OPTIMAL CONTROL OF AN SIR-INSPIRED MALNUTRITION MODEL: SUBCONSCIOUS & SOCIETAL

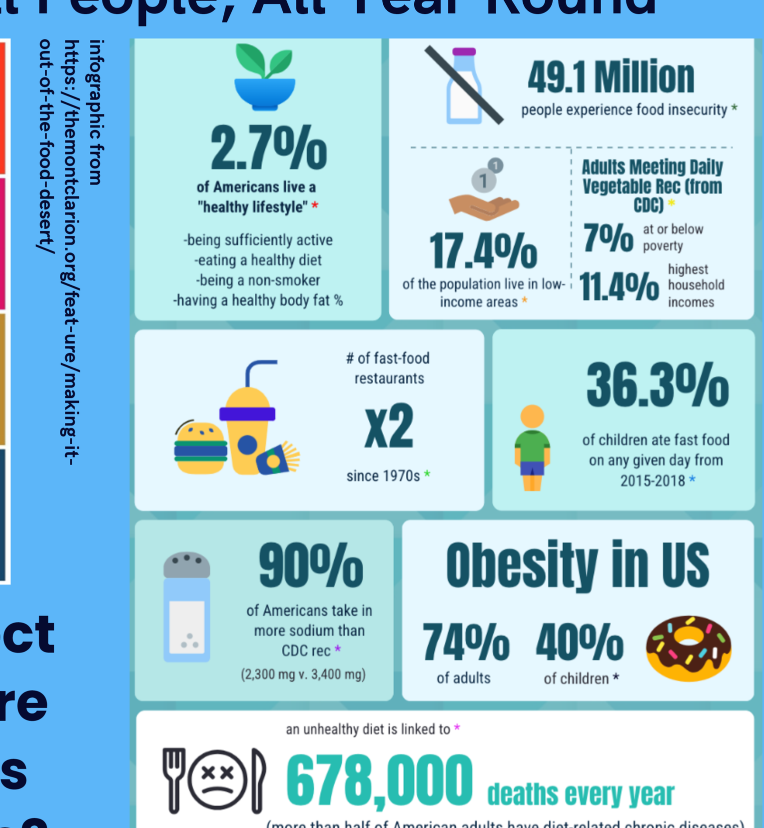
Dia Bonsu, Department of Mathematics, University of Maryland
Mentored by Dr. Padmanabhan Seshaiyer, George Mason University

INTRODUCTION

This research project uses differential equations based on an epidemiology ("SIR") model to study how malnutrition is comparable to an infectious disease, considering factors such as poverty and social and economic culture around processed foods. My analysis considers an "infected" population of malnourished people as a group of people who choose processed junk foods out of convenience or preference, but are not completely aware of their choice's severe impacts. This is a scientific investigation into how social conventions around food culture slowly work their way into everyday food choices. I am analyzing how education, different lifestyles, and conversations around food in the home impact the spread of "casual" and "normalized malnutrition" throughout the working middle class in the U.S. To scientifically model this, I consider an optimal control analysis to specifically analyze parental influence on food choices.

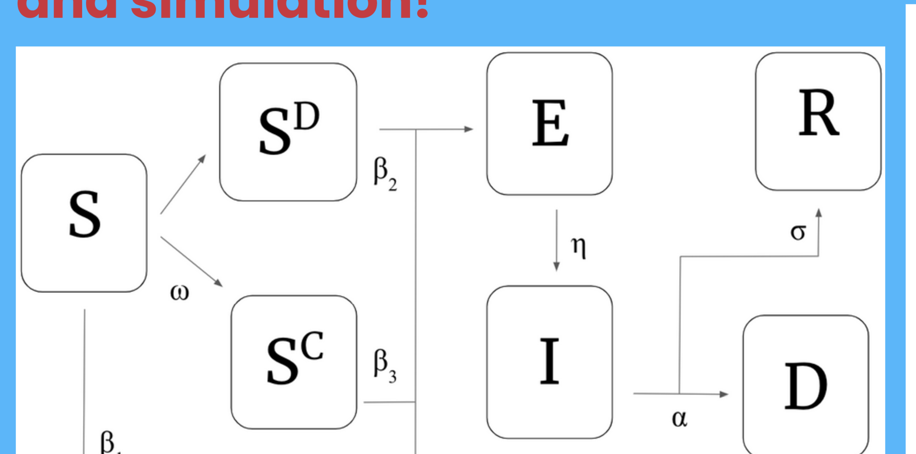
MOTIVATION & BACKGROUND

Sustainable Development—Zero Hunger: Access to Adequate Food and Healthy Diets, for All People, All Year Round



