

ABSTRACT

Title of Dissertation: CHARACTERIZATION OF GAP-ENGINEERED JOSEPHSON JUNCTIONS AND GATE FIDELITIES FOR A SUPERCONDUCTING QUBIT

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Quantum computing promises applications in physics, cryptography, material science, pharmaceuticals, and a wide range of other science. Superconducting qubits offer a possible platform for developing a quantum computer. To perform useful quantum computations, the coherence and control of present day superconducting qubits must be greatly improved. In this dissertation, I present two main results to improve the performance of transmon qubits.

For the first project, I fabricated and characterized the coherence of transmon devices with asymmetric superconducting gaps. Previous models suggested that devices with asymmetric superconducting gaps on either side of the Josephson junction can be designed to be less subject to loss from quasiparticle tunneling. To gap-engineer the Josephson junctions, I used Ti metal to proximitize and lower the superconducting gap of the Al counter-electrode. Unfortunately, the energy relaxation time constant for an Al/AlO_x/Al/Ti 3D transmon I fabricated and tested was $T_1 = 1 \mu\text{s}$, over two orders of magnitude shorter than the mea-

sured $T_1 = 134 \mu\text{s}$ of an Al/ AlO_x /Al 3D transmon with Al capacitor pads and the measured $T_1 = 143 \mu\text{s}$ of an Al/ AlO_x /Al 3D transmon with Ta capacitor pads. DC IV measurements of proximitized Josephson junctions showed a reduced superconducting gap, demonstrating that the gap-engineering in the Al/Ti layer was successful. However, these same IV measurements showed greatly increased excess current for voltage biases below the superconducting gap compared to my Al/ AlO_x /Al junctions. This suggests the addition of Ti caused the junction quality to worsen, potentially being a source of tunneling loss in the transmon devices. Intentionally adding oxygen disorder between the Al and Ti layers reduced the proximity effect and subgap current in DC measurements while increasing the relaxation time of a 3D transmon to $T_1 = 32 \mu\text{s}$.

Additionally, I designed an Al/ AlO_x /Al SQUID device to perform DC IV measurements of junctions with tunable total critical current. In a single junction, subgap tunneling features can be due to the critical current interacting with the environment, subgap quasiparticle processes, or other sources. Reducing the critical current allows these features to be differentiated and more accurately measure the effects from quasiparticle tunneling alone. Characterizing this device showed subgap tunneling features consistent with inelastic Cooper pair tunneling and quasiparticle transport via multiple Andreev reflection in a low transparency junction. This measurement technique could be used to further study gap-engineered junctions.

For the second project, I characterized an Al/ AlO_x /Al 2D transmon device with Ta features and performed high-fidelity single qubit gates. First, I used error amplifying pulse sequences to fine-tune the qubit gate pulses. I evaluated the gate error with randomized benchmarking. I characterized gates with Gaussian and cosine shaped pulses at a variety of

pulse lengths. Analyzing the pulse envelopes in the frequency domain and directly measuring leakage to the transmon's second excited state revealed that leakage from driving higher qubit transitions was a major source of gate error. Next, I characterized gates using a pulse shape designed by a physics informed neural network designed by Güngördü and Kestner and found improved gate error for 16 ns pulses achieving an average error per gate of $(3.36 \pm 0.03) \times 10^{-4}$. This outperformed errors of $(5.54 \pm 0.24) \times 10^{-4}$ for a cosine shaped pulse and $(3.93 \pm 0.12) \times 10^{-4}$ for a Gaussian shaped pulse of the same length. Further optimization of the pulse using predistortion or leakage reduction strategies may yield even greater performance.

CHARACTERIZATION OF GAP-ENGINEERED
JOSEPHSON JUNCTIONS AND GATE FIDELITIES
FOR A SUPERCONDUCTING QUBIT

by

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Dedication

To my dad, who would have been excited to read this.

Acknowledgements

Simply making it to UMD, let alone completing my Ph.D. here, took the support and care of an enormous number of people. I am beyond thankful for each and every one of them. To keep this section shorter than the dissertation that follows, I have only included only the briefest mention of some of these people and how they helped me.

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List of Abbreviations

- ADC** analog to digital converter. 96
- AWG** arbitrary waveform generator. 91, 94, 105, 182, 189
- BBQ** black box quantization. 58, 65, 66
- BCH** Baker-Campbell-Hausdorff. 49, 213, 219, 223
- BCS** Bardeen, Cooper, and Schrieffer. 4, 6, 7, 10, 13, 139, 147
- BFGS** Broyden-Fletcher-Goldfarb-Shanno. 183
- BTK** Blonder, Tinkham, and Klapwijk. 6, 20, 21
- cavity QED** cavity quantum electrodynamics. 41
- CPMG** Carr-Purcell-Meiboom-Gill. 116
- CPW** coplanar waveguide. 86, 150
- cQED** circuit quantum electrodynamics. 41, 42, 44, 58, 59
- CW** continuous wave. 91, 100
- DRAG** Derivative Reduction by Adiabatic Gate. 118, 120, 121, 174, 184, 187
- EPG** error per gate. 122, 126, 175, 187, 192, 195
- FPGA** field programmable gate array. 96
- HAADF** high-angular annular dark-field. 153
- HEMT** high electron mobility transistor. 89, 90
- HFSS** High Frequency Simulation Software. 63, 64
- IR** infrared. 91
- LPG** leakage per gate. 175

MAR Multiple Andreev Reflection. 4–6, 23, 53, 140, 147, 158, 164, 169, 194, 204, 206

NMP N-Methyl-2-pyrrolidone. 71, 85

PCB printed circuit board. 98, 128

PID proportional integral derivative. 88

PINN physics informed neural network. 4, 5, 170, 179, 181, 182, 185–189, 195

PSD power spectral density. 92, 173, 175, 176, 184, 186

QSHO quantum simple harmonic oscillator. 33

RF radio frequency. 154, 166, 167

SMA SubMiniature version A. 97, 98

SNR signal-to-noise ratio. 90

SQUID Superconducting QUantum Interference Device. 5, 29, 86, 147, 158–160, 162, 163, 167, 194, 210

STEM scanning transmission electron microscopy. 153, 155

TLS two-level system. 55, 56, 58, 61, 140, 197, 199, 201

TWPA travelling-wave parametric amplifier. 89–91

VNA vector network analyzer. 64, 91, 100

WKB Wentzel-Kramers-Brillouin. 15, 17

Chapter 1

Introduction

In 1981, Richard Feynman gave a talk on using quantum systems to simulate quantum physics [1]. In this talk, he introduced the idea of a quantum computer as distinct from a classical computer. Specifically, he argued the binary coding of a classical computer would make it difficult and resource intensive to simulate quantum systems. Instead he proposed using a quantum system to simulate an analogous quantum system can have significant benefits.

Quantum mechanics describes the behavior of atomic scale systems, and quantum computation requires the precise manipulation and measurement of these quantum systems. The field of quantum computing seeks to develop, engineer, and control quantum systems and use these systems to perform quantum computations and simulate problems of interest.

1.1 The Potential of Quantum Computers

Though discussed around the time Feynman coined the term, quantum computation gained serious academic, industry, and societal interest with the publication of Peter Shor's factoring algorithm [2]. This algorithm showed a theoretical exponential speed-up in factoring using a quantum computer vs a classical computer. Prime factoring features prominently in modern RSA cryptography implying that a sufficiently powerful quantum computer could decrypt

most modern secure communication, generating serious industry and government interest. While the reality of implementing cryptologically relevant algorithms is still far in the future, interest in quantum algorithms with computational advantage, as well as simulations with current or upcoming hardware abounds [3–5].

1.2 Building Blocks of a Quantum Computer

The essential components of a quantum computer are called qubits, short for quantum bits. In principle, a qubit can be physically implemented with any quantum system with two addressable quantum energy levels. DiVincenzo laid out five criteria required for qubit operation [6, 7]:

1. A scalable physical system with well-defined qubit
2. Reliable state preparation
3. Long decoherence times compared to computation time
4. Accurate quantum gate operations, including entangling gates
5. Ability to measure the quantum state

A wide range of possible qubit systems have been experimentally demonstrated including, but not limited to, trapped ions [8], photons [9], neutral atoms [10], semi-conductor spins [7, 11], and superconducting circuits [12]. My dissertation work has involved superconducting qubits, lithographically defined circuits of superconducting metal which exhibit quantum-mechanical properties. Unlike most other realizations of quantum computers, superconducting circuits are macroscopic objects up to millimeters in size. This makes the systems especially vulnerable to loss mechanisms via coupling to the environment around them.

1.3 The Need to Improve Qubit Performance

Qubit decoherence or control errors limit the effective length of a quantum computation that can be performed. While there have been a number of reports of “quantum supremacy” or “quantum utility” [13–16], where a quantum computer is claimed to outperform a classical computer, there have been a corresponding number of refutations of the details of these claims [17–20]. Implementation of ground-breaking, or even practical, quantum algorithms requires more operations than are achievable on modern devices. While theoretical efforts in quantum error correction provide paths to robust qubit operation [21], these error-corrected logical qubits are made up of physical qubits and individual controls. The fidelity requirements on these constituent components are right around the break even point for these error-correcting codes to be beneficial [22, 23]. Therefore, improving qubit lifetimes and control fidelity is paramount for achieving useful quantum computers.

My dissertation work concerns increasing both qubit lifetimes and improving qubit gate fidelity. My first main project involved the study of gap-engineered transmons. Transmons with different superconducting gaps on either side of the Josephson junction, such that the gap energy difference is greater than the qubit frequency, should be protected from relaxation due to quasiparticle tunneling from the low-gap side to the high-gap side [24–26]. Additionally, gap-engineered transmons have been recently shown to be resistant to quasiparticle bursts caused by cosmic ray impacts [27, 28]. To test these ideas, I fabricated devices with large gap energy differences utilizing the superconducting proximity effect in AlTi superconducting bilayers. Unfortunately, I found that other loss sources dominated these devices and measurements of DC junction characteristics, resonator quality factors, and qubit relaxation times helped disentangle the sources of loss in superconducting devices with AlTi bilayers.

My second project involved improving transmon qubit control. For this work, I tuned and characterized high-fidelity single qubit gates [29, 30] on a transmon device. I found that the power spectral density of a given pulse at the next highest transmon transition frequency

was proportional to the error of a given gate, suggesting that leakage to higher transmon levels was a significant source of error in my gates. Finally, I implemented a new pulse shape designed by a physics informed neural network (PINN) for high-fidelity transmon gates [31]. This pulse performed better than the Gaussian or cosine pulse shapes commonly used by the superconducting qubit community. Further optimization of these pulses may lead to improved qubit control and application of the PINN framework to other control sequences may lead to other improved qubit gates.

1.4 Dissertation Outline

In Chapter 2, I cover the basic physics of superconductivity theory and junction tunneling theory. In addition to the standard quasiparticle tunneling, Josephson tunneling, and Bardeen, Cooper, and Schrieffer (BCS) theory, I discuss more complex tunneling theories including Multiple Andreev Reflection (MAR) and inelastic Cooper pair tunneling in Josephson junctions.

In Chapter 3, I introduce the mathematics of a qubit and how this can be implemented in superconducting circuits with details of the transmon qubit including its energy structure, coupling, and common loss mechanisms.

In Chapter 4, I consider the design of a 3D transmon and simulation techniques I used to verify these designs.

In Chapter 5, I discuss the steps I followed to fabricate a transmon device, covering a range of fabrication steps and techniques used for different devices that I describe in this dissertation.

In Chapter 6, I discuss the hardware and supporting technologies required to perform superconducting qubit measurements, including dilution refrigeration, microwave thermalization and filtering, signal readout, and device packaging.

In Chapter 7, I cover the series of measurements I used to characterize my qubits and

control gates. This includes spectroscopic measurements to identify energy level spacings of the transmon and resonant cavity, time domain measurements to characterize the coherence of my qubits, and gate tuning and benchmarking used to tune and evaluate high-fidelity qubit gates.

In Chapter 8, I introduce the techniques used to measure the DC conductance properties of tunnel junctions and give an example using these techniques to characterize an NIS tunnel junction and show the device’s functionality as an electron thermometer.

In Chapter 9, I discuss my analysis of energy loss in gap-engineered transmon devices with AlTi electrodes. Using the superconducting proximity effect, I fabricated junctions, resonators, and qubits with reduced and asymmetric superconducting gaps. Measurements of the three types of devices suggest that lossy oxides of Ti and alloys of Al and Ti contribute significant loss to these devices.

In Chapter 10, I discuss my measurements of the differential conductance of a SQUID device and distinguish supercurrent tunneling features from quasiparticle tunneling features by suppressing the critical current of the device with external magnetic flux. This reveals features consistent with MAR for a low transparency junction.

In Chapter 11, I discuss experiments on single-qubit pulse fidelities. I measure gate errors for two standard pulse shapes and find the error and leakage per gate depend on the power spectral density of the pulse at the next-highest transmon transition, suggesting leakage reduction strategies as a path to improving gate fidelity. Additionally, I implement a pulse designed by a physics informed neural network (PINN) and demonstrate a small improvement in fidelity over the common pulse shapes with average error per gate of 3.4×10^{-4} for a 16 ns qubit gate.

Finally, I conclude with a summary of my main results and some suggestions for future work.

Chapter 2

Tunneling Theory

A Josephson junction is formed when a tunnel barrier exists between two superconducting electrodes such that Cooper pairs can tunnel through the barrier. Josephson junctions are a key component for superconducting qubits, and the associated physics is instrumental in understanding the behavior, limitations, and loss in these qubits.

This chapter covers the physics of a Josephson junction relevant to my measurements and the superconducting qubit field. I first cover the basics of the microscopic BCS theory of superconductivity including calculations of the superconducting gap and the excitations of the superconducting state, superconducting quasiparticles. I then discuss the critical temperature, T_C , below which superconductivity occurs for the case of a standard superconductor and for a bilayer of two superconductors with different gaps.

Next, I discuss different junction tunneling processes. For junctions between superconductors, tunneling processes involving both quasiparticles and pairs must be considered. I begin by describing quasiparticle tunneling, also known as Gaiever tunneling [32]. I next present two generalizations of quasiparticle tunneling for tunnel barriers with finite transparencies, Blonder, Tinkham, and Klapwijk (BTK) theory [33] and Multiple Andreev Reflection (MAR) theory [34].

For Cooper pair tunneling theory, I will explain the Josephson relations, inelastic scat-

tering in P(E) theory, and modulation of the critical current. In total this chapter should provide a background of the physics of superconductors and Josephson junctions for understanding the experiments of this dissertation.

2.1 BCS Theory

Superconductivity was discovered in 1911 by Heike Onnes shortly after he developed a process to liquify He. The solid mercury he measured had no resistance and expelled all magnetic field when cooled below 4 K [35]. In 1957, Bardeen, Cooper, and Schrieffer (BCS) proposed a microscopic theory for superconductivity where a phonon-mediated pairing potential allowed electrons to form Cooper pairs between other electrons with opposite momentum and spin. This pairing allows the electrons to condense into a single macroscopic quantum state with the property of zero electrical resistance [36]. BCS theory was able to predict important measured properties of the superconducting state including dissipationless DC current, expulsion of magnetic field, and the superconducting energy gap. Here I review the BCS pairing Hamiltonian and occupation probabilities as it is relevant to superconducting junction tunneling and quasiparticle excitations.

2.1.1 BCS Pairing Hamiltonian

The BCS superconducting state can be derived from the Hamiltonian of a normal metal electrons experiencing an attractive potential. I follow the solution by canonical transformation shown in Ref. [37]. Starting from the BCS single-particle pairing Hamiltonian,

$$\mathcal{H} = \sum_{k\sigma} n_{k\sigma} \epsilon_k + \sum_{k,l} V_{kl} c_{k\uparrow}^\dagger c_{-k\downarrow}^\dagger c_{-l\downarrow} c_{l\uparrow} \quad (2.1)$$

where $c_{k\sigma}^\dagger$ ($c_{k\sigma}$) is the creation (annihilation) operators for an electron with momentum k and spin σ , $n_{k\sigma}$ is the occupation of electrons with given spin and momentum, $\epsilon_k = \hbar^2 k^2 / 2m$ is

the kinetic energy of the electrons, V_{kl} is the pairing potential between k and l states. First, the number of particles is controlled by examining $\mathcal{H} - \mu\hat{n}$ where the chemical potential, μ , times the number operator, $\hat{n}$, has been subtracted from the energy. Then $\xi_k = \epsilon_k - \mu$ is defined as the electron's energy relative to μ . Assuming the superconductor is in charge equilibrium, μ is equal to the Fermi energy, E_F , of the metal.

Next, taking the mean field approximation

$$c_{-k\downarrow}c_{k\uparrow} = b_k + (c_{-k\downarrow}c_{k\uparrow} - b_k) \quad (2.2)$$

$$b_k = \langle c_{-k\downarrow}c_{k\uparrow} \rangle \quad (2.3)$$

yields the BCS model Hamiltonian

$$\mathcal{H}_M = \sum_{k\sigma} \xi_k c_{k\sigma}^\dagger c_{k\sigma} - \sum_k \left(\Delta_k c_{-k\downarrow}^\dagger c_{k\uparrow}^\dagger + \Delta_k c_{-k\downarrow} c_{k\uparrow} - \Delta_k b_k^* \right) \quad (2.4)$$

Here

$$\Delta_k = - \sum_l V_{k,l} b_l \quad (2.5)$$

which will later be identified as the superconducting energy gap. It is clear in Eq. 2.4 that the electronic states have paired up with other states of opposite momentum and spin.

To diagonalize this Hamiltonian in momentum space, Bogolyubov and Valetin introduced quasiparticle operators γ_{k0} , γ_{k1} such that [38, 39]

$$c_{k\uparrow} = u_k^* \gamma_{k0} + v_k \gamma_{k1}^\dagger, \quad c_{-k\downarrow} = -v_k^* \gamma_{k0} + u_k \gamma_{k1}^\dagger \quad (2.6)$$

$$\gamma_{k0}^\dagger = u_k^* c_{k\uparrow}^\dagger - v_k^* c_{-k\downarrow}, \quad \gamma_{k1}^\dagger = u_k^* c_{-k\downarrow}^\dagger v_k^* c_{k\uparrow} \quad (2.7)$$

where u_k and v_k are complex rotation amplitudes with the normalization constraint $|u_k|^2 + |v_k|^2 = 1$.

Substituting the Bogolyubov operators, Eq. 2.6, into the model Hamiltonian, Eq. 2.4,

and setting $|v_k|^2 = 1 - |u_k|^2 = \frac{1}{2}(1 - \xi_k/E_k)$ diagonalizes the Hamiltonian by canceling any non-particle conserving terms. The resulting Hamiltonian can be written diagonally in the new quasiparticle excitation basis as

$$H_M = \sum_k (\xi_k - E_k + \Delta_k b_k^*) + \sum_k E_k (\gamma_{k0}^\dagger \gamma_{k0} + \gamma_{k1}^\dagger \gamma_{k1}) \quad (2.8)$$

where $E_k = \sqrt{\xi_k^2 + \Delta_k^2}$ is the quasiparticle excitation energy. The second sum in Eq. 2.8 shows that the quasiparticles have a minimum excitation energy Δ_k , which is the BCS pairing potential. However, since the ground state is made of paired electrons, excitations from the ground state are created two at a time costing a total energy of $2\Delta_k$.

2.1.2 Superconducting Gap and Occupation Amplitudes

From the choice of u_k and v_k the definition of the energy gap (Eq. 2.5) can be written at zero temperature as

$$\Delta_k = - \sum_l V_{kl} \langle c_{-l\downarrow} c_{l\uparrow} \rangle = - \sum_l V_{kl} u_l^* v_l \langle 1 - \gamma_{l0}^\dagger \gamma_{l0} - \gamma_{l1}^\dagger \gamma_{l1} \rangle \quad (2.9)$$

$$= - \sum_l V_{kl} u_l^* v_l = - \frac{1}{2} \sum_l \frac{\Delta_l}{E_l} V_{kl} \quad (2.10)$$

which gives the self-consistency condition for the pairing energy for particles at momentum k, l . If V_{kl} is assumed to be constant up to a cutoff and 0 otherwise, the energy gap is independent of k and also constant up to the cutoff, allowing the subscript to be dropped and calling Δ the superconducting energy gap. The self-consistency condition can be used to calculate a numerical value for the gap [37]. By rewriting the final sum in Eq. 2.10 as an integral to the cutoff energy, $\hbar\omega_C$, the pairing energy, Δ , can be evaluated as

$$\Delta \approx 2\hbar\omega_C e^{-1/N(0)V} \quad (2.11)$$

where $N(0)$ is the normal density of states at the Fermi level and ω_C is the Debye frequency of the metal.

For a classic BCS superconductor, including the superconducting materials used in this dissertation, the gap is independent of electron momentum, this is called *s*-wave pairing, drawing further analogue between atomic orbitals and pairing wavefunctions. In some materials, the superconducting gap may depend non-trivially on the electron momentum. The behavior of the pairing potential must accommodate the Fermi statistics of electron exchange; therefore, the pairing wavefunction must be symmetric (antisymmetric) when the spin state of the pair is antisymmetric (symmetric) [40].

With the energy gap, u_k and v_k can be found as a function of energy. As seen in Fig. 2.1, at energies, ϵ_k , well below Fermi energy, the occupation v_k is near unity while u_k is near zero. This reverses for k far above the Fermi-wavevector. For this reason the semiconductor picture of excitations in the superconductor is often adopted [41]. This picture considers v_k^2 the occupation probability for a hole-like excitation and u_k^2 the occupation probability for an electron-like excitation. This way of thinking about the superconducting excitations should not be confused with actual electrons and holes semiconducting materials. For states near E_F , the quasiparticle excitation has finite electron-like and hole-like occupation probability, meaning at these energies the excitations are a superposition of an electron and hole in the superconductor.

2.1.3 Superconducting Density of States

The $\gamma_k^\dagger$ operators correspond to creation operators for quasiparticles with momentum k and energy $E_k = \sqrt{\xi_k^2 + \Delta_k^2}$. The density of states of excitations in the superconductor, N_S , can be found by connecting to the density of states of the electrons in the normal metal, N_n ,

$$N_S(E)dE = N_n(\xi)d\xi \quad (2.12)$$

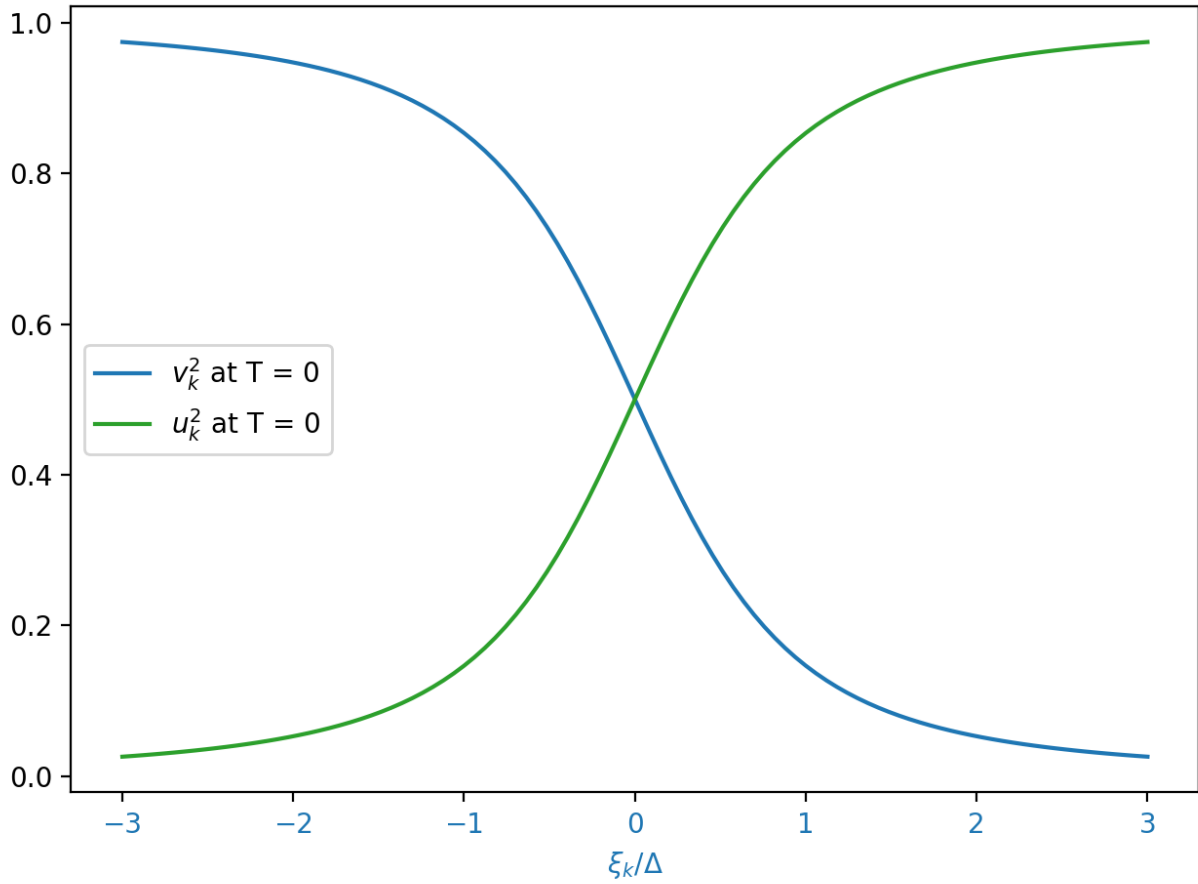


Figure 2.1: Plot of occupation probabilities u_k^2 and v_k^2 around E_F . As energy increases, v_k^2 smoothly decreases and u_k^2 smoothly increases, reaching equal occupations at $\xi_k = 0$. Near this crossing point, excitations in the superconductor consist of a superposition of electron- and hole-like excitations.

Therefore the ratio of superconducting to normal density of states can be calculated

$$\frac{N_S}{N_N} = \frac{d\xi}{dE} = \begin{cases} \frac{|E|}{\sqrt{E^2 - \Delta^2}}, & \text{for } |E| > \Delta \\ 0, & \text{for } |E| \leq \Delta \end{cases} \quad (2.13)$$

where the excitations have an energy gap between $\pm\Delta$ around the Fermi energy. On the energy scales of the gap, N_N can be approximated as constant implying a diverging number of available states at $E = \pm\Delta$.

2.1.4 Critical Temperature

Until this point the superconductor has been assumed to be at $T = 0$. At finite temperatures, the quasiparticle excitations take on a Fermi-Dirac distribution,

$$f(E) = \frac{1}{1 + e^{-E/k_B T}} \quad (2.14)$$

where k_B is Boltzman's constant and T is the temperature of the superconductor. Finite temperature modifies Eq. 2.10 such that

$$\Delta_k = - \sum_l V_{kl} u_l^\dagger v_l (1 - 2f(E_l)) = -\frac{1}{2} \sum_l \frac{\Delta_l}{E_l} V_{kl} \tanh(E_l/2k_B T) \quad (2.15)$$

By definition as $T \rightarrow T_C$, $\Delta(T) = 0$. In this limit and with weak coupling, Eq. 2.15 can be rewritten in integral form giving

$$k_B T_C = 1.13 \hbar \omega_c e^{-1/N(0)V} = \frac{\Delta(0)}{1.764} \quad (2.16)$$

where the relation to Δ is obtained from Eq. 2.11. This prediction of the relationship between the zero-temperature gap and critical temperature is a key result of the BCS theory and is numerically accurate for many materials [37].

2.1.5 Proximity Effect

When a superconductor is placed in contact with a normal metal, the coherent electron interactions extend into the normal metal. Therefore, a thin normal-metal film placed on a superconductor can be “imbued” with superconductivity [42]. This is called the proximity effect. I used the proximity effect to produce junction electrodes with variable gaps. Here I will explain how to calculate the critical temperature of such a bilayer. Then using Eq. 2.16, I can estimate the effective gap of a superconducting bilayer and make predictions of junction behavior.

The Usadel equation generalizes BCS theory for inhomogeneous, diffusive (dirty) superconductors [43]. These equations have been applied to the case of normal-superconductor (NS) and superconductor-superconductor (SS') bilayers [44–46]. Here I follow Zhao, *et al.* deriving the T_C for S-S' bilayers of two superconductors using the Usadel equation.

The Usadel equation parameterizes the superconducting properties of the metal with complex angle $\theta(x, E)$ which depends on both position and energy in the material. The Usadel equation is written in terms of θ as

$$\frac{\hbar D_S}{2} \frac{\partial^2 \theta}{\partial x^2} + iE \sin \theta + \Delta(x) \cos \theta = 0 \quad (2.17)$$

where $D_S = \sigma_S / N_S e^2$ is the diffusion constant for the electrons with the normal density of states in the metal, N_S , σ_S is the conductivity of the metal, and $\Delta(x)$ is the now position dependent superconducting energy gap. $\theta = \pi/2$ corresponds to paired electrons and $\theta = 0$ corresponds to normal metal electrons. The energy gap can be defined

$$\Delta(x) = N_S V_{0,S} \int_0^{\hbar \omega_{D,S}} \tanh \left[\frac{E}{2k_B T} \right] \text{Im}(\sin \theta) dE \quad (2.18)$$

where $V_{0,S}$ is the pair interaction potential, and $\omega_{D,S}$ is the Debye frequency in the metal.

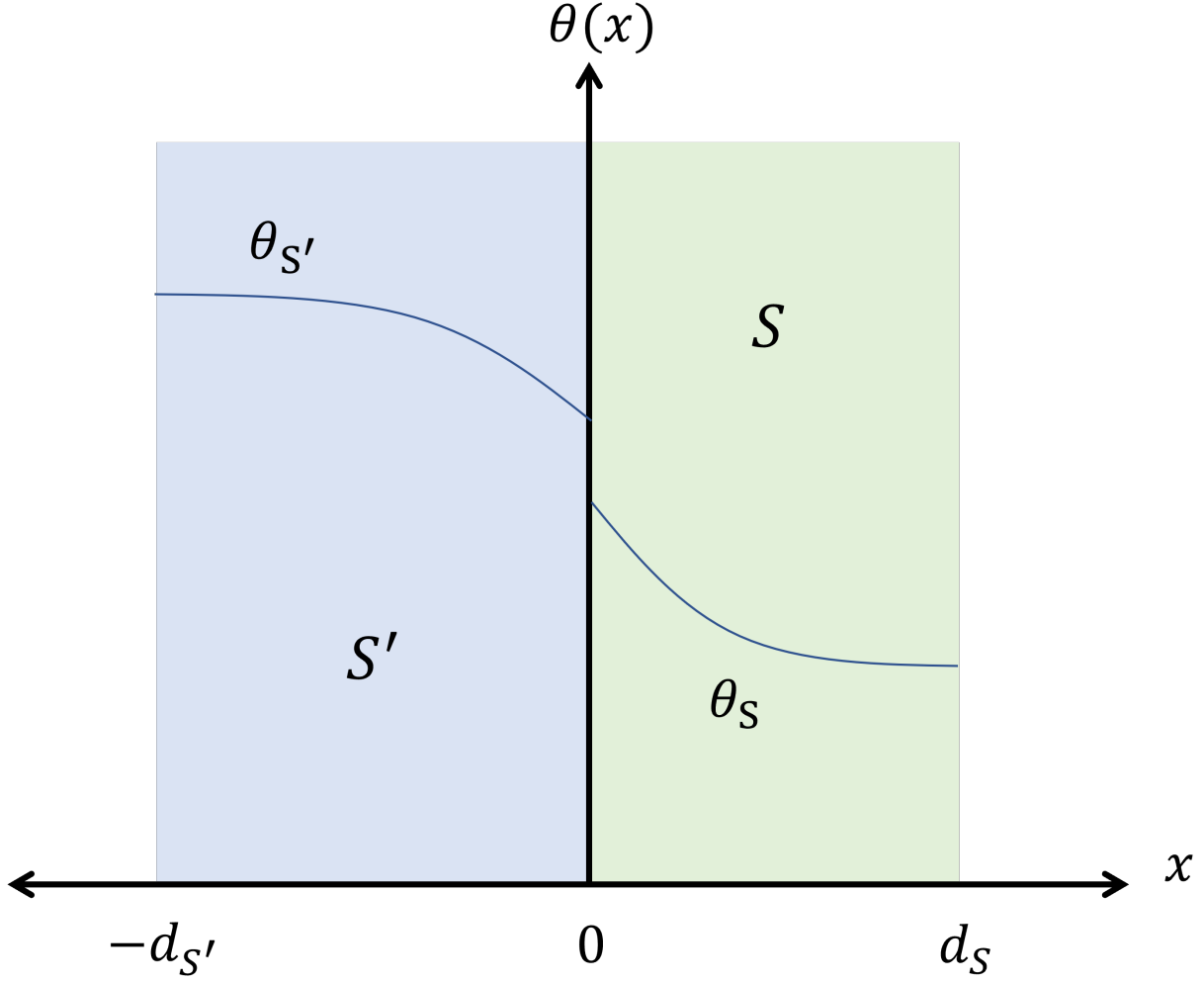


Figure 2.2: Schematic of the Usadel parameter in a superconducting bilayer. For a thin bilayer, $\theta_{S,S'}$ is approximated by a quadratic centered at $x = d_S$ and $x = -d_{S'}$ respectively. There is a discontinuity in both θ and θ' at $x = 0$ described by Eq. 2.22.

The final ingredient for using the Usadel equations is the interface condition

$$\sigma_r \frac{\partial \theta_r}{\partial x} = \sigma_l \frac{\partial \theta_l}{\partial x} = \frac{1}{R_B A} \sin(\theta_l - \theta_r) \quad (2.19)$$

where R_B is the interface resistance and A is the interface area.

These conditions are sufficient for deriving many properties using the Usadel equation. Here for the S-S' bilayer, two additional approximations are made. First, $\partial^2 \theta / \partial x^2$ can be approximated as a constant when θ varies slowly over the bilayer. Second, near T_C , $\theta \ll 1$ because superconductivity is weak when finding T_C [44, 47]. For a bilayer with the first

superconductor, S, in the range $x = (-d_{S'}, 0)$ and the second, S', in $x = (0, d_S)$, yielding three interface equations

$$\frac{\partial \theta_{S'}}{\partial x} = 0 \quad \text{at } x = -d_{S'} \quad (2.20)$$

$$\frac{\partial \theta_S}{\partial x} = 0 \quad \text{at } x = d_S \quad (2.21)$$

$$\sigma_{S'} \frac{2d_{S'}}{\hbar D_{S'}} (\Delta_{S'} + iE\theta_{S'}) = \sigma_S \frac{2d_S}{\hbar D_S} (\Delta_S + iE\theta_S) = \frac{1}{R_B A} (\theta_S - \theta_{S'}) \quad \text{at } x = 0 \quad (2.22)$$

Figure 2.2 shows a schematic of the Usadel approach in the bilayer, adapted from [47]. Next, Eq. 2.22 can be rearranged to express $\theta_{S,S'}$ in terms of Δ_S and $\Delta_{S'}$. Using the small θ approximation in Eq. 2.18 leads to self-consistency equations for the gaps in both layers at $T = T_C$

$$\Delta_S = [\alpha \Delta_S + \beta \Delta_{S'}] \quad (2.23)$$

$$\Delta_{S'} = [\alpha' \Delta_{S'} + \beta' \Delta_S] \quad (2.24)$$

where α, β are integral factors which can be numerically solved for T_C of an arbitrary S-S' bilayer.

For more general calculations using the Usadel equation, I refer the reader to the Yale Boulder school Usadel notes [48].

2.2 WKB Picture

A simple semi-classical model for quantum tunneling is the Wentzel-Kramers-Brillouin (WKB) approximation (see Fig. 2.3). In this picture, the quantum state is written as a plane wave

$$\Psi(x) \propto A_0 e^{ix/\hbar \sqrt{2m(E-V(x))}} \quad (2.25)$$

with A_0 being some initial amplitude of the state with energy E in a potential V . For a tunnel barrier starting at $x = 0$ and extending to $x = a$, the potential is written

$$V(x) = V_0[\Theta(x) - \Theta(x - a)] \quad (2.26)$$

where $\Theta(x)$ is the Heaviside step function and V_0 is the barrier height. A particle with positive energy outside the barrier propagate as a plane-wave. However, if the barrier height exceeds the energy ($V_0 > E$), the wave function experiences exponential decay inside the barrier.

$$\Psi_{\text{barrier}} = A_0 e^{-x/\hbar\sqrt{2m(V_0-E)}} \quad (2.27)$$

Once this state has left the tunnel barrier, it propagates with reduced amplitude

$$\Psi(x > a) = \left[A_0 e^{-a/\hbar\sqrt{2m(V_0-E)}} \right] e^{ix/\hbar\sqrt{2mE}} \quad (2.28)$$

The result in Eq. 2.28 can be used to estimate a junction's tunneling matrix element $\langle l | \hat{T} | r \rangle$ for a quasiparticle tunneling from state $|l\rangle$ on the left of the junction to state $|r\rangle$ on the right of the junction. Estimating the states and Hamiltonian using the WKB picture yields

$$\left| \langle l | \hat{T} | r \rangle \right| \approx \frac{|\Psi(x > a)|}{|\Psi(x < 0)|} = e^{-a/\hbar\sqrt{2m(V_0-E)}} \quad (2.29)$$

Setting $V_0 = 3.5$ eV as half the energy gap of Al_2O_3 [49], the thickness of the tunnel barrier $a = 1$ nm, $m = 9.1 \times 10^{-31}$ kg as the mass of an electron, and $E = 200$ μeV as the energy of a quasiparticle at the superconducting gap edge, the resulting matrix element is 4.1×10^{-5} . This can be used to predict the junction normal state resistance, R_n , and depends exponentially on the barrier thickness.

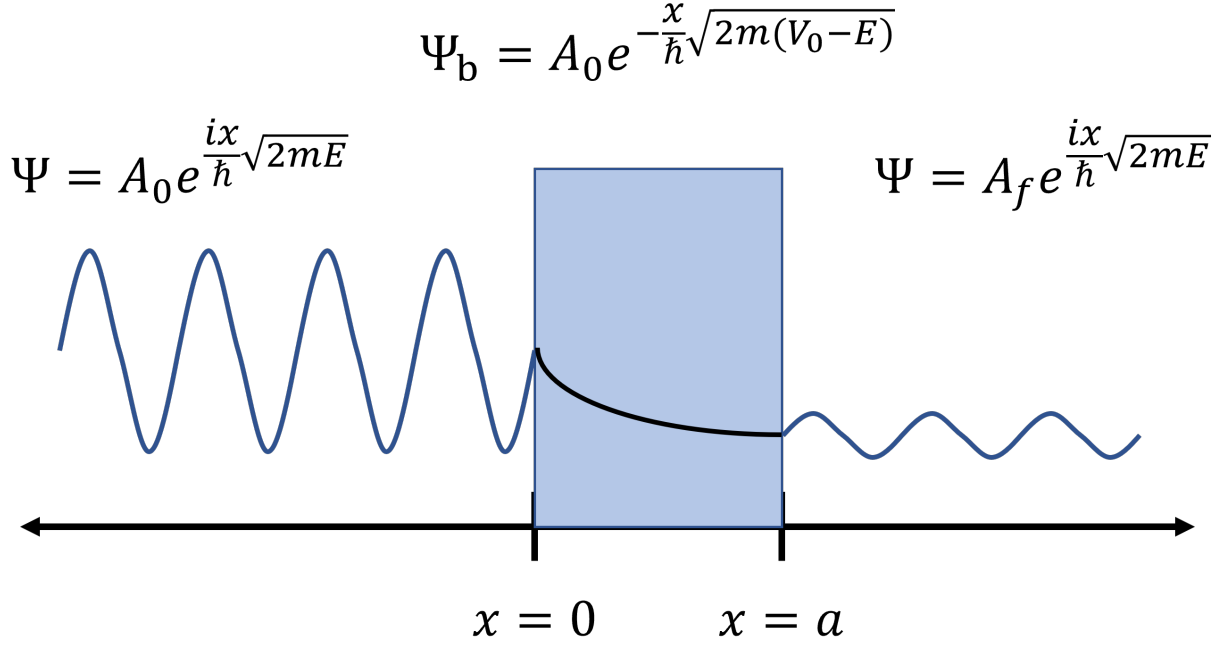


Figure 2.3: Cartoon of a quantum state tunneling under the WKB approximation. The state is a plane wave propagating from the left. In the barrier (blue), the sign of the total energy changes causing the state to exponentially decay. After the barrier, the state is again a propagating wave with reduced amplitude.

2.3 Quasiparticle Tunneling

Now I move beyond the simple WKB picture to actual physics models of tunneling processes in junctions. The transmission of quasiparticles is one important process in any tunnel junction. The tunneling physics that I cover will relate to large junction tunneling as my measurements were of mesoscopic junctions consisting of multiple quantum conducting channels.

2.3.1 Giaever Tunneling

Giaever tunneling describes the conduction of current, I , as a function of bias, V , through a tunnel barrier separating two conductors at temperature, T , [32, 37]

$$I(V) = \frac{4\pi e}{\hbar} \int_{-\infty}^{\infty} |M|^2 N_L(E - eV) N_R(E) [g_L(E - eV, T) - g_R(E, T)] dE \quad (2.30)$$

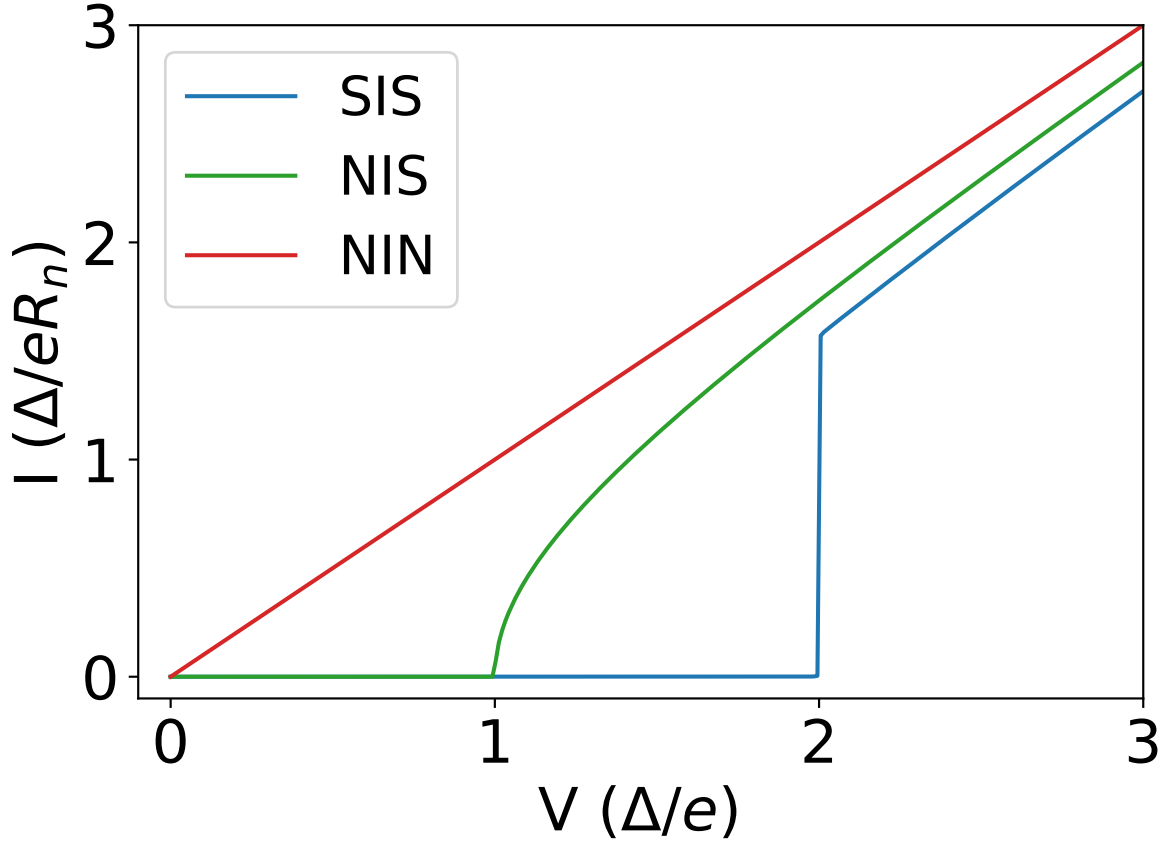


Figure 2.4: IV tunneling characteristics for junctions with the three different electrode permutations at $T = 0$ from Giaever theory. NIN junctions (red) behave like a resistor. NIS junctions (green) have a sharp increase from $I = 0$ when $V = \Delta$. SIS junctions (blue) have a discontinuous jump at voltage equal to the sum of the two superconducting gaps.

Here M is a phenomenological tunneling matrix element between electron states, $N_{L(R)}$ is the density of states of quasiparticles in the left (right) electrodes, and $g_{L(R)}$ are the occupation functions of quasiparticle as a function of the energy, E , and temperature, T , of the quasiparticle states. V is the voltage across the barrier. Here I consider tunneling for all possibilities of superconducting (S) and normal (N) electrodes separated by an insulating (I) barrier: *i.e.* NIN, NIS, and SIS tunneling.

For normal metals under small biases and low temperatures, the density of states is

approximately constant in energy. Therefore the tunneling between two normal metals is

$$I(V) = \frac{1}{eR_n} \int_{-\infty}^{\infty} [f(E - eV, T) - f(E, T)] dE = \frac{V}{R_n} \quad (2.31)$$

where $N_{L(R),0}$ is the normal density of states of the left (right) electrodes at the Fermi level, $f(E, T)$ is the Fermi-Dirac distribution function at a given energy and temperature, and $1/R_n = \frac{4\pi e^2}{h} |M|^2 N_{L,0} N_{R,0}$ is the effective resistance of the junction. Since the Fermi distributions are symmetric about their center energy, the integral simply becomes the difference in the energies of the distributions, eV . A tunnel barrier functions as a resistance for normal metal tunneling (see Fig. 2.4).

An NIS junction has one superconducting lead and one normal metal lead (section 8.2). Eq. 2.30 now has one normal, energy-independent density of states, $N_{L,0}$ and one BCS density of states (Eq. 2.13) yielding

$$I(V) = \frac{2}{eR_n} \int_{\Delta}^{\infty} \frac{E}{\sqrt{E^2 - \Delta_R^2}} [f(E - eV, T) - f(E, T)] dE \quad (2.32)$$

for positive V and electron distributions in thermal equilibrium. At low temperatures, $k_B T \ll \Delta$, the Fermi distributions can be approximated as step functions, and Eq. 2.32 can be written as

$$I(V) = \frac{2}{eR_n} \int_{\Delta}^{eV} \frac{E}{\sqrt{E^2 - \Delta^2}} dE = \frac{1}{R_n} \sqrt{V^2 - (\Delta/e)^2} \quad (2.33)$$

for $eV > \Delta$. Ignoring junction leakage, $I = 0$ for $V < \Delta/e$. The tunneling characteristic for an NIS junction is much different from NIN tunneling. There is no tunneling for voltages below Δ/e because the gap in the superconducting density of states (Eq. 2.13) prevents the tunneling of charges from the normal metal into the superconductor. At biases directly above $V = \Delta$, there is a steep increase in current due to the large density of states. Tunneling behavior approaches the typical NIN behavior at large biases (see Fig. 2.4). At

finite temperatures, the sharp kink at $eV = \Delta$ is smoothed out by the Fermi distributions. This temperature dependence makes NIS tunnel junctions an accurate low-temperature thermometer (section 8.2.2).

For an SIS junction, the tunneling of quasiparticles depends on the superconducting gaps of the two electrodes and the distribution functions of the excitations in each electrode. For SIS junctions with BCS density of states and thermal Fermi-Dirac occupation probabilities, Eq. 2.30 becomes

$$I(V) = \frac{1}{eR_n} \int_{-\infty}^{\infty} \frac{|E - eV|}{\sqrt{(E - eV)^2 - \Delta_R^2}} \frac{|E|}{\sqrt{E^2 - \Delta_L^2}} [f(E - eV, T) - f(E, T)] dE \quad (2.34)$$

where the superconducting gaps on opposite sides of the junction Δ_R, Δ_L can take different values.

If the temperature is well below the gap such that $k_B T \ll \Delta_{R,L}$ then $I = 0$ for biases $V < (\Delta_R + \Delta_L)/e$. At $V = \Delta_R + \Delta_L$ a discontinuous increase in current occurs to $I \approx V/R_n$ for higher voltages (Fig. 2.4). At finite temperatures, a thermal peak appears at $V = |\Delta_R - \Delta_L|/e$ due to thermally excited carriers. Note that the Giaever process in SIS junctions is the *tunneling* characteristic of *quasiparticle* excitations only and is separate from Andreev reflection (2.3.2) or supercurrent from tunneling Cooper pairs (2.4.1) which both involve Cooper pairs.

2.3.2 Blonder, Tinkham, Klapwijk Tunneling for Finite Barriers

When an electron in a normal metal is incident on a superconducting film, the process of Andreev reflection is possible [37]. In this process, the electron retro-reflects from the interface as a hole with opposite spin and a Cooper pair is transmitted into the superconductor.

Considering Andreev reflection and traditional tunneling processes at an NS interface, Blonder, Tinkham, and Klapwijk (BTK) generalized Giaever tunneling to account for partially transmissive tunneling barriers in superconducting junctions [33]. To do this, the

tunneling Hamiltonian Eq. 2.4 was solved on both sides of the NS interface giving four quantum tunneling conditions allowing the calculation of the process probabilities. BTK theory calculates four different probabilities for an arbitrary tunnel barrier strength Z :

- A corresponding to the probability of Andreev reflection,
- B corresponding to the probability of ordinary reflection,
- C corresponding to the transmission probability without branch crossing (*e.g.* electron tunneling into an electron state),
- D corresponding to the transmission probability with a branch crossing (*e.g.* electron tunneling into a hole state with opposite momentum).

Fig. 2.5 shows the A and B amplitudes for different barrier heights $Z = \sqrt{|r|^2/|t|^2}$ where r , t are the reflection and transmission amplitudes for electrons incident on the barrier. $Z = 0$ corresponds to a totally transparent interface experiencing only Andreev reflection [50]. $Z = \infty$ corresponds to a totally reflective barrier and the tunneling limit (section 2.3.1).

Summing these probabilities gives the total junction current as

$$I = \frac{1}{eR_n} \int_{-\infty}^{\infty} [g(E - eV) - g(E)](1 + A(E) + B(E))dE \quad (2.35)$$

where g is the electron distribution function in the normal metal, and A (B) is the Andreev (ordinary) reflection probability. For details on the derivation and tunneling processes, see [33, 37, 51].

2.3.3 Multiple Andreev Reflection

Averin and Bardas developed a model for Andreev reflection processes in SIS junctions. In their model of an SIS junction, the two superconducting electrodes sandwich a normal transmission channel with a partially transmitting barrier, for which they were able to derive

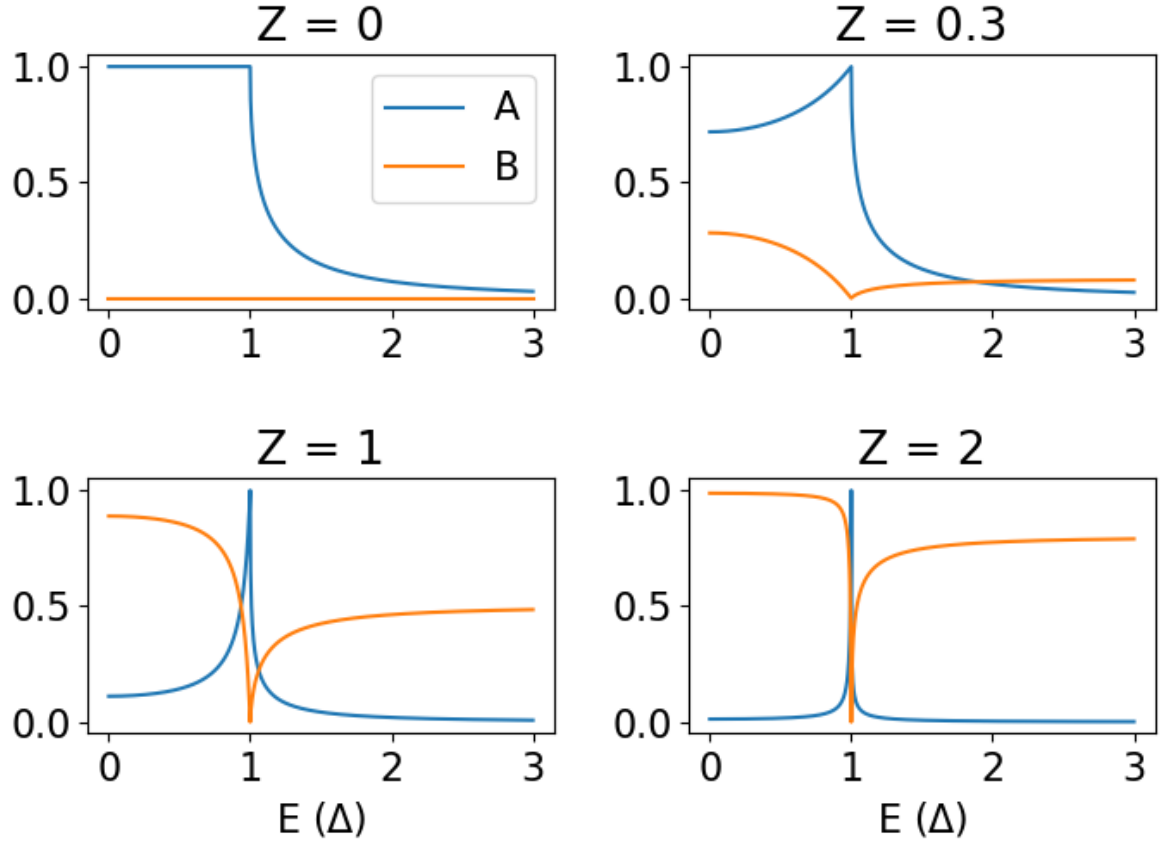


Figure 2.5: Andreev (A, blue) and ordinary (B, orange) reflection amplitudes vs incident electron energy for SIN junctions with varying barrier strengths Z . As the barrier strength increases, ordinary reflection becomes the dominant process.

the resulting tunneling characteristics for a finite-barrier Josephson junctions. In the normal channel, electrons can undergo a series of Andreev reflections from the NS interfaces. Because of this, the tunneling theory is called Multiple Andreev Reflection (MAR) theory [34].

More explicitly, the junction is modeled as a superconductor region (I), followed by a normal region (II), a partially transmissive barrier, then a second normal region (III), and finally a superconductor region (IV) (Fig. 2.7). When an electron within the barrier is incident on a superconducting electrode ($\text{II} \rightarrow \text{I}$ or $\text{III} \rightarrow \text{IV}$) the process has a complex reflection amplitude

$$a(E) = \frac{1}{\Delta} \begin{cases} E - \text{sgn}(E)\sqrt{E^2 - \Delta^2}, & |E| > \Delta \\ E - i\sqrt{\Delta^2 - E^2}, & |E| < \Delta \end{cases} \quad (2.36)$$

where E is the energy of the electron relative to the Fermi-energy, and Δ is the superconducting energy gap of the relevant electrode. This is simply Andreev reflection at an NS interface.

The center barrier ($\text{II} \leftrightarrow \text{III}$), is characterized by a scattering matrix

$$S_{el} = S_h^* = \begin{pmatrix} r & t \\ t & -r^*t/t^* \end{pmatrix} \quad (2.37)$$

where the transmission amplitude, t , squared is the transmission probability, D , of the link $|t|^2 = D$ and $|r|^2 = 1 - |t|^2$ is the reflection probability. The Andreev reflection process in Eq. 2.36 and the ordinary reflection process from Eq. 2.37 (see Fig. 2.6) must be satisfied with a self-consistent wavefunction.

Writing the total wavefunction using the planewave ansatz

$$\psi = \begin{cases} \sum_n A_n e^{ikx} + B_n e^{-ikx}, & \text{Region II} \\ \sum_n C_n e^{ikx} + a_{2n+1} D_n e^{-ikx}, & \text{Region III} \end{cases} \quad (2.38)$$

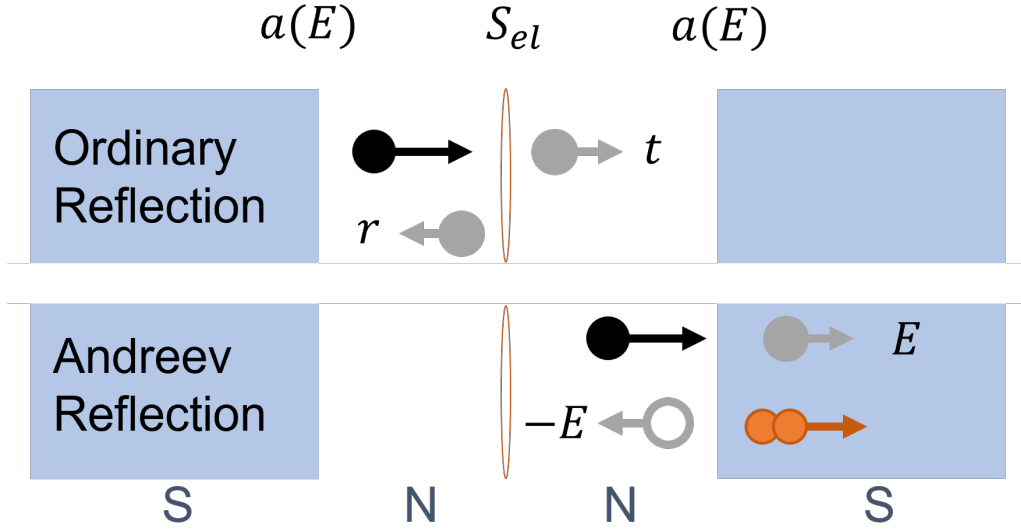


Figure 2.6: The two processes involved in MAR theory. Solving both simultaneously yields the current due to MAR processes. The incident electron (black) can have two outcomes (grey) for each process. **Top:** ordinary scattering off a tunnel barrier characterized by scattering matrix S_{el} . An electron incident on the barrier from the right can be reflected as an electron with probability $|r|^2$ or transmitted as an electron with probability $|t|^2$. **Bottom:** Andreev reflection at an NS interface characterized by amplitude $a(E)$. The electron can be transmitted into the superconducting electrode with probability $1 - a(E)$ if it has energy above the superconducting gap. The electron can also be reflected as a hole with opposite energy and momentum with probability $a(E)$, adding a Cooper pair to the superconductor (orange pair).

where the sum is taken over the number of reflections n , with V as the applied voltage across the junction, and $a_n = a(E + neV)$ adjusts the amplitude of the counter-propagating wave to account for Andreev reflection off the barrier. Averin and Bardas then used the scattering matrix to get a recursive relationship between the wave amplitudes of different n . The series can be truncated at some finite n_{max} and recursively solved for the coefficients given the superconducting gaps of each electrode and the applied voltage.

With the wavefunctions solved, the probability current density $j(E, t) = \hbar/2mi(\psi^*\nabla\psi - \psi\nabla\psi^*)$ can be calculated with electron mass, m , yielding the current flowing to the left as

$$I_L(t) = \frac{em}{2\pi\hbar^2k} \int_{-\infty}^{\infty} j(E, t) f_R(E, T) dE \quad (2.39)$$

Here f_R is the distribution function of the electrons in the right electrode. Combining the left and right flowing currents allowed Averin and Bardas to write the total current in Fourier components

$$I(t) = \sum_K I_K e^{i2KeVt/\hbar} \quad (2.40)$$

where I_K is

$$\begin{aligned} I_K = \frac{e}{\pi\hbar} & \left[eV\delta_{K0} - \int_{-\infty}^{\infty} dE \tanh(E/k_bT) \right. \\ & \left(\sqrt{1 - |a_0|^2} (a_{2K}A_K^* + a_{-2K}A_{-K}) \right. \\ & \left. \left. + \sum_n (1 + a_{2n}a_{2(n+K)}^*) (A_nA_{n+K}^* - B_nB_{n+K}^*) \right) \right] \end{aligned} \quad (2.41)$$

The DC current is the 0th Fourier component, $K = 0$. Detailed derivation of the current characteristics for asymmetric gapped junctions are similar and can be found in work by Liao [51].

These IV curves differ from the Gaiever tunneling result, showing finite current steps at biases $eV = \Delta_{R,L}/n$ for integer n . These current steps have a height proportional to the junction transparency, D , and the number of reflections, n . The superconducting gaps

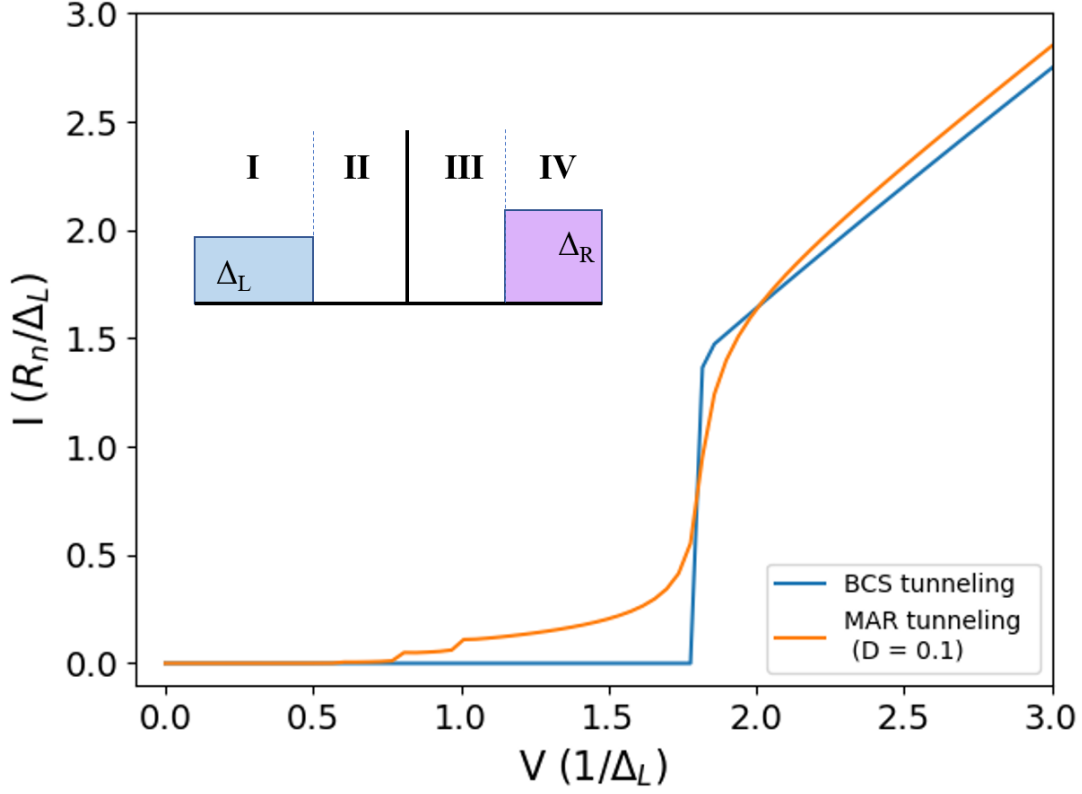


Figure 2.7: Example theoretical, normalized current-voltage characteristics for BCS theory (blue) and MAR theory (orange) for a tunnel junction with asymmetric gaps ($\Delta_R = 0.75\Delta_L$). A junction transparency of $D = 0.1$ was used for the MAR simulation. While the Gaijever theory has exponentially small current in the subgap, the MAR processes yields finite current steps at integer fractions of the individual superconducting gaps. Only the first step of each electrode is visible here with straight scale at low transparencies. Inset: the four regions of the junction used to model MAR effects. Regions I and IV are the superconducting electrodes. The junction barrier consists of two normal regions (II and III) with a finite transmitting barrier separating them.

$\Delta_{R,L}$, junction transparency D , electron temperature, T , and normal tunneling resistance, R_n , completely determine the tunneling characteristic in this model.

2.4 Cooper Pair Tunneling

Despite Bardeen's initial incredulity, Cooper pairs in a superconductor can tunnel across a thin barrier. I will outline some of the relevant measurable behaviors of the supercurrent given by this pair tunneling.

2.4.1 Josephson Relations

The tunneling process of pairs results in the following physics known as the Josephson relations [52, 53]

$$I_S = I_C \sin \varphi \quad (2.42)$$

$$\frac{d\varphi}{dt} = \frac{2e}{\hbar} V \quad (2.43)$$

Here the DC Josephson relation, Eq. 2.42 says the lossless supercurrent through the tunnel junction is proportional to the sine of the difference of the phase of the superconducting wavefunction across the junction, φ , times a maximum critical current, I_C . For a constant, non-zero φ , the junction will experience a supercurrent even with no applied voltage. Eq. 2.43, the AC Josephson equation, states the phase difference across the junction evolves proportional to the voltage across the junction. These relations are important in understanding the physics of a superconducting qubit circuit.

2.4.2 P(E) Theory

Cooper pairs can interact with the environment when tunneling across the junction. P(E) theory describes inelastic Cooper pair tunneling across a junction [54]. The theory describes

charging in Coulomb blockade measurements [55] and critical current measurements in small junctions [54]. For a given electromagnetic environment for a tunnel junction, the $P(E)$ function describes a tunneling Cooper pair which emits energy E to the environment and is calculated

$$P(E) = \frac{1}{2\pi\hbar} \int_{-\infty}^{\infty} \exp \left[J(t) + \frac{i}{\hbar} Et \right] dt \quad (2.44)$$

where J is the phase correlation function. Given this function, the supercurrent characteristic is given by

$$I(V) = \frac{\pi\hbar I_C^2}{4e} [P(2eV) - P(-2eV)] \quad (2.45)$$

where I_C is the critical current of the junction decoupled from the environment [54, 56]. Ingold and Grabert [54] rearranged the emission probability into a more useful form

$$P(E) = I(E) + \int_{-\infty}^{\infty} K(E, \omega) P(E - \hbar\omega) d\omega \quad (2.46)$$

where I is the inhomogeneity of the total circuit (junction plus environment) corresponding to the Lorentzian $P(E)$ of an ohmic environment at high temperatures.

$$I(E) = \frac{1}{\pi\hbar} \frac{D}{D^2 + (E/\hbar)^2} \quad (2.47)$$

where $D = (\pi k_B T / \hbar) (\text{Re } Z(0) / R_Q)$ is the magnitude of the asymptotic, ohmic behavior of the junction, with the DC resistance $\text{Re } Z(0)$, and the resistance quantum, $R_Q = h/2e^2$. K is the integral kernel

$$K(E, \omega) = \frac{E/\hbar}{D^2 + E^2/\hbar^2} k(\omega) + \frac{D}{D^2 + E^2/\hbar^2} \kappa(\omega) \quad (2.48)$$

where

$$k(\omega) = \frac{1}{1 - e^{-\hbar\omega/k_B T}} \frac{\text{Re } Z(\omega)}{R_Q} - \frac{k_B T}{\hbar\omega} \frac{\text{Re } Z(0)}{R_Q} \quad (2.49)$$

and κ is the Laplace transform of k .

For a given a rf-environment, the kernel can be calculated for arbitrary E . Then Eq. 2.46 can be solved numerically starting from the initial guess $P(E) = I(E)$ until relative corrections from the last term in Eq. 2.46 become small. Finally, the current can be calculated from Eq. 2.45 using the $P(E)$ calculated.

2.4.3 SQUIDS

A DC Superconducting QUantum Interference Device (SQUID) consists of two Josephson junctions in parallel forming a SQUID loop. The superconducting loop must be threaded by an integer number of flux quanta, $n\Phi_0 = nh/2e$, resulting in a circulating current and generating an interference in the phase difference between the two junctions [57].

In this dissertation, I used an applied magnetic flux to suppress the critical current measured across the SQUID. For an ideal, symmetric DC SQUID with $\beta = 2LI_{C,0}/\Phi_0 \ll 1$, the critical current is modulated by the external flux through the SQUID loop by

$$I_C = 2I_{C,0} \left| \cos \left(\pi \frac{\Phi_{ext}}{\Phi_0} \right) \right| \quad (2.50)$$

where $I_{C,0}$ is the critical current of one of the junctions in the loop.

Chapter 3

Qubit Theory

3.1 Qubit Basics

A qubit is composed of a two-level system which can be put in a quantum *superposition*. To be useful in quantum computation it must also be able to be *entangled* with other qubits. The two levels are called $|0\rangle$ (the ground state) and $|1\rangle$ (the excited state). An arbitrary single qubit state can then be written as

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle = \cos(\theta/2) |0\rangle + \sin(\theta/2)e^{i\phi} |1\rangle \quad (3.1)$$

where α and β are complex amplitudes which are normalized ($|\alpha|^2 + |\beta|^2 = 1$). The state can also be parameterised with two angles θ, ϕ , where $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$.

The qubit state can be represented on the Bloch sphere (Fig. 3.1) where θ is the polar angle and ϕ is the azimuthal angle for the polar coordinates representing the sphere and $|0\rangle$ is the north pole of the sphere while $|1\rangle$ corresponds to the south pole [58].

