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Joint Scheduling and Routing for Ad-hoc Networks Under Channel State Uncertainty

by Anna Pantelidou, Anthony Ephremides and Andre L. Tits

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Anna Pantelidou, Anthony Ephremides and André L. Tits
Department of Electrical and Computer Engineering
University of Maryland College Park, MD 20742
{apantel, etony, andre}@umd.edu

Abstract

Obtaining the stable throughput region of a wireless network, and a policy that achieves this throughput, has attracted the interest of the research community in the past years. A major simplifying assumption in this line of research has been to assume that the network control policy has full access to the current channel conditions at every time a decision is made. However, in practice one may only estimate the actual conditions of the wireless channel process, and hence the network control policy can at most have access to an estimate of the channel which can in fact be highly inaccurate. In this work we determine a stationary joint link activation and routing policy based on a weighted version of the "back-pressure" algorithm that maximizes the stable throughput region of time-varying wireless networks with multiple commodities by having access to only a possibly inaccurate estimate of the true channel state. We further show optimality of this policy within a broad class of stationary, non-stationary, and even anticipative policies under certain mild conditions. The only restriction is that policies in this class have no knowledge on the current true channel state, except what is available through its estimate.

Keywords — Cross layer design, Scheduling and Routing, Stable throughput, Channel state uncertainty

1 Introduction

An important criterion to measure performance is the rate at which data are delivered to their destinations, while guaranteeing that the queues at the network nodes don't grow without bound. This is what we generally call *stable throughput* of the network. Under stability the throughput rates coincide with the arrival rates of the traffic entering the network. The main objective is to identify the maximum set of achievable stable arrival rates, or otherwise to characterize the *stability region* of the network and further provide a network control policy that achieves these rates.

In this paper, we restrict our attention to the joint scheduling, and routing network control policies that maximize the stable throughput region of time-varying wireless networks. There exists a rich literature on the subject of maximum stable throughput (See e.g., [1], [2], [3], [4].).

Specifically in [1], a scheduling policy that achieves maximum stable throughput in single-hop time-varying networks is identified. Moreover, in [2], the authors characterize the stability region of static, multi-hop radio networks with multiple commodities, and propose a centralized, stationary, scheduling and routing rule, commonly referred as the "back-pressure", that achieves maximum stable throughput. The "back-pressure" policy forwards the traffic through the network from queues with high loads to queues with lower loads and achieves stability by load-balancing the queues in the network. Furthermore, the authors in [2] show that their proposed policy is at least as good as any stationary policy. Under the assumption that a scheduled transmission is always successful, they prove that their policy performs at least as well as any non-stationary policy with respect to maximizing the stable throughput region of the network. In fact, the "back-pressure" algorithm of [2] has been shown to maximize the stable throughput region under a variety of conditions. In a previous work of ours, [3], we proved optimality of a policy inspired by the back-pressure algorithm of [2] within the set of all stationary policies in the more general setting of wireless networks with *time-varying topologies*. Further, [3] also differs from [2] in that our proposed policy gives priority to each commodity according to a preassigned commodity weight. In both [2], and [3], it is assumed that links are imperfect and that a scheduled transmission may fail, based on a link failure probability, which is independent of the identity, and the number of the simultaneously activated links. Finally, in another related study, [4], a joint scheduling, routing, and power control policy, also inspired by the back-pressure algorithm, is proposed that maximizes the stable throughput region of time-varying wireless networks. The authors in [4] consider a time-varying process of perfect channels, i.e., a transmission through a link is always successful.

However, in practice the channel conditions can only be estimated, and hence exact knowledge of the current channel state is likely to be unavailable. Furthermore, in cases when the channel process varies fast with time (e.g., fast fading) or when the propagation delay of the feedback channel is large (See e.g., [5], [6].) this estimate may be highly inaccurate. Hence, the channel state at the time of a scheduling or routing decision is significantly different from the state at which the actual transmissions take place. The effect of this discrepancy in the channel state may be two folded; first, certain scheduled transmissions are going to fail, and second, transmissions through certain links which would be successful if scheduled, are not activated. Naturally, this situation will affect the set of stabilizable rates and will result in a smaller stability region that is a subset of the stability region under perfect links or under perfect channel estimation.

In this paper, we are interested in capturing the effect of imperfect channel estimation and characterize the maximum achievable stable throughput region. We also obtain a policy that maximizes the stable throughput region under this setting. Towards this end, this work is different from [4], and generalizes [2], and [3], in that we consider policies with knowledge of only an *estimate* of the true channel state. Specifically, we propose a stationary, joint scheduling, and routing policy for *multi-hop, time-varying* networks that maximizes the stable throughput region of the network by having access to only a, perhaps highly inaccurate, *estimate* of the current channel state. Our proposed policy, inspired by the "back-pressure" idea of [2], is shown to be optimal within a broad class of stationary, and non-stationary, even anticipative policies. We improve on the results of [2], and [3] in two aspects. First, we show that our proposed policy performs at least as well in terms of stable throughput as a large class of policies that do not have more information on the current true channel state than our policy and where this information is limited to be given through an estimate of the channel state. In contrast with [2], this result holds even when scheduled transmissions are not guaranteed to succeed. Second, our model of uncertainty in the channel state is more sophisticated than the simplistic model used in [2], and [3] in two respects: (i) the existence of a link is explicitly modeled through the Signal to Interference plus Noise Ratio criterion imposed by the physical layer, and (ii) our model accounts for the fact that the probability of success of a transmission is affected by the interference caused by other nearby concurrent transmissions.

The rest of the paper is organized as follows. In Section 2 we present the network model. Section 3 discusses the problem of stable throughput maximization under channel state uncertainty. Section 4 defines a large class of stationary, non-stationary, perhaps anticipative policies. The results of optimality of our proposed policy within this broad class of policies with respect to stable throughput maximization are presented in Section 5. Finally, Section 6 concludes the paper.

2 Model Formulation

We consider slotted time and a wireless network consisting of N, possibly mobile, nodes each of which is equipped with a single transceiver. We denote by $\mathcal{N} = \{1, 2, \ldots, N\}$ the set of all nodes in the network. Each node $n \in \mathcal{N}$ transmits at a fixed power level P_n.

We also consider a set $\mathcal{J} = \{1, 2, \ldots, J\}$ of distinct commodities of traffic with packet lengths equal to one time slot. The number of exogenous packet arrivals of commodity j at node n during time slot t is denoted by $A_{nj}(t)$. We let $\mathbf{A}^j(t)$ denote the N-vector $(A_{nj}(t) : n = 1, 2, \ldots, N)$ of arrivals of the j^{th} commodity during time slot t at every node in the network, and $\mathbf{A}(t)$ denote the $N \times J$ matrix $(A_{nj}(t), n = 1, 2, \ldots N, j = 1, 2, \ldots J)$ of arrivals in time slot t at every node n and for every commodity j. Traffic of commodity $j \in \mathcal{J}$ is routed in a multi-hop fashion through the network until it reaches *any* node in a set of *exit nodes* for that commodity, $V_j \subset \mathcal{N}$, where it exits the network. For any commodity $j' \neq j$, the sets $V_{j'}$, and V_j may overlap. We further assume that there are no exogenous arrivals of a particular commodity at the exit nodes of that commodity, i.e., $A_{nj}(t) = 0$ for all $n \in V_j,\ j \in \mathcal{J}$.

At each node n there exist J infinite capacity buffers, each holding separately the packets of a particular commodity $j \in \mathcal{J}$ that have reached node n. We denote the queue size for commodity j at node n *at the end* of time slot t by $X_{nj}(t)$. At time slot 0 the queue sizes at all nodes are arbitrary but finite, i.e., $X_{nj}(0) \geq 0$ for every node $n \in \mathcal{N}$, and commodity $j \in \mathcal{J}$. Moreover, the queue size at each exit node $n \in V_j$ of some commodity j, and for all time slots $t \geq 0$ satisfies $X_{nj}(t) = 0$. Finally, for every commodity $j \in \mathcal{J}$ we denote by $\mathbf{X}^j(t)$ the N-vector $(X_{nj}(t), n = 1, 2, \ldots N)$ of queue sizes of the j^{th} commodity at every node in the network at the end of time slot t, and by $\mathbf{X}(t)$ the $N \times J$ matrix $(X_{nj}(t), n = 1, 2, \ldots N, j = 1, 2, \ldots J)$ of queue sizes of every commodity at every node in the network at the end of time slot t. The set of possible values of $\mathbf{X}(t)$, i.e., the state space of the process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$, is denoted by $\mathcal{X}$.

The channel process $\{\mathbf{S}(t)\}_{t=1}^{\infty}$ defines the channel conditions between any pair of nodes in the network and is assumed to change only at the beginning of each time slot $t \in \{1, 2, \ldots\}$. Specifically, at time slot t, the channel state $\mathbf{S}(t) = \{(G_{(n,m)}(t), N_{o(m)},\ \forall n, m \in \mathcal{N}\}$ is characterized by the path loss $G_{(n,m)}(t)$ between each pair of nodes n, m, as well as the noise power, $N_{o(m)}$, at each receiving node m. A fundamental aspect of our model that contrasts it from prior work of [1], [3], and [4] is that at the beginning of each time slot t the network controller has access only to an *estimate* $\hat{\mathbf{S}}(t) = \{(\hat{G}_{(n,m)}(t), \hat{N}_{o(m)}(t),\ \forall n, m \in \mathcal{N}\}$ of the current channel state $\mathbf{S}(t)$. The *estimated* channel state $\hat{\mathbf{S}}(t)$ during slot t is characterized by the *estimated* path loss $\hat{G}_{(n,m)}(t)$ between each pair of nodes n, m, and the *estimated* noise power $\hat{N}_{o(m)}(t)$ at each receiving node m. Note that although the noise power $N_{o(m)}$ is time invariant, its estimate $\hat{N}_{o(m)}(t)$ depends on time, since as time progresses we may naturally get a monotonically improving estimate.

We further assume that the state space of the *true* and *estimated* channel processes is a finite set of cardinality K, which is naturally assumed to be common for both $\{\mathbf{S}(t)\}_{t=1}^{\infty}$, and $\{\hat{\mathbf{S}}(t)\}_{t=1}^{\infty}$. For example, that would be the case if we consider node mobility that is restricted to occur only among points of a finite grid. We denote this common set by $\mathcal{S} = \{\mathbf{S}^{(1)}, \mathbf{S}^{(2)}, \ldots, \mathbf{S}^{(K)}\}$. We will further denote by $\mathcal{K} = \{1, 2, \ldots, K\}$ the set of indices that label the elements of $\mathcal{S}$.

At every time slot t, a (unidirectional) link $\ell = (n, m)$ from node n to node m under the true channel state $\mathbf{S}(t) \in \mathcal{S}$ is defined to exist, if the Signal to Noise Ratio (SNR) at m exceeds a certain, non-negative, threshold θ_m, i.e.,

$$\text{SNR}(\ell, t) := \frac{P_n \; G_{(n,m)}(t)}{N_{o(m)}} \geq \theta_m. \tag{1}$$

We denote the source node n of link ℓ by $s(\ell)$, and its destination node m by $d(\ell)$. Given the time variability of the channel conditions, and the fact that nodes are mobile, the total number of links, L, can be as large as $N \times (N-1)$. We denote by $\mathcal{L} = \{1, 2, \ldots, L\}$ the set of indices of all links in the network.

