive slots are grouped to form a frame with duration T seconds. We assume that M_v voice terminals and M_d data terminals use the shared communication medium based on the PRMA protocol [1], [22], [23], [24]. In this assumption, voice and data terminals are separate terminals. However, a system with terminals having both voice and data capabilities can also be considered.

1.2.1 PRMA Voice Subsystem

A speech activity detector is used for voice terminals to detect when a speaker is silent or talking. During a talkspurt, speech information gathered at a terminal in $N\tau$ seconds is assembled in one packet. Duration of a talkspurt is much larger than a packet size. Therefore, first packet of the talkspurt is followed by new packets every $N\tau$ seconds. In other words, a speech terminal generates one voice packet every N time slots. During the talkspurt, the voice terminal generates speech information with the source rate R_s bits/s and the channel bit rate is R_c bits/s. The packet header is H bits. Therefore, N, the number of slots in one frame is:

$$N = \lfloor \frac{R_c T}{R_s T + H} \rfloor$$

Let t_1 and t_2 be mean duration of talkspurt and silent gap, respectively. Assume that these mean durations are much larger that τ. Hence, σ_v, probability that a silent gap is terminated in a time slot, and γ, probability of a talkspurt ending in a time slot, are as follows [23], [24]:

$$\sigma_v = 1 - \exp(-T/Nt_2),$$

$$\gamma = 1 - \exp(-T/Nt_1).$$

In the PRMA protocol, it is assumed that all transitions happen at end of a time slot. A speech terminal in its silence gap is in a silent (SIL) state. When a talkspurt starts, the speech terminal transitions to contending state (CON). A terminal in CON state, if it has permission, contends for reservation by transmitting a speech packet on an unreserved time slot. Permission is generated at each contending terminal, with a fixed probability p_v, and is independent for each terminal. A transmission of a contending terminal is successful if: (1) the time slot is unreserved, (2) speech terminal has permission to transmit, (3) no other contending voice terminal has permission to transmit, and (4) no backlogged data terminal has permission to transmit.

The access point (such as a base station) transmits result of the contention in that time slot to all the terminals in a feedback message. A successful transmission will grant the speech terminal a reservation of that time slot. The terminal will transition to state RES_{N-1}. In the same time slot in the next frame, the terminal will send next speech packet with no contention (like TDMA). The probability that a talkspurt ends in a particular frame is given by [23], [24]

$$\gamma_f = 1 - (1 - \gamma)^N \approx N\gamma.$$

When the talkspurt ends, the terminal transitions back to the SIL state.

Since speech packets need prompt delivery, each packet can only tolerate a maximum delay. Packets that wait more than this maximum permissible delay are dropped. A contending voice terminal contends for reservation by transmitting first packet of its speech message on available time slots, if it has permission. The voice terminal will drop the first packet if it has not been able to successfully transmit the packet before the permissible delay. Then, the terminal contends for reservation with next packet in the message. The process continues until the terminal successfully transmits a packet. A main performance measurement of the PRMA protocol is packet drop probability (P_{drop}) defined as the average number of packets dropped versus the average number of packets in a voice message. A packet drop probability of less than one percent is acceptable for the PRMA protocol in order to cause a minimal degradation in the speech quality. Therefore, one important parameter that is evaluated is capacity of a PRMA carrier, defined as the maximum number of user terminals that can share the channel with a packet drop probability of less than one percent. Another important performance measure for the voice subsystem in the PRMA protocol is throughput, defined as the average number of packets successfully transmitted per slot [23], [24].

1.2.2 PRMA Data Subsystem

Modeling and characterizing data subsystem is more difficult than voice subsystem because data traffic can vary from a short message (such as an e-mail) to a large data message (such as transfer of large files). Because of this difficulty, in this dissertation, we consider a simple data traffic model similar to models used in previous studies of slotted ALOHA and PRMA.

We assume that data packet generation at each data terminal is Poisson with a slot arrival rate of σ_d. Also for simplicity, we assume that each data terminal has a buffer of one packet long. Although, results are easily extendable to buffers with infinite capacity. A data terminal is called backlogged, BLK , if its buffer is not empty. A backlogged data terminal transmits its packet on an unreserved time slot, if it has permission to transmit. Permission is generated according to a permission probability p_d which is the same for all data terminals and is generated independently from other terminals. Because the speech packets need prompt delivery, permission probability for speech terminals is assumed to be larger than that of data terminals, $p_v \geq p_d$. A data packet's transmission is successful if (1) time slot is unreserved, (2) data terminal has permission to transmit, (3) no other backlogged data terminal has permission, and (4) no contending voice terminal has permission.

The access point (such as the base station) provides feedback to all terminals regarding outcome of transmission of this data terminal. A successful transmission does not grant the data terminal a slot reservation. Important performance parameters for the data subsystem can include average throughput and average delay. Average delay is defined from the time a packet arrives in the data terminal to the time it is successfully transmitted.

1.2.3 PRMA - System Model

Nanda *et al.* showed that PRMA system can be modeled by a Markov chain [23], [24]. Figures 1.1 and 1.2 show Markov chain models for each terminal in voice and data subsystem, respectively. The number of voice terminals in contending mode is denoted by c, r is the number of voice terminals holding reservations, s_v (s_d) is the number of voice (data) terminals in silent mode, and b is the number of backlogged data terminals. Transition probabilities for the Markov model can be written such that state of the system is the number of terminals in each terminal

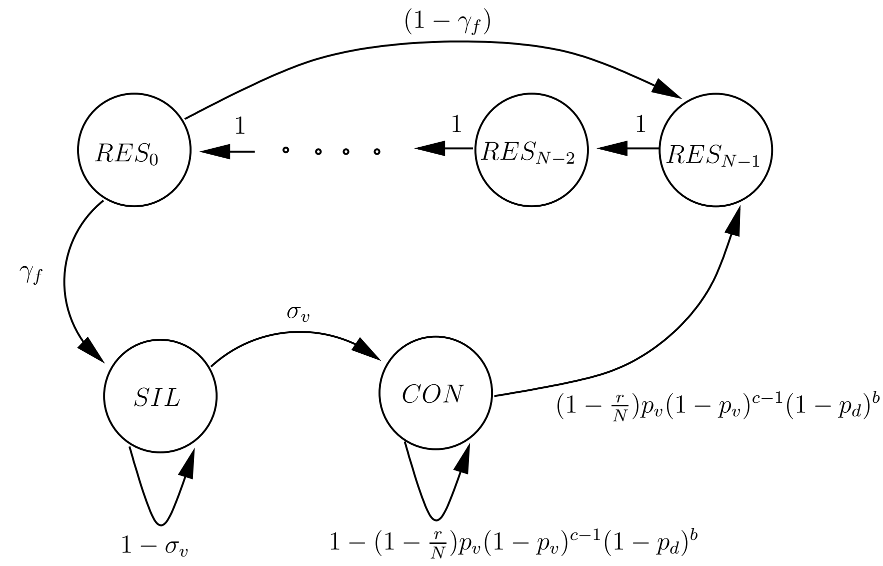

Figure 1.1: Markov chain model for PRMA voice subsystem.

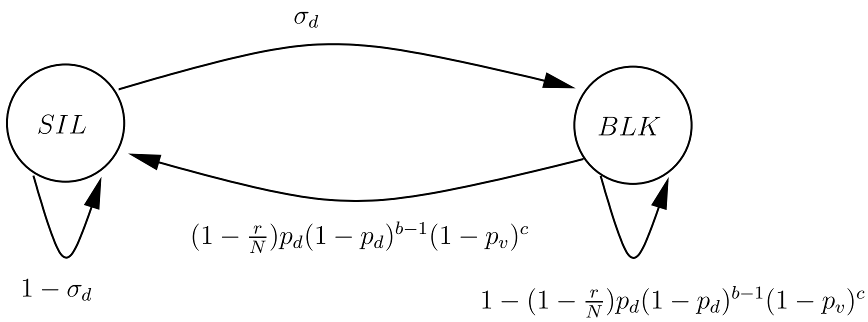

Figure 1.2: Markov chain model for PRMA data subsystem.

state. Presentation of the transition probability matrix for the PRMA system is omitted in this chapter, but revised transition probabilities (considering control schemes) are presented in the next chapter.

Although systems employing the PRMA protocol can be modeled with Markov chain, Markov analysis of the Markov model becomes very difficult as the number of speech and data terminals increases. Therefore, Nanda *et al.* [23] suggested using equilibrium point analysis (EPA) as a tool to investigate behavior of the protocol at steady state as described by Tasaka [17]. At equilibrium, it is assumed that the expected number of terminals exiting a state is equal to the expected number of terminals entering that state. This is the main idea behind the EPA. In order to find equilibrium equations for PRMA Voice-Data system, we consider each subsystem separately. The state of the system at equilibrium consists of the expected number of contending voice terminals (C), silent voice terminals (S_v), voice terminals in reservation mode (R), backlogged data terminals (B), and silent data terminals (S_d). For the voice subsystem, at SIL state, the expected number of voice terminals leaving this state is equal to the expected number of terminal entering this state,

$$(\frac{R}{N})\gamma_f = S_v\sigma_v.$$

Similarly at CON,

$$S_v\sigma_v = (1 - \frac{R}{N})Cp_v w_v(C, B),$$

here $\quad w_v(C,B) = \begin{cases} (1-p_v)^{C-1}(1-p_d)^B & C \geq 1 \\ (1-p_d)^B & C < 1 \end{cases}.$

Also, it is noted that the total number of voice terminals is fixed. Therefore, $S_v + C + R = M_v$. Equilibrium equations, as noted above, can be simplified to one equation with two states C and B.

$$F_1(C, B) = M_v - C - (\frac{\gamma_f}{\sigma_v} + N)(\frac{Cp_v w_v(C, B)}{\gamma_f + Cp_v w_v(C, B))} = 0, \qquad (1.1)$$

and

$$R = (\frac{N\sigma_v}{\gamma_f + N\sigma_v})(M_v - C).$$

Further, equilibrium equation for data subsystem is determined by equating flow out of the BLK state to flow into this state:

$$S_d\sigma_d = (1 - \frac{R}{N})Bp_d w_d(C, B).$$

Here $\quad w_d(C,B) = \begin{cases} (1-p_d)^{B-1}(1-p_v)^C & B \geq 1 \\ (1-p_v)^C & B < 1 \end{cases}.$

Using the fact that the total number of data terminals is fixed ($B + S_d = M_d$), and by substituting R, the above-noted equilibrium equation simplifies to

$$F_2(C, B) = M_d - B - (\frac{\gamma_f}{\sigma_d})(\frac{Bp_d w_d(C, B)}{\gamma_f + Cp_v w_v(C, B))} = 0. \tag{1.2}$$

Equilibrium points of the system are roots of the following two equations:

$$F_1(C, B) = 0, \qquad F_2(C, B) = 0.$$

1.2.4 Performance Measures in PRMA Protocol

In this subsection we consider important performance measures for the PRMA protocol namely packet drop probability, system throughput, and delay.

1.2.4.1 Packet Drop Probability

As previously mentioned, an important performance measure concerning voice terminals in the PRMA protocol is packet drop probability. A voice terminal in contending state will drop all packets that are waiting more than D time slots. In order to calculate the drop probability, the probability that a terminal obtains a reservation j slots after the beginning of the talkspurt is calculated. No packets is dropped if $j \leq D$. But if $j > D$, one packet is dropped, plus one packet for each additional frame (N slots) that the terminal waits for reservation. Since all the studies are at equilibrium, packet drop probability is calculated at steady state [23], [24]:

$$P_{drop} = \gamma_f \frac{\upsilon^D}{1-(1-\gamma_f)\upsilon^N}, \tag{1.3}$$

here υ is probability of no successful transmission:

$$\upsilon = \upsilon(C, R, B) = 1-(1-\frac{R}{N})p_v(1-p_v)^C(1-p_d)^B.$$

1.2.4.2 System Throughput

Another important performance measure of the PRMA protocol is average throughput defined as portion of time slots in one frame that successfully carry packets from terminals to the access point (such as a base station). Like packet drop probability, average throughput is studied at equilibrium [23], [24].

$$\eta = \frac{R}{N} + (M_d - B)\sigma_d. \tag{1.4}$$

1.2.4.3 Data Packet Delay

Data packet delay is defined as average waiting time for a data packet from the time it is generated in a silent data terminal until it is successfully transmitted. Here, it is assumed that each data terminal has a one-packet buffer. Therefore, data packet delay at equilibrium is [24]:

$$W_{av} = \frac{1}{(1-\frac{R}{N})Bp_d w_d(C, B)}. \tag{1.5}$$

1.2.5 Other PRMA Protocols Studied in the Literature:

Although, the focus of our research in this dissertation is on bifurcations and stability issues of the pure PRMA protocol, many variations of the PRMA protocol have been studied in the literature. It is contemplated that bifurcation analysis of this dissertation can be extended to these variations. This subsection summarizes a subset of other protocols that have been based on PRMA protocol. Further, we summarize control schemes introduced for the PRMA protocol in the literature and briefly compare with our studies.

1. PRMA with Transmission Errors: In the PRMA protocol proposed by Goodman [1], it is assumed that transmission channel is error free. However, some researchers have focused their attention on modeling the PRMA scheme over random packet error uplink channels [25] (again assuming that the downlink channel is error free). Packet header errors may cause access point (such as a base station) to be unable to decode a header of a received packet correctly and interpret result of a transmission as collision or an event that no packet was transmitted, even if the terminal is in a reservation state and has packets to transmit. Thus, the access point can announce an unsuccessful packet reception. In this case, a packet header transmission error causes a reservation terminal to lose its reservation prematurely. Hence, terminal needs to start contending for another reservation and risk packet dropping while waiting. It is noted that, in this dissertation, we also extend our bifurcation analysis and control to the PRMA protocols over random packet error channels.

2. IPRMA: Integrated packet reservation multiple access provides a reservation mechanism for both speech and data terminals [26]. In IPRMA, speech terminals are allowed to contend for reservation slots on a frame-by-frame basis while data terminals may reserve multiple slots across a frame to increase throughput. These enhancements lead to fewer collisions, which results in improvement in overall system performance.

3. Joint CDMA-PRMA Protocol: The joint CDMA-PRMA was first proposed by Brand and Aghvami as an extension to PRMA protocol for an uplink channel in a cellular communication system [27]. The joint CDMA-PRMA channel is organized into time-slots, which, in turn, are grouped into frames in the same way as in PRMA. Each user spreads its data with short direct sequences before accessing the channel such that several users can share a time slot using code division multiple access.

4. MD-PRMA: Multidimensional PRMA was proposed as media access control (MAC)protocol of wireless communication uplink channel [28]. MD-PRMA can be viewed as an extension to PRMA or a generalization to joint CDMA-PRMA, which embraces both code-division PRMA and frequency-division PRMA. In conventional PRMA, time is divided into slots, but in MD-PRMA slots are not only defined in time domain but also in an additional domain, either the "frequency domain" or the "code domain". Increasing number of slots in one frame in this way increases efficiency of multiplexing.

5. Effect of Mobility: Packet reservation multiple access is a scheme to transmit

a mixture of voice and data packets in micro-cells. An advantage of PRMA is that it needs little central control. A voice terminal that moves to another cell loses its reservation. Therefore, it needs to contend with other terminals to transmit its remaining packets. The terminal also needs to register with the new base station. This delay, which is modeled as a fixed delay by researchers, may force the terminal to drop voice packets, thereby, degrading its performance. Researchers have proposed models to capture effect of voice terminal mobility and have studied its effect on packet drop probability of the PRMA [29] and [30].

6. PRMA for Multimedia Wireless System: There are studies on the PRMA scheme to extend this media access control (MAC) protocol for multimedia traffic [31]. The main issue in designing a MAC protocol for multimedia traffic is to guarantee different quality of service parameters for different types of traffic while, at the same time, achieving high throughput. These studies propose an efficient MAC protocol that integrates voice, data, and real time variable bit rate video by reserving a number of time slots at the beginning of a frame for video packets and letting the other video packets contend with voice and data packets for the rest of time slots in that frame.

7. Exponential back off scheme for slotted ALOHA protocol: Jeong *et al.* introduced an exponential back off scheme for slotted ALOHA protocol in local wireless environment [32]. They considered a deferred first transmission (DFT) mode of slotted ALOHA where retransmission probability is adjusted at the

end of each time slot based on received feedback (idle, success, collision) from the base station and an exponential function. Retransmission probability is multiplied by $1/q$, 1, or q, if previous time slot was idle, successful transmission, or collision, respectively. Further, as an example of a slotted ALOHA system, Jeong *et al.* illustrated simulation results for a PRMA voice system with the exponential back of scheme for $q = 0.5$. As part of this dissertation, we introduce, model, and analytically study effects of a general price based control on the PRMA protocol. The general price based control is a more general control scheme that the exponential back off scheme can be considered as a special case of. Further, we analytically study equilibrium equations of controlled PRMA voice system, PRMA voice and data system, and PRMA voice system with delay, and provide conditions for bifurcation control for these system.

8. MD-PRMA with prioritized Bayesian broadcast: Brand *et al.* in [27] revisited pseudo-Bayesian broadcast estimation of [33] for calculating transmission permission probability for slotted ALOHA systems. In pseudo-Bayesian broadcast it is assumed that probability values can be approximated reasonably well by a Poisson distribution and therefore, mean of the Poisson distribution is needed to be estimated and optimum permission probability would be inverse of the mean. The mean is updated at the end of each time slot based on feedback information, by decrementing by 1 in case of an idle slot or successful transmission or is incremented by $(e-2)^{-1}$ in case of collision. Further, the mean is

calculated based on the updated mean and an estimated value of arrival rate. Brand *et al.* illustrated simulation results for a MD-PRMA voice system and PRMA voice and data system with and without acknowledgement delays employing the pseudo-Bayesian scheme. However, as mentioned above, as part of this dissertation, the introduced general price based control is a more general control scheme that can include the pseudo-Bayesian scheme. Further, another goal of this dissertation is to analytically model and study equilibrium behavior of PRMA systems and determine conditions for bifurcation control for these system.

1.3 Packet Reservation Multiple Access with Hindering States

Packet Reservation Multiple Access with Hindering States (PRMA-HS) was first proposed by Re *et al.* [34] as a medium access control scheme for *low earth orbit-mobile satellite systems* (LEO-MSSs). This protocol, which is designed to support both voice and data traffic in LEO-MSSs, is a modified version of PRMA protocol. PRMA protocol was first proposed for terrestrial microcellular networks. However, its interesting features have motivated many researchers to investigate its applicability to LEO-MSS [34], [35], [36], and [37].

Since in terrestrial microcellular round trip delay is much lower than packet transmission time, terminals in a PRMA system are able to receive outcome of their transmissions (feedback signal) almost immediately. This in not true for LEO-MSSs and therefore, large round trip delay (RTD) reduces efficiency of PRMA protocol in

LEO-MSSs [34]. In MSS, a user terminal stops contending while waiting for result of its transmission. This information is received after a round trip delay (RTD). Therefore, the user terminal has fewer attempts before the maximum tolerable delay is reached and as a result, packet dropping probability increases. It is usually assumed that RTD is equal to the maximum RTD and is always less than a frame time. Therefore, a user terminal knows result of its transmission before beginning of the same slot in the next frame.

Limitations of the PRMA protocol in LEO-MSS, motivated researchers to introduce PRMA-HS protocol as a modification to PRMA. In PRMA-HS the user terminal contends for available time slots while it is waiting for the outcome of its first attempt (waiting time). The first successful attempt by the user terminal is recorded in a database by the satellite in order to ignore successive successful transmission attempts by the same terminal in its waiting time. After the first successful transmission, the terminal enters a block of hindering states HIN, which are used to model the waiting time.

1.3.1 PRMA-HS Voice Subsystem

When a talkspurt starts, a voice terminal in the silent state, SIL, transitions to a contending state CON. A terminal in CON state, contends for reservation by transmitting a speech packet on an available time slot, if it has permission to transmit. A contending terminal successfully transmits its packet if (1) the time slot is unreserved, (2) it has permission to transmit, (3) no other speech packets are transmitted simultaneously, and (4) no data packets are transmitted at that time slot. When the contending terminal successfully transmits a packet, an access point (such as a satellite), using a feedback message, informs all the terminals that the time slot is reserved. Terminals will receive this feedback message after RTD time slots (the round trip delay). We assume that $RTD = N/d$ time slots, where d is an integer which is a divisor of N. During this waiting time, the contending voice terminal can still attempt transmissions. But these transmissions can only harm other contending voice terminals, since the first successful attempt of the voice terminal is recorded at the base station, and other successive successful transmissions will be ignored. After a voice terminal has successfully transmitted its first voice packet, the waiting time to receive the positive feedback is modeled by the hindering states HIN states. For the voice terminal, CON and HIN states are indistinguishable. After N/d time slots, the voice terminal enters a series of $N - N/d$ slots in state RES'. In this state, the terminal waits until its reserved time slot arrives. If the terminal has no more packets to transmit, it enters the SIL state. Otherwise, it enters the RES state. It transmits one speech packet in its reserved time slot in every frame. The terminal transmits its last packet at the end of the talkspurt and moves to SIL state.

1.3.2 PRMA-HS Data Subsystem

Researchers have studied different data traffic sources for the PRMA-HS protocol. These sources can include web traffic, email traffic, and multimedia. However, it this dissertation, we assume that data terminals, similar to the PRMA data subsystem, cannot reserve time slots and they have a buffer with one-packet capacity. A data terminal with no packet to transmit is in silent state SIL. With probability σ_d a data packet arrives and the terminal moves to backlogged mode, BLK. The data terminal in BLK tries to transmit its packet on available time slots. It will be successful if the time slot is available, it has permission to transmit, no other data packet is transmitted , and no speech packet is transmitted. After the successful transmission, the data terminal enters a series of $N - N/d$ hindering states, HIN, which models the waiting time. Any successful transmissions during the waiting time is ignored by the base station. After RTD time slots, the data terminal receives the feedback signal and moves back to silent mode.

1.3.3 PRMA-HS System Model

Re *et al.* in [34] and Benelli *et al.* in [35] showed that the behavior of the PRMA-HS protocol can be modeled as a Markov process. Figures 1.3 and 1.4 show the Markov model for each voice and data terminal, respectively. The number of contending voice terminals is c, the number of voice terminals with reservations is r, h_v is the number of voice terminals in hindering state, b is the number of data terminals in BLK mode, and h_d is the number of data terminals in hindering state. s_v and s_d are the number of silent voice and data terminals, respectively.

Unfortunately, because of large state space, precise analysis of the Markov process is very complex. Instead, Re *et al.* [34] and Benelli *et al.* [35] used an equilibrium point analysis (EPA) as described by Tasaka [17]. In this subsection,

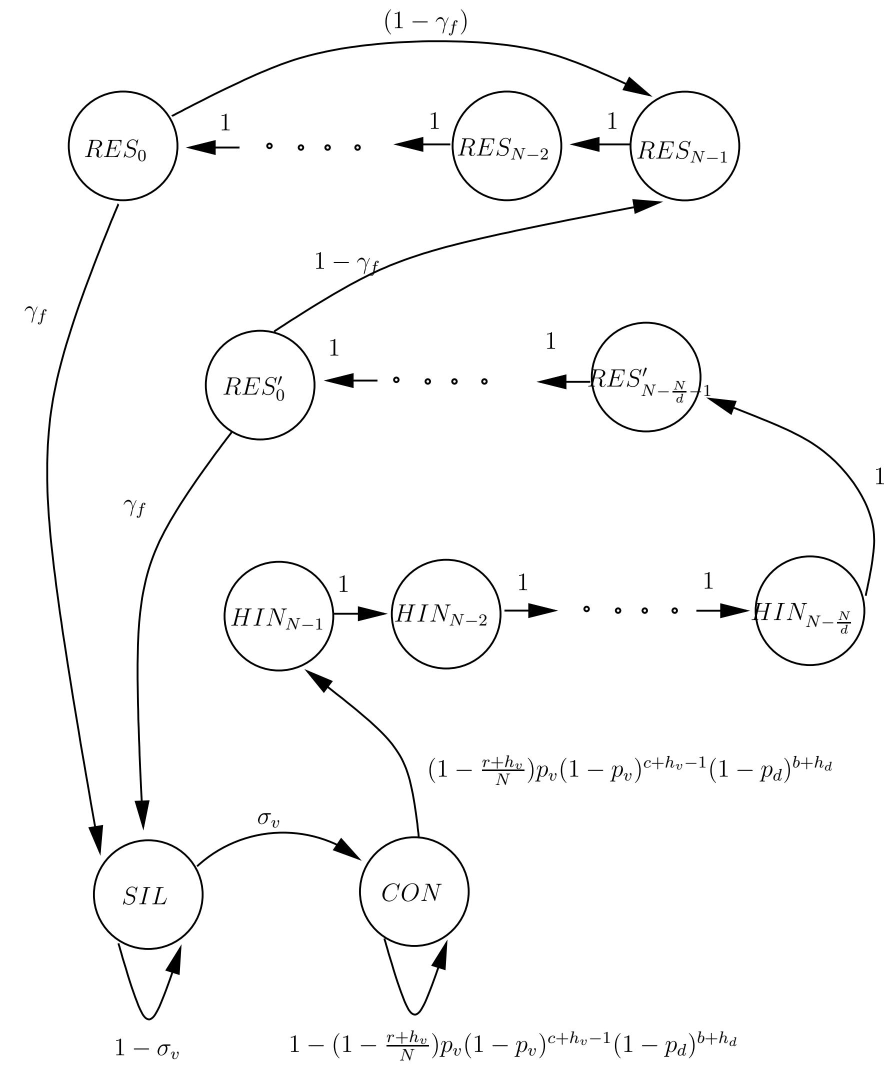

Figure 1.3: Markov chain model for PRMA-HS voice subsystem.

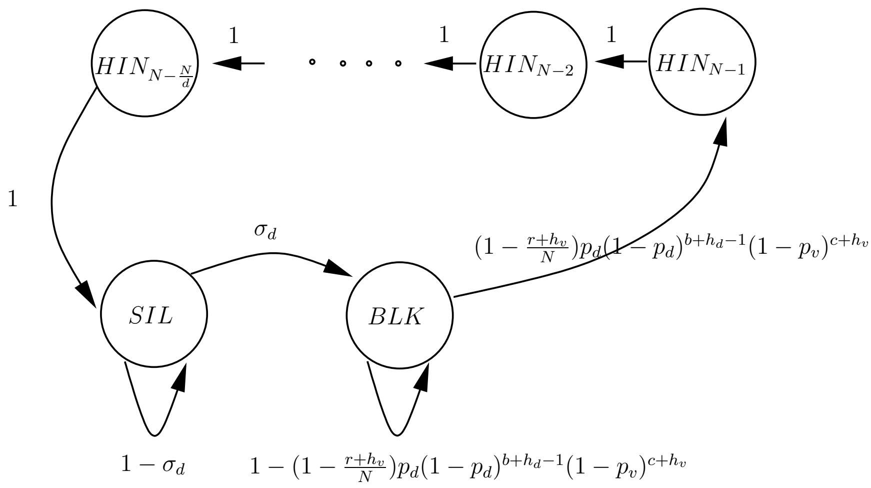

Figure 1.4: Markov chain model for PRMA-HS data subsystem.

we briefly present equilibrium equations of the PRMA-HS system. Let equilibrium values of state variables be denoted by (C, R^*, H_v, B, H_d). First, we consider the voice subsystem. Notice that at equilibrium, the number of voice terminals in each state HIN_i is $\frac{H_v}{(N/d)}$ for $i = N-1, ..., N-N/d$. In the same way, the number of terminals at equilibrium in state RES'_i is $\frac{R'}{N-(N/d)}$ for $i = N-N/d-1, ..., 0$. It is easy to show that

$$\frac{H_v}{N/d} = \frac{R'}{N-N/d}.$$

Also, the number of voice terminals in each state RES_i is $\frac{R}{N}$ for $i = 0, ..., N-1$. We define $R^* = R + R'$ as number of voice terminals in RES and RES'. Notice that

$$\frac{R^* + H_v}{N} = \frac{R}{N} + \frac{R'}{N-N/d}.$$

Equilibrium equation at SIL is found by equating flow out of the state to flow into the state:

$$(\frac{R^* + H_v}{N})\gamma_f = S_v\sigma_v.$$

Similarly at CON:

$$S_v\sigma_v = (1 - \frac{R^* + H_v}{N})Cp_v w_v(C, B).$$

Here $\quad w_v(C,B) = \begin{cases} (1-p_v)^{C+H_v-1}(1-p_d)^{B+H_d} & C \geq 1 \\ (1-p_v)^{H_v}(1-p_d)^{B+H_d} & C < 1 \end{cases}$.

At RES_{N-1}:

$$\frac{R}{N} = (\frac{R^* + H_v}{N})(1 - \gamma_f).$$

Also, $S_v + C + R^* + H_v = M_v$. Above equilibrium equations can be simplified as:

$$F_1(C, B) = M_v - C - (\frac{\gamma_f}{\sigma_v} + N)(\frac{Cp_v w_v(C, B)}{\gamma_f + Cp_v w_v(C, B)}) = 0, \tag{1.6}$$

here

$$H_v = \gamma_f(\frac{N}{d})\omega(M_v - C), \qquad R^* = (d - \gamma_f)(\frac{N}{d})\omega(M_v - C).$$

In the same way, equilibrium equations for data subsystem can be found by equating the expected number of data terminals leaving a state to the expected number of terminals entering that state. Equilibrium condition at the SIL is:

$$S_d\sigma_d = (\frac{d}{N})H_d,$$

and equilibrium equation at BLK can be written as:

$$S_d\sigma_d = (1 - \frac{R^* + H_v}{N})Bp_d w_d(C, B).$$

Here $\quad w_d(C,B) = \begin{cases} (1-p_d)^{B+H_d-1}(1-p_v)^{C+H_v} & B \geq 1 \\ (1-p_d)^{H_d}(1-p_v)^{C+H_v} & B < 1 \end{cases}$. Since $S_d + B + H_d = M_d$:

$$F_2(C, B) = M_d - B - (\frac{N}{d} + \frac{1}{\sigma_d})\gamma_f(\frac{Bp_d w_d(C, B)}{\gamma_f + Cp_v w_v(C, B)}) = 0, \tag{1.7}$$

here

$$H_d = (\frac{N\sigma_d}{N\sigma_d + d})(M_d - B).$$

Therefore, equilibrium points of the PRMA-HS voice-data system are solutions of the following equations:

$$F_1(C, B) = 0, \qquad F_2(C, B) = 0.$$

1.3.4 Performance Measures in PRMA-HS Protocol

Drop Probability - As in the PRMA model, voice packet drop probability at equilibrium can be defined as [23], [24], [34], and [35]:

$$P_{drop} = \gamma \frac{\upsilon^D}{1 - (1 - \gamma)\upsilon^N},$$

here υ is probability of no successful transmission:

$$\upsilon = \upsilon(C, R^*, H_v, B, H_d) = 1 - (1 - \frac{R^* + H_v}{N})p_v(1 - p_v)^{C+H_v}(1 - p_d)^{B+H_d} \quad (1.8)$$

System Throughput - Another important performance measure of the PRMA-HS protocol is average throughput defined as portion of time slots in one frame that successfully carry packets from terminals to an access point. Like drop probability, average throughput is studied at equilibrium [23], [24].

$$\eta = \frac{R^* + H_v}{N} + (M_d - B - H_d)\sigma_d. \quad (1.9)$$

Data Packet Delay - As stated earlier in the PRMA protocol, an important performance measure for the data subsystem in PRMA-HS is the data packet average delay. It is assumed that each data terminal has a one-packet buffer and that the data packet delay at equilibrium is:

$$W_{av} = \frac{1}{(1 - \frac{R^* + H_d}{N}) B p_d w_d(C, B)}. \tag{1.10}$$

1.3.5 Other PRMA-HS Protocols Studied in Literature

As part of this dissertation, our focus is to study nonlinear instability and bifurcation control of pure PRMA-HS protocol. However, it is contemplated that the analysis of this dissertation can be extend to other variations of the PRMA-HS protocol. Next, we briefly summarize a subset of protocols introduced in literature that are based on the PRMA-HS protocol.

1. MPRMA: MPRMA is a modified version of the PRMA protocol for both voice and data terminals [38]. The voice subsystem is exactly the same as the voice subsystem in PRMA-HS. However, the data subsystem is modified. When a contending data terminal successfully transmits its first packet (request packet), this packet is stored in a buffer on a satellite to form a queue of data terminals that need to transmit. A controller on board of the satellite manages the data terminals' requests. The controller assigns an available time slot in next frame (not reserved by voice terminals) to a data terminal according to an access probability. Hence, depending on activity of voice terminals, a variable number of time slots is assigned to data terminals. This policy is particularly suitable for available bit-rate (ABR) like data traffic.

2. S-PRMA: Another extension of the PRMA protocol that has drawn satellite

communication researchers' attention is a protocol named satellite PRMA (S-PRMA) [39]. The focus is again on the uplink of a two-way LEO satellite wireless network. The carrier is divided into slots and N slots are grouped together as one frame. At the end of each time slot, the satellite broadcasts a feedback to acknowledge status of that time slot. In order to maintain a good performance, round trip delay should be less than the frame time, a condition that is satisfied by most LEO satellites. Also it is assumed that both uplink and downlink channels are error free. This protocol is very similar to the PRMA-HS with following differences:

- Each unreserved time slot is divided into two sets of mini-slots. These two sets are devoted to voice and data terminals.
- Mini-slots in each set are contended among the associated (voice or data) terminals. User terminals in the contending state uniformly choose one mini-slot in their set if they have permission to transmit.
- If a voice terminal successfully transmits a request, the satellite grants it the use of an available time slot for the time the terminal needs.
- If a data terminal successfully sends a request, its request is stored in a virtual first in first out (FIFO) queue in the satellite.
- Any time slot that is left unreserved by voice terminals is granted to the data terminal with its request at the head of the queue, with some probability.

3. CD-PRMA-HS: CD-PRMA-HS is application of the PRMA-HS scheme to a

hybrid time code-division air interface [37] and [40]. As in the PRMA-HS protocol, in the CD-PRMA-HS scheme channel is divided into slots and N slots are grouped together as a frame. Voice terminals acquire reservation on a talkspurt basis. Whereas data terminals must acquire a reservation on a datagram basis. If a datagram arrives in a data terminal while it has a reservation with the satellite, the data terminal maintains its reservation until its buffer is empty. One difference between CD-PRMA-HS and PRMA-HS is that in CD-PRMA-HS, reservation is based on slot codes, meaning that each time slot is further divided into codes. Therefore, transmission attempts are random not only in time but also in the code domain. Collisions happen if more than two mobile terminals randomly choose one slot code. It is assumed that orthogonal codes are used in downlink and joint detection is used in uplink so that the intracell interference has a negligible impact on the signal-to-interference ratio. Voice and data terminals choose slot codes independently and with different probabilities. Voice terminal permission probability is greater than data terminal because voice terminals have a higher service priority than data terminals.

4. Dynamic Reservation PRMA-HS: In this modification of PRMA-HS, each time frame is divided into three parts: 1) reservation mini-slots, 2) slots for voice traffic, and 3) slots for data traffic [41]. Number of slots and mini-slots in each part is calculated dynamically. Generally, slots are assigned in following order subject to availability: slots for voice packets based on their reserva-

tion, reservation mini-slots for voice packet based on estimated number of voice terminals, slots for data traffic based on registered slot requirements, and reservation mini-slots for data terminals from residual capacity. Also, permission probabilities for both voice and data terminals are calculated dynamically based on a frame-based Bayesian algorithm for the delayed feedback environment.

1.4 Outline of Dissertation

In this chapter, we briefly reviewed stability issues of ALOHA systems and background information on PRMA and PRMA-HS protocols. In Chapter 2, we study equilibrium behavior of voice-only PRMA and PRMA-HS systems over error-free and random error channels, we study price based control and state estimation-based control and study equilibrium behavior of the controlled system, and we also study effects of using multiple power levels at terminals and capture at an access point on the bifurcations of the systems. In Chapter 3, we present equilibrium studies of voice+data PRMA systems and extend our control analysis of Chapter 2 to the voice-data system. Also, we briefly review extension of the price based control scheme to voice-data PRMA-HS system. In this chapter, we also compare and discuss the studied control schemes with two previously presented control schemes for a PRMA-HS voice+data system using simulations. In Chapter 4 we present equilibrium studies of a finite buffered finite users slotted ALOHA system with price based control. Chapter 5 includes equilibrium studies of framed PRMA voice-only and voice+data systems with the price based control. Finally, Chapter 6 concludes the dissertation.

Chapter 2

Equilibrium Analysis and Control for PRMA and PRMA-HS Protocols for Voice Terminals

As discussed before, nonlinear instability (such as bistability) and bifurcations are noticed in random access protocols such as ALOHA-like protocols. In this chapter, we focus our analysis on PRMA and PRMA-HS protocols. We consider a system of only voice terminals that employs either PRMA or PRMA-HS as its access protocol. We propose different control schemes and analytically study equilibrium behavior of the system with the proposed schemes. We prove that, under some conditions, these control schemes can control nonlinear instabilities of the system by either completely eliminating bifurcations or delaying bifurcations and therefore, depending on situation, we can achieve an expanded operating range. More information on bifurcation control can be found in [42].

In this chapter, we start with the PRMA protocol and first introduce a General Price Based Control for the PRMA voice system. We model the PRMA system with the controller using a Markov model and we analyze the Markov model. Further, we analyze specific variations of the General Price Based Control to study equilibrium behavior of the system. We can prove that, under some conditions, these specific variations can control bifurcations that occur in the system. Also, we study the control scheme for a PRMA system that operates over a random error channel.

Moreover, we study effects of capture phenomenon (using multiple power levels) on controlling bifurcations in PRMA system. Later in this chapter, we extend our analysis to the PRMA-HS voice only system.

2.1 General Price Based Control for PRMA Protocol

The general Price Based Control studied in this dissertation is, in part, motivated by the price based rate control scheme studied by Yuen and Marbach in [43] and [44]. Their work was also motivated by the popularity of wireless local area networks using the IEEE 802.11 standard for channel access, and is similar to price based rate control schemes for wireline network. They showed that the proposed rate control mechanism achieves a sustainable throughput as the number of nodes increases and (under appropriate assumptions) there exists a unique operating point.

General Price Based Control (GPBC) operates based on feedback information sent back from an access point (such as a base station). As discussed before, at the end of each time slot, the access point informs all terminals on status of that time slot. If time slot n is reserved, feedback information from base station indicates whether same time slot in next frame (time slot $N + n$) will be still reserved or not. A reserved time slot can become idle if terminal reserving that time slot has transmitted all its packets. If time slot n is not reserved, feedback information from the access point indicates whether no transmission, a successful transmission, or collision occurred during that time slot.

In this section we assume that channel is error free. Later, we assume that the channel is a "random packet error channel", where only errors that corrupt packet headers can occur randomly in the uplink channel [25]. Terminals use feedback information from the access point to adjust their permission probability. We assume that the permission probability, p_v, is a function of a control signal u. Control signal is updated at end of each time slot based on following:

$$u_{n+1} = \begin{cases} u_n & \text{if slot } n \text{ is reserved and reservation is kept,} \\ u_n + \phi & \text{if slot } n \text{ is reserved and reservation is lost,} \\ [u_n - \alpha I(Z_n = 0) + \beta I(Z_n = 1) + \xi I(Z_n \geq 2)]^+ & \text{if slot } n \text{ is not reserved.} \end{cases} \tag{2.1}$$

Here α, ξ, and ϕ are positive real numbers and β is a real number. $[x]^+$ denotes $\max(0, x)$. Random variable Z_n indicates the number of packets that are transmitted at the beginning of time slot n. Permission probability, p_v, is updated at the end of each time slot based on new value of control signal u.

Assumption 2.1. We assume that permission probability $p_v(u)$ is continuous, bounded ($0 \leq p_v(u) \leq 1$), and strictly decreasing in u ($u \in [0, +\infty)$). Furthermore, there exists a positive constant u_{max} such that $p_v(u) = 0$ when $u \geq u_{max}$.

Based on update equation (2.1) and assumption 2.1, if time slot n is not reserved and there is no packet transmission at this time slot ($Z_n = 0$), control signal decreases and permission probability is increased. If there is a collision at time slot n ($Z_n \geq 2$), control signal is increased by ξ and permission probability is decreased. In the case of a successful packet transmission at time slot n ($Z_n = 1$), depending on β, permission probability is either increased or decreased. Further, if time slot n is reserved but it becomes free, control signal is increased by ϕ. In other cases, control signal is unchanged.

PRMA system with voice terminals with General Price Based Control can be modeled by a Markov chain which extends the Markov model pointed out in chapter 1. State of system at the beginning of time slot n is given by $X_n = (c_n, r_n, u_n)$. Here c is the number of voice terminals in contending mode, r is the number of voice terminals in reservation, and u is control signal. Without loss of generality, it can be assumed that the Markov chain starts at initial state $X_0 = (c_0, r_0, u_0) = (0, 0, 0)$, $c \in \{0, 1, 2, \cdots, M_v\}$, $r \in \{0, 1, 2, \cdots, N\}$, and $u \in \Gamma = \{\min(u_{MAX}, [\phi e - \alpha a + \beta b + \xi d]^+) | a, b, d, e \in Z_+\}$. Here $u_{MAX} = u_{max} + \max(N\phi, \beta, \xi)$. However, it should be noticed that the state space of the system $\aleph$ is only a subset of $\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, N\} \times \Gamma$. Because, at least, the total number of contending and reserving voice terminals at each time slot can not be higher than the total number of voice terminals. Also, for $r_n = N$, control signal could only take values greater than or equal to $[\beta]^+$. Note that state space $\aleph$ is countable.

The transition probabilities for this Markov chain are:

$$Pr(c_{n+1} = c', r_{n+1} = r', u_{n+1} = u' | c_n = c, r_n = r, u_n = u) = \tag{2.2}$$

- $f_v(c' - c; M_v - c - r, \sigma_v)(r/N)\gamma_f$

  if $c \leq c' \leq M_v - r, r' = r - 1, u' = u + \phi$

- $f_v(c' - c; M_v - c - r, \sigma_v)(r/N)(1 - \gamma_f)$

  if $c \leq c' \leq M_v - r, r' = r, u' = u$

- $f_v(c' - c; M_v - c - r, \sigma_v)(1 - r/N)(1 - p_v)^c$

if $c \le c' \le M_v - r, r' = r, u' = u + \max(-u, -\alpha)$

- $f_v(c'-c; M_v - c - r, \sigma_v)(1 - r/N)(1 - (1-p_v)^c - cp_v(1-p_v)^{c-1})$

  if $c \le c' \le M_v - r, r' = r, u' = u + \xi$

- $f_v(c'-c+1; M_v - c - r, \sigma_v)(1 - r/N)cp_v(1-p_v)^{c-1}$

  if $c - 1 \le c' \le M_v - r - 1, r' = r + 1, u' = u + \max(-u, \beta)$

- 0 Otherwise

Here, $f_v(k; n, \sigma_v) = \begin{pmatrix} n \\ k \end{pmatrix} \sigma_v^k (1-\sigma_v)^{n-k}$

2.2 Price Based Control

In this section, we study a special case of the General Price Based Control introduced above. We assume $\phi = 0$, therefore, control signal is updated only during available time slots.

Proposition 2.1. *The Markov chain defined on $\aleph$ through (2.2) (for $\phi = 0$) is irreducible and aperiodic.*

The irreducibility of the Markov chain is proved, in the Appendix A, by showing that all states in the state space communicate with each other.

Proposition 2.2. *Under Assumption 2.1, the Markov chain given by (2.2) (for $\phi = 0$) is positive recurrent.*

In order to show that the Markov chain is positive recurrent, as shown in Appendix A, it is necessary to find a non-negative Lyapunov function $V(c, r, u)$ that satisfies following mean drift criteria [45], [46], [47], [48], [49]:

Proposition 2.3. *An irreducible and aperiodic Markov chain defined on a countable state space Σ is positive recurrent if and only if there exists a non-negative function $V(i), i \in \Sigma, \epsilon > 0, b < +\infty$ and a finite set Ω such that:*

$$E(\Delta V(i)) = E(V(j) - V(i)|i) \leq -\epsilon \qquad i \notin \Omega$$

$$E(\Delta V(i)) < b \qquad i \in \Omega$$

Since state space of the PRMA system with Price Based Control is large, analysis using transition probabilities is very difficult. Therefore, stationary behavior of the system is analyzed using equilibrium point analysis (EPA). In EPA it is assumed that the system is in equilibrium, therefore, any change in states of the system is zero. One-step expected change (mean drift) of control signal at state (c, r, u) is defined as follows:

$$d(c, r, u) = E(u_{n+1} - u_n | c_n = c, r_n = r, u_n = u).$$

By using definition of control signal in (2.1), with assumption that $\phi = 0$, expected drift is determined as following:

$$\begin{aligned} d(c, r, u) = & (\max(-\alpha, -u) - \xi)(1 - r/N)(1 - p_v)^c \\ & + (\max(\beta, -u) - \xi)(1 - r/N) c p_v w_v(c, u) \\ & + \xi(1 - r/N). \end{aligned}$$

Here $\quad w_v(c,u) = \begin{cases} (1-p_v(u))^{c-1} & c \geq 1 \\ 1 & c < 1 \end{cases}$. Relaxed drift equation is:

$$d(c,r,u) = -(\alpha+\xi)(1-r/N)(1-p_v)^c + (\beta-\xi)(1-r/N)cp_v w_v(c,u) + \xi(1-r/N). \tag{2.3}$$

As discussed in chapter 1 regarding the equilibrium equations of the PRMA voice system and considering the relaxed expected drift equation of the control parameter at equilibrium, a point (C,R,U) is called an equilibrium point of the PRMA voice system, if

$$\begin{aligned} &(M_v - C - R)\sigma_v - (1-R/N)Cp_v(U)w_v(C,U) = 0, \\ &(R/N)\gamma_f - (1-R/N)Cp_v(U)w_v(C,U) = 0, \\ &-(\alpha+\xi)(1-p_v(U))^C + (\beta-\xi)Cp_v(U)w_v(C,U) + \xi = 0. \end{aligned} \tag{2.4}$$

Here, C, R, and U are equilibrium values of expected values of the states of the system. Since set of equations (2.4) is nonlinear, it is not easy to find the conditions for the control parameters (α,β,ξ) to ensure the uniqueness of the operating point of the controlled PRMA system. Lemma 2.1 below defines two sets of conditions for set of equations (2.4) to have a unique fixed point.

Remark 2.1. *Based on the first two equations of the set of equations (2.4), it can be shown that* $R = \min(N, N\omega(M_v - C))$*, where* $\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}$*. Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$*,* $R \in [0,N]$*, and* $p_v \in [0,1]$*. However, it can easily be shown, using the set of equation (2.4), that* $C = 0$*,* $M_v - \frac{1}{\omega}$*,* $C = M_v$*,* $R = 0$*,* $R = N$*,*

$p_v = 0$, $p_v = 1$ *(for* $\beta = \xi$ *as considered in Lemma 2.1), or a combination thereof, cannot be solutions to the set of equilibrium equations (2.4).*

Lemma 2.1. *There exists a set of control parameters* (α, β, ξ) *for which the set of equations (2.4) has a unique solution in* (C, U) *and the system has a single operating point if any of conditions* $(1a)$, $(1b)$, *or* (2) *below hold:*

$(1a)$ $\quad M_v \geq 1 + \frac{1}{\omega}$ *and* $-\omega\gamma_f < \frac{\xi}{\alpha+\xi}[(\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v - \frac{1}{\omega}} + 1)\exp(-\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v - \frac{1}{\omega}}) - 1]$,

$(1b)$ $\quad M_v < 1 + \frac{1}{\omega}$ *and either*

$$\frac{(M_v - 1)\omega\gamma_f}{1 - (M_v - 1)\omega} \leq \frac{\xi}{\alpha + \xi} M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1), \qquad \textit{or}$$

$$-\omega\gamma_f < 1 - \frac{\xi}{\alpha+\xi} + \ln(\frac{\xi}{\alpha+\xi}),$$

(2) $\quad -1 + \frac{\xi}{\alpha+\xi} - \ln(\frac{\xi}{\alpha+\xi}) < \frac{\omega}{\gamma_f}[\frac{\xi}{\alpha+\xi} M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1) + \gamma_f]^2.$

Proof. We will show the existence of the control parameters (α, β, ξ) with $\beta = \xi$ such that the conditions holds. We first prove the lemma for conditions $(1a)$ and $(1b)$. Given $\beta = \xi$, set of equations (2.4) can be simplified as follows

$$F_1(C,U) = Cp_v w_v(C, U) - \frac{(M_v - C)\omega\gamma_f}{1 - (M_v - C)\omega} = 0, \tag{2.5}$$

$$F_2(C,U) = -(\alpha + \xi)(1 - p_v)^C + \xi = 0, \tag{2.6}$$

here $\omega = \frac{\sigma_v}{\gamma_f + \sigma_v N}$ and $p_v = p_v(U)$. Function $F_1(C, U)$ is derived by solving the first two equations of the set of equations (2.4) for R and then substituting R in any of the first two equations. The number of terminals in reservation state is $R = \max(N, N\omega(M_v - C))$. However, as discussed earlier, $R = N$ cannot be an operating point of the system. Therefore, it is assumed that $\max(0, M_v - \frac{1}{\omega}) < C < M_v$ such that $R < N$. Now let us define following new functions

$$f(C,U) = Cp_v w_v(C,U), \quad g(C) = \frac{(M_v - C)\omega\gamma_f}{1-(M_v - C)\omega}.$$

Solving equation (2.6) for p_v and then substituting it in $f(C,U)$, we have

$$f(C) = \begin{cases} \frac{\xi}{\alpha+\xi} C(\exp(\frac{-1}{C}\ln(\frac{\xi}{\alpha+\xi})) - 1) & C \geq 1 \\ C(1-\exp(\frac{1}{C}\ln(\frac{\xi}{\alpha+\xi}))) & C < 1 \end{cases} \tag{2.7}$$

Therefore, fixed point(s) of equations (2.5) and (2.6) is same as fixed point(s) of $f(C) = g(C)$. It is easy to show that first and second derivatives of $f(C)$ in the range of $\max(0, M_v - \frac{1}{\omega}) < C < M_v$ are as follows

$$\begin{cases} \frac{df}{dC} < 0 & C \geq 1 \\ \frac{df}{dC} > 0 & C < 1 \end{cases}, \quad \begin{cases} \frac{d^2f}{dC^2} > 0 & C \geq 1 \\ \frac{d^2f}{dC^2} < 0 & C < 1. \end{cases}$$

It can also be shown that $\frac{dg}{dC} < 0$ and $\frac{d^2g}{dC^2} > 0$, for this range of C. Also, notice that

$$f(M_v) = \frac{\xi}{\alpha+\xi} M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1) > g(M_v) = 0.$$

Based on these facts, in order to prove that $f(C) = g(C)$ has exactly one solution, for the conditions stated in the lemma, we consider following cases:

- **Case 1 -** $M_v \geq 1+\frac{1}{\omega}$: In this case, $g(C)$ is strictly decreasing and as $C \to M_v - \frac{1}{\omega}^+$, $g(C) \to +\infty$. Also, $f(C)$ is strictly decreasing, $f(M_v - \frac{1}{\omega}) < g((M_v - \frac{1}{\omega})^+)$, and $f(M_v) > g(M_v)$. We define $h(c) = g(c) - f(c)$. As discussed above, $h(M_v - \frac{1}{\omega}) > 0$ and $h(M_v) < 0$. Therefore, if we choose the control parameters such that $\frac{dh}{dC} < 0$, then $h(C) = 0$ will have a unique solution. As discussed above, both functions $f(C)$ and $g(C)$ have positive second order derivatives, therefore,

$$\frac{df}{dC}(M_v - \frac{1}{\omega}) < \frac{df}{dC} < \frac{df}{dC}(M_v),$$
$$\frac{dg}{dC}(M_v - \frac{1}{\omega}) < \frac{dg}{dC} < \frac{dg}{dC}(M_v).$$

Hence, if the control parameters are chosen such that $\frac{dg}{dC}(M_v) < \frac{df}{dC}(M_v - \frac{1}{\omega})$, then $\frac{dh}{dC} < 0$ and $h(C) = 0$ will have a unique solution.

- **Case 2 -** $\frac{1}{\omega} \leq M_v < 1 + \frac{1}{\omega}$: In order to make sure that there exist only one solution, we chose the control parameters (α, β, ξ) such that either $\frac{dg}{dC}(M_v) < \frac{df}{dC}(1)$ or $g(1) < f(M_v)$.

- **Case 3 -** $0 < M_v < \frac{1}{\omega}$: In this case $g(C)$ is positive, strictly decreasing, with positive second derivative. Also, $f(C)$ is positive, strictly, increasing with negative second order derivative for $0 \leq C < 1$, and strictly decreasing with positive second order derivative for $1 \leq C \leq M_v$. In same way as Case 2, we choose control parameters such that $\frac{dg}{dC}(M_v) < \frac{df}{dC}(1)$ or $g(1) < f(M_v)$. It is easy to show that in this case $f(C) = g(C)$ has exactly one solution. Figure 2.1 shows both functions $f(C)$ and $g(C)$ for this case where $g(1) < f(M_v)$.

Now we prove the results assuming condition (2) holds. Given $\beta = \xi$, we simplify the set of three equations (2.4) as the following single equation:

$$h(C) = -M_v + C + \frac{1}{\omega}\frac{f(C)}{\gamma_f + f(C)} = 0. \tag{2.8}$$

Here $f(C)$ is the same as defined earlier in equation (2.7). It can easily be shown, using equation (2.7), that $h(0) = -M_v$, $h(M_v - \frac{1}{\omega}) < 0$, and $h(Mv) > 0$. The first

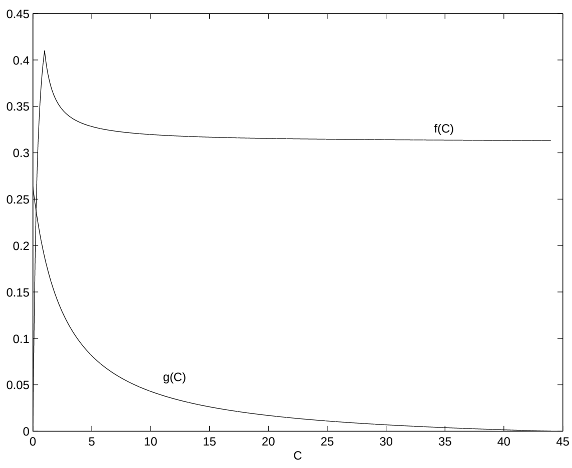

Figure 2.1: Functions $f(C)$ and $g(C)$ for Case 3 and $g(1) < f(M_v)$.

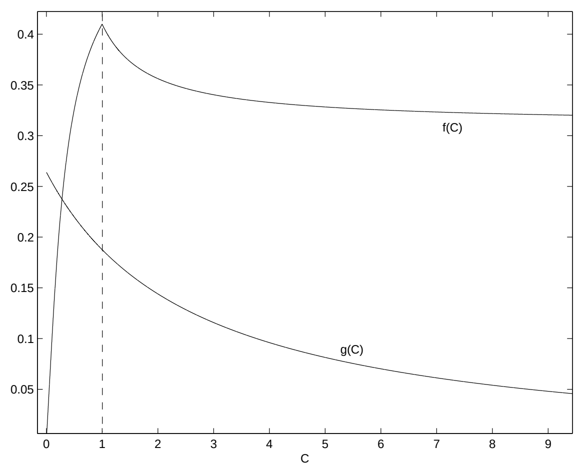

Figure 2.2: Functions $f(C)$ and $g(C)$ for Case 3 and $g(1) < f(M_v)$, which is zoomed.

derivative of $h(C)$ is

$$h'(C) = 1 + \frac{\gamma_f}{\omega}\frac{f'(C)}{(\gamma_f + f(C))^2},$$

where $f'(C) = \frac{df}{dC}$ and $h'(C) = \frac{dh}{dC}$. We showed earlier in the proof that $f'(C) > 0$ for $C < 1$, and therefore, $h'(C) > 0$ for $C < 1$. So $h(C)$ is negative for $C = \max(0, M_v - \frac{1}{\omega})$, is positive for $C = M_v$, and has a positive slope for $C < 1$. Next, we show that, under condition (2), the slope is also positive for $1 \leq C \leq M_v$, which will result in a unique solution to $h(C) = 0$ within interval $\max(0, M_v - \frac{1}{\omega}) \leq C \leq M_v$. Consider then the case $C \geq 1$ and we will show that if condition (2) is satisfied, then $h'(C) > 0$. Let us define $h_1(C) = -f'(C)$ and $h_2(C) = \frac{\omega}{\gamma_f}(\gamma_f + f(C))^2$.

It is noted that $h_1(C) > 0$ and $h_1'(C) < 0$ for $C \geq 1$. Therefore, $h_1(M_v) \leq h_1(C) \leq h_1(1)$.

It can also be shown that $h_2(C) > 0$ and $h_2'(C) < 0$ for $C \geq 1$. Therefore, $h_2(M_v) \leq h_2(C) \leq h_2(1)$.

Hence, if the control parameters are chosen such that $h_1(1) < h_2(M_v)$ (in other words, $-f'(1) < \frac{\omega}{\gamma_f}(\gamma_f + f(M_v))^2)$, then $h'(C) > 0$. Therefore, $h(C) = 0$ has a unique solution in the interval $0 \le C \le M_v$.

It can easily be shown, using equation (2.6), that p_v is one-to-one function of C. Also, as mentioned before, p_v is also a one-to-one function of U (for $0 < p_v < 1$). Therefore, it can easily be shown that for a given C, there exists a unique U. Therefore, under conditions stated in the lemma, the set of equations (2.5) and (2.6) or equation (2.8) has a unique solution in $0 < C < M_v$ and $0 < U < u_{max}$. $\square$

Designing the Control: In order to ensure that the controlled system has a unique operating point, we select the control parameters based on Lemma 2.1. We choose $\beta = \xi$ arbitrary positive real number. Then, α is chosen such that other condition of this lemma is satisfied.

Also, in order to be able to use the relaxed form of expected drift (in the set of equations (2.4)), we make a small change in assumption 2.1. Permission probability $p_v(u)$ is continuous and bounded ($0 \le p_v(u) \le 1$) function of control signal (u). Here, we assume $p_v(u) = 1$ for $u \in [0, \max(\alpha, -\beta)]$, strictly decreasing for $u \in (\max(\alpha, -\beta), u_{max})$, and $p_v(u) = 0$ for $u \in [u_{max}, +\infty)$. In this case, since equilibrium value of permission probability is less than 1, equilibrium value of control signal u will be greater than both α and $-\beta$ and therefore, relaxed expected drift equation at equilibrium can be used.

Also, the control parameters can be chosen such that the number of contending terminals at equilibrium equals to a pre-chosen value. This, enables a system designer to select system parameters such that at equilibrium, system operates at a given load in order to meet a certain throughput, delay, or drop probability criteria.

2.3 Control Using State Estimation

In this section, we propose another control scheme based on an estimation of state of the PRMA system. As we show later, this control scheme is another special case of the General Price Based Control introduced earlier.

As discussed before, state of the PRMA system at each time slot n is (c_n, r_n). At the beginning of each time slot, the number of voice terminals that have slot reservation (r) is known to all terminals in the system. However, the number of contending voice terminals is not known. As it is shown below, we can choose permission probability as a function of the number of contending voice terminals to maximize system throughput or minimize packet drop probability of the system. However, since the number of contending voice terminals is not known, we use an estimated value. The estimated number of contending terminals is calculated based on the number of voice terminals with a reservation.

As discussed earlier, average throughput is the number of time slots that carry one packet. At equilibrium, the average throughput is:

$$\eta = \frac{R}{N} = \frac{R}{N}(1 - \gamma_f) + (1 - R/N)Cp_v w_v(C)$$

Therefore, derivative of η with respect to permission probability p_v is:

$$\frac{d\eta}{dp_v} = \begin{cases} (1 - \frac{R}{N})C(1 - p_v)^{C-2}(1 - Cp_v) & C \geq 1 \\ (1 - \frac{R}{N})C & C < 1. \end{cases}$$

Hence, depending on equilibrium value of C, maximum throughput happens at:

$$p_v^* = \begin{cases} \frac{1}{C} & C \geq 1 \\ 1 & C < 1. \end{cases} \tag{2.9}$$

Further, as discussed before, a PRMA voice terminal drops all packets that wait longer than D time slots for a reservation. Average packet drop probability at equilibrium was shown to be

$$P_{drop} = \gamma_f \frac{\nu^D}{1-(1-\gamma_f)\nu^N}, \qquad \nu = \nu(C, R, p_v) = 1 - (1 - \frac{R}{N})p_v(1-p_v)^C.$$

Derivative of P_{drop} with respect to permission probability p_v is

$$\frac{dP_{drop}}{dp_v} = -\frac{dP_{drop}}{d\nu}(1 - \frac{R}{N})(1-(C+1)p_v)(1-p_v)^{C-1}.$$

Here $\frac{dP_{drop}}{d\nu} = \gamma_f \frac{D\nu^{D-1}(1-(1-\gamma_f)\nu^N)+(1-\gamma_f)N\nu^{D+N-1}}{(1-(1+\gamma_f)\nu^N)^2} > 0$. Therefore, minimum packet drop probability occurs at:

$$p_v^* = \frac{1}{C+1}. \tag{2.10}$$

If permission probability is chosen as indicated in equations (2.9) or (2.10), system throughput is maximized or system packet drop probability is minimized. However, as discussed above, the number of contending voice terminals is not known to the system. Therefore, here we use the estimated number of contending terminals to be used with equations (2.9) or (2.10). As discussed before, at equilibrium, the number of contending terminals is a function of the number terminals with reservation $C = \max(0, M_v - \frac{R}{N\omega})$.

At the beginning of time slot $n+1$, the number of contending voice terminals is estimated as $\hat{c}_n = \max(0, M_v - \frac{r_n}{N\omega})$ and the permission probability is updated (depending on maximizing throughput or minimizing packet drop probability) as

$$p_{v_{n+1}} = \begin{cases} \frac{1}{\hat{c}_n} & \hat{c}_n \geq 1 \\ 1 & \hat{c}_n < 1, \end{cases} \qquad\qquad p_{v_{n+1}} = \frac{1}{\hat{c}_n + 1}.$$

Next, we show that we can model control with state estimation as a special case of General Price Based Control. We define control signal $u_n = N - r_n$. Therefore, dynamics of control signal can be written as following

$$u_{n+1} = \begin{cases} u_n & \text{if slot } n \text{ is reserved and reservation is kept,} \\ u_n + 1 & \text{if slot } n \text{ is reserved and reservation is lost,} \\ [u_n - I(Z_n = 1)]^+ & \text{if slot } n \text{ is not reserved.} \end{cases}$$

In other words, control using state estimation is a special case of General Price Based Control with $\phi = 1$, $\alpha = \xi = 0$, and $\beta = -1$. Therefore, as discussed before in analysis of General Price Based Control, the PRMA system with state estimation can be modeled by a Markov chain.

