TECHNICAL RESEARCH REPORT

SYSTEMS RESEARCH CENTER

Supported by the National Science Foundation Engineering Research Center Program (NSFD CD 8803012), Industry and the University

Operational Theory of Design and Manufacturing

by I. Pandelidis

TR 90-6

OPERATIONAL THEORY OF
DESIGN AND MANUFACTURING

Ioannis Pandelidis
Assistant Research Scientist
Engineering Research Center
University of Maryland

1 Abstract

An operational theory of design and manufacturing is developed which formalizes the design problem and addresses some of its major issues. We first formulate the problem in its most general form and then present a single design axiom. The axiom in effect asserts that the optimum design process minimizes the total expected cost of design and manufacturing. Since the cost is a direct function of the solution procedure, our focus is shifted from the product to the solution procedure. The design axiom is analogous to the principle of minimum energy in physics. In this case the energy equivalent is the total cost and the path of minimum energy is the trajectory of the proposed design vector defines the path. The design axiom then states that the optimum design will follow a certain optimum trajectory of minimum expected cost.

One of the direct benefits in formulating the problem in operational terms is that a wealth results form systems theory may now be used to derive usefull methodologies for the design/manufacturing process. In the present paper the consequences of the axiom are delineated and general principles of design are derived as a direct outcome of the design axiom. The general principles available to the designer can now be defended on a single principle, thus making possible the rational comparison of alternative schemes.

2 Introduction

Although a coherent scientific theory of design has not yet been developed , some research work has already been conducted on this field. Asimow [1] and Woodson [2] focused on the development of rules based on the intuitive knowledge of experienced designers. More recently, Kim and Suh [3] postulated some scientific principles on which the theory of design can be founded. Their "axiomatic" approach is based on the "uncoupling" and the "minimum entropy" axiom . Furthermore, they presented corrolaries and additional minor axioms for specific processes and applications. Their approach starts from qualitative descriptions, which in many cases, can be translated into quantitative measures for comparison.

The present work follows a different approach. We first formulate the problem based on input-output mappings. This operational representation renders explicit the iterative nature of design and manufacturing. It also permits a wealth of results from systems theory to be directly used with design and manufacturing problems . We then present a single design axiom that identifies the optimum design procedure . According to the axiom this procedure minimizes, at each iteration, the total expected cost of design and manufacturing, including the cost of the design procedure itself.

The implications of this axiom under some specific constraints are explored and both quantitative and qualitative design rules are derived.

3 Operator formulation of the iterative design/manufacturing process

In this section it will be shown that an operational view of the design/manufacturing (D/M) process combined with a control system representation, will clarify the issues of design and manufacturing. Since this approach is closely related to systems theory, many fundamental findings of this latter area can be used to guide us in the search for the principles required to develop the science of

D/M.

We first define the spaces Ψ and Ω as the input and output space respectively. The input space represents the set of all possible inputs of the design process while the output space represents the set of all possible observed outputs of the process . The performance specifications of the desired design are represented as a subset in the output space, $T \subset \Psi$. Then both the design and manufacturing processes can be viewed as mappings from an input space to an output space. During the design/manufacturing of a shaft, for example, the length and diameter of the shaft are inputs in the design stage while the depth of cut , feed and spindle speed are inputs in the manufacturing stage. However, the strength and surface finish will be in the output space.

Let P_k be a mapping $P_k : \Omega \rightarrow \Psi$ that relates the chosen design vector $u(k)$ to the resulting functional parameters $y(k)$ at the kth iteration; i.e.

$$y(k) = P_k(u(k)); u(k) \in \Omega, y(k) \in \Psi \tag{1}$$

. where k is the iteration index. The domain of P_k is the entire input space, which includes all design and manufacturing inputs. However its representation may be in terms of lower dimensional operators which are restrictions of operator P to subsets of the input space. For example, the diagonal operator $P : R^2 \rightarrow R^2$ given by

$$col[y_1, y_2] = Acol[x_1, x_2]$$

where

$$a_{11} = 2, a_{12} = a_{21} = 0, a_{22} = 3$$

may be represented by two lower dimensional operators P', P'', such that $P' : R \rightarrow R, y_1 = 2x_1$ and $P'' : R \rightarrow R, y_2 = 3x_2$

We also define a "controller" mapping $Q_k : \Psi^2 \times \Omega \rightarrow \Omega$ such that Q_k operates at the kth iteration on the given output $y(k)$, the desired set T, and $u(k)$ to produce a proposed solution $u(k+1)$. Therefore Q_k is defined by

$$u(k+1) = Q_k(y(k), T, u(k)) \tag{2}$$

$$y(k) = P(u(k)) \tag{3}$$

A noniterative design is a special case of the above formulation where the proposed solution does not depend on the actual output. We will call this case an open loop design.

A block diagram of the closed loop system is given in Figure 1. Let the set of all mappings Q from the output space to the input space be denoted by $\mathbf{Q}$ and the set of all mappings P from the input space to the output space be defined as $\mathbf{P}$. The relation of the four identified spaces, namely $\Omega, \Psi, \mathbf{Q}$ and $\mathbf{P}$, is shown in Figure 2. Some of the operations that P and Q may represent are listed in Table 1.

