

KINETIC PARAMETERS OF THE UNIVERSITY OF MARYLAND REACTOR
BY THE INTERVAL-DISTRIBUTION METHOD

by
Simon Christian Simonson, III

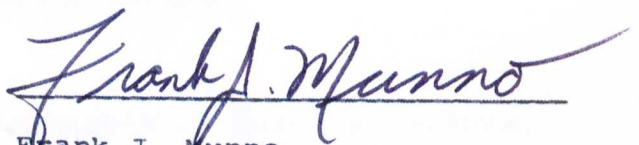
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Title of Thesis: Kinetic Parameters of the University of
Maryland Reactor by the Interval-Distri-
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Name of Candidate: Simon Christian Simonson, III
Master of Science, 1976

Thesis and Abstract Approved:



Frank J. Munno
Professor and Acting Director
Nuclear Engineering Program

Date Approved:

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ABSTRACT

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Thesis directed by: Frank J. Munno, Professor and Acting Director, Nuclear Engineering Program

The Rossi alpha of the University of Maryland Reactor was measured at criticality and at shutdown by the Babala interval-distribution method. At criticality, $\alpha = 188.8 \pm 4.7 \text{ sec}^{-1}$, and at shutdown, $\alpha = 1026.6 \pm 4.1 \text{ sec}^{-1}$. The shutdown reactivity was found to be $\rho/\beta = -4.44 \pm 0.14$ dollars.

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TABLE OF CONTENTS

	Page
LIST OF TABLES	iv
LIST OF FIGURES	v
I. INTRODUCTION	1
II. THEORY	2
A. Reactor Kinetics Equation	
B. Interval-Distribution Equation	
III. EXPERIMENTS	9
A. Equipment	
B. Experiments	
C. Data Reduction	
IV. RESULTS AND DISCUSSION	31
A. Choice of Parameters	
B. The Critical Rossi Alpha	
C. The Shutdown Reactivity	
D. Detector Efficiency and Prompt Multiplication	
V. CONCLUSIONS	39
LIST OF REFERENCES	41

LIST OF TABLES

Table	Page
I. Summary of Experiment Conditions	14
II. Experimental Results	33
III. Solutions for α_c	35
IV. Values of Derived Parameters	37

LIST OF FIGURES

Figure	Page
1. Block Diagram of Equipment	12
2. Core Geometry and Detector Location	13
3. Detector Pulse-Height Spectra	15
4. Interval Distributions for Experiments	17

near the core. The distribution depends on the reactor kinetic parameters, particularly the prompt alpha. By fitting the probability distribution function to the data the kinetic parameters can be derived.

The advantages of the interval-distribution method include the ability to measure the kinetic parameters in many different conditions, both at shutdown and at power. The instrumentation is standard and available. However, the available instrumentation imposes a limit in the present case to very low power levels at criticality. Otherwise the count rate becomes too high of the detector may be damaged. However, the parameters at criticality can be determined from measurements at subcritical states. In addition, the interval-distribution method provides a good measure of the total negative reactivity.

In this thesis the interval-distribution method is applied to the measurement of the prompt alpha at criticality and shutdown, and related parameters are derived.

I. INTRODUCTION

The interval-distribution method of Babala (1967b) has many advantages for measuring the kinetic properties of the University of Maryland Reactor. The technique essentially consists in determining the distribution of the intervals between successive counts from a neutron detector placed near the core. The distribution depends on the reactor kinetics parameters, particularly the Rossi alpha. By fitting the probability distribution function to the data the kinetic parameters can be derived.

The advantages of the interval-distribution method include the ability to measure the kinetic parameters in many different conditions, both at shutdown and at power. The instrumentation is standard and available. However, the available instrumentation imposes a limit in the present case to very low power levels at criticality. Otherwise the count rate becomes too high or the detector may be damaged. However, the parameters at criticality may be determined from measurements at subcritical states. At shutdown, the interval-distribution method provides a good measure of the total negative reactivity.

In this thesis the interval-distribution method is applied to the measurement of the Rossi alpha at criticality and shutdown, and related parameters are derived.

II. THEORY

A. Reactor Kinetics Equation

Let us derive the reactor kinetics equation in the approximation of a single energy group and without spatial dependence (the point-reactor model).

Let ℓ = neutron lifetime. Then the probability that one neutron will be lost during a time interval Δt is

$$p_{\text{loss}} \Delta t = \frac{\Delta t}{\ell} . \quad (1)$$

Let k = prompt neutron multiplication constant, β = fraction of delayed neutrons. Then the probability of returning a neutron after a loss is

$$\begin{aligned} p_{\text{gain}} \Delta t &= k (1 - \beta) \Delta t p_{\text{loss}} \\ &= k (1 - \beta) \frac{\Delta t}{\ell} . \end{aligned} \quad (2)$$

If there are N neutrons in the system, the net change is

$$\Delta N = N k (1 - \beta) \frac{\Delta t}{\ell} - N \frac{\Delta t}{\ell} . \quad (3)$$

Taking the limit as $\Delta t \rightarrow 0$,

$$\frac{dN}{dt} = \frac{k (1 - \beta) - 1}{\ell} N = -\alpha N, \quad (4)$$

where the Rossi alpha is defined as

$$\begin{aligned}\alpha &= \frac{1 - k(1 - \beta)}{\ell} \\ &= \frac{\beta - \rho}{\Lambda},\end{aligned}\tag{5}$$

where

$$\begin{aligned}\Lambda &= \frac{\ell}{k} = \text{prompt-neutron generation time,} \\ \rho &= \frac{(k - 1)}{k} = \text{reactivity.}\end{aligned}$$

The parameter α describes the growth or decay of the prompt neutron population in the reactor, according to the solution of Equation (4):

$$N = N_0 e^{-\alpha t}.\tag{6}$$

B. Interval Distribution Equation

Since the parameter α governs the kinetic behavior of the prompt neutron population, the time between the arrival of successive pulses in a neutron detector placed in the reactor must be related to α . Let us consider the probability distribution of such counts.

The difference between a fission chain reaction and a random (Poissonian) source is that neutrons belonging to a chain occur close together, in clumps. Therefore both the number of short intervals and the number of long intervals are greater for the chain reactor than for the random source.

Let us first consider the probability of detecting one neutron and then a second belonging to the same chain (cf.

Uhrig, 1970, chapt. 3). Let F = fission rate. Then the probability of fission in dt_0 at time $t = t_0$ is

$$p(t_0) dt_0 = F dt_0. \quad (7)$$

The probability of detecting a neutron in time Δt_1 at $t = t_1$ due to fission at $t = t_0$, i.e., the first neutron in the chain, is

$$p(t_1) \Delta t_1 = \epsilon \nu v \Sigma_f e^{-\alpha(t_1-t_0)} \Delta t_1, \quad (8)$$

where

ϵ = detector efficiency,

ν = number of neutrons produced in a fission,

v = mean neutron velocity,

Σ_f = macroscopic fission cross section,

and $v \Sigma_f$ is the average fission rate per unit neutron density.

The probability of detecting a second neutron in the chain in Δt_2 at $t = t_2$ is

$$p(t_2) \Delta t_2 = \epsilon (\nu - 1) v \Sigma_f e^{-\alpha(t_2-t_0)} \Delta t_2, \quad (9)$$

where the number of neutrons has been reduced to $\nu - 1$ by the absorption of a neutron in the first detection.

The combined probability is given by

$$\begin{aligned} p(t_1, t_2) \Delta t_1 \Delta t_2 &= \int_{-\infty}^{t_1} p(t_1) \Delta t_1 p(t_2) \Delta t_2 p(t_0) dt_0 \\ &= \int_{-\infty}^{t_1} F \epsilon^2 \frac{1}{\nu (\nu - 1)} (\nu \Sigma_f)^2 e^{-\alpha(t_1+t_2-t_0)} \Delta t_1 \Delta t_2 dt_0 \end{aligned}$$

$$= F \epsilon^2 \overline{\nu(\nu-1)} \frac{(\nu \Sigma_f)^2}{2\alpha} e^{-\alpha(t_2-t_1)} \Delta t_1 \Delta t_2, \quad (10)$$

where $\overline{\nu(\nu-1)}$ indicates an average over the numbers of neutrons emitted per fission. If we substitute the expressions,

$$\ell = \frac{1}{\nu \Sigma_a} = \text{mean neutron lifetime,}$$

where

$$\Sigma_a = \text{macroscopic absorption cross section,}$$

and

$$k = \frac{\nu \Sigma_f}{\Sigma_a},$$

then

$$\begin{aligned} p_c(t_1, t_2) \Delta t_1 \Delta t_2 &= \\ &= F \epsilon^2 \frac{\overline{\nu(\nu-1)}}{2 \bar{\nu}^2} \frac{k^2 e^{-\alpha(t_2-t_1)}}{\ell(1-k)} \Delta t_1 \Delta t_2 \\ &= F \epsilon B e^{-\alpha(t_2-t_1)} \Delta t_1 \Delta t_2, \end{aligned} \quad (11)$$

in which we have substituted an expression for the correlated neutron detection rate,

$$B = \frac{\epsilon D k^2}{2 \alpha \ell^2},$$

where

$$D = \frac{\overline{\nu(\nu-1)}}{\bar{\nu}^2} = \text{Diven fission parameter.}$$

In ^{235}U , $D = 0.796$.

The next step is to determine the probability, $p_0(t)$, that no count will occur in a time interval t . The expression

for $p_0(t)$ has been derived by Pál (1962, 1963), by Zolotukhin and Mogil'ner (1963), and by Babala (1967a). These authors have assumed steady-state point-kinetic reactor theory with a single energy group and no delayed neutrons.

Of the three, Babala's derivation is the most accessible. He uses considerations similar to those above in the framework of probability generating functions to derive

$$p_0(t) dt = \exp \left\{ - \left[\frac{2 A t}{\gamma + 1} + \frac{A}{B} \ln \frac{(\gamma + 1)^2 - (\gamma - 1)^2 e^{-\alpha \gamma t}}{4 \gamma} \right] \right\}, \quad (12)$$

where

$A = F \epsilon$ = detector counting rate,

$\gamma = (1 + Y)^{1/2}$ = correlation parameter,

$Y = \frac{\epsilon D k^2}{(1 - k)^2}$ = relative variance of prompt neutrons.

Y is the difference between the variance-to-mean ratio of the prompt-neutron counts, $(\overline{c^2} - \bar{c}^2)/\bar{c}$, where c is the count rate, and the Poissonian variance-to-mean ratio, \bar{c}/\bar{c} (cf. Uhrig, 1970).

