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Selecting an Optimization Model for Product Development

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ABSTRACT

Design optimization is an important engineering design activity. When used during the development of a new product, the overall profitability of that product depends upon the quality of the solution that the optimization model returns as well as the time and cost of using optimization. There exist many different ways to use optimization. The design engineer wants to select the most appropriate optimization model to create a profitable design. This paper discusses this meta-design (or meta-reasoning) problem and presents a method for selecting the best (most profitable) optimization model from a set of candidate optimization models. The approach allows multiple ways to handle uncertainty about the optimization models. We demonstrate the approach by considering the optimization of a universal electric motor.

1. Nomenclature

$\underline{F}_{qi}, \overline{F}_{qi}$ = lower and upper cumulative probability bounds for solution quality for M_i
$\underline{F}_{ti}, \overline{F}_{ti}$ = lower and upper cumulative probability bounds for time for M_i
M = the set of candidate optimization models
C_D = design cost of a motor design
C_K = capacity cost of a motor design
C_L = labor cost of a motor design
C_M = material cost of a motor design
L_{i1}, L_{i2} = lower bounds for the p-box for solution quality for optimization model M_i
M_i = i^{th} optimization model
R = vector of performance attributes for a motor design
U_{i1}, U_{i2} = upper bounds for the p-box for solution quality for optimization model M_i

W = vector of weights for scaling performance attribute deviations
Z = vector of performance attribute targets
c_i = cost of optimization model M_i
d = demand for a motor design
p = price for a motor design
q_i = solution quality of optimization model M_i
q_{ai}, q_{bi} = lower and upper bounds on solution quality for optimization model M_i
s = population size
t_i = time for optimization model M_i
t_{ai}, t_{bi} = lower and upper bounds on time for optimization model M_i
y_i = half-width of uniform distribution for solution quality for optimization model M_i
n = number of candidate optimization models
$\bullet$ = term by term vector multiplication
$\| \|_2$ = l_2 norm
Π_i = profitability of optimization model M_i
$\underline{\Pi}_i, \overline{\Pi}_i$ = lower and upper bounds on the expected profitability of M_i
Ψ_k = beta value of the k^{th} customer attribute
π = profitability of a motor design
ν = attraction value for a motor design

2. Introduction

Design optimization is an important engineering design activity. In general, design optimization determines values for design variables such that an objective function is optimized while performance and other constraints are satisfied [1]. The use of design optimization in engineering design continues to increase, driven by more powerful software packages and the formulation of new design optimization problems motivated by the decision-based design framework [2].

Like other types of modeling, formulating a design optimization model is a subjective process that requires engineering judgment and technical skills. In a given design situation, there are likely to be many variables, parameters, constraints, and criteria related to different performance attributes, costs, and customer preferences. Thus, there are a variety of relevant optimization models from which to choose. Moreover, a large problem can be decomposed into smaller problems, and, instead of a traditional optimization model, heuristics and rules of thumb can be applied to further simplify matters.

Consider, for example, the design of a motor. The overall product development objective is to maximize the profitability of the motor, which depends upon attributes such as the motor's mass and efficiency. These attributes are functions of the motor design variables. Given sufficient information about how the design variables affect the attributes and how the attributes affect profitability, a design engineer can formulate an integrated design optimization problem to maximize profitability (such a problem is sometimes called an "enterprise model"). On the other hand, the design engineer could, as a heuristic for maximizing profitability, choose to minimize mass and maximize efficiency and then formulate the corresponding optimization problem. This simpler optimization problem does not require detailed knowledge about how mass and efficiency affect profitability. Other approaches are possible.

In practice, designers rely upon their experience and insight to choose an appropriate design optimization model, especially in complex multidisciplinary optimization settings [3]. As more options become available, selecting the most appropriate approach becomes more difficult. Enterprise models promise to identify superior designs by accounting for a variety of engineering, cost, manufacturing and marketing concerns. Still, the advantages of better designs must be weighed against the time and cost of formulating and solving these models. Some formulations may require gathering additional data to create the needed relationships or estimate the necessary parameters in the model. Increasing development time and cost can significantly reduce a product's profitability [4]. Heuristics, rules of thumb, low-fidelity approximations, and decomposition can simplify and accelerate the product development process, leading quickly and cheaply to a more profitable product.