Given the above definitions we construct a product space $\mathbf{Z} = \mathbf{P} \times \mathbf{Q}$ such that the point $Z_k = (P_k, Q_k) \in \mathbf{Z}$ is a particular choice of a $P_k \in \mathbf{P}$ and $Q_k \in \mathbf{Q}$ at the kth iteration. The first terms of the implementation sequence $P(k) = \{P_1, P_2, ..., P_N\}$ represent some type of evaluation which may be as simple as the evaluation of a formula or as complex as the implementation of a simulation program such as a finite element package with the proposed design vector u^k as the input at the kth iteration. In practice the evaluations at the first iterations are more approximate and the latter evaluations are more accurate and therefore more inexpensive. For example an approximate ballpark evaluation is typically less expensive in computational effort than a simulation which is likely to be implemented in later design iterations. In a similar fashion, a design operator sequence is also given as a function of the iteration index k by $Q(k) = \{Q_1, Q_2, ..., Q_N\}$ The design operators Q_i represent the steps of the design process, whereby the results y^i of the previous iteration are compared with the target set T and a proposed new solution is generated. Depending on the type of problem at hand, the action of Q_i may be the inversion of some matrix, the generation of a random point in the indicated search space, or some other similar operation.

3.1 Example 1: P, Q fixed

Finding the roots of an equation by the bisection method

Consider the nonlinear operator $P : R \rightarrow R$ defined by $y = P(u)$, where $y \in R, u \in R$. Given the target space $T = y \in R : |y| \leq d$ find a solution $u \in R$ such that $y = P(u) \in T$, using the bisection method.

This is a special case of the general formulation where both the operators P and Q are fixed for the entire iteration sequence. The operator Q stands for the bisection method operator, where the upper and lower limits of the solution interval are given at iteration stage k by $z_u(k), z_l(k)$ respectively. The root of the equation is found by applying the following algorithm :

$$z_u(0) = z_u^0, z_e(0) = z_e^0 \ \mathrm{P}(\mathrm{z}_u^0)\mathrm{P}(\mathrm{z}_e^0) < 0 \tag{4}$$

$$u(k+1) = Q(y(k), T, u(k)) = \frac{z_l(k) + z_u(k)}{2} \tag{5}$$

$$z_u(k+1) = u(k+1) \ and \ z_l(k+1) = u(k+1)\{\text{if } P(u(k+1))Pz_e(k)) > 0$$

to

(7)

$$z_u(k+1) = z_u(k) \ and \ z_l(k+1) = z_l(k) \ if \ P(u(k+1)) < 0$$

The iteration sequence is initiated at $k = 0$ and stops for some $k = k_0$ when the target space T has been reached. The solution is therefore given by $u = u(k_0)$. One interpretation of the above algorithm is that u is the solution of the difference equation 3 at iteration (time) k_0.

3.2 Example 2: P fixed, Q not fixed

Combination of Newton Raphson and steepest descent A more general case will be considered here. The problem is again to find the roots of the nonlinear operator P , but we are to select the appropriate solution procedure . Let us denote the bisection method by Q_B and the Newton-Raphson method by Q_N. The Newton-Raphson method is also of the form

$$u(k+1) = Q_N(u(k), y(k)) = u(k) - y(k)/y'(k) \tag{8}$$

$$y(k) = P(u(k)) \tag{9}$$

where $y'(k)$ is the first derivative of y evaluated at $u(k)$. A plausible sequence $Z(k)$ of operators might turn out to be for instance

$$Z(k) = \{(Q_1, P_1), (Q_2, P_2)), (Q_3, P_3)\} = \{(Q_B, P), (Q_B, P), (Q_N, P)\}. \quad (10)$$

An illustration of the selection of an appropriate controller operator Q for a fixed plant P is given be by Economou [4].

3.3 The supervisory controller operator

The last example, illustrated that an appropriate operator pairs had to be selected at each iteration. We will represent that decision process by a 'supervisory' operator. This operator may describe either our own decision process or, in the case of computer environments, the rules embodied at the highest command level about the use of Q and P operators. Let a "supervisor" operator

$$Q^* : \Psi^2 \times \Omega \to \mathbf{Z} \quad (11)$$

denote the mapping which, based on the functional considerations and the available design space, chooses a particular pair of operators P_k and Q_k at each iteration k. We will define an iteration invariant procedure as the special case where $P_{k+1} = P_k$ and $Q_{k+1} = Q_k$ for all k. A block diagram of the overall design process is shown in Figure 3 depicting two loops, the supervisory-loop and the design loop.

Given P_k, Q_k a proposed solution $u(k+1)$ is calculated at the kth iteration, which in turn produces an output $y(k+1)$ by the following procedure:

$$Z(k+1) = [P_{k+1}, Q_{k+1}] = Q^*(u(k), y(k); T) \quad (12)$$

$$u(k+1) = Q_k(u(k), y(k); T) \quad (13)$$

$$y(k) = P_k(u(k)) \quad (14)$$

The solution $u(k), y(k), Z(k)$ of the set of difference equations (8, 9,10) is given as a function of the iteration index k, represents the set of rules by which the operators are chosen at each iteration step. It is noted that this set of rules embodies the design and manufacturing strategy that a particular system follows in solving a D/M problem. The concept of a system represents some well defined entity that interacts with the environment according to some internal rules. Thus a system might be a computer environment, an automatic control system, or a human being.

The next problem is to determine the optimum set of rules that a particular system must embody in order to deal with the D/M problem. This problem is addressed in the latter sections of this paper.