The pulse-to-pulse time interval distribution describes the probability, $p(t) dt$, that after a pulse at time $t = 0$, the next pulse arrives between t and $t + dt$. I.e., $p(t) dt$ is the conditional probability of no count in the interval 0 to t , $p_0(t)$, and the probability of a count in dt at t , $p_c(dt)$.

In Poissonian statistics, this probability distribution of time intervals between counts is given by

$$p(t) dt = p_0(t) p_c(dt) = e^{-At} A dt, \quad (13)$$

where A = count rate.

Babala and Ogrin (1967) point out that $p_0(t)$ represents a probability that after a time $t = 0$, chosen at random, the first count arrives at a time greater than t . The distribution function of the arrival time is therefore

$$F(t) = 1 - p_0(t). \quad (14)$$

The frequency distribution is then

$$f(t) = \frac{\partial F(t)}{\partial t} = -\frac{\partial p_0(t)}{\partial t}. \quad (15)$$

The quantity $f(t) dt$ is the interval distribution, giving the probability that after a time $t = 0$, chosen at random, the first count arrives in dt about t .

If the system is stationary, $f(t)$ is symmetrical. That is, the probability that a count is registered in dt at $t = 0$ and is followed by an empty interval of length t is also $f(t) dt$.

If A is the count rate, then $A dt$ is the probability of a count in dt . The conditional probability $p_{co}(t)$ of an empty interval of length t after a count at $t = 0$ is given by

$$p_{co}(t) = \frac{f(t) dt}{A dt}. \quad (16)$$

This is also the probability that an interval between two counts is greater than t . Then the distribution function

of the arrival time t for the first count after a count at $t = 0$ is

$$\Phi(t) = 1 - p_{CO}(t). \quad (17)$$

The corresponding frequency function, or count-to-count interval distribution, is

$$\varphi(t) = p(t) = \frac{\partial \Phi(t)}{\partial t} = -\frac{\partial p_{CO}(t)}{\partial t}. \quad (18)$$

Explicitly, this expression, which forms the basis of the present work, is

$$\begin{aligned} p(t) &= \frac{1}{A} \frac{\partial^2 p_O(t)}{\partial t^2} \\ &= p_O(t) \frac{4A [(\gamma+1) + (\gamma-1) e^{-\alpha\gamma t}]^2 + 16 B \gamma^2 e^{-\alpha\gamma t}}{[(\gamma+1)^2 - (\gamma-1)^2 e^{-\alpha\gamma t}]^2}. \end{aligned} \quad (19)$$

As discussed by Pacilio (1969), Equation (19) has several useful limiting forms. For very small values of the correlation parameter γ , i.e., $\gamma \simeq 1$,

$$p(t) \simeq e^{-At} (A + B e^{-\alpha t}). \quad (20)$$

This is the case for very low detection efficiency. For large values of the ratio A/B , i.e., at high power,

$$p(t) \simeq p_O(t) 4A \left[\frac{(\gamma+1) + (\gamma-1) e^{-\alpha\gamma t}}{(\gamma+1)^2 + (\gamma-1)^2 e^{-\alpha\gamma t}} \right]^2, \quad (21)$$

from which the reactor parameters may still be found.

III. EXPERIMENTS

A. Equipment

The University of Maryland Reactor is a swimming-pool type with a TRIGA core loaded with 20 percent ^{235}U -enriched uranium, rated at 250 kW. A horizontal through-tube passes 10 cm in front of the core in its central plane.

A 45-cm $^{10}\text{BF}_3$ proportional ionization counter (Wood Counter Laboratory No. G 20533) was placed in the through-tube next to the core. Its position was adjusted to give the maximum detection efficiency. It was operated at 2300 v.

The pulses from the BF_3 counter were amplified in a Tennelec No. TC 203 BLR linear amplifier at a gain of 2 K. They were then fed into a timing scaler (Tennelec No. TC 444). The discriminator was set at 1.00 - 1.10 v to eliminate the gamma background. The gamma discrimination is discussed later.

From the timing scaler the pulses were fed into a pulse generator (BNC Model 8010) external trigger input. The trigger output was set to approximately 4 v and fed to a counter-timer (Tennelec No. TC 545) in the counter mode to obtain the total number of counts detected in each experiment.

The positive output from the pulse generator was fed into a wave-train generator of local manufacture that actuated the store and reset functions of a multichannel analyzer (Northern Scientific, Inc., No. 710) with 1024 channels.

The channel advance of the multichannel analyzer was governed by a second pulse generator (BNC Model 8010) set at a pulse rate of approximately 50 kHz. Rate calibration was obtained by connecting the trigger output to the counter-timer before the start and after the end of each experiment. The positive pulse output was fed into a wave-train generator of local manufacture that was connected to the multichannel analyzer channel advance.

The length of the channel-advance wave train reduces the time each channel is open to receive a pulse. This blank time was measured at 3.54 μ sec with an oscilloscope.

The first channel is further reduced by a reset time of 2.7 μ sec. Also, since a pulse arrives at a random time, the time until the first clock pulse arrives after reset is also random. This requires a correction to the first channel counts. However, since the correction depends on the shape of the measured distribution curve, it was decided to ignore the first channel altogether.

The output from the multichannel analyzer, consisting of the counts in each channel, was recorded on magnetic tape with a Kennedy Incremental Tape Recorder No. 1600, which was connected to the multichannel analyzer through a

system coupler (Vidar No. 653-03) and a digital level converter (Fazi Industries, Inc.).

A block diagram of the experimental instrumentation is given in Figure 1. Figure 2 shows the geometry of the core and the location of the detector.

The experiment works this way: The channels advance in the multichannel analyzer until a pulse arrives from the detector. This pulse activates the store and reset functions. A count is stored in the current channel, and the channel sweep is reset to zero, from which it immediately begins again. The interval between two pulses is given by the number of channels (minus one) times the channel-advance period. If no pulse arrives before the channel advance passes the last channel, no. 1023, a count is stored in the passed channel, no. 0.

Discrimination against gamma rays is achieved in the following manner: The $B F_3$ proportional counter produces a pulse lasting on the order of a microsecond for each ionization event. Both gammas and neutrons produce ionization, the neutrons by the $^{10}B(n,\alpha)^7Li$ reaction, the gammas mainly by Compton electron scattering. The (n,α) reaction produces much greater ionization than that of a gamma. Hence the neutron pulses are larger, and one may discriminate against gammas by rejecting pulses smaller than a certain value.

In order to find this value, a pulse-height spectrum (PHS) was obtained with the discriminator on the timing

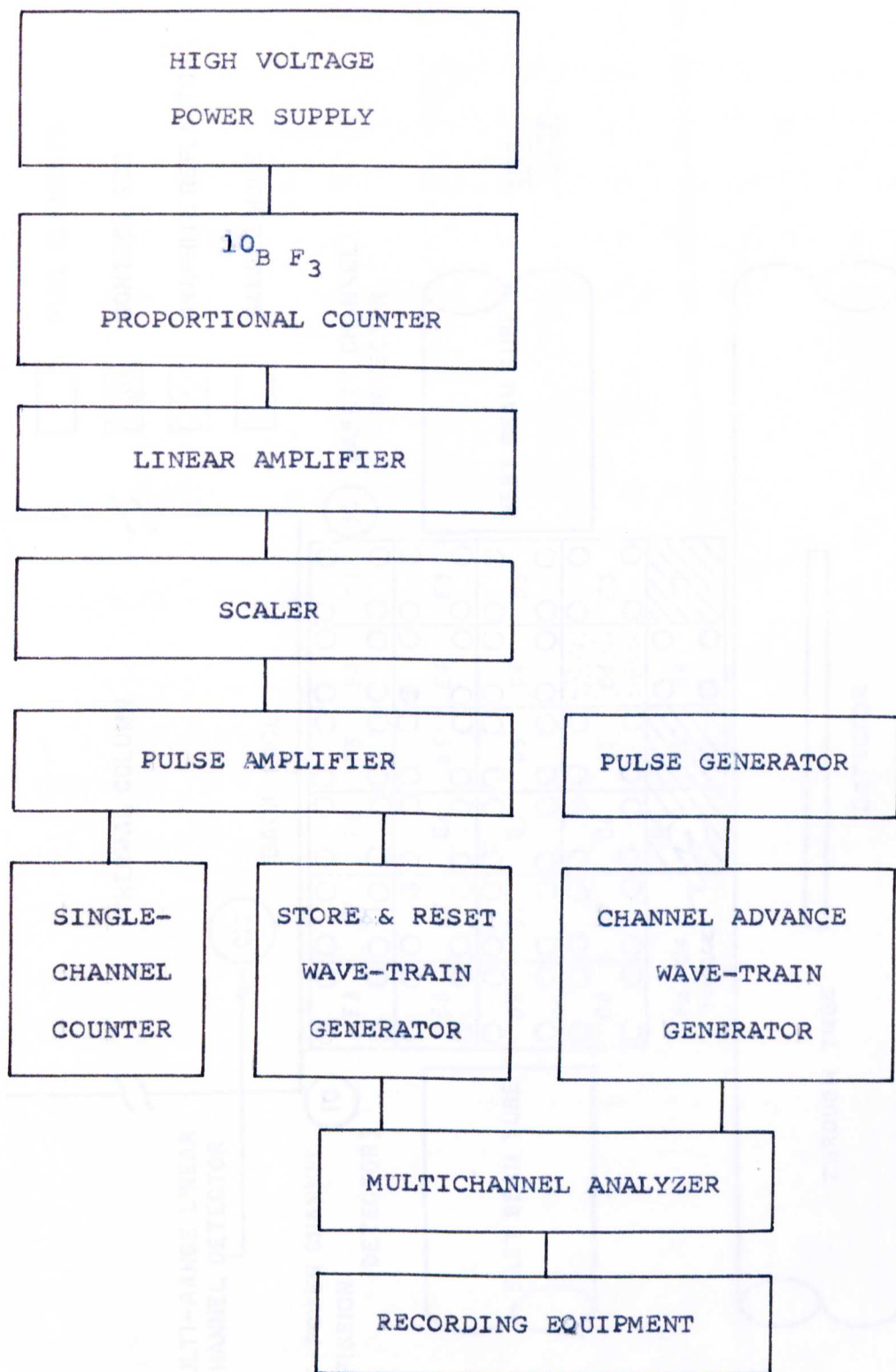


Fig. 1.--Block diagram of equipment.

