ABSTRACT

Title of Dissertation: STRATEGIC MONETIZATION AND UPGRADING DECISIONS FOR MOBILE APPLICATIONS

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The mobile applications (apps) market has been growing steadily, propelled by rapid technological developments and consumer adoption of smartphones and tablet personal computers. In this dissertation research, I study app publishers' strategic monetization and upgrading decisions. The first essay studies app publishers' dynamic forward-looking decisions on offering different versions of an app: free, paid, or both (i.e., freemium), and investigates alternative commission schemes which could benefit both app publishers and an app platform. My findings lead to recommendations on how one may improve the current commission structure to achieve mutual benefits for both the platform and app publishers. The second essay examines strategic upgrading decisions of mobile apps by taking into consideration of their interconnections with versioning decisions and between the free and paid versions of an app. Our joint model of versioning and upgrading decisions provides estimates of various revenues and costs associated with the two decisions, and our policy simulations based on the model estimates examine the soundness of certain current practices and identifies opportunities to improve app publishers' profits, the app distribution platform' revenue, and the eco-system payoff. This dissertation research provides a range of policy recommendations to key players in the mobile app industry.

STRATEGIC MONETIZATION AND UPGRADING DECISIONS FOR MOBILE APPLICATIONS

by

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Chapter 1: Introduction

The mobile applications (apps) market has been growing steadily, propelled by rapid technological developments and consumer adoption of smartphones and tablet personal computers. The key players of the mobile app market consist of app publishers, app distribution platforms, and consumers, which together form interconnected eco-systems. Each player's decisions have impact on the other players' decisions in the marketplace, as well as the revenues, profits, and welfare of all parties involved. Despite the importance of a thorough understanding of the interconnections in their decisions, there has been very little systematic research on the topic. In this dissertation research, I tackle the challenge by focusing on app publishers' strategic monetization and upgrading decisions, and thus the title of the dissertation is "Strategic Monetization and Upgrading Decisions for Mobile Applications," under which I have been working on two essays.

The first essay studies app publishers' dynamic forward-looking decisions on offering different versions of an app: free, paid, or both (i.e., freemium), and investigates alternative commission schemes which could benefit both app publishers and an app platform. We build a dynamic discrete choice structural model which incorporates both direct and indirect revenue sources of mobile apps and captures inter-temporal and cross-version dynamics of mobile app demand. We employ the Bayesian IJC estimation technique and estimate an app publisher monthly revenue via a single advertising network, free and paid apps' expected revenue from in-app purchases, a monthly fixed cost of offering each app version, and variable cost associated with new downloads. Also, our Bayesian BLP demand model reveals a variety of inter-temporal and cross-version effects between free and paid apps. A main objective of the essay is to investigate alternative fee structures imposed by app distribution platforms via policy simulations based on our dynamic structural model. We find that reducing the current commission rate, or charging a higher commission rate on direct revenues and a lower commission rate on indirect revenues increases the number of apps on the platform, and thereby increases the ecosystem payoff and creates profit-sharing opportunities. We also explore rent plus commission fee structures and find similar profit-sharing opportunities driven by the increase in the number of paid apps. These findings lead to recommendations on how one may improve the current commission structure to achieve mutual benefits for both the platform and app publishers.

The second essay examines upgrading decisions of mobile apps in conjunction with their versioning decisions and identifies opportunities to improve app publishers' profits and the app distribution platform' revenue. We build a joint model of app publishers' upgrading and versioning decisions that incorporates various revenue and cost sources associated with these decisions, and a demand model of app downloads that takes into account cross-effects of versioning and upgrading decisions between the free and paid versions. Based on the demand model, among other things, we find that upgrading one version of an app increases its demand while decreasing the demand for the other version, if it is available. We also find a contemporaneous cannibalization effect between the free and paid versions. These effects speak to the importance of jointly studying the upgrading decisions between the free and paid

version of an app. Our joint model of versioning and upgrading decisions provides estimates of various revenues and costs associated with the two decisions. We find that the free version generates a similar amount of revenue from in-app purchases and from a single ad-network, while the paid version generates much higher revenue from in-app purchases than the free version does. Also, not surprisingly, we find that monthly fixed costs and upgrading costs of paid apps are higher than those of free apps. Based on the model estimation results, our policy simulations examine the soundness of certain current practices and identifies opportunities to improve app publishers' profits, the app distribution platform' revenue, and the eco-system payoff.

Chapter II: Essay 1

Free, Paid, or Freemium: Dynamic Monetization Decisions for Mobile Apps

II.1 INTRODUCTION

The mobile applications (apps) market has been growing steadily, propelled by rapid technological developments and consumer adoption of smartphones and tablets. As of 2016, a total of about two million mobile apps are available on Google Play[1] and Apple's App Store[2]. The mobile app market is still rapidly expanding and annual total downloads are expected to exceed 260 billion by 2017[3]. App publishers have used a variety of monetization strategies. Some apps are available only in a paid version. In addition to the direct revenue from app downloads, their publishers may earn revenues from in-app purchases (by selling enhanced virtual weapons, fancier avatar clothes, or progress to the next level of a game). Other apps are free to download, and their publishers expect to make money through in-app purchases and in-app advertising. In recent years, the "freemium" strategy has become increasingly popular, under which an app publisher simultaneously provides a free version with limited functionality as well as a paid version with full functionality. The paid version usually does not contain in-app advertising, but in-app purchases are often available and an important revenue source for both the free and paid versions.

Despite the importance of monetization strategies for app publishers and distribution platforms, little is known about how app publishers make strategic decisions, most importantly regarding versioning and pricing. A versioning decision in this article refers to offering a free, paid or both versions of an app. Why are some apps available only in a paid version, some only in a free version, yet others are

[1] http://www.statista.com/statistics/266210/number-of-available-applications-in-the-google-play-store/
[2] http://www.zdnet.com/article/ios-versus-android-apple-app-store-versus-google-play-here-comes-the-next-battle-in-the-app-wars/
[3] http://www.gartner.com/newsroom/id/2592315

available in both? Why do some publishers offer the free version first and then introduce a paid version later, while others offer the paid version first and then add a free version? How do publishers decide to eliminate a free, paid, or both versions of an app? How do app publishers set the price of their paid apps? Do the free and paid versions stimulate future demand for each other or do they cannibalize each other? Are there ways to improve the current commission structure that would benefit both app publishers and a platform? We intend to address these questions by conducting an empirical investigation of app publishers' monetization decisions, including decisions on which versions to offer in any given time period, as well as the decisions on the price for the paid version. Here, versions are defined as "free only", "paid only", "freemium" (i.e., both), and neither.

Other than the one-time direct revenue from paid app downloads, in-app purchase and advertising revenues from an app are generated over the course of its life time, and the profitability of an app version also needs to be evaluated over its life time. This means that app publishers' version decisions are innately forward-looking in nature. Publishers need to compare the attractiveness of the options based on not only their current-period profits but those expected in the future. Therefore, we build a dynamic structural model that incorporates app publishers' forward-looking decision processes and multiple monetization strategies (i.e., direct revenues from paid version downloads and indirect revenues from in-app purchases and in-app advertising). In addition, we formulate a demand model, which is built upon users' utility of app downloads, that forms the basis of app publishers' expectations of demand dynamics over time and across app versions (stimulating demand vs.

cannibalization of each other). For the empirical analyses, we have compiled a unique dataset from various sources that consists of 584 apps with 766 unique app-version combinations spanning a 14-month period.

The main objectives of this research are: 1) to gain a deeper understanding of the various trade-offs involved in publishers' app monetization decisions, and 2) to provide policy recommendations on how to improve commission structures of app distribution platforms. By incorporating various monetization strategies into publishers' profit functions and allowing publishers to make monetization decisions based on the expected life-time profit of a versioning option, our proposed model can identify the trade-offs facing publishers, such as revenues vs. costs, direct vs. indirect revenues, fixed vs. variable costs, and current vs. future profits. Google Play and other platforms have thus far charged a 30% flat commission rate on both direct revenues from paid app sales and indirect revenues from in-app purchases. Since the parameters of our dynamic structural model are invariant to changes in the commission rate, we are able to conduct counterfactual policy simulations of alternative commission structures even though the commission rate was fixed in the data (cf. Chintagunta, Rossi, Erdem, and Wedel 2006). The simulation analyses enable us to identify opportunities where the payoffs for both the app distribution platform and publishers could be increased, and to make actionable recommendations on how to improve the current commission structures of app platforms to achieve mutual benefits for publishers and platforms.

The rest of the paper is organized as follows. After providing a review of relevant prior research in Section 2, we describe the institutional setting and the data collection process in Section 3. In Section 4, we present the specifications of our proposed model. We then explain our identification strategy and model estimation procedures in Section 5. The estimation results are reported in Section 6, followed by the counterfactual policy simulations in Section 7. In the final section, we summarize the contributions of our study and discuss its managerial insights.

II.2 RELATED LITERATURE

Four streams of research are relevant to the present study, on 1) versioning strategies of information goods, 2) free version information goods, 3) demand for mobile apps, and 4) two-sided markets. We provide a brief review of each research stream below.

Versioning strategies for information goods have been studied extensively using analytical/game-theoretic approaches. Sundararajan (2004) examines nonlinear pricing of information goods. He finds that a limited-feature version should be provided when the cost of versioning is lower, while a high-priced version with all possible features should be offered when the cost of versioning is high. Bhargava and Choudhary (2008) derive an optimal versioning strategy for a monopolistic seller, and find that versioning is advantageous when the optimal market share of a lower-quality version exceeds the optimal market share of the high-quality version, when both are offered alone. In these studies, “versioning” means to offer products that are vertically differentiated in terms of quality. Dogan and colleagues (2010) construct a two-period model and study how word-of-mouth relates the features of the product in the first period to its demand in the second period. In the context of the movie industry, the work by Calzada and Valletti (2012) suggests a versioning strategy for film distributors that involves a simultaneous release of the theatrical and video versions rather than a sequential release, if the studio controls both exhibition and distribution channels and if the versions are not perfect substitutes. The game-theoretical research in this stream of literature provides valuable insights into mobile app versioning strategies which invite empirical testing. The empirical modeling framework we develop in the present study intends to accomplish this. It provides a quantifiable assessment of revenues and costs of versioning options, customer base extension, advertising revenue, and incorporates potential word-of-mouth and cannibalization effects between versions.

A second stream of literature on versioning decisions addresses the free version. Lambrecht et al. (2014) provide a general review of the monetization strategies used by firms who sell digital goods online. Haruvy and Prasad (2001) build game theoretical models to derive the optimal freeware quality in the presence of network externalities. They find that a only having a low quality of the free version maximizes revenue. If the free version's quality is fair, cannibalization may wipe out the positive effect on market penetration. Jiang and Sarkar (2009) study the speed of diffusion, and show that a free version increases the net present value of future paid version downloads. Prasad et al. (2003) consider the free version's potential to generate revenue via advertising. Their analysis suggests a hybrid strategy of pay-per-view and advertiser-supported strategies, rather than pure strategies. Kumar and Sethi (2009) investigate a similar issue in a dynamic setting. Most works in this research stream also rely on game-theoretical approaches. An exception is Pauwels and Weiss (2008), who conduct several empirical studies looking at online firms' transition from free to fee. Their findings suggest that the transition needs to be triggered after the free version subscription starts showing signs of saturation.

There has been only a limited number of studies that directly investigate the demand for mobile apps. Carare (2012) investigates the impact of bestseller rank on the next day demand for an app using data of apps sold on Apple's App Store. This study shows that the consumers' willingness-to-pay increases if an app is ranked among the top 100 best-selling apps. Bresnahan et al. (2014) study supply-side decisions. They examine app developers' static choice between Google Play and Apple's App Store platforms. They find that the preference of app publishers does not differ between these two platforms and that it is better for popular apps to publish on both. Ghose and Han (2014) estimate the demand for mobile apps using an aggregate structural demand model following the BLP framework (Berry, Levinsohn, and Pakes 1995). They find amongst others that app age has a negative effect. The demand component of our model will build upon this nascent stream of research.

Lastly, our research relates to the literature on two-sided markets, where two groups of users interact through a platform and one group's decisions affect the other group's decisions (Rysman 2009). Similar to the platforms studied in the literature, e.g., video game platforms (Zhou 2014), TV networks (Gil and Riera-Crichton 2012), and credit card networks (Rochet and Torile 2003), mobile app platforms attract two user groups (i.e., mobile app publishers and app users), and their decisions are dependent upon each other. App publishers' versioning and pricing decisions depend on app users' downloads, and users consider characteristics that include the price of apps when they decide to download mobile apps. One of the main findings in the extensive theoretical literature on pricing decision in two-sided markets is that the pricing of one side not only depends on its own demand and costs but also the other side's participation decision (Rochet and Tirole 2003, 2006, Weyl 2010). Our model builds upon this literature and adds to it by considering multiple monetization strategies.

II.3 INSTITUTIONAL SETTING AND DATA

The apps in our data were Android apps published on Google Play[4], which was established in 2008 and is currently the largest and fastest growing platform for mobile apps. Over two million apps were published on it in 2016, up from 1.5 Million in 2015. Together with the App Store, Google Play accounts for over 90% of worldwide app revenues[5]. Table II-1 shows some basic statistics for these and three other major platforms.

Insert Table II-1 about here

A platform like Google Play provides an ecosystem that enables app publishers to develop apps compatible with different mobile devices and to distribute them to consumers. Publishers decide the monetization strategy for their apps. They can choose to offer free and/or paid versions, to provide in-app purchases, to plant

[4] https://play.google.com/store

[5] http://www.zdnet.com/article/ios-versus-android-apple-app-store-versus-google-play-here-comes-the-next-battle-in-the-app-wars/

third party ad-networks[6] in free apps, and set a price of the paid version. Once an app is listed on the platform and users start downloading it, the platform keeps track of download statistics and in-app purchases. The main revenue source of a platform consists of commissions it collects based on the revenues generated by apps. Like all other major platforms, Google Play charges a 30% flat commission rate on direct revenues from paid app downloads and on indirect revenues from in-app purchases. App publishers' in-app advertising revenues are not subject to commissions paid to the platform. Google Play collects revenues generated by apps and makes payments to publishers each month after taking its commissions. Google announced that it paid out $7 billion in 2014[7].

We have compiled a dataset on Android apps published on Google Play, including their monthly download volumes, in-app purchases, in-app ad-networks, retail prices, and app characteristics. The main body of our data was collected by scraping the website of the mobile app market research firm Priori Data (http://prioridata.com), which publicized the top 150 free or paid apps in terms of download volumes each month. We used these lists in the first six months (December 2011 to May 2012) to construct a basket of 584 apps, and then tracked the download volumes and other data of their free and/or paid versions every month in the next 14 months (June 2012 – July 2013). The resultant dataset consists of 766 unique app-

[6] A mobile ad network is a company that connects advertisers to mobile app publishers that host their advertisements.

[7] http://www.androidpolice.com/2015/02/26/google-announces-arrival-ads-play-store-searches-7-billion-paid-devs-last-year/

version combinations, 416 of which were free[8]. We used AppBrain.com and AppAnnie.com to find the publishing date of each app version, from which we calculated its age (in months) in each month of the data period. The number of in-app networks in each free app was derived by analyzing its' compressed package file (i.e., .apk file) using the AdRisk method (Grace et al. 2012) and the Addons Detector. Both methods detect 100 unique ad-networks, of which 41 were used by the apps in our data. We use the number of ad-networks as a proxy for the amount of ads users see[9]. Table II-2 presents the notation and descriptive statistics of key variables, and Figure II-1 shows the distributions of app ages and cumulative download volumes in our data.

Insert Table II-2 about here

Insert Figure II-1 about here

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[8] If an app offered both the free and paid versions (e.g., Angry Bird Space and Angry Bird Space Premium), we counted it as one app and two unique app-version combinations.

[9] The most important reason for app publishers to add another ad-network is to provide a different ad format (e.g., full screen ad, top of the screen banner, video, interstitial ad, etc.) that is not supported by the current ad-networks. Thus, the more ad-networks are used, the larger the number of ads that are shown. In addition, apps allow multiple ad-networks to serve a single ad format to increase the fill rate, which is defined as the number of ads shown to users divided by the number of ad requests sent by an app to its ad-networks. Therefore, the number of ads shown increases proportionally with the number of ad-networks.

In the data, 183 apps offered both free and paid versions at least once, 233 apps published a free version only, while 166 apps offered only a paid version. 97 apps (17%) changed their versioning options at least once during the data collection period. Table II-3 summarizes the transition pattern of the versioning decisions from time t - 1 to t. It shows that the versioning decision for a given app can change over time, but that for many apps/times it does not.

Insert Table II-3 about here

II.4 MODEL FORMULATION

The free and paid versions of an app are likely to influence each other's demand not only in the current period but also in future periods. This means that a publisher's versioning decision at any given time does not consist of two independent binary decisions, but rather is a choice out of four possible options: free only, paid only, freemium, and neither. A dynamic model is needed to account for the notion that a publisher's versioning decision in a given time period will take into account the future profit streams of all four versioning options.

II.4.1 Timing of Publishers' Decisions

The timing of publishers' decisions in our model is depicted in Figure II-2. At the beginning of each time period t for the publisher of app j in category $c(j)$:

1. The publisher observes the cumulative download volumes $Q_{F,j,t-1}$ and $Q_{P,j,t-1}$, which reflect the customer base for the free (F) and paid (P) version up to the previous time.
2. The publisher anticipates the download volumes of the free and paid versions in the current month ($\tilde{q}_{F,j,t}$, $\tilde{q}_{P,j,t}$, $\tilde{q}_{F|P,j,t}$, and $\tilde{q}_{P|F,j,t}$[10]) and their cumulative download volumes by month *t* ($\tilde{Q}_{F,j,t}$, $\tilde{Q}_{P,j,t}$, $\tilde{Q}_{F|P,j,t}$, and $\tilde{Q}_{P|F,j,t}$), assuming that they would be offered.
3. It then assesses the one-period profit of each versioning option based on the optimal retail price of the paid version, the number of in-app ad networks in the free version ($\bar{a}_j$), and in-app purchases from both versions ($\bar{b}_{F,j}$ and $\bar{b}_{P,j}$). The publisher also observes version specific random profit shocks, $\varepsilon_{C_{j,t}}$, where $C_{j,t} = C(S^1_{j,t}, S^2_{j,t}) \in \{N, F, P, B\}$ indicates the four versioning options: neither (N), free only (F), paid only (P), and both/freemium (B).
4. Based on the above information, the publisher anticipates the discounted (with a factor $\delta \in (0, 1)$) total future profit of each versioning option, and chooses the option that offers the highest discounted life-time profit.
5. Based on a publisher's versioning decision, consumers download an app-version, make in-app purchases, and the platform gets paid with a commission rate *r* on the resulting direct sales of paid apps and indirect revenues from in-app sales.

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[10] The $\tilde{q}_{F,j,t}$ and $\tilde{q}_{P,j,t}$ indicate the expected monthly download volumes of the free and paid versions when that version is offered alone without its counterpart version, whereas $\tilde{q}_{F|P,j,t}$, and $\tilde{q}_{P|F,j,t}$ denote the expected download volumes of the free, respectively the paid version when both versions are offered simultaneously.

