False Vacuum at the Heart of Black Holes

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It is proposed that the evaporation of black holes via Hawking radiation may conclude at densities and temperatures no higher than those invoked during the inflationary stage of common cosmological models, thus avoiding the need for quantum gravity or physics at the Planck length in the analysis. The false vacuum associated with cosmological inflation might produce sufficient antigravity to halt the approach to a singularity inside black holes, as matter there is compressed to conditions where a phase transition to false vacuum should occur. This conjecture is supported by a very simplified spherical model of a stationary nonsingular black hole containing some false vacuum in a central part of the spacetime well inside the Schwarzschild event horizon.

I. FALSE VACUUM INSIDE BLACK HOLES

False Vacuum, matter with $T^{\mu\nu} = -\rho g^{\mu\nu}$, plays two important roles in current routinely used cosmological models. A strongly interacting false vacuum phase of matter at high densities [1, esp. Ch.8] is invoked in inflationary models of the early universe, and a false vacuum phase of some matter with very weak non-gravitational interactions [2] is invoked as the predominant component of the current universe. Intellectual curiosity about the fate of matter that collapses into the interior of black holes then forces one to consider that the high density false vacuum phase may reappear as this matter is compressed and heated toward the Schwarzschild $r = 0$ singularity in its future. But false vacuum in the Einstein equations produces anti-gravity, predicting, e.g., that the present expansion of the universe is accelerating. The non-singular black hole model presented here, although not yet a dynamics of black hole formation and evaporation, suggests that a phase transition to false vacuum at high densities inside a black hole might prevent the formation of a singularity. This could provide a very different zero-order starting point for many black hole questions such as their evaporation via Hawking radiation, the unitarity of quantum field theories on black hole spacetimes, and the location of the large information deficit represented by black hole entropy. And it suggests that many aspects of these questions could fall in the domain of quantum field theory in curved spacetime, without requiring as a prerequisite a quantum theory of gravity or laws of physics valid at the Planck length.

II. A NONSINGULAR BLACK HOLE

An ongoing attempt to devise nonsingular black holes was initially motivated by their possible applications in numerical relativity. There their usefulness would be in avoiding some computational problems associated with singularities while only violating the laws of physics inside the black holes (where this has no effect on the radiation emitted from, e.g., inspiralling black hole binaries). These efforts, however, have led also to nonsingular black hole models where familiar energy conditions are violated, but in important regions only by familiar, if baffling, matter—false vacuum (a.k.a. dark energy, cosmological constant).

The proposed example of a nonsingular black hole is the metric

$$ds^2 = -dt^2 + \left(dx^i + x^i\sqrt{\frac{2M}{r^3}}dt\right)^2 \tag{2.1}$$

where a Euclidean sum of squares occurs on the space indices. With the choice $M(r) = \text{const}$ where $r^2 = x^k x^k$ this is the Schwarzschild spacetime with the Panlevé-Gullstrand choice [3, §3.3.3] of time coordinate. My choice for a nonsingular example uses a different $M(r)$ in the form

$$M(r) = \int_0^r \rho(r)\, 4\pi r^2 dr \tag{2.2}$$

which will require a nonzero stress-energy tensor in the Einstein equations. The case I find most interesting is where ρ is chosen to be constant in a central region and then smoothly tapered to zero while well inside the event horizon of the Schwarzschild black hole. As an example choose

$$\rho(r) = \frac{3}{8\pi}\begin{cases} 1 & \text{if } 0 \le r \le 5/4 \\ 1 - 12(r-5/4)^2 & \\ \quad + 16(r-5/4)^3 & \text{if } 5/4 \le r \le 7/4 \\ 0 & \text{if } 7/4 \le r \quad . \end{cases} \tag{2.3}$$

This gives $M(r) = r^3/2$ and $\sqrt{2M(r)/r^3} = 1$ in the region $0 \le r \le 5/4$ so the metric is clearly nonsingular there. The taper to $M = \text{const}$ can be made C^∞ if desired, but this example uses a polynomial that gives a C^2 metric. The unit of length used in equation (2.3) was chosen to put the deSitter anti-horizon at $r = 1$ and results, via the details of (2.3), in a total mass of $M = 549/320 = 1.715625000$ so the Schwarzschild event horizon is located at $r = 3.43125$, well outside the region $r \leq 1.75$ occupied by the magic matter.

