ABSTRACT

Title of Dissertation: ESSAYS ON PRODUCTION NETWORKS AND INTERNATIONAL MACROECONOMICS

Alvaro Nicolas Silva Uribe
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Dissertation Directed by: Professor Şebnem Kalemli-Özcan
Department of Economics

This dissertation includes three chapters on the role of production networks in (international) macroeconomics.

In the first chapter, "*Inflation in Disaggregated Small Open Economies*," I study the consequences of these production networks for our understanding of inflation in small open economies. I show that the production network alters the elasticity of the consumer price index (CPI) to changes in sectoral technology, factor prices, and import prices. Sectors can import and export directly but also indirectly through domestic intermediate input-output linkages. Indirect exporting dampens the inflationary pressure from domestic forces, such as adverse sectoral technology shocks and increases in factor prices. In contrast, indirect importing increases the inflation sensitivity to import price changes. Computing these CPI elasticities requires knowledge of the production network structure as these do not coincide with typical sufficient statistics used in the literature, such as sectoral sales-to-GDP ratios (Domar weights), factor shares, or imported consumption shares. Using input-output tables, I provide empirical evidence that adjusting CPI

elasticities for indirect exports and imports matters quantitatively for small open economies. I then use the model to illustrate the importance of production networks during the recent COVID-19 inflation in Chile and the United Kingdom.

In Chapter 2, "*Business Cycle Asymmetry and Input-Output Structure: The Role of Firm-to-Firm Networks*" co-authored with Jorge Miranda-Pinto and Eric R. Young, we study the network origins of business cycle asymmetries using cross-country and administrative firm-level data. At the country level, we document that countries with a larger number of non-zero intersectoral linkages (denser networks) display a more negatively skewed cyclical component of output. At the firm level, firms with more suppliers and customers display a more negatively skewed distribution of their output growth. To rationalize these findings, we construct a multisector model with input-output linkages. We show that the relationship between output skewness and network density naturally arises once we consider non-linearities in production. In an economy with low production flexibility, denser production structures imply that relying on more inputs becomes a risk that further amplifies the effects of negative productivity shocks. On the contrary, when firms display high production flexibility, having more inputs to choose from becomes an opportunity to diversify the effects of negative productivity shocks. We calibrate the model using our rich firm-to-firm network Chilean data and show that (i) more connected firms experience larger declines in output in response to a COVID-19 shock, and (ii) the cross-sectional distribution of output growth in the model displays a fatter left tail during downturns. The previous result is shaped by the interplay between production complementarities and network interconnectedness rather than by the asymmetry of the shocks. The size of the shock determines the strength of the relationship between degrees and output decline, highlighting the importance of non-linearities and the limitations of local approximations.

In Chapter 3, "*Commodity Prices and Production Networks in Small Open Economies*" co-authored with Petre Caraiani, Jorge Miranda-Pinto, and Juan Olaya-Agudelo, we study the role of domestic production networks in the transmission of commodity price fluctuations in small open economies. We provide empirical evidence of strong propagation of commodity price changes to quantities produced in domestic sectors that supply intermediate inputs to commodity sectors (*upstream* propagation) and muted propagation to sectors using commodities as intermediate inputs (*downstream* propagation). We develop a small open economy production network model to explain these transmission patterns. We show that the domestic production network is crucial for shaping the propagation of commodity prices. The two key mechanisms that rationalize the evidence are (i) the foreign demand channel and (ii) the input-output substitution channel. These two channels amplify the *upstream* propagation of commodity price changes by increasing the demand for non-commodity inputs, and, at the same time, they mitigate the *downstream* cost channel by allowing firms to use relatively cheaper primary inputs in production.

ESSAYS ON PRODUCTION NETWORKS AND INTERNATIONAL MACROECONOMICS

by

Alvaro Nicolas Silva Uribe

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Advisory Committee:
Professor Şebnem Kalemli-Özcan, Chair
Professor Pierre de Leo
Professor Thomas Drechsel
Ph.D. Julian di Giovanni
Professor Liu Yang

Dedication

A Luna y mi familia.

Acknowledgments

We always want to write this kind of text because it means we are closing a stage in our lives. To me, the motivation is the opposite: I would like to thank many people for helping me open this new door. Let me get to it.

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Chapter 1: Inflation in Disaggregated Small Open Economies

1.1 Introduction

In 2022, inflation reached 8 percent in the United States, its highest level in 40 years. The picture was similar on the other side of the Atlantic: Euro Area inflation was 8.4 percent, the highest since its creation. Explanations include shocks to commodity prices (Blanchard and Bernanke, 2023; Gagliardone and Gertler, 2023), sectoral demand changes (Ferrante et al., 2022), fiscal stimulus (Bianchi et al., 2023; di Giovanni et al., 2023b), and supply chain disruptions (di Giovanni et al., 2022, 2023a; Comin et al., 2023). As shown in Figure 1.1, high inflation was not restricted to these two economies: the median small open economy experienced an inflation rate of around 10 percent in 2022. However, inflation in this group of countries has been less studied during the current episode. This paper attempts to fill this gap using both theory and data.

My starting point is the multi-sector and multi-factor production network closed economy model in Baqaee and Farhi (2019b). It provides a useful benchmark to analyze inflation during macroeconomic shocks such as COVID-19, a combination of sectoral and aggregate shocks. Given my focus on small open economies, I augment this model to feature imports and exports at the sectoral level, adapting the production network model to the small open economy case. I use the model to study how the consumer price index (CPI) reacts to changes in sectoral technology, factor prices, and import prices, going from the micro to the macro level.

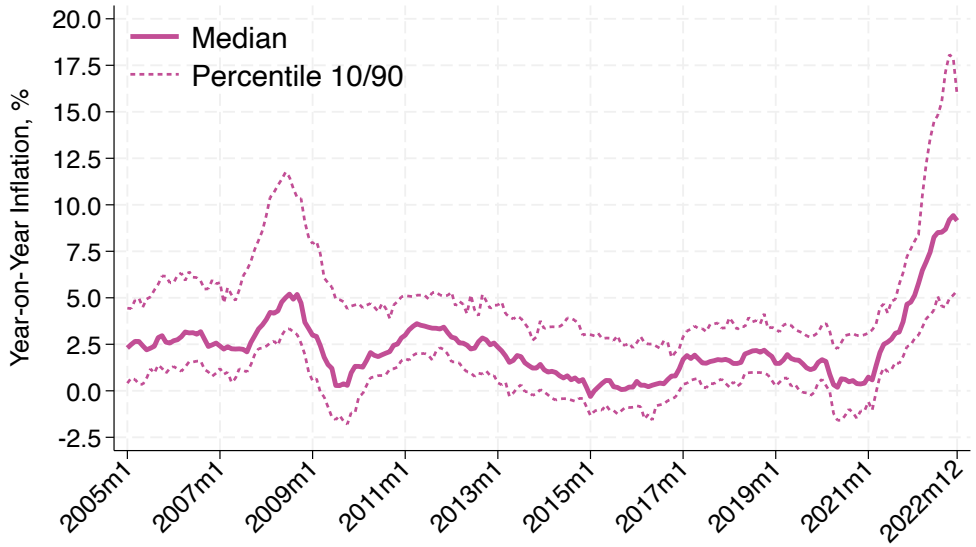
I show that openness and production networks affect our understanding of inflation in small open economies via two distinct channels. On the one hand, exporting, either directly or indirectly through other economic producers, dampens the effect of sectoral technology shocks and factor price changes relative to a closed economy. On the other hand, direct importing gives rise to the problem of imported inflation as the domestic consumer's basket now contains imported goods. On top of this channel, production networks imply that domestic goods are manufactured using imported inputs indirectly. As a result, production networks amplify imported inflation.[1] Uncovering these effects and quantifying their importance is only possible when both openness and network linkages are explicitly considered.

The key economic intuition is that opening up the economy is one of the ways to break the link between what the country produces and what is consumed by domestic consumers. In an efficient closed economy — an economy without distortions — with intersectoral linkages and domestic final consumption only, everything produced is consumed by the domestic consumer. *Network-adjusted domestic consumption*, by which I mean domestic consumption adjusted by domestic production network linkages, is thus equivalent to sales in the closed economy.[2] That domestic households consume everything produced, directly or indirectly, is one of the key building blocks of why the production network structure is irrelevant to first-order for macroeconomic outcomes such as real GDP or welfare in closed economies. This irrelevance result is a useful benchmark. It allows us to use the ratios of sectoral sales to nominal GDP (the so-called "Do-

[1] This channel is distinct from inflation resulting from imported intermediate goods, as models can have intermediate goods without intersectoral linkages. See Svensson (2000) for an early analysis of imported inflation via intermediate goods.

[2] This definition is deliberately reminiscent of the network-adjusted labor share introduced in Baqaee (2015).

Figure 1.1: CPI Inflation in Small Open Economies.



Note: The figure shows the median inflation rate (solid lines) and 10th and 90th percentile (dashed lines). Small open economies are economies that represent less than 5 percent of world GDP and have a trade openness larger than 30 percent of GDP. See section 1.3.1 for more details. Plot shows an unbalanced panel of 46 small open economies over time. Source: Bank for International Settlements.

mar weights") and factor payments to nominal GDP (factor shares) as sufficient statistics for the pass-through of sectoral technology changes or factor price changes to the CPI, respectively.[3] Increases in sectoral technology decrease consumer inflation by the Domar weight of the sector, while increases in factor prices increase inflation by the factor share.

I show that this irrelevance result no longer holds for consumer inflation in small open economies, without the need for second-order approximations, as in Baqaee and Farhi (2019b),

[3] As I show in the theory section, this can be thought of as a corollary of Hulten's theorem (Hulten, 1978a) but for the CPI rather than for real GDP. Recall that for real GDP, Hulten's theorem states that in an efficient closed economy with inelastic factor supplies, the first-order effect of sectoral technology on real GDP is given by the Domar weights, and the first-order effect of changes in factor supply is given by its factor share.

or distortions, as in Baqaee and Farhi (2020) and Bigio and La'o (2020a).[4] The reason is as follows. Consider first the impact of sectoral technology shocks on the CPI. In a small open economy, there are two final uses for goods produced within borders: domestic consumption or exports. Unlike the closed economy case, sectoral sales do not map to the network-adjusted domestic consumption for two reasons: (i) direct exports and (ii) indirect exports through domestic production network linkages. Instead, sectoral sales map to network-adjusted domestic consumption *plus* network-adjusted exports. Since what matters for the CPI is network-adjusted domestic consumption, one must subtract network-adjusted exports from sectoral sales. Hence, relative to a closed economy, a consumer is weakly less exposed to changes in sectoral technology. Importantly, we require knowledge of the domestic production network structure to compute these network-adjusted domestic consumption measures.

A similar intuition holds for how factor price changes affect CPI. The relevant statistic here is the *network-adjusted domestic factor share*: how much of each factor is embedded in goods consumed domestically after considering domestic production network linkages (in the spirit of the domestic factor demand concept in Adao et al. (2022)). The total amount of a factor available in the economy can be "consumed" by domestic or foreign consumers,[5] with the production network potentially reshaping these patterns. While factor shares are sufficient statistics in the closed economy, in the small open economy with production networks we need to subtract from

[4]Strictly speaking, those papers I cited sought ways to break Hulten's theorem in closed economies, which refer to quantities meaning the effect of sectoral technology changes or distortions on real GDP. However, since, as a corollary of Hulten's theorem, we can back out changes in CPI, I referenced them here.

[5]Here, consumers do not directly consume factors but goods. Given that goods are ultimately made of factors of production, we can think of consumers implicitly consuming them. This notion can be found in the reduced factor demand system proposed by Adao et al. (2017).

factor shares the fraction of each factor that is exported either directly or indirectly via production networks. This means that relative to a closed economy, the domestic consumer is weakly less exposed to changes in factor prices.

The effect of import prices on the CPI, on the other hand, is amplified in a small open economy with production networks relative to a small open economy without production networks. The relevant statistics here are *network-adjusted import consumption shares*. Since the domestic consumer directly imports, the direct consumption share captures part of the exposure to import price changes. However, if there are intersectoral linkages across producers, domestic good producers may end up importing intermediate inputs either directly, by buying from abroad, or indirectly, by buying from domestic sectors that buy from abroad or that buy from sectors that buy from abroad, and so on. This means that the imported content of domestically produced goods increases in the presence of production networks. To the extent that domestic goods increase their reliance on imported intermediate goods, so does the domestic consumer. Thus, the domestic consumer's exposure to import prices must account for both direct and indirect exposure, which are encapsulated in the *network-adjusted import consumption shares*.

Guided by the model, I turn to the data to measure the importance of these production network adjustments. I find that these adjustments matter quantitatively using data from the World Input-Output Tables. I illustrate these adjustments by focusing on the three sources of variation I have considered so far: sectoral technology, factor prices, and import prices.

First, consider the electricity sector in the United Kingdom (UK). The Domar weight of this sector is around 5.95 percent. Once we consider direct exports (but not indirect exports), the relevant ratio for the pass-through to CPI decreases to 5.90 percent, a negligible change. This is expected, as the UK electricity sector hardly exports directly to other countries. Yet, when

considering indirect exporting, the network-adjusted domestic consumption share decreases to 4.4 percent, a 25 percent decrease relative to the Domar weight benchmark. This is because other export-heavy sectors in the UK use electricity as a production input either directly or indirectly. Thus, Domar weights would overestimate the impact of a change in productivity in the UK electricity sector on the domestic CPI.

Second, consider the role of wage changes in the CPI. In a closed economy, the labor share is the relevant statistic for how wage changes pass through to the CPI. In the data, the labor share for the average small open economy is around 57 percent. However, the small open economy model with production networks suggests that we need to subtract from the labor share the portion that is exported directly or indirectly. After accounting for network-adjusted exports, this average labor share decreases to 39 percent. This means that the same increase in domestic wages has a 32 percent lower impact on inflation in a small open economy relative to a closed economy.

Finally, let me consider the role of import prices. In the data, the average small open economy exhibits a direct import consumption share of around 17 percent of its total expenditure. Yet, on average, the network-adjusted import consumption share is 30 percent. This implies that the impact of import prices on domestic inflation is (almost) twice what would be implied by a measure ignoring indirect linkages.

In the last section of the paper, I use the model to analyze the recent inflation in two small open economies: Chile and the United Kingdom. I chose these two countries as (i) they fit into the small open economy definition, (ii) they have experienced high inflation in recent years, and (iii) they allow me to compute and contrast between emerging and developed markets. Using these countries, I show that network adjustment on exports and imports provides a quantitative

improvement in matching data moments over both closed economy models and small open economy models without production networks.

Between 2020 and 2022, the average annual inflation rate in Chile was 6.13 percent, with a standard deviation of 3.89. A quantitative closed economy model with production networks implies an average inflation rate of 0.98 percent with a standard deviation 9.69. A small open economy model without production networks delivers an average inflation rate of 1.45 percent with a standard deviation 6.88. Finally, the small open economy model with production networks delivers an average inflation rate of 2.41 with a standard deviation 6.67. Overall, the small open economy with production networks better matches the mean and the standard deviation.

For the United Kingdom, average inflation rate was 3.69 percent over the same period, with a standard deviation of 3.11. The closed economy model with production networks implies an average inflation rate of 2.27 percent with a standard deviation 2.57. The small open economy model without network adjustments exhibits an average inflation rate of 2.72 percent with a standard deviation of 2.64. The small open economy model with production networks shows an average inflation rate of 3.21 percent with a standard deviation 3.00. As in the Chilean case, the production network coupled with openness helps to get closer to the date moments of United Kingdom inflation.

The measurement and application sections illustrate that the required network adjustments in a small open economy matter for our understanding of inflation not only as a theoretical curiosity but also in practice.

Related Literature. This paper relates to several strands of the literature. The first one studies inflation in closed economies with production networks (Basu, 1995; La'o and Tahbaz-Salehi,

2022; Guerrieri et al., 2022; Baqaee and Farhi, 2022b; Luo and Villar, 2023; Afrouzi and Bhattarai, 2022; Ferrante et al., 2022; Rubbo, 2023; Minton and Wheaton, 2023; di Giovanni et al., 2023a,b; Lorenzoni and Werning, 2023).[6] These studies consistently find that the interaction between sectoral price/wage rigidities and production networks is key to understanding the behavior of inflation, which has implications for the conduct of monetary policy, such as what inflation rate to target. This paper focuses on how introducing production networks in a small open economy model helps us to understand the pass-through of different shocks to inflation. Although there is no price rigidity in the model, and thus I cannot speak about the optimal conduct of monetary policy, I contribute to this literature by showing that the production network can have a first-order impact on inflation beyond its role in the sales share distribution without the need for any distortions.

Second, this paper relates to the literature on inflation in small open economies. In the second part of the 20th century, Latin America experienced episodes of high and persistent inflation, a term later coined as "chronic inflation". In response, there was extensive literature during the 1990s on how to best control chronic inflation and the impact of different nominal and real policy rules in small open economies (see Calvo and Végh, 1995; Calvo et al., 1995; Calvo and Végh, 1999, and especially the last one, for an overview of this earlier literature). Modern treatments that introduce New Keynesian features such as sticky prices and monopolistic competition into small open economy models include Gali and Monacelli (2005) and Faia and Monacelli (2008)[7].

[6]There is also extensive literature on multisector models with sticky prices that do not necessarily feature a production network structure, so I omit them from the main text. For earlier contributions, see Woodford (2003) and the references therein.

[7]There is also a large literature focusing on two or more countries. My work is not directly related to these models as I focus on small open economies. I refer the interested reader to Corsetti and Pesenti (2007) and Corsetti et al.

These models have been augmented to include intersectoral linkages and applied to understand the recent inflationary episode, focusing on the United States (Comin and Johnson, 2020; Comin et al., 2023). Relative to this literature, I explicitly analyze the role of production networks on inflation for small open economies. I show how the production network interacts with trade, affecting how domestic and foreign shocks ultimately affect CPI inflation both theoretically and quantitatively. Moreover, the fact that I use a first-order approximation allows me to consider unrestricted intersectoral linkages, in the sense of not needing to assume any functional forms for production or utility. Finally, the model also features multiple factors of production, while the previous models typically focus on only one factor (labor).

Finally, in focusing on the role of network-adjusted exports and imports, this paper contributes to the literature on indirect trade (Huneeus, 2018; Adao et al., 2022; Dhyne et al., 2021, 2023; Muñoz, 2023). This literature focuses on the firm-level consequences of indirect trade, which is equivalent to my trade network-adjustments. For example, Dhyne et al. (2021) use Belgian firm-to-firm level transaction data and find that the relevant concept for a firm sales' exposure to international markets is total exports (network-adjusted exports), while its exposure in costs is total imports (network-adjusted import share). My contribution to this literature is to embed indirect trade into a small open economy model to analyze how it matters for aggregate inflation.

(2010) for an overview of such models. Recent literature focusing on inflation using multi-country and multi-sector models include, for example, Auer et al. (2019) and Ho et al. (2022) during non-Covid-19 times and di Giovanni et al. (2022) and Andrade et al. (2023) during COVID-19.

Outline. The rest of this paper is structured as follows. Section 1.2 describes the model I use to understand inflation in small open economies. Section 1.3 measures the importance of production networks and trade for inflation comparing across sectors and countries. Section 1.4 applies the model to Chile and the United Kingdom. Finally, Section 1.5 concludes.

1.2 A Small Open Economy Model with Production Networks

Environment. There is a set of domestically produced goods that I denote by N with typical element i. These goods can be consumed domestically, used as intermediate inputs by other domestic sectors, and exported. I denote the imported goods set by M, with typical element m. These imported goods can be used as intermediate inputs to produce domestic goods or as final consumption. Finally, there is a set F of factors with typical element f.

Notation. I denote matrices and vectors using **bold** i.e., $\boldsymbol{Y}$. I denote the transpose of a matrix as $\boldsymbol{Y}^T$. Unless otherwise noted, vectors are always column vectors. For example, the vector of Domar weights, defined below, is $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, ..., \lambda_N)^T$. Log changes are expressed as $\mathrm{d} \log Y = \hat{Y}$.

Table 1.1 shows the different shares and matrices that are key for the analysis. I use a bar over a variable for shares based on *total expenditure*, while GDP-based measures do not contain a bar.

1.2.1 Households

There is a representative household that consumes domestically produced goods and foreign goods. It has an instantaneous utility function that I denote by $U(\boldsymbol{C}_D, \boldsymbol{C}_M)$, where $\boldsymbol{C}_D =$

Table 1.1: Definitions

Name	Notation	Expression	Goods/Factors
GDP-based			
Domar Weight	λ_i	$\frac{P_i Q_i}{GDP}$	for $i \in N$
Consumption Share	b_i	$\frac{P_i C_i}{GDP}$	for $i \in N$
Imported Consumption Share	b_m	$\frac{P_m C_m}{GDP}$	for $m \in M$
Export Share	x_i	$\frac{P_i X_i}{GDP}$	for $i \in N$
Factor Shares	$\boldsymbol{\Lambda}$	$\Lambda_f = \frac{W_f L_f}{GDP}$	for $f \in F$
Expenditure-based			
Domar Weight	$\bar{\lambda}_i$	$\frac{P_i Q_i}{E}$	for $i \in N$
Consumption Share	$\bar{b}_i$	$\frac{P_i C_i}{E}$	for $i \in N$
Imported Consumption Share	$\bar{b}_m$	$\frac{P_m C_m}{E}$	for $m \in M$
Export Share	$\bar{x}_i$	$\frac{P_i X_i}{E}$	for $i \in N$
Factor Shares	$\bar{\boldsymbol{\Lambda}}$	$\bar{\Lambda}_f = \frac{W_f L_f}{E}$	for $f \in F$
Sector-level Shares			
Input-Output Matrix	$\boldsymbol{\Omega}$	$\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$	$j \in N$
Leontief-Inverse Matrix	$\boldsymbol{\Psi}_D = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$	$\boldsymbol{\Psi}_{ij} = \sum_{s=0}^{\infty} \boldsymbol{\Omega}_{ij}^s$	$i, j \in N$
Factor Spending Matrix	$\boldsymbol{A}$	$a_{if} = \frac{W_f L_{if}}{P_i^D Q_i}$	$i \in N; f \in F$
Intermediate Import Spending Matrix	$\boldsymbol{\Gamma}$	$\Gamma_{im} = \frac{P_m M_{im}}{P_i Q_i}$	$i \in N; m \in M$

$\{C_i\}_{i \in N}$ denotes the vector of domestically-produced goods consumption and $\boldsymbol{C}_M = \{C_m\}_{m \in M}$ is the vector of foreign goods consumption. These consumption vectors have associated vector prices $\boldsymbol{P}_D = \{P_i\}_{i \in N}$ and $\boldsymbol{P}_M = \{P_m\}_{m \in M}$. Unless otherwise stated, all prices are denomi-

nated in local currency. I assume the utility function $U(.)$ is homogeneous of degree one in its arguments. The representative consumer also owns all factors of production and supplies them inelastically at the given factor prices.

Given a vector of prices, both domestically produced and foreign goods, the cost-minimization problem satisfies

$$PC = \min_{\boldsymbol{C}_D, \boldsymbol{C}_M} \sum_{i \in N} P_i C_i + \sum_{m \in M} P_m C_m \text{ subject to } U(\boldsymbol{C}_D, \boldsymbol{C}_M) \geq \bar{U}. \tag{1.1}$$

Solving this problem delivers a price index that is a function of good prices. I denote this price index by $P = P(\boldsymbol{P}_D, \boldsymbol{P}_M)$. As a reminder, notice that up to a first-order approximation, changes in this price index satisfy

$$\widehat{P} = \bar{\boldsymbol{b}}_D^T \widehat{\boldsymbol{P}}_D + \bar{\boldsymbol{b}}_M^T \widehat{\boldsymbol{P}}_M, \tag{1.2}$$

where

$$\bar{\boldsymbol{b}}_D = \{\bar{b}_i\} = \frac{P_i C_i}{E}; \quad \bar{\boldsymbol{b}}_M = \{\bar{b}_m\} = \frac{P_m C_m}{E}; \quad E = \boldsymbol{P}_D^T \boldsymbol{C}_D + \boldsymbol{P}_M^T \boldsymbol{C}_M = PC,$$

are the expenditure share on domestically produced goods ($\bar{b}_i$), imported goods ($\bar{b}_m$), and total expenditure (E), respectively.

The consumer budget constraint reads

$$PC + T = \sum_{f \in F} W_f L_f + \sum_{i \in N} \Pi_i,$$

where T is an *exogenous* net transfer to the rest of the world as in Baqaee and Farhi (2022b). In Appendix A.2, I provide a justification for having such a force in the current model using a two-period model without changing the main results.

1.2.2 Sectors

There is a representative firm in each i sector that produces according to the following production function

$$Q_i = Z_i F^i \left(\{L_{if}\}_{f \in F}, \{M_{ij}\}_{j \in N}, \{M_{im}\}_{m \in M} \right), \tag{1.3}$$

where Z_i is a sector-specific productivity, L_{if} is demand for factor f by firm i, M_{ij} represents intermediate input demand for good $j \in N$ by firm i, and M_{im} represents input demand for imported good $m \in M$. We can write cost-minimization firm i as

$$TC_i = \min_{\{L_{if}\}_{f=1}^{F}, \{M_{ij}\}_{j \in N}, \{M_{im}\}_{m \in M}} \sum_{f \in F} W_f L_{if} + \sum_{j \in N} P_j M_{ij} + \sum_{m \in M} P_m^M M_{im}^M$$

$$\text{subject to } Z_i F^i \left(\{L_{if}\}_{f \in F}, \{M_{ij}\}_{j \in N}, \{M_{im}\}_{m \in M} \right) \geq \bar{Q}_i.$$

This delivers a marginal cost function that only depends on prices and technology due to the constant returns to scale assumption. In particular,

$$MC_i = MC_i(Z_i, \boldsymbol{P}_D, \boldsymbol{P}_M, \boldsymbol{W}), \tag{1.4}$$

where $\boldsymbol{W} = \{W_f\}_{f \in F}$ is a vector of factor prices.

We can get conditional factor and intermediate input demand by applying Shephard's lemma to the optimized total cost, TC_i, such that

$$\frac{\partial MC_i}{\partial W_f} Q_i = L_{if} \quad \text{for each } f \in F, \tag{1.5}$$

$$\frac{\partial MC_i}{\partial P_j} Q_i = M_{ij} \text{ for each } j \in N, \tag{1.6}$$

$$\frac{\partial MC_i}{\partial P_m} Q_i = M_{im} \text{ for each } m \in M. \tag{1.7}$$

Due to constant returns to scale and perfectly competitive good and factor markets, each firm i makes zero profit:

$$P_i Q_i = \sum_{f \in F} W_f L_{if} + \sum_{j \in N} P_j M_{ij} + \sum_{m \in M} P_m M_{im} \quad \text{for all } i \in N. \tag{1.8}$$

1.2.3 Equilibrium

Market clearing conditions for good and factor markets satisfy

$$Q_i = C_i + X_i + \sum_{j \in N} M_{ji} \qquad \text{for each } i \in N. \tag{1.9}$$

Equation (1.9) is the good market clearing condition. I assume X_i is exogenous as in Adao et al. (2022) so that a price clearing the market always exists for each domestically produced good even if it is exported.

Since this is a real model, nominal prices are indeterminate unless I supplement one additional equation. To do so, I impose the following

$$PC \leq \mathcal{M} = E,$$

where $\mathcal{M}$ is the money supply that I take as exogenous in what follows. This is a cash-in-advance constraint used, for example, in La'o and Tahbaz-Salehi (2022) and Afrouzi and Bhattarai (2022).[8] We can think of this restriction as the small open economy's central bank effectively pinning down total nominal expenditure (E), providing an exogenous nominal anchor. It is apparent that the central bank, conditional on knowing C, which is determined by *real variables*, can

[8]It can be shown that this "constraint" is isomorphic to a model with money in the utility function that is separable from aggregate consumption. The cash-in-advance constraint thus serves no other purpose than pinning down nominal variables without affecting real allocations in this model.

implement any price level, P, that it desires consistent with C. This model features the classical dichotomy, where real variables are determined independently of the nominal side.[9] Under these assumptions, one should interpret the results as highlighting the role of production networks for the consumer price index, conditional on an exogenous central bank monetary policy.

Similar to Baqaee and Farhi (2019a), I define an equilibrium in this economy using a dual approach in which feasible and equilibrium allocations are found by taking as given factor prices $\boldsymbol{W}$ and a level of expenditure, E, as follows

1. Given sequences $(\boldsymbol{W}, \boldsymbol{P}_D, \boldsymbol{P}_M, \boldsymbol{\Pi})$ and exogenous parameters (T), the household chooses $(\boldsymbol{C}_D, \boldsymbol{C}_M)$ to maximize its utility subject to its budget constraint.

2. Given $(\boldsymbol{W}, \boldsymbol{P}_D, \boldsymbol{P}_M)$ and production technologies, firms choose $(\boldsymbol{L}_i, \boldsymbol{M}_i)$ to minimize their cost of production.

3. Given $\boldsymbol{X}$, goods markets clears.

4. The cash-in-advance constraint holds with equality $PC = \mathcal{M} = E$

1.2.4 Characterizing Changes in the Price Index

Having defined the environment, optimality, and equilibrium conditions, I can now study changes in the consumer price index, $\widehat{P}$. Inflation here consists of a log-linear approximation around the initial price level equilibrium. The purpose of the model is to distill whether and how the production network may matter for inflation, which in the model is a cross-sectional statement rather than a dynamic statement. "Inflation" in this context can thus be understood in the space

[9] The converse is not true as real shocks can affect nominal variables. See Végh (2013) chapter 5, especially footnote 11.

rather than the time dimension. This concept has been used, for example, to study inflation in the US during the COVID-19 period (Baqaee and Farhi, 2022b; di Giovanni et al., 2022), and the role of sticky prices in production networks (La'o and Tahbaz-Salehi, 2022; Baqaee and Rubbo, 2023).

The following result characterizes how the consumer price index reacts to changes in exogenous variables.

Proposition 1. Consider a perturbation $(\widehat{\boldsymbol{Z}}, \widehat{\boldsymbol{W}}, \widehat{\boldsymbol{P}}_M)$ around some initial equilibrium. Up to a first order, changes in the aggregate price index, $\widehat{P}$, satisfy

$$\widehat{P} = -\left(\bar{\boldsymbol{\lambda}}^T - \widetilde{\boldsymbol{\lambda}}^T\right)\widehat{\boldsymbol{Z}} + \left(\bar{\boldsymbol{\Lambda}}^T - \widetilde{\boldsymbol{\Lambda}}^T\right)\widehat{\boldsymbol{W}} + (\bar{\boldsymbol{b}}_M^T + \widetilde{\boldsymbol{b}}_M^T)\widehat{\boldsymbol{P}}_M, \tag{1.10}$$

where

$$\widetilde{\boldsymbol{\lambda}}^T = \bar{\boldsymbol{x}}^T\boldsymbol{\Psi}_D; \qquad \widetilde{\boldsymbol{\Lambda}}^T = \bar{\boldsymbol{x}}^T\boldsymbol{\Psi}_D\boldsymbol{A}; \qquad \widetilde{\boldsymbol{b}}_M^T = \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}_D\boldsymbol{\Gamma}$$

Proof. See Appendix A.1.1. ■

The above expression highlights how opening up the economy to goods trade and introducing a production network structure alter the usual prediction of closed economy models. I now proceed with some illustrations that provide intuition for this expression.

Illustration 1: Closed economy. The following proposition characterizes CPI in a closed economy.

Proposition 2. In a closed economy, equation (1.10) reduces to

$$\widehat{P} = -\boldsymbol{\lambda}^T\widehat{\boldsymbol{Z}} + \boldsymbol{\Lambda}^T\widehat{\boldsymbol{W}},$$

Proof. See Appendix A.1.2. ■ Proposition 2 shows the exact form of changes in the CPI in a closed economy (see Baqaee and Farhi, 2022b). Intuitively, CPI changes are a weighted average of changes in productivity (weighted by the Domar weights, $\boldsymbol{\lambda}$) and factor prices (weighted by the factor shares, $\boldsymbol{\Lambda}$).

Equation (1.10) extends this for small open economies with production networks. There are four differences between the closed economy expression and equation (1.10). First, of course, inflation now depends on import price changes. Second, the Domar weights and factor shares in equation (1.10) are now based on expenditure rather than on nominal GDP. This distinction arises in small open economies that feature trade imbalances, in which the income from domestic production need not equal what they consume. Since what matters for the CPI is what domestic consumers spend, nominal expenditure is the relevant object for dividing sales and factor payments.

Third, the effect of sectoral productivity changes on the CPI is dampened relative to a closed economy or a small open economy without production networks. To see this, note that the relevant statistic for the effect of sectoral productivity changes on the CPI is $\bar{\boldsymbol{\lambda}}^T - \widetilde{\boldsymbol{\lambda}}^T$, and thus the Domar weight $\bar{\boldsymbol{\lambda}}^T$ is no longer the sufficient statistic for understanding how sectoral productivity changes affect CPI. Importantly, the relevant elasticity requires adjusting the expenditure-based Domar weight $\bar{\boldsymbol{\lambda}}$ by substracting $\widetilde{\boldsymbol{\lambda}}^T = \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D$. This adjustment comes from the fact that what matters is the domestic consumer's exposure to changes in sectoral productivity.

To be precise, let me write the price index as a function of domestic and imported goods prices, i.e., $P = \mathcal{P}(\boldsymbol{P}_D, \boldsymbol{P}_M)$. Suppose there is a change in the productivity of sector k, $\widehat{Z}_k$, with no changes in factor or import prices. This shock impacts all domestic goods prices due to input-output linkages. Its propagation to the CPI is a tale of two elasticities. First, how exposed

is the consumer to changes in domestic good prices $\frac{\partial \log \mathcal{P}}{\partial \log P_i}$, for all i. By the envelope theorem, this elasticity is simply the consumption share of the good at the initial equilibrium, $\bar{b}_i$. Second, the impact depends on how productivity passes through to each domestic goods price, $\frac{\partial \log P_i}{\partial \log Z_k}$. This last term is simply given by $-\Psi_{ik}$, which measures the sensitivity of the price of good i to a change in productivity of sector k after taking into account all direct and indirect linkages via the production network. Collecting all these pieces, we can write:

$$\widehat{P} = \sum_{i \in N} \underbrace{\frac{\partial \log \mathcal{P}}{\partial \log P_i}}_{=\bar{b}_i} \underbrace{\frac{\partial \log P_i}{\partial \log Z_k}}_{=-\Psi_{ik}} \widehat{Z}_k = -\bar{b}_D^T \mathbf{\Psi}_{(:,\,k)} \widehat{Z}_k,$$

where $\mathbf{\Psi}_{(:,\,k)}$ is the kth column of the Leontief-inverse matrix $\mathbf{\Psi}$. Note that again, the reason why $\bar{\boldsymbol{b}}_D^T \mathbf{\Psi}$ is not equivalent to the vector of sales share is precisely because this is not the relevant exposure of the domestic consumer in the presence of input-output linkages and international trade.

Third, the effect of factor prices on the CPI is also dampened relative to the closed economy benchmark or a small open economy without production networks. Although the logic is similar to that of how productivity changes pass through CPI, I analyze the factor price case in detail in the next example, as it also allows me to relate equation (1.10) to a well-known concept in the trade literature: the factor content of exports.

Illustration 2: Domestic factor demand and the factor content of exports. This example helps to illustrate how the fact that exports used domestic factors lowers the sensitivity of prices to changes in domestic factor prices. Equation (1.10) highlights a tension between domestic factor demand and the factor content of exports, in the spirit of Adao et al. (2022). When an economy exports, some of its factors of production end up meeting foreign demand which, everything else

equal, reduces *domestic factor demand*. These factors meet foreign demand because they are used to produce domestic goods that are exported. As a result, factor price changes put less pressure on the price index[10]. Moreover, this channel is in place whenever an economy exports to the rest of the world, even if there is no production network. To see this, notice that factor payments to a given factor f can be written as

$$W_f L_f = \sum_{i \in N} W_f L_{if} = \sum_{i \in N} a_{if} \lambda_i. \tag{1.11}$$

Without intermediate inputs, the Domar weight of each sector, λ_i, is simply that sector's share in total final demand

$$Q_i = C_i + X_i \Longrightarrow \lambda_i = b_i + x_i. \tag{1.12}$$

Combining equations (1.11) and (1.12), I get

$$W_f L_f - \underbrace{\sum_{i \in N} a_{if} x_i}_{\text{Factor Content of Exports}} = \underbrace{\sum_{i \in N} a_{if} b_i}_{\text{Domestic Factor Demand}}. \tag{1.13}$$

This equation shows the tension: a rise in exports – higher x_i – must be balanced out by a fall in domestic factor demand on the right-hand side, conditional on aggregate payments to factor f being constant. This is one of the mechanisms through which exports cause domestic factor prices to put less pressure on domestic consumer prices.

When we allow for a production network and trade, sectors that do not export much directly (have low x_i) could end up exporting indirectly via other producers. The case I analyzed above, without a production network, is a particular case in which $\mathbf{\Omega} = \mathbf{0}_{N \times N}$ and thus $\mathbf{\Psi_D} = \boldsymbol{I}$.

[10]Though the factor content of exports is already well known in the trade literature, I am unaware of previous work linking this precise notion to inflation.

What this suggests is that in the presence of intermediate input linkages, what matters for the price index changes is not just how much each sector exports directly, $\boldsymbol{x}^T$, but also how much it exports indirectly through intermediate input linkages, $\boldsymbol{x}^T\boldsymbol{\Psi}_D$ (see Dhyne et al., 2021). This mechanism also affects how much each factor ends up being exported and how much factor price changes are passed through to the CPI, since $\boldsymbol{x}^T\boldsymbol{\Psi}_D\boldsymbol{A}$ represents the factor content of exports when there are intermediate input linkages across sectors and -$\left(\bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T\boldsymbol{\Psi}_D\right)$ is the relevant "Domar weight" for the pass-through of sectoral technology shocks to inflation.

Illustration 3: Import price changes with intersectoral linkages and the network-adjusted import consumption share. This example illustrates that intersectoral linkages amplify the influence of import price changes on inflation. In the presence of intermediate input linkages and imported intermediate inputs, the *direct* import consumption shares $\bar{\boldsymbol{b}}_M^T$ are *not* a sufficient statistic for the effect of import prices on the CPI. To see this, fix factor prices and assume no productivity shocks, $\widehat{\boldsymbol{W}} = \mathbf{0}_F$ and $\widehat{\boldsymbol{Z}} = \mathbf{0}_N$. Then

$$\widehat{P} = \underbrace{\left(\bar{\boldsymbol{b}}_M^T + \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}_D\boldsymbol{\Gamma}\right)}_{\text{Network-adjusted import consumption share}} \widehat{\boldsymbol{P}}_M$$

As was the case for the factor content of exports, this equation shows the importance of *network-adjusted import consumption shares*. While domestic consumers purchase imports directly as final consumption ($\bar{\boldsymbol{b}}_M$), they also consume imports indirectly by purchasing domestically produced goods that directly or indirectly use imported intermediate inputs. This channel is captured by the second term on the right-hand side, $\bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}_D\boldsymbol{\Gamma}$, which captures the total import content of each domestically produced good when we account for intermediate input linkages. Intuitively, a rise in the price of import good m raises the marginal cost of a given producer h by Γ_{hm}. This rise in the marginal cost implies that P_h raises. This increase in P_h, through intermediate input link-

ages, raises the price of (say) good i, by Ψ_{ih}, which denotes the exposure of sector i to changes in the price of sector h after taking into account intermediate input linkages. This increase in the price of good i, in turn, is passed through to the consumer price index via $\bar{b}_i$.

Additional Models. I provide two additional models in Appendix A.2 and A.3. In Appendix A.2, I provide a detailed two-period model of a small open economy to show that the simplified model presented here shares the same intuition. The key idea follows Baqaee and Farhi (2022b), who in turn build on Krugman (1998) and Eggertsson and Krugman (2012), where we can separate a dynamic problem into two sub-periods: the present and the future. All action happens in the present, while the future can be taken as given. Shocks occur during the present and last only for that period, wherein the "future", the economy returns to its initial no-shock equilibrium. Conditional on this interpretation, the model in this section is isomorphic to a multi-period model.

In Appendix A.3, I provide a three-sector (exportable, importable, and non-tradable) canonical small open economy dynamic model, as in Chapter 8 of Uribe and Schmitt-Grohé (2017). There, I embed a production network structure and show that the results also hold in that environment. In working out these additional models, I contribute to the literature by effectively embedding production networks into a small open economy setup and studying its consequences for inflation.

1.2.5 An alternative representation of factor markets: from factor prices to factor supplies

Factor price changes on the right-hand side of the equation (1.10) are exogenous and thus can be considered primitives in my exercise. However, typical neoclassical models treat factor prices as endogenous and factor supply as exogenous. Writing the problem considering factor prices as given simplified the intuition for the main result of this paper. It also allowed me to differentiate the proximate causes of inflation between my model and a closed economy with or without production networks.[11] I now show that the same intuition holds if we reverse the ordering, treating factor prices as endogenous outcomes and factor supply as exogenous objects.

1.2.5.1 Solving in terms of factor supply quantities

The key difference when solving for factor prices as endogenous objects is that we need to introduce factor market clearing conditions introduced below:

$$\sum_{i \in N} L_{if} = \bar{L}_f \quad \forall f \in F,$$

where the left-hand side is factor demand, and the right-hand side is factor supply. In what follows, I assume that factor supplies, $\bar{L}_f$, are exogenous.

Recall that the expenditure-based share of factor f can be written as

$$\bar{\Lambda}_f = \frac{W_f \bar{L}_f}{\mathcal{M}},$$

where I have imposed the factor market clearing condition and the cash-in-advance constraint.

[11] This is also the route followed by Baqaee and Farhi (2019a) when studying aggregation in disaggregated economies via *aggregate cost functions* rather than aggregate production functions.

Thus, changes in factor prices can be written as

$$\widehat{W}_f = \widehat{\bar{\Lambda}}_f + \widehat{\mathcal{M}} - \widehat{\bar{L}}_f, \tag{1.14}$$

which in vector form is

$$\widehat{\boldsymbol{W}} = \widehat{\bar{\boldsymbol{\Lambda}}} + \mathbf{1}_F \widehat{\mathcal{M}} - \widehat{\bar{\boldsymbol{L}}}.$$

Intuitively, factor prices can go up because (i) demand is *reallocated* toward that factor, as captured by $\widehat{\bar{\boldsymbol{\Lambda}}}$; (ii) aggregate demand is going up ($\widehat{\mathcal{M}}$); and (iii) there is a decrease in (inelastic) factor supply ($\widehat{\bar{\boldsymbol{L}}}$). As shown in the proposition below, this decomposition allows me to write changes in the price index as a function of sectoral and aggregate shocks and also changes in these expenditure-based factor shares.