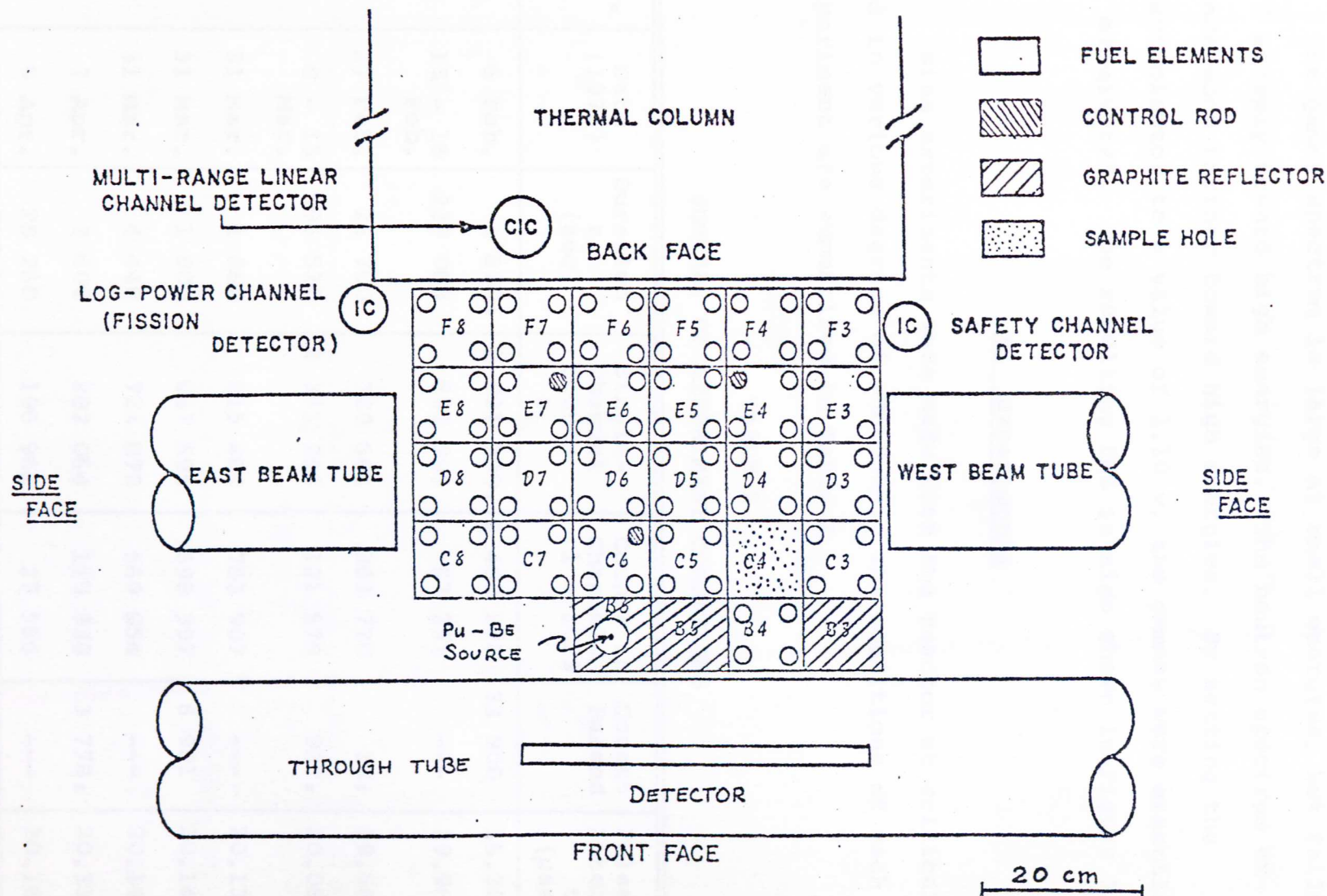


Fig. 2.--Core geometry and detector location.

scaler set to a low value of 0.5 v. This is shown in Figure 3. The gamma spectrum is large at small energies, but falls off steeply toward high energies. The neutron spectrum extends much farther toward high energies. By setting the discriminator to a value of 1.10 v, the gammas were essentially eliminated. The resulting PHS is also shown in Figure 3.

B. Experiments

Nine experiments were made with the reactor at critical and in various degrees of shutdown. The conditions of each experiment are summarized in Table I.

TABLE I
SUMMARY OF EXPERIMENT CONDITIONS

No.	Date (1976)	Duration t_t (sec)	Single- Channel Counts c_t	Counts in Channels 1 - 1023	Counts Passed	Pulse Interval t (μ sec)
1	6 Feb.	1 620:	1 931 239:	1 480 220	33 900	16.39
2	13 - 16 Feb.	239 063	692 995	67 291	---	19.98
3	27 Feb.	26 700	720 601	363 720	18:	19.98
4	8 - 15 Mar.	596 520	1 361 965	121 578	937:	20.06
5	31 Mar.	1 860	915 469	753 907	---	20.13
6	31 Mar.	1 800	847 595	698 397	6 901	20.14
7	31 Mar.	4 440	723 879	569 854	---	20.14
8	7 Apr.	3 600	282 064	179 338	3 778:	20.33
9	9 Apr.	28 260	196 961	27 556	---	20.16

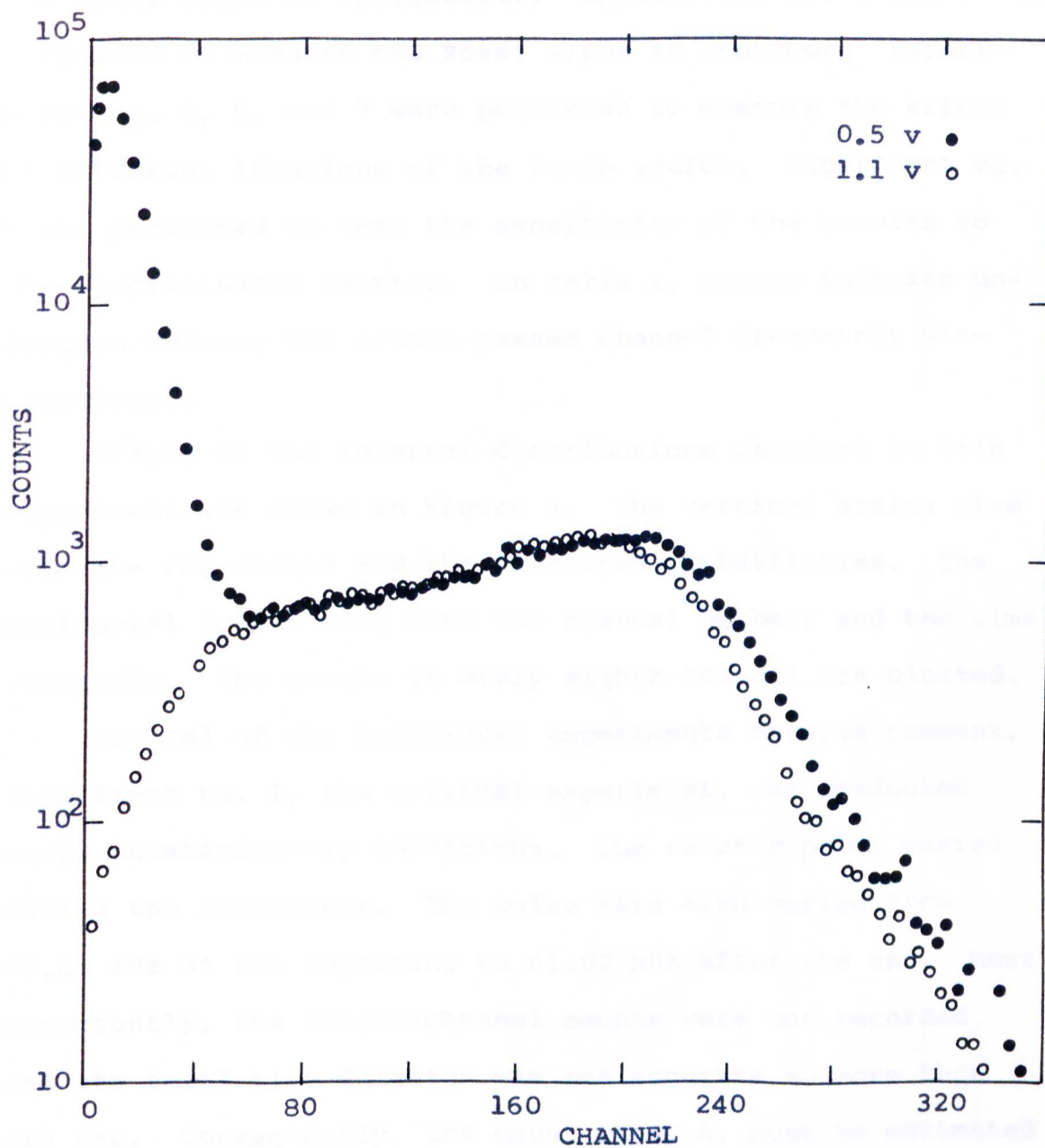


Fig. 2.--Detector pulse-height spectra at two discriminator settings.

Experiments no. 1, 3, and 8 were performed to measure the Rossi alpha at criticality. Experiments no. 2 and 4 were performed to measure the Rossi alpha at shutdown. Experiments no. 5, 6, and 7 were performed to measure the effect of different locations of the Pu-Be source. Experiment no. 9 was performed to test the sensitivity of the results to the discriminator setting. In Table I, colons indicate uncertain values; the counts-passed channel frequently malfunctioned.

Graphs of the interval distributions obtained in each experiment are shown in Figure 4. The vertical scales give both the raw counts and the corrected probabilities. The horizontal scales give both the channel numbers and the time intervals. The counts in every eighth channel are plotted.

Several of the individual experiments deserve comment. Experiment no. 1, the critical experiment, was conducted under unsatisfactory conditions. The reactor power varied during the experiment. The pulse rate also varied from 60.85 kHz at the beginning to 61.02 kHz after the end. Most importantly, the single-channel counts were not recorded, and the total time duration was not accurate to more than ± 15 sec. Consequently, the count rate, A , must be estimated from the multichannel analyzer data.

In experiment no. 2, the count rate varied from 3.3 sec^{-1} at the start to 2.83 sec^{-1} at the end. In experiment no. 3, the count rate at the start was estimated to be 30 sec^{-1} ; over the entire experiment it averaged 27.0 sec^{-1} .

