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## Derivation of Fuzzy Rules for Parameter Free Pid Gain Tuning

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# DERIVATION OF FUZZY RULES FOR PARAMETER FREE PID GAIN TUNING

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**Abstract.** In this paper, rules for a fuzzy logic based PID tuner (expert) for stable dominant pole plants with large rise times are derived. Applications of the expert to separator temperature control, and to pH control are presented. It is observed that the expert can successfully tune the PID gains without requiring any process identification.

**Key Words.** Fuzzy control; PID control; process control; expert control; parameter free tuning

## 1. INTRODUCTION

It is often seen that control experts tune the parameters of a controller according to error versus time curves based on their knowledge and experience, rather than on complicated control algorithms. Their tuning actions seem to be based on relations between the shape of the response curve and the parameters of the controller, rather than on explicit process models. This kind of tuning, if realizable, is captivating because of its independence from an accurate process model.

Following Tolle and Ersü (1992), one identifies two performance criteria that the expert needs to satisfy: (1) As with humans, satisfactory learning requires frequent repetition of the same effort, so the system is improved by being restarted from the same initial conditions again and again; (2) Important for technical control problems is the ability to stabilize the control loop in the first trial, however, with relatively bad performance in general.

## 2. STATEMENT OF THE PROBLEM

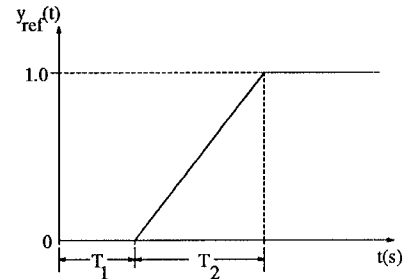The problem can be stated as follows. Given a stable plant



Fig. 1: Reference response

$$P(s) = \frac{Y(s)}{E(s)}$$

$$= \frac{K}{(\tau_1 s + 1)(\tau_2 s + 1)\cdots(\tau_n s + 1)} \quad (1)$$

where $\tau_1 \gg max(1, \tau_i)$, $i = 2, 3, \ldots, n$, along with a PID controller

$$C(s) = \frac{R(s)}{Y(s)} = K_C + \frac{K_I}{s} + K_D s \quad (2)$$

develop a strategy to instantaneously tune $K_C$, $K_I$ and $K_D$, based on observation of the plant output $y(t)$, and the set-point $s(t)$, to get a *good* response to setpoint changes. A good response, means that the closed-loop response approximates a given reference response. The latter is specified by two parameters, $T_1$ and $T_2$, and is illustrated in Fig. 1. Here $T_1$ can be thought of as dead time and $T_2$ as the open-loop response time.

This strategy deviates from the current trend, where effort is made to obtain information about the plant by carrying out data analysis and modelling. A review of these techniques can be found in Koivo and Tanttu (1991). Note that gain scheduling is an immediate consequence of the proposed tuning strategy.

## 3. DERIVATION OF THE RULE BASE

In the current context, one wants to compensate for rise time, and the settling time of the closed-loop response while ensuring that the system remains stable. Hence, it is natural to consider two sets of rules. The first set ($R_1$) deals with rise time compensation, whereas the second set ($R_2$) deals with reducing the settling time and stabilizing the closed-loop system. When a set-point change is detected, $R_1$ is activated. Once the response reaches the steady state, $R_1$ is switched off, and $R_2$ is activated. The input to $R_1$ is the error $e_1(t)$. The inputs to $R_2$ are: (i) the error $e_2(t)$ and (ii) the rate of change of the error $\frac{d}{dt}e_2(t)$. $e_1(t)$ and $e_2(t)$ are calculated as

$$e_1(t) = y_{ref}(t) - \frac{y(t) - s_{old}}{s_{new} - s_{old}} \qquad (3)$$

$$e_2(t) = s_{new} - y(t) \qquad (4)$$

where $y(t)$ is the observed plant response, $y_{ref}(t)$ is the reference response, $s_{old}$ is the previous set-point value, and $s_{new}$ is the current set-point value.