The fact that the wireless medium is a shared resource poses limitations on the set of nodes that may successfully transmit simultaneously. Hence, not every subset of links in $\mathcal{L}$ can be concurrently activated. In order to take the physical layer access constraints into account, appropriate medium access control schemes need to be introduced. In this paper, we focus on conflict free scheduling. Towards this end, we define an *activation vector* to be any L-element binary vector, each entry of which corresponds to a (unidirectional) link. At any time slot t, the entries of this vector are equal to one for those links that are concurrently activated at time slot t, and zero for all other links. We also require that an activation vector complies with the single transceiver assumption. This assumption implies that simultaneous transmission and reception from the same node as well as receiving/transmitting simultaneously from/to multiple nodes are not allowed. We further define an activation vector $\mathbf{c}$ to be *valid* with respect to some channel state $\mathbf{S}(t)$ if for every link $\ell \in \mathcal{L}$ such that the ℓ^{th} entry c_ℓ of $\mathbf{c}$ satisfies $c_\ell = 1$, the SINR criterion as shown in Equation (2)

$$\text{SINR}^{\,\mathbf{c}}(\ell, t) := \frac{P_{s(\ell)} \; G_{(s(\ell),d(\ell))}(t)}{N_{o(d(\ell))} + \sum_{\substack{\ell' \in \mathcal{L} \setminus \{\ell\} \\ \text{s.t. } c_{\ell'} = 1}} P_{s(\ell')} \; G_{(s(\ell'),d(\ell))}(t)} \geq \theta_{d(\ell)}, \tag{2}$$

is satisfied with $c_{\ell'}$ being the ℓ'^{th} entry of $\mathbf{c}$. The criterion of Equation (2) implies that the corresponding transmissions through all links $\ell \in \mathcal{L}$ with $c_\ell = 1$ will be successful under channel state $\mathbf{S}(t)$. Similarly, the estimated SINR criterion under $\hat{\mathbf{S}}(t)$ can be written as

$$\widehat{\text{SINR}}^{\,\mathbf{c}}(\ell, t) := \frac{P_{s(\ell)} \; \hat{G}_{(s(\ell),d(\ell))}(t)}{\hat{N}_{o(d(\ell))}(t) + \sum_{\substack{\ell' \in \mathcal{L} \setminus \{\ell\} \\ \text{s.t. } c_{\ell'} = 1}} P_{s(\ell')} \hat{G}_{(s(\ell'),d(\ell))}(t)} \geq \theta_{d(\ell)}. \tag{3}$$

Note that due to the inaccuracy of the estimate, an activation vector selected at time slot t may be valid with respect to the estimated channel state $\hat{\mathbf{S}}(t)$ at slot t, but not valid with respect to the true channel state $\mathbf{S}(t)$, and vice versa.

For every possible channel state $\mathbf{S}^{(k)} \in \mathcal{S}$ where $k \in \mathcal{K}$, we denote by $\mathcal{T}_k$ the *constraint set* of $\mathbf{S}^{(k)}$, i.e., the set of all *valid* activation vectors with respect to $\mathbf{S}^{(k)}$. Note that for every activation vector $\mathbf{c}' \in \{0, 1\}^L$ that is componentwise smaller than some vector $\mathbf{c} \in \mathcal{T}_k$, i.e., $\mathbf{c}' \leq \mathbf{c}$, it follows that $\mathbf{c}' \in \mathcal{T}_k$. This is natural because for any collection of links that jointly satisfy the SINR criteria of Equations (2) - (3), these criteria will still be satisfied by switching off certain transmissions. From the above observation it follows trivially that for every $k \in \mathcal{K}$ the $\mathbf{0}$-vector is also a valid activation vector for each channel state $\mathbf{S}^{(k)} \in \mathcal{S}$.

For each commodity j, consider a process $\{\mathbf{E}^j(t)\}_{t=1}^{\infty}$ that for every time slot t gives the link activations for packets of commodity j. In other words for every time slot t the vector $\mathbf{E}^j(t)$ is an L-element binary vector, the entries of which are equal to one for those links that are simultaneously activated and packets of commodity j are transmitted through them, and are equal to zero otherwise. Further, for every time slot t we define $\mathbf{E}(t) := \sum_{j=1}^{J} \mathbf{E}^j(t)$. The process $\{\mathbf{E}(t)\}_{t=1}^{\infty}$ corresponds to the overall link activations for every time slot t and it is such that whenever the at time slot t the estimated channel process is in state $\mathbf{S}^{(k)}$, the vector $\mathbf{E}(t)$ is a valid link activation vector with respect to $\mathbf{S}^{(k)}$. This means that $\mathbf{E}(t)$ is a vector from the constraint set $\mathcal{T}_k$, i.e, $\mathbf{E}(t) \in \mathcal{T}_k$. We call the process $\{\mathbf{E}^j(t)\}_{t=1}^{\infty}$ an *activation process*. Recall that the constraint set has the property that for any vector in the constraint set, any other vector that is smaller component-wise must be in the constraint set as well. Since $\mathbf{E}(t) \in \mathcal{T}_k$, the aforementioned property implies that for every commodity j the corresponding vector $\mathbf{E}^j(t)$ is also a valid activation vector with respect to $\mathbf{S}^{(k)}$, i.e., it satisfies $\mathbf{E}^j(t) \in \mathcal{T}_k$. Further, we require that for each commodity j, a vector $\mathbf{E}^j(t)$ must be such that its ℓ^{th} component, $(\mathbf{E}^j(t))_\ell$, takes the value zero for all those time slots t that the queue size at source node of the link, $s(\ell)$, for commodity j is equal to zero at the time of the link activation, i.e., $X_{s(\ell)j}(t-1) = 0$. We say that every such process $\{\mathbf{E}(t)\}_{t=1}^{\infty}$ is an *admissible policy*, and the process $\{\mathbf{E}^j(t),\ j \in \mathcal{J}\}_{t=1}^{\infty}$ is an *admissible policy corresponding to the* j^{th} *commodity*. Unless otherwise specified all the policies we consider are valid.

Further, for every time slot t where $\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}$ for some $k \in \mathcal{K}$, and for any activation vector $\mathbf{c} \in \mathcal{T}_k$, we construct the $L \times L$ diagonal indicator matrix $\mathbf{Q}^{\mathbf{c}}(t)$, whose ℓ^{th} diagonal entry, $(\mathbf{Q}^{\mathbf{c}}(t))_\ell$, satisfies

$$(\mathbf{Q}^{\mathbf{c}}(t))_\ell = \begin{cases} 1, \text{ if } \Big(\text{SINR}^{\mathbf{c}}(\ell,t) \geq \theta_{d(\ell)}, \widehat{\text{SINR}^{\mathbf{c}}}(\ell,t) \geq \theta_{d(\ell)}\Big) \text{ or} \\ \qquad \Big(\text{SINR}^{\mathbf{c}}(\ell,t) < \theta_{d(\ell)}, \widehat{\text{SINR}^{\mathbf{c}}}(\ell,t) < \theta_{d(\ell)}\Big), \\ 0, \text{ otherwise}. \end{cases} \tag{4}$$

Intuitively, for any given activation vector $\mathbf{c} \in \mathcal{T}_k$, and estimated channel state $\mathbf{S}^{(k)}$, the ℓ^{th} entry of the matrix $\mathbf{Q}^{\mathbf{c}}(t)$ takes the value one only when the estimator estimates the channel correctly in the sense that the values of the corresponding SINRs under both the *true*, and *estimated* channel state lie on the same side of the inequality. Note that whether $(\mathbf{Q}^{\mathbf{c}}(t))_\ell$ is equal to one or zero depends on the overall link activations given by the vector $\mathbf{c}$. In the ideal case of perfect channel estimation, the matrix $\mathbf{Q}^{\mathbf{c}}(t)$ is the identity matrix, i.e., $\mathbf{Q}^{\mathbf{c}}(t) = \text{I}$, for every time slot t where the estimated channel state is in state $\mathbf{S}^{(k)}$ for some $k \in \mathcal{K}$, and for any activation vector $\mathbf{c} \in \mathcal{T}_k$.

Also, for every commodity j we define the matrix $\mathbf{R}^j$ as an $N \times L$ matrix that denotes the changes in the queue sizes after a successful link activation. The (n,ℓ) entry, $R^j_{n\ell}$, of this matrix equals

$$R^j_{n\ell} = \begin{cases} 1, & \text{if } n = d(\ell) \notin V_j, \\ -1, & \text{if } n = s(\ell), \\ 0, & \text{otherwise}. \end{cases} \tag{5}$$

Note that $R^j_{n\ell} = 0$ when $n = d(\ell) \in V_j$, as packets of commodity j arriving at n exit the system. Overall, the above yields the following dynamic equation for the queue sizes

$$\mathbf{X}^j(t+1) = \mathbf{X}^j(t) + \mathbf{R}^j\ \mathbf{Q}^{\mathbf{E}(t+1)}(t+1)\ \mathbf{E}^j(t+1) + \mathbf{A}^j(t+1),\ \ t \geq 0. \tag{6}$$

Throughout this paper we make use of the following assumption on the input processes.

Assumption 1 *(a) The triplet* $\{\mathbf{S}(t), \hat{\mathbf{S}}(t), \mathbf{A}(t)\}_{t=1}^{\infty}$ *is i.i.d. over time and independent of* $\mathbf{X}(0)$*. (b) The arrival process has finite second moments, i.e.,* $\mathbb{E}[\mathbf{A}(t)^2] < \infty$*.*

Assumption 1 (a) guarantees that each of the processes $\{\mathbf{S}(t)\}_{t=1}^{\infty}$, $\{\hat{\mathbf{S}}(t)\}_{t=1}^{\infty}$, and $\{\mathbf{A}(t)\}_{t=1}^{\infty}$ are individually i.i.d, and hence have a stationary distribution. In particular, the probability $p_{\hat{\mathbf{S}}}(k)$ of the occurrence of *estimated* channel state $\mathbf{S}^{(k)} \in \mathcal{S}$, given by

$$p_{\hat{\mathbf{S}}}(k) := P[\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}], \quad \forall k \in \mathcal{K}, \tag{7}$$

does not depend on t. Without loss of generality, we assume that

$$p_{\hat{\mathbf{S}}}(k) > 0, \quad \forall\, k \in \mathcal{K}. \tag{8}$$

Indeed, all our results are probabilistic in nature, and are not affected if we discard sample paths corresponding to a nullset of outcomes. Moreover, from Assumption 1(a) it follows that although the processes are i.i.d. in time, for any particular time slot t they can be correlated among themselves. For example, the true, and estimated channel states $\mathbf{S}(t)$, and $\hat{\mathbf{S}}(t)$ are naturally correlated but not $\mathbf{S}(t)$ and $\hat{\mathbf{S}}(t-1)$.

From Assumption 1(b), it follows that the first moments of the arrival process $\{\mathbf{A}(t)\}_{t=1}^{\infty}$ are also finite, i.e., $\lambda_{nj} := \mathbb{E}[A_{nj}(t)]$, where the quantity λ_{nj} corresponds to the arrival rate of commodity j at node n. We also denote by $\boldsymbol{\lambda}$ the *arrival rate matrix* $(\lambda_{nj},\ n = 1, 2, \ldots N,\ j = 1, 2, \ldots J)$ of arrival rates at every node in the network, and for every commodity. Finally, for each commodity $j \in \mathcal{J}$ we write $\boldsymbol{\lambda}^j$ for the N-vector $\boldsymbol{\lambda}^j = (\lambda_{nj},\ n = 1, 2, \ldots N)$ of arrivals of the j^{th} commodity at every node in the network. All arrival rates in our model are measured in terms of packets per time slot.

The nomenclature defined so far is summarized through an example in Figure 1, where we consider a network of 3 nodes, i.e., $\mathcal{N} = \{1, 2, 3\}$. Nodes 1 and 2 transmit at a fixed powers P_1 and P_2 respectively. We consider that the channel conditions are such that we have two possible channel states, namely $\mathcal{S} = \{\mathbf{S}^{(1)}, \mathbf{S}^{(2)}\}$. On the left side of the figure, we give the possible links that can be established under channel state $\mathbf{S}^{(1)}$ and on the right side of the figure we give the set of possible links under channel state $\mathbf{S}^{(2)}$. Specifically, when the estimated channel state is $\mathbf{S}^{(1)}$, there exist two possible links, namely links 1, and 2, where a "link" satisfies the SNR criterion of Equation (1) and when it is $\mathbf{S}^{(2)}$ no connectivity exists among the nodes. Hence, $\mathcal{L} = \{1, 2\}$. Further, although both links 1 and 2 are in $\mathcal{L}$, we assume that they cannot be activated simultaneously due to the fact that they do not jointly satisfy the physical layer constraints of SINR. Specifically, we assume that at most one of them can be activated at any given time. Since the constraint set $\mathcal{T}_k$ for channel state $\mathbf{S}^{(k)}$ contains all the valid activation vectors with respect to $\mathbf{S}^{(k)}$, we have that $\mathcal{T}_1 = \{[0, 0], [0, 1], [1, 0]\}$ and $\mathcal{T}_2 = \{[0, 0]\}$. There exist two commodities of traffic in the network, i.e., $\mathcal{J} = \{1, 2\}$. $A_{11}(t)$ and $A_{22}(t)$ denote the arrivals in packets per slot, during time slot t, of commodity 1 at node 1, and of commodity 2 at node 2 respectively. We assume that packets of each commodity exit the network at node 3, i.e., $V_j = \{3\}$, for $j = 1, 2$. At every node in the network, there exist two infinite capacity buffers, that hold separately the packets of each commodity. We indicate the queue size of commodity 1 at node 2 at the end of time slot t by $X_{21}(t)$, and the queue size of commodity 2 at the same node by $X_{22}(t)$. Note that, due to the estimation errors, the policy may schedule e.g., link 1 assuming that the current channel state is $\mathbf{S}^{(1)}$ when in fact the current state is $\mathbf{S}^{(2)}$ and hence the scheduled transmission through link 1 will fail.

