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Collision Resolution Algorithms for Networks with Spread-Spectrum Capture Capability

by

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COLLISION RESOLUTION ALGORITHMS FOR NETWORKS WITH SPREAD-SPECTRUM CAPTURE CAPABILITY

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ABSTRACT

In networks which employ spread-spectrum signaling, capture and correct reception of one out of several packets transmitted to a common receiver by contending users is possible. For the case in which all the contending users employ the same spread-spectrum code, the probability of acquiring and retaining capture is evaluated via accurate approximations and tight bounds for direct-sequence, and frequency-hopped spread-spectrum signaling formats. From this probabilistic capture model, a (deterministic) threshold capture model is also derived.

Both these models of capture are then incorporated into the binary tree collision resolution algorithm which takes advantage of the capture capability provided by the spread-spectum signaling. Stable throughputs are evaluated for two types of feedback: (i) feedback with capture (4-ary), in which case the receiver can distinguish between capture and success slots (as well as between idle slots and collision slots as in the ternary feedback non-capture case), and (ii) feedback without capture (ternary) in which case the receiver can not distinguish between capture and success slots.

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I. INTRODUCTION

Previous studies of Collision Resolution Algorithms (CRA) for Random Access communications in a slotted ALOHA type broadcasting network, [1]-[4], make the assumption that whenever two or more packets are transmitted in the same time slot, then neither of them is correctly received at the common receiver. Recently, a number of new results appeared in the literature that departed from the above assumption and considered the possibility of successful reception of packets even when two or more packets are transmitted in the same time slot [5]-[6]. These results are based on the assumption of dominance of group(s) of nodes in the network over other nodes in the network. The concept of dominant group(s) could characterize situations with power capture [7]-[8] (e.g. the users of the dominant group(s) transmit with more power or are closer to the common receiver than the users of a nondominant group(s)).

In this paper we consider Random Access Systems (RAS) which use spread-spectrum modulation techniques and therefore take advantage of the possibility of successful reception of packets, even when two or more packets are transmitted in the same slot. This corresponds to the situation where all transmitters are equally capable of capturing the receiver; this is termed *delay (or code) capture* [10] and is described in section II-B.

In particular, we investigate the basic binary tree CRA of Capetanakis [1], and Tsybakov and Mikhailov [2] (CTMCRA) in a slotted ALOHA type broadcasting network with *code capture* capability. This algorithm is known to be very flexible and insensitive to channel errors [3].

It should be emphasized that our intention is not to make a comparison between conventional RAS's and RAS's with capture capability. Rather, given that our system has a capture capability (i.e., spread-spectrum RAS), the intent is to determine the performance of the system. In section II, we describe the basic model of system, and the types of feedback information available to the nodes of the system. A discussion of capture property of spread-spectrum systems follows. In section III, CTMCRA for feedback with capture is presented. In section IV, CTMCRA for feedback without capture is examined. In section V, the evaluation of maximal stable throughput is discussed. A discussion about the performance of different models, introduced in III-IV, is followed.

II. THE SYSTEM MODEL

A. General assumptions

We consider the random-accessing by many transmitters of a common receiver under the following assumptions :

(i) Transmission of information is in form of packets. The forward channel to the receiver is considered to be time-slotted collision type, AWGN channel, but we assume that the system operates at high signal-to-noise ratio, so that AWGN does not put any severe limitation on the correct reception of packets. This means that the primary source of packet error is the presence of interference due to other users in the same time slot. We assume that each packet to be transmitted fits into one time unit (slot) for transmission. All transmitters are synchronized, in the sense that the reception of each transmission starts at an integer time and ends before the next integer time.

(ii) The feedback channel from the common receiver is a noiseless broadcast channel that informs the transmitters immediately at the end of each slot, of what happened during that slot. The possible events that may occur during each slot are as follows :

a) Idle slot - no user (node) is transmitting during that slot, in which case the slot is idle.

b) Success slot - exactly one node uses the channel, in which case its packet is successfully received.

c) Collision slot - two or more nodes use the channel, but none of the individual transmitted packets can be reconstructed at the receiver. All packets must be retransmitted at some later time.

d) Capture slot - two or more nodes use the channel as in c), but in this case we assume that one of the packets captures the channel and, therefore, is successfully received at the receiver. All other packets involved in this capture slot can not be reconstructed at the receiver and must be retransmitted at some later time. The collection of nodes which must retransmit their packets after a capture slot forms a *capture set* [5].

If any of the above events a), b), c), or d) occurs, the receiver broadcasts the feedback messages LACK, ACK, NACK, or CAPT respectively.

(iii) Propagation delays are negligible, so that the feedback information for slot i can be used to determine who should transmit in slot i+1.

The basic CTMCRA (with no capture capability) is as follows[3] : After a collision, all transmitters involved, flip a binary fair coin, those flipping 0 retransmit in the very next slot, those flipping 1 retransmit in the next slot after the collision (if any) among those flipping 0 has been resolved. No new packets may be transmitted until after the initial collision has been resolved. We consider the channel-access protocol known as *obvious blocked-access* protocol. This means that a transmitter sends a new packet in the first slot following the resolution of all collisions that had occured prior to the arrival of the packet.

We distinguish between two possible feedback information channels [5]. In the first case, the receiver is able to distinguish between success and capture slots, i.e., it is able to detect that while it has received a packet, at least one more packet was simultanously transmitted. In this case the receiver broadcasts CAPT to all nodes whenever a capture slot occurs. We assume that each packet carries the identity number of its transmitter, so that when a capture slot occurs, receiver sends the CAPT feedback message along with the identity number of the transmitter whose packet captured the receiver, and hence correctly received. This assumption clarifies the confusion among nodes in the *capture set* of whose packet was successfully received. This 4-ary feedback is called, feedback with capture (FWC). In fact, in this case the LACK message is redundant, and the receiver need not distinguish between idle and success slots. That is, the receiver can broadcast ACK whenever a slot is either idle or successful.

The second feedback information channel is called, feedback without capture (FWOC). In FWOC, when a capture slot occurs, the receiver is not able to distinguish between capture and successful reception, and hence sends an ACK message, along with the identity number of successfully received packet, whenever a capture or success slot occurs.

Two important performance measures that characterize a CRA (with *blocked -access protocol*) are the average collision resolution interval length, conditioned on the number of packets involved in the initial collision and the maximum attainable throughput. The Collision Resolution Interval (CRI) is defined as the time elapsed from an initial collision (capture) until it is resolved. The maximum attainable throughput is defined as the maximum allowable arrival rate of new packets into the system for which the system is stable.

B. Capture phenomena.

Capture phenomena characterizes the ability of a receiver to successfully receive a packet (with nonzero probability) even though part or all of the packet arrives at the receiver overlapped in time by other packets. The basic mechanism for capture is the ability of the receiver to synchronize with and lock on to one packet and subsequently reject other overlapping packets as noise [9].

Systems which use spread-spectrum modulation techniques may exhibit the *delay capture* phenomenon. When the nodes of a network employ the same spread-spectrum code that does not repeat within a packet duration, then the packets would be strongly correlated over each data symbol if they arrived at a receiver simultanously, but would be pseudo-orthogonal if they arrived with a time offset. In general, there is a vulnerable period at the beginning of a packet, denoted by T_a and called *acquisition interval*, during which collision with the same portion of another packet results in the loss of both [9].

The probability of capture for the *delay capture* model described above, for different spread-spectrum modulation techniques [i.e., direct-sequence (DS/SS), frequency-hopped (FH/SS), and hybrid (DS-FH/SS)] is derived in our work of [11]. Next, we give a brief description of these derivations.

The probability of capture is defined as

$$P_c(n) \equiv Pr(1 \text{ user captured} \mid n \text{ users contend}) \quad ; \quad n \geq 2.$$

$P_c(n)$ can be decomposed into two parts; the probability of acquisition of capture, $P_a(n)$, and the probability of retaining capture, denoted by $P_r(n)$ (i.e., $P_c(n) = P_a(n)\, P_r(n)$). The probability of acquisition for the *delay capture* has been already derived by Gronemeyer and Davis [10]. By introducing a time of arrival randomization procedure, with parameter T_u, to eliminate discrimination as a function of range, $P_a(n)$ is given by:

$$P_a(n) = \begin{cases} 1 & n = 1 \\ (1-Q)^n & n \geq 2 \\ 0 & \mathit{otherwise} \end{cases}$$

where $Q \equiv \dfrac{T_a}{T_u}$, is called the capture ratio [10].

