Not So Fast: Comments on "Estimates of Performance and Cost for Boost Phase Intercept" presented to the Marshall Institute's Washington Roundtable on Science and Public Policy by Greg Canavan on 24 September 2004.

Dr. Greg Canavan's paper, "Estimates of Performance and Cost for Boost Phase Intercept," [http://www.marshall.org/article.php?id=262] examines some implications of constellation size and interceptor cost and weight for the total costs and feasibility of a space-based boost-phase interceptor (SBI) system. The paper argues, in general, that a "concentrated" system, that is, one that is tailored to defend against missiles launched from a small geographic area, can be substantially cheaper than is currently believed. North Korea might be considered "small." The paper states that reductions in expected cost come about from a combination of lower estimates of SBIs mass, lower estimates of individual satellite cost, and a constellation that requires fewer interceptors because it covers only a restricted range of latitudes.

We believe that mass and cost estimates are wrong and the simple model of satellite coverage exaggerates the effect of concentration. All the errors together lead to an extreme underestimation of the cost. The paper's SBI masses are based on unproven and very optimistic estimates of kill vehicle masses; its per satellite costs are based on unrealistic learning curve performance; and a more accurate model of satellite orbits shows that the benefits of concentration are somewhat smaller than the paper's simple model suggests.

The total mass of the system in orbit is critical because launching mass into orbit is expensive. The mass of the constellation is just the number of required interceptors times the mass of each interceptor. The interceptor consists of a "kill vehicle," which actually maneuvers and collides with the enemy rocket, the booster rocket that propels the kill vehicle from its orbit to the general area where the intercept will occur, and a "life jacket" that provides the maintenance functions for the interceptor while it waits in orbit but is not required for intercept, so is left in orbit when the intercept begins.

Defense system designers do not have to worry about making the kill vehicle too small and too light. Even a one kilogram projectile, closing with a thrusting booster rocket at several kilometers per second, will disable the rocket if it scores a hit. So the goal is to get the kill vehicle mass as low as technically feasible while still being able to maneuver well enough to hit it. Several estimates have been made of kill vehicle mass. Canavan uses masses reported in the American Physical Society (APS), *Boost Phase Intercept Systems for National Missile Defense* [http://www.aps.org/public_affairs/popa/reports/nmd03.cfm] and the Congressional Budget Office (CBO) *Alternatives for Boost-Phase Missile Defense*]http://www.cbo.gov/showdoc.cfm?index=5679&sequence=0] as baselines for comparison with emphasis on the latter because they have the smallest mass.

One must keep in mind that when the paper repeatedly refers to the "CBO" kill vehicle, these are not designs that CBO developed or endorses as feasible. Like any other

CBO study, their space-based interceptor study takes the form of "if one believes X, then the costs will be Y…" with consideration of a range of Xs. The lowest kill vehicle masses used by CBO were provided by Lawrence Livermore National Laboratory (LLNL). Thus, every time the paper refers to the "CBO interceptor," it really means the interceptor proposed by LLNL.

CBO cites a briefing from LLNL in November of 2003 that used a kill vehicle mass of 30 kg. [See footnote 4 on p. 24 of CBO report.] While the CBO report does not explicitly distinguish between fueled and unfueled masses (often called "wet" and "dry" weights), it is clear from the context that they consider the Livermore kill vehicle to be 30 kg fully fueled. CBO notes [p. 25] that "…producing a 30-kg kill vehicle with BPI performance would require a technological leap in miniaturization." The APS study also states that the Livermore Advanced Technology Kill Vehicle "…remains largely a conceptual design at this point; it has not been built or flight-tested" [APS p. 250] and might be ready for testing in a decade. More importantly, however, the mass of the Livermore interceptor, reported to APS in a briefing in September of 2001 [see APS footnote 160, cited on p. 250, appearing on p. 255], is 23 kg *dry*. With fuel, the total mass is 67 kg [APS Table 14.2, p. 253]—or more than twice what was reported to CBO just over two years later—and it is *the heavier* kill vehicle that the APS believes is challenging. Because launch costs to low Earth orbit are about $20,000/kg, launch costs are a substantial fraction of total system cost and the weight of the kill vehicle is critical.