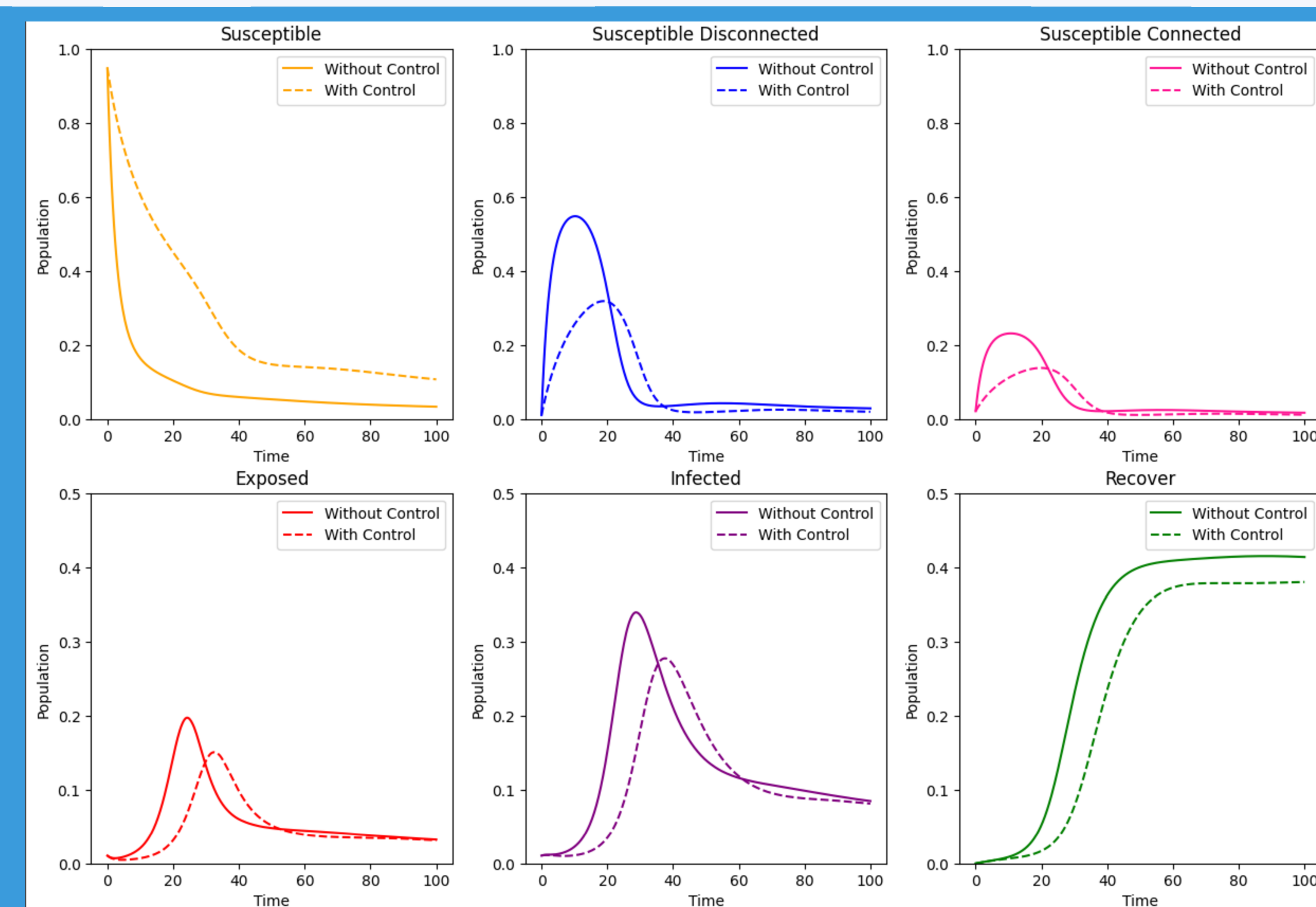
How does human interaction affect our food choices? Can we compare malnutrition to a social, infectious disease? How can we address this?