Proposition 3. Consider a perturbation $(\widehat{\mathcal{M}}, \mathrm{d}T, \widehat{\boldsymbol{Z}}, \widehat{\boldsymbol{P}}_M, \widehat{\boldsymbol{X}}, \widehat{\bar{\boldsymbol{L}}})$ around some initial equilibrium. Up to a first order, changes in the aggregate price index, $\widehat{P}$, satisfy

$$\widehat{P} = -\left(\bar{\boldsymbol{\lambda}}^T - \tilde{\boldsymbol{\lambda}}^T\right)\widehat{\boldsymbol{Z}} - \underbrace{\tilde{\boldsymbol{\Lambda}}^T \widehat{\bar{\boldsymbol{\Lambda}}} - \left(\bar{\boldsymbol{\Lambda}}^T - \tilde{\boldsymbol{\Lambda}}^T\right)\widehat{\bar{\boldsymbol{L}}} + \frac{\mathrm{d}T}{\mathcal{M}} + \left(1 - \tilde{\boldsymbol{\Lambda}}^T \mathbf{1}_F\right)\widehat{\mathcal{M}}}_{\text{Factor Price Changes}}$$
$$+ \left(\bar{\boldsymbol{b}}_M^T + \tilde{\boldsymbol{b}}_M^T\right)\widehat{\boldsymbol{P}}_M, \tag{1.15}$$

Proof. See Appendix A.1.3. ■

Proposition 3 is an *ex-post* sufficient statistics results in the spirit of Baqaee and Farhi (2022a) since there is still one endogenous vector that needs to be solved for, namely $\widehat{\bar{\boldsymbol{\Lambda}}}$. Conditional on knowing this vector, we can compute the response of the CPI to changes in the other primitives. Note that in a closed economy, this term would be zero, since in this case $\tilde{\boldsymbol{\Lambda}} = \mathbf{0}_F$ and thus factor share reallocation would not have any first-order effect on inflation.

Note that the only difference relative to the model with exogenous factor price changes is that we are now mapping factor price changes to other exogenous objects ($\widehat{\mathcal{M}}, \mathrm{d}T, \widehat{\widehat{\boldsymbol{L}}}$). As in Baqaee and Farhi (2022b), decreases in factor supply are inflationary because they increase factor prices conditional on factor demands. The impact on inflation of money supply changes, $\widehat{\mathcal{M}}$, is dampened relative to the closed economy because factor prices have less pass through to inflation. Increases in net transfers to the rest of the world, $\mathrm{d}T$, also increase CPI inflation because, conditional on money supply, they increase nominal GDP and thus increase factor prices. In this sense, factor price changes are "sufficient statistics" for how money supply and net transfer changes affect the CPI.

A few additional remarks regarding Proposition 3 are in order. First, note that sectoral export demands, $\boldsymbol{X}$, do not appear directly in this equation. It means that it can only affect inflation through its effect on $\widehat{\widehat{\boldsymbol{\Lambda}}}$. Second, as I show in Appendix A.4, $\widehat{\widehat{\boldsymbol{\Lambda}}}$ can be found by solving a linear system of equations. This system of equations depends on primitives, the production network structure, and the elasticities of substitution for producers and consumers. Thus, solving for $\widehat{\widehat{\boldsymbol{\Lambda}}}$ requires us to take a stand on the values of elasticities of substitution of producers across different inputs and of consumers across different goods. Perhaps more important than this is that through this endogenous vector, elasticities of substitution matter to a first-order for CPI inflation in small open economies. Hence, even with a simple, sufficient statistics framework, elasticities of substitution matter to a first-order for inflation in small open economies, a result that contrasts with the closed-economy benchmark.

1.3 The empirical relevance of adjustments of CPI elasticity

This section examines the quantitative relevance of the proposed production network adjustments for inflation across small open economies. I start by describing the data sources and how I classify countries as small open economies. I then present three different results. First, I focus on the network-adjusted domestic consumption shares, which are the relevant elasticities for the pass-through of sectoral technology shocks to inflation. Second, I examine the adjustments to labor shares once we account for indirect exports. Finally, I compare direct and network-adjusted import consumption shares.

1.3.1 Data

Although network-adjusted shares require more information than sales and factor shares, they are still easily computable from available data. In this subsection, I briefly describe the necessary data to compute them.

Input-Output Tables. I calculate the objects $(\boldsymbol{\Omega}, \bar{\boldsymbol{x}}, \bar{\boldsymbol{b}}_D, \bar{\boldsymbol{b}}_M, \bar{\boldsymbol{\lambda}})$ using domestic input-output tables from the World Input-Output database release 2016, the latest available.

Penn World Tables (PWT). I use version PWT 10.01. This dataset contains income, input, output, and productivity information between 1950 and 2019 for 183 countries. This database is freely available to download at https://www.rug.nl/ggdc/productivity/pwt/?lang=en.

Using this database, I construct two measures. First, I denote the share of world GDP

accounted for by country c as α_c. Formally,

$$\alpha_c = \frac{nGDP_c}{nGDP_W}, \quad nGDP_W = \sum_{c \in C} nGDP_c$$

I measure $nGDP_c$ using the series *cgdpo*, which corresponds to the Output-side real GDP at current PPP's (in 2017 US$ millions).

To measure trade openness, I use the series csh_x and csh_m in the PWT. The first corresponds to the ratio of merchandise exports over nominal GDP, while the second corresponds to imports over nominal GDP at current PPP's. I define the trade openness of country c as

$$\text{Openness}_c = \frac{\text{Exports}_c + \text{Imports}_c}{nGDP_c} = csh\_x_c - csh\_m_c,$$

where the last line follows since in the data $csh\_m_c = -\frac{\text{Imports}_c}{nGDP_c}$

Classifying Small Open Economies. I apply two criteria to separate countries into small and non-small open economies according to the data. First, an economy is *small* if $\alpha_c \leq 0.05$. Second, an economy is *open* if $\text{Openness}_c \geq 0.3$. A country is a small open economy if it satisfies both conditions.

1.3.2 Results

In this subsection, I compare the network-adjusted objects with their closed economy and no-network adjustment counterparts whenever possible. All cross-sectional plots are based on the year 2014 unless stated otherwise.

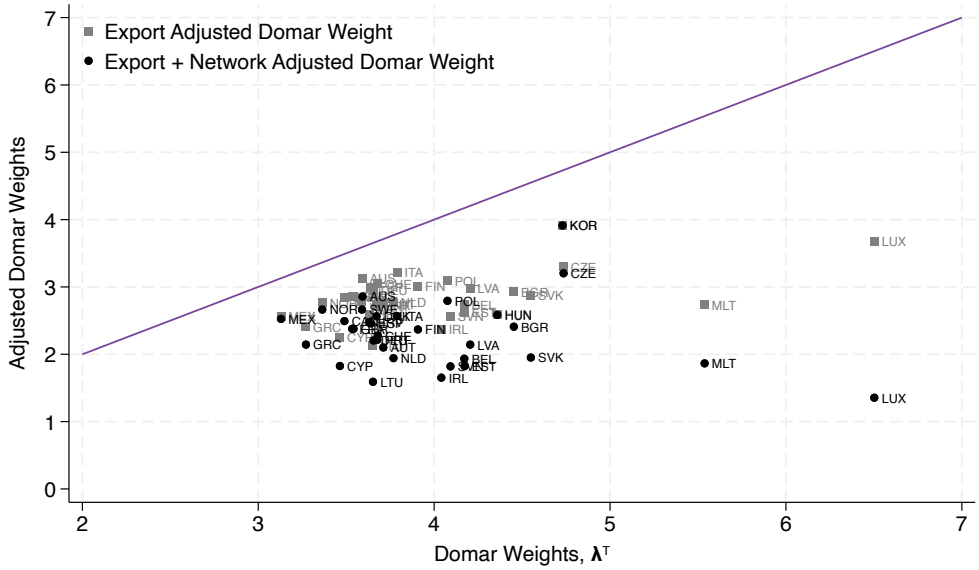
1.3.2.1 Network-adjusted domestic consumption shares

I start by showing results for network-adjusted domestic consumption shares $\bar{\boldsymbol{\lambda}} - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D$. Figure 1.2 shows three scenarios for the average sector in small open economies in panel (a) and for non-small open economies in panel (b). The x-axis shows the unadjusted Domar weights, while the y-axis shows the adjusted objects. Light squares in these figures refer to the export-adjusted weights, while dark points are export-network-adjusted weights. Thus, the dark points in these plots are the network-adjusted domestic consumption shares. As we can see, the adjustments are stronger for small open economies than non-small open economies. Moreover, we can see that the average Domar weight in small open economies is around 4 percent; it decreases to around 2.84 percent with the export adjustment and to around 2.31 percent when adjusting for network-adjusted exports. This is a non-negligible change. It suggests that the inflation impact of a given sized sectoral productivity shock in an average-sized sector will be dampened by around 50 percent for the average small open economy relative to the closed economy benchmark.

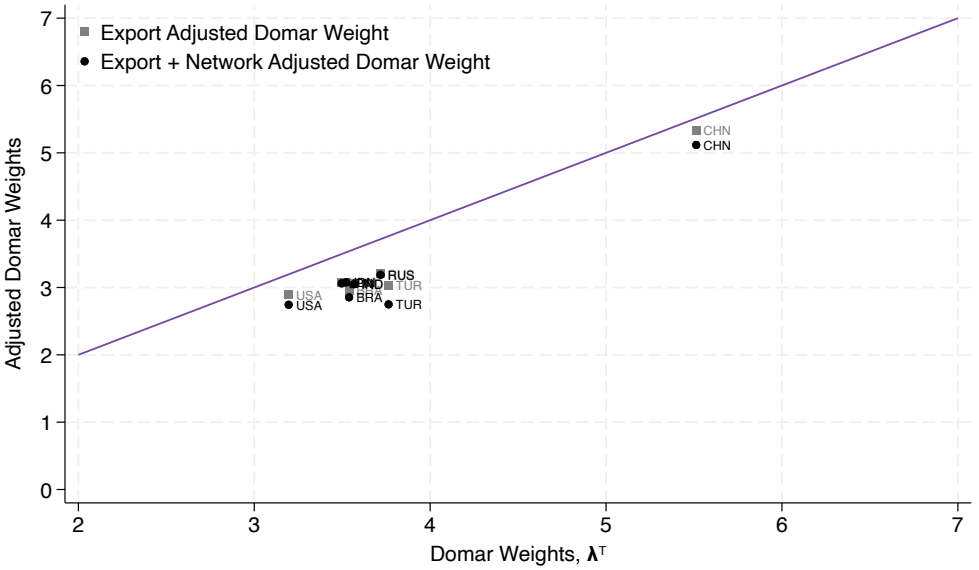
To provide a more concrete example, Figure 1.3 shows the three sectors for which network-export adjustment is the largest in the United Kingdom: administrative support, legal and accounting, and electricity, gas, and water. The latter sector is illustrative. Its Domar weight is around 5.95 percent. This number goes down only to 5.9 percent when we subtract direct exports. For all practical purposes, this means this sector is non-tradable. Once we consider indirect exports, however, the network-adjusted consumption share decreases to 4.4 percent. This illustrates how indirect linkages are quantitatively relevant and provide information beyond the direct export share.

Figure 1.2: Export and Network-Export adjusted Domar weights.

(a) Small Open Economies

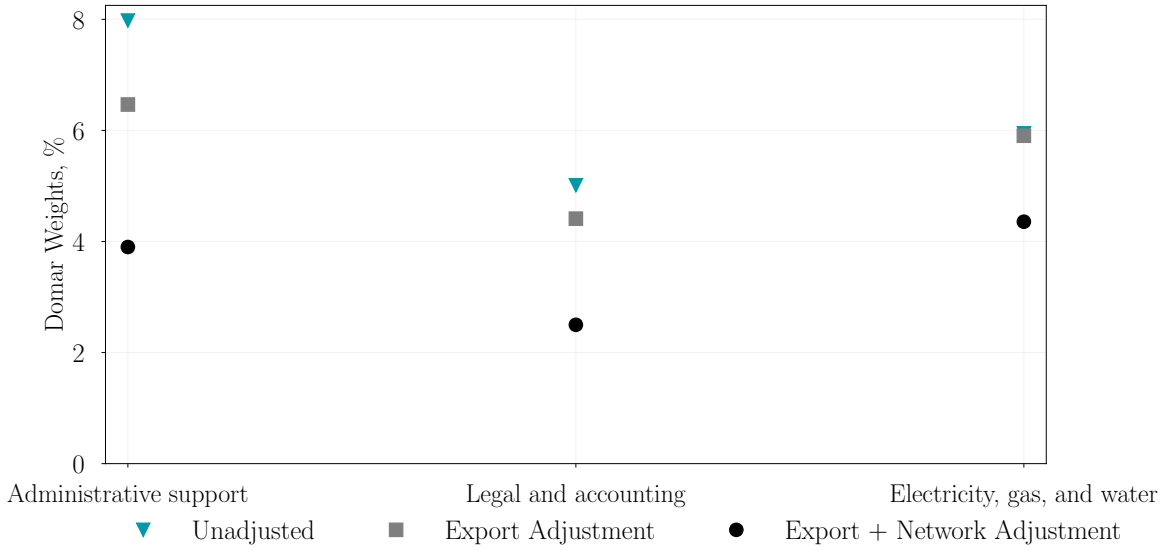


(b) Non-Small Open Economies



Note: This figure shows the average Domar weight for each country. The x-axis corresponds to the average Domar weight computed as in the closed economy model, $\boldsymbol{\lambda}^T$. The gray squares subtract only for direct exports i.e. $\bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T$. The black circles further consider the production network structure, $\bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D$. Panel (a) shows the results for small open economies, while Panel (b) shows the results for non-small open economies.

Figure 1.3: Three sectors with largest adjustments: United Kingdom.



Note: This figure shows the three sectors with the largest export network-adjusted share for the United Kingdom.

Regression framework. Which sectors and countries are most affected by network adjustments? To answer this question, I estimate the following cross-sectional regression

$$y_{sc} = \alpha_s + \alpha_c + \varepsilon_{cs}, \tag{1.16}$$

where y_{sc} represents the difference between a measure for the small open economy with a production network relative to the small open economy without networks for a given country c and sector s. α_s is a sector-specific fixed effect, α_c is a country-specific fixed effect, and ε_{cs} is an error term. From this regression, I get estimates of sector and country-specific fixed effects. Notice that these are identified up to a normalization, which in my case is that $\sum_{s \in S} \hat{\alpha}_s = 0$ and $\sum_{c \in C} \hat{\alpha}_c = 0$. All fixed effects are interpreted as deviations from the average fixed effects.

In Panel (a) of Figure 1.4, I show the country-fixed effect estimates when the left-hand side variable is the difference between the network-adjusted export share and the direct export

share. Note that these country-fixed estimates tell us the average difference between these shares — as a fraction of aggregate expenditure — across sectors within a country. We can see that the countries with the largest adjustments are Luxembourg, Slovak Republic, Malta, and Latvia, while countries with the smaller adjustments are Korea, Hungary, and Mexico. These numbers indicate that the export sectors of the latter economies do not rely much on inputs from the domestic economy, not exporting much indirectly.[12]

Panel (b) does the same exercise for the sector-fixed effect estimates. These estimates tell us the average difference between shares across countries within a sector. Financial services is the sector with the largest average production network adjustment. Note that the Electricity, Gas, and Water (EGSA) is the seventh sector with the largest difference, while Legal and accounting is the sixth sector. Thus, the examples cited above are not specific to the United Kingdom but have consistently large network adjustments across countries. Intuitively, these sectors are important suppliers for domestic sectors that export directly or indirectly.

These exercises suggest that accounting for the production network is important in computing inflation elasticities and that the adjustment varies substantially across countries and sectors.

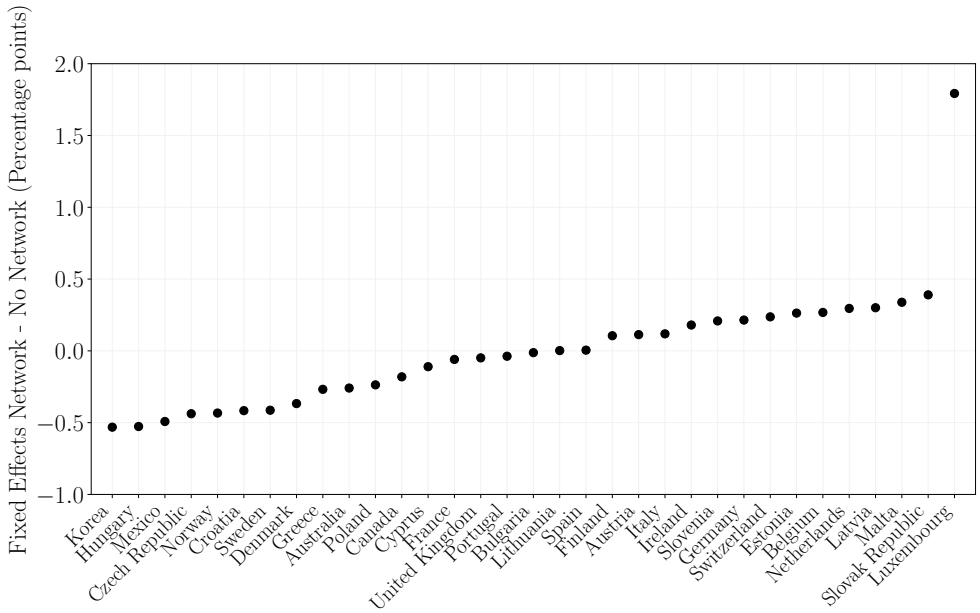
1.3.2.2 Network-adjusted domestic factor demand

I now conduct a similar exercise to examine the importance of network adjustments for factor shares. First, I study how the aggregate labor share in different countries varies depending on the export and network export adjustment. I then consider how sector-specific labor shares vary when considering direct and indirect exports.

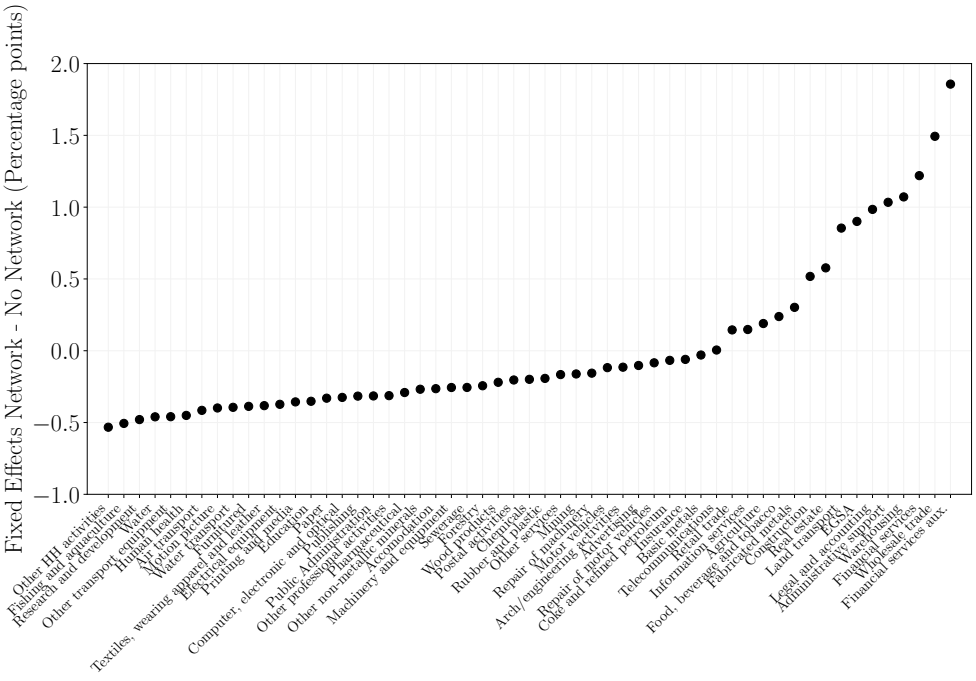
[12] Remember that it does not need to be confused with a country that does not export at all.

Figure 1.4: Country and sector fixed effects: export-network adjusted - export adjusted.

(a) Country Fixed Effects



(b) Sector Fixed Effects



Note: This figure shows fixed effects from estimating equation (1.16) where the dependent variable is the difference between the network-adjusted export share and the direct export share. Panel (a) shows country fixed effects estimates, while Panel (b) shows sector fixed effects.

Labor share. Figure 1.5 shows the labor share for different economies on the x-axis and the network export-adjusted labor share on the y-axis. Black diamons shows small open economies and gray circles represents non-small open economies. As we can see, the adjustments are again significant for small open economies but not for non-small open economies. The average labor share across non-small open economies is 53 percent, while the network export-adjusted labor share is 50 percent, a negligible change. In contrast, the average labor share in small open economies is around 57 percent, while the network export-adjusted labor share is only 39 percent. This means the impact of a given wage increase will be 32 percent lower in a small open economy with production networks relative to an otherwise similar closed economy.

Sector-specific labor shares. I now conduct a similar exercise to that of export and network-adjusted export measures above. Here, I consider the dependent variable to be the difference between the network-adjusted labor content of exports relative to the non-network-adjusted labor content of exports.

This exercise illustrates the heterogeneity across sectoral labor markets. Before, I considered the aggregate labor share. However, this aggregate labor share is a weighted average of what happens at the sectoral level. It can be a misleading statistic for certain questions, especially in an environment such as COVID-19, where sectoral labor markets were hit differently.

Panel (a) of Figure 1.6 shows the results for the country-fixed effects, while Panel (b) shows the same but for sector-fixed effects. Apart from Luxembourg, the ranking differs from the network-adjusted domestic consumption share in Figure 1.4. Interestingly, countries where sector-specific labor shares adjusted the most due to the domestic production network are the Netherlands, Slovenia, and Germany, while the ranking at the bottom stays the same. This says

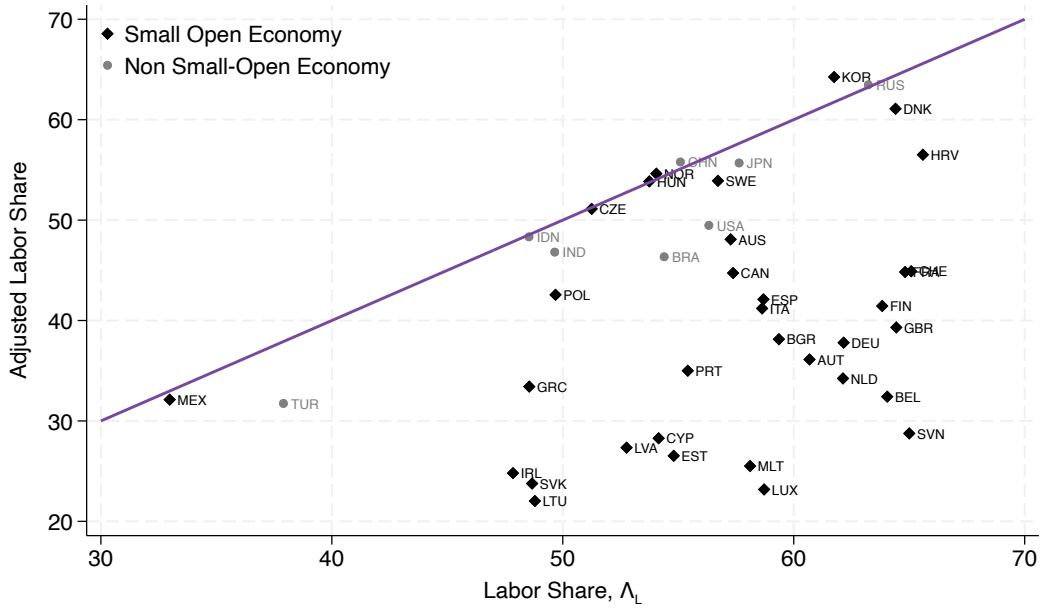
that Germany exhibits an average production network adjustment of sector-specific labor shares 0.15 percentage points larger than the adjustment for the average country.

Turning to the sector fixed-effects results, the sectors with the largest production network adjustment are Legal and Accounting, Wholesale Trade, and Administrative support. Legal and accounting, for example, has an average adjustment 0.6 percentage points larger than the average sector. Since the 0.6 percentage point is an average across all countries, consider the Legal and Accounting sector in Germany as a concrete example. The share of this sector's labor on nominal GDP is around 2.6 percent of GDP. It goes down to 2.3 percent when we subtract exports and to 0.8 percent when we consider the domestic production network structure. Thus, ignoring the production network adjustment would significantly overstate how much wage changes sector pass-through to the CPI.

1.3.2.3 Network-adjusted import consumption shares

As a final empirical exercise, I consider import consumption shares. Figure 1.7, provides a scatterplot of these shares across economies. On the x-axis, I show the direct import consumption share, while on the y-axis, I show the network-adjusted import consumption share. The average direct import consumption share across non-small open economies is 6.7 percent. It increases to 9.3 percent when considering the production network structure. While it increases by almost 3 percentage points, this change is small relative to the one I find for small open economies. The average direct import consumption share across small open economies is around 17 percent. This number goes up to 30 percent when considering the production network structure. This is a 13 percentage points increase. It suggests that the pass-through from import price changes to

Figure 1.5: Labor share adjustments for different countries.



Note: This figure shows the average labor share on the x-axis and the export-network adjusted labor share for small open economies in black diamonds and non-small open economies in gray circles.

inflation (almost) doubles when we introduce intersectoral linkages.

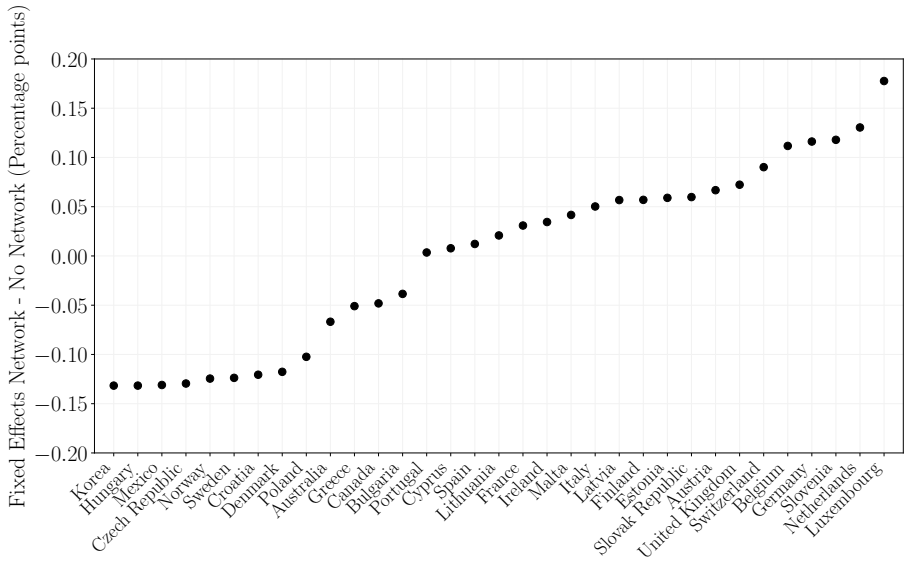
1.4 The evolution of inflation in Chile and United Kingdom during COVID-19

In this section, I use the model to study CPI inflation during the COVID-19 episode in Chile and the United Kingdom.

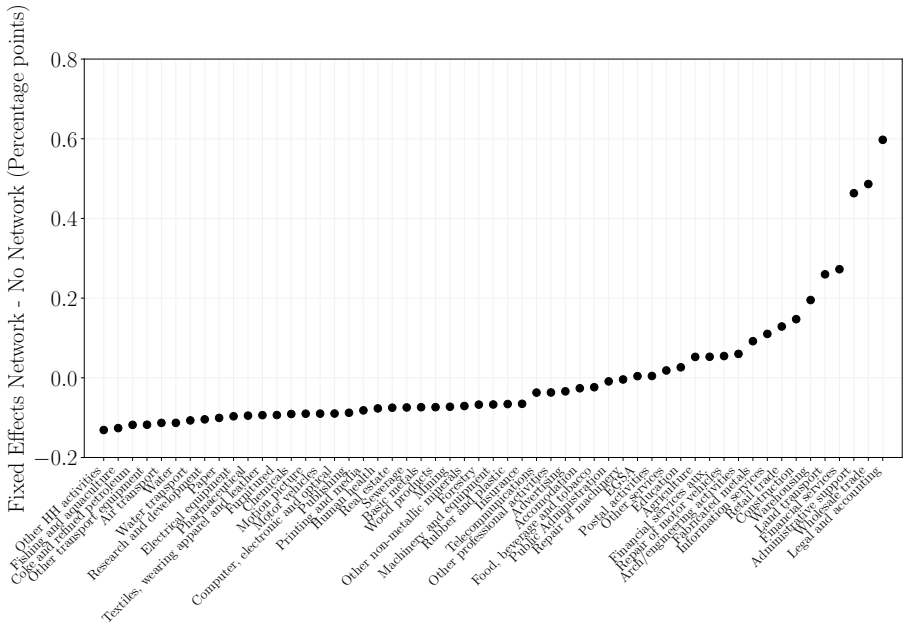
This empirical examination requires more data relative to the previous section. While the earlier section showed information on the CPI elasticities and compared these across countries and sectors using the WIOT alone, this section requires taking a stand on the processes $(\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$, which are not readily available for most countries worldwide. Therefore, I picked Chile and the United Kingdom, countries with all the necessary information to construct $(\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$

Figure 1.6: Country and sector fixed effects: export-network adjusted sector-specific factor shares - export adjusted sector-specific factor shares.

(a) Country Fixed Effects

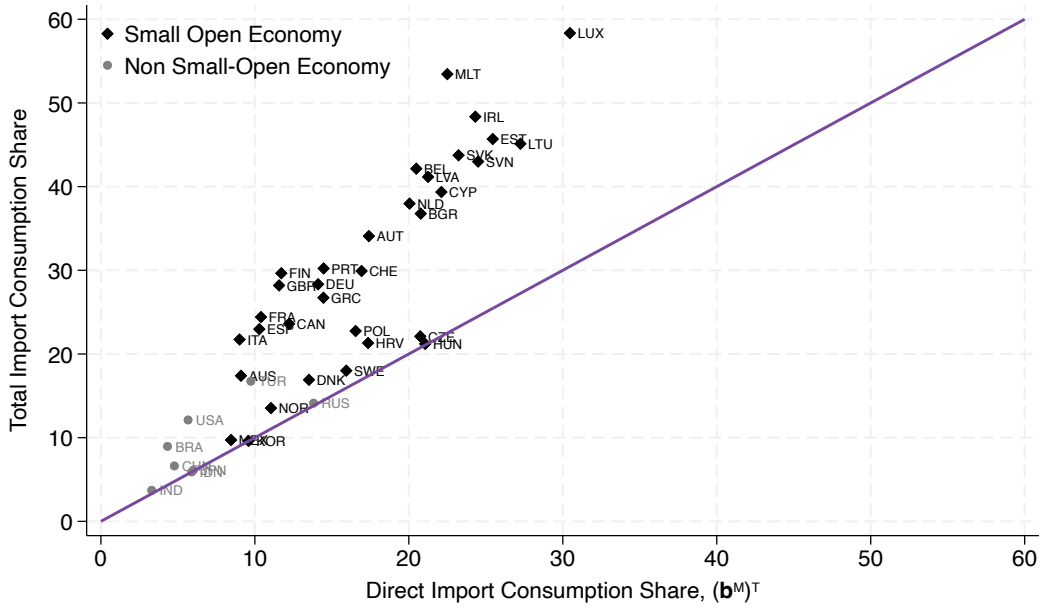


(b) Sector Fixed Effects



Note: This figure shows the fixed effects when the dependent variable is the difference between the network-adjusted sector-specific factor shares and the direct export share adjusted sector-specific factor shares. Panel (a) shows this difference for the country fixed effects estimates, while Panel (b) does the same for sector fixed effects.

Figure 1.7: Direct and Network-Adjusted import consumption shares.



Note: This figure shows the direct import consumption share on the x-axis and the network-adjusted import consumption share on the y-axis. Small open economies are the black diamonds, and non-small open economies are the gray circles.

that also belong to the small open economy category.

This is an ex-post exercise using existing data to analyze the past behavior of inflation between 2020 and 2022. Yet, Proposition 1 is helpful for forecasting inflation and is thus a valuable tool for policymakers in small open economies. Provided that we have forecast information on the processes $(\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$, we can combine this information with input-output tables to get an estimate of inflation. The accuracy of this exercise will depend on the accuracy of elasticities and that of the forecasted series. Throughout this section, I focus on the former, as it is the main point of this paper.

In what follows, I first describe the data. Then, I show how I map the model to the data.

Finally, I discuss the results for Chile and the United Kingdom.

1.4.1 Data

1.4.1.1 Chile

Input-Output Tables. Since Chilean data is unavailable from the WIOT, I resort to Chilean National Accounts. I use *Compilacion de Referencia* for year 2013. The structure is similar to that of the WIOT, having information on input-output linkages, final uses, and factor payments. Moreover, it is quite disaggregated, containing information for up to 171 industries. I collapse this data to a 17-sector classification due to data availability on sectoral wages. This 17-sector classification is equivalent to SIC2.

Sectoral Productivity. The ideal measure of productivity from the model is total factor productivity (TFP). However, TFP measures are hard to come by, especially at high frequencies and at the sectoral level. To circumvent this problem, I proxy sectoral TFP using sectoral labor productivity. I collect data on real GDP for the same 17 sectors and divide by total sectoral employment. Real GDP and sectoral employment data come from the Central Bank of Chile (CBCh) and are available quarterly from 1996 to 2022.

Sectoral Wages. I source sectoral nominal wages from the Chilean National Institute of Statistics (INE) series *Indice de Remuneraciones Nominal*. This database is available monthly from January 2016 to December 2022. To be consistent with the productivity data, I collapsed this data to a quarterly frequency.

Import Prices. I use the import price index available from the CBCh quarterly from 2013 to 2022.

1.4.1.2 United Kingdom

Input-Output Tables. I source data from the WIOT domestic tables as in the previous empirical section. I collapse these input-output tables into 20 industries to be consistent with the data on sectoral wages.

Sectoral Productivity. I sourced data from the Office for National Statistics (ONS) of the United Kingdom. I downloaded quarterly estimates of labor productivity from the *Flash productivity* report.[13] This contains information for up to 17 industries.

Sectoral Wages. I source this data from the ONS of the United Kingdom. In particular, I use the dataset `EARN03`. This contains monthly information on average weekly earnings for around 20 industries. This dataset is available from 2000 to 2022.

Import Prices. I use the import price index from the ONS (series `GD74`, dataset: `MM22`). This series is available at different frequencies. I use quarterly information from 2009 until 2022.

1.4.2 Mapping the model to the data

Before showing the results, a few remarks are in order. Since the model is static, all inherent inflation dynamics will combine the dynamics of exogenous variables and their interaction with the CPI elasticities.

[13] This data can be downloaded freely from the "*Flash productivity by section*" section at the ONS here.

First, I take all series and compute their level deviations from their value in 2018Q4. Formally, the sources of variation I feed in to construct implied inflation from the different models take the following form

$$\hat{y}_t = y_t - y_{2018Q4},$$

where y_t represents (the log) of any time series and y_{2018Q4} is its value in 2018Q4. Notice that each vector now has a t subscript as they are deviations from 2018Q4 at each time t.

In the above equation, I call the deviation $\hat{y}_t$ a "shock". This differs from a structurally identified shock because I feed variation directly from the data, taking it as given. With this caveat in mind and throughout this section, I refer to these $\hat{y}_t$ simply as shocks.

Using this procedure I construct counterparts to $\boldsymbol{\theta}_t = (\widehat{\boldsymbol{W}}_t, \widehat{\boldsymbol{P}}_{Mt}, \widehat{\boldsymbol{Z}}_t)$ in the model. I measure factor prices as sectoral wages. I assume segmented labor markets such that there are different wages across sectors to capture better the behavior of labor markets during the COVID-19 episode, as highlighted in the recent literature (Baqaee and Farhi, 2022b; di Giovanni et al., 2022, 2023a,b). Since I cannot observe sector-specific prices for other factors, such as capital or land, I assume that those factor prices did not change over the sample period.

CPI inflation in the data π_t, when t refers to a quarter, is

$$\pi_t = \log P_t - \log P_{t-4}$$

Combining the model and shocks, I have $\widehat{P}_t^{\text{Model}}$ as

$$\widehat{P}_t^{\text{Model}} = -\sum_{i \in N} \mathcal{R}_i^{CPI,Z} \widehat{Z}_{it} + \sum_{f \in F} \mathcal{R}_f^{CPI,W} \widehat{W}_{ft} + \mathcal{R}_M^{CPI,M} \widehat{P}_{Mt}.$$

Note that here $(\mathcal{R}_i^{CPI,Z}, \mathcal{R}_f^{CPI,W}, \mathcal{R}_M^{CPI,M})$ stand for the responses of the CPI to changes in sectoral technology, factor prices, and import price, respectively. These objects are *model-dependent*

and thus will be different when considering the closed economy model, the small open economy model without production networks, and the small open economy with production networks.

Finally, inflation *from the model* is

$$\pi_t^{\text{Model}} = \widehat{P}_t^{\text{Model}} - \widehat{P}_{t-4}^{\text{Model}}.$$

The approach of taking log differences relative to some initial point is the most transparent because it does not modify the data much, relative to other alternatives such as standard detrending procedures.

1.4.3 Results

In this subsection, I compare inflation implied by the models, π_t^{Model}, and that in the data.

Figures 1.8 and 1.9 show inflation in the data and the one implied by the model for Chile and the United Kingdom for 2020–2022, respectively. To highlight the distinct role of production networks and openness, I consider three models: a closed economy model (Closed, pink triangles), a small open economy without production networks (SOE no Network, green $*$), and a small open economy with production networks (SOE - Network, orange circles). I plot the model's numbers using symbols rather than lines to emphasize the absence of dynamics within the model apart from those generated by the shocks I am feeding in.

Although the empirical exercise is fairly simple, it captures the data patterns well and more significantly for the small open economy with a production network in Chile and the United Kingdom.

As pointed out, the model has no intrinsic dynamics: all the action over time comes from the dynamics in $\boldsymbol{\theta}_t$. A more meaningful comparison is to compare the moments implied by the

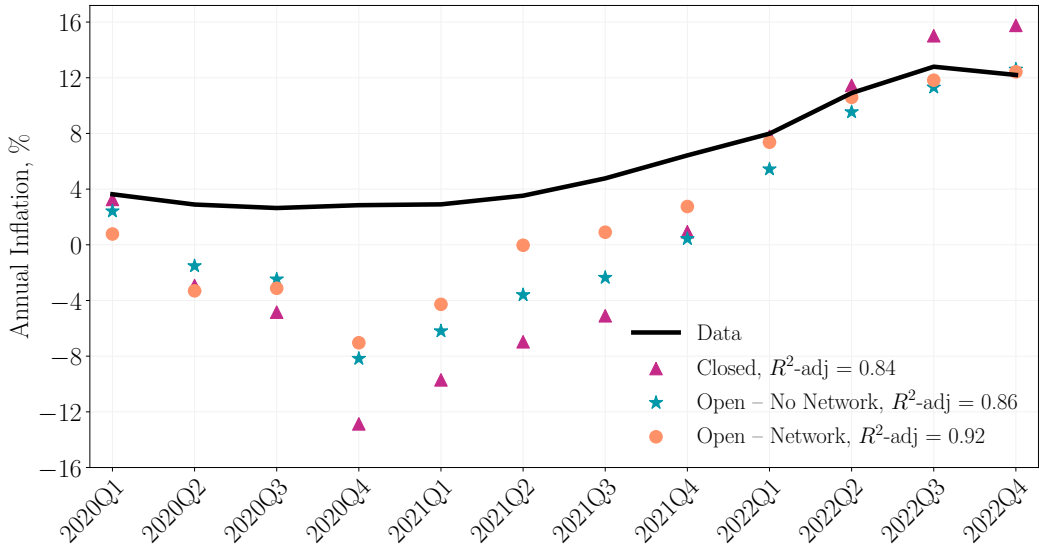
model and those in the data. Table 1.2 does precisely this and shows the first two moments of inflation in the data and the model. Panel (a) is for Chile, while Panel (b) shows the United Kingdom.

The average annual inflation in Chile between 2020 and 2022 was 6.13 percent, with a standard deviation of 3.89. The closed economy model delivers substantially lower mean inflation (0.98) and higher standard deviation (9.69) relative to the data. We can see that the sole introduction of a small open economy aspect, without production networks, gets us in the right direction as it exhibits a larger mean relative to the closed economy benchmark (1.45) and a lower standard deviation (6.88). The small open economy with production networks gets us closer to the data, with an average inflation of 2.41 and a standard deviation of 6.67.

In the United Kingdom, the average inflation was 3.69 percent, almost half that of Chile during the same period, with a standard deviation of 3.11. The closed economy benchmark generates again too little average inflation (2.27 vs. 3.31) but now too low a standard deviation (2.57 vs. 3.11). As was the case for Chile, introducing the small open economy aspect put us in the right direction: inflation is higher on average (2.72) and has a higher standard deviation (2.64). Considering production networks again improves the results: the model exhibits an even higher mean (3.21) and standard deviation (3.00).

To sum up, this exercise suggests that a small open economy model with production networks better matches inflation moments during 2020 – 2022 than a closed economy model and a small open economy model without production networks. To be fair, the small open economy with production networks should indeed do better than the other two as it adds a piece of realism missing from these other models, namely, intersectoral linkages. The question is how much. The results here are suggestive evidence that this is quantitatively relevant. Of course, the stylized

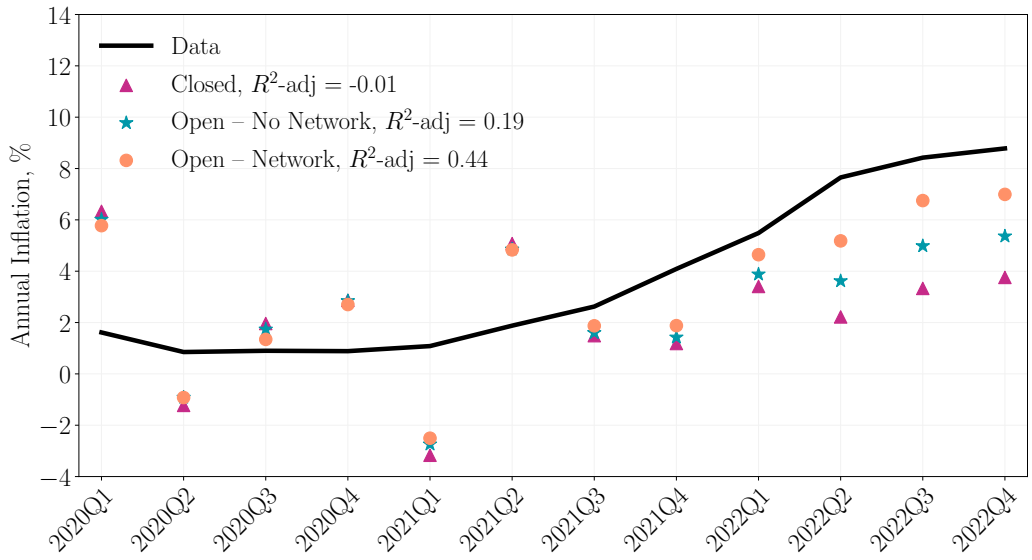
Figure 1.8: Chile Inflation under different models.



