# MODELLING OF HEURISTIC EVALUATION StRATEGIES <br> IN GAME PLAYING: LINEAR AND CONFIGURAL <br> EFFECTS IN OTHELLO 

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## APPROVAL SHEET

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# of Dissertation: Modelling of Heuristic Evaluation Strategies in Game Playing: Linear and Configural effects in Othellc 

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Psychological research on problem solving began with Thorndike's work on trial and error learning with cats, dogs, and monkeys. Kohler later initiated research with apes which convinced rim trat problems coula be solvea with insight. Through the 1940's, the study of ruman problem solving focused on general principles (following the Gestalt tradition) and $S-R$ mechanisms to explain how people solve problems.

The advent of computer technology in the 1950's spurrea research in artificial intelligence, game playing, ana problem solving. Formal definitions of problems outlined the components of a problem, constituting the problem representation. This provided a framework for computer scientists to mechanize problem solving with algoritrms of
search. Computer scientists met with success in aeveloping programs to work on well-defined problems, such as games ana puzzles, where the components of the problem representation are easily stated. Once the representation is adoptea, solution is a matter of search.

It has been shown that the efficiency of mechanized search is aided by the use of a "heuristic evaluation function" (Nilsson, 1971), which has a form similar to psychological models applied in research on ruman decision making ana juagment (Slovic and Lichtenstein, 1972). Samuel (1959), used a regression model of human judgment based on the knowleage of skilled checkers players in order to proauce a heuristic evaluation function for a checkers playing program. Another model which can also be used to proviae a heuristic evaluation function is based on Anaerson's (1962) technique of functional measurement. This approach allows estimation of subjective scale values for the levels of information components relevant to playing a game.

In contrast to these linear models, Edgell (1978) has arguea that people can utilize configural information when making judgments, an issue which has been avoiaed by most decision modelling research. Samuel (1967) showed that use of configural infermation by a heuristic evaluation function can augment the skill of a checkers playing program, but the question of whether human players use such information was
not researched.

This paper reports one pilot experiment and two other experiments which were conducted to investigate whetrer people do use configural information when evaluating alternative moves in a game situation. The effects of game experience, learning, and training on use of configural information were examined. In addition, the research was conducted in a game playing situation in order to adaress the issue of ecological valiaity (Neisser, 1976) in psychological researcr. As Newell and Simon (1972) rave argued, a good psychological theory of how a good chess player plays chess should play good chess.

## DEDICATION

To My Parents, Robert V. Phillips, Jr., and Nancy Phillips.

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I thank my advisor, Dr. Kent L. Norman, for his encouragement, support, and precious advice. In addition, I thank the members of my committee, Dr. Nancy Anderson, Dr. Dave Horton, Dr. Chuck Rieger, and Dr. Sig Soli, for their guidance.

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## CHAPTER 1

## Introduction

How does a person learn to play a good game of chess or checkers, or Othello, for that matter? More specifically, how does one learn what constitutes a good move? What factors are involved and how does one use them to evaluate potential moves? This research will attempt to answer these questions. In doing so, a comparison will be made between models of human judgment based on algebraic information integration (Anderson, 1974) and models based on configural pattern learning (Edgell, 1978). These models postulate that people use decision functions which are based on available information about the alternatives. The information integration approach assumes that people evaluate attributes of an alternative independently and then integrate these evaluations to arrive at an overall impression of the alternative. The configural models suggest that people gain an impression of an alternative by responding holistically to the total configuration of all available information. This research will be set in a typical game-playing situation in order to model the decision strategies of players as they gain experience in
the game of othello.

It is suggested that modelling human decision functions is an important part of studying human problem solving behavior in general. An overview of problem solving research will be presented and the role of decision modelling will be discussed. An important contribution, examined in the present research, is the ability to measure the development of evaluation skills as individuals gain experience in a problem domain. With experience, people learn to judge the goodness of alternatives by evaluating their attributes. Models of a person's subjective decision or evaluation function can be constructed and tested at various stages of the person's experience. This type of research can indicate how people develop and modify their decision strategies as they gain experience in solving problems.

## A Brief Overview of Some Problem Solving Research

Discussion on problem solving dates as far back as 300 A.D., the time of the Greek mathematician Pappus (Polya, 1945). In the seventh book of his "Collectiones", Pappus describes an area of study he calls "analyomenos", which can be interpreted as "art of solving problems", or even "heuristics", a term which will be defined later in a formal
context. The study of problem-solving behavior in psychology probably began in 1898 with Thorndike's work with cats, dogs, and later monkeys (Thorndike, 1898/1970). This was part of his dissertation on animal intelligence. In a typical experiment, a cat or dog was placed inside a "puzzle-box". Careful observations were made of the attempts of animals to escape from a box by activating a latching mechanism. With experience, an animal got better at puzzle-box tasks, and Thorndike hypothesized a build-up of associative strength as model for this learning process. His conclusion was that animals simply learn by the chance formations of associations in their random experience and that they show no evidence of inferential reasoning. Thorndike defines reasoning as "... the function of reaching conclusions by the perception of relations, comparison, and inference ... " (p.184). Even today it is not unreasonable to conceive of much problem solving and thinking occurring from the effects that follow a trial and error process.

Kohler (1927), of the Gestalt tradition, undertook research on the "mentality" of apes for two reasons. The first reason was to ascertain whether apes can behave with intelligence and insight under conditions which require such behavior. The second reason was to investigate the pure nature of intelligent acts, chiefly by observing the types of behavior and errors produced by apes. Kohler chose apes as the
subjects for his research because of their close phylogenetic proximity to humans, compared to that of ats, dogs, and monkeys used by Thorndike. Kohler's definition of "intelligence" (p. 3) was:
"As experience shows, we do not speak of behavior as being intelligent when human beings or animals attain their objective by a direct unquestionable route which clearly arises naturally out of their organization. But we tend to speak of intelligence when, circumstances having blocked the obvious course, the human being or animal takes a roundabout path, so meeting the situation."

Kohler conducted a series of studies which convinced him that apes were capable of insight in the solution of problems. The typical scenario begins with the ape introduced to a banana which is out of reach. The ape attempts to obtain the fruit with behaviors which have been appropriate in other situations, but which are no longer fruitful (grabbing, jumping, etc.). At this point, the animal paces the room anxiously. Suddenly, the animal's behavior changes completely (insight has occurred), and the ape quickly solves the problem (eg., it pulls a crate beneath the hanging banana, jumps on the crate, and grabs the fruit). In trial and error learning, an animal slowly gets better at a specific task. Kohler argued that his apes had gained insight because when they were again exposed to
the same situation they performed the task perfectly.

Kohler rejected the associationistic doctrine, on the grounds that it cannot predict the finding of insight. He states that problems are solved from a grasp of the structure of the problem and from knowledge of the relationships between elements of the problem.

Problem solving research through the l940's and l950's focused on Gestalt-oriented theories, following the work of Kohler (Maier, 1940), and $S$ - R theories, following the work of Thorndike and Hull. Maier (1940), characterized a problem situation by the fact that it blocks behavior. He describes a problem operationally: "If a situation is presented to an animal and elicits neither native nor acquired responses which remove the animal from the situation and if in addition the motivation is such as to demand such removal, then the animal is confronted with a problem." Maier cites the "string problem", among others, to argue the existence of certain behavior mechanisms used in solving problems.

The string problem requires the individual to tie the ends of two strings together, where each string is hanging from the ceiling. However, the strings are separated such that the individual cannot grasp the two strings at the same time. The solutions, Maier lists, fall into four patterns,
(1) extending the reach by the use of a pole or some other object, (2) increasing the length of one of the strings by fastening an extension to it, (3) anchoring one string at a mid-point between the strings, (4) fastening a weight to one string, and thus constructing a pendulum.

The possibility of each solution can be manipulated by leaving certain objects in the room with the subject. Maier postulates three separate behavior mechanisms which can lead to the solution of such problems. The first, called "variability of behavior", is a trial and error process whereby the individual tries various approaches to solving the problem in an apparently random fashion. The second, called "equivalence relations", is a mechanism whereby individuals try solutions which have been useful in similar problems. Finally, he argues for a process called "spontaneous integration", in which a problem is solved by the "integration" of two or more past experiences. He claims that spontaneous integration is different from the others because it involves a "new combination" of objects (the use of a pair of pliers tied to the end of one string).

Many of the problem situations studied in the 1940's and l950's were similar to that of Maier's, in that a novel function or operation for an object provided the solution. Duncker (1945) found that the use of a box as a physical support in the solution of a problem was unlikely to be
discovered by people who had witnessed the box being used as a container. The problem is solved because of a new way of thinking about the problem or the available materials. Insight problems are resolved when the appropriate structure or problem representation has been found. Maier explained that the inadequate design of problems had been responsible for much confusion in the psychological investigation of problem solving. He stated, "Each experimenter chose his own pet problems and then generalized on the nature of problem solving. Thus the use of certain problems led to the notion that problem solving was a matter of trial and error and could be reduced to learning; transfer of training experiments supported the notion of generalization and concept formation; and detour experiments supported the insight hypothesis."

Maltzman (1955) probably epitomized the approach of S-R theorists in the study of problem solving. He drew heavily on the terminology of hull and on the concept of "habit family hierarchies" to explain how people select which approaches to try in solving problems such as the string problem. A habit family hierarchy is a priority ordering of separate responses which are evoked by a stimulus. If an organism is not reinforced for the first response in the hierarchy, it generates the second response and later responses until it is reinforced or the set of responses in
the hierarchy has been exhausted. Where the stimulus is a complex problem situation, Maltzman hypothesized that the responses are alternate approaches to solving the problem.

Most of this early work did not generate testable hypotheses about problem solving skills, and as Maier pointed out, the conclusions drawn from the research were related more to the type of problem chosen, rather than psychological principles. In modern terms, the information processing done by the subjects of these experiments would best be called problem representation, rather than solution finding. Problem representation refers to the process of developing a framework for finding a problem solution. This will be defined formally in a later section.

In the l950's, computer scientists began work on programming computers to play games intelligently and to solve logic problems. Shannon (1950) reported methods for programming a computer for playing chess, while Newell and Simon (1956) and Newell and Shaw (1957) developed "The Logic Theory Machine", followed by the "General Problem Solver" or GPS (Newell and Simon, 1961). The success of these and similar programs was in part due to, and in part the cause of research on the formal representation of problems (Nilsson, 1971). This success also encouraged a new look at the elements of a theory of human problem solving (Newell, Shaw, \& Simon, 1958). This theory views the human as an active


#### Abstract

processor of information, rather than an organism constrained to respond mechanically according to $S-R$ associations.


The ability of computer programs to begin to emulate human intelligence was largely due to the fact that they worked in well-defined problem domains, such as games and puzzles where the problem structure and legal transformation rules are given. With the development of formalizations of problems came opportunities for psychology to address new issues of human problem solving. By studying the behavior of humans in well-defined problem domains, such as the game of chess, and logic puzzles, research on problem solving could focus on psychological skills involving decision processes and judgment.

Scientists in computer science and psychology soon found themselves working on similar problems, natural language comprehension, problem solving, pattern recognition processes, and others. However, differences in the philosophies of artificial intelligence (AI) scientists and cognitive psychologists have begun to become a cause for criticisms between the two areas (Miller, 1978). Miller's thesis is that computer scientists are interested in developing adequate theories which describe problem solving processes, done by machine. To the extent that the machine emulates human performance, the theory on which machine
performance is based is also accepted as a model of human thought processes. He places psychologists in the role of theory demonstrators, rather than developers. Psychologists's main efforts, according to Miller, are directed toward demonstrating the correctness of one theory versus another with the collection of empirical data. According to Miller, computer scientists are claimed to believe that no adequate theories exist because most psychological research is done in a vacuum, and that psychologists see the work of computer science as little more than the development of programs which may be interesting, but which offer no insight toward human thought processes.

Miller's analysis, though it may seem a bit pessimistic, includes the suggestion that scientists in both areas might profit from an examination of their own approaches to scientific investigation. It may be appropriate for some psychologists to embark on a bit more theory development while it may behoove some $A I$ scientists to make more of an effort at theory demonstration.

Newell and Simon (1972) outline their comprehensive theory of problem solving. According to them, a good information processing theory of good human chess players should play good chess. They also mention that their theory posits internal mechanisms of great extent and complexity, and
their research is an endeavor to "make contact" between those mechanisms and the visible evidences of problem solving. They also add that their theory is non-statistical in the sense that it is not easily testable with routine statistics. Their philosophical orientation matches that of Miller's analysis of the AI scientist, or theory developer.

In fact, the philosophical disparities between $A I$ and cognitive psychology are not dissimilar to those between associationistic and Gestalt psychology. Associationists endeavored to prove the existence of each new theoretical mechanism with empirical data while Gestaltists posited broad perceptual principles and sought to make contact between them and the visible evidences of human perception. The justification for this approach used by Gestaltists and AI scientists is the same. The topics under consideration, it is claimed, are much too complex for development of comprehensive theories using the empirical approach.

The contemporary study of problem solving may be roughly split into two main areas dealing with judgment and search. These two areas have been researched by theory demonstrators and theory developers, respectively. There exists a large body of psychological research and methodology in the area of human judgment and decision making to provide a foundation for research on judgment in problem solving situations. This type of research has followed the
empirical approach. On the other hand, there exists a great deal of work in artificial intelligence on sophisticated search methods used in mechanized problem solving. Computer scientists (such as Newell and Simon) have based their models of human search processes on their knowledge of the information processing requirements for implementing search on machines, as well as on psychological work on the capacities and limits of humans as information processors.

Elegant formalizations of problem solving have been developed as part of the work in artificial intelligence. An examination of one formalization, the state-space representation (SSR) will help to define the nature and inter-relation of judgment and search in the formal problem solving process. This will also provide a framework for reviewing the contributions of specific research in the study of problem solving.

In the following sections, the SSR will be discussed, along with work in cognitive psychology and AI. It will be shown that one vital part of the problem solving process (the heuristic evaluation function) may best be studied following the lines of work established by cognitive psychologists on human decision making and information integration processes. Multiple regression models and information integration models will be reviewed as possible models for human evaluation functions. Research in a game-playing
situation will be proposed, and some pilot data will be presented.

## State-space Representations

A state-space representation consists of four elements (1) a set of state descriptions, (2) an initial state, or a set of initial states, (3) a goal state, or a set of goal states, and (4) a set of operators, which transform one state or a set of states into a single state. Formally, the first three elements may be sets of infinite size, while there must be a finite number of operators. Nilsson (1971) and Winston (1977) offer more detailed accounts of SSRs.

Any problem is solved in two major steps. First, a problem representation must be adopted. That is, the four elements must be defined. Secondly, the solution of the problem is found through continual applications of the operators to eventually transform an initial state into a goal state. This process is called search.

As an example, consider a 2 X 2 sliding tile puzzle. Figure 1 depicts the four elements in a possible SSR for a 2 X 2 puzzle problem.

Given the initial position, the problem is to successively move tiles which border on the blank position into that space until the configuration of numbered tiles matches that
(1) State description: A $2 \times 2$ array of cells, individually labelled with permutations of the elements (1, 2, 3, -).
(2) Initial state:

(3) Operators: Interchange the blank position (hyphen) with either orthogonally adjacent tile.
(4) Goal state:


Figure 1. The four components of a problem representation for the $2 \times 2$ sliding tile puzzle.
in the goal state description. Now that the four elements of SSRs have been defined for this problem, a state-space graph can be constructed, and it is shown in Figure 2.

