ABSTRACT

Title of Thesis: **A METHOD FOR INTEGRATING MULTI-REGION FLEXIBLE-ROUTE BUS SERVICES**

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A flexible route bus system, serving an area and providing pick-up and drop-off at the users' doorsteps may be more effective than a conventional bus system in areas with relatively low demand density. Thus, it may reduce the cost per trip and avoid the time and cost to users of accessing bus stops. However, depending on the demand density and circuity of routes, passengers might experience longer travel times and longer wait times. The objective of the thesis is to combine individual many-to-one flexible route services operating in multiple regions, which serve both internal and external demand, and optimize them into an integrated flexible route bus service that offers complete many-to-many transit services for large urban and suburban regions. A total cost function is formulated, and an optimized headway is found for the system. The sensitivity analyses evaluate the influence of input variables on the headway and system effectiveness measures.

INTEGRATION OF COORDINATED MULTI-REGION FLEXIBLE ROUTE BUS SERVICES

By

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LIST OF VARIABLES

Notation	Description	Baseline Input Value	Units
$InteA_i$	Area of region i	-	mi^2
C	Vehicle operating cost	75	$/bus hr
C_i^s	Supplier vehicle cost for coordinated bus transfers in region i	-	$/hr
C_i^t	Total cost function for coordinated bus transfer for region i	-	$/hr
C_{sys}^t	Total cost function for the flexible bus route system	-	$/hr
C_i^u	User in-vehicle cost for region i	-	$/hr
C_i^w	User wait cost for region i	-	$/hr
$C_i^{f,j}$	User transfer cost from region i to j	-	$/hr
d	Delay (per stop)	0.0008	hrs
D_i	Total delay at stops in region i	-	hrs
g	Average group size per stop	1.2	-
h	Headway	-	hrs
J_i	Line haul distance for region i	10	miles
k	Stein constant	1.15	-
L_i	Roundtrip tour length of bus in region i	-	miles
L_i^p	Roundtrip tour length of passenger in region i	-	miles
n_i	Stops per bus tour in region i	-	-
N_i	Fleet size for region i	-	-
P_i	Population of region i	-	-

q_i	Trips generated per unit area (in both directions)	-	trips/mi^2hr
Q_i	Total demand/hour (both directions combined)	-	passengers/hr
Q_i^{in}	Internal demand for region i	-	Passengers/hr
Q_i^t	Total demand/bus round trip	-	passengers/trip
R_i^b	Round trip time of bus in region i (including delay at each stop)	-	hrs
R_i^p	Round trip time of passenger for region i (including delay at each stop)	-	hrs
T_{ij}	Demand from region i to j; denotes O-D pair in the O-D matrix	-	Passengers/hour
u	User wait cost	15	$/pass hr
V	Average speed within regions	25	miles/hr
v	In-vehicle cost	10	$/hr
W	Speed on line haul distance, W= yV	50	miles/hr
y	Non-stop ratio = non-stop speed/ local speed	2	-

1. INTRODUCTION

1.1 Motivation

In rural or suburban areas, where either no public transport services exist or are limited to servicing select areas for limited times due to low demand, flexible route bus transit systems can be a cost-effective option for providing transit services. They provide the convenience of pick-ups and drop-offs at the passengers' doorsteps. Flexible bus route service can be helpful in connecting low density areas to central transit hubs and be a low-cost solution towards improving public transportation and its reliability. This study examines how a system that integrates multiple many-to-one flexible route services can be used to effectively serve large urban areas with many-to-many demand patterns. A central terminal serves here as a transfer terminal for buses serving different regions. This terminal may be located, for example, in the central business district (CBD) of an urban area or at a major ground transportation terminal.

Each region in the network is at some distance from the terminal which is called the line haul distance (Kim and Schonfeld, 2014)(1). The buses start from the terminal, travel the line haul distance, service the route and travel the same line haul distance back to the terminal on each tour. The buses serve three types of demand, namely, intra-region, inter-region and terminal bound demand. Based on the demand type, the bus tour length and the round-trip time can vary for each type of demand.

1.2 Contribution

The study aims to combine individual flexible bus route services operating in multiple regions within a larger area, with both internal and external demand and serving many-to-one demand only, into an integrated flexible route bus service that offers many-to-many travel options. The proposed system is a stand-alone closed system which provides complete transit services without relying on other modes, within the area considered. The system is assumed to include n regions with varying characteristics and a generalized total cost function for the flexible bus system is proposed. Each region is connected to the central terminal by a flexible-route with headway h, which may be considered one module of a more comprehensive and integrated system. In addition to inter-region travel, the total demand also includes intra-region and terminal-bound users. The terminal bound users travel to the terminal and back to their region. They are assumed to either exit the station at the terminal or choose to travel by another mode of transport from that point. The supplier and user costs associated with the different types of travel undertaken by the users in the network are formulated separately, based on the travel characteristics accompanying that type of travel. The generalized formulation provides the combined total cost for all the regions being considered which can then be minimized to find a common headway for the entire system. The model minimizes the total system cost as well as the average cost per trip by optimizing the headway for the entire network.

The flexible route bus services model is a complete solution based on the modular concept, taking many-to-one travel in each module and combining the modules to provide transit service from everywhere to everywhere within the system. Flexible route bus services can be the answer to providing cost-effective transportation in areas where conventional services might not work. This study helps in addressing travel demands of both types of users: users who want to travel to a destination within the service area of their bus service and the users who need to transfer at a station to travel further to other areas. In addition, it also proposes a coordinated headway to reduce wait times for passengers and costs associated with it. Further, some applications of the formulation are shown with numerical examples in the following sections.

1.3 Scope

Integrating flexible route bus services operating in different regions involves many parameters. To limit the scope of the thesis, it is assumed that the demand is uniformly distributed within each region and does not vary over time. Each region is at a distance called the line haul distance from the terminal and a bus travels the line haul distance to get to the region from the terminal and back.

A sensitivity analysis of the input variables such as line haul distance, supplier and user costs, and demand density provides us with insights into what affects the average cost of the system and can be used to assess the feasibility of flexible route services in a study area. Since flexible routing is preferable in areas with low demand, the results obtained from the numerical analyses can be compared to the costs of operating a taxi service or a conventional bus service in that area to check whether the demand warrants flexible route services. This study helps explore the possibility of designing a flexible route bus network in larger areas with low demand, which are connected to a central terminal that facilitates transfers for them to travel to other destinations within and outside the network.

1.4 Thesis Overview

The rest of the thesis has been arranged as follows. Chapter 2 discusses relevant research that is closely related to this thesis. The literature review provides an overview of the previous studies that have been conducted on flexible route transit systems and coordinated transfers and discusses how this thesis' contributions advance the existing literature. Chapter 3 develops a generalized formulation for a total cost function for a system of flexible route buses that are serving multiple regions, while keeping in mind the types of travel and the parameters associated with the travel. Chapter 4 shows the results obtained by minimizing the total cost for a four region (3 region, 1 terminal) and seven region (6 region, 1 terminal) network. It also shows how the input variables can affect the average cost per passenger and the headway through a sensitivity analysis. Chapter 5 presents the conclusions drawn and from the study conducted in this thesis and possible future extensions.

CHAPTER 2 : LITERATURE REVIEW

The concept of providing flexible route bus services in both urban and suburban regions with low demand has been successfully studied in earlier studies conducted on this topic. Some of the studies examine some form of an integration of both conventional and flexible bus services to provide better connectivity to user (Kim and Schonfeld, 2014 and Fu, 2002), and Nourbakhsh and Ouyang, 2011 design a flexible bus route network for a hub-and-spoke grid network. However, on surveying the literature on flexible route bus systems and timed transfers, it is found that studies on many-to-many services exist for small vehicles shared by very few riders but not for mass transit services considered in this thesis, in which flexible-route buses serve the internal demand within defined regions as well many-to-many demand patterns across wide areas.

Kim and Schonfeld (2014) discuss the difference in characteristics of conventional bus services and flexible bus services. While conventional services can carry more passengers at low average cost per trip, flexible services can offer door-to-door services in low demand density areas. The study proposes integrating both conventional and flexible bus services and then compares the overall system cost to that of a purely conventional or flexible bus system. The regions being considered are further divided into rectangular zones and the cost and headway calculations have been done for each zone. They find that the flexible services are preferable in scenarios with shorter line haul distances and lower demand densities. A threshold analysis between the two services was also done for length of region, value of time and operating costs. The analysis was done for scenarios with and without timed transfer coordination and it was found that with timed transfers, the cost of the proposed integrated system can be further reduced. This study does not take into consideration any intra-zonal demand.

Ting and Schonfeld (2005) aimed to "optimize the headway and the slack time jointly in a multiple hub transit network with timed transfers". This is achieved by optimizing the total system operating cost for the network. The headway of all the terminals and their corresponding slack times are optimized simultaneously using a heuristic algorithm. For the coordinated routes, headways are taken as integer multiples of a base cycle value so as to ensure simultaneous arrivals of buses at the transfer station. It is established in the paper that headway coordination is preferable in low demand scenarios, despite the addition of slack times in the operation schedule. The benefits of a coordinated system depend on the transfer demand at the terminal, among other demand characteristics. The paper focuses on coordinating headways at all terminals for a conventional bus system. It does not explore headway coordination for a flexible route bus system and its related characteristics.