Based on classical control theory (see Kuo, 1991), the following rules are postulated for the manipulation of the dominant closed-loop poles. $R_1$:

1. If $e_1(t)$ is positive, move the dominant closed-loop poles towards the imaginary axis, and away from the real axis.

2. If $e_1(t)$ is negative, move the dominant closed-loop poles away from the imaginary axis and towards the real axis.

$R_2$:

1. If $e_2(t)$ is not *small*, or if $\frac{d}{dt}e_2(t)$ is not *zero*, move the dominant closed-loop poles away from the imaginary axis, and towards the real axis.

2. If $e_2(t)$ is *small*, and if $\frac{d}{dt}e_2(t)$ is *zero*, move the dominant closed-loop poles away from the real axis, and towards the imaginary axis. This rule is incorporated to prevent the response from getting overdamped.

### 3.1. Variation of the Controller Zeroes

Consider the relationship between the controller zeroes, and the dominant closed-loop poles. Note that the characteristic equation of the closed-loop system is given by

$$1 + \frac{N(s + z_1)(s + z_2)}{s(s + \lambda_1)(s + \lambda_2) \cdots (s + \lambda_n)} = 0 \qquad (5)$$

where $-z_1$, $-z_2$ are the controller zeroes; $-\lambda_i$, $i = 1 \ldots n$ are the open-loop plant poles; and $N = KK_D$ is the loop gain, where $K$ is the plant gain, and $K_D$ is the derivative mode gain.

*First order plant:*
$$P(s) = \frac{K}{s + \lambda} \qquad (6)$$

$\Rightarrow$

$$1 + C(s)P(s) = 1 + \frac{N(s + z_1)(s + z_2)}{s(s + \lambda)} = 0 \quad (7)$$

Let $z_1, z_2 = \alpha \pm j\beta$, and the roots of equation(7) be $s_1, s_2 = \nu \pm j\mu$.
$\Rightarrow$

$$\frac{d\nu}{d\alpha} = -\frac{N}{1 + N} \qquad (8)$$

$$\frac{d\mu}{d\alpha} = \frac{1}{(1 + N)} \times$$
$$\frac{N(2\alpha - \lambda)}{\sqrt{4N(\alpha^2 + \beta^2)(1 + N) - (\lambda + 2N\alpha)^2}} \qquad (9)$$

$$\frac{d\nu}{d\beta} = 0 \qquad (10)$$

$$\frac{d\mu}{d\beta} =$$
$$\frac{2N\beta}{\sqrt{4N(\alpha^2 + \beta^2)(1 + N) - (\lambda + 2N\alpha)^2}} \qquad (11)$$

Hence, if $\alpha \leq \frac{\lambda}{2}$ then

$$\frac{d\nu}{d\alpha} \leq 0, \quad \frac{d\mu}{d\alpha} \leq 0, \quad \frac{d\nu}{d\beta} = 0, \quad \frac{d\mu}{d\beta} \geq 0 \qquad (12)$$

*Second order plant:*
$$P(s) = \frac{K}{(s + \lambda_1)(s + \lambda_2)} \qquad (13)$$

$\Rightarrow$

$$1 + C(s)P(s) = 1 + \frac{N(s + z_1)(s + z_2)}{s(s + \lambda_1)(s + \lambda_2)} = 0 \quad (14)$$

Let $z_1, z_2 = \alpha \pm j\beta$, and the closed-loop poles be $s_1, s_2 = \nu \pm j\mu$ and $s_3 = \gamma$ with $\gamma < \nu < 0$.
$\Rightarrow$

$$\frac{d\nu}{d\alpha} = \frac{N(\gamma + \alpha)}{(\gamma - \nu)^2 + \mu^2} \qquad (15)$$

$$\frac{d\mu}{d\alpha} = \frac{N(\mu^2 - (\gamma - \nu)(\nu + \alpha))}{\mu((\gamma - \nu)^2 + \mu^2)} \qquad (16)$$

$$\frac{d\nu}{d\beta} = \frac{\beta N}{(\gamma - \nu)^2 + \mu^2} \qquad (17)$$

$$\frac{d\mu}{d\beta} = \frac{-\beta N(\gamma - \nu)}{\mu((\gamma - \nu)^2 + \mu^2)} \qquad (18)$$