The paper also introduces much lower costs than CBO for building the kill vehicles, or "KVs." It points out that "For CBO data base costs, KV costs dominate SBI cost," [p. 2] and argues that the CBO study errs by basing its kill vehicle costs estimates on individual military satellites. The paper states that a better model is the Iridium system of telephone satellites that were "mass produced." Specifically, "IRIDIUM does not compute the cost of individual satellites because production showed significant learning, which caused later satellites to cost a small fraction of the early ones." [p. 18] Seventy two Iridium satellites were produced for a cost per pound of satellite that is about 12% of the cost of the military satellites used as the CBO baseline.

By mass-producing the satellites (or anything, for that matter), the per unit costs should come down. This is often referred to as a "learning curve." The literature on learning curves is extensive and complex. Obviously a subject of much interest to industry, a great deal of effort has gone into uncovering the determinants of learning curves. Some studies show that learning curves depend on the type of industry and others show they also depend on management skill. Some reduction in cost results from true learning, that is, just avoiding mistakes and developing some tricks of the trade. Other reductions in cost come about because large production runs allow investment in capital equipment that reduces unit costs if the capital costs can be shared over a large enough number of units.

There are several ways to express reductions in unit cost as the number of units increases. The most common way is to express the reduction in average cost with a second lot of the same size as a first lot. For example, if the average cost of 200 widgets is 90% as much as with a production of 100 widgets, then the widget production is said to have a 90% learning curve. To a good approximation, for many types of manufacture, the average cost after the next lot of 200 will be 90% of the first two hundred, and so on (this is just to say that the "learning" is a logarithmic function of the number of units). Most observed learning curves are in the range of 85-95% (100% represents no learning). [See NASA's tutorial on learning curves, including a learning curve calculator, at http://www1.jsc.nasa.gov/bu2/learn.html .]

For learning to account for the reduction in cost between a single military satellite and the 72 Iridium satellites, the learning curve would have to be 72%. This learning curve is at the extreme range observed in any type of production and is significantly better than the 85% estimated by NASA for aerospace industry. Therefore, even if this high learning curve occurred during Iridium production, it is an anomaly, not a general example. In fact, there are few truly unique, one-of-a-kind military satellites—they typically are part of a series—thus the basis costs for military satellites used in the paper already include some degree of "learning," meaning that the paper is further exaggerating the learning effects in Iridium manufacture. It is far more likely that the difference in cost represents fundamental differences between the performance, specifications, and missions of commercial and military satellites.

Finally, the paper points out that the system does not need uniform satellite coverage over the Earth but can concentrate the coverage over a small area, reducing the total number of interceptors. Specifically, if we focus only on North Korea, a constellation with peak concentration at that latitude will be much smaller than one covering a larger area. This is true, but at several points the paper uses approximations or assumptions that overestimate the effect.

Figure 10 in the Canavan paper shows that SBIs in a constellation inclined at 42.5 degrees (the northern latitude of North Korea) will spend 11% of their orbital period within a latitude "band" 3.3 degrees wide, with 42.5 degrees as its northern edge. (We cannot reproduce the author's numbers from the paper alone but it appears that he uses the 3.3 degree band as covering North Korea. Figure 10 shows the fraction of SBI from constellations inclined at 43 and 30 degrees "in 3.3-degree bands, which are roughly the width needed to cover trajectories of missiles from North Korea." [p.25] North Korea runs from 38 to 42.5 degrees, or 4.5 degrees, not 3.3. This is important because later Canavan seems to reduce the mass of the interceptor based on its being able to just barely cover a 3.3 degree band from edge to edge.) Note this concentration effect is feasible only against missiles launched from a small country like North Korea. Iran is much larger, running between 25 and 40 degrees latitude and a constellation that covered Iran would clearly be much larger than one covering North Korea.