Abkowitz et al (1987) explored the concept of timed transfers and its implementation and evaluation. It studies the effect of different timed transfer scenarios namely, unscheduled, scheduled, waiting and double holding. The study showed that route characteristics were the main influencers which evaluating the feasibility of a timed transfer system and its preferred strategy for the transfers. It is established that whenever there is incompatibility between the headways, scheduled transfers are more effective as compared to an unscheduled case. However, the paper explores the various transfer scenarios but does not consider the effects of any regional characteristics like area and demand (if any) on the algorithms.

Nourbakhsh and Ouyang (2011) presented an alternative flexible-route transit system with each bus servicing passengers in a predefined area, with the collection of such areas forming a hybrid structure resembling hub-and-spoke and grid networks. They compared the proposed system characteristics with fixed route transit and taxi systems so as to analyze the performance of this system. Instead of proposing a numerical formulation, this paper articulates the system's operating performance into analytical functions using select design variables which can affect the system's efficiency. The system has two main costs: agency costs and user costs. Any internal demand is not addressed. The flexible route system provides various advantages in low demand areas by eliminating the need for the passengers to walk to the bus station and increased passenger safety.

Chang and Schonfeld (1993) use analytic optimization models to jointly optimize route length, route spacing and headways in an urban area bus system. The paper also proposes guidelines to divide the area into smaller zones which are serviced by a bus system with fixed routes and schedules. Each service area is serviced by buses operating from a single transportation terminal which is located at an optimal distance (express distance) from the area. The supply and demand characteristics of the area are assumed to vary over time and the headways are optimized for each time period. . However, the study does not explore a flexible bus route system and does not consider any internal demand while optimizing the headway and route length.

Chowdhury and Chien (2002) study inter-modal transfer coordination. A rapid transit mode is considered which has a number of feeder routes connecting with it at several transfer stations. In such a scenario, coordinating the schedules of the different modes can significantly reduce the delays while transferring at the connecting stations. The objective total cost comprises of supplier and user costs. To minimize the total cost, four different stages with increasing degrees of coordination are considered - no coordination, bus route coordination at isolated transfer stations, rail-bus coordination at isolated stations and finally, network-wide rail-bus coordination. An analytical approach is used for optimizing headways for uncoordinated schedules and for optimal coordinated headways, a numerical search algorithm has been used. In cases with low demand and long rail headways, coordination at all stations is desirable while for high demand cases, coordination at all stations might not be necessary. The study does not consider any internal demand or many-to-many travel patterns.

Chien and Schonfeld (1998) consider a rail transit route and its connecting feeder bus routes in an urban area. The total cost is formulated for an integrated rail and feeder bus network and is then minimized using an iterative method for optimization. The demand follows many-to-many travel patterns and non-elastic. The study optimizes the bus route spacing, bus headways and bus stop spacing for the network. The bus routes are parallel to each other and perpendicular to the rail line. The formulations obtained in the paper can be effectively used to design rail networks and transit services by optimizing the decision variables involved. However, it might not be practical to run feeder buses parallel to each other in all scenarios. The study does not try to integrate the bus services serving different regions and hubs.

Chandra and Quadrifoglio (2013) develop an analytical model to compute the terminal-to-terminal cycle length of a demand responsive feeder bus service. A rectangular service area being serviced by a single shuttle has been considered with the terminal located on the edge of the same area. Calculation of an optimal duration for cycle length from one terminal to another is done using an analytical queuing method and uses demand and geometrical parameters of the area as inputs. This is a limited model studying a very small area, hence, compared to this thesis, it is relatively limited in its scope.

Fu (2002) studies flex-route transit, which is "a hybrid of conventional fixed-route transit and demand-responsive paratransit service". It considers a conventional bus network in place for general bus users and demand-responsive, flexible route service for paratransit bus users to provide them with door-to-door pick-ups and drop-offs. The focus is to establish the best possible way to operate such a system and the relationship between various design parameters and the system performance. However, the flexible route considered is not entirely flexible, with standard spacing between stops and the bus system offers connectivity from the doorstep of the user to the conventional grid bus stops. It does not separately address the paratransit demand that opts to use the bus service to travel within the region that is being serviced by the flexible bus system.

Alshalalfah et al (2011) investigate the effects of introduction of flex-route bus services instead of a conventional bus system in a suburb of Toronto, Canada. The impact of the change is observed through simulations of operational performance in both types of bus systems and through comparing the performance parameters for both. The conventional bus stops are considered the hard constraints with mandatory bus stoppage on each which the demand-responsive stops are made only when required. The study establishes that replacing a fixed-stop service with a flexible one would be preferable in low demand areas but does not consider integrating the flexible bus operation of one region with other similar systems that might exist in nearby regions. The area under study is a small one and the observations made based on the results obtained might not correctly reflect the user needs for the entire city.

Previously, Kim and Schonfeld (2014) jointly optimized the headway for a combination of a conventional and flexible bus service. To further add to the already existing research, this thesis combines many-to-one flexible route bus services in different regions which are a part of a larger area and proposes an integrated flexible route bus model that can be used for many-to-many travel demand patterns. It addresses three types of travel demand in the network: intra-region, region to terminal and inter-region. The demand of each region can be divided into these three types by making an Origin-Destination matrix. The model computes the total cost of the system, which is then minimized to optimize the headway for the integrated flexible bus service system. Sensitivity analyses of the optimized headway with respect to other input variables have also been conducted.

CHAPTER 3: METHODOLOGY

The flexible bus route system being considered in this study has multiple regions connected to a central terminal that serves as a transfer station to all the passengers travelling from one region to another. Since formulating a total cost for a network with multiple regions being served through independent routes can have many complex conditions to consider, the following assumptions have been made while formulating the costs in this chapter.

3.1 Assumptions

The following assumptions were made in order to reduce the complexity of the problem and to keep the study consistent with other related studies (Kim and Schonfeld, 2014).

(a) The demand depends on population of the region and is uniformly distributed over space and time within each area.

(b) The input values for the variables have been suitably assumed or borrowed from existing similar studies (Kim and Schonfeld, 2014).

(c) Layover times are not considered in the total cost formulation.

(d) The external costs are negligible.

(e) There are multiple regions and the area of each region i can be calculated as $A_i = Q_i/q_i$.

(f) The distance of the terminal from a region i is called line-haul distance J_i and buses travel this distance at non-stop speed of yV_i to the edge of the region.

(g) Length of the bus tour within a region i, is approximated according to Stein (1978) as $L_i = k\sqrt{niAi}$ where the area of each region is assumed to be "fairly compact and fairly convex" and k = 1.15 for the space assumed here. (Daganzo, 1984)

(h) Buses have preset schedules and flexible routes so as to minimize the length of each bus tour, L_i.

(i) Headways are the same for all regions so as avoid delays at the transfer station (terminal).

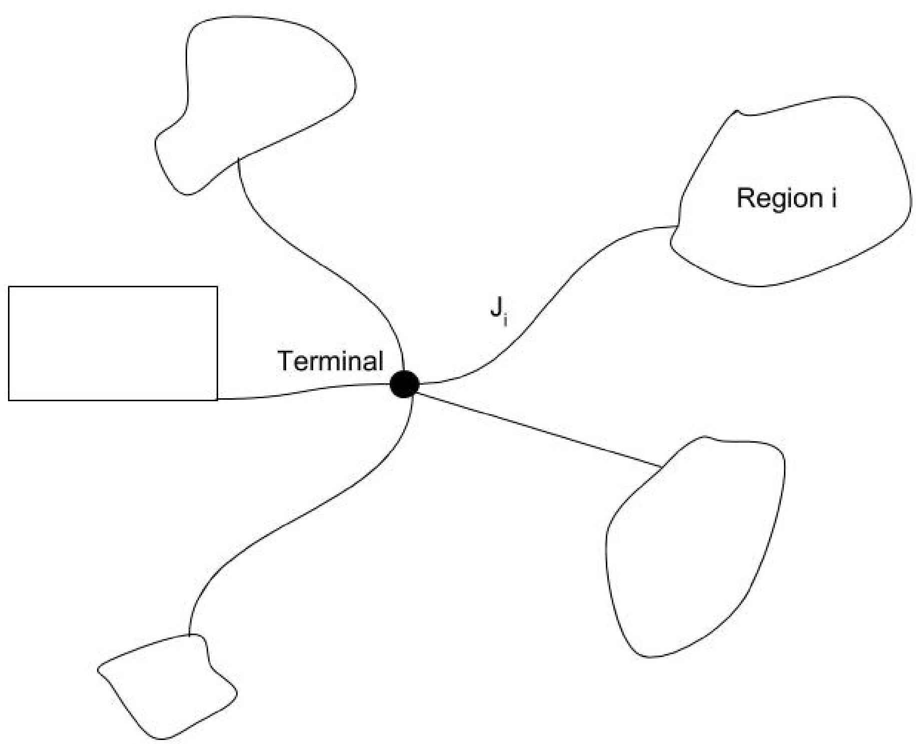


Fig 1. Transfer terminal and local regions network

3.2 FLEXIBLE ROUTE BUS SYSTEM WITH COORDINATED TRANSFERS

Flexible route bus systems offer door-to-door pick up and drop off services, thereby reducing the user access costs and cost of waiting for the arrival of the bus at the stop. In this study, as shown in Fig. 1, the departure and arrival of all the buses serving multiple regions is from a central terminal, serving as a transfer station. Buses travel a line haul distance J to go to the terminal from each region. The line haul distance to the terminal can be different for each region.