We can address this through mathematical modeling, analysis, and simulation!



$$
\begin{aligned}
\frac{dS}{dt} &= \Lambda - P(t)S(t)(\chi_1+\chi 2) - \beta_1 S(t)I(t) - \mu S(t)\\
\frac{dS^D}{dt} &= \chi_1 P(t)S(t) - \beta_2 S^D(t)I(t) - \mu S^D(t)\\
\frac{dS^C}{dt} &= \chi_2 P(t)S(t) - \beta_3 S^C(t)I(t) - \mu S^C(t)\\
\frac{dE}{dt} &= \beta_1 S(t)I(t) + \beta_2 S^D(t)I(t) + \beta_3 S^C(t)I(t) - \eta E(t) \; - \mu E(t)\\
\frac{dI}{dt} &= \eta E(t) - (I(t)(\sigma+\alpha+\mu))\\
\frac{dD}{dt} &= \alpha I(t)\\
\frac{dR}{dt} &= \sigma I(t) - \mu R(t)
\end{aligned}
$$

NON-LINEAR DYNAMICS



Parameters for Complex Model		
Parameter	Meaning	Value
β_1	Transmission Rate for all N	0.08
β_2	Transmission Rate for Disconnected Community	0.45
β_3	Transmission Rate for Connected Community	0.3
χ_1	Effect 1 of Parental Parental	0.238

χ_2	Effect 2 of Parental Influence	0.095
η	Rate at which exposed individuals become infected	0.366
σ	Recovery Rate	0.08
α	Death by Malnutrition Rate	0.05
λ	Birth Rate for all N	0.09
μ	Normal Non-Malnutrition-Related Death Rate	0.01

OPTIMAL CONTROL & R0

Objective Function:

$$J[P] = \int_0^T (I(t) + E(t) - A(S(t) + S^D(t) + S^C(t)) + BP(t))\, dt$$

Reproductive number found by the Next Generation Matrix Method

$$\mathcal{R}_0 = \frac{\eta(\beta_1 N + \beta_2 N^D + \beta_3 N^C)}{(\eta+\mu)(\sigma+\alpha+\mu)}$$

Create adjoint equations

$$
\begin{aligned}
H = {} & I(t) + E(t) - A(S(t) + S^D(t) + S^C(t)) + BP(t)\\
& + \lambda_1(\Lambda - P(t)S(t)(\chi_1 + \chi 2) - \beta_1 S(t)I(t) - \mu S(t))\\
& + \lambda_2 \chi_1 P(t)S(t) - \beta_2 S^D(t)I(t) - \mu S^D(t))\\
& + \lambda_3 \chi_2 P(t)S(t) - \beta_3 S^C(t)I(t) - \mu S^C(t))\\
& + \lambda_4(\beta_1 S(t)I(t) + \beta_2 S^D(t)I(t) + \beta_3 S^C(t)I(t) - \eta E(t) \; - \mu E(t))\\
& + \lambda_5(\eta E(t) - I(t)(\sigma+\alpha+\mu))\\
& + \lambda_6(\alpha I(t))\\
& + \lambda_7(\sigma I(t) - \mu R(t))
\end{aligned}
$$

$$
\begin{aligned}
\lambda_1' &= \frac{-\partial H}{\partial S} = -[-A + \lambda_1(-\chi_1 - \chi_2)P - \beta_1 I - \mu) + \lambda_2\chi_1 P + \lambda_3\chi_2 P + \lambda_4\beta_1 I]\\
\lambda_2' &= \frac{-\partial H}{\partial S^D} = -[-A + \lambda_2(-\beta_2 I - \mu) + \lambda_4\beta_2 I]\\
\lambda_3' &= \frac{-\partial H}{\partial S^C} = -[-A + \lambda_3(-\beta_3 I - \mu) + \lambda_4\beta_3 I]\\
\lambda_4' &= \frac{-\partial H}{\partial E} = -[1 + \lambda_4(-\eta - \mu) + \lambda_5\eta]\\
\lambda_5' &= \frac{-\partial H}{\partial I} = -[1 - \lambda_1\beta_1 S - \lambda_2\beta_2 S^D - \lambda_3\beta_3 S^C + \lambda_4(\beta_1 S + \beta_2 S^D + \beta_3 S^C) - \lambda_5(\sigma+\alpha+\mu) + \lambda_6\alpha + \lambda_7\sigma]\\
\lambda_6' &= \frac{-\partial H}{\partial D} = 0\\
\lambda_7' &= \frac{-\partial H}{\partial R} = \lambda_7\mu
\end{aligned}
$$