The APS effort started out considering only land-based boost-phase interceptors and later added analysis of space-based systems. The APS study team was well aware of the concentration effect, noting that "For any given orbit, satellite coverage will be concentrated at the same latitude as the inclination of the orbit, leaving the equatorial region underpopulated, as discussed in [80]. (The inclination of an orbit is the angle that it makes with the equator.) Although that concentration would be beneficial if the defense needs only to intercept missiles from a very narrow latitude band, such as from North Korea, it makes complete coverage over a wide range of latitudes more difficult to achieve." [APS p. 108] The APS study team also noticed that complete coverage at the equator gave fairly constant double coverage at the latitude of North Korea. [David Mosher, personal communication.]

Comparing APS Fig 6.3 [p. 109] and Canavan's Fig 11 [p. 26] shows that Canavan calculates substantially fewer interceptors even without concentration, but half of this reduction is due to seeking only single coverage, that is, only one interceptor is within range at any moment. That is, the system described in the paper is effective against a single North Korean missile. An obvious countermeasure would be to build a second missile that could be launched at the same time. The APS study does not concentrate over the latitude of North Korea and calculates a constellation "sufficient to cover any point in space and time in the region between approximately 30 and 45 degrees North latitude with an average of two interceptors and a minimum of one." [APS p. 110]

The Canavan paper uses the effective radius, r, of the area where the interceptor can engage the missile to be half the width of the band 3.3 degree band, so it just reaches from edge to edge. It then points out that "At their northmost latitude all SBI are headed due east, which produces a ring of eastward moving SBI on a line of latitude λ equal to the constellation inclination." Thus, all that is needed for complete coverage over any point in the latitude band is to have SBIs at that latitude separated by 2r. We can calculate the circumference of the Earth at that latitude and divide by 2r to get the number of satellites needed in that latitude band. If 11% of all the satellites are in the band, we can divide the number in the latitude band by 0.11 to get the total number of satellites. This approximation assumes that the coverage of the SBIs is a square moving around the band when it is in fact a circle. The circle covers only about three quarters of a square it fits in. This is not a big problem but shows the way to another source of error with this model. The 11% value is the fraction of time the SBI is anywhere within the band but the approximation assumes that the SBI is traveling along the center of the band at all times that it is within the band. In fact, the SBI will have some coverage over the band even before it enters the band because of the range of the interceptor will reach into the band before the SBI arrives. On the other hand, when the interceptor first barely enters the band, only half of its coverage is within the band, and half is lying south of the band. That is, it is covering only about 40% of the "square" assumed by the simple coverage model. At the very northernmost point of its orbit, the SBI is still "inside the band," but just grazing the top and again only half its effective coverage lies within the band, with the other half now north of the band.

The qualitative critique above suggested the need for more rigorous numerical calculations. The details of a MathCad worksheet for calculating satellite coverage are shown in the appendix. These calculations are based on equations found in "Earth Coverage by Satellites in Circular Orbits," by Alan R. Washburn, Department of Operations Research, Naval Postgraduate School.
[http://spica.or.nps.navy.mil/searchdocs/classnotes/Coverage.pdf]

Washburn's paper considers coverage of the Earth by low orbit communications satellites, but the mathematics applies directly to the problem of coverage by space-based interceptors. In particular, Fig 1 of Washburn defines the geometry used in the appendix. The MathCad calculation finds solutions to Washburn equation 8, 9, and 10. These equations express the distance between the satellite and the target. (Since the Earth and the satellite orbits are both assumed circular, it is convenient to measure distances in angles, using the center of the Earth as the origin of the coordinate system.) Once the formula for satellite-target separation is known, it is easy to find the fraction of the time that any one interceptor is within range of the target. The inverse of that number is the number of satellites needed to have one interceptor always within range.

The number of satellites needed depends on the range of each interceptor. If each one can cover a greater area, fewer are needed in total to cover any given target. The range that the interceptor can reach during the boost time of the targeted missile is almost directly proportional to the change in speed, or delta V, available to the interceptor. (The "almost" is because the time taken to get up to speed, depending on the acceleration, must also be taken into account.) But the higher delta V comes with greater fuel weight, and thus greater total interceptor weight. There is, therefore, a direct tradeoff between the number of interceptors and the weight of each individual interceptor and this relationship between the interceptor speed and the number of required interceptors is critical because it determines, along with the interceptor mass relationship, the minimum total weight for the system.