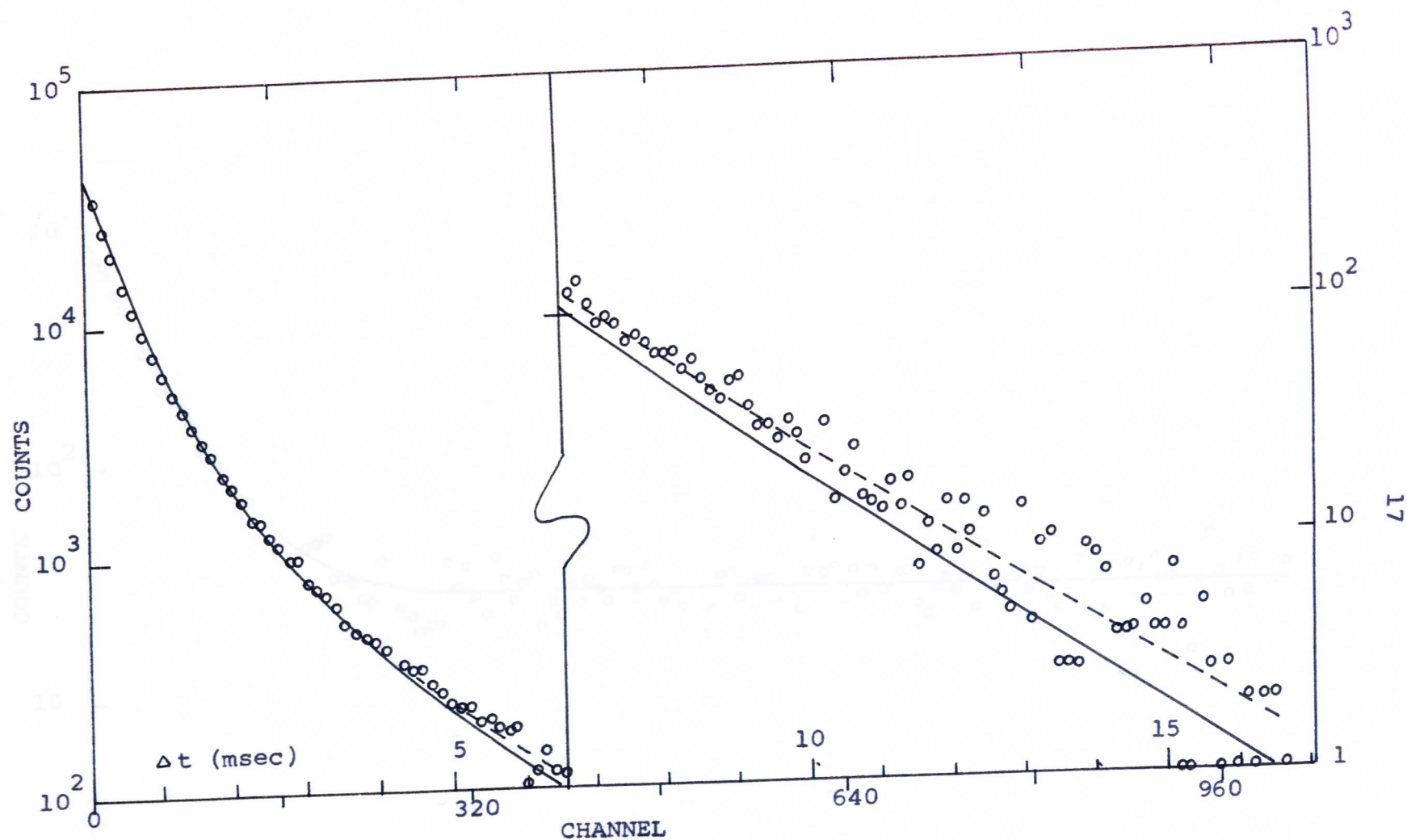


Fig. 4a.--Interval distribution for experiment no. 1. Two-parameter solid, three-parameter dashed.

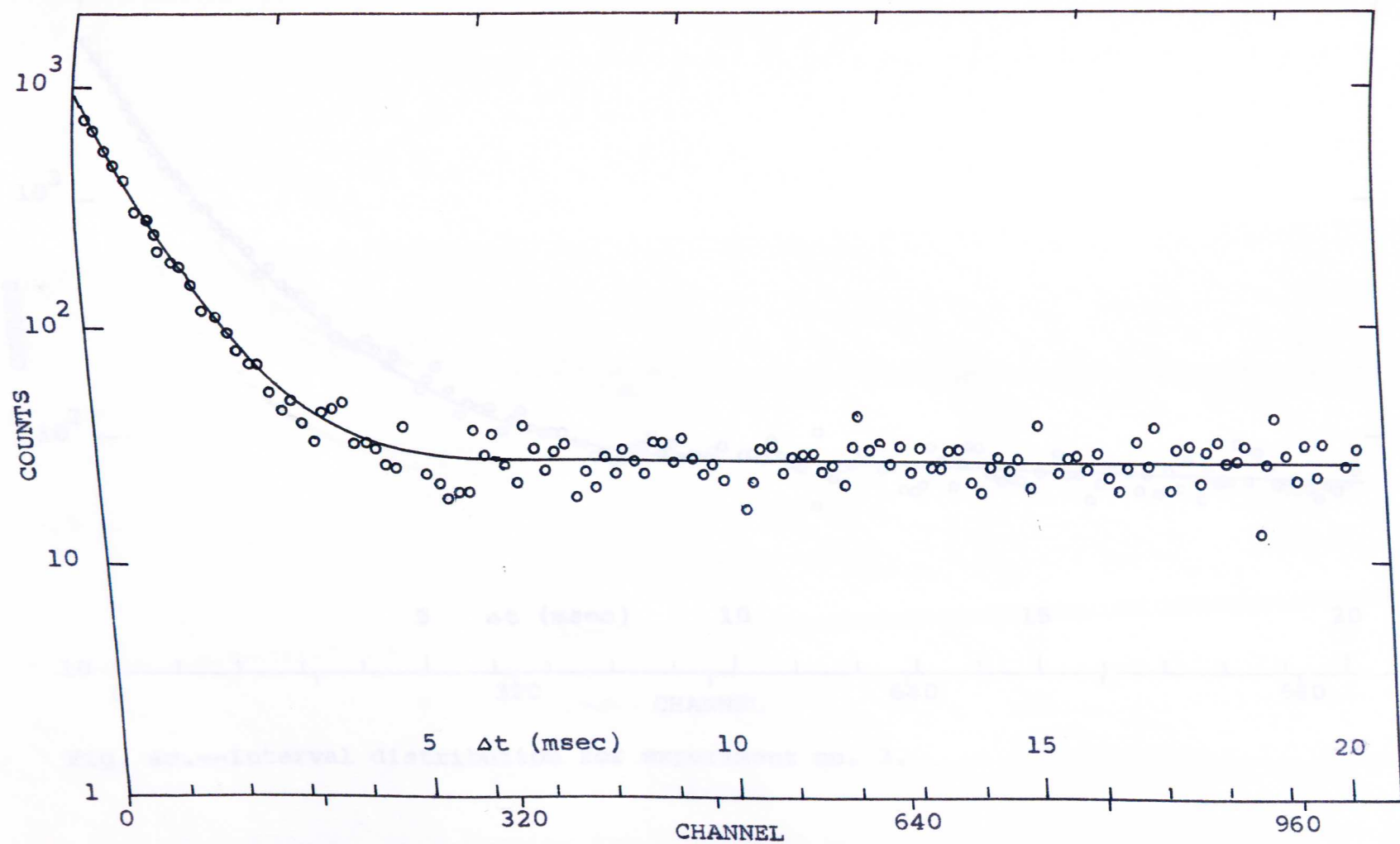


Fig. 4b.--Interval distribution for experiment no. 2.

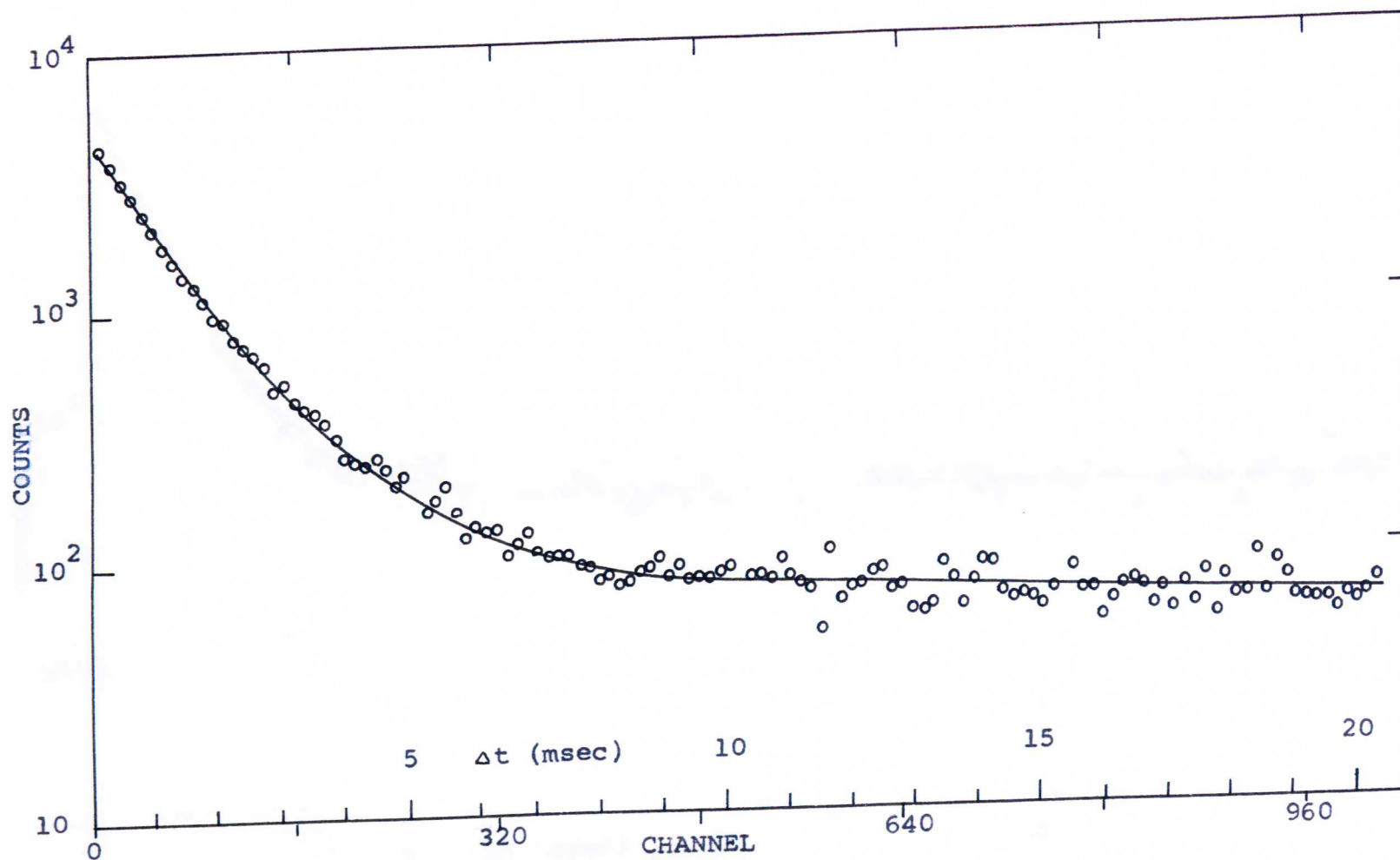


Fig. 4c.--Interval distribution for experiment no. 3.

