TECHNICAL RESEARCH REPORT

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T.R. 96-7

INSTITUTE FOR SYSTEMS RESEARCH

Sponsored by
the National Science Foundation
Engineering Research Center Program,
the University of Maryland,
Harvard University,
and Industry

Blind MIMO FIR Channel Identification Based on Second-Order Statistics with Multiple Signals Recovery*

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ABSTRACT

To separate and recover multiple signals in data communications, antenna arrays and acoustic sensor arrays, the impulse responses of multiple-input/multiple-output (MIMO) channels have to be identified explicitly or implicitly. This paper deals with the blind identification of MIMO FIR channels based on second-order statistics of the channel outputs. We first investigate the identifiability of MIMO FIR channels, and obtain a necessary and sufficient condition for MIMO FIR channels to be identifiable up to a unitary matrix using second-order statistics. Then, we extend the identification algorithms for single-input/multiple-output (SIMO) FIR channels, such as the algebraic algorithm and the subspace algorithm to the identification of MIMO FIR channels. We also present the application of the above blind identification algorithms to the separation of multiple signals in digital communication systems. Finally, we demonstrate effectiveness of the algorithms presented in this paper by computer simulations.

KEY WORDS

blind identification, parameter estimation, multiple-input/multiple-output channel, multiple signal separation

* The work was supported in part by the NSF grants MIP9309506 and MIP9457397.

I. Introduction

Adaptive antenna arrays have been recognized as an important technology to be used in wireless communication systems to increase the capacity and improve the quality of communication services. In wireless communication systems, each sensor may receive a superposition of several input signals with linear distortion, which can be modeled as *multiple-input/multiple-output* (MIMO) systems. The MIMO systems are also encountered in other engineering fields including speech processing, sonar array processing, and in the analysis of biological systems.

When the impulse responses of MIMO systems are obtained, linear equalizers [15], [27] or decision feedback equalizers [1], [4] can be used to remove intersymbol interference, suppress co-channel interference or crosstalk, and recover the original signals. However, in most cases, the impulse responses of MIMO systems are unknown, and there is no reference or training signal available. Therefore, blind identification becomes an important technique to estimate the parameters of MIMO systems.

Blind single-input/single-output (SISO) system identification is usually based on the higher-order statistics of the channel output [8], [16], [20]. For the systems with cyclostationary inputs or single-input/multiple-output (SIMO) systems, several recent papers have presented identification methods based on second-order statistics of the channel outputs. Among them, [10], [13], [19], [21] dealt with the blind identifiability and identification of SIMO FIR channels. Tong *et al* [18] first investigated the blind identification of FIR channels and proposed a singular value decomposition (SVD) method to identify FIR channels under a special rank condition. Li and Ding[10] further proved that this special rank condition is equivalent to the identifiability condition of FIR channel based on second-order cyclostationary statistics. Tugnait [21] also studied the identifiability of SIMO FIR channels from the point of view of common zeros of polynomials. Recently, subspace methods[13], [17] have been proposed to identify FIR channels using cyclostationary statistics of the channel output, which exploits the orthogonality of the noise-subspace and signal-subspace and the structure of filtering matrix. Van der Veen, *et al*, first investigated multiple signal separation and recovery in MIMO systems in [22]. They proposed a blind estimation method for multiple signals using decision-directed principle together with signal-subspace property

This paper deals with the blind identification of MIMO FIR channels using second-order statistics and its application to multiple signal separation. In this paper, we are going to establish a necessary and sufficient condition for MIMO FIR systems to be identified up to an ambiguity unitary matrix using second-order statistics. We also generalize two identification algorithms for SIMO FIR channels to the identification of MIMO FIR channels. To apply these algorithms to multiple signal separation and recovery, the ambiguity unitary matrix has to be estimated. Hence, we also propose an ambiguity matrix estimation algorithm.

The rest of this paper is organized as follows. In Section II, we describe the mathematical model of MIMO systems. We then discuss the identifiability of MIMO FIR channels based on second-order statistics of the channel output in Section III. Next, in Section IV, we prove an *algebraic identification theorem* and a *subspace identification theorem*, which are the extensions of TXK's algorithm[19] and subspace algorithm[13] to MIMO FIR channel identification. Then, we investigate the application of the MIMO identification algorithms in multiple signal separation, and present an ambiguity matrix estimation algorithm in Section V. Finally, Computer simulation is presented in Section VI to demonstrate the effectiveness of these algorithms.

II. Model Description

The mathematical model of d-input/M-output MIMO systems can be illustrated as in Figure 1. The d sequences $s_1[n], \cdots, s_d[n]$ are sent through linear channels $h_{ij}[n]$ for $i = 1, \cdots, M$ and $j = 1, \cdots, d$. Hence, the channel output vector $\mathbf{x}[n]$ can be expressed in time-domain as

$$\mathbf{x}[n] = H[n] * \mathbf{s}[n], \tag{1}$$

where we have used the following notations

$$\mathbf{x}[n] \triangleq \begin{pmatrix} x_1[n] \\ \vdots \\ x_M[n] \end{pmatrix}, \tag{2}$$

$$H[n] \triangleq \begin{pmatrix} h_{11}[n] & \dots & h_{1d}[n] \\ \vdots & \vdots & \vdots \\ h_{M1}[n] & \dots & h_{Md}[n] \end{pmatrix}, \tag{3}$$

and

$$\mathbf{s}[n] \triangleq \begin{pmatrix} s_1[n] \\ \vdots \\ s_d[n] \end{pmatrix}. \tag{4}$$

Equation (1) can also be written in Z-domain as

$$\mathbf{x}(z) = H(z)\mathbf{s}(z), \tag{5}$$

where $\mathbf{x}(z)$, $\mathbf{s}(z)$ and $H(z)$ are the $Z-$transform of $\mathbf{x}[n]$, $\mathbf{s}[n]$, and $H[n]$, respectively. For MIMO FIR channels, $H(z)$ is a ploynomial matrix. The system described above is a single-input/multiple-output (SIMO) system when $d = 1$.

Let the maximum length of $h_{ij}[n]$ be L, then the input-output relation of MIMO FIR systems can also be described in matrix form as

$$\mathbf{x}_K[n] = \mathcal{H}_K\mathbf{s}_K[n], \tag{6}$$

where $\mathbf{x}_K[n]$ and $\mathbf{s}_K[n]$ are respectively defined as

$$\mathbf{x}_K[n] \triangleq \begin{pmatrix} \mathbf{x}[n] \\ \vdots \\ \mathbf{x}[n+K-1] \end{pmatrix}, \quad \mathbf{s}_K[n] \triangleq \begin{pmatrix} \mathbf{s}[n-L+1] \\ \vdots \\ \mathbf{s}[n+K-1] \end{pmatrix}, \tag{7}$$

and $\mathcal{H}_K$ is a $KM \times (L+K-1)d$ block Toeplitz matrix defined as

$$\mathcal{H}_K \triangleq \begin{pmatrix} H[L-1] & H[L-2] & \cdots & H[0] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & H[L-1] & \ddots & \ddots & H[0] & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & H[L-1] & \cdots & \cdots & H[0] \end{pmatrix}, \tag{8}$$

which is sometimes called *generalized Sylvester matrix*. By means of (6), we are going to develop an algebraic blind identification algorithm for MIMO FIR channels, which can be viewed as the generalization of TXK's algorithm [19] for SIMO FIR channels.