Proposition 2.4. *The Markov chain defined on $\aleph$ through (2.2) (for $\phi = 1$, $\alpha = \xi = 0$, and $\beta = -1$) is irreducible, aperiodic, and positive recurrent.*

Detailed proof of Proposition 2.4 is presented in the Appendix B. Next, we use Equilibrium Point Analysis to study equilibrium behavior of the system. We show if some conditions on system parameters are met, the PRMA system with state estimation control scheme has unique equilibrium point.

2.3.1 Maximizing Throughput - EPA

In the first case we consider, permission probability is chosen as function of control signal to maximize average throughput of the system. At equilibrium

$$p_v(U) = \begin{cases} \frac{1}{C} & C \geq 1 \\ p_{v_{max}} & C < 1. \end{cases}$$

Parameter $p_{v_{max}}$ can be chosen to be very close to 1. Equilibrium equations of the system are written as follows:

$$(M_v - C)\gamma_f\omega - (1 - (M_v - C)\omega)W_v(C) = 0,$$
$$W_v(C) = \begin{cases} (1 - \frac{1}{C})^{C-1} & C \geq 1 \\ p_{v_{max}}C & C < 1, \end{cases} \tag{2.11}$$

Equilibrium equation (2.11) is nonlinear and following lemma defines one sufficient condition on system parameters to have a unique fixed point.

Remark 2.2. *Equation (2.11) is derived with consideration that* $R = \min(N, N\omega(M_v - C))$*, where* $\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}$*. Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$ *and* $R \in [0, N]$*. However, it can easily be shown that* $C = 0$*,* $C = M_v - \frac{1}{\omega}$*,* $C = M_v$*,* $R = 0$*,* $R = N$*, or a combination thereof, cannot be solutions to equilibrium equation (2.11).*

Lemma 2.2. *Equilibrium equation (2.11) has a unique solution in* C *and the system has a single operating point if any of conditions* (1) *or* (2) *below hold:*

(1) $\quad M_v > \frac{1}{\omega} + 1$ *and* $-\gamma_f\omega < (\frac{1}{M_v - \frac{1}{\omega}} + \ln(1 - \frac{1}{M_v - \frac{1}{\omega}}))(1 - \frac{1}{M_v - \frac{1}{\omega}})^{M_v - \frac{1}{\omega} - 1}$,

(2) $\quad M_v < \frac{1}{\omega} + 1$ *and* $\frac{(M_v - 1)\omega\gamma_f}{1 - (M_v - 1)\omega} < (1 - \frac{1}{M_v})^{M_v - 1}$.

Proof. We define $g(C) = \frac{(M_v - C)\omega\gamma_f}{1-(M_v-C)\omega}$. Also, as mentioned before

$$W_v(C) = \begin{cases} (1-\frac{1}{C})^{C-1} & C \geq 1 \\ p_{v_{max}} C & C < 1, \end{cases}$$

Therefore, roots of equation (2.11) are same as fixed points of $g(C) = W_v(C)$ in the range of $\max(0, M_v - \frac{1}{\omega}) < C < M_v$. It is easy to show that first and second derivatives of $W_v(C)$ are

$$\begin{cases} \frac{dW_v}{dC} < 0 & C \geq 1 \\ \frac{dW_v}{dC} = p_{v_{max}} > 0 & C < 1 \end{cases}, \begin{cases} \frac{d^2W_v}{dC^2} > 0 & C \geq 1 \\ \frac{d^2W_v}{dC^2} = 0 & C < 1. \end{cases}$$

It can be shown that $\frac{dg}{dC} < 0$ and $\frac{d^2g}{dC^2} > 0$, for this range of C ($\max(0, M_v - \frac{1}{\omega}) < C < M_v$). Also, notice that

$$W_v(M_v) = (1 - \frac{1}{M_v})^{M_v - 1} > g(M_v) = 0.$$

Based on these facts, in order to derive conditions such that $W_v(C) = g(C)$ has exactly one solution, we consider following different cases:

- **Case 1 -** $M_v > 1 + \frac{1}{\omega}$: In this case, $g(C)$ is strictly decreasing, positive, and with positive second order derivative. Also, $W_v(C)$ is positive, strictly decreasing, with positive second order derivative, $W_v(M_v - \frac{1}{\omega}) < g((M_v - \frac{1}{\omega})^+)$, and $W_v(M_v) > g(M_v)$. We define $h(C) = g(C) - W_v(C)$ and we can show that if $h(C)$ is strictly decreasing, then $h(C) = 0$ has a unique solution. Since both $g(C)$ and $W_v(C)$ have positive second order derivatives, therefore

$$\frac{dW_v}{dC}(M_v - \frac{1}{\omega}) < \frac{dW_v}{dC} < \frac{dW_v}{dC}(M_v),$$
$$\frac{dg}{dC}(M_v - \frac{1}{\omega}) < \frac{dg}{dC} < \frac{dg}{dC}(M_v).$$

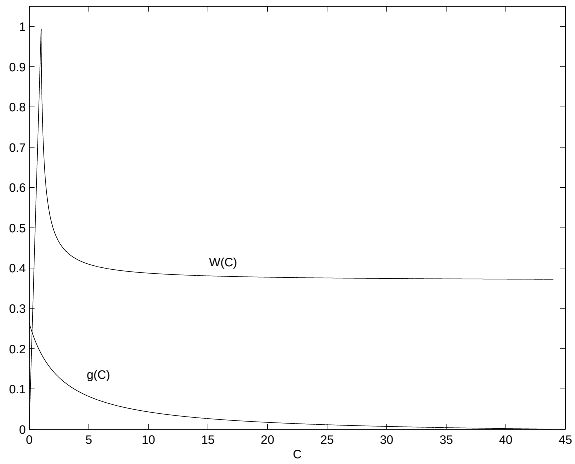

Figure 2.3: Functions $W(C)$ and $g(C)$ for Case 3 where $g(1) < f(M_v)$ and $p_{v_{max}} = .99$.

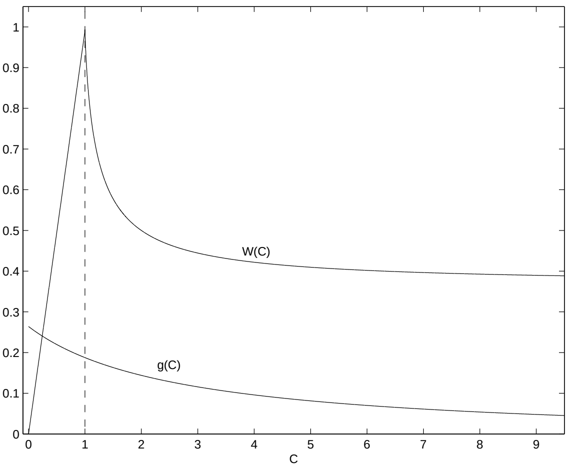

Figure 2.4: Figure 2.3 zoomed in.

Hence, if $\frac{dg}{dC}(M_v) < \frac{dW_v}{dC}(M_v - \frac{1}{\omega})$ then $h(C)$ is strictly decreasing and $h(C) = 0$ has a unique solution.

- **Case 2 -** $\frac{1}{\omega} < M_v < 1 + \frac{1}{\omega}$: If $g(1) < W_v(M_v)$, it is guaranteed that the equation has a unique solution.

- **Case 3 -** $0 < M_v < \frac{1}{\omega}$: In this case $g(C)$ is positive, strictly decreasing, with positive second derivative. Also, $W_v(C)$ is positive, strictly increasing, and with zero second order derivative for $0 < C < 1$. Further, $W_v(C)$ is strictly decreasing with positive second order derivative for $1 \leq C \leq M_v$. In the same way as Case 2, if $g(1) < W_v(M_v)$, it is easy to show that $W_v(C) = g(C)$ has exactly one solution. Figure 2.3 shows both functions $W_v(C)$ and $g(C)$ for this case where $g(1) < W_v(M_v)$.

Therefore, under conditions defined in the lemma, the system has a unique operating point in the interval of $0 < C < M_v$. □

If the parameters of the system are chosen based on Lemma 2.2, it can be shown that the unique equilibrium of the system is a stable equilibrium point. We illustrate the stability based on the fact that small changes in the equilibrium point will force the system back to the equilibrium point. It can be shown that the difference between "inflow" and "outflow" for number of contending terminals is $(1-(M_v-C)\omega)(g(C)-W_v(C))$, where $g(C)$ and $W_v(C)$ are as defined in Lemma 2.2. As discussed earlier $(1-(M_v-C)\omega)>0$, therefore, if number of contending terminals is increased slightly above its equilibrium value, the difference between "inflow" and "outflow" will be negative. Hence, the number of contending terminals will decrease. In the same manner, if number of contending terminals is decreased slightly below its equilibrium value, the difference between "inflow" and "outflow" will be positive, hence, the number of contending terminals is increased. Therefore, it can be shown that small changes in the unique equilibrium will force the system back to the equilibrium.

2.3.2 Minimizing Packet Drop Probability - EPA

Here, permission probability is chosen as function of control signal to minimize average packet drop probability of the system. At equilibrium $p_v(U)=\frac{1}{C+1}$. Equilibrium equations of the system are written as follows:

$$(M_v-C)\gamma_f\omega-(1-(M_v-C)\omega)W_v(C)=0,$$
$$W_v(C)=\begin{cases}\frac{C}{C+1}(1-\frac{1}{C+1})^{C-1} & C\geq 1\\ \frac{C}{C+1} & C<1,\end{cases} \tag{2.12}$$

Remark 2.3. *Equation (2.12) is derived with consideration that* $R = \min(N, N\omega(M_v - C))$*, where* $\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}$*. Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$ *and* $R \in [0, N]$*. However, it can easily be shown that* $C = 0$*,* $M_v - \frac{1}{\omega}$*,* $C = M_v$*,* $R = 0$*,* $R = N$*, or a combination thereof, cannot be solutions to equilibrium equation (2.12).*

The following Lemma summarizes one sufficient for parameters of the system to have a unique solution under this controller.

Lemma 2.3. *Equilibrium equation (2.12) has a unique solution in* C *and the system has a single operating point if any of conditions* (1) *or* (2) *below hold:*

(1) $M_v > \frac{1}{\omega} + 1$ *and* $-\gamma_f\omega < (\frac{1}{M_v+1-\frac{1}{\omega}} + \ln(1 - \frac{1}{M_v+1-\frac{1}{\omega}}))(1 - \frac{1}{M_v+1-\frac{1}{\omega}})^{M_v - \frac{1}{\omega}}$,

(2) $M_v < \frac{1}{\omega} + 1$ *and either*

$$\frac{(M_v - 1)\omega\gamma_f}{1 - (M_v - 1)\omega} < (1 - \frac{1}{M_v + 1})^{M_v}, \quad or$$

$$-\gamma_f\omega < 0.5(0.5 + \ln(.5)).$$

Proof. We define $g(C) = \frac{(M_v - C)\omega\gamma_f}{1-(M_v - C)\omega}$. Therefore, roots of equation (2.12) are same as fixed points of $g(C) = W_v(C)$. It is easy to show that first and second derivatives of $W_v(C)$ are

$$\begin{cases} \frac{dW_v}{dC} < 0 & C \geq 1 \\ \frac{dW_v}{dC} > 0 & C < 1 \end{cases}, \begin{cases} \frac{d^2W_v}{dC^2} > 0 & C \geq 1 \\ \frac{d^2W_v}{dC^2} < 0 & C < 1. \end{cases}$$

It is noted that $\max(0, M_v - \frac{1}{\omega}) < C < M_v$. As shown before, $\frac{dg}{dC} < 0$ and $\frac{d^2g}{dC^2} > 0$, for this range of C. Also, notice that

$$W_v(M_v) = (1 - \frac{1}{M_v + 1})^{M_v - 1} > g(M_v) = 0.$$

Based on these facts, in order to derive conditions such that $W_v(C) = g(C)$ has exactly one solution, we consider following different cases:

- **Case 1 -** $M_v > 1 + \frac{1}{\omega}$: In this range, $g(C)$ is strictly decreasing and positive. Also, $W(C)$ is positive, $W_v(M_v - \frac{1}{\omega}) < g((M_v - \frac{1}{\omega})^+)$, and $W_v(M_v) > g(M_v)$. Therefore, similar to proof of previous Lemma, if $\frac{dg}{dC}(M_v) < \frac{dW_v}{dC}(M_v - \frac{1}{\omega})$, then it is easy to show that $W_v(C) = g(C)$ has exactly one solution.

- **Case 2 -** $\frac{1}{\omega} < M_v < 1 + \frac{1}{\omega}$: If $g(1) < W_v(M_v)$ or $\frac{dg}{dC}(M_v) < \frac{dW_v}{dC}(1^+)$, it is guaranteed that the equation has a unique solution.

- **Case 3 -** $0 < M_v < \frac{1}{\omega}$: In this case $g(C)$ is positive, strictly decreasing, and its second derivative is positive. Also, $W_v(C)$ is positive, strictly increasing, and with negative second order derivative for $0 < C < 1$. Further, $W_v(C)$ is strictly decreasing with positive second order derivative for $1 \leq C \leq M_v$. In the same way as Case 2, if $g(1) < W_v(M_v)$ or $\frac{dg}{dC}(M_v) < \frac{dW_v}{dC}(1^+)$, then it is easy to show that $W_v(C) = g(C)$ has exactly one solution.

Therefore, under conditions defined in the lemma, the system has a unique operating point in the interval of $0 < C < M_v$. It can easily be shown that $C = 0$ or $C = M_v$ cannot be the operating point of the system.

Therefore, under conditions defined in the lemma, the system has a unique operating point in the interval of $0 < C < M_v$. $\square$

2.4 Bifurcation Control Using Multiple Transmission Power Levels

In this section, we examine effects of power capture phenomenon in controlling bifurcations in the PRMA system. We will show that using multiple transmission power levels at terminals and using capture effect at the access point (such as the base station) can control bifurcations by either completely eliminating the bifurcations or postponing them for higher values of bifurcation parameter. Hence, power capture phenomenon can increase capacity of the PRMA system by allowing more voice terminals sharing common communication medium.

Power capture in wireless networks is possible because, when packet collision occurs, there are multiple packets arriving at a receiver at the same time which the receiver may be able to decode a packet with highest power. Therefore, terminals can transmit their packets at multiple discrete power levels and a packet with highest power level may be captured. In perfect power capture model, it is assumed that a packet is captured at a receiver if and only if it has the highest power among all other packets that overlap it. In a more accurate model, a packet is captured if its signal-to-interference-plus-noise ratio is greater than a decodability threshold [50], [51], [52].

A voice terminal in CON state, contends for a reservation by transmitting a speech packet on an available time slot, if it has permission to send. In this section, we consider m different power levels $P_1 > P_2 > \cdots > P_m$. Assume that packet lengths in different power levels are the same. A contending terminal with permission to transmit, will choose a power level P_i with probability q_i, $i \in \{1, 2, .., m\}$. In this section, we assume the perfect power capture model. Also, we assume that distance between the terminals and the base station are equal. Hence, received power and transmission power are interchangeable. A contending terminal will successfully transmit its packet on a time slot if 1) the time slot is unreserved, 2) terminal has permission to transmit, and 3) no other speech packets with equal or higher power levels are transmitted simultaneously. When the contending terminal successfully transmits a packet, base station, using a feedback message, informs all the terminals that the time slot is reserved. The terminal with reservation enters the reservation state, RES. It will transmit a speech packet in its reserved time slot in every frame. Since the terminal has successfully reserved the time slot, it is assumed that this terminal will send the rest of its packets with the lowest power level, P_m. The terminal transmits its last packet at the end of the talkspurt and moves to SIL state.

We study equilibrium behavior of the PRMA system using equilibrium point analysis. Equilibrium equations of the system can be written as follows

$$F(C) = M_v - C - (\frac{\gamma_f}{\sigma_v} + N)(\frac{Cp_vW_v(C)}{\gamma_f + Cp_vW_v(C)}), \tag{2.13}$$

with $W_v(C) = \sum_{h=1}^{m} v_h(C)$, and

$$v_h(C) = \begin{cases} q_h(1 - p_v \sum_{t=1}^{h} q_t)^{C-1} & C \geq 1 \\ q_h & C < 1 \end{cases} .$$

In an $\underline{X} - \alpha$ state-control space, a simple static bifurcation of a fixed point of (2.13) is said to occur at $(\underline{X}_0; \alpha_c)$ if following conditions are satisfied [53]:

- $\underline{F}(\underline{X}_0; \alpha_c) = \underline{0}$,

- $D_X\underline{F}$ has a zero eigenvalue while all of its other eigenvalues have nonzero real parts at $(\underline{X}_0; \alpha_c)$.

First condition ensures that the considered solution is a fixed point of equation (2.13), and the second condition implies that this fixed point is a nonhyperbolic fixed point. Let $\underline{F}_\alpha$ be the derivative of $\underline{F}$ with respect to the control parameter α and construct the matrix $[D_X\underline{F} \mid \underline{F}_\alpha]$. At a saddle-node bifurcation, $\underline{F}_\alpha$ does not belong to the range of matrix $D_X\underline{F}$. In other words, matrix $[D_X\underline{F} \mid \underline{F}_\alpha]$ has a rank of n at saddle-node bifurcation points. Considering n to be the number of the states of the system.

A Simple static bifurcation of the fixed points of equation (2.13) happens at $(C_0; M_{v_c})$, if $F(C_0; M_{v_c}) = 0$ and $F_C(C_0; M_{v_c}) = 0$. Here $F_C(\cdot\ ;\cdot)$ is the derivative of $F(\cdot\ ;\cdot)$ with respect to C.

$$F_C(C) = -1 - (\frac{\gamma_f}{\sigma_v} + N)(\frac{\gamma_f(W(C) + Cp_vW_C(C))}{(\gamma_f + Cp_vW(C))^2}), \tag{2.14}$$

with $W_C(C) = \sum_{h=1}^{m} w_{h_C}(C)$, and

$$w_{h_C}(C) = \begin{cases} q_h(1 - p_v \sum_{t=1}^{h} q_t)^{C-1} \ln(1 - p_v \sum_{t=1}^{h} q_t) & C \geq 1 \\ 0 & C < 1 \end{cases}.$$

Equation (2.14) is only in terms of C. Roots of this equation, gives critical values of C_0 where bifurcations happen. The critical values of the control parameter is found by solving $F(C_0; M_{v_c}) = 0$ for M_{v_c}. Since $F_{M_v}(\cdot\ ;\cdot) = 1$, then $[F_C \mid F_{M_v}]$ at bifurcation points $(C_0; M_{v_c})$ has a rank of one. Therefore, these bifurcation points are saddle-nodes.

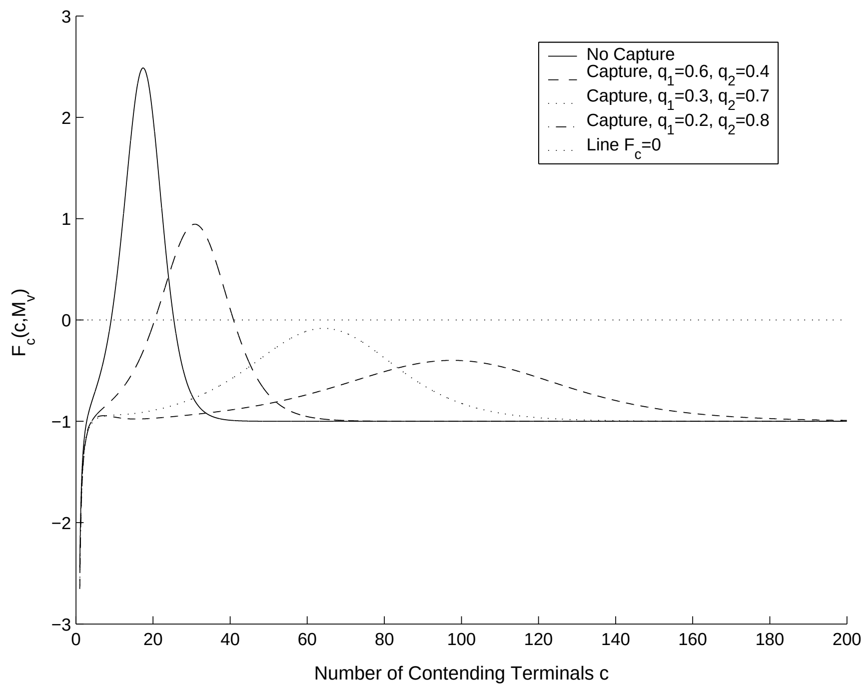

Figure 2.5: F_C vc. C

Figure 2.5 illustrates $F_C(C, M_v)$ as a function of C for the PRMA system with voice terminal with and without power capture. In the case that power capture is not used, $F_C(C, M_v)$ has two roots. Figure 2.5 shows that using power capture, equation $F_C(C, M_v)$ will either have no root or its roots are at higher values of C_0. Higher values of C_0 results in higher values for the critical control parameter M_{v_c}. Therefore, the bifurcations are delayed for the higher values of the control parameter.

2.5 Performance Analysis of PRMA Over Random Error Channel

Previously, we studied Markov model and equilibrium point equations of the PRMA system with General Price Based Control. We assumed that packet collisions are the only source of error in the system. Here, we study effects of General Price Based Control on the PRMA system over “random packet error channels” [25]. We assume an uplink channel where only errors that corrupt packet header are considered [25]. The reason for this assumption is that these errors directly affect behavior of the PRMA system [25]. If packet header error occurs while a terminal is in reservation mode, it may lose its reservation before a message is completely transmitted. Hence, voice terminal is moved back to contending state and it contends again for rest of the packets in the message. Therefore, the terminal may face more packet droppings. Also, if packet header error happens while a contending voice terminal sends a packet on an available time slot with no collision, base station cannot decode the message and cannot grant the terminal a reserved time slot. Therefore, random packet error channel directly affects the PRMA system. As discussed in [25], it is assumed that packet header errors occur randomly, independent of each other and with a fixed probability Δ. Next, we introduce the Markov chain model for the PRMA system with the General Price Based Control scheme over the random packet error channel.

2.5.1 General Price Based Control

Figure 2.6 illustrates Markov model for the PRMA system over random packet error channel. Here, we assume that when a contending voice terminal transmits a packet on an available time slot without collision, it will reserve that time slot if no packet header error happens. If there is a header packet error, the access point

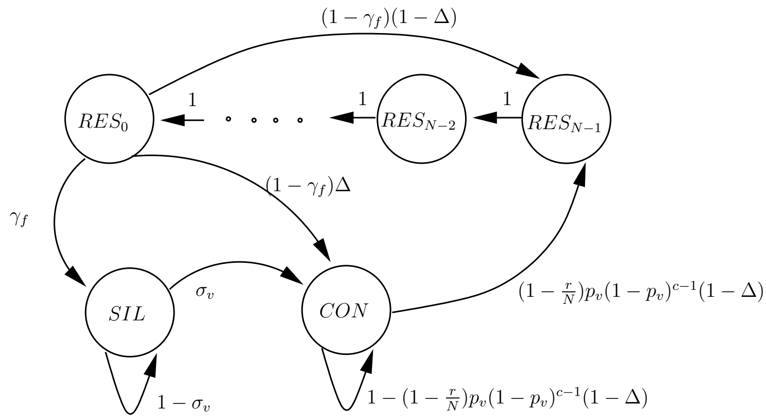

Figure 2.6: Markov Chain Model for PRMA Voice System over Random Packet Error Channel.

(such as the base station) interprets the error either as a collision or as an event that no packet was transmitted [25]. When there is a header error in a contending packet, we assume that with a fixed probability q the access point sends back a collision feedback and with probability $1-q$ it sends an idle feedback. We define the following events: $A_n = \{$at an available time slot n, one packet transmitted with error - base station assumed idle$\}$ and $B_n = \{$at an available time slot n, one packet transmitted with error - base station assumed collision$\}$.

Therefore, update algorithm (2.1) for control signal is changed as follows:

$$u_{n+1} = \begin{cases} u_n & \text{if slot } n \text{ is reserved and reservation is kept,} \\ u_n + \phi & \text{if slot } n \text{ is reserved and reservation is lost,} \\ [u_n - \alpha I(Z_n = 0 \vee A_n) + \beta I(Z_n = 1 \wedge \textit{no error}) + \xi I(Z_n \geq 2 \vee B_n)]^+ & \\ & \text{if time slot } n \text{ is not reserved.} \end{cases} \tag{2.15}$$

Transition probabilities for this Markov chain can be written as follows:

$$Pr(c_{n+1} = c', r_{n+1} = r', u_{n+1} = u' | c_n = c, r_n = r, u_n = u) = \tag{2.16}$$

- $f_v(c' - c; M_v - c - r, \sigma_v)(r/N)\gamma_f$

  if $c \leq c' \leq M_v - r, r' = r - 1, u' = u + max(-u, \phi)$

- $f_v(c' - c - 1; M_v - c - r, \sigma_v)(r/N)(1 - \gamma_f)\Delta$

  if $c + 1 \leq c' \leq M_v - r + 1, r' = r - 1, u' = u + max(-u, \phi)$

- $f_v(c' - c; M_v - c - r, \sigma_v)(r/N)(1 - \gamma_f)(1 - \Delta)$

  if $c \leq c' \leq M_v - r, r' = r, u' = u$

- $f_v(c' - c; M_v - c - r, \sigma_v)(1 - r/N)((1 - p_v)^c + (1 - q)\Delta c p_v (1 - p_v)^{c-1}$

  if $c \leq c' \leq M_v - r, r' = r, u' = u + \max(-u, -\alpha)$

- $f_v(c' - c + 1; M_v - c - r, \sigma_v)(1 - r/N)c p_v (1 - p_v)^{c-1}(1 - \Delta)$

  if $c - 1 \leq c' \leq M_v - r - 1, r' = r + 1, u' = u + \max(-u, \beta)$

- $f_v(c' - c; M_v - c - r, \sigma_v)(1 - r/N)(1 - (1 - p_v)^c - (1 - q\Delta)c p_v (1 - p_v)^{c-1}$

  if $c \leq c' \leq M_v - r, r' = r, u' = u + \xi$

- 0 Otherwise.

  Here, $f_v(k; n, \sigma_v) = \begin{pmatrix} n \\ k \end{pmatrix} \sigma_v^k (1 - \sigma_v)^{n-k}$.

It should be mentioned that the Markov chain modeling the PRMA system over the random packet error channel has the same state space $\aleph$ as before.

2.5.2 Price Based Control

A special case of General Price Based Control (over random packet error channel) is when $\phi = 0$.

Proposition 2.5. *The Markov chain defined on $\aleph$ through equation (2.16) (for $\phi = 0$) is irreducible and aperiodic. Also Under Assumption 2.1, the Markov chain is positive recurrent.*

Proof of Proposition 2.5 is similar to proof of Propositions 2.1 and 2.2 and therefore, the details of the proof are omitted. Next, we analyze performance of the PRMA system with price based control scheme over the random packet error channel using equilibrium point approach. Similar analysis as in previous sections can be used. A point (C, R, U) is an equilibrium point of the system if:

$$
\begin{aligned}
&(M_v - C - R)\sigma_v + (R/N)(1-\gamma_f)\Delta - (1 - R/N)Cp_v(U)w_v(C,U)(1-\Delta) = 0 \\
&(R/N)\gamma_f - (M_v - C - R)\sigma_v = 0 \\
&(-\alpha(1-q)\Delta + \beta(1-\Delta) - \xi(1-q\Delta))Cp_v(U)w_v(C,U) - (\alpha+\xi)(1-p_v(U))^C + \xi = 0
\end{aligned}
\tag{2.17}
$$

Set of equations (2.17) defines equilibrium equations of the PRMA system with voice terminals over the random error channel with price based control scheme. The goal in using price based scheme is to control bifurcations that might occur in the number of equilibrium points of the system and therefore, to increase capacity of the system. The following lemma introduces two different sets of conditions for the control parameters as sufficient conditions on uniqueness of the operating point of the system.

Remark 2.4. *Based on the first two equations of the set of equations (2.17), it can be shown that* $R = \min(N, N\omega(M_v - C))$*, where* $\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}$*. Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$*,* $R \in [0, N]$*, and* $p_v \in [0, 1]$*. However, it can easily be shown, using the set of equation (2.17), that* $C = 0$*,* $M_v - \frac{1}{\omega}$*,* $C = M_v$*,* $R = 0$*,* $R = N$*,* $p_v = 0$*,* $p_v = 1$ *(for* $\beta = \frac{\alpha(1-q)\Delta + \xi(1-q\Delta)}{1-\Delta}$ *as considered in Lemma 2.4), or a combination thereof, cannot be solutions to the set of equilibrium equations (2.17).*

Lemma 2.4. *There exists a set of control parameters* (α, β, ξ) *for which the set of equations (2.17) has a unique solution in* (C, U) *and the system has a single operating point if any of conditions* $(1a)$*,* $(1b)$*, or* (2) *below hold:*

$(1a)$ $\quad Mv > 1 + \frac{1}{\omega}$ *and* $-\frac{\omega\gamma_f'}{(1-\Delta)} < \frac{\xi}{\alpha+\xi}[(\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v - \frac{1}{\omega}} + 1)\exp(-\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v - \frac{1}{\omega}}) - 1]$,

$(1b)$ $\quad Mv < 1 + \frac{1}{\omega}$ *and either*

$$\frac{(M_v - 1)\omega\gamma_f'}{(1-\Delta)(1-(M_v-1)\omega)} \leq \frac{\xi M_v}{\alpha+\xi}(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1), \qquad or$$

$$-\frac{\omega\gamma_f'}{(1-\Delta)} < 1 - \frac{\xi}{\alpha+\xi} + \ln(\frac{\xi}{\alpha+\xi}),$$

(2) $\quad -1 + \frac{\xi}{\alpha+\xi} - \ln(\frac{\xi}{\alpha+\xi}) < \frac{\omega}{\gamma_f'(1-\Delta)}[(1-\Delta)\frac{\xi}{\alpha+\xi}M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1) + \gamma_f']^2$,

here $\gamma_f' = \gamma_f + (1-\gamma_f)\Delta$.

The existence of the control parameters (α, β, ξ) with $\beta = \frac{\alpha(1-q)\Delta + \xi(1-q\Delta)}{1-\Delta}$ such that the conditions hold can be proved in a similar manner to proof of Lemma 2.1. Next, we extend our control analysis of the PRMA system over random packet error channel to two other special cases of the General Price Based Control.

2.5.3 State Estimation-Based Control

Maximizing Throughput - Control using state estimation over random packet error channel is a special case of General Price Based Control for $\phi = 1$, $\alpha = \xi = 0$, and $\beta = -1$. Similar to proof of Proposition 2.4, Markov chain defined on $\aleph$ through equation (2.16) (for $\phi = 1$, $\alpha = \xi = 0$, and $\beta = -1$) is irreducible, aperiodic, and under Assumption 2.1, positive recurrent.

As discussed previously regarding state estimation control scheme, first, we consider a case where permission probability is chosen as function of control signal to maximize average throughput of the system. At equilibrium

$$p_v(U) = \begin{cases} \frac{1}{C} & C \geq 1 \\ p_{v_{max}} & C < 1. \end{cases}$$

Parameter $p_{v_{max}}$ can be chosen to be very close to 1. Equilibrium equations of the system are written as follows:

$$\begin{aligned} &(M_v - C)\omega(\gamma_f + (1-\gamma_f)\Delta) - (1-(M_v - C)\omega)W_v(C)(1-\Delta) = 0, \\ &W_v(C) = \begin{cases} (1-\frac{1}{C})^{C-1} & C \geq 1 \\ p_{v_{max}}C & C < 1, \end{cases} \end{aligned} \tag{2.18}$$

Lemma 2.5. *Equilibrium equation (2.18) has a unique solution in C and the system has a single operating point if any of conditions* (1) *or* (2) *below hold:*

(1) $M_v > \frac{1}{\omega}+1$ *and* $-\frac{\omega(\gamma_f+(1-\gamma_f)\Delta)}{(1-\Delta)} < (\frac{1}{M_v-\frac{1}{\omega}}+\ln(1-\frac{1}{M_v-\frac{1}{\omega}}))(1-\frac{1}{M_v-\frac{1}{\omega}})^{M_v-\frac{1}{\omega}-1}$,

(2) $M_v < \frac{1}{\omega}+1$ *and* $\frac{(M_v-1)\omega(\gamma_f+(1-\gamma_f)\Delta)}{1-(M_v-1)\omega(1-\Delta)} < (1-\frac{1}{M_v})^{M_v-1}$.

Proof is straight forward and very similar to proof of lemma 2.2.

Minimizing Packet Drop Probability **-** Next, as discussed before, permission probability is chosen as function of control signal to minimize average packet drop probability of the (errorless) system. At equilibrium $p_v(U) = \frac{1}{C+1}$. Equilibrium equations of the system are written as follows:

$$(M_v - C)\omega(\gamma_f + (1-\gamma_f)\Delta) - (1 - (M_v - C)\omega)W_v(C)(1-\Delta) = 0,$$

$$W_v(C) = \begin{cases} \frac{C}{C+1}(1 - \frac{1}{C+1})^{C-1} & C \geq 1 \\ \frac{C}{C+1} & C < 1, \end{cases} \tag{2.19}$$

Lemma 2.6. *Equilibrium equation (2.19) has a unique solution in* C *and the system has a single operating point if any of conditions* (1) *or* (2) *below hold:*

(1) $\quad M_v > \frac{1}{\omega} + 1$ *and*

$$-\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{(1-\Delta)} < (\frac{1}{M_v + 1 - \frac{1}{\omega}} + \ln(1 - \frac{1}{M_v + 1 - \frac{1}{\omega}}))(1 - \frac{1}{M_v + 1 - \frac{1}{\omega}})^{M_v - \frac{1}{\omega}},$$

(2) $\quad M_v < \frac{1}{\omega} + 1$ *and either*

$$-\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{(1-\Delta)} < 0.5(0.5 + \ln(0.5)), \qquad \textit{or}$$

$$\frac{(M_v - 1)\omega(\gamma_f + (1-\gamma_f)\Delta)}{1 - (M_v - 1)\omega(1-\Delta)} < (1 - \frac{1}{M_v + 1})^{M_v - 1}.$$

Proof is straight forward and very similar to proof of lemma 2.3.

2.6 Numerical Results

In this section we compare our analytical results with numerical. We consider a PRMA voice only system with following parameters [23]: $R_c = 720,000$ bits/s,

$R_s = 32,000$ bits/s, $T = 0.016$ s, $H = 64$ bits, $D_{max} = 0.032$ s, $t_1 = 1.00$ s, and $t_2 = 1.35$ s.

Figures 2.7, 2.8, and 2.9 show packet drop probability (P_{drop}) for the system when M_v is taken as bifurcation parameter and no control is used. In these figures, permission probability is fixed at $p_v = 0.2$, $p_v = 0.3$, and $p_v = 0.4$, respectively.

Next, we design a price based control for the same PRMA system. We choose $\beta = \xi = 1$. Then, α is chosen to be $\alpha = 0.114$ such that the number of contending terminals for $M_v = 40$ is $C = 0.3037$. Figure 2.10 shows bifurcation diagram for the packet drop probability, when M_v is bifurcation parameter. We can see that using price based control, bifurcations are eliminated. Moreover, capacity of the PRMA system has increased to 40 from 39 when $p_v = 0.2$, 28 when $p_v = 0.3$, and from 21 when $p_v = 0.4$.

Further, we design state estimation control schemes. Figure 2.11 illustrates bifurcation diagram for packet drop probability, when state estimation control scheme is based on maximum throughput. Here, it is assumed that $p_{v_{max}} = 0.9$. We can observe that bifurcations are eliminated and capacity of the PRMA system is increased to 44. Figure 2.12 illustrates bifurcation diagram for packet drop probability, when state estimation control scheme is based on minimum packet drop probability. It can be observed that bifurcations are controlled and capacity of the system is 44.

Now we consider the same voice PRMA system over random packet error channel. Figure 2.13 shows bifurcation diagram for packet drop probability when error probability (Δ) is bifurcation parameter, $M_v = 25$, and $p_v = 0.3$. This figure illustrates that only for small values of Δ system has an acceptable drop probability (less than 0.01). Also figure 2.14 shows bifurcation diagrams for drop probability when M_v is chosen as bifurcation parameter and for two different values of Δ. The plus signs show packet drop probability for $\Delta = 0.05$ and the points are for $\Delta = 0.01$. It can be noticed that for $\Delta = 0.01$ capacity of the system is 27 and for $\Delta = 0.05$ capacity is 24.

Next, we use price based control with same parameters as before, $\beta = \xi = 1$ and $\alpha = 0.114$. Figure 2.15 shows bifurcation diagram for packet drop probability for both $\Delta = 0.01$ and $\Delta = 0.05$. It is seen that capacity of the system is increased to 29 and 37 for $\Delta = 0.01$ and $\Delta = 0.05$, respectively. Here we have assumed that $q = 1$.

Also, figure 2.16 illustrates effects of state estimation control scheme (based on maximum throughput) on nonlinear behavior of the PRMA system. If error probability is $\Delta = 0.01$, bifurcations of the operating points of the system is completely eliminated and capacity of the PRMA system is increased to 43. However, for the case where $\Delta = 0.05$, although the bifurcations are not completely eliminated (PRMA system has three equilibrium points at $M_v = 44$), but they are controlled by delaying the bifurcations. In this case, capacity of the system is increased to 41.

Figure 2.17 illustrates effects of state estimation control (based on minimizing packet drop probability) on behavior of the PRMA system over random packet error channel for $\Delta = 0.01$ and $\Delta = 0.05$. Bifurcations are controlled and capacity of the system is $M_v = 43$ for $\Delta = 0.01$ and $M_v = 41$ or $\Delta = 0.05$.

Finally, figure 2.18 illustrates a bifurcation diagram for packet drop probability when the PRMA system utilizes power capture with two power levels. The highest

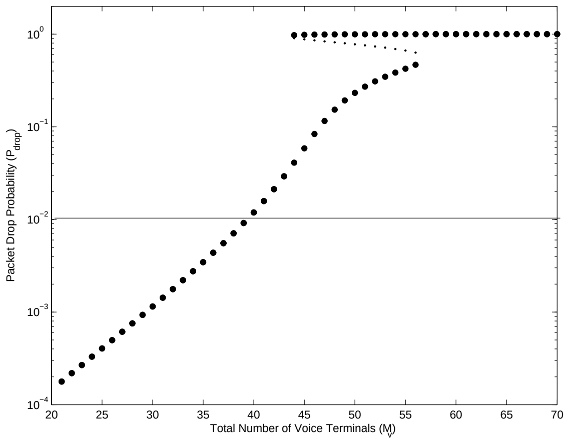

Figure 2.7: Bifurcation diagram for packet drop probability with no control ($p_v = 0.2$)

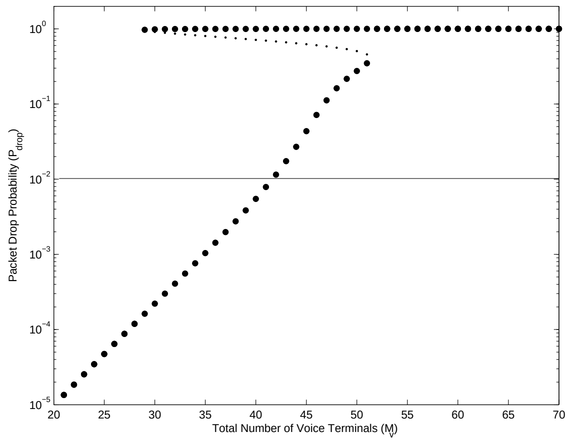

Figure 2.8: Bifurcation diagram for packet drop probability with no control ($p_v = 0.3$)

power level is chosen with probability $q_1 = 0.2$ and the terminals choose the lowest power level with probability $q_2 = 0.8$. Figure 2.18 shows that bifurcations are completely controlled by the capture effect and capacity is increased to 41. Here, it is assumed that $p_v = 0.3$ Use of multiple power levels is not limited to controlling bifurcations in equilibrium points of the PRMA system when M_v is the bifurcation parameter. Figures 2.19 and 2.20 show that capture effect eliminates bifurcation when permission probability p_v is the bifurcation parameter. In this case M_v is fixed at 40.

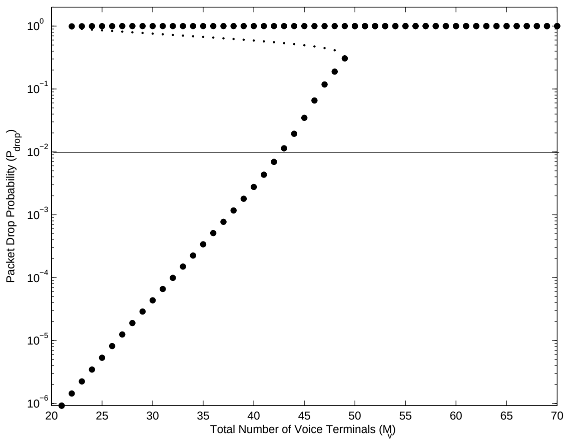

Figure 2.9: Bifurcation diagram for packet drop probability with no control ($p_v = 0.4$)

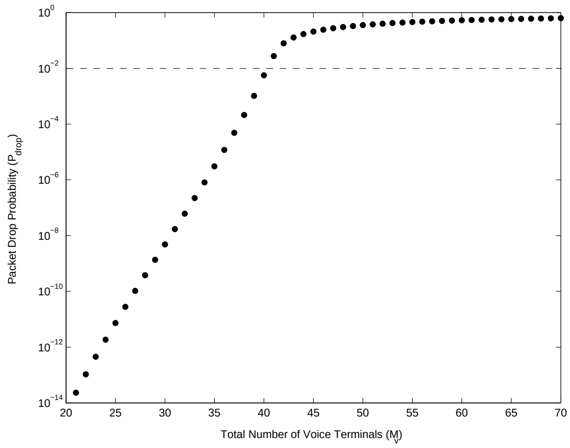

Figure 2.10: Bifurcation diagram for packet drop probability with the price based control ($\alpha = 0.114$, $\beta = \xi = 1$)

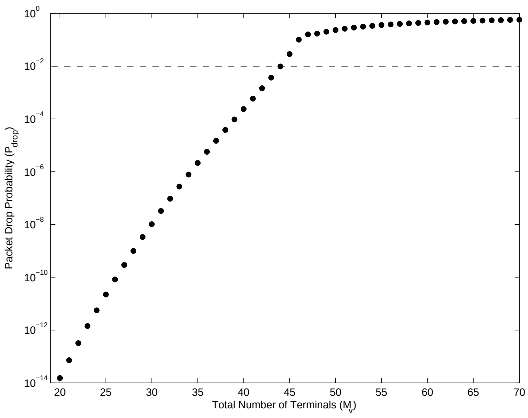

Figure 2.11: Bifurcation diagram for packet drop probability with state estimation control (maximizing throughput) ($p_{v_{max}} = 0.9$)

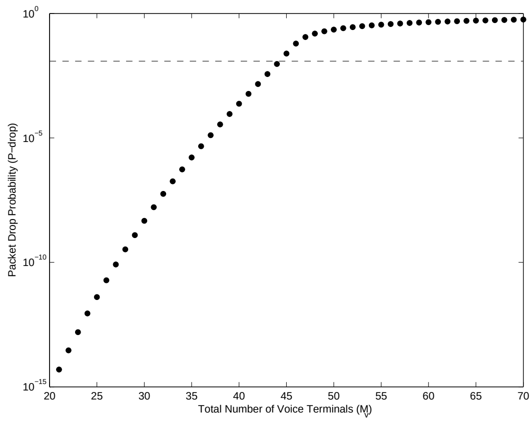

Figure 2.12: Bifurcation diagram for packet drop probability with state estimation control (minimizing P_{drop})

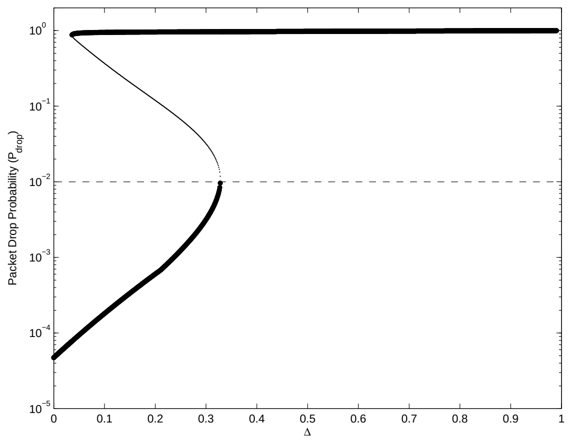

Figure 2.13: Bifurcation diagram for packet drop probability with no control (Δ is bifurcation parameter, $p_v = 0.3$, $M_v = 25$)

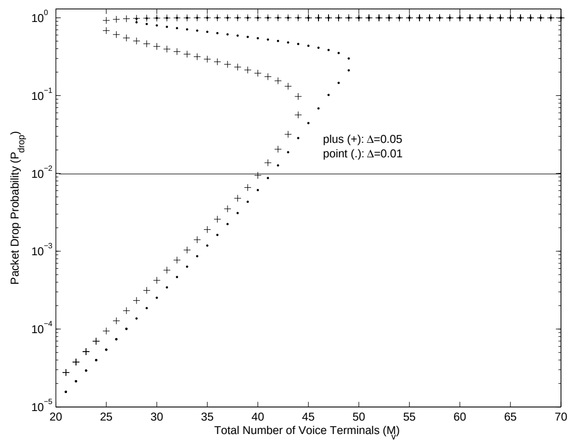

Figure 2.14: Bifurcation diagram for packet drop probability with no control ($p_v = 0.3$ and two different values of Δ)

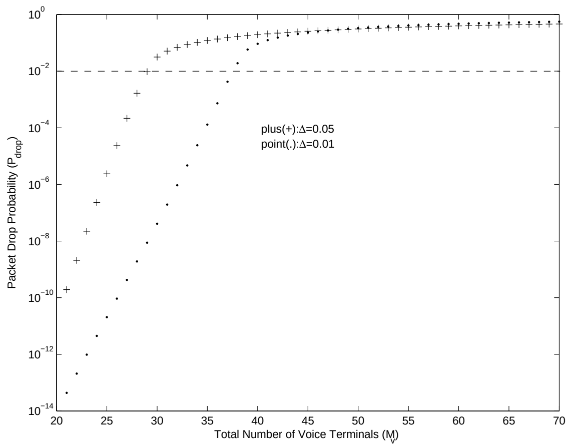

Figure 2.15: Bifurcation diagram for packet drop probability with control ($\Delta = 0.05$, $\Delta = 0.01$, $\alpha = 0.114$, $\beta = \xi = 1$)

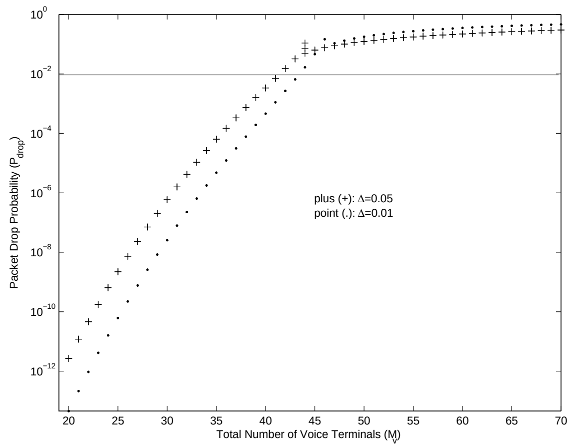

Figure 2.16: Bifurcation diagram for packet drop probability with state estimation control (maximizing throughput) ($\Delta = 0.05$, $\Delta = 0.01$, $p_{v_{max}} = 0.9$)

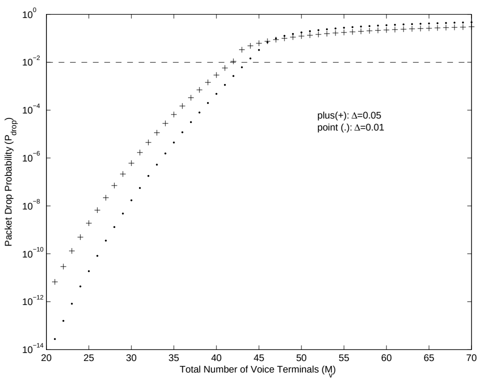

Figure 2.17: Bifurcation diagram for packet drop probability with state estimation control (minimizing P_{drop}) ($\Delta = 0.05$ and $\Delta = 0.01$)

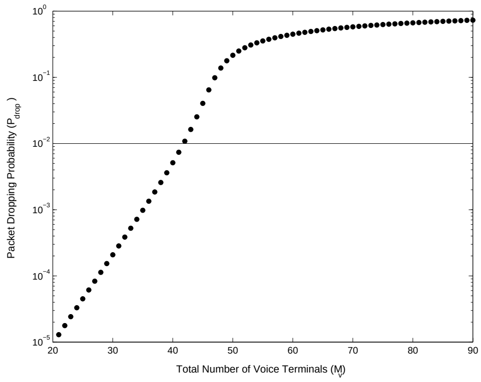

Figure 2.18: Bifurcation diagram for packet dropping probability with power capture ($q_1 = 0.2$ and $q_2 = 0.8$)

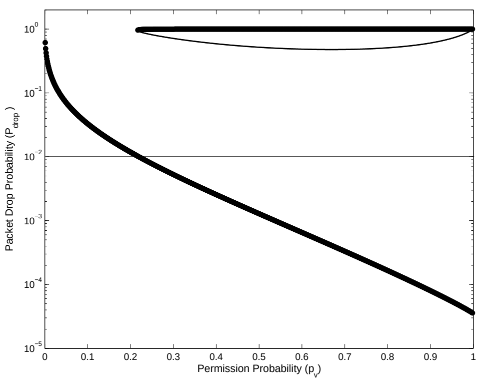

Figure 2.19: Bifurcation diagram for packet drop probability with no capture

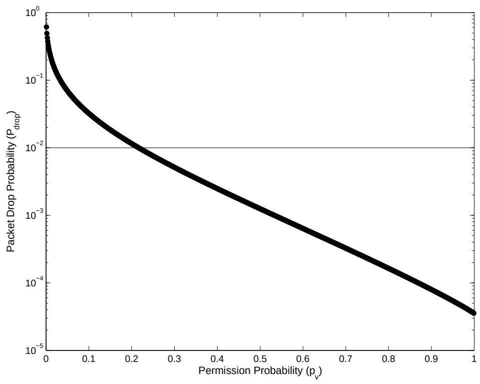

Figure 2.20: Bifurcation diagram for packet drop probability at equilibrium with power capture ($q_1 = 0.2$ and $q_2 = 0.8$)

2.7 General Price Based Control for PRMA-HS protocol

Continuing with a system that only employs voice terminals, we further, consider a PRMA-HS protocol. As discussed earlier in chapter 1, in reviewing the PRMA-HS protocol, round trip delay RTD is defined as time that takes for a terminal to receive feedback information from time it has transmitted a packet. For the PRMA-HS protocol, it is assumed that RTD is less than duration of a frame. We can define round trip delay as $RTD = \frac{N}{d}$ times slots, assuming d is an integer devisor of N. For the sake of simplicity in notation, in modeling the PRMA-HS system using Markov chain, we assume that $d = N$ and therefore, $RTD = 1$ time slot. However, our analysis can be expanded to any value of round trip delay. For equilibrium point analysis of the system, however, we ease the assumption of $RTD = 1$, and we provide sufficient conditions for bifurcation control for any value of RTD less than or equal to a frame size.

In the PRMA-HS protocol, at the end of each time slot (for example time slot n), terminals receive a feedback message regarding status of RTD time slot ago (time slot $n - RTD$). The status message can indicate that if a reserved time slot $n - RTD$ is still reserved or now it is free. The feedback message can indicate that if an available time slot $n - RTD$ had no transmission, one successful transmission, or collision. Terminals use this feedback information to adjust their permission probability. We assume that permission probability p_v is a function of a control signal u. Control signal is updated at the end of each time slot based on equation (2.20).

$$
u_{n+1} = \begin{cases} u_n & \text{if slot } n-1 \text{ is reserved and reservation is kept,} \\ u_n + \phi & \text{if slot } n-1 \text{ is reserved and reservation is lost,} \\ [u_n - \alpha I(Z_{n-1} = 0) + \beta I(Z_{n-1} = 1 \wedge by\ a\ CON\ ) & \\ \qquad + \xi I((Z_{n-1} \geq 2) \vee (Z_{n-1} = 1 \wedge by\ a\ HIN))]^{+} & \\ & \text{if slot } n-1 \text{ is not reserved.} \end{cases} \tag{2.20}
$$

Here ϕ, α, and ξ are positive real numbers and β is a real number. $[x]^+$ denotes $\max(0, x)$. Random variable Z_n is the number of packets that are transmitted at beginning of time slot n. Permission probability p_v is updated at end of each time slot based on new value of the control signal u.

Assumption 2.2. We assume that permission probability $p_v(u)$ is continuous, bounded ($0 \leq p_v(u) \leq 1$), and strictly decreasing in u ($u \in [0, +\infty)$). Furthermore, there exists a positive constant u_{max} such that $p_v(u) = 0$ when $u \geq u_{max}$.

Based on update equation (2.20), if time slot $n-1$ is reserved and its reservation is kept, control signal u is unchanged. However, if reservation of time slot $n-1$ is lost, control signal u is increased by ϕ and as a result, permission probability is decreased. In the case that time slot $n-1$ is not reserved, if there is no packet transmission at time slot $n-1$ ($Z_{n-1} = 0$), control signal decreases and as a result, permission probability is increased. If there is a collision at time slot $n-1$ ($Z_{n-1} \geq 2$) or a successful transmission by a terminal in HIN state ($Z_{n-1} = 1 \wedge by\ a\ HIN$), control signal is increased by ξ and therefore, permission probability is decreased. For a successful packet transmission at time slot $n-1$ by a contending/backlogged terminal ($Z_{n-1} = 1 \wedge by\ a\ CON$), depending on value of β, permission probability is either increased or decreased.

To model the PRMA-HS protocol with General Price Based Control, choosing the right system state is important. The state of the system is chosen to be $X_n = (Y_{n-1}, Y_n)$, here $Y_n = (c_n, r_n, h_{v_n}, u_n)$ and

$$c \in \{0, 1, 2, \cdots, M_v\}, r \in \{0, 1, 2, \cdots, N\}, h_v \in \{0, 1\},$$

$$u \in \Gamma = \{\min(u_{MAX}, [f\phi - a\alpha + e\beta + d\xi]^+) | f, a, e, d \in \mathbb{Z}_+\}.$$

Here $u_{MAX} = u_{max} + \max(N\phi + \beta, \xi)$. Then state space $\aleph$ is a subset of

$$(\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, N\} \times \{0, 1\} \times \Gamma)^2,$$

with following constraints. If $((c, r, h_v, u), (c_1, r_1, h_{v_1}, u_1)) \in \aleph$

- $c + r + h_v \leq M_v$ and $c_1 + r_1 + h_{v_1} \leq M_v$
- $r + h_v \leq N$ and $r_1 + h_{v_1} \leq N$
- $c - 1 \leq c_1 \leq M_v - r - h_v$
- $r - 1 \leq r_1 \leq r + 1$
- For $h_v = 1$, control signal $u_1 \geq [\beta]^+$
- For $r = N$, control signal $u \geq [\beta]^+$ and $u_1 \geq [\beta]^+$

State space is countable and transition probabilities are written as follows:

$$Pr(X_{n+1} = X_1 | X_n = X) = \tag{2.21}$$

- $V(c_1, c, r, h_v)(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f(\frac{r+h_v}{N} + \Theta + \Theta^{nc})$

  if $c \le c_1 \le M_v - h_v - r, r_1 = r - 1, h_{v_1} = h_v, u_1 = u + \phi$

- $V(c_1 + 1, c, r, h_v)(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f \Theta^c$

  if $c - 1 \le c_1 \le M_v - h_v - r - 1, r_1 = r - 1, h_{v_1} = h_v + 1, u_1 = u + \phi$

- $V(c_1, c, r, h_v)(\frac{r_{-1}+h_{v_{-1}}}{N})(1 - \gamma_f)(\frac{r+h_v}{N} + \Theta + \Theta^{nc})$

  if $c \le c_1 \le M_v - h_v - r, r_1 = r, h_{v_1} = h_v, u_1 = u$

- $V(c_1 + 1, c, r, h_v)(\frac{r_{-1}+h_{v_{-1}}}{N})(1 - \gamma_f)\Theta^c$

  if $c - 1 \le c_1 \le M_v - h_v - r - 1, r_1 = r, h_{v_1} = h_v + 1, u_1 = u$

- $V(c_1, c, r, h_v)\Theta^{nc}_{-1}(\frac{r+h_v}{N} + \Theta + \Theta^{nc})$

  if $c \le c_1 \le M_v - h_v - r, r_1 = r, h_{v_1} = h_v, u_1 = [u - \alpha]^+$

- $V(c_1 + 1, c, r, h_v)\Theta^{nc}_{-1}\Theta^c$

  if $c - 1 \le c_1 \le M_v - h_v - r - 1, r_1 = r, h_{v_1} = h_v + 1, u_1 = [u - \alpha]^+$

- $V(c_1, c, r, h_v)\Theta^{c}_{-1}(\frac{r+h_v}{N} + \Theta + \Theta^{nc})$

  if $c \le c_1 \le M_v - h_v - r, r_1 = r + 1, h_{v_1} = h_v - 1, u_1 = [u + \beta]^+$

- $V(c_1 + 1, c, r, h_v)\Theta^{c}_{-1}\Theta^c$

  if $c - 1 \le c_1 \le M_v - h_v - r - 1, r_1 = r + 1, h_{v_1} = h_v, u_1 = [u + \beta]^+$

- $V(c_1, c, r, h_v)\Theta_{-1}(\frac{r+h_v}{N} + \Theta + \Theta^{nc})$

  if $c \le c_1 \le M_v - h_v - r, r_1 = r, h_{v_1} = h_v, u_1 = u + \xi$

- $V(c_1+1,c,r,h_v)\Theta_{-1}\Theta^c$

  if $c-1 \le c_1 \le M_v - h_v - r - 1, r_1 = r, h_{v_1} = h_v + 1, u_1 = u + \xi$

- 0 Otherwise

here

$$X_1 = ((c,r,h_v,u),(c_1,r_1,h_{v_1},u_1)),$$

$$X = ((c_{-1},r_{-1},h_{v_{-1}},u_{-1}),(c,r,h_v,u)),$$

$$V(c_1,c,r,h_v) = \begin{pmatrix} M_v - c - r - h_v \\ \\ c_1 - c \end{pmatrix} \sigma_v^{c_1-c}(1-\sigma_v)^{M_v-c_1-r-h_v},$$

$$\Theta^c = (1-\frac{r+h_v}{N})cp_v(1-p_v)^{c+h_v-1},$$

$$\Theta^{nc} = (1-\frac{r+h_v}{N})(1-p_v)^{c+h_v},$$

$$\Theta = (1-\frac{r+h_v}{N}) - \Theta^{nc} - \Theta^c,$$

$$\Theta^c_{-1} = (1-\frac{r_{-1}+h_{v_{-1}}}{N})c_{-1}p_{v_{-1}}(1-p_{v_{-1}})^{c_{-1}+h_{v_{-1}}-1},$$

$$\Theta^{nc}_{-1} = (1-\frac{r_{-1}+h_{v_{-1}}}{N})(1-p_{v_{-1}})^{c_{-1}+h_{v_{-1}}},$$

$$\Theta_{-1} = (1-\frac{r_{-1}+h_{v_{-1}}}{N}) - \Theta^{nc}_{-1} - \Theta^c_{-1},$$

$$p_v = p_v(u), \quad p_{v_{-1}} = p_v(u_{-1}).$$

2.8 Price Based Control

In this section we study a special case of General Price Based Control assuming $\phi = 0$. Therefore, control signal is only updated during an available time slot.

Proposition 2.6. *The Markov chain defined on $\aleph$ through (2.21) (considering $\phi = 0$) is irreducible, aperiodic, and positive recurrent.*

Detailed proof is presented in the Appendix C. Because of large size of the state space, studying the PRMA-HS system with priced base control using transition probabilities is very difficult. Therefore, stationary behavior of the system is analyzed using equilibrium point analysis (EPA). In EPA it is assumed that the system is in equilibrium, therefore, any change in any state is zero. One-step expected change (mean drift) of the control signal at state X is defined as:

$$d(X) = E(u_{n+1} - u_n | X_n = X).$$

Using definition of control signal in (2.20) (assuming $\phi = 0$), expected drift is determined as:

$$\begin{aligned} d(X) =& (\max(-\alpha, -u) - \xi)(1 - \frac{r_{-1} + h_{v_{-1}}}{N})(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}} \\ &+ (\max(\beta, -u) - \xi)(1 - \frac{r_{-1} + h_{v_{-1}}}{N})c_{-1}p_{v_{-1}}(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}-1} \\ &+ \xi(1 - \frac{r_{-1} + h_{v_{-1}}}{N}) \end{aligned}$$

Or relaxed drift equation is:

$$\begin{aligned} d_r(X) =& -(\alpha + \xi)(1 - \frac{r_{-1} + h_{v_{-1}}}{N})(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}} \\ &+ (\beta + \xi)(1 - \frac{r_{-1} + h_{v_{-1}}}{N})[c_{-1}p_{v_{-1}}(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}-1} \\ &+ \xi(1 - \frac{r_{-1} + h_{v_{-1}}}{N}) \end{aligned} \tag{2.22}$$

As discussed in chapter one with respect to equilibrium equation of the PRMA-HS system, note that at equilibrium, the number of voice terminals in each state

HIN_i is $\frac{Hv}{(N/d)}$ for $i = N-1, \cdots, N-N/d$. In the same way, the number of terminals, at equilibrium, in state $RES_i^{'}$ is $\frac{R'}{N-(N/d)}$ for $i = N - N/d - 1, \cdots, 0$. It is easy to show that

$$\frac{H_v}{N/d} = \frac{R'}{N - N/d}.$$

Also, the number of voice terminals in each state RES_i is $\frac{R}{N}$ for $i = 0, \cdots, N-1$. We define $R^* = R + R'$ as the number of voice terminals in RES and $RES^{'}$. Notice that

$$\frac{R^* + Hv}{N} = \frac{R}{N} + \frac{R'}{N - N/d}.$$

A point (C, R^*, H_v, U) is called an equilibrium point, if

$$\begin{aligned}
&(M_v - C - R^* - H_v)\sigma_v - (1 - \frac{R^* + H_v}{N})Cp_v w_v(C) = 0,\\
&(\frac{R^* + H_v}{N})\gamma_f - (M_v - C - R^* - H_v)\sigma_v = 0,\\
&(\frac{R^* + H_v}{N})\gamma_f - (\frac{d}{N})H_v = 0,\\
&(\alpha + \xi)(1 - p_v)^{C+H_v} - (\beta - \xi)Cp_v w_v(C) - \xi = 0.
\end{aligned} \tag{2.23}$$

Here $w_v(C) = \begin{cases} (1 - p_v(U))^{C+H_v-1} & C \geq 1 \\ (1 - p_v(U))^{H_v} & C < 1 \end{cases}$. Also, it is noted that for the equilibrium point analysis we consider a round trip delay of N/d time slots. Set of equations (2.24) modeling equilibrium equations of the system can be simplified as:

$$\begin{aligned}
F_1(C, U) =& M_v - C - (\frac{1}{\omega})(\frac{Cp_v w_v(C)}{\gamma_f + Cp_v w_v(C)}) = 0,\\
F_2(C, U) =& (\alpha + \xi)(1 - p_v)^{C+H_v} - (\beta - \xi)Cp_v w_v(C) - \xi = 0.
\end{aligned} \tag{2.24}$$

We analyze conditions on the control parameters (α, β, ξ) such that equilibrium equations (2.24) have a unique operating point and in lemma below we present two conditions for uniqueness of operating points of the system.