II.4.2 Objective Functions

An app publisher's objective in any given time period *t* is to choose the versioning option that maximizes the net present value of the future profit streams derived from all versioning options over an infinite time horizon. The publisher of app *j* makes a sequence of such decisions over time, $\{C_{j,t}\}_{t=1}^{\infty}$, to maximize life-time profit, where future profits are discounted with a factor δ:

$$V_\theta(S_{j,t}^1, S_{j,t}^2, \varepsilon_{i,t}) = \sup_{\{C_{j,\tau}\}_{\tau=t}^{\infty}} E\left\{\sum_{\tau=t}^{\infty} \delta^{\tau-t}\, \Pi_{j,\tau}\big(C_{j,\tau}; S_{j,\tau}^1, S_{j,\tau}^2, \varepsilon_{j,\tau}\big) \big| S_{j,t}^1, S_{j,t}^2, \varepsilon_{j,t}, \theta\right\}. \quad \text{(II-1)}$$

Insert Figure II-2 about here

The version decisions depend on the vectors of state variables $S_{j,t}^1$ and $S_{j,t}^2$ (described below). Let $\Pi_{j,t}\big(C_{j,t}; S_{j,t}^1, S_{j,t}^2, \varepsilon_{j,t}\big)$ denote the current-period profit, to be specified later, and θ be the unknown parameters. The optimal versioning decision for app *j* at time *t* is provided by the solution to Bellman's equation (Rust 1987):

$$\begin{aligned} V_\theta(S_{j,t}^1, S_{j,t}^2, \varepsilon_{j,t}) &= \max_{C_{j,t}}\left\{V_\theta^{C_{j,t}}(S_{j,t}^1, S_{j,t}^2, \varepsilon_{j,t})\right\} \\ &= \max_{C_{j,t}}\left\{\Pi_{j,t}\left(C_{j,t}; S_{j,t}^1, S_{j,t}^2, \varepsilon_{C_{j,t},j,t}, \theta\right) + \delta E_{\tilde{S}_{j,t}^1, \tilde{\varepsilon}_{j,t}}\left[V_\theta(\tilde{S}_{j,t}^1, \tilde{S}_{j,t}^2, \tilde{\varepsilon}_{j,t}) | S_{j,t}^1, S_{j,t}^2, C_{j,t}\right]\right\}. \end{aligned} \quad \text{(II-2)}$$

The version-specific value function $V_\theta^{C_{j,t}}(S_{j,t}^1, S_{j,t}^2, \varepsilon_{j,t})$ is implicitly defined in equation (II-2). The expectation, $E_{\tilde{S}_{j,t}^1, \tilde{\varepsilon}_{j,t}}[\cdot]$, is taken over the future values of the state variables $\tilde{S}_{j,t}^1$ and the random shocks $\tilde{\varepsilon}_{j,t}$, and is discounted with a factor δ to yield the expected net present value of profit at t+1, given optimal decisions at each time period from t+1 to the infinite future:

$$E_{\tilde{S}^1_{j,t},\tilde{\varepsilon}_{j,t}}\left[V_\theta\left(\tilde{S}^1_{j,t},\tilde{S}^2_{j,t},\tilde{\varepsilon}_{j,t}\right)|S^1_{j,t},S^2_{j,t},C_{j,t}\right]$$
$$= \int_{\tilde{S}^1_{j,t}}\int_{\tilde{\varepsilon}_{j,t}} V_\theta\left(\tilde{S}^1_{j,t},\tilde{S}^2_{j,t},\tilde{\varepsilon}_{j,t}\right) p\left(d\tilde{S}^1_{j,t},d\tilde{\varepsilon}_{j,t}\middle|C_{j,t},S^1_{j,t},S^2_{j,t},\varepsilon_{j,t},\theta\right). \tag{II-3}$$

We proceed to specify the profit functions for the four versioning options.

II.4.3 Single-period profit functions

We incorporate various sources of revenues and costs in the profit functions, including the direct revenue of sales of the paid version, indirect revenue from in-app purchases for the paid version, indirect revenue from in-app purchases and in-app advertising for the free version, monthly fixed costs of offering each app version, and costs associated with new customers. The publisher's ex-ante single-period profit functions for the four versioning options are specified as:

$$\Pi_{N,j,t} = \varepsilon_{N,j,t}, \tag{II-4a}$$

$$\Pi_{F,j,t} = (1-r)\lambda_{1,F}\bar{b}_{F,j}\tilde{Q}_{F,j,t} + \lambda_2\bar{a}_j\,\tilde{Q}_{F,j,t} - \gamma_{1,F,c} - \gamma_2\tilde{q}_{F,j,t} + \varepsilon_{F,j,t}, \tag{II-4b}$$

$$\Pi_{P,j,t} = (1-r)\left(P^*_{j,t}\tilde{q}_{P,j,t} + \lambda_{1,P}\bar{b}_{P,j}\tilde{Q}_{P,j,t}\right) - \gamma_{1,P,c} - \gamma_2\tilde{q}_{P,j,t} + \varepsilon_{P,j,t}, \tag{II-4c}$$

$$\begin{aligned}\Pi_{B,j,t} = {} & (1-r)\left(P^*_{j,t}\tilde{q}_{P|F,j,t} + \lambda_{1,F}\bar{b}_{F,j}\tilde{Q}_{F|P,j,t} + \lambda_{1,P}\bar{b}_{P,j}\tilde{Q}_{P|F,j,t}\right) \\ & + \lambda_2\bar{a}_j\tilde{Q}_{F|P,j,t} - \gamma_{1,F,c} - \gamma_{1,P,c} - \gamma_2\left(\tilde{q}_{P|F,j,t} + \tilde{q}_{F|P,j,t}\right) + \varepsilon_{B,j,t}.\end{aligned} \tag{II-4d}$$

The interpretation of the revenue terms in these equations is as follows. First, the expected direct revenue from paid version downloads is $(1-r)P^*_{j,t}\tilde{q}_{P,j,t}$, where r is the commission rate charged by the platform. Second, the expected revenue from in-app purchases for free apps equals $(1-r)\lambda_{1,F}\bar{b}_{F,j}\tilde{Q}_{F,j,t}$ and that for paid apps is $(1-r)\lambda_{1,P}\bar{b}_{P,j}\tilde{Q}_{P,j,t}$. The parameters $\lambda_{1,\mathrm{F}}$ and $\lambda_{1,\mathrm{P}}$ represent the (unobserved) monthly revenue per download of a free or paid app, respectively. Third, the expected revenue from in-app advertising is $\lambda_2\bar{a}_j\ \tilde{Q}_{F,j,t}$, where the parameter λ_2 represents the (unobserved) average monthly revenue generated per ad-network per download. All revenue parameters are constrained to be positive.

The publisher incurs two types of costs. First, there are monthly fixed costs of offering an app version, which equal $\gamma_{1,F,c}$ for the free version and $\gamma_{1,P,c}$ for the paid version. These costs are specific to app category c and may include ongoing development and support, licensing, backup, bandwidth, infrastructure, office space and utilities, marketing, legal and insurance costs, etc.. These costs are likely one of the most heterogeneous components in the profit function (e.g., game requires back-end server operations, more frequent upgrading and debugging, while information apps pay licensing fee to data providers, which result in higher fixed cost than for other apps). We use a hierarchical Bayesian framework to incorporate such (unobserved) heterogeneity in fixed costs across app categories. In addition, there are costs of serving expected new customers in period *t*, which equal $\gamma_2\tilde{q}_{F,j,t}$ for the free version and $\gamma_2\tilde{q}_{P,j,t}$ for the paid version, where the parameter γ_2 represents the (unobserved) cost associated with each expected new download. These costs may be due to promotions and customer acquisition, server expansion, and technical support and service for new customers.

Finally, we make the standard assumption that the random terms $\varepsilon_{C_{j,t}}$ in the profit functions follow i.i.d. Gumbel distributions with the location parameter equal to 0 and the scale parameter σ, which leads to the multinomial logit form for the decision probabilities. In conventional discrete choice models, one has to fix $\sigma = 1$ for identification purposes because the utilities are unobserved and scale-free (Ben-Akiva and Lerman 1985, p.104). The random terms in our model, however, represent random shocks in profit functions. Since revenues, costs, and profits are measured in US dollars and are thus not scale-free, we estimate the scale parameter σ from the data (σ is constrained to be positive).

II.4.4 Demand Models

At the beginning of each period, a publisher needs to predict the would-be demand for each version of its app in order to evaluate profitability and to make a versioning decision based on that evaluation. We adopt a Bayesian aggregate demand model (Ishihara and Ching, 2012; Zhou 2014). We specify a utility function that represents an individual app user's preference for downloading an app version. We account for endogeneity of price via an instrumental variable approach, in which we use the average price of all other paid apps published by the same publisher in a future time period outside of the data period (i.e., February 2016) as the instrument. This variable is correlated with the price of a paid app due to a common underlying cost structure, but is unlikely to be correlated with unobserved shocks in the demand for any given app. We control for differences in the download volumes between game, social and other app categories by including a vector of dummy variables d. An app user *i*'s utilities of downloading the free and paid versions are formally specified as:

$$\begin{aligned} U_{i,t}^F &= X_t^F \beta_i^F + \eta_t^F + \xi_{i,t}^F \\ &= \beta_{0,i}^F + \beta_{1,i}^F G_{F,t} + \beta_{2,i}^F G_{P,t} + \phi_{1,i}^F \log\left(Q_{F,t-1}\right) \qquad \text{(II-5a)} \\ &+ \phi_{2,i}^F \log\left(Q_{P,t-1}\right) + \kappa_i^F I(C_t = B) + \beta_{3:4,i}^{F'} d + \eta_t^F + \xi_{i,t}^F, \end{aligned}$$

$$
\begin{aligned}
U_{i,t}^{P} &= X_t^P \beta_i^P + \eta_t^P + \xi_{i,t}^P \\
&= \beta_{0,i}^P + \beta_{1,i}^P G_{P,t} + \beta_{2,i}^P G_{F,t} + \beta_{3,i}^P \hat{P}_t^* + \phi_{1,i}^P \log\left(Q_{F,t-1}\right) \qquad \text{(II-5b)} \\
&+ \phi_{2,i}^P \log\left(Q_{P,t-1}\right) + \kappa_i^P I(C_t = B) + \beta_{4:5,i}^{P'} d + \eta_t^P + \xi_{i,t}^P,
\end{aligned}
$$

where $G_{F,t}$ and $G_{P,t}$ are the age of the free and paid app, respectively; $I(C_t = B)$ is an indicator variable with 1 indicating both versions being offered, and 0 otherwise; η_t^F and η_t^P are aggregated demand shocks, which follow Normal distributions with mean 0 and standard deviations ρ^F and ρ^P, respectively, that affect all app users at time t[11]; and $\xi_{i,t}^F$ and $\xi_{i,t}^P$ are idiosyncratic demand shocks which follow i.i.d. Gumbel distributions with location parameter 0 and scale parameter 1.

Figure II-3 provides a conceptual diagram of the inter-temporal and contemporaneous cross- effects between free and paid versions that the demand model captures. The parameters $\phi_{1,i}^F$ and $\phi_{2,i}^P$ measure the effects of the past cumulative download volume on the current download utility for each app version, respectively, and capture time dependence in app demand through word-of-mouth and social network influence. The parameters $\phi_{2,i}^F$ and $\phi_{1,i}^P$ measure the inter-temporal cross-over effects between the download utilities of the two versions, with positive signs indicating demand stimulation through word-of-mouth, and negative signs indicating inter-temporal cannibalization. The parameters κ_i^F and κ_i^P capture the effects of the presence of one version on the download utility for the other in a given time period. We expect their signs to be negative, which would reflect contemporaneous cannibalization.

[11] These terms control for global updates of Android operating system, app platforms' server issues, and/or platform-wide promotions that are not observed in the data.

Individual users' preference parameters follow standard Normal distributions $\beta_i^F \sim N(\bar{\beta}_F, \Sigma_F)$ and $\beta_i^P \sim N(\bar{\beta}_P, \Sigma_{\mathrm{P}})$, respectively. We integrate individual level probability of downloading free and paid versions over the distributions of $\beta_{i,t}^F$ and $\beta_{i,t}^P$, respectively, to obtain the shares s_t^F and s_t^P:

$$s_t^F = \int \frac{\exp(X_t^F \beta_i^F + \eta_t^F)}{1 + \exp(X_t^F \beta_i^F + \eta_t^F)} \zeta(\beta_i^F | \bar{\beta}^F, \Sigma_{\mathrm{F}}) \mathrm{d}\beta_i^F, \tag{II-6a}$$

$$s_t^P = \int \frac{\exp(X_t^P \beta_i^P + \eta_t^P)}{1 + \exp(X_t^P \beta_i^P + \eta_t^P)} \zeta(\beta_i^P | \bar{\beta}^P, \Sigma_{\mathrm{P}}) \mathrm{d}\beta_i^P. \tag{II-6b}$$

Note that individual users can download none, one or both versions of each app in the same month, and that therefore the shares of paid and free versions do not need to sum to one. This formulation accounts for competition via the outside option. Download volumes of each version are obtained by multiplying the shares by the total monthly download volumes of each version on the platform:

$$\tilde{q}_{F,t} = Q_M^F \cdot s_t^F, \tag{II-7a}$$

$$\tilde{q}_{P,t} = Q_M^P \cdot s_t^P, \tag{II-7b}$$

where Q_M^F and Q_M^P are the sums of all free and paid apps' monthly download volumes on the platform, respectively.

Insert Figure II-3 about here

II.4.5 Optimal Pricing

At each time period, an app publisher sets the ex-ante retail price of its paid app for the upcoming period to maximize monthly profits of the paid version. We derive the optimal price based on the demand model and the profit functions in Appendix. The resulting optimal price is a function of all parameters in the utility function of the paid version, the state variables in $S_{j,t}^{1}$, the platform's commission rate, the marginal cost of new app users, and the revenue generated via in-app purchases. The optimal price is:

$$P_t^* = -\frac{1}{\beta_3^P}\left[1 + W_n\left(e^{\frac{\beta_3^P \cdot \gamma_2}{(1-r)} - \beta_3^P \cdot \bar{b}_P \cdot \lambda_{1,P} + \psi_t - 1}\right)\right] - \bar{b}_P \cdot \lambda_{1,P} + \frac{\gamma_2}{(1-r)}, \qquad \text{(II-8)}$$

where ψ_t is the mean utility of downloading the paid version net of the price effect (i.e., $\psi_t \equiv U_t^P - \beta_3^P P_t$).

II.4.6 State variables and their transition processes

The model contains two sets of state variables, $S_{j,t}^{1} \in \{\tilde{Q}_{F,j,t}, \tilde{Q}_{P,j,t}, \tilde{Q}_{F|P,j,t}, \tilde{Q}_{P|F,j,t} \tilde{G}_{F,j,\tau}, \tilde{G}_{P,j,\tau}\}$ and $S_{j,t}^{2} \in \{P_{j,t}^{*}, \tilde{q}_{F,j,t}, \tilde{q}_{P,j,t}, \tilde{q}_{F|P,j,t}, \tilde{q}_{P|F,j,t}\}$. The version-specific app ages $(\tilde{G}_{F,j,t}, \tilde{G}_{P,j,t})$ are discrete state variables with a deterministic transition process. They increase by one every month after their introduction:

$$\begin{aligned} \tilde{G}_{F,j,t} &= \tilde{G}_{F,j,t-1} + 1, \\ \tilde{G}_{P,j,t} &= \tilde{G}_{P,j,t-1} + 1. \end{aligned} \qquad \text{(II-9)}$$

The cumulative download volumes $(\tilde{Q}_{F,j,t}, \tilde{Q}_{P,j,t}, \tilde{Q}_{F|P,j,t}, \tilde{Q}_{P|F,j,t})$ are continuous state variables, and their transition processes depend upon the previous cumulative download volumes, the publisher's versioning decision at time *t* -1, and the expected download volumes at time *t*. For the free version:

$$\tilde{Q}_{F,j,t} = \tilde{Q}_{F,j,t-1} + \tilde{q}_{F,j,t}, if\ C_{j,t-1} = F,$$
$$\tilde{Q}_{F|P,j,t} = \tilde{Q}_{F,j,t-1} + \tilde{q}_{F|P,j,t}, if\ C_{j,t-1} = B, \quad \text{(II-10a)}$$
$$\tilde{Q}_{F,j,t} = \tilde{Q}_{F,j,t-1}, if\ C_{j,t-1} \in \{N, P\}.$$

Similarly, for the paid version:

$$\tilde{Q}_{P,j,t} = \tilde{Q}_{P,j,t-1} + \tilde{q}_{P,j,t}, if\ C_{j,t-1} = P,$$
$$\tilde{Q}_{P|F,j,t} = \tilde{Q}_{P,j,t-1} + \tilde{q}_{P|F,j,t}, if\ C_{j,t-1} = B, \quad \text{(II-10b)}$$
$$\tilde{Q}_{P,j,t} = \tilde{Q}_{P,j,t-1}, if\ C_{j,t-1} \in \{N, F\}.$$

The optimal retail price of a paid app at time t ($P^*_{j,t}$) and the predicted monthly download volumes ($\tilde{q}_{F,j,t}$, $\tilde{q}_{P,j,t}$, $\tilde{q}_{F|P,j,t}$, and $\tilde{q}_{P|F,j,t}$) depend on the state variables in $S^1_{j,t}$ as described in the previous sections and on the publisher's versioning decision at $t-1$.

Finally, in-app purchase items are a fundamental part of the initial design of a free or paid app and the number of ad networks for an app varies hardly across time periods in the data. We therefore treat the number of in-app advertising networks for app j, $\bar{a}_j$, and the presence of in-app purchase items for app j, $\bar{b}_{C,j}$, as static and deterministic.

II.4.7 Priors

We use a diffuse prior for all parameters in the utility functions of the demand model. In the profit functions, prior distributions of the heterogeneous fixed cost parameters are $N(\mu_{\gamma_{1,F}}, \sigma^2_{\gamma_{1,F}})$ and $N(\mu_{\gamma_{1,F}}, \sigma^2_{\gamma_{1,F}})$ with diffuse hyper-prior distributions. Prior uncertainty on the revenue parameters, cost of new downloads parameter, and the scale parameter in the profit functions are reflected in diffuse prior distributions. We use half-Normal prior distributions to constrain these parameters to be positive. The discount parameter δ is constrained to between zero and one through the re-parametrization $= \frac{\exp(\delta^*)}{1+\exp(\delta^*)}$, where a theory-based Normal prior is specified on δ^* such that δ *a priori* has a 95% probability to be in the interval (0.90, 0.99).

II.5 IDENTIFICATION AND ESTIMATION

II.5.1 Identification

All parameters in our model are identifiable through specific variations in the data. Table II-4 lists the sources of identification for each parameter. For the two cost parameters $\gamma_{1,F}$ and $\gamma_{1,P}$ (monthly fixed costs) and γ_2 (costs associated with new customers), the three options of offering either or both versions of an app (i.e., $C_{i,t} \in \{F, P, B\}$) would have dominated the option of not publishing an app (i.e., $C_{i,t} = N$) at all times if both fixed costs were zero, and thus observations of the "no app" option in the data allow identification of these parameters. Specifically, $\gamma_{1,F}$ and $\gamma_{1,P}$ are identified via variation in download volumes across the age of an app in conjunction with the frequency of withdrawal decisions. The magnitude of the fixed costs does not change with app age, whereas the monthly download volume decreases with age after a certain point for most apps. As a result, as age increases the fixed costs becomes the dominant cost component compared to the cost associated with new customers. Therefore, a higher frequency of withdrawal among older apps relative to newer apps is consistent with a higher value of $\gamma_{1,F}$ and $\gamma_{1,P}$, whereas similar withdrawal frequencies across age would imply a lower value of $\gamma_{1,F}$ and $\gamma_{1,P}$. Relative differences in withdrawal timing of free and paid versions helps separate identification of $\gamma_{1,F}$ and $\gamma_{1,P}$. Specifically, a higher value of $\gamma_{1,F}$ than of $\gamma_{1,P}$ is consistent with earlier withdrawal of free apps than paid apps, whereas later withdrawal of free apps than paid apps is consistent with a higher value of $\gamma_{1,P}$ than of $\gamma_{1,F}$.

The variation in timing of withdrawal between high-priced apps and low-priced apps among the ones with no in-app purchases identifies the parameter γ_2. When a paid app does not offer in-app purchases, previous downloads do not generate any revenues. Each new download, however, contributes to app profit by the unit contribution margin ($= P_{j,t} - \gamma_2$). A low value of γ_2 relative to the average magnitude of $P_{j,t}$ has little impact on the unit contribution margin and the direct profit ($= (P_{j,t} - \gamma_2) \cdot q_{p,i,t}$). However, if the value of γ_2 is high enough, it lowers the unit contribution margin and the direct profits, which lowers the incentive of publishing an app. This impact is stronger for low-priced apps than for high-priced apps. Therefore, a low value of γ_2 is consistent with little differences in withdrawal timing between high- and low-priced apps. Earlier withdrawal among low-priced than high-priced app is consistent with a high value of γ_2.