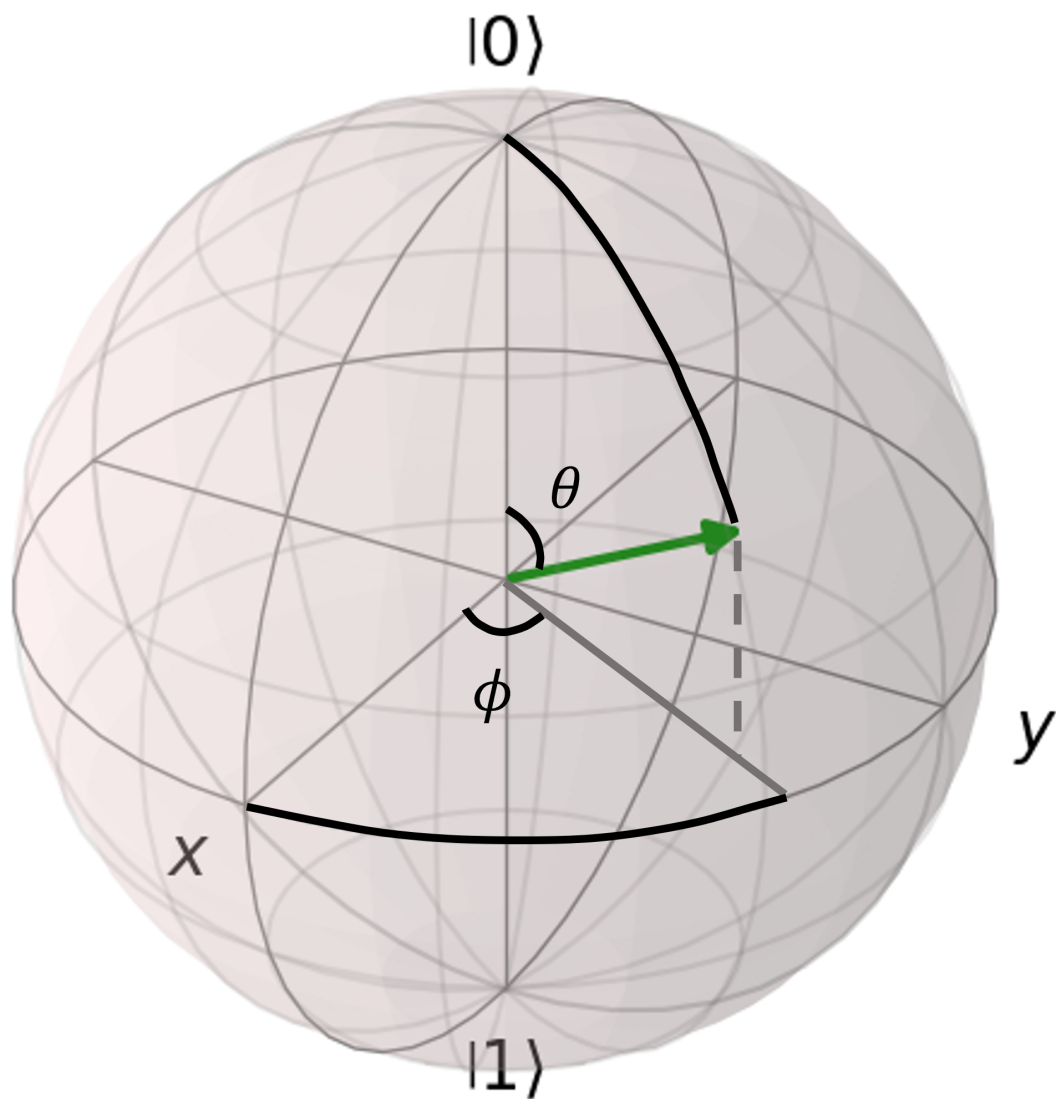


Figure 3.1: Bloch sphere with a superposition qubit state in green. This state is characterized by the two angles θ and ϕ shown.

3.2 Qubits from Superconducting Circuits

Superconducting circuits are one platform to implement a qubit as described above. Here I derive the Hamiltonian for the transmon, a common superconducting qubit that I used for my qubit measurements.

3.2.1 Resonant Circuits

An inductor with inductance L and capacitor with capacitance C connected together forms a simple circuit that can exhibit quantum properties (Fig. 3.2a) [59–61]. The circuit follows the differential equations

$$v = q/C \tag{3.2}$$

$$v = \frac{dI}{dt}L \tag{3.3}$$

$$\text{with } I = -\frac{dq}{dt} \tag{3.4}$$

with v being the voltage across the capacitor or inductor and I being the current in the circuit, which can be simply rearranged into the second-order differential equation for an undriven, undamped resonator

$$\frac{d^2q}{dt^2}LC + q = 0 \tag{3.5}$$

The solutions are $q(t) = q_0 e^{\pm i\omega_r t}$ where $\omega_r = 1/\sqrt{LC}$ is the resonant angular frequency of the circuit which is half the rate the capacitor and inductor exchange energy in the circuit. Initial charge q_0 depends on the initial conditions of the resonator.

The total energy stored in the LC circuit can be written in terms of the inductor flux $\Phi = IL$ as

$$E = \frac{q^2}{2C} + \frac{\Phi^2}{2L} \tag{3.6}$$

Note that the charge, q , is the canonical momentum associated with the flux coordinate Φ .

At this point, the charge and flux can be promoted to quantum operators and the system

Hamiltonian can be written [62]

$$\hat{H}_r = \frac{\hat{q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}. \quad (3.7)$$

Considering the circuit in terms of total excitation number is more useful when coupling multiple bosonic systems (e.g. coupling the resonator to a qubit) [63]. Therefore, next non-Hermitian operators are introduced

$$\hat{a} = \hat{q}\sqrt{\frac{1}{2C}} + i\hat{\Phi}\sqrt{\frac{1}{2L}} \quad (3.8)$$

$$\hat{a}^\dagger = \hat{q}\sqrt{\frac{1}{2C}} - i\hat{\Phi}\sqrt{\frac{1}{2L}} \quad (3.9)$$

which are the lowering and raising operators, respectively. Next, by writing $\hat{q}$ and $\hat{\Phi}$ in terms of these new bosonic operators, the Hamiltonian can be diagonalized

$$\hat{H}_r = \omega_r(\hat{a}^\dagger\hat{a} + 1/2) = \hbar\omega_r(\hat{n} + 1/2) \quad (3.10)$$

where the energy is clearly proportional to the number of photons in the resonator given by $\hat{n}$. The remaining $1/2$ term corresponds to vacuum fluctuations in the QSHO, which will give only a relative energy offset when looking at an uncoupled resonator.

A superconducting microwave resonator can be cooled to its ground state in a dilution refrigerator with base temperature, $T_{sys} \approx 20 \text{ mK}$

$$\Delta E = \hbar\omega_r = \hbar/\sqrt{LC} \approx h \cdot 5 \text{ GHz} = k_B \cdot 240 \text{ mK} > k_B T_{sys} \quad (3.11)$$

3.2.2 Josephson Junctions in Circuits

While a QSHO can exhibit interesting quantum behavior, the equal spacing of the energy levels makes isolating only two levels to form a qubit difficult in this system. However, this can be remedied by adding some non-linearity to the circuit. In superconducting circuits,

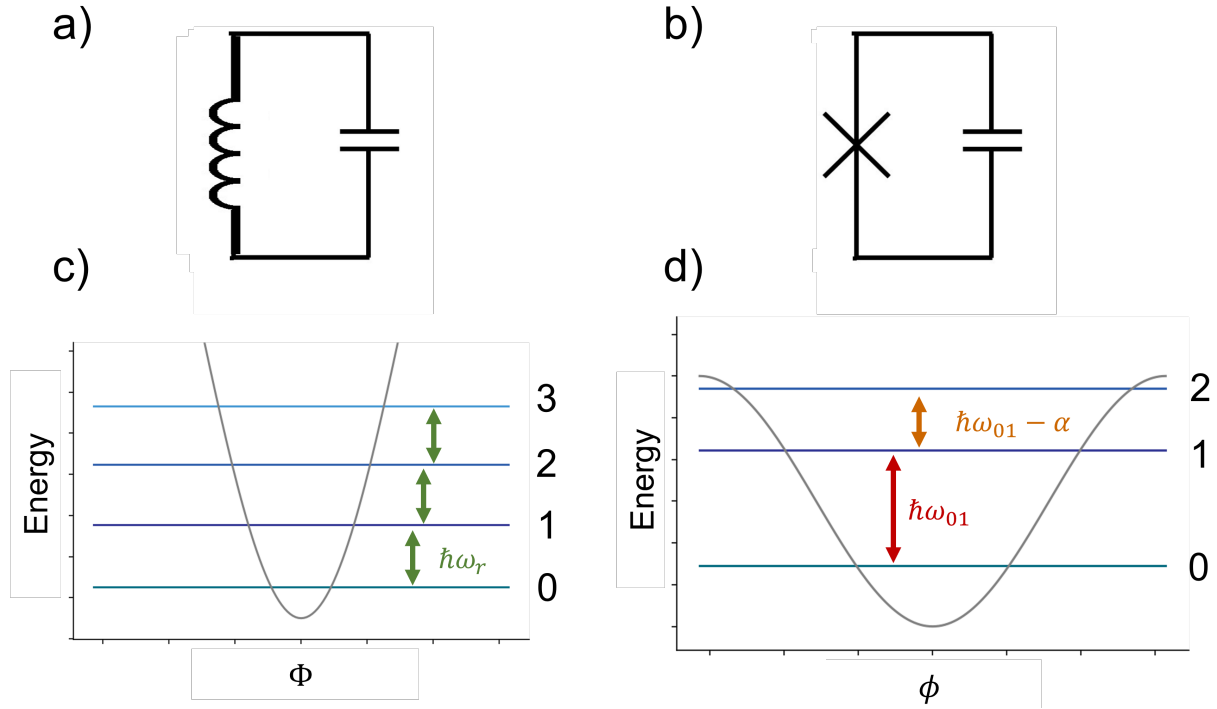


Figure 3.2: Circuit diagrams of a resonator and transmon. a) Linear LC oscillator with an inductor and capacitor in parallel. b) Transmon qubit. Similar to the LC resonant circuit but with the inductor replaced by a Josephson Junction. The junction functions as a non-linear inductor with effective inductance depending on total current. c) Quantized energy levels of the LC oscillator with equal spacing, $\hbar\omega_r$ between all energy levels. d) Energy levels of the transmon. Here the non-linearity of the junction gives a finite anharmonicity between ω_{01} and $\omega_{12} = \omega_{01} - \alpha$. The difference in energy level spacing allows any chosen two levels to be addressed as a qubit.

this non-linearity comes from the physics of Cooper pair tunneling in a Josephson tunnel junction [53]. Rewriting the Josephson relations defined in section 2.4.1, these equations can be combined to get the Josephson inductance, L_J . Taking the time derivative of the DC Josephson relation and using the AC Josephson relation for $d\phi/dt$ yields

$$\frac{dI}{dt} = I_c \cos \phi \frac{d\phi}{dt} = \frac{2\pi I_c}{\Phi_0} \cos \phi V = V/L_J \quad (3.12)$$

where I_c is the junction critical current, $\Phi_0 = h/2e$ is the superconducting magnetic flux quantum, and I, V, ϕ are the current, voltage, and superconducting phase difference across the junction respectively. The remaining terms in Eq. 3.12 can be written as an inductance because the current derivative of a junction depends on the voltage bias, similar to an inductor. Note that, the magnitude of this inductive term depends on the phase difference and can take a different value at larger currents. This results in a circuit element with non-linear inductive behavior. Combined with a capacitor, a non-linear oscillator is formed.

Rearranging the Josephson relations again, the Josephson energy can be calculated

$$U_J = \int IV dt = \frac{-I_c \Phi_0}{2\pi} \cos \phi \approx \frac{I_c \Phi_0}{2\pi} (-1 + \phi^2/2 - \phi^4/4) \quad (3.13)$$

the cosine potential energy has been expanded to fourth order in ϕ , showing that to lowest order, the potential energy of the junction is identical to a physical inductor while higher order corrections will give the transmon finite anharmonicity between levels (Fig. 3.2).

3.2.3 Transmon Hamiltonian

A transmon consists of a Josephson junction and a capacitor in parallel. This is the same circuit as an unbiased phase or charge qubit. Initial charge qubits, such as the Cooper pair box, showed quantum coherence [12, 64], but were extremely sensitive to offset drift and $1/f$ charge noise [65, 66]. To mitigate the effect of charge noise on the qubit, Koch, *et al.* proposed the transmon qubit by adding a large shunting capacitor to the Cooper pair box

[67].

The Hamiltonian of transmon circuit can be written

$$\hat{H}_q = \frac{(\hat{q} - q_g)^2}{2C} - \frac{I_c \Phi_0}{2\pi} \cos \hat{\phi} \quad (3.14)$$

$$= \frac{2e^2}{C} (\hat{n} - n_g)^2 - \frac{I_c \Phi_0}{2\pi} \cos \hat{\phi} \quad (3.15)$$

$$= 4E_C (\hat{n} - n_g)^2 - E_J \cos \hat{\phi} \quad (3.16)$$

where $\hat{n} = \hat{q}/2e$, charge number operator, gives the number of Cooper pairs on one capacitor plate of the transmon, $E_C = e^2/2C$ is the single-electron charging energy, and $E_J = I_c \Phi_0/2\pi$ is the junction tunneling energy. Additionally, the gate charge, q_g , and gate charge number, $n_g = q_g/2e$, are continuous charge offsets, historically controlled with bias gates. Now n_g is a slowly varying parameter determined by the charge environment near the transmon. The transmon's tunneling energy is determined physically by the junction's critical current while the charging energy is determined by the total capacitance of the circuit.

Written from the circuit representation, the transmon Hamiltonian, Eq. 3.16, involves terms in both the phase, $\hat{\phi}$, and charge number, $\hat{n}$, bases. However, the charge and phase are a conjugate pair of variables, allowing the Hamiltonian to be written in the phase basis

$$\hat{H}_q = -4E_C \left(i \frac{\partial}{\partial \hat{\phi}} + n_g \right)^2 - E_J \cos \hat{\phi} \quad (3.17)$$

Now that the Hamiltonian is expressed in a single variable, the transmon eigenstates can be found by solving the time-independent Schrödinger equation. By rotating $\hat{H}_q$ such that $n_g = 0$, the solutions become Mathieu functions [68], special functions which also arise in the mathematics for inverted pendulums, train-track stability, and elliptical bongos [69]. The eigenenergies for the system are Mathieu characteristic numbers, $a_k(-2E_J/E_C)$, where k is a counter specifying the set of solutions for a given n_g [68]. This solution is useful for obtaining exact transmon energy spectra.

If one wants to avoid these elegant but complex mathematics, $\hat{H}_q$ can be written instead in the charge basis,

$$\hat{H}_q = 4E_C(\hat{n} - n_g)^2 - E_J/2 \sum_n |n\rangle \langle n+1| + |n+1\rangle \langle n| \quad (3.18)$$

Eq. 3.18 has quadratic charging energies diagonal in $\hat{n}$ and hopping terms adjacent to the diagonal given by half the tunneling energy. For a device with $E_J/E_C \approx 1$ (known as a Cooper pair box, an early superconducting qubit), the eigenstates have significant population in only a few charge levels so it is possible to truncate Eq. 3.18 to $|n_g| < 10$ [12, 64]. However, due to the the transmon's low charging energy, truncation at $|n_g| < 80$ must be used for accurate calculations.

By numerically diagonalizing H_q in the charge basis using SCQubits [70], the energy spectrum vs. offset charge in Fig. 3.4 is shown for $E_J/E_C = 2, 10$, and 50 . For lower E_J/E_C values the energies depend strongly on n_g . The change in energy $E_{01}(n_g = 0) - E_{01}(n_g = 1/2)$ defines the charge dispersion for a transmon. Small changes in background charge can shift the qubit energy levels, causing dephasing proportional to the charge dispersion (see 3.6.2). From asymptotics of the Mathieu functions, the charge dispersion is exponentially suppressed with increasing E_J/E_C motivating the transmon regime for charge qubits [67].

In the transmon regime with $E_J/E_C > 20 - 50$, the cosine potential is large leading to strongly localized phase for transmon eigenstates. The lowest energy eigenstate stays trapped in the cosinusoidal well shown in Fig. 3.3a. Conversely, the charge number is delocalized and eigenstates can have contributions in a large range of charge numbers (Fig. 3.3b).

3.2.4 Diagonalizing the Transmon

For further analytical work on the transmon, it is useful to introduce transmon raising ($b^\dagger$) and lowering (b) operators similar to Eq. 3.8 for the simple harmonic oscillator. Following

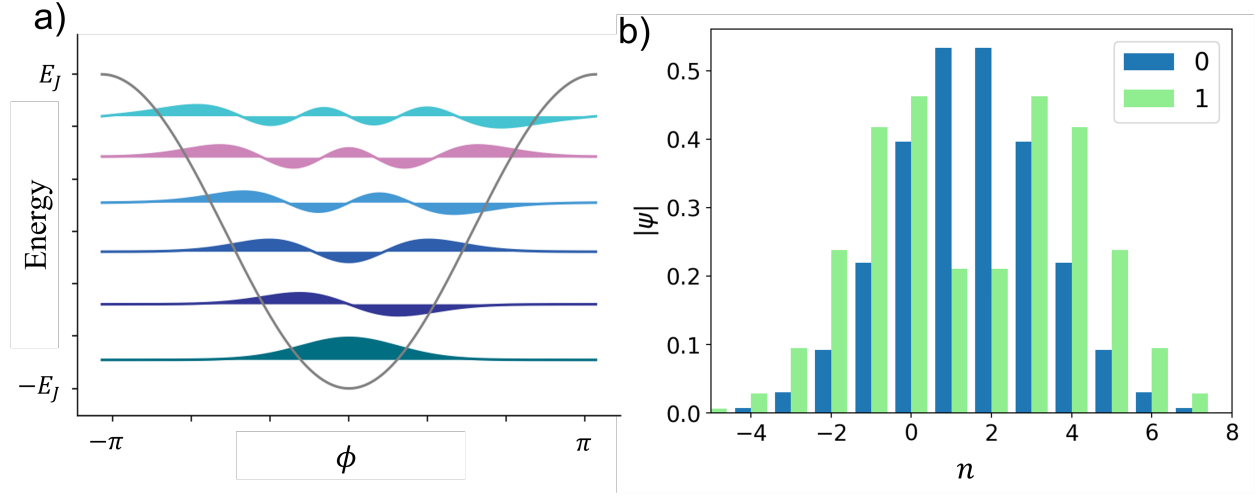


Figure 3.3: **a)** Transmon eigenstates in the phase basis. Shown are the wavefunctions as functions of the phase difference, ϕ , offset by the level's energy. Also shown is the cosine potential which results in anharmonic energy levels. **b)** The absolute value of the first two eigenstates in the charge basis with the amplitudes at each integer charge.

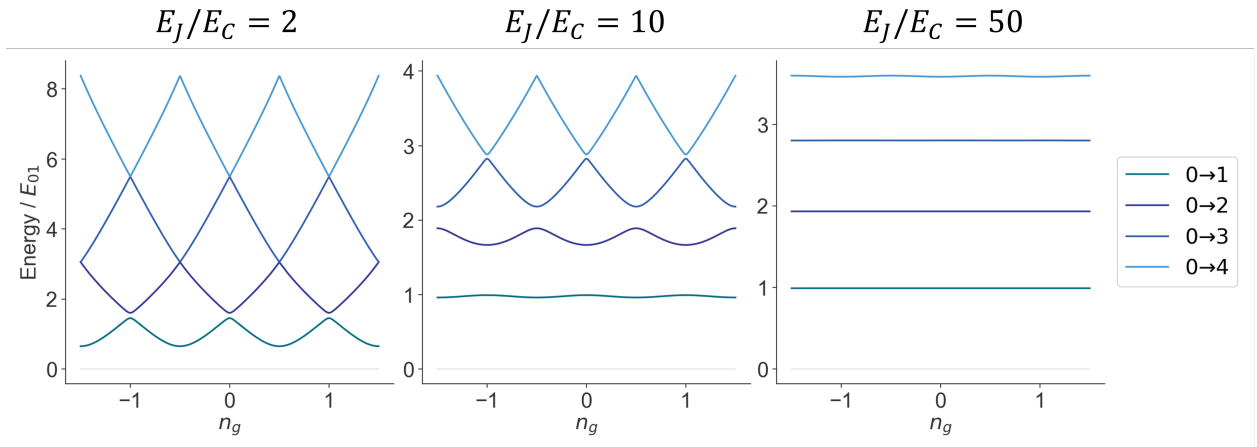


Figure 3.4: Charge qubit spectrum normalized by E_{01} vs. n_g for different E_J/E_C ratios. As this ratio increases, the charge dispersion within a given level decreases exponentially. The transmon qubit has large E_J/E_C on the order of the right-most plot to decrease charge dispersion.

[60, 63]

$$\hat{b}^\dagger = \frac{1}{2} \left(\frac{E_J}{2E_C} \right)^{1/4} \hat{\phi} + i \left(\frac{2E_C}{E_J} \right)^{1/4} \hat{n} \quad (3.19)$$

$$\hat{b} = \frac{1}{2} \left(\frac{E_J}{2E_C} \right)^{1/4} \hat{\phi} - i \left(\frac{2E_C}{E_J} \right)^{1/4} \hat{n} \quad (3.20)$$

$$\hat{\phi} = \left(\frac{2E_C}{E_J} \right)^{1/4} (\hat{b} + \hat{b}^\dagger) \quad (3.21)$$

$$\hat{n} = \frac{i}{2} \left(\frac{E_J}{2E_C} \right)^{1/4} (\hat{b} - \hat{b}^\dagger) \quad (3.22)$$

Here the hats on the b operators have been dropped for convenience. Performing a Taylor expansion on the cosine term in H_q (Eq. 3.16) to fourth order in $\hat{\phi}$ gives

$$\hat{H}_q \approx 4E_C(\hat{n} - n_g)^2 + \frac{E_J}{2} \hat{\phi}^2 - \frac{E_J}{24} \hat{\phi}^4 \quad (3.23)$$

$$= \sqrt{8E_J E_C} (b^\dagger b + \frac{1}{2}) - \frac{E_C}{12} (b^\dagger + b)^4 \quad (3.24)$$

$$\approx \left(\sqrt{8E_J E_C} - E_C \right) (b^\dagger b + \frac{1}{2}) - \frac{E_C}{2} b^\dagger b^\dagger b b \quad (3.25)$$

$$= \hbar\omega_q (b^\dagger b + \frac{1}{2}) - \frac{\alpha}{2} b^\dagger b^\dagger b b \quad (3.26)$$

In Eq. 3.24, the quartic term was expanded and the rotating-wave approximation (RWA) was used to drop number non-conserving terms, each term was normal ordered resulting in Eq. 3.26. For details see C.2.

In Eq. 3.26 the transmon frequency and anharmonicity are approximated using typical transmon limit formulas

$$\hbar\omega_q \approx \sqrt{8E_J E_C} - E_C, \quad \alpha = (E_2 - E_1) - (E_1 - E_0) \approx -E_C \quad (3.27)$$

In the transmon limit, the charge dispersion can be approximated from Mathieu asymp-

otics as

$$\begin{aligned}\epsilon_m &= E_m(n_g = 1/2) - E_m(n_g = 0) \\ &= (-1)^m E_C \frac{2^{4m+5}}{m!} \sqrt{2/\pi} \left(\frac{E_J}{2E_C} \right)^{m/2+3/4} e^{-\sqrt{8E_J/E_C}}\end{aligned}\quad (3.28)$$

The motivation for going to the transmon regime is the exponential suppression of the charge dispersion in E_J/E_C at the expense of slowly decreasing anharmonicity [67]. The results in Eq. 3.27 and Eq. 3.28 are approximations in the $E_J/E_C \gg 1$ limit so more accurate results from the Mathieu solutions [68] or numerical diagonalization [70] are often needed.

3.3 Controlling the Qubit

A microwave drive at (or near) the qubit frequency was used in order to perform operations and characterizations on a qubit device. The Hamiltonian formalism for a qubit drive is presented here.

3.3.1 Hamiltonian of a Driven, Multi-level Transmon

A multi-level transmon with an applied classical drive at frequency ω_d has the Hamiltonian

$$H = H_J + H_D = \hbar \sum_j \omega_j |j\rangle \langle j| + \frac{\hbar \Omega_q}{2} (b e^{i\omega_d t} + b^\dagger e^{-i\omega_d t}) \quad (3.29)$$

where the undriven Hamiltonian is written in terms of the transmon eigenstates $|j\rangle$ at eigenenergies $\hbar\omega_j$ (e.g. $\omega_1 - \omega_0 = \omega_q$ and $\omega_2 - \omega_1 = \omega_q - \alpha$). The drive strength is characterized by the rate of Rabi flopping, Ω_q , caused by the qubit drive and the expectation values of the ladder operators b , $b^\dagger$ on the different transmon levels.

Eq. 3.29 is rotated into the frame of the drive by applying a time-dependent unitary

transformation

$$U = e^{i\omega_d t \sum_j j|j\rangle\langle j|} \quad (3.30)$$

which yields for a resonantly driven transmon

$$H' = \hbar \sum_j (\omega_j - j\omega_d) + \frac{\hbar\Omega_q}{2}(b + b^\dagger) \quad (3.31)$$

(see C.3 for details).

3.3.2 Phase Control of the Qubit Drive

In the previous section 3.3.1, a continuous qubit drive was modeled. In many applications, short microwave pulses with controlled amplitude and phase are applied to control the qubit. In this case, Eq. 3.31 can be generalized following [61] in the lab frame

$$H_d = \frac{\hbar\Omega_q}{2} [I(t)(be^{i\omega_d t} + b^\dagger e^{-i\omega_d t}) + Q(t)(be^{i\omega_d t} - b^\dagger e^{-i\omega_d t})] \quad (3.32)$$

where I , Q are the in-phase and out-of-phase time dependent drive amplitudes and Eq. 3.33 approximates the transmon as a two level system. In the frame of the drive

$$H'_d = \frac{\hbar\Omega_q}{2} [I(t)(b + b^\dagger) + Q(t)(b - b^\dagger)] = \frac{\hbar\Omega_q}{2} (I(t)\sigma_x + Q(t)\sigma_y) \quad (3.33)$$

This formalism will be used to treat pulse shaping and phase error pulse correction 7.5.2.

3.4 Coupling the Qubit to a Cavity

The field of circuit quantum electrodynamics (cQED) was adapted from the AMO field of cavity quantum electrodynamics (cavity QED). In cavity QED an atom was dropped into an optical [71] or microwave [72] cavity and the properties of the atom were probed optically via photons stored inside the cavity.

Circuit QED encompasses much of the same physics, but implemented in mesoscopic superconducting circuits [63, 73]. In cQED a superconducting resonator (section 3.2.1) or cavity is used in place of the optical cavity. The resonator is then coupled to the artificial atom, a.k.a. the transmon qubit (section 3.2.3).

3.4.1 Jaynes-Cummings Hamiltonian

In many applications, cQED architecture is used to perform non-demolition qubit readout. This is achieved by capacitively coupling a transmon to a resonator resulting in the coupled Hamiltonian

$$H_{JC} = H_r + H_q + H_I \quad (3.34)$$

$$= \hbar\omega_r(a^\dagger a + \frac{1}{2}) + \hbar \sum_{j=0} \omega_j |j\rangle \langle j| + \hbar \sum_{i,j} g_{i,j} (|i\rangle \langle j| + |j\rangle \langle i|)(a + a^\dagger) \quad (3.35)$$

where the interaction term models the interaction between the qubit and resonator [59, 60]. The resonator is approximated as a single oscillator mode with frequency ω_r .

To further simplify, Eq. 3.26 is used to approximate the transmon levels. Additionally, coupling to only adjacent levels is assumed (see discussion of Eq. 3.46) and the rotating wave approximation is applied and terms which do not conserve photon number are dropped giving a simplified

$$H_{JC} = \hbar\omega_r(a^\dagger a + \frac{1}{2}) + \hbar\omega_q b^\dagger b + \frac{E_C}{2} b^\dagger b^\dagger b b + \hbar g(ab^\dagger + a^\dagger b) \quad (3.36)$$

Here the transmon Hamiltonian is written in the basis of transmon eigenstates. The interaction term represents the swapping of photons between the transmon and resonator when the two systems are resonant.

Often, it is convenient to consider only the first two levels of the transmon, approximating the device as an ideal qubit. In this case only terms up to $j = 1$ are kept and rewritten in

terms of Pauli matrices, giving

$$H = \hbar\omega_r(a^\dagger a + \frac{1}{2}) + \frac{1}{2}\hbar\omega_q\sigma_z + \hbar g(a\sigma^+ + a^\dagger\sigma^-) \quad (3.37)$$

Here $\sigma_\pm = \sigma_x \pm i\sigma_y$.

This Hamiltonian is block diagonal in 2x2, equi-excitation blocks, with all other elements 0. For a fixed excitation number n , the block can be diagonalized to find the dressed energies

$$E_{g,n} = \hbar n\omega_r - \frac{\hbar}{2}\sqrt{(\omega_r - \omega_q)^2 + 4ng^2} \quad (3.38)$$

$$E_{e,n-1} = \hbar n\omega_r + \frac{\hbar}{2}\sqrt{(\omega_r - \omega_q)^2 + 4ng^2} \quad (3.39)$$

where a constant offset of $\omega_r/2$ has been dropped.

3.4.2 Dispersive Jaynes-Cummings

Typical cQED devices operate with large detuning $\omega_r - \omega_j \gg g$. In this limit the Hamiltonian can be expanded perturbatively in g/Δ where $\Delta = \omega_q - \omega_r$ [63]. The formal method for going about this diagonalization is using the Schrieffer-Wolff transformation [60, 74]. Details can be found in C.5.

Diagonalizing the block-diagonal JC Hamiltonian, Eq. 3.37, gives the dispersive Jaynes-Cummings Hamiltonian

$$H = \hbar(\tilde{\omega}_r + \chi\sigma_z)a^\dagger a + \frac{\hbar}{2}\tilde{\omega}_q\sigma_z \quad (3.40)$$

$$= \hbar\tilde{\omega}_r a^\dagger a + \frac{\hbar}{2}(\tilde{\omega}_q + 2\chi a^\dagger a)\sigma_z \quad (3.41)$$

where $\tilde{\omega}_r = \omega_r - \chi_{12}/2$ is the dressed cavity frequency and $\tilde{\omega}_q = \omega_q + \chi_{01}$ is the dressed qubit frequency with $\chi_{ij} = g_{ij}^2/(\omega_r - \omega_{ij})$ [75]. The term $\hbar\chi\sigma_z a^\dagger a$ is the dispersive interaction

between the cavity and qubit. In Eq. 3.40 this interaction causes a shift in the cavity frequency by 2χ dependent on the qubit state. This shift *forms the basis for cQED qubit readout* (section 7.2).

In Eq. 3.41, the interaction term is grouped with the qubit, showing that the qubit frequency is shifted by 2χ for each photon in the coupled cavity. This can lead to cavity photon dephasing (section 3.6.3) in the qubit if the number of cavity photons fluctuates (*e.g.* from thermal noise photons), but also provides a method to measure the cavity photon number [63, 76]. Interestingly, during cQED experiments, both the dressed and bare cavity frequencies are regularly measured, but only the dressed qubit frequency is measured.

In the dispersive limit, the dressed eigenvectors can be written

$$|\overline{g, n}\rangle = |g, n\rangle - \frac{\sqrt{n}g}{\Delta} |e, n-1\rangle \quad (3.42)$$

$$|\overline{e, n-1}\rangle = |e, n-1\rangle + \frac{\sqrt{n}g}{\Delta} |g, n\rangle \quad (3.43)$$

which can be understood as mainly the qubit (cavity) states perturbed by a small amount of the cavity (qubit) state.

3.4.3 Higher order χ shifts

The Schrieffer-Wolff transform can be generalized as [77, 78] to the multi-level transmon and obtain the following

$$H = \hbar\omega_r(a^\dagger a + \frac{1}{2}) + \hbar \sum_{j=0} (\omega_j + \Lambda_j) |j\rangle \langle j| + \hbar \sum_{j=0} \chi_j a^\dagger a |j\rangle \langle j| \quad (3.44)$$

The first term is the Hamiltonian for the bare resonator. The second term is the transmon Hamiltonian with Lamb shift $\Lambda_k = \sum_{j=0} \chi_{jk}$. The final term is the interaction Hamiltonian between the transmon and resonator which can be seen as a shift on either the transmon or resonator frequency. The frequency shift on the resonator from the transmon's j th state is

given by

$$\chi_j = \sum_{k=0} (\chi_{jk} - \chi_{kj}) \quad (3.45)$$

where $\chi_{jk} = g_{jk}^2 / \Delta_{jk}$ and $\Delta_{jk} = (\omega_j - \omega_k) - \omega_r$ [78].

While it is often sufficient to take the harmonic approximation such that

$$g_{jk} = \begin{cases} \sqrt{j+1}, & \text{if } j - k = 1 \\ 0, & \text{otherwise} \end{cases} \quad (3.46)$$

it is more accurate to note that for a resonator capacitively coupled to a transmon, this g comes from matrix elements in the charge basis. So while the approximation is close and transitions between j and $j \pm 2n$ are forbidden for integer n , the matrix elements between states differing by odd numbers are finite. This can lead to significant changes to the dispersive shifts from Eq. 3.45 when the relevant detunings (e.g. $\Delta_{j,j+3}$) are small. An example of the higher order χ shifts affecting qubit measurement and readout is given in section 7.2.1.

3.5 Density Matrix Formalism

In this section the density matrix will be introduced as a tool modeling for loss in a qubit. When considering ensembles of quantum states (*e.g.* repeated measurements) and decoherence, the density matrix formalism provides a generalization of the qubit state vector, Eq. 3.1.

Any ensemble of quantum states can be expressed as

$$\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i| \quad (3.47)$$

where p_i is the probability of the system having quantum state ψ_i [79]. The density matrix

for a qubit in a pure state (Eq. 3.52) is given by

$$|\psi\rangle\langle\psi| = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{pmatrix} \quad (3.48)$$

The expectation value of operator A for a general density matrix is $\text{tr}(\rho A)$. With these two properties, it is possible to evaluate

$$\text{tr}(\rho) = \sum_i p_i \text{tr}(|\psi_i\rangle\langle\psi_i|) = \sum_i p_i = 1 \quad (3.49)$$

which is the first property of a density matrix: the trace of the density matrix is 1.

The expectation value of an arbitrary state $|\varphi\rangle$

$$\langle\varphi|\rho|\varphi\rangle = \sum_i p_i |\langle\varphi|\psi_i\rangle|^2 \geq 0 \quad (3.50)$$

A density matrix is a positive operator (*i.e.* all eigenvalues are positive).

Finally looking at the square of the density matrix,

$$\text{tr}(\rho^2) = \sum_i p_i^2 |\psi_i\rangle\langle\psi_i|\psi_i\rangle\langle\psi_i| = \sum_i p_i^2 \quad (3.51)$$

$$\text{tr}(\rho^2) = 1 \quad \text{iff} \quad \exists j : p_j = 1 \text{ and } p_{k \neq j} = 0 \quad (3.52)$$

Where Eq. 3.51 has ρ written in its eigenbasis. This property qualifies that ρ represents a pure state if and only if the $\text{tr}(\rho^2) = 1$.

3.5.1 Bloch-Redfield Decoherence

Wangsness, Bloch, and Redfield developed a formalism for noise affecting a qubit [61, 80–82]. Under this model, when the qubit is coupled to a noisy thermal bath with no long-term correlations, the decoherence of the qubit can be modeled by T_1 and T_ϕ error processes. The

T_1 errors are called relaxation errors and correspond to a σ_x operation on the qubit from the environment. Noise sources which can spontaneously absorb (emit) a photon from (to) the qubit will relax (excite) the qubit unexpectedly and cause a T_1 error. Dephasing errors, or T_ϕ errors, correspond to σ_z operations removing coherence from the qubit. Noise sources which adjust the qubit frequency will give T_ϕ errors. The coherence of a qubit state can be effected by both relaxation and dephasing errors which in total are called T_2 coherence errors.

The total density matrix for a single qubit state starting in a pure state and under the effects of Bloch-Redfield decoherence is

$$\rho(t) = \begin{pmatrix} 1 + (|\alpha|^2 - 1)e^{-\Gamma_1 t} & \alpha\beta^*e^{-\Gamma_2 t} \\ \alpha^*\beta e^{-\Gamma_2 t} & |\beta|^2 e^{-\Gamma_1 t} \end{pmatrix} \quad (3.53)$$

where the diagonal terms decay to the $|0\rangle$ state at a rate Γ_1 and the off-diagonal coherences decay away at a rate $\Gamma_2 = \Gamma_1/2 + \Gamma_\phi$ [61]. If desired, this time-dependent density matrix can be derived from the master equation dynamics explained below in section 3.5.2.

3.5.2 Master Equation Dynamics

When considering evolution of the density matrix with decoherence, I used the Lindblad-Kossakowski master equation which states that the density matrix evolves according to its commutator with the Hamiltonian and completely dissipative operators [75, 83, 84]

$$i\hbar \frac{\partial \rho}{\partial t} = -[\rho, H] + \mathcal{L}[\rho] \quad (3.54)$$

where the Liouvillian super-operator, $\mathcal{L}$, gives the dissipation dynamics of a jump operator, A , by

$$\mathcal{L}[\rho] = \Gamma_A \left(A\rho A^\dagger - \frac{1}{2}A^\dagger A\rho - \frac{1}{2}\rho A^\dagger A \right) \quad (3.55)$$

where the loss rate due to the given jump operator is Γ_A . For example, T_1 relaxation errors in a qubit can be accurately modeled as this formalism and σ^- as the jump operator. For an excellent review of Lindblad-Kossakowski evolution in a qubit, see [85].

3.6 Qubit Decoherence

Any physical quantum system will be coupled to its environment. These couplings provide loss channels where information or excitations can leave or enter the computational subspace. In this section commonly considered loss mechanisms for superconducting qubits will be considered.

3.6.1 Purcell Effect

Originally described by Edward Purcell [86], coupling a two level system to a resonator can enhance the spontaneous emissions rate by effectively increasing the density of states which the system can decay into. In this case, the interaction between the qubit and the cavity provides a relaxation pathway for the qubit increasing its spontaneous emission rate. The interaction here is a T_1 relaxation rate for a transmon coupled to a cavity. Here two quick derivations of the standard dispersive limit Purcell decay approximation commonly used will be presented. The first uses Fermi's golden rule for an intuitive approximation. The second presents a more formal operator derivation. Finally, I briefly discuss the limitations of analytical Purcell loss calculations.

Derivation from Fermi's Golden Rule

The rate the transmon qubit will relax from the excited to ground state due to interaction with the cavity (section 3.4.2) can then be written simply from Fermi's golden rule

$$\Gamma_{eg}^a = \frac{2\pi}{\hbar} N(\omega) |\langle 0, g | \lambda \hat{a} | \overline{0, e} \rangle|^2 \quad (3.56)$$

Here $N(\omega)$ is the density of states of the bath which the resonator relaxes into and λ is the coupling strength between the resonator and the bath. Here, the cavity has a decay rate $\kappa = 2\pi/\hbar N(\omega)|\lambda|^2$ [63].

Starting from the Eq. 3.34, the dressed eigenstates (Eq. 3.42) for a single excitation are written

$$|\overline{g, 1}\rangle = |g, 1\rangle - \frac{g}{\Delta} |e, 0\rangle \quad (3.57)$$

$$|\overline{e, 0}\rangle = |e, 0\rangle + \frac{g}{\Delta} |g, 1\rangle \quad (3.58)$$

Therefore, in the dressed $|\overline{e, 0}\rangle$ state, there is probability g^2/Δ^2 of having 1 photon in the cavity. This state decays with the rate κ yielding the Purcell relaxation rate

$$\Gamma_{ge}^a = \kappa \frac{g^2}{\Delta^2} \quad (3.59)$$

Operator Derivation

For a cavity relaxing to a bath, the relevant operator is $H_\kappa = \sum_k \lambda_k a c_k^\dagger = \lambda a$, lumping all the bath dynamics (characterized by c operators) into a single coupling rate λ . This relaxation is in the uncoupled basis so it needs to be transformed into the dressed basis. Using the Schrieffer-Wolff rotation in section C.5.1

$$U = e^{iS}, \quad S = \frac{g}{\Delta} (a^\dagger \sigma_- - a \sigma_+) \quad (3.60)$$

to transform H_κ with the Baker-Campbell-Hausdorff (BCH) formula.

$$H'_\kappa = H_\kappa + [S, H_\kappa] + \dots = \lambda \hat{a} - \frac{g}{\Delta} \lambda \sigma_- + \dots \quad (3.61)$$

Again, we're interested in

$$\Gamma_\kappa = \frac{2\pi}{\hbar} N(\omega) |\langle 0, g | H'_\kappa | 0, e \rangle|^2 \quad (3.62)$$

$$\approx \frac{2\pi}{\hbar} N(\omega) \lambda^2 \frac{g^2}{\Delta^2} \quad (3.63)$$

$$= \kappa \frac{g^2}{\Delta^2} \quad (3.64)$$

The first order correction is the main term that contributes in Fermi's golden rule. I have notes calculating other terms to higher levels but it was not particularly informative or difficult, just tedious.

Limitations in a Physical Systems

The Purcell loss will have contributions from all the modes of a microwave cavity. In some regimes, the multi-mode Purcell effect can be an important source of relaxation. However, considering this naively, the parallel loss channels lead to a diverging loss rate and a more careful analysis is required [60]. In a physically realized circuit, the loss can calculate $\Gamma_\kappa = \text{Re}[Y(\omega_q)]/C_\Sigma = 1/RC$ where $Y(\omega_q) = 1/Z(\omega)$ is the admittance of the circuit seen by the transmon at the transmon's frequency. Often the admittance is simulated with a finite-element solver such as HFSS [87] including contributions up to the limit of the solver, then the Purcell loss is calculated to confirm a design is not limited by this loss mechanism (section 4.2.2).

3.6.2 Dephasing from Charge Dispersion

Section 3.2.4 examined how the transmon's charge dispersion, or how the transition frequencies vary with offset charge n_g . Due to quasiparticle tunneling (section 3.6.4) and slow background charge drifts, n_g can change randomly. A transmon with large E_J/E_C , Eq. 3.28

gives the frequency sensitivity to n_g as

$$\frac{d\omega_q}{dn_g} = \epsilon_1 \frac{\pi}{\hbar} \sin(2\pi n_g) \quad (3.65)$$

Fluctuations of n_g can have $1/f$ behavior at low frequencies [61], and so for long times the offset charge will explore it's full periodic range with $\delta n_g > 1$. Therefore a rough bound on the dephasing becomes

$$T_\phi < \frac{1}{\Delta\omega_q} = \frac{\hbar}{\epsilon_1} \quad (3.66)$$

while more complex modeling involving charge noise spectral densities can give tighter results.

3.6.3 Cavity Photon Dephasing

Another source of dephasing from the structure of cQED is cavity induced dephasing. From Eq. 3.41, the qubit frequency is dependent on the number of photons in the cavity. The total shift of the qubit frequency is given by

$$\Delta\omega_{q, Stark} = 2\chi n \quad (3.67)$$

where n is the photon number in the cavity [88].

From this, the dephasing due to the cavity in the dispersive limit ($\chi \ll \Delta$) is

$$\Gamma_\phi \approx \frac{4\chi^2}{\kappa} \bar{n} \quad (3.68)$$

where κ is the decay rate of photons in the cavity [61, 89]. Dephasing from cavity photons makes it especially important to properly thermalize the cavity input lines [76, 88, 90].

3.6.4 Quasiparticle Loss

Quasiparticles are excitations of the superconducting condensate (section 2.1). These quasiparticles present another source of loss in a transmon device [91]. When a quasiparticle tunnels across the transmon junction, the quasiparticle can absorb energy from the qubit, causing relaxation from the excited state to the ground state (Fig. 3.5a). Additionally, at higher temperatures, quasiparticles with energies well above the gap can emit a photon while tunneling and excite the qubit. Together, these two effects give the quasiparticle relaxation pathway for the qubit.

Following Catelani et al. [91], I assume equal superconducting gaps $\Delta_1 = \Delta_2$ and that the quasiparticles have a thermal distribution with temperature below the qubit frequency $k_B T \ll \hbar\omega_q \ll 2\Delta_1$. In this case the quasiparticle relaxation rate is

$$\Gamma_{1 \rightarrow 0} = \left| \langle 0 | \sin(\hat{\phi}/2) | 1 \rangle \right|^2 S_{qp}(\omega_{01}) = \frac{E_C}{\hbar\omega_{01}} S_{qp}(\omega_{01}) \quad (3.69)$$

where S_{qp} is the quasiparticle tunneling noise spectrum. This formula can be used to find the loss for an arbitrary distribution function given its noise spectrum.

S_{qp} can be written in terms of the quasiparticle density as

$$S_{qp}(\omega) \approx \frac{\omega}{2\pi} \left(\frac{2\Delta}{\hbar\omega} \right)^{3/2} x_{qp} \quad (3.70)$$

where $x_{qp} = n_{qp}/2\nu_0\Delta$ is the quasiparticle density normalized by the density of paired electrons. $\Gamma_{1 \rightarrow 1}$ then simplifies to

$$\Gamma_{1 \rightarrow 0} = \frac{E_c}{2\pi\hbar} \left(\frac{2\Delta}{\hbar\omega} \right)^{3/2} x_{qp} \quad (3.71)$$

The quasiparticle noise spectral density can also be calculated with the junction tunneling

at the qubit energy [92].

$$S_{qp}(\omega_{01}) = \frac{e}{2\pi} [I_{qp}(\hbar\omega_{01}/e) + I_{qp}(-\hbar\omega_{01}/e)] \quad (3.72)$$

Equation 2.34 gives the net current across a junction $I_{net} = I_{RL} - I_{LR}$ but can be modified to give the total current due to quasiparticle tunneling. For a single quasiparticle tunneling through a low-transparency barrier,

$$I_{qp}(V) = I_{RL} + I_{LR} \quad (3.73)$$

$$= \frac{1}{eR_n} \int_{-\infty}^{\infty} \frac{|E - eV|}{\sqrt{(E - eV)^2 - \Delta_R^2}} \frac{|E|}{\sqrt{E^2 - \Delta_L^2}} \\ * [f_R(E - eV, T) + f_L(E, T) - 2f_R(E - eV, T)f_L(E, T)] dE \quad (3.74)$$

If quasiparticles are present in the electrodes, their tunneling should be suppressed in a junction with asymmetric gaps when $|\Delta_R - \Delta_L| > \hbar f_{01}$ where the difference in superconducting gap energies, Δ_R, Δ_L is greater than the qubit frequency f_{01} . At low temperatures, any quasiparticles present should have energy close to the gap energy. Therefore any qps on the low-gap side require more than $\hbar f_{01}$ of energy to tunnel to the high-gap side of the junction (Fig. 3.5b).

Modifications for other, more exotic tunneling processes such as Multiple Andreev Reflection (MAR) can be made to Eq. 3.73 to calculate the loss due to those processes.

3.6.5 Thermal Quasiparticles

Due to the non-zero operating temperature of the dilution refrigerator and environment of the qubit, the qubit has thermally distributed quasiparticle excitations [37]. In thermal equilibrium, quasiparticles lie near the gap edge and their density is exponentially suppressed at low temperature.

With this new assumption, the thermal distribution of superconducting quasiparticles

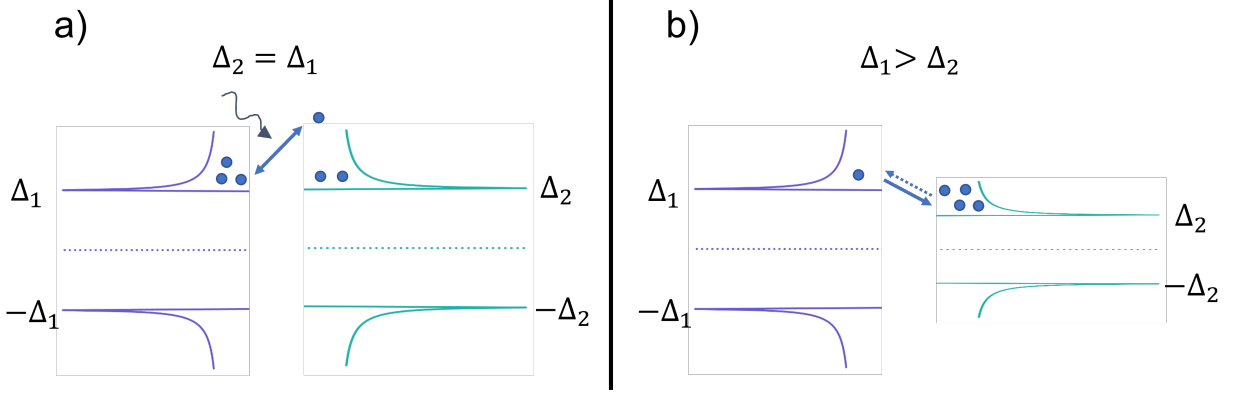


Figure 3.5: Energy gap diagrams of Josephson junctions. **a)** a typical junction with equal superconducting gaps allows quasiparticles at the junction to tunnel across and absorb a photon from the qubit causing relaxation of the qubit state. **b)** In a gap engineered junction, quasiparticles on the low-gap side require sufficient energy to tunnel to the high-gap side, suppressing quasiparticle tunneling in the large difference limit $|\Delta_1 - \Delta_2| > hf_{01}$. Quasiparticles will tend to accumulate in the low-gap side to minimize their energies.

can be written

$$x_{qp} = \sqrt{2\pi k_B T / \Delta} e^{-\Delta/k_B T} \quad (3.75)$$

Plugging into Eq. 3.71 finds that the relaxation rate can be expressed by

$$\Gamma_{1 \rightarrow 0} = \frac{2E_C \Delta}{\hbar} \sqrt{\frac{k_B T}{\pi(\hbar\omega)^3}} e^{-\Delta/k_B T} \quad (3.76)$$

It is worth noting the exponential decrease in relaxation rate with decreasing temperature. In Al films at base temperature of the dilution refrigerator, 20 mK, the normalized thermal quasiparticle density $x_{qp} \approx 10^{-51}$ and the expected relaxation time for a typical qubit due to thermal quasiparticles far longer than the age of the universe.

3.6.6 Non-Equilibrium Quasiparticles

Quasiparticles in transmons have been hypothesized to come from many sources including normal tunneling in Josephson junctions [93], IR radiation on the device [24–26, 94], cosmic ray scattering in the chip [27, 95, 96], mechanical modes [97], and other excitations in the system.

No matter their source, these non-equilibrium quasiparticles will, by definition, have a distribution different from the thermal quasiparticles in Eq. 3.75. One popular modification is arrived at by assuming the quasiparticles reach a thermal distribution at an effective temperature, T^* , above the temperature of the thermal phonon bath [24, 26, 98]. This model arises when pair-breaking phonons persist in the superconductor and limit the quasiparticle density decay time [98]. For a general effective temperature,

$$n_{neqp}^2 = n_{thqp}^2 \left(\frac{T^*}{T} \right)^3 \left[\int_{X_G^*}^{\infty} \frac{x^2}{e^x - 1} dx / \int_{X_G}^{\infty} \frac{x^2}{e^x - 1} dx \right] \quad (3.77)$$

where $X_G = 2\Delta(T)/k_B T$ and $X_G^* = 2\Delta(T^*)/k_B T^*$ are the thermal and effective thermal energies normalized. As long as $k_B T^* \ll \Delta$ remains, this effective temperature defines a non-equilibrium quasiparticle density

$$n_{neqp} = \sqrt{2\pi k_B T^* / \Delta(T^*)} e^{-\Delta(T^*)/k_B T^*} \quad (3.78)$$

These densities are found to be much higher than expected thermal densities. Measured quasiparticle densities, x_{QP} , range from around 10^{-9} [26] to 10^{-5} [99] corresponding to 100 - 200 mK effective temperatures. An alternative model introduces a chemical potential, μ , for the quasiparticles in the superconductor [100].

Additionally, if there is any gap asymmetry between electrodes on either side of the Josephson junction, the quasiparticles can redistribute, giving complicated loss properties as equilibrium is reached by the quasiparticles rearranging to the lower gap material [24, 25].

3.6.7 Two-Level System Loss

A final loss mechanism commonly considered in superconducting qubits is two-level system (TLS) loss. Somewhat confusingly, both the qubit level system and this microscopic loss mechanism are referred to as two-level systems. To differentiate, I will use the abbreviation

TLS exclusively to refer to the loss source.

In dielectric materials and interfaces, the stored electric energy is partially lost due to TLSs in the dielectrics [101, 102]. The loss from an ensemble of TLSs can be characterized by the loss tangent of a given material

$$\tan \delta = \frac{\text{Im}(\epsilon)}{\text{Re}(\epsilon)} \quad (3.79)$$

where ϵ is the complex permittivity constant of the dielectric material when electric field is applied [103]. The loss tangent gives the portion of power lost due to energy stored in the dielectric material meaning the dielectric loss for the device can be written as

$$\Gamma_{TLS} = \omega_q \sum_i P_i \tan \delta_i \quad (3.80)$$

where the sum is over all the lossy dielectrics in the system with participation ratios P_i quantifying the portion of the energy stored in a given material. Normally, these include the dielectric sapphire substrate, oxide interfaces of metal layers [104, 105], and even packaging material [106].

Dielectric loss arises in low temperature dielectrics from two-level systems. A TLS can be thought of as a charge moving in a double-well potential characterized by the energy difference between the two trapped states and the energy of the tunneling barrier. From this microscopic picture, Gao [107–109] derived the loss of an ensemble of TLSs as

$$\Gamma_{TLS} = \Gamma_{TLS,0} \frac{\tanh(\hbar\omega_q/2k_B T)}{\sqrt{1 + \bar{\Omega}^2 T_1 T_2}} = \Gamma_{TLS,0} \frac{\tanh(\hbar\omega_q/2k_B T)}{\sqrt{1 + \bar{E}^2 C \frac{\tanh(\hbar\omega_q/2k_B T)}{k_B T}}} \quad (3.81)$$

Here $\bar{\Omega}$ is the Rabi rate of the TLS under the applied field, and T_1 and T_2 are the coherence times for the TLS [110, 111], $\bar{E}$ is the electric field strength, C is a coupling constant between the qubit and TLS, and $\Gamma_{TLS,0}$ is the loss at zero temperature and field for the TLS. This theory predicts a temperature and power dependence of the measured TLS loss.

Measurements of low-loss resonators with $Q_i > 10^5$ do not follow the power dependence in Eq. 3.81 so the power dependence is modified with a weaker power dependence [109]. These changes to the model often have tenuous or no justification making the details of TLS loss models phenomenological. Non-linear behavior of quasiparticles has been proposed as an alternative explanation for the measured power dependence of loss in superconducting devices [112].

Chapter 4

Device Design

The unparalleled flexibility of superconducting qubits is one of the fields greatest strengths, enabling a wide range of circuits, experiments, and science. However, since a wide range of cavities, qubits, couplings, readouts, controls, and more are possible, careful design work must be done to ensure a given device is well suited for an experiment. A device designed to test fast state readout will make a terrible design for testing T_1 limitations whose design, in-turn, would be terrible for testing 2-qubit gates. In this chapter, I cover the principles of designing a 3D cavity-transmon system. I then discuss the design of the 3D transmons I fabricated. Next I cover finite element simulation of a 3D cQED architecture. Finally, I cover black box quantization (BBQ) to extract simulated qubit parameters before any devices are fabricated.

4.1 3D Transmon Design

A 3D transmon system consists of a transmon with large dipole moment placed in a 3 dimensional conducting cavity. Implementing cQED architecture in a 3D geometry has many benefits including providing a high quality cavity for readout, a superconducting enclosure to shield the qubit from electromagnetic fields, and a large qubit electromagnetic mode volume leading to dispersed electric field and lower TLS loss [113].

4.1.1 Circuit Design

The circuit diagram for my 3D transmon devices is shown in Fig. 4.1a and resembles any cQED transmon system [63]. The transmon consists of a capacitively shunted Josephson junction coupled to a cavity. A coupling capacitor C_g controls the coupling strength between the transmon and resonator in the Jaynes-Cummings Hamiltonian (Eq. 3.36). The cavity is modeled as a single mode resonator and is coupled to an input transmission line through an input coupling capacitor, C_{in} , and an output transmission line through an output coupling capacitor, C_{out} .

4.1.2 Cavity Design

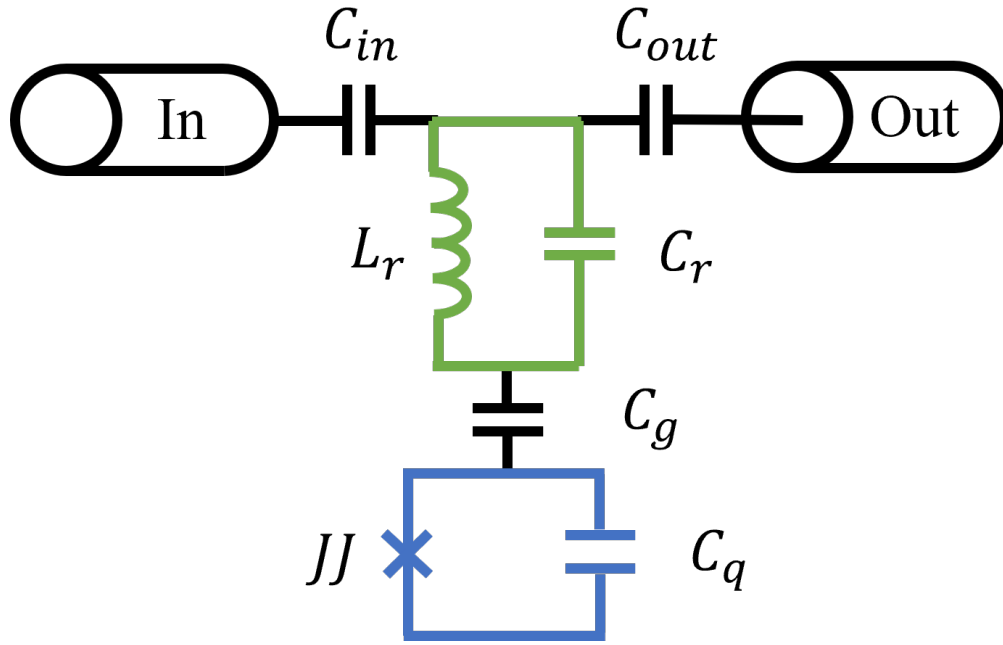
In my physical system, an Al 3D cavity serves as the readout resonator (Fig. 4.1b). A 35 mm by 20.5 mm by 5 mm cavity was machined out of 6063 Al. The lowest fundamental resonant mode of the cavity is the TE_{101} mode at approximately 7.8 GHz [114].

To excite and measure the cavity, two SMA coupling ports were connected to the refrigerator wiring. The conducting center-pins of the non-magnetic SMA feed-through connectors enter the cavity and form the coupling capacitors between the cavity and external transmission line.

A physical cavity has multiple resonant frequencies. For my qubit experiments, the cavity was designed so the dominant interaction with the qubit was with the fundamental resonant TE_{101} mode. Since the interaction between the qubit and a given cavity mode scales like $\chi \propto 1/(\omega_i - \omega_q)$, this is accomplished by fabricating a transmon with transition frequency below the first cavity mode, reducing the effect of higher modes.

A sapphire chip with the transmon device is placed in a slot in the center of the cavity. The transmon pads create a dipole moment aligned with the cavity mode electric field. This gives relatively strong coupling C_g to the primary mode and also reduces coupling to higher order modes with different electric field configurations (*e.g.* the TE_{102} has a node at the qubit position) [114].

(a)



(b)

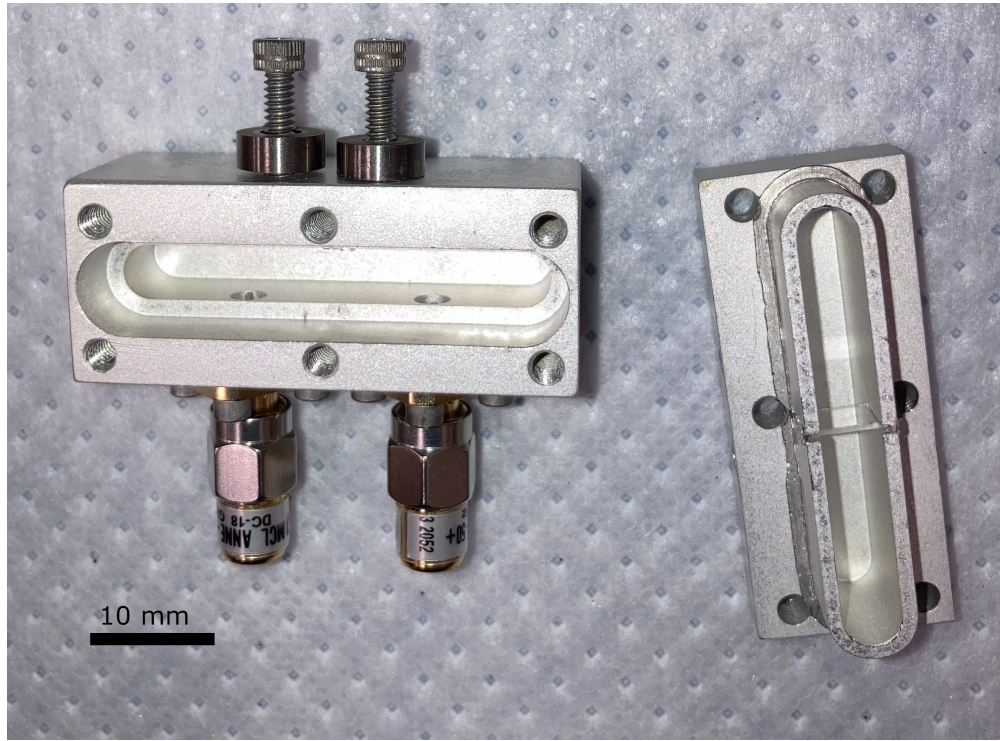


Figure 4.1: Circuit model **(a)** and photograph **(b)** of a 3D transmon. **a)** Transmon (blue) consists of a Josephson junction (marked JJ) and capacitor (C_q). Cavity is modeled as a single mode LC resonator with L_r and C_r . The transmon and cavity are coupled by capacitance C_g . The cavity is coupled to an external transmission line via coupling capacitors C_{in} and C_{out} . **b)** Al 3D cavity with transparent sapphire transmon chip in the center of the right half and two coupling ports on the left half.

4.1.3 Transmon Design

The design of a 3D transmon consists of two large pads forming the capacitor and a small junction connecting the two pads (Fig. 4.2). For my transmon, the capacitor pads were designed to have $E_C/h \approx 190$ MHz and $g/2\pi \approx 100$ MHz. These values were verified by simulation and previous device measurements [24].

The junction electrodes were designed to be 150 nm x 150 nm. The size is chosen to be large enough for repeatable fabrication, but as small as otherwise possible to minimize the number of anomalous two-level system in the junction [107]. Next, a desired transmon transition frequency is chosen, usually between 2-6 GHz. This range is below the readout cavity and far enough detuned to be in the dispersive regime (section 3.4.2). Qubits with higher frequencies in this range will have larger readout signals and less charge dispersion of the transmon levels, but lower frequency qubits have been found to have longer coherence times [24, 115].

For a designed E_C and qubit frequency f_q , the required E_J can be estimated from Eq. 3.27 as $E_J \approx h(f_q + E_C)^2/8E_C$. Using the Ambegaokar-Baratoff relation for a symmetric gap, low-temperature tunnel junction [116], the target normal-state resistance is given by

$$R_n = \frac{\Delta\Phi_0}{4eE_J} \quad (4.1)$$

These approximations provide a good design target for fabrication. The qubit frequency range of 2-6 GHz and $\Delta_{Al} \approx 200$ μeV corresponds to junction resistance $R_n = 24 - 6$ $k\Omega$. For the process outlined in section 5.3, this resistance range was achieved with an oxidation dose of 10 - 100 mbar·min. In general, multiple rounds of test junctions should be fabricated and measured because the oxygen dose's relation to junction resistance changes over time due to uncertainties in control parameters.

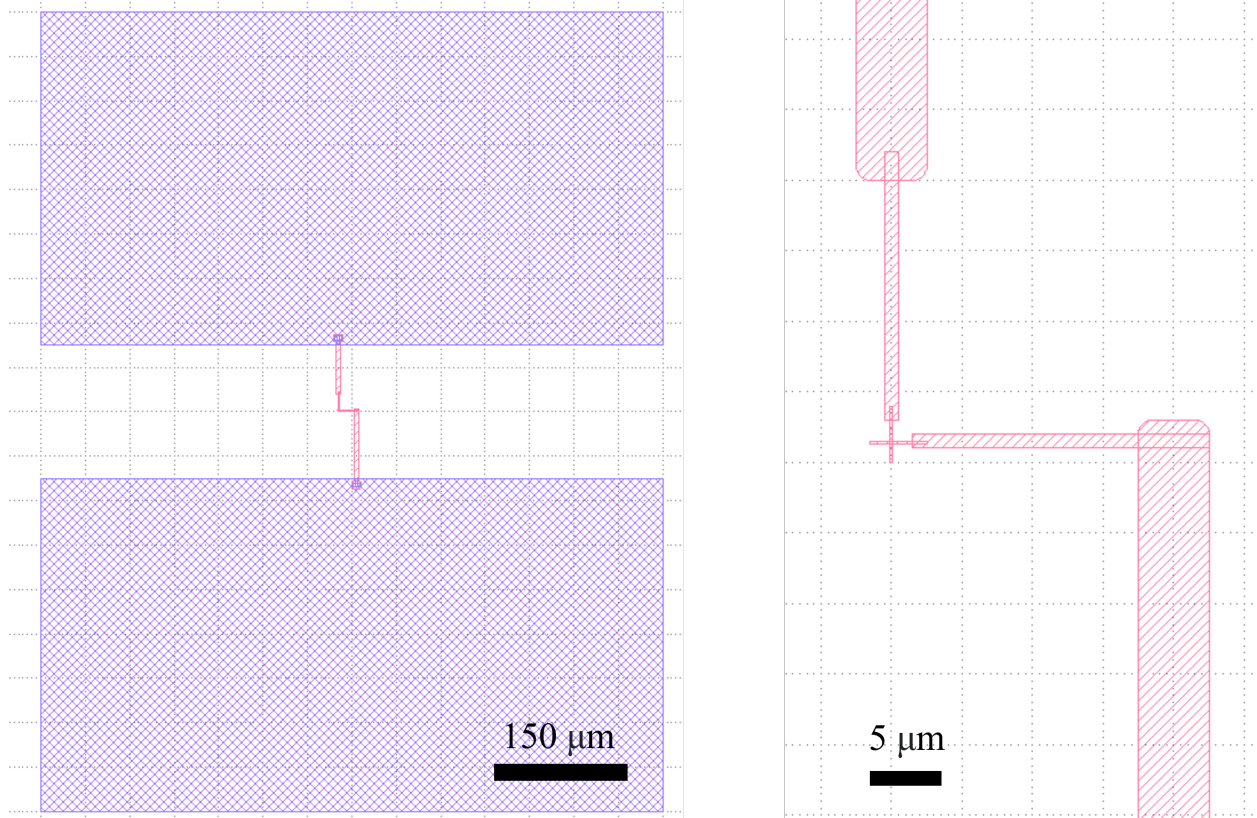


Figure 4.2: Transmon design pattern. *Left* is the full 3D transmon pattern. Large pads shown in purple serve both as the transmons shunting capacitor and also as a dipole coupling antenna to the cavity mode. *Right* is a zoom in of the junction with thin lines forming a nominally 150 nm x 150 nm junction fabricated using the Manhattan style (section 5.3.2).

Junction Capacitance Contribution

It is interesting to check the assumption that the junction capacitance is negligible for transmon junctions. For a 5 GHz transmon with $E_J/E_C = 50$, the total $I_C = 26$ nA and $C_{tot} = C_J + C_{sh} = 73$ fF for the qubit can be extracted from Eqs. 3.27, where C_J is the capacitance directly from the junction and C_{sh} is the shunt capacitance. A typical critical current density for small, double-angle Josephson junctions was around $0.7 \mu A/\mu m^2$. This implies a junction area of $0.037 \mu m^2$, slightly larger than my designed junction area. The work in [117] estimated the capacitance per unit area of similar junctions to be 100 ± 25 pF/ μm^2 meaning this qubit's junction would have a capacitance of approximately 3.6 fF. The junction then would contribute 4% of the total capacitance of the transmon. Therefore in the transmon regime, the total capacitance is largely dominated by the capacitance of the external capacitor, but the approximate junction capacitance should be considered for accurate calculations.

4.2 Simulation

Before fabricating or measuring a device, many properties can be verified via electromagnetic simulations. Having robust simulations of a device gives confidence that the device will perform correctly and builds a collection of predicted properties that can be used to check device measurement.

4.2.1 Electro-Magnetic Simulations

To verify the microwave properties of the cavity, I performed simulations in High Frequency Simulation Software (HFSS) a software package from Ansys to perform finite element EM simulations.