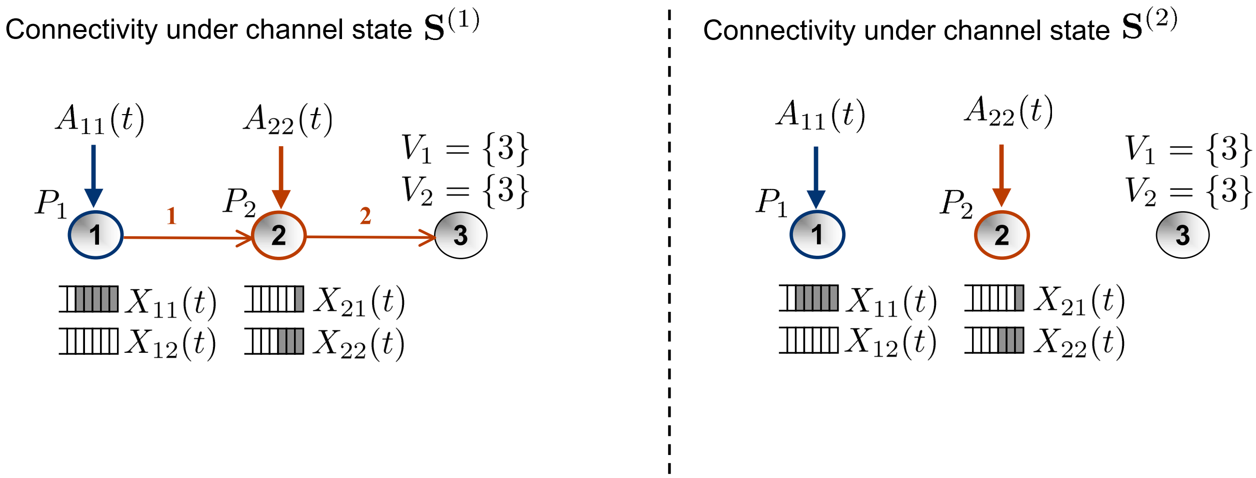

Figure 1: The possible connectivities of a 3 node network under 2 possible channel states, $\mathbf{S}^{(1)}$ and $\mathbf{S}^{(2)}$.

3 Stable throughput maximization under channel state uncertainty

In this section, we consider a policy that maximizes the stable throughput region of the network by making use of *only* an estimate of the true channel state. Our policy is built upon the "back-pressure" idea in [2]. As its name suggests, this policy attempts to maximize the stable throughput by spreading the traffic from the more congested to the less congested areas in the network. Accordingly, the policy we introduce activates the nodes of the network in a way that the weighted queue sizes for every commodity j will be kept as close to equal as possible, while at the same time the constraints imposed by the physical layer are being satisfied. Since the physical layer information available to our policy is limited due to the uncertainty in the channel state, our policy will try to maximize the stable throughput region of the network, within a broad class of policies, by having access to only an estimate of the channel conditions.

The routing component of the introduced policy resembles the so-called "hot-potato" routing approach in which nodes simply unload packets to neighboring nodes with smaller queue loads ([7]). In fact, in our model, the route any packet follows is determined by the link activation schedule that aims at maximizing the stable throughput region of the network. Hence, although an individual packet may follow a circuitous route towards one of its exit nodes, the overall characteristics of the routes are expected to be reasonable, albeit non-optimal. Since our objective is to achieve maximum stable throughput, this sort of routing is legitimate. No other routing will increase the stable throughput region, although it may decrease the delay that packets of the different commodities experience in the network.

The introduced policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ is parameterized by a weight assignment $\mathbf{w} = (w_j, j = 1, 2 \ldots, J)$, where w_j is a positive weight assigned to each commodity j. Packets corresponding to a commodity of a larger weight are given priority over the others, by being scheduled, and routed through the network more frequently. For every given weight vector $\mathbf{w}$, the *stationary policy* $\mathbf{E}(t) := \boldsymbol{\pi}_0^{\mathbf{w}}(t)$ is a certain J-tuple of mappings $\boldsymbol{\pi}_0^{\mathbf{w}j} : \mathcal{X} \times \mathcal{S} \to \{0,1\}^L$, each corresponding to a commodity j, and where $\mathbf{E}^j(t) := \boldsymbol{\pi}_0^{\mathbf{w}j}(t)$. So, we also have that $\boldsymbol{\pi}_0^{\mathbf{w}} = \sum_{j=1}^{J} \boldsymbol{\pi}_0^{\mathbf{w}j}$. For every time slot t, the quantity $\boldsymbol{\pi}_0^{\mathbf{w}j}(t)$ indicates the link activations for packets of commodity j, and $\boldsymbol{\pi}_0^{\mathbf{w}}(t)$ gives the overall link activations in the network.

We proceed by specifying the stable throughput maximizing policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ in detail. Given the current queue size matrix $\mathbf{x} \in \mathcal{X}$, weight assignment $\mathbf{w}$, and activation vector $\mathbf{c} \in \mathcal{T}_k$, for every estimated channel state $\mathbf{S}^{(k)}$, let

$$\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x}) := -w_j\, \tilde{\mathbf{Q}}_k^{\mathbf{c}}\, \mathbf{R}^{j\top}\mathbf{x}^j, \quad k \in \mathcal{K},\ j \in \mathcal{J},\ \mathbf{c} \in \mathcal{T}_k, \tag{9}$$

where

$$\tilde{\mathbf{Q}}_k^{\mathbf{c}} := \mathbb{E}\left[\mathbf{Q}^{\mathbf{c}}(t) \mid \hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}\right]. \tag{10}$$

From this definition it follows that the matrix $\tilde{\mathbf{Q}}_k^{\mathbf{c}}$ is an $L \times L$ diagonal matrix. Its ℓ^{th} diagonal entry $(\tilde{\mathbf{Q}}_k^{\mathbf{c}})_\ell$ gives the conditional probability that both the estimated, and true SINR values corresponding to ℓ lie at the same side of the inequality, provided that the overall link activations in the network are determined through the activation vector $\mathbf{c}$ and the estimated channel state is $\mathbf{S}^{(k)}$. For any given link ℓ, our model allows this probability to be dependent on the concurrent transmissions. For example, this probability is expected to be higher when link ℓ is the only link activated than when link ℓ is activated along with other concurrent nearby transmissions. Also, Assumption 1(a) guarantees that the matrix $\tilde{\mathbf{Q}}_k^{\mathbf{c}}$ for every $k \in \mathcal{K}$ and $\mathbf{c} \in \mathcal{T}_k$, defined in Equation (10), is time invariant.

Since the queue size x_{nj} is equal to zero whenever $n \in V_j$, it follows that the ℓ^{th} component $\left(\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x})\right)_\ell$ of $\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x})$ is the weighted queue size difference

$$\left(\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x})\right)_\ell = w_j(\tilde{\mathbf{Q}}_k^{\mathbf{c}})_\ell(x_{s(\ell)j} - x_{d(\ell)j}). \tag{11}$$

For every link $\ell \in \mathcal{L}$, let

$$(\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x}))_\ell := \max_{j\in\mathcal{J}} \left(\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x})\right)_\ell, \tag{12}$$

and

$$\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x}) := \left(\, (\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x}))_\ell,\ \ \ell = 1, \ldots, L\right). \tag{13}$$

Finally, define

$$(j_k^\star(\mathbf{x}))_\ell := \ \arg\ \max_{j\in\mathcal{J}} \left\{\left(\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}j}(\mathbf{x})\right)_\ell\right\}, \tag{14}$$

to be the maximizer in Equation (12), and also let

$$\mathbf{c}_k^\star(\mathbf{x}) := \ \arg\ \max_{\mathbf{c}\in\mathcal{T}_k} \left\{\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x})^\top \mathbf{c}\right\}. \tag{15}$$

Recall that the entries of every valid activation vector $\mathbf{c} \in \mathcal{T}_k$ are either 0 or 1, with 1 indicating activation of the corresponding link. Hence $\mathbf{D}_{k\mathbf{c}}^{\mathbf{w}}(\mathbf{x})^\top\mathbf{c}$ is a partial sum of weighted queue size differences over all the links, maximized over all the elements of the constraint set $\mathcal{T}_k$. If there exist more than one maximizer in Equation (15) ties are resolved arbitrarily provided that a link ℓ will be left inactive whenever the corresponding maximum weighted difference associated with that link is 0. Furthermore, if there exist more than one maximizer in Equation (14), ties are resolved arbitrarily. With the above in hand, and in the spirit of the optimal policy of [2], our proposed policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ is such that its ℓ^{th} entry $(\boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)}))_\ell$ is given by

$$(\boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)}))_\ell = \begin{cases} 1, & j = (j_k^\star(\mathbf{x}))_\ell, (\mathbf{c}_k^\star(\mathbf{x}))_\ell = 1, \text{ and } x_{s(\ell)j} > 0, \\ 0, & \text{otherwise}, \end{cases} \tag{16}$$

where $(\mathbf{c}_k^\star(\mathbf{x}))_\ell$ is the ℓ^{th} entry of the vector $\mathbf{c}_k^\star(\mathbf{x})$. When a link ℓ is activated, i.e., $\left(\boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x}, \mathbf{S}^{(k)})\right)_\ell = 1$, the policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ will select for transmission through that link a packet of one of the classes j that achieves the "max" in Equation (14). Note that from Equations (14), (15), and (16) the policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ also satisfies

$$\left(\mathbf{D}_k^{\mathbf{w}}(\mathbf{x})^\top - \mathbf{D}_k^{\mathbf{w}j}(\mathbf{x})^\top\right) \boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)}) = 0. \tag{17}$$

Note that the matrix $\tilde{\mathbf{Q}}_k^{\mathbf{c}}$ is all the information our policy has regarding the current channel conditions as shown through Equations (11), (14), and (15). The policy employs this information by giving a higher preference to those links for which both the true and the estimated SINRs lie at the same side of the inequality. Specifically, the policy will have the tendency to activate links that have a higher chance of successful transmission.

Clearly, for every commodity j we have that $\boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)}) \in \mathcal{T}_k$. Note further that for every link ℓ that is activated, a packet of a single commodity j is transmitted, and hence there will exist a single $\boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)})$ that satisfies $(\boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x}, \mathbf{S}^{(k)}))_\ell = 1$. From this observation it follows that $\boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x}, \mathbf{S}^{(k)}) \in \mathcal{T}_k$. The above, along with the fact that the policy leaves a link ℓ inactive whenever the maximum weighted difference over that link is 0, guarantees that $\boldsymbol{\pi}_0^{\mathbf{w}}$ satisfies the conditions for being an admissible policy. In Section 5, we will show the maximizing property of this policy under the following mild assumption.

Assumption 2 *Let $n' \in \mathcal{N}$ be a node such that for some $n \in \mathcal{N}$, $j \in \mathcal{J}$ with $\lambda_{nj} > 0$ there exists a sequence of links $\{\ell_i\}_{i=1}^m \in \mathcal{L}$, with $s(\ell_1) = n$, $d(\ell_i) = s(\ell_{i+1}),\ i = 1, \dots, m-1$, and $d(\ell_m) = n'$ such that $\forall\ i = 1, \dots, m$*

$$P[\mathrm{SNR}(\ell_i, t) \geq \theta_{d(\ell_i)},\ \text{and}\ \widehat{\mathrm{SNR}}(\ell_i, t) \geq \theta_{d(\ell_i)}] > 0, \tag{18}$$

where $\mathrm{SNR}(\ell, t)$ is obtained through Equation (1), and $\widehat{\mathrm{SNR}}(\ell, t)$ is defined similarly as

$$\widehat{\mathrm{SNR}}(\ell, t) := \frac{P_{s(\ell)}\ \hat{G}_{(s(\ell), d(\ell))}(t)}{\hat{N}_{o(d(\ell))}(t)}. \tag{19}$$

Then, there exists a node $n'' \in V_j$, and a sequence of links $\{\ell'_i\}_{i=1}^{m'} \in \mathcal{L}$ with $s(\ell'_1) = n'$, $d(\ell'_i) = s(\ell'_{i+1}),\ i = 1, \dots, m'-1$, and $d(\ell'_{m'}) \in V_j$ such that Equation (18) holds with $\{\ell_i\}_{i=1}^m$ replaced by $\{\ell'_i\}_{i=1}^{m'}$.