The graph is a group of inter-connected state descriptions. Each state description is called a node, and the doubleheaded arrows connecting them are called edges. The initial state is labelled at the top of the graph, and the goal state is also labelled. Each arrow signifies the application of an operator to transform one state into another. The double-heads indicate that an operator may be applied to return to a state which was just left. Trees, which constitute a subset of SSRs, are structures in which such retraction is not allowed. They will be the main structures discussed in later sections.

Any path from the initial state to the goal state is called a solution path. When an operator is applied to a state to produce a new state, the former state is called the parent and the latter is called the successor state.

The choice of this $S S R$ for this problem was based on its clarity as an example. The problem of choosing a good representation in general is unsolved and lightly researched. Nilsson (1971) provides some detailed comments on this topic. Depending on the choice of state descriptions and operators, what might be a practically insurmountable


Figure 2. The state-space graph for the sliding
tile puzzle.
tile puzzle.
problem under one SSR could turn out to be trivial under another.

Newell and Simon (1972) give an interesting example with a game called "Number Scrabble". In this game, two players label nine slips of paper with the numbers one through nine. Players take turns in drawing the slips, without replacement, from the pool. Players know which numbers they are drawing, and the object is to be the first player with three slips whose total is 15. With experience, players cannot get a feel for a strategy. However, if the game is presented under another representation, players have little difficulty in improving their games. Figure 3 shows one possible representation for this game.

When the numbers are arranged in a $3 \times 3$ magic square, as shown in the figure, one can compare the operation of drawing a number to that of placing the drawer's mark in the appropriate position of the diagram. The numbers are so arranged that the sum across any row, column, or diagonal is 15, and it becomes clear that "Number Scrabble" is a convoluted version of Tic-Tac-Toe.

Choosing the best representation for a problem depends largely on the capacities and limitations of the problem solver that will be working on the problem. Even among computers, the differing architectures will dictate the


Figure 3. Magic square representation for the game 'Number Scrabble'
relative efficiencies of possible SSRs. On the most conventional large computer systems, the operation of multiplication is performed by successive additions. On the Cray-l, perhaps the most powerful computer of the 1970 's, multiplication is best performed by looking up the answer in multiplication tables. The problem of representation is so involved that $A I$ research has best been directed toward developing efficient methods to search state-space graphs in general. These techniques will be discussed in the following section.

## State-space search

In this section, some important $A I$ work concerning statespace search techniques will be discussed. The "heuristic evaluation function" will also be discussed, which will provide the major link between the science of cognitive psychology and artificial intelligence.

Nilsson (l971) provides a detailed account of the two major search methods, breadth-first and depth-first. For clarity of presentation, it is advisable to concentrate on "trees" rather than graphs. Trees form a subset of graphs because each state can have only one parent state. A glance at Figure 2 will show that any state may have two parent states, depending on how the graph is traversed (i.e., each state is pointed to by two arrows), so this is not a tree.

In a breadth-first search, a tree is searched by first generating all successors to the initial state. Then, for each of these nodes in the first generation, its successors are generated, creating a second generation. Then, the third generation is created, and so on. When the successors for a state are about to be generated, the parent state is checked to see if it is a goal state. If so, search is terminated with success.

In a depth-first search, an operator is applied to the initial state to generate a single successor. Then, a single successor node is generated from this first generation node to yield a second generation node, and so on until an mth generation node is generated, where $m$ is a pre-set depth limit. If the depth limit is reached or if it becomes impossible to generate another successor for a node because all have been generated, then another successor is generated from its parent.

The breadth-first and depth-first searches are called admissible because if a solution node exists in the tree, it will be found. Note that these two algorithms make disparate demands on the processor. Consider a small tree, associated with the game of Tic-Tac-Toe. This tree has nine successors to the initial state (the first player may choose nine positions to play in), each of these has eight successors, and each of these has seven, and so on. In order to search
the first five generations of this tree for a solution state (a player has won the game), the breadth-first algorithm requires that the processor store 9!/4! (l5l20) states in memory. The depth-first method requires the storage of only six states, one for the initial state and one for each generation searched.

Both of these methods are called blind search methods because the procedure for choosing which successor to generate is done independently from any information gathered during the search.

## Heuristic Evaluation Functions

The use of a heuristic evaluation function allows the assignment of a numerical value to each generated state, which should reflect the probability that the state is on a solution path to the goal state. With this function as a guide, the most promising nodes may be evaluated first, and search time may be reduced dramatically. This is where the role of judgment comes into play. It has been shown that employment of heuristic evaluation functions can speed up the search by reducing the number and extent of "blind alleys" pursued during search, without sacrificing the important property of admissibility.

To illustrate a potential heuristic evaluation function typical of those used in AI, consider a $4 \times 4$ puzzle problem
approached with the use of a heuristic evaluation function. One possible function is a distance measure, whose value is the sum of the distances of each tile from its desired position in the goal state. The heuristic procedure for searching the space is simple. First, generate all of the successors to the initial state. To each successor, associate its distance measure, as calculated above. pick the state with the minimum distance value for expansion. Continue to expand all un-expanded nodes, and always pick the one with the minimum distance value.

But how are these functions determined? Usually, the programmer draws on personal experience in the problem domain. This state of affairs is a major invitation to psychological modelling research. For any given problem domain, it is possible to model the "heuristic evaluation function", or decision model, of human problem solvers with empirical research.

Samuel (1959, 1967), in two landmark papers concerning machine learning in the game of checkers, performed some psychological modelling of master checker players as part of his work. His model for their heuristic evaluation function was

$$
\underline{R}=\underline{w}(1) * \underline{x}(1)+\underline{w}(2) * \underline{x}(2)+\ldots+\underline{w}(p) * \underline{x}(p),
$$

where the $x^{\prime} s$ are information components, such as center
control, piece advantage, and mobility, and the w's are regression weights which prescribe optimal weighting for the components. Samuel (1959) ran hundreds of game simulations which performed the mechanical adjustment of the weights, having surveyed master players for their opinions on relevant dimensions. The program played a very good game of checkers, and Samuel's work is now a classic in the AI literature.

Later, Samuel (1967) revised his model of feature evaluation to include interaction components for the game dimensions, which necessitated further game simulations to estimate the weights for these interaction terms. However, no empirical data were collected, and it is not clear whether master checker players do respond configurally. In addition, Samuel did not model the scale values that master checker players perceive for various levels of the information components, an endeavor that has recently been pursued by psychologists working in information integration theory.

## Subjective Evaluation Functions

Information integration is a general term for the process performed when someone makes a judgment based on one or more components of information relevant to the judgment. Brunswick (1956) proposed the lens model, which considers the human to be an active processor of information. This
model is based on the assumptions of multiple regression, which Samuel (1959, 1967) used in his work. Brunswick noted that many phenomena in the real world, for example amount of rainfall, can be predicted by a number of variables, such as cloud density, temperature, and barometric pressure, allowing for some error. An empirical multiple regression equation can be calculated for these variables, which will form a basis for predicting rainfall. In addition, human observers can be exposed to these variables and asked to form rainfall predictions based on them. From their responses, it is possible to derive a best guess at what their personal prediction equation is, thereby deriving a model of their cognitive prediction mechanism.

Anderson (1962, 1970) was not willing to accept some of the assumptions of the multiple regression model, namely that people perceive the predictive values (cue validities) of the predictors to be on an interval scale. He has studied various judgment tasks and proposed different algebraic models which describe how people scale and combine information components to produce judgments.

One of these models, which will also be used for the present research, is called the constant weight averaging model. According to this model, the response is computed as a weighted average of the subjective scale values of relevant information.

$$
\begin{equation*}
\underline{R}=\underline{w}(1) * \underline{s}(1)+\underline{w}(2) * \underline{s}(2)+\ldots+\underline{w}(\underline{p}) * \underline{s}(\underline{p}), \tag{2}
\end{equation*}
$$

where $\underline{R}$ is a response and there are $p$ information components relevant to a decision. The weight of component $\underline{p}$ is $\underline{w}(\underline{p})$, constant across all levels of the component, and $s(p)$ is a subjective scale value which will vary according to the particular presented level of component $p$.

Very recently, Fox (1980) distinguished between static and dynamic models of decision making, a distinction which roughly parallels that of judgment versus search. Dynamic or process models of decision making should, according to Fox (1980), incorporate a representation of dynamic components, such as transfer of control between cognitive mechanisms. They should also include consideration of the capacity limitations of working memory and representation of the structure of memory.

Static models describe how humans weight and combine various components of information before making a decision. For example, Anderson (1962) describes an experiment wherein subjects are asked to rate the "likability" of hypothetical people when subjects are provided with adjectives describing the people. The adjectives had been pre-scaled individually on their "likability", and the purpose of the study was to model how subjects map the input information onto a response scale with information integration theory. slovic and Lichenstein (1972) also review algebraic models of cue
learning and information integration based on the multiple regression model. A major criticism of information integration theory and most multiple regression models is that they have tended to ignore the possibility that people respond to patterns or "configurality" of the relevant information, rather than consider the components independently. Edgell (l978) presents some work which indicates that people can combine or perceive information components in a configural manner.

Norman (l974) pioneered research which involved the training of people to respond in accordance with a fixed algebraic model of information integration. Other research (Norman and Phillips, Note l) explored how subjects may learn the subjective scale values or weights of a constant weight averaging model (CWAM) with different transfer effects, dependent on training condition.

In many real-life situations, people are confronted with decisions which are amenable to solution with the CWAM. However, adjustment of the scale values and weights is done along with experience in many decision situations. Often, the problem solver does not know what the optimal weights and scale values are, but does have a good idea about the relevance of various information components (Dawes, 1979).

Such is the case when a person is learning how to play a new
game. At some point in deciding the best move, the player must decide on the promise or value of alternative game situations which might result. It is hypothesized that a linear decision model might adequately model the cognitive process of evaluating the promise or "goodness" of a game situation. As a player gains more experience with a game, it is hypothesized that he or she is going to change the decision function to produce more ecologically valid evaluations (ones which will allow winning moves to be judged most valuable).

The popular game Othello was chosen for the following research. The reader is referred to Appendix A for background on the history and rules of the game. Othello was preferred over chess for several reasons. It is shorter in duration, with even serious games lasting less than an hour. It is easy to learn. The average number of available moves is around ten, as opposed to 40 or so in chess. Othello was preferred to both chess and checkers for the interesting property of its end-game. Rather than having more available moves as the game draws to a close, the player has fewer. Thus, a computer program has a brute-force advantage over the human in the end-game. Use of the game Othello also provides an opportunity for testing the ecological validity of decision models acquired in this research. Once the parameters for a decision model have been estimated, the
model can be incorporated into an Othello playing program as a heuristic evaluation function, and performance of the model in the problem domain can be assessed.

A pilot experiment was performed in which subjects rated the "goodness" of hypothetical Othello moves which were described solely by information on the position of the played piece and the number of pieces captured by the move. The intent of this research was to investigate the existence of configural effects and to identify changes in the component effects which occur with learning (See Appendix B for details of the experiment).

A factorial presentation of nine position types paired with three levels of pieces captured or flipped (1, 3, or 5) allowed estimation of parameters in an analys of variance model:

$$
\begin{equation*}
\underline{Y}=p(\underline{i})+\underline{f}(\underline{j})+p \underline{f}(\underline{i}, \underline{i}) \tag{3}
\end{equation*}
$$

where $\underline{Y}$ is the rated goodness for a move, $p(\underline{i})$ is the effect of the move's position, $f(\underline{j})$ is the effect of the number of pieces flipped by the move, and $p f(\underline{i}, \underline{i})$ is the interaction effect, or the effect of a specific configuration of position and pieces flipped.

Seven non-Othello players participated and were taught the rules of the game at the outset of the experiment. A booklet of 54 hypothetical moves (two factorial presentations of the
$9 \times 3$ design), was filled out by each subject after learning the rules of the game. Each subject then played two games of Othello against a computer program which chose its moves randomly, and they filled out another booklet after each game to determine the effects of game experience on their use of the information components.

In individual subject analyses, it was found that the component effects of Position and Pieces flipped were significant. Also, individual differences were found for these two components. It was found that some effects changed with game experience, and that individual subjects changed their assessment of pieces Flipped differently over game experience.

Analysis of configural effects revealed that some subjects did respond to configural information, but they did not do so consistently in each booklet.

An analysis of variance was performed over all subjects. This indicated that the components of Position and Flips were significant, but the interaction term was not.

These results indicate that when the responses of subjects are modelled with a linear model, changes in the parameters of the model can be found due to learning. In addition, the use of configural information by beginning othello players is almost non-existent, or at best inconsistent.

Also, as part of the pilot study, data was gathered from the third nationally ranked Othello player (M.W.) in order to estimate parameters for his response model. Then, a heuristic evaluation function based on his model and a heuristic evaluation function based on the model for the average subject were pitted against each other in a series of Othello games. All of these games were won by the evaluation function of the ranked player. This provided a form of ecological validation for the modelling process.

## CHAPTER 2

## Experiment I

The pilot experiment found that beginning othello players change their use of information on the position of a move and the pieces flipped by an othello move with game experience. However, the decision modelling was done in an analysis of variance paradigm, rather than in the context of modern psychological decision theory. Experiment $I$ was conducted in order to allow for complete parameter estimates of a constant weight averaging model (CWAM) and to investigate the effects of adding a third information component to the design, Countermoves. Countermoves refers to the number of alternative moves available to the opponent on a given turn. The employment of three information components allows for the use of an experimental design which provides data adequate to obtain parameter estimates in a CWAM (Norman, l975).

In this design, subjects are asked to base their opinion of a move using information for two information components in any given judgment. A complete set of problems is composed of three subdesigns, each presenting all combinations of the levels of a pair of information components.

Each participant played a total of three games in this experiment, and each subject responded to four booklets of problems. This allowed parameter estimates for decision models at four stages of experience, and it provided additional data for a test of configural decision models. The three information components, Position, Pieces Flipped, and Countermoves, are all frequent dimensions of discrimination suggested by good othello players in the literature (viz. The Othello Quarterly). Since the information components are based on those suggested by Othello players, rather than artificial dimensions suggested by the experimenter, it is felt that data collected on these dimensions will accurately portray the decision models used in real play. It may be that one or more dimensions are used configurally, while another is evaluated in an additive fashion.

To keep the number of responses at a reasonable level, the component of Position had five levels, rather than the nine used in the pilot experiment. These corresponded to five classes of board position which are mutually exclusive and exhaustive subsets of the 64 Othello board positions. Figure 4 replicates a diagram and accompanying legend presented to the subjects during their instructions.

This grouping of positions was based on discussions of

| $C$ | $A$ | $S$ | $S$ | $S$ | $S$ | $A$ | $C$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $A$ | $A$ | $L$ | $L$ | $L$ | $L$ | $A$ | $A$ |
| $S$ | $L$ | $M$ | $M$ | $M$ | $M$ | $L$ | $S$ |
| $S$ | $L$ | $M$ | $M$ | $M$ | $M$ | $L$ | $S$ |
| $S$ | $L$ | $M$ | $M$ | $M$ | $M$ | $L$ | $S$ |
| $S$ | $L$ | $M$ | $M$ | $M$ | $M$ | $L$ | $S$ |
| $A$ | $A$ | $L$ | $L$ | $L$ | $L$ | $A$ | $A$ |
| $C$ | $A$ | $S$ | $S$ | $S$ | $S$ | $A$ | $C$ |

C CORNER
A ANGLE
S SIDE
M MIDDLE
L LANE

Figure 4. Classification of the board positions into five types.
similar and dissimilar strategic values of the positions in the Othello literature. The corner positions are most valuable, because they are "stable" once occupied (they cannot be captured). The angle positions are least valuable because a move there, adjacent to a corner, may provide the opponent with an opportunity to play into the corner. Moves in a middle position are usually considered neutral. Based on discussion with tournament champions and casual othello players, the casual players tend to prefer side positions over lane positions. Champions, however, do not seem to show a strong preference.