Following [75, 85, 114], I imported the CAD design into HFSS and manually added the coupling pins, sapphire chip, and transmon capacitor. The two cavity ports were set as

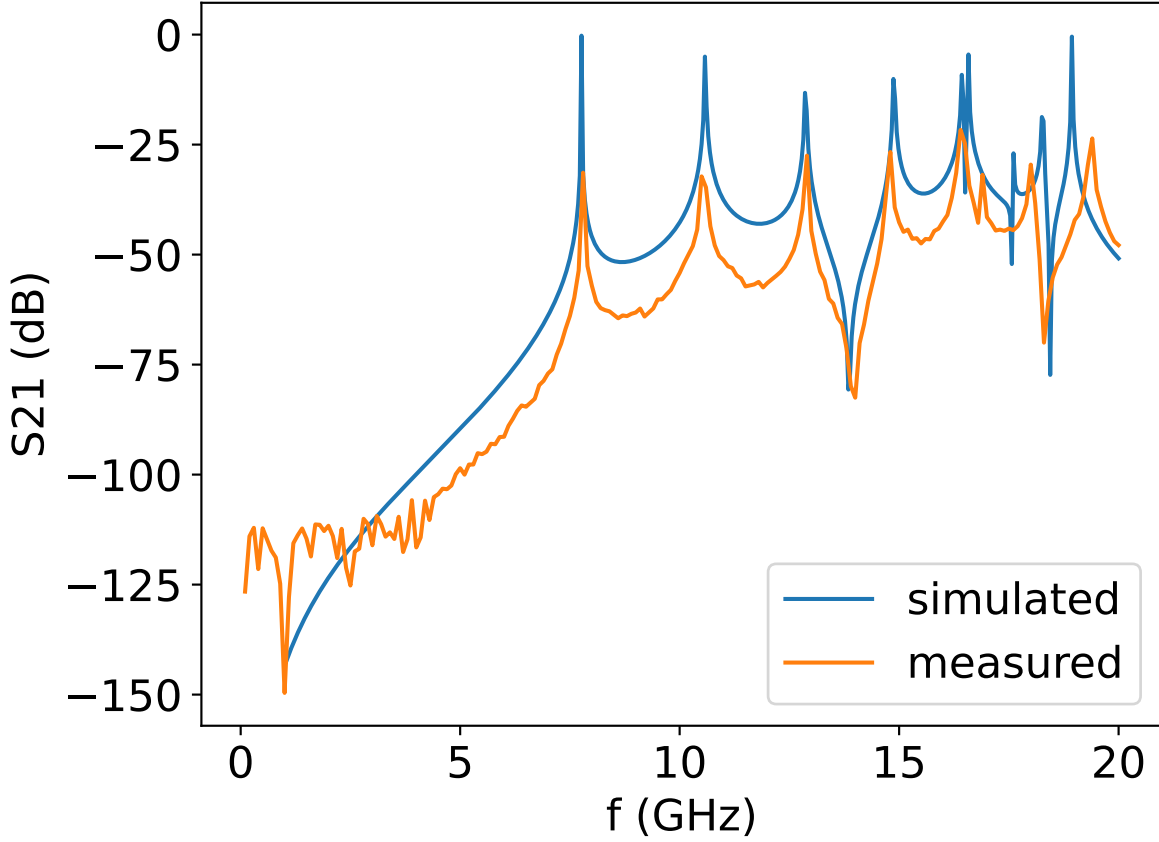


Figure 4.3: Simulation and measurement at room temperature of the 3D cavity S_{21} modes of the cavity shown in Fig. 4.1b. The HFSS simulations and VNA measurement of the cavity at room temperature give very close agreement on cavity mode frequencies. Discrepancies in amplitudes are likely from conductive losses not present in the simulation.

50 Ω waveports and the transmon junction were simulated as a high impedance (10 $G\Omega$) lumped element port connecting the capacitors. This setup allows me to simulate the total impedance seen by the qubit by examining the impedance at the port located at the junction.

First, I verified the cavity resonances were simulated correctly. Fig. 4.3 shows the transmitted signal (S_{21}) vs frequency for lossless simulated data (blue) and room temperature measured data (orange). At 7.8 GHz the S_{21} has a strong peak corresponding to the cavity's fundamental resonant mode. The higher modes are also observed to be in good agreement between simulation and measurement suggesting the simulation is being performed accurately.

4.2.2 Black Box Quantization

Finite element microwave simulation can also be used to predict transmon properties using a semi-classical technique called black box quantization (BBQ). This method takes simulation data of the electromagnetic environment of the device (in my case, the resonator and the qubit capacitor) and inserts the linear behavior of the junction to calculate the first few energy levels and couplings [118].

Practically, I take the simulated admittance at the junction port (section 4.2.1), and add the first order junction admittance

$$Y_{tot}(\omega) = Y_{sim}(\omega) + Y_J(\omega) = Y_{sim}(\omega) + \omega C_J - 1/\omega L_J \quad (4.2)$$

where C_J is the parallel plate capacitance for the junction (section 4.1.3), and L_J is the Josephson inductance (Eq. 3.12).

With the linear behavior of the junction added, the resonant mode (*i.e.* the qubit frequency) will occur at the frequency where the admittance has a node (Fig. 4.4). Other circuit behavior including lowest order couplings and Purcell loss can be calculated from the simulated impedance data [85, 118]. The full impedance simulation is the method of obtaining the Purcell effect loss with least assumptions [60] (section 3.6.1). However, for qubit designs which have large detuning from the cavity and, hence, a small Purcell decay rate, the resolution of the simulation can reduce the accuracy of the results.

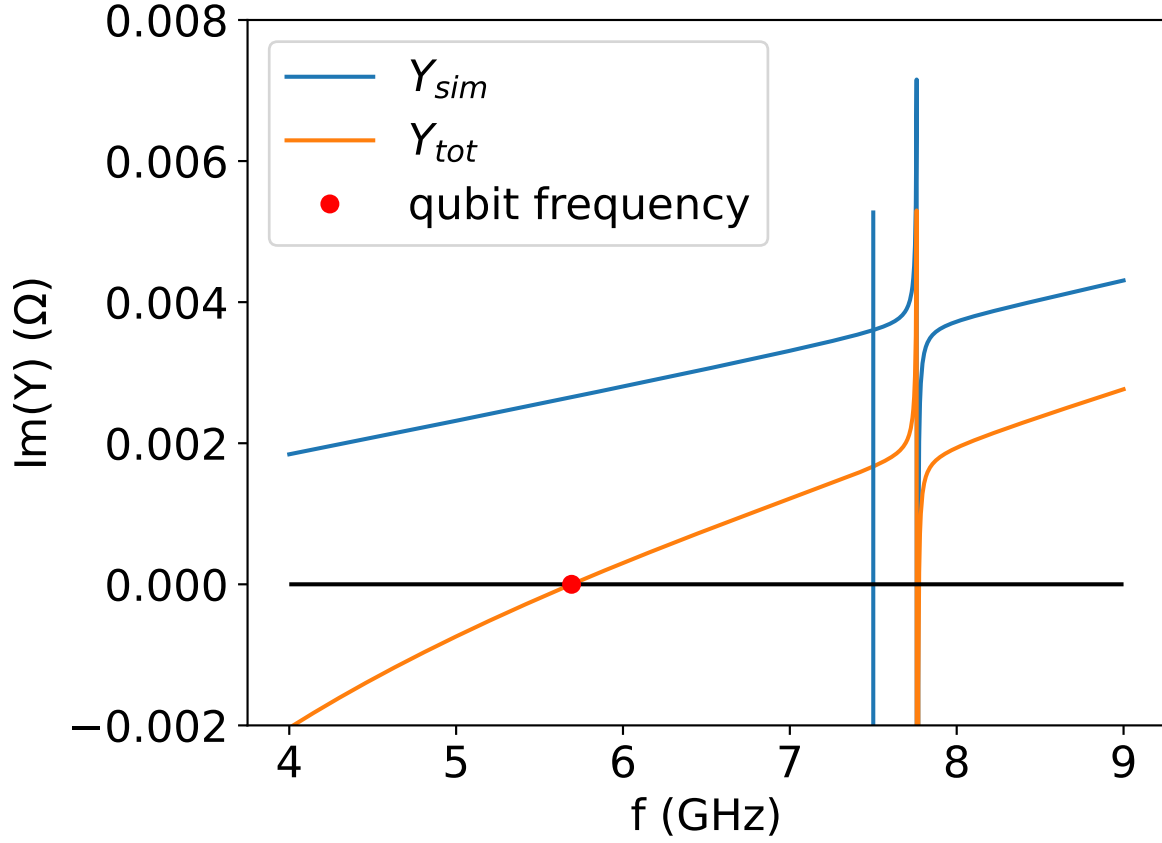


Figure 4.4: BBQ results for a 3D transmon. The imaginary part of the admittance vs. frequency for the qubit rf-environment (blue) and rf-environment plus linear qubit behavior (orange). Nodes of the total impedance predict resonant frequencies of the circuit. In this case the lower node corresponds to the qubit frequency.

Chapter 5

Device Fabrication

5.1 Nanofabrication

5.1.1 Working in a cleanroom

Nanofabrication is a field of precise chemistry and optics to form customizable, repeatable mesoscopic electronic devices. Following the specifics of a nanofabrication process allows the creation of devices with great control and accuracy. However, for a grad student in a research group, fabrication is a lot like baking and even more like Hogwarts potions class. Following the developed recipe is essential, but at times, personal intuition on sample rinsing or scrawled notes from the half-blood prince about how to agitate the device in solution may also be extremely helpful. Learning the details of a fabrication recipe (ideally alongside an experienced user) allows the successful fabrication of devices. Keeping an open and critical mind to the large number of factors affecting a recipe will allow one to test and debug a recipe when things go wrong. With this perspective, this chapter discusses the processes I used to fabricate superconducting qubits.

5.2 Optical Lithography

The first specific processes that I cover is optical lithography. Optical lithography allows the patterning of a large area (in my case a chip $5 - 10\text{ mm}$ on a side) with precise features down to single μm scales. This is used to fabricate large 3D transmon pads as well as complex 2D multi-qubit and resonator chips. For single layer patterns of features achievable by the optical exposure tools, this is much faster and repeatable than e-beam writing.

5.2.1 Patterning Methods

There are two main methods I used to pattern metal with optical lithography: liftoff and etching. The order of process steps for the two is fairly different. For liftoff, the substrate is cleaned, resist is spun, exposed, and developed, metal is deposited, resist and extraneous metal are removed - forming the pattern. For etching, the substrate is cleaned, then metal is deposited and cleaned, resist is patterned and developed, metal is etched completely away - forming the pattern, and finally, resist is removed. I next cover etching patterns for Ta qubits in full detail. The following steps can basically be done in the adjusted order to perform a liftoff process.

Etching the films allows the high-quality growth of a film on a cleaned substrate with molecular beam epitaxy (MBE) or sputtering before any lithography, often leading to cleaner interfaces and higher quality films. However, most etches involve strong acids which may be incompatible with any previous fabrication steps so may not be possible and require proper safety precautions. Often etch processing is used to pattern the large features of a device.

Liftoff processes can create multiple shifted patterns of the same mask by manipulating deposition angles which is used to make standard Josephson junctions (section 5.3). Liftoff also involves one fewer step than etching. However, liftoff can leave trace undeveloped resist between the substrate and deposited metal giving dirtier devices. Additionally, liftoff patterns are incompatible with sputtering films and must only be used in a metal evaporation

process.

5.2.2 Substrate Cleaning

I started with a $0.550\ \mu\text{m}$ thick, three-inch diameter HEM grown c-plane sapphire wafer. First, I sequentially rinsed the wafer in acetone, methanol, and isopropanol to remove any dust or soluble residue then blow dried the wafer with N_2 gas. This wafer was then cleaned in a piranha solution of 3:1 ratio of H_2SO_4 and H_2O_2 on a $70\ ^\circ\text{C}$ hotplate for 15 minutes, then rinsed in running DI water for 10-20 minutes. The piranha clean should remove any residual organics from the wafer before processing begins. The wafer was then rinsed with isopropanol to remove the water and blown dry with filtered nitrogen gas. Extra care should be taken and all the specific safety procedures should be followed when working with strong acids like those used in piranha solution.

5.2.3 Metal Deposition

For my Ta metal devices, the clean and deposition were done by STAR Cryoelectronics foundry services. Ta sputtering was done by the company and with a thickness of 100 – 200 nm of α -phase Ta on sapphire.

For my Al devices, the Al ground planes were deposited in the Plassys MEB 550 S (section 5.3.7). When depositing the Al ground plane, 100 nm of metal was deposited at a nominal rate of 0.5 nm/s with the substrate heated to $750\ ^\circ\text{C}$.

5.2.4 Resist patterning

To etch the metal, one must have photoresist with the desired pattern to serve as a mask for the etch.

First, I spin positive photoresist OIR 908-35. The wafer was placed on the resist spinner and a few mL of resist was taken from the resist jar with a pipette and added to the center

of the wafer. The wafer was then spun at 3000 RPM for 60 s. For all spins, the spinner was ramped up (down) between 500 RPM and full speed in 5 s at the beginning (end) of the process. The resist was pre-baked on a 90 °C hotplate for 60 s.

Next the wafer was placed in the exposure tool. For the devices discussed in this dissertation, a Heidelberg MLA150 Advanced Maskless Aligner was used. This tool takes a .gds file as input and raster-writes the specified pattern. I wrote with the 405 nm laser at 550 power level and -5 defocus for my devices.

The technique allowed relatively quick writing of large patterns and easy pattern adjustment between runs. Earlier devices made during my dissertation were patterned on the GCA i-line wafer stepper. This tool uses a pre-defined metal mask to expose a wafer in under a second but is more suited for larger scale production than device design and testing.

After the optical pattern was exposed, the sample was post-baked on a 120 °C hotplate for 60 s. To develop the resist, the sample was dipped in OPD 4262 developer for 60 s and agitated aggressively. To stop the develop, the sample was dipped in DI water and agitated for 30-60 s, then rinsed under a DI water spigot and blow dried.

Finally, the sample was visually inspected under an optical or confocal microscope. For most ground plane patterns, features were visible with these microscopes and any critical features were checked.

5.2.5 Etching

To pattern the deposited metal and form devices, I then etched the metal through the photoresist mask. Ta devices were patterned primarily with a wet etch by dipping the sample in a 1:1:1 solution of HF:HNO₃:H₂O for 7-8 s [119]. This etchant removes Ta very quickly and so some practice and comparisons with microscope images were done to know what the sample looks like in etchant when completed. Diluting the etchant with 1 part DI water, 2 parts etchant will approximately halve the etch rate, but may attack the resist at a higher rate.

Al devices were etched in Al Etchant 80-15-3-2 for 145 s for 100 nm films [85].

5.2.6 Resist Removal

Next the resist mask was removed from the device. The resist was removed using PG Remover, 1165 Remover, or any other NMP based resist stripper. A beaker of remover was poured and heated at 80 °C. The device was placed into the solution, covered, and left for 60-180 minutes. For additional cleaning, the device was moved to another heated bath of remover for 10 minutes after the main liftoff was done in the first bath. Next, the sample was dipped in isopropanol for 1-2 min then blown dry with nitrogen.

5.2.7 Metal Cleaning

Since tantalum is an inert metal, an aggressive clean was done on the metal after it was patterned to remove any resist residues.

I repeated the piranha clean outlined in section 5.2.2 to remove any remaining resist or other organics on the device.

After these steps, the Ta patterning was completed (see Fig. 5.1) and next the small Josephson junctions could be added to form qubits.

I note that x-ray photoelectron spectroscopy has shown evidence of sulphur on Ta and Nb surfaces after a piranha clean [120].

For Al ground-planes, such aggressive procedures will etch the metal so simple acetone, methanol, isopropanol rinses were performed instead.

5.3 Junction Fabrication

The Josephson junction is a critical component to most superconducting qubits, providing nonlinear response in the circuit enabling level isolation. For a transmon, the Josephson junction's response forms a slightly anharmonic resonator (section 3.2.3). For this reason,

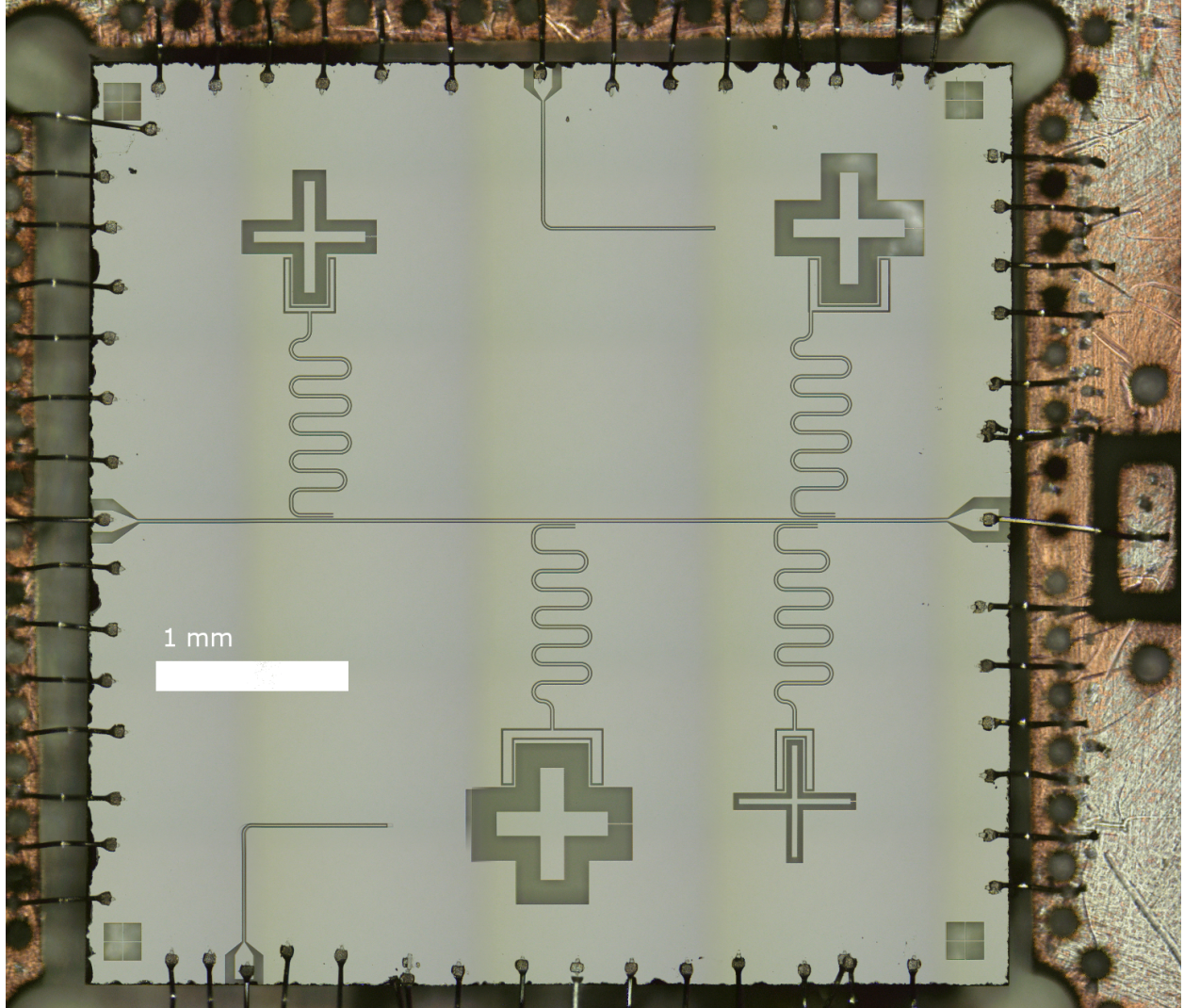


Figure 5.1: Micrograph of packaged chip **Q1**. This chip has four different qubits (crosses) coupled to resonators which are in turn connected to a central transmission line. The metal patterns visible on the chip are patterned Ta using the techniques in 5.2. Results on this device are discussed in Chp. 11. **Q1** is the qubit in the top left. Three other qubits were designed on this device but did not survive processing.

much of the fabrication processing for superconducting qubits focuses on how to make clean, reproducible Josephson junctions.

In general, for typical qubit devices and all the junctions used in this thesis, the objective was to create two layers of aluminum metal connected through an aluminum oxide barrier. To create a controllable tunnel barrier, all three of these layers were made in the same chamber without exposing to the atmosphere in between. First aluminum was deposited, then a controlled amount of oxygen was let into the chamber to form a native oxide, and then the final layer of aluminum was deposited. Ideally, then the barrier is then ‘locked in’ and will not change drastically when exposed to air. Details of the process I used will be given below.

For the junction parameters used in conventional transmon qubits, the critical current must be small, on the order of 100 nA. Therefore, for typical barrier thicknesses, the overlap area between the two aluminum layers must be very small, on the order of 10^{-14} m², or a square overlap with 100 nm on a side. To define this small area, electron beam lithography was used to pattern a junction. The tri-layer junctions were formed by two common techniques: Dolan bridge junctions and Manhattan junctions, the details of each are explained below 5.3.1. At the beginning of my dissertation work, it was standard in the group to use Dolan bridge junctions for all devices. However, part way through, these devices started to have a high failure rate when using the new e-beam writer (section 5.3.5). It was found that the Dolan bridge was cracking under strain similar to [121] and shorting the junctions (Fig. 5.2). Since this problem was immune to many fabrication adjustments we attempted, I chose to implement junctions using the Manhattan technique. These junctions had much higher fabrication success rates and are now the standard method for junction processing in the group. Below I outline and compare both techniques.

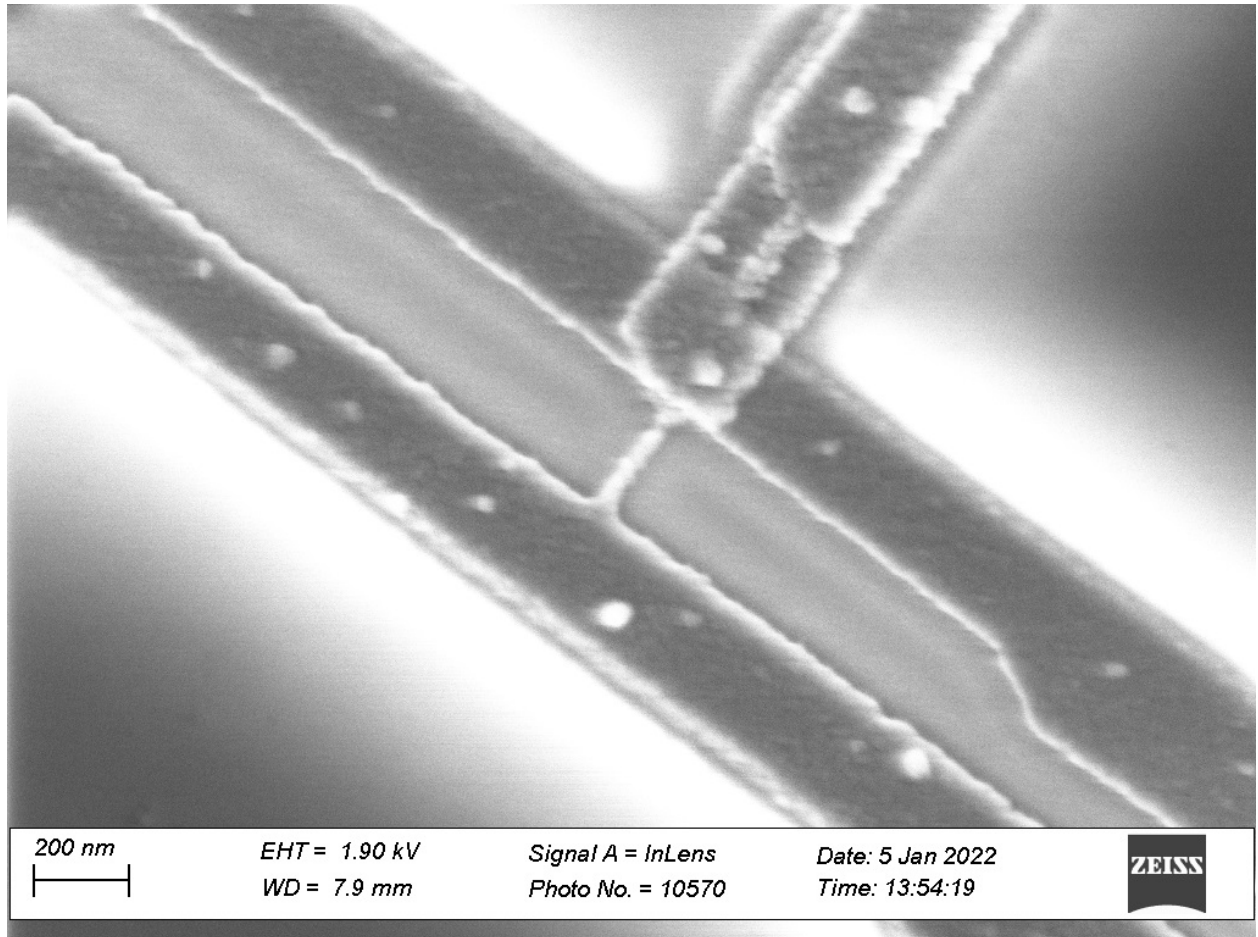


Figure 5.2: SEM image of a cracked Dolan bridge junction. The two parallel lines and one perpendicular line are Al fingers forming the junction. However, a small amount of Al is seen connecting the two parallel Al bars. This feature shorts the junction and renders the qubit non-functioning.

5.3.1 Fulton-Dolan Bridge Technique

The Fulton-Dolan bridge technique is a common way to create small Josephson junctions. Here, a bilayer of e-beam resist (section 5.3.3) is patterned with two features nearly in contact, but leaving a small bridge in the resist between them [122]. Next, the bottom layer is developed with significant undercut so that it is removed from underneath this area forming the titular “bridge”, Fig. 5.3. By evaporating at a ‘tilt’ angle through this resist mask, the Al metal can be deposited underneath the bridge. Before the next deposition, the Al is oxidized, forming the junction barrier. Depositing at a second, opposite tilt angle can form a junction underneath the bridge where the junction size is determined by the size of the defined bridge feature and the angles used for deposition. Different Dolan designs can remove the dependence of the junction area on the deposition angle at the cost of a slightly less supported bridge.

The Dolan bridge is simple to design and forms junctions with very low footprint. This makes them appealing for large arrays of junctions in devices like the fluxionum [123]. However, the suspended resist is very fragile and prone to breaking. In our process this led to less reliable junction fabrication with the Dolan bridge technique. However, the addition of stress-relief regions to the junction design have been shown to improve junction yield [121].

5.3.2 Manhattan Junction Technique

An alternative process for forming small junctions is called the Manhattan design [124, 125]. This technique uses a thick bilayer of resist, developed in a cross pattern and does not involve any bridge (Fig. 5.5). The thin bottom layer is developed with an undercut for later metal liftoff. Next, metal is deposited with both a ‘tilt’ and ‘rotation’ angle. When the rotation is set so one of the arms of the cross is aligned with the tilt axis, metal will be deposited along the parallel arm. If the width of the features, $w < t \cdot \tan \theta$, where t is the resist thickness and θ is the deposition angle, then no metal will be deposited along the perpendicular arm.

With these restrictions satisfied, first deposition is done aligned to one arm of the cross,

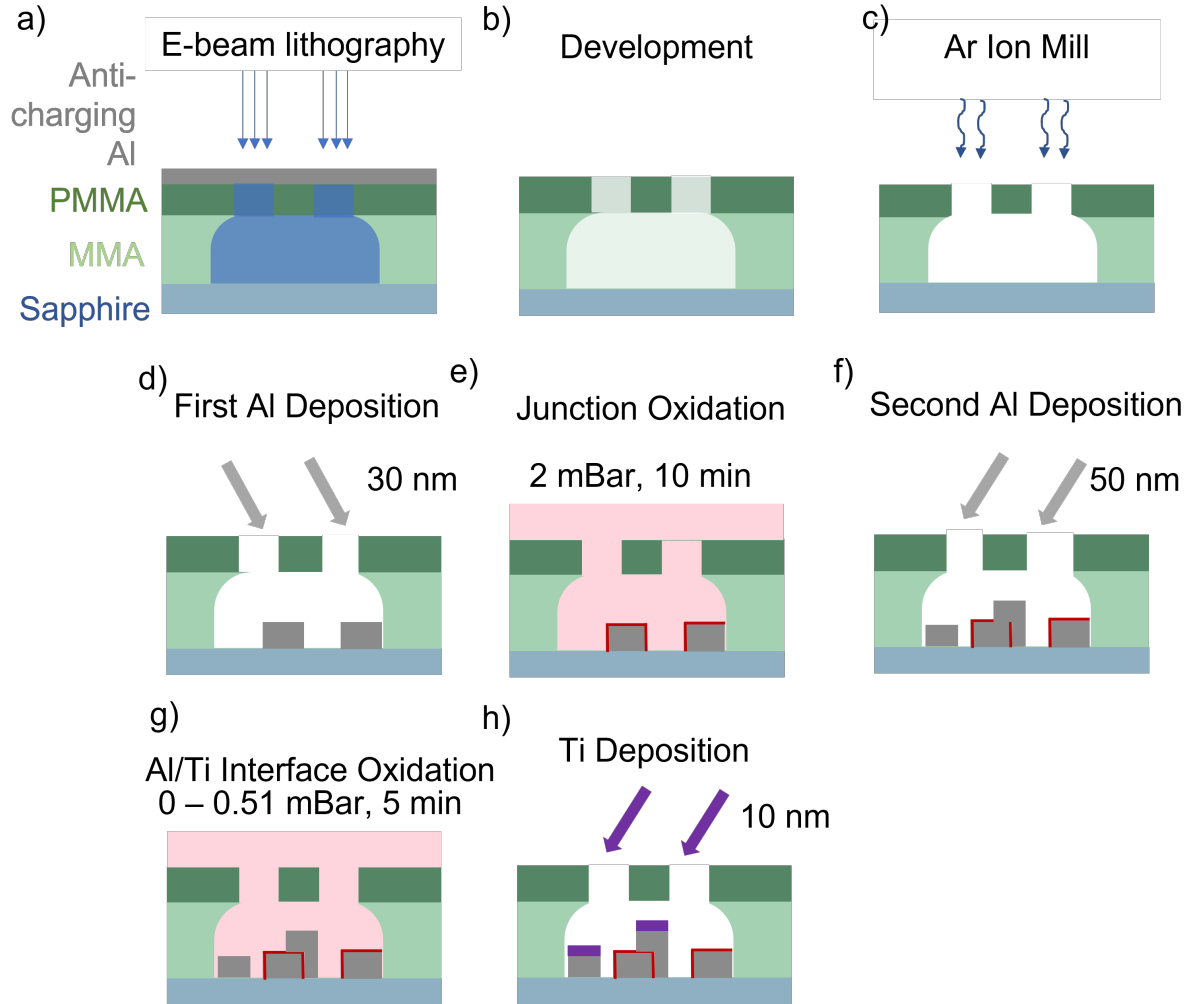


Figure 5.3: **a-f)** Process for making a Dolan bridge Josephson junction. Steps are identical but geometry differs for a manhattan-style Josephson junction. **g, h)** Additional steps used in making gap-engineered junctions (section 9.2).

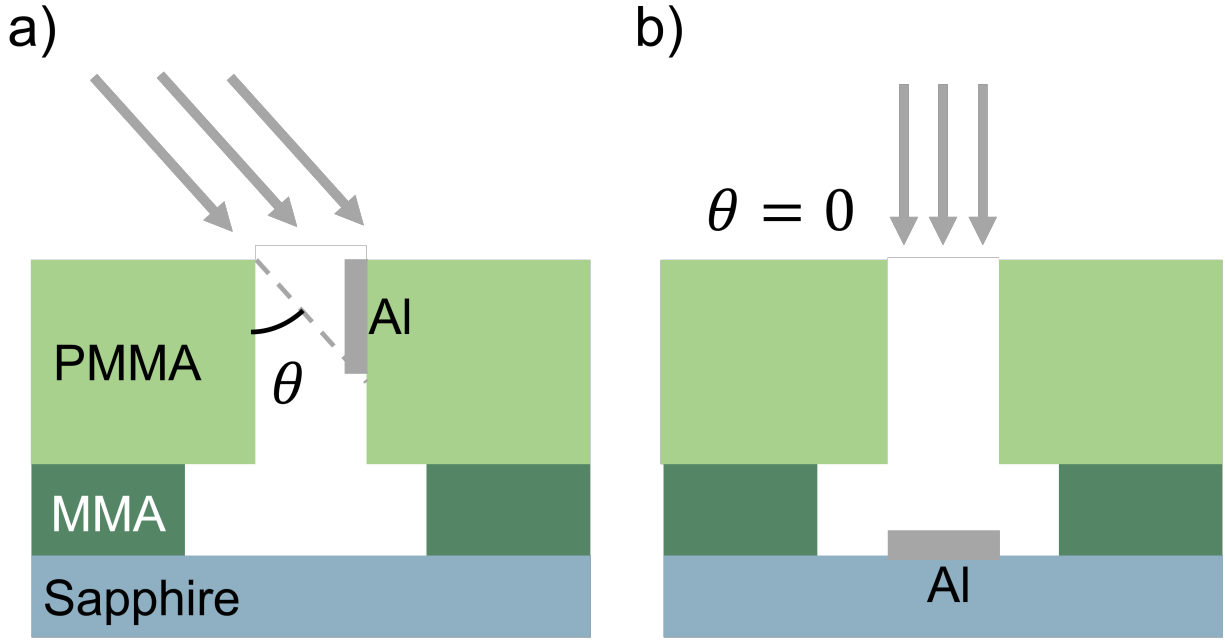


Figure 5.4: Diagram of angle-dependent deposition used in forming Manhattan-style Josephson junctions. Shown here is the cross section of the resist from the side, looking “down the street”. a) when metal is deposited at a large enough angle, θ , the metal hits the resist wall and is removed during liftoff. b) when the metal is evaporated directly overhead, it is deposited on the substrate and forms a metal finger. Using two perpendicular “streets” or arms and rotating the wafer, one street can be deposited while the other is not. This is leveraged to form the tunnel junction.

then the Al is oxidized, and finally the second deposition is performed aligned to the second arm. To visualize this, and explain the name of the technique, picture the sun rising in a city with many tall buildings (Manhattan). The streets which run toward and away from the sun will be lit, while the perpendicular streets will be shaded by the buildings. Then waiting until noon when the sun has moved 90 degrees in the sky, the other streets will be sunny while the first streets will be shaded. A junction is formed at the “intersection” where the two arms meet.

Using additional angles and Al depositions, this technique can be used to bandage junctions to a ground plane in a single lithography step [126], preventing an aggressive ion mill at the junction.

5.3.3 E-beam Resist Stack

To pattern a Josephson junction, I first spun a bilayer of electron-beam (e-beam) sensitive resist. First, I made sure the wafer has been thoroughly cleaned from any previous processing steps such as the ground plane clean described in section 5.2.7. If the wafer had been sitting out for a week or longer, a 30 s acetone dip followed by a spray bottle rinse with acetone, then methanol, then isopropanol and blown dry was used to clean the wafer. This general clean is good for removing dust, fingerprints, or other contaminants in the cleanroom.

After the wafer was cleaned, I spun MMA EL6 at 2000 RPM for 60 s on the wafer, producing a layer approximately 200 nm thick. Then I baked the wafer for 5 minutes at 180 °C on a hotplate. Next PMMA A9 was spun at 4000 RPM for 60 s to give around a 900 nm thick top layer. Again the wafer was baked at 180 °C for 5 minutes. However, if I planned to write large features ($\approx 50 \mu m$ max dimension) with the e-beam tool, I increased this bake time to 30 minutes to harden the resist and prevent excessive undercut during development.

After the bilayer of resist, an anti-charging layer was needed to allow the e-beam write in the next step to be precise. For this, traditionally I would remove the device from the cleanroom and deposit ≈ 10 nm of Al in a thermal evaporator. However, it is easier, simpler, and cleaner to instead use aquaSAVE or DisCharge as an anti-charging layer. Note, while DisCharge can be later removed with isopropanol or DI water, aquaSAVE is generally insoluble in isopropanol or acetone. For the anti-charging layer on my devices, I spun aquaSAVE at 3000 RPM for 60 s.

Finally, if dicing the wafer or letting sit for more than 12 hrs, I spun protective resist. The typical protective resist is FSC-M spun at 2000 RPM for 60 s then baked at 120 °C for 2 mins. This thick resist should provide good protection from damage outside the cleanroom.

This resist stack was used for making Manhattan-style junctions. For Dolan bridges, I used a thick lower layer of MMA 8.5 EL11 spun at 1000 RPM for 60 s followed by a 180 °C bake for 5 minutes then ZEP 502A diluted with anisole at a 2.3 weight ratio. This top

ZEP layer was spun at 5000 RPM for 60 s then baked at 180 °C for 5 minutes. Finally an anti-charging layer and protective resist were added.

The resist can be spun on an individual 5 mm x 5 mm chip but there will be significant edge beads increasing the thickness near the edges and possibly affecting the resist thickness throughout. I generally had success writing junctions after this but with less precision.

5.3.4 Dicing

Often, resist was spun on an entire wafer or a quarter wafer, but I would only write a pattern on a single chip. Therefore, at this point I diced the wafer. Blue resist was always spun onto the device before dicing, see 5.3.3. A Disco-DAD321 dicing saw was used with a CA-010-270-080 H diamond blade cutting fed at 0.75 mm/s and 22,000 RPM. After dicing, the protective blue resist was removed with a 30 s acetone dip, rinse in acetone, methanol, isopropanol, then blown dry.

There is also a new laser dicer at LPS worth considering for dicing wafers.

5.3.5 E-beam Writing

Next, the pattern was written using an e-beam writer. For most of my devices, I wrote using an Elionix ELS-G100 system. This tool directly writes patterns with a 100 kV electron beam. While this tool is powerful and robust in its exposure process, it is ideal to perform a dose test at a minimum when starting a new recipe. I wrote junctions with 1 nA beam current and total area dose of 2000 $\mu C/cm^2$.

5.3.6 Development

Once the pattern was written, I returned to the cleanroom to develop the resist. I cleaned off anti-charging resist. For the Al layer, I dipped the device in OPD 4262 for 60 s, then rinsed in DI water for 30 s. For aquaSAVE, the device was dipped in DI water for 60 s then

rinsed with DI water.

Next, the e-beam bi-layer was developed in an MIBK:IPA mixture in a 1:3 ratio. The sample was dipped in the developer and agitated for 60 s. To stop development, the chip was dipped or rinsed in isopropanol. Development time of 35 s was better if large features were also written with the e-beam, but if only small features were written this sometimes underdeveloped the resist.

After development, imaging of the junction with the confocal microscope allowed me to catch errors in lithography and confirm the pattern has been accurately written. An image of a junction pattern written for a 3D transmon is shown in Fig. 5.5a.

5.3.7 Deposition

The quality of the junction deposition can strongly effect the coherence of the qubit [126–128]. Additionally the presence of even trace amounts of magnetic elements, lossy dielectrics, or normal metals can contribute to energy loss for the qubit [106, 129]. To limit exposure to other elements and have greater control of the junction deposition process, I deposited junctions in a Plassys MEB 550 S e-beam evaporator. This system has two vacuum chambers separated by a gate valve. The lower chamber contains the metal targets and e-beam evaporation system. In my devices, I deposited from the Al and Ti targets. The upper vacuum chamber, or load lock, holds the sample mounted on a stage with both planetary (rotational) and tilt control, allowing me to make either Dolan bridge or Manhattan junctions in the evaporator. Additionally, the load lock is used for ion milling and oxidation, meaning any gas used in these processes only fills the upper chamber. This system is designed for junction fabrication and is used solely for depositing Al, Ti, and other materials commonly used with superconducting qubits.

After development, I removed the chip from the cleanroom and mounted it in the Plassys (Fig. 5.6). For my qubits or other high coherence devices, the load-lock was pumped down for 5-12 hr to reach base pressure. The heat straps were turned on for the first few hours of

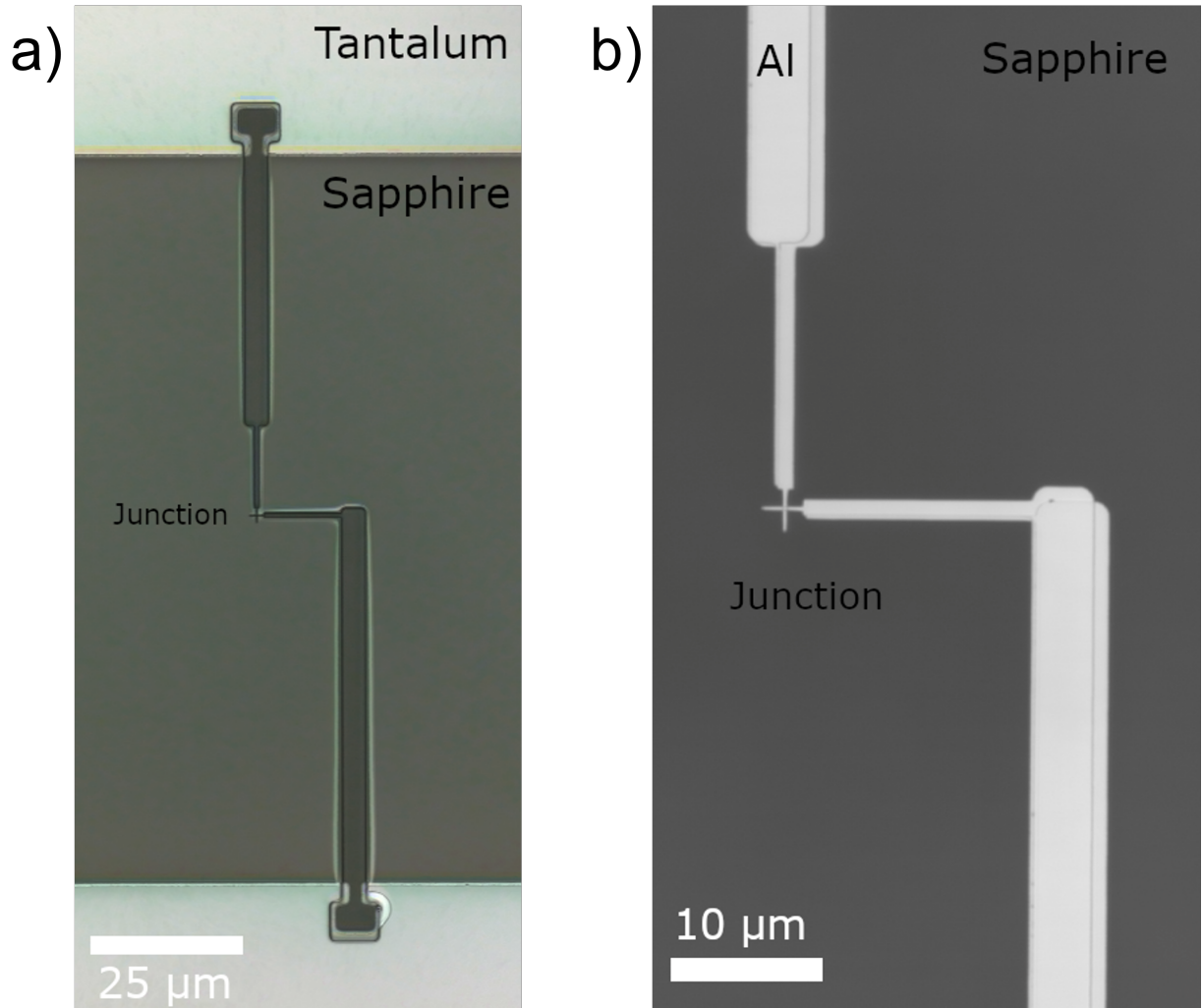


Figure 5.5: Confocal images of junction fabrication **a)** Resist written and developed for a junction in a 3D Ta device, **Q2**. The junction pattern has a faint halo from undercut in the MMA lower layer. **b)** Al junction deposited on sapphire. This Manhattan-style junction is formed in the center-left where two Al fingers cross.

this pump-down to remove more volatiles (*e.g.* water vapor) from the load-lock surface and achieve a lower base pressure.

If the junction needed to contact other metal, an aggressive ion-mill was done to remove the surface oxide from the base metal. Otherwise, a lighter ion-clean was done to remove any organics remaining on the substrate surface after development. Surface damage and reduced qubit performance has been reported when performing a strong ion-mill where the junction is eventually deposited [130–132]. The ion mill in the evaporator I used was a KRI KDC 40 Ion Source with a 2-3° divergent beam.

To run the ion-mill, first Ar was bled into the chamber at 20 sccm for 8 min to clean the lines before milling. Next, the flow was adjusted to 9 sccm and the beam was turned on. For the light ion clean the beam parameters were 250 V beam voltage, 8 mA beam current, and 50 V acceleration voltage. For the aggressive clean the beam parameters were 400 V beam voltage, 20 mA beam current, and 75 V acceleration voltage. After the beam had successfully started, the Ar flow rate was gradually ramped down to 4 sccm to reach a chamber pressure around 6×10^{-4} mBar during milling. Higher chamber pressure reduced the mean free path of the ions and greatly reduced the milling rate [85]. The ion mill was run for 60 s total. The aggressive ion mill was done in four 15 s sections with 30 s breaks to prevent the resist from melting under the high energy beam.

Next, before deposition, I attempted to lower the pressure in the deposition system by using a getter. I evaporated Ti into the chamber while leaving the shutter closed so none reaches the substrate. The Ti grabs oxygen and other gases still in the chamber and reduced the pressure further to as low as $2 \cdot 10^{-8}$ mBar. Ti was deposited at nominally 0.2 nm/s for 2 minutes.

After adjusting the angles for the first deposition, 30 nm of Al was deposited at a nominal rate of 0.5 nm/s controlled by the Plassys’ PID evaporation rate control system. A slow ramp up of beam current helped to prevent “bursts” from the melting metal which give large pressure spikes and can cause the deposition to fail when the Al is heated too quickly.

Next, after depositing the first metal layer, oxygen was added to the load lock to form a thin AlO_x tunnel barrier for the junction. Changes in critical current density for a constant oxygen dose were observed over the time scale of months. This meant tuning junction resistance was required periodically before making final devices. In general 1-5 mBar of pure O₂ pressure for 5-15 mins has been used to get 150 nm x 150 nm junctions with $R_n = 10\text{ k}\Omega$, however the final resistance varied by an order of magnitude during the work for this dissertation.

After the oxidation time, the chamber was evacuated again. At this point occasionally the Plassys screen showed the “open door” symbol on the load lock. To fix this error, the blue pressure sensor on the top lid of the load lock must be hit with one’s palm. I found no other solution to this problem. Once the pressure is below 0.1 mBar, the turbo begin pumping on the load lock. The time to pump to this point was marked down as it effectively increases the oxygen exposure. Note, after the turbo opens, the pressure quickly returns near the minimum value, suggesting that the reason the initial pump-down takes many hours was the difficulty of removing water vapor and other films slowly outgassing into the chamber.

For the second junction layer, the Plassys sample stage was rotated to the correct second angle. Then 50 nm of Al was deposited at a rate of 0.5 nm/s after slowly ramping to the target deposition rate.

For gap-engineered devices in section 9.2, I deposited Ti at the same angle as the second Al deposition. An optional oxidation of up to 0.5 mBar for 5 min formed a disordered barrier between the two metals. The Ti was then evaporated at a rate of 0.2 nm/s and a thickness of 5-30 nm.

After the final deposition, a passivation oxidation was performed. I let 40 mBar into the load-lock for 20 min. Here I was attempting to form an oxide on the surface in a controlled environment instead of exposing the junction to the air directly. Due to the good natural oxide of Al, this may be an optional step. After the O₂ has been pumped out, the load-lock was vented and the sample was taken back to the cleanroom for liftoff.

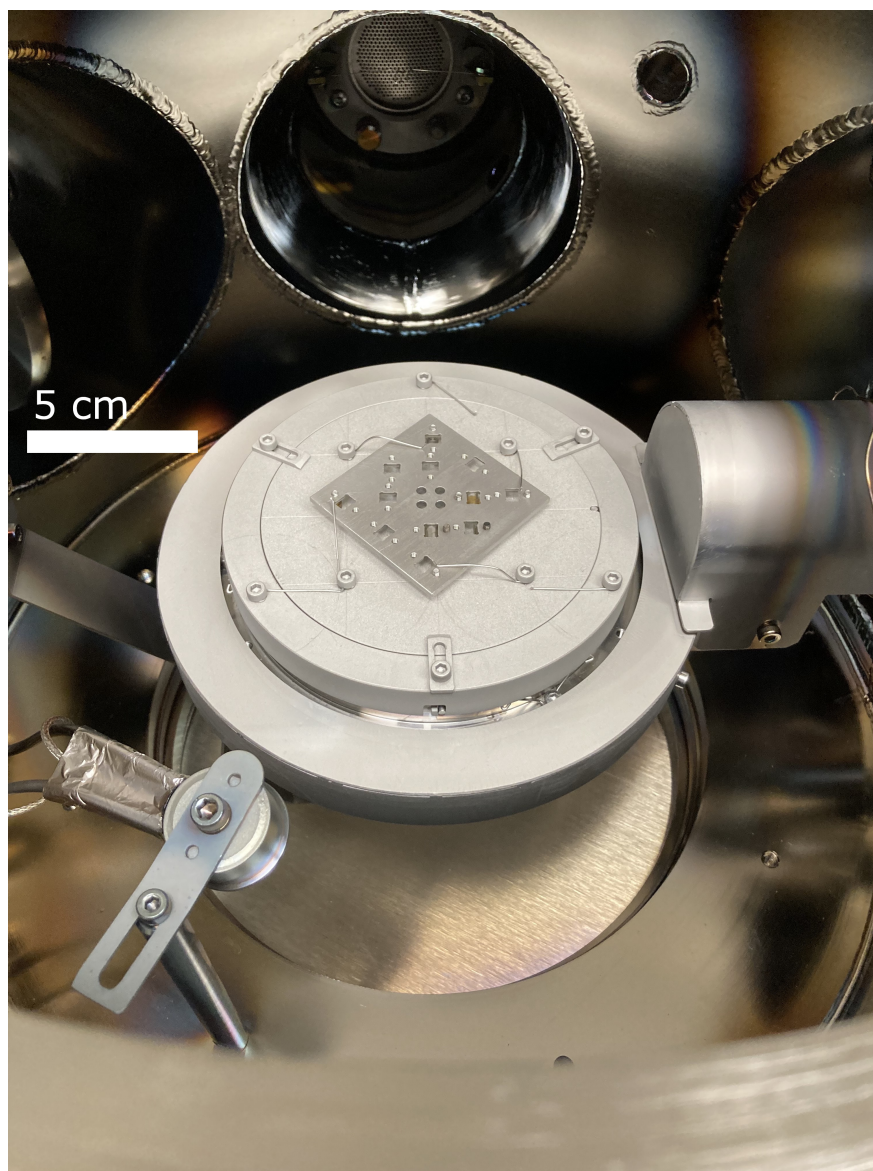


Figure 5.6: Devices mounted in the Plassys evaporator for junction deposition. Center is the samples mounted on a plate for 5 mm x 7 mm chips on the sample stage. This stage can perform planetary rotation (circular stage rotating symmetrically about its center) and also tilt (arm on right rotating, causing circular stage to tilt). Top center is the ion gun used for Ar ion-milling. Bottom left is the crystal monitor which measures deposition rate.

5.3.8 Liftoff

We are ready for liftoff. To liftoff the extraneous Al and e-beam resist from the device, a solution of N-Methyl-2-pyrrolidone (NMP) based remover was heated on a cleanroom hotplate to 80 °C. The chip was added to the heated NMP and soaked for 15 min. Next, the beaker of NMP was removed from the hotplate, but the device was left in the NMP solution for 3-12 hrs to fully remove residual resist. Next, the chip was agitated to remove any large pieces of Al. Then the chip was moved to a fresh beaker of NMP and let sit for 10 minutes before being dipped in isopropanol and rinsed with the isopropanol spray bottle.

Typically, the junctions were imaged with a high-resolution optical or confocal microscope after liftoff was completed to verify that the junction cross was visible and looked as designed (Fig. 5.5b).

During and after liftoff, it was important to consider electro-static discharge (ESD). The insulating sapphire substrate can easily build up enough charge to destroy a junction on the surface. I avoided sliding the sapphire over other non-conducting surfaces whenever possible. Finished junctions were transported in conducting packaging and any probes were discharged before contacting metal on the chip's surface.

5.4 Measured Device Table

Listed below is Table 5.1 of the devices measured in this dissertation and the methods used to fabricate them. Further details are provided in relevant chapters.

5.5 Conclusion

Although superconducting qubits use relatively few lithography steps, the processing can be complicated and following the detailed recipes is crucial for reliable success in fabrication. In this chapter, I outlined the steps and recipes I used for making devices measured in this

Table 5.1: Table of devices reported on in this disseration. The device name, type of device, and metal and technique for the large features and junctions are listed.

Name	Type	Large metal features	Junctions
Q1	2D transmon	Wet etched Ta	Al/AlO _x /Al Manhattan
Q2	3D transmon	Wet etched Ta	Al/AlO _x /Al Manhattan
Q3	3D transmon	Liftoff Al	Al/AlO _x /Al Manhattan
Q4	3D transmon	Liftoff AlTi	Al/AlO _x /AlTi Dolan
Q5	3D transmon	Liftoff AlTi	Al/AlO _x /Al/AlO _y /Ti Manhattan
S1	SQUID	Liftoff Al	Al/AlO _x /Al Manhattan
N1	NIS junction	Liftoff AlAg	Al/AlO _x /Al _ε /Ag
R1	CPW resonators	Liftoff Al	
R2	CPW resonators	Liftoff Ti	
R3	CPW resonators	Liftoff TiAl	

thesis. A final device is shown in Fig. 5.1. Fabrication details may help avoid problems in fabricating similar devices.

Chapter 6

Qubit Measurement Setup

In this chapter I cover the setup I used in measuring qubit devices. I first discuss the dilution refrigerator and how it enables solid-state experiments below 20 mK. Next, I explain the setup needed for low-temperature microwave measurements including necessary components, signal generation, and readout. Finally, I briefly cover device packaging for a 3D transmon qubit.

6.1 Dilution Refrigerators

Superconducting qubit experiments require operation at extremely low, sub-Kelvin, temperatures to satisfy $k_B T \ll \hbar \omega_q$, thus reducing thermal excitations of the qubit (section 3.2). A dilution refrigerator uses a combination of refrigeration technologies to reach sample-stage temperatures below 20 mK (Fig. 6.1).

A two-stage pulse-tube cooler cool the first two cryogenic stages to nominally 50 K and 3 K plates from room temperature (300 K) [133]. After the other three lower stages have reached 3 K, He_3 , He_4 mixture is condensed, helium circulation begins, and the dilution refrigeration process is started. This cooling method pumps liquid He_3 into a dilute mixture of He_3 in He_4 which separates into a He_3 rich phase and He_3 poor phase. The latent heat of He_3 passing from the concentrated to the dilute phase pulls heat from the mixing chamber (10-20 mK) stage. The He_3 is pumped out of this dilute phase at the still plate (0.7 K stage)

to room temperature and then is recondensed to continually repeat this process [134]. An intermediate 100 mK cold plate acts as a heat exchanger to cool He flowing to the mixing chamber. The dilution process on the Leiden CF-450 and CF-1400 I used generated mixing chamber cooling power at 200 mK of approximately 450 μ W and 1.4 mW respectively [114]. The device is mounted on the base/mixing chamber plate.

Some of my experiments were conducted at elevated temperatures with temperatures up to $T = 1\text{ K}$. To increase the temperature of the mixing chamber, a PID controlled resistive heater warmed the mixing chamber plate. To reach temperatures above 500 mK I would also collect some of the dilution mixture to reduce the cooling power.

The qubit device I measured was surrounded by two open-ended Amumetal cylinders connected to the mixing chamber plate at ≈ 20 mK to reduce stray magnetic fields on the device and provide radiation shielding. To shield the lower temperature stages from radiation from higher temperature stages, copper cans were placed on the cold plate, and still plate. Aluminum cans were bolted to the 3 K stage and 50 K stage for shielding at those temperatures.

6.2 Microwave Measurements

Microwave signals use standard 50 Ω impedance lines to maximize power handling and minimize input loss. Impedance matching at connections and loads is crucial to reduce reflections and frequency dependent effects on the signals [135].

The qubit and the resonator frequencies for my devices were all designed between 2 GHz and 8 GHz. Resonant microwave tones were used to address, control, and readout my devices. In nearly all microwave experiments, signals are generated at room temperature. For $k_B T \gg \hbar\omega$, the thermal (Johnson-Nyquist) noise voltage magnitude squared of a signal when $T \gg \hbar f/k_B T$ is given by

$$\overline{V_n^2} = 4k_B T R B \quad (6.1)$$

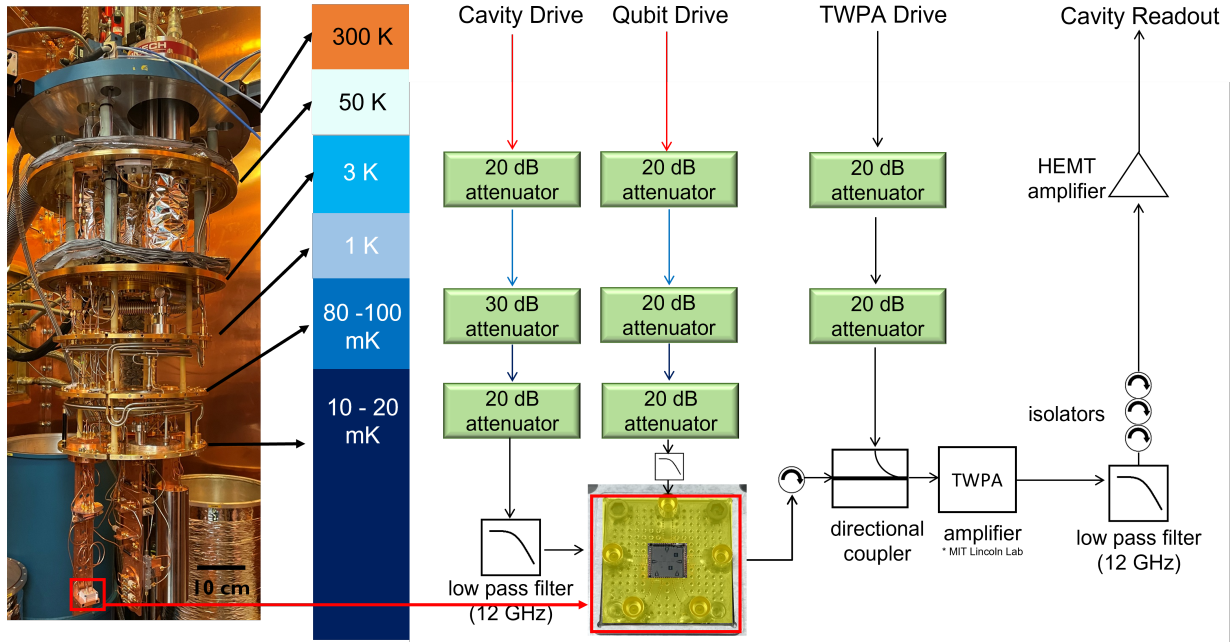


Figure 6.1: Dilution refrigerator image and microwave measurement setup. On the left the image of the open dilution refrigerator shows 6 plates at different temperatures. Stages at progressively lower temperatures allows the final mixing-chamber stage to reach temperatures below 20 mK. Not shown are two Amumetal magnetic shielding cans that enclose the sample and the copper and aluminum cans which isolate each stage from radiation from higher stages. The measurement circuit to the right shows components required for low-temperature microwave measurements. Attenuators thermalize signals from room temperature. Filters eliminate noise from high frequencies. TWPA and HEMT amplifiers increase the signal magnitude for readout at room temperature.

where T is the signal temperature, R is the resistance of the noise source, and B is the bandwidth the signal is sampled. This voltage noise causes finite cavity photon number leading to dephasing (section 3.6.3) and spontaneous excitation or relaxation of the qubit. Therefore, multiple stages of microwave attenuators and filters are used to thermalize the signals to lower temperatures and reduce the noise seen by the device [136].

For my measurements, the input microwave drive line for the cavity had 3 stages of thermalization using attenuators with 20 dB of attenuation (1/100th of the input power being transmitted) at 3 K, 30 dB at 100 mK, and 20 dB at base temperature (Fig. 6.1). Finally, a K&L 12 GHz low pass filter was added to reduce higher frequency noise incident on the cavity. The qubit drive line was filtered similarly, but with 10 dB less total attenuation. Reducing the attenuation enabled strong qubit drives and, therefore, faster qubit gates (section 11.4).

Readout of single-photon microwave signals require multi-stage amplification. Since the final signal-to-noise affected by the additional noise introduced by the first amplifier in a chain, adding a low-noise amplifier at low temperature can greatly increase the signal-to-noise ratio (SNR). In my measurement setup, a travelling-wave parametric amplifier (TWPA) was used as a first stage amplifier at 20 mK [137]. This amplifier uses a four-wave mixing process to coherently amplify signals. The pump drive is added on another attenuated line through a Marki Microwave C20-0226 directional coupler. A Quinstar 18 dB microwave isolator prevents the TWPA drive from leaking through the coupler input port and affecting the device. The signal is then filtered and goes through another 3 isolators before being amplified by a Cosmic Microwave cryogenic low-noise high electron mobility transistor (HEMT) amplifier at the 3 K stage. At room temperature, the signal is amplified again by a room temperature amplifier before being measured.

6.2.1 Infrared filters and shielding

Recently, reducing infrared (IR) radiation near the qubit sample has been shown to reduce background quasiparticle levels [26]. To better shield my devices from radiation a few improvements to the refrigerator setup were made. First, adding a copper shield to the mixing chamber stage further blocks radiation from hotter stages. Second, measurements I performed on a Ta-based 3D transmon showed adding Quantum Microwave IR Eccosorb CR110 filters improved the relaxation times for that device (section A.2.1). Limiting quasiparticle generation from IR radiation could reduce background quasiparticle densities [26] and improve the transmon's lifetime.

Further investigation and possible addition of IR absorbing paint [24, 26] is needed to better understand the effect of high-frequency radiation on the qubit.

6.3 Signal Generation

Both cavity, qubit, and TWPA are driven with microwave signal generators (Fig. 6.1). These specialized electronics generate tones at microwave frequencies between 1-12 GHz with frequency precision sometimes exceeding 1 part-per-trillion [138].

I used different microwave signal generators including, Rhode & Schwarz SGS100a, Agilent E8257D, Agilent E8267D, Agilent 83650B, and HP 83732B. For vector controlled microwave signals used in section 11.4, I used the vector generator option of the R&S SGS100a modulated by a Tabor WX2184C AWG. Pulsed measurements were gated by SRS DG645 delay generators. CW measurements were taken with an Agilent E5071C VNA. For details on pulse alignment and sequencing see [85].

6.3.1 Signal Mixing

Vector generators allow full control of the phase and amplitude of the microwave drives applied to the qubit. This control can be leveraged to drive the qubit with complex control

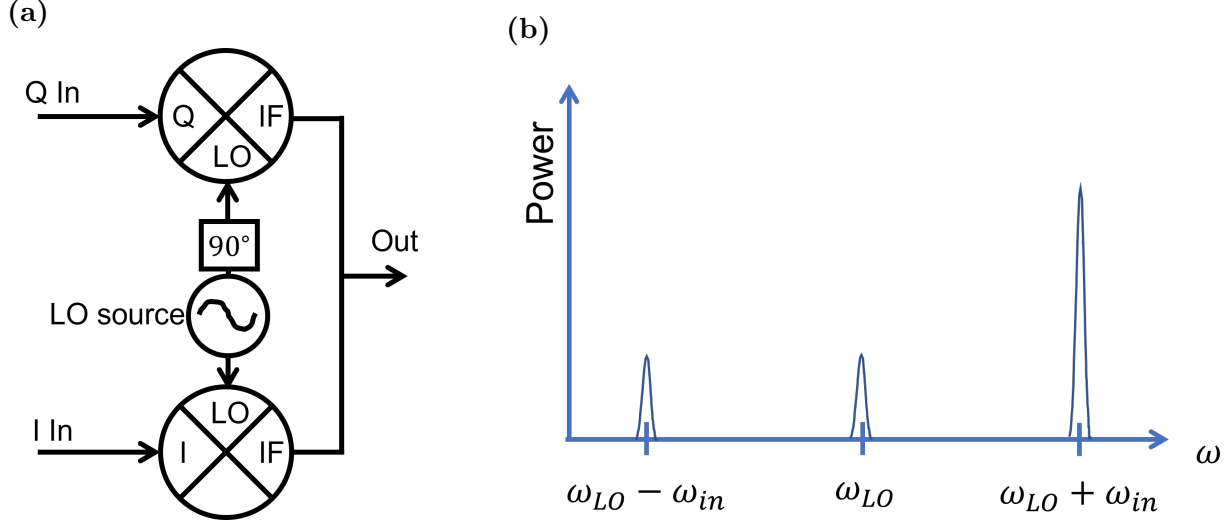


Figure 6.2: IQ signal mixing diagram and power output. **a)** Diagram of an IQ mixer. Signals from the I and Q input ports are multiplied by an LO tone which is shifted 90° before mixing with Q. These signals are combined and output. **b)** Schematic of the resulting PSD after upper-side-band modulation. Here the drive power is at $\omega_{LO} + \omega_{in}$. Non-idealities in the mixers, setup, and applied signals leave a small amount of power in the LO and lower-side-band peaks.

sequences (section 11.6). Here I provide a quick overview on IQ modulation

To control the qubit with a pulse with arbitrary in-phase and quadrature envelope $[S_I(t), S_Q(t)]$, the signal $S(t) = S_I(t) \cos(\omega_q t) - S_Q(t) \sin(\omega_q t)$ is applied, where ω_q is the qubit transition frequency. An IQ mixer (Fig. 6.2a) multiplies an LO signal and an envelope signal generating

$$S_{out} = S_{env} \times S_{LO} = S_I(t) \cos(\omega_{LO} t) - S_Q(t) \sin(\omega_{LO} t) \quad (6.2)$$

The most straight-forward way to get the desired signal is to set $\omega_{LO} = \omega_q$ and apply DC envelopes $[S_I(t), S_Q(t)]$ to the input “I” and “Q” ports of the mixer.

However, to reduce $1/f$ noise at the I, Q input ports, it is better to apply an input signal with a significant frequency component, $\omega_{in} \approx 100$ MHz [30]. It is useful to take the following ansatz for a given $\omega_{in} > 0$:

$$S_{in,I}(t) = S_I(t) \cos(\omega_{in}t) - S_Q(t) \sin(\omega_{in}t) \quad (6.3)$$

$$S_{in,Q}(t) = S_I(t) \sin(\omega_{in}t) + S_Q(t) \cos(\omega_{in}t) \quad (6.4)$$

Note, this is simply a 2D-rotation of $[S_I(t), S_Q(t)]$ by angle $\omega_{in}t$.

When this pulse is applied to the ports of the mixer, the mixer outputs the signal

$$s(t) = S_{in,I}(t) \cos(\omega_{LO}t) - S_{in,Q}(t) \sin(\omega_{LO}t) \quad (6.5)$$

$$\begin{aligned} &= S_I(t) \cos(\omega_{in}t) \cos(\omega_{LO}t) - S_Q(t) \sin(\omega_{in}t) \cos(\omega_{LO}t) \\ &\quad - S_I(t) \sin(\omega_{in}t) \sin(\omega_{LO}t) - S_Q(t) \cos(\omega_{in}t) \sin(\omega_{LO}t) \end{aligned} \quad (6.6)$$

$$\begin{aligned} &= S_I(t)/2[\cos((\omega_{in} + \omega_{LO})t) + \cos((\omega_{in} - \omega_{LO})t)] \\ &\quad - S_Q(t)/2[\sin((\omega_{in} + \omega_{LO})t) + \sin((\omega_{in} - \omega_{LO})t)] \\ &\quad - S_I(t)/2[\cos((\omega_{in} - \omega_{LO})t) - \cos((\omega_{in} + \omega_{LO})t)] \\ &\quad - S_Q(t)/2[\sin((\omega_{in} + \omega_{LO})t) - \sin((\omega_{in} - \omega_{LO})t)] \end{aligned} \quad (6.7)$$

$$= S_I(t) \cos((\omega_{in} + \omega_{LO})t) - S_Q(t) \sin((\omega_{in} + \omega_{LO})t) \quad (6.8)$$

where all four sine and cosine product formulas have been used such that many terms cancel.

Finally, simply by setting $\omega_{LO} = \omega_q - \omega_{in}$ the desired signal in Eq. 6.2 is obtained. Since the LO frequency was set below the qubit transition frequency and the upper frequency modulated peak drives the qubit, this is called “upper-side-band mixing” (Fig. 6.2b). Switching to lower-side-band mixing, where the LO is above the qubit frequency, is done simply by making ω_{in} negative or, equivalently, adjusting the direction of the rotation matrix used in Eq. 6.3.

6.3.2 Mixer Impairments

Limitations of actual electronics introduce imperfections in IQ-mixer operation [139]. When operating in single-side-band mode, these imperfections introduce spurious peaks in the frequency spectrum of the output signal. Luckily, the R&S SGS100a has internal components to correct for these non-idealities by adjusting the mixer impairment settings. Figure 6.3 shows the five common mixer impairments I tuned. To tune the mixer, I input $S_I(t) = \sin(\omega_{IF}t)$ and $S_Q(t) = \cos(\omega_{IF}t)$ as continuous waves from the AWG. Then I examined the microwave generator output signal on a EXA Signal Analyzer in spectrum analyzer mode. The display of the signal generator resembles Fig. 6.2b. I first tuned the I offset and Q offset parameters to minimize the peak at ω_{LO} . Next I tuned the IQ imbalance and quadrature offset to reduce the spurious side-band. These two were iterated for more precise tuning. However, I found -70 dBc compared to the $\omega_{LO} - \omega_{IF}$ is easily obtainable for these two peaks. Impairment settings can be stable on the timescale of days or weeks, but if the temperature of the room is unstable or new electronics are added to the measurement rack, retuning was performed.

Additional peaks due to non-linear mixing are also often apparent at $\omega_{LO} + 2\omega_{IF}$ [139]. I also observed other peaks on the spectrum analyzer that I did not identify. When tuned properly, I did not find any of the spurious peaks to be limiting of my qubit drive performance in lower-side-band mode.

6.4 Heterodyne Readout

Readout of the qubit-cavity system involves extracting the amplitude and phase response of the cavity transmission signal (section 7.2). For my measurements, this is done with a 10 MHz heterodyne signal measurement.