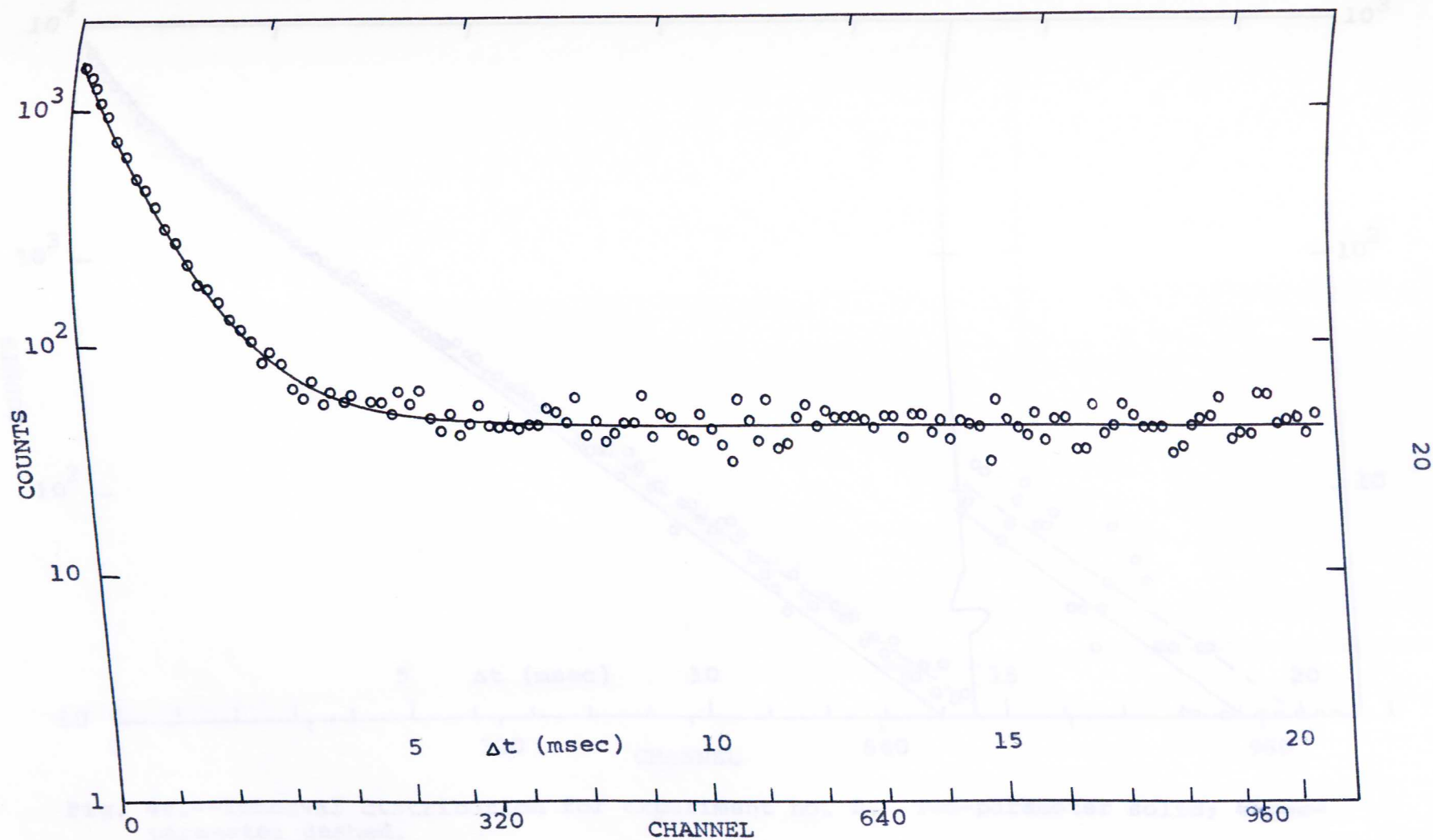


Fig. 4d. Interval distribution for experiment no. 4.

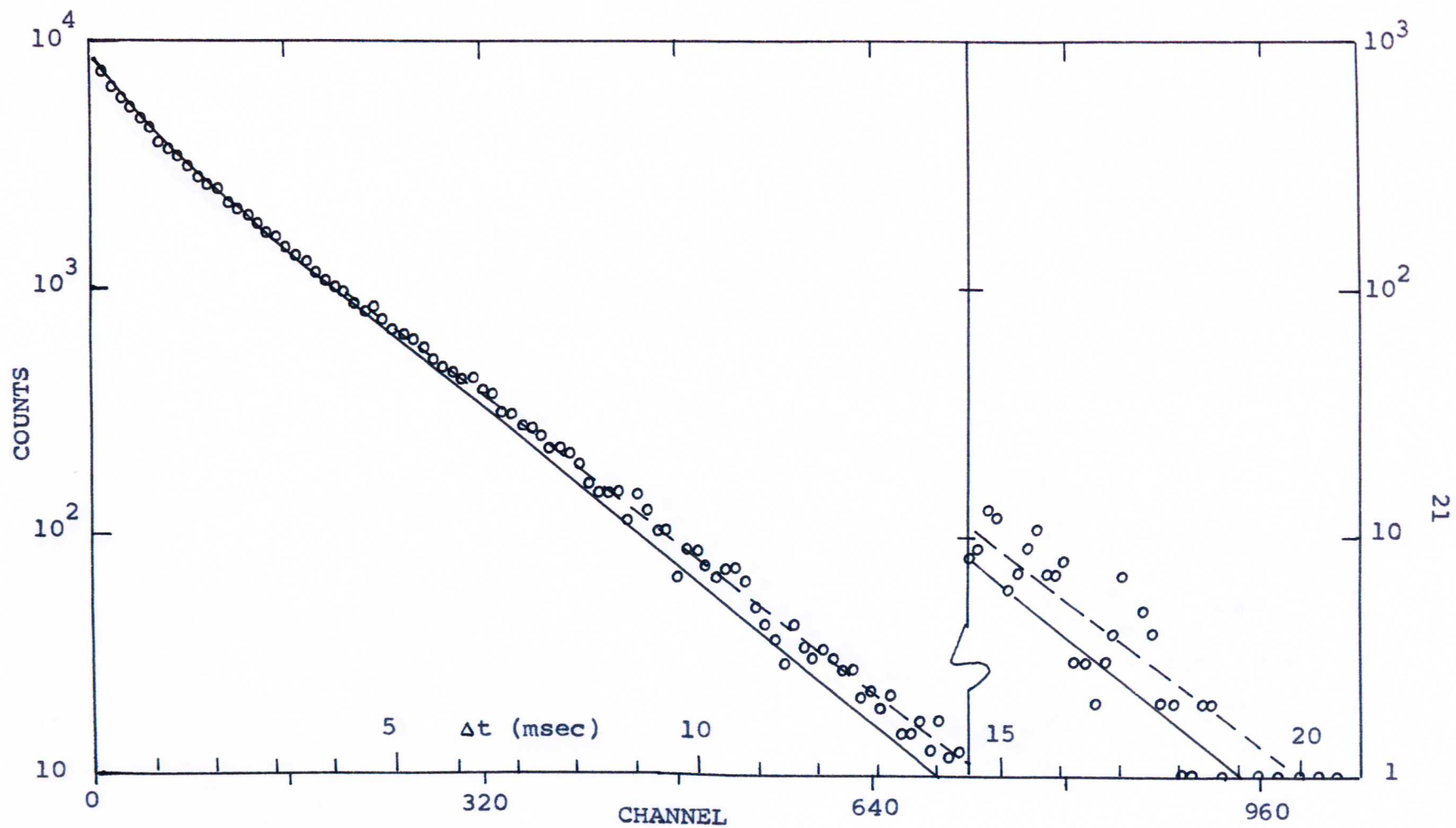


Fig. 4e.--Interval distribution for experiment no. 5. Two-parameter solid; three-parameter dashed.

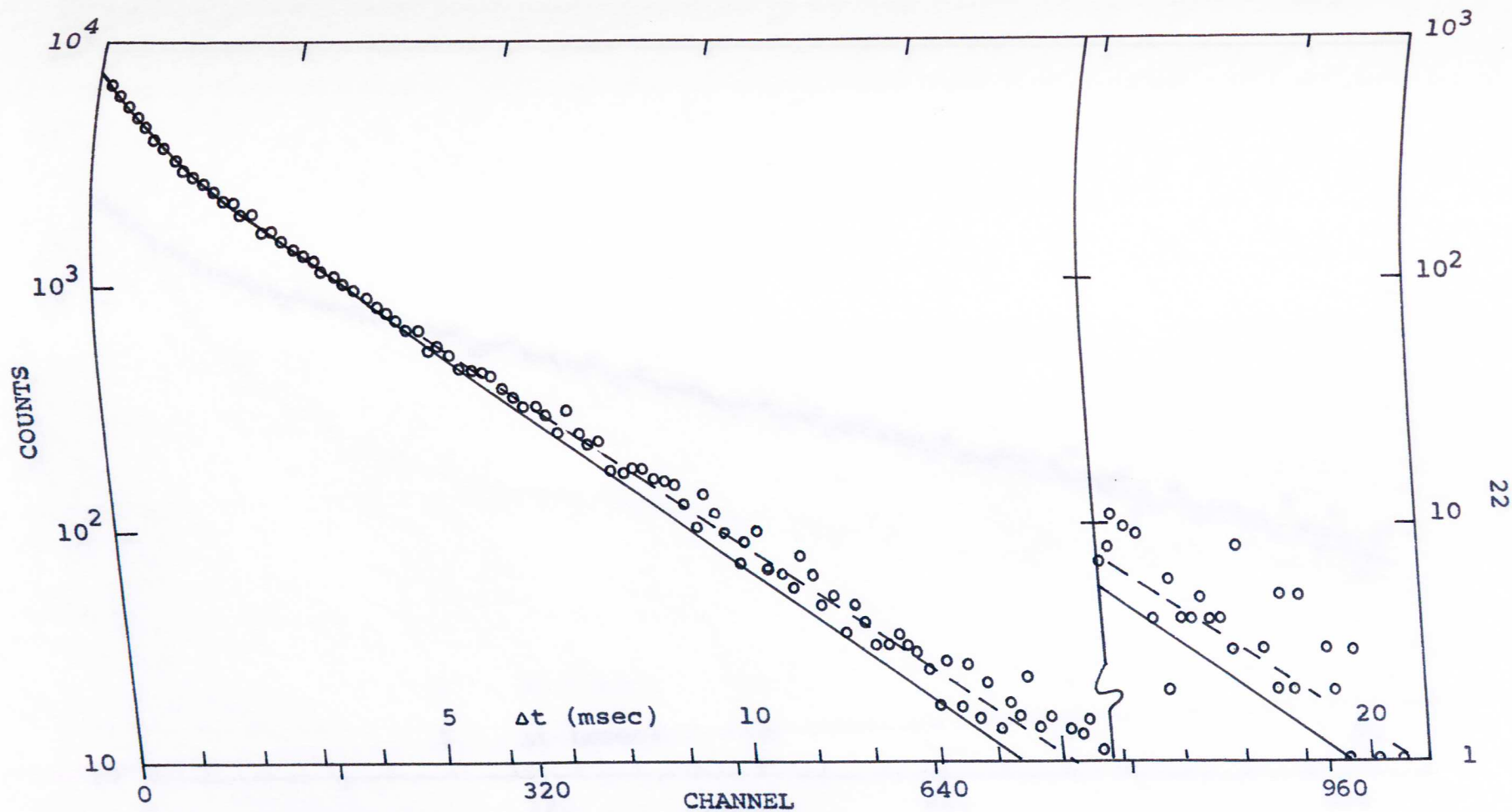


Fig. 4f.--Interval distribution for experiment no. 6. Two-parameter solid, three-parameter dashed.