3.4 Example 3: P, Q not fixed

Injection molding

In this design/manufacturing problem different implementation and controller operators may be selected at each iteration in order to arrive at some acceptable design parameters. Injection molding is a process where it is difficult, if not impossible, to differentiate design from manufacturing . We will show how we can formulate the D/M process in terms of the operational design methodology.

Let us look at the design of a mold for a rectangular piece that has to meet a set of functional constraints (targets). The width and length of the rectangular piece is given and we must choose the design variables for the filling process; i.e., the mold temperature u_1, melt temperature u_2, and filling time u_3 as well as the thickness of the part u_4. The output variables are the maximum temperature difference y_1, the maximum shear stress y_2 and the maximum shear rate y_3.

Note that the considerations in this case include both design and processing parameters, since the manufacturability of the part is strongly dependent upon both. In fact the processing parameters such as melt temperature , mold temperature and fill time are used as design parameters in the design process, and therefore may be easily classified as either process or design parameters depending on our perspective .

Three possible implementation operators are available; i.e., $\mathbf{P} = \{P', P'', P'''\}$. The operator P' is a commercial software package used for approximate filling analysis , P'' is a software package for finite element filling simulation, and P''' is the actual experiment designed to test the design.

The implementation operators are mappings from the design space to the functional space, so that

$$P : \Omega \rightarrow \Psi \tag{15}$$

where

$$\Omega = R^4, \Psi = R^3. \tag{16}$$

The P operator is given by $y(k) = P(u(k))$ where $y \in R^3, u \in R^4$, and k denotes the iteration index.

We also have three synthesis operators, each corresponding to one implementation operator denoted as $\mathbf{Q} = \{Q', Q'', Q'''\}$. The operator Q' is an optimization software package for use with P' that yields approximate design parameters [5]. Q'' is an optimization package that is used with the finite element simulation P'', [6], and Q''' is the methodology implemented in the experiment represented by P'''. The operators Q are mappings from the functional space to the design space , $Q : \Psi \rightarrow \Omega$, defined by

$$u(k+1) = Q(y(k), r) \tag{17}$$

The function of the supervisory operator Q^* is to select the appropriate sequence of the operator pairs (Q_i, P_i) at each iteration step in the solution process. A typical sequence in this case would be the operator sequence

$$(Q_k, P_k) = (Q', P') \tag{18}$$

for $k = 0, 1, ..., n_1$

$$(Q_k, P_k) = (Q'', P'') \tag{19}$$

for $k = n_1 + 1, ..., n_2$ and

$$(Q_N, P_N) = (Q''', P''') \tag{20}$$

where $N = n_2 + 1$ is the final iteration.

4 The Optimum Design/Manufacturing Axiom:

The operator Q^* represents our solution procedure for a particular problem; i.e., it represents the set of rules and principles that guide the design and manufacturing process. The following design axiom asserts that there is an optimum Q^*_{opt} and defines the associated criterion of optimality. It is postulated that the optimum set of embodied rules minimizes the expected overall cost of the D/M procedure at each iteration stage.

Let the output functional space be Ψ and the input functional space be Ω . Let the target functional space $T \subset \Psi$, and consider a set of analysis operators $\mathbf{P}$, as well as a set of design operators $\mathbf{Q}$. Define $Z(k) = \{Z_i\}$ where $Z_i = (P_i, Q_i) \in \mathbf{P} \times \mathbf{Q}$, and the space of all such sequences as $\mathbf{Z(k)}$, so that $Z(k) \in \mathbf{Z(k)}$. Each element Z_k^{opt} of the optimum design and manufacturing sequence $Z^{opt}(k)$ satisfies for each iteration k, the following condition:

$$Z_k^{opt} = \{Z_k \in \mathbf{Z} : C(Z_k^{opt}) = min_{Z_k \in \mathbf{Z}} E[C_T(Z(k)|k)]\}. \qquad (21)$$

where $E[C_T(Z(k)|k)]$ represents the expected total cost of implementing the design and manufacturing operator sequence $Z(k)$ given all the available information about the total system to the iteration $k-1$. The above formulation of the design axiom thus includes consideration of the expected cost of the search for a solution, the implementation as well as the selection process of the operator sequence associated with the supervisory controller.

The minimum cost axiom is analogous to an axiom in physics about objects following the path of minimum energy, where the energy equivalent is the total cost and the trajectory of the proposed design vector defines the path. The design axiom then states that the optimum design will follow a certain optimum trajectory of minimum expected cost.

The extent to which a set of supervisory rules achieves this optimization goal, will define the goodness of that approach. In other words the axiom a measure by which expert systems for design/manufacturing or different design methodologies may be judged as to their effectiveness .

5 Consequences of the design axiom

The minimum expected total cost be seen as a goal in the high level command of an expert system for design/manufacturing. This system judges the appropriateness of the design rules according to the axiom. All experiential or deductive rules that are in accordance with the goal of minimizing the expected cost of the D/M process are acceptable and desirable. Prior knowledge must be incorporated based on this principle. Rules on decomposition, minimization of search space, disturbance minimization, e.t.c. will be thus seen as rules that conform to the axiom and therefore will be part of the rule base of the system. In what follows some of the necessary conditions for an optimum D/M process will be presented.