The purpose of heterodyne readout is to reduce a difficult to digitize 5-8 GHz signal to a lower frequency 10 MHz signal preserving the phase and amplitude. The down-conversion

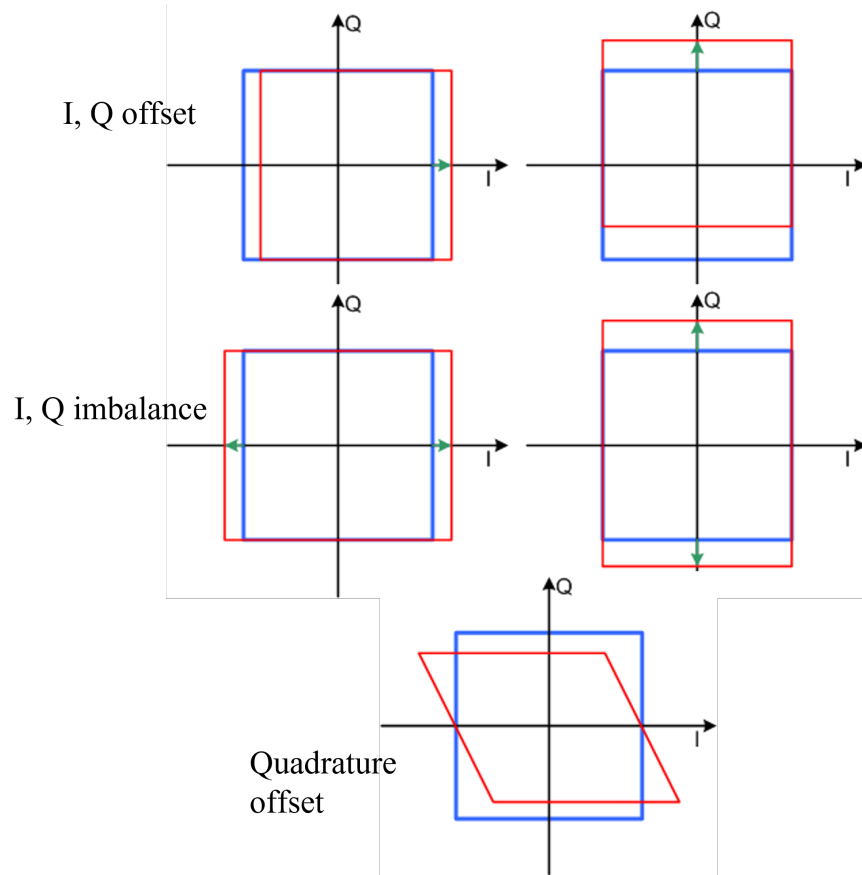


Figure 6.3: Common IQ impairments corrected by the R&S SGS100a. Ideal IQ modulation is shown in blue and error modulations are shown in red. I, Q offsets shift the modulation center from the origin and will generate error signals at f_{LO} . I, Q imbalance improperly scales signals in the I and Q directions. Quadrature offset refers to I and Q signals not being totally orthogonal. Both imbalance and quadrature offset will give error signals at the opposite side-band ($f_{LO} - f_{IF}$ for upper-side-band mixing).

in this readout scheme is performed by a single-port mixer. Similar to the individual mixers in Fig. 6.2a, this component is responsible for multiplying two input signals. For example for $S_{RF}(t) = a \cos(\omega_{RF}t + \phi)$ and $S_{LO}(t) = b \cos(\omega_{LO}t)$

$$S_{IF} = S_{RF} \times S_{LO} = \frac{ab}{2} [\cos((\omega_{RF} - \omega_{LO})t + \phi) + \cos((\omega_{RF} + \omega_{LO})t + \phi)] \quad (6.9)$$

Eq. 6.9 allows the generation of signals at the sum and difference frequencies of two signals. Heterodyne readout involves setting $\omega_{LO} = \omega_{RF} - \delta\omega$ detuned from the signal a small amount $\delta\omega$ (here 10 MHz). This generates a high frequency signal at $2\omega_{RF} - \delta\omega$, which is filtered out by the electronics, as well as a slowly varying signal at frequency $\delta\omega$. The difference frequency is made large enough to avoid $1/f$ noise at low frequencies, but small enough to be easily converted by an ADC to a digital signal to be processed. Advances in FPGA technology have allowed new electronics capable of directly digitizing signals above 10 GHz [140], but I did not get the chance to use such devices in my measurements.

Figure 6.4 shows the detailed circuit used to perform readout of a 5-10 GHz signal. In the circuit, a generated RF signal goes through a C320520 MAC Technology directional coupler, sending approximately 1% of the signal to serve as a reference tone, while the rest is sent to the device. An LO signal is split with a C32056 directional coupler then the branches go through either two Mini-Circuits VHF-2700A or two SHP-1000+ high-pass filters before each connecting to the LO port of a Marki MI-0310LA double-balanced mixer. The signal (one reference and one from the device) are connected to the RF port of the mixer. The IF output of the reference mixer is filtered by a Mini-Circuits SLP-21.4+ low-pass filter to remove the sum frequency signal before being digitized by an Agilent U1082A digitizer. The IF output of the signal mixer is filtered by a Picosecond 5915 low-pass filter then amplified one final time by a Mini-Circuits ZFL-400LN+ amplifier before digitization.

Once the signals have been digitized, the amplitude and phase of the signal relative to the reference is calculated in software, yielding the total microwave response of the measurement

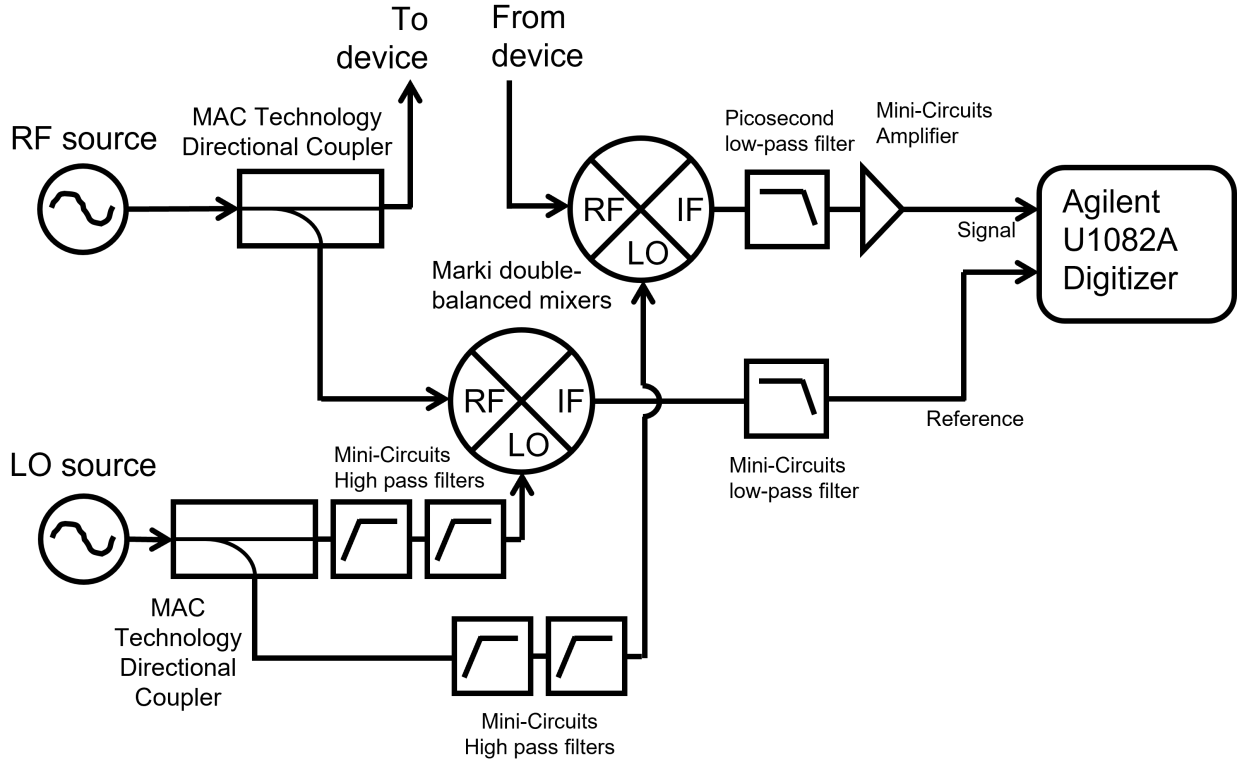


Figure 6.4: Heterodyne measurement circuit diagram. This circuit mixes two microwave drives which results in a 10 MHz signal for digitization and readout. A signal tone travels through the measurement circuit (Fig. 6.1). A phase and amplitude reference tone undergoes similar mixing without leaving the measurement box. The final digitized signals are compared to extract the amplitude and phase response of the device.

circuit. Then comparison between separate measurements can detect shifts in response due to changes in the cavity and qubit system (section 7.2). Referencing the signal measurement eliminates spurious amplitude or phase measurements due to fluctuations in the amplitude or phase of the generators.

6.5 Devices Packaging and Mounting

Now that I have covered the measurement setup and control electronics, I briefly discuss the device hardware packaging and mounting. To package a 3D transmon, a 35 mm by 20.5 mm by 5 mm pocket is machined in the Al to obtain the designed resonant cavity (Fig. 4.1b). Pins from SubMiniature version A (SMA) feed-through connectors on the edge of the cavity

form the coupling capacitors between the cavity and external transmission line. The length of the pins were trimmed to control C_{in} and C_{out} . For my cavities, the resonator decay rate $\kappa \approx \omega_r/10,000$ was dominated by the output coupling, C_{out} . A sapphire chip with the transmon device is placed in a slot in the center of the cavity and held in place with indium metal. The transmon couples to the cavity mode electric field. Details on cavity design and fabrication can be found in [75].

After the transmon was added, the cavity was closed and an indium O-ring was added to seal any seams with a superconducting connection. The cavity was then mounted on the dilution refrigerator mixing chamber and connected to input and output readout lines via its SMA connectors (Fig. 4.1b).

Packaging for a 2D device requires a separate PCB and package housing [85]. The details of the packaging can have strong effects on the loss of packaged devices [106].

6.6 Conclusion

In this chapter I described the necessary setup for measuring a superconducting qubit. Dilution refrigerator technology is required to reach temperatures below 20 mK. Details of the microwave measurement circuit are important to obtain high-fidelity readout and control of the qubit device. Signal generation and readout use microwave components to generate precision signals. Finally, the proper device packaging can mitigate losses to the environment.

Chapter 7

Qubit Characterization and Control

7.1 Introduction

Characterizing a qubit is the process of determining experimental parameters of the device. These may differ from designed parameters. Simple differences occur such as the qubit transition frequency changing due to imprecise junction resistance during fabrication. Also more complicated properties such as total qubit relaxation rate changing device to device for a multitude of reasons. Well-established characterization measurements and protocols allow the device parameters to be measured. Determining actual qubit parameters is a prerequisite for most qubit experiments and often comparing parameters (*e.g.* relaxation times) between devices comprise important types of experiments.

7.2 Cavity Spectroscopy

Since the qubit is read-out through the coupled cavity (section 3.4.2), first cavity spectroscopy must be performed to determine the cavity frequency. A microwave tone is sent into the cavity and the complex transmission, $S_{21} = V_{out}/V_{in}$ is measured. By sweeping the frequency, a resonant feature can be found corresponding to the cavity (Fig. 7.1). When the applied microwave power is high (*e.g.* > 0 dBm in Fig. 7.1), the resonator oscillates at its

“bare” frequency, $f_{bare} = 1/2\pi\sqrt{LC}$, where L and C are the effective resonator inductance and capacitance. At lower drive power, the cavity frequency is shifted by the interaction with the qubit resulting in a “dressed” frequency, f_{dr} , (Eq. 3.42). At intermediate powers, the system behaves non-linearly and the resonant frequency varies with applied power. The dressed frequency depends on the qubit state (Eq. 3.40). When the qubit is excited (bottom of Fig. 7.1), the cavity frequency shifts by 2χ . Due to the form of the coupling term in Eq. 3.4.2, measuring the cavity state can be mapped to a measurement of σ_Z of the qubit. Measuring whether the cavity frequency is in the g-dressed state or e-dressed state is the basis for qubit state readout.

Initially, I performed CW cavity spectroscopy with a VNA to quickly find the resonance and estimate the bare and dispersive frequencies. Next, I repeated this measurement using heterodyne measurement (section 6.4) with microwave pulses of a fixed length (usually $t_m = 5 \mu s$). Practically to read out the qubit, a single point in frequency and power were driven and the cavity response at that point was measured. Due to the dispersive shift, the cavity response will have a difference in amplitude or phase in the e-state vs. the g-state. I choose an initial readout point to perform subsequent measurements near the dispersive g frequency and at a power just below the non-linear regime. The e-state cavity map can be taken by repeating this measurement after a π -pulse frequency, amplitude, and time has been found. Then a readout point can be chosen based on the maximum separation between g and e cavity response.

Readout at the high-power resonant peak is also possible because non-linearities change the onset power of the bare peak [141]. The bare peak begins at lower power when the qubit is in the e state than the g state. This creates a region of differing cavity response in this high-power regime as well. I usually choose dispersive, low-power readout to avoid any unintended effects from high cavity drive powers [85, 142].

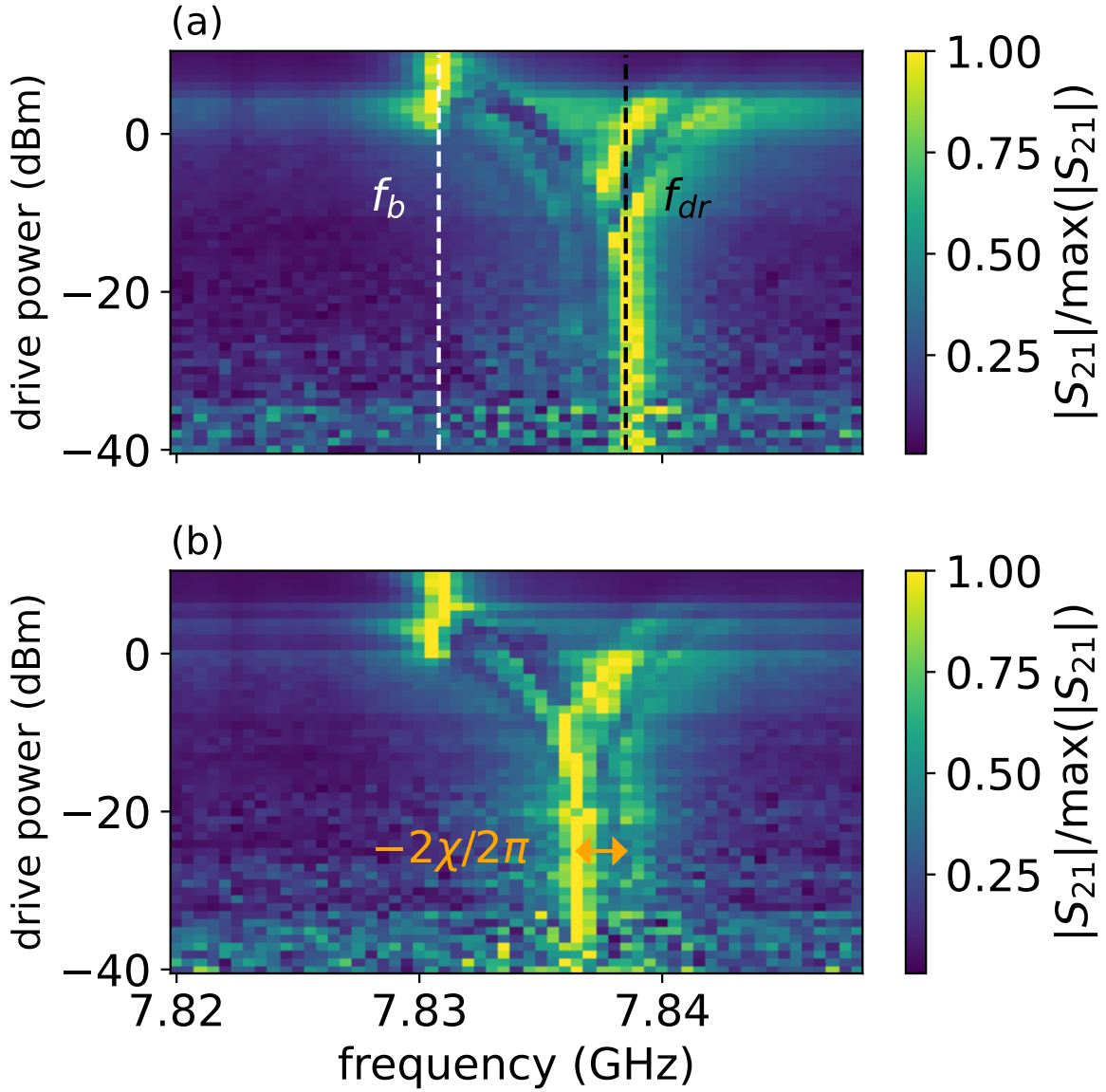


Figure 7.1: False color map of the pulsed resonator response vs. frequency and power for a 3D transmon. **a)** The transmission response of the cavity for the qubit in the g state and **(b)** e state is shown normalized at each drive power. For a 3D cavity, resonance corresponds to a peak in the transmission amplitude. At high power, the resonant frequency corresponds to the resonator bare frequency. At intermediate power, the cavity-qubit system is non-linear. At lower powers (< -20 dBm), the resonator frequency corresponds to the dressed state. The dressed frequency shifts by $2\chi/2\pi \approx 2$ MHz when the qubit goes from the g to e state. A smaller g peak is visible in the e state scan and visa-versa suggesting finite thermal qubit population during this measurement.

7.2.1 Example of Anomalous Cavity Shifts

In section 3.4.3, I explained that higher transmon levels produce anomalous χ shifts. This effect can be observed in cavity spectroscopy. To illustrate this, consider the following system. A qubit is coupled to a resonator with the parameters given in table 7.1. Fig. 7.2 shows measured pulsed spectroscopy of the cavity with the qubit in the $|g\rangle$, $|e\rangle$, and $|f\rangle$ states. The figure shows transmission of the cavity vs. frequency and power. At the highest powers, the cavity transmission has a resonant dip at its bare frequency. At low powers, the cavity frequency is shifted to the dressed frequency which is dependent on the state of the coupled qubit. Using this data I experimentally extracted total effective chi shifts $\chi_{g,e,f}/2\pi$ as $f_{dr,g} - f_{bare} = 610$ MHz, $f_{dr,e} - f_{bare} = 640$ MHz, and $f_{dr,f} - f_{bare} = 426$ MHz respectively. Using the scQubits package [70] to find the coupling g 's between the transmon levels then using the formalism above, these dispersive shifts can be accurately predict within 0.5% as $f_{dr,g} - f_{bare} = 610$ MHz, $f_{dr,e} - f_{bare} = 640$ MHz, and $f_{dr,f} - f_{bare} = 427$ MHz.

Note, this transmon is highly atypical with $|\chi_e| > |\chi_g|$. This is due to the $1 \rightarrow 4$ transition being nearly resonant with the cavity, giving a strong higher-order correction to the χ_e shift. Without accounting for this matrix element, the predicted shift would be $\chi_e/2\pi \approx 474$ MHz, significantly different from the measured value. The total $2\chi = \chi_e - \chi_g$ shift will be predicted to have the wrong sign. This example shows the importance of calculating the chi shifts precisely when experimental data does not match lower order approximations.

Full diagonalization of a coupled Jaynes-Cummings Hamiltonian using the charge basis (Eq. 3.18) can give more accurate results than the dispersive approximation. Since some transitions must be nearly resonant to observe the anomalous behavior, estimating g and higher level states using the dispersive approximation is inaccurate with the picture described above. However, Eq. 3.45 can still produce novel behavior that the commonly used first order approximations do not.

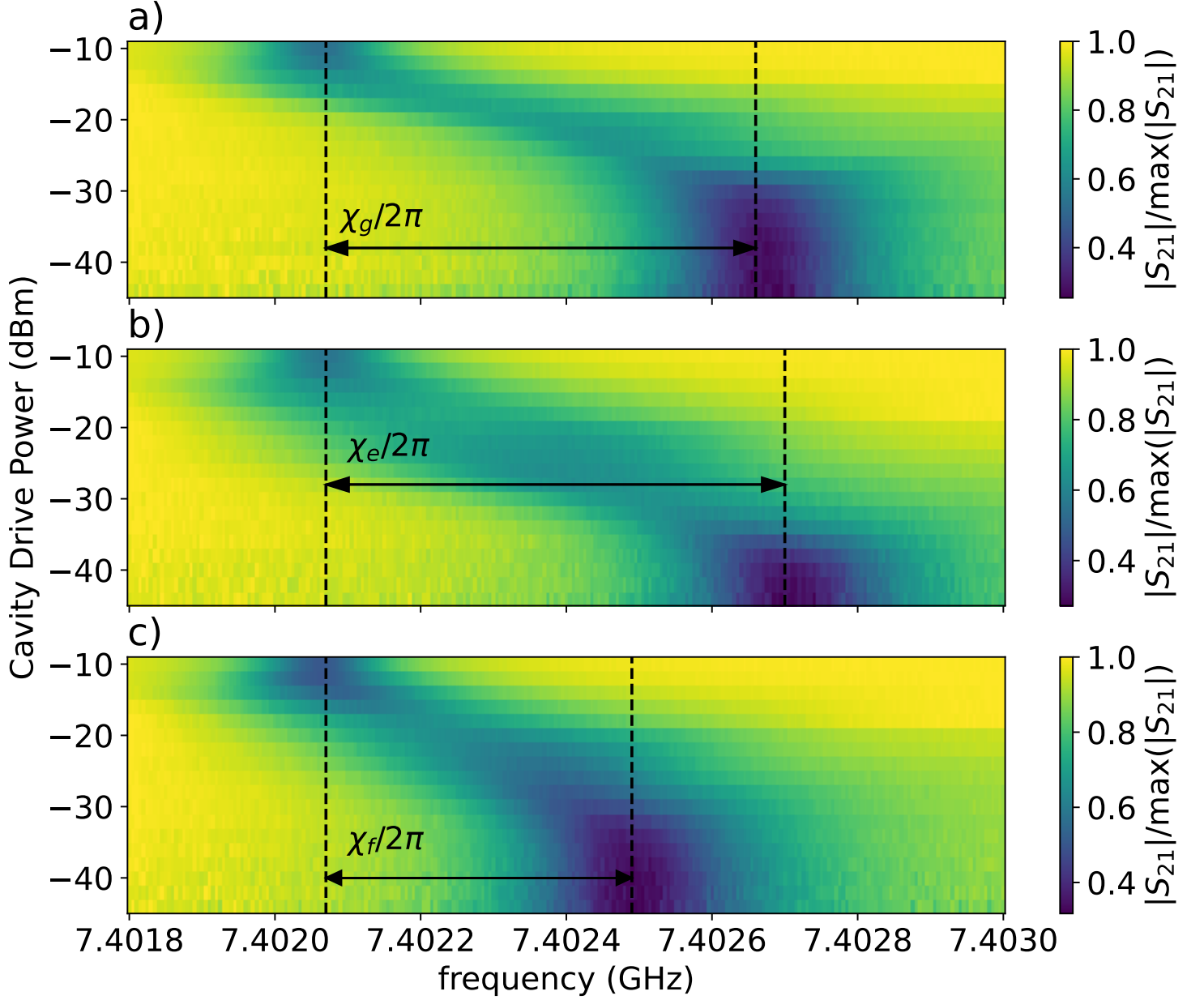


Figure 7.2: Measurement of the cavity dispersive shifts. **a)** Pulsed cavity spectroscopy with the qubit in the g , **(b)** e , and **(c)** f state. The cavity shows a dispersive shift between the bare cavity frequency at high power to the dressed frequency at low power. Unexpectedly for a transmon, the e -state dispersive shift is greater than the g -state shift. This is due to a near resonance of the cavity and the $1 \leftrightarrow 4$ qubit transition. This effect can be qualitatively predicted by Eq. 3.45 and simulated precisely by direct diagonalization of Eq. 3.34.

Table 7.1: Relevant parameters for a system showing the importance of higher level corrections to χ_e

Param	Value
f_q	2.747 GHz
f_r	7.402 GHz
E_J/h	7.73 GHz
E_C/h	135 MHz
$g/2\pi$	51.95 MHz

7.3 Qubit Spectroscopy

After initial cavity spectroscopy is performed, qubit spectroscopy is performed to determine the qubit frequency. The measurement sequence shown in Fig. 7.3 is performed with alternating driven, M_{dr} , and ground state, M_g , measurements separated by t_w . The cavity measurement is performed at the frequency, power, and length as discussed in section 7.2. Each measurement results in an amplitude and phase signal from the cavity. The amplitude and phase are converted into IQ voltage values, $M_{j,I}$, $M_{j,Q}$ for $j = dr, g$. t_w should be chosen to be greater than $10 \cdot T_1$ to fully reset the qubit to the ground state before any pulse sequence begins.

To excite the qubit for spectroscopy, a long drive pulse is applied. The length of this pulse is set longer than the qubit's coherence time to drive the qubit into a mixed state, otherwise Rabi fringes could be visible, complicating the spectroscopy. The frequency of the drive pulse is stepped. When the drive frequency is resonant with a qubit transition, the cavity response will change. The total response of the cavity is calculated as the distance in IQ space between the two measurements

$$D = \left| \vec{M}_{dr} - \vec{M}_g \right| = \sqrt{(M_{dr,I} - M_{g,I})^2 + (M_{dr,Q} - M_{g,Q})^2} \quad (7.1)$$

Drive detuning changes the magnitude of the steady-state qubit response following the generalized Rabi angle $D = D_{max} \Omega / \sqrt{\Omega^2 + \delta^2}$ where D_{max} is the response on resonance, Ω is the on-resonance Rabi driving rate, and $\delta = |f_q - f_{01}|$ is the drive detuning.

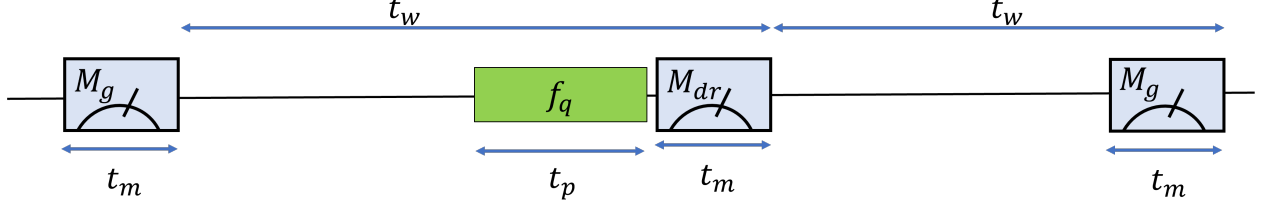


Figure 7.3: Qubit spectroscopy pulse sequence. To perform qubit spectroscopy, I alternate measurements of the cavity with (without) exciting the qubit, M_{dr} (M_g). Before the driven measurement, the qubit is excited with a single-frequency pulse with length $t_p > T_2$ to drive the qubit into the mixed state. The measurement reset time, $t_w \approx 10T_1$ allows the qubit to naturally relax into the ground state.

Figure 7.4 shows a sample qubit spectroscopy scan where f_q and the amplitude of the qubit drive are varied. At large drive powers three separate spectroscopic peaks were observed corresponding to $f_q = f_{01}$, $f_q = f_{02}/2$, and $f_q = f_{03}/3$ from right to left. At large drive powers, multi-photon processes allow the qubit to be excited to higher states using the intermediate excited states. However, as the pulse drive power decreases, the multi-photon peaks quickly disappear. The widest peak can be fit to obtain the qubit frequency.

7.4 Time Domain Measurements

Spectroscopy of the cavity and qubit involve driving the system into a steady state. Qubit manipulations and further characterization requires time domain control. This is done with precision delay generators and AWGs controlling the timing of microwave pulses from the generators.

7.4.1 Normalization and Population Extraction

For these measurements, I generated the populations from cavity transmission signals with a scheme here called 2 or 3 state normalization. For 2 state normalization, each data point consists of 3 measurements of the cavity transmission, M_g of the g state, M_e of the e state, and M_m of the measurement of interest (Fig. 7.5a). Subtly, when M_i is averaged, it is

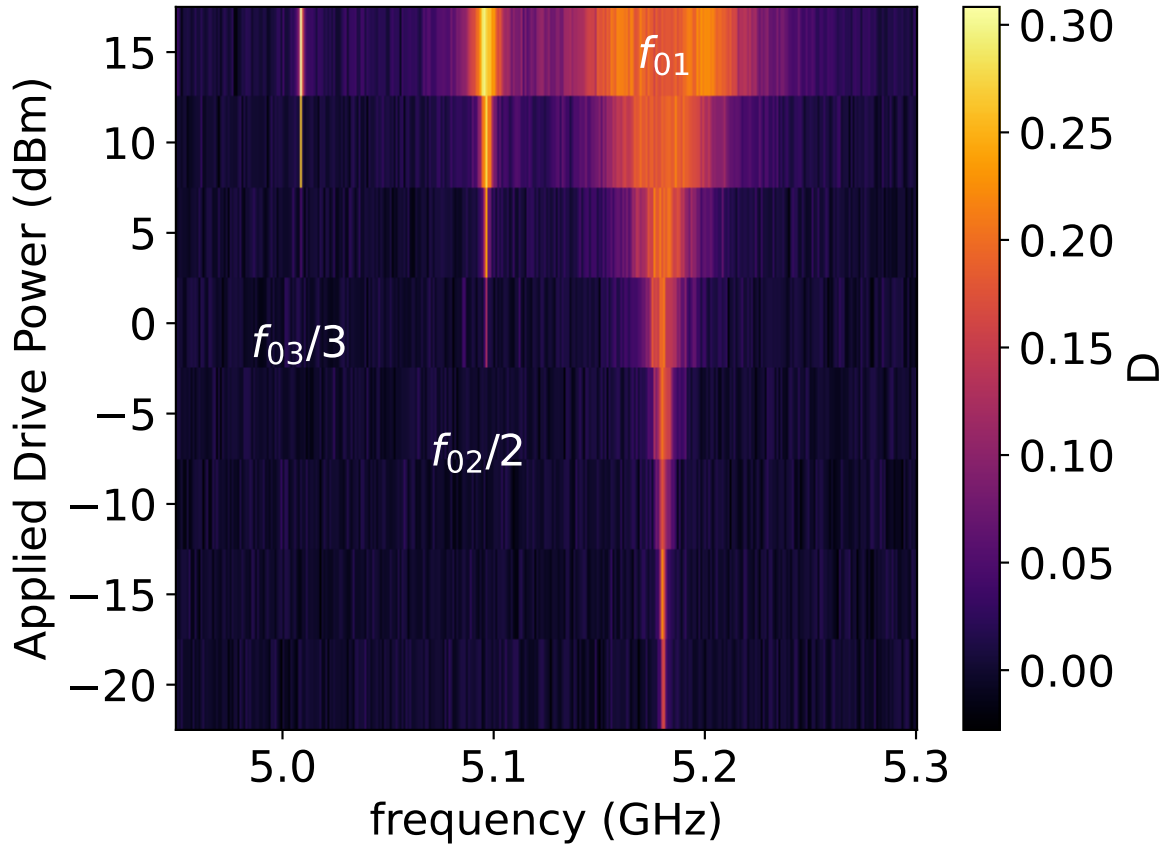


Figure 7.4: Qubit spectroscopy measurement. Here the frequency and power of the excitation drive is stepped and the resulting displacement of the cavity caused by the qubit is plotted. Three bright peaks are visible. From right to left these correspond to the qubit's f_{01} , $f_{02}/2$, and $f_{03}/3$ frequencies. When the drive is resonant with one of the fractional frequencies, a multi-photon process excites the qubit to a higher excited state. Since these processes require multiple photons, their peaks fall off very quickly as the power decreases.

a measurement the corresponding thermal state with finite population, $P_i = e^{-hf_i/k_B T}/Z$ where the canonical partition function $Z = \sum_i e^{-hf_i/k_B T}$. When the qubit is well thermalized at low temperatures, this effect can be ignored.

Averaged measurements are assigned a probability based on the projection of the measurement vector onto the e state vector

$$P_e = \frac{(\vec{M}_m - \vec{M}_g) \cdot (\vec{M}_e - \vec{M}_g)}{(\vec{M}_e - \vec{M}_g)^2} \quad (7.2)$$

When measuring $P_e \approx 0$ or 1, noise in the measurement can give negative probabilities using this formula. To limit these non-physical probabilities, measurement can be averaged further to reduce the noise.

If single shot measurement is performed, individual measurements are recorded and binned. This scheme avoids non-physical negative probabilities but also has minimum measurable g state population limited by measurement assignment fidelity and thermal populations. For $n > 26$ measurements, the central limit theorem [143] states both averaged and single shot readout schemes give the same expected population value.

The pulse sequence for 3 state normalization is shown in Fig. 7.5b. This sequence is identical to 2 state normalization but adds a measurement of the f state to extract the population of the first three levels of the transmon. This is useful for qutrit measurements or measuring excitations out of the qubit subspace. A measured point is assigned probabilities satisfying

$$\vec{M}_m - \vec{M}_g = P_e(\vec{M}_e - \vec{M}_g) + P_f(\vec{M}_f - \vec{M}_g) \quad (7.3)$$

as shown in Fig. 7.7. The three state normalization equation clearly simplifies to Eq. 7.2 when $P_f = 0$ and the state is in the g, e manifold. The sequence is named 3 state normalization because of the four total pulses characterizing the populations in the first three transmon levels. The three transmon states are measured for the normalization, while the fourth pulse is the measurement itself.

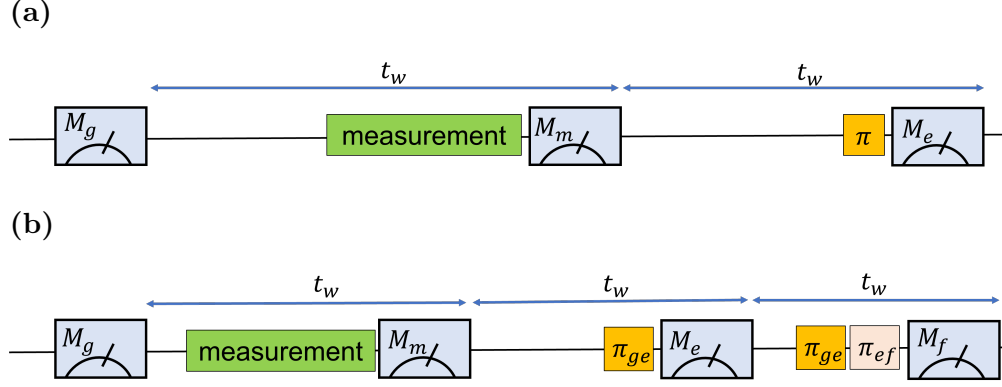


Figure 7.5: Measurement normalization pulse sequences (not to scale). **a)** The 2 state normalization sequence. First the g state is measured, then the measurement sequence is performed (green), the qubit is reset via natural relaxation, and finally the e state is measured. Repeating the sequence allows a measurement to be averaged. This sequence calibrates measurement instability at the expense of total measurement time. **b)** The 3 state normalization sequence. Similar to **(a)**, but adds a measurement of the f state to account for leakage out of the qubit subspace and second excited state population.

7.4.2 Rabi Oscillations

The first time domain measurement typically performed in the characterization process is Rabi oscillations. This “flopping” between two states is a key behavior of a driven two-level system [12, 144]. A Rabi oscillation measurement is done by increasing the time (or amplitude) of the qubit drive, which increases the total angle the qubit state rotates on the Bloch sphere (Fig. 7.8a). Usually this is done over many multiples of 2π to see clear oscillations.

Fig. 7.9 shows a typical measurement of Rabi oscillations on a transmon qubit. The 2 state normalization results in oscillations between 0 and 1 of the measured excited state population, P_e . Fitting the data for $t_r \ll T'$, where T' is the Rabi decay time,

$$P_e(t_r) = \frac{1}{2}[1 - \cos(\Omega t_r + \phi)] \quad (7.4)$$

allows extraction of the Rabi rate, Ω with a (usually small) phase offset ϕ . For a given Rabi amplitude, A_r , a Rabi oscillation allows the extraction of the π -pulse time, $t_\pi = \pi/\Omega$.

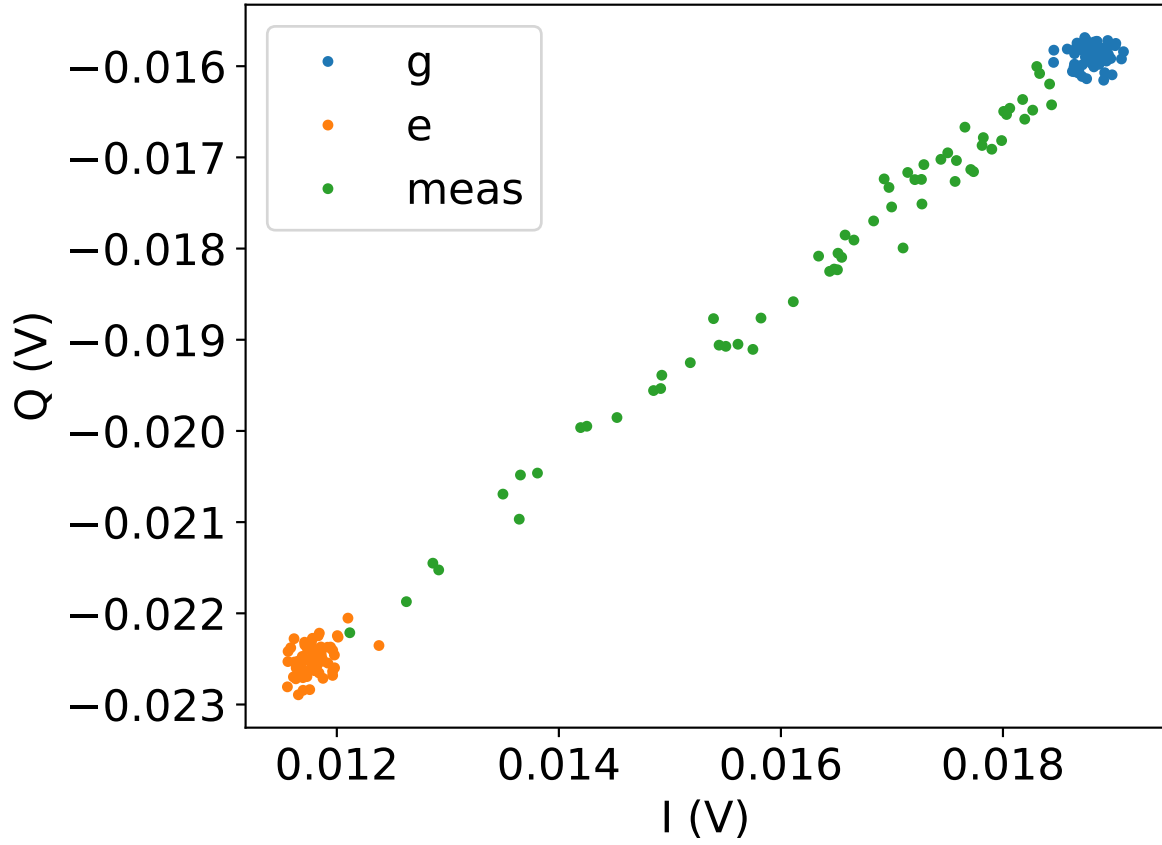


Figure 7.6: Sample measurement using 2 state normalization. The g , e , and measurement (here a T_1 decay) are shown in IQ space. The measurement points clearly lie on the vector path between the g and e normalization measurements.

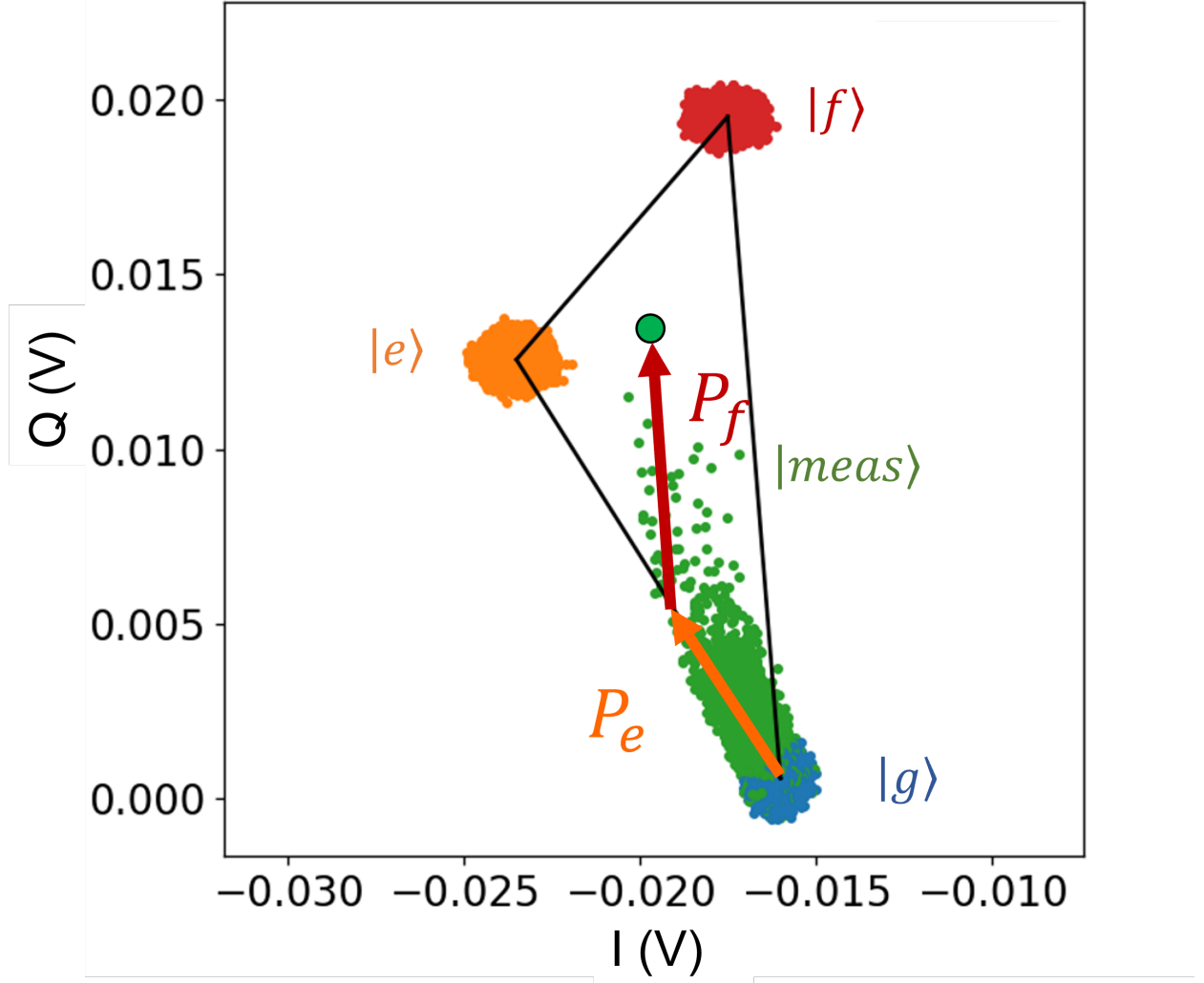


Figure 7.7: Sample randomized benchmarking measurement using 3 state normalization. Hypothetical marked point (large green circle) can be made as a linear combination of $\vec{M}_e - \vec{M}_g$ and $\vec{M}_f - \vec{M}_g$ vectors with the scalar values representing the respective populations.

For Rabi measurements with t_r on the same order as T_1 , the decoherence of the qubit reduces the oscillation amplitude by a factor of $e^{-t_r/T'}$ where the Rabi decay time, T' is given approximately by $1/T' = 1/2T_1 + 1/2T_2$.

7.4.3 T_1 Relaxation

With a calibrated π -pulse, I next measure the transmon relaxation time, T_1 . The pulse sequence consists of performing a π -pulse to put the qubit in the excited state, waiting for delay time, t_d , then measuring the average expectation probability (Fig. 7.8b). For time-independent loss channels, Eq. 3.53 predicts an exponential decay of $P_e = P_0 e^{-t_d/T_1} + P_{res}$, where the initial population $P_0 + P_{res}$ and residual population P_{res} account for experimental offsets and allow more accurate extraction of the decay rate. Fitting measured data to this exponential gives an experimental T_1 time (Fig. 7.10).

The measured T_1 was then used to adjust the initialization wait time such that $t_w \geq 10T_1$.

7.4.4 Ramsey Measurements

Next, a Ramsey measurement is done to characterize the decoherence rates of my qubits [145]. For this sequence, a $\pi/2$ -pulse detuned from the qubit frequency by $\delta < \Omega$ rotates the qubit state from the ground state to the equator of the Bloch sphere. The qubit state is allowed to precess for time t_d . A final $\pi/2$ -pulse with the same detuning as the initial pulse is performed. For $t_d = 0$ this pulse will move the qubit state to the excited state. However if the state has precessed a total angle π the final pulse will move the state to the ground state. Oscillations between these two states due to the precession are called Ramsey oscillations. Performing a $\pi/2$ -pulse immediately before measurement effectively maps the qubit measurement of σ_Z to a measurement of σ_X .

Fitting the Ramsey oscillations to

$$P_e(t_d) = e^{-t_d/T_2^*} \left[\frac{1}{2}(1 + \cos \delta t_d) \right] \quad (7.5)$$

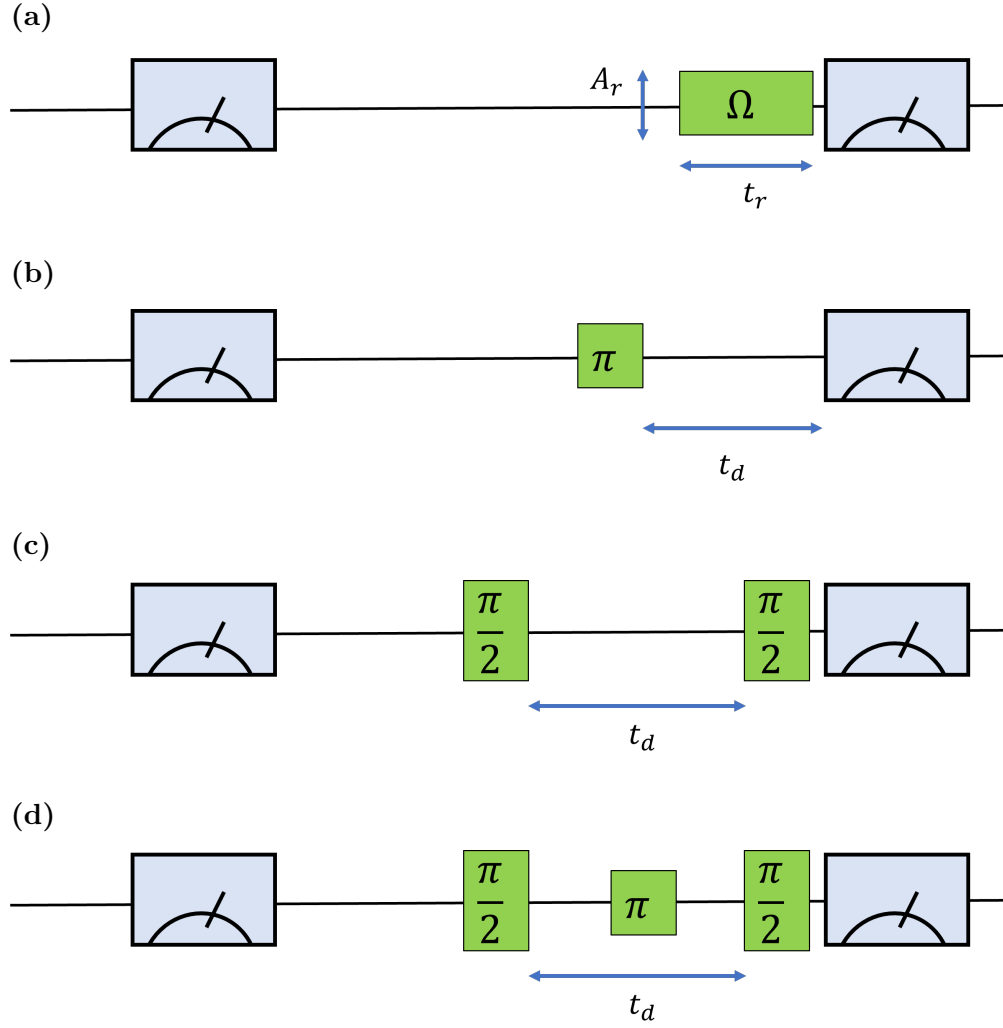


Figure 7.8: Time domain measurement pulse sequences. The “measurement” portion of a 3 or 4 pulse normalization measurement (Fig. 7.5a) for the typical time domain characterization experiments. **a)** A Rabi pulse sequence. Here a resonant Rabi pulse is applied and the time or amplitude is swept, changing the total rotation applied to the qubit. A specific amplitude and time are calibrated to perform a π -pulse. **b)** A T_1 pulse sequence which performs a π -pulse to put the qubit in the excited state, waits a delay time, and measures the average excitation probability. **c)** A Ramsey pulse sequence. A $\pi/2$ -pulse brings the qubit state to the equator of the Bloch sphere where decoherence processes move the qubit toward the center axis of the Bloch sphere. A final $\pi/2$ -pulse is performed to map the qubit measurement to a measurement of σ_x , characterizing the coherences in the density matrix (Eq. 3.53). **d)** A Hahn-echo pulse sequence. The Hahn-echo measurement is very similar to a Ramsey measurement but with a π -pulse in the middle of the delay time. This “flips” the qubit state causing any inhomogeneous dephasing processes to cancel out over the full duration.

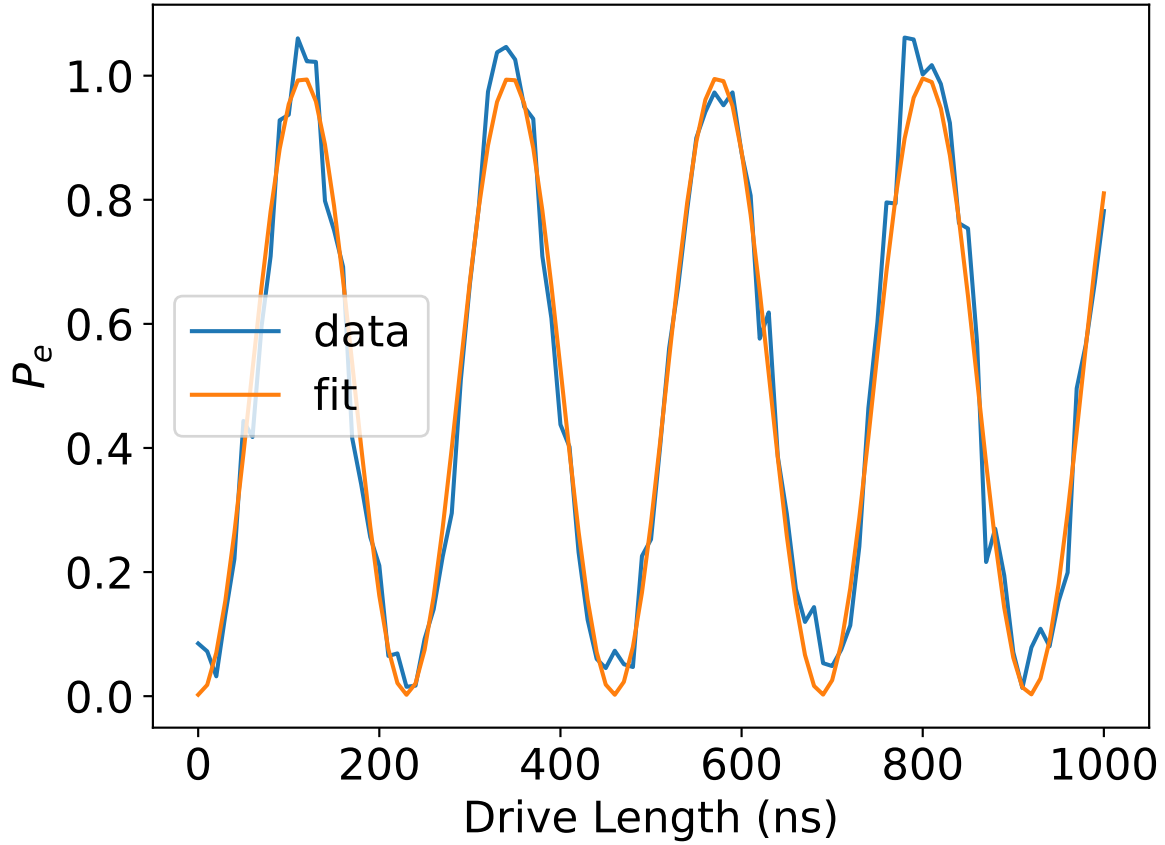


Figure 7.9: Rabi oscillations measured on a transmon qubit (blue) and oscillatory fit (orange). The excited state population of the qubit for a range Rabi drive times is plotted. Fitting to a sinusoid yields the Rabi rate and π -pulse time (here 105 ns).

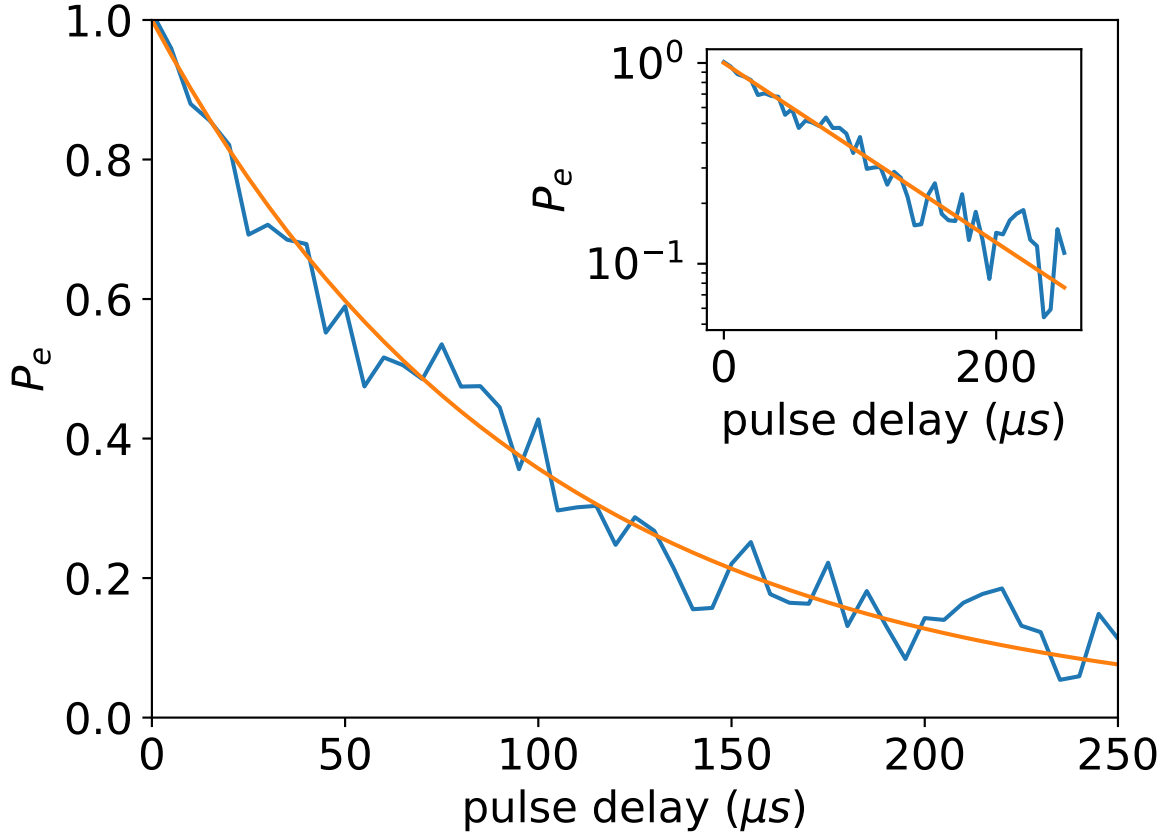


Figure 7.10: Plot of a T_1 relaxation measurement of device **Q1**. Plotted is the average excited state probability vs. delay time between exciting the qubit to the e state and the cavity measurement (blue). The qubit state relaxes back to the ground state at a constant rate $1/T_1$ with $T_1 = 97.1 \mu s$ (orange). Inset is the same data and fit on a semilog scale.

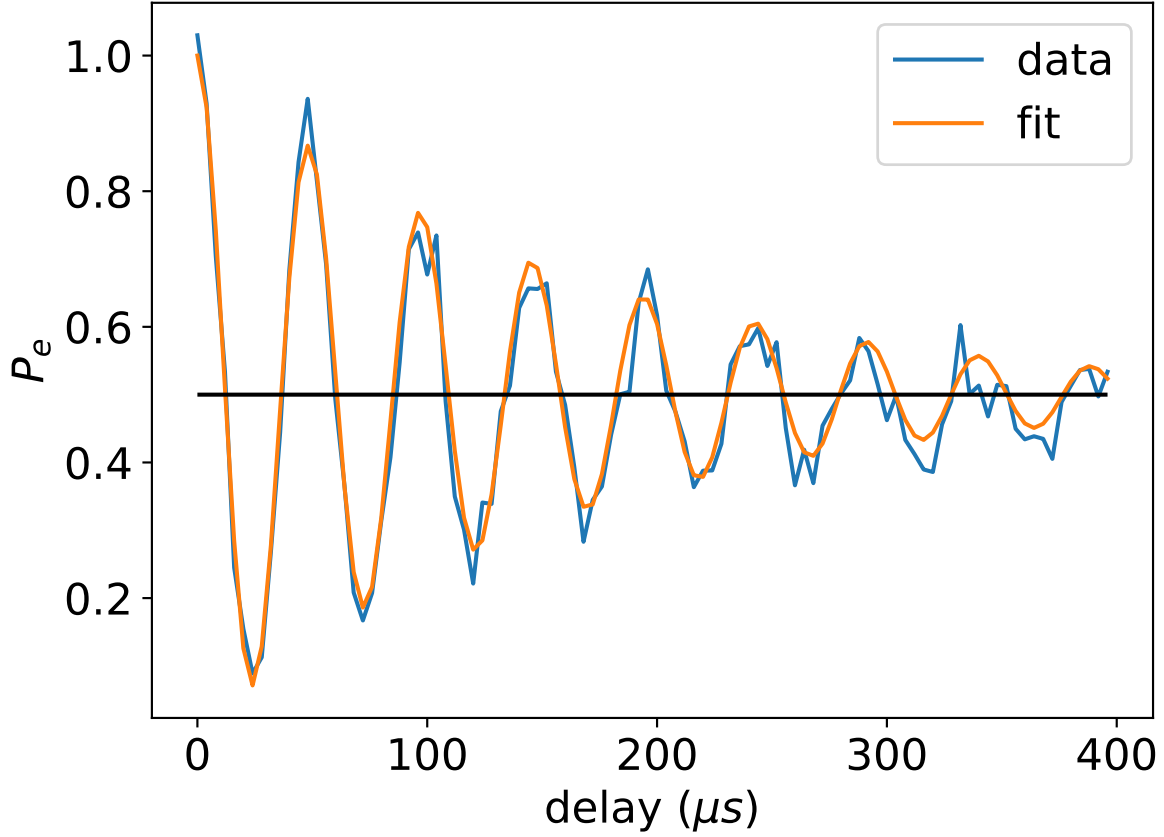


Figure 7.11: Plot of a Ramsey oscillation measurement of **Q1**. The measured final state after a Ramsey measurement vs. variable delay time (blue). The qubit state precesses with delay time creating oscillations in the measurement. Decoherence processes reduce the amplitude of the oscillations at longer times. Fitting the data shown yields $T_2^* = 157 \mu s$ and $\delta = 20.58 kHz$ (orange).

yields the Ramsey decoherence time, T_2^* and the detuning δ (Fig. 7.11). T_2^* measures the decoherence of the qubit including inhomogeneous noise causing shot-to-shot variations in coherence. Measuring the detuning from Ramsey oscillations allowed me to determine the qubit frequency much more accurately than qubit spectroscopy.

7.4.5 Hahn-Echo Measurements

The final qubit characterization measurement that I review here is the Hahn-echo measurement [146]. Using the same sequence as a Ramsey measurement but with an additional π -

pulse in the middle of the delay time, homogeneous (low) frequency errors will be “echoed” out (Fig. 7.8d). Here I apply all three pulses about the same axis on the Bloch sphere, while the echo pulses are often performed around orthogonal axes. For my measurement, the Hahn-echo experiment sees an exponential rise from the ground state to the mixed state $P_e(t_d) = 1/2(1 - e^{-t_d/T_2})$ with decoherence time T_2 fitted experimentally (Fig. 7.12). The Hahn-echo decay will have a non-exponential decay if strongly affected by $1/f$ noise processes [61].

Addition π -pulses can be added to cancel higher frequency noise. These sequences are called Carr-Purcell-Meiboom-Gill (CPMG) sequences after two pairs of authors that discovered the sequences in different contexts [147, 148]. Noise spectroscopy can be performed by measuring the decoherence at a range of pulses [61, 149].

7.5 High-Fidelity Gate Tuning

For the general characterization techniques covered above, a maximum of three qubit gates are used. For even simple quantum algorithms, hundreds of quantum gates must be performed. Fine tuning quantum gates is extremely important for more complex measurements. Here I cover the measurements I used to tune single qubit gates and characterize their fidelity. The tuning measurements I performed largely follow Greene [30]. These tuning measurements are all implementations of error amplifying sequences, where a specific gate error is amplified by the gate sequence allowing subsequent tuning and minimization. The concept of error amplifying sequences can be applied to most errors and is not limited to single qubit gates.

7.5.1 π -Pulse Error Amplification

The first tuning measurement I cover is π -pulse tuning. In section 7.4.2, a π -pulse was tuned by measuring Rabi oscillations. This technique doesn’t account for the rise and fall of a pulse

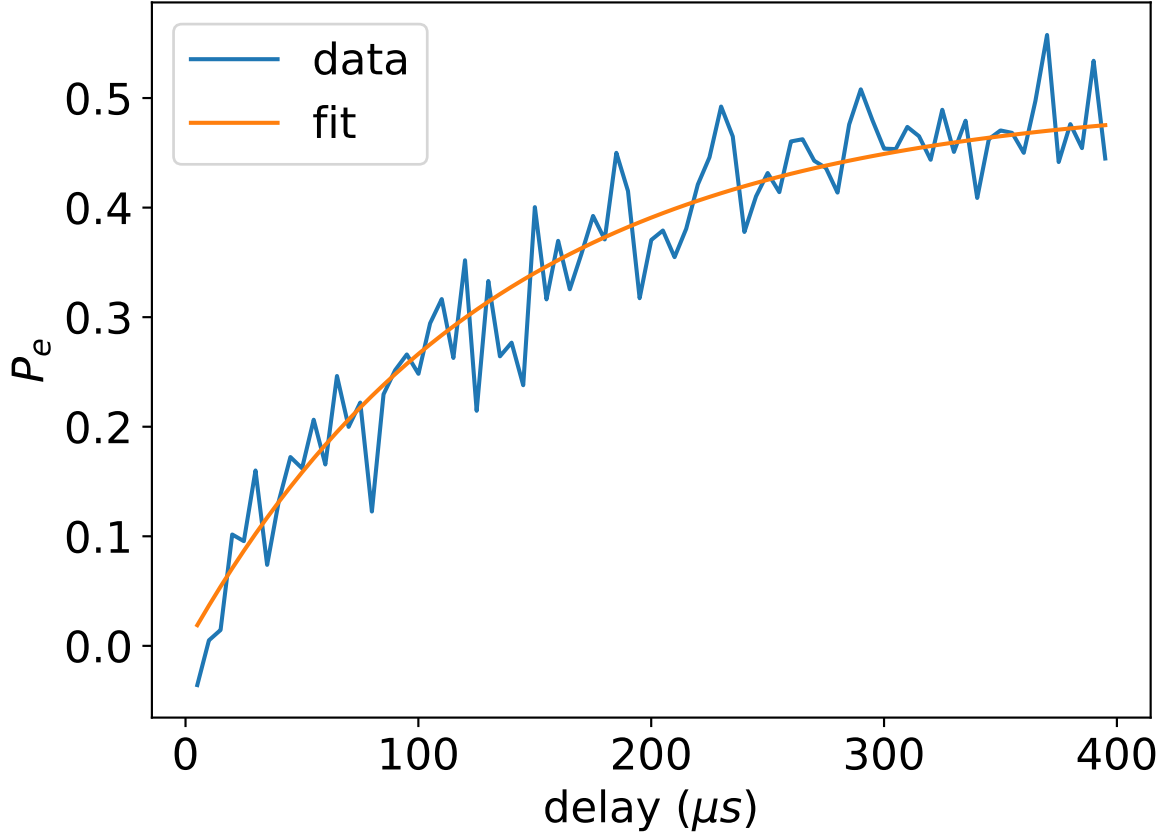


Figure 7.12: Plot of a Hahn-echo measurement of **Q1**. The measured (blue) and fit (orange) excited state probability as a function of delay time is shown. As the qubit experiences decoherence, the final state approaches the fully mixed state with $P_e = 0.5$. The curve shown has decoherence decay time of $T_2 = 131 \mu s$.

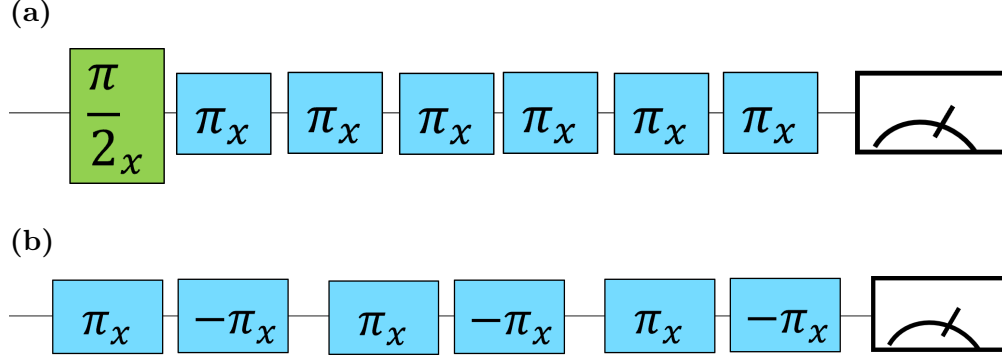


Figure 7.13: Pulse tuning sequences for tuning single-qubit gates. **a)** Error-amplifying sequence for measuring amplitude errors of a π -pulse. The state is moved to the equator of the Bloch sphere by a $\pi/2$ -pulse, then repeated π -pulses will amplify small over- or under-rotations from the π -pulse. **b)** Error-amplifying sequence for phase errors (*i.e.* σ_Z errors) used to tune the DRAG amplitude. Alternating π and $-\pi$ -pulses cause the phase errors to add. Repeating this pulse pair amplifies the phase error.

shape or any non-linearity of the drives in the system [150]. To accurately tune a π -pulse, I used an amplitude error amplifying sequence (Fig. 7.13a). First, a $\pi/2$ -pulse is applied since $\sigma_Z = 0$ on the Bloch sphere maximizes the measurement's sensitivity to amplitude errors. Next, a series of π -pulses are applied along the same Bloch sphere axis as the initial $\pi/2$ -pulse. For perfect pulses the qubit state will remain on the equator, simply flipping sides of the Bloch sphere. However, any over or under rotation from the π -pulse will be multiplied by the number of pulses. By sweeping the amplitude of the pulse for a series of pulses, fringes are observed (Fig. 7.14). The center of the fringes marks the optimal amplitude. Fitting to the function

$$P_e(A, n) = \frac{1}{2} \left(1 - \cos \left[\pi \left(\frac{A}{A_0} n + 1/2 \right) \right] \right) \quad (7.6)$$

with applied amplitude A , optimal amplitude, A_0 , and pulse number n can yield a precise value for the π -pulse amplitude. Even though the calibration measurement is done out to only 10-100 π -pulses, the final pulse with fitted amplitude can be accurate to much greater pulse numbers.