In this thesis, the total cost for a flexible bus route system which integrates flexible bus routes in multiple regions is formulated, and it includes three types of demands (intra-region, inter-region and terminal bound) considered which calculating the user costs. Further, it is minimized by optimizing the headway to increase the performance of the system. To simplify the problem, first the total cost function for just the terminal and one region (1R-1T) is formulated, which is then generalized based on the region sensitive parameters. For simplification and better understanding, the total cost function has been expanded for a three-region and one terminal system (3R-1T) further in this section.

For 1R-1T, the bus runs a round trip from the terminal to the region i and back to the terminal. The total cost consists of supplier costs and user costs, namely, user in-vehicle cost and user waiting cost. Since it is a flexible route, there is no user access cost involved. Since there are no transfers involved here, there is no transfer cost either.

The total cost function for a flexible route transit service for multiple regions is formulated and the values of decision variables, including headway and area are optimized in the next section. Some considerations affecting the formulation of the costs associated with different types of travels discussed above are:

Intra-region travels : The fraction of demand with origin and destination both within the region are intra-region demand. The average distance travelled by an intra-region user is equal to half the average length of bus route within that region.

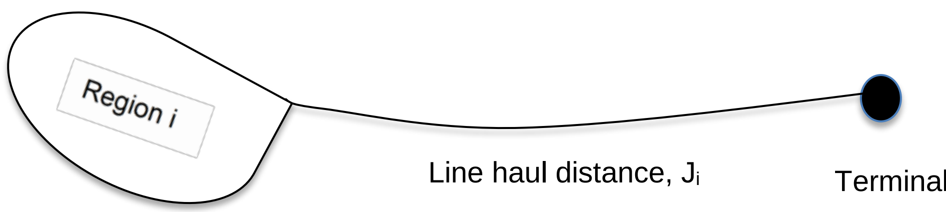
Inter-region travels : For inter-region travels, the users travel to the terminal for transfers and board on the bus that is bound for the region they want to travel to. Hence, they travel the entire line haul distance and half the average length of bus route inside the region while travelling to and back from the terminal respectively. To calculate the total cost of travel per passenger from region i to region j, the cost of travel from region i to the terminal and the cost of travel from terminal to region j are calculated. All inter-region demand passes through the terminal.

Region to terminal : While this fraction of demand originates in the regions that are a part of the system being considered, it is assumed that on reaching the terminal, this demand segment either travels to regions which are outside the flexible bus route system being considered, or continues its travel using another mode of transport. In such a case, the user is considered to travel between the region and the terminal only. To elaborate on the design of a flexible route network, we shall first formulate the total cost for one region and one terminal network and then generalize the total cost function for a network with multiple regions.

3.2.1 One-Region and One-Terminal (1R-1T)

To simplify the problem, initially only one region being serviced is considered, with transfers happening at the terminal and the costs associated with operating the flexible bus route between the region and the terminal is considered.

Fig. 2 One Region- One Terminal



The total cost formulation for one region and one terminal includes the bus operating cost, user in-vehicle cost and user wait cost and is expressed as:

$$C_i^t = C_i^s + C_i^v + C_i^w \tag{1}$$

Here,

Supplier Vehicle Cost for region i, $C_i^s = N_i . C$ (2)

N_i is the fleet size for region i. For cases where the round-trip time is larger than the headway, there needs to be more than one bus servicing the route. To calculate the number of buses required to service a region :

$$N_i = \frac{R_i^b}{h} \quad (3)$$

The user in-vehicle costs and the user wait costs are individually calculated for the three different types of trips made by the users in region i - users travelling within the region, users going from the region to the terminal and the users travelling to other regions. For this, we divide the demand into the two types of destinations, Q_i^{in} for intra region travel, Q_i^t for users travelling to the terminal. Q_i^t includes all users travelling to the terminal as well as to the other regions in the network. The demand travelling to the terminal and the demand going to other regions are combined since the users have to transfer at the terminal regardless of their final destination. In this case, since there is only one region, we shall consider intra-region demand and terminal bound demand only.

Therefore, for region i,

User in-vehicle cost, $C_i^v = vQ_i^{in}R_i^p + vQ_i^tR_i^b$ (4)

User wait cost, $C_i^w = u(Q_i^{in} + Q_i^t)h$ (5)

Hence, the total cost function for 1R-1T can be expressed as:

$$C_i^t = \frac{R_i^b}{h}C + v\,Q_i^{in}R_i^p + vQ_i^t\,R_i^b + u(Q_i^{in} + Q_i^t)h \quad (6)$$

Now, Q_i is the total demand generated in the region i per hour and can be shown as a sum of intra-regional demand, Q_i^{in} and terminal-bound demand, Q_i^t

$$Q_i = Q_i^{in} + Q_i^t \quad (7)$$

It is obtained from an O-D matrix generated for the network, where the demand for each O-D pair is shown as:

$$T_{ij} = 0.002\,\sqrt{P_i}*\sqrt{P_j} \quad (8)$$

In this case, P_j denotes the population of terminal. The total demand per hour, Q_i can also be expressed as:

$$Q_i = q_i \cdot A_i \tag{9}$$

Where, q_i is the demand density of region i and A_i denotes the area of region i.

Since a bus would have to travel the entire line haul distance, J_i to get to region i to make pick-ups and drop offs and then again return to the terminal by travelling the line haul distance, the round-trip time of the bus, R_i^b is expressed as

$$R_i^b = \frac{L_i}{V} + \frac{2J_i}{W} + D_i \tag{10}$$

Here, L_i is the length of a bus trip within region i and D_i is the total delay experienced by a bus serving region i on each tour. V and W are the speeds of the bus within region i and the line haul distance respectively.

Following Stein (1978), the flexible bus tour distance, L_i and the total delay on stops in region i, D_i can be expressed in terms of n_i, the number of stops made in region I and A_i, the area of region i as follows:

$$L_i = k\sqrt{n_i A_i} \tag{11}$$

$$D_i = n_i d \tag{12}$$

Round trip time is a sum of distance travelled by the bus in one cycle and the total time spent by the bus in making stops in the region. The number of stops made per trip are denoted by n_i and can be computed by dividing the total demand, Q_i by g, the group size of passengers boarding at each stop, as shown in Eqn. (12).

$$n_i = \frac{Q_i}{g} \tag{13}$$

It is apparent from Eqns. (9) to (13) that with variations in demand, the values of area A_i, length of tour, L_i and the number of stops made in one single tour, n_i also change. Also, the line haul distance, J_i can be different for each region. Keeping the above in mind, we can expand Eqn. (6) as:

$$C_i^t = \frac{C}{h}(L_i/V + 2.J_i/W + D_i) + v\,Q_i^{in}(L_i/V + D_i) + v\,Q_i^t\,(\frac{L_i}{V} + \frac{2J_i}{W} + D_i) + u(\,Q_i^{in} + Q_i^t)h \quad (14)$$

$$C_i^t = \left(\frac{L_i}{V} + D_i\right).\left(\frac{C}{h} + v \cdot Q_i\right) + \frac{2.J_i}{W}.(\frac{C}{h} + vQ_i^t) + uQ_ih \quad (15)$$

Eqn. (14) and (15) give us the total cost formulation for a network with one-region and one terminal.

Dividing the Total Cost C_i^t by the total demand for region i gives us the Average Cost per passenger per hour for the region. The total cost function in Eqn. (6) is further minimized with respect to headway to find the optimal headway for the network:

$$\frac{\partial C_i^t}{\partial h} = -\left(\frac{1}{h^2}\right)\left(R_i^b C\right) + uQ_i = 0 \quad (16)$$

$$h^2 = (Li/V + 2Ji/W + n_i d)\frac{C}{(uQi)} \quad (17)$$

$$h^2 = \frac{k.C}{(\,v\sqrt{q_i g})} + \frac{2Ji.C}{(uW\,Q_i)} + \frac{n_i.d.C}{(uQ_i)} \quad (18)$$

$$h^{*} = \left\{ k \frac{C}{v\sqrt{q_i g}} + 2Ji.\frac{C}{uWQ_i} + n_i d \frac{C}{u\,Q_i} \right\}^{0.5} \tag{19}$$

The optimized headway, h^* provides a common headway for the entire system. The formulated Total Cost can also be minimized by using optimization models. The numerical analysis of the above-mentioned equations and their corresponding results is provided in the subsequent chapters.