5.1 Necessary conditions on the P operator

The question of choice of an appropriate model to represent the input-output mapping is one of great importance that cannot be addressed fully in this paper. For the following results, an infinite cost will be assigned to the case where we fail to reach a solution in a finite number of iterations. In practice, a large finite cost may be assigned instead, so that the iterations will terminate if the total cost exceeds the preassigned level. Note that this is in accordance with the principle of minimization of the expected cost. The results for a finite failure cost are similar in nature, and will not be discussed. The representation of the input- output mapping by a P operator is called an external model representation. An internal model representation, called the state space representation, will be discussed here because of its relevance in dealing with issues of controllability and observability in a system.

A state space representation of the operator P is a pair $\{A, C\}$ such that the mapping

$$P = C \circ A \tag{22}$$

where

$$A : U \to X \tag{23}$$

and

$$C : X \to Y. \tag{24}$$

X is called the state space of the system. Under this representation, the target space X_T is defined in terms of the state space rather than the output space, so that $X_T \subset X$

Definitions: A set of states $X_1 \subseteq X$ is controllable if for any given state $x \in X_1 \exists$ an input $u \in U$ such that $x = Au$. The controllable space X_c is defined as the maximum controllable set and is equal to the image of AU, that is $X_c = \{x \in X : x = Au, \forall u \in U\}$. Note that every controllable set is a subset of X_c. A system will be controllable if $X_c = X$, that is if the whole state space is controllable.

Theorem: Given a set of available inputs U the input operator A, and a set of desired states X_d, a necessary condition for finite D/M cost is that $X_d \subseteq AU$

Proof: The proof is by contradiction. Let a desired target state $x \in X_d$ be such that $x \notin AU$, then there is no possible input that will yield the desired state, and thus the D/M cost will be infinite by definition.

This condition, states that in order to reach the desired targer set X_d , both the available design input space must be sufficiently large, and the input operator must have sufficient rank, in order to reach the desired target set X_d. If a particular state is not reachable using the available inputs, then the iteration process will never terminate, and the total D/M cost will be infinite.

Definition:

A set $X_2 \subseteq X$ is observable if for all $x \in X_2 \exists$ a unique $y \in Y$ such that

$y = Cx$. The observable space X_o is the largest observable set, so that if a set X_2 is observable, then $X_2 \subseteq X_o$. A system is observable if $X_o = X$.

A set $X_2 \subseteq X$ is minimaly observable if for all $y \in Y \ \exists$ a distance function $d(y, T)$ such that

$$\begin{aligned} d(y,T) &= 0 \quad y \in T \\ &= 1 \quad y \notin T \end{aligned} \tag{25}$$

The observable space X_{mo} is the largest minimally observable set, so that a set X_2 is minimally observable, then $X_2 \subseteq X_{mo}$. A system is minimally observable if $X_{mo} = X$.

The next theorem states the minimum conditions for termination of an iteration loop, is that the evaluation function must have the ability to determine whether the target has or has not been reached. Note that this is a requirement for an iterative process and not an open loop D/M procedure, since the open loop process does not depend upon the observed outputs, and thus it will always terminate at iteration index k=1.

Theorem:

Given the desired target space X_T, let the selection procedure of Q^* terminate when $d(y, T) = 0$. A necessary condition for a finite termination time for an iterative process (finite D/M cost) is that $X_T \subseteq X_{mo}$.

Proof:

Let $x \in X_T$ and $x \notin X_{mo}$. Then the condition for termination of the iteration loop will never be met because we will not be able to infer from the available output that we have indeed reached the target. Thus the D/M cost will be infinite.

Injection Molding Example:

One of the benefits of the observability and controllability criteria, is that they include the necessary actions to control the process or to develop an appropriate design procedure. In injection molding the weld lines, which are created by opposing melt fronts must not violate structural and aesthetic considerations. In this case, the available P'' operator (see example 4.1 as P'') provided the isochrones of filling time in the mold. It did not provide, however, direct results on the location of the weld lines; i.e., the weld line states were unobservable under this operator. A new algorithm was developed that identified the location of the weld lines making these states observable under the new modified operator. The next issue that had to be addressed was controllability. Under the original package (operator Q' in example 4.1) used for molding conditions optimization, the effect of the molding conditions on the weld line location was negligible; i.e. the weld line location states were considered uncontrollable. A new variable, the gate location, was included in order to make the weld line locations controllable.

5.2 Necessary conditions for Q and QP operators

We will now state some necessary conditions on the controller operator Q and the mapping QP. The first condition below states that the range of the operator Q must have at least one point which lies in the preimage of the target set denoted by $P^{-1}T$.

Theorem: The necessary condition that operator Q must satisfy for a finite D/M is given by

$$R(Q) \cap P^{-1}T \neq \emptyset \tag{26}$$

Definition The iteration sequence $\{y_k\}$ is eventually in T if for some $N < \infty, y_N \in T$. Althoug this definition is the set theoretic representation of convergence to a set, it does not require that the sequence remains in T after N because the termination criterion will stop the sequence at N.

The following theorem will be stated without proof.

Theorem: A necessary condition for a finite D/M cost is that the iteration sequence $\{y_k\}, whichisdefined$ defined by $(P(k), Q(k)$ and the initial conditions, is eventually in T.