We consider the problem of selecting the most profitable design optimization model. Because optimization is only one way to make a decision, we will use the term "model" in a very general way. The "model" may be a heuristic or some other procedure that is used to generate a solution. It is not limited to quantitative mathematical relationships, and it might involve generating a set of alternatives from which the designer selects a solution based on unspoken criteria.

In general, the problem of selecting a design optimization model is a specific example of the meta-design problem [5] that is, in turn, a type of meta-reasoning problem. The meta-design problem is the problem of designing the design process, which requires determining the best set of design activities to perform. The selection of a design optimization model is an especially interesting and important meta-design problem because it directly affects the value of the resulting solution.

This paper specifically addresses the question of selecting design optimization models for product development. A significant feature of our approach is its consideration of the impact of this decision on the time required to complete the product development process and, furthermore, how those delays affect the product's profitability. This approach can be generalized to consider other meta-design problems as well.

3. Related Work

Although design optimization is an important tool for engineering design decision-making, designers do not always explicitly formulate and solve design optimization problems. They often make their decisions in other ways. In general, bounded rationality implies that limits on the amount of information and computational power that are available make complete optimization infeasible. Thus, satisficing can be an appropriate strategy. Similarly, Gigerenzer et al. [6] have shown that people can make good decisions with little information by using simple rules or heuristics and that this approach is rational considering the resources and time spent to make the decision and the impossibility of completely removing uncertainty in our dynamic universe.

Given a variety of ways to approach a design problem, then, a designer must decide which approach to take. The term "meta-reasoning" refers to thinking about which action to take next when searching for a solution [7]. Actions are contingent on multiple attributes: the amount of time required, the quality of the solution returned, the certainty of the solution being satisfactory, and the usefulness of a partial solution (if the action is interrupted). Each action has some utility based on the value of time and whether it leads to a better action. A rational meta-reasoning strategy is to perform the action with the maximal expected utility until there exist none with a positive utility. At that point, commit to the best action found so far.

The term "meta-design" refers to meta-reasoning about engineering design. Designing the design process is also known as design process planning. O'Donovan et al. [8] review approaches to design process planning that focus on the tasks that need to be performed and approaches for sequencing them, which more closely resembles scheduling than the problem of selecting the best way to perform a task. Nogal et al. [9] presented an approach for constructing a decision support system that solves meta-design problems for selecting design alternatives to evaluate, determining when to perform sensitivity analysis, and guiding the iterative design process. The objective is to maximize the user's multiattribute utility function for the design. Their example utility function included cost, weight, and deflection. Time was not included as a consideration.

Roser et al. [10] present a method for evaluating the impact that potential design changes have on the profitability of a product. In particular, they consider the additional costs due to the direct cost of the change, costs of the delay caused by the change, and increases in unit cost. They do not consider any change in sales as a result of the change because their approach focuses on the removal of a serious defect that must be resolved (or else the product is not produced and sold). Radhakrishnan and McAdams [11] discuss the selection of engineering analysis models, such as those used to estimate the stiffness of a mechanical component. A model is selected to maximize a utility function that is based on estimates of the error and the cost of each candidate model. Time and profitability are not considered in the approach.

Another important meta-design problem is the general problem of information gathering ("should I get more information?"), which leads to the well-known concept of value of information [12]. More recently, Ling et al. [13] present an approach for estimating the value of information when the needed probabilities are imprecise.

Finally, decomposition is an important part of product development [14], and selecting the appropriate decomposition is a type of meta-design problem. For example, Cramer et al. [15] discuss different decomposition techniques for multidisciplinary aerospace design optimization.