The probability of retaining capture pertains to the rejection of the interfering packets as noise, and therefore depends on the SS modulation scheme and correlation properties of the signature sequences or hopping patterns. We model the signature sequence (or hopping pattern) being used by all users, as a random SS code (e.g., random binary sequence for DS/SS and random hopping pattern for FH/SS) and use techniques similar to those used for the evaluation of error probabilities in the case of multiple-access interference to calculate the probability of retaining capture. For DS/SS or hybrid (FH-DS) SS, the exact calculation of this probability is intracable, so we use appropriate approximations; for FH/SS we obtain a tight upper bound. It is shown in [11], that $P_r(n)$ can be written as

$$P_r(n) = (1 - P_{es}(n))^L \tag{1}$$

$P_{es}(n)$ is the probability of symbol error in the presense of $n-1$ interfering users, and L is the number of symbols per packet. Using the signal-to-noise ratio method of [12], we can approximate $P_{es}(n)$ by

$$P_{es}(n) = Q\,[SNR\,(n)] \tag{2}$$

where $SNR\,(n)$ is the average signal-to-noise ratio at the output of the correlation receiver for a DS/SS system with n users and $Q(.)$ is the complementary error function. It has been found in [11] that for binary coherent DS/SS systems employing random signature sequence and time of arrival randomization,

$$SNR\,(n) = \left[\left(\frac{2E_b}{N_0}\right)^{-1} + \frac{(n-1)m_\psi}{2N}\left\{2 + (1-\frac{1}{N})\Delta_\tau\right\}\right]^{-\frac{1}{2}} \tag{3}$$

where N is the number of code chips per data bit, m_ψ is the waveshaping parameter ($m_\Psi = \frac{1}{3}$ for rectangular chip pulse), E_b is signal energy per data bit, N_0 is the AWGN density, and Δ_τ is defined as the probability of differential delay between two packets being less than the bit duration; i.e.,

$$\Delta_\tau = P(\tau_k - \tau_i \leq T \mid \tau_k - \tau_i > T_a;\ \text{for } all\ k \neq i)$$

In above, we have made the assumption that symbol errors within a packet are independent. This is not true in the DS-SS case, because of strong correlation of symbols, due to the fact that all nodes use the same SS code.

In FH/SS case, we assume that all nodes in the network use the same memoryless hopping pattern and the capture interval, T_a, is equal to the dwell time of the FH system. Now, given that the receiver has acquired capture of the first packet arriving at the front end of the receiver, all the other packets arriving later have differential delays of greater than T_a with the first packet. This means that the probability of a hit from an interferring packet is the same as the case of multi-user interference. Assuming that the number of frequency bins, q, is much greater than 1 ($q >> 1$), the probability of a hit is given by [13]:

$$P_h \approx \frac{1}{q}(1 + \frac{\log_2 M}{N_b})$$

where N_b is the number of data bits per dwell-time. Given that n users simultaneously transmit in a given slot, the probability of a hit from k users ($0 \leq k \leq n-1$) is given by

$$P_h(k) = \binom{n-1}{k} P_h^k (1 - P_h)^{n-1-k} \quad ; \quad 0 \leq k \leq n-1$$

Therefore, the average probability of symbol error is given by

$$P_{es}(n) = \sum_{k=0}^{n-1} P_h(k)\, P_{e\,|\,h}(k) \tag{4}$$

where $P_{e\,|\,h}(k)$ is the conditional probability of error given k full hits from other users have occured. In this case proper interleaving of symbols would achieve the independence of errors. Figures 1.-2. present the probability of capture for DS/SS and FH/SS systems with different parameters.

We incoporate the capture effect in the CTMCRA in two forms. According to the first model, if no more than Γ packets transmit in a given slot, then one packet is captured and correctly received, while the others should be retransmitted. If more than Γ nodes contend, there will be a collision event and all of them must be retransmitted. Γ is called the *capture capacity* of the system. We refer to this model as *deterministic capture model* (DCM). The capture parameter, Γ, can be obtained directly from the probability of capture by imposing a hard constraint (upper bound) on the tolerable probability of no capture.

According to the second model, called *the probabilistic capture model* (PCM), with probability $P_c(n)$, one out of n contending packets is captured and correctly received. $P_c(n)$ is a monotone decreasing function of n. This means that for $n > K_u$, K_u a positive integer obtained from some performance index, the probability of capture falls below an acceptable level, say P_{cu}. For this reason, we carry out the performance evaluation of the CRA under the assumption that no capture occurs when $n > K_u$; i.e., set $P_c(n) = 0$ whenever $P_c(n) < P_{cu}$.

III. CTMCRA FOR FEEDBACK WITH CAPTURE

In feedback with capture (FWC), where the receiver can distinguish between capture and success slots, we assume that whenever a capture event occurs all packets belonging to a capture set must be retransmitted and eventually be correctly received at the common receiver before any other packets may be (re)transmitted.

Two possible actions might be taken by nodes from the capture set that are involved in a capture slot. One possibility is to perform the exact CTMCRA, i.e., flip a coin, etc. We shall call this scheme 1. The other possibility is to transmit during the slot immediately following the capture slot and then continue to perform the CTMCRA. We call this scheme 2 [5].

A. DCM - FWC

Let the random variable X denote the number of packets transmitted in the first slot of a CRI, and let Y denote the length of the same CRI. We define Y_n as the (random) length of time required by the CRA to resolve a contention among n nodes. Then the conditional mean of the CRI length is given by

$$L_n \equiv E(Y_n) = E(Y \mid X = n)$$

It is easy to see that given X, Y depends only on the results of the coin tosses performed internally in the algorithm, and hence independent of the arrival process of new packets into the system.

First, considering scheme 1, it is obvious that $Y_0 = Y_1 = 1$. When $n \geq 2$, we have two possibilities : i) $2 \leq n \leq \Gamma$; or ii) $n > \Gamma \geq 2$.

In the first case (i), a capture occurs in the first slot of CRI, and the remaining n-1 nodes form a capture set and flip their binary coin.

From Figure 3(a), we see that Y_n is given by

$$Y_n = 1 + Y_{S_{n-1}} + Y_{n-S_{n-1}-1} \tag{5a}$$

where S_n is a binomially distributed random variable with PMF

$$P(S_n = i) \equiv P_n(i) = \binom{n}{i} 2^{-n} \quad ; \quad 0 \le i \le n \tag{6}$$

Using (5a) and (6), we find the conditional mean of the CRI as

$$L_n = 1 + 2 \sum_{i=0}^{n-1} L_i \, P_{n-1}(i) \quad ; \quad 2 \le n \le \Gamma \tag{5b}$$

where we have used the fact that $P_n(i) = P_n(n-i)$.

In the second case (ii), the first slot of the CRI is a collision slot (see Figure 3(b)), and the recursion relation for L_n is given by

$$L_n = [1 + 2 \sum_{i=0}^{n-1} L_i P_n(i)] \, / \, [1 - 2 \, P_n(n)] \quad ; \quad n > \Gamma \tag{5c}$$

Equation (5c) is same as the recursion relation given in [3] for basic CTMCRA. The only difference is that for $2 \le i \le \Gamma$, the values of L_i are obtained by the recursion relation of (5b).

Regarding scheme 2, we notice that when $2 \le n \le \Gamma$, the first slot of CRI is a capture slot, and therefore by rule of scheme 2, all the following slots, except the last one, are capture slots. The last slot of the CRI will be a success slot. This gives us

$$L_n = n \qquad \text{for } 2 \le n \le \Gamma. \tag{7}$$

When $n > \Gamma$, recursion relation of (5c) holds. Hence, (7) along with (5c) give us the relations for calculation of L_n for scheme 2.

The first few values of L_n for schemes 1- 2 are given in Table 1., for different values of the capture capacity. As expected, scheme 2 outperforms scheme 1. This follows from the fact that in scheme 1, when $2 \le n \le \Gamma$, there is always a possibility that all the nodes in a capture set flip 1, which leads to a wasted idle slot. On the other hand, in scheme 2, for $2 \leq n \leq \Gamma$, there is no randomness in actions of the nodes in a capture set, i.e., all the nodes in the capture set transmit their packets following a capture slot, and therefore no slot will be wasted.

B. PCM - FWC

According to this model, when $n \geq 2$, with probability $P_c(n)$, one out of n contending packets is captured and correctly received.

Considering scheme 1, given that $n \geq 2$ users contend in the first slot of some CRI, with probability $P_c(n)$, we have the situation in Figure 3(a); while with probability $[1-P_c(n)]$, we have the situation in Figure 3(b). Hence, Y_n is given by

$$Y_n = 1 + P_c(n)\,[Y_{S_{n-1}} + Y_{n-S_{n-1}-1}] + (1 - P_c(n))\,[Y_{S_n} + Y_{n-S_n}].$$

Taking the expectation and simplifying, we obtain L_n as:

$$L_n = \frac{1 + 2\,P_c(n) \sum_{i=0}^{n-1} L_i \; P_{n-1}(i) + 2\,(1 - P_c(n)) \sum_{i=0}^{n-1} L_i \; P_n(i)}{1 - 2\,(1 - P_c(n))\,P_n(n)} \tag{8}$$

where again $L_0 = L_1 = 1$. Equation (8) is the desired recursion relation for the calculation of L_n.