Note: This figure shows inflation in the data and the one implied by the different models. The black line is the data. The pink triangles correspond to the closed economy model. The green $*$ are the small open economy model without production networks, and the orange circles correspond to the small open economy model with production networks.

model has many missing parts, but remarkably, such a stylized exercise matches these inflation data moments well.

Figure 1.9: United Kingdom Inflation under different models.



Note: This figure shows inflation in the data and the one implied by the different models. The black line is the data. The pink triangles correspond to the closed economy model. The green $*$ are the small open economy model without production networks, and the orange circles correspond to the small open economy model with production networks.

1.5 Conclusion

I study inflation in small open economies with production networks. Theoretically and empirically, I show that production networks matter for the effect of sectoral technology shocks, factor prices, and import prices on CPI inflation. I argue that the reason why the interaction of trade and production network matters is because opening up the economy is one of the ways to break the equivalence between what is produced within borders and what is consumed by the domestic consumer, for whom the CPI is relevant. Once we break that relationship, the production network amplifies this discrepancy via indirect linkages: Non-exporters become indirect exporters, while non-importers become indirect importers. This ultimately affects the exposure

Table 1.2: Average Inflation, 2020 – 2022.

	Panel (a): Chile		Panel (b): United Kingdom	
	Mean	Std. Dev.	Mean	Std. Dev
Data	6.13	3.89	3.69	3.11
Model				
Closed	0.98	9.69	2.27	2.57
SOE no Network	1.45	6.88	2.72	2.64
SOE - Network	2.41	6.67	3.21	3.00

Note: This table shows the mean and standard deviation of inflation for the data and the different models. The Closed model uses the implied elasticities as if we consider the economies as closed. The SOE no network model considers the elasticities in a small open economy that does not feature any production network. Finally, the SOE - Network model accounts for network linkages.

of the domestic consumer to a different set of shocks. The production network thus has a first order impact on inflation, with sales and factor shares no longer being sufficient statistics to study how changes in sectoral technology or factor prices pass through to inflation, as it would be the case in a closed economy.

In a small open economy with production networks, indirect exporting dampens domestic shocks relative to an otherwise equivalent closed economy. Foreign shocks, such as import price shocks, are amplified relative to an otherwise equivalent small open economy without production networks. Which channels dominate at the aggregate level depends on the production

network structure and is, in the end, a quantitative question. I show that the production network adjustments on both the export and import side matter quantitatively across a set of small open economies. I apply the small open economy model with production networks to the recent inflationary episode in Chile and the United Kingdom. Including production networks helps better match the mean and variance of inflation of these countries between 2020 and 2022.

Chapter 2: Business Cycle Asymmetry and Input-Output Structure: The Role of Firm-to-Firm Networks

(with Jorge Miranda-Pinto and Eric R. Young. Published at the *Journal of Monetary Economics, 2023.*)

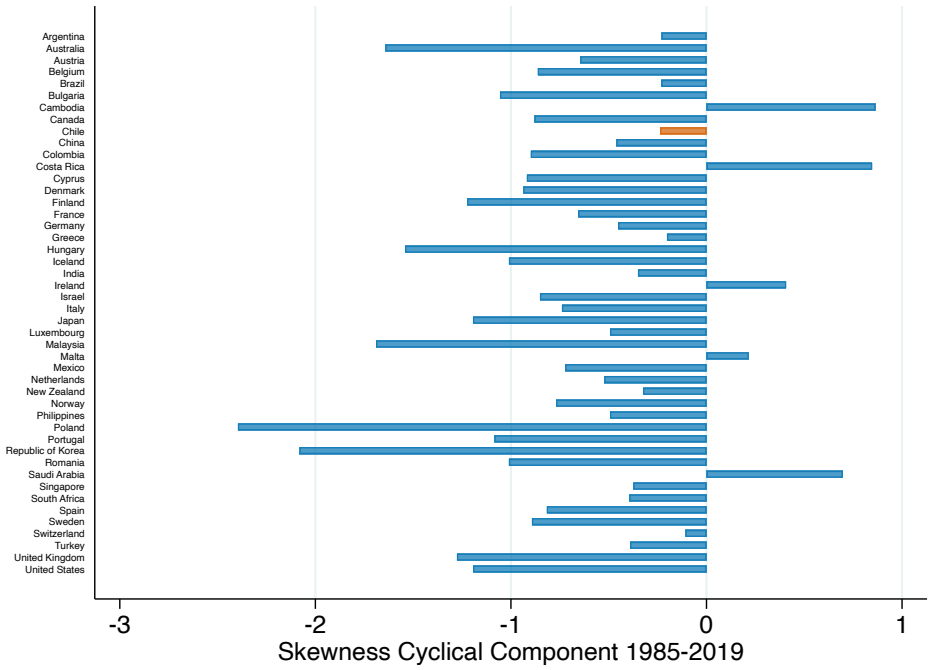
2.1 Introduction

It is a general fact that recessions are shorter and more severe than expansions, i.e. they are "sharper". This asymmetry leads to a negatively-skewed distribution of real GDP growth as documented in, for example, Ordonez (2013). Figure 2.1 shows this asymmetry for the cyclical component of real GDP for a sample of 46 countries during 1985-2019. Out of the 46 countries, 43 display negatively-skewed business cycles. The primary explanation for this fact in the literature is the existence of financial constraints (Ordonez, 2013; Jensen et al., 2020). In this paper, we offer a different explanation for the asymmetry based on the empirical importance of sectoral shocks and the structure of input-output connections.

We use sectoral input-output data for 46 countries and firm-to-firm network data for the Chilean economy to study the role of production networks in shaping the magnitude of macroeconomic downturns. In the cross-country data, we document strong correlations between the skewness of the cyclical component of real GDP — our measure of the downturn's severity—and

input-output structure (density of the network). We then use the firm-level network data to investigate to which extent production linkages at the firm level relate to firm-level output growth skewness and firm-level output declines during COVID-19.

Figure 2.1: Skewness of Cyclical Component of Real GDP (1985 – 2019)



Note: This figure plots the skewness of countries' annual cyclical component of real GDP for the period 1985-2019. We compute the cyclical component following Hamilton (2018) and estimate it as the residual (ε_t) from a regression of the form $y_t = \beta_0 + \beta_1 y_{t-2} + \beta_2 y_{t-3} + \varepsilon_t$ country-by-country, where y_t is log real GDP at time t.

Using OECD domestic input-output data, we show that — controlling for other important cross-country characteristics — countries in which more input-output connections are active (denser networks) display more negatively-skewed business cycles, as expressed by a more negative skewness of the cyclical component of real GDP for the period 1985-2019. Our estimates imply that if a country with a network density of 0.69 (the average in the sample) were to increase

its active links by 10 percentage points (to 0.79), the skewness of the cyclical component of real GDP would decrease from -0.68 to -0.98. To put the numbers into perspective, a country with a skewness of -0.68 (e.g., Italy) experiences an average percent decline in real GDP of -2.3 percent, while a country with a skewness of -0.98 (e.g., Portugal) displays an average downturn of -3.5 percent.

We then use administrative data on Chilean firm-to-firm transactions to investigate the relationship between firm-level interconnectedness, as described by the total degrees of the firm—defined as the total number of suppliers and customers the firm has—and firm-level resilience to negative shocks. In particular, we measure firm-level output (sales and employment) growth skewness and show that, controlling for covariates, firms with a larger number of suppliers and customers display a more negatively skewed distribution of output growth.[1] This relationship hides an important asymmetry as the negative relationship is mainly driven by the firms with negative skewness. The group of firms with positive skewness instead displays a positive, albeit smaller, relationship between degrees and output skewness. We then study the performance of more interconnected firms during COVID-19. We show that, controlling for other firm-level covariates, firms with more connections during COVID-19 experienced larger declines in sales and employment in 2020q2. We also find that during the recovery period, more connected firms were able to grow slightly faster than less connected firms.

We explain our evidence and quantify the role of networks in a production network model

[1] Our baseline specification uses the sum of buyers and suppliers as our preferred measure of firms' interconnectedness. We also studied the relationship between output skewness and the number of suppliers (indegree) or buyers (outdegree) separately. Our results are robust to use the sum of these measures or each one separately. See Section 2.3.3 for more details.

with N firms connected through intermediate input purchases. Our approach follows Baqaee and Farhi (2019a) closely, in which non-linearities in production can generate asymmetric business cycles out of symmetric idiosyncratic technology shocks. We start by extending the analysis in Baqaee and Farhi (2019a) and show the role of network density—the number of active input-output links—in amplifying or mitigating negative productivity shocks.[2] To do so, we study two networks that only differ in the number of positive input-output links and show that more connections amplify the adverse effects of negative productivity shocks, creating a more negative skewness of output if inputs are gross complements; the opposite holds if firms have a higher flexibility in substituting their inputs. The intuition behind our results lies in the strength of sectoral/firm-level price and quantity adjustments. In our model, price adjustments depend only on the network structure to a first order. Quantity adjustments, also to a first order, depend on the network structure and on the flexibility in substituting inputs. If inputs are gross complements, a negative productivity shock in the dense network generates a Baumol-cost disease mechanism in which the sector hit by the negative shock becomes larger in the economy relative to a more sparse network.

We then perform two quantitative exercises to understand the drivers of macroeconomic skewness and firm-level responses to negative productivity shocks. First, we calibrate our model economy to match the production network of the 46 countries in our sample. The model-implied skewness of log real GDP is negative in all countries and displays significant cross-country het-

[2]In our model, as in Carvalho et al. (2020), productivity shocks propagate upstream and downstream in the network. Therefore, indegrees and outdegrees both determine the exposure of firms to shocks.

erogeneity[3]. Moreover, we show that the model delivers a relationship between network density and skewness that is qualitatively similar to that in the data, although not quantitatively.

Our second quantitative exercise uses the firm-to-firm network structure of the Chilean economy before COVID-19. We investigate the ability of the model to deliver the non-linear relationship between firm-level (in and out) degrees (the number of customers and suppliers) and output growth before, during, and after COVID-19. We calibrate the decline in firm-level productivity using the annual percent change in revenue labor productivity. The model can deliver a relationship between degrees and output growth that is very similar to that in the data. We show that the magnitude of the shock is crucial for delivering these facts and that conditional on the size of the shock, the relationship between degrees and output growth is stronger for negative productivity shocks than for positive ones. Therefore, the model's internal propagation is strong enough to deliver a procyclical cross-sectional skewness of output growth, even if productivity shocks are symmetric. Finally, the concavity of aggregate output in this economy reconciles two seemingly contradictory facts: at the firm level, about half of the firms display positive skewness of output growth and half negative, while at the aggregate level, the economy has negatively-skewed output growth.

Contribution to the literature. The results in our paper highlight the risks of production interconnectedness when the economy is hit by large shocks. Our paper contributes to the literature that underlines the role of financial frictions in generating asymmetric business cycles (e.g., Ordonez (2013) and Jensen et al. (2020)). We instead propose a mechanism that relies on

[3]Note that 41 out of 46 countries in our sample features negative skewness of the cyclical component of real GDP.

production non-linearities and the structure of the production network.[4]

Our paper also contributes to the literature that studies the role of production network density in determining the level of GDP (Herskovic, 2018), GDP growth (Acemoglu and Azar, 2020), and the volatility of GDP (Miranda-Pinto, 2021a). Unlike the previous papers, we focus on the non-linear effects of large negative shocks to productivity and the role of firm-to-firm networks. Our paper is more closely related to Baqaee and Farhi (2019a) and Dew-Becker (2022). Similar to Dew-Becker (2022), we complement Baqaee and Farhi (2019a) by focusing on a particular network statistic (network density) and on higher-order moments of GDP. Compared to these papers, our contribution is twofold. First, we provide empirical evidence both at the country level and at the firm level that highlights the role of production networks in driving business cycle asymmetries. Second, our firm-to-firm network data allow us to test the model's predictions in the cross-section at a more granular level during COVID-19. We show that the size of the shock is crucial to deliver the observed relationship between firm-level degrees and economic resilience during COVID-19, as measured by the firm-level decline in output. Therefore, we highlight the importance of non-linearities and the need for global methods (or higher-order approximations) to solve models of intersectoral linkages and CES production technologies.

Our paper also connects to Salgado et al. (2019), who investigate the sources of the asymmetry in the cross-sectional distribution of firm-level output growth. We see our work as complementary to theirs. While they argue that shocks to the skewness of firm-level TFP shocks are important in recessions, we instead emphasize the non-linear role of firm-level network structures

[4]Our mechanism is likely to be amplified by the presence of financial frictions. In an environment like the one developed by Bigio and La'O (2020b), Miranda-Pinto and Zhang (2020) show that trade credit linkages can generate asymmetric effects of financial shocks along the supply chain.

in amplifying (large) negative productivity shocks.

2.2 Cross-country evidence

In this section, we analyze the cross-sectional relationship between countries' cyclical component of real GDP asymmetry, as measured in Figure 2.1, and countries' input-output structure. We collect data on real GDP (domestic currency) for the period 1985-2019 and real GDP per capita (at chained PPPs in 2017 US dollars) in 1985 from the Penn World Tables version 9.0. To measure the details of the production network structure across countries, we use the OECD input-output database. This dataset contains input-output data for about 60 countries at a level of disaggregation of 45 sectors for the period 1995-2018. Our final sample has 46 countries, of which about half are developed and half are emerging countries. We use the input-output data from 1995, which is the earliest available in the OECD database.

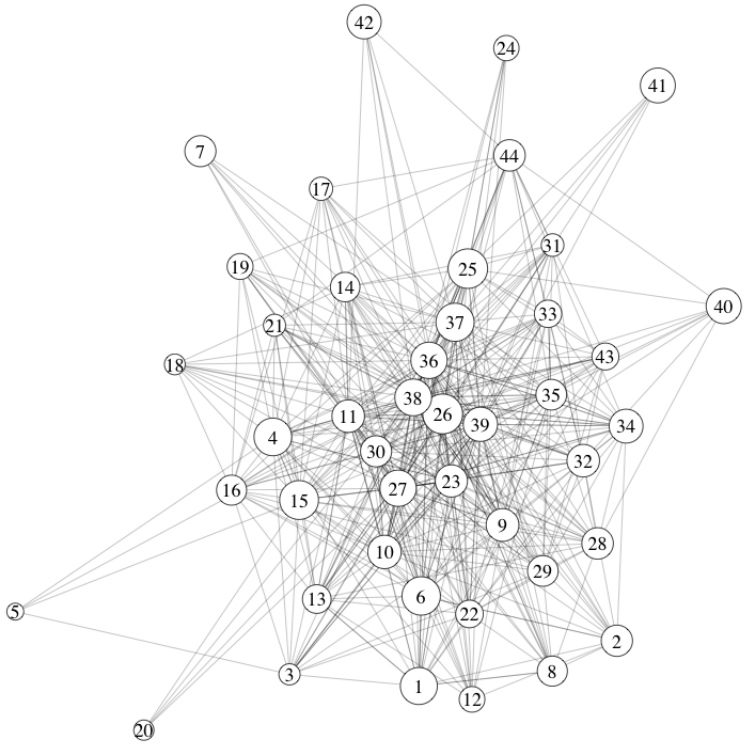
Figure 2.2 depicts the input-output network for Chile. In this figure, an arrow from sector j to sector i represents intermediate inputs flowing from j to i. The size of the nodes is determined by the size of the sector, in terms of gross output. The figure highlights the network features we focus on in this paper: the significant heterogeneity in sectoral degrees (number of suppliers and clients) and the level of production interconnectedness in the economy. We measure interconnectedness using network density as in, for example, Miranda-Pinto (2021a). In particular, our measure of density is

$$\text{Density} = \frac{\sum_{i=1}^{N}\sum_{i=j}^{N}\mathbf{1}(\omega_{ij} > 0) - N}{N(N-1)};$$

where the numerator counts the number of non-zero off-diagonal input-output links in the economy, and the denominator sums all the possible off-diagonal input-output connections in the

economy. Thus, density measures the fraction of feasible connections that are active in an economy. As an example, the Chilean production network displays a density of 69%.[5]

Figure 2.2: Input-Output Structure: Chile in 1995



Note: This figure shows the structure of inter-sectoral linkages for the Chilean economy in 1995 using the 45 sectors classification in the OECD input-output data revision 4. A node is a sector and the size of the node depends on the sectoral gross output.

Our focus in this section is the cross-country correlation between production network density and the skewness of the cyclical component of real GDP. We measure the cyclical component

[5]We use a very small threshold to define a non-zero input-output link. In particular, we use a value of 0.1% for the ratio between a specific intermediate input expenditure and total intermediate input expenditure by each sectoral pair.

of real GDP following Hamilton (2018), which hereafter we will call the Hamilton filter. We run the following regression country-by-country:

$$y_t = \beta_0 + \beta_1 y_{t-2} + \beta_2 y_{t-3} + \varepsilon_t,$$

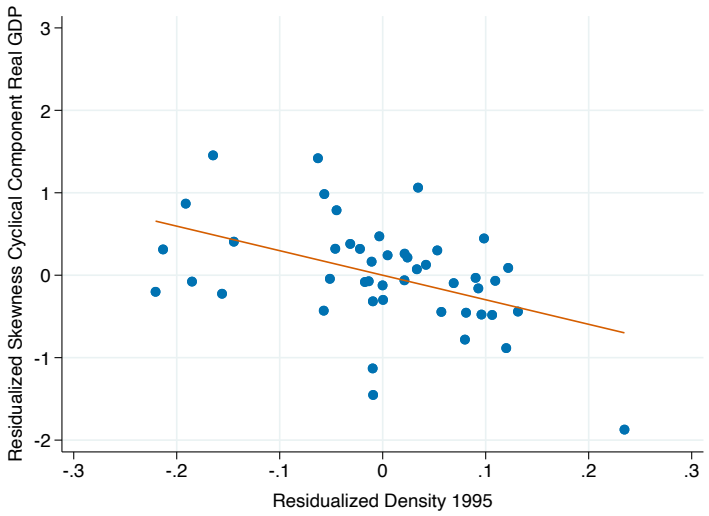
where y_t is the log of real GDP at time t and ε_t is our measure of the cyclical component. As discussed in Appendix A, we chose the Hamilton filter over linear detrending or the Hodrick-Prescott filter because it better captures downturns in the data, a must for our exercise.

To measure the cross-country correlation between density and skewness, we control for other possible network moments such as the mean, standard deviation, and skewness of the weighted outdegrees and indegrees.[6] In addition, we control for other development measures such as the GDP per capita in 1985 and the volatility of the cyclical component during the period.

Figure 2.3 shows a scatter plot of the residual skewness of the cyclical component of real GDP and network density after removing any variation coming from our controls. As is apparent from this figure, the cross-country correlation between the cyclical component's skewness and density is strongly negative, meaning that countries with more connections feature on average lower skewness of their cyclical component of real GDP. Since, on average, countries exhibit negative skewness of their cyclical component, this result means that countries with more interconnected production networks tend to exhibit more negatively skewed business cycles.

[6]The *weighted outdegree* of a sector i is the sum of the shares of sector i sales to sector j as a fraction of sector j's total output, for all j. The *weighted in-degree* of a sector i is the sum of the shares of sector i's purchases of intermediate inputs from sector j as a fraction of sector i's total output, for all j.

Figure 2.3: Cross-Country Production Network Density and Skewness



Note: This figure plots the residualized skewness of the cyclical component of real GDP between 1985 – 2019 and the residualized network density in 1995. These are residualized using the following controls: GDP per capita in 1985, the volatility of the cyclical component of real GDP, skewness of the weighted outdegree, and the weighted indegree distribution in 1995. See Table B1 in the Appendix for magnitudes of the correlation between the two variables.

2.3 Firm-level evidence

In this section, we present evidence that relates firm-level output (sales and employment) growth asymmetry to firm-level network structures. We start by describing our data and then perform two empirical exercises. First, we study the relationship between firm-level networks and firm-level output growth skewness. Second, we investigate the relationship between firm-level networks and output growth during COVID-19.

2.3.1 Data Description and Sample construction

We combine four different administrative datasets collected by the Chilean tax authority (Servicio de Impuestos Internos, SII) that provide detailed firm-level information based on firms' tax ID numbers anonymized for research purposes.[7] [8] We use data on firms' total sales from the monthly and annual tax declarations for the period 2005-2020. In particular, the F29 form keeps track of monthly sales, while the F22 form has final information on annual sales for the tax payments. We also use firm-level information on employment from the form DJ1887. Finally, we use data at the firm-to-firm transaction level from the electronic transaction system implemented since 2014.[9] These data cover the universe of formal firms in Chile, and reporting is mandatory for all firms since mid 2018.

We combine the monthly sales data in F29 with the annual sales data in F22 because the high-frequency data in F29 is more likely to suffer from misreporting problems. For example, firms could misreport sales in May but report sales in June as sales from May and June. On the other hand, the F22 annual form, which is the official data for tax purposes, should be less susceptible to misreporting, both because it is also an official document and because it is at a lower frequency. Therefore, we use the high-frequency sales data only for firms whose sales

[7]This study was developed within the scope of the research agenda conducted by the Central Bank of Chile (CBC) in the economic and financial affairs of its competence. The CBC has access to anonymized information from various public and private entities by virtue of collaboration agreements signed with these institutions.

[8]The information contained in the databases of the Chilean IRS is of a tax nature originating in self-declarations of taxpayers presented to the Service; therefore, the veracity of the data is not the responsibility of the Service.

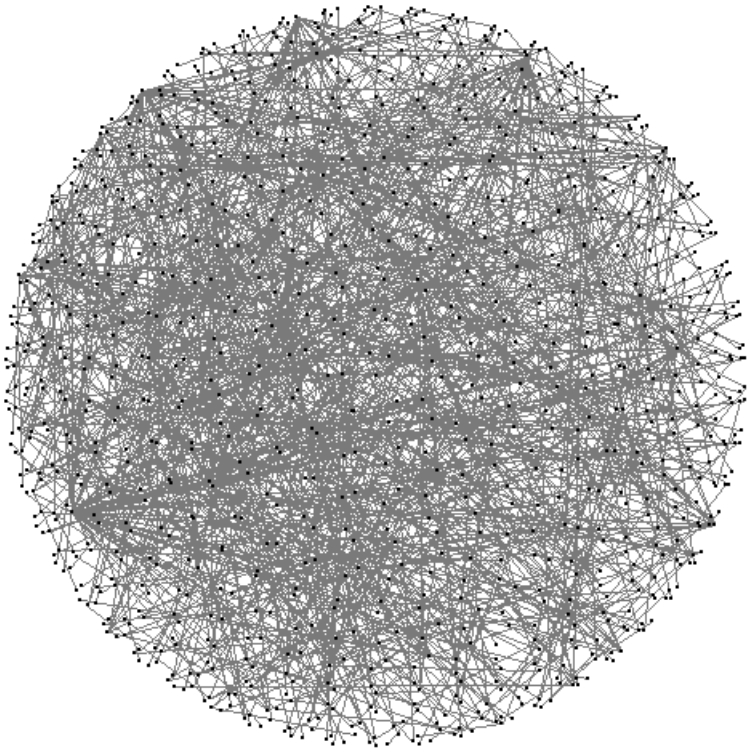
[9]To secure the privacy of workers and firms, the CBC mandates that the development, extraction, and publication of the results should not allow the identification, directly or indirectly, of natural or legal persons. All the analysis was implemented by the authors and did not involve nor compromise the IRS.

reported in the F29 represent between $90 - 110$ percent of the sales reported in the F22. In addition, we drop the bottom and top 1 percent of the observations in terms of sales growth. For this group of firms, we also obtain their total employment data from the DJ1887 tax declaration form, which requires firms to report their wage bill and the number of employees. We then aggregate these monthly data to quarterly for the period 2005q1-2020q4. For firms' sales, we simply add sales across months in a given quarter. For employment, we aggregate it by taking the simple average across months in a given quarter. For the firm-to-firm transaction data, we add up all sales for a given pair of firms (i, j) across all months in a given quarter. As a final filter, we keep firms with five or more employees that are present at least 20 quarters for the period 2005q1-2020q4. To visualize how detailed our firm-to-firm data is, Figure 2.4 plots a random sample of 2000 firms in 2019q4. For visualization purposes, we plot the links representing at least 10% of firms' total intermediate input purchases.

From the firm-to-firm transaction data used to construct Figure 2.4 we measure the unweighted indegree of firm i as the number of firms that supply a positive amount of goods or services to firm i, while we measure the unweighted outdegree of firm i as the number of firms that buy a positive amount of the output produced by firm i. We calculate the total degrees as the sum of indegrees and outdegrees at the firm level. This *total degree* is our preferred measure for assessing how many connections each firm has throughout the paper, and henceforth, we will call it *degree*. Although there are compelling reasons for using either the indegree or outdegree as the correct measure of connectedness for a given firm, it turns out that both play a role in shaping the firm-level responses we focus on — sales and employment — as we discuss in the Section 2.4.

We also consider different moments of the distribution of firms' weighted indegrees and outdegrees, which we use as controls in our regressions and to calibrate our quantitative exercises

Figure 2.4: Chilean Firm-to-Firm Network: Random Sample of 2000 firms



Note: This figure plots the firm-to-firm network in 2019q4, for a random sample of 2000 Chilean firms. A dot represents a firm, and each edge is an intermediate input sale that represents at least 10% of the client's total intermediate input purchases.

below. In particular, we obtain the average indegree as the ratio between firm i's total expenditure on other firms' output and firm i's total sales $\Omega_{iM} = \frac{\sum_{j=1}^{N} P_j M_{ij}}{P_i Q_i}$. We then obtain the input-output shares as $\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$.[10]

[10]We also follow an alternative approach to measure firm-level linkages. In particular, instead of using firm-to-firm linkages, we use disaggregated industry classifications (170 industries) to measure degrees at the firm-to-industry level. The advantage of this approach is that it can better describe different intermediate inputs in the production process (e.g., metal vs glass) rather than different varieties of the same intermediate input (e.g., glass type A and

Table 2.1 presents the main descriptive statistics. We report average sales, the average number of employees, degrees, volatility, and skewness of output growth. Two main facts stand out. First, network degrees display significant heterogeneity. Second, about half of the firms in our sample display a negatively skewed distribution of output growth.

Table 2.1: Descriptive statistics

	Mean	SD	p25	p50	p75	Obs.
Sales 1st period (millions)	314.290	4,770.680	12.186	34.507	96.921	68,885
Average employees	40.330	189.481	6.486	11.400	24.564	68,885
Degree 1st period	40.654	64.748	10.000	20.000	41.000	68,885
Standard deviation sales growth	0.500	0.282	0.278	0.438	0.676	68,885
Standard deviation employment growth	0.405	0.308	0.192	0.313	0.521	68,875
Skewness sales growth	-0.103	0.966	-0.489	-0.046	0.374	68,885
Skewness employment growth	0.048	1.230	-0.487	0.066	0.629	68,855

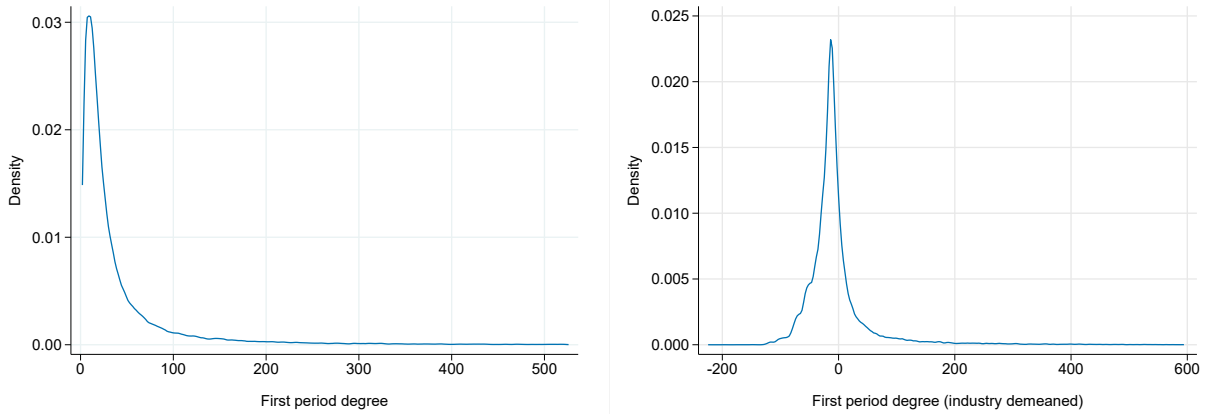
Note: This table presents basic descriptive statistics on output, network, volatility of output growth, and skewness of output growth. Sales first period and degree first period correspond to the sales and degrees that firms display either at the beginning of the period in 2005q1 or whenever the firm enters the sample (as long as it meets the requirement of 20 quarters in the sample).

glass type B that differ little and are simply sold by competitor firms). The results are very similar, which is why we prefer to use firm-to-firm linkages throughout the paper.

2.3.2 Firm-level networks

Here, we provide more information on the distribution of firm-level degrees. Figure 2.5, panel a, shows that degrees have a thick right tail. A relatively small number of firms are very well connected. Indeed, the average degree is twice as large as the median degree. Similar results hold when we consider the average degree over the period.

Figure 2.5: Network degrees distribution



Note: This figure presents the kernel distribution of firm-level degrees the first time these firms report data on linkages. Panel (a) plots the distribution of the raw data, while panel (b) reports the firm degree subtracting the industry average (170 industries classification).

Our firm-level network data allows us to study the heterogeneity in linkages within narrowly-defined industries. Panel (b) of Figure 2.5 shows that there is substantial heterogeneity in degrees across firms, even after removing industry-fixed effects. Consider the following two examples: bakery products and hotels. There are $1,509$ companies in the bakery products industry with an average first-period degree of 26.9 links, a standard deviation of 32.8, and a skewness of 7.2. In comparison, in the hotels industry there are $1,376$ companies with an average first-period degree

of 43.9 links, a standard deviation, and a skewness of 58.7 and 3.7, respectively.

We now study the relationship between firm size and linkages in the cross-section. To do so, we run the following regression

$$\log \text{Degree}_i = \alpha + \alpha_I + \beta_s \log \text{Sales}_i + \boldsymbol{\gamma}' \boldsymbol{X}_i + \varepsilon_i, \tag{2.1}$$

where $\log \text{Degree}_i$ is the average number of degrees (number of customers and suppliers) that a given firm i has during our analyzed period. α is a constant term, α_I is an industry fixed-effect, and $\boldsymbol{X}_i$ contains firm-level controls such as the average intermediate input and export share (as a share of total sales). ε_i is an error term. The parameter of interest is β_s, which is the elasticity of degrees with respect to changes in sales.

Unsurprisingly, in Table 2.2 we find that larger firms have more connections. In the cross-section, the elasticity of indegree with respect to sales is 0.30, meaning that moving up 1 percent in the distribution of firm size is associated with a 0.3 percent increase in the degree. The R^2 of the regression is 0.25, and once we control for other firm-level characteristics and sector fixed effects the R^2 of the regression only increases to 0.38. The positive relationship between degrees and size is consistent with the implications of multisector models with intersectoral linkages, as in Acemoglu et al. (2012a). However, there is a significant portion of the variation in degrees (62 percent) that is not accounted for the cross-sectional variation in firm-level observables that we use. This result leads us to believe that the structure of firm-level linkages provides valuable information beyond the firm size and industry fixed-effects that we analyze in the next sections.

Table 2.2: Size and Interconnectedness

	Dep. Var: log Degree		
	(1)	(2)	(3)
log Sales	0.333***	0.338***	0.301***
	(0.003)	(0.003)	(0.003)
Observations	64,642	64,642	64,642
R-squared	0.250	0.251	0.386
Controls	No	Yes	Yes
Sector FE	No	No	Yes

Note: This table reports the OLS coefficient of a regression in which the dependent variable is log degrees and the independent variable is log sales. Controls include average intermediate input share and the average export share (as a share of sales). Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

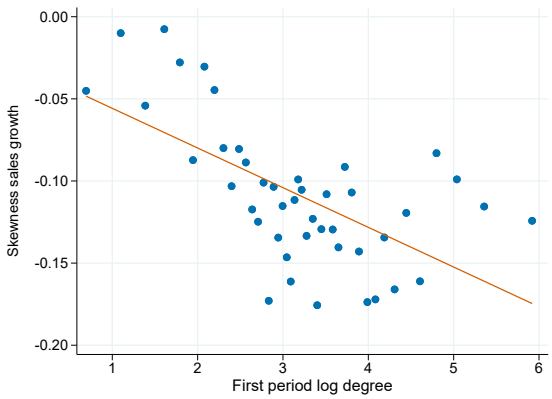
2.3.3 Firm-level output asymmetry and networks

In this subsection, we investigate the connection between firm-level asymmetry and network structure.

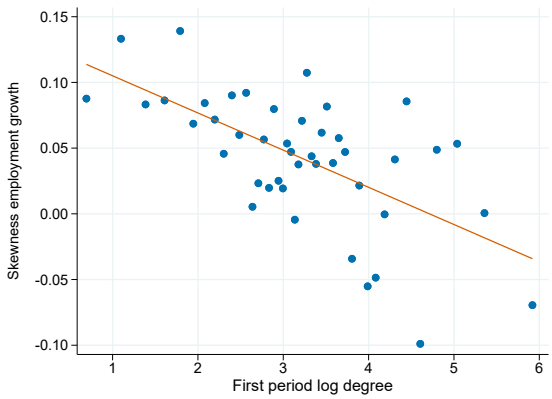
Figure 2.6 provides a first glance at the unconditional relationship between degrees and output growth skewness, measured using either sales (panel (a)) and employment (panel (b)). We observe a clear negative one: more interconnected firms display larger declines in output than

Figure 2.6: Output asymmetry and firm level networks

(a) Sales Growth Skewness



(b) Employment Growth Skewness



Note: These figures plot the binscatter plots, using 50 bins, for log degrees in the x-axis and the skewness of firm-level sales growth (panel a) the skewness of employment growth (panel b).

less connected firms.

We now study the relationship between firm-level networks and firms' asymmetry in output growth in more detail. To do so, we take advantage of the cross-sectional heterogeneity in firm-level output growth skewness for the whole sample and the average number of linkages (indegree, outdegree, and the sum of both). In particular, we estimate the following equation:

$$Y_i = \alpha + \alpha_I + \beta_d \log \text{Degree}_i + \boldsymbol{\gamma}' \boldsymbol{X}_i + \varepsilon_i, \tag{2.2}$$

where Y_i may represent either sales growth skewness or employment growth skewness during the period for a given firm i. Our parameter of interest is β_d which provides the relationship between log degree and the two skewness measures. We measure the degree of a firm using the average number of connections a firm has during the period, including both buyer and seller relationships.[11] The parameter α_I represents industry fixed effects that we include to account for the

[11] We also studied the relationship between output skewness and the number of suppliers (indegree) or buyers (out-

fact that some industries might be naturally exposed to more skewed shocks. The $\boldsymbol{X}_i$ represents firm-level characteristics and include size (in terms of sales), export share, and intermediate input share. Finally, ε_i is an error term.

In Tables 2.3 and 2.4 we report the results of estimating Equation 2.2 for both sales growth skewness and employment growth skewness, respectively. Also, we partition the number of firms into those that exhibit either negative or positive skewness and run the regressions for those subsamples. The results show that the negative relationship between degrees and skewness only holds for the group of firms with negative skewness. Columns (1) to (3) in Table 2.3 show that more interconnected firms display a more negative skewness of output (sales and employment) growth, even after controlling for firm-level characteristics and industry-fixed effects. In columns (4) to (6), we observe that the opposite holds for the group of positively skewed firms.

2.3.4 Firm-level resilience and networks during downturns

In this subsection, we follow Salgado et al. (2019) and study the asymmetry in the cross-sectional distribution of output growth and show how the firm-level network structure could provide insights into this asymmetry. Figure 2.7 shows that during macroeconomic downturns the distribution of firm-level output growth displays a fatter left tail, consistent with Salgado et al. (2019).[12]

We now zoom into the COVID-19 crisis. COVID-19 is the only recession for which we

degree) for each firm separately. The results indicate that each degree displays a negative and statistically significant relationship with output skewness. Since they do not add information separately, we used the total degree to measure network interconnectedness. As we will see later, the model we present indeed displays a similar role for both the indegree and the outdegree, which is why the total degree provides enough information.

[12] Similar results hold for the size-weighted distribution of output growth.

Table 2.3: Sales Growth Skewness and Network Degrees

	Dep. Var: Sales Growth Skewness					
	Negative Skewness			Positive Skewness		
	(1)	(2)	(3)	(4)	(5)	(6)
Log degree 1st	-0.091***	-0.056***	-0.048***	0.040***	0.040***	0.044***
	(0.004)	(0.005)	(0.005)	(0.003)	(0.004)	(0.004)
Observations	36,534	34,216	34,209	32,309	29,841	29,834
R-squared	0.016	0.031	0.129	0.005	0.043	0.072
Controls	No	Yes	Yes	No	Yes	Yes
Sector FE	No	No	Yes	No	No	Yes

Note: This table reports the OLS coefficient of a regression in which the dependent variable is the skewness of sales growth and the independent variable is log degree. Controls include log sales, intermediate input share, and export share. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

have firm-to-firm network data and COVID-19 represents an ideal event study for at least three reasons. First, COVID-19 is a very large shock, which is therefore more likely to activate non-linear effects. Second, COVID-19 is a sectoral productivity shock that had heterogeneous effects across industries (contact services vs non-contact goods and services) and firms (small vs large firms).[13] Third, during COVID-19 in Chile, financial conditions actually improved due to the implementation of several policies aimed at supporting the most-affected firms (see, Albagli et al.,

[13]There is an important demand component to the COVID-19 shock as well, due to lockdowns and other restrictions on business operations.

Table 2.4: Employment Growth Skewness and Network Degrees

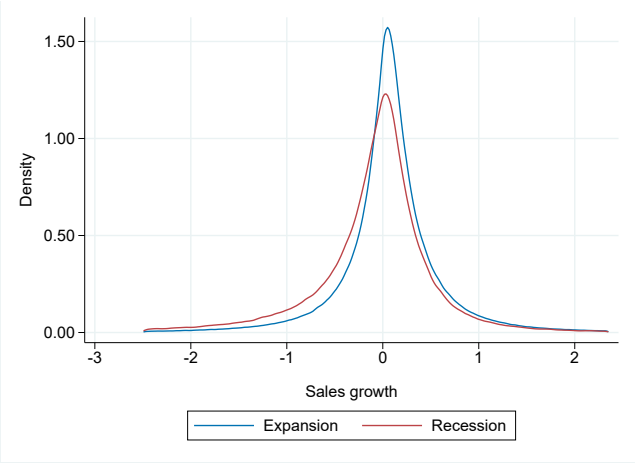
	Dep. Var: Employment Growth Skewness					
	Negative Skewness			Positive Skewness		
	(1)	(2)	(3)	(4)	(5)	(6)
Log degree 1st	-0.073***	-0.070***	-0.050***	0.035***	0.069***	0.070***
	(0.005)	(0.006)	(0.006)	(0.004)	(0.005)	(0.005)
Observations	31,992	30,329	30,324	36,859	33,709	33,705
R-squared	0.007	0.032	0.094	0.002	0.014	0.036
Controls	No	Yes	Yes	No	Yes	Yes
Sector FE	No	No	Yes	No	No	Yes

Note: This table reports the OLS coefficient of a regression in which the dependent variable is the skewness of employment growth and the independent variable is log degree. Controls include log sales, intermediate input share, and export share. Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

2021, for detailed evidence). Hence, COVID-19 represents a situation where the complementary hypothesis of financial frictions driving business cycle asymmetries may be relatively muted. As we highlight in Section 2.5.2, we propose that the large negative productivity shocks induced by COVID-19 were amplified via production interconnectedness.

We show that, consistent with Figure 2.7, firm-level output growth skewness declined significantly during COVID-19: a significantly larger number of firms experienced declines in sales and employment. In particular, Figure 2.8 shows a large increase in the mass of firms in the left tail of the distribution of sales growth during 2020q2, compared to 2019q2. But who exactly

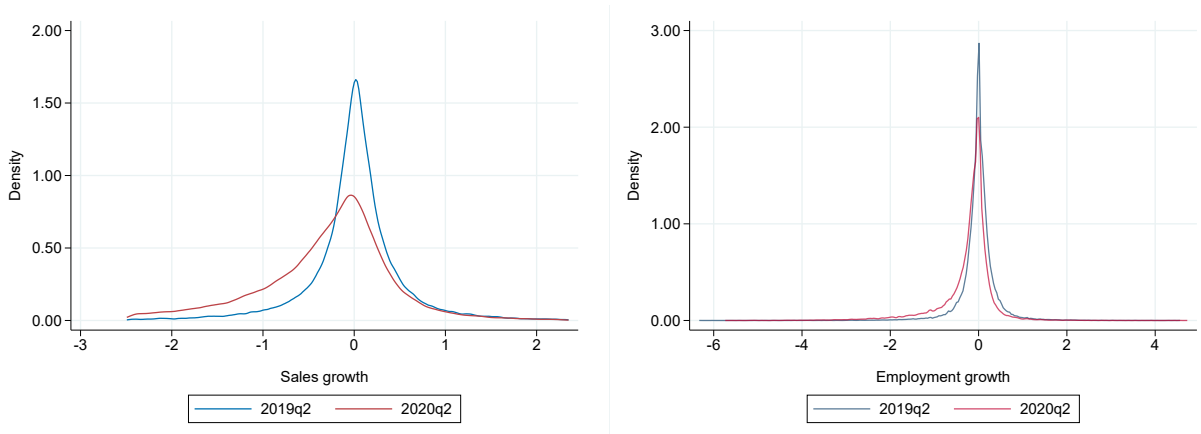
Figure 2.7: Output growth distribution recessions and expansions



Note: This figure presents the kernel density of sales growth during expansions and recessions. Recessions are defined as the firm-quarter observations in 2009 (GFC) and 2020 (COVID-19), while expansions are all the other firm-quarter observations.