3.2.2 Generalized Total Cost for multiple regions

In this section a generalized total cost function is formulated for a flexible bus route network with multiple regions (Fig 1). Similar to Eqn. (1) in section 3.2.1, we start by adding the supplier costs and the user costs for all the regions. Since there are n regions being considered in this system, we shall find the total cost of the network by adding all the associated supplier and user costs for all n regions.

$$\sum_{i=1}^{n} C_i^t = \sum_{i=1}^{n} C_i^s + \sum_{i=1}^{n} C_i^v + \sum_{i=1}^{n} C_i^w \tag{20}$$

Again, the user in-vehicle costs and the user wait costs are individually calculated for the three different types of trips made by the users for all the regions - users travelling within the region, users going from the region to the terminal and the users travelling to other regions. The demand travelling to the terminal and the demand going to other regions are combined since the users have to transfer at the terminal regardless of their final destination. It has been assumed that the headway (h), supplier cost per passenger (C) and the user cost per passenger (v, u) are same for all the regions being considered. However, the demand mentioned in Eqns. (4) to (6) varies from region to region depending on the population of the respective regions. It is apparent from Eqns. (9) to (13) that with variations in demand, the values of area A_i, length of tour, L_i and the number of stops made in one single tour, n_i also change. Also, the line haul distance, J_i can be different for each region.

Therefore, the combined costs for all n regions can be shown as:

Supplier vehicle cost: $$C_i^s = \sum_{i=1}^{n} N_i \, C \tag{21}$$

User in-vehicle cost: $$C_i^v = v.\left\{\sum_{i=1}^{n}\left(Q_i^{in}.R_i^p\right) + \sum_{i=1}^{n} Q_i^t \cdot R_i^b\right\} \tag{22}$$

User wait cost: $$C_i^w = u.h.\sum_{i=1}^{n} Q_i \tag{23}$$

The round-trip time, tour length and delay per trip of each bus in all the regions can be calculated as shown in Eqns. (10) to (13).

Now, combining Eqns. (21) to (23), we can rewrite Eqn. (20) as:

$$C_{sys}^t = \sum_{i=1}^{n} C_i^t = \sum_{i=1}^{n} N_i \,.\, C + v.\left\{\sum_{i=1}^{n}\left(Q_i^{in}.R_i^p\right) + \sum_{i=1}^{n} Q_i^t \cdot R_i^b\right\} + u.h.\sum_{i=1}^{n} Q_i \tag{24}$$

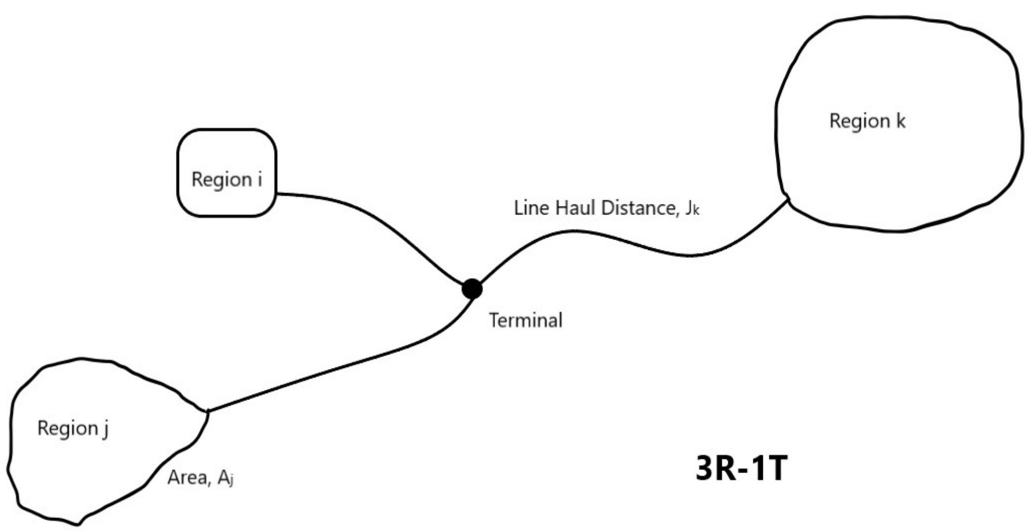
This is a generalized model for a flexible route bus system with users travelling within a region and a central terminal serving as a transfer station for passengers travelling from one region to other. Another example of using the model to serve a network with multiple regions, in this case, with three regions and one terminal, is presented below.

3.2.3 Three Region and One Terminal - Coordinated Transfers (3R-1T)

Similarly to the 1R-1T formulation for Total Cost, in this case we consider supplier costs, user in-vehicle cost including the delay costs and the user wait cost for all the three regions and combine them to find the total cost for the entire network. There is no transfer delay cost since the transfers are coordinated at the terminal.

By following Eqn.(24), we can formulate a total cost function for an entire city or zone by dividing it into multiple smaller regions like above. The demand distribution for the three types of travel (intra-region, region to terminal and inter-region) for the network can be obtained by making an O-D matrix as shown in Chapter 4.

Fig. 3 Three region and one terminal network



Based on Eqn. (24), the Total Cost for the network shown in Fig. 3 can be expressed as:

$$C_{ijk}^{t} = C_{i}^{t} + C_{j}^{t} + C_{k}^{t} \tag{25}$$

This can be further written as:

$$C_{ijk}^{t} = C_{i}^{s} + C_{i}^{v} + C_{i}^{w} + C_{j}^{s} + C_{j}^{v} + C_{j}^{w} + C_{k}^{s} + C_{k}^{v} + C_{k}^{w} \tag{26}$$

Following Eqn. (6), the Total Cost for 3R-1T for regions I, j and k is:

$$C_{ijk}^{t} = R_{i}^{b}.C/h + vQ_{i}^{in} R_{i}^{p} + v\, Q_{i}^{t} R_{i}^{b} + uQ_{i}\, h + R_{j}^{b}.C/h + vQ_{j}^{in} R_{j}^{p} + v\, Q_{j}^{t} R_{j}^{b} + uQ_{j} h + R_{k}^{b}.C/h + v\, Q_{k}^{in} R_{k}^{p} + v\, Q_{k}^{t} R_{k}^{b} + u\, Q_{k}\, h \tag{27}$$

Similar to Eqns. (16) to (19), we can optimize the highway for the network in Fig. 3 by minimizing the total cost function in Eqn. (27). The optimal headway is the same for all regions to ensure coordinated transfers for all passengers at the terminal.

CHAPTER 4 : RESULTS

This chapter explores the proposed flexible route coordinated bus system through numerical examples. The model takes into consideration 3R-1T and 6R-1T systems being serviced by flexible route buses, exploring internal travel, travel from a region to the terminal and from one region to another. The total cost is the sum of the supplier and user costs for the system. The solutions are calculated numerically, followed by optimization of the headway.

The sensitivity analyses are conducted by using the results obtained and the variation of the average cost per passenger per hour and the headway, with respect to the line haul distance, demand density, user wait cost, vehicle operating cost and user in-vehicle cost. It is expected that user costs will affect the average cost more as compared to other input parameters. The headway is more likely to be affected the most by user wait cost and demand density since a variation in both is directly proportional to the headway of the system.

4.1 EXAMPLE 1: THREE REGIONS AND ONE TERMINAL

Table 2 shows the O-D demand matrix for the three-region network along with the population distribution of each region. The values of the matrix are calculated by using Eqn. (8), which assumes that the demand is dependent on the population density of the regions, P_i and P_j:

$$T_{ij} = 0.002 \sqrt{P_i} * \sqrt{P_j}$$

The values obtained from Eqn. (8) are used as the base inputs for total cost and have been mentioned in Table 2. Table 3 shows the other input variables and their corresponding values used in the computation of the total cost.