4. Optimization Model Selection Procedure

We now present a procedure for selecting the optimization model that has the best impact on the product's profitability, taking into account not only the cost and quality of the product design that results from solving the model but also the cost and time needed to formulate and solve the optimization problem.

Our procedure has the following steps, which are described in the following paragraphs:

1. Identify the candidate optimization models.
2. Estimate the time, cost, and solution quality of each candidate model.
3. Evaluate the profitability of each candidate model.
4. Select the most profitable model.
5. Formulate and solve the selected model.
6. Develop the product design that results from the selected model.

Identifying the candidate optimization models requires thinking about the different ways the designer can solve the design problem. As mentioned before, there may be many variables, parameters, constraints and objectives to consider. For instance, in a discussion of the aeroelastic optimization problem, Cramer et al. [15] identify various formulations, including minimizing weight subject to a constraint on drag, minimizing drag subject to a constraint on weight, minimizing some combination of drag and weight, or minimizing operating cost (which both drag and weight affect). Panchal et al. [5] present four strategies for designing a linear cellular alloy, and Logan [16] presents two different approaches to a multidisciplinary aircraft design optimization problem.

Formulating and solving the optimization model may be only part of the effort requrired. Post-optimal parametric studies on bounds or weights may be needed [1]. Recall that the "model" may not be a model at all. Instead, it may be a heuristic or some other procedure that is used to generate a solution. Moreover, some of the candidates could be combinations of the other candidates, where two or more optimization models are used in parallel (similar to the idea of set-based design).

Let $\boldsymbol{M} = \{M_1, \ldots, M_n\}$ be the set of all candidate models. Estimating the time, cost, and solution quality of each candidate model can be a difficult task, especially if the candidates include optimization approaches that are unfamiliar to the designer. Although cost and time measurements are generally straightforward, describing the quality of the solution that the candidate model generates is more subtle and could require multiple attributes (such as unit cost and different performance metrics). The quality measures used should be related to the value of the product. We will let c_i, t_i, and q_i be the cost, time, and solution quality of each candidate model M_i. Moreover, we initially assume that there is no uncertainty in these estimates. Later, we will consider the case when uncertainty exists.

Evaluating the profitability of each candidate model requires a method for relating each candidate's cost, time, and solution quality to its profitability. If available, one could use a sophisticated approach that determines sales, revenue, and costs based on the attributes of the solution. A less sophisticated but still useful approach is to take advantage of a product profit model or the tradeoff rules derived from a product profit model [4]. In general, the profitability should increase as the solution quality improves, decrease as the time-to-market is delayed by the time needed to formulate and solve model M_i, and decrease due to the cost of formulating and solving model M_i. Conceptually, this can be expressed as follows:

$$\Pi_i = f\left(t_i, c_i, q_i\right)$$

Selecting the most profitable model is simply identifying the model M_{i^*} that has the greatest profitability:

$$i^* = \arg\max_{i=1,\ldots,n} \left\{\Pi_i\right\}$$

The last two steps follow clearly from there.

5. Example

A universal electric motor example originally developed by Simpson [17] will be used to demonstrate the model selection procedure. Simpson used this example to demonstrate new techniques in product family design. The following example ignores the product family aspect and deals with only a single motor design.

The optimization model for the universal electric motor problem includes nine design variables, four customer attributes, twenty-three intermediate engineering attribute calculations, six constraints, and seven fixed engineering parameters. Except for the intermediate engineering attributes (which are listed in Appendix A), the equations used to represent the universal electric motor problem will be listed here. The derivations of the equations and other background information on universal electric motors can be found in [17]. The nomenclature and equations for the design variables, fixed model parameters, customer attributes, and constraints are listed below.