Figure 2.8: Output growth distribution during COVID-19

(a) Sales Growth (b) Employment Growth



Note: This figure presents the kernel density of sales and employment growth before (2019q2) and during COVID-19 (2020q2).

were these firms? To answer this question, we run the following cross-sectional regression at each quarter t:

$$\Delta \log y_i(t) = \alpha(t) + \beta(t) \log degree_i^{2017q4} + \Gamma(t) \cdot controls(t) + \epsilon_i(t), \tag{2.3}$$

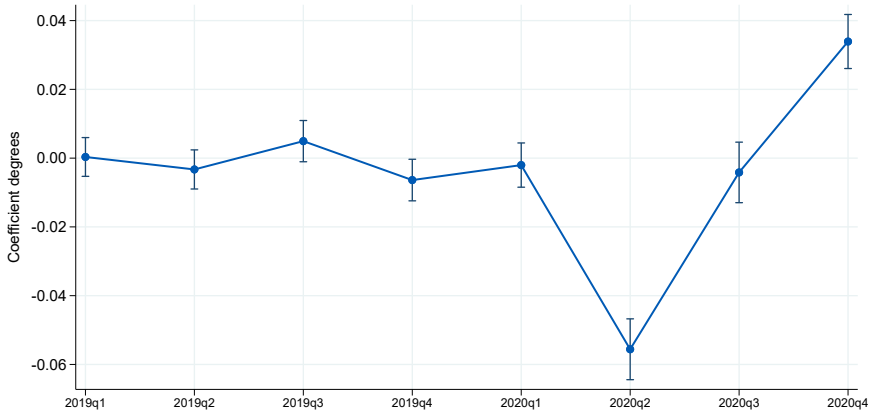
where $\beta(t)$ measures the importance of firm-level degrees in determining performance at time t compared to other firms. We include controls for industry fixed effects, sales, intermediate input shares, and export shares.

In Figure 2.9 we plot the OLS-estimated coefficient $\beta(t)$ in Equation 2.3 at different quarters in 2019 and 2020. We can see that the structure of firm-level networks was a relevant predictor of firm performance during COVID-19, even after controlling for intermediate input share, export share, sales, weighted degree measures, and industry fixed effects. A firm with a degree 10% larger (4 extra links compared to the average in Table 2.1) displayed a decline in output growth that was 0.55 percentage points larger. Interestingly, the coefficient of log sales is positive and equal to 0.04, indicating that a firm with sales 10% larger had 0.4 percentage points of larger (or less negative) sales growth. Hence, larger firms were more resilient to COVID-19, and, conditional on size, those that were more connected were less resilient. In the next section, we construct a production network model that helps us understand the cross-country and firm-level facts we document on the relationship between output skewness and network structure.

2.4 Theory

We consider a general equilibrium closed economy environment with a representative consumer, N producers, and F factors of production based on Baqaee and Farhi (2019a). This economy features no distortions or frictions. We describe each block in turn.

Figure 2.9: Coefficient of regressing output growth against log degrees



Note: This figure plots the OLS estimated $\beta(t)$ from Equation 2.3.

Notation. Throughout, we use **bold** to denote vectors and matrices. For any vector/matrix $\boldsymbol{X}$, we use $\boldsymbol{X}^T$ for its transpose.

2.4.1 Representative Consumer

The representative consumer has preferences over the N different goods according to the utility function

$$U\left(C_1, C_2, ..., C_N\right) \tag{2.4}$$

where C_i represents consumption of good i. We assume $U\left(\cdot\right)$ is homothetic.

The representative consumer owns all F factors of production and supplies them inelastically to producers. Denoting the price of factor f as W_f, the representative consumer budget constraint is then

$$\sum_{i=1}^{N} P_i C_i \leq \sum_{f=1}^{F} W_f L_f + \sum_{i=1}^{N} \Pi_i \tag{2.5}$$

where Π_i is firm i's profit.

Consumer's problem. Taking as given good and factor prices $(\boldsymbol{P}, \boldsymbol{W})$, and factor supplies $\bar{\boldsymbol{L}}$ the representative consumer chooses a sequence $\boldsymbol{C}$ of consumption demands to maximize (2.4) subject to (2.5). The solution to this problem delivers *Marshallian demands* for each good i as a function of prices and factor supplies i.e. $C_i = C_i\left(\boldsymbol{P}, \boldsymbol{W}, \bar{\boldsymbol{L}}\right)$ for all $i = 1, 2, ..., N$. We denote the optimal vector by $\boldsymbol{C}^*$ and let $Y = U\left(\boldsymbol{C}^*\right)$ to be the maximum utility.

2.4.2 Producers

There are N producers indexed by $i = 1, 2, ..., N$. Each producer i produces quantity Q_i using factors $\{L_{if}\}_{f=1}^{F}$ and intermediate inputs from other producers $\{M_{ij}\}_{j=1}^{N}$. Here L_{if} represents the demand for factor f by producer i, and M_{ij} represents producer's i demand for good j. Each producer has access to a producer-specific production function that satisfies

$$Q_i = A_i F^i\left(\{L_{if}\}_{f=1}^{F}, \{M_{ij}\}_{j=1}^{N}\right) \tag{2.6}$$

where A_i is a Hicks-neutral technology level. We assume that the $F^i(\cdot)$ is constant returns to scale.

Given good and factor prices, the total cost of producer i is

$$TC_i = \sum_{f=1}^{F} W_f L_{if} + \sum_{j=1}^{N} P_j M_{ij}. \tag{2.7}$$

Producer's problem. Taking as given good and factor prices $(\boldsymbol{P}, \boldsymbol{W})$, each producer i minimizes (2.7) subject to (2.6). The solution to this problem delivers conditional demand for all inputs (both factors and intermediate goods), that are functions of prices, technology and quantities i.e. $L_{if} = L_{if}(P, W, Q_i, A_i)$ and $M_{ij} = M_{ij}(P, W, Q_i, A_i)$.

One consequence of our assumptions is that total costs can be written as

$$TC_i = TC_i\left(\boldsymbol{W}, \boldsymbol{P}, A_i, Q_i\right) = MC_i\left(\boldsymbol{W}, \boldsymbol{P}, A_i\right) Q_i, \tag{2.8}$$

so that total costs are linear in the quantity, Q_i. This result is a consequence of the constant returns to scale assumption.

By *Shephard's Lemma*, we can write conditional demands as

$$L_{if} = \frac{\partial TC_i}{\partial W_f} = \frac{\partial MC_i}{\partial W_f} Q_i \quad \text{for all } f = 1, 2, ..., F \tag{2.9}$$

$$M_{ij} = \frac{\partial TC_i}{\partial P_j} = \frac{\partial MC_i}{\partial P_j} Q_i \quad \text{for all } j = 1, 2, ..., N \tag{2.10}$$

2.4.3 Equilibrium

To close the model, we need to specify the market clearing conditions for both good and factor markets.

$$Q_i = C_i + \sum_{j=1}^{N} M_{ji} \quad \text{for all } i = 1, 2, ..., N \tag{2.11}$$

$$\bar{L}_f = \sum_{i=1}^{N} L_{if} \quad \text{for all } f = 1, 2, ..., F \tag{2.12}$$

Equation 2.11 are the goods market clearing conditions, while Equation 2.12 are the factor market clearing conditions.

2.4.4 Useful Definitions.

We now define some objects that are going to be key for our analysis.

We let $\boldsymbol{\Omega}$ to be the *input-output matrix* of this economy, with typical element

$$\boldsymbol{\Omega} = \{\Omega_{ij}\} = \frac{P_j M_{ij}}{P_i Q_i} \quad \text{for all } i, j = 1, ..., N$$

Typical element Ω_{ij} measures how much producer i spends on good j, $P_j M_{ij}$, as a fraction of i's sales, $P_i Q_i$. Here P_i is the price of good i, Q_i is the quantity sold of good i, and M_{ij} is how much producer i buys of the quantity of good j. In other words, it is a measure of the importance of producer j (column, seller) as a *supplier* to producer i (row, buyer).

With some abuse of notation, we also define the producer's i expenditure on factor f as a fraction of its sales

$$\Omega_{if} = \frac{W_f L_{if}}{P_i Q_i}.$$

We define $\mathbf{\Psi}$ as the *Leontief-Inverse matrix*, of dimension $N \times N$, as

$$\mathbf{\Psi} = (I - \mathbf{\Omega})^{-1} = \sum_{s=0}^{\infty} \mathbf{\Omega}^s \text{ with typical element } \{\Psi_{ij}\}. \tag{2.13}$$

Ψ_{ij} denotes how important is producer j as a *direct and indirect* supplier to producer i.

On the consumption side, we define the vector of consumption shares, $\boldsymbol{b}$, as

$$\boldsymbol{b} = \{b_i\} = \frac{P_i C_i}{GDP},$$

where C_i represents the consumption of good i.

Since there are F factors of production, we define their shares of *Nominal Gross Domestic Product* (GDP) as

$$\Lambda_f = \frac{W_f \bar{L}_f}{GDP}; \quad \sum_{f=1}^{F} \Lambda_f = 1; \quad \sum_{f=1}^{F} W_f \bar{L}_f = GDP,$$

where W_f is the price of factor f, $\bar{L}_f$ is the equilibrium factor f quantity, which in this model coincides with the factor supply endowment, $\bar{L}_f$. The second result is a restatement of the first result and follows from the fact that everything in this economy is produced out of factors. Therefore, the total value added (GDP) should equal factor payments.

We let λ_i denote the *Domar weight* of producer i in total value added:

$$\lambda_i = \frac{P_i Q_i}{GDP}.$$

In the presence of intermediate goods in production, the Domar weight is the relevant size statistic for each producer's contribution to total value added (Domar, 1961; Hulten, 1978b).

2.4.5 Aggregate Impact of Sectoral Technology Shocks

We now focus on how the production network structure can affect aggregate output skewness. The following is a re-statement of a previous result in Baqaee and Farhi (2019a):

Proposition 4 (Macroeconomic Impact of Sectoral Technology Shocks)**.** To a second-order approximation, around an equilibrium defined by output $\bar{Y}$ and a Domar weights vector $\bar{\boldsymbol{\lambda}}$ of size $N \times 1$, the macroeconomic effect of sectoral technology shocks on real GDP, Y_t, is given by

$$\log Y_t = \log \bar{Y} + \sum_{i=1}^{N} \bar{\lambda}_i (\log A_{it} - \log \bar{A}_i) \left(1 + \frac{1}{2} \frac{d \log \bar{\lambda}_i}{d \log A_{it}} (\log A_{it} - \log \bar{A}_i)\right), \tag{2.14}$$

where we assume that sectoral technological changes are uncorrelated.

The above proposition highlights that sectoral technology shocks can have meaningful second-order effects on real GDP provided that the Domar weight $\bar{\lambda}_i$ reacts to changes in productivity $\log A_i$. How much $\bar{\lambda}_i$ reacts to the productivity shock depends on two key concepts: the elasticity of substitution of each producer and the production network structure. On the one hand, the elasticity of substitution is key for determining the sign of $d\bar{\lambda}_i$ (the direction of the response). The production network structure, on the other hand, provides the quantitative bite that makes the direction, given by the elasticity of substitution, stronger or weaker.

To better get the intuition for this result, consider a model with only two sectors and one factor of production that we call labor. We assume labor is the numeraire. Define the Allen-Uzawa elasticity of substitution between any pair of inputs (k, h) by a given producer j (θ^j_{kh}), as

$$\theta^j_{kh} = \frac{\frac{\partial \log M_{jk}}{\partial \log P_h}}{\Omega_{jh}}. \tag{2.15}$$

This elasticity of substitution is a share-weighted demand elasticity since $\frac{\partial \log M_{jk}}{\partial \log P_h}$ is the *constant-output* response of demand of producer j for good k when we change the price of good h.

Consider now a technology shock to a producer n such that $d \log A_n > 0$. The change in the Domar weight of a producer i change in response to this shock is given by

$$\frac{\mathrm{d}\lambda_i}{\mathrm{d}\log A_n} = \sum_{j=1}^{2} \lambda_j \Phi_j \left(\Psi_{(i)}, \Psi_{(n)}\right)$$

$$\Phi_j \left(\Psi_{(i)}, \Psi_{(n)}\right) = -\sum_{k=1}^{2}\sum_{h=1}^{2} \Omega_{jk} \left(\delta_{kh} + \left(\theta^j_{kh} - 1\right)\Omega_{jh}\right) \Psi_{ki}\Psi_{hn}$$

$$\delta_{kh} = 1 \text{ if } k = h \quad \text{and } 0 \text{ otherwise}$$

where we use $\Psi_{(i)}$ to denote the ith column of the Leontief-inverse matrix $\mathbf{\Psi}$. $\Phi_j \left(\Psi_{(i)}, \Psi_{(n)}\right)$ is what Baqaee and Farhi (2019a) call the *input-output substitution operator*. This operator is important because it records how each producer j redirects expenditures towards sector i after a change in sector productivity n. To fix ideas, take a given producer j. In the presence of intermediate input linkages, an increase in the technology of sector n translates into a price change of good h of -Ψ_{hn}. Following this decrease in the price of good h, sector j may reallocate its expenditure from other goods k towards good h. How much it does so is measured by $\Omega_{jk}\left(\delta_{kh} + \left(\theta^j_{kh} - 1\right)\Omega_{jh}\right)$ that provides the partial equilibrium change in expenditure share of producer j on good k when we vary the price of good h i.e., $\frac{\partial \Omega_{jk}}{\partial \log P_h}$. This effect is then transmit-

ted *upstream* in the production network (from the buyer to the seller) from producer k, the one that producer j was redirecting expenditure towards/from, to producer i by the element Ψ_{ki} that records how important is producer i as a seller to producer k. This chain of reasoning holds for all producers j that potentially demand good i. The final effect of producer j on producer i is weighted by the size of sector j, i.e $\lambda_j \Phi_j(\Psi_{(i)}, \Psi_{(n)})$.

As a final remark, we highlight that the previous simple example suggests that Domar weight responses depend both on the supply side and the demand side of the economy. This result means that both the roles of each firm as a supplier (outdegree) and buyer (indegree) matter for sales responses and thus shape its cross-sectional distribution. This dependence provides a rationale for using outdegrees and indegrees to explain the cross-sectional outcomes we studied in the firm-level empirical evidence.

Simple Quantitative Example. We now conduct a simple quantitative exercise to illustrate how this measure affects aggregate output skewness. Imagine a two-sector world where

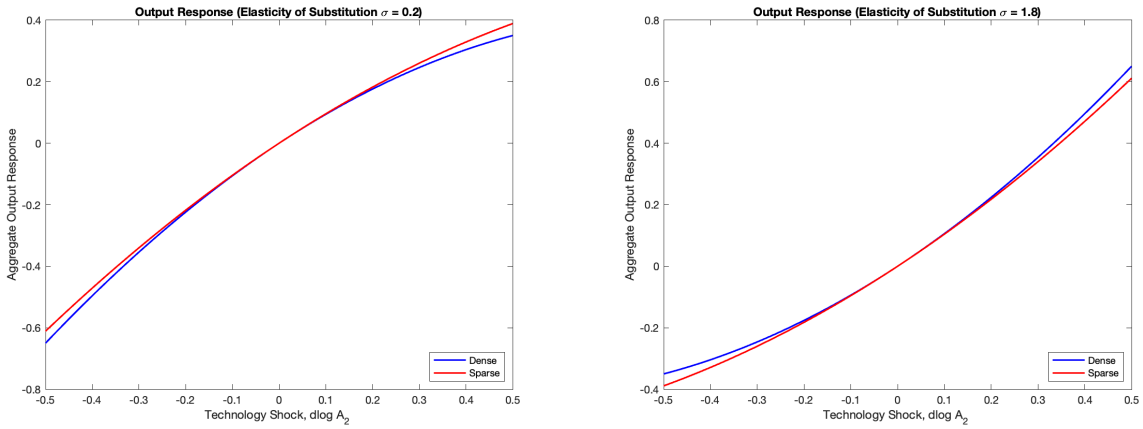
$$\mathbf{\Omega}^{sparse} = \begin{bmatrix} 0 & (1-a) \\ (1-a) & 0 \end{bmatrix} \tag{2.16}$$

$$\mathbf{\Omega}^{dense} = \begin{bmatrix} (1-a)/2 & (1-a)/2 \\ (1-a)/2 & (1-a)/2 \end{bmatrix}. \tag{2.17}$$

Figure 2.10 shows the aggregate output response to a technology shock in sector 2, for two different elasticities of substitution $\sigma = 0.2$ and $\sigma = 1.8$, and the two different network structures detailed in Equations (2.16) and (2.17) assuming $a = 0.5$. We construct this exercise by changing expenditure distribution on intermediate inputs while keeping the aggregate expenditure on intermediates, the consumption shares, and the Domar weights equal in both cases. If inputs are gross complements, as shown by Baqaee and Farhi (2019a), aggregate output is a concave

Figure 2.10: Aggregate Output Response to a Technology Shock in Sector 2

(a) $\sigma = 0.2$ (b) $\sigma = 1.8$

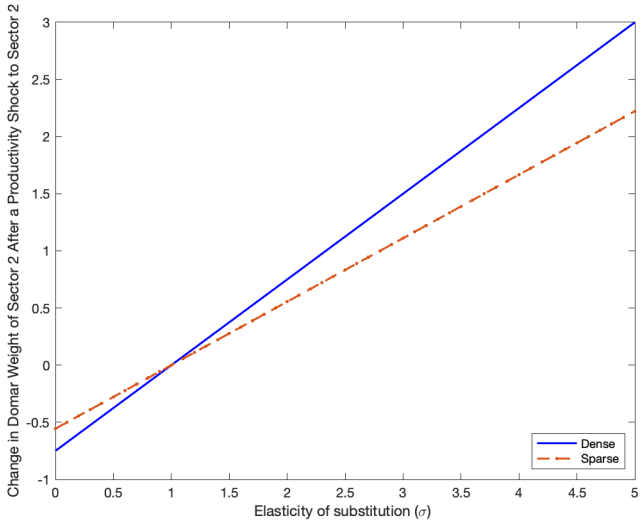


Note: This figure plots aggregate output responses to a technology shock in sector 2 for different elasticities of substitution and different network structures. The blue line shows the aggregate output responses with a dense network structure, while the red line shows the aggregate output responses with a sparse network structure.

function of productivity, implying that negative shocks are amplified and positive shocks are attenuated. Our contribution here is to demonstrate that the dense network displays a stronger concavity of aggregate output compared to the sparse network. In panel (b), we observe that the opposite holds when inputs are substitutes in production. In that case, negative shocks are mitigated, while positive shocks are amplified. Both effects are stronger in the dense network.

Figure 2.11 explains the intuition behind the results in Figure 2.10. When inputs are gross complements, positive productivity shocks to sector 2 shrink the sector's size. The decline in the Domar weight of sector 2 is larger in the dense network compared to the sparse, which explains the stronger concavity in production observed for the more interconnected network. Exactly the opposite holds when the elasticity $\sigma > 1$. This result is akin to Baumol's cost disease. Under

Figure 2.11: Changes in Domar Weight of Sector 2 after a positive technology shock



Note: This figure shows $d\lambda_2/d\log A_2$ for different values of the elasticity of substitution σ.

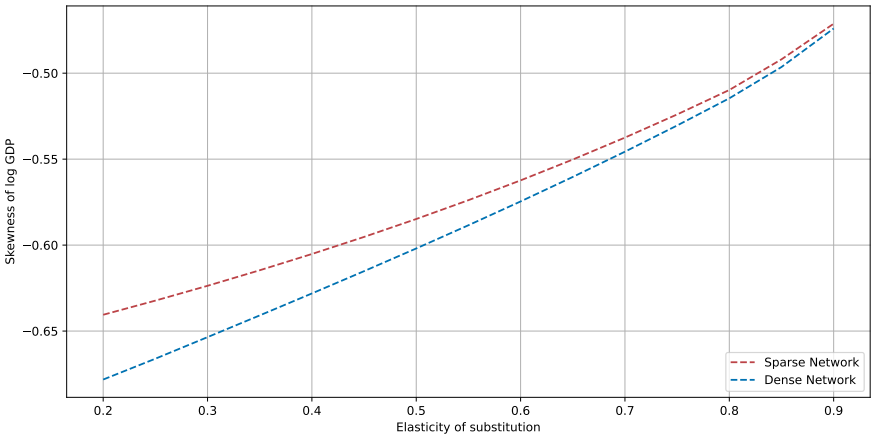
complementarities, the sector in which productivity declines becomes larger in the economy, which further amplifies the negative effect of the initial shock.

Finally, in Figure 2.12, we show the implications for the skewness of log real GDP in both networks. As we observe, the smaller the elasticity, the more negative the skewness of output in the dense network compared to the sparse one.

2.5 Quantitative Exercises

In this section, we calibrate a production network model with non-unitary elasticities of substitution between inputs and constant returns to scale in production, as in Baqaee and Farhi (2019a), Miranda-Pinto (2021a), and Carvalho et al. (2021a). We perform two quantitative exercises. First, we use the OECD industry-to-industry production network for the 46 countries

Figure 2.12: Simulated Skewness as a Function of the Elasticity



Note: This figure shows the simulated skewness of log real GDP in the dense and sparse networks, and the value of the elasticity of substitution σ in the x-axis. Productivity shocks follow a normal distribution with a mean of 1.5 and a standard deviation of 0.25. The skewness reported is the skewness of 20,000 simulations. This corresponds to the global solution of the model.

in our sample from section 2.2. Our goal is to show how the model can generate the observed relationship between macroeconomic skewness and network density. In our second exercise, we use the firm-to-firm network data for the Chilean economy from Section 2.3 to study the ability of our model to generate the cross-sectional patterns that relate firm-level networks to firm-level output asymmetry. Here, we also use the firm-to-firm calibration to evaluate the ability of our model to generate a negatively skewed distribution of real GDP.[14] A key parameter in our quantitative exercises is the elasticity of substitution between inputs. We use an elasticity of substitution between inputs σ of 0.55 from Fujiy et al. (2022) for both calibrations.[15]

[14]Note that real GDP in this model is stationary by construction and thus is the model counterpart of the cyclical component we analyze in the empirical section.

[15]The authors' estimate for the elasticity between inputs in India during COVID-19 lies within the range of the estimates from Boehm et al. (2019a) and Atalay (2017a). Although Miranda-Pinto (2021a) and Miranda-Pinto and

2.5.1 Intersectoral linkages and skewness across countries

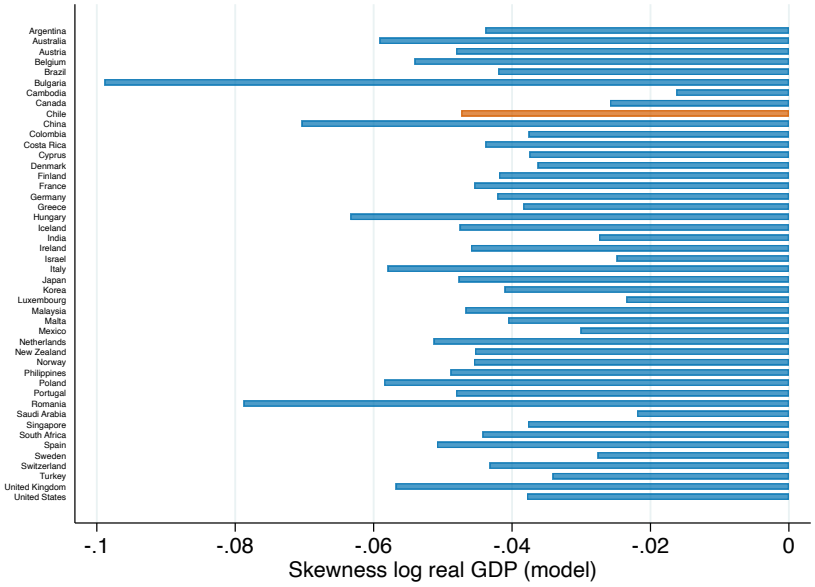
We use the OECD input-output data for the 46 countries in our sample for the year 1995 to calibrate the model input-output shares Ω_{ij}, labor shares Ω_{if}, and consumption shares b_i. We follow Baqaee and Farhi (2019a) and assume that sectoral productivity A_i follows a log-normal distribution with mean $-\Sigma_{ii}/2$ and standard deviation Σ_{ii}. For simplicity, we assume $\Sigma_{ii} = 12\%$ for all i. We obtain the model-implied skewness of log real GDP using the global solution of the model and simulating 5,000 draws of productivity. Figure 2.13 depicts the skewness of log real GDP across countries implied by the model. Two observations are consistent with the empirical evidence in Section 2.2.

First, the model can deliver a negative skewness for almost all countries, and the implied skewness shows significant heterogeneity. However, the model falls short of replicating the level of skewness. This shortfall is not surprising given that we are not targetting the level and that our calibration assumes common volatility of productivity shocks across sectors and countries. Hence, we are not leveraging the potential heterogeneity in cross-country sectoral productivity and sectoral elasticities. Indeed, Chile's skewness in this calibration is -0.023, while the firm-to-firm network calibration in the next section can generate a skewness of -0.3, which is more in line with the observed value in Figure 2.1.

Second, as we observe in Figure 2.14, the model is able to deliver the observed negative relationship between log real GDP skewness and production network density we document in section 2, Figure 2.3.

Young (2022a) find substantial sectoral heterogeneity in production elasticities, to focus on the role of networks and complementarities, we assume homogeneous elasticities.

Figure 2.13: Skewness of real GDP (model)



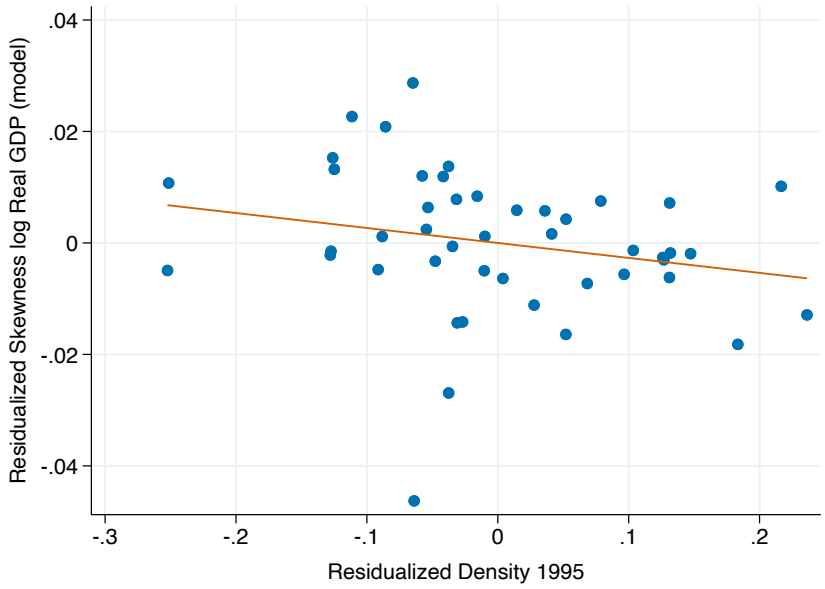
Note: This figure plots the model implied skewness of countries' log real GDP using 5,000 draws from the model. Shocks to sectoral productivity are iid log-normal with a standard deviation of 12%.

2.5.2 Firm-to-firm network in Chile

We construct our firm-to-firm production network ($\mathbf{\Omega}$) using the detailed transaction data from Chile used in section 2.4. We also calculate value-added input shares a (capital and labor). Our calibration assumes that the input-output shares and value-added input shares in 2019q2 describe the steady state of the economy. Hence, an element Ω_{ij} is the ratio between intermediate inputs that firm i spends on firm j's output as a fraction of firm i's total sales. The vector of value-added input shares is one minus the share of intermediate inputs in gross output. The vector of consumption shares $\boldsymbol{b}$ is assumed to be symmetric for all sectors $1/N$. We make this decision due to data limitations and to focus on the role of the production network.

Our sample of firms is substantially smaller than the sample of firms we had in previous

Figure 2.14: Model Implied Relationship Production Network Density and Skewness



Note: This figure plots the residualized skewness of log real GDP simulated from the model using 5,000 draws and the residualized network density in 1995. These are residualized using the following controls: the skewness of the weighted outdegree and the skewness of the weighted indegree.

sections. To be consistent with our model (no entry/exit and exogenous linkages), we choose the network of firms that display active linkages and positive sales throughout 2019q2-2020q2. We also keep firms with labor productivity data for at least eight quarters. Our final sample includes $N = 16,255$ firms.

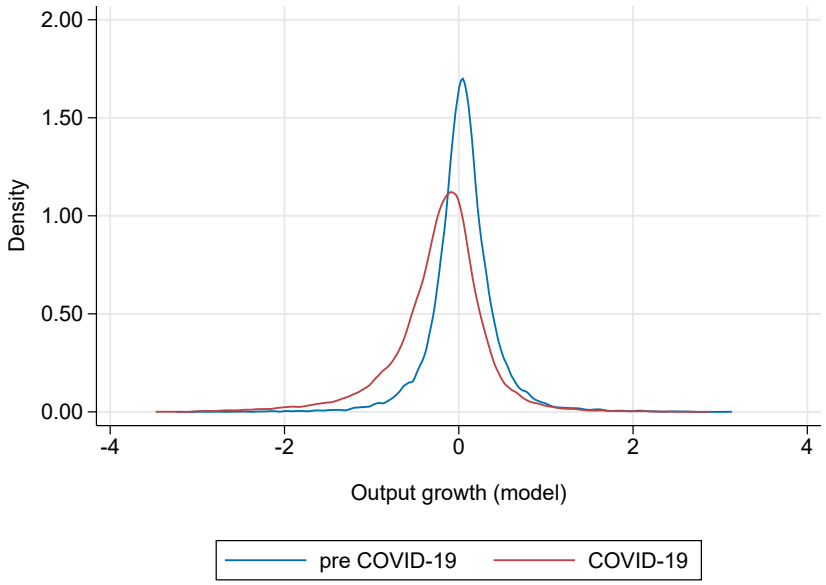
We calibrate firm-level productivity using data on firm-level revenue labor productivity $LP_{it} = \frac{P_{it}Q_{it}}{E_{it}}$, where $P_{it}Q_{it}$ and E_{it} are, respectively, total sales and total employment of firm i at time t. We compute changes in productivity relative to the previous year's quarter. For example, the percent change in labor productivity during COVID-19 is $\Delta \log A_{i,2020q2} = \log LP_{i,2020q2} - \log LP_{i,2019q2}$. Therefore, we map our model to the data through the evolution of

firm-level productivity. Our first goal is to investigate whether the model can generate the pro-cyclical skewness of output growth we observe in the data. Figure 2.15 plots the model-implied cross-sectional distribution of output growth pre-COVID-19 (2020q1) and during COVID-19 (2020q2). The cross-sectional distribution of output growth becomes substantially more asymmetric during COVID-19. While output growth skewness in 2020q1 was 0.04, it declined to -0.77 during 2020q2. This result is not driven by the skewness of the shock to productivity but entirely due to the mechanism we highlight in the paper: large negative shocks are further amplified by complementarities in production and the network structure. Indeed, as observed in Table B2 in the Appendix, during 2020q2, the average percent change in productivity is -0.04 (compared to 0.05 in 2020q1), and the skewness of cross-sectional productivity growth is -0.10 (compared to -0.09 in 2020q1).

We now investigate the role of firm-to-firm linkages in amplifying the decline in output growth during COVID-19. To do so, we study the ability of the model to generate the empirical pattern between log degrees and output growth previously documented in Figure 2.9 from estimating Equation 2.3. Figure 2.16 plots the model implied coefficient $\beta(t)$ in Equation 2.3 for different quarters in 2019 and 2020. We also plot in the same figure the coefficients and confidence intervals of the empirical estimates of $\beta(t)$. Our results show a pattern similar to that in the data. More interconnected firms saw larger declines in output during COVID-19 and also recovered faster. During other quarters, when the size of the shocks is smaller, the relationship between output growth and degrees vanishes, which emphasizes the role of non-linearities in the model.

Finally, we study the ability of the model to generate aggregate skewness. To do so, we use the time series of firm-level productivity to measure firm-level volatility of shocks. We follow

Figure 2.15: Model implied distribution of output growth distribution

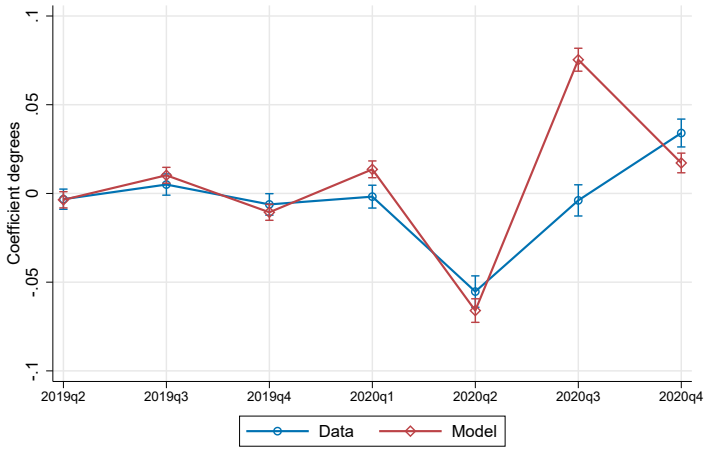


Note: This figure plots the kernel density of output growth from the model calibrated to 2020q1 (pre-COVID-19) and 2020q2 (COVID-19). Each quarter in the model corresponds to a different level of firm-level productivity, calibrated from the data.

Baqaee and Farhi (2019a) and assume that firm-level productivity is iid and log-normally distributed, with mean $-\varsigma_i/2$ and standard deviation ς_i. We measure the skewness of log real GDP from $S = 10$ simulations of $T = 100$ periods each.[16] The average skewness over the simulations is -0.31, which is significantly larger than the one implied by the industry-to-industry calibration (-0.047) for Chile and much closer to that in Figure 2.1.

[16]The solution of the model entails inverting a square matrix of dimension $N \approx 17,000$, implying that each simulation takes a considerable amount of time.

Figure 2.16: Coefficient of regressing output growth against log degrees



Note: This figure plots the OLS estimated $\beta(t)$ from $d\log q_i(t) = \alpha + \beta(t)\log degree_i + \gamma(t)\log sales_i + \epsilon_i(t)$, using the model implied $d\log q_i$. Each quarter in the model corresponds to a different level of firm-level productivity, calibrated from the data.

2.6 Conclusion

We showed that denser networks are related to business cycle asymmetries in the data – across countries, sectors, or firms, entities with more connections experience more negatively-skewed distributions of economic output. In our model, this correlation is driven by the concavity of aggregate output with respect to productivity, which complements existing results on the sources of skewed business cycles (such as financial constraints) and highlights the importance of solving models globally.

While our model is efficient, our results have policy implications that we intend to explore in future work. For example, frictions at the sectoral level can amplify sectoral shocks inefficiently, as in Bigio and La'O (2020b) and Miranda-Pinto and Young (2022a). The gains of industrial policies in such environments that reallocate inputs across sectors will hinge on how

connected the network is (Liu, 2019), and therefore also whether that network structure amplifies or dampens negative shocks. Since negative skewness imposes larger costs of fluctuations on households, understanding its source is important for assessing the welfare costs of cycles as well.

Chapter 3: Commodity Prices and Production Networks in Small Open Economies

(with Petre Caraiani, Jorge Miranda-Pinto and Juan Olaya-Agudelo)

3.1 Introduction

This paper analyzes the propagation of commodity price changes through domestic production networks in small open economies in terms of both quantities and prices. We take advantage of two stylized facts. First, the commodity sectors (mining, agriculture, and food sectors) are central sectors in small open economies, both as suppliers and buyers of intermediate inputs, which gives them a potential role as a source of supply and demand shock propagation. Second, commodity price fluctuations are only mildly correlated across sectors within a country. Therefore, as commodity prices are exogenous to non-commodity sectors in the economies analyzed, we have an ideal scenario to study the propagation of sectoral commodity price changes along the production chain.

First, we provide empirical evidence of a strong upstream propagation of quantities—to sectors providing intermediate inputs to commodity sectors—of commodity price changes in a sample of nine small open economies for the period 1995-2009. In a production network setup, in which each sector buys intermediate inputs from other sectors, commodity price booms (busts) generate an increase (decrease) in intermediate input demand. We also show that while commod-

ity price increases increase the price of downstream sectors—that is, those sectors buying from the commodity sector to produce their output—they have no real effect on the output of downstream sectors. Thus, commodity price fluctuations appear to propagate mainly as a demand-side shock in small open economies.

We then developed a static small open economy model featuring a domestic production network to explain these empirical patterns. Our model features labor and capital that are supplied inelastically. While capital is specific to the commodity sector, labor is used for all sectors of the economy and can move cost-free. Both domestic factors and goods markets are competitive, and representative firms in each sector display constant returns to scale in production. Importantly, the commodity sector supplies goods to domestic firms and consumers at home and abroad and makes production decisions; that is, it uses labor, capital, and domestic intermediate inputs in production. The commodity price is exogenously determined in international markets and driven by foreign demand.

Our model highlights four mechanisms by which commodity price changes propagate to non-commodity sectors via the domestic production network. There is one supply-side channel and three demand-side channels. We label the supply side component *cost push channel* because costs for non-commodity producers using the commodity as input increase following an increase in the commodity price. Consequently, the prices of these downstream sectors increase, generating further downstream propagation of the initial shock to other producers. The second is the *foreign demand channel*, where higher commodity exports, induced by an increase in foreign demand for the commodity sector, increase the commodity sector's demand for factors and intermediate inputs in production, pushing up production in non-commodity sectors. The key to this channel is the commodity sector's role as a buyer of non-commodity sectors, both directly

and indirectly, via domestic production networks. The third is the *domestic demand channel*: changes in commodity prices affect the total available expenditure for domestic consumers, affecting demand for all non-commodity sectors, akin to the well-known wealth effect of commodity prices. The extent to which each non-commodity sector is affected by this channel depends on its exposure to domestic consumer expenditure changes, which considers both direct and indirect linkages through production networks. The final mechanism is the *input-output substitution channel*, where all sectors reallocate their demand towards (away from) other sectors in response to changes in good and factor prices. The latter channel crucially depends on the elasticities of substitution at the consumer and producer levels and the production network structure.

In the last part of the paper, we use a simplified version of our model to shed light on the qualitative and quantitative importance of these channels. In a simple calibration exercise using Australian sectoral data as a benchmark, we point to the essential role of demand-side channels in explaining the upstream effect on quantities that we find in the empirical results. The quantitative exercise can also feature a muted downstream propagation channel in quantities, which is in line with the empirical results. In this case, the elasticity of substitution between intermediates and labor plays a crucial role, as it allows industries to rely less on relatively more expensive intermediates.

Related Literature and Contribution. This paper contributes to two strands of literature. We relate to the now extensive literature on the propagation and macroeconomic effects of commodity price fluctuations (e.g. Corden and Neary, 1982; Mendoza, 1995; Kose, 2002; Drechsel and Tenreyro, 2018; Benguria et al., 2020; Cao and Dong, 2020; Allcott and Keniston, 2018; Kohn et al., 2021; Romero, 2022; González, 2022). We contribute to this literature by providing empirical evidence on the role of domestic production networks in propagating commodity price

changes to other sectors of the economy. The closest papers to ours are Allcott and Keniston (2018) and Benguria et al. (2020), which study the effects of commodity booms on manufacturing industries that locate upstream and downstream of commodities. We contribute to these papers on the following fronts.

First, while these papers define manufacturing industries upstream or downstream to commodities using indicator variables, we consider all direct and indirect production linkages across all producers, including service sectors. Second, we consider a broader set of commodities and countries. Third, we study the effects of short-run fluctuations of commodity prices on non-commodity sectors' prices and production, while their focus is on low-frequency fluctuations in commodity prices (Benguria et al., 2020) and commodity endowments (Allcott and Keniston, 2018). In looking separately at prices and quantities, we can better dissect the transmission mechanisms of commodity prices. Fourth, unlike previous papers that only control for production linkages outside the model, our theoretical model directly speaks and interprets the implications of our empirical results.

We provide a model that precisely highlights the mechanisms by which commodity price fluctuations can affect the cross-sectional distribution of gross output and prices with an arbitrary production network structure. In particular, our model sheds light on the role of non-unitary production elasticities in amplifying the upstream propagation and dampening the downstream propagation of commodity price changes on sectoral output. To rationalize the data, our model suggests that elasticities of substitution among inputs ought to be larger than one. At the annual frequency, which is the frequency we consider, several studies suggest that elasticities are larger than one (e.g. Carvalho et al., 2021b; Miranda-Pinto, 2021b; Huneeus, 2020; Nakano

and Nishimura, 2023).[1] Although recent estimates of elasticities are below one (e,g., Boehm et al., 2019b; Barrot and Sauvagnat, 2016), these elasticities are usually short-run estimates at the monthly or quarterly frequency.[2] Further, we show that the well-known wealth effect of commodity price fluctuations has an important (upstream) network propagation component.

We also contribute to the literature on production networks and business cycle fluctuations (e.g., Horvath, 1998; Foerster et al., 2011; Acemoglu et al., 2012b; Atalay, 2017b; Baqaee and Farhi, 2019b, 2021; Miranda-Pinto, 2021b; vom Lehn and Winberry, 2020; Carvalho et al., 2021b), which has mainly focused on closed-economy models. We highlight that, in a small open economy, commodity price changes can have important effects on prices and output quantities and are largely propagated through input-output linkages. As in Carvalho et al. (2021b) and Luo (2020) that emphasize the upstream and downstream propagation of productivity shocks and financial shocks, respectively, we show that commodity prices can strongly propagate to upstream and downstream sectors.

Finally, our paper emphasizes that the cross-sectional distribution of output can respond to commodity price changes while real gross domestic product — measured at constant prices — can stay constant. Thus, from a macroeconomic perspective, our paper exploits the result that commodity price fluctuations generate movements along the production possibility frontier but do not shift it. The particular point in the production possibility frontier the economy ends up

[1] Atalay (2017b) shows that the elasticity of substitution between labor and intermediates is close to one, while the elasticity among intermediate inputs is closer to zero. Miranda-Pinto (2021b) uses the same data in Atalay (2017b) and finds that production elasticities are highly heterogeneous among sectors. While manufacturing industries have elasticities similar to those estimated by Atalay (2017b), service sectors present elasticities that exceed one.

[2] Boehm et al. (2023) and Peter and Ruane (2023) highlights the importance of the horizon in estimating trade elasticities.

after the commodity price shock depends on elasticities of substitution and network exposures and is what this paper is concerned about. In addition, our paper provides a laboratory to explore the role of elasticities of substitution among inputs in matching salient facts of the transmission of shocks via domestic production networks in line with recent literature and our discussion above (e.g., Boehm et al., 2019b; Miranda-Pinto, 2021b; Carvalho et al., 2021b; Miranda-Pinto and Young, 2022b).

3.2 Stylized Facts

In this section, we present two stylized facts regarding commodity sectors. First, commodity sectors are central in the domestic production network. Second, commodity price changes strongly commove across countries but present a very small correlation across sectors within countries.

We first define what we mean by commodity sectors. To do so, we combine data on commodity goods' exports from Fernández et al. (2018) and input-output data from the WIOD. We use the WIOD data as, unlike the OECD input-output data, it contains sectoral information on production and prices, separately. For more details on data sources and definitions please refer to our Appendix C.1. We match each commodity good to one of the 34 industries in the World Input-Output Database (WIOD). Table C.2 in our Appendix C.2 provides a detailed mapping between goods and sectors in the WIOD data.[3] The three commodity sectors in the WIOD are Agriculture, Forestry, and Fishing; Mining and Quarrying; and Food Products, Beverages, and Tobacco.