Applying scheme 2 to this model; whenever a capture slot occurs, all the users in the capture set transmit their packets in the very next slot. Therefore,

$$Y_n = P_c(n)\,[1 + Y_{n-1}] + (1 - P_c(n))\,[1 + Y_{S_n} + Y_{n-S_n}] \;\;;\quad n \geq 2.$$

and L_n is given by :

$$L_n = [1+P_c(n)\,L_{n-1} + 2\,(1-P_c(n)) \sum_{i=0}^{n-1} L_i \; P_n(i)] \,/\, [1 - 2\,(1-P_c(n))\,P_n(n)] \tag{9}$$

Here again, for the same reason stated in III-A, scheme 2 outperforms scheme 1.

IV. CTMCRA FOR FEEDBACK WITHOUT CAPTURE

In feedback without capture (FWOC), the receiver cannot distinguish between capture and success slots. Here, the problem is to determine what action should be taken by the nodes from the capture set that transmit a packet and receive an ACK with a different node identity number. Only these nodes from the capture set know that the receiver has captured a packet, while other nodes consider the capture slot as a success one.

We consider two different courses of action, introduced in [5], that can be taken by the nodes from the capture set.

A. DCM - FWOC1 ; "Wait for Partial Conflict Resolution" Scheme

In this scheme, a CRI that corresponds to a given initial collision is divided into several parts. The first part ends when all nodes involved in the initial collision, except those nodes from capture sets that were involved in a capture slot in the first part, have successfully transmitted their packets. Those nodes from capture sets wait until the first part ends, and then retransmit their packets. These packets are called *residual packets.* The second part is dedicated to resolve conflicts (if any) among these nodes. Nodes from the capture sets of the second part (if any) will retransmit their packets in the third part and so on. The initial conflict is finally resolved when an empty part is detected (i.e., a single idle slot). An example of this scheme is given in Figure 4.

Denoting by L_n, the conditional mean of a CRI of conflict multiplicity n, and by $\tilde{L}_n$, the conditional mean of the first part of the same CRI; L_n can be written as

$$L_n = \tilde{L}_n + \sum_{m=0}^{n-1} Q_n(m)\, L_m \qquad ; \quad n \geq 1, \tag{10}$$

with $L_0 = 1$. $Q_n(m)$ is defined as

$$Q_n(m) \equiv Pr(\text{exactly } m \text{ residual packets} \mid n \text{ packets in initial collision}).$$

The conditional mean of the first part of CRI is given by:

$$\tilde{L}_n = 1 \quad ; \quad n \leq \Gamma \tag{11a}$$

$$\tilde{L}_n = [1 + 2\sum_{i=0}^{n-1} \tilde{L}_i P_n(i)] \,/\, [1 - 2P_n(n)] \quad ; \quad n > \Gamma. \tag{11b}$$

Now, depending on the initial number of colliding packets, Γ, and results of actions taken by nodes in the first part, the number of residual packets which must be retransmitted in the second part can be between 0 and $n-1$. It is easy to see that $Q_0(0) = 1$; and for $0 < n \leq \Gamma$;

$$Q_n(m) = \begin{cases} 1 \ ; & m = n-1 \\ 0 \ ; & otherwise \end{cases} \tag{12a}$$

When $n > \Gamma$, there is a collision in the first slot of first part of CRI. With probability $P_n(i)$, i nodes flip 0 and retransmit in the next slot; that is a CRI of i nodes begins. With probability $Q_i(l)$, $0 \leq l \leq i$, the number of residual packets of this CRI is exactly l. Also, with probability $Q_{n-i}(m-l)$, the number of residual packets of CRI for remaining $n-i$ packets is $m-l$. Therefore the probability that the number of residual packets at the end of first part is exactly m is given by

$$Q_n(m) = \sum_{i=0}^{n} \sum_{l=0}^{\min(i,m)} P_n(i) Q_i(l) Q_{n-i}(m-l) \tag{12b}$$

Simplifying the above, we get

$$Q_n(m) = \frac{\sum_{i=1}^{n-1} \sum_{l=0}^{min(i,m)} P_n(i) Q_i(l) Q_{n-i}(m-l)}{1-2P_n(n)} \quad ; \quad 0 \leq m \leq n-1 \tag{12c}$$

It is easy to see that for $n \leq \Gamma$, $L_n = n+1$.

B. DCM - FWOC2 ; "Send in the Next Slot" Scheme.

In this scheme, whenever a node detects that it belongs to a capture set, it retransmit in the next slot. Here, a CRI consists of two parts. The first part evolves according to the rules of the CRA. The second part is dedicated to retransmission of packets that were involved in a capture event during the last slot of the first part. An example of this scheme is given in Figure 5.

Here again, $Q_n(m)$ represents the probability of exactly m residual packets, given that n packets were involved in the initial collision. When $n \leq \Gamma$, the initial conditions of $Q_n(m)$ are same as part A.

For $n > \Gamma$, there is a collision in the first part of the CRI. With probability $P_n(i)$, i nodes flip "zero" and retransmit in the next slot; that is a CRI of i nodes begins. With probability $Q_i(l)$, $0 \leq l \leq i$, the number of residual packets of this CRI is exactly l. Therefore, with probability $Q_i(l)$, the next CRI begins with $n-i+l$ nodes. Hence, with probability $Q_{n-i+l}(m)$, we have m residual packets at the end of the first part of CRI.

It is important to observe that the number of residual packets is less than or equal to $\Gamma-1$. This is due to the fact that the residual packets are those packets that were involved in a capture slot in the last time slot of preceding CRI part.

From the above observation, we can obtain a recursion formula for residual probabilities. This can be written as

$$Q_n(m) = \sum_{i=0}^{n} \sum_{l=0}^{i} P_n(i) Q_i(l) Q_{n-i+l}(m) \quad ; \quad 0 \leq m \leq \Gamma-1, \tag{13a}$$

and $Q_n(m)=0$ for $m \geq \Gamma$. Using the above relation and initial conditions, we can simplify the recursion to get

$$Q_n(0) = \frac{Q_{n-1}(0)\sum_{i=1}^{\Gamma} P_n(i) + \sum_{i=\Gamma+1}^{n-1} \sum_{l=0}^{\Gamma-1} P_n(i) Q_i(l) Q_{n-i+l}(0) + P_n(n) Q_n(1)}{1-2P_n(n)} \tag{13b}$$

$$Q_n(m) = \frac{Q_{n-1}(m)\sum_{i=1}^{\Gamma} P_n(i) + \sum_{i=\Gamma+1}^{n} \sum_{l=0}^{\Gamma-1} P_n(i) Q_i(l) Q_{n-i+l}(m)}{1-P_n(n)} \quad ; \quad 1 \le m \le \Gamma-1 \tag{13c}$$

It is easy to see that for $i<j$, $Q_n(i)$ depends on $Q_n(j)$, and therefore, for calculation of these probabilities, we start at $m=\Gamma-1$ and move down to $m=0$.

Once we obtained the probabilities of residual packets, we can proceed to calculation of conditional mean of CRI length. It is easy to see that (10) still holds, although $\tilde{L}_n$ has a different formulation. Since $L_n = n+1$ for $n \le \Gamma$, and $Q_n(m)$, $0 \le m \le \Gamma-1$, is the probability mass function of the number of residual packets, we have

$$L_n = 1 + \tilde{L}_n + \sum_{m=1}^{\Gamma-1} m \; Q_n(m) \quad ; \quad n \ge 1. \tag{14}$$

Furthermore, $\tilde{L}_n = 1$ for $n \le \Gamma$; and for $n > \Gamma$:

$$\tilde{L}_n = \sum_{i=0}^{n} [1 + \tilde{L}_i + \sum_{m=0}^{i} Q_i(m) \tilde{L}_{n-i+m}] P_n(i) \quad ; \quad n > \Gamma. \tag{15a}$$

Simplifying the above, we obtain the recursion relation for $\tilde{L}_n$:

$$\tilde{L}_n = [1 + \sum_{i=1}^{n-1} P_n(i) [\tilde{L}_i + \sum_{m=0}^{min(i,\Gamma-1)} Q_i(m) \tilde{L}_{n-i+m}] + 2\, P_n(n)] \,/\, [1 - 2\, P_n(n)] \tag{15b}$$

C. PCM - FWOC1 ; "Wait for Partial Conflict Resolution" Scheme

In PCM, with probability $P_c(n)$, one out of n contending packets is captured. Following the rules of this scheme, we observe that for the first part of the CRI, we have:

$$\tilde{L}_n = P_c(n) + (1 - P_c(n)) \sum_{i=0}^{n} (1 + \tilde{L}_i + \tilde{L}_{n-i}) P_n(i) \tag{16a}$$

where $\tilde{L}_0 = \tilde{L}_1 = 1$. After simplification, we have

$$\tilde{L}_n = \frac{1+2(1-P_c(n))\sum_{i=0}^{n-1}\tilde{L}_i P_n(i)}{1-2(1-P_c(n))P_n(n)} \quad ; \quad n \geq 2. \tag{16b}$$

To calculate the probability of residual packets from the first part, we know the following initial conditions:

$$Q_0(0) = Q_1(0) = 1 \tag{17a}$$

and given that a capture has occurred

$$Q_n(m \mid c) = \begin{cases} 1 \; ; & m = n-1 \\ 0 \; ; & otherwise \end{cases} \tag{17b}$$

Therefore, for $n \geq 2$ and $0 \leq m \leq n-1$:

$$Q_n(m) = P_c(n)\,\delta_{m,n-1} + (1-P_c(n))\sum_{i=0}^{n}\sum_{l=0}^{min(i,m)} P_n(i)Q_i(l)Q_{n-i}(m-l) \tag{17c}$$

where $\delta_{i,j}$ is the Kronecker delta defined to be 1 if $i = j$, and 0 if $i \neq j$. Simplifying the above recursion equation gives the desired relation for the computation of residual probabilities:

$$Q_n(m) = \frac{P(n)\,\delta_{m,n-1} + (1-P(n))\sum_{i=1}^{n-1}\sum_{l=0}^{min(i,m)} P_n(i)Q_i(l)Q_{n-i}(m-l)}{1-2\,(1-P(n))\,P_n(n)} \tag{17d}$$

Finally, the conditional mean of the CRI length is obtained using (10).