Hence, if $\alpha \approx 0$, and $\mu^2 + \nu^2 < \nu\gamma$ one obtains

$$\frac{d\nu}{d\alpha} < 0, \quad \frac{d\mu}{d\alpha} < 0, \quad \frac{d\nu}{d\beta} > 0, \quad \frac{d\mu}{d\beta} > 0 \qquad (19)$$

A sufficient condition for the above is $\alpha \approx 0$, and $\mu^2 + \nu^2 < -\nu\lambda_2$.

*Higher order plants:* Similar relations hold for higher order plants. For example, in the case of a third order plant with real poles one gets

$$\frac{d\nu_1}{d\alpha} < 0, \quad \frac{d\mu_1}{d\alpha} < 0, \quad \frac{d\nu_1}{d\beta} > 0, \quad \frac{d\mu_1}{d\beta} > 0 \qquad (20)$$

provided $\nu_1^2 + \mu_1^2 < \frac{-\nu_1\lambda_2}{2}$, and $\alpha \approx 0$; where $\nu_1 \pm j\mu_1$ are the dominant closed-loop poles and $-\lambda_2$ is the middle plant pole.

Since one is dealing with dominant pole plants having large rise times, one expects these relationships in equations(12), (19) and (20) to hold. Based on these, the rules for manipulating the controller zeroes($z_i$) are derived.

$R_1$:
1. If $e_1(t)$ is positive, then increase $\mathrm{Re}(z_i)$, and increase $\mathrm{Im}(z_i)$ in magnitude.

2. If $e_1(t)$ is negative, then decrease $\mathrm{Re}(z_i)$, and decrease $\mathrm{Im}(z_i)$ in magnitude.

$R_2$:
1. If $e_2(t)$ is not *small* or *zero*, or $\frac{d}{dt}e_2(t)$ is not *zero*, then decrease $\mathrm{Re}(z_i)$.

2. If $e_2(t)$ is *small*, and $\frac{d}{dt}e_2(t)$ is *zero*, then increase $\mathrm{Re}(z_i)$, and increase $\mathrm{Im}(z_i)$ in magnitude.

### 3.2. Variation of the Loop Gain (N)

As observed above, $R_2$ stabilizes the system with respect to unstable dominant poles by decreasing $\mathrm{Re}(z_i)$. Another cause of instability could be dead-time. To stabilize the system with respect to dead-time, one needs to reduce the loop gain ($N$). Based on this observation, and the fact that a larger loop gain forces the dominant closed-loop poles towards the controller zeroes, the following rules are proposed for manipulating the loop gain.

$R_1$:
1. If $e_1(t)$ is *positive large*, increase $N$.

2. If $e_1(t)$ is *negative large*, decrease $N$.

$R_2$:
1. If $e_2(t)$ is greater than *small*, or $\frac{d}{dt}e_2(t)$ is not *zero*, then decrease $N$. This rule stabilizes the system against dead-time.

2. If $e_2(t)$ is *small*, and $\frac{d}{dt}e_2(t)$ is *zero*, then increase $N$. This rule aids in decreasing the damping.

Next consider the manipulation of the PID gains to obtain the proposed variations in the controller zeroes and the loop gain.

### 3.3. PID Gain Variation

The variation in the PID gains is derived under the following assumptions:

1. The PID gain variation $\Delta K_C$, $\Delta K_D$, and $\Delta K_I$ can be expressed as

$$\Delta K_C = \rho_{K_C}[K_{C_{max}} - K_{C_{min}}]\delta_{K_C}$$
$$= \rho_{K_C} R_{K_C} \delta_{K_C}$$
$$\Delta K_D = \rho_{K_D}[K_{D_{max}} - K_{D_{min}}]\delta_{K_D}$$
$$= \rho_{K_D} R_{K_D} \delta_{K_D}$$
$$\Delta K_I = \rho_{K_I}[K_{I_{max}} - K_{I_{min}}]\delta_{K_I}$$
$$= \rho_{K_I} R_{K_I} \delta_{K_I}$$

where $\delta_{K_C}$, $\delta_{K_D}$, $\delta_{K_I}$ are the defuzzified output from the fuzzy logic controller.