Assumption 2 is an assumption on sufficient connectivity of the network. Specifically it requires that for any node that may receive traffic of a particular commodity, there should also exist a downstream path of links to some exit node for that commodity under both the true, and estimated channel states.

3.1 System stability

The state of our system is driven by the process of the queue sizes. In this section, we show that under Assumption 1(a), and policy $\boldsymbol{\pi}_0^{\mathbf{w}}$, the queue size process defined by Equation (6), i.e., the state of our system, evolves according to a homogeneous Markov Chain. Our aim is to show that this Markov Chain is stable, and thus derive network stability for as large a set of arrival rates as possible.

Proposition 1 *Under Assumption 1(a), the process $\{\mathbf{X}(t)\}_{t=0}^\infty$ generated by Equation (6) with $\mathbf{E}^j(t) = \boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{X}(t-1), \hat{\mathbf{S}}(t))$ for every $j \in \mathcal{J}$ is a homogeneous Markov chain. Furthermore, $\mathbf{X}(t)$ is independent of $(\mathbf{S}(t'), \hat{\mathbf{S}}(t'), \mathbf{A}(t'))$ for all $t' > t \geq 0$.*

The result in the above proposition is a direct consequence of the fact that any process defined by a recurrence equation driven by white noise input, with initial value independent of the input, is Markov (See, e.g., [8, Theorem 2.1].).

A usual definition for stability of an irreducible Markov Chain is that the Markov Chain is positive recurrent. When the Markov Chain is not guaranteed to be irreducible, a more general definition for stability needs to be employed. Following [2], we adopt the following definition for stability of a (not necessarily irreducible) homogeneous Markov Chain.

Definition 1 *[2] Let $\{Y(t)\}_{t=0}^{\infty}$ be a Markov Chain with, possibly empty, transient class $\mathcal{Y}$, and recurrent communicating classes $\mathcal{Z}_i$, $i = 1, 2, \ldots$. Then $\{Y(t)\}_{t=0}^{\infty}$ is* stable *if*

$$P\left[\min\{\tau \geq 0 \,:\, Y(\tau) \notin \mathcal{Y}\} < \infty \,|\, Y(0) = y\right] = 1, \,\forall\, y \in \mathcal{Y},$$

and all states $z \in \cup_{i=1}^{\infty} \mathcal{Z}_i$ are positive recurrent.

We will say that the network is stable if the state process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ is stable, as defined in Definition 1.

4 A broad class of policies under channel state uncertainty

In this section, we introduce a general class of policies, $\mathcal{E}$. Our objective will be to compare the performance of the members in $\mathcal{E}$ to $\boldsymbol{\pi}_0^{\mathbf{w}}$ with respect to maximizing the stable throughput region of the network. This comparison will be performed in Section 5.

In order to specify the class $\mathcal{E}$ we define $n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(t; k, \mathbf{c}, \mathbf{Q})$ to be the number of time slots in the interval $[0, t]$ that the estimated channel state is in state $\mathbf{S}^{(k)}$, the activation vector $\mathbf{E}(t)$ takes value $\mathbf{c} \in \mathcal{T}_k$, and the matrix $\mathbf{Q}^{\mathbf{E}(t)}(t)$ is equal to $\mathbf{Q} \in \mathcal{Q}$. Here $\mathcal{Q}$ is the set of all $L \times L$ diagonal matrices whose diagonal is in the set $\{0, 1\}^L$. Also, we define $n_{\hat{\mathbf{S}}\mathbf{E}}(t; k, \mathbf{c})$ to be the number of time slots in the interval $[0, t]$ that the estimated channel state is $\mathbf{S}^{(k)}$, and the activation vector $\mathbf{E}(t)$ takes value $\mathbf{c} \in \mathcal{T}_k$. We define the set $\mathcal{E}$ as follows. We say that a policy $\{\mathbf{E}(t)\}_{t=1}^{\infty}$ belongs to $\mathcal{E}$ if for every $k,\ k' \in \mathcal{K}$, and time slot $t \in \{1, 2, \ldots\}$ the following is true

$$P[\mathbf{S}(t) = \mathbf{S}^{(k')} | \hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}, \mathbf{E}(t) = \mathbf{c}] = P[\mathbf{S}(t) = \mathbf{S}^{(k')} | \hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}], \tag{20}$$

and for every $k \in \mathcal{K}$, activation vector $\mathbf{c} \in \mathcal{T}_k$, and matrix $\mathbf{Q} \in \mathcal{Q}$ the following is true

$$\frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(t; k, \mathbf{c}, \mathbf{Q})}{n_{\hat{\mathbf{S}}\mathbf{E}}(t; k, \mathbf{c})} \to \frac{P[\mathbf{Q}^{\mathbf{c}}(t) = \mathbf{Q}, \hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}, \mathbf{E}(t) = \mathbf{c}]}{P[\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}, \mathbf{E}(t) = \mathbf{c}]}, \text{ almost surely as } t \to \infty, \tag{21}$$

when $n_{\hat{\mathbf{S}}\mathbf{E}}(t; k, \mathbf{c}) \neq 0$ as $t \to \infty$. Note that if $n_{\hat{\mathbf{S}}\mathbf{E}}(t; k, \mathbf{c}) = 0$ as $t \to \infty$, then the corresponding activation vector $\mathbf{c}$ is not used by the policy. In such a case, this activation vector can be eliminated from its constraint set. Recall that the constraint set is the set of all valid activation vectors with respect to the current channel state estimate.

Equation (20) is a natural condition which requires that at any time slot t, $\mathbf{E}(t)$, and the true channel state $\mathbf{S}(t)$ are conditionally independent given the estimate $\hat{\mathbf{S}}(t)$. In other words, all policies $\{\mathbf{E}(t)\}_{t=1}^{\infty}$ we may consider have no more information on the true channel state $\mathbf{S}(t)$ than the stationary policy $\boldsymbol{\pi}_0^{\mathbf{w}}$. Naturally, a policy that has additional information regarding the true channel state at time slot t can potentially exploit this knowledge, and for example avoid collisions by not scheduling the corresponding nodes. Also, Equation (21) is natural and it is in spirit similar to regular ergodicity conditions. From Equations (20) and (21) we may easily deduce that

$$\frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(t;k,\mathbf{c},\mathbf{Q})}{n_{\hat{\mathbf{S}}\mathbf{E}}(t;k,\mathbf{c})} \to P[\mathbf{Q}^{\mathbf{c}}(t)=\mathbf{Q}|\hat{\mathbf{S}}(t)=\mathbf{S}^{(k)}], \tag{22}$$

where from Assumption 1(a), $P[\mathbf{Q}^{\mathbf{c}}(t)=\mathbf{Q}|\hat{\mathbf{S}}(t)=\mathbf{S}^{(k)}]$ is independent of time t. Note also that the set $\mathcal{E}$ includes all the stationary policies since for stationary policies both Equations (20) and (21) are being satisfied. It may further include some *non-stationary*, as well as *anticipative* policies as long as they comply with the conditions for being in set $\mathcal{E}$. Finally, we remind the reader that anticipative network control policies are all those policies that have knowledge on the future values of the quantities that affect the evolution of the state process, driven by Equation (6).

4.1 The notion of intermittent boundedness

When the policy $\{\mathbf{E}(t)\}_{t=1}^{\infty}$ belongs to the class $\mathcal{E}$, the resulting queue size process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ generated by Equation (6) is not necessarily a Markov Chain. Therefore, the stability definition according to Definition 1 is not applicable anymore. Instead, we will make use of a weaker notion of stability, that of intermittent boundedness.

Definition 2 *The random process $\{Y(t)\}_{t=0}^{\infty}$ is almost surely intermittently bounded, if there exists a subset W of the sample space, with $P[W]=1$, such that for every $\omega \in W$ there exists a sequence $\{t_i\}_{i=1}^{\infty}$, and a finite $Y_{\max}$ for which $|Y(\omega,t_i)| < Y_{\max},\ \forall\ i=1,2,\ldots$, where $Y(\omega,t)$ denotes the sample path of the process $\{Y(t)\}_{t=0}^{\infty}$ corresponding to outcome ω. Further, $\{Y(t)\}_{t=0}^{\infty}$ is said to be intermittently bounded with positive probability, if there exists a subset W of the sample space, with $P[W]>0$, such that for every $\omega \in W$ there exists a sequence $\{t_i\}_{i=1}^{\infty}$, and a finite $Y_{\max}$ for which $|Y(\omega,t_i)| < Y_{\max},\ \forall\ i=1,2,\ldots$.*

5 Optimality of the proposed policy

In this section we will prove optimality of the policy introduced in Section 3 with respect to maximizing the stable throughput region of the network under uncertainty in the channel state. We will first define some sets of rates that are important in our proofs.

In a stable network, traffic at any given node $n \in \mathcal{N}$ cannot accumulate without bound. Hence, stability can be viewed through the concept of *flow conservation*, namely that for any commodity the sum of departing flows at any node, except for the exit nodes for this commodity, must be equal to the sum of arriving flows for this commodity. Therefore, we define the set of *feasible* arrival rates $\mathbf{\Lambda}$ as

$$\mathbf{\Lambda} = \left\{ \boldsymbol{\lambda} \in \mathbb{R}_+^{NJ} : \exists \mathbf{f}_k^j \in \mathbb{R}_+^L, \text{ such that } \quad \boldsymbol{\lambda}^j = -\mathbf{R}^j \sum_{k=1}^{K} p_{\hat{\mathbf{S}}}(k)\mathbf{f}_k^j, \text{ and } \sum_{j=1}^{J} \mathbf{f}_k^j \in \mathrm{co}(\tilde{\mathcal{Q}}_k) \right\}, \tag{23}$$

where $\tilde{\mathcal{Q}}_k = \{\tilde{\mathbf{Q}}_k^{\mathbf{c}}\ \mathbf{c},\ \mathbf{c} \in \mathcal{T}_k\}$, $\mathbf{f}_k^j$ are flow vectors of the j^{th} commodity under estimated channel state $\mathbf{S}^{(k)}$ and $\mathrm{co}(\cdot)$ denotes the convex hull of a set. Further, let the stable throughput region $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$ under $\boldsymbol{\pi}_0^{\mathbf{w}}$ be defined as

$$\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}} = \Big\{\text{The set of arrival rates } \boldsymbol{\lambda} \text{ such that for all processes } \Big\{\mathbf{S}(t), \hat{\mathbf{S}}(t), \mathbf{A}(t)\Big\}_{t=1}^{\infty}, \text{ satisfying Assumptions 1, and 2, where } \boldsymbol{\lambda} = \mathbb{E}[\mathbf{A}(t)], \text{ the network is stable under } \boldsymbol{\pi}_0^{\mathbf{w}}.\Big\}$$

We also denote by $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$ the following set of rates

$$\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}} = \Big\{\text{The set of rates } \boldsymbol{\lambda} \text{ such that for all processes } \Big\{\mathbf{S}(t), \hat{\mathbf{S}}(t), \mathbf{A}(t)\Big\}_{t=1}^{\infty}, \text{ satisfying Assumptions 1, and 2, where } \boldsymbol{\lambda} = \mathbb{E}[\mathbf{A}(t)], \text{ the process of the queue sizes is almost surely intermittently bounded under } \boldsymbol{\pi}_0^{\mathbf{w}}.\Big\}$$

Finally, to compare with $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, and $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, we introduce the set of arrival rates $\widetilde{\mathbf{C}}^p_{\mathcal{E}}$ as

$$\widetilde{\mathbf{C}}^p_{\mathcal{E}} = \Big\{\text{ The set of rates } \boldsymbol{\lambda} \text{ such that for some processes } \Big\{\mathbf{S}(t), \hat{\mathbf{S}}(t), \mathbf{A}(t)\Big\}_{t=1}^{\infty}, \text{ satisfying Assumption 1 where } \boldsymbol{\lambda} = \mathbb{E}[\mathbf{A}(t)], \text{ the process of the queue sizes is intermittently bounded with positive probability under some policy } \{\mathbf{E}(t)\}_{t=1}^{\infty} \in \mathcal{E}.\Big\}$$

Note that although the requirement for an arrival rate being in $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$ is that the process of the queue sizes is stable under $\boldsymbol{\pi}_0^{\mathbf{w}}$, the set of arrival rates $\widetilde{\mathbf{C}}^p_{\mathcal{E}}$ only requires that the queue size process satisfies the weak notion of intermittent boundedness with positive probability.