One group of novice subjects played a program which picked its moves randomly (Group N-Ran), while another played a program which played a better game of Othello (Group N-Max).

Since the object of the game is to acquire a majority of the discs by the end of the game, it is hypothesized that beginners will prefer moves which capture more pieces. However, it is also hypothesized that as subjects gain experience in the game, they will weight the factor of Pieces Flipped as less important, and they may even preter moves which capture fewer pieces. This is because a piece majority in the beginning or middle of a game often results in a loss of a choice of good moves in the end game.

Frey (1980) analyzed verbal protocols of people learning to
play Othello, and he posited that people learned positional values with game experience. This research will also allow a test of that hypothesis.

It was also hypothesized that, depending on the skill of the opponent, novice players would learn to use the information components differently in their judgments.

Three highly ranked othello players also responded with their values for a set of hypothetical moves. Since they are skilled othello players, it was hypothesized that they would utilize both linear and configural information in their judgments.

Norman (1980) has stated that if information is combined according to a weighted averaging model, then certain relations will hold among the relative effects of the information components in each subdesign. Specifically, the rule states,

$$
\begin{align*}
& \underline{m}(\underline{A}, A \times B) / \underline{m}(\underline{B}, A X B) \\
& \quad \star \underline{m}(\underline{B}, B X C) / \underline{m}(\underline{C}, B X C) \\
& \quad  \tag{4}\\
& \quad \underline{m}(\underline{C}, A X C) / \underline{m}(\underline{A}, A X C)=1,
\end{align*}
$$

where, for example, $\underline{m}(\underline{A}, A X B)$ represents the magnitude of effect of factor $A$ in the $A X B$ subdesign. This product rule can also be re-stated by substituting the mean square error of a factor for the magnitude of effect. This rule follows from the assertion that the ratio of effects between any two
attributes is a constant irrespective of informational context. This assertion also implies the weaker statement that the relations between the magnitudes of effect of the factors in the subdesigns will suggest a transitive ordering of the importance of the information components. The data collected in this experiment will be used to test these statements.

In this experiment, all games played by the subjects were recorded to enable tests of the predictive validity of linear and configural judgment models.

To test the ecological validity of the CWAM, the responses of one champion player (M.W.) will be modelled, along with the first set of responses for every novice player, with a CWAM. These models will then be used as the heuristic evaluation functions for an othello playing program which will pit one model against another. It is hypothesized that the model based on the responses by M.W. will win a majority of games.

## Method

Subjects. Two groups of six subjects and one group of three subjects participated. Subjects in Group N-Max and Group N -Ran were University of Maryland undergraduates participating for course credit. The three subjects in Group M (Master players) were those of seven players who
responded to a booklet of 78 problems mailed to them. All of these players were among the 12 best Othello players in the United States, according to the U.S.O.A. rating system.

Apparatus. All subjects received test "booklets", each presenting information on three components of information and requiring the subject to rate the "goodness" of hypothetical moves. Each test booklet consisted of two repetitions of the three possible subdesigns (a) Position by Pieces Flipped (b) position by Countermoves (c) pieces Flipped by Countermoves.

Computer runs acquired information on the distribution of the number of available moves and Pieces Flipped for the first 46 moves of an Othello game. A series of Othello games was played in a Monte Carlo fashion on Challenger-lp microcomputer, wherein moves were chosen randomly. Appendix C describes the analysis in more detail. This was done to allow representative levels of the information components to be used.

The component of pieces Flipped had three levels (1, 2, 4). The component of Countermoves also had three levels $(5,10$, 15). The component of position had five levels, corresponding to five classes of positions which have similar strategic values. A complete set of all problems in these subdesigns number 39 , and a test booklet therefore had

78 problems.

Each test booklet was presented on a Challenger lp microcomputer to participants in Groups $N-R a n$ and $N-M a x$. Problems were randomly ordered within two blocks of 39. Champion players received printed booklets. Figure 5 displays a sample problem, as it appeared in the printed booklet.

The format of each problem was identical for subjects who viewed them on the video monitor, except for minor features. For these subjects an upward-pointing arrow appeared under the line scale at the middle position. Subjects could move the arrow to the left (Bad) end of the scale by pressing the left shift key on the Challenger's keyboard. Likewise, the arrow could be moved toward the right (Good) end of the scale by pressing the right shift key. When the subject had positioned the arrow at the desired place, a touch of the "escape" key registered the response and triggered the presentation of the next problem. The BASIC program used for presentation of these problems is listed in Appendix $C$. Appendix $D$ contains instructions to the subjects on the use of the response scale.

Subjects in groups $N-R a n$ and $N$-Max also played a game of Othello against the Challenger. A listing of the strong program is in Appendix $C$. Briefly, the strong program


Figure 5. Sample problem in Experiment 1
captures a minimum number of pieces during the beginning and middle game (moves one through 46), and a maximum during the end game. It also avoids angle positions and prefers corner positions over all others. In addition, it checks those of the opponent's countermoves adjacent to any move position under consideration.

The Othello board was presented on a 62.8 cm . (diagonally measured) video monitor as an $8 \times 8$ matrix of white dots ("period" characters) on a black background. The letters "A" through "H" were used to label the columns, and the numbers one through eight were used to label the rows. The white pieces were represented by a graphics character resembling a white diamond, and the black piece was represented by a black circle, with a white lining. The program began by presenting the initial configuration of the othello board. After a pause of seven seconds, the program, which always played Black (Black moves first), made its first move. A black disc blinked on and off three times in the chosen position, after which each captured white disc was converted to a black disc. The pause for each blink and conversion was approximately. 5 seconds. The player indicated the position of a move by keying first the appropriate letter and then the number of the position of the move. The play of the move was illustrated by the program in the same manner described above.

The program, upon completion of a game, printed the message "AN INTERESTING GAME!", and then displayed the final number of discs possessed by the human opponent and itself.

Procedure. The experiment was performed in two sessions, the first lasting for approximately 70 minutes and the second for 110 minutes. The second session was always performed within 48 hours of the first session. Figure 6 is a schematic of the partitioning of subjects into groups and of the sequence of events for each subject.

On the first day of experimentation, subjects were trained to play othello and were screened for further participation in the experiment. The experimenter first asked all subjects whether they had ever played Othello or heard anyone discuss strategy for the game. Only naive subjects were allowed to participate. The subjects began the experiment by reading a section on learning to play Othello from an introductory book on the game. The experimenter had censored any mention of strategy in this text. The experimenter then showed the subject a diagram of an Othello board in a mid-game situation. The experimenter pointed out empty squares on the diagram, and the subject responded by pointing out which enemy pieces would be captured by a move to the position, if any. If none could be captured, the subjects had been told to identify the position as an illegal move. After the subjects had correctly responded on


Figure 6. Sequence of events for novice subjects in Experiment 1.
five consecutive trials, this testing procedure was terminated.

Subjects were then scheduled to rate the goodness of 78 hypothetical othello moves. The experimenter read each subject the instructions shown in Appendix $D$ and responded to any of the subject's questions. The subject was shown how to adjust and register a response by running the "booklet" program through some sample problems.

After the set of 78 problems was completed, subjects took a six minute rest while the experimenter recorded the data and loaded the othello playing program into the microcomputer. Subjects were told that they were to play the computer in a game of othello. They were told that it was possible to beat the program, but it was also possible that they would lose, and they were instructed to try their best. The game typically lasted 25 minutes.

After the game, those subjects who lost the game were thanked for their participation and told that the experiment was over. Those subjects who won were scheduled to return from 24 to 48 hours later. They were asked not to discuss the experiment with anyone, nor to play a game of Othello. Three subjects were not invited back because they were not native English speakers, so they had difficulty understanding the instructions, and they could not finish
the session in a reasonable period of time. Seven subjects did not win their game, and and they were not asked to continue.

Upon returning for their second day, subjects were reacquainted with the nature of the information components. They were also reminded of the procedure for responding to the "booklet" program and allowed to ask any questions about their task. Subjects responded to the 78 problems and then played two games of Othello against the computer. Each game was succeeded with a set of 78 problems. Each game was immediately followed by a four minute break and the first two booklets were each followed by a six minute break. This time was required to record the acquired data and to set up the next computer program.

On the second day, subjects in Group N-Ran played both of their games against a program which picked its moves randomly, while subjects in Group $N-M a x$ played a stronger program. All subjects in Group N-Max lost both games in the second session. Only one subject in Group $N$-Ran lost a game during the second session.

## Results

Analyses were performed to examine the use of the information components by the beginning players when rating the hypothetical moves. In this section, results are
reported as significant at the $p=.05$ level. A test of the transitivity of the importance of the information components was conducted. Analyses were also performed to test the use of linear and configural information of subjects in actual play. Finally, a test of the ecological validity of using a CWAM was performed.

Beginners' use of information. Three analyses of variance were performed on each of the four booklets for each player. These analyses were performed on the position $X$ pieces Flipped, Position $X$ Countermoves, and Countermoves X Pieces Flipped subdesigns, respectively. Appendix $E$ contains a table of the magnitudes of effects found in these analyses. Table 1 reports the number of analyses in which the main effects and interactions were found significant in each of the three subdesigns.

Table l shows that roughly half of the analyses on main effects were significant, while only nine of 144 tests of interactions were significant. Since seven of these tests were expected to be significant by chance alone, this supports the hypothesis that subjects tended to use linear rather than configural models. At least for those subjects who showed no configural effects, the linear assumption of the constant weight averaging model seems justified.


Table 1. Summary of significant effects found in the individual booklet analyses.

Champions' use of information. Analyses of variance were also performed on the data for the three champion players (M.W., M.S., and J.C.). Two of the players (M.W. and M.S.) showed significant configural effects for the Countermoves $X$ Position and Pieces Flipped by Position interactions, respectively. These analyses further showed that M.W. and J.C. used the linear components of Pieces Flipped, Position, and Countermoves in their responses. Subject M.S. used only the components of Position and Countermoves in his judgments. Figure 7 displays the Position $X$ Pieces Flipped means for M.S., and also for J.C. The pattern of means for J.C. is similar to that of M.S:, though this interaction was not significant $\quad(\underline{F}(8,8)=3.30$, MSe. $=.775$, $p=.056)$. The pattern of these interactions suggests that champion Othello players weight the information on Pieces Flipped only minimally when the move is to corner or angle position. They utilize information on pieces Flipped more fully when a move is being made to other positions.

All champions also evaluated moves which capture fewer pieces as being more valuable. The effect of pieces flipped was not significant for subject M.S., but his means are included in Figures 8 and 9. The graphs show that player M.W. had a stronger weighting for pieces Captured, as the range of his means is larger.


PIECES
FLIPPED
$\begin{array}{ll}1 & \square \\ 2 & \square \\ 4 & \square\end{array}$

Figure 7.
The Position $X$ pieces Flipped interaction for subject M.S. (top panel) and J.C. (bottom).


Figure 8 champions in the $F \times P$ subdesign.


Figure 9. Mean rated goodness for pieces flipped
by champions in the $c \times p$ subdesign.

The effects of game experience. All data for each beginning player was grouped to perform analyses of variance to test learning effects for each subject. Since booklets were filled out at four stages of experience, the factor of Booklet had four levels in this analysis. Changes in the use of linear or configural information are therefore confirmed by an interaction with the factor of Booklet. For each novice subject, three analyses of variance were performed for each of the three informational subdesigns crossed with the factor of Booklet. Tables 2 and 3 summarize the results by indicating significant effects for the subjects in each subdesign. The learning effects are shown by interactions with the factor of Booklet and the information components of Position, Pieces Flipped, and Countermoves. Subjects tended to weight the factors of position and Countermoves as more important as they gained game experience. Also, people tended to weight the factor of pieces flipped as less important in determining the goodness of an othello move. More of the subjects who played the stronger opponent showed learning effects than those who played the weaker opponent, which indicates that they are more often changing their decision functions.

These analyses revealed significant learning effects for many subjects, and they also revealed that four of the subjects did respond to configural information over


Table 2. Significant effects for subjects who played the weak opponent.


Table 3. Significant effects for subjects who played the strong opponent.
booklets.
The effects of opponent's skill. To test the effects of Opponent (random vs. strong), analyses of variance were performed on the last booklet over all subjects in Groups $N-R a n$ and N-Max. Interaction of any information component with the factor of opponent support the hypothesis that people may change their decision functions dependent on type of game experience. Summary tables of these analyses appear in Appendix E.

In the position $X$ pieces Flipped subdesign, the opponent $X$ Position interaction was significant $(\underline{F}(4,48)=3.02$, MS. $=1.93$, $\mathrm{p}=.027$ ). This interaction is shown in Figure 10. It appears that both groups agreed on the absolute values of corner and angle positions, but they were relatively higher for those who played the strong program, compared to the other positional values. This is compatible with the notion that people who played the stronger program became more aware of the strategic value of the corner. However, it seams that the group did not learn, over three games of experience, the danger of the angle position. This is commensurate with Frey's (1980) statement that beginning players first learn the value of the corner position, and they later learn the danger of the angle position. In the Position $X$ Countermoves subdesign, the Countermoves $X$


MS. $=5.52, \underline{p}=.039$ ). Figure 11 portrays this interaction. This indicates that people who played the strong program became more aware of the importance of limiting the opponent's countermoves.

In the Countermoves $X$ pieces Flipped subdesign, the opponent $X$ Pieces Flipped interaction was significant $(\underline{F}(2,24)=3.82$, MS. $=1.87$, $\mathrm{P}=.036$ ). Figure 12 shows this interaction. Players who played the stronger program believed that it was better to capture more pieces. Apparently, the idea that it is better to capture fewer pieces does not emerge for players with only three games of experience.

Tests of the product rule. For each subdesign analysis on each of the 48 booklets, the magnitude of effect for the two main components were calculated (Finer, 1962). However, in 20 booklets, at least one of the main factors had a magnitude of effect of zero, eliminating these data from consideration for a test of the product rule. Alternatively, the mean square error for each factor was used. Tables 4 and 5 list these mean squares, along with their product as computed by equation 4. The product rule predicts a value of 1.0. The distribution function for these products is unknown (Norman, 1980); consequently, a goodness of fit test cannot be offered.
However, a weaker prediction can be tested. The weighted


Figure 10. The Opponent $x$ Pieces Flipped interaction in the Position $X$ Countermoves subdesign in Experiment $I$.


Figure 11. The Opponent $X$ Countermoves interaction in the Position $X$ Countermoves subdesign in Experiment 1.


Figure 12. Opponent $X$ Pieces Flipped interaction in the Countermoves $X$ Pieces Flipped subdesign in Experiment 1.