It is now easy [4] to calculate the Einstein tensor and discover via $G^{\mu\nu} = 8\pi T^{\mu\nu}$ what sort of matter would be needed to produce this nonsingular black hole. One finds, in spherical coordinates, with

$$ds^2 = -dt^2 + \left(dr + dt\sqrt{2M(r)/r}\right)^2 + r^2 d\Omega^2 \tag{2.4}$$

that $G^\mu{}_\nu$ is diagonal with $G^t{}_t = G^r{}_r = -2M'/r^2$ and with $G^\theta{}_\theta = G^\varphi{}_\varphi = -M''/r$. Translated to the needed stress-energy tensor where $-T^t{}_t$ is the energy density and the other diagonal elements are the stresses (or anisotropic pressures) it gives

$$T^\mu{}_\nu = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & P_r & 0 & 0 \\ 0 & 0 & P_\perp & 0 \\ 0 & 0 & 0 & P_\perp \end{pmatrix} \quad . \tag{2.5}$$

Here $\rho(r)$ is the same ρ introduced in equation (2.2) while $P_r = -\rho$, $P_\perp = -\rho - \frac{1}{2}\rho' r$. For the choice of ρ in equation (2.3) these values are plotted in Figure 1.

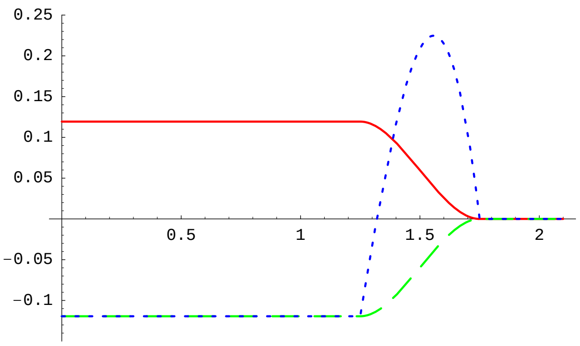

FIG. 1. The distribution of the matter defined by equations (2.3) and (2.5). The solid curve is the energy density $\rho(r)$, while the dashed curve is the radial stress $P_r(r)$ in the same units. The dotted curve is $P_\perp(r)$, the transverse stress. Note that for $r \leq 5/4$ the stress energy tensor reduces to a false vacuum condition (cosmological 'constant') since $P_r = P_\perp = -\rho$.

A further calculation of the Riemann tensor (using GRTensor) in the region where $\rho(r) = 3/8\pi = \text{const}$ gives the result

$$R^{\mu\nu}{}_{\alpha\beta} = \delta^\mu_\alpha \delta^\nu_\beta - \delta^\mu_\beta \delta^\nu_\alpha \tag{2.6}$$

showing that this part of the spacetime has constant unit curvature, and is therefore a sector of the deSitter cosmology. Although neither the properties of the False Vacuum matter in this part of the spacetime, nor the local geometry and curvature, define any preferred rest frame, the boundary conditions in the full metric ($M(r) = \text{const}$ at large r) do. Information (or small debris) falling into the black hole from distant regions will allow apparatus in the deSitter region to locate the origin and its velocity, much as the velocity of the solar system can be seen from cosmic microwave radiation even though this radiation has a negligible influence on the current spacetime curvature.

There are three distinct regions in this spacetime based on the different properties of matter assumed. The inner "heart" of the black hole is the region occupied by false vacuum with $\rho = \text{const}$. This is surrounded by an "impedance matching" region where the properties of the needed matter are even more unfamiliar; the foundations for such matter pose the greatest challenge to the development of this model. Note, however, that because of the condition $P_r = -\rho$, the stress-energy tensor of this matter has no unique timelike eigenvector, and thus shares with false vacuum the property that its 4-velocity cannot be defined, thus evading any concern that it may be moving faster than light. Outside this matter is a conventional Schwarzschild black hole, including its horizon somewhere outside the central matter.

A different set of three regions is based on causality considerations as explored below.

III. CAUSALITY AND HORIZONS

The inward and outward edges of the light cones in the metric (2.1, 2.2) are given by the radial null vectors

$$\begin{aligned} \ell &= \partial_t + (x^i/r)(1 - \sqrt{2M/r})\partial_i \\ &\text{and} \\ n &= \partial_t - (x^i/r)(1 + \sqrt{2M/r})\partial_i \end{aligned} \tag{3.1}$$

so one sees that the entire light cone tips inward (toward smaller r) whenever $2M(r)/r > 1$, and then rights itself near the origin where $2M/r \approx (8\pi/3)\rho(0)r^2 \to 0$. Since $2M(r)/r$ is zero both at infinity and at the origin, it must reach a maximum in between. We assume this is at a value greater than unity so that there is a conventional black hole event horizon. In the simple case considered here where $2M(r)/r$ falls monotonically away from its maximum there are just two horizons where the Killing vector ∂_t is null and $2M(r)/r = 1$. Our working model (2.3) places the inner horizon in the deSitter region where its properties can more easily be explored.