Insert Table II-4 about here

The timing and frequency of introductions of paid apps with in-app purchases, relative to those without, enable identification of $\lambda_{1,P}$ (paid apps' indirect revenue from in-app purchases). The two revenue parameters for free apps, $\lambda_{1,F}$ (from in-app purchases) and λ_2 (from in-app advertising), are separately identified utilizing orthogonal variation in the number of in-app ad networks (a_i) and the presence of in-app purchase items ($b_{F,i}$). Specifically, applying a similar logic as for $\lambda_{1,P}$ but now considering free apps without in-app advertising, the time and frequency of introductions of free apps with in-app purchases, relative to those without, enable the identification of $\lambda_{1,F}$. For the identification of the parameter λ_2 (free apps' indirect revenue from in-app advertising), consider only free apps that did not offer in-app purchases. Variation in the timing and frequency of offering paid apps with in-app ad networks along with variation in the variable a_i allows the identification of λ_2. The identification of the discount factor, δ, relies on the functional form of the dynamic program; the scale parameter of the Gumbel distribution, σ, is identified by the scale of the expected profits.

II.5.2 Estimation

The likelihood of the proposed forward looking model factors in the likelihood contribution of the versioning decisions $\mathrm{C} \equiv \{C_{j,t}, \forall j, t\}$ given the state transitions and the likelihood contribution of the transitions among states $\mathrm{S}^1 \equiv \{S_{j,t}^1, \forall j, t\}$ and $\mathrm{S}^2 \equiv \{S_{j,t}^2, \forall j, t\}$. Therefore, the state transition probabilities can be estimated separately. They are based on the demand model and, assuming a Normal distribution for the demand shocks, we use the MCMC approach developed by Jiang et al. (2009) for estimation.

The likelihood for the parameters of the forward looking model (collected in θ), conditional upon the state transitions, is:

$$L(\theta|S^1, S^2, C) = \prod_j \prod_t \prod_v \Pr\left(C_{j,t} = v \middle| S^1_{j,t}, S^2_{j,t}, \theta\right)^{I(C_{j,t}=v)}, \tag{II-11}$$

where the choice probability that the publisher of app *i* chooses versioning option *j* at time *t* is:

$$\Pr\left(C_{j,t} = v \middle| S^1_{j,t}, S^2_{j,t}, \theta\right) = \frac{exp\left\{\sigma V^v_\theta\left(S^1_{j,t}, S^2_{j,t}\right)\right\}}{\sum_{k=1}^4 exp\left\{\sigma V^k_\theta\left(S^1_{j,t}, S^2_{j,t}\right)\right\}}. \tag{II-12}$$

We estimate the model using the MCMC algorithm developed by Imai, Jain, and Ching (IJC 2009). The IJC algorithm avoids having to solve the Bellman equation at each iteration of the MCMC process by using a non-parametric approximation of the value function based on its values in the previous MCMC iterations (Imai et al., 2009). We use a kernel-based approach for the value function approximation, which enables us to deal with a high dimensional state space consisting of continuous state variables (i.e., $\tilde{Q}_{F,j,t}, \tilde{Q}_{P,j,t}$) as well as discrete state variables whose transitions are deterministic (i.e., $\tilde{G}_{F,j,t}, \tilde{G}_{P,j,t}$). We use Gaussian kernel densities $K_h(\cdot)$ with bandwidths $h = \left(h\theta, hQ_{F,j,t}, hQ_{P,j,t}, hG_{F,j,t}, hG_{P,j,t}\right)$. The expected value function approximation at the *w*-th draw of the parameters θ^w, using $m = 1, \ldots, M$ stored prior value functions is:

$$E_{S^{1'}_{j,t}, \varepsilon'_{j,t}}\left[V_{\theta^w}\left(S^{1'}_{j,t}, S^{2'}_{j,t}, \varepsilon'_{j,t}\right)|S^1_{j,t}, C_{j,t}\right] \approx \sum_{m=w-M}^{w-1} V_{\theta^m}\left(\hat{S}^1_m, \hat{S}^2_m, \varepsilon_m\right) \times \left[\frac{K_{h\theta}(\theta^w, \theta^k) \cdot K_{hQ_{F,j,t}}\left(Q_{F,j,t}, \hat{Q}^k_{F,j,t}\right) \cdot K_{hQ_{P,j,t}}\left(Q_{P,j,t}, \hat{Q}^k_{P,j,t}\right) \cdot K_{hG_{F,j,t}}\left(G_{F,j,t}, \hat{G}^k_{F,j,t}\right) \cdot K_{hG_{P,j,t}}\left(G_{P,j,t}, \hat{G}^k_{P,j,t}\right)}{\sum_{l=w-M}^{w-1} K_{h,\theta}(\theta^w, \theta^l) \cdot K_{h,Q_{F,j,t}}\left(Q_{F,j,t}, \hat{Q}^l_{F,j,t}\right) \cdot K_{h,Q_{P,j,t}}\left(Q_{P,j,t}, \hat{Q}^l_{P,j,t}\right) \cdot K_{h,G_{F,j,t}}\left(G_{F,j,t}, \hat{G}^l_{F,j,t}\right) \cdot K_{h,G_{P,j,t}}\left(G_{P,j,t}, \hat{G}^l_{P,j,t}\right)}\right] \tag{II-13}$$

where $\hat{S}^1_l \in \left\{\hat{Q}_{F,j,t}, \hat{Q}_{P,j,t}, \hat{G}_{F,j,\tau}, \hat{G}_{P,j,\tau}\right\}$ and $\hat{S}^2_l \in \left\{\hat{P}^*_{j,t}, \hat{q}_{F,j,t}, \hat{q}_{P,j,t}\right\}$ are the predicted current values of the two sets of state variables at θ^w. The Metropolis-Hasting algorithm consists of the following steps:

1. At iteration *w*, draw candidate parameters θ^{*w} from a (half-) Normal proposal density $\theta^{*r} \sim q(\theta^{*w}|\theta^{w-1})$.
2. Non-parametrically approximate value functions $E_{S_{j,t}^{1'},\varepsilon'_{j,t}}\left[V_{\theta^{*w}}\left(S_{j,t}^{1'}, S_{j,t}^{2'}, \varepsilon'_{j,t}\right)|S_{j,t}^{1}, C_{j,t}\right]$ at θ^{*w} and $E_{S_{j,t}^{1'},\varepsilon'_{j,t}}\left[V_{\theta^{w-1}}\left(S_{j,t}^{1'}, S_{j,t}^{2'}, \varepsilon'_{j,t}\right)|S_{j,t}^{1}, C_{j,t}\right]$ at θ^{w-1} using the stored values $\left\{E_{S_l^1,\varepsilon_l}\left[V_{\theta^{*l}}\left(\hat{S}_l^1, \hat{S}_l^2, \varepsilon_l\right)\right]\right\}_{l=w-M}^{w-1}$.
3. Calculate decision-specific value functions $V_{\theta^{*w}}^{C_{j,t}}\left(S_{j,t}^{1}, S_{j,t}^{2}, \varepsilon_{j,t}\right)$ and $V_{\theta^{w-1}}^{C_{j,t}}\left(S_{j,t}^{1}, S_{j,t}^{2}, \varepsilon_{j,t}\right)$ using the approximations of the value functions in step 2.
4. Calculate $L(\theta^{*w}|S^1, S^2, C)$ and $L(\theta^{w-1}|S^1, S^2, C)$ using equation (II-11).
5. Accept the proposed parameters with probability:

$$\lambda = \min\left\{\frac{g(\theta^{*w})L\left(\theta^{*w}\middle|S^1, S^2, C\right)q(\theta^{w-1}|\theta^{*w})}{g(\theta^{w-1})L\left(\theta^{w-1}\middle|S^1, S^2, C\right)q(\theta^{*w}|\theta^{w-1})}, 1\right\}. \tag{II-14}$$

   where $g(\theta^{*w})$ is the joint prior distribution of the parameters.
6. Draw $\hat{S}_w^1$ from uniform distributions whose support covers the range of the variable in question, and $\hat{S}_w^2$ from the empirical distributions of the variables. Update the value function once, using the Bellman equation and the estimated state transition probabilities, to obtain $E_{S_w^1,\varepsilon_w}\left[V_{\theta^{*w}}\left(\hat{S}_w^1, \hat{S}_w^2, \varepsilon_w\right)\right]$, go to step 1.

We carried out the estimation process using R Version 3.1.2 on an Intel Xeon Processor E5-2680 v2 (2.8 GHz). We parallelized the likelihood computation using twenty logical processors to reduce the estimation time. We ran all the MCMC chains for 50,000 iterations, with a burn-in of 30,000 and thinning of 1 in 10. Convergence was monitored using the diagnostics of Geweke (1992), which indicated convergence by the end of the burn-in.

II.6 MODEL ESTIMATION RESULTS

II.6.1 Model Comparisons and Predictive Accuracy

We compare the fit of the proposed model with several alternative models: 1) a dynamic versioning choice model with homogeneous fixed-cost parameters; 2) a myopic model of versioning decisions based on the single-period profit functions with fixed-cost heterogeneity, and 3) a myopic decision model with heterogeneous fixed-cost parameters. We use the Gelfand-Dey (1994) log-marginal likelihood for model comparison. The results are reported in Table II-5, along with the predicted frequencies of each of the app versions.

The proposed model has a higher Gelfand-Dey (1994) log-marginal density than each of the three benchmark models. The table shows that including fixed-cost heterogeneity has a major impact on model fit. It also shows that the fit of the myopic models is worse than that of the forward looking models. In addition, by failing to consider the future profit stream for each versioning option, predictions based on the myopic model almost completely miss the occurrence of the "no app" option, while the forward looking models yield more reasonable predictions. These results speak to the importance of taking into account the forward-looking nature of app publishers' decisions, as well as heterogeneity.

Insert Table II-5 about here

Table II-6 presents the first order transition pattern predicted by the proposed model. The most prominent pattern observed in the data is high persistence of decisions over time. The proposed model captures this pattern fairly well. Because of a large installed base (i.e., cumulative download volumes) and (heterogeneous) fixed costs, the relative magnitudes of the profit functions of different versions is quite stable over time, which explains the persistence of versioning decisions for many apps.

Insert Table II-6 about here

II.6.2 Demand Model

The estimates and standard deviations (SDs) of the parameters of the utility functions are presented in Table II-7. First, positive coefficients of (log-transformed) prior cumulative download volumes in both utility functions indicate that the installed base of either version increases the current utility of downloading both versions of the app. Prior download volumes most strongly impact the current utility for the same version, and the effect is larger for paid than for free apps. In addition, the cross effect is also larger for the paid version, showing that it more strongly stimulates the demand for the free version than vice versa. This suggests that positive word-of-mouth generated from either version can stimulate future demand. We conjecture that because users of the paid version are more involved and experience the full functionality of the app, the word-of-mouth that they generate has stronger effects. Nevertheless, the presence of either version in a given time period decreases the contemporaneous download utility of the other version, which indicates cannibalization.

In terms of other variables' contribution to the download utility, the (instrumented) price of a paid app has a large negative effect (-3.26) on the download utility of the paid version, as expected. The negative coefficients of the age variables in both utility functions indicate decreasing utility over the app version life-cycles and shows app users' preference for newer apps over older ones. Note, however, that the estimation data starts after a six-month initialization period when all apps in our data were already on the market. Since for many apps the data may not include the period of peak downloads nor the periods before it, app age should be seen as a control variable instead of comprehensively capturing the entire app life-cycle. We observe differences in the download utility across app categories, with game apps having the highest download utility (baseline) and social apps having lower download utilities.

Insert Table II-7 about here

II.6.3 Dynamic Versioning Decision Model

The parameter estimates of the dynamic versioning model are provided in Table II-8. We convert the monthly revenue and variable cost estimates into dollars per 1,000 downloads (RPM and CPM for revenues and costs, respectively)[12]. We find that in-app purchases generate about five times higher RPM than in-app advertising for free apps ($0.312 RPM vs. $0.067 RPM). The revenue from in-app purchases for paid apps is considerably higher still ($169 RPM). As for the cost estimates, the fixed costs of paid versions are about ten times as high as those of free versions ($590 vs. $58); the standard deviations of the heterogeneity distributions indicate considerable differences in fixed costs between apps. The variable costs associated with new downloads is about $3 CPM. The estimated discount factor is 0.933, which is in the range of what has been found in field experiments (Yao et al. 2012) and empirical research (e.g. Hitsch 2006)[13].

Insert Table II-8 about here

Table II-9 and Figure II-4 provide the distributions and the summary statistics of free and paid app profits in July 2012, respectively, based on the revenue and cost estimates in Table II-8. The distributions of profits are highly skewed; the median of

[12] The medians of free version and paid version cumulative download volumes are $2M and $55K, respectively.

[13] We have also estimated a model with the discount factor fixed at the commonly used value of 0.95, which does not significantly change the results of the model estimation.

the paid versions' monthly profit is about ten times as high as that of free versions ($6,212 vs. $614)[14].

Insert Table II-9 about here

Insert Figure II-4 about here

II.7 SIMULATION OF COMMISSION POLICIES

A main objective of this research is to investigate the implications of alternative commission structures on the payoffs of the platform and the app publishers in order to make policy recommendations. We do this through counterfactual simulations in which we vary the commission imposed by the app platform, which affects publishers' demand predictions, app price setting, app version profits, and versioning decisions. The parameters of our model represent the primitives of publishers' profit functions and app users' utility functions that are invariant to changes in the commission policy, and can therefore be used to evaluate how publishers would make

[14] Although industry estimates vary, our finding is consistent with industry reports (VisionMobile, 2014). Our median revenue estimate of the free apps is in the top 35% and the median of the paid apps is in the top 15% of the distribution reported for Android, which is reasonable given our data collection strategy.

decisions under alternative commission policies, even though the commission did not vary in the data (cf. Chintagunta et al. 2006).

Google Play and other app platforms currently charge a 30% flat commission rate on direct and indirect revenues. (There is no commission on advertising revenues which are handled between app publishers and ad networks). We study three sets of commission policies: 1) uniform commission rates on all revenues, 2) differential commission rates on direct vs. indirect revenues, 3) a rent plus commission on all revenues. For each scenario, we change the commission structure in the profit functions (Equations II-4b to II-4d) and evaluate them across the posterior distributions of the parameters to compute the expected revenue for the platform, total profit for all app publishers, average profit per app, and the ecosystem payoff[15]. We outline the key findings below.

II.7.1 Different uniform commission rates

First, we examine scenarios in which the commission on all revenues uniformly increases from 10% to 50% in steps of 10%. Table II-10 summarizes the incremental profit/revenue/payoff under each scenario, relative to the current policy of 30%.

Insert Table II-10 about here

As expected, the platform's revenue increases with the commission rate, while

[15] The ecosystem payoff is the sum of platform revenue and total app publisher profit.

app publishers' total profit decreases with it. An increase in the commission rate is associated with a decrease of the number of apps on the platform. Of the apps that drop out 90% are paid apps. The opposite occurs when the commission rate is lower than the current level. More importantly, the ecosystem payoff increases as the commission rate drops, because the gain in the profits for app publishers is larger than the loss in the platform's revenue. These findings suggest an opportunity for profit sharing. The platform could reduce its commission rate in exchange for app publishers agreeing to collectively compensate the platform's loss in such a way that both the platform and app publishers will be better off than under the current policy.

II.7.2 Differential commissions on direct and indirect revenues

The above analyses show that a change in the commission rate has an asymmetric effect on app publishers' decisions to offer free vs. paid apps because of their different monetization strategies. We therefore study the impact of differential commission rates on direct vs. indirect revenues, varying them from 10% to 50% in the next policy simulation. Table II-11 shows the results of scenarios in which, relative to the current policy, the commission rate on direct revenue is raised and that on indirect revenues is lowered, or vice versa. The results reveal that platform revenue increases if the commission on indirect revenue is raised, while app profits increase if it is lowered even if commissions on direct revenue are raised at the same time. The payoff for the entire ecosystem increases when the commission rate on direct revenues is raised and the commission rate on indirect revenues is lowered, relative to the current commission rate. This is accompanied by an increase in the number of both free and paid apps on the platform.

Insert Table II-11 about here

These findings point to another profit-sharing opportunity. The higher ecosystem payoff under a higher commission rate on direct and a lower one on indirect revenues is driven by higher profits for app publishers. Moreover, the results in Table II-11 suggest that around a 40% commission on direct and 20% on indirect revenues the ecosystem payoff is highest. Follow-up analyses reveal that a 35% commission on direct and 25% on indirect revenue indeed yield a lower ecosystem payoff ($9512). Therefore, the platform may be able to take advantage of an optimal differential commission in this region, under a profit sharing agreement with publishers.

II.7.3 Rent plus commission

The previous two recommendations rely on profit sharing agreements between the platform and app publishers. We next examine a rent-plus-commission scheme which may provide an alternative mechanism for transfer of revenue between the platform and app publishers (Cachon 2003). We vary the monthly rent from $10 to $500 and the commission rate from 30% to 10%, resulting in 30 scenarios[16], of which we report four in Table II-12.

[16] Combinations of 29%, 25%, 20%, 15%, and 10% commissions with $10, $30, $50, $100, $200, and $500 rent.

Insert Table II-12 about here

Across all scenarios investigated there was none in which both app profits and platform revenue increase relative to the current situation. All of the four scenarios in Table II-12, however, present profit sharing opportunities. In addition, scenarios with a $30/month rent and 10% or 20% commission also increase the ecosystem revenue and create profit-sharing opportunities. Scenarios with a low commission rates (10%-20%) and lower rents (below $50/month) generally increase the number of smaller paid apps on the platform.

Follow-up analyses reveal that the increase in ecosystem payoff under the various scenarios with uniform, differential and rent-plus-commission policies that present revenue sharing opportunities, is mostly driven by an increase in the number of smaller apps on the platform (in terms of download volume). Larger apps rarely change their versioning decisions in response to changes in the commission rate. Therefore, these policies change the revenue/profit split between the platform and the larger app publishers. Importantly, lower commission rates on indirect revenues and lower uniform commission rates even if combined with a modest rent increase the number of smaller apps, which generates a surplus ecosystem payoff that creates profit sharing opportunities. It thus seems that whichever of the three types of policies is preferred to create profit sharing opportunities, stimulating the growth of the platform in terms of smaller startup apps should be a critical component of the considerations, and may also positively affect consumer welfare.

The theoretical literature studying vertical relationships between firms, such as the platform and the publisher, finds that when both firms have price-setting power the markets in question suffer from double marginalization. This arises because the firms in question do not set price to maximize their combined profits, but to each optimize their own profits (Spengler 1950). The literature shows that contractual agreements between the firms, such as two-par tariffs and profit sharing, may overcome this problem (Cachon 2003, Cachon and Lariviere 2005). Our counterfactual studies reveal that in a two-sided market a profit sharing agreement can mitigate this problem, but that a two-part tariff (rent plus commission) is an insufficiently powerful mechanism when used alone without an additional revenue sharing contract.

II.8 CONTRIBUTIONS, MANAGERIAL INSIGHTS, AND FUTURE DIRECTIONS

We have developed a dynamic structural model of app publishers' monetization strategies that captures their forward-looking decision processes and incorporates the key revenue sources and costs in their profit functions. We compiled a unique dataset that suited the purposes of this research. This research has made the following contributions:

(1) We have proposed a comprehensive dynamic structural model of app publishers' monetization decisions and demand predictions which outperforms various alternative models, including a myopic model and a model which does not

allow heterogeneity across apps.

(2) Our empirical analyses have provided in-depth knowledge of the various trade-offs of costs and revenues arising from different sources that app publishers face when making monetization decisions.

(3) Our study reveals several opportunities to improve the commission scheme currently widely adopted in the industry and provides policy recommendations that would lead to mutual benefits for both the app platform and app publishers via profit-sharing.

We summarize the key model estimation results below.