2. The output scaling factors are equal i.e. $\rho_{K_C} = \rho_{K_D} = \rho_{K_I} = \rho$.

3. The PID gains are of the same order of magnitude as their ranges $(R_{K_C}, R_{K_D}, R_{K_I})$.

*Influence on $N$:*
$$N = K K_D \qquad (21)$$

$\Rightarrow$

$$\Delta N \approx K\Delta K_D = \rho K R_{K_D} \delta_{K_D} \qquad (22)$$

*Influence on the controller zeroes:* An order of magnitude analysis is presented here. Let $z_1 = \alpha + j\beta$.
$\Rightarrow$

$$\alpha = \frac{-K_C}{2K_D} \qquad (23)$$

$$\beta = \frac{K_C}{2K_D}\sqrt{\frac{4K_D K_I}{K_C^2} - 1} \qquad (24)$$

$\Rightarrow$

$$\Delta\alpha \approx$$

$$\frac{1}{2K_D}\left(\frac{K_C}{K_D}\rho_{K_D} R_{K_D}\delta_{K_D} - \rho_{K_C} R_{K_C}\delta_{K_C}\right) \qquad (25)$$

| Table 1 Rule set $R_1$ | | | |
|---|---|---|---|
| $e_1(t)$ | $\Delta_{K_C}$ | $\Delta_{K_D}$ | $\Delta_{K_I}$ |
| PL | NL | PS | PL |
| PM | NM | Z | PM |
| PS | NS | Z | PS |
| Z | Z | Z | Z |
| NS | PS | Z | NS |
| NM | PM | Z | NM |
| NL | PL | NS | NL |

**Table 2 Rule set $R_2$**

| | $\frac{de_2(t)}{dt}$ | | | | | | |
|---|---|---|---|---|---|---|---|
| $e_2(t)$ | PL | PM | PS | Z | NS | NM | NL |
| PL | Z | Z | Z | Z | PS | PS | PM |
| PM | Z | Z | Z | Z | PS | PM | PM |
| PS | Z | Z | Z | NS | PS | PM | PM |
| Z | Z | Z | Z | Z | Z | Z | Z |
| NS | PM | PM | PS | NS | Z | Z | Z |
| NM | PM | PM | PS | Z | Z | Z | Z |
| NL | PM | PS | PS | Z | Z | Z | Z |

$$\Delta_{K_C}$$

| | $\frac{de_2(t)}{dt}$ | | | | | | |
|---|---|---|---|---|---|---|---|
| $e_2(t)$ | PL | PM | PS | Z | NS | NM | NL |
| PL | Z | Z | Z | Z | NM | NM | NL |
| PM | Z | Z | Z | Z | NM | NL | NL |
| PS | Z | Z | Z | PS | NM | NL | NL |
| Z | Z | Z | Z | Z | Z | Z | Z |
| NS | NL | NL | NM | PS | Z | Z | Z |
| NM | NL | NL | NM | Z | Z | Z | Z |
| NL | NL | NM | NM | Z | Z | Z | Z |

$$\Delta_{K_D}$$

| | $\frac{de_2(t)}{dt}$ | | | | | | |
|---|---|---|---|---|---|---|---|
| $e_2(t)$ | PL | PM | PS | Z | NS | NM | NL |
| PL | Z | Z | Z | Z | NS | NS | NM |
| PM | Z | Z | Z | Z | NS | NM | NM |
| PS | Z | Z | Z | PS | NS | NM | NM |
| Z | Z | Z | Z | Z | Z | Z | Z |
| NS | NM | NM | NS | PS | Z | Z | Z |
| NM | NM | NM | NS | Z | Z | Z | Z |
| NL | NM | NS | NS | Z | Z | Z | Z |

$$\Delta_{K_I}$$

By assumption 3, one has $\frac{K_C R_{K_D}}{K_D} \approx R_{K_C}$. Hence

$$\Delta\alpha \approx \frac{\rho R_{K_C}(\delta_{K_D} - \delta_{K_C})}{2K_D} \qquad (26)$$