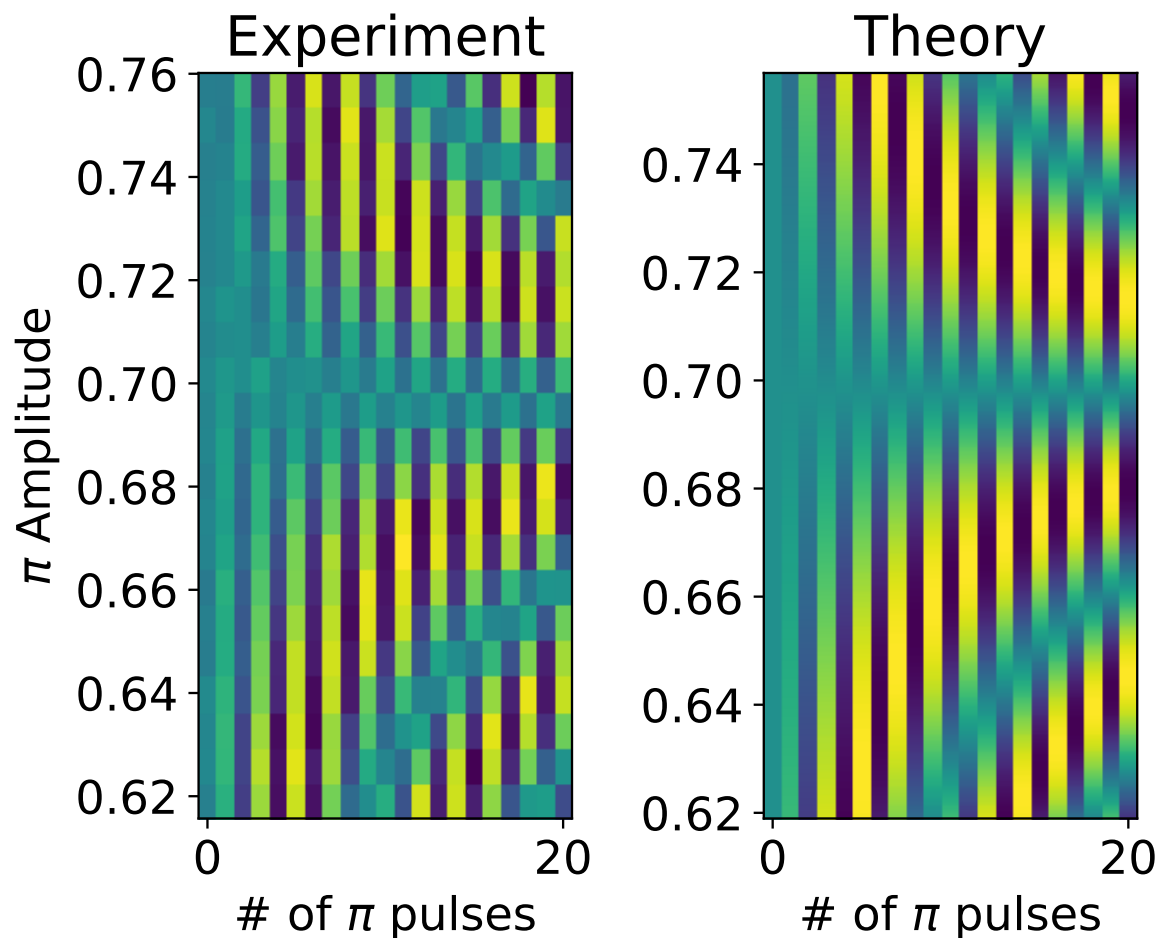


Figure 7.14: Experimental data and fitted simulation of a π -pulse amplitude tuning experiment. *Left* is data taken using pulse sequence in Fig. 7.13a sweeping the amplitude of the π -pulse. Amplitude errors cause increasing oscillation magnitude with increasing pulse number. Center of the fringe corresponds to the optimal π -pulse amplitude. *Right* is theory from expected errors using Eq. 7.6. Good match between experiment and theory verifies an accurate extracted amplitude from the fit.

7.5.2 DRAG Scheme

For short qubit pulses, the pulse is wide in the frequency domain. These pulses can directly drive higher order transitions of the transmon such as the $f_{ef} = f_{ge} - \alpha$ transition from the first to second excited states which is only detuned a small amount, α , from the ground to excited state transition (Eq. 3.27). Aside from directly driving this higher order transition, a strong qubit pulse can cause an AC Stark shift on the e to f transition, changing the qubit frequency during the pulse, and causing coherent dephasing errors. To mitigate these effects, the superconducting qubit community commonly uses Derivative Reduction by Adiabatic Gate (DRAG) [29, 151, 152]. This technique corrects phase errors from a pulse by applying an out-of-phase drive, $S_Q(t)$, to the main pulse, $S_I(t)$, such that

$$S_Q(t) = \frac{-1}{2\alpha} \frac{d}{dt} S_I(t) \quad (7.7)$$

with qubit anharmonicity α (Fig. 7.15). By applying an auxiliary drive out-of-phase from the main driving term, a phase error is induced which corrects the error from the AC Stark shift. DRAG can be adjusted to reduce leakage errors simply by multiplying the out of phase component by 2; however, in this case the phase errors must be corrected by other means [153].

This simple version of the DRAG scheme was sufficient to correct the majority of the gate errors in my qubit control experiments. However, more complex schemes which correct both leakage and phase errors or correct errors to higher order exist. Although DRAG can be used to significantly improve gate performance, further improvements of gate tuning will be necessary for transmon qubits [29, 153, 154].

7.5.3 DRAG Tuning

DRAG amplitude has a well defined theoretical value from Eq. 7.7. However, phase errors from other sources in an experiment will shift the optimal amplitude for the DRAG drive

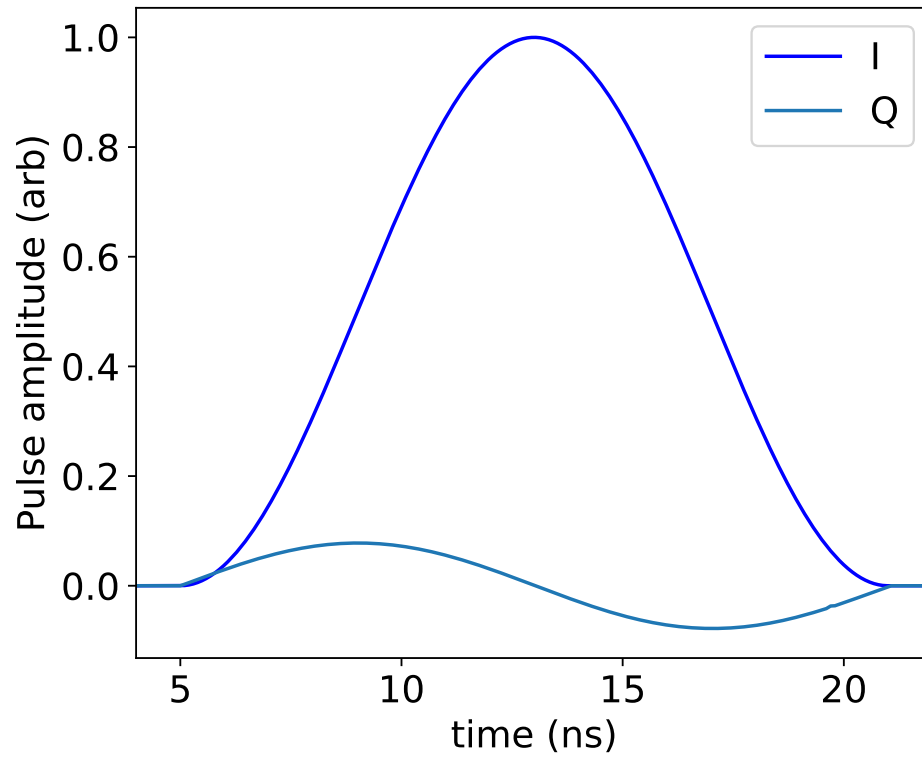


Figure 7.15: Raised cosine pulse envelope. The in-phase (I) and out-of-phase (Q) components of the drive envelope are shown. Here the Q drive is proportional to the derivative of the I drive to cancel phase errors using the DRAG protocol.

from $-1/2\alpha$. Therefore, I performed experiments to tune the amplitude of the DRAG experimentally.

Phase errors are canceled to first order by repeated π -pulses. By approximating a π -pulse with a phase error by two rotations in series, $X_\pi Z_\phi$, a sequence of π -pulses can be written

$$X_\pi Z_\phi X_\pi Z_\phi \dots X_\pi Z_\phi = \underbrace{X_\pi Z_\phi X_\pi Z_\phi}_{Z_{-\phi} Z_\phi} \underbrace{X_\pi Z_\phi X_\pi Z_\phi}_{Z_{-\phi} Z_\phi} \dots X_\pi Z_\phi \quad (7.8)$$

$$= Z_{-\phi} Z_\phi Z_{-\phi} \dots Z_\phi = \mathbb{1} \quad (7.9)$$

for an even number of pulses. Clearly this sequence does not amplify phase errors, however, the simple adjustment of reversing the direction of every-other π -pulse thereby applying repeated π , $-\pi$ pulse pairs will amplify the phase error.

$$X_\pi Z_\phi X_{-\pi} Z_{-\phi} \dots X_{-\pi} Z_{-\phi} = n Z_\phi \quad (7.10)$$

To tune the DRAG amplitude, I start with the qubit in the ground state and apply a series of π , $-\pi$ pulses (Fig. 7.13b). By performing the measurement with a range of DRAG amplitudes, a fringe pattern from the amplified error again appears (Fig. 7.16). Fitting the measured data to

$$P_e(A, n) = \frac{1}{2}(1 - \cos[\pi((A - A_0)sn)]) \quad (7.11)$$

where A is the DRAG amplitude applied, A_0 is the ideal DRAG amplitude, n is the number of pulse pairs, and s is a phase error scaling constant. Fitting the experimental data can yield a precise value for the DRAG amplitude.

7.5.4 Gate Benchmarking

After tuning the qubit gates, the quantity of interest is the remaining error per gate (EPG), or the chance the gate introduces a computational error. I used randomized benchmarking to measure the error per gate [155]. Random gates were chosen from a specified gate-set (here

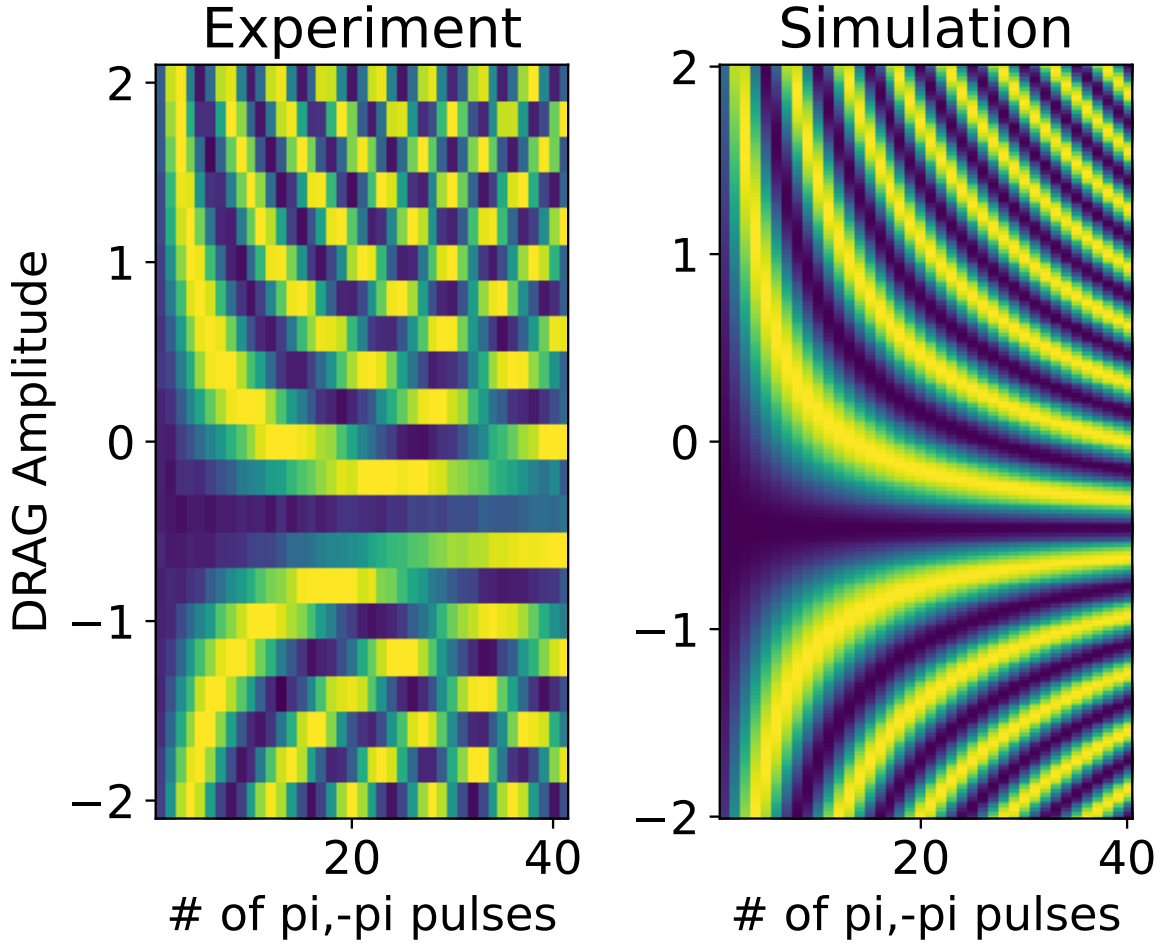


Figure 7.16: Experimental data and fitted simulation of a DRAG amplitude tuning experiment showing P_e vs DRAG amplitude and number of π , $-\pi$ pulses. *Left* is data taken using pulse sequence in Fig. 7.13b sweeping the amplitude of the out-of-phase component. Phase errors cause increasing oscillation magnitude with increasing pulse number. Center of the fringe corresponds to the optimal DRAG amplitude. *Right* is theory from expected errors using Eq. 7.11. Good match between experiment and theory verifies an accurate extracted amplitude from the fit. Upward bend in the experimental data is an artifact from aliasing.

$\pi/2$ -pulses in the +X, -X, +Y, and -Y directions) and applied to the qubit. The intended qubit state was tracked in software and a final gate restored the qubit to the ground state if no errors occurred. The measurement was repeated for a range of total gate number (m) and different randomizations (>10 randomizations at each m). The gate errors were modeled together as a depolarizing channel with probability ϵ of moving the qubit state to the fully mixed state [77].

An error per gate metric was determined by fitting the ground state probability, P_g , after the benchmarking pulse sequences of varied pulse number, m , to.

$$P_g = \frac{1}{2}(1 + (1 - \epsilon)^{m+1}) \quad (7.12)$$

where ϵ is the fitted error per gate. For large m the form approaches an exponential decay to $P_g = 1/2$.

Benchmarking measurements can also be expressed in terms of sequence fidelity defined as

$$\mathcal{F} = 2 \frac{P_g}{P_g + P_e + P_f} - 1 \quad (7.13)$$

For $\epsilon \ll 1$, the fidelity exponentially decays with $\mathcal{F} = 1$ corresponding to the ideal benchmarking sequence and $\mathcal{F} = 0$ corresponding to the fully mixed state.

Figure 7.17a shows results from a sample benchmarking experiment done with 3 state normalization (section 7.4.1). Extracted $P_{g,e,f}$ are plotted vs. the number of gates applied. The spread in the measured data is due to two effects: first, measurement noise adds a fixed error to each measurement. Additionally, for longer sequences coherent errors can add or cancel depending on the random sequence chosen. Similar to Hahn-echo measurements (section 7.4.5), certain gate sequences can cancel coherent errors. Effectively, the coherent errors from random gates perform a random walk on a sphere [156] and create another source of variance in the measured benchmarking signals which increases with greater number of gates.

Figure 7.17b shows the corresponding fidelity. Computational fidelity defined as $\mathcal{F}_C = 2P_g/(P_g + P_e) - 1$ is also calculated and fit to show the impact of leakage on gate error.

Leakage per gate characterizes how much on average an individual gate causes the qubit state to leave the computational subspace. Leakage was fit from the extracted f -state population with

$$p_f(m) = \frac{l}{l+s} (1 - e^{-(l+s)m}) + p_{f0} e^{-(l+s)m} \quad (7.14)$$

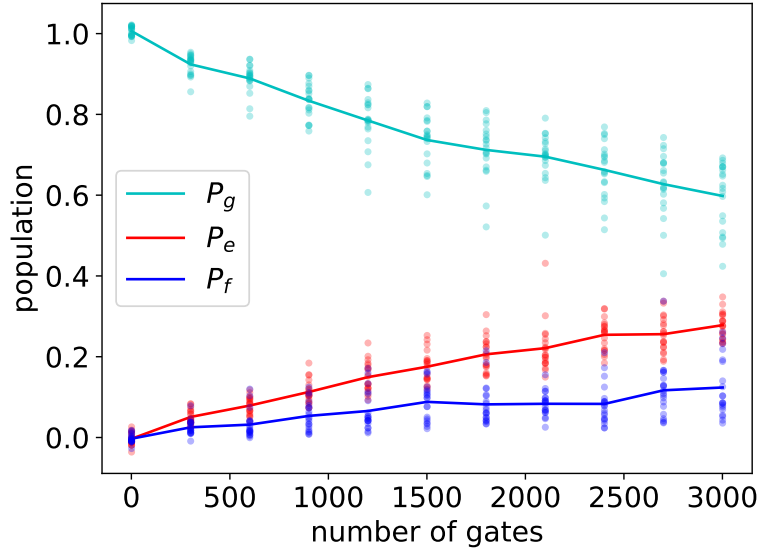
by fitting l , the leakage per gate. Here $s = \Gamma_{fe} t_g$ is the $e \leftrightarrow f$ decay per gate and m is the number of pulses [150].

I should also note that interleaved randomized benchmarking inserts a gate of interest in-between each random gate and allows the extraction of the error of the specific gate [157]. Purity benchmarking performs state tomography, measuring σ_X , σ_Y , and σ_Z after each sequence to measure the length of the state vector in the Bloch sphere and distinguish coherent and incoherent errors [158]. Other benchmarking sequences exist to measure specific properties of the gates. For my experiments, randomized benchmarking and the overall gate error were the most important metrics.

7.6 Conclusion

In this chapter I covered the full qubit characterization process from first measurements of the readout resonator to gate randomized benchmarking. In my experiments, initially spectroscopy is used to determine key parameters about the resonator and qubit. Time domain measurements characterize properties of the qubit and device. I performed precision gate tuning experiments with specific error amplifying sequences to tune qubit gates which were then tested with randomized benchmarking experiments.

(a)



(b)

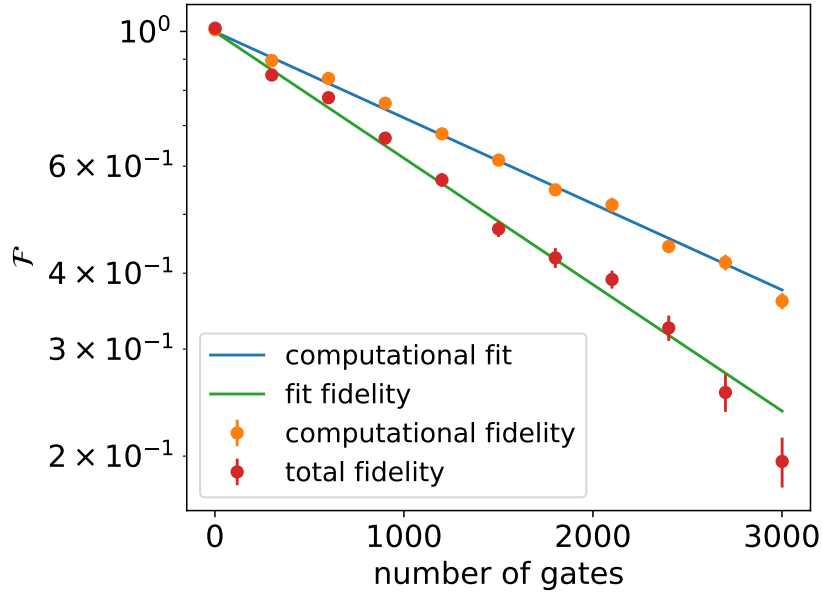


Figure 7.17: Data from a randomized benchmarking experiment of **Q1**. **a)** Populations extracted using the 3 state normalization method vs. the number of random $\pi/2$ gates applied. Data shown is benchmarking of 16 ns raised cosine pulses. 20 random sequences were measured at each gate number. Each measurement was averaged 2000 times. Error in the measured voltages gives a spread in the population measurements, leading to non-physical, negative extracted probabilities in individual measurements. However, sufficiently averaged measurements will have probabilities between 0 and 1. **b)** Corresponding sequence fidelity from Eq. 7.13. Circles show measured fidelities. Error bars correspond to the standard deviation of the estimation of the mean and are smaller than the points. Extracted error per gate is 3.36×10^{-4} .

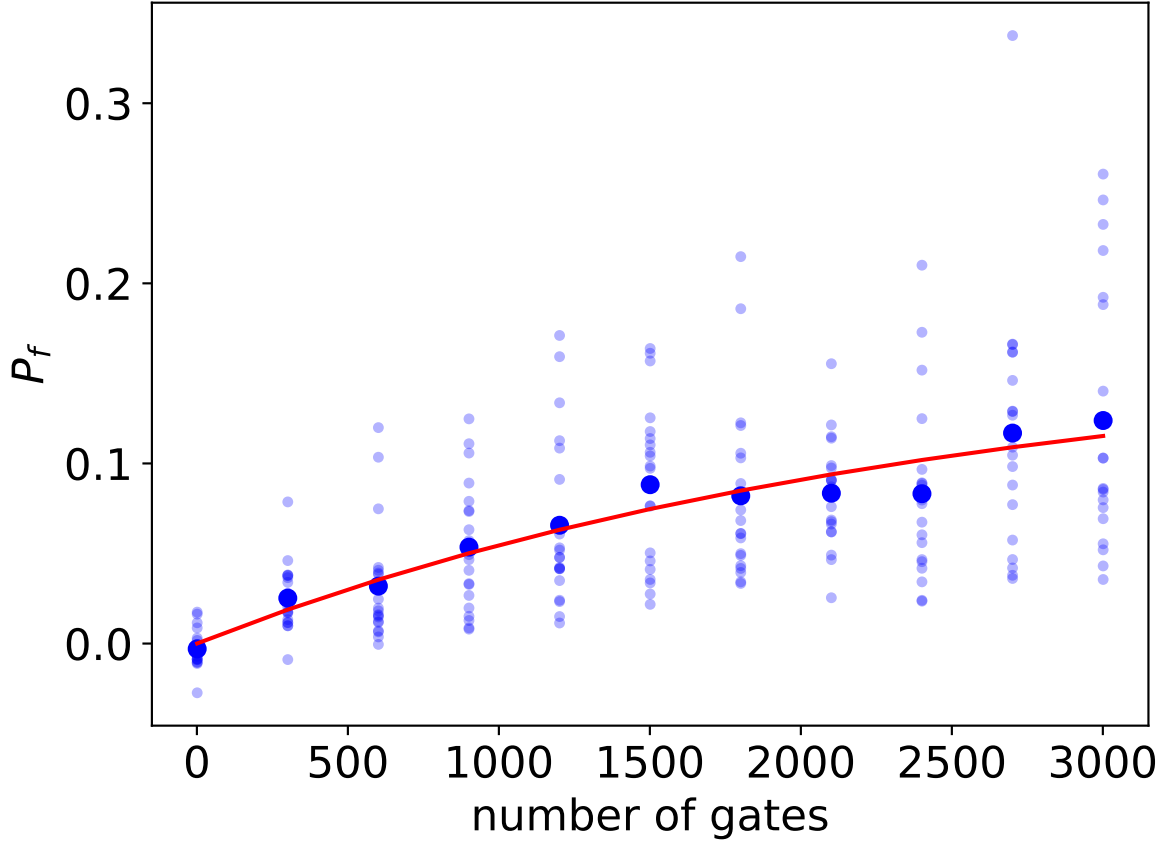


Figure 7.18: Leakage data (blue) and fitting (red) for **Q1** showing f -state probability *vs* random gate sequence length. Data is the same as Fig. 7.17a zoomed in to low population. Red curve shows fit to Eq. 7.14. Extracted leakage per gate is 2.14×10^{-5} .

Chapter 8

Conductance Measurement Setup

DC transport characteristics of junctions can potentially play an important role in explaining qubit relaxation [26, 91, 99]. Being able to accurately measure these characteristics can provide information on junction tunneling behavior and inform material and qubit designs. In this chapter, I explain the basics of low-noise transport measurements and how I performed IV and dI/dV measurements of my junctions. I also provide an example of how I used this setup to characterize NIS junctions and extract an effective electron temperature of those junctions.

8.1 Low Current Measurements

To perform the junction DC measurements, the device was glued using GE varnish to a copper package and then wire-bonded using Al wire to four separate leads on a PCB with either an RC low-pass, an LC low-pass filter, or no filters. Each measurement line then went through a combined π -powder low-pass filter on the mixing chamber plate before going to room temperature using thermal coax cables (Fig. 8.3).

Since these small junctions had small critical currents ($I_C \approx 10$ nA), reducing measurement noise was crucial for obtaining clean junction characteristics. Trying to minimize ground loops and using floating bias sources and ammeters was important for my measure-

ments. Additionally, performing the IV measurements in the copper shielded room surrounding the dilution fridge reduced noise from the environment. Any low frequencies in the experiment were chosen to be far from 60 Hz and its harmonics. In general, the height and sharpness of the main quasiparticle tunneling conductance peak measured in section 8.1.2 was a good heuristic for the noise in the measurement and was used to compare different setups.

8.1.1 IV Measurements

Four wire measurements provide an accurate way to characterize the current-voltage characteristic of the junction by eliminating the effect of lead resistance in the measurement. For my current-biased measurement (Fig. 8.1a), a Yokogawa GS200 voltage source was connected to the I+ wire through a bias resistor $R_b \gg R_J$. The I- wire returned current to a Keysight B2981B picoammeter with a floating ground triax connection. I used a Keysight 34465A digital multimeter to measure the voltage drop across the junction connected via the V+ and V- wires.

I took current biased measurements of Josephson junctions to measure the switching current. One measurement of the switching current of a junction is shown in Fig. 8.2a. Here points with increasing current were taken. A series of points at $V = 0$ show the supercurrent of the junction and a hysteretic region. The current value where the voltage increases sharply is called the switching current, I_{sw} , and is less than the junction's critical current due to fluctuations, noise, and macroscopic quantum tunneling.

Voltage-biased measurements are performed with the same setup as current-biased measurements, but the bias resistor is replaced with a voltage divider circuit (Fig. 8.1b). The voltage divider is made simply of a T structure with a large series resistor R_{ser} and a small shunting resistor R_{sh} connected to the junction. To precisely probe the subgap tunneling of Al-based junctions, smaller voltage steps than the 100 nV minimum step size from the Yokogawa were required. The voltage divider reduces the voltage applied to the junction by

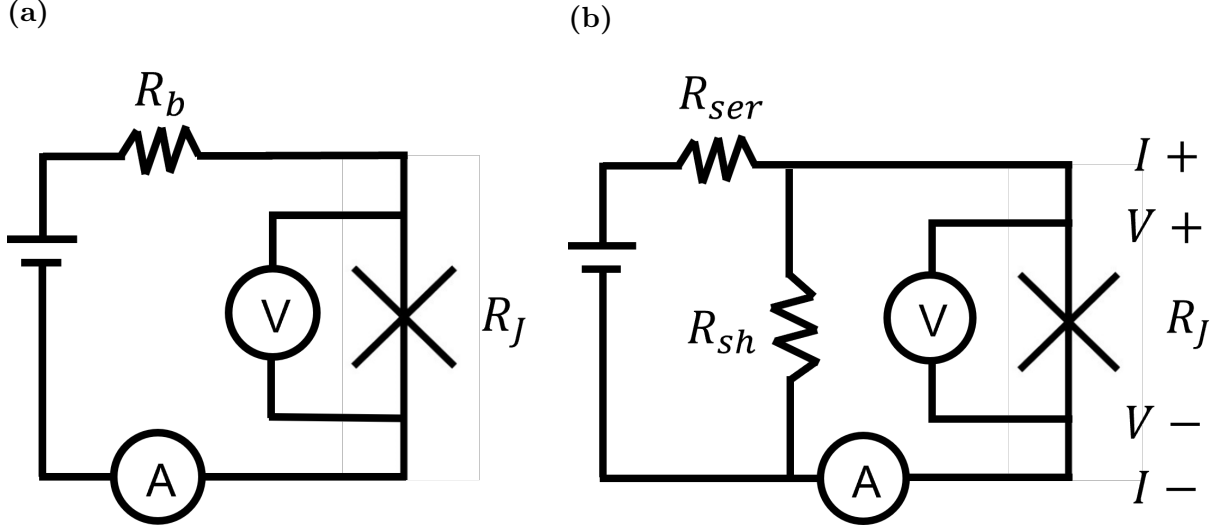


Figure 8.1: Current and voltage-biased IV measurement diagrams. **a)** Junction under a current bias. A voltage source is put through a bias resistor $R_b \gg R_J$. The current through the junction $I = V/R_b$ is measured with an in-circuit ammeter. Voltage across the junction is measured through two separate wires by a voltmeter. **b)** Junction under current bias. A voltage divider circuit made up of R_{ser} and R_{sh} reduce the minimum current step that can be applied to the junction.

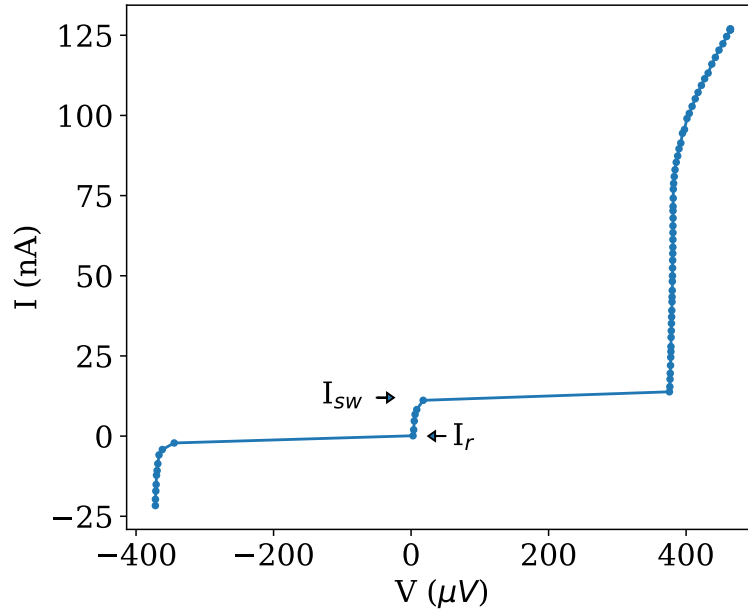
a factor $R_{ser}/R_{sh} = 1000$. This requires $R_{sh} \ll R_J, R_{ser}$.

Voltage biased measurements of the junction allow direct measurement of the subgap current (Fig. 8.2b). Unlike the current-biased measurement, very little of the critical current branch is visible in the voltage-biased measurement. Instead, the measurement can capture many points in the junction subgap with the bias voltage below the sum of the superconducting gaps.

8.1.2 Differential Conductance Measurements

Four-wire differential conductance measurements were performed by applying a combined DC and low frequency 13 Hz AC voltage excitation and measuring the AC current with a lock-in amplifier through the I+/I- leads. The voltage across the V+/V- leads of the junction was amplified with a PARC pre-amplifier before being digitized to measure V as well as input to an additional lock-in amplifier to measure the AC voltage.

(a)



(b)

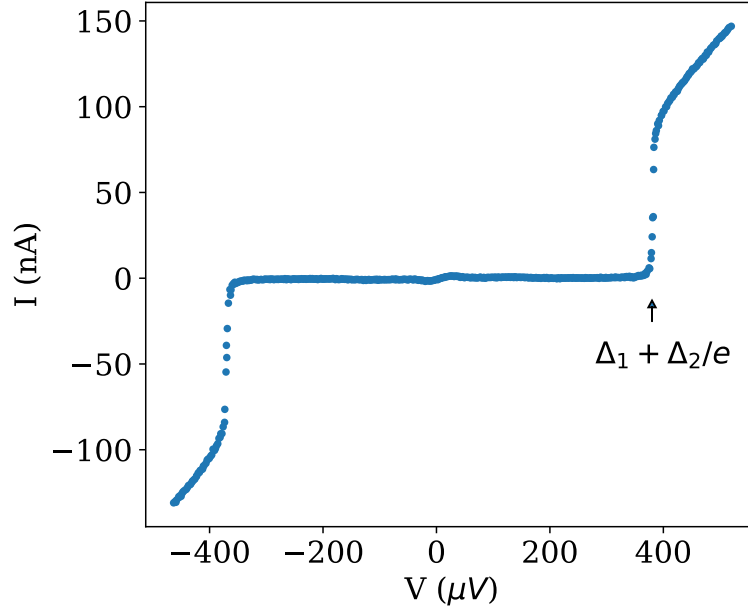


Figure 8.2: Current and voltage-biased IV measurements of **S1**, an Al/AlO_x/Al junction measured at $T \approx 20$ mK. **a)** Current-biased IV curve showing hysteretic junction behavior and zero-bias supercurrent. As the current is increased from the largest negative values toward 0, the junction is in a finite voltage state until a low retrapping current. When increasing the current from 0, the junction switches to the finite voltage state at the switching current $I_{sw} > 0$. **b)** Voltage-biased IV curve showing subgap current behavior for the same device. A small critical current feature is seen at low voltages. The intermediate voltage points, $V < \Delta_1 + \Delta_2$ show low subgap current in the junction.

Fig. 8.4 shows a sample dI/dV vs. V curve of **S1**, an Al/AlO_x/Al junction measured at $T \approx 20$ mK. Large, sharp differential conductance peaks were measured at $|V| = 380 \mu\text{V}$, corresponding to the onset of quasiparticle-quasiparticle tunneling, with conductance values more than four orders of magnitude greater than the minimum resolvable subgap conductance. At the highest bias values, the differential conductance is constant with a value corresponding to the normal state tunnel conductance $1/R_n$. In the subgap region at biases less than $380 \mu\text{V}$, the differential conductance is much lower as quasiparticles do not tunnel via Giaever tunneling (Eq. 2.34). Near zero bias, there are conductance peaks due to supercurrent processes (section 10.2.5). The large resolvable range of differential conductance was made possible by the direct measurement of dV , filter, and minimizing other measurement noise.

8.2 NIS Measurements

As a check of my measurement setup, I performed dI/dV conductance measurements on an Al/AlO_x/Al _{δ} /Ag junction, **N1**. The junction was fabricated in a turbo-pumped thermal evaporator. The Al and Ag were in separate wound tungsten boats for evaporation. After accounting for the evaporation angle difference between the two boats, I was able to fabricate a Dolan bridge style NIS junction (section 5.3.1). First, I deposited 30 nm of Al followed by a 2 mBar oxidation for 10 min. Then I performed a thin 2 nm Al _{δ} deposition to not disrupt the Al/AlO_x/Al barrier structure [129]. Finally 50 nm of Ag is deposited completing the NIS junction. The thin second Al layer does not superconduct if properly covered by the thick normal metal Ag layer.

8.2.1 dI/dV Measurements and Density of States

Once the device is packaged and cooled down in the dilution refrigerator, I performed conductance measurements. I took a single-direction voltage sweep. Then I adjusted the mixing

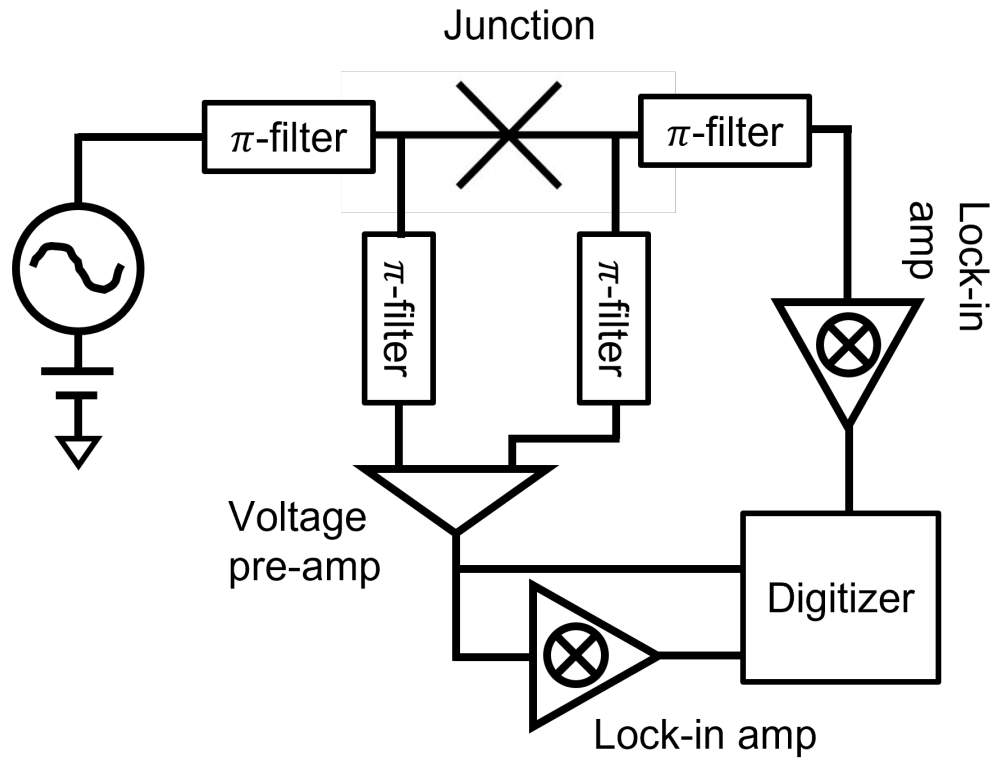


Figure 8.3: Measurement setup diagram for differential conductance measurements. A combined AC and DC voltage source is used to bias the junction. In a filtered 4-wire measurement, the DC voltage is measured with a voltage preamplifier and digitizer. The AC voltage is measured with a lock-in amplifier connected to the voltage amplifier. The AC current is measured with a separate lock-in amplifier. The signals can be used to create a $\delta I / \delta V$ vs. V plot accurately approximating the differential conductance for a range of DC voltage biases.

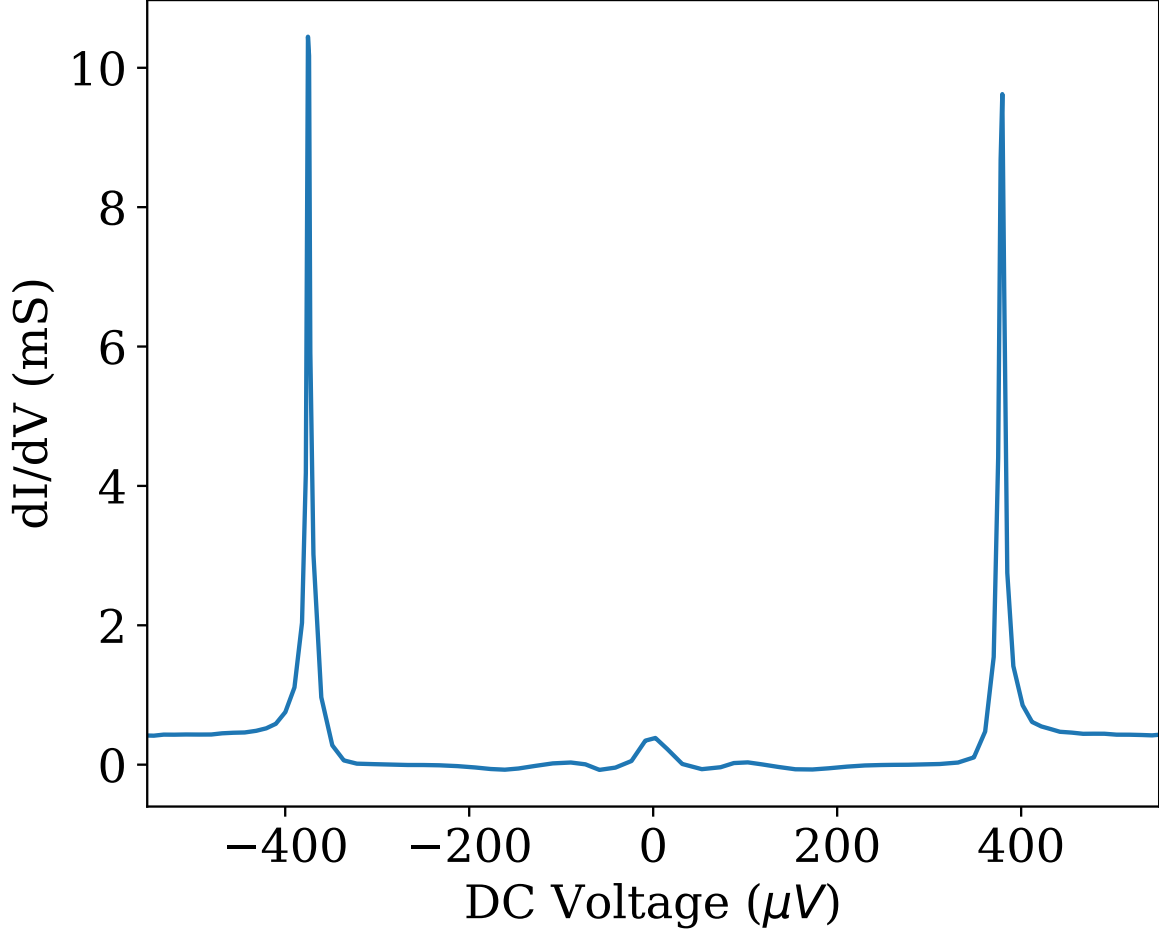


Figure 8.4: Sample differential conductance measurement of a Josephson junction **S1**. Here dI/dV vs. V curve was measured for a SIS tunnel junction. At low biases, sub-gap conductance is low but some critical current conductance features are visible. At $V = (\Delta_1 + \Delta_2)/e \approx 380 \mu\text{V}$, sharp conductance peaks with maximum values more than 4 orders of magnitude above the subgap conductance are measured. At the largest biases, the junction has a constant normal-state tunneling conductance $1/R_n$.

chamber temperature with a mounted resistive heater. After the temperature was stabilized, I performed another conductance measurement. I repeated this process for mixing chamber temperatures between 20 mK and 1.0 K (Fig. 8.5).

The conductance values at the lowest temperatures were near 0 in the subgap with the average differential conductance, $\overline{dI/dV}$, being much smaller than the noise fluctuations in this region. At the value of the superconducting gap, there is a large differential conductance spike due to the diverging density of states (Eq. 2.13). Above this peak, the conductance approaches the normal state tunnel conductance $1/R_n$.

8.2.2 NIS Junctions as a Thermometer

As the temperature increases (positive y-axis offset in Fig. 8.5), the conductance peak widens. This broadening is due to the width of the Fermi-Dirac distribution function of the electrons being proportional to temperature which smears the measured density of states. The data is fit to the derivative of the NIS Giaever tunneling theory, Eq. 2.32. The temperature, T , superconducting gap, Δ , and tunnel resistance, R_n are used as fitting parameters. At the largest temperatures measured, the fitted gap also decreases. Any sharp conductance features are mostly washed out at these temperatures as seen in Fig. 8.5. Therefore it becomes increasingly inaccurate to fit the gap and temperature when $T \approx T_C$.

Here the NIS junction functions as another thermometer on the mixing chamber. From 70 mK to 800 mK, the extracted temperatures from conductance fitting agree well with measurements of the calibrated CMN inductive thermometer which measures the phonon temperature on the mixing chamber plate. Below this range, the electron and phonon temperatures are not in thermal equilibrium and the electrons have a measurably higher temperature [159].

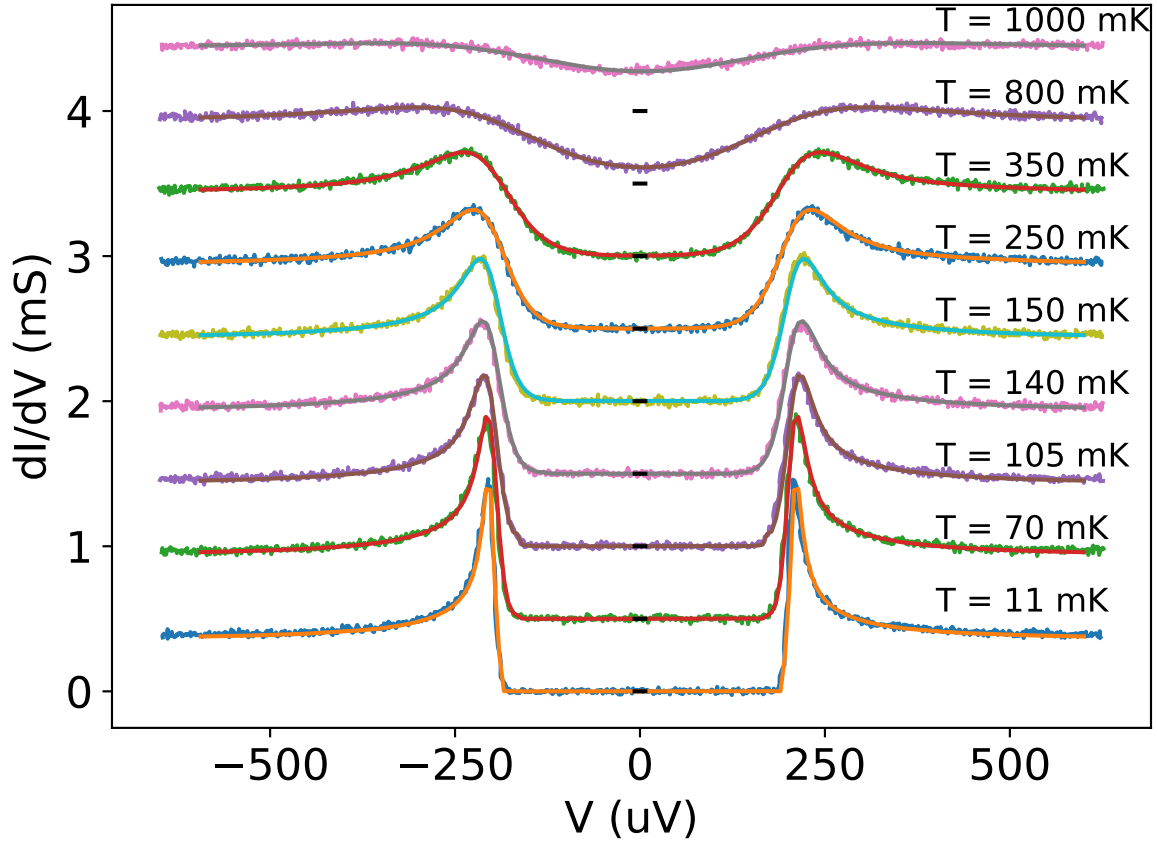
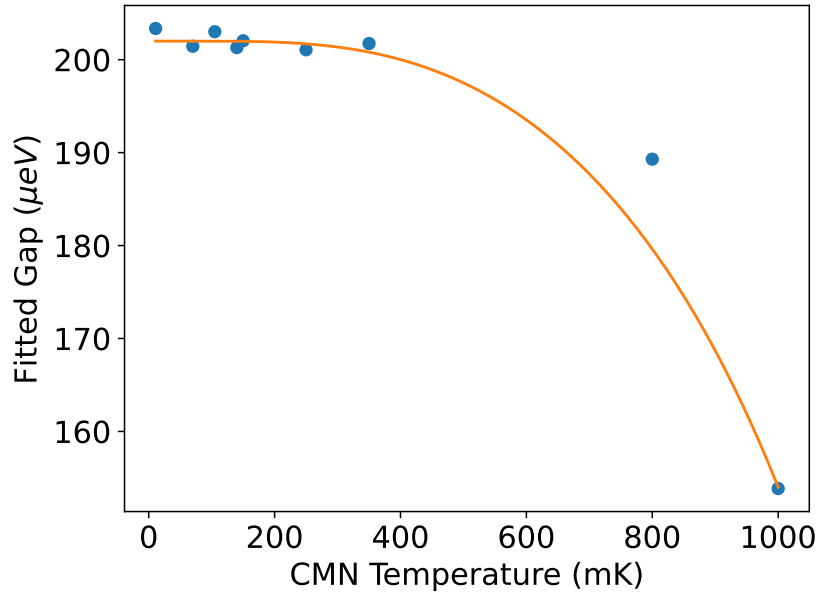


Figure 8.5: Conductance dI/dV vs. voltage bias V measurements of NIS junction **N1** at a range of temperatures. Nine separate conductance scans are shown offset in increasing temperature. Overlaid on each set of data is the fitted Giaever tunneling theory (Eq. 2.32). Zero conductance values are marked for each curve in black. As the temperature increases, the conductance peak widens due to the broadening Fermi-Dirac distribution of the electrons in the normal electrode.

(a)



(b)

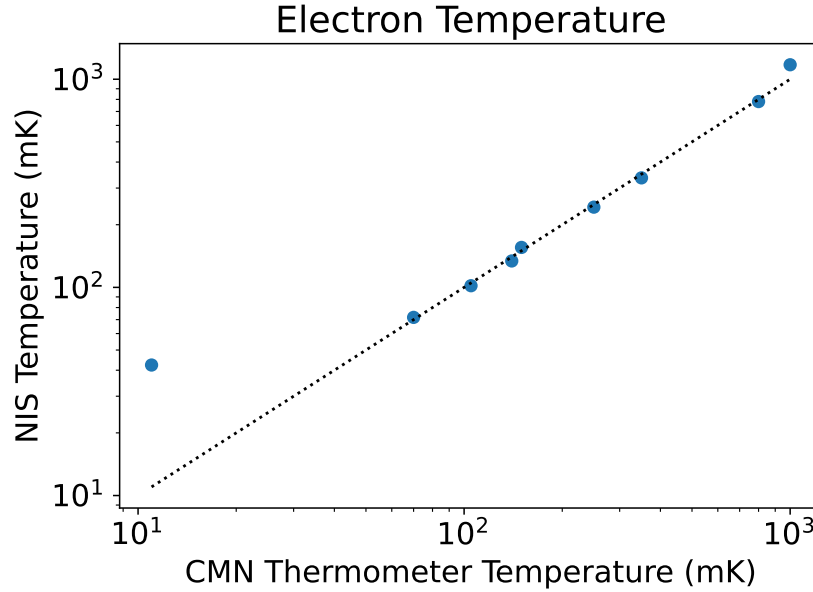


Figure 8.6: Extracted gaps and electron temperatures plotted vs. mixing chamber temperature. **a)** Extracted superconducting gap of the Al electrode obtained from the conductance fits in Fig. 8.5 plotted vs. the measured mixing chamber temperature. Solid curve is the BCS prediction for the temperature dependence of the gap. **b)** Fitted electron temperature vs. mixing chamber temperature measured by the CMN inductive thermometer. Dotted line shows $T_{NIS} = T_{CMN}$. The two temperature measurements have good agreement until below 70 mK where the electrons and phonons are no longer in thermal equilibrium with each other. Lowest fitted electron temperature is 42 mK. Additionally the two measured temperatures do not disagree for the highest temperature point. This may be due to a weak conductance signature near T_C

8.3 Conclusion

In this chapter I described the measurement setup I used to take DC measurements of junction tunnel characteristics. I used a combination of current-biased and voltage-biased IV measurements as well as differential conductance measurements to characterize junctions (sections 8.2, 9.4, 10.2.2). Specifically here I gave an example how precision differential conductance measurements of an NIS junction can function as a thermometer for electron temperature.

Chapter 9

Loss in Al/Ti Devices

9.1 Motivation and Gap-Engineered Transmons

In a transmon qubit, intrinsic losses come predominantly from the coupling of the qubit to two-level systems in lossy dielectrics [101, 102, 105] and non-equilibrium quasiparticle loss channels [91, 94, 95, 99, 160]. Quasiparticles in superconducting qubits can tunnel across the Josephson junction, absorbing a photon, and relaxing the qubit. The decay rate from the qubit's excited state $|e\rangle$ to the ground state $|g\rangle$ as a result of quasiparticles tunneling across the qubit junction can be written as in Eq. 3.69 as

$$\Gamma_{e \rightarrow g} = \frac{E_C}{\hbar f_{ge} e} \left(I_{\rightarrow}(f_{ge}) + I_{\leftarrow}(f_{ge}) \right). \quad (9.1)$$

Here E_C is the charging energy and $(I_{\rightarrow}(f_{ge}) + I_{\leftarrow}(f_{ge}))$ is the sum of the quasiparticle current in both directions across the junction at a bias equivalent to the qubit frequency f_{ge} [92]. $I_{\rightarrow}, I_{\leftarrow}$ are determined by the specifics of the Josephson tunnel barrier and the density of quasiparticles n_{qp} in the electrodes. For a BCS superconductor in thermal equilibrium, n_{qp} decreases exponentially with decreasing temperature, T , [37] resulting in a reduced quasiparticle current and therefore reduced $\Gamma_{e \rightarrow g}$. While theoretically, Al below $T < 50$ mK should have $n_{qp} < 10^{-12} \mu\text{m}^{-3}$; experiments have reported nonequilibrium quasiparticles with

densities of $n_{qp} \sim 0.001 - 10 \mu\text{m}^{-3}$ [26, 66, 99, 112, 161–164] (section 3.6.6).

Recently, T_1 measurements of 3D transmons have shown an increase in T_1 as T was increased from 20 mK to 150 mK [24]. As an alternative explanation to multi-parameter TLS loss models [108, 109], the superconducting gaps $\Delta_{1,2}$ for the two Al layers forming the Al/AlOx/Al junction were modeled to be slightly different, such that for $T < 100$ mK, quasiparticles accumulate into the lower-gap electrode resulting in a decrease in T_1 [24]. This model, which is distinct from quasiparticle trapping models, suggests that increasing the $|\Delta_1 - \Delta_2| > \hbar f_{ge}$ should increase T_1 , even in the presence of nonequilibrium quasiparticles. Recent work using oxygen-doped Al has shown long lived transmons informed by these predictions [115]. Other work has also discussed the improvement in the performance of similar devices by gap-engineering the junction [25, 66, 165] although others have seen no improvement in gap engineered devices [166].

With this motivation, I fabricated and characterized junctions and transmon qubits with asymmetric superconducting energy gaps across the junction. In particular, the superconducting gap of the junction’s larger-volume top electrode was lowered via the proximity effect by evaporating a thin layer of Ti on top of Al. Because the superconducting energy gap of Ti is lower than that of Al, the effective superconducting order parameter of an Al/Ti bilayer should have a value less than that of an Al film.

In section 5.3 I described the fabrication of the Al/Ti bilayer with and without an additional thin disordered AlO_y layer added in between the Al and Ti layers. The theoretical critical temperature depends not only on the order parameters and coherence lengths of the two films but also on the AlO_y interface barrier resistance [45–47, 167].

In this chapter, I apply the Usadel equation from section 2.1.5 to calculate the proximity effect for the films. I then discuss DC characterization of these junctions. Junctions with the Ti capping layer directly on the Al display a reduced energy gap but also a significant amount of tunneling current in the subgap region. I model the IV’s, including some of subgap features, using a MAR theory and extract junction parameters. Next, I discuss

lifetime measurements of 3D transmons with these electrode structures. In particular, the Ti capping layer displays a substantial shorter lifetime of $T_1 = 1 \mu s$ when compared to standard Al/AlO_x/Al junctions with $T_1 = 130 \mu s$ while introduction of AlO_y disorder between the Al and Ti layers can partially restore the qubit T_1 to $32 \mu s$.

9.2 Fabrication Details

For this work, a variety of small area ($\sim 150 \text{ nm} \times 150 \text{ nm}$) low critical-current junctions were fabricated either by the Dolan bridge [122] or Manhattan technique [125]. To fabricate the junctions (Fig. 5.3, section 5.3), a lift-off stencil was created using an electron-beam lithography system in a bilayer electron-beam resist with an undercut allowing metal to be deposited at an angle on the substrate.

After developing the resist, multiple metal depositions and oxidations were performed either in a Plassys MED 550 S electron-beam evaporator or using a tungsten wound basket in a turbo-pumped thermal evaporator. For each junction, 30 nm of Al was first deposited, followed by a static oxidation with a fixed dose of 0.8 - 3 mBar O₂ for 10 minutes to form the insulating tunnel barrier. Following the first oxidation, the chamber was evacuated and a secondary metal counter-electrode was deposited at a different angle. The composition of this top electrode was 50 nm Al for the standard Al/AlO_x/Al junctions, 45 nm Al followed by 5 nm of Ti for the Al/Ti bilayer, or 40 nm Al followed by an additional optional oxidation to add disorder and a deposition of 20 nm Ti for the disordered Al/AlO_y/Ti bilayer. The Al films were deposited at a rate of 0.5 nm/s; the Ti films were deposited at 0.2 nm/s. Next, the extraneous metal on the top of the resist was lifted off in a hot bath of NMP.

Table 9.1 has a summary of the devices that were fabricated in this study. Because the contact pad areas were approximately 10^7 times larger than the junction areas, I argue each of these devices is effectively a small, single Josephson junction with asymmetric electrodes. A fabricated device is shown in Fig. 9.1.

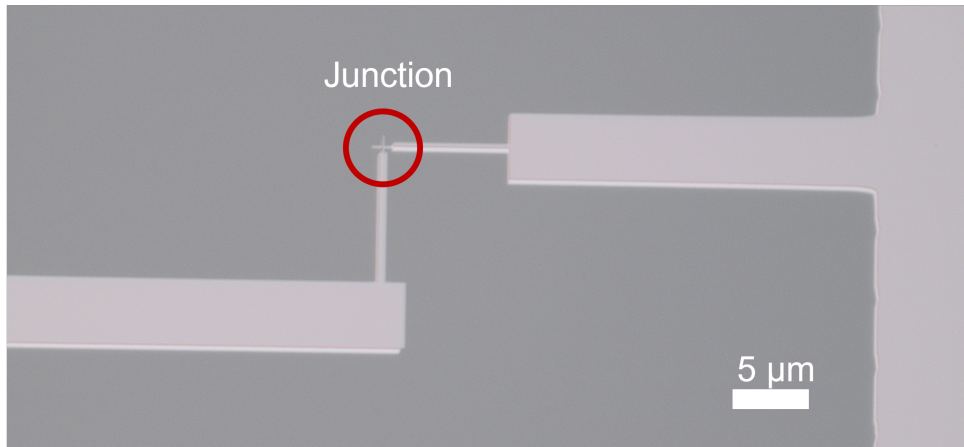
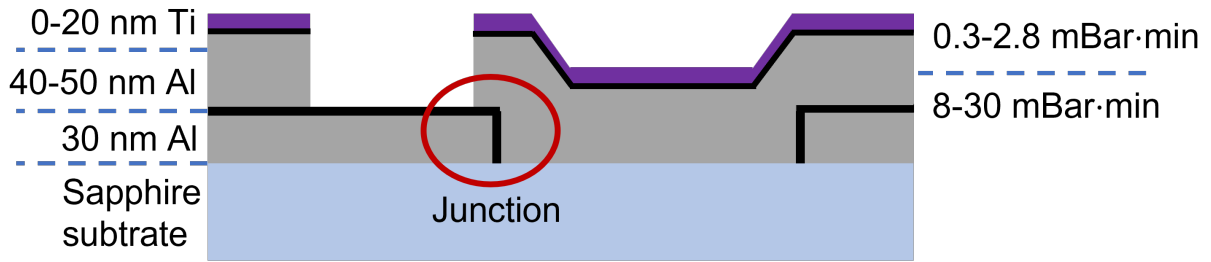


Figure 9.1: Diagram and image of a gap engineered junction device. *Top* is a cross section diagram of the junction. A thin Al layer forms the first electrode and a Al-Ti bilayer forms the second electrode. The junction is formed across the oxide tunnel barrier where the two electrodes meet. *Bottom* is an optical micrograph of a fabricated Al-Ti junction. Darkest color is sapphire substrate while lighter color is metal. The Al metal has an even lighter silver color and can just be seen on the bottom edge of the pattern due to the Manhattan depositions. The junction (circled in red) is the smallest feature where the two electrode are connected.

Table 9.1: Extracted junction parameters. Comparison of measured superconducting gap for the top counter-electrode Δ_2 and normal state resistances for different junction compositions. The counter-electrode gap Δ_2 is extracted from the quasiparticle-quasiparticle tunneling peak assuming the bottom electrode $\Delta_1 = 190 \mu\text{eV}$. Adding Ti to the counter-electrode reduces Δ_2 while adding an AlO_y barrier greater than 0.3 mbar·min oxygen exposure between the Al and Ti layers restores Δ_2 .

Device Name	Junction Composition	O_2 disorder (mbar·min)	Δ_2 (μeV)	R_n ($k\Omega$)
J0	Al/ AlO_x /Al	-	200	18.6
J1	Al/ AlO_x /Al	-	200	8.3
J2	Al/ AlO_x /Al/Ti	-	100	6.3
J3	Al/ AlO_x /Al/Ti	-	140	7.0
J4	Al/ AlO_x /Al/Ti	-	110	8.5
J5	Al/ AlO_x /Al/ AlO_y /Ti	0.3	120	2.6
J6	Al/ AlO_x /Al/ AlO_y /Ti	0.6	230	8.8
J7	Al/ AlO_x /Al/ AlO_y /Ti	0.6	220	8.5
J8	Al/ AlO_x /Al/ AlO_y /Ti	1.8	220	9.0
J9	Al/ AlO_x /Al/ AlO_y /Ti	2.8	220	8.5

9.3 AlTi Bilayers and the Proximity Effect

To verify the gap engineering via superconducting proximity effect was working, I fabricated different test bilayers. For these bilayers, a Ti layer was deposited followed by an Al layer on top. This order was different from the junctions and may have reduced Ti oxides in these T_C test devices.

The data from the four devices is shown in Fig. 9.2. I performed a four-wire resistance measurement to extract T_C of each film then calculated the expected gap using Eq. 2.10. These experimentally measured gap values are compared with theory calculations using Eq. 2.11. While the Usadel theory does not closely match with the experiment, the gap does change with thickness ratio meaning gap engineering with the proximity effect is possible. The film precise thickness and purity and quality of the layer interface are important in determining T_C [43, 66, 115]. Additionally, AlTi alloys with different conducting properties may form between these two metals and may explain the discrepancy between theory and

experiment [168].

9.4 Junction Measurements

To perform low-temperature measurements, the devices were mounted and cooled on a cryogen-free Leiden dilution refrigerator with a base temperature below 20 mK. To perform the junction DC measurements, the device was glued using GE varnish to a copper package and then wire-bonded using Al wire to four separate leads on a PCB with either a RC low-pass, LC low-pass filter, or no filters. Each measurement line then went through a combined π -powder low-pass filter on the mixing chamber plate before going to room temperature using thermal coax cables. Four-wire differential conductance measurements were performed by applying a combined DC and low frequency 13 Hz AC voltage excitation and measuring the AC current with a lock-in amplifier. Using the additional two wires across the junction, the voltage was amplified with a PARC pre-amplifier before being digitized or measured with an additional lock-in amplifier (section 8.1.2). After the data were acquired, the dI/dV measurements were numerically integrated resulting in the IV curves presented below.

To quantify the superconducting gap electrodes and characterize the quality of the junction, I performed DC measurements. Figure 9.3 shows integrated IV curves for three representative junctions: **J1** with no Ti (blue, dotted), **J3** with an Al/Ti bilayer counterelectrode (orange, solid), and **J6** with an added AlO_y disorder layer between the Al and Ti layers (green, dot-dashed). These junctions were nominally identical to the measured transmon junctions.

For the Al/ AlO_x /Al junctions, the current sharply increases at $V = 380 \mu\text{V}$ by approximately 25 nA. This is associated with quasiparticle-quasiparticle tunneling at a voltage bias $V = (\Delta_1 + \Delta_2)/e$ corresponding to the sum of the two superconducting gaps. Above this bias, the junction shows ohmic tunneling.

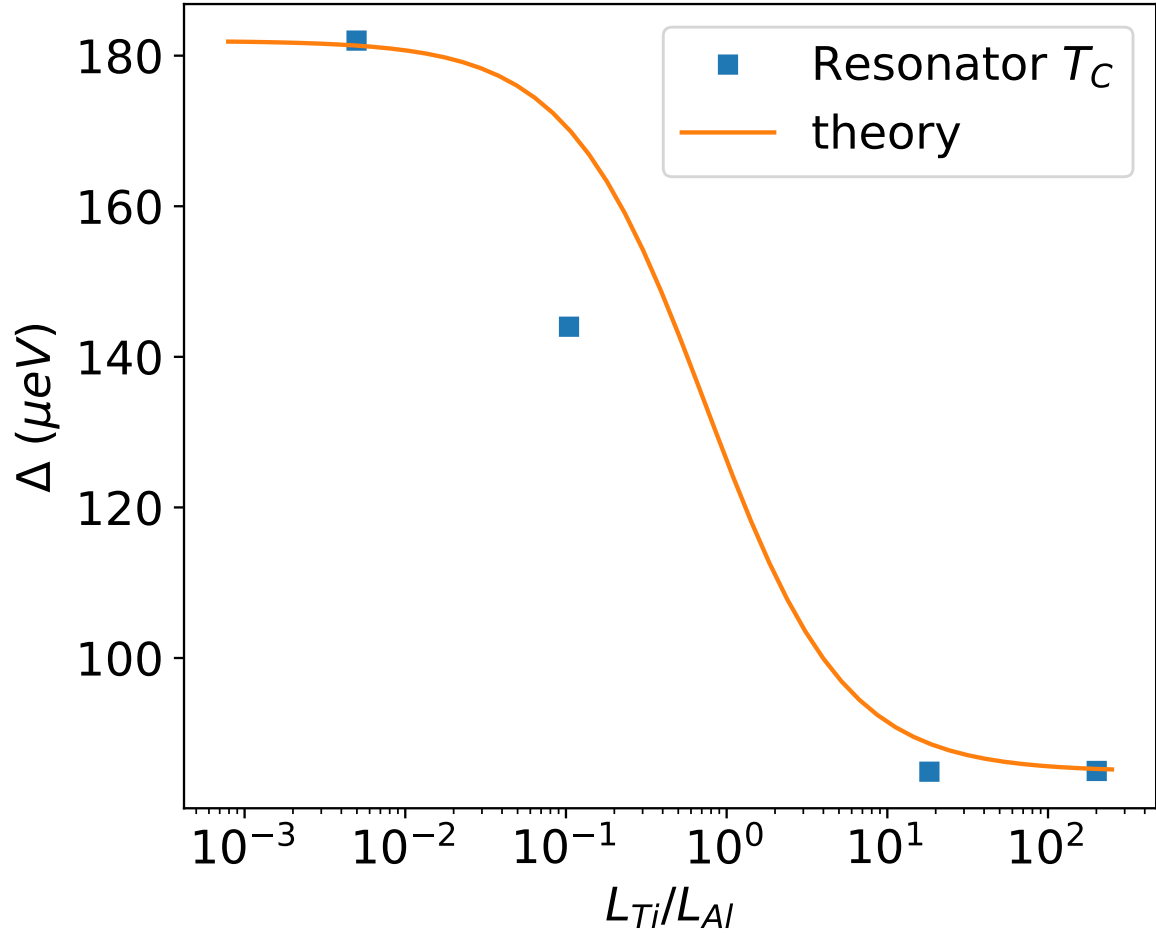


Figure 9.2: Proximity effect on AlTi bilayers. The superconducting gap extracted from critical temperature measurements versus the relative thickness of the Ti and Al metal layers. Four test structures were co-deposited with resonators in section 9.6. Theory fit shown in orange corresponds to Usadel calculations from section 2.1.5. Mismatch between theory and measured points may be due to Al and Ti alloys formed at the interface affecting the properties of the bilayers.

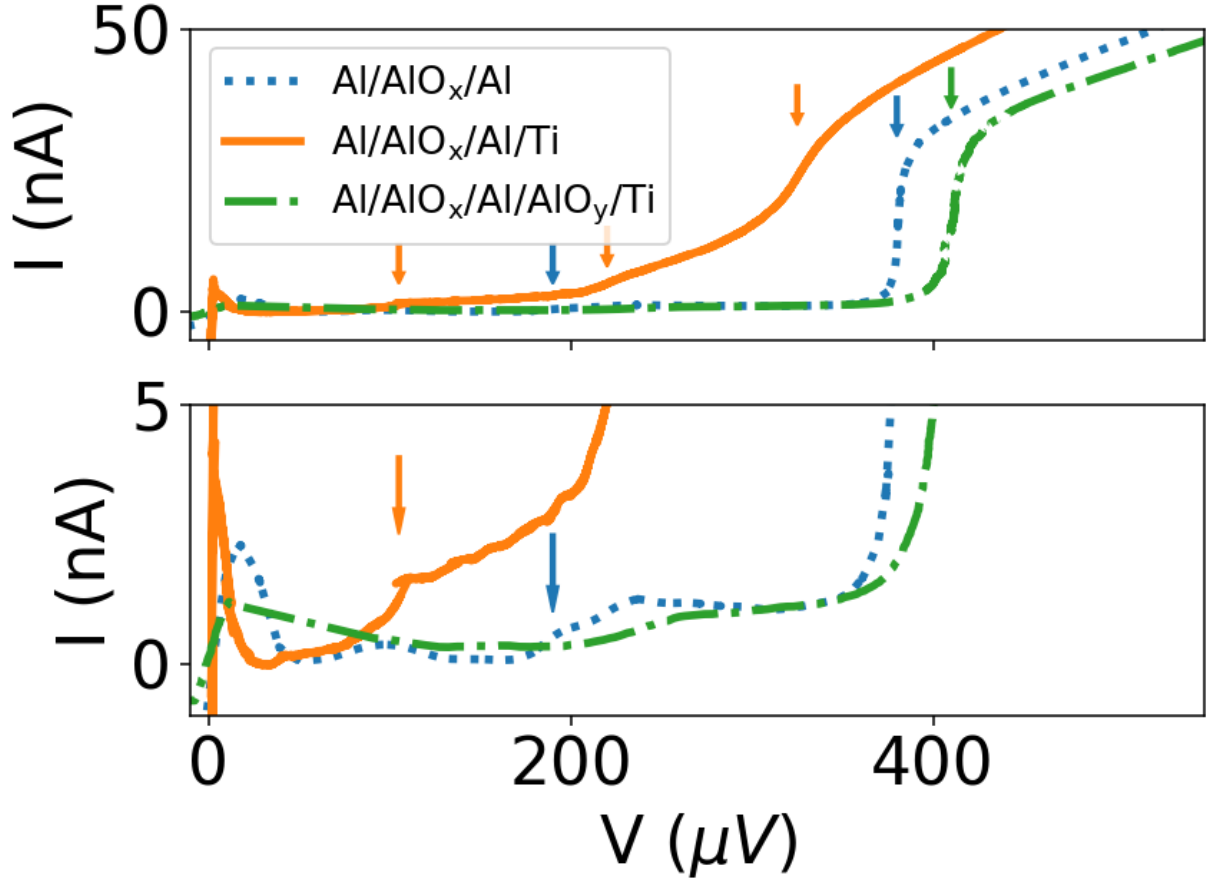


Figure 9.3: IV traces of three Josephson junctions, **J1**, **J3**, and **J6** at 20 mK with different counter-electrode structures (bottom plot is the same data as the top plot but with reduced current scale). The dotted blue trace is a standard $\text{Al}/\text{AlO}_x/\text{Al}$ transmon junction. The solid orange trace is data for an $\text{Al}/\text{AlO}_x/\text{Al}/\text{Ti}$ junction (note reduction in V to observe quasiparticle-quasiparticle tunneling due to the Al/Ti bilayer counter-electrode). The dot-dashed green curve is a measurement of a $\text{Al}/\text{AlO}_x/\text{Al}/\text{AlO}_y/\text{Ti}$ junction with an additional AlO_y disorder layer between the Al and Ti bilayer. Arrows mark visible steps at expected Δ_1 , Δ_2 , or $\Delta_1 + \Delta_2$ values.