Remark 2.5. *Based on the first two equations of the set of equations (2.23), it can be shown that* $R^* + H_v = \min(N, N\omega(M_v - C))$*, where* $\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}$*. Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$*,* $R^* + H_v \in [0, N]$*, and* $p_v \in [0, 1]$*. However, it can easily be shown, using the set of equation (2.23), that* $C = 0$*,* $M_v - \frac{1}{\omega}$*,* $C = M_v$*,* $R^* + H_v = 0$*,* $R^* + H_v = N$*,* $p_v = 0$*,* $p_v = 1$ *(for* $\beta = \xi$ *as considered in Lemma 2.7), or a combination thereof, cannot be solutions to the set of equilibrium equations (2.23).*

Lemma 2.7. *We define* $f^+(C) = \frac{\xi}{\alpha+\xi} C(\exp(\frac{-\ln(\frac{\xi}{\alpha+\xi})}{C+H_v}) - 1)$*. Here* $H_v = \gamma_f \omega(\frac{N}{d})(M_v - C)$*. There exists a set of control parameters* (α, β, ξ) *for which the set of equations (2.24) has a unique solution in* (C, U) *and the system has a single operating point if*

$$\frac{1}{\ln(\frac{\xi}{\alpha+\xi})}(\exp(\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v}) - 1) < \frac{1 - (N/d)\gamma_f \omega}{M_v}, \quad \textit{and}$$

$$\ln(\frac{\xi}{\alpha+\xi}) < \frac{-2\gamma_f(N/d)\omega M_v(1 + \gamma_f(N/d)\omega(M_v - 1))}{1 - \gamma_f(N/d)\omega}, \quad \textit{and}$$

any of conditions (1a), (1b), *or* (2) *below hold:*

(1*a*) $\quad Mv \geq 1 + \frac{1}{\omega}$ *and* $-\gamma_f \omega < \frac{df^+}{dC}(M_v - \frac{1}{\omega})$,

(1*b*) $\quad Mv \leq 1 + \frac{1}{\omega}$ *and either*

$$-\frac{(M_v - 1)\omega\gamma_f}{1 - (M_v - 1)\omega} \leq \frac{\xi}{\alpha+\xi} M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1), \quad \textit{or}$$

$$-\gamma_f \omega < \frac{df^+}{dC}(1),$$

(2) $$\frac{\xi}{\alpha+\xi}[(1 + \frac{(1 - \gamma_f(N/d)\omega)\ln(\frac{\xi}{\alpha+\xi})}{(1 + \hat{H}_v)^2})\exp(-\frac{1}{1 + \hat{H}_v}\ln(\frac{\xi}{\alpha+\xi})) - 1] <$$

$$\frac{\omega}{\gamma_f}[\frac{\xi}{\alpha+\xi} M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1) + \gamma_f]^2,$$

here, $\hat{H}_v = \gamma_f(N/d)\omega(Mv-1)$.

Proof. We will show the existence of the control parameters (α, β, ξ) with $\beta = \xi$ such that the conditions hold. First, we prove the lemma for conditions $(1a)$ and $(1b)$. Let us define the following new functions

$$f(C,U) = Cp_v w_v(C), \quad g(C) = \frac{(M_v - C)\omega\gamma_f}{1-(M_v-C)\omega}.$$

Equilibrium point(s) of equation (2.24) are the same as fixed point(s) of $f(C,U) = g(C)$. Solving $F_2(C,U) = 0$ in equation (2.24) for p_v and substituting it in $f(C,U)$, we will have

$$f(C) = \begin{cases} \frac{\xi}{\alpha+\xi}C(\exp(\frac{-\ln(\frac{\xi}{\alpha+\xi})}{C+H_v}) - 1) & C \geq 1 \\ C(1-\exp(\frac{\ln(\frac{\xi}{\alpha+\xi})}{C+H_v}))\exp(\frac{H_v\ln(\frac{\xi}{\alpha+\xi})}{C+H_v}) & C < 1 \end{cases} \tag{2.25}$$

It is easy to show that if the following two conditions are satisfied then $\frac{df}{dC} < 0$ and $\frac{d^2f}{dC^2} > 0$, respectively, in the range of $1 \leq C < M_v$.

$$\frac{1}{\ln(\frac{\xi}{\alpha+\xi})}(\exp(\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v}) - 1) < \frac{1-(N/d)\gamma_f\omega}{M_v} \tag{2.26}$$

$$\ln(\frac{\xi}{\alpha+\xi}) < \frac{-2\gamma_f(N/d)\omega M_v(1+\gamma_f(N/d)\omega(M_v-1))}{1-\gamma_f(N/d)\omega} \tag{2.27}$$

Also, we can numerically show that $\frac{df}{dC} > 0$ in the range of $0 < C < 1$. We define $b = \gamma_f(N/d)\omega$ and $a = \frac{\xi}{\alpha+\xi}$. It can be noticed that $0 < a, b < 1$. We can show that for the values of $0 < a < 1$ and $0 < b < 1$ and $0 < M_v < 201$, $\frac{df}{dC} > 0$ for $C < 1$.

Also, note that $\max(0, M_v - \frac{1}{\omega}) < C < M_v$. In this range of C, we can show that $\frac{dg}{dC} < 0$ and $\frac{d^2g}{dC^2} > 0$. Also, notice that

$$f(M_v) = \frac{\xi}{\alpha+\xi}M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1) > g(M_v) = 0.$$

Based on these facts in order to prove that $f(C) = g(C)$ has exactly one solution, for the conditions stated in the lemma, we consider different cases:

- **Case 1 -** $M_v > 1 + \frac{1}{\omega}$: In this case, $g(C)$ is strictly decreasing with positive second derivative. Also, if the above-noted conditions for first and second derivatives of f are satisfied, then $f(C)$ is strictly decreasing, with positive second derivative, $f(M_v - \frac{1}{\omega}) < g((M_v - \frac{1}{\omega})^+)$, and $f(M_v) > g(M_v)$. Since second derivative of both $f(C)$ and $g(C)$ are positive, if we chose control parameters such that $\frac{dg}{dC}(M_v) < \frac{df}{dC}(M_v - \frac{1}{\omega})$, then we can show that, for the desired range of C, $\frac{dg}{dC} < \frac{df}{dC}$. Therefore, $g(C) = f(C)$ has a unique solution.

- **Case 2 -** $M_v < 1 + \frac{1}{\omega}$: In this case, if the above-noted conditions for first and second derivatives of f are satisfied, then $f(C)$ is strictly decreasing, with positive second derivative for $1 < C < M_v$ and f is strictly increasing for $\max(0, M_v - \frac{1}{\omega}) < C \leq 1$. Therefore, the control parameters are chosen such that either $\frac{dg}{dC}(M_v) < \frac{df}{dC}(1)$ or $g(1) < f(M_v)$.

Now we prove the result assuming condition (2) holds. Given $\beta = \xi$, we simplify the set of two equations (2.24) as the following single equation

$$h(C) = -M_v + C + \frac{1}{\omega}\frac{f(C)}{\gamma_f + f(C)} = 0. \tag{2.28}$$

Here $f(C)$ is the same as defined earlier in the proof. It can easily be shown using equation (2.25) that $h(0) = -M_v$, $h(M_v - \frac{1}{\omega}) < 0$, and $h(M_v) > 0$. The first derivative of $h(C)$

$$h'(C) = 1 + \frac{\gamma_f}{\omega}\frac{f'(C)}{(\gamma_f + f(C))^2},$$

where $f'(C) = \frac{df}{dC}$ and $h'(C) = \frac{dh}{dC}$. We showed earlier in the proof that $f'(C) > 0$ for $C < 1$, and therefore, $h'(C) > 0$ for $C < 1$. So $h(C)$ is negative for $C = \max(0, M_v - \frac{1}{\omega})$, is positive for $C = M_v$, and has a positive slope for $C < 1$. Next we show that the slope is also negative for $1 \geq C < M_v$, which will result in a unique solution to $h(C) = 0$ in the interval of $\max(0, M_v - \frac{1}{\omega}) < C < M_v$. Consider the case $1 \leq C < M_v$ and we will show that if condition (2) is satisfied, then $h'(C) > 0$. Let us define $h_1(C) = -f'(C)$ and $h_2(C) = \frac{\omega}{\gamma_f}(\gamma_f + f(C))^2$.

Under conditions set in this lemma, it is noted that $h_1(C) > 0$ and $h_1'(C) < 0$ for $C \geq 1$. Therefore, $h_1(M_v) \leq h_1(C) \leq h_1(1^+)$.

It can also be shown that $h_2(C) > 0$ and $h_2'(C) < 0$ for $C \geq 1$. Therefore, $h_2(M_v) \leq h_2(C) \leq h_2(1^+)$.

Hence, if the control parameters are chosen such that $h_1(1^+) < h_2(M_v)$ (in other words, $-f'(1^+) < \frac{\omega}{\gamma_f}(\gamma_f + f(M_v))^2$), then $h'(C) > 0$. Therefore, $h(C) = 0$ has a unique solution in the interval of $0 \leq C \leq M_v$.

It can easily be shown, using equation $F_2(C, U)$, that p_v is one-to-one function of C. Also, as mentioned before, p_v is a one-to-one function of U (for $0 < p_v < 1$). Therefore, it can easily be shown that for a given C, there exists a unique U. Therefore, under conditions stated in the lemma, the set of equations (2.24) or equation (2.28) has a unique solution in $0 < C < M_v$ and $0 < U < u_{max}$. □

Designing the Control: In order to ensure that the controlled system has a unique operating point, we select the control parameters based on Lemma 2.7. We choose $\beta = \xi$ as arbitrary positive real number. Then α is chosen such that other conditions of this lemma is satisfied.

Also, in order to be able to use the relaxed form of the expected drift (equation (3.3)), we make a small change in assumption 2.2. Permission probability $p_v(u)$ is continuous and bounded ($0 \leq p_v(u) \leq 1$). But we assume that $p_v(u) = 1$ for $u \in [0, \max(\alpha, -\beta)]$, strictly decreasing for $u \in (\max(\alpha, -\beta), u_{max})$, and $p_v(u) = 0$ for $u \in [u_{max}, +\infty)$. In this case, since equilibrium value of permission probability is less than 1, equilibrium value of the control signal u will be greater than both α and $-\beta$ and therefore, relaxed expected drift equation at the equilibrium could be used.

2.9 Control Using State Estimation

Next, we consider a control scheme based on the state estimation control we introduced in previous sections. As discussed before, the number of voice terminals with reservation is the only state of the system that is known to all terminals in the system. Therefore, we choose the control signal $u_n = N - r_n$. Assuming that round trip delay is 1 time slot, dynamics of control signal can be written as following

$$u_{n+1} = \begin{cases} u_n & \text{if slot } n-1 \text{ is reserved and reservation is kept,} \\ u_n + 1 & \text{if slot } n-1 \text{ is reserved and reservation is lost,} \\ [u_n - I(Z_{n-1} = 1) \;\; by \;\; a \;\; CON \;\; VT]^+ & \text{if slot } n-1 \text{ is not reserved.} \end{cases}$$

Later in this section, we define permission probability as a function of the control signal to maximize throughput of the system or minimize packet drop probability. It is noted that control using state estimation is a special case of General

Price Based Control with $\phi = 1$, $\alpha = \xi = 0$, and $\beta = -1$. Therefore, as discussed before in analysis of General Price Based Control, the PRMA-HS Voice system with state estimation can be modeled by a Markov chain.

Proposition 2.7. *The Markov chain defined on $\aleph$ through (2.21) (for $\phi = 1$, $\alpha = \xi = 0$, $\beta = -1$, and considering the system only employs voice terminals) is irreducible, aperiodic, and positive recurrent.*

Proof of Proposition 2.7 is presented in the Appendix D. Next, we consider Equilibrium Point Analysis to study equilibrium behavior of the system. We show if some conditions on system parameters are met, the system with state estimation control scheme has unique equilibrium point. First, permission probability is chosen as a function of control signal such that average throughput of the system is maximized. Second, we choose permission probability as a function of control signal such that average packet drop probability is minimized.

Maximizing Throughput - EPA: Average throughput is the number of time slots that carry one packet: $\eta = \frac{R^* + H_v}{N}$. By using equation (2.23) and definition of the average throughput at equilibrium:

$$\eta = \frac{R^* + H_v}{N}(1 - \gamma_f) + (1 - \frac{R^* + H_v}{N})Cp_v w_v(C).$$

Depending on equilibrium value of C, maximum throughput at equilibrium happens at:

$$p_v^* = \begin{cases} \frac{1}{C+H_v} & C \geq 1, \\ \frac{1}{1+H_v} & C < 1. \end{cases}$$

Given $C = M_v - \frac{d}{N\omega(d-\gamma_f)}R^*$ and $H_v = \frac{\gamma_f}{d-\gamma_f}R^*$, equilibrium equation of the controlled system can be written as:

$$\frac{(M_v - C)\omega\gamma_f}{1-(M_v-C)\omega} - W(C) = 0, \tag{2.29}$$

$$W(C) = \begin{cases} \frac{C}{C+H_v}(1-\frac{1}{C+H_v})^{C+H_v-1} & C \geq 1, \\ \frac{C}{1+H_v}(1-\frac{1}{1+H_v})^{H_v} & C < 1, \end{cases}$$

and

$$\omega = \frac{\sigma_v}{N\sigma_v + \gamma_f}, \qquad H_v = \gamma_f\omega(N/d)(M_v - C).$$

Remark 2.6. *Equation (2.29) is derived with consideration that* $R^* + H_v = \min(N, N\omega(M_v - C))$*. Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$ *and* $R \in [0, N]$*. However, it can easily be shown that* $C = 0$, $M_v - \frac{1}{\omega}$, $C = M_v$, $R^* + H_v = 0$, $R^* + H_v = N$, *or a combination thereof, cannot be solutions to equilibrium equation (2.29).*

Lemma 2.8. *Let us define* $f^+(C) = (1-\frac{1}{C+H_v})^{C+H_v-1}$*. Here* $H_v = \gamma_f\omega(N/d)(M_v - C)$*. Also, we define* $\tilde{C} = M_v - \frac{1}{\omega}$*. Set of equations (2.29) has a unique solution in* C *and the system has a single operating point if any of conditions* (1) *or* (2) *below hold:*

(1) $M_v > \frac{1}{\omega} + 1$ *and* $-\gamma_f\omega < \frac{df^+}{dC}(\tilde{C})$,

(2) $M_v < \frac{1}{\omega} + 1$ *and either*

$$-\gamma_f\omega < \frac{df^+}{dC}(1) \quad \text{or} \quad \frac{(M_v-1)\omega\gamma_f}{1-(M_v-1)\omega}(1+\hat{H}_v) < (1-\frac{1}{M_v})^{M_v-1}.$$

Here $\hat{H}_v = \gamma_f\omega(N/d)(M_v - 1)$.

Proof. We can rewrite set of equations (2.29) as following

$$(\frac{(M_v - C)\omega\gamma_f}{1-(M_v - C)\omega})(\frac{\max(1,C)+H_v}{C}) - f(C) = 0.$$

Here

$$f(C) = \begin{cases} (1-\frac{1}{C+H_v})^{C+H_v-1} & C \geq 1, \\ (1-\frac{1}{1+H_v})^{H_v} & C < 1, \end{cases}$$

and we define the following new functions

$$h(C) = \frac{\max(1,C)+H_v}{C}, \quad g(C) = \frac{(M_v - C)\omega\gamma_f}{1-(M_v - C)\omega}.$$

Therefore, it is noted that solutions of set of equations (2.29) is the same as roots of $g(C)h(C) = f(C)$. It is easy to show that:

$$\begin{cases} \frac{df}{dC} < 0 & C \geq 1, \\ \frac{df}{dC} > 0 & C < 1. \end{cases} \qquad \begin{cases} \frac{d^2 f}{dC^2} > 0 & C \geq 1, \\ \frac{d^2 f}{dC^2} > 0 & C < 1. \end{cases}$$

Also, $\frac{d(g(C)h(C))}{dC} < 0$ and $\frac{d^2(g(C)h(C))}{dC^2} > 0$. Note that $\max(0, M_v - \frac{1}{\omega}) < C < M_v$. Finally,

$$f(M_v) = (1-\frac{1}{M_v})^{M_v - 1} > g(M_v)h(M_v) = 0.$$

Based on these facts and in order to prove that $g(C)h(C) = f(C)$ has exactly one solution, for the conditions stated in the lemma, we consider different cases:

- **Case 1 -** $M_v > 1 + \frac{1}{\omega}$: In this case $g(C)h(C)$ is positive, strictly decreasing, and with positive second derivative. Also, $f(C)$ is positive, strictly decreasing, with positive second derivative, $g((M_v - \frac{1}{\omega})^+)h((M_v - \frac{1}{\omega})^+) > f(M_v - \frac{1}{\omega})$, and $f(M_v) > g(M_v)$. Since second derivative of both $f(C)$ and $g(C)h(C)$ are positive, if the system parameters are chosen such that $\frac{d(gh)}{dC}(M_v) < \frac{df}{dC}(\tilde{C})$, then $\frac{d(g(C)h(C))}{dC} < \frac{df(C)}{dC}$ for the desired range of C and therefore, $g(C)h(C) = f(C)$ has a unique solution.

- **Case 2 -** $M_v < 1 + \frac{1}{\omega}$: In this case, if system parameters are chosen such that either $\frac{d(gh)}{dC}(M_v) < \frac{df}{dC}(1)$ or $g(1)h(1) < f(M_v)$ it is guaranteed that the system has a unique operating point.

$\square$

Minimizing Packet Drop Probability - EPA: Average packet drop probability at equilibrium is

$$P_{drop} = \gamma_f \frac{\nu^D}{1-(1-\gamma_f)\nu^N},$$

here $\nu = \nu(C, R^*, H_v, p_v) = 1 - (1 - \frac{R^*+H_v}{N})p_v(1-p_v)^{C+H_v}$. It can be shown that packet drop probability is minimized if

$$p_v^* = \frac{1}{C + H_v + 1}$$

Given $C = M_v - \frac{d}{N\omega(d-\gamma_f)}R^*$ and $H_v = \frac{\gamma_f}{d-\gamma_f}R^*$, equilibrium equation of the controlled system can be written as:

$$\frac{(M_v - C)\omega\gamma_f}{1-(M_v-C)\omega} - W(C) = 0, \tag{2.30}$$

$$W(C) = \begin{cases} \frac{C}{C+H_v+1}(1 - \frac{1}{C+H_v+1})^{C+H_v-1} & C \geq 1, \\ \frac{C}{C+H_v+1}(1 - \frac{1}{C+H_v+1})^{H_v} & C < 1. \end{cases}$$

and

$$\omega = \frac{\sigma_v}{N\sigma_v + \gamma_f}, \qquad H_v = \gamma_f\omega(N/d)(M_v - C).$$

Remark 2.7. *Equation (2.30) is derived with consideration that* $R^* + H_v = \min(N, N\omega(M_v - C))$. *Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$*and* $R \in [0, N]$. *However, it can easily be shown that* $C = 0$, $M_v - \frac{1}{\omega}$, $C = M_v$, $R^* + H_v = 0$, $R^+H_v = N$, *or a combination thereof, cannot be solutions to equilibrium equation (2.30).*

Lemma 2.9. *Let us define* $f^+(C) = (1 - \frac{1}{C+H_v+1})^{C+H_v-1}$. *Here* $H_v = \gamma_f \omega (N/d)(M_v - C)$. *Also, we define* $\tilde{C} = M_v - \frac{1}{\omega}$. *Set of equations (2.30) has a unique solution in* C *and the system has a single operating point if any of conditions* (1) *or* (2) *below:*

(1) $\quad M_v > \frac{1}{\omega} + 1 : -\gamma_f \omega < \frac{df^+}{dC}(\tilde{C})$,

(2) $\quad M_v < \frac{1}{\omega} + 1$ *and either*

$$-\gamma_f \omega < \frac{df^+}{dC}(1) \quad or \quad \frac{(M_v - 1)\omega\gamma_f}{1 - (M_v - 1)\omega}(2 + \hat{H}_v) < (1 - \frac{1}{M_v + 1})^{M_v - 1}.$$

Here $\hat{H}_v = \gamma_f \omega (N/d)(M_v - 1)$.

Proof of Lemma 2.9 is very similar to proof of Lemma 2.8.

2.10 Performance Analysis of PRMA-HS Voice Only system over Random Packet Error Channel

In previous sections, we studied a PRMA-HS system employing voice terminals on an uplink channel without any error. In this section, we assume that uplink channel is a "random packet error channel" [25] and we study the PRMA-HS system with the General Price Based Control. The analysis of this section closely follows study of PRMA system with General Price Based Control over random error channel.

2.10.1 General Price Based Control

Figures 3.1 and 3.2 of chapter 3 illustrate general Markov models for the PRMA-HS system over random packet error channel (employing both voice and data terminals). In this section, we assume that when a contending voice terminal transmits a packet on an available time slot without any collision, it reserves that time slot if no packet header error happens. If there is a header packet error, access point (such as base station and/or satellite) interprets the error either as a collision or as an event that no packet was transmitted [25]. If there is a packet header error in a contending packet, we assume that with a fixed probability q the base station sends a collision feedback and with probability $1-q$ it sends an idle feedback. We define following events: $A_n = \{$at an available time slot $n, 1$ packet transmitted with error - access point assumed idle$\}$ and $B_n = \{$at an available time slot $n, 1$ packet transmitted with error - access point assumed collision$\}$.

Therefore, control signal update algorithm (2.20) is changed as follows. As in pervious sections, and only for notation purposes, we assume that $d = N$ and therefore, round trip delay is one time slot.

$$u_{n+1} = \begin{cases} u_n & \text{if slot } n-1 \text{ is reserved and reservation is kept,} \\ u_n + \phi & \text{if slot } n-1 \text{ is reserved and reservation is lost,} \\ [u_n - \alpha I(Z_{n-1} = 0 \vee A_{n-1}) + \beta I(Z_{n-1} = 1 \wedge no\ error) & \\ \quad + \xi I(Z_{n-1} \geq 2 \vee B_{n-1}]^+ & \text{if slot } n-1 \text{ is not reserved.} \end{cases} \tag{2.31}$$

Note that in random packet error channel, control signal at time slot $n+1$

is decreased by α if time slot $n-1$ was either idle or a successful transmission by a contending or hindering terminal contained a packet header error and the access point assumed an idle time slot. Also, control signal is increased by ξ if either time slot $n-1$ had a collision or a packet header error in a successful transmission by a contending or hindering terminal, which was assumed to be collision by the access point. In the case of a successful transmission with no packet header error, control signal is updated by β. If time slot $n-1$ was reserved and the reservation is lost (either because of packet header error or because all packet of voice message are transmitted) control signal is increased by ϕ.

Transition probabilities for this Markov chain can be written as before considering packet random error. It is noted that the Markov chain modeling the PRMA-HS system over random packet error channel has the same state space $\aleph$ as PRMA-HS system.

2.10.2 Price Based Control

An special case of General Price Based Control is for $\phi = 0$. Transition probabilities for this Markov chain can be written as before considering packet random error. It is noted that the Markov chain modeling the PRMA-HS system over random packet error channel has the same state space $\aleph$ as PRMA-HS system. We can prove that this Markov model is irreducible, aperiodic, and under Assumption 2.2, the Markov chain is positive recurrent.

Next, we study performance of the PRMA-HS system with price based control scheme over random packet error channel using the equilibrium point approach. Same analysis as in previous section can be used. A point (C, R^*, H_v, U) is an equilibrium point of the system if:

$$
\begin{aligned}
&(M_v - C - R^* - H_v)\sigma_v + (\frac{R^* + H_v}{N})(1-\gamma_f)\Delta \\
&\qquad\qquad\qquad - (1 - \frac{R^* + H_v}{N})Cp_v w_v(C)(1-\Delta) = 0, \\
&(\frac{R^* + H_v}{N})\gamma_f - (M_v - C - R^* - H_v)\sigma_v = 0, \\
&(R/N) - (\frac{R^* + H_v}{N})(1-\gamma_f)(1-\Delta) = 0, \\
&(-\alpha(1-q)\Delta + \beta(1-\Delta) - \xi(1-q\Delta))(C + H_v)p_v w_v(C) \\
&\qquad\qquad\qquad - (\alpha+\xi)(1-p_v)^{C+H_v} + \xi = 0.
\end{aligned} \tag{2.32}
$$

Here $p_v = p_v(U)$, $R^* = R + R'$, and $\frac{R'}{N - N/d} = \frac{H_v}{N/d}$. Next, we derive a set of sufficient conditions on the control parameters such that this system has a unique equilibrium point.

Lemma 2.10. *We define* $f^+(C) = \frac{\xi}{\alpha+\xi}C(\exp(\frac{-\ln(\frac{\xi}{\alpha+\xi})}{C+H_v})-1)$. *Here* $H_v = \gamma_f\omega(\frac{N}{d})(M_v - C)$. *There exists a set of control parameters* (α, β, ξ) *for which the set of equations (2.32) has a unique solution in* (C, U) *and the system has a single operating point if any of conditions* $(1a)$, $(1b)$, *or* (2) *below:*

$(1a)$ $\quad Mv \geq 1 + \frac{1}{\omega}$ *and*

$$
\begin{aligned}
&\frac{1}{\ln(\frac{\xi}{\alpha+\xi})}(\exp(\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v}) - 1) < \frac{1 - (N/d)\gamma_f\omega}{M_v}, \quad \textit{and} \\
&\ln(\frac{\xi}{\alpha+\xi}) < \frac{-2\gamma_f(N/d)\omega M_v(1 + \gamma_f(N/d)\omega(M_v - 1))}{1 - \gamma_f(N/d)\omega}, \quad \textit{and} \\
&\frac{-\omega\gamma_f'}{1-\Delta} < \frac{df^+}{dC}(\tilde{C}),
\end{aligned}
$$

(1b) $\quad Mv \le 1 + \frac{1}{\omega}$ *and*

$$\frac{1}{\ln(\frac{\xi}{\alpha+\xi})}(\exp(\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v}) - 1) < \frac{1-(N/d)\gamma_f\omega}{M_v}, \quad and$$

$$\ln(\frac{\xi}{\alpha+\xi}) < \frac{-2\gamma_f(N/d)\omega M_v(1+\gamma_f(N/d)\omega(M_v-1))}{1-\gamma_f(N/d)\omega}, \quad and \quad either$$

$$\frac{(M_v-1)\omega\gamma_f}{1-(M_v-1)\omega} \le \frac{\xi}{\alpha+\xi}M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1), \quad or$$

$$\frac{-\omega\gamma_f'}{1-\Delta} < \frac{df^+}{dC}(1),$$

here, $\tilde{C} = M_v - \frac{1}{\omega}$,

$$(2) \qquad \frac{1}{\ln(\frac{\xi}{\alpha+\xi})}(\exp(\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v}) - 1) < \frac{1-(N/d)\gamma_f\omega}{M_v}, \quad and$$

$$\ln(\frac{\xi}{\alpha+\xi}) < \frac{-2\gamma_f(N/d)\omega M_v(1+\gamma_f(N/d)\omega(M_v-1))}{1-\gamma_f(N/d)\omega}, \quad and$$

$$\frac{\xi}{\alpha+\xi}[(1+\frac{(1-\gamma_f(N/d)\omega)\ln(\frac{\xi}{\alpha+\xi})}{(1+\tilde{H}_v)^2})\exp(-\frac{1}{1+\tilde{H}_v}\ln(\frac{\xi}{\alpha+\xi})) - 1] <$$

$$\frac{\omega}{\gamma_f'(1-\Delta)}[(1-\Delta)\frac{\xi}{\alpha+\xi}M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1) + \gamma_f']^2,$$

here, $\tilde{H}_v = \gamma_f(N/d)\omega(Mv-1)$ *and* $\gamma_f' = \gamma_f + (1-\gamma_f)\Delta$.

Proof of Lemma 2.10 and exitance of the control parameters (α, β, ξ) with $\beta = \frac{\alpha(1-q)\Delta + \xi(1-q\Delta)}{1-\Delta}$ such that the conditions hold is shown similar to proof of Lemma 2.7.

2.10.3 State Estimation-Based Control

In this subsection we consider state estimation-based control over random packet error channel, a special case of General Price Based Control for $\phi = 1$, $\alpha = \xi = 0$, and $\beta = -1$.

Maximizing Throughput - EPA: Depending on equilibrium value of C, permission probability is chosen as following to maximize throughput of the system:

$$p_v^* = \begin{cases} \frac{1}{C+H_v} & C \geq 1, \\ \frac{1}{1+H_v} & C < 1. \end{cases}$$

Equilibrium equations of the system can be written as follows

$$(M_v - C)\omega(\gamma_f + (1-\gamma_f)\Delta) - (1 - (M_v - C)\omega)W_v(C)(1-\Delta) = 0,$$

$$W(C) = \begin{cases} \frac{C}{C+H_v}(1 - \frac{1}{C+H_v})^{C+H_v-1} & C \geq 1, \\ \frac{C}{1+H_v}(1 - \frac{1}{1+H_v})^{H_v} & C < 1. \end{cases} \tag{2.33}$$

Lemma 2.11. *Let us define* $f^+(C) = (1 - \frac{1}{C+H_v})^{C+H_v-1}$. *Here* $H_v = \omega(\gamma_f + (1-\gamma_f)\Delta)(N/d)(M_v - C)$. *Also, we define* $\tilde{C} = M_v - \frac{1}{\omega}$. *Set of equations (2.33) has a unique solution in* C *and the system has a single operating point if any of conditions* (1) *or* (2) *below hold:*

(1) $\quad M_v > \frac{1}{\omega} + 1$ *and* $-\frac{(\gamma_f+(1-\gamma_f)\Delta)\omega}{1-\Delta} < \frac{df^+}{dC}(\tilde{C})$,

(2) $\quad M_v < \frac{1}{\omega} + 1$ *and either*

$$-\frac{(\gamma_f + (1-\gamma_f)\Delta)\omega}{1-\Delta} < \frac{df^+}{dC}(1) \quad or$$
$$\frac{(M_v-1)\omega(\gamma_f + (1-\gamma_f)\Delta)}{(1-(M_v-1)\omega)(1-\Delta)}(1+\hat{H}_v) < (1 - \frac{1}{M_v})^{M_v-1}.$$

Here $\hat{H}_v = \omega(\gamma_f + (1-\gamma_f)\Delta)(N/d)(M_v - 1)$.

Proof is similar to proof of Lemma 2.8.

Minimizing Packet Drop Probability - EPA: Permission probability is chosen as following to minimize packet drop probability of the system:

$$p^* = \frac{1}{C + H_v + 1}.$$

Equilibrium equations of the system can be written as follows

$$(M_v - C)\omega(\gamma_f + (1-\gamma_f)\Delta) - (1 - (M_v - C)\omega)W_v(C)(1-\Delta) = 0,$$
$$W(C) = \begin{cases} \frac{C}{C+H_v+1}(1 - \frac{1}{C+H_v+1})^{C+H_v-1} & C \geq 1, \\ \frac{C}{C+H_v+1}(1 - \frac{1}{C+H_v+1})^{H_v} & C < 1. \end{cases} \quad (2.34)$$

Lemma 2.12. *Let us define* $f^+(C) = (1 - \frac{1}{C+H_v})^{C+H_v-1}$. *Here* $H_v = \omega(\gamma_f + (1-\gamma_f)\Delta)(N/d)(M_v - C)$. *Also, we define* $\tilde{C} = M_v - \frac{1}{\omega}$. *Set of equations (2.34) has a unique solution in* C *and the system has a single operating point if any of conditions* (1) *or* (2) *below hold:*

(1) $M_v > \frac{1}{\omega} + 1$ *and* $-\frac{(\gamma_f+(1-\gamma_f)\Delta)\omega}{1-\Delta} < \frac{df^+}{dC}(\tilde{C})$,

(2) $M_v < \frac{1}{\omega} + 1$ *and either*

$$-\frac{(\gamma_f + (1-\gamma_f)\Delta)\omega}{1-\Delta} < \frac{df^+}{dC}(1) \quad \textit{or}$$
$$\frac{(M_v-1)\omega(\gamma_f + (1-\gamma_f)\Delta)}{(1-(M_v-1)\omega)(1-\Delta)}(2 + \hat{H}_v) < (1 - \frac{1}{M_v+1})^{M_v-1}.$$

Here $\hat{H}_v = \omega(\gamma_f + (1-\gamma_f)\Delta)(N/d)(M_v - 1)$.

Proof is similar to proof of Lemma 2.9.

2.11 Numerical Results

Consider a PRMA-HS voice only system with following parameters [23]: $R_c = 720,000$ bits/s, $R_s = 32,000$ bits/s, $T = 0.016$ s, $H = 64$ bits, $RTD = 1$ time slot, $D_{max} = 0.032$ s, $t_1 = 1.00$ s, and $t_2 = 1.35$ s. Figures 2.21, 2.22, and 2.23 illustrate packet drop probability (P_{drop}) for the system when M_v is taken as bifurcation parameter and no control is used. In these figures, permission probability is fixed at $p_v = 0.2$, $p_v = 0.3$, and $p_v = 0.4$, respectively.

Next, we design a price based control for the same PRMA-HS voice system and compare bifurcation diagrams. We choose $\beta = \xi = 1$. The control parameter α is chosen to be $\alpha = 1.04$ such that conditions in Lemma 2.7 is satisfied. Figure 2.24 shows bifurcation diagram for packet drop probability, when M_v is bifurcation parameter. We can see that by using the price based control, bifurcations are eliminated. Moreover, capacity of the PRMA-HS system has increased to 42 from 38 when $p_v = 0.2$, 28 when $p_v = 0.3$, and 21 when $p_v = 0.4$.

Further, we design state estimation control schemes for PRMA-HS system with voice terminals. Figure 2.25 illustrates bifurcation diagram for packet drop probability, when state estimation control scheme is based on maximum throughput. We can observe that bifurcations are eliminated and capacity of the PRMA system is increased to 43. Figure 2.26 illustrates bifurcation diagram for packet drop probability, when state estimation control scheme is based on minimum packet drop probability. It can be observed that bifurcations are controlled and capacity of the system is 43.

Next, we consider the same voice PRMA-HS system over random packet error channel. Figure 2.27 shows bifurcation diagram for packet drop probability when error probability (Δ) is the bifurcation parameter, $M_v = 25$, and $p_v = 0.3$. This figure shows that only for small values of Δ system has an acceptable drop probability (less than 0.01). Also, figure 2.28 shows bifurcation diagrams for drop probability when M_v is chosen as the bifurcation parameter and for two different values of Δ.

Plus signs show packet drop probability for $\Delta = 0.05$ and points are for $\Delta = 0.01$. It can be noticed that for $\Delta = 0.01$ capacity of the system is 27 and for $\Delta = 0.05$ capacity is 24.

Further, we use the price based bifurcation control with the same parameters as before, $\beta = \xi = 1$ and $\alpha = 1.04$. Figure 2.29 shows bifurcation diagram for the packet drop probability for both $\Delta = 0.01$ and $\Delta = 0.05$ when M_v is the bifurcation parameter. Capacity of the system is increased to 41 and 38 for $\Delta = 0.01$ and $\Delta = 0.05$, respectively. Here we assumed that $q = 1$.

Figure 2.30 illustrates effects of state estimation control scheme (based on maximum throughput) on nonlinear behavior of the PRMA-HS system. If error probability is $\Delta = 0.01$, bifurcations of the operating points of the system is completely eliminated and capacity of the PRMA-HS system is increased to 42. However, for the case where $\Delta = 0.05$, although the bifurcations are not completely eliminated (PRMA-HS voice system has three equilibrium points at $M_v = 42$), but they are controlled by delaying the bifurcations. In this case, capacity of the system is increased to 39. Figure 2.31 illustrates effects of state estimation control (based on minimizing packet drop probability) on behavior of the PRMA-HS system over random packet error channel for $\Delta = 0.01$ and $\Delta = 0.05$. Bifurcations are controlled and capacity of the system is $M_v = 43$ for $\Delta = 0.01$ and $M_v = 41$ for $\Delta = 0.05$.

2.12 Summary and Future Lines of Work

In this chapter, we studied the equilibrium behavior of voice systems employing PRMA or PRMA-HS protocols as their medium access control scheme. We studied how small changes in system parameters can transfer a system with one operating point to a system with several operating points, therefore, limiting capacity of the system. Further, we introduced price based control and state estimation-based control and studied the effects of these control schemes on the equilibrium behavior of PRMA and PRMA-HS voice systems. We derived sufficient conditions on control and system parameters such that the controlled systems will have unique equilibrium points. The controlled systems were analyzed over error-free and random error channels. Also, we considered the voice system with PRMA and PRMA-HS that employs multiple transmission power at the terminals and capture effect at the access point and studied the effects of multiple power levels and capture on bifurcations of the equilibrium points of the system. Future lines of research can include extending the analysis of introduced control schemes to other variations of PRMA protocol for voice systems, studying a dynamic control scheme based on state observation in addition to stat estimation, and extending the bifurcation analysis using multiple power levels to more accurate models of capture that deals with signal-to-interface-plus-noise ration and different distances of terminals from the access point. Also, similar price based control can be used adjust rate of generating voice messages and/or the average number of voice packets in a message. Further, as future work, robustness of the introduced controlled schemes can be studied considering variations in system parameters.

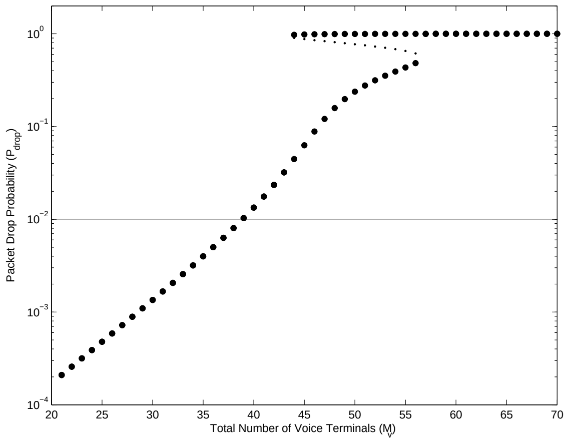

Figure 2.21: Bifurcation diagram for packet drop probability with no control ($p_v = 0.2$)

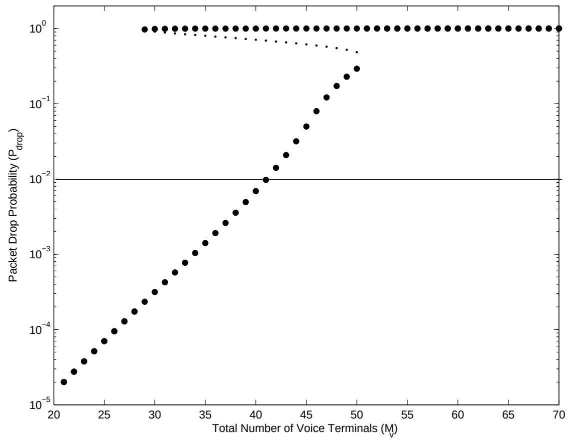

Figure 2.22: Bifurcation diagram for packet drop probability with no control ($p_v = 0.3$)

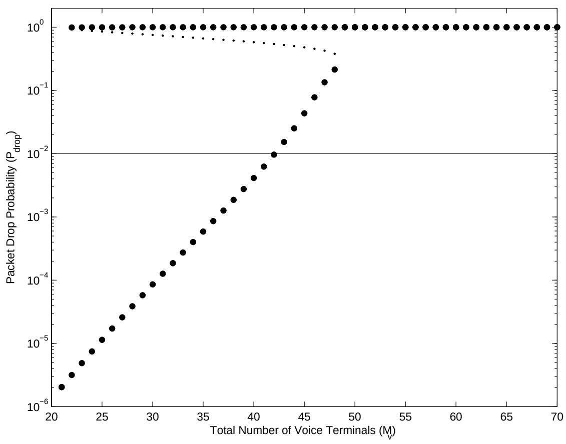

Figure 2.23: Bifurcation diagram for packet drop probability with no control ($p_v = 0.4$)

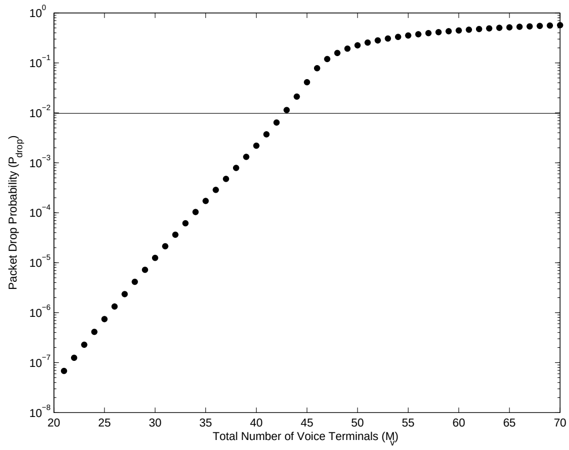

Figure 2.24: Bifurcation diagram for packet drop probability with the price based control ($\alpha = 1.04$, $\beta = \xi = 1$)

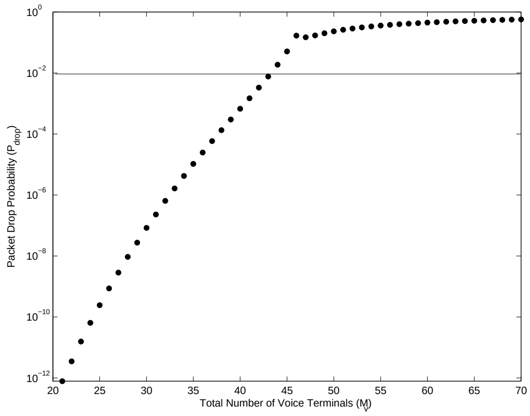

Figure 2.25: Bifurcation diagram for packet drop probability with state estimation control (maximizing throughput)

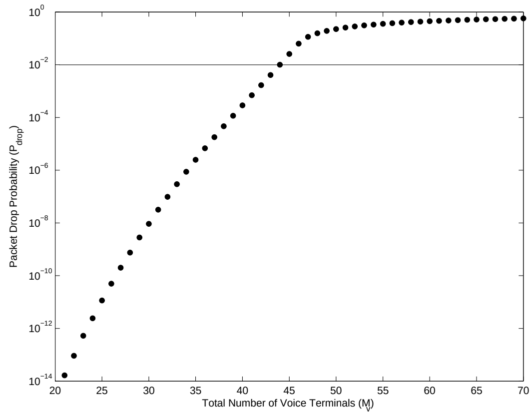

Figure 2.26: Bifurcation diagram for packet drop probability with state estimation control (minimizing P_{drop})

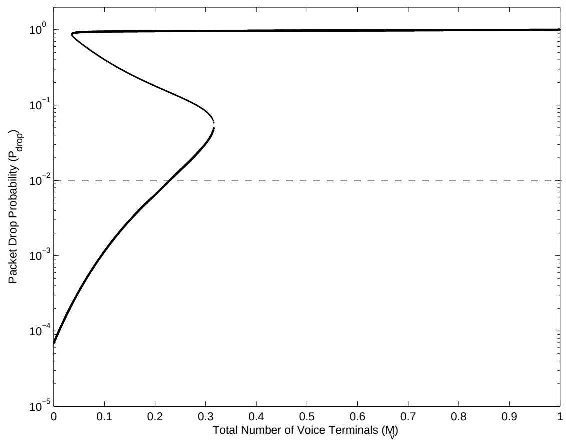

Figure 2.27: Bifurcation diagram for packet drop probability no control (Δ is bifurcation parameter, $p_v = 0.3$, $M_v = 25$)

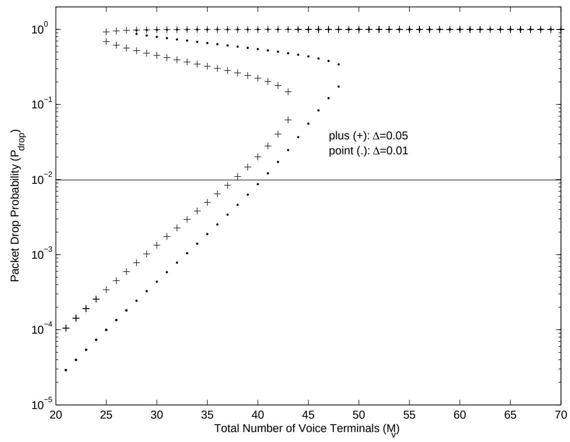

Figure 2.28: Bifurcation diagram for packet drop probability with no control ($p_v = 0.3$ and two different values for Δ)

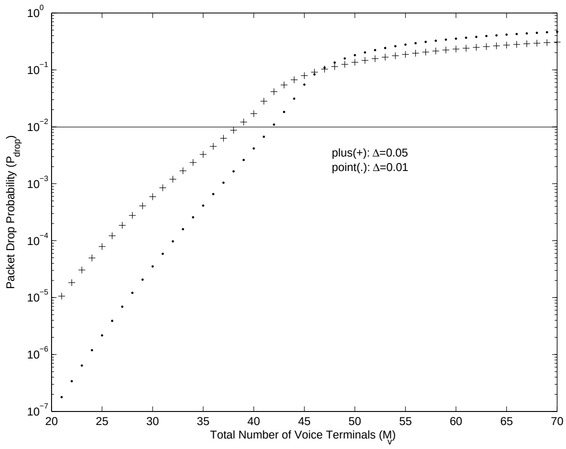

Figure 2.29: Bifurcation diagram for packet drop probability with control ($\Delta = 0.05$, $\Delta = 0.01$, $\alpha = 1.04$, $\beta = \xi = 1$)

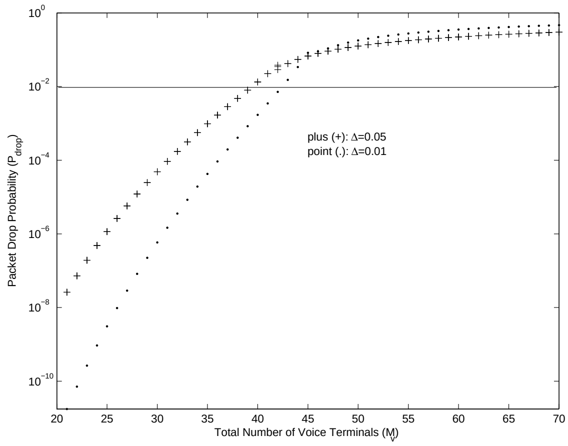

Figure 2.30: Bifurcation diagram for packet drop probability with state observation control (maximizing throughput) ($\Delta = 0.05$ and $\Delta = 0.01$)

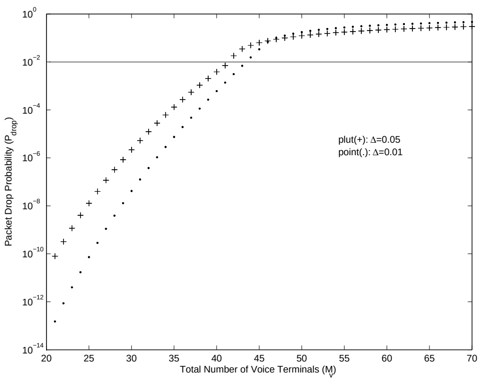

Figure 2.31: Bifurcation diagram for packet drop probability with state estimation control (minimizing P_{drop}) for $\Delta = 0.05$ and $\Delta = 0.01$.

Chapter 3

Equilibrium Analysis and Control for PRMA and PRMA-HS Protocols for Voice and Data Terminals

Focus of our analysis in previous chapter was on a system employing voice terminals using the PRMA or PRMA-HS protocols. However, in this chapter, we expand our analysis to a system that employs both voice and data terminals. We study nonlinear instabilities of the PRMA and PRMA-HS protocols and introduce a General Price Based Control.

We start this chapter by modeling the PRMA-HS protocol with General Price Based Control as a Markov chain. Further, we consider the PRMA protocol as a special case of the PRMA-HS protocol (with round trip delay of zero) and analytically study stability of specific variations of the PRMA voice-data system using equilibrium point analysis. Also, we consider an uplink channel of the system with random packet error, and study our control schemes over this channel.

3.1 General Price Based Control

As discussed in previous chapters, we define round trip delay for PRMA-HS protocol as $RTD = \frac{N}{d}$ times slots, assuming d is an integer devisor of N. For the sake of simplicity in notation, for modeling the system based on Markov chain, we assume that $d = N$ and therefore, $RTD = 1$ time slot. However, our analysis can be expanded to any value of round trip delay and equilibrium equation analysis is based on more general values of delay.

In the PRMA-HS protocol, at the end of each time slot (for example, time slot n), terminals receive a feedback message regarding status of RTD time slot ago (time slot $n - RTD$). Voice and data terminals use this feedback information to adjust their permission probability. We assume that permission probabilities, p_v and p_d, are functions of a control signal u. Control signal is updated at the end of each time slot based on equation (3.1).

$$u_{n+1} = \begin{cases} u_n & \text{if slot } n-1 \text{ is reserved and reservation is kept,} \\ u_n + \phi & \text{if slot } n-1 \text{ is reserved and reservation is lost,} \\ [u_n - \alpha I(Z_{n-1} = 0) + \beta I(Z_{n-1} = 1 \wedge by\ a\ CON\ or\ BLK) & \\ \quad + \xi I((Z_{n-1} \geq 2) \vee (Z_{n-1} = 1 \wedge by\ a\ HIN))]^+ & \\ & \text{if slot } n-1 \text{ is not reserved.} \end{cases} \tag{3.1}$$

Here ϕ, α, and ξ are positive real numbers and β is a real number. $[x]^+$ denotes $\max(0, x)$. Random variable Z_n shows the number of packets that are transmitted at beginning of time slot n. Permission probabilities, p_v and p_d, are updated at end of each time slot based on new value of the control signal u.

Assumption 3.1. We assume that permission probabilities $p_v(u)$ and $p_d(u)$ are continuous, bounded ($0 \leq p_v(u), p_d(u) \leq 1$), and strictly decreasing in u ($u \in$

$[0, +\infty)$). Furthermore, there exists a positive constant u_{max} such that $p_v(u) = p_d(u) = 0$ when $u \geq u_{max}$.

Based on update equation (3.1), if time slot $n-1$ is reserved and its reservation is kept, control signal u is unchanged. However, if reservation of time slot $n-1$ is lost, control signal u is increased by ϕ and as a result, permission probabilities are decreased. In the case that time slot $n-1$ is not reserved, if there is no packet transmission at time slot $n-1$ ($Z_{n-1} = 0$), control signal decreases and as a result, permission probabilities are increased. If there is a collision at time slot $n-1$ ($Z_{n-1} \geq 2$) or a successful transmission by a terminal in HIN state ($Z_{n-1} = 1 \wedge by\ a\ HIN$), control signal is increased by ξ and therefore, permission probabilities are decreased. For a successful packet transmission at time slot $n-1$ by a contending/backlogged terminal ($Z_{n-1} = 1 \wedge by\ a\ CON\ or\ BLK$), depending on value of β, permission probabilities are either increased or decreased.

We define system state as $X_n = (Y_{n-1}, Y_n)$ in order to model the PRMA-HS protocol with General Price Based Control. Here

$$Y_n = (c_n, r_n, h_{v_n}, b_n, h_{d_n}, u_n),$$

$$c \in \{0, 1, 2, \cdots, M_v\}, r \in \{0, 1, 2, \cdots, N\}, h_v \in \{0, 1\},$$

$$b \in \{0, 1, 2, \cdots, M_d\}, h_d \in \{0, 1\},$$

$$u \in \Gamma = \{\min(u_{MAX}, [f\phi - a\alpha + e\beta + d\xi]^+) | f, a, e, d \in \mathbb{Z}_+\}.$$

Here $u_{MAX} = u_{max} + \max(N\phi + \beta, \xi)$. Then state space $\aleph$ is a subset of

$$(\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, N\} \times \{0, 1\} \times \{0, 1, 2, \cdots, M_d\} \times \{0, 1\} \times \Gamma)^2,$$

with following constraints. If $((c, r, h_v, b, h_d, u), (c_1, r_1, h_{v_1}, b_1, h_{d_1}, u_1)) \in \aleph$

- $c + r + h_v \leq M_v$ and $c_1 + r_1 + h_{v_1} \leq M_v$
- $b + h_d \leq M_d$ and $b_1 + h_{d_1} \leq M_d$
- $r + h_v \leq N$ and $r_1 + h_{v_1} \leq N$
- $c - 1 \leq c_1 \leq M_v - r - h_v$
- $b - 1 \leq b_1 \leq M_d - h_d$
- $r - 1 \leq r_1 \leq r + 1$
- $h_v + h_d \leq 1$ and $h_{v_1} + h_{d_1} \leq 1$
- For $h_v = 1$ or $h_d = 1$, control signal $u_1 \geq [\beta]^+$
- For $r = N$, control signal $u \geq [\beta]^+$ and $u_1 \geq [\beta]^+$

State space is countable and transition probabilities are written as follows:

$$Pr(X_{n+1} = X_1 | X_n = X) = \tag{3.2}$$

- $V(c_1, c, r, h_v)D(b_1, b, h_d)(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f(\frac{r+h_v}{N} + \Theta + \Theta^{cb})$

  if $c \leq c_1 \leq M_v - h_v - r, b \leq b_1 \leq M_d - h_d, r_1 = r - 1, h_{v_1} = h_v, h_{d_1} = h_d, u_1 = u + \phi$

- $V(c_1 + 1, c, r, h_v)D(b_1, b, h_d)(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f\Theta^c$

  if $c-1 \leq c_1 \leq M_v - h_v - r - 1, b \leq b_1 \leq M_d - h_d, r_1 = r - 1, h_{v_1} = h_v + 1, h_{d_1} = h_d, u_1 = u + \phi$

- $V(c_1, c, r, h_v)D(b_1+1, b, h_d)(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f\Theta^b$

  if $c \leq c_1 \leq M_v - h_v - r, b-1 \leq b_1 \leq M_d - h_d - 1, r_1 = r-1, h_{v_1} = h_v, h_{d_1} = h_d + 1, u_1 = u + \phi$

- $V(c_1, c, r, h_v)D(b_1, b, h_d)(\frac{r_{-1}+h_{v_{-1}}}{N})(1-\gamma_f)(\frac{r+h_v}{N} + \Theta + \Theta^{cb})$

  if $c \leq c_1 \leq M_v - h_v - r, b \leq b_1 \leq M_d - h_d, r_1 = r, h_{v_1} = h_v, h_{d_1} = h_d, u_1 = u$

- $V(c_1+1, c, r, h_v)D(b_1, b, h_d)(\frac{r_{-1}+h_{v_{-1}}}{N})(1-\gamma_f)\Theta^c$

  if $c-1 \leq c_1 \leq M_v - h_v - r - 1, b \leq b_1 \leq M_d - h_d, r_1 = r, h_{v_1} = h_v + 1, h_{d_1} = h_d, u_1 = u$

- $V(c_1, c, r, h_v)D(b_1+1, b, h_d)(\frac{r_{-1}+h_{v_{-1}}}{N})(1-\gamma_f)\Theta^b$

  if $c \leq c_1 \leq M_v - h_v - r, b-1 \leq b_1 \leq M_d - h_d - 1, r_1 = r, h_{v_1} = h_v, h_{d_1} = h_d + 1, u_1 = u$

- $V(c_1, c, r, h_v)D(b_1, b, h_d)\Theta^{cb}_{-1}(\frac{r+h_v}{N} + \Theta + \Theta^{cb})$

  if $c \leq c_1 \leq M_v - h_v - r, b \leq b_1 \leq M_d - h_d, r_1 = r, h_{v_1} = h_v, h_{d_1} = h_d, u_1 = [u-\alpha]^+$

- $V(c_1+1, c, r, h_v)D(b_1, b, h_d)\Theta^{cb}_{-1}\Theta^c$

  if $c-1 \leq c_1 \leq M_v - h_v - r - 1, b \leq b_1 \leq M_d - h_d, r_1 = r, h_{v_1} = h_v + 1, h_{d_1} = h_d, u_1 = [u-\alpha]^+$

- $V(c_1, c, r, h_v)D(b_1+1, b, h_d)\Theta^{cb}_{-1}\Theta^b$

  if $c \leq c_1 \leq M_v - h_v - r, b-1 \leq b_1 \leq M_d - h_d - 1, r_1 = r, h_{v_1} = h_v, h_{d_1} = h_d + 1, u_1 = [u-\alpha]^+$

- $V(c_1, c, r, h_v)D(b_1, b, h_d)\Theta^c_{-1}(\frac{r+h_v}{N} + \Theta + \Theta^{cb})$

  if $c \leq c_1 \leq M_v - h_v - r, b \leq b_1 \leq M_d - h_d, r_1 = r + 1, h_{v_1} = h_v - 1, h_{d_1} = h_d, u_1 = [u + \beta]^+$

- $V(c_1 + 1, c, r, h_v)D(b_1, b, h_d)\Theta^c_{-1}\Theta^c$

  if $c - 1 \leq c_1 \leq M_v - h_v - r - 1, b \leq b_1 \leq M_d - h_d, r_1 = r + 1, h_{v_1} = h_v, h_{d_1} = h_d, u_1 = [u + \beta]^+$

- $V(c_1, c, r, h_v)D(b_1 + 1, b, h_d)\Theta^c_{-1}\Theta^b$

  if $c \leq c_1 \leq M_v - h_v - r, b - 1 \leq b_1 \leq M_d - h_d - 1, r_1 = r + 1, h_{v_1} = h_v - 1, h_{d_1} = h_d + 1, u_1 = [u + \beta]^+$

- $V(c_1, c, r, h_v)D(b_1, b, h_d)\Theta^b_{-1}(\frac{r+h_v}{N} + \Theta + \Theta^{cb})$

  if $c \leq c_1 \leq M_v - h_v - r, b \leq b_1 \leq M_d - h_d, r_1 = r, h_{v_1} = h_v, h_{d_1} = h_d - 1, u_1 = [u + \beta]^+$

- $V(c_1 + 1, c, r, h_v)D(b_1, b, h_d)\Theta^b_{-1}\Theta^c$

  if $c - 1 \leq c_1 \leq M_v - h_v - r - 1, b \leq b_1 \leq M_d - h_d, r_1 = r, h_{v_1} = h_v + 1, h_{d_1} = h_d - 1, u_1 = [u + \beta]^+$

- $V(c_1, c, r, h_v)D(b_1 + 1, b, h_d)\Theta^b_{-1}\Theta^b$

  if $c \leq c_1 \leq M_v - h_v - r, b - 1 \leq b_1 \leq M_d - h_d - 1, r_1 = r, h_{v_1} = h_v, h_{d_1} = h_d, u_1 = [u + \beta]^+$

- $V(c_1, c, r, h_v)D(b_1, b, h_d)\Theta_{-1}(\frac{r+h_v}{N} + \Theta + \Theta^{cb})$

  if $c \leq c_1 \leq M_v - h_v - r, b \leq b_1 \leq M_d - h_d, r_1 = r, h_{v_1} = h_v, h_{d_1} = h_d, u_1 = u + \xi$

- $V(c_1+1,c,r,h_v)D(b_1,b,h_d)\Theta_{-1}\Theta^c$

  if $c-1 \le c_1 \le M_v-h_v-r-1, b \le b_1 \le M_d-h_d, r_1=r, h_{v_1}=h_v+1, h_{d_1}=h_d, u_1=u+\xi$

- $V(c_1,c,r,h_v)D(b_1+1,b,h_d)\Theta_{-1}\Theta^b$

  if $c \le c_1 \le M_v-h_v-r, b-1 \le b_1 \le M_d-h_d-1, r_1=r, h_{v_1}=h_v, h_{d_1}=h_d+1, u_1=u+\xi$

- 0 Otherwise

here

$$X_1=((c,r,h_v,b,h_d,u),(c_1,r_1,h_{v_1},b_1,h_{d_1},u_1)),$$

$$X=((c_{-1},r_{-1},h_{v_{-1}},b_{-1},h_{d_{-1}},u_{-1}),(c,r,h_v,b,h_d,u)),$$

$$V(c_1,c,r,h_v)=\begin{pmatrix} M_v-c-r-h_v \\ c_1-c \end{pmatrix}\sigma_v^{c_1-c}(1-\sigma_v)^{M_v-c_1-r-h_v},$$

$$D(b_1,b,h_d)=\begin{pmatrix} M_d-b-h_d \\ b_1-b \end{pmatrix}\sigma_d^{b_1-b}(1-\sigma_d)^{M_d-b_1-h_d},$$

$$\Theta^c=(1-\frac{r+h_v}{N})cp_v(1-p_v)^{c+h_v-1}(1-p_d)^{b+h_d},$$

$$\Theta^b=(1-\frac{r+h_v}{N})bp_d(1-p_d)^{b+h_d-1}(1-p_v)^{c+h_v},$$

$$\Theta^{cb}=(1-\frac{r+h_v}{N})(1-p_v)^{c+h_v}(1-p_d)^{b+h_d},$$

$$\Theta=(1-\frac{r+h_v}{N})-\Theta^{cb}-\Theta^c-\Theta^b,$$

$$\Theta^c_{-1}=(1-\frac{r_{-1}+h_{v_{-1}}}{N})c_{-1}p_{v_{-1}}(1-p_{v_{-1}})^{c_{-1}+h_{v_{-1}}-1}(1-p_{d_{-1}})^{b_{-1}+h_{d_{-1}}},$$

$$\Theta^b_{-1}=(1-\frac{r_{-1}+h_{v_{-1}}}{N})b_{-1}p_{d_{-1}}(1-p_{d_{-1}})^{b_{-1}+h_{d_{-1}}-1}(1-p_{v_{-1}})^{c_{-1}+h_{v_{-1}}},$$

$$\Theta^{cb}_{-1} = (1 - \frac{r_{-1} + h_{v_{-1}}}{N})(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}}(1 - p_{d_{-1}})^{b_{-1}+h_{d_{-1}}},$$

$$\Theta_{-1} = (1 - \frac{r_{-1} + h_{v_{-1}}}{N}) - \Theta^{cb}_{-1} - \Theta^{c}_{-1} - \Theta^{b}_{-1},$$

$$p_v = p_v(u), \quad p_{v_{-1}} = p_v(u_{-1}), \quad p_d = p_d(u), \quad p_{d_{-1}} = p_d(u_{-1}).$$

3.2 Price Based Control

In this section, we study a special case of General Price Based Control assuming $\phi = 0$. Therefore, control signal is only updated during an available time slot. Because of large size of the state space, studying the PRMA-HS system with priced base control using transition probabilities is very difficult. Therefore, stationary behavior of the system is analyzed using equilibrium point analysis (EPA). In EPA it is assumed that the system is in equilibrium, therefore, any change in any state is zero. One-step expected change (mean drift) of the control signal at state X is defined as: $d(X) = E(u_{n+1} - u_n | X_n = X)$. By using definition of control signal in (3.1) (assuming $\phi = 0$), expected drift is determined as:

$$\begin{aligned} d(X) = {} & (\max(-\alpha, -u) - \xi)(1 - \frac{r_{-1} + h_{v_{-1}}}{N})(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}}(1 - p_{d_{-1}})^{b_{-1}+h_{d_{-1}}} \\ & +(\max(\beta, -u) - \xi)(1 - \frac{r_{-1} + h_{v_{-1}}}{N})[c_{-1}p_{v_{-1}}(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}-1}(1 - p_{d_{-1}})^{b_{-1}+h_{d_{-1}}} \\ & +b_{-1}p_{d_{-1}}(1 - p_{d_{-1}})^{b_{-1}+h_{d_{-1}}-1}(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}}] + \xi(1 - \frac{r_{-1} + h_{v_{-1}}}{N}). \end{aligned}$$

Or relaxed drift equation is:

$$\begin{aligned} d_r(X) = & -(\alpha + \xi)(1 - \frac{r_{-1} + h_{v_{-1}}}{N})(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}}(1 - p_{d_{-1}})^{b_{-1}+h_{d_{-1}}} \\ & +(\beta + \xi)(1 - \frac{r_{-1} + h_{v_{-1}}}{N})[c_{-1}p_{v_{-1}}(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}-1}(1 - p_{d_{-1}})^{b_{-1}+h_{d_{-1}}} \\ & +b_{-1}p_{d_{-1}}(1 - p_{d_{-1}})^{b_{-1}+h_{d_{-1}}-1}(1 - p_{v_{-1}})^{c_{-1}+h_{v_{-1}}}] + \xi(1 - \frac{r_{-1} + h_{v_{-1}}}{N}). \end{aligned} \tag{3.3}$$

Now we derive equilibrium equations of the system. As discussed in chapter 1 with respect to equilibrium equation of the PRMA-HS system, note that at equilibrium, the number of voice terminals in each state HIN_i is $\frac{Hv}{(N/d)}$ for $i = N-1, \cdots, N-N/d$. In the same way, the number of terminals, at equilibrium, in state RES_i' is $\frac{R'}{N-(N/d)}$ for $i = N-N/d-1, \cdots, 0$. It is easy to show that

$$\frac{H_v}{N/d} = \frac{R'}{N-N/d}.$$

Also, the number of voice terminals in each state RES_i is $\frac{R}{N}$ for $i = 0, \cdots, N-1$. We define $R^* = R + R'$ as the number of voice terminals in RES and RES'. Notice that

$$\frac{R^* + Hv}{N} = \frac{R}{N} + \frac{R'}{N-N/d}.$$

A point (C, R^*, H_v, B, H_d, U) is called an equilibrium point, if

$$\begin{aligned}
&(M_v - C - R^* - H_v)\sigma_v - (1 - \frac{R^* + H_v}{N})Cp_v w_v(C,B) = 0,\\
&(\frac{R^* + H_v}{N})\gamma_f - (M_v - C - R^* - H_v)\sigma_v = 0,\\
&(\frac{R^* + H_v}{N})\gamma_f - (\frac{d}{N})H_v = 0,\\
&(M_d - B - H_d)\sigma_d - (1 - \frac{R^* + H_v}{N})Bp_d w_d(C,B) = 0,\\
&(M_d - B - H_d)\sigma_d - (\frac{d}{N})H_d = 0,\\
&(\alpha + \xi)(1-p_v)^{C+H_v}(1-p_d)^{B+H_d} - (\beta - \xi)(Cp_v w_v(C,B) + Bp_d w_d(C,B)) - \xi = 0,
\end{aligned} \tag{3.4}$$

here $$w_v(C,B) = \begin{cases} (1-p_v(U))^{C+H_v-1}(1-p_d(U))^{B+H_d} & C \geq 1 \\ (1-p_v(U))^{H_v}(1-p_d(U))^{B+H_d} & C < 1 \end{cases},$$

and $$w_d(C,B) = \begin{cases} (1-p_d(U))^{B+H_d-1}(1-p_v(U))^{C+H_v} & B \geq 1 \\ (1-p_d(U))^{H_d}(1-p_v(U))^{C+H_v} & B < 1 \end{cases}.$$

Set of equations (3.4) can be simplified as following:

$$\begin{aligned} F_1(C,B,U) =& M_v - C - (\frac{1}{\omega})(\frac{Cp_v w_v(C,B)}{\gamma_f + Cp_v w_v(C,B)}) = 0, \\ F_2(C,B,U) =& M_d - B - (\frac{N}{d} + \frac{1}{\sigma_d})\gamma_f(\frac{Bp_d w_d(C,B)}{\gamma_f + Cp_v w_v(C,B)}) = 0, \\ F_3(C,B,U) =& (\alpha+\xi)(1-p_v)^{C+H_v}(1-p_d)^{B+H_d} \\ & - (\beta-\xi)(Cp_v w_v(C,B) + Bp_d w_d(C,B)) - \xi = 0, \end{aligned} \tag{3.5}$$

here $\omega = \frac{\sigma_v}{\gamma_f + \sigma_v N}$, $p_v = p_v(U)$, $p_d = p_d(U)$, and:

$$\begin{aligned} H_v &= \gamma_f(\frac{N}{d})\omega(M_v - C), \\ R^* &= (d-\gamma_f)(\frac{N}{d})\omega(M_v - C), \\ H_d &= (\frac{N\sigma_d}{N\sigma_d + d})(M_d - B). \end{aligned}$$

In the next few sections, we focus our analysis on the PRMA protocol with voice and data terminals, therefore, assuming round trip delay is negligible.