Similarly

$$\Delta\beta \approx \frac{1}{\sqrt{\frac{4K_D K_I}{K_C^2} - 1}}\{\frac{\rho R_{K_C}}{2K_D}(\delta_{K_D} - \delta_{K_C})$$

$$+ \frac{\rho R_{K_I}}{K_C}(\delta_{K_I} - \delta_{K_D})\} \qquad (27)$$

Since, the rules for manipulating $N$, $\alpha$, and $\beta$ have been determined, one can relate $\delta_{K_D}$, $\delta_{K_C}$, and $\delta_{K_I}$ from equations(22), (26) and (27).

The inputs and the outputs are divided into 7 fuzzy classes. Namely, 1)Positive Large(PL), 2)Positive Medium(PM), 3)Positive Small(PS), 4)Zero(Z), 5)Negative Small(NS), 6)Negative Medium(NM), and 7)Negative Large(NL). Furthermore, let the the outputs share a common fuzzy membership function $\Delta$, and denote by $\Delta_{K_C}$, $\Delta_{K_D}$, and $\Delta_{K_I}$ their individual output classes obtained after rule evaluation from which $\delta_{K_C}$, $\delta_{K_D}$ and $\delta_{K_I}$ are obtained after defuzzification.

Based on the analysis presented above, one obtains the rule bases, shown in table 1 for $R_1$, and in table 2 for $R_2$. The number of rules on $R_2$ have been almost halved, by requiring that the expert take action only when the response is returning to it's steady state value. The membership functions were determined after simulation studies with a second order plant. These are shown in Fig. 2. It should be noted that the abscissas of the membership functions have been normalized to lie between -1 and +1. Hence, the inputs to the fuzzy logic controller should be scaled accordingly. Suggested scaling factors are 1 for $e_1(t)$, $\frac{0.2}{s_{new} - s_{old}}$ for $e_2(t)$, and $\frac{0.2}{T_2}$ for $\frac{d}{dt}e_2(t)$. However, these values are not binding.

## 4. APPLICATIONS

To verify the validity of the rule base, two applications are presented.

### 4.1 Separator Temperature Control

The separator is part of a larger plant (Tennessee Eastman Test Problem, Vogel and Downs 1990) comprising of a reactor, condensor, separator, stripper and a recycle compressor. It should be noted that i) there is dead time present, ii) the plant has an unknown number of poles, iii) there is measurement noise, and iv) there are restrictions on the manipulated variable i.e. the condensor cooling water flow valve. The reference response has $T_1 = 80$ seconds, and $T_2 = 300$ seconds. A step change from 80.109 deg C to 85 deg C is chosen.

Two initial responses are considered: (i) The initial PID settings give a very large rise time; and (ii) the initial settings result in an oscillatory response. Figure 3(top) illustrates the initial response for the case of large rise time. Figure 3(bottom) illustrates the response after the expert has tuned the PID gains. Figure 4 illustrates the variation in the steady state gains from iteration to iteration.

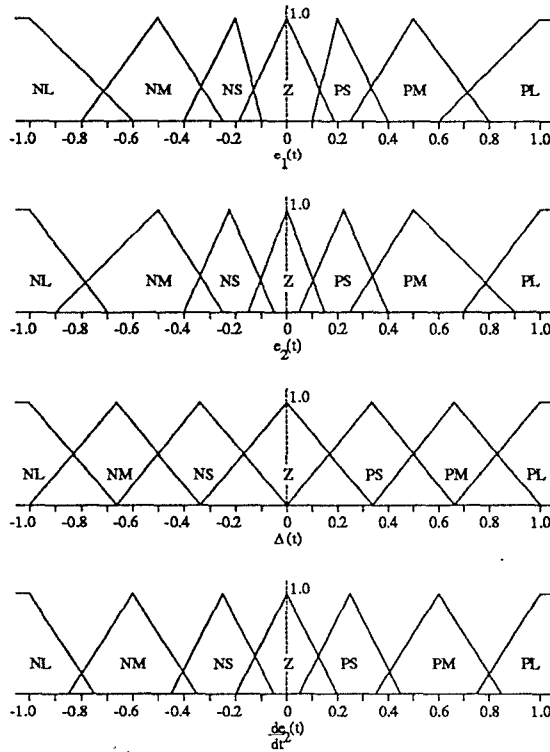Similarly, Fig. 5(top) illustrates the initially os-