In the subgap region below $V = (\Delta_1 + \Delta_2)/e$, there are a number of small subgap tunneling features revealing deviations from BCS tunneling characteristics for a low transparency barrier. At the lowest voltage biases, the measured current slowly decreases. Dissipation from Josephson oscillations in a lossy radio-frequency (RF) environment can give rise to subgap features when the junction sees finite, real impedance at the Josephson frequency. This effect is likely the cause of the anomalous tunneling observed at small biases corresponding to Josephson oscillations from 2 - 20 GHz and could be reduced by measuring a biased SQUID (section 2.4.3) or otherwise suppressing the junction critical current [169]. In the following chapter, I present tunneling measurements of a junction with a SQUID configuration allowing the coherent critical current to be modulated and distinguished from quasiparticle tunneling features (section 10.2.5).

Subgap current steps at individual gap values can also appear in junctions with finite transparency due to MAR [34, 51]. MAR theory expands quasiparticle tunneling to junctions with non-zero electron transmission amplitude, t (section 2.3.3). In this case, a quasiparticle transmitted through the tunnel barrier can Andreev reflect off the superconducting interface repeatedly between the two superconducting electrodes. In tunneling characteristics, these multiple reflections give rise to steps in the measured current at biases equal to integer fractions of the individual electrode gaps, $\Delta_{1,2}/n$ as well as certain sums and differences of these gap fractions [34]. The height of these steps is proportional to the junction transparency $D = |t|^2$ raised to the power of the number of reflections n . The MAR theory can be used to model the IV data and extract D and $\Delta_{1,2}$ by matching the features in the theory to the measured IV's. Small gap asymmetry is expected due to differences in film thickness [170, 171], but self-heating, noise, and the nature of some of these novel junctions limit the resolution of this technique. At $V \approx 190 \mu\text{V}$, the curve in Fig. 9.3 displays an approximate 1 nA step in the current, which I associate with the superconducting gap of the Al electrodes.

Changing the counter-electrode to an Al/Ti bilayer modifies many aspects of the IV characteristics (see the solid curve in Fig. 9.3). The onset of quasiparticle-quasiparticle

tunneling occurs at a reduced voltage bias such that the sum of the superconducting gaps $\Delta_1 + \Delta_2 = 310 \mu\text{eV}$. Because the fabrication of the first layer for all of my devices was nominally identical, the reduction in the sum of the gaps is attributed to reduction of the gap in the top electrode (see Table 9.1), implying that the addition of a thin Ti layer reduces the gap of the structure by nearly $100 \mu\text{eV}$. Steps at $220 \mu\text{eV}$ and $105 \mu\text{eV}$ (see arrows in Fig. 9.3) match these proposed gap values. Another striking feature of the IV data for the Al/Ti bilayer junctions is the gradual increase in the current by about 15 nA in the subgap region between $100 \mu\text{eV}$ and $300 \mu\text{eV}$, which is significantly more than the other junctions.

From the MAR theory, the increased current suggests the junction barrier has higher transparency which could lead to greater qubit relaxation rates. Both Al and O can diffuse with Ti layers [172], potentially introducing significant changes to the top electrode or the junction barrier [173].

Attempting to reduce this subgap current, new devices were fabricated with an additional thin AlO_y disordered layer between the Al and Ti layers. The oxygen doses for this process were less than the dose to form the junction tunnel barrier. For the lowest dose of $0.3 \text{ mBar} \cdot \text{min}$, the IV characteristics resemble Al/ AlO_x /Al/Ti junctions with no added disorder. However, when the oxygen dose is increased to $0.6 \text{ mBar} \cdot \text{min}$, the sum of the gaps and low junction transparency of the initial Al/ AlO_x /Al junctions are recovered (see green curve of Fig. 9.3). Further IV characterization of junctions with disorder doses between $0.3 \text{ mBar} \cdot \text{min}$ and $0.6 \text{ mBar} \cdot \text{min}$ is still required to determine if this bilayer technique can be used to fabricate junctions with both a low transparency and the large desired gap asymmetry ($|\Delta_1 - \Delta_2| > 20 \mu\text{eV}$). Table 9.1 shows the extracted top electrode gaps for all of the measured junctions.

9.5 Transmon Measurements

To perform qubit energy relaxation measurements, I fabricated a transmon qubit on a sapphire substrate and mounted in a 3D rectangular cavity machined out of an aluminum-6063 alloy (section 4.1). The bare resonant frequency of the cavity was approximately at 7.8 GHz and a loaded quality factor, limited by the output coupler, was $Q_L \simeq 12,000$. The cavity was mounted in two Amumetal 4K magnetic shields on the mixing chamber plate of the CF-1400 Leiden dilution refrigerator (section 6.2). To measure T_1 , a π -pulse was used to initially excite the qubit from the ground state, followed by a variable delay before measuring the qubit state through dispersive cavity readout [73]. Measurements were averaged 500-5000 times (section 7.4.3). The total acquisition time of a single decay curve varied between one and ten minutes. $\overline{T}_1$ values are averaged from 10's of individual curves at base temperature.

I fabricated three separate 3D transmons to investigate how these different junctions perform as qubits. The fabricated devices consisted of **Q3**, an Al/AlO_x/Al transmon with no Ti layer, **Q4**, an Al/AlO_x/Al/Ti transmon with a nominally clean interface between the Al and Ti, and **Q5** an Al/AlO_x/Al/AlO_y/Ti transmon with an oxygen dose of 0.6 mbar · min between the Al and Ti depositions. The Al/AlO_x/Al/Ti device **Q4** was fabricated using the Dolan bridge process and had frequency $f_{ge} = 5.1$ GHz while the Al/AlO_x/Al transmon **Q3** and the Al/AlO_x/Al/AlO_y/Ti transmon **Q5** were fabricated using the Manhattan process and had $f_{ge} = 2.7$ GHz and 4.0 GHz respectively.

The standard Al/AlO_x/Al transmon with no Ti, **Q3**, performed the best with a mean energy relaxation time $\overline{T}_1 = 134 \mu\text{s}$ at base temperature. When the temperature of the fridge was increased to $T = 140$ mK, the relaxation time increased to $\overline{T}_1 = 210 \mu\text{s}$ before decreasing at higher temperatures (see Fig. 9.4). This temperature dependence of T_1 is consistent with previous results [24].

The Al/AlO_x/Al/Ti transmon, **Q4**, with no additional AlO_y barrier had the worst lifetime with $\overline{T}_1 = 1 \mu\text{s}$. This device's coherence was significantly degraded from the standard Al/AlO_x/Al devices and the DC IV's showed a large amount of subgap current (Fig. 9.4,

Fig. 9.3). From these observations, the junction barrier may be compromised by this specific fabrication procedure potentially due to the ability of Al, Ti, and O to diffuse between layers changing material properties of the electrode [172]. To attempt to remove this issue, the next device implemented an AlO_y barrier in between the Al and Ti bilayer.

The $\text{Al}/\text{AlO}_x/\text{Al}/\text{AlO}_y/\text{Ti}$ transmon, **Q5**, with the additional barrier had a relaxation time with $\bar{T}_1 = 32 \mu\text{s}$. The addition of the disordered layer not only reduced the current in the sub-gap region of the IV's (Fig. 9.3), but also increased the T_1 when compared to the Al/Ti bilayer with no disorder. T_1 was still degraded compared to the $\text{Al}/\text{AlO}_x/\text{Al}$ transmon, but further measurements are necessary to determine device-to-device variability of the disorder bilayered transmons.

9.6 Resonator Measurements

Finally, to test the loss of the Ti film, I fabricated a device with quarter-wave coplanar waveguide (CPW) resonators, **R2**, made from a 200 nm thick Ti film using a liftoff process (Fig. 9.5). The low-temperature single photon internal quality factors of these resonators were measured to be $Q_i \simeq 3000$ resulting in an internal photon lifetime of $\kappa^{-1} = 100 \text{ ns}$, a value even smaller than the $\text{Al}/\text{AlO}_x/\text{Al}/\text{Ti}$ transmon qubits (Fig. 9.6).

Unlike Al, Ti does not form a self-limiting oxide [172, 174]. This could result in a greater amount of dielectric loss or surface resistance loss and thus a greater rate of energy decay as observed in the devices with Ti presented here.

To test the effect of Ti surface oxides, I fabricated similar resonators, **R3**, from a bilayer of 200 nm of Ti capped by 10 nm of Al. The single photon internal quality factors of these capped resonators reached $Q_i \simeq 50k$ (Fig. 9.6). By reducing the Ti exposure to atmosphere, the superconducting loss improved.

For comparison, I fabricated resonators from 200 nm of Al, **R1**. The measured high power internal quality factors of these Al resonators was above 3×10^6 (Fig. 9.6). Using

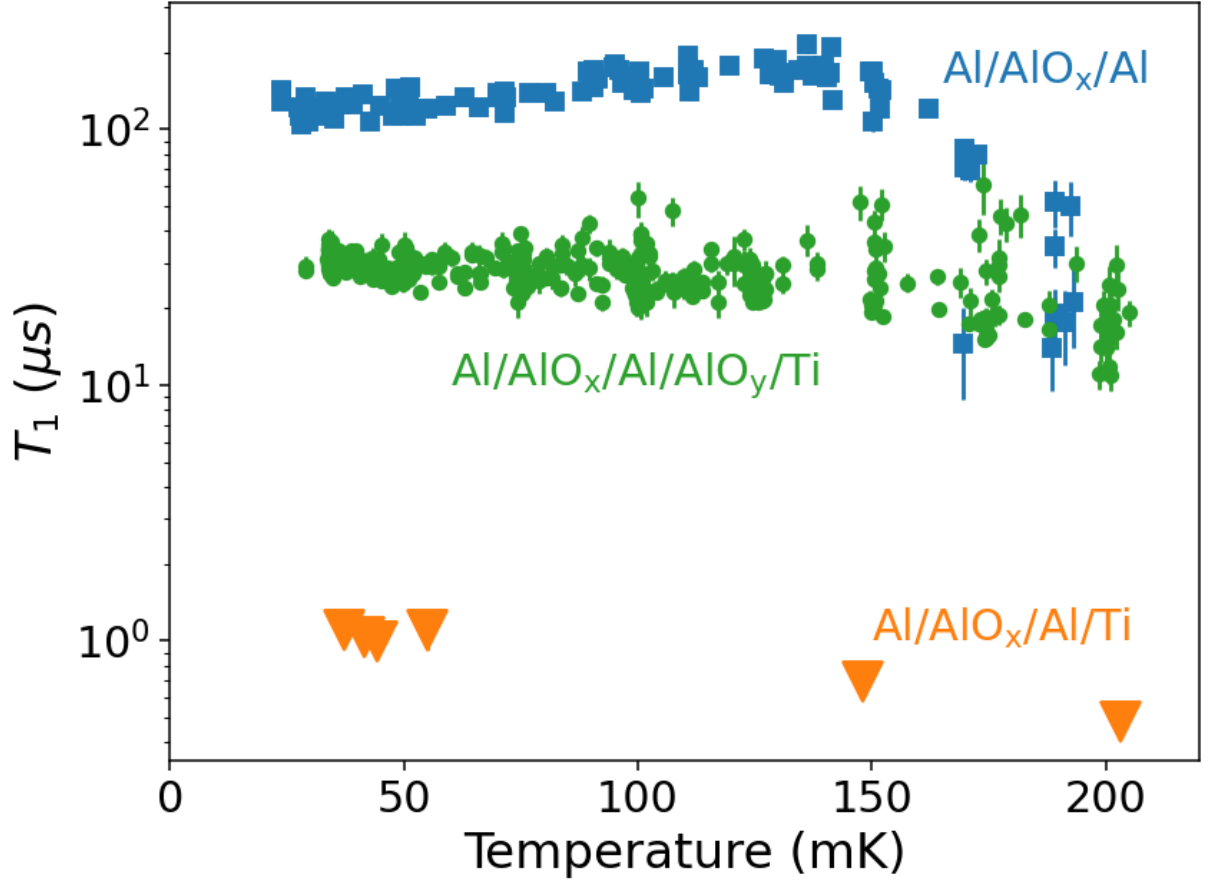


Figure 9.4: Repeated T_1 measurements of three different 3D transmon qubits, **Q3,4,5** versus temperature, T . The blue squares show data of **Q3**, a standard $\text{Al}/\text{AlO}_x/\text{Al}$ qubit. The orange triangle data is taken on **Q4** with a bilayer of Al/Ti as the counter-electrode. The green circle data is from **Q5** which had an additional thin disordered AlO_y (0.6 mbar $\cdot$ min O_2 dose) in between the Al and Ti counter-electrode. The fluctuations for each qubit are approximately 10% of the respective average T_1 time.

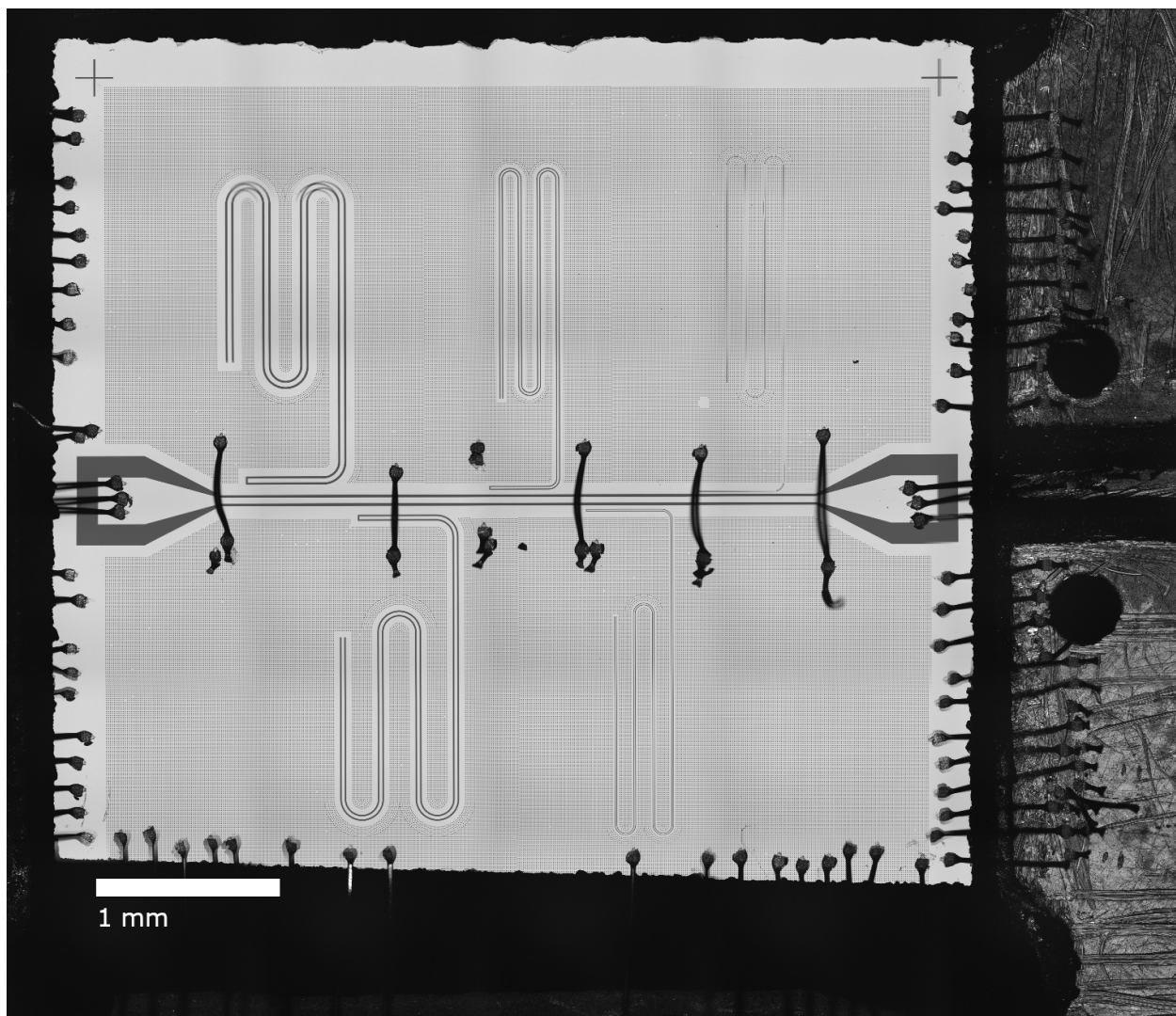


Figure 9.5: Laser micrograph of Ti resonators, **R2**. Five resonators with different widths and couplings fabricated on a sapphire chip in 200 nm of ebeam evaporated Ti metal. Quality factor measurements of R2_3 (center, top) are shown in Fig. 9.6.

an identical design and fabrication process, high quality resonators are possible simply by removing Ti metal from the device.

9.7 TEM Data

One of the Al/AlO_x/Al/AlO_y/Ti samples I fabricated was imaged by Edwin Supple at the Colorado School of Mines using high-angular annular dark-field (HAADF) scanning transmission electron microscopy (STEM). This measurement involved careful preparation of a thin cross-section slice of the junction metal. Then the electron properties of the sample can be investigated with precise spacial resolution. Fig. 9.7a shows the HAADF STEM map of the cross section. Color is added to locate elements identified by electron scattering. A line-cut of the map is taken and the relative intensities of the 3 present elements are shown in Fig. 9.7b. This data informed and confirmed the understanding of the material composition of the junction. A spike in oxygen is visible forming a tunnel barrier between the two aluminum layers. On the right, the Al has diffused into the Ti metal. Additionally, a very large amount of oxide has formed on the top surface (largest distance) of the Ti layer. Both the intermixing of Al and Ti and the large amount of TiO_x are likely sources of material loss in these devices.

9.8 Conclusions

By depositing a thin Ti film on top of Al counter-electrode, the Al is proximitized and the superconducting gap is reduced from approximately $\Delta = 200 \mu\text{eV}$ to $100 \mu\text{V}$ for the Al/Ti bilayer. This bilayer was used to create low critical current asymmetric superconducting gap Josephson junctions. DC differential conductance measurements at 20 mK were performed to compare these junctions to standard Al/AlO_x/Al junctions as well as devices with additional AlO_y disorder placed in between the Al and Ti layer. 3D transmon qubits with added Ti had reduced energy relaxation times with the Al/Ti device having the smallest $T_1 = 1.0 \mu\text{s}$. The

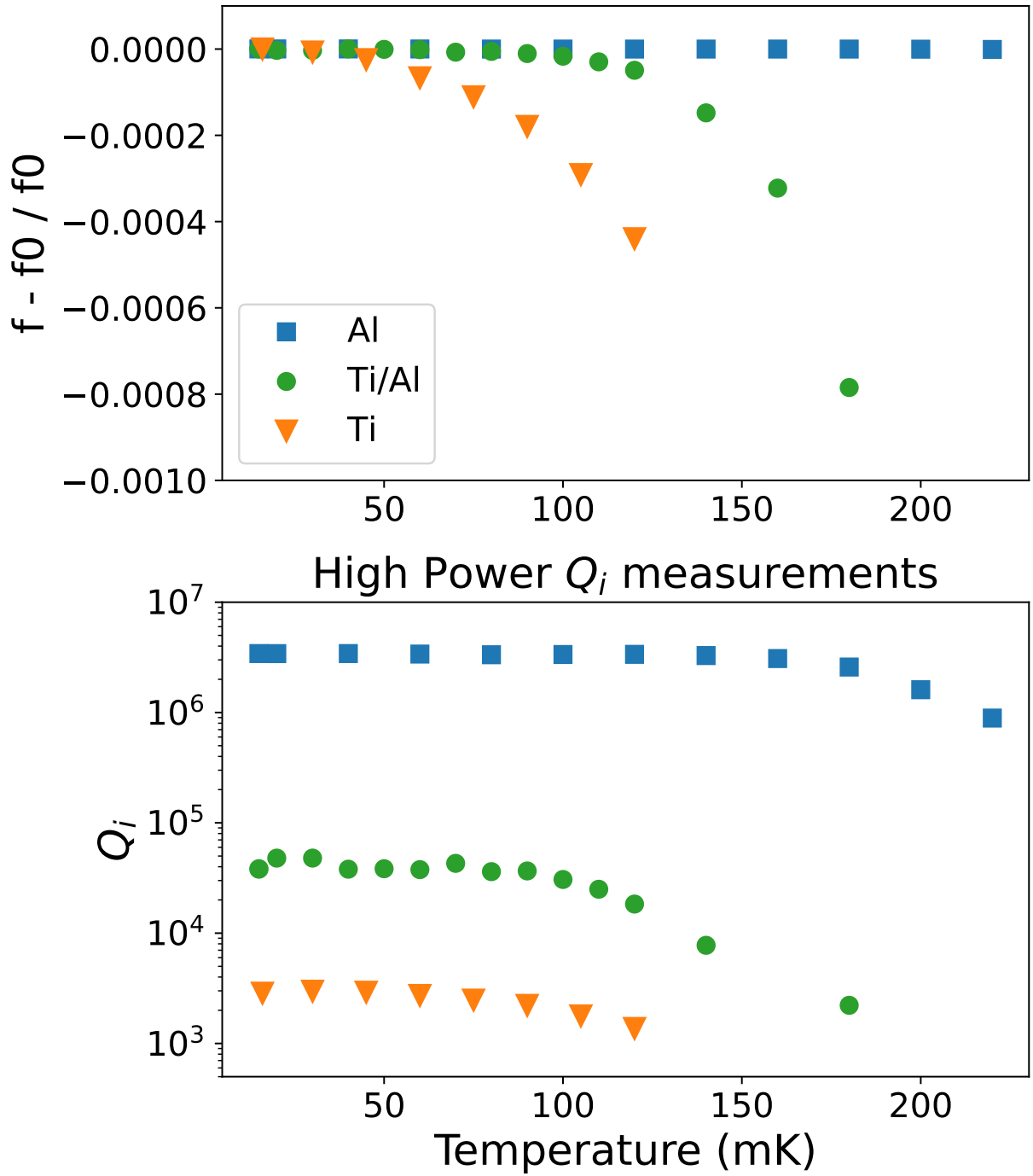
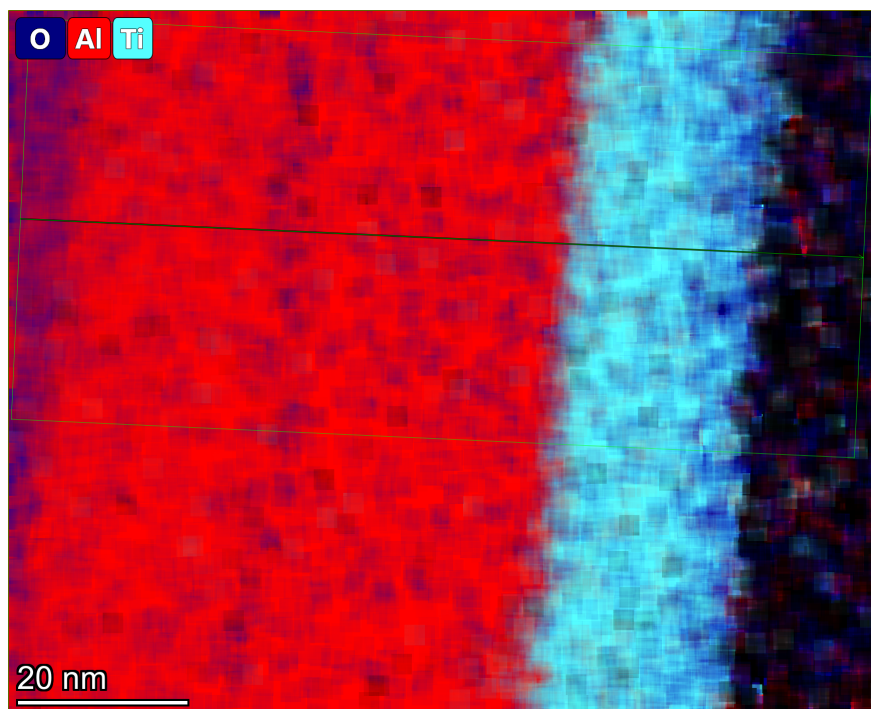


Figure 9.6: Comparison between Al, Ti, and capped Ti resonators (**R1,2,3**). Resonator fractional frequency shift and internal quality factors were extracted for three devices vs mixing chamber temperature. The Q factors for the Al resonators were the highest measured. Capped Ti resonators had Q 's measured more than an order of magnitude lower, suggesting the Ti metal has significantly worse microwave loss than Al. Finally, the uncapped Ti resonators had by far the worst performance suggesting that oxides of Ti also provide a significant loss channel in RF devices. The temperature value where the data diverges from the zero-temperature values shows the resonators have large differences in superconducting gaps.

(a)



(b)

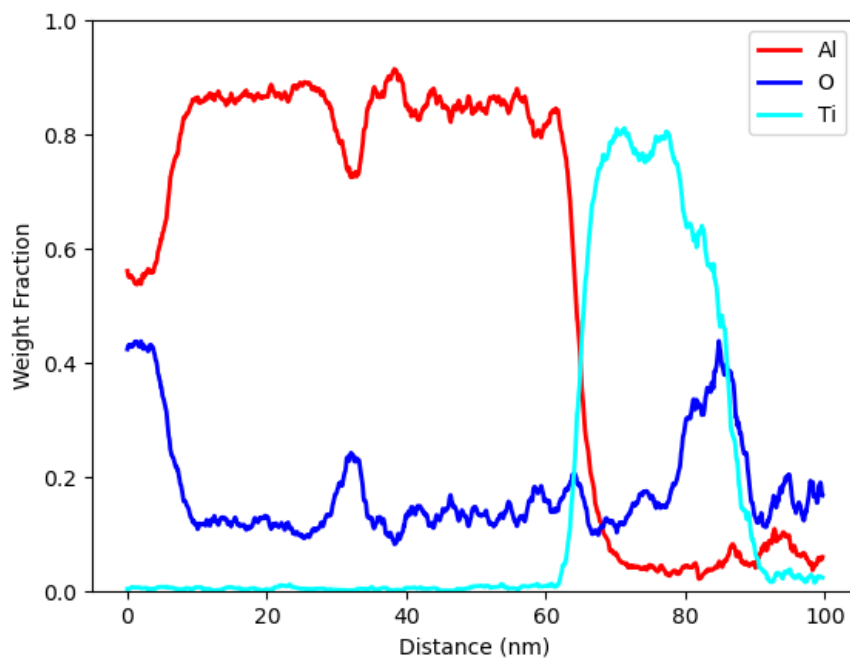


Figure 9.7: STEM data of an Al/AlO_x/Al/AlO_y/Ti junction. **a)** an elemental map of the junction cross-section. A side view of the two Al layers separated by an O tunnel barrier and capped by a Ti layer is visible. **b)** A line-cut of element intensities from (a). The tunnel barrier appears as a spike in O content at a distance of 30 nm. The scan suggests diffusion of Al into Ti and a thick TiO_x layer at the top of the junction.

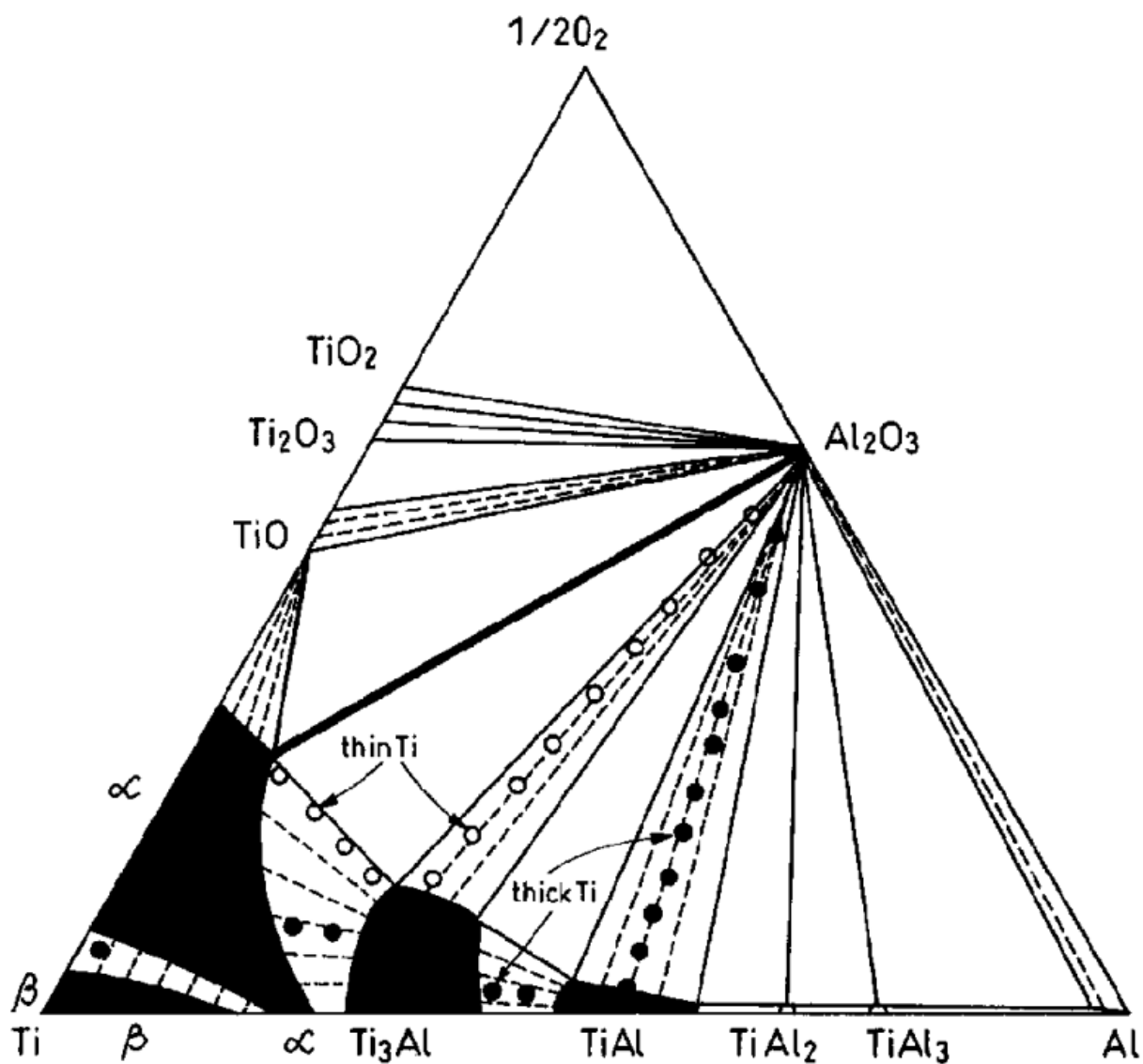


Figure 9.8: Experimentally determined isothermal AlTiO phase diagram at 1100 °C. A huge variety of alloys and separate phases are present. Mixing Al and Ti in a low-loss device may not be desirable or possible. Reprinted from [168] with license from Springer Link.

transmon fabricated with AlO_y disorder between the Al and Ti showed intermediate lifetimes up to $T_1 = 32 \mu\text{s}$. Overall poor performance of Ti in devices due to lossy phases of the AlTi seen in Fig. 9.8 where a huge variety of compounds can be formed when Al, Ti, and O are mixed creating a multitude of lossy phases [168]. Future work could explore burying the Ti under other metal layers to reduce oxidation, fabricating asymmetric junctions with other superconducting materials (such as Bi), as well as investigating the material composition used for these devices.

Chapter 10

Cooper Pair and Quasiparticle Tunneling in SQUIDs

10.1 Motivation

In previous junction conductance measurements, subgap tunneling features were observed. It is difficult to convincingly distinguish between quasiparticle and Cooper pair tunneling processes in a dI/dV measurement of a single Josephson junction. Specifically, MAR features characterized in section 9.4 may be obscured by Cooper pair tunneling effects and extracted transparencies may be inaccurate. However, with a SQUID junction, the critical current can be modulated and coherent Cooper pair tunneling features can be suppressed. By performing tunneling measurements on a SQUID device, I was able to distinguish and model quasiparticle and Cooper pair processes in qubit-like junctions.

10.2 SQUID Device Measurements

I fabricated and measured SQUID **S1** shown in Fig. 10.1. The device consisted of two Josephson junctions on opposite sides of a square SQUID loop approximately $20\text{ }\mu\text{m}$ on a side. To modulate the SQUID critical current (section 2.4.3), the applied external flux

was controlled by injecting current in a separate line adjacent to the loop. The device was fabricated using ebeam lithography and a standard Manhattan Al deposition process formed the junctions, lines, and bondpads (section 5.3). I performed conductance measurements with a four-wire differential setup (section 8.1.1). I controlled the flux bias with a Yokogawa GS100 voltage source through a 10 k Ω bias resistor in series the low impedance flux bias line.

10.2.1 Critical Current Modulation

To begin the characterization of **S1**, I first measured the switching current, I_{sw} . In a current biased IV measurement with applied I_{bias} , starting from $I_{bias} = 0$, an ideal junction is in a supercurrent state with $V_J = 0$. As I_{bias} is increased, I measured a consistent value, $I_{bias} = I_{sw}$, where the junction voltage switches from 0 to a value near $(\Delta_1 + \Delta_2)/e$ (Fig. 8.2a). Slow measurement speed, finite device temperature, macroscopic quantum tunneling, and noise of the measurement means the switching point is not an accurate measurement of the true junction critical current because fluctuations can cause the junction to switch into a finite voltage state for $I < I_C$ [37]. However, the switching current should be proportional to critical current and shows strong modulation with applied flux (Fig. 10.2). Modulating the applied flux with flux bias current, I_Φ , reveals bias values where $I_{sw} \approx 0$ consistent with a symmetric SQUID with small loop inductance. However, the shape of the modulation I observed appears nearly sinusoidal where SQUID theory predicts $I_{sw} \propto |\cos(\Phi)|$ (Fig. 10.2). Additionally, there is a flux offset shifting the zero-nodes of I_{sw} to be non-symmetric about zero flux. For the SQUID area of approximately $A = 400 \mu m^2$ this corresponds to a field of $B \approx 2.1 \mu T$. A DC flux offset from the environment is not surprising as this device was not measured in a cryoperm magnetic shield. Modeling of strongly damped SQUIDs with thermal fluctuations show modulation shapes similar to measured data [57]. Alternatively flux noise will also round the measured switching current curve. The flux bias, I_Φ , can reduce the critical current to near 0 allowing me to perform the intended measurements on

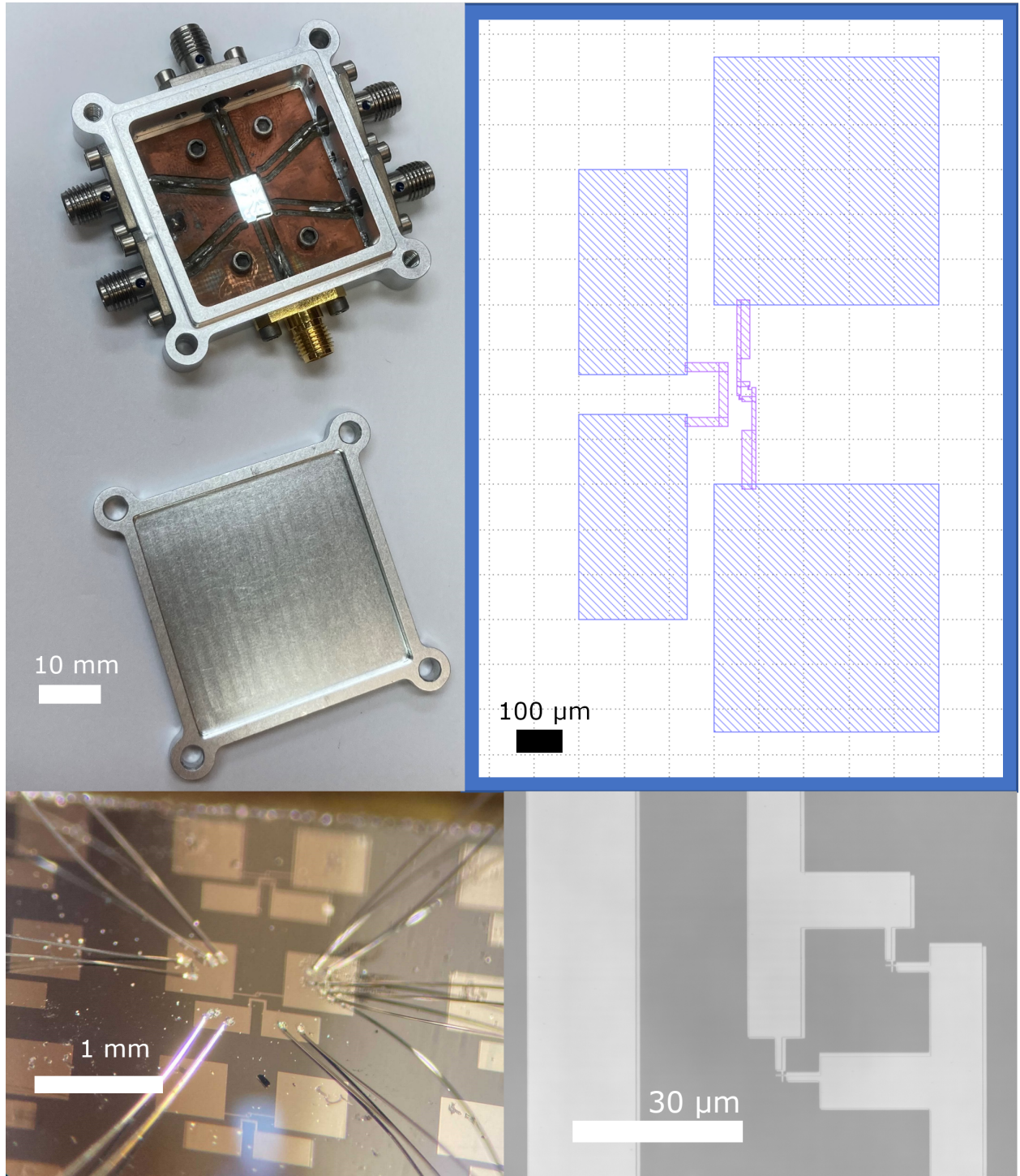


Figure 10.1: Images of SQUID device **S1**. *Top right* is the design of the SQUID device with flux bias line on the left and SQUID loop on the right. *Bottom right* is a optical micrograph of the SQUID loop which is approximately $20\ \mu\text{m}$ on a side. The loop consists of two Manhattan style junctions. Dark color is Si substrate, light color is Al metal. *Bottom left* is an image of the packaged device wire-bonded for 4 point dI/dV measurement and 2-wire flux tuning. Each connection is made with two wirebonds for redundancy. *Top left* is an image of the SQUID device and its package.

this device.

10.2.2 Differential Conductance Measurements

Using the calibration of the critical current from section 10.2.1, I performed differential conductance measurements as outlined in section 8.1.2 for multiple flux bias currents, I_Φ between -0.15 and +0.05 mA near the minimum critical current of the **S1** junction. The data has a clear critical current peak at zero voltage bias with symmetric, oscillatory features at larger voltage biases (Fig. 10.3a). The magnitude of the critical current oscillates with applied flux bias and has a minimum at $I_\Phi = -0.10$ mA, matching the minimum bias current in Fig. 10.2.

When examining the data at even smaller conductance values, a conductance peak at around $200 \mu V$ is visible. This feature appears constant for a few curves (red, green, purple, brown, orange) of the smaller bias values (Fig. 10.3b). For large critical currents, this feature is partially washed out by the large oscillations. Since this feature is not strongly dependent on critical current, it is likely a quasiparticle tunneling feature.

10.2.3 P(E) Theory for Subgap Critical Current

When considering tunneling of Cooper pairs in an electrical environment, using P(E) theory (section 2.4.2) has been used to model sub-gap tunneling features [54, 175].

Following the theory outlined in Sect. 2.4.2 and [54], I calculated the P(E) function for a junction in parallel with an RLC resonant circuit with a resonant frequency ω_e (Fig. 10.4). For $\omega_e/2\pi = 380$ GHz parallel resonance at an effective temperature of 60 mK, I plot the numerically solved P(E) function in Fig. 10.5a. At junction biases $V \approx n125\mu V = n\hbar\omega_e/e$ for positive integer n , the $P(E)$ amplitude of Cooper pair tunneling has a peak as n resonant photons can be emitted into the resonance allowing the pair to tunnel. The high frequency resonance could come from a strongly coupled mode on the chip or in the package. Finally,

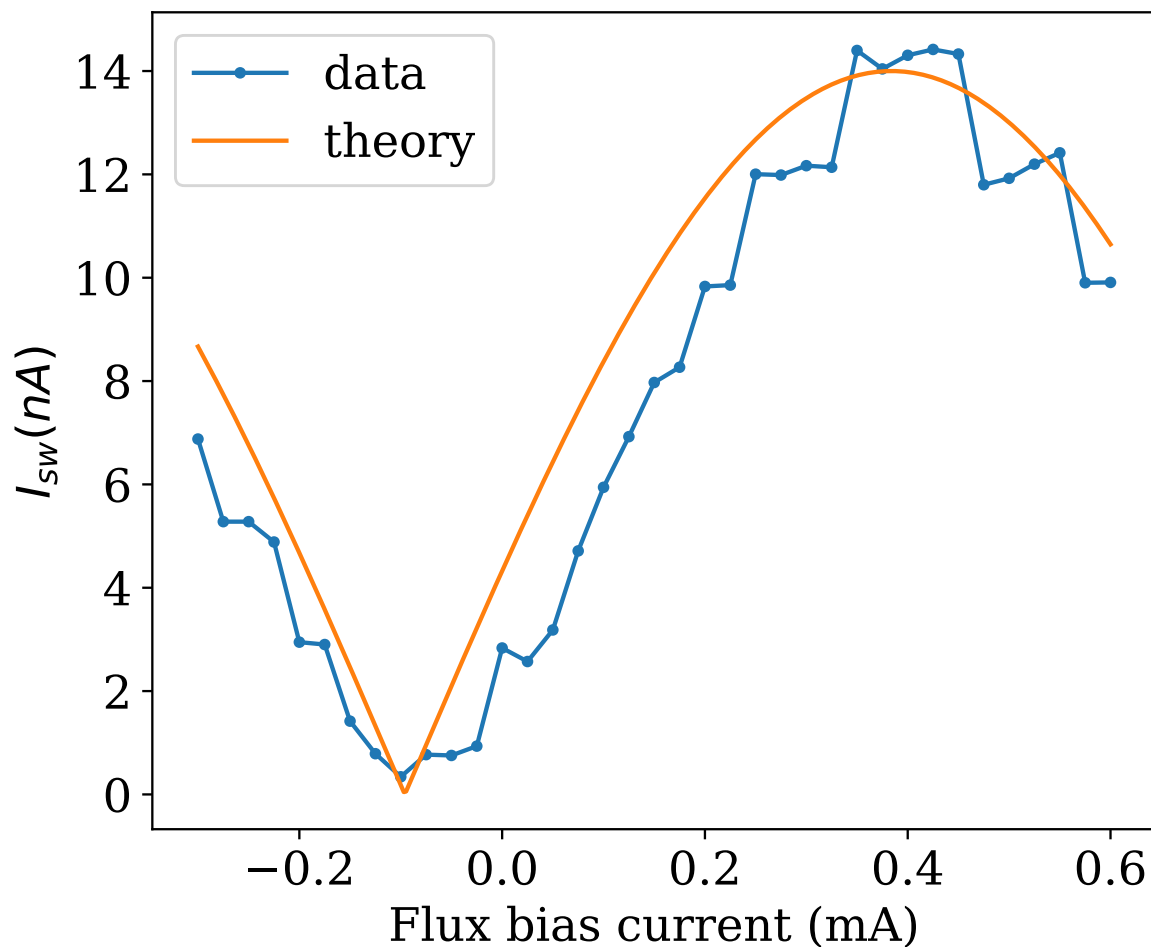
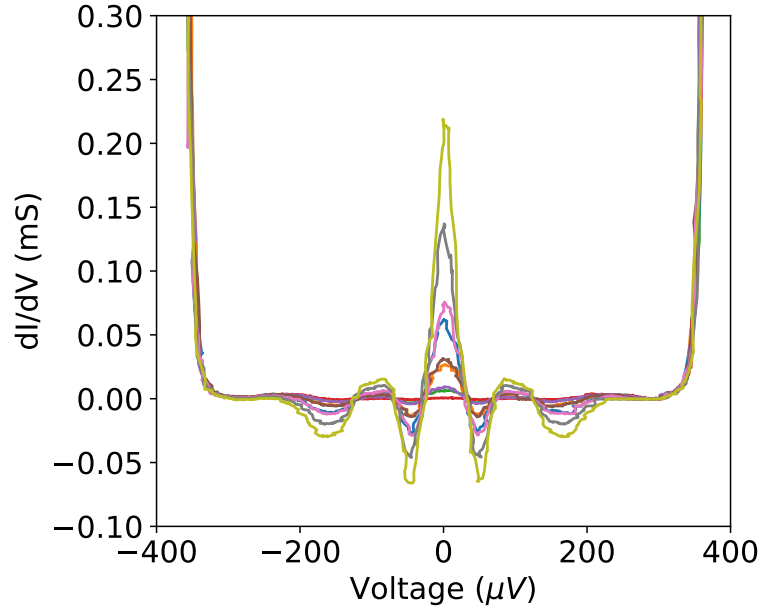


Figure 10.2: Measurement of SQUID flux modulation. Blue points are measured switching, I_{sw} currents for different flux bias current, I_{Φ} . I_{sw} modulates nearly to 0 suggesting a symmetric SQUID with small loop inductance. Orange curve shows ideal low-screening SQUID behavior for a $0.4 \Phi_0$ flux offset and $0.96 \text{ mA}/\Phi_0$ flux bias. Strong damping, thermal noise, and flux noise are potential sources of the rounding of I_{sw} .

(a)



(b)

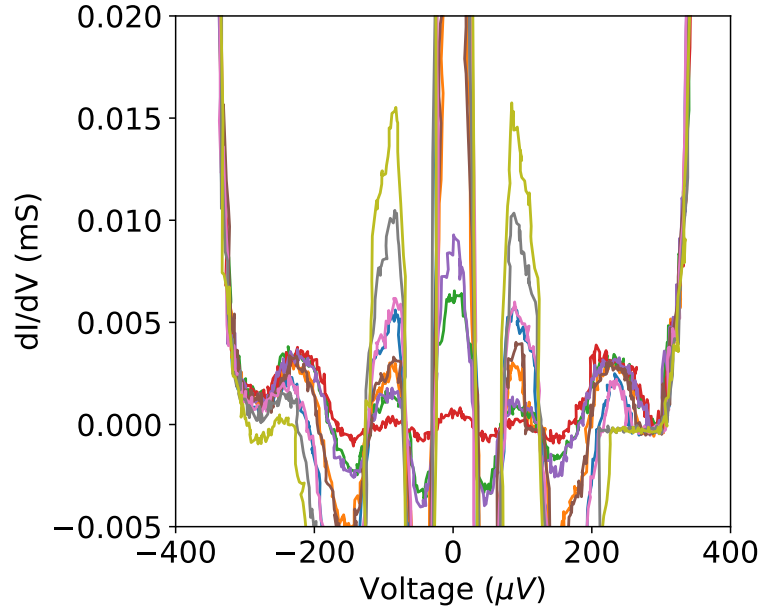


Figure 10.3: dI/dV vs. V at different I_C . The flux bias of the SQUID is stepped. At each flux bias, a differential conductance curve is taken. Conductance data shown consists of a range of critical currents around the minimum critical current. **a)** A zero-bias critical current peak is prominent in the conductance data. This peak changes magnitude as the critical current is changed. Oscillatory features at high bias are also apparent. **b)** The same data as a) with reduced the y axis scale. A conductance peak near $200 \mu V$ is visible for multiple curves with the lowest critical current. Small oscillations exist for all curves suggesting a finite minimum critical current.

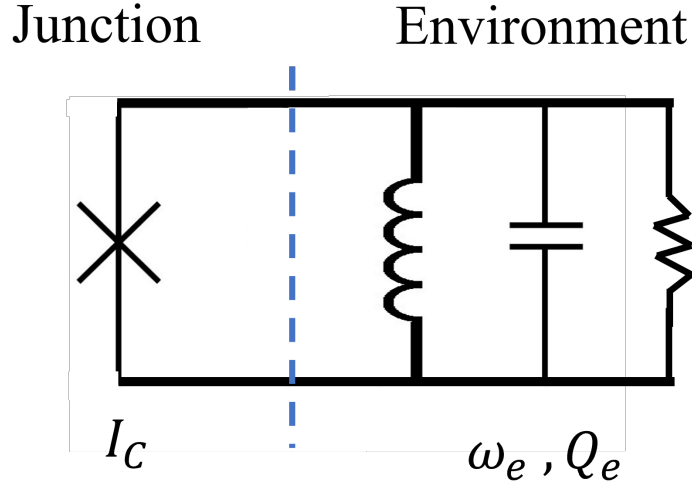


Figure 10.4: Diagram of the $P(E)$ model. The junction with critical current I_C is connected to an environmental impedance, here a simple RLC resonance with resonant frequency ω_e and quality factor Q_e . Interactions with this environment allows Cooper pairs to tunnel at finite junction biases with proportionally to the corresponding function $P(E)$.

the $P(E)$ function is convolved with a thermal broadening term

$$P_C(E) = \frac{1}{\sqrt{4\pi E_C k_B T}} \exp\left[\frac{-E^2}{4E_C k_B T}\right] \quad (10.1)$$

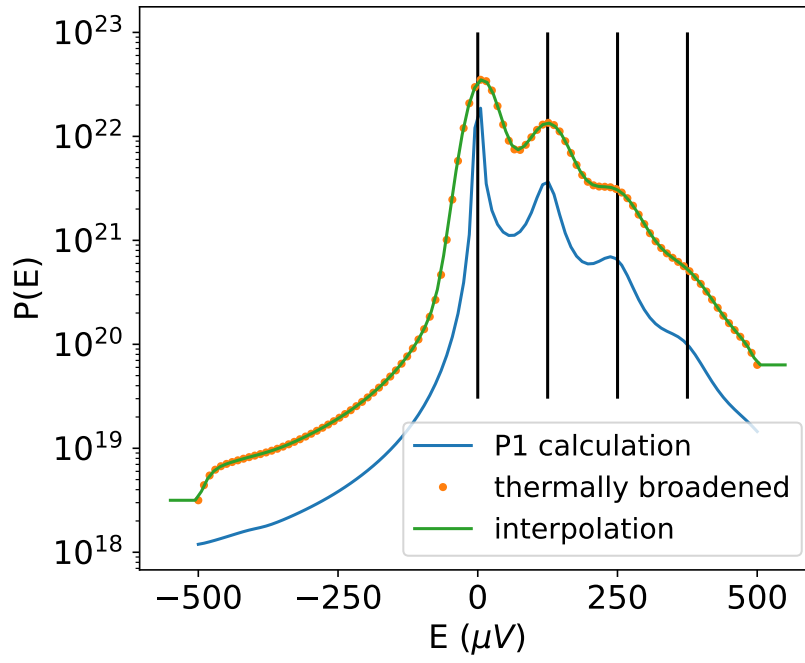
to account for the finite temperature of the system, with $E_C = e^2/2C$, the charging energy of the environmental mode.

Using Eq. 2.45 the characteristic current from $P(E)$ can be calculated. The critical current (Fig. 10.5b) has a strong conductance feature at zero bias from the typical critical current but also oscillating sub-peaks.

10.2.4 Andreev Steps

The conductance peak at 200-230 μV is near the expected gap value for the junction electrode. MAR theory (section 2.3.3) predicts conductance peaks at voltage biases equal to the two gaps, suggesting a possible source of these features (Fig. 2.7). I performed numerical MAR IV calculations (Fig. 2.7) and varied the junction transparency, D , and junction elec-

(a)



(b)

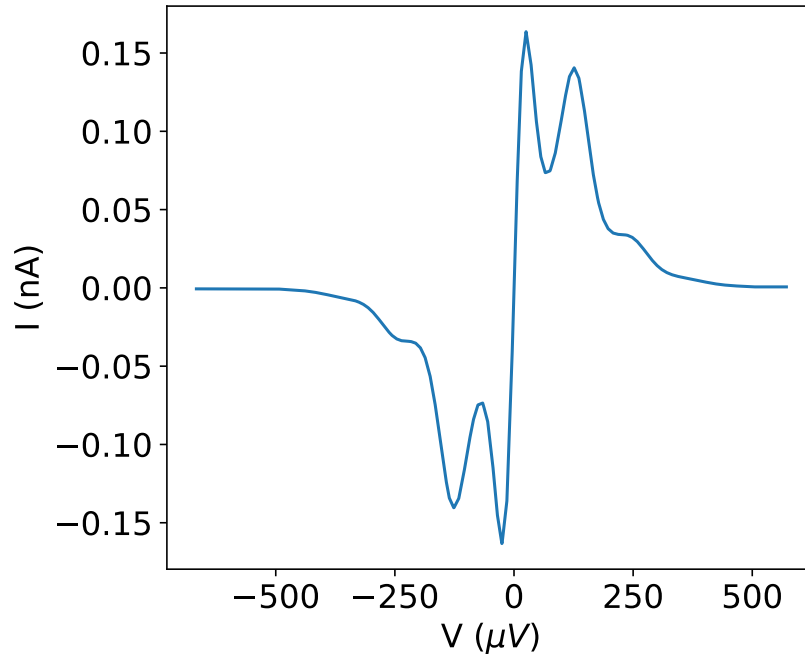


Figure 10.5: $P(E)$ theory simulation results. **a)** Numerical $P(E)$ functions calculated. Solution to the integral equation Eq. 2.46 is in blue. Calculated thermally broadened probability function from Eq. 10.1 is in orange. Final interpolated function is shown in green (overlapping orange final $P(E)$). Vertical lines mark integer values of ω_e . **b)** Cooper pair critical current, I , versus voltage, V , for the given system calculated from Eq. 2.45.

trode gaps, $\Delta_{1,2}$ to match features in the dI/dV curve and obtain reasonable limits on these parameters.

10.2.5 Total Conductance Fitting

To model the data in Fig. 10.3 I used a combination of P(E) theory and MAR theory to incorporate both supercurrent and quasiparticle features. I generated numeric dI/dV curves for Cooper pairs tunneling using P(E) theory and quasiparticles tunneling using MAR theory. Then I summed the two conductances and applied Gaussian broadening with a width $\sigma = 2.5 \mu V$ (width similar to the measured quasiparticle-quasiparticle peak width) to represent voltage noise in the measurement. This broadening primarily affected the MAR conductance data as the P(E) conductance curve was already thermally broadened

The data at $I_\Phi = -0.1$ mA corresponding to minimum I_C and my model curves are shown in Fig. 10.6 with my best parameters (Table 10.1). The combined theory matches many of the measured subgap tunneling features. The critical current dependent oscillations are modeled by the P(E) theory while the peak at $200 \mu V$ is replicated by the MAR theory. The extracted junction transparency from the MAR theory is $D = 0.002$. All data traces in Fig. 10.3a can be similarly modeled with this theory by adjusting *only* the critical current parameter in the P(E) model.

The low transparency means the tunnel junction approximation for this junction is well-founded and MAR effects should play a small role in junction behavior. By estimating the loss from MAR tunneling as resistive loss at the qubit frequency, $R = \frac{\hbar f/e}{I(\hbar f/e)}$, in a series RLC circuit, the T_1 limit of this resonance from MAR processes would be greater than 50 hours.

The critical current shows oscillations suggesting an environmental resonance around 380 GHz. While the RF environment is important for qubit loss [60, 106], resonances at these high frequencies should have little effect on the qubit.

At higher biases, the theory diverges. Junction self-heating may cause a reduction in the

Table 10.1: Parameters used in modeling dI/dV data shown in Fig. 10.6. To match other curves in Fig. 10.3, only the I_C parameter is adjusted and good agreement is obtained.

Parameter	Value	Name
R_n	2.5 k Ω	junction resistance
I_C	4.5 nA	critical current
Δ_1	220 μ eV	gap of first electrode
Δ_2	198 μ eV	gap of second electrode
D	0.002	junction transparency
T	60 mK	electron temperature
$\omega_e/2\pi$	380 GHz	environmental resonant frequency
Q_e	4	resonant quality factor

measured gap at larger currents (section B.1), explaining this discrepancy. The measured large conducting peak occurs at $V = (\Delta_1 + \Delta_2)/e \approx 390 \mu V$ bounding the smallest Al superconducting gap value of one of the electrode to $\Delta \leq 185 \mu eV$. For thermally evaporated thin films, this is an uncharacteristically low gap providing more evidence that the measured gap could be thermally suppressed [66, 115, 167, 170, 171].

10.3 Conclusion

I fabricated, measured, and modeled a SQUID to investigate the coherent Cooper pair tunneling and subgap quasiparticle tunneling pathways in Josephson junctions. Cooper pair tunneling features in the IV characteristic change with varying critical current while quasiparticle features should be independent of I_C .

I isolated features in Al/AlO_x/Al SQUID IV measurements that resemble $P(E)$ inelastic pair tunneling and modeled differential conductance features with this theory. The differential conductance at low voltage biases, $|V| < (\Delta_1 + \Delta_2)/e$ shows oscillations with suggesting an environmental resonance coupled to the junction. While the RF environment is important for qubit loss [60, 106], resonances at these high frequencies are likely due to the leads as these effects are not observed in a qubit.

Additionally, measurements showed a quasiparticle tunneling peak near $V = \Delta_{1,2}$ for

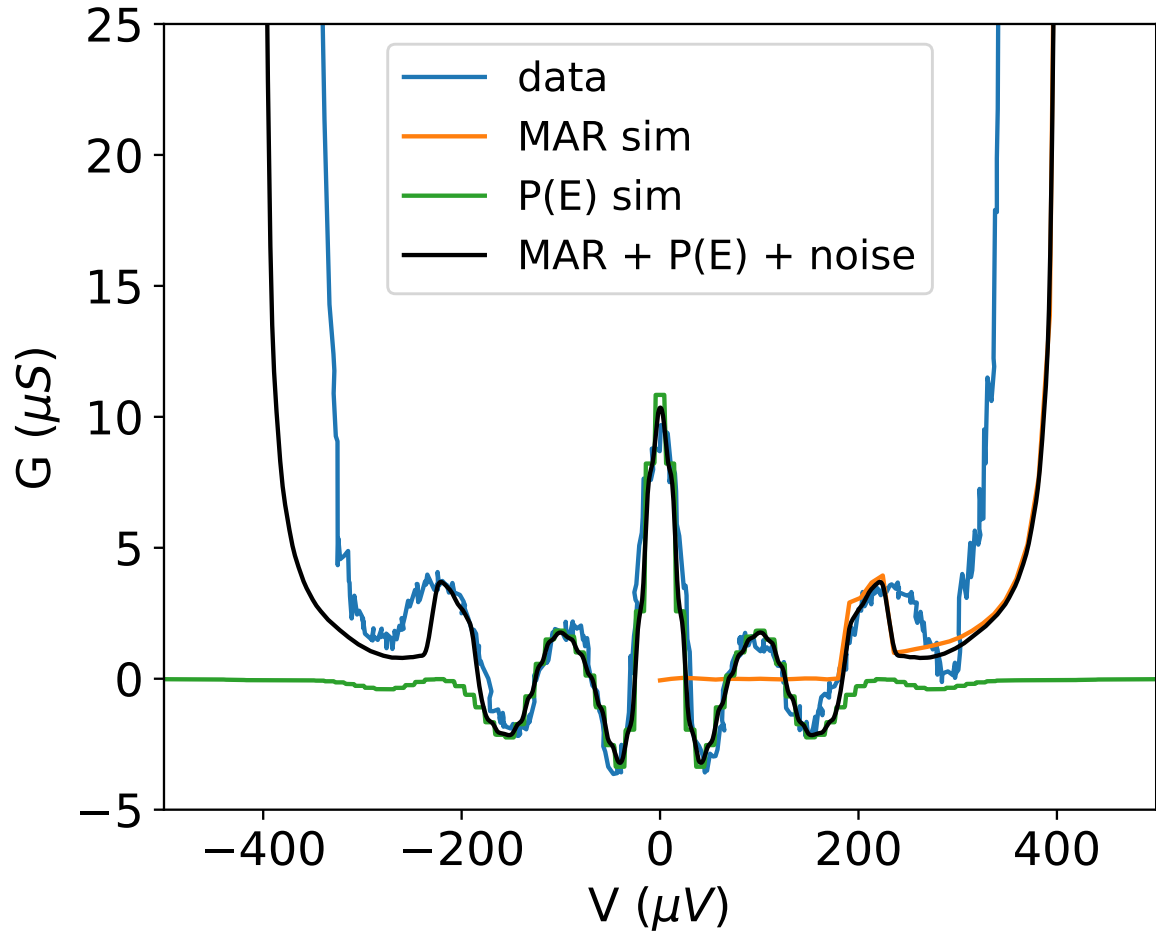


Figure 10.6: Conductance model compared to SQUID measurement of **S1**. The blue trace is a dI/dV vs V measurement at the lowest I_C taken from Fig. 10.3. The green trace is my $P(E)$ model of the critical current features associated with Cooper pair tunneling. The orange trace is my MAR theory associated with quasiparticle tunneling at $eV = \Delta_{1,2}$. The sum of the two models shown in red has good agreement with the data in the subgap region. Discrepancies above $250 \mu V$ may be due to junction self-heating and a distribution of superconducting gaps in different electrode grains. All data traces in Fig. 10.3a can be similarly modeled with this theory by adjusting *only* the critical current parameter in the $P(E)$ model.

the Al junction electrodes. I modeled this feature with MAR tunneling theory. The extracted junction transparency of $D = 0.002$ is much less than 1. Loss due to finite junction transparency would provide very low relaxation rates in typical qubit devices with similar junctions.

Although the measured features can be reproduced using the P(E) and MAR theories, the behavior of small junctions is very complicated and the RF environment can affect the behavior in ways separate from P(E) theory [37, 176]. Further measurement and theoretical work is necessary to determine the source of the measured features more confidently.

Both MAR and P(E) theory were developed for modeling IV characteristics of single tunnel junctions. Future work should be performed to generalize the theories to SQUID geometries. Additionally, the junction loss in a qubit and tunneling for that junction at the qubit frequency are related but distinct. The characterization of the junction conductance gives an idea of junction behavior but the theory must be extended to give hard limits on qubit coherence.

Chapter 11

Single-Qubit Pulse Optimization

11.1 Motivation

Decreasing qubit gate time and increasing gate fidelity results in an increase of the length of any quantum operations that can be performed on a quantum computer. This is especially important for superconducting qubits where the coherence time is only a few orders of magnitude longer than the gate time, resulting in significant errors. In this chapter, I present two main experiments to understand and improve single qubit gate performance. I discuss pulse shaping and my measurements of gate error for different pulse shapes and lengths, finding that the error of a gate was strongly impacted by the frequency component of the pulse at the transmon $|e\rangle \leftrightarrow |f\rangle$ transition. Next, I discuss how I designed, simulated, and tested a new type of pulse, a physics informed neural network (PINN) pulse [31]. Finally, I show some measurements I used to investigate possible pulse distortion in the measurement setup and conclude with suggestions for future directions in pulse optimization.

11.2 Experimental Setup

Here I quickly outline the experimental setup and device parameters used for the pulse tuning experiments. The device, **Q1**, used is shown in Fig. 5.1. The qubit was characterized as

Table 11.1: Parameters for qubit device **Q1** used in the pulse tuning experiments of this chapter. T_1 and T_2 values are mean values from repeated measurements over 15 hr.

Parameter	Value	Name
f_C	7.402 GHz	bare cavity frequency
f_{ge}	2.780 GHz	qubit frequency
α	150 MHz	transmon anharmonicity
T_1	77.2 μs	relaxation time
T_2	125 μs	Hahn-echo decoherence time
T_2^*	105 μs	Ramsey decoherence time
g	61 MHz	qubit-cavity coupling
E_J/E_C	58	transmon energy ratio

outlined beginning in section 7.1. The extracted qubit parameters are shown in table 11.1.

11.3 Pulse Envelopes

In section 3.3.2, I presented the Hamiltonian for a qubit driven on resonance through a capacitively coupled line (Eq. 3.33)

$$H'_d = \frac{\hbar\Omega_q}{2} (I(t)\sigma_x + Q(t)\sigma_y) \quad (11.1)$$

For an ideal qubit, a drive envelope with $[I(t), Q(t)]$ with total area $\int I dt = \pi/\Omega_q$ will perform an X π -pulse [61]. However, due to the small anharmonicity of the transmon, performing a short pulse at $f_d = f_{ge}$ may result in a measurable spectral power at the relatively close $f_{ef} = f_{ge} - \alpha$. Power at f_{ef} will cause leakage of the transmon population out of the computational subspace and, therefore, gate errors. Therefore, an experiment performing fast gates must be concerned with the frequency spectrum of their drive envelope given by the Fourier transform of the envelope function

$$\mathcal{I}(f) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} I(t) e^{-i2\pi f t} dt \quad (11.2)$$

The common method to reduce the frequency component at $f = f_{ef}$ is pulse shaping.

Pulse shaping is simply the process of choosing a pulse shape, $I(t)$, that has a well behaved Fourier spectrum, $\mathcal{I}(f)$.

One common envelope is the Gaussian pulse, given by

$$I(t) = A_0 e^{-t^2/2\sigma^2} \quad (11.3)$$

where A_0 is the pulse amplitude and σ is the pulse width. This shape is often used because the Fourier transform of the Gaussian is

$$\mathcal{I}(f) = \frac{A_0}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-t^2/2\sigma^2 - i2\pi ft} dt = A_0 \sigma e^{-2\pi^2 f^2 \sigma^2} \quad (11.4)$$

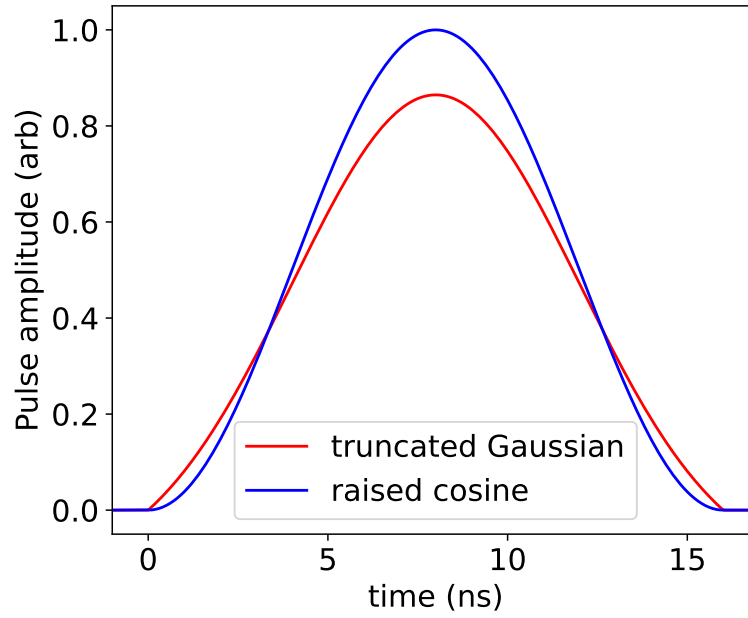
which also takes a Gaussian shape. Therefore, the spectral power will fall off exponentially with detuning, strongly limiting the power at frequencies detuned from the drive further than $1/\sigma$. For example, my qubit with $\alpha = 150 \text{ MHz}$ driven by a Gaussian pulse with $\sigma = 4 \text{ ns}$ would have drive power at f_{ef} at least a factor of $8 \cdot 10^{-4}$ smaller than the driving power at f_{ge} .

Finite duration pulses are necessary for qubit operations since orthogonal rotations do not commute. Additionally, the finite bandwidth of the pulse generation limits the rate of change of the pulse shape so pulses with continuous $I(t), Q(t)$ are desirable. To satisfy these criteria, the Gaussian shape were truncated in time at $\pm 2\sigma$ from the center and shifted down for a final pulse envelope

$$I(t) = \begin{cases} A' \left[e^{-t^2/2\sigma^2} - e^{-(2\sigma)^2/2\sigma^2} \right] & \text{for } |t| < 2\sigma \\ 0 & \text{for } |t| \geq 2\sigma \end{cases} \quad (11.5)$$

where $A' = 4/\varsigma\sqrt{2\pi}$ is the amplitude for a π -pulse where $\varsigma = 0.7386$ is found from a Gaussian integral or Erf error function. The pulse is continuous at all points. The truncated Gaussian pulse used is shown in Fig. 11.1a.