Let $\mathrm{ri}(\cdot)$ denote the relative interior of a set. The following theorem states our main result. The proof can be found in the Appendix.

Theorem 1 *The set $\boldsymbol{\Lambda}$ is a convex polytope. Furthermore, for* all *weight assignments $\mathbf{w} = (w_j, j = 1, 2, \dots, J)$, with $w_j > 0$ for every commodity $j \in \mathcal{J}$, the following relationships hold*

$$\mathrm{ri}(\boldsymbol{\Lambda}) \subseteq \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}} \subseteq \widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}} \subseteq \widetilde{\mathbf{C}}^p_{\mathcal{E}} \subseteq \boldsymbol{\Lambda}. \tag{24}$$

We proceed to give some more insight into the meaning of this theorem. From Equation (24) it follows that for all weight assignments $\mathbf{w}$, the rate regions $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, and $\widetilde{\mathbf{C}}^p_{\mathcal{E}}$ are all squeezed between the convex polytope $\boldsymbol{\Lambda}$, and its relative interior. Hence, the sets of rates $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, and $\widetilde{\mathbf{C}}^p_{\mathcal{E}}$ can differ by at most points on the relative boundary of $\boldsymbol{\Lambda}$, and therefore they are almost identical sets. In fact, this implies that for any rate, except perhaps for a few rates in the relative boundary of $\boldsymbol{\Lambda}$, that cannot be stabilized by our introduced stationary policy $\boldsymbol{\pi}_0^{\mathbf{w}}$, there exists no policy in the large class $\mathcal{E}$ that can even make the process of the queue sizes intermittently bounded with some positive probability.

As an example, by utilizing Equation (23), in Figure 2 we depict the stability region for the example network presented in Figure 1. Here, it is assumed that the channel estimation is such that the matrices $\tilde{\mathbf{Q}}_1^{[0,0]^T}$, $\tilde{\mathbf{Q}}_1^{[0,1]^T}$, $\tilde{\mathbf{Q}}_1^{[1,0]^T}$ are all equal to a diagonal matrix with diagonal entries given by 0.5, while the values of $\tilde{\mathbf{Q}}_2^{[0,0]^T}$ are immaterial due to the fact that there are no links available under channel state $\mathbf{S}^{(2)}$. Further, we assumed that the stationary probabilities of the estimated channel states $\hat{\mathbf{S}}^{(1)}$ and $\hat{\mathbf{S}}^{(2)}$ are both equal to 0.5, i.e., $p_{\hat{\mathbf{S}}}(1) = p_{\hat{\mathbf{S}}}(2) = 0.5$. As discussed above, the set of stable achievable rates may differ from $\boldsymbol{\Lambda}$ by only the relative interior of $\boldsymbol{\Lambda}$, which is the union of three line segments shown in Figure 2. Further, in Figure 2 we also provide the stability region of the network under perfect channel estimation, obtained by replacing $\tilde{\mathbf{Q}}_1^{[0,0]^T}$, $\tilde{\mathbf{Q}}_1^{[0,1]^T}$, and $\tilde{\mathbf{Q}}_1^{[1,0]^T}$ with the identity matrix in Equation (23). It is evident that the channel estimation errors have a significant impact on the stability region.

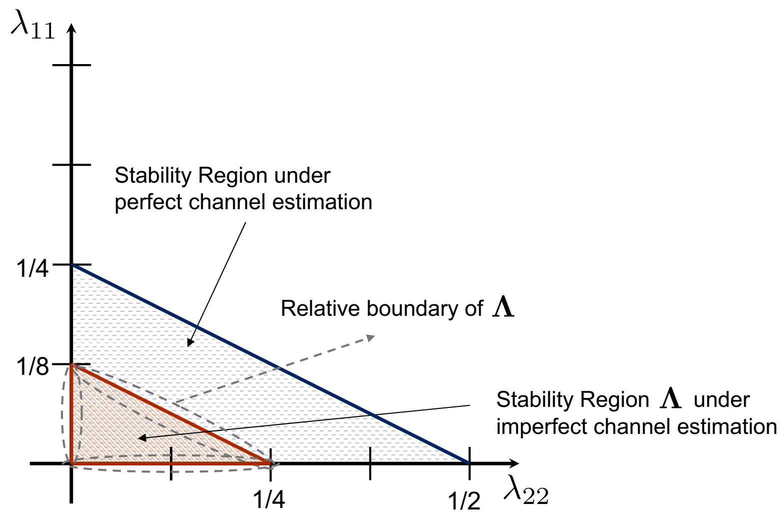

Figure 2: Stability region of the network presented in Figure 1 under perfect and imperfect channel estimation.

6 Conclusions

In this paper, we characterized the stability region of a network with multiple commodities in which the true channel state cannot be known by the network control policy. We introduced a joint scheduling, and routing policy that assigns weights of preference to each commodity and attempts to maximize the stable throughput region of a time-varying network, independently of the weight assignment, while having access only to a possibly inaccurate estimate of the true channel process. We characterized the common set of stable arrival rates that this policy supports, and proved its optimality with respect to maximizing the stable throughput region of the network within a broad class of stationary, non-stationary, and possibly anticipative policies, under some mild conditions. We finally verified that the network stability region can be considerably smaller than the corresponding stability region under perfect channel estimation.

Appendix
Proof of Theorem 1

In this section we are going to prove each individual inclusion relationship of Theorem 1. The third inclusion, that is $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}} \subseteq \widetilde{\mathbf{C}}^p_{\mathcal{E}}$, follows trivially from the definitions of the sets $\widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, and $\widetilde{\mathbf{C}}^p_{\mathcal{E}}$. Next, we prove the three remaining inclusions, namely that (i) $\mathrm{ri}(\mathbf{\Lambda}) \subseteq \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, (ii) $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}} \subseteq \widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, and (iii) $\widetilde{\mathbf{C}}^p_{\mathcal{E}} \subseteq \mathbf{\Lambda}$.

(i)
Proof of $\mathrm{ri}(\mathbf{\Lambda}) \subseteq \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$

Consider a rate $\boldsymbol{\lambda} \in \mathrm{ri}(\mathbf{\Lambda})$. We show that $\boldsymbol{\lambda} \in \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$, i.e., that this rate is stabilized by our proposed policy $\boldsymbol{\pi}_0^{\mathbf{w}}$. We make use of Extended Foster's Theorem ([2]), which provides a sufficient condition for stability.

Theorem 2 **(Extended Foster Theorem)** *Consider a Homogenous Markov Chain $\{Y(t)\}_{t=0}^{\infty}$ with state space $\mathcal{Y}$. Suppose there exists a real valued, function $V : \mathcal{Y} \to \mathbb{R}$, that is bounded from below, such that*

$$\mathbb{E}[V(Y(t+1)) \mid Y(t) = y] < \infty, \ \forall y \in \mathcal{Y}, \tag{25}$$

and such that for some $\epsilon > 0$, and some finite subset $\mathcal{Y}_0$ of $\mathcal{Y}$

$$\mathbb{E}[V(Y(t+1)) - V(Y(t)) \mid Y(t) = y] < -\epsilon, \quad \forall\, y \notin \mathcal{Y}_0 \tag{26}$$

Then, $\{Y(t)\}_{t=0}^{\infty}$ is stable in the sense of Definition 1.

We will show that the process of the queue sizes $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ satisfies the conditions of this theorem. For compactness of notation, we use t^+ to denote $t+1$. Given $\mathbf{w} > 0$, and $\mathbf{x} \in \mathcal{X}$, let $V(\mathbf{x}) := \sum_{j=1}^{J} w_j {\mathbf{x}^j}^\top \mathbf{x}^j$, be a candidate Lyapunov function. We show that, with $V(\cdot)$ thus defined under policy $\boldsymbol{\pi}_0^{\mathbf{w}}$, and given any process $\{\mathbf{A}(t)\}_{t=1}^{\infty}$, such that $\mathbb{E}[\mathbf{A}(t)] = \boldsymbol{\lambda}$, the process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ given by Equation (6) with $\mathbf{E}^j(t) = \boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{X}(t-1), \hat{\mathbf{S}}(t))$ for all $j \in \mathcal{J}$ satisfies the conditions of Theorem 2.

First, it is immediate that $\mathbb{E}[V(\mathbf{X}(t^+)) \mid \mathbf{X}(t) = \mathbf{x}] < \infty, \forall \mathbf{x} \in \mathcal{X}$. To see this, let $\mathbf{x} \in \mathcal{X}$, and let

$$\mathbf{G}^j(t) := \mathbf{x}^j + \mathbf{R}^j \mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x}, \hat{\mathbf{S}}(t))}(t) \boldsymbol{\pi}^j(\mathbf{x}, \hat{\mathbf{S}}(t)) + \mathbf{A}^j(t). \tag{27}$$

Note that for every t the matrix $\mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x}, \hat{\mathbf{S}}(t))}(t)$ is a function of $\mathbf{S}(t)$, and $\hat{\mathbf{S}}(t)$. Since by Proposition 1, the variables $\mathbf{S}(t^+)$, $\hat{\mathbf{S}}(t^+)$, $\mathbf{A}(t^+)$ are independent of $\mathbf{X}(t)$, Equation (6) yields

$$\mathbb{E}[V(\mathbf{X}(t^+)) \mid \mathbf{X}(t) = \mathbf{x}] = \sum_{j=1}^{J} w_j \mathbb{E}\left[\mathbf{G}^j(t^+)^\top \mathbf{G}^j(t^+)\right], \tag{28}$$

which is finite for all $\mathbf{x}$ since from Assumption 1 (b) the process $\{\mathbf{A}(t)\}_{t=1}^{\infty}$ is assumed to have finite second moments, and further the policy $\boldsymbol{\pi}^j(\mathbf{x}, \hat{\mathbf{S}}(t^+))$, as well as the process $\{\mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x}, \hat{\mathbf{S}}(t))}(t)\}_{t=1}^{\infty}$ take values in finite sets. This in fact holds independently of the choice of stationary policy $\boldsymbol{\pi}$, and of the arrival rate $\boldsymbol{\lambda}$. To complete the proof, we show that, when policy $\boldsymbol{\pi}_0^{\mathbf{w}}$ is used, there exists a finite set $\mathcal{X}_0$ such that Equation (26) holds. For compactness of notation, we define

$$\Delta V(\mathbf{x}) := \mathbb{E}\left[V(\mathbf{X}(t^+)) - V(\mathbf{X}(t)) \mid \mathbf{X}(t) = \mathbf{x}\right].$$

We first prove two lemmas that will be useful in proving the desired result.

Lemma 1 *Given any policy $\boldsymbol{\pi}$, arrival rate $\boldsymbol{\lambda}$, and queue size matrix $\mathbf{x} \in \mathcal{X}$, the Markov Chain $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ given by Equation* (6) *satisfies*

$$\Delta V(\mathbf{x}) \leq 2\left(\sum_{j=1}^{J} w_j {\mathbf{x}^j}^\top \boldsymbol{\lambda}^j - \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{j=1}^{J} \mathbf{D}_{k\boldsymbol{\pi}(\mathbf{x}, \mathbf{S}^{(k)})}^{\mathbf{w}j}(\mathbf{x})^\top\, \boldsymbol{\pi}^j(\mathbf{x}, \mathbf{S}^{(k)})\right) + B, \tag{29}$$

where B does not depend on $\mathbf{x}$.