<u>Design Variables</u>

N_c	Number of turns of wire on the armature
N_s	Number of turns of wire on the stator, per pole
A_{aw}	Cross sectional area of armature wire [mm^2]
A_{sw}	Cross sectional area of stator wire [mm^2]
r_o	Outer radius of the stator [m]
t_s	Thickness of the stator [m]
I	Electric current [Amperes]
L	Stack length [m]

<u>Fixed Engineering Parameters</u>

l_g	Length of air gap = 7.0×10^{-4} m
V_t	Terminal voltage = 115 V
ρ	Resistivity of copper = 1.69×10^{-8} Ohms•m
μ_o	Permeability of free space = $4\pi \times 10^{-7}$ H/m
p_{st}	Number of stator poles = 2
C_{copper}	Cost of copper = 2.2051 \$/kg
C_{steel}	Cost of steel = 0.882 \$/kg

Customer Attributes

T	Torque [Nm], calculated as follows: $T=K\varphi I$
P	Power [W], calculated as follows: $P=P_{in}-P_{out}$
η	Efficiency [%], calculated as follows: $\eta = P/P_{in}$
M	Mass [kg], calculated as follows: $M=M_w+M_s+M_a$

Constraints and Bounds

$H \le 5000$ A turns/m

$r_o > t_s$

$P = 300$ W

$T = 0.05$ Nm

$\eta \ge 0.15$

$M \le 2$ kg

Table 1: Bounds on Design Variables.

Bounds on the Design Variables								
	N_c	N_s	A_{aw}	A_{sw}	r_o	t_s	I	L
LB	100	1	0.01	0.01	0.01	0.0005	0.1	0.01
UB	1500	500	1.0	1.0	0.1	0.01	6	0.2
Unit	turns	turns	mm^2	mm^2	m	m	A	m

The cost equations were originally derived in Wassenaar and Chen [18]. We simplified the equations slightly. The design cost C_D is assumed to be fixed at \$500,000 while the material cost C_M, labor cost C_L, and capacity cost C_K vary with demand and engineering attributes.

$$C_D = 500{,}000$$

$$C_M = d\left(M_w C_{copper} + \left(M_s + M_r\right) C_{steel}\right)$$

$$C_L = \frac{3}{7} C_M$$

$$C_K = 50\left(\left(d - 500{,}000\right)/1000\right)^2$$

To predict demand, we used discrete choice analysis (DCA) and synthetic spline functions for customer preference. The total demand (d) is the population size (s) multiplied by the probability that a consumer will select a particular design (i.e. estimated market share). We set s = 1,000,000. The following equation shows the common DCA equations developed in [19, 20].

$$d = se^{v}\left[1+e^{v}\right]^{-1}$$
$$v = \Psi_1(M)+\Psi_2(\eta)+\Psi_3(p)$$

The attraction value v is calculated from the following spline functions for the mass, efficiency, and price attributes:

$$\Psi_1(M) = 0.5(1-M)$$
$$\Psi_2(\eta) = \eta - 0.5$$
$$\Psi_3(p) = \frac{25-4p}{15}$$

The profit π of a motor design is a function of the demand (*d*), price (*p*), and the costs discussed above.

$$\pi = dp - \left(C_D + C_M + C_L + C_K\right)$$

Step 1: Identifying Candidate Optimization Models

Five different models (M1, M2, M3, M4, and M5) were created using the motor example in order to have a basis for comparing the information requirements and solution quality. In general, M1, M2, M3, and M4 optimize product performance, while M5 optimizes profit using an enterprise model that combines the marketing and engineering disciplines. The following equations list the objective functions for each model.

$$
\begin{aligned}
&M1:\ \ Min\, f = M - \eta \\
&\qquad P = 300W, T = 0.05Nm \\
&M2:\ \ Min\, f = M - \eta + \left\| W \circ \left(R - Z \right) \right\|_2 \\
&\qquad Z = \left[300\ \ 0.05 \right] \\
&\qquad W = \left[1\ \ 6000 \right] \\
&M3:\ \ Min\, f = M \\
&\qquad P = 300W, T = 0.05Nm \\
&M4:\ \ Min\, f = -\eta \\
&\qquad P = 300W, T = 0.05Nm \\
&M5:\ \ Min\, f = -\pi + \left\| W \circ \left(R - Z \right) \right\|_2 \\
&\qquad Z = \left[300\ \ 0.05 \right] \\
&\qquad W = \left[1\ \ 6000 \right]
\end{aligned}
\tag{6}
$$