The above results are summarized in the following thoerem:

Theorem: The neccesary conditions for a D/M procedure to have a finite termination time are: There exists a nonempty set $X_1 \subseteq X_d$ where X_d is the target set, such that X_1 is both controllable and minimally observable. In addition, the iteration sequence must eventually be in T.

5.3 Decoupling and Minimum Entropy

Minimization of the expected total cost of the D/M process includes both the cost of search for an adequate solution, and the cost of implementing this solution. A consequence of that requirement is the development of appropriate rules for the reduction in the search costs . From this perspective, the axiomatic rules for decoupling and minimum entropy postulated by Suh and Kim [3] may be also interpreted as general rules for the reduction of the expected search and implementation costs.

6 The role of disturbances in the design and manufacturing process

The model presented for the design and manufacturing process will now be generalized to include the effects of the disturbances to the system. It will be seen that one of the implications of the D/M axiom which will be discussed in this section will be the requirement for robust D/M processes for systems with large expected disturbances. We will distinguish the disturbances associated with any design/manufacturing process according to the following classification (see Figure 4):

a. Load disturbances (External Load Noise): These are the external inputs to the system, that is they belong in the input space Ω, which affect the output variables in question, and are considered noncontrollable for various reasons such as ecause they are unpredictable, or because they are too difficult or too expensive to incorporate as control inputs. Given an operator pair (Q, P), a load disturbance signal d will satisfy the following conditions: $d \in \Omega, d \in N(P)^{\perp}, d \notin R(Q)$. where R(Q), the range of Q, defines the controllable space, and the complement of the null space, $N(P)^{\perp}$, defines the space of inputs that have a nonzero output.

b. Parameter Disturbances (Internal Noise): The output varies due to the variation of the structure and parameters of the plant. The plant disturbance is represented as a variation in the operator mapping. For linear mappings, this will be represented as ΔP; i.e., $P = P' + \Delta P$, where P' is the nominal operator.

c. Measurement Disturbance (External Measurement Noise): The measured output signal is contaminated by measurement noise. Then the evaluation function in the Q operator is faulty since the inputs to this function are unreliable. Given a pair of operators (P, Q), a measurement noise disturbance n satisfies the following conditions: $n \in \Psi, n \in N(Q)^{\perp}$.

6.1 Example: Injection molding manufacturing process

Some concrete examples of the classification of the disturbances will be presented for the case of injection molding [7].

a. Load Disturbances: The material parameter variations, such as the viscocity variation from shot to shot or from batch to batch, are disturbances that we have little control over, at least on the manufacturing level. Thus these disturbances, which affect the quality of the plastic part, are considered as load disturbances. The ambient conditions such as humidity and temperature will also be considered as load disturbances when they are not under strict control.

b. Parameter disturbances: The natural variation of the machine control functions fall under this category. The variation of the actual ram velocity profile from shot to shot reflects the capabilities of that particular machine to hold the machine variables close to the reference values. Typically machines with closed loop controls have a much smaller sensitivity to parameter disturbances.

c. Noise disturbances: The ability of the installed measurement devices to accurately reflect the true values of the variables been monitored, determines the level of noise disturbances in the system. In the injection molding example for instance, the barrel temperature transducers are noisy measurement devices of the melt temperature, a variable which we typically are interested in controlling.

6.2 Robust D/M process

Minimization of the expected cost of D/M implies that the selection of the P and Q operators must reduce the risk of a nonconvergent iteration sequence. During the design stage, for example, the constraint that the proposed must be successfully manufactured imposes the conditions of robustness in the design process. A robust design reduces the effects of the disturbances on the functional outputs of interest, and by this action reduces the expected total cost. This problem is addressed presently by the 'robust design' and the 'design for manufacturing' philosophies.

Two approaches to increase the disturbance rejection of the system will be discussed here. Given a 'plant' operator P, this goal is achieved by the appropriate selection of inputs u such that the sensitivity of the functional outputs y to u, $S_u^y >> S_d^y$, where d denotes the disturbance. Such an approach is advocated by the Taguchi methodology of design [8]. Another method for minimizing disturbances is the implementation of feedback, where the effect of load disturbances is minimized by maximizing the sensitivity of the controller Q to the measured error.

6.3 Application : Tolerance Setting for design parameters

A design model P is given by the relation $P : R^2 \to R$ where

$$y = \alpha_1 x_1 + \alpha_2 x_2 \tag{27}$$

where x_1, x_2 are independent random variables, and the target is defined by $T = \{y \in R : V(y) \leq r\}$ where $V(y)$ is the variance of y. We are asked to choose the appropriate variances of the random inputs in order to meet the target output. Since

$$V(y) = \alpha_1^2 V(x_1) + \alpha_2^2 V(x_2) \tag{28}$$

, an infinite number of solutions can satisfy the requirements. However in order to satisfy the D/M axiom, we wish to minimize the expected total cost of the design. In the absence of any specific cost information , we will assume that the expected cost of the design C is

$$C = \frac{\beta_1}{V(x_1)} + \frac{\beta_2}{V(x_2)} \tag{29}$$

with $\beta_1 = \beta_2$. Equation 29 expresses the experiencial knowledge that, in general , the tolerances of the design inputs are inversely related to cost. The minimization of the Equation 29 subject to the constraints given by T and Equation 28, give the optimum input variances. Note that had the prior information been more specific, let us say for example $\beta_1 = 10\beta_2$, we would have arrived at a different design . The variance of x_1 will now be greater than in the first case, since the cost of reducing that variance is comparatively greater.