3.3 PRMA Voice and Data

In this section we consider a PRMA system that employs both voice and data terminals. Here, we assume that round trip delay is negligible and from now on in this chapter we assume that permission probabilities for both voice and data terminals are the same, $p_v = p_d = p$.

Proposition 3.1. *The Markov chain defined on $\aleph$ through (3.2) (considering $\phi = 0$,* *the system employs voice and data terminals, and round trip delay is negligible) is irreducible, aperiodic, and positive recurrent.*

Detailed proof is presented in the Appendix E. Set of equations (3.4) modeling equations of the PRMA-HS voice-data system can be simplified as following for the PRMA voice-data system considered in this section

$$
\begin{aligned}
&(M_v - C - R)\sigma_v - (1 - \frac{R}{N})Cpw(C,B) = 0,\\
&(\frac{R}{N})\gamma_f - (M_v - C - R)\sigma_v = 0,\\
&(M_d - B)\sigma_d - (1 - \frac{R}{N})Bpw(C,B) = 0,\\
&(\alpha + \xi)(1-p)^{C+B} - (\beta - \xi)(C+B)pw(C,B) - \xi = 0.
\end{aligned} \tag{3.6}
$$

here $w(C,B) = \begin{cases} (1-p(U))^{C+B-1} & C+B \geq 1 \\ 1 & C+B < 1 \end{cases}$ .Set of equations (3.6) can be further simplified as following:

$$
\begin{aligned}
F_1(C,B,U) =& M_v - C - (\frac{1}{\omega})(\frac{Cpw(C,B)}{\gamma_f + Cpw(C,B)}) = 0,\\
F_2(C,B,U) =& M_d - B - (\frac{\gamma_f}{\sigma_d})(\frac{Bpw(C,B)}{\gamma_f + Cpw(C,B)}) = 0,\\
F_3(C,B,U) =& (\alpha + \xi)(1-p)^{C+B} - (\beta - \xi)(C+B)pw(C,B) - \xi = 0,
\end{aligned} \tag{3.7}
$$

Remark 3.1. *Based on the first two equations of the set of equations (3.6), it can be shown that* $R = \min(N, N\omega(M_v - C))$, *where* $\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}$. *Also, based on first and third equations of the set of equations (3.6), it can easily be shown that* $B(C) = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$. *Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$, $R \in [0, N]$, $B \in [0, M_d]$, *and* $p \in [0,1]$. *However, it can easily be shown, by using the set of equation (3.6),* *that* $C = 0$, $M_v - \frac{1}{\omega}$, $C = M_v$, $B = 0$, $B = M_d$, $R = 0$, $R = N$, $p = 0$, $p = 1$ *(for* $\beta = \xi$ *as considered in Lemma 3.1), or a combination thereof, cannot be solutions to the set of equilibrium equations (3.6).*

Lemma 3.1. *Let us define* $f^+(C) = \frac{\xi}{\alpha+\xi}(C+B)(\exp(\frac{-1}{C+B}\ln(\frac{\xi}{\alpha+\xi})) - 1)$. *Here* $B(C) = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$. *There exists a set of control parameters* (α, β, ξ) *for which the set of equations (3.7) has a unique solution in* (C, U) *and the system has a single operating point if any of conditions* (1), (2), *or* (3) *below hold:*

(1) $\frac{\gamma_f \omega}{\sigma_d} < 1$, $M_v - \frac{1}{\omega} > \hat{C}$, *and* $-\gamma_f \omega(1 + \frac{M_d}{M_v}) < \frac{df^+}{dC}(\tilde{C})$,

(2) $\frac{\gamma_f \omega}{\sigma_d} < 1$, $M_v - \frac{1}{\omega} < \hat{C}$, *and either*

$$-\gamma_f \omega(1 + \frac{M_d}{M_v}) < \frac{df^+}{dC}(\hat{C}), \quad \textit{or}$$
$$\frac{(M_v - \hat{C})\omega\gamma_f + (M_d - \hat{B})\sigma_d}{1 - (M_v - \hat{C})\omega} \leq \frac{\xi}{\alpha+\xi}(M_v + M_d)(\exp(\frac{-1}{M_v + M_d}\ln(\frac{\xi}{\alpha+\xi})) - 1),$$

(3) $\frac{\gamma_f \omega}{\sigma_d} \geq 1$, $M_v < \frac{1}{\omega} + \hat{C}$ *and*

$$\frac{(M_v - \hat{C})\omega\gamma_f + (M_d - \hat{B})\sigma_d}{1 - (M_v - \hat{C})\omega} \leq \frac{\xi}{\alpha+\xi}(M_v + M_d)(\exp(\frac{-1}{M_v + M_d}\ln(\frac{\xi}{\alpha+\xi})) - 1).$$

Here $\tilde{C} = M_v - \frac{1}{\omega}$, $\hat{C} + \hat{B} = 1$, *and* $\hat{B} = B(\hat{C})$.

Proof. We will show the existence of control parameters (α, β, ξ) with $\beta = \xi$ such that the conditions hold. We define following new functions

$$f(C, U) = (C + B)pw(C, U) \qquad g(C) = \frac{(M_v - C)\omega\gamma_f + (M_d - B)\sigma_d}{1 - (M_v - C)\omega}.$$

It is easy to show that equilibrium points of set of equations (3.7) are same as fixed points of $f(C, U) = g(C)$. Note that $R = \min(N, N\omega(M_v - C))$ and $B =$

$\frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$. Solving $F_3(C, U) = 0$ in equation (3.7) for p (assuming $\beta = \xi$) and substituting it in $f(C, U)$, we have

$$f(C) = \begin{cases} \frac{\xi}{\alpha+\xi}(C+B)(\exp(\frac{-1}{C+B}\ln(\frac{\xi}{\alpha+\xi})) - 1) & C + B \geq 1 \\ (C+B)(1 - \exp(\frac{1}{C+B}\ln(\frac{\xi}{\alpha+\xi}))) & C + B < 1 \end{cases}$$

It is easy to show that first and second derivatives of $f(C)$ are as follows

$$\frac{df}{dC} < 0 \quad for \quad C + B \geq 1 \quad and \quad \frac{df}{dC} > 0 \quad for \quad C + B < 1.$$

And if $\frac{\gamma_f \omega}{\sigma_d} < 1$

$$\frac{d^2 f}{dC^2} > 0 \quad for \quad C + B \geq 1 \quad and \quad \frac{d^2 f}{dC^2} < 0 \quad for \quad C + B < 1.$$

Note that $\max(0, M_v - \frac{1}{\omega}) < C < M_v$. Also, in this range of values of C and for $\frac{\gamma_f \omega}{\sigma_d} < 1$, it is easy to show that $\frac{dg}{dC} < 0$ and $\frac{d^2 g}{dC^2} > 0$. Also, note that

$$f(M_v) = \frac{\xi}{\alpha+\xi}(M_v + M_d)(\exp(-\frac{1}{(M_v + M_d)}\ln(\frac{\xi}{\alpha+\xi})) - 1) > g(M_v) = 0.$$

We define $\hat{C}$ such that $B(\hat{C}) + \hat{C} = 1$. Since B is a strictly increasing function of C, $\hat{C}$ is unique. Further, for simplification in notations, we define $\tilde{C} = M_v - \frac{1}{\omega}$. Now based on the above-noted facts, in order to prove that $f(C) = g(C)$ has exactly one solution, for the conditions stated in the lemma, we consider different cases:

- **Case 1:** $\frac{\gamma_f \omega}{\sigma_d} < 1$

    1. $M_v > \hat{C} + \frac{1}{\omega}$: In this case we only consider $M_v - \frac{1}{\omega} < C < M_v$. In this range, $g(C)$ is positive and strictly decreasing with positive second derivative. Also, $f(C)$ is positive, strictly decreasing, with positive second derivative, $f(M_v - \frac{1}{\omega}) < g((M_v - \frac{1}{\omega})^+)$, and $f(M_v) > g(M_v)$. Therefore,

if the control parameters are chosen such that $g(C) - f(C)$ is strictly decreasing in the range of C, then $g(C) - f(C) = 0$ has a unique solution. Since both functions $g(C)$ and $f(C)$ have positive second derivatives, we choose the control parameters such that $\frac{dg}{dC}(M_v) < \frac{df}{dC}(\tilde{C})$.

2. $\frac{1}{\omega} < M_v < \hat{C} + \frac{1}{\omega}$: In this case, in order to make sure that there exist only one solution, we chose the control parameters (α, β, ξ) such that either $\frac{dg}{dC}(M_v) < \frac{df}{dC}(\hat{C})$ or $g(\hat{C}) < f(M_v)$.

3. $0 < M_v < \frac{1}{\omega}$: In this case, $g(C)$ is positive, strictly decreasing, and with positive second derivative. Also, $f(C)$ is positive, strictly increasing for $0 < C < \hat{C}$, and strictly decreasing for $1 \leq C \leq M_v$. In the same way, we choose control parameters such that either $\frac{dg}{dC}(M_v) < \frac{df}{dC}(\hat{C})$ or $g(\hat{C}) < f(M_v)$. It is easy to show that in this case $f(C) = g(C)$ has exactly one solution.

- **Case 2:** $\frac{\gamma_f \omega}{\sigma_d} \geq 1$

  - $0 < M_v < \frac{1}{\omega} + \hat{C}$: In this case, $g(C)$ is positive and strictly decreasing. Also, $f(C)$ is positive, strictly increasing for $0 < C < \hat{C}$, and strictly decreasing for $\hat{C} \leq C \leq M_v$. We choose control parameters such that $g(\hat{C}) < f(M_v)$.

It can easily be shown, by using $F_3(C, B, U)$ of equation (3.7), that p is one-to-one function of C. Also, as mentioned before, p is also a one-to-one function of U (for $0 < p < 1$). Therefore, it can easily be shown that for a given C, there exists a unique U. Therefore, under conditions stated in the lemma, the set of equations (3.7) has a unique solution in $0 < C < M_v$ and $0 < U < u_{MAX}$. $\square$

3.4 State Estimation-Based Control

Next, we consider a control scheme based on the state estimation control (as introduced in previous chapter) for the PRMA-HS voice-data system. As discussed before, the number of voice terminals with reservation (or the number of reserved time slots) is the only state of the system that is known to all terminals in the system. Therefore, we choose the control signal $u_n = N - r_n$. Assuming that round trip delay is 1 time slot, dynamics of control signal can be written as following

$$u_{n+1} = \begin{cases} u_n & \text{if slot } n-1 \text{ is reserved and reservation is kept,} \\ u_n + 1 & \text{if slot } n-1 \text{ is reserved and reservation is lost,} \\ [u_n - I(Z_{n-1} = 1) \ \ by \ \ a \ \ CON \ \ VT]^+ & \text{if slot } n-1 \text{ is not reserved.} \end{cases}$$

Later in this section, we define permission probabilities as functions of control signal to maximize throughput of the system or minimize packet drop probability. It is noted that control using state estimation is a special case of General Price Based Control with $\phi = 1$, $\alpha = \xi = 0$, and $\beta = -1$. Therefore, as discussed before in analysis of General Price Based Control, the PRMA-HS Voice-Data system with state estimation can be modeled by a Markov chain.

Next, we consider Equilibrium Point Analysis to study equilibrium behavior of the PRMA voice-data system. We show if some conditions on system parameters are met, the system with state estimation control scheme has unique equilibrium point. However, in order to analytically prove that the state estimation scheme can control bifurcations that happen in the system, we consider the special cases of PRMA Voice and Data system.

3.4.1 PRMA Voice and Data System

In this subsection, we consider a PRMA system that employs both voice and data terminals. We assume the round trip delay is negligible and for mathematical simplicity, we assume that permission probabilities for both voice and data terminals are the same $p_v = p_d = p$. First, we consider a case where permission probability is chosen such that system throughput is maximized. Later, we consider a case where permission probability is chosen to minimize packet drop probability.

Before analyzing equilibrium equations of the system, we consider the Markov chain modeling the system.

Proposition 3.2. *The Markov chain defined on $\aleph$ through (3.2) (for $\phi = 0$, considering the system employs voice and data terminals, and round trip delay is negligible) is irreducible, aperiodic, and positive recurrent.*

Proof is presented in Appendix F.

Maximizing Throughput - EPA: Average system throughput at equilibrium for the PRMA Voice-Data system is

$$\eta = \frac{R}{N}(1-\gamma_f) + (1 - R/N)(C+B)pw(C,B),$$

here $\quad w(C,B) = \begin{cases} (1-p)^{C+B-1} & C+B \geq 1 \\ 1 & C+B < 1. \end{cases}$

Depending on the equilibrium value of $C+B$, maximum throughput occurs at:

$$p^* = \begin{cases} \frac{1}{C+B} & C+B \geq 1 \\ 1 & C+B < 1. \end{cases}$$

Given $C = M_v - (1 + \frac{\gamma_f}{N\sigma_v})R$ and $B = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$, equilibrium equations are as follows

$$\frac{(M_v - C)\gamma_f \omega + (M_d - B)\sigma_d}{(1 - (M_v - C)\omega)} - W(C) = 0, \tag{3.8}$$

$$W(C) = \begin{cases} (1 - \frac{1}{C+B})^{C+B-1} & C+B \geq 1 \\ (C+B)p_{max} & C+B < 1, \end{cases}$$

Parameter p_{max} can be chosen to be very close to 1. The following Lemma summarizes one sufficient condition for system parameters such that the PRMA voice-data system will have a unique operating point.

Remark 3.2. *Equation (3.8) is derived with consideration that* $R = \min(N, N\omega(M_v - C))$ *and* $B = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$, *where* $\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}$. *Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$, $B \in [0, M_d]$, *and* $R \in [0, N]$. *However, it can easily be shown that* $C = 0$, $C = M_v - \frac{1}{\omega}$, $C = M_v$, $B = 0$, $B = M_d$, $R = 0$, $R = N$, *or a combination thereof, cannot be solutions to equilibrium equation (3.8).*

Lemma 3.2. *Let us define* $W^+(C) = (1 - \frac{1}{C+B})^{C+B-1}$. *Here* $B(C) = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$. *Set of equations (3.8) has a unique solution in* C *and the system has a single operating point if any of conditions* (1), (2), *or* (3) *below hold:*

(1) $\quad \frac{\gamma_f \omega}{\sigma_d} \geq 1,\ M_v < \hat{C} + \frac{1}{\omega},\ and$

$$\frac{(M_v - \hat{C})\omega\gamma_f + (M_d - \hat{B})\sigma_d}{1 - (M_v - \hat{C})\omega} < (1 - \frac{1}{M_v + M_d})^{M_v + M_d - 1},$$

(2) $\quad \frac{\gamma_f \omega}{\sigma_d} < 1,\ M_v > \hat{C} + \frac{1}{\omega},\ and\ -\gamma_f\omega(1 + \frac{M_d}{M_v}) < \frac{dW^+}{dC}(\tilde{C}),$

(3) $\quad \frac{\gamma_f \omega}{\sigma_d} < 1,\ M_v < \hat{C} + \frac{1}{\omega},\ and\ either$

$$-\gamma_f\omega(1 + \frac{M_d}{M_v}) < \frac{dW^+}{dC}(\hat{C}), \quad or$$
$$\frac{(M_v - \hat{C})\omega\gamma_f + (M_d - \hat{B})\sigma_d}{1 - (M_v - \hat{C})\omega} < (1 - \frac{1}{M_v + M_d})^{M_v + M_d - 1},$$

Here $\tilde{C} = M_v - \frac{1}{\omega},\ \hat{C} + \hat{B} = 1,\ and\ \hat{B} = B(\hat{C}).$

Proof. We define $g(C) = \frac{(M_v - C)\omega\gamma_f + (M_d - B)\sigma_d}{1 - (M_v - C)\omega}$. Solutions of set of equations (3.8) are the same as fixed points of $g(C) = W(C)$. It is easy to show that

$$\begin{cases} \frac{dW}{dC} < 0 & C + B \geq 1 \\ \frac{dW}{dC} > 0 & C + B < 1 \end{cases} \qquad \begin{cases} \frac{d^2W}{dC^2} > 0 & C + B \geq 1 \\ \frac{d^2W}{dC^2} < 0 & C + B < 1. \end{cases}$$

Also, it can be shown that

$$\frac{dg}{dC} < 0, \quad and \quad \frac{d^2g}{dC^2} > 0 \quad if \quad \frac{\gamma_f\omega}{\sigma_d} < 1.$$

Note that $\max(0, M_v - \frac{1}{\omega}) < C < M_v$ and

$$W(M_v) = (1 - \frac{1}{M_v + M_d})^{M_v + M_d - 1} > g(M_v) = 0.$$

We define $\hat{C}$ such that $B(\hat{C}) + \hat{C} = 1$. Since $B(C)$ is a strictly increasing function of C, it is easy to show that $\hat{C}$ is unique. Also, in order to simplify notations, we define $\tilde{C} = M_v - \frac{1}{\omega}$. Now based on these facts and in order to derive conditions such that $W(C) = g(C)$ has exactly one solution, we consider following different cases:

- **Case 1:** $\frac{\gamma_f \omega}{\sigma_d} < 1$

  1. $M_v > \hat{C} + \frac{1}{\omega}$: In this case, $g(C)$ is positive and strictly decreasing with a positive second derivative. Also, $W(C)$ is positive and strictly decreasing with positive second derivative, $W(M_v - \frac{1}{\omega}) < g((M_v - \frac{1}{\omega})^+)$, and $W(M_v) > g(M_v)$. Therefore, similar to previous uniqueness Lemmas, if the system parameters are chosen such that $\frac{dg}{dC}(M_v) < \frac{dW}{dC}(\tilde{C})$, it can be shown that $\frac{dg}{dC} < \frac{dW}{dC}$ for the desirable range of C and therefore, $g(C) = W(C)$ has a unique solution.

  2. $M_v < \hat{C} + \frac{1}{\omega}$: In this case, if $\frac{dg}{dC}(M_v) < \frac{dW}{dC}(\tilde{C})$ or $g(\hat{C}) < W(M_v)$, it is guaranteed that the $g(C) = W(C)$ has a unique solution.

- **Case 2:** $\frac{\gamma_f \omega}{\sigma_d} \geq 1$

  1. $\frac{1}{\omega} < M_v < \hat{C} + \frac{1}{\omega}$: In this case, if $g(\hat{C}) < W(M_v)$, it is guaranteed that the equation has a unique solution.

  2. $0 < M_v < \frac{1}{\omega}$: In this case, $g(C)$ is positive and strictly decreasing. Also, $W(C)$ is positive, strictly increasing for $0 < C < \hat{C}$, and strictly decreasing for $\hat{C} \leq C \leq M_v$. In the same way as above, if $g(\hat{C}) < W(M_v)$, it is easy to show that $W(C) = g(C)$ has exactly one solution.

It can easily be shown that B is a one-to-one function of C. Therefore, set of equations (3.8) has a unique solution in (C, B) for $\max(0, M_v - \frac{1}{\omega}) < C < M_v$ and $0 < B < M_d$. □

Minimizing Packet Drop Probability - EPA Average packet drop probability at equilibrium is

$$P_{drop} = \gamma_f \frac{\nu^D}{1-(1-\gamma_f)\nu^N},$$

where $\nu = \nu(C, R, B, p) = 1 - (1 - \frac{R}{N})p(1-p)^{C+B}$. It can be shown that packet drop probability is minimized if

$$p^* = \frac{1}{C+B+1}$$

Given $C = M_v - (1 + \frac{\gamma_f}{N\sigma_v})R$ and $B = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$, equilibrium equations are as follows

$$\frac{(M_v - C)\gamma_f\omega + (M_d - B)\sigma_d}{(1-(M_v - C)\omega)} - W(C) = 0, \tag{3.9}$$

$$W(C) = \begin{cases} \frac{C+B}{C+B+1}(1 - \frac{1}{C+B+1})^{C+B-1} & C+B \geq 1 \\ \frac{C+B}{C+B+1} & C+B < 1, \end{cases}$$

Lemma 3.3. *Let us define* $W^+(C) = (1 - \frac{1}{C+B+1})^{C+B}$. *Here* $B(C) = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$. *Set of equations (3.9) has a unique solution in* C *and the system has a single operating point if any of conditions* (1), (2), *or* (3) *below hold:*

(1) $\frac{\gamma_f \omega}{\sigma_d} \geq 1$, $M_v < \hat{C} + \frac{1}{\omega}$, *and*

$$\frac{(M_v - \hat{C})\omega\gamma_f + (M_d - \hat{B})\sigma_d}{1 - (M_v - \hat{C})\omega} < (1 - \frac{1}{M_v + M_d + 1})^{M_v + M_d},$$

(2) $\frac{\gamma_f \omega}{\sigma_d} < 1$, $M_v > \hat{C} + \frac{1}{\omega}$, *and* $-\gamma_f\omega(1 + \frac{M_d}{M_v}) < \frac{dW^+}{dC}(\tilde{C})$,

(3) $\frac{\gamma_f \omega}{\sigma_d} < 1$, $M_v < \hat{C} + \frac{1}{\omega}$, *and either*

$$-\gamma_f\omega(1 + \frac{M_d}{M_v}) < \frac{dW^+}{dC}(\hat{C}), \quad \text{or}$$

$$\frac{(M_v - \hat{C})\omega\gamma_f + (M_d - \hat{B})\sigma_d}{1 - (M_v - \hat{C})\omega} < (1 - \frac{1}{M_v + M_d + 1})^{M_v + M_d},$$

Here $\tilde{C} = M_v - \frac{1}{\omega}$, $\hat{C} + \hat{B} = 1$, *and* $\hat{B} = B(\hat{C})$.

Proof is similar to proof of Lemma 3.2.

3.5 Bifurcation Control Using Multiple Transmission Power Levels

In this section, we examine effects of power capture phenomenon in controlling bifurcations in the PRMA-HS system employing voice and data terminals. We will show that using multiple transmission power levels at terminals and using capture effect at base station, we can control bifurcations by postponing them for higher values of system parameters. Hence, power capture phenomenon can increase capacity of the PRMA-HS system by allowing more voice and data terminals sharing common communication medium.

Equilibrium equations for the PRMA-HS system can be written as following

$$\begin{aligned} F_1(\underline{X}) &= M_v - C - (\frac{\gamma_f}{\sigma_v} + N)(\frac{Cp_vW_v(C,H_v,B,H_d)}{\gamma_f + Cp_vW_v(C,H_v,B,H_d)}) = 0, \\ F_2(\underline{X}) &= M_d - B - (\frac{N}{d} + \frac{1}{\sigma_d})\gamma_f(\frac{Bp_dW_d(C,H_v,B,H_d)}{\gamma_f + Cp_vW_v(C,H_v,B,H_d)}) = 0. \end{aligned} \tag{3.10}$$

here

$$H_v = \gamma_f(\frac{N}{d})\omega(M_v - C), \qquad H_d = (\frac{N\sigma_d}{N\sigma_d + d})(M_d - B), \qquad \underline{X} = (C \quad B),$$

$$W_v(C,H_v,B,H_d) = \sum_{h=1}^{m} v_h(C,H_v,B,H_d),$$

$$W_d(C,H_v,B,H_d) = \sum_{h=1}^{m} d_h(c,h_v,b,h_d),$$

$$v_h(C,H_v,B,H_d) = \begin{cases} q_h(1 - p_v\sum_{t=1}^{h} q_t)^{C+H_v-1}(1 - p_d\sum_{t=1}^{h} q_t)^{B+H_d} & C \geq 1, \\ q_h(1 - p_v\sum_{t=1}^{h} q_t)^{H_v}(1 - p_d\sum_{t=1}^{h} q_t)^{B+H_d} & C < 1, \end{cases}$$

$$d_h(C,H_v,B,H_d) = \begin{cases} q_h(1 - p_v\sum_{t=1}^{h} q_t)^{C+H_v}(1 - p_d\sum_{t=1}^{h} q_t)^{B+H_d-1} & B \geq 1, \\ q_h(1 - p_v\sum_{t=1}^{h} q_t)^{C+H_v}(1 - p_d\sum_{t=1}^{h} q_t)^{H_d} & B < 1. \end{cases}$$

Note that round trip delay (RTD) is $\frac{N}{d}$ time slots. Effects of using multiple transmission power levels at terminals and using capture effect at base station on the stability of the system is examined in the numerical results section of this chapter.

3.6 Performance Analysis of PRMA and PRMA-HS Voice-Data over Random Packet Error Channel

In previous sections, we considered a PRMA-HS system employing voice and data terminals on an uplink channel where collisions were only source of error. In this section, we assume that uplink channel is a "random packet error channel" [25] and we study the PRMA-HS system with the General Price Based Control. The analysis of this section closely follows study of PRMA system with General Price Based Control over random error channel in previous chapter.

3.6.1 General Price Based Control

Figures 3.1 and 3.2 illustrate Markov models for the PRMA-HS system over random packet error channel. In this section, we assume that when a contending voice terminal transmits a packet on an available time slot without any collision, it reserves that time slot if no packet header error happens. If there is a header packet error, the access point (base station and/or satellite) interprets the error either as a collision or as an event that no packet was transmitted [25]. If there is a packet header error in a contending packet, we assume that with a fixed probability q the base station sends a collision feedback and with probability $1-q$ it sends an idle feedback. We define following events: $A_n = \{$at an available time slot $n, 1$ packet transmitted with error - access point assumed idle$\}$ and $B_n = \{$at an available time slot $n, 1$ packet transmitted with error - access point assumed collision$\}$.

Therefore, control signal update algorithm (3.1) is changed as follows. As in pervious sections, and only for notation purposes, we assume that $d = N$ and therefore, round trip delay is one time slot.

$$u_{n+1} = \begin{cases} u_n & \text{if slot } n-1 \text{ is reserved and reservation is kept,} \\ u_n + \phi & \text{if slot } n-1 \text{ is reserved and reservation is lost,} \\ [u_n - \alpha I(Z_{n-1} = 0 \vee A_{n-1}) + \beta I(Z_{n-1} = 1 \wedge \mathit{no\ error}) & \\ \quad + \xi I(Z_{n-1} \geq 2 \vee B_{n-1}]^+ & \text{if slot } n-1 \text{ is not reserved.} \end{cases} \tag{3.11}$$

Note that in random packet error channel, control signal at time slot $n+1$ is decreased by α if time slot $n-1$ was either idle or a successful transmission by a contending or hindering terminal contained a packet header error and the base station assumed an idle time slot. Also, control signal is increased by ξ if either time slot $n-1$ had a collision or a packet header error in a successful transmission by a contending or hindering terminal, which was assumed to be collision by the base station. In the case of a successful transmission with no packet header error, control signal is updated by β. If time slot $n-1$ was reserved and the reservation is lost (either because of packet header error or because all packet of voice message are transmitted) control signal is increased by ϕ.

Transition probabilities for this Markov chain can be written as before considering packet random error. It is noted that the Markov chain modeling the

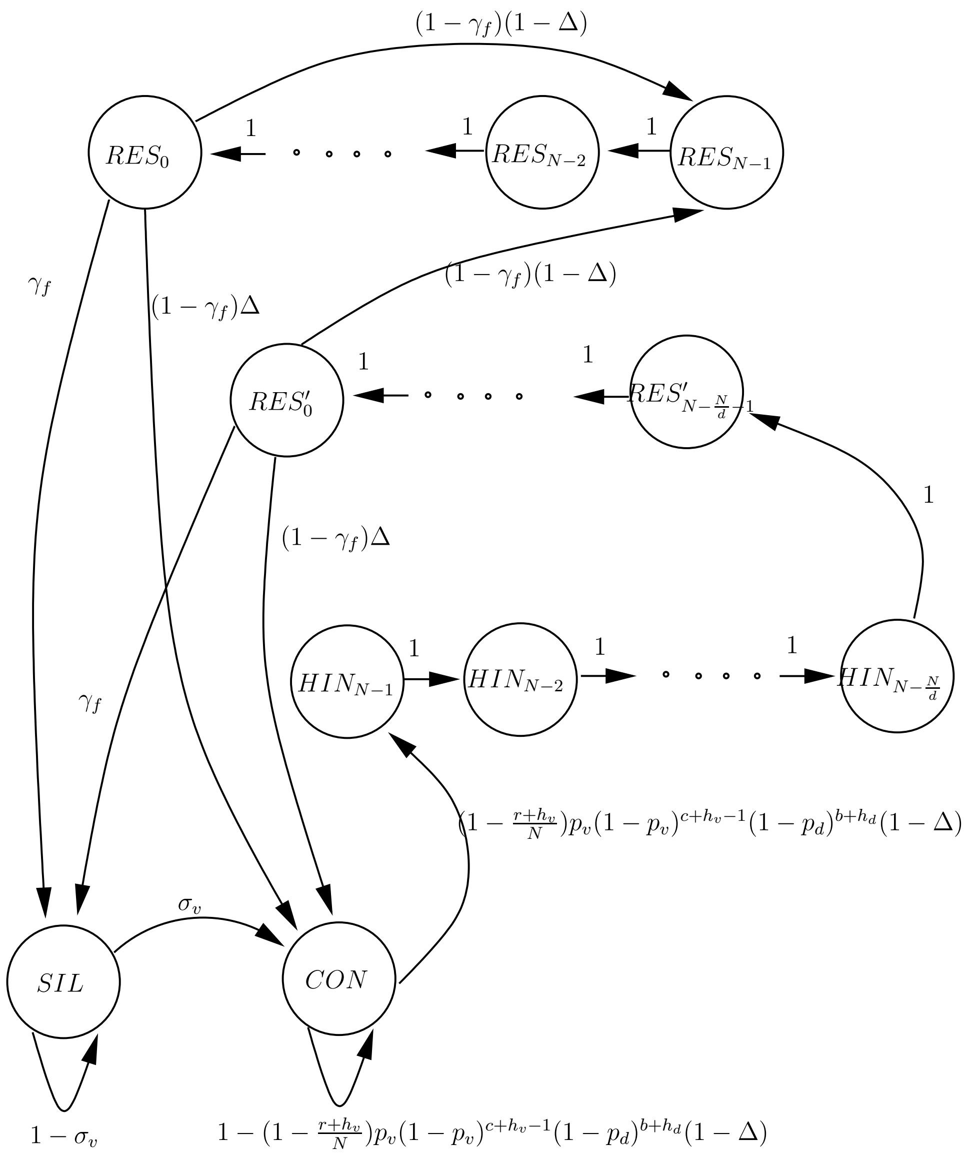

Figure 3.1: Markov Chain Model for PRMA-HS Voice Subsystem over Random Packet Error Channel.

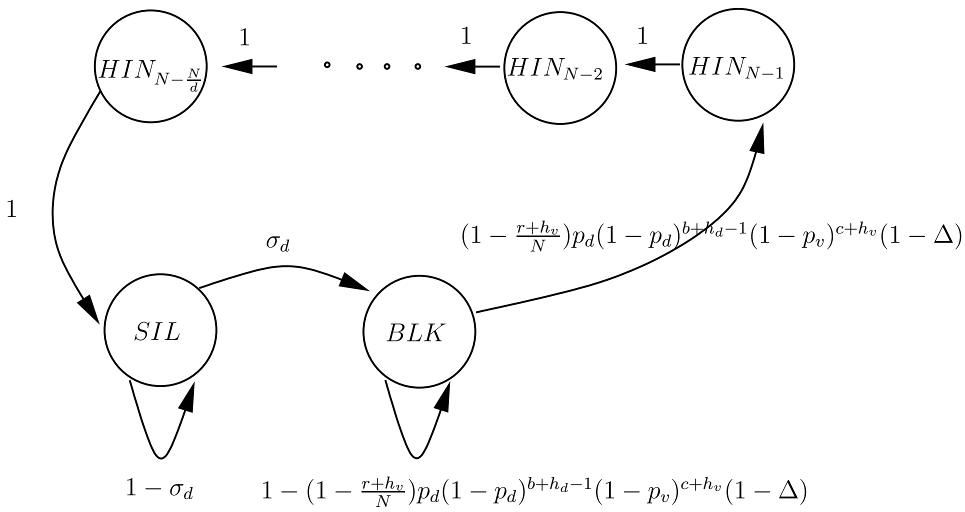

Figure 3.2: Markov Chain Model for PRMA-HS Data Subsystem over Random Packet Error Channel.

PRMA-HS system over random packet error channel has the same state space $\aleph$ as PRMA-HS system.

3.6.2 Price Based Control

An special case of General Price Based Control is for $\phi = 0$. Transition probabilities for this Markov chain can be written as before considering packet random error. It is mentioned that the Markov chain modeling the PRMA-HS system over random packet error channel has the same state space $\aleph$ as PRMA-HS system. We can prove that this Markov model is irreducible, aperiodic, and under Assumption 3.1, the Markov chain is positive recurrent.

Next, we study performance of the PRMA-HS system with price based control scheme over random packet error channel using the equilibrium point approach. Same analysis as in previous sections can be used. A point (C, R^*, H_v, B, H_d, U) is an equilibrium point of the system if:

$$
\begin{aligned}
&(M_v - C - R^* - H_v)\sigma_v + (\frac{R^* + H_v}{N})(1-\gamma_f)\Delta \\
&\qquad\qquad\qquad - (1 - \frac{R^* + H_v}{N})Cp_v w_v(C,B)(1-\Delta) = 0, \\
&(\frac{R^* + H_v}{N})\gamma_f - (M_v - C - R^* - H_v)\sigma_v = 0, \\
&(R/N) - (\frac{R^* + H_v}{N})(1-\gamma_f)(1-\Delta) = 0, \\
&(M_d - B - H_d)\sigma_d - (1 - \frac{R^* + H_v}{N})Bp_d w_d(C,B)(1-\Delta) = 0, \\
&(M_d - B - H_d)\sigma_d - (\frac{d}{N})H_d = 0, \\
&(-\alpha(1-q)\Delta + \beta(1-\Delta) - \xi(1-q\Delta))[(C+H_v)p_v w_v(C,B) + (B+H_d)p_d w_d(C,B)] \\
&\qquad - (\alpha+\xi)(1-p_v)^{C+H_v}(1-p_d)^{B+H_d} + \xi = 0.
\end{aligned}
\tag{3.12}
$$

Here $p_v = p_v(U)$, $p_d = p_d(U)$, $R^* = R + R'$, and $\frac{R'}{N-N/d} = \frac{H_v}{N/d}$. Next, we consider a specific system, PRMA system with voice and data terminals. We derive a set of conditions on the control parameters such that these systems have a unique equilibrium point.

3.6.3 PRMA Voice and Data

Here, we consider a PRMA system that employs both voice and data terminals. We assume that round trip delay is negligible and from now on in this subsection we assume that permission probabilities for both voice and data terminals are the same, $p_v = p_d = p$. Set of equations (3.12) for a PRMA system with voice and data terminals is summarized as following

$$
\begin{aligned}
&(M_v - C - R)\sigma_v + (R/N)(1-\gamma_f)\Delta - (1-R/N)Cpw(C,B,U)(1-\Delta) = 0,\\
&(R/N)\gamma_f - (M_v - C - R)\sigma_v = 0,\\
&(M_d - B)\sigma_d - (1-R/N)Bpw(C,B,U)(1-\Delta) = 0,\\
&(-\alpha(1-q)\Delta + \beta(1-\Delta) - \xi(1-q\Delta))(C+B)pw(C,B,U)\\
&\qquad\qquad - (\alpha+\xi)(1-p)^{C+B} + \xi = 0. \qquad (3.13)
\end{aligned}
$$

Here $p = p(U)$ and $w(C,B,U) = \begin{cases} (1-p(U))^{C+B-1} & C+B \geq 1 \\ 1 & C+B < 1. \end{cases}$

Remark 3.3. *Based on the second equation of the set of equations (3.13), it can be shown that* $R = \min(N, N\omega(M_v - C))$, *where* $\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}$. *Also, based on first and third equations of the set of equations (3.6), it can easily be shown that* $B(C) = \frac{M_d C}{(M_v - C)\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{\sigma_d} + C}$. *Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$, $R \in [0, N]$, $B \in [0, M_d]$, *and* $p \in [0,1]$. *However, it can easily be shown, using the set of equation (3.13), that* $C = 0$, $M_v - \frac{1}{\omega}$, $C = M_v$, $B = 0$, $B = M_d$, $R = 0$, $R = N$, $p = 0$, $p = 1$ *(for* $\beta = \frac{\alpha(1-q)\Delta + \xi(1-q\Delta)}{1-\Delta}$ *as considered in Lemma 3.4), or a combination thereof, cannot be solutions to the set of equilibrium equations (3.13).*

Lemma 3.4. *Let us define* $f^+(C) = \frac{\xi}{\alpha+\xi}(C+B)(\exp(\frac{-1}{C+B}\ln(\frac{\xi}{\alpha+\xi})) - 1)$. *Here* $B(C) = \frac{M_d C}{(M_v - C)\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{\sigma_d} + C}$. *There exists a set of control parameters* (α, β, ξ) *for which the set of equations (3.13) has a unique solution in* (C, U) *and the system has a single operating point if any of conditions* (1), (2) *or* (3) *below hold:*

(1) $\frac{\gamma_f \omega}{\sigma_d} < 1$, $M_v - \frac{1}{\omega} > \hat{C}$, *and* $-\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{1-\Delta}(1 + \frac{M_d}{M_v}) < \frac{df^+}{dC}(\tilde{C})$,

(2) $\frac{\gamma_f \omega}{\sigma_d} < 1$, $M_v - \frac{1}{\omega} < \hat{C}$, *and either*

$$-\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{1-\Delta}(1 + \frac{M_d}{M_v}) < \frac{df^+}{dC}(\hat{C}). \quad or$$

$$\frac{(M_v - \hat{C})\omega(\gamma_f + (1-\gamma_f)\Delta) + (M_d - \hat{B})\sigma_d}{(1-\Delta)(1-(M_v - \hat{C}))\omega} \leq$$

$$\frac{\xi}{\alpha+\xi}(M_v + M_d)(\exp(\frac{-1}{M_v + M_d}\ln(\frac{\xi}{\alpha+\xi})) - 1),$$

(3) $\frac{\gamma_f \omega}{\sigma_d} \geq 1$, $M_v < \frac{1}{\omega} + \hat{C}$, *and*

$$\frac{(M_v - \hat{C})\omega(\gamma_f + (1-\gamma_f)\Delta) + (M_d - \hat{B})\sigma_d}{(1-\Delta)(1-(M_v - \hat{C}))\omega} \leq$$

$$\frac{\xi}{\alpha+\xi}(M_v + M_d)(\exp(\frac{-1}{M_v + M_d}\ln(\frac{\xi}{\alpha+\xi})) - 1).$$

Here $\tilde{C} = M_v - \frac{1}{\omega}$, $\hat{C} + \hat{B} = 1$, *and* $\hat{B} = B(\hat{C})$.

Proof of Lemma 3.4 is similar to proof of Lemma 3.1 considering $\beta = \frac{\alpha(1-q)\Delta + \xi(1-q\Delta)}{1-\Delta}$.

3.6.4 State Estimation-Based Control

In this subsection, we consider state estimation-based control over random packet error channel for a PRMA voice-data system, a special case of General Price Based Control for $\phi = 1$, $\alpha = \xi = 0$, and $\beta = -1$.

Maximizing Throughput - EPA: Depending on the equilibrium value of $C + B$, maximum throughput occurs at:

$$p^* = \begin{cases} \frac{1}{C+B} & C + B \geq 1 \\ p_{max} & C + B < 1. \end{cases}$$

Equilibrium equations can be written as:

$$\frac{(M_v - C)\omega(\gamma_f + (1-\gamma_f)\Delta) + (M_d - B)\sigma_d}{(1-(M_v - C)\omega)(1-\Delta)} - W(C) = 0,$$
$$W(C) = \begin{cases} (1 - \frac{1}{C+B})^{C+B-1} & C + B \geq 1 \\ (C+B)p_{max} & C + B < 1, \end{cases} \tag{3.14}$$

here

$$\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}, \qquad B = \frac{M_d C}{(M_v - C)\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{\sigma_d} + C}.$$

Remark 3.4. *Equation (3.14) is derived with consideration that* $R = \min(N, N\omega(M_v - C))$ *and* $B = \frac{M_d C}{(M_v - C)\frac{\omega(\gamma_f+(1-\gamma_f)\Delta)}{\sigma_d} + C}$*, where* $\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}$*. Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{1}{\omega}), M_v]$*,* $B \in [0, M_d]$*, and* $R \in [0, N]$*. However, it can easily be shown that* $C = 0$*,* $C = M_v - \frac{1}{\omega}$*,* $C = M_v$*,* $B = 0$*,* $B = M_d$*,* $R = 0$*,* $R = N$*, or a combination thereof, cannot be solutions to equilibrium equation (3.14).*

Lemma 3.5. *Let us define* $W^+(C) = (1 - \frac{1}{C+B})^{C+B-1}$*. Here* $B(C) = \frac{M_d C}{(M_v - C)\frac{\omega(\gamma_f+(1-\gamma_f)\Delta)}{\sigma_d} + C}$*. Set of equations (3.14) has a unique solution in* C *and the system has a single operating point if any of conditions* (1)*,* (2)*, or* (3) *below hold:*

(1) $\frac{\gamma_f \omega}{\sigma_d} \geq 1$*,* $M_v < \hat{C} + \frac{1}{\omega}$*, and*

$$\frac{(M_v - \hat{C})\omega(\gamma_f + (1-\gamma_f)\Delta) + (M_d - \hat{B})\sigma_d}{(1-(M_v - \hat{C})\omega)(1-\Delta)} < (1 - \frac{1}{M_v + M_d})^{M_v + M_d - 1},$$

(2) $\frac{\gamma_f \omega}{\sigma_d} < 1$*,* $M_v > \hat{C} + \frac{1}{\omega}$*, and* $-\frac{\omega(\gamma_f+(1-\gamma_f)\Delta)}{1-\Delta}(1 + \frac{M_d}{M_v}) < \frac{dW^+}{dC}(\tilde{C})$*,*

(3) $\frac{\gamma_f \omega}{\sigma_d} < 1$, $M_v < \hat{C} + \frac{1}{\omega}$, *and either*

$$-\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{1-\Delta}(1 + \frac{M_d}{M_v}) < \frac{dW^+}{dC}(\hat{C}), \quad or$$
$$\frac{(M_v - \hat{C})\omega(\gamma_f + (1-\gamma_f)\Delta) + (M_d - \hat{B})\sigma_d}{(1-(M_v - \hat{C})\omega)(1-\Delta)} < (1 - \frac{1}{M_v + M_d})^{M_v+M_d-1},$$

Here $\tilde{C} = M_v - \frac{1}{\omega}$, $\hat{C} + \hat{B} = 1$, *and* $\hat{B} = B(\hat{C})$.

Lemma 3.5 can be proved similar to proof of Lemma 3.2.

Minimizing Packet Drop Probability - EPA: Permission probability is chosen such that packet drop probability (of the error free system) is minimized:

$$p^* = \frac{1}{C+B+1}.$$

Equilibrium equations can be written as:

$$\frac{(M_v - C)\omega(\gamma_f + (1-\gamma_f)\Delta) + (M_d - B)\sigma_d}{(1-(M_v - C)\omega)(1-\Delta)} - W(C) = 0,$$
$$W(C) = \begin{cases} (\frac{C+B}{C+B+1})(1 - \frac{1}{C+B+1})^{C+B-1} & C+B \geq 1 \\ \frac{C+B}{C+B+1} & C+B < 1, \end{cases} \tag{3.15}$$

here

$$\omega = \frac{\sigma_v}{\gamma_f + N\sigma_v}, \qquad B = \frac{M_d C}{(M_v - C)\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{\sigma_d} + C}.$$

Lemma 3.6. *Let us define* $W^+(C) = (1 - \frac{1}{C+B+1})^{C+B}$. *Here* $B(C) = \frac{M_d C}{(M_v - C)\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{\sigma_d} + C}$. *Set of equations (3.15) has a unique solution and the system has a single operating point under either condition* (1), (2), *or* (3) *below:*

(1) $\frac{\gamma_f \omega}{\sigma_d} \geq 1$, $M_v < \hat{C} + \frac{1}{\omega}$, *and*

$$\frac{(M_v - \hat{C})\omega(\gamma_f + (1-\gamma_f)\Delta) + (M_d - \hat{B})\sigma_d}{(1-(M_v - \hat{C})\omega)(1-\Delta)} < (1 - \frac{1}{M_v + M_d + 1})^{M_v+M_d},$$

(2) $\frac{\gamma_f \omega}{\sigma_d} < 1,\ M_v > \hat{C} + \frac{1}{\omega},\ and\ -\frac{\omega(\gamma_f+(1-\gamma_f)\Delta)}{1-\Delta}(1+\frac{M_d}{M_v}) < \frac{dW^+}{dC}(\tilde{C}),$

(3) $\frac{\gamma_f \omega}{\sigma_d} < 1,\ M_v < \hat{C} + \frac{1}{\omega},\ and\ either$

$$-\frac{\omega(\gamma_f + (1-\gamma_f)\Delta)}{1-\Delta}(1+\frac{M_d}{M_v}) < \frac{dW^+}{dC}(\hat{C}), \quad or$$
$$\frac{(M_v - \hat{C})\omega(\gamma_f + (1-\gamma_f)\Delta) + (M_d - \hat{B})\sigma_d}{(1-(M_v-\hat{C})\omega)(1-\Delta)} < (1 - \frac{1}{M_v + M_d + 1})^{M_v + M_d},$$

$Here\ \tilde{C} = M_v - \frac{1}{\omega},\ \hat{C} + \hat{B} = 1,\ and\ \hat{B} = B(\hat{C}).$

Proof is similar to proof of Lemma 3.3.

3.7 Numerical Results

Next, we consider a PRMA voice and data system with following parameters [23]: $R_c = 720,000$ bits/s, $R_s = 32,000$ bits/s, $T = 0.016$ s, $H = 64$ bits, $d = 1$, $D_{max} = 0.032$ s, $t_1 = 1.00$ s, $t_2 = 1.35$ s, and $\sigma_d = 0.0002$. Figures 3.3, 3.4, and 3.5 illustrate packet drop probability (P_{drop}) for the system when M_v is taken as bifurcation parameter, and no control is used. In these figures, permission probability is fixed at $p = 0.1$, $p = 0.2$, $p = 0.3$, and $M_d = 25$. Also, figure 3.6 illustrates bifurcation diagram for P_{drop} when M_d is bifurcation parameter, $M_v = 30$, and $p = 0.2$.

We design a price based control for the same voice-data PRMA system and compare bifurcation diagrams. We choose $\beta = \xi = 1$. Control parameter α is chosen to be $\alpha = 0.0532$ such that the number of contending terminals for $M_v = 32$ is $C = 0.3376$. $M_v = 32$ is the capacity of the system for fixed value of $p = 0.1$ where the number of contending voice terminals is $C = 0.3376$. Figure 3.7 shows bifurcation diagram for packet drop probability for M_v as bifurcation parameter. We can see that by using the price based control, bifurcations are eliminated. Moreover, capacity of the PRMA system has increased to $M_v = 32$ and $M_d = 25$ from $M_v = 20$ and $M_d = 25$ when $p_v = 0.2$ and from $M_v = 5$ and $M_d = 25$ when $p_v = 0.3$.

Also, we consider effects of the price based control on the PRMA system when M_d is chosen as bifurcation parameter. Figure 3.8 is bifurcation diagram for controlled PRMA system with $\alpha = 0.0644$ and $\beta = \xi = 1$. The control parameter α is chosen such that at $M_v = 30$, $M_d = 57$, and $p = 0.1$ the number of contending voice terminals is 0.2805. $M_v = 30$ and $M_d = 57$ is capacity of uncontrolled system.

Further, we design a control scheme based on above-noted state estimation minimizing system throughput. We define $p_{v_{max}} = 0.9$. Figure 3.9 shows bifurcation diagram for packet drop probability. In this figure, the total number of voice terminals (M_v) is chosen as bifurcation parameter. It is observed that bifurcations in operating points of the system is eliminated and capacity of the system is increased to $M_v = 43$ and $M_d = 25$. Capacity of the PRMA system has increased from $M_v = 32$ and $M_d = 25$ when $p_v = 0.1$, $M_v = 20$ and $M_d = 25$ when $p_v = 0.2$, and from $M_v = 5$ and $M_d = 25$ when $p_v = 0.3$. Also, figure 3.10 shows bifurcation diagram for packet drop probability of the controlled system as M_d is bifurcation parameter. It is noted that in this case bifurcations are not completely eliminated. However, the voice-data system is controlled in a sense that bifurcations are delayed for larger values of the bifurcation parameter. Capacity of the system is increased to $M_v = 30$ and $M_d = 679$ from $M_v = 30$ and $M_d = 14$ when $p_v = 0.2$.

Figure 3.11 illustrates bifurcation diagram for a system with state estimation control scheme minimizing packet drop probability. It is noted that bifurcations are eliminated and capacity of the system is $M_v = 43$ and $M_d = 25$. Also, figure 3.12 shows bifurcation diagram (for M_d as bifurcation parameter) for the system with state estimation control scheme minimizing P_{drop}. Bifurcations are delayed and capacity of the system is $M_v = 30$ and $M_d = 676$.

Next, we consider the same PRMA voice-data system over random packet error channel. Figure 3.13 shows bifurcation diagram for drop probability when M_v is chosen as bifurcation parameter and for two different values of Δ. Plus signs show packet drop probability for $\Delta = 0.1$ and points are for $\Delta = 0.01$. In this case $M_d = 25$ and $p = 0.2$. For $\Delta = 0.01$ capacity of the system is $M_v = 19$ and $M_d = 25$ and for $\Delta = 0.1$ capacity is $M_v = 18$ and $M_d = 25$.

We use the price based control with $\alpha = 0.125$ and $\beta = \xi = 1$ as the control parameters. This value of α is chosen such that for $M_v = 39$, $M_d = 25$, and $p_d = 0.2$, the number of contending terminals is $C = 0.3837$ which results in maximum packet drop probability of less than 0.01. Figure 3.14 shows bifurcation diagrams for packet drop probability for both $\Delta = 0.01$ and $\Delta = 0.1$ when M_v is bifurcation parameter. It is seen that capacity of the system is increased to $M_v = 36$ and $M_d = 25$ for $\Delta = 0.01$ and $M_v = 21$ and $M_d = 25$ for $\Delta = 0.1$. Here we assumed that $q = 1$.

Figure 3.15 shows effects of the state estimation control scheme (maximizing throughput) on nonlinear behavior of the PRMA voice-data system. If error probability is $\Delta = 0.01$, although bifurcations of the operating points of the system is not completely eliminated (PRMA system has three equilibrium points at $M_v = 46$) but it is delayed for larger values of bifurcation parameter. Also, capacity of the

PRMA system is increased to $M_v = 43$ and $M_d = 25$. In the case where $\Delta = 0.1$, bifurcations are also not completely eliminated (PRMA system has three equilibrium points for $M_v = 39 - 42$), but they are controlled by delaying the bifurcations. In this case, capacity of the system is increased to $M_v = 38$ and $M_d = 25$.

Figure 3.16 illustrates bifurcation diagrams for the system with state estimation control (minimizing packet drop probability) over random packet error channel. In the case $\Delta = 0.01$, capacity of the system is $M_v = 43$ and $M_d = 25$. For the case $\Delta = 0.1$, capacity of the system is $M_v = 37$ and $M_d = 25$.

Figure 3.17 illustrates another exemplary bifurcation diagram for the PRMA voice-data system with $p_d = 0.044$, $p_v = 0.3$, $M_d = 36$, and M_v is the bifurcation parameter. The capacity of the system is $M_v = 23$ and $M_d = 36$ simultaneous voice and data terminals. Next, we examiner effects of using two power levels at the terminals and capture at the access point on the stability of the system. Figure 3.18 illustrates this case where each of the power levels is chosen with probability 0.5. It can be observed that although bifurcations are not completely eliminated, they occur at larger values of the bifurcation parameter and the capacity of the system is increased to $M_v = 43$ and $M_d = 36$ simultaneous voice and data terminals.

Further, we studied the effects of the studied control schemes (price based control and state estimation-based control (maximizing throughput)) by simulating the probabilistic behavior of a PRMA-HS voice+data system. We considered a system with both voice and data terminals that share a common communication medium using a PRMA-HS protocol, therefore, round trip delay is not negligible. Also, in order to compare the behavior of the studied control schemes with two previously presented control schemes, we used the exponential scheme introduced in [32] and the Bayesian scheme of [27] in the simulations. We assumed similar system parameters are in effect, $R_c = 720,000$ bits/s, $R_s = 32,000$ bits/s, $T = 0.016$ s, $H = 64$ bits, $D_{max} = 0.032$ s, $t_1 = 1.00$ s, $t_2 = 1.35$ s, and $\sigma_d = 0.0002$. We considered $M_d = 25$ to be fixed and for two different values of the round trip delay (1 and 10 time slots) we simulated the behavior of the system for different values of total number of voice terminals (M_v). Table below represents the capacity of the system without a controller, with price based control, state estimation-based control (maximizing throughput), and Bayesian and exponential schemes. Figures 3.19 and 3.20 summarize the packet drop probability (P_{drop}) derived from simulations for different values of M_v for delay of 1 and 10 time slots. It is noted that differences that might exist between simulations and analysis can be the result of approximations in deriving packet drop probability expressions.

Control Scheme	Capacity (delay=1 slot)	Capacity (delay=10 slots)
No control ($p_v = 0.1$)	$M_v = 27$	$M_v = 19$
Price Based Control	$M_v = 35$	$M_v = 29$
Maximize η	$M_v = 33$	$M_v = 27$
Bayesian	$M_v = 31$	$M_v = 28$
Exponential	$M_v = 34$	$M_v = 16$

3.8 Summary and Future Lines of Work

In this chapter we focused our analysis on a voice and data system that employs PRMA as the medium access protocol. We studied the equilibrium behavior of the system and analyzed the effects of the price based control and the state estimation-based control on bifurcations of the system. These analysis were both over error-free and random error channels and we derived some sufficient conditions on system and the control parameters to guarantee uniqueness of operating point of the system. We further extended the multiple power level and capture effect to the PRMA voice and data system. For the analysis of this chapter, we assumed that permission probabilities for voice and data terminals are the same. For future line of research, a more general case can be considered. Further, future lines of research can include extending the analysis of this chapter to variations of voice-data PRMA system, more specifically, voice-data PRMA-HS systems, studying a dynamic control scheme based on state observation in addition to stat estimation, and extending the bifurcation analysis using multiple power levels to more accurate models of capture that deals with signal-to-interface-plus-noise ration and different distances of terminals from the access point. Also, for future work, the robustness of the control schemes can be studied, for example, when the system parameters are not exactly known. Further, similar price based control can be used to control rate of generation of voice and/or data messages and also to control the average number of voice packets in a voice message.