Proof: From Equation (28), and the definition of our candidate Lyapunov function we have

$$\begin{aligned}\Delta V(\mathbf{x}) &= \sum_{j=1}^{J} w_j \mathbb{E}\left[\left(\mathbf{X}^j(t^+) - \mathbf{X}^j(t)\right)^\top \left(\mathbf{X}^j(t^+) + \mathbf{X}^j(t)\right) \mid \mathbf{X}(t) = \mathbf{x}\right] \\ &= \sum_{j=1}^{J} w_j \mathbb{E}\left[\left(\mathbf{X}^j(t^+) - \mathbf{X}^j(t)\right)^\top \left(2\mathbf{X}^j(t) + \mathbf{X}^j(t^+) - \mathbf{X}^j(t)\right) \mid \mathbf{X}(t) = \mathbf{x}\right] \\ &= 2\sum_{j=1}^{J} w_j \left({\mathbf{x}^j}^\top \mathbb{E}\left[\mathbf{X}^j(t^+) - \mathbf{X}^j(t) \mid \mathbf{X}(t) = \mathbf{x}\right]\right) \\ &+ \sum_{j=1}^{J} w_j \mathbb{E}\left[(\mathbf{X}^j(t^+) - \mathbf{X}^j(t))^\top (\mathbf{X}^j(t^+) - \mathbf{X}^j(t)) \mid \mathbf{X}(t) = \mathbf{x}\right].\end{aligned}$$

By using Equation (6) we obtain

$$\begin{aligned}\Delta V(\mathbf{x}) = 2\sum_{j=1}^{J} &\left(w_j \mathbf{x}^{j\top} \mathbb{E}\left[\mathbf{R}^j \mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t^+))}(t^+)\boldsymbol{\pi}^j(\mathbf{x},\hat{\mathbf{S}}(t^+)) + \mathbf{A}^j(t^+) \mid \mathbf{X}(t) = \mathbf{x}\right]\right) \\ &+ \sum_{j=1}^{J} w_j \mathbb{E}\left[\left(\mathbf{R}^j \mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t^+))}(t^+)\boldsymbol{\pi}^j(\mathbf{x},\hat{\mathbf{S}}(t^+)) + \mathbf{A}^j(t^+)\right)^\top \right. \\ &\quad \left. \left(\mathbf{R}^j \mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t^+))}(t^+)\boldsymbol{\pi}^j(\mathbf{x},\hat{\mathbf{S}}(t^+)) + \mathbf{A}^j(t^+)\right) |\mathbf{X}(t) = \mathbf{x}\right].\end{aligned}$$

Since $\{\mathbf{A}(t)\}_{t=1}^{\infty}$ is stationary, and has finite first and second moments, and the policy $\boldsymbol{\pi}^j(\mathbf{x},\hat{\mathbf{S}}(t^+))$, as well as the process $\{\mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t))}(t)\}_{t=1}^{\infty}$, where $\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t)) = \sum_{j=1}^{J} \boldsymbol{\pi}^j(\mathbf{x},\hat{\mathbf{S}}(t))$, take values in finite sets, the second term is finite and bounded for every $j \in \mathcal{J}$ by a quantity independent of the queue size matrix $\mathbf{x}$, and time slot t. Hence for every $\mathbf{x} \in \mathcal{X}$,

$$\Delta V(\mathbf{x}) \leq 2\sum_{j=1}^{J} \left(w_j \mathbf{x}^{j\top} \mathbb{E}\left[\mathbf{R}^j \mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\hat{\mathbf{S}}(t^+))}(t^+)\boldsymbol{\pi}^j(\mathbf{x},\hat{\mathbf{S}}(t^+)) + \mathbf{A}^j(t^+) \mid \mathbf{X}(t) = \mathbf{x}\right]\right) + B$$

for some B independent of $\mathbf{x}$, and t. Further by making use of Proposition 1, namely that $\mathbf{A}(t^+)$ is independent of $\mathbf{X}(t)$, and using conditional expectations it follows that

$$\begin{aligned}\Delta V(\mathbf{x}) &\leq 2\sum_{j=1}^{J} w_j \mathbf{x}^{j\top}\boldsymbol{\lambda}^j + B \\ &+ \quad 2\sum_{j=1}^{J} w_j \mathbf{x}^{j\top}\mathbf{R}^j \sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k)\mathbb{E}\left[\mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})}(t^+)|\mathbf{X}(t) = \mathbf{x}, \hat{\mathbf{S}}(t^+) = \mathbf{S}^{(k)}\right]\boldsymbol{\pi}^j(\mathbf{x},\mathbf{S}^{(k)}).\end{aligned}$$

Using Equation (10), and the fact that $\mathbf{Q}^{\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})}(t^+)$, and $\hat{\mathbf{S}}(t^+)$ are independent of $\mathbf{X}(t)$ we obtain

$$\Delta V(\mathbf{x}) \;\leq\; 2\sum_{j=1}^{J} w_j \mathbf{x}^{j\top}\boldsymbol{\lambda}^j - 2\sum_{j=1}^{J}\sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k)\left(-w_j \tilde{\mathbf{Q}}_k^{\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})}\mathbf{R}^{j\top}\mathbf{x}^j\right)^\top \boldsymbol{\pi}^j(\mathbf{x},\mathbf{S}^{(k)}) + B. \quad (30)$$

Finally, by using Equation (9), the above equation becomes

$$\Delta V(\mathbf{x}) \leq 2\left(\sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j - \sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{j=1}^{J} \mathbf{D}^{\mathbf{w}j}_{k\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})}(\mathbf{x})^\top \boldsymbol{\pi}^j(\mathbf{x},\mathbf{S}^{(k)})\right) + B,$$

which completes the proof.

■

When an arrival rate $\boldsymbol{\lambda}$ belongs to $\mathrm{ri}(\boldsymbol{\Lambda})$, a useful upper bound can be obtained on the first term in the parenthesis of Equation (29), by means of the following lemma.

Lemma 2 *Let $\boldsymbol{\lambda} \in \mathrm{ri}(\boldsymbol{\Lambda})$. Then there exist nonnegative scalars $\mu'^{\mathbf{c}}_k$, for all $\mathbf{c} \in \mathcal{T}_k$, $k \in \mathcal{K}$, with $\sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k < 1$, such that, for all $\mathbf{x} \in \mathcal{X}$,*

$$\sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j \leq \sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k \mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top \mathbf{c}. \tag{31}$$

Proof: Let rate $\boldsymbol{\lambda} \in \mathrm{ri}(\boldsymbol{\Lambda})$. Then $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}$, as $\mathrm{ri}(\boldsymbol{\Lambda}) \subseteq \boldsymbol{\Lambda}$. Hence, with reference to Equation (23) there exists a scalar $\delta > 1$, and non-negative flow vectors $\mathbf{f}^j_k \in \mathbb{R}^L_+$ such that

$$\boldsymbol{\lambda}^j = -\mathbf{R}^j \sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k) \mathbf{f}^j_k, \tag{32}$$

and where $\delta \sum_{j=1}^{J} \mathbf{f}^j_k \in \mathrm{co}(\tilde{\mathcal{Q}}_k)$ i.e., for some $\mu^{\mathbf{c}}_k \geq 0$ such that $\sum_{\mathbf{c}\in\mathcal{T}_k} \mu^{\mathbf{c}}_k = 1$ we have

$$\delta \sum_{j=1}^{J} \mathbf{f}^j_k = \sum_{\mathbf{c}\in\mathcal{T}_k} \mu^{\mathbf{c}}_k \tilde{\mathbf{Q}}^{\mathbf{c}}_k \mathbf{c}. \tag{33}$$

Note that from Equation (33) it follows that, for all $j \in \mathcal{J}$, and $k \in \mathcal{K}$, we have

$$(\mathbf{f}^j_k)_\ell = 0\,, \quad \forall \ell \notin \mathbf{S}^{(k)}. \tag{34}$$

Using Equation (32), and the fact each of the vectors $\mathbf{f}^j_k$ are non-negative component-wise we can write

$$\begin{aligned} \sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j &\leq \sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{j=1}^{J} \left(\max_{j\in\mathcal{J}} \left(-w_j \mathbf{x}^{j\top} \mathbf{R}^j\right) \mathbf{f}^j_k \right) \\ &= \sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k) \max_{j\in\mathcal{J}} \left(-w_j \mathbf{x}^{j\top} \mathbf{R}^j\right) \sum_{\mathbf{c}\in\mathcal{T}_k} \frac{\mu^{\mathbf{c}}_k}{\delta} \tilde{\mathbf{Q}}^{\mathbf{c}}_k \mathbf{c}, \end{aligned} \tag{35}$$

where Equation (35) follows by making use of Equation (33). Let $\mu'^{\mathbf{c}}_k := \frac{\mu^{\mathbf{c}}_k}{\delta}$. By definition, $\mu'^{\mathbf{c}}_k \geq 0$. Also, since $\sum_{\mathbf{c}\in\mathcal{T}_k} \mu^{\mathbf{c}}_k = 1$, and $\delta > 1$, it follows that $\sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k < 1$. Further, Equation (35) can be written as

$$\begin{aligned} \sum_{j=1}^{J} w_j \mathbf{x}^{j\top} \boldsymbol{\lambda}^j &\leq \sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k \max_{j\in\mathcal{J}} \left(\left(-w_j \tilde{\mathbf{Q}}^{\mathbf{c}}_k \mathbf{R}^{j\top} \mathbf{x}^j\right)^\top \right) \mathbf{c} \\ &= \sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k) \sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k \mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top \mathbf{c}, \end{aligned} \tag{36}$$

where Equation (36) follows by making use of Equations (9), (12), and (13). This completes the proof of Lemma 2.

■

We proceed to finalize the proof of the claim that $\mathrm{ri}(\boldsymbol{\Lambda}) \subseteq \mathbf{C}_{\boldsymbol{\pi}_0^{\mathbf{w}}}$. From Lemmas 1 and 2 we conclude that, given $\boldsymbol{\lambda} \in \mathrm{ri}(\boldsymbol{\Lambda})$, there exist nonnegative scalars $\mu'^{\mathbf{c}}_k$, for all $\mathbf{c} \in \mathcal{T}_k$, and $k \in \mathcal{K}$, with $\sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k < 1$, such that, for all $\mathbf{x} \in \mathcal{X}$, and all stationary policies $\boldsymbol{\pi}$,

$$\Delta V(\mathbf{x}) \leq 2\sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k)\left(\sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k \mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top\mathbf{c} - \sum_{j=1}^{J} \mathbf{D}^{\mathbf{w}j}_{k\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)})}(\mathbf{x})^\top \boldsymbol{\pi}^j(\mathbf{x},\mathbf{S}^{(k)})\right) + B. \tag{37}$$

So far $\boldsymbol{\pi}$ was an arbitrary stationary policy. We now focus on the policy $\boldsymbol{\pi}_0^{\mathbf{w}}$. In view of the fact that $\boldsymbol{\pi}(\mathbf{x},\mathbf{S}^{(k)}) = \sum_{j=1}^{J} \boldsymbol{\pi}^j(\mathbf{x},\mathbf{S}^{(k)}) \in \mathcal{T}_k$, from Equation (17), and of the definition of $\boldsymbol{\pi}_0^{\mathbf{w}}$, we obtain

$$\begin{aligned}
\sum_{j=1}^{J} \mathbf{D}^{\mathbf{w}j}_{k\boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x},\mathbf{S}^{(k)})}(\mathbf{x})^\top \boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x},\mathbf{S}^{(k)}) &= \mathbf{D}^{\mathbf{w}}_{k\boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x},\mathbf{S}^{(k)})}(\mathbf{x})^\top \sum_{j=1}^{J} \boldsymbol{\pi}_0^{\mathbf{w}j}(\mathbf{x},\mathbf{S}^{(k)}) \\
&= \mathbf{D}^{\mathbf{w}}_{k\boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x},\mathbf{S}^{(k)})}(\mathbf{x})^\top \boldsymbol{\pi}_0^{\mathbf{w}}(\mathbf{x},\mathbf{S}^{(k)}) \\
&= \max_{\mathbf{c}\in\mathcal{T}_k}\{\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top\mathbf{c}\}.
\end{aligned}$$

By substituting into Equation (37), we get

$$\begin{aligned}
\Delta V(\mathbf{x}) &\leq B + 2\sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k)\left(\sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k \mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top\mathbf{c} - \max_{\mathbf{c}\in\mathcal{T}_k}\{\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top\mathbf{c}\}\right) \\
&\leq B - 2\sum_{k\in\mathcal{K}} p_{\hat{\mathbf{S}}}(k) \max_{\mathbf{c}\in\mathcal{T}_k}\{\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top\mathbf{c}\}\left(1 - \sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k\right) \\
&\leq B - \rho \max_{k\in\mathcal{K}} \max_{\mathbf{c}\in\mathcal{T}_k}\{\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top\mathbf{c}\},
\end{aligned}$$

where from Equation (7), and the fact that $\sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k < 1$

$$\rho := 2\min_{k\in\mathcal{K}}\left(p_{\hat{\mathbf{S}}}(k)\left(1 - \sum_{\mathbf{c}\in\mathcal{T}_k} \mu'^{\mathbf{c}}_k\right)\right) > 0.$$

Now, let $\mathbf{x} \in \mathcal{X}$, with $\mathbf{x} \neq \mathbf{0}$, and suppose $\mathbf{X}(t) = \mathbf{x}$. Choose a node n, and a commodity j such that

$$x_{nj} > 0.$$

The Markov property of $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ implies that

$$\Delta V(\mathbf{x}) = \mathbb{E}\left[V(\mathbf{X}(t^+)) - V(\mathbf{X}(t)) \mid \mathbf{X}(t) = \mathbf{x}, \mathbf{X}(0) = \mathbf{0}\right].$$