[3]In our Appendix C.1 we also provide information on the sample of countries we use from the WIOD and the definition of the variables.

Fact 1: Commodity sectors are central sectors in the production network. We describe the network centrality of commodity sectors using standard centrality measures that capture how connected the sectors I am connected to and how connected the sectors that are connected to the sectors that I am connected to, etc. To that end, we analyze commodity sectors' customer and supplier centrality following Acemoglu et al. (2016).[4] We measure the supplier or *downstream* centrality of a given sector i as

$$Supplier_i = \sum_{j=1}^{N} \Psi_{ji}, \tag{3.1}$$

where Ψ_{ij} is an element of the Leontieff-Inverse matrix defined as

$$\boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} = \sum_{s=0}^{\infty} \boldsymbol{\Omega}^s$$

where $\boldsymbol{I}$ is an identity matrix of size equal to the size of $\boldsymbol{\Omega}$. An element of $\boldsymbol{\Omega}$ is $\Omega_{ji} = P_i M_{ji}/P_j Q_j$. This represents the share of intermediates that sector i supplies to sector j ($P_i M_{ji}$) as a fraction of sector j's sales ($P_j Q_j$). This shows the direct importance of producer j as a supplier to producer i. An element Ψ_{ji} then records the importance of producer i as a supplier to producer j after considering both direct and indirect linkages. This intuition is precisely highlighted by the last equality in the equation above, where $\boldsymbol{\Psi}$ is an infinite sum of direct and indirect linkages across

[4]These definitions are slightly different from the notions of downstreamness and upstreamness highlighted in the global value chains literature (see Antras and Chor, 2021). Their measure of upstreamness shows how important other sectors are as buyers to a given sector i. In our case, customer centrality comes from the importance of sector i as a buyer to other sectors. This difference is expected because we focus on how shocks propagate, as in Acemoglu et al. (2016), while Antras and Chor (2021) focuses on the distance of each sector to final demand and primary factors. Our concept is closer to the Katz-Bonacich centrality used in the production networks literature. See Carvalho (2014) for an overview, especially footnote 11.

producers. Therefore, $Supplier_i$ adds across all buyers of good i and measures its importance as a *supplier* to the economy after taking into account direct and indirect linkages.

We then measure the customer or *upstream* centrality of a sector i as

$$Customer_i = \sum_{j=1}^{N} \widetilde{\Psi}_{ij}, \tag{3.2}$$

where $\widetilde{\Psi}_{ij}$ is an element of the following matrix

$$\widetilde{\boldsymbol{\Psi}} = (\boldsymbol{I} - \boldsymbol{M})^{-1} = \sum_{s=0}^{\infty} \boldsymbol{M}^s$$

where $\boldsymbol{I}$ is an identity matrix of size equal to the size of $\boldsymbol{M}$. An element of $\boldsymbol{M}$ is $m_{ij} = P_j M_{ij}/P_j Q_j$. This represents the share of the sector's j sales that the sector i accounts for. This shows the direct importance of producer i as a buyer to producer j. An element $\widetilde{\Psi}_{ij}$ then records the importance of producer i as a *buyer* to producer j after considering both direct and indirect linkages. $Customer_i$ adds across all suppliers to sector i and measures sector i's importance as a *buyer* to the economy after considering direct and indirect linkages.

Figure 3.1 plots the domestic network structure of Australia in 1995, using input-output data from the WIOD database. Each node (circle) is a different sector in the economy, and the node's size represents how important that sector is in the network based on the network centralities defined above. Panel (a) shows the network in which each node's size describes the customer centrality of the sector—this is, how much output of other sectors a given sector uses, directly and indirectly—, while in panel (b), the node size is based on each sector's supplier centrality—how much of a given sector output is used as input by other sectors, directly and indirectly.

We observe in Figure 3.1 that commodity sectors were central sectors in the domestic production network of Australia in 1995. In particular, panel (a) shows that the food (3) sector is one

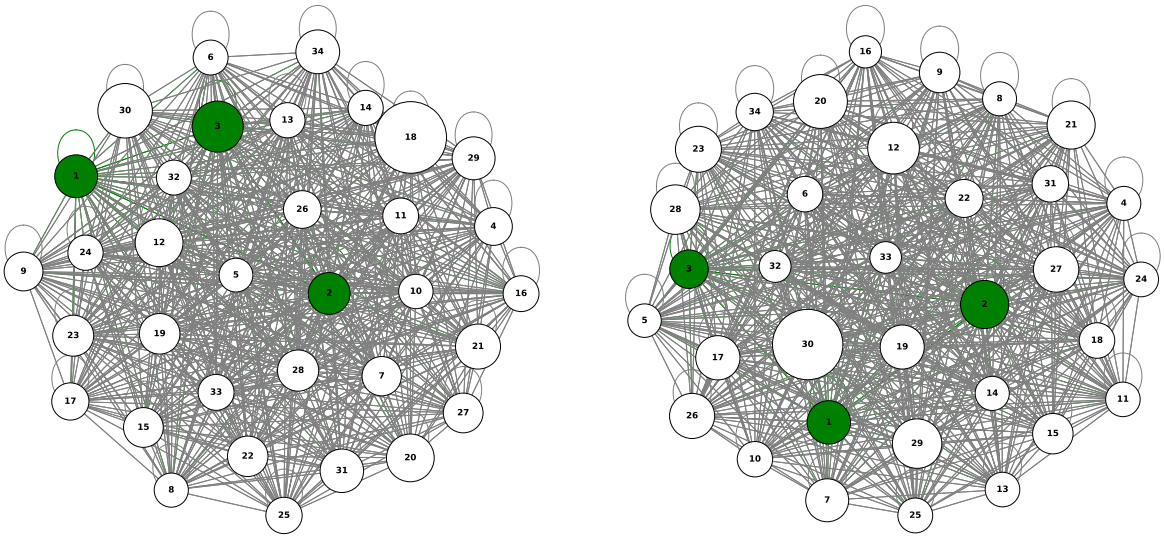


Figure 3.1: Domestic Production Network Australia

Note: This figure shows the domestic production network of Australia (WIOD Input-Output data) for 2011 at the sector level (ISIC rev. 3). An arrow from sector j to sector i represents intermediate inputs flowing from j o i. Each node (circle) is a different sector in the economy, and the size of the node represents how important that sector is as a direct and indirect buyer (panel a) and supplier (panel b) of intermediate inputs. The labels in the nodes are linked to sectors in Table C.1 of our Appendix.

of the sectors with the largest customer centrality. Panel (b) also shows that mining is one of the most central sectors in its direct and indirect supply of intermediates inputs.

To describe the relative importance of commodity sectors in the domestic production network of small open economies, we report in Table 3.1 the ranking of the customer and supplier propagation centrality for the three commodity sectors, with respect to all the other sectors in the economy (a total of 34 in the WIOD data). The main takeaway from Table 3.1 is that for all the countries in our sample, at least one of the commodity sectors (many times 2 of them) is a central customer and/or a central supplier (top-10) in the domestic production network.

Table 3.1: Ranking of Network Centrality of Commodity Sectors in 1995

	Customer Centrality			*Supplier Centrality*		
Country	Agric.	Mining	Food	Agric.	Mining	Food
Australia	10	11	3	13	6	17
Bulgaria	2	8	1	2	9	13
Brazil	14	25	2	7	14	10
Canada	6	18	3	4	10	15
Denmark	6	33	1	8	17	11
India	9	25	6	3	9	23
Lithuania	1	33	3	2	34	9
Mexico	10	18	1	7	1	15
Russia	3	6	2	5	3	14
Average	**7**	**20**	**2**	**6**	**11**	**14**

Note: This table presents, for each country and commodity sector, the customer and supplier network centrality. Source: WIOD Input-Output database, 1995.

Fact 2: Sectoral commodity price changes are only mildly correlated across sectors, within a country. We first describe the process of constructing sectoral indexes of commodity prices.

(i) We use the export data Fernández et al. (2018) and calculate, for each country, the share of each commodity good in its sectoral group, be it agriculture, mining, or food sectors. Then, we multiply each sector-country weight by the monthly commodity price.

(ii) The outcome from step (i) is a matrix of country-specific monthly commodity price index that we deflate using the US Consumer Price Index (CPI).

(iii) We take the average across months within each quarter by year.

We now investigate the correlation between sectoral commodity price growth within countries. As highlighted in Fernández et al. (2018), commodity fluctuations strongly commove across countries. Indeed, the cross-country correlation between commodity price changes in Agriculture and Forestry, Mining and Quarrying, and Foods Products and Beverage sectors are 0.85, 0.65, and 0.5, respectively. However, as shown in Table 3.2, the growth rate of commodity prices present a small correlation across sectors within countries. The average cross-country correlation between agriculture and mining commodity price growth is 0.57; the average cross-country correlation between agriculture and food commodity prices is 0.16; and the average cross-country correlation between mining and foods commodity prices is -0.13.[5]

Table 3.2: Average Pairwise Correlation across Commodity Prices

	Correlation
Agriculture/Mining	0.57
Agriculture/Food	0.16
Mining/Food	-0.13

Note: This table presents the cross-country average of the within-country pairwise correlations among the log change of sectoral commodity prices.

[5] Figure C.1 to Figure C.3 in our Appendix depict sectoral commodity price growth for countries in our sample. Besides confirming Fact 2 (low within-country correlation across commodity prices), these figures show substantial volatility of commodity prices over time.

3.3 Commodity prices via production networks

In this section, we study the network effects of commodity price changes on non-commodity sectors.[6] Our identification assumption is that commodity prices determined in international markets are exogenous to non-commodity sectors in these small open economies.[7] Our empirical specification follows Acemoglu et al. (2016) and Carvalho et al. (2021b). In particular, we estimate

$$y_{ict} = \delta_t + \alpha_{i,c} + \delta_{c,t} + \phi_1 Upstream_{ict} + \phi_2 Downstream_{ict} + \boldsymbol{\nu}' \boldsymbol{X}_{ict-1} + \epsilon_{ict}, \tag{3.3}$$

where y_{ict} is sector i's output or prices in country c at time t. δ_t represent year fixed effects, $\alpha_{i,c}$ are country-sector fixed-effects, and $\delta_{c,t}$ are a full set of country-time fixed effects. $Upstream_{ict}$ and $Downstream_{ict}$ are our network spillover measures, which we explain in the next subsection, and vary at the sector-country-year level. $\boldsymbol{X}_{ict-1}$ is a $H \times 1$ vector of lagged controls, including the dependent variable and our network spillover measures. Finally, ϵ_{ict} is an error term.

Theoretically, as shown in Acemoglu et al. (2016), supply-side shocks (e.g., productivity

[6] While there is an important literature emphasizing the different effects of demand-side vs. supply-side shocks to commodity prices (e.g., Kilian, 2009; Aastveit et al., 2023), our goal is to study the propagation mechanisms of changes in commodity prices, regardless of the source of shock. Nevertheless, our results will speak to the potential sources of commodity prices. In particular, demand-side shocks to commodity prices propagate more strongly to sectors upstream to commodities, while supply-side shocks to commodity prices propagate more strongly to sectors downstream to commodities.

[7] We could instead estimate commodity price shocks using the SVAR approach proposed by Schmitt-Grohé and Uribe (2018). However, this approach reduces the amount of observations in our dataset by 20%. Still, the results using SVAR shocks are similar to the results presented next and are available upon request.

shocks) should mainly propagate to downstream industries through an intermediate input cost channel. In our case, if the Downstream shock reflects increased production cost from increased commodity prices, we expect $\phi_2 < 0$. On the other hand, an increase in commodity prices can boost exports and, therefore, increase demand for domestic inputs, implying $\phi_1 > 0$. We include a full set of fixed effects to account for different sources of variation. We first include a full set of year-fixed effects that account for any common differences across years. Then, we use a set of country-sector fixed effects, to account for any time-invariant differences idiosyncratic to each country-pair ($\alpha_{i,c}$). Finally, we add country-time fixed effects to account for differences between countries over time ($\delta_{c,t}$).

In the next subsection, we define and briefly explain the network spillover measures that shape the effects of commodity prices on non-commodity sectoral output in a small open economy. These network measures are an application of the strategy in Acemoglu et al. (2016)—developed to understand the propagation of productivity shocks and government spending shocks in a closed economy—for the context of a small open economy subject to commodity price fluctuations.

3.3.1 Measuring Network Spillovers

We now outline our network spillover measures along the lines of Acemoglu et al. (2016). We denote a commodity sector by $k \in \mathcal{K}$ where $\mathcal{K}$ is the set of commodity sectors. We denote non-commodity sectors by either i or j, where $i, j = 1, ..., N$ with N the total number of non-commodity sectors.

The downstream effect of commodity price fluctuations that is to those *buying* from the

commodity sector either directly or indirectly through input-output linkages, in sector i in the country c at time t is

$$Downstream_{ict} = \sum_{k \in \mathcal{K}} \left(\Psi_{ikc} - \mathbf{1}_{i=k} \right) \cdot \tilde{p}_{kct}, \tag{3.4}$$

where Ψ_{ikc} stands for the importance of the commodity sector k in supplying intermediate inputs to sector i both directly and indirectly through the domestic production network of country c, $\mathbf{1}_{i=k}$ is an indicator variable that takes the value of 1 when $i = k$ and zero otherwise. $\tilde{p}_{kct}$ corresponds to commodity price growth for commodity sector k in the country c at time t.

The upstream effect of commodity price fluctuations, that is from the commodity sectors to that *selling* to it, from sector k to sector i in the country c at time t is measured as

$$Upstream_{ict} = \sum_{k \in \mathcal{K}} \left(\tilde{\Psi}_{kic} - \mathbf{1}_{i=k} \right) \cdot \tilde{p}_{kct}, \tag{3.5}$$

where $\tilde{\Psi}_{kic}$ stands for the direct and indirect importance of commodity sector k as a buyer to sector i.

As pointed out in Acemoglu et al. (2016), the production networks literature is usually ambiguous about what upstream or downstream means. In this paper, we strictly follow their approach in that upstream or downstream refers to how shocks are propagated throughout the network structure and not by the sectors' position. A graphical representation of this idea is in Figure 3.2 below, where we plot two sectors k and i where sector k supplies to sector i. Here, the shock to sector k (the supplier) propagates *downstream*, while a shock to sector i (the buyer) propagates *upstream*.

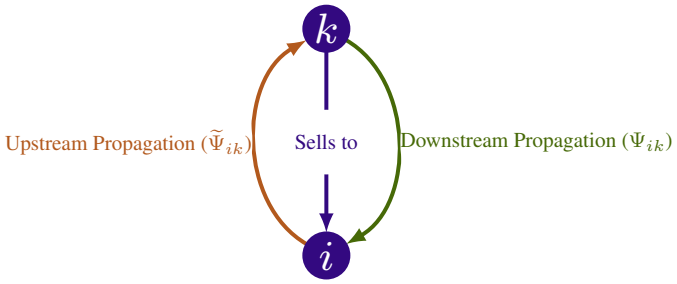


Figure 3.2: Upstream and Downstream Propagation

Note: This figure shows the propagation of shocks along the production network where we remove all other nodes and focus on total propagation (both direct and indirect). Downstream propagation from seller k to buyer i (Ψ_{ik}) and upstream propagation from buyer i to seller k ($\tilde{\Psi}_{ik}$). This illustrates the construction of measures in equations (3.4) and (3.5).

3.3.2 Network propagation

We now present empirical evidence on the transmission mechanism of commodity price fluctuations via production networks using the WIOD database. The WIOD database has an important advantage compared to the OECD database: it reports sectoral quantity and price indexes, allowing us to better study the channels in which commodity price changes affect quantities and prices. Instead, the OECD data only reports nominal data (in US dollars) for sales, value-added and intermediate input use. To construct the Upstream and Downstream network effects defined in Equation (3.4) and Equation (3.5) we use the input-output structure in 1995.

Table 3.3 presents the results of estimating Equation (3.3) using quantity and price indexes for gross output. All regressions include one lag of the dependent variable. To ease the interpretation of our coefficients, we standardized our Upstream and Downstream measures to have a unit standard deviation. We first focus on the effects on sectors selling to the commodity sector

($Upstream_{ict}$). Columns (1) to (3) show that real commodity price fluctuations positively affect the gross output of non-commodity sectors. In particular, in column (3)—where we control for a year, country-sector, and country-year fixed effects—a one standard deviation increase in commodity prices generates a 0.72 percent (1.3 percent) increase in the sectoral gross output quantity index, on impact (cumulative). We find no evidence of downstream ($Downstream_{ict}$) effects on quantities of commodity price changes. Columns (4) to (6) show that, despite the muted downstream effect on quantities, we observe a strong downstream propagation of commodity prices to the price of non-commodity sectors, with no upstream propagation. A one standard deviation increase in commodity prices generates a 0.82 percent impact increase in the sectoral gross output price index (1.97 percent cumulative) of non-commodity sectors downstream to commodities. In our Appendix, Table C.5, we show that the same results hold when we exclude Russia from our sample.

The empirical evidence in this section points to strong upstream propagation of commodity prices, alongside a muted downstream propagation, on the quantity produced by non-commodity industries. At the same time, we find a strong increase in the price of industries that are downstream from the commodity sector with no effect on upstream industries. In the next section, we build a theoretical model of a small open economy with production networks and a commodity sector that rationalizes the findings we document in this section. In particular, we ask how can a small open economy model with production networks, in which the commodity sector is a central supplier and user of intermediate inputs (as documented in Table 3.1), rationalize the large upstream propagation and muted downstream propagation of commodity price fluctuations.

Table 3.3: Network Effects of Commodity Price Changes on Non-Commodity Sectors

	Panel (a): Quantity			Panel (b): Prices		
	(1)	(2)	(3)	(4)	(5)	(6)
Upstream$_{ict}$	0.0067**	0.0080**	0.0072***	0.0004	0.0067	0.0019
	(0.0031)	(0.0031)	(0.0022)	(0.0067)	(0.0075)	(0.0024)
Upstream$_{ict-1}$	0.0027	0.0055	0.0058***	-0.0171	-0.0008	-0.0003
	(0.0035)	(0.0037)	(0.0020)	(0.0137)	(0.0070)	(0.0018)
Downstream$_{ict}$	0.0022	0.0018	-0.0007	0.0104*	0.0099**	0.0082***
	(0.0017)	(0.0016)	(0.0012)	(0.0054)	(0.0049)	(0.0026)
Downstream$_{ict-1}$	-0.0020	-0.0020	-0.0024**	0.0074	0.0090**	0.0115***
	(0.0015)	(0.0015)	(0.0011)	(0.0058)	(0.0039)	(0.0023)
Accumulated Upstream	0.0094**	0.0135***	0.0130***	-0.0167	0.0058	0.0016
	(0.0044)	(0.0047)	(0.0032)	(0.0159)	(0.0092)	(0.0035)
Accumulated Downstream	0.0002	-0.0001	-0.0031	0.0178***	0.0189***	0.0197***
	(0.0023)	(0.0024)	(0.0019)	(0.0097)	(0.0073)	(0.0036)
Observations	3906	3906	3906	3906	3906	3906
Within R^2	0.924	0.777	0.766	0.959	0.737	0.694
Year F.E.	Yes	Yes	Yes	Yes	Yes	Yes
Country $\times$ Sector F.E.		Yes	Yes		Yes	Yes
Country $\times$ Year F.E.			Yes			Yes

Note: This table presents OLS regressions using sectoral log quantity (columns 1 to 3) and log price index (columns 4 to 6) as the dependent variable. The independent variables also include one lag of the dependent variable. Double clustered country-year standard errors in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

3.4 Theory

Setup.

Our model features a representative consumer that consumes $N+1$ goods in a static setting.

Each $N+1$ sectors produce using constant returns to scale production function. Sectors up to sector N produce using labor and intermediate inputs. Sector $N+1$ produces using labor, intermediate inputs, and capital. Importantly, sector $N+1$ good price is *exogenously* given. All factor payments are rebated back to the household.

Notation and Definitions.

We use **bold** to denote vectors and matrices. For any matrix $\boldsymbol{X}$, we use $\boldsymbol{X}^T$ for its transpose.

We now define some objects that are going to be key for the analysis.

We let $\boldsymbol{\Omega}$ to be the *input-output matrix* of this economy, with typical element

$$\boldsymbol{\Omega} = \{\Omega_{ij}\} = \frac{P_j M_{ij}}{P_i Q_i} \quad \text{for all } i, j = 1, ..., N+1.$$

This typical element states how much producer i spend on good j, $P_j M_{ij}$, as a fraction of i's sales, $P_i Q_i$. Here P_i is the price of good i, Q_i is the quantity sold of good i, and M_{ij} is how much producer i buys of the quantity of good j.

With some abuse of notation, we also define producer's i expenditure on factor $f = \{L, K\}$ i.e. expenditure on labor and capital, respectively as

$$\Omega_{iL} = \frac{W L_i}{P_i Q_i}; \qquad \Omega_{iK} = \frac{R K_i}{P_i Q_i}.$$

We define $\boldsymbol{\Psi}$ as the *Leontieff-Inverse matrix* that satisfy

$$\boldsymbol{\Psi} = (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} = \sum_{s=0}^{\infty} \boldsymbol{\Omega}^s \text{ with typical element } \{\Psi_{ij}\}.$$

Notice that this is defined over the $N+1$ goods and does not incorporate spending on factors. This matrix captures both the direct and indirect linkages across producers. For instance, Ψ_{ij} denotes how important producer j as a *direct and indirect* supplier to producer i.[8]

On the consumption side, we define the vectors of final domestic consumption, $\boldsymbol{b}$, as follows

$$\boldsymbol{b} = \{b_i\} = \frac{P_i C_i}{GDP},$$

where C_i represents home consumption of good i.

Since there are two factors of production, capital, and labor, we define their shares on *Nominal Gross Domestic Product* (GDP) as

$$\Lambda_L = \frac{WL}{GDP}; \quad \Lambda_K = \frac{RK}{GDP}; \quad \Lambda_L + \Lambda_K = 1.$$

where W and R are the wage rate (labor price) and the rental rate (capital price), L is the equilibrium labor quantity, and K is the capital equilibrium quantity. The last result above follows from the fact that everything in this economy is produced out of factors, and therefore total value added (GDP) should equal factor payments.

Finally, we let λ_i to denote the *Domar weight* of producer i on total value added i.e.

$$\lambda_i = \frac{P_i Q_i}{GDP}.$$

[8]There are some regularity conditions that $\boldsymbol{\Omega}$ must satisfy to be able to write in this way. We note, however that they are seldom satisfied since $\sum_{j=1}^{N+1} \Omega_{ij} < 1$, and so is a sub-stochastic matrix. This implies its spectral radii are less than one. Then by the *Neumann Series Lemma*, the result follows. See Sargent and Stachurski (2022), pp 12-16, for a more formal discussion on these issues.

In the presence of intermediate goods in production, this is the relevant size statistic of each producer on total value added.

3.4.1 Representative Consumer.

We assume a representative consumer that owns both factors of production that it supplies inelastically to producers in the economy. We denote these inelastic supplies as $\bar{L}$ and $\bar{K}$. It has a utility function defined over the $N+1$ goods, $U(\boldsymbol{C})$. We introduce an additional income source in the budget constraint of the consumer that we label $\bar{Y}$, a net transfer to the rest of the world as in Baqaee and Farhi (2021). The purpose of this is two-fold. First, it allows the economy to exhibit exports in equilibrium. Second, it will allow us to break the relationship between expenditure and production side. Later, we will make this object a function of the commodity price. From the household perspective, though, this is taken as given.

Taking good and factor prices, $(\boldsymbol{P}, W, R)$, together with $\bar{Y}$ as given, the representative consumer solves the following program

$$\max_{\boldsymbol{C}} \quad U(\boldsymbol{C}) \quad \text{s.t.} \quad \sum_{i=1}^{N+1} P_i C_i \leq W\bar{L} + R\bar{K} - \bar{Y} = E, \tag{3.6}$$

where we use E as a short-cut for *total expenditure*. The solution to this program delivers consumption schedules that are a function of prices and the additional income source i.e. $\boldsymbol{C} = \boldsymbol{C}(\boldsymbol{P}, W, R, \bar{Y})$.

3.4.2 Non-Tradable Sectors

Gross output in sector $i = 1, 2, ..., N$, Q_i, is produced according to the following production function

$$Q_i = Z_i F_i(L_i, \{M_{ij}\}_{j=1}^{N+1}), \tag{3.7}$$

where Z_i is a producer-specific shock, $F_i(.)$ is a constant-returns to scale function. We use a subscript i to index this production function to allow for the possibility of different production functions across producers. L_i is labor demand of producer i and M_{ij} is intermediate demand for good j by producer i.

Cost-minimization implies that the marginal cost of production, MC_i can be written as

$$P_i = MC_i(W, \boldsymbol{P}; Z_i). \tag{3.8}$$

This implies that the marginal cost is a function of the wage rate, the price of all goods $\boldsymbol{P} = (P_1, P_2, ..., P_{N+1})$ and its own productivity, Z_i. Its equality to its good price, P_i, then follows from profit maximization. In a nutshell, this is just another way to write the zero-profit condition of each producer $i = 1, 2, ..., N$.

To get conditional demands for labor and each intermediate input, we can simply differentiate the marginal cost function

$$L_i = Q_i \frac{\partial MC_i(.)}{\partial W},$$
$$M_{ij} = Q_i \frac{\partial MC_i(.)}{\partial P_j}.$$

3.4.3 Commodity Sector

The commodity sector gross output Q_{N+1} is produced according to the following production function

$$Q_{N+1} = Z_{N+1} F_{N+1}(L_{N+1}, K_{N+1}, \{M_{N+1,j}\}_{j=1}^{N+1}).$$

Similar to non-tradable sectors, cost minimization delivers the commodity sector's marginal costs, which in equilibrium must coincide with the commodity sector price

$$P_{N+1} = MC_{N+1}(W, R, \boldsymbol{P}; Z_{N+1}), \tag{3.9}$$

which is exogenously given for the small open economy.[9]

We assume that the net transfer to the rest of the world ($\bar{Y}$), which in equilibrium corresponds to net exports, is a function of the commodity price. In particular,

$$\log \bar{Y} = \phi \log P_{N+1},$$

where ϕ shapes the nature of the commodity price shock in a simple fashion. A positive ϕ implies that an increase in the commodity price generates an increase in net exports, which is akin to a demand-driven shock to commodity prices. On the other hand, a negative ϕ is akin to a supply-driven shock to commodity prices.

[9] By the law of one price $P_{N+1} = \mathcal{E} P^*_{N+1}$, in which $\mathcal{E}$ is the nominal exchange rate. Without loss of generality, we assume $\mathcal{E} = 1$. Alternatively, we could assume the exchange rate to be the numeraire of the economy and express all prices in foreign currency. This model is slightly more complex but has the same qualitative implications for the propagation of commodity prices along the production network.

3.4.4 Equilibrium.

The following conditions characterize the equilibrium in our model

$$
\begin{aligned}
Q_i &= C_i + \sum_{j=1}^{N+1} M_{ji} \qquad \forall i = 1, ..., N, \\
P_{N+1}Q_{N+1} &= P_{N+1}C_{N+1} + \sum_{j=1}^{N+1} P_{N+1}M_{j,N+1} + \bar{Y}, \\
\bar{L} &= \sum_{i=1}^{N+1} L_i, \\
\bar{K} &= K_{N+1},
\end{aligned}
$$

The first row shows the market clearing condition in non-tradable goods markets. The second row shows the aggregate resource constraint in this economy that follows by combining the consumer's budget constraint with the non-tradable market clearing conditions. The last two rows refer to the labor market and capital market clearing respectively.

Multiplying each non-tradable goods market clearing condition by each good price and dividing by GDP, we arrive at a market clearing condition in terms of observables

$$\lambda_i = b_i + \sum_{j=1}^{N+1} \Omega_{ji}\lambda_j.$$

We can also divide by GDP the aggregate resource constraint to get

$$\lambda_{N+1} = b_{N+1} + \sum_{j=1}^{N+1} \Omega_{j,N+1}\lambda_j + \frac{\bar{Y}}{GDP} \quad \text{for } i = 1, ..., N.$$

Stacking all these conditions in a vector and inverting the system delivers the following

$$\boldsymbol{\lambda} = \boldsymbol{\Psi}^T \left(\boldsymbol{b} + \boldsymbol{e}_{N+1} \frac{\bar{Y}}{GDP} \right),$$

where $\boldsymbol{e}_{N+1}$ is a unit vector with its $N+1$ element equal to one and all the rest equal to zero.

Similarly, we can express the factor markets clearing conditions conveniently in terms of aggregate factor shares as

$$\Lambda_L = \sum_{i=1}^{N+1} \Omega_{i,L} \lambda_i,$$

$$\Lambda_K = \Omega_{N+1,K} \lambda_{N+1},$$

$$\Lambda_L + \Lambda_K = 1.$$

3.4.5 Comparative Statics.

In what follows, we consider a perturbation of the commodity price, $\mathrm{d} \log P_{N+1}$, and study how this affects good and factor prices, $(\boldsymbol{P}, W, R)$ together with quantities produced, $\boldsymbol{Q}$. In all our comparative exercises below, we assume that technology and factor supplies are fixed. Therefore, $d \log \boldsymbol{Z} = \mathbf{0}$ and $d \log \bar{K} = d \log \bar{L} = 0$. This means that real GDP in our economy does not change with changes in commodity prices, as both sectoral technology and factor supplies are exogenous and fixed. This is a generalization of the two-sector result in Kehoe and Ruhl (2008). We provide a formal derivation of this statement in Proposition 8 in the appendix. This allows us to solely focus on the cross-sectional responses of output and prices in different sectors to commodity price changes and their propagation throughout the production network.

3.4.5.1 Prices.

Totally differentiating Equation (3.8) and Equation (3.9) and using *Shephard's lemma*, yields that up to a first-order approximation price changes should satisfy

$$d\log P_i = \Omega_{iL} d\log W + \sum_{j=1}^{N+1} \Omega_{ij} d\log P_j \quad \text{for all } i = 1, 2, ..., N,$$

$$d\log P_{N+1} = \Omega_{N+1,L} d\log W + \Omega_{N+1,K} d\log R + \sum_{j=1}^{N+1} \Omega_{N+1,j} d\log P_j,$$

where

$$\Omega_{iL} = \frac{WL_i}{P_i Q_i} = \frac{WL_i}{TC_i}, \qquad \Omega_{iK} = \frac{RK_i}{P_i Q_i} = \frac{RK_i}{TC_i} \qquad \text{for all } i = 1, 2, ..., N+1,$$

is how much producer i spends on either labor and capital as a fraction of its sales, $P_i Q_i$, which due to the constant returns to scale assumption of the production function equals total costs, $P_i Q_i = TC_i$.

The above system of equations is $N+1$ equations in $N+2$ unknowns $(d\log \boldsymbol{P}, d\log W, d\log R)$. Up to choosing a numeraire, we can solve for domestic price changes as a function of commodity price changes. We let the nominal wage W be the numeraire. The following proposition characterizes these responses as we show in Appendix C.3.

Proposition 5 (Price Responses to a Commodity Price Change)**.** Consider a perturbation of the commodity price, $d\log P_{N+1}$. Up to a first-order approximation, changes in good prices satisfy

$$d\log P_i = \frac{\widetilde{\Omega}_{i,K}}{\widetilde{\Omega}_{N+1,K}} d\log P_{N+1} = \frac{\Psi_{i,N+1}}{\Psi_{N+1,N+1}} d\log P_{N+1}, \tag{3.10}$$

where $\widetilde{\Omega}_{i,K} = \sum_{j=1}^{N+1} \Psi_{ij}\Omega_{jK} = \Psi_{i,N+1}\Omega_{N+1,K}$ are the network-adjusted usage of capital of producer i and Ψ_{ij} represents how important is sector j as a supplier, both directly and indirectly, to sector i.

Proof. See Appendix C.3. ■

This equation states that all prices increase proportionally to their exposure to the commodity sector. Intuitively, a rise in the commodity sector price raises the marginal cost of all producers. The relevant exposure to the commodity sector in the presence of intermediate input linkages is $\Psi_{i,N+1}$. It measures how important the commodity sector is as a *supplier* to sector i after considering both direct and indirect linkages. As with productivity shocks in the production network literature, commodity price fluctuations propagate downstream to other prices in the economy.

3.4.5.2 Quantities.

To solve for changes in gross output, $d\log Q_i$, we use the definition of the Domar weight, λ_i, and totally differentiate it to get

$$d\log Q_i = d\log \lambda_i + d\log GDP - d\log P_i. \tag{3.11}$$

We already know $d\log P_i$, so we are left to determine changes in nominal GDP, GDP, and the Domar weights.

Since labor is the numeraire, we can write changes in nominal GDP as

$$\Lambda_L d\log W + \Lambda_K d\log R = d\log GDP \Longrightarrow \Lambda_K d\log R = d\log GDP.$$

Therefore, nominal GDP rises in proportion to changes in the rental rate R. We can then link changes in nominal GDP to changes in the commodity price by using the zero profit condition in the commodity sector that links the rental rate with the commodity price shock

$$d\log P_{N+1} = \widetilde{\Omega}_{N+1,K} d\log R.$$

Hence, nominal GDP changes depend on the change in the commodity price as

$$d\log GDP = \frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}} d\log P_{N+1}. \tag{3.12}$$

We are left to determine changes in the Domar weight. Before doing so, we need to introduce a version of the *Input-Output Substitution Operator* first introduced in Baqaee and Farhi (2019b) applied directly to our context. In particular, in response to a change in the commodity price $\mathrm{d}\log P_{N+1}$, producer j substitutes to/away from sector i as

$$\begin{aligned}\Phi_j(i, N+1) = &\frac{1}{\widetilde{\Omega}_{N+1,K}} \sum_{k=1}^{N+1}\sum_{h=1}^{N+1} (\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh})\Psi_{ki}\Omega_{jk}\widetilde{\Omega}_{h,K}, \\ &+ \frac{1}{\widetilde{\Omega}_{N+1,K}} \sum_{k=1}^{N+1} \Psi_{ki}\Omega_{jk}(\theta^j_{kK} - 1)\Omega_{jK},\end{aligned} \tag{3.13}$$

where $\delta_{kh} = 1$, whenever $k = h$, and zero otherwise. θ^j_{kh} is the Allen-Uzawa elasticity of substitution for producer j between any two pair of inputs (k, h), where k is an intermediate input and h can be other intermediate inputs or factors, defined as

$$\theta^j_{kh} = \frac{\frac{\partial \log M_{jk}}{\partial \log P_h}}{\Omega_{jh}}. \tag{3.14}$$

We discuss the intuition for this operator below. At this point, the substitution operator is helpful because it allows us to write changes in Domar weights in response to a change in the commodity price compactly, as the following proposition shows.

Proposition 6 (Changes in Domar Weights.)**.** Up to a first-order approximation, changes in the Domar weight of sector i, $d\log\lambda_i$, following a commodity price shock $\mathrm{d}\log P_{N+1}$, satisfy

$$\begin{aligned}\frac{\mathrm{d}\log\lambda_i}{\mathrm{d}\log P_{N+1}} = &\frac{1}{\lambda_i}\left(\sum_{j=1}^{N+1}\lambda_j\Phi_j(i, N+1) + \sum_{k=1}^{N+1} b_k\Psi_{ki}\frac{\mathrm{d}\log E}{\mathrm{d}\log P_{N+1}} + \sum_{k=1}^{N+1}\Psi_{ki}\delta_{k,N+1}\frac{\bar{Y}}{GDP}\frac{\mathrm{d}\log\bar{Y}}{\mathrm{d}\log P_{N+1}}\right) \\ &- \frac{d\log GDP}{\mathrm{d}\log P_{N+1}}.\end{aligned} \tag{3.15}$$

Proof. See Appendix C.3. ■

The last proposition follows by just differentiating the market clearing condition of each good. To get this result, we impose Cobb-Douglas preferences for domestic consumers. Four terms govern changes in Domar weights. The first three terms are related to changes in sales of good i, while the last term is a mechanical effect of an increase in aggregate *nominal* value added for given sales of producer i. The first term on the right-hand side represents *substitution* that occurs at the level of the firm/sector that is then propagated *upstream* to other sectors, i.e., from buyers to suppliers. These substitution patterns are captured by the input-substitution operator $\Phi_j(i, N+1)$, which can be thought of as a measure of *expenditure switching*. This input-output substitution operator comprises three steps that occur when a commodity price shock hits the economy that we explain next.

The first step is that following a positive shock to the commodity sector price, the price of good h increases by $\widetilde{\Omega}_{h,K}/\widetilde{\Omega}_{N+1,K}$.[10] This initial price change occurs because of downstream propagation on costs, as we already showed in Proposition 5.

In response to a change in the price of good h, each producer j may substitute away/towards other intermediate goods or to factors of production. If, for example, it substitutes away/towards to some other intermediate good k, it does so by $\Omega_{jk}(\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh})$, which represents how much the expenditure share of producer j on k respond to a change in the price of good h $(\partial\Omega_{jk}/\partial \log P_h)$. This depends on the direct exposure of producer j to both k and h, and also on the elasticity of substitution between k and h, a term captured by the Allen-Uzawa elasticity of substitution, θ^j_{kh}. If goods have a high degree of substitutability, meaning that $\theta^j_{kh} > 1$, an

[10] When good h is capital (K), this measure is simply $1/\widetilde{\Omega}_{N+1,K}$ as the network-adjusted capital share of the commodity sector is enough to pin down the rental rate changes in general equilibrium.

increase in the price of good h increases the expenditure share of producer j on good k: producer j thus substitute away from producer h to k in such a way that its expenditure share on k increase. If goods k and h have a low degree of substitutability ($\theta^j_{kh} < 1$), then producer j cannot reallocate its input demand by that much, which in turn decreases its expenditure share in good k, i.e., it cannot get away from the price increase in good h, and it is forced to decrease is expenditure share on good k as a result. This is the second step.

How does this ultimately affect producer i? The third step answers this question by tying the substitution that each producer j is doing towards/away other producers k in the economy. The key term in this final step is the element of the Leontief-inverse Ψ_{ki} that represents how important producer i is as a supplier to producer k. This last term shows the *upstream propagation* of substitution since it goes from k (buyer) to i (seller). While the initial shock in this economy was propagated downstream in the network structure, these different demand substitution patterns propagated upstream in the production network.

The second and third terms in the equation represent how changes in total expenditure, induced by changes in domestic expenditure $\mathrm{d}\log E$ and $\mathrm{d}\log \bar{Y}$, propagate upstream to sector i. Since the logic is the same for both, we explain it here for domestic demand. An increase in domestic expenditure raises, up to first order, consumption of all goods that the consumer buys directly. These increases in demand translate into increases in intermediate input demand by all sectors. In turn, this means that the relevant statistic for exposure to final changes in demand is not just the direct exposure of sector i to final demand (b_i) but rather a network-adjusted measure, $\tilde{b}_i$, that consider the fact that changes in demand for other sectors also affect sector i via input-output linkages.

The last term, $\mathrm{d}\log GDP/\mathrm{d}\log P_{N+1}$ enters with a negative sign and captures the idea that

keeping everything else constant, if total value added in this economy goes up, the Domar weight of each sector should decline proportionally. This follows by the definition of the Domar weights.

We are now ready to characterize changes in gross output.

Proposition 7 (Changes in Gross Output, $d\log Q_i$)**.** Up to a first-order approximation, changes in gross output following a commodity price shock, $\mathrm{d}\log P_{N+1}$, satisfy

$$\frac{\mathrm{d}\log Q_i}{\mathrm{d}\log P_{N+1}} = \underbrace{\sum_{j=1}^{N+1}\frac{\lambda_j}{\lambda_i}\Phi_j(i,N+1)}_{\text{Input-Output Substitution}} + \underbrace{\frac{\tilde{b}_i}{\lambda_i}\left(\frac{GDP}{E}\frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}} - \frac{\bar{Y}}{E}\phi\right)}_{\text{Domestic Demand}} + \underbrace{\frac{\Psi_{N+1,i}}{\lambda_i}\frac{\bar{Y}}{GDP}\phi}_{\text{Foreign Demand}} - \underbrace{\frac{\widetilde{\Omega}_{i,K}}{\widetilde{\Omega}_{N+1,K}}}_{\text{Cost Push}} \tag{3.16}$$

where

$$\tilde{b}_i = \sum_{k=1}^{N+1}\Psi_{ki}b_k,$$

$$\tilde{b}_i + \Psi_{N+1,i} = \lambda_i, \quad \text{for all } i = 1,2,...,N+1$$

Proof. See Appendix C.3. ■

Intuitively, changes in the gross output of producer i can be inferred from changes in its sales and its price. The first three terms on the right-hand side correspond to the change in sales and represent how consumers, both final and intermediates, of good i react to an exogenous change in the commodity price. We already discussed these terms in a general way when stating Proposition 6. Here, we impose more structure that allows us to separate changes in domestic expenditure from changes in net transfers to the rest of the world.

Let us first focus on the domestic demand component. When the commodity price increases, nominal GDP increases due to higher demand by the commodity sector. This change in nominal GDP then propagates upstream throughout the network structure affecting all sectors. In a way, this effect captures a form of the well-known *wealth effect*: keeping everything else

equal, the economy now has more income to spend. This idea is captured by the first term in the domestic demand component $\frac{\widetilde{b}_i}{\lambda_i}\frac{GDP}{E}\frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}}$, which depends on how much of total sales by producer i ended up meeting final domestic demand ($\widetilde{b}_i/\lambda_i$), how important is nominal GDP in total domestic expenditure at the initial equilibrium (GDP/E), and the relative importance of the commodity sector in the economy ($\Lambda_K/\widetilde{\Omega}_{N+1,K} = \lambda_{N+1}/\Psi_{N+1,N+1}$). At the same time, the increase in the commodity price increases net transfers to the rest of the world, $\bar{Y}$. Everything else equal, this effectively reduces the domestic economy's expenditure on the goods it produces, pushing down domestic demand as a whole. There are many possible interpretations for this reduced-form device of net transfers to the rest of the world. One intuitive explanation is to think of $\bar{Y}$ as interest rate payments on foreign assets at the steady state of a dynamic small open economy model, open to international financial markets. To support exports at the steady state, the economy is effectively borrowing from the rest of the world. For the current account to be balanced, exports and interest rate payments on these assets must go hand in hand. By increasing exports, interest payments go up because the economy is now borrowing more from abroad in the new equilibrium, thus reducing expenditure on domestically produced goods, a form of expenditure switching towards the foreign economy.[11] This is why the second term on the domestic demand component ($\frac{\widetilde{b}_i}{\lambda_i}\frac{\bar{Y}}{E}\phi$) enters with a negative sign. What effect dominates is, in the end, a quantitative matter.