D. PCM - FWOC2 ; "Send in the Next Slot" Scheme

Extending the results of parts B and C, we can easilty see that

$$Q_n(m) = P_c(n)\delta_{m,n-1} + (1-P_c(n))\sum_{i=0}^{n} P_n(i)\sum_{l=0}^{i} Q_i(l)Q_{n-i+l}(m) \quad ; \quad n \geq 2 \tag{18}$$

where $Q_0(0)=Q_1(0)=1$, and $Q_n(m)=0$ for $m > n$.

We can also write the recursion equation for the conditional mean of the first part of CRI as

$$\tilde{L}_n = P_c(n) + (1 - P_c(n)) \sum_{i=0}^{n} \{1 + \tilde{L}_i + \sum_{m=0}^{i} Q_i(m) \tilde{L}_{n-i+m}\} P_n(i) \quad ; \quad n \geq 2 \qquad (19)$$

where $\tilde{L}_0 = \tilde{L}_1 = 1$. Again, the overall conditional mean of CRI length is given by (10).

V. MAXIMAL STABLE THROUGHPUT FOR POISSON PACKET ARRIVALS

Let the packet generation process be a stationary Poisson point process with parameter λ. Denoting by Y_i, the length of the i-th CRI, it can be easily shown that $\{Y_i\}$ is a Markov chain. The condition for the ergodicity of this chain leads to obtaining a linear upper bound on L_n [3], [14]; i.e.,

$$L_n \leq \alpha_u . n + \beta \tag{20}$$

where α_u and β are constants depending on the parameters of RAS (e.g., capture parameters, bias of binary coins, etc.). The stability of CRA is then gauranteed whenever $\lambda < 1 / \alpha_u \equiv \lambda_u^*$. This means that the asymptotic values of CRI length and number of packets waiting for (re)transmission remains finite as long as the new packet process rate, λ, is less than λ_u^*.

It is tempting to write α_u as

$$\alpha_u = \lim_{n \to \infty} \frac{L_n}{n}$$

Although $\frac{L_n}{n}$ is bounded as n approaches ∞, it is known that it has a fluctuating (periodic) component [15], and the above limit does not exist (for a proof see [16]). Figures 6.-10. present this behavior of L_n, as we have superimposed the plot of $L_n - L_{n-1}$ over plot of $\frac{L_n}{n}$. As can be seen, this behavior varies for different schemes considered. It is observed that as the capture capacity of algorithm increases, the distinction between $\frac{L_n}{n}$ and $L_n - L_{n-1}$ becomes more prominent and influences the tightness of linear bound on L_n.

We may therefore define α_u as

$$\alpha_u = \limsup_{n \to \infty} \frac{L_n}{n} \tag{21}$$

The analytical derivation of α_u seems to be intracable, so once we evaluate L_n, we use numerical computation to find the "best" upper bound on α_u and therefore the maximal stable throughput of system. In particular, for a given integer M, we search for the smallest constant α_{uM} such that

$$L_n \leq \alpha_{uM} \ n - 1 \quad ; \quad \text{for } all \quad n \geq M \tag{22}$$

As in [3], we have let $\beta = -1$.

Table 2. gives the maximal stable throughput for DCM and the two different models of feedback. In the case of FWC, scheme 2 always performs better than scheme 1; but, in case of FWOC, for smaller number of contending nodes, scheme 1 performs better than scheme 2, while scheme 2 outperforms scheme 1 for larger number of contending nodes. This behavior can be observed from the inspection of Table 3. For example, for $\Gamma = 4$, scheme 1 is a better option up to 329 contending nodes. As can be seen from Table 3., the increase in capture capacity of the system results in superior performance of the "partial resolution" scheme for larger number of contending users. The reason is that in the "send in the next slot" scheme, for larger values of capture capacity, after each capture slot, it is more likely to have a collision slot which in turn prolongs the CRI. Examination of Tables 4.-5. reveals the fact that as the capture capacity of the system increases, the performance of the two schemes approachs to a same limit. For example, in PCM-FWOC1-2 with 16-ary SFH system and $P_{cu} = 10^{-2}$ (i.e., $K_u = 39$), the performance of the two schemes are practically the same.

The FWOC schemes are more realistic in the sense that the receiver need not distinguish between success and capture slot, however, they need to distinguish between success and idle slot for the algorithms to function properly (i.e., for transmitters to detect the end of a CRI).

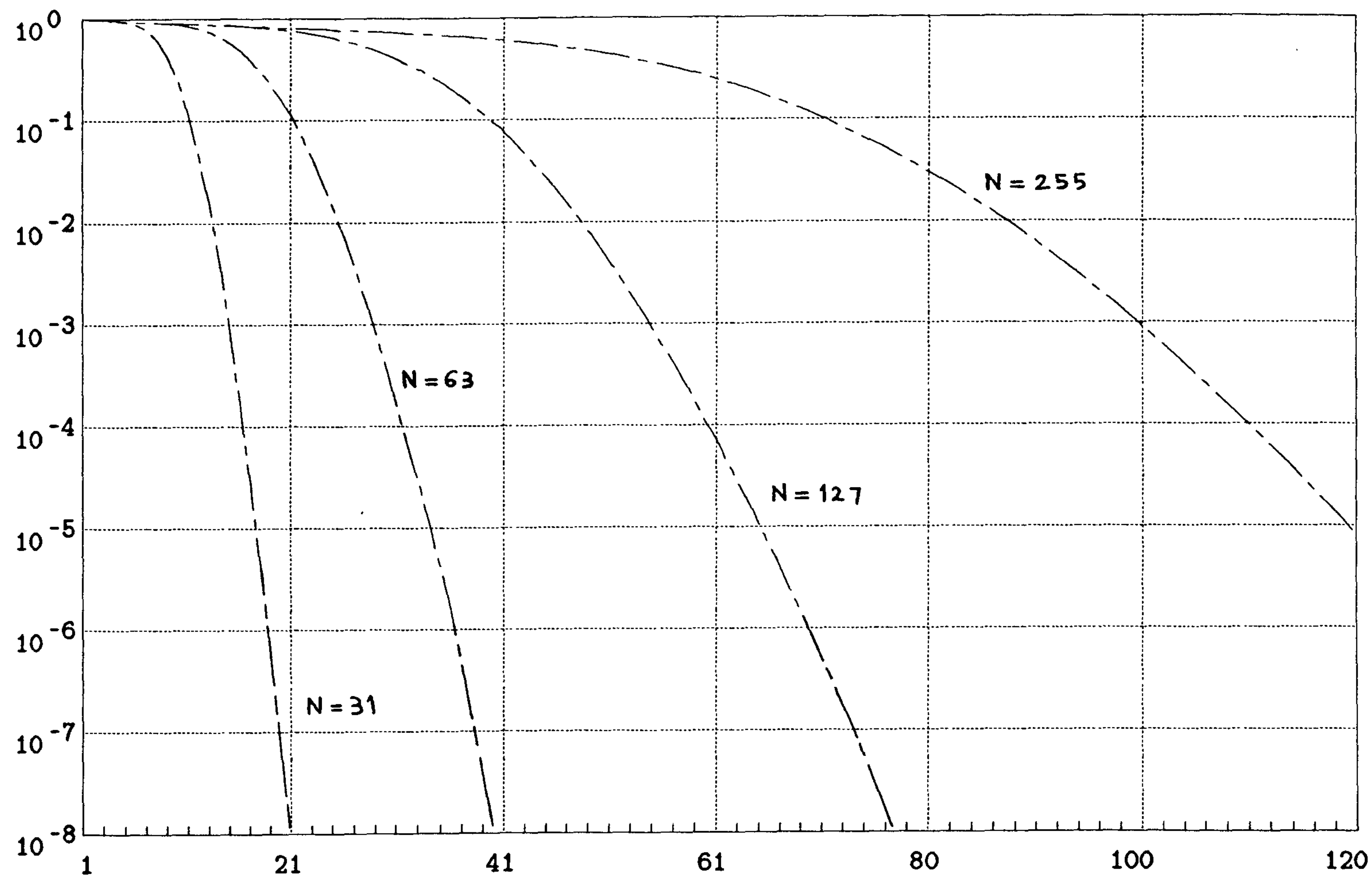

Figure 1. Probability of Capture vs. number of contending nodes for BPSK DS/SS.
($\frac{E_b}{N_0} = 16$ dB, L = 1000 bits/packet, $Q = 0.01, m_\psi = \frac{1}{3}$)