Fig. 2: Fuzzy membership functions



Fig. 3: Separator: very slow initial response
(top), final response(bottom)

cillatory response. Figure 5(middle) shows the response during the first application of the expert, and Fig. 5(bottom) shows the final response.

Thus, it is observed that the expert correctly tunes the gains of the PID controller.

### 4.2 pH Control

Consider an application of the expert to a plant with nonlinearity in its output. The system chosen is the one considered by Tolle and Ersü (1992). The reference response has $T_1 = 20$ seconds, and $T_2 = 200$ seconds. A step change in the pH from 1 to 9 is considered. Figure 6(top) shows the initial response, and Fig. 6(bottom) shows the response after the expert has tuned the PID gains.

## 5. BUILDING ON THE EXPERT

The behaviour of the expert maybe modified to suit one's needs. One such modification is considered here.

### 5.1. Overshoot Control
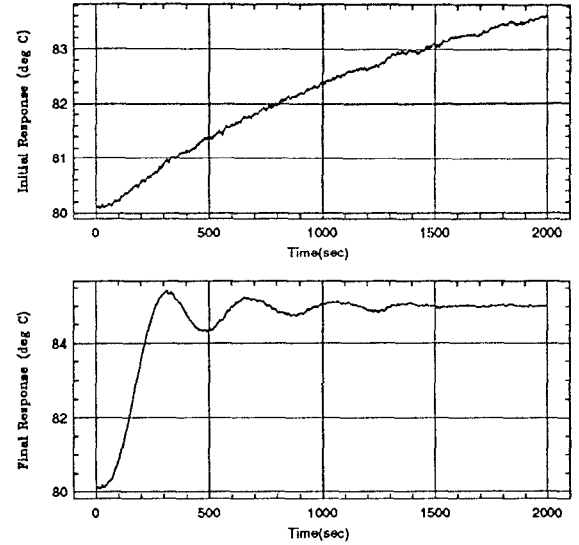
So far, no attempt has been made to control the

amount of overshoot. In fact, the rules have been developed so that there is a greater emphasis on rise time compensation. For some applications, the control of overshoot maybe more important.

As an application, consider separator temperature control with the liquid level in the separator increased by 20%. the reference response is not modified. In order to meet the rise time requirement, the response is highly underdamped. Figure 7 illustrates the final response obtained when the expert was applied with overshoot control, with a maximum allowable overshoot of 5%. It is observed that the response meets the overshoot and rise time specifications.

## 6. CONCLUSION

Although, a lot of work has been done in the area of tuning PID gains, most of it is based on the analysis of the plant response, and on parameter estimation. In this paper, it is shown that for the class of stable, dominant pole plants with large rise times, it is not necessary to carry out data analysis and parameter estimation. There are inherent properties of this class that can be exploited to design a learning controller which can learn on-line. This could be thought of as being analogous to humans, who possess the ability to tune PID parameters without necessarily carrying out data analysis or identification. The results show that the rule base derived performs as required. Furthermore, one could also derive a similar rule base for PI controllers, since these are the controllers most commonly found in the
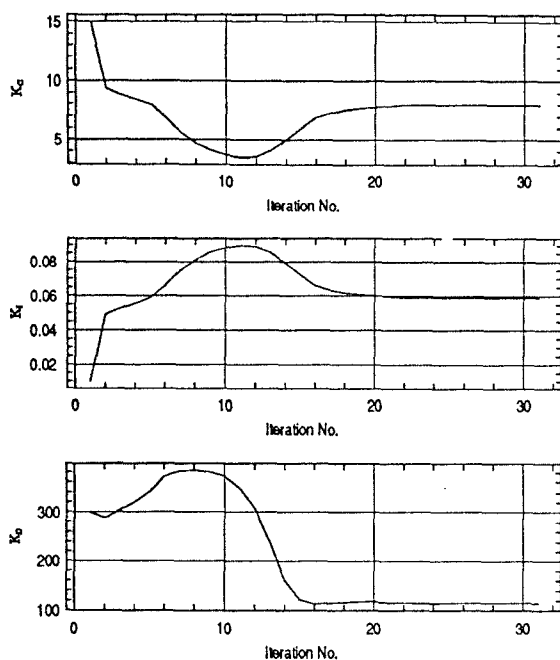
Fig. 4: Separator(initially slow): steady state
gains versus iteration number