Of future interest will be the limiting case where the two horizons here coincide, i.e., where $2M(r)/r$ achieves a maximum exactly at the value 1. This case should arise as the outer event horizon shrinks due to Hawking radiation. For then the two horizons would evolve toward each other and annihilate, freeing the central matter to decay into the surrounding external world where we live.

Although our working model is patched together of three regions with different types of matter, all respect the time independence and spherical symmetry assumptions. Consequently constants of motion in the geodesic equations will allow us to relate geodesics in the various regions without necessarily studying in detail their behaviors in the impedance matching region. Of particular interest are the negative energy geodesics in the region between the horizons where the ∂_t Killing vector is spacelike. These negative energy states should allow for quantum particle creation in which pairs of zero total energy are created, with the positive energy particle emerging in the region outside the Schwarzschild event horizon, or inside the deSitter anti-horizon, while the negative energy particle is confined to the region between the horizons. This process is particularly easy for the deSitter horizon, since the positive energy particle from a pair created between the horizons naturally (classically) falls in through the deSitter horizon rather than having to originally materialize there.

For this model a most important consequece of negative energy geodesics is that pairs of particles with zero total energy can move on geodesics which contribute, via their transverse velocities, to a transverse pressure. This may be useful in constructing a physical picture of the matter distribution in the impedance matching region where the required transverse stresses greatly exceed the net matter energy.

IV. GEODESICS AND MAGIC MATTER

Geodesics can be studied through the variation principle $\delta I = 0$ with $I = \int (p_\mu \, dx^\mu - \mathcal{H} d\lambda)$ where $2\mathcal{H} = g^{\mu\nu}(x) p_\mu p_\nu + m^2$ and $\mathcal{H} = 0$ is imposed as an initial condition. The solutions $x^\mu(\lambda), p_\mu(\lambda)$ will have a path parameter λ related to proper time by $m \, d\lambda = d\tau$ in the timelike case, while λ will be an affine parameter leading to $p_{\mu;\nu} p^\nu = 0$ also in the case of null geodesics. Several different coordinate systems can be used here, and the relations between the momenta in each are found when the above action integral is converted from one to another. Among the coordinate systems of interest are the "rectangular" coordinates of equation (2.1) where

$$2\mathcal{H} = -\left(p_t - p_i x^i \sqrt{2M/r^3}\right)^2 + p_i \, p_i + m^2 \qquad (4.1)$$

and the related spherical coordinates with

$$2\mathcal{H} = -\left(p_t - p_r \sqrt{\frac{2M}{r}}\right)^2 + p_r^2 + \frac{L^2}{r^2} + m^2 \qquad (4.2)$$

where $L^2 \equiv p_\theta^2 + (p_\varphi^2 / \sin^2\theta)$ will be a constant of motion. In the deSitter region one also uses the coordinates $X^i = x^i \exp(t)$ which yield a conventional deSitter metric form

$$ds^2 = -dt^2 + \exp(-2t) dX^i \, dX^i \qquad (4.3)$$

so

$$2\mathcal{H} = -P_0^2 + \exp(2t) P_i P_i + m^2 \quad . \qquad (4.4)$$

In equation (4.2) one defines the conserved energy E as $E = -p_t$ to be appropriate at $r \to \infty$. Then in the Hamilton equation $dt/d\lambda = \partial\mathcal{H}/\partial p_t$ one finds

$$\frac{dt}{d\lambda} = E + p_r \sqrt{\frac{2M}{r}} = \sqrt{p_r^2 + (L^2/r^2) + m^2} > 0 \qquad (4.5)$$

and demands positivity so that the geodesic is followed along the future light cone. [The hypersurfaces $t = $ const are always spacelike in these metrics, so the future light cone is always in the sense of increasing t.] This positivity of $dt/d\lambda$ can be maintained, however, with negative E if the second term has p_r sufficiently positive. But $\sqrt{p_r^2 + (L^2/r^2) + m^2} > |p_r|$ so the inequality in (4.5) implies

$$E > |p_r| \left[1 - \sqrt{2M/r}\right] \qquad (4.6)$$

and $E < 0$ is only possible if both $p_r > 0$ and $2M/r > 1$. One finds that $E < 0$ can be satisfied (at least by some null rays) throughout the region between the horizons.

With the aid of the constants of motion P_i from the Hamiltonian of equation (4.4) one can find all the geodesics [5, §8.11] [6] [7, Appendix C] in the deSitter region explicitly. All null geodesics have the form $x^i = n^i + e^{-t}(x_0^i - n^i)$ with $n^i = P_i/P$ and $P = \sqrt{P_k \, P_k} > 0$. Thus all null geodesics in the deSitter region, and hence all null geodesics inside the Schwarzschild event horizon in the working model (2.3), end up approaching the deSitter horizon as $t \to \infty$ in the sense that $r \to 1 = n^i n^i$. The energy of such geodesics is found from $E = -P_i X^i - P_0$ to be $E = P n^i (n^i - x_0^i)$ and is positive only for those null rays that enter the core, the interior of the deSitter anti-horizon, before returning to that horizon.