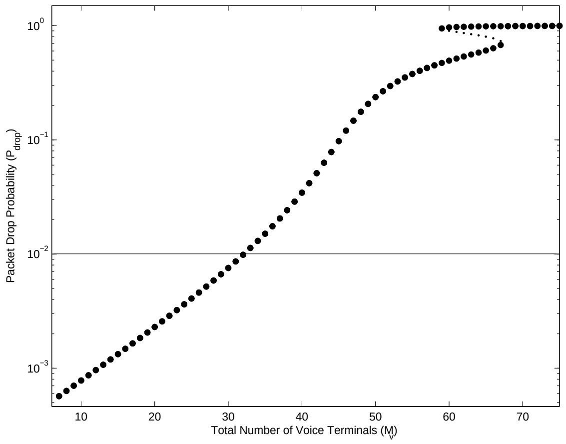

Figure 3.3: Bifurcation diagram for packet drop probability with no control ($p = 0.1$ and M_v is bifurcation parameter)

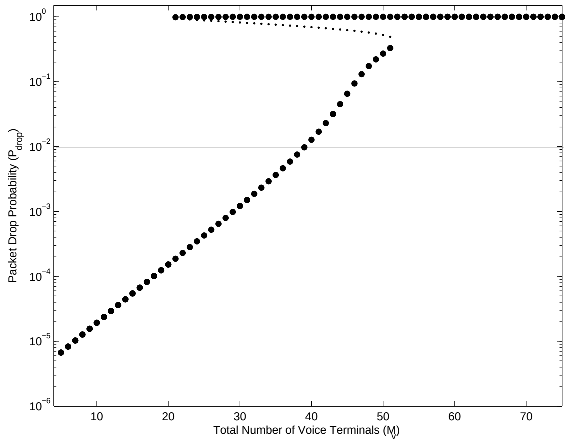

Figure 3.4: Bifurcation diagram for the packet drop probability with no control ($p = 0.2$ and M_v is bifurcation parameter)

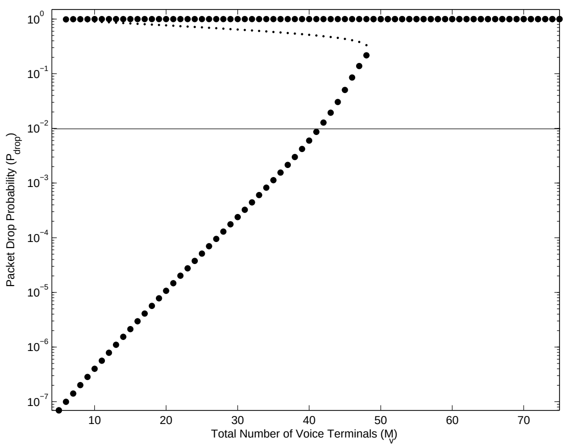

Figure 3.5: Bifurcation diagram for packet drop probability with no control ($p = 0.3$ and M_v is bifurcation parameter)

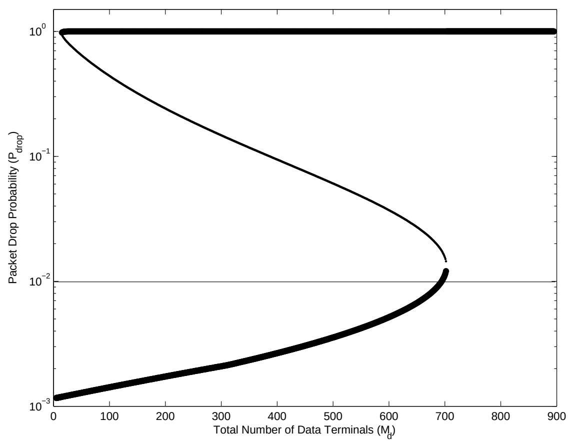

Figure 3.6: Bifurcation diagram for the packet drop probability with no control ($p = 0.2$ and M_d is bifurcation parameter)

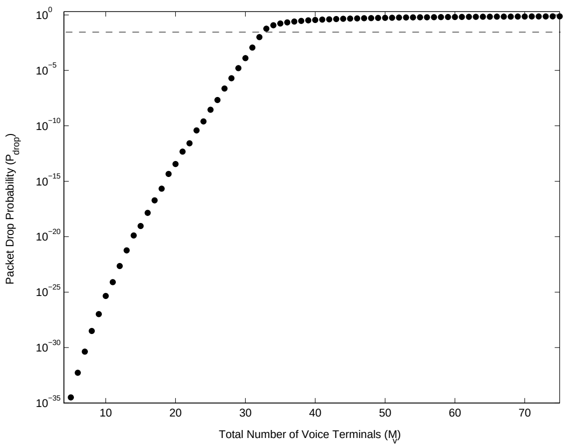

Figure 3.7: Bifurcation diagram for packet drop probability with price based bifurcation control ($\alpha = 0.0532$, $\beta = \xi = 1$, M_v bifurcation parameter)

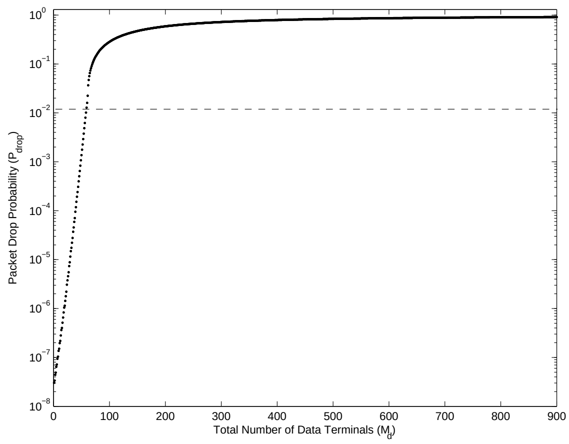

Figure 3.8: Bifurcation diagram for packet drop probability with the price based bifurcation control ($\alpha = 0.0644$, $\beta = \xi = 1$, M_d bifurcation parameter)

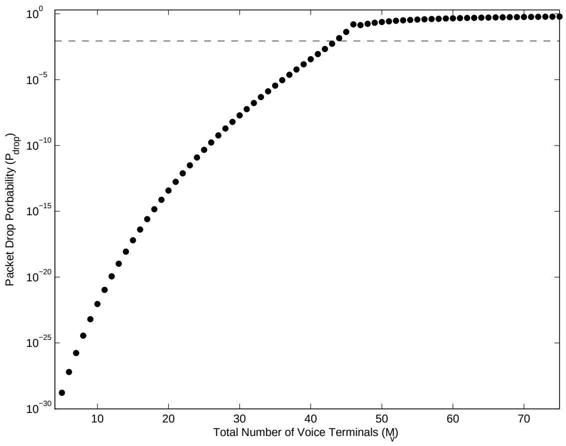

Figure 3.9: Bifurcation diagram for packet drop probability with state estimation control (maximizing throughput) ($p_{max} = 0.9$ and M_v bifurcation parameter)

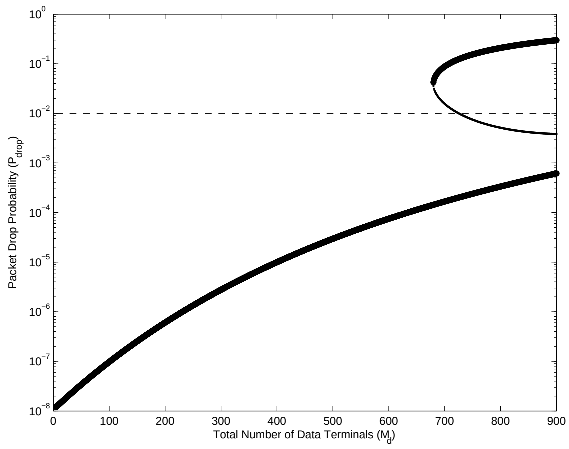

Figure 3.10: Bifurcation diagram for packet drop probability with state estimation control (maximizing throughput)($p_{max} = 0.9$ and M_d is bifurcation parameter)

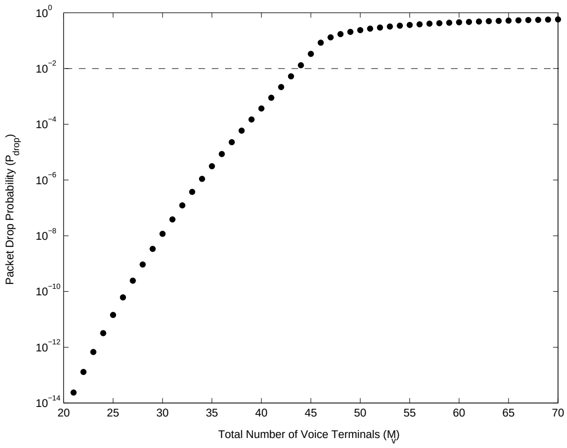

Figure 3.11: Bifurcation diagram for packet drop probability with state estimation control (minimizing P_{drop}) (M_v bifurcation parameter)

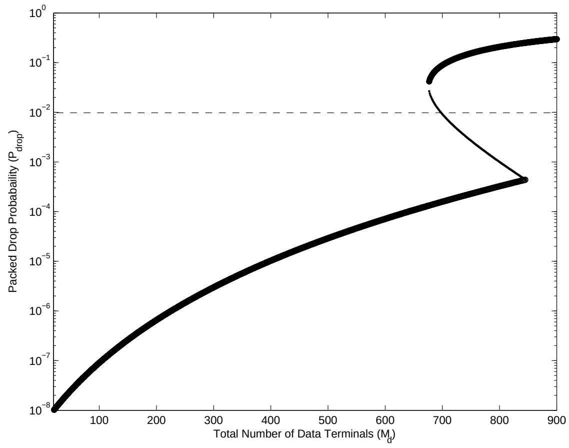

Figure 3.12: Bifurcation diagram for packet drop probability with state estimation control (minimizing P_{drop}) (M_d bifurcation parameter)

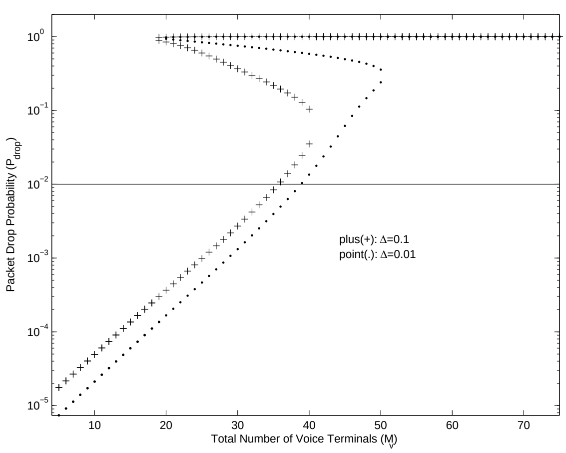

Figure 3.13: Bifurcation diagram for packet drop probability with no control ($p = 0.2$ and two different values for Δ)

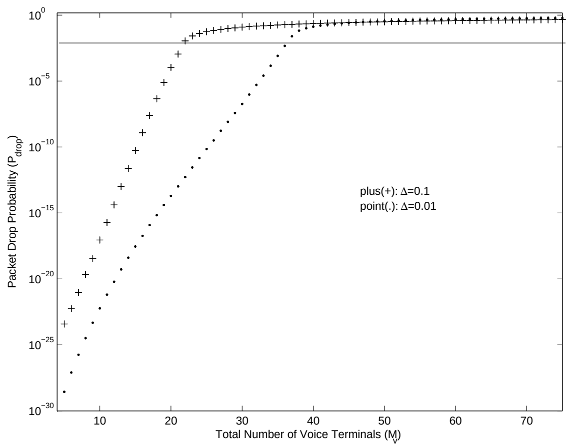

Figure 3.14: Bifurcation diagram for packet drop probability with control ($\Delta = 0.1$, $\Delta = 0.01$, $\alpha = 0.0532$, $\beta = \xi = 1$)

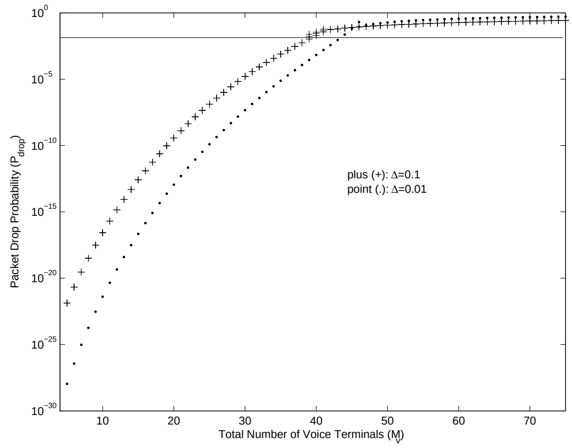

Figure 3.15: Bifurcation diagram for packet drop probability with state estimation control (maximizing throughput) ($\Delta = 0.1$, $\Delta = 0.01$, M_v bifurcation parameter, $p_{max} = 0.9$)

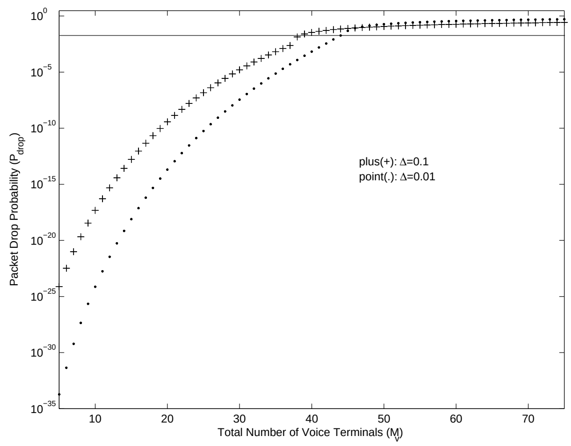

Figure 3.16: Bifurcation diagram for packet drop probability with state estimation control (minimizing P_{drop}) ($\Delta = 0.1$, $\Delta = 0.01$, M_v bifurcation parameter)

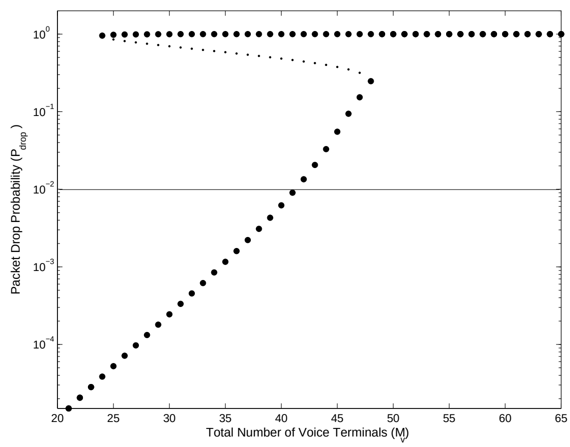

Figure 3.17: Bifurcation diagram for packet drop probability with no control ($p_v = 0.3$, $p_d = 0.044$, $M_d = 36$, M_v bifurcation parameter, $p_{max} = 0.9$)

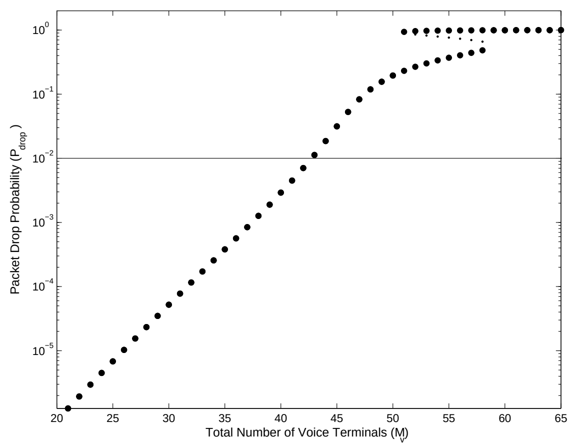

Figure 3.18: Bifurcation diagram for packet drop probability with capture effect ($q_1 = q_2 = 0.5$)

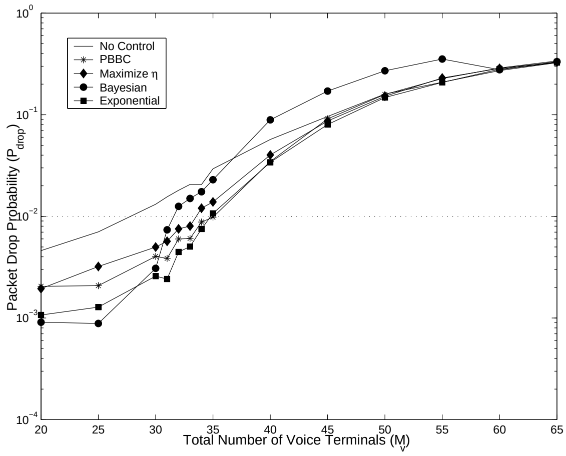

Figure 3.19: Simulation results for PRMA-HS voice+data system with delay of 1 time slot

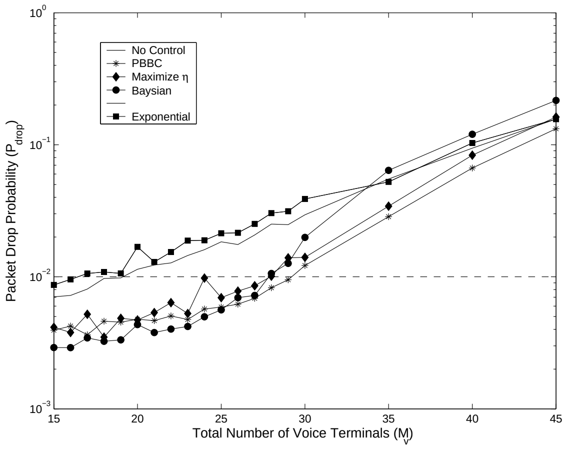

Figure 3.20: Simulation results for PRMA-HS voice+data system with delay of 10 time slot

Chapter 4

Stability Analysis for Finite Buffered Slotted ALOHA Protocol Using Tagged User Approach

As discussed before, bistable behavior in slotted ALOHA protocol is an important phenomenon. Changes in system parameters, transmits a system with a stable equilibrium point to a system with two stable equilibrium points, one with short delay and other one congested. Carleial and Hellman first noticed the bistable behavior of ALOHA type systems [15]. They demonstrated and analyzed this important aspect of the dynamics of ALOHA protocol. At the same time, Kleinrock and Lam showed the same bistable behavior in ALOHA protocol using an input-output packet flow balance principle [14]. That is, the system possesses two statistically stable equilibrium points, one in a desirable low-delay region, and the other in an undesirable high-delay region. Tasaka studied R-ALOHA using equilibrium point analysis [17]. The dynamic behavior of the protocol was studied and it was shown that under high traffic, the system has multiple equilibrium points. Onozato and Noguchi developed a new tool for performance evaluation of a multiaccess communication system. They gave an explicit analytical description of a cusp catastrophe in a computer communication system [18] and showed that sudden changes in behavior of slotted ALOHA system, which can be observed in throughput, average delay, and the average number of backlogged terminals, are induced by smooth alteration of the control parameters.

Also Fayolle *et al.* [54] showed that the slotted ALOHA channel with infinite population, poisson arrivals, and fixed retransmission probabilities is unstable. Many researchers studied the unstable behavior of the slotted ALOHA system and proposed retransmission control schemes in order to stabilize the channel. Fayolle *et al.* [54] assumed that the number of blocked terminals is exactly known to all terminals. They used this information to adaptively change retransmission probability. Clare [55] and Rivest [33] updated retransmission probability based on the estimate of number of blocked terminals. They used idle, success, and collision feedback to compute this estimate. Hajek and Van Loon [56] estimated retransmission probability directly based on the idle, success, and collision feedback through a scheme called stochastic approximation.

In this chapter, we study bistable behavior of the slotted ALOHA protocol with finite population of users and finite buffer and we examine a retransmission control algorithm based on the price based control scheme, which, in part, is motivated by the price based rate control scheme studied by Yuen and Marbach in [43] and [44]. In this chapter, we assume that a finite number of terminals share a common communication medium. Slotted ALOHA protocol is used as the medium access protocol. Also, in this system each terminal has a finite buffer. In general, Markov analysis is the only available approach for exact analysis of multiple access systems. But Markov analysis is especially difficult to use for exact analysis of buffered systems because of large dimension of its state space. Therefore, approximation techniques have been introduced to analyze buffered multiple access systems.

One approximation approach introduced by Wan and Sheikh [57, 58, 59], is Tagged User Approach (TUA). In this technique it is assumed that communication channel is symmetric, meaning that statistical behavior of each user in the system is the same. This assumption helps the Markov analysis by reducing dimension of the state space describing the system. The basic idea in TUA is to assume that each user always operates at its own equilibrium probability distribution, independently of other terminals, though its equilibrium probability distribution is determined by the behavior of all users in the system because of interfering queue problem. Hence, in this approach, the performance of the system is studied by analyzing the behavior of an arbitrarily chosen terminal named as tagged user. In this chapter, we study bifurcations in equilibrium probability distribution of the slotted ALOHA protocol with finite user and finite buffer using the TUA. Also, we analyze the price based control for the slotted ALOHA protocol using the same approach.

4.1 System Model

We consider a slotted ALOHA system with finite users population M. Each user has a finite buffer L. Time is divided into slots and each time slot is equal to a packet transmission time. Also, we assume that round trip delay is negligible, therefore, a terminal will know transmission status of a packet immediately after it had finished the transmission. We assume that the communication channel is noise free. A packet arrives into a terminal's queue in a time slot with probability λ and no packet arrives in a time slot with probability $1-\lambda$. If an arriving packet finds the buffer full, it is dropped. Also, in this chapter we assume that packet transmissions follow defer first transmission (DFT). Meaning that there is no difference between new packets and backlogged packets and they are transmitted with a permission probability p.

Since it is assumed that the channel is symmetric and all users have identical statistical behavior, therefore, in the tagged user approach, an arbitrary user is chosen and we observe its behavior. By using this approach, it is implied that all users in the system are working at their equilibrium probability distribution. Therefore, probability that a busy tagged terminal (with non-empty buffer) successfully transmits a packet given it has permission to transmit is

$$p_s = (1 - p(1 - p_0))^{M-1}. \tag{4.1}$$

Here p_0 is probability that a terminal has an empty buffer. Now our focus will be on the tagged user's buffer. Transmission of a packet in the tagged user's queue will be successful with probability pp_s and will fail with probability $1 - pp_s$. Therefore, service time for its queue is geometrically distributed with mean $1/pp_s$. Hence, as Wan and Sheik show in [58], the user's queue can be treated as a $Geo/Geo/1/K$ queue. Figure 4.1 shows Markov model for the tagged user's buffer. Since in a $Geo/Geo/1/K$ queue, total probability flow through any closed boundary must be zero, following relations are easy to obtain:

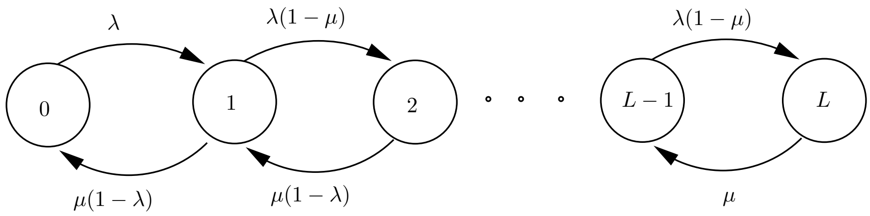

Figure 4.1: Probability flow chart of the buffer.

$$\lambda p_0 = \mu(1-\lambda)p_1,$$

$$\vdots$$

$$\lambda(1-\mu)p_i = \mu(1-\lambda)p_{i+1},$$

$$\vdots$$

$$\lambda(1-\mu)p_{L-1} = \mu p_L, \tag{4.2}$$

here $i = 1, 2, \cdots, L-2$, and μ is the mean service time:

$$\mu = pp_s = p(1-p(1-p_0))^{M-1}. \tag{4.3}$$

From (4.2), p_i, $i = 1, 2, \cdots, L-1$ can be expressed in terms of p_0:

$$p_i = [\frac{\lambda(1-\mu)}{\mu(1-\lambda)}]^{i-1}\frac{\lambda}{\mu(1-\lambda)}p_0, \qquad p_L = [\frac{\lambda(1-\mu)}{\mu(1-\lambda)}]^{L-1}\frac{\lambda}{\mu}p_0.$$

In order to calculate p_0, it is noted that sum of above probabilities should be 1:

$$p_0 = (\frac{\mu}{\mu-\lambda} - \frac{\lambda^2}{\mu(\mu-\lambda)}[\frac{\lambda(1-\mu)}{\mu(1-\lambda)}]^{L-1})^{-1}. \tag{4.4}$$

Equations (4.3) and (4.4) are sufficient to solve two unknown variables μ and p_0.

4.2 Stability Analysis

Equilibrium point(s) of the slotted ALOHA system with finite users and finite buffer is (are) solution(s) of the equations (4.3) and (4.4) in μ and p_0. Unfortunately, these two equations are nonlinear equations and an analytical solution seems to be very difficult. Therefore, numerical approach is used to find equilibrium point(s). Wan and Sheikh in [58] show that number and stability of equilibrium points changes as parameters of the system change. They show that for a range of parameters, the system has only one stable equilibrium point which corresponds to a well-behaved system (uncongested system). In this case, the system is globally stable. When parameters change, the slotted ALOHA system, has two stable and one unstable equilibrium points. One of the stable equilibrium points corresponds to high value of idle probability p_0 (desirable operating point [58]). But other stable fixed point corresponds to a system with small values of p_0 (saturation point [58]). Due to disturbances, the slotted ALOHA system tends to oscillate between these two equilibrium points. For further values of system parameters, the system possesses one stable equilibrium point but at an undesirable region.

Figure 4.2 shows an exemplary bifurcation diagram for idle probability, p_0, when the total number of users in the system, M, changes. It shows that when the number of users is less than 96, the slotted ALOHA system with a finite buffer size of $L = 5$, has only one equilibrium point. $\lambda = 0.0034$ and permission probability $p = 0.025$ are fixed. For these values of M, system has a globally stable equilibrium point which corresponds to high values of idle probability and also, high throughput.

But for values of $96 \leq M \leq 107$, figures 4.2 and 4.3 show that system possesses three equilibrium points. Smallest and largest values of p_0 correspond to locally stable equilibrium points and middle point is an unstable fixed point. When M is increased ($M > 107$), once again the system has only one globally stable equilibrium point with small values for p_0, which corresponds to a congested system with many collisions. Figure 4.3 shows channel throughput defined as

$$S_{out} = MS_{\mu} = M\mu(1 - p_0),$$

here S_u is one terminal throughput, defined as the number of packets transmitted successfully in one time slot. Since the channel is shared by M users with identical statistical behavior, channel throughput is $S_{out} = MS_u$ [58].

Also, figures 4.4 and 4.5 show exemplary bifurcation diagrams for user idle probability (p_0) and channel throughput (S_{out}) when permission probability, p, is the bifurcation parameter and the total number of terminals M is fixed at 100. Figures 4.6 and 4.8 show bifurcation diagrams for idle probability p_0 when queue size L or arrival rate λ are bifurcation parameters, respectively. In the first case $\lambda = 0.0034$, $p = 0.025$, and $M = 100$. In the second case $p = 0.025$, $L = 5$, and $M = 100$.

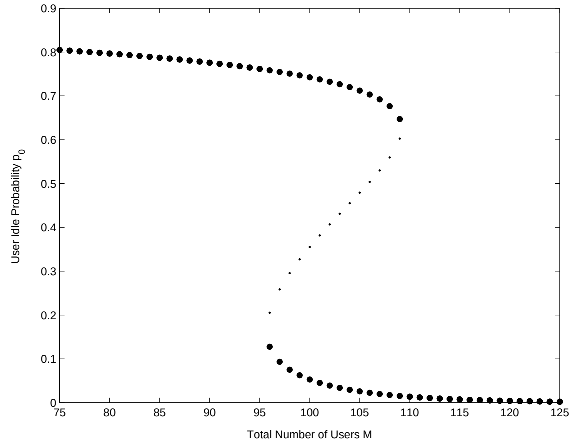

Figure 4.2: Bifurcation diagram for user idle probability (p_0) when the total number of users (M) is bifurcation parameter. $p = 0.025$, $\lambda = 0.0034$, $L = 5$.

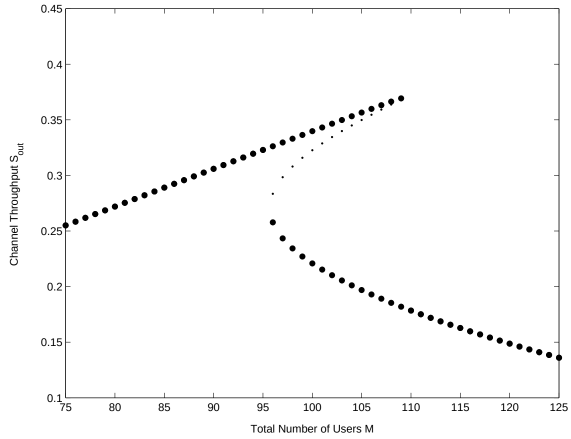

Figure 4.3: Bifurcation diagram for channel throughput (S_{out}) when the total number of users (M) is bifurcation parameter. $p = 0.025$, $\lambda = 0.0034$, $L = 5$.

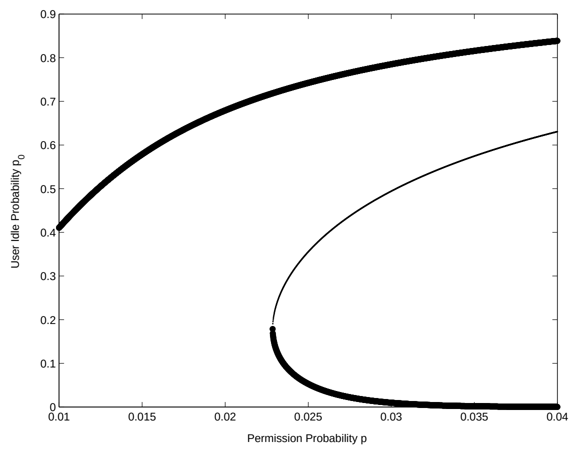

Figure 4.4: Bifurcation diagram for user idle probability (p_0) when permission probability (p) is bifurcation parameter. $M = 100$, $\lambda = 0.0034$, $L = 5$.

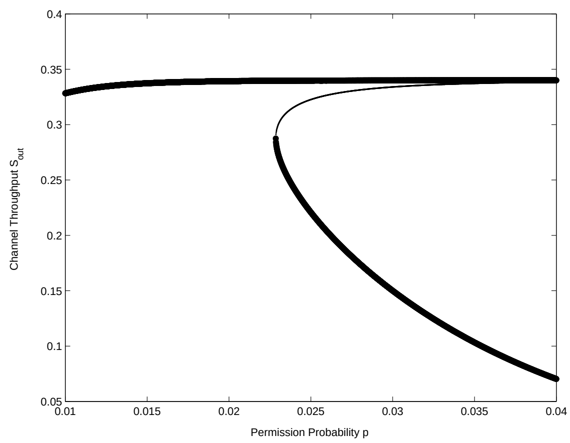

Figure 4.5: Bifurcation diagram for channel throughput (S_{out}) when permission probability (p) is bifurcation parameter. $M = 100$, $\lambda = 0.0034$, $L = 5$.

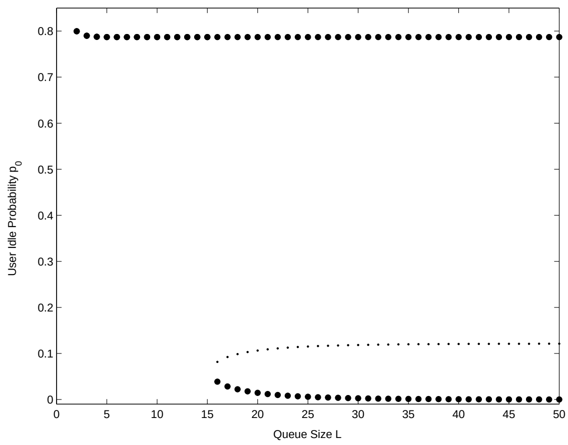

Figure 4.6: Bifurcation diagram for user idle probability (p_0) when queue size (L) is bifurcation parameter. $M = 100$, $\lambda = 0.0034$, $p = 0.025$.

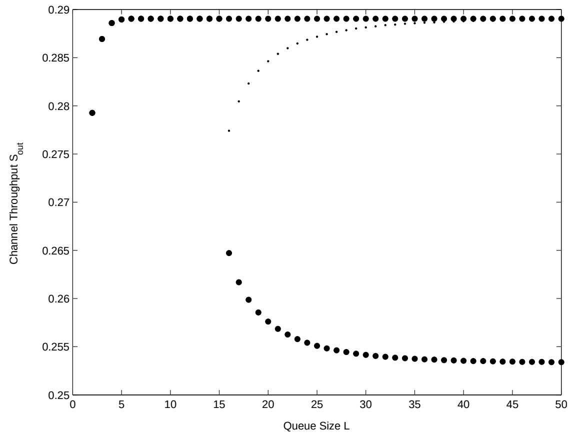

Figure 4.7: Bifurcation diagram for channel throughput (S_{out}) when queue size (L) is bifurcation parameter. $M = 100$, $\lambda = 0.0034$, $p = 0.025$.

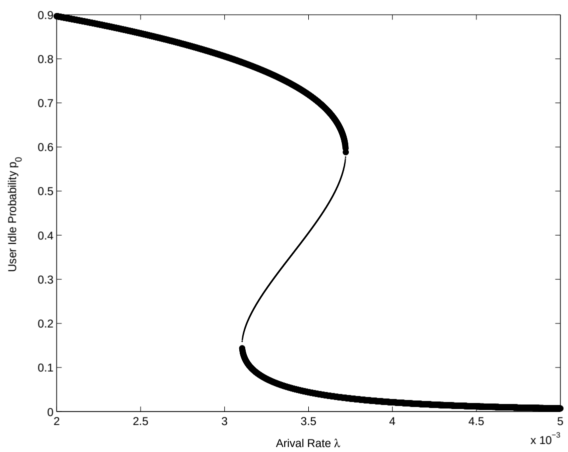

Figure 4.8: Bifurcation diagram for user idle probability (p_0) when arrival rate (λ) is bifurcation parameter. $M = 100$, $p = 0.025$, $L = 5$.

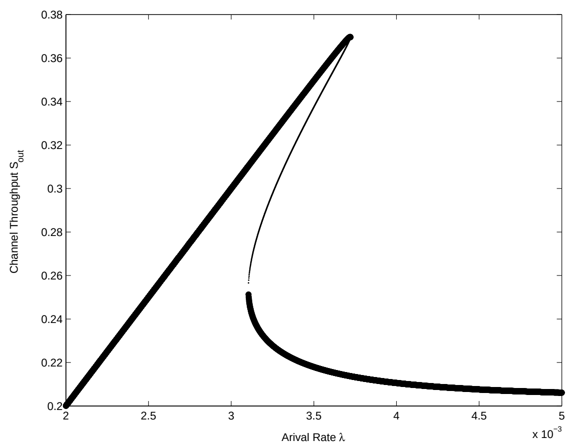

Figure 4.9: Bifurcation diagram for channel throughput (S_{out}) when arrival rate (λ) is bifurcation parameter. $M = 100$, $p = 0.025$, $L = 5$.

4.3 Price Based Control

As we discussed in previous section, the slotted ALOHA system with the finite number of users and finite capacity buffer shows a bifurcation phenomenon when a parameter of the system changes.

Yuen and Marbach in [43] and [44] use a price-based rate control for slotted ALOHA system with infinite users and buffer capacity of one packet. They show that by controlling arrival rate (and, in addition, controlling transmission probability) they can stabilize the system. In this section, a price based control, based on Yuen and Marbach's work, is introduced to eliminate bifurcations in the slotted ALOHA system. The goal is to adaptively change permission probability at the end of each time slot based on outcome of packet transmissions in that time slot. Permission probability, p, is a function of control signal u.

Assumption 4.1. We assume permission probability $p(u)$ is continuous, bounded ($0 \leq p(u) < 1$), and strictly decreasing ($u \in [0, +\infty)$). Furthermore, there exists a positive constant u_{max} such that $p(u) = 0$ when $u \geq u_{max}$.

The control signal u is updated at each time slot using following equation [43]:

$$u_{n+1} = [u_n - \alpha I(Z_n = 0) + \beta I(Z_n = 1) + \gamma I(Z_n \geq 2)]^+. \tag{4.5}$$

At the end of each time slot, users know the outcome of their transmission. Z_n is a random variable corresponding to the number of transmission in time slot n. The control parameters α and γ are positive real numbers and β is a real number. Also, $[x]^+ = \max(0, x)$.

At time slot $n+1$, value of control signal increases if a collision happens in time slot n. If no user transmits in time slot n, value of control signal is decreased. In the case of a successful transmission, depending on whether the system is conservative or aggressive, value of control signal is decreased or increased.

In order to study effects of the price based control on the slotted ALOHA system with finite users and finite buffer, the tagged user approach is used. In this approach, it is assumed that the system is at the equilibrium. Hence, expected change in control signal is zero.

$$
\begin{aligned}
&pr(Z_n = 0) = (1 - p(1 - p_0))^M, \\
&pr(Z_n = 1) = Mp(1 - p_0)(1 - p(1 - p_0))^{M-1}, \\
&pr(Z_n \geq 2) = 1 - Mp(1 - p_0)(1 - p(1 - p_0))^{M-1} - (1 - p(1 - p_0))^M. \qquad (4.6)
\end{aligned}
$$

Therefore,

$$
\begin{aligned}
E(u_{n+1} - u_n | u_n = u, p_0) = {} & \max(-\alpha, -u)(1 - p(1 - p_0))^M, \\
& + \max(\beta, -u)Mp(1 - p_0)(1 - p(1 - p_0))^{M-1}, \\
& + \gamma(1 - Mp(1 - p_0)(1 - p(1 - p_0))^{M-1} - (1 - p(1 - p_0))^M), \qquad (4.7)
\end{aligned}
$$

here $p = p(u)$. State (p_0^*, u^*) is an operating point of the system if it is a solution to equilibrium equations of the system. Consider a relaxed case, the equilibrium equations of the slotted ALOHA system follow

$$0 = -(\alpha + \gamma)(1 - p(1 - p_0))^M + (\beta - \gamma)Mp(1 - p_0)(1 - p(1 - p_0))^{M-1} + \gamma, \qquad (4.8)$$

$$\mu = pp_s = p(1 - p(1 - p_0))^{M-1}, \qquad (4.9)$$

$$p_0 = (\frac{\mu}{\mu - \lambda} - \frac{\lambda^2}{\mu(\mu - \lambda)}[\frac{\lambda(1 - \mu)}{\mu(1 - \lambda)}]^{L-1})^{-1}. \qquad (4.10)$$

Proposition 4.1. *Let Assumption 4.1 hold, then equation (4.8) has a unique solution in $y = p(1 - p_0)$.*

Proof. In order to prove Proposition 4.1, following definitions are necessary:

$$y = p(1 - p_0),$$

$$d_u(y) = -(\alpha + \gamma)(1 - y)^M + (\beta - \gamma)My(1 - y)^{M-1} + \gamma.$$

Derivative of $d_u(y)$ is given by

$$d'_u(y) = M(1 - y)^{M-2}[-(\alpha + \gamma + M\beta - \gamma))y + \alpha + \beta].$$

Hence, value of y that makes the derivative zero is found as follows

$$y^* = \frac{\alpha + \beta}{\alpha + \gamma + M(\beta - \gamma)}.$$

Now to prove the proposition, four different cases are considered.

Case (1): $0 \leq \gamma \leq \beta$: In this case,

$$0 < y^* < 1 \qquad \textit{and} \qquad d_u(y^*) = [\frac{(M-1)(\beta - \gamma)}{\alpha + \gamma + M(\beta - \gamma)}]^{M-1}(\beta - \gamma) + \gamma > \gamma > 0.$$

Note that $d_u(1) = \gamma > 0$ and $d'_u(y) < 0$ for$y \in (y^*, 1]$. Therefore, in this range of y, $d_u(y)$ is positive and strictly decreasing and hence, $d_u(y) = 0$ does not have any solution. However, $d_u(0) = -\alpha < 0$ and $d'_u(y) > 0$ for $y \in [0, y^*)$. Therefore, in this range of values of y, $d_u(y)$ is strictly increasing from $-\alpha < 0$ to $d_u(y^*) > 0$ and hence, there exists exactly one solution for $d_u(y) = 0$ for $y \in [0, y^*)$.

Case (2): $\frac{M-1}{M}\gamma - \frac{\alpha}{M} \leq \beta \leq \gamma$: In this case $1 < y^*$. Therefore, for $y \in [0, 1]$, $d'_u(y) > 0$ and $d_u(y)$ is an strictly increasing function of y from $d_u(0) = -\alpha < 0$ to $d_u(1) = \gamma > 0$. Hence, $d_u(y) = 0$ for $y \in [0, 1]$ has exactly one solution.

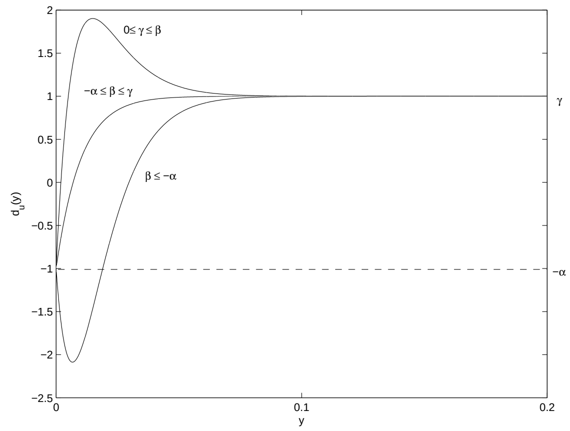

Figure 4.10: Drift function $d_u(y)$ for different values of β.

Case (3): $-\alpha \leq \beta \leq \frac{M-1}{M}\gamma - \frac{\alpha}{M}$: In this case $y^* < 0$. Hence, for $y \in [0,1]$, $d'_u(y) > 0$. Therefore, $d_u(y)$ in this range is strictly increasing and has exactly one zero.

Case (4): $\beta \leq -\alpha$: In this case, and also

$$0 < y^* < 1, \qquad \alpha+\gamma < -(\beta-\gamma), \qquad \frac{(M-1)(\beta-\gamma)}{\alpha+\gamma+M(\beta-\gamma)} < 1.$$

Therefore, $d_u(y^*) = [\frac{(M-1)(\beta-\gamma)}{\alpha+\gamma+M(\beta-\gamma)}]^{M-1}(\beta-\gamma)+\gamma < (\beta-\gamma)+\gamma = \beta \leq -\alpha < 0$. For $y \in [0, y^*)$, $d'_u(y) < 0$ and therefore, $d_u(y)$ is strictly decreasing from $-\alpha$ to $d_u(y^*) < -\alpha$. However, for $y \in (y^*, 1]$, $d'_u(y) > 0$ and $d_u(y)$ is strictly increasing from $d_u(y^*) < -\alpha < 0$ to $d_u(1) = \gamma > 0$. Therefore, in this range, $d_u(y)$ has exactly one zero.

Figure 4.10 summarizes drift function $d_u(y)$ for different cases of β. It is clear that cases 2 and 3 can be combined to one case where $-\alpha \leq \beta \leq \gamma$. $\square$

Therefore, Proposition 4.1 proves that equation (4.8) has exactly one solution for any given control parameters (α, β, γ). Solution to $d_u(y) = 0$ is called $\tilde{y}$.

$$d_u(\tilde{y}) = 0, \qquad\qquad \tilde{y} = p(1 - p_0).$$

System throughput is defined as the average number of packets successfully transmitted in each time slot. Wan and Sheikh [58] suggested that since all packets that arrive in a terminal with a queue which is not full will be transmitted (no packet is dropped), therefore, throughput for a single terminal can be defined as follows

$$S_u = \lambda(1 - p_L),$$

here p_L is probability that terminal queue is full and λ is average packet arrival rate. Also, single terminal throughput can be defined as average service rate (μ) times probability that the terminal is busy $(1 - p_0)$. In other words,

$$S_u = \mu(1 - p_0).$$

Or the service rate, μ, can be written as $\mu = \frac{S_u}{1-p_0}$. Therefore, using equation (4.9), throughput at a point $\tilde{y}$ is

$$\tilde{S}_u = p(1 - p_0)(1 - p(1 - p_0))^{M-1} = \tilde{y}(1 - \tilde{y})^{M-1}.$$

Proposition 4.2. *For any given value of arrival rate* $0 \leq \lambda \leq 1$ *and queue size* $L \geq 0$*:*

$$(\mu - \lambda)(\mu - L) + \lambda(1 - \mu)(1 - F(\mu)) \leq 0, \qquad \forall \;\; 0 \leq \mu \leq 1, \tag{4.11}$$

here

$$F(\mu) = \frac{\lambda}{\mu}[\frac{\lambda(1-\mu)}{\mu(1-\lambda)}]^{L-1}.$$

Proof. This proposition can be proved numerically. Numerical analysis shows that for any given value of arrival rate $0 \leq \lambda \leq 1$ and queue size $L \geq 0$, equation (4.11) has a maximum at $\mu = \lambda$. At this point, equation (4.11) equals to zero. Therefore, for any other value of $0 \leq \mu \neq \lambda \leq 1$ this equation is always negative. $\square$

Proposition 4.3. *Let Assumption 4.1 hold and $0 \leq \lambda \leq 1$, for given control parameters (α, β, γ), set of equations (4.8), (4.9), and (4.10) has exactly one solution.*

Proof. In order to prove this proposition the following definitions are necessary:

$$F(\mu) = \frac{\lambda}{\mu}[\frac{\lambda(1-\mu)}{\mu(1-\lambda)}]^{L-1},$$

$$g(\mu) = \frac{\mu}{\mu-\lambda} - \frac{\lambda^2}{\mu(\mu-\lambda)}\frac{\lambda}{\mu}[\frac{\lambda(1-\mu)}{\mu(1-\lambda)}]^{L-1} = \frac{\mu}{\mu-\lambda} - \frac{\lambda^2}{\mu-\lambda}F(\mu).$$

Therefore, using above definitions and also equation (4.10), probability that the queue is full, p_L, can be rewritten as follows

$$p_L = [\frac{\lambda(1-\mu)}{\mu(1-\lambda)}]^{L-1}\frac{\lambda}{\mu}p_0 = \frac{F(\mu)}{g(\mu)}.$$

Since $\mu = \mu(p_0)$ is a function of p_0, using equation (4.9), probability p_L is rewritten as a function of p_0 as follows

$$p_L(p_0) = \frac{F(\mu(p_0))}{g(\mu(p_0))}.$$

Note that solution to set of equations (4.8), (4.9), and (4.10) is the same as solution to following equation

$$\tilde{S}_u = \lambda(1 - p_L(p_0)).$$

In this proof, define $S_{load} = \lambda(1 - p_L(p_0))$. Derivative of S_{load} with respect to p_0 is

$$\frac{dS_{load}}{dp_0} = -\lambda\frac{\frac{dF}{d\mu}g - \frac{dg}{d\mu}F}{(g(\mu(p_0)))^2}\frac{d\mu}{dp_0}.$$

Here

$$\mu = \frac{\tilde{S}_u}{1-p_0} \Rightarrow \frac{d\mu}{dp_0} = \frac{\tilde{S}_u}{(1-p_0)^2} > 0 \qquad for \;\; 0 < p_0 < 1.$$

Also,

$$g(\mu) = \frac{\mu}{\mu-\lambda} - \frac{\lambda^2}{\mu-\lambda}F(\mu) \Rightarrow \frac{dg(\mu)}{d\mu} = \frac{-\lambda}{(\mu-\lambda)^2} - \lambda[\frac{-F(\mu)}{(\mu-\lambda)^2} + \frac{\frac{dF}{d\mu}}{\mu-\lambda}].$$

And,

$$\frac{dF(\mu)}{d\mu} = \frac{\mu-L}{\mu(1-\mu)}F(\mu).$$

Therefore,

$$\begin{aligned}\frac{dF}{d\mu}g - \frac{dg}{d\mu}F &= \frac{1}{(\mu-\lambda)^2}[\mu(\mu-\lambda)\frac{dF}{d\mu} + \lambda F(1-F)] \\ &= \frac{F(\mu)}{(\mu-\lambda)^2(1-\mu)}[(\mu-\lambda)(\mu-L) + \lambda(1-\mu)(1-F(\mu))].\end{aligned}$$

Using Proposition 4.2 and the fact that $F(\mu) \geq 0$, it is easy to see that $\frac{dF}{d\mu}g - \frac{dg}{d\mu}F \leq 0$. Hence

$$\frac{dS_{load}}{dp_0} \geq 0.$$

Therefore, $S_{load} = \lambda(1-p_L)$ is an increasing function of p_0. Hence, if $S_{load}(0) \leq \tilde{S}_u \leq S_{load}(1-\tilde{y})$ then $S_{load}(p_0) = \tilde{S}_u$ has exactly one solution. (It is obvious that $0 \leq p_0 \leq 1-\tilde{y}$) □

Proposition 4.4. *Function $g(\mu)$ is defined as before*

$$g(\mu) = \frac{\mu}{\mu-\lambda} - \frac{\lambda^2}{\mu(\mu-\lambda)}[\frac{\lambda(1-\mu)}{\mu(1-\lambda)}]^{L-1}, \tag{4.12}$$

is a decreasing function of μ for $0 < \mu < 1$.

Proof. In order to prove the proposition, derivative of $g(\mu)$ with respect to μ is found

$$g'(\mu) = \frac{\lambda}{(\mu-\lambda)^2}[-1 + \frac{\lambda^L(1-\mu)^{L-2}}{\mu^{L+1}(1-\lambda)^{L-1}}((L-\mu)(\mu-\lambda)+\mu(1-\mu))].$$

Define $h(\mu)$ as follows

$$h(\mu) = \frac{\lambda^L(1-\mu)^{L-2}}{\mu^{L+1}(1-\lambda)^{L-1}}((L-\mu)(\mu-\lambda)+\mu(1-\mu)).$$

Derivative of $h(\mu)$ with respect to μ is

$$h'(\mu) = \frac{\lambda^L(1-\mu)^{L-3}}{\mu^{L+2}(1-\lambda)^{L-1}}(\mu-\lambda)(-2\mu^2+4L\mu-L^2-L).$$

For $0 < \mu < 1$, the part $(-2\mu^2+4L\mu-L^2-L)$ is always negative. Therefore, for $0 < \mu < \lambda$, derivative $h'(\mu)$ is positive and for $\lambda < \mu < 1$ it is negative. Hence, point $\mu = \lambda$ is maximum of $h(\mu)$ for $0 < \mu < 1$. Therefore, it is safe to say that

$$h(\mu) \leq h(\mu = \lambda) = 1 \qquad\qquad 0 < \mu < 1.$$

It is easy to show

$$g'(\mu) \leq 0 \qquad\qquad 0 < \mu < 1.$$

Therefore, $g(\mu)$ is a decreasing function of μ for $0 < \mu < 1$. Define $f(p_0) = \frac{1}{g(\mu(p_0))}$, where $\mu = \frac{\tilde{S}_u}{1-p_0}$.

$$f'(p_0) = \frac{df}{dp_0} = -\frac{\frac{d\mu}{dp_0}\frac{dg}{d\mu}}{(g(\mu(p_0)))^2} = -\frac{\tilde{S}_u g'(\mu)}{(1-p_0)^2(g(\mu))^2} \geq 0 \qquad 0 < p_0 < 1.$$

Hence, $f(p_0)$ is an increasing function of p_0 for $0 < p_0 < 1$. Next, limit values of $f(p_0)$ when p_0 goes to 0 and 1 is calculated.

$$f(p_0) = [\frac{\tilde{S}_u}{\tilde{S}_u - \lambda(1-p_0)} - \frac{\lambda^2(1-p_0)^2}{\tilde{S}_u(\tilde{S}_u-\lambda(1-p_0))}[\frac{\lambda(1-p_0-\tilde{S}_u)}{\tilde{S}_u(1-\lambda)}]^{L-1}]^{-1}$$

Now the limits are found

$$\lim_{p_0 \to 0^+} f(p_0) = [\frac{\tilde{S}_u}{\tilde{S}_u - \lambda} - \frac{\lambda^2}{\tilde{S}_u(\tilde{S}_u - \lambda)}[\frac{\lambda(1-\tilde{S}_u)}{\tilde{S}_u(1-\lambda)}]^{L-1}]^{-1} > 0$$

Note that $\tilde{S}_u = \lambda(1 - p_L)$ and since $0 \leq p_L \leq 1$, then $\tilde{S}_u \leq \lambda$. Also,

$$\lim_{p_0 \to 1^-} f(p_0) = 1.$$

$\square$

4.4 Designing the Controller

In this section we study the design of the controller through selecting the set of control parameters (α, β, γ). Given a desired operating throughput $\tilde{S}_u$, the control parameters are found such that the system will have a unique operating point with the given throughput. However, it is noted that we proved that the slotted ALOHA system with the price based control has a unique operating point if the equilibrium value of the control signal is greater than the maximum of α and $-\beta$ ($u > \max\{\alpha, -\beta\}$ - the relaxed drift equation). Therefore, in designing the controller, we choose the parameters such that equilibrium value of the control parameter will satisfy the above criteria. To do so we can consider two different cases based on one-terminal arrival rate, λ. Notice that maximum one user throughput is $\frac{1}{M}(1 - \frac{1}{M})^{M-1}$ which happens when $y = \frac{1}{M}$.

In this section we make a small change in the Assumption 4.1. We define continuous function $p(u)$ such that $p(u) = 1$ for $0 \leq u \leq \max\{\alpha, -\beta\}$. And for $\max\{\alpha, -\beta\} < u \leq u_{max}$, $p(u)$ is strictly decreasing. Furthermore, $p(u) = 0$ for $u \geq u_{max}$. Now we define two different cases:

Case 1: $\lambda \geq \frac{1}{M}(1-\frac{1}{M})^{M-1}$ - we first find $\tilde{y}$ such that $\tilde{S_u} = \tilde{y}(1-\tilde{y})^{M-1}$. The control parameters α and γ are arbitrarily chosen to be small positive real numbers. The control parameter β is calculated as follows

$$\beta = \frac{(\alpha+\gamma)(1-\tilde{y})^M + \gamma M\tilde{y}(1-\tilde{y})^{M-1} - \gamma}{M\tilde{y}(1-\tilde{y})^{M-1}} \tag{4.13}$$

It is easy to show that the controlled system has a unique operating point at $\tilde{y}$. And the system has no operating points for the values of $u < \max\{\alpha, -\beta\}$.

Case 2: $0 < \lambda < \frac{1}{M}(1-\frac{1}{M})^{M-1}$ - For λ in this range, since the maximum allowable one user throughput is $S_u = \lambda$, we consider two sperate cases. For a given operating point $\tilde{S_u} = \lambda$ it is easy to show that using the above algorithm for choosing the control parameters will result in a system with unique operating point. However, for the case where the given operating point $\tilde{y} < \lambda$, we change the algorithm in order to make sure that the operating point of the system happens only for $u > \max\{\alpha, -\beta\}$. As before, we choose positive parameter γ arbitrarily. We use $\tilde{y}$ to calculate β and α as follows

$$-\beta = \alpha = \gamma\{\frac{1 - M\tilde{y}(1-\tilde{y})^{M-1} - (1-\tilde{y})^M}{M\tilde{y}(1-\tilde{y})^{M-1} + (1-\tilde{y})^M}\}$$

This ensures that the equilibrium value of the control parameter is greater than $\max\{\alpha, -\beta\}$. Therefore, the system has a unique operating point at $\tilde{y}$.

4.5 Simulation Results

In this section we examine effects of the price based control on a finite population slotted ALOHA with finite buffer, numerically and through simulations. We consider a slotted ALOHA system with the finite number of users M and finite buffer $L = 5$. Also, arrival rate is $\lambda = 0.0034$ and permission probability is $p = 0.025$. Bifurcation diagrams for idle probability p_0 and throughput S_{out} are shown in figures 4.2 and 4.3, respectively, for $75 \leq M \leq 125$. In order to design the price based control, we choose $\alpha = 1$ and $\gamma = 1$ arbitrarily. The control parameter $\beta = 2.5138$ is calculated using equation (4.13) such that at $M = 75$ one user throughput is at its maximum value $S_u = \lambda = 0.0034$. Figure 4.11 illustrates that bifurcation is eliminated completely. In this range of M, system throughput is almost fixed at $S_{out} = 0.2547$. Figure 4.12 illustrates bifurcation diagram for one user throughput S_u. Further, we examine accuracy of our numerical analysis with respect to simulations. For each value of M we run a simulation 5 times with an initial empty queue for all the terminals and 5 times with a full initial queue for all the terminals. Each run of the simulation consists of 80000 time slots. We calculate an average channel throughput. Also,

$$p(u) = \begin{cases} 1 & 0 \leq u \leq \max\{\alpha, -\beta\} \\ \{(1 - \frac{u - \max\{\alpha, -\beta\}}{150})^3\}^+ & u > \max\{\alpha, -\beta\} \end{cases}$$

Where $\{x\}^+ = \max\{0, x\}$. Figure 4.13 illustrates a comparison between numerical analysis and simulations. Diamonds show numerical results for channel throughput for the uncontrolled system using TUA, which is the same as figure 4.3. These diamonds show stable and unstable fixed points of the system. Crosses show the result of simulations on the uncontrolled system. We can see that at beginning and end of the graph simulation results follow numerical results closely. However, in the region where numerical analysis show three equilibrium points, simulation results are different than numerical. The reason is that actual system is oscillating between two stable fixed points and therefore, result throughput is an average of throughput at those two stable fixed points. Finally the asterisks show simulation results for channel throughput for the controlled system. We can see that average throughput is almost fixed for this range of M and is very close to average throughput calculated numerically $S_{out} = 0.2547$.

Also, we look at effects of the price based control on the system when queue size L is the bifurcation parameter. Figures 4.6 and 4.7 illustrate bifurcation diagrams with no control for idle probability and system throughput. A controller is designed by choosing $\alpha = 1$ and $\gamma = 1$ arbitrarily and $\beta = 1.2$ such that for a system with $N = 100$ terminals, one user throughput is at its maximum value, $S_u = \lambda = 0.0034$. Bifurcation diagrams for idle probability is shown in Figures 4.14. It is clear that bifurcations are eliminated completely. Also, it can be observed that system throughput is fixed as queue size changes. This is the result of using the price based control. Since the control parameters are chosen such that the system will operate at a pre-given throughput.

4.6 Summary

In this chapter revisited the bistable behavior of finite buffered finite user slotted ALOHA system. We used a tagged user approach to study equilibrium behavior of the system. We applied the price based control proved that, under some conditions, the controlled system has a unique operating point.

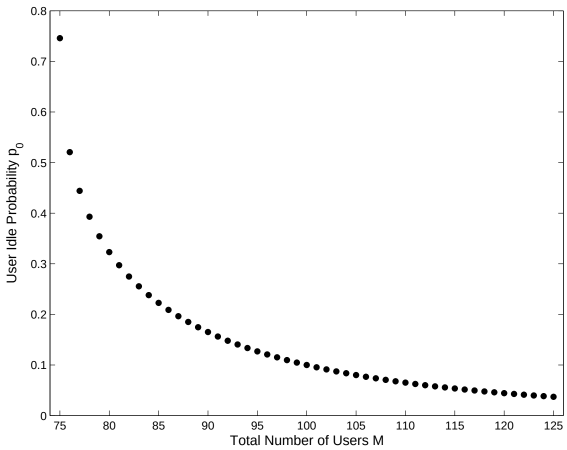

Figure 4.11: Bifurcation diagram for user idle probability (p_0) with price based bifurcation, the total number of users (M) bifurcation parameter, $\lambda = 0.0034$, and $L = 5$.

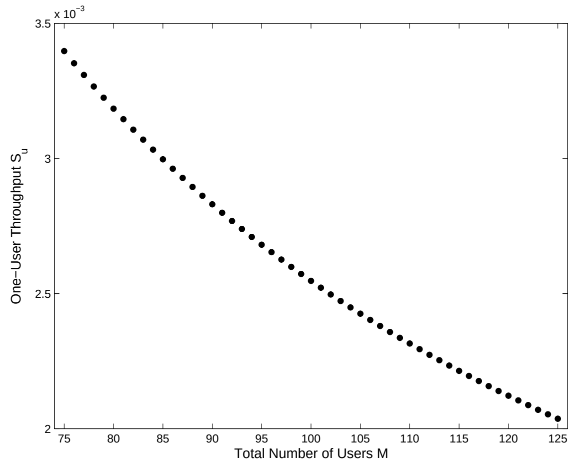

Figure 4.12: Bifurcation diagram for channel throughput (S_u) with price based bifurcation, the total number of users (M) bifurcation parameter, $\lambda = 0.0034$, and $L = 5$.

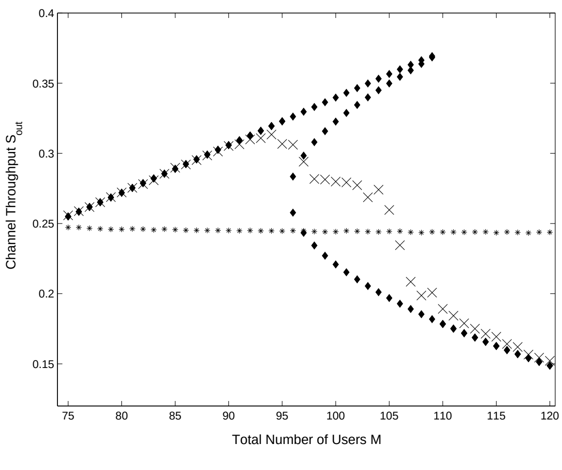

Figure 4.13: Analytical vs. Simulation - Diamonds are analytical results (solutions to equilibrium equations) and Crosses are simulation results

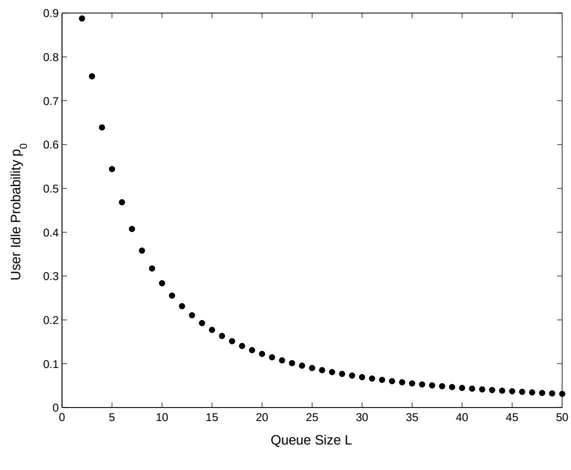

Figure 4.14: Bifurcation diagram for user idle probability (p_0) with price based bifurcation, queue size (L) bifurcation parameter, $\lambda = 0.0034$, and $M = 100$.

Chapter 5

Equilibrium Analysis and Control for Framed PRMA Protocol for Voice and Data Terminals

5.1 Introduction

In our analysis of PRMA protocols in previous chapters, we assumed that feedback information is sent to terminals as soon as packets are received at an access point (such as a base station). In the PRMA protocols, feedback information is transmitted at the end of each time slot. However, it this chapter, we consider a PRMA protocol (with no round trip delay) and we assume that feedback information is only sent at the end of a frame. There are several reasons to consider transmitting feedback information at the end of frames, instead of end of time slots. Convenience for the base station and saving in signalling (such as feedback signals overhead) can be considered few exemplary improvements of framed PRMA over PRMA.