(a)



(b)

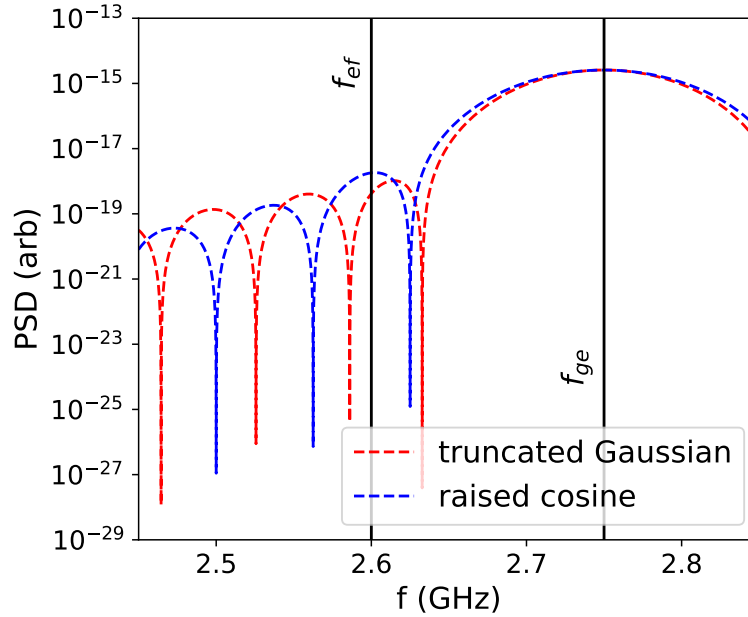


Figure 11.1: Plots of two common pulse shapes. **(a)** Time domain pulse envelope for 16 ns truncated Gaussian and raised cosine pulses. Pulse amplitudes are scaled in experiment to perform a $\pi/2$ -pulse on the qubit. Both shapes are continuous pulses, but the Gaussian shaped pulse has a discontinuity in its derivative at $t = \pm 2\sigma$ from the center of the pulse. **(b)** Fourier power spectral density of the two pulse shapes. The peak power for both pulses is at the qubit driving frequency, f_{ge} . Due to the pulses being finite in time, the PSD has zeroes. Changing the pulse length to align these zeroes with the higher order transmon transition f_{ef} can reduce gate leakage out of the computational subspace.

The raised-cosine pulse shape is another commonly used pulse envelope for superconducting qubit control. For this pulse, the envelope shape (Fig. 11.1a) is defined by

$$\begin{cases} I(t) = \frac{A_0}{2}[\cos(\pi t/2\sigma) + 1] & \text{for } |t| < 2\sigma \\ 0 & \text{for } |t| \geq 2\sigma \end{cases} \quad (11.6)$$

Often the choice between these two common pulse shapes is made with consideration of the amplitude at the $|e\rangle \leftrightarrow |f\rangle$ transition frequency [30]. In the next section, I will look at this choice in more detail.

11.3.1 Zeroes of Finite Pulses

A finite pulse will have zeroes in its spectrum. Since a finite pulse is a pulse shape times a box function in time domain, in frequency domain the pulse will be a sinc function convolved with a function with equal width. The position of these zeroes, however, depends on the pulse shape (Fig. 11.1b). When trying to minimize the frequency component at a detuned $|e\rangle \rightarrow |f\rangle$ frequency, these zeroes can give an advantage. The addition of DRAG to a pulse (section 7.5.2) will shift these zeroes in frequency. Placing a pole at the $|e\rangle \leftrightarrow |f\rangle$ frequency was an original motivation for DRAG implementation [29, 152].

In general, the choice between a truncated Gaussian and a raised cosine or any pulse shape can be made by evaluating the PSD for the specific pulse. For the given pulse shape choices (optionally with DRAG to reduce phase errors), one evaluates the Fourier transform at the $|e\rangle \leftrightarrow |f\rangle$ frequency and choose the pulse with a minimum at f_{ef} .

11.4 Leakage and Error vs. Pulse Length

Driving the $|e\rangle \rightarrow |f\rangle$ transition will lead to leakage out of the computational $|g\rangle, |e\rangle$ subspace. However, how this affects gate error is not straightforward. I ran a series of gate benchmarking experiments with different pulse lengths and pulse shapes to experimentally

test these pulse design choices. For truncated Gaussian and raised cosine pulses with pulse lengths between 13 ns and 29 ns, I followed the gate tuning procedure outlined in section 7.5 and measured the gate error and leakage using 3 state normalized randomized benchmarking (section 7.5.4).

The measured leakage per gate vs. pulse length is shown in Fig. 11.2. Measured leakage per gate values are shown as data points with least-squares error bars of leakage fit function Eq. 7.14 with the e to f state decay time $s = 50 \mu s$. The solid lines are theory curves given by

$$l(t_{gate}) = C |\mathcal{I}(\omega_{ef}, t_{gate}, S)|^2 + l_0 \quad (11.7)$$

where the leakage, $l(t_{gate})$ is proportional to the power spectral density (PSD), $|\mathcal{I}|^2$, and a constant background leakage, l_0 . The PSD is a function of the pulse shape and length (Eq. 11.2) and is evaluated at the leakage transition frequency, f_{ef} . Both scaling constant C and offset l_0 are phenomenological and chosen values are shown in Table 11.2. Note, these parameters only scale the y-axis values. The gate lengths at which a minima and maxima occur for LPG are totally determined by the PSD.

Both pulse shapes exhibit clear oscillations in leakage matching the oscillations in the PSD. Therefore, as expected, leakage is proportional to a pulses spectral power at the qubit's $|e\rangle \leftrightarrow |f\rangle$ frequency. The simple PSD leakage model matches data well for pulse lengths $t_{gate} > 15$ ns. However for both pulse shapes, the leakage per gate begins to diverge with pulse length decreasing below 15 ns. While the model does predict diverging leakage below $t_{gate} = 12$ ns, the cause of the experimental divergence at longer pulse lengths is not known and is a topic for future study. Identifying and mitigating this leakage source could improve qubit gate speed.

The measured error per gate vs pulse length is shown in Fig. 11.3. My model of the error is

$$\epsilon(t_{gate}) = 3 \cdot l(t_{gate}) + \epsilon_{inc} \quad (11.8)$$

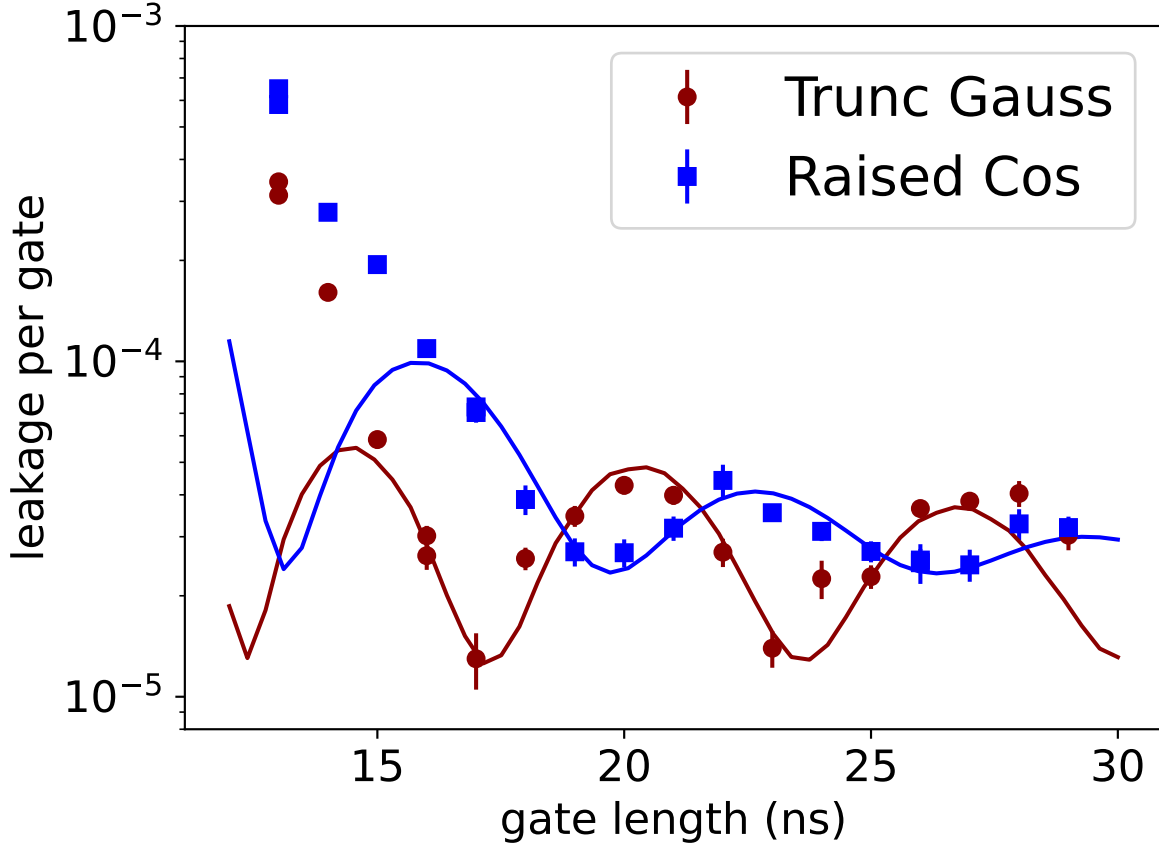


Figure 11.2: Leakage dependence on length for two pulse types. Measured leakage data points and error bars are shown for a range of pulse lengths. Solid lines are leakages predicted by Eq. 11.7 using parameters in Table 11.2. Oscillations in leakage follow the oscillations in spectral power at the transmon's ω_{ef} frequency. At shorter pulse lengths another source of leakage dominates.

Table 11.2: Parameters from data and fits in Figs. 11.2 and 11.3. C scales the pulse PSD to a leakage value. l_0 is residual leakage likely from noise and drift or small miscalibration which may differ between pulses. The minimum leakage was obtained for the cosinusoidal pulses at $t_{gate} = 27$ ns and the minimum error was obtained for $t_{gate} = 20$ ns. The minimum leakage and error were obtained for the Gaussian pulses at $t_{gate} = 17$ ns.

Pulse Shape	min(LPG)	min(EPG)	C	l_0
Truncated Gaussian	$1.3 \cdot 10^{-5}$	$2.52 \cdot 10^{-4}$	$1.7 \cdot 10^{11}$ Hz/W	$1.2 \cdot 10^{-5}$
Raised Cosine	$2.5 \cdot 10^{-5}$	$3.02 \cdot 10^{-4}$	$1.2 \cdot 10^{11}$ Hz/W	$2.3 \cdot 10^{-5}$

where $\epsilon_{inc} = 1 - \exp(-t_{gate}/T_2)$ is the dephasing that would occur during the gate length. A conservative estimate of ϵ_{inc} with $T_2 = 100 \mu s$ is shown in Fig. 11.3 (red and blue curves). $l(t_{gate})$ is the leakage function obtained above. Measured error per gate points and error bars shown are obtained from a least squares fit to Eq. 7.12. I emphasize that, given the leakage function, this error model's only free parameter is the phenomenological 3 scaling factor. Examining Fig. 11.3, the model follows the oscillating trend in the data and predicts the errors for $t_{gate} > 16$ ns. At lower gate lengths, the error per gate still follows my model using the measured leakage (dotted lines in Fig. 11.3). Small disagreements with the model may be due to miscalibration or instabilities between measurements. The possibility of reducing gate error provides more incentive to identify and eliminate this unknown short gate leakage source at short gate times.

The model's factor of 3 is phenomenological. Simple Markovian leakage out of the computational subspace would give $\epsilon_{leak} = l$. However the transmon $|f\rangle$ state will not function like a Markovian bath for the qubit. More detailed simulation may be able to produce this factor and would be a good future research direction.

The calculation of incoherent error are often performed with Kraus operators [177]. This formalism gives a different form for the incoherent error due to relaxation with characteristic time T_1 and dephasing with T_ϕ of

$$\epsilon_{inc} = \frac{t_{gate}}{3} \left(\frac{1}{T_1} + \frac{1}{T_\phi} \right) \quad (11.9)$$

I did not explore deviations from this formula.

From these measurements, I conclude single qubit gates on this device are limited by leakage processes. The error per gate is roughly 3 times the leakage per gate over a range of pulse lengths, error rates, and pulse types. Exploring more modern methods to reduce leakage may provide substantial improvement to gate errors. These include using DRAG to reduce leakage while correcting phase errors with virtual Z gates or pulse detuning (section

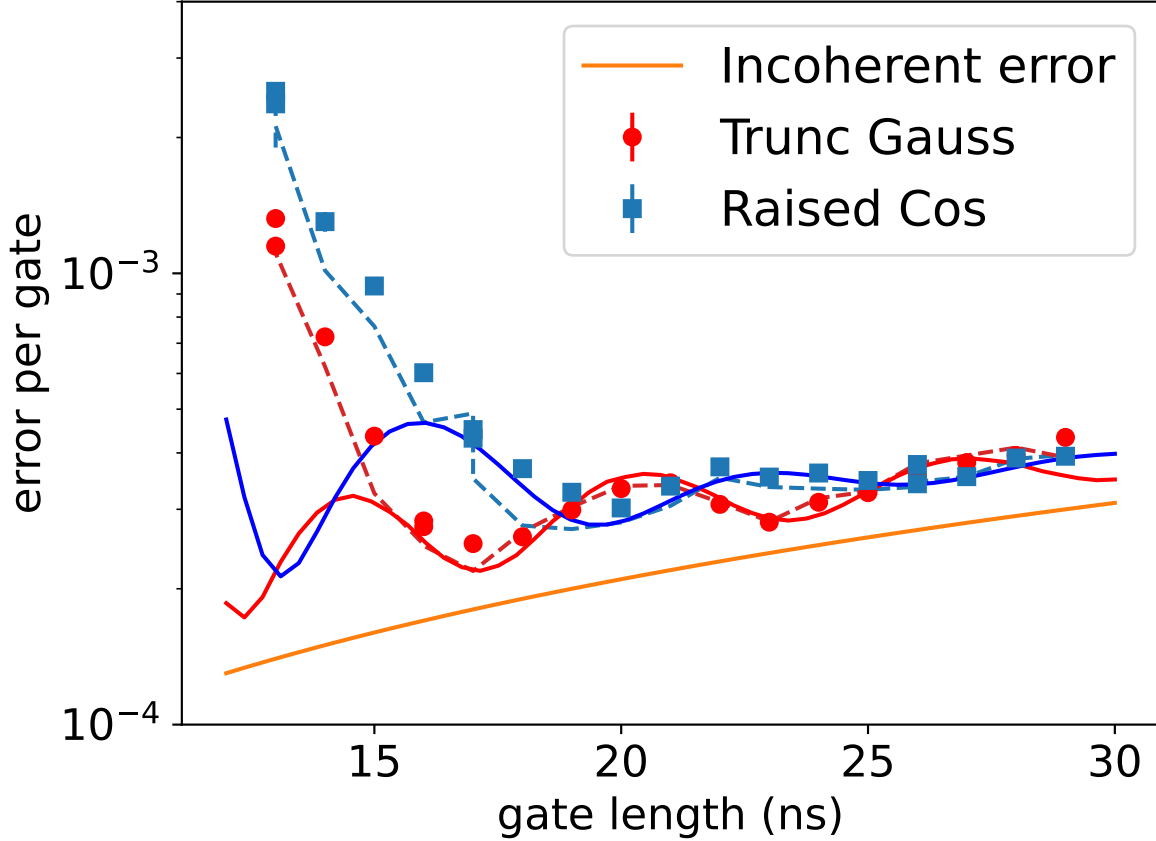


Figure 11.3: Measured error per gate vs. gate length for truncated Gaussian and raised cosine pulses. The measured error rates are shown with points and error bars. Blue and red solid lines are the error predicted from the PSD error model, Eq. 11.8. The model fits errors for $t_{gate} > 16$ ns. Dashed lines are the same model as the solid lines applied to experimentally measured leakage value shown in Fig. 11.2 emphasizing that the error dependence on leakage is the same for all pulse values. The orange solid line is the incoherent decay from the qubit's finite coherence time, T_2 . For this device, gate errors are limited by leakage and decoherence.

7.5.2) [153, 154]. More complex pulse shapes may also lead to less leakage [31, 178, 179]. Finally, investigating this dependence for a variety of qubit frequencies and anharmonicities could add more insight to the error.

11.5 Optimized Pulses

I now present a second experiment involving optimizing qubit gate fidelity using a new pulse shape. Significant studies have gone into designing optimal pulses for single qubit gates. The early developments of pulse shaping involved analytically correcting specific errors [29, 152]. However, with the advent of modern machine learning optimization techniques more brute force methods can be used to generate optimized qubit pulses [31, 179, 180].

I chose to implement and test pulses based on the technique in [31] so I quickly go over the pulse design framework presented there.

11.5.1 PINN Framework

The proposal by Güngördü and Kestner uses a physics informed neural network (PINN) to design a smooth pulse for a desired qubit control gate [31]. Here I quickly go through their framework, deriving the cost function for a transmon gate robust to noise in the drive amplitude.

Starting with a full Hamiltonian, $H = H_C + H_\epsilon$ composed of a control and error Hamiltonian, the system dynamics can be written

$$U(t) = U_\epsilon^\dagger(t)U_C(t) \tag{11.10}$$

as a combination of the incoherent and coherent dynamics. Using the interaction picture, the coherent system evolves via the Schrödinger equation

$$i\hbar\dot{U}_C(t) = H_C(t, \mathbf{p})U_C(t) \tag{11.11}$$

where the H_C is parameterized by time-dependent control parameters $\mathbf{p}$.

The noise affects the system as follows

$$i\hbar\dot{U}_\epsilon(t) = \left[U_C^\dagger(t) H_\epsilon(t, \mathbf{p}) U_C(t) \right] U_\epsilon(t) \quad (11.12)$$

This can be used to define the strength of the noise where

$$\int_0^T \|U_C^\dagger(t) H_\epsilon(t, \mathbf{p}) U_C(t)\|/\hbar dt \ll \pi \quad (11.13)$$

is the weak noise limit. This can be thought of as the error term driving the qubit only a small distance on the Bloch sphere.

U_ϵ can be expanded for weak errors as

$$U_\epsilon(t) = \exp(-i/\hbar \epsilon \mathcal{E}(t)) = \exp\left(-i/\hbar \epsilon \int_0^t U_C^\dagger(t') H_{\epsilon,N} U_C(t') dt'\right) \quad (11.14)$$

where the noise strength, ϵ , has been separated from the normalized error Hamiltonian. A robust qubit pulse will have dynamics insensitive to ϵ meaning the integral, $\mathcal{E}$, in Eq. 11.14 will vanish.

The cost function can now be constructed based on accurate dynamics and noise robustness. For a desired unitary rotation U_0 , the cost function is written

$$\mathcal{C} = \left(1 - \frac{\text{tr}\left([PU_C(t)P^\dagger]U_0^\dagger\right)}{D}\right) + \left(\frac{w\epsilon^{max}}{\hbar D} \|P\mathcal{E} - \text{tr}(P\mathcal{E})/D\|\right)^2 \quad (11.15)$$

where P is the projection onto the computational subspace with dimension D (*e.g.* $D = 2$ for a qubit), ϵ^{max} is the maximum considered noise, and w is an additional parameter swept to avoid local minima. The first term of $\mathcal{C}$ is the infidelity $(1 - \mathcal{F})$ of the operation and minimizing it optimizes the control to give the correct operation in the computational subspace. The second term is the noise sensitivity of the logical subspace and will generate

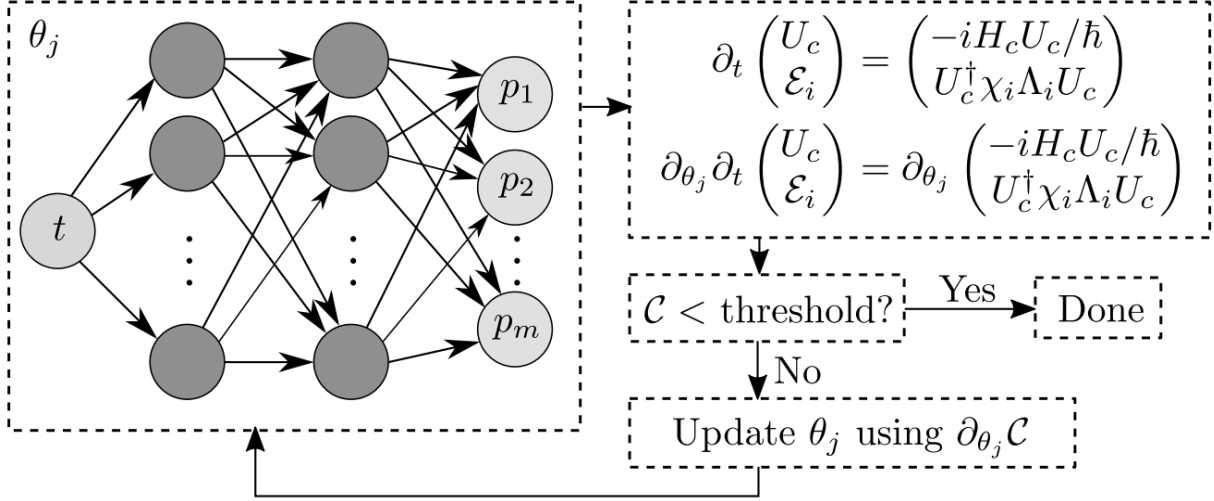


Figure 11.4: Flowchart of the PINN pulse design taken from [31] with permission (copyright APS 2022). The neural network generates $\mathbf{p}$, the cost and its gradient is evaluated, and new neural network weights are generated. This is repeated a given number of cycles or until the cost falls below a preset threshold.

robust control when minimized. This cost function is fed into a deep neural network which finds an optimum $\mathbf{p}$ to minimize $\mathcal{C}$ (Fig. 11.4). This formalism can be applied to an arbitrary quantum system and can be easily generalized to multiple noise terms. Details of the optimization process can be found in [31].

11.5.2 PINN Pulse for Single Qubit Gates

Performing single qubit gates on this transmon qubit provides a simple testbed for the PINN pulse framework outlined above. I write the control Hamiltonian of a resonantly driven multi-level transmon (section 3.3.1)

$$H_C(t) = \frac{\alpha}{2} a^\dagger a^\dagger a a + \frac{\Omega(t)a + \Omega^*(t)a^\dagger}{2} \quad (11.16)$$

with a being the destruction operator for the transmon, α the anharmonicity, and Ω the drive envelope. The parameters $\mathbf{p}(t)$, generated from the neural net, determine the complex drive strength Ω as a function of time. The σ_x and σ_y driving envelopes described in section

3.3.2 are written $I(t) = \text{Re } \Omega(t)$ and $Q(t) = i \text{Im } \Omega(t)$.

The error Hamiltonian is proportional to the errors to which the pulse will be robust. In this case, pulse amplitude errors correspond to

$$H_{\epsilon,N} = (a + a^\dagger) + i(a - a^\dagger) \quad (11.17)$$

or X and Y rotations on the Bloch sphere.

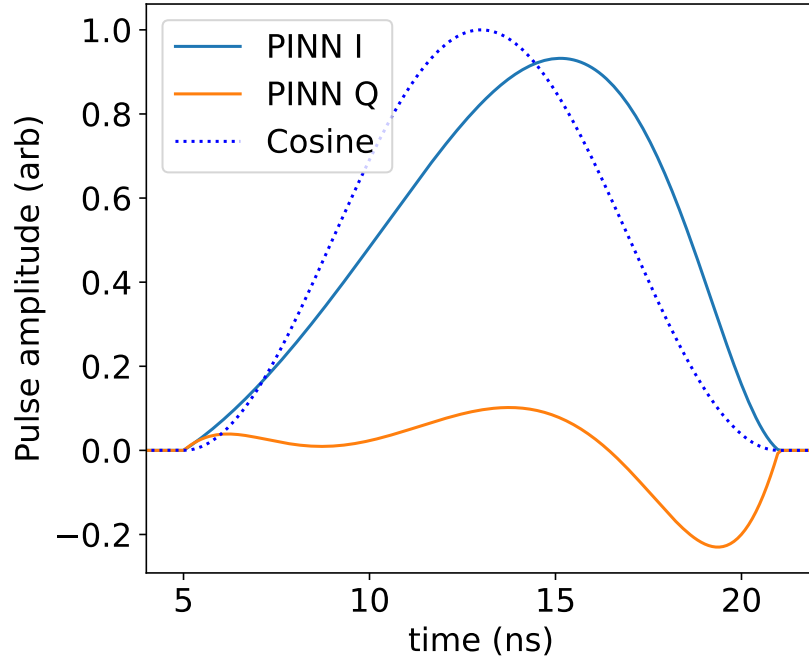
For device **Q1** with parameters in table 11.1, the 16 ns $X_{\pi/2}$ PINN pulse after 320 iterations is shown in Fig. 11.5a. An example of cost function evaluations with subsequent optimization iterations is shown in Fig. 11.5b. After the optimized pulse shape has been determined, the shape is approximated with a Chebyshev expansion to be easily and quickly numerically output in testing. This enables use of the tuned pulse shape in software environments without the neural network infrastructure (such as the AWG). For implementation, I chose a PINN pulse without any error robustness by removing the second term from Eq. 11.15 during optimization. When tested experimentally, these pulses outperformed pulses optimized for robustness and fidelity. Potentially the available control of two drive quadratures is too simple to achieve both high-fidelity and robustness.

The optimized shape from the PINN pulse depends on the desired unitary, U_0 , the desired noise robustness, $H_{\epsilon,N}$, the qubit anharmonicity, α , and the pulse length, t_{gate} . For a fixed frequency qubit, once a pulse has been decided upon and designed, the pulse can be used on the given system without the need for re-optimization in changing noise environments unlike other error mitigation schemes [181]. Scaling the length by the anharmonicity even allows the pulse to be used between different qubits or systems.

11.5.3 Pulse Simulation

To verify the design of the PINN pulse, I performed pulse fidelity simulations in QuTiP [182]. I simulated $\pi/2$ -pulses for square, raised cosine, truncated Gaussian, and PINN pulse shapes.

(a)



(b)

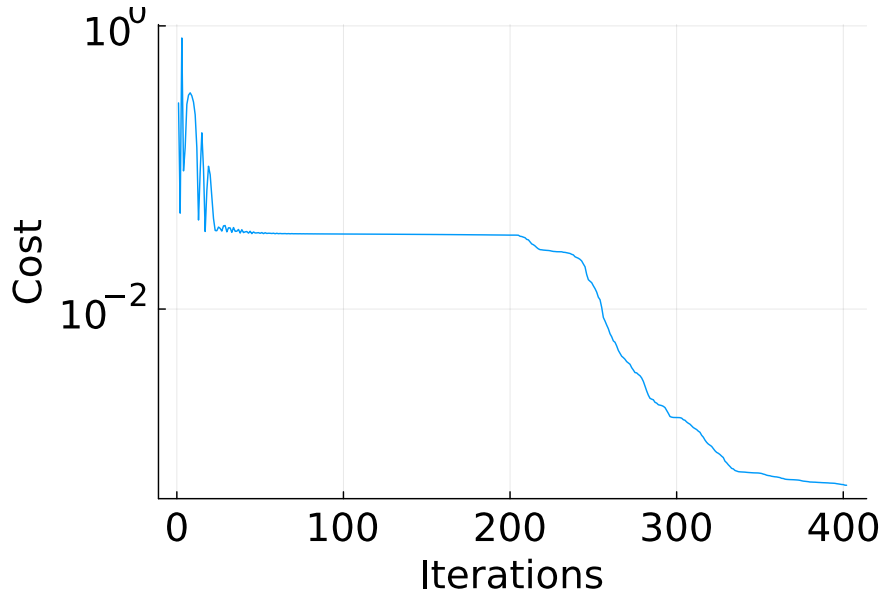


Figure 11.5: The single qubit PINN pulse. **(a)** The final optimized pulse shape for a 16 ns $\pi/2$ -pulse starting at $t = 5$ ns. Cosine shape is shown for reference. The shape is qualitatively similar to the pulse shapes shown in Fig. 11.1a but has small tuned differences. **(b)** Example cost function value over the optimization iterations. From 1-200 iterations the AMSGrad process is used to avoid local minima. From 200-400, Broyden-Fletcher-Goldfarb-Shanno (BFGS) optimization is used to further refine the shape.

In the simulations, I evolved the initial state $|0\rangle$ toward the desired state $|+\rangle = \frac{1}{2}(|0\rangle + |1\rangle)$. The system was modeled as a 6 level transmon with Hamiltonian from Eq. 3.26. Gates are applied using the on-resonance drive Hamiltonian Eq. 11.16 with $I(t), Q(t)$ corresponding to the pulse shape. Simulations were done with the Schrödinger equation and do not account for incoherent dynamics. Therefore in this simulation, errors arise from leakage to higher levels or phase errors from Stark shifts on the higher transmon levels [29]. Results are shown in Fig. 11.6.

At zero time, the error for all gates is $1/2$ because of the overlap of the initial $|0\rangle$ and desired $|+\rangle$ states. As time increases, the pulse drives the qubit toward the $|+\rangle$ state and the error decreases. After the pulse is completed (grey vertical line in Fig. 11.6), the final gate error is visible. A simple 16 ns square pulse results in a final gate error of 1.0×10^{-2} while the raised cosine and truncated Gaussian pulses obtain much lower errors of 8.4×10^{-5} and 1.3×10^{-5} respectively. However, the PINN pulse performs the best with a simulated 5.3×10^{-6} gate error. At this pulse length, the PINN pulse has approximately half the error of the truncated Gaussian pulse shape and an order of magnitude better fidelity than the raised cosine pulse shape.

Additionally, I examine the PSD of the simulated pulse shapes (Fig. 11.7). All pulses have a maximum power at the qubit resonance f_{ge} . Lower frequency components at f_{ef} will cause less leakage for a given pulse. The raised cosine, truncated Gaussian, and PINN pulses have asymmetric spectra because of the quadrature driving (from DRAG or DRAG-like component of the PINN pulse). This effect reduces the frequency at the higher level transition. The PINN pulse has a minimum near the f_{ef} frequency, suggesting that the optimizer has “learned” the importance of avoiding this frequency through the simulation iterations.

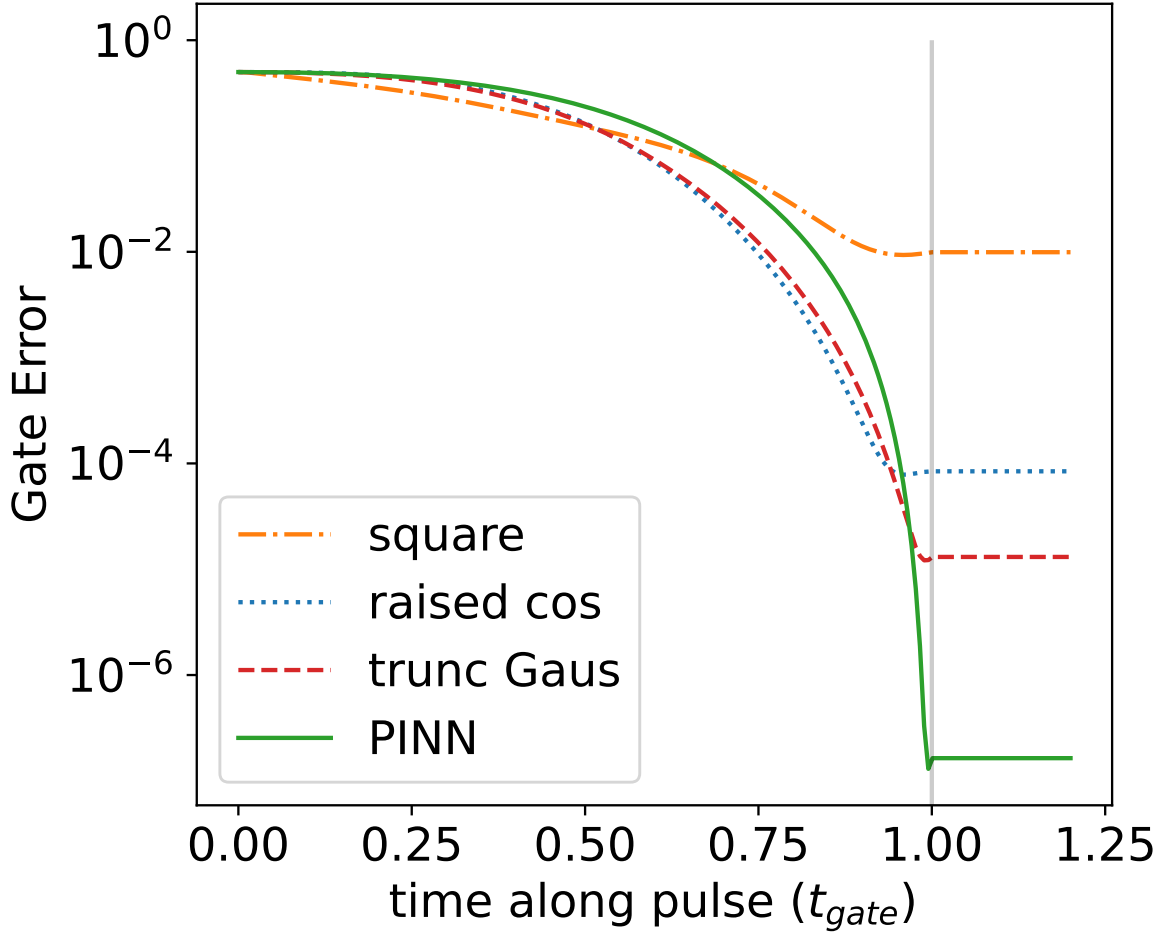


Figure 11.6: Simulation of gate errors from applying a $\pi/2_y$ rotation. Traces of gate error while the pulse is applied are shown for four pulse shapes. Time at 0 corresponds to the start of the pulse and the grey vertical line corresponds to the end of the pulse. Gate error decreases with increasing time as the pulse drives the qubit and the state moves toward the desired state. Leakage and phase errors from higher levels result in imperfect pulses. Pulse shapes which minimize these errors have lower final error. In these simulations, PINN pulses have the smallest error.

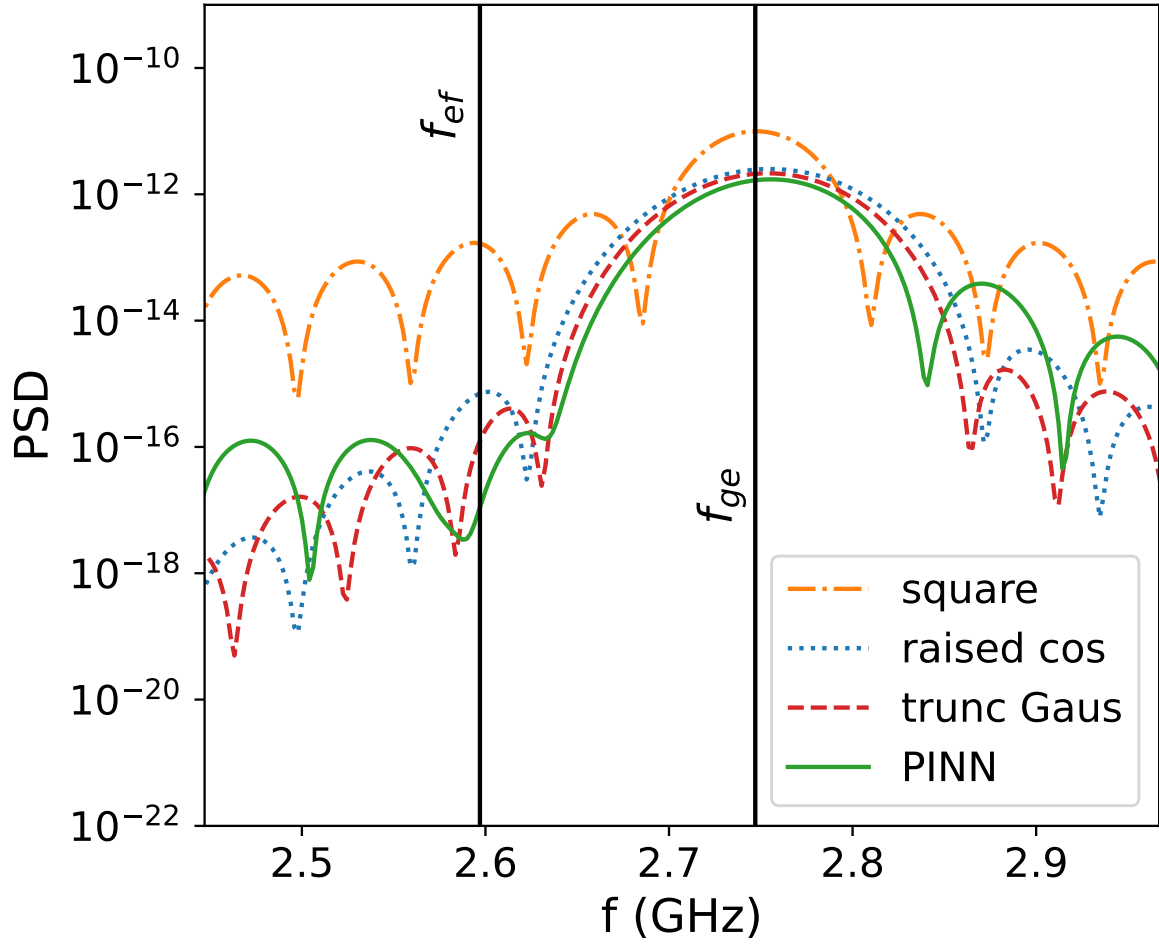


Figure 11.7: PSD of PINN pulse. The frequency components of the pulses simulated in Fig. 11.6 are shown. The PINN pulse has a modified shape to align one of the zeroes with f_{ef} reducing leakage to decrease errors during the pulse.

11.6 Implementation

After the desired pulse shape is designed and verified, uploading the pulse to the AWG and using it for qubit manipulations is as easy as any other pulse shape.

I performed a 3 state normalized randomized benchmarking measurement using $\pi/2$ PINN pulses (section 7.5.4) measuring the g, e, and f state populations after a benchmarking sequence. The PINN pulse had an average error per gate of $(3.36 \pm 0.03) \times 10^{-4}$ and average leakage per gate of $(2.14 \pm 0.056) \times 10^{-5}$ (Fig. 11.8). The measured error per gate is roughly 60 times larger than the simulated error. This is partially due to incoherent errors during the gate, however, at least half of this error cannot be explained by decoherence and instead may be due to pulse distortions, charge dispersion, or other factors not modeled.

For comparison, I took similar benchmarking experiments with truncated Gaussian and raised cosine pulses on the same day. This allows comparing the pulse fidelities while reducing any effects of slow drifts in the system. The 16 ns raised cosine pulse tested had an average error per gate of $(5.54 \pm 0.24) \times 10^{-4}$ and average leakage per gate of $(8.60 \pm 0.39) \times 10^{-5}$. The PINN pulse has better performance. The 16 ns truncated Gaussian pulse tested had an average error per gate of $(3.93 \pm 0.12) \times 10^{-4}$ and average leakage per gate of $(3.24 \pm 0.20) \times 10^{-5}$. The performance of the PINN pulse here is still slightly better than the Gaussian shape. I note that the truncated Gaussian is expected to perform better than the raised cosine while the PINN pulse is expected to perform best from simulations in 11.5.3 and results from section 11.4.

In this experiment, the quadrature component of the PINN pulses was set to the designed ideal amplitude. Performing DRAG calibration measurements (section 7.5.3) measured that phase errors were minimized by scaling the out-of-phase component by 1.12. This small difference from the designed scaling, 1, suggests either a limitation of the DRAG tuning protocol or a small distortion of the pulse reaching the qubit, or perhaps both. I note that the PINN pulse with the quadrature component scaled by this factor performed worse than the ideal PINN pulse. However, in previous measurements where the calibrated quadrature

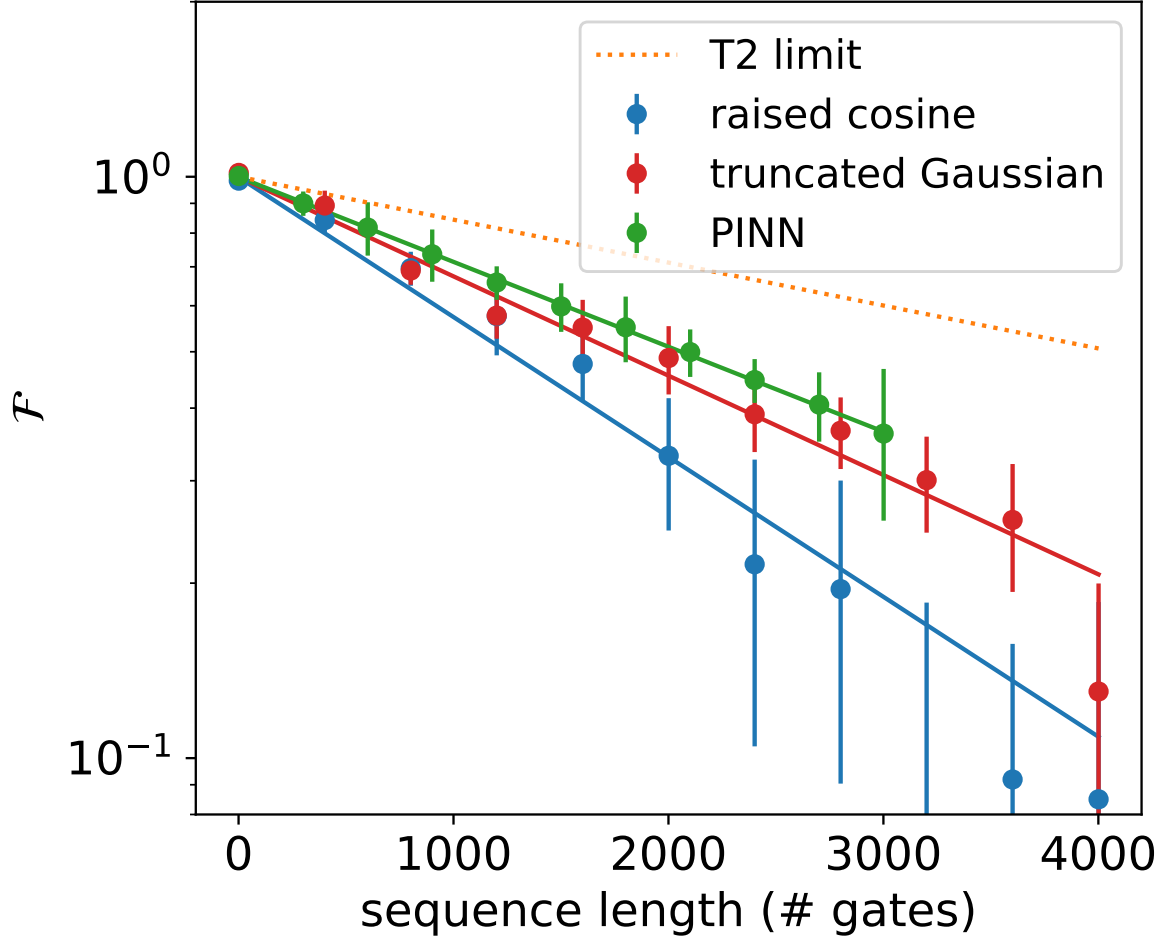


Figure 11.8: Benchmarking results comparison between raised cosine (blue), truncated Gaussian (red), and PINN pulses (green) for 16 ns pulses. Here the experimental sequence fidelity (Eq. 7.13) vs. number of applied gates in the benchmarking gate sequence is plotted for the three different pulse shapes studied. Data points represent the mean fidelity for all the trials performed and error bars are the standard deviations of the trial results at the given gate sequence length. Solid lines are fits for average error per gate. The dotted line shows the decoherence for a given number of gates for a qubit with $T_2 = 100 \mu s$ coherence time. While gate errors are on the same order as the coherence limit, larger coherent errors are observed compared to simulations (Fig. 11.6). Raised cosine pulses have the worst performance at 16 ns, while PINN and truncated Gaussian pulses have similar performance with the PINN pulses performing slightly better.

scaling is even further from 1, the ideal PINN pulse performed worse. More simulation and testing of the pulse behavior is needed to fully understand this discrepancy.

All pulses fall short of the simulated fidelity. Decoherence and tuning inaccuracy will deviate pulse behavior from the optimal performance, but pulse distortion (section 11.7) is also likely partially to blame for the drop in performance. The incoherent error, ϵ_{inc} in Eq. 11.8, is approximately $7.9 \cdot 10^{-5}$ for a 16 ns pulse on this device. Nearly a quarter the error for the truncated Gaussian and PINN pulses can be attributed to decoherence error. A final error source may be related to the excess leakage seen for pulses shorter than 15 ns in section 11.4.

11.7 Pulse Distortion

Nonlinear microwave components [150], standing wave modes [183], and other nonidealities [184] can cause distortion to microwave pulses going to the qubit. Pulse distortion has been observed in many qubit experiments and pulse predistortion has been shown to improve pulse fidelity [178, 183–185]. Because the precise shape of the PINN pulse produces the improved performance, identifying and counteracting pulse distortion may improve the performance of the PINN pulses.

Based on data on devices before **Q1**, reflections may be present in the qubit drive line. Accounting for a reflection from a mismatched component inspired the pulse pre-distortion scheme I designed and outline here. For a desired pulse train $P(t)$ (*e.g.* a train of PINN $\pi/2$ -pulses), I designed a pulse $A(t)$ that will give the desired shape after it is reflected. More formally

$$P(t) = A(t) + bA(t - d) \quad (11.18)$$

for complex reflection amplitude, b , and time delay, d , for the reflection to reach the qubit. The function A is easily solved for by discretizing the time axis into points $t_i = n\Delta t$. Here, for practical purposes, I take the time step, Δt , to be the inverse of the AWG sampling rate

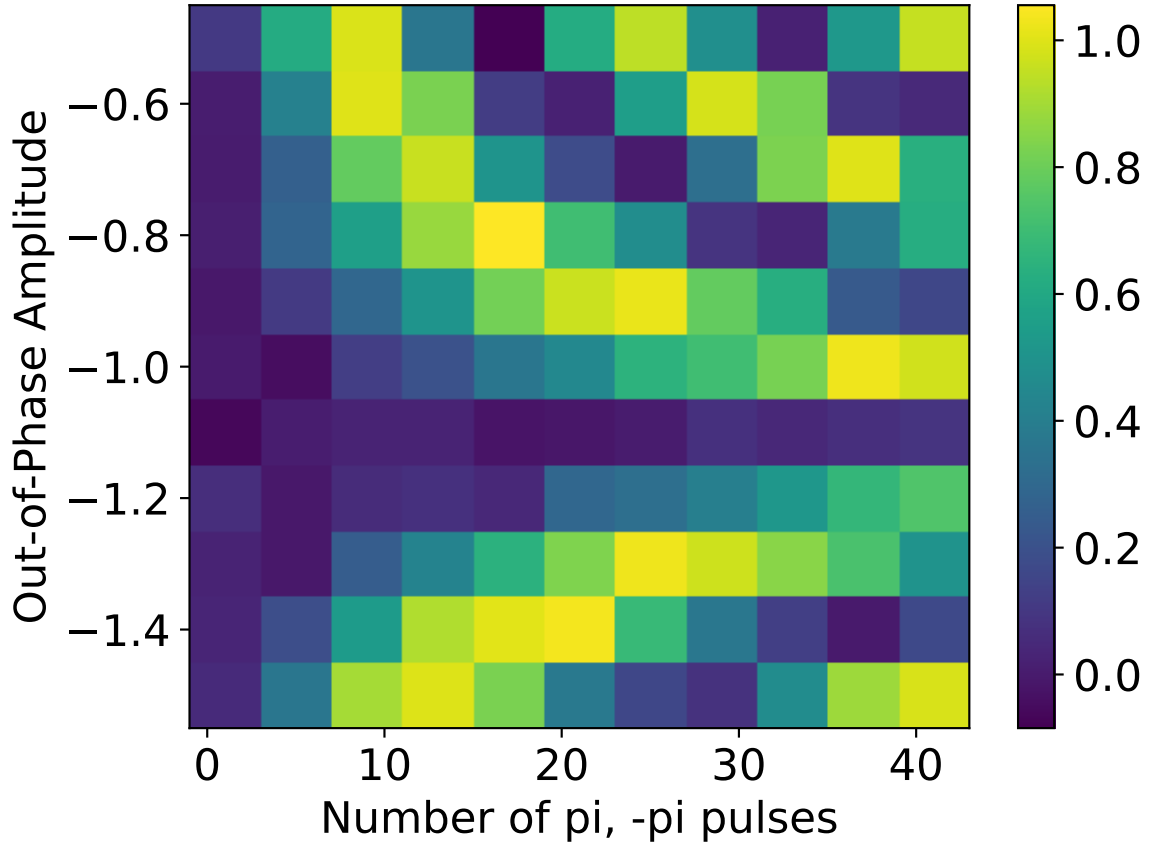


Figure 11.9: DRAG calibration measurement run on 16 ns PINN pulses on device **Q1**. Here measuring the phase errors while sweeping the out-of-phase amplitude can give the ideal amplitude (section 7.5.3). Here the measurement gives a fitted quadrature amplitude of -1.12. The PINN pulse is designed for this scaling value to be -1 for an ideal qubit. The source of this discrepancy could be pulse distortion.

(1 / (2 GHz) in my experiments). The delay, d , must be an integer number of Δt . I then use the following procedure to build $A(t_i)$ at each discrete time point:

1. For the first time point, $t_0 = 0$, set $A(0) = P(0)$
2. Then set each point separated by d inductively by the point before, $A(nd) = P(nd) - b \cdot A((n-1)d)$. Note that the complex b mixes the I and Q components of P .
3. Repeat step 2 for points until nd is greater than the total sequence length.
4. Return to $A(t_1) = P(t_1)$ then repeat step 2 for all $A(nd + t_1)$, stepping n until the end of the sequence is reached
5. Repeat step 4 for increasing initial points until $t_j > d$. At this point $A(t)$ should be defined everywhere P is defined!

An example of this pre-distortion scheme is shown in Fig. 11.10. The scheme accurately reproduces the desired pulse up to an error with length d . The magnitude of the error is proportional to b and the final d points of P , so for short delays and pulses that return to 0 (including all pulses in this chapter), the error is small. Additionally, the error is constant while the pulse sequence length increases. This scheme can be generalized to multiple reflections.

Below is a simple python function to generate the pre-distorted pulse.

```
def build_dist_pulse(P, d, b):
    # gives the predistorted pulse that will give the
    # desired pulse P after a delay of d samples and
    # with complex reflection amplitude b
    P = np.append(P, np.zeros(d))
    A = np.zeros(len(P), dtype = complex)
    for jj in range(d):
        A[jj] = P[jj]
```

```

for n in range(1, len(A)//d-1):
    A[n * d + jj] = P[n * d + jj] - b * A[(n-1) * d + jj]

return A[: -d]

```

11.7.1 Applying Distorted Pulses

By amplifying phase errors for different distortion parameters b and d , I can adjust the optimal DRAG amplitude. However, measured gate error did not improve and, in fact, worsened significantly with measured EPG of $(3.5 \pm 0.4) \times 10^{-3}$, an order of magnitude larger error. This may be due to the fact that the main distortions in the system during this measurement were not reflection errors. Generalized pulse pre-distortion as in [178, 184] may yield better results. Additionally, this pulse distortion scheme could be tested directly by purposefully introducing a mismatch at a defined point in the measurement line.

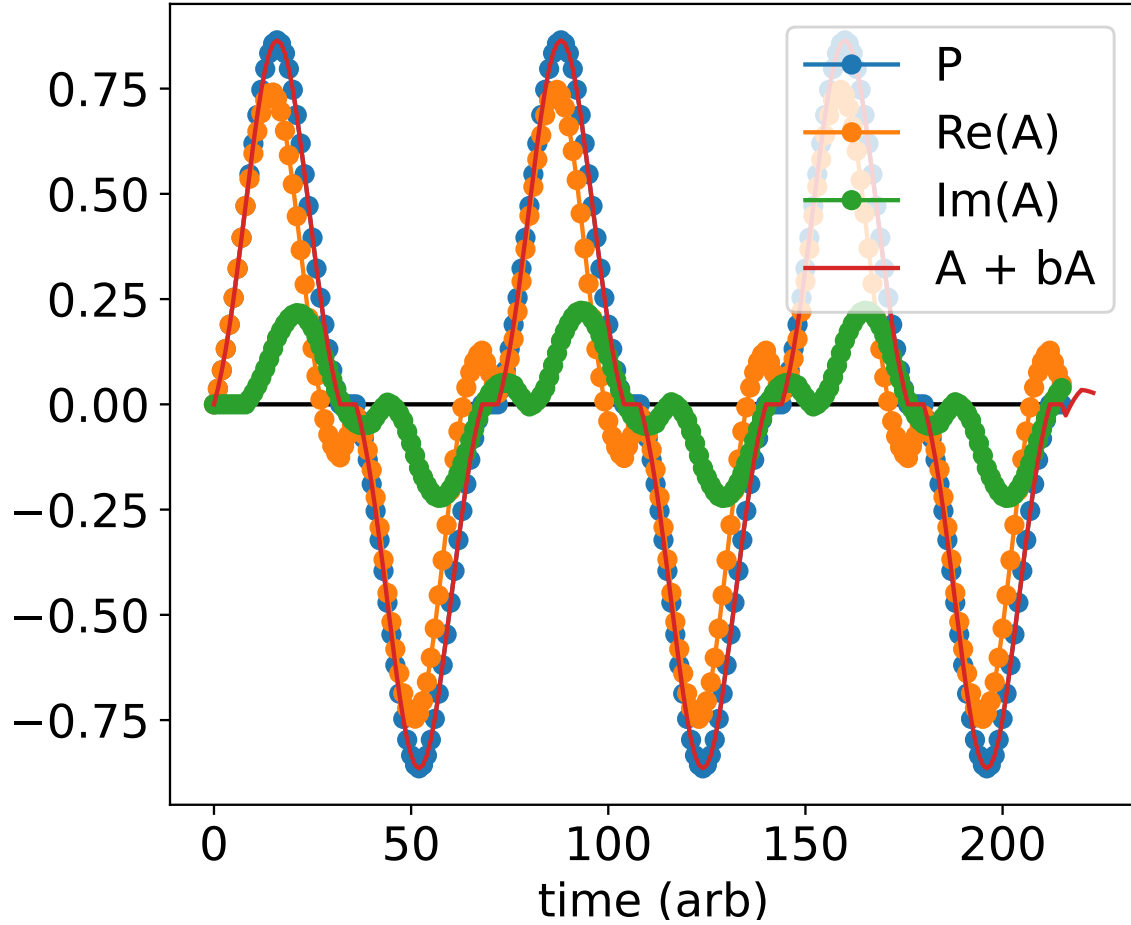


Figure 11.10: Pulse pre-distortion scheme example. Pulse trace amplitudes are shown vs. time. The desired pulse sequence (blue) here is a train of truncated Gaussians with alternating signs. The generated pulse has real (orange) and imaginary (green) components due to complex b . The resulting pulse with added reflection of magnitude $b = 0.445e^{-0.85i}$ and delay $d = 8$ units is shown in red. The red resulting pulse matches the desired pulse exactly everywhere except the end of the pulse sequence.

Chapter 12

Conclusion

There are two main results I have presented in my dissertation. The first result relates to devices where I used Ti to lower the superconducting gap of Al via the superconducting proximity effect and fabricate Josephson junctions with asymmetric electrode gaps. From measurements of Al/AlO_x/AlTi qubits and AlTi resonators, I observed significant loss in these devices compared to comparable all Al devices. I also found increased subgap tunneling when measuring IV characteristics of Al/AlO_x/AlTi Josephson junctions. A likely source of microwave loss comes from oxides forming on the Ti. Ti does not form a self limiting oxide like in Al thin films, allowing thick lossy TiO layers to form on exposed Ti metal. Additionally, the phase diagram of Al, Ti, and O contains many different alloys with unknown properties any or all of which could be a source of loss. Ti has a large affinity for oxygen potentially disrupting the tunnel barrier structure in measured junctions. Although the proximity effect is an effective way of gap-engineering, materials with better properties should be chosen. A possible option is Bi which has no compounds with Al. Bi films should be buried underneath Al because Bi also does not form a self-limiting oxide.

Additional investigation of tunneling in a SQUID device revealed tunneling features independent on the SQUID critical current. These features are consistent with MAR and suggest a junction transparency below 0.005. In this range the junction transparency would

not limit the coherence of transmon qubits. Performing these SQUID measurements with different junctions could accurately characterize their tunneling properties and verify the analysis given here.

The second main result of my dissertation pertains to the use of PINN pulses for precision single-qubit control. Using these designed pulses, I measured improved performance over raised cosine or truncated Gaussian pulse shapes. The PINN pulses had an average error per gate of $(3.36 \pm 0.03) \times 10^{-4}$ and an average leakage per gate of $(2.14 \pm 0.056) \times 10^{-5}$. Gate fidelity was found to be limited by leakage and decoherence. More advance techniques [153, 154, 178] may be able to reduce the pulse leakage and improve fidelity with little extra design work.

Applying the PINN pulse framework to design a single-qubit gate served as a proof of principle. Likely this framework will be even more useful in systems with more complex Hamiltonians such as driving 2-qubit gates or controlling qudit systems of higher dimension.

Appendix A

Appendix - Ta Qubit Coherence

A.1 Motivation

Initial results using Ta as ground plane material for transmon qubits showed improvements in T_1 times [119, 186]. Although recent measurements have achieved similar coherence times in Nb [187] and Al [188] with a better understanding of the metal cleans, Ta is still a material of interest.

In this chapter I present a handful of measurements of Ta qubits I performed or assisted with. The studies of Ta qubits at LPS are still ongoing.

A.2 Ta 3D transmon

For the following experiments, I fabricated a 3D transmon with large capacitor pads made of Ta and an Al/AlO_x/Al junction, **Q2**. This device was packaged and measured as described in section 6.5.

A.2.1 T₁ vs T

Two geometries were measured, 3D transmons in a physical cavity and 2D planar devices. For each geometry, a device with Al and Ta large metal features were measured. All devices had Al/AlO_x/Al junctions. Shown data is from the best device made at LPS of a given geometry and material that was measured. The Ta 3D transmon was device **Q2**. The Al 3D transmon was device **Q3**.

Repeated T_1 measurements were performed (section 7.4.3) for each device as the mixing chamber temperature was swept between base temperature, $T < 20$ mK and $T > 170$ mK. At the highest temperatures, thermal quasiparticles in the junction reduce the measured T_1 time. Behavior of T_1 vs. temperature can potentially be modeled by quasiparticle tunneling in asymmetric gapped junctions [24, 25] or TLS thermal saturation [108, 109].

At base temperature, the 3D devices have longer relaxation times than the 2D devices. Additionally, the Ta devices have slightly longer relaxation times than the Al devices. These measurements are interesting but single device measurements are not sufficient to claim one material being better than another for making superconducting qubits.

A.2.2 Stark-Shifted T₁s

Recent experiments have observed frequency dependent relaxation times in fixed frequency 2D transmons [189, 190]. By applying an AC Stark drive, the effective qubit frequency can be shifted into or out of resonance with a TLS. I performed these measurements for a 3D Ta device.

Any energy level experiences a Stark shift from drive Hamiltonian, H_D , at frequency, ω_c [189, 191],

$$\Delta E_i = \sum_j \left(\frac{|\langle i | H_D | j \rangle|^2}{E_i - E_j - \hbar\omega_c} + \frac{|\langle i | H_D | j \rangle|^2}{E_i - E_j + \hbar\omega_c} \right) \quad (\text{A.1})$$

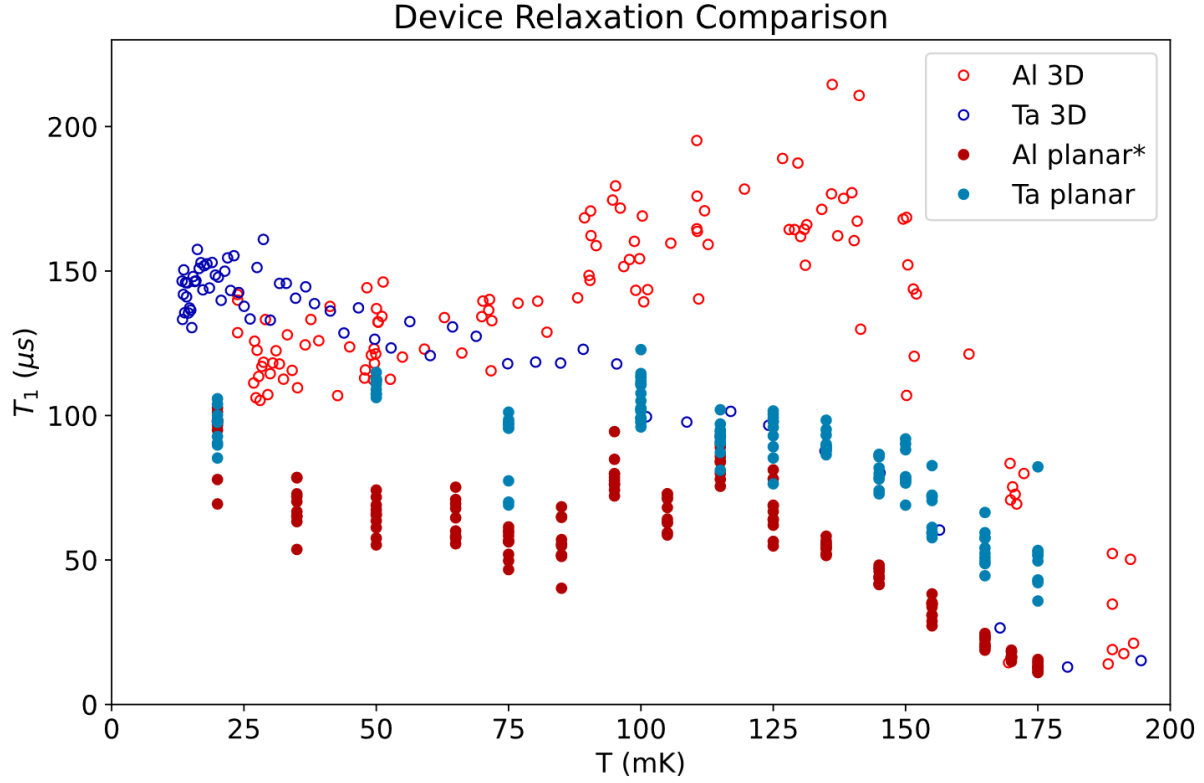


Figure A.1: T_1 relaxation values measured at different temperatures for four different qubits. Overall, 3D relaxation times exceed 2D relaxation times. Relaxation times for Ta devices were greater than Al devices except for the 3D Al device at elevated temperature (see [24]). At temperatures above 140 mK, the relaxation times of all devices decrease due to the increasing density of thermal quasiparticles in the junction. These are measurements of single devices so further devices should be studied before drawing conclusions on material effects.

For a drive given by Eq. 3.31, the shift on the first two energy levels can be written

$$\Delta E_g \approx \frac{\hbar^2 \Omega_q^2 / 4}{E_g - E_e + \hbar \omega_c} = \frac{\Omega_q^2}{4(-\omega_{ge} + \omega_c)} \quad (\text{A.2})$$

$$\Delta E_e \approx \frac{\hbar^2 \Omega_q^2 / 4}{E_e - E_g - \hbar \omega_c} + \frac{2\hbar^2 \Omega_q^2 / 4}{E_e - E_f + \hbar \omega_c} = \frac{\Omega_q^2}{4(\omega_{ge} - \omega_c)} + \frac{2\Omega_q^2}{4(-\omega_{ef} + \omega_c)} \quad (\text{A.3})$$

where the lowest three transmon energies are written, in order, E_g , E_e , and E_f , with qubit frequency, ω_{ge} , second transmon transition, ω_{ef} , and on-resonance Rabi rate Ω_q . In this calculation I have ignored the effects of the coupled cavity.

From these shifts, the qubit frequency is Stark shifted by

$$\Delta \omega_{ge} = \Delta E_e - \Delta E_g = \frac{\Omega_q^2}{4} \left[\frac{1}{-\Delta_S} + \frac{2}{\Delta_S + \alpha} - \frac{1}{\Delta_S} \right] = \frac{-\alpha \Omega_q^2}{2\Delta_S(\Delta_S + \alpha)} \quad (\text{A.4})$$

with the cavity-qubit detuning, Δ_S , and transmon anharmonicity α .

I verified this relation by performing Stark-shifted Ramsey measurements. Fig. A.2a shows the pulse sequence which is identical to the Ramsey measurement (Fig. 7.8c) but with an additional Stark shift drive that is applied for the entire precession of the Ramsey. The qubit frequency is shifted only during the Ramsey precession. Extracting the qubit frequency from the Ramsey oscillation frequency matches Eq. A.4.

I perform interleaved shifted and non-shifted T_1 measurements on a 3D Ta transmon. The measurement pulses used in the two state normalization sequence (section 7.4.1) now consists of two measurements: a T_1 measurement at the shifted qubit frequency and a T_1 measurement at the natural qubit frequency. There is no noticeable difference between the decay rates. I do not observe large changes in decay rate with detuning. This contrasts to data in 2D devices. My 3D transmon may have fewer TLSs strongly coupled to the qubit.

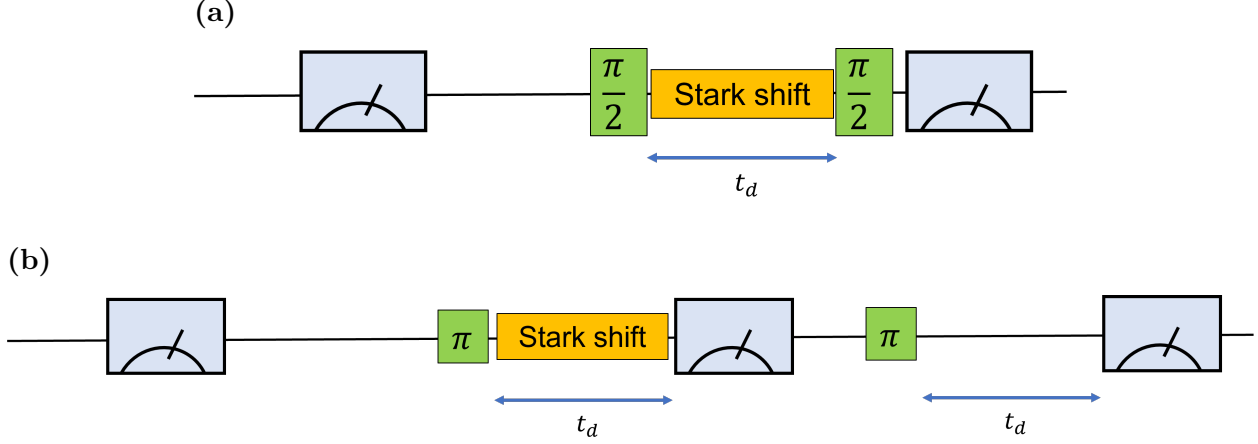
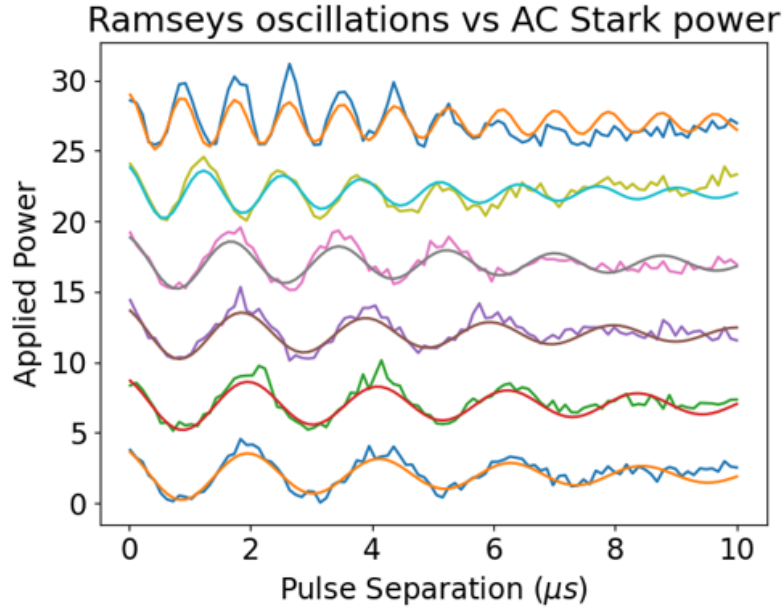


Figure A.2: Pulse sequences for AC Stark-shifted qubit measurements. **a)** The Stark-shifted Ramsey measurement. A shifting affects the qubit during the precession of the Ramsey measurement. **b)** Stark-shifted T_1 measurement pulse sequence. Here two T_1 measurements are taken, one shifted and the other at the transmons natural frequency. The additional measurement accounts for other shifts in the qubit's relaxation.

A.2.3 IR Filter Improvements

I measured the coherence of the same Ta 3D transmon with and without added IR filters on the cavity input and output lines on four separate cooldowns, alternating with the filters added and removed. These filters were inserted to the microwave drive lines on the mixing chamber stage directly before the Ammumetal shields. For each cooldown, after the device had gotten cold, repeated T_1 measurements were taken and extracted decay constants are plotted in Fig. A.4. The average relaxation time of the qubit was significantly longer with the filters in place. Previously, these filters have been used to reduce IR radiation incident on the transmon which has reduced the measured non-equilibrium quasiparticle density [26]. In my device, the addition of the filters led to a direct improvement in T_1 . This suggests radiation and likely quasiparticles are a significant source of relaxation in this device.