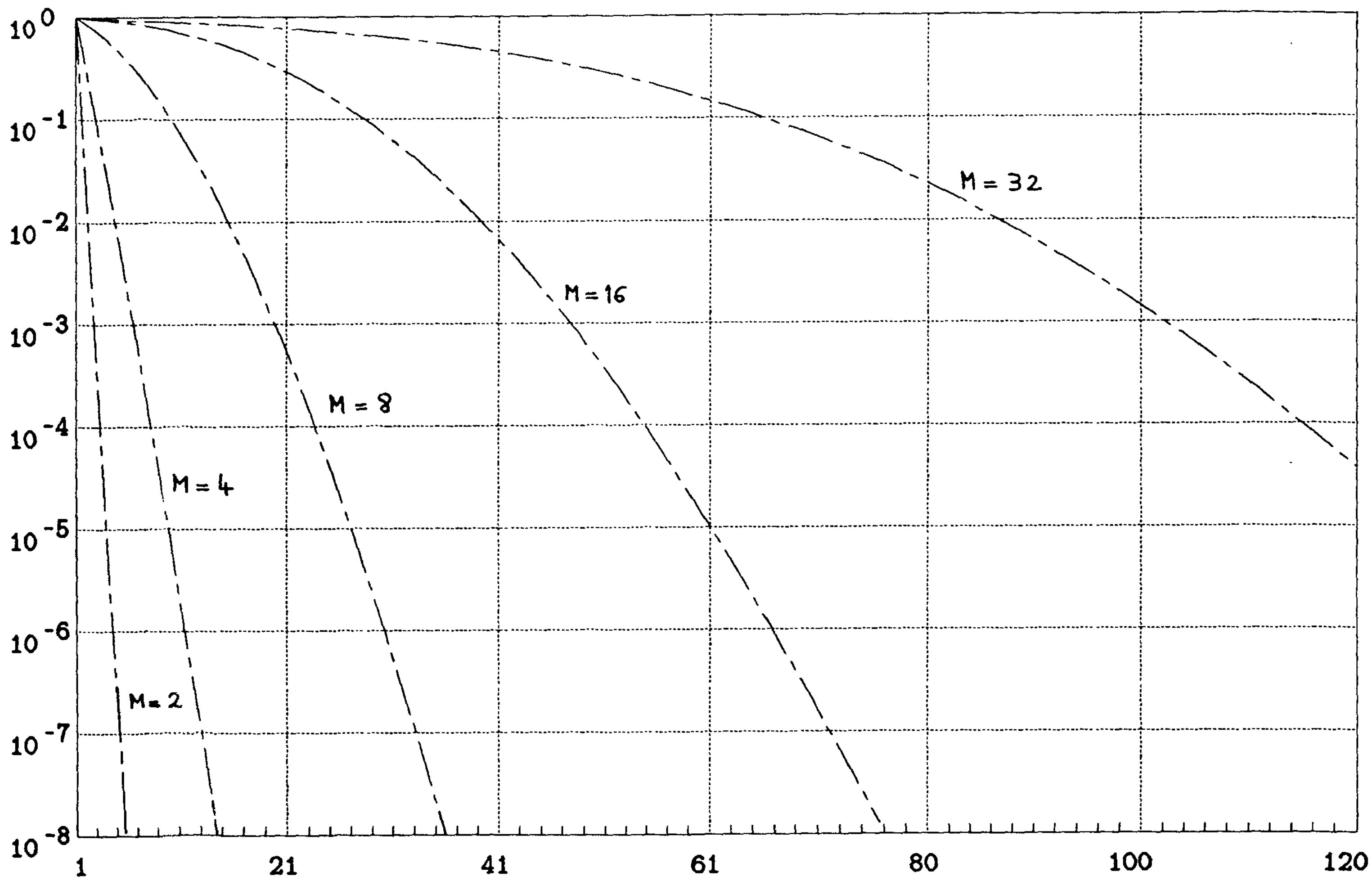

Figure 2. Probability of Capture vs. number of contending nodes for MFSK SFH/SS.
$\left(\frac{E_b}{N_0} = 16 \text{ dB}, \text{L} = \frac{1000}{\log_2 M}, \; Q = 0.01 \right)$

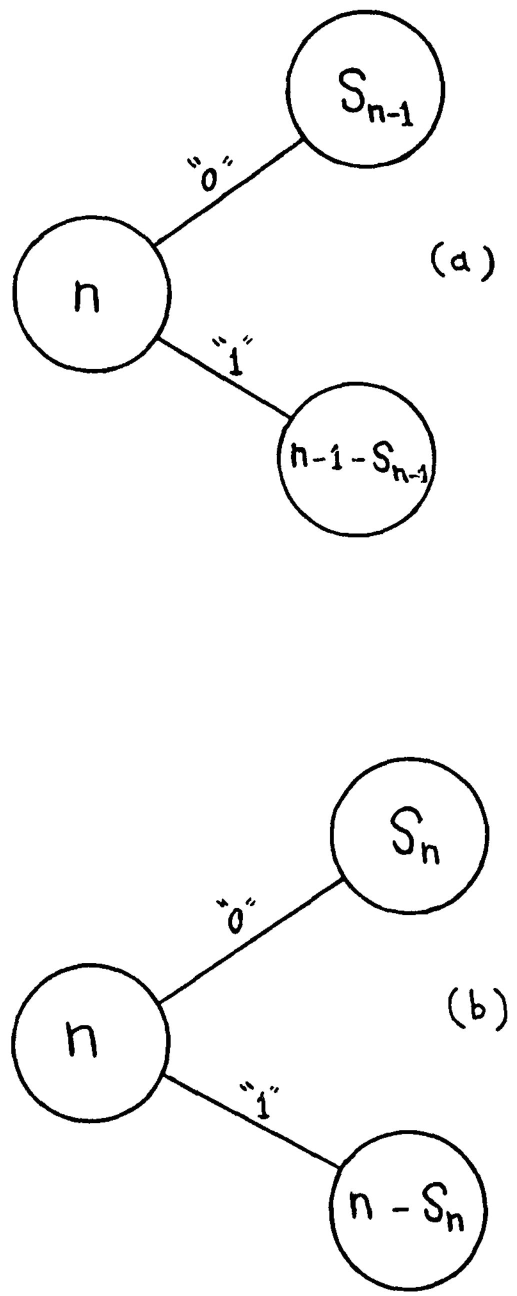

Fig. 3: General action of the CTMCRA in first slot of CRI,

(a) $2 \leq n \leq \Gamma$, (b) $n > \Gamma \geq 2$.

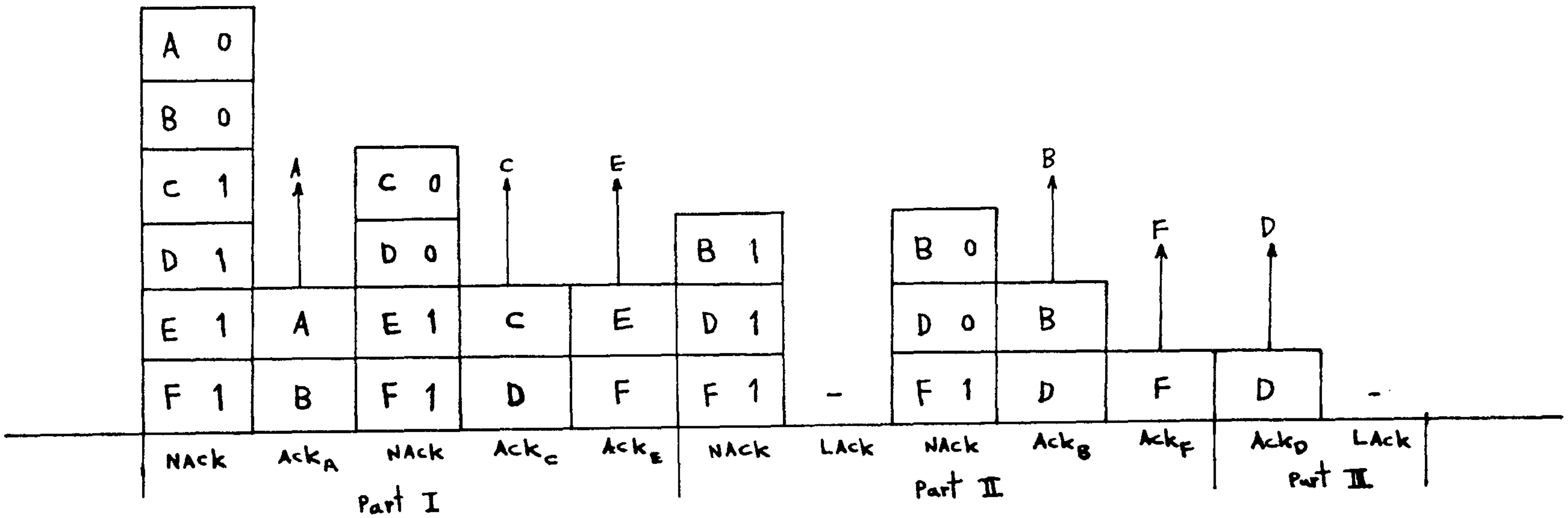

Fig. 4: Example of DCM-FWOC1 ; $\Gamma = 2$.