Further, for framed PRMA, we assume that each contending terminal (a non-reserved terminal with packet to transmit) can only transmit one packet during each frame. This assumption may improve stability of the system. Several researches have studied frame slotted ALOHA and its applications [61], [62], [63], [64], and [65]. Also, framed PRMA was introduced as a contention TDMA (C-TDMA) protocol in [66].

In this chapter, we study the nonlinear behavior of framed PRMA at equilibrium and apply price based control scheme introduced in previous chapters to analyze stability of the system.

5.2 Price Based Control - Framed PRMA Voice-only

In this section we model framed PRMA that only employs voice terminals. A voice terminal is in silent state if it has no packets to transmit. With probability σ_v a new voice message is generated. We assume that transmission attempts for first packet of the voice message starts at the beginning of the next frame and therefore, we assume that state transitions occur at the beginning of each time frame. Based on feedback information received at the end of each frame, the terminals, at the beginning of next frame, have exact knowledge of the number of reserved time slots (in that frame) and outcomes of transmissions in previous frame. The voice terminals in contending state, at the beginning of a frame, if they have permission to transmit their packets, randomly choose an "available" time slot of the frame and will transmit their packet at that time slot. If the transmission was successful, the successful terminal reserves that time slot and can transmit the reminder of its packets at that time slot in next frames without contention.

As part of feedback information, terminals are informed of the number of collisions, successful transmissions, and idle time slots (time slots with no transmission) in a time frame. Each terminal updates its control signal based on this information. Packet transmission for contending terminals is based on the control signal. We assume that the control signal u is the same for all the terminals and is updated at the end of time frame n as following:

$$u_{n+1} = [u_n - \alpha w_n + \xi v_n]^+. \tag{5.1}$$

Here, the control parameters α and ξ are positive real numbers. $[x]^+$ denotes $\max(0, x)$. Random variable w_n indicates the number of (non-reserved) time slots in frame n with no transmission. Also, random variable v_n indicates the number of time slots in frame n with one or more transmissions. Here, we assume that times slots with successful transmissions and time slots with collisions have similar effect on the control signal u.

Next, we consider how the control signal u can affect transmission behavior of contending terminals. For the framed PRMA with price based control, we assume all contending terminals have permission to transmit their packets, at the beginning of time frame n. However, we assume that the number of "available" time slots for the contending terminals is controlled by the control signal u. Assume that at the beginning of time frame n, the number of reserved time slots is given by r_n. We define δ_n as the number of "available" time slots for contending voice terminals as following:

$$\delta_n = \begin{cases} \lceil \frac{N - r_n}{\rho(u_n)} \rceil & \text{if} \quad r_n < N, \\ 1 & \text{if} \quad r_n = N. \end{cases}$$

Assumption 5.1. We assume that $\rho(u)$ is continuous, bounded ($0 \leq \rho(u) \leq N$), and strictly decreasing in u ($u \in [0, +\infty)$). Furthermore, there exists a positive constant u_{max} such that $\rho(u) = 0$ when $u \geq u_{max}$.

At the beginning of a time frame n, contending voice terminals calculate the control signal u (or receive update control signal from the access point), determine the parameter $\rho(u)$ (or receive the updated parameter $\rho(u)$), and randomly choose one time slot out of δ_n "available" times slots. If $\delta_n \leq N - r_n$, the contending voice terminals transmit in their chosen time slot. However, if $\delta_n > N - r_n$, only contending voice terminals that have chosen time slots less than $N - r_n$ can transmit their packets. Other contending terminal will not transmit. This case is similar to having permission probability less than 1 where only terminals that have permission to transmit choose one free time slot.

Considering dynamic behavior of framed PRMA with price based control, state of the system at the beginning of time frame n is given by $X_n = (c_n, r_n, u_n)$. Without loss of generality, it can be assumed that the system starts at initial state $X_0 = (c_0, r_0, u_0) = (0, 0, 0)$. Also, $c \in \{0, 1, 2, \cdots, M_v\}$, $r \in \{0, 1, 2, \cdots, N\}$, and $u \in \Gamma = \{\min(u_{MAX}, [-\alpha a + \xi d]^+)|a, d \in Z_+\}$. Where $u_{MAX} = u_{max} + \max(N\xi)$. However, it is noted that state space $\aleph$ is only a subset of $\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, N\} \times \Gamma$. Because, at least, the total number of contending and reserving voice terminals at each time slot could not be higher than total the number of voice terminals. Also, for $r_n = N$, control signal could only take values greater than or equal to ξ. Note that state space $\aleph$ is countable. Time evolution of the states of the system is as following:

$$c_{n+1} = c_n + y_n - q_n, \qquad r_{n+1} = r_n + q_n - z_n,$$

here,

y_n : the number of terminals transitioned from SIL to CON,

q_n : the number of terminals transitioned from CON to RES,

z_n : the number of terminals transitioned from RES to SIL.

1) y_n : Random variable y_n represents the number of terminals in SIL that generate a new message in frame n. Given the number of contending and reserved voice terminals are known, statistics of y_n is

$$Pr(y_n = y | c_n = c, r_n = r) = \begin{pmatrix} M_v - c - r \\ y \end{pmatrix} \sigma_v^y (1 - \sigma_v)^{M_v - c - r - y},$$

here $0 \leq y \leq M_v - c - r$. Further, the average number of terminals that transition from SIL to CON at the end of frame n, given the number of contending and reserved terminals are known, is determined as following:

$$E(y_n | c_n = c, r_n = r) = (M_v - c - r)\sigma_v.$$

2) z_n : Random variable z_n presents the number of terminals in reservation state that move to silent state by transmitting all their packets. As noted, we assume that the communication channel is error free. Given the number of contending and reserved terminals are known, statistics of z_n is

$$Pr(z_n = z | c_n = c, r_n = r) = \begin{pmatrix} r \\ z \end{pmatrix} \gamma_f^z (1 - \gamma_f)^{r - z},$$

here $0 \leq z \leq r$. Further, the average number of terminals that transition from RES to SIL at the end of frame n, given the number of contending and reserved terminals are known, is determined as following:

$$E(z_n|c_n = c, r_n = r) = r\gamma_f.$$

3) q_n : Random variable q_n presents the number of contending voice terminals that have successfully transmitted packets in frame n. As noted, at the beginning of each time frame, each contending terminal determines (or receives) parameter $\rho(u_n)$ based on received feedback information. Next, the terminal calculates (or receives) the number of "available" time slots $\delta_n = \lceil \frac{N-r_n}{\rho(u_n)} \rceil$, and it randomly chooses one of δ_n "available" slots. As discussed, if $\delta_n \leq N - r_n$, chosen time slots exists in time frame n and the terminal transmits its packet at that time slot. However, if $\delta_n > N - r_n$, and the chosen time slot does not belong to the time frame, the terminal will not transmit its packet.

Collisions happen if more than two contending terminals choose the same time slot. As shown in [67], statistics for q_n can be determined as following

$$Pr(q_n = q|c_n = c, r_n = r, u_n = u)$$
$$= \begin{cases} \sum_{i=q}^{\min(N-r,c)} \begin{pmatrix} N-r \\ i \end{pmatrix} \begin{pmatrix} i \\ q \end{pmatrix} \begin{pmatrix} c \\ i \end{pmatrix} i!(-1)^{i-q} \frac{(N-r-pi)^{c-i}}{(N-r)^c} p^i, & \text{if} \quad \delta > N-r, \\ \sum_{i=q}^{\min(N-r,c)} \begin{pmatrix} N-r \\ i \end{pmatrix} \begin{pmatrix} i \\ q \end{pmatrix} \begin{pmatrix} c \\ i \end{pmatrix} i!(-1)^{i-q} \frac{(\delta-i)^{c-i}}{\delta^c}, & \text{if} \quad \delta \leq N-r. \end{cases}$$

where $p \equiv \frac{N-r}{\delta}$ and $0 \leq q \leq \min(N-r, c)$.

Further, as shown in [66], the number of contending terminals with successful transmission can be given by

$$q_n = \sum_{i=1}^{c_n} \chi_i,$$

where χ_i is a random variable with values belonging to $\{0, 1\}$. Random variable χ_i is 1 if ith contending terminal succeed (no other contending terminal transmits in the slot chosen by the ith terminal among $\min(\delta_n, N - r_n)$ available time slots). Therefore, probability of success for ith terminal is

$$Pr(\chi_i = 1 | c_n = c, r_n = r, u_n = u) = \begin{cases} (1 - \frac{1}{\delta})^{c-1} & \text{if} \quad \delta \leq N - r, \\ \frac{N-r}{\delta}(1 - \frac{1}{\delta})^{c-1} & \text{if} \quad \delta > N - r, \end{cases}$$

here $\delta = \delta(u_n)$. Therefore, the expected number of successful contending terminals (given the number of contending and reserved terminals and control signal is known) is given by

$$\begin{aligned} E(q_n | c_n = n, r_n = r, u_n = u) &= E[\sum_{i=1}^{c} \chi_i | c_n = n, r_n = r, u_n = u] \\ &= \begin{cases} c(1 - \frac{1}{\delta})^{c-1} & \text{if} \quad \delta \leq N - r, \\ c\frac{N-r}{\delta}(1 - \frac{1}{\delta})^{c-1} & \text{if} \quad \delta > N - r. \end{cases} \end{aligned}$$

Next we consider expected change of control signal. One step expected change (mean drift) of control signal at state (c, r, u) is:

$$\begin{aligned} d(c, r, u) =& E(u_{n+1} - u_n | c_n = c, r_n = r, u_n = u), \\ &= \max(-u, -\alpha E(w_n | c_n = c, r_n = r, u_n = u) \\ &+ \xi E(v_n | c_n = c, r_n = r, u_n = u)). \end{aligned}$$

Relaxed drift equation is:

$$
\begin{aligned}
d(c,r,u) &= -\alpha E(w_n|c_n = c, r_n = r, u_n = u) + \xi E(v_n|c_n = c, r_n = r, u_n = u),\\
&= \begin{cases} -\alpha\delta(1-\frac{1}{\delta})^c + \xi(\delta - \delta(1-\frac{1}{\delta})^c) & \text{if} \quad \delta \le N-r,\\ -\alpha(N-r)(1-\frac{1}{\delta})^c + \xi(N-r-(N-r)(1-\frac{1}{\delta})^c) & \text{if} \quad \delta > N-r. \end{cases}
\end{aligned}
$$

Now, we study behavior of framed PRMA using equilibrium point analysis. It is assumed that any change in states of the system at equilibrium is zero. Similar to the PRMA system and based on the averages determined above, equilibrium equations of the framed PRMA system are derived. A point (C,R,U) is called an equilibrium point, if

$$
\begin{aligned}
&(M_v - C - R)\sigma_v - C((\frac{N-R}{\delta})I(\delta > N-R) + I(\delta \le N-R))w_v(C,U) = 0,\\
&R\gamma_f - C((\frac{N-R}{\delta})I(\delta > N-R) + I(\delta \le N-R))w_v(C,U) = 0,\\
&-(\alpha+\xi)(1-\frac{1}{\delta})^C + \xi = 0.
\end{aligned} \tag{5.2}
$$

Here $\quad w_v(C,U) = \begin{cases} (1-\frac{1}{\delta})^{C-1} & C \ge 1 \\ 1 & C < 1 \end{cases}$ and $\delta = \delta(U) = \lceil \frac{N-R}{\rho(U)} \rceil$.

In order to determine solution of set of equations (5.2), for given system and control parameters, we assume two different cases (1) $\delta > N-R$ and (2) $\delta \le N-R$. For each case, we present sufficient conditions for the control parameter such that the system has a unique operating point (solution of the set of equations (5.2) is unique). The analysis do not examine whether the unique solution satisfies the assumption made.

Remark 5.1. *Based on the first two equations of the set of equations (5.2), it is easy to show that* $R = \min(N, \omega(M_v - C))$, *where* $\omega = \frac{\sigma_v}{\gamma_f + \sigma_v}$. *Therefore,* *when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{N}{\omega}), M_v]$, $R \in [0, N]$, *and* $\delta \in [1, \infty)$. *However, it can easily be shown, using the set of equation (5.2) and considering each of the cases* $\delta > N - R$ *and* $\delta \leq N - R$, *that* $C = 0$, $M_v - \frac{N}{\omega}$, $C = M_v$, $R = 0$, $R = N$ *(resulting in* $\delta = 1 > N - R = 0$*),* $\delta = 1$, $\delta \to +\infty$, *or a combination thereof, cannot be solutions to the set of equilibrium equations (5.2).*

Lemma 5.1. *Assuming that* $\delta > N - R$, *there exists a set of control parameters* (α, ξ) *for which the set of equations (5.2) has a unique solution in* (C, U) *and the system has a single operating point if any of conditions* $(1a)$, $(1b)$, *or* (2) *below hold:*

$(1a)$ $\quad M_v \geq 1 + \frac{N}{\omega}$ *where* $\omega = \frac{\sigma_v}{\gamma_f + \sigma_v}$, *and*

$$-\frac{\omega\gamma_f}{N} < \frac{\xi}{\alpha+\xi}[(\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v - \frac{N}{\omega}} + 1)\exp(-\frac{\ln(\frac{\xi}{\alpha+\xi})}{M_v - \frac{N}{\omega}}) - 1],$$

$(1b)$ $\quad M_v \leq 1 + \frac{N}{\omega}$ *and either*

$$\frac{(M_v - 1)\omega\gamma_f}{N - (M_v - 1)\omega} \leq \frac{\xi}{\alpha+\xi} M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1), \qquad \textit{or}$$

$$-\frac{\omega\gamma_f}{N} < 1 - \frac{\xi}{\alpha+\xi} + \ln(\frac{\xi}{\alpha+\xi}),$$

(2) $\quad -1 + \frac{\xi}{\alpha+\xi} - \ln(\frac{\xi}{\alpha+\xi}) \leq \frac{\omega}{N\gamma_f}[\frac{\xi}{\alpha+\xi} M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1) + \gamma_f]^2.$

Proof. We will show the existence of control parameters (α, ξ) such that the conditions hold. We first consider conditions $(1a)$ and $(1b)$ and prove that under these conditions the system has a unique operating point. Assuming that $\delta > N - R$, set of equations (5.2) can be simplified as follows

$$F_1(C,U) = C\frac{1}{\delta}w_v(C,U) - \frac{(M_v - C)\omega\gamma_f}{N - (M_v - C)\omega} = 0, \tag{5.3}$$

$$F_2(C,U) = -(\alpha+\xi)(1 - \frac{1}{\delta})^C + \xi = 0, \tag{5.4}$$

here $\max(0, M_v - \frac{N}{\omega}) < C < M_v$ and $R = \omega(M_v - C)$. Now let us define following new functions

$$f(C,U) = C\frac{1}{\delta}w_v(C,U), \quad g(C) = \frac{(M_v - C)\omega\gamma_f}{N - (M_v - C)\omega}.$$

Solving equation (5.4) for δ and then substituting it in $f(C,U)$, we have

$$f(C) = \begin{cases} \frac{\xi}{\alpha+\xi}C(\exp(\frac{-1}{C}\ln(\frac{\xi}{\alpha+\xi})) - 1) & C \geq 1, \\ C(1 - \exp(\frac{1}{C}\ln(\frac{\xi}{\alpha+\xi}))) & C < 1. \end{cases} \tag{5.5}$$

Fixed point(s) of equations (5.3) and (5.4) is same as fixed point(s) of $f(C) = g(C)$. For the given range of C, it is easy to show that first and second derivatives of $f(C)$ are as follows

$$\begin{cases} \frac{df}{dC} < 0 & C \geq 1 \\ \frac{df}{dC} > 0 & C < 1 \end{cases}, \begin{cases} \frac{d^2f}{dC^2} > 0 & C \geq 1 \\ \frac{d^2f}{dC^2} < 0 & C < 1. \end{cases}$$

It can also be shown that $\frac{dg}{dC} < 0$ and $\frac{d^2g}{dC^2} > 0$ for the given range of C. Also, notice that

$$f(M_v) = \frac{\xi}{\alpha+\xi}M_v(\exp(-\frac{1}{M_v}\ln(\frac{\xi}{\alpha+\xi})) - 1) > g(M_v) = 0.$$

Based on these facts, in order to prove that $f(C) = g(C)$ has exactly one solution, for the conditions $(1a)$ and $(1b)$ stated in the lemma, we consider following cases:

- **Case 1 -** $M_v > 1 + \frac{N}{\omega}$: For $M_v - \frac{N}{\omega} < C \leq M_v$, $g(C)$ is strictly decreasing and positive. $f(C)$ is positive and strictly decreasing with positive second order derivatives. Also, $f(M_v - \frac{N}{\omega}) < g((M_v - \frac{N}{\omega})^+)$, and $f(M_v) > g(M_v)$. We define $h(C) = g(C) - f(C)$. As mentioned, $h(M_v - \frac{N}{\omega}) > 0$ and $h(M_v) < 0$. Therefore, if we choose the control parameters such that $\frac{dh}{dC} < 0$, then $h(C) = 0$ will have a unique solution. Since both functions $f(C)$ and $g(C)$ have positive second order derivatives, then

$$\frac{df}{dC}(M_v - \frac{N}{\omega}) < \frac{df}{dC} < \frac{df}{dC}(M_v),$$
$$\frac{dg}{dC}(M_v - \frac{N}{\omega}) < \frac{dg}{dC} < \frac{dg}{dC}(M_v).$$

Hence, if the control parameters are chosen such that $\frac{dg}{dC}(M_v) \leq \frac{df}{dC}(M_v - \frac{N}{\omega})$, then $\frac{dh}{dC} < 0$ for the range of C and $h(C) = 0$ has a unique solution.

- **Case 2 -** $\frac{N}{\omega} < M_v < 1 + \frac{N}{\omega}$: In order to make sure that there exists only one solution, we chose the control parameters (α, ξ) such that either $\frac{dg}{dC}(M_v) \leq \frac{df}{dC}(1)$ or $g(1) < f(M_v)$.

- **Case 3 -** $0 < M_v < \frac{N}{\omega}$: In this case $g(C)$ is positive, strictly decreasing, with positive second derivative. Also, $f(C)$ is positive, strictly, increasing with negative second order derivative for $0 \leq C < 1$, and strictly decreasing with positive second order derivative for $1 \leq C \leq M_v$. In same way as Case 2, we choose control parameters so $\frac{dg}{dC}(M_v) \leq \frac{df}{dC}(1)$ or $g(1) < f(M_v)$. It is easy to show that in this case $f(C) = g(C)$ has exactly one solution.

Next we prove the result assuming condition (2) holds. The set of three equations (5.2) can be simplified, under assumption $\delta > N - R$, as the following single equation:

$$h(C) = -M_v + C + \frac{N}{\omega} \frac{f(C)}{\gamma_f + f(C)} = 0. \tag{5.6}$$

Here, $f(C)$ is the same as defined earlier in the proof. It can easily be shown using equation (5.5) that $h(0) = -M_v$, $h(M_v - \frac{N}{\omega}) < 0$, and $h(M_v) > 0$. The first derivative of $h(C)$ is

$$h'(C) = 1 + \frac{N\gamma_f}{\omega}\frac{f'(C)}{(\gamma_f + f(C))^2},$$

where $f'(C) = \frac{df}{dC}$ and $h'(C) = \frac{dh}{dC}$. We showed earlier in the proof that $f'(C) > 0$ for $0 \le C < 1$. Therefore, for this range of C, $h'(C) > 0$. So $h(C)$ is negative for $C = \max(0, M_v - \frac{N}{\omega})$, is positive for $C = M_v$, as has a positive slope for $0 \le C < 1$. Next we show that the slope is also positive for $1 \le C \le M_v$, which will imply that a unique solution for $h(C) = 0$ exist within interval $0 \le C \le M_v$. Consider the case $1 \le C \le M_v$ and we will show that if condition (2) is satisfied, then $h'(C) > 0$. We define $h_1(C) = -f'(C)$ and $h_2(C) = \frac{\omega}{N\gamma_f}(\gamma_f + f(C))^2$.

It is noted that $h_1(C) > 0$ and $h_1'(C) < 0$ for $C \ge 1$. Therefore, $h_1(M_v) < h_1(C) < h_1(1)$.

Also, $h_2(C) > 0$ and $h_2'(C) < 0$ for $C \ge 1$. Therefore, $h_2(M_v) < h_2(C) < h_2(1)$.

Hence, if the control parameters are chosen such that $h_1(1) \le h_2(M_v)$ (in other words, $-f'(1) < \frac{\omega}{N\gamma_f}(\gamma_f + f(M_v))^2$), then $h'(C) > 0$. Therefore, $h(C) = 0$ has a unique solution.

It can easily be shown, using equation (5.4), that δ is one-to-one function of C. Further, for the equilibrium studies we assume that $\delta = \frac{N-R}{\rho(U)}$. As mentioned before, ρ is also a one-to-one function of u (for $0 < \rho < N$). Therefore, it can easily be shown that for a given C, there exists a unique U. Therefore, under conditions stated in the lemma, the set of equations (5.3) and (5.4) or equation (5.6) has a unique solution in $0 < C < M_v$ and $0 < U < u_{max}$. □

Lemma 5.2. *Assuming that $\delta \leq N - R$, there exist a set of control parameters (α, ξ) for which the set of equations (5.2) has a unique solution in (C, U) and the system has a single operating point if $\frac{\xi}{\xi+\alpha} > e^{-1}$.*

Proof. Under the assumption of this lemma, set of equations (5.2) can be simplified as follows

$$F_1(C,U) = -M_v + C + \frac{1}{\omega\gamma_f} C w_v(C, U) = 0, \tag{5.7}$$

$$F_2(C,U) = -(\alpha + \xi)(1 - \frac{1}{\delta})^C + \xi = 0, \tag{5.8}$$

here $R = \omega(M_v - C)$, and $\max(0, M_v - \frac{N}{\omega}) < C \leq M_v$. Now let us define following new function

$$f(C, U) = C w_v(C, U).$$

Solving equation (5.8) for δ and then substituting it in $f(C, U)$, we have

$$f(C) = \begin{cases} \frac{\xi}{\alpha+\xi} C \exp(\frac{-1}{C} \ln(\frac{\xi}{\alpha+\xi})) & C \geq 1, \\ C & C < 1. \end{cases}$$

with first derivative of

$$\frac{df}{dC} = \begin{cases} \frac{\xi}{\alpha+\xi} \exp(\frac{-1}{C} \ln(\frac{\xi}{\alpha+\xi}))(1 + \frac{\ln(\frac{\xi}{\alpha+\xi})}{C}) & C \geq 1, \\ 1 & C < 1. \end{cases}$$

Equation 5.7 can be rewritten as $F_1(C) = -M_v + C + \frac{1}{\omega\gamma_f} f(C) = 0$. It is noted that $F_1(0) = -M_v$, $F_1(M_v - \frac{N}{\omega}) < 0$ and $F_1(M_v) > 0$. Also, derivative of $F_1(C)$ is

$$\frac{dF_1}{dC} = 1 + (\frac{1}{\omega\gamma_f})\frac{df}{dC}.$$

For $C < 1$ it is easy to show that $\frac{dF_1}{dC} > 0$. However, for $C \geq 1$, if the control parameters are chosen such that $1+\frac{\ln(\frac{\xi}{\alpha+\xi})}{C} > 0$, then it is easy to show that $\frac{dF_1}{dC} > 0$. So $F_1(C)$ is negative for $C = \max(0, M_v - \frac{N}{\omega})$, is positive for $C = M_v$, and has a positive slope for $\max(0, M_v - \frac{N}{\omega}) < C \leq M_v$. Therefore, $F_1(C) = 0$ has a unique solution for this range of C. We will show that if $\frac{\xi}{\alpha+\xi} > e^{-1}$ then $1 + \frac{\ln(\frac{\xi}{\alpha+\xi})}{C} > 0$ for $C \geq 1$.

$$\begin{aligned}
&\frac{\xi}{\alpha+\xi} > e^{-1},\\
&\ln(\frac{\xi}{\alpha+\xi}) > -1,\\
&-\ln(\frac{\xi}{\alpha+\xi}) < 1 \leq C,\\
&\frac{-\ln(\frac{\xi}{\alpha+\xi})}{C} < 1,\\
&1+\frac{\ln(\frac{\xi}{\alpha+\xi})}{C} > 0.
\end{aligned}$$

Therefore, if $\frac{\xi}{\alpha+\xi} > e^{-1}$ then $\frac{dF_1}{dC} > 0$. Hence, $F_1(C) = 0$ will have a unique operating point.

It can easily be shown, using equation (5.8), that δ is one-to-one function of C. Further, for the equilibrium studies we assume that $\delta = \frac{N-R}{\rho(U)}$. As mentioned before, ρ is also a one-to-one function of u (for $0 < \rho < N$). Therefore, it can easily be shown that for a given C, there exists a unique U. Therefore, under conditions stated in the lemma, the set of equations (5.7) and (5.8) has a unique solution in $0 < C < M_v$ and $0 < U < u_{max}$. □

In order to be able to use the relaxed form of expected drift, we make a small change in assumption 5.1. The update equation (5.1) reproduced here is $u_{n+1} =$

$[u_n - \alpha w_n + \xi v_n]^+$. As noted before, $\rho(u)$ is continuous and bounded $(0 \leq \rho(u) \leq N)$ function of control signal u. However, here, we assume $\rho(u) = N$ for $u \in [0, N\alpha]$, strictly decreasing for $u \in (N\alpha, u_{max})$, and $\rho(u) = 0$ for $u \in [u_{max}, +\infty)$.

For $u < N\alpha$, $\rho(u) = N$ and $\delta = 1$. Hence, if at the beginning of time frame n the value of control signal u_n is less than $N\alpha$, then only one time slot is available for contention in that time frame. If the number of contending voice terminals at the beginning of time frame n is more than zero ($c_n > 0$), the available time slot will have at least one transmission. However, if no terminal is contending ($c_n = 0$), the available time slot will be idle. For a given state $(c_n, r_n, u_n) = (c, r, u)$, the update control signal is as following:

$$\begin{cases} u_{n+1} = u_n + \xi & \text{if} \quad c_n > 0, \\ u_{n+1} = [u_n - \alpha]^+ & \text{if} \quad c_n = 0. \end{cases}$$

However, at equilibrium, it can be shown that the value of contending terminals is greater than zero ($C > 0$). Therefore, the expected drift of the control signal at equilibrium is $\xi > 0$ (if equilibrium value of control signal is less than $N\alpha$). Hence, it can be confirmed that equilibrium value of control signal cannot be less than $N\alpha$ and therefore, relaxed expected drift for control signal can be used.

It is noted that the control parameters can be chosen such that the number of contending terminals at equilibrium equals to a pre-chosen value. This, enables a system designer to select system parameters such that at equilibrium, system works at a given load in order to meet a certain throughput, delay, or drop probability criteria.

5.3 Price Based Control - Framed PRMA Protocol for Voice and Data Terminals

In this section we consider a framed PRMA system that employs both voice and data terminals. As we studied in the PRMA voice and data system, we assume that voice and data terminal share the same communication medium. Behavior of voice terminals is the same as discussed before. We assume that data terminals have a buffer with capacity of one packet, however, our analysis is easily extendable to buffers with higher capacities. Also, we assume that data terminals, unlike voice terminals, can not reserve time slots. At the beginning of each time frame, each terminal with packets to transmit (contending voice terminals and backlogged data terminals) randomly chooses an "available" time slot for transmission. At the beginning of each frame, terminals have knowledge of how many time slots in that frame are reserved and therefore, how many time slots are free. Depending on a control signal (as explained later), "available" time slots are either a subset of free time slots or include free time slots.

If the umber of "available" time slots are less than free time slots, the difference is left without any transmission. However, if the number of "available" time slots are greater than the number of free time slots, if the chosen time slot is after the end of free slots, the terminal that has chosen that slot will not transmit.

Therefore, a terminal successfully transmits at a chosen time slot, if the chosen time slot is before the end of free time slot and other terminals have not chosen that time slot. A successful voice terminal reserves that time slot in future frames.

However, a successful data terminal enters a silent state and awaits generation of new data packets.

At the end of each time frame, the base station transmits feedback information to the terminals indicating outcome of transmissions in that frame. Therefore, voice and data terminals are informed of the number of collisions, successful transmissions, and idle time slots ("available" time slots with no transmission) in that time frame.

Each terminal updates a control signal u based on this information. The number of "available" time slots in next frame is further updated based on the control signal and the updated number of free time slots in next frame. The control signal u is updated at the end of time frame n as following:

$$u_{n+1} = [u_n - \alpha w_n + \xi v_n]^+ \tag{5.9}$$

Here, the control parameters α and ξ are positive real number. $[x]^+$ denotes $\max(0, x)$. Random variable w_n indicates the number of "available" time slots in frame n with no transmission. Also, random variable v_n indicates the number of time slots in frame n with one or more transmissions.

At the beginning of frame n, the number of "available" time slots δ_n is updated based on control signal u_n and the number of free time slots $N - r_n$ as following:

$$\delta_n = \begin{cases} \lceil \frac{N-r_n}{\rho(u_n)} \rceil & \text{if} \quad r_n < N, \\ 1 & \text{if} \quad r_n = N. \end{cases}$$

Assumption 5.2. We assume that parameter $\rho(u)$ is continuous, bounded ($0 \leq \rho(u) \leq N$), and strictly decreasing in u ($u \in [0, +\infty)$). Furthermore, there exists a positive constant u_{max} such that $\rho(u) = 0$ when $u \geq u_{max}$.

Considering dynamic behavior of framed PRMA with voice and data terminal and with price based control, state of the system at the beginning of time slot n is given by $X_n = (c_n, r_n, b_n, u_n)$. Without loss of generality, it can be assumed that the system starts at initial state $X_0 = (c_0, r_0, b_0, u_0) = (0, 0, 0, 0)$. Also, $c \in \{0, 1, 2, \cdots, M_v\}$, $r \in \{0, 1, 2, \cdots, N\}$, $b \in \{0, 1, 2, \cdots, M_d\}$, and $u \in \Gamma = \{\min(u_{MAX}, [-\alpha a + \xi d]^+) | a, d \in Z_+\}$. Where $u_{MAX} = u_{max} + \max(N\xi)$. However, it is noted that state space $\aleph$ is only a subset of $\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, N\} \times \{0, 1, 2, \cdots, M_d\} \times \Gamma$. Because, at least, the total number of contending and reserving voice terminals at each time slot could not be higher than the total number of voice terminals. Also, for $r_n = N$, control signal could only take values greater than or equal to ξ. Note that state space $\aleph$ is countable.

Time evolution of the states of the system is as following:

$$c_{n+1} = c_n + y_{c_n} - q_{c_n}, \qquad r_{n+1} = r_n + q_{c_n} - z_{c_n}, \qquad b_{n+1} = b_n + y_{b_n} - q_{c_n}.$$

Where,

y_{c_n} :	the number of voice terminals transitioned from SIL to CON,
q_{c_n} :	the number of voice terminals transitioned from CON to RES,
z_{c_n} :	the number of voice terminals transitioned from RES to SIL,
y_{b_n} :	the number of data terminals transitioned from SIL to BLK,
q_{b_n} :	the number of data terminals transitioned from BLK to SIL.

1) y_{c_n} : Random variable y_{c_n} represents the number of voice terminals in SIL that generate a new message in frame n. Given the number of contending and

reserved voice terminals and backlogged data terminals are known, statistics of y_{c_n} is:

$$Pr(y_{c_n} = y_c | c_n = c, r_n = r, b_n = b) = \begin{pmatrix} M_v - c - r \\ y_c \end{pmatrix} \sigma_v^{y_c} (1 - \sigma_v)^{M_v - c - r - y_c},$$

here $0 \leq y_c \leq M_v - c - r$. Further, the average number of voice terminals that transition from SIL to CON during time frame n (given control signal, the number of contending and reserved voice terminals, and the number of backlogged data terminals are known) is determined as following:

$$E(y_{c_n} | c_n = c, r_n = r, b_n = b) = (M_v - c - r)\sigma_v.$$

2) z_{c_n} : Random variable z_{c_n} presents the number of voice terminals in reservation state that move to silent state by transmitting all their packets. Statistics of z_{c_n} is:

$$Pr(z_{c_n} = z_c | c_n = c, r_n = r, b_n = b) = \begin{pmatrix} r \\ z_c \end{pmatrix} \gamma_f^{z_c} (1 - \gamma_f)^{r - z_c},$$

here $0 \leq z_c \leq r$. Further, the average number of voice terminals that transition from RES to SIL during time frame n is determined as following:

$$E(z_{c_n} | c_n = c, r_n = r, b_n = b) = r\gamma_f.$$

3) q_{c_n} : Random variable q_{c_n} presents the number of contending voice terminals that have successfully transmitted packets in frame n. As noted, at the beginning of each time frame, each contending terminal updates parameter $\rho(u_n)$ based on received feedback information, determines the number of "available" time slots δ_n, and randomly chooses one of the "available" time slots.

If $\delta_n \leq N - r_n$, the chosen time slot is in time frame n. However, $N - r_n - \delta_n$ time slots are left without contention and transmission. In case $\delta_n > N - r_n$, if the chosen time slot does not belong to time frame n, the terminal that have chosen that time slot, will not transmit. This is similar to the assumption that terminal does not have permission to transmit.

In order to determine the statistics for q_{c_n}, we take the following approach. As mentioned before, we assumed that each voice or data terminals have permission probability of 1 and randomly chooses one of the unreserved time slots to transmit. We define random variable q_n representing the number of terminals (voice and data) that successfully transmit. Statistics for q_n can be determined, as shown in [66] and [67], to be

$$Pr(q_n = q | c_n = c, r_n = r, b_n = b, u_n = u)$$
$$= \begin{cases} \sum_{i=q}^{\min(N-r,c+b)} \begin{pmatrix} N-r \\ i \end{pmatrix} \begin{pmatrix} i \\ q \end{pmatrix} \begin{pmatrix} c+b \\ i \end{pmatrix} i!(-1)^{i-q} \frac{(N-r-pi)^{c+b-i}}{(N-r)^{c+b}} p^i & \text{if } \delta > N - r, \\ \sum_{i=q}^{\min(N-r,c+b)} \begin{pmatrix} N-r \\ i \end{pmatrix} \begin{pmatrix} i \\ q \end{pmatrix} \begin{pmatrix} c+b \\ i \end{pmatrix} i!(-1)^{i-q} \frac{(\delta-i)^{c+b-i}}{\delta^{c+b}} & \text{if } \delta \leq N - r, \end{cases}$$

here $p \equiv \frac{N-r}{\delta}$ and $0 \leq q \leq \min(N-r, c+b)$. Since both voice and data terminals behave in the same manner in choosing a time slot from the pool of the "available" time slots, statistics for q_{c_n} given the number of successful transmission q_n can be determined as follows

$$Pr(q_{c_n} = q_c | c_n = c, r_n = r, b_n = b, u_n = u, q_n = q) = \begin{pmatrix} q \\ q_c \end{pmatrix} (\frac{c}{c+b})^{q_c} (\frac{b}{c+b})^{q-q_c},$$

here $0 \leq q_c \leq \min(q, c)$.

Further, as shown in [66], the number of contending terminals with successful transmission can be given by

$$q_{c_n} = \sum_{i=1}^{c_n} \chi_{c_i},$$

here χ_{c_i} is a random variable with values belonging to $\{0, 1\}$. Random variable χ_{c_i} is 1 if ith contending voice terminal succeed (no other terminal transmits in the slot chosen by the ith terminal among $min(\delta_n, N - r_n)$ available time slots). Therefore, probability of success for ith terminal is

$$Pr(\chi_{c_i} = 1 | c_n = c, r_n = r, b_n = b, u_n = u) = \begin{cases} (1 - \frac{1}{\delta})^{c+b-1} & \text{if} \quad \delta \leq N - r, \\ \frac{N-r}{\delta}(1 - \frac{1}{\delta})^{c+b-1} & \text{if} \quad \delta > N - r, \end{cases}$$

here $\delta = \delta(u_n)$. Therefore, the expected number of successful contending terminals (given the number of contending, backlogged, and reserved terminals and control signal is known) is:

$$\begin{aligned} E(q_{c_n} | c_n = c, r_n = r, b_n = b, u_n = u) &= E[\sum_{i=1}^{c} \chi_{c_i} | c_n = n, r_n = r, b_n = b, u_n = u] \\ &= \begin{cases} c(1 - \frac{1}{\delta})^{c+b-1} & \text{if} \quad \delta \leq N - r, \\ c\frac{N-r}{\delta}(1 - \frac{1}{\delta})^{c+b-1} & \text{if} \quad \delta > N - r. \end{cases} \end{aligned}$$

4) y_{b_n} : Random variable y_{b_n} represents the number of data terminals in SIL that generate a new packet in frame n. Statistics of y_{b_n} is:

$$Pr(y_{b_n} = y_c | c_n = c, r_n = r, b_n = b) = \begin{pmatrix} M_d - b \\ y_b \end{pmatrix} \sigma_d^{y_b} (1 - \sigma_d)^{M_d - b - y_b},$$

here $0 \le y_b \le M_d - b$. Further, the average number of data terminals that transition from SIL to BLK during time frame n is determined as following:

$$E(y_{b_n}|c_n = c, r_n = r, b_n = b) = (M_d - b)\sigma_d.$$

5) q_{b_n} : Random variable q_{b_n} presents the number of backlogged data terminals that have successfully transmitted packets in frame n. Similar to q_{c_n}, statistics for q_{b_n} can be determined, given the number of successful transmission q_n, as follows

$$Pr(q_{b_n} = q_b|c_n = c, r_n = r, b_n = b, u_n = u, q_n = q) = \begin{pmatrix} q \\ q_b \end{pmatrix} (\frac{b}{c+b})^{q_b} (\frac{c}{c+b})^{q-q_b},$$

here $0 \le q_b \le \min(q, c)$.

Further, as shown in [66], the number of backlogged data terminals with successful transmission can be given by

$$q_{b_n} = \sum_{i=1}^{b_n} \chi_{b_i},$$

here χ_{b_i} is a $\{0, 1\}$-random variable. Random variable χ_{b_i} is 1 if ith backlogged data terminal succeed (no other terminal transmits in the slot chosen by the ith terminal among $min(\delta_n, N - r_n)$ available time slots). Therefore, probability of success for ith terminal is

$$Pr(\chi_{b_i} = 1|c_n = c, r_n = r, b_n = b, u_n = u) = \begin{cases} (1 - \frac{1}{\delta})^{c+b-1} & \text{if} \quad \delta \le N - r, \\ \frac{N-r}{\delta}(1 - \frac{1}{\delta})^{c+b-1} & \text{if} \quad \delta > N - r, \end{cases}$$

here $\delta = \delta(u_n)$. Therefore, the expected number of successful backlogged data terminals (given the number of contending, backlogged, and reserved terminals and control signal is known) is given by

$$E(q_{b_n}|c_n = c, r_n = r, b_n = b, u_n = u) = E[\sum_{i=1}^{b} \chi_{b_i}|c_n = n, r_n = r, b_n = b, u_n = u]$$
$$= \begin{cases} b(1-\frac{1}{\delta})^{c+b-1} & \text{if} \quad \delta \leq N-r, \\ b\frac{N-r}{\delta}(1-\frac{1}{\delta})^{c+b-1} & \text{if} \quad \delta > N-r. \end{cases}$$

Based on the above statistics, one step expected change (mean drift) of control signal at state (c, r, b, u) is:

$$\begin{aligned} d(c,r,b,u) =& E(n_{n+1} - u_n|c_n = c, r_n = r, b_n = n, u_n = u) \\ =& \max(-u, -\alpha E(w_n|c_n = c, r_n = r, b_n = b, u_n = u) \\ &+ \xi E(v_n|c_n = c, r_n = r, b_n = b, u_n = u)). \end{aligned}$$

Relaxed drift equation is:

$$\begin{aligned} &d(c,r,b,u) = \\ &\quad -\alpha E(w_n|c_n = c, r_n = r, b_n = b, u_n = u) + \xi E(v_n|c_n = c, r_n = r, b_n = b, u_n = u) \\ &\quad = \begin{cases} -\alpha\delta(1-\frac{1}{\delta})^{c+b} + \xi(\delta - \delta(1-\frac{1}{\delta})^{c+b}) & \text{if} \quad \delta \leq N-r, \\ -\alpha(N-r)(1-\frac{1}{\delta})^{c+b} + \xi(N-r-(N-r)(1-\frac{1}{\delta})^{c+b}) & \text{if} \quad \delta > N-r. \end{cases} \end{aligned}$$

Next, we study the behavior of the framed PRMA voice and data system using equilibrium point analysis. A point (C, R, B, U) is called an equilibrium point, if

$$\begin{aligned} &(M_v - C - R)\sigma_v - C((\frac{N-R}{\delta})I(\delta > N-R) + I(\delta \leq N-R))w(C,B,U) = 0, \\ &R\gamma_f - C((\frac{N-R}{\delta})I(\delta > N-R) + I(\delta \leq N-R))w(C,B,U) = 0, \\ &(M_d - B)\sigma_d - B((\frac{N-R}{\delta})I(\delta > N-R) + I(\delta \leq N-R))w(C,B,U) = 0, \\ &-(\alpha+\xi)(1-\frac{1}{\delta})^{C+B} + \xi = 0. \end{aligned} \tag{5.10}$$

Here $w(C, B, U) = \begin{cases} (1-\frac{1}{\delta})^{C+B-1} & C+B \geq 1 \\ 1 & C+B < 1 \end{cases}$ and $\delta = \delta(U)$. Similar to the framed PRMA voice system, we consider two different cases: (1) $\delta > N - R$ and (2) $\delta \leq N - R$. For each case we determine solution to the set of equations (5.10) and determine if the solution satisfies the assumption made in the case.

Remark 5.2. *Based on the first two equations of the set of equations (5.10), it is easy to show that* $R = \min(N, \omega(M_v - C))$, *where* $\omega = \frac{\sigma_v}{\gamma_f + \sigma_v}$. *Also, based on the first and second equations of the set of equations (5.10), it is easy to show that* $B(C) = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$. *Therefore, when the equilibrium equations of the system is considered, it is assumed that* $C \in [\max(0, M_v - \frac{N}{\omega}), M_v]$, $B \in [0, M_d]$, $R \in [0, N]$, *and* $\delta \in [1, \infty)$. *However, it can easily be shown, using the set of equation (5.10) and considering each of the cases* $\delta > N - R$ *and* $\delta \leq N - R$, *that* $C = 0$, $M_v - \frac{N}{\omega}$, $C = M_v$, $B = 0$, $B = M_d$, $R = 0$, $R = N$ *(resulting in* $\delta = 1 > N - R = 0$*),* $\delta = 1$, $\delta \to +\infty$, *or a combination thereof, cannot be solutions to the set of equilibrium equations (5.10).*

Lemma 5.3. *Assuming* $\delta > N - R$, *let us define*

$$f^{+}(C) = \frac{\xi}{\alpha + \xi}(C + B)(\exp(\frac{-1}{C+B}\ln(\frac{\xi}{\alpha+\xi})) - 1).$$

Here $B(C) = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$. *There exists a set of control parameters* (α, ξ) *for which the set of equations (5.10) has a unique solution in* (C, U) *and the system has a single operating point if any of condition* (1), (2), *or* (3) *below hold:*

(1) $\frac{\gamma_f \omega}{\sigma_d} < 1$, $M_v - \frac{N}{\omega} > \hat{C}$, *and* $-\frac{\gamma_f \omega}{N}(1 + \frac{M_d}{M_v}) < \frac{df^{+}}{dC}(\tilde{C})$,

(2) $\frac{\gamma_f \omega}{\sigma_d} < 1$, $M_v - \frac{N}{\omega} < \hat{C}$, *and either*

$$-\frac{\gamma_f \omega}{N}(1 + \frac{M_d}{M_v}) < \frac{df^+}{dC}(\hat{C}), \quad or$$

$$\frac{(M_v - \hat{C})\omega\gamma_f + (M_d - \hat{B})\sigma_d}{N - (M_v - \hat{C})\omega} \leq \frac{\xi}{\alpha + \xi}(M_v + M_d)(\exp(\frac{-1}{M_v + M_d}\ln(\frac{\xi}{\alpha + \xi})) - 1),$$

(3) $\frac{\gamma_f \omega}{\sigma_d} \geq 1$, $M_v < \frac{N}{\omega} + 1$, *and*

$$\frac{(M_v - \hat{C})\omega\gamma_f + (M_d - \hat{B})\sigma_d}{N - (M_v - \hat{C})\omega} \leq \frac{\xi}{\alpha + \xi}(M_v + M_d)(\exp(\frac{-1}{M_v + M_d}\ln(\frac{\xi}{\alpha + \xi})) - 1).$$

Here, $\tilde{C} = M_v - \frac{N}{\omega}$, $\hat{C} + \hat{B} = 1$, *and* $\hat{B} = B(\hat{C})$.

Proof. We define following new functions

$$f(C, U) = (C + B)\frac{1}{\delta}w(C, B, U) \qquad g(C) = \frac{(M_v - C)\omega\gamma_f + (M_d - B)\sigma_d}{N - (M_v - C)\omega}.$$

Note that $R = \min(N, \omega(M_v - C))$, $B = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$, and $\max(0, M_v - \frac{N}{\omega}) < C < M_v$. Solving relaxed expected drift of control signal in equation (5.10) for δ and substituting it in $f(C, U)$, we have

$$f(C) = \begin{cases} \frac{\xi}{\alpha+\xi}(C + B)(\exp(\frac{-1}{C+B}\ln(\frac{\xi}{\alpha+\xi})) - 1) & C + B \geq 1 \\ (C + B)(1 - \exp(\frac{1}{C+B}\ln(\frac{\xi}{\alpha+\xi}))) & C + B < 1 \end{cases}$$

It is easy to show that equilibrium points of set of equations (5.10) are same as fixed points of $f(C) = g(C)$. First and second derivatives of $f(C)$ are as follows

$$\frac{df}{dC} < 0 \;\; for \;\; C + B \geq 1 \quad and \quad \frac{df}{dC} > 0 \;\; for \;\; C + B < 1.$$

And if $\frac{\gamma_f \omega}{\sigma_d} < 1$

$$\frac{d^2 f}{dC^2} > 0 \;\; for \;\; C + B \geq 1 \quad and \quad \frac{d^2 f}{dC^2} < 0 \;\; for \;\; C + B < 1.$$

Also, for the range of C mentioned above and for $\frac{\gamma_f \omega}{\sigma_d} < 1$, first and second derivatives of $g(C)$ are as follows $\frac{dg}{dC} < 0$ and $\frac{d^2 g}{dC^2} > 0$. Also, note that

$$f(M_v) = \frac{\xi}{\alpha + \xi}(M_v + M_d)(\exp(-\frac{1}{(M_v + M_d)}\ln(\frac{\xi}{\alpha + \xi})) - 1) > g(M_v) = 0.$$

We define $\hat{C}$ such that $B(\hat{C}) + \hat{C} = 1$. Since B is a strictly increasing function of C, $\hat{C}$ is unique. Further, for simplification in notations, we define $\tilde{C} = M_v - \frac{N}{\omega}$. Now based on the above-noted facts, in order to prove that $f(C) = g(C)$ has exactly one solution, for the conditions stated in the lemma, we consider different cases:

- **Case 1:** $\frac{\gamma_f \omega}{\sigma_d} < 1$

    1. $M_v > \hat{C} + \frac{N}{\omega}$: In this case we only consider $M_v - \frac{N}{\omega} < C \leq M_v$. In this range, $g(C)$ is positive and strictly decreasing with positive second derivative. Also, $f(C)$ is positive, strictly decreasing with positive second derivative, $f(M_v - \frac{N}{\omega}) < g((M_v - \frac{N}{\omega})^+)$, and and $f(M_v) > g(M_v)$. Therefore, if the control parameters are chosen such that $g(C) - f(C)$ is strictly decreasing in the range of C, then $g(C) - f(C) = 0$ has a unique solution. Since both functions $g(C)$ and $f(C)$ have positive second derivatives, we choose the control parameters such that $\frac{dg}{dC}(M_v) < \frac{df}{dC}(\tilde{C})$.

    2. $\frac{N}{\omega} < M_v < \hat{C} + \frac{N}{\omega}$: In this case, in order to make sure that there exist only one solution, we chose the control parameters (α, ξ) such that either $\frac{dg}{dC}(M_v) < \frac{df}{dC}(\hat{C})$ or $g(\hat{C}) < f(M_v)$.

    3. $0 < M_v < \frac{N}{\omega}$: In this case, $g(C)$ is positive, strictly decreasing, and with positive second derivative. Also, $f(C)$ is positive, strictly increasing for

$0 < C < \hat{C}$, and strictly decreasing for $\hat{C} \leq C \leq M_v$. In the same way, we choose control parameters such that either $\frac{dg}{dC}(M_v) < \frac{df}{dC}(\hat{C})$ or $g(\hat{C}) < f(M_v)$. It is easy to show that in this case $f(C) = g(C)$ has exactly one solution.

- **Case 2:** $\frac{\gamma_f \omega}{\sigma_d} \geq 1$

  - $0 < M_v < \hat{C} + \frac{N}{\omega}$: In this case, $g(C)$ is positive, strictly decreasing. Also, $f(C)$ is positive, strictly increasing for $0 < C < \hat{C}$, and strictly decreasing for $\hat{C} \leq C \leq M_v$. We choose control parameters such that $g(\hat{C}) < f(M_v)$.

Therefore, set of equations (5.10) has a unique solution for the case $\delta > N - R$.

It can easily be shown that δ is one-to-one function of C. Further, for the equilibrium studies we assume that $\delta = \frac{N-R}{\rho(U)}$. As mentioned before, ρ is also a one-to-one function of u (for $0 < \rho < N$). Therefore, it can easily be shown that for a given C, there exists a unique U. Therefore, under conditions stated in the lemma, the set of equations (5.10) for the case $\delta > N - R$ has a unique solution in $0 < C < M_v$ and $0 < U < u_{max}$. $\square$

Lemma 5.4. *Assuming $\delta \leq N - R$, set of equations (5.10) has a unique solution and the system has a single operating point if $\frac{\xi}{\alpha+\xi} > e^{-1}$.*

Proof. Under the assumption of this lemma, set of equation (5.10) can be simplified as follows

$$F_1(C,B,U) = -(M_v - C)\gamma_f\omega - (M_d - B)\sigma_d + (C+B)w(C,B,U) = 0,$$

$$F_2(C,B,U) = -(\alpha+\xi)(1-\frac{1}{\delta})^{C+B} + \xi = 0.$$

Here $w(C,B,U) = \begin{cases} (1-\frac{1}{\delta})^{C+B-1} & C+B \geq 1 \\ 1 & C+B < 1 \end{cases}$, $R = \min(N, \omega(M_v - C))$, $B = \frac{M_d C}{(M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C}$, and $\max(0, M_v - \frac{N}{\omega}) < C < M_v$. We define following new function

$$f(C,U) = (C+B)w(C,B,U).$$

Solving $F_2(C,B,U)$ for δ and substituting it in $f(C,U)$, we have

$$f(C) = \begin{cases} \frac{\xi}{\alpha+\xi}(C+B)\exp(\frac{-1}{C+B}\ln(\frac{\xi}{\alpha+\xi})) & C+B \geq 1, \\ C+B & C+B < 1, \end{cases}$$

with first derivative of

$$\frac{df}{dC} = \begin{cases} \frac{\xi}{\alpha+\xi}\exp(\frac{-1}{C+B}\ln(\frac{\xi}{\alpha+\xi}))(1+B')(1+\frac{\ln(\frac{\xi}{\alpha+\xi})}{C+B}) & C+B \geq 1, \\ 1+B' & C+B < 1. \end{cases}$$

Here, $B' = \frac{dB}{dC} = \frac{M_v M_d \frac{\gamma_f \omega}{\sigma_d}}{((M_v - C)\frac{\gamma_f \omega}{\sigma_d} + C)^2} > 0$. Further, we rewrite equation $F_1(C,B,U) = 0$ as following

$$F_1(C) = -(M_v - C)\gamma_f\omega - (M_d - B)\sigma_d + f(C) = 0.$$

Note that $F_1(0) = -M_v - M_d$ and $F_1(M_v) > 0$. Also, first derivative of $F_1(C)$ is

$$\frac{dF_1}{dC} = \gamma_f\omega + \sigma_d + \frac{df}{dC}.$$

For $C+B < 1$, it is easy to show that $\frac{dF_1}{dC} > 0$. However, for $C+B \geq 1$, if the control parameters are chosen such that $(1+B')(1+\frac{\ln(\frac{\xi}{\alpha+\xi})}{C+B}) > 0$, then it is easy to show that $\frac{dF_1}{dC} > 0$. Next, we will show that if $\frac{\xi}{\alpha+\xi > e^{-1}}$ then $(1+B')(1+\frac{\ln(\frac{\xi}{\alpha+\xi})}{C+B}) > 0$ for $C + B \geq 1$.

$$\frac{\xi}{\alpha+\xi} > e^{-1},$$

$$\ln(\frac{\xi}{\alpha+\xi}) > -1,$$

$$-\ln(\frac{\xi}{\alpha+\xi}) < 1 \leq C + B,$$

$$\frac{-\ln(\frac{\xi}{\alpha+\xi})}{C+B} < 1,$$

$$1 + \frac{\ln(\frac{\xi}{\alpha+\xi})}{C} > 0,$$

$$(1+B')(1+\frac{\ln(\frac{\xi}{\alpha+\xi})}{C}) > 0.$$

Therefore, if $\frac{\xi}{\alpha+\xi} > e^{-1}$ then $\frac{dF_1}{dC} > 0$. Hence, $F_1(C) = 0$ will have a unique operating point.

It can easily be shown that δ is one-to-one function of C. Further, for the equilibrium studies we assume that $\delta = \frac{N-R}{\rho(U)}$. As mentioned before, ρ is also a one-to-one function of u (for $0 < \rho < N$). Therefore, it can easily be shown that for a given C, there exists a unique U. Therefore, under conditions stated in the lemma, the set of equations (5.10) for the case $\delta > N - R$ has a unique solution in $0 < C < M_v$ and $0 < U < u_{max}$.

□

5.4 Numerical Results

In this section we verify our analysis in previous sections with numerical results. First, we consider a Framed PRMA system that employs only voice terminals.

Using numerical analysis (based on the equilibrium equations of the system), we illustrate nonlinear behavior and bifurcations of the system and further, we illustrate effects of the price based control. Next in this section, we consider a Framed PRMA system that employs both voice and data terminals.

A Framed PRMA system with voice terminals with following parameters are considered ([66]) $\sigma_v = 0.0055$, $\gamma_f = 0.05$, $N = 25$, and $D = 75$. Figures 5.1, 5.3, and 5.5 illustrate bifurcation diagrams for the Framed PRMA system with permission probability of $p_v = 1$. It is noticed that although the number of voice terminals less than 213 results in packet drop probability of less that 0.01, however, because of the bifurcations that occurs, the capacity of the system is limited to 170.

Next, we design a price based control and study the behavior of the Frame PRMA system with the controller. In order to design the control parameters, we choose $\xi = 1$. Further, we calculate $\alpha = 0.35$ such that packet drop probability in the case where $\delta < N - R$ is less than 0.01. Figures 5.2, 5.4, and 5.6 illustrate bifurcation diagrams for the Framed PRMA system with the price based control. The bifurcation diagram for packet drop probability illustrate that although bifurcations are not completely eliminated, however, they are controlled by being delayed for larger values of the bifurcation parameter. Also, it is noted that the capacity of the system in increased from 170 simultaneous voice terminals to 210.

Now a Framed PRMA system with voice and data terminals with following parameters is considered ([66]-in part) $\sigma_v = 0.0055$, $\sigma_d = 0.002$, $\gamma_f = 0.05$, $N = 25$, and $D = 75$. Figures 5.7, 5.9, 5.11, and 5.13 illustrate bifurcation diagrams for the Framed PRMA system with permission probability of $p_v = 1$, fixed $M_d = 100$,

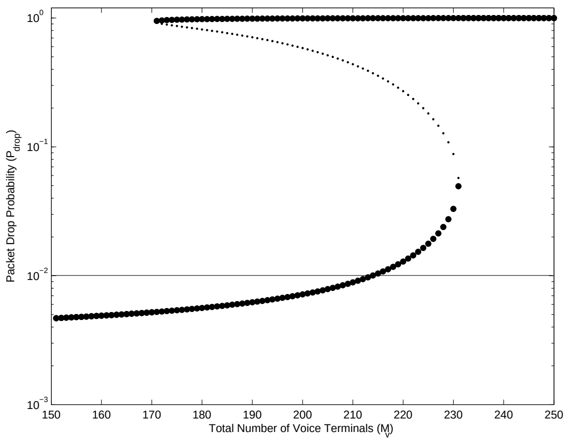

Figure 5.1: Bifurcation diagram for packet drop probability with no control ($p_v = 1$ and M_v is bifurcation parameter)

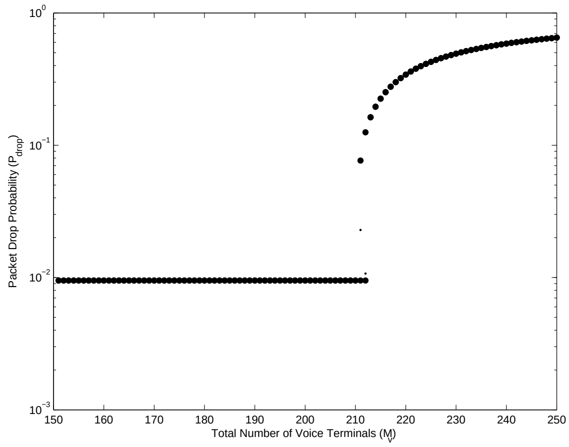

Figure 5.2: Bifurcation diagram for the packet drop probability with PBBC ($\alpha = 0.35$, $\xi = 1$, M_v is bifurcation parameter)

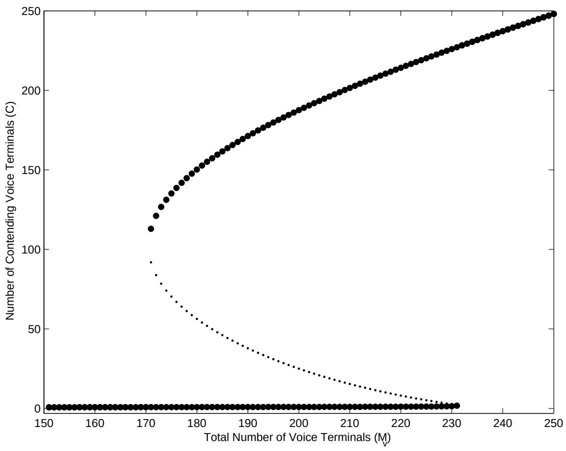

Figure 5.3: Bifurcation diagram for the number of contending terminals with no control ($p_v = 1$ and M_v is bifurcation parameter)

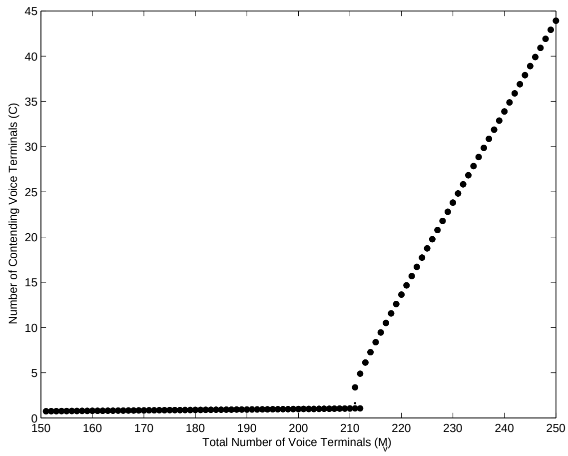

Figure 5.4: Bifurcation diagram for the number of contending terminals with PBBC ($\alpha = 0.35$, $\xi = 1$, M_v is bifurcation parameter)

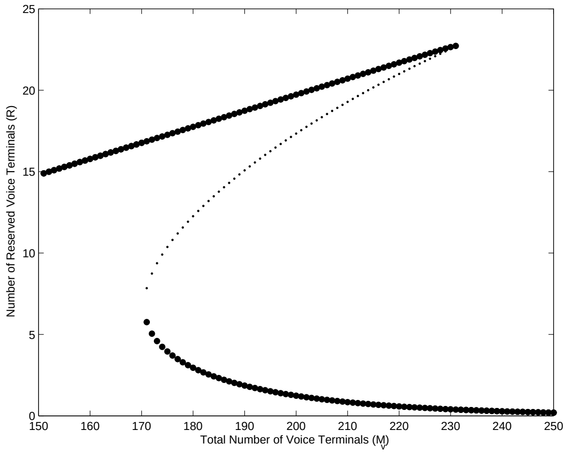

Figure 5.5: Bifurcation diagram for the number of reserved terminals with no control ($p_v = 1$ and M_v is bifurcation parameter)

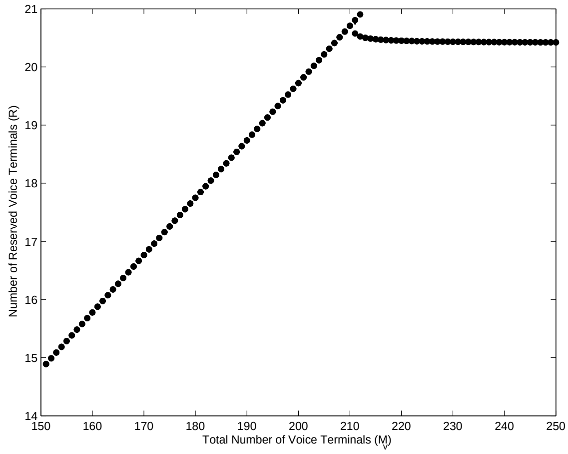

Figure 5.6: Bifurcation diagram for the number of reserved terminals with PBBC ($\alpha = 0.35$, $\xi = 1$, M_v is bifurcation parameter)

and M_v as bifurcation parameter. It is noticed that although the number of voice terminals less than 206 results in packet drop probability of less that 0.01, however, because of the bifurcations that occurs, the capacity of the system is limited to $M_v = 108$ and $M_d = 100$.

Further, figures 5.15, 5.17, 5.19, and 5.21 illustrate bifurcation diagrams for the Framed PRMA system with permission probability of $p_v = 1$, fixed $M_v = 100$, and M_d as bifurcation parameter. It is noticed that although the number of data terminals less than 1570 results in packet drop probability of less that 0.01, however, because of the bifurcations that occurs, the capacity of the system is limited to $M_v = 100$ and $M_d = 111$.

