

ABSTRACT

Title of Thesis: The Natural Response of Uniform and Nonuniform Plates
in Air and Partially Submerged in a Quiescent Water Body

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The free vibration of three aluminum plates (.4 m wide, 1.08 m long) oriented horizontally is studied experimentally under two fluid conditions, one with the plate surrounded by air, called the Air case, and the other with the bottom plate surface in contact with a large undisturbed pool of water, called the Half-Wet case. Measurements of the out-of-plane deflection of the upper surfaces of the plates are made using cinematic Digital Image Correlation (DIC) over the center portion of the surface and optical tracking of the center point. Three plate geometries and boundary conditions are studied: A uniform plate with 6.35 mm thickness pinned at the two opposite narrow ends (designated UP), a uniform plate with 4.83 mm thickness simply supported at one narrow end and clamped at the opposite end (UC), and a stepped plate with thickness varying from 12.7 mm to 6.35 mm along its 1.08 m length pinned at two opposite narrow ends (SP). The plate's free response is induced using an impact hammer at three locations along the center-line of the plate. Video frames of the motion of the upper surface of the plate are collected from stereoscopic cameras and processed using DaVis-Strainmaster and MATLAB to extract

full-field displacements as a function of time. Two-degree-of-freedom displacements of the plate center are also collected by tracking a target attached to the center of the plate's lower surface. Time and frequency response plots are presented for comparison between the Half-Wet and Air cases and analysis of their dynamics. It is found that the added mass of the water results in lower measured natural frequencies and modified mode shapes. In the Air case, these results are compared to mode shapes/frequencies produced in Creo Simulate and found to agree. Further experiments are discussed.

THE NATURAL RESPONSE OF UNIFORM AND NONUNIFORM PLATES
IN AIR AND PARTIALLY SUBMERGED IN A QUIESCENT WATER BODY

by

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List of Symbols and Abbreviations

f_n, ω_n	Natural Frequency
ζ	Damping Ratio
ω_d	Damped Natural Frequency $\omega\sqrt{1 - \zeta^2}$
L	Plate Length (Leading Edge to Trailing Edge)
W	Plate Width
B	Constant
α	Plate aspect ratio ($\frac{b}{a}$)
h	Plate Thickness
x, y, z	Coordinates along plate length, width, depth.
E	Plate Elastic Modulus
ν	Poisson's Ratio
ρ	Volume Density
g	Gravity Constant
ν	Poisson's Ratio
D	Plate Flexural Rigidity $D = \frac{Eh^3}{12(1-\nu^2)}$
$w(x, y)$	Plate Surface Displacement along z
ψ, η	Dummy variables
$q(t)$	Time function
$\Phi(x, y)$	Mode shape field
DIC	Digital Image Correlation

Chapter 1: Motivation and Literature Review

1.1 Motivation

The free vibration of structures immersed in a fluid are of interest to many fields of engineering. This includes, submarine, airplane, and ship design. Additionally, these structures are often exposed to asymmetric fluid loading. For example, slamming of hull structures is by nature not symmetric as one side of the hull is in contact with water and the other with the air of the interior of the ship. An important parameter in fluid structure interaction is added mass. Added mass also has a number of applications in engineering. In aerodynamic elastic plate flutter, for example, solutions exist when the added mass is known [1]. In hull design applications, study of added mass can guide more optimised designs of slamming vessels subject to high-amplitude and transient free vibrations. The present study is conducted as part of a larger study of a rectangular elastic plate slamming on a water surface. These studies are intended to model real-world hull slamming and to study the forces and displacements. This oblique slamming against a quiescent water surface is discussed by Wang in [2]. Displacement measurement in Wang's study was limited to a select number of probes along the centerline of the plate. This present study serves to develop a model for the dynamic behavior of the complicated motions inherent to the fluid-structure interaction of elastic plate slamming. It also serves to explore Digital Image Correlation as a superior measurement technique for full field displacements of the aforementioned plate

undergoing slamming.

1.2 Literature Review

The effect of added mass on vibrating structures immersed in a fluid has been a subject of interest in the literature. Yadykin [1] explored the influence of aspect ratio on single mode vibration of a cantilever plate, finding higher frequency modes and lower aspect ratio plates contributed to lower added mass coefficients.

Mylan [3] explored transforming the coupled fluid-structure equations into a variational form and used a Rayleigh-Ritz method to develop numerical solutions of mode shapes of a floating plate.

Nokob and Yeung [4] studied the added mass of thin plates of various shapes, and studied the effect of hole size on the added mass. They used a discrete Galarkin and potential flow model and found evidence that 3-D models of the plate undergoing heave, roll, and pitch were more accurate at modeling added mass than simple algebraic methods typically found in the literature.

Uğurlu in [5] used a finite element method and potential flow model to determine the mode shapes and natural frequencies of a vertical, fully-simply-supported rectangular plate supported by an elastic foundation in contact with a varying height of a fluid. They found lower frequencies as the fluid height increased and stronger coupling between dry modes as the fluid height was decreased.

Zhang in [6] used both single camera Digital Image Correlation and Particle Image Velocimetry (PIV) to study impulsive hydrodynamic loading on a plate with one side submerged. They found the out-of-plane deflection was dominated by the first dry mode of vibration. They found that

this simple deflection response was, however, not representative of the complex time-evolution of the pressure on the plate. They found a high pressure region that alternated in time between the center of the plate and the clamped plate edge.

An experimental study of plate vibration in the specific case of water in contact with one side was performed by Liao ([7] and [8]). They employed amplitude-fluctuation electronic speckle pattern interferometry for a plate subject to forced vibration while fixed to a water tank wall. In their study, the tank was filled to various water heights. In this study they were able to plot contours for a large range of mode shapes. They observed a decrease in natural frequencies as the immersed area of the plate increased. In their study, they used these modes as experimental evidence for a numerical model in which these half-wet mode shapes can be modeled as a series of dry mode shapes each multiplied by a contribution coefficient:

$$W_{\text{half-wet}}(x, y) = \sum C(x_i, y_j) W_{\text{dry}}(x_i, y_i) \quad (1.1)$$

Lucey and Carpenter [9] studied the instability of compliant rectangular panels with a spring foundation in potential flow. They found the divergence speed for these panels could be determined accurately only considering the fundamental bending mode of the panel. They developed a relationship between critical stream-wise mode number, panel foundation stiffness, and transverse mode number. Increased foundation stiffness was shown to increase the divergence of the streamwise mode number.

While the literature has presented results pertaining to modes for various fluid/plate systems, limited work has investigated the effects of boundary conditions and changes in plate geometry as done in the present study.

1.3 Relevant Theory

1.3.1 Notes on plate theory

Modeling the free vibration of rectangular plates has long been present in the literature. Warburton [10] published the first comprehensive study of solutions for plate mode shapes of various boundary conditions. In terms of plate vibration in general, [11]'s 1969 text remains the definitive resource on the topic. Additionally, there is some study using thin wing theory to examine aeroelastic properties in [12]. Some basic review of elastic plate dynamics is presented.

Warburton [10] used beam functions to develop equations for the expected mode shapes of a plate in free vibration. His discussion considers rectangular plates free at two edges and simply supported at the other two edges. Using the characteristic functions given by [10], we can obtain frequencies and mode shapes for a plate system approximating the plate system used in the dry experiment. We can write the partial differential equation for the out-of-plane plate displacement $w(x, y, t)$ as:

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{dw}{dy} + \frac{12\rho(1-\nu^2)}{Eh^2} \frac{\partial^2 w}{\partial t^2} = 0 \quad (1.2)$$

In general, this equation cannot be solved exactly, so approximate solutions are sought. One approach is to assume the solution for plate displacement $w(x, y, t)$ in free vibration to be the infinite series :

$$\sum \sum X_n(x) Y_m(y) \sin(\omega_n m t) \quad (1.3)$$

where X_n, Y_m are 1-D elastic beam solution functions that satisfy the edge boundary conditions. Indices n, m correspond to the number of nodal lines (lines of zero displacement) along the x, y

axis respectively, including the boundaries. Contour plots of these modes and their associated frequencies are shown in Figure 1.1. Also noted in the literature is the limitations on [10]'s accuracy. A more accurate estimation of the plate's frequency would be found by solving the minimization problem for unknown constants and unknown frequencies:

$$\frac{\partial V}{\partial A_i} - \frac{\omega^2 \rho h}{2} \frac{\partial}{\partial A_i} \int \int w^2 dx dy = 0 \quad (1.4)$$

Where A_i is an unknown constant, and V is the plate potential energy. These more accurate estimates of the mode shapes are found to each be a linear combination of the modes generated using beam functions as detailed by Young in [13].

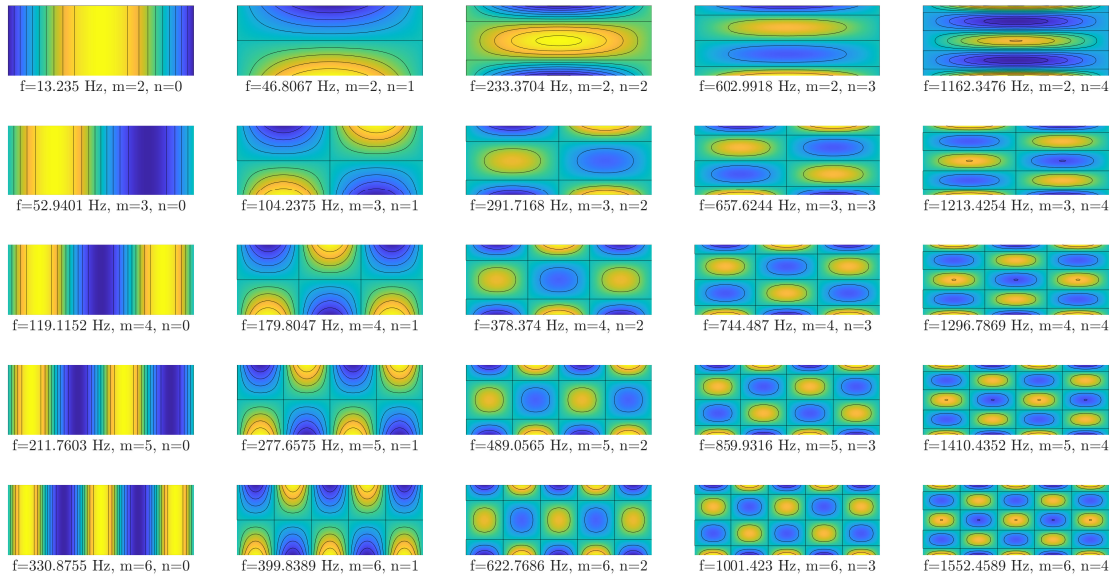


Figure 1.1: F-SS-F-SS modes for a thin rectangular plate using beam functions.

1.3.2 Notes on Digital Image Correlation

Digital image correlation (DIC) is an optical technique for calculating displacement fields by correlating image intensity fields between a reference image with the structure in a know state and an image with the structure in a deformed state. Since the 1980s many DIC algorithms have been developed; however, they all follow the same general principals as outlined in works such as those performed by Peters [14]. A brief summary of the procedure for calculating 2-D in plane displacements detailed in [14] is presented.

Suppose there is a relationship between the intensities of a subset of the undeformed and deformed of two (digital) images. Let the intensity of the undeformed subset be $I(x, y)$ and the deformed subset be $I'(x'_1, x'_2)$. We map this to a new coordinate system (an affine transformation) as:

$$x'_i = x_i + U_i + \frac{\partial U_i}{\partial x_j} \Delta x_j \quad i = 1, 2; j = 1, 2 \quad (1.5)$$

Where U_i is the translation of the center of an image subset to the center of the deformed subset, and Δx_i is the static distance between two subset centers. If we presume the intensity of a selected undeformed subset is correlated with an unknown subset, we can attempt to correlate with surrounding subsets. One such correlation method is the least squares correlation:

$$C_r(U_i, \frac{\partial U_i}{\partial x_j}) = \sum (I'(x'_1, x'_2) - I(x_1, x_2))^2 \quad (1.6)$$

Where C_r represents the correlation between the undeformed subset with the r -th candidate deformed subset. The values for $U_i, \frac{\partial U_i}{\partial x_j}$ which produce a minimum C_r can be considered the displacements of the valid deformed subset. For 3-D displacements, displacements found from

two (or more) images of the same object of interest must be triangulated with reference to some calibration target. In order to provide sufficient contrast between undeformed subsets of images of the object of interest, and avoid spurious correlation between unrelated subsets a randomized speckle pattern must be applied to the structure under study. Digital Image Correlation is considered to measure the displacements of this speckle pattern. There are benefits to Digital Image Correlation as opposed to other deflection measurement techniques. As this is a nearly purely optical technique, there is limited interference with the physics of the structure under study. Digital Image Correlation is also able to gather a large area of displacements at a large number of grid points, limited only by the camera resolution and frame rate. Historically, DIC has been limited by technological constraint in sampling rate (In [14], Peters measured 1 Hz vibrations of a rod using a "high-speed" camera operating at 100 FPS) and computation time for calculating the intensity correlations. Today, however, DIC is well suited to take advantage of modern high-speed camera hardware. Compute time is still a limitation given the large quantity of correlation calculations that must be resolved.

Chapter 2: Experimental Details

2.1 Experimental Setup

This section includes descriptions of the water tank, the carriage, the three aluminum plates, the plate mounting configurations, the DIC measurement system and the optical tracking measurement system as well as the experimental procedures and conditions in the free vibration experiments.

2.1.1 The Towing Tank and Carriage

The experiments are conducted in a 13.4 m long, 2.45 m wide, and 1.33 m tall towing tank (Figure 2.1) that consists of a steel frame that supports a system of 35 mm-thick clear acrylic panels that form the inner sidewalls and bottom surfaces of the water tank. Two types of experiments are conducted, dry and half-wet. During "half-wet" experiments, the tank is filled to a depth of 1 m, with the water surface submerging the bottom half of the plate thickness. During "dry" experiments, the tank is empty. A two-axis (horizontal and vertical motion) carriage system is mounted on a separated steel frame located along one of the long side walls of the tank. This carriage system is typically used in plate-slamming experiments, for example by Wang in [15], and is depicted in Figures 2.1 and 2.2 of the present document. The carriage support structure and

the tank are separate structures that are mounted to the laboratory floor but not to each other. In the present experiments, the carriage remains stationary and the plate to be tested is attached to the bottom of the carriage using one of two possible mounting configurations at the $x = -L/2$ and $x = L/2$ denoted as the “trailing edge” and “leading edge”, respectively, where L is plate length (1080 mm for all plates). The plate edges are free at $y = -W/2$ and $y = W/2$, where W is the width (406 mm) of all plates. The carriage includes a six-component force-moment dynamometer consisting of four three-component force sensors mounted in a rectangular aluminum frame structure. The plate mounting systems described below are attached to the dynamometer via thick aluminum plates that are bolted to the bottoms of the four force sensors, see Figure 2.2. In order to make the present free vibration measurements appropriate for interpretation of data from plate slamming experiments, which data is either published in [15] or in papers now in preparation, the plates and plate mounting systems used in the present experiments are identical those in the plate slamming experiments. The reader is cautioned that since the carriage is designed with a compromise between lightness and rigidity, its residual flexibility may modify the plate vibration modes compared to those that might be found with the plate mounted on a highly rigid structure. Because the present results are intended to inform the plate slamming experiments, this possibility of a coupled vibration effect is an intended part of the experimental design. A photograph of the entire experimental setup can be found in Figure 2.3.

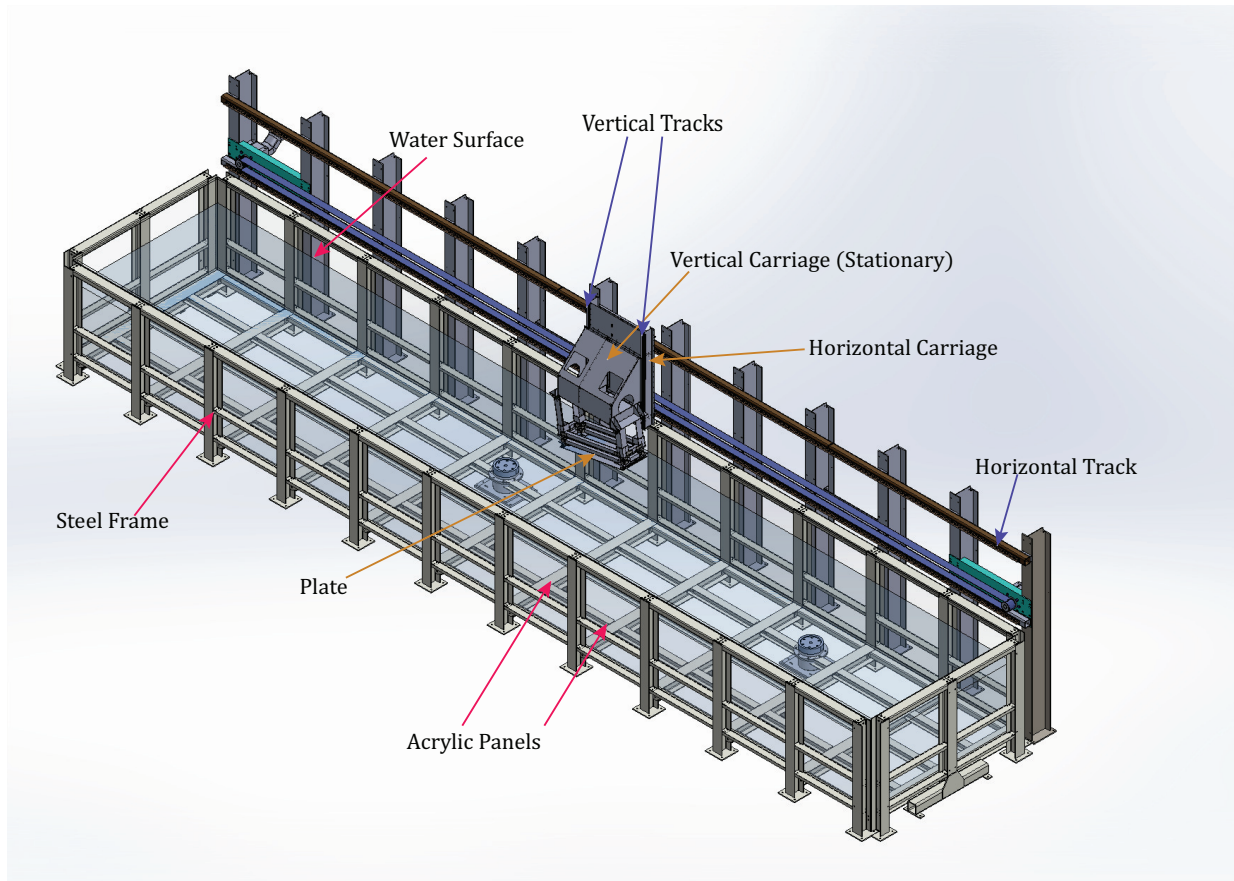


Figure 2.1: A schematic of the carriage and towing tank (Figure created by Samuel Lee). The tank is 13.4 m long, 2.45 m wide, and 1.33 m tall.