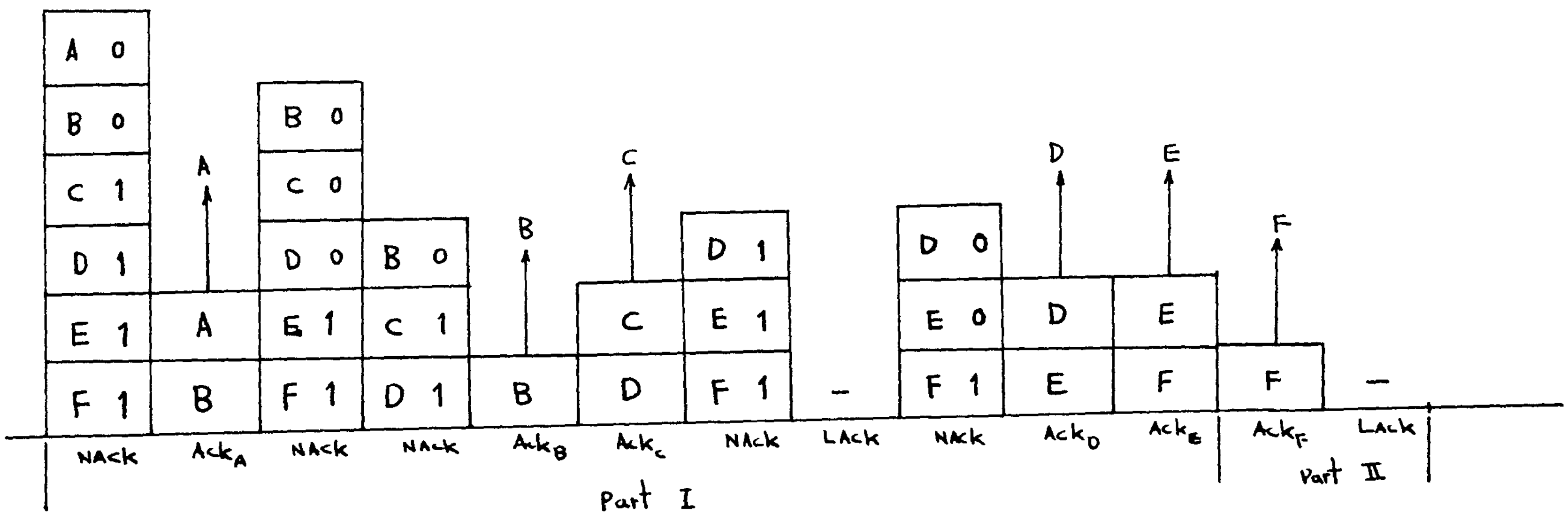

Fig. 5: Example of DCM-FWOC2 ; $\Gamma = 2$.

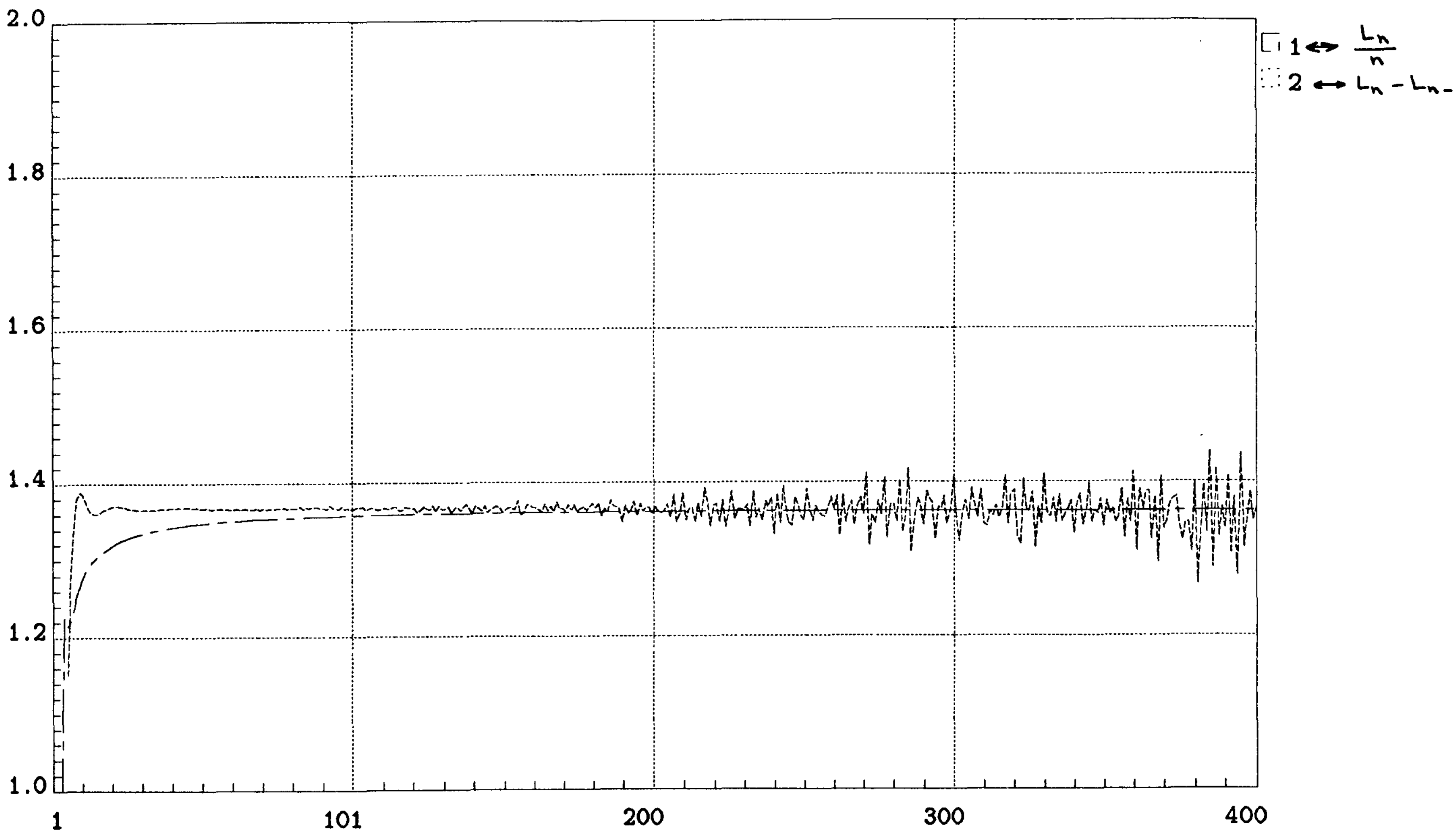

Figure 6. The fluctuation behavior of L_n ; DCM-FWC, scheme 2, $\Gamma = 4$.