We label the third term *foreign demand* since it follows from changes in net transfers to the

[11] Another explanation is to interpret $\bar{Y}$ as unmodeled imports of final goods. The increase in exports implies that imports must also rise to preserve the trade balance and allow the possibility of exporting more in equilibrium. However, this deviates demand from domestically produced goods to foreign-produced goods, again a form of expenditure switching.

rest of the world. Again, for intuition purposes, think of net transfer to the rest of the world as exports. When the commodity price increases, everything else equal, the economy would like to export more. The only sector that can export is the commodity sector in our model. To produce more, the commodity sector requires domestically produced intermediate inputs, pushing their demand up and thus increasing gross output in these sectors. How much this channel matters for each producer, i, depends on how much it, directly and indirectly, supplies to the commodity sector $\Psi_{N+1,i}$, highlighting the upstream property of this channel.

The final term, the price of good i, fully characterizes the goods supply side of this market. Since the marginal cost of sector i pins down the price of good i in general equilibrium, conditional on factor prices and technology, it encompasses all relevant information on the supply of good i.

3.5 Quantitative exploration

Our theoretical model in the previous section highlighted four channels by which a change in the commodity price affects the gross output of non-commodity sectors. All these channels depend on the direct and indirect network linkages between non-commodity sectors and the commodity sector. Moreover, these channels are a function of the degree of substitutability among production inputs and the price elasticity of demand for commodity goods.

In this section, we provide a quantitative illustration of the model. To do so, we must take a stand on the different production functions of each producer. We assume that each producer has a nested CES function. They substitute among intermediate inputs with an elasticity of substitution equal to ϵ, which can potentially be different across producers. They also substitute between the

intermediate input bundle and value-added with an elasticity of substitution equal to σ. These production elasticities play a crucial role in our results. Rather than use one value for each, we will illustrate the importance of different combinations of σ and ϵ in shaping the propagation of commodity price changes to non-commodity sectors output. We provide a detailed description of the model structure in Appendix C.4.

Our goal is to understand the empirical evidence in Section 3.3, which highlights a strong positive upstream propagation and a muted downstream propagation of commodity price fluctuations on the gross output.

3.5.1 The Case of Non-Unitary Elasticities in Production

To better understand the role of production elasticities in shaping the output responses to commodity prices, we re-write the key terms of the input-output substitution operator in Proposition 7, based on Equation (3.13), as a function of CES production elasticities as follows

$$\left(\frac{\mathrm{d}\log Q_i}{\mathrm{d}\log P_{N+1}}\right)^{\text{Term 1}} = \sum_{j=1}^{N+1}\frac{\lambda_j}{\lambda_i}\left(\sum_{h=1}^{N+1}\sum_{k=1}^{N+1}\Omega_{jk}\left[(\varepsilon_j-1)\left(\Omega_{jh}^{M}-\delta_{kh}\right)-(\sigma_j-1)\Omega_{jh}^{M}\Omega_{jL}\right]\Psi_{ki}\frac{\Psi_{h,N+1}}{\Psi_{N+1,N+1}}\right). \tag{3.17}$$

where

$$\Omega_{jk}=\frac{P_kM_{jk}}{P_jQ_j};\quad \Omega_{jL}=\frac{WL_j}{P_jQ_j};\quad \Omega_{jh}^{M}=\frac{P_hM_{jh}}{P_j^{M}M_j}.$$

When the term in Equation (3.17) is negative, firms in sector j substitute away from sector h's intermediates because firms in sector h use commodity goods, now more expensive, as inputs in production. These effects are mediated by $\Psi_{h,N+1}$, which describes the importance of the commodity sector as a supplier of intermediate inputs to sector h and by $\frac{\lambda_j}{\lambda_i}\Psi_{ki}$ which is the importance of a given sector k as a buyer to sector i (upstream propagation). All else equal, this generates a decline in intermediate input demand for good i, thus decreasing its gross output.

On top of the network structure, production elasticities are important in determining the strength of the input substitutability channel in amplifying or dampening the upstream and downstream propagation of commodity prices on non-commodity sectors' output. To analyze the role of production elasticities, we study the two possible cases for the term in brackets in Equation (3.17). When $k \neq h$, so $\delta_{kh} = 0$, the term in brackets becomes

$$\Omega_{jh}^{M}\big((\varepsilon_j - 1) - (\sigma_j - 1)\Omega_{jL}\big).$$

In this case, when flexibility between intermediates is high and either (i) flexibility between intermediates and labor is low or (ii) the labor share is small, increases in commodity prices tend to increase the demand for non-commodity output i. When $k = h$, so $\delta_{kh} = 1$, the term in brackets becomes

$$-(\varepsilon_j - 1)(1 - \Omega_{jh}^{M}) - (\sigma_j - 1)\Omega_{jh}^{M}\Omega_{jL}.$$

In this case, demand for sector i's output can increase when ϵ_j and σ_j are low or when ϵ_j is low but $\Omega_{jL} \approx 0$. In both cases, the importance of sector h (the commodity client) as a supplier to sector j (Ω_{jh}^{M}) is crucial for the quantitative importance of these propagation channels.

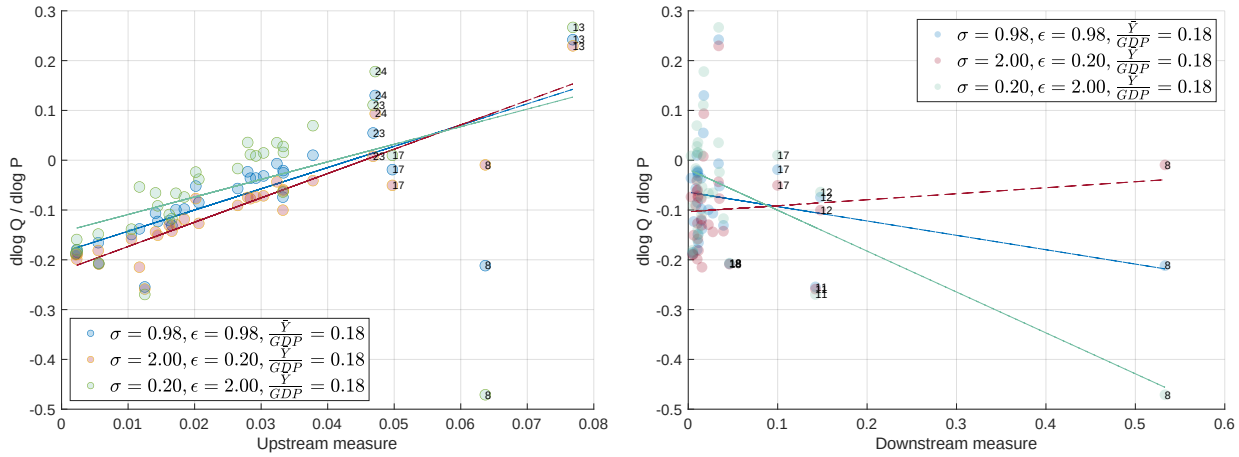
3.5.2 A Calibrated Example

We calibrate our model to match the Australian production structure in 1995. We calibrate the input-output parameters Ω_{ji}^{M} and Ω_{jL} assuming the economy starts at a symmetric equilibrium with $P_j = 1$ for all j. This way, we have that $\Omega_{ji}^{M} = \frac{P_i M_{ji}}{P_j^M M_j}$ and $1 - \Omega_{jL} = \frac{P_j^M M_j}{P_j Q_j}$ equal the observed intermediate input shares. We calibrate the consumption shares $\boldsymbol{b}$ and $\boldsymbol{b}^*$ using observed data on sectoral consumption expenditure in 1995 and exports, respectively. Finally, we assume

that the commodity price is exogenous and $\log \bar{Y} = \phi \log P_{N+1}$, where ϕ is the sensitivity of the transfer to the rest of the world to the change in the commodity price. We assume that $\phi = 2$. In our calibrated example, we treat all sectors as non-tradable sectors, except the commodity sector. We calibrate our model using mining as the only commodity sector to highlight how input-output linkages interact with production elasticities for different commodity industries.

We first investigate how production elasticities shape the relationship between sectoral output response to commodity price shocks and each sector's upstream and downstream exposure to the commodity sector. Figure 3.3 plots in the vertical axis the model implied log change in non-commodities sectoral output from a change in the commodity price. The horizontal axis in panel (a) shows the upstream distance of each sector to the commodity sector, while panel (b) shows the downstream distance of each sector to the commodity sector. Regarding the upstream exposure to commodities, we observe that high substitutability between inputs (high σ or ε) can amplify the upstream propagation of commodity prices (panel (a) of Figure 3.3). For example, a high ε makes the output of two important upstream sectors to mining, sectors 13 (Machinery, Nec) and 24 (Water Transport), increase even more. Interestingly, the opposite occurs with sector 8 (Coke, Refined Petroleum, and Nuclear Fuel), which sees a larger reduction in output when ε is large. The reason is that while sector 8 is a relatively important supplier of mining, it is also, by far, the most important client of mining. Therefore, as we observe in panel (b), for the case of $\varepsilon > 1$, sector 8 is the sector with the largest output decline. The increase in mining price strongly propagates downstream to sector 8, increasing its marginal cost and reducing its production. Here we can see how the interaction between production elasticities and network connections to the commodity sector are crucial in shaping the transmission of commodity price fluctuations.

In Panel (b) of Figure 3.3, we observe that production elasticities are very important in



(a) Upstream

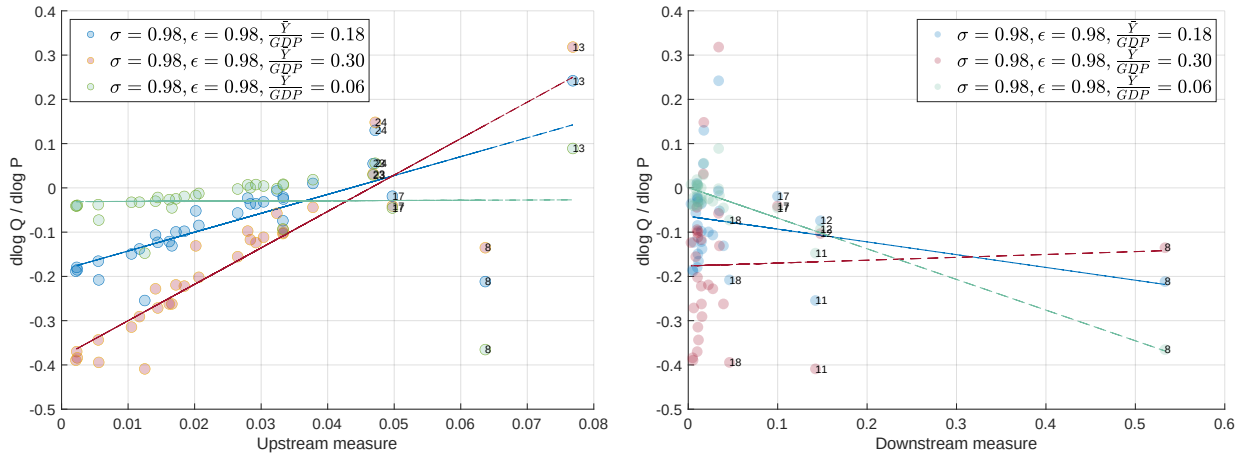
(b) Downstream

Figure 3.3: Sectoral Propagation of a Mining Price Shock: The Role of Elasticities

Note: This figure shows the model-implied relationship between sectors' Downstream (right-panel) and Upstream (left-panel) exposure to the mining sector, using Equation (3.1) and (3.2), respectively. The vertical axis measures the output response to an increase in the commodity price based on Proposition 7. The size of each node indicates the magnitude of the cost-push channel (right panel) and the input-output substitution channel (left panel). Labels in each node are linked to sectors in Table C.1 of our Appendix.

shaping the relationship between downstreamness to commodity and the output response to an increase in commodity prices. As we discussed above, when intermediate inputs are highly substitutable (high ε), the downstream propagation of commodity prices can be further amplified. Nevertheless, when firms are flexible in substituting labor and intermediates (high σ), the downstream propagation of commodity prices is completely muted. Intuitively, a high σ allow firms that are downstream to commodities to use more labor which is now relatively cheaper. Hence, deviating from Cobb-Douglas production technologies is important to understand the empirical observations on the transmission of commodity prices in small open economies.

In Figure 3.4, we study the importance of the foreign demand channel in rationalizing



(a) Upstream (a) Downstream

Figure 3.4: Sectoral Propagation of a Mining Price Shock: the Role of the Export Share

Note: This figure shows the model-implied relationship between sectors' Downstream (right-panel) and Upstream (left-panel) exposure to the mining sector, using Equation (3.1) and (3.2), respectively. The vertical axis measures the output response to an increase in the commodity price based on Proposition 7. The size of each node indicates the magnitude of the cost-push channel (right panel) and the input-output substitution channel (left panel). Labels in each node are linked to sectors in Table C.1 of our Appendix.

our evidence. We solve our model for different values of $\frac{\bar{Y}}{GDP}$, the importance of commodity exports for the small open economy. In this case, we fix production elasticities to be close to Cobb-Douglas. We observe that low values of $\frac{\bar{Y}}{GDP}$ would generate a counterfactual relationship between the upstreamness and downstreamness of the commodity sector and the sectoral output effect of commodity prices. In particular, that calibration would generate muted upstream and strong downstream propagation. A large value of $\frac{\bar{Y}}{GDP}$ can rationalize the evidence of strong upstream propagation and muted downstream propagation.[12]

[12]Tables C.3 and C.4 in Appendix C.2.1 provide regressions results for different exercises using log output change as the dependent variable and upstreamness to commodity (Table C.3) or downstreamness to commodity (Table C.4)

3.6 Conclusion

We study how sectoral commodity price fluctuations propagate through domestic production networks in small open economies. We provide empirical evidence and a theoretical model that highlight the role of production elasticities and commodity export intensity in shaping the propagation of commodity price changes along the production chain. We first show that commodity sectors are central sectors, both as sellers and buyers, in the domestic production network of small open economies. We then show, empirically and theoretically, that the propagation of sectoral commodity price fluctuations to non-commodity sectors' output has an important production network component. We find that the gross output of non-commodity upstream sectors, those sectors supplying intermediate inputs to commodity sectors, largely respond to commodity price shocks. In contrast, we find evidence of muted downstream propagation, to those buying intermediate inputs from commodity sectors.

We develop a small open economy model featuring domestic production networks and characterize the transmission channels of commodity price changes through production chains. We highlight three upstream channels (from intermediate input demand, domestic household income, and foreign demand) and one downstream channel (increased input costs). We show that the elasticities of substitution between inputs and the importance of commodity exports on total GDP are crucial in amplifying or dampening the upstream and downstream channels. In particular, the demand-side effects operate as follows. An increase in commodity price can increase demand for domestic intermediate inputs (upstream propagation) from changes in foreign demand or domestic demand ("wealth effect"). The role of production elasticities works as follows. When

as independent variables.

sectors present high substitutability between intermediate inputs, a higher commodity price increases demand for non-commodity intermediates, which generates an increase in the production of upstream sectors. On the other hand, when sectors present high substitutability between intermediates and labor, increases in commodity prices have smaller effects on the marginal costs of sectors downstream of commodities. Thus, there exists a dampened effect on quantities of industries downstream to commodity sectors. The

All in all, our results highlight the importance of the production network in propagating commodity price fluctuations throughout the economy.

Appendix A: Appendix to Chapter 1

A.1 Proofs

A.1.1 Proof of Proposition 1

By definition, changes in the aggregate price index satisfy

$$\widehat{P} = \bar{\boldsymbol{b}}_D^T \widehat{\boldsymbol{P}}_D + \bar{\boldsymbol{b}}_M^T \widehat{\boldsymbol{P}}_M \tag{A.1}$$

By Shephard's lemma the vector of domestic prices can be written as

$$\widehat{\boldsymbol{P}}_D = -\widehat{\boldsymbol{Z}} + \boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Omega}\widehat{\boldsymbol{P}}_D + \boldsymbol{\Psi}_D \boldsymbol{\Gamma} \widehat{\boldsymbol{P}}_M,$$

where this result follows since in equilibrium $MC_i = P_i$ for all $i = 1, 2, ..., N$.

Inverting this system yields

$$\widehat{\boldsymbol{P}}_D = -\boldsymbol{\Psi}_D \widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}_D \boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Psi}_D \boldsymbol{\Gamma} \widehat{\boldsymbol{P}}_M$$

Using the definitions and equilibrium results

$$\bar{\boldsymbol{b}}_D^T \widehat{\boldsymbol{P}}_D = \bar{\boldsymbol{b}}_D^T \left[-\boldsymbol{\Psi}_D \widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}_D \boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Psi}_D \boldsymbol{\Gamma} \widehat{\boldsymbol{P}}_M \right] \tag{A.2}$$

Thus CPI changes can be written as

$$\widehat{P} = -\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \boldsymbol{A}\widehat{\boldsymbol{W}} + (\bar{\boldsymbol{b}}_M^T + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi}_D \boldsymbol{\Gamma}) \widehat{\boldsymbol{P}}_M \tag{A.3}$$

To get the expression in the text, simply note that goods market clearing condition and the definition of factor shares can be written as

$$\bar{\boldsymbol{\lambda}}^T = (\bar{\boldsymbol{b}}_D^T + \bar{\boldsymbol{x}}^T)\boldsymbol{\Psi}_D \Longrightarrow \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}_D = \bar{\boldsymbol{\lambda}}^T - \bar{\boldsymbol{x}}^T\boldsymbol{\Psi}_D$$

$$\bar{\boldsymbol{\Lambda}}^T = \bar{\boldsymbol{\lambda}}^T\boldsymbol{A} \Longrightarrow \bar{\boldsymbol{b}}_D^T\boldsymbol{\Psi}_D\boldsymbol{A} = \bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T\boldsymbol{\Psi}_D\boldsymbol{A}.$$

Replacing these expressions with changes in the CPI index yields the result.

A.1.2 Proof of Proposition 2

To prove this proposition, notice that Hulten's theorem in an efficient closed economy with inelastic factor supplies implies that changes in real GDP must satisfy

$$\widehat{Y} = \boldsymbol{\lambda}^T\widehat{\boldsymbol{Z}} + \boldsymbol{\Lambda}^T\widehat{\boldsymbol{L}}.$$

In turn, changes in factor supply have to comply $\widehat{\boldsymbol{L}} = -\widehat{\boldsymbol{W}} + \widehat{nGDP}\mathbf{1}_F + \widehat{\boldsymbol{\Lambda}}$, which upon replacing this expression above I get

$$\widehat{Y} = \boldsymbol{\lambda}^T\widehat{\boldsymbol{Z}} + \widehat{nGDP} - \boldsymbol{\Lambda}^T\widehat{\boldsymbol{W}}.$$

The definition of changes in nominal GDP has a quantity and a price component. The quantity component refers to $\widehat{Y}$ (real GDP). The price component is the GDP deflator, which is equivalent to CPI in a closed economy[1]. This allows us to write changes in nominal GDP as

$$\widehat{nGDP} = \widehat{Y} + \widehat{CPI} \Longrightarrow \widehat{CPI} = \widehat{nGDP} - \widehat{Y}$$

[1]This provided that we interpret CPI broadly to include all final uses different from intermediate inputs, such as investment, government expenditure, and so on.

Combining the above with the changes in real GDP as a function of productivity, factor prices, and nominal GDP, I get

$$\widehat{CPI} = \widehat{nGDP} - (\boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \widehat{nGDP} - \boldsymbol{\Lambda}^T \widehat{\boldsymbol{W}}) = -\boldsymbol{\lambda}^T \widehat{\boldsymbol{Z}} + \boldsymbol{\Lambda}^T \widehat{\boldsymbol{W}},$$

which was the desired expression.

A.1.3 Proof of Proposition 3

As stated in the text

$$\widehat{\boldsymbol{W}} = \widehat{\bar{\boldsymbol{\Lambda}}} + \mathbf{1}_F \widehat{\mathcal{M}} - \widehat{\bar{\boldsymbol{L}}}.$$

Multiplying the above expression by the weight on wages in CPI from equation (1.10), I get

$$\left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \widehat{\boldsymbol{W}} = \left(\bar{\boldsymbol{\Lambda}}^T - \bar{\boldsymbol{x}}^T \boldsymbol{\Psi}_D \boldsymbol{A}\right) \left(\widehat{\bar{\boldsymbol{\Lambda}}} + \mathbf{1}_F \widehat{\mathcal{M}} - \widehat{\bar{\boldsymbol{L}}}\right)$$

Now note that, in general, the budget constraint of a consumer in a small open economy can be written as

$$\mathcal{M} + T = GDP \Longrightarrow \widehat{nGDP} = \frac{\mathcal{M}}{nGDP} \widehat{\mathcal{M}} + \frac{\mathrm{d}T}{nGDP}.$$

By definition,

$$\Lambda_f = \bar{\Lambda}_f \frac{\mathcal{M}}{nGDP} \Longrightarrow \widehat{\bar{\Lambda}}_f = \widehat{\Lambda}_f - \widehat{\mathcal{M}} + \widehat{nGDP},$$

$$\bar{\boldsymbol{\Lambda}}^T \mathbf{1}_F = \sum_{f \in F} \bar{\Lambda}_f = \frac{nGDP}{\mathcal{M}} = \frac{\mathcal{M} + T}{\mathcal{M}} \Longrightarrow \bar{\boldsymbol{\Lambda}}^T \bar{\boldsymbol{\Lambda}} = \sum_{f \in F} \mathrm{d}\bar{\Lambda}_f = \mathrm{d}\left(1 + \frac{T}{\mathcal{M}}\right) = \frac{\mathrm{d}T}{\mathcal{M}} - \frac{T}{\mathcal{M}} \widehat{\mathcal{M}}$$

Then,

$$\begin{aligned}
\left(\bar{\mathbf{\Lambda}}^T - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A}\right) \widehat{\boldsymbol{W}} &= \bar{\mathbf{\Lambda}}^T \hat{\bar{\mathbf{\Lambda}}} - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A} \hat{\bar{\mathbf{\Lambda}}} + \left(\bar{\mathbf{\Lambda}}^T - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A}\right) \mathbf{1}_F \widehat{\mathcal{M}} - \left(\bar{\mathbf{\Lambda}}^T - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A}\right) \hat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} - \frac{T}{\mathcal{M}} \widehat{\mathcal{M}} - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A} \hat{\bar{\mathbf{\Lambda}}} + \left(\bar{\mathbf{\Lambda}}^T - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A}\right) \mathbf{1}_F \widehat{\mathcal{M}} - \left(\bar{\mathbf{\Lambda}}^T - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A}\right) \hat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A} \hat{\bar{\mathbf{\Lambda}}} + \left(\left(\bar{\mathbf{\Lambda}}^T - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A}\right) \mathbf{1}_F - \frac{T}{\mathcal{M}}\right) \widehat{\mathcal{M}} - \left(\bar{\mathbf{\Lambda}}^T - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A}\right) \hat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A} \hat{\bar{\mathbf{\Lambda}}} + \left(\frac{\mathcal{M}+T}{\mathcal{M}} - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A} \mathbf{1}_F - \frac{T}{\mathcal{M}}\right) \widehat{\mathcal{M}} - \left(\bar{\mathbf{\Lambda}}^T - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A}\right) \hat{\bar{\boldsymbol{L}}} \\
&= \frac{\mathrm{d}T}{\mathcal{M}} + \left(1 - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A} \mathbf{1}_F\right) \widehat{\mathcal{M}} - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A} \hat{\bar{\mathbf{\Lambda}}} - \left(\bar{\mathbf{\Lambda}}^T - \bar{\boldsymbol{x}}^T \mathbf{\Psi}_D \boldsymbol{A}\right) \hat{\bar{\boldsymbol{L}}}
\end{aligned}$$

Replacing this expression into equation (1.10), I get

$$\begin{aligned}
\widehat{P} = &- \left(\bar{\boldsymbol{\lambda}}^T - \tilde{\boldsymbol{\lambda}}^T\right) \widehat{\boldsymbol{Z}} - \underbrace{\tilde{\mathbf{\Lambda}}^T \hat{\bar{\mathbf{\Lambda}}} - \left(\bar{\mathbf{\Lambda}}^T - \tilde{\mathbf{\Lambda}}^T\right) \hat{\bar{\boldsymbol{L}}} + \frac{\mathrm{d}T}{\mathcal{M}} + \left(1 - \tilde{\mathbf{\Lambda}}^T \mathbf{1}_F\right) \widehat{\mathcal{M}}}_{\text{Factor price changes}} \\
&+ \left(\bar{\boldsymbol{b}}_M^T + \bar{\boldsymbol{b}}_D^T \mathbf{\Psi}_D \mathbf{\Gamma}\right) \widehat{\boldsymbol{P}}_M, \qquad \text{(A.4)}
\end{aligned}$$

which is the expression in the main text.

A.2 Two period model

In this section, I justify using the one-period model in the main text. The intuition from the main text is unchanged in this more complicated model.

Suppose there are two periods, 0 and 1. There is no uncertainty. The consumer has preferences over the consumption bundle in both periods according to some utility function $U(C)$. The consumer can also access an internationally traded bond that pays a nominal interest rate i_t^*. This nominal interest rate is *exogenous* from the perspective of the small open economy.

The consumer's budget constraint at times 0 and 1 read

$$P_0C_0 + \mathcal{E}_0B_0 = (1 + i^*_{-1})\mathcal{E}_0B_{-1} + nGDP_0,$$

$$P_1C_1 + \mathcal{E}_1B_1 = (1 + i^*_0)\mathcal{E}_1B_0 + nGDP_1,$$

where P_t is the price index at time t, C_t is consumption at time t, B_t denotes asset holdings in foreign currency at time t, $\mathcal{E}_t$ is the nominal exchange rate at time t defined as local currency per unit of foreign currency, and $nGDP_t$ denotes nominal GDP at time t.

Combining the two budget constraints gives the intertemporal budget constraint

$$P_0C_0 + \frac{P_1C_1}{\frac{\mathcal{E}_1}{\mathcal{E}_0}(1 + i^*_0)} = (1 + i^*_{-1})\mathcal{E}_0B_{-1} + nGDP_0 + \frac{nGDP_1}{\frac{\mathcal{E}_1}{\mathcal{E}_0}(1 + i^*_0)}$$

Under perfect mobility of capital flows, we have the *no-arbitrage condition*[2]

$$(1 + i_0) = (1 + i^*_0)\frac{\mathcal{E}_1}{\mathcal{E}_0}. \tag{A.5}$$

Where given perfect foresight, there is no expectation regarding the future level of the exchange rate. I can also get this condition by adding a domestic bond in zero net supply. This does not change any of the conclusions below.

The no-arbitrage condition is important for the small open economy as it clearly illustrates the fact that the Central bank has two instruments to set the nominal interest rate i_0: it can either choose i_0 directly and let the exchange rate $\mathcal{E}_0$ adjust. Or it can pick $\mathcal{E}_0$ and let the nominal interest rate accommodate to comply with that rule.

We can thus rewrite the maximization problem as solving the following program

$$\max_{C_0,C_1} \quad U(C_0) + \beta U(C_1) \quad \text{s.t} \quad P_0C_0 + \frac{P_1C_1}{(1 + i_0)} = (1 + i^*_{-1})\mathcal{E}_0B_{-1} + nGDP_0 + \frac{nGDP_1}{(1 + i_0)} \tag{A.6}$$

[2]Under uncertainty, this is simply the uncovered interest parity condition (UIP).

Letting λ be the multiplier on the intertemporal budget constraint, I have

$$U'(C_0) = \lambda P_0$$

$$\beta U'(C_1) = \lambda \frac{P_1}{(1+i_0)}$$

Combining both equations

$$\frac{U'(C_0)}{P_0} = \beta \frac{U'(C_1)}{P_1}(1+i_0)$$

Assume $U(C) = \log C$, as in Golosov and Lucas (2007) and Baqaee and Farhi (2022b), so that the intertemporal elasticity of substitution is unitary. Then

$$\beta P_0 C_0 (1+i_0) = P_1 C_1$$

As argued in Baqaee and Farhi (2022b), who in turn relied on an argument made in Krugman (1998) and Eggertsson and Krugman (2012), we can understand this model as assuming that anything happening at $t = 1$ is labeled as "future". The assumption here is isomorphic to an infinite horizon model where an unexpected shock happens at $t = 0$, and the economy returns to the long-run equilibrium from $t = 1$ onwards. For all practical purposes, this means I assume that any variables at $t = 1$ are exogenously given. Then, from the Euler equation, I have

$$P_0 C_0 = \frac{P_1 C_1}{\beta(1+i_0)} = \frac{P_1 C_1}{\beta(1+i_0^*)} \frac{\mathcal{E}_0}{\mathcal{E}_1}$$

Therefore, since $(\beta, i_0^*, P_1 C_1, \mathcal{E}_1)$ are exogenous, the nominal exchange rate $\mathcal{E}_0$ is determined via the no-arbitrage condition, equation (A.5). This equation provides a value for current expenditure in local currency, $P_0 C_0$.

For simplicity, suppose $B_{-1} = 0$. Replacing the Euler equation in the intertemporal budget

constraint.

$$
\begin{aligned}
P_0C_0 + \frac{P_1C_1}{(1+i_0)} &= nGDP_0 + \frac{nGDP_1}{(1+i_0)} \\
P_0C_0 + \frac{\beta P_0C_0(1+i_0)}{(1+i_0)} &= nGDP_0 + \frac{nGDP_1}{(1+i_0)} \\
P_0C_0 &= \frac{1}{(1+\beta)}\left(nGDP_0 + \frac{nGDP_1}{(1+i_0)}\right) \\
P_0C_0 &= \frac{1}{(1+\beta)}\left(nGDP_0 + \frac{nGDP_1}{(1+i_0^*)}\frac{\mathcal{E}_0}{\mathcal{E}_1}\right).
\end{aligned}
$$

Note that given $(\mathcal{E}_0, P_0C_0, nGDP_1, i_0)$ from the Euler equation and no-arbitrage condition, the latter equation pins down $nGDP_0$.

A.2.1 Solving for consumption at time 0.

Before solving for consumption, let me introduce the real exchange rate, $\mathcal{Q}_0$ as

$$
\mathcal{Q}_0 = \frac{\mathcal{E}_0 P_0^F}{P_0},
$$

where P_0^F is the rest of the world price index, which is exogenous from the perspective of the small open economy. Following, Schmitt-Grohé et al. (2022), I can write this foreign price index as $P_0^F = \mathcal{P}^F(\{P^*_{m,0}\}_{m\in M}, \{P^*_{j,0}\}_{j\in N^*})$, where M is the same set of goods that are imported by the small open economy and N^* is the set of all other goods consumed abroad by the foreign economy. Since I assume that all prices P_k^* for $k \in M \cup N^*$ are exogenous from the perspective of the small open economy, this allows me to write

$$
\frac{P_0^F}{P^*_{m_0,0}} = \mathcal{P}^F(1, \{P^*_{m,0}/P^*_{0,0}\}_{m\in M\ 0}, \{P^*_{j,0}/P^*_{0,0}\}_{j\in N^*})
$$

Using this in the definition of the real exchange rate and the law of one price for imported goods $P_{m,0} = \mathcal{E}_0 P^*_{m,0}$ for all $m \in M$, then

$$\mathcal{Q}_0 = \frac{P_0^F / P^*_{m_0,0}}{P_0 / P_{m_0,0}}$$

Since the numerator is exogenous, the real exchange rate can be written as the relative price of CPI to one of the imported goods $P_{m_0,0}$. Let set $P_0^F / P^*_{m_0,0} = 1$, then

$$\mathcal{Q}_0 = \frac{P_{m_0,0}}{P_0}$$

Note that consumption at time 0 is only a function of this real exchange rate since from the Euler equation

$$C_0 = \frac{P_1 C_1}{\beta \mathcal{E}_1 (1 + i_0^*)} \frac{\mathcal{E}_0}{P_0} = \frac{P_1 C_1}{\beta \mathcal{E}_1 (1 + i_0^*)} \mathcal{Q}_0 = \frac{E_1}{\beta \mathcal{E}_1 (1 + i_0^*)} \mathcal{Q}_0$$

Changes in the real exchange rate represent changes in all prices relative to the imported good m_0.

$$\widehat{P}_0 - \widehat{P}_{m_0,0} = \sum_{i \in N} \frac{P_i C_i}{PC} (\widehat{P}_i - \widehat{P}_{m_0,0}) + \sum_{m \in M} \frac{P_m C_m}{PC} (\widehat{P}_m - \widehat{P}_{m_0,0}) = \bar{\boldsymbol{b}}_D^T (\widehat{\boldsymbol{P}}_D - \mathbf{1}_N \widehat{P}_{m_0,0}) + \bar{\boldsymbol{b}}_M^T (\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0})$$

$$\widehat{P}_0 - \widehat{P}_{m_0,0} = \bar{\boldsymbol{b}}_D^T (-\boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \boldsymbol{\Psi} \boldsymbol{A} (\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + \boldsymbol{\Psi} \boldsymbol{\Gamma} (\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0})) + \bar{\boldsymbol{b}}_M^T (\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0})$$

$$\widehat{P}_0 - \widehat{P}_{m_0,0} = -\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} (\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T) (\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0})$$

$$\widehat{Q}_0 = -(\widehat{P}_0 - \widehat{P}_{m_0,0}) = \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} (\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) - (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T) (\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0})$$

It follows that consumption changes satisfy

$$\begin{aligned}
\widehat{C}_0 &= \widehat{E}_1 - \widehat{\beta} - \widehat{\mathcal{E}}_1 - \widehat{(1 + i_0^*)} + \widehat{Q}_0 \\
&= \widehat{E}_1 - \widehat{\beta} - \widehat{\mathcal{E}}_1 - \widehat{(1 + i_0^*)} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} (\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) - (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T) (\widehat{\boldsymbol{P}}^*_M - \mathbf{1}_M \widehat{P}^*_{m_0,0}),
\end{aligned}$$

this illustrates how consumption at time 0 is not pinned down by real GDP, Y_0, as is the case in the closed economy model. Rather, it is pinned down by the real exchange rate, $\mathcal{Q}_0$, which in turn depends on factor prices in units of good m_0.

Using the fact that $P_0C_0 = E_0$ is given once I set either i_0 or $\mathcal{E}_0$, then changes in the price index satisfy

$$\begin{aligned}
\widehat{P}_0 &= \widehat{E}_0 - \widehat{C}_0 \\
&= \widehat{E}_0 - \widehat{E}_1 + \widehat{\beta} + \widehat{\mathcal{E}}_1 + \widehat{(1+i_0^*)} - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*) \\
&= \widehat{\mathcal{E}}_1 + \widehat{(1+i_0^*)} - \widehat{(1+i_0)} - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A}(\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0}) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T)(\widehat{\boldsymbol{P}}_M^* - \mathbf{1}_M \widehat{P}_{m_0,0}^*)
\end{aligned}$$

From real wages to aggregate demand and inelastic labor supply changes. To solve the model in terms of factor quantities, let me define real wages, in terms of imported good m_0, as a function of these, factor shares, and the import price (denominated in foreign currency)

$$\begin{aligned}
\widehat{W}_{f,0} - \widehat{P}_{m_0,0} &= \widehat{\bar{\Lambda}}_f - \widehat{\bar{L}}_{f,0} + \widehat{E}_0 - \widehat{P}_{m_0,0} \\
&= \widehat{\bar{\Lambda}}_f - \widehat{\bar{L}}_{f,0} + (\widehat{E}_0 - \widehat{\mathcal{E}}_0) - \widehat{P}_{m_0,0}^* \\
\widehat{\boldsymbol{W}} - \mathbf{1}_F \widehat{P}_{m_0,0} &= \widehat{\bar{\boldsymbol{\Lambda}}} - \widehat{\bar{\boldsymbol{L}}} + \mathbf{1}_F((\widehat{E}_0 - \widehat{\mathcal{E}}_0) - \widehat{P}_{m_0,0}^*),
\end{aligned}$$

where the second line follows from the law of one price.

Note that expenditure denominated in foreign currency is exogenous from the Euler equation

$$\frac{E_0}{\mathcal{E}_0} = \frac{E_1}{\mathcal{E}_1} \frac{1}{\beta(1+i_0^*)}.$$

Thus $(\widehat{E}_0 - \widehat{\mathcal{E}}_0)$ represents an *aggregate demand shifter*. It increases if the consumer becomes more impatient (β declines), expenditure in the future increases ($E_1 = P_1C_1$), the interest

rate in foreign currency goes up, $(1+i_0^*)$, or the exchange rate in the future goes up, $\mathcal{E}_1$. These two last terms induce the consumer to move away from consumption towards saving in the traded bond, thus decreasing aggregate consumption at $t=0$.

Set $\widehat{P}^*_{m0,0} = 0$ to simplify the exposition. Combining expenditure at time 0 in foreign currency with the expression for CPI changes, I get

$$\widehat{P}_0 = \widehat{\mathcal{E}}_0 - \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}} + \bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \left(\widehat{\bar{\boldsymbol{\Lambda}}} - \widehat{\bar{\boldsymbol{L}}} + \mathbf{1}_F (\widehat{E}_0 - \widehat{\mathcal{E}}_0) \right) + (\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T) \widehat{\boldsymbol{P}}^*_M,$$

which can be written as

$$\begin{aligned} \widehat{P}_0 = & \underbrace{\widehat{\mathcal{E}}_0}_{\text{Nominal Anchor}} + \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \mathbf{1}_F (\widehat{E}_0 - \widehat{\mathcal{E}}_0)}_{\text{Aggregate demand shifter}} - \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \widehat{\boldsymbol{Z}}}_{\text{Technology effects}} + \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \widehat{\bar{\boldsymbol{\Lambda}}}}_{\text{Factor share reallocation}} \\ & - \underbrace{\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{A} \widehat{\bar{\boldsymbol{L}}}}_{\text{Factor supplies}} + \underbrace{(\bar{\boldsymbol{b}}_D^T \boldsymbol{\Psi} \boldsymbol{\Gamma} + \bar{\boldsymbol{b}}_M^T) \widehat{\boldsymbol{P}}^*_M}_{\text{Import price channel}}, \end{aligned} \tag{A.7}$$

where I used the fact the no-arbitrage condition, implies that changes in the nominal exchange rate at time 0 can be written as

$$\widehat{(1+i_0)} = \widehat{(1+i_0^*)} + \widehat{\mathcal{E}}_1 - \widehat{\mathcal{E}}_0 \Longrightarrow \widehat{\mathcal{E}}_0 = \widehat{(1+i_0^*)} + \widehat{\mathcal{E}}_1 - \widehat{(1+i_0)}.$$

This provides a nominal anchor at time 0.

A.2.2 Mapping the net transfer, T.

We can use the two-period model above to justify the exogenous net transfer in the static setup T. To see this, write

$$\begin{aligned} T &= nGDP_0 - P_0 C_0 = (1+\beta) P_0 C_0 - \frac{nGDP_1}{(1+i_0^*)} \frac{\mathcal{E}_0}{\mathcal{E}_1} - P_0 C_0 \\ T &= \beta P_0 C_0 - \frac{nGDP_1}{(1+i_0^*)} \frac{\mathcal{E}_0}{\mathcal{E}_1} = \beta P_0 C_0 - \beta \frac{nGDP_1}{P_1 C_1} P_0 C_0 = \beta P_0 C_0 \left(1 - \frac{nGDP_1}{P_1 C_1} \right), \end{aligned}$$

this net transfer is positive or negative depending on whether nominal GDP in the future is higher or lower than future expenditure. This ultimately hinges on the difference between future consumption and income since if $nGDP_1/P_1C_1 > 1$; this ratio is negative, meaning $T < 0$. A negative net transfer means the economy receives resources at time 0 that do not come from their own production at time 0. In an intertemporal model, this comes from future resources. In a static model, this should come from the rest of the world. The converse also holds. Of course, if the steady state features no assets holding, this equation collapses to $T = 0$. To see why, note the budget constraint satisfies:

$$PC\left(1+\frac{1}{(1+i)}\right) = \mathcal{E}i^*B + nGDP\left(1+\frac{1}{(1+i)}\right),$$

If $B = 0$, then

$$PC = nGDP,$$

and therefore $T = 0$.

A.3 A Small Open Economy Dynamic Model with Production Networks

In this appendix, I briefly explore how the intuition of the model in the main text extends to a small open economy *dynamic* environment.

Environment. The model is a variant of the canonical importable, exportable, and non-tradable model (MXN) as in Chapter 8 of Uribe and Schmitt-Grohé (2017), where I remove capital from the model.

Time is discrete, indexed by t, and runs forever. Households consume exportables, importables, and non-tradables. Non-tradable and exportable goods are produced using labor and

intermediate inputs from other sectors. This intermediate input-output structure is one of the novelties of the model.

Financial markets are incomplete. The domestic household has access to two bonds. The first is a domestic bond denominated in local currency and pays a nominal interest rate i_t. The second is a foreign bond denominated in foreign currency and pays a nominal interest rate i_t^*. The small open economy takes this latter foreign interest rate as given.

Household. The household owns labor and consumes the three goods. Labor is supplied inelastically. The consumption aggregator is of the CES form

$$C = \left(b_N^{\frac{1}{\chi}} C_N^{\frac{\chi-1}{\chi}} + b_M^{\frac{1}{\chi}} C_M^{\frac{\chi-1}{\chi}} + (1 - b_N - b_M)^{\frac{1}{\chi}} C_X^{\frac{\chi-1}{\chi}} \right)^{\frac{\chi}{\chi-1}}, \tag{A.8}$$

where C_N is consumption of non-tradables, C_M is consumption of importables, and C_X is consumption of the exportable good. (b_N, b_M) are the expenditure shares on non-tradable and importables at the symmetric price steady-state. In turn, $1 - b_N - b_M$ is the expenditure share on exportable goods. Finally, χ is the elasticity of substitution across the different goods.

I solve the household's problem in two steps. In the first step, I solve for the dynamic path of $\{C_t, B_t, B_t^*\}_{t=0}^{\infty}$. Conditional on knowing the path of C_t, we can solve for (C_{Nt}, C_{Mt}, C_{Xt}) at each instant t. This way of solving the problem allows me to simplify the exposition and is the route followed in, for example, Chapter 4 of Obstfeld and Rogoff (1996).[3]

The dynamic problem of the household is as follows. Taking as given paths of prices $\{P_t, W_t, \mathcal{E}_t\}$, interest rates $\{i_t, i_t^*\}$, and labor endowment $\{\bar{L}_t\}$, the consumer solves the following

[3] Specifically, see Section 4.4.1. of Obstfeld and Rogoff (1996).

program

$$\max_{\{C_t, B_t, B_t^*\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma} - 1}{1-\sigma}$$

$$\text{subject to } P_t C_t + \mathcal{E}_t B_t^* + B_t \leq W_t \bar{L}_t + (1 + i_{t-1}^*)\mathcal{E}_t B_{t-1}^* + (1 + i_{t-1}) B_{t-1},$$

where P_t is the consumer's price index, B_t^* is assets holdings of a foreign bond, B_t is assets holding of a domestic bond, $\mathcal{E}_t$ is the nominal exchange rate defined as units of home currency per unit of foreign currency, W_t is the wage rate, $(1 + i_t)$ is the gross interest rate in the domestic bond and $(1 + i_t^*)$ is the gross interest rate in the foreign bond. $\beta \in (0, 1)$ is a discount factor.