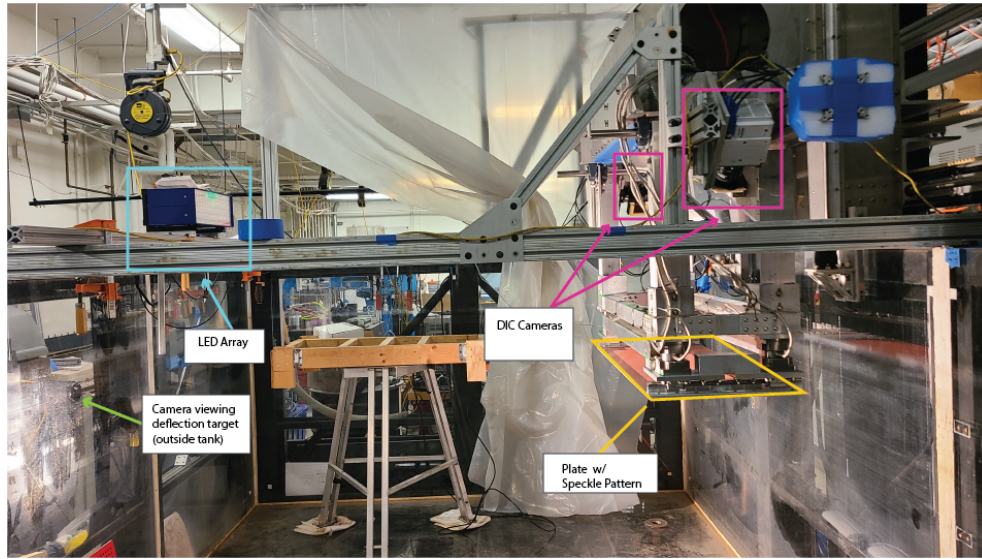


Figure 2.3: Photograph of the experimental setup. The camera for this image is located inside the towing tank and is pointing horizontally toward the near 2.45-m wide end of the tank. The full tank width is shown with the carriage and plate on the right.

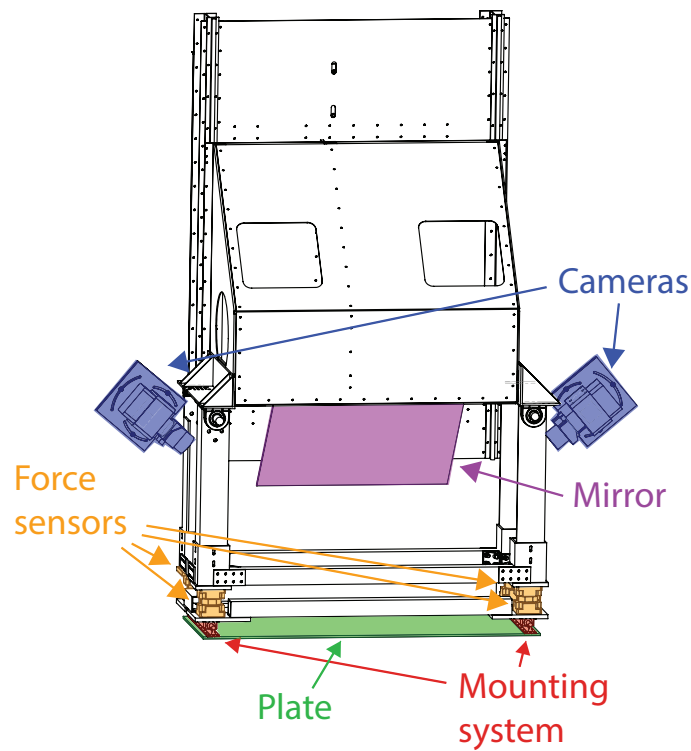


Figure 2.2: A schematic diagram of the carriage, aluminum plate mounted with pinned supports, force sensors and stereo DIC system is shown. See Figure 2.4 for mounting system details.

2.1.2 Plates and Mounting Systems

Three rectangular (L by W) aluminum plates/mounting system pairs are used in the present experiments: 1.) A plate with constant thickness h mounted with pinned supports at the leading and trailing edges (Henceforth, referred to as the Uniform-Pinned or UP plate.); 2.) A plate with constant thickness mounted with a clamped support at the leading edge and a simple support at the trailing edge (Henceforth, referred to as the Uniform-Clamped or UC plate) ; and 3.) A plate with thickness varying in 10 steps from the leading to the trailing edges mounted with pinned supports at the leading and trailing edges (Henceforth, referred to as the Stepped-Pinned or SP plate.). The Uniform-Pinned plate has a nominal thickness of $h = 6.35$ mm and is made of 6061 Aluminum (density $\rho = 2700 \text{ kgm}^{-3}$, elastic modulus $E = 68.9 \text{ GPa}$, and Poisson's ratio $\nu = 0.33$). The Uniform-Clamped plate has a nominal thickness of $h = 4.76$ mm and is also made of 6061 Aluminum. The Stepped-Pinned plate has a nominal thickness of $h = 12.7$ mm at its leading edge and decreases in 10 equal steps until reaching a final nominal thickness of $h = 6.35$ mm at the trailing edge. The Stepped-Pinned plate is made of 5083 Aluminum ($\rho = 2660 \text{ kgm}^{-3}$, $E = 71.0 \text{ GPa}$, and $\nu = 0.33$).

As mentioned above, two distinct mounting systems are used in this experiment. Each mounting system is secured to the plate by nut/bolt fasteners through two rows of holes drilled every 50.8 mm across the width of the plate. In the pinned support cases, the plates are connected to the lower frame of the dynamometer using a system consisting of two bearings and a linear shaft (SRB2-08SS-016, PBC Linear) at the leading and trailing ends of the dynamometer frame; see Figure 2.4(a). In this configuration, one bearing is centered on and bolted to the bottom of each of the four load cell units and a shaft is bolted to the plate via a T-mount structure at each

end of the plate. This mounting system allows the leading and trailing edges of the plate to have free rotation about transverse horizontal axes located 32 mm inward of the plate's width- W edges and 28.6 mm above the plate's upper surface, see Figure 2.4(a). In the clamped simply-supported mounting system, see Figure 2.4(b), the leading edge of the plate is bolted to a stiff aluminum plate that is in turn bolted to the two force sensors. At the trailing edge, one of the above-described shafts is bolted to the plate and the shaft is positioned to slide along spacer blocks that are bolted to the two force sensors. The shaft is held coincident to the spacer blocks by a retaining spring. This system allows free rotation and translation at the shaft-spacer block contact line.

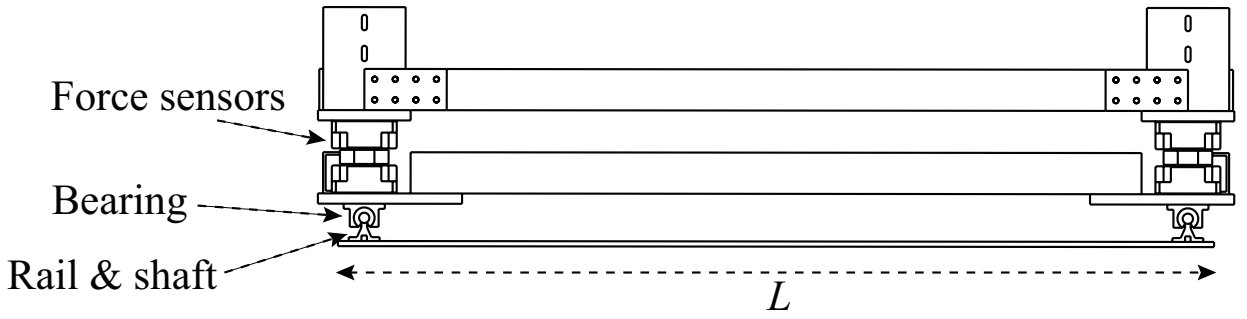
Following the procedures outlined in [15], the plate thicknesses were measured with an ultrasonic thickness gauge (GE CL5 Krautkramer, with Alpha2-DFR probe) at locations on a 7-by-6 grid on the uniform thickness plates and a 5-by-2 grid on each thickness of the stepped plate. The measurement resolution of the ultrasonic meter is 0.001 mm. The measured average thicknesses of the uniform plates were $h = 6.431 \pm 0.008$ mm and 4.806 ± 0.009 mm and the thicknesses of the ten steps of the third plate are given in table 2.1.

Table 2.1: Measured average thicknesses (h) and standard deviations (σ) of the 10 steps of the stepped plate.

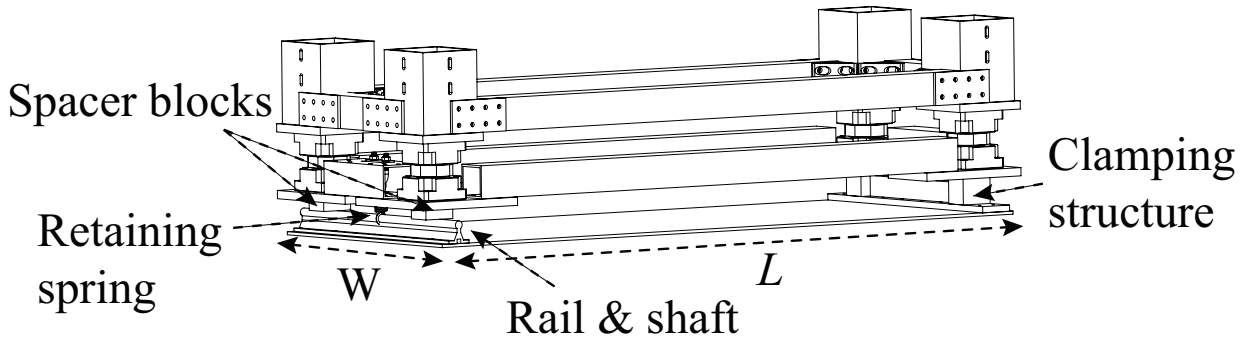
Step	h (mm)	σ (mm)
1	12.723	0.006
2	11.987	0.013
3	11.274	0.008
4	10.577	0.005
5	9.814	0.019
6	9.044	0.025
7	8.330	0.014
8	7.664	0.012
9	7.011	0.012
10	6.292	0.012

2.1.3 Plate Deflection Measurements

The temporal histories of the out-of-plane deflection of the plates are measured using cinematic Digital Image Correlation (DIC) and a target tracking system. For the DIC measurements, the top side of each plate is detailed with a speckle pattern consisting of fluorescent paint applied using a custom made paint roller. Stereo movies of the plate motion are taken with two Phantom VEO4K 990L cameras (8-mega pixel sensors) with each camera mounted on a separate beam that crosses the width of the tank and is clamped to the tank's steel frame, one camera near the leading edge of the plate and one near the trailing edge. This mounting system isolates the cameras from the plate motion. The cameras are placed approximately 800 mm from the plate center and rotated so that the optical axis of the 24 mm lens on each camera is 40 degrees below horizontal and intersects with the center of the plate. A Scheimpflug adapter is used to mount the lens, and is adjusted so that the object focal plane is coincident with the static surface of the plate. Each lens is outfitted with with a 530 nm high-pass filter to block light wavelengths emitted by the LED array described below while passing light emitted by the fluorescent paint. A sample image of the speckle pattern as seen by the left DIC camera is given in Figure 2.5a. The cameras capture gray-scale images at a resolution of 4096 by 1728 pixels simultaneously at 1,200 frames/sec, with a maximum capture time of 5.5 sec. Due to line-of-sight blockage by the carriage components, the DIC system records the motion only in a rectangular region of the plate as shown in Figure 2.6. The length (L_D) and width (B_D) of the DIC measurement window on the plate surface are approximately 648 mm and 240 mm, respectively.

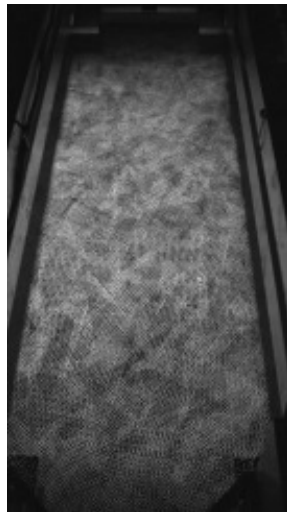


(a) Pinned plate mounting system



(b) Clamped plate mounting system

Figure 2.4: Schematic diagrams of the pinned and clamped plate mounting systems are shown in (a) and (b), respectively.



(a)

Figure 2.5: In (a), an example image of the speckle pattern applied to the plate is shown. In (b), an example image of the speckle pattern applied to the plate as seen by the left DIC camera is shown.

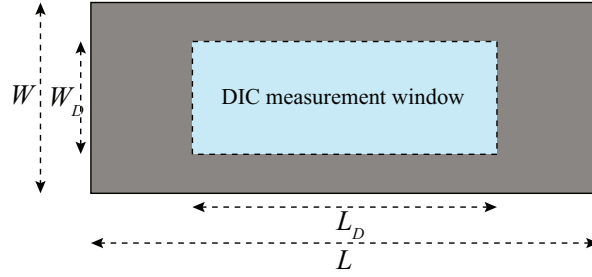


Figure 2.6: The DIC measurement window is shown relative to the plate dimensions where $L_D = 648$ mm, $W_D = 240$ mm, $L = 1079.5$ mm, and $W = 406.4$ mm.

Lighting of the plate's upper surface in the DIC measurement is provided by a pulsed LED Array Module (LaVision LED-300) that is setup to project the light horizontally and is mounted to the tank's steel frame, off to the side and above the level of the plate, see Figure 2.3. An acrylic mirror mounted to the carriage redirects light from this array downward onto the plate surface. The camera frame capture and the pulsing of the LED array are synchronized and controlled by a LaVision PTU-X.

For the target-tracking deflection measurements, a small target (approximately 2 cm by 2 cm) consisting of a white surface covered with black dots was affixed to a 3D-printed L-shaped bracket. This bracket was glued to the center of the plate's bottom surface such that the normal to the plane of the target's dotted white surface was horizontal and directed perpendicular to the long axis of the plate. The position of the target was recorded with a third camera (Phantom V640 (4-mega pixel sensor)) that is mounted outside of the tank to its steel frame. The camera is synchronized with the two cameras used for the DIC measurements. Light is provided by a photo flood light. A sample image from the target tracking camera is given in Figure 2.7

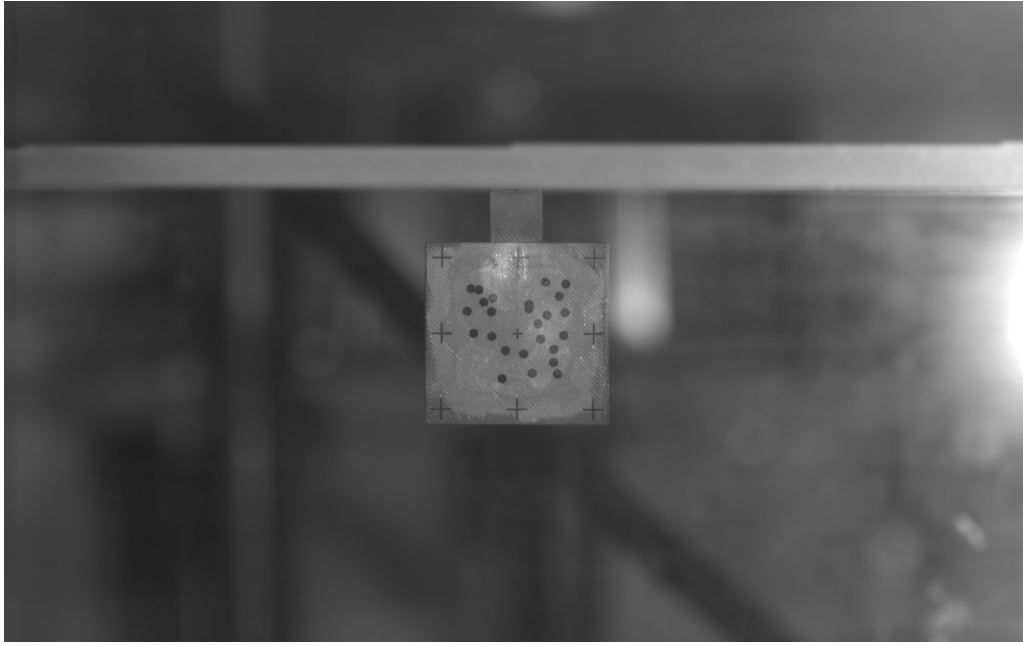


Figure 2.7: A sample image of the deflection target as seen by the camera used for the target-tracking deflection measurement.

2.1.4 Control System and the Impact Hammer

The control and recording system for the free vibration experiments is depicted in Figure 2.8. A PCB Piezoelectronics Model 086C01 impact hammer is used to excite the plate and provide a low-voltage signal proportional to the force during impact. This signal is sent to an Arduino Uno prototyping platform and an N-Scope Oscilloscope recording system. The Arduino sends a 5-volt digital start signal to a BNC Model 577 delay box. The delay box then sends a trigger signal to a LaVision PTU-X Timing Unit that controls the three cameras and the LED.