Using his simple model, Canavan calculated the relationship between interceptor speed and total interceptor number and shows his results in Fig 11 [p. 26]. We recalculated the same relationship with the more rigorous model and found that the simple model works fairly well, but there are two significant differences.

First, our calculated result shows that the number of required interceptors is more sensitive to the booster burn time than the simple model suggests. Canavan writes, "Thus, uniform and concentrated coverage require roughly equal numbers of SBI for liquid missiles, but uniform coverage requires about twice as many SBI for solid missiles. That is because concentrated coverage scales on missile burn time as $1/T$, while uniform coverage scales as $1/T^2$. The ratio of uniform to concentrated coverage scales as $1/T$ which is a significant penalty for solid missiles." [p. 25] This result of Canavan's calculation would be critical, because it suggests that using concentrated coverage overthrows the widely-held assumption that a key countermeasures to any boost-phase interceptor defense is to reduce the vulnerable boost time by moving from liquid to solid boosters. It appears, however, that the concentrated constellation's apparently reduced sensitivity to ICBM booster burn time is an artifact of simplifications in the Canavan model. This means that fast-burning solid-propellant boosters remain a challenging countermeasure even to concentrated space-based boost-phase interceptor defense.

Second, at the low delta Vs that Canavan determines are optimal, the simple model underestimates the number of interceptors required. At higher delta V, the simple model actually overestimates. That is, the curve describing the required number of interceptors as a function of the interceptor delta V is steeper than predicted by the simple model. The net effect of this different sensitivity is that the optimal delta V is not the 2 to 2.5 km/sec (depending on whether liquid or solid fuel boosters are being attacked) that Canavan calculates but 3 to 3.5 km/sec. [The paper's data on total mass is apparently contained in fig. 12, which seems to be accidentally omitted, but the text and fig. 13 make clear where the minima lie. Note also, that the calculations in the Appendix do not include the weight of the in-orbit "lifejacket."] At the higher optimal delta V, the reach of the interceptors increases and the benefit of concentrating the interceptors is less. This particular difference between the simple model and the analytical model is not the most significant one because the minima are quite flat and the precise point of the minimum does not affect total mass on orbit much.

Canavan's discussion of reducing the number of interceptors in a constellation of space-based missile defense interceptors compared to previous studies relies heavily on the well-known principle that giving up on coverage near the equator allows a constellation to concentrate coverage at a higher latitude. The simplifications of the Canavan coverage model exaggerate the benefit. More accurate calculations show the benefits of concentrations, of course, but they are somewhat less than predicted by the simple model. In any case, the differences between the simple model and the analytical model are small compared to the difference between single and double coverage and much smaller than the effects of hopeful estimates of kill vehicle weight and cost.

In summary: With a combination of (1) extremely optimistic kill vehicle mass estimates, (2) SBI cost estimates based on unrealistic learning curve values, (3) a somewhat exaggerated benefit of interceptor concentration based on a simplified model of satellite coverage, and (4) single rather than double coverage of North Korea, the author shows large reductions in the cost of a defensive space-based interceptor constellation defending against North Korea compared to previous estimates.

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Absentee Ratio for Fixed ICBM Burn-out Latitude and SBI Inclination

First define some basic parameters:

$G := 6.672 \cdot 10^{-11}$ $\quad M_e := 5.9736 \cdot 10^{24}$ $\quad \mu := (G \cdot M_e) \cdot 10^{-9}$ $\quad \mu = 398559$

$R_e := 6378.1$ equatorial radius (km) $\quad R_m := 6371.0$ mean radius (km)

$g := \dfrac{\mu}{R_m^{\,2}}$ $\quad g = 0.00982$

$S_e(t) := 86164.09055 + 0.00015 \cdot (t - 1900)$ $\quad S_e(2004) = 86164.106$ seconds per day