7 Conclusions:

This paper presented an operational theory of design and manufacturing. The iterative D/M process was seen as the sequencial application of analysis and synthesis operators P and Q respectively. The sequencing of these operators was done either by a high level set of rules in a computer or by the designer/manufacturer himself. This set of rules were represented by the supervisory level operator Q^*.

We presented a single design/manufacturing axiom, which stated that the optimum set of supervisory level rules at each iteration level minimizes the total expected cost of the D/M process. To the extent that the set of rules of a given system is in accordance with that goal, this system is closer to the optimum. The D/M axiom can be applied both quantitatively as well as qualitatively.

Necessary conditions on the selection of the P and Q operators were stated as a consequence of this axiom, and some design rules that have appeared in the literature were interpreted as rules that are in accordance with the D/M axiom. Such rules help the designer or the computer system in which they are incorporated, to minimize the expected total costs. There is indeed a wealth of rigorous results derived from systems theory that may be brought to bear on the D/M process, particularly when the input-output spaces and the corresponding mapping functions are restricted to some special classes.

8 Acknowledgements

The invaluable comments and editing effort of my colleague and friend Ioannis Minis are gratefully acknowledged .

References

[1] Asimow, M., "Introduction to Design, ", Prentice- Hall Inc., N.J., 1962.

[2] Woodson, T.T., "Introduction of Engineering Design, " Magraw-Hill, N.Y. 1966.

[3] Kim, S.H. and Suh, N.P. "Mathematical Foundations for Manufacturing", *Journal of Engineering for Industry*, Vol. 109, pp. 213-218, 1987.

[4] Economou, C.G. "An operator theory approach to nonlinear controller design", *Ph.D. thesis*, Cal. Institute of Technology, Pasadena, CA, 1985.

[5] Pandelidis, I.O. , Zou, Q., "Determination of Approximate Feasible Molding Space for Injection Molding", *Conference Proceedings of Annual Technical Conference ANTEC of the Society of Plastics Engineers*, pp.233-236, April 1988.

[6] Pandelidis, I.O., Zou, Q., Lingard, T., "Optimization of Gate Location and Operational Molding Conditions for Injection Molding", *Conference Proceedings of Annual Technical Conference ANTEC of the Society of Plastics Engineers*, pp. 219-222, April 1988.

[7] Agrawal, A.R., Pandelidis, I.O., Pecht, M., "Injection Molding Process Control:A Review", *Polymer Engineering and Science Journal*, Vol. 27, No.18, pp.1345-1357, Mid-October, 1987.

[8] Barker, T.B., "Quality Engineering by Design, The Taguchi Approach", *ASQC Quality Congress Transaction*, Baltimore, 1985.

Table 1

Q **operator**	P **operator**
Synthesis	Analysis
Controller	Plant
Hypothesis Generation	Hypothesis Testing
Proposal	Implementation