Table 4. Mean squares of main effects and products for subjects in Group N -Max

| Subject | Book | EP |  | Subdesign$C P$ |  | CF |  | Product |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | F | P | C | P | ${ }^{C}$ | F |  |
| MS | 1 | 72.9 | $2 \cdot t$ | 104.0 | 6.0 | 88.2 | 50.4 | c.e. ${ }^{\text {c }}$ |
|  | 2 | 74.1 | 2.9 | 07.7 | 4.9 | 54.2 | 40.4 | $<.48$ |
|  | 3 | 53.2 | 7.8 | 00.3 | 7.0 | 31.5 | 43.2 | -50 |
| * | 4 | 46.0 | $5 \cdot 0$ | bs. 9 | 13.1 | 33.2 | 42.0 | 1.70 |
|  | 1 | 144.4 | 25.9 | 207.7 | 52.6 | 176.4 | 126.1 | 1.53 |
| MN | 2 | 53.0 | 28.7 | 185.4 | 25.9 | 96.2 | 80.7 | - 31 |
|  | 3 | 31.3 | 34.8 | 185.2 | 8.4 | 146.7 | 88.4 | . 07 |
|  | 4 | 22.6 | 344.9 | 28.2 | 310.9 | 63.7 | 35.1 | 1.31 |
| BA | 1 | 32.9 | 10.9 | 123.1 | 11.1 | 47.2 | 41.2 | - 31 |
|  | 2 | 62.8 | 20.1 | 119.2 | 30.2 | 96.0 | 45.5 | $1 \cdot 67$ |
|  | 3 | 42.7 | 18.9 | 130.1 | 29.2 | 104.4 | 56.7 | -93 |
|  | 4 | 128.4 | 24.2 | 82.4 | 42.9 | 70.7 | 77.1 | c. 53 |
| DM | 1 | 17.0 | 10.3 | 288.3 | 6.8 | 174.5 | 4.2 | $1 \cdot 62$ |
|  | 2 | 22.0 | 5.0 | 621.0 | 1.7 | 318.5 | 13.5 | - 22. |
|  | 3 | 32.4 | $2 \cdot 6$ | 585.8 | 1.0 | 343.5 | 18.7 | - 39 |
|  | 4 | 18.0 | 9.1 | 329.7 | 15.2 | 213.5 | 3.5 | 〕.56 |
| MR | 1 | 2.5 | 18.6 | 22.6 | 21.1 | 33.2 | 1.5 | c. 78 |
|  | 2 | 1.0 | 25.7 | b. 4 | 30.6 | 55.5 | - 5 | 24.47 |
|  | 3 | . 9 | $4 \cdot 3$ | 65.2 | 12.9 | 47.4 | . 7 | $2 \cdot 80$ |
| BR | 4 | . 4 | $2 \cdot 8$ | 74.1 | 13.3 | 50.2 | - 5 | 2.57 |
|  | 1 | 45.9 | $9 \cdot 8$ | 1.2 | 22.8 | 7.7 | 35.1 | 14.95 |
|  | 2 | 2.1 | 11.1 | 4.0 | 37.3 | . 4 | 11.5 | . 06 |
|  | 3 | 10.2 | 33.8 | . 6 | 47.3 | 1.6 | 19.4 | 1.96 |
|  | 4 | 16.5 | 45.9 | - 2 | 66.9 | - 1 | 10.7 | - E7 |

Table 5. Mean squares of main effects and products for subjects in Group N-Ran.
$-61-$

| Case | $F: P$ | $C: P$ | C:F | Ordering |
| :---: | :---: | :---: | :---: | :---: |
| 1. | $F>P$ | C)P | $C>F$ | $C>F>P$ |
| 2. | $F>P$ | C>P | $C<F$ | $F>C>P$ |
| 3. | $F>P$ | $C<P$ | C) $F$ | Intransitive |
| 4. | $F>P$ | $C<P$ | $C<F$ | $F>P>C$ |
| 5. | $F<P$ | C>P | $\mathrm{C}>\mathrm{F}$ | C) ${ }_{\text {c }}$ F |
| 6. | $F<P$ | C) P | $C<F$ | Intransitive |
| 7. | $F<P$ | $C<P$ | C) F | $P>C>F$ |
| $\varepsilon$. | $F<P$ | $C<P$ | $C<F$ | $P>F>C$ |

Table 6. Transitivity implications for all possible inequality relations between mean squares in the subdesigns.
averaging model predicts a transitivity of effects for the information components in the subdesigns. Table 6 presents an exhaustive list of all eight possible inequality relations among mean squares in three subdesigns, along with their transitivity implications. Only two of the eight patterns suggest intransitivity. Tables 4 and 5 were interpreted with this information to yield a transitivity implication for each booklet. These are listed in Table 7. Assuming the mean squares are randomly distributed, 25 percent of the cases, or 12 of 48 , would be expected to be intransitive. However, two cases were dismissed from consideration because the values of the mean square error for two information components were equal. A binomial test reveals that the two intransitive cases are significantly fewer than would be expected, thus supporting the transitivity hypothesis.

Behavioral predictions from judgment models. The above analyses have shown that subjects do use the information components when judging the "goodness" of Othello moves. The transitivity results also indicate that these data in the three subdesigns can be combined to arrive at a decision model based on all three information components. However, a predictive test of the linear and configural judgment models must be made to show that subjects may be using these models in actual play. A model was constructed for a

| SUBJECT | Group N-RAN |  | Group N-MAX |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | BOOKLET | TRANSITIVITY RELATION | SUBJECT | BOOKLET | TRANSITIVITY RELATION |
| MS | 1 | C c $^{\text {F }}$ P $P$ | PS | 1 | $C>F>P$ |
|  | 2 | C)F>P |  | 2 | Intransitive |
|  | 3 | $F>C>P$ |  | 3 | $C>P>F$ |
|  | 4 | $F>C>P$ |  | 4 | $C>P>F$ |
| MN | 1 | C)FっP | KK | 1 | $P>C>F$ |
|  | 2 | C $\$ F $>P$ |  | 2 | $C>P>F$ |
|  | 3 | $C>P>F$ |  | 3 | $C>P>F$ |
|  | 4 | $P>C>F$ |  | 4 | $P>C>F$ |
| BA | 1 | C)F>P | DS | 1 | $F>C>P$ |
|  | 2 | $C$ C ${ }^{\text {P }} P$ |  | 2 | $C>F>P$ |
|  | 3 | C)F>P |  | 3 | $C>F>P$ |
|  | 4 | $F) C>P$ |  | 4 | $P>C>F$ |
| DM | 1 | $C>F>P$ | DG | 1 | $F>C>P$ |
|  | 2 | C)F>P |  | 2 | --- |
|  | 3 | $C>F>P$ |  | 3 | F>C ${ }^{\text {P }}$ P |
|  | 4 | $C>F>P$ |  | 4 | $P>F>C$ |
| MR | 1 | C>P>F | MS | 1 | $F>C>P$ |
|  | 2 | $C>P>F$ |  | 2 | $F>C>P$ |
|  | 3 | $C>P>F$ |  | 3 | $C>F>P$ |
|  | 4 | C $\triangle P$ P $F$ |  | 4 |  |
| BR | 1 | $F>P>C$ | SS | 1 | $F>C>P$ |
|  | 2 | $F>P>C$ |  | 2 | Intransitive |
|  | 3 | $F>P>C$ |  | 3 | C>F $>$ P |
|  | 4 | $F>P>C$ |  | 4 | $C>P>F$ |

Table 7. Priority ordering for information components among booklets in Experiment 1.
booklet of the form,

$$
\begin{align*}
R & =(\underline{f}(\underline{i})+\underline{f}(\underline{i})) / 2 \\
& +\left(\underline{p}(\underline{j})+\underline{p}^{\prime}(\underline{j})\right) / 2 \\
& +\left(\underline{c}(\underline{k})+\underline{c}^{\prime}(\underline{k})\right) / 2 \\
& +\underline{f p}(\underline{i}, \underline{j})+\underline{c} p(\underline{k}, \underline{j})+\underline{c}(\underline{k}, \underline{i}),
\end{align*}
$$

where $p(\underline{i})$ is the calculated effect for level $\underline{i}$ of factor $p$ (Position) in one subdesign (FXP or CXP) and $P^{\prime}(\underline{i})$ is the same effect in the other subdesign. The $\underline{f}$ and $\subseteq$ terms are similarly defined. The term $f \underline{p}(\underline{i}, \underline{j})$ is the calculated cell effect of level $i$ on the component of pieces Flipped with level $j$ on the component of Position in the FXP subdesign. The CP and Cf terms are similarly defined. This will be referred to as the configural model because the interaction terms are included in the equation. The same model, excluding the interaction terms, will be referred to as the linear model.

For the first stage of the prediction analyses, the linear model was used. A computer program analyzed a booklet of data and computed the first three terms in equation 5. The program followed Othello games between subjects and their computer opponents, reproducing each game during the analysis. It computed a heuristic value, according to the linear model, for every alternative move on a subject's turn. For every game, it based the linear model on the
booklet of responses which the subject had completed immediately before that game. Since only three levels of Pieces Flipped and Countermoves were used in the design of the booklets, the effects for non-represented levels of these factors were interpolated and extrapolated. For the factor of pieces Flipped, the effect of flipping three or more pieces was computed from the slope of the effects for flipping two and four pieces. For the factor of Countermoves, the effect for leaving ll or more countermoves was computed along the slope for the effects of ten and 15 countermoves. The effect for leaving nine or fewer countermoves was likewise calculated from the effects for leaving five and ten countermoves.

A measure was derived to reflect the fit of the subject's choice on a turn with the choice which followed from application of the linear model to all alternatives. A simple proportion measure was considered first. To calculate this, the range of the heuristic values for the alternative moves on a turn would be calculated. Then, the heuristic value of the move which the subject actually chose would be converted to the proportion of the range it covered. In the worst case, where the subject chose the worst move according to the linear model, the proportion score would be zero. In the best case, the score would be one.

However, the number of alternatives on any turn varies, and a proportion score measure would tend to over emphasize the fit of a subject's choice when there is a small number of alternatives. Instead, proportion scores were computed for all of the alternatives. The score for the goodness of fit of the subject's choice was computed as the proportion score for the chosen move minus the average of all of the proportion scores. This adjusted the theoretical mean value (under the null hypothesis) for the subject's choice score to zero, and served to temper the scores for choices made with a small number of alternatives. For example, consider a situation in which a subject has four moves with heuristic values of 15, 20, 25, and 35. The range of values is 20, and the proportion scores for the moves are $0.0,0.25,0.50$, and 1.0, respectively. The average proportion score is 0.44 , and the computed score for the third move is $0.5-0.44=0.06$.

Prediction analyses were performed for three games for each subject, over the first 46 moves of each game. The end-game was not included because subjects had been informed when responding to the booklet of hypothetical moves to consider moves only made in the beginning and middle of a game. On those turns for which the heuristic value of all moves were tied, a choice score was not computed. Since every other move is usually the subject's, 23 or fewer choice scores were computed per game. The mean of these was then tested,

Using a t-test, for its significance from zero. The mean Choice score, along with its standard deviation and t-value is included in Table 8 for every analysis. There appears to be no systematic relation between the means and their standard deviations, so a transformation of the choice scores is not necessary.

In almost every individual analysis, the predictive validity Of a linear decision model was substantiated.

Analyses were also performed to test for the predictive validity of configural effects. For any given turn, a choice Score was computed according to the linear model, and one was also computed according to the full model in equation 5 , or the contigural model. Then, a difference score was computed by subtracting the choice score of the linear model from that of the configural model. Again certain approximations had to be made in the heuristic model because the configural effects were measured at only a finite number of levels in the booklets. Referring to equation 5 , the factor of pieces flipped is indexed by $\underline{i}$ for the main effects and interaction effects. Consider the number of Pieces flipped by a move to be represented by the variable n. For the purpose of indexing the interaction terms, a value of one for $\underline{n}$ was regarded as a one for $\underset{\sim}{i}$. values of two and three on $\underline{n}$ were interpreted as a value of two on $\underset{i}{ }$.


Table 8. Prediction scores and t-values for the linear model for three games for six subjects who played the weak program (first 18 table entries) and for those who played the strong program (last 18 entries).

Values of four or more were transformed to an 1 value of three (level three is four pieces captured). Similarly, if the number of countermoves was seven or less, a $\underline{k}$ value of $l$ was used to index interaction terms. When number of countermoves were greater than 12 , or between eight and 12 , k values of 3 and 2 were used, respectively.

If the subjects were using configural information, then the difference score would be expected to be greater than zero. Table 9 displays the results of t-tests performed on these difference scores.

It can be seen that the hypothesis that configural information was used by the subjects in making their decisions receives no general support.

Ecological validity of linear judgment models. Finally, the decision models should be subjected to a test of their ecological validity. It has been shown that linear models are adequate to predict the behavior of subjects when playing the game Othello, so the next step is to objectively evaluate the validity of the models when used as heuristic evaluation functions.

A parameter estimation program, SIMILE, (Norman, 1979) was used to analyze the booklets for the subjects. It produced parameter estimates for the CWAM (equation 2) using Position, Pieces Flipped, and Countermoves as the


Table 9. Increased prediction scores and t-values for the configural models for three games for six subjects who played the weak program (first 18 entries) and for six subjects who played the strong program (last 18 entries).
information components. In addition, the parameters for a CWAM were estimated based on the booklet filled out by M.W. Even though M.W. showed configural effects, a CWAM was used to capture the main effects in M.W.'s decision model.

A series of 24 Othello games was played to test the ecological validity of using a CWAM as a model of a decision function. An Othello program was prepared which pitted the CWAM of each of the 12 subjects against that of M.W. twice. Once M.W. played Black, and once M.W. played White. The program also avoided moves which would offer the opponent a corner on the next turn. In addition, the program discarded both CWAMs as of move 50 and played perfect end-games for both sides. Table 10 reports the results of these games.

The CWAM modelled on the responses of M.W. won 21 of 24 games, tieing once. A simple binomial test is powerful enough to reject the null hypothesis that the modelling is not ecologically valid. The CWAM proved to be a good theory in that the model for a good Othello player played good Othello. Alternatively, it may be considered to be a good theory of decision functions because the model for a novice player played novice-level Othello.

|  |  |  |
| :---: | :---: | :---: |
|  | M.W. |  |
| Subject | White | Black |
| DM | $28-36$ | $43-21$ |
| MS | $23-41$ | $33-31$ |
| MN | $24-40$ | $51-13$ |
| BA | $14-50$ | $51-13$ |
| MR | $37-27$ | $32-32$ |
| BR | $16-45$ | $41-23$ |
| PS | $22-42$ | $51-13$ |
| KK | $20-44$ | $47-17$ |
| DS | $20-44$ | $45-19$ |
| DG | $27-37$ | 4420 |
| $M S$ | $30-33$ | $41-23$ |
| SS | $13-51$ | $27-37$ |
|  |  |  |

Table 10. Game scores from games of a model of M.W. versus models of novice players. Black's score is recorded first.

## Discussion

The subject analyses revealed that four of 12 completely new Othello players made use of configural information when judging the goodness of othello moves. In addition, two of three champion players also used configural information in their judgments. It is clear that some people can and do consider the attributes of a move configurally in their judgment of its goodness. However, the prediction analyses revealed that though some people may use configural information in their judgments, they may not be using that information in their decision making.

The experiment was successful in verifying a number of hypotheses about how people learn the values of moves in the game. Individual analyses found significant changes in the judgments of players with game experience, depending on all three of the information components. The between subjects manipulation of the opponent's skill also was an effective one. Those subjects who played the stronger opponent realized the importance of Countermoves and the importance of the corner position, compared to those who played the weaker opponent. Both of these strategy developments are more in line with the evaluation functions used by good players.

Finally, the ecological validation analyses confirmed that

CWAMs based on judgment data from beginning players and a very good player can stand on their own in the problem domain.

This experiment proved to be a successful attempt to measure individual strategy changes adopted by subjects as they gain experience in a problem domain and as a function of the strategy of their opponent in a game playing situation.