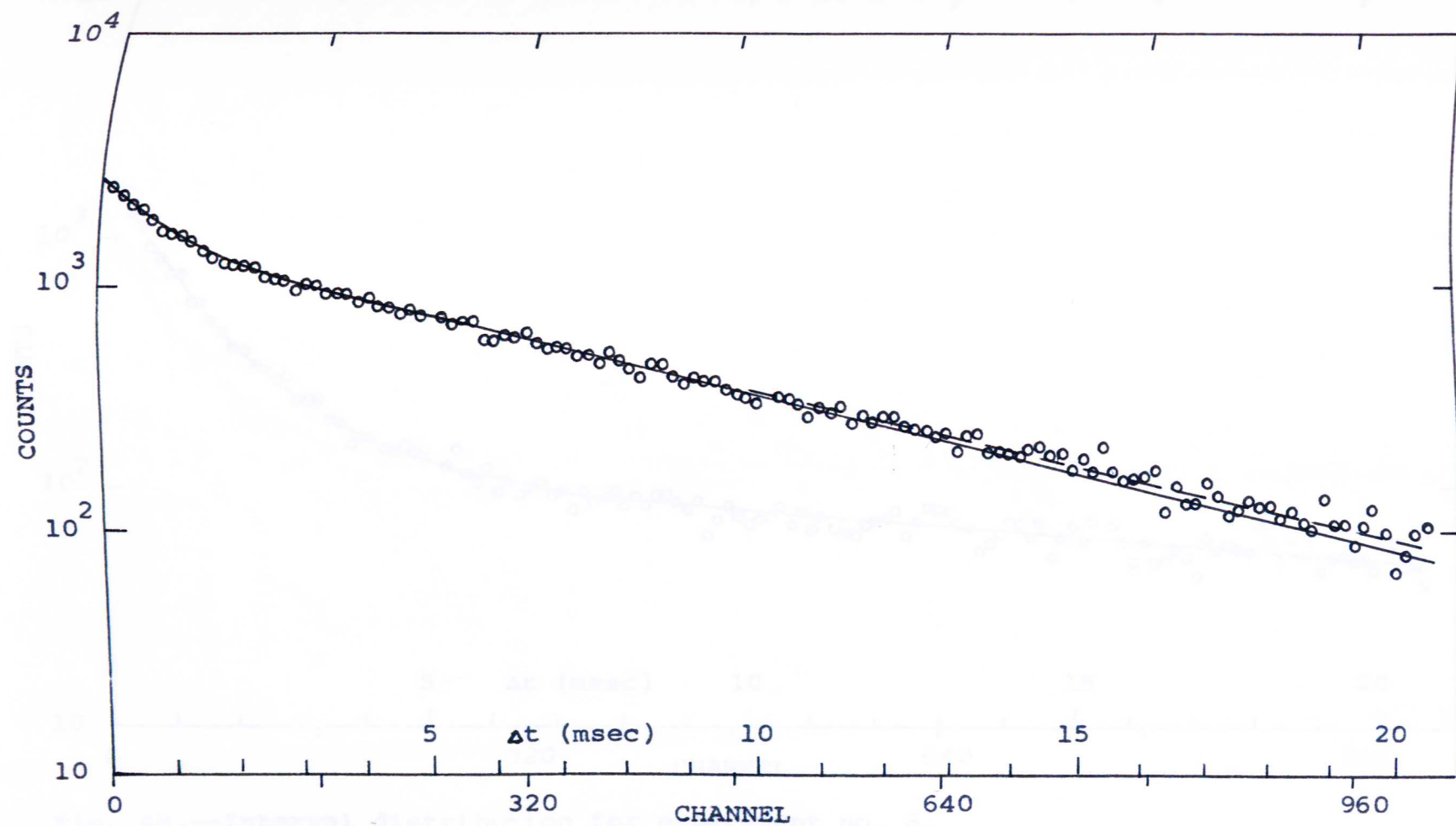


Fig. 4g.--Interval distribution for experiment no. 7. Two-parameter solid, three-parameter dashed.

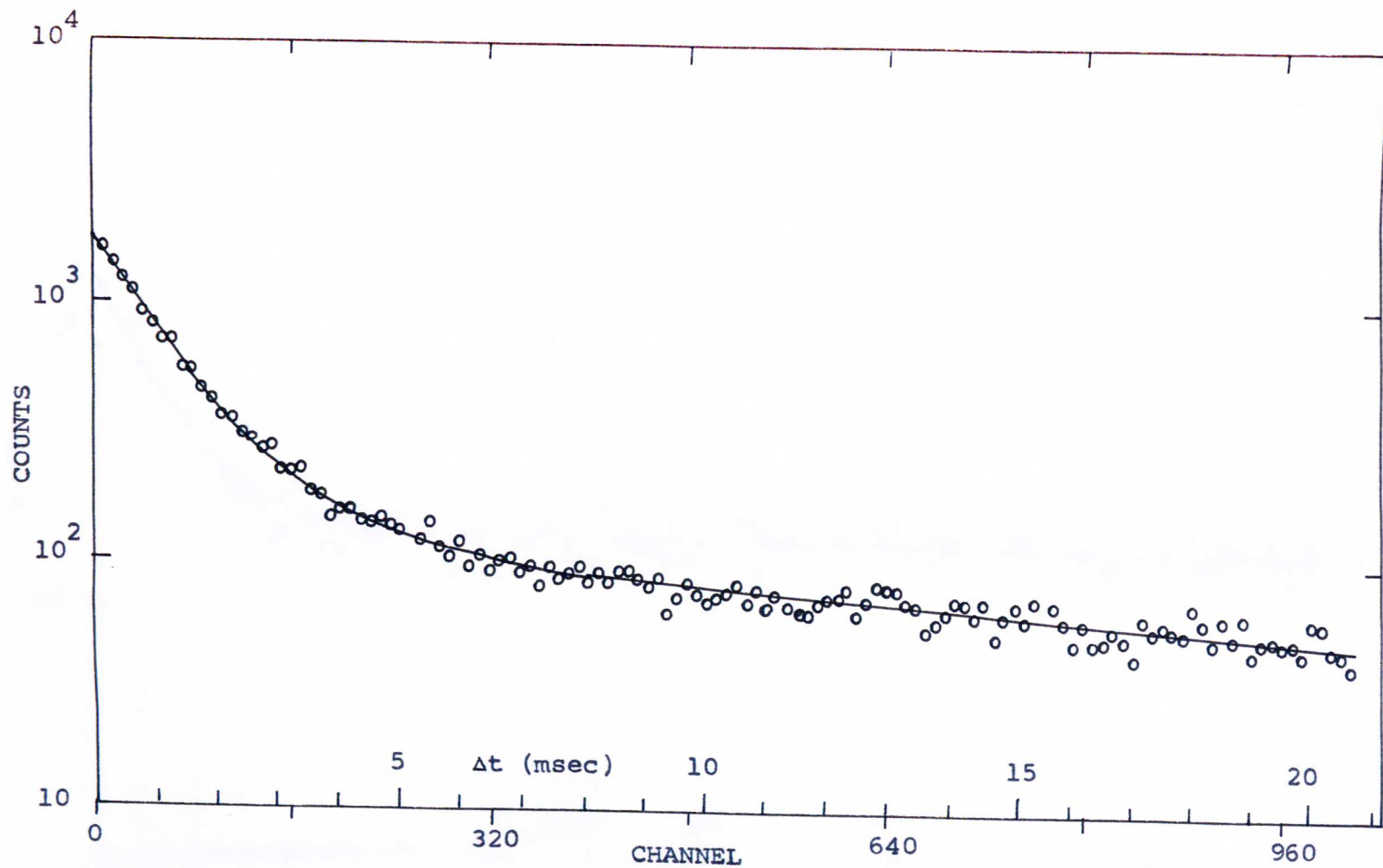


Fig. 4h.--Interval distribution for experiment no. 8.