Next, we design a price based control and study the behavior of the Frame PRMA system with the controller. In order to design the control parameters, we choose $\xi = 1$. Further, we calculate $\alpha = 0.35$ such that packet drop probability in the case where $\delta < N-R$ is less than 0.01. Figures 5.8, 5.10, 5.12, and 5.14 illustrate bifurcation diagrams for the Framed PRMA system with the price based control with M_v as bifurcation parameter. The bifurcation diagram for packet drop probability illustrate that although bifurcations are not completely eliminated, however, they are controlled by being delayed for larger values of the bifurcation parameter. Also, it is noted that the capacity of the system in increased from $M_v = 108$ and $M_d = 100$ simultaneous voice terminals to $M_v = 203$ and $M_d = 100$.

Figures 5.16, 5.18, 5.20, and 5.22 illustrate bifurcation diagrams for the Framed PRMA system with the price based control with M_d as bifurcation parameter. The bifurcation diagram for packet drop probability illustrate that although bifurcations are not completely eliminated, however, they are controlled by being delayed for larger values of the bifurcation parameter. Also, it is noted that the capacity of the system in increased from $M_v = 100$ and $M_d = 111$ simultaneous voice terminals to $M_v = 100$ and $M_d = 1482$.

5.5 Summary and Future Lines of Work

In this chapter we studied a voice system and a voice-data system that employ Framed PRMA. We assumed that instead of transmitting feedback information during each time slot, like PRMA system, the feedback information regarding the status of transmissions are send at the end of each time frame. We studied the equilibrium behavior of the system and analyzed the effects of the price based control on bistability of the system. For each of voice and voice-data system, we further derived

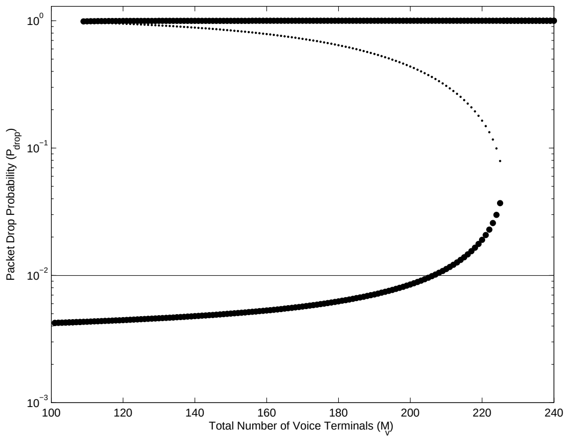

Figure 5.7: Bifurcation diagram for packet drop probability with no control ($p = 1$, $M_d = 100$, M_v is bifurcation parameter)

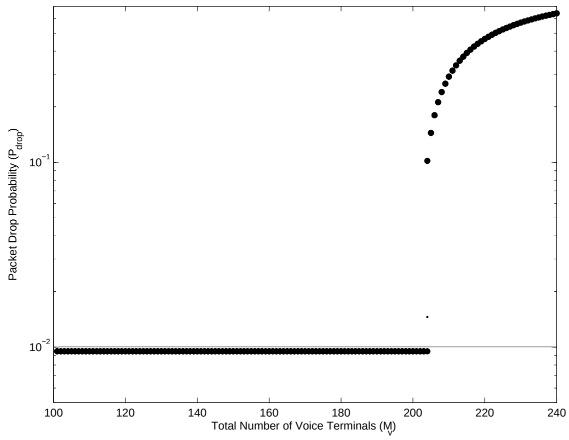

Figure 5.8: Bifurcation diagram for the packet drop probability with PBBC ($\alpha = 0.35$, $\xi = 1$, $M_d = 100$, M_v is bifurcation parameter)

sufficient conditions on the control parameters under each condition $\delta leq N - R$ or $\delta > N - R$ such that the controlled system has a unique equilibrium point. Future lines of research can include extending theses analysis to determine sufficient conditions for uniqueness of the equilibrium point of the system, studying a dynamic bifurcation control based on state observation, and extending state estimation bifurcation control, as discussed in previous chapters, to the Framed PRMA.

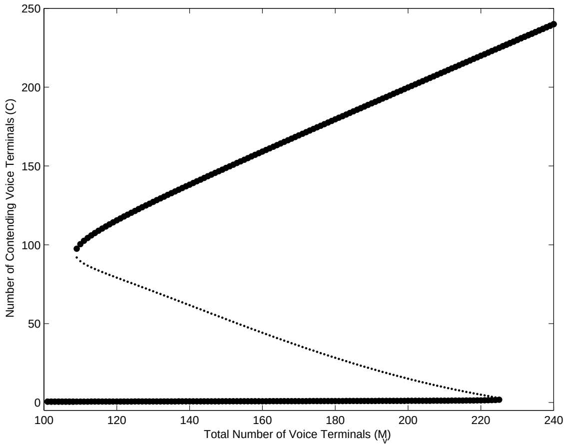

Figure 5.9: Bifurcation diagram for the number of contending terminals with no control ($p = 1$, $M_d = 100$, M_v is bifurcation parameter)

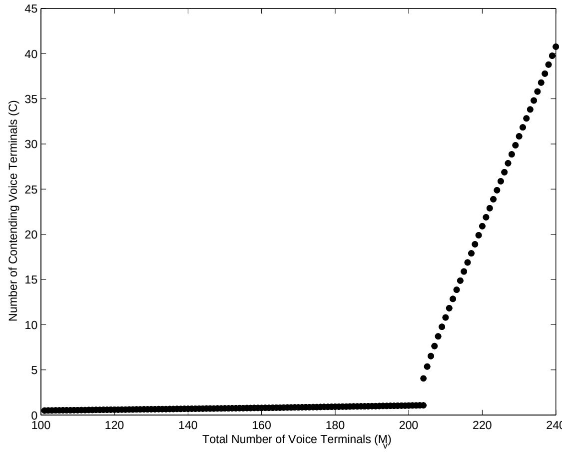

Figure 5.10: Bifurcation diagram for the number of contending terminals with PBBC ($\alpha = 0.35$, $\xi = 1$, $M_d = 100$, M_v is bifurcation parameter)

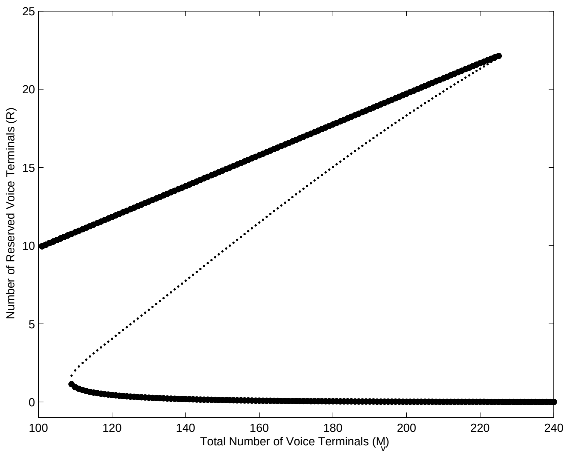

Figure 5.11: Bifurcation diagram for the number of reserved terminals with no control ($p = 1$, $M_d = 100$, M_v is bifurcation parameter)

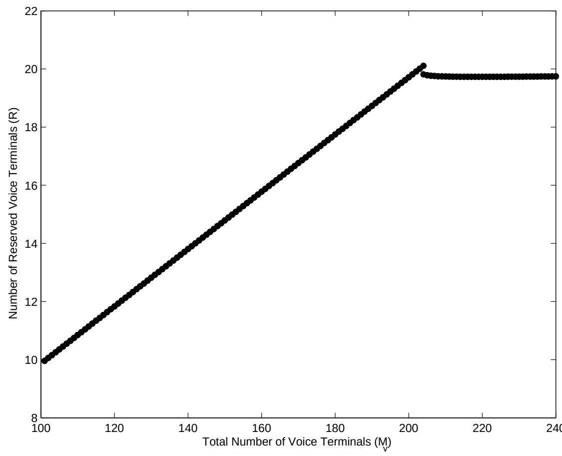

Figure 5.12: Bifurcation diagram for the number of reserved terminals with PBBC ($\alpha = 0.35$, $\xi = 1$, $M_d = 100$, M_v is bifurcation parameter)

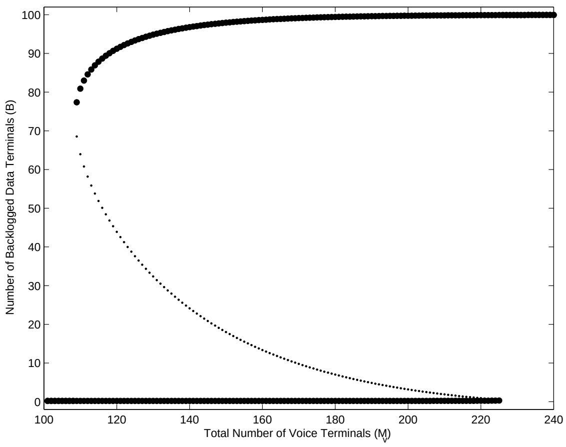

Figure 5.13: Bifurcation diagram for the number of backlogged data terminals with no control ($p = 1$, $M_d = 100$, M_v is bifurcation parameter)

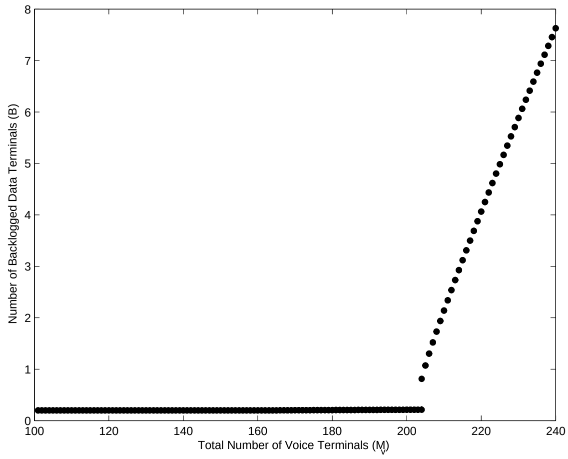

Figure 5.14: Bifurcation diagram for the number of backlogged data terminals with PBBC ($\alpha = 0.35$, $\xi = 1$, $M_d = 100$, M_v is bifurcation parameter)

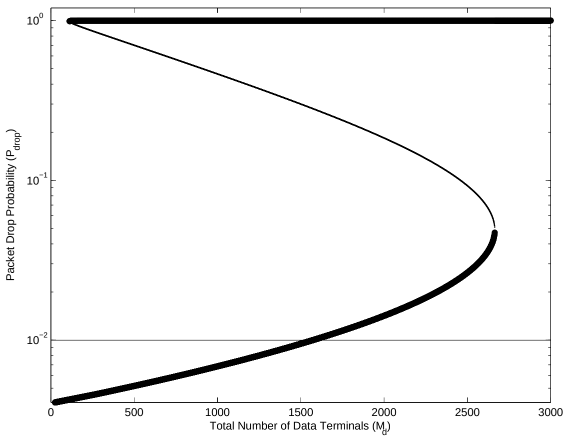

Figure 5.15: Bifurcation diagram for packet drop probability with no control ($p = 1$, $M_v = 100$, M_d is bifurcation parameter)

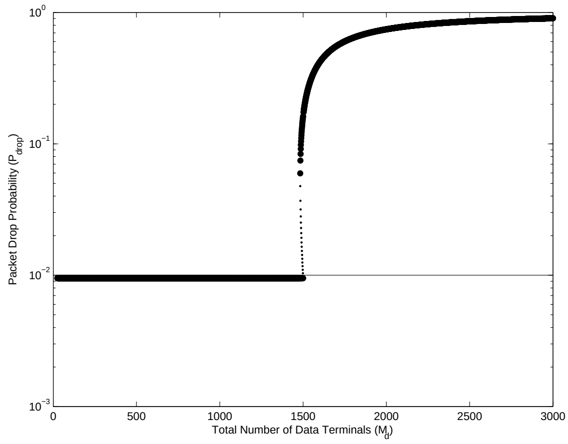

Figure 5.16: Bifurcation diagram for the packet drop probability with PBBC ($\alpha = 0.35$, $\xi = 1$, $M_v = 100$, M_d is bifurcation parameter)

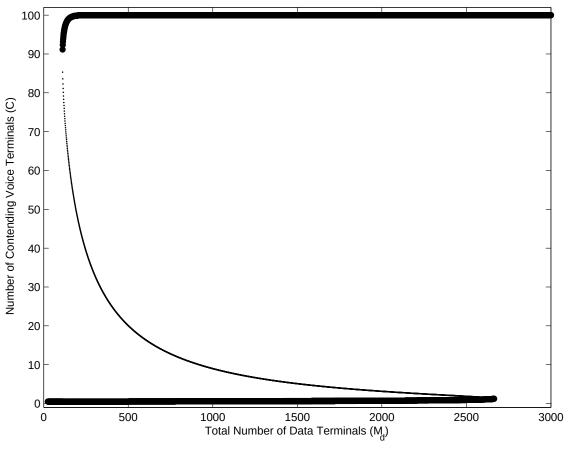

Figure 5.17: Bifurcation diagram for the number of contending terminals with no control ($p = 1$, $M_v = 100$, M_d is bifurcation parameter)

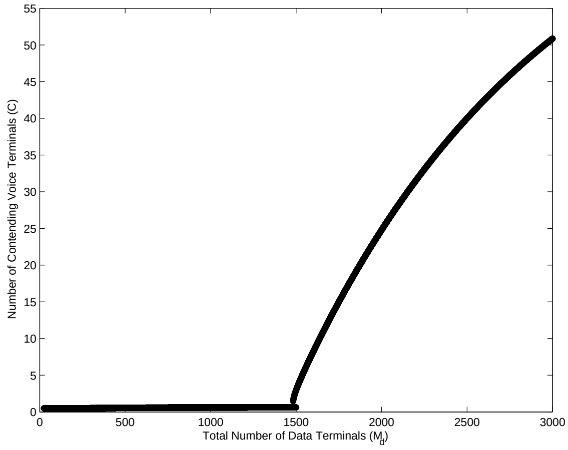

Figure 5.18: Bifurcation diagram for the number of contending terminals with PBBC ($\alpha = 0.35$, $\xi = 1$, $M_v = 100$, M_d is bifurcation parameter)

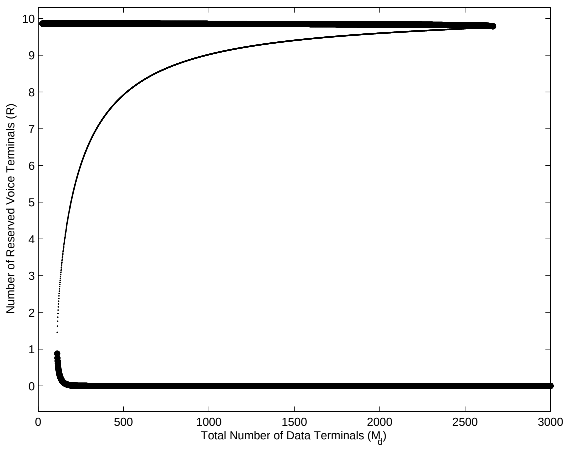

Figure 5.19: Bifurcation diagram for the number of reserved terminals with no control ($p = 1$, $M_v = 100$, M_d is bifurcation parameter)

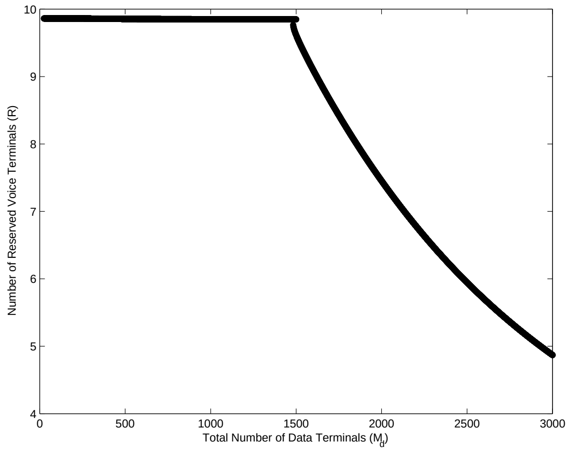

Figure 5.20: Bifurcation diagram for the number of reserved terminals with PBBC ($\alpha = 0.35$, $\xi = 1$, $M_v = 100$, M_d is bifurcation parameter)

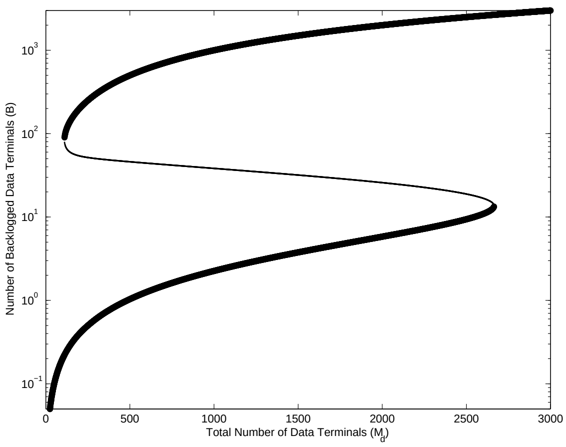

Figure 5.21: Bifurcation diagram for the number of backlogged data terminals with no control ($p = 1$, $M_v = 100$, M_d is bifurcation parameter)

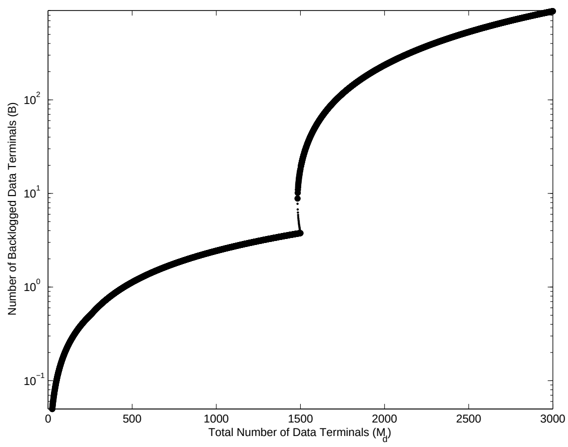

Figure 5.22: Bifurcation diagram for the number of backlogged data terminals with PBBC ($\alpha = 0.35$, $\xi = 1$, $M_v = 100$, M_d is bifurcation parameter)

Chapter 6

Conclusion

We studied equilibrium behavior and control for design of Packet Reservation Multiple Access (PRMA) protocols. In Chapter 2, we considered a system of voice terminals that employs PRMA as the medium access scheme. We studied the equilibrium behavior of the system and illustrated how small changes in system parameters can dramatically change the equilibrium behavior of the system. Further, we studied a price based control scheme for the PRMA system that updates permission probability of voice terminals based on a feedback information received from an access point indicating result of transmission in previous time slot. Contributions of this chapter include Markov analysis of the price based controlled system and analytical sufficient conditions guaranteeing a unique equilibrium point for the controlled system. We further introduced a state estimation-based controller, which updates permission probability based on estimate of one of the states of system to maximize throughput or minimize packet drop probability. Among contributions of this chapter are analytical sufficient conditions guaranteeing a unique equilibrium point for the controlled system. We also considered using multiple levels of transmission power at terminals and capture effect at the access point and studied its effects on bifurcations of the PRMA system. These are studied both for an error-free channel and a random error channel.

Moreover, we extended our analysis of the PRMA voice system to the PRMA with hindering states (PRMA-HS) in Chapter 2. For the PRMA-HS system, unlike the PRMA system, round trip delay plays a significant role in modeling the system. We studied the equilibrium behavior of the PRMA-HS voice system without and with the control schemes introduced earlier for both an error-free channel and a random error channel. Among contributions of this chapter are analytical sufficient conditions guaranteeing a unique equilibrium point for the controlled system.

In Chapter 3, we studied a general price based control for voice and data system employing PRMA-HS. We studied the equilibrium behavior of a PRMA voice-data system with and without price based control and state estimation-based control over error-free and random error channels. In Chapter 3, a Markov model of the controlled system (under some conditions) is analyzed and analytical sufficient conditions for system and control parameters of the controlled systems are derived such that the controlled system posses a unique operating point.

In Chapter 4, we revisited a finite terminals finite buffered slotted ALOHA system and we studied equilibrium effects of a price based control on bistability of the system using the tagged user approach. Among contributions of this chapter, we illustrated that bifurcations of the system can be controlled by appropriately choosing the control parameters.

Finally, in chapter 5, we considered a Framed PRMA system with voice only terminals and voice and data terminals. Unlike the PRMA system that feedback information is transmitted to the terminals at the end of each time slot, in Framed PRMA it is assumed that feedback information for a frame is transmitted at the end of a time frame. We extended the price based control scheme to the Framed PRMA and studied the equilibrium behavior of the controlled system.

Appendix A

Proof of Propositions 2.1, 2.2

In order to prove that Markov chain (2.2) (for $\phi = 0$) is positive recurrent, first we show that the Markov chain's state space is countable. Then, the irreducibility and aperiodicity of the chain is examined.

A.1 Proof of Proposition 2.1 - Markov Chain is Irreducible and Aperiodic

A.1.1 Countable State Space

The state (c, r, u) belongs to the following state space:

$$
\begin{aligned}
(\{0,1,2,\cdots,M_v\} \times & \{0,1,2,\cdots,N-1\} \times U \cup V) \\
& \cup (\{0,1,2,\cdots,M_v\} \times \{N\} \times U' \cup V'),
\end{aligned}
$$

here

$$
\begin{aligned}
U &= \{u | u = -\alpha a + \beta b + \xi d \geq 0; a, b, d \in \mathbb{Z}_+\}, \\
V &= \{v | v = u_0 - \alpha a + \beta b + \xi d > 0; a, b, d \in \mathbb{Z}_+\}, \\
U' &= \{u' | u' = -\alpha a + \beta b + \xi d \geq [\beta]^+; a, b, d \in \mathbb{Z}_+\}, \\
V' &= \{v' | v' = u_0 - \alpha a + \beta b + \xi d > [\beta]^+; a, b, d \in \mathbb{Z}_+\}.
\end{aligned}
$$

Without loss of generality, it is assumed that $u_0 \notin U$, therefore, sets U and V (U' and V') are disjoint. The state space of the Markov chain is a subset of the following set:

$$\aleph = \{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, N\} \times U \cup V.$$

Using the fact that every subset of a countable set is also countable, to show that the state space is countable it suffices to show that the set $\aleph$ is countable. Consider the general case where α, β, and ξ are just arbitrary real numbers. Then there exists a one-to-one correspondence between $u \in U$ (or $v \in V$) and the corresponding triplet (a, b, d). Now since set $\{(a, b, d) | a, b, d \in \mathbb{Z}_+\}$ is enumerable, we conclude that U and V, and hence the state space, are countable sets.

A.1.2 Irreducible State Space

As far as the stability analysis is concerned, only absorbing communication sets are relevant. We show that set of states with control starting at $u_0 \notin U$ is a non-absorbing set. Suppose $v = u_0 - \alpha a \in V$ where $a = \lfloor \frac{u_0}{\alpha} \rfloor$ when $\frac{u_0}{\alpha} \notin \mathbb{Z}$, and $a = \lfloor \frac{u_0}{\alpha} \rfloor - 1$ when $\frac{u_0}{\alpha} \in \mathbb{Z}$. Consider two different cases:

1) $r \in \{0, 1, 2, \cdots N-1\}$: Transition probability from state (c, r, v) to state $(c, r, 0)$ is: $P((c, r, v), (c, r, 0)) = (1 - \sigma_v)^{M_v - c - r}(1 - \frac{r}{N})(1 - p_v(v))^c > 0$. But $0 \in U$ ($0 \notin V$), hence there exists some $m > 0$ such that:

$$P^m((c_0, r_0, u_0), (c_0, r_0, 0)) = P^{m-1}((c_0, r_0, u_0), (c_0, r_0, v))P((c_0, r_0, v), (c_0, r_0, 0)) > 0$$

2) $r = N$: The probability of reaching state $(c, N, [\beta]^+)$ from state (c, N, v) for any $(c, r) \in \{0, 1, 2, \cdots, M_v\} \times \{N\}$ is:

$$
\begin{aligned}
P((c,N,v),(c,N,[\beta]^+)) =& P((c,N,v),(c,N-1,v)) \times P((c,N-1,v)(c,N-1,0)) \\
& \times P((c,N-1,0),(c,N,[\beta]+)) \\
=& (1-\sigma_v)^{M_v-c-N}\gamma_f(1-\sigma_v)^{M_v-c-N+1}(1-\frac{N-1}{N})(1-p_v(v))^c \\
& (M_v-c-N+1)\sigma_v(1-\sigma_v)^{M_v-c-N} \\
& (1-\frac{N-1}{N})cp_v(0)(1-p_v(0))^{c-1} \\
& ((1-\sigma_v)^{M_v-c-N+1}(\frac{N-1}{N})(1-\gamma_f))^{2N-2} \\
& > 0
\end{aligned}
$$

But $[\beta]^+ \notin V$, hence $P((c_0, N, u_0), (c_0, N, [\beta]^+)) > 0$

Therefore, set V is non-absorbing. As a result, only irreducibility of the following subset of the state space is examined:

$$(\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, N-1\} \times U) \cup (\{0, 1, 2, \cdots, M_v\} \times \{N\} \times U').$$

Lemma A.1. *Suppose (c, r, u) and (c, r, v) are two states in the state space $(\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, N-1\} \times U) \cup (\{0, 1, 2, \cdots, M_v\} \times \{N\} \times U')$. Then (c, r, u) and (c, r, v) communicate ($(c, r, u) \leftrightarrow (c, r, v)$).*

Proof. Consider two different cases:

Case 1: $(c, r, u), (c, r, v) \in \{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, N-1\} \times U$

Suppose $m = \lceil \frac{u}{\alpha} \rceil$ and $v = -\alpha a + \beta b + \xi d$. Then according to the transition probabilities:

$$P((c,r,u),(c,r,v)) \geq P((c,r,u),(c,r,0))P((c,r,0),(c,r,\xi d))$$
$$P((c,r,\xi d),(c,r,\beta b+\xi d))P((c,r,\beta b+\xi d),(c,r,v)).$$

In order to show that two states (c,r,u) and (c,r,v) communicate, it should be shown that transition probability from one to the other is positive.

1) $(c,r,u) \to (c,r,0)$: Define $x = \lfloor \frac{m}{N-r} \rfloor$

$$P((c,r,u),(c,r,0)) \geq$$
$$\prod_{i=0}^{x-1} (\prod_{k=0}^{N-r-1} P((c,r,[u-(i(N-r)+k)\alpha]^{+}),(c,r,[u-(i(N-r)+k+1)\alpha]^{+})))$$
$$(P((c,r,[u-(i+1)(N-r)\alpha]^{+}),(c,r,[u-(i+1)(N-r)\alpha]^{+})))^{r}$$
$$\times \prod_{k=0}^{m-x(N-r)-1} P((c,r,[u-(x(N-r)+k)\alpha]^{+}),(c,r,[u-(x(N-r)+k+1)\alpha]^{+}))$$

$$= \prod_{i=0}^{x-1} \prod_{k=0}^{N-r-1} (1-\sigma_v)^{M_v-c-r}(1-r/N)(1-p_v([u-(i(N-r)+k)\alpha]^{+}))^{c}$$
$$((1-\sigma_v)^{M_v-c-r}(r/N)(1-\gamma_f))^{xr}$$
$$\times \prod_{k=0}^{m-x(N-r)-1} (1-\sigma_v)^{M_v-c-r}(1-r/N)(1-p_v([u-(x(N-r)+k)\alpha]^{+}))^{c}$$
$$> 0.$$

2) $(c,r,0) \to (c,r,\xi d)$: Define $y = \lfloor \frac{d}{N-r} \rfloor$

$$
\begin{aligned}
&P((c,r,0),(c,r,\xi d)) \geq \\
&\prod_{i=0}^{y-1}\prod_{k=0}^{N-r-1} P((c,r,(i(N-r)+k)\xi),(c,r,(i(N-r)+k+1)\xi)) \\
&(P((c,r,(i+1)(N-r)\xi),(c,r,(i+1)(N-r)\xi)))^{r} \\
&\times \prod_{k=0}^{d-y(N-r)-1} P((c,r,(y(N-r)+k)\xi),(c,r,(y(N-r)+k+1)\xi))
\end{aligned}
$$

$$
\begin{aligned}
&= \prod_{i=0}^{y-1}\prod_{k=0}^{N-r-1} (1-\sigma_v)^{M_v-c-r}(1-r/N)(1-cp_v(1-p_v)^{c-1}-(1-p_v)^{c}) \\
&((1-\sigma_v)^{M_v-c-r}(r/N)(1-\gamma_f))^{yr} \\
&\times \prod_{k=0}^{d-y(N-r)-1} (1-\sigma_v)^{M_v-c-r}(1-r/N)(1-cp_v(1-p_v)^{c-1}-(1-p_v)^{c}) \\
&> 0.
\end{aligned}
$$

3) $(c,r,\xi d) \to (c,r,\beta b+\xi d)$: Define $z = \lfloor \frac{b}{N} \rfloor$. In this section of proof we consider a case where $r \geq b - Nz$. However, proof is similar if $r < b - Nz$.

$$
\begin{aligned}
&P((c,r,\xi d),(c,r,\beta b+\xi d)) \geq \\
&\prod_{k=0}^{r-1} P((c,r-k,\xi d),(c,r-k-1,\xi d)) \\
&\times \prod_{i=0}^{z-2}\prod_{k=0}^{N-1} P((c,k,\xi d+(iN+k)\beta),(c,k+1,\xi d+(iN+k+1)\beta)) \\
&\prod_{k=0}^{N-1} P((c,N-k,\xi d+(i+1)N\beta),(c,N-k-1,\xi d+(i+1)N\beta))
\end{aligned}
$$

$$\times \prod_{k=0}^{N-1} P((c,k,\xi d+((z-1)N+k)\beta),(c,k+1,\xi d+((z-1)N+k+1)\beta))$$
$$\times \prod_{k=0}^{N+b-zN-r-1} P((c,N-k,\xi d+zN\beta),(c,N-k-1,\xi d+zN\beta))$$
$$\times \prod_{k=N+b-zN-r}^{N-1} P((c,r-b+zN,\xi d+zN\beta),(c,r-b+zN,\xi d+zN\beta))$$
$$\times \prod_{k=0}^{b-Nz-1} P((c,r-b+zN+k,u_1),(c,r-b+zN+k+1,u_2))$$
$$> 0.$$

Where $u_1 = \xi d+(zN+k)\beta, \quad u_2 = \xi d+(zN+k+1)\beta.$

4) $(c,r,\beta b+\xi d) \to (c,r,-\alpha a+\beta b+\xi d)$: Define $w = \lfloor \frac{a}{N-r} \rfloor$

$$P((c,r,\beta b+\xi d),(c,r,-\alpha a+\beta b+\xi d)) \geq$$
$$\prod_{i=0}^{w-1}\prod_{k=0}^{N-r-1} P((c,r,\xi d+\beta b-(i(N-r)+k)\alpha),(c,r,\xi d+\beta b-(i(N-r)+k+1)\alpha))$$
$$(P((c,r,\xi d+\beta b-(i+1)(N-r)\alpha),(c,r,\xi d+\beta b-(i+1)(N-r)\alpha)))^r \times$$
$$\prod_{k=0}^{a-w(N-r)-1} P((c,r,\xi d+\beta b-(w(N-r)+k)\alpha),(c,r,\xi d+\beta b-(w(N-r)+k+1)\alpha))$$

$$= \prod_{i=0}^{w-1}\prod_{k=0}^{N-r-1} (1-\sigma_v)^{M_v-c-r}(1-r/N)(1-p_v(\xi d+\beta b-(i(N-r)+k)\alpha))^c$$
$$\times ((1-\sigma_v)^{M_v-c-r}(r/N)(1-\gamma_f))^{wr}$$
$$\times \prod_{k=0}^{a-w(N-r)-1} (1-\sigma_v)^{M_v-c-r}(1-r/N)(1-p_v(\xi d+\beta b-(w(N-r)+k)\alpha))^c$$
$$> 0.$$

Case 2: $(c, r, u), (c, r, v) \in \{0, 1, 2, \cdots, M_v\} \times \{N\} \times U'$

Suppose $m = \lceil \frac{u}{\alpha} \rceil$ and $v = -\alpha a + \beta b + \xi d$ then according to the transition probabilities:

$$\begin{aligned} P((c,N,u),(c,N,v)) \geq & P((c,N,u),(c,N-1,0))P((c,N-1,0),(c,N-1,\xi d)) \\ & P((c,N-1,\xi d),(c,N-1,\beta(b-1)+\xi d)) \\ & P((c,N-1,\beta(b-1)+\xi d),(c,N,v)) \end{aligned}$$

Now we show that state (c, N, v) is reachable from stat (c, N, u) with positive probability.

1) $(c, N, u) \to (c, N-1, 0)$:

$$\begin{aligned} & P((c,N,u),(c,N-1,0)) \geq \\ & P((c,N,u),(c,N-1,u)) \prod_{i=0}^{m-1} P((c,N-1,[u-i\alpha]^+),(c,N-1,[u-(i+1)\alpha]^+)) \\ & (P((c,N-1,[u-(i+1)\alpha]^+),(c,N-1,[u-(i+1)\alpha]^+)))^{N-1} \\ & = (1-\sigma_v)^{M_v-c-N}\gamma_f \prod_{i=0}^{m-1}(1-\sigma_v)^{M_v-c-N+1}(1-\frac{N-1}{N})(1-p_v(u-i\alpha))^c \\ & ((1-\sigma_v)^{M_v-c-N+1}\frac{N-1}{N}(1-\gamma_f))^{m(N-1)} > 0 \end{aligned}$$

2) $(c, N-1, 0) \to (c, N-1, \xi d)$:

$$\begin{aligned} & P((c,N-1,0),(c,N-1,\xi d)) \geq \\ & \prod_{i=0}^{d-1} P((c,N-1,i\xi),(c,N-1,(i+1)\xi))(P((c,N-1,(i+1)\xi),(c,N-1,(i+1)\xi)))^{N-1} \end{aligned}$$

$$= \prod_{i=0}^{d-1}(1-\sigma_v)^{M_v-c-N+1}(1-\frac{N-1}{N}(1-cp_v(i\xi)(1-p_v(i\xi))^{c-1}-(1-p_v(i\xi))^c))$$

$$((1-\sigma_v)^{M_v-c-N+1}\frac{N-1}{N}(1-\gamma_f))^{d(N-1)}$$

$$> 0$$

3) $(c, N-1, \xi d) \rightarrow (c, N-1, \beta(b-1)+\xi d)$. Define $y = \lfloor \frac{b-1}{N} \rfloor$

$$P((c, N-1, \xi d), (c, N-1, \beta(b-1)+\xi d)) \geq$$

$$\prod_{k=0}^{N-2} P((c, N-1-k, \xi d), (c, N-1-k-1, \xi d))$$

$$\times \prod_{i=0}^{y-2}\prod_{k=0}^{N-1} P((c, k, (iN+k)\beta+\xi d), (c, k+1, (iN+k+1)\beta+\xi d))$$

$$\prod_{k=0}^{N-1} P((c, N-k, (i+1)N\beta+\xi d), (c, N-k-1, (i+1)N\beta+\xi d))$$

$$\times \prod_{k=0}^{N-1} P((c, k, ((y-1)N+k)\beta+\xi d), (c, k+1, ((y-1)N+k+1)\beta+\xi d))$$

$$\times \prod_{k=0}^{b-yN-1} P((c, N-k, yN\beta+\xi d), (c, N-k-1, yN\beta+\xi d))$$

$$\times \prod_{k=b-yN}^{N-1} P((c, N-b+yN, yN\beta+\xi d), (c, N-b+yN, yN\beta+\xi d))$$

$$\times \prod_{k=0}^{b-yN-2} P((c, N-b+yN+k, u_3), (c, N-b+yN+k+1, u_4))$$

$$> 0.$$

Where $u_3 = (yN+k)\beta+\xi d, \quad u_4 = (yN+k+1)\beta+\xi d.$

4) $(c, N-1, \beta(b-1)+\xi d) \to (c, N, -\alpha a+\beta b+\xi d)$

$$P((c, N-1, \beta(b-1)+\xi d),(c, N, -\alpha a+\beta b+\xi d)) \geq$$

$$\prod_{i=0}^{a-1} P((c, N-1, -i\alpha+\beta(b-1)+\xi d),(c, N-1, -(i+1)\alpha+\beta(b-1)+\xi d))$$

$$(P((c, N-1, -(i+1)\alpha+\beta(b-1)+\xi d),(c, N-1, -(i+1)\alpha+\beta(b-1)+\xi d)))^{N-1}$$

$$\times P((c, N-1, -\alpha a+\beta(b-1)+\xi d),(c, N, v))$$
$$= \prod_{i=0}^{a-1} (1-\sigma_v)^{M_v-c-N+1}(1-\frac{N-1}{N})(1-p_v(-i\alpha+\beta(b-1)+\xi d))^c$$

$$\times ((1-\sigma_v)^{M_v-c-N+1}(N-1/N)(1-\gamma_f))^{a(N-1)}$$

$$\times (M_v-c-N+1)\sigma_v(1-\sigma_v)^{M_v-c-N}(1-\frac{N-1}{N})cp_v(1-p_v)^{c-1}$$

$$> 0$$

So far we proved that the arbitrary state (c, r, v) is reachable from any state (c, r, u) with positive probability $((c, r, u) \to (c, r, v))$. Since two states are chosen arbitrarily, then $(c, r, u) \leftrightarrow (c, r, v)$ or both states communicate. □

Now in this part, all states that communicate with $(0, 0, 0)$ are examined. It is assumed that $0 \leq r < N$. The case where $r = N$ is very similar.

1) $(c, r, u) \to (0, 0, 0)$: Consider two different cases:

A) $N - r > c$

$$P((c, r, u),(0, 0, [u-\alpha(N-c-r)+\beta c]^+)) \geq$$
$$\prod_{k=0}^{c-1} P((c-k, r+k, [u+k\beta]^+),(c-k-1, r+k+1, [u+(k+1)\beta]^+))$$
$$\times \prod_{k=0}^{N-c-r-1} P((0, r+c, [u-k\alpha+\beta c]^+),(0, r+c, [u-(k+1)\alpha+\beta c]^+))$$

$$\times \prod_{k=0}^{r+c-1} P((0, r+c-k, u_5), (0, r+c-k-1, u_5))$$

$$> 0$$

where $u_5 = [u - \alpha(N-c-r) + \beta c]^+$. Using lemma A.1, $(0,0,u_5) \leftrightarrow (0,0,0)$.

B) $N - r \leq c$. Define $x = \lfloor \frac{c-(N-r)}{N} \rfloor$.

$$P((c,r,u),(0,0,[u-\alpha(N-c+(N-r)+xN)+\beta c]^+)) \geq$$

$$\prod_{k=0}^{N-r-1} P((c-k, r+k, [u+k\beta]^+), (c-k-1, r+k+1, [u+(k+1)\beta]^+))$$

$$\times \prod_{i=0}^{x-1} \prod_{k=0}^{N-1} P((c-(i+1)N+r, N-k, [u_6]^+), (c-(i+1)N+r, N-k-1, [u_6]^+))$$

$$\prod_{k=0}^{N-1} P((c_4-k, k, [u_6+k\beta]^+), (c_4-k-1, k+1, [u_6+(k+1)\beta]^+))$$

$$\times \prod_{k=0}^{N-1} P((c-(N-r)-xN, N-k, [u_7]^+), (c-(N-r)-xN, N-k-1, [u_7]^+))$$

$$\times \prod_{k=0}^{c-(N-r)-xN-1} P((c_5-k, k, [u_7+k\beta]^+), (c_5-k-1, k+1, [u_7+(k+1)\beta]^+))$$

$$\times \prod_{k=0}^{N-c+(N-r)+xN-1} P((0, c_5, [u-k\alpha+\beta c]^+), (0, c_5, [u-(k+1)\alpha+\beta c]^+))$$

$$\times \prod_{k=0}^{c-(N-r)-xN-1} P((0, c_5-k, u_8), (0, c_5-k-1, u_8))$$

$$> 0$$

where

$$u_6 = u + (N - r + iN)\beta \qquad c_4 = c - (i+1)N + r$$

$$u_7 = u + (N - r + xN)\beta \qquad c_5 = c - (N-r) - xN$$

$$u_8 = [u - \alpha(N - c + (N-r) + xN) + \beta c]^+$$

According to Lemma A.1, $(0, 0, [u - \alpha(N - c + (N - r) + xN) + \beta c]^+) \leftrightarrow (0, 0, 0)$

2) $(0, 0, 0) \to (c, r, u)$

- $(0, 0, 0) \to (0, 0, u)$: Immediate result of Lemma A.1.

- $(0, 0, 0) \to (c, r, u)$: Using Lemma A.1, it is obvious that there exist a $m_1 > 0$ such that: $P^{m_1}((c, r, [\beta r + \xi]^+), (c, r, u)) > 0$. Therefore:

$$
\begin{aligned}
&P \geq P((0,0,0),(c+r,0,0)) \times P((c+r,0,0),(c+r,0,\xi)) \\
&\times \prod_{k=0}^{r-1} P((c+r-k,k,[k\beta+\xi]^+),(c+r-k-1,k+1,[(k+1)\beta+\xi]^+)) \\
&\times P^{m_1}((c,r,[\beta r+\xi]^+),(c,r,u)) \\
&> 0
\end{aligned}
$$

The above suggest that the state space is an absorbing communication set and therefore, irreducible.

A.1.3 Aperiodic State Space:

It remains to show that the Markov chain (2.2) defined on the state space is aperiodic. Since the Markov chain is irreducible, it suffices to show the aperiodicity for a single state (c, r, u). Any state (c, r, u) communicates with $(0, 0, 0)$. Therefore, there exists $m_1 > 0$ and $m_2 > 0$ such that $P^{m_1}((c, r, u), (0, 0, 0)) > 0$ and $P^{m_2}((0, 0, 0), (c, r, u)) > 0$. Let $m = m_1 + m_2$. Then:

$$P^m((c,r,u),(c,r,u)) \geq P^{m_1}((c,r,u),(0,0,0))P^{m_2}((0,0,0),(c,r,u)) > 0$$

$$P^{m+1}((c,r,u),(c,r,u)) \geq P^{m_1}((c,r,u),(0,0,0))P((0,0,0),(0,0,0))P^{m_2}((0,0,0),(c,r,u)) > 0$$

where $P((0,0,0),(0,0,0)) = (1-\sigma_v)^{M_v} > 0$. Hence the period for any state (c,r,u) is 1 and the chain is aperiodic.

A.2 Proof of Proposition 2.2 - Markov Chain is Positive Recurrent

So far we proved that the state space is countable, irreducible, and aperiodic. In this section, using the Proposition 2.3, we show that the Markov chain is positive recurrent and hence, it has a unique stationary probability distribution. Following non-negative Lyapunov function is chosen:

$$V(c,r,u) = u + Kr(1-\sigma_v + f(u))^c$$

where:

- $K > 0$ is any real constant,
- $f(.) : \mathbb{R} \to [0,1)$ is non-increasing, continuous, and bounded,
- there exists $\hat{u} > u_{max}$ such that $f(u) = 0$ for $u \geq \hat{u}$.

Note that $V(c,r,u)$ satisfies the requirement that it is non-negative. Following definition of simplifying notations is also necessary:

$$p_v = p_v(u) \qquad f = f(u) \qquad f_\alpha = f(u-\alpha)$$

$$f_\beta = f(u+\beta) \qquad f_\xi = f(u+\xi)$$

Expected drift of the Lyapunov function is a calculated as follows:

$$\begin{aligned}E(\Delta V|(c,r,u)) =& E(V(c_{n+1},r_{n+1},u_{n+1})-V(c_n,r_n,u_n)|(c_n,r_n,u_n)=(c,r,u))\\ =& E(u_{n+1}-u_n|(c,r,u))\\ &+KE(r_{n+1}(1-\sigma_v+f(u_{n+1}))^{c_{n+1}}-r_n(1-\sigma_v+f(u_n))^{c_n}|(c,r,u))\end{aligned}$$

First $E(r_{n+1}(1-\sigma_v+f(u_{n+1}))^{c_{n+1}}|(c,r,u))$ is calculated:

$$\begin{aligned}&E(r_{n+1}(1-\sigma_v+f(u_{n+1}))^{c_{n+1}}|(c,r,u))=\\ &-(1-\sigma_v+f)^c(1-\sigma_v^2+\sigma_v f)^{M_v-c-r}(\frac{r}{N})\gamma_f\\ &+r(1-\sigma_v+f)^c(1-\sigma_v^2+\sigma_v f)^{M_v-c-r}(\frac{r}{N})\\ &+r(1-\sigma_v+f_\alpha)^c(1-\sigma_v^2+\sigma_v f_\alpha)^{M_v-c-r}(1-\frac{r}{N})(1-p_v)^c\\ &+(r+1)(1-\sigma_v+f_\beta)^{c-1}(1-\sigma_v^2+\sigma_v f_\beta)^{M_v-c-r}(1-\frac{r}{N})cp_v(1-p_v)^{c-1}\\ &+r(1-\sigma_v+f_\xi)^c(1-\sigma_v^2+\sigma_v f_\xi)^{M_v-c-r}(1-\frac{r}{N})(1-(1-p_v)^c-cp_v(1-p_v)^{c-1}).\end{aligned}$$

Hence:

$$\begin{aligned}&E(\Delta V|(c,r,u))=\\ &-(\alpha+\xi)(1-\frac{r}{N})(1-p_v)^c+(\beta-\xi)(1-\frac{r}{N})cp_v(1-p_v)^{c-1}+\xi(1-\frac{r}{N})\\ &-Kr(1-\sigma_v+f)^c\\ &-K(1-\sigma_v+f)^c(1-\sigma_v^2+\sigma_v f)^{M_v-c-r}(\frac{r}{N})\gamma_f\\ &+Kr(1-\sigma_v+f)^c(1-\sigma_v^2+\sigma_v f)^{M_v-c-r}(\frac{r}{N})\\ &+Kr(1-\sigma_v+f_\alpha)^c(1-\sigma_v^2+\sigma_v f_\alpha)^{M_v-c-r}(1-\frac{r}{N})(1-p_v)^c\\ &+K(r+1)(1-\sigma_v+f_\beta)^{c-1}(1-\sigma_v^2+\sigma_v f_\beta)^{M_v-c-r}(1-\frac{r}{N})cp_v(1-p_v)^{c-1}\\ &+Kr(1-\sigma_v+f_\xi)^c(1-\sigma_v^2+\sigma_v f_\xi)^{M_v-c-r}(1-\frac{r}{N})(1-(1-p_v)^c-cp_v(1-p_v)^{c-1}).\end{aligned}$$

In order to prove that the Markov chain 2.2 is positive recurrent, a finite set Ξ is found such that for states outside this set, expected drift in Lyapunov function, $V(c,r,u)$ is negative. Consider $u \geq \hat{u} - \min(-\alpha, \beta)$. Then $p_v = f = f_\alpha = f_\beta = f_\xi = 0$:

$$\begin{aligned}
E(\Delta V|(c,r,u)) = &- K(1-\sigma_v)^c(1-\sigma_v^2)^{M_v-c-r}(\frac{r}{N})\gamma_f \\
&+ (Kr(1-\sigma_v)^c(1-\sigma_v^2)^{M_v-c-r} - Kr(1-\sigma_v)^c)(\frac{r}{N}) \\
&+ (Kr(1-\sigma_v)^c(1-\sigma_v^2)^{M_v-c-r} - Kr(1-\sigma_v)^c - \alpha)(1-\frac{r}{N}) \\
= &- K(1-\sigma_v)^c(1-\sigma_v^2)^{M_v-c-r}(\frac{r}{N})\gamma_f \\
&- Kr(1-\sigma_v)^c(1-(1-\sigma_v^2)^{M_v-c-r}) - \alpha(1-\frac{r}{N}) \\
\leq &- K(1-\sigma_v)^c(1-\sigma_v^2)^{M_v-c-r}(\frac{r}{N})\gamma_f - \alpha(1-\frac{r}{N}) \\
\leq &- K(1-\sigma_v)^{M_v}(1-\sigma_v^2)^{M_v}(\frac{r}{N})\gamma_f - \alpha(1-\frac{r}{N}) \\
= &- (K(1-\sigma_v)^{M_v}(1-\sigma_v^2)^{M_v}\gamma_f)(\frac{r}{N}) - \alpha(1-\frac{r}{N}) \\
\leq &- \min(K(1-\sigma_v)^{M_v}(1-\sigma_v^2)^{M_v}\gamma_f, \alpha)
\end{aligned}$$

Hence take $\epsilon = \min(K(1-\sigma_v)^{M_v}(1-\sigma_v^2)^{M_v}\gamma_f, \alpha)$. Now consider the case where $u < \hat{u} - \min(-\alpha, \beta)$, then:

$$\begin{aligned}
&E(\Delta V|(c,r,u)) \leq \\
&(\beta-\xi)(1-\frac{r}{N})cp_v(1-p_v)^{c-1} + \xi(1-\frac{r}{N}) \\
&+ Kr(1-\sigma_v+f)^c(1-\sigma_v^2+\sigma_v f)^{M_v-c-r} \\
&+ Kr(1-\sigma_v+f_\alpha)^c(1-\sigma_v^2+\sigma_v f_\alpha)^{M_v-c-r} \\
&+ K(r+1)(1-\sigma_v+f_\beta)^{c-1}(1-\sigma_v^2+\sigma_v f_\beta)^{M_v-c-r}
\end{aligned}$$

$$+ Kr(1-\sigma_v + f_\xi)^c (1-\sigma_v^2 + \sigma_v f_\xi)^{M_v - c - r}$$

$$\leq K(4N+1)4^{M_v} + \max(\beta, \xi)$$

Hence, we take $b = K(4N+1)4^{M_v} + \max(\beta, \xi)$ and define Ξ as:

$$\Xi = \{0, 1, \cdots, M_v\} \times \{0, 1, \cdots, N\} \times \{u | 0 \leq u = -a\alpha + b\beta + d\xi \leq \hat{u} - \min(-\alpha, \beta)\}$$

Hence:

$$E(\Delta V | (c, r, u)) \leq -\epsilon \qquad (c, r, u) \notin \Xi$$

$$E(\Delta V | (c, r, u)) \leq b \qquad (c, r, u) \in \Xi$$

and Markov chain (2.2) is positive recurrent.

Appendix B

Proof of Proposition 2.4 - Markov Chain is Irreducible, Aperiodic, and Positive Recurrent

State (c, r, u) belongs to following state space:

$$\aleph = \{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, N\} \times \{0, 1, 2, \cdots, N\}$$

Note that $c + r \leq M_v$, $u = N - r$, and $\aleph$ is a countable set. Since a subset of a countable set is also countable, thereby, the state space of the system is also countable. Next, we show that all states in the state space communicate. Since we have $r + u = N$, our focus is to show that $(c, r) \leftrightarrow (c', r')$. First, we show $(c, r) \rightarrow (0, 0)$. Consider two cases.

Case 1 - $c + r \leq N$

$$\begin{aligned}
&P((c, r), (0, 0)) \geq \\
&\prod_{k=0}^{c-1} P((c-k, r+k), (c-k-1, r+k+1)) \\
&\times \prod_{k=0}^{r-1} P((0, r+c-k), (0, r+c-k-1)) \\
&\times \prod_{k=0}^{N-c-r-1} P((0, c), (0, c)) \\
&\times \prod_{k=0}^{c-1} P((0, c-k), (0, c-k-1))
\end{aligned}$$

$$
\begin{aligned}
&= \prod_{k=0}^{c-1} (1-\sigma_v)^{M_v-c-r}(1-(r+k)/N)(c-k)p_v(1-p_v)^{c-k-1} \\
&\times \prod_{k=0}^{r-1} (1-\sigma_v)^{M_v-c-r+k}((r+c-k)/N)\gamma_f \\
&\times \prod_{k=0}^{N-c-r-1} (1-\sigma_v)^{M_v-c}(1-c/N) \\
&\times \prod_{k=0}^{c-1} (1-\sigma_v)^{M_v-c+k}((c-k)/N)\gamma_f \\
&> 0
\end{aligned}
$$

Case 2 - $c + r > N$. We define $x = \lfloor \frac{c-N+r}{N} \rfloor$. Since $c - N + r - xN \leq N$ based on case 1 above $(c - N + r - xN, 0) \to (0, 0)$. Therefore, here we show that $(c, r) \to (c - N + r - xN, 0)$.

$$
\begin{aligned}
&P((c,r),(c-N+r-xN,0)) \geq \\
&\prod_{k=0}^{N+r-1} P((c-k,r+k),(c-k-1,r+k+1)) \\
&\times \prod_{i=0}^{x-1} \prod_{k=0}^{N-1} P((c-N+r-iN,N-k),(c-N+r-iN,N-k-1)) \\
&\prod_{k=0}^{N-1} P((c-N+r-iN-k,k),(c-N+r-iN-k-1,k+1)) \\
&\times \prod_{k=0}^{N-1} P((c-N+r-xN,N-k),(c-N+r-xN,N-k-1))
\end{aligned}
$$

$$= \prod_{k=0}^{N+r-1} (1-\sigma_v)^{M_v-c-r}(1-(r+k)/N)(c-k)p_v(1-p_v)^{c-k-1}$$
$$\times \prod_{i=0}^{x-1}\prod_{k=0}^{N-1} (1-\sigma_v)^{M_v-c-r+k+iN}((N-k)/N)\gamma_f$$
$$\prod_{k=0}^{N-1} (1-\sigma_v)^{M_v-c-r+(i+1)N}(1-k/N)c_1p_v(1-p_v)^{c_1-1}$$
$$\times \prod_{k=0}^{N-1} (1-\sigma_v)^{M_v-c-r+(x+1)N}((N-k)/N)\gamma_f$$
$$> 0,$$

here $c_1 = c - N + r - iN - k$.

Next, we show $(0,0) \to (c', r')$:

$$P((0,0),(c',r')) \geq$$
$$P((0,0),(c'+r',0)) \times \prod_{k=0}^{r'} P((c'+r'-k,k),(c'+r'-k-1,k+1))$$

$$= \begin{pmatrix} M_v \\ c'+r' \end{pmatrix} \sigma_v^{c'+r'}(1-\sigma_v)^{M_v-c'-r'}$$
$$\times \prod_{k=0}^{r'} (1-\sigma_v)^{M_v-c'-r'}(1-k/N)(c'+r'-k)p_v(1-p_v)^{c'+r'-k-1}$$
$$> 0,$$

Therefore, $(c,r) \leftrightarrow (c',r')$. Hence, the state space is an absorbing communication set and irreducible. Next, we show that the Markov chain is aperiodic. As we showed above, any state (c,r) communicates with $(0,0)$. Therefore, there exists m_1 and m_2 such that $P^{m_1}((c,r),(0,0)) > 0$ and $P^{m_2}((0,0),(c,r)) > 0$. We define

$m = m_1 + m_2$. Also note that $P((0,0),(0,0)) = (1-\sigma_v)^{M_v}$. Therefore:

$$P^m((c,r),(c,r)) > P^{m_1}((c,r),(0,0))P^{m_2}((0,0),(c,r)) > 0,$$

$$P^{m+1}((c,r),(c,r)) > P^{m_1}((c,r),(0,0))P((0,0),(0,0))P^{m_2}((0,0),(c,r)) > 0.$$

Hence, period for each state is 1 and therefore, the Markov chain is aperiodic. So far we showed that Markov chain defined on $\aleph$ through (2.2) (for $\phi = 1$, $\alpha = \xi = 0$, and $\beta = -1$) is irreducible and aperiodic. Further, since the state space is finite, the Markov chain is positive recurrent and a unique stationary probability distribution exists.

Appendix C

Proof of Proposition 2.6

In this Appendix, we consider the PRMA-HS Voice system employing price based control and we show that state space defining the Markov chain for this system is countable and the Markov chain is aperiodic, irreducible, and positive recurrent. As discussed before, for the PRMA-HS Voice system we assume that round trip delay is one time slot. This assumption is merely for mathematical notations and the proposition can be proved for any round trip delay (in terms of the number of time slots) less than a frame.

Countable State Space **-** State (Y, Y_1) belongs to a state space that is a subset of

$$\aleph = (\{0, 1, 2, \cdots, M_v\} \times \{0, 1\} \times \{0, 1, 2, \cdots, N\} \times U \cup V)^2,$$

here

$$Y = (c, r, h_v, u) \qquad Y_1 = (c_1, r_1, h_{v_1}, u_1),$$

$$U = \{u | u = -\alpha a + \beta e + \xi d \geq 0; a, e, d \in \mathbb{Z}_+\},$$

$$V = \{v | v = u_0 - \alpha a + \beta e + \xi d > 0; a, e, d \in \mathbb{Z}_+\}.$$

Without loss of generality, it is assumed that $u_0 \notin U$ therefore, sets U and V are disjoint. Using the fact that every subset of a countable set is also countable, to show that the state space is countable it suffices to show that the set $\aleph$ is countable. Consider the general case where α, β, and ξ are just arbitrary real numbers. Then there exists a one-to-one correspondence between $u \in U$ (or $v \in V$) and the corresponding triplet (a, e, d). Now since set $\{(a, e, d)|a, e, d \in \mathbb{Z}_+\}$ is enumerable, we conclude that U and V, and hence the state space, are countable sets.

Irreducible State Space - As far as the stability analysis is concerned, only absorbing communication sets are relevant. As proved in the PRMA Voice only system (and as will be proved for PRMA Voice-Data system), it is easy to show that set of states with controls starting at $u_0 \notin U$ is a non-absorbing set. Therefore, sets V is non-absorbing. As a result, only irreducibility of the following subset of the state space is examined:

$$(\{0, 1, 2, \cdots, M_v\} \times \{0, 1\} \times \{0, 1, 2, \cdots, N\} \times U)^2.$$

However, as mentioned before, at least the following constraints exist on states of the system which belong to a subset of $\aleph$

- $c + r + h_v \leq M_v$ and $c_1 + r_1 + h_{v_1} \leq M_v$
- $r + h_v \leq N$ and $r_1 + h_{v_1} \leq N$
- $c - 1 \leq c_1 \leq M_v - r - h_v$
- $r - 1 \leq r_1 \leq r + 1$
- $h_v + h_{v_1} \leq 1$
- For $h_v = 1$, control signal $u_1 \geq [\beta]^+$
- For $r = N$, control signal $u \geq [\beta]^+$ and $u_1 \geq [\beta]^+$

Next, we show that if $X_u = ((c, r, h_v, u), (c_1, r_1, h_{v_1}, u_1))$ and $X_v = ((c, r, h_v, v), (c_1, r_1, h_{v_1}, v_1))$ are two states in the state space, then X_u and X_v communicate ($X_u \leftrightarrow X_v$).

Note that since $c, r, h_v, c_1, r_1, h_{v_1}, u, u_1, v, v_1$ are chosen such that X_u and X_v are valid states, it is obvious that (c_1, r_1, h_{v_1}, u_1) is reachable from (c, r, h_v, u) in one time slot and (c_1, r_1, h_{v_1}, v_1) is reachable from (c, r, h_v, v). Therefore, it suffices to show that (c, r, h_v, v) is reachable from (c, r, h_v, u).

If $h_v = 0$, using similar proof for PRMA voice only system, it is easy to show that $(c, r, 0, v)$ is reachable from $(c, r, 0, u)$.

If $h_v = 1$ and $h_{v_1} = 0$, with positive probability point $(c, r, 1, u)$ transitions to point $(c_1, r + 1, 0, [u + \beta]^+)$ in next time slot. Based on proof of PRMA voice only system, point $(c + 1, r, 0, v)$ is reachable from $(c_1, r + 1, 0, [u + \beta]^+)$ (in the PRMA voice only system we proved that states in the absorbing state space communicate with each other). Point $(c + 1, r, 0, v)$ transitions to point $(c, r, 1, v)$ in one time slot with positive probability (a contending voice terminal successfully transmits its voice packet).

If $h_v = 1$ and $h_{v_1} = 1$, with positive probability point $(c, r, 1, u)$ transitions to point $(c_1, r + 1, 1, [u + \beta]^+)$ in next time slot and can transition to point $(c_1, r + 2, 0, [u + 2\beta]^+)$. As proved in the PRMA voice only system that all the states communicate, point $(c_1, r+2, 0, [u+2\beta]^+)$ can transition to point $(c+1, r, 0, v)$ with positive probability that can transition to $(c, r, 1, v)$ in one time slot.

Therefore, we can show that $X_u \rightarrow X_v$. Since X_u and X_v are chosen arbitrarily, it can be shown that $X_v \rightarrow X_u$ and thus, $X_u \leftrightarrow X_v$.

Further, for a given state $X_u = ((c, r, h_v, u), (c_1, r_1, h_{v_1}, u_1))$ in the state space, it is easy to prove that $X_u \leftrightarrow O$ where $O = ((0, 0, 0, 0), (0, 0, 0, 0))$.

If the system is at state X_u, during time all contending terminals can make reservation and eventually all the reserved terminal lose their reservation until the number of contending and reserved voice terminals is zero. Therefore, $X_u \rightarrow ((0, 0, 0, u_2), (0, 0, 0, u_3))$. However, as shown above $((0, 0, 0, u_2), (0, 0, 0, u_3)) \rightarrow O$. Thus, $X_u \rightarrow O$.

In order to show $O \rightarrow X_u$, we again focus on $(0, 0, 0, 0)$ and (c, r, h_v, u), because $c, r, h_v, c_1, r_1, h_{v_1}, u, u_1$ are chosen such that point (c_1, r_1, h_{v_1}, u_1) is reachable from (c, r, h_v, u) in one time slot. If $h_v = 0$, we proved in the PRMA voice only system that $(c, r, 0, u)$ is reachable from $(0, 0, 0, 0)$. If $h_v = 1$, using the PRMA voice only system, we can again show that point $(0, 0, 0, 0)$ can transition to point $(c+1, r, 0, u)$ with positive probability and point $(c + 1, r, 0, u)$ can transition to point $(c, r, 1, u)$ in one time slot with positive probability. Therefore, X_u is reachable form O.

Thus $X_u \leftrightarrow O$.

The above derivations proves that the state space is an absorbing communication set and therefore, irreducible.