Theorem: Optimal control is given by:

$$P(t) = \frac{-\phi_2(t)}{\phi_1(t)}$$

$$\phi_1(t) = \Lambda\chi_1[-\lambda_1(\chi_1+\chi_2) + \lambda_2\chi_1 + \lambda_3\chi_2] + \Lambda\chi_2[-\lambda_1(\chi_1+\chi_2) + \lambda_2\chi_1 + \lambda_3\chi_2]$$

$$
\begin{aligned}
\phi_2(t) = {} & \{(A - \lambda_2(-\beta_2 I - \mu) - \lambda_4\beta_2 I)(\beta_2 - \beta_1)\chi_1 + (A - \lambda_3(-\beta_3 I - \mu) - \lambda_4\beta_3 I)(\beta_3 - \beta_1)\chi_2\\
& -[1 + \lambda_4(-\eta - \mu) + \lambda_5\eta][\chi_1(\beta_1 - \beta_2) + \chi_2(\beta_1 - \beta_3)]\}SI + \{\lambda_2\chi_1(\beta_2 - \beta_1) + \lambda_3\chi_2(\beta_3 - \beta_1)\\
& +\lambda_4[\chi_1(\beta_1 - \beta_2) + \chi_2(\beta_1 - \beta_3)]\}\{(\Lambda - PS(\chi_1 + \chi 2) - \beta_1 SI - \mu S)I + S(\eta E - I(\sigma+\alpha+\mu))\}\\
& -\Lambda\chi_1[\lambda_2(\beta_2 I + \mu) - \lambda_4\beta_2 I - \lambda_1(\beta_1 I + \mu) + \lambda_4\beta_1 I] - \Lambda\chi_2[\lambda_3(\beta_3 I + \mu) - \lambda_4\beta_3 I - \lambda_1(\beta_1 I + \mu) + \lambda_4\beta_1 I]
\end{aligned}
$$

FORWARD-BACKWARD SWEEP ALGORITHM

1. Make an initial guess for P over the interval.
2. Using the initial condition $S(0) = S_0$, $S^D(0) = S_0^D$, $S^C(0) = S_0^C$, $E(0) = E_0$, $I(0) = I_0$, $D(0) = D_0$, $R(0) = R_0$, and the stored values for P, solve the compartments forward in time according to their differential equations in the optimality system.
3. Check the transversality condition $\lambda_i(T) = 0$ and the stored values for P, S, S^D, S^C, E, I, D, and R, solve λ_i backward in time according to its differential equation in the optimality system.
4. Update the control by entering the new S, S^C, S^D, E, I, D, R, and λ_i values into the characterization of P.
5. Check convergence. If values of the variables in this iteration and the last iteration are negligibly small, output the current values as solutions. If not, go back to Step 2 and repeat until convergence.

CONCLUSION/FUTURE WORK

From preliminary tests for values of P we find that parental influence has an impact on all compartments. Therefore, it can be posited that malnutrition in the U.S. is indeed societal, and parental influence has the power to significantly impact an individual's food choices. More experiments need to be conducted to clarify this effect. This will be done through the forward-backward sweep algorithm to minimize the objective function, showing the influence of the control on the non-linear dynamics, and conducting more studies.

REFERENCES

Harvard Law School Students. Choosing Big Food: Who is responsible for widespread malnutrition in America?Systemic Justice Journal: Vol. 1 https://systemicjustice.org/article/choosing-big-food-who-is-responsible-for-widespread-malnutrition-in-america/ Population health management, 14(6), 307–315. https://doi.org/10.1089/pop.2010.0071

Joshi, H. R., Lenhart, S., Sanjukta, H., & Agusto, F. (2015). Optimal Control of an SIR Model With Changing Behavior Through and Education Campaign. Electronic Journal of Differential Equations, 50, 1–14. http://ejde.math.txstate.edu

Mahmood, L., Flores-Barrantes, P., Moreno, L. A., Manios, Y., & Gonzalez-Gil, E. M. (2021). The Influence of Parental Dietary Behaviors and Practices on Children's Eating Habits. Nutrients, 13(4), 1138. https://doi.org/10.3390/nu13041138

Ohajunwa, C., Kumar, K., & Seshaiyer, P. (2020). Mathematical modeling, analysis, and simulation of the COVID-19 pandemic with explicit and implicit behavioral changes. Computational and MathematicalBiophysics, 8(1), 216-232.

[Savage, J. S., Fisher, J. O., & Birch, L. L. (2007). Parental influence on eating behavior: conception to adolescence. The Journal of law, medicine & ethics : a journal of the American Society of Law, Medicine & Ethics, 35(1), 22–34. https://doi.org/10.1111/j.1748-720X.2007.00111.x