Table 2. O-D Matrix for regions i, j, k and terminal

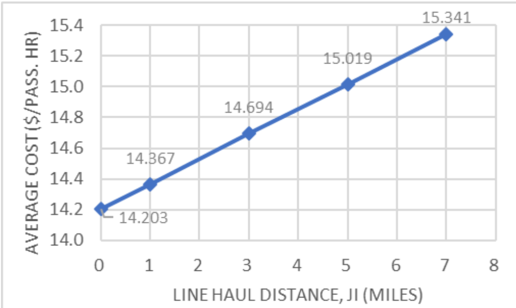
poplulation		Region	**i**	**j**	**k**	**terminal**
i	4000	**i**	6.0	6.9	7.7	9.8
j	6500	**j**	6.9	8.0	11.3	11.3
k	8000	**k**	7.7	8.9	10.0	12.6
terminal	10000	**terminal**	9.8	11.3	12.6	16.0

Table 3. Input values and costs for three-region network

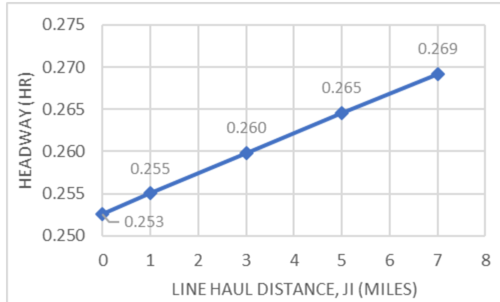
	Region		
	i	j	k
Total Supplier vehicle cost $/hr	133.174	179.089	177.691
Total User In-vehicle cost ($/hr)	134.168	218.999	226.012
Total User wait cost ($/hr)	127.383	156.994	164.451
Bus operation cost, C ($/hr)	50	50	50
Line haul distance, J (miles)	3	5	7
Express speed, yV (mph)	50	50	50
Speed, V (mph)	25	25	25
User In-vehicle cost, v ($/hr)	10	10	10
User wait cost, u ($/hr)	15	15	15
Average delay at each stop, d (hrs)	0.0017	0.0017	0.0017
Demand density, q (passengers/sq. mile/hr)	10	10	10
Intra-region demand, Q_i^{in} (passengers/hr)	6	8	10
Terminal bound demand, Q_i^t (passengers/hr)	9.8	11.31	12.65
Inter-region demand, Q_{ij} (passengers/hr)	14.67	18.24	16.69

For the sensitivity analysis, average cost is calculated for different values of line haul distance, user wait cost, vehicle operating cost, demand density and use in-vehicle cost. The headway is also optimized while varying the same parameters. The values obtained from the analyses are shown in Table 4, where the average cost and the headway have been plotted against the variations in line haul distances of two of the regions, i and k (Table 4a-4d), bus operating cost (Table 4e-4f), user in-vehicle cost (Table 4g-4h), user wait cost (Table 4i-4j) and the demand density (Table 4k-4l).

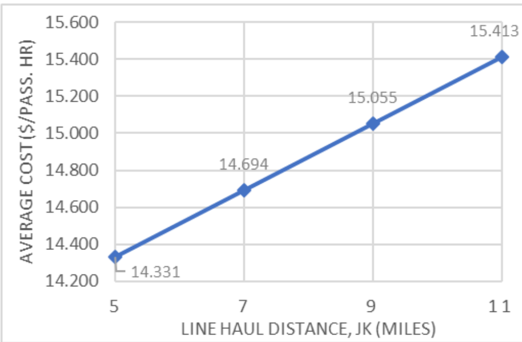
Table 4. Sensitivity analysis results



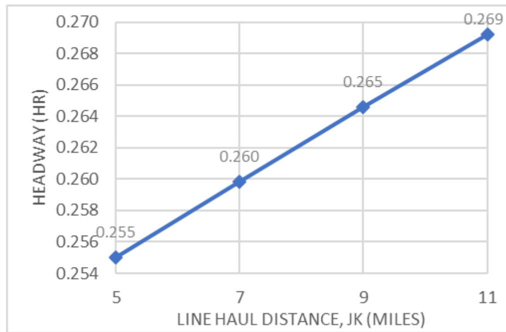
4(a).



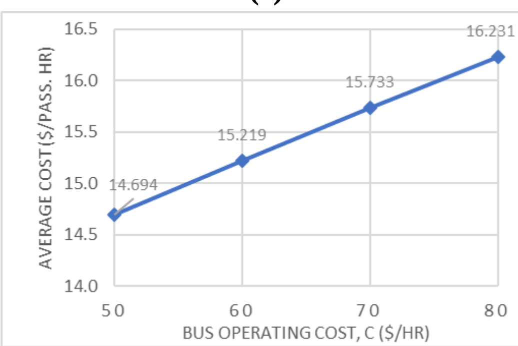
4(b).



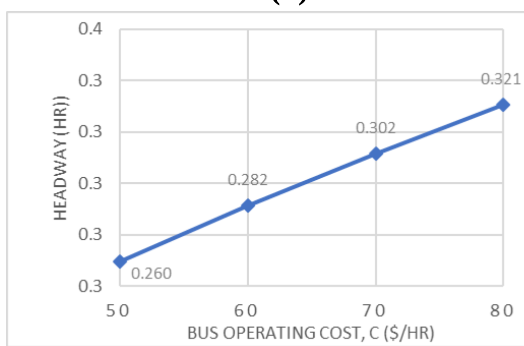
4(c).



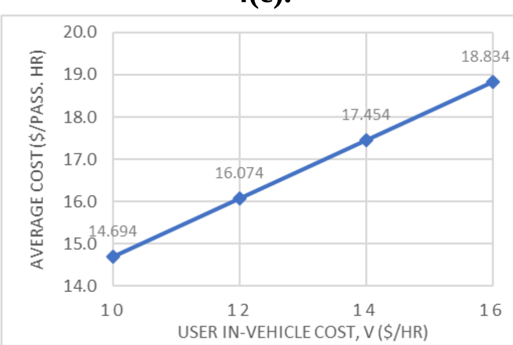
4(d).



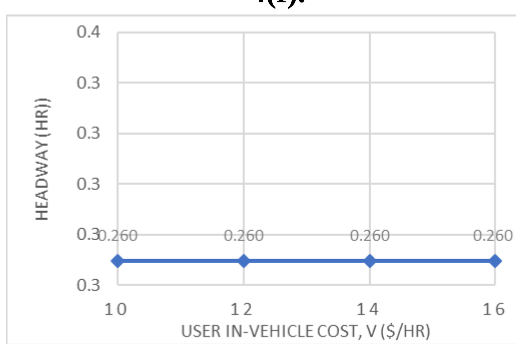
4(e).



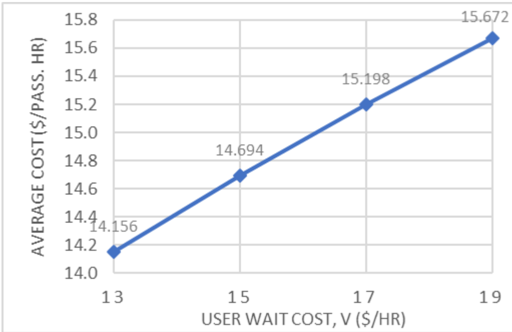
4(f).



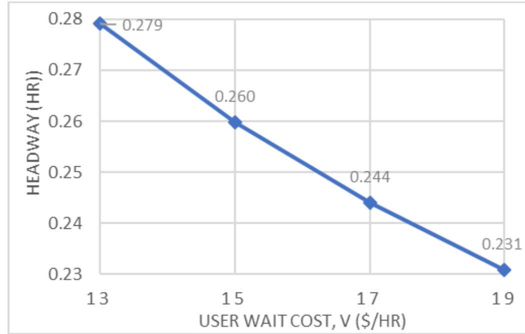
4(g).



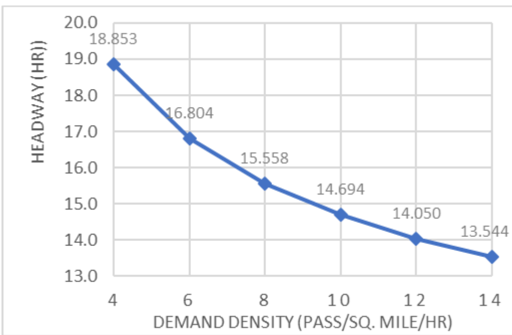
4(h).



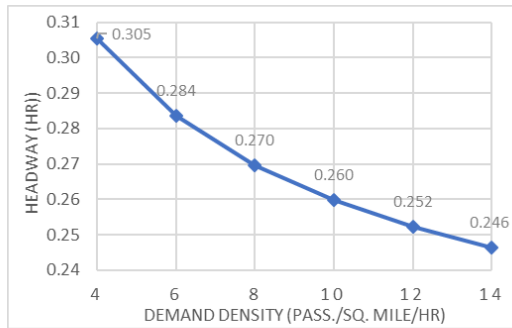
4(i).



4(j).



4(k).



4(l).