2.1.5 Procedures

In preparation for the hammer impact, the DIC and target tracking system is turned on with the LED emitting light pulses and images moving continuously through the camera memories at a

frame rate of 1,200 images per second. When the trigger signal from the impact hammer/Arduino reaches the cameras, they save the 100 images before the trigger and the 6,500 images after the trigger. Of these, 2600 images are retained for processing.

Each plate is tested in two conditions, one with the plate entirely in air (designated as the Air cases) and one with the lower surface of the plate in contact with the water and the upper surface in contact with air (designated as the Half-Wet cases). The plate is struck on the bottom side in the Air cases and the top side for the Half-Wet cases. Each plate is struck at three distinct points along the its longitudinal center-line ($y = 0$): halfway between the trailing edge and the midpoint, the midpoint and halfway between the midpoint and the leading edge. However, for the midpoint-Air condition the plate is struck at approximately 50 mm to the port-side from the center-line ($y = -50$ mm) since the deflection target occupies the center of underside of the plate. Measurements for each strike location are performed twice for both the Air and Half-Wet conditions for a total of 36 distinct trials.

2.2 Data Processing

The DIC image sequences are processed with DaVis DIC StrainMaster software to produce 3-DoF displacement fields of the plate's upper surface with respect to the stationary plate reference obtained using the images of the plate just before impact. Maps of the plate out of plane deflection on a grid of 104-by-39 data points are produced from each frame pair.

In the target tracking method, the dot pattern is assumed to move in two degrees of freedom in the plane of the target, which is perpendicular to the optical axis of the lens. In-house MATLAB scripts are used to process the target tracking images to produce out-of-plane plate deflections.

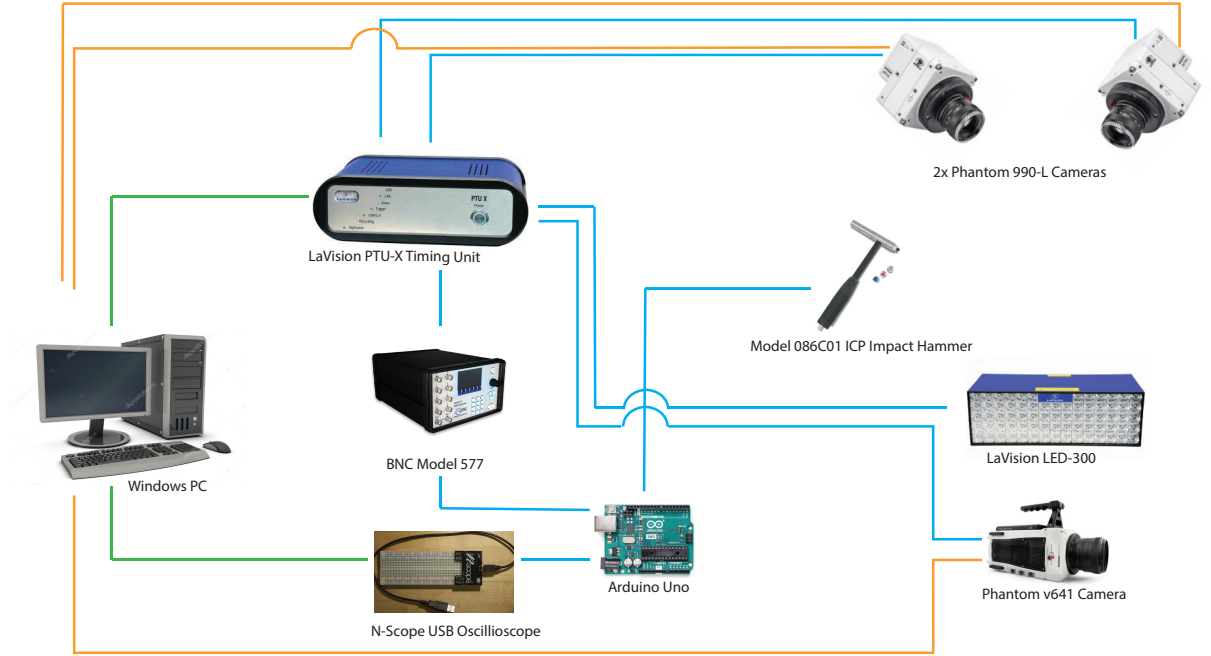


Figure 2.8: Cabling diagram - Orange = Ethernet , Blue = BNC, Green = USB

These scripts produce intensity correlation maps in x and y image coordinates of the reference image taken with the plate stationary relative to the image taken at a given time after hammer impact. Sub-pixel accuracy methods are also employed to obtain the translation that produces the best correlation and therefore the most accurate target deflection.

The out of plane deflection measurement measured by the target tracking method, which is used routinely in the Hydrodynamics Laboratory, was used to verify the accuracy of the DIC system, which was used for the first time in the present experiments. To this end, the deflection versus time record and the associated power spectrum at the plate center as measured by the target tracking method is compared to similar data measured by the DIC method. Results for a single trial in the Dry configuration are shown for the Uniform-Clamped, Uniform-Pinned and Stepped-Pinned plates cases in Figure 2.9(a), (b) and (c), respectively. As can be seen from the figure, the

records of deflection versus time and the frequencies and peak heights of the dominant vibration modes as measured by the two methods are nearly identical for all three plates. Since the target tracking methods is simpler than the DIC method and well used in the associated slamming tests, this agreement is taken as an indication that the DIC system produces accurate results.

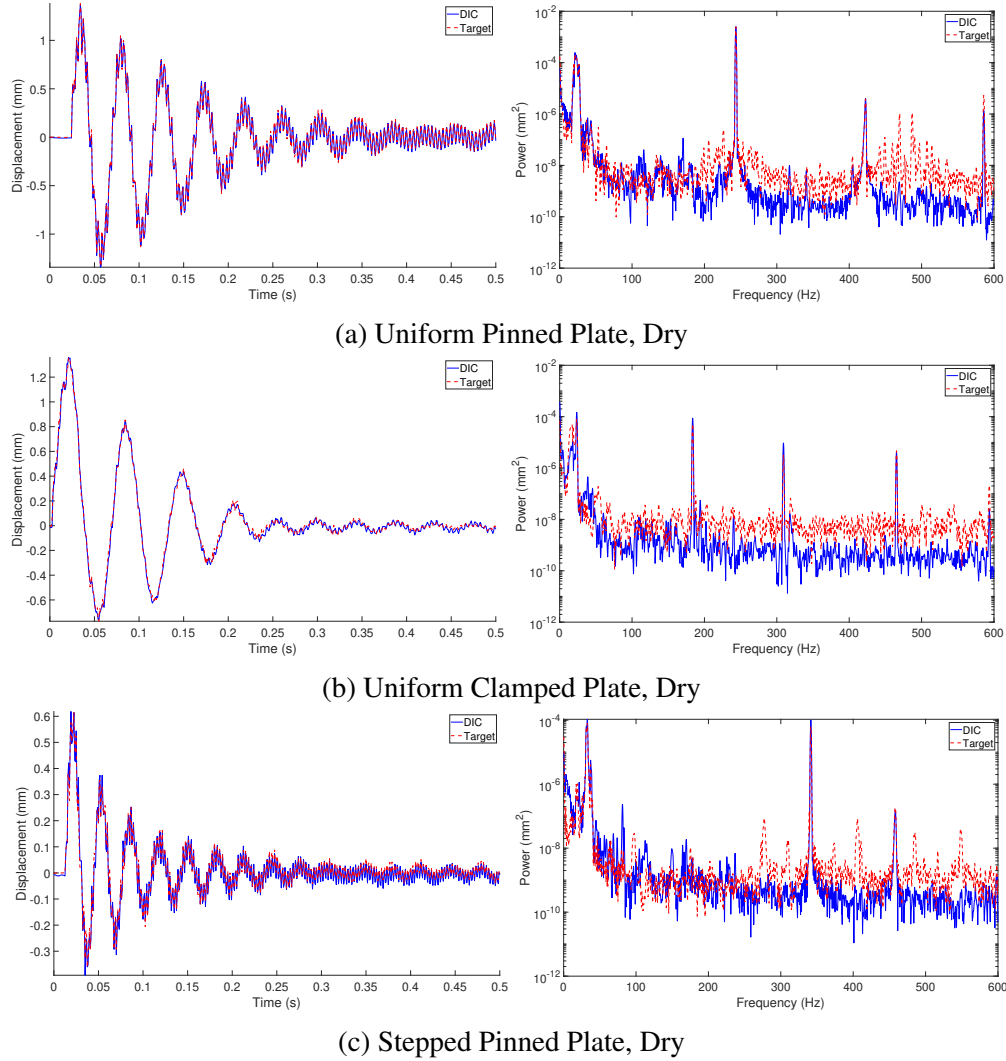


Figure 2.9: Comparison of deflection versus time at the plate center and corresponding power spectra as measured by the DIC and Target Tracking methods. Results for the three plates in the Dry condition and tapped at the plate center are shown.

Chapter 3: Results and Discussion

In this section, the experimental are presented and discussed. Focus is given to the determination of the natural frequencies and corresponding mode shapes measured in each impact trial. The measured natural frequencies and mode shapes in the Dry cases are also compared to those generated using a finite element method, as well as those found from simple Rayleigh beam function methods.

3.1 Data Processing Methods and Mode Shape Construction

Digital image correlation generates a 2-D grid of out-of-plane deflections (w) as a function of time, $w(x, y, t)$, where, as mentioned above x and y are coordinates along the plate length and width, respectively, with the coordinate system origin at the center of the plate's upper surface. Processing this data to obtain natural frequencies and mode shapes is discussed first with the help of Figure 3.1. In the first step of the processing, the average power spectrum for each run is calculated. This is accomplished by first obtaining the spectral power versus frequency curve, by using the `pspectrum` function in MATLAB, from the record of deflection versus time at each DIC grid point, see Figure 3.1(a). The set of these spectra are then combined by averaging the spectral power at each frequency over values at all of the grid points to obtain the average spectrum, Figure 3.1(b). From these average spectra, a number of discrete peak frequencies are

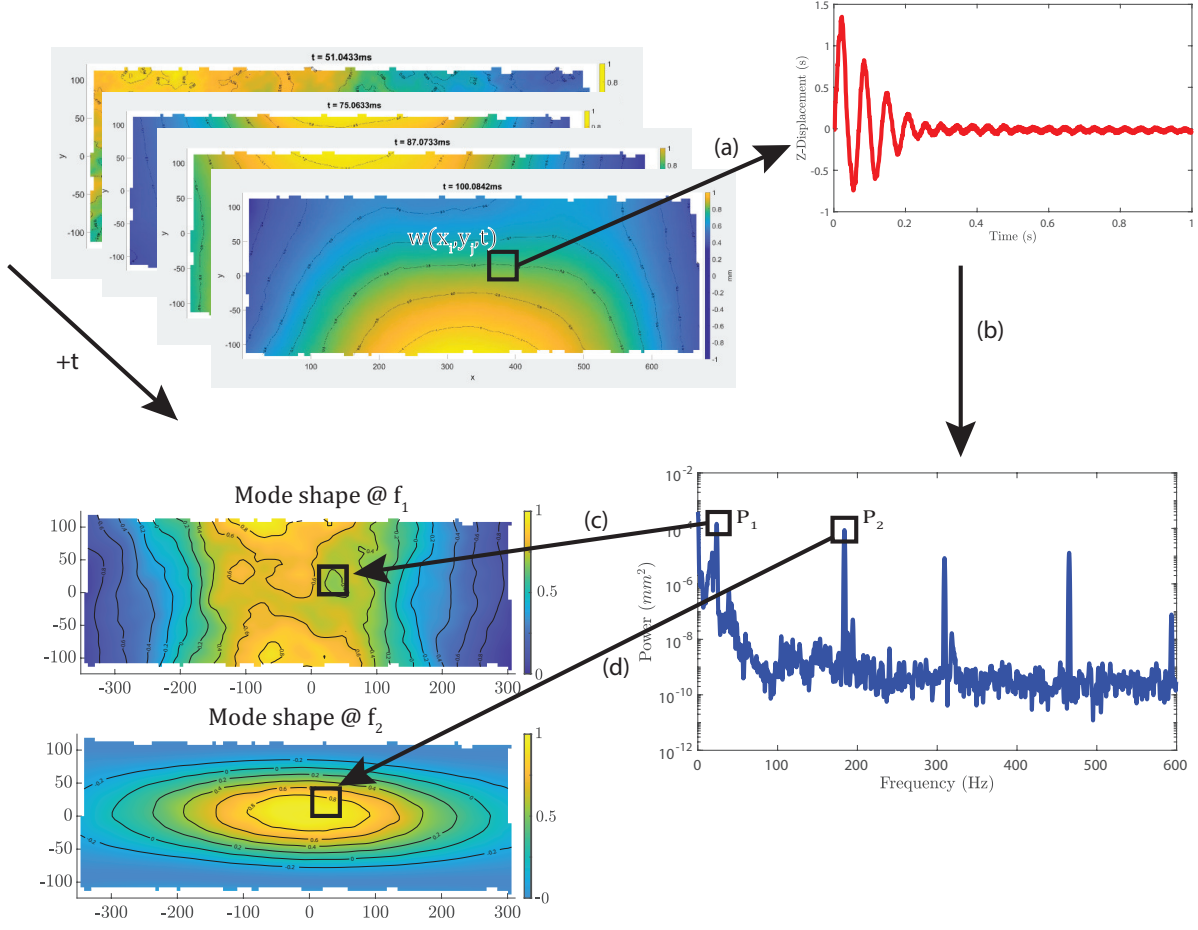


Figure 3.1: Mode shape construction method from DIC experiments. (a): Generate displacement-time plot at some x_i, y_j (b): Transform signal to power-frequency domain (c): Determine peak height and plot this height on an $x-y$ contour plot. (d): Repeat step (c) for next power-freq peak. (e): Repeat steps (a)-(d) for all x_i, y_j grid points

identified and taken as the frequencies of the various natural vibration modes of the plate. Mode shapes are then computed as a contour map on the $x-y$ plane of the square root of the spectral intensity at the selected frequency from the spectrum at each $x-y$ position, see Figures 3.1(c) and (d).

The most detailed results presented in the following are for span-wise “beam” bending mode at a low frequency and a “concentric” bending mode at a higher frequency since these modes appear to dominate the response during the slamming experiments, which are discussed

in other publications, see for example [\[15\]](#). In this processing, `pspectrum` is configured to compute the spectra to a maximum frequency of $f = 600$ Hz with a resolution of $\delta f = 1.5$ Hz. This transform function configuration is discussed in more detail in [4.5](#).

3.2 Natural Frequencies and Mode Shapes

The natural frequencies and mode shapes for each plate for each trial are presented in this section. The presentation is split into three sections, one for each of the 3 plate-boundary systems. Each section presents one set of line plots, a table and two sets of contour plots with the following information. The line plots show the average spectral power versus frequency for each experimental trial. The table contains the values of three natural frequencies (f_L , f_C and f_a) measured for each trial, as well as the maximum spectral power at each of those frequencies (P_L , P_C and P_a). In addition, the damping ratio for the first mode is presented. The two sets of contour plots show the mode shapes for the three frequencies with the first set of mode shapes for the Dry plate experiments and the second for the Half-Wet plate experiments. In both sets of contour plots, each row of plots is from the same experimental trial. From low to high frequency, these are the modes vibrating at the f_L (Longitudinal bending mode), f_C (Circular mode) and f_a (alternate mode) frequencies, respectively. For this analysis, the longitudinal and circular mode peaks are chosen based on the mode shapes. The frequency f_a typically corresponds to the peak with the third highest magnitude. However, in some trials, depending on the location of the hammer strike, f_a is the frequency of the mode with the second highest magnitude. The shape of the mode at frequency f_a is not always consistent from trial to trial.

3.2.1 Uniform-Pinned Plate Results

In this subsection, the results from the DIC measurements for the uniform plate with pinned-pinned boundary conditions are presented. The data is presented in the frequency spectrum plots in Figure 3.2, the numerical values of the modal frequencies, amplitudes and damping ratios

in Table 3.1 and the Dry and Half-Wet mode shape plots in Figures 3.3 and 3.4, respectively.

The Uniform-Pinned plate's frequency spectrum contains a range of excited natural frequencies with various amplitudes for both the Dry and Half-Wet conditions, see Figure 3.2. As the hammer impact location changes, the frequencies of the peaks remain the same but their amplitudes change, as expected. The spectral amplitudes also change from impact to impact at the same location, probably due to changes in impact impulse. Across the entire data set, we observe two dominate modes of vibration. The lowest frequency mode, $f_L = 26.7$ Hz, is shown below to be a the fundamental bending mode along the plate length and is called the longitudinal or f_L mode. This is the second highest spectral peak for cases with the hammer impacts at the plate center. The mode with the highest peak, frequency $f_C = 243.6$ Hz, is shown below to have a circular modal shape and is called the concentric or f_C mode. Typically, other modes appear as “modified” versions of these types of modes, i.e. more span wise bending peaks or more concentric-shaped hill peaks. Of these *alternative* modes, a high-power mode (f_a) was chosen and displayed for each trial.

The Dry mode shapes at the frequencies f_L , f_C and f_a are shown in Figure 3.3. The longitudinal bending mode, while noisy, has constant modal lines roughly oriented parallel to the plate width, a maximum amplitude line along the plate width at the half-length point ($x = 0$) and nodal lines presumed to be only at the constrained boundaries. It is likely that this mode shape varies somewhat at the two long free edges of the plate ($y = \pm W/2$). The source of the noise in the f_L mode is not fully understood though it is expected to be associated with the fact that this mode has a broad peak in the spectral plots in Figure 3.2 and that the shape of this peak in the local spectra varies with position over the plate surface. Thus, the amplitude of the local spectrum at the frequency of the peak in the average spectrum varies slightly from location to location in

the x - y plane. The Dry case trials with hammer impact at roughly $x = \pm L/2$ present a second higher order longitudinal bending mode ($f_a = 54.8$ Hz), see Figure 3.2(a), (b), (e) and (f), with a modal shape consisting of two peaks and a nodal line at $x = 0$, see Figures 3.3(c), (f), (o) and (r). For the center impact, we chose a higher order concentric mode ($f_a = \text{Hz}$) with a nodal line along $y = 0$. These contours are not adjusted for phase (sign) and it is assumed that across nodal lines the sign of the contour value would actually change to reveal the true mode shape.