- The demand model estimation result shows that, although the presence of one version decreases the download utility of the other version in a given month, the cumulative customer base of the free and paid versions increases the download utility of both versions. This provides more empirical support for social influence in the app market as conjectured by prior research. Although intuition would suggest that prior downloads of the free version stimulates current decisions of the paid version most, our results reveal the opposite.
- We find that in-app purchases generate higher revenue than in-app advertising for free apps. The revenue from in-app purchases for paid apps is considerably higher than that for free apps. And the fixed costs of paid apps are higher than those of free apps.

In addition to a rich set of empirical results, this research provides valuable and actionable insights for app industry analysts, app distribution platforms, and app publishers. The key managerial insights and policy recommendations are:

- Reducing the commission rate from the current 30% could increase the total payoff for the app eco-system by increasing the number of apps on the platform and generating higher app profits, which together outweigh the loss in platform revenue, thus creating profit-sharing opportunities that would benefit the app platform and publishers.
- Charging a higher commission rate to direct revenues from paid apps and a lower commission rate to indirect revenues from in-app purchases could also create a profit-sharing opportunity by increasing total app publishers' profits more than decreasing the app platform's revenue.
- A rent-plus-commission policy serves the same purpose: across a wide range of rents and commissions, it has the potential to create a profit-sharing opportunity, where app publishers' share some of their incremental profits with the platform.

The research methodology and framework we proposed in this study can be used to examine other policy changes and current events in the app industry. For example, the Google+ gaming platform recently changed to a lower commission rate of 5% on in-game purchases. Our analysis suggests that this would lead to increase in number of apps on the platform and higher ecosystem payoffs than the current policy.

A few limitations of the present study can be addressed in future research. First, we have not fully incorporated app-level heterogeneity in the model due to data limitations. For example, variables such as app quality, which we do not have in our data but could be related to versioning costs, can be added to our demand model and potentially improve the model fit. This can also enable the counterfactual simulation analyses to test how higher quality apps react differently to policy changes from lower quality apps. In addition, the monthly fixed cost parameters of the supply-side model can be made app-specific rather than category-specific if there is more variation in app publishers' versioning decisions than the case in our current data. Then the counterfactual simulation studies can provide customized suggestions to individual app publishers.

Our demand model implicitly accounts for the competition between apps and app-versions by including an outside option along with the option of downloading the focal app-version to app users. An alternative demand model could be specified as a multinomial choice among all apps and app-versions in a user's consideration set, so that the parameter estimates quantify the direct effect of an app on competing apps' downloads. However, such a model would require careful delineation of the consideration set to avoid the problem of having an excessive number of parameters in the model. Alternatively, we could indirectly incorporate the effect of competition between apps within a category by specifying the monthly fixed cost as a function of category size (i.e., number of apps in the same category). We leave it as a direction for future research to fully incorporate competition among apps.

Other limitations of the current work are that it has focused on one platform and does not address competition between platforms and the fact that we assume that app publishers maximize monthly profit of paid version instead of maximizing its net present value. We believe that these are important topics for future research, which we hope will be stimulated by the present study.

Chapter III: Essay 2

Strategic Mobile App Upgrading Decisions across Versions

III.1 INTRODUCTION

Over the lifetime of a mobile app, its publisher often releases multiple upgrades. Each upgraded version may enhance the functionality of existing features and add new features. Releasing upgrades is a common strategy used by app publishers to rejuvenate the appeal of an app and to fend off competitions. Most app distribution platforms, such as Google Play and the App Store, have the policy that such upgrades should be made available to current users for free. Therefore, the costs that publishers incur in upgrading their apps need to be recovered from new users that the upgraded app attracts as well as the incremental indirect revenues (such as from in-app purchases and advertising) brought about by an upgrade. Some app publishers only offer free apps, others only offer paid apps, yet others offer both versions at the same time. Because the different versions have different monetization sources and associated costs, they may involve different considerations and strategies when it comes to upgrading decisions. For example, upgrading a paid app could lead to incremental direct revenues from the boost of its sales, while upgrading a free app must have been motivated by other benefits such as incremental in-app purchases or advertising revenues. In addition, releasing an upgrade of one version is likely to affect not only its own demand but also the demand for the other version, if it is also available, and thus impact the revenues of both versions. Such cross-version effects need to be taken into consideration when studying app publishers' upgrading decisions.

While software upgrading (e.g., MS Office 2013 to MS Office 2016) and versioning[17] (e.g., Home, Standard, and Professional Versions of MS Office 2016) decisions have been studied separately by prior research (e.g., Ellison and Fudenberg 2000, and Bhargava and Choudhary 2008), to our knowledge no study has investigated upgrading decisions in combination with app publishers' versioning decision. In this study, we examine how these two decisions are interconnected for app publishers who are profit maximizers. We assemble a unique dataset of versioning and upgrading decisions of 572 apps on Google Play. Our empirical investigation is built upon a joint model of versioning and upgrading decisions that incorporates various revenue and cost sources associated with these decisions, and a demand model of app downloads that takes into account cross-effects of versioning and upgrading between the free and paid versions. Based on the model estimation results, we conduct several policy simulations with the purpose of examining the soundness of certain current practices and identifying opportunities to improve app publishers' profits, the app distribution platform' revenue, and the eco-system payoff.

Our study will help app publishers, app platforms, and app market researchers answer the following questions: 1) How does the upgrading decision of one version impact the upgrading decisions of the other version and what are the revenue and profit consequences for the app publishers and platform? 2) When the free version is present, would a publisher be better off by upgrading it or introducing a paid version? 3) When both versions are present, should the paid, the free, or both versions be

[17] Consistent with the terminology in Lee, Zhang, and Wedel (2017), the versioning decision refers to an app publisher's decision over offering the free version only, paid version only, or both versions at a given time. Lee, Zhang, and Wedel (2017) is the working paper based on my first essay.

upgraded? 4) What would be the impact on a platform’s revenue, app publishers’ profits, and the app ecosystem payoff if app publishers charge existing owners the retail prices for upgrades of their paid apps? 5) How much payoff could an app platform expect by investing in its infrastructure and support capability to help app publishers reduce costs of upgrading apps?

Mobile app publishers release upgrades of their apps via mobile app distribution platforms such as Google Play and the App Store. App upgrades bring new contents, add features, or improve usability[18]. On the major app platforms, these app upgrades are offered for free to users who own a copy of the app regardless of whether the copy was downloaded for free or was paid for. For users, the free upgrade policy generates higher levels of satisfaction, but for publishers it means that the costs of the upgrade have to be offset by incremental sales from new users or through incremental indirect revenues. In contrast, the pricing practice of packaged desktop software, which is also a class of digital products sharing many similarities with mobile apps, is quite different. When a software company releases an upgrade, existing users usually have to pay for it in order to use the upgraded version. For example, a user who owns a copy of Microsoft Office 2013 needs to pay a fee if she wants to use the upgraded version Microsoft Office 2016. An objective of this study is to examine the soundness of such current practices.

[18] The app publishers also release bug fixing updates for stable app performance. We distinguish app upgrades from bug fixing updates. These updates do not contain additional features and are mere responses to app crashes and users’ complaints. We focus on app upgrades in this study and ignore bug fixing updates, because the latter are unlikely to be deliberated and strategic decisions by app publishers.

App upgrades are costly to their publishers, because they need to utilize a variety of resources to develop and launch upgrades (e.g., R&D, human resources, new servers, etc.). Upgrades are introduced only if the expected incremental revenues generated by the upgrades are greater than their costs. Revenues and costs, however, are very different between free and paid apps. Some app publishers only offer a paid version of their apps. These apps generate direct revenues when users download them from a platform. In addition, paid apps can offer in-app purchases, which provide additional indirect revenues. If an upgrade of a paid version increases the expected download volume, and the expected (direct and/or indirect) revenues generated by the incremental downloads are greater than the upgrade costs, the app publisher should launch an upgrade.

Some app publishers only offer the free version of their apps. Although free apps do not generate direct revenues, they generate indirect revenues via in-app purchases and advertising. In addition to generating revenues via in-app purchases, a publisher can monetize a free app by advertising products via working with mobile ad-networks. App publishers show in-app mobile ads by planting programming codes provided by mobile ad-networks within their apps' source code. App publishers are compensated by the advertisers via ad-networks anytime an app user watches the ads, clicks the ad banners, or makes purchases via the links in the ads. If the expected incremental revenues via the two indirect revenue sources generated by the upgrades are greater than the costs of upgrades, app publishers should release upgrades.

Other app publishers offer both free and paid versions simultaneously. They monetize the apps via direct revenues from paid version downloads and indirect revenue sources (i.e., free in-app purchases, paid in-app purchases, free app advertising). If the combined incremental revenues of the two versions generated by an upgrade of either or both versions are greater than the upgrade costs, the app publishers should release the corresponding upgrade(s). Note that upgrading decisions are not simply conditional upon the versioning decisions: versioning and upgrading decisions are closely related and jointly made. For example, an app publisher could consider offering a paid version alongside of the existing free version instead of offering an upgrade of the free version, if the payoff of the former is higher than the latter option. A publisher who offers both free and paid versions might cancel the free version if that is more profitable than keeping it when she releases an upgrade of the paid version. This scenario is possible because of the potential cannibalization between the two versions and a potential negative impact of a paid version upgrade on the free version’s demand. Because an upgrade of one version could impact the profitability of the other version, upgrading both versions simultaneously could be more profitable than releasing an upgrade of either version only, under certain circumstances. The supply- and demand-side models that we develop in this study allow us to incorporate these complex and intricate connections of the versioning and upgrading decisions between the free and paid versions of an app.

In the next section, we review the relevant studies in the literature. We then describe our data collection procedure and provide summary statistics, provide our model specification, summarize estimation results, and conduct policy simulation in the following sections. In the last section, we discuss the policy implications and future research directions.

III.2 RELATED LITERATURE

Software upgrade decisions and versioning decisions have been studied in the literature extensively, but separately. In this section, we summarize the key findings in the relevant research streams, and outline how our study is connected to the prior research as well as how we contribute to the literature.

A group of studies examines strategic upgrading decisions of software products. Releasing upgrades after the initial introduction has frequently been observed in packaged software products, which have motivated marketing researchers to study the incentives of using upgrading strategies. For example, Ellison and Fudenberg (2000) provide a reason for why a software supplier releases more upgrades than its social optimal. They find that the trade-off between the revenue generated by the marginal customers who purchase upgraded software and the negative externality caused by them sets the frequency of upgrades, and it could decrease social welfare. Dogan and colleagues (2011) find that the demand-side uncertainty affects the supply side decision (i.e., upgrade timing) and positive word-of-mouth could help software providers handle the demand variability. Further, Mehra et al. (2014) examines the relation between the product life-cycle and upgrade intervals and show that the upgrade interval and a product's life-cycle are positively related. Also, Bala and Carr (2009) analyze optimal upgrade pricing of a software publisher by considering users' upgrade costs and hardware compatibility.

Prior studies in this research stream provide insights into upgrading strategies primarily based on analytical and game theoretical models. Our empirical framework incorporates the insights from the previous literature and extends it by considering unique aspects of the product market we examine in the present study (i.e., the mobile app market). There are several key differences between packaged software and the context we study. First, existing owners of packaged software usually have to pay a price for a software upgrade and this is also a common set-up in the theoretical models that studies software upgrades. In contrast, in the mobile app market, publishers are not allowed to charge prices to existing owners for their upgrades. Some studies do not exclude the potential of releasing free upgrades such as Viard (2007), who builds an infinite horizon dynamic model of successive software generations to explain incentives of monopolistic firms to release upgrades and why publishers set different prices for consumer groups in various adoption stages. The price could go down to zero in his setting, but it does not include a comprehensive discussion on the free release of upgrades. It calls for studies that examine the free release of upgrades, and we empirically investigate the free upgrading strategy of app publisher using a unique dataset. Our model also incorporates factors the theoretical literature has considered (such as consumer heterogeneity and the optimal pricing of full product) to thoroughly examine the strategic upgrade decisions of mobile app publishers.

Also, most studies in upgrading literature implicitly assume simultaneous upgrades of multiple versions. For example, Dogan and colleagues (2011) build a game-theoretic model that can explain the upgrade decisions of software product like Microsoft Windows (e.g., upgrade from Windows XP to Vista) and it is implicitly assumed in the model that Windows XP Home edition and Professional Editions are upgrades at the same time. Potential sequential/asynchronous upgrades of two versions are not incorporated into their model. In the present study, we build a joint model of upgrading and versioning decisions and allow app publishers to separately upgrade different versions while taking into the interconnections in the decisions. In addition to drawing insights from studies on software upgrading decisions, we incorporate findings from the theoretical literature on versioning decisions into our empirical models. For example, Raghunathan (2000) focuses on the introduction of multiple versions of a packaged software and derives the optimal introduction strategies (i.e., simultaneous vs. sequential introduction) under different conditions based on cannibalization between versions and consumer characteristics. We consider consumer heterogeneity and cannibalization between the free and paid versions of an app in our models. Also, our model incorporates differential costs for different versioning decisions, because Sundararajan (2004) finds that the versioning cost is an important factor for software publishers' decision. He concludes that offering a high-priced full-feature version is advantageous when the cost of versioning is high. Also, Bhargava and Choudhary (2008) suggest that a monopolist should offer both low- and high-quality versions if the low-quality version's market share is larger than the market share of a high-quality version when both versions are offered alone. Calzada and Valletti (2012) focus on channel distribution issues in the film industry where a film distributor can offer both theatrical and video versions to the market. They find that simultaneous releases of both versions are more desirable than sequential releases of one and then the other, when the two versions do not perfectly substitute each other. Our proposed model would allow us to empirically test the generalizability of these findings from theoretical research to the mobile app market.

When it comes to versioning decisions, another characteristic of the mobile app market is that app publishers often offer a full-feature paid version along with a limited-feature free version at the same time. The latter is usually called a “freemium” strategy by the mobile app industry. Limited-feature free versions are provided by app publishers possibly because they expect some free version users will purchase the paid versions in the future. Also, free version apps can generate revenues by showing in-app ads and selling in-app purchase items. Most studies in the versioning literature have focused on products other than mobile apps and thus underestimated the indirect revenues generation opportunities of the free version. We briefly describe the stream of research that examines free version information goods here. Lambrecht and colleagues (2014) provide an excellent general review on multiple monetization methods of digital goods using advertising and selling information and contents. They call for research studying trade-offs in choosing mobile app’s revenue models, and we answer the call by studying mobile app’s optimal pricing and revenue generation from in-app purchases and advertising. The study by Prasad and colleagues (2003) finds that a mixed strategy of offering ad-supported and pay-per-view options is advantageous over the pure strategy of offering either. Our empirical model considers indirect revenues from in-app purchases and in-app advertising generated by free apps. Several studies have focused on examining the interplays between free and paid versions of a product. For example, Haruvy and Prasad (2001) incorporate network externalities to derive the boundary condition for the quality of free version in the market with both versions of a product. They find that the quality of the free version has to be low enough to encourage customers who appreciate high product qualities to purchase the paid version, and it has to be high enough to give incentives to the other customers to try the free version. Jiang and Sarkar (2009) consider the speed of diffusion and show that the net present value of future downloads of a paid version is increased by a free version. Kumar and Sethi (2008) extend the previous literature by building a dynamic model and find optimal level of advertisement and subscription fee. While most studies in this stream of literature are theoretical, an exception if the study by Pauwels and Weiss (2008) who empirically examine the transition from "free to fee" by an online content provider firm. They suggest that firms initiate the transition from "free to fee" when the subscription of the free version starts showing saturation.

As stated above, software upgrade decisions and versioning decisions have been studied separately in the literature. The two decisions could very well be interconnected, and our study intends to address methodological and managerial issues that arise when the two decisions are made jointly. To summarize, we study upgrading strategies in the presence of software versions and provide managerial implications to the app publishers and app platforms. Specifically, we propose a joint model of upgrading and versioning decisions, which in turn is built upon a demand model that takes into account the cross-effects of versioning and upgrading decisions between the free and paid versions of mobile apps. Our model provides estimates of various revenue and cost components associated with the versioning and upgrading decisions. Furthermore, utilizing the model estimation results, we conduct counterfactual simulations to provide policy recommendations for improving app publishers' profits, an app platforms' revenue, and the total payoff of an app ecosystem.

III.3 DATA COLLETION AND DESCRIPTIVE ANALYSIS

We have compiled a dataset of Android apps published on Google Play. Google Play is a platform which facilitates publishing and upgrading of apps by their publishers and the distribution of these apps to consumers. Developers can choose to offer free and/or paid versions, and set the price of a paid version. While the initial version of an app is vetted before it is published, Google Play imposes few barriers towards releasing app upgrades, and new app editions can be published quickly. The platform takes a 30% commission on revenues from paid app downloads and in-app purchases made from both free and paid apps. It collects revenues generated by the apps, withholds its commissions, and makes monthly payments to app publishers. App publishers' in-app advertising revenues are not subject to commissions because they work directly with ad networks outside any app distribution platforms. As to app upgrades, Google play's policy is that app publishers should make them available for free to existing customers, if they choose to upgrade either a free or paid app. Therefore, the platform does not make any revenue from app upgrades under the current policy.

The data were collected by scraping the websites of mobile app market research firms Priori Data (http://prioridata.com), App Annie (http://appannie.com), and App Brain (http://appbrain.com). Priori Data publishes monthly reports listing the top 150 free or paid apps in terms of monthly download volumes. We used six months of their reports (December 2011 to June 2012) to construct an initial basket of 572 apps[19]. We then collected detailed data of the apps in this basket every month for 13 months from June 2012 to July 2013. The key variables in the data include upgrading and versioning decisions, download volumes, time since last upgrade, and revenue source variables such as the presence of in-app purchase items, number of ad-networks, and retail prices of paid apps. We use the number of ad-networks in an app as a proxy for the amount of in-app advertising. To extract the list of ad-networks used by apps from the compressed packages files (i.e., .apk file), we used the AdRisk method (Grace et al. 2012) and the Addons Detector (see Lee, Zhang, and Wedel 2017 for more details).

The upgrading decision of apps is identified by the change in the identifier assigned to app editions. A unique sequence of three numbers separated by periods (e.g., 2.3.7) is commonly used by app publishers as an identifier of the app's edition. Following conventions in the software and app industries (called Semantic Versioning[20]), we code an increment in the first or second digit of an app edition number as an upgrade. A change in the third digit is regarded as a release of a bug-fixing patch, and thus is omitted since it is unlikely a strategic decision by an app publisher and usually does not include additional functionalities of an app.

We present the key variables and their summary statistics in Table III-1, and show distributions of monthly download volumes and number of upgrades in Figure III-1. In the data, 38% of the apps have offered only a free version, 28% of them have

[19] We dropped 12 apps from the basket due to missing data.
[20] http://semver.org/

offered only a paid version, and 31% of them have offered both versions simultaneously at least once. The majority of apps have released an upgrade at least once, with a higher percentage of free apps doing so than paid apps: 64% of free apps vs. 55% of paid apps. In the 13-month time period of our estimation data, there were about 2.6 upgrades per app for free apps, and 1.5 upgrades per app for paid apps on average. The average time between two consecutive upgrades was about 3 months for free apps, and 5 months for paid apps. Free apps had a much higher monthly download volume (297,100) than paid apps (3,888). 35% of free apps sold in-app purchase items, and the percentage was substantially higher than that of paid apps (19%). The free apps worked with about 2.5 unique ad-networks on average, and the average retail price of paid apps was $3.10. Interestingly, free apps are upgraded more frequently when the free version is offered alone than when it is accompanied by a paid version (3.14 vs. 1.88 upgrades on average), while the opposite pattern holds for paid apps which have more upgrades when the paid version is offered alone than when accompanied by a free version (2.59 vs. 1.99 upgrades on average). This suggests that the upgrading and versioning decisions are interrelated, and supports our motivation to build a joint model of versioning and upgrading decisions.