Letting $\beta^t \mu_t$ be the Lagrange multiplier on the flow budget constraint, we get the following first-order conditions

$$C_t : C_t^{-\sigma} = \lambda_t P_t, \tag{A.9}$$

$$B_t : \lambda_t = \beta(1 + i_t)\mathbb{E}_t \lambda_{t+1}, \tag{A.10}$$

$$B_t^* : \lambda_t = \beta(1 + i_t^*)\mathbb{E}_t \lambda_{t+1} \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}, \tag{A.11}$$

plus the budget constraint.

Conditional on C, the intratemporal problem solves the following program (ignoring time subscripts)

$$\min_{C_N, C_M, C_X} P_N C_N + P_M C_M + P_X C_X \text{ subject to } C \geq \bar{C}. \tag{A.12}$$

That is, taking as given prices (P_N, P_M, P_X), a consumption aggregator function C and a level of aggregate consumption $\bar{C}$; the household minimizes its expenditure to achieve $\bar{C}$.

The conditional demands that solve this problem are

$$C_N = b_N \left(\frac{P_N}{P}\right)^{-\chi} C; \quad C_M = b_M \left(\frac{P_M}{P}\right)^{-\chi} C; \quad C_X = (1 - b_N - b_M) \left(\frac{P_X}{P}\right)^{-\chi} C, \tag{A.13}$$

where the price index P satisfies

$$P = (b_N P_N^{1-\chi} + b_M P_M^{1-\chi} + (1 - b_N - b_M) P_X^{1-\chi})^{\frac{1}{1-\chi}} \tag{A.14}$$

CPI Inflation is thus defined as

$$\pi_t = \log P_t - \log P_{t-1} \tag{A.15}$$

Production Side. There are two producing sectors: non-tradable (N) and exportable (X). I omit time indices whenever it causes no confusion.

Gross output of sector $i \in \{N, X\}$ is of the CES form

$$Q_i = Z_i \left(a_i^{\frac{1}{\sigma_i}} L_i^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - a_i)^{\frac{1}{\sigma_i}} M_i^{\frac{\sigma_i - 1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i - 1}}, \tag{A.16}$$

where L_i is labor demand and M_i is an intermediate input bundle. σ_i is the elasticity of substitution between labor and intermediate inputs. Z_i is a sector-specific productivity level. These productivity levels are exogenous. Finally, a_i represents the labor share in total costs (sales) at the symmetric price equilibrium.

The intermediate input bundle (M_i) aggregates non-tradable (M_{iN}) and tradable (M_{iT}) intermediate inputs according to another CES layer

$$M_i = \left(\omega_i^{\frac{1}{\varepsilon_i}} M_{iN}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} + (1 - \omega_i)^{\frac{1}{\varepsilon_i}} M_{iT}^{\frac{\varepsilon_i - 1}{\varepsilon_i}} \right)^{\frac{\varepsilon_i}{\varepsilon_i - 1}}, \tag{A.17}$$

where ε_i is the elasticity of substitution between non-tradable and tradable intermediate inputs. ω_i represents the expenditure share on non-tradable intermediates out of total intermediate input spending. Therefore, $1 - \omega_i$ is the expenditure share on tradable goods.

The tradable intermediate input bundle combines the exportable good and the imported input

$$M_{iT} = \left(\omega_{iX}^{\frac{1}{\varepsilon_i^T}} M_{iX}^{\frac{\varepsilon_i^T - 1}{\varepsilon_i^T}} + (1 - \omega_{iX})^{\frac{1}{\varepsilon_i^T}} M_{iM}^{\frac{\varepsilon_i^T - 1}{\varepsilon_i^T}} \right)^{\frac{\varepsilon_i^T}{\varepsilon_i^T - 1}}, \tag{A.18}$$

where ε_i^T is the elasticity of substitution between importable and exportable goods. ω_{iX} is the expenditure share on exportable goods as a share of intermediate spending on tradable goods (both exportable and importable).

Cost minimization at each CES layer delivers the following conditional demands

$$L_i = a_i \left(\frac{W}{MC_i} \right)^{-\sigma} Z_i^{\sigma_i - 1} Q_i; \quad M_i = (1 - a_i \left(\frac{P_i^I}{MC_i} \right)^{-\sigma} Z_i^{\sigma_i - 1} Q_i \tag{A.19}$$

$$M_{iN} = \omega_i \left(\frac{P_N}{P_i^I} \right)^{-\varepsilon_i} M_i; \quad M_{iT} = (1 - \omega_i) \left(\frac{P_i^T}{P_i^I} \right)^{-\varepsilon_i} M_i \tag{A.20}$$

$$M_{iX} = \omega_{iX} \left(\frac{P_X}{P_i^T} \right)^{-\varepsilon_i^T} M_{iT}; \quad M_{iM} = (1 - \omega_{iX}) \left(\frac{P_M}{P_i^T} \right)^{-\varepsilon_i^T} M_{iT} \tag{A.21}$$

with marginal costs and price indices

$$MC_i = Z_i^{-1} \left(a_i W^{1-\sigma_i} + (1 - a_i)(P_i^I)^{1-\sigma_i} \right)^{\frac{1}{1-\sigma_i}}, \tag{A.22}$$

$$P_i^I = \left(\omega_i P_N^{1-\varepsilon_i} + (1 - \omega_i)(P_i^T)^{1-\varepsilon_i} \right)^{\frac{1}{1-\varepsilon_i}}, \tag{A.23}$$

$$P_i^T = \left(\omega_{iX} P_X^{1-\varepsilon_i^T} + (1 - \omega_{iX})(P_M)^{1-\varepsilon_i^T} \right)^{\frac{1}{1-\varepsilon_i^T}}, \tag{A.24}$$

where MC_i stands for marginal cost, P_i^I is the price index of the intermediate input bundle, and P_i^T is the price index of the tradable intermediate input bundle.

Law of one price. I assume the law of one price holds for the exportable and importable good. This means

$$P_{Xt} = P_{Xt}^* \mathcal{E}_t \quad P_{Mt} = P_{Mt}^* \mathcal{E}_t \tag{A.25}$$

Nominal Anchor. Households require local currency to buy the consumption bundles. I introduce this notion as a cash-in-advance constraint. I impose this as an additional aggregate equation

$$\mathcal{M}_t = P_t C_t, \tag{A.26}$$

where $\mathcal{M}_t$ is an *exogenous* money supply. Since prices are fully flexible and there are no market distortions, imposing this constraint does not affect relative prices and optimal allocations. It allows me to pin down the price level and, as a by-product inflation. This is akin to a nominal GDP targeting rule, which in the open economy is an *expenditure* targeting rule instead.[4]

Exogenous processes. In the model there are 6 exogenous processes $(Z_{Nt}, Z_{Xt}, P^*_{Xt}, P^*_{Mt}, \mathcal{M}_t, \bar{L}_t)$. For the purposes of the exercise, I explore changes in (Z_{Nt}, P^*_{Mt}). I assume all processes follow AR(1) in logs. That is

$$\log Z_{Nt} = \rho_{Z_N} \log Z_{Nt-1} + \nu^N_t \tag{A.27}$$

$$\log P^*_{Mt} = \rho_{P_M} \log P^*_{Mt-1} + \nu^{P_M}_t, \tag{A.28}$$

where $(\nu^N_t, \nu^{P_M}_t)$ are disturbances.

Equilibrium. Since the domestic bond is traded only within the country, we have that $B_t = 0$. The non-tradable market clearing condition and the labor market clearing condition is

$$Q_{Nt} = C_{Nt} + M_{NNt} + M_{XNt} \tag{A.29}$$

$$\bar{L}_t = L_{Nt} + L_{Xt} \tag{A.30}$$

[4] I can also specify a model with money in the utility function. As long as money and aggregate consumption preferences are separable, none of the results I present here change when using such a specification. This illustration closely follows the main text.

Combining these three conditions into the consumer's budget constraint, we can write the law of motion for foreign assets as

$$B_t^* = (1 + i_{t-1}^*)B_{t-1}^* - \frac{1}{\mathcal{E}_t}\left(P_{Xt}(C_{Xt} + M_{XXt} + M_{NXt} - Q_{Xt}) + P_{Mt}(C_{Mt} + M_{NMt} + M_{XMt})\right) \quad \text{(A.31)}$$

I induce stationarity in this foreign asset position using a debt-elastic interest rate device, as in Schmitt-Grohé and Uribe (2003). This means

$$i_t^* = \bar{i}^* + \psi(e^{\bar{B}^* - B_t^*} - 1), \quad \text{(A.32)}$$

where $\bar{i}^*$ and $\bar{B}^*$ are steady-state values of the interest rate and foreign assets.

A.3.1 Calibration and Scenarios

A period is a year. To assess the role of the production network structure, I consider two different scenarios that vary the exposure and dependence of the exportable and non-tradable sectors to each other via input-output linkages.

In particular, I consider the following specifications:

1. Island: $(\omega_N, \omega_X, \omega_{NX}, \omega_{XX}) = (1, 0, 0, 0.5)$.

2. Intersectoral linkages: $(\omega_N, \omega_X, \omega_{NX}, \omega_{XX}) = (0, 1, 1, 0.5)$.

The first scenario ignores intersectoral linkages and treats both sectors as islands isolated from each other in the input-output structure. The non-tradable sector only buys intermediate input from the same sector, while the exportable sector buys from itself and the imported good. The idea of this scenario is to shut down intersectoral linkages. The second scenario assumes the non-tradable sector only uses tradable intermediate inputs, while the exportable sector only uses non-tradable intermediate inputs. Importantly, these two scenarios play around with the

intermediate expenditure share distribution while keeping the total intermediate expenditure share constant at $1 - a_i = 0.33$ in both sectors.

Table A.1 shows the calibrated shares and parameters kept fixed in both scenarios that I borrowed from the literature. Since the consumption share on tradables (exportables plus importables) is 0.3, I use the estimate expenditure share on importables (relative to tradable expenditure) from Table 8.2 in Uribe and Schmitt-Grohé (2017), which is $\chi_m = 0.898$ (in their notation). This implies that the consumption expenditure on importables as a share of total expenditure equals $b_M = 0.3 \times 0.898 = 0.27$. I set all elasticities in production and consumption to be Cobb-Douglas. I do this to highlight the first-order mechanisms, as it is well known that under low elasticities of substitution, the production network amplifies negative shocks on quantities and mitigates positive shocks Baqaee and Farhi (2019b), and thus production networks matter beyond sales shares to a second-order. All remaining parameters are standard in the small open economy literature.

A.3.2 Results

I explore how inflation reacts to a one percent shock negative productivity in the non-tradable sector (ν_t^N) and a positive import price shock (ν_t^M). I solve the model using a first-order approximation around the non-stochastic steady state to be comparable with the model in the main text. Panel (a) of Figure A.1 shows the response of inflation to a negative productivity shock in the non-tradable sector, while panel (b) of Figure A.1 does the same for a positive import price shock. The solid purple line shows the implied impulse-response function for inflation in a model with intersectoral linkages. In contrast, the pink dashed line shows the impulse-response function

Table A.1: Calibration

Parameters	Value	Description	Source
Shares			
$a_N = a_X$	0.66	Labor Share	Benigno et al. (2013)
b_N	0.70	Consumption share on non-tradables	Bianchi (2011)
b_X	0.03	Consumption share on exportable	Table 8.2 (Uribe and Schmitt-Grohé, 2017)
b_M	0.27	Consumption share on importable	Table 8.2 (Uribe and Schmitt-Grohé, 2017)
Elasticities			
χ	1	Elasticity of substitution in consumption	Cobb-Douglas specification
σ	2	Intertemporal Elasticity of Substitution	Table 8.2 (Uribe and Schmitt-Grohé, 2017)
$\sigma_N = \sigma_X$	1	Elasticity between value-added and intermediates	Cobb-Douglas specification
$\varepsilon_N = \varepsilon_X$	1	Elasticity across intermediates	Cobb-Douglas specification
$\varepsilon_N^T = \varepsilon_X^T$	1	Elasticity across tradable intermediates	Cobb-Douglas specification
Other Parameters			
$\rho_{Z_N} = \rho_{P_M}$	0.53	AR(1) non-tradable productivity and import price	Table 7.1 (Uribe and Schmitt-Grohé, 2017)
$\bar{B}^*$	0	Steady-state foreign assets position	Zero trade balance
ψ	0.000742	Interest rate sensitivity to foreign assets	Schmitt-Grohé and Uribe (2003)
$\bar{i}^*$	0.04	Steady-state foreign interest rate	Bianchi (2011)
β	$\frac{1}{(1+i^*)} = 0.9615$	Discount Factor	

for the island model.

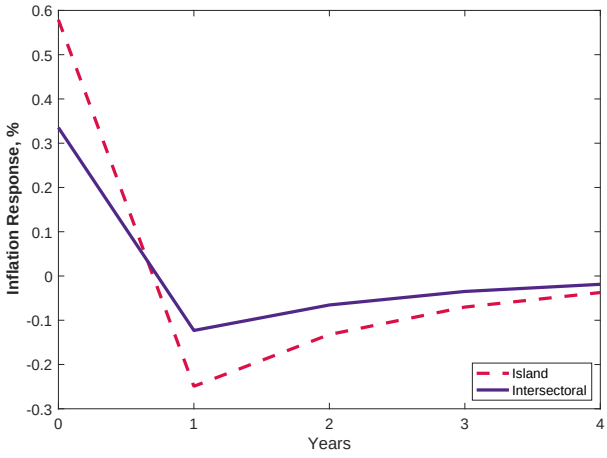
The dynamic model confirms the intuition of the static model. Panel (a) shows that a decrease in productivity generates inflation, which is lower in an economy with intersectoral linkages (solid line) due to indirect trade. A positive import price shock, on the other hand, increases inflation, more so in a model with intersectoral linkages due to indirect trade. This highlights that the production network matters to a first-order for the productivity pass-through to inflation. Importantly, Domar weights are no longer sufficient statistics for the productivity pass-through to inflation, as they are equal in both scenarios by construction.

The persistence in the dynamic responses is given by shocks' persistence as the model has no inherent dynamics from the production network structure. Of course, the model does have dynamics from the foreign asset position and thus is richer relative to the static model in the main section. Moreover, a one-time shock at time 0 generates inflation in period 0 but requires deflation thereafter for the price level to recover its steady-state level in the long run.

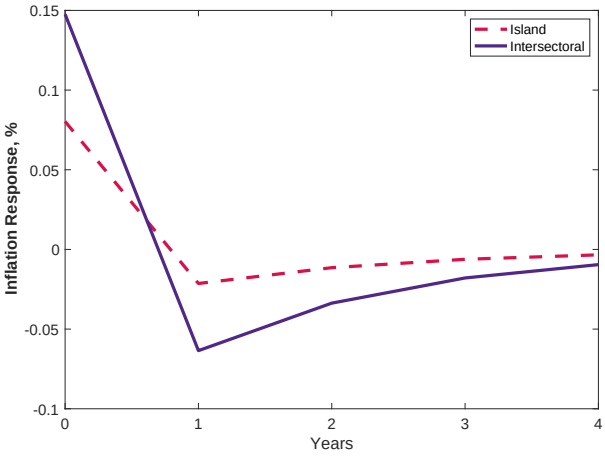
Finally, inflation volatility due to productivity shocks in the non-tradable sector is lower under the intersectoral model than the island model. The opposite is true when we consider import price shocks. These inflation impulse responses are consistent with the results in Section 1.4, where the small open economy model with production networks decreases inflation volatility in Chile and increases it for the UK, as the production network adjustments put more weight on import prices and less on productivity shocks.

Figure A.1: Inflation Impulse Responses

(a) Negative Productivity Shock in Non-Tradable Sector



(b) Import Price Shock



Note: This figure shows the response of inflation to a one percent negative productivity shock (panel a) and a one percent positive import price shock (panel b). The solid purple line is for the model that considers intersectoral linkages, while the pink dashed line is the model that assumes an island production network structure.

A.4 Shares

In this appendix I explicitly solve for $\widehat{\bar{\mathbf{\Lambda}}}$. Before doing so, I need to define several objects on the consumption and production sides.

A.4.1 Consumption

Let the consumer's share expenditure on good $i \in N \cup M$ be b_i and

$$\bar{b}_i = \frac{P_i C_i}{E}$$

Let the price elasticity of demand be $\varepsilon_{ik}^C = \frac{\partial \log C_i}{\partial \log P_k}$ and $\delta_{ik} = 1$ if $k = i$ and zero otherwise. This last element is usually called the Kronecker-delta. Log-differentiating the shares and using the homotheticity assumption, I have

$$\widehat{\bar{b}}_i = \sum_{k \in N \cup M} (\delta_{ik} + \varepsilon_{ik}^C - \bar{b}_i)\widehat{P}_k = \sum_{k \in N \cup M} \phi_{ik}^C \widehat{P}_k$$

$$\mathrm{d}\bar{b}_i = \bar{b}_i \sum_{k \in N \cup M} \phi_{ik}^C \widehat{P}_k \qquad \text{for } i \in N \cup M$$

where $\phi_{ik}^C = (\delta_{ik} + \varepsilon_{ik}^C - \bar{b}_k)$ represents the elasticity of consumption share on good i, b_i, in response to a change in the price of good k, P_k.

Note that we must have

$$\sum_{i \in N \cup M} \mathrm{d}\bar{b}_i = 0 \Longrightarrow \sum_{i \in N \cup M} \bar{b}_i \sum_{k \in N \cup M} \phi_{ik}^C \widehat{P}_k = 0,$$

for any changes in prices. It thus follows that

$$\sum_{i \in N \cup M} \bar{b}_i \phi_{ik}^C = 0 \quad \text{for all } k \in N \cup M$$

For further reference, it proves useful to define changes in domestic expenditure shares as

$$\mathrm{d}\bar{\boldsymbol{b}}_D = diag(\bar{\boldsymbol{b}}_D)(\boldsymbol{\Phi}_D^C \widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M^C \widehat{\boldsymbol{P}}_M),$$

where $\boldsymbol{\Phi}_D^C$ is an $N \times N$ matrix with typical element ϕ_{ij}^C with $i, j \in N$, and $\boldsymbol{\Phi}_M^C$ is an $N \times M$ matrix with typical element ϕ_{im}^C, and $i \in N$, $m \in M$.

A.4.2 Production

On the production side, I must define an operator similar to the above but for each $i \in N$ producer. In addition, the decision of the representative firms also depends on factor prices $\widehat{W}_f$. With this in mind, define the expenditure share of producer i on input $j \in N \cup M$ as

$$\Omega_{ij} = \frac{P_j M_{ij}}{P_i Q_i}$$

Define

$$\phi_{jk}^i = \frac{\partial \log \Omega_{ij}}{\partial \log P_k} \quad \text{for } i = 1, ..., N; \quad k = 1, ..., N \cup M \cup F$$

Then

$$\begin{aligned}
\widehat{\Omega}_{ij} &= \widehat{P}_j + \sum_{k \in N \cup M} \varepsilon_{jk}^i \widehat{P}_k + \sum_{f \in F} \varepsilon_{jf}^i \widehat{W}_f - \sum_{k \in N \cup M} \Omega_{ik} \widehat{P}_k - \sum_{f \in F} a_{if} \widehat{W}_f \\
&= \sum_{k \in N \cup M} (\delta_{jk} + \varepsilon_{jk}^i - \Omega_{ik}) \widehat{P}_k + \sum_{f \in F} (\varepsilon_{jf}^i - a_{if}) \widehat{W}_f \\
\widehat{\Omega}_{ij} &= \sum_{k \in N \cup M} \phi_{jk}^i \widehat{P}_k + \sum_{f \in F} \phi_{jf}^i \widehat{W}_f \\
\mathrm{d}\Omega_{ij} &= \Omega_{ij} \left(\sum_{k \in N \cup M} \phi_{jk}^i \widehat{P}_k + \sum_{f \in F} \phi_{jf}^i \widehat{W}_f \right)
\end{aligned}$$

where

$$\varepsilon^i_{jk} = \frac{\partial \log M_{ij}}{\partial \log P_k}$$

$$\phi^i_{jk} = \delta_{jk} + \varepsilon^i_{jk} - \Omega_{ik}$$

$$\phi^i_{jf} = \varepsilon^i_{jf} - a_{if}$$

represents the elasticity of expenditure share on good j by producer i, Ω_{ij}, when there is a change in either good $k \in N \cup M$ or factors $f \in F$.

By a similar logic, I can write the change in expenditure share of producer i on factor f as

$$\mathrm{d}a_{if} = a_{if}\left(\sum_{k\in N\cup M}(\varepsilon^i_{fk} - \Omega_{ik})\widehat{P}_k + \sum_{f'\in F}(\delta_{ff'} + \varepsilon^i_{ff'} - a_{if'})\widehat{W}_{f'}\right)$$

$$\mathrm{d}a_{if} = a_{if}\left(\sum_{k\in N\cup M}\phi^i_{fk}\widehat{P}_k + \sum_{f'\in F}\phi^i_{ff'}\widehat{W}_{f'}\right)$$

where again

$$\varepsilon^i_{fk} = \frac{\partial \log L_{if}}{\partial \log P_k}$$

$$\varepsilon^i_{ff'} = \frac{\partial \log L_{if}}{\partial \log W_{f'}}$$

$$\phi^i_{fk} = \varepsilon^i_{fk} - \Omega_{ik}$$

$$\phi^i_{ff'} = \delta_{ff'} + \varepsilon^i_{ff'} - a_{if'}$$

The first two rows represent the demand elasticity of factor f relative to a change in other good prices (first row) or factors of production (second row).

The last two rows represent the elasticity of expenditure share of producer i on factor f relative to either good or factor price changes. When these are positive, then expenditure changes increase after a change in other input prices, meaning that the producer substitutes away from

those price increases towards factor f. If this term is negative, then the expenditure share in factor f declines with a change in other input prices: it means that it has to move resources away from factor f towards those goods that are seeing an increase in their price. This is the complementarity in the production case, and it arises with low elasticities of substitution (low ε's).

For imported intermediate, I can construct the same as

$$\mathrm{d}\Gamma_{im} = \Gamma_{im}\left(\sum_{k\in N}(\varepsilon^i_{mk} - \Omega_{ik})\widehat{P}_k + \sum_{m'\in M}(\delta_{mm'} + \varepsilon^i_{mm'} - \Gamma_{im'})\widehat{P}_{m'} + \sum_{f\in F}(\varepsilon^i_{mf} - a_{if})\widehat{W}_f\right)$$

$$\mathrm{d}\Gamma_{im} = \Gamma_{im}\left(\sum_{k\in N}\phi^i_{mk}\widehat{P}_k + \sum_{m'\in M}\phi^i_{mm'}\widehat{P}_{m'} + \sum_{f\in F}\phi^i_{mf}\widehat{W}_f\right)$$

where again

$$\varepsilon^i_{mk} = \frac{\partial \log M_{im}}{\partial \log P_k}$$

$$\varepsilon^i_{mm'} = \frac{\partial \log M_{im}}{\partial \log P_{m'}}$$

$$\varepsilon^i_{mf} = \frac{\partial \log M_{im}}{\partial \log W_f}$$

$$\phi^i_{mk} = (\varepsilon^i_{mk} - \Omega_{ik})$$

$$\phi^i_{mm'} = (\delta_{mm'} + \varepsilon^i_{mm'} - \Gamma_{im'})$$

$$\phi^i_{mf} = (\varepsilon^i_{mf} - a_{if})$$

A.4.3 Market clearing conditions and substitution patterns

Goods market clearing conditions. Recall that the market clearing conditions for the N domestic goods can be written as

$$Q_i = C_i + X_i + \sum_{j\in N} M_{ji}$$

In nominal terms and dividing by expenditure, I have

$$\frac{P_i Q_i}{E} = \frac{P_i C_i}{E} + \frac{P_i X_i}{E} + \sum_{j \in N} \frac{P_i M_{ji}}{P_j Q_j} \frac{P_j Q_j}{E}$$

Define *expenditure-based* ratios with a bar i.e. $\bar{\lambda}_i = \frac{P_i Q_i}{E}$. Then,

$$\bar{\lambda}_i = \bar{b}_i + \bar{x}_i + \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j$$

Differentiating this expression, I have

$$\mathrm{d}\bar{\lambda}_i = \mathrm{d}\bar{b}_i + \mathrm{d}\bar{x}_i + \sum_{j \in N} (\mathrm{d}\Omega_{ji} \bar{\lambda}_j + \Omega_{ji} \mathrm{d}\bar{\lambda}_j)$$

Now, recall from shares and making some changes of indices

$$\mathrm{d}\Omega_{ji} = \Omega_{ji} \left(\sum_{k \in N \cup M} \phi_{ik}^j \widehat{P}_k + \sum_{f \in F} \phi_{if}^j \widehat{W}_f \right)$$

Then, the third term on the right-hand side can be written as

$$\begin{aligned}
\sum_{j \in N} \mathrm{d}\Omega_{ji} \bar{\lambda}_j &= \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \left(\sum_{k \in N \cup M} \phi_{ik}^j \widehat{P}_k + \sum_{f \in F} \phi_{if}^j \widehat{W}_f \right) \\
&= \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \sum_{k \in N \cup M} \phi_{ik}^j \widehat{P}_k + \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \sum_{f \in F} \phi_{if}^j \widehat{W}_f \\
&= \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \sum_{k \in N} \phi_{ik}^j \widehat{P}_k + \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \sum_{m \in M} \phi_{im}^j \widehat{P}_m + \sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \sum_{f \in F} \phi_{if}^j \widehat{W}_f \\
&= \sum_{k \in N} \underbrace{\left[\sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \phi_{ik}^j \right]}_{\equiv \phi_{ik}} \widehat{P}_k + \sum_{m \in M} \underbrace{\left[\sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \phi_{im}^j \right]}_{\equiv \phi_{im}} \widehat{P}_m + \sum_{f \in F} \underbrace{\left[\sum_{j \in N} \Omega_{ji} \bar{\lambda}_j \phi_{if}^j \right]}_{\equiv \phi_{if}} \widehat{W}_f \\
&= \sum_{k \in N} \phi_{ik} \widehat{P}_k + \sum_{m \in M} \phi_{im} \widehat{P}_m + \sum_{f \in F} \phi_{if} \widehat{W}_f
\end{aligned}$$

A useful thing about writing this in this way, is that I can write this in matrix form

$$\mathrm{d}\boldsymbol{\Omega}^T \boldsymbol{\lambda} = \boldsymbol{\Phi}_D \widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M \widehat{\boldsymbol{P}}_M + \boldsymbol{\Phi}_F \widehat{\boldsymbol{W}}$$

where $\mathbf{\Phi}$ represents *direct substitution matrices*. A version of these substitution matrices appears in Baqaee and Farhi (2019b), although they consider both direct and indirect substitution. Each column represents the changing price and the rows represent where intermediate input demand is going. This only takes into account *first-round effects* and does not consider any input-output linkages beyond the direct exposure (or path of order 1). The next step is to recompute these matrices using the Leontief-inverse. To see this, note that the differentiated form of the market clearing condition write the problem as

$$\begin{aligned}
\mathrm{d}\bar{\boldsymbol{\lambda}} &= \boldsymbol{\Psi}^T(\mathrm{d}\bar{\boldsymbol{b}}_D + \mathrm{d}\bar{\boldsymbol{x}} + \boldsymbol{\Phi}_D\widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M\widehat{\boldsymbol{P}}_M + \boldsymbol{\Phi}_F\widehat{\boldsymbol{W}}) \\
&= \boldsymbol{\Psi}^T\left(diag(\bar{\boldsymbol{b}}_D)(\boldsymbol{\Phi}_D^C\widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M^C\widehat{\boldsymbol{P}}_M) + \mathrm{d}\bar{\boldsymbol{x}} + \boldsymbol{\Phi}_D\widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_M\widehat{\boldsymbol{P}}_M + \boldsymbol{\Phi}_F\widehat{\boldsymbol{W}}\right) \\
\mathrm{d}\bar{\boldsymbol{\lambda}} &= \boldsymbol{\Psi}^T\left(diag(\bar{\boldsymbol{b}}_D)\boldsymbol{\Phi}_D^C + \boldsymbol{\Phi}_D\right)\widehat{\boldsymbol{P}}_D + \left(diag(\bar{\boldsymbol{b}}_D)\boldsymbol{\Phi}_M^C + \boldsymbol{\Phi}_M\right)\widehat{\boldsymbol{P}}_M + \boldsymbol{\Psi}^T\mathrm{d}\bar{\boldsymbol{x}} + \boldsymbol{\Psi}^T\boldsymbol{\Phi}_F\widehat{\boldsymbol{W}}
\end{aligned}$$

Factor shares changes. We need F more equations, which come from the factor market clearing conditions

$$\bar{L}_f = \sum_{i\in N} L_{if}$$

Write this in share form

$$\bar{\Lambda}_f = \frac{W_f\bar{L}_f}{E} = \sum_{i\in N}\frac{W_fL_{if}}{P_iQ_i}\frac{P_iQ_i}{E} = \sum_{i\in N} a_{if}\bar{\lambda}_i$$

In differential form,

$$\mathrm{d}\Lambda_f = \sum_{i\in N}\mathrm{d}a_{if}\bar{\lambda}_i + \sum_{i\in N} a_{if}\mathrm{d}\bar{\lambda}_i$$

Using the expression for changes in factor usage at the producer level, I have

$$\begin{aligned}
\mathrm{d}\Lambda_f &= \sum_{i\in N}\left[a_{if}\left(\sum_{k\in N\cup M}\phi^i_{fk}\widehat{P}_k + \sum_{f'\in F}\phi^i_{ff'}\widehat{W}_{f'}\right)\right]\bar{\lambda}_i + \sum_{i\in N} a_{if}\mathrm{d}\bar{\lambda}_i \\
\mathrm{d}\Lambda_f &= \sum_{i\in N}\left[a_{if}\left(\sum_{k\in N}\phi^i_{fk}\widehat{P}_k + \sum_{m\in M}\phi^i_{fm}\widehat{P}_m + \sum_{f'\in F}\phi^i_{ff'}\widehat{W}_{f'}\right)\right]\bar{\lambda}_i + \sum_{i\in N} a_{if}\mathrm{d}\bar{\lambda}_i
\end{aligned}$$

Taking each of the terms on the right-hand side, I can write

$$\sum_{i\in N} a_{if} \sum_{k\in N} \phi^i_{fk} \widehat{P}_k \bar{\lambda}_i = \sum_{k\in N} \underbrace{\left(\sum_{i\in N} a_{if} \phi^i_{fk} \bar{\lambda}_i \right)}_{\equiv \phi_{fk}} \widehat{P}_k = \sum_{k\in N} \phi_{fk} \widehat{P}_k$$

$$\sum_{i\in N} a_{if} \sum_{f'\in F} \phi^i_{ff'} \widehat{W}_{f'} \bar{\lambda}_i = \sum_{f'\in F} \underbrace{\left(\sum_{i\in N} a_{if} \bar{\lambda}_i \phi^i_{ff'} \right)}_{\equiv \phi_{ff'}} \widehat{W}_{f'} = \sum_{f'\in F} \phi_{ff'} \widehat{W}_{f'}$$

$$\sum_{i\in N} a_{if} \sum_{m\in M} \phi^i_{fm} \widehat{P}_m \bar{\lambda}_i = \sum_{m\in M} \underbrace{\left(\sum_{i\in N} a_{if} \bar{\lambda}_i \phi^i_{fm} \right)}_{\equiv \phi_{fm}} \widehat{P}_m = \sum_{m\in M} \phi_{fm} \widehat{P}_m$$

Replacing this into the differential form for the factor share

$$\mathrm{d}\Lambda_f = \sum_{k\in N} \phi_{fk} \widehat{P}_k + \sum_{f'\in F} \phi_{ff'} \widehat{W}_{f'} + \sum_{m\in M} \phi_{fm} \widehat{P}_m + \sum_{i\in N} a_{if} \mathrm{d}\bar{\lambda}_i$$

In matrix form,

$$\mathrm{d}\bar{\boldsymbol{\Lambda}} = \boldsymbol{\Phi}^F_D \widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}^F_F \widehat{\boldsymbol{W}} + \boldsymbol{\Phi}^F_M \widehat{\boldsymbol{P}}_M + \boldsymbol{A}^T \mathrm{d}\bar{\boldsymbol{\lambda}}$$

Export share changes. Since export demand is exogenous, I can write

$$\bar{x}_i = \frac{P_i X_i}{\mathcal{M}} \Longrightarrow \mathrm{d}\bar{x}_i = \bar{x}_i (\widehat{P}_i + \widehat{X}_i - \widehat{\mathcal{M}}),$$

which staking into a vector form

$$\mathrm{d}\bar{\boldsymbol{x}} = diag(\bar{\boldsymbol{x}})(\widehat{\boldsymbol{P}}_D + \widehat{\boldsymbol{X}} - \mathbf{1}_N \widehat{\mathcal{M}}) \tag{A.33}$$

A.4.4 Putting all equations together

$$\mathrm{d}\bar{\boldsymbol{\Lambda}} = \boldsymbol{\Phi}_D^F \widehat{\boldsymbol{P}}_D + \boldsymbol{\Phi}_F^F \widehat{\boldsymbol{W}} + \boldsymbol{\Phi}_M^F \widehat{\boldsymbol{P}}_M + \boldsymbol{A}^T \mathrm{d}\bar{\boldsymbol{\lambda}} \qquad (F \text{ equations}, F+N+F+N \text{ unknowns})$$

$$\mathrm{d}\bar{\boldsymbol{\lambda}} = \boldsymbol{\Psi}^T \left(diag(\bar{\boldsymbol{b}}_D)\boldsymbol{\Phi}_D^C + \boldsymbol{\Phi}_D \right) \widehat{\boldsymbol{P}}_D + \left(diag(\bar{\boldsymbol{b}}_D)\boldsymbol{\Phi}_M^C + \boldsymbol{\Phi}_M \right) \widehat{\boldsymbol{P}}_M + \boldsymbol{\Psi}^T \mathrm{d}\bar{\boldsymbol{x}} + \boldsymbol{\Psi}^T \boldsymbol{\Phi}_F \widehat{\boldsymbol{W}} \qquad (N \text{ equations}, N \text{ add. unknowns})$$

$$\widehat{\boldsymbol{P}}_D = -\boldsymbol{\Psi}\widehat{\boldsymbol{Z}} + \boldsymbol{\Psi}\boldsymbol{A}\widehat{\boldsymbol{W}} + \boldsymbol{\Psi}\boldsymbol{\Gamma}\widehat{\boldsymbol{P}}_M \qquad (N \text{ equations, no new unknowns})$$

$$\mathrm{d}\bar{\boldsymbol{\Lambda}} = diag(\bar{\boldsymbol{\Lambda}}) \left(\widehat{\boldsymbol{W}} + \widehat{\boldsymbol{L}} - \mathbf{1}_F \widehat{\mathcal{M}} \right) \qquad (F \text{ equations, no new unknowns})$$

$$\mathrm{d}\bar{\boldsymbol{x}} = diag(\bar{\boldsymbol{x}})(\widehat{\boldsymbol{P}}_D + \widehat{\boldsymbol{X}} - \mathbf{1}_N \widehat{\mathcal{M}}) \qquad (N \text{ equations, no new unknowns})$$

This is a system of $2F+3N$ unknowns: $(\mathrm{d}\bar{\boldsymbol{\Lambda}}, \widehat{\boldsymbol{W}}, \mathrm{d}\bar{\boldsymbol{\lambda}}, \widehat{\boldsymbol{P}}_D, \mathrm{d}\bar{\boldsymbol{x}})$ on the same number of equations, and thus it pins down all necessary objects.

Note that the distribution of factor shares ($\mathrm{d}\bar{\boldsymbol{\Lambda}}$) and Domar weights ($\mathrm{d}\bar{\boldsymbol{\lambda}}$) changes crucially depends on the substitution matrices ($\boldsymbol{\Phi}'s$ matrices). As a result, to the extent that substitution patterns are encapsulated in these matrices, they affect aggregate inflation in the small open economy via changing factor shares. It is in this sense that elasticities of substitution matter for inflation in this model, something that does not hold in the closed economy as these terms cancel out to a first-order.

Appendix B: Appendix to Chapter 2

B.1 Cross-Country Evidence: Real GDP Detrending

Here, we compare the performance of the Hamilton filter against two commonly used detrending procedures: Hodrick-Prescott Filter and linear detrending. We use a smoothing parameter equal to 100 for the Hodrick-Prescott filter.

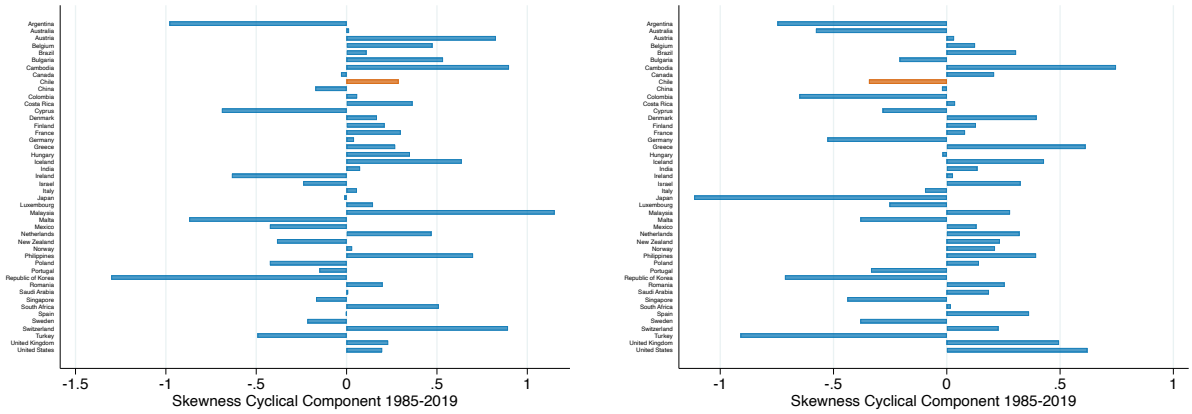
Figure B.1 shows the cyclical component for the Chilean real GDP under the three different scenarios. We note that the Hamilton filter performs better than the other two procedures for our purposes as it better captures crises. For example, the Hamilton filter (pink line) captures the decline in economic activity around 1998 due to the Asian Financial Crisis that badly hit the Chilean economy, among other emerging markets economies, while the other two detrending procedures view the crisis as merely a decline towards the mean.

The fact that the Hamilton filter better captures crises is not particular to the Chilean economic data but holds more broadly in the cross-section. In Figure B.2, we plot an analog to Figure 2.1 — where we used the Hamilton filter — but using the other two methods. As it is clear from the figure, both methods produce inconsistent results, generating negative skewness of the cyclical component of real GDP only on around half of the countries in our sample. In contrast, the Hamilton filter captures the skewed business cycles characteristic of real GDP.

Figure B.1: Chilean Real GDP Cyclical Components under different detrending procedures

Note: The figures shows the cyclical component of real GDP for Chile between 1985 – 2019 using three different sets of detrending procedures. The solid blue line uses a Hodrick-Prescott filter with a smooth parameter equal to 100. The orange dashed line shows the cyclical component when using linear detrending. Finally, the dot-solid pink line shows the cyclical component using the Hamilton filter.

Figure B.2: Cross-Country Skewness of Real GDP Cyclical Components



(a) Hodrick-Prescott Filter

(b) Linear Detrending

Note: Panel (a) shows the skewness of the cyclical component of (log) real GDP when using the Hodrick-Prescott filter with a smooth parameter equal to 100. Panel (b) shows the skewness of (log) real GDP using a linear detrending procedure.

B.2 Additional Tables

Table B.1: Cross-Country Relationship between Real GDP Cyclical Component Skewness and Network Density

	(1)	(2)
Density 1995	-2.766***	-2.977***
	(0.912)	(1.074)
Obs.	46	46
R^2	0.220	0.260
Controls	No	Yes

Note: This table reports the OLS coefficient of running a regression between real GDP cyclical component skewness as the dependent variable. Column 1 uses network density in 1995 as an independent variable. Column 2 uses the following controls: GDP per capita in 1985, the volatility of the cyclical component of real GDP, and the skewness of the outdegree and indegree distribution in 1995.

Table B.2: Annual labor productivity growth

	2019q2	2019q3	2019q4	2020q1	2020q2	2020q3	2020q4
Mean	0.01	0.04	0.02	0.05	-0.04	0.07	0.11
Median	0.02	0.04	0.02	0.05	-0.03	0.07	0.11
St. Dev	0.27	0.27	0.27	0.28	0.35	0.35	0.32
Skewness	-0.04	-0.09	-0.06	-0.09	-0.10	-0.18	-0.15
Observations	16,938	16,938	16,938	16,938	16,938	16,938	16,938

Note: This table presents the descriptive statistics of annual labor revenue labor productivity growth at different quarters. The sample of firms corresponds to that used in Section 3 for our firm-to-firm network calibration.

Appendix C: Appendix to Chapter 3

C.1 Data Sources and Definitions

Macroeconomic Aggregates

The data for the estimation of commodity price changes are obtained from the following sources: Fernández et al. (2018) is used for the sectoral commodity price index.

Input-Output Table Database

WIOD Data. Our main database is the World Input-Output database (Timmer et al., 2015), release 2013. It provides information on intersectoral and cross-country final and intermediate flows for 40 countries and 35 sectors classified according to the International Standard Industrial Classification Revision 3 (ISIC Rev. 3). These tables match the 1993 version of the SNA. We use the sectoral data on quantities (gross output, value-added, number of employees, and capital) and price indexes for the period 1995-2011(2009) in the National IO tables. The sample of small open economies with data on commodity prices and WIOD input-output data includes the following countries: Australia, Bulgaria, Brazil, Canada, Denmark, India, Lithuania, Mexico, and Russia.

This dataset is freely available here https://www.rug.nl/ggdc/valuechain/wiod/wiod-2013-release.

Commodity Data

To measure sectoral linkages to the *commodity sector* we use detailed information on each country's commodity bundle composition from Fernández et al. (2018). There is a total of 44 commodities classified according to the Harmonized System (HS) 1992 – 4 digits. We separate commodities into 3 groups: Agriculture, Hunting, Forestry, and Fishing; Mining and Quarrying; and Food Products, Beverages, and Tobacco.