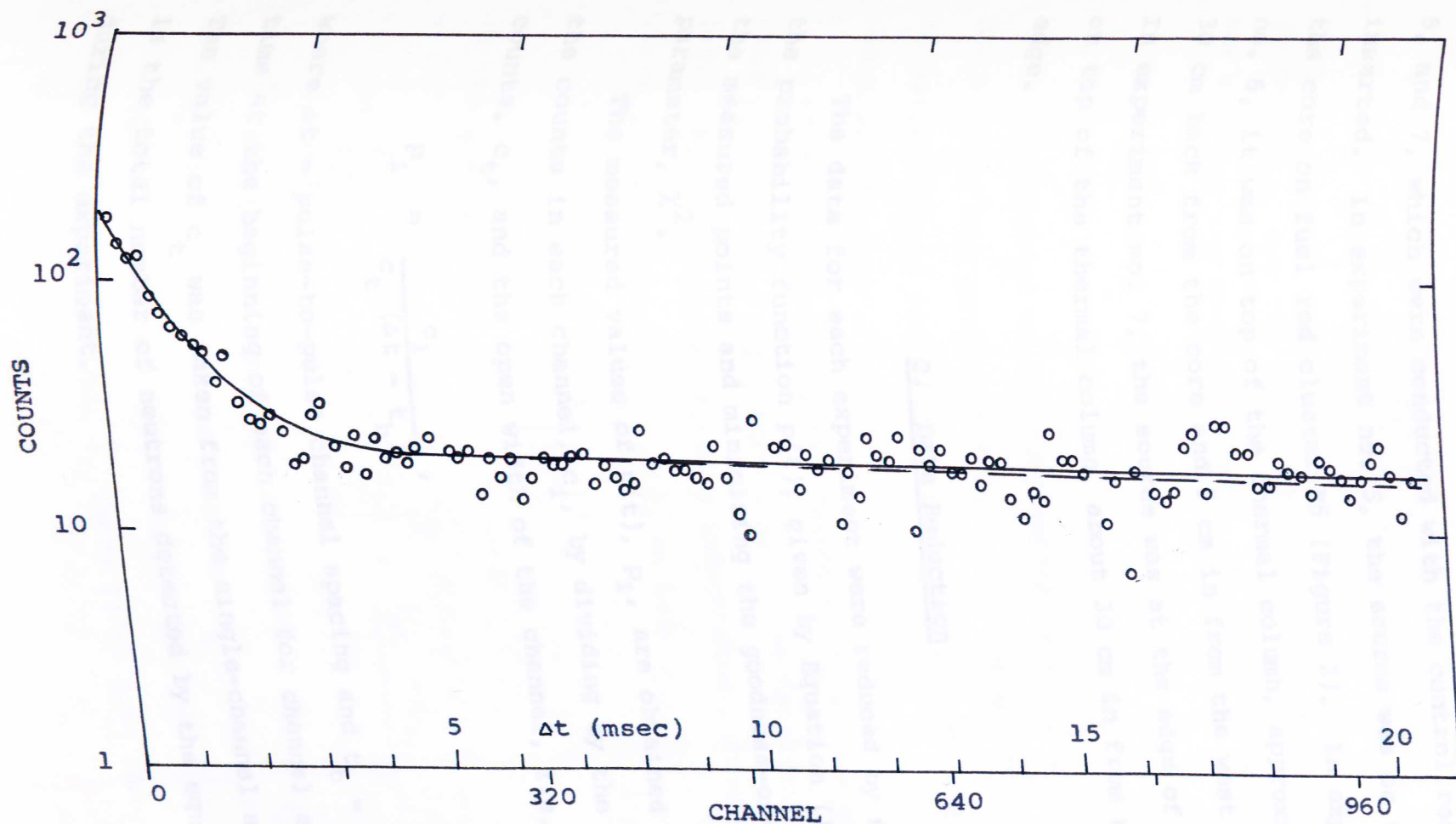


Fig. 41.--Interval distribution for experiment no. 9. Two-parameter solid, three-parameter dashed.

The source position was varied for experiments no. 5, 6, and 7, which were conducted with the control rods fully inserted. In experiment no. 5, the source was on top of the core on fuel rod cluster F5 (Figure 2). In experiment no. 6, it was on top of the thermal column, approximately 30 cm back from the core and 5 cm in from the west edge. In experiment no. 7, the source was at the edge of the pool on top of the thermal column, about 30 cm in from the west edge.

C. Data Reduction

The data for each experiment were reduced by fitting the probability function $p(t)$, given by Equation (19), to the measured points and minimizing the goodness-of-fit parameter, χ^2 .

The measured values of $p(t)$, p_i , are obtained from the counts in each channel, c_i , by dividing by the total counts, c_t , and the open width of the channel, i.e.,

$$p_i = \frac{c_i}{c_t (\Delta t - t_b)} \quad (22)$$

where Δt = pulse-to-pulse channel spacing and t_b = blanked time at the beginning of each channel for channel advance. The value of c_t was taken from the single-channel scaler; it is the total number of neutrons detected by the equipment during the experiment.

The measure of goodness of fit, χ^2 , is defined as

$$\chi^2 = \sum_{i=1}^N \frac{[p_i - p(t_i)]^2}{\sigma_i^2}, \quad (23)$$

where the σ_i are the uncertainties in the measured points.

For testing the goodness of fit, we use the reduced χ_m^2 ,

$$\chi_m^2 = \frac{\chi^2}{m}, \quad (24)$$

where m = number of free parameters, $N - n$. In this case $N = 1023$ or the number of non-zero channels, and n = number of parameters in the fitting function, either 2 or 3, as discussed later.

In this experiment, the values of σ_i may be calculated from p_i , Equation (22), according to the propagation of errors. For a function, f , of m independent parameters, x_1, \dots, x_m , i.e., $f = f(x_1, \dots, x_m)$, we have

$$\sigma_f^2 = \sum_{i=1}^m \sigma_{x_i}^2 \left(\frac{\partial f}{\partial x_i} \right)^2. \quad (25)$$

In the present case,

$$\left(\frac{\sigma_i}{p_i} \right)^2 = \left(\frac{\sigma_{c_i}}{c_i} \right)^2 + \left(\frac{\sigma_{c_t}}{c_t} \right)^2 + \frac{\sigma_{\Delta t}^2 + \sigma_{t_b}^2}{(\Delta t - t_b)^2}. \quad (26)$$

The errors in the counts are statistical, i.e., $\sigma_{c_i}^2 = c_i$, and $\sigma_{c_t}^2 = c_t$. The errors in Δt and t_b are estimated to be at most $0.1 \mu\text{sec}$. The last term is then less than 7×10^{-5} , except for experiment no. 1, where it is less than 1.2×10^{-4} .

The second term on the right-hand side is less than 5×10^{-6} from Table I. The first term on the right-hand side has its minimum in the first channels. From Figure 4 it can be seen that the lower limit of this minimum value is 1.1×10^{-4} except for experiment no. 1, where it is 2.5×10^{-5} . Hence the dominant error term is σ_{c_i} , except for the first 70 channels of experiment no. 1. Therefore I have taken $\sigma_i = p_i^2 / c_i$. A check of this procedure is described in the next section.

The method adopted to minimize χ^2 is described by Bevington (1969). It combines a gradient search for the path of steepest descent of χ^2 with a method of linearizing the fitting function near the minimum.

If we regard χ^2 as a curved surface in the n -parameter hyperspace, we may seek a minimum by proceeding along the gradient. This works well far away from the minimum, but convergence is slow as the minimum is approached. However, near the minimum, the fitting function can be expanded in a first-order Taylor's series without much error. Then χ^2 can be found at the minimum by setting its partial derivatives with respect to each of the parameter increments equal to zero.

If $p_1(t)$ is the value of $p(t)$ at the starting point, and if we expand $p(t)$ in its parameters, then

$$\begin{aligned}
 \frac{\partial \chi^2}{\partial \Delta a_k} &= -2 \sum_{i=1}^N \frac{1}{\sigma_i^2} \left\{ p_i - p_1(t_i) - \sum_{j=1}^n \left[\frac{\partial p_1(t_i)}{\partial a_j} \Delta a_j \right] \right\} \frac{\partial p_1(t_i)}{\partial a_k} \\
 &= 0.
 \end{aligned} \tag{27}$$

This yields a set of n simultaneous equations, which may be expressed in matrix form,

$$(\beta) = (\Delta a) [\alpha] , \quad (28)$$

where

$$\begin{aligned} \beta_k &= -\frac{1}{2} \frac{\partial \chi^2}{\partial a_k} \\ &= \sum_{i=1}^N \frac{1}{\sigma_i^2} \left[p_i - p_1(t_i) \right] \frac{\partial p_1(t_i)}{\partial a_k} , \end{aligned} \quad (29)$$

and

$$\alpha_{jk} = \frac{1}{2} \frac{\partial^2 \chi^2}{\partial a_j \partial a_k} = \sum_{i=1}^N \frac{1}{\sigma_i^2} \frac{\partial p_1(t_i)}{\partial a_j} \frac{\partial p_1(t_i)}{\partial a_k} . \quad (30)$$

The matrix elements α_{jk} give the curvature of χ^2 in the parameter space.

The search algorithm replaces the diagonal elements, α_{jj} , with $\alpha_{jj}(1 + \lambda)$. If λ is large, the diagonal dominates, and the method tends to a gradient search. If λ is small, the method tends to the first-order expansion. The solution is found by inverting the α matrix,

$$(\Delta a) = (\beta) [\epsilon] , \quad (31)$$

where $[\epsilon] = [\alpha]^{-1}$. The procedure is iterated, varying λ , until χ^2 converges on a minimum.

At the minimum, the diagonal elements of $[\epsilon]$ give the errors in the parameters, i.e.,

$$\sigma_{a_j} = \epsilon_{jj} . \quad (32)$$

This follows from the definition of σ_{a_j} in terms of the propagation of errors,

$$\sigma_{a_j}^2 = \sum_{i=1}^N \sigma_i^2 \left(\frac{\partial a_j}{\partial p_i} \right)^2. \quad (33)$$

The validity of this expression can be seen most clearly if we suppose near the minimum the variation of χ^2 with respect to each parameter is independent of the others. Then both $[\alpha]$ and $[\epsilon]$ are diagonal, and from Equation (30),

$$\sigma_{a_j}^2 = \frac{1}{\alpha_{jj}} = \epsilon_{jj}. \quad (34)$$

As Bevington (1969) shows, in the more general case, similar considerations give the same result, that the error matrix is the inverse of the curvature matrix.

IV. RESULTS AND DISCUSSION

A. Choice of Parameters

In reducing the data one has a choice of the number of parameters to regard as independent or undetermined. Equation (19) for the interval distribution contains four parameters, namely, A , B , α , and γ . Babala and Ogrin (1967) solved for all four independently. However, γ is a function of B and α . It seems hard to justify solving for γ separately. The better procedure would appear to be to solve for B and α and compute γ .

The mean count rate, A , is also determined by the contents of the single-channel scaler, c_t , and the total time of the experiment, t_t :

$$A = \frac{c_t}{t_t} . \quad (35)$$

In principle, therefore, A is determined. However, there are two sources of error in converting to count rates, or probabilities, from the channel counts (in addition to the statistical error in each number). These errors are due to the inconstancy of the count rate and the inconstancy of the

channel advance rate. The first is due to variations in the reactor state, e.g., decay of the delayed neutron precursors (important in long shutdown runs) and start of experiments before the fission rate has reached equilibrium with the source (important in short runs). Consequently, it may be advisable to solve for A in Equation (19). The second source of error is variation in the pulse rate, due possibly to line voltage fluctuations, which can only be monitored crudely during data acquisition. This is estimated to give an error of less than $0.1 \mu\text{sec}$ in Δt , and it is inconsequential in the present experiments. It could be a point of concern, however, for high count rates in measurements at power.

To allow for error in the determination of A , two solutions were made to each set of data, one for the two parameters B and α , the other for the three parameters A , B , and α . The results are shown in Table II.

In the cases where there is a substantial difference in the value of A between the two-parameter fit and the three-parameter fit, namely, experiments no. 1, 2, 5, 6, and 7, there is reason to suspect the count rate varied during the experiment. This was due to relatively long-term source variations in experiment no. 2 and to lack of steady-state conditions in the shorter experiments, no. 1, 5, 6, and 7.