Insert Table III-1 about here

Insert Figure III-1 about here

III.4 MODEL

III.4.1 A Joint Model of Versioning and Upgrading Decisions

We propose a joint model of versioning and upgrading decisions. We formulate the versioning decision $C_{j,t}$ as a multinomial choice decision between four options: None (N), Free version only (F), Paid version only (P), or a freemium option with both versions offered (B). We formulate the upgrading decision as a conditional choice problem, conditioning on the versioning decision. If $C_{j,t}$ = F or P, the upgrading decision $U_{j,t}|C_{j,t}$ is a binary choice decision of yes (Y) or no (N). If $C_{j,t}$ = B, the upgrading decision $U_{j,t}|C_{j,t}$ is a multinomial choice decision among: no upgrading (UN), upgrading only the free version (UF), upgrading only the paid version (UP), and upgrading both versions (UB). We formulate the simultaneous versioning and upgrading decisions as a joint logit model (see Figure III-2) in which the choice probabilities are functions of app publishers' profits derived from the decision options in question. The individual components specified in the following.

Insert Figure III-2 about here

III.4.1.1 Profit functions

We decompose the total profit of a given versioning and upgrading option into two parts: a common profit component for each of the versioning options {N, F, P, B} respectively, and a unique conditional profit component associated with the upgrading decision. We incorporate various sources of revenues and costs in the profit functions for the versioning and upgrading options.

The common profit component for each of the versioning options is specified as:

$$\Pi_{j,t}^{N} = \varepsilon_{j,t}^{N}, \tag{III-1}$$

$$\Pi_{j,t}^{F} = V_{j,t}^{F} + \varepsilon_{j,t}^{F} = r\lambda_{1,F}\bar{\iota}_{F,j}\tilde{D}_{j,t}^{F} + \lambda_2\bar{a}_j\,\tilde{D}_{j,t}^{F} - \gamma_{1,F} - \gamma_2\tilde{d}_{j,t}^{F} + \varepsilon_{j,t}^{F}, \tag{III-2}$$

$$\Pi_{j,t}^{P} = V_{j,t}^{P} + \varepsilon_{j,t}^{P} = r\left(P_{j,t}^{*}\tilde{d}_{j,t}^{P} + \lambda_{1,P}\bar{\iota}_{P,j}\tilde{D}_{j,t}^{P}\right) - \gamma_{1,P} - \gamma_2\tilde{d}_{j,t}^{P} + \varepsilon_{j,t}^{P}, \tag{III-3}$$

$$\begin{aligned}\Pi_{j,t}^{B} = V_{j,t}^{B} + \varepsilon_{j,t}^{B} &= r\left(P_{j,t}^{*}\tilde{d}_{j,t}^{P|F} + \lambda_{1,F}\bar{\iota}_{F,j}\tilde{D}_{j,t}^{F|P} + \lambda_{1,P}\bar{\iota}_{P,j}\tilde{D}_{j,t}^{P|F}\right) + \lambda_2\bar{a}_j\tilde{D}_{j,t}^{F|P} \\ &\quad -\gamma_{1,F} - \gamma_{1,P} - \gamma_2\left(\tilde{d}_{j,t}^{F|P} + \tilde{d}_{j,t}^{P|F}\right) + \varepsilon_{j,t}^{B},\end{aligned} \tag{III-4}$$

where $\tilde{d}_{j,t}^{F}$, $\tilde{d}_{j,t}^{P}$, $\tilde{d}_{j,t}^{F|P}$, and $\tilde{d}_{j,t}^{P|F}$ are the publisher's expected monthly download volumes of the free and paid versions, and $\tilde{D}_{j,t}^{F}$, $\tilde{D}_{j,t}^{P}$, $\tilde{D}_{j,t}^{F|P}$, and $\tilde{D}_{j,t}^{P|F}$ are the expected cumulative download volumes of the free and paid versions, for the free-only, paid-only, and free-and-paid (both) versioning options, respectively. They are computed based on the demand functions (to be described in the next section). Similar to the profit function specification in Lee, Zhang, and Wedel (2017), r is the fraction of the revenue retained by a publisher, i.e., one minus the known commission rate charged by the platform. The presence of in-app purchase items for app j is denoted by a dummy variable $\bar{\iota}_{C,j}$. The number of in-app advertising networks for app j is denoted by $\bar{a}_j$. $P_{j,t}^{*}$ is the (optimal) price that would be charged by the app publisher for the paid version if it is offered[21]. The monthly fixed cost of offering the free version is $\gamma_{1,F}$, and it is $\gamma_{1,P}$ for the paid version, which include costs related to research and development, licensing, server maintenance, marketing, legal, and insurance, etc. The parameter γ_2 represents the variable cost associated with each expected new download, which is related to promotions, customer acquisition, server expansion, and technical support. In addition, the parameters $\lambda_{1,\mathrm{F}}$ and $\lambda_{1,\mathrm{P}}$ represent the monthly revenue per download from in-app purchases, for a free app and a paid app, respectively. The parameter λ_2 represents the monthly revenue generated per ad-network per download of a free app. These parameters reflect app publishers' costs and revenues that are known to them in their decision making but unknown to the researcher, and will be estimated from the data.

The unique conditional profit components associated with the upgrading decisions (no upgrading, upgrading the free version, upgrading the paid version, and upgrading both versions) are specified as follows.

$$\Pi_{j,t}^{UN} = \xi_{j,t}^{UN}, \tag{III-5}$$

$$\Pi_{j,t}^{UF} = V_{j,t}^{UF} + \xi_{j,t}^{UF} = r\lambda_{1,F}\bar{\iota}_{F,j}\widetilde{\Delta}_{j,t}^{F} + \lambda_2\bar{a}_j\,\widetilde{\Delta}_{j,t}^{F} - \gamma_{1,UF} - \gamma_2\widetilde{\Delta}_{j,t}^{F} + \xi_{j,t}^{UF}, \tag{III-6}$$

$$\Pi_{j,t}^{UP} = V_{j,t}^{UP} + \xi_{j,t}^{UP} = r\left(P_{j,t}^{*}\widetilde{\Delta}_{j,t}^{P} + \lambda_{1,P}\bar{\iota}_{P,j}\widetilde{\Delta}_{j,t}^{P}\right) - \gamma_{1,UP} - \gamma_2\widetilde{\Delta}_{j,t}^{P} + \xi_{j,t}^{UP}, \tag{III-7}$$

$$\begin{aligned}\Pi_{j,t}^{UB} = V_{j,t}^{UB} + \xi_{j,t}^{UB} &= r\left(\lambda_{1,F}\bar{\iota}_{F,j}\widetilde{\Delta}_{j,t}^{F|P} + P_{j,t}^{*}\widetilde{\Delta}_{j,t}^{P|F} + \lambda_{1,P}\,\bar{\iota}_{P,j}\widetilde{\Delta}_{j,t}^{P|F}\right) + \lambda_2\bar{a}_j\,\widetilde{\Delta}_{j,t}^{F|P} \\ &\quad -\gamma_{1,UF} - \gamma_{1,UP} - \gamma_2(\widetilde{\Delta}_{j,t}^{F|P} + \widetilde{\Delta}_{j,t}^{P|F}) + \xi_{j,t}^{UB},\end{aligned} \tag{III-8}$$

where $\widetilde{\Delta}_{j,t}^{F}$, $\widetilde{\Delta}_{j,t}^{P}$, $\widetilde{\Delta}_{j,t}^{F|P}$, and $\widetilde{\Delta}_{j,t}^{P|F}$ are the changes in the expected download volumes of the free and paid versions resulting from an upgrade of the free-only, paid-only, and

[21] We assume that a publisher would charge the price for a paid app to maximize her expected profit. The derivation of the optimal price is provided in a later section.

free-and-paid (both) versioning options, respectively. The parameter $\gamma_{1,UF}$ is the cost of upgrading the free version, and $\gamma_{1,UP}$ is the cost of upgrading the paid version. These parameters capture the costs of research and development, marketing, and technical support associated with upgrading an app, which are assumed to be realized in the month when an upgrade is launched. Other parameters appearing in equations (III-5) to (III-8) are defined the same as described before. Random shocks in revenues are captured by the error terms $\xi_{j,t}^{UN}$, $\xi_{j,t}^{UF}$, $\xi_{j,t}^{UP}$, and $\xi_{j,t}^{UB}$.

A publisher's ex-ante single-period total profit functions are the sum of the respective common and unique profit components listed above. Specifically, the profits for the eight versioning and upgrading options are:

$$\Pi_{j,t}^{F\_N} = \Pi_{j,t}^{F} + \Pi_{j,t}^{UN} = V_{j,t}^{F} + \varepsilon_{j,t}^{F} + \xi_{j,t}^{UN} = V_{j,t}^{F} + \varepsilon_{j,t}^{F\_N}, \tag{III-9}$$

$$\Pi_{j,t}^{F\_Y} = \Pi_{j,t}^{F} + \Pi_{j,t}^{UF} = V_{j,t}^{F} + V_{j,t}^{UF} + \varepsilon_{j,t}^{F} + \xi_{j,t}^{UF} = V_{j,t}^{F} + V_{j,t}^{UF} + \varepsilon_{j,t}^{F\_Y}, \tag{III-10}$$

$$\Pi_{j,t}^{P\_N} = \Pi_{j,t}^{P} + \Pi_{j,t}^{UN} = V_{j,t}^{P} + \varepsilon_{j,t}^{P} + \xi_{j,t}^{UN} = V_{j,t}^{P} + \varepsilon_{j,t}^{P\_N}, \tag{III-11}$$

$$\Pi_{j,t}^{P\_Y} = \Pi_{j,t}^{P} + \Pi_{j,t}^{UP} = V_{j,t}^{P} + V_{j,t}^{UP} + \varepsilon_{j,t}^{P} + \xi_{j,t}^{UP} = V_{j,t}^{P} + V_{j,t}^{UP} + \varepsilon_{j,t}^{P\_Y}, \tag{III-12}$$

$$\Pi_{j,t}^{B\_N} = \Pi_{j,t}^{B} + \Pi_{j,t}^{UN} = V_{j,t}^{B} + \varepsilon_{j,t}^{B} + \xi_{j,t}^{UN} = V_{j,t}^{B} + \varepsilon_{j,t}^{B\_N}, \tag{III-13}$$

$$\Pi_{j,t}^{B\_F} = \Pi_{j,t}^{B} + \Pi_{j,t}^{UF} = V_{j,t}^{B} + V_{j,t}^{UF} + \varepsilon_{j,t}^{B} + \xi_{j,t}^{UF} = V_{j,t}^{B} + V_{j,t}^{UF} + \varepsilon_{j,t}^{B\_F}, \tag{III-14}$$

$$\Pi_{j,t}^{B\_P} = \Pi_{j,t}^{B} + \Pi_{j,t}^{UP} = V_{j,t}^{B} + V_{j,t}^{UP} + \varepsilon_{j,t}^{B} + \xi_{j,t}^{UP} = V_{j,t}^{B} + V_{j,t}^{UP} + \varepsilon_{j,t}^{B\_P}, \tag{III-15}$$

$$\Pi_{j,t}^{B\_B} = \Pi_{j,t}^{B} + \Pi_{j,t}^{UB} = V_{j,t}^{B} + V_{j,t}^{UB} + \varepsilon_{j,t}^{B} + \xi_{j,t}^{UB} = V_{j,t}^{B} + V_{j,t}^{UB} + \varepsilon_{j,t}^{B\_B}. \tag{III-16}$$

Following the set-up of a joint logit model[22], we assume that the random terms $\varepsilon_{j,t}^{N}$,

[22] We have tested a nested-logit formulation of the versioning and upgrading decisions in which upgrading decisions are assumed to be nested within the versioning decisions. This model does not

$\varepsilon_{j,t}^{F\_N}$, $\varepsilon_{j,t}^{F\_Y}$, $\varepsilon_{j,t}^{P\_N}$, $\varepsilon_{j,t}^{P\_Y}$, $\varepsilon_{j,t}^{B\_N}$, $\varepsilon_{j,t}^{B\_F}$, $\varepsilon_{j,t}^{B\_P}$, and $\varepsilon_{j,t}^{B\_B}$ follow Gumbel distributions with location parameter 0 and scale parameter μ.

III.4.1.2 Choice Probabilities & Likelihood Functions

Denote the versioning decision outcome variable for app *j* at time *t* by $C_{j,t}$, where $C_{j,t} = c \in \{N, F, P, B\}$, and the upgrading decision outcome variable by $U_{j,t}^{C} = u \in \{Y, N, UN, UF, UP, UB\}$ conditional on $C_{j,t} \in \{F, P, B\}$. Then the likelihood function of the parameters (collected in Θ) is:

$$L(\Theta|data) = \prod_j \prod_t \prod_c \prod_u \Pr\left(C_{j,t} = c\middle|\Theta\right)^{I(C_{j,t}=c)} \Pr\left(U_{j,t}^{C} = u\middle|\Theta, C_{j,t} = c\right)^{I\left(U_{j,t}^{C}=u\right)}, \quad \text{(III-17)}$$

Based on the assumptions on the random terms, we obtain the marginal and conditional choice probabilities of the versioning and upgrading decisions (see Ben-Akiva and Lerman 1985, pages 278-285).

The marginal probability of choosing versioning outcome $C_{j,t} = c$ is:

$$\Pr\left(C_{j,t} = N\middle|\Theta\right) = \frac{1}{1 + \exp\left[\mu\left(V^F + \frac{1}{\mu}IV^F\right)\right] + \exp\left[\mu\left(V^P + \frac{1}{\mu}IV^P\right)\right] + \exp\left[\mu\left(V^B + \frac{1}{\mu}IV^B\right)\right]}, \quad \text{(III-18)}$$

$$\Pr\left(C_{j,t} = F\middle|\Theta\right) = \frac{\exp\left[\mu\left(V^F + \frac{1}{\mu}IV^F\right)\right]}{1 + \exp\left[\mu\left(V^F + \frac{1}{\mu}IV^F\right)\right] + \exp\left[\mu\left(V^P + \frac{1}{\mu}IV^P\right)\right] + \exp\left[\mu\left(V^B + \frac{1}{\mu}IV^B\right)\right]}, \quad \text{(III-19)}$$

offer better performance than the joint logit model, and therefore we present the latter as our model formulation here.

$$\Pr\left(C_{j,t} = P \middle| \Theta\right)$$

$$= \frac{\exp\left[\mu\left(V^P + \frac{1}{\mu} IV^P\right)\right]}{1 + \exp\left[\mu\left(V^F + \frac{1}{\mu} IV^F\right)\right] + \exp\left[\mu\left(V^P + \frac{1}{\mu} IV^P\right)\right] + \exp\left[\mu\left(V^B + \frac{1}{\mu} IV^B\right)\right]}, \tag{III-20}$$

$$\Pr\left(C_{j,t} = B \middle| \Theta\right)$$

$$= \frac{\exp\left[\mu\left(V^B + \frac{1}{\mu} IV^B\right)\right]}{1 + \exp\left[\mu\left(V^F + \frac{1}{\mu} IV^F\right)\right] + \exp\left[\mu\left(V^P + \frac{1}{\mu} IV^P\right)\right] + \exp\left[\mu\left(V^B + \frac{1}{\mu} IV^B\right)\right]}, \tag{III-21}$$

where the IV terms indicate the inclusive values, which are computed as:

$$IV^F = \ln[1 + \exp(\mu V_{j,t}^{UF})],$$

$$IV^P = \ln[1 + \exp(\mu V_{j,t}^{UP})],$$

$$IV^B = \ln[1 + \exp(\mu V_{j,t}^{UF}) + \exp(\mu V_{j,t}^{UP}) + \exp(\mu V_{j,t}^{UB})].$$

The conditional probability of the upgrading decision $U_{j,t} = u$, conditional on $c \in \{F, P, B\}$ is:

$$\Pr\left(U_{j,t}^F = Y \middle| C_{j,t} = F; \Theta\right) = \frac{\exp\{\mu V_{j,t}^{UF}\}}{1 + \exp\{\mu V_{j,t}^{UF}\}} \tag{III-22}$$

$$\Pr\left(U_{j,t}^P = Y \middle| C_{j,t} = P; \Theta\right) = \frac{\exp\{\mu V_{j,t}^{UP}\}}{1 + \exp\{\mu V_{j,t}^{UP}\}} \tag{III-23}$$

$$\Pr\left(U_{j,t}^B = UF \middle| C_{j,t} = B; \Theta\right) = \frac{\exp\{\mu V_{j,t}^{UF}\}}{1 + \exp\{\mu V_{j,t}^{UF}\} + \exp\{\mu V_{j,t}^{UP}\} + \exp\{\mu V_{j,t}^{UB}\}}, \tag{III-24}$$

$$\Pr\left(U_{j,t}^B = UP \middle| C_{j,t} = B; \Theta\right) = \frac{\exp\{\mu V_{j,t}^{UP}\}}{1 + \exp\{\mu V_{j,t}^{UF}\} + \exp\{\mu V_{j,t}^{UP}\} + \exp\{\mu V_{j,t}^{UB}\}}, \tag{III-25}$$

$$\Pr\left(U_{j,t}^B = UB \middle| C_{j,t} = B; \Theta\right) = \frac{\exp\{\mu V_{j,t}^{UB}\}}{1 + \exp\{\mu V_{j,t}^{UF}\} + \exp\{\mu V_{j,t}^{UP}\} + \exp\{\mu V_{j,t}^{UB}\}} \tag{III-26}$$

In a standard joint logit model, which is built on unit-free utility functions, the scale parameter μ needs to be fixed to 1 for identification purposes. But, since profits are not unit free, we need to estimate the scale parameter in our model.

III.4.3 Demand Models

We adopt a Bayesian aggregate demand model of app downloads. We account for potential endogeneity of price via an instrumental variable approach by using the average log price of all other paid apps published by the same publisher at a future time point outside of the data period (i.e., February 2017) as the instrument. An app user *i*'s utilities of downloading the free and paid versions are:

$$\begin{aligned} H_{i,j,t}^{F} &= X_{j,t}^{F}\beta_i^{F} + \eta_{j,t}^{F} + \xi_{i,j,t}^{F} \\ &= \beta_{0,i}^{F} + \beta_{1,i}^{F}U_{\mathrm{j,t}}^{\mathrm{F}} + \beta_{2,i}^{F}U_{\mathrm{j,t}}^{\mathrm{P}} + \beta_{3,i}^{F}T_{j,t}^{F} + \kappa_i^{F}B_{j,t} + \eta_{j,t}^{F} + \xi_{i,j,t}^{F}, \end{aligned} \tag{III-27}$$

$$\begin{aligned} H_{i,j,t}^{P} &= X_{j,t}^{P}\beta_i^{P} + \eta_{j,t}^{P} + \xi_{i,j,t}^{P} \\ &= \beta_{0,i}^{P} + \beta_{1,i}^{P}U_{\mathrm{j,t}}^{\mathrm{F}} + \beta_{2,i}^{P}U_{\mathrm{j,t}}^{\mathrm{P}} + \beta_{3,i}^{P}T_{j,t}^{P} + \beta_{4,i}^{P}\hat{P}_{j,t}^{*} + \kappa_i^{P}B_{j,t} + \eta_{j,t}^{P} + \xi_{i,j,t}^{P}, \end{aligned} \tag{III-28}$$

where indicator variables $U_{\mathrm{j,t}}^{\mathrm{F}}$ and $U_{\mathrm{j,t}}^{\mathrm{P}}$ equal 1 if the free or paid version releases an upgrade at time *t*, and 0 otherwise; $T_{j,t}^{F}$ and $T_{j,t}^{P}$ are the length of time since the last upgrade of the free or paid version, respectively (measured in months); $B_{j,t}$ is an indicator variable with 1 indicating both versions being offered, and 0 otherwise; $\eta_{j,t}^{F}$ and $\eta_{j,t}^{P}$ are aggregated demand shocks affecting all app users at time *t* (for examples, updates of the Android operating system, or the platform's server issues), which are assumed to follow Normal distributions with mean 0 and standard deviations ρ^{F} and ρ^{P}, respectively; and $\xi_{i,j,t}^{F}$ and $\xi_{i,j,t}^{P}$ are idiosyncratic demand shocks which follow i.i.d. Gumbel distributions with location parameter 0 and scale parameter 1.