The Half-Wet mode shapes, see 3.4, strongly resemble the modes in the Dry case; however, the natural frequencies are reduced. This is expected as the fluid loading introduces an added mass effect (discussed section 4.3). The frequencies of the longitudinal and concentric bending modes in the Half-Wet case are $f_L = 8.9$ Hz and $f_C = 131.7$ Hz, respectively, which are 33 and 54 percent, respectively, of their values in the Dry case. For the off-center impact cases, the highest power additional mode was a concentric mode with two nodal lines parallel to the x -axis and a nodal line along the y -axis at $x = 0$, see Figure 3.4(c), (f), (o) and (r). The longitudinal bending mode with a nodal line at $x = 0$ and frequency $f_a = 54.8$ Hz found in the off-axis impact Air trails was not observed in any Half-Wet trial.

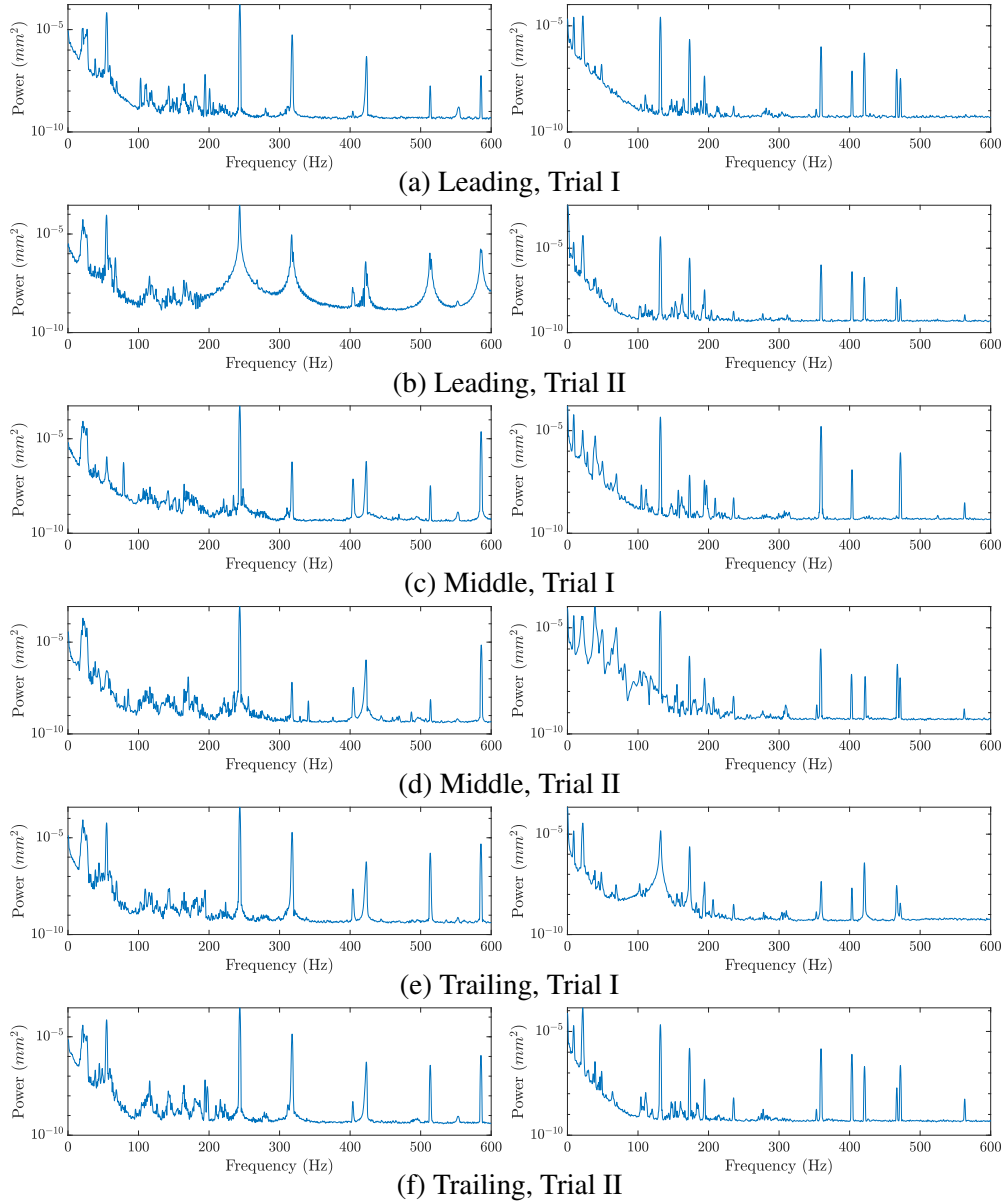


Figure 3.2: Average frequency spectrum for each trial for the Uniform-Pinned plate. Left column: Dry runs, Right column: Half-Wet runs. In the captions for each row of plots, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2.

Run	f_L	P_L	f_C	P_C	f_a	P_a	ζ_L	Notes
Dry Leading I	27.11	2.03E-05	243.7	6.1119E-04	54.8	1.30E-04	0.092	
Dry Leading II	26.67	1.80E-05	243.4	9.5114E-04	54.7	1.72E-04	0.0788	
Dry Middle I	26.81	6.40E-05	243.5	1.8048E-03	585.6	6.27E-05	0.079	
Dry Middle II	26.67	9.62E-05	243.4	2.6764E-03	585.9	1.80E-05	0.0574	
Dry Trailing I	26.37	2.34E-05	243.7	1.175E-03	54.8	1.10E-04	0.0721	
Dry Trailing II	26.81	1.93E-05	243.7	9.1241E-04	54.8	1.37E-04	0.0898	
H-W Leading I	8.94	4.82E-05	131.7	9.15E-05	173.2	8.61E-06	0.1293	
H-W Leading II	8.79	4.54E-05	131.7	1.623E-04	173.2	9.81E-06	0.1225	
H-W Middle I	8.94	1.03E-04	131.7	1.5866E-04	359.6	4.02E-05	0.1137	
H-W Middle II	8.94	7.01E-05	131.6	2.0418E-04	359.0	2.54E-06	0.1325	
H-W Trailing I	8.79	3.02E-05	132.2	5.12E-05	173.2	9.21E-06	0.1249	
H-W Trailing II	8.79	3.52E-05	131.7	7.33E-05	173.0	6.00E-06	0.1349	

Table 3.1: Frequency and spectral power for three natural frequencies from the average spectra of the Uniform-Pinned plate. In the Run column, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2 and the Roman numeral refer to the run number of each test. In addition, the damping factor (ζ_L) for mode 1 is given.

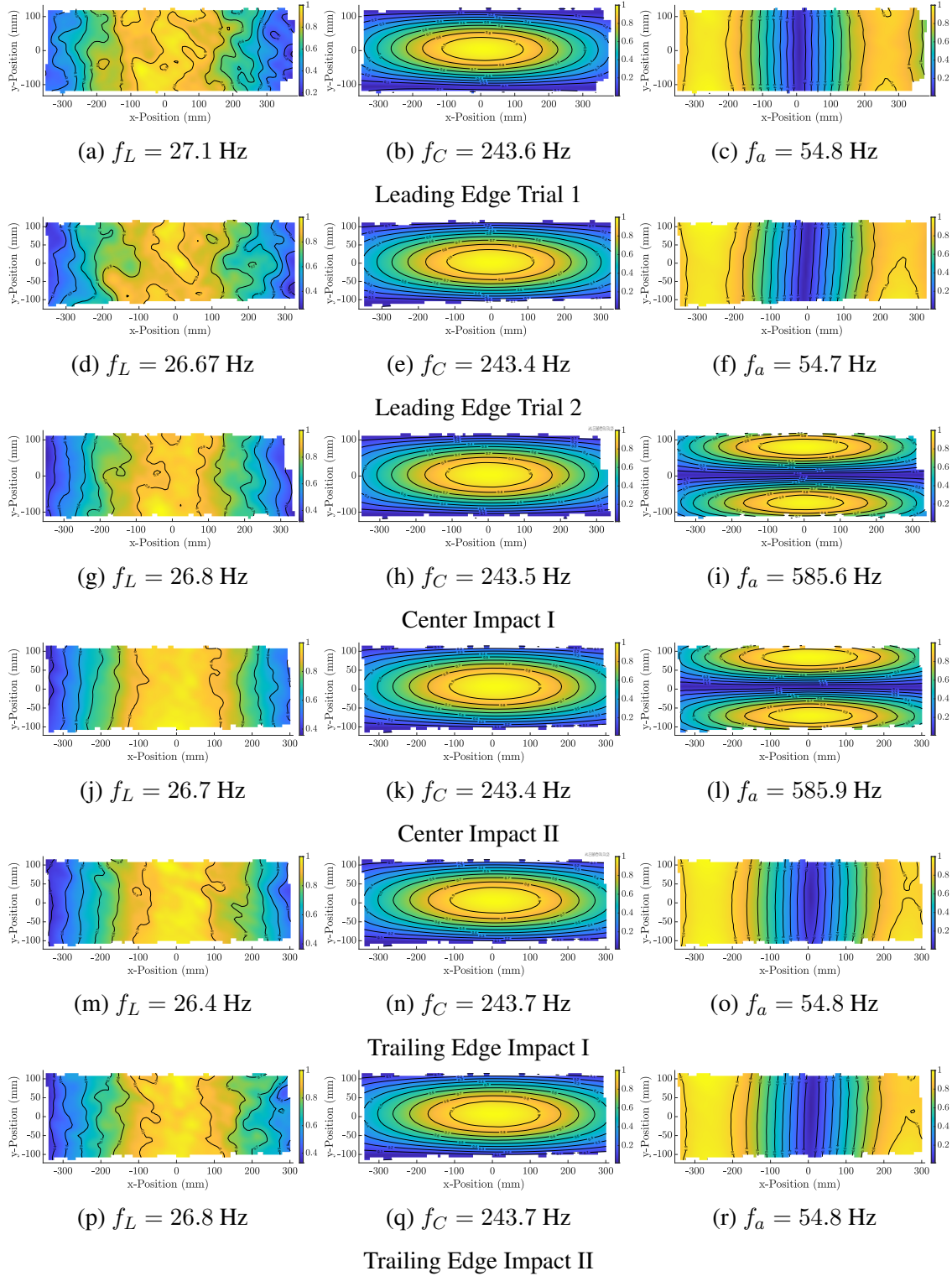


Figure 3.3: Mode shapes for frequencies f_L (left column), f_C (middle column) and f_a (right column) for the Uniform-Pinned Air case. In the captions for each row of plots, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2.

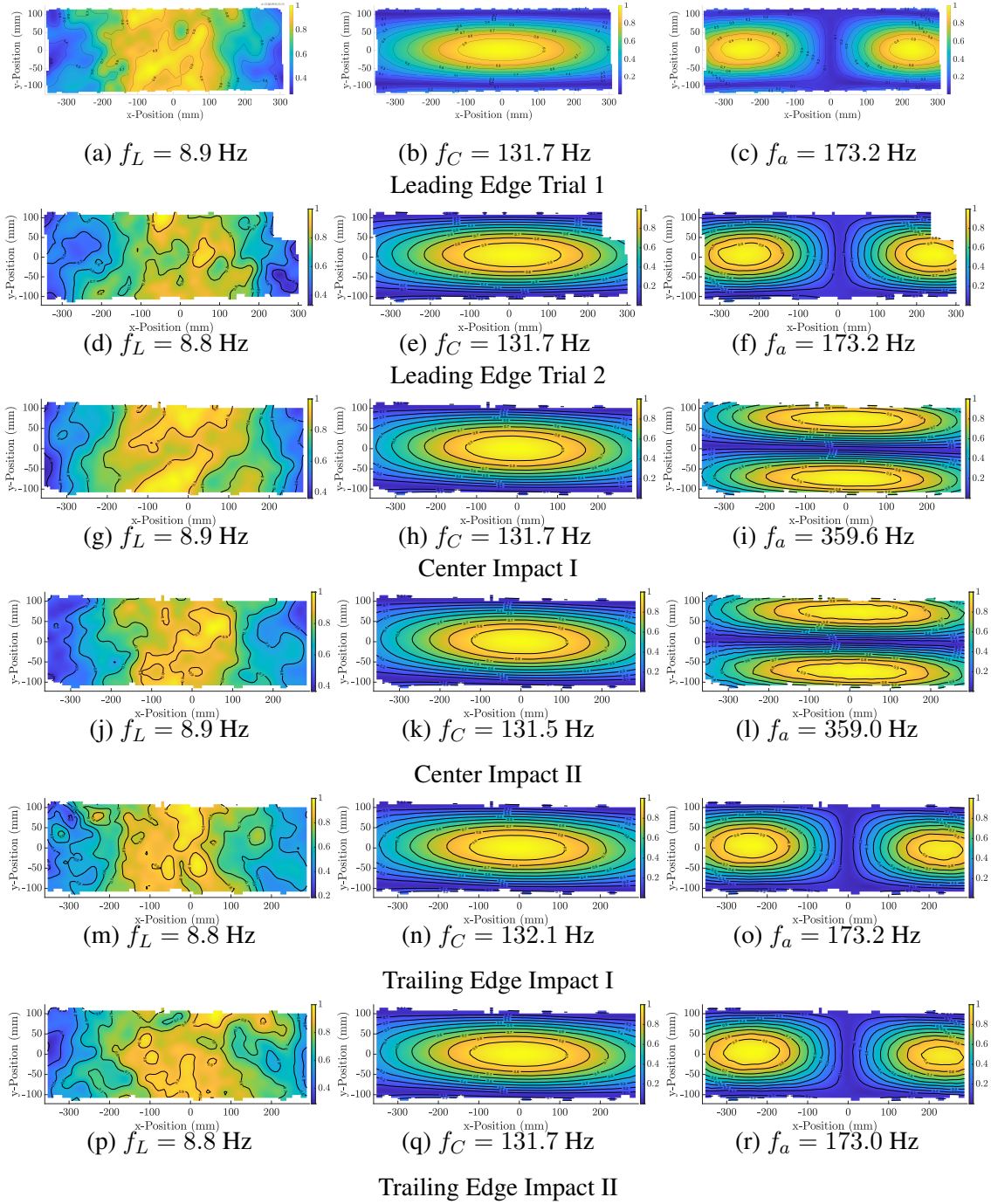


Figure 3.4: Mode shapes for frequencies f_L (left column), f_C (middle column) and f_a (right column) for the Uniform-Pinned Half-Wet case. In the captions for each row of plots, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2.

3.2.2 Uniform Clamped (UC)

In this subsection, the results from the DIC measurements for the uniform plate with clamped-simple boundary conditions (the Uniform-Clamped plates) are presented. The data is presented in the frequency spectrum plots in Figure 3.5, the numerical values of the modal frequencies, amplitudes and damping ratios in Table 3.2, and the mode shape plots for and the Dry and Half-Wet cases in Figures 3.6 and 3.8, respectively. The shapes of additional modes are presented in Figure 3.7.

The frequency spectra for the Uniform Clamped plate trials qualitatively resemble those seen in the Uniform Pinned plate trials; however, the frequencies and amplitudes of the peaks have changed. To elucidate the effects of the asymmetric boundary conditions on the natural frequencies and modal structure of the plate, a symmetric longitudinal bending mode (frequency f_L), a concentric bending mode (frequency f_C) and an asymmetric longitudinal bending mode (frequency f_a) have been chosen for examination. The mode shapes for the f_L mode in the Air case are shown in the first column of Figure 3.6. This mode has the same shape as the f_L mode for the Uniform-Pinned plate response and is consistently present in all trials. Its frequency, $f_L = 23$ Hz, is a bit lower than that for the Uniform-Pinned plate in the Air case, $f_L = 26.5$ Hz. This lower frequency is consistent with the idea that compared to the UP case, the reduced stiffness afforded by the thinner plate and the simple support at the trailing edge has overcome the increased stiffness due to the clamped support at the leading edge. The comparison between the shapes and frequencies of the f_L modes in the two plates carries over to the comparisons of the f_C modes, where the modal frequency dropped from approximately $f_C = 243$ Hz for the UP plate to $f_C = 183$ Hz for the UC plate. In all the UC Air trials, the f_C mode has one of the largest

amplitudes while the f_L mode amplitude is small.

The asymmetric longitudinal bending mode has one of the largest amplitudes in all trials. As can be seen from the mode plots in 3.6 the peak is between the plate center ($x = 0$) and the trailing edge ($x = L/s$). This mode's frequency (f_a) is lower than the symmetric longitudinal mode, $f_a = 15.9$ Hz. This mode is not present in the Uniform-Pinned trials and is likely a result of the decreased stiffness at the trailing edge due to the simple boundary condition.

The Half-Wet modes, similar to the Uniform-Pinned case, resemble the Dry modes. These modes occur at lower frequencies. As can be seen in the right column of Figure 3.5, these low frequency peaks are more isolated than the Dry peaks. The symmetric longitudinal bending mode $f_L = 6.8$ Hz mode decreased from the Dry mode by 70%. This is lower than Half-Wet longitudinal mode in the UP case at 8.9 Hz. This mode becomes very distorted when the plate is struck at $x = -L/2$. (Figure 3.8 (m),(p)). This warrants further investigation of complicated physics at the simple boundary.

The concentric bending mode had a frequency $f_C = 89.5$ Hz which decreased from the Dry case by 50%. This mode had nearly the same power as the longitudinal mode, with both modes having maximum power of order 10^{-4} across all trials. This is changed from the Dry case where the concentric mode had the highest power.