If we define a $KM \times (N-K+1)$ block Toeplitz matrix $\mathcal{X}_K$ by

$$\mathcal{X}_K = (\mathbf{x}_K[0], \mathbf{x}_K[1], \cdots, \mathbf{x}_K[N-K]) \tag{9}$$

and an $(L+K-1)d \times (N-K+1)$ block Toeplitz matrices $\mathcal{S}_K$ by

$$\mathcal{S}_K = (\mathbf{s}_k[0], \mathbf{s}_k[1], \cdots, \mathbf{s}_k[N-K]), \tag{10}$$

then

$$\mathcal{X}_K = \mathcal{H}_K\mathcal{S}_K, \tag{11}$$

From the characteristics of the signal-subspace and noise-subspace of $\mathcal{X}_K$ and the block Toeplitz structure of $\mathcal{H}_K$, we will obtain a subspace identification algorithm, which generalizes the subspace method proposed by Moulines, *et al* in [13].

III. Identifiability using Second-oreder Statistics

Since the rank of generalized Sylvester matrix $\mathcal{H}_k$ plays an important role in the blind identification of MIMO FIR channels, we will reveal the relationship between its singularity and reducibility of polynomial matrix $H(z)$. First, we give the definition of the MIMO FIR channel identification based on second-order statistics of the channel outputs.

A. The Definition of MIMO Channel Identification

In most cases, blind channel (system) identification algorithms can only estimate the channels up to some ambiguity.

For the blind channel identification and equalization of single-input/single-output (SISO) channels, the delay and the phase of the gain of the impulse response can not be estimated since $s[n]$ and $e^{\jmath\theta}s[n-n_d]$ for any integer n_d and $\theta \in [-\pi, \pi]$ share the same (second- and higher-order) statistics.

For the blind identification of SIMO FIR channels using second-order statistics of the channel outputs, the algorithms presented in [13], [19] can identify the impulse response up to a constant factor. In fact, this constant factor is unable to be identified by means of second-order statistics of the channel output.

For MIMO FIR systems, if the system inputs $s_i[n]$ are independent identically distributed (i.i.d.) for different i and n, then $\mathbf{s}[n]$ and $\mathbf{u}\mathbf{s}[n-n_d]$ for any integer n_d and $d \times d$ unitary matrix $\mathbf{u}$ have the same second-order statistics. Hence, we can at most identify the impulse responses of MIMO FIR channels up to a delay and an ambiguity unitary matrix. Therefore, an MIMO FIR system is said to be *identified by means of second-order statistics of the channel outputs*, if we can find an $\widehat{H}[n]$ such that

$$\widehat{H}[n] = H[n-n_d]\mathbf{u}^H. \tag{12}$$

for all integer n, some integer n_d, and unitary matrix $\mathbf{u}$. Hence, the *mean square error* (MSE) of identified channel should be defined as

$$MSE \triangleq \min_{n_d,\mathbf{u}} \sum_n \| H[n] - \hat{H}[n+n_d]\mathbf{u} \|_F^2, \tag{13}$$

where $\| \, . \, \|_F$ denotes the Frobenius norm.

B. A Necessary and Sufficient MIMO FIR Channel Identification Condition

Not all MIMO FIR channels can be identified using second-order statistics of the channel outputs. For SIMO FIR channels, which is a special case of MIMO FIR channels when $d = 1$, a necessary and sufficient condition for them to be identifiable by using second-order statistics of the channel output is that there is no common zero among each subchannel [10], [19], [21]. However, for general MIMO FIR channels, necessary and sufficient identification condition is unknown yet. Here, we establish a necessary and sufficient condition for $\mathcal{H}_K$ to be of full (column) rank, which, as we will show, is also a necessary and sufficient condition for MIMO FIR channels to be identifiable by using second-order statistics of the channel outputs.

First we give a relevant definition. An $M \times d$ $(M > d)$ polynomial matrix $H(z)$ is said to be *irreducible* if there is no $d \times d$ polynomial matrix $R(z)$, with non-constant determination, such that $H(z) = \tilde{H}(z)R(z)$, where $\tilde{H}(z)$ is an $M \times d$ polynomial matrix.

The following lemma will establish the relationship between the non-singularity of the Sylvester matrix $\mathcal{H}_K$ and the irreducibility of the polynomial matrix $H(z)$.

Lemma 1: Let $H[L-1]$ be of full (column) rank, then $\mathcal{H}_K$ is of full (column) rank for all $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$, if and only if $H(z)$ is irreducible.

Proof: Since $H[L-1]$ is of full rank, it has a full rank $d \times d$ minor. Without lose of generality, we assume that $C[L-1] = (h_{ij}[L-1])_{i,j=1}^d$ is of full rank. Then there is a constant matrix E with $\det(E) = 1$ such that $C[L-1]E$ is upper triangular. Let $C[n] = (h_{ij}[n])_{i,j=1}^d$ for $n = 0, \cdots, L-2$, and

$$C(z) = \sum_{n=0}^{L-1} C[n] z^n, \tag{14}$$

then

$$\begin{aligned} \det\{C(z)\} &= \det\{C(z)E\}\det\{E^{-1}\} \\ &= \det\{C(z)E\}. \end{aligned} \tag{15}$$

Hence, the degree of $\det\{C(z)\}$ is $\deg\{\det\{C(z)\}\} = (L-1)d$. Then from Corollary 1 of [2], $H(z)$ is irreducible if and only if for $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$,

$$\begin{aligned} \mathrm{rank}\{\mathcal{H}_K\} &= Kd + \deg\{\det\{C(z)\}\} \\ &= (K+L-1)d, \end{aligned} \tag{16}$$

which implies that $\mathcal{H}_K$ is of full column rank for $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$. ■

From Lemma 1, if $\mathcal{H}_K$ for some $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$ is of full (column) rank, then $\mathcal{H}_K$ for any $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$ is of full (column) rank.

It is proved in [9] that $H(z)$ is irreducible if and only if it is of full (column) rank for all $z \in \mathcal{C}$. Therefore, the non-singularity of $\mathcal{H}_K$ for all integer $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$ is also equivalent to $H(z)$ being of full (column) rank for all $z \in \mathcal{C}$.

From Lemma 1, we can conclude that if $\mathcal{H}_K$ for some $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$ is of full (column) rank, then $\mathcal{H}_K$ for all $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$ are also of full (column) rank.