The parameters κ_i^{F} and κ_i^{P} capture the effects of the presence of one version on the utility of the other version in the same time period. We expect their signs to be negative, which would reflect contemporaneous cannibalization. $\beta_{1,i}^F$ and $\beta_{2,i}^P$ capture the effects of releasing an upgrade on the demand for the free or paid version, respectively, and we expect them to be positive. In addition, $\beta_{2,i}^F$ and $\beta_{1,i}^P$ capture the cross-effects of releasing an upgrade of the paid version on the demand of the free version, and vice versa, and we expect them to be negative. Further, $\beta_{3,i}^F$ and $\beta_{3,i}^P$ capture the potential delaying effects of an update over time, and a negative sign would confirm such patterns.

We make the standard assumption that individual users' preference parameters $\beta_i = (\beta_i^F \cup \beta_i^P)$ follows a Normal distribution $\beta_i \sim N(\bar{\beta}, \Sigma)$. We integrate the individual level probability of downloading the free or paid version over the distribution of β_i to obtain the shares of downloads for the two versions, $s_{j,t}^F$ and $s_{j,t}^P$[23]:

$$s_{j,t}^F = \int \frac{o_{j,t}^F \cdot \exp\left(X_{j,t}^F \beta_i^F + \eta_{j,t}^F\right)}{1 + o_{j,t}^F \cdot \exp\left(X_{j,t}^F \beta_i^F + \eta_{j,t}^F\right) + o_{j,t}^P \cdot \exp\left(X_{j,t}^P \beta_i^P + \eta_{j,t}^P\right)} N\left(\beta_i \middle| \bar{\beta}, \Sigma\right) \mathrm{d}\beta_i, \tag{III-29}$$

$$s_{j,t}^P = \int \frac{o_{j,t}^P \cdot \exp\left(X_{j,t}^P \beta_i^P + \eta_{j,t}^P\right)}{1 + o_{j,t}^F \cdot \exp\left(X_{j,t}^F \beta_i^F + \eta_{j,t}^F\right) + o_{j,t}^P \cdot \exp\left(X_{j,t}^P \beta_i^P + \eta_{j,t}^P\right)} N\left(\beta_i \middle| \bar{\beta}, \Sigma\right) \mathrm{d}\beta_i. \tag{III-30}$$

Here, $o_{j,t}^F$ and $o_{j,t}^P$ are indicator variables that equal 1 if the focal version is available, and zero otherwise. The share of downloads represents the monthly downloads of a given app version as a share of the total market. The download volume of each version of an app can be obtained as $\tilde{d}_{j,t}^F = D_M \cdot s_{j,t}^F$, and $\tilde{d}_{j,t}^P = D_M \cdot s_{j,t}^P$, where D_M is the total monthly download volume of all apps on the platform.

[23] $\tilde{\rho}_F^2$ and $\tilde{\rho}_P^2$ are functions of the standard deviations ρ^F and ρ^P, of the demand shocks, respectively. See Jiang, Manchanda, and Rossi (2009) for details.

The incremental demand expected by the publisher to be generated from releasing an upgrade of the free version can be calculated as $\widetilde{\Delta}_{j,t}^{F} = \tilde{d}_{j,t}^{F}\left(U_{j,t}^{F} = 1\right) - \tilde{d}_{j,t}^{F}\left(U_{j,t}^{F} = 0\right)$, and its impact on the paid version as $\widetilde{\Delta}_{j,t}^{P|F} = \tilde{d}_{j,t}^{P}\left(U_{j,t}^{F} = 1\right) - \tilde{d}_{j,t}^{P}\left(U_{j,t}^{F} = 0\right)$. Conversely, the incremental demand expected by the publisher to be generated from releasing an upgrade of the paid version is $\widetilde{\Delta}_{j,t}^{P} = \tilde{d}_{j,t}^{P}\left(U_{j,t}^{P} = 1\right) - \tilde{d}_{j,t}^{P}\left(U_{j,t}^{P} = 0\right)$, and its impact on the free version is $\tilde{d}_{j,t}^{F}\left(U_{j,t}^{P} = 1\right) - \tilde{d}_{j,t}^{F}\left(U_{j,t}^{P} = 0\right)$. These quantities are plugged into the profit functions of equations (III-5) to (III-8) when estimating the versioning and upgrading decision model.

III.4.3 Optimal Price

At each time period, we assume that a publisher sets the would-be retail price of its paid app, if it is to be offered, such that it maximizes the publisher's monthly profit. In addition, publishers are assumed to use the mean utilities obtained from equations (III-27) and (III-28) in determine their optimal prices. It can be shown that the optimal price is then (see Lee, Zhang, and Wedel 2017 for the derivation under a similar set-up):

$$P_t^* = -\frac{1}{\beta_6^P}\left[1 + W_n\left(\exp\left(\frac{\left(\frac{\beta_6^P\gamma_2}{r} - \beta_6^P\bar{b}_P\lambda_{1,P} + \Omega_t - 1\right)}{1 + \bar{H}_t^F}\right)\right)\right] + \frac{\gamma_2}{r} - \bar{b}_P\lambda_{1,P} \qquad \text{(III-31)}$$

where Ω_t is the mean utility of downloading the paid version net of the price effect (i.e., $\Omega_t \equiv \bar{H}_t^P - \beta_4^P P_t$), and $W_n(\cdot)$ is the Lambert-W function. The optimal price is a function of all parameters in the demand utility function of both free and paid versions, the commission rate, the marginal cost of new app users, and the revenue generated via in-app purchases.

III.5 MODEL ESTIMATION RESULTS

III.5.1 Model Comparisons

We compare the proposed joint model of upgrading and versioning decisions with three restricted models, based on the Gelfand-Dey (1994) log-marginal density (LMD) and the hit rate. The purpose of the comparison is to investigate the model assumptions on app publishers' revenue and cost structures, which affect both the versioning and upgrading decisions. Specifically, we test whether 1) the indirect revenues from in-app purchases for the free and paid versions are different, 2) the monthly fixed costs of free version and paid version are different, and 3) app publishers incur different costs for upgrading the free and paid versions. Therefore, the two in-app purchase revenue parameters of the free version and paid version are restricted to be the same in the first benchmark model. In the second benchmark model, the monthly cost parameters for the free and paid versions are restricted to be the same. The third model restricts the upgrade costs of the free and paid versions to be the same. Table III-2 presents the results. The proposed model outperforms all three benchmark models in terms of the Gelfand-Dey (1994) log-marginal density and offers at least slightly better hit rate than the benchmarks. These results show the importance of distinguishing the indirect revenue generation from in-app purchases between free and paid versions, because they affect not only the versioning decision but also the upgrade decisions for both versions of apps. The fact that the proposed model outperforms the second and third benchmark model is especially noteworthy, which demonstrates that the monthly costs and the upgrading costs between free and paid version are significantly different. The model comparisons thus testify to the importance of jointly modeling the versioning and upgrading decisions from a common underlying revenue and cost structure.

Insert Table III-2 about here

III.5.2 Demand Model Estimation Result

We report the estimation result of the demand model parameters in Table III-3. All variables show “significant” effects, as none of the credible intervals of the parameters covers zero. We highlight the key findings below. First, an upgrade of the free version increases its demand and decreases the demand for the paid version. Similarly, an upgrade of the paid version increases its demand and decreases the demand for the free version. Second, the demand for an app decreases with the time since its last upgrade, which may indicate wear-out of the novelty of an upgrade. The effect appears similar in magnitude for both versions of apps (-2.33 and -2.42 for the free and paid, respectively). Third, we find that the presence of one version of an app decreases the demand for the other version in the same time period, which indicates contemporaneous cannibalization effects between the two versions. The coefficient of the (instrumented) price of the paid version is has a negative sign, and shows that the demand for a paid app decreases with its price, as expected.

Insert Table III-3 about here

III.5.3 Upgrading and Versioning Decision Model Estimation Result

The estimation results of the joint model of upgrading and versioning decisions are presented in Table III-4. The estimated monthly revenue from the free version's in-app purchases per 1,000 downloads (RPM) is $2.58. A single ad-network generates a similar amount of revenue for a free app each month ($2.85 RPM). The paid version generates much higher RPM from in-app purchases than the free version does ($2,623.44 vs. $2.58). This is likely due to enhanced functionality of a paid app in comparison to its free counterpart, and thus much higher participation and willingness to pay for in-app items among paid app users. As to the cost parameters, the monthly fixed cost of the paid version is about twice as large as that of the free version ($14,913 vs. $6,988). Similarly, the upgrade cost of the paid version is substantially higher than that of the free version ($45,633 vs. $15,958). These higher costs for the paid version are consistent with that fact that it costs more to support additional features or enhanced functionalities. The variable cost per 1,000 downloads is about $13.29.

Insert Table III-4 about here

In addition, we provide the histogram of the optimal price calculated based on the parameter estimates and Equation (III-31), in Figure III-3. Note that the optimal price is a function of versioning and upgrading decisions and the demand model and joint decision model parameters. In our analysis, the estimated optimal price for paid apps ranges from \$0.42 and \$13.0, which is consistent with the distribution of actual app prices in the data.

Insert Figure III-3 about here

III.6 POLICY SIMULATIONS

An objective of this study is to examine the soundness of current pratices and explore opporutnities to improve app publishers' profits, app platforms' revenues, and the payoff for an app ecosystem.

In contrast to packaged software, a unique feature of mobile app upgrades is that existing users do not need to pay for obtaining the upgrades, even in the case of paid apps. Does this practice make sense? An alternative policy is to charge prices for upgraded versions of paid apps. Is it worth pursuing?

Mobile app upgrades can benefit not only their publishers but also increase an app platform's revenues, yet the costs of developing and launching app upgrades are primarily born by the publishers. A potential opportuntiy for increasing mutual gains from app upgrades is for the app platform to invest in its infrastructure, such as efficient integrated development environment (IDE) and software development kits (SDK), to help app publishers reduce upgrade costs. Is it worth such investment? How much benefit can an app platform expect to achieve through such initiatives?

The answers to these managerial questions depend on how each proposed action of interest would affect app publishers' profits and an app platform's revenues. We address these questions via conducting counter-factual simulation analyses. Specially, we simulate app publishers' versioning and upgrading decisions and estimate the resultant app publisher profits, app platform revenue, and eco-system payoff[24], under the following scenarios: 1) app publishers charge retail prices to existing owners for upgrades of paid apps; 2) the app platform helps publishers reduce upgrade costs by offering efficient integrated development environment (IDE) and software development kits (SDK). The simulation analyses are conducted based on the estimated primitive parameters in the demand and joint decision models, and the parameters are invariant to policy changes in the data generation environment (cf. Chintagunta et al. 2006).

III.6.1. Charge Existing Owners Retail Prices for Upgrades of Paid Apps

Major app platforms require app publishers to make their upgrades available for free to the existing owners of their apps, even in the case of paid apps. Is this a sound policy? What may happen if publishers are allowed to charge retail prices for upgrades of their paid apps[25]? If publishers charge retail prices for upgrades, should

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[24] Like in Lee, Zhang, and Wedel (2017), we define the eco-system payoff as the sum of total publisher profits and the app platform's revenue.

[25] We do not consider the scenario of changing upgrades of free apps into paid apps, which would involve versioning changes by the set-up of our model.

they continue to support previous editions or only the new edition? We explore the scenarios in which app publishers charge for upgrades of paid version to the previous owners. In each scenario, we assume that a certain percentage of the customer base is willing to pay for an upgrade. Then we explore two options: 1) the publisher would not support previous editions of a paid app after it launches an upgrade, and 2) the app publisher continues to support the immediate previous edition in addition to the upgraded edition. We vary the percentage of customers who are willing to upgrade from 1% to 25% and test seven scenarios within that range under each option, and simulate the impact on app publishers' total profit and the platform's revenue. We report the simulation result in Table III-5 and III-6.

When app publishers only support the upgraded edition, the app publishers' total profit, platform revenue, and ecosystem payoff are lower than these under the current practice in the first five scenarios (1% to 15% of existing owners willing to pay for an upgrade), indicating that the loss in in-app purchases due to reduced customer base outweighs the gain in the direct revenue from selling upgrades. When 20% of existing customers pay for an upgrade, the sum of all apps' profit is higher than that in the current practice, but the loss of the platform's revenue is larger than the gain of all app publishers' profit, so the app publishers are not able to fully compensate the loss of the platform to make both parties better-off. Both platform and app publishers are better-off when we further increase the percentage of existing customers who are willing to pay for an upgrade to 25%. The result shows that the app ecosystem has a potential to improve its payoff by charging a price for upgrades and promoting the paid upgrades to the users of previous editions. To attract a large number of existing users (i.e., 25% or more) to purchase upgrades, the new app editions should provide attractive new features and substantially improve functionalities, and the publishers need to promote them well. In addition, the app platform can support such efforts of app publishers by providing efficient promotion tools or by lending ad space on the platform's web pages.

We further explore what may happen if we relax the assumption that app publishers do not supporting previous app editions. To see the qualitative changes, we assume the simplest form of extension in which app publishers continue to support only the immediate previous edition in addition to the new edition, which means that both user groups would generate in-app purchase revenues. The result is summarized in Table III-6. In the first five scenarios, both app publishers' profit and the platform's revenue are still lower than under those under the current practice, even though the loss becomes less severe as the percentage of existing owners willing to pay for an upgrade goes up. Under the scenario where 20% users pay for an upgrade, both app publishers and the platform would benefit from the policy change. Note that it would only benefit app publishers in the previous set of analysis under option 1 (see Table III-5). This result suggests that supporting the users of previous editions can help the platform and the publishers to find opportunities for mutual benefits. App publishers' effort to create an environment for software engineers to develop standardized/consistent and upward compatible programming codes for successive app editions could help them lower the cost of supporting previous app editions. Also, the platform might want to provide app publishers with efficient development environment and user management tools to encourage them to support their previous app editions.

In conclusion, the current policy of not charging for upgrades is desirable if only a small number of current customers are willing to pay for upgrades. Our analysis shows that flexibility in the policy could benefit both the app platform and app publishers. Depending on the appeal of an app and the characteristics of its customers, there are situations where charging retail prices for paid app upgrades could bring about mutual benefits, and thus it would pay off for app platforms and publishers to work together to identify such opportunities.

III.6.2 Reducing Costs of Upgrades via Efficient IDE/SDK

By reducing the costs of developing and launching upgrades, app publishers would have more incentives to do so, which in turn could benefit not only themselves but also app platforms. One way app platforms can help reduce upgrade costs for publishers is to invest in efficient app upgrading environment. For example, Google provides an integrated development environment (IDE) for Android platform called Android Studio for Android app developers. It comes with Android software development kit (SDK), which includes a number of Android app related tools. Google has been adding new features to Android Studio and Android SDK since its release in 2013. How much should app platforms spend on continued improvements of such systems? We use Google as a case study, and examine the expected benefit in terms of the platform’s revenue, total app publisher profit, and eco-system payoff, assuming that such investment would reduce the upgrading costs of the free or paid version by 1) 50% or 2) 75%[26].

We report the result in Table III-7. In almost all scenarios, there is a substantial increase in the platform's revenue and the total profit of app publishers, and all scenarios yield higher ecosystem payoffs than that under the current practice. The incremental revenue for the platform under each scenario would suggest the upper bound for the amount worth investing in infrastructure improvements aimed at reducing upgrading costs for app publishers, in the absence of profit-sharing agreements with the latter. If the two parties agree to share the incremental eco-system payoffs, this upper bound could go even higher. The result of exercise lends support to app platforms' continuing investments in establishing and improving their IDE and SDK.

III.7 CONCLUSIONS AND DISCUSSION

The mobile app market is growing continuously, and it becomes increasingly important to understand mobile app publishers' strategic considerations when it comes to make decisions on offering free, paid, or freemium versions and releasing upgrades. In this study, we have examined strategic upgrade decisions of mobile apps in connection with their versioning decisions. The versioning and upgrading decisions could be closely interconnected, and we have addressed the managerial and methodological challenges arise when the two decisions are made jointly. We have

[26] Our focus here is to examine the worthiness of a platform's investment in IDE or SDK. The numerical results of the simulations can be interpreted as the expected outcomes from any initiatives that would reduce the upgrades costs by 50% or 75%.

collected a unique dataset that includes app publishers' versioning and upgrading decisions in addition to various revenue sources over 13 months, and the dataset is well suited for the empirical model we have developed to tackle the challenges.

We contribute to the literature by studying the upgrading and versions decisions jointly and across the free and paid versions of an app. The two decisions have been studied separately by prior research, and we provide novel managerial insights by taking into account the strategic consideration of making these decisions jointly and across app versions. To this end, we have proposed a joint model of versioning and upgrading decisions that considers various revenue and cost sources associated with the two decisions for free and paid apps. The comparison of our proposed model with several nested benchmark models shows the importance of taking into account the differential revenue sources and cost components associated with the free vs. paid version. Further, the estimation result of our demand model reveals the impact of upgrading one version of an app on the demand of both versions and the cannibalization between the two versions. The estimation result of the joint decision models has expanded our knowledge on the revenue and costs associated with the two decisions. Finally, our policy simulations have tested the soundness of the current policy of app distribution platforms and explored opportunities to increase the mutual benefits of app publishers and app platform by relaxing a restrictive policy of app platform and providing additional support to app publishers.

The key results of our model estimations are summarized below.

- Upgrading one version of an app increases the demand for the same version, but decreases the demand for the counterpart version.

- The demand for an app decreases with the time since its last upgrade, which shows the wear-out effect of an upgrade and suggests the need for releasing additional upgrades over time. The magnitude of the wear-out effect of free and paid versions appears to be similar.
- The presence of one version decreases the demand for the other version, which indicates contemporaneous cannibalization effects between the two versions.
- The estimated monthly revenues from in-app purchases and from a single ad-network are similar for free apps, but paid apps generate much higher revenues from in-app purchases than their free version counterparts do.
- The monthly fixed cost and upgrading cost for the paid version is much greater than these costs for the free version.

In addition, we have conducted several counter-factual simulation analyses in order to examine the soundness of the current policy concerning the pricing of upgrades and the effort to provide efficient app upgrading environment for app publishers. The key findings from these policy simulations are:

- The platform revenue, total app profit, and ecosystem payoff could increase if a sufficiently high percentage of current customers are willing to pay for upgrades, while the current policy of not charging prices for upgrades is desirable if the percentage falls below such a threshold. The app platform should provide flexibility in its pricing policy regarding upgrades and work with app publishers to identify opportunities where charging for upgrades could bring mutual benefits. And app publishers should carefully assess the trade-off between the gain in direct revenues from selling upgrades vs. loss in indirect revenues due to loss of customer base, as well as between the costs of supporting previous editions of an app vs. the indirect revenues resulted from such efforts.

- Platforms' efforts to help reduce upgrading costs for app publishers, such as investing in efficient app upgrading environment, can benefit both app publishers and an app platform. The efficient upgrading environment promotes more upgrades of app publishers than the current level, increases app publishers' revenues, which in turns helps a platform collect additional commission revenues. Our simulation analysis lends support to platforms' investments in establishing and continuously improving their efficient app upgrading environment, and provides numerical estimates for the upper bound of the amount of such investments.

The findings of our study provide valuable insights to app publishers and the app platforms. In addition, the methodology we developed can be applied to other software markets that have similar two-sided structures between a platform and publishers and monetization methods (e.g., console game, PC software).

Although the proposed model incorporates complex dynamics related to the versioning and upgrading decisions and considers various revenue and cost structures, we have not fully incorporated the app users' behavioral changes over time and after the release of upgrades, partly due to the data limitation. Nonetheless, the joint decision model has the potential to incorporate those behavioral changes. For example, we can consider linear and quadratic effects of apps' age in the demand model to explain the churning of app users. In addition, an upgrade might stimulate idle users to reuse the apps. We can account for these behavioral changes by relaxing the current assumptions on the revenue parameters in the joint decision model. For the purpose of model parsimony, we currently constrain the revenue parameters in the common profit function of versioning decisions and the conditional profit functions of upgrading decisions to be the same. Instead, the two sets of parameters in the two profit functions could be allowed to differ. The comparison between the two sets parameters could then explain the potential behavior changes of app users after app upgrades mentioned above. We plan to test the models that account for the app users' behavioral changes in future research.