M1 seeks to maximize efficiency while minimizing mass. The equality constraints shown force the torque and power to meet the specifications. In M2, the equality constraints on torque and power are removed. Instead, the objective function penalizes deviations from the torque and power requirements. The vector $R = [P\ T]$ describes the power and torque of the motor design, while the vector $Z = [300\ 0.05]$ represents the power and torque requirements. (The vector $W = [1\ 6000]$ is used to scale the deviations.) M3 minimizes mass, and M4 maximizes efficiency. Again, the equality constraints shown force the torque and power to meet the specifications. Finally, M5 is a multidisciplinary optimization incorporating marketing decisions simultaneously with engineering decisions. The all-at-once (AAO) approach seeks to maximize profit while penalizing deviations from the torque and power requirements. The penalty function was used because attempts to maximize profit with equality constraints on torque and power failed to find any feasible solutions.

Step 2: Estimating Time, Cost, and Solution Quality

For the purposes of this example we will estimate the time and cost of each model as follows. We assume that the power and torque requirements are already known.

M1 requires gathering information on how the design variables affect the mass and efficiency, formulating the problem, and solving it. Say this takes 90 days and costs $100,000.

M2 requires everything done for M1 but also requires solving the problem multiple times with different weights in the objective function. Say this takes 100 days and costs $105,000.

M3 and M4 require the same time and cost as M1 because they require the same information and effort.

M5 requires everything done for M1 as well as a marketing survey (which costs $100,000) to determine how customer demand depends upon the mass and efficiency attributes. In addition, the optimization problem is more complex, and it also requires solving the problem multiple times with different weights in the objective function. Say this takes 150 days and costs $200,000.

Evaluating solution quality can be a complex issue, especially for multi-objective optimization problems. Wu and Azarm [21] developed a set of five metrics that can be used to evaluate the quality of a solution set in a multi-objective optimization. For this example, the solution quality will be based on the actual achievable profit of each solution. Because the goal of this paper is to explore the selection of optimization models, we compute the quality of the actual solutions for the various models rather than estimate it as described in Step 2 above (Radhakrishnan and McAdams [11] proceed in the same way to demonstrate their methodology for comparing design models). The five models were solved using the *fmincon* function included in the MATLAB optimization toolbox. Within each model various input parameters were changed, such as weighting coefficients and initial solutions. Seven initial solutions [22] were used for each of the five setups. The feasibility of each initial solution was determined by entering the values into a spreadsheet model to check for constraint violation prior to running any optimizations.

The results of models M1 through M4 were entered into a price optimization to determine the solution quality of each design. The solution quality of M5 is determined simultaneously with the design variables in the all-at-once optimization. Table 2 shows the actual solution quality of each of the five models.

Step 3: Determining Profitability of Each

At this point, we estimate the profitability of each model as follows:

$$\Pi_i = q_i - 1000t_i - c_i \tag{7}$$

This simple profitability estimate capture the essential relationships of product development. Namely, increasing solution quality increases profitability, increasing development time reduces profitability, and additional costs reduce profitability. Table 2 displays the results of all five models.

Table 2: Example of Time, Cost, Solution Quality and Profitability Estimates

Motor Design (T = 0.05 Nm)				
Model M_i	Estimated Parameters			Π_i ($)
	t_i (days)	c_i ($)	q_i ($)	
M1	90	100,000	2,691,980	2,501,980
M2	100	105,000	2,249,784	2,044,784
M3	90	100,000	2,314,632	2,124,632
M4	90	100,000	1,445,608	1,255,608
M5	200	200,000	2,641,217	2,241,217

Step 4: Select the Most Profitable Model

As indicated in Table 2, the most profitable model is M1. Of the five available models a designer should choose the M1 modeling approach for the motor design and use it to determine the final design variable values (Steps 5 and 6).