Lemma 2: For any square polynomial matrix $R(z)$ with non-constant determination, there exists a square polynomial matrix $\widehat{R}(z)$ satisfying the following two conditions:

(i). $R(z)$ and $\widehat{R}(z)$ are related by

$$R(z)R_*(z) = \widehat{R}(z)\widehat{R}_*(z), \tag{17}$$

where for any matrix $A(z) = \sum_n A_n z^{-n}$, $A_*(z)$ is defined as

$$A_*(z) = \sum_n A_n^H z^n. \tag{18}$$

(ii). There is no constant matrix E, of which all elements are complex-constants, such that

$$R(z) = \widehat{R}(z)E, \tag{19}$$

Proof: Without lost of generality, we may assume that $R(z)$ is minimum-phase, that is, the zeros of $\det\{R(z)\}$ are inside the unit circle. Let

$$G(z) = R(1/z)R_*(1/z), \tag{20}$$

then from [14], there is a minimum-phase matrix polynomial $\bar{R}(z)$ suct that

$$G(z) = \bar{R}(z)\bar{R}_*(z). \tag{21}$$

Therefore, $\widehat{R}(z) = \bar{R}(1/z)$ is a maximum-phase matrix polynomial satisfying condition (i).

Furthermore, if there is a constant matrix E such that

$$R(z) = \widehat{R}(z)E, \tag{22}$$

then

$$\det\{R(z)\} = \det\{\widehat{R}(z)\}\det\{E\}. \tag{23}$$

Since the left-hand side of the above identity is a polynomial with zeros inside the unite circle, while the right-hand side is with zeros outside the unit circle, such a contradiction proves the non-existence of E, and therefore, $R(z)$ must satisfy Condition (ii). ■

From the above two lemmas, we are now able to establish the following identifiability theorem.

Theorem 1: For an MIMO FIR channel with length L and $H[L-1]$ being of full (column) rank, the necessary and sufficient condition for such a channel to be identifiable up to an ambiguity unitary matrix by using second-order statistics of the channel outputs is that $H(z)$ is irreducible.

Proof: For an MIMO FIR channel with $H(z)$ being irreducible, from Lemma 1, its corresponding Sylvester matrix $\mathcal{H}_K$ is of full rank for all $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$. From Theorem 2 and Theorem 3 proved in the next section, the MIMO FIR channel can be identified up to an ambiguity unitary matrix.

On the other hand, if $H(z)$ is reducible, then there exist an $M \times d$ polynomial matrix $\tilde{H}(z)$ and a $d \times d$ polynomial matrix $R(z)$ with non-constant determination, such that

$$H(z) = \tilde{H}(z)R(z), \tag{24}$$

From Lemma 2, there is a polynomial matrix $\widehat{R}(z)$ satisfying

$$R(z)R_*(z) = \widehat{R}(z)\widehat{R}_*(z), \tag{25}$$

and Condition (ii) in Lemma 2. Let

$$\widehat{H}(z) = \tilde{H}(z)\widehat{R}(z), \tag{26}$$

then

$$H(z)H_*(z) = \widehat{H}(z)\widehat{H}_*(z). \tag{27}$$

The identity (27) implies that the outputs of two MIMO FIR channels $H(z)$ and $\widehat{H}(z)$ share the same second-order statistics. Hence, there is no algorithm which can identify the MIMO FIR channel based only on second-order statistics of the MIMO FIR channel outputs. ■

From Theorem 1, we can conclude the *identifiability condition* of MIMO FIR channels as the following:

An MIMO FIR channel is said to satisfy the *identifiability condition* if

(i). $H[L-1]$ is of full (column) rank with L being the length of the MIMO FIR channel, and

(ii). $H(z)$ is irreducible, or equivalently, $H(z)$ is of full (column) rank for all $z \in \mathcal{C}$.

When $d = 1$, an MIMO system becomes a SIMO system. In this case, Theorem 1 reduces to a special case and can be stated as: An SIMO FIR channel is identifiable using second-order statistics of the channel outputs if and only if $H[L-1]$ is nonzero and there is no common zero among the M subchannels, which has been presented in [10].

IV. Blind Identification Algorithms Based on Second-Order Statistics

In this section, we will develop blind identification algorithms for MIMO FIR channels. They generalize the algebraic algorithm proposed in [19] and subspace algorithm proposed in [13] to the identification of MIMO FIR channels.

A. Algebraic Identification

Assume that the channel input vector $\mathbf{s}[n]$ is i.i.d. with zero-mean and unit-variance, that is

$$R_s[m] \triangleq E\{\mathbf{s}[n]\mathbf{s}^H[n+m]\} = \delta[m]I_d, \tag{28}$$

where $\delta[m]$ is the Kronecker delta function and I_d is a $d \times d$ identity matrix. From (6) and (28), we have

$$R_{x_K}[0] \triangleq E\{\mathbf{x}_K[n]\mathbf{x}_K^H[n]\} = \mathcal{H}_K\mathcal{H}_K^H, \tag{29}$$

and

$$R_{x_K}[1] \triangleq E\{\mathbf{x}_K[n]\mathbf{x}_K^H[n+1]\} = \mathcal{H}_K J\mathcal{H}_K^H, \tag{30}$$

where J is an $(L+K-1)d \times (L+K-1)d$ matrix defined as

$$J \triangleq \begin{pmatrix} \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ I_d & \ddots & \ddots & \mathbf{0} \\ \vdots & \ddots & \ddots & \vdots \\ \mathbf{0} & \cdots & I_d & \mathbf{0} \end{pmatrix}. \tag{31}$$

Using the algebraic property of $R_{x_K}[0]$ and $R_{x_K}[1]$, the MIMO channels can be identified up to an ambiguity unitary matrix, which is stated as follows.

Theorem 2: For digital communication systems with MIMO FIR channels satisfying the identifiability condition, if the channel input vector $\mathbf{s}[n]$ is i.i.d. with zero-mean and unit-variance, then, based on $R_{x_K}[0]$ and $R_{x_K}[1]$ for any $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$, the impulse response matrices of MIMO FIR channels $H[n]$ for $n = 0, \cdots, L-1$ can be uniquely determined up to an ambiguity unitary matrix.

Proof: Assume there is a matrix $\tilde{\mathcal{H}}_K$ satisfying

$$\mathcal{H}_K \mathcal{H}_K^H = \tilde{\mathcal{H}}_K \tilde{\mathcal{H}}_K^H, \tag{32}$$

and

$$\mathcal{H}_K J \mathcal{H}_K^H = \tilde{\mathcal{H}}_K J \tilde{\mathcal{H}}_K^H. \tag{33}$$

From (32), there exists a unitary matrix Q such that [19]

$$\tilde{\mathcal{H}}_K = \mathcal{H}_K Q. \tag{34}$$

If the MIMO channels satisfy the identifiability condition, $\mathcal{H}_K$ is of full rank for all $K \geq \lceil \frac{(L-1)d}{M-d} \rceil$. Therefore, substituting (34) into (33), we have

$$Q J Q^H = J, \tag{35}$$

or

$$QJ = JQ. \tag{36}$$

Partitioning Q into $(L+K-1)^2$ $d \times d$ square matrices, by means of (36), we can show that Q must be block diagonal, i.e.,

$$Q = \begin{pmatrix} \mathbf{q} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{q} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{q} \end{pmatrix} \tag{37}$$

Since Q is unitary, $\mathbf{q}$ is also a $d \times d$ unitary matrix. Hence, from (34), $H[n]$ can be determined up to an ambiguity unitary matrix $\mathbf{q}$. ■

The above theorem demonstrates the blind identifiability of MIMO FIR channels based on second-order statistics of the channel outputs for those MIMO FIR channels satisfying the identifiability condition.