In Table 4a, the average cost per passenger of region i is observed while varying the line haul distance of the region starting from 0 up to 8 miles. The average cost steadily increases from 14.203 $/pass hr to 15.341 $/pass.hr, thereby indicating that as the distances between the terminal and the regions grow, the system costs increase too. Similar trend is observed when the line haul distance of region k is varied with respect to the average cost in Table 4c. With the variations in the line haul distance from 0 to 8 miles, an increase in the headway is also noted from 0.253 to 0.269 with respect to J_i and 0.255 to 0.269 with respect to J_k. This can be attributed to the fact that as the system costs increase, the headway of the buses shall also increase, so as to keep the system operating costs to a minimum possible while providing reliable bus service. This same concept is reinforced in the graph plotted in Table 4(f), where an increase in the bus operating cost leads to an increase in the headway as well. In Table 4(e), since the bus operating cost is a component of the total cost, it is obvious that an increase in the former leads to an increase in the average cost per passenger. Table 4(g) and 4(i) show a plot of the average cost versus the user in-vehicle cost and user wait cost respectively. With a change in the user in-vehicle cost from 10 $/hr to 16 $/he, the average cost changes from 14.694 to 18.834 $/pass hr. While with user wait cost changing from 13 $/hr to 19 $/hr, the average cost goes from 14.156 to 15.762 $/pass hr. It is clear from these values that a change of one unit in the user in-vehicle cost affects the average cost more strongly than the user wait cost. Table 4(h) shows that variations in the user in-vehicle cost does not affect the headway but an increase in the user wait cost leads to a decrease in the headway (Table 4(j)), so as to offset the increase in the total cost caused by the increase in the wait cost. Table 4(k) shows that low demand densities increase the average cost per passenger, hence leading to a higher total cost of the system. As the demand density increases, the average cost starts reducing, leading to a reduction in the headway (Table 4(l)) as well. Smaller headways mean increased frequency of buses in the region. The frequency needs to be increased to accommodate the increased demand generated per square mile of the region.

4.2 EXAMPLE 2: SIX REGIONS AND ONE TERMINAL (6R-1T)

In this example, a system with six regions being served by the flexible route bus service with transfers at one single terminal is being considered. We shall again compute the cost components for all the regions separately and then combine them to find the total system cost. The input demand is shown in Table 5 while the input values and the computed cost components for each region are shown in Table 6.

Table 5. Demand O-D matrix for 6R-1T

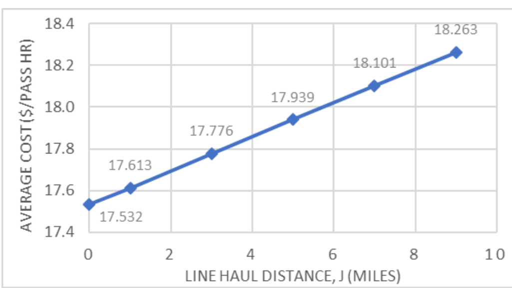
Region	**i**	**j**	**k**	**l**	**m**	n	terminal	poplulation
i	4.0	4.9	6.0	5.3	6.9	7.2	8.5	2000
j	4.9	6.0	7.3	6.5	8.5	8.8	10.4	3000
k	6.0	7.3	9.0	7.9	10.4	10.8	12.7	4500
l	5.3	6.5	7.9	7.0	9.2	9.5	11.2	3500
m	6.9	8.5	10.4	10.4	9.2	12.0	12.5	6000
n	7.2	8.8	10.8	9.5	12.5	13.0	15.3	6500
terminal	8.5	10.4	12.7	11.2	14.7	15.3	18.0	9000

Table 6. Input values and results for 6R-1T

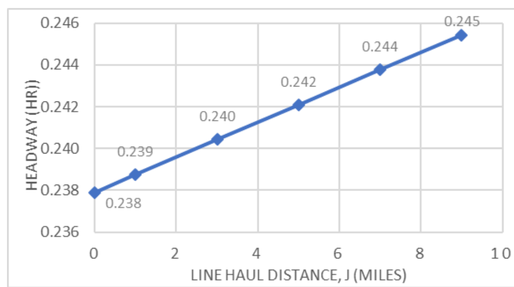
	Region					
	i	j	k	l	m	n
Total Supplier vehicle cost $/hr	145.176	182.382	226.082	194.996	152.879	289.955
Total User In-vehicle cost ($/hr)	295.691	452.683	683.794	522.685	495.188	1039.798
Total User wait cost ($/hr)	154.412	189.115	231.618	204.268	133.688	278.370
Bus operation cost, C ($/hr)	50	50	50	50	50	50
Line haul distance, J (miles)	2	3	4	3	5	7
Express speed, yV (mph)	50	50	50	50	50	50
Speed, V (mph)	25	25	25	25	25	25
User In-vehicle cost, v ($/hr)	10	10	10	10	10	10
User wait cost, u ($/hr)	15	15	15	15	15	15
Average delay at each stop, d (hrs)	0.001667	0.001667	0.001667	0.001667	0.001667	0.001667
Demand density, q (passengers/sq. mile/hr)	10	10	10	10	10	10

Similar to section 4.1, a sensitivity analysis is also performed for 6R-1T system. The individual results for each region are shown in Table 6. The average cost per passenger for the entire integrated system is minimized to obtain a common headway for all the regions. The average cost and the headway are then calculated by varying the values of line haul distance, supplier and user costs and the demand density, which, in this case, are assumed to be the same for all the regions. Table 7 shows the trends that are obtained by plotting the results with respect to the variables mentioned.

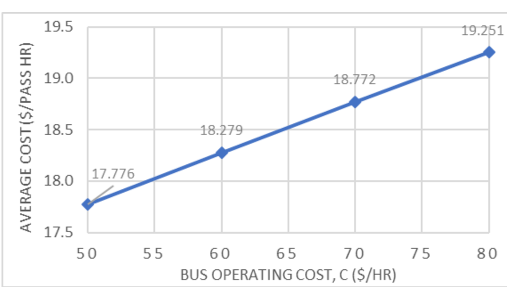
Table 7. Sensitivity analysis for 6R-1T



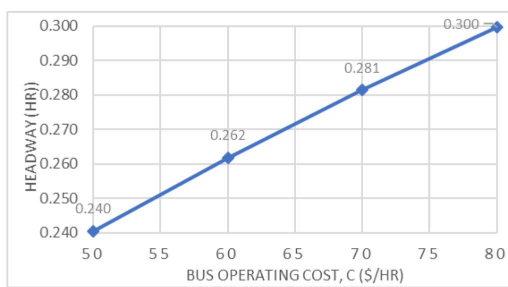
7(a)



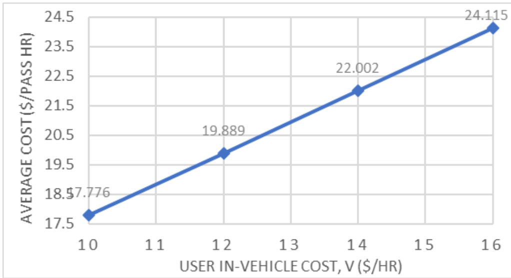
7(b)



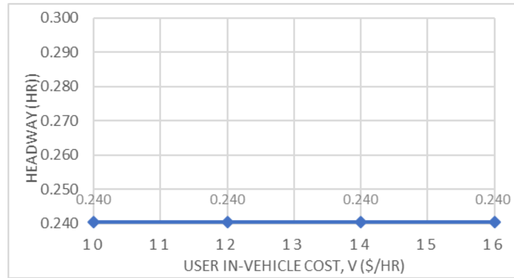
7(c)



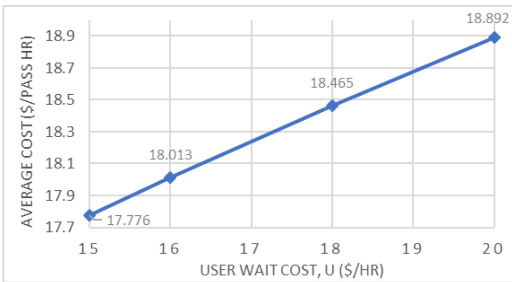
7(d)



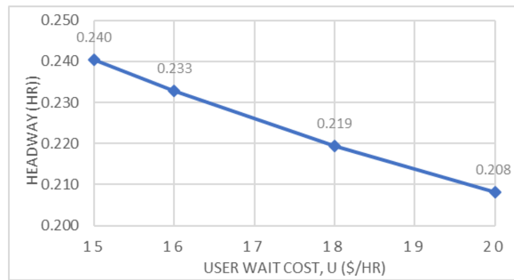
7(e)



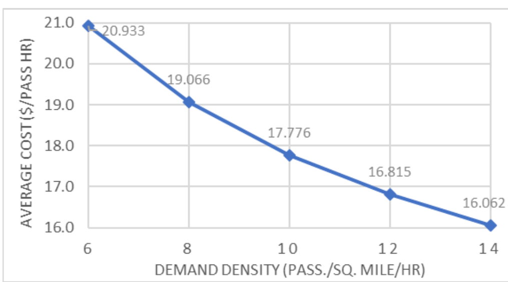
7(f)



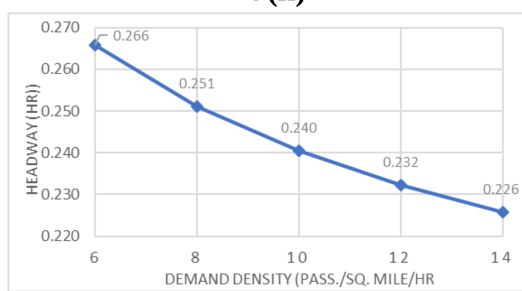
7(g)



7(h)



7(i)



7(j)