We call $r = 1$ an anti-horizon because it attracts null rays rather than repelling them as does the Schwarzschild horizon. Thus no null rays (except the horizon generators) remain near the Schwarzschild horizon, while no null ray inside the Schwarzschild horizon can avoid the deSitter anti-horizon. An important point to note here, however, is that the sector of the deSitter universe used here is collapsing rather than expanding as in most discussions which use this metric in cosmology. Note also that timelike and null geodesics in the deSitter region reach $t = \infty$ at finite values of the affine parameter λ. This is due to the limitations of the coordinate system, and these geodesics can be continued in other representations of deSitter spacetime. But we anticipate that effects from the evaporation of the black hole will also play an important role in the fate of these geodesics, so we do not study them further here.

Magic Matter is by definition something that will divert spacetime evolution away from singularities. It therefore has unusual properties. One candidate for this job is a distribution of free particles or waves, many in negative energy states, in the region between the two horizons where $2M/r > 0$. Matter models of this kind (without the negative energy states) have been used in spherical symmetry by Einstein [8] and by Wheeler [9,10]. Thus we inspect the possibities of a stress-energy tensor which is the sum of terms $p^\mu p_\nu$ for null vectors p_μ in the region between the horizons. For a single term $T^\mu{}_\nu = g^{\mu\alpha} p_\alpha p_\nu$ we find from the metric in equation (4.2) that $T^\theta{}_\theta = p_\theta^2/r^2$ and $T^\varphi{}_\varphi = p_\varphi^2/r^2 \sin^2\theta$, as indications of the transverse stresses, are positive for all momenta. But $-T^t{}_t = E\sqrt{p_r^2 + L^2/r^2}$ and $T^r{}_r = p_r[p_r - \sqrt{2M/r}\sqrt{p_r^2 + L^2/r^2}]$ as indications of energy density and radial pressure, can both be negative when $p_r > 0$ and $2M/r > 0$ allow negative particle energy E. It is therefore not impossible that some mixture of positive and negative energy particles being created in the region between the horizons could allow at least an evolving nonsingular black hole somewhat similar to the stationary example of Figure 1.

V. OPEN QUESTIONS

The classic Gibbons and Hawking work [11] on thermal radiation associated with deSitter horizons needs to be reconsidered for this application in view of the very different assumptions appropriate here. Most importantly, our deSitter sector is collapsing rather than expanding, so matter in this region can see the entire past including the outside of the Schwarzschild horizon. Thus entropy here does not accrue due to lack of information about the outside. But, also, there is a preferred rest frame based on the exterior of the black hole, so that any thermal radiation would not necessarily be produced by the motion of the observer, but could be real enough to make a contribution to the stress-energy tensor in Einstein's equations. In addition to exploration of modified Hawking radiation in deSitter space under these conditions, one would like to see such studies where the vacuum exterior was also included. For such studies a limiting case of the metric from equation 2.3 could be appropriate where the density drops discontinuously from $\rho(0)$ to zero at some radius r_b. From the equation $P_\perp = -\rho - \frac{1}{2} r\, d\rho/dr$ required by the Einstein equations one then has a high compression shell ("geodesic dome") with $P_\perp = +(3/8\pi) r_b \delta(r - r_b)$ sustaining the tension of the false vacuum to its interior. Since both the positive and negative energy particles, created quantum mechanically in the region between the horizons, fall toward the core inside the deSitter anti-horizon, it may be that these pairs have no gravitational effect inside the anti-horizon, but only enter the Einstein equations just outside the anti-horizon which negative energy particles are unable to cross. As such back-reaction from the quantum pairs is considered, a more general spherically symmetric metric may be required. In stationary cases changing $r^2 d\Omega^2$ in equation (2.4) to $R(r)^2 d\Omega^2$ gives the most general stationary spherically symmetric metric. In time dependent cases it will likely be necessary to relax the condition $g^{00} = -1$ in order to avoid coordinate singularities. Also open is the question whether negative energy particles created under the influence of the de Sitter anti-horizon may, when the horizons are close, give stimulated emission of particles outside the Schwarzschild horizon.

Of course explicit models of nonlinear scalar fields converging to produce nonsingular black holes would be very desireable, and studies are underway in collaboration with Scott Hawley in the hope of constructing such models numerically.

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