Using Theorem 2, we can develop the following *algebraic identification algorithm*:

Step 1. Estimate $R_{x_K}[m]$ for $m = 0,\ 1$ by

$$\widehat{R}_{x_K}[m] = \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{x}_K[n]\mathbf{x}_K^H[n+m]. \tag{38}$$

Step 2. Find R by

$$R = FR_{x_K}[1]F^H, \tag{39}$$

where

$$F = \Sigma^{-1}U_s^H, \tag{40}$$

$$\Sigma = \mathrm{diag}(\sigma_1, \cdots, \sigma_{(L+K-1)d}), \tag{41}$$

$$U_s = [\mathbf{u}_1, \cdots, \mathbf{u}_{(L+K-1)d}], \tag{42}$$

with σ_i^2 are the nonzero eigenvalues of $R_{x_K}[0]$ corresponding to the eigenvectors $\mathbf{u}_i$ for $i = 1, \cdots,\ (L+M-1)d$.

Step 3. Estimate $\mathcal{H}_K$ by

$$\widehat{\mathcal{H}}_K = U_s\Sigma Q, \tag{43}$$

where

$$Q = (\mathbf{r}, R\mathbf{r}, \cdots, R^{L+K-2}\mathbf{r}), \tag{44}$$

with $\mathbf{r} = (\mathbf{r}_1, \cdots, \mathbf{r}_d)$ and $\mathbf{r}_i$ for $i = 1, \cdots, d$ being the left singularvectors of R corresponding to the only d zero singular-values.

To confirm the above algorithm, we only require to prove that $\widehat{\mathcal{H}}_K$ satisfies constraints (29) and (30) according to Theorem 2. From (29), there is a unitary matrix V such that

$$\mathcal{H}_K = U_s\Sigma V, \tag{45}$$

where U_s and Σ are defined as in (41) and (42). Hence, R in (39) can be expressed as

$$R = VJV^H, \tag{46}$$

where J is defined as in (31). If we partition V into $L+K-1$ $(L+K-1)d \times d$ matrices so that

$$V = (\mathbf{v}_1, \cdots, \mathbf{v}_{L+K-1}), \tag{47}$$

then from (46),

$$R\mathbf{v}_i = \mathbf{v}_{i+1}, \tag{48}$$

and

$$\mathbf{v}_1^H R = \mathbf{0}. \tag{49}$$

From the construction of R in Step 2, the rank of the null-space of R is d. Since $\mathbf{r}^H R = \mathbf{0}$ from Step 3, there must exist a $d \times d$ unitary matrix $\mathbf{q}$ such that

$$\mathbf{r} = \mathbf{v}_1 \mathbf{q}. \tag{50}$$

Hence, according to (48) and (50), Q defined in (44) can be expressed as

$$\begin{aligned} Q &= (\mathbf{v}_1\mathbf{q}, R\mathbf{v}_1\mathbf{q}, \cdots, R^{L+K-2}\mathbf{v}_1\mathbf{q}) \\ &= (\mathbf{v}_1\mathbf{q}, \mathbf{v}_2\mathbf{q}, \cdots, \mathbf{v}_{L+K-1}\mathbf{q}) \\ &= V(\mathbf{q} \otimes I_{L+K-1}), \end{aligned} \tag{51}$$

where $\otimes$ denotes the Kronecker product of two matrices. Hence,

$$QQ^H = VV^H, \text{ and } QJQ^H = VJV^H, \tag{52}$$

which imply that $\widehat{\mathcal{H}}_K$ in (43) satisfies constraints (29) and (30).

B. Subspace Identification

The algebraic algorithm developed in the previous section exploits the structure of $R_{x_K}[0]$ and $R_{x_K}[1]$. The crucial assumption there is that the channel inputs are i.i.d.. Here, we will develop a subspace algorithm for MIMO FIR channel identification, which does not require the i.i.d. assumption of input symbols.

In what follows, we will assume that $\mathcal{S}_K$ is of full row-rank, which means that the vector sequence $\mathbf{s}[n]$ is *persistently exciting of order* K [3]. It relaxes the i.i.d. assumption for input vector sequence.

Theorem 3: For digital communication systems with MIMO FIR channels satisfying the identifiability condition, let the channel input vector sequence $\mathbf{s}[n]$ is persistently exciting of order K. For any $K \geq \lceil \frac{(L-1)d}{M-d} \rceil + 1$, if there are a Sylvester matrix $\tilde{\mathcal{H}}$ and a matrix $\tilde{\mathcal{S}}$ with the same dimension as $\mathcal{H}_K$ and $\mathcal{S}_K$ respectively, such that $\mathcal{X}_K = \tilde{\mathcal{H}}\tilde{\mathcal{S}}$, then $\tilde{\mathcal{H}} = \mathcal{H}_K(\mathbf{p} \otimes I_{L+K-1})$ for some $d \times d$ nonsingular ambiguity matrix $\mathbf{p}$. Futhermore, if $\tilde{\mathcal{H}}$ satisfies the constraints (29), then $\mathbf{p}$ is unitary.

Proof: Let $\tilde{\mathcal{H}}\tilde{\mathcal{S}} = \mathcal{X}_K$. Since $\mathcal{H}_K\mathcal{S}_K = \mathcal{X}_K$, then

$$\mathcal{H}_K = \tilde{\mathcal{H}}P, \tag{53}$$

with $P = \tilde{\mathcal{S}}\mathcal{S}_K^H(\mathcal{S}_K\mathcal{S}_K^H)^{-1}$. As the MIMO channels satisfying identifiability condition, both $\mathcal{H}_K$ and $\mathcal{H}_{K-1}$ are of full rank. Hence, P is of full rank and

$$\tilde{\mathcal{H}} = \mathcal{H}_K P^{-1}. \tag{54}$$

If we partition P^{-1} into $(L+K-1)^2$ $d \times d$ square matrix $\mathbf{p}_{ij}$ for $i, j = 1, \cdots L+K-1$, the first d columns of both sides of (54) can be expressed as

$$\begin{pmatrix} \tilde{H}[L-1] \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{pmatrix} = \left(\begin{array}{c|c} H[L-1] & * \\ \hline \mathbf{0} & \\ \vdots & \mathcal{H}_{K-1} \\ \mathbf{0} & \end{array}\right) \begin{pmatrix} \mathbf{p}_{11} \\ \mathbf{p}_{12} \\ \vdots \\ \mathbf{p}_{1\ L+K-1} \end{pmatrix}. \tag{55}$$