Aperiodic State Space- In the same way as the PRMA voice only system it can be proved that the Markov chain is aperiodic considering:

$$P(((0, 0, 0, 0), (0, 0, 0, 0)), ((0, 0, 0, 0), (0, 0, 0, 0))) = (1 - \sigma_v)^{M_v} > 0.$$

Markov Chain is Positive Recurrent- So far we proved that the state space is countable and the Markov chain is irreducible and aperiodic. Here, using the

Proposition 2.3, we show that the Markov chain is positive recurrent and hence, it has a unique stationary probability distribution. Following non-negative Lyapunov function is chosen:

$$V(X) = u + Kr(1 - \sigma_v + f(u))^c$$

where:

- $X = ((c_{-1}, r_{-1}, h_{v_{-1}}, u_{-1}), (c, r, h_v, u))$
- $K > 0$ is any real constant,
- $f(.) : \mathbb{R} \to [0, 1)$ is non-increasing, continuous, and bounded,
- there exists $\hat{u} > u_{max}$ such that $f(u) = 0$ for $u \geq \hat{u}$.

Note that $V(c, r, b, u)$ satisfies the requirement that it is non-negative. Following definition of simplifying notations is also necessary:

$$p_v = p_v(u) \qquad p_{v_{-1}} = p_v(u_{-1}) \qquad f = f(u)$$

$$f_\alpha = f(u - \alpha) \qquad f_\beta = f(u + \beta) \qquad f_\xi = f(u + \xi)$$

Expected drift of the Lyapunov function is a calculated as follows:

$$\begin{aligned} E(\Delta V|X) =& E(V(X_{n+1}) - V(X_n)|X_n = X) \\ =& E(u_{n+1} - u_n|X) \\ &+ KE(r_{n+1}(1 - \sigma_v + f(u_{n+1}))^{c_{n+1}} - r_n(1 - \sigma_v + f(u_n))^{c_n}|X) \end{aligned}$$

First $E(r_{n+1}(1 - \sigma_v + f(u_{n+1}))^{c_{n+1}}|X)$ is calculated:

$$E(r_{n+1}(1-\sigma_v+f(u_{n+1}))^{c_{n+1}}-r_n(1-\sigma_v+f(u_n))^{c_n} \mid X_n=X)=$$

$$(r-1)(1-\sigma_v+f)^c(1-{\sigma_v}^2+\sigma_v f)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f(\frac{r+h_v}{N}+\Upsilon+\Upsilon^{Nc})$$

$$+(r-1)(1-\sigma_v+f)^{c-1}(1-{\sigma_v}^2+\sigma_v f)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f\Upsilon^c$$

$$+r(1-\sigma_v+f)^c(1-{\sigma_v}^2+\sigma_v f)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})(1-\gamma_f)(\frac{r+h_v}{N}+\Upsilon+\Upsilon^{Nc})$$

$$+r(1-\sigma_v+f)^{c-1}(1-{\sigma_v}^2+\sigma_v f)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})(1-\gamma_f)\Upsilon^c$$

$$+r(1-\sigma_v+f_\alpha)^c(1-{\sigma_v}^2+\sigma_v f_\alpha)^{M_v-c-r-h_v}\Upsilon_{-1}^{Nc}(\frac{r+h_v}{N}+\Upsilon+\Upsilon^{Nc})$$

$$+r(1-\sigma_v+f_\alpha)^{c-1}(1-{\sigma_v}^2+\sigma_v f_\alpha)^{M_v-c-r-h_v}\Upsilon_{-1}^{Nc}\Upsilon^c$$

$$+(r+1)(1-\sigma_v+f_\beta)^c(1-{\sigma_v}^2+\sigma_v f_\beta)^{M_v-c-r-h_v}\Upsilon_{-1}^c(\frac{r+h_v}{N}+\Upsilon+\Upsilon^{Nc})$$

$$+(r+1)(1-\sigma_v+f_\beta)^{c-1}(1-{\sigma_v}^2+\sigma_v f_\beta)^{M_v-c-r-h_v}\Upsilon_{-1}^c\Upsilon^c$$

$$+r(1-\sigma_v+f_\xi)^c(1-{\sigma_v}^2+\sigma_v f_\xi)^{M_v-c-r-h_v}\Upsilon_{-1}(\frac{r+h_v}{N}+\Upsilon+\Upsilon^{Nc})$$

$$+r(1-\sigma_v+f_\xi)^{c-1}(1-{\sigma_v}^2+\sigma_v f_\xi)^{M_v-c-r-h_v}\Upsilon_{-1}\Upsilon^c.$$

Where

$$X_1=((c,r,h_v,b,h_d,u),(c_1,r_1,h_{v_1},b_1,h_{d_1},u_1)),$$

$$\Upsilon^c=(1-\frac{r+h_v}{N})cp_v(1-p_v)^{c+h_v-1},$$

$$\Upsilon^{Nc}=(1-\frac{r+h_v}{N})(1-p_v)^{c+h_v},$$

$$\Upsilon=(1-\frac{r+h_v}{N})-\Upsilon^{Nc}-\Upsilon^c,$$

$$\Upsilon_{-1}^c=(1-\frac{r_{-1}+h_{v_{-1}}}{N})c_{-1}p_{v_{-1}}(1-p_{v_{-1}})^{c_{-1}+h_{v_{-1}}-1},$$

$$\Upsilon_{-1}^{Nc}=(1-\frac{r_{-1}+h_{v_{-1}}}{N})(1-p_{v_{-1}})^{c_{-1}+h_{v_{-1}}},$$

$$\Upsilon_{-1}=(1-\frac{r_{-1}+h_{v_{-1}}}{N})-\Upsilon_{-1}^{Nc}-\Upsilon_{-1}^c.$$

Hence $E(\Delta V \mid X)$ is:

$$
\begin{aligned}
&E(\Delta V \mid X) = \\
&- K(1-\sigma_v+f)^c(1-{\sigma_v}^2+\sigma_v f)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f(\frac{r+h_v}{N}+\Upsilon+\Upsilon^{Nc}) \\
&- K(1-\sigma_v+f)^{c-1}(1-{\sigma_v}^2+\sigma_v f)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f\Upsilon^c \\
&+ Kr(1-\sigma_v+f)^c(1-{\sigma_v}^2+\sigma_v f)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})(\frac{r+h_v}{N}+\Upsilon+\Upsilon^{Nc}) \\
&+ Kr(1-\sigma_v+f)^{c-1}(1-{\sigma_v}^2+\sigma_v f)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})\Upsilon^c \\
&+ Kr(1-\sigma_v+f_\alpha)^c(1-{\sigma_v}^2+\sigma_v f_\alpha)^{M_v-c-r-h_v}\Upsilon_{-1}^{Nc}(\frac{r+h_v}{N}+\Upsilon+\Upsilon^{Nc}) \\
&+ Kr(1-\sigma_v+f_\alpha)^{c-1}(1-{\sigma_v}^2+\sigma_v f_\alpha)^{M_v-c-r-h_v}\Upsilon_{-1}^{Nc}\Upsilon^c \\
&+ K(r+1)(1-\sigma_v+f_\beta)^c(1-{\sigma_v}^2+\sigma_v f_\beta)^{M_v-c-r-h_v}\Upsilon_{-1}^c(\frac{r+h_v}{N}+\Upsilon+\Upsilon^{Nc}) \\
&+ K(r+1)(1-\sigma_v+f_\beta)^{c-1}(1-{\sigma_v}^2+\sigma_v f_\beta)^{M_v-c-r-h_v}\Upsilon_{-1}^c\Upsilon^c \\
&+ Kr(1-\sigma_v+f_\xi)^c(1-{\sigma_v}^2+\sigma_v f_\xi)^{M_v-c-r-h_v}\Upsilon_{-1}(\frac{r+h_v}{N}+\Upsilon+\Upsilon^{Nc}) \\
&+ Kr(1-\sigma_v+f_\xi)^{c-1}(1-{\sigma_v}^2+\sigma_v f_\xi)^{M_v-c-r-h_v}\Upsilon_{-1}\Upsilon^c \\
&- Kr(1-\sigma_v+f)^c-(\alpha+\xi)\Upsilon_{-1}^{Nc}+(\beta-\xi)\Upsilon_{-1}^c+(1-\frac{r_{-1}+h_{v_{-1}}}{N})\xi.
\end{aligned}
$$

In order to prove that the Markov chain is positive recurrent, a finite set Ξ is found such that for states not in that set expected drift in Lyapunov function is negative. Hence, consider two cases:

- $u_{-1} > \hat{u} - 2\min(-\alpha, \beta)$: In this case: $p_{v_{-1}} = p_v = f = f_\alpha = f_\beta = f_\xi = 0$.

Expected drift in the Lyapunov function is:

$$
\begin{aligned}
E(\Delta V \mid X) = &- K(1-\sigma_v)^c(1-{\sigma_v}^2)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f \\
&+ Kr(1-\sigma_v)^c(1-{\sigma_v}^2)^{M_v-c-r-h_v} - Kr(1-\sigma_v)^c \\
&- \alpha(1-\frac{r_{-1}+h_{v_{-1}}}{N}) \\
= &- K(1-\sigma_v)^c(1-{\sigma_v}^2)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f \\
&- Kr(1-\sigma_v)^c(1-(1-{\sigma_v}^2)^{M_v-c-r-h_v}) \\
&- \alpha(1-\frac{r_{-1}+h_{v_{-1}}}{N}) \\
\leq &- K(1-\sigma_v)^c(1-{\sigma_v}^2)^{M_v-c-r-h_v}(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f \\
&- \alpha(1-\frac{r_{-1}+h_{v_{-1}}}{N}) \\
\leq &- K(1-\sigma_v)^{M_v}(1-{\sigma_v}^2)^{M_v}(\frac{r_{-1}+h_{v_{-1}}}{N})\gamma_f \\
&- \alpha(1-\frac{r_{-1}+h_{v_{-1}}}{N}) \\
\leq &- \min(K(1-\sigma_v)^{M_v}(1-{\sigma_v}^2)^{M_v}\gamma, \alpha).
\end{aligned}
$$

Hence we take $\epsilon = \min(K(1-\sigma_v)^{M_v}(1-{\sigma_v}^2)^{M_v}\gamma_f, \alpha)$.

- $u_{-1} \leq \hat{u} - 2\min(-\alpha, \beta)$: In this case, upper bound for expected drift of Lyapunov function is found as following:

$$
\begin{aligned}
E(\Delta V \mid X) \leq &Kr(1-\sigma_v+f)^c(1-{\sigma_v}^2+\sigma_v f)^{M_v-c-r-h_v} \\
&+Kr(1-\sigma_v+f)^{c-1}(1-{\sigma_v}^2+\sigma_v f)^{M_v-c-r-h_v} \\
&+Kr(1-\sigma_v+f_\alpha)^c(1-{\sigma_v}^2+\sigma_v f_\alpha)^{M_v-c-r-h_v} \\
&+Kr(1-\sigma_v+f_\alpha)^{c-1}(1-{\sigma_v}^2+\sigma_v f_\alpha)^{M_v-c-r-h_v}
\end{aligned}
$$

$$+ K(r+1)(1-\sigma_v+f_\beta)^c(1-{\sigma_v}^2+\sigma_v f_\beta)^{M_v-c-r-h_v}$$

$$+ K(r+1)(1-\sigma_v+f_\beta)^{c-1}(1-{\sigma_v}^2+\sigma_v f_\beta)^{M_v-c-r-h_v}$$

$$+ Kr(1-\sigma_v+f_\xi)^c(1-{\sigma_v}^2+\sigma_v f_\xi)^{M_v-c-r-h_v}$$

$$+ Kr(1-\sigma_v+f_\xi)^{c-1}(1-{\sigma_v}^2+\sigma_v f_\xi)^{M_v-c-r-h_v}$$

$$+ \max(\beta,\xi)$$

$$\leq 2K(4N+1)4^{M_v} + \max(\beta,\xi)$$

Hence take $b = 2K(4N+1)4^{M_v} + \max(\beta,\xi)$.

Define

$$\Psi = \{0,1,\cdots,M_v\} \times \{0,1,\cdots,N\} \times \{0,1\}$$

$$\Psi_u = \{u \mid u = -a\alpha + e\beta + d\xi, u < \hat{u} - 2\min(-\alpha,\beta), a,e,d \in \mathbb{Z}_+\}.$$

Take finite set Ξ to be $\Xi = (\Psi \times \Psi_u) \times (\Psi \times \Psi_u)$. For the values of ϵ, b, and Ξ stated before:

$$E(\Delta V \mid X_n = X) \leq -\epsilon \qquad X \notin \Xi,$$

$$E(\Delta V \mid X_n = X) \leq b \qquad X \in \Xi.$$

Therefore, the Markov chain is positive recurrent.

Appendix D

Proof of Proposition 2.7

State $(Y, Y_1) = ((c, r, h_v, u), (c_1, r_1, h_{v_1}, u_1))$ belongs to a state space that is a subset of:

$$\aleph = (\{0, 1, 2, \cdots, M_v\} \times \{0, 1\} \times \{0, 1, 2, \cdots, N\} \times \{0, 1, 2, \cdots, N\})^2.$$

Note that $u = N - r$, $u_1 = N - r_1$, and $\aleph$ is a countable set. Since a subset of a countable set is also countable, thereby, the state space of the system is also countable.

Next, we show that all states in the state space communicate. Since we have $r + u = N$ and $r_1 + u_1 = N$, our focus is to show that $((c, r, h_v), (c_1, r_1, h_{v_1})) \leftrightarrow ((c', r', h'_v), (c'_1, r'_1, h'_{v_1}))$. However, as discussed above, it is noted that values of Y, Y_1, Y', Y'_1 are chosen such that Y_1 is reachable from Y in one time slot and Y'_1 is reachable from Y' in one time slot (where $(Y', Y'_1) = ((c', r', h'_v), (c'_1, r'_1, h'_{v_1})))$. Therefore, it suffices to show that $(c, r, h_v) \leftrightarrow (c', r', h'_v)$.

In order to prove that $(c, r, h_v) \leftrightarrow (c', r', h'_v)$, we show that point $O = (0, 0, 0)$ is reachable form every point (c, r, h_v) with positive probability and also, every point (c, r, h_v) is reachable from O.

Based on the proofs presented for the PRMA voice only system and the fact that hindering states (such as $h_v, h'_v, h_{v_1}, h'_{v_1}$) only model delays, it is easy to show that $(c, r, h_v) \leftrightarrow (0, 0, 0)$, and therefore, $(Y, Y_1) \leftrightarrow (Y', Y'_1)$.

Further, in the same way as the PRMA voice only system it can be proved that the Markov chain is aperiodic considering:

$$P(((0,0,0,N),(0,0,0,N)),((0,0,0,N),(0,0,0,N))) = (1-\sigma_v)^{M_v} > 0.$$

So far we showed that Markov chain defined on $\aleph$ through (3.2) (for $\phi = 1$, $\alpha = \xi = 0$, $\beta = -1$, and $RTD = 1$) is irreducible and aperiodic. Further, since the state space is finite, the Markov chain is positive recurrent and a unique stationary probability distribution exists.

Appendix E

Proof of Proposition 3.1

As discussed before, for the PRMA Voice-Data system we assume that round trip delay is negligible. However, we ease the assumption that $p_v = p_d$.

Countable State Space **-** State (c, r, b, u) belongs to the following state space:

$$(\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, M_d\} \times \{0, 1, 2, \cdots, N-1\} \times U \cup V)$$
$$\cup\, (\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, M_d\} \times \{N\} \times U' \cup V'),$$

here

$$U = \{u | u = -\alpha a + \beta e + \xi d \geq 0; a, e, d \in \mathbb{Z}_+\},$$
$$V = \{v | v = u_0 - \alpha a + \beta e + \xi d > 0; a, e, d \in \mathbb{Z}_+\},$$
$$U' = \{u' | u' = -\alpha a + \beta e + \xi d \geq [\beta]^+; a, e, d \in \mathbb{Z}_+\},$$
$$V' = \{v' | v' = u_0 - \alpha a + \beta e + \xi d > [\beta]^+; a, e, d \in \mathbb{Z}_+\}.$$

Without loss of generality, it is assumed that $u_0 \notin U$ therefore, sets U and V (U' and V') are disjoint. The state space of the Markov chain is a subset of the following set:

$$\aleph = (\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, M_d\} \times \{0, 1, 2, \cdots, N\} \times U \cup V).$$

Using the fact that every subset of a countable set is also countable, to show that the state space is countable it suffices to show that the set $\aleph$ is countable. Consider the general case where α, β, and ξ are just arbitrary real numbers. Then there exists a one-to-one correspondence between $u \in U$ (or $v \in V$) and the corresponding triplet (a, e, d). Now since set $\{(a, e, d)|a, e, d \in \mathbb{Z}_+\}$ is enumerable, we conclude that U and V, and hence the state space, are countable sets.

Irreducible State Space - As far as the stability analysis is concerned, only absorbing communication sets are relevant. As proved in the PRMA Voice only system, it is easy to show that set of states with controls starting at $u_0 \notin U$ is a non-absorbing set. Therefore, sets V and V' are non-absorbing. As a result, only irreducibility of the following subset of the state space is examined:

$$\begin{aligned}&(\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, M_d\} \times \{0, 1, 2, \cdots, N-1\} \times U)\\&\cup (\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, M_d\} \times \{N\} \times U').\end{aligned}$$

Next, we show that if (c, r, b, u) and (c, r, b, v) are two states in the state space $(\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, M_d\} \times \{0, 1, 2, \cdots, N-1\} \times U) \cup (\{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, M_d\} \times \{N\} \times U')$, then (c, r, b, u) and (c, r, b, v) communicate $((c, r, b, u) \leftrightarrow (c, r, b, v))$.

As discussed in the PRMA Voice only system, we can consider two different cases where either $r < N$ or $r = N$. Here, we consider the case where $r < N$. The case $r = N$ is similar and can be easily proved.

Assume $m = \lceil \frac{u}{\alpha} \rceil$ and $v = -\alpha a + \beta b + \xi d$. Then according to the transition probabilities:

$$P((c,r,b,u),(c,r,b,v)) \geq P((c,r,b,u),(c,r,b,0))P((c,r,b,0),(c,r,b,\xi d))$$
$$P((c,r,b,\xi d),(c,r,b,\beta b+\xi d))P((c,r,b,\beta b+\xi d),(c,r,b,v)).$$

In order to show that two states (c,r,b,u) and (c,r,b,v) communicate, it should be shown that transition probability from one to the other is positive.

1) $(c,r,b,u) \to (c,r,b,0)$: Define $x = \lfloor \frac{m}{N-r} \rfloor$

$$P((c,r,b,u),(c,r,b,0)) \geq$$
$$\prod_{i=0}^{x-1} (\prod_{k=0}^{N-r-1} P((c,r,b,[u-(i(N-r)+k)\alpha]^+),(c,r,b,[u-(i(N-r)+k+1)\alpha]^+)))$$
$$(P((c,r,b,[u-(i+1)(N-r)\alpha]^+),(c,r,b,[u-(i+1)(N-r)\alpha]^+)))^r$$
$$\times \prod_{k=0}^{m-x(N-r)-1} P((c,r,b,[u-(x(N-r)+k)\alpha]^+),(c,r,b,[u-(x(N-r)+k+1)\alpha]^+))$$

$$= \prod_{i=0}^{x-1} \prod_{k=0}^{N-r-1} (1-\sigma_v)^{M_v-c-r}(1-\sigma_d)^{M_d-b}(1-r/N)(1-p_v(u_1))^c(1-p_d(u_1))^b$$
$$((1-\sigma_v)^{M_v-c-r}(1-\sigma_d)^{M_d-b}(r/N)(1-\gamma_f))^{xr}$$
$$\times \prod_{k=0}^{m-x(N-r)-1} (1-\sigma_v)^{M_v-c-r}(1-\sigma_d)^{M_d-b}(1-r/N)(1-p_v(u_2))^c$$
$$> 0.$$

Here $u_1 = [u-(i(N-r)+k)\alpha]^+$ and $u_2 = [u-(x(N-r)+k)\alpha]^+$.

2) $(c,r,b,0) \to (c,r,b,\xi d)$: Define $y = \lfloor \frac{d}{N-r} \rfloor$

$$
\begin{aligned}
&P((c,r,b,0),(c,r,b,\xi d)) \geq \\
&\prod_{i=0}^{y-1} \prod_{k=0}^{N-r-1} P((c,r,b,(i(N-r)+k)\xi),(c,r,b,(i(N-r)+k+1)\xi)) \\
&(P((c,r,b,(i+1)(N-r)\xi),(c,r,b,(i+1)(N-r)\xi)))^r \\
&\times \prod_{k=0}^{d-y(N-r)-1} P((c,r,b,(y(N-r)+k)\xi),(c,r,b,(y(N-r)+k+1)\xi)) \\
&> 0
\end{aligned}
$$

3) $(c,r,b,\xi d) \to (c,r,b,\beta b + \xi d)$: Define $z = \lfloor \frac{e}{N} \rfloor$. Here, we consider a case where $r \geq e - Nz$. However, proof is similar if $r < e - Nz$.

$$
\begin{aligned}
&P((c,r,b,\xi d),(c,r,b,\beta b + \xi d)) \geq \\
&\prod_{k=0}^{r-1} P((c,r-k,b,\xi d),(c,r-k-1,b,\xi d)) \\
&\times \prod_{i=0}^{z-2} \prod_{k=0}^{N-1} P((c,k,b,\xi d + (iN+k)\beta),(c,k+1,b,\xi d + (iN+k+1)\beta)) \\
&\prod_{k=0}^{N-1} P((c,N-k,b,\xi d + (i+1)N\beta),(c,N-k-1,b,\xi d + (i+1)N\beta)) \\
&\times \prod_{k=0}^{N-1} P((c,k,b,\xi d + ((z-1)N+k)\beta),(c,k+1,b,\xi d + ((z-1)N+k+1)\beta)) \\
&\times \prod_{k=0}^{N+b-zN-r-1} P((c,N-k,b,\xi d + zN\beta),(c,N-k-1,b,\xi d + zN\beta))
\end{aligned}
$$

$$\times \prod_{k=N+b-zN-r}^{N-1} P((c, r-e+zN, b, \xi d + zN\beta), (c, r-e+zN, b, \xi d + zN\beta))$$

$$\times \prod_{k=0}^{b-Nz-1} P((c, r-e+zN+k, b, u_3), (c, r-e+zN+k+1, b, u_4))$$

$$> 0.$$

Here $u_3 = \xi d + (zN + k)\beta, \quad u_4 = \xi d + (zN + k + 1)\beta.$

4) $(c, r, b, \beta e + \xi d) \to (c, r, b, -\alpha a + \beta e + \xi d)$: Define $w = \lfloor \frac{a}{N-r} \rfloor$

$$P((c, r, b, \beta e + \xi d), (c, r, b, -\alpha a + \beta e + \xi d)) \geq$$

$$\prod_{i=0}^{w-1} \prod_{k=0}^{N-r-1} P((c, r, b, \xi d + \beta e - (i(N-r) + k)\alpha), (c, r, b, \xi d + \beta e - (i(N-r) + k + 1)\alpha))$$

$$(P((c, r, b, \xi d + \beta e - (i+1)(N-r)\alpha), (c, r, b, \xi d + \beta e - (i+1)(N-r)\alpha)))^r \times$$

$$\prod_{k=0}^{a-w(N-r)-1} P((c, r, b, \xi d + \beta e - (w(N-r) + k)\alpha), (c, r, b, \xi d + \beta e - (w(N-r) + k + 1)\alpha))$$

$$= \prod_{i=0}^{w-1} \prod_{k=0}^{N-r-1} (1-\sigma_v)^{M_v - c - r} (1-\sigma_d)^{M_d - b} (1 - r/N)(1 - p_v(u_5))^c (1 - p_d(u_5))^b$$

$$\times ((1-\sigma_v)^{M_v - c - r} (1-\sigma_d)^{M_d - b} (r/N)(1-\gamma_f))^{wr}$$

$$\times \prod_{k=0}^{a-w(N-r)-1} (1-\sigma_v)^{M_v - c - r} (1-\sigma_d)^{M_d - b} (1 - r/N)(1 - p_v(u_6))^c (1 - p_d(u_6))^b$$

$$> 0.$$

Here $u_5 = \xi d + \beta e - (i(N-r) + k)\alpha$ and $\xi d + \beta e - (w(N-r) + k)\alpha$. As mentioned above, for the case $r = N$, the proof of communication between states of the state space is very similar to the same case of PRMA Voice only system.

So far we proved that the arbitrary state (c, r, b, v) is reachable from any state (c, r, b, u) with positive probability $((c, r, b, u) \to (c, r, b, v))$. Since two states are chosen arbitrarily, then $(c, r, b, u) \leftrightarrow (c, r, b, v)$ or both states communicate.

Now in this part, all states that communicate with $(0, 0, 0, 0)$ are examined. It is assumed that $0 \leq r < N$. The case where $r = N$ is very similar.

1) $(c, r, u) \rightarrow (0, 0, 0)$: Consider two different cases:

A) $N - r > c$. Define $y = \lfloor \frac{N}{b} \rfloor$.

$$
\begin{aligned}
&P((c, r, b, u), (0, 0, 0, [u - 2\alpha(N - c - r) + \beta(c + b)]^+)) \geq \\
&\prod_{k=0}^{c-1} P((c - k, r + k, b, [u + k\beta]^+), (c - k - 1, r + k + 1, b, [u + (k + 1)\beta]^+)) \\
&\times \prod_{k=0}^{N-c-r-1} P((0, r + c, b, [u - k\alpha + \beta c]^+), (0, r + c, b, [u - (k + 1)\alpha + \beta c]^+))
\end{aligned}
$$

$$
\begin{aligned}
&\times \prod_{k=0}^{r+c-1} P((0, r + c - k, u_7), (0, r + c - k - 1, u_7)) \\
&\times \prod_{k=0}^{N-c-r-1} P((0, 0, b, [u_7 - k\alpha]^+), (0, 0, b, [u_7 - (k + 1)\alpha]^+)) \\
&\times \prod_{i=0}^{y-1} \prod_{k=0}^{N-1} P((0, 0, b_1 - k, [u_8 + (iN + k)\beta]^+), (0, 0, b_1 - k - 1, [u_8 + (iN + k + 1)\beta]^+)) \\
&\times \prod_{k=0}^{b-yN-1} P((0, 0, b_2 - k, [u_8 + (yN + k)\beta]^+), (0, 0, b_2 - k - 1, [u_8 + (yN + k + 1)\beta]^+)) \\
&> 0,
\end{aligned}
$$

here $u_7 = [u - \alpha(N - c - r) + \beta c]^+$, $u_8 = [u - 2\alpha(N - c - r) + \beta c]^+$, $b_1 = b - iN$, and $b_2 = b - yN$. As proved earlier, it is easy to show that $(0, 0, 0, [u_8 + b\beta]^+) \leftrightarrow (0, 0, 0, 0)$.

$B)$ $N-r \leq c$. Define $x = \lfloor \frac{c-(N-r)}{N} \rfloor$ and $y = \lfloor \frac{N}{b} \rfloor$.

$$P((c,r,b,u),(0,0,b,[u-2\alpha(N-c+(N-r)+xN)+\beta(c+b)]^{+})) \geq$$

$$\prod_{k=0}^{N-r-1} P((c-k,r+k,b,[u+k\beta]^{+}),(c-k-1,r+k+1,b,[u+(k+1)\beta]^{+}))$$

$$\times \prod_{i=0}^{x-1}\prod_{k=0}^{N-1} P((c-(i+1)N+r,N-k,b,[u_9]^{+}),(c-(i+1)N+r,N-k-1,b,[u_9]^{+}))$$

$$\prod_{k=0}^{N-1} P((c_4-k,k,b,[u_9+k\beta]^{+}),(c_4-k-1,k+1,b,[u_9+(k+1)\beta]^{+}))$$

$$\times \prod_{k=0}^{N-1} P((c-(N-r)-xN,N-k,b,[u_{10}]^{+}),(c-(N-r)-xN,N-k-1,b,[u_{10}]^{+}))$$

$$\times \prod_{k=0}^{c-(N-r)-xN-1} P((c_5-k,k,b,[u_{10}+k\beta]^{+}),(c_5-k-1,k+1,b,[u_{10}+(k+1)\beta]^{+}))$$

$$\times \prod_{k=0}^{N-c+(N-r)+xN-1} P((0,c_5,b,[u-k\alpha+\beta c]^{+}),(0,c_5,b,[u-(k+1)\alpha+\beta c]^{+}))$$

$$\times \prod_{k=0}^{c-(N-r)-xN-1} P((0,c_5-k,b,u_{11}),(0,c_5-k-1,b,u_1 1))$$

$$\times \prod_{k=0}^{N-c+(N-r)+xN-1} P((0,0,b,[u_{11}-k\alpha]^{+}),(0,0,b,[u_{11}-(k+1)\alpha]^{+}))$$

$$\times \prod_{i=0}^{y-1}\prod_{k=0}^{N-1} P((0,0,b_1-k,[u_{12}+(iN+k)\beta]^{+}),(0,0,b_1-k-1,[u_{12}+(iN+k+1)\beta]^{+}))$$

$$\times \prod_{k=0}^{b-yN-1} P((0,0,b_2-k,[u_{12}+(yN+k)\beta]^{+}),(0,0,b_2-k-1,[u_{12}+(yN+k+1)\beta]^{+}))$$

$$> 0,$$

here

$$u_9 = u + (N - r + iN)\beta \qquad c_4 = c - (i+1)N + r$$

$$u_{10} = u + (N - r + xN)\beta \qquad c_5 = c - (N - r) - xN$$

$$u_{11} = [u - \alpha(N - c + (N - r) + xN) + \beta c]^+ \qquad b_1 = b - iN$$

$$u_{12} = [u - 2\alpha(N - c + (N - r) + xN) + \beta c]^+ \qquad b_2 = b - yN$$

Also, as proved before, $(0,0,0,[u-2\alpha(N-c+(N-r)+xN)+\beta(c+b)]^+) \leftrightarrow (0,0,0,0)$

2) $(0,0,0,0) \to (c,r,b,u)$

- $(0,0,0,0) \to (0,0,0,u)$: Immediate result of previous discussions.
- $(0,0,0,0) \to (c,r,b,u)$: It is obvious that there exist a $m_1 > 0$ such that: $P^{m_1}((c,r,b,[\beta r+\xi]^+),(c,r,b,u)) > 0$. Therefore:

$$\begin{aligned}
P &\geq P((0,0,0,0),(c+r,0,b,0)) \times P((c+r,0,b,0),(c+r,0,b,\xi)) \\
&\times \prod_{k=0}^{r-1} P((c+r-k,k,b,[k\beta+\xi]^+),(c+r-k-1,k+1,b,[(k+1)\beta+\xi]^+)) \\
&\times P^{m_1}((c,r,b,[\beta r+\xi]^+),(c,r,b,u)) \\
&> 0
\end{aligned}$$

The above derivations suggest that the state space is an absorbing communication set and therefore, irreducible.

Aperiodic State Space- It remains to show that the Markov chain defined on the state space is aperiodic. Since the Markov chain is irreducible, it suffices to show the aperiodicity for a single state (c,r,b,u). Any state (c,r,b,u) communicates with $(0,0,0,0)$. Therefore, there exists $m_1 > 0$ and $m_2 > 0$ such that

$P^{m_1}((c,r,b,u),(0,0,0)) > 0$ and $P^{m_2}((0,0,0),(c,r,b,u)) > 0$. Let $m = m_1 + m_2$. Then:

$$P^m((c,r,b,u),(c,r,b,u)) \geq P^{m_1}((c,r,b,u),(0,0,0))P^{m_2}((0,0,0,0),(c,r,b,u)) > 0$$

$$P^{m+1}((c,r,b,u),(c,r,b,u)) \geq$$

$$P^{m_1}((c,r,b,u),(0,0,0,0))P((0,0,0,0),(0,0,0,0))P^{m_2}((0,0,0,0),(c,r,b,u)) > 0$$

where $P((0,0,0,0),(0,0,0,0)) = (1-\sigma_v)^{M_v}(1-\sigma_d)^{M_d} > 0$. Hence the period for any state (c,r,b,u) is 1 and the chain is aperiodic.

Markov Chain is Positive Recurrent- So far we proved that the state space is countable and the Markov chain is irreducible and aperiodic. Here, using the Proposition 2.3, we show that the Markov chain is positive recurrent and hence, it has a unique stationary probability distribution. Following non-negative Lyapunov function is chosen:

$$V(c,r,b,u) = u + Kr(1 - \sigma_v + f(u))^c$$

here:

- $K > 0$ is any real constant,
- $f(.) : \mathbb{R} \to [0,1)$ is non-increasing, continuous, and bounded,
- there exists $\hat{u} > u_{max}$ such that $f(u) = 0$ for $u \geq \hat{u}$.

Note that $V(c,r,b,u)$ satisfies the requirement that it is non-negative. Following definition of simplifying notations is also necessary:

$$p_v = p_v(u) \qquad p_d = p_d(u) \qquad f = f(u)$$

$$f_\alpha = f(u-\alpha) \qquad f_\beta = f(u+\beta) \qquad f_\xi = f(u+\xi)$$

Expected drift of the Lyapunov function is a calculated as follows:

$$E(\Delta V|(c,r,b,u)) =$$

$$E(V(c_{n+1}, r_{n+1}, b_{n+1}, u_{n+1}) - V(c_n, r_n, b_n, u_n)|(c_n, r_n, b_n, u_n) = (c,r,b,u)) =$$

$$E(u_{n+1} - u_n|(c,r,b,u))$$

$$+ KE(r_{n+1}(1-\sigma_v + f(u_{n+1}))^{c_{n+1}} - r_n(1-\sigma_v + f(u_n))^{c_n}|(c,r,b,u))$$

First $E(r_{n+1}(1-\sigma_v + f(u_{n+1}))^{c_{n+1}}|(c,r,b,u))$ is calculated:

$$E(r_{n+1}(1-\sigma_v + f(u_{n+1}))^{c_{n+1}}|(c,r,b,u)) =$$

$$-(1-\sigma_v+f)^c(1-\sigma_v^2+\sigma_v f)^{M_v-c-r}(\frac{r}{N})\gamma_f$$

$$+ r(1-\sigma_v+f)^c(1-\sigma_v^2+\sigma_v f)^{M_v-c-r}(\frac{r}{N})$$

$$+ r(1-\sigma_v+f_\alpha)^c(1-\sigma_v^2+\sigma_v f_\alpha)^{M_v-c-r}(1-\frac{r}{N})(1-p_v)^c(1-p_d)^b$$

$$+ (r+1)(1-\sigma_v+f_\beta)^{c-1}(1-\sigma_v^2+\sigma_v f_\beta)^{M_v-c-r}(1-\frac{r}{N})cp_v(1-p_v)^{c-1}(1-p_d)^b$$

$$+ r(1-\sigma_v+f_\beta)^c(1-\sigma_v^2+\sigma_v f_\beta)^{M_v-c-r}(1-\frac{r}{N})bp_d(1-p_d)^{b-1}(1-p_v)^c$$

$$+ r(1-\sigma_v+f_\xi)^c(1-\sigma_v^2+\sigma_v f_\xi)^{M_v-c-r}$$

$$(1-\frac{r}{N})(1-(1-p_v)^c(1-p_d)^b - cp_v(1-p_v)^{c-1}(1-p_d)^b - bp_d(1-p_d)^{b-1}(1-p_v)^c)$$

Hence:

$$
\begin{aligned}
&E(\Delta V|(c,r,b,u)) = \\
&-(\alpha+\xi)(1-\frac{r}{N})(1-p_v)^c(1-p_d)^b + \xi(1-\frac{r}{N}) \\
&+(\beta-\xi)(1-\frac{r}{N})(cp_v(1-p_v)^{c-1}(1-p_d)^b + bp_d(1-p_d)^{b-1}(1-p_v)^c) \\
&-Kr(1-\sigma_v+f)^c \\
&-K(1-\sigma_v+f)^c(1-\sigma_v^2+\sigma_v f)^{M_v-c-r}(\frac{r}{N})\gamma_f \\
&+Kr(1-\sigma_v+f)^c(1-\sigma_v^2+\sigma_v f)^{M_v-c-r}(\frac{r}{N}) \\
&+Kr(1-\sigma_v+f_\alpha)^c(1-\sigma_v^2+\sigma_v f_\alpha)^{M_v-c-r}(1-\frac{r}{N})(1-p_v)^c(1-p_d)^b \\
&+K(r+1)(1-\sigma_v+f_\beta)^{c-1}(1-\sigma_v^2+\sigma_v f_\beta)^{M_v-c-r}(1-\frac{r}{N})cp_v(1-p_v)^{c-1}(1-p_d)^b \\
&+Kr(1-\sigma_v+f_\beta)^c(1-\sigma_v^2+\sigma_v f_\beta)^{M_v-c-r}(1-\frac{r}{N})bp_d(1-p_d)^{b-1}(1-p_v)^c \\
&+Kr(1-\sigma_v+f_\xi)^c(1-\sigma_v^2+\sigma_v f_\xi)^{M_v-c-r} \\
&\quad(1-\frac{r}{N})(1-(1-p_v)^c(1-p_d)^b - cp_v(1-p_v)^{c-1}(1-p_d)^b - bp_d(1-p_d)^{b-1}(1-p_v)^c)
\end{aligned}
$$

In order to prove that the Markov chain is positive recurrent, a finite set Ξ is found such that for states outside this set, expected drift in Lyapunov function, $V(c,r,b,u)$ is negative. Consider $u \geq \hat{u} - \min(-\alpha,\beta)$. Then $p_v = p_d = f = f_\alpha = f_\beta = f_\xi = 0$:

$$
\begin{aligned}
E(\Delta V|(c,r,b,u)) = &-K(1-\sigma_v)^c(1-\sigma_v^2)^{M_v-c-r}(\frac{r}{N})\gamma_f \\
&+(Kr(1-\sigma_v)^c(1-\sigma_v^2)^{M_v-c-r} - Kr(1-\sigma_v)^c)(\frac{r}{N}) \\
&+(Kr(1-\sigma_v)^c(1-\sigma_v^2)^{M_v-c-r} - Kr(1-\sigma_v)^c - \alpha)(1-\frac{r}{N})
\end{aligned}
$$

$$
\begin{aligned}
=& - K(1-\sigma_v)^c(1-\sigma_v^2)^{M_v-c-r}(\frac{r}{N})\gamma_f \\
& - Kr(1-\sigma_v)^c(1-(1-\sigma_v^2)^{M_v-c-r}) - \alpha(1-\frac{r}{N}) \\
\leq& - K(1-\sigma_v)^c(1-\sigma_v^2)^{M_v-c-r}(\frac{r}{N})\gamma_f - \alpha(1-\frac{r}{N}) \\
\leq& - K(1-\sigma_v)^{M_v}(1-\sigma_v^2)^{M_v}(\frac{r}{N})\gamma_f - \alpha(1-\frac{r}{N}) \\
=& - (K(1-\sigma_v)^{M_v}(1-\sigma_v^2)^{M_v}\gamma_f)(\frac{r}{N}) - \alpha(1-\frac{r}{N}) \\
\leq& - \min(K(1-\sigma_v)^{M_v}(1-\sigma_v^2)^{M_v}\gamma_f, \alpha)
\end{aligned}
$$

Hence take $\epsilon = \min(K(1-\sigma_v)^{M_v}(1-\sigma_v^2)^{M_v}\gamma_f, \alpha)$. Now consider the case where $u < \hat{u} - \min(-\alpha, \beta)$, then:

$$
\begin{aligned}
& E(\Delta V|(c,r,b,u)) \leq \\
& (\beta-\xi)(1-\frac{r}{N})(cp_v(1-p_v)^{c-1}(1-p_d)^b + bp_d(1-p_d)^{b-1}(1-p_v)^c) \\
& + Kr(1-\sigma_v+f)^c(1-\sigma_v^2+\sigma_v f)^{M_v-c-r} \\
& + Kr(1-\sigma_v+f_\alpha)^c(1-\sigma_v^2+\sigma_v f_\alpha)^{M_v-c-r} \\
& + K(r+1)(1-\sigma_v+f_\beta)^{c-1}(1-\sigma_v^2+\sigma_v f_\beta)^{M_v-c-r} \\
& + Kr(1-\sigma_v+f_\xi)^c(1-\sigma_v^2+\sigma_v f_\xi)^{M_v-c-r} \\
\leq& K(4N+1)4^{M_v} + \max(\beta,\xi).
\end{aligned}
$$

Therefore, we take $b' = K(4N+1)4^{M_v} + \max(\beta,\xi)$ and define Ξ as:

$$
\begin{aligned}
\Xi =& \{0,1,\cdots,M_v\} \times \{0,1,\cdots,M_v\} \times \{0,1,\cdots,N\} \\
& \times \{u | 0 \leq u = -a\alpha + e\beta + d\xi \leq \hat{u} - \min(-\alpha,\beta)\}
\end{aligned}
$$

Hence:

$$E(\Delta V|(c,r,b,u)) \leq -\epsilon \qquad\qquad (c,r,b,u) \notin \Xi$$

$$E(\Delta V|(c,r,b,u)) \leq b' \qquad\qquad (c,r,b,u) \in \Xi$$

and Markov chain is positive recurrent.

Appendix F

Proof of Proposition 3.2

State (c, r, b, u) belongs to following state space:

$$\aleph = \{0, 1, 2, \cdots, M_v\} \times \{0, 1, 2, \cdots, M_d\} \times \{0, 1, 2, \cdots, N\} \times \{0, 1, 2, \cdots, N\}.$$

Note that $c + r \leq M_v$, $u = N - r$, and $\aleph$ is a countable set. Since a subset of a countable set is also countable, thereby, the state space of the system is also countable. Next, we show that all states in the state space communicate. Since we have $r + u = N$, our focus is to show that $(c, r, b) \leftrightarrow (c', r', b')$. First, we show $(c, r, b) \rightarrow (0, 0, 0)$. Consider two cases.

Case 1 - $c + r \leq N$. Define $y = \lfloor \frac{b}{N} \rfloor$

$$\begin{aligned}
&P((c, r, b), (0, 0, 0)) \geq \\
&\prod_{k=0}^{c-1} P((c-k, r+k, b), (c-k-1, r+k+1, b)) \\
&\times \prod_{k=0}^{r-1} P((0, r+c-k, b), (0, r+c-k-1, b)) \\
&\times \prod_{k=0}^{N-c-r-1} P((0, c, b), (0, c, b)) \\
&\times \prod_{k=0}^{c-1} P((0, c-k, b), (0, c-k-1, b)) \\
&\times \prod_{i=0}^{y-1} \prod_{k=0}^{N-1} P((0, 0, b-iN-k), (0, 0, b-iN-k-1)) \\
&\times \prod_{k=0}^{b-yN-1} P((0, 0, b-yN-k), (0, 0, b-yN-k-1))
\end{aligned}$$

$$
\begin{aligned}
&= \prod_{k=0}^{c-1} (1-\sigma_v)^{M_v-c-r}(1-\sigma_d)^{M_d}(1-(r+k)/N)(c-k)p_v(1-p_v)^{c-k-1}(1-p_d)^b \\
&\times \prod_{k=0}^{r-1} (1-\sigma_v)^{M_v-c-r+k}(1-\sigma_d)^{M_d}((r+c-k)/N)\gamma_f \\
&\times \prod_{k=0}^{N-c-r-1} (1-\sigma_v)^{M_v-c}(1-\sigma_d)^{M_d}(1-c/N)(1-bp_d(1-p_d)^{b-1}) \\
&\times \prod_{k=0}^{c-1} (1-\sigma_v)^{M_v-c+k}(1-\sigma_d)^{M_d-b}((c-k)/N)\gamma_f \\
&\times \prod_{i=0}^{y-1}\prod_{k=0}^{N-1} (1-\sigma_v)^{M_v}(1-\sigma_d)^{M_d-b+iN+k}(b-iN-k)p_d(1-p_d)^{b-iN-k-1} \\
&\times \prod_{k=0}^{b-yN-1} (1-\sigma_v)^{M_v}(1-\sigma_d)^{M_d-b+yN+k}(b-yN-k)p_d(1-p_d)^{b-yN-k-1} \\
&> 0
\end{aligned}
$$

Case 2 - $c+r > N$. We define $x = \lfloor \frac{c-N+r}{N} \rfloor$ and $y = \lfloor \frac{b}{N} \rfloor$. Since $c-N+r-xN \leq N$ based on case 1 above $(c-N+r-xN, 0, b) \to (0,0,0)$. Therefore, here we show that $(c,r,b) \to (c-N+r-xN, 0, b)$.

$$
\begin{aligned}
&P((c,r,b),(c-N+r-xN,0,b)) \geq \\
&\prod_{k=0}^{N+r-1} P((c-k,r+k,b),(c-k-1,r+k+1,b)) \\
&\times \prod_{i=0}^{x-1}\prod_{k=0}^{N-1} P((c-N+r-iN,N-k,b),(c-N+r-iN,N-k-1,b)) \\
&\prod_{k=0}^{N-1} P((c-N+r-iN-k,k,b),(c-N+r-iN-k-1,k+1,b)) \\
&\times \prod_{k=0}^{N-1} P((c-N+r-xN,N-k,b),(c-N+r-xN,N-k-1,b))
\end{aligned}
$$

$$= \prod_{k=0}^{N+r-1} (1-\sigma_v)^{M_v-c-r}(1-\sigma_d)^{M_d-b}(1-(r+k)/N)(c-k)p_v(1-p_v)^{c-k-1}(1-p_d)^b$$
$$\times \prod_{i=0}^{x-1}\prod_{k=0}^{N-1} (1-\sigma_v)^{M_v-c-r+k+iN}(1-\sigma_d)^{M_d-b}((N-k)/N)\gamma_f$$
$$\prod_{k=0}^{N-1} (1-\sigma_v)^{M_v-c-r+(i+1)N}(1-\sigma_d)^{M_d-b}(1-k/N)c_1p_v(1-p_v)^{c_1-1}(1-p_d)^b$$
$$\times \prod_{k=0}^{N-1} (1-\sigma_v)^{M_v-c-r+(x+1)N}(1-\sigma_d)^{M_d-b}((N-k)/N)\gamma_f$$
$$> 0,$$

here $c_1 = c - N + r - iN - k$. Next, we show $(0,0,0) \to (c', r', b')$:

$$P((0,0,0),(c',r',b')) \geq$$
$$P((0,0,0),(c'+r',0,b')) \times \prod_{k=0}^{r'} P((c'+r'-k,k,b'),(c'+r'-k-1,k+1,b'))$$

$$= \begin{pmatrix} M_v \\ c'+r' \end{pmatrix} \sigma_v^{c'+r'}(1-\sigma_v)^{M_v-c'-r'} \times \begin{pmatrix} M_d \\ b' \end{pmatrix} \sigma_d^{b'}(1-\sigma_d)^{M_d-b'}$$
$$\times \prod_{k=0}^{r'} (1-\sigma_v)^{M_v-c'-r'}(1-\sigma_d)^{M_d-b'}(1-k/N)(c'+r'-k)p_v(1-p_v)^{c'+r'-k-1}(1-p_d)^{b'}$$
$$> 0.$$

Therefore, $(c,r,b) \leftrightarrow (c',r',b')$. Hence, the state space is an absorbing communication set and irreducible. Next, we show that the Markov chain is aperiodic. As we showed above, any state (c,r,b) communicates with $(0,0,0)$. Therefore, there exists m_1 and m_2 such that $P^{m_1}((c,r,b),(0,0,0)) > 0$ and $P^{m_2}((0,0,0),(c,r,b)) > 0$. We define $m = m_1+m_2$. Also note that $P((0,0,0),(0,0,0)) = (1-\sigma_v)^{M_v}(1-\sigma_d)^{M_d-b}$.

Therefore:

$$P^m((c,r,b),(c,r,b)) > P^{m_1}((c,r,b),(0,0,0))P^{m_2}((0,0,0),(c,r,b)) > 0,$$

$$\begin{aligned} P^{m+1}((c,r,b),(c,r,b)) &> P^{m_1}((c,r,b),(0,0,0))P((0,0,0),(0,0,0))P^{m_2}((0,0,0),(c,r,b)) \\ &> 0. \end{aligned}$$

Hence, period for each state is 1 and therefore, the Markov chain is aperiodic.

So far we showed that Markov chain defined on $\aleph$ through (3.2) (for $\phi = 1$, $\alpha = \xi = 0$, $\beta = -1$, and $RTD = 0$) is irreducible and aperiodic. Further, since the state space is finite the Markov chain is positive recurrent and a unique stationary probability distribution exists.

Bibliography

[1] D.J. Goodman, R.A. Valenzuela, K.T. Gayliard, and B. Ramamurthi, "Packet reservation multiple access for local Wireless Communications," IEEE Trans. Commun., **37**, pp. 885-890, Aug. 1989.

[2] N. Abramson, "The ALOHA system - Another Alternative for Computer Communications," *Jonit Comput. Conf. AFIPS Conf. Proc.,* vol. 37, Montvale, NJ, AFIPS Press., 1970.

[3] Y.C. Jenq, "On the Stability of Slotted ALOHA Systems", IEEE Trans. on Comm, **28**, no. 11, pp. 1936-1939, 1980.

[4] S. Ghez, S. Verdu, and S. C. Schwartz, "Stability Properties of Slotted ALOHA with Multipacket Recepection Capability," *IEEE Trans. Auto. Cont.,* vol. 33, no. 7, pp.640-649, July 1988.

[5] R. Rom and M. Sidi *Multiple Access Protocols* Springer-Verlog New York Inc., New York, 1990.

[6] A. El-Sherif, A. Kwasinski, A. K. Sadek, and K. J. R. Liu, "Content-Aware Multiple Access Protocol for Cooperative Packet Speech Communications," IEEE Trans. on Wireless Comm. Vol.8, No. 2, Feb. 2009, pp. 995-1005.

[7] B. R. Qazi, H. Alshaer, and J. M. H. Elmirghani, “Analysis and Design of a MAC Protocol and Vehicular Traffice Simulator for Multimedia Communications on Motorways,” Accepted to IEEE Trans. on Vehicular Technology, July 2008.

[8] B. R. Qazi, A. A. de Grado Vivero, and J. M. H. Elmirghani, “TINA: The INtelligent Airport - A Proposed Architecture,” Proceedings of IWCWC, Leipzig, Germany, June 2009.

[9] B. R. Qazi, A. A. de Grado Vivero, O. Y. K. Alani, and J. M. H. Elmirghani, “Pico Cellular MAC Protocol for Multimedia Communication,” International Conference on Advanced Information Networking and Applications Workshops (WAINA), 26-29 May 2009.

[10] Y. Gao, H. Tian, L. Sun, and H. Xu, “Research on the Access Network and MAC Technique for Beyond 3G System,” IEEE Trans. on Wireless Comm. Vol. 14, No. 2, April 2007, pp. 57-61.

[11] S. Saboji and C. B. Akki, “A Survey of Fourth Generation (4G) Wireless Network Models,” Journal of Telecommunications Management, Vol. 2, No. 1, March 2009, pp. 77-91.

[12] R. Rettberg, “A Brief Simulation of the Dynamics of an ALOHA system with Slots,” ARPA Satellite Syste. Note 11, 1972.

[13] R. Metacalfe, "Steady-State Analysis of a Slotted and Controlled ALOHA System with Blocking," 6th Hawaii Int. Conf. on Syst. Sci., 1973.

[14] L. Kleinrock and S. S. Lam, "Packet switching in a multiaccess broadcast channel: performance evaluation," *IEEE Trans. Comm.,* vol. 23, no. 4, pp. 410-423, Apr. 1975.

[15] A. B. Carleial and M. E. Hellman, "Bistable behavior of ALOHA-type systems," *IEEE Trans. Comm.,* vol. 23, no. 4, pp. 401-410, Apr. 1975.

[16] S.S. Lam, "Packet Switching Networks: A Performance Analysis of the R-ALOHA Protocol," IEEE Trans. on Comp., **29**, no. 7, pp. 596-603, 1980

[17] S. Tasaka, "Stability and performance of the R-ALOHA packet broadcast system," *IEEE Trans. Comp.,* vol. 32, no. 8, pp. 717-725, Aug. 1983.

[18] Y. Onozato and S. Noguchi, "On the Thrashing Cusp in Slotted ALOHA Systems," IEEE Trans. on Comm., **33**, no. 11, pp. 1171-1182, 1985.

[19] Y. Onozato and J. Liu and S. Noguchi, "Stability of a Slotted ALOHA System with Capture Effect," IEEE Trans. on Veh. Tech., **38**, no. 1, pp. 31-36, 1989.

[20] K. Sakakibara, H. Muta, and Y. Yuba, "The effect of limiting the number of retransmission trials on the stability of slotted ALOHA systems," IEEE Trans. Veh. Technol., **49**, no. 4, pp 1449-1453, Jul. 2000

[21] K. Sakakibara and T. Seto and D. Yoshimura and J. Yamakita, "Effect of Exponential Backoff Scheme and Retransmission Cutoff on the Stability of Frequency-Hopping Slotted ALOHA Systems," IEEE Trans. on Wireless Comm., **2**, no. 4, pp. 714-722, 2003.

[22] D.J. Goodman and S.X. Wei, "Efficiency of Packet Reservation Multiple Access,"IEEE Trans. on Vehic. Tech.,**40**, pp. 170-176, 1991.

[23] S. Nanda, D. J. Goodman, and U. Timor, "Performance of PRMA: a packet voice protocol for cellular systems," *IEEE Trans. Veh. Technol.,* vol. 40, no. 3, pp 584-598, Aug. 1991.

[24] S. Nanda, "Stability evaluation and design of the PRMA joint voice data system," *IEEE Trans. Comm.,* vol. 42, no. 5, pp 2092-2104, May 1994.

[25] H. Qi and R. Wyrwas, "Performance Analysis of Joint Voice-Data PRMA Over Random Packet Error Channels," IEEE Trans. on Veh. Tech., **45**, no. 2, pp. 332-345, 1996.

[26] G. Wu and K. Muhumoto and A. Fukuda, "Analysis of an Integrated Voice and Data Transmission System Using Packet Reservation Multiple Access," IEEE Trans. on Veh. Tech., **43**, no. 2, pp. 289-297, 1994.

[27] A.E. Brand and A.H. Aghvami, "Performance of a Joint CDMA/PRMA Protocol for Mixed Voice/Data Transmission for Third Generation Mobile Com-

munication," IEEE Trans. on Sel. Areas in Comm., **14**, no. 9, pp. 1698-1707, 1996.

[28] A.E. Brand and A.H. Aghvami, "Mutidimensional PRMA with Prioritized Bayesian Broadcast-A MAC Strategy for Multiservice Traffic over UMTS," IEEE Trans. on Veh. Tech., **47**, no. 4, pp. 1148-1161, 1998.

[29] N.B. Mehta and A.J. Goldsmith, "Effect of Mobility on PRMA," IEEE Trans. on Comm., **50**, no. 3, pp. 400-405, 2002.

[30] D. Park, J. Bang and S. Tekinay, "Handoff Effect on Packet Reservation Multiple Access (PRMA) in Microcellular System," IEEE Trans. on Veh. Tech., **52**, no. 5, pp. 1232-1241, 2003

[31] S. Elnoubi and A. M. Alsayh, "A Packet Reservation Multiple Access (PRMA)-Based Algorithm for Multimedia Wireless System," IEEE Trans. on Veh. Tech., **43**, no. 1, pp. 215-222, 2004.

[32] D.G. Jeong and W.S. Jeon, "Performance of an Exponential Backoff Scheme for Slotted-ALOHA Protocol in Local Wireless Environment," IEEE Trans. on Vehic. Tech., **44**, no.3, pp. 470-479, 1995.

[33] R. L. Rivest, "Network Control by Bayesian Broadcast," IEEE Trans. Inform. Theory, **IT-33**, no. 3, pp. 323–328, 1987.

[34] E. D. Re, R. Fantacci, G. Giambene, and W. Sergio, “Performance analysis of an improved PRMA protocl for low Earth orbit-mobile satellite systems,” IEEE Trans. Veh. Technol., **48**, no. 3, May 1999.

[35] G. Benelli and R. Fantacci and G. Giambene and C. Ortolani, “Voice and Data Transmissions with a PRMA-Like Protocol in High Propagation Delay Cellular Systems,” IEEE Trans. on Vehic. Tech., **49**, no. 6, pp. 2126-2147, 2000.

[36] G. Benelli and R. Fantacci and G. Giambene and C. Ortolani, “Performance Analysis of a PRMA Protocol Suitable for Voice and Data Transmissions in Low Earth Orbit Mobile Satellite Systems,” IEEE Trans. on Wire. Comm., **1**, no. 1, pp. 156-168, Jan. 2002.

[37] R. Fantacci and T. Pecorella and G. Giambene and F. Bossoli, “Stability and Performance Analysis of a MAC Protocol for a Time-Code Air Interface in LEO Mobile Satellite Systems,” IEEE Trans. on Vehic. Tech., **52**, no. 3, pp. 607-621, 2003.

[38] R. Fantacci and G. Giambene and R. Angioloni, “A Modified PRMA Protocol for Voice and Data Transmissions in Low Earth Orbit Mobile Satellite Systems,” IEEE Trans. on Vehic. Tech., **49**, no. 5, pp. 1856-1876, 2000.

[39] R. Fantaci, T. Pecorella, and I. Habib, “Proposal and performance evaluation of an efficient multiple-access protocol for LEO satellite packet networks,” *IEEE J. Select. Areas Comm.,* vol. 22, no. 3, Apr. 2004.

[40] G. Giambene and E. Zoli, "Stability Analysis of an Adaptive Packet Access Schem for Mobile Communication Systems with High Propagation Delays," Intern. Journal of Satell. Comm., **20**, pp. 1-27, 2000.

[41] H.C.B. Chan and J. Zhang and H. Chen, "A Dynamic Reservation Protocol for LEO Mobile Satellite Systems," IEEE Journal on Sel. Areas in Comm., **22**, no. 3, pp. 559-573, 2004.

[42] E.H. Abed, H.O. Wang and A. Tesi, "Control of Bifurcatuins and Cahos" in W.S. Levine, Editor, *The Control Handbook* CRC Press, Boca Raton, FL, 1995, pp. 951966.

[43] C. Yuen and P. Marbach, "Price-Based Rate Control in Random Access Networks," IEEE/ACM Trans. Networking, **13**, no. 5, pp 1027-1040, Oct. 2005.

[44] C. Yuen, "Rate Control with Pricing in Contention-Based Access Networks," Master of Science Thesis, Department of Computer Science, University of Toronto, 2002.

[45] H.J. Kushner *Stochastic Stability And Control* Academic Press, New York, 1967.

[46] S.P. Meyn and R.L. Tweedie *Markov Chains and Stochastic Stability* Springer-Verlag, London, 1993.

[47] A.G. Pakes, "Some Conditions for Ergodicity and Recurrence of Markov Chain," Operations Research, **17**, pp. 1058-1061, (1969).

[48] F.G. Foster, "On the Stochastic Matrices Associated with Certain Queuing Processes," Annals. of Math. Stat, **24**, no. 3, pp. 355-360, (1953).

[49] T.B. Crabill, "Sufficient Conditions for Positive Recurrence and Recurrence of Specially Structured Markov Chains," Operations Research, **16**, pp.858-867, (1968).

[50] J. J. Metzner, "On improving utilization in ALOHA networks," IEEE Trans. on Communications, **24**, pp. 447-448, Apr. 1976.

[51] C. T. Lau and C. Leung, "Capture models for mobile packet radio networks," IEEE Trans. Comm., **40**, no. 5, pp. 917-925, May 1992.

[52] W. Luo and A. Ephremides, "Power levels and packet lengths in random multiple access,"IEEE Trans. Infor. Theo., **48**, no. 1, Jan. 2002.

[53] A.H. Nayfeh and B. Balachandran *Applied Nonlinear Dynamics, Analytical, Computational, and Experimental Methods* John Wiley and Sons, Inc., New York, 1995.

[54] G. Fayolle, E. Gelenbe, and J. Labetoulle, "Stability and Optimal Control of Packet Switching Broadcast Channel," Assoc. Comput Machinery, **24**, no. 3, pp. 375–386, 1977.

[55] L. P. Clare, "Control Procedures for Slotted ALOHA Systems that Achieve Stability," Proc. ACM SIGCOMM '86 Symp., Commun. Arch. and Protocols,

Aug. 1986.

[56] B. Hajek and T. V. Loon, "Decentralized Dynamic Control of a Multiaccess Broadcast Channel," IEEE Trans. on Auto. Cont., **27**, no. 3, pp. 559-569, 1982.

[57] A.U. Sheikh, T. Wan, and Z. Alakhdhar, "A Unified Approach to Analyze Multiple Access Protocols for Buffere Finite Users," Journal of Network and Computer Applications, **27**, pp. 49-76, 2004.

[58] T. Wan and A.U. Sheikh, "Performance and Stability Analysis of Buffered Slotted ALOHA Protocols Using Tagged User Approach," IEEE Trans. on Vehic. Tech., **49**, no. 2, pp. 582-593, (2000).

[59] T. Wan, A.U. Sheikh, Y. Gu, and A. Atweri, "Performance Analysis of PRMA with Integration of Voice and Buffered Data Users," The 9th IEEE Intern. Symposium on Personal, Indoor and Mobile Radio Communication, **2**, pp. 581-585, (1998).

[60] S.S. Lam, *Packet Switching in a Multiaccess Broadcast Channel with Application to Sattelite Communication in a Computer Network*, Ph.D. Thesis, University of California, Los Angeles, Department of Computer Science (1974).

[61] H. Okada, Y. Nakanishi, and Y. Igarashi, "Analysis of framed ALOHA channel in satellite packet switching networks," Proc. Fourth Int. Conf. Comput. Commun., Sept. 1978, pp. 617-622.

[62] F. C. Schoute, "Dynamic frame length ALOHA," IEEE Trans. Commun., vol. COM., **31**, pp. 565-568, Apr. 1983.

[63] R. Du, H. Okada, H. Nakanishi, H. Sanada, and Y. Tezuka, "Performance evaluation and optimization of ALOHA scheme with capture effect," Conf. Rec. GLOBECOM87, Nov. 1987, pp. 555-560.

[64] J. E. Wieselthier, A. Ephremides, and L. A. Michaels, "An exact analysis and performance evaluation of framed ALOHA with capture," IEEE Trans. Commun., vol. 37, no. 2, pp. 125137, Feb. 1989.

[65] Wen-Tzu Chen, "An Accurate Tag Estimate Method for Improving the Performance of an RFID Anticollision Algorithm Based on Dynamic Frame Length ALOHA," IEEE Transactions on Automation Science and Engineering, Vol. 6, No. 1, January 2009

[66] G. Pierobon, A. Zanella, and A. Salloum, "Contention-TDMA Protocol: Performance Evalutation," IEEE Transactions on Vehicular Technology, Vol. 51, No. 4, July 2002.

[67] A. Zanella, *Analysis and Modeling of Wireless Data Networks*, Ph.D. Thesis, Univerita degli Studi di Padova, Italy, Dipartimento di Electronica e Informatica (2001).