The observations from the sensitivity analyses done for 3R-1T system are echoed in the analysis for 6R-1T. An increase in the line haul distance of region j causes an increase in the average cost (Table 7a) since the bus covers a longer distance to get to the region, thereby accumulating both supplier as well as user costs. The sensitivity analysis has been done for line haul distance of only one region since a change in line haul distance of any of the regions would produce a similar effect on the average cost as this one. The headway also increases with an increase in the line haul distance (Table 7b). Similarly, bus operating cost (Table 7c), user in-vehicle cost (Table 7e) and the user wait cost (Table 7g) directly affect the average cost. Hence, as their values increase, the average cost increases. An increase in the supplier cost (Table 7d) also affects the headway in a similar way. The user in-vehicle cost does not affect the headway (table 7f) since the user wait time is not affected by the users sitting inside the bus. If the user wait cost per passenger per hour increases, then in order to minimize the costs, the headway will have to decrease. In Table 7(h), an increase of $5 per passenger per hour to the wait cost results in the reduction of headway by almost 2 minutes from 0.240 hrs. to 0.208 hrs. Demand density represents the demand per square unit area and, in this study, it affects the area of the corresponding region. For the same overall demand, a lower demand density leads to a larger area being serviced, which then results in longer bus routes and higher operating costs. This is also reflected in Table 7(i) where the average cost declines from 20.933 $/pass hr to 16.062 $/ pass hr whereas the demand density increases from 6 pass/sq. mile/hr to 14 pass./sq. mile/hr. The headway decreases when the demand density increases (Table j) since more frequent bus departures are needed to serve the demand in the region.

Through these sensitivity analyses we find that generally, while the average cost per passenger decreases with an increase in the demand density, it increases when the line haul distance is increased, as well as with an increase in the supplier and user costs. This behavior is in conjunction with the expected results since it reinforces the assumption that the longer the distances the buses cover, the more the system operating costs would be. The increase in the costs directly affects the total cost as well. The headway is found to increase with an increase in the line haul distance and the vehicle operating cost. The resultant decrease in the headway with an increase in the demand density and the increase in the user wait cost, both, were reflected uniformly in both the examples as well. With an increasing line haul distance, it takes the buses longer to complete their round-trip journey, hence leading to longer headways between the cycles. An increase in the demand density warrants an increased frequency of buses to avoid longer wait times for passengers. However, the headway remains unaffected by the user in-vehicle costs.

4.3 EXAMPLE 3: SUBURBAN AREA FLEXIBLE ROUTE BUS SYSTEM PLANNING APPROACH

The previous sections of this chapter numerically explore three and six region systems with one terminal each. While the results obtained give a realistic outline of the behavior patterns of average cost and headway within the limits of the assumptions made, the layout assumed in Fig. 3 might not be entirely practical. The regions are spaced out over the area with no possibility of transfer of passengers among regions without going through the terminal. A slight modification in the total cost function and the input variables can accommodate additional constraints that might get introduced in the problem due to the layout of the region being serviced by the flexible route bus service.

To demonstrate how some additional constraints can be introduced in the formulation, this example assumes a small part of a suburban area and its distribution of three regions, as shown in Fig. 4.

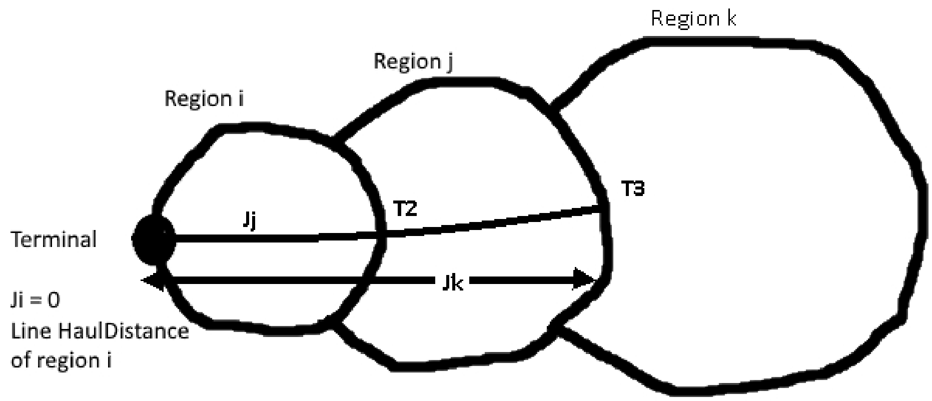


Fig. 4 Region distribution model for a suburban area

The regions share boundaries with each other and the distance between terminal and region i is zero, which means that region i is right next to the terminal. Also, the line haul distance for region j, J_j goes right through region i and line haul distance of region k, J_k goes right through both region i and region j. A user going from region k to region j goes all the way to the terminal and then transfers to go to region j. This leads to unnecessary delays and extra travel time for the passenger. To eliminate this problem, the line haul distances are lined together and a facility to transfer at the starting point of each region, T2 and T3, is provided in this case. This means that a user travelling from region k to region j can travel just up to point T2 and transfer to the bus serving region j directly instead of travelling all the way to the terminal for transfer. This reduces the travel time of the passenger and also decongests the terminal by redirecting a fraction of the passengers to the local transfer points.

Another constraint that is introduced in this problem is that the line haul distance of a region depends on the line haul distances of the regions from the terminal. The area of each region has been assumed to be fairly compact in shape, thereby making the length and width of a region to be almost equal. Hence, the line haul distance of region j, J_j can be assumed to be equal to the square root of region i, and the line haul distance of region k ,J_k is a sum of the square root of area of region i and square root of region j.

While minimizing the total cost of this system, the headway and the areas of the three regions are also jointly optimized. The line haul distance of a region is dependent on the areas of the regions preceding it. The inputs in the numerical analysis of this case are computed a little differently as compared to the preceding examples. Most of the calculations are the same as in previous examples, except for the travel between region j and region k. In this example, a user travelling from region j travels to T2, transfer to the bus going to region k and travel to its final destination in region k. A similar route is followed by a user travelling from region k to region j. The travels between the rest of the region combinations stay the same in this case. The input variables used for calculating the total cost and the results obtained are shown in Table 8.

Table 8. Input values and results for suburban area system

	Region		
	i	j	k
RESULTS			
Total Cost for the region ($/hr)	183.647	237.883	425.359
Total Supplier vehicle cost $/hr	58.475	79.677	125.918
Total User In-vehicle cost ($/hr)	54.837	80.333	183.579
Total User wait cost ($/hr)	70.334	77.873	115.862
Area of each region, A (sq. miles)	1.144	1.554	2.363
INPUTS	i	j	k
Bus operation cost, C ($/hr)	50	50	50
Line haul distance, J (miles)	0	1.044	2.303
Express speed, W = γV (mph)	50	50	50
Speed, V (mph)	25	25	25
User In-vehicle cost, v ($/hr)	10	10	10
User wait cost, u ($/hr)	15	15	15
Average delay at each stop, d (hrs)	0.00333	0.00333	0.00333
Demand density, q (passengers/sq. mile/hr)	10	10	10

The average cost obtained for each region is first individually minimized while optimizing the area of that region. This way, first the area of region i is optimized by minimizing the average cost per trip per hour for region i and then subsequently the areas of region j and region k are optimized with respect to their average cost per trip per hour. This is followed by optimization of the headway with respect to the average cost per trip of the entire system. Reasonably optimized values of the areas of the three regions and headway are obtained after carrying out the above process iteratively. In this case, three iterations were done to converge on the optimized values for the four variables. The line haul distances of regions j and k and are dependent on the areas of the regions preceding each region. This concept can be extended to add more regions to the system.

The values of line haul distances and areas obtained in Table 8 confirm that with an increase in the distance of a region from the terminal, the area of the region being serviced by a flexible bus route system also expands. Providing transfers at points T2 and T3 not only leads to shorter travel times as compared to transferring at the terminals but can also reduce the operation costs in some cases. This case exhibits the application of the model proposed in this thesis to integrate multiple regions with many-to-one flexible route services integrated into one many-to-many flexible route bus service.

4.4 EXAMPLE 4 - INDEPENDENTLY OPTIMIZED HEADWAYS V/S COORDINATED HEADWAY IN 6R-1T SYSTEM

A coordinated headway is assumed for the entire system in all the examples discussed in this chapter so far. However, to further examine the effect of operating the buses with independent headways for different routes, a system with six regions and one terminal is explored. In this case, all the regions of the system have their own independent headway, demand density and line haul distance. The focus is to investigate the effect of coordinating the headway within the system and to check whether it is beneficial to operate a flexible route bus system with a single coordinated headway or not. To do this, two cases are considered:

1. Combined total cost and single optimized headway for the system.
2. Individual total cost and optimized headway for each separate region.

The input values for both the cases, as mentioned in Table 9, stay the same.