Comparing both sides of the above identity, we have $\mathbf{p}_{1j} = \mathbf{0}$ for $j > 1$ since $\mathcal{H}_{K-1}$ is of full rank. Similarly, comparing the i-th d columns of both sides of (54), we can conclude that $\mathbf{p}_{ij} = \mathbf{0}$ for all $j > i$. Using similar procedure to both sides of (54) from the last d columns to the first d columns, we will have $\mathbf{p}_{ij} = \mathbf{0}$ for $i > j$. Therefore, $\mathbf{p}_{ij} \neq \mathbf{0}$ only if $i = j$. Since $H[L-1]$ is of full rank,

$$\mathbf{p}_{11} = \cdots = \mathbf{p}_{L+K-1\ L+K-1} = \mathbf{p}, \tag{56}$$

with $\mathbf{p}$ being full-rank. Hence,

$$\tilde{\mathcal{H}} = \mathcal{H}_K(\mathbf{p} \otimes I_{L+K-1}). \tag{57}$$

If

$$\tilde{\mathcal{H}}\tilde{\mathcal{H}}^H = \mathcal{H}_K\mathcal{H}_K^H, \tag{58}$$

then

$$(\mathbf{p} \otimes I_{L+K-1})(\mathbf{p} \otimes I_{L+K-1})^H = I_{(L+K-1)d}, \tag{59}$$

so $\mathbf{p}\mathbf{p}^H = I_d$, which implies that the ambiguity matrix $\mathbf{q}$ is unitary. ∎

Compared with the algebraic identification theorem, subspace identification theorem makes full use of the Sylvester structure of the channel matrix $\mathcal{H}_K$. Without using the i.i.d property of the channel input, the channel can be identified up to a $d \times d$ constant ambiguity matrix $\mathbf{p}$. Theorem 3 is in fact the extension of Theorem 2 in [13] to the identification of MIMO FIR channels.

Using the similar procedures to those in [13], a subspace algorithm has been developed in [23] for the identification of MIMO FIR channels satisfying the identifiability condition. Theorem 3 establishes the theoretical foundation for the subspace algorithm.

C. Remarks

1. The algebraic algorithm exploits the algebraic property of the correlation matrix of the channel output. Since this algebraic property relies on the i.i.d assumption of the channel input, the algebraic algorithm needs more symbols to estimate channels. On the other hand, the subspace algorithm uses the Sylvester structure of channel matrix based on the exciting persistence of the channel input vector sequence, hence, it requires less symbols. This fact will be confirmed by computer simulation results in Section VI.
2. Since the rank of $\mathcal{H}_K$ is $(K + L - 1)d$, as indicated in [23], the number of signals d can be estimated by changing K and observing the increase of the rank of $\mathcal{H}_K$. Once d is obtained, L can be estimated easily.
3. Compared with the algorithm in [22], the blind identification algorithms presented in this paper require less computation.

V. Multiple Signals Recovery

Once the impulse responses of MIMO channels are known, the optimum filter in [1], [4], [27] can be applied to recover the multiple signals. As indicated in Section III, using second-order statistics, the MIMO FIR channels can be identified up to an ambiguity matrix $\mathbf{q}_o$, that is, we can find $\widehat{H}[n]$, such that,

$$\widehat{H}[n] = H[n]\mathbf{q}_o. \tag{60}$$

Without loss of generality, we assume that $\mathbf{q}_o$ is a unitary matrix. As shown in Figure 2, if an MIMO filter with Z-transform

$$F(z) = (\widehat{H}^H(z)\widehat{H}(z))^{-1}\widehat{H}^H(z) \tag{61}$$

is applied, then the Z-transform of the filter output $\mathbf{y}[n]$ is

$$\begin{aligned} \mathbf{y}(z) &= F(z)\mathbf{x}(z) \\ &= \mathbf{q}_o^{-1}\mathbf{s}(z). \end{aligned} \tag{62}$$

To remove the effect of the ambiguity matrix, we perform a linear transform $\mathbf{q} = [q_{ij}]$ to the MIMO filter output $\mathbf{y}[n]$ for each time n. To recover $s_i[n]$ for $i = 1, \cdots, d$ up to a scaler, $\mathbf{q}$ must satisfy

$$\mathbf{q}\mathbf{q}_o = PD, \tag{63}$$

where P is a $d \times d$ permutation matrix and D is a diagonal matrix defined as

$$D \triangleq \mathrm{diag}\{e^{j\theta_1}, \cdots, e^{j\theta_d}\}, \tag{64}$$

with $\theta_i \in [-\pi,\ \pi]$ for $i = 1, \cdots, d$. The following theorem will give an ambiguity matrix estimation algorithm based on up to forth-order cumulants of the MIMO filter output $\mathbf{y}[n]$.

Theorem 4: Assume that

(i). channel input $s_i[n]$'s satisfy

$$E\{s_i[n]\} = 0, \quad E\{s_i[n]^2\} = 0 \tag{65}$$

and $s_i[n]$ and $s_j[n]$ are independent for all $i \neq j$, and

(ii). $\mathbf{y}[n]$ satisfies Equation (62) with $\mathbf{q}_o$ being unitary.

Then, if a unitary matrix $\mathbf{q}$ minimizes $\sum_{ij\ i\neq j} |R_{ij}|$, it will satisfy (63), where

$$\begin{aligned} R_{ij} &\triangleq Cum(z_i, z_i^*, z_j, z_j^*) \\ &= \sum_{k_1,k_2,k_3,k_4} q_{ik_1}q_{ik_2}^*q_{jk_3}q_{jk_4}^*Cum(y_{k_1}, y_{k_2}^*, y_{k_3}, y_{k_4}^*), \end{aligned} \tag{66}$$

with $Cum(x_1, x_2, x_3, x_4)$ being the cumulant of random variables x_1, x_2, x_3 and x_4 defined as

$$\begin{aligned} Cum(x_1,x_2,x_3,x_4) \triangleq\ & E\{x_1x_2x_3x_4\} - E\{x_1x_2\}E\{x_3x_4\} \\ & -E\{x_1x_3\}E\{x_2x_4\} - E\{x_1x_4\}E\{x_2x_3\}. \end{aligned} \tag{67}$$

Proof: Let $O = \mathbf{q}\mathbf{q}_o = [o_{ij}]$, then from Figure 2,

$$\begin{aligned} \mathbf{z}[n] &= \mathbf{q}\mathbf{y}[n] \\ &= O\mathbf{s}[n]. \end{aligned} \tag{68}$$

or

$$z_i[n] = \sum_{i=1}^{d} o_{ij}s_j[n]. \tag{69}$$

Since both $\mathbf{q}$ and $\mathbf{q}_o$ are unitary, so is O. From (67), the direct calculation yields

$$|R_{ij}| = |m_4 - 2m_2^2| \sum_{k=1}^{d} |o_{ik}|^2|o_{jk}|^2. \tag{70}$$

where

$$E\{|s_i[n]|^2\} = m_2, \quad \text{and} \quad E\{|s_i[n]|^4\} = m_4. \tag{71}$$