In addition, we could further incorporate heterogeneity of app publishers to improve the model's performance in terms of explaining their upgrading and versioning decisions. Specifically, we could allow the revenue and cost parameters of the joint decision model to vary across apps or app-categories, if data variation permits us to do so. Accounting for unobserved heterogeneity between apps or categories will likely improve model fit and enable us to provide customized recommendations to app publishers or to the apps in a certain category. Also, adding the variables we discussed in the first essay such as app size and quality to the demand model would allow us to reveal different patterns of app publishers' reactions to policy changes via simulation analyses. Further, app publishers could be forward-looking when they make upgrading decisions. For example, they might expect to recoup the cost of releasing an upgrade not in a single period but in multiple periods in the future. As a direction for future research, we suggest to build a forward-looking model considering app publishers' dynamic considerations and heterogeneity, and to compare the model performance and findings with those in the present study.

Appendices

A.1. Optimal Pricing of Mobile Apps

An app publisher sets the price of its paid app to maximize monthly profit. Expected profit of the paid version at time period t is:

$$\Pi_t = (1-r)\big(P_t\tilde{q}_{P,t} + \lambda_{1,P}\bar{b}_P\tilde{Q}_{P,t}\big) - \gamma_{1,P} - \gamma_2\tilde{q}_{P,t} + \varepsilon_{P,t} \quad \text{(A.1-1)}$$

$$= (1-r)\big(P_t\tilde{q}_{P,t} + \lambda_{1,P}\,\bar{b}_P(Q_{P,t-1} + \tilde{q}_{P,t})\big) - \gamma_{1,P} - \gamma_2\tilde{q}_{P,t} + \varepsilon_{P,t} \quad \text{(A.1-2)}$$

Predicted monthly download volume $\tilde{q}_{P,t}$ based on the demand model parameters β is:

$$\tilde{q}_{P,t}(P_t, \boldsymbol{x_t}|\beta) = \mathrm{Q}_{\mathrm{M}}^{\mathrm{P}} \cdot S_{P,t}(P_t, \boldsymbol{x_t}|\beta) \quad \text{(A.2)}$$

where P_t is the price of paid version, $\boldsymbol{x_t}$ is a vector of app characteristics including app age, download volume, and category. $\mathrm{Q}_{\mathrm{M}}^{\mathrm{P}}$ is the market size of the paid version and $S_{P,t}(P_t, \boldsymbol{x_t}|\beta)$ is the share of the paid version. $\beta = [\bar{\beta}, \Sigma, \tau]$ are the demand model parameters and $\beta_i \sim N(\bar{\theta}, \Sigma)$ and $\eta_{\mathrm{t}} \sim \mathrm{N}(0, \tau^2)$. Now,

$$S_{P,t}(P_t, \boldsymbol{x_t}|\beta) = \iint \frac{\exp\big([P_t, \boldsymbol{x_t}] \times \beta^i + \eta_t\big)}{1 + \exp([P_t, \boldsymbol{x_t}] \times \beta^i + \eta_t)} \phi\big(\beta^i\big|\bar{\beta}, \Sigma\big) p(\eta_t|\tau) d\beta^i d\eta \quad \text{(A.3-1)}$$

(A.3-2)

$$= \iint \frac{\exp\big(\mu_t(P_t, \boldsymbol{x_t}|\bar{\beta}) + \eta_t + [P_t, \boldsymbol{x_t}] \times v_i\big)}{1 + \exp\big(\mu_t(P_t, \boldsymbol{x_t}|\bar{\beta}) + \eta_t + [P_t, \boldsymbol{x_t}] \times v_i\big)} \phi(v_i|\boldsymbol{0}, \Sigma) p(\eta_t|\tau) dv d\eta$$

where μ_t is mean utility, η_t is a common demand shock across households, and $[P_t, x_t] \times v_i$ is individual level deviation from the mean utility. The mean utility of downloading a paid app is a function of age and cumulative download volumes of app-versions, app category, presence of counterpart version, and price of paid version app:

$$\mu_t = \beta_0^P + \beta_1^P G_t^P + \beta_2^P G_t^F + \beta_3^P P_t + \phi_1^P \log\left(Q_{F,t-1}\right) + \phi_2^P \log\left(Q_{P,t-1}\right) + \kappa^P I(C_t = B) + \beta_{4:5}^{P\prime} d \quad \text{(A.4)}$$

We plug the download volume (A.2) into the profit equation (A.1-1):

$$\Pi_t = (1-r)\left(P_t \mathrm{Q}_\mathrm{M}^\mathrm{P} S_{P,t}(P_t, \boldsymbol{x_t}|\beta) + \lambda_{1,P}\, \bar{b}_P (Q_{P,t-1} + \mathrm{Q}_\mathrm{M}^\mathrm{P} S_{P,t}(P_t, \boldsymbol{x_t}|\beta)\right) - \gamma_1 Q_{P,t-1} - \gamma_2 \mathrm{Q}_\mathrm{M}^\mathrm{P} S_{P,t}(P_t, \boldsymbol{x_t}|\beta) + \varepsilon_{P,t}. \quad \text{(A.5)}$$

The price vector satisfies the FOC as follows:

$$\mathrm{Q}_\mathrm{M}^\mathrm{P}\left[(1-r)\left\{S_{P,t}(P_t^*, \boldsymbol{x_t}|\beta) + P_t^* S'_{P,t}(P_t^*, \boldsymbol{x_t}|\beta) + \lambda_{1,P} \bar{b}_P S'_{P,t}(P_t^*, \boldsymbol{x_t}|\beta)\right\} - \gamma_2 S'_{P,t}(P_t^*, \boldsymbol{x_t}|\beta)\right] = 0 \quad \text{(A.6)}$$

where

$$\frac{dS_{P,t}(P_t^*, \boldsymbol{x_t}|\beta)}{dP_t} = \iint \beta_5^P \cdot S_{P,i,t} \cdot \left(1 - S_{P,i,t}\right) \phi(v_i|\mathbf{0}, \Sigma) p(\eta_t|\tau) dv d\eta,$$

$$S_{P,i,t} = \frac{\exp\left(\mu_t(P_t^*, \boldsymbol{x_t}|\bar{\beta}) + \eta_t + [P_t^*, \boldsymbol{x_t}] \times v_i\right)}{1 + \exp\left(\mu_t(P_t^*, \boldsymbol{x_t}|\bar{\beta}) + \eta_t + [P_t^*, \boldsymbol{x_t}] \times v_i\right)}.$$

Because there is no closed-form solution for the optimal price, we need to solve it numerically. However, implementing this step within IJC estimation makes model estimation computationally infeasible. Therefore, we assume that app publishers set the price based on the mean utility μ_t. This reduces the computational burden significantly, and it is not an unreasonable assumption that was also made by Hitsch (2006). The share of the paid version based on the mean utility, $\tilde{S}_{P,t}$:

$$\tilde{S}_{P,t}\left(P_t, \boldsymbol{x_t}|\bar{\beta}\right) = \frac{\exp\left(\mu_t(P_t, \boldsymbol{x_t}|\bar{\beta})\right)}{1 + \exp\left(\mu_t(P_t, \boldsymbol{x_t}|\bar{\beta})\right)}. \quad \text{(A.7)}$$

Now, the expected profit of the paid version based on $\tilde{S}_{P,t}$ is:

$$\Pi_t = (1-r)\big(P_t \mathrm{Q}_{\mathrm{M}}^{\mathrm{P}} \tilde{S}_{P,t}(P_t, \boldsymbol{x_t}|\bar{\beta}) + \lambda_{1,P}\, \bar{b}_P(\tilde{Q}_{P,t-1} + \mathrm{Q}_{\mathrm{M}}^{\mathrm{P}} \tilde{S}_{P,t}(P_t, \boldsymbol{x_t}|\bar{\beta})\big) \tag{A.8}$$

$$-\gamma_1 Q_{P,t-1} - \gamma_2 \mathrm{Q}_{\mathrm{M}}^{\mathrm{P}} \tilde{S}_{P,t}(P_t, \boldsymbol{x_t}|\bar{\beta}) + \varepsilon_{P,t}.$$

The price vector satisfies the FOC as follows:

$$\Pi_t = (1-r)\big(P_t \mathrm{Q}_{\mathrm{M}}^{\mathrm{P}} \tilde{S}_{P,t}(P_t, \boldsymbol{x_t}|\bar{\beta}) + \lambda_{1,P}\, \bar{b}_P(\tilde{Q}_{P,t-1} + \mathrm{Q}_{\mathrm{M}}^{\mathrm{P}} \tilde{S}_{P,t}(P_t, \boldsymbol{x_t}|\bar{\beta})\big) \tag{A.9}$$

$$-\gamma_1 Q_{P,t-1} - \gamma_2 \mathrm{Q}_{\mathrm{M}}^{\mathrm{P}} \tilde{S}_{P,t}(P_t, \boldsymbol{x_t}|\bar{\beta}) + \varepsilon_{P,t}.$$

where

$$\frac{d\tilde{S}_{P,t}(P_t^*, \boldsymbol{x_t}|\bar{\beta})}{dP_t} = \beta_3^P \cdot \tilde{S}_{P,t} \cdot \big(1 - \tilde{S}_{P,t}\big).$$

The optimal price P_t^* is then derived as follows:

$$P_t^* = -\frac{1}{\beta_3^P}\left[1 + W_n\left(e^{\frac{\beta_3^P \cdot \gamma_2}{(1-r)} - \beta_3^P \cdot \bar{b}_P \cdot \lambda_{1,P} + \psi_t - 1}\right)\right] - \bar{b}_P \cdot \lambda_{1,P} + \frac{\gamma_2}{(1-r)} \tag{A.10}$$

Tables

Table II-1 Description of Major Mobile App Distribution Platforms

Platform	Google Play	App Store	BlackBerry World	Windows Phone Store	Amazon Appstore
Founded in	Oct. 2008	Jul. 2008	Apr. 2009	Oct. 2010	Mar. 2011
OS	Android	iOS	BlackBerry OS	Windows	Fire OS, Android
# Apps in 2016[a]	2,200,000	2,000,000	234,500	669,000	600,000
Developer Fee	$25	$99/Year	Free	$19/Year	$99/Year
Commission Rate	30%	30%	30%	30%	30%

[a] http://www.statista.com/statistics/276623/number-of-apps-available-in-leading-app-stores/

Table II-2 Summary Statistics of Key Variables in the Data

Variable Description	Notation	Mean	SD
Free App Cumulative Download Volume	$Q_{F,i,t}$	5,855,001	8,988,584
Paid App Cumulative Download Volume	$Q_{P,i,t}$	103,936	140,828
Free App Monthly Download Volume	$q_{F,i,t}$	321,631	649,759
Paid App Monthly Download Volume	$q_{P,i,t}$	4,005	8,441
Free App with In-app Purchases (1=yes,0=no)	$b_{F,i}$	0.3668	0.4820
Paid App with In-app Purchases (1=yes,0=no)	$b_{P,i}$	0.1932	0.3949
Number of Ad-Networks (Free)	a_i	2.5325	2.3507
Retail Price (Paid, in $)	$P_{i,t}$	3.1816	2.8292
Free App Age (in months)	$G_{F,i,t}$	19.50	10.83
Paid App Age (in months)	$G_{P,i,t}$	19.70	9.62

Table II-3 Versioning Decision Transition Patterns in the Data

	Time	*T*			
Versioning Decision		No App	Free Only	Paid Only	Freemium
t-1	No App	519	13	11	2
	Free Only	52	2670	0	8
	Paid Only	30	2	2123	21
	Freemium	6	5	13	2066

Table II-4 Identification of Parameters

Parameters	Identified by variation in:
$\gamma_{1,F}$	Observed withdrawal decisions of free apps for which download volumes decrease with app age; observed timing of entry decision relative to paid apps;
$\gamma_{1,\mathrm{P}}$	Observed withdrawal decisions of paid apps for which download volumes decrease with app age; observed timing of entry decision relative to free apps;
γ_2	Observed withdrawal decisions of paid apps without in-app purchases in conjunction with app price;
$\lambda_{1,P}$	Introduction/withdrawal decisions by paid apps with in-app purchases relative to those without in-app purchases, and Versioning decisions across paid apps that vary in cumulative download volumes, among apps whose paid version offers in-app purchases;
$\lambda_{1,F}$	Introduction/withdrawal decisions by free apps with in-app purchases but without in-app advertising, relative to apps without these features, and Versioning decisions across free apps that vary in cumulative download volumes, among apps that offer in-app purchases but not advertising;
λ_2	Timing of introduction of free and paid versions and withdrawal of free versions, among apps that do not offer in-app purchases, and Timing of introduction/withdrawal of the free version across apps with different download volumes and/or different number of ad-networks.

Table II-5 Model Comparisons

Model	Proposed: Forward-Looking Heterogeneous	Benchmark Models (1) Forward-Looking Homogeneous	(2) Myopic Heterogeneous	(3) Myopic Homogeneous
Gelfand-Dey LMD	-7247.9	-7480.2	-7265.7	-7472.8
Actual Decisions (N)	Predicted Decisions (N)			
No app: 607	387	789	0	1
Free only: 2690	2861	3424	3145	4061
Paid only: 2147	2085	2419	2180	2572
Both: 2097	2208	909	2216	907

Table II-6 First Order Versioning Transition Patterns based on Proposed Model

Time		*T*			
Versioning Decision		No App	Free Only	Paid Only	Freemium
t-1	No App	55	337	87	66
	Free Only	51	2243	11	425
	Paid Only	152	53	1529	442
	Freemium	129	228	458	1275

Table II-7 Parameter Estimates of the Demand Model

Variable	Parameter	$U_{i,t}^{F}$	$U_{i,t}^{P}$
Intercept	$\beta_{F(P),0}$	**-13.16**[a] (0.22)[b]	**-8.41** (0.14)
$G_{F,i,t}$	$\beta_{F(P),1}$	**-12.31** (1.27)	**-6.68** (0.32)
$G_{P,i,t}$	$\beta_{F(P),2}$	-0.69 (0.84)	**-1.64** (0.4)
$log(Q_{F,i,t-1})$	$\phi_{F(P),1}$	**6.57** (0.40)	**0.38** (0.19)
$log(Q_{P,i,t-1})$	$\phi_{F(P),2}$	**1.09** (0.33)	**10.13** (0.29)
$I(C_t = B)$	$\kappa_i^{F(P)}$	**-1.31** (0.58)	**-1.78** (0.35)
$\hat{P}_{i,t}$	$\beta_{P,3}$	**-**	**-3.26** (0.24)
$Other$	$\beta_{F(P),3(4)}$	0.27 (0.22)	**-1.11** (0.27)
$Social$	$\beta_{F(P),4(5)}$	0.09 (0.44)	**-1.31** (0.62)

[a] The bold font indicates that the 95% credible interval does not contain zero.
[b] Standard deviation of each parameter estimate is provided in the parentheses

Table II-8 Parameters Estimates of the Versioning Decision Model

Parameter	Interpretation	Posterior Mean (SD)[a]
$\lambda_{1,F}^{m}$	Monthly in-app purchase RPM ($), free version	**0.312** (0.061)
$\lambda_{1,P}^{m}$	Monthly in-app purchase RPM ($), paid version	**169.231** (3.260)
λ_{2}^{m}	Monthly advertising RPM per ad-network ($)	**0.067** (0.009)
γ_{1}^{F}	Monthly fixed cost, free version	57.796 (53.935)
γ_{1}^{P}	Monthly fixed cost, paid version	**589.847** (136.942)
γ_{2}^{m}	Monthly CPM ($) for expected new downloads	**0.343** (0.093)
σ_F	Heterogeneity SD of fixed cost, free version	**154.755** (80.520)
σ_P	Heterogeneity SD of fixed cost, paid version	**292.083** (163.891)
σ	Scale parameter	**552.122** (1.482)
δ	Discount factor	**0.933** (0.009)

[a] The bold font indicates that the 95% credible interval does not contain zero.

Table II-9 Free and Paid Version Profits in July 2012 Based on Parameter Estimates

Version	1^{st} Quantile	Median	Mean	3^{rd} Quantile	Max.
Free	\$244	\$613	\$1,409	\$1,465	\$21,360
Paid	\$2,545	\$6,212	\$13,160	\$14,960	\$121,400

Table II-10 Payoffs under Uniform Commission Rates on All Revenue Sources

Commission	Platform Revenue	# Apps/Month (Free/Paid)	Total App Profit	Avg. App Profit	Ecosystem Payoff
Current	*7,591,032*	*697 (372/325)*	*20,464,691*	*29,361*	*28,055,722*
10%	-5,047,246	+16 (+2/+14)	+5,077,180	+6,462	+29,935
20%	-2,512,255	+8 (+1/+7)	+2,535,141	+3,263	+3,263
40%	+2,473,335	-10 (-1/-9)	-2,523,537	-3,246	-50,202
50%	+4,859,822	-22 (-2/-20)	-5,027,837	-6,492	-168,016

Payoffs under Differential Commissions on Direct vs. Indirect Revenues

Commissions		Platform	# Apps/Month	All App	Avg. App	Ecosystem
Dir.	Ind.	Revenue	(Free/Paid)	Profit	Profit	Payoff
Current		*7,591,032*	*697 (372/325)*	*20,464,691*	*29,361*	*28,055,722*
50%	10%	-4,815,408	+21 (+8/+13)	+4,824,734	+5,861	+9,326
40%	20%	-2,390,694	+11 (+4/+7)	+2,407,146	+2,944	+16,452
20%	40%	+2,352,115	-12 (-4/-8)	-2,393,993	-2,981	-41,877
10%	50%	+4,630,468	-27 (-8/-19)	-4,770,729	-5,937	-140,261

Table II-12 Payoffs under Rent plus Commissions on All Revenues

Commissions		Platform	# Apps/Month	All App	Avg. App	Ecosystem
Rent	Com	Revenue	(Free/Paid)	Profit	Profit	Payoff
Current		*7,591,032*	*697 (372/325)*	*20,464,691*	*29,361*	*28,055,722*
$10	10%	-4,953,751	+12 (0/+12)	+4,983,070	+6,531	+29,320
$10	20%	-2,419,186	+5 (-1/+6)	+2,441,262	+3,268	+22,076
$50	10%	-4,604,446	0 (-7/+7)	+4,619,684	+6,628	+15,238
$50	20%	-2,078,292	-7 (-8/+1)	+2,079,374	+3,311	+1,082

Table III-1 Key Variables and Summary Statistics

Variable Description	Notation	Mean	SD
Number of Free App Upgrades	$U_{j,t}^{F}$	2.45	3.07
Number of Paid App Upgrades	$U_{j,t}^{P}$	1.44	1.88
Free App Time Since Last Upgrade (months)	$T_{F,i,t}$	6.58	5.90
Paid App Time Since Last Upgrade (months)	$T_{P,i,t}$	8.81	7.09
Free App Monthly Download Volume	$d_{j,t}^{F}$	297,100	606,312
Paid App Monthly Download Volume	$d_{j,t}^{P}$	3,888	8,097
Free App with In-app Purchases (1=yes, 0=no)	$i_{F,j}$	0.373	0.484
Paid App with In-app Purchases (1=yes, 0=no)	$i_{P,j}$	0.191	0.393
Number of Ad-Networks (Free)	a_j	2.49	2.34
Retail Price (Paid, in $)	$P_{j,t}$	3.10	2.77

Table III-2 Model Comparisons

Model		Proposed Model	Benchmark Models		
			(1)	(2)	(3)
			$\lambda_{1,\mathrm{F}} = \lambda_{1,P}$	$\gamma_{1,\mathrm{F}} = \gamma_{1,P}$	$\gamma_{\mathrm{UF}} = \gamma_{UP}$
Gelfand-Dey LMD		-11812.9	-12797.5	-11837.7	-12003.8
Hit Rate		51.7%	31.5%	51.0%	50.9%
Actual Decisions (N)		Predicted Decisions (N)			
$C_{j,t} = N$	600	1119	1262	1035	990
$C_{j,t} = F;\ U_{j,t}^{F} = \mathrm{N}$	1955	1723	1724	1560	1756
$C_{j,t} = F;\ U_{j,t}^{F} = \mathrm{Y}$	623	330	338	378	395
$C_{j,t} = P;\ U_{j,t}^{P} = \mathrm{N}$	1901	1436	1249	1538	1354
$C_{j,t} = P;\ U_{j,t}^{P} = \mathrm{Y}$	184	220	179	214	321
$C_{j,t} = B;\ U_{j,t}^{B} = \mathrm{N}$	1619	1671	1768	1777	1873
$C_{j,t} = B;\ U_{j,t}^{B} = \mathrm{UF}$	110	525	453	545	243
$C_{j,t} = B;\ U_{j,t}^{B} = \mathrm{UP}$	97	195	226	189	313
$C_{j,t} = B;\ U_{j,t}^{B} = \mathrm{UB}$	181	51	71	34	25