The asymmetric mode discussed previously does not appear in the Half-Wet case. As such, Figure 3.8 rightmost column displays a different mode. This mode is the highest power alternative mode for the Half-Wet case.

Similar to the Uniform Pinned case, the highest power alternative mode (frequency f_a) across all trials is a mode with two nodal lines in the horizontal and a nodal line in the vertical, $f_a = 117$ Hz. This is observed in the third column of Figure 3.8.

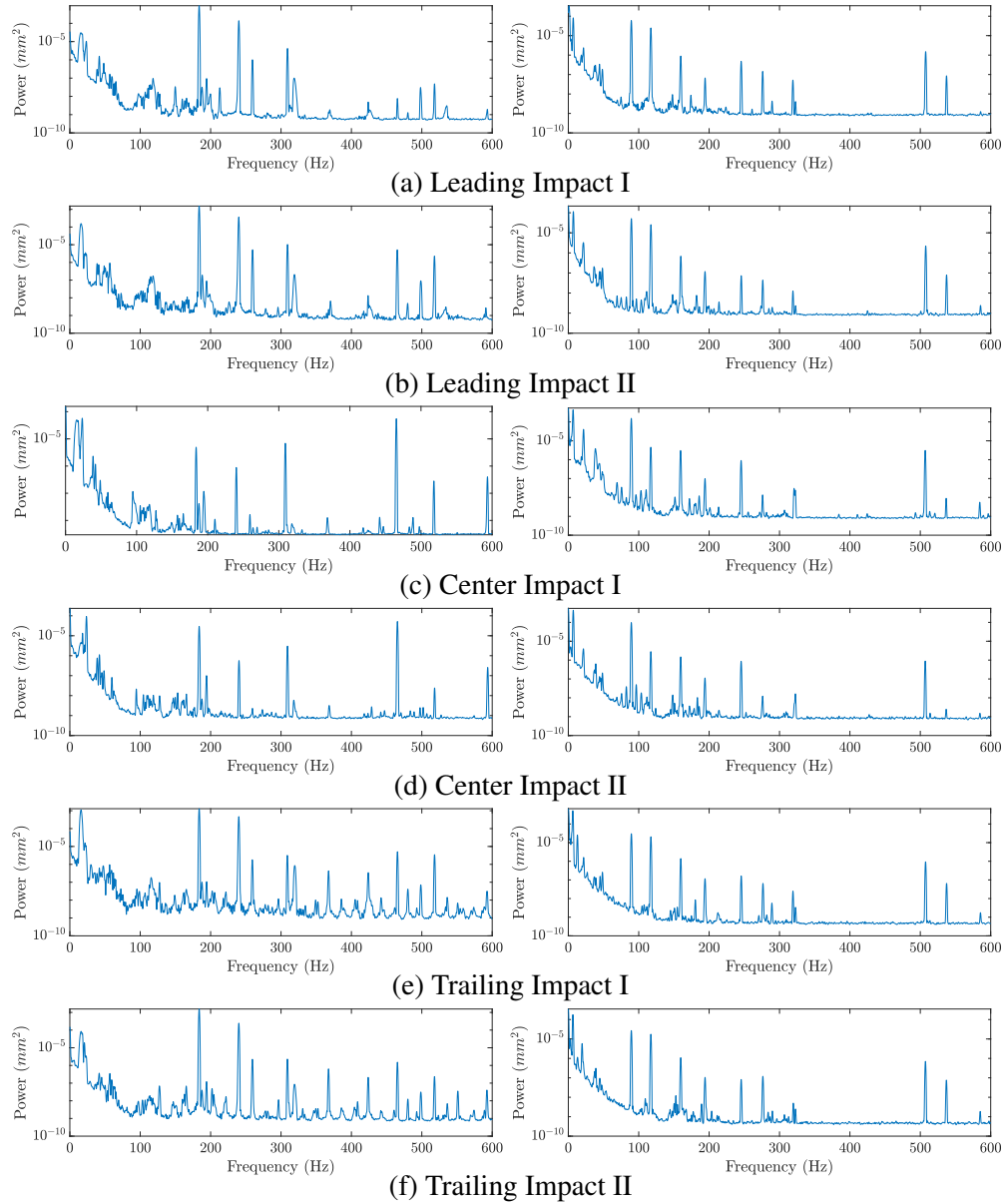


Figure 3.5: Average frequency spectrum for each trial for the Uniform-Clamped plate. Left column: Dry runs, Right column: Half-Wet runs. In the captions for each row of plots, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2.

Run	f_L	P_L	f_C	P_C	f_a	P_a	ζ_L	Notes
Dry Leading I	23.44	2.13E-05	183.74	3.1312E-03	15.68	6.3E-05	0.09	
Dry Leading II	22.27	6.92E-06	183.74	5.37E-03	15.97	2.9E-04	0.07	
Dry Middle I	23.59	1.085E-04	183.74	1.64E-05	15.8	9.0E-05	0.09	
Dry Middle II	23.59	1.69E-04	183.74	9.14E-05	15.97	1.024E-05	0.1	
Dry Trailing I	21.68	2.77E-05	183.74	3.93E-03	15.82	1.855E-03	0.05	
Dry Trailing II	23.15	8.18E-06	183.74	4.65E-03	15.97	1.4685E-04	0.07	
H-W Leading I	7.03	1.6677E-04	89.52	2.1466E-04	117.22	8.98E-05	0.15	
H-W Leading II	7.18	2.2542E-04	89.52	1.806E-04	117.36	9.09E-05	0.17	
H-W Middle I	6.74	8.221E-04	89.52	5.65003E-04	117.36	1.06E-05	.114	
H-W Middle II	7.03	7.8236E-04	89.52	3.5914E-04	117.22	1.06E+05	.12	
H-W Trailing I	6.30	1.1011E-03	89.52	1.1074E-04	117.22	7.46E-05	.097	
H-W Trailing II	6.45	3.8419E-04	89.52	1.0037E-04	117.22	6.43E-05	.12	

Table 3.2: Frequency and spectral power for three natural frequencies from the average spectra of the Uniform-Clamped plate. In the Run column, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2 and the Roman numeral refer to the run number of each test. In addition, the damping factor (ζ_L) for mode 1 is given.

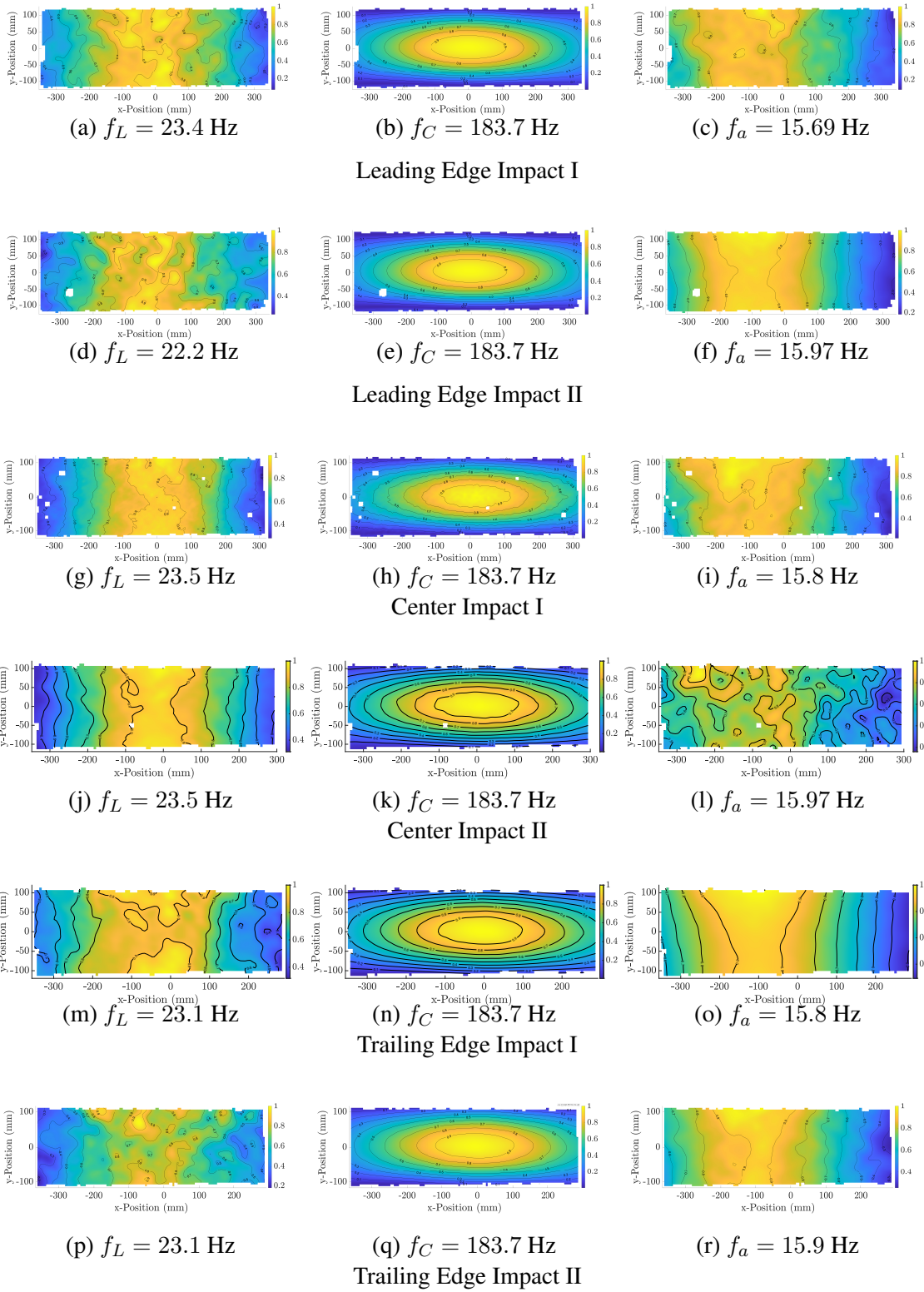
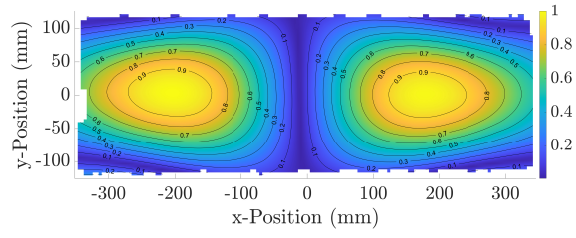
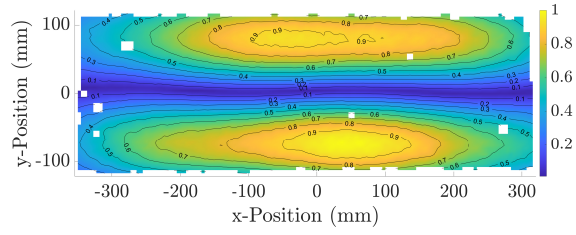


Figure 3.6: Mode shapes for frequencies f_L (left column), f_C (middle column) and f_a (right column) for the Uniform-Clamped Air case. In the captions for each row of plots, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2.



(a) $f = 240$ Hz



(b) $f = 465.3$ Hz

Figure 3.7: Other Dry Uniform Clamped Modes

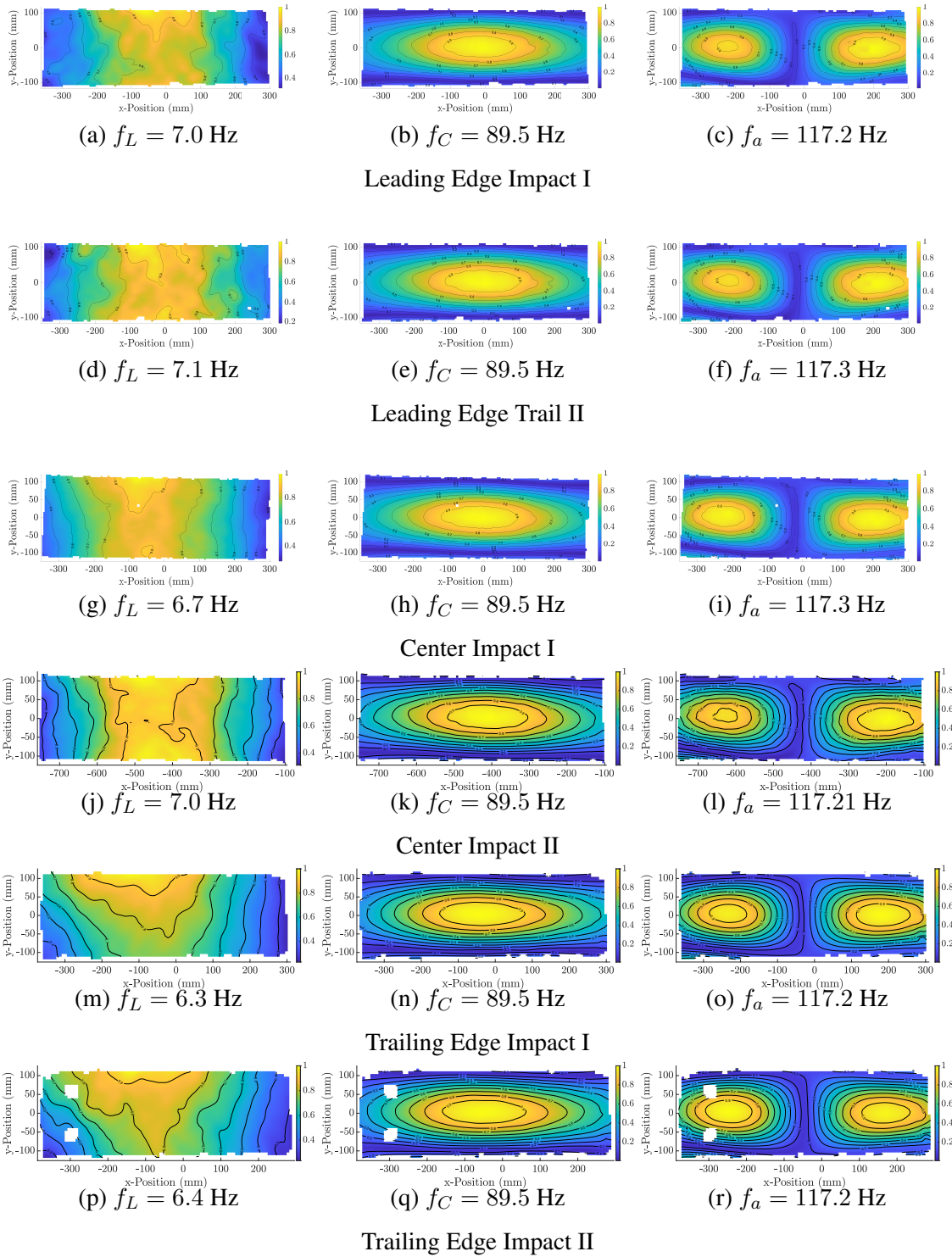


Figure 3.8: Mode shapes for frequencies f_L (left column), f_C (middle column) and f_a (right column) for the Uniform-Clamped Half-Wet case. In the captions for each row of plots, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2.

3.2.3 Stepped Pinned Plate (SP)

In this subsection, the results from the DIC measurements for the stepped plate with pinned-pinned boundary conditions are presented. The data is presented in the frequency spectrum plots in Figure 3.9, the numerical values of the modal frequencies, amplitudes and damping ratios in Table 3.3 and the Dry and Half-Wet mode shape plots in Figures 3.10 and 3.11, respectively.

The frequency spectrum for the Stepped Pinned plate shows differences from the previous plates. As seen in Figure 3.9, there are substantially fewer frequency peaks in the average power spectra in both the Dry and Half-Wet cases for all impact locations. In 3.10, the first column contains a longitudinal bending mode (frequency f_L), the second column contains a concentric bending mode (frequency f_C) and the third column contains an alternative high-power mode (frequency f_a) in the cases where one is available. Above $f_C = 243.5$ Hz, only two frequencies have clearly defined peaks, the highest of these is selected for f_a . The SP measured natural frequencies were higher than those of the UP plate. This behavior is stronger in the Half-Wet case, with peaks located above 200 Hz of very low maximum amplitude ($10E - 8 \text{ mm}^2$).

The Stepped-Pinned plate displayed the largest variation in mode shape behavior among the three plates. Similar to the previous plates discussed we see two primary bending modes, once concentric mode (frequency f_C) and one longitudinal mode (frequency f_L). We note that both of these mode shapes present maximum power at a location towards the trailing edge of the plate rather than at the plate center as in the UP plate. This location shift expected since the thinner (trailing edge) end of the plate has a lower rigidity. The aforementioned longitudinal and concentric modes in the SP case strongly resemble the modes in the UP and UC cases. This shift in the position of maximum power was not the same across all modes, the Dry fundamental

$f_L = 31.6$ Hz mode shifted only 50 mm, and the $f_C = 341.5$ Hz shifted approximately 230 mm. One alternative mode is shown in the third column (Figure 3.10). This mode has a frequency $f_a = 458.3$ Hz, higher than similar modes seen in the UP case ($f = 317$ Hz) and UC case ($f = 240$ Hz). However, this mode is shifted 100 mm toward the trailing edge from the center.

The Half-Wet modes are shown in Figure 3.11. The Half-Wet longitudinal mode frequency decreased from $f_L = 31.6$ Hz in the Dry case to $f_L = 12.6$ Hz, and the concentric mode frequency decreased from $f = 341.5$ Hz to $f_C = 200$ Hz. Both modes had the same shift in peak position that was observed in the Dry case. Two alternative modes are presented. The alternative modes shown in the Center Impact rows are the same as the alternative mode in the Dry case, but at the lower frequency of $f_a = 277$. The alternative mode for the impact toward the trailing edge, subplots (m) and (p), appears to be a shifted version of the mode seen in the center impacts for the Dry UP trials (see Figure 3.3. This mode's concentric peaks have shifted 300 mm toward the trailing edge from the center.