Then

$$\begin{aligned} \sum_{ij\ i\neq j} |R_{ij}| &= |m_4 - 2m_2^2| \sum_{ij\ i\neq j} \sum_{k=1}^{d} |o_{ik}|^2|o_{jk}|^2 \\ &= |m_4 - 2m_2^2| \sum_{i=1}^{d}\sum_{k=1}^{d} |o_{ik}|^2 \sum_{j\ j\neq i} |o_{jk}|^2 \\ &= |m_4 - 2m_2^2| \sum_{i,k=1}^{d} |o_{ik}|^2(1 - |o_{ik}|^2) \end{aligned} \tag{72}$$

Hence, $\sum_{ij\ i\neq j} |R_{ij}|$ attains one of its minima if and only if the matrix $[|o_{ij}|^2]$ satisfies

$$[|o_{ij}|^2] = P, \tag{73}$$

for some $d \times d$ permutation matrix P. Hence,

$$O = [o_{ij}] = PD, \quad \text{or} \quad \mathbf{q}\mathbf{q}_o = PD, \tag{74}$$

for some diagonal matrix D defined as in (64). ■

TABLE I
THE PARAMETERS OF THE MULTIPATH CHANNEL

Sources	i	α_i	τ_i	p_i
1st	1	-10°	0.0	$1.0e^{-2.72\jmath}$
	2	-2°	0.3	$0.8e^{0.61\jmath}$
	3	-120°	1.2	$0.4e^{0.93\jmath}$
	4	160°	2.1	$0.4e^{1.64\jmath}$
2nd	1	10°	0.5	$1.0e^{-3.07\jmath}$
	2	15°	0.9	$0.9e^{1.17\jmath}$
	3	-40°	1.5	$0.5e^{-1.06\jmath}$
	4	150°	2.8	$0.3e^{1.83\jmath}$

VI. Computer Simulation Results

A Monte Carlo simulation example has been conducted to illustrate MIMO FIR channel identification algorithms with application to QAM digital communication systems.

We have modified the simulation example in [22] for QAM digital communication systems. In our simulation, the digital signals $s_i[n]$'s are independent of each other for any different n or i, and they are uniformly distributed over $\{\pm 1 \pm \jmath\}$. The digital signals are modulated by raised-cosin shaping filters with roll-off factor 0.35 and truncated to a length of six symbol periods. The modulated signals are received by 2 sensors spaced by half wavelength. The simulated channel consists of four paths per signal and each path is specified by angle-of-arrival α_i, delay τ_i, and complex gain p_i as shown in Table I. From Table I, the maximum channel length is $L = 9$. The channel noise is complex white Gaussian with zero-mean and variance determined by the signal-to-noise ratio (SNR).

The signal from each sensor is 5-time oversampled, resulting $M = 10$ in the MIMO channel model shown as in Figure 1. The algebraic and subspace algorithms are used respectively to estimate the impulse responses of the multiple-input/multiple-output (MIMO) FIR channel. Although the maximum length of the MIMO FIR channel is $L = 9$, our simulation indicates that the estimation obtains the optimum performance when $\widehat{L} = 8$. The estimation performance is measured by the mean-square-error (MSE) defined in (13) in Section III. Figure 3 and 4 demonstrate the MSE's of both algebraic algorithm and subspace algorithm via SNR and the number of symbols respectively.

100 independent trials have been conducted under the same simulation scenario to obtain these MSE's. From these figures, when $SNR \geq 40dB$ and the number of samples $N \geq 500$, the estimation can attain satisfying performance. As indicated in Section IV, the subspace algorithm has better performance than the algebraic algorithm.

From the discussion of Section III, we can identify the MIMO FIR channel impulse responses up to an ambiguity matrix based soly on second-order statistics. To remove the ambiguity and separate the signals, we have used the algorithm presented in Section V. Figure 5 and 6 demonstrate the eye-patterns before and after the ambiguity matrix removing. The subspace algorithm is used to estimate the impulse responses of the MIMO channel $\{\widehat{H}[n]\}$ based on 2000 symbols when $SNR = 40dB$. From Figure 6, the subspace algorithm, together with ambiguity matrix estimation method, can effectively separate and recover the input digital signals.

VII. Conclusions

We have investigated the blind identification of MIMO FIR channels based on the second-order statistics of the channel outputs. A necessary and sufficient condition for MIMO FIR channels to be identified up to an ambiguity matrix has been established. An algebraic identification algorithm and a subspace identification algorithm have been developed. Using these algorithms, the MIMO FIR channels satisfying the identifiability condition can be estimated up to an ambiguity matrix. Since the ambiguity matrix can not be identified using second-order statistics, we also presented an ambiguity matrix estimation method using forth-order cumulant. With this ambiguity matrix estimation method, the MIMO channel identification algorithms developed in this paper can be applied to array processing in mobile radio communication systems to increase the communication capacity.

References

[1] M. Abdulrahman and D. D. Falconer, "Cyclostationary crosstalk suppression by decision feedback equalization on digital subscriber loops," *IEEE Journal on Selected Areas in Comm.*, JSAC-10:640-649, April 1992.

[2] R. R. Bitmead, S. Y. Kung, B. D. O. Anderson, and T. Kailath, "Greatest common divisors via generalized Sylvester and Bezout matrices," *IEEE Trans. on Automatic Control*, AC23:1043-1047, December 1978.

[3] F. Gustafsson and B. Wahlberg, "Blind equalization by direct examination of the input sequence," *IEEE Trans. on Comm.*, vol.43, July 1995.

[4] A. Duel-Hallen, "Equalizers for multiple input/multiple output channels and PAM systems with cyclostationary input sequences," *IEEE Journal on Selected Areas in Comm.*, JSAC-10:630-639, April 1992.