## Chapter 3

## Experiment II

Experiment II was conducted to investigate some issues that arise from the results of Experiment I. It was found that subjects formed different opinions on the goodness of moves, depending on their opponent's strategy. However, this was confounded with the subject's experience of winning or losing a game. Another question which arises from Experiment I is whether people can use configural information during play. Although it was found that people can make configural judgments, no support was found for the hypothesis that they used them during their games.

Experiment II investigates the effects of training a person to use certain othello strategies on the person's judgment rule. Norman and Phillips (Note l) have shown that people can be trained, with feedback, to respond in accordance with an attribute weighting. In this experiment, subjects are taught to use a decision rule, and they incorporate it into their strategy as they play a game of othello. After the game, they registered their judgments to the set of 78 hypothetical moves used in Experiment I. Subjects were told to respond with their personal opinions on the values of moves, rather than using the training rule. All of the
subjects played a program which was capable of playing the last 11 moves of an othello game perfectly. For half of the subjects, the program did play a perfect end game which enabled the program to beat those subjects in the game. For the other half, the program always chose end game moves which, if followed by perfect subsequent play from the human opponent, would result in a win by the human. If no such move existed, the computer played its worst move based on the assumption of perfect subsequent play. This feature was programmed to enable half of the subjects to win their games, but without their detecting that the program was purposely letting them win.

Three decision rules were taught in the experiment, and this was a between subjects factor. Figures 13 through 15 depict the three decision rules that were taught to the subjects, along with the decision rules used by the computer opponents. All rules required that subjects evaluate a move based on position, pieces captured, and Countermoves.
Subjects were asked to give each alternative move a point value according to the rule and then to always pick the move With the highest point value. Rule 1 was configural because the number of pieces captured either added to or detracted from a move's value, dependent on the position of the move. Rules 2 and 3 were both linear rules which differed in the relative weightings of the information components. It was


Figure 13. Training Rule 1. The computer opponent used the same rule except that for moves to the corner and side positions it subtracted one point for every piece captured and it added one point for every piece captured for moves to the lane or middle positions.
(1) Position
Corner
Angle
$\frac{\text { Value }}{10}$
Lane
0
Side
2
Middle
6
4
(2) Pieces captured: Subtract one point from the value of a move for every piece it captures.
(3) Countermoves: Subtract one point from the value of a move for every countermove which would be available for your opponent.

Figure 14. Training Rule 2. The computer opponent for this rule used the same rule except that it added one half point to the value of a move for every piece it captured.
(1)

| Position | Value |
| :--- | ---: |
| Corner | 20 |
| Angle | 0 |
| Lane | 4 |
| Side | 12 |
| Middle | 8 |

(2) Pieces captured: Subtract one point from the value of a move for every piece it captures.
(3) Countermoves: Subtract one point from the value of a move for every countermove which would be available for your opponent.

Figure 15. Training Rule 3. The computer opponent for this rule used the same rule except that it added one point for every piece that was captured, and subtracted two points for every countermove.


#### Abstract

hypothesized that differences in the use of the information components would be found between winning and losing subjects. It was also hypothesized that group differences would exist for the judgment data which would support the notion that subjects adopt their training rules into their judgment model. The strategy of the computer opponent was also held constant for subjects who learned Rules 2 and 3. This was done so that any differences in judgment models could be attributed to training strategy, rather than opponent's strategy.


## Method

Subjects. Subjects were 36 college students and college graduates who participated for their own playing experience. All subjects had played at least three games of othello and were aware that the corner and angle positions are the most and least valuable positions, respectively. The average number of games of experience for the subjects was 15.

Apparatus. All subjects received test booklets of 78 problems, identical to those given to the champion players in Experiment $I$.

All subjects also played a game of Othello on a Decwriter terminal against a program on the Univac 1100/40 at the University of Maryland. The board was printed at the beginning of each move, with "B", "W", and "-" representing
blacis and white pieces and unoccupied positions. The columns were labelled "A-H", and the rows were labelled "l$8^{\prime \prime}$, as in Experiment $I$. The computer program for subjects in the Win condition played its moves strictly according to the computer decision rule for the first 40 moves of the game. At that point, it began to add one point to the value of a move for every countermove it offered. At move 52, as described earlier, it chose its best move which would allow the human opponent to win. That is, if the computer had a choice between a number of losing moves, it would pick the best one. The computer program for subjects in the Lose condition also followed the computer decision rule, except that it also avoided any move which would give the human a corner on the next turn. As of move 50 , it played a perfect end game.

Procedure. Subjects were run in groups ranging in size from one to five. They were all told that they would play a game of Othello against the computer, and that they would be asked to play according to a specific decision rule, rather than the way they would normally play. The instructions appear in Appendix $D$. The experimenter explained one rule to the group and entertained any questions before getting the subjects started on their individual terminals. After playing their game, subjects responded to a booklet of 78 hypothetical moves, and they were then debriefed and thanked
> for their participation. All subjects who had lost their games were asked if they noticed anything unusual about the way the computer played the end game. No one guessed that the program intentionally lost.

## Results

All results are reported significant at the $\underline{p}=.05$ level. Analyses of variance tables appear in Appendix E.

Effects of training on performance. As in Experiment $I$, prediction analyses were performed. In these analyses, game performance was predicted by the training rule for each subject. These analyses confirmed that subjects could incorporate the training rule into game play. These analyses also found that three of the twelve subjects who were trained on the configural rule did make significant use of configural information while playing their games. Tables 11 and 12 display these results.

Effects of game outcome and training. Separate analyses of variance were performed for each Rule to determine the effect of winning on the use of each information component. For Rule 1 , the effect of Outcome $X$ Position was significant in both subdesigns in which it was tested. People who had lost tended to rate the angle positions more negatively than people who won. Figures 16 and 17 portray these interactions. The Outcome $X$ Pieces Flipped interaction was
$-83-$


Table II. Prediction scores and t-values for the linear model
for subjects learning Rule 2 (top panel) and Rule 3
(lower panel).
$84-$

| $N$ | Score | SD (Mean) | T |
| :---: | :---: | :---: | :---: |
| 22 | . 29 | . 07 | 4.06 |
| 22 | . 40 | . 07 | 4 |
| 22 | +35 | .07 | 4.77 |
| 22 | . 33 | . 08 | 4.23 |
| 22 | . 37 | . 08 | 4.50 |
| 22 | . 36 | . 07 | $5.2 \times$ |
| 22 | . 33 | . 07 | 4.50 |
| 22 | . 25 | . 08 | 3.24 |
| 22 | . 31 | . 08 | 3.78 |
| 19 | . 17 | . 11 | 1.59 |
| 22 | . 13 | . 08 | 1.80 |
| 22 | .29 | . 07 | 4.17 |
|  | - |  |  |
| 29 | . 03 | . 03 | . 93 |
| $\cdots 2$ | . 01. | .02 | . Ai |
| 22 | . 02. | . 02 | . 70 |
| 22 | - 04 | .02 | 2.35 |
| 23 | . 0 | -08 | 2.04 |
| 22 | . 0.1 | . 01 | 1.18 |
| 22 | $\cdots .02$ | . 06 | $\cdots .64$ |
| 22 | . 00 | . 02 | $\cdots .04$ |
| 22 | . 05 | . 02 | 2.67 |
| 19 | . 84 | .06 | . 6.6 |
| 22 | . 00 | . 62 | $\cdots .04$ |
| 2 | . 00 | . 04 | . 07 |

Table 12. Prediction scores and t-values for the linear (upper panel) and configural (lower panel) models for subjects training under Rule 1.


Figure 16. The Outcome $X$ Position interaction for subjects learning Rule 1 in the Position $X$ Pieces Flipped subdesign


Figure 17. The Outcome $x$ Position interaction for subjects
learning Rule 1 in the Position $X$ Countermoves
subdesign.


Figure 18. Outcome $X$ Pieces Flipped interaction for subjects learning Rule 1 in the Countermoves $X$ Pieces Flipped subdesign.
significant in the Countermoves $X$ Pieces Flipped subdesign. This is shown in figure 18. Losing players also tended to evaluate more pieces flipped as valuable, while winners did the opposite.

In addition, the Position $X$ Pieces Flipped interaction was significant for these groups, as would be expected if they had adopted their training rule. However, the Countermoves $X$ Position interaction was also significant, which was not an expected training effect. The factor of pieces Flipped was significant in neither subdesign, which would be expected from application of the training rule. The factors of Position and Countermoves were significant in both subdesigns.

For Rule 2, the factor of Outcome did not interact with any information components. All main effects were significant in all subdesigns, and subjects rated pieces Flipped negatively, as would their training rule. However, the Position $X$ pieces Flipped and Position $X$ Countermoves interactions were significant.

For Rule 3, the Outcome $x$ Position interaction was significant in both tests, with the pattern of the means similar to that for Rule l. These interactions are shown in Figures 19 and 20. Losing players tended to rate corners as more valuable than winners, and angles as less valuable. The

Outcome $x$ Countermoves interaction was significant in the Countermoves $x$ position subdesign, with losers rating more countermoves as less negative than winners. This is shown in Figure 21. All main effects of information components were significant. No interactions between information components were significant.

The above analyses confirmed that game outcome does affect the judgment of the players concerning the goodness of Othello moves.

Differential training effects: An analysis of variance was Conducted over groups learning Rules 1 and 2 to determine if training rule or opponent's strategy affected the judgment $O f$ the player. The interaction of Rule $x$ position and Rule $X$ Pieces Flipped was significant in both cases, as seen in Figures 22 and 23 , respectively. The patterns of the means for these interactions are both as would be expected if the Subjects had adopted the training rule. Corner and angle positions were rated approximately equal for the two groups, but subjects who used the configural rule rated side moves more positively and middle positions more negatively. This may be due to the fact that subjects in the configural group always added points to side positions for pieces flipped and always subtracted points from middle and lane positions. Since a move always flips at least one piece, the configural Subjects may have acquired higher values for side positions


Position

Figure 19. Outcome $X$ Position interaction for subjects learning Rule 3 in the Position X Pieces Flipped subdesign.


Figure 20. The Cutcome $X$ Position interaction for subjects learning Rule 3 in the Position $X$ Countermoves subdesign.


Figure 21. The Outcome $X$ Countermoves interaction for subjects $X$ Position subdesign. learning Rule 3 i


Position

Figure 22. Position $X$ Rule interaction for Rules 1 and 2 in
the Position $X$ Pieces Flipped subdesign.


Figure 23. Rule $X$ Pieces Flipped interaction for subjects learning Rules 1 and 2 in the Position $X$ Pieces Flipped subdesign
and lower values for middle and lane positions.

The pattern of means for Rule $x$ pieces Flipped, as shown in Figure 23, also meets with easy explanation. Means for one, two, and four pieces flipped for the configural group were 10.4, 10.7, and 10.9, in the Position X Pieces Flipped subdesign, and means for Rule 2 subjects were 11.8, 11.3, and 10.5 . Since Rule 1 had an overall weight of zero for Pieces flipped, it may be that subjects relied on the strategy that more is better. Subjects who were trained on Rule 2 used a negative weight for pieces flipped, in accordance with the rule.

Three analyses of variance were also performed over the groups learning Rules 2 and 3. Since both groups played opponents which used the same strategy, differences between these groups can be attributed to the training rules used. The Rule $X$ position interaction is significant in both subdesigns, as is the Rule $x$ Countermoves interaction.

These interaction patterns also have intuitive explanations. For the Rule $X$ position interaction, pictured in Figure 24, subjects in both groups weighted corner and angle positions approximately equal. since subjects already have ideas about the values of corner and angle positions (it was a requirement for participation), it is expected that their values be approximately equal across training conditions.

Subjects who learned Rule 3 ..earned Position is twice as important as was learned by subjects under rule 2. The values of the side position was the second highest while that for the lane position was second lowest. Therefore, if subjects under Rule 3 do weight positions more heavily, their ratings for a side position should be higher and their ratings for lane positions should be lower than those under Rule 2. Ratings for side and lane positions under Rule 2 were 12.86 and 9.08 , while they were 14.74 and 8.37 under Rule 3 training.

The Rule $x$ Countermoves interactions, plotted in Figure 25, have a similar explanation. It may be that since the factor of Position received twice as much importance under Rule 3 , the factor of countermoves received less weight. This is borne out in the judgment data by the pattern of the means.

## Discussion

This experiment also proved successful in supporting hypotheses about how people adjust their strategies in a game playing situation. Specifically, the technique of measuring the move evaluation of subjects between games has provided a sensitive measure of strategy shifts which result from game experience, success at winning, and training.

This experiment also substantiated the use of configural information in judgments by subjects in this experiment.


Figure 24. The Rule $X$ Position interaction for subjects learning rules 2 and 3 in the Position X Pieces Flipped (upper) and Position $X$ Countermoves (lower) subdesigns.


Countermoves

Figure 25. Rule $X$ Countermove interaction for subjects who learné Rules 2 and 3 in subdes Flipped (bottom). and Countermoves $X$ Pieces

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Evidence was also found that some subjects could use configural information in the game situation when trained to do so.
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With training under certain decision rules, across game outcomes, subjects do adopt aspects of the training rules in their later judgments. This result is promising because it indicates that training can have a direct effect on changing a problem solver's judgment model. It has been shown that models of champion othello players can be derived, and it has also been shown that through training subjects can begin to incorporate the training rule. A game situation provides a workable context wherein unskilled problem solvers can develop ecologically valid judgment models automatically with computer feedback.

## Chapter 4

## General Discussion

This research was conducted to address issues that involve problem solving, decision modelling, and ecological validation of psychological research. In the area of problem solving, Newell and Simon (1972) have stated that one area in need of study is the development of problem solving skills. Work in artificial intelligence (Samuel, 1959) has indicated that machine "learning" can occur as an adjustment of the parameters in a heuristic evaluation function. Therefore, this research was designed to identify a problem solving skill in humans which involves their ability to change their judgment models. The results of the pilot experiment and both major experiments not only verified that changes in the judgment models occur, but they also showed that they can be manipulated both through training and by more subtly controlling a player's opponent and the game outcome.

One intent of this research was to show that psychological methods of modelling static evaluation functions are adequate to produce ecologically valid heuristic evaluation functions. The produced functions not only have a
scientific basis in psychology, but they also have practical applications in that they may improve the performance of mechanized problem solvers. One of the practical contributions of this research is its demonstration that decision modelling research has practical applications to artificial intelligence and also to the training of problem solving skills in humans. Skilled problem solvers can supply researchers with the dimensions relevant to their evaluation function and psychological measurement can be applied to determine the component effects and interactions. Models can be derived which use the same weights and scale values as human problem solvers. From these, machines may be programmed to use the models to increase machine problem solving skill. For example, the electronic games popular today may play more challenging games when they are programmed with heuristic evaluation functions based on data collected from skilled players. In fact, "Electronic Othello" (copyright, 1980, CBS Toys), is based on a version of the "strong Program" used in Experiment 3. This program uses a heuristic evaluation function modelled after the champion player, M. W., who participated in Experiment 2.

This research explored the effects of some factors influencing the judgment models of problem solvers. It was shown that linear judgment models can be used to predict the behavior of subjects during game playing. These predictions,
however, were based on an analysis of move values at a one ply level. That is, the simple assumption was made that subjects look only at their move choices without regard to potential countermoves by their opponents. Vo Neman and Morgenstern (1947) have prescribed the minimax method for evaluating the value of a move, based on possible opponent responses. Subjects probably also learn to use this search method for two person games, so it may be that the best prediction for the behavior of subjects would involve estimating their evaluation functions and also gaining information on the depth of a subject's look ahead. A prediction could then be computed by using the derived judgment model in conjunction with a minimax search. This prediction would be even better than those made in the present research.