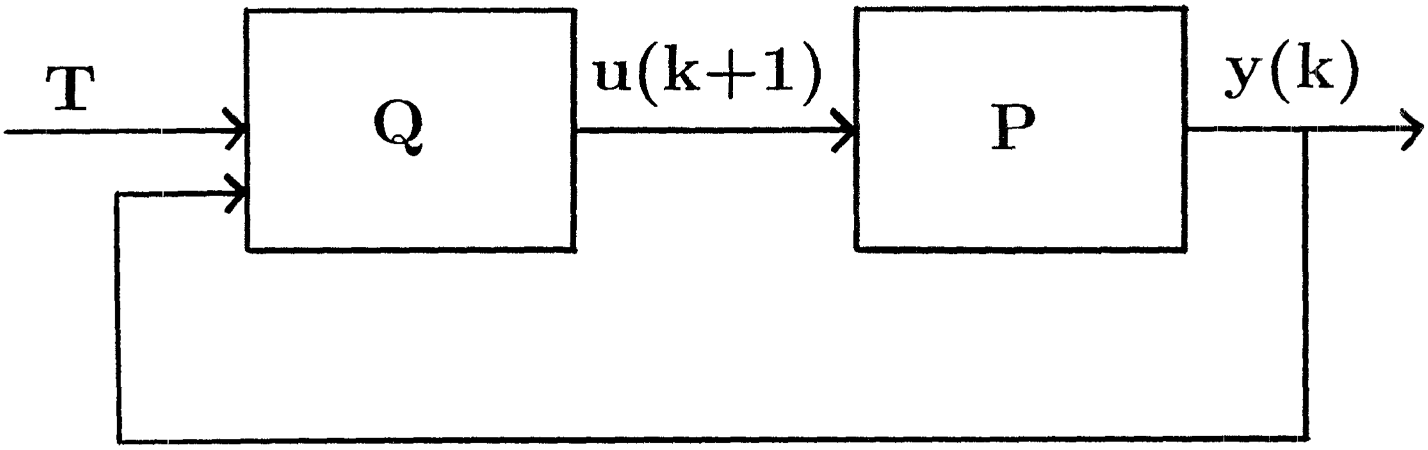

Figure 1

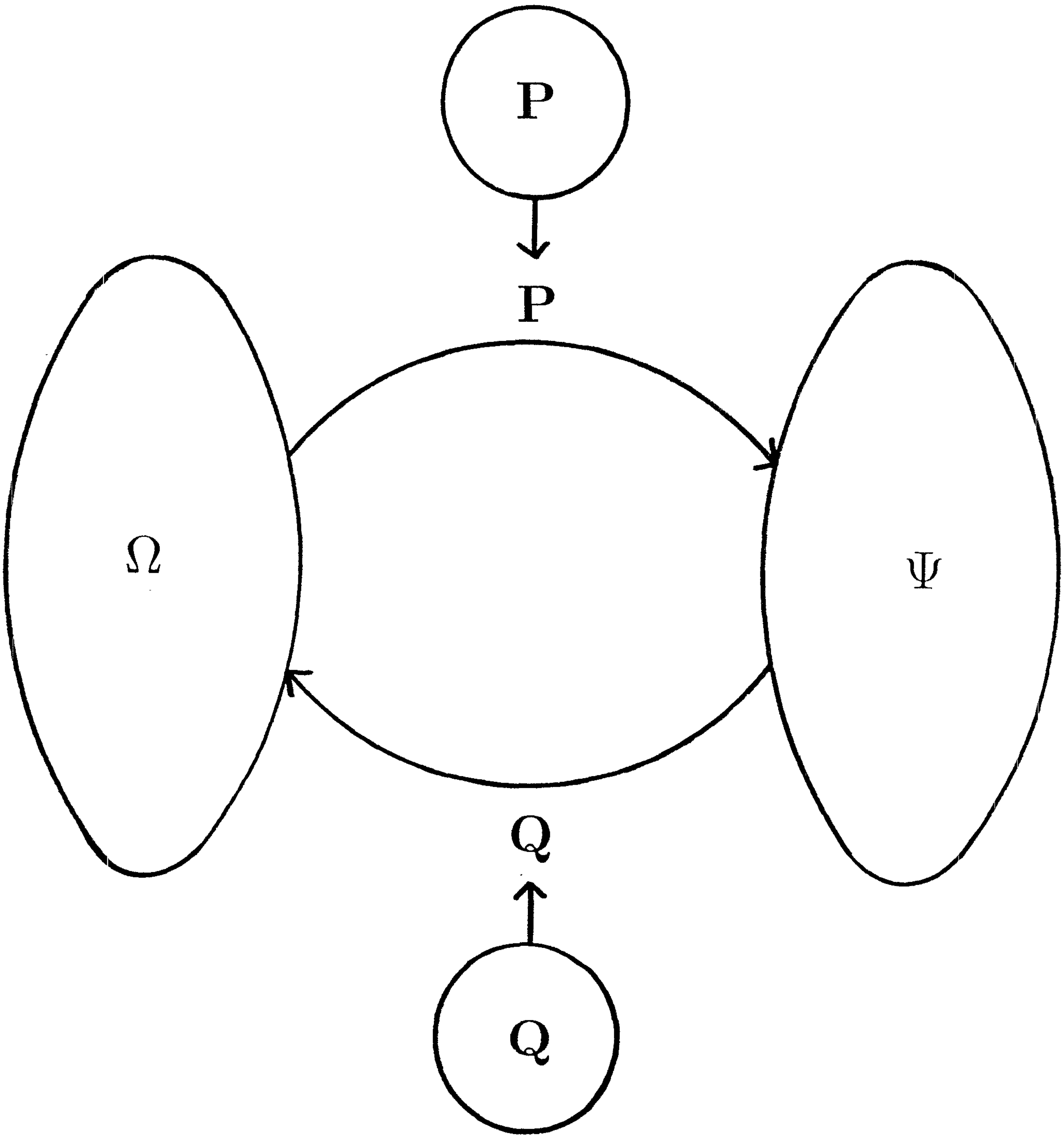

Figure 2