Define a frozen satellite orbit (i.e., one with a repeating ground trace) with altitude of about 300 km:

$m := 111$ number of orbits before ground trace is repeated

$n := 7$ number of days before ground trace is repeated

$Q := \dfrac{m}{n}$ $\quad Q = 15.857$ ground trace parameter (orbits per day)

$P := \dfrac{S_e(2004)}{Q}$ $\quad P = 5434$ orbital period (s)

$a := \sqrt[3]{\mu \cdot \left(\dfrac{P}{2 \cdot \pi}\right)^2}$ $\quad a = 6680$ semi-major axis of circular orbit with period P (km)

$h := a - R_e$ $\quad h = 302$ altitude of satellite orbit (km)

Calculate the maximum SBI range r (km), given SBI burn-out velocity v (km/s) and ICBM burnout time t_{bo} (s), assuming constant SBI acceleration of c (g) and a SBI launch delay of t_d (s):

$r(v, t_{bo}, t_d, c) := v \cdot \left(t_{bo} - t_d - \dfrac{v}{2 \cdot c \cdot g}\right)$ maximum SBI flyout distance (km)

Calculate the maximum intercept angle α from satellite to an ICBM at burnout altitude h_{bo} (the angle formed by the intercept point, the center of the Earth, and the satellite at time of SBI launch):

$$\alpha(v, t_{bo}, t_d, c, h, h_{bo}) := \operatorname{acos}\left[\frac{a^2 + (R_e + h_{bo})^2 - r(v, t_{bo}, t_d, c)^2}{2 \cdot a \cdot (R_e + h_{bo})}\right]$$

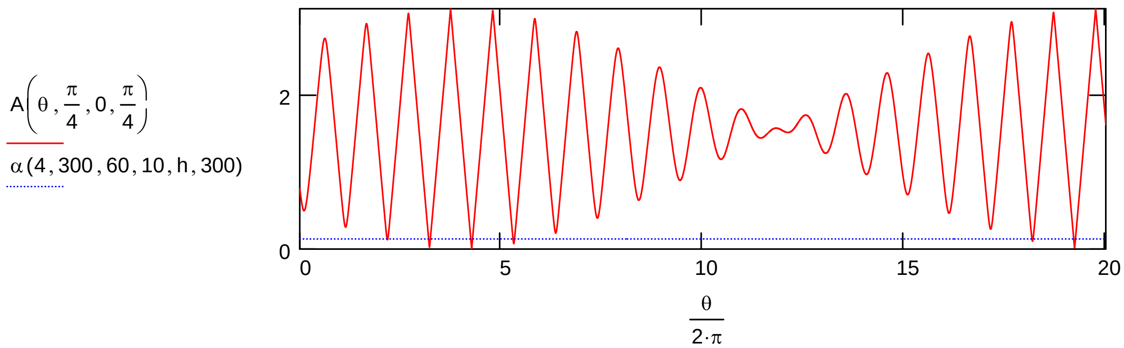
Calculate the actual angle A between the satellite and the potential intercept point as a function of time:

$$A(\theta, \psi, \phi, i) := \operatorname{acos}\left[\cos(\psi) \cdot \cos\left(\phi + \frac{\theta}{Q}\right) \cdot \cos(\theta) + \left(\cos(\psi) \cdot \sin\left(\phi + \frac{\theta}{Q}\right) \cdot \cos(i) + \sin(\psi) \cdot \sin(i)\right) \cdot \sin(\theta)\right]$$

where θ measures time by angle in satellite orbit, ψ is the latitude of the target, ϕ measures the initial longitude of the target on the fixed celestial sphere, and i is the inclination of the orbit.