Trade-offs between judgment and search. Any judgment takes time to make, as does the act of considering future possible sequences of play in a game. If a judgment requires a long time, then a player will not be able to evaluate numerous game possibilities, and the person will therefore be limited to considering shorter sequences of play. Game programmers always strive to strike a happy medium between the accuracy of an evaluation function and the time it takes to make an evaluation.

People, who also face the same trade-off as information processors, probably change their evaluation functions by establishing new heuristic dimensions with experience. Simon and Gilmartin (1973) showed that skilled chess players organized their perception of chess positions based on interrelations among chess pieces. The development of this type of perception was much less advanced in beginning players. The ability to perceive many board aspects as one chunk decreases memory load and speeds up processing, thus allowing more of the human's processing capacity to be used in search. With experience, .skilled players learn what aspects of a move should be chunked. Perhaps even a beginner could play at a champion level if he or she were trained to chunk aspects into some useful dimension, so far unknown.

Humans are better than machines in games such as Chess and Go. It has been suggested that part of this discrepancy may be due to the fact that skilled problem solvers have developed more ecologically valid evaluation functions than game programmers have implemented. It may also be that they have more flexible and effective search strategies than computer programs. While a computer can always look a few plies ahead, in a complete search, humans may learn what lines of play to ignore. One possible method for studying the search process is to program a computer to aid the subject in searching possible lines of play. Especially in
the game of othello, a program which will allow the user to play possible game sequences before deciding on a move would be a useful research tool.

Linear and configural models. A major issue in decision modelling which was examined in this research concerned whether people do or can respond to configurations of information components. Evidence was found which indicated that some subjects do base their judgments of the goodness of a move on configural information. However, in the first experiment, even though four of 12 subjects did use configural information in their judgments, no evidence was found that these people based their move choices on configural information while they actually played the game. In fact, for most subjects in the first experiment, predictions of move choice which included configural information were worse than predictions based on a linear model alone. These results may be due to the possibility that subjects are basing their move selection on factors other than position, pieces Flipped, and Countermoves. Some Othello players attend to such factors as "clusteredness of pieces" and "having an even number of pieces on a side".

The models of information integration theory are very compatible with the types of evaluation functions used in AI work in problem solving. For this reason, this research was conducted to extend the study of static decision models to a
dynamic game-playing situation. Fox (1980) and others have focused their research on process models of decision making. These describe judgment as a process which operates over time. While static models describe the mapping of information components onto a response scale, process models seek to describe the sequence of steps followed by the human information processor while implementing a mapping.

The measurement technique used in Experiment $I$ could be a useful tool for gathering data to test process models. Since subjects register their responses by moving an arrow across a line scale, data on their pauses may add insight to their processes of information integration. If subjects are combining two information components configurally, then a response may occur as a continuous movement of the arrow to the scale point of the judgment. If subjects combine two information components linearly, their responses may occur first as a movement of the arrow to an intermediate scale point which represents the independent contribution of one information component. This would be followed by a pause, as subjects process information relevant to the second component, and then another arrow movement reflecting the contribution of the second component may occur. Informal observations of subject behavior during Experiment $I$ indicate that a judgment is broken down by subjects into component judgments.

Advantages of modelling the opponent. Learning evaluation functions for playing games is unlike learning the weights in a regression equation in cue validity studies. In cue validity studies, there is an optimal weight for information components in the equation which will maximize the accuracy of prediction. To predict or evaluate the goodness of a game situation depends not only on the game under study, but also on the strategy of one's opponent.

Othello, like chess and checkers, is a two-person zero-sum game. One player's win is the other's loss (Luce and Raiffa, l957). However, in games where players cannot foresee the final consequence of a move, it is necessary to introduce heuristic evaluation functions to estimate the probability of a win. Since both players da not necessarily use the same function, one player's perceived advantage is not necessarily the other player's loss in terms of their respective evaluation functions.

In Luce and Raiffa's terminology, the games mentioned above are strictly competitive, but individual moves may not be. That is, since players cannot foresee a win or loss in the early stages of a game like Othello, they do not compete for a total win. Rather, each tries to attain the highest possible heuristic value in minor skirmishes. If a player can model the opponent's heuristic evaluation function, and if it is different than his or her own, then the player may
seek to make moves which entice the opponent to cooperate by following his or her own function. This type of modelling would be useful in winning a game, and it would also reduce processing time in deciding on a move.

It is clear that players do actively attempt to model the strategy of the opponent. The verbal commentary of even beginning Othello players is filled with remarks about the computer opponent's inferred reasons for choosing a move. Surprisingly, these remarks seemed just as frequent for players who played a random opponent as for those who played a strong one.

This type of modelling can be measured by asking subjects to respond to hypothetical moves as they believe their computer opponent would do. This opponent modelling process might also account for the fact that in triads of players it is not uncommon that player A beats player $B, B$ beats $C$, and $C$ beats A. This may be, in Luce and Raiffa's terms, that players win by "exploiting the opponent's weaknesses". Of course, in order to do this, one must have a model of the opponent's strategy.

Summary. This research touched on issues in problem solving, decision modelling, and validation of psychological research. It was shown that some subjects may use configural models in their judgment, and it was further

Shown that modelling a subject's decision function may predict a subject's choices in a game situation. A game situation was used because it provides a well-defined problem solving domain, while also providing a situation where subjects are motivated to perform complex information processing. since problem solving can be split into the areas of judgment and search, research on judgment models can be applied across various areas of problem solving to produce a basis for training skilled problem solvers and increasing the effectiveness of machine problem solving.

Psychological research can provide scientifically valid techniques for establishing heuristic evaluation functions in AI. Other issues of problem solving and decision modelling also prove amenable to exploration with game playing research. Some of these issues involve development Of search strategies, the development of perceptual skills, and modelling of an opponent's strategy.

## Appendix A - History and Background of Othello

In the $1890^{\prime} \mathrm{s}$, an English mathematician by the name of Sullivan created a game he called "Reverse". An article in the London rimes described how one could fashion pieces for the game and play on an ordinary chess or checkers board. The rules differ slightly from othello in that the starting pattern could be any configuration of two black and two white pieces in the center four squares. In addition, rather than the players sharing the pool of 60 pieces, each was limited to 30 pieces once the starting position was set. In the early 1900's two other Englishmen applied for patents for special boards on which to play this game. Since it is impossible to patent a game's rules, these men patented new playing boards. At about this same time, the name of the game was altered to "Reversion", and a series of articles on strategy was authored by Lewis Waterman and appeared in "The Queen", a London based women's magazine. Revers enjoyed a period of popularity in England until it mysteriously lost its favor with the public in the early 1900's.

In 1973, a Japanese chemist rediscovered the game and named it othello, because like the play it is filled with "dramatic reversals". Its popularity in Japan has been growing since 1976. Through the summer of 1979, 25 million sets have been sold in Japan, and over five million sets have been sold in the United States.

In December, 1978, the United States Othello Association (USOA) was formed in Washington, D. C. (Note 2). This association published the Othello Quarterly, and it works closely with CBS Toys, the United States licensee of the game, to coordinate and officiate yearly competitions on the local, regional, and national levels.

## Playing the Game

Othello is played on a chess-like $8 \times 8$ board, except that all squares are green, bordered by black lines. The pieces are 64 discs which are black on one side and white on the other. Figure 26 shows the starting Othello board for every game. The four bold dots on the board, as well as the starting pattern, distinguish Othello from similar games.

Black always plays first. Black picks up one of the unplayed discs and places it (black side up) in an empty board position such that it captures one or more white discs. To capture, black plays adjacent to a white disc, with some restrictions. First, an imaginary ray from black's played piece must travel through an adjacent white disc and continue to travel only through white discs. If the ray hits a blank square or the edge of the board, it is terminated and none of the white pieces is captured. If the ray hits a black piece (an anchor piece), the ray is terminated, and all of the white pieces in a direct line between the played


Figure 26. The starting pattern for othello.
piece and anchor piece are captured. When a disc is captured, it is not removed from the board, but it is just turned over to the other color. The disc or discs which are captured are said to have been "outflanked", in othello terminology, by the played piece.

A played piece may be directly adjacent to up to eight enemy pieces, so it is possible to capture pieces in eight directions simultaneously. In fact, the player must capture all capturable pieces from a play. If a player cannot play so as to capture the enemy, then the turn is lost. A player must capture the enemy if possible on each turn. The game is over when neither player can play on the board, and the player with the most pieces on the final board is the winner. Figure 27 shows some plays for white and their capturing rays.

Playing time ranges from about twenty minutes to an hour. Since four board positions are occupied at the outset of the game, 60 are left, allowing a maximum of 60 moves in a game. Two methods are generally used to record the moves in a game, tournament transcription and coordinate transcription (Phillips, 1979). The convention shown in Figure 27, of labelling the columns "A - $H$ ", and of labelling the rows "l - 8", provides coordinates for every board position. The coordinate transcription of a game consists of a list of at most 60 coordinates corresponding to the played positions.


Figure 27. Four possible moves for black (designated by crosses), and their capturing rays.

Some games may end before move (or play) 60 because it is possible that neither player will have a play before the board is filled.

The tournament transcription is an $8 \times 8$ matrix representing the Othello board. The entry in any cell contains the move number at which that position was occupied at the beginning of the game.

With experience, Othello players learn that certain board positions are more valuable to occupy than others. Hasegawa (1977), in his book on Othello strategy, makes use of a schema for designating each of the ten "unique" board positions on a board. Since, unlike chess or checkers, the player does not move with any set "direction" in mind, rotations of the board do not affect the strategic value of the board position.

## Appendix B - Pilot Experiment

## Method

Subjects. Subjects were 7 undergraduates enrolled in introductory psychology courses at the University of Maryland. They participated for course credit. They had never played the game of Othello before.

Procedure and Apparatus The experimenter instructed all participants on the rules of Othello and on how to make moves in the game. Participants then observed the first 15 moves of an Othello game in order to confirm their knowledge on how to play. They were allowed to ask the experimenter about any of the rules which were not clear.

All participants then received 54 descriptions of hypothetical Othello moves. The descriptions were incomplete in that they did not reveal the entire configuration of the Othello board. Each of the problems presented a diagram of an Othello board with four asterisks in the center squares and a piece placed in one of the 60 remaining positions. A message appeared under the board which supplied the coordinate location of the piece and the number of pieces that it captured. Subjects were told that the asterisks were placed to provide a frame of reference, and that they did not represent pieces of any particular color.

Each booklet was constructed of two blocks of 27 problems. Within each block, each of the nine unique board positions was tested at three levels of pieces flipped (one, three or five pieces). Figure 28 shows the nine unique othello positions. These map onto all unoccupied board positions through the operations of reflection and rotation. This nine by three factorial combination accounts for the 27 problems in each block. The particular board position representing a unique position was chosen randomly, and the problems were ordered randomly in each block for every participant.

Subjects were asked to rate each hypothetical move on a scale from one to twenty for its "goodness". Instructions to the subjects told them to imagine that the moves occurred toward the middle of a game, and that each move was from a different game. They were warned that no other pieces on the board would be presented and that they would not know the positions of the captured pieces. They were told to give a value of twenty to the best possible move, a value of one to the worst possible move, and a value around ten or eleven for average moves.

Participants in Group 1 then played an Othello game against a program which picked its moves at random. They then filled out another problem booklet, played the program a second time, and finally filled out a third booklet.


Figure 28
The nine unique othello positions.

## Results

Figure 29 shows the mean rated goodness of a move by seven participants for all 27 combinations of position and pieces captured. To demonstrate the lack of a Position $X$ Pieces Flipped interaction, the points in this figure are connected even though histograms would be correct.

An analysis of variance was conducted on four factors, Position (9 levels), Pieces Flipped (3 levels), Games Played (3 levels), and Subjects (7 subjects). The replications factor was repetitions of problems in each booklet. Position and number of Pieces Flipped were both significant, which indicates that subjects were using those components of information to evaluate the "goodness" of a move. The factor of Games Played also had a main effect. Subjects tended to rate moves more positively with more game experience. Mean ratings were 11.4, 12.4, and 12.6 with zero, one, and two games of experience, respectively. The analysis also shows that subjects responded differently.

The lack of an interaction between Position and pieces Flipped indicates that the "average subject" was evaluating the information components in an additive fashion, in accordance with the linear model.

The factor of Subjects interacted significantly with Position and pieces flipped, as well as with the interaction


Unique Position

Figure 29
Mean rated goodnes of moves for all 27 combinations of Position $X$ Pieces Flipped.
between Position and Pieces Flipped.

Games Played interacted with subjects. This was not unexpected. Games Played did not interact with the Position by Subject interaction, but it did interact with the pieces Flipped by Subject interaction.

Individual subject analyses were performed in order to assess the appropriateness of a linear model on an individual basis. The analysis did not detect the effect of any interaction for the information components of Position and Pieces Flipped for any subject.

## Discussion

A factorial presentation of levels of two information components was used to measure subjects' evaluations of the goodness of moves in the game of Othello. The analysis indicated that subjects responded to the two information components presented (Position and Pieces Flipped). There was no interaction between these two components for the group, in accordance with the prediction of a linear model. However, there was a significant Subjects $X$ Position $X$ Pieces Flipped interaction, indicating that some subjects may have evaluated the information components in a nonadditive manner. If so, this can be taken as evidence
that some subjects did respond to the information components in a configural manner. However, as reported in the individual subject analyses, for no participant was the Position $X$ Pieces Flipped interaction significant.

The Subjects $X$ Games Played X Pieces Flipped interaction was also significant, providing evidence that some subjects adjusted their ratings for pieces Flipped as they gained experience in the game. This finding is important, because it indicates that some of the learning that occurs with problem solving experience can be described as changes in the effects of information components in a linear decision model.

The individual subject analyses provide confirmation of a linear model used by six of seven subjects. One subject showed no evidence of using the information components at all. Three subjects used only the dimension of pieces Flipped in their judgments. Three subjects used both the dimensions of Position and Pieces Flipped in their judgments. No subject responded to a consistent configuration of Position and Pieces Flipped, as this interaction is not significant for any subject.

However, two subjects did maintain a significant Position $X$ Pieces Flipped $X$ Games interaction, indicating that they may have responded differently to configurations of the two
information components across games. This result is not predicted by a linear model, and it is not consistent with a particular configural model. It might be argued that these two subjects were experimenting with configural models and that perhaps with more game experience they would respond according to a specific configural model.

The fact that six of seven subjects responded to the dimension of pieces Flipped, while only three used the information on Position indicates that pieces flipped may be a more salient or useful dimension.

In these individual analyses, the Pieces Flipped $X$ Games interaction was significant for four subjects. This result indicates, as in the overall analysis, that a linear model can be used to detect parameter changes by subjects learning the game. Therefore, this paradigm may be employed in order to test specific hypotheses concerning the way subjects adjust their decision strategies with experience. Two subjects also obtained a significant Position $X$ Games interaction, indicating that they also adjusted their responses to Position with game experience.

In summary, the experiment demonstrated that beginning Othello players rate the goodness of Othello moves in accordance with a linear model over games. However, two subjects responded configurally within games, but they did
not settle on using any consistent configural information across games. The hypothesis that subjects' learning in the game of othello could be measured by changes in their linear decision model was supported. This last result is particularly promising because it allows for testing hypotheses about how subjects may adjust their decision strategies with experience.