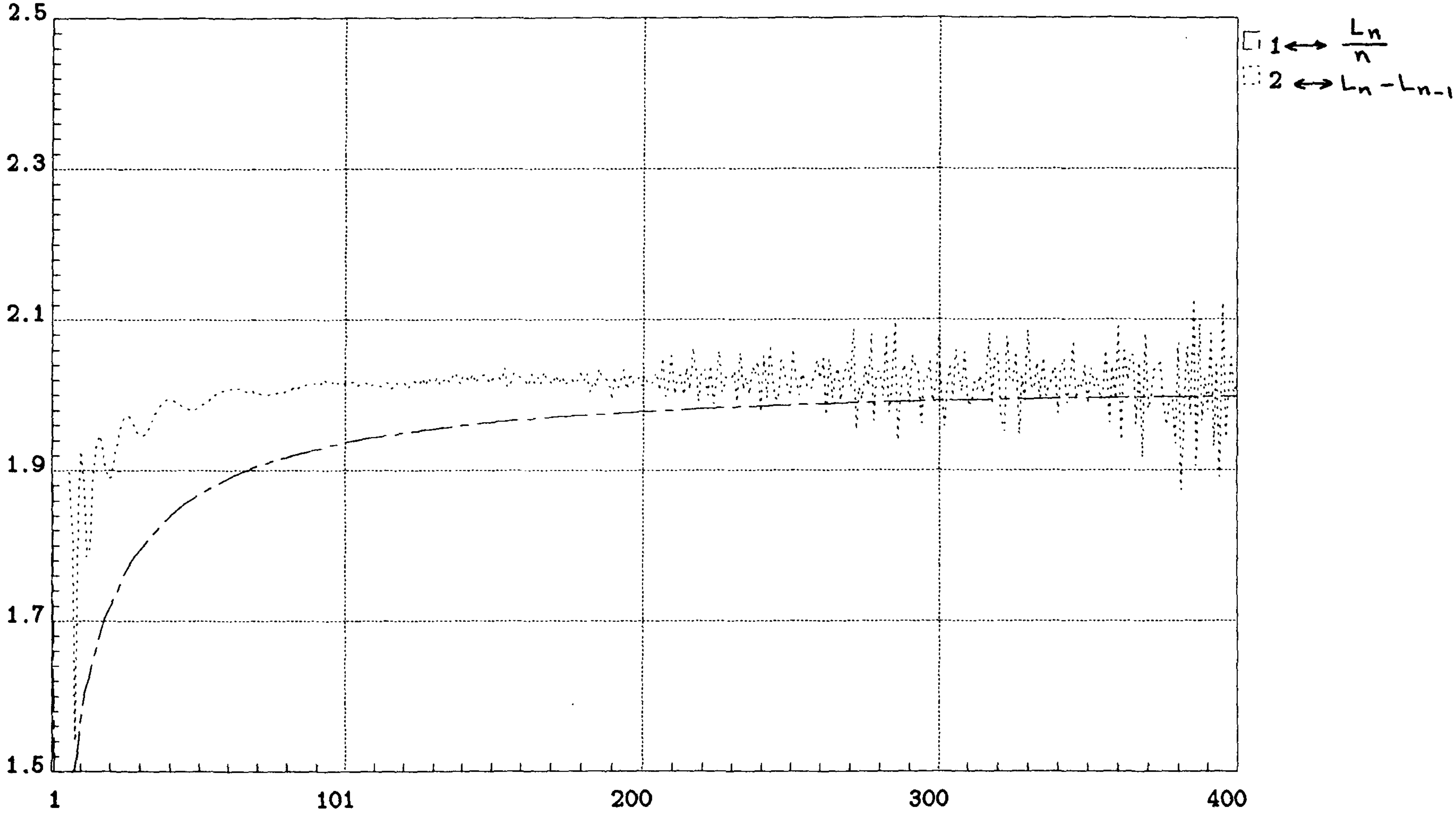

Figure 7. The fluctuation behavior of L_n ; DCM-FWOC1, $\Gamma = 4$.

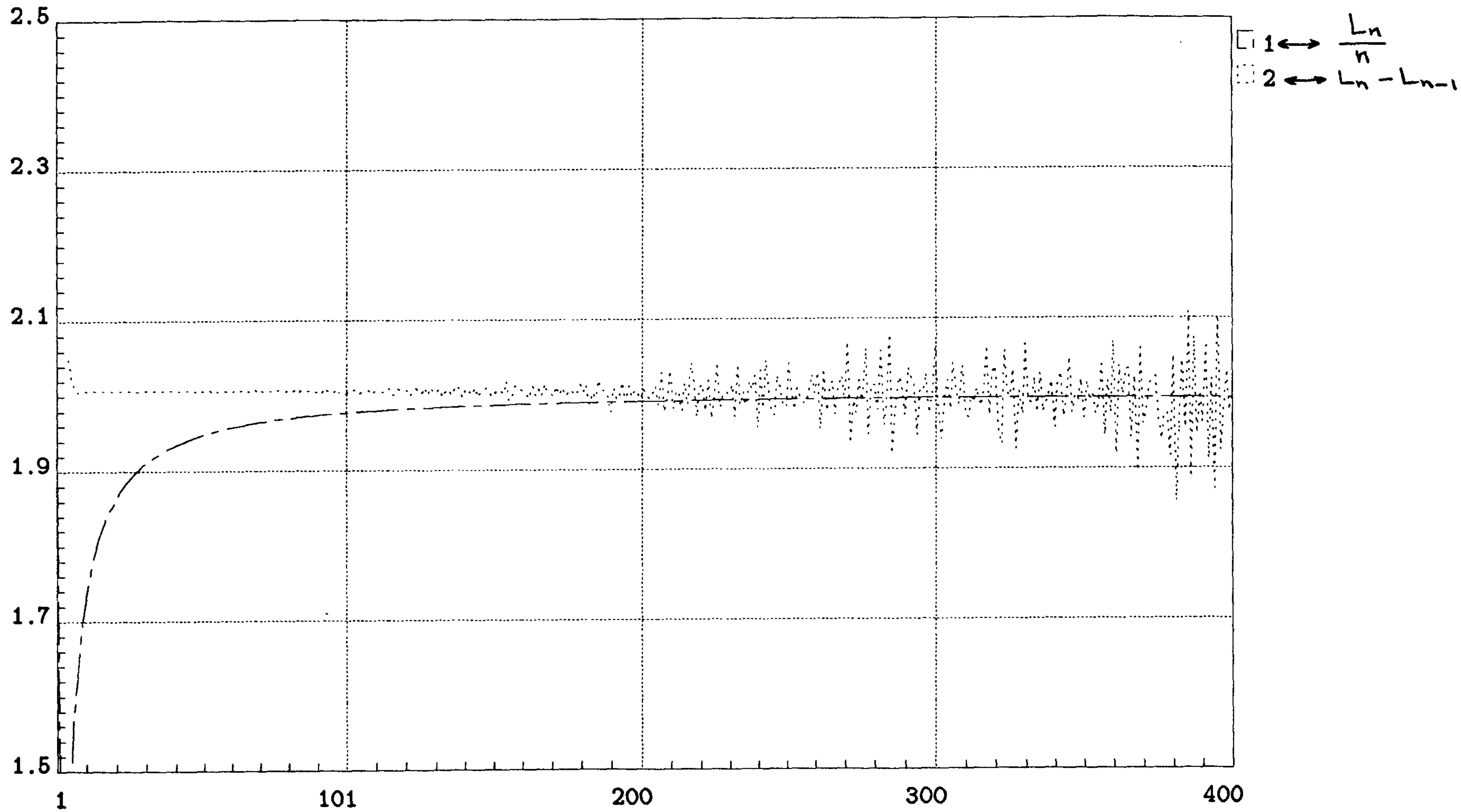

Figure 8. The fluctuation behavior of L_n ; DCM-FWOC2, $\Gamma = 4$.