Below is a plot of A as a function of time (the number of SBI orbits) together with illustrative value of α; when A is less than α, the SBI is within range of the ICBM burnout point:

$\theta := 0, 0.01 .. 2\cdot\pi\cdot 20$



A proper calculation of the fraction of time that satellite is within range of target would integrate the function if(A(θ) < α,1,0) from θ = 0 to 2πm, where m is the number of unique orbits:

$$f(\psi, i, v, t_{bo}, t_d, c, h_{bo}) := \frac{\int_0^{2\cdot\pi\cdot m} \text{if}\left(A(\theta, \psi, 0, i) \le \alpha(v, t_{bo}, t_d, c, h, h_{bo}), 1, 0\right) d\theta}{2\cdot\pi\cdot m} \qquad TOL \equiv 0.001$$

Unfortunately, the function is too poorly behaved to be evaluated numerically with MathCad, so we'll have to use a discrete function:

$\theta := 0, 0.01 .. 2\cdot\pi\cdot m$

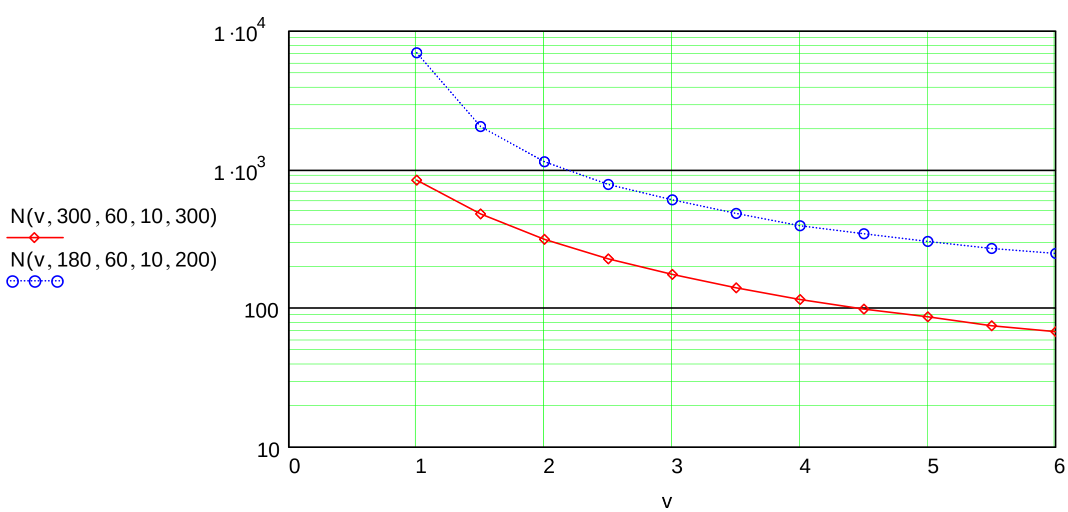
$$f(\psi, i, v, t_{bo}, t_d, c, h_{bo}) := \frac{\left(\sum_{\theta} \text{if}\left(A(\theta, \psi, 0, i) \le \alpha(v, t_{bo}, t_d, c, h, h_{bo}), 1, 0\right)\right)}{\sum_{\theta} 1}$$

$$f\left(\frac{\pi}{4}, \frac{\pi}{4}, 4, 300, 60, 10, 300\right) = 0.0086316$$

The minimum number of satellites required to provide continuous coverage of a point on Earth is 1/f:

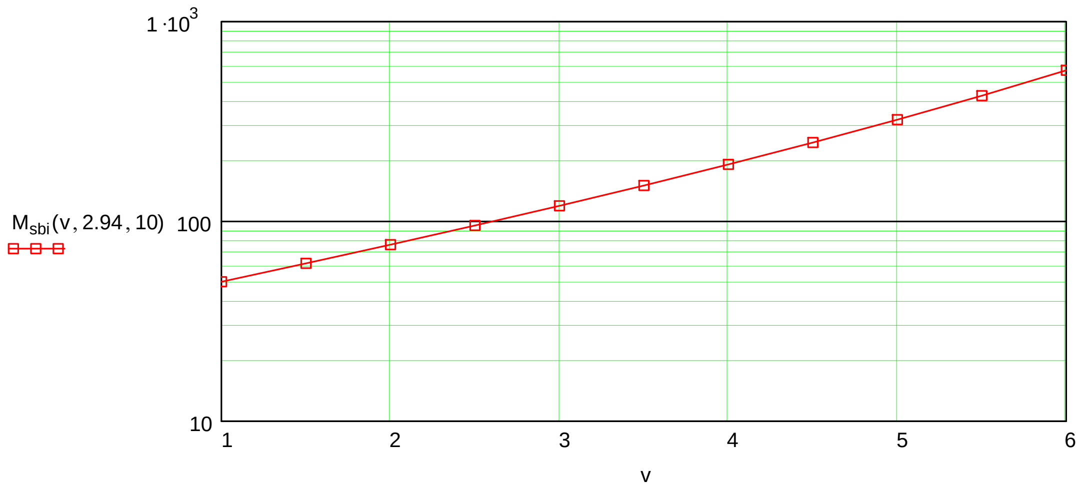
$$N(v, t_{bo}, t_d, c, h_{bo}) := \text{ceil}\left(f\left(\frac{\pi}{4}, \frac{\pi}{4}, v, t_{bo}, t_d, c, h_{bo}\right)^{-1}\right) \qquad N(4.5, 300, 60, 10, 300) = 99$$