## Appendix C - Monte Carlo Analyses

A series of 200 Othello games was played on Challenger lp microcomputer in order to gather data on the distribution of number of available moves and number of pieces captured by a move. From this information, representative levels of the information components of Pieces Captured and Countermoves were chosen for the hypothetical moves presented to subjects in Experiment $I$. The program chose moves randomly, and played only the first 46 moves of a game because subjects were told that hypothetical moves occurred only in the beginning and middle of a game. Figure 30 displays the frequency polygon for the number of available moves and Figure 31 displays the $\log (2)$ frequency polygon for the number of pieces flipped (captured) by a move.

Listings 1 and 2 show the BASIC programs used to gather booklet data and to play a "strong" Othello strategy, respectively.


Number of Available Moves

Figure 30. The frequencies of having zero to 24 available moves in Othello on a given turn. (measured
over 200 games)


Figure 31. The frequencies for capturing one to eleven discs while playing othello. (Measured over 200 games of

## - REMZ3MAYBOOKLETS

1J DIMVE ( 8,8 )
15 IIMDC(3),F(3),GM(11,2)
1uJ REM
104 DIMRN(39),ST(3),VL(5), RS(78), SS(14)
1US INPUTMSUBJECT NUMEER AND NAME?":SNSS
11: FORI=1TOSN:R=RND(1):NEXT
115 gosubs 20
11
118 FORN $=11039$
119 GOSUEG00
12: gOSUEGCJ:GOSUBSEC:NEXTN
13: NEXTR
135 FRINT*HIT SHIFT TO RECORDN
14: : IfFEEK (57100) $=2546010140$
15: SAVE:PRINTSS:PRINTSS
155 FORI=1TO78:PRINTRS(I):NEXT
10: G0T0135
2JFCRI=1T038:R1=INT(RND $(1) *(39-1)+1)$
21: $T=R N(R 1+I): R N(R 1+I)=R N(I): R N(I)=T: N \times T I: R E T U G N$
25: READT:FORI=1TOT:NS =NS + $9: S S(N S)=S S(N-9)$

255 READTS
¿̄: FORA=1 TOLENSTS):SS(NS)=SS(NS) 1
27: POKESS(NS),ASC(MIOS(TS, J, 1)) :NEXT:NXT:RETURA
$28=F O R J=S S(T-1)+1 T O S S(T): P=P E E X(J): P R I T C H R S(P):: N E X T: R E T U R N$

Listing 1. The "booklet" program.
300 REF
3．5 DATA1，2，4，4，2，3，3，5，5，5
11：FORI＝1TOム：FOR」＝1TOム：：I1＝9－I：J1＝9－J：EADV
$315 V E(I, J)=V: V E(I, d 1)=V: V B(J, I)=V: V B(d, I)=V$
32：VE（I1，J1）＝V：VB（J1，I1）＝V：VB（d，I1）＝V：B（I9，d）＝V：NEXTJ：のEXTI
325 LP＝54181：FF＝LF＋1C
332 FS＝7：DO：PC＝1NT（FS／256）：FOKE134，FC：FKE133，PS－253＊PC
$334 P C=P S+39: M E=P C+39: S S(0)=M E+39: N S=J$
336 $F(1)=F S: F(2)=P C: F(3)=M B$
34：ST（1）＝3：ST（2）＝8：ST（3）＝11
25：T＝O：FORI＝1T05：FORJ＝1T03
$352 T=T+1: T 2=T+15: F O K E F S+T, I: P O K E F C+T, J F O K E M B+T, J$
355 POKEPS＋T2，I：POKEPC＋T2，O：POKEME＋T2．J
358 NEXIJ：NEXTI：T＝3＿：FORI＝1TO3：FORJ＝9T3：T＝T＋1
37：PCKEPS＋T，U：POKEPC＋T，I：POKEMB＋T，J：NETJ：NEXTI
275 FORI＝1TO39：RN（I）＝I：NEXTI
38：DATAコ．MPOSITION＝＂．＂FIECES CAPIURED＝，COUNTERFOVES＝
？ 81 DATA5，＂CORNER＂，＂ANGLE＂，＂LANE＂，＂SIDE，＂MIDDLE＂
382 DATA2，＂ONE＂，＂THON，＂FOUR＂
383 DATA3，＂FIVE＂，＂TEN＂，＂FIFTEEN＂
385 FORK＝1TO4：GOSUE 25 ：$:$ ：NEXT：RETURA
LJC PRINT：PRINT
4.2 PRINT＂ ABCDEGG $H^{+1}$
$4 \div 5$ T＝PEEX（F（1）＋RN（N））
41：$F O R I=1$ TO8：PRINT：PRINTI：$=F O R J=9 T 08$

```
42E IFVL(I,J)=TTHENFRINT**N:=GOTC430
425 PRINT"."';
43. PRIATM m::NEXTA:FRINT:NEXTI
44: RETURIr
5OD AR=16:PS=10:RP=57100
5\5 POKEFF,32:T=10
59: PPP=LP+T:POKEPP.16
E91 OR=_:18=57100:17=254
51: FORI=1T0PS:I9=PEEK(18):IFI9<)ITTHENR=19:NEX1
513 IFDR=222THENS2C
514 IFDK=2 SUANDT>CTHENT=T-1:FOKEPF,32:GTO51:
515 IfOR=252AHDT<22THENT=T+1:POKEPP,3%
596 GOTC59%
52. RS(K*39+RN(N))=T:RETURN
G_O REP
CJS FORI=1TO 3:C=PEEK(RN(N)+F(I).)
9JS IFC=:60T092U
9U8 PRINT:PRINT
99:T=1:GOSUB28U:T=ST(I)+C:GOSUE28C
92: NEXTI:PRINT:FRINT
93: POKELP-32.216:POKELP-12,218
935FORI=LP-31TOLP-13:FOKEI,148:NEXT1
94:-PRINT"BAD":SPC(14):"6000*:
93. RETURN
    999 ENU
```


## 5 REMごSMAY SMARTPROG

1J DIME（100），IN（8），FT（2），MB（8），CB（8）．
15 DIMDC（3），F（3），GM（11，2）

23 OM＝NM：GOSUO5G70
24 IFNF＜16THENEG＝1
25 IFNF＝：GOT030
28 IFNMD：OROM＞OGOTO23
3」 FRINT＂INTERESTING GAME：＂

35 FRINT＂HIT SHIFT TO RECORD＂
26 IFFEEK（571．0）$=254$ GOT036
4J JAVE：FORI＝MO＋GOTOMS＋1STEP－1：PRINTPEEK（I）：NEXI
42 j0T035
1Jこ：FRINT：PRINT：PRINT＂ABCGEFGH＂
1，1：FORI＝2T09：PRINT：PRINT：PRINTI－1：：T＝（1－1）＊10：FORJ＝2TCS

105：FORI＝1TOE：FRINT：NEXT：RETURN
$12:=R=I N T(M V / 1 \dot{C}): C=M V-R * 10-1: P O K E H P+R * 64+C * 2, T$
122：FOR12＝1TOZOO：NEXT：RETURN
ZJUE FORI＝1T0100：E（I）＝O：NEXTI
このこ5 FOKE11，T：FOKE12．253
く」1：$B(45)=1: E(46)=-9: B(55)=-1: E(56)=1$

$2 j 32 D C(1)=2: D C(3)=2$
iv34 MC＝78CO

```
2335 F=INT(MO/256):FOKE134,F:FOKE133,MO-P* 256
235C P=1:M1=MJ+60:M2=M1+40:m3=M2+3:M4=M3+1 20
E36i MS=M4+6 O:CP=O:HF=53447
2:75 POKEM2+1,226:POKEM2+2.46:POKEM2+3.232
2US0 FORI=M3+1TOM3+1EJ=FOKEI,J:NENT
22:C T=ME:FORR=1TOR:FORC=2T09:T=T+1:BP=R*1 O+C
222こ FOKET,BF:NEXT:NEXT:NF=64
23i0 FORI=9TO10:B(I)=2:E(90+1)=2:G((1-9)*1:49)=a:B(I*1E)=2:NEXTI
235. FORI=OTOT:MB(1+1)=INT(2-1+.5)=NEXTI
236: FORI=1TO4:CB(1)=ME(4+1):CR(4+1)=ME(1):NEXT]
```



```
244こ MV=55:GOSUB8 ICC:MV=56:60SUE8:OC
25ここ DATA255,12,19,82,89,88,23,28,73,78
25%5 DATA1 08,13,18,22,29,108,72,75,83,88
251- OATA925.32.62.39.69
2515 DATA125,14,17,84,87
2596 DATA124,15,16,85,86
2517 DATA124,42,52,49,59
252% FORI=1T0100:POKEMS+I,127:NEXTI
253: FORS=1T0B:READV:FORI=1T04
< 54: READT:POKEM5 +T,V:NEXT:NEXT:RETURN
28CC NM=C:FT(U)=M1:LV=1:V=0:GV= 1:OO:MN= O
28:5 HV=-1
281. FORS2=M4+1TOM4+CF:MV=FEEK(J2)
```

$2815 V=\operatorname{PEEK}(M 5+K V)-128: N M=N M+1: 6 C S U B 3: 00$
282CIFFFくFT（U）＋2THENNM＝NM－1：GOTOく850
2821 RV＝RND（1）：IFRV＞HVTMENHV＝RV：TT＝MV
2825 IFF $=-1$ Hengosub 2900
285：NEXT」2：IFNM＜2THENMV＝GM（1．1）：RETURN
2855 IFF＝1THENRETURN
$286: \times 2=-1000=6 \mathrm{G}(11,2)=\times 2$
2862 FORI2＝1TOGN：MV＝GM（12．1）：V＝GM（I2，2）：GOSUB55 ic
2864 1FV1－V2＜＝x260T02868
$2860 \times 2=v 1-v 2: M M=M v$

2895 NEXTI2：MV＝MM：RETURN
$\overline{Z 9 C G} 6 N=N M: I F N M>10$ THENGN $=11$
$29 \because 5 \operatorname{GM}(G N, 1)=M V=G M(G N, 2)=V$
791．IFAM＝1THENRETURN
292：FOFI2＝GN－1TO1STEF－1：IFGM（I2．2））＝VTHENRETURA
793： $\operatorname{GM}(12+1,1)=\operatorname{GM}(12,1): \operatorname{GM}(12+1,2)=G M(12,2)$

300 －$F P=F T(L V-1)+1: P O K E F P, M V: P K=P E E K(M 3+M V)$
？コ1に IfG（MV）＜＞OTHENRETURN

$3 J 3$ 亿 $11=I N(1): M=M V+11: O F=F P: T V=C$


？ 355 IFEG＝1ANDPV＞－15THENPV＝ABS（PV）

3JGC $T V=T V+P V: M=M+11: G 0 T 03040$
?1:C IFE (M)<>PTHENFP=OF=GOT03190
$31: 5 \quad v=v+i v$
31CO NEATI:FT(LV)=FP:RETURN
4JO: $M=M N: L V=1: \operatorname{GOSUB} 3000$


LJE3 MEXTI:FORI=M1+1TOFT(1):T=PEEK(M2+P+2)
4JI: MY=FEEK(I):B(KV)=F:GOSUBIZCL:NEXII
4 ${ }^{\prime} 11$ NC=FT(1)-M1-1
$4 J i S$ DC( $2+F)=D C(Z+F)+N C+1=D C(2-F)=D C(Z-P)-N C$
4,26 NC=FT(1)-1-M1:MV=MN:GOSUBR CCL:RETURN
〈11: $T V=$ PEEK (M):TV=TV+2*(T28-TV):POKEM.TV:NEXTI :RETURN
537
5j72 IFP=1THENGOSUR6000:60T05140
5074 gosub 28 0u
5160 JFNM= GTHENRETURN
$5145 M N=R$
515: GOSUE 3000:GOSUE4COO:RETURN

55 : $C$ IFFPRMT + 2THENPRINTNHAT?N:END
S5:5 FORS=1T08:MV=LM+IN(J)
551: $V=F E E K(M 5+M V)-128: 1 F B(M V)<>560705 \leq 55$

552：OD＝PEEK（M3＋MV）：LV＝2：POKEM3＋MV•MB（1）：60SUE3（00
553：FOKEM 3＋MV，OD：IFFT（1）＋2）FT（2）GOTO5555
55う5 1FV＞V2THENVZ＝V
554：1TET（2）－FT（1）＞DC（2－F）＋FT（1）－K9THENV2＝3こ0
5555 NEXTJ：MV＝LM：GOSUO5900： $\mathrm{B}(\mathrm{MV})=$ ：
556．$E=P: P=-P: L V=1: R E T U R N$
59：：FORI＝M1＋1TOFT（1）：B（FEEK（1））＝F：NEXTI：RETURN
EJこう GOSUG 28 OU：IFNM＝CTHENRETURN
EJE：Fq＝54 16 O：FOKEF1．232：FOKEF1＋1．232
t」Sc $X=U S R(X): K E=P E E K(531): M V=K E-63$
EJЯ5 FOKEF1，Kも：IFMV＞9ORNVく2GOTO6ここて
6Jご $\mathrm{X}=\mathrm{USR}(\mathrm{X}): T=$ FEEK（531）：FOKEP1＋1，T：T＝T－4を
GJこO IFT＞EORT＜1GOTOOGO2

6」5こ KN＝MV＝RETURN
EJSE POKEMO＋NF，MV：NF＝NF－1
$8 \mathrm{j} 32 \mathrm{FORI}=\mathrm{ML}+1$ TOM4 + CF：T＝FEEK（I）：IET＝MVGOTO8：34
عコこミ NEXT1：GOTO8040
EJこム T2＝FEEK（M4＋CF）：POKE1，T2：FOKEK4＋CF，T
$E J 30 \quad C P=C P-1$
8J4J FORI＝1T08：M＝MV＋IN（I）：IFB（M）＜＞0GOT08100
8．J5：T＝A3 + M：PK＝FEEK（T）：IFFK＝STHENCF＝CF＋1：POKEM4＋CF，K
8J6：PK＝PKORMG（1）：POKET，PK
81：C NEXTI：RETURN

## Appendix D -Instructions for Experiments

Subjects were read the following instructions before responding to their first set of 78 problems in Experiment I.

Instructions for Experiment I

You are going to be asked for your personal opinion on the values of Othello moves. The information that you are given about a move will not be complete. That is, you will \#not\# be shown a board configuration pointing out a possible move. Rather, you will be given information on only a few attributes of the move.

For example, you might be asked your opinion of the "goodness" of an Othello move which captures three pieces and is made on the side of the board. This information could describe a number of completely different moves, made in hundreds of different situations. Your judgment should reflect your general opinion of the goodness of any move of this description. This type of judgment may seem unnatural because the provided information is incomplete. However, psychological research indicates that these types of questions can add insight to our understanding of human decision processes, but only if they are answered thoughtfully.

You will be asked to rate the goodness of 78 Othello moves on a T.V. screen with the aid of a small computer. Each of these hypothetical moves is completely independent, so do \#not\# assume that they follow each other in a game. Please assume that each move is from a different othello game and that each move occurs somewhere in the beginning or middle of a game, not in the last fifteen moves of a game.

Three attributes of othello moves will be used to describe them. Before you give your rating of moves, you should understand what each attribute is.

The first attribute is the number of pieces captured by a move. This refers to the number of enemy pieces which a more captures. Each of the moves you are about to rate will capture one, two, or four enemy pieces. Of course, during a game, a move might capture some other number of pieces, but for all of the hypothetical moves you are about to see, either one, two, or four pieces will be captured by the move.