6. Addressing Estimate Uncertainty

There may be uncertainty in the estimates of the cost and time needed to formulate and solve each candidate model and in the estimates of solution quality. No matter what profitability evaluation technique is used, these uncertainties cause uncertainty in the profitability measure Π_i. Moreover, the profitability evaluation technique may have error due to its abstraction, which introduces certain approximations, and the actual profitability of the resulting design is a random variable that is affected by a large number of exogenous variables, including the cost of raw materials and labor, economic conditions that affect demand, and many other such factors.

Research on decision-making under uncertainty has yielded a large number of various techniques whose suitability depends upon the amount of information that the designer has about the distribution of possible outcomes.

If there is sufficient information about the probabilities for each possible outcome for each candidate model as well as the designer's risk attitude, the designer can employ decision analysis techniques to determine the expected utility of each candidate model and select the optimization model that maximizes expected utility [23]. Since this is a well-known topic, no further information needs to be provided here. However, if little information about the probabilities is available, the designer could use imprecise probabilities to find an imprecise probability distribution for each candidate model's profitability.

To illustrate this approach, which is less well-known, consider the example from Section 5, which required estimates for the cost, time, and solution quality of each model. Suppose a design engineer recognizes that there exists substantial uncertainty in the time and solution quality estimates. However, he is unable to provide precise probability distributions on these quantities (which we assume are independent). Instead, for model M_i, he is willing to say only that the time will be in a certain interval $\left[t_{ai}, t_{bi}\right]$ and the solution quality (profitability) has a uniform distribution with a width of $2y_i$ but the mean may be anywhere in the interval $\left[q_{ai}, q_{bi}\right]$. We can model each of these imprecise probability distributions as a p-box (see, for example, [24]). Each p-box is specified by two functions: a lower cumulative probability bound and an upper cumulative probability bound.

The following functions are the lower and upper cumulative probability bounds for the time (see also Figure 1):

$$\underline{F}_{ti}(x) = \begin{cases} 0, x < t_{bi} \\ 1, x \geq t_{bi} \end{cases}$$

$$\overline{F}_{ti}(x) = \begin{cases} 0, x < t_{ai} \\ 1, x \geq t_{ai} \end{cases}$$

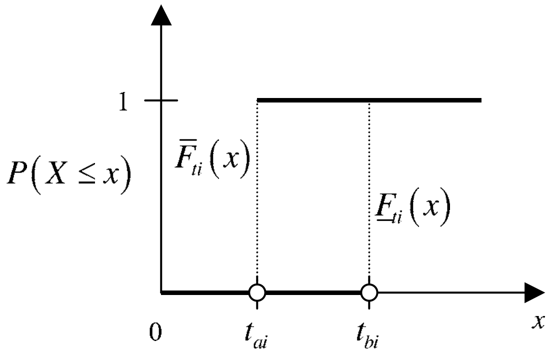

Figure 1: Probability Bounds for Time

To define the p-box for the solution quality, we will need the following values:

$$
\begin{aligned}
L_{i1} &= q_{ai} - y_i \\
U_{i1} &= q_{ai} + y_i \\
L_{i2} &= q_{bi} - y_i \\
U_{i2} &= q_{bi} + y_i
\end{aligned}
$$

The following functions are the lower and upper cumulative probability bounds for the solution quality (see also Figure 2):

$$
\underline{F}_{qi}(x) = \begin{cases} 0, x < L_{i2} \\ \dfrac{x - L_{i2}}{2y_i}, L_{i2} \le x \le U_{i2} \\ 1, x \ge U_{i2} \end{cases}
$$

$$
\overline{F}_{qi}(x) = \begin{cases} 0, x < L_{i1} \\ \dfrac{x - L_{i1}}{2y_i}, L_{i1} \le x \le U_{i1} \\ 1, x \ge U_{i1} \end{cases}
$$