$v := 1, 1.5 .. 6$



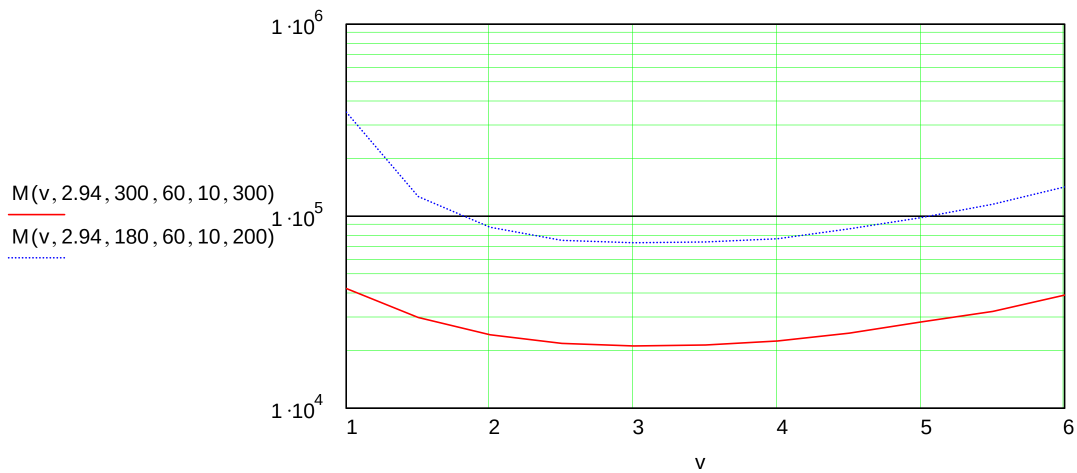
$$M_{sbi}(v, v_e, c) := 30 \cdot \left[\frac{1.05 \cdot e^{\frac{v}{2 \cdot v_e}}}{1 - \left(0.087 + 0.1 + \frac{0.0009 \cdot v_e \cdot c \cdot g}{v}\right) \cdot \left(e^{\frac{v}{2 \cdot v_e}} - 1\right)} \right]^2$$

Canavan's Eq. 6 for 2-stage SBI with penalties for payload, fuel, and thrust, assuming constant acceleration c and KV mass of 30 kg; fig. 3 for his option 5:



$$M(v, v_e, t_{bo}, t_d, c, h_{bo}) := N(v, t_{bo}, t_d, c, h_{bo}) \cdot M_{sbi}(v, v_e, c)$$

Total constellation mass



v =	$\frac{M(v, 2.94, 180, 60, 10, 200)}{1000} =$	$\frac{M(v, 2.94, 300, 60, 10, 300)}{1000} =$
1.0	347.9	41.9
1.5	126.4	29.6
2.0	87.5	24.1
2.5	74.8	21.7
3.0	72.7	21.1
3.5	73.4	21.3
4.0	76.2	22.4
4.5	85.8	24.6
5.0	98.1	28.1
5.5	115.3	31.9
6.0	142.0	38.8