(a)



(b)

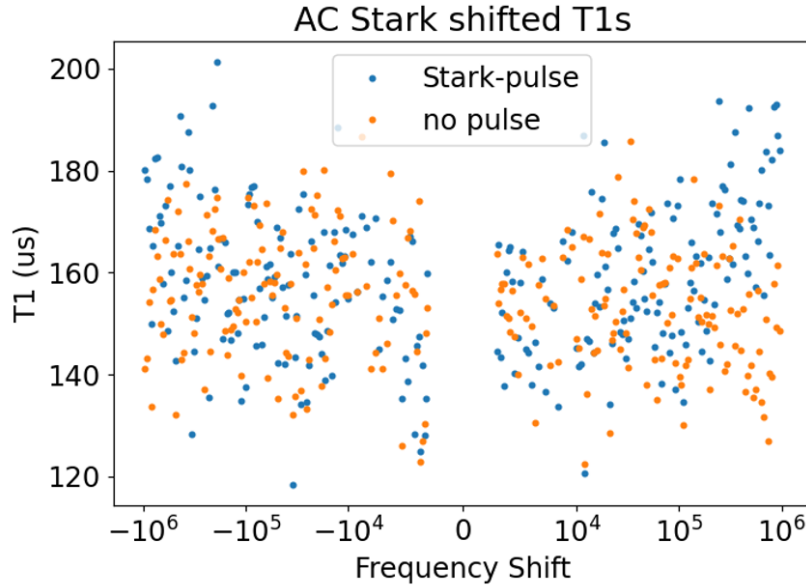


Figure A.3: Stark-shifted data on a Ta 3D transmon, **Q2**. **a)** Stark-shifted Ramsey measurements with increasing shifting powers being offset. As the tone power increases, the detuning of the $\pi/2$ -pulse and the qubit increases leading to higher oscillation frequency. Fitted Ramsey frequencies matches Eq. A.4. **b)** Stark-shifted T_1 s vs. frequency shift. The natural frequency decay rates (orange) show little difference from the shifted decay rates (blue). Strongly coupled nearby TLS are not visible in this measurement in contrast to data in [189, 190].

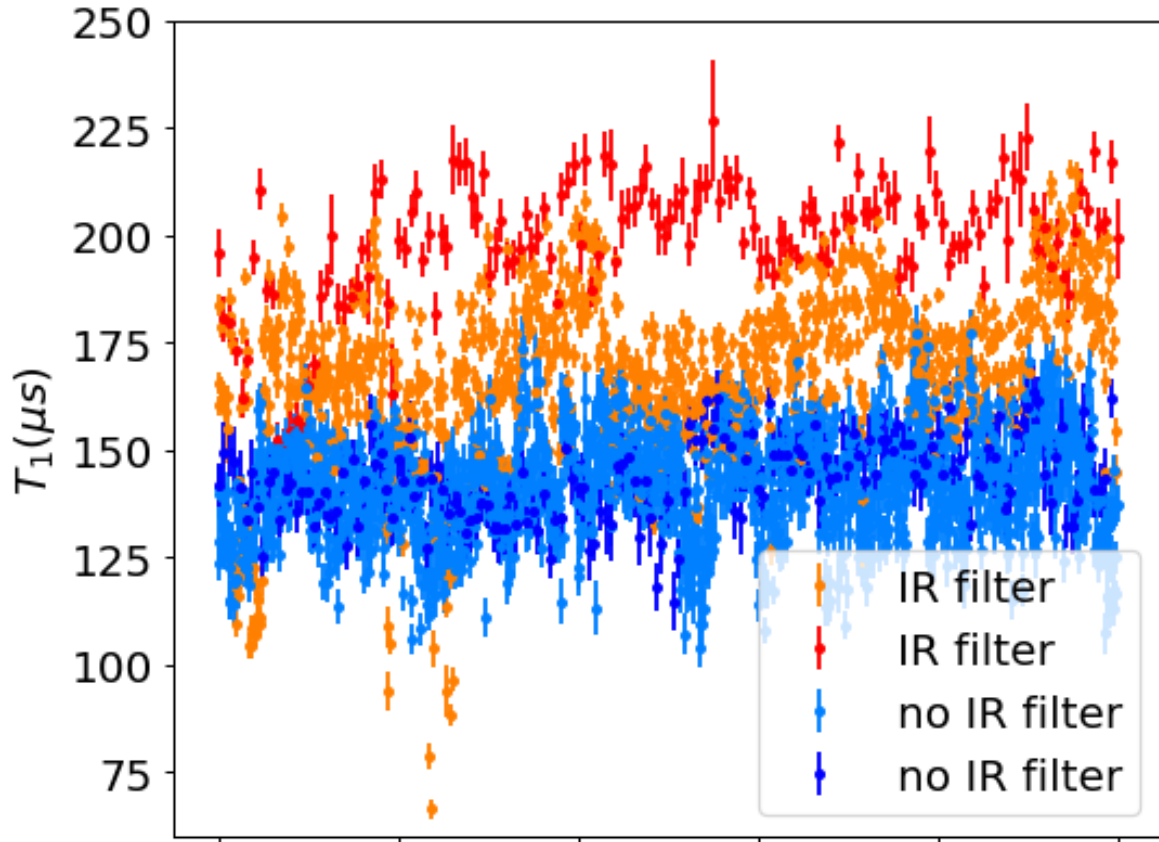


Figure A.4: Repeated T_1 measurements for the Ta 3D transmon, **Q2**. Extracted relaxation time constants are plotted vs. measurement number for repeated measurements taken on the timescale of ≈ 10 hr for each device. The average relaxation times for the filtered qubit are visibly higher. Cooldowns with and without filters were alternated eliminating slow T_1 drift as the source of this difference.

Appendix B

Appendix - Junction Self Heating Model

B.1 Junction Self-Heating

Current in a junction dissipates power

$$P = IV \tag{B.1}$$

For supercurrent, $V = 0$ implying $P = 0$. However, quasiparticle tunneling processes do dissipate power. This energy generated at the junction has the potential to heat the electrodes and modify the junction tunneling characteristics. Specifically, at higher temperatures the superconducting gap decreases with increasing temperature. Various processes can dissipate the heat and remove this power from the junction such as electron-phonon coupling [159, 192] or quasiparticle diffusion [85, 193]. A steady state is established between the junction power and cooling processes and a constant temperature is observed.

B.2 Thermal IV Model

To model current-voltage characteristics accounting for junction self heating, I developed the following model. Before starting, a specific cooling process (or set of processes) is chosen and the the power removed from the system as a function of temperature, $P_c(T)$, is calculated for a relevant range of temperatures (*e.g.* 0 to T_C). Additionally, a tunneling process (or set of tunneling processes) must be chosen (*e.g.* SIS tunneling, MAR theory, etc.)

1. Start at $V = 0, I = 0$
2. Calculate next current value
 - (a) Calculate $P_h = IV$
 - (b) Find T_{eq} where the steady state is reached, $P_c(T_{eq}) = IV$
 - (c) Find $\Delta_{i,j}(T_{eq})$ for the gaps in the junction
 - (d) Calculate $I(V, T_{eq}, \Delta_{i,j}(T_{eq}))$
 - (e) step V by a small amount and repeat step 2
3. Stop when current has been calculated on the desired voltage range

Applying this model to SIS tunnel junctions generally yields very low temperatures in the subgap region, then a large spike in temperature near the normal tunneling current jump. For MAR tunneling with low transparency, the large jump in temperature means the large conductance peak at $eV = \Delta_1 + \Delta_2$ occurs at a lower voltage than the sum of the individual subgap peaks at $eV = \Delta_{1,2}$.

B.3 Cooling Models

Here I introduce three cooling models I examined. Each yield a different cooling function, $P_c(T)$.

B.3.1 Normal Electron-Phonon Coupling

Normal metal electrons can dissipate power through their coupling to the phonons of the material. The total power is given by

$$P_{e-ph} = \Sigma V (T_{el}^5 - T_{ph}^5) \quad (\text{B.2})$$

where V is the volume of material, Σ is the electron-phonon coupling constant, and $T_{el(ph)}$ is the temperature of the electrons (phonons). In Al, $\Sigma \approx 2.5 \times 10^8 \text{W}/\text{m}^3 \text{K}^5$. At base temperature, $T_{ph} = 20 \text{ mK}$. Temperature vs. power is plotted in Fig. B.1.

B.3.2 Quasiparticle Diffusion

Hot quasiparticles will also transfer heat via diffusion. The heat energy flow for any ideal gas follows the thermal gradient, which in one dimension gives

$$j_U = -\kappa \frac{dT}{dx} \quad (\text{B.3})$$

Assuming the quasiparticles are non-interacting electrons they follow Eq. B.3 with thermal conductance

$$\kappa_{el} = \frac{\pi^2 k_B^2}{3m_e v_f} n T l \quad (\text{B.4})$$

where m_e is the electron effective mass, v_f is the Fermi-velocity, n is the number density of quasiparticles, and l is the mean free path in the metal. The total heat power down a conducting lead is then

$$P = -\frac{\kappa_{el} A}{L} \frac{dT}{dx} \quad (\text{B.5})$$

For power generated at the junction, the quasiparticles diffuse down the leads with cross-sectional area, A and length, L . For a electron heat bath at T_{ph} at the transmon pads, the

temperature along the leads and find the equilibrium temperature at the junction

$$T_{el}(x=0) = \sqrt{T_{ph}^2 + \frac{PL}{2\kappa_{el}A}} \quad (\text{B.6})$$

Next, I assume that κ_{el}/nT is constant in temperature and that n follows the thermal distribution

$$n_{thqp}(T) = 4N_0 \int_{\Delta}^{\infty} D(E)f(E,T)dE \quad (\text{B.7})$$

where N_0 is the normal electron density of states at the Fermi energy, D is the superconducting density of states (Eq. 2.13), and f is the Fermi-Dirac distribution.

Numerically solving Eq. B.6 gives a steady-state temperature (Fig. B.1).

B.3.3 Superconducting Electron-Phonon Coupling

Nikolić, *et al.* [192], calculate the thermal conductance per unit volume for electron-phonon coupling in a superconductor. Eschewing the details, for a chosen volume, the total cooling power can be calculated, resulting in a $P(T)$ for electron-phonon coupling in superconductors (see [192] for details). Cooling resulting from electron-phonon coupling in a superconductor has a qualitatively similar shape as electron-phonon cooling in a normal metal (Fig. B.1).

B.4 Examples

Here I apply the junction heating theory to two examples.

First, an Al/AlO_x/Al single junction is shown with MAR theory fitted (Fig. B.2). Here the $eV = \Delta_1 + \Delta_2$ conductance peak occurs near 380 μV however two potential MAR subpeaks occur at $\Delta_1 = 220\mu eV$ and $\Delta_2 = 184\mu eV$. To explain the discrepancy in the sum, the junction could heat to $T \approx 700 \text{ mK}$, reducing the gaps of the electrodes.

Second, the data shown in section 10.2.5 is examined using the heating model. Using the thermal IV theory similarly to above, the measured differential conductance is compared

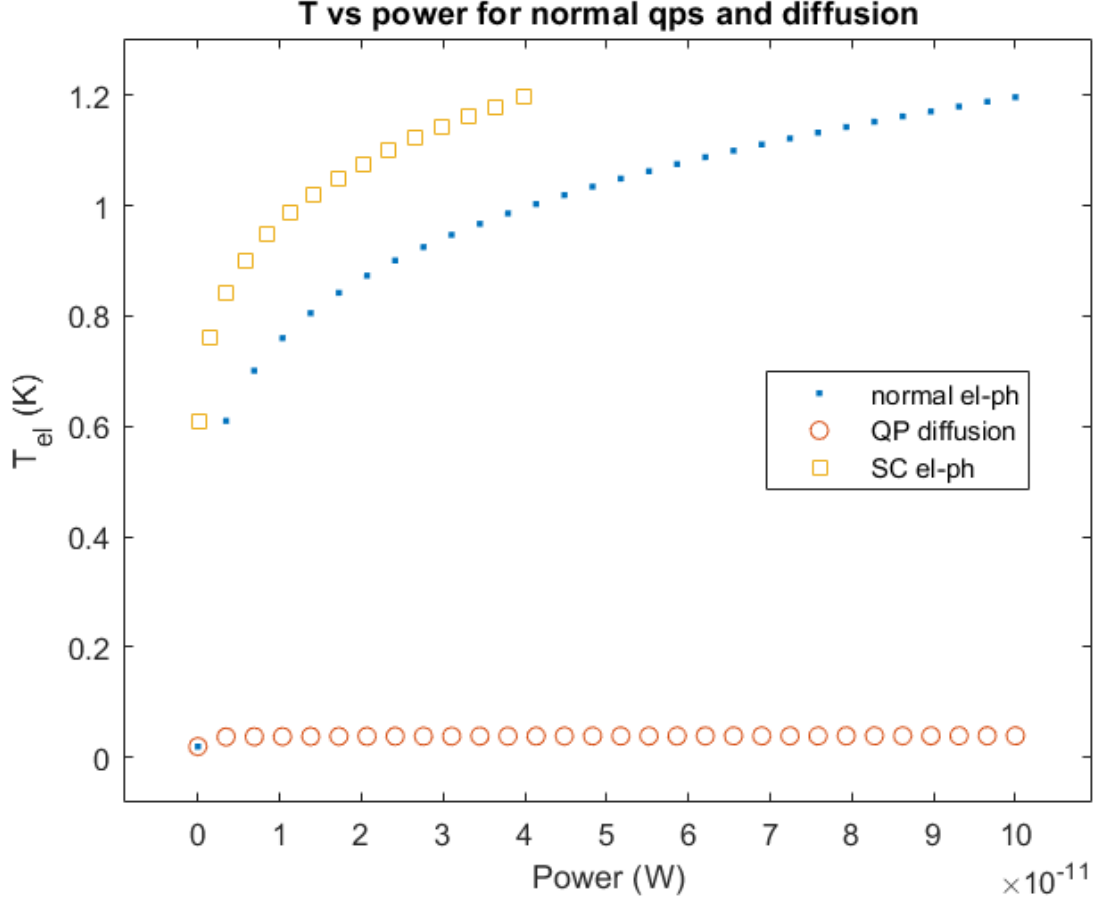
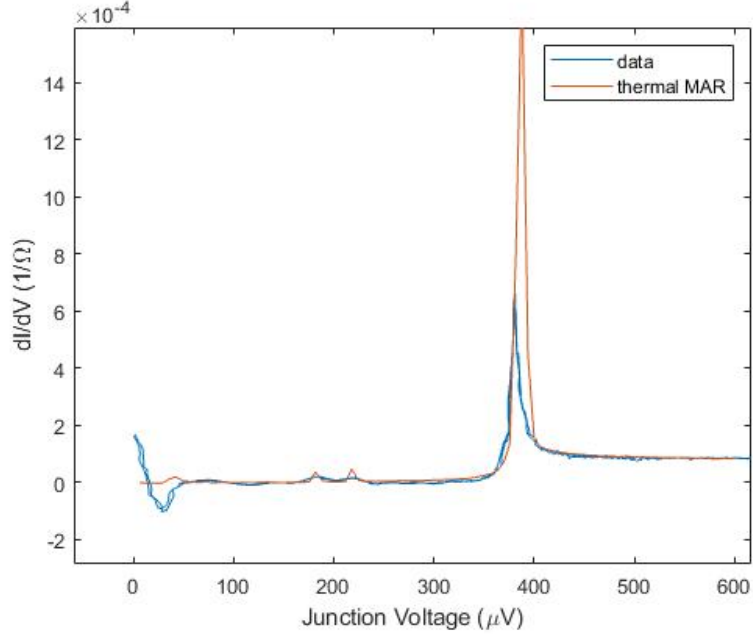


Figure B.1: Comparison of cooling processes considered for thermal IV calculations. Quasi-particle diffusion along a $45 \mu m$ Al lead with $n_{qp} = 10/\mu m^3$ is shown in red circles (Eq. B.6). The number of quasiparticles increases exponentially with temperature greatly limiting the temperature seen by the theory. In reality the assumption likely fails that the transmon pads stay at $20 mK$ when power is generated at the junction. In blue dots is the effective cooling from normal metal electron-phonon coupling (Eq. B.2) in a volume 51 times the junction volume ($V_J = 3.2e - 3 \mu m^3$). Yellow squares show electron-phonon cooling in a superconductor of the same volume from [192]. The cooling power is less than the normal metal as expected.

(a)



(b)

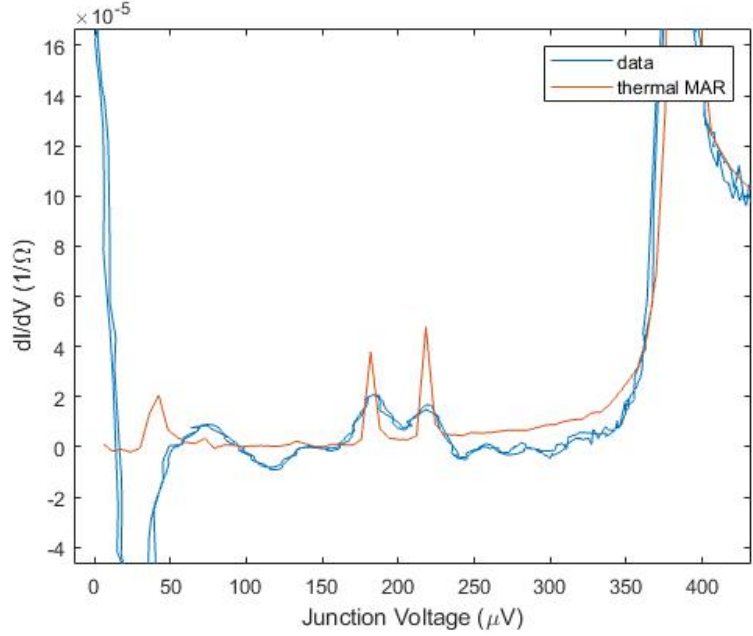


Figure B.2: Differential conductance measurements compared with MAR theory with junction heating. Data and theory are identical in (a) and (b) but are shown on different scales. Subgap conductance peaks and main normal conductance peaks match with the theory. This is not possible for theory with constant $\Delta_i(V)$. In this model, as the dissipated power increases, the junction gaps decrease. Parameters used from this model were Volume = $200V_J = 0.64 \mu m^{-3}$, $D = 0.04$, $\Delta_1(T = 0) = 220 \mu eV$, $\Delta_2(T = 0) = 184 \mu eV$, $R_n = 13.3 k\Omega$. Voltage noise may explain the broader peaks in measured data.

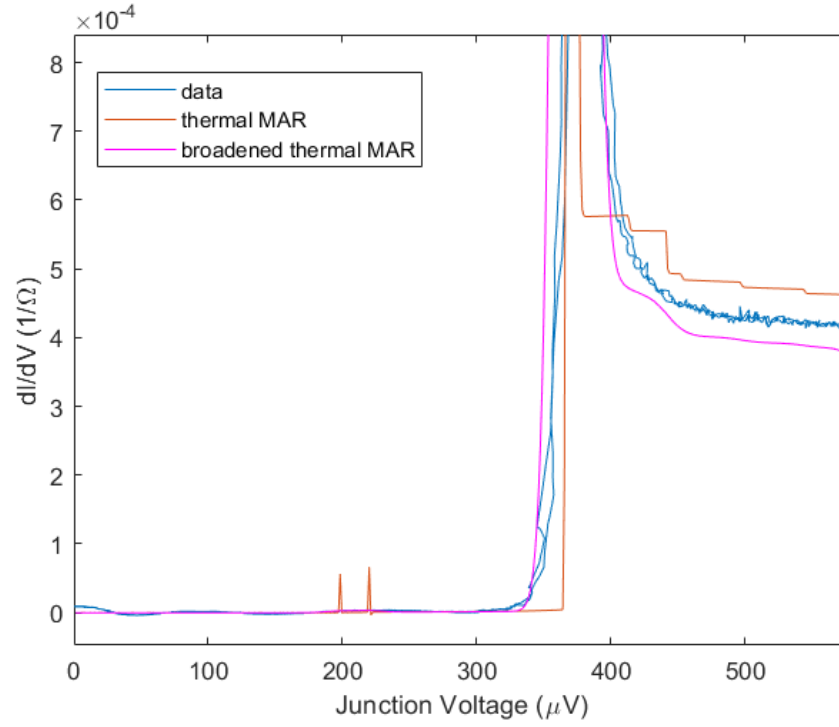
to MAR theory with junction self-heating with and without voltage noise broadening (Fig. B.3). Here I added a power offset of -150 fW for the theory to replicate the low subgap current measured. This power could be due to a diffusion effect.

B.5 Conclusion

In this section I described a model to account for junction self-heating during current-voltage or differential conductance measurements. This model sequentially updates the temperature at the junction for modeled I, V points allowing the tunneling properties of the junction to change with temperature. This model can be used to replicated measured data of two junction measurements shown above.

However, the thermal modeling considered here is very crude and is likely a vast simplification of the thermal behavior in the junction and surrounding metal. This leads to mostly arbitrary cooling volumes being chosen in the models. Potentially more advanced modeling combining both diffusion and electron-phonon cooling could give a more realistic picture. Additionally, because of the iterative solving used by this model, the resulting IV curve depends on the step size of the model. This is not the case in other static models.

(a)



(b)

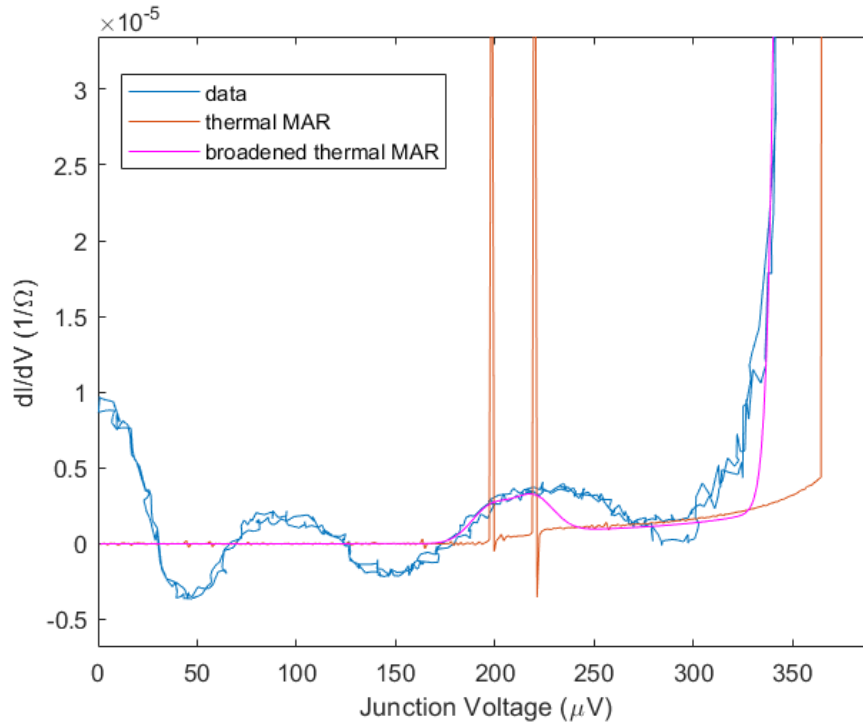


Figure B.3: Junction self-heating model applied to SQUID differential conductance data. Data (blue) shows peaks in similar locations to theory (red). Adding Gaussian voltage noise to the theory (magenta) matches the shape of the data reasonably well. Parameters used in the fit were Volume = $60V_J = 0.19\mu m^{-3}$, $D = 0.002$, $R_n = 2.4 k\Omega$, $\Delta_1(T = 0) = 220 \mu eV$, $\Delta_2(T = 0) = 198 \mu eV$, $P_{offset} = -150 fW$, $\sigma_V = 10 \mu eV$.

Appendix C

Appendix - cQED Calculation and Derivations

Presented here are some more details calculations of calculations in considering cQED systems. Though this is not an exhaustive list and not all aspects are worked out equally, these detailed methods will hopefully give a reader the tools to tackle similar problems.

C.1 Common tools and techniques

C.1.1 Time-Dependent Unitary Transformations

Early students of quantum mechanics (or advanced students of classical mechanics [194]) learn to perform unitary transformations on Hamiltonians

$$H' = U H U^\dagger \tag{C.1}$$

where U is a time independent constant transformation.

However, when working with qubit systems, we need the more general form for time-

dependent transformations (often rotating unitaries). The general form

$$H' = UHU^\dagger - i\hbar\dot{U}U^\dagger \quad (\text{C.2})$$

which clearly reduces to Eq. C.1 for time independent transforms.

The derivation is simple [194]. Beginning with the Schrödinger equation

$$\dot{\psi} = -\frac{i}{\hbar}H\psi \quad (\text{C.3})$$

We would like to rotate state ψ by a unitary U (with $U^\dagger U = \mathbb{1}$ by definition) and calculate how our Hamiltonian H transforms in this rotated frame. To do this, realize what we are really interested in is how our state evolves in our new frame

$$\frac{d}{dt}(U\psi) = U\dot{\psi} + \dot{U}\psi \quad (\text{C.4})$$

by the product rule.

Next, we can find the $U\dot{\psi}$ term by applying U to Eq. C.3, and inserting the identity to get the state $U\psi$ that we want.

$$U\dot{\psi} = -\frac{i}{\hbar}UHU^\dagger U\psi \quad (\text{C.5})$$

Now we can evaluate Eq. C.4 with another convenient identity

$$\frac{d}{dt}(U\psi) = U\dot{\psi} + \dot{U}\psi \quad (\text{C.6})$$

$$= -\frac{i}{\hbar}UHU^\dagger U\psi + \dot{U}U^\dagger U\psi \quad (\text{C.7})$$

$$= -\frac{i}{\hbar}[UHU^\dagger + i\hbar\dot{U}U^\dagger]U\psi \quad (\text{C.8})$$

$$= -\frac{i}{\hbar}H'U\psi \quad (\text{C.9})$$

to get our transformed Hamiltonian

C.1.2 Baker-Campbell-Hausdorff

The Baker-Campbell-Hausdorff (BCH) formula lets us expand many Hamiltonian transformations. It can be derived by applying unitary transformations as described above in C.1.1 by one with much patience.

BCH says

$$e^X Y e^{-X} = \sum_m \frac{1}{m!} \underbrace{[X, [X, \dots, [X, Y] \dots]]}_{m\text{-nested}} \equiv \sum_m \frac{1}{m!} [X, Y]_m. \quad (\text{C.10})$$

Or in cQED it often takes the form

$$H' = U H U^\dagger = e^X H e^{-X} \approx H + [X, H] + \frac{1}{2} [X, [X, H]] \quad (\text{C.11})$$

to lowest order. Here the transformation must be able to be written in the form $U = e^X$.

C.2 Diagonalizing the Transmon Hamiltonian

Starting from Eq. 3.24

$$\hat{H}_q \approx \sqrt{8E_J E_C} (b^\dagger b + \frac{1}{2}) - \frac{E_C}{12} (b^\dagger + b)^4 \quad (\text{C.12})$$

This quartic term can be expanded into all 16 terms, but we can quickly drop terms which have unequal numbers of raising and lowering operators due to the rotating-wave approximation (RWA). When evolving a state with this Hamiltonian in the lab frame, each term will pick up a factor oscillating at the qubit frequency, $\hat{b} \rightarrow \hat{b} e^{-i\omega_q t}$. Terms in $\hat{H}_q$ with equal numbers of raising and lowering operators will have these terms cancel, terms with unequal numbers will have a fast oscillation remain. The RWA states that these fast oscillating terms

will average to 0 for dynamics over a reasonable time. Therefore, under RWA we expand the final term in Eq. 3.24 as

$$(b^\dagger + b)^4 \rightarrow b^\dagger b^\dagger bb + b^\dagger bb^\dagger b + b^\dagger bbb^\dagger + bbb^\dagger b^\dagger + bb^\dagger bb^\dagger + bb^\dagger b^\dagger b. \quad (\text{C.13})$$

We normal order all of them using $[b, b^\dagger] = 1$ or $bb^\dagger = 1 + b^\dagger b$:

$$b^\dagger b^\dagger bb = b^\dagger b^\dagger bb, \quad (\text{C.14})$$

$$b^\dagger bb^\dagger b = b^\dagger b^\dagger bb + b^\dagger b, \quad (\text{C.15})$$

$$b^\dagger bbb^\dagger = b^\dagger bb^\dagger b + b^\dagger b = b^\dagger b^\dagger bb + 2b^\dagger b. \quad (\text{C.16})$$

$$bbb^\dagger b^\dagger = b^\dagger bbb^\dagger + 2bb^\dagger = b^\dagger b^\dagger bb + 2b^\dagger b + 2bb^\dagger = b^\dagger b^\dagger bb + 4b^\dagger b + 2, \quad (\text{C.17})$$

$$bb^\dagger bb^\dagger = b^\dagger bbb^\dagger + bb^\dagger = b^\dagger b^\dagger bb + 2b^\dagger b + bb^\dagger = b^\dagger b^\dagger bb + 3b^\dagger b + 1, \quad (\text{C.18})$$

$$bb^\dagger b^\dagger b = b^\dagger bb^\dagger b + b^\dagger b = b^\dagger b^\dagger bb + 2b^\dagger b \quad (\text{C.19})$$

giving that

$$(b + b^\dagger)^4 = 6b^\dagger b^\dagger bb + 12b^\dagger b + 3. \quad (\text{C.20})$$

allowing us to write the normal-ordered transmon Hamiltonian

$$\hat{H}_q = \left(\sqrt{8E_J E_C} - E_C \right) \left(b^\dagger b + \frac{1}{2} \right) - \frac{E_C}{2} b^\dagger b^\dagger bb = \hbar\omega_q \left(b^\dagger b + \frac{1}{2} \right) - \frac{\alpha}{2} b^\dagger b^\dagger bb \quad (\text{C.21})$$

C.2.1 Validity of Rotating Wave Approximation

We can now look in more detail about the RWA. Defining $\omega_q = \sqrt{8E_J E_C} - E_C$, we transform Eq. 3.24 into the rotating frame

$$H' = U H U^\dagger + i\dot{U} U^\dagger, \quad (\text{C.22})$$

$$\text{where } U = e^{i\omega_q t b^\dagger b} \quad (\text{C.23})$$

The $i\dot{U}U^\dagger$ term cancels with the $\omega_q b^\dagger b$, and conjugating the interaction term with U yields

$$U(b^\dagger + b)^4 U^\dagger = (Ub^\dagger U^\dagger + UbU^\dagger)^4 = (e^{-i\omega_q t} b^\dagger + e^{i\omega_q t} b)^4. \quad (\text{C.24})$$

It is easy to see that, in this frame, the terms with equal numbers of $b^\dagger$ and b do not rotate, whereas the other terms rotate at frequencies $\pm 2\omega_q, \pm 4\omega_q$. We can throw these terms away as long as the coupling constant, $E_C/\sqrt{E_C E_J}$, is “small enough.” To define how small, if we look in the rotated frame at the terms we’re throwing away and do similar normal ordering to above, we get

$$e^{-i4\omega_q t} b^\dagger b^\dagger b^\dagger b^\dagger + 4e^{-i2\omega_q t} b^\dagger b^\dagger b^\dagger b + 6e^{-i2\omega_q t} b^\dagger b^\dagger + e^{i4\omega_q t} b b b b + 4e^{i2\omega_q t} b^\dagger b b b - 6e^{i2\omega_q t} b b \quad (\text{C.25})$$

In order to throw away any particular term proportional to some factor $e^{\pm i\alpha\omega_q t}\beta$ (with $\alpha > 0$ by definition), we need the rotation speed to be much bigger than the coupling constant, or

$$\alpha\omega_q \gg \frac{E_C}{12}\beta \implies \omega_q \gg \frac{E_C}{12} \frac{\beta}{\alpha} \quad (\text{C.26})$$

For small excitation numbers of the transmon, the most stringent condition given in C.25 is $\beta = 6, \alpha = 2$ or $\omega_q \gg E_C/4$. Note that in terms of our “Transmon regime,” this becomes

$$\sqrt{8E_J E_C} - E_C \gg \frac{E_C}{4} \implies \frac{E_J}{E_C} \gg \frac{25}{128} \sim \frac{1}{5} \quad (\text{C.27})$$

$E_J/E_C = 50$ is well within this regime.

C.3 Rotating Driven Qubit Hamiltonian

In 3.3.1, we rotate into the frame of the drive by applying a time-dependent unitary transformation

$$U = e^{i\omega_d t \sum_j j|j\rangle\langle j|} \quad (\text{C.28})$$

giving

$$H' = U H U^\dagger + i \dot{U} U^\dagger \quad (\text{C.29})$$

$$\begin{aligned} &= e^{i\omega_d t \sum_j j |j\rangle \langle j|} \sum_j \hbar \omega_j |j\rangle \langle j| e^{-i\omega_d t \sum_j j |j\rangle \langle j|} \\ &\quad + e^{i\omega_d t \sum_j j |j\rangle \langle j|} \frac{\hbar \Omega_q}{2} (b e^{i\omega_d t} + b^\dagger e^{-i\omega_d t}) e^{-i\omega_d t \sum_j j |j\rangle \langle j|} \\ &\quad + i(\omega_d \sum_j j |j\rangle \langle j|) \end{aligned} \quad (\text{C.30})$$

$$= \hbar \sum_j (\omega_j - j\omega_d) + \frac{\hbar \Omega_q}{2} (b + b^\dagger) \quad (\text{C.31})$$

where the first term in the last line is the first and third lines of the previous expression where the commutation relations are trivial. The second term can be derived by writing

$$b = \sum_k \sqrt{k+1} |k\rangle \langle k+1| \quad (\text{C.32})$$

and collapsing the sums

$$e^{i\omega_d t \sum_j j|j\rangle\langle j|} b e^{i\omega_d t} e^{-i\omega_d t \sum_l l|l\rangle\langle l|} \quad (\text{C.33})$$

$$= e^{i\omega_d t \sum_j j|j\rangle\langle j|} \sum_k \sqrt{k+1} |k\rangle \langle k+1| e^{i\omega_d t} e^{-i\omega_d t \sum_l l|l\rangle\langle l|} \quad (\text{C.34})$$

$$= e^{i\omega_d t \sum_j j+1|j\rangle\langle j|} \sum_k \sqrt{k+1} |k\rangle \langle k+1| e^{-i\omega_d t \sum_l l|l\rangle\langle l|} \quad (\text{C.35})$$

$$= e^{i\omega_d t \sum_j j+1|j\rangle\langle j|} \sum_k \sqrt{k+1} |k\rangle \langle k+1| e^{-i\omega_d t (k+1)|k+1\rangle\langle k+1|} \quad (\text{C.36})$$

$$= \sum_k e^{i\omega_d t (k+1)|k\rangle\langle k|} \sqrt{k+1} |k\rangle \langle k+1| e^{-i\omega_d t (k+1)|k+1\rangle\langle k+1|} \quad (\text{C.37})$$

$$= \sum_k \sqrt{k+1} |k\rangle \langle k+1| e^{i\omega_d t k+1|k+1\rangle\langle k+1|} e^{-i\omega_d t (k+1)|k+1\rangle\langle k+1|} \quad (\text{C.38})$$

$$= \sum_k \sqrt{k+1} |k\rangle \langle k+1| \quad (\text{C.39})$$

$$= b \quad (\text{C.40})$$

as expected, twice using the property $\sum_k \langle k|l\rangle = \delta_{kl}$. The derivation for $b^\dagger$ is similar.

C.4 RWA for JC Hamiltonian

If we assume that the transmon is only a qubit, we have the Jaynes-Cummings Hamiltonian, Eq. 3.37

$$H = \omega_r \left(a^\dagger a + \frac{1}{2} \right) + \frac{\omega_q}{2} \sigma_z + g \sigma_x (a + a^\dagger) \quad (\text{C.41})$$

Assume that $\omega_r \sim \omega_q$ such that $|\omega_r - \omega_q| \ll |\omega_r + \omega_q|$, and also assume that $|\omega_r + \omega_q| \sim 2|\omega_r| \gg g$. This is the rotating-wave approximation (RWA). Then the Transmon qubit Hamiltonian in C.41 becomes

$$H = \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma_z + g(a \sigma_+ + a^\dagger \sigma_-), \quad (\text{C.42})$$

where we have dropped the counter-rotating terms $a^\dagger\sigma_+$ and $a\sigma_-$ which do not conserve particle number (and hence couple different particle-number sectors).

Let's derive this RWA approximation because it is somewhat instructive. We will rotate the Hamiltonian by a unitary U

$$U = e^{it(\omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z)} = e^{it\omega_r a^\dagger a} e^{it\frac{\omega_q}{2}\sigma_z}. \quad (\text{C.43})$$

In this frame,

$$H' = U H U^\dagger + i\dot{U} U^\dagger \quad (\text{C.44})$$

$$= e^{it(\omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z)} \left[\omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z + g\sigma_x(a + a^\dagger) \right] e^{-it(\omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z)} + i(i(\omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z)) \quad (\text{C.45})$$

$$= \omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z + e^{it(\omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z)} g\sigma_x(a + a^\dagger) e^{-it(\omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z)} - (\omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z) \quad (\text{C.46})$$

$$= e^{it(\omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z)} g\sigma_x(a + a^\dagger) e^{-it(\omega_r a^\dagger a + \frac{\omega_q}{2}\sigma_z)} \quad (\text{C.47})$$

$$= g \left(e^{it\frac{\omega_q}{2}\sigma_z} \sigma_x e^{-it\frac{\omega_q}{2}\sigma_z} \right) \left(e^{it\omega_r a^\dagger a} (a + a^\dagger) e^{-it\omega_r a^\dagger a} \right) \quad (\text{C.48})$$

We have used commutativity of various terms in the Hamiltonian/exponential and skipped over the simple calculation of $\dot{U}U^\dagger$. From here, we use the BCH formula to calculate each parenthetical separately. The first we just use the well-known result regarding rotations of Pauli operators to get that

$$e^{it\frac{\omega_q}{2}\sigma_z} \sigma_x e^{-it\frac{\omega_q}{2}\sigma_z} = \cos(\omega_q t) \sigma_x + i \sin(\omega_q t) \sigma_y \quad (\text{C.49})$$

$$= \frac{e^{i\omega_q t} + e^{-i\omega_q t}}{2} \sigma_x + \frac{e^{i\omega_q t} - e^{-i\omega_q t}}{2} i\sigma_y \quad (\text{C.50})$$

$$= e^{i\omega_q t} \frac{\sigma_x + i\sigma_y}{2} + e^{-i\omega_q t} \frac{\sigma_x - i\sigma_y}{2} \quad (\text{C.51})$$

$$= e^{i\omega_q t} \sigma_+ + e^{-i\omega_q t} \sigma_- \quad (\text{C.52})$$

Next, we recall that $[a^\dagger a, a] = -a$ and $[a^\dagger a, a^\dagger] = a^\dagger$ and that BCH says

$$e^X Y e^{-X} = \sum_m \frac{1}{m!} \underbrace{[X, [X, \dots, [X, Y] \dots]]}_{m\text{-nested}} \equiv \sum_m \frac{1}{m!} [X, Y]_m. \quad (\text{C.53})$$

Thus

$$e^{it\omega_r a^\dagger a} (a + a^\dagger) e^{-it\omega_r a^\dagger a} = \sum_m \frac{(it\omega_r)^m}{m!} [a^\dagger a, (a^\dagger + a)]_m \quad (\text{C.54})$$

$$= \sum_m \frac{(it\omega_r)^m}{m!} [a^\dagger a, a^\dagger]_m + \sum_m \frac{(it\omega_r)^m}{m!} [a^\dagger a, a]_m \quad (\text{C.55})$$

$$= \sum_m \frac{(it\omega_r)^m}{m!} a^\dagger + \sum_m \frac{(it\omega_r)^m}{m!} (-1)^m a \quad (\text{C.56})$$

$$= e^{i\omega_r t} a^\dagger + e^{-i\omega_r t} a. \quad (\text{C.57})$$

So in total,

$$H' = g(e^{i\omega_q t} \sigma_+ + e^{-i\omega_q t} \sigma_-)(e^{i\omega_r t} a^\dagger + e^{-i\omega_r t} a) \quad (\text{C.58})$$

$$= g \left(e^{i(\omega_q + \omega_r)t} \sigma_+ a^\dagger + e^{i(\omega_q - \omega_r)t} \sigma_+ a + e^{-i(\omega_q - \omega_r)t} \sigma_- a^\dagger + e^{-i(\omega_q + \omega_r)t} \sigma_- a \right). \quad (\text{C.59})$$

If we now assume that the counterrotating terms, i.e. those with frequency $\pm(\omega_q + \omega_r)$ average out to zero, then we get

$$H'_{\text{RWA}} = g \left(e^{i(\omega_q - \omega_r)t} \sigma_+ a + e^{-i(\omega_q - \omega_r)t} \sigma_- a^\dagger \right). \quad (\text{C.60})$$

Now let's move back to the original frame by using $U^\dagger = e^{-it(\omega_r a^\dagger a + \frac{\omega_q}{2} \sigma_z)}$.

$$H_{\text{RWA}} = U^\dagger H'_{\text{RWA}} U + i\dot{U}^\dagger U \quad (\text{C.61})$$

$$= g e^{-it(\omega_r a^\dagger a + \frac{\omega_q}{2} \sigma_z)} \left(e^{i(\omega_q - \omega_r)t} \sigma_+ a + e^{-i(\omega_q - \omega_r)t} \sigma_- a^\dagger \right) e^{it(\omega_r a^\dagger a + \frac{\omega_q}{2} \sigma_z)} + \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma_z. \quad (\text{C.62})$$

The key is using the BCH formula again on each of the terms. We again have

$$e^{-it\omega_r a^\dagger a} a e^{it\omega_r a^\dagger a} = \sum_m \frac{(-it\omega_r)^m}{m!} [a^\dagger a, a]_m \quad (\text{C.63})$$

$$= \sum_m \frac{(-it\omega_r)^m}{m!} (-1)^m a \quad (\text{C.64})$$

$$= e^{i\omega_r t} a, \quad (\text{C.65})$$

and

$$e^{-it\omega_r a^\dagger a} a^\dagger e^{it\omega_r a^\dagger a} = \sum_m \frac{(-it\omega_r)^m}{m!} [a^\dagger a, a^\dagger]_m \quad (\text{C.66})$$

$$= \sum_m \frac{(-it\omega_r)^m}{m!} a^\dagger \quad (\text{C.67})$$

$$= e^{-i\omega_r t} a^\dagger. \quad (\text{C.68})$$

Also, as a reminder:

$$[\sigma_z, \sigma_\pm] = \frac{1}{2} [\sigma_z, \sigma_x \pm i\sigma_y] \quad (\text{C.69})$$

$$= \frac{1}{2} [\sigma_z, \sigma_x] \pm \frac{1}{2} i [\sigma_z, \sigma_y] \quad (\text{C.70})$$

$$= \frac{1}{2} 2i\sigma_y \pm \frac{1}{2} i(-2i\sigma_x) \quad (\text{C.71})$$

$$= i\sigma_y \pm \sigma_x \quad (\text{C.72})$$

$$= \pm 2\sigma_\pm. \quad (\text{C.73})$$

Thus

$$e^{-it\frac{\omega_q}{2}\sigma_z}\sigma_{\pm}e^{it\frac{\omega_q}{2}\sigma_z} = \sum_m \frac{(-it\omega_q/2)^m}{m!} [\sigma_z, \sigma_{\pm}]_m \quad (\text{C.74})$$

$$= \sum_m \frac{(-it\omega_q/2)^m}{m!} (\pm 2)^m \sigma_{\pm} \quad (\text{C.75})$$

$$= \sum_m \frac{(\mp it\omega_q)^m}{m!} \sigma_{\pm} \quad (\text{C.76})$$

$$= e^{\mp it\omega_q} \sigma_{\pm}. \quad (\text{C.77})$$

Notice that, as expected, the phase associated with each term cancels things perfectly such that we arrive at our result, Eq. 3.37.

$$H_{\text{RWA}} = \omega_r a^\dagger a + \frac{\omega_q}{2} \sigma_z + g(\sigma_+ a + \sigma_- a^\dagger). \quad (\text{C.78})$$

The block-diagonal Hamiltonian can be written in equi-excitation blocks, with all other elements 0.

$$H_{\text{RWA}} = \begin{bmatrix} -\omega_q/2 & 0 & 0 & \dots & & \\ 0 & \omega_q/2 & g & & & \\ 0 & g & \omega_r - \omega_q/2 & & & \\ \vdots & & & \ddots & & \\ & & & & (n-1)\omega_r + \omega_q/2 & g\sqrt{n} \\ & & & & g\sqrt{n} & n\omega_r - \omega_q/2 \\ & & & & & & \ddots \end{bmatrix} \quad (\text{C.79})$$

For a fixed excitation number n , we can diagonalize to find the dressed energies

$$E_{\overline{g,n}} = n\omega_r - \frac{1}{2}\sqrt{(\omega_r - \omega_q)^2 + 4ng^2} \quad (\text{C.80})$$

$$E_{\overline{e,n-1}} = n\omega_r + \frac{1}{2}\sqrt{(\omega_r - \omega_q)^2 + 4ng^2} \quad (\text{C.81})$$

where we have dropped a constant offset of $\omega_r/2$.

C.5 Dispersive JC Hamiltonian

To derive the Jaynes-Cummings Hamiltonian in the dispersive limit, $\Delta = (\omega_r - \omega_q) \gg g$, also called the weak-coupling limit, we start from the Eq. 3.37. This is done via the Schrieffer-Wolff transform. For an outline for Eq. 3.36, the multi-level transmon JC Hamiltonian, see [60]. This approach is also a generalization of the approach in the exercises in [63].

C.5.1 Schrieffer-Wolff Calculations

Here we will detail the Schrieffer-Wolff calculations for the Jaynes-Cummings Hamiltonian (for the qubit). We will follow the prescription detailed in Ref. [74]. The idea is that we have a Hamiltonian H that has multiple sub-blocks, and we want to perturbatively diagonalize H to remove the interactions between sub-blocks. We have

$$H' = U^\dagger H U \qquad H' = e^S H e^{-S}, \quad (\text{C.82})$$

where S is the generator for U . We will write $H = H_0 + H_1$, with H_0 block diagonal and H_1 block off-diagonal, which we assume to have some small coupling associated it. We essentially want to get rid of H_1 perturbatively in this small coupling strength. Say that we can write

$$H = H_0 + H_1 = H_{\text{Block Diagonal}} + H_{\text{Off-Diagonal}}, \quad (\text{C.83})$$

such that H_0 separates us into sectors of interest and H_1 couples these sectors. If we look at H' using BCH, we see that

$$H' = \sum_{n=0}^{\infty} \frac{1}{n!} [S, H]_n = \underbrace{H_0 + H_1}_{n=0} + \underbrace{[S, H_0] + [S, H_1]}_{n=1} + \underbrace{\frac{1}{2}[S, [S, H_0]] + \frac{1}{2}[S, [S, H_1]]}_{n=2} + \sum_{n=3}^{\infty} \frac{1}{n!} [S, H]_n. \quad (\text{C.84})$$

Let's imagine that we select $[S, H_0] = -H_1$. Then we get

$$H' = H_0 + H_1 + (-H_1) + [S, H_1] + \frac{1}{2}[S, (-H_1)] + \frac{1}{2}[S, [S, H_1]] + \dots \quad (\text{C.85})$$

$$= H_0 + \frac{1}{2}[S, H_1] + \frac{1}{2}[S, [S, H_1]] + \dots \quad (\text{C.86})$$

The selection $[S, H_0] = -H_1$ ensures that S and H_1 are the same order. Thus, we get that $H' = H_0$ up to second order corrections in S . That is, we have effectively rotated out the first order contribution of H_1 to H . Additionally, everything after and including the third term is third order or higher in S . Thus, to second order, we have that

$$H_{\text{eff}} = H_0 + \frac{1}{2}[S, H_1]. \quad (\text{C.87})$$

After the RWA, the single-qubit transmon Hamiltonian is

$$H = \underbrace{\omega_r a^\dagger a + \frac{1}{2}\omega_q \sigma_z}_{H_0} + \underbrace{g(a^\dagger \sigma_- + a \sigma_+)}_{H_1}. \quad (\text{C.88})$$

It will be useful to tabulate a few commutation relations for later convenience

$$[a, a^\dagger] = 1, \quad [a^\dagger a, a] = -a, \quad [a^\dagger a, a^\dagger] = a^\dagger \quad (\text{C.89})$$

$$[\sigma_z, \sigma_+] = 2\sigma_+, \quad [\sigma_z, \sigma_-] = -2\sigma_-, \quad [\sigma_+, \sigma_-] = -\sigma_z \quad (\text{C.90})$$

Now, we can use our commutation relations to derive

$$\eta = [H_0, H_1] = \omega_r g a^\dagger \sigma_- - \omega_r g a \sigma_+ - g \omega_q a^\dagger \sigma_- + g \omega_q a \sigma_+. \quad (\text{C.91})$$

We know, then, that S exists in the span of the two operators that compose η , i.e.

$$S = A a^\dagger \sigma_- + B a \sigma_+. \quad (\text{C.92})$$

From here, we need to determine A and B . To do that, we use $[S, H_0] = -H_1$ such that

$$[S, H_0] = -A \omega_r a^\dagger \sigma_- + B \omega_r a \sigma_+ + A \omega_q \sigma_- a^\dagger - B \omega_q a \sigma_+ = -g a^\dagger \sigma_- - g a \sigma_+ = -H_1. \quad (\text{C.93})$$

Thus, we see that

$$A = \frac{g}{\omega_r - \omega_q}, \quad (\text{C.94})$$

$$B = \frac{g}{\omega_q - \omega_r} = -A. \quad (\text{C.95})$$

Having actually determined A and B , it remains to determine H_{eff} by calculating $[S, H_1]$. Note that this is a bit more complicated because it will require us to use the commutator formula

$$[AB, CD] = A[B, C]D + [A, C]BD + CA[B, D] + C[A, D]B. \quad (\text{C.96})$$

Specifically, we have that

$$[S, H_1] = Ag[a^\dagger \sigma_-, a \sigma_+] + Bg[a \sigma_+, a^\dagger \sigma_-] = 2Ag[a^\dagger \sigma_-, a \sigma_+]. \quad (\text{C.97})$$

We have that

$$[a^\dagger \sigma_-, a \sigma_+] = \underbrace{a^\dagger [\sigma_-, a] \sigma_+}_{=0} + \underbrace{[a^\dagger, a] \sigma_- \sigma_+}_{=-\sigma_- \sigma_+} + \underbrace{a a^\dagger [\sigma_-, \sigma_+]_{=a a^\dagger \sigma_z}}_{=a a^\dagger \sigma_z} + \underbrace{a [a^\dagger, \sigma_+] a}_{=0} \quad (\text{C.98})$$

$$= -\sigma_- \sigma_+ + a a^\dagger \sigma_z, \quad (\text{C.99})$$

$$[S, H_1] = \frac{2g^2}{\omega_r - \omega_q} (-\sigma_- \sigma_+ + a a^\dagger \sigma_z) = \frac{2g^2}{\omega_r - \omega_q} (-\sigma_- \sigma_+ + \sigma_z + a^\dagger a \sigma_z). \quad (\text{C.100})$$

We then observe that

$$\sigma_- \sigma_+ = |0\rangle \langle 1| 1\rangle \langle 0| = |0\rangle \langle 0| = \frac{\mathbb{1} + \sigma_z}{2} \quad (\text{C.101})$$

to express

$$[S, H_1] = \frac{2g^2}{\omega_r - \omega_q} \left(\frac{1}{2} \sigma_z - \frac{\mathbb{1}}{2} + a^\dagger a \sigma_z \right) = \frac{g^2}{\omega_r - \omega_q} (\sigma_z - \mathbb{1} + 2a^\dagger a \sigma_z) = \frac{g^2}{\omega_q - \omega_r} (\mathbb{1} - \sigma_z - 2a^\dagger a \sigma_z). \quad (\text{C.102})$$

Thus,

$$H_{\text{eff}} = H_0 + \frac{1}{2} [S, H_1] = a^\dagger a \left(\omega_r + \frac{g^2}{\omega_q - \omega_r} \sigma_z \right) + \frac{\sigma_z}{2} \left(\omega_q - \frac{g^2}{\omega_q - \omega_r} \right) \quad (\text{C.103})$$

$$= a^\dagger a (\omega_r + \chi \sigma_z) + \frac{\sigma_z}{2} (\omega_q - \Lambda) \quad (\text{C.104})$$

We have now arrived at the dispersive Jaynes-Cummings Hamiltonian, Eq. 3.40. Writing for clarity the Lamb shift

$$\Lambda = \chi_{0,1} = -\frac{g^2}{2\Delta} \quad (\text{C.105})$$

and the dispersive shift

$$\chi = -\frac{g^2}{\Delta} \quad (\text{C.106})$$

To see the Lamb shifts on the cavity, one can take more terms in the expansion.

Appendix D

Appendix - Measurement Code in Python

D.1 Purpose of Measurement Code

Rutherford's gold foil experiment is taught to every highschool science student, but (allegedly) all the data was taken graduate students hand counting detector traces for hours or days or months. Luckily, modern experiments have a huge amount of automation built in. In our qubit experiments, the measurement computer and measurement code can setup a dozen instruments to administer a complex pulse sequence, record thousands of individual measurements, and analyze this raw data to tell us something about the qubits all in under a second. Due to the complexity, variety, and speed that qubit measurements can take it is important to have good control code. What "good control code" means in the context of superconducting qubit experiments has many parts:

- Be able to accurately and quickly setup and analyze a given experiment
- Be able to easily setup a diverse range of experiments and have the implementation of new experiments be straightforward

- Be reproducible and reliable
- Be intuitive to tweak, adjust, and rerun
- Be easy to understand and learn for new users
- etc.

Although the Palmer lab python control code certainly does not fully achieve all of these goals, hopefully some of these motivations can be seen in the code structure and can inspire any future changes and improvements.

In the end, the code should allow a user to sit at the computer and run the experiments they need to run.

D.2 Basic Flow of a Measurement

This section will give a general overview of how a measurement works using different types of files and what is input and output. Specifics and examples will be provided in later sections.

To run a measurement, we run the *driver file*. This “drives” the measurement, pulling all the other pieces of code together and executing the measurement. This file starts by importing various packages and an important `measurement parameters` file. This stores the parameters for the measurement like the frequencies, powers, pulse lengths, etc. This should specify instruments that will be used, identified with their remote addresses, and the settings we will apply to those instruments. Each instrument is associated with its own *instrument class* which translates the settings we send into commands the instrument can follow. How the file system is setup is shown in Fig. D.1.

Next, any specific settings to this specific measurement are loaded into the settings dictionary. At this point we have a list of the instruments and settings to apply but haven’t actually applied them yet.

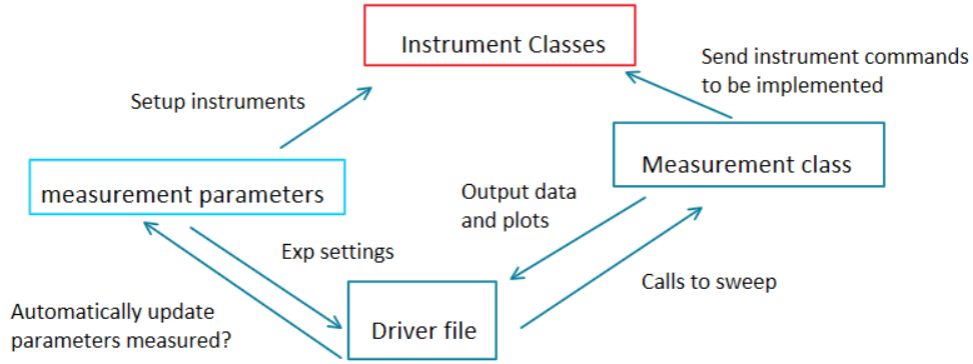


Figure D.1: Outline of the code hierarchy and how files interact. The instrument classes are written for each instrument used and contains translations from object properties to SCPI commands written to the instruments. The Measurement class holds instruments and settings, applies settings, sweeps and acquires data, demodulates, plots, saves, and outputs results. It sends commands to the instruments during operations. The measurement parameters is a setup/device specific file that holds the parameters shared between measurements. This file also sets up the instrument classes for any instrument being used in a measurement. The driver file is the file actually run. It uses the measurement parameters to load the experimental settings, then uses the measurement class to do a variety of sweeps. In the future the driver files may update the measurement parameter file as we do device characterization.

In general, the settings in the driver file may need to be set for each measurement (e.g. specify the length of the Rabi scan). Settings in the measurement parameters should only be changed when something is updated about the measurement (e.g. the qubit frequency is remeasured by Ramsey giving a new, more precise value).

To do this we create a *measurement object* from the **Measurement** class. This object is setup to help perform the measurement. It has functions to apply settings, sweep parameters, make plots, and more. In general, how we will use it will go as follows: create a Measurement object, change relevant object parameters (e.g. `verbose = True`), set parameters, sweep parameter, optionally repeat or do more sweeps or analysis.

The measurement object will by default generate plots of what is being swept. More advanced plots have to be created manually. The measurement object will automatically save the data at the specified path for further analysis!

D.3 Example

Next, we will go through an example driver file and the files it uses. We start with the file `T1_mp.py`. This is a driver file for doing a T_1 measurement using measurement parameters.

First a quick overview of the experiment and setup will help explain the purpose of the code and steps we are trying to implement. A T_1 measurement involves applying a π pulse, waiting a variable delay time, and measuring the qubit state (see 7.4.3). We need to implement this T_1 measurement while repeatedly measuring the g and e states for normalization. We have a cavity and local oscillator generator setup for heterodyne cavity readout 6.4, a qubit generator for applying π pulses, and two delay generators for triggering the pulses. Lastly, we have a DAC card in the computer to digitize, average, and save the data.

The driver file begins with a brief comment header and imports. The first three are standard python packages and the last two import the Measurement class and the measurement parameters file.

```
import numpy as np
import time
import os
from pulsed_measurements.Measurement import Measurement
import measurement_parameters.mp_e2_nov_2023 as mp
# import parameters from the mp file
settings, instruments = mp.get_settings(apply_settings=True)
```

Next, we call `get_settings` which imports the settings and instruments listed in the measurement parameters file.

```
# import parameters from the mp file
settings, instruments = mp.get_settings(apply_settings=True)
```

Specifically, these are the settings which are the same between multiple measurements, like the readout frequency and power, pi pulse length and power, etc. The output of this

function is two dictionaries. The first is settings which is a dictionary of dictionaries, each one holding the settings for one instrument. This keep the settings better organized but requires an extra index to access. The `sweep` dictionary contains settings not used by a specific instrument but instead that describe the sweep we are doing. Often these are used to calculate settings for instruments.

Back in the measurement file, the import of the global measurement parameters is followed by setting a few measurement specific settings for the T_1 sweep.

```
settings['sweep']['filename'] = "T1_scan_"
settings['sweep']['save_path'] = *folder*
settings['sweep']['save_formats'] = ['csv', 'pkl']
settings['sweep']['start'] = 0
settings['sweep']['stop'] = 600e-6
settings['sweep']['step'] = 2.5e-6
settings['sweep']['repeated'] = False
```

These setup the filename (which will be appended with a timestamp by default), the folder to save in, the formats to save the data, the delay times for the T_1 , and if to repeat the measurement on loop. These will be slightly adjusted for different measurements. For example, a Ramsey measurement will need a detuning parameter which will need to adjust the `qd_ssg['freq']` setting.

Next, we need to setup the pulses on the delay generators. These are currently used in all our experiments run with home electronics (e.g. not quantum machines, QICK, etc.). They trigger and gate the generators to precisely time the pulses sent to the qubits. In the `measurement_parameters` file, we setup the first delay generator which triggers the cavity pulses and we call `cddg`.

```
# cavity pulse generator settings
cddg_settings['burst']['count'] = 3
# time between individual pulses.
```



```

cddg_settings['burst']['period'] = 900e-6
# 1 / time between bursts of pulses.
cddg_settings['trig_rate'] = 1 / (cddg_settings['burst']['count'] \
    * cddg_settings['burst']['period'] + 5e-6)
cddg_settings['A'] = sweep_settings['cavity_pulse_delay']
cddg_settings['B'] = sweep_settings['cavity_pulse_length']
# setup EF for triggering DAQ card
cddg_settings['E'] = sweep_settings['cavity_pulse_delay']
cddg_settings['F'] = sweep_settings['cavity_pulse_length']

```

This sets up 3 pulses to measure g , e , and T_1 . The pulses happen every 900 μs with a 5 μs buffer for the DAQ card processing to work. This is our typical normalization setup to remove any measurement drift from the setup at the expense of longer measurement times. This cavity delay generator should be setup to trigger another delay generator for the qubit pulses, called `qddg`.

The positioning of the pulses is unique for each measurement because it determines what experiment we are doing. For this reason, the pulse setup for the `qddg` need to be specified in each driver file. For the T_1 measurement the pulse setup looks as follows

```

# setup pulses
e_pulse_delay = settings['sweep']['cavity_pulse_delay'] \
    + 2 * settings['cddg']['burst']['period'] \
    - settings['sweep']['qubit_pulse_delay'] \
    - settings['sweep']['pi_pulse_length']
settings['qddg']['C'] = e_pulse_delay
settings['qddg']['D'] = settings['sweep']['pi_pulse_length']
settings['qddg']['A'] = e_pulse_delay \
    - settings['cddg']['burst']['period']
settings['qddg']['B'] = settings['sweep']['pi_pulse_length']

```

```

# set other pulses to overlap and not interfere
settings['qddg']['E'] = e_pulse_delay
settings['qddg']['F'] = settings['sweep']['pi_pulse_length']
settings['qddg']['G'] = e_pulse_delay
settings['qddg']['H'] = settings['sweep']['pi_pulse_length']

```

Here we setup the CD pulse to do the e state normalization and the AB pulse to do the T_1 measurement while the other pulses do nothing.

Next, we have to specify what settings will be swept

```

# setup sweep
num_steps = (settings['sweep']['stop']
             - settings['sweep']['start']) \
             \ settings['sweep']['step'] + 1
delay_range = np.linspace(settings['sweep']['start'],
                           settings['sweep']['stop'], int(num_steps))
sweep_insts = [instruments['qddg']]
sweep_params = ["A"]
sweep_range = [settings['cddg']['burst']['period'] \
               + settings['sweep']['cavity_pulse_delay'] - delay_range \
               - settings['sweep']['qubit_pulse_delay'] \
               - settings['sweep']['pi_pulse_length']]

```

In this section we generate a range of delays for the π pulse based on the parameters at the beginning of the file and the alignment for the `qddg`. Next we set the very important parameters `sweep_insts`, `sweep_params`, and `sweep_range`. These will be passed to the sweep function later so must be setup precisely. The `sweep_insts` is a list of instrument handles, here the `qddg` but it can be any number of instruments. The `sweep_params` is a

list of parameters that need to be swept. The first parameter must be a setting on the first instrument, etc. If multiple parameters are swept on a single instrument, pass that instrument multiple times in the `sweep_insts` list. Finally, `sweep_params` is a list of the values to sweep over. Each entry in this list is a iterable (list, numpy array, etc.) of values that the corresponding setting will be looped through. Again, the first iterable will be applied to the first setting and first instrument, etc. Keep in mind this is a 1D sweep. Multiple settings will all be moved together. This is to allow measurements like Rabis where both the beginning and length of a pulse must be swept together. It does not enable 2D sweeps, instead these are implemented within a driver file.

Now finally we can perform the sweep.

```
# Perform Sweep
t1_meas = Measurement(instruments , settings)
t1_meas.verbose = True
t1_meas.print_each_n = 100
t1_meas.set_default_params()
t1_meas.set_all_params()
time.sleep(2)
data_dict = t1_meas.sweep(sweep_insts , sweep_params , sweep_range)
```

In the first line of code, we create an object we name `t1_meas` from the `Measurement` class. This object performs all of the actual operations and parameter sweeping. But first, we change a few of the class parameters, here `verbose`, `print_each_n`. These and others can be adjusted for user preference and improved transparency of code operation.

Next, we set default parameters. This has a predefined list of parameters for specific instruments coded within the `Measurement` class. These are only settings which never need to change except in rare circumstances. After, we set the remaining parameters which tries to apply settings to every instrument in our `instruments` dictionary with the corresponding `settings` dictionary.

Finally, we pause for 2 seconds to make sure the instruments have settled, then we perform the sweep! The last line sweeps the specified parameter(s), takes data, does demodulation and some basic analysis, plots and saves data. Additionally, the results will be available in python in the `data_dict` dictionary after the measurement is complete.

Additionally, at the end of this file we have a while loop for repeated T_1 measurement.

```
if settings[ 'sweep' ][ 'repeated' ]:
    t1_meas.verbose = False
    t1_meas.print_each_n = 100
    sweep_num = 1
    while True:
        print(f"Sweep_{sweep_num}_completed")
        t1_meas.make_plots = True
        try:
            data_dict = t1_meas.sweep(sweep_insts ,
                                      sweep_params , sweep_range)
        except KeyboardInterrupt:
            print("Stopping_via_keyboard_interrupt")
            break
        sweep_num = sweep_num + 1
```

Having a loop like this is useful for any experiment we may want to repeatedly measure. Alternatively, if you want a 2D sweep of two parameters, building a for loop along the second parameter is the best way to do this. The Measurement class also helps do complicated measurements where the result of the first measurement (e.g. qubit frequency) determine the setup of the next measurement (e.g. QP tunneling measurement).

D.4 Instrument Classes

Here we go over a sample instrument class `AgilentE8257D` used to control one of the rf generators. Additionally, we go into the `SCPIinst` class for more understanding on how the instruments are setup.

To work in the control code framework and be easily controlled with the `Measurement` class, each instrument needs a class which allows us to make objects from that class. These objects have properties which correspond to the instruments address on the local network and it's settings. By setting and getting these properties, we set and read the corresponding settings on the instrument. For most instruments, this is most easily done by inheriting from the parent class `SCPIinst`. This is a parent class written by Martin for making instrument classes which communicate via SCPI commands. This class overwrites the python inbuilt `__setattr__` and `__getattr__`. The new functionality means setting object attributes to a value assigns that value to the physical instrument. For example, if I want to set the generator frequency to 5.1 GHz , I write `qd_ssg.freq = 5.1e9` and I am done!

Simple classes may be easier to implement by specifying getters and setters for the necessary properties (e.g. the `AttenuatorBox` class). More complicated classes can take advantage of the `SCPImodule` and `SCPIproperty` classes which are related to `SCPIinst` but are more modular. An instrument class using these is `WX2184C`, the slow AWG controller.

Below is the entirety of the `AgilentE8257D` class. It is fairly short to control everything we need for this generator, due to the usefulness of the `SCPIinst` class.

```
class AgilentE8257D(SCPIinst):
    errorcmd = 'SYST:ERR?'
    commandlist = {}
    commandlist['core'] = {}
    commandlist['Pulse'] = {}
```

```

core = {}
core[ 'Freq' ] = 'FREQ:CW' # in Hz
core[ 'Power' ] = 'POWER' # in dBm
core[ 'Output' ] = 'OUTP:STAT' # ON, OFF, 0, 1
core[ 'ALC' ] = ':POW:ALC' # ON, OFF, 0, 1

Pulse = {}
Pulse[ 'delay' ] = ':PULM:INT:DEL' # delay of pulse
Pulse[ 'pulse_width' ] = ':PULM:INT:PWID' # width of the pulse
Pulse[ 'source' ] = ":PULM:SOUR"
Pulse[ 'output_state' ] = "OUTP:STAT"
Pulse[ 'pulse_mode_state' ] = "PULM:STAT"
Pulse[ 'mod' ] = "OUTP:MOD"

commandlist[ 'core' ] = core
commandlist[ 'Pulse' ] = Pulse

def __init__(self, address, reset = True):
    super().__init__(address,
        self.commandlist, self.errorcmd, reset = reset)

def pulse_mode_init(self, internal_trigger = False):
    # set SSG to be triggered by the external slow AWG
    self.inst.write("OUTP:STAT_ON")
    self.inst.write("PULM:STAT_ON")
    self.inst.write("OUTP:MOD_ON")
    if internal_trigger:

```

```

        self.inst.write(":PULM:SOUR:INT_TRIG")
        self.inst.write(":PULM:SOUR_INT")
    else:
        self.inst.write(":PULM:SOUR_EXT")

```

In the first line class definition, we inherit from `SCPIinst`. Then we need to define an error command and a list of core commands for the instrument. The `core` dictionary has the commands for the instrument with keys corresponding to case-sensitive attributes for using those commands. The commands themselves are always taken out of the remote programming guide for the instrument. These SCPI commands are a fairly universal way of controlling scientific instruments remotely. After making a generator object from this class, running `gen.freq = 5.1e9` will set that attribute and send the SCPI command to the generator "FREQ:CW 5.1e9" which will set the generator frequency. Typing `gen.freq` will get the attribute and query the instrument with "FREQ:CW?" which will return the current generator frequency.

Next, we add pulsed commands in a separate command module, `Pulse`. These are additional commands which share a common theme, in this case, pulse mode. These function similarly but are set or gotten with a command like `gen.Pulse.source` specifying the module.

In the class, the `core` and `Pulse` command dictionaries are then added to the command list. Next, we have our initialization which just calls the initialization for `SCPIinst`. This initialization requires the command list and error command we setup above.

Finally, this class has a class method `pulse_mode_init` which runs a series of commands to setup pulse modulation for the generator. We call this in the measurment class under `set_default_settings`. Otherwise the generator will be outputting in CW.

D.5 Prerequisites

On a new setup, a few things will need to be done.

This code is managed on a local git repository in the 1202S server. Regularly syncing the code with this repo makes sure the code is the same between measurement setups and is backed-up in case we lose a machine. To do this download and run the Git installer. Follow steps usually with all default options. Check by running "Git Bash" in the start menu

This code is written in python. To run it use any python setup you like. For beginners, I recommend Anaconda. Simply run the installer and this will give you a python install and access to Spyder and Jupyter as well as the other packaged programs.

The code is dependent on a few public python packages (like numpy, pyvisa, etc.). To install these run the "conda install *package*" command in an anaconda prompt terminal, or check the package installation guide online.

Next, clone the local git repository (currently at //1202S/GIT/control-code). Run the config file to add the control code to your python PATH. After this the code should be good to run!

After making modifications or additions to the code. Please push the changes to the repository. Refer to the tutorial from Git to learn how, specifically the Making Changes section.

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