Table III-3 Parameter Estimates of the Demand Model

Variables	Parameters for Free Apps ($H^F_{i,j,t}$)	Posterior Mean (SD)	Parameters for Paid Apps ($H^P_{i,j,t}$)	Posterior Mean (SD)
Intercept	$\bar{\beta}^F_0$	**-7.53**[a] (0.048)	$\bar{\beta}^P_0$	**-11.14** (0.082)
$U^{\mathrm{F}}_{\mathrm{j,t}}$	$\bar{\beta}^F_1$	**1.32** (0.071)	$\bar{\beta}^P_1$	**-0.8** (0.069)
$U^{\mathrm{P}}_{\mathrm{j,t}}$	$\bar{\beta}^F_2$	**-1.07** (0.095)	$\bar{\beta}^P_2$	**0.9** (0.094)
$T^F_{j,t}$ or $T^P_{j,t}$	$\bar{\beta}^F_3$	**-2.33** (0.005)	$\bar{\beta}^P_3$	**-2.42** (0.005)
$I(C_t = B)$	$\bar{\kappa}^F$	**-1.09** (0.058)	$\bar{\kappa}^P$	**-0.33** (0.058)
$\hat{P}^*_{j,t}$	-	-	$\bar{\beta}^P_4$	**-1.27** (0.031)

Table III-4 Parameter Estimates of the Upgrading and Versioning Decision Model

Parameter	Interpretation	Posterior Mean (SD)[a]
$\lambda_{1,F}^{m}$	Monthly in-app purchase RPM (\$), free version	2.58[b] (0.47)
$\lambda_{1,P}^{m}$	Monthly in-app purchase RPM (\$), paid version	2,623.44 (33.26)
λ_{2}^{m}	Monthly advertising RPM per ad-network (\$)	2.85 (0.13)
$\gamma_{1,F}$	Monthly fixed cost, free version	6,988.34 (591.63)
$\gamma_{1,P}$	Monthly fixed cost, paid version	14,913.96 (519.83)
γ_{2}^{m}	Monthly CPM (\$) for expected new downloads	13.29 (0.59)
γ_{UF}	Upgrade cost, free version	15,958.37 (1,237.79)
γ_{UP}	Upgrade cost, paid version	45,633.08 (1,191.28)
μ	Scale parameter ($\times 10^3$)	0.06 (0.02)

[a] Posterior standard deviation of each parameter estimate is provided in parentheses

Table III-5 Charging Retail Prices for Upgrades of Paid Apps (Option 1: No Support of Previous Editions)

% Users willing to pay	Platform Revenue	Total App Profit	Ecosystem Payoff	# (Version, Upgrade) / Month								
				(N)	(F,N)	(F,Y)	(P,N)	(P,Y)	(B,N)	(B,F)	(B,P)	(B,B)
Current	$96,307,114	$258,859,076	$355,166,190	88	136	26	113	17	131	41	15	4
1%	-$5,220,770	-$6,180,463	-$11,401,233	-1	3	-1	1	-11	10	1	-1	-1
2%	-$5,153,756	-$5,694,245	-$10,848,001	-3	6	-1	2	-11	6	1	0	0
5%	-$4,923,468	-$5,209,630	-$10,133,098	-2	4	-1	0	-10	10	1	-2	-1
10%	-$4,513,069	-$3,746,541	-$8,259,610	-4	2	1	0	-10	8	5	-2	-1
15%	-$3,738,374	-$5,551,865	-$9,290,239	-3	-1	0	-2	-7	7	4	0	0
20%	-$1,366,786	$451,304	-$915,482	-4	-1	0	0	-1	4	2	0	0
25%	$1,190,151	$6,428,119	$7,618,270	-4	0	1	-6	5	5	-1	1	0

Table III-6 Charging Retail Prices for Upgrades of Paid Apps (Option 2: Support Immediate Previous Edition)

% Users willing to pay	Platform Revenue	Total App Profit	Ecosystem Payoff	# (Version, Upgrade) / Month								
				(N)	(F,N)	(F,Y)	(P,N)	(P,Y)	(B,N)	(B,F)	(B,P)	(B,B)
Current	$96,307,114	$258,859,076	$355,166,190	88	136	26	113	17	131	41	15	4
1%	-$4,478,239	-$5,470,965	-$9,949,203	-2	4	0	-1	-9	10	0	-1	0
2%	-$4,549,914	-$5,842,981	-$10,392,894	-3	3	1	1	-8	9	-1	-1	-1
5%	-$3,910,466	-$4,627,033	-$8,537,499	-2	4	-1	0	-8	4	4	-1	-1
10%	-$3,344,394	-$3,996,320	-$7,340,714	-3	1	-1	0	-7	8	3	0	0
15%	-$1,475,772	-$1,507,417	-$2,983,189	-6	1	2	-2	-2	6	1	0	0
20%	$1,329,948	$4,375,212	$5,705,160	-8	3	1	-3	5	2	0	0	0
25%	$4,650,880	$12,806,852	$17,457,732	-4	1	1	-8	12	-3	2	0	-1

Table III-7 Payoffs by Reducing Upgrade Costs

Upgr. Cost Reduction	Platform Revenue	Total App Profit	Ecosystem Payoff	# (Version, Upgrade) / Month								
				(N)	(F,N)	(F,Y)	(P,N)	(P,Y)	(B,N)	(B,F)	(B,P)	(B,B)
Current	$96,307,114	$258,859,076	$355,166,190	88	136	26	113	17	131	41	15	4
Free: 50%	+$275,274	+$410,224	+$685,498	-11	-5	+13	-6	-1	-8	+19	-1	+1
Free: 75%	+$775,147	+$1,857,745	+$2,632,892	-10	-17	+19	-13	0	-14	+34	-2	+2
Paid: 50%	+$6,549,083	-$2,357,528	+$4,191,555	-14	-12	-5	-27	+36	-15	-4	+33	+8
Paid: 75%	+$12,516,694	+$13,866,192	+$26,382,886	-24	-29	-5	-40	+71	-42	-11	+65	+15

Figures

Figure II-1 Distributions of app ages and cumulative download volumes.

Figure II-1a

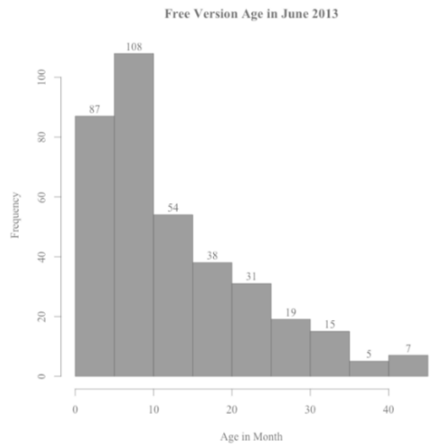


Figure II-1b

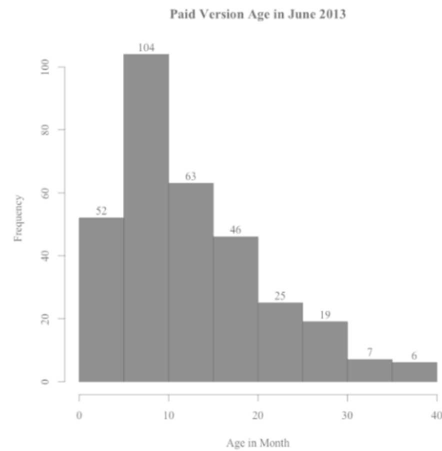


Figure II-1c

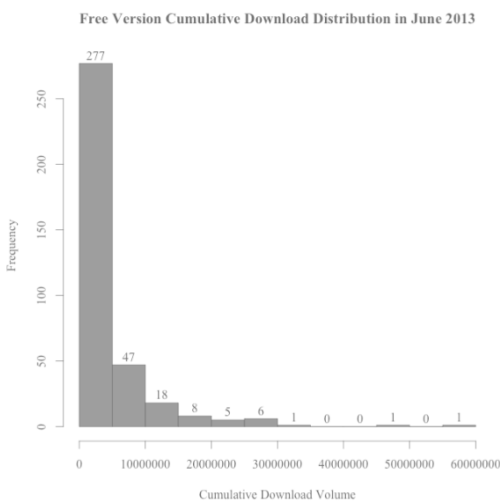


Figure II-1d

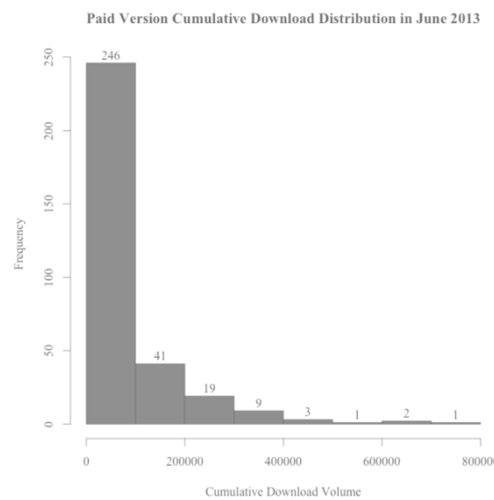


Figure II-2 Timing of the App Publishers' Decision Process

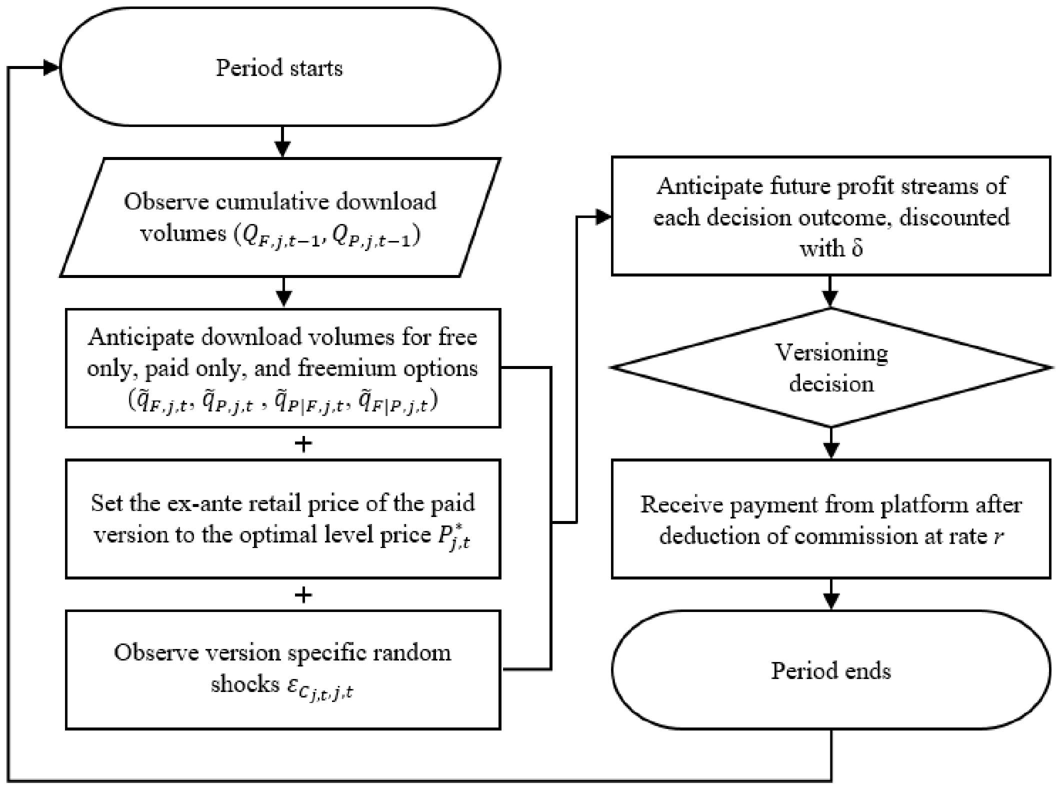


Figure II-3 Cross-Over Effects Captured in the Demand Model via Utility Functions

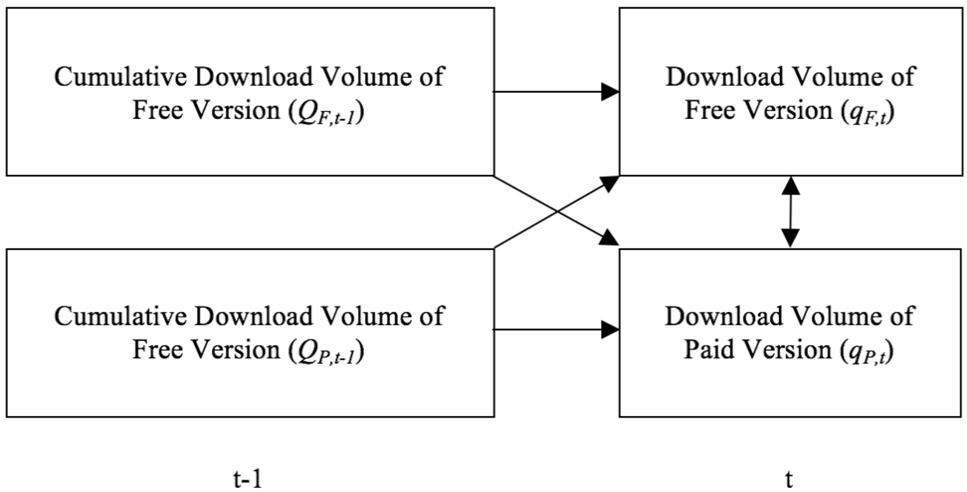


Figure II-4 Distributions of App Profits

Figure II-4a

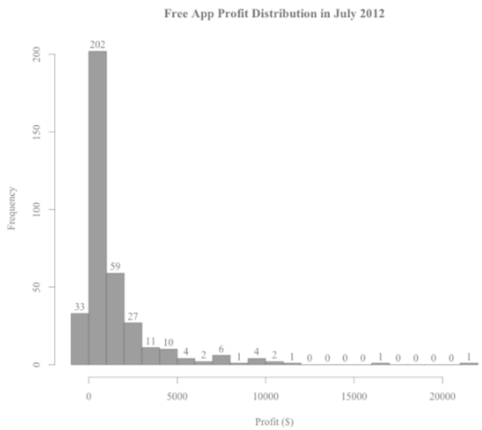


Figure II-4b

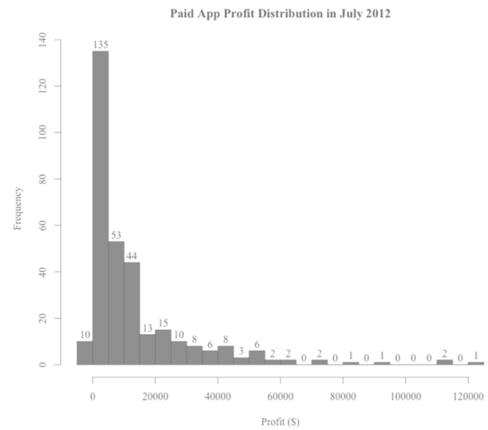


Figure III-1 Distributions of Download Volumes and Number of Upgrades

Figure III-1a

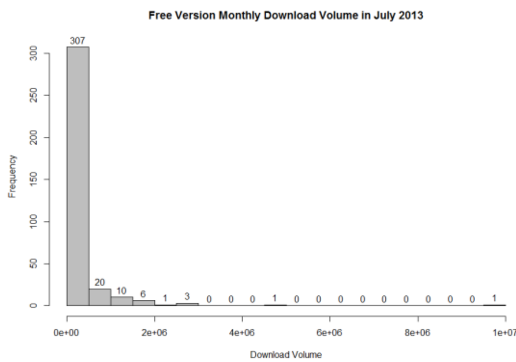


Figure III-1b

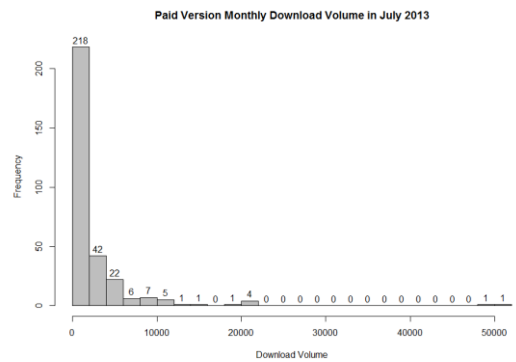


Figure III-1c

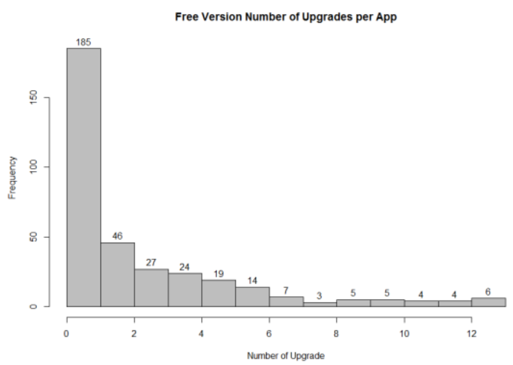


Figure III-1d

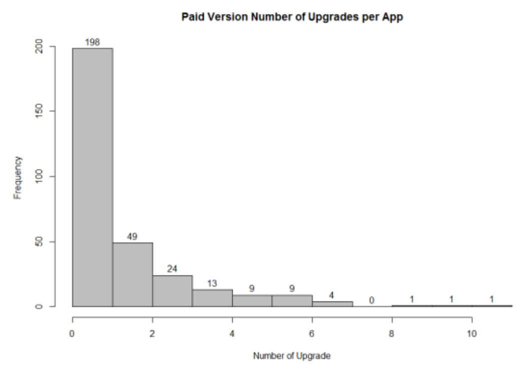


Figure III-2 App Publisher's Versioning and Upgrading Decision Structure

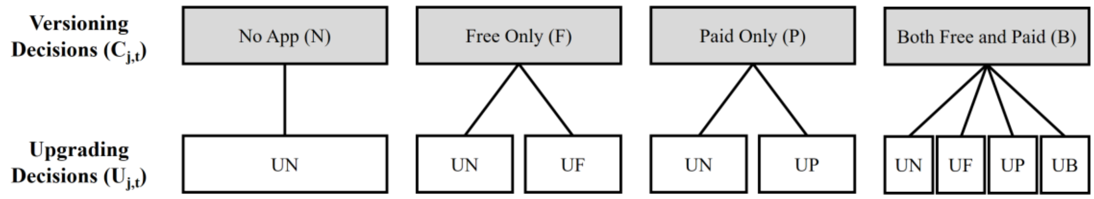
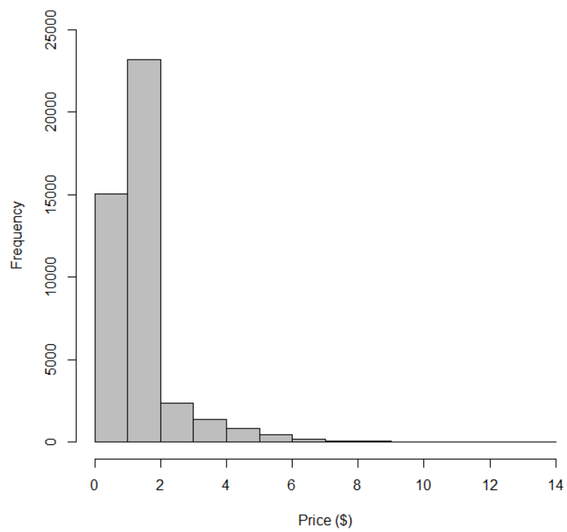


Figure III-3 Optimal Price Distribution



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