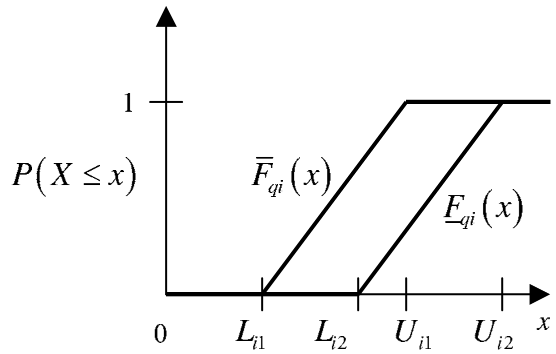

Figure 2: Probability Bounds for Time

The distribution of Π_i is imprecise. (One can show that its p-box has a shape similar to the one shown in Figure 2.) Therefore, the expected profitability of the model is unknown. However, we can set bounds $\underline{\Pi}_i, \overline{\Pi}_i$ on the expected profitability based on the above. Based on our definition of model profitability, introduced in Section 5, the lower bound on the expected profitability can be found by determining the lower bound on the expected value of the solution quality and the upper bound on the expected value of time. The lower bound on the expected value of the solution quality is q_{ai}. The upper bound on the expected value of the solution quality is t_{bi}. Likewise, the upper bound on the expected profitability comes from determining the upper bound on the expected value of the solution quality (which is q_{bi}) and the lower bound on the expected value of time, which is t_{ai}. Therefore,

$$\underline{\Pi}_i = q_{ai} - 1000 t_{bi} - c_i$$
$$\overline{\Pi}_i = q_{bi} - 1000 t_{ai} - c_i$$

For the purposes of this example suppose the design engineer can provide the following values:

1. For M1, the time will be between 80 and 100 days, and the mean solution quality will be between \$2,500,000 and \$2,900,000.
2. For M2, the time will be between 90 and 110 days, and the mean solution quality will be between \$2,000,000 and \$2,500,000.

3. For M3, the time will be between 80 and 100 days, and the mean solution quality will be between \$2,100,000 and \$2,500,000.
4. For M4, the time will be between 80 and 100 days, and the mean solution quality will be between \$1,000,000 and \$1,800,000.
5. For M5, the time will be between 140 and 200 days, and the mean solution quality will be between \$2,600,000 and \$2,700,000.

Table 3 summarizes the information and shows the upper and lower bounds on the expected profitability of each model.

Table 3: Example of Expected Profitability Bounds

Model	Bounds on mean solution quality (\$)		Interval for time (days)		Bounds on expected model profitability (\$)	
	Lower	Upper	Min	Max	Lower	Upper
M1	\$2,500,000	\$2,900,000	80	100	\$2,300,000	\$2,720,000
M2	\$2,000,000	\$2,500,000	90	110	\$1,785,000	\$2,305,000
M3	\$2,100,000	\$2,500,000	80	100	\$1,900,000	\$2,320,000
M4	\$1,000,000	\$1,800,000	80	100	\$800,000	\$1,620,000
M5	\$2,600,000	\$2,700,000	140	200	\$2,200,000	\$2,360,000

Although M4 is clearly dominated, the other four models are pairwise nondominated. The decision is indeterminate. However, in order to select a single model, the design engineer must use a decision policy. Many such policies exist. If the design engineer prefers the model with the best worst-case performance, he will choose the one that maximizes $\underline{\Pi}_i$. (This risk-averse policy is also known as the Γ-maximin solution policy.) In this example, M1 has the largest lower bound on expected profitability.

7. A General Approach to Meta-Design Problems

Although this paper has focused on the selection of an optimization model for a design problem, the approach presented above can be generalized to other meta-design problems. It is certainly relevant to decisions about the architecture of a design process, since this is what decomposes the overall design problem into a network of decisions, design activities, and information flow [5, 14, 25]. In any case, the basic idea is the same: identify alternatives, estimate the time, cost, and solution quality of each alternative, and select the one that leads to the most profit.