chemical process industry.

## 7. ACKNOWLEDGEMENTS

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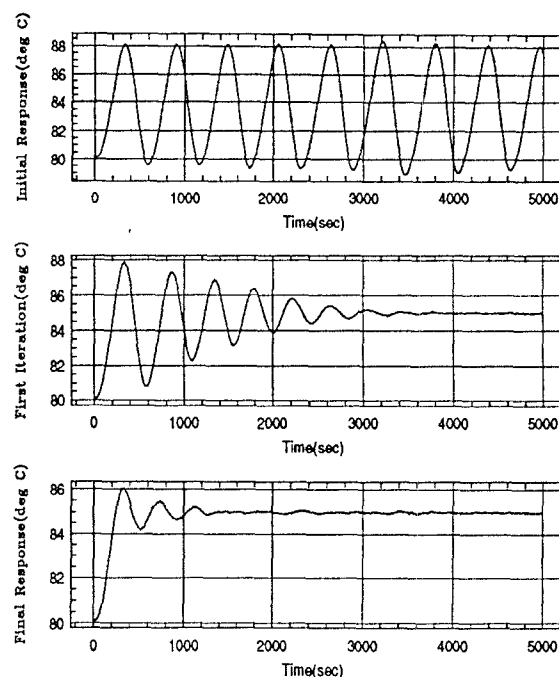paper 24a, Chicago, IL.

Fig. 5: Separator: oscillatory initial response
(top), first iteration response (middle),
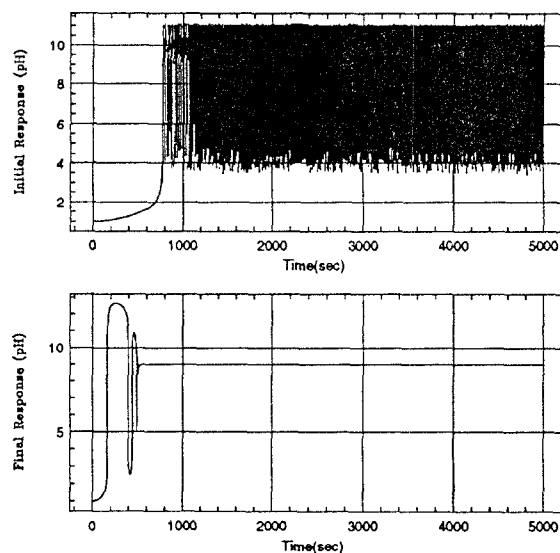final response (bottom)



Fig. 6: pH neutralization: initial response (top),
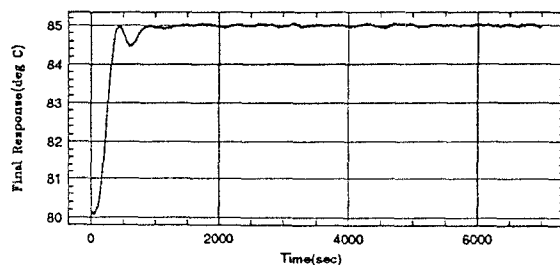final response (bottom)



Fig. 7: Separator(liquid level change): final re-
sponse with overshoot control