The second attribute is the position of the move. There are, of course, 64 positions on the Othello board. However, for the purposes of this research, five \#types\# of board position have been defined. Please refer to the diagram which indicates these five position types.

The third attribute is countermoves. Your opponent will have a certain number of possible countermoves. Each of the moves you are about to rate will leave your opponent with five, ten, or fifteen countermoves on his/her next turn. This does not indicate exactly what positions your opponent can move to, nor does it tell you how many pieces your opponent will be able to capture. The attribute of countermoves simply tells you the number of possible legal moves your opponent would have available if you made the hypothetical move.

For every hypothetical move, you will be given information on only two of the three attributes. This does not indicate whether the missing attribute is good or bad for that move. You simply will not be given information on one attribute.

## Instructions on the Response Scale

For each hypothetical move, you will be asked to rate the "goodness" of a move on a scale which goes from good to bad. You will be presented with a line scale with the left end marked "BAD" and the right end marked "GOOD". You are to express your opinion of the goodness of a move by indicating a point on the line scale which corresponds to your judgment. For example, if you choose a point toward the middle of the scale, you believe that the move is average in
value. If you choose a point toward the left end of the scale, you believe that the move is bad. Finally, if you choose a point toward the right end of the scale, you believe that the move is good.

Please be careful to save the ends of the scales only for the moves you think are terrible or fantastic. Usually, you will not be using the ends of the scale.

## Instructions for Experiment II

You are going to be asked to play a game of Othello against a computer program and then give your opinions on the "goodness" of certain Othello moves. It is possible to win against the program, but it is also possible that you will lose.

However, when you play your game, you will be asked to choose your moves according to a certain rule, rather than the way you would normally play. Whenever it is your turn, you are to evaluate every one of your alternative moves on the basis of a decision rule, and then choose the move with the highest value. First of all, the rule classifies the positions on the othello board into five basic types, with different strategic values. (Subjects were given a map of the othello board with the five position types, and the
positions were explained to them).

Now that you are familiar with the types of positions, please also refer to your second sheet as I explain how you are to use your decision rule (Subjects were shown the appropriate decision rule, as shown in Chapter 3, and they were allowed to ask questions).

Now, you will play your game against the computer (subjects played a game and were instructed on the use of the response scale before they filled out their booklet).

## Appendix E Summary Tables

This Appendix contains the analyses of variance sumary tables referenced in the text. In addition, it contains tables with estimates of the magnitudes of effect for each factor and interaction term in the booklet data from Experiment I. In the ANOVA tables, position, pieces Flipped, and Countermoves are designated $P, F$, and $C$, respectively. The contents of this appendix are the following.

1. Magnitudes of Effects
a. Subjects playing the weak program, subdesign Position X Pieces Flipped
b. Subjects who played the weak program, subdesign Position $X$ Countermoves
c. Subjects who played the weak program, subdesign Pieces Flipped X Countermoves
d. Subjects who played the strong program, subdesign Position X Pieces Flipped
e. Subjects who played the strong program, subdesign Position $X$ Countermoves
f. Subjects who played the strong program, subdesign Pieces Flipped $X$ Countermoves2. Analysis of Variance Tablesa. Opponent $X$ Position $X$ Pieces Flippedb. Opponent $X$ Position $X$ Countermovesc. Opponent $X$ Countermoves $X$ Pieces Flipped

| Subject |  | $F$ | $P$ | $P \times F$ |
| :---: | :---: | :---: | :---: | :---: |
| M.S. |  | 2.11 | 0 | 0 |
|  | 2 | 2.20 | . 57 | 0 |
|  | 3 | 1.88 | .73 | 0 |
|  | 4 | 1.75 | . 77 | 28 |
| M.N. | 1 | 3.01 | 1.70 | 0 |
|  | 2 | 1.86 | 1.92 | $0^{57}$ |
|  | 3 | 1.20 | 1.99 | 0 |
|  | 4 | 1.08 | 6.68 | 0 |
| B. A. | 1 | 1.48 | 1.16 | . 41 |
|  | 2 | 1.95 | 1.54 | -90 |
|  | 3 | 1.65 | 1.57 | 1.08 |
|  | 4 | 2.84 | 1.74 | . 70 |
| D.M. | 1 | . 75 | 1.06 | .$^{90}$ |
|  | 2 | 1.21 | . 63 | 0 |
|  | 3 | 1.44 | .$^{32}$ | . 41 |
|  | 4 | 1.08 | 1.09 | . 41 |
| M.R. |  | . 29 | 1.24 | . 07 |
|  | 2 |  | 1.76 | .$^{07}$ |
|  | 3 | . 2 | . 75 | - 38 |
|  | 4 | 0 | . 21 | . 38 |
| B.R. |  | 1.74 | 1.06 | . 42 |
|  | 2 | . 28 | .71 | .$^{81}$ |
|  | 3 | . 81 | 2.03 |  |
|  | 4 | 1.04 | 2.47 | 0 |

Magnitudes of effects for subjects playing the weak program in the Position $X$ Pieces Flipped subdesign.

| Subject | Book |  |  | fect |
| :---: | :---: | :---: | :---: | :---: |
|  |  | C | $P$ | $C \times P$ |
| M.S. | 1 | 2.57 | . 76 | ${ }^{0} .27$ |
|  | 3 | 2.08 | .70 .85 | . 27 |
|  | 3 | 1.88 | .85 1.18 | $\stackrel{-}{0}$ |
|  | 4 | 1.92 | 1.18 | 0 |
| M.N. | 1 | 3.84 | 2.20 | 1.38 |
|  | 2 | 3.41 | 1.30 | $0^{77}$ |
|  | 3 | 3.50 | ${ }^{0} 6.40$ | . 89 |
|  | 4 | . 98 | 6.40 |  |
| B. A. |  | 2.82 | 1.11 | 0 |
|  | 2 | 2.81 | 1.90 | $0^{38}$ |
|  | 3 | 2.87 | 1.71 2.37 | 1.09 |
|  | 4 | 2.27 | 2.37 |  |
| D.M. |  | 4.26 | . 71 | 1.09 |
|  | 2 | 6.43 | 4.00 | .33 .80 |
|  | 3 | 6.24 | 0 1.25 | 0 |
|  | 4 | 4.52 | 1.25 |  |
| M.R. |  | 1.10 | 1.34 | 0 |
|  | 2 | . 51 | 1.89 | ${ }^{0} 43$ |
|  | 3 | 2.05 | 1.23 1.23 | . .66 |
|  | 4 | 2.16 | 1.23 |  |
| B.R. |  |  | 0 | 0 |
|  | 2 | . 5 | 2.13 | 0 |
|  | 3 | . 16 | 2.49 | .33 .24 |
|  | 4 | 0 | 2.98 |  |

Magnitudes of effects for subjects playing the weak program in the Position $X$ Countermoves subdesign.


Magnitudes of effects for subjects playing the weak program in the pieces Flipped $X$ Countermoves subdesign.

| Subject | Booklet |  |  | Effect |
| :---: | :---: | :---: | :---: | :---: |
|  |  | $F$ | P | FXP |
| P.S. | 1 | 1.26 | 0 | 1.05 |
|  | 2 | 1.66 2.13 | 3.46 4.07 | . 69 |
|  | 4 | 1.96 | 4.83 | 0 |
| K.K. | 1 | 0 | 2.43 | 0 |
|  | 2 | 1.14 | 1.80 2.57 | 0 |
|  | 4 | $\dot{0}^{\text {a }}$ | 3.18 | . 37 |
| S.G. | 1 | 1.03 | 0 | . 09 |
|  | 2 | 1.53 | 1.56 |  |
|  | 3 | . 45 | 2.31 | . 71 |
|  | 4 | 0 | 2.11 |  |
| D.S. | 1 | 2.95 | 0 | . 48 |
|  | 2 | 2.52 | 1.45 | $0^{68}$ |
|  | 3 | 1.39 | 1.65 | 0 |
|  | 4 | . 87 | 1.78 | 0 |
| D.G. | 1 | . 97 | 0 | 0 |
|  | 2 | . 97 | . 16 | ${ }_{0} 75$ |
|  | 3 | 1.93 .52 | 1. 69 | $0^{75}$ |
|  | 4 | . 52 | 1.38 |  |
| M.S. | 1 | 3.22 | . 71 | 0 |
|  | 2 | 1.85 | . 37 | 0 |
|  | 3 | 2.0 | . 58 | . 80 |
|  | 4 | 1.11 | . 13 | . 46 |

Magnitudes of effects for subjects playing the strong program in the Position $X$ pieces Flipped subdesign.

| Subject | Booklet |  |  | Effect |
| :---: | :---: | :---: | :---: | :---: |
|  |  | C | P | $C \times P$ |
| P. $5 \cdot$ | 1 | 3.92 | 2.02 | ${ }_{1} 1.42$ |
|  | 2 | 3.10 4.35 | 2.97 | . 52 |
|  | 4 | 3.86 | 5.34 | 1.78 |
| K.K. | 1 | 1.28 | 1.90 | . 61 |
|  | 2 | 1.57 | 1.15 | $\cdot^{35}$ |
|  | 3 | 1.40 | 1.50 |  |
|  | 4 | 1.27 |  |  |
| S.G. | 1 | . 09 | . 59 |  |
|  | 2 | . 44 | 1.24 | $0^{37}$ |
|  | 3 | $\dot{0}^{38}$ | 1.85 1.90 | . 48 |
|  | 4 | 0 |  |  |
| D. 5. | 1 | 2.55 | $\mathrm{i}^{21} 37$ | .75 .29 |
|  | 2 | 3.75 | 1.37 | $0_{0}$ |
|  | 3 | 2.78 | 1.11 | . 15 |
|  | 4 | 2.71 | 2.75 | . 15 |
| D.G. | 1 | 1.00 | . 47 |  |
|  | 2 | 1.65 | .25 | . 21 |
|  | 3 | 1.87 | 0 | . 32 |
|  | 4 | 1.14 | 1.86 | . 58 |
| M.S. | 1 | 3.5 | 0 | . 40 |
|  | 2 | 3.62 | 1.27 | 1.00 |
|  | 3 | 4.11 |  |  |

Magnitudes of effects for subjects playing the strong program in the Position $X$ Countermoves subdesign.

| Subject |  | Booklet |  | Effect |
| :---: | :---: | :---: | :---: | :---: |
|  |  | C | F | CXF |
| P.S. | 1 | 4.11 | 3.46 | 1.15 |
|  | 2 | 3.85 | 3.89 | 1.88 |
|  | 3 | 2.81 | 1.29 | . 88 |
|  | 4 | 4.73 | 3.54 | .16 |
| K.K. |  | 2.25 | 1.21 | . 31 |
|  | 2 | 1.39 | . 47 | . 43 |
|  | 3 | 1.71 | . 42 | . 11 |
|  | 4 | 1.97 | 0 | . |
| S.G. | 1 | 0 | . 79 | . 94 |
|  | 2 | . 27 | 2.22 |  |
|  | 3 | . 27 | . 67 | . 19 |
|  | 4 | 0 | . 22 | 0 |
| D.S. | 1 | 1.48 | 2.06 | 0 |
|  | 2 | 2.15 | 2.04 | 0 |
|  | 3 | 2.32 | . 79 | . 58 |
|  | 4 | 1.43 | 1.09 | 0 |
| D.G. | 1 | 1.0 | 1.12 | . 38 |
|  | 2 | 1.1 | 1.0 | 0 |
|  | 3 | . 93 | 1.19 | 0 |
|  | 4 | . 70 | . 97 | . 50 |
| M.S. | 1 | 2.53 | 3.01 | 1.96 |
|  | 2 | 1.74 | 2.44 | 1.22 |
|  | 3 | 2.94 | 2.76 | . 72 |
|  | 4 | 2.47 | 2.53 | 0 |

Magnitudes of effects for subjects playing the strong program in the Pieces Flipped by Countermoves subdesign.

| SUUACE OF VARIATIOH | SUMS OF SQUARES | DEGREES OF FREEDOM | mean SGUARES |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| , | 455.67222 | 2 | 227.83611 | F2.24) ${ }^{\text {F }}$ (.48) $=$ | $76.020 \mathrm{P}=$ | . 000 |
| P | 1324.20536 | 4 | $331: 05139$ |  | 171.73: $81, \mathrm{P}=$ | :599 |
| RP | 15:17778 | 1 | -10000 |  |  |  |
| $\stackrel{R}{\text { F }}$ | 6.21667 | $\frac{2}{4}$ | 3. 10833 |  |  |  |
| ${ }^{\text {pr }}$ | 9.31111 | 8 | 1.1638 |  |  |  |
| ${ }_{5}{ }^{\text {Pr }}$ | 491.18889 | 5 | $9 \mathrm{P} \cdot 23778$ | F5012 $=$ | 48.18, ${ }^{\text {4.75 }}$ P | .001 |
| Fs | 142.3278 | 10 | 14.23278 | FiU:24 ${ }_{\text {F }}{ }^{\text {c }}$ | 35:44, P P | -000 |
|  | 1366.36111 | 20 | 6:90639 | F4U.06) $=$ | 1.20. $P=$ | . 231 |
| FPS | 115.25556 | 5 | 2.47333 |  |  |  |
| KRS | 41.71067 | 10 | 4.17167 |  |  |  |
| PHS | 30.82778 | 40 | 1.57306 |  |  |  |
| FPRS | 92:92222 | 1 | 74:71112 | F1:12) | 36.64: ${ }^{\text {P9, }}$ P |  |
| Fo | 17:572 $52 \frac{1}{2}$ | 2 | A.78611 | F2,24) ${ }_{\text {F }}(2,48)$ | $\frac{3.931}{P}{ }^{\text {P }}$ | .027 |
| Ho | 23.31667 | 8 | -.75833 | FB,96) | $\cdots \mathrm{P}=$ | . 959 |
| FPO | 1:11il | 1 | 1.11111 |  |  |  |
| ${ }_{\text {ROO}}$ | 12.33849 | 2 | 6.16944 |  |  |  |
| H\%ó | . 69444 | 4 | 2.17944 |  |  |  |
| Fpro | 23.35556 | 8 | 32.96444 | F5.12) $=$ | 16.17: P |  |
| 50 | 164.82222 | 10 | 8.16944 | Fiu: ${ }^{4}$ ) E | 2.73: P | .002 |
| $\begin{array}{r}\text { +50 } \\ \\ \\ \hline\end{array}$ | 273.65040 | 20 | 13.68259 | F2u:96) 三 | :49: $\mathrm{p}^{\text {P }}$ | :820 |
| Fryo | 74.16607 | 40 | $\frac{1}{2.1777 R}$ |  |  |  |
| HSO | 10.88889 | 10 | 1.16611 |  |  |  |
| ${ }^{2} \mathrm{HSO}$ | 51.47223 | 20 | $2: 51384$ |  |  |  |
| FBrso | 116.47978 | 40 | 2.01194 |  |  |  |
| Efr betheen | 24.46667 | 12 | 2.038899 |  |  |  |
| EんK F | 72:93333 | $2_{4}^{4}$ | 1:9377 |  |  |  |
| EKK | 232.0066 | 96 | 2.41796 |  |  |  |
| tutal | 5048.78489 | 359 |  |  |  |  |
|  |  |  |  |  |  |  |

Opponent $X$ position $X$ Pieces Flipped
analysis.


Opponent $x$ position $x$ Countermoves analysis


Opponent X Countermoves X Pieces Flipped analysis.

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