C.2 Additional Tables and Figures

C.2.1 Tables

Table C.1: Sectors in WIOD Database

Sector Number	Sector Name
1	Agriculture, Hunting, Forestry and Fishing
2	Mining and Quarrying
3	Food, Beverages, and Tobacco
4	Textiles and Textile Products
5	Leather, Leather, and Footwear
6	Wood and Products of Wood and Cork
7	Pulp, Paper, Paper, Printing, and Publishing
8	Coke, Refined Petroleum and Nuclear Fuel
9	Chemicals and Chemical Products
10	Rubber and Plastics
11	Other Non-Metallic Mineral
12	Basic Metals and Fabricated Metal
13	Machinery, Nec
14	Electrical and Optical Equipment
15	Transport Equipment
16	Manufacturing, Nec; Recycling
17	Electricity, Gas and Water Supply
18	Construction
19	Sale, Maintenance and Repair of Motor Vehicles and Motorcycles
20	Wholesale Trade and Commission Trade
21	Retail Trade
22	Hotels and Restaurants
23	Inland Transport
24	Water Transport
25	Air Transport
26	Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies
27	Post and Telecommunications
28	Financial Intermediation
29	Real Estate Activities
30	Renting of M&Eq and Other Business Activities
31	Public Admin and Defence; Compulsory Social Security
32	Education
33	Health and Social Work
34	Other Community, Social and Personal Services

Table C.2: Commodities and WIOD Industries.

Commodity	HS Code	Industry
Beef	201	Agriculture, hunting, forestry and fishing
Pork	203	Agriculture, hunting, forestry and fishing
Lamb	204	Agriculture, hunting, forestry and fishing
Chicken	207	Agriculture, hunting, forestry and fishing
Fish	301	Agriculture, hunting, forestry and fishing
Fish Meal	304	Agriculture, hunting, forestry and fishing
Shrimp	306	Agriculture, hunting, forestry and fishing
Bananas	803	Agriculture, hunting, forestry and fishing
Coffee	901	Agriculture, hunting, forestry and fishing
Tea	902	Agriculture, hunting, forestry and fishing
Wheat	1001	Agriculture, hunting, forestry and fishing
Barley	1003	Agriculture, hunting, forestry and fishing
Corn	1005	Agriculture, hunting, forestry and fishing
Rice	1006	Agriculture, hunting, forestry and fishing
Soybeans	1201	Agriculture, hunting, forestry and fishing
Groundnuts	1202	Agriculture, hunting, forestry and fishing
Wool	1505	Agriculture, hunting, forestry, and fishing
Sugar	1701	Agriculture, hunting, forestry and fishing
Cocoa	1801	Agriculture, hunting, forestry and fishing
Natural Rubber	4001	Agriculture, hunting, forestry, and fishing
Hides	4101	Agriculture, hunting, forestry and fishing
Hard Log	4401	Agriculture, hunting, forestry and fishing
Soft Log	4403	Agriculture, hunting, forestry and fishing
Hard Swan	4407	Agriculture, hunting, forestry and fishing
Soft Swan	4408	Agriculture, hunting, forestry and fishing
Cotton	5201	Agriculture, hunting, forestry and fishing
Iron	2601	Mining and quarrying
Copper	2603	Mining and quarrying
Nickel	2604	Mining and quarrying
Aluminum	2606	Mining and quarrying
Lead	2607	Mining and quarrying
Zinc	2608	Mining and quarrying
Tin	2609	Mining and quarrying
Coal	2701	Mining and quarrying
Crude Oil	2709	Mining and quarrying
NatGas	2711	Mining and quarrying
Uranium	2844	Mining and quarrying
Gold	7108	Mining and quarrying
Soybean Meal	1208	Food products, beverages and tobacco
Soy Oil	1507	Food products, beverages and tobacco
Olive Oil	1509	Food products, beverages and tobacco
Palm Oil	1511	Food products, beverages and tobacco
Sun Oil	1512	Food products, beverages and tobacco
Coconut Oil	1513	Food products, beverages and tobacco

Table C.3: Calibration Exercises: The Role of Upstreamness

	Calibration: $(\sigma, \epsilon, \phi, \frac{\bar{Y}}{GDP})$				
	(0.98, 0.98, 2, 0.18)	(2, 0.2, 2, 0.18)	(0.2, 2, 2, 0.18)	(0.98, 0.98, 2, 0.30)	(0.98, 0.98, 2, 0.06)
Upstream	4.26***	4.89***	3.53***	8.23***	0.05
	(0.7205)	(0.4083)	(1.2552)	(0.7278)	(0.7805)
Observations	31	31	31	31	31

Note: This table shows a cross-sectional regression of the form $\mathrm{d}\log Q_i/\mathrm{d}\log P_{N+1} = \beta_0+\beta_1 Upstream_i+\varepsilon_i$. The dependent variable is the change in quantities implied by Proposition 7, where $Upstream_i$ refers to how upstream and upstream sector i is from the commodity sector.

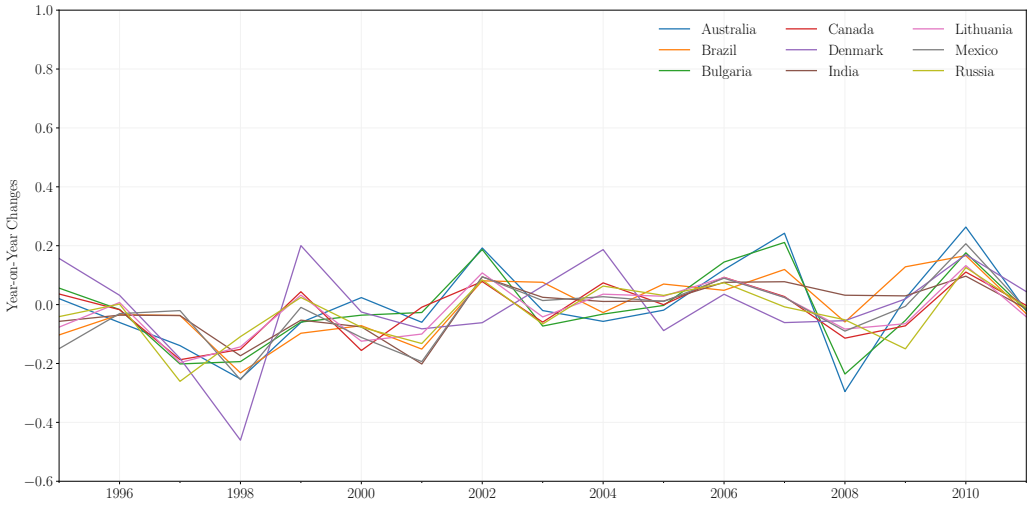
Table C.4: Calibration Exercises: The Role of Downstreamness

	Calibration: $(\sigma, \epsilon, \phi, \frac{\bar{Y}}{GDP})$				
	(0.98, 0.98, 2, 0.18)	(2, 0.2, 2, 0.18)	(0.2, 2, 2, 0.18)	(0.98, 0.98, 2, 0.30)	(0.98, 0.98, 2, 0.06)
Downstream	-0.29	0.12	-0.82***	0.06	-0.69***
	(0.1889)	(0.1811)	(0.2101)	(0.3102)	(0.0628)
Observations	31	31	31	31	31

Note: This table shows a cross-sectional regression of the form $\mathrm{d}\log Q_i/\mathrm{d}\log P_{N+1} = \beta_0+\beta_1 Downstream_i+\varepsilon_i$. The dependent variable is the change in quantities implied by Proposition 7, where $Downstream_i$ refers to how downstream sector i is from the commodity sector.

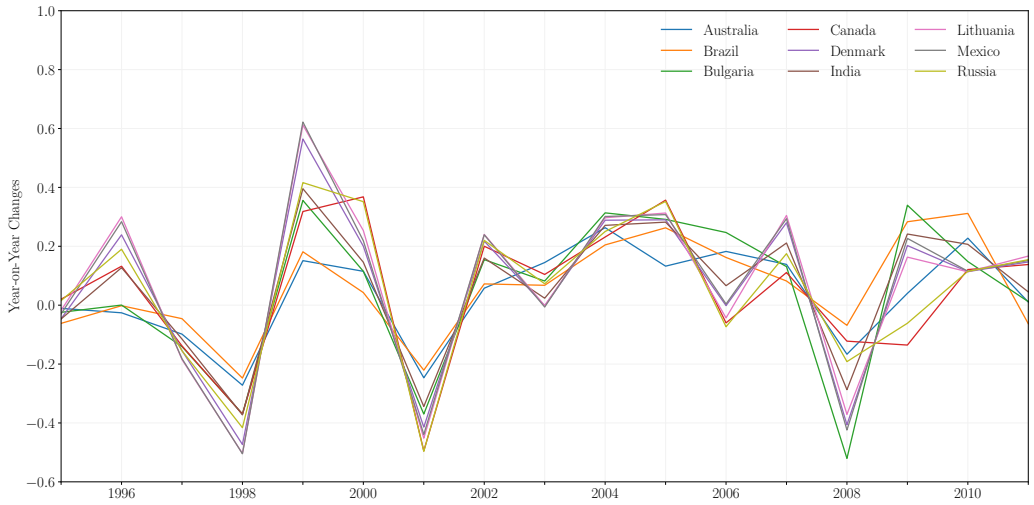
C.2.2 Extra Figures

Figure C.1: Commodity Price Changes: Agriculture and Forestry



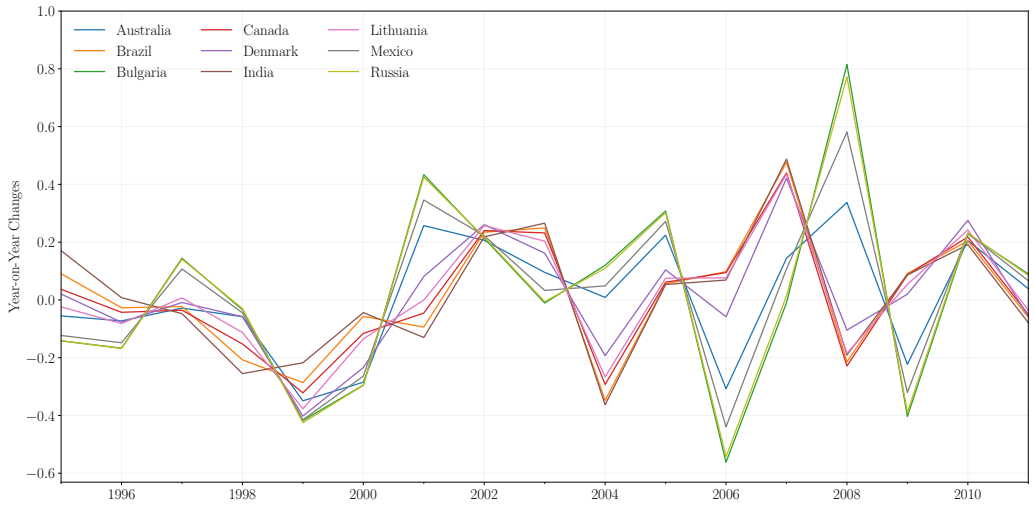
Note: This figure plots commodity price year-on-year changes for the agriculture and forestry sector in our sample of 9 countries in the WIOD database.

Figure C.2: Commodity Price Changes: Mining and Quarrying



Note: This figure plots commodity price year-on-year changes for the mining and quarrying sector in our sample of 9 countries in the WIOD database.

Figure C.3: Commodity Price Changes: Food Products and Beverages



Note: This figure plots commodity price year-on-year changes for the food and beverages sector in our sample of 9 countries in the WIOD database.

C.3 Proofs

Proof of Proposition 5. Starting from price changes, we have

$$d\log P_i = \Omega_{iL} d\log W + \sum_{j=1}^{N+1} \Omega_{ij} d\log P_j - d\log Z_i \quad \text{for all } i = 1, 2, ..., N$$

$$d\log P_{N+1} = \Omega_{N+1,L} d\log W + \Omega_{N+1,K} d\log R + \sum_{j=1}^{N+1} \Omega_{N+1,j} d\log P_j - d\log Z_{N+1}$$

where we define $\mathbf{\Omega}_K = (0, 0, ..., \Omega_{N+1,K})$ is a $(N+1) \times 1$ vector where its first N elements are 0 because non-tradable sectors do not use capital directly.

Using the wage as the numeraire, $d\log W = 0$, and stacking the system into matrix/vector form, we have

$$d\log \boldsymbol{P} = \mathbf{\Omega} d\log \boldsymbol{P} + \mathbf{\Omega}_K d\log R - d\log \boldsymbol{Z}$$

Setting $d\log \boldsymbol{Z} = \mathbf{0}$ and inverting the system we arrive

$$d\log \boldsymbol{P} = \mathbf{\Psi}\mathbf{\Omega}_K d\log R \implies d\log P_i = \Psi_{i,N+1}\Omega_{N+1,K} d\log R \text{ for all } i = 1, 2, ..., N+1 \quad \text{(C.1)}$$

Note that we can write the above expression as

$$d\log \boldsymbol{P} = \widetilde{\mathbf{\Omega}}_K d\log R$$

where we define the typical element of $\widetilde{\mathbf{\Omega}}_K = \{\widetilde{\Omega}_{iK}\} = \{\Psi_{i,N+1}\Omega_{N+1,K}\}$, that represents the *network-adjusted* capital share of producer i.

We now make use of the fact that $d\log P_{N+1}$ is exogenously given to express changes in the rental rate, $d\log R$, as an explicit function of it since

$$d\log P_{N+1} = \widetilde{\Omega}_{N+1,K} d\log R \implies d\log R = \frac{1}{\widetilde{\Omega}_{N+1,K}} d\log P_{N+1}$$

Replacing this expression into Equation (C.1), we get

$$d\log P_i = \frac{\widetilde{\Omega}_{i,K}}{\widetilde{\Omega}_{N+1,K}} d\log P_{N+1} \tag{C.2}$$

which completes the proof. ■

Proof of Proposition 6. Start from the market clearing condition and aggregate resource constraints in terms of Domar weights and matrix form

$$\boldsymbol{\lambda} = \boldsymbol{\Psi}^T \left(\boldsymbol{b} + \boldsymbol{e}_{N+1} \frac{\bar{Y}}{GDP} \right) \tag{C.3}$$

Totally differentiating this expression

$$d\boldsymbol{\lambda} = \underbrace{d\boldsymbol{\Psi}^T \left(\boldsymbol{b} + \boldsymbol{e}_{N+1} \frac{\bar{Y}}{GDP} \right)}_{\text{Changes in IO matrix given Demand Shares}} + \underbrace{\boldsymbol{\Psi}^T \left(d\boldsymbol{b} + \boldsymbol{e}_{N+1} d\left(\frac{\bar{Y}}{GDP} \right) \right)}_{\text{Changes in Demand Shares given IO linkages}} \tag{C.4}$$

We now totally differentiate the definition of the Leontieff-Inverse, $\boldsymbol{\Psi}$, to map its changes to changes in the IO matrix, $\boldsymbol{\Omega}$

$$\begin{aligned}
\boldsymbol{\Psi}^T &= (\boldsymbol{I} - \boldsymbol{\Omega}^T)^{-1} \\
\boldsymbol{\Psi}^T(\boldsymbol{I} - \boldsymbol{\Omega}^T) &= \boldsymbol{I} \\
\boldsymbol{\Psi}^T - \boldsymbol{\Psi}^T\boldsymbol{\Omega}^T &= \boldsymbol{I} \\
d\boldsymbol{\Psi}^T - d\boldsymbol{\Psi}^T\boldsymbol{\Omega}^T - \boldsymbol{\Psi}^T d\boldsymbol{\Omega}^T &= \mathbf{0} \\
d\boldsymbol{\Psi}^T(\boldsymbol{I} - \boldsymbol{\Omega}^T) &= \boldsymbol{\Psi}^T d\boldsymbol{\Omega}^T \\
d\boldsymbol{\Psi}^T &= \boldsymbol{\Psi}^T d\boldsymbol{\Omega}^T \boldsymbol{\Psi}^T \\
d\boldsymbol{\Psi}^T(\boldsymbol{b} + \boldsymbol{b}^*) &= \boldsymbol{\Psi}^T d\boldsymbol{\Omega}' \underbrace{\boldsymbol{\Psi}^T(\boldsymbol{b} + \boldsymbol{b}^*)}_{=\boldsymbol{\lambda}} \\
d\boldsymbol{\Psi}^T(\boldsymbol{b} + \boldsymbol{b}^*) &= \boldsymbol{\Psi}^T d\boldsymbol{\Omega}^T \boldsymbol{\lambda}
\end{aligned}$$

Using this expression into Equation (C.4)

$$d\boldsymbol{\lambda} = \boldsymbol{\Psi}^T d\boldsymbol{\Omega}^T \boldsymbol{\lambda} + \boldsymbol{\Psi}^T \left(d\boldsymbol{b} + \boldsymbol{e}_{N+1} d\left(\frac{\bar{Y}}{GDP} \right) \right) \tag{C.5}$$

For a given producer i, we have

$$d\lambda_i = \sum_{k=1}^{N+1} \Psi_{ki} \sum_{j=1}^{N+1} \Omega_{jk} \lambda_j d\log \Omega_{jk} + \sum_{k=1}^{N+1} \Psi_{ki} \left(b_k d\log b_k + \delta_{k,N+1} \frac{\bar{Y}}{GDP} (\mathrm{d}\log \bar{Y} - \mathrm{d}\log GDP) \right) \tag{C.6}$$

We now focus on the last term on the right-hand side of the above equation. To focus on the propagation mechanisms from intermediate inputs, we assume that the home consumer has Cobb-Douglas preferences over goods. This means that consumption of each good k as a share of *total expenditure* is constant and independent of quantities and prices. Note, however, that b_k is a ratio with respect to the nominal GDP of the home country. As a result, they may respond to changes in both expenditure and GDP. The Cobb-Douglas preferences do not imply that these ratios are constant. To be more transparent, write

$$b_k = \frac{P_k C_k}{GDP} = \frac{P_k C_k}{E} \frac{E}{GDP}$$

where E represents total expenditure at home.

Log-differentiating the above expression

$$\mathrm{d}\log b_k = \underbrace{d\log \frac{P_k C_k}{E}}_{=0 \text{ due to Cobb-Douglas Preferences}} + \mathrm{d}\log \frac{E}{GDP} = \mathrm{d}\log E - \mathrm{d}\log GDP$$

Therefore, b_k will change, provided that there is a difference between changes in expenditure and changes in nominal GDP. Intuitively, under Cobb-Douglas preferences, expenditure in good k raises proportionally to changes in total expenditure. This raises the rator in $d\log E$ for all good k, while the denominator raises as changes in nominal GDP $d\log GDP$. Therefore, if expenditure

raises more than nominal GDP, the expenditure share on good k as a fraction of nominal GDP, $d\log b_k$, will raise.

Using these results into Equation (C.6), we get

$$d\lambda_i = \sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}(b_k d\log b_k + \delta_{k,N+1}\frac{\bar{Y}}{GDP}(d\log\bar{Y} - d\log GDP))$$

$$d\lambda_i = \sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}\left(b_k(d\log E - d\log GDP) + \delta_{k,N+1}\frac{\bar{Y}}{GDP}(d\log\bar{Y} - d\log GDP)\right)$$

$$d\lambda_i = \sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}(b_k d\log E + \delta_{k,N+1}\frac{\bar{Y}}{GDP}d\log\bar{Y}) - \underbrace{\sum_{k=1}^{N+1}\Psi_{ki}\left(b_k + \delta_{k,N+1}\frac{\bar{Y}}{GDP}\right)}_{=\lambda_i} d\log GDP$$

$$d\lambda_i = \sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}\left(b_k d\log E + \delta_{k,N+1}\frac{\bar{Y}}{GDP}d\log\bar{Y}\right) - \lambda_i d\log GDP$$

Upon rearranging

$$d\log\lambda_i = \frac{1}{\lambda_i}\left(\sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} + \sum_{k=1}^{N+1}\Psi_{ki}(b_k d\log E + \delta_{k,N+1}\frac{\bar{Y}}{GDP}d\log\bar{Y})\right)$$
$$- d\log GDP \tag{C.7}$$

We are now ready to construct the input-output substitution operator. Write the changes in expenditure shares

$$\begin{aligned}\mathrm{d}\log\Omega_{jk} &= \mathrm{d}\log P_k + \sum_{h=1}^{N+1+F}(\theta^j_{kh} - 1)\Omega_{jh}\mathrm{d}\log P_h \\ &= \delta_{kh}\mathrm{d}\log P_h + \sum_{h=1}^{N+1+F}(\theta^j_{kh} - 1)\Omega_{jh}\mathrm{d}\log P_h \\ \mathrm{d}\log\Omega_{jk} &= \sum_{h=1}^{N+1+F}(\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh})\mathrm{d}\log P_h \end{aligned} \tag{C.8}$$

where δ_{kh} is the Kronecker delta, equal to 1 if $k = h$ and zero otherwise. In that expression, we also define

$$\theta^j_{kh} = \frac{\varepsilon^j_{kh}}{\Omega_{jh}} \tag{C.9}$$

$$\varepsilon^j_{kh} = \frac{\partial\log M_{jk}}{\partial\log P_h} \tag{C.10}$$

where Equation (C.9) is the Allen-Uzawa elasticity for producer j between input k and h. Note that input h can be either a factor or an intermediate input, while input k is always an intermediate good. Equation (C.10) represents the constant-output elasticity of input demand of producer j of good k with respect to a change in the price of good/factor h.

We now use the model's structure to simplify the above expression. In particular, notice that we only have one factor, which price is changing: capital. Therefore, $F = 1$ and we can write Equation (C.8) as

$$\mathrm{d}\log\Omega_{jk} = \sum_{h=1}^{N+1}(\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh})\mathrm{d}\log P_h + (\theta^j_{kK} - 1)\Omega_{jK}\mathrm{d}\log R$$

At this point, we can use the result in Proposition 5 that relates changes in prices to changes in the commodity price and the one that links changes in the price of capital to changes in the commodity price as well. We rewrite those below

$$\mathrm{d}\log P_h = \frac{\widetilde{\Omega}_{h,K}}{\widetilde{\Omega}_{N+1,K}}\mathrm{d}\log P_{N+1} \tag{C.11}$$

$$\mathrm{d}\log R = \frac{1}{\widetilde{\Omega}_{N+1,K}}\mathrm{d}\log P_{N+1} \tag{C.12}$$

Plugging these expressions into the above expression, we get

$$\mathrm{d}\log\Omega_{jk} = \left(\sum_{h=1}^{N+1}(\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh})\frac{\widetilde{\Omega}_{h,K}}{\widetilde{\Omega}_{N+1,K}} + \frac{(\theta^j_{kK} - 1)\Omega_{jK}}{\widetilde{\Omega}_{N+1,K}}\right)\mathrm{d}\log P_{N+1} \tag{C.13}$$

We can replace Equation (C.13) into the first term on the right-hand side of Equation (C.7), to get

$$\begin{aligned}\sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j d\log\Omega_{jk} &= \sum_{k=1}^{N+1}\Psi_{ki}\sum_{j=1}^{N+1}\Omega_{jk}\lambda_j\left(\sum_{h=1}^{N+1}(\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh})\frac{\widetilde{\Omega}_{h,K}}{\widetilde{\Omega}_{N+1,K}} + \frac{(\theta^j_{kK} - 1)\Omega_{jK}}{\widetilde{\Omega}_{N+1,K}}\right)\mathrm{d}\log P_{N+1} \\ &= \sum_{j=1}^{N+1}\lambda_j\left(\sum_{k=1}^{N+1}\sum_{h=1}^{N+1}(\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh})\Psi_{ki}\Omega_{jk}\frac{\widetilde{\Omega}_{h,K}}{\widetilde{\Omega}_{N+1,K}} + \sum_{k=1}^{N+1}\Psi_{ki}\Omega_{jk}\frac{(\theta^j_{kK} - 1)\Omega_{jK}}{\widetilde{\Omega}_{N+1,K}}\right)\mathrm{d}\log P_{N+1}\end{aligned}$$

Let's define the following objects

$$\Phi_j(\Psi_{(:,i)}, \widetilde{\mathbf{\Omega}}_K) = \frac{1}{\widetilde{\Omega}_{N+1,K}} \sum_{k=1}^{N+1} \sum_{h=1}^{N+1} (\delta_{kh} + (\theta_{kh}^j - 1)\Omega_{jh}) \Psi_{ki} \Omega_{jk} \widetilde{\Omega}_{h,K}$$

$$\Phi_j(\Psi_{(:,i)}, \mathbf{\Omega}_K) = \frac{1}{\widetilde{\Omega}_{N+1,K}} \sum_{k=1}^{N+1} \Psi_{ki} \Omega_{jk} (\theta_{kK}^j - 1)\Omega_{jK}$$

$$\Phi_j(i, N+1) = \Phi_j(\Psi_{(:,i)}, \widetilde{\mathbf{\Omega}}_K) + \Phi_j(\Psi_{(:,i)}, \mathbf{\Omega}_K)$$

Hence, we have

$$\sum_{k=1}^{N+1} \Psi_{ki} \sum_{j=1}^{N+1} \Omega_{jk} \lambda_j \mathrm{d} \log \Omega_{jk} = \sum_{j=1}^{N+1} \lambda_j \Phi_j(i, N+1) \mathrm{d} \log P_{N+1} \tag{C.14}$$

where $\Phi_j(i, N+1)$ is a version of the *Input-Substitution Operator* defined by Baqaee and Farhi (2019b) applied to our small open economy environment. This operator captures how, in response to a change in the commodity price, producer j substitutes away/towards producer i both directly and indirectly through input-output linkages.

Therefore, changes in the Domar weights can be written as

$$d \log \lambda_i = \frac{1}{\lambda_i} \left(\sum_{j=1}^{N+1} \lambda_j \Phi_j(i, N+1) \mathrm{d} \log P_{N+1} \sum_{k=1}^{N+1} \Psi_{ki} \left(b_k d \log E + \delta_{k,N+1} \frac{\bar{Y}}{GDP} d \log \bar{Y} \right) \right)$$
$$- d \log GDP, \tag{C.15}$$

which completes the proof. ■

Proof of Proposition 7. To prove this result, we rearrange the result in Proposition 6 to get

$$d \log Q_i = d \log \lambda_i + d \log GDP - d \log P_i$$
$$= \frac{1}{\lambda_i} \left(\sum_{j=1}^{N+1} \lambda_j \Phi_j(i, N+1) \mathrm{d} \log P_{N+1} + \sum_{k=1}^{N+1} \Psi_{ki} \left(b_k d \log E + \delta_{k,N+1} \frac{\bar{Y}}{GDP} d \log \bar{Y} \right) \right) - d \log P_i$$

Using Proposition 5, we can rewrite this as

$$d\log Q_i = \left(\sum_{j=1}^{N+1}\frac{\lambda_j}{\lambda_i}\Phi_j(\Psi_{(:,i)},\Psi_{(:,N+1)}) - \frac{\widetilde{\Omega}_{i,K}}{\widetilde{\Omega}_{N+1,K}}\right)\mathrm{d}\log P_{N+1} + \sum_{k=1}^{N+1}\frac{\Psi_{ki}}{\lambda_i}b_k\mathrm{d}\log E + \sum_{k=1}^{N+1}\frac{\Psi_{ki}}{\lambda_i}\frac{\bar{Y}}{GDP}\delta_{k,N+1}d\log\bar{Y}$$

We are left to link $\mathrm{d}\log E$ and $\mathrm{d}\log\bar{Y}$ to $\mathrm{d}\log P_{N+1}$. For total expenditure, recall the following two equations

$$\mathrm{d}\log GDP = \frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}}\mathrm{d}\log P_{N+1},$$

$$E = GDP - \bar{Y}.$$

To proceed, we assume $\bar{Y}$ is a positive function of the commodity price. In particular,

$$\log\bar{Y} = \phi\log P_{N+1}.$$

Log-differentiating the expression for expenditure and using the above relationships, we get

$$\mathrm{d}\log E = \frac{GDP}{E}\mathrm{d}\log GDP - \frac{\bar{Y}}{E}\mathrm{d}\log\bar{Y} = \left(\frac{GDP}{E}\frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}} - \frac{\bar{Y}}{E}\phi\right)\mathrm{d}\log P_{N+1}$$

Hence, changes in domestic expenditure satisfies

$$\mathrm{d}\log E = \left(\frac{GDP}{E}\frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}} - \frac{\bar{Y}}{E}\phi\right)\mathrm{d}\log P_{N+1}$$

Replacing the expression for domestic expenditure and net transfers to the rest of the world as a function of the commodity price shock, we arrive at

$$\frac{\mathrm{d}\log Q_i}{\mathrm{d}\log P_{N+1}} = \underbrace{\sum_{j=1}^{N+1}\frac{\lambda_j}{\lambda_i}\Phi_j(i,N+1)}_{\text{Input-Output Substitution}} + \underbrace{\frac{\tilde{b}_i}{\lambda_i}\left(\frac{GDP}{E}\frac{\Lambda_K}{\widetilde{\Omega}_{N+1,K}} - \frac{\bar{Y}}{E}\phi\right)}_{\text{Domestic Demand}} + \underbrace{\frac{\Psi_{N+1,i}}{\lambda_i}\frac{\bar{Y}}{GDP}\phi}_{\text{Foreign Demand}} - \underbrace{\frac{\widetilde{\Omega}_{i,K}}{\widetilde{\Omega}_{N+1,K}}}_{\text{Cost Push}} \tag{C.16}$$

where

$$\tilde{b}_i = \sum_{k=1}^{N+1} \Psi_{ki} b_k,$$

$$\tilde{b}_i + \Psi_{N+1,i} = \lambda_i, \quad \text{for all } i = 1, 2, ..., N+1$$

This completes the proof. ■

Proposition 8. Changes in real GDP ($rGDP$) in the model satisfies

$$\mathrm{d}\log rGDP = \sum_{i=1}^{N+1} \lambda_i \mathrm{d}\log Z_i + \Lambda_L \mathrm{d}\log \bar{L} + \Lambda_K \mathrm{d}\log \bar{K},$$

and are thus independent of commodity price changes.

Proof of Proposition 8.

Consider the definition of nominal GDP. In our particular model,

$$GDP = \sum_{i=1}^{N+1} P_i Q_i - \sum_{i=1}^{N+1} \sum_{j=1}^{N+1} P_j M_{ij}.$$

Changes in nominal GDP thus satisfies

$$\mathrm{d}\log GDP = \underbrace{\sum_{i=1}^{N+1} \frac{P_i Q_i}{GDP} \left[\mathrm{d}\log Q_i - \sum_{j=1}^{N+1} \frac{P_j M_{ij}}{P_i Q_i} \mathrm{d}\log M_{ij} \right]}_{\text{Real GDP changes}} + \underbrace{\sum_{i=1}^{N+1} \frac{P_i Q_i}{GDP} \left[\mathrm{d}\log P_i - \sum_{j=1}^{N+1} \frac{P_j M_{ij}}{P_i Q_i} \mathrm{d}\log P_j \right]}_{\text{GDP deflator changes}}$$

$$\mathrm{d}\log GDP - \sum_{i=1}^{N+1} \lambda_i \left[\mathrm{d}\log P_i - \sum_{j=1}^{N+1} \Omega_{ij} \mathrm{d}\log P_j \right] = \sum_{i=1}^{N+1} \lambda_i \left[\mathrm{d}\log Q_i - \sum_{j=1}^{N+1} \Omega_{ij} \mathrm{d}\log M_{ij} \right] = \mathrm{d}\log rGDP,$$

where the last line defines $\lambda_i = P_i Q_i / GDP$ and $\Omega_{ij} = P_j M_{ij} / P_i Q_i$.

Consider the left-hand side

$$\mathrm{d}\log GDP = \Lambda_L(\mathrm{d}\log W + \mathrm{d}\log \bar{L}) + \Lambda_K(\mathrm{d}\log R + \mathrm{d}\log \bar{K}),$$

$$\mathrm{d}\log P_i - \sum_{j=1}^{N+1} \Omega_{ij}\mathrm{d}\log P_j = -\mathrm{d}\log Z_i + \Omega_{iL}\mathrm{d}\log W + \Omega_{iK}\mathrm{d}\log R \text{ for all } i = 1, 2, ..., N+1,$$

where the first line is just the definition of nominal GDP as factor payments, and the second line follows from Shephard's lemma. Moreover, following the main text, we define $\Lambda_L = W\bar{L}/GDP$, $\Lambda_K = R\bar{K}/GDP$, $\Omega_{iL} = WL_i/P_iQ_i$ and $\Omega_{iK} = RK_i/P_iQ_i$.

Combining both results, we have that

$$\begin{aligned}\mathrm{d}\log GDP - \sum_{i=1}^{N+1} \lambda_i \left[\mathrm{d}\log P_i - \sum_{j=1}^{N+1} \Omega_{ij}\mathrm{d}\log P_j\right] &= \Lambda_L(\mathrm{d}\log W + \mathrm{d}\log \bar{L}) + \Lambda_K(\mathrm{d}\log R + \mathrm{d}\log \bar{K}) \\ &\quad - \sum_{i=1}^{N+1} \lambda_i \left[-\mathrm{d}\log Z_i + \Omega_{iL}\mathrm{d}\log W + \Omega_{iK}\mathrm{d}\log R\right] \\ &= \sum_{i=1}^{N+1} \lambda_i \mathrm{d}\log Z_i + \Lambda_L \mathrm{d}\log \bar{L} + \Lambda_K \mathrm{d}\log \bar{K}\end{aligned}$$

And so changes in real GDP ($rGDP$)

$$\mathrm{d}\log rGDP = \sum_{i=1}^{N+1} \lambda_i \mathrm{d}\log Z_i + \Lambda_L \mathrm{d}\log \bar{L} + \Lambda_K \mathrm{d}\log \bar{K},$$

are independent of the commodity price changes. Since we assume for our exercise that $\mathrm{d}\log Z_i = 0$ for all $i = 1, 2, ..., N+1$ and $\mathrm{d}\log \bar{L} = \mathrm{d}\log \bar{K} = 0$, then $\mathrm{d}\log rGDP = 0$. This completes the proof. ■

C.4 Model Structure

We assume that each producer possesses a nested CES structure.

C.4.1 Non-Tradable Producers

Each non-tradable producer $j = 1, 2, ..., N$ combines intermediate inputs and labor to produce gross output, Q_j, according to the following production function

$$Q_j = Z_j \left(a_j^{\frac{1}{\sigma_j}} L_j^{\frac{\sigma_j - 1}{\sigma_j}} + (1 - a_j)^{\frac{1}{\sigma_j}} M_j^{\frac{\sigma_j - 1}{\sigma_j}} \right)^{\frac{\sigma_j}{\sigma_j - 1}}$$

where Z_j is the productivity level, L_j is labor demand, M_j is an intermediate input bundle, and σ_j is the elasticity of substitution between labor and the intermediate input bundle. a_j is the share of labor in total sales of producer i.

The intermediate input bundle combines intermediate inputs from each producer j, such that

$$M_j = \left(\sum_{k=1}^{N+1} \omega_{jk}^{\frac{1}{\varepsilon_j}} M_{jk}^{\frac{\varepsilon_j - 1}{\varepsilon_j}} \right)^{\frac{\varepsilon_j}{\varepsilon_j - 1}}$$

where ω_{jk} is the share of intermediate input i on the cost of the intermediate input bundle and M_{jk} is the demand by producer j from producer k. ε_j is the elasticity of substitution among intermediate inputs by producer j.

Cost minimization implies the following conditional demands, marginal costs, and inter-

mediate input price index

$$L_j = a_j \left(\frac{W}{MC_j}\right)^{-\sigma_j} Z_j^{\sigma_j - 1} Q_j$$

$$M_j = (1 - a_j) \left(\frac{P_j^M}{MC_j}\right)^{-\sigma_j} Z_j^{\sigma_j - 1} Q_j$$

$$M_{jk} = \omega_{jk} \left(\frac{P_k}{P_j^M}\right)^{-\varepsilon_j} M_j = \omega_{jk}(1 - a_j) P_j^{-\varepsilon_j} (P_j^M)^{\varepsilon_j - \sigma_j} MC_j^{\sigma_j} Z_j^{\sigma_j - 1} Q_j$$

$$MC_j = Z_j \left(a_j W^{1-\sigma_i} + (1 - a_j)(P_j^M)^{1-\sigma_i}\right)^{\frac{1}{1-\sigma_i}}$$

$$P_j^M = \left(\sum_{k=1}^{N+1} \omega_{jk} P_k^{1-\varepsilon_j}\right)^{\frac{1}{1-\varepsilon_j}}$$

C.4.2 Commodity Sector

The only difference between the commodity sector ($j = N+1$) and the rest of the sectors is that it also uses capital. Otherwise, the structure is the same. In particular, it produces according to the following production function

$$Q_j = Z_j \left(a_j^{\frac{1}{\sigma_j}} V_j^{\frac{\sigma_j - 1}{\sigma_j}} + (1 - a_j)^{\frac{1}{\sigma_j}} M_j^{\frac{\sigma_j - 1}{\sigma_j}}\right)^{\frac{\sigma_j}{\sigma_j - 1}}$$

where the intermediate input bundle is the same as before. This commodity sector also combines labor and capital according to the following aggregator

$$V_j = \left(d_j^{\frac{1}{\nu_j}} L_j^{\frac{\nu_j - 1}{\nu_j}} + (1 - d_j)^{\frac{1}{\nu_j}} K_j^{\frac{\nu_j - 1}{\nu_j}}\right)^{\frac{\nu_j}{\nu_j - 1}}$$

Here, K_j is capital demand, and ν_j is the elasticity of substitution between labor and capital for the commodity sector.

As before, conditional demands, marginal costs, and price indices satisfy

$$
\begin{aligned}
L_j &= d_j \left(\frac{W}{P_j^V}\right)^{-\nu_j} V_j \\
K_j &= d_j \left(\frac{W}{P_j^V}\right)^{-\nu_j} V_j \\
V_j &= a_j \left(\frac{P_j^V}{MC_j}\right)^{-\sigma_j} Z_j^{\sigma_j - 1} Q_j \\
M_j &= (1 - a_j) \left(\frac{P_j^M}{MC_j}\right)^{-\sigma_j} Z_j^{\sigma_j - 1} Q_j \\
M_{jk} &= \omega_{jk} \left(\frac{P_k}{P_j^M}\right)^{-\varepsilon_j} M_j = \omega_{jk}(1 - a_j) P_j^{-\varepsilon_j} (P_j^M)^{\varepsilon_j - \sigma_j} MC_j^{\sigma_j} Z_j^{\sigma_j - 1} Q_j \\
MC_j &= Z_j \left(a_j (P_j^V)^{1-\sigma_i} + (1 - a_j)(P_j^M)^{1-\sigma_i}\right)^{\frac{1}{1-\sigma_i}} \\
P_j^M &= \left(\sum_{k=1}^{N+1} \omega_{jk} P_k^{1-\varepsilon_j}\right)^{\frac{1}{1-\varepsilon_j}} \\
P_j^V &= \left(d_j W^{1-\nu_i} + (1 - d_j) R^{1-\nu_i}\right)^{\frac{1}{1-\nu_i}}
\end{aligned}
$$

C.4.3 Allen-Uzawa Elasticities of Substitution

From these conditional demands, we can compute each producer's Allen-Uzawa elasticities of substitution between any two pairs of inputs, (k, h), θ_{kh}^j, as follows

$$
\begin{aligned}
M_{jk} &= P_k^{-\varepsilon_j} (P_j^M)^{\varepsilon_j - \sigma_j} MC_j^{\sigma_j} Z_j^{\sigma_j - 1} Q_j \\
\frac{\partial \log M_{jk}}{\partial \log P_h} &= -\varepsilon_j \delta_{kh} + (\varepsilon_j - \sigma_j) \frac{\partial \log P_j^M}{\partial \log P_h} + \sigma_j \frac{\partial \log MC_j}{\partial \log P_h} \\
\frac{\partial \log M_{jk}}{\partial \log P_h} &= -\varepsilon_j \delta_{kh} + (\varepsilon_j - \sigma_j) \underbrace{\frac{P_h M_{jh}}{P_j^M M_j}}_{=\Omega_{jh}^M} + \sigma_j \underbrace{\frac{P_h M_{jh}}{TC_j}}_{=\Omega_{jh}} \\
\theta_{kh}^j = \frac{\frac{\partial \log M_{jk}}{\partial \log P_h}}{\Omega_{jh}} &= -\frac{\varepsilon_j \delta_{kh}}{\Omega_{jh}} + (\varepsilon_j - \sigma_j) \frac{\Omega_{jh}^M}{\Omega_{jh}} + \sigma_j
\end{aligned}
$$

where δ_{kh} is the Kronecker delta that equal 1 if $k = h$, and 0, otherwise. $h = 1, 2, ..., N + 1 \cup \{L, K\}$. Hence, P_h includes both good and factor prices to save on notation.

An object of interest is $\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh}$ because this enters directly in the definition of the input-output substitution operator. We can write this term compactly using the above expression for the Allen-Uzawa elasticity of substitution

$$\delta_{kh} + (\theta^j_{kh} - 1)\Omega_{jh} = (\varepsilon_j - 1)\left(\Omega^M_{jh} - \delta_{kh}\right) + (\sigma_j - 1)\left(\Omega_{jh} - \Omega^M_{jh}\right)$$

Table C.5: Network Effects of Commodity Price Changes on Non-Commodity Sectors Without Russia

	Panel (a): Quantity			Panel (b): Prices		
	(1)	(2)	(3)	(4)	(5)	(6)
Upstream$_{ict}$	0.0055	0.0071**	0.0081***	-0.0060	0.0062	0.0026
	(0.0034)	(0.0033)	(0.0024)	(0.0067)	(0.0060)	(0.0022)
Upstream$_{ict-1}$	0.0043	0.0071*	0.0054**	-0.0210	-0.0034	-0.0008
	(0.0038)	(0.0040)	(0.0022)	(0.0153)	(0.0055)	(0.0018)
Downstream$_{ict}$	0.0019	0.0015	-0.0005	0.0034	0.0071**	0.0070***
	(0.0016)	(0.0015)	(0.0013)	(0.0052)	(0.0036)	(0.0025)
Downstream$_{ict-1}$	-0.0014	-0.0015	-0.0023**	0.0042	0.0073**	0.0107***
	(0.0015)	(0.0014)	(0.0012)	(0.0056)	(0.0033)	(0.0023)
Accumulated Upstream	0.0098	0.0142**	0.0135***	-0.0269	0.0028	0.0018
	(0.0052)	(0.0053)	(0.0036)	(0.0185)	(0.0060)	(0.0032)
Accumulated Downstream	0.0005	0.0001	-0.0028	0.0075	0.0144**	0.0177***
	(0.0023)	(0.0024)	(0.0020)	(0.0102)	(0.0049)	(0.0036)
Observations	3472	3472	3472	3472	3472	3472
Within R^2	0.932	0.800	0.789	0.958	0.702	0.720
Year F.E.	Yes	Yes	Yes	Yes	Yes	Yes
Country × Sector F.E.		Yes	Yes		Yes	Yes
Country × Year F.E.			Yes			Yes

Note: This table presents OLS regressions using sectoral log quantity (columns 1 to 3) and log price index (columns 4 to 6) as the dependent variable. The independent variables also include one lag of the dependent variable. Double clustered country-year standard errors in parentheses. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

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