Hence, without loss of generality, assume that the queue size process at time slot 0 satisfies $\mathbf{X}(0) = \mathbf{0}$. Since $X_{nj}(t) = x_{nj} > 0$, and $X_{nj}(0) = 0$, there must exist a sequence of links in $\mathcal{L}$ from some node n', with $\lambda_{n'j} > 0$, to node n that satisfy Assumption 2. Further, Assumption 2 then implies that there exist links

$\ell_i \in \mathcal{L}$, $i = 1, \ldots, z$, for some z, satisfying $0 < z < N$, such that $n = s(\ell_1)$, and nodes $n_1, \ldots, n_z$, such that $d(\ell_1) = n_1$, $s(\ell_{i+1}) = n_i$, $d(\ell_{i+1}) = n_{i+1}$, $i = 1, \ldots, z-1$, and $n_z \in V_j$. For notational simplicity, also let $n_0 := n$. Since $x_{n_z j} = 0$, whenever $n_z \in V_j$, we can write

$$x_{nj} = \sum_{i=1}^{z} (x_{n_{i-1}j} - x_{n_i j}) \leq z \max_{i,j} (x_{n_{i-1}j} - x_{n_i j}). \tag{38}$$

It follows that there exists some link $\ell_{i^\star}$ for which the above queue size difference through it, is maximized for some commodity $j^\star \in \mathcal{J}$. Let $n_{i^\star - 1} = s(\ell_{i^\star})$, and $n_{i^\star} = d(\ell_{i^\star})$. Then, from Equation (38) we have

$$x_{n_{i^\star-1}j^\star} - x_{n_{i^\star}j^\star} \geq \frac{x_{nj}}{z} \geq \frac{x_{nj}}{N}. \tag{39}$$

Recall that $\ell_i \in \mathcal{L}$ for all $i = 1, \ldots, z$. Further, let $k^\star$ be such that $\ell_{i^\star}$ satisfies Equation (1) under the estimated channel state $\hat{\mathbf{S}}(t) = \mathbf{S}^{(k^\star)}$. Let $\mathrm{e}_{\ell_{i^\star}} \in \mathbb{R}^L$ be a vector with its ${\ell_{i^\star}}^{\text{th}}$ component equal to 1, and with all other components equal to 0. Then, from the property of the constraint set it follows that $\mathrm{e}_{\ell_{i^\star}} \in \mathcal{T}_{k^\star}$. Also, it follows from Equations (12) and (13) that

$$\begin{aligned}
\max_{k \in \mathcal{K}} \max_{\mathbf{c} \in \mathcal{T}_k} \{\mathbf{D}^{\mathbf{w}}_{k\mathbf{c}}(\mathbf{x})^\top \mathbf{c}\} &\geq \max_{\mathbf{c} \in \mathcal{T}_{k^\star}} \{\mathbf{D}^{\mathbf{w}}_{k^\star \mathbf{c}}(\mathbf{x})^\top \mathbf{c}\} \\
&\geq \mathbf{D}^{\mathbf{w}}_{k^\star \mathrm{e}_{\ell_{i^\star}}}(\mathbf{x})^\top \mathrm{e}_{\ell_{i^\star}} = \left(\mathbf{D}^{\mathbf{w}}_{k^\star \mathrm{e}_{\ell_{i^\star}}}(\mathbf{x})\right)_{\ell_{i^\star}} \geq \left(\mathbf{D}^{\mathbf{w}j^\star}_{k^\star \mathrm{e}_{\ell_{i^\star}}}(\mathbf{x})\right)_{\ell_{i^\star}},
\end{aligned}$$

where $\left(\mathbf{D}^{\mathbf{w}j^\star}_{k^\star \mathrm{e}_{\ell_{i^\star}}}(\mathbf{x})\right)_{\ell_{i^\star}}$ is the ${\ell_{i^\star}}^{\text{th}}$ entry of the vector $\mathbf{D}^{\mathbf{w}j^\star}_{k^\star \mathrm{e}_{\ell_{i^\star}}}(\mathbf{x})$. In view of Equations (11), and (39), it follows that

$$\max_{k \in \mathcal{K}} \max_{\mathbf{c} \in \mathcal{T}_k} \{\mathbf{D}^{\mathbf{w}}_{k}(\mathbf{x})^\top \mathrm{e}_{\ell_{i^\star}}\} \geq w_{j^\star} (\tilde{\mathbf{Q}}^{\mathrm{e}_{\ell_{i^\star}}}_{k^\star})_{\ell_{i^\star}} (x_{n_{i^\star-1}j^\star} - x_{n_{i^\star}j^\star}) \geq \frac{w_{\min}\, \tilde{q}_{\min}\, x_{nj}}{N},$$

where $(\tilde{\mathbf{Q}}^{\mathrm{e}_{\ell_{i^\star}}}_{k^\star})_{\ell_{i^\star}}$ is the $\ell^{\text{th}}_{i^\star}$ diagonal entry of the matrix $\tilde{\mathbf{Q}}^{\mathrm{e}_{\ell_{i^\star}}}_{k^\star}$, while

$$w_{\min} := \min_{j \in \mathcal{J}} w_j > 0,$$

and, in view of Assumption 2,

$$\tilde{q}_{\min} > 0.$$

Note that the entries $w_{\min}$ and $\tilde{q}_{\min}$ do not depend on $\mathbf{x}$. Overall, we have

$$\Delta V(\mathbf{x}) \leq B - \frac{\rho\, w_{\min}\, \tilde{q}_{\min}\, x_{nj}}{N}$$

so that, given any $\epsilon > 0$,

$$\Delta V(\mathbf{x}) < -\epsilon, \quad \forall \mathbf{x} \notin \mathcal{X}_0 := \left\{ \mathbf{x} \in \mathcal{X} \;:\; x_{nj} \leq \frac{N(B+\epsilon)}{\rho\, w_{\min}\, \tilde{q}_{\min}} \right\}.$$

Since vectors in $\mathcal{X}$ have integer components, the set $\mathcal{X}_0$ is finite, and the proof is complete.

■

(ii)
Proof of $\mathbf{C}_{\boldsymbol{\pi}_0^{\mathrm{w}}} \subseteq \widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathrm{w}}}$

Consider an arrival rate $\boldsymbol{\lambda} \in \mathbf{C}_{\boldsymbol{\pi}_0^{\mathrm{w}}}$. In order to prove that $\boldsymbol{\lambda} \in \widetilde{\mathbf{C}}^1_{\boldsymbol{\pi}_0^{\mathrm{w}}}$, we need to show that stability according to Definition 1 implies intermittent boundedness with probability 1. We proceed by giving a theorem that gives a sufficient condition for intermittent boundedness of a Markov Chain.

Theorem 3 *Let $\{Y(t)\}_{t=0}^{\infty}$ be a Markov Chain, with $\mathcal{Y}$ the, possibly empty, set of its transient states. If $\{Y(t)\}_{t=0}^{\infty}$ almost surely exits the set of transient states in finite time, i.e. if*

$$P\left[\min\{\tau \geq 0 \;:\; Y(\tau) \notin \mathcal{Y}\} < \infty \,|\, Y(0) = y\right] = 1, \quad \forall y \in \mathcal{Y} \tag{40}$$

(which holds vacuously when $\mathcal{Y}$ is empty), then $\{Y(t)\}_{t=0}^{\infty}$ is intermittently bounded with probability 1.

Proof: Consider the Markov Chain $\{Y(t)\}_{t=0}^{\infty}$ that satisfies Equation (40). Then with probability 1, the Markov Chain $\{Y(t)\}_{t=0}^{\infty}$ will be eventually confined within a single recurrent class. It follows (e.g. from Theorem 7.3 in Chapter 2 of [8]) that, with probability 1, some (recurrent) state will be visited infinitely many times. Hence, there exists a set W, that is a subset of the sample space Ω, i.e. $W \subseteq \Omega$, with $P[W] = 1$ such that for every event $\omega \in W$, there exist a state y, and a sequence $\{t_i\}_{i=1}^{\infty}$, such that in the sample path ω the process satisfies

$$Y(\omega, t_i) = y, \; \forall i = 1, 2, \ldots.$$

Hence, by Definition 2 it follows that $\{Y(t)\}_{t=0}^{\infty}$ is intermittently bounded with probability 1.

■

A direct consequence of Theorem 3 is Corollary 1, that we state next.

Corollary 1 *Let $\{Y(t)\}_{t=0}^{\infty}$ be a stable Markov Chain. Then, $\{Y(t)_{t=0}^{\infty}$ is intermittently bounded with probability* 1.

From Corollary 1, the desired result follows.

(iii)
Proof of $\widetilde{\mathbf{C}}^p_{\mathcal{E}} \subseteq \boldsymbol{\Lambda}$

We need to show that if $\boldsymbol{\lambda} \in \widetilde{\mathbf{C}}^p_{\mathcal{E}}$ then $\boldsymbol{\lambda} \in \boldsymbol{\Lambda}$. We start by introducing the notation required for our proof. We define the random variable $n_{\hat{\mathbf{S}}}(t;k)$ to be the number of time slots τ in the interval $[0,t]$ during which $\hat{\mathbf{S}}(\tau)$ takes the value $\mathbf{S}^{(k)}$. Moreover, we denote by $\{n_{\hat{\mathbf{S}}}(\omega,t;k)\}_{t=1}^{\infty}$, $\{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega,t;k,\mathbf{c})\}_{t=1}^{\infty}$, $\{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega,t;k,\mathbf{c},\mathbf{Q})\}_{t=1}^{\infty}$ the sample path ω of the corresponding processes (Recall that the processes $\{n_{\hat{\mathbf{S}}\mathbf{E}}(t;k,\mathbf{c})\}_{t=1}^{\infty}$, $\{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(t;k,\mathbf{c},\mathbf{Q})\}_{t=1}^{\infty}$ are defined in Section 4.). Finally by $\{\mathbf{A}(\omega,t)\}_{t=1}^{\infty}$, $\{\hat{\mathbf{S}}(\omega,t)\}_{t=1}^{\infty}$, $\{\mathbf{E}(\omega,t)\}_{t=1}^{\infty}$, $\{\mathbf{Q}^{\mathbf{c}}(\omega,t)\}_{t=1}^{\infty}$, and $\{\mathbf{X}(\omega,t)\}_{t=1}^{\infty}$ we denote each of the sample paths ω of the respective processes.

Since $\boldsymbol{\lambda} \in \widetilde{\mathbf{C}}^p_{\mathcal{E}}$, there exists a policy $\{\mathbf{E}(t)\}_{t=1}^{\infty} \in \mathcal{E}$, and an i.i.d. process $\{\mathbf{S}(t), \hat{\mathbf{S}}(t), \mathbf{A}(t)\}_{t=1}^{\infty}$ such that $\mathbb{E}[\mathbf{A}(t)] = \boldsymbol{\lambda}$. In particular

$$P\left[\omega : \lim_{t\to\infty} \frac{1}{t}\sum_{\tau=1}^{t} \mathbf{A}^j(\omega,\tau) = \boldsymbol{\lambda}^j\right] = 1, \quad \forall j \in \mathcal{J}, \tag{41}$$

$$P\left[\omega : \lim_{t\to\infty} \frac{n_{\hat{\mathbf{S}}}(\omega,t;k)}{t} = p_{\hat{\mathbf{S}}}(k)\right] = 1, \quad \forall k \in \mathcal{K}. \tag{42}$$

Furthermore, from Equation (22) we have that

$$P\left[\omega : \lim_{t\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega,t;k,\mathbf{c},\mathbf{Q})}{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega,t;k,\mathbf{c})} = P[\mathbf{Q}^{\mathbf{c}}(t) = \mathbf{Q}|\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}]\right] = 1. \tag{43}$$

Also, since the process $\{\mathbf{X}(t)\}_{t=0}^{\infty}$ is intermittently bounded with positive probability it follows that

$$P\left[\omega : \mathbf{X}(\omega,\tau_i) < \mathbf{X}_{\max}, \text{ for some finite } \mathbf{X}_{\max}, \text{ and for some sequence } \{\tau_i\}_{i=1}^{\infty}\right] > 0. \tag{44}$$