[5] Z. Ding and Y. Li, "On channel identification based on second-order cyclic spectra," *IEEE Transactions on Signal Processing*, vol.42, pp1260-1264, May 1994
[6] W. A. Gardner, "Exploitation of redundancy in cyclostationary signals," *IEEE Signal Processing Magazine*, pp. 814-836, April 1991.
[7] W.A.Gardner, "A new method of channel identification," *IEEE Trans. on Comm*, COM-39:813-817, August 1991.
[8] D. Hatzinakos and C. L. Nikias. Blind equalization using a tricepstrum based algorithm. *IEEE Trans. on Communications*, 39:669–682, May 1991.
[9] T. Kailath, *Linear Systems*, Prentice-Hall, Englewood Cliffs, N. J., 1980.
[10] Y. Li and Z. Ding, "Blind channel identification based on second-order cyclostationary statistics," *Proc. ICASSP*, 1993, vol.4, pp81-84.
[11] Y. Li and Z. Ding, "ARMA system identification based on second-order cyclostationarity," *IEEE Transactions on Signal Processing*, vol.42, pp 3483-3494, December 1994.
[12] Y. Li and K. J. R. Liu, "On blind equalization of MIMO channels," accepted by *ICC'96*, Dallas, Texas, June 1996.
[13] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Transactions on Signal Processing*, vol.43, pp516-525, Feb. 1995.
[14] D. N. P. Murthy, "Factorization of discrete-process spectral matrices," *IEEE Trans. on Inf. Theory*, pp 693-696, September 1973.
[15] B. R. Petersen and D. D. Falconer, "Suppression of adjacent-channel, co-channel, and intersymbol interference by equalizers and linear combiners," *IEEE Trans. on Comm.*, COM-42, pp3109-3118, December 1994.
[16] M. Rangoussi and G. B. Giannakis, "FIR modeling using log-bispectra: Weighted least-squares algorithms and performance analysis," *IEEE Trans. on Circuits and Systems*, vol.38, pp. 281-296, March 1991.
[17] B. Samapth, Y. Li, and K. J. R. Liu, 'A subspace based blind identification and equalization algorithm,' accepted by *ICC'96*, Dallas, Texas, June 1996.
[18] L. Tong, G. Xu and T. Kailath, "A new approach to blind identification and equalization of multipath channels," *Proc. 25th Asilomar Conf. on Signal, Systems, Computers*, 1991, pp856-860
[19] L. Tong, G. Xu and T. Kailath, "Blind identification and equalization based on second-order statistics: a time domain approach,"*IEEE Transactions on Information Theory*, vol.40, pp340-349, March 1994
[20] J. K. Tugnait. Identification of linear stochastic systems via second and fourth-order cumulant matching. *IEEE Trans. Information Theory*, IT-33:393–407, May 1987.
[21] J. K. Tugnait, "On blind identifiability of multipath channels using fractional sampling and second-order cyclostationary statistics," *IEEE Trans. on Inf. Theory*, vol.41 pp 308-311, January 1995.
[22] A. van der Veen, S. Talwar, and A. Paulraj, "Blind estimation of multiple digital signals transmitted over FIR channels," *IEEE SP Lett.*, vol.2, May 1995.
[23] A. van der Veen, S. Talwar, and A. Paulraj, "Blind identification of FIR channels carrying multiple finite alphabet signals," *Proceedings of ICASSP'95*, vol.2, pp1213-1216, May 1995.
[24] E. Weinstein, A. V. Oppenheim, M. Feder, and J. R. Buck, "Iterative and sequential algorithms for multisensor signal enhancement," *IEEE Transactions on Signal Processing*, SP-42:846-859, April 1994.
[25] E. Weinstein, A. Swami, G. Giannakis, and S. Shamsunder, "Multichannel ARMA processes," *IEEE Transactions on Signal Processing*, SP-42:898-913, April 1994.
[26] W. A. Wolovich, *Linear Multivariable Systems*. Springer-Verlag, New York Inc., 1974.
[27] J. Yang and S. Roy, "On joint transmitter and receiver optimization for multiple-input-multiple-output (MIMO) transmission systems," *IEEE Trans. on Comm.*, COM-42, pp3221-3231, December 1994.
[28] D. Yellin and E. Weinstein, "Criteria for multichannel signal separation," *IEEE Transactions on Signal Processing*, SP-42:2158-2168, August 1994.

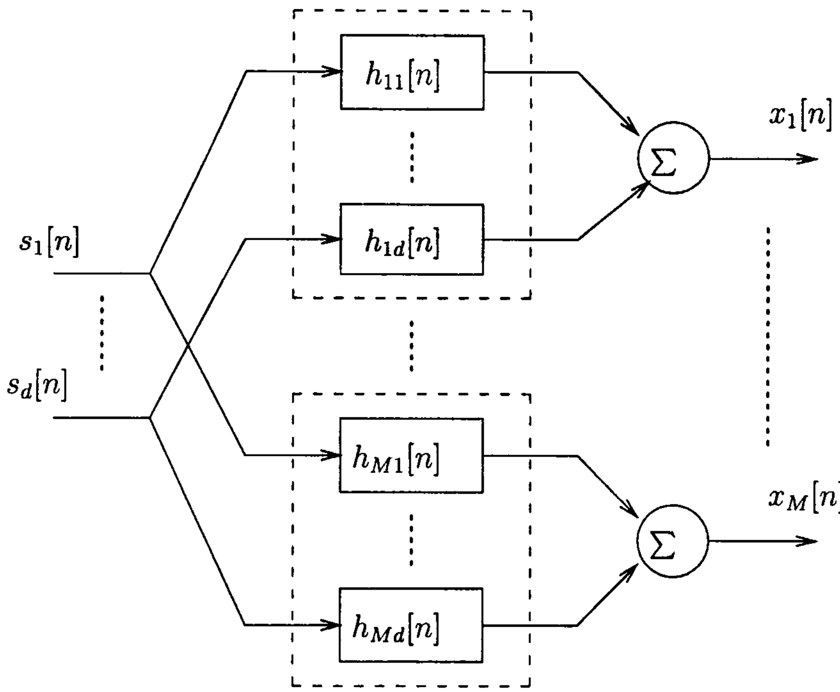

Fig. 1. MIMO system.

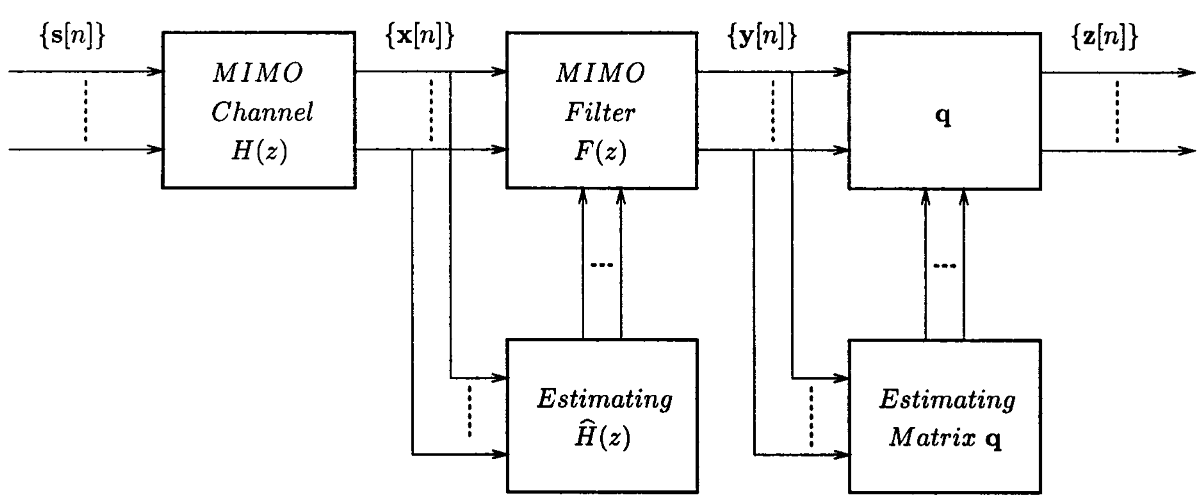

Fig. 2. MIMO channel equalizer.