Some runs did not produce significant 3rd highest power mode shape. The frequency plots in 3.9 show some peaks at low frequencies, however our processing was unable to resolve these peaks into adequate contour plots, suggesting this is either noise or residual carriage vibration.

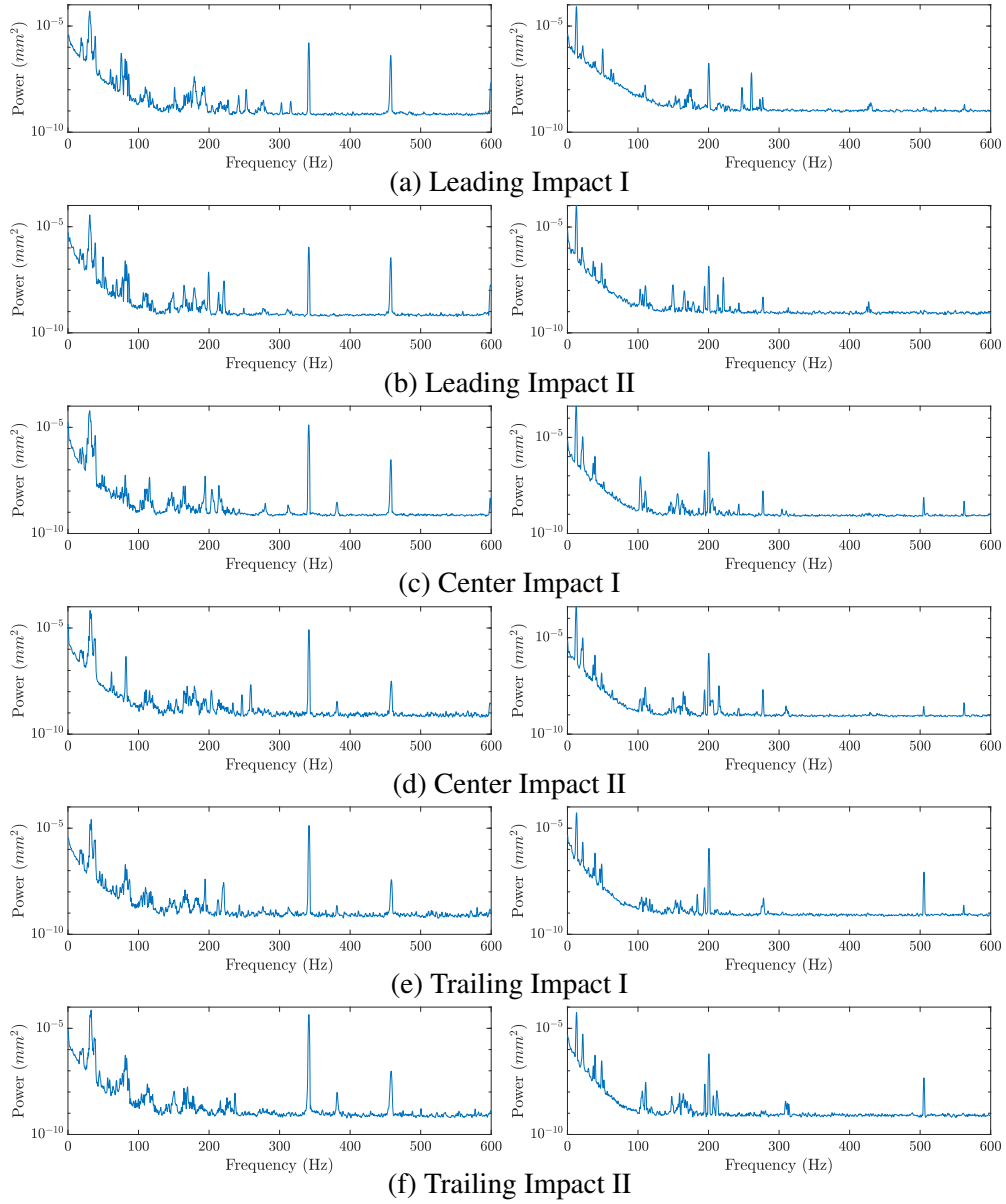


Figure 3.9: Average frequency spectrum for each trial for the Stepped-Pinned plate. Left column: Dry runs, Right column: Half-Wet runs. In the captions for each row of plots, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2.

Run/Impact Location	f_L	P_L	f_C	P_C	f_a	P_a	ζ_L	Notes
Dry Leading I	30.7692	8.53E-05	341.3919	6.09E-06	457.6	1.4E-06	.0495	
Dry Leading II	30.9158	5.94E-05	341.3919	4.12E-06	457.6	5.17E-06	.0514	
Dry Middle I	30.7692	1.01E-04	341.3919	4.80E-05	457.7289	1.07E-06	.0514	
Dry Middle II	31.3553	1.12E-04	341.5385	3.34E-05	458.315	1.55E-07	.0528	
Dry Trailing I	32.967	4.01E-05	341.685	5.40E-05	N/A	N/A	.0548	Distorted f_a mode
Dry Trailing II	32.674	1.15E-04	341.5385	1.79E-04	458.6	3.8E-07	.0725	
H-W Leading I	12.7473	1.44E-04	200.293	8.12E-07	N/A	N/A	.035	
H-W Leading II	12.6007	1.70E-04	200.293	6.99E-07	N/A	N/A	.0338	
H-W Middle I	12.4542	6.98E-04	200.293	8.85E-06	277.2161	8.21E-08	.032	
H-W Middle II	12.4542	6.28E-04	200.293	7.44E-06	277.2161	9.37E-08	.0317	
H-W Trailing I	12.8938	8.48E-05	200.5861	5.46E-06	505.4945	5.76E-07	.0317	
H-W Trailing II	12.8938	9.21E-05	200.4396	3.11E-06	505.348	3.41E-07	.0326	

Table 3.3: Frequency and spectral power for three natural frequencies from the average spectra of the Stepped-Pinned plate. In the Run column, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2 and the Roman numeral refer to the run number of each test. In addition, the damping factor (ζ_L) for mode 1 is given.

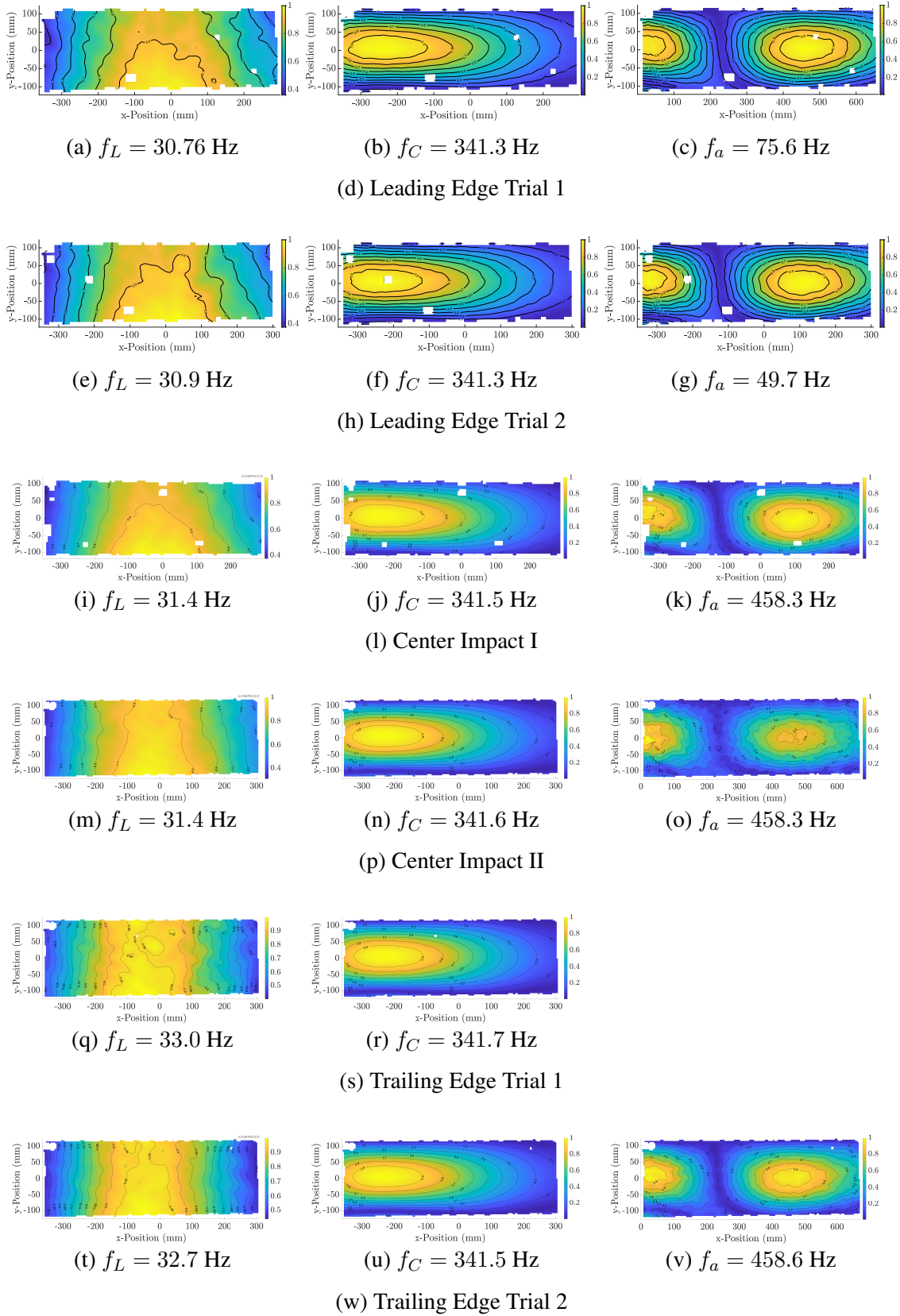


Figure 3.10: Mode shapes for frequencies f_L (left column), f_C (middle column) and f_a (right column) for the Stepped-Pinned Air case. In the captions for each row of plots, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2.

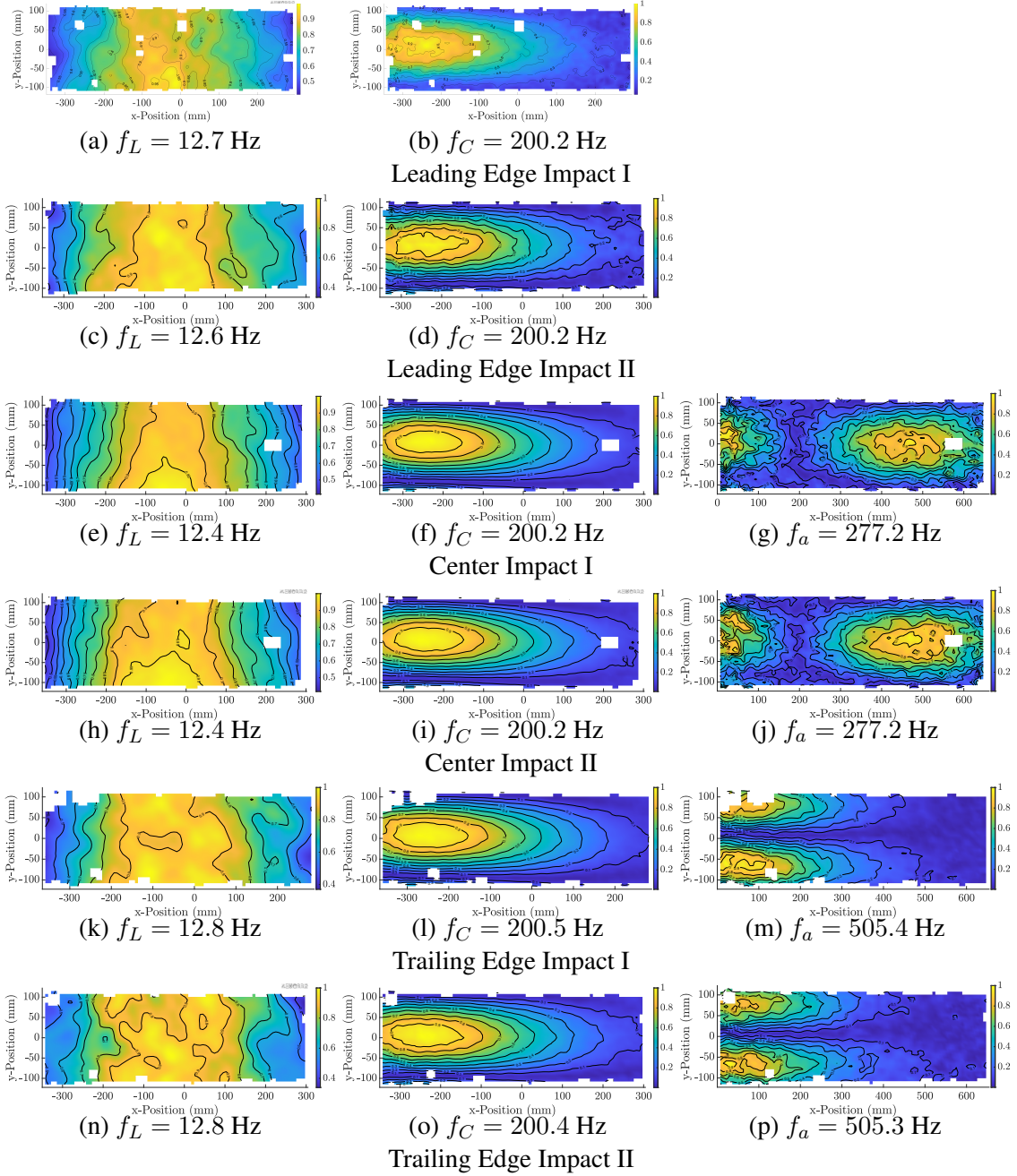


Figure 3.11: Mode shapes for frequencies f_L (left column), f_C (middle column) and f_a (right column) for the Stepped-Pinned Half-Wet case. In the captions for each row of plots, the words Leading, Middle and Trailing refer to the location of the impulse imparted by the impact hammer as discussed in Chapter 2.

Chapter 4: Discussion & Analysis

In this chapter, derived quantities are computed and discussed in relation to the experimental results presented in the previous chapter. The discussion is divided into sections with numerical calculations of mode shapes and natural frequencies for the Dry plate cases presented Section 4.1, damping in Section 4.2 and added mass in Section 4.3. This chapter ends with a discussion of the uncertainties and sources of error in the experimental study in Section 4.5.

4.1 Finite Element Method and Results

Finite element models for the Dry cases are developed and used to calculate the natural frequencies and corresponding mode shapes for each of the three plates in a vacuum. These cases are assumed equivalent to the Air cases in the experiments, given the low density of air compared to that of aluminum. The finite element models are produced in Creo Simulate using constrained, undamped modal analysis. Mode shapes for frequencies up to 600 Hz are computed. Each plate model contains the plate itself and the mounting hardware that is bolted to the plate as seen in Figure 4.1. The motion constraints imposed by the attachments to the carriage (pinned rails in the Uniform-Pinned and Stepped-Pinned plates, and clamped and simple supports in the Uniform-Clamped plates) are imposed through the numerical constraints in the code. Element size and count are provided in Table 4.1. These models are simplified and do not capture any transient

behavior, assume uniform material properties, and ignore any physics due to the fasteners and misalignment.

Plate	Max Element Size [mm]	Element Shape	Element Count
Uniform Pinned (UP)	10	Tet	54130
Uniform Clamped-Simply Supported (UC)	10	Tet	57279
Stepped Pinned (SP)	10	Tet	87936

Table 4.1: Finite element details

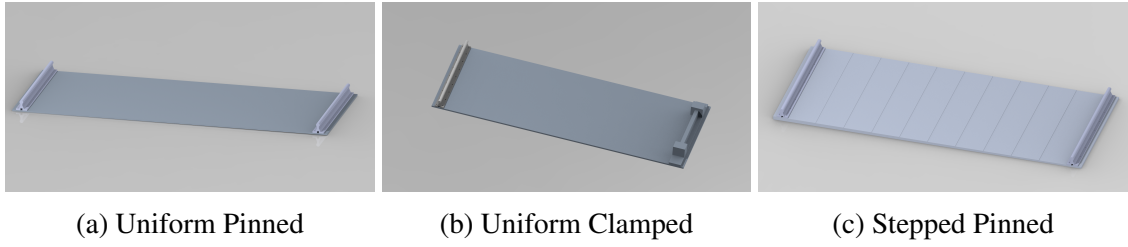


Figure 4.1: Modal Finite element models of the three plate/mounting systems from the experiments discussed in Chapter 3.

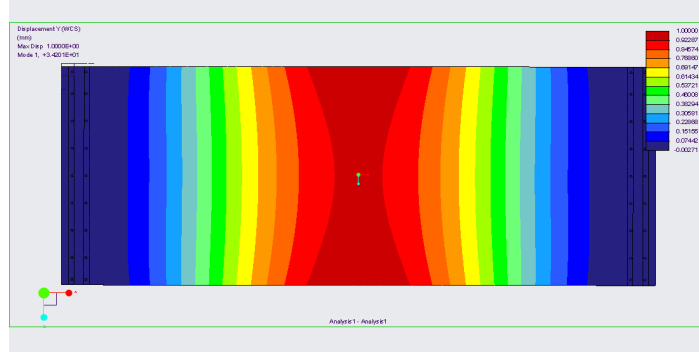
The frequencies of the longitudinal bending mode and the concentric mode, f_L and f_C , respectively for the three plates are given in Table 4.2. Included in this table are the frequencies from Finite Elements and the experimental values. The values of f_L from the calculations are approximately a factor of 30% higher than the values from the experiments, while the finite element and experimental values nearly agree for the concentric mode. These higher frequencies are expected since discrete finite element meshes add stiffness as compared to a real world continuous system. When comparing the mode shapes from the finite element calculations to those from the experiments, it should be noted that the finite element mode shapes are plotted with contour levels ranging from -1 to 1 while the experimentally determined mode shapes are plotted with contours ranging from 0 to 1, i.e., they are the magnitude of the modal function rather than the function itself. By comparing the mode shape contour plots for the Uniform-Pinned

plate from the finite element calculations, see Figure 4.2, to the those from the experiments, see Figure 3.3, one can see that the DIC measurements and the finite element calculations are in very good agreement. Similar conclusions can be drawn for the Uniform-Clamped plate and the Stepped-Pinned plate by comparing the mode shapes in Figure 4.3 to those in Figure 3.6 and the mode shapes in Figure 4.4 to those in Figure 3.10, respectively. Since the DIC results cover only a limited inner rectangle on the plate surface, see Figure 2.6, this agreement lends some confidence to using the computed mode shape contours to fill in the experimentally determined mode shapes to the edges of the plate.