Table 9. Input values for independent and combined headway cases

		Region					
		i	j	k	l	m	n
avg. group size per stop, g		1.2	1.2	1.2	1.2	1.2	1.2
vehicle operating cost, C	$/bus hr	50	50	50	50	50	50
Line haul distance, Ji	miles	2	3	4	5	6	7
Speed on line haul distance, W	miles/hr	50	50	50	50	50	50
Avg. speed in regions, V	miles/hr	25	25	25	25	25	25
In-vehicle cost, v	$/hr	10	10	10	10	10	10
User wait cost, u	$/pass hr	15	15	15	15	15	15
Stein constant, k		1.15	1.15	1.15	1.15	1.15	1.15
delay (per stop), d	hrs	0.00333	0.00333	0.00333	0.00333	0.00333	0.00333
demand density, qi	trips/mi^2hr	20	17	14	12	8	5
Area of region	mi^2	3.673	4.151	4.365	4.492	4.782	5.762
Stops/tour (bus)		61.211	58.810	50.931	44.917	31.883	24.009
total demand/hour	passengers/hr	73.45	70.57	61.12	53.90	38.26	28.81

For case 1, each region has its own dedicated flexible route bus system with its own independent headway. The total cost consists of both supplier and user costs. An additional user cost, user transfer (wait) cost is considered in this case since the headways are not coordinated throughout the system. Here, the average wait time of a passenger is assumed as half the headway of the next route to be taken by the passenger. The total cost formulation for individual region i can be expressed as a sum of the supplier cost, user in-vehicle cost, user wait cost and user transfer cost respectively:

$$C_i^t = C_i^s + C_i^v + C_i^w + C_i^{f,j} \tag{28}$$

The total cost can be calculated for each of the six regions by following Eqn. (28). As the distance of the region from the terminal increases, the demand density decreases and the headway of the bus increases.

For case 2, the total cost formulation will be the same as Eqn. (20), where the number of regions, n = 6. The input values for this case will be the same as mentioned in Table 9. The total cost and the common headway for the system will be optimized collectively. A comparison of the results obtained from case 1 and case 2 will give an idea about which case is preferable over the other one in this scenario. The results obtained for both the cases are enumerated in Table 10.

Table 10. Results for Independent and combined headway cases

Case 1 : Independent headway							
Region	Total Supplier vehicle cost	Total User In-vehicle cost	Total user wait cost	Total user transfer cost	Total system cost	Average cost	Headway (hrs)
i	231.61	704.844	231.611	85.235	1253.3	17.063	0.210
j	234.028	715.856	234.03	81.686	1265.598	17.934	0.221
k	215.768	606.347	215.77	72.086	1109.97	18.161	0.235
l	201.371	526.67	201.37	63.627	993.039	18.424	0.249
m	161.972	337.1	161.97	54.342	715.387	18.698	0.282
n	139.537	254.007	139.54	36.628	569.709	19.774	0.323
Total					**5907.003**		
Case 2 : Coordinated headway							
Region	Total Supplier vehicle cost	Total User In-vehicle cost	Total user wait cost	Total user transfer cost	Total system cost	Average cost of system	Common Headway (hrs)
i	199.229	704.844	269.255	0	1173.328		0.244
j	211.716	715.856	258.692	0	1186.264		0.244
k	207.808	606.347	224.034	0	1038.188		0.244
l	205.236	526.67	197.579	0	929.485		0.244
m	187.065	337.1	140.246	0	664.411		0.244
n	184.362	254.007	105.61	0	543.98		0.244
Total					**5535.656**	**16.975**	

As can be seen from the results in Table 10, providing independent headways to each region leads to an increase in the total cost of the system. The total cost of the system in case 1 with independent headway for each region is $5907.033/hr, which is $371.347/hr higher than that of case 2, where there is a single headway being used for the entire system. User transfer costs are introduced in case 1 since the headways are uncoordinated and an average user has to wait at the transfer terminal for the arrival of the bus that will take him to his final destination. This primarily leads to an increase in the user costs, in the form of user transfer costs. The average wait time of a user going from region i to region j will be equal to half the headway of region j. Providing a coordinated headway eliminates the user transfer costs from the formulation and also reduces the overall average cost per passenger for the system. Cases such as above can provide insights into the feasibility and benefits of using a coordinated headway as compared to uncoordinated headways, depending on factors such as the demand density, distance from the terminal, area of each region and the related user costs.

CHAPTER 5: CONCLUSIONS

Flexible route bus services can be used to provide cost-effective transportation in areas where conventional services are not viable. This study helps in addressing travel demands of two types of users: internal users who want to travel to a destination within the service area of their bus service and external users who need to transfer at a station to travel further to other areas. The focus of the thesis is the design of a many-to-many flexible route bus system that integrates multiple regions being served from a central terminal in the system, with many-to-one travel demand in each region, while taking care of both internal and external demand in the same system. The entire system has a coordinated headway to eliminate any waiting time for the passengers transferring at the terminal. It not only addresses inter-region travel but integrates intra-region travel into the design model. A numerical method is used to optimize the headway and minimize the average cost per passenger. It is established through numerical examples and sensitivity analyses that for a flexible route bus system, suitable headway, line haul distance and other associated inputs can be determined based on the above design. The distance of a region from the terminal also affects the overall operation costs and can lead to larger headways between buses. The supplier costs and the user costs directly affect the headway of the system. The coordinated transfers also provide a degree of comfort to the user. In this flexible route design, no passenger going from one region to another must make more than one transfer per direction. An example to demonstrate the application of the proposed model within a suburban area is also mentioned. It is noted that the region closest to the terminal has the smallest area while the one farthest from the terminal is larger in area. This shows that with an increase in the line haul distance, the area of region being serviced by the flexible route bus also expands. Further, this design can be compared to a taxi service operating within a city or urban area to determine the preferable service type.

5.1 Extensions and future work

The model proposed in this thesis may be applied for different O-D patterns. A further study can be done to test the applicability and the limitations of the proposed model in a larger urban or suburban area than discussed so far. To get more realistic results, slack times and uncoordinated headways may be modelled. Also, using different headways for different regions, which can be multiples of a base cycle, might provide a degree of flexibility in bus arrivals and lead to longer headways on some routes, which may reduce operation costs. In case of significant changes in demand during different times of the day, variations in demand over time can also be considered.

The example of a suburban area divided into three regions in example 4.3 can be further extended, similar to the layout in the example, to include a larger area. The regions can stretch out both radially and circumferentially (as shown in Fig. 5), where lateral transfers are also provided to enable users to travel between regions without passing through the terminal.

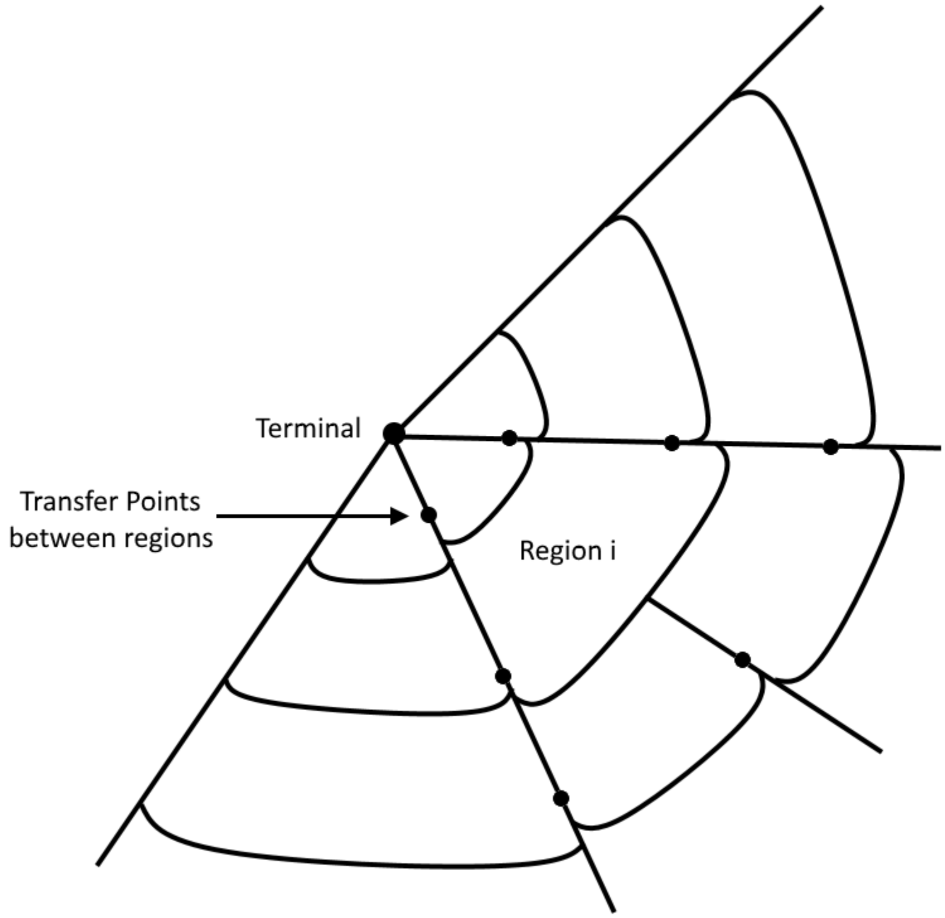


Fig. 5 Example of radial extension of regions in a suburban area

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