Since the events in Equations (41), (42) and (43) have probability 1, and the event in Equation (44) has a positive probability, their intersection will have a positive probability. Hence, it follows that the 4 events have a non-empty common intersection. We first fix an outcome ω' that belongs to this common intersection, and once ω' is selected, we identify an $\mathbf{X}_{\max}$, and a sequence $\{t_i\}_{i=1}^{\infty}$ as specified by Equation (44). We have

$$\lim_{i\to\infty} \frac{1}{t_i}\sum_{\tau=1}^{t_i} \mathbf{A}^j(\omega',\tau) = \boldsymbol{\lambda}^j \tag{45}$$

$$\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}{t_i} = p_{\hat{\mathbf{S}}}(k) \tag{46}$$

$$\lim_{t\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega',t;k,\mathbf{c},\mathbf{Q})}{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega',t;k,\mathbf{c})} = P[\mathbf{Q}^{\mathbf{c}}(t) = \mathbf{Q}|\hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}] \tag{47}$$

$$\mathbf{X}(\omega',t_i) < \mathbf{X}_{\max}, \quad \text{for some } \mathbf{X}_{\max}, \ \forall i = 1,2,\ldots. \tag{48}$$

We now proceed to first sum both sides of Equation (6) from time slot 0 to t_i for some $i = 1,2,\ldots$, and cancel the identical terms. Then, by dividing both sides of the resulting equation by t_i we obtain

$$\frac{1}{t_i}\mathbf{X}^j(\omega',t_i) = \frac{1}{t_i}\mathbf{X}^j(\omega',0) + \frac{1}{t_i}\sum_{\tau=1}^{t_i}\mathbf{R}^j\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau) + \frac{1}{t_i}\sum_{\tau=1}^{t_i}\mathbf{A}^j(\omega',\tau). \tag{49}$$

From (48), we have

$$\lim_{i\to\infty}\frac{1}{t_i}\mathbf{X}^j(\omega',t_i) = 0, \tag{50}$$

and

$$\lim_{i\to\infty}\frac{1}{t_i}\mathbf{X}^j(\omega',0) = 0. \tag{51}$$

Taking the limit in Equation (49) as $i \to \infty$, and by using Equations (45), (50) and (51) we obtain

$$
\begin{aligned}
\boldsymbol{\lambda}^j &= -\lim_{i\to\infty}\left\{\frac{1}{t_i}\sum_{\tau=1}^{t_i}\mathbf{R}^j\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)\right\}\\
&= -\lim_{i\to\infty}\left\{\mathbf{R}^j\sum_{k\in\mathcal{K}}\frac{1}{t_i}\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)\right\}\\
&= -\lim_{i\to\infty}\left\{\mathbf{R}^j\sum_{k\in\tilde{\mathcal{K}}}\frac{1}{t_i}\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)\right\}, \qquad (52)
\end{aligned}
$$

where

$$\tilde{\mathcal{K}} = \left\{k\in\mathcal{K}\text{ s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}\text{ for some }\tau\in\{1,\ldots,\infty\}\right\}.$$

Thus, for $k\in\tilde{\mathcal{K}}$, and for i large enough it follows that $n_{\hat{\mathbf{S}}}(\omega',t_i;k)>0$. Without loss of generality (by redefining the sequence $\{t_i\}_{i=1}^{\infty}$ if necessary), assume that $n_{\hat{\mathbf{S}}}(\omega',t_i;k)>0$ for all $k\in\tilde{\mathcal{K}}$, and $i=1,2,\ldots$. Then, Equation (52) can be written as

$$\boldsymbol{\lambda}^j = -\lim_{i\to\infty}\left\{\mathbf{R}^j\sum_{k\in\tilde{\mathcal{K}}}\frac{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}{t_i}\frac{1}{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)\right\}. \qquad (53)$$

Note that $\mathbf{E}^j(\omega',\tau)\in\mathcal{T}_k$ whenever $\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}$. Also, for every time slot τ, the matrix $\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)$ is a diagonal matrix, whose diagonal entries take values in the set $\{0,1\}$. Therefore, it is also true that the product $\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\ \mathbf{E}^j(\omega',\tau)\in\mathcal{T}_k$. Also, since

$$\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}\frac{1}{n_{\hat{\mathbf{S}}}(\omega',t_i;k)} = \frac{1}{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}1 = \frac{1}{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}\,n_{\hat{\mathbf{S}}}(\omega',t_i;k)=1,$$

we have that for every $i\in\{1,\ldots\}$, $j\in\mathcal{J}$, and $k\in\tilde{\mathcal{K}}$,

$$\frac{1}{n_{\hat{\mathbf{S}}}(\omega',t_i;k)}\sum_{\substack{\tau\in\{1,\ldots,t_i\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)\in\mathrm{co}(\mathcal{T}_k).$$

Since $\tilde{\mathcal{K}}$ is a finite set, and since for every k, the set $\mathrm{co}(\mathcal{T}_k)$ is a compact set, there exists a subsequence $\{t_{i_\ell}\}_{\ell=1}^{\infty}$, and vectors $\mathbf{f}_k^j$ such that

$$\lim_{\ell\to\infty}\left\{\frac{1}{n_{\hat{\mathbf{S}}}(\omega',t_{i_\ell};k)}\sum_{\substack{\tau\in\{1,\ldots,t_{i_\ell}\}\\ \text{s.t. }\hat{\mathbf{S}}(\omega',\tau)=\mathbf{S}^{(k)}}}\mathbf{Q}^{\mathbf{E}(\omega',\tau)}(\omega',\tau)\mathbf{E}^j(\omega',\tau)\right\}=\mathbf{f}_k^j, \qquad (54)$$

for all $j\in\mathcal{J}, k\in\tilde{\mathcal{K}}$. Hence from Equations (46), (53) and (54) we obtain

$$\boldsymbol{\lambda}^j = -\mathbf{R}^j\sum_{k\in\tilde{\mathcal{K}}}p_{\hat{\mathbf{S}}}(k)\mathbf{f}_k^j, \quad \forall k\in\tilde{\mathcal{K}}. \qquad (55)$$

Finally, by letting the corresponding $L \times 1$ vector $\mathbf{f}_k^j$ be the $\mathbf{0}$-vector, whenever $k \in K \setminus \tilde{\mathcal{K}}$ we conclude that

$$\boldsymbol{\lambda}^j = -\mathbf{R}^j \sum_{k \in \mathcal{K}} p_{\hat{\mathbf{S}}}(k) \mathbf{f}_k^j, \quad \forall k \in \mathcal{K}. \tag{56}$$

Clearly, $\mathbf{f}_k^j \in \mathbb{R}_+^L$ for every $k \in \mathcal{K}$ and $j \in \mathcal{J}$. To complete the proof we need to show that $\sum_{j=1}^J \mathbf{f}_k^j \in \mathrm{co}(\tilde{\mathcal{Q}}_k)$ for every $k \in \mathcal{K}$. We consider two cases.

1. $k \in \mathcal{K} \setminus \tilde{\mathcal{K}}$: For every $k \in \mathcal{K} \setminus \tilde{\mathcal{K}}$, we have that

$$\sum_{j=1}^J \mathbf{f}_k^j \in \mathrm{co}(\tilde{\mathcal{Q}}_k), \tag{57}$$

   since $\mathbf{0} \in \mathcal{T}_k$ for every $k \in \mathcal{K}$.

2. $k \in \tilde{\mathcal{K}}$: From Equation (54), and since $\mathbf{E}(\omega', \tau) = \sum_{j=1}^J \mathbf{E}^j(\omega', \tau)$, for all $k \in \tilde{\mathcal{K}}$ we have

$$\begin{aligned}
\sum_{j=1}^J \mathbf{f}_k^j &= \lim_{i \to \infty} \left\{ \sum_{\substack{\tau \in \{1, \ldots, t_i\} \\ \text{s.t. } \hat{\mathbf{S}}(\omega', \tau) = \mathbf{S}^{(k)}}} \frac{1}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \mathbf{Q}^{\mathbf{E}(\omega', \tau)}(\omega', \tau) \mathbf{E}(\omega', \tau) \right\} \\
&= \lim_{i \to \infty} \left\{ \frac{1}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \sum_{\mathbf{c} \in \mathcal{T}_k} \sum_{\mathbf{Q} \in \mathcal{Q}} \sum_{\substack{\tau \in \{1, \ldots, t_i\} \\ \text{s.t. } \hat{\mathbf{S}}(\omega', \tau) = \mathbf{S}^{(k)}, \\ \mathbf{E}(\omega', \tau) = \mathbf{c}, \\ \mathbf{Q}^{\mathbf{c}}(\omega', \tau) = \mathbf{Q}}} \mathbf{Q}\ \mathbf{c} \right\} \\
&= \lim_{i \to \infty} \left\{ \sum_{\mathbf{c} \in \mathcal{T}_k} \sum_{\mathbf{Q} \in \mathcal{Q}} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \mathbf{Q}\ \mathbf{c} \right\} \\
&= \lim_{i \to \infty} \left\{ \sum_{\mathbf{c} \in \mathcal{T}_k} \sum_{\mathbf{Q} \in \mathcal{Q}} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} \frac{t_i}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \mathbf{Q}\ \mathbf{c} \right\}.
\end{aligned} \tag{58}$$

   Since each of the terms involved in the sum are non-negative, and since the outer limit exists, it follows that each of the product terms in the limit are bounded. Further, since $\frac{n_{\hat{\mathbf{S}}}(\omega', t_i; k)}{t_i}$ converges to a non-zero value, we may extract a converging subsequence such that $\lim_{i \to \infty} \left\{ \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} \right\}$ exists, and therefore

$$\sum_{j=1}^J \mathbf{f}_k^j = \sum_{\mathbf{c} \in \mathcal{T}_k} \sum_{\mathbf{Q} \in \mathcal{Q}} \lim_{i \to \infty} \left\{ \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} \right\} \frac{1}{p_{\hat{\mathbf{S}}}(k)}\ \mathbf{Q}\ \mathbf{c}. \tag{59}$$

   Note also that $\lim_{i \to \infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{t_i}$ exists and can be written as a finite sum of existing limits as

$$\lim_{i \to \infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{t_i} = \lim_{i \to \infty} \sum_{\mathbf{Q} \in \mathcal{Q}} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} = \sum_{\mathbf{Q} \in \mathcal{Q}} \lim_{i \to \infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i}, \tag{60}$$

where we made use of the fact that the limit $\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i}$ exists. As discussed in Section 4, for all $\mathbf{c} \in \mathcal{T}_k$, the quantity $n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c}) \neq 0$ as $t \to \infty$. Hence, we can write

$$\lim_{i\to\infty} \left\{ \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} \right\} = \lim_{i\to\infty} \left\{ \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} \frac{n_{\hat{\mathbf{S}}}(\omega', t_i; k)}{t_i} \right\}. \tag{61}$$

It follows from Equations (46) and (60) that

$$\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)} = \frac{\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{t_i}}{\lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}}(\omega', t_i; k)}{t_i}}$$

exists. Let this limit be equal to

$$\gamma_k^{\mathbf{c}} := \lim_{i\to\infty} \frac{n_{\hat{\mathbf{S}}\mathbf{E}}(\omega', t_i; k, \mathbf{c})}{n_{\hat{\mathbf{S}}}(\omega', t_i; k)}. \tag{62}$$

From Equations (46), (47) and (62) it follows that the individual limits in Equation (61) exist. Hence, it can be written as

$$\lim_{i\to\infty} \left\{ \frac{n_{\hat{\mathbf{S}}\mathbf{E}\mathbf{Q}}(\omega', t_i; k, \mathbf{c}, \mathbf{Q})}{t_i} \right\} = P[\mathbf{Q}^{\mathbf{c}}(t) = \mathbf{Q} | \hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}] \, \gamma_k^{\mathbf{c}} \, p_{\hat{\mathbf{S}}}(k). \tag{63}$$

By replacing Equation (63) in Equation (59) we get

$$\begin{aligned} \sum_{j=1}^{J} \mathbf{f}_k^j &= \sum_{\mathbf{c}\in\mathcal{T}_k} \sum_{\mathbf{Q}\in\mathcal{Q}} \gamma_k^{\mathbf{c}} \, P[\mathbf{Q}^{\mathbf{c}}(t) = \mathbf{Q} | \hat{\mathbf{S}}(t) = \mathbf{S}^{(k)}] \; \mathbf{Q} \; \mathbf{c} \\ &= \sum_{\mathbf{c}\in\mathcal{T}_k} \gamma_k^{\mathbf{c}} \tilde{\mathbf{Q}}_k^{\mathbf{c}} \mathbf{c}, \end{aligned} \tag{64}$$

where Equation (64) follows by employing Equation (10). Consequently, it follows that

$$\sum_{j=1}^{J} \mathbf{f}_k^j \in \mathrm{co}(\tilde{\mathcal{Q}}_k),$$

and the proof is complete.

■

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