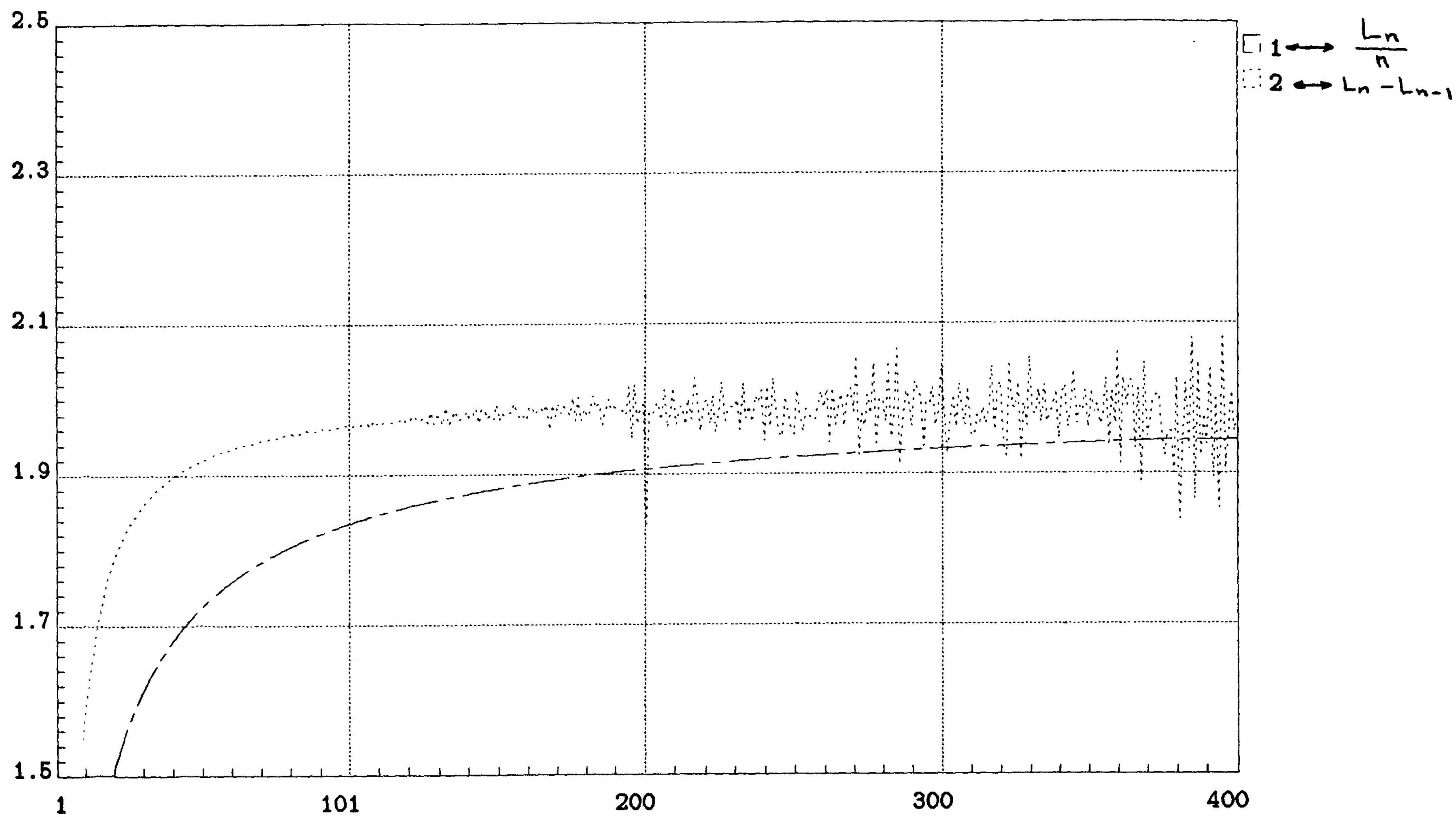

Figure 9. The fluctuation behavior of L_n; PCM-FWOC1, DS/SS with $N = 31$, $K_u = 12$.

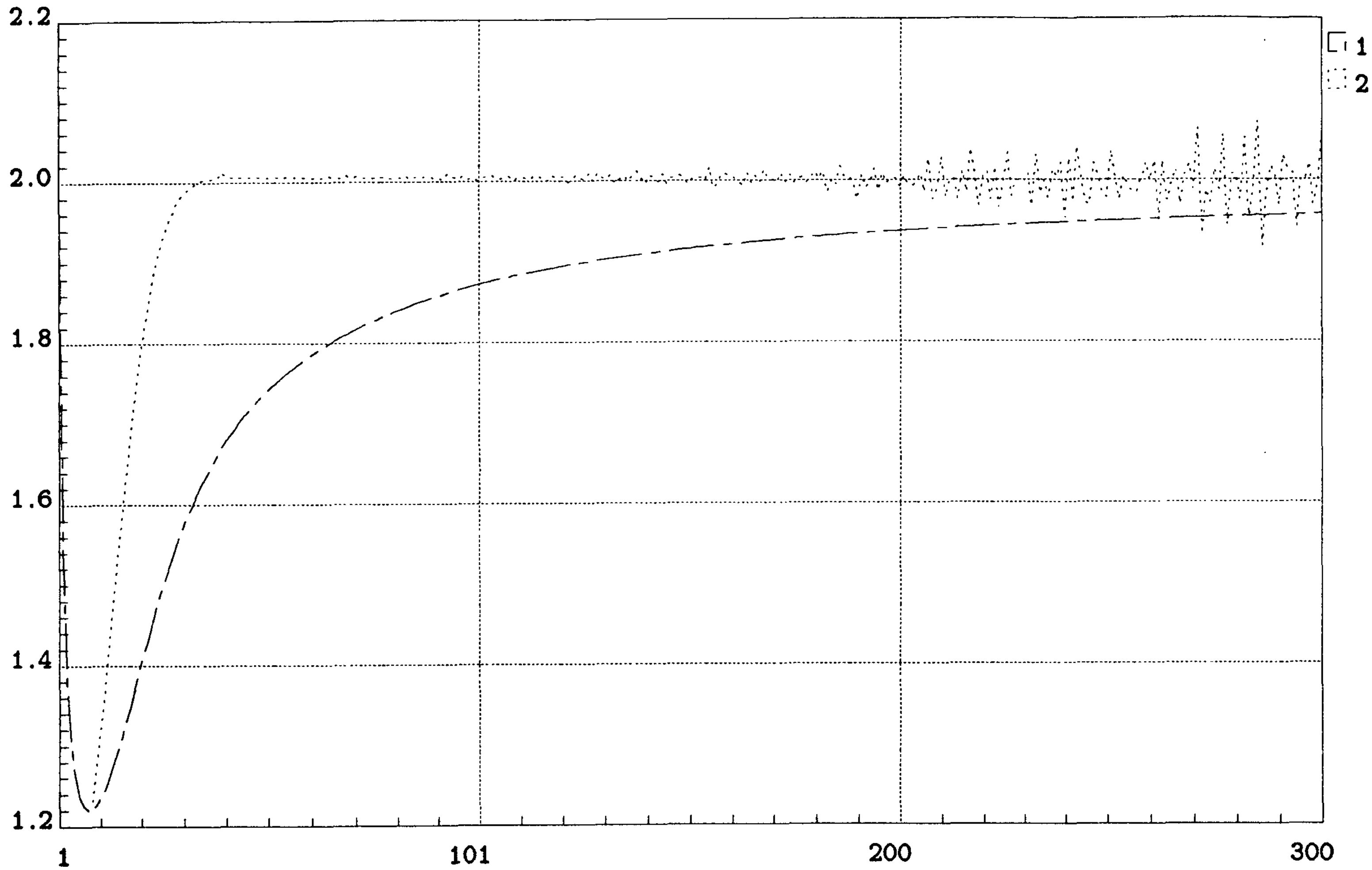

Figure 10. The fluctuation behavior of L_n ;
PCM-FWOC2, SFH/SS with $q = 100$, $M = 16$, $K_u = 39$.

Table 1. Expected CRI length for different values of capture capacity, (DCM-FWC ; $L_n^i \equiv L_n$ for scheme i)

$\Gamma = 2$		
n	L_n^1	L_n^2
2	3.0000	2.0000
3	5.6667	4.6667
4	7.6667	6.2381
5	9.8000	7.9905
6	11.9720	9.8015
7	14.1472	11.6203
8	16.3165	13.4321
9	18.4808	15.2367
10	20.6427	17.0375
$\Gamma = 3$		
n	L_n^1	L_n^2
2	3.0000	2.0000
3	4.0000	3.0000
4	6.7143	5.2857
5	8.3714	6.5619
6	10.1595	7.9889
7	12.0145	9.4877
8	13.8928	11.0083
9	15.7716	12.5275
10	17.6433	14.0381
$\Gamma = 4$		
n	L_n^1	L_n^2
2	3.0000	2.0000
3	4.0000	3.0000
4	5.2500	4.0000
5	7.8833	6.1333
6	9.3565	7.2839
7	10.9491	8.5522
8	12.6263	9.8962
9	14.3472	11.2768
10	16.0812	12.6665

Table 2. Values of the coefficient α_{uM}; upper bound on maximal stable throughput.

DCM-FWC1			
Γ	M	α_{uM}	λ_u^*
2	5	2.1646	0.4620
3	6	1.8648	0.5363
4	7	1.7158	0.5830

DCM-FWC2			
Γ	M	α_{uM}	λ_u^*
2	5	1.8042	0.5543
3	6	1.5064	0.6638
4	7	1.3695	0.7301

Table 4. Values of the coefficient α_{uM};
upper bound on maximal stable throughput.

DCM-FWOC1			
Γ	M	α_{uM}	λ_u^*
2	5	2.2468	0.4450
3	6	2.0750	0.4820
4	7	2.0202	0.4950

DCM-FWOC2			
Γ	M	α_{uM}	λ_u^*
2	5	2.0885	0.4788
3	6	2.0162	0.4959
4	7	1.9994	0.5001

Table 5. Values of the coefficient α_{uM}; upper bound on maximal stable throughput.

PCM-FWC1 (**DS/SS**, $K_u = 12$)		
M	α_{uM}	λ_u^*
13	1.5185	0.6585

PCM-FWC2 (**DS/SS**, $K_u = 12$)		
M	α_{uM}	λ_u^*
13	1.2383	0.8076

PCM-FOWC1 (**DS/SS**, $K_u = 12$)		
M	α_{uM}	λ_u^*
13	2.0271	0.4932

PCM-FOWC2 (**DS/SS**, $K_u = 12$)		
M	α_{uM}	λ_u^*
13	2.0234	0.4942

PCM-FOWC1 (**SFH/SS**, $K_u = 39$)		
M	α_{uM}	λ_u^*
40	1.9962	0.5009

PCM-FOWC2 (**SFH/SS**, $K_u = 39$)		
M	α_{uM}	λ_u^*
40	1.9962	0.5009

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