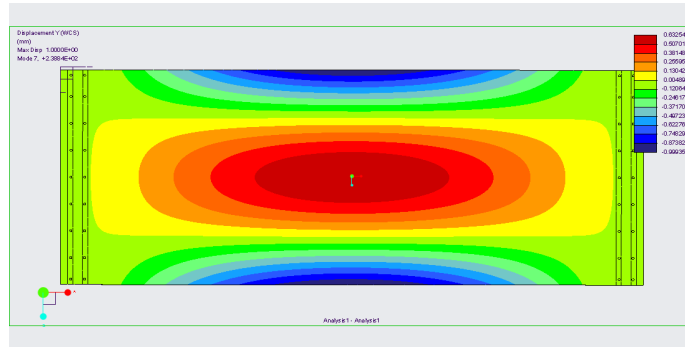
Given the above described general agreement between the finite element and experimental mode shape results, a few points from detailed comparisons are worth mentioning. First, while the clamped end was modeled with a zero-displacement boundary condition, the simply-supported end was less true to the experimental boundary since the spring-prismatic condition was not modeled. Regardless, the UC Finite Elements modes agree with the experimental modes, however the experimental results show a greater shift in the location of the peak displacement. Additionally, we do not observe the lower frequency longitudinal bending mode we observed in the dry case (See Figure 3.6). Second, the Stepped Pinned-Plate Finite Elements modes appear to agree with experimental modes with some limitations. Of particular interest was how the nodal lines and mode maximums shifted with respect to the uniform plates. These results are mixed, the f_L mode maximum appears to shift, if at all, to the thicker end of the plate, contrary to experimental (and expected) results. The higher modes however agree well with the higher experimental modes, with a shift in concentric modal peaks to the thinner edge of the plate. The f_a mode produced by the finite element method shows not only a shift but also a elongation in the higher modes, due to the nodal lines no longer being distributed symmetrically along the axis of the plate.

Plate	Experimental		Finite Elements		Warburton-Rayleigh	
	f_L (Hz)	f_C (Hz)	f_L (Hz)	f_C (Hz)	f_L (Hz)	f_C (Hz)
UP	$26.7 \pm .2$	$243.5 \pm .1$	34.2	238.8	13.2	233.4
UC	$23.0 \pm .8$	183.8 ± 0	27.07	182.7	N/A	N/A
SP	31.6 ± 1	$341.4 \pm .1$	43.7	345	N/A	N/A

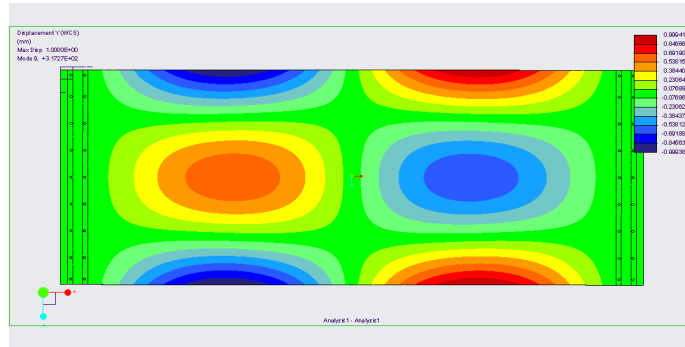
Table 4.2: Plate Natural Frequency Comparison



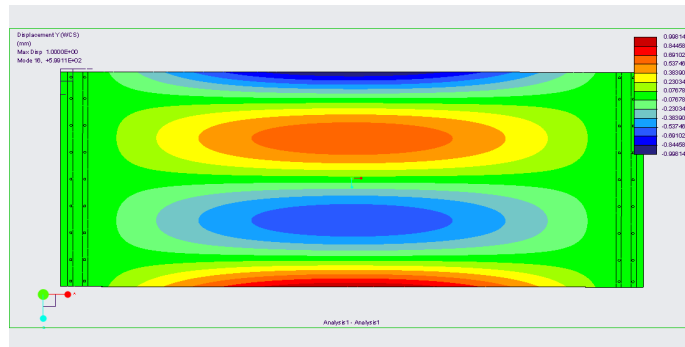
(a)



(b)

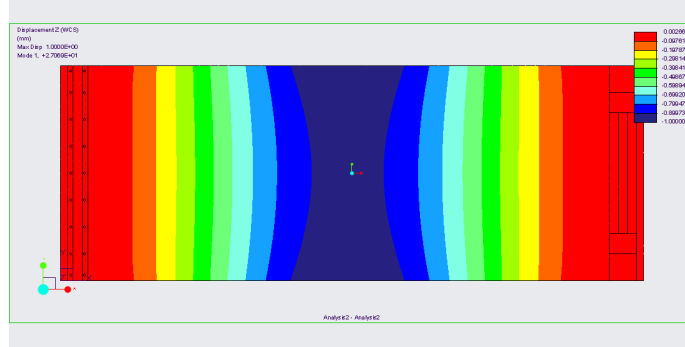


(c)

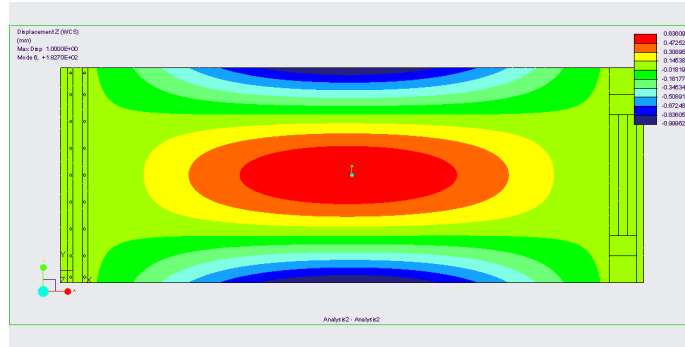


(d)

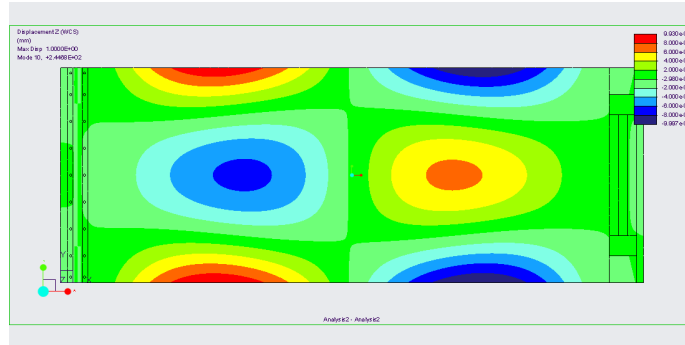
Figure 4.2: Mode shapes from finite element calculations for the Uniform-Pinned plate in Air. The finite element model is depicted in Figure 4.1(a).



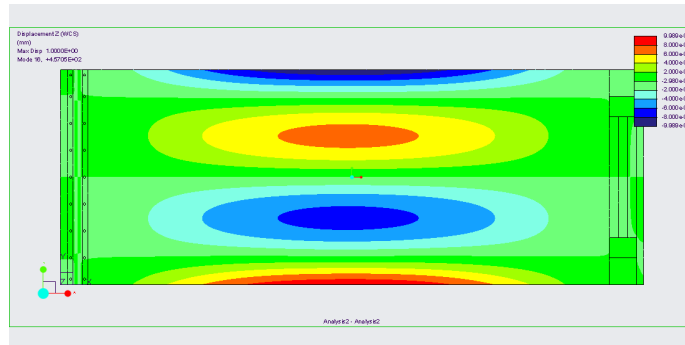
(a)



(b)

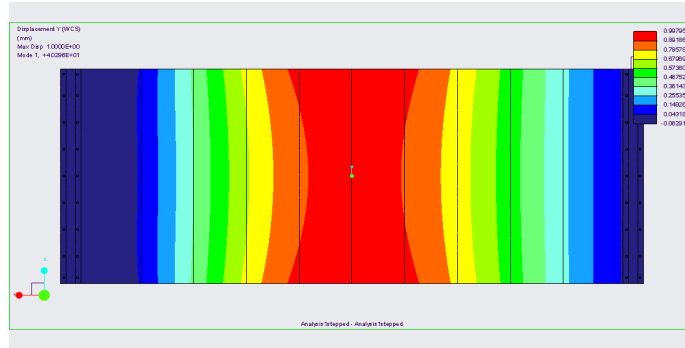


(c)

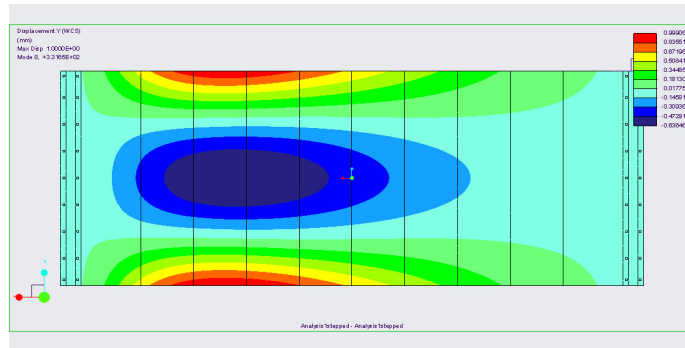


(d)

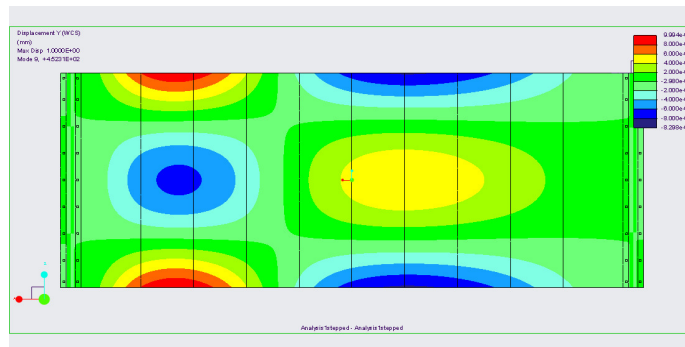
Figure 4.3: Mode shapes from finite element calculations for the Uniform-Clamped plate in Air. The finite element model is depicted in Figure 4.1(b).



(a)



(b)



(c)

Figure 4.4: Mode shapes from finite element calculations for the Stepped-Pinned plate in Air. The finite element model is depicted in Figure 4.1(c).

4.2 Damping Ratios

The damping ratios, ζ , are determined from the deflection versus time curves measured with the target tracking method and are tabulated in tables 3.1, 3.2 and 3.3. (The deflection target data (as opposed to DIC at the plate center) was used because these data sequences were longer.) Damping ratios are determined as follows: The displacement time record of the centerpoint deflection (As obtained from the displacement target) is filtered at a selected damped natural frequency ω_d . This is a frequency determined from the power-frequency plots. We introduce A decaying sinusoid of the form:

$$w_{0,0,i}(t) = B_i e^{-\omega_i \zeta_i t} \sin(\omega_{d,i} t), \quad (4.1)$$

Where ω_i is an unknown undamped (circular) natural frequency, $\omega_{d,i}$ is the known damped (circular) natural frequency, B_i is the amplitude, and ζ_i is the desired damping ratio. This sinusoid is then fitted to the filtered data via a least squares method using the *MATLAB Optimization Toolbox*. When filtering for the f_L frequencies, this function fits the data well and the resulting values of the damping ratio are fairly repeatable from trial to trial. At the higher frequencies, f_C and f_a , the filtered deflection curves had such low damping that attempting to fit Equation 4.1 to curves was unreliable. Thus, only the damping ratios for the f_L mode are included herein. The resulting values of ζ_L are given in Tables 3.1, 3.2 and 3.3.

4.3 Added Mass

Two methods of calculating added mass were used in this study. The methods and the resulting added mass values are presented and discussed below.

4.3.1 Method I

In this method, the deflection the mode shapes from the DIC measurements is used directly to compute the added mass at each dominant frequency. This method is based on the work of [1] and [9]. In [1], the derivation starts by setting the rate of change of the kinetic energy (E) of an incompressible, inviscid fluid equal to the rate of doing work by the pressure (ΔP) on the moving boundaries

$$\frac{dE}{dt} = m \int_0^a \int_0^b \left(\frac{dw}{dt} \frac{d^2w}{dt^2} \right) dx dy = - \int_0^a \int_0^b \left(\frac{dw}{dt} \right) \Delta P dx dy \quad (4.2)$$

where w is the fluid-plate interface deflection, ΔP is the pressure on the plate surface and m is the added mass. The integrations, after linearization, are over the undisturbed plate surface. The middle term is obtained by using boundary integral methods to convert the integration of the local kinetic energy over the fluid volume to an integral over the boundary of the surface of the volume. In order to compute the term on the right, we employ the formulation of the unsteady Bernoulli equation pressure on a flat plate as presented in [9] for a stationary bulk fluid as

$$\Delta P(x, y, t) = -\frac{\rho}{2\pi} \int_0^a \int_0^b \left(\frac{1}{\sqrt{(x-\psi)^2 + (y-\eta)^2}} \frac{d^2w}{dt^2} \Big|_{(x,y)=(\psi,\eta)} \right) d\psi d\eta \quad (4.3)$$

While previous studies ([9], [1]) applied numerical models for the mode shape and time dependant behavior, this study will use the fields generated from DIC. We define i -th 2-D mode shape field as $\Phi_i(x, y)$. We assume the total displacement in time (neglecting damping effects) takes the form:

$$w(x, y, t) = \sum_{i=1}^{\infty} w_i(x, y, t) = \sum_{i=1}^{\infty} q_i(t) \Phi_i(x, y) \quad (4.4)$$

This summation allows us to write the kinetic energy equation for mass for the i -th mode as:

$$-\frac{dq_i}{dt} \int_0^a \int_0^b \Phi_i(x, y) \Delta P dx dy = m_i \frac{dq_i}{dt} \frac{d^2 q_i}{dt^2} \int_0^a \int_0^b \Phi_i^3(x, y) dx dy \quad (4.5)$$

And the pressure¹:

$$\Delta P = -\frac{d^2 q_i(t)}{dt^2} \frac{\rho_{\text{fluid}}}{2\pi} \int_0^a \int_0^b \frac{\Phi_i^2(x, y) \Big|_{(x,y)=(\psi,\eta)}}{\sqrt{(x-\psi)^2 + (y-\eta)^2}} d\psi d\eta \quad (4.6)$$

We substitute equation 4.6 into 4.5. We divide and cancel out all time dependant terms to arrive at:

$$m_i = \frac{\int_0^a \int_0^b (\Phi_i(x, y) * \frac{\rho}{2\pi} \int_0^a \int_0^b \frac{\Phi_i(x, y)^2 \Big|_{(x,y)=(\psi,\eta)}}{\sqrt{(x-\psi)^2 + (y-\eta)^2}} d\psi d\eta) dx dy}{\int_0^a \int_0^b \Phi_i^3(x, y) dx dy} \quad (4.7)$$

This added mass has units of mass per unit area. We normalize this quantity by the fluid (water) density and plate length a ²

$$M_i = m \left(\frac{\rho_{\text{fluid}} a}{2\pi} \right)^{-1} \quad (4.8)$$

¹ A feature of this pressure formulation is the singularity at the intersection $x = \psi$ and $y = \eta$.

² Selection of the length a in this case is for comparison to studies with the plate length aligned with a bulk flow velocity see [1].

The results for these added mass values are plotted with respect to frequency in figure 4.5. In these calculations, $\Phi_i(x, y)$ is assumed to be the height of the i -th mode at x, y . These modes are converted to be unitless and have height in the range [0-1]. Integrals are calculated using the trapezoidal rule. This added mass estimation is limited in accuracy as we only have a limited measurement window of the plate surface. Additionally, our relationship for the added mass and frequency is incomplete as we are only able to plot frequencies detectable by DIC after impact. One point of concern is the phase difference in the modes across nodal lines. Specifically, should we consider a mode shape $\phi(x, y)$ to be purely positive or incorporate the change in phase? For convenience we have selected to use purely positive values for change in phase as this will result in purely positive values for the added mass.

4.3.2 Method II

A lumped parameter method for calculating added mass is presented. We consider each mode shape to behave like an independent SDOF undamped oscillator with total plate mass m and some stiffness k

$$m\ddot{w} + kw = 0 \quad (4.9)$$

This ODE has a solution of the form

$$w(t) = B \sin(2\pi f_{\text{dry},n} t + \phi) \quad (4.10)$$

Where the natural frequency $2\pi f_{\text{dry},n}^2 = \frac{k}{m}$. We consider the stiffness to remain constant between the half-wet and dry cases. We introduce half-wet natural frequencies as:

$$(2\pi f_{\text{HW}})^2 = \frac{k}{m + m_a} \quad (4.11)$$

Where m_a is the added mass due to the fluid loading. We solve for the stiffness and write the following relationship:

$$m_a = \frac{f_{\text{dry},n}^2 m}{f_{\text{HW}}^2} - m \quad (4.12)$$

Thus, if the dry and half-wet frequencies are known, we can solve for the added mass. This process is repeated for each mode. These results are presented in table 4.3. Also presented in this table are the method I added masses for comparison (converted to kg).

Plate	Mode number n	Added Mass Method I (kg)	Added Mass Method II (kg)
UP	Mode <i>L</i>	71.3	60.9
	Mode <i>C</i>	58.6	18.189
UC	Mode <i>L</i>	71.9	58.8
	Mode <i>C</i>	59.4	18.11

Table 4.3: Added mass for two dominate modes, two methods.