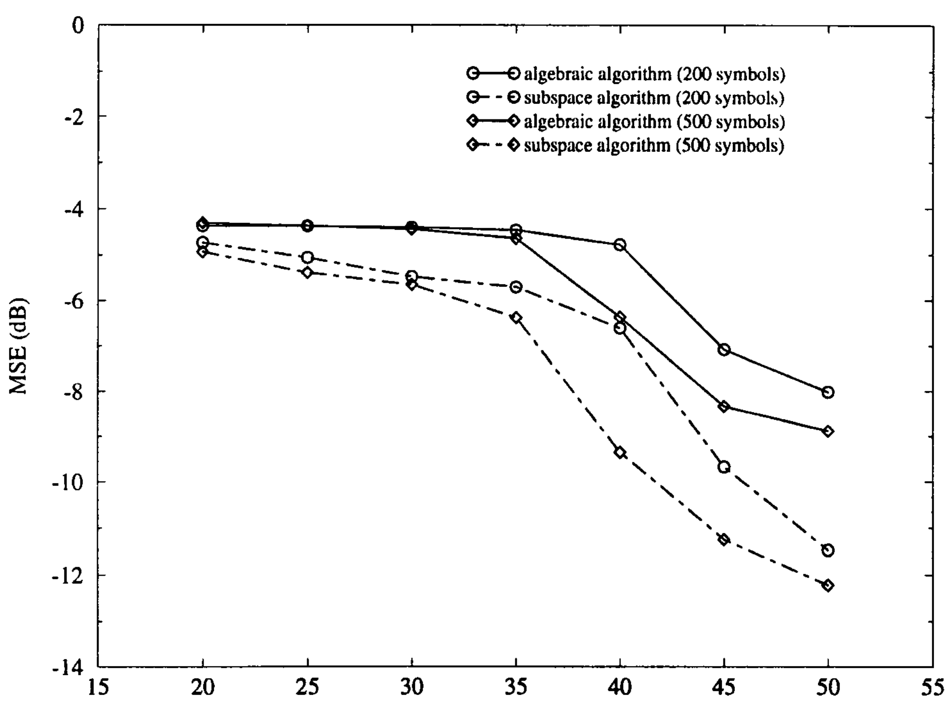

Fig. 3. The MSE of estimated channel impulse responses via signal-to-noise ratio (SNR) based on 100 ensemble trials.

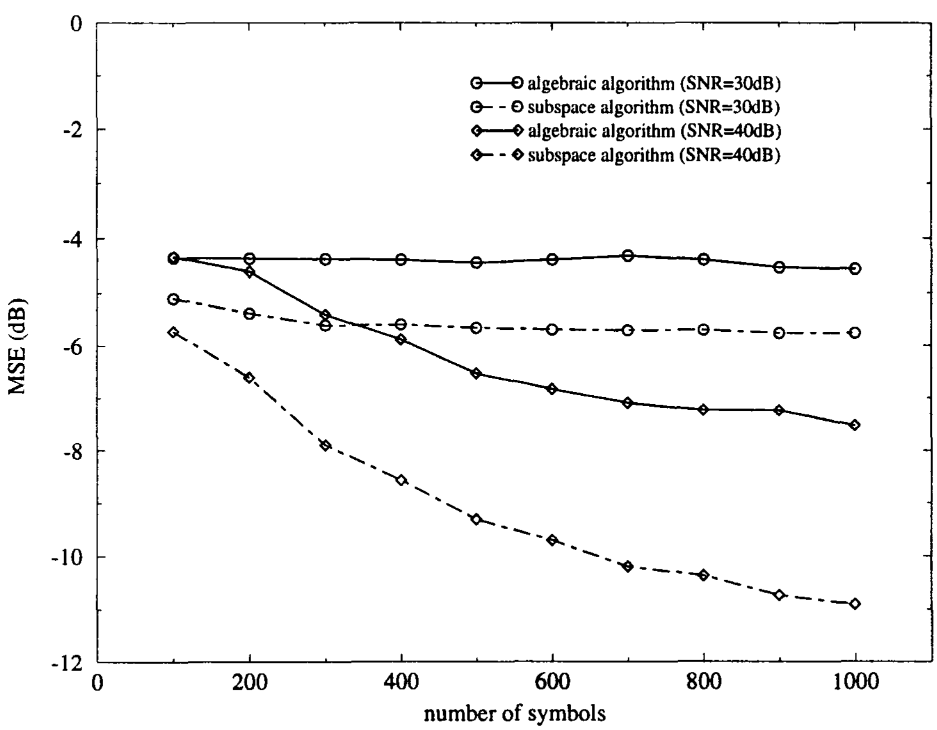

Fig. 4. The MSE of estimated channel impulse responses via the number of symbols used in the estimation based on 100 ensemble trials.

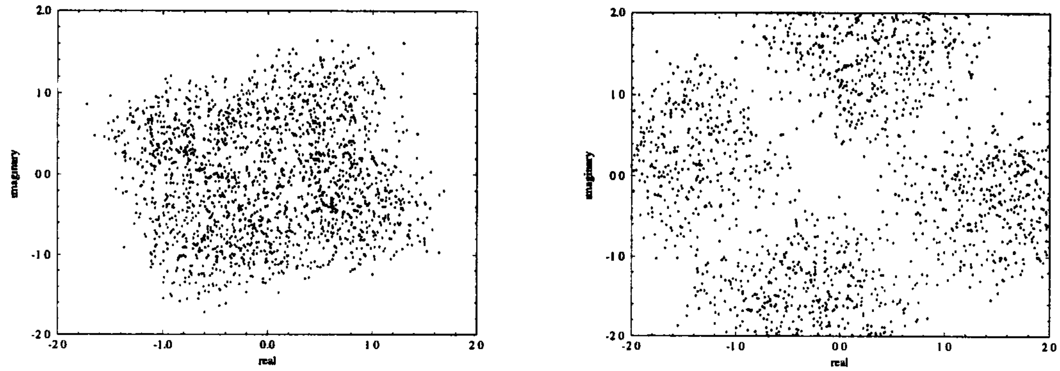

Fig. 5. 2000 MIMO fiter outputs, SNR=40dB.

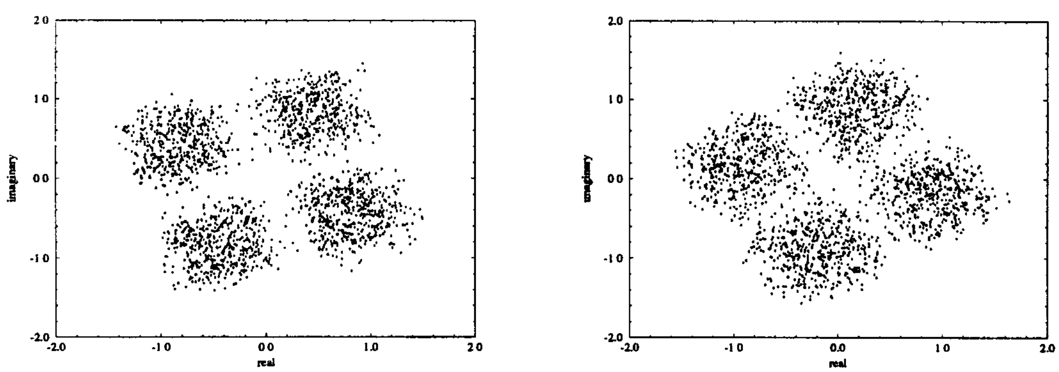

Fig. 6. 2000 MIMO equalizer outputs, SNR=40dB.