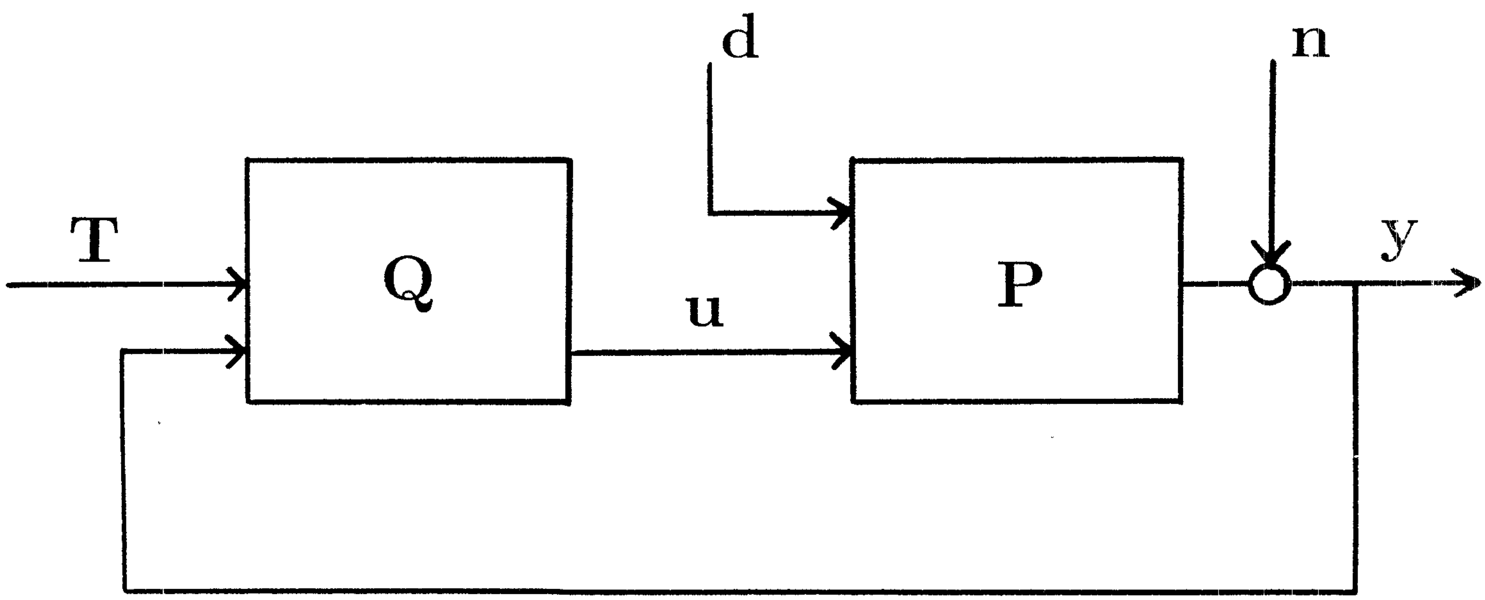

Figure 3

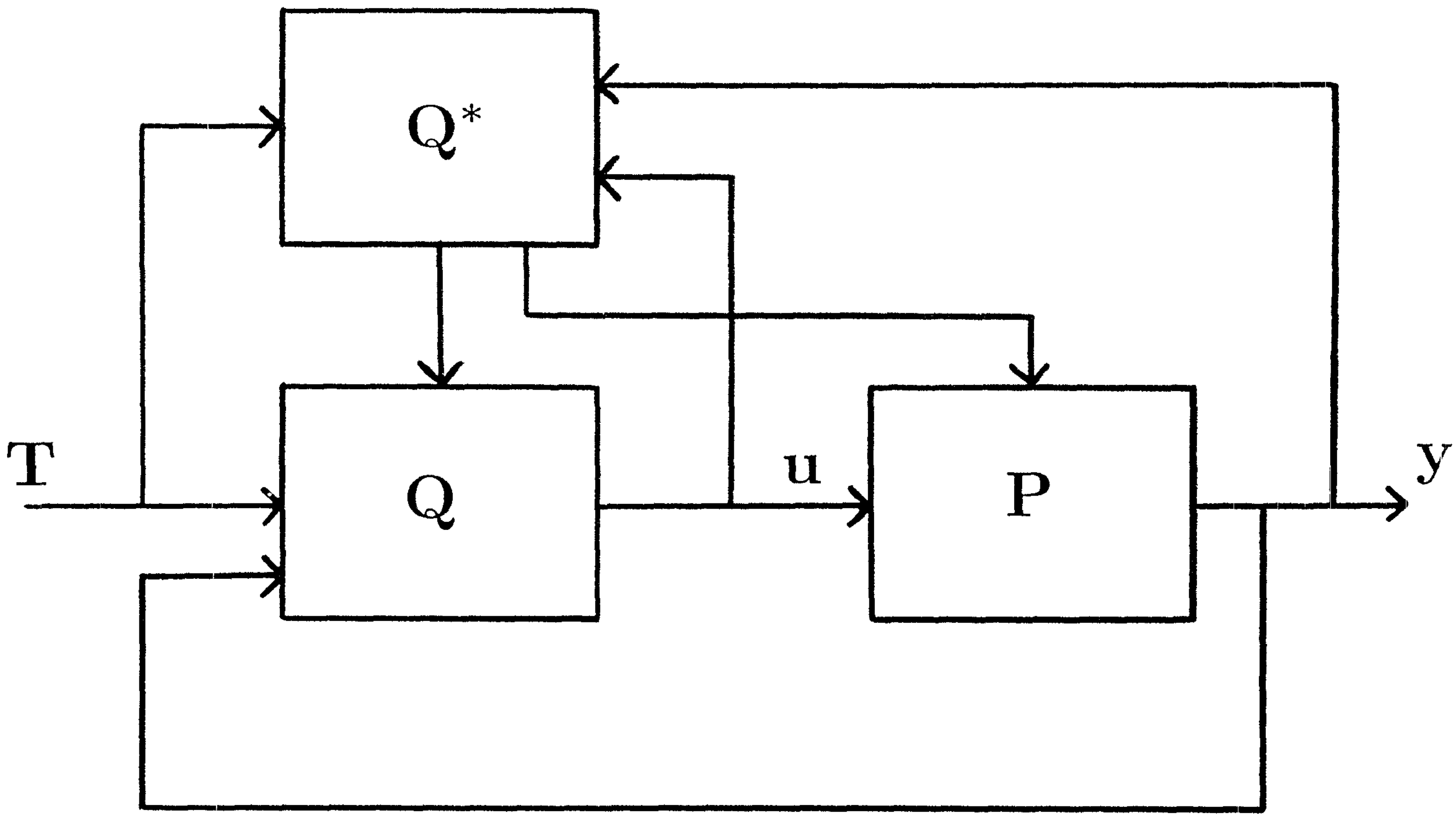

Figure 4