4.3.3 Added Mass Discussion

Comparing the two methods, the added mass computed using Method I is always higher than that computed using Method II. The values from the two methods are relatively close for the longitudinal modes and far apart for the concentric modes. The effects of the limited integration area and the assumption of only positive mode contours on these results has not been explored. It is noted that the limited measurement area of the plate (Figure 2.6) will cause inaccuracy with

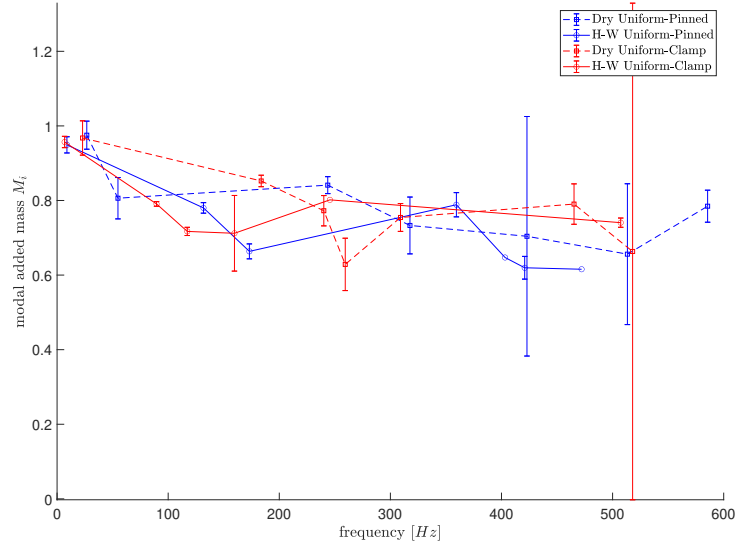


Figure 4.5: Added mass for dry and wet cases vs modal frequency

added masses computed with method I, however without a mode shape of the full plate area it is difficult to determine the nature of this error. In the case of the longitudinal bending mode, given that the deflection being low towards the leading/trailing edges of the plate ($x = \pm L/2$), we believe the added mass per unit area calculated using method I is fairly accurate. Further added mass calculations likely introduce inaccuracy as important features are lost to the missing plate area. This includes the concentric bending mode's displacements near the free edges.

4.4 Power Response

This section explores the ratio of power between the two most dominate modes observed in this study, the longitudinal bending mode (frequency f_L) and the concentric bending mode (frequency f_L). Of interest in this study is the potential change in power response of these modes under our selection of geometries and fluid loading conditions. We define the maximum power of one of these modes as highest value on the mode contour plot. For the longitudinal bending

mode, this height is typically near center of one of the two free edges. We consider the max height of a particular contour plot to be P_i for the i -th natural frequency. We denote the quantity P^* as the ratio of the max power of the f_L mode and the f_C mode, or:

$$P^* = \frac{P_L}{P_C} \quad (4.13)$$

This quantity is an estimate of the relative power of these two dominate modes for a particular strike location of a particular trial; A higher P^* means for a particular trial, the longitudinal bending mode is more excited than the concentric bending mode. These quantities are then tabulated in table 4.4. Of interest is how this quantity changes along the strike location of the plate, and how it differs between the dry and half-wet conditions.

The results shown in Table 4.4 are now discussed. Three tables are presented, one for each plate, with each table showing P^* for different strike locations (leading, at $x = L/2$, center at $x = 0$ trailing at $x = -L/2$) separated into Dry and Half-Wet sections respectively. We can observe across all Dry cases, changing strike location has an influence on P^* . In the UC case, P^* is found to be high for center strikes (order 10^0) and low for leading edge and trailing edge strikes (order 10^{-3}) The SP results show a increasing P^* from the trailing edge to leading edge. In the UP case P^* at all strike locations remains the same order of magnitude (10^{-2}).

The asymmetric fluid loading appears to change what modes are excited given the same impact location on the same plate. The UC trials have equalized, P is the same order across all strike locations. In the SP case, we again observe P^* increase from, however all values have increased. In the UP case, P remains the same across strike locations, however P^* has increased overall at order 10^{-1} . Overall, we see the Half-Wet condition appears to increase P^* .

Plate	Impact	P^*	Plate	Impact	P^*
UP Dry	Leading I	3.32E-02	UC Dry	Leading I	6.81E-03
	Leading II	1.89E-02		Leading II	1.29E-03
	Center I	3.55E-02		Center I	6.64E+00
	Center II	3.60E-02		Center II	1.84E+00
	Trailing I	1.99E-02		Trailing I	7.05E-03
	Trailing II	2.11E-02		Trailing II	1.76E-03
UP Half-Wet	Leading I	5.27E-01	UC Half-Wet	Leading I	7.77E-01
	Leading II	2.80E-01		Leading II	1.25E+00
	Center I	6.49E-01		Center I	1.46E+00
	Center II	3.43E-01		Center II	2.18E+00
	Trailing I	5.90E-01		Trailing I*	9.94E+00
	Trailing II	4.80E-01		Trailing II*	3.83E+00

Plate	Impact	P^*
SP Dry	Leading I	1.40E+01
	Leading II	1.44E+01
	Center I	2.11E+00
	Center II	3.36E+00
	Trailing I	7.44E-01
	Trailing II	6.42E-01
SP Half-Wet	Leading I	1.77E+02
	Leading II	2.43E+02
	Center I	7.89E+01
	Center II	8.44E+01
	Trailing I	1.55E+01
	Trailing	2.96E+01

Table 4.4: Frequency Power Ratio $P^* = \frac{P_{\max}(f_L)}{P_{\max}(f_C)}$

One possible physical explanation for this is the observed lower added mass for higher frequency concentric bending modes (see 4.3) the same impact load resulting in smaller displacements due to a greater fluid inertial resistance than those at the higher half-wet frequencies.

4.5 Uncertainty and Sources of Error

DIC is an intensity measurement technique and as such our experiment is limited to measuring these differences as captured by both video cameras. Some of the video frames collected did not render the full scope of the DIC range due to outliers in the intensity field. This is noticeable on the stepped plate where the image appears "washed out" at certain locations in the image, resulting in poor viability of the speckle pattern. If the speckle pattern is not viable, local pixels do not produce sufficient contrast and that image region cannot be processed to produced a displacement. For the half-wet experiments, we chose to strike the plate from the same side visible to the cameras. This resulted in several issues. Typically, the first 300-400 frames did not have the speckle pattern entirely visible due to the impact hammer-shadow and as such limited the quantity of points in those frames that could process a deflection. With this in mind, however, some area of the measurement region was able to be processed for the entire duration of the experiment and the power-frequency plots of these points did not show to differ substantially to those using the limited measurement time.

For each data set, the concentric high frequency mode shape was smooth and agrees with the modes found by finite element and Rayleigh methods (See sec4.1), however the low frequency results were less coherent. Typically, the amplitude plots for each set would present two similar modes at close frequencies and a comparatively high noise floor. Results for the lower frequency bending modes display erratic and nonphysical behavior. The most obvious is the asymmetry across the x axis of the primary low frequency mode across all data sets. One discrepancy between the theory/simulation results is the appearance of two distinct frequency peaks at low frequencies. This is coupled with a high noise floor. This behavior can be observed in the power-

spectrum plots (Figures 3.2, 3.5, and 3.9). These two frequencies produce similar mode structures, although one was typically of a higher power. Attempts to optimize these results (increase the filter bandwidth/lower spectral leakage) did not result in more coherent and symmetric structures. We have a few explanations for the emergence of these spurious modes. We believe this mode could be a result of the non-idealized boundary conditions, as the plate vibrates with the carriage a/or the end conditions are skewed, altering the mode shape directly. The plate itself may also be asymmetrically damped. As observed in 2.9, lower frequency oscillations appear to die out very quickly, limiting the accuracy of a discrete Fourier transform due to the low sample size.

Frequency analysis of mode shapes also presents a number of trade offs in resolution and accuracy. The discrete Fourier transform can struggle to produce accurate power spectra, especially for highly transient signals with a low sampling rate. Tweaking the frequency resolution (or spectral leakage), along with including amplitudes for neighboring frequencies can produce mode shapes more typical of this type of geometry. Selecting a low transform resolution produces smoother curves, however this introduces uncertainty. Selecting a higher resolution (lower δf) can create spurious frequency peaks and be more influenced by signal noise. This can be seen in figure 4.6.

As mentioned in previous sections, Some modes (especially for the modes of lower frequencies) clearly present as random and non-symmetric. Additionally, there clearly are two (or more) distinct low frequency peaks. Resolving these peaks into mode shapes would typically generate nearly identical modes. While this suggests that selection of a lower transform resolution could consolidate these into a single "true" mode, this does not appear to be the case, as seen in fig 4.6. This suggests the cause of this behavior is more systemic to this experiment.

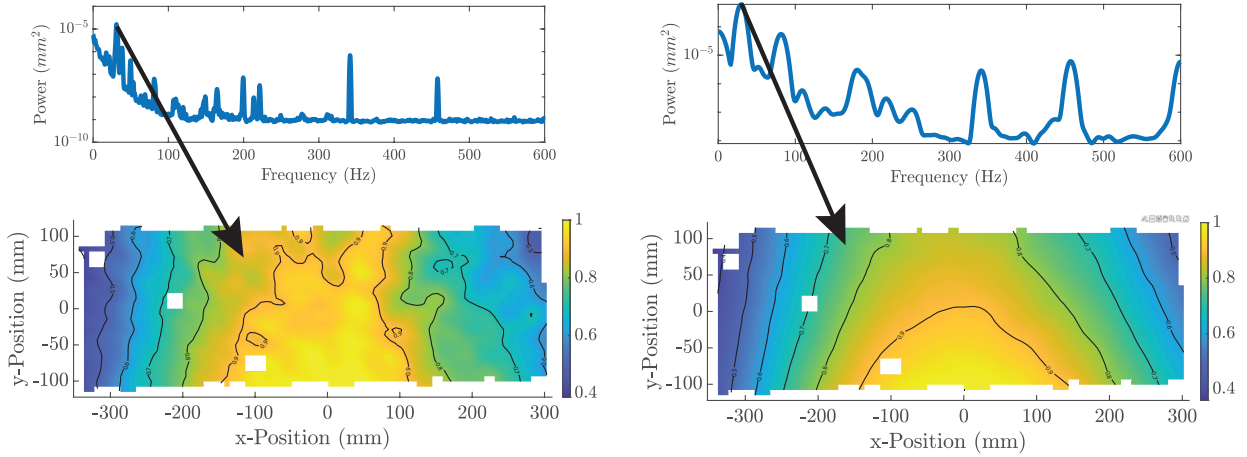


Figure 4.6: Comparison between alternative processing of the same dataset. Right, Processing dataset using a low frequency resolution ($\delta = 10Hz$), left is processing using a high frequency resolution ($\delta = 1.5Hz$)

4.6 Comparison with Literature and Previous Experiments

In [2], Wang presented a 1-D study showing non-linear, non-sinusoidal oscillatory behavior for the center of a half-wet Uniform Pinned-Pinned plate of 6.35-mm. In our trials of the same plate, geometry, we were unable to reproduce these results. Wang's maximum displacements were much greater than those found in this experiment, which could explain our inability to replicate this non-linear behavior. We did measure similar natural frequencies. Our results for Wang's dry cases are also comparable.

Chapter 5: Conclusions and Future Work

5.1 Conclusions

An experimental study of the free vibrations of elastic plates was presented. Three rectangular aluminum plates, all with width of $W = 406$ mm and length of $L = 1080$ mm and all attached to a stationary instrument carriage along the two ends of length W , were tested: 1.) A plate of uniform thickness with both W -length edges fixed to the carriage via a rail-and-bearing assembly to simulate a pinned mounting system, called the “Uniform Pinned” plate. 2.) A plate of uniform thickness with one W -length edge clamped and the opposite edge simply-supported, called the “Uniform Clamped” plate. 3.) A plate with a thickness of 6.3 mm at one W -length edge and increasing in 10 equal steps to 12.7 mm at the opposite edge and subject to the same pinned boundary conditions as the Uniform Pinned plate, called the “Stepped Pinned” plate. This experimental study was conducted in a 13.4-m-long by 2.45-m-wide towing tank. Two fluid loading conditions were explored. The first is a “Dry” condition where the tank is empty and the plate is surrounded by only air. The second is a “Half-Wet” condition where the tank is filled such that the bottom surface of the plate is submerged in water and the top surface is exposed to air.

In the experiments, the free vibration response of each plate was achieved by striking the plate with an impact hammer at one of three locations on the plate’s longitudinal centerline. Out-

of-plane displacements of the upper surface of the plates were measured using cinematic Digital Image Correlation (DIC). These displacements were processed to determine the natural response of the plate, its mode shapes and natural frequencies.

The Uniform Pinned plate response exhibited a variety of modes. While in air, the two highest power modes were a longitudinal bending mode at a low frequency, $(f_L)_{\text{Air}} = 26.7$ Hz, and a concentric bending mode at a higher frequency, $(f_C)_{\text{Air}} = 243.5$ Hz. With this plate in the Half-Wet condition, longitudinal and concentric bending modes with nearly the same shape as those in the Air case were observed; however these modes vibrated at lower frequencies than in air; $(f_L)_{\text{HW}} = 8.9$ Hz and $(f_W)_{\text{HW}} = 131.8$ Hz.

The Uniform Clamped plate had similar results. In air, the same longitudinal $((f_L)_{\text{Air}} = 23.0$ Hz) and concentric $((f_C)_{\text{Air}} = 183.8$ Hz) modes as those described earlier was observed. In addition to these modes, a new longitudinal mode centered towards the simple boundary of the plate was produced. This mode was observed at a lower frequency $((f_a)_{\text{Air}} = 15.9$ Hz). In the Half-Wet experiments, this asymmetric mode was not present. The Half-Wet condition produced the symmetric longitudinal mode and concentric mode at lower frequencies than in air, $(f_L)_{\text{HW}} = 6.8$ Hz and $(f_W)_{\text{HW}} = 89.5$ Hz respectively.

The Stepped Pinned plate was shown to have the highest natural frequencies among all plates. In air, we again saw the longitudinal $((f_L)_{\text{Air}} = 31.6$ Hz) and concentric $((f_C)_{\text{Air}} = 341.5$ Hz) bending modes seen in the Uniform Pinned case, however the Stepped Pinned modes shifted their maximums towards thinner plate edge (6.3 mm thickness). The longitudinal mode shifted only 50 mm, while the concentric mode shifted 250 mm. In the Half-Wet experiments, these same shifted modes were observed. These half-wet modes occurred at lower frequencies than in air with the longitudinal mode at $(f_L)_{\text{HW}} = 12.7$ Hz and the concentric mode at $(f_C)_{\text{HW}} =$

200.4 Hz.

The natural frequencies measured across these experiments are shown in Table 5.1. This table shows the average frequencies measured for a particular plate, bending mode, and fluid condition.

These studies produced mode shapes that are in agreement general with those found experimentally. The frequencies of the higher mode shapes were also in agreement with those found in the experiments, but the numerically determined frequencies of the low frequency bending mode were higher than those found experimentally. These studies also showed that the limited measurement window of the experiments resulted in a loss of accuracy in producing “complete” mode shapes.

Calculations for the added mass of each mode of vibration was explored with using two methods, an energy method and a lumped parameter method. These methods were found to agree for the added mass of the fundamental longitudinal bending mode; however, the limited measurement window of the experimental method introduced error in the added mass for the higher frequencies. Regardless, modes of higher frequencies were found to have lower added mass. The added mass, as computed by the lumped parameter method, are about 8 times the mass of the plate. While for the concentric mode, the added mass is about 2 to 3 times larger.

The relative power of the longitudinal and concentric modes was also explored. For all three plates, the longitudinal mode was found to have a greater power in the half-wet trials. For the Stepped Pinned plate, the longitudinal mode was found to have the highest power when the plate was impacted towards the thicker edge.

The results presented in this paper will be used to explore fluid-structure interaction physics in additional slamming experiments in the Hydrodynamics Laboratory.

	UP	UC	SP	% change in frequency from UP to SP
Dry $\bar{f}_L$	26.7	23.0	31.6	18
Dry $\bar{f}_C$	243.5	183.8	341.5	40
H-W $\bar{f}_L$	8.9	6.8	12.7	42
H-W $\bar{f}_C$	131.8	89.5	200.4	52

Table 5.1: Comparison of average ($\bar{f}$) natural frequency between SP and UP plates

5.2 Future Work

Future studies of elastic plates in contact with a water surface are discussed in this section. Previous studies on plate deflection in quiescent water surface slamming experiments were limited to a small number of deflection probes along the plate. A DIC measurement system could provide more detail regarding the nature of these deflections, particularly in terms of quantifying the elastic response in the frequency/mode domain, even given a limited measurement area. This study is especially useful considering the results of this study can be applied directly to the carriage system seen in [15].

One motivation for this study was initially to investigate nonlinearities in the half-wet case as seen by Wang in [15]. This study was unable to reproduce this behavior. This was speculated to be due to the vibrations generated with the impact hammer being of lower amplitude than those excited by Wang in his experiment. Further experiments could follow similar procedures as outlined in this experiment, however with a larger impulsive forcing.

Addressing limitations found in this study can also guide future research. While the free vibration experiment gave us insight into ways the plate responded to variation in the impact location, the results from this study were limited to study of modes excited by the impacts tested. A study interested in investigating a broader range of modes should consider a forced excitation

method, such as the piezoelectric-actuator method employed in [16] as low sample size (for any particular mode) and a small measurement window presented limitations in accurately measuring the added mass. A larger and/or more thoughtful sweep of strike locations could also excite modes of interest, which could be confirmed by a finite element study used to guide what locations to strike. Another limitation of this study is the lack of a strong fluid-coupling model for the plate vibration. Further work could apply models found in the literature, such as in Lucy [9] or Chen [7] for determining mode shapes and address their accuracy. This study performed experiments on an instrument carriage which introduced its response into our results, as well as limited the measurement area due to its geometry. While this coupled behavior was desired, a more general study could use a more rigid, idealized boundary structure with more care taken to observing a larger area of the plate.

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