As a check on the dominant error source in Equation (26), the data for experiment no. 3 were reduced twice

TABLE II

EXPERIMENTAL RESULTS

#	n	m	χ_m^2	A (sec ⁻¹)	B (sec ⁻¹)	α (sec ⁻¹)	γ
1	2	1007	3.678	1192.1±0.8	688.1±2.5	168.9±3.1	4.159±.151
	3	1006	2.160	1135.0±1.4	705.5±2.5	160.2±2.6	4.314±.033
2	2	1012	1.195	2.899±.003	80.01±.64	1030.1±7.3	1.145±.002
	3	1011	1.173	2.806±.019	79.56±.64	1022.0±7.4	1.145±.002
3	2	1013	1.226	26.99±.03	402.4±1.2	241.0±1.2	2.771±.007
	3	1012	1.210	27.66±.16	404.7±1.3	240.3±1.2	2.781±.007
4	2	1011	1.052	2.283±.002	79.37±.45	1024.0±5.0	1.144±.001
	3	1010	1.046	2.316±.001	79.54±.45	1027.1±5.1	1.144±.001
5	2	987	2.946	492.19±.51	83.6 ±2.1	1158 ±51	1.135±.007
	3	986	0.973	465.87±.56	81.1 ±2.0	1094 ±47	1.139±.006
6	2	995	2.913	470.89±.51	86.2 ±2.1	1140 ±49	1.141±.008
	3	994	1.135	445.88±.56	83.7 ±2.1	1087 ±45	1.144±.007
7	2	1015	1.800	163.04±.19	83.6 ±1.5	1214 ±31	1.129±.004
	3	1014	1.004	153.76±.33	81.2 ±1.4	1028 ±27	1.147±.004
8	2	1011	1.080	78.35±.15	330.3±1.9	292.6±2.8	2.349±.011
	3	1010	1.042	74.55±.59	326.5±1.9	292.3±2.7	2.338±.011
9	2	1016	1.030	6.970±.016	54.6±1.1	1065 ±21	1.098±.003
	3	1015	0.993	6.620±.056	53.7±1.1	1034 ±21	1.099±.003

again, with the first ten and the first 40 channels ignored. This experiment has the highest initial count rate among the more interesting experiments of this series, 5000 sec⁻¹. Consequently, the error of the first term in Equation (26) is about 2×10^{-4} in the first channel. But by the tenth channel it is 25 percent higher, and by the fortieth it is twice as high. The two-parameter results with the deleted channels are $B = 400.6 \pm 1.4 \text{ sec}^{-1}$, $\alpha = 241.1 \pm 1.1 \text{ sec}^{-1}$, and $B = 394.8 \pm 1.9 \text{ sec}^{-1}$, $\alpha = 241.7 \pm 1.2 \text{ sec}^{-1}$, ignoring

the first ten and the first 40 channels, respectively. The results are within the error bars of the previous solution for α . The gradual decrease in B reflects the loss of the channels with the greatest weight in determining B. These results indicate we may expect this and the other experiments to be dominated by the statistical errors in the counts (the only exception being experiment no. 1).

B. The Critical Rossi Alpha

In order to find the value of Rossi alpha for criticality, two methods are available. The first is the direct measurement with the reactor critical. The second is a linear extrapolation from a series of slightly subcritical measurements.

The disadvantage of the direct measurement at criticality is the effect of fluctuating power levels at the low power desired. In the present case, not only did that happen, but also the value of A was poorly measured.

The second method seems to offer several advantages. No adjustments of control rods are made once the reactor has been brought to the desired state. It is only necessary to wait for the reactor to come to equilibrium with the source (in its withdrawn position) before starting the data taking. This may take 15 minutes, but the time can be reduced by inserting the source only the minimum distance required to raise the flux enough to move the control rods.

Near criticality the reactivity worth of the control rods is linear with the fractional withdrawal, x . A solution for $\alpha(x) = a + b x$ gives α_c for the critical withdrawal, x_c .

Solutions were obtained to the two subcritical points as found by two-parameter and three-parameter fits to the data. These are shown in the first row of Table III.

TABLE III
SOLUTIONS FOR α_c (sec^{-1})

Points	2-Parameter	3-Parameter
Subcritical	189.3 ± 4.7	188.3 ± 4.7
Subcritical & Critical	177.5 ± 2.3	169.8 ± 2.1

Solutions including the critical points are shown in the second row.

Because of the unsatisfactory conditions under which α was measured at the critical point, I feel the greater weight should lie with the subcritical experiments. I adopt the mean of the two subcritical solutions, $\alpha_c = 188.8 \pm 4.7 \text{ sec}^{-1}$.

With the adopted value of α_c , the prompt neutron lifetime is $\ell = \beta/\alpha = 3.7 \pm 0.1 \times 10^{-5} \text{ sec}$, assuming the effective delayed neutron fraction is $\beta = 0.0070$ (University of Maryland Safety Analysis Report, 1973). The Safety Analysis Report gives $\ell = 3.9 \times 10^{-5} \text{ sec}$, implying $\alpha_c = 180 \text{ sec}^{-1}$.

The only other measurement of the critical Rossi alpha of the University of Maryland Reactor was by C. Alonso (1976, private communication). She used the rod-drop technique, analyzing the time-dependent flux as a sum of exponentials. Her result was $\alpha_c = 160 \text{ sec}^{-1}$, but the error is unknown.

C. The Shutdown Reactivity

The three shutdown experiments, no. 2, 4, and 9, give consistent values for the shutdown Rossi alpha for measurements over periods of a weekend, a week, and a day, respectively. The fits to the data as shown by the values of χ_m^2 are also reasonably satisfactory. The larger uncertainties in experiment no. 9 are a result of the small numbers of counts recorded in eight hours. The effect of the higher discriminator setting of 2.0 instead of 1.1 in experiment no. 9 is not apparent. Although the value of α is higher for the two-parameter fit, the large uncertainty gives it low weight.

The weighted mean of the two-parameter fits for the shutdown reactivity is $\alpha_s = 1027.4 \pm 4.0 \text{ sec}^{-1}$ (s.d. of the mean); that of the three-parameter fits is $\alpha_s = 1025.8 \pm 4.1 \text{ sec}^{-1}$. I adopt the mean, $\alpha_s = 1026.6 \pm 4.1 \text{ sec}^{-1}$.

The shutdown reactivity, ρ_s , is best expressed in the ratio ρ_s/β , since β is not measured in this experiment. The unit of ρ/β is the dollar. From Equation (5), the shutdown reactivity is given by the expression

$$\frac{\rho_s}{\beta} = 1 - \frac{\alpha_s}{\alpha_c} . \quad (36)$$

For the adopted values of α_c and α_s , $\rho_s/\beta = -4.44 \pm 0.14$ dollars. Expressed as a percentage of prompt multiplication by multiplying with $\beta = 0.0070$, $(k - 1)/k = -3.1 \pm 0.1$ percent.

D. Detector Efficiency and Prompt Multiplication

From the definition of B, the detector efficiency, ϵ , can be measured from the expression

$$\epsilon = \frac{2 B \alpha \ell^2}{D k^2} = \frac{2 B \beta^2}{D \alpha} \left(1 - \frac{\rho}{\beta} \right)^2 . \quad (37)$$

Based on the value $\beta = 0.0070$, the derived values of the detector efficiency and the prompt multiplication factor are as shown in Table IV, along with ρ/β .

TABLE IV
VALUES OF DERIVED PARAMETERS

No.	ρ/β	k	ϵ
3	$-0.276 \pm .032$	$0.9981 \pm .0002$	$3.36 \pm .12 \times 10^{-4}$
8	$-0.550 \pm .041$	$0.9962 \pm .0003$	$3.32 \pm .13 \times 10^{-4}$
2 + 4	$-4.43 \pm .14$	$0.9699 \pm .0009$	$2.82 \pm .10 \times 10^{-4}$
9	$-4.56 \pm .18$	$0.9691 \pm .0012$	$1.97 \pm .11 \times 10^{-4}$

The efficiency shows a decrease as the control rods are inserted. This could well be the result of the changing flux shape in the axial direction. The detector is located near the vertical center of the core, but the flux is depressed in the region above the bottoms of the control rods. Near criticality, this region is just above the level of the detector, and the detector should be exposed to a greater fraction of the flux than near shutdown.

Also, as expected, the efficiency for experiment no. 9 is lower than experiments no. 2 and 4. With the discriminator set high, fewer of the fissions are detected. This is equivalently expressed as a lower value of B for this experiment.

V. CONCLUSIONS

The Rossi alpha of the University of Maryland Reactor has been measured at criticality and at shutdown by the interval-distribution method. The results are $\alpha_c = 188.8 \pm 4.7 \text{ sec}^{-1}$ and $\alpha_s = 1026.6 \pm 4.1 \text{ sec}^{-1}$. The combination of these two values gives a shutdown reactivity $\rho_s/\beta = -4.44 \pm 0.14$ dollars.

According to the Safety Analysis Report, calculations show a total control rod worth of $\Delta k/k = -0.0827$ and an excess reactivity $\Delta k/k = 0.0204$. The net shutdown reactivity should therefore be $-.0623$, or -8.90 dollars. This is a factor 2 greater than that found here. The discrepancy should be resolved by further experiment, such as a rod-drop measurement of shutdown reactivity. The value of α_c found here agrees with that in the Safety Analysis Report, which is 180 sec^{-1} . The value of α_s has been measured consistently on three occasions of different duration and with different discriminator settings, so it should be reliable.

For a successful experiment the main condition to be met is a constant count rate. The most important instrumental condition is adequate discrimination against gammas. Care must be taken to achieve equilibrium before starting the experiment. The duration of the experiment must be accurately

timed. Finally, the total time of the channel sweep should be long enough to allow A to be measured but short enough to avoid correlations with the shortest group of delayed neutrons. The span of 20 msec used here was found to be a good value.

Under these conditions the method is found to be easy to apply. The experiments near criticality yield good results in periods of one hour for count rates on the order of 100 sec^{-1} . The shutdown experiments take days but require no supervision.

As a class laboratory experiment, these measurements can be a useful exercise. The shutdown experiments made with a low count rate on the order of 3 sec^{-1} can be analyzed by graphical methods alone. The experiments in which $A \approx B$ or in which $\gamma \gg 1$ require numerical methods for solution, but the techniques are readily available.

A series of experiments that could be useful as either an individual or a class project is the measurement of the reactivity worth of the individual control rods. In this case the reactor is operated in a subcritical condition with two rods fully inserted. The Rossi alpha is measured with the third rod at various stages of withdrawal. The method can yield a calibration over the full extent of the rod travel.

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