8. Summary

In this paper, we have introduced the problem of selecting a design optimization model and presented a procedure for solving the problem. The importance of this procedure is to demonstrate that one can approach the selection of an optimization model using a rigorous mathematical procedure instead of depending upon one's intuition. Furthermore, this paper shows that the choice of a design optimization model is a meta-design decision that affects the profitability of the product. It is not necessarily the case that the most sophisticated optimization model is the best choice.

In many cases, heuristics or rules of thumb can play a critical role when estimating the profitability of a model. For the motor example, Brochtrup shows that, as the torque requirement increases to T=0.50 Nm, the solution quality for M3 (which minimizes mass) exceeds all four other models [22].

It is possible to extend this approach to selecting a portfolio of optimization models. That is, given a set of distinct candidates, the designer may select more than one optimization model and use those in parallel, taking the best solution that results from the ones selected. The selection problem must find the optimal set of optimization models to use. Such an approach would be useful if there were a large number of feasible combinations. Otherwise, the approach presented here should be sufficient.

More generally, the procedure can be adapted for other meta-design choices, leading to better decisions about the choices that designers make.

Finally, this procedure could be used to guide the development of new optimization models that are designed for profitability.

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APPENDIX A

Intermediate Engineering Calculations

H	Magnetizing intensity [Ampere turns/m] $H=N_c I/(l_c+l_r+2l_g)$
l_c	Mean path length within the stator [m] $l_c = \pi(2r_o+t_s)/2$
l_r	Diameter of armature [m] $l_r=2(r_o-t_s-l_g)$
P_{in}	Input power [W] $P_{in}=V_t I$
P_{out}	Power losses due to copper and brushes [W] $P_{out}=I^2(R_a+R_s)+2I$
l_{aw}	Armature wire length [m] $l_{aw}=2L+4(r_o-t_s-l_g)N_c$
l_{sw}	Stator wire length [m] $l_{sw}=p_{st}(2L+4(r_o-t_s))N_s$
R_a	Armature wire resistance [Ohm] $R_a = \rho l_{aw}/A_{aw}$
R_s	Stator wire resistance [Ohm] $R_s = \rho l_{sw}/A_{sw}$
M_w	Mass of windings [kg] $M_w=(l_{aw}A_{aw}+l_{sw}A_{sw})\rho_{copper}$
M_s	Mass of stator [kg] $M_s = \pi L(r_o^2-(r_o-t_s)^2)\rho_{steel}$
M_a	Mass of armature [kg] $M_a = \pi L(r_o-t_s-l_g)^2\rho_{steel}$
K	Motor constant [dimensionless] $K=N_c/\pi$
$\Im$	Magneto magnetic force [A turns] $\Im = N_s I$
$\Re$	Total reluctance of the magnetic circuit [A turns/Wb] $\Re = \Re_s + \Re_a + 2\Re_g$
$\Re_s$	Reluctance of stator [A turns/Wb] $\Re_s = l_c/(2\mu_{steel}\mu_o A_s)$
$\Re_a$	Reluctance of armature [A turns/Wb] $\Re_a = l_r/(\mu_{steel}\mu_o A_a)$
$\Re_g$	Reluctance of one air gap [A turns/Wb] $\Re_g = l_g/(\mu_o A_g)$
A_s	Cross sectional area of stator [m^2] $A_s=t_s L$
A_a	Cross sectional area of armature [m^2] $A_a=l_r L$
A_g	Cross sectional area of air gap [m^2] $A_g=l_g L$
μ_{steel}	Relative permeability of steel [dimensionless]

$$\mu_{steel} = -0.2279H^2 + 52.411H + 3115.8 \qquad H \le 220$$
$$\mu_{steel} = 11633.5 - 1486.33\ln(H) \qquad 220 < H \le 1000$$
$$\mu_{steel} = 1000 \qquad H > 1000$$

φ	Magnetic flux [Wb] $\varphi = \Im/\Re$